

MATHEMATICS FOR WASTEWATER MANAGEMENT



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Mathematics for Wastewater Management (Brooks)

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Licensing

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CHAPTER OVERVIEW

1: Numbers, Decimals, and Fractions

[1.1: The Set of Real Numbers](#)

[1.2: Mathematical Operations](#)

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1.1: The Set of Real Numbers

The set of real numbers consists of the numbers that we use in everyday life. We will define the set of real numbers more precisely, but first, let's define some other sets of numbers.

Natural Numbers: The counting numbers starting with 1: 1, 2, 3, 4, ...

Whole Numbers: 0 and the counting numbers: 0, 1, 2, 3, 4, ...

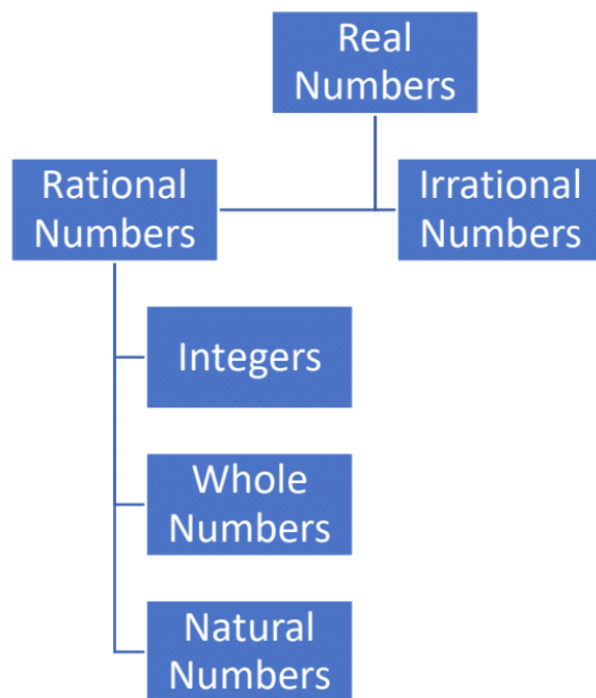
Integers: Positive and Negative Whole Numbers (includes 0); The whole numbers and their opposites: ..., -4, -3, -2, -1, 0, 1, 2, 3, 4, ...

Rational Numbers: A number that can be written as a fraction (integers, terminating decimals and repeating decimals)

Irrational Numbers: Numbers that cannot be written as a fraction (non-terminating decimals and non-repeating decimals)

Real Numbers: All rational and irrational numbers

Some numbers can be classified in many categories. The next diagram displays the relationship between the different sets of numbers.



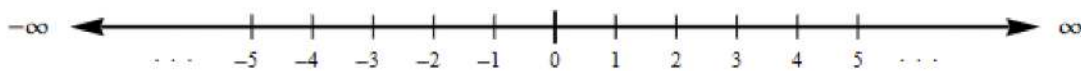
Example: Classify the following numbers

1. 2.718
2. $\sqrt{13}$
3. $\frac{24}{51}$
4. 101
5. $0.\overline{3}$
6. -5

See the video for the solutions.

The Real Number Line

We order real numbers on a number line based on their distance from 0. Positive numbers lie to the right of zero and Negative numbers lie to the left of zero. Each point on the number line corresponds to a real number.



Notice that -2 is represented by a distance of 2 units to the left of zero, the negative determines the side of zero while the 2 represents the distance from zero. Similarly, +2 is represented by a distance of 2 units to the right of zero, the positive indicates we move to the right of zero while the 2 represents the distance from zero. When writing positive numbers, it is customary not to write the “+” sign in the front.

Example: Plot the following numbers on a real number line

1. 2.718
2. $\sqrt{13}$
3. $\frac{24}{51}$
4. 101
5. $0.\overline{3}$
6. -5

See the video for a demonstration.

We determine the size of a number relative to another number using less than ($<$) or greater than ($>$) signs. Whichever number is further to the right, is larger, while a number that is further to the left is smaller.

We say “ a is less than b ” if a lies to the left of b on a real number line and we write this as: $a < b$.

We say “ a is greater than b ” if a lies to the right of b on a real number line and we write this as: $a > b$.

Examples: Use $<$ or $>$ to indicate the relationship between the stated numbers

1. $7 \underline{\hspace{1cm}} -3$
2. $5 \underline{\hspace{1cm}} 12$
3. $5.8 \underline{\hspace{1cm}} 5.7$
4. $-3.2 \underline{\hspace{1cm}} -3.1$
5. $-5 \underline{\hspace{1cm}} -12$

See the video for the solutions.

The Opposite of a Number and Absolute Value

Definition: The opposite of a real number, a , is $-a$.

We can think of the opposite of a real number as changing the sign of the real number.

Examples: The opposite of 5 is -5. The opposite of -7 is $-(-7) = 7$

Hence, $-(-7)$ can be read as “the opposite of -7” or “the opposite of the opposite of 7” or 7.

Definition: The absolute value of a real number is its distance from zero on a real number line. The notation for the absolute value uses two vertical bars on either side of the number, hence “the absolute value of a ” would be denoted as $|a|$. Since the absolute value represents a distance, its value is always positive.

Examples:

$|5| = 5$ since 5 is a distance of 5 from 0

$|-7| = 7$ since -7 is a distance of 7 from 0

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1.2: Mathematical Operations

The Addition of Integers and Decimals

Adding Numbers with the same signs

- Add the absolute values of the numbers and assign the common sign to the result
- NOTE: When the signs are both positive, add the values

Example: $23 + 18$

The common sign is +, so the answer will be positive, now we add the numbers to obtain

$$23 + 18 = 41 \quad (1.2.1)$$

Example: $-17 + (-31)$

The common sign is -, so the answer will be negative, now we add the absolute values of the numbers to obtain:

$$-17 + (-31) = \quad (1.2.2)$$

First, calculate the sum of the absolute values:

$$|-17| + |-31| = 17 + 31 = 48 \quad (1.2.3)$$

Next, assign the result the common sign, so the final answer is: -48.

$$-17 + (-31) = -48 \quad (1.2.4)$$

Adding Numbers with the Opposite Signs

- Take the absolute value of each number
- Subtract the smaller number from the larger number
- Assign the sign of the result based on the sign of the number with the larger absolute value
- NOTE: The sign of the larger absolute value wins. With practice, these steps will start to feel seamless.

Example: $21 + (-13)$

First, calculate the absolute value of each number:

$$|21| = 21 \text{ and } |-13| = 13 \quad (1.2.5)$$

Next, subtract the smaller number from the larger number:

$$21 - 13 = 8 \quad (1.2.6)$$

Next, assign the sign of the number whose absolute value was greater, in this case, the sign of 21 is the sign of the final answer (positive), hence

$$21 + (-13) = 8 \quad (1.2.7)$$

Scientific calculator keystrokes:

Example: $28 + (-54)$

First, calculate the absolute value of each number:

$$|28| = 28 \text{ and } |-54| = 54$$

Next, subtract the smaller number from the larger number:

$$54 - 28 = 26 \quad (1.2.8)$$

Next, assign the sign of the number whose absolute value was greater, in this case, the sign of -54 is the sign of the final answer (negative), hence

$$28 + (-54) = -26 \quad (1.2.9)$$

Short-cut: Now that we have gone through the formal process for adding integers, let's get the general idea of the process:

- If the signs are the same, combine the numbers and keep the sign
- If the signs are different, disregarding the signs, subtract the smaller number from the larger number, then keep the sign of the larger number

Example: $13 + (-29)$

Since 29 is larger than 13, we subtract 13 from 29 to get 16, and use the sign of 29 which is negative: -16

Example: $-28 + (-54)$

Since the signs are the same, the answer will have the same sign (negative), now combine the numbers $28 + 54 = 82$, so the final answer is -82

Similarly with decimal numbers:

Example: $-2.34 + (-5.4) =$

Since the signs are the same, we add the numbers: $2.34 + 5.4 = 7.74$ and keep the common sign (negative) to get -7.74

Subtraction of Integers and Decimals

When subtracting an integer or decimal, we add the opposite of the number following the subtraction.

Example: $15 - 7 =$

In this case, we can perform regular subtraction, however, we can also apply the subtraction rule of adding the opposite to obtain:

$$15 + (-7) = 8 \quad (1.2.10)$$

since 15 is larger than 7, we subtract and keep the sign of the larger number which is positive

Example: $3 - 12 =$

Remembering to add the opposite of 12, we get

$$3 + (-12) = -9 \quad (1.2.11)$$

Since the signs are different and 12 is larger than 3, we subtract $12 - 3$ to get 9, but use the sign of 12 which is negative to obtain -9

Likewise with decimal values:

Example: $8.23 - (-1.2) =$

We first add the opposite of -1.2: $8.23 + 1.2 =$

Next, we combine the numbers since they have same signs to obtain:

$$8.23 - (-1.2) = 8.23 + 1.2 = 9.43 \quad (1.2.12)$$

Multiplication and Division of Integers and Decimals

To multiply or divide integers and decimals, we use multiply the numbers and use the following rules for the sign:

- If two numbers have the same signs, the result is positive
- If two numbers have different (opposite) signs, the result is negative

Examples:

1. $(3)(-7) = -21$
2. $(-15)(-10) = 150$
3. $(-5.2)(2) = -10.4$
4. $(2)(7) = 14$
5. $(-25) \div (-5) = 5$
6. $(-12.8) \div (4) = -3.2$
7. $(250) \div (-10) = -25$

8. $(48) \div (6) = 8$

Addition, Subtraction, Multiplication, and Division of Integers and Decimals using a Scientific Calculator

There are many types of scientific calculators available in the retail market, on our personal computers, and even on our personal devices such as cell phones and tablets. As a result, let's first distinguish between two types of scientific calculators: A Display calculator and a non-display calculator. Some examples of display calculators are: TI30XIIS, TI-36X Pro, TI-30XS Multiview, and the Casio fx-350ES PLUS. Some examples of non-display calculators are TI-30Xa, most cell phone calculators, and most calculators on personal computers.

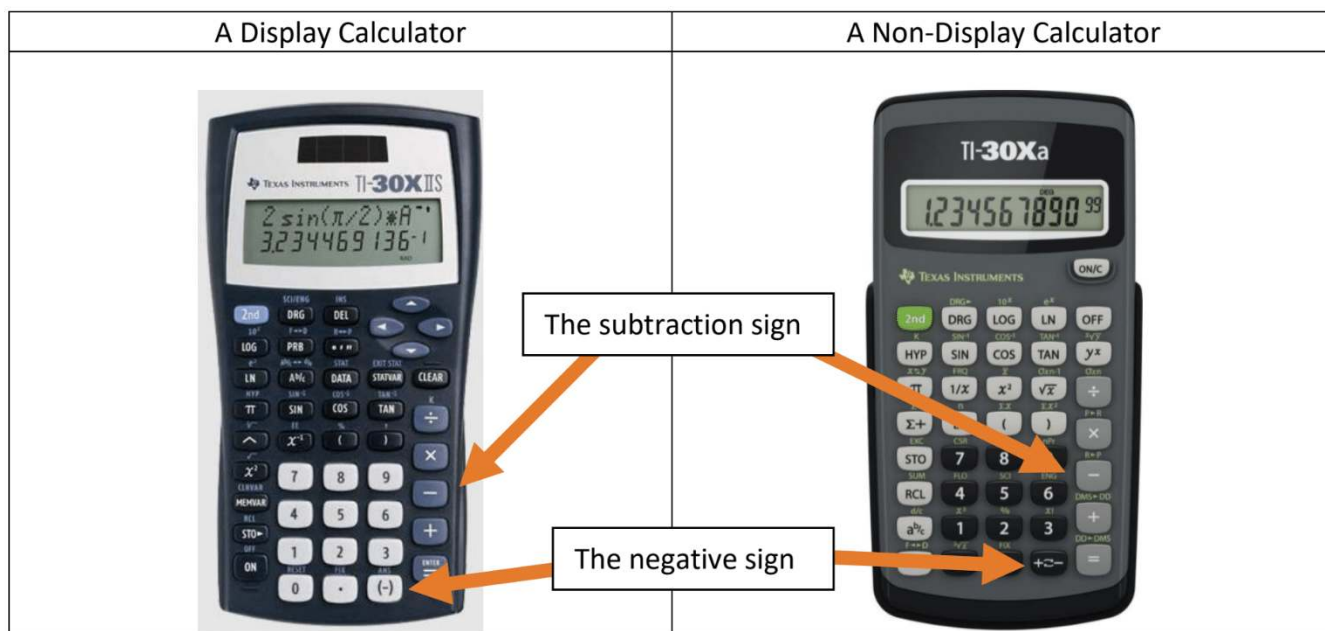
For our purposes, we will determine if a calculator is a "Display" calculator or a "non-display" calculator by doing the following:

Type $2 + 3$ on your calculator.

