TAMU_SA_UNIVERSITYPHYSIC



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TAMU-SA UNIVERSITY PHYSICS 2 2026

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1.1: Prelude to Vectors

Vectors are essential to physics and engineering. Many fundamental physical quantities are vectors, including displacement, velocity, force, and electric and magnetic vector fields. Scalar products of vectors define other fundamental scalar physical quantities, such as energy. Vector products of vectors define still other fundamental vector physical quantities, such as torque and angular momentum. In other words, vectors are a component part of physics in much the same way as sentences are a component part of literature.



Figure 1.1.1: A signpost gives information about distances and directions to towns or to other locations relative to the location of the signpost. Distance is a scalar quantity. Knowing the distance alone is not enough to get to the town; we must also know the direction from the signpost to the town. The direction, together with the distance, is a vector quantity commonly called the displacement vector. A signpost, therefore, gives information about displacement vectors from the signpost to towns. (credit: modification of work by “studio tdes”/Flickr)

In introductory physics, vectors are Euclidean quantities that have geometric representations as arrows in one dimension (in a line), in two dimensions (in a plane), or in three dimensions (in space). They can be added, subtracted, or multiplied. In this chapter, we explore elements of vector algebra for applications in mechanics and in electricity and magnetism. Vector operations also have numerous generalizations in other branches of physics.

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1.2: Scalars and Vectors (Part 1)

Learning Objectives

- Describe the difference between vector and scalar quantities.
- Identify the magnitude and direction of a vector.
- Explain the effect of multiplying a vector quantity by a scalar.
- Describe how one-dimensional vector quantities are added or subtracted.
- Explain the geometric construction for the addition or subtraction of vectors in a plane.
- Distinguish between a vector equation and a scalar equation.

Many familiar physical quantities can be specified completely by giving a single number and the appropriate unit. For example, “a class period lasts 50 min” or “the gas tank in my car holds 65 L” or “the distance between two posts is 100 m.” A physical quantity that can be specified completely in this manner is called a **scalar quantity**. Scalar is a synonym of “number.” Time, mass, distance, length, volume, temperature, and energy are examples of **scalar** quantities.

Scalar quantities that have the same physical units can be added or subtracted according to the usual rules of algebra for numbers. For example, a class ending 10 min earlier than 50 min lasts 50 min – 10 min = 40 min. Similarly, a 60-cal serving of corn followed by a 200-cal serving of donuts gives 60 cal + 200 cal = 260 cal of energy. When we multiply a scalar quantity by a number, we obtain the same scalar quantity but with a larger (or smaller) value. For example, if yesterday’s breakfast had 200 cal of energy and today’s breakfast has four times as much energy as it had yesterday, then today’s breakfast has 4(200 cal) = 800 cal of energy. Two scalar quantities can also be multiplied or divided by each other to form a derived scalar quantity. For example, if a train covers a distance of 100 km in 1.0 h, its speed is 100.0 km/1.0 h = 27.8 m/s, where the speed is a derived scalar quantity obtained by dividing distance by time.

Many physical quantities, however, cannot be described completely by just a single number of physical units. For example, when the U.S. Coast Guard dispatches a ship or a helicopter for a rescue mission, the rescue team must know not only the distance to the distress signal, but also the direction from which the signal is coming so they can get to its origin as quickly as possible. Physical quantities specified completely by giving a number of units (magnitude) and a direction are called **vector quantities**. Examples of vector quantities include displacement, velocity, position, force, and torque. In the language of mathematics, physical vector quantities are represented by mathematical objects called **vectors** (Figure 1.2.1). We can add or subtract two vectors, and we can multiply a vector by a scalar or by another vector, but we cannot divide by a vector. The operation of division by a vector is not defined.

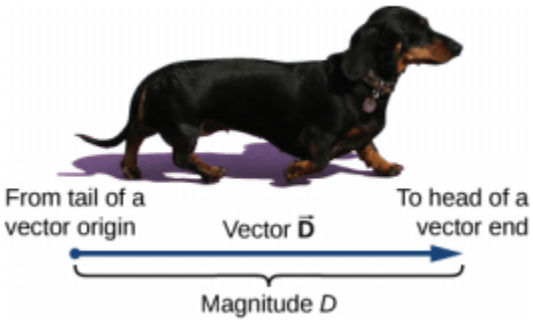


Figure 1.2.1: We draw a vector from the initial point or origin (called the “tail” of a vector) to the end or terminal point (called the “head” of a vector), marked by an arrowhead. Magnitude is the length of a vector and is always a positive scalar quantity. (credit: modification of work by Cate Sevilla)

Let’s examine vector algebra using a graphical method to be aware of basic terms and to develop a qualitative understanding. In practice, however, when it comes to solving physics problems, we use analytical methods, which we’ll see in the next section. Analytical methods are more simple computationally and more accurate than graphical methods. From now on, to distinguish between a vector and a scalar quantity, we adopt the common convention that a letter in bold type with an arrow above it denotes a vector, and a letter without an arrow denotes a scalar. For example, a distance of 2.0 km, which is a scalar quantity, is denoted by d = 2.0 km, whereas a displacement of 2.0 km in some direction, which is a vector quantity, is denoted by $\vec{d}$.

Suppose you tell a friend on a camping trip that you have discovered a terrific fishing hole 6 km from your tent. It is unlikely your friend would be able to find the hole easily unless you also communicate the direction in which it can be found with respect to your campsite. You may say, for example, “Walk about 6 km northeast from my tent.” The key concept here is that you have to give not one but two pieces of information—namely, the distance or magnitude (6 km) **and** the direction (northeast).

Displacement is a general term used to describe a change in position, such as during a trip from the tent to the fishing hole. Displacement is an example of a vector quantity. If you walk from the tent (location A) to the hole (location B), as shown in Figure 1.2.2, the vector $\vec{D}$, representing your **displacement**, is drawn as the arrow that originates at point A and ends at point B. The arrowhead marks the end of the vector. The direction of the displacement vector $\vec{D}$ is the direction of the arrow. The length of the arrow represents the **magnitude** D of vector $\vec{D}$. Here, D = 6 km. Since the magnitude of a vector is its length, which is a positive number, the magnitude is also indicated by placing the absolute value notation around the symbol that denotes the vector; so, we can write equivalently that D ≡ $|\vec{D}|$. To solve a vector problem graphically, we need to draw the vector $\vec{D}$ to scale. For example, if we assume 1 unit of distance (1 km) is represented in the drawing by a line segment of length u = 2 cm, then the total displacement in this example is represented by a vector of length d = 6u = 6(2 cm) = 12 cm , as shown in Figure 1.2.3. Notice that here, to avoid confusion, we used D = 6 km to denote the magnitude of the actual displacement and d = 12 cm to denote the length of its representation in the drawing.

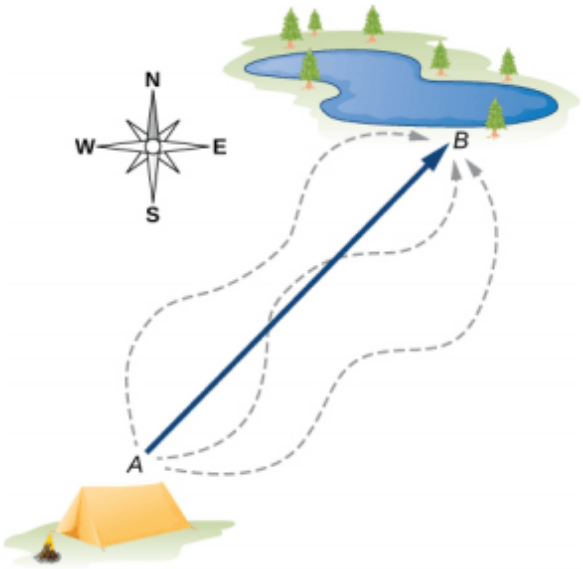


Figure 1.2.2: The displacement vector from point A (the initial position at the campsite) to point B (the final position at the fishing hole) is indicated by an arrow with origin at point A and end at point B. The displacement is the same for any of the actual paths (dashed curves) that may be taken between points A and B.

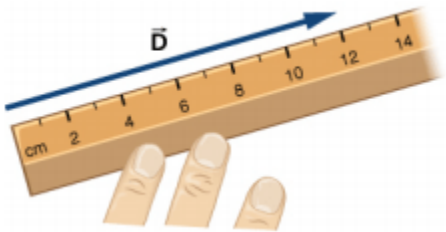


Figure 1.2.3: A displacement $\vec{D}$of magnitude 6 km is drawn to scale as a vector of length 12 cm when the length of 2 cm represents 1 unit of displacement (which in this case is 1 km).

Suppose your friend walks from the campsite at A to the fishing pond at B and then walks back: from the fishing pond at B to the campsite at A. The magnitude of the displacement vector $\vec{D}_{AB}$ from A to B is the same as the magnitude of the displacement vector $\vec{D}_{BA}$ from B to A (it equals 6 km in both cases), so we can write $\vec{D}_{AB} = \vec{D}_{BA}$. However, vector $\vec{D}_{AB}$ is not equal to vector $\vec{D}_{BA}$ because these two vectors have different directions: $\vec{D}_{AB} \neq \vec{D}_{BA}$. In Figure 2.3, vector $\vec{D}_{BA}$ would be represented by a vector with an origin at point B and an end at point A, indicating vector $\vec{D}_{BA}$ points to the southwest, which is exactly 180° opposite to the direction of vector $\vec{D}_{AB}$. We say that vector $\vec{D}_{BA}$ is **antiparallel** to vector $\vec{D}_{AB}$ and write $\vec{D}_{AB} = -\vec{D}_{BA}$, where the minus sign indicates the antiparallel direction.

Two vectors that have identical directions are said to be **parallel vectors**—meaning, they are **parallel** to each other. Two parallel vectors $\vec{A}$ and $\vec{B}$ are equal, denoted by $\vec{A} = \vec{B}$, if and only if they have equal magnitudes $|\vec{A}| = |\vec{B}|$. Two vectors with directions perpendicular to each other are said to be **orthogonal vectors**. These relations between vectors are illustrated in Figure 1.2.4.

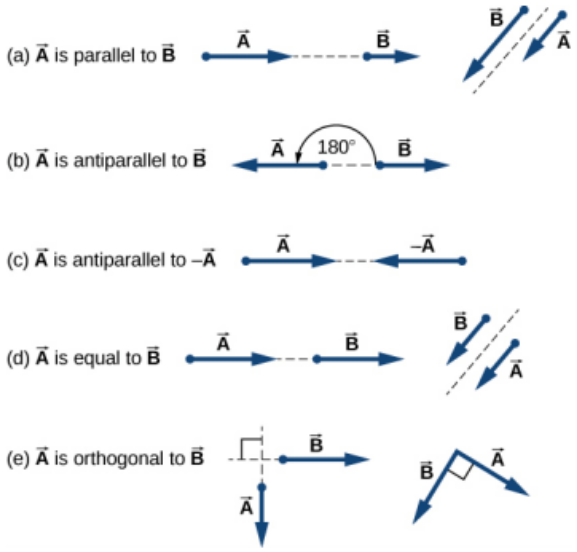


Figure 1.2.4: Various relations between two vectors $\vec{A}$ and $\vec{B}$. (a) $\vec{A} \neq \vec{B}$ because A ≠ B . (b) $\vec{A} \neq \vec{B}$ because they are not parallel and A ≠ B . (c) $\vec{A} \neq -\vec{A}$ because they have different directions (even though $|\vec{A}| = |-\vec{A}|$ = A) . (d) $\vec{A} = \vec{B}$ because they are parallel and have identical magnitudes A = B. (e) $\vec{A} \neq \vec{B}$ because they have different directions (are not parallel); here, their directions differ by 90° —meaning, they are orthogonal.

Exercise 2.1

Two motorboats named **Alice** and **Bob** are moving on a lake. Given the information about their velocity vectors in each of the following situations, indicate whether their velocity vectors are equal or otherwise.

a. **Alice** moves north at 6 knots and **Bob** moves west at 6 knots.
b. **Alice** moves west at 6 knots and **Bob** moves west at 3 knots.
c. **Alice** moves northeast at 6 knots and **Bob** moves south at 3 knots.
d. **Alice** moves northeast at 6 knots and **Bob** moves southwest at 6 knots.
e. **Alice** moves northeast at 2 knots and **Bob** moves closer to the shore northeast at 2 knots.

Algebra of Vectors in One Dimension

Vectors can be multiplied by scalars, added to other vectors, or subtracted from other vectors. We can illustrate these vector concepts using an example of the fishing trip seen in Figure 1.2.5.

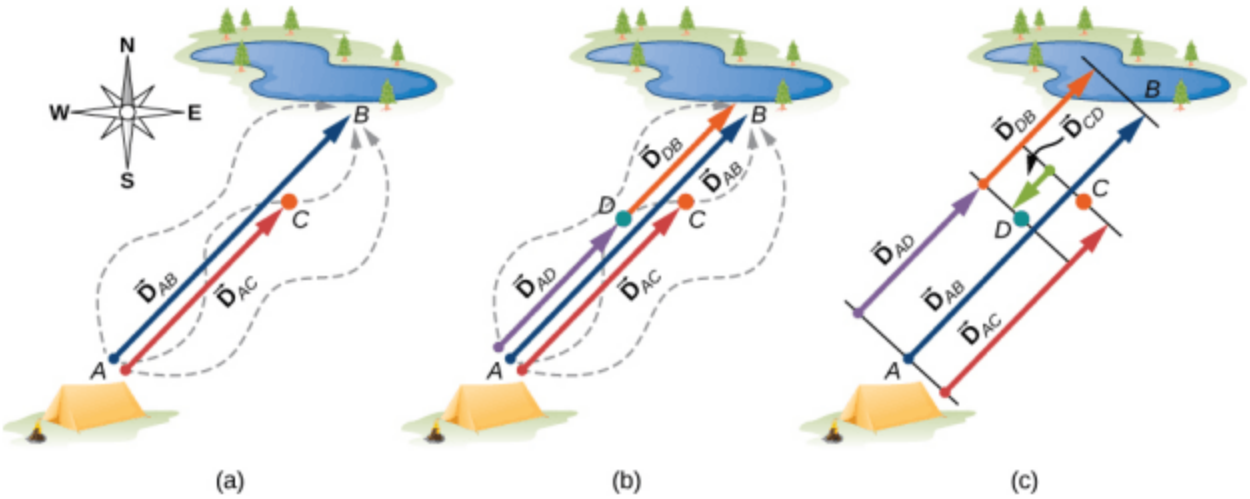


Figure 1.2.5: Displacement vectors for a fishing trip. (a) Stopping to rest at point C while walking from camp (point A) to the pond (point B). (b) Going back for the dropped tackle box (point D). (c) Finishing up at the fishing pond.

Suppose your friend departs from point A (the campsite) and walks in the direction to point B (the fishing pond), but, along the way, stops to rest at some point C located three-quarters of the distance between A and B, beginning from point A (Figure 1.2.5*a*). What is his displacement vector $\vec{D}_{AC}$ when he reaches point C? We know that if he walks all the way to B, his displacement vector relative to A is $\vec{D}_{AB}$, which has magnitude D_{AB} = 6 km and a direction of northeast. If he walks only a 0.75 fraction of the total distance, maintaining the northeasterly direction, at point C he must be 0.75 D_{AB} = 4.5 km away from the campsite at A. So, his displacement vector at the rest point C has magnitude D_{AC} = 4.5 km = 0.75 D_{AB} and is parallel to the displacement vector $\vec{D}_{AB}$. All of this can be stated succinctly in the form of the following **vector equation**:

$$\vec{D}_{AC} = 0.75\ \vec{D}_{AB}.$$

In a vector equation, both sides of the equation are vectors. The previous equation is an example of a vector multiplied by a positive scalar (number) α = 0.75. The result, $\vec{D}_{AC}$, of such a multiplication is a new vector with a direction parallel to the direction of the original vector $\vec{D}_{AB}$. In general, when a vector $\vec{D}_A$ is multiplied by a positive scalar α, the result is a new vector $\vec{D}_B$ that is parallel to $\vec{D}_A$:

$$\vec{B} = \alpha\vec{A} \tag{1.2.1}$$

The magnitude $|\vec{B}|$ of this new vector is obtained by multiplying the magnitude $|\vec{A}|$ of the original vector, as expressed by the **scalar equation**:

$$B = |\alpha|A. \tag{1.2.2}$$

In a scalar equation, both sides of the equation are numbers. Equation 1.2.2 is a scalar equation because the magnitudes of vectors are scalar quantities (and positive numbers). If the scalar α is **negative** in the vector equation Equation 1.2.1, then the magnitude $|\vec{B}|$ of the new vector is still given by Equation 1.2.2, but the direction of the new vector $\vec{B}$ is **antiparallel** to the direction of $\vec{A}$. These principles are illustrated in Figure 1.2.6*a* by two examples where the length of vector $\vec{A}$ is 1.5 units. When α = 2, the new vector $\vec{B} = 2\vec{A}$ has length B = 2A = 3.0 units (twice as long as the original vector) and is parallel to the original vector. When α = –2, the new vector $\vec{C} = -2\vec{A}$ has length C = |–2| A = 3.0 units (twice as long as the original vector) and is antiparallel to the original vector.

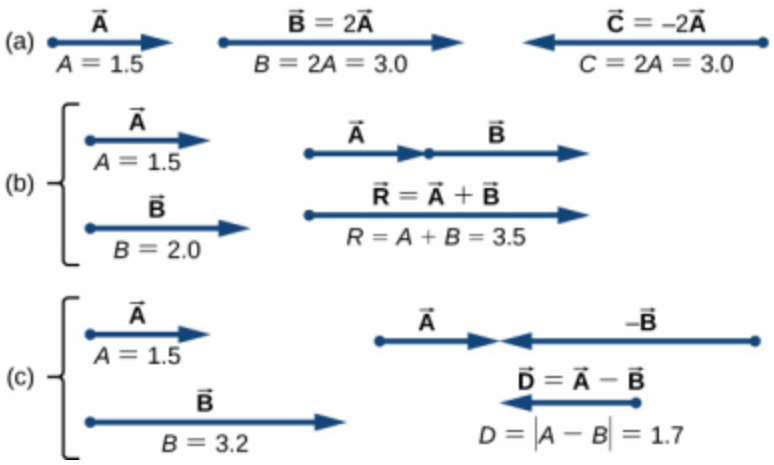


Figure 1.2.6: Algebra of vectors in one dimension. (a) Multiplication by a scalar. (b) Addition of two vectors ($\vec{R}$ is called the resultant of vectors ($\vec{A}$ and ($\vec{B}$). (c) Subtraction of two vectors ($\vec{D}$ is the difference of vectors ($\vec{A}$ and $\vec{B}$).

Now suppose your fishing buddy departs from point A (the campsite), walking in the direction to point B (the fishing hole), but he realizes he lost his tackle box when he stopped to rest at point C (located three-quarters of the distance between A and B, beginning from point A). So, he turns back and retraces his steps in the direction toward the campsite and finds the box lying on the path at some point D only 1.2 km away from point C (see Figure 1.2.5*b*). What is his displacement vector $\vec{D}_{AD}$ when he finds the box at point D? What is his displacement vector $\vec{D}_{DB}$ from point D to the hole? We have already established that at rest point C his displacement vector is $\vec{D}_{AC}$ = 0.75 $\vec{D}_{AB}$. Starting at point C, he walks southwest (toward the campsite), which means his new displacement vector $\vec{D}_{CD}$ from point C to point D is antiparallel to $\vec{D}_{AB}$. Its magnitude $|\vec{D}_{CD}|$ is D_{CD} = 1.2 km = 0.2 D_{AB}, so his second displacement vector is $\vec{D}_{CD}$ = –0.2 $\vec{D}_{AB}$. His total displacement $\vec{D}_{AD}$ relative to the campsite is the vector sum of the two displacement vectors: vector $\vec{D}_{AC}$ (from the campsite to the rest point) and vector $\vec{D}_{CD}$ (from the rest point to the point where he finds his box):

$$\vec{D}_{AD} = \vec{D}_{AC} + \vec{D}_{CD}. \tag{1.2.3}$$

The vector sum of two (or more vectors is called the **resultant vector** or, for short, the **resultant**. When the vectors on the right-hand-side of Equation 1.2.3 are known, we can find the resultant $\vec{D}_{AD}$ as follows:

$$\vec{D}_{AD} = \vec{D}_{AC} + \vec{D}_{CD} = 0.75\ \vec{D}_{AB} - 0.2\ \vec{D}_{AB} = (0.75 - 0.2)\vec{D}_{AB} = 0.55\vec{D}_{AB}. \tag{1.2.4}$$

When your friend finally reaches the pond at B, his displacement vector $\vec{D}_{AB}$ from point A is the vector sum of his displacement vector $\vec{D}_{AD}$ from point A to point D and his displacement vector $\vec{D}_{DB}$ from point D to the fishing hole: $\vec{D}_{AB}$ = $\vec{D}_{AD}$ + $\vec{D}_{DB}$ (see Figure 1.2.5*c*). This means his displacement vector $\vec{D}_{DB}$ is the difference of two vectors:

$$\vec{D}_{DB} = \vec{D}_{AB} - \vec{D}_{AD} = \vec{D}_{AB} + (-\vec{D}_{AD}). \tag{1.2.5}$$

Notice that a difference of two vectors is nothing more than a vector sum of two vectors because the second term in Equation 1.2.5 is vector $-\vec{D}_{AD}$ (which is antiparallel to $\vec{D}_{AD}$). When we substitute Equation 1.2.4 into Equation 1.2.5, we obtain the second displacement vector:

$$\vec{D}_{DB} = \vec{D}_{AB} - \vec{D}_{AD} = \vec{D}_{AB} - 0.55\ \vec{D}_{AB} = (1.0 - 0.55)\ \vec{D}_{AB} = 0.45\ \vec{D}_{AB}. \tag{1.2.6}$$

This result means your friend walked D_{DB} = 0.45 D_{AB} = 0.45(6.0 km) = 2.7 km from the point where he finds his tackle box to the fishing hole.

When vectors $\vec{A}$ and $\vec{B}$ lie along a line (that is, in one dimension), such as in the camping example, their resultant $\vec{R}$ = $\vec{A}$ + $\vec{B}$ and their difference $\vec{D}$ = $\vec{A}$ – $\vec{B}$ both lie along the same direction. We can illustrate the addition or subtraction of vectors by drawing the corresponding vectors to scale in one dimension, as shown in Figure 1.2.6.

To illustrate the resultant when $\vec{A}$ and $\vec{B}$ are two parallel vectors, we draw them along one line by placing the origin of one vector at the end of the other vector in head-to-tail fashion (see Figure ({1{.}2{.}\eatSpaces{6b} \endSpaces}\)). The magnitude of this

resultant is the sum of their magnitudes: R = A + B. The direction of the resultant is parallel to both vectors. When vector $\vec{A}$ is antiparallel to vector $\vec{B}$, we draw them along one line in either head-to-head fashion (Figure ({1{.}2{.}\eatSpaces{6c} \endSpaces}\)) or tail-to-tail fashion. The magnitude of the vector difference, then, is the **absolute value** D = |A – B| of the difference of their magnitudes. The direction of the difference vector $\vec{D}$ is parallel to the direction of the longer vector.

In general, in one dimension—as well as in higher dimensions, such as in a plane or in space—we can add any number of vectors and we can do so in any order because the addition of vectors is **commutative**,

$$\vec{A} + \vec{B} = \vec{B} + \vec{A}. \tag{1.2.7}$$

and **associative,**

$$(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C}). \tag{1.2.8}$$

Moreover, multiplication by a scalar is **distributive**:

$$\alpha_1\vec{A} + \alpha_2\vec{A} = (\alpha_1 + \alpha_2)\vec{A}. \tag{1.2.9}$$

We used the distributive property in Equation 1.2.4 and Equation 1.2.6.

When adding many vectors in one dimension, it is convenient to use the concept of a **unit vector**. A unit vector, which is denoted by a letter symbol with a hat, such as $\hat{u}$, has a magnitude of one and does not have any physical unit so that $|\hat{u}|$ ≡ u = 1. The only role of a unit vector is to specify direction. For example, instead of saying vector $\vec{D}_{AB}$ has a magnitude of 6.0 km and a direction of northeast, we can introduce a unit vector $\hat{u}$ that points to the northeast and say succinctly that $\vec{D}_{AB}$ = (6.0 km) $\hat{u}$. Then the southwesterly direction is simply given by the unit vector$-\hat{u}$. In this way, the displacement of 6.0 km in the southwesterly direction is expressed by the vector

$$\vec{D}_{BA} = (-6.0\ km)\ \hat{u}.$$

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1.3: Scalars and Vectors (Part 2)

Example 1.3.1: A Ladybug Walker

A long measuring stick rests against a wall in a physics laboratory with its 200-cm end at the floor. A ladybug lands on the 100-cm mark and crawls randomly along the stick. It first walks 15 cm toward the floor, then it walks 56 cm toward the wall, then it walks 3 cm toward the floor again. Then, after a brief stop, it continues for 25 cm toward the floor and then, again, it crawls up 19 cm toward the wall before coming to a complete rest (Figure 1.3.1). Find the vector of its total displacement and its final resting position on the stick.

Strategy

If we choose the direction along the stick toward the floor as the direction of unit vector $\hat{u}$, then the direction toward the floor is $+\hat{u}$ and the direction toward the wall is $-\hat{u}$. The ladybug makes a total of five displacements:

$$\vec{D}_1 = (15\ cm)(+\hat{u}),$$
$$\vec{D}_2 = (56\ cm)(-\hat{u}),$$
$$\vec{D}_3 = (3\ cm)(+\hat{u}),$$
$$\vec{D}_4 = (25\ cm)(+\hat{u}),\ and$$
$$\vec{D}_5 = (19\ cm)(-\hat{u}).$$

The total displacement $\vec{D}$ is the resultant of all its displacement vectors.

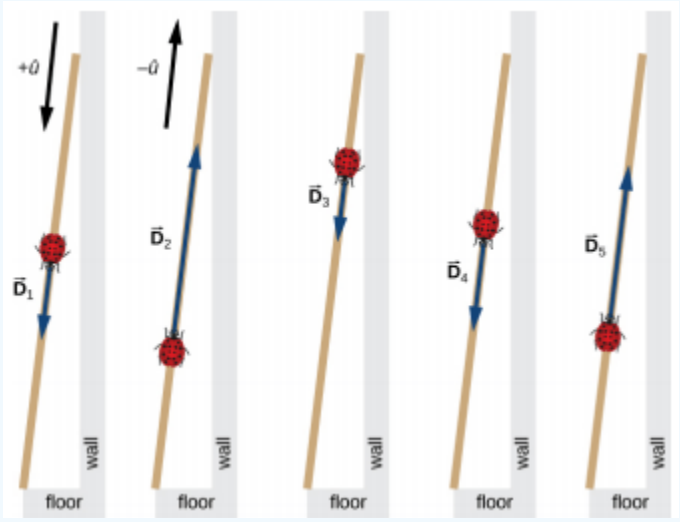


Figure 1.3.1: Five displacements of the ladybug. Note that in this schematic drawing, magnitudes of displacements are not drawn to scale. (credit: modification of work by "Persian Poet Gal"/Wikimedia Commons)

Solution

The resultant of all the displacement vectors is

$$\begin{aligned}\vec{D} &= \vec{D}_1 + \vec{D}_2 + \vec{D}_3 + \vec{D}_4 + \vec{D}_5 \\ &= (15\ cm)(+\hat{u}) + (56\ cm)(-\hat{u}) + (3\ cm)(+\hat{u}) + (25\ cm)(+\hat{u}) + (19\ cm)(-\hat{u}) \\ &= (15 - 56 + 3 + 25 - 19)cm\ \hat{u} \\ &= -32\ cm\ \hat{u}.\end{aligned}$$

In this calculation, we use the distributive law given by Equation 2.2.9. The result reads that the total displacement vector points away from the 100-cm mark (initial landing site) toward the end of the meter stick that touches the wall. The end that touches the wall is marked 0 cm, so the final position of the ladybug is at the (100 – 32) cm = 68-cm mark.

Exercise 2.2

A cave diver enters a long underwater tunnel. When her displacement with respect to the entry point is 20 m, she accidentally drops her camera, but she doesn't notice it missing until she is some 6 m farther into the tunnel. She swims back 10 m but cannot find the camera, so she decides to end the dive. How far from the entry point is she? Taking the positive direction out of the tunnel, what is her displacement vector relative to the entry point?

Algebra of Vectors in Two Dimensions

When vectors lie in a plane—that is, when they are in two dimensions—they can be multiplied by scalars, added to other vectors, or subtracted from other vectors in accordance with the general laws expressed by Equation 2.2.1, Equation 2..2.2, Equation 2.2.7, and Equation 2.2.8. However, the addition rule for two vectors in a plane becomes more complicated than the rule for vector addition in one dimension. We have to use the laws of geometry to construct resultant vectors, followed by trigonometry to find vector magnitudes and directions. This geometric approach is commonly used in navigation (Figure 1.3.2). In this section, we need to have at hand two rulers, a triangle, a protractor, a pencil, and an eraser for drawing vectors to scale by geometric constructions.



Figure 1.3.2: In navigation, the laws of geometry are used to draw resultant displacements on nautical maps.

For a geometric construction of the sum of two vectors in a plane, we follow the **parallelogram rule**. Suppose two vectors $\vec{A}$ and $\vec{B}$ are at the arbitrary positions shown in Figure 1.3.3. Translate either one of them in parallel to the beginning of the other vector, so that after the translation, both vectors have their origins at the same point. Now, at the end of vector $\vec{A}$ we draw a line parallel to vector $\vec{B}$ and at the end of vector $\vec{B}$ we draw a line parallel to vector $\vec{A}$ (the dashed lines in Figure 1.3.3). In this way, we obtain a parallelogram. From the origin of the two vectors we draw a diagonal that is the resultant $\vec{R}$ of the two vectors: $\vec{R} = \vec{A} + \vec{B}$ (Figure 1.3.3a). The other diagonal of this parallelogram is the vector difference of the two vectors $\vec{D} = \vec{A} - \vec{B}$, as shown in Figure 1.3.3b. Notice that the end of the difference vector is placed at the end of vector $\vec{A}$.

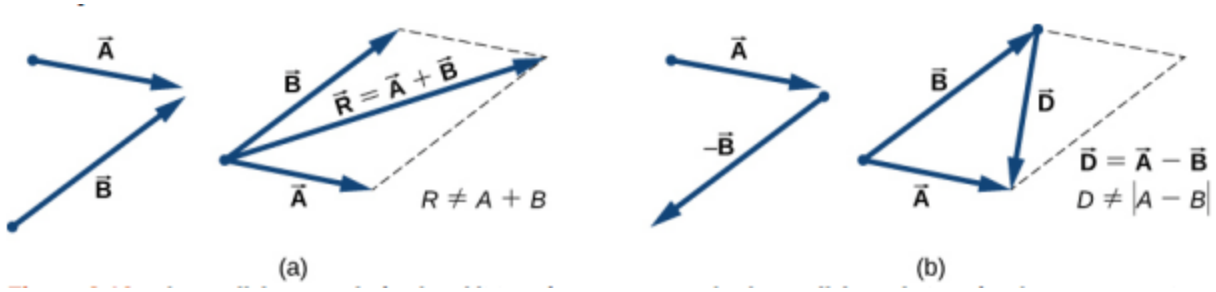


Figure 1.3.3: The parallelogram rule for the addition of two vectors. Make the parallel translation of each vector to a point where their origins (marked by the dot) coincide and construct a parallelogram with two sides on the vectors and the other two sides (indicated by dashed lines) parallel to the vectors. (a) Draw the resultant vector $\vec{R}$ along the diagonal of the parallelogram from the common point to the opposite corner. Length R of the resultant vector is not equal to the sum of the magnitudes of the two vectors. (b) Draw the difference vector $\vec{D} = \vec{A} - \vec{B}$ along the diagonal connecting the ends of the vectors. Place the origin of vector $\vec{D}$ at the end of vector $\vec{B}$ and the end (arrowhead) of vector $\vec{D}$ at the end of vector $\vec{A}$. Length D of the difference vector is not equal to the difference of magnitudes of the two vectors.

It follows from the parallelogram rule that neither the magnitude of the resultant vector nor the magnitude of the difference vector can be expressed as a simple sum or difference of magnitudes A and B, because the length of a diagonal cannot be expressed as a simple sum of side lengths. When using a geometric construction to find magnitudes $|\vec{R}|$ and $|\vec{D}|$, we have to use trigonometry laws for triangles, which may lead to complicated algebra. There are two ways to circumvent this algebraic complexity. One way is to use the method of components, which we examine in the next section. The other way is to draw the vectors to scale, as is done in navigation, and read approximate vector lengths and angles (directions) from the graphs. In this section we examine the second approach.

If we need to add three or more vectors, we repeat the parallelogram rule for the pairs of vectors until we find the resultant of all of the resultants. For three vectors, for example, we first find the resultant of vector 1 and vector 2, and then we find the resultant of this resultant and vector 3. The order in which we select the pairs of vectors does not matter because the operation of vector addition is commutative and associative (see Equation 2.2.7 and Equation 2.2.8). Before we state a general rule that follows from repetitive applications of the parallelogram rule, let's look at the following example.

Suppose you plan a vacation trip in Florida. Departing from Tallahassee, the state capital, you plan to visit your uncle Joe in Jacksonville, see your cousin Vinny in Daytona Beach, stop for a little fun in Orlando, see a circus performance in Tampa, and visit the University of Florida in Gainesville. Your route may be represented by five displacement vectors $\vec{A}$, $\vec{B}$, $\vec{C}$, $\vec{D}$, and $\vec{E}$, which are indicated by the red vectors in Figure 1.3.4. What is your total displacement when you reach Gainesville? The total displacement is the vector sum of all five displacement vectors, which may be found by using the parallelogram rule four times. Alternatively, recall that the displacement vector has its beginning at the initial position (Tallahassee) and its end at the final position (Gainesville), so the total displacement vector can be drawn directly as an arrow connecting Tallahassee with Gainesville (see the green vector in Figure 1.3.4). When we use the parallelogram rule four times, the resultant $\vec{R}$ we obtain is exactly this green vector connecting Tallahassee with Gainesville: $\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D} + \vec{E}$.

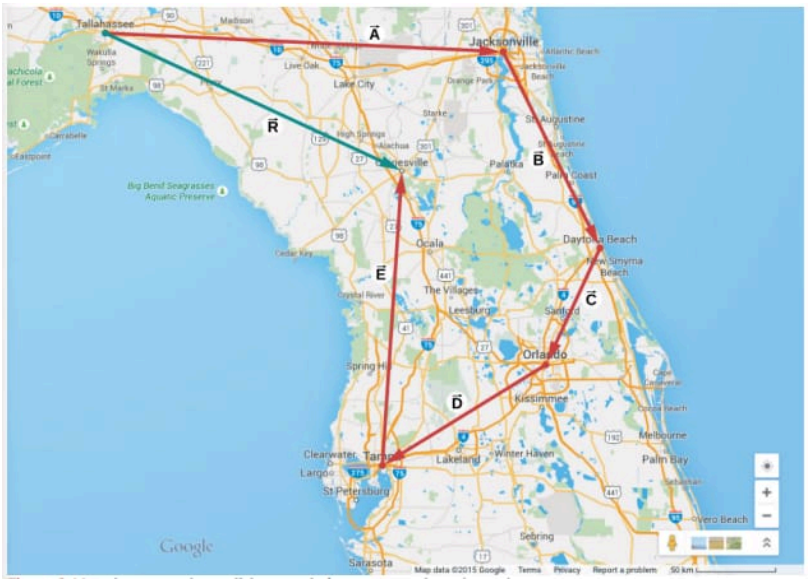


Figure 1.3.4: When we use the parallelogram rule four times, we obtain the resultant vector $\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D} + \vec{E}$, which is the green vector connecting Tallahassee with Gainesville.

Drawing the resultant vector of many vectors can be generalized by using the following tail-to-head geometric construction. Suppose we want to draw the resultant vector $\vec{R}$ of four vectors $\vec{A}$, $\vec{B}$, $\vec{C}$, and $\vec{D}$ (Figure 1.3.5a). We select any one of the vectors as the first vector and make a parallel translation of a second vector to a position where the origin ("tail") of the second vector coincides with the end ("head") of the first vector. Then, we select a third vector and make a parallel translation of the third vector to a position where the origin of the third vector coincides with the end of the second vector. We repeat this procedure until all the vectors are in a head-to-tail arrangement like the one shown in Figure 1.3.5. We draw the resultant vector $\vec{R}$ by connecting the origin ("tail") of the first vector with the end ("head") of the last vector. The end of the resultant vector is at the end of the last vector. Because the addition of vectors is associative and commutative, we obtain the same resultant vector regardless of which vector we choose to be first, second, third, or fourth in this construction.

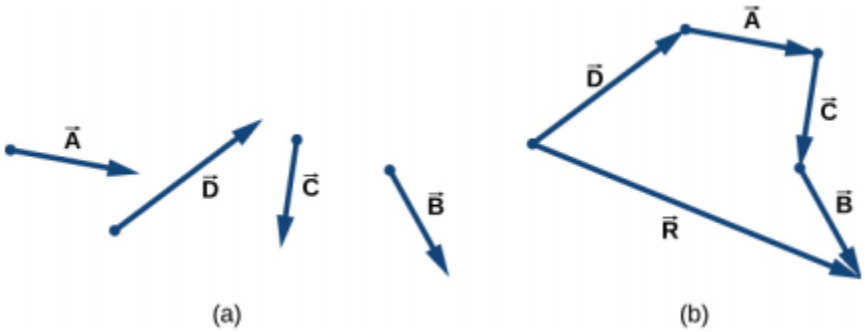


Figure 1.3.5: Tail-to-head method for drawing the resultant vector $\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$. (a) Four vectors of different magnitudes and directions. (b) Vectors in (a) are translated to new positions where the origin ("tail") of one vector is at the end ("head") of another vector. The resultant vector is drawn from the origin ("tail") of the first vector to the end ("head") of the last vector in this arrangement.

Example 1.3.2: Geometric Construction of the Resultant

The three displacement vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ in Figure 1.3.6 are specified by their magnitudes A = 10.0, B = 7.0, and C = 8.0, respectively, and by their respective direction angles with the horizontal direction α = 35°, β = –110°, and γ = 30°. The physical units of the magnitudes are centimeters. Choose a convenient scale and use a ruler and a protractor to find the following vector sums: (a) $\vec{R} = \vec{A} + \vec{B}$, (b) $\vec{D} = \vec{A} - \vec{B}$, and (c) $\vec{S} = \vec{A} - 3\vec{B} + \vec{C}$.

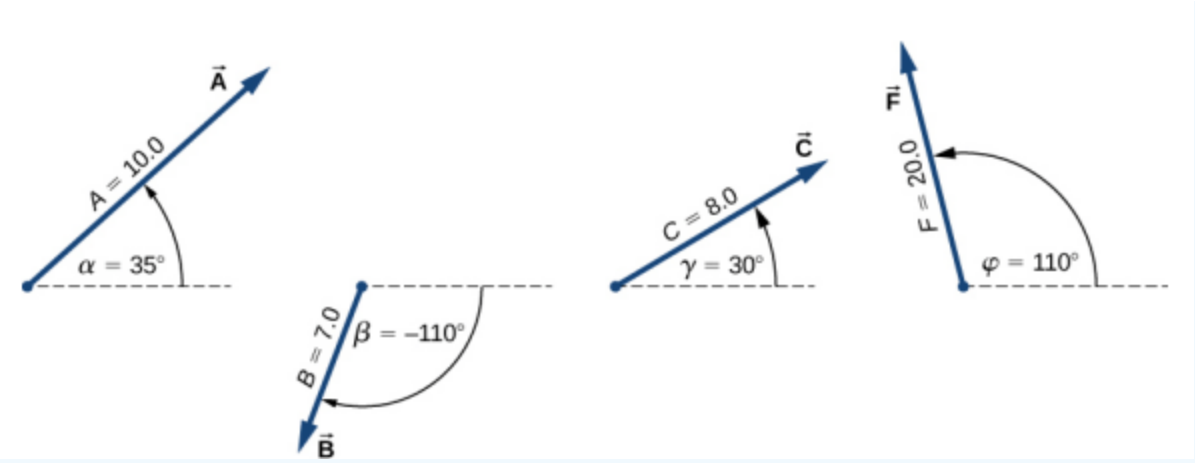


Figure 1.3.6: Vectors used in Example 1.3.2 and in the Exercise feature that follows.

Strategy

In geometric construction, to find a vector means to find its magnitude and its direction angle with the horizontal direction. The strategy is to draw to scale the vectors that appear on the right-hand side of the equation and construct the resultant vector. Then, use a ruler and a protractor to read the magnitude of the resultant and the direction angle. For parts (a) and (b) we use the parallelogram rule. For (c) we use the tail-to-head method.

Solution

For parts (a) and (b), we attach the origin of vector $\vec{B}$ to the origin of vector $\vec{A}$, as shown in Figure 1.3.7, and construct a parallelogram. The shorter diagonal of this parallelogram is the sum $\vec{A}$ + $\vec{B}$. The longer of the diagonals is the difference $\vec{A}$ – $\vec{B}$. We use a ruler to measure the lengths of the diagonals, and a protractor to measure the angles with the horizontal. For the resultant $\vec{R}$, we obtain R = 5.8 cm and θ_R ≈ 0°. For the difference $\vec{D}$, we obtain D = 16.2 cm and θ_D = 49.3°, which are shown in Figure 1.3.7.

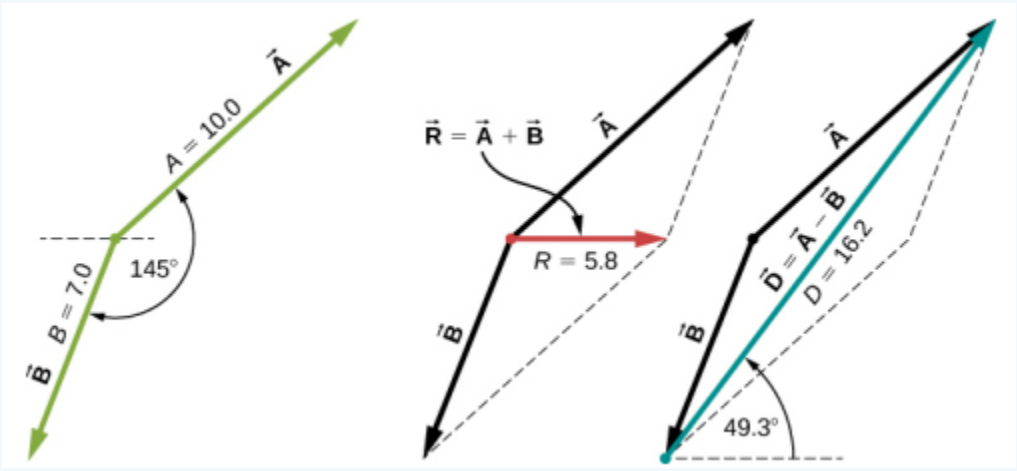


Figure 1.3.7: Using the parallelogram rule to solve (a) (finding the resultant, red) and (b) (finding the difference, blue).

For (c), we can start with vector –3 $\vec{B}$ and draw the remaining vectors tail-to-head as shown in Figure 1.3.8. In vector addition, the order in which we draw the vectors is unimportant, but drawing the vectors to scale is very important. Next, we draw vector $\vec{S}$ from the origin of the first vector to the end of the last vector and place the arrowhead at the end of $\vec{S}$. We use a ruler to measure the length of $\vec{S}$, and find that its magnitude is S = 36.9 cm. We use a protractor and find that its direction angle is θ_S = 52.9°. This solution is shown in Figure 1.3.8.

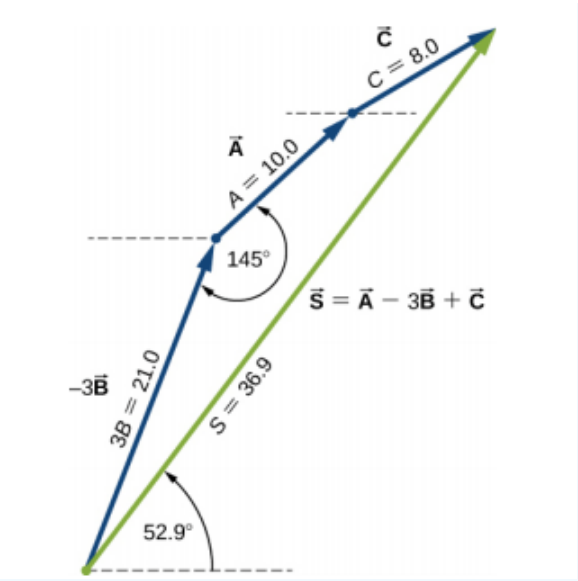


Figure 1.3.8: Using the tail-to-head method to solve (c) (finding vector $\vec{S}$, green).

Exercise 2.3

Using the three displacement vectors $\vec{A}$, $\vec{B}$, and $\vec{F}$ in Figure 1.3.6, choose a convenient scale, and use a ruler and a protractor to find vector $\vec{G}$ given by the vector equation $\vec{G} = \vec{A} + 2\vec{B} - \vec{F}$.

Simulation

Observe the addition of vectors in a plane by visiting this vector calculator and this PhET simulation.

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1.4: Coordinate Systems and Components of a Vector (Part 1)

Learning Objectives

- Describe vectors in two and three dimensions in terms of their components, using unit vectors along the axes.
- Distinguish between the vector components of a vector and the scalar components of a vector.
- Explain how the magnitude of a vector is defined in terms of the components of a vector.
- Identify the direction angle of a vector in a plane.
- Explain the connection between polar coordinates and Cartesian coordinates in a plane.

