

SPIRAL MODERN PHYSICS



Paul D'Alessandris
Monroe Community College

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Paul D'Alessandris

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CHAPTER OVERVIEW

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Contributors and Attributions

- [Paul D'Alessandris](#) ([Monroe Community College](#))

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1.1: Time Dilation

Einstein developed much of his understanding of relativity through the use of gedanken, or thought, experiments. In a gedanken experiment, Einstein would imagine an experiment that could not be performed due to technological limitations, and “perform” the experiment in his head. By analyzing the results of these experiments, he was lead to a deeper understanding of his theory.

In developing his understanding of the relativity of time, Einstein imagined what he considered the simplest possible clock, consisting of a mirror on the floor and a mirror on the ceiling of a train car, and a light pulse bouncing back and forth between the mirrors. Each reflection of the light pulse serves as a “tick” of the clock.

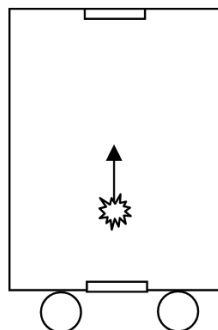


Figure 1.1.1: Einstein's simple "clock".

Now imagine two observers and the train car traveling to the right at speed v . A passenger on the train will see the light pulse travel a vertical path from floor to ceiling. An observer at rest on the earth, however, will see the train displaced to the right during the time it takes the pulse to travel from floor to ceiling, and hence see the light pulse follow an angled path.

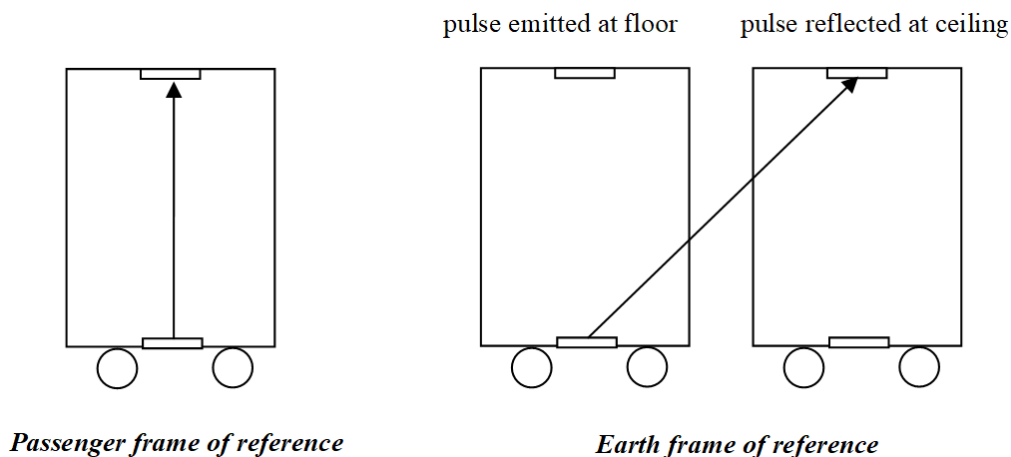


Figure 1.1.2: Einstein's simple "clock" in motion.

Let Δt represent the travel time from floor to ceiling measured on the earth, and Δt_o represent the travel time from floor to ceiling measured on the train. Based on these definitions, and the speed of the train (v), the distances in the diagram can be determined.

You do remember from your study of electromagnetism that the speed of electromagnetic waves is the same in all frames of reference, don't you?

By [Pythagoras' Theorem](#),

$$(c\Delta t)^2 = (v\Delta t)^2 + (c\Delta t_o)^2 \quad (1.1.1)$$

$$(c\Delta t)^2 - (v\Delta t)^2 = (c\Delta t_o)^2 \quad (1.1.2)$$

$$(c^2 - v^2)\Delta t^2 = c^2\Delta t_o^2 \quad (1.1.3)$$

$$\Delta t = \Delta t_o \sqrt{\frac{c^2}{c^2 - v^2}} = \Delta t_o \gamma \quad (1.1.4)$$

$$\left(1 - \frac{v^2}{c^2}\right) \Delta t = \Delta t_o \quad (1.1.4)$$

$$\Delta t = \frac{\Delta t_o}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (1.1.5)$$

Therefore, since the denominator is less than 1 for any speed $v < c$, the time interval between the “ticks” of the clock measured on the earth is greater than the time interval measured on the train. To earth-bound observers, the clock is ticking slower than normal. Einstein proposed that this is not a property of the clock, but rather a reflection of the fact that time itself passes slower for an object is in motion.

