

ELECTRICITY AND MAGNETISM



Jeremy Tatum
University of Victoria

University of Victoria
Electricity and Magnetism

Jeremy Tatum

This text is disseminated via the Open Education Resource (OER) LibreTexts Project (<https://LibreTexts.org>) and like the thousands of other texts available within this powerful platform, it is freely available for reading, printing, and "consuming."

The LibreTexts mission is to bring together students, faculty, and scholars in a collaborative effort to provide an accessible, and comprehensive platform that empowers our community to develop, curate, adapt, and adopt openly licensed resources and technologies; through these efforts we can reduce the financial burden born from traditional educational resource costs, ensuring education is more accessible for students and communities worldwide.

Most, but not all, pages in the library have licenses that may allow individuals to make changes, save, and print this book. Carefully consult the applicable license(s) before pursuing such effects. Instructors can adopt existing LibreTexts texts or Remix them to quickly build course-specific resources to meet the needs of their students. Unlike traditional textbooks, LibreTexts' web based origins allow powerful integration of advanced features and new technologies to support learning.



LibreTexts is the adaptable, user-friendly non-profit open education resource platform that educators trust for creating, customizing, and sharing accessible, interactive textbooks, adaptive homework, and ancillary materials. We collaborate with individuals and organizations to champion open education initiatives, support institutional publishing programs, drive curriculum development projects, and more.

The LibreTexts libraries are Powered by [NICE CXone Expert](#) and was supported by the Department of Education Open Textbook Pilot Project, the California Education Learning Lab, the UC Davis Office of the Provost, the UC Davis Library, the California State University Affordable Learning Solutions Program, and Merlot. This material is based upon work supported by the National Science Foundation under Grant No. 1246120, 1525057, and 1413739.

Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation nor the US Department of Education.

Have questions or comments? For information about adoptions or adaptations contact info@LibreTexts.org or visit our main website at <https://LibreTexts.org>.

This text was compiled on 06/09/2026

TABLE OF CONTENTS

Licensing

1: Electric Fields

- 1.1: Prelude to Electric Fields
- 1.2: Triboelectric Effect
- 1.3: Experiments with Pith Balls
- 1.4: Experiments with a Gold-leaf Electroscope
- 1.5: Coulomb's Law
- 1.6: Electric Field E
 - 1.6A: Field of a Point Charge
 - 1.6B: Spherical Charge Distributions
 - 1.6C: A Long, Charged Rod
 - 1.6D: Field on the Axis of and in the Plane of a Charged Ring
 - 1.6E: Field on the Axis of a Uniformly Charged Disc
 - 1.6F: Field of a Uniformly Charged Infinite Plane Sheet
- 1.7: Electric Field D
- 1.8: Flux
- 1.9: Gauss's Theorem

2: Electrostatic Potential

- 2.1: Introduction to Electrostatic Potentials
- 2.2: Potential Near Various Charged Bodies
 - 2.2A: Point Charge
 - 2.2B: Spherical Charge Distributions
 - 2.2C: Long Charged Rod
 - 2.2D: Large Plane Charged Sheet
 - 2.2E: Potential on the Axis of a Charged Ring
 - 2.2F: Potential in the Plane of a Charged Ring
 - 2.2G: Potential on the Axis of a Charged Disc
- 2.3: Electron-volts
- 2.4: A Point Charge and an Infinite Conducting Plane
- 2.5: A Point Charge and a Conducting Sphere
- 2.6: Two Semicylindrical Electrodes

3: Dipole and Quadrupole Moments

- 3.1: Introduction
- 3.2: Mathematical Definition of Dipole Moment
- 3.3: Oscillation of a Dipole in an Electric Field
- 3.4: Potential Energy of a Dipole in an Electric Field
- 3.5: Force on a Dipole in an Inhomogeneous Electric Field
- 3.6: Induced Dipoles and Polarizability
- 3.7: The Simple Dipole
- 3.8: Quadrupole Moment
- 3.9: Potential at a Large Distance from a Charged Body
- 3.10: A Geophysical Example

4: Batteries, Resistors and Ohm's Law

- 4.1: Introduction
- 4.2: Resistance and Ohm's Law
- 4.3: Resistance and Temperature
- 4.4: Resistors in Series
- 4.5: Conductors in Parallel
- 4.6: Dissipation of Energy
- 4.7: Electromotive Force and Internal Resistance
- 4.8: Power Delivered to an External Resistance
- 4.9: Potential Divider
- 4.10: Ammeters and Voltmeters
- 4.11: Wheatstone Bridge
- 4.12: Delta-Star Transform
- 4.13: Kirchhoff's Rules
- 4.14: Tortures for the Brain
- 4.15: Solutions, Answers or Hints to 4.14
- 4.16: Attenuators

5: Capacitors

- 5.1: Introduction
- 5.2: Plane Parallel Capacitor
- 5.3: Coaxial Cylindrical Capacitor
- 5.4: Concentric Spherical Capacitor
- 5.5: Capacitors in Parallel
- 5.6: Capacitors in Series
- 5.7: Delta-Star Transform
- 5.8: Kirchhoff's Rules
- 5.9: Problem for a Rainy Day
- 5.10: Energy Stored in a Capacitor
- 5.11: Energy Stored in an Electric Field
- 5.12: Force Between the Plates of a Plane Parallel Plate Capacitor
- 5.13: Sharing a Charge Between Two Capacitors
- 5.14: Mixed Dielectrics
- 5.15: Changing the Distance Between the Plates of a Capacitor
- 5.16: Inserting a Dielectric into a Capacitor
- 5.17: Polarization and Susceptibility
- 5.18: Discharging a Capacitor Through a Resistor
- 5.19: Charging a Capacitor Through a Resistor
- 5.20: Real Capacitors
- 5.21: More on E, D, P, etc
- 5.22: Dielectric material in an alternating electric field.