Vectors are usually described in terms of their components in a coordinate system. Even in everyday life we naturally invoke the concept of orthogonal projections in a rectangular coordinate system. For example, if you ask someone for directions to a particular location, you will more likely be told to go 40 km east and 30 km north than 50 km in the direction 37° north of east.

In a rectangular (Cartesian) xy-coordinate system in a plane, a point in a plane is described by a pair of coordinates (x, y). In a similar fashion, a vector $\vec{A}$ in a plane is described by a pair of its vector coordinates. The x-coordinate of vector $\vec{A}$ is called its x-component and the y-coordinate of vector $\vec{A}$ is called its y-component. The vector x-component is a vector denoted by $\vec{A}_x$. The vector y-component is a vector denoted by $\vec{A}_y$. In the Cartesian system, the x and y **vector components** of a vector are the orthogonal projections of this vector onto the x- and y-axes, respectively. In this way, following the parallelogram rule for vector addition, each vector on a Cartesian plane can be expressed as the vector sum of its vector components:

$$\vec{A} = \vec{A}_x + \vec{A}_y. \tag{1.4.1}$$

As illustrated in Figure 1.4.1, vector $\vec{A}$ is the diagonal of the rectangle where the x-component $\vec{A}_x$ is the side parallel to the x-axis and the y-component $\vec{A}_y$ is the side parallel to the y-axis. Vector component $\vec{A}_x$ is orthogonal to vector component $\vec{A}_y$.

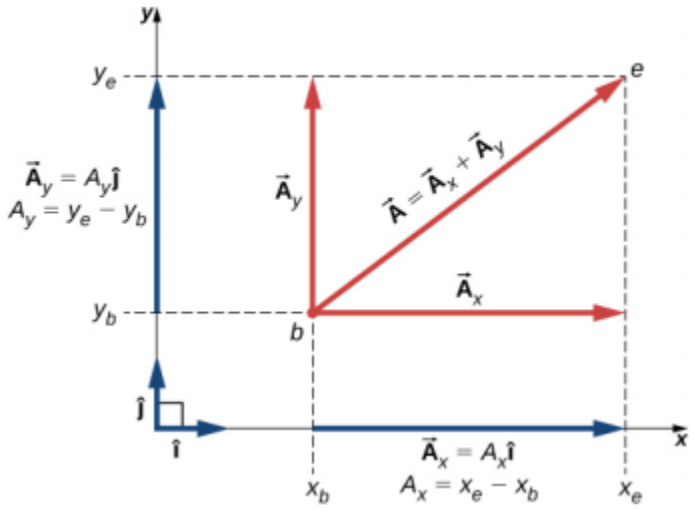


Figure 1.4.1: Vector $\vec{A}$ in a plane in the Cartesian coordinate system is the vector sum of its vector x- and y-components. The x-vector component $\vec{A}_x$ is the orthogonal projection of vector $\vec{A}$ onto the x-axis. The y-vector component $\vec{A}_y$ is the orthogonal projection of vector $\vec{A}$ onto the y-axis. The numbers A_x and A_y that multiply the unit vectors are the scalar components of the vector.

It is customary to denote the positive direction on the x-axis by the unit vector $\hat{i}$ and the positive direction on the y-axis by the unit vector $\hat{j}$. Unit vectors of the axes, $\hat{i}$ and $\hat{j}$, define two orthogonal directions in the plane. As shown in Figure 1.4.1, the x- and y- components of a vector can now be written in terms of the unit vectors of the axes:

$$\begin{cases} \vec{A}_x = A_x\hat{i} \\ \vec{A}_y = A_y\hat{j} \end{cases} \tag{1.4.2}$$

The vectors $\vec{A}_x$ and $\vec{A}_y$ defined by Equation 2.11 are the vector components of vector $\vec{A}$. The numbers A_x and A_y that define the vector components in Equation 1.4.2 are the **scalar components** of vector $\vec{A}$. Combining Equation 1.4.1 with Equation 1.4.2, we obtain **the component form of a vector**:

$$\vec{A} = A_x\hat{i} + A_y\hat{j}. \tag{1.4.3}$$

If we know the coordinates $b(x_b, y_b)$ of the origin point of a vector (where b stands for "beginning") and the coordinates e(x_e, y_e) of the end point of a vector (where e stands for "end"), we can obtain the scalar components of a vector simply by subtracting the origin point coordinates from the end point coordinates:

$$\begin{cases} A_x = x_e - x_b \\ A_y = y_e - y_b. \end{cases} \tag{1.4.4}$$

Example 1.4.1: Displacement of a Mouse Pointer

A mouse pointer on the display monitor of a computer at its initial position is at point (6.0 cm, 1.6 cm) with respect to the lower left-side corner. If you move the pointer to an icon located at point (2.0 cm, 4.5 cm), what is the displacement vector of the pointer?

Strategy

The origin of the xy-coordinate system is the lower left-side corner of the computer monitor. Therefore, the unit vector $\hat{i}$ on the x-axis points horizontally to the right and the unit vector $\hat{j}$ on the y-axis points vertically upward. The origin of the displacement vector is located at point b(6.0, 1.6) and the end of the displacement vector is located at point e(2.0, 4.5). Substitute the coordinates of these points into Equation 1.4.4 to find the scalar components D_x and D_y of the displacement vector $\vec{D}$. Finally, substitute the coordinates into Equation 1.4.3 to write the displacement vector in the vector component form.

Solution

We identify x_b = 6.0, x_e = 2.0, y_b = 1.6, and y_e = 4.5, where the physical unit is 1 cm. The scalar x- and y-components of the displacement vector are

$$D_x = x_e - x_b = (2.0 - 6.0)\ cm = -4.0\ cm, \tag{1.4.5}$$

$$D_y = y_e - y_b = (4.5 - 1.6)\ cm = +2.9\ cm. \tag{1.4.6}$$

The vector component form of the displacement vector is

$$\vec{D} = D_x\,\hat{i} + D_y\,\hat{j} = (-4.0\ cm)\,\hat{i} + (2.9\ cm)\,\hat{j} = (-4.0\,\hat{i} + 2.9\,\hat{j})\ cm. \tag{1.4.7}$$

This solution is shown in Figure 1.4.2.

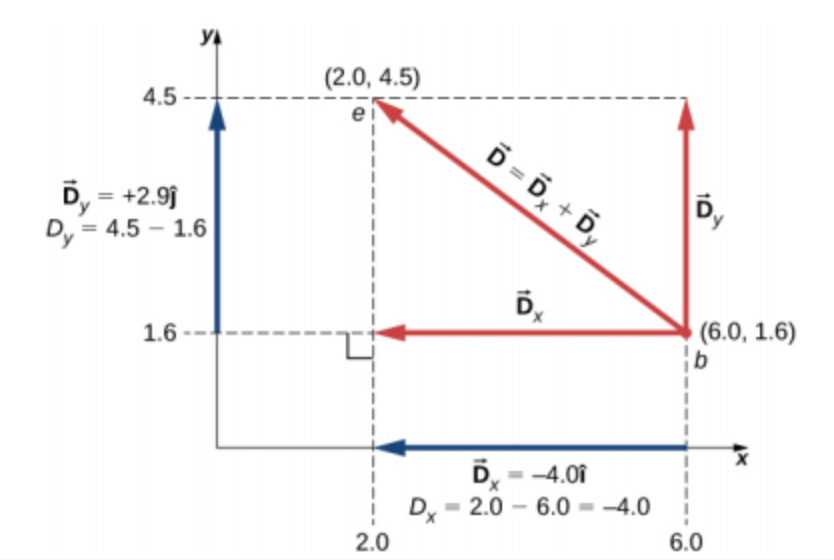


Figure 1.4.2: The graph of the displacement vector. The vector points from the origin point at b to the end point at e.

Significance

Notice that the physical unit—here, 1 cm—can be placed either with each component immediately before the unit vector or globally for both components, as in Equation 1.4.7. Often, the latter way is more convenient because it is simpler.

The vector x-component $\vec{D}_x$ = −4.0 $\hat{i}$ = 4.0$(-\hat{i})$ of the displacement vector has the magnitude $|\vec{D}_x|$ = |– 4.0||$\hat{i}$| = 4.0 because the magnitude of the unit vector is |$\hat{i}$| = 1. Notice, too, that the direction of the x-component is $-\hat{i}$, which is antiparallel to the direction of the +x-axis; hence, the x-component vector $\vec{D}_x$ points to the left, as shown in Figure 1.4.2. The scalar x-component of vector $\vec{D}$ is D_x = −4.0. Similarly, the vector y-component $\vec{D}_y = +2.9\hat{j}$ of the displacement vector has magnitude $|\vec{D}_y|$ = |2.9|| $\hat{j}$| = 2.9 because the magnitude of the unit vector is |$\hat{j}$| = 1. The direction of the y-component is $+\hat{j}$, which is parallel to the direction of the +y-axis. Therefore, the y-component vector $\vec{D}_y$ points up, as seen in Figure 1.4.2. The scalar y-component of vector $\vec{D}$ is D_y = + 2.9. The displacement vector $\vec{D}$ is the resultant of its two vector components.

The vector component form of the displacement vector Equation 1.4.7 tells us that the mouse pointer has been moved on the monitor 4.0 cm to the left and 2.9 cm upward from its initial position.

Exercise 2.4

A blue fly lands on a sheet of graph paper at a point located 10.0 cm to the right of its left edge and 8.0 cm above its bottom edge and walks slowly to a point located 5.0 cm from the left edge and 5.0 cm from the bottom edge. Choose the rectangular coordinate system with the origin at the lower left-side corner of the paper and find the displacement vector of the fly. Illustrate your solution by graphing.

When we know the scalar components A_x and A_y of a vector $\vec{A}$, we can find its magnitude A and its direction angle θ_A. The **direction angle**—or direction, for short—is the angle the vector forms with the positive direction on the x-axis. The angle θ_A is measured in the counterclockwise direction from the +x-axis to the vector (Figure 1.4.3). Because the lengths A, A_x, and A_y form a right triangle, they are related by the Pythagorean theorem:

$$A^2 = A_x^2 + A_y^2 \Leftrightarrow A = \sqrt{A_x^2 + A_y^2}. \tag{1.4.8}$$

This equation works even if the scalar components of a vector are negative. The direction angle θ_A of a vector is defined via the tangent function of angle θ_A in the triangle shown in Figure 1.4.3:

$$\tan\theta = \frac{A_y}{A_x} \Rightarrow \theta = \tan^{-1}\left(\frac{A_y}{A_x}\right). \tag{1.4.9}$$

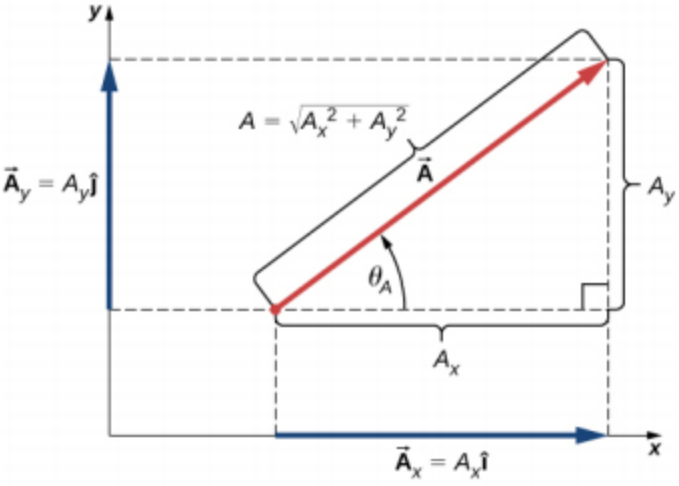


Figure 1.4.3: For vector $\vec{A}$, its magnitude A and its direction angle θ_A are related to the magnitudes of its scalar components because A, A_x, and A_y form a right triangle.

When the vector lies either in the first quadrant or in the fourth quadrant, where component A_x is positive (Figure 1.4.4), the angle θ in Equation 1.4.9) is identical to the direction angle θ_A. For vectors in the fourth quadrant, angle θ is negative, which means that for these vectors, direction angle θ_A is measured clockwise from the positive x-axis. Similarly, for vectors in the second quadrant, angle θ is negative. When the vector lies in either the second or third quadrant, where component A_x is negative, the direction angle is $\theta_A = \theta + 180°$ (Figure 1.4.4).

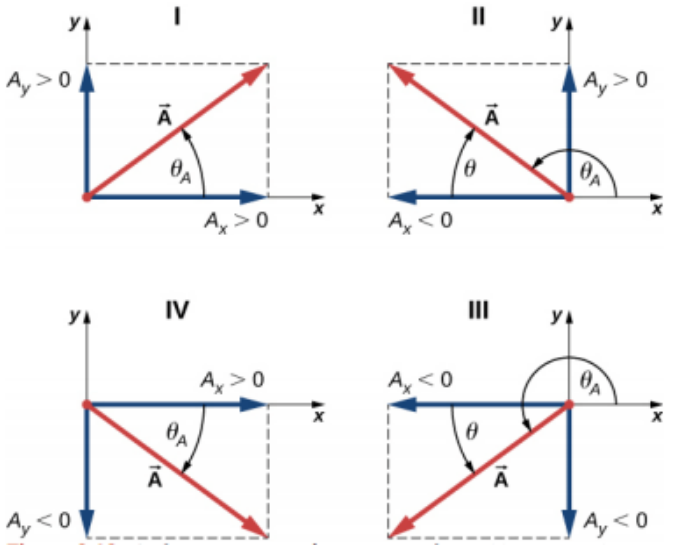


Figure 1.4.4: Scalar components of a vector may be positive or negative. Vectors in the first quadrant (I) have both scalar components positive and vectors in the third quadrant have both scalar components negative. For vectors in quadrants II and III, the direction angle of a vector is $\theta_A = \theta + 180°$.

Example 1.4.2: Magnitude and Direction of the Displacement Vector

You move a mouse pointer on the display monitor from its initial position at point (6.0 cm, 1.6 cm) to an icon located at point (2.0 cm, 4.5 cm). What is the magnitude and direction of the displacement vector of the pointer?

Strategy

In Example 1.4.1, we found the displacement vector $\vec{D}$ of the mouse pointer (see Equation 1.4.7). We identify its scalar components D_x = −4.0 cm and D_y = + 2.9 cm and substitute into Equation 1.4.8 and Equation 1.4.9 to find the magnitude D and direction θ_D, respectively.

Solution

The magnitude of vector $\vec{D}$ is

$$D = \sqrt{D_x^2 + D_y^2} = \sqrt{(-4.0\ cm)^2 + (2.9\ cm)^2} = \sqrt{(4.0)^2 + (2.9)^2}\ cm = 4.9\ cm. \tag{1.4.10}$$

The direction angle is

$$\tan\theta = \frac{D_y}{D_x} = \frac{+2.9\ cm}{-4.0\ cm} = -0.725 \Rightarrow \theta = \tan^{-1}(-0.725) = -35.9^o. \tag{1.4.11}$$

Vector $\vec{D}$ lies in the second quadrant, so its direction angle is

$$\theta_D = \theta + 180^o = -35.9^o + 180^o = 144.1^o. \tag{1.4.12}$$

Exercise 2.5

If the displacement vector of a blue fly walking on a sheet of graph paper is $\vec{D} = (-5.00\ \hat{i} - 3.00\ \hat{j})$ cm, find its magnitude and direction.

In many applications, the magnitudes and directions of vector quantities are known and we need to find the resultant of many vectors. For example, imagine 400 cars moving on the Golden Gate Bridge in San Francisco in a strong wind. Each car gives the bridge a different push in various directions and we would like to know how big the resultant push can possibly be. We have already gained some experience with the geometric construction of vector sums, so we know the task of finding the resultant by drawing the vectors and measuring their lengths and angles may become intractable pretty quickly, leading to huge errors. Worries like this do not appear when we use analytical methods. The very first step in an analytical approach is to find vector components when the direction and magnitude of a vector are known.

Let us return to the right triangle in Figure 1.4.3. The quotient of the adjacent side A_x to the hypotenuse A is the cosine function of direction angle θ_A, $A_x/A = \cos\theta_A$, and the quotient of the opposite side A_y to the hypotenuse A is the sine function of θ_A, $A_y/A = \sin\theta_A$. When magnitude A and direction θ_A are known, we can solve these relations for the scalar components:

$$\begin{cases} A_x = A\cos\theta_A \\ A_y = A\sin\theta_A. \end{cases} \tag{1.4.13}$$

When calculating vector components with Equation 1.4.13, care must be taken with the angle. The direction angle θ_A of a vector is the angle measured **counterclockwise** from the positive direction on the x-axis to the vector. The clockwise measurement gives a negative angle.

Example 1.4.3: Components of Displacement Vectors

A rescue party for a missing child follows a search dog named Trooper. Trooper wanders a lot and makes many trial sniffs along many different paths. Trooper eventually finds the child and the story has a happy ending, but his displacements on various legs seem to be truly convoluted. On one of the legs he walks 200.0 m southeast, then he runs north some 300.0 m. On the third leg, he examines the scents carefully for 50.0 m in the direction 30° west of north. On the fourth leg, Trooper goes directly south for 80.0 m, picks up a fresh scent and turns 23° west of south for 150.0 m. Find the scalar components of Trooper's displacement vectors and his displacement vectors in vector component form for each leg.

Strategy

Let's adopt a rectangular coordinate system with the positive x-axis in the direction of geographic east, with the positive y-direction pointed to geographic north. Explicitly, the unit vector $\hat{i}$ of the x-axis points east and the unit vector $\hat{j}$ of the y-axis points north. Trooper makes five legs, so there are five displacement vectors. We start by identifying their magnitudes and direction angles, then we use Equation 1.4.13 to find the scalar components of the displacements and Equation 1.4.3 for the displacement vectors.

Solution

On the first leg, the displacement magnitude is L_1 = 200.0 m and the direction is southeast. For direction angle θ_1 we can take either 45° measured clockwise from the east direction or 45° + 270° measured counterclockwise from the east direction. With the first choice, θ_1 = −45°. With the second choice, θ_1 = + 315°. We can use either one of these two angles. The components are

$$L_{1x} = L_1 \cos\theta_1 = (200.0\ m)\cos 315^o = 141.4\ m, \tag{1.4.14}$$

$$L_{1y} = L_1 \sin\theta_1 = (200.0\ m)\sin 315^o = -141.4\ m, \tag{1.4.15}$$

The displacement vector of the first leg is

$$\vec{L}_1 = L_{1x}\,\hat{i} + L_{1y}\,\hat{j} = (14.4\,\hat{i} - 141.4\,\hat{j})\ m. \tag{1.4.16}$$

On the second leg of Trooper's wanderings, the magnitude of the displacement is L_2 = 300.0 m and the direction is north. The direction angle is θ_2 = + 90°. We obtain the following results:

$$L_{2x} = L_2 \cos\theta_2 = (300.0\ m)\cos 90^o = 0.0, \tag{1.4.17}$$

$$L_{2y} = L_2 \sin\theta_2 = (300.0\ m)\sin 90^o = 300.0\ m, \tag{1.4.18}$$

$$\vec{L}_2 = L_{2x}\,\hat{i} + L_{2y}\,\hat{j} = (300.0\ m)\,\hat{j}. \tag{1.4.19}$$

On the third leg, the displacement magnitude is L_3 = 50.0 m and the direction is 30° west of north. The direction angle measured counterclockwise from the eastern direction is θ_3 = 30° + 90° = + 120°. This gives the following answers:

$$L_{3x} = L_3 \cos\theta_3 = (50.0\ m)\cos 120^o = -25.0\ m, \tag{1.4.20}$$

$$L_{3y} = L_3 \sin\theta_3 = (50.0\ m)\sin 120^o = +43.3\ m, \tag{1.4.21}$$

$$\vec{L}_3 = L_{3x}\,\hat{i} + L_{3y}\,\hat{j} = (-25.0\,\hat{i} + 43.3\,\hat{j})\ m. \tag{1.4.22}$$

On the fourth leg of the excursion, the displacement magnitude is L_4 = 80.0 m and the direction is south. The direction angle can be taken as either θ_4 = −90° or \(\theta_{4} = + 270°. We obtain

$$L_{4x} = L_4 \cos\theta_4 = (80.0\ m)\cos(-90^o) = 0, \tag{1.4.23}$$

$$L_{4y} = L_4 \sin\theta_4 = (80.0\ m)\sin(-90^o) = -80.0\ m, \tag{1.4.24}$$

$$\vec{L}_4 = L_{4x}\,\hat{i} + L_{4y}\,\hat{j} = (-80.0\ m)\,\hat{j}. \tag{1.4.25}$$

On the last leg, the magnitude is L_5 = 150.0 m and the angle is θ_5 = −23° + 270° = + 247° (23° west of south), which gives

$$L_{5x} = L_5 \cos\theta_5 = (150.0\ m)\cos 247^o = -58.6\ m, \tag{1.4.26}$$

$$L_{5y} = L_5 \sin\theta_5 = (150.0\ m)\sin 247^o = -138.1\ m, \tag{1.4.27}$$

$$\vec{L}_5 = L_{5x}\,\hat{i} + L_{5y}\,\hat{j} = (-58.6\,\hat{i} - 138.1\,\hat{j})\ m. \tag{1.4.28}$$

Exercise 2.6

If Trooper runs 20 m west before taking a rest, what is his displacement vector?

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1.5: Coordinate Systems and Components of a Vector (Part 2)

Polar Coordinates

To describe locations of points or vectors in a plane, we need two orthogonal directions. In the Cartesian coordinate system these directions are given by unit vectors $\hat{i}$ and $\hat{j}$ along the x-axis and the y-axis, respectively. The Cartesian coordinate system is very convenient to use in describing displacements and velocities of objects and the forces acting on them. However, it becomes cumbersome when we need to describe the rotation of objects. When describing rotation, we usually work in the **polar coordinate system**.

In the polar coordinate system, the location of point P in a plane is given by two **polar coordinates** (Figure 1.5.1). The first polar coordinate is the **radial coordinate** r, which is the distance of point P from the origin. The second polar coordinate is an angle φ that the radial vector makes with some chosen direction, usually the positive x-direction. In polar coordinates, angles are measured in radians, or rads. The radial vector is attached at the origin and points away from the origin to point P. This radial direction is described by a unit radial vector $\hat{r}$. The second unit vector $\hat{t}$ is a vector orthogonal to the radial direction $\hat{r}$. The positive + $\hat{t}$ direction indicates how the angle φ changes in the counterclockwise direction. In this way, a point P that has coordinates (x, y) in the rectangular system can be described equivalently in the polar coordinate system by the two polar coordinates (r, φ). Equation 2.4.13 is valid for any vector, so we can use it to express the x- and y-coordinates of vector $\vec{r}$. In this way, we obtain the connection between the polar coordinates and rectangular coordinates of point P:

$$\begin{cases} x = r\cos\varphi \\ y = r\sin\varphi \end{cases}. \tag{1.5.1}$$

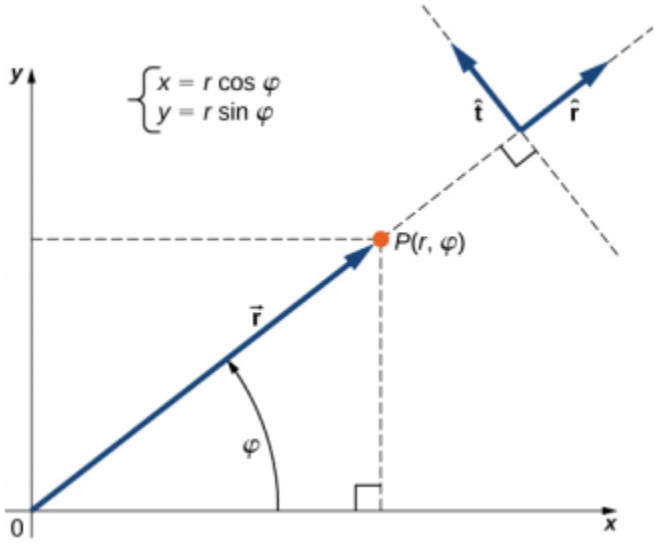


Figure 1.5.1: Using polar coordinates, the unit vector $\hat{r}$ defines the positive direction along the radius r (radial direction) and, orthogonal to it, the unit vector $\hat{t}$ defines the positive direction of rotation by the angle φ.

Example 1.5.1: Polar Coordinates

A treasure hunter finds one silver coin at a location 20.0 m away from a dry well in the direction 20° north of east and finds one gold coin at a location 10.0 m away from the well in the direction 20° north of west. What are the polar and rectangular coordinates of these findings with respect to the well?

Strategy

The well marks the origin of the coordinate system and east is the +x-direction. We identify radial distances from the locations to the origin, which are r_S = 20.0 m (for the silver coin) and r_G = 10.0 m (for the gold coin). To find the angular coordinates, we convert 20° to radians: $20° = \frac{\pi}{180}\frac{20}{} = \frac{\pi}{9}$. We use Equation 1.5.1 to find the x- and y-coordinates of the coins.

Solution

The angular coordinate of the silver coin is $\varphi_S = \frac{\pi}{9}$, whereas the angular coordinate of the gold coin is $\varphi_G = \pi - \frac{\pi}{9} = \frac{8\pi}{9}$. Hence, the polar coordinates of the silver coin are (r_S, φ_S) = (20.0 m, $\frac{\pi}{9}$) and those of the gold coin are (r_G, φ_G) = (10.0 m, \frac{8 \pi}{9}\)). We substitute these coordinates into Equation 1.5.1 to obtain rectangular coordinates. For the gold coin, the coordinates are

$$\begin{cases} x_G = r_G \cos\varphi_G = (10.0\ m)\cos\frac{8\pi}{9} = -9.4\ m \\ y_G = r_G \sin\varphi_G = (10.0\ m)\sin\frac{8\pi}{9} = 3.4\ m \end{cases} \Rightarrow (x_G, y_G) = (-9.4\ m, 3.4\ m). \tag{1.5.2}$$

For the silver coin, the coordinates are

$$\begin{cases} x_S = r_S \cos\varphi_S = (20.0\ m)\cos\frac{\pi}{9} = 18.9\ m \\ y_S = r_S \sin\varphi_S = (20.0\ m)\sin\frac{\pi}{9} = 6.8\ m \end{cases} \Rightarrow (x_S, y_S) = (18.9\ m, 6.8\ m). \tag{1.5.3}$$

Vectors in Three Dimensions

To specify the location of a point in space, we need three coordinates (x, y, z), where coordinates x and y specify locations in a plane, and coordinate z gives a vertical positions above or below the plane. Three-dimensional space has three orthogonal directions, so we need not two but three unit vectors to define a three-dimensional coordinate system. In the Cartesian coordinate system, the first two unit vectors are the unit vector of the x-axis $\hat{i}$ and the unit vector of the y-axis $\hat{j}$. The third unit vector $\hat{k}$ is the direction of the z-axis (Figure 1.5.2). The order in which the axes are labeled, which is the order in which the three unit vectors appear, is important because it defines the orientation of the coordinate system. The order x-y-z, which is equivalent to the order $\hat{i}$ - $\hat{j}$ - $\hat{k}$, defines the standard right-handed coordinate system (positive orientation).

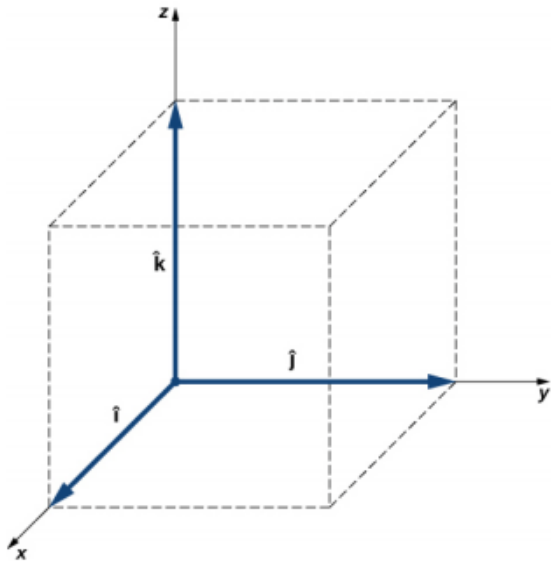


Figure 1.5.2: Three unit vectors define a Cartesian system in three-dimensional space. The order in which these unit vectors appear defines the orientation of the coordinate system. The order shown here defines the right-handed orientation.

In three-dimensional space, vector $\vec{A}$ has three vector components: the x-component $\vec{A}_x$ = A_x $\hat{i}$, which is the part of vector $\vec{A}$ along the x-axis; the y-component $\vec{A}_y$ = A_y $\hat{j}$, which is the part of $\vec{A}$ along the y-axis; and the z-component $\vec{A}_z$ = A_z $\hat{k}$, which is the part of the vector along the z-axis. A vector in three-dimensional space is the vector sum of its three vector components (Figure 1.5.3):

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}. \tag{1.5.4}$$

If we know the coordinates of its origin b(x_b, y_b, z_b) and of its end e(x_e y_e, z_e), its scalar components are obtained by taking their differences: A_x and A_y are given by

$$\begin{cases} A_x = x_e - x_b \\ A_y = y_e - y_b. \end{cases}$$

and the z-component is given by

$$A_z = z_e - z_b. \tag{1.5.5}$$

Magnitude A is obtained by generalizing Equation 2.4.8 to three dimensions:

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}. \tag{1.5.6}$$

This expression for the vector magnitude comes from applying the Pythagorean theorem twice. As seen in Figure 1.5.3, the diagonal in the xy-plane has length $\sqrt{A_x^2 + A_y^2}$ and its square adds to the square A_z^2 to give A^2. Note that when the z-component is zero, the vector lies entirely in the xy-plane and its description is reduced to two dimensions.

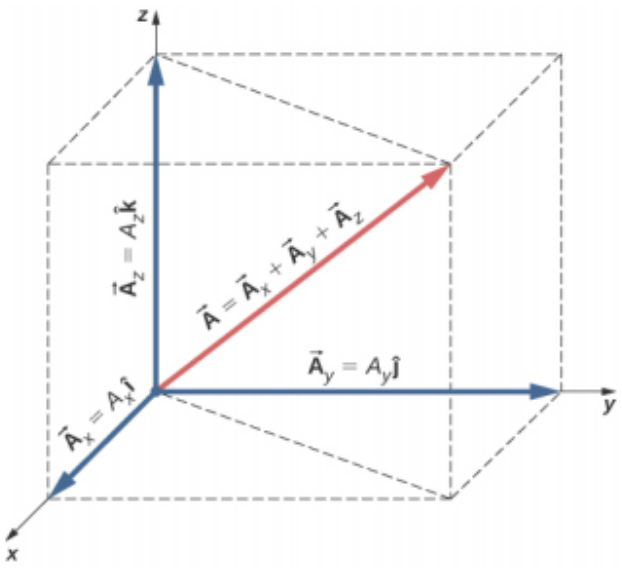


Figure 1.5.3: A vector in three-dimensional space is the vector sum of its three vector components.

Example 1.5.2: Takeoff of a Drone

During a takeoff of IAI Heron (Figure 1.5.4), its position with respect to a control tower is 100 m above the ground, 300 m to the east, and 200 m to the north. One minute later, its position is 250 m above the ground, 1200 m to the east, and 2100 m to the north. What is the drone's displacement vector with respect to the control tower? What is the magnitude of its displacement vector?



Figure 1.5.4: The drone IAI Heron in flight. (credit: SSgt Reynaldo Ramon, USAF)

Strategy

We take the origin of the Cartesian coordinate system as the control tower. The direction of the +x-axis is given by unit vector $\hat{i}$ to the east, the direction of the +y-axis is given by unit vector $\hat{j}$ to the north, and the direction of the +z-axis is given by unit vector $\hat{k}$, which points up from the ground. The drone's first position is the origin (or, equivalently, the beginning) of the displacement vector and its second position is the end of the displacement vector.

Solution

We identify b(300.0 m, 200.0 m, 100.0 m) and e(480.0 m, 370.0 m, 250.0m), and use Equation 2.4.4 and Equation 1.5.5 to find the scalar components of the drone's displacement vector:

$$\begin{cases} D_x = x_e - x_b = 1200.0\ m - 300.0\ m = 900.0\ m, \\ D_y = y_e - y_b = 2100.0\ m - 200.0\ m = 1900.0\ m, \\ D_z = z_e - z_b = 250.0\ m - 100.0\ m = 150\ m. \end{cases} \tag{1.5.7}$$

We substitute these components into Equation 1.5.4 to find the displacement vector:

$$\vec{D} = D_x\,\hat{i} + D_y\,\hat{j} + D_z\,\hat{k} = 900.0\,\hat{i} + 1900.0\,\hat{j} + 150.0\,\hat{k} = (0.90\,\hat{i} + 1.90\,\hat{j} + 0.15\,\hat{k})\ km. \tag{1.5.8}$$

We substitute into Equation 1.5.6 to find the magnitude of the displacement:

$$D = \sqrt{D_x^2 + D_y^2 + D_z^2} = \sqrt{(0.90\ km)^2 + (1.90\ km)^2 + (0.15\ km)^2} = 4.44\ km. \tag{1.5.9}$$

Exercise 2.7

If the average velocity vector of the drone in the displacement in Example 2.7 is $\vec{u}$ = (15.0 $\hat{i}$ + 31.7 $\hat{j}$ + 2.5 $\hat{k}$) m/s, what is the magnitude of the drone's velocity vector?

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1.6: Algebra of Vectors

Learning Objectives

- Apply analytical methods of vector algebra to find resultant vectors and to solve vector equations for unknown vectors.
- Interpret physical situations in terms of vector expressions.

Vectors can be added together and multiplied by scalars. Vector addition is associative (Equation 2.2.8) and commutative (Equation 2.2.7), and vector multiplication by a sum of scalars is distributive (Equation 2.2.9). Also, scalar multiplication by a sum of vectors is distributive:

$$\alpha(\vec{A}+\vec{B})=\alpha\vec{A}+\alpha B. \tag{1.6.1}$$

In this equation, α is any number (a scalar). For example, a vector antiparallel to vector $\vec{A}$ = $A_x\,\hat{i}$ + $A_y\,\hat{j}$ + $A_z\,\hat{k}$ can be expressed simply by multiplying $\vec{A}$ by the scalar α = –1:

$$-\vec{A}=A_x\hat{i}-A_y\hat{j}-A_z\hat{k}. \tag{1.6.2}$$

Example 1.6.1: Direction of Motion

In a Cartesian coordinate system where $\hat{i}$ denotes geographic east, $\hat{j}$ denotes geographic north, and $\hat{k}$ denotes altitude above sea level, a military convoy advances its position through unknown territory with velocity $\vec{v}$ = (4.0 $\hat{i}$ + 3.0 $\hat{j}$ + 0.1 $\hat{k}$) km/h. If the convoy had to retreat, in what geographic direction would it be moving?

Solution

The velocity vector has the third component $\vec{v}_z$ = (+ 0.1 km/h) $\hat{k}$, which says the convoy is climbing at a rate of 100 m/h through mountainous terrain. At the same time, its velocity is 4.0 km/h to the east and 3.0 km/h to the north, so it moves on the ground in direction $\tan^{-1}(3/4) \approx 37°$ north of east. If the convoy had to retreat, its new velocity vector $\vec{u}$ would have to be antiparallel to $\vec{v}$ and be in the form $\vec{u}=-\alpha\vec{v}$, where α is a positive number. Thus, the velocity of the retreat would be $\vec{u}$ = α(–4.0 $\hat{i}$ – 3.0 $\hat{j}$ – 0.1 $\hat{k}$) km/h. The negative sign of the third component indicates the convoy would be descending. The direction angle of the retreat velocity is $\tan^{-1}(-3\,\alpha/-4\,\alpha) \approx 37°$ south of west. Therefore, the convoy would be moving on the ground in direction 37° south of west while descending on its way back.

The generalization of the number zero to vector algebra is called the **null vector**, denoted by $\vec{0}$. All components of the null vector are zero, $\vec{0}$ = 0 $\hat{i}$+ 0 $\hat{j}$ + 0 $\hat{k}$, so the null vector has no length and no direction.

Two vectors $\vec{A}$ and $\vec{B}$ are **equal vectors** if and only if their difference is the null vector: $\vec{0}=\vec{A}-\vec{B}$ = ($A_x\,\hat{i}$ + $A_y\,\hat{j}$ + $A_z\,\hat{k}$) – ($B_x\,\hat{i}$ + $B_y\,\hat{j}$ + $B_z\,\hat{k}$) = (A_x – B_x) $\hat{i}$ + (A_y – B_y) $\hat{j}$ + (A_z – B_z) $\hat{k}$. This vector equation means we must have simultaneously A_x – B_x = 0, A_y – B_y = 0, and A_z – B_z = 0. Hence, we can write $\vec{A}=\vec{B}$ if and only if the corresponding components of vectors $\vec{A}$ and $\vec{B}$ are equal:

$$\vec{A}=\vec{B}\Leftrightarrow\begin{cases}A_x=B_x\\A_y=B_y\\A_z=B_z\end{cases}. \tag{1.6.3}$$

Two vectors are equal when their corresponding scalar components are equal. Resolving vectors into their scalar components (i.e., finding their scalar components) and expressing them analytically in vector component form (given by Equation 2.5.4) allows us to use vector algebra to find sums or differences of many vectors **analytically** (i.e., without using graphical methods). For example, to find the resultant of two vectors $\vec{A}$ and $\vec{B}$, we simply add them component by component, as follows:

$$\vec{R}=\vec{A}+\vec{B}=(A_x\,\hat{i}+A_y\,\hat{j}+A_z\,\hat{k})+(B_x\,\hat{i}+B_y\,\hat{j}+B_z\,\hat{k})=(A_x+B_x)\,\hat{i}+(A_y+B_y)\,\hat{j}+(A_z+B_z)\,\hat{k}. \tag{1.6.4}$$

In this way, using Equation 1.6.3, scalar components of the resultant vector $\vec{R}$ = R_x $\hat{i}$ + R_y $\hat{j}$ + R_z $\hat{k}$ are the sums of corresponding scalar components of vectors $\vec{A}$ and $\vec{B}$:

$$\begin{cases} R_x = A_x + B_x, \\ R_y = A_y + B_y, \\ R_z = A_z + B_z \end{cases}. \tag{1.6.5}$$

Analytical methods can be used to find components of a resultant of many vectors. For example, if we are to sum up N vectors $\vec{F}_1$, $\vec{F}_2$, $\vec{F}_3$, ... , $\vec{F}_N$, where each vector is $\vec{F}_k$ = F_{kx} $\hat{i}$ + F_{ky} $\hat{j}$ + F_{kz} $\hat{k}$, the resultant vector $\vec{F}_R$ is

$$\vec{F}_R = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + ... + \vec{F}_N = \sum_{k=1}^{N} \vec{F}_k = \sum_{k=1}^{N} \left(F_{kx}\hat{i} + F_{ky}\hat{j} + F_{kz}\hat{k} \right) = \left(\sum_{k=1}^{N} F_{kx} \right)\hat{i} + \left(\sum_{k=1}^{N} F_{ky} \right)\hat{j} + \left(\sum_{k=1}^{N} F_{kz} \right)\hat{k}. \tag{1.6.6}$$

Therefore, scalar components of the resultant vector are

$$\begin{cases} F_{Rx} = \sum_{k=1}^{N} F_{kx} = F_{1x} + F_{2x} + ... + F_{Nx} \\ F_{Ry} = \sum_{k=1}^{N} F_{ky} = F_{1y} + F_{2y} + ... + F_{Ny} \\ F_{Rz} = \sum_{k=1}^{N} F_{kz} = F_{1z} + F_{2z} + ... + F_{Nz}. \end{cases} \tag{1.6.7}$$

Having found the scalar components, we can write the resultant in vector component form:

$$\vec{F}_R = F_{Rx}\,\hat{i} + F_{Ry}\,\hat{j} + F_{Rz}\,\hat{k}. \tag{1.6.8}$$

Analytical methods for finding the resultant and, in general, for solving vector equations are very important in physics because many physical quantities are vectors. For example, we use this method in kinematics to find resultant displacement vectors and resultant velocity vectors, in mechanics to find resultant force vectors and the resultants of many derived vector quantities, and in electricity and magnetism to find resultant electric or magnetic vector fields.

In many physical situations, we often need to know the direction of a vector. For example, we may want to know the direction of a magnetic field vector at some point or the direction of motion of an object. We have already said direction is given by a unit vector, which is a dimensionless entity—that is, it has no physical units associated with it. When the vector in question lies along one of the axes in a Cartesian system of coordinates, the answer is simple, because then its unit vector of direction is either parallel or antiparallel to the direction of the unit vector of an axis. For example, the direction of vector $\vec{d}$ = –5 m $\hat{i}$ is unit vector $\hat{d}$ = – $\hat{i}$. The general rule of finding the unit vector $\hat{V}$ of direction for any vector $\vec{V}$ is to divide it by its magnitude V:

$$\hat{V} = \frac{\vec{V}}{V}. \tag{1.6.9}$$

We see from this expression that the unit vector of direction is indeed dimensionless because the numerator and the denominator in Equation 1.6.9 have the same physical unit. In this way, Equation 1.6.9 allows us to express the unit vector of direction in terms of unit vectors of the axes. Example 2.7.6 illustrates this principle.

Contributors

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1.7: Algebra of Vectors Examples

Example 1.7.1: Analytical Computation of a Resultant

Three displacement vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ in a plane (Figure 2.3.6) are specified by their magnitudes A = 10.0, B = 7.0, and C = 8.0, respectively, and by their respective direction angles with the horizontal direction α = 35°, β = –110°, and γ = 30°. The physical units of the magnitudes are centimeters. Resolve the vectors to their scalar components and find the following vector sums:

a. $\vec{R} = \vec{A} + \vec{B} + \vec{C}$,
b. $\vec{D} = \vec{A} - \vec{B}$, and
c. $\vec{S} = \vec{A} - 3\,\vec{B} + \vec{C}$.

Strategy

First, we use Equation 2.4.13 to find the scalar components of each vector and then we express each vector in its vector component form given by $\overrightarrow{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}}$. Then, we use analytical methods of vector algebra to find the resultants.

Solution

We resolve the given vectors to their scalar components:

$$\begin{cases} A_x = A\cos\alpha = (10.0\ cm)\cos 35^o = 8.19\ cm \\ A_y = A\sin\alpha = (10.0\ cm)\sin 35^o = 5.73\ cm \end{cases} \tag{1.7.1}$$

$$\begin{cases} B_x = B\cos\beta = (7.0\ cm)\cos(-110^o) = -2.39\ cm \\ B_y = B\sin\beta = (7.0\ cm)\sin(-110^o) = -6.58\ cm \end{cases} \tag{1.7.2}$$

$$\begin{cases} C_x = C\cos\gamma = (8.0\ cm)\cos(30^o) = 6.93\ cm \\ C_y = C\sin\gamma = (8.0\ cm)\sin(30^o) = 4.00\ cm \end{cases} \tag{1.7.3}$$

For (a) we may substitute directly into Equation 2.6.7 to find the scalar components of the resultant:

$$\begin{cases} R_x = A_x + B_x + C_x = 8.19\ cm - 2.39\ cm + 6.93\ cm = 12.73\ cm \\ R_y = A_y + B_y + C_y = 5.73\ cm - 6.58\ cm + 4.00\ cm = 3.15\ cm \end{cases} \tag{1.7.4}$$

Therefore, the resultant vector is $\vec{R} = R_x\hat{i} + R_y\hat{j} = (12.7\hat{i} + 3.1\hat{j})$cm. For (b), we may want to write the vector difference as

$$\vec{D} = \vec{A} - \vec{B} = (A_x\hat{i} + A_y\hat{j}) - (B_x\hat{i} + B_y\hat{j}) = (A_x - B_x)\hat{i} + (A_y - B_y)\hat{j}. \tag{1.7.5}$$

Hence the difference vector is $\vec{D} = D_x\hat{i} + D_y\hat{j} = (10.6\hat{i} + 12.3\hat{j})$cm.

For (c), we can write vector $\vec{S}$ in the following explicit form:

$$\vec{S} = \vec{A} - 3\vec{B} + \vec{C} = (A_x\hat{i} + A_y\hat{j}) - 3(B_x\hat{i} + B_y\hat{j}) + (C_x\hat{i} + C_y\hat{j}) = (A_x - 3B_x + C_x)\hat{i} + (A_y - 3B_y + C_y)\hat{j}. \tag{1.7.6}$$

Then, the scalar components of $\vec{S}$ are

$$\begin{cases} S_x = A_x - 3B_x + C_x = 8.19\ cm - 3(-2.39\ cm) + 6.93\ cm = 22.29\ cm \\ S_y = A_y - 3B_y + C_y = 5.73\ cm - 3(-6.58\ cm) + 4.00\ cm = 29.47\ cm \end{cases} \tag{1.7.7}$$

The vector is $\vec{S} = S_x\hat{i} + S_y\hat{j} = (22.3\hat{i} + 29.5\hat{j})$cm.

Significance

Having found the vector components, we can illustrate the vectors by graphing or we can compute magnitudes and direction angles, as shown in Figure 1.7.1. Results for the magnitudes in (b) and (c) can be compared with results for the same problems obtained with the graphical method, shown in Figure 2.3.7 and Figure 2.3.8. Notice that the analytical method produces exact results and its accuracy is not limited by the resolution of a ruler or a protractor, as it was with the graphical method used in Example 2.3.2 for finding this same resultant.

