

LECTURE NOTES ON KINEMATICS



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Licensing

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1: Velocity and Acceleration of a Point

Fix a right-handed (positively oriented) Cartesian coordinate system $Oxyz$ in physical space $\mathbb{R}^3$. The law of motion of a point M with position vector $\mathbf{r}_M(t) = \overrightarrow{OM}(t)$ is defined as the mapping

$$t \mapsto \mathbf{r}_M(t) = x(t)\mathbf{e}_x + y(t)\mathbf{e}_y + z(t)\mathbf{e}_z, \quad x(\cdot), y(\cdot), z(\cdot) \in C^2[0, T].$$

Here, $\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z$ denote the unit basis vectors of the corresponding coordinate axes.

The trajectory (path) of point M is the set of points

$$\{\mathbf{r}_M(t) \mid t \in [0, T]\} \subset \mathbb{R}^3,$$

which is usually a piecewise smooth curve.

The velocity and acceleration of point M relative to $Oxyz$ are defined as the vectors

$$\mathbf{v}_M = \frac{d\mathbf{r}_M}{dt} = \dot{\mathbf{r}}_M = \dot{x}(t)\mathbf{e}_x + \dot{y}(t)\mathbf{e}_y + \dot{z}(t)\mathbf{e}_z, \quad \mathbf{a}_M = \dot{\mathbf{v}}_M.$$

Unless specified otherwise, velocities and accelerations in what follows are considered relative to the frame $Oxyz$.

2: Kinematics of a Rigid Body

Definition 1:

Given a non-empty open set $D \subset \mathbb{R}^3$ and a pair of functions

$$\mathbf{r}_A \in C^2([0, T], \mathbb{R}^3), \quad U \in C^2([0, T], \text{SO}(3)), \quad U(0) = E, \quad \mathbf{r}_A(0) \in \bar{D}.$$

By definition, a rigid body consists of points B moving according to the law

$$\mathbf{r}_B(t) = \mathbf{r}_A(t) + U(t) (\mathbf{r}_B(0) - \mathbf{r}_A(0)), \quad t \in [0, T], \quad (1)$$

where the initial condition satisfies $\mathbf{r}_B(0) \in \bar{D}$.

The law of motion of point B remains unchanged if the position vector $\mathbf{r}_A$ in formula (1) is replaced by the position vector of any other point belonging to the rigid body.

Definition 2:

A vector $\mathbf{u}(t)$ is said to be attached (bound) to a rigid body if there exist two points P, N of the body and a scalar λ such that $\mathbf{u}(t) = \lambda (\mathbf{r}_P(t) - \mathbf{r}_N(t))$, $t \in [0, T]$.

Theorem 2.1: Euler's Formula

There exists a unique axial vector $\boldsymbol{\omega}$ such that for any two points A, B of the rigid body, the following equality holds:

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{AB}, \quad \mathbf{AB} = \mathbf{r}_B - \mathbf{r}_A.$$

Definition 3:

The vector $\boldsymbol{\omega}$ is called the angular velocity of the rigid body.

The vector $\boldsymbol{\varepsilon} = \dot{\boldsymbol{\omega}}$ is called the angular acceleration of the rigid body.

Proof of Theorem 1

Differentiating equality (1), we find

$$\mathbf{v}_B(t) = \mathbf{v}_A(t) + \dot{U}(t) (\mathbf{r}_B(0) - \mathbf{r}_A(0)) = \mathbf{v}_A(t) + \dot{U}(t) U^{-1}(t) (\mathbf{r}_B(t) - \mathbf{r}_A(t)).$$

The operator $\Omega = \dot{U}U^{-1} = \dot{U}U^*$ is skew-symmetric:

$$UU^* = E, \quad \frac{d}{dt} (UU^*) = \dot{U}U^* + U\dot{U}^* = 0 \implies \Omega^* = -\Omega.$$

From linear algebra [1], every skew-symmetric operator in $\mathbb{R}^3$ corresponds to a unique axial vector $\boldsymbol{\omega}$ such that

$$\Omega \mathbf{x} = \boldsymbol{\omega} \times \mathbf{x}, \quad \forall \mathbf{x} \in \mathbb{R}^3.$$

To show that the angular velocity vector is uniquely determined, suppose for contradiction that there exists another vector $\boldsymbol{\omega}'$ satisfying Euler's formula

$$v_B = v_A + \omega' \times AB$$

for any two points A, B of the rigid body. Subtracting one formula from the other yields

$$(\omega' - \omega) \times AB = 0.$$

It suffices to choose AB as a non-zero vector perpendicular to $\omega' - \omega$, reaching a contradiction.