6: The Magnetic Effect of an Electric Current

- 6.1: Introduction
- 6.2: Definition of the Amp
- 6.3: Definition of the Magnetic Field
- 6.4: The Biot-Savart Law
- 6.5: Magnetic Field Near a Long, Straight, Current-carrying Conductor
- 6.6: Field on the Axis and in the Plane of a Plane Circular Current-carrying Coil
- 6.7: Helmholtz Coils
- 6.8: Field on the Axis of a Long Solenoid

- 6.9: The Magnetic Field H
- 6.10: Flux
- 6.11: Ampère's Theorem
- 6.12: Boundary Conditions

7: Force on a Current in a Magnetic Field

- 7.1: Introduction
- 7.2: Force Between Two Current-carrying Wires
- 7.3: The Permeability of Free Space
- 7.4: Magnetic Moment
- 7.5: Magnetic Moment of a Plane, Current-carrying Coil
- 7.6: Period of Oscillation of a Magnet or a Coil in an External Magnetic Field
- 7.7: Potential Energy of a Magnet or a Coil in a Magnetic Field
- 7.8: Moving-coil Ammeter
- 7.9: Magnetogyric Ratio

8: On the Electrodynamics of Moving Bodies

- 8.1: Introduction to Electrodynamics of Moving Bodies
- 8.2: Charged Particle in an Electric Field
- 8.3: Charged Particle in a Magnetic Field
- 8.4: Charged Particle in an Electric and a Magnetic Field
- 8.5: Motion in a Nonuniform Magnetic Field
- 8.6: Appendix. Integration of the Equations

9: Magnetic Potential

- 9.1: Introduction to Magnetic Potential
- 9.2: The Magnetic Vector Potential
- 9.3: Long, Straight, Current-carrying Conductor
- 9.4: Long Solenoid
- 9.5: Divergence

10: Electromagnetic Induction

- 10.1: Introduction to Electromagnetic Induction
- 10.2: Electromagnetic Induction and the Lorentz Force
- 10.3: Lenz's Law
- 10.4: Ballistic Galvanometer and the Measurement of Magnetic Field
- 10.5: AC Generator
- 10.6: AC Power
- 10.7: Linear Motors and Generators
- 10.8: Rotary Motors
- 10.9: The Transformer
- 10.10: Mutual Inductance
- 10.11: Self Inductance
- 10.12: Growth of Current in a Circuit Containing Inductance
- 10.13: Discharge of a Capacitor through an Inductance
- 10.14: Discharge of a Capacitor through an Inductance and a Resistance
- 10.15: Charging a Capacitor through an Inductance and a Resistance
- 10.16: Energy Stored in an Inductance
- 10.17: Energy Stored in a Magnetic Field

11: Dimensions

12: Properties of Magnetic Materials

- 12.1: Introduction
- 12.2: Magnetic Circuits and Ohm's Law
- 12.3: Magnetization and Susceptibility
- 12.4: Diamagnetism
- 12.5: Paramagnetism
- 12.6: Ferromagnetism
- 12.7: Antiferromagnetism
- 12.8: Ferrimagnetism

13: Alternating Current

- 13.1: Alternating current in an inductance
- 13.2: Alternating Voltage across a Capacitor
- 13.3: Complex Numbers
- 13.4: Resistance and Inductance in Series
- 13.5: Resistance and Capacitance in Series
- 13.6: Admittance
- 13.7: The RLC Series Acceptor Circuit
- 13.8: The RLC Parallel Rejector Circuit
- 13.9: AC Bridges
 - 13.9A: The Owen Bridge
 - 13.9B: The Schering Bridge
 - 13.9C: The Wien Bridge
 - 13.9D: Bridge Solution by Delta-Star Transform
- 13.10: The Transformer
- 13.11: Root-mean-square values, power and impedance matching

14: Laplace Transforms

- 14.1: Introduction to Laplace Transforms
- 14.2: Table of Laplace Transforms
- 14.3: The First Integration Theorem
- 14.4: The Second Integration Theorem (Dividing a Function by t)
- 14.5: Shifting Theorem
- 14.6: A Function Times t^n
- 14.7: Differentiation Theorem
- 14.8: A First Order Differential Equation
- 14.9: A Second Order Differential Equation
- 14.10: Generalized Impedance
- 14.11: RLC Series Transient
- 14.12: Another Example

15: Maxwell's Equations

- 15.1: Introduction
- 15.2: Maxwell's First Equation
- 15.3: Poisson's and Laplace's Equations
- 15.4: Maxwell's Second Equation
- 15.5: Maxwell's Third Equation

- [15.6: The Magnetic Equivalent of Poisson's Equation](#)
- [15.7: Maxwell's Fourth Equation](#)
- [15.8: Summary of Maxwell's and Poisson's Equations](#)
- [15.9: Electromagnetic Waves](#)
- [15.10: Gauge Transformations](#)
- [15.11: Maxwell's Equations in Potential Form](#)
- [15.12: Retarded Potential](#)

[16: CGS Electricity and Magnetism](#)

- [16.1: Introduction](#)
- [16.2: The CGS Electrostatic System](#)
- [16.3: The CGS Electromagnetic System](#)
- [16.4: The Gaussian Mixed System](#)
- [16.5: Conversion Factors](#)
- [16.6: Dimensions](#)

[17: Magnetic Dipole Moment](#)

- [17.1: Introduction to Magnetic Dipole Moments](#)
- [17.2: The SI Definition of Magnetic Moment](#)
- [17.3: The Magnetic Field on the Equator of a Magnet](#)
- [17.4: CGS Magnetic Moment, and Lip Service to SI](#)
- [17.5: Possible Alternative Definitions of Magnetic Moment](#)
- [17.6: Thirteen Questions](#)
- [17.7: Additional Remarks](#)
- [17.8: Conclusion](#)

[18: Electrochemistry](#)

[Index](#)

[Glossary](#)

[Detailed Licensing](#)

Licensing

A detailed breakdown of this resource's licensing can be found in [Back Matter/Detailed Licensing](#).

CHAPTER OVERVIEW

1: Electric Fields

So, you might ask, if your primary interest in electricity is to understand how machines, instruments and electrical equipment work, is there any point in studying electricity from the very “academic” and abstract approach that will be used in these notes, completely divorced as they appear to be from the world of practical reality? The answer is that electrical engineers *more than anybody* must understand the basic scientific principles before they even begin to apply them to the design of practical appliances. So – do not even think of electrical engineering until you have a thorough understanding of the basic scientific principles of the subject.