Using Time Dilation

The half-life of a muon at rest is $2.2 \mu s$. One can store muons for a much longer time (as measured in the laboratory) by accelerating them to a speed very close to that of light and then keeping them circulating at that speed in an evacuated ring. Assume that you want to design a ring that can keep muons moving so fast that they have a laboratory half-life of $20 \mu s$. How fast will the muons have to be moving?

The time between two events, for example the “birth” and “death” of a muon, depends on who makes the measurements. Within any particular reference system, the familiar results of classical physics are valid. However, in comparing results between observers in different reference systems, a method of relating one observer’s measurements to another is needed.

Equation 1.1.5 for time dilation is typically written in the form

$$\Delta t = \gamma(\Delta t_o) \quad (1.1.6)$$

where

- Δt_o is the proper time, the time between two events in the frame of reference in which both events occur at the same spatial point,
- Δt is the time between the same two events in a different frame, moving at relative speed v ,
- and γ is the Lorentz factor, given by

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (1.1.7)$$

In the muon’s frame of reference, the muon’s are at rest and hence have a half-life of $2.2 \mu s$. Moreover, this is a proper time since in this frame of reference both the birth and death of the muons occur at the same point. (Since the muons are not moving, everything that happens to them happens at the same point.)

$$\Delta t = \gamma(\Delta t_o) \quad (1.1.8)$$

$$20 \mu s = \gamma(2.2 \mu s) \quad (1.1.9)$$

$$\gamma = \frac{20 \mu s}{2.2 \mu s} \quad (1.1.10)$$

$$= 9.09 \quad (1.1.11)$$

Using Equation 1.1.7

$$9.09 = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (1.1.12)$$

$$1 - \frac{v^2}{c^2} = \left(\frac{1}{9.09}\right)^2 \quad (1.1.13)$$

$$\frac{v^2}{c^2} = 1 - \left(\frac{1}{9.09}\right)^2 \quad (1.1.14)$$

$$\frac{v^2}{c^2} = 0.9879 \quad (1.1.15)$$

$$v = 0.994c \quad (1.1.16)$$

To get this level of time dilation, the muons have been moving at 99.4% the speed of light.

Time Dilation at Everyday Speeds

The cruising speed of a jet airplane is approximately 250 m/s relative to the earth. For each hour of earth-time that passes, how much less time passes for the passengers on the plane?

The time interval measured by the passengers is the proper time, Δt_o , so we start with Equation 1.1.5

$$\Delta t = \frac{\Delta t_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\Delta t_o = \Delta t \sqrt{1 - \frac{v^2}{c^2}} \quad (1.1.17)$$

$$\Delta t_o = (1 \text{ hr}) \sqrt{1 - \frac{250^2}{(3 \times 10^8)^2}} \quad (1.1.18)$$

$$\Delta t_o = (1 \text{ hr}) \sqrt{1 - 6.94 \times 10^{-13}} \quad (1.1.19)$$

Unfortunately, at this point my calculator tells me that $\Delta t_o = 1 \text{ hr}$, which is obviously not true. I need a better way to deal with the extremely small numbers that sometimes show up in relativity. One way to do this is with the *binomial approximation*.

Expressions of the form:

$$(1 + x)^n \quad (1.1.20)$$

where x is much less than 1 can be approximated by:

$$(1 + x)^n \approx 1 + nx \quad (1.1.21)$$

if $x \ll 1$

Using this approximation allows Equation 1.1.19 to be simplified:

$$\Delta t_o = (1 \text{ hr}) \sqrt{1 - 6.94 \times 10^{-13}} \quad (1.1.22)$$

$$\Delta t_o = (1 \text{ hr}) (1 - 6.94 \times 10^{-13})^{1/2} \quad (1.1.23)$$

$$\Delta t_o \approx (1 \text{ hr}) \left(1 - \left(\frac{1}{2} \right) 6.94 \times 10^{-13} \right) \quad (1.1.24)$$

$$\Delta t_o \approx 1 \text{ hr} - 3.47 \times 10^{-13} \text{ hr} \quad (1.1.25)$$

$$\Delta t_o \approx 1 \text{ hr} - 1.25 \times 10^{-9} \text{ s} \quad (1.1.26)$$

Thus, for every hour that passes on earth, 1.25 nanoseconds less time passes on the airplane! This may seem like an incredibly small amount of time (and it is) but this effect has been measured in numerous experiments.