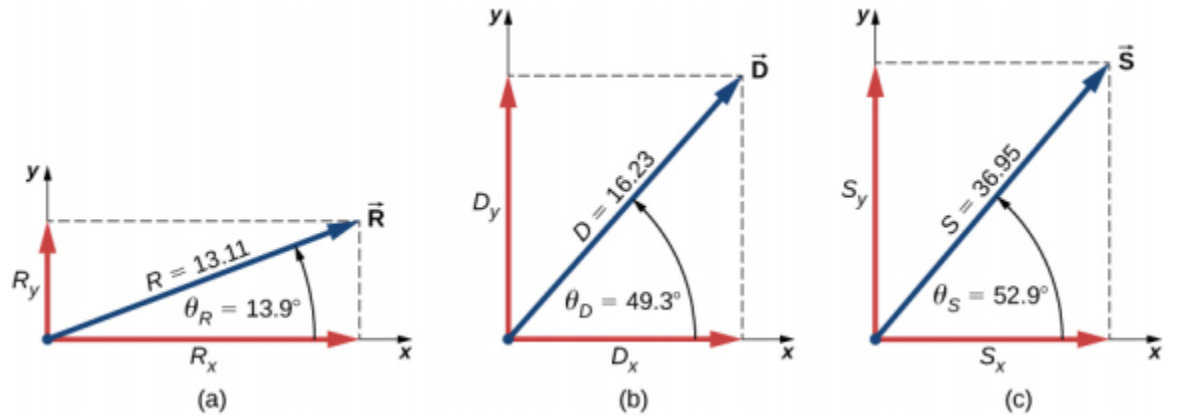


Figure 1.7.1: Graphical illustration of the solutions obtained analytically.

Exercise 2.8

Three displacement vectors $\vec{A}$, $\vec{B}$, and $\vec{F}$ (Figure 2.3.6) are specified by their magnitudes A = 10.00, B = 7.00, and F = 20.00, respectively, and by their respective direction angles with the horizontal direction α = 35°, β = –110°, and φ = 110°. The physical units of the magnitudes are centimeters. Use the analytical method to find vector $\vec{F} = \vec{A} + 2\,\vec{B} - \vec{F}$. Verify that G = 28.15 cm and that θ_G = –68.65°.

Example 1.7.2: The Tug-of-War Game

Four dogs named Astro, Balto, Clifford, and Dug play a tug-of-war game with a toy (Figure 1.7.2). Astro pulls on the toy in direction α = 55° south of east, Balto pulls in direction β = 60° east of north, and Clifford pulls in direction γ = 55° west of north. Astro pulls strongly with 160.0 units of force (N), which we abbreviate as A = 160.0 N. Balto pulls even stronger than Astro with a force of magnitude B = 200.0 N, and Clifford pulls with a force of magnitude C = 140.0 N. When Dug pulls on the toy in such a way that his force balances out the resultant of the other three forces, the toy does not move in any direction. With how big a force and in what direction must Dug pull on the toy for this to happen?

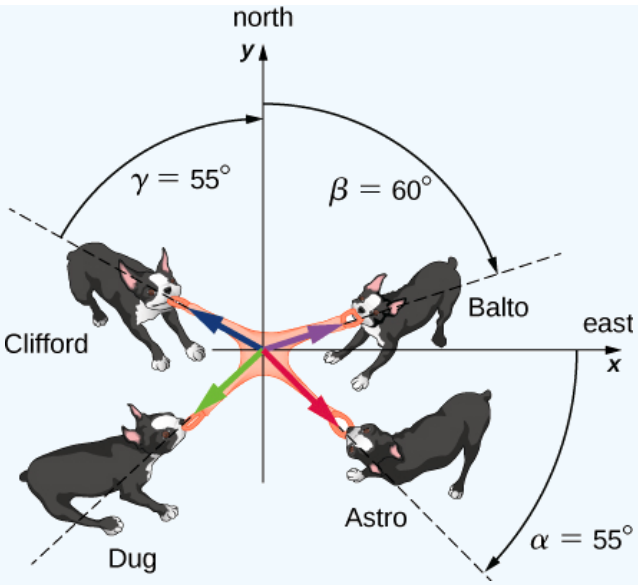


Figure 1.7.2: Four dogs play a tug-of-war game with a toy.

Strategy

We assume that east is the direction of the positive x-axis and north is the direction of the positive y-axis. As in Example 1.7.1, we have to resolve the three given forces — $\vec{A}$ (the pull from Astro), $\vec{B}$ (the pull from Balto), and $\vec{C}$ (the pull from Clifford)—into their scalar components and then find the scalar components of the resultant vector $\vec{R} = \vec{A} + \vec{B} + \vec{C}$. When the pulling force $\vec{D}$ from Dug balances out this resultant, the sum of $\vec{D}$ and $\vec{R}$ must give the null vector $\vec{D} + \vec{R} = \vec{0}$. This means that $\vec{D} = -\vec{R}$ so the pull from Dug must be antiparallel to $\vec{R}$.

Solution

The direction angles are $\theta_A = -\alpha = -55°$, $\theta_B = 90° - \beta = 30°$, and $\theta_C = 90° + \gamma = 145°$, and substituting them into Equation 2.4.13 gives the scalar components of the three given forces:

$$\begin{cases} A_x = A\cos\theta_A = (160.0\ N)\cos(-55^o) = +91.8\ N \\ A_y = A\sin\theta_A = (160.0\ N)\sin(-55^o) = -131.1\ N \end{cases} \tag{1.7.8}$$

$$\begin{cases} B_x = B\cos\theta_B = (200.0\ N)\cos 30^o = +173.2\ N \\ B_y = B\sin\theta_B = (200.0\ N)\sin 30^o = +100.0\ N \end{cases} \tag{1.7.9}$$

$$\begin{cases} C_x = C\cos\theta_C = (140.0\ N)\cos 145^o = -114.7\ N \\ C_y = C\sin\theta_C = (140.0\ N)\sin 145^o = +80.3\ N \end{cases} \tag{1.7.10}$$

Now we compute scalar components of the resultant vector $\vec{R} = \vec{A} + \vec{B} + \vec{C}$:

$$\begin{cases} R_x = A_x + B_x + C_x = +91.8\ N + 173.2\ N - 114.7\ N = +150.3\ N \\ R_y = A_y + B_y + C_y = -131.1\ N + 100.0\ N + 80.3\ N = +49.2\ N \end{cases} \tag{1.7.11}$$

The antiparallel vector to the resultant $\vec{R}$ is

$$\vec{D} = -\vec{R} = -R_x\hat{i} - R_y\hat{j} = (-150.3\hat{i} - 49.2\hat{j})N. \tag{1.7.12}$$

The magnitude of Dug's pulling force is

$$D = \sqrt{D_x^2 + D_y^2} = \sqrt{(-150.3)^2 + (-49.2)^2}N = 158.1\ N. \tag{1.7.13}$$

The direction of Dug's pulling force is

$$\theta = \tan^{-1}\left(\frac{D_y}{D_x}\right) = \tan^{-1}\left(\frac{-49.2\ N}{-150.3\ N}\right) = \tan^{-1}\left(\frac{49.2}{150.3}\right) = 18.1^o. \tag{1.7.14}$$

Dug pulls in the direction 18.1° south of west because both components are negative, which means the pull vector lies in the third quadrant (Figure 2.4.4).

Exercise 2.9

Suppose that Balto in Example 1.7.2 leaves the game to attend to more important matters, but Astro, Clifford, and Dug continue playing. Astro and Clifford's pull on the toy does not change, but Dug runs around and bites on the toy in a different place. With how big a force and in what direction must Dug pull on the toy now to balance out the combined pulls from Clifford and Astro? Illustrate this situation by drawing a vector diagram indicating all forces involved.

Example 1.7.3: Vector Algebra

Find the magnitude of the vector $\vec{C}$ that satisfies the equation 2 $\vec{A}$ – 6 $\vec{B}$ + 3 $\vec{C}$ = 2 $\hat{j}$, $\vec{A}$ = $\hat{i}$ – 2$\hat{k}$ and $\vec{B}$ = – $\hat{j}$ + $\frac{\hat{k}}{2}$.

Strategy

We first solve the given equation for the unknown vector $\vec{C}$. Then we substitute $\vec{A}$ and $\vec{B}$; group the terms along each of the three directions $\hat{i}$, $\hat{j}$, and $\hat{k}$; and identify the scalar components C_x, C_y, and C_z. Finally, we substitute into Equation 2.5.6 to find magnitude C.

Solution

$$\begin{aligned} 2\vec{A} - 6\vec{B} + 3\vec{C} &= 2\hat{j} \\ 3\vec{C} &= 2\hat{j} - 2\vec{A} + 6\vec{B} \\ \vec{C} &= \frac{2}{3}\hat{j} - \frac{2}{3}\vec{A} + 2\vec{B} \\ &= \frac{2}{3}\hat{j} - \frac{2}{3}(\hat{i} - 2\hat{k}) + 2\Big(-\hat{j} + \frac{\hat{k}}{2}\Big) \\ &= \frac{2}{3}\hat{j} - \frac{2}{3}\hat{i} + \frac{4}{3}\hat{k} - 2\hat{j} + \hat{k} \\ &= -\frac{2}{3}\hat{i} + \Big(\frac{2}{3} - 2\Big)\hat{j} + \Big(\frac{4}{3} + 1\Big)\hat{k} \\ &= -\frac{2}{3}\hat{i} - \frac{4}{3}\hat{j} + \frac{7}{3}\hat{k} \end{aligned} \tag{1.7.15}$$

The components are $C_x = -\frac{2}{3}$, $C_y = -\frac{4}{3}$, and $C_z = \frac{7}{3}$, and substituting into Equation 2.5.6 gives

$$C = \sqrt{C_x^2 + C_y^2 + C_z^2} = \sqrt{\Big(-\frac{2}{3}\Big)^2 + \Big(-\frac{4}{3}\Big)^2 + \Big(\frac{7}{3}\Big)^2} = \sqrt{\frac{23}{3}}. \tag{1.7.16}$$

Example 1.7.4: Displacement of a Skier

Starting at a ski lodge, a cross-country skier goes 5.0 km north, then 3.0 km west, and finally 4.0 km southwest before taking a rest. Find his total displacement vector relative to the lodge when he is at the rest point. How far and in what direction must he ski from the rest point to return directly to the lodge?

Strategy

We assume a rectangular coordinate system with the origin at the ski lodge and with the unit vector $\hat{i}$ pointing east and the unit vector $\hat{j}$ pointing north. There are three displacements: $\vec{D}_1$, $\vec{D}_2$, and $\vec{D}_3$. We identify their magnitudes as D_1 = 5.0 km , D_2 = 3.0 km , and D_3 = 4.0 km . We identify their directions are the angles θ_1 = 90°, θ_2 = 180°, and θ_3 = 180° + 45° = 225°. We resolve each displacement vector to its scalar components and substitute the components into Equation 2.6.5 to obtain the scalar components of the resultant displacement $\vec{D}$ from the lodge to the rest point. On the way back from the rest point to the lodge, the displacement is $\vec{B}$ = – $\vec{D}$. Finally, we find the magnitude and direction of $\vec{B}$.

Solution

Scalar components of the displacement vectors are

$$\begin{cases} D_{1x} = D_1 \cos\theta_1 = (5.0\ km)\cos 90^o = 0 \\ D_{1y} = D_1 \sin\theta_1 = (5.0\ km)\sin 90^o = 5.0\ km \end{cases} \tag{1.7.17}$$

$$\begin{cases} D_{2x} = D_2 \cos\theta_2 = (3.0\ km)\cos 180^o = -3.0\ km \\ D_{2y} = D_2 \sin\theta_2 = (3.0\ km)\sin 180^o = 0 \end{cases} \tag{1.7.18}$$

$$\begin{cases} D_{3x} = D_3 \cos\theta_3 = (4.0\ km)\cos 225^o = -2.8\ km \\ D_{3y} = D_3 \sin\theta_3 = (4.0\ km)\sin 225^o = -2.8\ km \end{cases} \tag{1.7.19}$$

Scalar components of the net displacement vector are

$$\begin{cases} D_x = D_{1x} + D_{2x} + D_{3x} = (0 - 3.0 - 2.8)km = -5.8\ km \\ D_y = D_{1y} + D_{2y} + D_{3y} = (5.0 + 0 - 2.8)km = +2.2\ km \end{cases} \tag{1.7.20}$$

Hence, the skier's net displacement vector is $\vec{D}$ = D_x $\hat{i}$ + D_y $\hat{j}$ = (–5.8 $\hat{i}$ + 2.2 $\hat{j}$)km . On the way back to the lodge, his displacement is $\vec{B}$ = – $\vec{D}$ = –(–5.8 $\hat{i}$+ 2.2 $\hat{j}$)km = (5.8 $\hat{i}$ – 2.2 $\hat{j}$)km. Its magnitude is B = $\sqrt{B_x^2 + B_y^2}$ = $\sqrt{(5.8)^2 + (-2.2)^2}$ km = 6.2 km and its direction angle is θ= $\tan^{-1}\left(\frac{-2.2}{5.8}\right)$ = –20.8°. Therefore, to return to the lodge, he must go 6.2 km in a direction about 21° south of east.

Significance

Notice that no figure is needed to solve this problem by the analytical method. Figures are required when using a graphical method; however, we can check if our solution makes sense by sketching it, which is a useful final step in solving any vector problem.

Example 1.7.5: Displacement of a Jogger

A jogger runs up a flight of 200 identical steps to the top of a hill and then runs along the top of the hill 50.0 m before he stops at a drinking fountain (Figure 1.7.3). His displacement vector from point A at the bottom of the steps to point B at the fountain is $\vec{D}_{AB}$ = (–90.0 $\hat{i}$ + 30.0 $\hat{j}$)m. What is the height and width of each step in the flight? What is the actual distance the jogger covers? If he makes a loop and returns to point A, what is his net displacement vector?

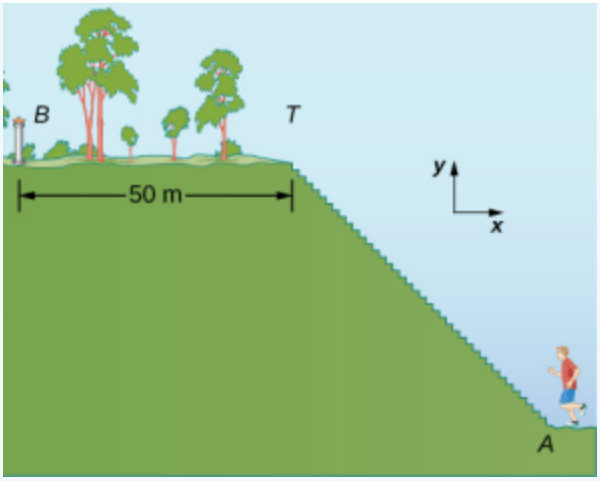


Figure 1.7.3: A jogger runs up a flight of steps.

Strategy

The displacement vector $\vec{D}_{AB}$ is the vector sum of the jogger's displacement vector $\vec{D}_{AT}$ along the stairs (from point A at the bottom of the stairs to point T at the top of the stairs) and his displacement vector $\vec{D}_{RB}$ on the top of the hill (from point T at the top of the stairs to the fountain at point B). We must find the horizontal and the vertical components of $\vec{D}_{TB}$. If each step has width w and height h, the horizontal component of $\vec{D}_{TB}$ must have a length of 200w and the vertical component must have a length of 200h. The actual distance the jogger covers is the sum of the distance he runs up the stairs and the distance of 50.0 m that he runs along the top of the hill.

Solution

In the coordinate system indicated in Figure 1.7.3, the jogger's displacement vector on the top of the hill is $\vec{D}_{RB}$ = (–50.0 m) $\hat{i}$. His net displacement vector is

$$\vec{D}_{AB} = \vec{D}_{AT} + \vec{D}_{TB}.$$

Therefore, his displacement vector $\vec{D}_{TB}$ along the stairs is

$$\begin{aligned}\vec{D}_{AT} &= \vec{D}_{AB} - \vec{D}_{TB} = (-90.0\hat{i} + 30.0\hat{j})m - (-50.0m)\hat{i}) = [(-90.050.0)hati + 30.0\hat{j})]m \\ &= (-40.0\hat{i} + 30.0\hat{j})m.\end{aligned} \tag{1.7.21}$$

Its scalar components are D_{ATx} = –40.0 m and D_{ATy} = 30.0 m. Therefore, we must have

$$200w = |-40.0|\text{m and } 200h = 30.0 \text{ m}.$$

Hence, the step width is w = $\frac{40.0\ m}{200}$ = 0.2 m = 20 cm, and the step height is w = $\frac{30.0\ m}{200}$ = 0.15 m = 15 cm. The distance that the jogger covers along the stairs is

$$\vec{D}_{AT} = \sqrt{\vec{D}_{ATx}^2 + \vec{D}_{ATy}^2} = \sqrt{(-40.0)^2 + (30.0)^2}m = 50.0\ m.$$

Thus, the actual distance he runs is D_{AT} + D_{TB} = 50.0 m + 50.0 m = 100.0 m. When he makes a loop and comes back from the fountain to his initial position at point A, the total distance he covers is twice this distance,or 200.0 m. However, his net displacement vector is zero, because when his final position is the same as his initial position, the scalar components of his net displacement vector are zero (Equation 2.4.4).

In many physical situations, we often need to know the direction of a vector. For example, we may want to know the direction of a magnetic field vector at some point or the direction of motion of an object. We have already said direction is given by a unit vector, which is a dimensionless entity—that is, it has no physical units associated with it. When the vector in question lies along one of the axes in a Cartesian system of coordinates, the answer is simple, because then its unit vector of direction is either parallel or antiparallel to the direction of the unit vector of an axis. For example, the direction of vector $\vec{d}$ = -5 m $\hat{i}$ is unit vector $\vec{d}$

= -$\hat{i}$. The general rule of finding the unit vector $\vec{V}$ of direction for any vector $\vec{V}$ is to divide it by its magnitude *V*:

$$\widehat{\mathbf{V}} = \frac{\vec{\mathbf{V}}}{V}. \tag{1.7.22}$$

We see from this expression that the unit vector of direction is indeed dimensionless because the numerator and the denominator in Equation 1.7.22 have the same physical unit. In this way, Equation 1.7.22 allows us to express the unit vector of direction in terms of unit vectors of the axes. The following example illustrates this principle.

Example 1.7.6: The Unit Vector of Direction

If the velocity vector of the military convoy in Example 2.6.1 is $\vec{v}$ = (4.000 $\hat{i}$ + 3.000 $\hat{j}$ + 0.100 $\hat{k}$)km/h, what is the unit vector of its direction of motion.

Strategy

The unit vector of the convoy's direction of motion is the unit vector $\hat{v}$ that is parallel to the velocity vector. The unit vector is obtained by dividing a vector by its magnitude, in accordance with Equation 1.7.22.

Solution

The magnitude of the vector $\vec{v}$ is

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2} = \sqrt{4.000^2 + 3.000^2 + 0.100^2} km/h = 5.001\ km/h.$$

To obtain the unit vector $\hat{v}$, divide $\vec{v}$ by its magnitude:

$$\begin{aligned} \hat{v} &= \frac{\vec{v}}{v} = \frac{(4.000\hat{i} + 3.00\hat{j} + 0.100\hat{k})km/h}{5.001\ km/h} \\ &= \frac{(4.000\hat{i} + 3.000\hat{j} + 0.1100\hat{k})}{5.001} \\ &= \frac{4.000}{5.001}\hat{i} + \frac{3.000}{5.001}\hat{j} + \frac{0.100}{5.001}\hat{k} \\ &= (79.98\hat{i} + 59.99\hat{j} + 2.00\hat{k}) \times 10^{-2}. \end{aligned} \tag{1.7.23}$$

Significance

Note that when using the analytical method with a calculator, it is advisable to carry out your calculations to at least three decimal places and then round off the final answer to the required number of significant figures, which is the way we performed calculations in this example. If you round off your partial answer too early, you risk your final answer having a huge numerical error, and it may be far off from the exact answer or from a value measured in an experiment.

Exercise 2.10

Verify that vector $\hat{v}$ obtained in Example 1.7.3 is indeed a unit vector by computing its magnitude. If the convoy in Example 2.6.1 was moving across a desert flatland—that is, if the third component of its velocity was zero—what is the unit vector of its direction of motion? Which geographic direction does it represent?

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1.8: Products of Vectors (Part 1)

Learning Objectives

- Explain the difference between the scalar product and the vector product of two vectors.
- Determine the scalar product of two vectors.
- Determine the vector product of two vectors.
- Describe how the products of vectors are used in physics.

A vector can be multiplied by another vector but may not be divided by another vector. There are two kinds of products of vectors used broadly in physics and engineering. One kind of multiplication is a **scalar multiplication of two vectors**. Taking a scalar product of two vectors results in a number (a scalar), as its name indicates. Scalar products are used to define work and energy relations. For example, the work that a force (a vector) performs on an object while causing its displacement (a vector) is defined as a scalar product of the force vector with the displacement vector. A quite different kind of multiplication is a **vector multiplication of vectors**. Taking a vector product of two vectors returns as a result a vector, as its name suggests. Vector products are used to define other derived vector quantities. For example, in describing rotations, a vector quantity called **torque** is defined as a vector product of an applied force (a vector) and its lever arm (a vector). It is important to distinguish between these two kinds of vector multiplications because the scalar product is a scalar quantity and a vector product is a vector quantity.

The Scalar Product of Two Vectors (the Dot Product)

Scalar multiplication of two vectors yields a scalar product.

Definition: Scalar Product (Dot Product)

The scalar product $\vec{A} \cdot \vec{B}$ of two vectors $\vec{A}$ and $\vec{B}$ is a number defined by the equation

$$\vec{A} \cdot \vec{B} = AB\cos\varphi, \tag{1.8.1}$$

where ϕ is the angle between the vectors (shown in Figure 1.8.1). The scalar product is also called the **dot product** because of the dot notation that indicates it.

In the definition of the dot product, the direction of angle φ does not matter, and φ can be measured from either of the two vectors to the other because $\cos\varphi = \cos(-\varphi) = cos(2\pi - \varphi)$. The dot product is a negative number when 90° < φ ≤ 180° and is a positive number when 0° ≤ ϕ < 90°. Moreover, the dot product of two parallel vectors is $\vec{A} \cdot \vec{B}$ = AB cos 0° = AB, and the dot product of two antiparallel vectors is $\vec{A} \cdot \vec{B}$ = AB cos 180° = –AB. The scalar product of two orthogonal vectors vanishes: $\vec{A} \cdot \vec{B}$ = AB cos 90° = 0. The scalar product of a vector with itself is the square of its magnitude:

$$\vec{A}^2 \equiv \vec{A} \cdot \vec{A} = AA\cos 0^o = A^2 \tag{1.8.2}$$

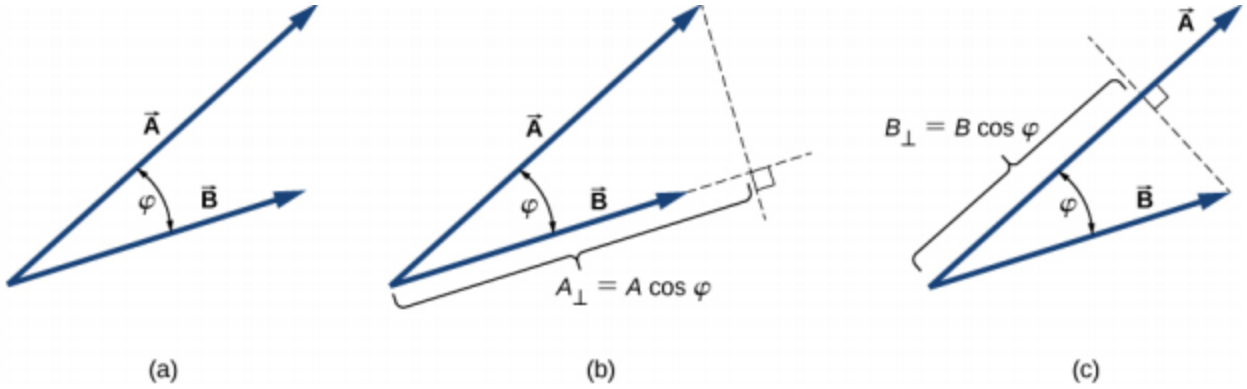


Figure 1.8.1: The scalar product of two vectors. (a) The angle between the two vectors. (b) The orthogonal projection $A_\perp$ of vector $\vec{A}$ onto the direction of vector $\vec{B}$. (c) The orthogonal projection $B_\perp$ of vector $\vec{B}$ onto the direction of vector $\vec{A}$.

Example 1.8.1: The Scalar Product

For the vectors shown in Figure 2.3.6, find the scalar product $\vec{A} \cdot \vec{F}$.

Strategy

From Figure 2.3.6, the magnitudes of vectors $\vec{A}$ and $\vec{B}$ are A = 10.0 and F = 20.0. Angle θ, between them, is the difference: $\theta = \varphi - \alpha$ = 110° – 35° = 75°. Substituting these values into Equation 1.8.1 gives the scalar product.

Solution

A straightforward calculation gives us

$$\vec{A} \cdot \vec{F} = AF \cos\theta = (10.0)(20.0) \cos 75^o = 51.76. \tag{1.8.3}$$

Exercise 2.11

For the vectors given in Figure 2.3.6, find the scalar products $\vec{A} \cdot \vec{B}$ and $\vec{B} \cdot \vec{C}$.

In the Cartesian coordinate system, scalar products of the unit vector of an axis with other unit vectors of axes always vanish because these unit vectors are orthogonal:

$$\hat{i} \cdot \hat{j} = |\hat{i}||\hat{j}| \cos 90^o = (1)(1)(0) = 0, \tag{1.8.4}$$

$$\hat{i} \cdot \hat{k} = |\hat{i}||\hat{k}| \cos 90^o = (1)(1)(0) = 0, \tag{1.8.5}$$

$$\hat{k} \cdot \hat{j} = |\hat{k}||\hat{j}| \cos 90^o = (1)(1)(0) = 0. \tag{1.8.6}$$

In these equations, we use the fact that the magnitudes of all unit vectors are one: $|\hat{i}| = |\hat{j}| = |\hat{k}|$ = 1. For unit vectors of the axes, Equation 1.8.2 gives the following identities:

$$\hat{i} \cdot \hat{i} = i^2 = \hat{j} \cdot \hat{j} = j^2 = \hat{k} \cdot \hat{k} = 1. \tag{1.8.7}$$

The scalar product $\vec{A} \cdot \vec{B}$ can also be interpreted as either the product of B with the projection $A_\parallel$ of vector $\vec{A}$ onto the direction of vector $\vec{B}$ (Figure 1.8.1(b)) or the product of A with the projection $B_\parallel$ of vector $\vec{B}$ onto the direction of vector $\vec{A}$ (Figure 1.8.1(c)):

$$\begin{aligned} \vec{A} \cdot \vec{B} &= AB \cos\varphi \\ &= B(A \cos\varphi) = BA_\parallel \\ &= A(B \cos\varphi) = AB_\parallel. \end{aligned} \tag{1.8.8}$$

For example, in the rectangular coordinate system in a plane, the scalar x-component of a vector is its dot product with the unit vector $\hat{i}$, and the scalar y-component of a vector is its dot product with the unit vector $\hat{j}$:

$$\begin{cases} \vec{A} \cdot \hat{i} = |\vec{A}||\hat{i}| \cos\theta_A = A\cos\theta_A = A\cos\theta_A = A_x \\ \vec{A} \cdot \hat{j} = |\vec{A}||\hat{j}| \cos(90^o - \theta_A) = A\sin\theta_A = A_y \end{cases} \tag{1.8.9}$$

Scalar multiplication of vectors is communtative,

$$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}, \tag{1.8.10}$$

and obeys the distributive law:

$$\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}. \tag{1.8.11}$$

We can use the commutative and distributive laws to derive various relations for vectors, such as expressing the dot product of two vectors in terms of their scalar components.

Exercise 2.12

For vector $\vec{A} = A_x\ \hat{i} + A_y\ \hat{j} + A_z\ \hat{k}$ in a rectangular coordinate system, use Equation 1.8.4 through Equation 1.8.11 to show that $\vec{A}\ \cdot \hat{i} = A_x \vec{A}\ \cdot\ \hat{j} = A_y$ and $\vec{A}\ \cdot\ \hat{k} = A_z$.

When the vectors in Equation 1.8.1 are given in their vector component forms,

$$\vec{A} = A_x\ \hat{i} + A_y\ \hat{j} + A_z\ \hat{k}\ and \vec{B} = B_x\ \hat{i} + B_y\ \hat{j} + B_z\ \hat{k}, \tag{1.8.12}$$

we can compute their scalar product as follows:

$$\begin{aligned} \vec{A} \cdot \vec{B} &= (A_x\ \hat{i} + A_y\ \hat{j} + A_z\ \hat{k}) \cdot (B_x\ \hat{i} + B_y\ \hat{j} + B_z\ \hat{k}) \\ &= A_x B_x\ \hat{i} \cdot\ \hat{i} + A_x B_y\ \hat{i} \cdot\ \hat{j} + A_x B_z\ \hat{i} \cdot\ \hat{k} \\ &+ A_y B_x\ \hat{j} \cdot\ \hat{i} + A_y B_y\ \hat{j} \cdot\ \hat{j} + A_y B_z\ \hat{j} \cdot\ \hat{k} \\ &+ A_z B_x\ \hat{k} \cdot\ \hat{i} + A_z B_y\ \hat{k} \cdot\ \hat{j} + A_z B_z\ \hat{k} \cdot\ \hat{k}. \end{aligned} \tag{1.8.13}$$

Since scalar products of two different unit vectors of axes give zero, and scalar products of unit vectors with themselves give one (see Equation 1.8.4 and Equation 1.8.7), there are only three nonzero terms in this expression. Thus, the scalar product simplifies to

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z. \tag{1.8.14}$$

We can use Equation 1.8.14 for the scalar product in terms of scalar components of vectors to find the angle between two vectors. When we divide Equation 1.8.1 by AB, we obtain the equation for cos φ, into which we substitute Equation 1.8.14:

$$\cos\varphi = \frac{\vec{A} \cdot \vec{B}}{AB} = \frac{A_x B_x + A_y B_y + A_z B_z}{AB}. \tag{1.8.15}$$

Angle φ between vectors $\vec{A}$ and $\vec{B}$ is obtained by taking the inverse cosine of the expression in Equation 1.8.15.

Example 1.8.2

Three dogs are pulling on a stick in different directions, as shown in Figure 1.8.2. The first dog pulls with force $\vec{F}_1$ = (10.0 $\hat{i}$ – 20.4 $\hat{j}$ + 2.0 $\hat{k}$)N, the second dog pulls with force $\vec{F}_2$ = (–15.0 $\hat{i}$ – 6.2 $\hat{k}$)N , and the third dog pulls with force $\vec{F}_3$ = (5.0 $\hat{i}$ + 12.5 $\hat{j}$)N . What is the angle between forces $\vec{F}_1$ and $\vec{F}_2$?

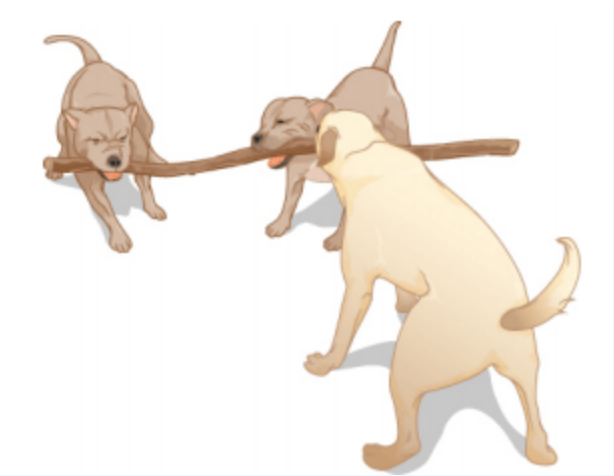


Figure 1.8.2: Three dogs are playing with a stick.

Strategy

The components of force vector $\vec{F}_1$ are F_{1x} = 10.0 N, F_{1y}= −20.4 N, and F_{1z} = 2.0 N, whereas those of force vector $\vec{F}_2$ are F_{2x} = −15.0 N, F_{2y} = 0.0 N, and F_{2z} = −6.2 N. Computing the scalar product of these vectors and their magnitudes, and substituting into Equation 1.8.15 gives the angle of interest.

Solution

The magnitudes of forces $\vec{F}_1$ and $\vec{F}_2$ are

$$F_1 = \sqrt{F_{1x}^2 + F_{1y}^2 + F_{1z}^2} = \sqrt{10.0^2 + 20.4^2 + 2.0^2} N = 22.8\ N \tag{1.8.16}$$

and

$$F_2 = \sqrt{F_{2x}^2 + F_{2y}^2 + F_{2z}^2} = \sqrt{15.0^2 + 6.2^2} N = 16.2\ N. \tag{1.8.17}$$

Substituting the scalar components into Equation 1.8.14 yields the scalar product

$$\begin{aligned} \vec{F}_1 \cdot \vec{F}_2 &= F_{1x}F_{2x} + F_{1y}F_{2y} + F_{1z}F_{2z} \\ &= (10.0\ N)(-15.0\ N) + (-20.4\ N)(0.0\ N) + (2.0\ N)(-6.2\ N) \\ &= -162.4\ N^2. \end{aligned} \tag{1.8.18}$$

Finally, substituting everything into Equation 1.8.15 gives the angle

$$\cos\varphi = \frac{\vec{F}_1 \cdot \vec{F}_2}{F_1 F_2} = \frac{-162.4\ N^2}{(22.8\ N)(16.2\ N)} = -0.439 \Rightarrow \varphi = \cos^{-1}(-0.439) = 116.0^o. \tag{1.8.19}$$

Significance

Notice that when vectors are given in terms of the unit vectors of axes, we can find the angle between them without knowing the specifics about the geographic directions the unit vectors represent. Here, for example, the +x-direction might be to the east and the +y-direction might be to the north. But, the angle between the forces in the problem is the same if the +x-direction is to the west and the +y-direction is to the south.

Exercise 2.13

Find the angle between forces $\vec{F}_1$ and $\vec{F}_3$ in Example 1.8.2.

Example 1.8.3: The Work of a Force

When force $\vec{F}$ pulls on an object and when it causes its displacement $\vec{D}$, we say the force performs work. The amount of work the force does is the scalar product $\vec{F} \cdot \vec{D}$. If the stick in Example 1.8.2 moves momentarily and gets displaced by vector $\vec{D}$ = (–7.9 $\hat{j}$ – 4.2 $\hat{k}$)cm, how much work is done by the third dog in Example 1.8.2?

Strategy

We compute the scalar product of displacement vector $\vec{D}$ with force vector $\vec{F}_3$ = (5.0 $\hat{i}$ + 12.5 $\hat{j}$)N, which is the pull from the third dog. Let's use W_3 to denote the work done by force $\vec{F}_3$ on displacement $\vec{D}$.

Solution

Calculating the work is a straightforward application of the dot product:

$$\begin{aligned} W_3 = \vec{F}_3 \cdot \vec{D} &= F_{3x}D_x + F_{3y}D_y + F_{3z}D_z \\ &= (5.0\ N)(0.0\ cm) + (12.5\ N)(-7.9\ cm) + (0.0\ N)(-4.2\ cm) \\ &= -98.7\ N \cdot cm. \end{aligned} \tag{1.8.20}$$

Significance

The SI unit of work is called the joule (J) , where 1 J = 1 N · m. The unit cm · N can be written as 10^{-2} m · N = 10^{-2} J, so the answer can be expressed as W_3 = –0.9875 J ≈ –1.0 J.

Exercise 2.14

How much work is done by the first dog and by the second dog in Example 1.8.2 on the displacement in Example 1.8.3?

Contributors and Attributions

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1.9: Products of Vectors (Part 2)

The Vector Products of Two Vectors (the Cross Product)

Vector multiplication of two vectors yields a vector product.

Vector Product (Cross Product)

The vector product of two vectors $\vec{A}$ and $\vec{B}$ is denoted by $\vec{A} \times \vec{B}$ and is often referred to as a cross product. The vector product is a vector that has its direction perpendicular to both vectors $\vec{A}$ and $\vec{B}$. In other words, vector $\vec{A} \times \vec{B}$ is perpendicular to the plane that contains vectors $\vec{A}$ and $\vec{B}$, as shown in Figure 1.9.1. The magnitude of the vector product is defined as

$$|\vec{A} \times \vec{B}| = AB \sin \varphi, \tag{1.9.1}$$

where angle φ, between the two vectors, is measured from vector $\vec{A}$ (first vector in the product) to vector $\vec{B}$ (second vector in the product), as indicated in Figure 1.9.1, and is between 0° and 180°.

According to Equation 1.9.1, the vector product vanishes for pairs of vectors that are either parallel (φ = 0°) or antiparallel (φ = 180°) because sin 0° = sin 180° = 0.

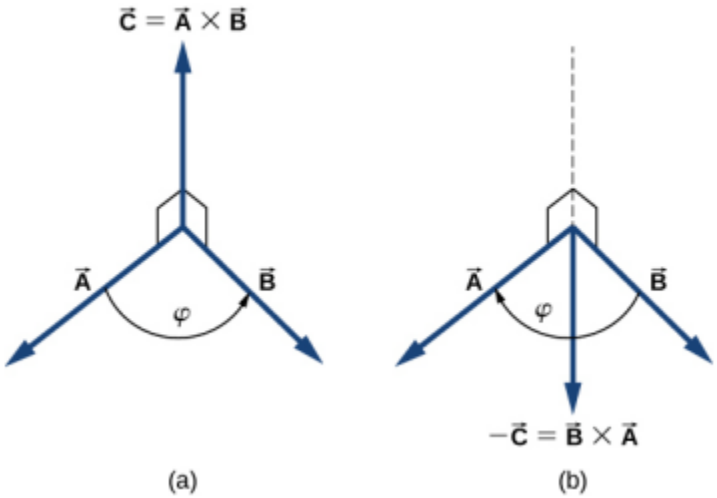


Figure 1.9.1: The vector product of two vectors is drawn in three-dimensional space. (a) The vector product $\vec{A} \times \vec{B}$ is a vector perpendicular to the plane that contains vectors $\vec{A}$ and $\vec{B}$. Small squares drawn in perspective mark right angles between $\vec{A}$ and $\vec{C}$, and between $\vec{B}$ and $\vec{C}$ so that if $\vec{A}$ and $\vec{B}$ lie on the floor, vector $\vec{B}$ points vertically upward to the ceiling. (b) The vector product $\vec{B} \times \vec{A}$ is a vector antiparallel to vector $\vec{A} \times \vec{B}$.

On the line perpendicular to the plane that contains vectors $\vec{A}$ and $\vec{B}$ there are two alternative directions—either up or down, as shown in Figure 1.9.1—and the direction of the vector product may be either one of them. In the standard right-handed orientation, where the angle between vectors is measured counterclockwise from the first vector, vector $\vec{A} \times \vec{B}$ points **upward**, as seen in Figure 1.9.1(a). If we reverse the order of multiplication, so that now $\vec{B}$ comes first in the product, then vector $\vec{B} \times \vec{A}$ must point **downward**, as seen in Figure 1.9.1(b). This means that vectors $\vec{A} \times \vec{B}$ and $\vec{B} \times \vec{A}$ are **antiparallel** to each other and that vector multiplication is **not** commutative but **anticommutative**. The **anticommutative property** means the vector product reverses the sign when the order of multiplication is reversed:

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}. \tag{1.9.2}$$

The **corkscrew right-hand rule** is a common mnemonic used to determine the direction of the vector product. As shown in

Figure 1.9.2, a corkscrew is placed in a direction perpendicular to the plane that contains vectors $\vec{A}$ and $\vec{B}$, and its handle is turned in the direction from the first to the second vector in the product. The direction of the cross product is given by the progression of the corkscrew.

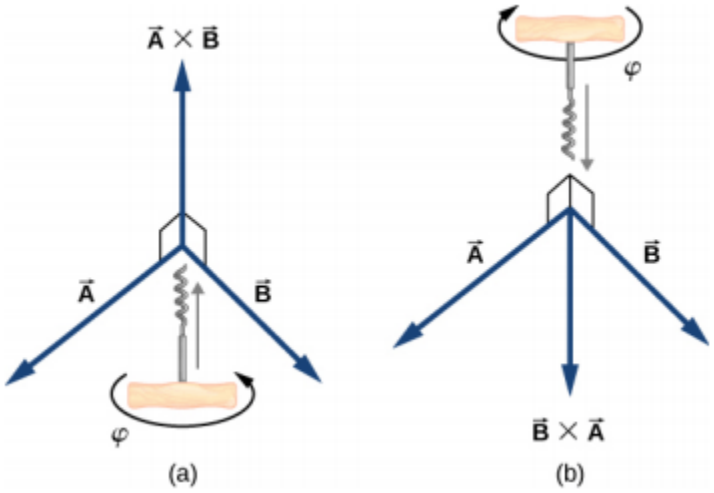


Figure 1.9.2: The corkscrew right-hand rule can be used to determine the direction of the cross product $\vec{A} \times \vec{B}$. Place a corkscrew in the direction perpendicular to the plane that contains vectors $\vec{A}$ and $\vec{B}$, and turn it in the direction from the first to the second vector in the product. The direction of the cross product is given by the progression of the corkscrew. (a) Upward movement means the cross-product vector points up. (b) Downward movement means the cross-product vector points downward.

Example 1.9.1: The Torque of a Force

The mechanical advantage that a familiar tool called a **wrench** provides (Figure 1.9.3) depends on magnitude F of the applied force, on its direction with respect to the wrench handle, and on how far from the nut this force is applied. The distance R from the nut to the point where force vector $\vec{F}$ is attached is called the **lever arm** and is represented by the radial vector $\vec{R}$. The physical vector quantity that makes the nut turn is called **torque** (denoted by $\vec{\tau}$), and it is the vector product of the lever arm with the force: $\vec{\tau} = \vec{R} \times \vec{F}$.

To loosen a rusty nut, a 20.00-N force is applied to the wrench handle at angle φ = 40° and at a distance of 0.25 m from the nut, as shown in Figure 1.9.3(a). Find the magnitude and direction of the torque applied to the nut. What would the magnitude and direction of the torque be if the force were applied at angle φ = 45°, as shown in Figure 1.9.3(b)? For what value of angle φ does the torque have the largest magnitude?

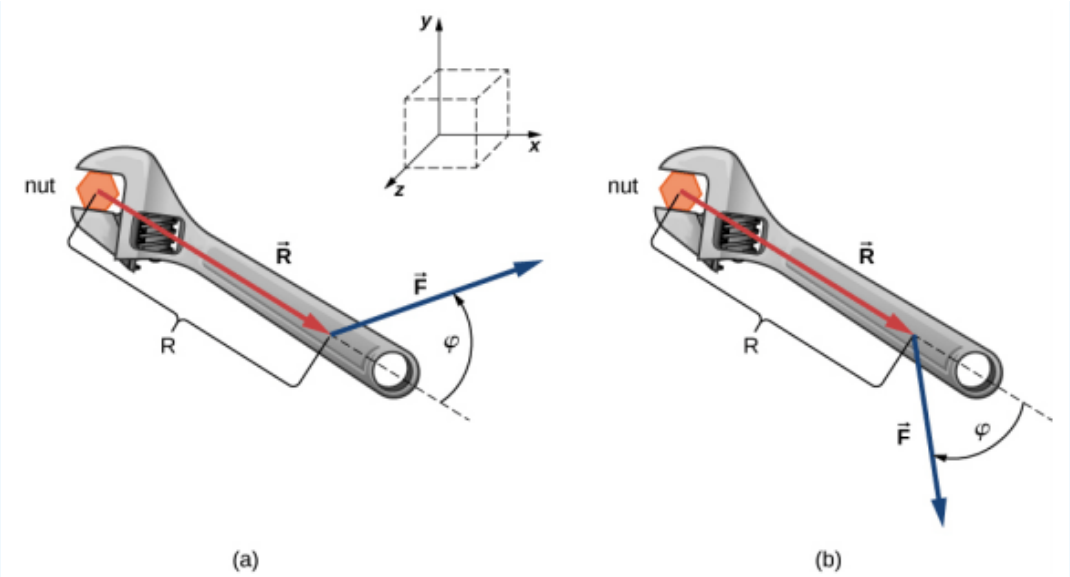


Figure 1.9.3: A wrench provides grip and mechanical advantage in applying torque to turn a nut. (a) Turn counterclockwise to loosen the nut. (b) Turn clockwise to tighten the nut.

Strategy

We adopt the frame of reference shown in Figure 1.9.3, where vectors $\vec{R}$ and $\vec{F}$ lie in the xy-plane and the origin is at the position of the nut. The radial direction along vector $\vec{R}$ (pointing away from the origin) is the reference direction for measuring the angle φ because $\vec{R}$ is the first vector in the vector product $\vec{\tau} = \vec{R} \times \vec{F}$. Vector $\vec{\tau}$ must lie along the z-axis because this is the axis that is perpendicular to the xy-plane, where both $\vec{R}$ and $\vec{F}$ lie. To compute the magnitude τ, we use Equation 1.9.1. To find the direction of $\vec{\tau}$, we use the corkscrew right-hand rule (Figure 1.9.2).

Solution

For the situation in (a), the corkscrew rule gives the direction of $\vec{R} \times \vec{F}$ in the positive direction of the z-axis. Physically, it means the torque vector $\vec{\tau}$ points out of the page, perpendicular to the wrench handle. We identify F = 20.00 N and R = 0.25 m, and compute the magnitude using Equation 1.9.1:

$$\tau = |\vec{R} \times \vec{F}| = RF \sin \varphi = (0.25\ m)(20.00\ N) \sin 40^o = 3.21\ N \cdot m. \tag{1.9.3}$$

For the situation in (b), the corkscrew rule gives the direction of $\vec{R} \times \vec{F}$ in the negative direction of the z-axis. Physically, it means the vector $\vec{\tau}$ points into the page, perpendicular to the wrench handle. The magnitude of this torque is

$$\tau = |\vec{R} \times \vec{F}| = RF \sin \varphi = (0.25\ m)(20.00\ N) \sin 45^o = 3.53\ N \cdot m. \tag{1.9.4}$$

The torque has the largest value when sin φ = 1, which happens when φ = 90°. Physically, it means the wrench is most effective—giving us the best mechanical advantage—when we apply the force perpendicular to the wrench handle. For the situation in this example, this best-torque value is τ_{best} = RF = (0.25 m)(20.00 N) = 5.00 N • m.