The theorem is proved.

Corollary 2.1

Suppose that at some instant $v_A = 0$ while $\omega \neq 0$. Then the set of points of the rigid body whose instantaneous velocities are zero forms a straight line passing through point A parallel to ω . This line is called the instantaneous axis of rotation. The velocities of all other points of the body at this instant are perpendicular to this axis.

The motion of a rigid body is called translational if the velocities of all its points are equal. The motion of a rigid body is translational if and only if $\omega = 0$.

Attach a right-handed orthonormal frame $e_1(t), e_2(t), e_3(t)$ to the rigid body.

Lemma 2.1

Lemma 1. The following formula holds ¹ :

$$\omega = (\dot{e}_2, e_3) e_1 + (\dot{e}_3, e_1) e_2 + (\dot{e}_1, e_2) e_3. \quad (2)$$

Indeed, rewrite Euler's formula in the form

$$\frac{d}{dt} AB = \omega \times AB \quad (3)$$

In particular, Poisson's formulas hold:

$$\dot{e}_i = \omega \times e_i, \quad i = 1, 2, 3. \quad (4)$$

Now, to verify the assertion of the lemma, it suffices to substitute (4) into the right-hand side of (2) and use the properties of the scalar triple product.

The lemma is proved.

Theorem 2.2: Rivals' Formula

The following formula holds:

$$a_B = a_A + \varepsilon \times AB + \omega \times (\omega \times AB).$$

Indeed, this formula is obtained by direct differentiation of Euler's formula using Eq. (3).

[1] Note that formula (2) as such cannot serve as the definition of angular velocity, since its right-hand side a priori depends on the choice of frame attached to the rigid body.

3: Rotation of a Rigid Body About a Fixed Axis

Consider an important special case. Suppose a rigid body rotates about a fixed axis Oz through an angle $\varphi = \varphi(t)$ such that for a certain attached right-handed frame $\{e_i\}$ we have $e_3 \equiv e_z$,

$$e_1 = \cos \varphi e_x + \sin \varphi e_y, \quad e_2 = \cos \varphi e_y - \sin \varphi e_x.$$

Differentiating the first equation yields:

$$\dot{e}_1 = \dot{\varphi} (-\sin \varphi e_x + \cos \varphi e_y) = \dot{\varphi} e_2.$$

Similarly:

$$\dot{e}_2 = -\dot{\varphi} e_1, \quad \dot{e}_3 = 0.$$

Substituting these formulas into (2), we find

$$\boldsymbol{\omega} = \dot{\varphi} e_3 = \dot{\varphi} e_z. \quad (5)$$

4: Kinematics of Relative Motion

Suppose a right-handed Cartesian coordinate system $Q\xi\eta\zeta$ moves relative to $Oxyz$ with angular velocity $\omega^e(t)$ and position vector $OQ = OQ(t)$. Note that a Cartesian coordinate system can be regarded as attached to a certain rigid body.

Introduce the notation

$$e_1 = e_\xi, \quad e_2 = e_\eta, \quad e_3 = e_\zeta.$$

Let $f(t)$ be any time-dependent vector. We can write

$$f = f_x(t)e_x + f_y(t)e_y + f_z(t)e_z = f^i(t)e_i. \quad (6)$$

The derivative of this vector relative to $Oxyz$ is, as above, defined as

$$\dot{f} = \dot{f}_x e_x + \dot{f}_y e_y + \dot{f}_z e_z;$$

while its derivative relative to the system $Q\xi\eta\zeta$ is defined as

$$\frac{\delta f}{\delta t} := \dot{f}^i e_i.$$

Lemma 4.2

The following formula holds:

$$\dot{f} = \frac{\delta f}{\delta t} + \omega^e \times f. \quad (7)$$

Indeed, this formula is obtained by direct differentiation of equality (6) using Poisson's formulas.