- If your screen displays the entire expression $2 + 3$, then you have a "Display" calculator.
- If your screen displays only the 3 after typing $2 + 3$, then you have a "non-display" calculator.

On Display calculators, we can usually enter the problem as it is written. On a non-display calculator, we usually must enter the problem in reverse order of operations (see Unit 2 for more information).

Let's revisit some of our earlier examples and calculate them on the scientific calculator. For negative numbers, be sure to use the negative symbol, $(-)$, which is generally at the bottom of the keyboard on a TI calculator and not the subtraction symbol, $-$, which is usually on the right side of the keyboard. Each calculator is different, so you may have to experiment with your calculator to determine the process.



To compute $-17 + (-31)$ use the following sequence of keystrokes to obtain -48 :

Display Calculator:	Non-display Calculator:
$(-) 17 + (-) 31 \text{ ENTER}$	$17 (-) + 31 (-)=$
Type the negative sign before the number	Type the negative number after the sign

To compute $8.23 - (-1.2) =$ use the following sequence of keystrokes to obtain 9.43:

Display Calculator	Non-display Calculator:
8.23 - (-) 1.2 ENTER	8.23 - 1.2 (-) =
Use the subtraction key, then type the negative sign before the number	Use the subtraction key then type the negative sign after the number

To compute $(-15)(-10) =$ use the following sequence of keystrokes to obtain 150:

Display Calculator	Non-display Calculator:
(-)15 X (-)10 ENTER	15 (-) X 10 (-) =
Type the negative sign before the number	Type the negative sign after the number

To compute $(250) \div (-10) =$ use the following sequence of keystrokes to obtain -25:

Display Calculator	Non-display Calculator:
250 $\div$ (-)10 ENTER	250 $\div$ 10 (-) =
Type the negative sign before the number	Type the negative sign after the number

Fractions

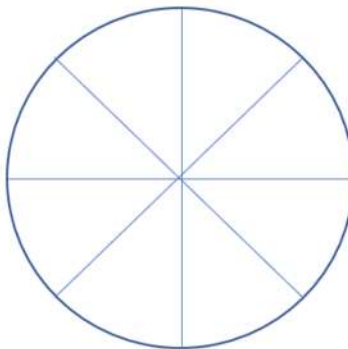
Fractions are real numbers that indicate a portion of a whole. A fraction consists of a numerator and denominator. The denominator represents the number of equal parts of an object and the numerator represents a portion of those equal parts. For example, if pipe is separated into 4 equal parts, $\frac{1}{4}$ would represent 1 out of 4 of the parts, $\frac{2}{4}$ would represent 2 out of the 4 parts, $\frac{3}{4}$ would represent 3 out of the 4 parts, and $\frac{4}{4} = 1$ would represent all 4 parts or the whole pipe. Likewise, $\frac{0}{4}$ would represent 0 of the 4 parts or nothing, hence $\frac{0}{4} = 0$.

The rectangle below is divided into two equal parts. Hence 1 of the pieces out of the 2 pieces would represent $\frac{1}{2}$.



The circle shown below is divided into 8 equal parts. 1 part would represent $\frac{1}{8}$. 2 parts would represent $\frac{2}{8}$ which would reduce to $\frac{1}{4}$ for a couple of reasons:

1. if we were to shade 2 of the 8 pieces, that would also represent 1 of 4 equal pieces and
2. $\frac{2}{8}$ can be reduced by dividing both the numerator (2) and the denominator (8) by the same number, which in this case would be 2, hence: $\frac{2}{8} = \frac{2 \div 2}{8 \div 2} = \frac{1}{4}$



The “bar” in a fraction also represents division, so $\frac{1}{4}$ would also mean $1 \div 4$. This can be interpreted as a whole divided into 4 equal parts or 1 of 4 equal parts.

Fractions have many applications. Fractions can also be used to represent units such as miles per hour or $\frac{\text{miles}}{\text{hour}}$. Fractions can also be used to denote ratios and proportions; we will see the use of fractions in these applications in a later unit.

Multiplying and Dividing Fractions

To multiply fractions:

Multiply the numerators and multiply the denominators.

To simplify, reduce by dividing like factors (numbers that divide into other numbers).

Examples:

1. $\frac{2}{3} \cdot \frac{5}{7}$

a. Multiply the numerators and multiply the denominators.

$$\frac{2}{3} \cdot \frac{5}{7} = \frac{10}{21}$$

Since 10 and 21 do not have any common factors, this is the simplified or reduced answer.

2. $\frac{22}{63} \cdot \frac{9}{24}$

a. Multiply the numerators and multiply the denominators.

$$\frac{22}{63} \cdot \frac{9}{24} = \frac{108}{1512}$$

b. Simplify by reducing the fraction (divide the numerator and denominator by the same number(s))

$$\frac{108}{1512} = \frac{108 \div 12}{1512 \div 12} = \frac{9}{126} = \frac{9 \div 9}{126 \div 9} = \frac{1}{14}$$

Another option for this problem is to reduce before multiplying:

$$\frac{12}{63} \cdot \frac{9}{24}$$

a. Reduce the numerators and denominators by dividing each by the same number

$$\frac{12 \div 12}{63 \div 9} \cdot \frac{9 \div 9}{24 \div 12} = \frac{1}{7} \cdot \frac{1}{2}$$

b. Multiply the numerators and denominators (multiply straight across)

$$\frac{1}{7} \cdot \frac{1}{2} = \frac{1}{14}$$

3. $\frac{24}{72} \cdot \frac{18}{30}$

Since the numbers in the fractions are large, let's reduce first, then multiply.

Determine numbers that divide into both the numerator and denominator:

9 divides into 18 and 72; 6 divides into 24 and 30 OR 6 divides into 18 and 30; 24 divides into 24 and 72

$$\frac{24}{72} \cdot \frac{18}{30} = \frac{24 \div 24}{72 \div 24} \cdot \frac{18 \div 9}{30 \div 9} = \frac{1}{3} \cdot \frac{2}{5} = \frac{4 \div 4}{8 \div 4} \cdot \frac{2}{5} = \frac{1}{2} \cdot \frac{2}{5} = \frac{1}{2 \div 2} \cdot \frac{2 \div 2}{5} = \frac{1}{5}$$

OR

$$\frac{24}{72} \cdot \frac{18}{30} = \frac{24 \div 24}{72 \div 24} \cdot \frac{18 \div 6}{30 \div 6} = \frac{1}{3} \cdot \frac{3}{5} = \frac{1}{3 \div 3} \cdot \frac{3 \div 3}{5} = \frac{1}{5}$$

To divide fractions: Invert the second fraction and change the operation to multiplication and continue as described above.

Example: $\frac{3}{5} \div \frac{9}{10}$

$$\frac{3}{5} \div \frac{9}{10} = \frac{3}{5} \cdot \frac{10}{9} = \frac{30}{45} = \frac{2}{3}$$

Example: $\frac{15}{7} \div \frac{25}{21}$

$$\frac{15}{7} \div \frac{25}{21} = \frac{15}{7} \cdot \frac{21}{25} = \frac{3}{1} \cdot \frac{3}{5} = \frac{9}{5}$$

Adding and Subtracting Fractions

With like denominators

To add or subtract fractions, the fractions must have the same denominator:

Step 1: Verify that the fractions are like fractions (fractions with the same denominator).

Step 2: Add or subtract the numerators and keep the same denominator.

Examples:

1. $\frac{2}{7} + \frac{3}{7} = \frac{2+3}{7} = \frac{5}{7}$
2. $\frac{14}{17} - \frac{5}{17} = \frac{14-5}{17} = \frac{9}{17}$

With unlike Denominators

Step 1: Rewrite the unlike fractions (fractions with different denominators) as like fractions (fractions with the same denominator) with the least common multiple as their new denominator. This new denominator is called the least common denominator (LCD).

To determine the least common denominator (LCD), find the smallest common multiple of the denominators, in other words, find the smallest number that both denominators divide into

Rewrite the unlike fractions as like fractions by multiplying the numerator and denominator of each fraction by the number that makes the denominator of each the least common denominator.

Step 2: Add or subtract the numerators and keep the common denominator.

Examples:

1. $\frac{2}{3} + \frac{1}{9} =$

First find the smallest number that both 3 and 9 will divide into, in this case it will be 9, so 9 is the LCD.

So, we need to multiply the first fraction by 3/3 to create a 9 in the denominator:

$$\frac{3}{3} \cdot \frac{2}{3} + \frac{1}{9} = \frac{6}{9} + \frac{1}{9} = \frac{7}{9}$$

2. $\frac{3}{7} + \frac{1}{4} =$

The least common denominator = 28 (since 28 is the smallest number that can be divided by both 7 and 4)

Next, multiply the numerator and denominator of 3/7 by 4/4 to create a denominator of 28 and multiply the numerator and denominator of 1/4 by 7/7 to create a denominator of 28, finally add the numerators and keep the common denominator.

$$\frac{4}{4} \cdot \frac{3}{7} + \frac{1}{4} \cdot \frac{7}{7} = \frac{12}{28} + \frac{7}{28} = \frac{19}{28}$$

NOTE: We add, subtract, multiply, and divide negative fractions similar to how we added, subtracted, multiplied and divided negative numbers in Section ____.

Percents

A percent is defined to be an amount based on 100. If we divide the word into two words, we have “per” and “cent”. Per means divide by and cent means 100, hence divide by 100 or out of 100.

Example: 54% means 54 out of 100

We will use percentages to calculate values in Unit ____.

Converting between different types of numbers (fractions, decimals, and percentages)

To convert from a fraction to a decimal, divide the numerator by the denominator.

To convert from a decimal to a fraction, determine the place of the last digit after the decimal, write the fraction as all the digits after the decimal over the last place of the decimal.

Example: 0.547 represents 547 thousandths since the last digit after the decimal is in the thousandths place, we have $547/1000$.

To convert from a fraction to a percent

Option 1: Convert the fraction to a decimal, then convert the decimal to a percent

Option 2: Use a proportion to write the fraction with a denominator of 100, then using the definition of a percent, the percent will be based on the numerator of the fraction with a denominator of 100. We will explore this option more in Unit ____ when we learn about proportions.

To convert from a decimal to a percent, multiply the decimal by 100 or move the decimal place two units to the right.

To convert from a percent to a decimal, divide the percent number by 100 or move the decimal place two units to the left.

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CHAPTER OVERVIEW

2: Powers, Roots, Order of Operations, and Evaluating Expressions

[2.1: Powers and Roots](#)

[2.2: Order of Operations](#)

[2.3: Evaluating Expressions](#)

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2.1: Powers and Roots

As we learned in Unit 1, we can use the following operations on real numbers: addition, subtraction, multiplication, division, and absolute value. In addition to these operations, we can also apply powers and roots.

Powers

Powers are as the notation for multiplying a number by itself multiple times. Another name for a power is an exponent. The power or exponent is denoted as a superscript.

$$a^n = \underbrace{(a \cdot a \cdot a \cdots a)}_{n \text{ times}} \quad (2.1.1)$$

In this notation, the a is called the “**base**” and the n is called the **power** (or **exponent**).

Examples:

1. $5^3 = 5 \cdot 5 \cdot 5 = 125$
2. $(-2)^4 = (-2)(-2)(-2)(-2) = 16$
3. $(2.3)^2 = (2.3)(2.3) = 5.29$
4. $(2/3)^5 = 2/3 \cdot 2/3 \cdot 2/3 \cdot 2/3 \cdot 2/3 = 32/243$

Roots

Roots are obtained by “undoing” a power. There are many types of roots, we will look at square roots, cube roots, and fourth roots.