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1.2: Length Contraction

If the time interval between two events depends on the relative motion of the observer, Einstein realized that the spatial separation between the events must also be observer-dependent. Consider a hypothetical spaceship journey from earth to a distant star. Assume the star is a distance L_o from the earth, as measured by stationary earth-bound observers. Therefore, the elapsed time for the spaceship to reach the star, as measured on earth, is

$$\Delta t = \frac{L_o}{v} \quad (1.2.1)$$

where v is the speed of the spaceship measured on earth.

By time dilation, however, the elapsed time for the spaceship to reach the star, as measured on the spaceship (a proper time), is

$$\Delta t_o = \frac{\Delta t}{\gamma} \quad (1.2.2)$$

$$= \frac{L_o}{v\gamma} \quad (1.2.3)$$

The distance the spaceship travels (L), as measured on the spaceship, is simply the product of its speed and the elapsed time measured on the ship

$$L = v\Delta t_o \quad (1.2.4)$$

$$= v \left(\frac{L_o}{v\gamma} \right) \quad (1.2.5)$$

$$= \frac{L_o}{\gamma} \quad (1.2.6)$$

Since gamma is greater than one, the distance between the earth and the star as measured on the ship is less than the distance as measured on the earth. To moving observers, the distance to the star shrinks.

Using Length Contraction

The star Vega is approximately 25 light-years from Earth (as measured by observers on Earth).

- How fast must a spaceship travel in order to reach Vega in 30 years, as measured on Earth?*
- How fast must a spaceship travel in order to reach Vega in 30 years, as measured on the spaceship?*

The distance between two events, for example leaving Earth and arriving at Vega, depends on who makes the measurements. Within any particular reference system, the familiar results of classical physics are valid. However, in comparing results between observers in different reference systems, a method of relating one observer's measurements to another is needed.

The formula for length contraction is

where

- L_o the proper length, the distance between two events in the frame of reference in which both events are at rest,
- and L is the distance between the same two events in a different frame, moving at relative speed v .

For part a, relativity theory is unnecessary. Both the distance and time measurements are made from the same frame of reference. Therefore, results from classical physics are valid.

(Note the use of the speed of light as a unit. Rather than substituting 3.0×10^8 m/s for c , simply leave c as a unit of velocity.)

For part b, the distance is measured in the Earth's frame while the time is in the frame of the spaceship. To solve part b, you must either convert the distance into the spaceship frame or the time into the Earth frame. You can convert the distance into the spaceship frame by realizing that the distance to Vega as measured on Earth is a proper length. Therefore,

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1.3: Lorentz Transformation

The time dilation and length contraction relationships actually have very limited applicability. To use the time dilation relationship, one of the two observers must measure a proper time, where the two events occur at exactly the same spatial point. To use the length contraction relationship, one of the two observers must measure a proper length, where the two events must be at rest with respect to the observer. What if you want to compare measurements concerning more general events? To do this requires the Lorentz Transformation, which allows you to transform the spacetime coordinates of an event in one inertial reference system to any other inertial reference system.

To derive the *Lorentz transformation*, imagine two inertial reference systems, labeled O and O' . Let the origins of O and O' overlap at time zero, and allow O' to move with speed u relative to O . (Therefore, at a later time t , the origins are separated by a distance ut). Call the direction of motion the x -direction.

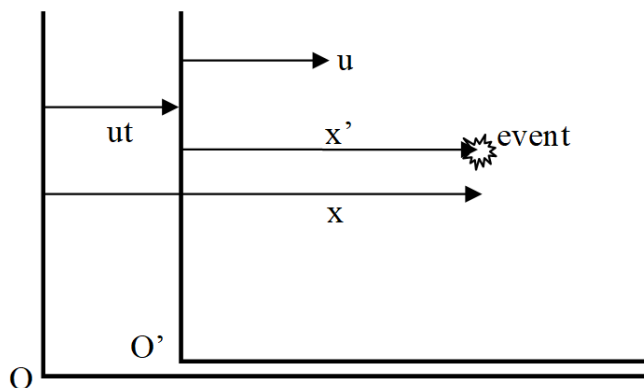


Figure 1.3.1:

Now imagine an event that occurs somewhere in spacetime. This event is located at position x relative to the O system, and position x' relative to the O' system. How are these two locations related?