Let us study the relative motion of a point M .

Definition \tag{4}:

Definition 4. The relative velocity of point M is defined as its velocity relative to $Q\xi\eta\zeta$:

$$v^r = \frac{\delta QM}{\delta t}.$$

The frame (transport) velocity of point M is the velocity of the point of the system $Q\xi\eta\zeta$ that coincides with M at the given instant. Notation: v^e .

The relative acceleration of point M is defined as its acceleration relative to $Q\xi\eta\zeta$:

$$a^r = \frac{\delta v^r}{\delta t}.$$

The frame (transport) acceleration of point M is the acceleration of the point of the system $Q\xi\eta\zeta$ that coincides with M at the given instant. Notation: a^e .

The Coriolis acceleration of point M is the vector $a^c = 2\omega^e \times v^r$.

Note that, in general, $a^e \neq \dot{v}^e$.

Theorem

The following addition formulas hold:

$$\mathbf{v}_M = \mathbf{v}^r + \mathbf{v}^e, \quad (8)$$

$$\mathbf{a}_M = \mathbf{a}^r + \mathbf{a}^e + \mathbf{a}^c. \quad (9)$$

Proof

Differentiate the identity

$$OM = OQ + QM,$$

using (7):

$$\mathbf{v}_M = \mathbf{v}_Q + \frac{\delta QM}{\delta t} + \boldsymbol{\omega}^e \times QM = \mathbf{v}_Q + \mathbf{v}^r + \boldsymbol{\omega}^e \times QM. \quad (10)$$

By Euler's formula,

$$\mathbf{v}_Q + \boldsymbol{\omega}^e \times QM = \mathbf{v}^e$$

Formula (8) is proved.

To verify formula (9), differentiate (10):

$$\frac{d}{dt}[\mathbf{a}_M = \mathbf{a}_Q + \frac{\delta \mathbf{v}^r}{\delta t} + \boldsymbol{\omega}^e \times \mathbf{v}_Q + \boldsymbol{\omega}^e \times \frac{\delta QM}{\delta t} + \boldsymbol{\omega}^e \times (\boldsymbol{\omega}^e \times QM)] = \mathbf{a}^e + \boldsymbol{\omega}^e \times \mathbf{v}_Q + \boldsymbol{\omega}^e \times (\boldsymbol{\omega}^e \times QM) = \mathbf{a}^e.$$

By Rivals' formula,

$$\mathbf{a}_Q + \boldsymbol{\omega}^e \times \mathbf{v}_Q + \boldsymbol{\omega}^e \times (\boldsymbol{\omega}^e \times QM) = \mathbf{a}^e.$$

The theorem is proved.

Theorem

The addition formula for angular velocity holds: $\boldsymbol{\omega} = \boldsymbol{\omega}^e + \boldsymbol{\omega}^r$.

Proof

Take arbitrary points A, B of the rigid body. Using the velocity addition formula (8):

$$\mathbf{v}_B = \mathbf{v}_A^r + \mathbf{v}_B^e.$$

Applying Euler's formula to the rigid body motion relative to $Q\xi\eta\zeta$:

$$\mathbf{v}_B^r = \mathbf{v}_A^r + \boldsymbol{\omega}^r \times AB.$$

Applying Euler's formula to the system $Q\xi\eta\zeta$ relative to $Oxyz$:

$$\mathbf{v}_B^e = \mathbf{v}_A^e + \boldsymbol{\omega}^e \times AB.$$

Combining these expressions gives:

$$\mathbf{v}_B = \mathbf{v}_A + (\boldsymbol{\omega}^r + \boldsymbol{\omega}^e) \times \mathbf{AB}.$$

By the uniqueness of angular velocity, we obtain the required assertion.