- The square root of a number, a , is b if $b^2 = a$
- The cube root of a number, a , is b if $b^3 = a$
- The fourth root of a number, a , is b if $b^4 = a$
- We can generalize this to say the n^{th} root of a number, a , is b if $b^n = a$

We use a radical symbol to represent the operation of a root: $\sqrt{\quad}$

- Square root symbol: $\sqrt{\quad}$
- Cube root symbol: $\sqrt[3]{\quad}$
- Fourth root symbol: $\sqrt[4]{\quad}$
- n^{th} root symbol: $\sqrt[n]{\quad}$

The number under a radical symbol is called the **radicand**.

Examples:

1. The square root of 9 is denoted as $\sqrt{9}$, since $3^2 = 9$, then $\sqrt{9} = 3$.
2. $\sqrt{49}$, since $7^2 = 49$, then $\sqrt{49} = 7$.
3. The cube root of 8 is denoted as $\sqrt[3]{8}$, since $2^3 = 8$, then $\sqrt[3]{8} = 2$.
4. $\sqrt{1.44} = 1.2$ since $(1.2)^2 = 1.44$
5. $\sqrt{\frac{36}{121}} = 6/11$ since $(\frac{6}{11})^2 = 36/121$

To compute a root:

- On a scientific display calculator, type the radical symbol first, then the radicand followed by enter.

Example: $\sqrt{1.44}$ the keystrokes would be $\sqrt{\quad} 1.44 \text{ Enter}$

- On a non-display calculator, type the radicand first, then the radical symbol (do not press =)

Example: $\sqrt{1.44}$ the keystrokes would be $1.44 \sqrt{\quad}$

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2.2: Order of Operations

The order of operations represents a mathematical agreement of the order in which calculations should be performed. The order is Grouping Symbols, Exponents, Multiplication and Division as they appear left to right, and Addition and Subtraction as they appear left to right. Traditionally, we use PEMDAS as the acronym to remember the order of operations using Parentheses, Exponents, Multiplication, Division, Addition, and Subtraction. To allow us to perform operations on more problems, let's consider Grouping Symbols as the first order since it would encompass parentheses, brackets, roots, absolute values, and fraction bars. So, let's learn the acronym GEMDAS, one way to remember the letters is to remember GEMDAS: Greater Education Makes Doctors and Scholars or Good (Environment) Efforts Minimizes Diseased Animals Swimming

Grouping Symbols-Exponents-Multiplication/Division-Addition/Subtraction

GEMDAS

✓ Example 2.2.1

Simplify $5 - 3(4 - 6 \div (-2))$.

Solution

In this problem, we start with the inner most grouping symbol which would be the parentheses around the -2, however, there is no operation with this set of parentheses, so we move to the other set of parentheses: $(4 - 6 \div (-2))$. Within this set of parentheses, the division must occur before the subtraction, so we compute $6 \div (-2) = -3$, so the parentheses would be

$$(4 - (-3))$$

Next, we can recall that we subtract integers by adding the opposite, so $(4 - (-3)) = 4 + 3 = 7$ continuing, we now have

$$5 - 3(4 - 6 \div (-2)) = 5 - 3(7)$$

Next, we perform the multiplication before the subtraction to obtain:

$$5 - 21 = -15$$

✓ Example 2.2.2

Simplify $2 + 3 \cdot \sqrt{21 - 5} - 8 + 4^2$.

Solution

In this problem, we start with the radical since it is a grouping symbol:

$$\sqrt{21 - 5} = \sqrt{16} = 4$$

So now we have:

$$2 + 3 \cdot 4 - 8 + 4^2$$

Next, we apply the power or exponent:

$$4^2 = 16$$

So, we now have:

$$2 + 3 \cdot 4 - 8 + 16$$

Next, we perform multiplication:

$$3 \cdot 4 = 12$$

So, we now have:

$$2 + 12 - 8 + 16$$

Finally, we perform addition and subtraction as it appears left to right to obtain:

$$2 + 12 - 8 + 16 = 14 - 8 + 16 = 6 + 16 = 22$$

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2.3: Evaluating Expressions

In the Water Industry, we use many formulas to make calculations. As a result, we need to know how to evaluate expressions, that is, substitute known values for variables, and simplify the expression using the order of operations.

✓ Example 2.3.1

If $x = 4$ and $y = 3$ evaluate $3xy^2 + 2\sqrt{x}$.

Solution

First, substitute the given values for x and y :

$$3(4)(3)^2 + 2\sqrt{4}$$

Next, perform order of operations to simplify:

$$3(4)(3)^2 + 2\sqrt{4} = 3(4)(9) + 2(2) = 108 + 4 = 112$$

✓ Example 2.3.2

If $a = 9$ and $b = 7$, evaluate $\frac{2\sqrt{a}+3}{b-5}$.

Solution

First, substitute the given values for a and b :

$$\frac{2\sqrt{9}+3}{7-5}$$

Next, simplify using order of operations:

$$\frac{2\cdot 3+3}{7-5} = \frac{6+3}{7-5} = \frac{9}{2}$$

Recall from Unit 1, we could convert this fraction to a decimal to obtain 4.5, however, the general rule of thumb is to use decimals only when you start with decimals, or the directions states to use decimals.

✓ Example 2.3.3

A water plant has a rectangular basin with a base that is in the shape of a square with sides of length x and the height of the basin is y . As a result, the volume is calculated using the following expression: x^2y . If the length of the base of the basin is 50 feet and the height of the basin is 20 feet, calculate the volume of the basin.

Solution

First, we substitute the values for x and y : $x^2y = (50 \text{ feet})^2 \cdot (20 \text{ feet})$

Next, we simplify the expression using order of operations to obtain:

$$(2500 \text{ ft}^2)(20 \text{ ft}) = 50,000 \text{ ft}^3 \quad (2.3.1)$$

✓ Example 2.3.4

To estimate the extra water a town needs to store for emergencies, we must multiply the population by 45 gallons, then add 42,000 gallons to account for what the fire fighters would need in case of a fire, and then multiply the result by a safety factor of 1.8.

- Write an expression for calculating the amount of extra water a town needs to store in case of an emergency.
- Use the expression to calculate the amount of extra water a town needs to store in case of an emergency if the population of the town is 48,000.

Solution

a. We begin by converting the English to math:

Multiply the population, let's call the population P , by 45 gallons: $45P$

Then add 42,000 gallons: $45P + 42,000$

Then multiply by 1.8: $1.8(45P + 42,000)$

Hence, the amount of extra water needed is $1.8(45P + 42,000)$

b. If the population is 48,000, we will replace P in the above expression with 48,000 to obtain: $1.8(45 \cdot 48,000 + 42,000)$
 $= 1.8(2,160,000 + 42,000) = 1.8(2,202,000) = 3,963,600$

Hence, the town needs to reserve 3,963,600 gallons of water in case of an emergency.

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CHAPTER OVERVIEW

3: Solving Equations and Formulas

3.1: Addition, Subtraction, Multiplication, and Division Properties of Equality

3.2: Solving a Formula for a Variable (Rewriting a Formula)

3.3: Solving Quadratic Equations of the Form- $(x-a)^2=c$

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3.1: Addition, Subtraction, Multiplication, and Division Properties of Equality

The addition, subtraction, multiplication, and division properties of equality allow us to add, subtract, multiply, or divide the same value on both sides of an equation, this guarantees the equation remains true (note, we cannot divide by zero).

Concept: We know this to be a true statement: $5 = 5$

The statement will remain true if we perform the same operation on both sides of the equation.

- Add 4 on both sides of the equation to obtain:

$$\begin{aligned}5 + 4 &= 5 + 4 \\9 &= 9\end{aligned}$$

- Subtract 10 on both sides of the original equation to obtain:

$$\begin{aligned}5 - 10 &= 5 - 10 \\-5 &= -5\end{aligned}$$

- Multiply by 2 on both sides of the original equation to obtain:

$$\begin{aligned}5 \cdot 2 &= 5 \cdot 2 \\10 &= 10\end{aligned}$$

- Divide by 5 on both sides of the original equation to obtain:

$$\begin{aligned}\frac{5}{5} &= \frac{5}{5} \\1 &= 1\end{aligned}$$

We use the Addition, Subtraction, Multiplication, and Division Properties of Equality to solve equations for a specified variable or unknown.

Process for Solving a Basic Linear Equation in One-Variable

- Isolate the variable by “undoing” the operation on the variable, that is, applying the opposite operation on both sides of the equation using the Properties of Equality

✓ Example 3.1.1

Solve for x : $x + 2 = 9$

Solution

Since 2 is being added to x , to isolate the x , we need to “undo” the addition of 2, the opposite of adding 2 is subtracting 2, so using the Subtraction Property of Equality, let’s subtract 2 on both sides of the equation to obtain:

$$x + 2 - 2 = 9 - 2 \quad (3.1.1)$$

$$x = 7 \quad (3.1.2)$$

✓ Example 3.1.2

Solve for x : $x - 7 = 13$

Solution

Since 7 is being subtracted from x , to isolate the x , we need to “undo” the subtraction of 7, the opposite of subtracting 7 is adding 7, so using the Addition Property of Equality, let’s add 7 on both sides of the equation to obtain:

$$x - 7 + 7 = 13 + 7 \quad (3.1.3)$$

$$x = 20 \quad (3.1.4)$$

✓ Example 3.1.3

Solve for x : $3x = 12$

Solution

Since x is being multiplied by 3, to isolate the x , we need to “undo” the multiplication by 3, the opposite of multiplying by 3 is dividing by 3, so using the Division Property of Equality, let’s divide by 3 on both sides of the equation to obtain:

$$\frac{3x}{3} = \frac{12}{3} \quad (3.1.5)$$

$$x = \frac{12}{3} = 4 \quad (3.1.6)$$

✓ Example 3.1.4

Solve for x : $\frac{x}{8} = 2$

Solution

Since x is being divided by 8, to isolate the x , we need to “undo” the division by 8. The opposite of dividing by 8 is multiplying by 8, so using the Multiplication Property of Equality, let’s multiply by 8 on both sides of the equation to obtain:

$$\frac{x}{8} \cdot 8 = 2 \cdot 8 \quad (3.1.7)$$

$$\frac{x}{8} \cdot \frac{8}{1} = 2 \cdot 8 \quad (3.1.8)$$

$$x = 16 \quad (3.1.9)$$

✓ Example 3.1.5

Solve for x : $\frac{1}{2}x = 5$

Solution

We can approach this problem in a couple of different ways,

Option 1: Read the problem as x is being multiplied by $\frac{1}{2}$, hence we can divide both sides by $\frac{1}{2}$ to isolate the variable, x .

$$\frac{\frac{1}{2}x}{(\frac{1}{2})} = \frac{5}{(\frac{1}{2})} \quad (3.1.10)$$

$$x = \frac{5}{(\frac{1}{2})} = \frac{5}{1} \cdot \frac{2}{1} = \frac{10}{1} = 10 \quad (3.1.11)$$

Option 2: Rewrite the problem or think of the problem as being read as x is being divided by 2 since $\frac{1}{2}x$ is equivalent to $\frac{x}{2}$, so we can multiply both sides of the equation by 2 to isolate x :

$$\frac{1}{2}x = 5 \quad (3.1.12)$$

$$\frac{x}{2} = 5 \quad (3.1.13)$$

$$\frac{x}{2} \cdot 2 = 5 \cdot 2 \quad (3.1.14)$$

$$x = 10 \quad (3.1.15)$$

Similarly, if we have a fraction times a variable, let’s say x , then we can multiply both sides of the equation by the reciprocal of the fraction (flip the fraction such that the numerator becomes the denominator, and the denominator becomes the numerator):

✓ Example 3.1.6

Solve for x: $\frac{2}{3}x = 7$

Solution

$$\frac{2}{3}x = 7 \quad (3.1.16)$$

$$\frac{3}{2} \cdot \frac{2}{3}x = \frac{3}{2} \cdot 7 \quad (3.1.17)$$

$$x = \frac{3}{2} \cdot \frac{7}{1} = \frac{21}{2} \quad (3.1.18)$$

Process for solving a linear equation in one variable with multiple operations

When solving a linear equation with multiple operations, we reverse the order of operations because we are “undoing” the original operations.