Significance

When solving mechanics problems, we often do not need to use the corkscrew rule at all, as we'll see now in the following equivalent solution. Notice that once we have identified that vector $\vec{R} \times \vec{F}$ lies along the z-axis, we can write this vector in terms of the unit vector $\hat{k}$ of the z-axis:

$$\vec{R} \times \vec{F} = RF \sin \varphi \hat{k}. \tag{1.9.5}$$

In this equation, the number that multiplies $\hat{k}$ is the scalar z-component of the vector $\vec{R} \times \vec{F}$. In the computation of this component, care must be taken that the angle φ is measured counterclockwise from $\vec{R}$ (first vector) to $\vec{F}$ (second vector) Following this principle for the angles, we obtain RF sin (+ 40°) = + 3.2 N • m for the situation in (a), and we obtain RF sin

(−45°) = −3.5 N • m for the situation in (b). In the latter case, the angle is negative because the graph in Figure 1.9.3 indicates the angle is measured clockwise; but, the same result is obtained when this angle is measured counterclockwise because +(360° − 45°) = + 315° and sin (+ 315°) = sin (−45°). In this way, we obtain the solution without reference to the corkscrew rule. For the situation in (a), the solution is $\vec{R} \times \vec{F}$ = + 3.2 N • m $\hat{k}$; for the situation in (b), the solution is $\vec{R} \times \vec{F}$ = −3.5 N • m $\hat{k}$.

Exercise 2.15

For the vectors given in Figure 2.3.6, find the vector products $\vec{A} \times \vec{B}$ and $\vec{C} \times \vec{F}$.

Similar to the dot product (Equation 2.8.10), the cross product has the following distributive property:

$$\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}. \tag{1.9.6}$$

The distributive property is applied frequently when vectors are expressed in their component forms, in terms of unit vectors of Cartesian axes. When we apply the definition of the cross product, Equation 1.9.1, to unit vectors $\hat{i}$, $\hat{j}$, and $\hat{k}$ that define the positive x-, y-, and z-directions in space, we find that

$$\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0. \tag{1.9.7}$$

All other cross products of these three unit vectors must be vectors of unit magnitudes because $\hat{i}$, $\hat{j}$, and $\hat{k}$ are orthogonal. For example, for the pair $\hat{i}$ and $\hat{j}$, the magnitude is $|\hat{i} \times \hat{j}|$ = ij sin 90° = (1)(1)(1) = 1. The direction of the vector product $\hat{i} \times \hat{j}$ must be orthogonal to the xy-plane, which means it must be along the z-axis. The only unit vectors along the z-axis are $-\hat{k}$ or $+\hat{k}$. By the corkscrew rule, the direction of vector $\hat{i} \times \hat{j}$ must be parallel to the positive z-axis. Therefore, the result of the multiplication $\hat{i} \times \hat{j}$ is identical to + $\hat{k}$. We can repeat similar reasoning for the remaining pairs of unit vectors. The results of these multiplications are

$$\begin{cases} \hat{i} \times \hat{j} = +\hat{k}, \\ \hat{j} \times \hat{k} = +\hat{i}, \\ \hat{k} \times \hat{i} = +\hat{j}. \end{cases} \tag{1.9.8}$$

Notice that in Equation 1.9.8, the three unit vectors $\hat{i}$, $\hat{j}$, and $\hat{k}$ appear in the cyclic order shown in a diagram in Figure 1.9.4(a). The cyclic order means that in the product formula, $\hat{i}$ follows $\hat{k}$ and comes before $\hat{j}$, or $\hat{k}$ follows $\hat{j}$ and comes before $\hat{i}$, or $\hat{j}$ follows $\hat{i}$ and comes before $\hat{k}$. The cross product of two different unit vectors is always a third unit vector. When two unit vectors in the cross product appear in the cyclic order, the result of such a multiplication is the remaining unit vector, as illustrated in Figure 1.9.4(b). When unit vectors in the cross product appear in a different order, the result is a unit vector that is antiparallel to the remaining unit vector (i.e., the result is with the minus sign, as shown by the examples in Figure 1.9.4(c) and Figure 1.9.4(d). In practice, when the task is to find cross products of vectors that are given in vector component form, this rule for the cross-multiplication of unit vectors is very useful.

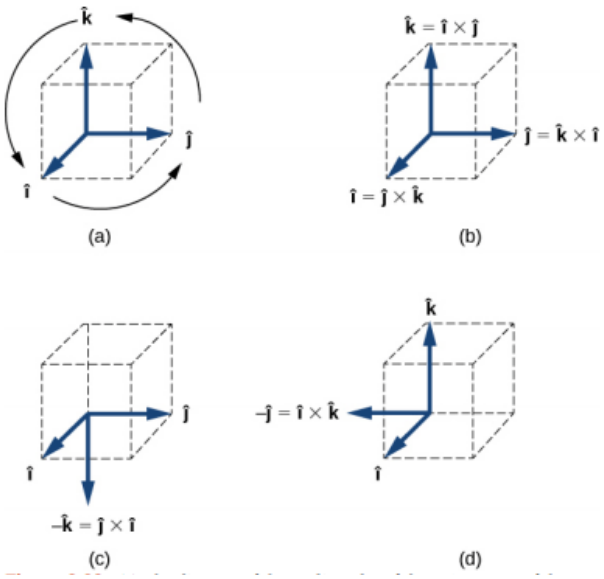


Figure 1.9.4: (a) The diagram of the cyclic order of the unit vectors of the axes. (b) The only cross products where the unit vectors appear in the cyclic order. These products have the positive sign. (c, d) Two examples of cross products where the unit vectors do not appear in the cyclic order. These products have the negative sign.

Suppose we want to find the cross product $\vec{A} \times \vec{B}$ for vectors $\vec{A} = A_x\,\hat{i} + A_y\,\hat{j} + A_z\,\hat{k}$ and $\vec{B} = B_x\,\hat{i} + B_y\,\hat{j} + B_z\,\hat{k}$. We can use the distributive property (Equation 1.9.6), the anticommutative property (Equation 1.9.2), and the results in Equation 1.9.7 and Equation 1.9.8 for unit vectors to perform the following algebra:

$$\begin{aligned}
\vec{A} \times \vec{B} &= (A_x\,\hat{i} + A_y\,\hat{j} + A_z\,\hat{k}) \times (B_x\,\hat{i} + B_y\,\hat{j} + B_z\,\hat{k}) \\
&= A_x\,\hat{i} \times (B_x\,\hat{i} + B_y\,\hat{j} + B_z\,\hat{k}) + A_y\,\hat{j} \times (B_x\,\hat{i} + B_y\,\hat{j} + B_z\,\hat{k}) + A_z\,\hat{k} \times (B_x\,\hat{i} + B_y\,\hat{j} + B_z\,\hat{k}) \\
&= A_xB_x\,\hat{i} \times \hat{i} + A_xB_y\,\hat{i} \times \hat{j} + A_zB_z\,\hat{i} \times \hat{k} \\
&+ A_yB_x\,\hat{j} \times \hat{i} + A_yB_y\,\hat{j} \times \hat{j} + A_yB_z\,\hat{j} \times \hat{k} \\
&+ A_zB_x\,\hat{k} \times \hat{i} + A_zB_y\,\hat{k} \times \hat{j} + A_zB_z\,\hat{k} \times \hat{k} \\
&= A_xB_x(0) + A_xB_y(+\hat{k}) + A_xB_z(-\hat{j}) \\
&+ A_yB_x(-\hat{k}) + A_yB_y(0) + A_yB_z(+\hat{i}) \\
&+ A_zB_x(+\hat{j}) + A_zB_y(-\hat{i}) + A_zB_z(0).
\end{aligned} \tag{1.9.9}$$

When performing algebraic operations involving the cross product, be very careful about keeping the correct order of multiplication because the cross product is anticommutative. The last two steps that we still have to do to complete our task are, first, grouping the terms that contain a common unit vector and, second, factoring. In this way we obtain the following very useful expression for the computation of the cross product:

$$\vec{C} = \vec{A} \times \vec{B} = (A_yB_z - A_zB_y)\,\hat{i} + (A_zB_x - A_xB_z)\,\hat{j} + (A_xB_y - A_yB_x)\,\hat{k}. \tag{1.9.10}$$

In this expression, the scalar components of the cross-product vector are

$$\begin{cases} C_x = A_yB_z - A_zB_y, \\ C_y = A_zB_x - A_xB_z, \\ C_z = A_xB_y - A_yB_x. \end{cases} \tag{1.9.11}$$

When finding the cross product, in practice, we can use either Equation 1.9.1 or Equation 1.9.10, depending on which one of them seems to be less complex computationally. They both lead to the same final result. One way to make sure if the final result is correct is to use them both.

Example 1.9.2: A Particle in a Magnetic Field

When moving in a magnetic field, some particles may experience a magnetic force. Without going into details—a detailed study of magnetic phenomena comes in later chapters—let's acknowledge that the magnetic field $\vec{B}$ is a vector, the magnetic force $\vec{F}$ is a vector, and the velocity $\vec{u}$ of the particle is a vector. The magnetic force vector is proportional to the vector product of the velocity vector with the magnetic field vector, which we express as $\vec{F} = \zeta\vec{u} \times \vec{B}$. In this equation, a constant ζ takes care of the consistency in physical units, so we can omit physical units on vectors $\vec{u}$ and $\vec{B}$. In this example, let's assume the constant ζ is positive. A particle moving in space with velocity vector $\vec{u}$ = −5.0 $\hat{i}$ − 2.0 $\hat{j}$+ 3.5 $\hat{k}$ enters a region with a magnetic field and experiences a magnetic force. Find the magnetic force $\vec{F}$ on this particle at the entry point to the region where the magnetic field vector is (a) $\vec{B}$ = 7.2 $\hat{i}$ − $\hat{j}$ − 2.4 $\hat{k}$ and (b) $\vec{B}$ = 4.5 $\hat{k}$. In each case, find magnitude F of the magnetic force and angle θ the force vector $\vec{F}$ makes with the given magnetic field vector $\vec{B}$.

Strategy

First, we want to find the vector product $\vec{u} \times \vec{B}$, because then we can determine the magnetic force using $\vec{F} = \zeta\vec{u} \times \vec{B}$. Magnitude F can be found either by using components, F = $\sqrt{F_x^2 + F_y^2 + F_z^2}$, or by computing the magnitude $|\vec{u} \times \vec{B}|$ directly using Equation 1.9.1. In the latter approach, we would have to find the angle between vectors $\vec{u}$ and $\vec{B}$. When we have $\vec{F}$, the general method for finding the direction angle θ involves the computation of the scalar product $\vec{F} \cdot \vec{B}$ and substitution into Equation 2.8.13. To compute the vector product we can either use Equation 1.9.10 or compute the product directly, whichever way is simpler.

Solution

The components of the velocity vector are u_x = −5.0, u_y = −2.0, and u_z = 3.5. (a) The components of the magnetic field vector are B_x = 7.2, B_y = −1.0, and B_z = −2.4. Substituting them into Equation 1.9.11 gives the scalar components of vector $\vec{F} = \zeta\vec{u} \times \vec{B}$:

$$\begin{cases} F_x = \zeta(u_y B_z - u_z B_y) = \zeta[(-2.0)(-2.4) - (3.5)(-1.0)] = 8.3\zeta \\ F_y = \zeta(u_z B_x - u_x B_z) = \zeta[(3.5)(7.2) - (-5.0)(-2.4)] = 13.2\zeta \\ F_z = \zeta(u_x B_y - u_y B_x) = \zeta[(-5.0)(-1.0) - (-2.0)(7.2)] = 19.4\zeta \end{cases} \tag{1.9.12}$$

Thus, the magnetic force is $\vec{F}$ = ζ(8.3 $\hat{i}$ + 13.2 $\hat{j}$ + 19.4 $\hat{k}$) and its magnitude is

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2} = \zeta\sqrt{(8.3)^2 + (13.2)^2 + (19.4)^2} = 24.9\zeta. \tag{1.9.13}$$

To compute angle θ, we may need to find the magnitude of the magnetic field vector

$$B = \sqrt{B_x^2 + B_y^2 + B_z^2} = \sqrt{(7.2)^2 + (-1.0)^2 + (-2.4)^2} = 7.6, \tag{1.9.14}$$

and the scalar product $\vec{F} \cdot \vec{B}$:

$$\vec{F} \cdot \vec{B} = F_x B_x + F_y B_y + F_z B_z = (8.3\zeta)(7.2) + (13.2\zeta)(-1.0) + (19.4\zeta)(-2.4) =. \tag{1.9.15}$$

Now, substituting into Equation 2.8.13 gives angle θ:

$$\cos\theta = \frac{\vec{F} \cdot \vec{B}}{FB} = \frac{0}{(18.2\zeta)(7.6)} = 0 \Rightarrow \theta = 90^o. \tag{1.9.16}$$

Hence, the magnetic force vector is perpendicular to the magnetic field vector. (We could have saved some time if we had computed the scalar product earlier.)

(b) Because vector $\vec{B}$ = 4.5 $\hat{k}$ has only one component, we can perform the algebra quickly and find the vector product directly:

$$\begin{aligned}\vec{F} &= \zeta\vec{u}\times\vec{B} = \zeta(-5.0\hat{i}-2.0\hat{j}+3.5\hat{k})\times(4.5\hat{k})\\ &= \zeta[(-5.0)(4.5)\hat{i}\times\hat{k}+(-2.0)(4.5)\hat{j}\times\hat{k}+(3.5)(4.5)\hat{k}\times\hat{k}]\\ &= \zeta[-22.5(-\hat{j})-9.0(+\hat{i})+0] = \zeta(-9.0\hat{i}+22.5\hat{j}).\end{aligned} \tag{1.9.17}$$

The magnitude of the magnetic force is

$$F=\sqrt{F_x^2+F_y^2+F_z^2}=\zeta\sqrt{(-9.0)^2+(22.5)^2+(0.0)^2}=24.2\zeta. \tag{1.9.18}$$

Because the scalar product is

$$\vec{F}\cdot\vec{B}=F_xB_x+F_yB_y+F_zB_z=(-9.0\zeta)(90)+(22.5\zeta)(0)+(0)(4.5)=0, \tag{1.9.19}$$

the magnetic force vector $\vec{F}$ is perpendicular to the magnetic field vector $\vec{B}$.

Significance

Even without actually computing the scalar product, we can predict that the magnetic force vector must always be perpendicular to the magnetic field vector because of the way this vector is constructed. Namely, the magnetic force vector is the vector product $\vec{F}$ = $\zeta\vec{u}\times\vec{B}$ and, by the definition of the vector product (see Figure 1.9.1), vector $\vec{F}$ must be perpendicular to both vectors $\vec{u}$ and $\vec{B}$.

Exercise 2.16

Given two vectors $\vec{A}=-\hat{i}+\hat{j}$ and $\vec{B}$ = 3 $\hat{i}$ – $\hat{j}$, find (a) $\vec{A}\times\vec{B}$, (b) |$\vec{A}\times(\vec{B}$|, (c) the angle between $\vec{A}$ and $\vec{B}$, and (d) the angle between $\vec{A}\times\vec{B}$ and vector $\vec{C}=\hat{i}+\hat{k}$.

In conclusion to this section, we want to stress that “dot product” and “cross product” are entirely different mathematical objects that have different meanings. The dot product is a scalar; the cross product is a vector. Later chapters use the terms **dot product** and **scalar product** interchangeably. Similarly, the terms **cross product** and **vector product** are used interchangeably.

Contributors

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1.10: Vectors (Answers)

Check Your Understanding

2.1. a. not equal because they are orthogonal;

b. not equal because they have different magnitudes;

c. not equal because they have different magnitudes and directions;

d. not equal because they are antiparallel;

e. equal.

2.2. 16 m; $\vec{D} = -16m\hat{u}$

2.3. G = 28.2 cm, $\theta_G = 291\,°$

2.4. $\vec{D} = (-5.0\hat{i} - 3.0\hat{j})cm$; the fly moved 5.0 cm to the left and 3.0 cm down from its landing site.

2.5. 5.83 cm, $211\,°$

2.6. $\vec{D} = (-20m)\hat{j}$

2.7. 35.1 m/s = 126.4 km/h

2.8. $\vec{G} = (10.25\hat{i} - 26.22\hat{j})cm$

2.9. D = 55.7 N; direction $65.7\,°$ north of east

2.10. $\hat{v} = 0.8\hat{i} + 0.6\hat{j}, 36.87\,°$ north of east

2.11. $\vec{A} \cdot \vec{B} = -57.3, \vec{F} \cdot \vec{C} = 27.8$

2.13. $131.9\,°$

2.14. $W_1 = 1.5J, W_2 = 0.3J$

2.15. $\vec{A} \times \vec{B} = -40.1\hat{k}$ or, equivalently, $|\vec{A} \times \vec{B}| = 40.1$, and the direction is into the page; $\vec{C} \times \vec{F} = +157.6\hat{k}$ or, equivalently, $|\vec{C} \times \vec{F}| = 157.6$, and the direction is out of the page.

2.16. a. $-2\hat{k}$,

b. 2,

c. $153.4\,°$,

d. $135\,°$

Conceptual Questions

1. scalar

3. answers may vary

5. parallel, sum of magnitudes, antiparallel, zero

7. no, yes

9. zero, yes

11. no

13. equal, equal, the same

15. a unit vector of the **x**-axis

17. They are equal.

19. yes

21. a. $C = \vec{A} \cdot \vec{B}$

b. $\vec{C} = \vec{A} \cdot \vec{B}$ or $\vec{C} = \vec{A} - \vec{B}$

c. $\vec{C} = \vec{A} \times \vec{B}$,

d. $\vec{C} = A\vec{B}$,

e. $\vec{C} + 2\vec{A} = \vec{B}$,

f. $\vec{C} = \vec{A} \times \vec{B}$,

g. left side is a scalar and right side is a vector,

h. $\vec{C} = 2\vec{A} \times \vec{B}$,

i. $\vec{C} = \vec{A}/B$,

j. $\vec{C} = \vec{A}/B$

23. They are orthogonal.

Problems

25. $\vec{h} = -49m\hat{u}$, 49 m

27. 30.8 m, $35.7\,^\circ$ west of north

29. 134 km, $80\,^\circ$

31. 7.34 km, $63.5\,^\circ$ south of east

33. 3.8 km east, 3.2 km north, 7.0 km

35. 14.3 km, $65\,^\circ$

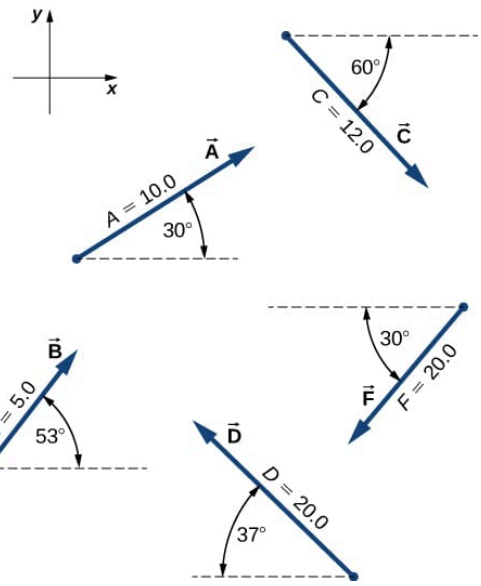
37. a. $\vec{A} = +8.66\hat{i} + 5.00\hat{j}$,

b. $\vec{B} = +3.01\hat{i} + 3.99\hat{j}$,

c. $\vec{C} = +6.00\hat{i} - 10.39\hat{j}$,

d. $\vec{D} = -15.97\hat{i} + 12.04\hat{j}$,

f. $\vec{F} = -17.32\hat{i} - 10.00\hat{j}$



39. a. 1.94 km, 7.24 km;

b. proof

41. 3.8 km east, 3.2 km north, 2.0 km, \(\vec{D}=(3.8\hat{i}+3.2\hat{j})km

43. $P_1(2.165m, 1.250m), P_2(-1.900m, 3.290m), 5.27m$

45. 8.60 m, $A(2\sqrt{5}m, 0.647\pi), B(3\sqrt{2}m, 0.75\pi)$

47. a. $\vec{A} + \vec{B} = -4\hat{i} - 6\hat{j}$, $|\vec{A} + \vec{B}| = 7.211, \theta = 236.3°$;

b. $\vec{A} - \vec{B} = -2\hat{i} + 2\hat{j}$, $|\vec{A} - \vec{B}| = 2\sqrt{2}, \theta = 135°$

49. a. $\vec{C} = (5.0\hat{i} - 1.0\hat{j} - 3.0\hat{k})m, C = 5.92m$;

b. $\vec{D} = (4.0\hat{i} - 11.0\hat{j} + 15.0\hat{k})m, D = 19.03m$.

51. $\vec{D} = (3.3\hat{i} - 6.6\hat{j})km, \hat{i}istotheeast, 7.34km, -63.5°$

53. a.$\vec{R} = -1.35\hat{i} - 22.04\hat{j}$,

b. $\vec{R} = -17.98\hat{i} + 0.89\hat{j}$

55. $\vec{D} = (200\hat{i} + 300\hat{j})yd$, D = 360.5 yd, $56.3°$ north of east; The numerical answers would stay the same but the physical unit would be meters. The physical meaning and distances would be about the same because 1 yd is comparable with 1 m.

57. $\vec{R} = -3\hat{i} - 16\hat{j}$

59. $\vec{E} = E\hat{E}, E_x = +178.9V/m, E_y = -357.8V/m, E_z = 0.0V/m, \theta_E = -tan^{-1}(2)$

61. a. $\vec{R}_B = (12.278\hat{i} + 7.089\hat{j} + 2.500\hat{k})km$, \($\vec{R}_D = (-0.262\hat{i} + 3.000\hat{k})km$;

b. $|\vec{R}_B - \vec{R}_D| = 14.414km$|R→B−R→D|=14.414km

63. a. 8.66,

b. 10.39,

c. 0.866,

d. 17.32

65. $\theta_i = 64.12\,°, \theta_j = 150.79\,°, \theta_k = 77.39\,°$

67. a. $-119.98\hat{k}$

b. $0\hat{k}$,

c. $+93.69\hat{k}$,

d. $-240.0\hat{k}$,

e. $+3.993\hat{k}$,

f. $-3.009\hat{k}$,

g. $+14.99\hat{k}$,

h. 0

69. a. 0,

b. 173,194,

c. +199,993$\hat{k}$

Additional Problems

71. a. 18.4 km and 26.2 km,

b. 31.5 km and 5.56 km

73. a. $(r, \varphi + \pi/2)$,

b. $(2r, \varphi + 2\pi)($,

c. $(3r, -\varphi)$

75. $d_{PM} = 33.12nmi = 61.34km, d_{NP} = 35.47nmi = 65.69km$

77. proof

79. a. 10.00 m,

b. $5\pi m$,

c. 0

81. 22.2 km/h, $35.8\,°$. south of west

83. 240.2 m, $2.2\,°$ south of west

85. $\vec{B} = -4.0\hat{i} + 3.0\hat{j}$ or $\vec{B} = 4.0\hat{i} - 3.0\hat{j}$

87. proof

Challenge Problems

89. $G_{\perp} = 2375\sqrt{17} \approx 9792$

91. proof

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1.11: Vectors (Exercises)

Conceptual Questions

2.1 Scalars and Vectors

1. A weather forecast states the temperature is predicted to be –5 °C the following day. Is this temperature a vector or a scalar quantity? Explain.
2. Which of the following is a vector: a person's height, the altitude on Mt. Everest, the velocity of a fly, the age of Earth, the boiling point of water, the cost of a book, Earth's population, or the acceleration of gravity?
3. Give a specific example of a vector, stating its magnitude, units, and direction.
4. What do vectors and scalars have in common? How do they differ?
5. Suppose you add two vectors $\vec{A}$ and $\vec{B}$. What relative direction between them produces the resultant with the greatest magnitude? What is the maximum magnitude? What relative direction between them produces the resultant with the smallest magnitude? What is the minimum magnitude?
6. Is it possible to add a scalar quantity to a vector quantity?
7. Is it possible for two vectors of different magnitudes to add to zero? Is it possible for three vectors of different magnitudes to add to zero? Explain.
8. Does the odometer in an automobile indicate a scalar or a vector quantity?
9. When a 10,000-m runner competing on a 400-m track crosses the finish line, what is the runner's net displacement? Can this displacement be zero? Explain.
10. A vector has zero magnitude. Is it necessary to specify its direction? Explain.
11. Can a magnitude of a vector be negative?
12. Can the magnitude of a particle's displacement be greater that the distance traveled?
13. If two vectors are equal, what can you say about their components? What can you say about their magnitudes? What can you say about their directions?
14. If three vectors sum up to zero, what geometric condition do they satisfy?

2.2 Coordinate Systems and Components of a Vector

15. Give an example of a nonzero vector that has a component of zero.
16. Explain why a vector cannot have a component greater than its own magnitude.
17. If two vectors are equal, what can you say about their components?
18. If vectors $\vec{A}$ and $\vec{B}$ are orthogonal, what is the component of $\vec{B}$ along the direction of $\vec{A}$? What is the component of $\vec{A}$ along the direction of $\vec{B}$?
19. If one of the two components of a vector is not zero, can the magnitude of the other vector component of this vector be zero?
20. If two vectors have the same magnitude, do their components have to be the same?

2.4 Products of Vectors

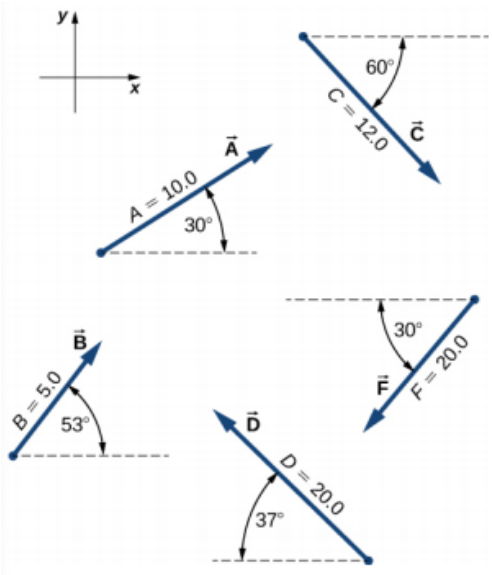
21. What is wrong with the following expressions? How can you correct them?
    a. $C = \vec{A}\vec{B}$,
    b. $\vec{C} = \vec{A}\vec{B}$,
    c. $C = \vec{A} \times \vec{B}$,
    d. $C = A\vec{B}$,
    e. $C + 2\vec{A} = B$,
    f. $\vec{C} = A \times \vec{B}$,
    g. $\vec{A} \cdot \vec{B} = \vec{A} \times \vec{B}$,
    h. $\vec{C} = 2\vec{A} \cdot \vec{B}$,
    i. $C = \vec{A}/\vec{B}$, and
    j. $C = \vec{A}/B$.

22. If the cross product of two vectors vanishes, what can you say about their directions?
23. If the dot product of two vectors vanishes, what can you say about their directions?
24. What is the dot product of a vector with the cross product that this vector has with another vector?

Problems

2.1 Scalars and Vectors

25. A scuba diver makes a slow descent into the depths of the ocean. His vertical position with respect to a boat on the surface changes several times. He makes the first stop 9.0 m from the boat but has a problem with equalizing the pressure, so he ascends 3.0 m and then continues descending for another 12.0 m to the second stop. From there, he ascends 4 m and then descends for 18.0 m, ascends again for 7 m and descends again for 24.0 m, where he makes a stop, waiting for his buddy. Assuming the positive direction up to the surface, express his net vertical displacement vector in terms of the unit vector. What is his distance to the boat?
26. In a tug-of-war game on one campus, 15 students pull on a rope at both ends in an effort to displace the central knot to one side or the other. Two students pull with force 196 N each to the right, four students pull with force 98 N each to the left, five students pull with force 62 N each to the left, three students pull with force 150 N each to the right, and one student pulls with force 250 N to the left. Assuming the positive direction to the right, express the net pull on the knot in terms of the unit vector. How big is the net pull on the knot? In what direction?
27. Suppose you walk 18.0 m straight west and then 25.0 m straight north. How far are you from your starting point and what is the compass direction of a line connecting your starting point to your final position? Use a graphical method.
28. For the vectors given in the following figure, use a graphical method to find the following resultants:
    a. $\vec{A}+\vec{B}$,
    b. $\vec{C}+\vec{B}$,
    c. $\vec{D}+\vec{F}$,
    d. $\vec{A}-\vec{B}$,
    e. $\vec{D}-\vec{F}$,
    f. $\vec{A}+2\vec{F}$,
    g. $\vec{A}-4\vec{D}+2\vec{F}$.



29. A delivery man starts at the post office, drives 40 km north, then 20 km west, then 60 km northeast, and finally 50 km north to stop for lunch. Use a graphical method to find his net displacement vector.
30. An adventurous dog strays from home, runs three blocks east, two blocks north, one block east, one block north, and two blocks west. Assuming that each block is about 100 m, how far from home and in what direction is the dog? Use a graphical method.
31. In an attempt to escape a desert island, a castaway builds a raft and sets out to sea. The wind shifts a great deal

during the day and he is blown along the following directions: 2.50 km and 45.0° north of west, then 4.70 km and 60.0° south of east, then 1.30 km and 25.0° south of west, then 5.10 km straight east, then 1.70 km and 5.00° east of north, then 7.20 km and 55.0° south of west, and finally 2.80 km and 10.0° north of east. Use a graphical method to find the castaway's final position relative to the island.

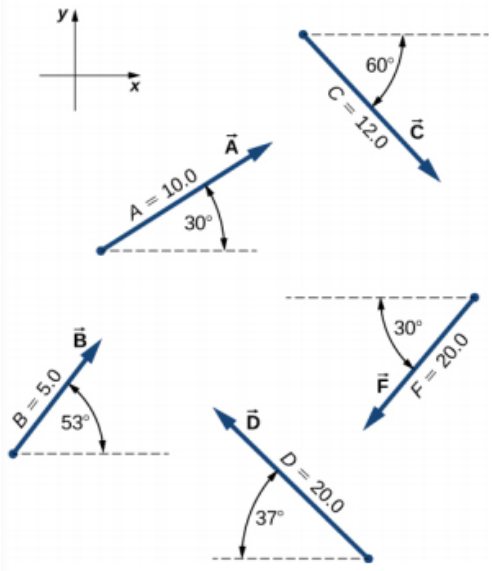
32. A small plane flies 40.0 km in a direction 60° north of east and then flies 30.0 km in a direction 15° north of east. Use a graphical method to find the total distance the plane covers from the starting point and the direction of the path to the final position.
33. A trapper walks a 5.0-km straight-line distance from his cabin to the lake, as shown in the following figure. Use a graphical method (the parallelogram rule) to determine the trapper's displacement directly to the east and displacement directly to the north that sum up to his resultant displacement vector. If the trapper walked only in directions east and north, zigzagging his way to the lake, how many kilometers would he have to walk to get to the lake?



34. A surveyor measures the distance across a river that flows straight north by the following method. Starting directly across from a tree on the opposite bank, the surveyor walks 100 m along the river to establish a baseline. She then sights across to the tree and reads that the angle from the baseline to the tree is 35°. How wide is the river?
35. A pedestrian walks 6.0 km east and then 13.0 km north. Use a graphical method to find the pedestrian's resultant displacement and geographic direction.
36. The magnitudes of two displacement vectors are A = 20 m and B = 6 m. What are the largest and the smallest values of the magnitude of the resultant $\vec{R} = \vec{A} + \vec{B}$?

2.2 Coordinate Systems and Components of a Vector

37. Assuming the +x-axis is horizontal and points to the right, resolve the vectors given in the following figure to their scalar components and express them in vector component form.



38. Suppose you walk 18.0 m straight west and then 25.0 m straight north. How far are you from your starting point? What is your displacement vector? What is the direction of your displacement? Assume the +x-axis is to the east.
39. You drive 7.50 km in a straight line in a direction 15° east of north. (a) Find the distances you would have to drive straight east and then straight north to arrive at the same point. (b) Show that you still arrive at the same point if the east and north legs are reversed in order. Assume the +xaxis is to the east.
40. A sledge is being pulled by two horses on a flat terrain. The net force on the sledge can be expressed in the Cartesian coordinate system as vector $\vec{F}$= (–2980.0 $\hat{i}$ + 8200.0 $\hat{j}$)N , where $\hat{i}$ and $\hat{j}$ denote directions to the east and north, respectively. Find the magnitude and direction of the pull.
41. A trapper walks a 5.0-km straight-line distance from her cabin to the lake, as shown in the following figure. Determine the east and north components of her displacement vector. How many more kilometers would she have to walk if she walked along the component displacements? What is her displacement vector?



42. The polar coordinates of a point are $\frac{4\pi}{3}$ and 5.50 m. What are its Cartesian coordinates?
43. Two points in a plane have polar coordinates P_1 (2.500 m, $\frac{\pi}{6}$) and P_2 (3.800 m, $\frac{2\pi}{3}$). Determine their Cartesian coordinates and the distance between them in the Cartesian coordinate system. Round the distance to a nearest centimeter.
44. A chameleon is resting quietly on a lanai screen, waiting for an insect to come by. Assume the origin of a Cartesian coordinate system at the lower left-hand corner of the screen and the horizontal direction to the right as the +x-direction. If its coordinates are (2.000 m, 1.000 m), (a) how far is it from the corner of the screen? (b) What is its location in polar coordinates?
45. Two points in the Cartesian plane are A(2.00 m, –4.00 m) and B(–3.00 m, 3.00 m). Find the distance between them and their polar coordinates.
46. A fly enters through an open window and zooms around the room. In a Cartesian coordinate system with three axes along three edges of the room, the fly changes its position from point b(4.0 m, 1.5 m, 2.5 m) to point e(1.0 m, 4.5 m, 0.5 m). Find the scalar components of the fly's displacement vector and express its displacement vector in vector component form. What is its magnitude?

2.3 Algebra of Vectors

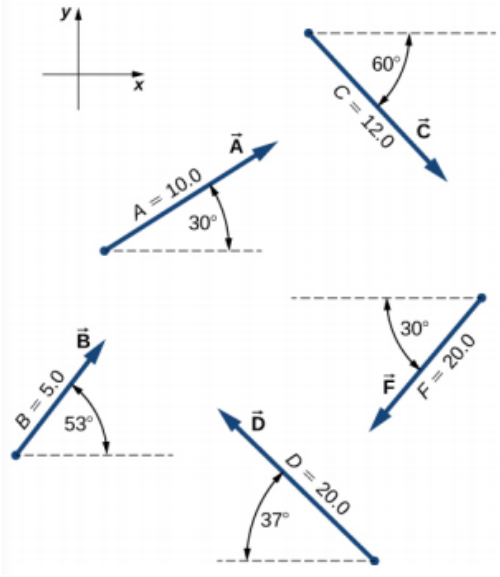
47. For vectors $\vec{B} = -\hat{i} - 4\hat{j}$ and $\vec{A} = -3\hat{i} - 2\hat{j}$, calculate (a) $\vec{A} + \vec{B}$ and its magnitude and direction angle, and (b) $\vec{A} - \vec{B}$ and its magnitude and direction angle.
48. A particle undergoes three consecutive displacements given by vectors $\vec{D}_1$ = (3.0 $\hat{i}$ – 4.0 $\hat{j}$ – 2.0 $\hat{k}$)mm, $\vec{D}_2$ = (1.0 $\hat{i}$ – 7.0 $\hat{j}$ + 4.0 $\hat{i}$)mm, and $\vec{D}_3$ = (–7.0 $\hat{i}$ + 4.0 $\hat{j}$ + 1.0 $\hat{k}$)mm. (a) Find the resultant displacement vector of the particle. (b) What is the magnitude of the resultant displacement? (c) If all displacements were along one line, how far would the particle travel?
49. Given two displacement vectors $\vec{A}$ = (3.00 $\hat{i}$ – 4.00 $\hat{j}$ + 4.00 $\hat{k}$)m and $\vec{B}$ = (2.00 $\hat{i}$ + 3.00 $\hat{j}$ – 7.00 $\hat{k}$)m, find the displacements and their magnitudes for (a) $\vec{C} = \vec{A} + \vec{B}$ and (b) $\vec{D} = 2\vec{A} - \vec{B}$.
50. A small plane flies 40.0 km in a direction 60° north of east and then flies 30.0 km in a direction 15° north of east. Use the analytical method to find the total distance the plane covers from the starting point, and the geographic direction of its displacement vector. What is its displacement vector?
51. . In an attempt to escape a desert island, a castaway builds a raft and sets out to sea. The wind shifts a great deal during the day, and she is blown along the following straight lines: 2.50 km and 45.0° north of west, then 4.70 km and 60.0° south of east, then 1.30 km and 25.0° south of west, then 5.10 km due east, then 1.70 km and 5.00° east of north, then 7.20 km and 55.0° south of west, and finally 2.80 km and 10.0° north of east. Use the analytical method to find the

resultant vector of all her displacement vectors. What is its magnitude and direction?

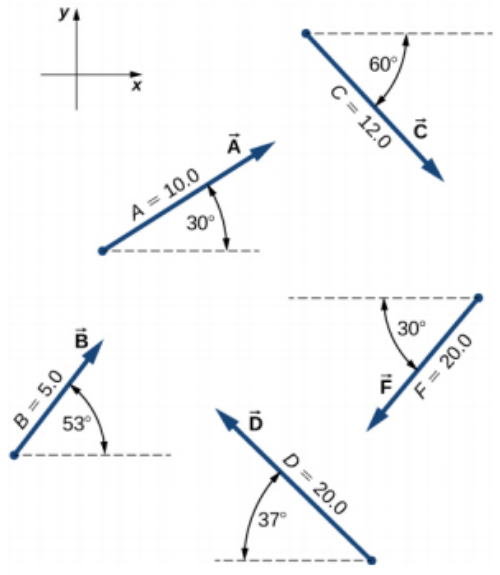
52. Assuming the +x-axis is horizontal to the right for the vectors given in the following figure, use the analytical method to find the following resultants:
    a. $\vec{A} + \vec{B}$,
    b. $\vec{C} + \vec{B}$,
    c. $\vec{D} + \vec{F}$,
    d. $\vec{A} - \vec{B}$,
    e. $\vec{D} - \vec{F}$,
    f. $\vec{A} + 2\vec{F}$,
    g. $\vec{C} - 2\vec{B} + 3\vec{F}$, and
    h. $\vec{A} - 4\vec{D} + 2\vec{F}$.
53. Given the vectors in the preceding figure, find vector $\vec{R}$ that solves equations (a) $\vec{D} + \vec{R} = \vec{F}$ and (b) $\vec{C} - 2\vec{D} + 5\vec{R} = 3\vec{F}$. Assume the +x-axis is horizontal to the right.
54. A delivery man starts at the post office, drives 40 km north, then 20 km west, then 60 km northeast, and finally 50 km north to stop for lunch. Use the analytical method to determine the following: (a) Find his net displacement vector. (b) How far is the restaurant from the post office? (c) If he returns directly from the restaurant to the post office, what is his displacement vector on the return trip? (d) What is his compass heading on the return trip? Assume the +x-axis is to the east.
55. An adventurous dog strays from home, runs three blocks east, two blocks north, and one block east, one block north, and two blocks west. Assuming that each block is about a 100 yd, use the analytical method to find the dog's net displacement vector, its magnitude, and its direction. Assume the +x-axis is to the east. How would your answer be affected if each block was about 100 m?
56. If $\vec{D}$ = (6.00 $\hat{i}$ – 8.00 $\hat{j}$m, $\vec{B}$ = (–8.00 $\hat{i}$ + 3.00 $\hat{j}$)m , and $\vec{A}$ = (26.0 $\hat{i}$ + 19.0 $\hat{j}$)m, find the unknown constants a and b such that a $\vec{D} + b\vec{B} + \vec{A} = \vec{0}$.
57. Given the displacement vector $\vec{D}$ = (3 $\hat{i}$ – 4 $\hat{j}$)m, find the displacement vector $\vec{R}$ so that $\vec{D}$ + $\vec{R}$ = –4D $\hat{j}$.
58. Find the unit vector of direction for the following vector quantities: (a) Force $\vec{F}$ = (3.0 $\hat{i}$ – 2.0 $\hat{j}$)N, (b) displacement $\vec{D}$ = (–3.0 $\hat{i}$ – 4.0 $\hat{j}$)m, and (c) velocity $\vec{v}$ = (–5.00 $\hat{i}$ + 4.00 $\hat{j}$)m/s.
59. At one point in space, the direction of the electric field vector is given in the Cartesian system by the unit vector $\hat{E} = \frac{1}{\sqrt{5}}\hat{i} - \frac{2}{\sqrt{5}}\hat{j}$. If the magnitude of the electric field vector is E = 400.0 V/m, what are the scalar components E_x, E_y, and E_z of the electric field vector $\vec{E}$ at this point? What is the direction angle θ_E of the electric field vector at this point?
60. A barge is pulled by the two tugboats shown in the following figure. One tugboat pulls on the barge with a force of magnitude 4000 units of force at 15° above the line AB (see the figure and the other tugboat pulls on the barge with a force of magnitude 5000 units of force at 12° below the line AB. Resolve the pulling forces to their scalar components and find the components of the resultant force pulling on the barge. What is the magnitude of the resultant pull? What is its direction relative to the line AB?
61. In the control tower at a regional airport, an air traffic controller monitors two aircraft as their positions change with respect to the control tower. One plane is a cargo carrier Boeing 747 and the other plane is a Douglas DC-3. The Boeing is at an altitude of 2500 m, climbing at 10° above the horizontal, and moving 30° north of west. The DC-3 is at an altitude of 3000 m, climbing at 5° above the horizontal, and cruising directly west. (a) Find the position vectors of the planes relative to the control tower. (b) What is the distance between the planes at the moment the air traffic controller makes a note about their positions?

2.4 Products of Vectors

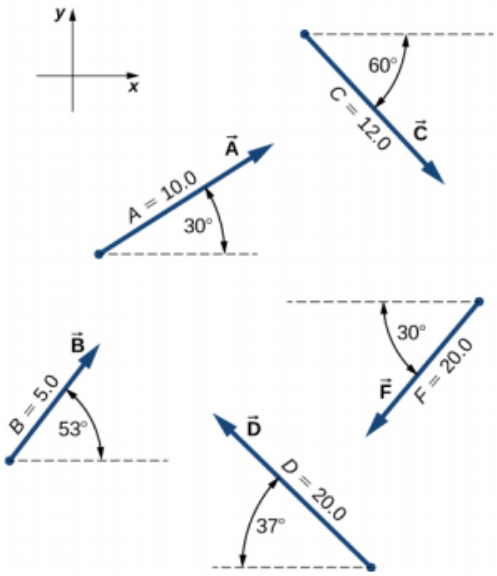
62. Assuming the +x-axis is horizontal to the right for the vectors in the following figure, find the following scalar products:
    a. $\vec{A} \cdot \vec{C}$,
    b. $\vec{A} \cdot \vec{F}$,
    c. $\vec{D} \cdot \vec{C}$,
    d. $\vec{A} \cdot (\vec{F} + 2\vec{C})$,
    e. $\hat{i} \cdot \vec{B}$,
    f. $\hat{j} \cdot \vec{B}$,
    g. $(3\hat{i} - \hat{j}) \cdot \vec{B}$ and
    h. $\hat{B} \cdot \vec{B}$.



63. Assuming the +x-axis is horizontal to the right for the vectors in the preceding figure, find (a) the component of vector $\vec{A}$ along vector $\vec{C}$, (b) the component of vector $\vec{C}$ along vector $\vec{A}$, (c) the component of vector $\hat{i}$ along vector $\vec{F}$, and (d) the component of vector $\vec{F}$ along vector $\hat{i}$.
64. Find the angle between vectors for
    a. $\vec{D}$ = (–3.0 $\hat{i}$ – 4.0 $\hat{j}$)m and $\vec{A}$ = (–3.0 $\hat{i}$ + 4.0 $\hat{j}$)m and
    b. $\vec{D}$ = (2.0 $\hat{i}$ – 4.0 $\hat{j}$ + $\hat{k}$)m and $\vec{B}$ = (–2.0 $\hat{i}$ + 3.0 $\hat{j}$ + 2.0 $\hat{k}$)m.
65. Find the angles that vector $\vec{D}$ = (2.0 $\hat{i}$ – 4.0 $\hat{j}$ + $\hat{k}$)m makes with the x-, y-, and z-axes.
66. Show that the force vector $\vec{D}$ = (2.0 $\hat{i}$ – 4.0 $\hat{j}$ + $\hat{k}$)N is orthogonal to the force vector $\vec{G}$ = (3.0 $\hat{i}$ + 4.0 $\hat{j}$+ 10.0 $\hat{k}$)N.
67. Assuming the +x-axis is horizontal to the right for the vectors in the following figure, find the following vector products:
    a. $\vec{A} \times \vec{C}$,
    b. $\vec{A} \times \vec{F}$,
    c. $\vec{D} \times \vec{C}$
    d. $\vec{A} \times (\vec{F} + 2\vec{C})$,
    e. $\hat{i} \times \vec{B}$,
    f. $\hat{j} \times \vec{B}$,
    g. $(3\hat{i} - \hat{j}) \times \vec{B}$ and
    h. $\hat{B} \times \vec{B}$.