You may be tempted to state that

$$x = x' + ut \quad (1.3.1)$$

however, this can't be correct because x and x' are measured in different reference systems. However, imagine that the event is the tip of a meterstick, fixed in O' , striking some object. Since x' is now a proper length in O' , it will appear contracted in O by the gamma factor. Therefore, the correct relationship between x and x' is

$$x = \frac{x'}{\gamma} + ut \quad (1.3.2)$$

rearranging yields

$$x' = \gamma(x - ut) \quad (1.3.3)$$

Since there is no relative motion in the y and z directions, these positions are the same in both coordinate systems

$$y' = y \quad (1.3.4)$$

$$z' = z \quad (1.3.5)$$

This completes the spatial part of the Lorentz transformation, but what about the temporal part? To determine how t and t' are related, now imagine that the event under investigation is the result of a light pulse, emitted from the origin when the two origins overlapped at time zero, striking some detector. Since the speed of light is the same in both systems, the distance measured in each system must be equal to the product of c and the elapsed time

$$x' = \gamma(x - ut) \quad (1.3.6)$$

$$ct' = \gamma\left(ct - u\frac{x}{c}\right) \quad (1.3.7)$$

$$t' = \gamma \left(t - u \frac{x}{c^2} \right) \quad (1.3.8)$$

Using the Lorentz Transformation

Inside of a spaceship zooming past earth at $0.5c$, I fire a laser (in the same direction as the ship's motion) and let it strike a mirror 10 m in front of the laser.

- What is the elapsed time measured on the earth between turning on the laser and the light striking the mirror?
- How far has the light traveled before hitting the mirror, as measured on earth?

Since neither the earth's observers nor the observers on the ship measure a proper time or a proper length between the two events (turning on the laser and the laser striking the mirror), a more general method of relating different observers' measurements is needed. This general method of relating measurements is the Lorentz Transformation. The Lorentz Transformation relates the coordinates of a spacetime event, (x, y, z, t) , measured in one frame to the coordinates of the same event in a frame moving with relative velocity u , (x', y', z', t') as follows:

$$x' = \gamma(x - ut)$$

$$y' = y$$

$$z' = z$$

$$t' = \gamma \left(t - \frac{ux}{c^2} \right)$$

These equations are written in a form that easily allows the determination of the primed coordinates from the unprimed. If the situation requires the inverse of this task, the equations can be easily inverted (by changing the sign of u and flipping the primed and unprimed notation) to yield

$$x = \gamma(x' - ut') \quad (1.3.9)$$

$$y = y' \quad (1.3.10)$$

$$z = z' \quad (1.3.11)$$

$$t = \gamma \left(t' + \frac{ux'}{c^2} \right) \quad (1.3.12)$$

Let the two coordinate systems overlap at the first event (the laser is fired). Thus, the position and time of the laser's firing is zero in both coordinate systems. We now must find the position and time of the second event (the laser strikes the mirror). This is relatively easy to determine in the frame of the spaceship (the primed frame):

$$x' = 10 \text{ m} \quad (1.3.13)$$

$$t' = \frac{10 \text{ m}}{c} \approx 3.33 \times 10^{-8} \text{ s} \quad (1.3.14)$$

Since we know the spacetime location of the event in the primed frame, the Lorentz Transformation allows us to *transform* this information into the earth frame. With $u = 0.5c$ ($\gamma = 1.155$),

$$t = \gamma \left(t' + \frac{ux'}{c^2} \right) \quad (1.3.15)$$

$$= 1.155 \left(\left(\frac{10}{c} \right) + \frac{(0.5c)(10)}{c^2} \right) \quad (1.3.16)$$

$$= \frac{17.3 \text{ m}}{c} = 5.7 \times 10^{-8} \text{ s} \quad (1.3.17)$$

and for the x-direction

$$x = \gamma(x' + ut') \quad (1.3.18)$$

$$= 1.155 \left(10 + (0.5c) \left(\frac{10}{c} \right) \right) \quad (1.3.19)$$

$$= 17.3 \text{ m} \quad (1.3.20)$$

The light travels 17.3 m and takes $5.78 \times 10^{-8}\text{ s}$ to strike the mirror in the earth's frame.

Contributors

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