Example:

1. Solve for x: $2x + 5 = 15$

Order of operations state to perform multiplication, then addition, so when solving, we will reverse this order so we will “undo” the addition first, then we will “undo” the multiplication

$$2x + 5 = 15$$

Step 1: “Undo the addition by 5” by subtracting 5 on both sides of the equation

$$2x + 5 - 5 = 15 - 5$$

$$2x = 10$$

Step 2: “Undo the multiplication by 2” by dividing by 2 on both sides of the equation

$$\frac{2x}{2} = \frac{10}{2}$$

$$x = 5$$

We can check our answer by substituting the value for x into the original equation and verifying that the equation is true: $(2(5) + 5 = 10 + 5 = 15)$, ✓

2. Solve for x: $\frac{2}{3}x + \frac{1}{5} = \frac{2}{7}$

$$\frac{2}{3}x + \frac{1}{5} = \frac{2}{7}$$

Step 1: Subtract $\frac{1}{5}$ on both sides of the equation

$$\frac{2}{3}x + \frac{1}{5} - \frac{1}{5} = \frac{2}{7} - \frac{1}{5}$$

$$\frac{2}{3}x = \frac{2}{7} - \frac{1}{5}$$

Step 2: Find an LCD to subtract the fractions on the right side:

$$\frac{2}{3}x = \frac{2}{7} \cdot \frac{5}{5} - \frac{1}{5} \cdot \frac{7}{7}$$

$$\frac{2}{3}x = \frac{10}{35} - \frac{7}{35}$$

$$\frac{2}{3}x = \frac{3}{35}$$

Step 3: Multiply both sides of the equation by the reciprocal of $\frac{2}{3}$ which would be $\frac{3}{2}$:

$$\frac{3}{2} \cdot \frac{2}{3}x = \frac{3}{2} \cdot \frac{3}{35}$$
$$x = \frac{9}{70}$$

Process for solving linear equations with parenthesis

When an equation contains parentheses, we can clear the parentheses using the distributive property.

Distributive Property: $a(b + c) = ab + ac$

Examples

1. Solve for m: $5(m + 3) - 2(7 - m) = 12$

Step 1: Apply the distributive property to clear the parentheses:

$$5m + 5(3) - 2(7) - 2(-m) = 12$$
$$5m + 15 - 14 + 2m = 12$$

Step 2: Combine like terms

$$5m + 2m + 15 - 14 + 1 = 12$$
$$7m + 1 = 12$$

Step 3: Isolate the variable by subtracting 1 on both sides, then dividing both sides by 7

$$7m + 1 - 1 = 12 - 1$$
$$7m = 11$$
$$\frac{7m}{7} = \frac{11}{7}$$
$$m = \frac{11}{7}$$

2. Solve for x: $-7(3 - x) + 11 = 2(x - 3)$

Step 1: Clear the parentheses by using the distributive property

$$-21 + 7x + 11 = 2x - 6$$

Step 2: Combine like terms

$$-10 + 7x = 2x - 6$$

Step 3: Isolate the variable by subtracting $2x$ on both sides of the equation and adding 10 on both sides of the equation

$$-10 + 10 + 7x - 2x = 2x - 2x - 6 + 10$$
$$5x = 4$$

Now, divide both sides by 5:

$$\frac{5x}{5} = \frac{4}{5}$$
$$x = \frac{4}{5}$$

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3.2: Solving a Formula for a Variable (Rewriting a Formula)

In the Water Industry, we use lots of formulas for computing area, volume, pressure, cost, and so much more. Sometimes, we need to rewrite a formula in terms of a different quantity.

When solving a formula for a particular variable, it is sometimes helpful to place a rectangle or a circle around the variable you are isolating since formulas contain many variables and we may lose our focus as to which variable we are isolating. We will proceed as we did for solving linear equations by “undoing” operations to isolate the variable.

✓ Example 3.2.1

The volume of a rectangular tank is given by $V = LWH$, rewrite the formula by solving for H .

Solution

To “undo” the multiplication of H by LW , we divide both sides of the equation by LW to obtain:

$$\frac{V}{LW} = \frac{LWH}{LW} \quad (3.2.1)$$

$$\frac{V}{LW} = H \quad (3.2.2)$$

✓ Example 3.2.2

The filtration rate is defined as $\text{Filtration Rate} = \frac{\text{Flow}}{\text{Surface Area}}$. Solve this equation for the Surface Area.

Solution

Step 1: Multiply both sides of the equation by Surface Area (to remove the Surface Area from the denominator. In other words, we cannot isolate Surface Area until it is no longer in the denominator)

$$(\text{Filtration Rate}) \cdot (\text{Surface Area}) = \frac{\text{Flow}}{\text{Surface Area}} \cdot (\text{Surface Area}) \quad (3.2.3)$$

$$(\text{Filtration Rate}) \cdot (\text{Surface Area}) = \text{Flow} \quad (3.2.4)$$

Step 2: Isolate the Surface Area by dividing both sides by Filtration Rate

$$\frac{(\text{Filtration Rate}) \cdot (\text{Surface Area})}{(\text{Filtration Rate})} = \frac{\text{Flow}}{(\text{Filtration Rate})} \quad (3.2.5)$$

$$\text{Surface Area} = \frac{\text{Flow}}{\text{Filtration Rate}} \quad (3.2.6)$$

This is helpful if we need to calculate the Surface Area and we are given the flow and the filtration rate.

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3.3: Solving Quadratic Equations of the Form- $(x-a)^2=c$

When solving equations of the form: $(x-a)^2 = c$, we can use the square root property which states to drop the square and take the positive and negative square root of the constant, c . Mathematically, we say the following:

If $(x-a)^2 = c$,

then $x-a = \pm\sqrt{c}$

next, add a on both sides of the equation to obtain

$x = a \pm \sqrt{c}$

✓ Example 3.3.1

Solve $(x-5)^2 = 16$

Solution

To solve, we first drop the square from the left side and take the positive and negative square root of 16 to obtain

$$x-5 = \pm\sqrt{16} \quad (3.3.1)$$

Next, we simplify the square root

$$x-5 = \pm 4 \quad (3.3.2)$$

Next, we isolate the variable, x by adding 5 on both sides of the equation:

$$x-5+5 = +5 \pm 4 \quad (3.3.3)$$

Such that after simplifying, we obtain two answers

$$x = 5 \pm 4 \quad (3.3.4)$$

$$x = 5 + 4 \text{ and } x = 5 - 4 \quad (3.3.5)$$

$$x = 9 \text{ and } x = 1 \quad (3.3.6)$$

✓ Example 3.3.1

Solve $3(x+2)^2 + 5 = 80$

Solution

To solve, we must first isolate the perfect square to create the form: $(x-a)^2 = c$, so we first subtract 5 on both sides of the equation, then divide both sides by 3

$$3(x+2)^2 + 5 - 5 = 80 - 5 \quad (3.3.7)$$

$$3(x+2)^2 = 75 \quad (3.3.8)$$

$$\frac{3(x+2)^2}{3} = \frac{75}{3} \quad (3.3.9)$$

$$(x+2)^2 = 25 \quad (3.3.10)$$

Next, we drop the square and take the positive and negative square root of 25 to obtain

$$x+2 = \pm\sqrt{25} \quad (3.3.11)$$

$$x+2 = \pm 5 \quad (3.3.12)$$

Next, we isolate the x by subtracting 2 on both sides of the equation

$$x + 2 - 2 = -2 \pm 5 \quad (3.3.13)$$

After simplifying, we obtain two answers

$$x = -2 \pm 5 \quad (3.3.14)$$

$$x = -2 + 5 \text{ and } x = -2 - 5 \quad (3.3.15)$$

$$x = 3 \text{ and } x = -7 \quad (3.3.16)$$

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CHAPTER OVERVIEW

4: Scientific Notation, Rounding, Conversions, and Dimensional Analysis

[4.1: Scientific Notation](#)

[4.2: Rounding](#)

[4.3: Dimensional Analysis](#)

Thumbnail: Expansion of small numbers expressed in scientific notation (CC BY SA 3.0 Unported; [Brian Brondel](#) via [Wikipedia](#))

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4.1: Scientific Notation

Scientific notation is used to make very large numbers such as 4,895,000,000 or very small numbers such as 0.0000073 easier to use. In the Water industry, we use very large numbers when referring to the volume of water in large tanks or very small numbers when referring to pollutants per gallon of water.

Scientific Notation Form:

$$\underbrace{(a \text{ decimal})}_{\text{between 0 and 10 (not including 10)}} \times 10^{\text{power}} \quad (4.1.1)$$

To convert from scientific notation to the actual number

- For a positive exponent on the 10, move the decimal place to the right the equivalent number of spaces as the power
- For a negative exponent on the 10, move the decimal place to the left the equivalent number of spaces as the absolute value of the power

✓ Example 4.1.1

Convert the scientific notation to the actual number: 8.735×10^7

Solution

Since the power of 10 is positive 7, we will move the decimal place 7 units to the RIGHT to obtain: 87,350,000

✓ Example 4.1.2

Convert the scientific notation to the actual number: 2.356×10^{-4}

Solution

Since the power of 10 is negative 4, we will move the decimal place 4 units to the LEFT to obtain: 0.2356

To convert a positive number to scientific notation

- Move the decimal place to the right of the first non-zero digit. This will be the decimal number portion of the scientific notation.
- If the decimal place was moved to the left, use a positive power of 10 based on the number of places the decimal was moved
- If the decimal place was moved to the right, use a negative power of 10 based on the number of places the decimal was moved

✓ Example 4.1.3

Convert 567,900,000 to scientific notation.

Solution

Currently, the decimal is understood to be after the last digit since this is a whole number, so move the decimal to the left 8 places so the decimal is between 5 and 6, hence the scientific notation is 5.679×10^8

✓ Example 4.1.4

Convert 0.00032 to scientific notation.

Solution

Move the decimal to the right 4 places so the decimal is between 3 and 2 to create a decimal number of 3.2 and the power of 10 would then be negative 4 since we moved the decimal to the right 4 places. The scientific notation would be 3.2×10^{-4}

Note

Numbers that are larger than 1 will have a positive power of 10 in scientific notation and numbers that are less than 1 (but still positive) will have a negative power of 10 in scientific notation.

Display of scientific notation on scientific calculators

To multiply and divide numbers in scientific notation, we can multiply (or divide) the decimals and multiply (or divide) the powers of 10, then simplify by rewriting into the proper scientific notation form. To do this, we will need to understand some rules of exponents.

Rule of Exponents

1. When multiplying expressions with the same base, we keep the base and add the exponents:

$$a^x \cdot a^y = a^{x+y} \quad (4.1.2)$$

2. When dividing expressions with the same base, we keep the base and subtract the exponents:

$$\frac{a^x}{a^y} = a^{x-y} \quad (4.1.3)$$

3. Negative exponents become positive exponents if we move the expression to the opposite side of a fraction:

$$a^{-x} = \frac{1}{a^x} \text{ and } \frac{1}{a^{-x}} = a^x \quad (4.1.4)$$

4. A non-zero expression raised to an exponent of zero is equivalent to 1:

$$a^0 = 1 \quad (4.1.5)$$

✓ Example 4.1.5

Multiply and simplify using scientific notation: $(2.45 \times 10^6)(3.23 \times 10^{-15})$

Solution

First, we can rearrange the multiplication to obtain:

$$(2.45 \times 10^6) (3.23 \times 10^{-15}) \quad (4.1.6)$$

$$= (2.45) (3.23) \times (10^6)(10^{-15}) \quad (4.1.7)$$

Next, we can multiply the decimals, then multiply the powers of 10 using an exponent rule:

$$= 7.9135 \times 10^{6+(-15)} \quad (4.1.8)$$

$$= 7.9135 \times 10^{-9} \quad (4.1.9)$$

Since the decimal value of 7.9135 is between 0 and 10 (not including 10), then this is the proper scientific notation form.