[1.1: Prelude to Electric Fields](#)

[1.2: Triboelectric Effect](#)

[1.3: Experiments with Pith Balls](#)

[1.4: Experiments with a Gold-leaf Electroscope](#)

[1.5: Coulomb's Law](#)

[1.6: Electric Field E](#)

[1.6A: Field of a Point Charge](#)

[1.6B: Spherical Charge Distributions](#)

[1.6C: A Long, Charged Rod](#)

[1.6D: Field on the Axis of and in the Plane of a Charged Ring](#)

[1.6E: Field on the Axis of a Uniformly Charged Disc](#)

[1.6F: Field of a Uniformly Charged Infinite Plane Sheet](#)

[1.7: Electric Field D](#)

[1.8: Flux](#)

[1.9: Gauss's Theorem](#)

Thumbnail: The electric field lines and equipotential lines for field of two point charges. (CC BY-SA 3.0; [Geek3](#) via [Wikipedia](#)).

This page titled [1: Electric Fields](#) is shared under a [CC BY-NC 4.0](#) license and was authored, remixed, and/or curated by [Jeremy Tatum](#) via [source content](#) that was edited to the style and standards of the LibreTexts platform.

1.1: Prelude to Electric Fields

This is the first in a series of chapters on electricity and magnetism. Much of it will be aimed at an introductory level suitable for first or second year students, or perhaps some parts may also be useful at high school level. Occasionally, as I feel inclined, I shall go a little bit further than an introductory level, though the text will not be enough for anyone pursuing electricity and magnetism in a third or fourth year honours class. On the other hand, students embarking on such advanced classes will be well advised to know and understand the contents of these more elementary notes before they begin.

The subject of electromagnetism is an amalgamation of what were originally studies of three apparently entirely unrelated phenomena, namely *electrostatic* phenomena of the type demonstrated with pieces of amber, pith balls, and ancient devices such as Leyden jars and Wimshurst machines; magnetism, and the phenomena associated with lodestones, compass needles and Earth's magnetic field; and current electricity – the sort of electricity generated by chemical cells such as Daniel and Leclanché cells. These must have seemed at one time to be entirely different phenomena. It wasn't until 1820 that Oersted discovered (during the course of a university lecture, so the story goes) that an electric current is surrounded by a magnetic field, which could deflect a compass needle. The several phenomena relating the apparently separate phenomena were discovered during the nineteenth century by scientists whose names are immortalized in many of the units used in electromagnetism – Ampère, Ohm, Henry, and, especially, Faraday. The basic phenomena and the connections between the three disciplines were ultimately described by Maxwell towards the end of the nineteenth century in four famous equations. This is not a history book, and I am not qualified to write one, but I strongly commend to anyone interested in the history of physics to learn about the history of the growth of our understanding of electromagnetic phenomena, from Gilbert's description of terrestrial magnetism in the reign of Queen Elizabeth I, through Oersted's discovery mentioned above, up to the culmination of Maxwell's equations.

This set of notes will be concerned primarily with a description of electricity and magnetism as natural phenomena, and it will be treated from the point of view of a “pure” scientist. It will *not* deal with the countless electrical devices that we use in our everyday life – how they work, how they are designed and how they are constructed. These matters are for electrical and electronics engineers. So, you might ask, if your primary interest in electricity is to understand how machines, instruments and electrical equipment work, is there any point in studying electricity from the very “academic” and abstract approach that will be used in these notes, completely divorced as they appear to be from the world of practical reality? The answer is that electrical engineers *more than anybody* must understand the basic scientific principles before they even begin to apply them to the design of practical appliances. So – do not even think of electrical engineering until you have a thorough understanding of the basic scientific principles of the subject.

This chapter deals with the basic phenomena, definitions and equations concerning *electric fields*.

This page titled [1.1: Prelude to Electric Fields](#) is shared under a [CC BY-NC 4.0](#) license and was authored, remixed, and/or curated by [Jeremy Tatum](#) via [source content](#) that was edited to the style and standards of the LibreTexts platform.

1.2: Triboelectric Effect

In an introductory course, the basic phenomena of electrostatics are often demonstrated with “pith balls” and with a “gold-leaf electroscope”. A *pith ball* used to be a small, light wad of pith extracted from the twig of an elder bush, suspended by a silk thread. Today, it is more likely to be either a ping-pong ball, or a ball of styrofoam, suspended by a nylon thread – but, for want of a better word, I’ll still call it a pith ball. I’ll describe the gold-leaf electroscope a little later.

It was long ago noticed that if a sample of amber (fossilized pine sap) is rubbed with cloth, the amber became endowed with certain apparently wonderful properties. For example, the amber would be able to attract small particles of fluff to itself. The effect is called the *triboelectric effect*. [Greek *τριβος* (rubbing) + *ήλεκτρον*(amber)] The amber, after having been rubbed with cloth, is said to bear an *electric charge*, and space in the vicinity of the charged amber within which the amber can exert its attractive properties is called an *electric field*.

Amber is by no means the best material to demonstrate triboelectricity. Modern plastics (such as a comb rubbed through the hair) become easily charged with electricity (provided that the plastic, the cloth or the hair, and the atmosphere, are *dry*). Glass rubbed with silk also carries an electric charge – but, as we shall see in the next section, the charge on glass rubbed with silk seems to be not quite the same as the charge on plastic rubbed with cloth.

This page titled [1.2: Triboelectric Effect](#) is shared under a [CC BY-NC 4.0](#) license and was authored, remixed, and/or curated by [Jeremy Tatum](#) via [source content](#) that was edited to the style and standards of the LibreTexts platform.

1.3: Experiments with Pith Balls

A pith ball hangs vertically by a thread. A plastic rod is charged by rubbing with cloth. The charged rod is *brought close to the pith ball without touching it*. It is observed that the charged rod weakly *attracts* the pith ball. This may be surprising – and you are right to be surprised, for the pith ball carries no charge. For the time being we are going to put this observation to the back of our minds, and we shall defer an explanation to a later chapter. Until then it will remain a small but insistent little puzzle.

We now *touch* the pith ball with the charged plastic rod. Immediately, some of the magical property (i.e. some of the electric charge) of the rod is transferred to the pith ball, and we observe that thereafter the ball is strongly *repelled* from the rod. We conclude that two electric charges repel each other. Let us refer to the pith ball that we have just charged as Ball A.