68. Find the cross product $\vec{A} \times \vec{C}$ for
    a. $\vec{A}$ = 2.0 $\hat{i}$ – 4.0 $\hat{j}$ + $\hat{k}$ and $\vec{C}$ = 3.0 $\hat{i}$ + 4.0 $\hat{j}$ + 10.0$\hat{k}$,
    b. $\vec{A}$ = 3.0 $\hat{i}$ + 4.0 $\hat{j}$ + 10.0 $\hat{k}$ and $\vec{C}$ = 2.0 $\hat{i}$ – 4.0 $\hat{j}$ + $\hat{k}$,
    c. $\vec{A}$ = –3.0 $\hat{i}$ – 4.0 $\hat{j}$ and $\vec{C}$ = –3.0 $\hat{i}$ + 4.0 $\hat{j}$, and
    d. $\vec{C}$ = –2.0 $\hat{i}$ + 3.0 $\hat{j}$ + 2.0 $\hat{k}$ and $\vec{A}$ = –9.0 $\hat{j}$.
69. For the vectors in the following figure, find (a) $(\vec{A} \times \vec{F} \cdot \vec{D})$, (b) $(\vec{A} \times \vec{F}) \cdot (\vec{A} \times \vec{C})$, and (c) $(\vec{A} \cdot \vec{F})(\vec{D} \times \vec{B})$.



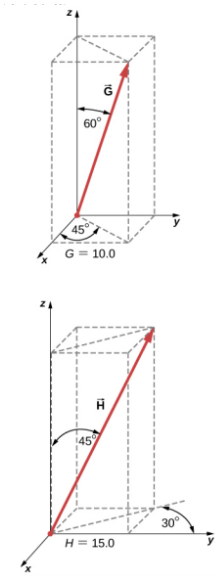
70. (a) If $\vec{A} \times \vec{F} = \vec{B} \times \vec{F}$, can we conclude $\vec{A} = \vec{B}$? (b) If $\vec{A} \cdot \vec{F} = \vec{B} \cdot \vec{F}$, can we conclude $\vec{A} = \vec{B}$? (c) If $F\vec{A} = \vec{B}F$, can we conclude $\vec{A} = \vec{B}$? Why or why not?

Additional Problems

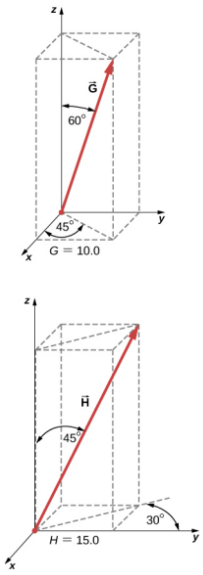
71. You fly 32.0 km in a straight line in still air in the direction 35.0° south of west. (a) Find the distances you would have to fly due south and then due west to arrive at the same point. (b) Find the distances you would have to fly first in a direction 45.0° south of west and then in a direction 45.0° west of north. Note these are the components of the displacement along a different set of axes—namely, the one rotated by 45° with respect to the axes in (a).
72. Rectangular coordinates of a point are given by (2, y) and its polar coordinates are given by (r, $\frac{\pi}{6}$). Find y and r.
73. If the polar coordinates of a point are (r,φ) and its rectangular coordinates are (x, y), determine the polar coordinates of the following points: (a) (–x, y), (b) (–2x, –2y), and (c) (3x, –3y).
74. Vectors $\vec{A}$ and $\vec{B}$ have identical magnitudes of 5.0 units. Find the angle between them if $\vec{A}$ + $\vec{B}$ = 5 2 $\hat{j}$.
75. Starting at the island of Moi in an unknown archipelago, a fishing boat makes a round trip with two stops at the islands of Noi and Poi. It sails from Moi for 4.76 nautical miles (nmi) in a direction 37° north of east to Noi. From Noi, it sails 69° west of north to Poi. On its return leg from Poi, it sails 28° east of south. What distance does the boat sail between Noi and Poi? What distance does it sail between Moi and Poi? Express your answer both in nautical miles and in kilometers. Note: 1 nmi = 1852 m.
76. An air traffic controller notices two signals from two planes on the radar monitor. One plane is at altitude 800 m and in a 19.2-km horizontal distance to the tower in a direction 25° south of west. The second plane is at altitude 1100 m and its horizontal distance is 17.6 km and 20° south of west. What is the distance between these planes?
77. Show that when $\vec{A}$ + $\vec{B}$ = $\vec{C}$, then $C^2 = A^2 + B^2 + 2AB \cos \varphi$, where φ is the angle between vectors $\vec{A}$ and $\vec{B}$.
78. Four force vectors each have the same magnitude f. What is the largest magnitude the resultant force vector may have when these forces are added? What is the smallest magnitude of the resultant? Make a graph of both situations.
79. A skater glides along a circular path of radius 5.00 m in clockwise direction. When he coasts around one-half of the circle, starting from the west point, find (a) the magnitude of his displacement vector and (b) how far he actually skated. (c) What is the magnitude of his displacement vector when he skates all the way around the circle and comes back to the west point?
80. A stubborn dog is being walked on a leash by its owner. At one point, the dog encounters an interesting scent at some

spot on the ground and wants to explore it in detail, but the owner gets impatient and pulls on the leash with force $\vec{F}$ = (98.0 $\hat{i}$ + 132.0 $\hat{j}$ + 32.0 $\hat{j}$)N along the leash. (a) What is the magnitude of the pulling force? (b) What angle does the leash make with the vertical?

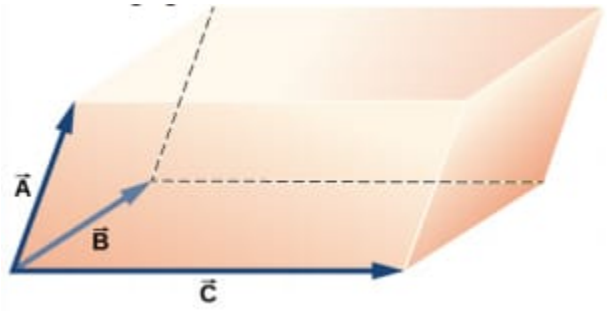
81. If the velocity vector of a polar bear is $\vec{u}$ = (–18.0 $\hat{i}$ – 13.0 $\hat{j}$)km/h , how fast and in what geographic direction is it heading? Here, $\hat{i}$ and $\hat{j}$ are directions to geographic east and north, respectively.
82. Find the scalar components of three-dimensional vectors $\vec{G}$ and $\vec{H}$ in the following figure and write the vectors in vector component form in terms of the unit vectors of the axes.



83. A diver explores a shallow reef off the coast of Belize. She initially swims 90.0 m north, makes a turn to the east and continues for 200.0 m, then follows a big grouper for 80.0 m in the direction 30° north of east. In the meantime, a local current displaces her by 150.0 m south. Assuming the current is no longer present, in what direction and how far should she now swim to come back to the point where she started?
84. A force vector $\vec{A}$ has x- and y-components, respectively, of –8.80 units of force and 15.00 units of force. The x- and y-components of force vector $\vec{B}$ are, respectively, 13.20 units of force and –6.60 units of force. Find the components of force vector $\vec{C}$ that satisfies the vector equation $\vec{A}$– $\vec{B}$ + 3 $\vec{C}$ = 0.
85. Vectors $\vec{A}$ and $\vec{B}$ are two orthogonal vectors in the xy-plane and they have identical magnitudes. If $\vec{A}$ = 3.0 $\hat{i}$ + 4.0 $\hat{j}$, find $\vec{B}$.
86. For the three-dimensional vectors in the following figure, find (a) $\vec{G} \times \vec{H}$, (b) $|\vec{G} \times \vec{H}|$, and (c) $\vec{G} \cdot \vec{H}$.

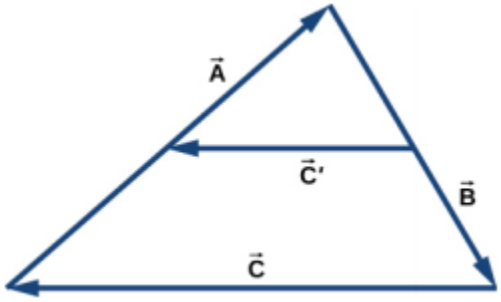


87. Show that $(\vec{B} \times \vec{C}) \cdot \vec{A}$ is the volume of the parallelepiped, with edges formed by the three vectors in the following figure.



Challenge Problems

88. Vector $\vec{B}$ is 5.0 cm long and vector $\vec{A}$ is 4.0 cm long. Find the angle between these two vectors when $|\vec{A} + \vec{B}|$ = 3.0 cm and $|\vec{A} - \vec{B}|$ = 3.0 cm.
89. What is the component of the force vector $\vec{G}$ = (3.0 $\hat{i}$ + 4.0 $\hat{j}$ + 10.0 $\hat{k}$)N along the force vector $\vec{H}$ = (1.0 $\hat{i}$ + 4.0 $\hat{j}$)N?
90. The following figure shows a triangle formed by the three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$. If vector $\vec{C}\,'$ is drawn between the midpoints of vectors $\vec{A}$ and $\vec{B}$, show that $\vec{C}\,' = \frac{\vec{C}}{2}$.



91. Distances between points in a plane do not change when a coordinate system is rotated. In other words, the magnitude of a vector is **invariant** under rotations of the coordinate system. Suppose a coordinate system S is rotated about its origin by angle φ to become a new coordinate system S′, as shown in the following figure. A point in a plane has coordinates (x, y) in S and coordinates (x′, y′) in S′.
    a. Show that, during the transformation of rotation, the coordinates in S′ are expressed in terms of the coordinates in S by the following relations:

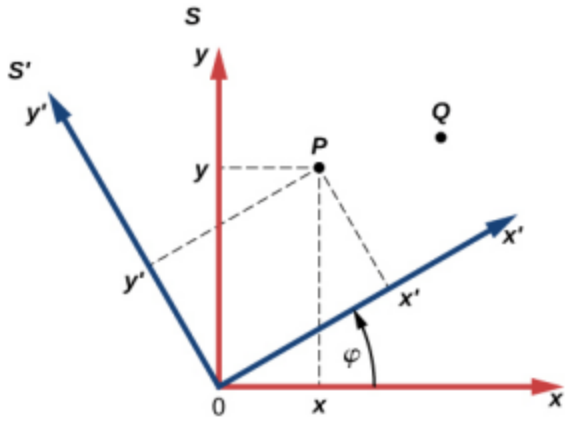
$$\begin{cases} x' = x\cos\varphi + y\sin\varphi \\ y' = -x\sin\varphi + y\cos\varphi \end{cases}. \tag{1.11.1}$$

    b. Show that the distance of point P to the origin is invariant under rotations of the coordinate system. Here, you have to show that

$$\sqrt{x^2 + y^2} = \sqrt{x'^2 + y'^2}. \tag{1.11.2}$$

    c. Show that the distance between points P and Q is invariant under rotations of the coordinate system. Here, you have to show that

$$\sqrt{(x_P - x_Q)^2 + (y_P - y_Q)^2} = \sqrt{(x'_P - x'_Q)^2 + (y'_P - y'_Q)^2}. \tag{1.11.3}$$



Contributors

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1.12: Vectors (Summary)

Key Terms

anticommutative property	change in the order of operation introduces the minus sign
antiparallel vectors	two vectors with directions that differ by 180°
associative	terms can be grouped in any fashion
commutative	operations can be performed in any order
component form of a vector	a vector written as the vector sum of its components in terms of unit vectors
corkscrew right-hand rule	a rule used to determine the direction of the vector product
cross product	the result of the vector multiplication of vectors is a vector called a cross product; also called a vector product
difference of two vectors	vector sum of the first vector with the vector antiparallel to the second
direction angle	in a plane, an angle between the positive direction of the x-axis and the vector, measured counterclockwise from the axis to the vector
displacement	change in position
distributive	multiplication can be distributed over terms in summation
dot product	the result of the scalar multiplication of two vectors is a scalar called a dot product; also called a scalar product
equal vectors	two vectors are equal if and only if all their corresponding components are equal; alternately, two parallel vectors of equal magnitudes
magnitude	length of a vector
null vector	a vector with all its components equal to zero
orthogonal vectors	two vectors with directions that differ by exactly 90°, synonymous with perpendicular vectors
parallel vectors	two vectors with exactly the same direction angles
parallelogram rule	geometric construction of the vector sum in a plane
polar coordinate system	an orthogonal coordinate system where location in a plane is given by polar coordinates
polar coordinates	a radial coordinate and an angle
radical coordinate	distance to the origin in a polar coordinate system
resultant vector	vector sum of two (or more) vectors
scalar	a number, synonymous with a scalar quantity in physics
scalar component	a number that multiplies a unit vector in a vector component of a vector
scalar equation	equation in which the left-hand and right-hand sides are numbers
scalar product	the result of the scalar multiplication of two vectors is a scalar called a scalar product; also called a dot product
scalar quantity	quantity that can be specified completely by a single number with an appropriate physical unit

tail-to-head geometric construction	geometric construction for drawing the resultant vector of many vectors
unit vector	vector of a unit magnitude that specifies direction; has no physical unit
unit vectors of the axes	unit vectors that define orthogonal directions in a plane or in space
vector	mathematical object with magnitude and direction
vector components	orthogonal components of a vector; a vector is the vector sum of its vector components
vector equation	equation in which the left-hand and right-hand sides are vectors
vector product	the result of the vector multiplication of vectors is a vector called a vector product; also called a cross product
vector quantity	physical quantity described by a mathematical vector—that is, by specifying both its magnitude and its direction; synonymous with a vector in physics
vector sum	resultant of the combination of two (or more) vectors

Key Equations

Multiplication by a scalar (vector equation)	$\vec{B} = \alpha\vec{A}$
Multiplication by a scalar (scalar equation for magnitudes)	$B = \lvert\alpha\rvert A$
Resultant of two vectors	$\vec{D}_{AD} = \vec{D}_{AC} + \vec{D}_{CD}$
Commutative law	$\vec{A} + \vec{B} = \vec{B} + \vec{A}$
Associative law	$(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C})$
Distributive law	$\alpha_1\vec{A} + \alpha_2\vec{A} = (\alpha_1 + \alpha_2)\vec{A}$
The component form of a vector in two dimensions	$\vec{A} = A_x\hat{i} + A_y\hat{j}$
Scalar components of a vector in two dimensions	$\begin{cases} A_x = x_e - x_b \\ A_y = y_e - y_b \end{cases}$
Magnitude of a vector in a plane	$A = \sqrt{A_x^2 + A_y^2}$
The direction angle of a vector in a plane	$\theta_A = \tan^{-1}\left(\frac{A_y}{A_x}\right)$
Scalar components of a vector in a plane	$\begin{cases} A_x = A\cos\theta_A \\ A_y = A\sin\theta_A \end{cases}$

Polar coordinates in a plane	$\begin{cases} x = r\cos\varphi \\ y = r\sin\varphi \end{cases}$
The component form of a vector in three dimensions	$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$
The scalar z-component of a vector in three dimensions	$A_z = z_e - z_b$
Magnitude of a vector in three dimensions	$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$
Distributive property	$\alpha(\vec{A} + \vec{B}) = \alpha\vec{A} + \alpha\vec{B}$
Antiparallel vector to $\vec{A}$	$-\vec{A} = A_x\hat{i} - A_y\hat{j} - A_z\hat{k}$
Equal vectors	$\vec{A} = \vec{B} \Leftrightarrow \begin{cases} A_x = B_x \\ A_y = B_y \\ A_z = B_z \end{cases}$
Components of the resultant of N vectors	$\begin{cases} F_{Rx} = \sum_{k=1}^{N} F_{kx} = F_{1x} + F_{2x} + ... + F_{Nx} \\ F_{Ry} = \sum_{k=1}^{N} F_{ky} = F_{1y} + F_{2y} + ... + F_{Ny} \\ F_{Rz} = \sum_{k=1}^{N} F_{kz} = F_{1z} + F_{2z} + ... + F_{Nz} \end{cases}$
General unit vector	$\hat{V} = \dfrac{\vec{V}}{V}$
Definition of the scalar product	$\vec{A}\cdot\vec{B} = AB\cos\varphi$
Commutative property of the scalar product	$\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$
Distributive property of the scalar product	$\vec{A}\cdot(\vec{B} + \vec{C}) = \vec{A}\cdot\vec{B} + \vec{A}\cdot\vec{C}$
Scalar product in terms of scalar components of vectors	$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$
Cosine of the angle between two vectors	$\cos\varphi = \dfrac{\vec{A}\cdot\vec{B}}{AB}$
Dot products of unit vectors	$\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$
Magnitude of the vector product (definition)	$\lvert\vec{A}\times\vec{B}\rvert = AB\sin\varphi$

Anticommutative property of the vector product	$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$
Distributive property of the vector product	$\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$
Cross products of unit vectors	$\begin{cases} \hat{i} \times \hat{j} = +\hat{k}, \\ \hat{j} \times \hat{l} = +\hat{i}, \\ \hat{l} \times \hat{i} = +\hat{j}. \end{cases}$
The cross product in terms of scalar components of vectors	$\vec{A} \times \vec{B} = (A_y B_z - A_z B_y)\hat{i} + (A_z B_x - A_x B_z)\hat{j} + (A_x B_y - A_y B_x)\hat{k}$

Summary

2.1 Scalars and Vectors

- A vector quantity is any quantity that has magnitude and direction, such as displacement or velocity.
- Geometrically, vectors are represented by arrows, with the end marked by an arrowhead. The length of the vector is its magnitude, which is a positive scalar. On a plane, the direction of a vector is given by the angle the vector makes with a reference direction, often an angle with the horizontal. The direction angle of a vector is a scalar.
- Two vectors are equal if and only if they have the same magnitudes and directions. Parallel vectors have the same direction angles but may have different magnitudes. Antiparallel vectors have direction angles that differ by 180°. Orthogonal vectors have direction angles that differ by 90°.
- When a vector is multiplied by a scalar, the result is another vector of a different length than the length of the original vector. Multiplication by a positive scalar does not change the original direction; only the magnitude is affected. Multiplication by a negative scalar reverses the original direction. The resulting vector is antiparallel to the original vector. Multiplication by a scalar is distributive. Vectors can be divided by nonzero scalars but cannot be divided by vectors.
- Two or more vectors can be added to form another vector. The vector sum is called the resultant vector. We can add vectors to vectors or scalars to scalars, but we cannot add scalars to vectors. Vector addition is commutative and associative.
- To construct a resultant vector of two vectors in a plane geometrically, we use the parallelogram rule. To construct a resultant vector of many vectors in a plane geometrically, we use the tail-to-head method.

2.2 Coordinate Systems and Components of a Vector

- Vectors are described in terms of their components in a coordinate system. In two dimensions (in a plane), vectors have two components. In three dimensions (in space), vectors have three components.
- A vector component of a vector is its part in an axis direction. The vector component is the product of the unit vector of an axis with its scalar component along this axis. A vector is the resultant of its vector components.
- Scalar components of a vector are differences of coordinates, where coordinates of the origin are subtracted from end point coordinates of a vector. In a rectangular system, the magnitude of a vector is the square root of the sum of the squares of its components.
- In a plane, the direction of a vector is given by an angle the vector has with the positive x-axis. This direction angle is measured counterclockwise. The scalar x-component of a vector can be expressed as the product of its magnitude with the cosine of its direction angle, and the scalar y-component can be expressed as the product of its magnitude with the sine of its direction angle.
- In a plane, there are two equivalent coordinate systems. The Cartesian coordinate system is defined by unit vectors $\hat{i}$ and $\hat{j}$ along the x-axis and the y-axis, respectively. The polar coordinate system is defined by the radial unit vector $\hat{r}$, which gives the direction from the origin, and a unit vector $\hat{t}$, which is perpendicular (orthogonal) to the radial direction.

2.3 Algebra of Vectors

- Analytical methods of vector algebra allow us to find resultants of sums or differences of vectors without having to draw them. Analytical methods of vector addition are exact, contrary to graphical methods, which are approximate.
- Analytical methods of vector algebra are used routinely in mechanics, electricity, and magnetism. They are important mathematical tools of physics.

2.4 Products of Vectors

- There are two kinds of multiplication for vectors. One kind of multiplication is the scalar product, also known as the dot product. The other kind of multiplication is the vector product, also known as the cross product. The scalar product of vectors is a number (scalar). The vector product of vectors is a vector.
- Both kinds of multiplication have the distributive property, but only the scalar product has the commutative property. The vector product has the anticommutative property, which means that when we change the order in which two vectors are multiplied, the result acquires a minus sign.
- The scalar product of two vectors is obtained by multiplying their magnitudes with the cosine of the angle between them. The scalar product of orthogonal vectors vanishes; the scalar product of antiparallel vectors is negative.
- The vector product of two vectors is a vector perpendicular to both of them. Its magnitude is obtained by multiplying their magnitudes by the sine of the angle between them. The direction of the vector product can be determined by the corkscrew right-hand rule. The vector product of two either parallel or antiparallel vectors vanishes. The magnitude of the vector product is largest for orthogonal vectors.
- The scalar product of vectors is used to find angles between vectors and in the definitions of derived scalar physical quantities such as work or energy.
- The cross product of vectors is used in definitions of derived vector physical quantities such as torque or magnetic force, and in describing rotations.

Contributors

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2: PHYSICS 1- REVIEW MOTION ALONG A STRAIGHT LINE

2.1: Prelude Motion Along a Straight Line

2.2: Position, Displacement, and Average Velocity

2.3: Instantaneous Velocity and Speed

2.4: Average and Instantaneous Acceleration

2.5: Motion with Constant Acceleration (Part 1)

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2.10: Motion Along a Straight Line (Summary)

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2.1: Prelude Motion Along a Straight Line

Our universe is full of objects in motion. From the stars, planets, and galaxies; to the motion of people and animals; down to the microscopic scale of atoms and molecules—everything in our universe is in motion. We can describe motion using the two disciplines of kinematics and dynamics. We study dynamics, which is concerned with the causes of motion, in Newton's Laws of Motion; but, there is much to be learned about motion without referring to what causes it, and this is the study of kinematics. Kinematics involves describing motion through properties such as position, time, velocity, and acceleration.



Figure 2.1.1: A JR Central L0 series five-car maglev (magnetic levitation) train undergoing a test run on the Yamanashi Test Track. The maglev train's motion can be described using kinematics, the subject of this chapter. (credit: modification of work by "Maryland GovPics"/Flickr)

A full treatment of **kinematics** considers motion in two and three dimensions. For now, we discuss motion in one dimension, which provides us with the tools necessary to study multidimensional motion. A good example of an object undergoing one-dimensional motion is the maglev (magnetic levitation) train depicted at the beginning of this chapter. As it travels, say, from Tokyo to Kyoto, it is at different positions along the track at various times in its journey, and therefore has displacements, or changes in position. It also has a variety of velocities along its path and it undergoes accelerations (changes in velocity). With the skills learned in this chapter we can calculate these quantities and average velocity. All these quantities can be described using kinematics, without knowing the train's mass or the forces involved.

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2.2: Position, Displacement, and Average Velocity

Learning Objectives

- Define position, displacement, and distance traveled.
- Calculate the total displacement given the position as a function of time.
- Determine the total distance traveled.
- Calculate the average velocity given the displacement and elapsed time.

When you're in motion, the basic questions to ask are: Where are you? Where are you going? How fast are you getting there? The answers to these questions require that you specify your position, your displacement, and your average velocity—the terms we define in this section.

Position

To describe the motion of an object, you must first be able to describe its **position** (x): **where it is at any particular time**. More precisely, we need to specify its position relative to a convenient frame of reference. A frame of reference is an arbitrary set of axes from which the position and motion of an object are described. Earth is often used as a frame of reference, and we often describe the position of an object as it relates to stationary objects on Earth. For example, a rocket launch could be described in terms of the position of the rocket with respect to Earth as a whole, whereas a cyclist's position could be described in terms of where she is in relation to the buildings she passes Figure 2.2.1. In other cases, we use reference frames that are not stationary but are in motion relative to Earth. To describe the position of a person in an airplane, for example, we use the airplane, not Earth, as the reference frame. To describe the position of an object undergoing onedimensional motion, we often use the variable x. Later in the chapter, during the discussion of free fall, we use the variable y.



Figure 2.2.1: These cyclists in Vietnam can be described by their position relative to buildings or a canal. Their motion can be described by their change in position, or displacement, in a frame of reference. (credit: Suzan Black)

Displacement

If an object moves relative to a frame of reference—for example, if a professor moves to the right relative to a whiteboard Figure 2.2.2—then the object's position changes. This change in position is called **displacement**. The word displacement implies that an object has moved, or has been displaced. Although position is the numerical value of x along a straight line where an object might be located, displacement gives the change in position along this line. Since displacement indicates direction, it is a vector and can be either positive or negative, depending on the choice of positive direction. Also, an analysis of motion can have many displacements embedded in it. If right is positive and an object moves 2 m to the right, then 4 m to the left, the individual displacements are 2 m and –4 m, respectively.

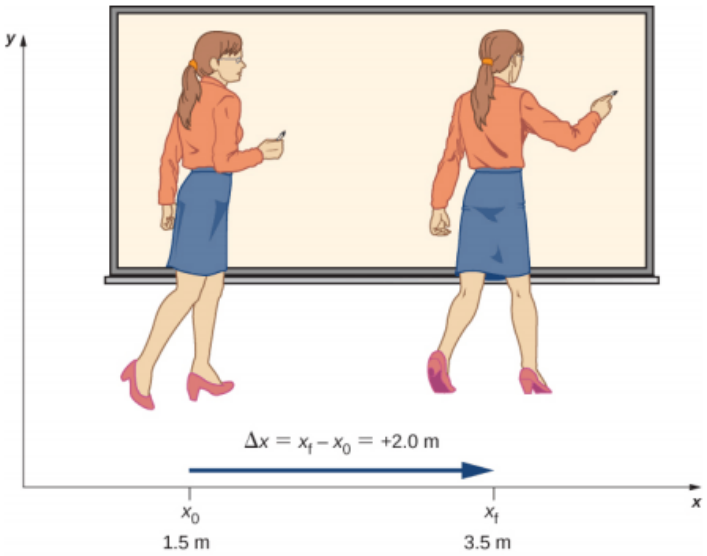


Figure 2.2.2: A professor paces left and right while lecturing. Her position relative to Earth is given by x. The +2.0-m displacement of the professor relative to Earth is represented by an arrow pointing to the right.

Displacement

Displacement Δx is the change in position of an object:

$$\Delta x = x_f - x_0, \tag{2.2.1}$$

where Δx is displacement, x_f is the final position, and x_0 is the initial position.

We use the uppercase Greek letter delta (Δ) to mean "change in" whatever quantity follows it; thus, Δx means **change in position** (final position less initial position). We always solve for displacement by subtracting initial position x_0 from final position x_f. Note that the SI unit for displacement is the meter, but sometimes we use kilometers or other units of length. Keep in mind that when units other than meters are used in a problem, you may need to convert them to meters to complete the calculation (see Appendix B).

Objects in motion can also have a series of displacements. In the previous example of the pacing professor, the individual displacements are 2 m and –4 m, giving a total displacement of –2 m. We define **total displacement** Δx_{Total}, as the **sum of the individual displacements**, and express this mathematically with the equation

$$\Delta x_{Total} = \sum \Delta x_i, \tag{2.2.2}$$

where δx_i are the individual displacements. In the earlier example,

$$\Delta x_1 = x_1 - x_0 = 2 - 0 = 2\ m. \tag{2.2.3}$$

Similarly,

$$\Delta x_2 = x_2 - x_1 = -2 - (2) = -4\ m. \tag{2.2.4}$$

Thus,

$$\Delta x_{total} = x_1 + x_2 = 2 - 4 = -2\ m. \tag{2.2.5}$$

The total displacement is 2 – 4 = –2 m to the left, or in the negative direction. It is also useful to calculate the magnitude of the displacement, or its size. The magnitude of the displacement is always positive. This is the absolute value of the displacement, because displacement is a vector and cannot have a negative value of magnitude. In our example, the magnitude of the total displacement is 2 m, whereas the magnitudes of the individual displacements are 2 m and 4 m.

The magnitude of the total displacement should not be confused with the distance traveled. Distance traveled x_{Total}, is the total length of the path traveled between two positions. In the previous problem, the **distance traveled** is the sum of the magnitudes of the individual displacements:

$$x_{total} = |x_1| + |x_2| = 2 + 4 = 6\ m. \tag{2.2.6}$$

Average Velocity

To calculate the other physical quantities in kinematics we must introduce the time variable. The time variable allows us not only to state where the object is (its position) during its motion, but also how fast it is moving. How fast an object is moving is given by the rate at which the position changes with time.

For each position x_i, we assign a particular time t_i. If the details of the motion at each instant are not important, the rate is usually expressed as the **average velocity** $\bar{v}$. This vector quantity is simply the total displacement between two points divided by the time taken to travel between them. The time taken to travel between two points is called the **elapsed time** Δt.

Average Velocity

If x_1 and x_2 are the positions of an object at times t_1 and t_2, respectively, then

$$\begin{aligned} Average\ velocity = \bar{v} &= \frac{Displacement\ between\ two\ points}{Elapsed\ time\ between\ two\ points} \\ \bar{v} &= \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}. \end{aligned} \tag{2.2.7}$$

It is important to note that the average velocity is a vector and can be negative, depending on positions x_1 and x_2.

Example 3.1: Delivering Flyers

Jill sets out from her home to deliver flyers for her yard sale, traveling due east along her street lined with houses. At 0.5 km and 9 minutes later she runs out of flyers and has to retrace her steps back to her house to get more. This takes an additional 9 minutes. After picking up more flyers, she sets out again on the same path, continuing where she left off, and ends up 1.0 km from her house. This third leg of her trip takes 15 minutes. At this point she turns back toward her house, heading west. After 1.75 km and 25 minutes she stops to rest.

a. What is Jill's total displacement to the point where she stops to rest?
b. What is the magnitude of the final displacement?
c. What is the average velocity during her entire trip?
d. What is the total distance traveled?
e. Make a graph of position versus time. A sketch of Jill's movements is shown in Figure 2.2.3.

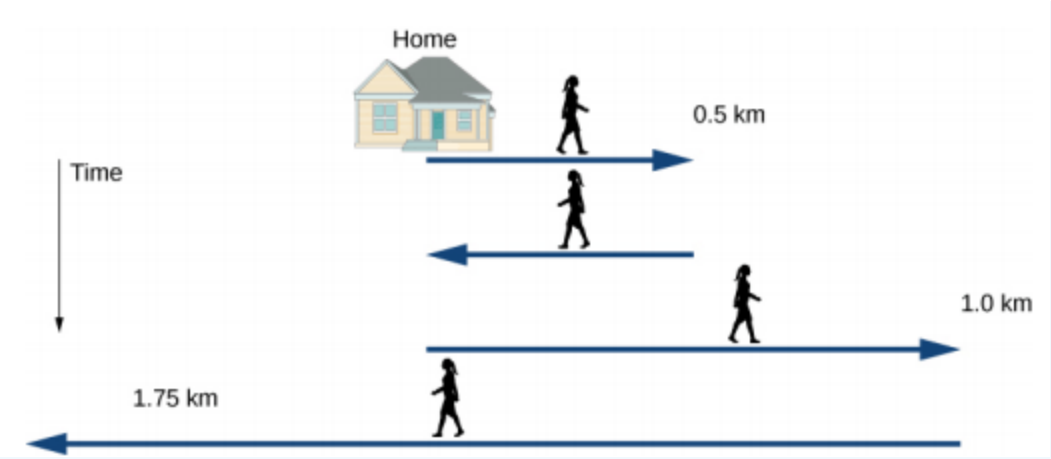


Figure 2.2.3: Timeline of Jill's movements.

Strategy

The problem contains data on the various legs of Jill's trip, so it would be useful to make a table of the physical quantities. We are given position and time in the wording of the problem so we can calculate the displacements and the elapsed time. We take east to be the positive direction. From this information we can find the total displacement and average velocity. Jill's home is the starting point x_0. The following table gives Jill's time and position in the first two columns, and the displacements are calculated in the third column.

Time t_i (min)	Position x_i (km)	Displacement Δx_i (km)
$t_0 = 0$	$x_0 = 0$	$\Delta x_0 = 0$
$t_1 = 9$	$x_1 = 0.5$	$\Delta x_1 = x_1 - x_0 = 0.5$
$t_2 = 18$	$x_2 = 0$	$\Delta x2 = x_2 - x_1 = -0.5$
$t_3 = 33$	$x_3 = 1.0$	$\Delta x_3 = x_3 - x_2 = 1.0$
$t_4 = 58$	$x_4 = -0.75$	$\Delta x_4 = x_4 - x_3 = -1.75$

Solution

a. From the above table, the total displacement is

$$\sum \Delta x_i = 0.5 - 0.5 + 1.0 - 1.75\ km = -0.75\ km. \tag{2.2.8}$$

b. The magnitude of the total displacement is |–0.75| km = 0.75 km.

c. $$Average\ velocity = \frac{Total\ displacement}{Elapsed\ time} = \bar{v} = \frac{-0.75\ km}{58\ min} = -0.013\ km/min \tag{2.2.9}$$

d. The total distance traveled (sum of magnitudes of individual displacements) is

$$x_{Total} = \sum |\Delta x_i| = 0.5 + 0.5 + 1.0 + 1.75\ km = 3.75\ km. \tag{2.2.10}$$

e. We can graph Jill's position versus time as a useful aid to see the motion; the graph is shown in Figure 2.2.4.

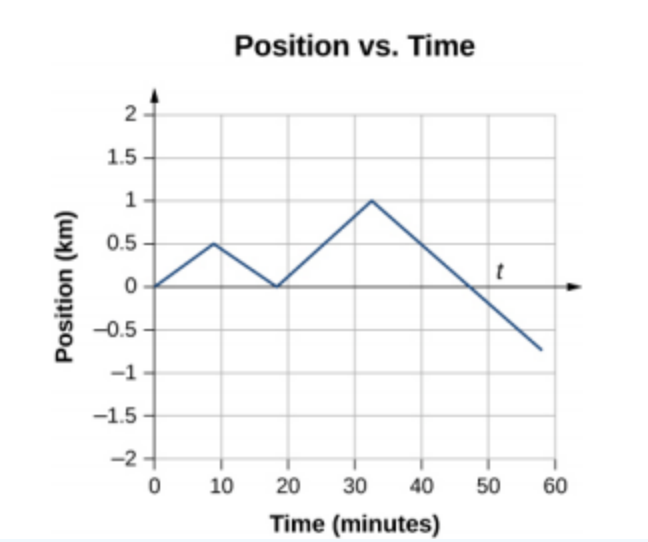


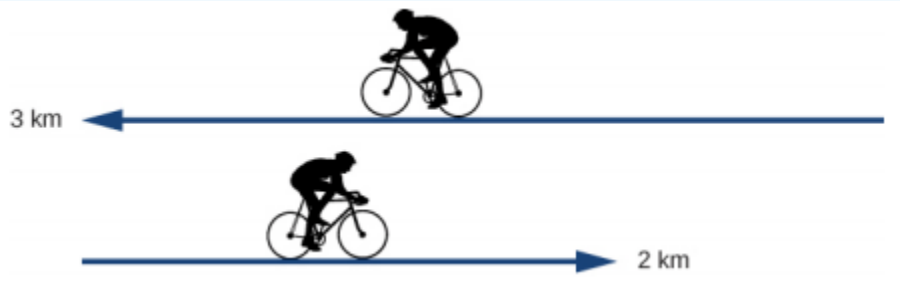
Figure 2.2.4: This graph depicts Jill's position versus time. The average velocity is the slope of a line connecting the initial and final points.

Significance

Jill's total displacement is −0.75 km, which means at the end of her trip she ends up 0.75 km due west of her home. The average velocity means if someone was to walk due west at 0.013 km/min starting at the same time Jill left her home, they both would arrive at the final stopping point at the same time. Note that if Jill were to end her trip at her house, her total displacement would be zero, as well as her average velocity. The total distance traveled during the 58 minutes of elapsed time for her trip is 3.75 km.

Exercise 3.1

A cyclist rides 3 km west and then turns around and rides 2 km east. (a) What is his displacement? (b) What is the distance traveled? (c) What is the magnitude of his displacement?



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2.3: Instantaneous Velocity and Speed

Learning Objectives

- Explain the difference between average velocity and instantaneous velocity.
- Describe the difference between velocity and speed.
- Calculate the instantaneous velocity given the mathematical equation for the velocity.
- Calculate the speed given the instantaneous velocity.

We have now seen how to calculate the average velocity between two positions. However, since objects in the real world move continuously through space and time, we would like to find the velocity of an object at any single point. We can find the velocity of the object anywhere along its path by using some fundamental principles of calculus. This section gives us better insight into the physics of motion and will be useful in later chapters.

Instantaneous Velocity

The quantity that tells us how fast an object is moving anywhere along its path is the **instantaneous velocity**, usually called simply **velocity**. It is the average velocity between two points on the path in the limit that the time (and therefore the displacement) between the two points approaches zero. To illustrate this idea mathematically, we need to express position x as a continuous function of t denoted by x(t). The expression for the average velocity between two points using this notation is $\bar{v} = \frac{x(t_2)-x(t_1)}{t_2-t_1}$. To find the instantaneous velocity at any position, we let $t_1 = t$ and $t_2 = t + \Delta t$. After inserting these expressions into the equation for the average velocity and taking the limit as $\Delta t \to 0$, we find the expression for the instantaneous velocity:

$$v(t) = \lim_{\Delta t \to 0} \frac{x(t+\Delta t) - x(t)}{\Delta t} = \frac{dx(t)}{dt}. \tag{2.3.1}$$

Instantaneous Velocity

The instantaneous velocity of an object is the limit of the average velocity as the elapsed time approaches zero, or the derivative of x with respect to t:

$$v(t) = \frac{d}{dt}x(t). \tag{2.3.2}$$

Like average velocity, instantaneous velocity is a vector with dimension of length per time. The instantaneous velocity at a specific time point t_0 is the rate of change of the position function, which is the slope of the position function x(t) at t_0. Figure 2.3.1 shows how the average velocity $\bar{v} = \frac{\Delta x}{\Delta t}$ between two times approaches the instantaneous velocity at t_0. The instantaneous velocity is shown at time t_0, which happens to be at the maximum of the position function. The slope of the position graph is zero at this point, and thus the instantaneous velocity is zero. At other times, t_1, t_2, and so on, the instantaneous velocity is not zero because the slope of the position graph would be positive or negative. If the position function had a minimum, the slope of the position graph would also be zero, giving an instantaneous velocity of zero there as well. Thus, the zeros of the velocity function give the minimum and maximum of the position function.

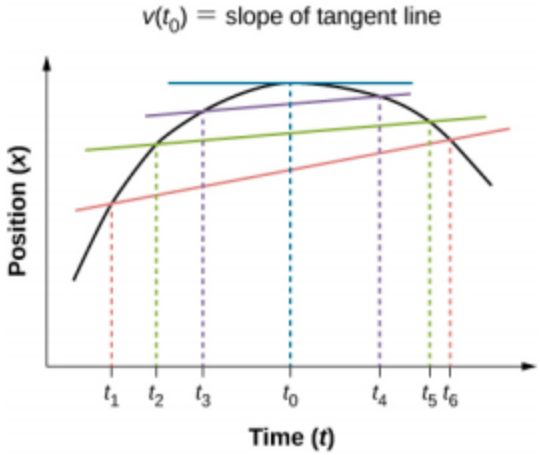


Figure 2.3.1: In a graph of position versus time, the instantaneous velocity is the slope of the tangent line at a given point. The average velocities $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$ between times $\Delta t = t_6 - t_1$, $\Delta t = t_5 - t_2$, and $\Delta t = t_4 - t_3$ are shown. When $\Delta t \rightarrow 0$, the average velocity approaches the instantaneous velocity at $t = t_0$.

Example 3.2: Finding Velocity from a Position-Versus-Time Graph

Given the position-versus-time graph of Figure 2.3.2, find the velocity-versus-time graph.

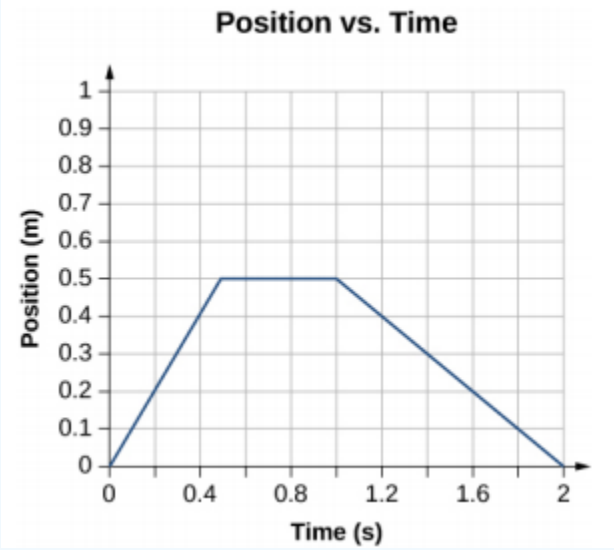


Figure 2.3.2: The object starts out in the positive direction, stops for a short time, and then reverses direction, heading back toward the origin. Notice that the object comes to rest instantaneously, which would require an infinite force. Thus, the graph is an approximation of motion in the real world. (The concept of force is discussed in Newton's Laws of Motion.)

Strategy

The graph contains three straight lines during three time intervals. We find the velocity during each time interval by taking the slope of the line using the grid.

Solution

Time interval 0 s to 0.5 s: $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{0.5\ m - 0.0\ m}{0.5\ s - 0.0\ s} = 1.0\ \ m/s$

Time interval 0.5 s to 1.0 s: $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{0.0\ m - 0.0\ m}{1.0\ s - 0.5\ s} = 0.0\ \ m/s$

Time interval 1.0 s to 2.0 s: $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{0.0\ m - 0.5\ m}{2.0\ s - 1.0\ s} = -0.5\ \ m/s$

The graph of these values of velocity versus time is shown in Figure 2.3.3.

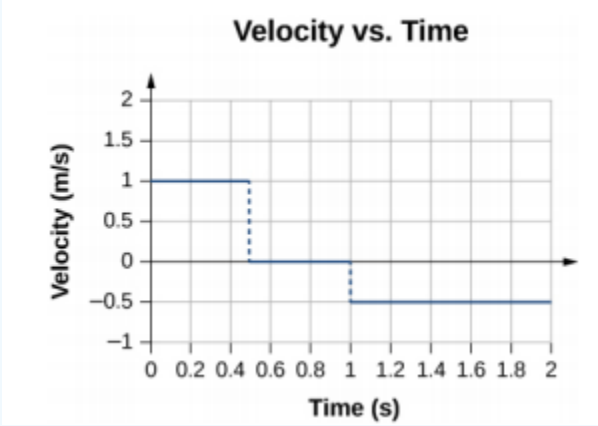


Figure 2.3.3: The velocity is positive for the first part of the trip, zero when the object is stopped, and negative when the object reverses direction.

Significance

During the time interval between 0 s and 0.5 s, the object's position is moving away from the origin and the position-versus-time curve has a positive slope. At any point along the curve during this time interval, we can find the instantaneous velocity by taking its slope, which is +1 m/s, as shown in Figure 2.3.3. In the subsequent time interval, between 0.5 s and 1.0 s, the position doesn't change and we see the slope is zero. From 1.0 s to 2.0 s, the object is moving back toward the origin and the slope is −0.5 m/s. The object has reversed direction and has a negative velocity.

Speed

In everyday language, most people use the terms speed and velocity interchangeably. In physics, however, they do not have the same meaning and are distinct concepts. One major difference is that speed has no direction; that is, speed is a scalar.

We can calculate the **average speed** by finding the total distance traveled divided by the elapsed time:

$$Average\ speed = \bar{s} = \frac{Total\ distance}{Elapsed\ time}. \tag{2.3.3}$$

Average speed is not necessarily the same as the magnitude of the average velocity, which is found by dividing the magnitude of the total displacement by the elapsed time. For example, if a trip starts and ends at the same location, the total displacement is zero, and therefore the average velocity is zero. The average speed, however, is not zero, because the total distance traveled is greater than zero. If we take a road trip of 300 km and need to be at our destination at a certain time, then we would be interested in our average speed.

However, we can calculate the **instantaneous speed** from the magnitude of the instantaneous velocity:

$$Instantaneous\ speed = |v(t)|. \tag{2.3.4}$$

If a particle is moving along the x-axis at +7.0 m/s and another particle is moving along the same axis at −7.0 m/s, they have different velocities, but both have the same speed of 7.0 m/s. Some typical speeds are shown in the following table.

Table 3.1 - Speeds of Various Objects

Speed	m/s	mi/h
Continental drift	10^{-7}	2×10^{-7}
Brisk walk	1.7	3.9

Speed	m/s	mi/h
Cyclist	4.4	10
Sprint runner	12.2	27
Rural speed limit	24.6	56
Official land speed record	341.1	763
Speed of sound at sea level	343	768
Space shuttle on reetry	7800	17,500
Escape velocity of Earth*	11,200	25,000
Orbital speed of Earth around the Sun	29,783	66,623
Speed of light in a vacuum	299,792,458	670,616,629
*Escape velocity is the velocity at which an object must be launched so that it overcomes Earth's gravity and is not pulled back toward Earth.		

Calculating Instantaneous Velocity

When calculating instantaneous velocity, we need to specify the explicit form of the position function x(t). If each term in the x(t) equation has the form of At^n where A is a constant and n is an integer, this can be differentiated using the power rule to be:

$$\frac{d\left(At^n\right)}{dt} = Ant^{n-1}. \tag{2.3.5}$$

Note that if there are additional terms added together, this power rule of differentiation can be done multiple times and the solution is the sum of those terms. The following example illustrates the use of Equation 2.3.5.