The theorem is proved.

5: Velocity and Acceleration of a Point in Polar Coordinates

Let a point M move in the Oxy plane, equipped with polar coordinates

$$x = r \cos \varphi, \quad y = r \sin \varphi.$$

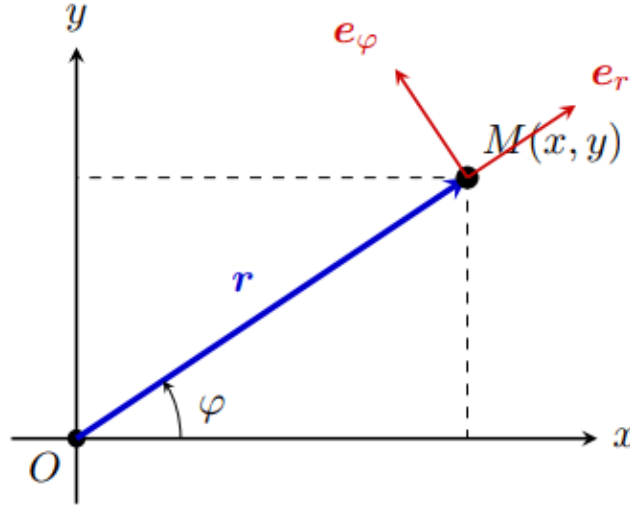


Figure 5.1: Polar coordinates and unit basis vectors.

By formula (5), the angular velocity of the frame $e_r e_\varphi e_z$ is $\omega = \dot{\varphi} e_z$. Hence, by Poisson's formulas,

$$\dot{e}_r = \omega \times e_r = \dot{\varphi} e_\varphi, \quad \dot{e}_\varphi = \omega \times e_\varphi = -\dot{\varphi} e_r.$$

Now the velocity and acceleration of point M in polar coordinates are computed by differentiating the position vector $OM = r e_r$:

$$v_M = \frac{d}{dt} OM = \dot{r} e_r + r \dot{\varphi} e_\varphi, \quad a_M = \frac{d}{dt} v_M = (\ddot{r} - r \dot{\varphi}^2) e_r + (r \ddot{\varphi} + 2\dot{r} \dot{\varphi}) e_\varphi.$$

6: Euler Angles and Kinematic Euler Equations

Euler angles define the position of a rigid body with a fixed point O relative to a fixed frame $Oxyz$. Attach a right-handed orthonormal frame $O\xi\eta\zeta$ to the rigid body.

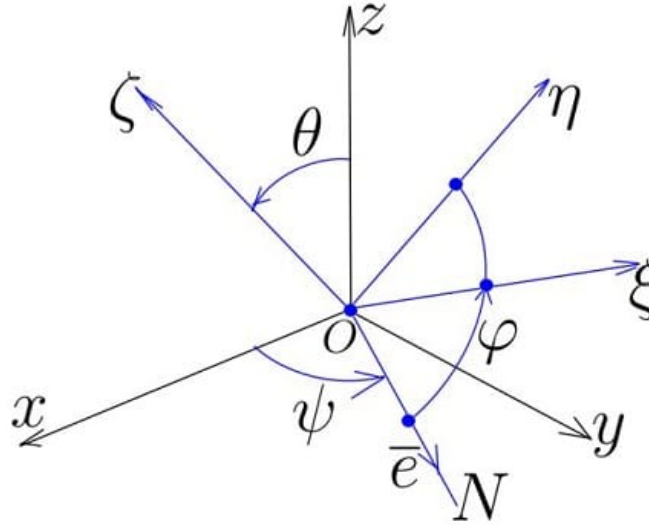


Figure 6.1: Euler angles (ψ, θ, φ) and the line of nodes ON .

The line of nodes ON lies at the intersection of the planes $O\xi\eta$ and Oxy , which are assumed not to coincide. The Euler angles

$$\theta \in (0, \pi), \quad \varphi \pmod{2\pi}, \quad \psi \pmod{2\pi}$$

specify the orientation (nutation, precession, and spin angle).