✓ Example 4.1.2

Multiply and simplify using scientific notation: $(8.7 \times 10^{-6})(2.5 \times 10^{12})$

Solution

First, we can rearrange the multiplication to obtain:

$$(8.7 \times 10^{-6})(2.5 \times 10^{12}) = (8.7) (2.5) \times (10^{-6})(10^{12}) \quad (4.1.10)$$

Next, we can multiply the decimals, then multiply the powers of 10 using an exponent rule:

$$= 21.75 \times 10^{-6+12} \quad (4.1.11)$$

$$= 21.75 \times 10^6 \quad (4.1.12)$$

Since the decimal value of 21.75 is not between 0 and 10 (not including 10), we need to convert it to proper scientific notation form and simplify further

$$21.75 \times 10^6 = (2.175 \times 10^1) \times 10^6 \quad (4.1.13)$$

$$= 2.175 \times 10^{1+6} \quad (4.1.14)$$

$$= 2.175 \times 10^7 \quad (4.1.15)$$

✓ Example 4.1.3

Divide and simplify using scientific notation: $\frac{4.125 \times 10^{13}}{7.5 \times 10^{-2}}$

Solution

First, we divide the decimals, then divide the powers of 10 using an exponent rule:

$$\frac{4.125 \times 10^{13}}{7.5 \times 10^{-2}} = \frac{4.125}{7.5} \times \frac{10^{13}}{10^{-2}} \quad (4.1.16)$$

$$= 0.55 \times 10^{13-(-2)} \quad (4.1.17)$$

$$= 0.55 \times 10^{15} \quad (4.1.18)$$

Since the decimal value of 0.55 is not between 0 and 10 (not including 10), we need to convert it to proper scientific notation form and simplify further

$$0.55 \times 10^{15} = (5.5 \times 10^{-1}) \times 10^{15} \quad (4.1.19)$$

$$= 5.5 \times 10^{-1+15} \quad (4.1.20)$$

$$= 5.5 \times 10^{14} \quad (4.1.21)$$

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4.2: Rounding

We have been using decimal numbers throughout our lives, but let's take some time to formalize the language so we can learn how to round decimals appropriately.

Reading decimal numbers:

Fraction	Decimal	Name
$\frac{1}{10}$	0.1	One-tenth
$\frac{1}{100}$	0.01	One-hundredth
$\frac{1}{1000}$	0.001	One-thousandth
$\frac{1}{10000}$	0.0001	One-ten-thousandth
$\frac{1}{100000}$	0.00001	One-hundred-thousandth

The place values to the right of the decimal are given the following names:

Tenths, hundredths, thousandths, ten-thousandths, hundred-thousandths, millionths, etc.

As a result, we read 5.643 as “five and six-hundredth-forty-three- thousandths since the last digit after the decimal ends in the thousandths place.

Examples:

1. 45.6 is read as “forty-five and six tenths”
2. In the number 543.7892, the 8 is in which place value? Since the 8 is two digits after the decimal point, the place value is hundredths.
3. 0.0143 is read as “one-hundred-forty-three ten-thousandths

For rounding decimals, to round to the nearest decimal place,

1. Consider the number to the right of the desired rounding place value
 1. If the digit is less than 5 (4 or less), we truncate at the desired rounding place value, that is, we drop all the digits after the desired rounding place value
 2. If the digit is 5 or more, we round up by adding 1 to the desired rounding place value before dropping all the remaining digits to the right

Examples:

1. Round 32.784 to the nearest hundredth. Since 8 is in the hundredths place, we consider the number immediately to the right (in the thousandths place), since 4 is less than 5, we drop all the digits after the hundredths place to obtain 32.78.
2. Round 32.786 to the nearest hundredth. Since 8 is in the hundredths place, we consider the number immediately to the right (in the thousandths place), since 6 is more than 5, we add 1 to 8 (in the hundredths place, this is called rounding up) and we drop all the digits after the hundredths place to obtain 32.79.
3. Round 6.48327 to the nearest thousandth. We start by consider the value of the digit immediately after the thousandths place, which is 2, since 2 is less than 5, we drop all the digits after the thousandths place to obtain 6.483 as the rounded number to the nearest thousandth.
4. Round 6.4592 to the nearest
 1. Tenth $\Rightarrow$ Since 5 immediately follows the tenths place, we round the 4 in the tenths place up one number to 5 (by adding 1 to 4) to obtain 6.5 as the rounded number to the tenths place.
 2. Hundredth $\Rightarrow$ Since 9 immediately follows the hundredths place, we round the 5 in the hundredths place up to 6 to obtain 6.46 as the rounded number to the hundredths place.

3. Thousandth $\Rightarrow$ Since 2 immediately follows the thousandths place, we round the 5 in the hundredths place up to 6 to obtain 6.46 as the rounded number to the hundredths place.
5. Round 6.4597 to the nearest thousandth (to three decimal places). Since the fourth digit is 5 or higher, we must round the third digit after the decimal up by adding 1, but if we add 1 to 9, we would get 10, so we think of 59 (the second and third places after the decimal and round it up by 1 to obtain 60, hence 6.4597 rounded to three decimal places or the nearest thousandth becomes 6.460. We must write the zero in the thousandth place in this case to show the rounding to three decimal places, without the zero on the end, the rounding would be incorrect.

Significant Figures

Significant figures are used to assure the accuracy of a calculation based on the numbers used within the calculation. When a problem doesn't specify how to round an answer or it is not implied in its context (such as money), we use significant figures.

The following digits are considered significant:

- Digits between 1-9. Example: 875 would consist of three significant figures.
- Any zeros between non-zero digits. Example: 20.07 and 5608 would both consist of four significant figures.
- The trailing zeros in a decimal number are significant

The following digits represent ambiguous cases, so the best method to determine the significant figures is to convert the number to scientific notation:

- Any zeros in numbers that do not contain a decimal point. Example: 530,000 depending on the situation, this could represent two significant figures or six significant figures, so let's convert it to scientific notation to obtain: 5.3×10^4 which would now represent two significant figures. In general, if the zeros are used to locate the decimal point, they are not significant figures, whereas if the zeros are being used for accuracy, they are included as significant figures.

The following digits are not significant:

- If a number is less than 1, the zeros that occur after the decimal point but before a non-zero digit. Example: 0.0075 would consist of two significant figures

When performing calculations, the number of significant figures of the final answer is equivalent to the fewest number of significant figures provided in the calculation.

Examples:

1. State the number of significant figures in each of the following numbers
 1. 3.402 $\Rightarrow$ 4 significant figures, the non-zero digits are counted since they are between 1-9 and the zero is counted since it is between two non-zero digits
 2. 0.000472 $\Rightarrow$ 3 significant figures; this number is less than 1, so the zeros after the decimal point but before the non-zero digits do not count, so we only count the digits between 1-9
 3. 8.0×10^4 $\Rightarrow$ 2 significant figures; we consider the decimal value (not the power of 10), the trailing zero counts since it is used to show accuracy of the number
 4. 452,000 $\Rightarrow$ ambiguous case, 3 to 6 significant figures depending on the context of the number
2. Round the following numbers to three significant figures
 1. 89.0146 $\Rightarrow$ Since the zero after the decimal point is significant since it is between two non-zero digits, we round to the nearest tenth in this case to obtain 89.0
 2. 0.005324 $\Rightarrow$ Since this number is less than 1, the zeros immediately to the right of the decimal point are not significant, so to round to three significant figures, we round to the nearest hundred-thousandth to obtain 0.00532
 3. 872.58 $\Rightarrow$ Since all the digits are significant, to round to three significant figures, we round to the nearest whole number (three digits from the left) to obtain 873.
 4. 8723.158 $\Rightarrow$ Since all the digits are significant, to round to three significant figures, we round to the nearest ten (three digits from the left) to obtain 8720.
 5. 65,207 $\Rightarrow$ Since all the digits are significant, to round to three significant figures, we round to the nearest hundred (three digits from the left) or 65,200.

3. Based on the given calculations, state the number of significant figures of the answer, then perform the calculation and round it based on the significant figures

1. $(4.5)(2.33)(6.232) \Rightarrow$ Two significant figures since 4.5 has the least number of significant figures.

$$(4.5)(2.33)(6.232) = 65.34252 \text{ which rounds to } 65$$

2. $\frac{(2.543)(3.516)}{0.01} \Rightarrow$ One significant figure since 0.01 has the least number of significant figures.

$$\frac{(2.543)(3.516)}{0.01} = \frac{8.941188}{0.01} = 894.1188 \text{ which rounds to } 900$$

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4.3: Dimensional Analysis

Often in the Water and Wastewater Operations, we must use the relationship between different quantities to determine the results of various calculations. In other words, we will need to perform calculations while also recognizing the changes in the measurements and dimensions to assure the measurement of the result is reasonable.

Conversions

In the Water Industry, we often convert between units of measurement such as gallons to liters or days to hours. To convert between units of measurement, we first need to know the conversion factors or equivalencies. These equivalencies are usually provided in a chart and over time, you will most likely start to learn the ones that are used often, much like we know there are 24 hours in a day or 7 days in a week or 12 feet in a yard.

When converting between units, the idea is to multiply by a form of 1. We will create a fraction that is equivalent to 1 by doing the following:

1. Set up the starting measurement as a fraction.
2. The unit we are removing is placed opposite the position of where it is currently
3. The “new” unit will be placed in the other position
4. Insert the equivalencies

Examples:

1. Convert 153 miles per hour to feet per hour. We start by writing the words into a fraction, namely, $\frac{153 \text{ miles}}{1 \text{ hour}}$. Next, since we want to convert miles to feet, we need to eliminate the miles, so we will multiply by a fraction with miles in the denominator (opposite of where the miles are currently) and since we need to replace the miles with feet, we will place the feet in the numerator (to replace the current miles), then insert the equivalencies, 5280 feet = 1 mile, hence we multiply the original measurement by $\frac{5280 \text{ feet}}{1 \text{ mile}}$

$$\frac{153 \text{ miles}}{1 \text{ hour}} \cdot \frac{5280 \text{ feet}}{1 \text{ mile}}$$

Next, we simplify by reducing or canceling the mile unit of measurement and multiply straight across to obtain:

$$\frac{153 \text{ miles}}{1 \text{ hour}} \cdot \frac{5280 \text{ feet}}{1 \text{ mile}} = \frac{153}{1 \text{ hour}} \cdot \frac{5280 \text{ feet}}{1} = \frac{807,840 \text{ feet}}{1 \text{ hour}} \text{ or } 807,840 \text{ feet per hour}$$

2. The Plant needs to treat every 60 gallons of water with 7 grams of chlorine. However, the chlorine is packaged in ounces. Convert the required amount from gallons/gram to gallons/ounce. We start by writing the given information as a fraction, namely, $\frac{60 \text{ gallons}}{7 \text{ grams}}$. Next, since we want to convert the grams to ounces and the grams are currently in the denominator, we create a fraction with the grams in the numerator and the ounces in the denominator (so that the grams are opposite each other) and then insert the equivalencies to obtain:

$$\begin{aligned} \frac{60 \text{ gallons}}{7 \text{ grams}} \cdot \frac{1 \text{ gram}}{0.03527396195 \text{ ounces}} &= \frac{60 \text{ gallons}}{0.246917734 \text{ ounces}} \\ \frac{60 \text{ gallons}}{7 \text{ grams}} \cdot \frac{1 \text{ gram}}{0.03527396195 \text{ ounces}} &= \frac{60 \text{ gallons}}{0.246917734 \text{ oz}} = 242.9959125 \text{ gal/oz} \end{aligned} \quad (4.3.1)$$

To convert between metric units, we can use the same process described above or we can use a prefix table and move the decimal point based on the direction and the number of places it takes to move from one metric unit of measure to the others based on the order of the sizes of the prefixes.

Let's begin by using a mnemonic to learn the basic six prefixes along with the base unit measurement and their order on the ladder.

“King Henry Died By Drinking Chocolate Milk”

This statement can help with remembering the order of six prefixes and the base unit.

KILO	HECTO	DEKA	BASE	DECI	CENTI	MILLI
------	-------	------	------	------	-------	-------

For the base, you can have any of the following units of measurement:

- Meters (length)
- Grams (weight)
- Liters (volume)
- Meters² (area)

To convert from one prefix to another,

1. Determine if we must move left or right
2. Determine the number of places to move on the ladder
3. Move the decimal place in the number the same number of places in the same direction as we moved on the prefix ladder.