Example 3.3: Instantaneous Velocity Versus Average Velocity

The position of a particle is given by $x(t) = 3.0t + 0.5t^3$ m.

a. Using Equation 2.3.2 and Equation 2.3.5, find the instantaneous velocity at t = 2.0 s.
b. Calculate the average velocity between 1.0 s and 3.0 s.

Strategy

Equation 2.3.2 give the instantaneous velocity of the particle as the derivative of the position function. Looking at the form of the position function given, we see that it is a polynomial in t. Therefore, we can use Equation 2.3.5, the power rule from calculus, to find the solution. We use Equation 2.3.4 to calculate the average velocity of the particle.

Solution

a. $v(t) = \frac{dx(t)}{dt} = 3.0 + 1.5t^2$ m/s. Substituting t = 2.0 s into this equation gives $v(2.0\ s) = [3.0 + 1.5(2.0)^2]$ m/s = 9.0 m/s.
b. To determine the average velocity of the particle between 1.0 s and 3.0 s, we calculate the values of x(1.0 s) and x(3.0 s):

$$x(1.0s) = \left[(3.0)(1.0) + 0.5(1.0)^3\right]m = 3.5\ m \tag{2.3.6}$$

$$x(3.0s) = \left[(3.0)(3.0) + 0.5(3.0)^3\right]m = 22.5\ m \tag{2.3.7}$$

Then the average velocity is

$$\bar{v} = \frac{x(3.0\ s) - x(1.0\ s)}{t(3.0\ s) - t(1.0\ s)} = \frac{22.5 - 3.5\ m}{3.0 - 1.0\ s} = 9.5\ m/s. \tag{2.3.8}$$

Significance

In the limit that the time interval used to calculate $\bar{v}$ goes to zero, the value obtained for $\bar{v}$ converges to the value of v.

Example 3.4: Instantaneous Velocity Versus Speed

Consider the motion of a particle in which the position is $x(t) = 3.0t - 3t^2$ m.

a. What is the instantaneous velocity at t = 0.25 s, t = 0.50 s, and t = 1.0 s?
b. What is the speed of the particle at these times?

Strategy

The instantaneous velocity is the derivative of the position function and the speed is the magnitude of the instantaneous velocity. We use Equation 2.3.2 and Equation 2.3.5 to solve for instantaneous velocity.

Solution

a. $v(t) = \frac{dx(t)}{dt} = 3.0 - 6.0t$ m/s
b. v(0.25 s) = 1.50 m/s, v(0.5 s) = 0 m/s, v(1.0 s) = −3.0 m/s
c. Speed = |v(t)| = 1.50 m/s, 0.0 m/s, and 3.0 m/s

Significance

The velocity of the particle gives us direction information, indicating the particle is moving to the left (west) or right (east). The speed gives the magnitude of the velocity. By graphing the position, velocity, and speed as functions of time, we can understand these concepts visually Figure 2.3.4. In (a), the graph shows the particle moving in the positive direction until t = 0.5 s, when it reverses direction. The reversal of direction can also be seen in (b) at 0.5 s where the velocity is zero and then turns negative. At 1.0 s it is back at the origin where it started. The particle's velocity at 1.0 s in (b) is negative, because it is traveling in the negative direction. But in (c), however, its speed is positive and remains positive throughout the travel time. We can also interpret velocity as the slope of the position-versus-time graph. The slope of x(t) is decreasing toward zero, becoming zero at 0.5 s and increasingly negative thereafter. This analysis of comparing the graphs of position, velocity, and speed helps catch errors in calculations. The graphs must be consistent with each other and help interpret the calculations.

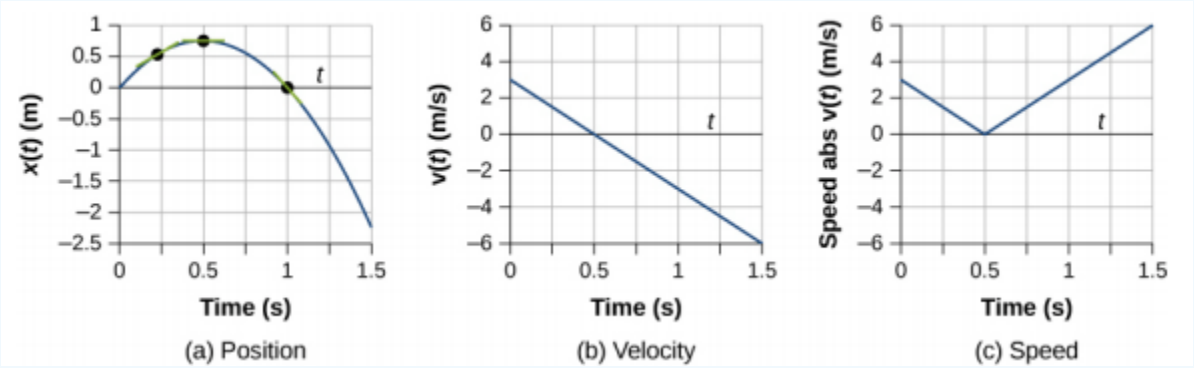


Figure 2.3.4: (a) Position: x(t) versus time. (b) Velocity: v(t) versus time. The slope of the position graph is the velocity. A rough comparison of the slopes of the tangent lines in (a) at 0.25 s, 0.5 s, and 1.0 s with the values for velocity at the corresponding times indicates they are the same values. (c) Speed: |v(t)| versus time. Speed is always a positive number.

Exercise 3.2

The position of an object as a function of time is $x(t) = -3t^2$ m. (a) What is the velocity of the object as a function of time? (b) Is the velocity ever positive? (c) What are the velocity and speed at t = 1.0 s?

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2.4: Average and Instantaneous Acceleration

Learning Objectives

- Calculate the average acceleration between two points in time.
- Calculate the instantaneous acceleration given the functional form of velocity.
- Explain the vector nature of instantaneous acceleration and velocity.
- Explain the difference between average acceleration and instantaneous acceleration.
- Find instantaneous acceleration at a specified time on a graph of velocity versus time.

The importance of understanding acceleration spans our day-to-day experience, as well as the vast reaches of outer space and the tiny world of subatomic physics. In everyday conversation, to **accelerate** means to speed up; applying the brake pedal causes a vehicle to slow down. We are familiar with the acceleration of our car, for example. The greater the acceleration, the greater the change in velocity over a given time. Acceleration is widely seen in experimental physics. In linear particle accelerator experiments, for example, subatomic particles are accelerated to very high velocities in collision experiments, which tell us information about the structure of the subatomic world as well as the origin of the universe. In space, cosmic rays are subatomic particles that have been accelerated to very high energies in supernovas (exploding massive stars) and active galactic nuclei. It is important to understand the processes that accelerate cosmic rays because these rays contain highly penetrating radiation that can damage electronics flown on spacecraft, for example.

Average Acceleration

The formal definition of acceleration is consistent with these notions just described, but is more inclusive.

Average Acceleration

Average acceleration is the rate at which velocity changes:

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{t_f - t_0}, \tag{2.4.1}$$

where $\bar{a}$ is **average acceleration**, v is velocity, and t is time. (The bar over the a means **average** acceleration.)

Because acceleration is velocity in meters divided by time in seconds, the SI units for acceleration are often abbreviated m/s^2—that is, meters per second squared or meters per second per second. This literally means by how many meters per second the velocity changes every second. Recall that velocity is a vector—it has both magnitude and direction—which means that a change in velocity can be a change in magnitude (or speed), but it can also be a change in direction. For example, if a runner traveling at 10 km/h due east slows to a stop, reverses direction, continues her run at 10 km/h due west, her velocity has changed as a result of the change in direction, although the **magnitude** of the velocity is the same in both directions. Thus, acceleration occurs when velocity changes in magnitude (an increase or decrease in speed) or in direction, or both.

Acceleration as a Vector

Acceleration is a vector in the same direction as the **change** in velocity, Δv. Since velocity is a vector, it can change in magnitude or in direction, or both. Acceleration is, therefore, a change in speed or direction, or both.

Keep in mind that although acceleration is in the direction of the change in velocity, it is not always in the direction of motion. When an object slows down, its acceleration is opposite to the direction of its motion. Although this is commonly referred to as **deceleration** Figure 2.4.1, we say the train is accelerating in a direction opposite to its direction of motion.



Figure 2.4.1: A subway train in Sao Paulo, Brazil, decelerates as it comes into a station. It is accelerating in a direction opposite to its direction of motion. (credit: Yusuke Kawasaki)

The term **deceleration** can cause confusion in our analysis because it is not a vector and it does not point to a specific direction with respect to a coordinate system, so we do not use it. Acceleration is a vector, so we must choose the appropriate sign for it in our chosen coordinate system. In the case of the train in Figure 2.4.1, acceleration is in the **negative direction in the chosen coordinate system**, so we say the train is undergoing negative acceleration.

If an object in motion has a velocity in the positive direction with respect to a chosen origin and it acquires a constant negative acceleration, the object eventually comes to a rest and reverses direction. If we wait long enough, the object passes through the origin going in the opposite direction. This is illustrated in Figure 2.4.2.

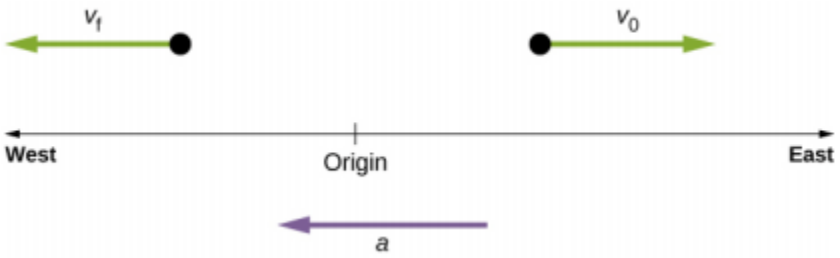


Figure 2.4.2: An object in motion with a velocity vector toward the east under negative acceleration comes to a rest and reverses direction. It passes the origin going in the opposite direction after a long enough time.

Example 3.5: Calculating Average Acceleration: A Racehorse Leaves the Gate

A racehorse coming out of the gate accelerates from rest to a velocity of 15.0 m/s due west in 1.80 s. What is its average acceleration?



Figure 2.4.3: Racehorses accelerating out of the gate. (credit: Jon Sullivan)

Strategy

First we draw a sketch and assign a coordinate system to the problem Figure 2.4.4. This is a simple problem, but it always helps to visualize it. Notice that we assign east as positive and west as negative. Thus, in this case, we have negative velocity.

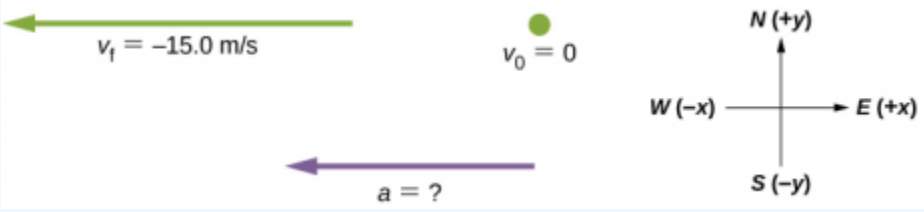


Figure 2.4.4: Identify the coordinate system, the given information, and what you want to determine.

We can solve this problem by identifying Δv and Δt from the given information, and then calculating the average acceleration directly from the equation $\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{t_f - t_0}$.

Solution

First, identify the knowns: $v_0 = 0$, $v_f = -15.0$ m/s (the negative sign indicates direction toward the west), $\Delta t = 1.80$ s. Second, find the change in velocity. Since the horse is going from zero to –15.0 m/s, its change in velocity equals its final velocity:

$$\Delta v = v_f - v_0 = v_f = -15.0\ m/s. \tag{2.4.2}$$

Last, substitute the known values (Δv and Δt) and solve for the unknown $\bar{a}$:

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{-15.0\ m/s}{1.80\ s} = -8.33\ m/s^2. \tag{2.4.3}$$

Significance

The negative sign for acceleration indicates that acceleration is toward the west. An acceleration of 8.33 m/s^2 due west means the horse increases its velocity by 8.33 m/s due west each second; that is, 8.33 meters per second per second, which we write as 8.33 m/s^2. This is truly an average acceleration, because the ride is not smooth. We see later that an acceleration of this magnitude would require the rider to hang on with a force nearly equal to his weight.

Exercise 3.3

Protons in a linear accelerator are accelerated from rest to 2.0×10^7 m/s in 10^{-4} s. What is the average acceleration of the protons?

Instantaneous Acceleration

Instantaneous acceleration a, or **acceleration at a specific instant in time**, is obtained using the same process discussed for instantaneous velocity. That is, we calculate the average velocity between two points in time separated by Δt and let Δt approach zero. The result is the derivative of the velocity function v(t), which is **instantaneous acceleration** and is expressed mathematically as

$$a(t) = \frac{d}{dt}v(t). \tag{2.4.4}$$

Thus, similar to velocity being the derivative of the position function, instantaneous acceleration is the derivative of the velocity function. We can show this graphically in the same way as instantaneous velocity. In Figure 2.4.5, instantaneous acceleration at time t_0 is the slope of the tangent line to the velocity-versus-time graph at time t_0. We see that average acceleration $\bar{a} = \frac{\Delta v}{\Delta t}$ approaches instantaneous acceleration as Δt approaches zero. Also in part (a) of the figure, we see that velocity has a maximum when its slope is zero. This time corresponds to the zero of the acceleration function. In part (b), instantaneous acceleration at the

minimum velocity is shown, which is also zero, since the slope of the curve is zero there, too. Thus, for a given velocity function, the zeros of the acceleration function give either the minimum or the maximum velocity

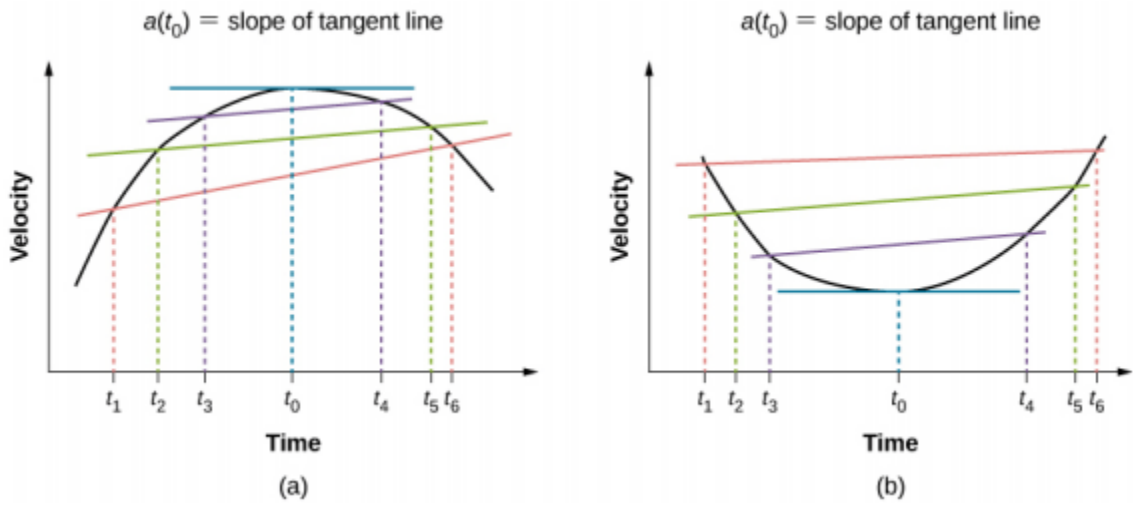


Figure 2.4.5: In a graph of velocity versus time, instantaneous acceleration is the slope of the tangent line. (a) Shown is average acceleration $\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{t_f - t_0}$ between times $\Delta t = t_6 - t_1$, $\Delta t = t_5 - t_2$, and $\Delta t = t_4 - t_3$. When $\Delta t \rightarrow 0$, the average acceleration approaches instantaneous acceleration at time t_0. In view (a), instantaneous acceleration is shown for the point on the velocity curve at maximum velocity. At this point, instantaneous acceleration is the slope of the tangent line, which is zero. At any other time, the slope of the tangent line—and thus instantaneous acceleration—would not be zero. (b) Same as (a) but shown for instantaneous acceleration at minimum velocity.

To illustrate this concept, let's look at two examples. First, a simple example is shown using Figure 3.3.4(b), the velocityversus-time graph of Example 3.3, to find acceleration graphically. This graph is depicted in Figure 2.4.6(a), which is a straight line. The corresponding graph of acceleration versus time is found from the slope of velocity and is shown in Figure 2.4.6(b). In this example, the velocity function is a straight line with a constant slope, thus acceleration is a constant. In the next example, the velocity function is has a more complicated functional dependence on time.

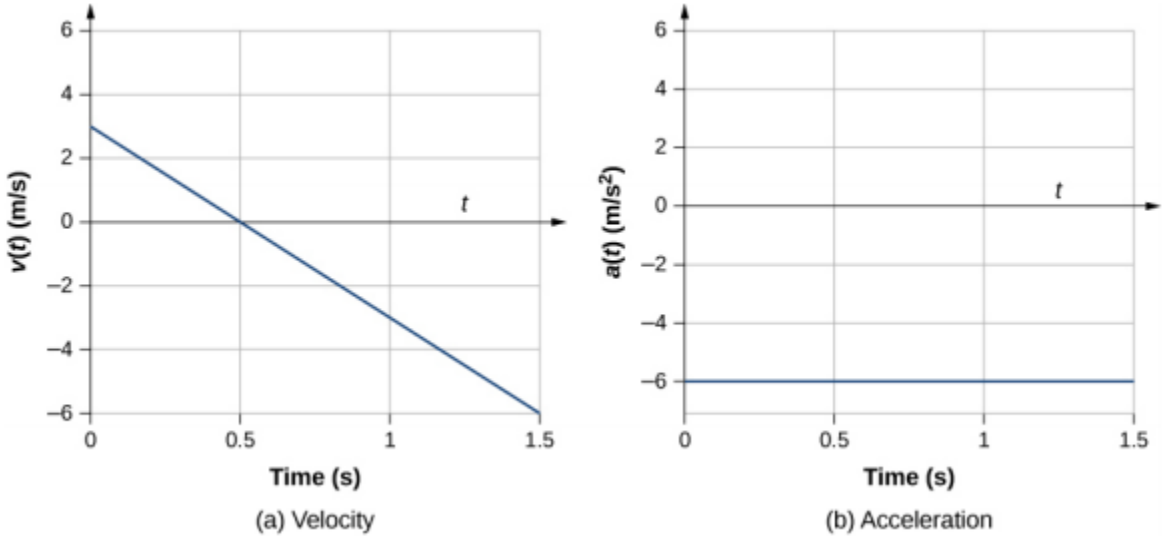


Figure 2.4.6: (a, b) The velocity-versus-time graph is linear and has a negative constant slope (a) that is equal to acceleration, shown in (b).

If we know the functional form of velocity, v(t), we can calculate instantaneous acceleration a(t) at any time point in the motion using Equation 2.4.4.

Example 3.6: Calculating Instantaneous Acceleration

A particle is in motion and is accelerating. The functional form of the velocity is $v(t) = 20t - 5t^2$ m/s.

a. Find the functional form of the acceleration.
b. Find the instantaneous velocity at t = 1, 2, 3, and 5 s.
c. Find the instantaneous acceleration at t = 1, 2, 3, and 5 s.
d. Interpret the results of (c) in terms of the directions of the acceleration and velocity vectors.

Strategy

We find the functional form of acceleration by taking the derivative of the velocity function. Then, we calculate the values of instantaneous velocity and acceleration from the given functions for each. For part (d), we need to compare the directions of velocity and acceleration at each time.

Solution

a. $a(t) = \frac{dv(t)}{dt} dv(t)\ dt = 20 - 10t\ m/s_2$
b. v(1 s) = 15 m/s, v(2 s) = 20 m/s, v(3 s) = 15 m/s, v(5 s) = –25 m/s
c. $a(1\ s) = 10 m/s^2$, $a(2\ s) = 0 m/s^2$, $a(3\ s) = -10 m/s^2$, $a(5\ s) = -30 m/s^2$
d. At t = 1 s, velocity v(1 s) = 15 m/s is positive and acceleration is positive, so both velocity and acceleration are in the same direction. The particle is moving faster.

At t = 2 s, velocity has increased to v(2 s) = 20 m/s , where it is maximum, which corresponds to the time when the acceleration is zero. We see that the maximum velocity occurs when the slope of the velocity function is zero, which is just the zero of the acceleration function.

At t = 3 s, velocity is v(3 s) = 15 m/s and acceleration is negative. The particle has reduced its velocity and the acceleration vector is negative. The particle is slowing down.

At t = 5 s, velocity is v(5 s) = –25 m/s and acceleration is increasingly negative. Between the times t = 3 s and t = 5 s the particle has decreased its velocity to zero and then become negative, thus reversing its direction. The particle is now speeding up again, but in the opposite direction.

We can see these results graphically in Figure 2.4.7.

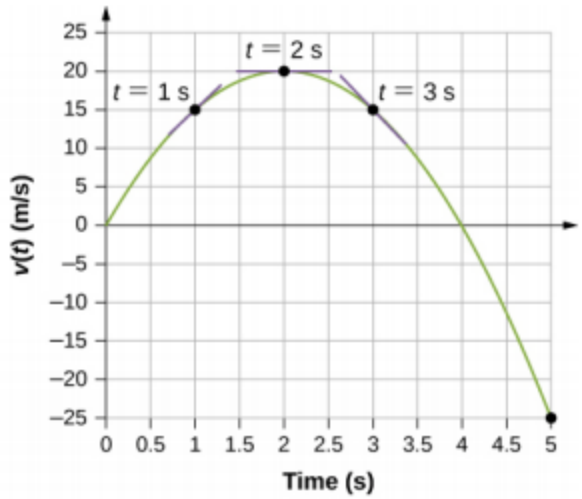


Figure 2.4.7: (a) Velocity versus time. Tangent lines are indicated at times 1, 2, and 3 s. The slopes of the tangents lines are the accelerations. At t = 3 s, velocity is positive. At t = 5 s, velocity is negative, indicating the particle has reversed direction. (b) Acceleration versus time. Comparing the values of accelerations given by the black dots with the corresponding slopes of the tangent lines (slopes of lines through black dots) in (a), we see they are identical.

Significance

By doing both a numerical and graphical analysis of velocity and acceleration of the particle, we can learn much about its motion. The numerical analysis complements the graphical analysis in giving a total view of the motion. The zero of the acceleration function corresponds to the maximum of the velocity in this example. Also in this example, when acceleration is positive and in the same direction as velocity, velocity increases. As acceleration tends toward zero, eventually becoming negative, the velocity reaches a maximum, after which it starts decreasing. If we wait long enough, velocity also becomes negative, indicating a reversal of direction. A real-world example of this type of motion is a car with a velocity that is increasing to a maximum, after which it starts slowing down, comes to a stop, then reverses direction.

Exercise 3.4

An airplane lands on a runway traveling east. Describe its acceleration.

Getting a Feel for Acceleration

You are probably used to experiencing acceleration when you step into an elevator, or step on the gas pedal in your car. However, acceleration is happening to many other objects in our universe with which we don't have direct contact. Table 3.2 presents the acceleration of various objects. We can see the magnitudes of the accelerations extend over many orders of magnitude.

Table 3.2 - Typical Values of Acceleration

(credit: Wikipedia: Orders of Magnitude (acceleration))

Acceleration	Value (m/s^2)
High-speed train	0.25
Elevator	2
Cheetah	5
Object in a free fall without air resistance near the surface of Earth	9.8
Space shuttle maximum during launch	29
Parachutist peak during normal opening of parachute	59
F16 aircraft pulling out of a dive	79
Explosive seat ejection from aircraft	147
Sprint missile	982
Fastest rocket sled peak acceleration	1540
Jumping flea	3200
Baseball struck by a bat	30,000
Closing jaws of a trap-jaw ant	1,000,000
Proton in the large Hadron collider	1.9×10^9

In this table, we see that typical accelerations vary widely with different objects and have nothing to do with object size or how massive it is. Acceleration can also vary widely with time during the motion of an object. A drag racer has a large acceleration just after its start, but then it tapers off as the vehicle reaches a constant velocity. Its average acceleration can be quite different from its instantaneous acceleration at a particular time during its motion. Figure $2.4.8$ compares graphically average acceleration with instantaneous acceleration for two very different motions.

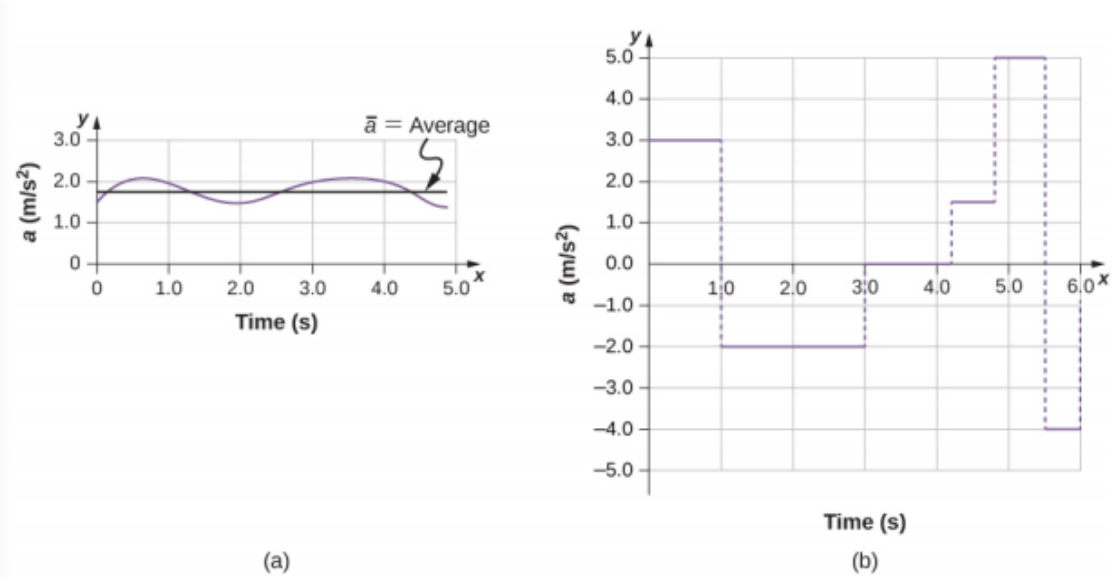


Figure 2.4.8: Graphs of instantaneous acceleration versus time for two different one-dimensional motions. (a) Acceleration varies only slightly and is always in the same direction, since it is positive. The average over the interval is nearly the same as the acceleration at any given time. (b) Acceleration varies greatly, perhaps representing a package on a post office conveyor belt that is accelerated forward and backward as it bumps along. It is necessary to consider small time intervals (such as from 0–1.0 s) with constant or nearly constant acceleration in such a situation.

Simulation

Learn about position, velocity, and acceleration graphs. Move the little man back and forth with a mouse and plot his motion. Set the position, velocity, or acceleration and let the simulation move the man for you. Visit this link to use the moving man simulation.

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2.5: Motion with Constant Acceleration (Part 1)

Learning Objectives

- Identify which equations of motion are to be used to solve for unknowns.
- Use appropriate equations of motion to solve a two-body pursuit problem.

You might guess that the greater the acceleration of, say, a car moving away from a stop sign, the greater the car's displacement in a given time. But, we have not developed a specific equation that relates acceleration and displacement. In this section, we look at some convenient equations for kinematic relationships, starting from the definitions of displacement, velocity, and acceleration. We first investigate a single object in motion, called single-body motion. Then we investigate the motion of two objects, called **two-body pursuit problems**.

Notation

First, let us make some simplifications in notation. Taking the initial time to be zero, as if time is measured with a stopwatch, is a great simplification. Since elapsed time is $\Delta t = t_f - t_0$, taking $t_0 = 0$ means that $\Delta t = t_f$, the final time on the stopwatch. When initial time is taken to be zero, we use the subscript 0 to denote initial values of position and velocity. That is, x_0 **is the initial position** and v_0 **is the initial velocity**. We put no subscripts on the final values. That is, t **is the final time**, x **is the final position**, and v **is the final velocity**. This gives a simpler expression for elapsed time, $\Delta t = t$. It also simplifies the expression for x displacement, which is now $\Delta x = x - x_0$. Also, it simplifies the expression for change in velocity, which is now $\Delta v = v - v_0$. To summarize, using the simplified notation, with the initial time taken to be zero,

$$\Delta t = t \tag{2.5.1}$$

$$\Delta x = x - x_0 \tag{2.5.2}$$

$$\Delta v = v - v_0, \tag{2.5.3}$$

where the subscript 0 denotes an initial value and the absence of a subscript denotes a final value in whatever motion is under consideration.

We now make the important assumption that acceleration is constant. This assumption allows us to avoid using calculus to find instantaneous acceleration. Since acceleration is constant, the average and instantaneous accelerations are equal—that is,

$$\bar{a} = a = constant. \tag{2.5.4}$$

Thus, we can use the symbol a for acceleration at all times. Assuming acceleration to be constant does not seriously limit the situations we can study nor does it degrade the accuracy of our treatment. For one thing, acceleration is constant in a great number of situations. Furthermore, in many other situations we can describe motion accurately by assuming a constant acceleration equal to the average acceleration for that motion. Lastly, for motion during which acceleration changes drastically, such as a car accelerating to top speed and then braking to a stop, motion can be considered in separate parts, each of which has its own constant acceleration.

Displacement and Position from Velocity

To get our first two equations, we start with the definition of average velocity:

$$\bar{v} = \frac{\Delta x}{\Delta t}. \tag{2.5.5}$$

Substituting the simplified notation for Δx and Δt yields

$$\bar{v} = \frac{x - x_0}{t}. \tag{2.5.6}$$

Solving for x gives us

$$x = x_0 + \bar{v}t, \tag{2.5.7}$$

where the average velocity is

$$\bar{v} = \frac{v_0 + v}{2}. \tag{2.5.8}$$

The equation $\bar{v} = \frac{v_0+v}{2}$ reflects the fact that when acceleration is constant, v is just the simple average of the initial and final velocities. Figure 2.5.1 illustrates this concept graphically. In part (a) of the figure, acceleration is constant, with velocity increasing at a constant rate. The average velocity during the 1-h interval from 40 km/h to 80 km/h is 60 km/h:

$$\bar{v} = \frac{v_0 + v}{2} = \frac{40\ km/h + 80\ km/h}{2} = 60\ km/h. \tag{2.5.9}$$

In part (b), acceleration is not constant. During the 1-h interval, velocity is closer to 80 km/h than 40 km/h. Thus, the average velocity is greater than in part (a).

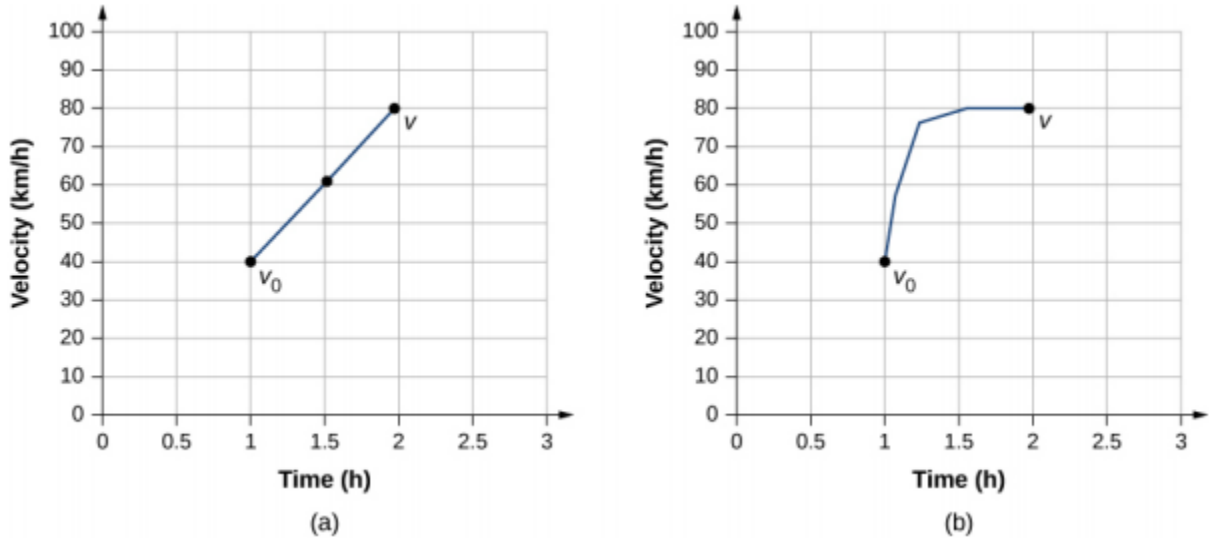


Figure 2.5.1: (a) Velocity-versus-time graph with constant acceleration showing the initial and final velocities v_0 and v. The average velocity is $\frac{1}{2}$ (v_0 + v) = 60 km/h. (b) Velocity-versus-time graph with an acceleration that changes with time. The average velocity is not given by $\frac{1}{2}$(v_0 + v), but is greater than 60 km/h.

Solving for Final Velocity from Acceleration and Time

We can derive another useful equation by manipulating the definition of acceleration:

$$a = \frac{\Delta v}{\Delta t}. \tag{2.5.10}$$

Substituting the simplified notation for Δv and Δt gives us

$$a = \frac{v - v_0}{t}\ (constant\ a). \tag{2.5.11}$$

Solving for v yields

$$v = v_0 + at\ (constant\ a). \tag{2.5.12}$$

Example 3.7: Calculating Final Velocity

An airplane lands with an initial velocity of 70.0 m/s and then decelerates at 1.50 m/s^2 for 40.0 s. What is its final velocity?

Strategy

First, we identify the knowns: v_0 = 70 m/s, a = –1.50 m/s^2, t = 40 s.

Second, we identify the unknown; in this case, it is final velocity v_f.

Last, we determine which equation to use. To do this we figure out which kinematic equation gives the unknown in terms of the knowns. We calculate the final velocity using Equation 2.5.12, $v = v_0 + at$.

Solution

Substitute the known values and solve:

$$v = v_0 + at = 70.0\ m/s + (-1.50\ m/s^2)(40.0\ s) = 10.0\ m/s. \tag{2.5.13}$$

Figure 2.5.2 is a sketch that shows the acceleration and velocity vectors.



Figure 2.5.2: The airplane lands with an initial velocity of 70.0 m/s and slows to a final velocity of 10.0 m/s before heading for the terminal. Note the acceleration is negative because its direction is opposite to its velocity, which is positive.

Significance

The final velocity is much less than the initial velocity, as desired when slowing down, but is still positive (see figure). With jet engines, reverse thrust can be maintained long enough to stop the plane and start moving it backward, which is indicated by a negative final velocity, but is not the case here.

In addition to being useful in problem solving, the equation $v = v_0 + at$ gives us insight into the relationships among velocity, acceleration, and time. We can see, for example, that

- Final velocity depends on how large the acceleration is and how long it lasts
- If the acceleration is zero, then the final velocity equals the initial velocity ($v = v_0$), as expected (in other words, velocity is constant)
- If a is negative, then the final velocity is less than the initial velocity

All these observations fit our intuition. Note that it is always useful to examine basic equations in light of our intuition and experience to check that they do indeed describe nature accurately.

Solving for Final Position with Constant Acceleration

We can combine the previous equations to find a third equation that allows us to calculate the final position of an object experiencing constant acceleration. We start with

$$v = v_0 + at. \tag{2.5.14}$$

Adding v_0 to each side of this equation and dividing by 2 gives

$$\frac{v_0 + v}{2} = v_0 + \frac{1}{2}at. \tag{2.5.15}$$

Since $\frac{v_0+v}{2} = \bar{v}$ for constant acceleration, we have

$$\bar{v} = v_0 + \frac{1}{2}at. \tag{2.5.16}$$

Now we substitute this expression for $\bar{v}$ into the equation for displacement, x = x_0 + $\bar{v}$t, yielding

$$x = x_0 + v_0 t + \frac{1}{2}at^2 \ (constant\ a). \tag{2.5.17}$$

Example 3.8: Calculating Displacement of an Accelerating Object

Dragsters can achieve an average acceleration of 26.0 m/s^2. Suppose a dragster accelerates from rest at this rate for 5.56 s Figure 2.5.3. How far does it travel in this time?



Figure 2.5.3: U.S. Army Top Fuel pilot Tony “The Sarge” Schumacher begins a race with a controlled burnout. (credit: Lt. Col. William Thurmond. Photo Courtesy of U.S. Army.)

Strategy

First, let’s draw a sketch Figure 2.5.4. We are asked to find displacement, which is x if we take x_0 to be zero. (Think about x_0 as the starting line of a race. It can be anywhere, but we call it zero and measure all other positions relative to it.) We can use the equation $x = x_0 + v_0 t + \frac{1}{2}at^2$ when we identify v_0, a, and t from the statement of the problem.

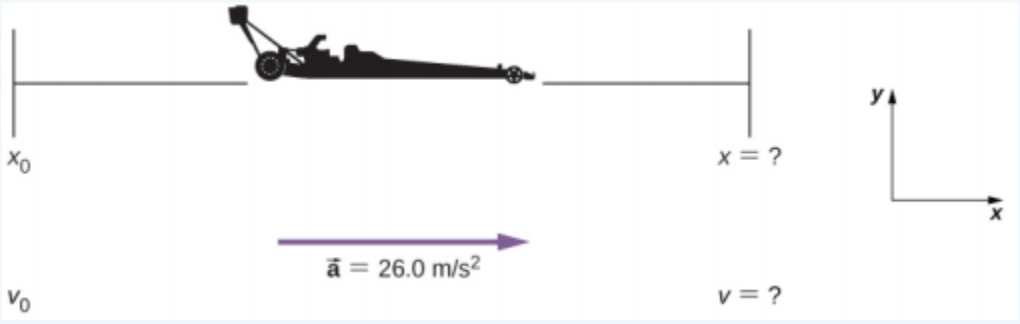


Figure 2.5.4: Sketch of an accelerating dragster.

Solution

First, we need to identify the knowns. Starting from rest means that $v_0 = 0$, a is given as 26.0 m/s^2 and t is given as 5.56 s.

Second, we substitute the known values into the equation to solve for the unknown:

$$x = x_0 + v_0 t + \frac{1}{2}at^2. \tag{2.5.18}$$

Since the initial position and velocity are both zero, this equation simplifies to

$$x = \frac{1}{2}at^2. \tag{2.5.19}$$

Substituting the identified values of a and t gives

$$x = \frac{1}{2}(26.0\ m/s^2)(5.56\ s)^2 = 402\ m. \tag{2.5.20}$$

Significance

If we convert 402 m to miles, we find that the distance covered is very close to one-quarter of a mile, the standard distance for drag racing. So, our answer is reasonable. This is an impressive displacement to cover in only 5.56 s, but top-notch dragsters can do a quarter mile in even less time than this. If the dragster were given an initial velocity, this would add another term to the distance equation. If the same acceleration and time are used in the equation, the distance covered would be much greater.

What else can we learn by examining the equation $x = x_0 + v_0 t + \frac{1}{2}at^2$? We can see the following relationships:

- Displacement depends on the square of the elapsed time when acceleration is not zero. In Example 3.8, the dragster covers only one-fourth of the total distance in the first half of the elapsed time.
- If acceleration is zero, then initial velocity equals average velocity ($v_0 = \bar{v}$) , and $x = x_0 + v_0 t + \frac{1}{2}at^2$ becomes $x = x_0 + v_0 t$.

Solving for Final Velocity from Distance and Acceleration

A fourth useful equation can be obtained from another algebraic manipulation of previous equations. If we solve $v = v_0 + at$ for t, we get

$$t = \frac{v - v_0}{a}. \tag{2.5.21}$$

Substituting this and $\bar{v} = \frac{v_0 + v}{2}$ into $x = x_0 + \bar{v}t$, we get

$$v^2 = v_0^2 + 2a(x - x_0)\ (constant\ a). \tag{2.5.22}$$

Example 3.9: Calculating Final Velocity

Calculate the final velocity of the dragster in Example 3.8 without using information about time.

Strategy

The equation $v^2 = v_0^2 + 2a(x - x_0)$ is ideally suited to this task because it relates velocities, acceleration, and displacement, and no time information is required.

Solution

First, we identify the known values. We know that $v_0 = 0$, since the dragster starts from rest. We also know that $x - x_0 = 402$ m (this was the answer in Example 3.8). The average acceleration was given by a = 26.0 m/s^2. Second, we substitute the knowns into the equation $v^2 = v_0^2 + 2a(x - x_0)$ and solve for v:

$$v^2 = 0 + 2(26.0\ m/s^2)(402\ m). \tag{2.5.23}$$

Thus,

$$v^2 = 2.09 \times 10^4\ m/s^2 \tag{2.5.24}$$

$$v = \sqrt{2.09 \times 10^4\ m^2/s^2} = 145\ m/s. \tag{2.5.25}$$

Significance

A velocity of 145 m/s is about 522 km/h, or about 324 mi/h, but even this breakneck speed is short of the record for the quarter mile. Also, note that a square root has two values; we took the positive value to indicate a velocity in the same direction as the acceleration.

An examination of the equation $v^2 = v_0^2 + 2a(x - x_0)$ can produce additional insights into the general relationships among physical quantities:

- The final velocity depends on how large the acceleration is and the distance over which it acts.
- For a fixed acceleration, a car that is going twice as fast doesn't simply stop in twice the distance. It takes much farther to stop. (This is why we have reduced speed zones near schools.)

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2.6: Motion with Constant Acceleration (Part 2)

Putting Equations Together

In the following examples, we continue to explore one-dimensional motion, but in situations requiring slightly more algebraic manipulation. The examples also give insight into problem-solving techniques. The note that follows is provided for easy reference to the equations needed. Be aware that these equations are not independent. In many situations we have two unknowns and need two equations from the set to solve for the unknowns. We need as many equations as there are unknowns to solve a given situation.

Summary of Kinematic Equations (constant a)

$$x = x_0 + \bar{v}t \tag{2.6.1}$$

$$\bar{v} = \frac{v_0 + v}{2} \tag{2.6.2}$$

$$v = v_0 + at \tag{2.6.3}$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2 \tag{2.6.4}$$

$$v^2 = v_0^2 + 2a(x - x_0) \tag{2.6.5}$$

Before we get into the examples, let's look at some of the equations more closely to see the behavior of acceleration at extreme values. Rearranging $v = v_0 + at$, we have

$$a = \frac{v - v_0}{t}. \tag{2.6.6}$$

From this we see that, for a finite time, if the difference between the initial and final velocities is small, the acceleration is small, approaching zero in the limit that the initial and final velocities are equal. On the contrary, in the limit t → 0 for a finite difference between the initial and final velocities, acceleration becomes infinite.

Similarly, rearranging $v^2 = v_0^2 + 2a(x - x_0)$, we can express acceleration in terms of velocities and displacement:

$$a = \frac{v^2 - v_0^2}{2(x - x_0)}. \tag{2.6.7}$$

Thus, for a finite difference between the initial and final velocities acceleration becomes infinite in the limit the displacement approaches zero. Acceleration approaches zero in the limit the difference in initial and final velocities approaches zero for a finite displacement.

Example 3.10: How Far Does a Car Go?

On dry concrete, a car can decelerate at a rate of 7.00 m/s^2, whereas on wet concrete it can decelerate at only 5.00 m/s^2. Find the distances necessary to stop a car moving at 30.0 m/s (about 110 km/h) on (a) dry concrete and (b) wet concrete. (c) Repeat both calculations and find the displacement from the point where the driver sees a traffic light turn red, taking into account his reaction time of 0.500 s to get his foot on the brake.

Strategy

First, we need to draw a sketch Figure 2.6.1. To determine which equations are best to use, we need to list all the known values and identify exactly what we need to solve for.

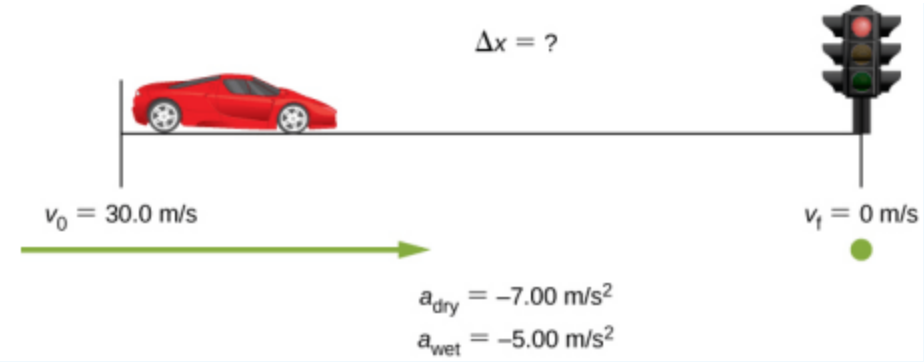


Figure 2.6.1: Sample sketch to visualize deceleration and stopping distance of a car.