Let frame I denote the coordinate system associated with the axes ON and Oz ;
 Let frame II denote the coordinate system associated with the axes ON and $O\xi$;
 By formula (5), the angular velocity of frame I relative to $Oxyz$ is

$$\omega_1 = \dot{\psi} e_z;$$

the angular velocity of frame II relative to frame I is

$$\omega_2 = \dot{\theta} e, \quad e = \cos \varphi e_\xi - \sin \varphi e_\eta;$$

the angular velocity of $O\xi\eta\zeta$ relative to frame II is

$$\omega_3 = \dot{\varphi} e_\zeta.$$

By Theorem 4, the angular velocity of frame $O\xi\eta\zeta$ relative to $Oxyz$ is

$$\omega = \omega_1 + \omega_2 + \omega_3.$$

Notice that the plane $O\zeta z$ is perpendicular to the line ON , so

$$e_z = \cos \theta e_\zeta + \sin \theta (e_\zeta \times e).$$

Thus, we obtain the kinematic Euler equations:

$$\begin{aligned}\boldsymbol{\omega} &= (\dot{\psi} \sin \theta \sin \varphi + \dot{\theta} \cos \varphi) \mathbf{e}_\xi \\ &\quad + (\dot{\psi} \sin \theta \cos \varphi - \dot{\theta} \sin \varphi) \mathbf{e}_\eta \\ &\quad + (\dot{\psi} \cos \theta + \dot{\varphi}) \mathbf{e}_\zeta.\end{aligned}$$

7: APPENDIX

7.1: A Useful Fact

7.2: Kinematics of the Frenet-Serret Frame

7.3: Rolling Without Slipping

7.4: Instantaneous Screw Motion of a Rigid Body

7.5: Rotation Matrix of a Rigid Body

7.1: A Useful Fact

The projections of the velocities of two points A and B of a rigid body onto the line connecting them are equal:

$$(v_A, e) = (v_B, e), \quad e = \frac{AB}{|AB|}.$$

7.2: Kinematics of the Frenet-Serret Frame

Let a curve γ be parameterized by its position vector $\mathbf{r} = \mathbf{r}(s)$, where s is the arc length parameter:

$$\frac{d}{ds}\mathbf{r} = \mathbf{r}', \quad |\mathbf{r}'| \equiv 1$$

Assume that the vectors $\boldsymbol{\tau} = \mathbf{r}'$ and $\mathbf{r}''$ are linearly independent. Note that $\mathbf{r}' \perp \mathbf{r}''$.

Define the curvature of the curve by $k(s) = |\mathbf{r}''|$.

The Frenet-Serret formulas are:

$$\boldsymbol{\tau}'(s) = k(s)\mathbf{n}(s), \quad \mathbf{n}'(s) = -k(s)\boldsymbol{\tau}(s) - \kappa(s)\mathbf{b}(s), \quad \mathbf{b}'(s) = \kappa(s)\mathbf{n}(s).$$

Here $\mathbf{b} = \boldsymbol{\tau} \times \mathbf{n}$ is the binormal vector.

Expand the position vector in the Frenet frame:

$$\mathbf{r} = x\boldsymbol{\tau} + y\mathbf{n} + z\mathbf{b}.$$

Differentiating this formula and substituting the result into $\mathbf{r}' = \boldsymbol{\tau}$, we obtain useful relations:

$$x' = ky + 1, \quad y' = -kx - \kappa z, \quad z' = \kappa y.$$

Suppose the Frenet frame $M\boldsymbol{\tau}\mathbf{n}\mathbf{b}$ is constructed at a point M moving along the curve according to $\mathbf{r}_M = \mathbf{r}(s(t))$. Then the angular velocity of the Frenet frame is calculated by formula (2):

$$\boldsymbol{\omega} = \dot{s}(-\kappa\boldsymbol{\tau} + k\mathbf{b}), \quad \left(\frac{d}{dt} = \dot{s} \frac{d}{ds} \right).$$

In particular, if the curve is planar ($\kappa = 0$), then from formula (5) we find $k(s) = \varphi'$.