NOTE: Mega is three units to the left of Kilo and Micro is three units to the right of Milli

Examples:

1. Convert 674.325 centimeters to dekameters

Using the ladder, to move from centi to deka, we must move LEFT 3 places, hence, we must move the decimal place 3 places to the left to obtain 0.674325 dekameters.

2. Convert 67 hectoliters to milliliters.

Using the ladder, to move from hector to milli, we must move RIGHT 5 places, hence, we must move the decimal place 5 places to the right, keeping in mind that for a whole number, the decimal is understood to be after the last digit, 67 hectoliters convert to 6,700,000 milliliters.

3. Convert 6.5 kilograms to megagrams.

Since mega is 3 places to the left of kilo, we must move the decimal place 3 places to the left so 6.5 kilograms becomes 0.0065 megagrams.

4. Convert 8765 centimeters to meters.

Remembering that the base represents meters, we need to move from centi to the base (meters) which would be LEFT by 2 places, so move the decimal two places to the left to convert 8765 centimeters to 87.65 meters.

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CHAPTER OVERVIEW

5: Ratios, Proportions, and Percentages

[5.1: Ratios](#)

[5.2: Proportions](#)

[5.3: Percentages](#)

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5.1: Ratios

A ratio is a comparison of two numerical quantities in the same units that is usually expressed as a fraction.

Examples:

1. The ratio of 2 meters to 7 meters is $\frac{2m}{7m} = \frac{2}{7}$.
2. There are 200,000 gallons of water in a rectangular basin and 400,000 gallons of water in a cylindrical basin.
 1. What is the ratio of the number of gallons of water in the rectangular basin to the cylindrical basin?

$$\frac{200,000 \text{ gallons}}{400,000 \text{ gallons}} = \frac{2}{4} = \frac{1}{2}$$

2. What is the ratio of the number of gallons of water in the cylindrical basin to the rectangular basin?

$$\frac{400,000 \text{ gallons}}{200,000 \text{ gallons}} = \frac{4}{2} = \frac{2}{1} = 2$$

3. The price of (some chemical) recently increased from \$10.80/case to \$13.50/case. Find the ratio of the increase in price to the original price.

The increase in price is determined by subtracting the original price from the ending price: increase in price = \$13.50 - \$10.80 = \$2.70.

The ratio of the increase in price to the original price = $\frac{\$2.70}{\$10.80} = 0.25$ or $\frac{1}{4}$

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5.2: Proportions

A proportion determines the relationship between two variables as it relates to the magnitude or size of the variables. A proportional relationship shows that two ratios are equivalent such that as one amount changes, another amount changes by a relative amount.

To solve proportion problems, we set up an equation such that two ratios are equivalent:

$$\frac{\text{One characteristic from one item}}{\text{A second characteristic from the same item}} = \frac{\text{the same first characteristic from a second item}}{\text{the same second characteristic from the second item}} \quad (5.2.1)$$

✓ Example 5.2.1

Seven containers of a chemical costs \$125. If the cost of the containers is proportional, meaning the unit price does not change, determine the cost of 12 containers of the same chemical.

Solution

Let's begin by setting up the fractions like in Equation 5.2.1:

$$\frac{\text{Number of containers}}{\text{Cost of those containers}} = \frac{\text{Number of another set of containers}}{\text{Cost of those containers}}$$

$$\frac{7 \text{ containers}}{\$125} = \frac{12 \text{ containers}}{x}$$

To solve for x we can use one of two methods:

Method 1: Solve for x by isolating the x (using information learned in Unit 3)

Multiply both sides of the equation by x :

$$\frac{7}{125}x = 12$$

Multiply both sides of the equation by \$125:

$$7x = 12 \cdot 125$$

$$7x = 1500$$

Divide both sides of the equation by 7 containers:

$$x = \frac{1500}{7} = \$214.29$$

Method 2: Use cross multiplication: Multiply the numerator of the first fraction by the denominator of the second fraction and set it equal to the multiplication of the numerator of the second fraction by the denominator of the first fraction.

$$(7 \text{ containers}) \cdot (\$x) = (12 \text{ containers}) \cdot (\$125)$$

$$7x = 1500$$

$$x = \frac{1500}{7}$$

$$= \$214.29$$

Hence, the cost of the 12 containers is \$214.29.

✓ Example 5.2.2

We want to estimate the number of fish in a pond. Suppose we capture 280 fish, tag them, and throw them back into the pond. After a couple of days, we go back to the pond and capture 420 fish, of which 28 are tagged. Estimate the number of fish in the pond.

✓ Example 5.2.3

We wish to mix a solution of hypochlorite and water by dissolving 4.8 pounds of hypochlorite in 70 gallons of water. For the same concentration, how many pounds of hypochlorite should we dissolve in 20 gallons of water?

Solution

Let's begin by setting up the fractions like in Equation 5.2.1:

$$\begin{aligned}\frac{\text{Number of pounds of hypochlorite}}{\text{Number of gallons of water}} &= \frac{\text{Number of pounds of hypochlorite}}{\text{Number of gallons of water}} \\ \frac{4.8 \text{ lbs}}{70 \text{ gal}} &= \frac{x \text{ lbs}}{20 \text{ gal}} \\ (4.8 \text{ lbs})(20 \text{ gal}) &= (70 \text{ gal})(x \text{ lbs}) \\ 96 &= 70x \\ \frac{96}{70} &= x \\ x &= 1.4 \text{ lbs}\end{aligned}$$

Hence, we need 1.4 pounds of hypochlorite to dissolve in 20 gallons of water to create the same concentration.

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5.3: Percentages

In Unit 1, we learned how to convert between fractions, decimals, and percentages. We also learned that a percent calculated

$$\text{Percent} = \frac{\text{Part}}{\text{Whole}} \cdot 100\% \quad (5.3.1)$$

In this unit, we will use percentages in applications. We will concentrate on the following types of calculations which can be derived by rewriting the formula above (as learned in Unit 3):

- Calculating a percent of an amount: $(\text{percentage})(\text{whole}) = \text{part}$
- Determine the percent based on the part and the whole: $\frac{\text{Part}}{\text{Whole}} \cdot 100\% = \text{Percent}$
- A value is a certain percent of a number: $\text{whole} = \frac{\text{part}}{\text{percent}}$

Examples:

1. 52% of 560 is what number?

Since “of” means multiplication, we have $(52\%)(560) = \text{a number}$, so let’s convert 52% to a decimal by dividing 52 by 100 or by moving the decimal place two units to the left to obtain 0.52, hence $(52\%)(560) = (0.52)(560) = 291.2$. As a result, 52% of 560 is 291.2.

2. 32 is what percent of 420?

In this problem, we need to calculate the percent and we have the part, which is 32, and the whole, which is 420. Hence $\text{Percent} = \frac{32}{420} \cdot 100\% = 0.076 \cdot 100\% = 7.6\%$ (using two significant figures; see Unit 5).

3. 23 is 12% of what number? Round the final answer to the nearest hundredth. (Unit 5)

In this problem, we are given the part and the percent and need to determine the whole, so we use: $\text{whole} = \frac{\text{part}}{\text{percent}} = \frac{23}{12\%} = \frac{23}{0.12} = 191.67$

Percentages can be used in the Water Industry in many ways. Below are a few applications of percentages and ratios.

- The volatile content of raw sludge can be calculated as a rate, proportion, or a percent. Generally, we will use a percent to represent the volatile content of raw sludge by calculating the reduction of volatile matter using the following formula:

$$\text{Volatile Solids Reduction} (= \frac{(\text{Value IN}) - (\text{Value OUT})}{(\text{Value IN}) - [(\text{Value IN})(\text{Value OUT})]} \cdot 100\% \quad (5.3.2)$$

When the volatile solids reduction is between 50%-60%, it is a good or healthy anaerobic digester. Any amount less than 50% is not reducing the volatile solids enough. NOTE: The fraction portion of this formula represents the rate or proportion. By multiplying by 100% we obtain the percentage. Also, note that the Value IN and Value OUT are decimal values between 0 and 1.

Example: The volatile content of raw sludge is 68.2% while the digester sludge is 55.6%. Calculate the volatile solids reduction and determine if the reduction is a healthy amount.

$$\text{Volatile Solids Reduction} (= \frac{(\text{Value IN}) - (\text{Value OUT})}{(\text{Value IN}) - [(\text{Value IN})(\text{Value OUT})]} \cdot 100\% \quad (5.3.3)$$

$$= \frac{0.682 - 0.556}{0.682 - (0.682)(0.556)} \cdot 100\% \quad (5.3.4)$$

$$= \frac{0.126}{0.682 - 0.379} \cdot 100\% \quad (5.3.5)$$

$$= \frac{0.126}{0.303} \cdot 100\% \quad (5.3.6)$$

$$= 0.4158 \cdot 100\% = 41.6\% \quad (5.3.7)$$

Since 41.6% is not between 50% and 60%, this is considered to not be a good reduction of waste solids.

- The efficiency of water can be calculated using the following formula:

$$\text{Efficiency} = \frac{(\text{Value IN} - \text{Value OUT})}{\text{Value IN}} \cdot 100\% \quad (5.3.8)$$

where the Value IN corresponds to the water received from another water system through intertie and Value OUT corresponds to water supplied to another water system through an intertie. [Intertie as defined by Merriam- Webster Dictionary, “an interconnection permitting passage of current between two or more electric utility systems” such as in a share federal-state water system.

NOTE: The fraction portion of this formula represents the rate or proportion. By multiplying by 100% we obtain the percentage.

Example: Calculate the efficiency (as a percent) of removing BOD (Biological Oxygen Demand) at a wastewater treatment plant if the influx of BOD is 215 mg per liter and the outflow is 18 mg per liter.

$$\text{Efficiency} = \frac{(\text{Value IN} - \text{Value OUT})}{\text{Value IN}} \cdot 100\% \quad (5.3.9)$$

$$= \frac{(215 \frac{\text{mg}}{\text{L}} - 18 \frac{\text{mg}}{\text{L}})}{215 \frac{\text{mg}}{\text{L}}} \cdot 100\% \quad (5.3.10)$$

$$= \frac{197 \frac{\text{mg}}{\text{L}}}{215 \frac{\text{mg}}{\text{L}}} \cdot 100\% \quad (5.3.11)$$

$$= 0.916 \cdot 100\% \quad (5.3.12)$$

$$= 91.6\% \quad (5.3.13)$$

Hence, the efficiency of removing BOD at the wastewater treatment plant is 91.6%.

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CHAPTER OVERVIEW

6: Geometry

[6.1: Definitions](#)

[6.2: Examples](#)

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6.1: Definitions

In the Water Industry, often it will be important to calculate perimeter, circumference, area, volume, or surface area (lateral area) of various objects. Let's start by defining each geometric calculation.

Definitions

Perimeter: The linear measurement or distance around the outside of a two-dimensional object such as a square, rectangle, triangle, etc. The units are expressed as a linear measurement such as inches, feet, yards, miles, millimeters, centimeters, meters, kilometers, etc.

$$P = \text{length of side 1} + \text{length of side 2} + \text{length of side 3} + \text{length of side 4} + \dots \quad (6.1.1)$$

Circumference: The linear measurement around the outside of a circle. The units are expressed as a linear measurement such as inches, feet, yards, miles, millimeters, centimeters, meters, kilometers, etc. NOTE: Circumference is the name we use to represent the perimeter of a circle.

$$C = \pi(\text{diameter}) \text{ or } C = 2\pi(\text{radius}) \quad (6.1.2)$$

Area (or Surface Area or Lateral Area): The measurement of the amount of space inside a 2-dimensional object or on the surface of a 3-dimensional object. The units are expressed as square linear units such as square inches (in²), square meters (m²), square feet (ft²), etc.

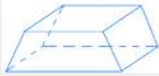
The formula for area depends on the object, so let's start with the area of some common objects:

Geometric Measurement	Formula	Shape
Area of a rectangle	$A = (\text{length})(\text{width})$	
Area of a triangle	$A = \frac{1}{2}(\text{base})(\text{height})$	
Area of a circle	$A = \pi(\text{radius})^2$ or $A = (0.785)(\text{diameter})^2$	
Area of a trapezoid	$A = \frac{1}{2}(\text{base 1} + \text{base 2})(\text{height})$	

Volume: The measurement of the amount of space inside a 3-dimensional object. The units are expressed as cubic linear units such as cubic inches (in³), cubic meters (m³), cubic feet (ft³), etc.