Solution

a. First, we need to identify the knowns and what we want to solve for. We know that v_0 = 30.0 m/s, v = 0, and a = –7.00 m/s^2 (a is negative because it is in a direction opposite to velocity). We take x_0 to be zero. We are looking for displacement Δx, or x – x_0. Second, we identify the equation that will help us solve the problem. The best equation to use is

$$v^2 = v_0^2 + 2a(x - x_0). \tag{2.6.8}$$

This equation is best because it includes only one unknown, x. We know the values of all the other variables in this equation. (Other equations would allow us to solve for x, but they require us to know the stopping time, t, which we do not know. We could use them, but it would entail additional calculations.) Third, we rearrange the equation to solve for x:

$$x - x_0 = \frac{v^2 - v_0^2}{2a} \tag{2.6.9}$$

and substitute the known values:

$$x - 0 = \frac{0^2 - (30.0\ m/s)^2}{2(-7.00\ m/s^2)}. \tag{2.6.10}$$

Thus,

$$x = 64.3\ m\ on\ dry\ concrete. \tag{2.6.11}$$

b. This part can be solved in exactly the same manner as (a). The only difference is that the acceleration is –5.00 m/s^2. The result is

$$x_{wet} = 90.0\ m\ on\ wet\ concrete. \tag{2.6.12}$$

c. When the driver reacts, the stopping distance is the same as it is in (a) and (b) for dry and wet concrete. So, to answer this question, we need to calculate how far the car travels during the reaction time, and then add that to the stopping time. It is reasonable to assume the velocity remains constant during the driver's reaction time. To do this, we, again, identify the knowns and what we want to solve for. We know that $\bar{v}$ = 30.0 m/s, $t_{reaction}$ = 0.500 s, and $a_{reaction}$ = 0. We take $x_{0\text{-reaction}}$ to be zero. We are looking for $x_{reaction}$. Second, as before, we identify the best equation to use. In this case, x = x_0 + $\bar{v}$t works well because the only unknown value is x, which is what we want to solve for. Third, we substitute the knowns to solve the equation:

$$x = 0 + (30.0\ m/s)(0.500\ s) = 15.0\ m. \tag{2.6.13}$$

This means the car travels 15.0 m while the driver reacts, making the total displacements in the two cases of dry and wet concrete 15.0 m greater than if he reacted instantly. Last, we then add the displacement during the reaction time to the displacement when braking (Figure 2.6.2),

$$x_{braking} + x_{reaction} = x_{total}, \tag{2.6.14}$$

and find (a) to be 64.3 m + 15.0 m = 79.3 m when dry and (b) to be 90.0 m + 15.0 m = 105 m when wet.

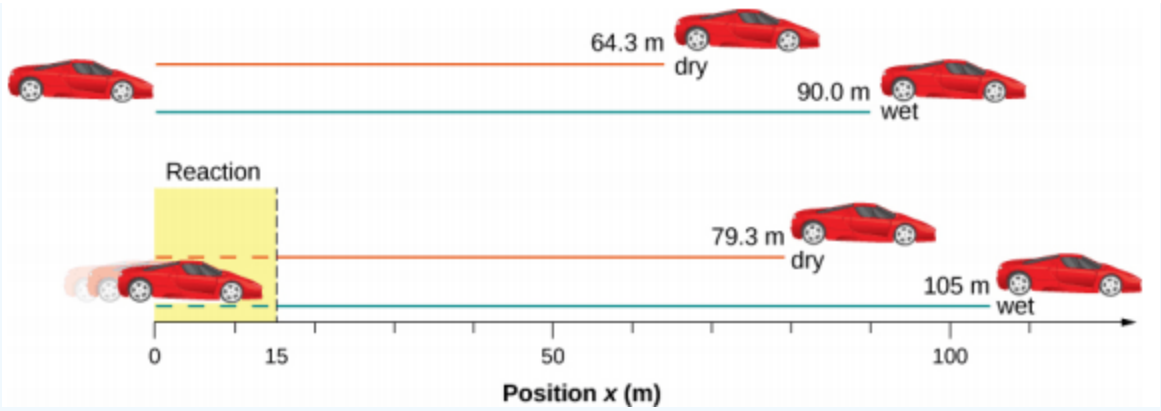


Figure 2.6.2: The distance necessary to stop a car varies greatly, depending on road conditions and driver reaction time. Shown here are the braking distances for dry and wet pavement, as calculated in this example, for a car traveling initially at 30.0 m/s. Also shown are the total distances traveled from the point when the driver first sees a light turn red, assuming a 0.500-s reaction time.

Significance

The displacements found in this example seem reasonable for stopping a fast-moving car. It should take longer to stop a car on wet pavement than dry. It is interesting that reaction time adds significantly to the displacements, but more important is the general approach to solving problems. We identify the knowns and the quantities to be determined, then find an appropriate equation. If there is more than one unknown, we need as many independent equations as there are unknowns to solve. There is often more than one way to solve a problem. The various parts of this example can, in fact, be solved by other methods, but the solutions presented here are the shortest.

Example 3.11: Calculating Time

Suppose a car merges into freeway traffic on a 200-m-long ramp. If its initial velocity is 10.0 m/s and it accelerates at 2.00 m/s^2, how long does it take the car to travel the 200 m up the ramp? (Such information might be useful to a traffic engineer.)

Strategy

First, we draw a sketch Figure 2.6.3. We are asked to solve for time t. As before, we identify the known quantities to choose a convenient physical relationship (that is, an equation with one unknown, t.)

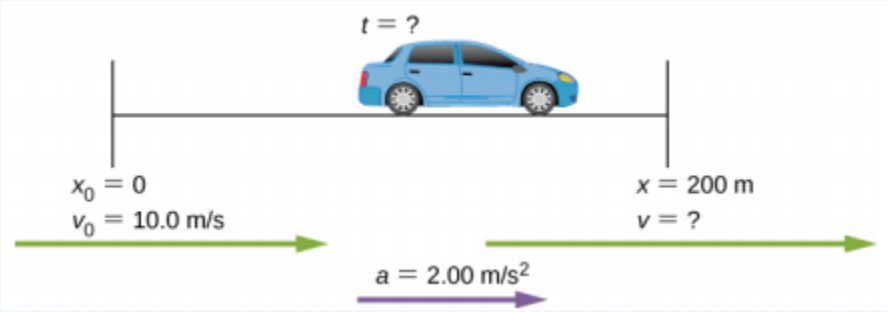


Figure 2.6.3: Sketch of a car accelerating on a freeway ramp.

Solution

Again, we identify the knowns and what we want to solve for. We know that $x_0 = 0$, $v_0 = 10$ m/s, a = 2.00 m/s^2, and x = 200 m.

We need to solve for t. The equation $x = x_0 + v_0t + \frac{1}{2}at^2$ works best because the only unknown in the equation is the variable t, for which we need to solve. From this insight we see that when we input the knowns into the equation, we end up with a quadratic equation.

We need to rearrange the equation to solve for t, then substituting the knowns into the equation:

$$200\ m = 0\ m + (10.0\ m/s)t + \frac{1}{2}(2.00\ m/s^2)t^2. \tag{2.6.15}$$

We then simplify the equation. The units of meters cancel because they are in each term. We can get the units of seconds to cancel by taking t = t s, where t is the magnitude of time and s is the unit. Doing so leaves

$$200 = 10t + t^2. \tag{2.6.16}$$

We then use the quadratic formula to solve for t,

$$t^2 + 10t - 200 = 0 \tag{2.6.17}$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \tag{2.6.18}$$

which yields two solutions: t = 10.0 and t = −20.0. A negative value for time is unreasonable, since it would mean the event happened 20 s before the motion began. We can discard that solution. Thus,

$$t = 10.0\ s. \tag{2.6.19}$$

Significance

Whenever an equation contains an unknown squared, there are two solutions. In some problems both solutions are meaningful; in others, only one solution is reasonable. The 10.0-s answer seems reasonable for a typical freeway on-ramp.

Exercise 3.5

A manned rocket accelerates at a rate of 20 m/s^2 during launch. How long does it take the rocket to reach a velocity of 400 m/s?

Example 3.12: Acceleration of a Spaceship

A spaceship has left Earth's orbit and is on its way to the Moon. It accelerates at 20 m/s^2 for 2 min and covers a distance of 1000 km. What are the initial and final velocities of the spaceship?

Strategy

We are asked to find the initial and final velocities of the spaceship. Looking at the kinematic equations, we see that one equation will not give the answer. We must use one kinematic equation to solve for one of the velocities and substitute it into another kinematic equation to get the second velocity. Thus, we solve two of the kinematic equations simultaneously.

Solution

First we solve for v_0 using $x = x_0 + v_0t + \frac{1}{2}at^2 = \frac{1}{2}t^2$:

$$x - x_0 = v_0t + \frac{1}{2}at^2 = \frac{1}{2}t^2 \tag{2.6.20}$$

$$1.0 \times 10^6\ m = v_0(120.0\ s) + \frac{1}{2}(20.0\ m/s^2)(120.0\ s)^2 \tag{2.6.21}$$

$$v_0 = 7133.3\ m/s. \tag{2.6.22}$$

Then we substitute v_0 into $v = v_0 + at$ to solve for the final velocity:

$$v = v_0 + at = 7133.3\ m/s + (20.0\ m/s^2)(120.0\ s) = 9533.3\ m/s. \tag{2.6.23}$$

Significance

There are six variables in displacement, time, velocity, and acceleration that describe motion in one dimension. The initial conditions of a given problem can be many combinations of these variables. Because of this diversity, solutions may not be easy as simple substitutions into one of the equations. This example illustrates that solutions to kinematics may require solving two simultaneous kinematic equations.

With the basics of kinematics established, we can go on to many other interesting examples and applications. In the process of developing kinematics, we have also glimpsed a general approach to problem solving that produces both correct answers and insights into physical relationships. The next level of complexity in our kinematics problems involves the motion of two interrelated bodies, called **two-body pursuit problems**.

Two-Body Pursuit Problems

Up until this point we have looked at examples of motion involving a single body. Even for the problem with two cars and the stopping distances on wet and dry roads, we divided this problem into two separate problems to find the answers. In a **two-body pursuit problem**, the motions of the objects are coupled—meaning, the unknown we seek depends on the motion of both objects. To solve these problems we write the equations of motion for each object and then solve them simultaneously to find the unknown. This is illustrated in Figure 2.6.4.

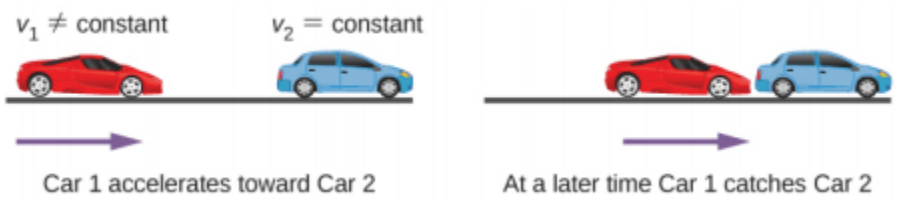


Figure 2.6.4: A two-body pursuit scenario where car 2 has a constant velocity and car 1 is behind with a constant acceleration. Car 1 catches up with car 2 at a later time.

The time and distance required for car 1 to catch car 2 depends on the initial distance car 1 is from car 2 as well as the velocities of both cars and the acceleration of car 1. The kinematic equations describing the motion of both cars must be solved to find these unknowns.

Consider the following example.

Example 3.13: Cheetah Catching a Gazelle

A cheetah waits in hiding behind a bush. The cheetah spots a gazelle running past at 10 m/s. At the instant the gazelle passes the cheetah, the cheetah accelerates from rest at 4 m/s^2 to catch the gazelle. (a) How long does it take the cheetah to catch the gazelle? (b) What is the displacement of the gazelle and cheetah?

Strategy

We use the set of equations for constant acceleration to solve this problem. Since there are two objects in motion, we have separate equations of motion describing each animal. But what links the equations is a common parameter that has the same value for each animal. If we look at the problem closely, it is clear the common parameter to each animal is their position x at a later time t. Since they both start at $x_0 = 0$, their displacements are the same at a later time t, when the cheetah catches up with the gazelle. If we pick the equation of motion that solves for the displacement for each animal, we can then set the equations equal to each other and solve for the unknown, which is time.

Solution

a. Equation for the gazelle: The gazelle has a constant velocity, which is its average velocity, since it is not accelerating. Therefore, we use Equation 3.5.7 with $x_0 = 0$:

$$x = x_0 + \bar{v}t = \bar{v}t. \tag{2.6.24}$$

Equation for the cheetah: The cheetah is accelerating from rest, so we use Equation 3.5.17 with $x_0 = 0$ and $v_0 = 0$:

$$x = x_0 + v_0 t + \frac{1}{2}at^2 = \frac{1}{2}at^2. \tag{2.6.25}$$

Now we have an equation of motion for each animal with a common parameter, which can be eliminated to find the solution. In this case, we solve for t:

$$x = \bar{v}t = \frac{1}{2}at^2 \tag{2.6.26}$$

$$t = \frac{2\bar{v}}{a}. \tag{2.6.27}$$

The gazelle has a constant velocity of 10 m/s, which is its average velocity. The acceleration of the cheetah is 4 m/s^2. Evaluating t, the time for the cheetah to reach the gazelle, we have

$$t = \frac{2\bar{v}}{a} = \frac{2(10)}{4} = 5\ s. \tag{2.6.28}$$

b. To get the displacement, we use either the equation of motion for the cheetah or the gazelle, since they should both give the same answer. Displacement of the cheetah:

$$x = \frac{1}{2}at^2 = \frac{1}{2}(4)(5)^2 - 50\ m. \tag{2.6.29}$$

Displacement of the gazelle:

$$x = \bar{v}t = 10(5) = 50\ m. \tag{2.6.30}$$

We see that both displacements are equal, as expected.

Significance

It is important to analyze the motion of each object and to use the appropriate kinematic equations to describe the individual motion. It is also important to have a good visual perspective of the two-body pursuit problem to see the common parameter that links the motion of both objects.

Exercise 3.6

A bicycle has a constant velocity of 10 m/s. A person starts from rest and begins to run to catch up to the bicycle in 30 s when the bicycle is at the same position as the person. What is the acceleration of the person?

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2.7: Free Fall

Learning Objectives

- Use the kinematic equations with the variables y and g to analyze free-fall motion.
- Describe how the values of the position, velocity, and acceleration change during a free fall.
- Solve for the position, velocity, and acceleration as functions of time when an object is in a free fall.

An interesting application of Equation 3.3.2 through Equation 3.5.22 is called **free fall**, which describes the motion of an object falling in a gravitational field, such as near the surface of Earth or other celestial objects of planetary size. Let's assume the body is falling in a straight line perpendicular to the surface, so its motion is one-dimensional. For example, we can estimate the depth of a vertical mine shaft by dropping a rock into it and listening for the rock to hit the bottom. But "falling," in the context of free fall, does not necessarily imply the body is moving from a greater height to a lesser height. If a ball is thrown upward, the equations of free fall apply equally to its ascent as well as its descent.

Gravity

The most remarkable and unexpected fact about falling objects is that if air resistance and friction are negligible, then in a given location all objects fall toward the center of Earth with the **same constant acceleration, independent of their mass**. This experimentally determined fact is unexpected because we are so accustomed to the effects of air resistance and friction that we expect light objects to fall slower than heavy ones. Until Galileo Galilei (1564–1642) proved otherwise, people believed that a heavier object has a greater acceleration in a free fall. We now know this is not the case. In the absence of air resistance, heavy objects arrive at the ground at the same time as lighter objects when dropped from the same height Figure 2.7.1.

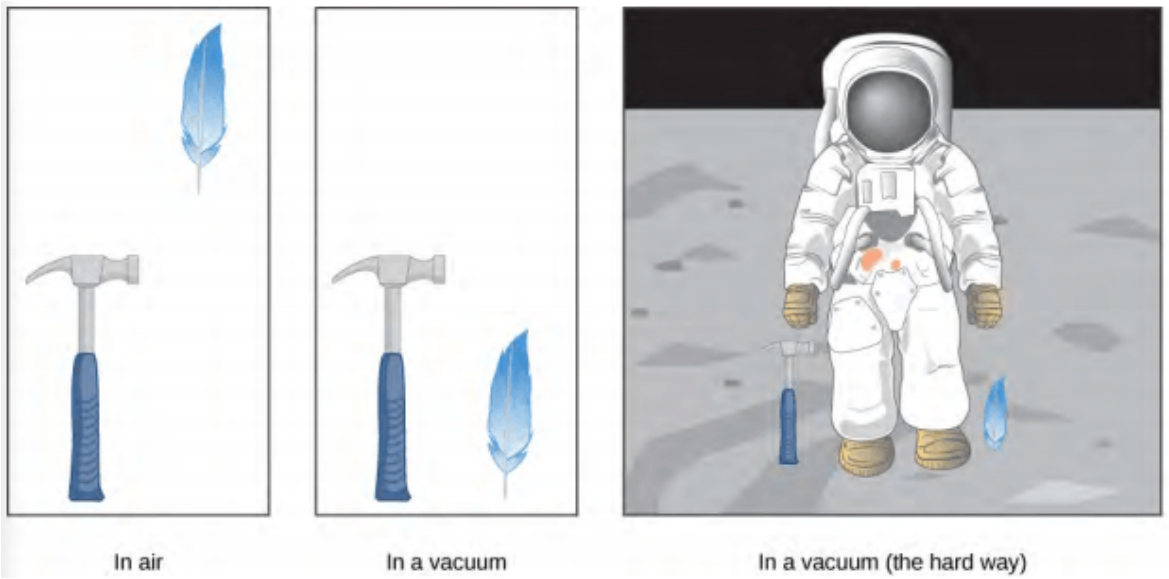


Figure 2.7.1: A hammer and a feather fall with the same constant acceleration if air resistance is negligible. This is a general characteristic of gravity not unique to Earth, as astronaut David R. Scott demonstrated in 1971 on the Moon, where the acceleration from gravity is only 1.67 m/s^2 and there is no atmosphere.

In the real world, air resistance can cause a lighter object to fall slower than a heavier object of the same size. A tennis ball reaches the ground after a baseball dropped at the same time. (It might be difficult to observe the difference if the height is not large.) Air resistance opposes the motion of an object through the air, and friction between objects—such as between clothes and a laundry chute or between a stone and a pool into which it is dropped—also opposes motion between them.

For the ideal situations of these first few chapters, an object **falling without air resistance or friction** is defined to be in **free fall**. The force of gravity causes objects to fall toward the center of Earth. The acceleration of free-falling objects is therefore

called **acceleration due to gravity**. Acceleration due to gravity is constant, which means we can apply the kinematic equations to any falling object where air resistance and friction are negligible. This opens to us a broad class of interesting situations.

Acceleration due to gravity is so important that its magnitude is given its own symbol, g. It is constant at any given location on Earth and has the average value

$$g = 9.81\ m/s^2\ (or\ 32.2\ ft/s^2). \tag{2.7.1}$$

Although g varies from 9.78 m/s^2 to 9.83 m/s^2, depending on latitude, altitude, underlying geological formations, and local topography, let's use an average value of 9.8 m/s^2 rounded to two significant figures in this text unless specified otherwise. Neglecting these effects on the value of g as a result of position on Earth's surface, as well as effects resulting from Earth's rotation, we take the direction of acceleration due to gravity to be downward (toward the center of Earth). In fact, its direction **defines** what we call vertical. Note that whether acceleration a in the kinematic equations has the value +g or –g depends on how we define our coordinate system. If we define the upward direction as positive, then a = –g = –9.8 m/s^2, and if we define the downward direction as positive, then a = g = 9.8 m/s^2.

One-Dimensional Motion Involving Gravity

The best way to see the basic features of motion involving gravity is to start with the simplest situations and then progress toward more complex ones. So, we start by considering straight up-and-down motion with no air resistance or friction. These assumptions mean the velocity (if there is any) is vertical. If an object is dropped, we know the initial velocity is zero when in free fall. When the object has left contact with whatever held or threw it, the object is in free fall. When the object is thrown, it has the same initial speed in free fall as it did before it was released. When the object comes in contact with the ground or any other object, it is no longer in free fall and its acceleration of g is no longer valid. Under these circumstances, the motion is one-dimensional and has constant acceleration of magnitude g. We represent vertical displacement with the symbol y.

Kinematic Equations for Objects in Free Fall

We assume here that acceleration equals –g (with the positive direction upward).

$$v = v_0 - gt \tag{2.7.2}$$

$$y = y_0 + v_0 t - \frac{1}{2} g t^2 \tag{2.7.3}$$

$$v^2 = v_0^2 - 2g(y - y_0) \tag{2.7.4}$$

Problem-Solving Strategy: Free Fall

1. Decide on the sign of the acceleration of gravity. In Equation 2.7.2 through Equation 2.7.4, acceleration g is negative, which says the positive direction is upward and the negative direction is downward. In some problems, it may be useful to have acceleration g as positive, indicating the positive direction is downward.
2. Draw a sketch of the problem. This helps visualize the physics involved.
3. Record the knowns and unknowns from the problem description. This helps devise a strategy for selecting the appropriate equations to solve the problem.
4. Decide which of Equation 2.7.2 through Equation 2.7.4 are to be used to solve for the unknowns.

Example 3.14: Free Fall of a Ball

Figure 2.7.2 shows the positions of a ball, at 1-s intervals, with an initial velocity of 4.9 m/s downward, that is thrown from the top of a 98-m-high building. (a) How much time elapses before the ball reaches the ground? (b) What is the velocity when it arrives at the ground?

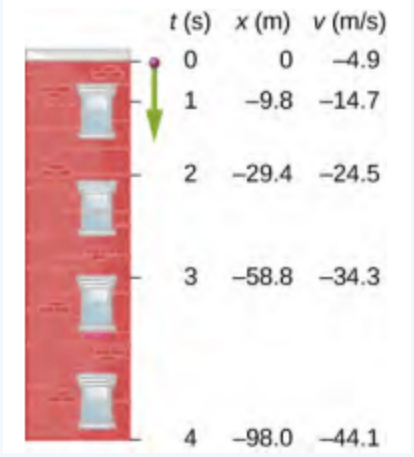


Figure 2.7.2: The positions and velocities at 1-s intervals of a ball thrown downward from a tall building at 4.9 m/s.

Strategy

Choose the origin at the top of the building with the positive direction upward and the negative direction downward. To find the time when the position is –98 m, we use Equation 2.7.3, with $y_0 = 0$, $v_0 = -4.9$ m/s, and g = 9.8 m/s^2.

Solution

a. Substitute the given values into the equation:

$$y = y_0 + v_0 t - \frac{1}{2} g t^2 \tag{2.7.5}$$

$$-98.0\ m = 0 - (4.9\ m/s)t - \frac{1}{2}(9.8\ m/s^2)t^2. \tag{2.7.6}$$

This simplifies to

$$t^2 + t - 20 = 0. \tag{2.7.7}$$

This is a quadratic equation with roots t = –5.0 s and t = 4.0 s. The positive root is the one we are interested in, since time t = 0 is the time when the ball is released at the top of the building. (The time t = –5.0 s represents the fact that a ball thrown upward from the ground would have been in the air for 5.0 s when it passed by the top of the building moving downward at 4.9 m/s.)

b. Using Equation 2.7.2, we have

$$v = v_0 - gt = -4.9\ m/s - (9.8\ m/s^2)(4.0\ s) = -44.1\ m/s. \tag{2.7.8}$$

Significance

For situations when two roots are obtained from a quadratic equation in the time variable, we must look at the physical significance of both roots to determine which is correct. Since t = 0 corresponds to the time when the ball was released, the negative root would correspond to a time before the ball was released, which is not physically meaningful. When the ball hits the ground, its velocity is not immediately zero, but as soon as the ball interacts with the ground, its acceleration is not g and it accelerates with a different value over a short time to zero velocity. This problem shows how important it is to establish the correct coordinate system and to keep the signs of g in the kinematic equations consistent.

Example 3.15: Vertical Motion of a Baseball

A batter hits a baseball straight upward at home plate and the ball is caught 5.0 s after it is struck Figure 2.7.3. (a) What is the initial velocity of the ball? (b) What is the maximum height the ball reaches? (c) How long does it take to reach the maximum height? (d) What is the acceleration at the top of its path? (e) What is the velocity of the ball when it is caught? Assume the ball is hit and caught at the same location.

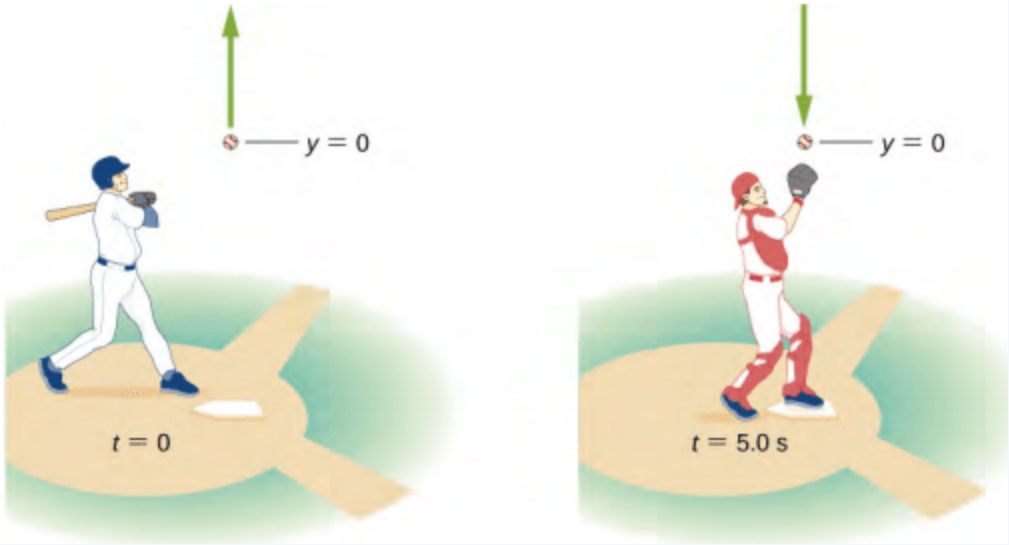


Figure 2.7.3: A baseball hit straight up is caught by the catcher 5.0 s later.

Strategy

Choose a coordinate system with a positive y-axis that is straight up and with an origin that is at the spot where the ball is hit and caught.

Solution

a. Equation 2.7.3 gives

$$y = y_0 + v_0 t - \frac{1}{2} g t^2 \tag{2.7.9}$$

$$0 = 0 + v_0(5.0\ s) - \frac{1}{2}(9.8\ m/s^2)(5.0\ s)^2. \tag{2.7.10}$$

which gives v_0 = 24.5 m/sec.

b. At the maximum height, v = 0. With v_0 = 24.5 m/s, Equation 2.7.4 gives

$$v^2 = v_0^2 - 2g(y - y_0) \tag{2.7.11}$$

$$0 = (24.5\ m/s^2) - 2(9.8\ m/s^2)(y - 0) \tag{2.7.12}$$

or

$$y = 30.6\ m. \tag{2.7.13}$$

c. To find the time when v = 0 , we use Equation 2.7.2:

$$v = v_0 - gt \tag{2.7.14}$$

$$0 = 24..5\ m/s - (9.8\ m/s^2)t. \tag{2.7.15}$$

This gives t = 2.5 s. Since the ball rises for 2.5 s, the time to fall is 2.5 s.

d. The acceleration is 9.8 m/s^2 everywhere, even when the velocity is zero at the top of the path. Although the velocity is zero at the top, it is changing at the rate of 9.8 m/s^2 downward.

e. The velocity at t = 5.0 s can be determined with Equation 2.7.2:

$$\begin{aligned} v &= v_0 - gt \\ &= 24.5\ m/s - 9.8\ m/s^2(5.0\ s) \\ &= -24.5\ m/s. \end{aligned} \tag{2.7.16}$$

Significance

The ball returns with the speed it had when it left. This is a general property of free fall for any initial velocity. We used a single equation to go from throw to catch, and did not have to break the motion into two segments, upward and downward. We are used to thinking of the effect of gravity is to create free fall downward toward Earth. It is important to understand, as illustrated in this example, that objects moving upward away from Earth are also in a state of free fall.

Exercise 3.7

A chunk of ice breaks off a glacier and falls 30.0 m before it hits the water. Assuming it falls freely (there is no air resistance), how long does it take to hit the water? Which quantity increases faster, the speed of the ice chunk or its distance traveled?

Example 3.16: Rocket Booster

A small rocket with a booster blasts off and heads straight upward. When at a height of 5.0 km and velocity of 200.0 m/s, it releases its booster. (a) What is the maximum height the booster attains? (b) What is the velocity of the booster at a height of 6.0 km? Neglect air resistance.



Figure 2.7.4: A rocket releases its booster at a given height and velocity. How high and how fast does the booster go?

Strategy

We need to select the coordinate system for the acceleration of gravity, which we take as negative downward. We are given the initial velocity of the booster and its height. We consider the point of release as the origin. We know the velocity is zero at the maximum position within the acceleration interval; thus, the velocity of the booster is zero at its maximum height, so we can use this information as well. From these observations, we use Equation 2.7.4, which gives us the maximum height of the booster. We also use Equation 2.7.4 to give the velocity at 6.0 km. The initial velocity of the booster is 200.0 m/s.

Solution

a. From Equation 2.7.4, $v^2 = v_0^2 - 2g(y - y_0)$. With v = 0 and y_0 = 0, we can solve for y:

$$y = \frac{v_0^2}{-2g} = \frac{(2.0 \times 10^2\ m/s)^2}{-2(9.8\ m/s^2)} = 2040.8\ m. \tag{2.7.17}$$

This solution gives the maximum height of the booster in our coordinate system, which has its origin at the point of release, so the maximum height of the booster is roughly 7.0 km.

b. An altitude of 6.0 km corresponds to y = 1.0 x 10^3 m in the coordinate system we are using. The other initial conditions are y_0 = 0, and v_0 = 200.0 m/s. We have, from Equation 2.7.4,

$$v^2 = (200.0\ m/s)^2 - 2(9.8\ m/s^2)(1.0 \times 10^3\ m) \Rightarrow v = \pm 142.8\ m/s. \quad (2.7.18)$$

Significance

We have both a positive and negative solution in (b). Since our coordinate system has the positive direction upward, the +142.8 m/s corresponds to a positive upward velocity at 6000 m during the upward leg of the trajectory of the booster. The value v = –142.8 m/s corresponds to the velocity at 6000 m on the downward leg. This example is also important in that an object is given an initial velocity at the origin of our coordinate system, but the origin is at an altitude above the surface of Earth, which must be taken into account when forming the solution.

Simulation

Visit this site to learn about graphing polynomials. The shape of the curve changes as the constants are adjusted. View the curves for the individual terms (for example, y = bx) to see how they add to generate the polynomial curve.

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2.8: Finding Velocity and Displacement from Acceleration

Learning Objectives

- Derive the kinematic equations for constant acceleration using integral calculus.
- Use the integral formulation of the kinematic equations in analyzing motion.
- Find the functional form of velocity versus time given the acceleration function.
- Find the functional form of position versus time given the velocity function.

This section assumes you have enough background in calculus to be familiar with integration. In Instantaneous Velocity and Speed and Average and Instantaneous Acceleration we introduced the kinematic functions of velocity and acceleration using the derivative. By taking the derivative of the position function we found the velocity function, and likewise by taking the derivative of the velocity function we found the acceleration function. Using integral calculus, we can work backward and calculate the velocity function from the acceleration function, and the position function from the velocity function.

Kinematic Equations from Integral Calculus

Let's begin with a particle with an acceleration a(t) is a known function of time. Since the time derivative of the velocity function is acceleration,

$$\frac{d}{dt}v(t) = a(t), \tag{2.8.1}$$

we can take the indefinite integral of both sides, finding

$$\int \frac{d}{dt}v(t)dt = \int a(t)dt + C_1, \tag{2.8.2}$$

where C_1 is a constant of integration. Since $\int \frac{d}{dt}v(t)dt = v(t)$, the velocity is given by

$$v(t) = \int a(t)dt + C_1. \tag{2.8.3}$$

Similarly, the time derivative of the position function is the velocity function,

$$\frac{d}{dt}x(t) = v(t). \tag{2.8.4}$$

Thus, we can use the same mathematical manipulations we just used and find

$$x(t) = \int v(t)dt + C_2, \tag{2.8.5}$$

where C_2 is a second constant of integration.

We can derive the kinematic equations for a constant acceleration using these integrals. With a(t) = a, a constant, and doing the integration in Equation 2.8.3, we find

$$v(t) = \int adt + C_1 = at + C_1. \tag{2.8.6}$$

If the initial velocity is $v(0) = v_0$, then

$$v_0 = 0 + C_1. \tag{2.8.7}$$

Then, $C_1 = v_0$ and

$$v(t) = v_0 + at, \tag{2.8.8}$$

which is Equation 3.5.12. Substituting this expression into Equation 2.8.5 gives

$$x(t) = \int (v_0 + at)dt + C_2. \tag{2.8.9}$$

Doing the integration, we find

$$x(t) = v_0 t + \frac{1}{2}at^2 + C_2. \tag{2.8.10}$$

If x(0) = x_0, we have

$$x_0 = 0 + 0 + C_2. \tag{2.8.11}$$

so, $C_2 = x_0$. Substituting back into the equation for x(t), we finally have

$$x(t) = x_0 + v_0 t + \frac{1}{2}at^2. \tag{2.8.12}$$

which is Equation 3.5.17.

Example 3.17: Motion of a Motorboat

A motorboat is traveling at a constant velocity of 5.0 m/s when it starts to decelerate to arrive at the dock. Its acceleration is $a(t) = -\frac{1}{4}$ t m/s^2. (a) What is the velocity function of the motorboat? (b) At what time does the velocity reach zero? (c) What is the position function of the motorboat? (d) What is the displacement of the motorboat from the time it begins to decelerate to when the velocity is zero? (e) Graph the velocity and position functions.

Strategy

(a) To get the velocity function we must integrate and use initial conditions to find the constant of integration. (b) We set the velocity function equal to zero and solve for t. (c) Similarly, we must integrate to find the position function and use initial conditions to find the constant of integration. (d) Since the initial position is taken to be zero, we only have to evaluate the position function at t = 0 .

Solution

We take t = 0 to be the time when the boat starts to decelerate.

a. From the functional form of the acceleration we can solve Equation 2.8.3 to get v(t):

$$v(t) = \int a(t)dt + C_1 = \int -\frac{1}{4}tdt + C_1 = -\frac{1}{8}t^2 + C_1. \tag{2.8.13}$$

At t = 0 we have v(0) = 5.0 m/s = 0 + C_1, so C_1 = 5.0 m/s or v(t) = 5.0 m/s – $\frac{1}{8}$ t^2.

b. v(t) = 0 = 5.0 m/s – $\frac{1}{8}$ t^2 (\Rightarrow\) t = 6.3 s

c. Solve Equation 2.8.5:

$$x(t) = \int v(t)dt + C_2 = \int (5.0 - \frac{1}{8}t^2)dt + C_2 = 5.0t - \frac{1}{24}t^3 + C_2. \tag{2.8.14}$$

At t = 0, we set x(0) = 0 = x_0, since we are only interested in the displacement from when the boat starts to decelerate. We have

$$x(0) = 0 = C_2. \tag{2.8.15}$$

Therefore, the equation for the position is

$$x(t) = 5.0t - \frac{1}{24}t^3. \tag{2.8.16}$$

d. Since the initial position is taken to be zero, we only have to evaluate x(t) when the velocity is zero. This occurs at t = 6.3 s. Therefore, the displacement is

$$x(6.3) = 5.0(6.3) - \frac{1}{24}(6.3)^3 = 21.1\ m. \tag{2.8.17}$$

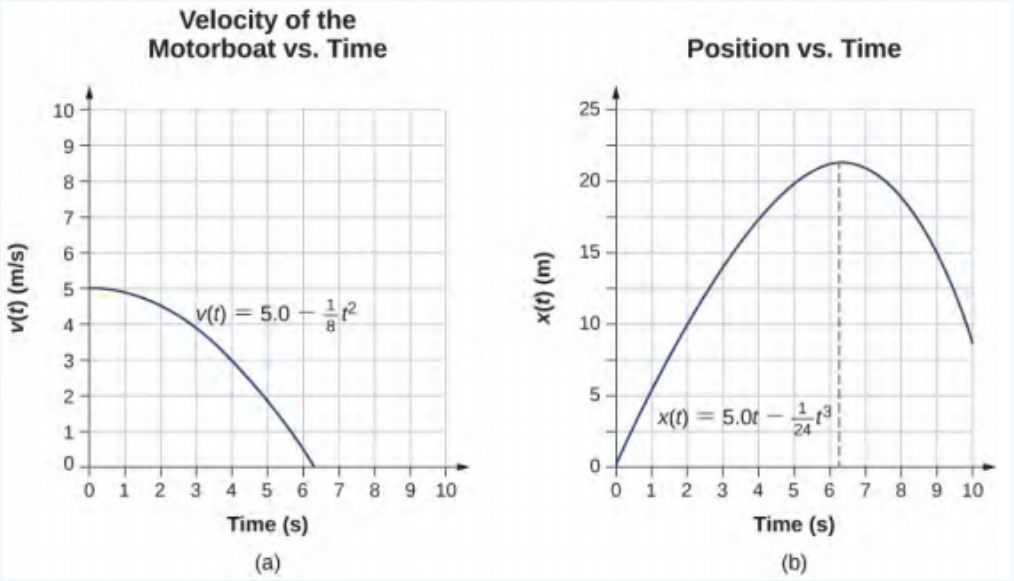


Figure 2.8.1: (a) Velocity of the motorboat as a function of time. The motorboat decreases its velocity to zero in 6.3 s. At times greater than this, velocity becomes negative—meaning, the boat is reversing direction. (b) Position of the motorboat as a function of time. At t = 6.3 s, the velocity is zero and the boat has stopped. At times greater than this, the velocity becomes negative—meaning, if the boat continues to move with the same acceleration, it reverses direction and heads back toward where it originated.

Significance

The acceleration function is linear in time so the integration involves simple polynomials. In Figure 2.8.1, we see that if we extend the solution beyond the point when the velocity is zero, the velocity becomes negative and the boat reverses direction. This tells us that solutions can give us information outside our immediate interest and we should be careful when interpreting them.

Exercise 3.8

A particle starts from rest and has an acceleration function $a(t) = \left(5 - \left(10\frac{1}{s}\right)t\right)\frac{m}{s^2}$. (a) What is the velocity function? (b) What is the position function? (c) When is the velocity zero?

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2.9: Motion Along a Straight Line (Exercises)

Conceptual Questions

3.1 Position, Displacement, and Average Velocity

1. Give an example in which there are clear distinctions among distance traveled, displacement, and magnitude of displacement. Identify each quantity in your example specifically.
2. Under what circumstances does distance traveled equal magnitude of displacement? What is the only case in which magnitude of displacement and displacement are exactly the same?
3. Bacteria move back and forth using their flagella (structures that look like little tails). Speeds of up to 50 μm/s (50 x 10^{-6} m/s) have been observed. The total distance traveled by a bacterium is large for its size, whereas its displacement is small. Why is this?
4. Give an example of a device used to measure time and identify what change in that device indicates a change in time.
5. Does a car's odometer measure distance traveled or displacement?
6. During a given time interval the average velocity of an object is zero. What can you say conclude about its displacement over the time interval?

3.2 Instantaneous Velocity and Speed

7. There is a distinction between average speed and the magnitude of average velocity. Give an example that illustrates the difference between these two quantities.
8. Does the speedometer of a car measure speed or velocity?
9. If you divide the total distance traveled on a car trip (as determined by the odometer) by the elapsed time of the trip, are you calculating average speed or magnitude of average velocity? Under what circumstances are these two quantities the same?
10. How are instantaneous velocity and instantaneous speed related to one another? How do they differ?

3.3 Average and Instantaneous Acceleration

11. Is it possible for speed to be constant while acceleration is not zero?
12. Is it possible for velocity to be constant while acceleration is not zero? Explain.
13. Give an example in which velocity is zero yet acceleration is not.
14. If a subway train is moving to the left (has a negative velocity) and then comes to a stop, what is the direction of its acceleration? Is the acceleration positive or negative?
15. Plus and minus signs are used in one-dimensional motion to indicate direction. What is the sign of an acceleration that reduces the magnitude of a negative velocity? Of a positive velocity?

3.4 Motion with Constant Acceleration

16. When analyzing the motion of a single object, what is the required number of known physical variables that are needed to solve for the unknown quantities using the kinematic equations?
17. State two scenarios of the kinematics of single object where three known quantities require two kinematic equations to solve for the unknowns.

3.5 Free Fall

18. What is the acceleration of a rock thrown straight upward on the way up? At the top of its flight? On the way down? Assume there is no air resistance.
19. An object that is thrown straight up falls back to Earth. This is one-dimensional motion. (a) When is its velocity zero? (b) Does its velocity change direction? (c) Does the acceleration have the same sign on the way up as on the way down?

20. Suppose you throw a rock nearly straight up at a coconut in a palm tree and the rock just misses the coconut on the way up but hits the coconut on the way down. Neglecting air resistance and the slight horizontal variation in motion to account for the hit and miss of the coconut, how does the speed of the rock when it hits the coconut on the way down compare with what it would have been if it had hit the coconut on the way up? Is it more likely to dislodge the coconut on the way up or down? Explain.
21. The severity of a fall depends on your speed when you strike the ground. All factors but the acceleration from gravity being the same, how many times higher could a safe fall on the Moon than on Earth (gravitational acceleration on the Moon is about one-sixth that of the Earth)?
22. How many times higher could an astronaut jump on the Moon than on Earth if her takeoff speed is the same in both locations (gravitational acceleration on the Moon is about on-sixth of that on Earth)?

3.6 Finding Velocity and Displacement from Acceleration

23. When given the acceleration function, what additional information is needed to find the velocity function and position function?

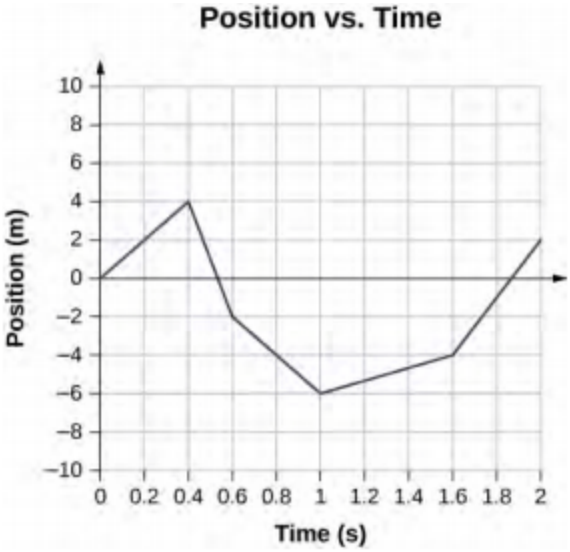
Problems

3.1 Position, Displacement, and Average Velocity

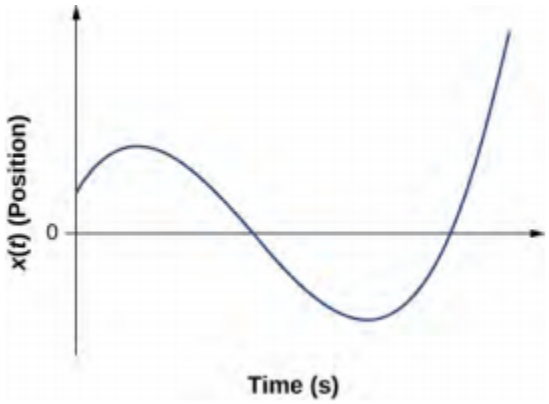
24. Consider a coordinate system in which the positive x axis is directed upward vertically. What are the positions of a particle (a) 5.0 m directly above the origin and (b) 2.0 m below the origin?
25. A car is 2.0 km west of a traffic light at t = 0 and 5.0 km east of the light at t = 6.0 min. Assume the origin of the coordinate system is the light and the positive x direction is eastward. (a) What are the car's position vectors at these two times? (b) What is the car's displacement between 0 min and 6.0 min?
26. The Shanghai maglev train connects Longyang Road to Pudong International Airport, a distance of 30 km. The journey takes 8 minutes on average. What is the maglev train's average velocity?
27. The position of a particle moving along the x-axis is given by $x(t) = 4.0 - 2.0t$ m. (a) At what time does the particle cross the origin? (b) What is the displacement of the particle between t = 3.0 s and t = 6.0 s?
28. A cyclist rides 8.0 km east for 20 minutes, then he turns and heads west for 8 minutes and 3.2 km. Finally, he rides east for 16 km, which takes 40 minutes. (a) What is the final displacement of the cyclist? (b) What is his average velocity?
29. On February 15, 2013, a superbolide meteor (brighter than the Sun) entered Earth's atmosphere over Chelyabinsk, Russia, and exploded at an altitude of 23.5 km. Eyewitnesses could feel the intense heat from the fireball, and the blast wave from the explosion blew out windows in buildings. The blast wave took approximately 2 minutes 30 seconds to reach ground level. (a) What was the average velocity of the blast wave? b) Compare this with the speed of sound, which is 343 m/s at sea level.

3.2 Instantaneous Velocity and Speed

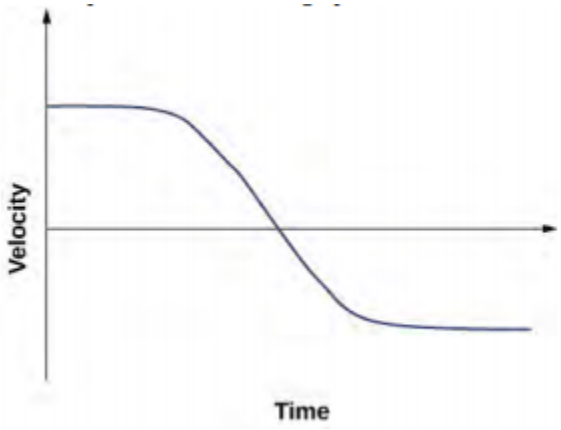
30. A woodchuck runs 20 m to the right in 5 s, then turns and runs 10 m to the left in 3 s. (a) What is the average velocity of the woodchuck? (b) What is its average speed?
31. Sketch the velocity-versus-time graph from the following position-versus-time graph.