Now let's do exactly the same experiment with the glass rod that has been rubbed with silk. We bring the charged glass rod close to an uncharged Ball B. It initially attracts it weakly – but we'll have to wait until [Section 3.6](#) for an explanation of this unexpected behavior. However, as soon as we *touch* Ball B with the glass rod, some charge is transferred to the ball, and the rod thereafter repels it. So far, no obvious difference between the properties of the plastic and glass rods.

But... now bring the *glass* rod close to *Ball A*, and we see that Ball A is strongly *attracted*. And if we bring the *plastic* rod close to *Ball B*, it, too, is strongly *attracted*. Furthermore, Balls A and B *attract each other*.

We conclude that there are *two kinds* of electric charge, with exactly opposite properties. We arbitrarily call the kind of charge on the glass rod and on Ball B *positive* and the charge on the plastic rod and Ball A *negative*. We observe, then, that *like* charges (i.e. those of the *same* sign) *repel* each other, and *unlike* charges (i.e. those of *opposite* sign) *attract* each other.

This page titled [1.3: Experiments with Pith Balls](#) is shared under a [CC BY-NC 4.0](#) license and was authored, remixed, and/or curated by [Jeremy Tatum](#) via [source content](#) that was edited to the style and standards of the LibreTexts platform.

1.4: Experiments with a Gold-leaf Electroscope

A gold-leaf electroscope has a vertical rod R attached to a flat metal plate P. Gold is a malleable metal which can be hammered into extremely thin and light sheets. A light gold leaf G is attached to the lower end of the rod.

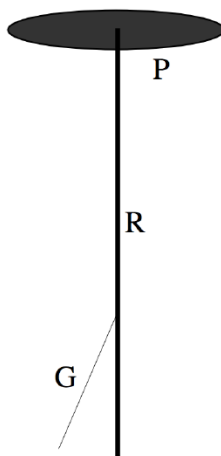


FIGURE I.1

If the electroscope is positively charged by touching the plate with a positively charged glass rod, G will be repelled from R, because both now carry a positive charge.

You can now experiment as follows. Bring a positively charged glass rod close to P. The leaf G diverges further from R. We now know that this is because the metal (of which P, R and G are all composed) contains *electrons*, which are negatively charged particles that can move about more or less freely inside the metal. The approach of the positively charged glass rod to P attracts electrons towards P, thus increasing the excess positive charge on G and the bottom end of R. G therefore moves away from R.

If on the other hand you were to approach P with a negatively charged plastic rod, electrons would be repelled from P down towards the bottom of the rod, thus reducing the excess positive charge there. G therefore approaches R.

Now try another experiment. Start with the electroscope uncharged, with the gold leaf hanging limply down. (This can be achieved by touching P briefly with your finger.) Approach P with a negatively charged plastic rod, but don't touch. The gold leaf diverges from R. Now, briefly touch P with a finger of your free hand. Negatively charged electrons run down through your body to ground (or earth). Don't worry – you won't feel a thing. The gold leaf collapses, though by this time the electroscope bears a positive charge, because it has lost some electrons through your body. Now remove the plastic rod. The gold leaf diverges again. By means of the negatively charged plastic rod and some deft work with your finger, you have induced a positive charge on the electroscope. You can verify this by approaching P alternately with a plastic (negative) or glass (positive) rod, and watch what happens to the gold leaf.

This page titled [1.4: Experiments with a Gold-leaf Electroscope](#) is shared under a [CC BY-NC 4.0](#) license and was authored, remixed, and/or curated by [Jeremy Tatum](#) via [source content](#) that was edited to the style and standards of the LibreTexts platform.

1.5: Coulomb's Law

If you are interested in the history of physics, it is well worth reading about the important experiments of Charles Coulomb in 1785. In these experiments he had a small fixed metal sphere which he could charge with electricity, and a second metal sphere attached to a vane suspended from a fine torsion thread. The two spheres were charged and, because of the repulsive force between them, the vane twisted round at the end of the torsion thread. By this means he was able to measure precisely the small forces between the charges, and to determine how the force varied with the amount of charge and the distance between them.

From these experiments resulted what is now known as *Coulomb's Law*. Two electric charges of like sign repel each other with a force that is proportional to the product of their charges and inversely proportional to the square of the distance between them:

$$F \propto \frac{Q_1 Q_2}{r^2} \quad (1.5.1)$$

Here Q_1 and Q_2 are the two charges and r is the distance between them.

We could in principle use any symbol we like for the constant of proportionality, but in standard SI (Système International) practice, the constant of proportionality is written as $\frac{1}{4\pi\epsilon}$ so that Coulomb's Law takes the form

$$F = \frac{1}{4\pi\epsilon} \frac{Q_1 Q_2}{r^2} \quad (1.5.2)$$

Here ϵ is called the *permittivity* of the medium in which the charges are situated, and it varies from medium to medium. The permittivity of a *vacuum* (or of “free space”) is given the symbol ϵ_0 . Media other than a vacuum have permittivities a little greater than ϵ_0 . The permittivity of air is very little different from that of free space, and, unless specified otherwise, I shall assume that all experiments described in this chapter are done either in free space or in air, so that I shall write Coulomb's Law as

$$F = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{r^2} \quad (1.5.3)$$

You may wonder – why the factor 4π ? In fact it is very convenient to define the permittivity in this manner, with 4π in the denominator, because, as we shall see, it will ensure that all formulas that describe situations of spherical symmetry will include a 4π , formulas that describe situations of cylindrical symmetry will include 2π , and no π will appear in formulas involving uniform fields. Some writers (particularly those who favour cgs units) prefer to incorporate the 4π into the definition of the permittivity, so that Coulomb's law appears in the form $F = Q_1 Q_2 / (\epsilon_0 r^2)$, though it is standard SI practice to define the permittivity as in Equation 1.5.3. The permittivity defined by Equation 1.5.3 is known as the “rationalized” definition of the permittivity, and it results in much simpler formulas throughout electromagnetic theory than the “unrationalized” definition.

The SI unit of charge is the *coulomb*, C. Unfortunately at this stage I cannot give you an exact definition of the coulomb, although, if a current of 1 amp flows for a second, the amount of electric charge that has flowed is 1 coulomb. This may at first seem to be very clear, until you reflect that we have not yet defined what is meant by an amp, and that, I'm afraid, will have to come in a much later chapter.