The general formula for volume is: $\text{Volume} = (\text{Area of a surface})(\text{height or length})$

The formula for volume depends on the object, so let's start with the volume of some common objects:

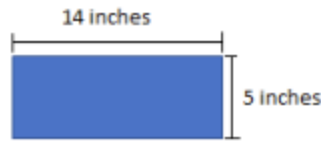
Geometric Measurement	Formula	Shape
Volume of a Rectangular Box or Prism	$V = (\text{length})(\text{width})(\text{height})$	
Volume of a Cylinder	$V = \pi(\text{radius})^2(\text{height})$ or $V = (0.785)(\text{diameter})^2(\text{height})$	
Volume of a Cone	$V = \frac{1}{3}\pi(\text{radius})^2(\text{height})$ or $V = \frac{1}{3}(0.785)(\text{diameter})^2(\text{height})$	
Volume of a Sphere	$V = \frac{4}{3}\pi(\text{radius})^3$ or $V = \frac{2}{3}(0.785)(\text{diameter})^3$	
Volume of a Triangular Prism	$V = (\text{Area of the triangular surface})(\text{depth})$ or $V = \frac{1}{2}(\text{base})(\text{height})(\text{depth or width})$	
Volume of a Trapezoidal Prism	$V = (\text{Area of the trapezoidal surface})(\text{depth})$ or $V = \frac{1}{2}(\text{base 1} + \text{base 2})(\text{height})(\text{depth or width})$	

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6.2: Examples

1. Calculate the perimeter (or circumference if appropriate) and area of the following shapes:

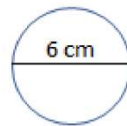
a. Calculate the perimeter and area of the rectangle using the figure below.



$$\begin{aligned}\text{Perimeter} &= \text{sum of the lengths of the sides} \\ &= 14 \text{ in} + 5 \text{ in} + 14 \text{ in} + 5 \text{ in} = 2(14 \text{ in}) + 2(5 \text{ in}) \\ &= 28 \text{ in} + 10 \text{ in} = 38 \text{ in}\end{aligned}$$

$$\text{Area} = (\text{length}) \times (\text{width}) = (14 \text{ in})(5 \text{ in}) = 70 \text{ in}^2$$

b. Calculate the circumference and area of the circle using the figure below.

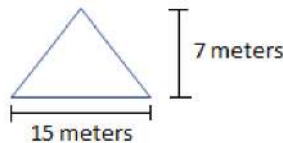


$$\text{Circumference} = \pi(\text{diameter}) = \pi(6 \text{ cm}) = (3.14)(6 \text{ cm}) = 18.84 \text{ cm}$$

$$\text{Area} = (0.785)(\text{diameter})^2 = (0.785)(6 \text{ cm})^2 = (0.785)(36 \text{ cm}^2) = 28.26 \text{ cm}^2$$

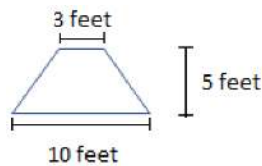
c. Calculate the area of the figures below.

i. A triangle with height of 7 meters and a base of 15 meters.



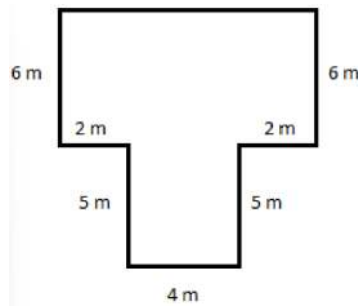
$$\text{Area} = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(15 \text{ m})(7 \text{ m}) = \frac{1}{2}(105 \text{ m}^2) = 52.5 \text{ m}^2$$

ii. A trapezoid with a height of 5 feet and bases of 3 feet and 10 feet.



$$\begin{aligned}\text{Area} &= \frac{1}{2}(\text{base 1} + \text{base 2})(\text{height}) \\ &= \frac{1}{2}(3 \text{ ft} + 10 \text{ ft})(5 \text{ ft}) \\ &= \frac{1}{2}(13 \text{ ft})(5 \text{ ft}) \\ &= \frac{1}{2}(65 \text{ ft}^2) = 32.5 \text{ ft}^2\end{aligned}$$

2. Calculate the perimeter and area of the following figure



a. Calculate the perimeter.

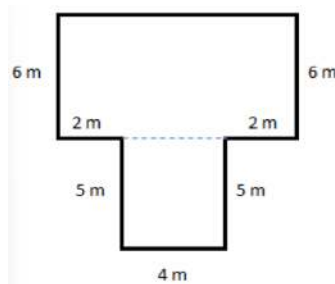
Since the perimeter is the distance around the outer edges, we can calculate the distance across the top edge to be the sum of the lower horizontal edges, namely $2\text{ m} + 4\text{ m} + 2\text{ m} = 8\text{ m}$

Next, add the lengths of the outer edges to obtain

$$\begin{aligned}\text{Perimeter} &= 8\text{ m} + 6\text{ m} + 2\text{ m} + 5\text{ m} + 4\text{ m} + 5\text{ m} + 2\text{ m} + 6\text{ m} \\ &= 8\text{ m} + 2(6\text{ m}) + 2(2\text{ m}) + 2(5\text{ m}) + 4\text{ m} \\ &= 8\text{ m} + 12\text{ m} + 4\text{ m} + 10\text{ m} + 4\text{ m} \\ &= 38\text{ m}\end{aligned}$$

b. Calculate the area.

Using a dashed line, divide the figure into two rectangles by drawing a horizontal line through the figure at the 2 m markings



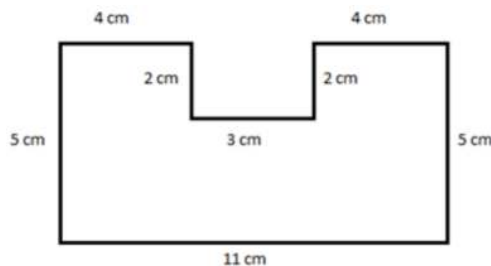
Next, find the area of each figure and add the results to obtain the area of the original figure.

$$\text{The area of the upper rectangle} = A_1 = (8\text{ m})(4\text{ m}) = 32\text{ m}^2$$

$$\text{The area of the lower rectangle} = A_2 = (8\text{ m})(2\text{ m}) = 16\text{ m}^2$$

$$\text{Total Area} = A_1 + A_2 = 32\text{ m}^2 + 16\text{ m}^2 = 48\text{ m}^2$$

3. Calculate the perimeter and area of the figure below.



a. Calculate the perimeter.

Since the perimeter is the distance around the outer edges, we add the lengths of the edges to obtain

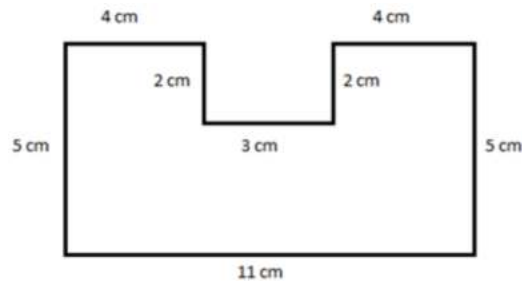
$$\begin{aligned}\text{Perimeter} &= 4\text{ cm} + 2\text{ cm} + 3\text{ cm} + 2\text{ cm} + 4\text{ cm} + 5\text{ cm} + 11\text{ cm} + 5\text{ cm} \\ &= 2(4\text{ cm}) + 2(2\text{ cm}) + 3\text{ cm} + 2(5\text{ cm}) + 11\text{ cm} \\ &= 8\text{ cm} + 4\text{ cm} + 3\text{ cm} + 10\text{ cm} + 11\text{ cm} \\ &= 36\text{ cm}\end{aligned}$$

b. Calculate the area.

We can compute the area of this figure in one of two ways, either using addition or by using subtraction. Let's investigate both methods.

i. Using Addition

Divide the figure into three rectangles by drawing a dashed line from the left side of the figure through to the right side of the figure, connecting at the 3 cm line.



Calculate the area of each rectangle and add the results; let's notice that the two upper rectangles are the same size

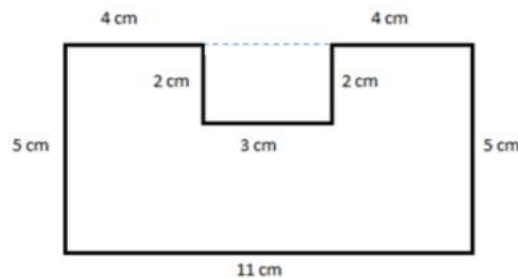
$$\text{Area of small rectangle} = (4 \text{ cm})(2 \text{ cm}) = 8 \text{ cm}^2$$

$$\text{Area of large rectangle} = (3 \text{ cm})(11 \text{ cm}) = 33 \text{ cm}^2$$

$$\text{Total area} = 8 \text{ cm}^2 + 8 \text{ cm}^2 + 33 \text{ cm}^2 = 49 \text{ cm}^2$$

ii. Using Subtraction

Find the area of the outer rectangle and subtract the area of the rectangular hole, so now let's draw a horizontal dashed line across the top opening of the figure



Next, let's find the area of the larger outer edge rectangle

$$\text{Area of large rectangle} = (5 \text{ cm})(11 \text{ cm}) = 55 \text{ cm}^2$$

$$\text{Area of the rectangular hole} = (2 \text{ cm})(3 \text{ cm}) = 6 \text{ cm}^2$$

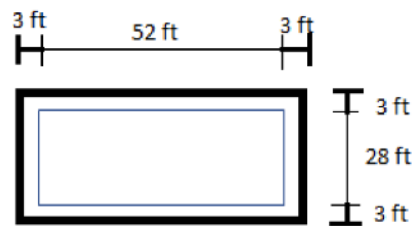
$$\begin{aligned} \text{Total Area} &= \text{Area of large outer rectangle} - \text{Area of the rectangular hole} \\ &= 55 \text{ cm}^2 - 6 \text{ cm}^2 = 49 \text{ cm}^2 \end{aligned}$$

c. A rectangular pool measures 28 feet by 52 feet. Surrounding (and bordering) the basin is a path 3 feet wide. Find the area of the path.

First, let's draw a figure to depict the scenario. Let's start by drawing a rectangle to represent the pool.



Next, let's draw the 3 ft wide border around the pool to represent the path. The path is represented by the shaded region.



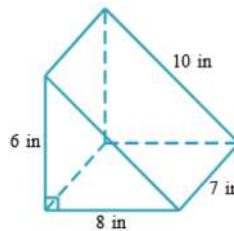
The outer edge of the path creates a larger rectangle whose length is the length of the pool plus twice the width of the path or $52 \text{ ft} + 2(3 \text{ ft}) = 58 \text{ ft}$. The width of the outer edge of the path is the width of the pool plus twice the width of the path or $28 \text{ ft} + 2(3 \text{ ft}) = 34 \text{ ft}$.

To obtain the area of the path, find

the area of the outer edge - the area enclosed by the pool = area of the path

$$\begin{aligned}\text{Area of the path} &= (58 \text{ ft})(34 \text{ ft}) - (28 \text{ ft})(52 \text{ ft}) \\ &= 1972 \text{ ft}^2 - 1456 \text{ ft}^2 \\ &= 516 \text{ ft}^2\end{aligned}$$

d. Calculate the lateral area (surface area) of a triangular prism as shown below



There are five sides or surfaces to this triangular prism. Two of the sides are triangles of the same size, namely with a height of 6 inches and a base of 8 inches and the other three sides are rectangles, the left face is 6 inches by 7 inches rectangle, the bottom face is 8 inches by 7 inches rectangle, and the right-side face is 10 inches by 7 inches rectangle.