The velocity and acceleration of point M in such motion are found by

$$\mathbf{v}_M = \dot{\mathbf{r}}_M = \mathbf{r}'\dot{s} = \dot{s}\boldsymbol{\tau}, \quad \mathbf{a}_M = \ddot{s}\boldsymbol{\tau} + \dot{s}^2 k\mathbf{n}.$$

7.3: Rolling Without Slipping

Suppose a convex rigid body rolls without slipping with angular velocity ω on a fixed plane.

Let P denote the point of the body touching the plane at the given instant. Rolling without slipping by definition means that $v_P = 0$.

Let M denote the contact point on the plane. As the body moves, the contact point traces a curve on the plane.

Theorem

The following formula holds: $a_P = -\omega \times v_M$.

Proof

Note the following formula:

$$\frac{dv^e}{dt} = a^e + \omega^e \times v^r. \quad (11)$$

It is obtained by differentiating formula (8):

$$a_M = \frac{\delta v^r}{\delta t} + \omega^e \times v^r + \frac{dv^e}{dt} = a^r + \omega^e \times v^r + \frac{dv^e}{dt}$$

and comparing this result with (9).

Attach a coordinate system $Q\xi\eta\zeta$ to the body. Then

$$\mathbf{v}_M = \mathbf{v}_M^r + \mathbf{v}_M^e = \mathbf{v}_M^r. \quad \text{[notag]}$$

Here we used the definition of frame velocity and the no-slip condition: $v_M^e = v_P = 0$.

Formula (11) yields:

$$0 = \frac{d}{dt} v_M^e = a_M^e + \omega \times v_M^r, \quad a_M^e = a_P$$

The theorem is proved.

7.4: Instantaneous Screw Motion of a Rigid Body

In conclusion, in connection with Corollary 1, note that in the general case, at any instant there exists a point B in the rigid body (or its rigid extension) such that $v_B \parallel \omega$. If $\omega \neq 0$, such points form a straight line parallel to ω (the instantaneous screw axis). Point B can be found from

$$AB = \frac{\omega \times v_A}{|\omega|^2}.$$

If the body undergoes planar motion (i.e., the velocities of all its points are parallel to a fixed plane), its angular velocity is perpendicular to this plane, and point B is the instantaneous center of rotation ($v_B = 0$).

7.5: Rotation Matrix of a Rigid Body

Let u be an arbitrary vector attached to a rigid body moving with angular velocity $\omega(t)$. Let $u(t)$ denote the column of coordinates of vector u in the fixed system $Oxyz$. We have

$$u(t) = X(t)u_0, \quad u(0) = u_0,$$

where $X(t)$ is a 3×3 orthogonal matrix with $X(0) = E$.

Suppose that the frame $Q\xi\eta\zeta$ attached to the rigid body coincides with $Oxyz$ at the initial instant.

In particular, this means that the coordinate column of vector u in $Q\xi\eta\zeta$ is equal to u_0 at all times. Moreover, if v is the coordinate column of a vector v in $Oxyz$, then $X^{-1}(t)v$ is the coordinate column of vector v in $Q\xi\eta\zeta$.

The formula

$$\dot{u} = \omega \times u \tag{12}$$

is invariant and follows from formula (3).

Suppose the angular velocity vector ω is given by its coordinate column ω in the frame $Q\xi\eta\zeta$. Then we can write formula (12) in $Q\xi\eta\zeta$ coordinates:

$$X^{-1}\dot{u}(t) = \omega \times u_0 = \Omega u_0,$$

where

$$\Omega = \begin{pmatrix} 0 & -\omega_\zeta & \omega_\eta \\ \omega_\zeta & 0 & -\omega_\xi \\ -\omega_\eta & \omega_\xi & 0 \end{pmatrix}, \quad \omega = \begin{pmatrix} \omega_\xi \\ \omega_\eta \\ \omega_\zeta \end{pmatrix}.$$

Finally, we obtain:

$$X^{-1}\dot{X}u_0 = \Omega u_0 \implies \dot{X} = X\Omega.$$

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