Until then, I can give you some small indications. For example, the charge on an electron is about -1.6022×10^{-19} C, and the charge on a proton is about $+1.6022 \times 10^{-19}$ C. That is to say, a collection of 6.24×10^{18} protons, if you could somehow bundle them all together and stop them from flying apart, amounts to a charge of 1 C. A *mole* of protons (i.e. 6.022×10^{23} protons) which would have a mass of about one gram, would have a charge of 9.65×10^4 C, which is also called a *faraday* (which is *not at all* the same thing as a *farad*).

[The current definition of the coulomb and the amp, which will be given in [Chapter 6](#), requires some knowledge of electromagnetism. However, it is likely that, in 2018, the coulomb will be redefined in such a manner that the magnitude of the charge on a single electron is exactly 1.60217×10^{-19} C.]

The charges involved in our experiments with pith balls, glass rods and gold-leaf electrosopes are very small in terms of coulombs, and are typically of the order of nanocoulombs.

The permittivity of free space has the approximate value

$$\epsilon_0 = 8.8542 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}.$$

Later on, when we know what is meant by a “farad”, we shall use the units F m^{-1} to describe permittivity – but that will have to wait until section 5.2.

You may well ask how the permittivity of free space is measured. A brief answer might be “by carrying out experiments similar to those of Coulomb”. However – and this is rather a long story, which I shall not describe here – it turns out that since we today define the metre by *defining* the speed of light, c , to be exactly $2.997\,925\,58 \times 10^8 \text{ m s}^{-1}$, the permittivity of free space has a *defined value*, given, in SI units, by

$$4\pi\epsilon_0 = \frac{10^7}{c^2}$$

It is therefore not necessary to *measure* ϵ_0 any more than it is necessary to *measure* c . But that, as I say, is a long story.

From the point of view of *dimensional analysis*, electric charge cannot be expressed in terms of M, L and T, but it has a dimension, Q, of its own. (This assertion is challenged by some, but this is not the place to discuss the reasons. I may add a chapter, eventually, discussing this point much later on.) We say that the dimensions of electric charge are Q.

I shall strongly advise the reader to work out and make a note of the dimensions of every new electric or magnetic quantity as it is introduced.

tonnes or nearly a million tonnes.

times as strong as gravitational forces – but such a statement out of context is rather meaningless. For example, the gravitational force between Earth and Moon is much more than the electrostatic force (if any) between them, and cosmologists could make a good case for saying that the strongest forces in the Universe are gravitational.

The ratio of the permittivity of an insulating substance to the permittivity of free space is its *relative permittivity*, also called its *dielectric constant*. The dielectric constants of many commonly-encountered insulating substances are of order “a few”. That is, somewhere between 2 and 10. Pure water has a dielectric constant of about 80, which is quite high (but bear in mind that most water is far from pure and is not an insulator.) Some special substances, known as *ferroelectric* substances, such as strontium titanate SrTiO_3 , have dielectric constants of a few hundred.

This page titled [1.5: Coulomb's Law](#) is shared under a [CC BY-NC 4.0](#) license and was authored, remixed, and/or curated by [Jeremy Tatum](#) via [source content](#) that was edited to the style and standards of the LibreTexts platform.

SECTION OVERVIEW

1.6: Electric Field E

The region around a charged body within which it can exert its electrostatic influence may be called an *electric field*. In principle, it extends to infinity, but in practice it falls off more or less rapidly with distance. We can define the *intensity* or *strength* E of an electric field as follows. Suppose that we place a small test charge Q in an electric field. This charge will then experience a force. The ratio of the force to the charge is called the *intensity of the electric field*, or, more usually, simply the *electric field*. Thus I have used the words “electric field” to mean either the region of space around a charged body, or, quantitatively, to mean its intensity. Usually it is clear from the context which is meant, but, if you wish, you may elect to use the longer phrase “intensity of the electric field” if you want to remove all doubt. The field and the force are in the same direction, and the electric field is a vector quantity, so the definition of the electric field can be written as

$$\mathbf{F} = Q\mathbf{E} \quad (1.6.1)$$

The SI units of electric field are newtons per coulomb, or N C^{-1} . A little later, however, we shall come across a unit called a *volt*, and shall learn that an alternative (and more usual) unit for electric field is volts per metre, or V m^{-1} . The dimensions are $\text{MLT}^{-2}\text{Q}^{-1}$.

You may have noticed that I supposed that we place a “small” test charge in the field, and you may have wondered why it had to be small, and how small. The problem is that, if we place a large charge in an electric field, this will change the configuration of the electric field and hence frustrate our efforts to measure it accurately. So – it has to be sufficiently small so as not to change the configuration of the field that we are trying to measure. How small is that? Well, it will have to mean infinitesimally small. I hope that is clear! (It is a bit like that pesky particle of negligible mass m that keeps appearing in mechanics problems!)

We now need to calculate the intensity of an electric field in the vicinity of various shapes and sizes of charged bodies, such as rods, discs, spheres, and so on.

1.6A: Field of a Point Charge

1.6B: Spherical Charge Distributions

1.6C: A Long, Charged Rod

1.6D: Field on the Axis of and in the Plane of a Charged Ring

1.6E: Field on the Axis of a Uniformly Charged Disc

1.6F: Field of a Uniformly Charged Infinite Plane Sheet

This page titled [1.6: Electric Field E](#) is shared under a [CC BY-NC 4.0](#) license and was authored, remixed, and/or curated by [Jeremy Tatum](#) via [source content](#) that was edited to the style and standards of the LibreTexts platform.

1.6A: Field of a Point Charge

It follows from equation 1.5.3 and the definition of electric field intensity that the electric field at a distance r from a point charge Q is of magnitude

$$E = \frac{Q}{4\pi\epsilon_0 r^2}. \quad (1.6.2)$$

This can be written in vector form:

$$\mathbf{E} = \frac{Q}{4\pi\epsilon_0 r^2} \hat{\mathbf{r}} = \frac{Q}{4\pi\epsilon_0 r^3} \mathbf{r}. \quad (1.6.3)$$

Here $\hat{\mathbf{r}}$ is a unit vector in the radial direction, and $\mathbf{r}$ is a vector of length r in the radial direction.

This page titled [1.6A: Field of a Point Charge](#) is shared under a [CC BY-NC 4.0](#) license and was authored, remixed, and/or curated by [Jeremy Tatum](#) via [source content](#) that was edited to the style and standards of the LibreTexts platform.