To calculate the lateral area or surface area, we find the area of all the surfaces and add the results

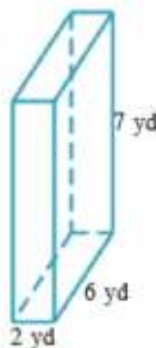
$$\text{Area of the two triangles} = 2 \left[\frac{1}{2} (8 \text{ in})(6 \text{ in}) \right] = 48 \text{ in}^2$$

$$\begin{aligned}\text{Area of the three rectangles} &= (6 \text{ in})(7 \text{ in}) + (8 \text{ in})(7 \text{ in}) + (10 \text{ in})(7 \text{ in}) \\ &= 42 \text{ in}^2 + 56 \text{ in}^2 + 70 \text{ in}^2 \\ &= 168 \text{ in}^2\end{aligned}$$

$$\text{Lateral area} = 48 \text{ in}^2 + 168 \text{ in}^2 = 216 \text{ in}^2$$

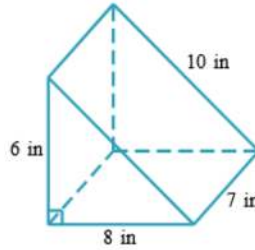
More examples using volume of three-dimensional objects

1. Calculate the volume of the rectangular prism with dimensions 2 yds X 6 yds X 7 yds.



$$\text{Volume} = (\text{length})(\text{width})(\text{height}) = (2 \text{ yds})(6 \text{ yds})(7 \text{ yds}) = 84 \text{ yds}^3$$

2. Calculate the volume of a triangular prism as shown below.



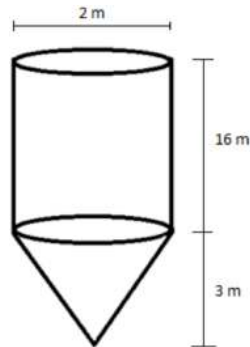
$$V = \frac{1}{2} (\text{base}) (\text{height}) (\text{depth or width})$$

$$V = \frac{1}{2} (8 \text{ in}) (6 \text{ in}) (7 \text{ in}) = 168 \text{ in}^3$$

3. Calculate the volume of a sphere with a diameter of 15.6 mm.

$$V = \frac{2}{3} (0.785) (\text{diameter})^3 = \frac{2}{3} (0.785) (15.6 \text{ mm})^3 = \frac{2}{3} (0.785) (3796.416 \text{ mm}^3) = 1986.8 \text{ mm}^3$$

4. Calculate the volume of the object below (not drawn to scale)



Total Volume = Volume of the cylinder + Volume of the cone

$$\begin{aligned} \text{Volume of cylinder} &= (0.785) (\text{diameter})^2 (\text{height}) \\ &= (0.785) (2 \text{ m})^2 (16 \text{ m}) \\ &= (0.785) (4 \text{ m}^2) (16 \text{ m}) \\ &= (0.785) (64 \text{ m}^3) \\ &= 50.24 \text{ m}^3 \end{aligned} \tag{6.2.1}$$

$$\begin{aligned} \text{Volume of cone} &= \frac{1}{3} (0.785) (\text{diameter})^2 (\text{height}) \\ &= \frac{1}{3} (0.785) (2 \text{ m})^2 (3 \text{ m}) \\ &= \frac{1}{3} (0.785) (4 \text{ m}^2) (3 \text{ m}) \\ &= \frac{1}{3} (0.785) (12 \text{ m}^3) \\ &= 3.14 \text{ m}^3 \end{aligned} \tag{6.2.2}$$

$$\begin{aligned} \text{Total Volume} &= \text{Volume of the cylinder} + \text{Volume of the cone} \\ &= 50.24 \text{ m}^3 + 3.14 \text{ m}^3 \\ &= 53.38 \text{ m}^3 \end{aligned} \tag{6.2.3}$$

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CHAPTER OVERVIEW

7: Reading and Interpreting Graphs

In the Water Industry, an important skill is reading and interpreting graphs and charts. There are many types of graphs and charts such as pie charts, bar graphs, histograms, line graphs, dials, gauges, etc. When reading and interpreting graphs, it is very important to read the labels on the graph or chart and to consider the units (if any).

[7.1: Types of Graphs](#)

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7.1: Types of Graphs

Line graphs: Line graphs are commonly used in the water and wastewater industry. They are quick and easy to create and read. It is important to read the labels on the horizontal and vertical axes and to read the graph from left to right. To read information on a line graph, locate a point on the line graph, find the corresponding value on the horizontal and vertical axes.

Bar graphs: A bar graph can consist of vertical or horizontal bars that can display numerical or categorical data. To read a bar graph, for each category, locate the height (or length) of the bar and read the scale or value. Some bar graphs will consist of double bars or multiple bars for each category. This allows information to be compared at a quick glance.

Pie charts: A pie chart is generally used to show percentages or proportions of categories as they relate to each other. Each wedge in a pie chart represents a category and the size of the wedge represents the percentage or proportion of that category.

Semi-log or log-log graphs: A semi-log graph has logarithmic scales on one axis and a numeric scale on the other axis. A log-log graph has logarithmic scales on both the horizontal and vertical axes. They can be used for showing the flow rate.

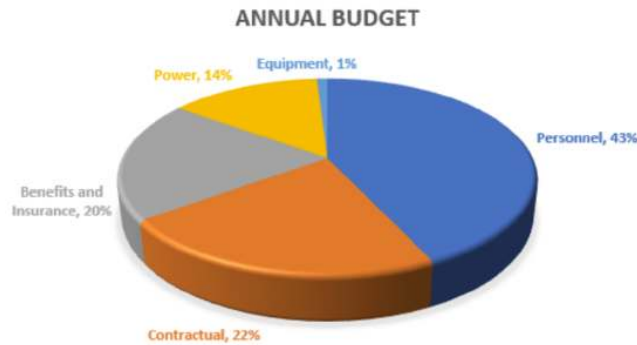
When reading a graph, be sure to pay close attention to the scale of the axes and/or picture. The markings on the axes or picture indicate the size of the numerical values depicted on the graph.

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7.2: Examples

Examples:

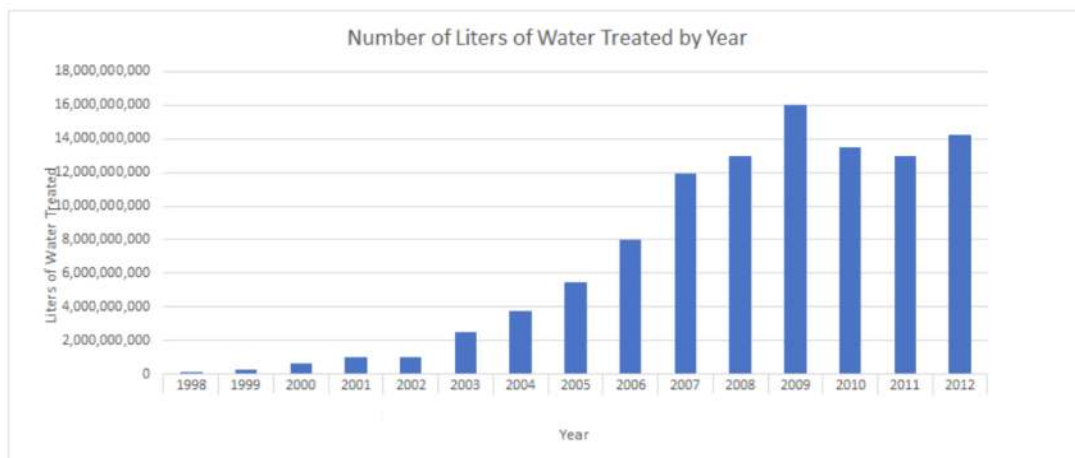
1. A city's sanitary sewer budget for the year is \$350,000. The budget has been divided into five categories: Personnel, contractual, power, benefits and insurance, and equipment. Based on the pie chart, which category has the larger proportion of the budget been allocated to? Which category has the least allocation? Which two categories have similar allocations? What dollar amount has been allocated for power?



Solution

We can visually see that the largest part of the pie is allocated to personnel at 43% and the smallest part of the pie is allocated to equipment at 1%. The two pieces of the pie that are similar in size are for contractual and benefits and insurance which represent 22% and 20% respectively. It is also important to note that these percentages are based on the \$350,000 budget. Hence the amount of money allocated for power is 14% of \$350,000 or $.14(\$35,000) = \$4900$.

2. Using the bar graph below, answer the following questions:
 - a. What general information is provided in the bar graph?
 - b. Which year had the most liters of treated water?
 - c. Which year had the least liters of treated water?
 - d. In which year was 8,000,000,000 liters of water treated?
 - e. The amount of treated water increased between which years?
 - f. The amount of treated water decreased between which years?
 - g. Which years had approximately the same number of treated liters of water?



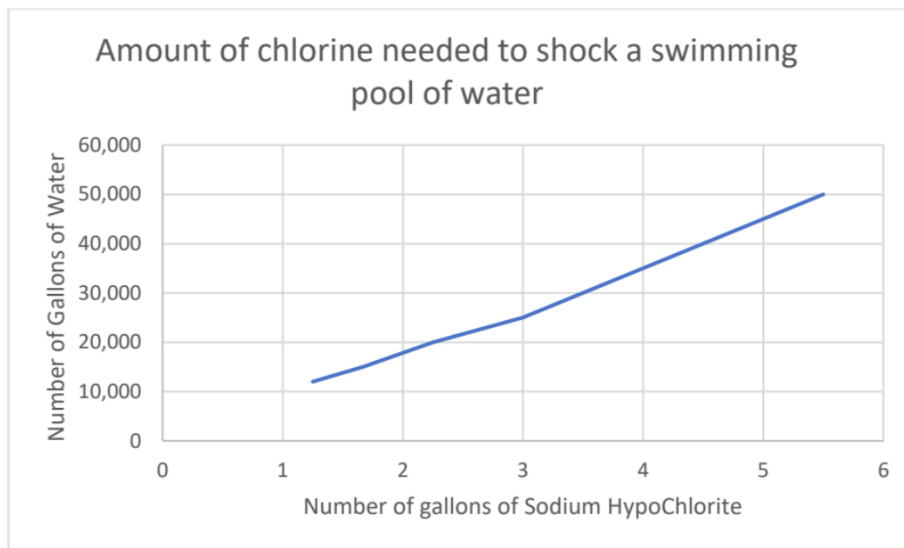
Solution

- a. What general information is provided in the bar graph? The number of liters of water treated each year between 1998 and 2012

- b. Which year had the most liters of treated water? 2009
- c. Which year had the least liters of treated water? 1998
- d. In which year was 8,000,000,000 liters of water treated? 2006
- e. The amount of treated water increased between which years? 1998-2009
- f. The amount of treated water decreased between which years? 2009-2011
- g. Which years had approximately the same number of treated liters of water? 2008 and 2011

3. Using the line graph below, answer the following questions:

- a. What general information is provided in the line graph?
- b. The number of gallons of water in the pool appear to increase, decrease, or remain the same as the number of gallons of Sodium HypoChlorite increases?
- c. How many gallons of Sodium HypoChlorite are needed to shock 25,000 gallons of water?
- d. How many gallons of water can 4 gallons of Sodium HypoChlorite shock?
- e. What is the maximum point on the graph and what does it represent?



Solution

- a. What general information is provided in the line graph? The number of gallons of Sodium HypoChlorite (a type of chlorine) that are needed to shock a certain number of gallons of water in a swimming pool.
- b. The number of gallons of water in the pool appear to increase, decrease, or remain the same as the number of gallons of Sodium HypoChlorite increases? Since the line graph is going up from left to right, the number of gallons of water in the pool is increasing as the number of gallons of Sodium HypoChlorite used increases.
- c. How many gallons of Sodium HypoChlorite are needed to shock 25,000 gallons of water? Reading the vertical axis, we can find 25,000 gallons of water, move to the right to find the point on the graph and read the corresponding number on the horizontal axis, which is 3 gallons of Sodium HypoChlorite.
- d. How many gallons of water can 4 gallons of Sodium HypoChlorite shock? Reading the horizontal axis, we can find 4 gallons of Sodium HypoChlorite, go up the graph to find the point on the graph and read the corresponding number on the vertical axis, which is 35,000 gallons of water.
- e. What is the maximum point on the graph and what does it represent? The highest point on the graph is at (5, 50000). Note: the horizontal number is written first in a point and the vertical axis number is written second. This point means we need 5 gallons of Sodium HypoChlorite to shock 50,000 gallons of swimming pool water.

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