1.6B: Spherical Charge Distributions

I shall not here give calculus derivations of the expressions for electric fields resulting from spherical charge distributions, since they are identical with the derivations for the gravitational fields of spherical mass distributions in the Classical Mechanics “book” of these physics notes, provided that you replace mass by charge and G by $-1/(4\pi\epsilon_0)$. See Chapter 5, subsections 5.4.8 and 5.4.9 of Celestial Mechanics. Also, we shall see later that they can be derived more easily from Gauss’s law than by calculus. I shall, however, give the *results* here.

At a distance r from the centre of a *hollow spherical shell* of radius a bearing a charge Q , the electric field is *zero* at any point *inside* the sphere (i.e. for $r < a$). For a point *outside* the sphere (i.e. $r > a$) the field intensity is

$$E = \frac{Q}{4\pi\epsilon_0 r^2}. \quad (1.6.4)$$

This is the same as if all the charge were concentrated at a point at the centre of the sphere.

If you have a *spherically-symmetric distribution of charge* Q contained within a spherical volume of radius a , this can be considered as a collection of nested hollow spheres. It follows that at a point *outside* a spherically-symmetric distribution of charge, the field at a distance r from the centre is again

$$E = \frac{Q}{4\pi\epsilon_0 r^2}. \quad (1.6.5)$$

That is, it is the same as if all the charge were concentrated at the centre. However, at a point *inside* the sphere, the charge beyond the distance r from the centre contributes zero to the electric field; the electric field at a distance r from the centre is therefore just

$$E = \frac{Q_r}{4\pi\epsilon_0 r^2}. \quad (1.6.6)$$

Here Q_r is the charge within a radius r . If the charge is uniformly distributed throughout the sphere, this is related to the total charge by $Q_r = (\frac{r}{a})^3 Q$, where Q is the total charge. Therefore, for a uniform spherical charge distribution the field inside the sphere is

$$E = \frac{Qr}{4\pi\epsilon_0 a^3}. \quad (1.6.7)$$

That is to say, it increases linearly from centre to the surface, where it reaches a value of $\frac{Q}{4\pi\epsilon_0 a^2}$, whereafter it decreases according to equation 1.6.5.

It is not difficult to imagine some electric charge distributed (uniformly or otherwise) throughout a finite spherical volume, but, because like charges repel each other, it may not be easy to realize this idealized situation in practice. In particular, if a *metal* sphere is charged, since charge can flow freely through a metal, the self-repulsion of charges will result in all the charge residing on the *surface* of the sphere, which then behaves as a hollow spherical charge distribution with zero electric field within.

This page titled [1.6B: Spherical Charge Distributions](#) is shared under a [CC BY-NC 4.0](#) license and was authored, remixed, and/or curated by [Jeremy Tatum](#) via [source content](#) that was edited to the style and standards of the LibreTexts platform.

1.6C: A Long, Charged Rod

A long rod bears a charge of λ coulombs per metre of its length. What is the strength of the electric field at a point P at a distance r from the rod?

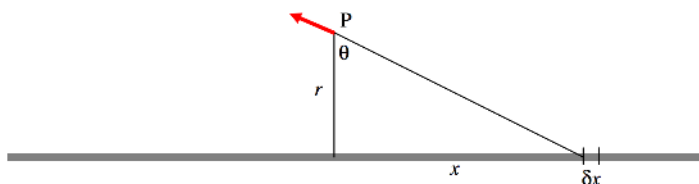


FIGURE 1.2

Consider an element δx of the rod at a distance $(r^2 + x^2)^{1/2}$ from the rod. It bears a charge $\lambda \delta x$. The contribution to the electric field at P from this element is $\frac{1}{4\pi\epsilon_0} \cdot \frac{\lambda \delta x}{r^2 + x^2}$ in the direction shown. The radial component of this is $\frac{1}{4\pi\epsilon_0} \cdot \frac{\lambda \delta x}{r^2 + x^2} \cos \theta$. But $x = r \tan \theta$, $\delta x = r \sec^2 \theta \delta \theta$ and $r^2 + x^2 = r^2 \sec^2 \theta$. Therefore the radial component of the field from the element δx is $\frac{\lambda}{4\pi\epsilon_0 r} \cos \theta \delta \theta$. To find the radial component of the field from the entire rod, we integrate along the length of the rod. If the rod is infinitely long (or if its length is much greater than r), we integrate from $\theta = -\pi/2$ to $+\pi/2$, or, what amounts to the same thing, from 0 to $\pi/2$, and double it. Thus the radial component of the field is

$$E = \frac{2\lambda}{4\pi\epsilon_0 r} \int_0^{\pi/2} \cos \theta \delta \theta = \frac{\lambda}{2\pi\epsilon_0 r}. \quad (1.6.8)$$

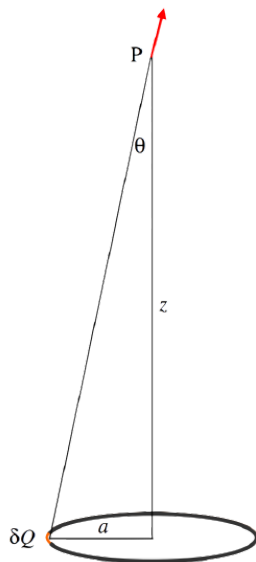
The component of the field parallel to the rod, by considerations of symmetry, is zero, so Equation 1.6.8 gives the total field at a distance r from the rod, and it is directed radially away from the rod.

Notice that Equation 1.6.4 for a spherical charge distribution has $4\pi r^2$ in the denominator, while Equation 1.6.8, dealing with a problem of cylindrical symmetry, has $2\pi r$.

This page titled [1.6C: A Long, Charged Rod](#) is shared under a [CC BY-NC 4.0](#) license and was authored, remixed, and/or curated by [Jeremy Tatum](#) via [source content](#) that was edited to the style and standards of the LibreTexts platform.

1.6D: Field on the Axis of and in the Plane of a Charged Ring

Field on the axis of a charged ring.



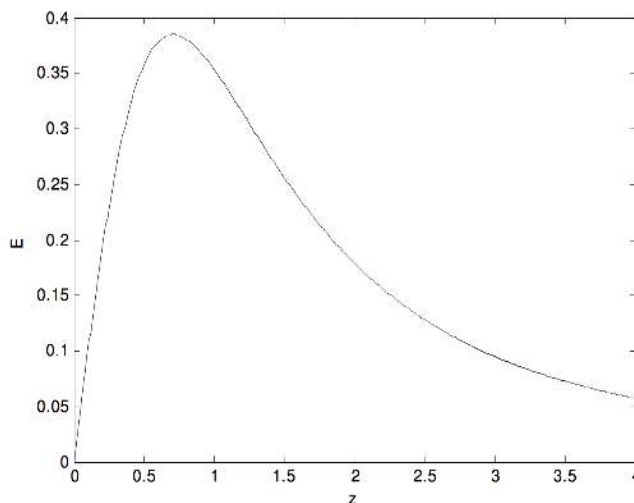
Ring, radius a , charge Q . Field at P from element of charge $\delta Q = \frac{\delta Q}{4\pi\epsilon_0(a^2 + z^2)^{3/2}}$. Vertical component of this $= \frac{\delta Q \cos \theta}{4\pi\epsilon_0(a^2 + z^2)^{3/2}}$.
 $= \frac{\delta Q z}{4\pi\epsilon_0(a^2 + z^2)^{3/2}}$. Integrate for entire ring:

$$\text{Field } E = \frac{Q}{4\pi\epsilon_0} \frac{z}{(a^2 + z^2)^{3/2}}.$$

In terms of dimensionless variables:

$$E = \frac{z}{(1 + z^2)^{3/2}}. \quad (1.6D.1)$$

where E is in units of $\frac{Q}{4\pi\epsilon_0 a^2}$, and z is in units of a .



From calculus, we find that this reaches a maximum value of $\frac{2\sqrt{3}}{9} = 0.3849$ at $z = 1/\sqrt{2} = 0.7071$.

It reaches half of its maximum value where $\frac{z}{(1 + z^2)^{3/2}} = \frac{\sqrt{3}}{9}$.

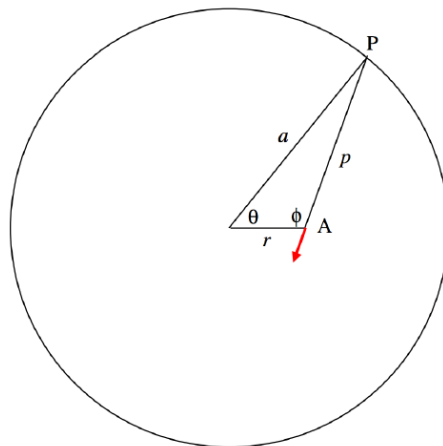
That is, $3 - 72Z + 9Z^2 + 3Z^2 = 0$, where $Z = z^2$.

The two positive solution are $Z = 0.041889$ and 3.596267 .

That is, $z = 0.2047$ and 1.8964 .

Field in the plane of a charged ring.

We suppose that we have a ring of radius a bearing a charge Q . We shall try to find the field at a point in the plane of the ring and at a distance r ($0 \leq r < a$) from the centre of the ring.



Consider an element $\delta\theta$ of the ring at P. The charge on it is $\frac{Q\delta\theta}{2\pi}$. The field at A from this element of charge is

$$\frac{1}{4\pi\epsilon_0} \cdot \frac{Q\delta\theta}{2\pi} \cdot \frac{1}{a^2 + r^2 - 2ar\cos\theta} = \frac{Q}{4\pi\epsilon_0 \cdot 2\pi a^2} \cdot \frac{\delta\theta}{b - c\cos\theta}, \quad (1.6D.2)$$

where $b = 1 + r^2/a^2$ and $c = 2r/a$. The component of this toward the centre is

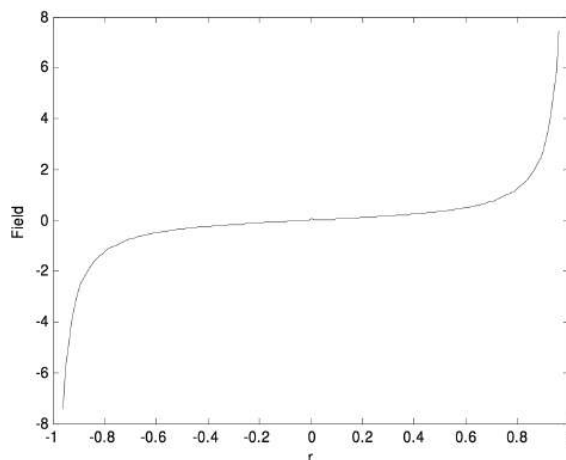
$$-\frac{Q}{4\pi\epsilon_0 \cdot 2\pi a^2} \cdot \frac{\cos\phi\delta\theta}{b - c\cos\theta}.$$

To find the field at A due to the entire ring, we must express ϕ in terms of θ , r and a , and integrate with respect to θ from 0 to 2π (or from 0 to π and double it). The necessary relations are

$$p^2 = a^2 + r^2 - 2ar\cos\theta \quad (1.6D.3)$$

$$\cos\phi = \frac{r^2 + p^2 - a^2}{2rp}. \quad (1.6D.4)$$

The result of the numerical integration is shown below, in which the field is expressed in units of $Q/(4\pi\epsilon_0 a^2)$ and r is in units of a .



This page titled [1.6D: Field on the Axis of and in the Plane of a Charged Ring](#) is shared under a [CC BY-NC 4.0](#) license and was authored, remixed, and/or curated by [Jeremy Tatum](#) via [source content](#) that was edited to the style and standards of the LibreTexts platform.

1.6E: Field on the Axis of a Uniformly Charged Disc

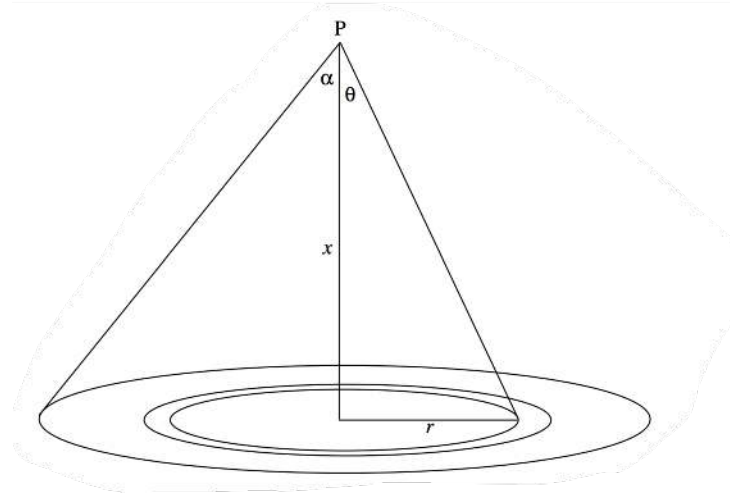


FIGURE 1.3

We suppose that we have a circular disc of radius a bearing a surface charge density of σ coulombs per square metre, so that the total charge is $Q = \pi a^2 \sigma$. We wish to calculate the field strength at a point P on the axis of the disc, at a distance x from the centre of the disc.

Consider an elemental annulus of the disc, of radii r and $r + \delta r$. Its area is $2\pi r \delta r$ and so it carries a charge $2\pi \sigma r \delta r$. Using the result of subsection 1.6.4, we see that the field at P from this charge is

$$\frac{2\pi \sigma r \delta r}{4\pi \epsilon_0} \cdot \frac{x}{(r^2 + x^2)^{3/2}} = \frac{\sigma x}{2\epsilon_0} \cdot \frac{r \delta r}{(r^2 + x^2)^{3/2}}. \quad (1.6E.1)$$

But $r = x \tan \theta$, $\delta r = x \sec^2 \theta \delta \theta$ and $(r^2 + x^2)^{1/2} = x \sec \theta$. Thus the field from the elemental annulus can be written

$$\frac{\sigma}{2\epsilon_0} \sin \theta \delta \theta. \quad (1.6E.2)$$

The field from the entire disc is found by integrating this from $\theta = 0$ to $\theta = \alpha$ to obtain

$$E = \frac{\sigma}{2\epsilon_0} (1 - \cos \alpha) = \frac{\sigma}{2\epsilon_0} \left(1 - \frac{x}{(a^2 + x^2)^{1/2}} \right). \quad (1.6.11)$$

This falls off monotonically from $\sigma/(2\epsilon_0)$ just above the disc to zero at infinity.

This page titled [1.6E: Field on the Axis of a Uniformly Charged Disc](#) is shared under a [CC BY-NC 4.0](#) license and was authored, remixed, and/or curated by [Jeremy Tatum](#) via [source content](#) that was edited to the style and standards of the LibreTexts platform.

1.6F: Field of a Uniformly Charged Infinite Plane Sheet

All we have to do is to put $\alpha = \pi/2$ in equation 1.6.10 to obtain

$$E = \frac{\sigma}{2\epsilon_0}. \quad (1.6.12)$$

This is independent of the distance of P from the infinite charged sheet. The electric field lines are uniform parallel lines extending to infinity.

Summary

Point charge Q : $E = \frac{Q}{4\pi\epsilon_0 r^2}.$ (1.6F.1)

Hollow Spherical Shell: $E =$ zero inside the shell, (1.6F.2)

$E = \frac{Q}{4\pi\epsilon_0 r^2}$ outside the shell (1.6F.3)

Infinite charged rod : $E = \frac{\lambda}{2\pi\epsilon_0 r}.$ (1.6F.4)

Infinite plane sheet : $E = \frac{\sigma}{2\epsilon_0}.$ (1.6F.5)

This page titled [1.6F: Field of a Uniformly Charged Infinite Plane Sheet](#) is shared under a [CC BY-NC 4.0](#) license and was authored, remixed, and/or curated by [Jeremy Tatum](#) via [source content](#) that was edited to the style and standards of the LibreTexts platform.

1.7: Electric Field D

We have been assuming that all “experiments” described have been carried out in a vacuum or (which is almost the same thing) in air. But what if the point charge, the infinite rod and the infinite charged sheet of [Section 1.6](#) are all immersed in some medium whose permittivity is not ϵ_0 , but is instead ϵ ? In that case, the formulas for the field become

$$E = \frac{Q}{4\pi\epsilon r^2}, \quad \frac{\lambda}{2\pi\epsilon r}, \quad \frac{\sigma}{2\epsilon}$$

There is an ϵ in the denominator of each of these expressions. When dealing with media with a permittivity other than ϵ_0 it is often convenient to describe the electric field by another vector, $\mathbf{D}$, defined simply by

$$\mathbf{D} = \epsilon \mathbf{E} \tag{1.7.1}$$

In that case the above formulas for the field become just

$$D = \frac{Q}{4\pi r^2}, \quad \frac{\lambda}{2\pi r}, \quad \frac{\sigma}{2}$$

The dimensions of D are Q L^{-2} , and the SI units are C m^{-2} .

This may seem to be rather trivial, but it does turn out to be more important than it may seem at the moment.

Equation [1.7.1](#) would seem to imply that the electric field vectors $\mathbf{E}$ and $\mathbf{D}$ are just vectors in the same direction, differing in magnitude only by the scalar quantity ϵ . This is indeed the case *in vacuo* or in any isotropic medium – but it is more complicated in an anisotropic medium such as, for example, an orthorhombic crystal. This is a crystal shaped like a rectangular parallelepiped. If such a crystal is placed in an electric field, the magnitude of the permittivity depends on whether the field is applied in the x -, the y - or the z -direction. For a given magnitude of E , the resulting magnitude of D will be different in these three situations. And, if the field $\mathbf{E}$ is not applied parallel to one of the crystallographic axes, the resulting vector $\mathbf{D}$ will not be parallel to $\mathbf{E}$. The permittivity in Equation [1.7.1](#) is a *tensor* with nine components, and, when applied to $\mathbf{E}$ it changes its direction as well as its magnitude.

However, we shan’t dwell on that just yet, and, unless specified otherwise, we shall always assume that we are dealing with a vacuum (in which case $\mathbf{D} = \epsilon_0 \mathbf{E}$) or an isotropic medium (in which case $\mathbf{D} = \epsilon \mathbf{E}$). In either case the permittivity is a scalar quantity and $\mathbf{D}$ and $\mathbf{E}$ are in the same direction.

This page titled [1.7: Electric Field D](#) is shared under a [CC BY-NC 4.0](#) license and was authored, remixed, and/or curated by [Jeremy Tatum](#) via [source content](#) that was edited to the style and standards of the LibreTexts platform.