32. Sketch the velocity-versus-time graph from the following position-versus-time graph.



33. Given the following velocity-versus-time graph, sketch the position-versus-time graph.

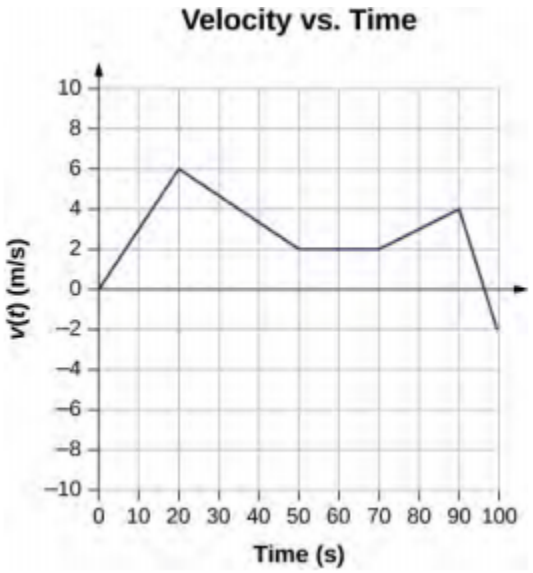


34. An object has a position function x(t) = 5t m. (a) What is the velocity as a function of time? (b) Graph the position function and the velocity function. 35. A particle moves along the x-axis according to $x(t) = 10t - 2t^2$ m. (a) What is the instantaneous velocity at t = 2 s and t = 3 s? (b) What is the instantaneous speed at these times? (c) What is the average velocity between t = 2 s and t = 3 s?
35. **Unreasonable results**. A particle moves along the x-axis according to $x(t) = 3t^3 + 5t$. At what time is the velocity of

the particle equal to zero? Is this reasonable?

3.3 Average and Instantaneous Acceleration

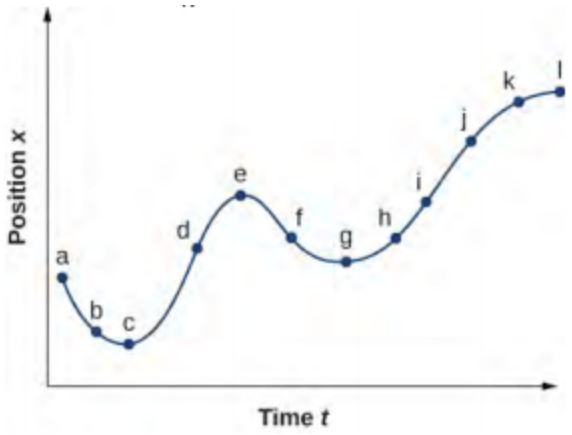
37. A cheetah can accelerate from rest to a speed of 30.0 m/s in 7.00 s. What is its acceleration?
38. Dr. John Paul Stapp was a U.S. Air Force officer who studied the effects of extreme acceleration on the human body. On December 10, 1954, Stapp rode a rocket sled, accelerating from rest to a top speed of 282 m/s (1015 km/h) in 5.00 s and was brought jarringly back to rest in only 1.40 s. Calculate his (a) acceleration in his direction of motion and (b) acceleration opposite to his direction of motion. Express each in multiples of g (9.80 m/s^2) by taking its ratio to the acceleration of gravity.
39. Sketch the acceleration-versus-time graph from the following velocity-versus-time graph.



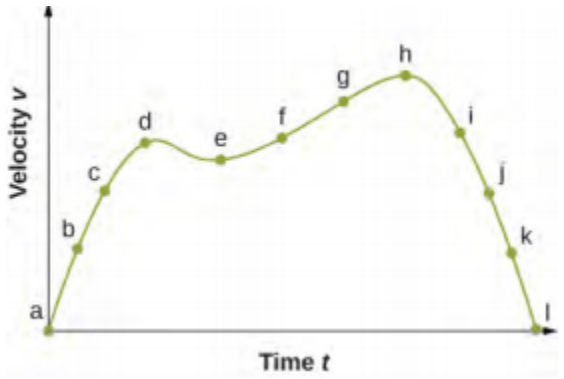
40. A commuter backs her car out of her garage with an acceleration of 1.40 m/s^2. (a) How long does it take her to reach a speed of 2.00 m/s? (b) If she then brakes to a stop in 0.800 s, what is her acceleration?
41. Assume an intercontinental ballistic missile goes from rest to a suborbital speed of 6.50 km/s in 60.0 s (the actual speed and time are classified). What is its average acceleration in meters per second and in multiples of g (9.80 m/s^2)?
42. An airplane, starting from rest, moves down the runway at constant acceleration for 18 s and then takes off at a speed of 60 m/s. What is the average acceleration of the plane?

3.4 Motion with Constant Acceleration

43. A particle moves in a straight line at a constant velocity of 30 m/s. What is its displacement between t = 0 and t = 5.0 s?
44. A particle moves in a straight line with an initial velocity of 0 m/s and a constant acceleration of 30 m/s^2. If x = 0 at t = 0, what is the particle's position at t = 5 s?
45. A particle moves in a straight line with an initial velocity of 30 m/s and constant acceleration 30 m/s^2. (a) What is its displacement at t = 5 s? (b) What is its velocity at this same time?
46. (a) Sketch a graph of velocity versus time corresponding to the graph of displacement versus time given in the following figure. (b) Identify the time or times (t_a, t_b, t_c, etc.) at which the instantaneous velocity has the greatest positive value. (c) At which times is it zero? (d) At which times is it negative?



47. (a) Sketch a graph of acceleration versus time corresponding to the graph of velocity versus time given in the following figure. (b) Identify the time or times (ta , tb, tc , etc.) at which the acceleration has the greatest positive value. (c) At which times is it zero? (d) At which times is it negative?



48. A particle has a constant acceleration of 6.0 m/s^2. (a) If its initial velocity is 2.0 m/s, at what time is its displacement 5.0 m? (b) What is its velocity at that time?
49. At t = 10 s, a particle is moving from left to right with a speed of 5.0 m/s. At t = 20 s, the particle is moving right to left with a speed of 8.0 m/s. Assuming the particle's acceleration is constant, determine (a) its acceleration, (b) its initial velocity, and (c) the instant when its velocity is zero.
50. A well-thrown ball is caught in a well-padded mitt. If the acceleration of the ball is 2.10×10^4 m/s^2, and 1.85 ms ($1 ms = 10^{-3}$ s) elapses from the time the ball first touches the mitt until it stops, what is the initial velocity of the ball?
51. A bullet in a gun is accelerated from the firing chamber to the end of the barrel at an average rate of 6.20×10^5 m/s^2 for 8.10×10^{-4} s. What is its muzzle velocity (that is, its final velocity)?
52. (a) A light-rail commuter train accelerates at a rate of 1.35 m/s^2. How long does it take to reach its top speed of 80.0 km/h, starting from rest? (b) The same train ordinarily decelerates at a rate of 1.65 m/s^2. How long does it take to come to a stop from its top speed? (c) In emergencies, the train can decelerate more rapidly, coming to rest from 80.0 km/h in 8.30 s. What is its emergency acceleration in meters per second squared?
53. While entering a freeway, a car accelerates from rest at a rate of 2.04 m/s^2 for 12.0 s. (a) Draw a sketch of the situation. (b) List the knowns in this problem. (c) How far does the car travel in those 12.0 s? To solve this part, first identify the unknown, then indicate how you chose the appropriate equation to solve for it. After choosing the equation, show your steps in solving for the unknown, check your units, and discuss whether the answer is reasonable. (d) What is the car's final velocity? Solve for this unknown in the same manner as in (c), showing all steps explicitly.
54. **Unreasonable results** At the end of a race, a runner decelerates from a velocity of 9.00 m/s at a rate of 2.00 m/s^2. (a) How far does she travel in the next 5.00 s? (b) What is her final velocity? (c) Evaluate the result. Does it make sense?
55. Blood is accelerated from rest to 30.0 cm/s in a distance of 1.80 cm by the left ventricle of the heart. (a) Make a sketch of the situation. (b) List the knowns in this problem. (c) How long does the acceleration take? To solve this part, first identify the unknown, then discuss how you chose the appropriate equation to solve for it. After choosing the equation,

show your steps in solving for the unknown, checking your units. (d) Is the answer reasonable when compared with the time for a heartbeat?

56. During a slap shot, a hockey player accelerates the puck from a velocity of 8.00 m/s to 40.0 m/s in the same direction. If this shot takes 3.33 x 10^{-2} s , what is the distance over which the puck accelerates?
57. A powerful motorcycle can accelerate from rest to 26.8 m/s (100 km/h) in only 3.90 s. (a) What is its average acceleration? (b) Assuming constant acceleration, how far does it travel in that time?
58. Freight trains can produce only relatively small accelerations. (a) What is the final velocity of a freight train that accelerates at a rate of 0.0500 m/s^2 for 8.00 min, starting with an initial velocity of 4.00 m/s? (b) If the train can slow down at a rate of 0.550 m/s^2, how long will it take to come to a stop from this velocity? (c) How far will it travel in each case?
59. A fireworks shell is accelerated from rest to a velocity of 65.0 m/s over a distance of 0.250 m. (a) Calculate the acceleration. (b) How long did the acceleration last?
60. A swan on a lake gets airborne by flapping its wings and running on top of the water. (a) If the swan must reach a velocity of 6.00 m/s to take off and it accelerates from rest at an average rate of 0.35 m/s^2, how far will it travel before becoming airborne? (b) How long does this take?
61. A woodpecker's brain is specially protected from large accelerations by tendon-like attachments inside the skull. While pecking on a tree, the woodpecker's head comes to a stop from an initial velocity of 0.600 m/s in a distance of only 2.00 mm. (a) Find the acceleration in meters per second squared and in multiples of g, where g = 9.80 m/s^2. (b) Calculate the stopping time. (c) The tendons cradling the brain stretch, making its stopping distance 4.50 mm (greater than the head and, hence, less acceleration of the brain). What is the brain's acceleration, expressed in multiples of g?
62. An unwary football player collides with a padded goalpost while running at a velocity of 7.50 m/s and comes to a full stop after compressing the padding and his body 0.350 m. (a) What is his acceleration? (b) How long does the collision last?
63. A care package is dropped out of a cargo plane and lands in the forest. If we assume the care package speed on impact is 54 m/s (123 mph), then what is its acceleration? Assume the trees and snow stops it over a distance of 3.0 m.
64. An express train passes through a station. It enters with an initial velocity of 22.0 m/s and decelerates at a rate of 0.150 m/s^2 as it goes through. The station is 210.0 m long. (a) How fast is it going when the nose leaves the station? (b) How long is the nose of the train in the station? (c) If the train is 130 m long, what is the velocity of the end of the train as it leaves? (d) When does the end of the train leave the station?
65. **Unreasonable results** Dragsters can actually reach a top speed of 145.0 m/s in only 4.45 s. (a) Calculate the average acceleration for such a dragster. (b) Find the final velocity of this dragster starting from rest and accelerating at the rate found in (a) for 402.0 m (a quarter mile) without using any information on time. (c) Why is the final velocity greater than that used to find the average acceleration? (**Hint**: Consider whether the assumption of constant acceleration is valid for a dragster. If not, discuss whether the acceleration would be greater at the beginning or end of the run and what effect that would have on the final velocity.)

3.5 Free Fall

66. Calculate the displacement and velocity at times of (a) 0.500 s, (b) 1.00 s, (c) 1.50 s, and (d) 2.00 s for a ball thrown straight up with an initial velocity of 15.0 m/s. Take the point of release to be $y_0 = 0$.
67. Calculate the displacement and velocity at times of (a) 0.500 s, (b) 1.00 s, (c) 1.50 s, (d) 2.00 s, and (e) 2.50 s for a rock thrown straight down with an initial velocity of 14.0 m/s from the Verrazano Narrows Bridge in New York City. The roadway of this bridge is 70.0 m above the water.
68. A basketball referee tosses the ball straight up for the starting tip-off. At what velocity must a basketball player leave the ground to rise 1.25 m above the floor in an attempt to get the ball?
69. A rescue helicopter is hovering over a person whose boat has sunk. One of the rescuers throws a life preserver straight down to the victim with an initial velocity of 1.40 m/s and observes that it takes 1.8 s to reach the water. (a) List the knowns in this problem. (b) How high above the water was the preserver released? Note that the downdraft of the helicopter reduces the effects of air resistance on the falling life preserver, so that an acceleration equal to that of gravity is reasonable.
70. **Unreasonable results** A dolphin in an aquatic show jumps straight up out of the water at a velocity of 15.0 m/s. (a) List the knowns in this problem. (b) How high does his body rise above the water? To solve this part, first note that the final velocity is now a known, and identify its value. Then, identify the unknown and discuss how you chose the appropriate equation to solve for it. After choosing the equation, show your steps in solving for the unknown, checking units, and discuss whether the answer is reasonable. (c) How long a time is the dolphin in the air? Neglect any effects resulting

from his size or orientation.

71. A diver bounces straight up from a diving board, avoiding the diving board on the way down, and falls feet first into a pool. She starts with a velocity of 4.00 m/s and her takeoff point is 1.80 m above the pool. (a) What is her highest point above the board? (b) How long a time are her feet in the air? (c) What is her velocity when her feet hit the water?
72. (a) Calculate the height of a cliff if it takes 2.35 s for a rock to hit the ground when it is thrown straight up from the cliff with an initial velocity of 8.00 m/s. (b) How long a time would it take to reach the ground if it is thrown straight down with the same speed?
73. A very strong, but inept, shot putter puts the shot straight up vertically with an initial velocity of 11.0 m/s. How long a time does he have to get out of the way if the shot was released at a height of 2.20 m and he is 1.80 m tall?
74. You throw a ball straight up with an initial velocity of 15.0 m/s. It passes a tree branch on the way up at a height of 7.0 m. How much additional time elapses before the ball passes the tree branch on the way back down?
75. A kangaroo can jump over an object 2.50 m high. (a) Considering just its vertical motion, calculate its vertical speed when it leaves the ground. (b) How long a time is it in the air?
76. Standing at the base of one of the cliffs of Mt. Arapiles in Victoria, Australia, a hiker hears a rock break loose from a height of 105.0 m. He can't see the rock right away, but then does, 1.50 s later. (a) How far above the hiker is the rock when he can see it? (b) How much time does he have to move before the rock hits his head?
77. There is a 250-m-high cliff at Half Dome in Yosemite National Park in California. Suppose a boulder breaks loose from the top of this cliff. (a) How fast will it be going when it strikes the ground? (b) Assuming a reaction time of 0.300 s, how long a time will a tourist at the bottom have to get out of the way after hearing the sound of the rock breaking loose (neglecting the height of the tourist, which would become negligible anyway if hit)? The speed of sound is 335.0 m/s on this day.

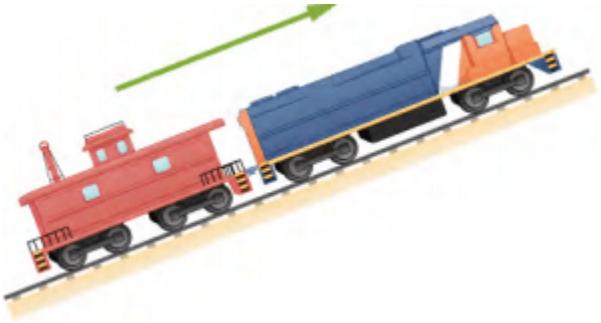
3.6 Finding Velocity and Displacement from Acceleration

78. The acceleration of a particle varies with time according to the equation $a(t) = pt^2 - qt^3$. Initially, the velocity and position are zero. (a) What is the velocity as a function of time? (b) What is the position as a function of time?
79. Between $t = 0$ and $t = t_0$, a rocket moves straight upward with an acceleration given by $a(t) = A - Bt^{1/2}$, where A and B are constants. (a) If x is in meters and t is in seconds, what are the units of A and B? (b) If the rocket starts from rest, how does the velocity vary between $t = 0$ and $t = t_0$? (c) If its initial position is zero, what is the rocket's position as a function of time during this same time interval?
80. The velocity of a particle moving along the x-axis varies with time according to $v(t) = A + Bt^{-1}$, where A = 2 m/s, B = 0.25 m, and $1.0\ s \le t \le 8.0\ s$. Determine the acceleration and position of the particle at t = 2.0 s and t = 5.0 s. Assume that $x(t = 1\ s) = 0$.
81. A particle at rest leaves the origin with its velocity increasing with time according to $v(t) = 3.2t$ m/s. At 5.0 s, the particle's velocity starts decreasing according to $[16.0 - 1.5(t - 5.0)]$ m/s. This decrease continues until t = 11.0 s, after which the particle's velocity remains constant at 7.0 m/s. (a) What is the acceleration of the particle as a function of time? (b) What is the position of the particle at t = 2.0 s, t = 7.0 s, and t = 12.0 s?

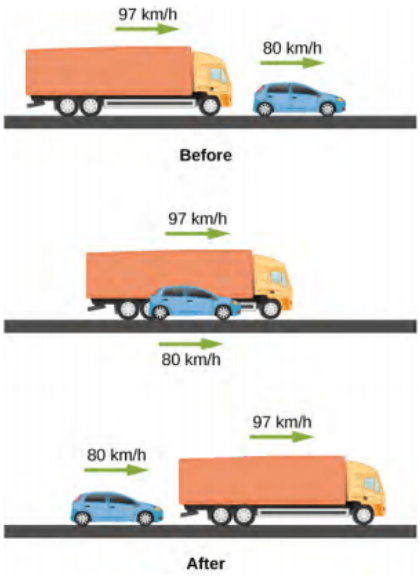
Additional Problems

82. Professional baseball player Nolan Ryan could pitch a baseball at approximately 160.0 km/h. At that average velocity, how long did it take a ball thrown by Ryan to reach home plate, which is 18.4 m from the pitcher's mound? Compare this with the average reaction time of a human to a visual stimulus, which is 0.25 s.
83. An airplane leaves Chicago and makes the 3000-km trip to Los Angeles in 5.0 h. A second plane leaves Chicago one-half hour later and arrives in Los Angeles at the same time. Compare the average velocities of the two planes. Ignore the curvature of Earth and the difference in altitude between the two cities.
84. **Unreasonable Results** A cyclist rides 16.0 km east, then 8.0 km west, then 8.0 km east, then 32.0 km west, and finally 11.2 km east. If his average velocity is 24 km/ h, how long did it take him to complete the trip? Is this a reasonable time?
85. An object has an acceleration of $+1.2\ cm/s^2$. At t = 4.0 s , its velocity is −3.4 cm/s. Determine the object's velocities at t = 1.0 s and t = 6.0 s.
86. A particle moves along the x-axis according to the equation $x(t) = 2.0 - 4.0t^2$ m. What are the velocity and acceleration at t = 2.0 s and t = 5.0 s?
87. A particle moving at constant acceleration has velocities of 2.0 m/s at t = 2.0 s and −7.6 m/s at t = 5.2 s. What is the acceleration of the particle?

88. A train is moving up a steep grade at constant velocity (see following figure) when its caboose breaks loose and starts rolling freely along the track. After 5.0 s, the caboose is 30 m behind the train. What is the acceleration of the caboose?



89. An electron is moving in a straight line with a velocity of 4.0×10^5 m/s. It enters a region 5.0 cm long where it undergoes an acceleration of 6.0×10^{12} m/s^2 along the same straight line. (a) What is the electron's velocity when it emerges from this region? b) How long does the electron take to cross the region?
90. An ambulance driver is rushing a patient to the hospital. While traveling at 72 km/h, she notices the traffic light at the upcoming intersections has turned amber. To reach the intersection before the light turns red, she must travel 50 m in 2.0 s. (a) What minimum acceleration must the ambulance have to reach the intersection before the light turns red? (b) What is the speed of the ambulance when it reaches the intersection?
91. A motorcycle that is slowing down uniformly covers 2.0 successive km in 80 s and 120 s, respectively. Calculate (a) the acceleration of the motorcycle and (b) its velocity at the beginning and end of the 2-km trip.
92. A cyclist travels from point A to point B in 10 min. During the first 2.0 min of her trip, she maintains a uniform acceleration of 0.090 m/s^2. She then travels at constant velocity for the next 5.0 min. Next, she decelerates at a constant rate so that she comes to a rest at point B 3.0 min later. (a) Sketch the velocity-versus-time graph for the trip. (b) What is the acceleration during the last 3 min? (c) How far does the cyclist travel?
93. Two trains are moving at 30 m/s in opposite directions on the same track. The engineers see simultaneously that they are on a collision course and apply the brakes when they are 1000 m apart. Assuming both trains have the same acceleration, what must this acceleration be if the trains are to stop just short of colliding?
94. A 10.0-m-long truck moving with a constant velocity of 97.0 km/h passes a 3.0-m-long car moving with a constant velocity of 80.0 km/h. How much time elapses between the moment the front of the truck is even with the back of the car and the moment the back of the truck is even with the front of the car?



95. A police car waits in hiding slightly off the highway. A speeding car is spotted by the police car doing 40 m/s. At the instant the speeding car passes the police car, the police car accelerates from rest at 4 m/s^2 to catch the speeding car. How long does it take the police car to catch the speeding car?

96. Pablo is running in a half marathon at a velocity of 3 m/s. Another runner, Jacob, is 50 meters behind Pablo with the same velocity. Jacob begins to accelerate at 0.05 m/s^2. (a) How long does it take Jacob to catch Pablo? (b) What is the distance covered by Jacob? (c) What is the final velocity of the Jacob?
97. **Unreasonable results** A runner approaches the finish line and is 75 m away; her average speed at this position is 8 m/s. She decelerates at this point at 0.5 m/s^2. How long does it take her to cross the finish line from 75 m away? Is this reasonable?
98. An airplane accelerates at 5.0 m/s^2 for 30.0 s. During this time, it covers a distance of 10.0 km. What are the initial and final velocities of the airplane?
99. Compare the distance traveled of an object that undergoes a change in velocity that is twice its initial velocity with an object that changes its velocity by four times its initial velocity over the same time period. The accelerations of both objects are constant.
100. An object is moving east with a constant velocity and is at position x_0 at time $t_0 = 0$. (a) With what acceleration must the object have for its total displacement to be zero at a later time t? (b) What is the physical interpretation of the solution in the case for $t \rightarrow \infty$?
101. A ball is thrown straight up. It passes a 2.00-m-high window 7.50 m off the ground on its path up and takes 1.30 s to go past the window. What was the ball's initial velocity?
102. A coin is dropped from a hot-air balloon that is 300 m above the ground and rising at 10.0 m/s upward. For the coin, find (a) the maximum height reached, (b) its position and velocity 4.00 s after being released, and (c) the time before it hits the ground.
103. A soft tennis ball is dropped onto a hard floor from a height of 1.50 m and rebounds to a height of 1.10 m. (a) Calculate its velocity just before it strikes the floor. (b) Calculate its velocity just after it leaves the floor on its way back up. (c) Calculate its acceleration during contact with the floor if that contact lasts 3.50 ms (3.50×10^{-3} s) (d) How much did the ball compress during its collision with the floor, assuming the floor is absolutely rigid?
104. **Unreasonable results**. A raindrop falls from a cloud 100 m above the ground. Neglect air resistance. What is the speed of the raindrop when it hits the ground? Is this a reasonable number?
105. Compare the time in the air of a basketball player who jumps 1.0 m vertically off the floor with that of a player who jumps 0.3 m vertically.
106. Suppose that a person takes 0.5 s to react and move his hand to catch an object he has dropped. (a) How far does the object fall on Earth, where g = 9.8 m/s^2? (b) How far does the object fall on the Moon, where the acceleration due to gravity is 1/6 of that on Earth?
107. A hot-air balloon rises from ground level at a constant velocity of 3.0 m/s. One minute after liftoff, a sandbag is dropped accidentally from the balloon. Calculate (a) the time it takes for the sandbag to reach the ground and (b) the velocity of the sandbag when it hits the ground.
108. (a) A world record was set for the men's 100-m dash in the 2008 Olympic Games in Beijing by Usain Bolt of Jamaica. Bolt "coasted" across the finish line with a time of 9.69 s. If we assume that Bolt accelerated for 3.00 s to reach his maximum speed, and maintained that speed for the rest of the race, calculate his maximum speed and his acceleration. (b) During the same Olympics, Bolt also set the world record in the 200-m dash with a time of 19.30 s. Using the same assumptions as for the 100-m dash, what was his maximum speed for this race?
109. An object is dropped from a height of 75.0 m above ground level. (a) Determine the distance traveled during the first second. (b) Determine the final velocity at which the object hits the ground. (c) Determine the distance traveled during the last second of motion before hitting the ground.
110. A steel ball is dropped onto a hard floor from a height of 1.50 m and rebounds to a height of 1.45 m. (a) Calculate its velocity just before it strikes the floor. (b) Calculate its velocity just after it leaves the floor on its way back up. (c) Calculate its acceleration during contact with the floor if that contact lasts 0.0800 ms (8.00×10^{-5} s) (d) How much did the ball compress during its collision with the floor, assuming the floor is absolutely rigid?
111. An object is dropped from a roof of a building of height h. During the last second of its descent, it drops a distance h/3. Calculate the height of the building.

Challenge Problems

112. In a 100-m race, the winner is timed at 11.2 s. The second-place finisher's time is 11.6 s. How far is the second-place finisher behind the winner when she crosses the finish line? Assume the velocity of each runner is constant throughout the race.
113. The position of a particle moving along the x-axis varies with time according to $x(t) = 5.0t^2 - 4.0t^3$ m. Find (a) the velocity and acceleration of the particle as functions of time, (b) the velocity and acceleration at t = 2.0 s, (c) the time at which the position is a maximum, (d) the time at which the velocity is zero, and (e) the maximum position.

114. A cyclist sprints at the end of a race to clinch a victory. She has an initial velocity of 11.5 m/s and accelerates at a rate of 0.500 m/s^2 for 7.00 s. (a) What is her final velocity? (b) The cyclist continues at this velocity to the finish line. If she is 300 m from the finish line when she starts to accelerate, how much time did she save? (c) The second-place winner was 5.00 m ahead when the winner started to accelerate, but he was unable to accelerate, and traveled at 11.8 m/s until the finish line. What was the difference in finish time in seconds between the winner and runner-up? How far back was the runner-up when the winner crossed the finish line?
115. In 1967, New Zealander Burt Munro set the world record for an Indian motorcycle, on the Bonneville Salt Flats in Utah, of 295.38 km/h. The one-way course was 8.00 km long. Acceleration rates are often described by the time it takes to reach 96.0 km/h from rest. If this time was 4.00 s and Burt accelerated at this rate until he reached his maximum speed, how long did it take Burt to complete the course?

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2.10: Motion Along a Straight Line (Summary)

Key Terms

acceleration due to gravity	acceleration of an object as a result of gravity
average acceleration	the rate of change in velocity; the change in velocity over time
average speed	the total distance traveled divided by elapsed time
average velocity	the displacement divided by the time over which displacement occurs
displacement	the change in position of an object
distance traveled	the total length of the path traveled between two positions
elapsed time	the difference between the ending time and the beginning time
free fall	the state of movement that results from gravitational force only
instantaneous acceleration	acceleration at a specific point in time
instantaneous speed	the absolute value of the instantaneous velocity
instantaneous velocity	the velocity at a specific instant or time point
kinematics	the description of motion through properties such as position, time, velocity, and acceleration
position	the location of an object at a particular time
total displacement	the sum of individual displacements over a given time period
two-body pursuit problem	a kinematics problem in which the unknowns are calculated by solving the kinematic equations simultaneously for two moving objects

Key Equations

Displacement	$\Delta x = x_f - x_i$
Total displacement	$\Delta x_{Total} = \sum \Delta x_i$
Average velocity	$\overline{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{x_2 - x_1}{t_2 - t_1}$
Instantaneous velocity	$v(t) = \dfrac{dx(t)}{dt}$
Average speed	$\overline{s} = \dfrac{Total\ distance}{Elapsed\ time}$
Instantaneous speed	$Instantaneous\ speed = \lvert v(t) \rvert$
Average acceleration	$\overline{a} = \dfrac{\Delta v}{\Delta t} = \dfrac{v_f - x_0}{t_f - t_0}$

Instantaneous acceleration	$a(t) = \frac{dv(t)}{dt}$
Position from average velocity	$x = x_0 + \bar{v}t$
Average velocity	$\bar{v} = \frac{v_0 + v}{2}$
Velocity from acceleration	$v = v_0 + at\ (constant\ a)$
Position from velocity and acceleration	$x = x_0 + v_0 t + \frac{1}{2}at^2\ (constant\ a)$
Velocity from distance	$v^2 = v_0^2 + 2a(x - x_0)\ (constant\ a)$
Velocity of free fall	$v = v_0 - gt(positive\ upward)$
Height of free fall	$y = y_0 + v_0 t - \frac{1}{2}gt^2$
Velocity of free fall from height	$v^2 = v_0^2 - 2g(y - y_0)$
Velocity from acceleration	$v(t) = \int a(t)dt + C_1$
Position from velocity	$x(t) = \int v(t)dt + C_2$

Summary

3.1 Position, Displacement, and Average Velocity

- Kinematics is the description of motion without considering its causes. In this chapter, it is limited to motion along a straight line, called one-dimensional motion.
- Displacement is the change in position of an object. The SI unit for displacement is the meter. Displacement has direction as well as magnitude.
- Distance traveled is the total length of the path traveled between two positions.
- Time is measured in terms of change. The time between two position points x_1 and x_2 is $\Delta t = t_2 - t_1$. Elapsed time for an event is $\Delta t = t_f - t_0$, where t_f is the final time and t_0 is the initial time. The initial time is often taken to be zero.
- Average velocity $\bar{v}$ is defined as displacement divided by elapsed time. If x_1, t_1 and x_2, t_2 are two position time points, the average velocity between these points is

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}. \tag{2.10.19}$$

3.2 Instantaneous Velocity and Speed

- Instantaneous velocity is a continuous function of time and gives the velocity at any point in time during a particle's motion. We can calculate the instantaneous velocity at a specific time by taking the derivative of the position function, which gives us the functional form of instantaneous velocity v(t).

- Instantaneous velocity is a vector and can be negative.
- Instantaneous speed is found by taking the absolute value of instantaneous velocity, and it is always positive.
- Average speed is total distance traveled divided by elapsed time.
- The slope of a position-versus-time graph at a specific time gives instantaneous velocity at that time.

3.3 Average and Instantaneous Acceleration

- Acceleration is the rate at which velocity changes. Acceleration is a vector; it has both a magnitude and direction. The SI unit for acceleration is meters per second squared.
- Acceleration can be caused by a change in the magnitude or the direction of the velocity, or both.
- Instantaneous acceleration a(t) is a continuous function of time and gives the acceleration at any specific time during the motion. It is calculated from the derivative of the velocity function. Instantaneous acceleration is the slope of the velocity-versus-time graph.
- Negative acceleration (sometimes called deceleration) is acceleration in the negative direction in the chosen coordinate system.

3.4 Motion with Constant Acceleration

- When analyzing one-dimensional motion with constant acceleration, identify the known quantities and choose the appropriate equations to solve for the unknowns. Either one or two of the kinematic equations are needed to solve for the unknowns, depending on the known and unknown quantities.
- Two-body pursuit problems always require two equations to be solved simultaneously for the unknowns.

3.5 Free Fall

- An object in free fall experiences constant acceleration if air resistance is negligible.
- On Earth, all free-falling objects have an acceleration g due to gravity, which averages $g = 9.81\ m/s^2$.
- For objects in free fall, the upward direction is normally taken as positive for displacement, velocity, and acceleration.

3.6 Finding Velocity and Displacement from Acceleration

- Integral calculus gives us a more complete formulation of kinematics.
- If acceleration a(t) is known, we can use integral calculus to derive expressions for velocity v(t) and position x(t).
- If acceleration is constant, the integral equations reduce to Equation 3.12 and Equation 3.13 for motion with constant acceleration.

Contributors and Attributions

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3: PHYSICS 1- REVIEW MOTION IN TWO AND THREE DIMENSIONS

3.1: Prelude to Motion in Two and Three Dimensions

3.2: Displacement and Velocity Vectors

3.3: Acceleration Vector

3.4: Projectile Motion

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3.7: Motion in Two and Three Dimensions (Exercises)

3.8: Motion in Two and Three Dimensions (Summary)

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3.1: Prelude to Motion in Two and Three Dimensions

To give a complete description of kinematics, we must explore motion in two and three dimensions. After all, most objects in our universe do not move in straight lines; rather, they follow curved paths. From kicked footballs to the flight paths of birds to the orbital motions of celestial bodies and down to the flow of blood plasma in your veins, most motion follows curved trajectories.



Figure 3.1.1: The Red Arrows is the aerobatics display team of Britain's Royal Air Force. Based in Lincolnshire, England, they perform precision flying shows at high speeds, which requires accurate measurement of position, velocity, and acceleration in three dimensions. (credit: modification of work by Phil Long)

Fortunately, the treatment of motion in one dimension in the previous chapter has given us a foundation on which to build, as the concepts of position, displacement, velocity, and acceleration defined in one dimension can be expanded to two and three dimensions. Consider the Red Arrows, also known as the Royal Air Force Aerobatic team of the United Kingdom. Each jet follows a unique curved trajectory in three-dimensional airspace, as well as has a unique velocity and acceleration. Thus, to describe the motion of any of the jets accurately, we must assign to each jet a unique position vector in three dimensions as well as a unique velocity and acceleration vector. We can apply the same basic equations for displacement, velocity, and acceleration we derived in Motion Along a Straight Line to describe the motion of the jets in two and three dimensions, but with some modifications—in particular, the inclusion of vectors.

In this chapter we also explore two special types of motion in two dimensions: projectile motion and circular motion. Last, we conclude with a discussion of relative motion. In the chapter-opening picture, each jet has a relative motion with respect to any other jet in the group or to the people observing the air show on the ground.

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3.2: Displacement and Velocity Vectors

LEARNING OBJECTIVES

- Calculate position vectors in a multidimensional displacement problem.
- Solve for the displacement in two or three dimensions.
- Calculate the velocity vector given the position vector as a function of time.
- Calculate the average velocity in multiple dimensions.

Displacement and velocity in two or three dimensions are straightforward extensions of the one-dimensional definitions. However, now they are vector quantities, so calculations with them have to follow the rules of vector algebra, not scalar algebra.

Displacement Vector

To describe motion in two and three dimensions, we must first establish a coordinate system and a convention for the axes. We generally use the coordinates x, y, and z to locate a particle at point $P(x, y, z)$ in three dimensions. If the particle is moving, the variables x, y, and z are functions of time (t):

$$x = x(t) \quad y = y(t) \quad z = z(t). \tag{3.2.1}$$

The position vector from the origin of the coordinate system to point P is $\vec{r}(t)$. In unit vector notation, introduced in Coordinate Systems and Components of a Vector, $\vec{r}$(t) is

$$\vec{r}(t) = x(t)\,\hat{i} + y(t)\,\hat{j} + z(t)\,\hat{k}. \tag{3.2.2}$$

Figure 3.2.1 shows the coordinate system and the vector to point P, where a particle could be located at a particular time t. Note the orientation of the x, y, and z axes. This orientation is called a right-handed coordinate system and it is used throughout the chapter.

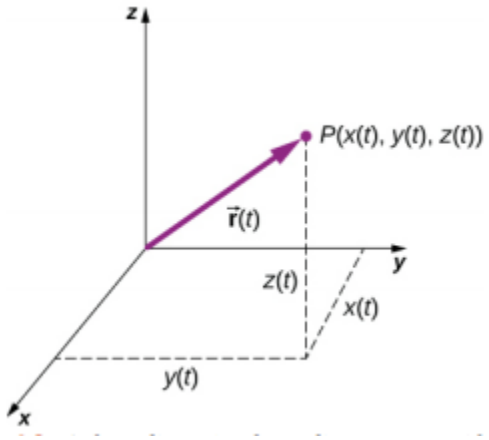


Figure 3.2.1: A three-dimensional coordinate system with a particle at position $P(x(t), y(t), z(t))$.

With our definition of the position of a particle in three-dimensional space, we can formulate the three-dimensional displacement. Figure 3.2.3 shows a particle at time t_1 located at P_1 with position vector $\vec{r}(t_1)$. At a later time t_2, the particle is located at P_2 with position vector $\vec{r}(t_2)$. The displacement vector $\Delta\vec{r}$ is found by subtracting $\vec{r}(t_1)$ from $\vec{r}(t_2)$:

$$\Delta\vec{r} = \vec{r}(t_2) - \vec{r}(t_1). \tag{3.2.3}$$

Vector addition is discussed in Vectors. Note that this is the same operation we did in one dimension, but now the vectors are in three-dimensional space.

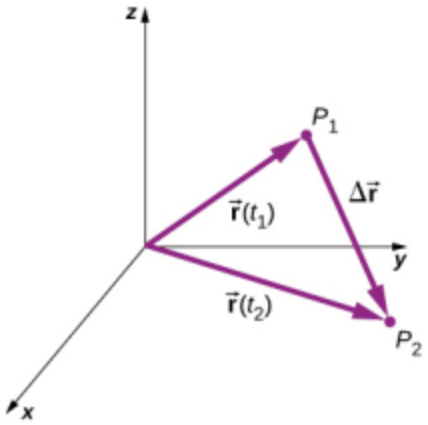


Figure 3.2.2: The displacement $\Delta\vec{r} = \vec{r}(t_2) - \vec{r}(t_1)$ is the vector from P_1 to P_2.

The following examples illustrate the concept of displacement in multiple dimensions

Example 4.1: Polar Orbiting Satellite

A satellite is in a circular polar orbit around Earth at an altitude of 400 km—meaning, it passes directly overhead at the North and South Poles. What is the magnitude and direction of the displacement vector from when it is directly over the North Pole to when it is at −45° latitude?

Strategy

We make a picture of the problem to visualize the solution graphically. This will aid in our understanding of the displacement. We then use unit vectors to solve for the displacement.

Solution

Figure 3.2.3 shows the surface of Earth and a circle that represents the orbit of the satellite. Although satellites are moving in three-dimensional space, they follow trajectories of ellipses, which can be graphed in two dimensions. The position vectors are drawn from the center of Earth, which we take to be the origin of the coordinate system, with the y-axis as north and the x-axis as east. The vector between them is the displacement of the satellite. We take the radius of Earth as 6370 km, so the length of each position vector is 6770 km.

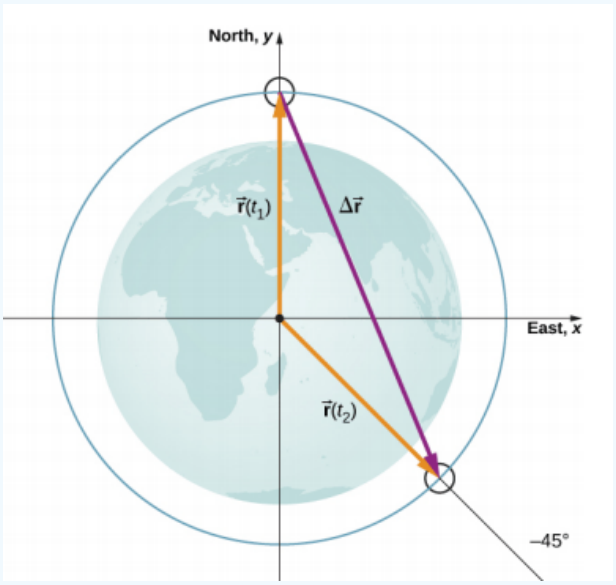


Figure 3.2.3: Two position vectors are drawn from the center of Earth, which is the origin of the coordinate system, with the y-axis as north and the x-axis as east. The vector between them is the displacement of the satellite.

In unit vector notation, the position vectors are

$$\vec{r}(t_1) = 6770.\ km\ \hat{j}$$
$$\vec{r}(t_2) = 6770.\ km(\cos(-45°))\ \hat{i} + 6770.\ km(\sin(-45°))\ \hat{j}.$$

Evaluating the sine and cosine, we have

$$\vec{r}(t_1) = 6770.\hat{j}$$
$$\vec{r}(t_2) = 4787\ \hat{i} - 4787\ \hat{j}.$$

Now we can find $\Delta\vec{r}$, the displacement of the satellite:

$$\Delta\vec{r} = \vec{r}(t_2) - \vec{r}(t_1) = 4787\ \hat{i} - 11,557\ \hat{j}.$$

The magnitude of the displacement is

$$|\Delta\vec{r}| = \sqrt{(4787)^2 + (-11,557)^2} = 12,509\ km.$$

The angle the displacement makes with the x-axis is

$$\theta = \tan^{-1}\left(\frac{-11,557}{4787}\right) = -67.5^o.$$

Significance

Plotting the displacement gives information and meaning to the unit vector solution to the problem. When plotting the displacement, we need to include its components as well as its magnitude and the angle it makes with a chosen axis—in this case, the x-axis (Figure 3.2.4).

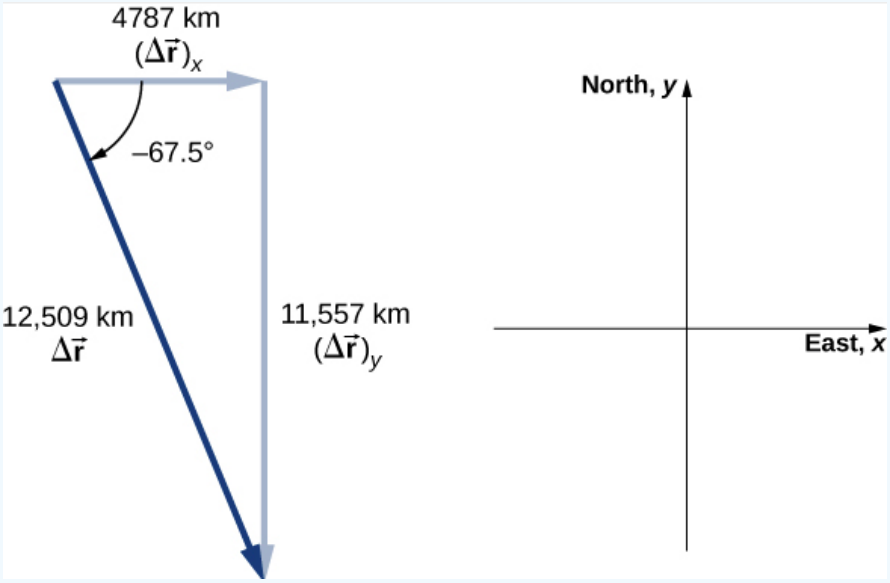


Figure 3.2.4: Displacement vector with components, angle, and magnitude.

Note that the satellite took a curved path along its circular orbit to get from its initial position to its final position in this example. It also could have traveled 4787 km east, then 11,557 km south to arrive at the same location. Both of these paths are longer than the length of the displacement vector. In fact, the displacement vector gives the shortest path between two points in one, two, or three dimensions.

Many applications in physics can have a series of displacements, as discussed in the previous chapter. The total displacement is the sum of the individual displacements, only this time, we need to be careful, because we are adding vectors. We illustrate this concept with an example of Brownian motion.

Example 4.2: Brownian Motion

Brownian motion is a chaotic random motion of particles suspended in a fluid, resulting from collisions with the molecules of the fluid. This motion is three-dimensional. The displacements in numerical order of a particle undergoing Brownian motion could look like the following, in micrometers (Figure 3.2.5):

$$\Delta\vec{r}_1 = 2.0\ \hat{i} + \hat{j} + 3.0\hat{k} \tag{3.2.4}$$

$$\Delta\vec{r}_2 = -\hat{i} + 3.0\ \hat{k} \tag{3.2.5}$$

$$\Delta\vec{r}_3 = 4.0\ \hat{i} - 2.0\ \hat{j} + \hat{k} \tag{3.2.6}$$

$$\Delta\vec{r}_4 = -3.0\ \hat{i} + \hat{j} + 3.0\ \hat{k}. \tag{3.2.7}$$

What is the total displacement of the particle from the origin?

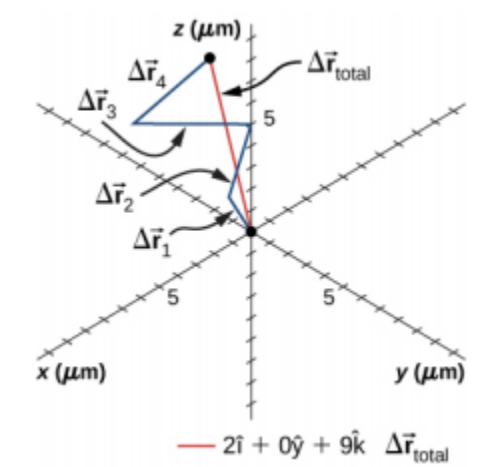


Figure 3.2.5: Trajectory of a particle undergoing random displacements of Brownian motion. The total displacement is shown in red.

Solution

We form the sum of the displacements and add them as vectors:

$$\begin{aligned}\Delta\vec{r}_{Total} &= \sum \Delta\vec{r}_i = \Delta\vec{r}_1 + \Delta\vec{r}_2 + \Delta\vec{r}_3 + \Delta\vec{r}_4 \\ &= (2.0 - 1.0 + 4.0 - 3.0)\ \hat{i} + (1.0 + 0 - 2.0 + 1.0)\ \hat{j} + (3.0 + 3.0 + 1.0 + 2.0)\ \hat{k} \\ &= 2.0\ \hat{i} + 0\ \hat{j} + 9.0\ \hat{k}\ \mu m.\end{aligned} \tag{3.2.8}$$

To complete the solution, we express the displacement as a magnitude and direction,

$$|\Delta\vec{r}_{Total}| = \sqrt{2.0^2 + 0^2 + 9.0^2} = 9.2\mu m, \quad \theta = \tan^{-1}\left(\frac{9}{2}\right) = 77^o, \tag{3.2.9}$$

with respect to the x-axis in the xz-plane.

Significance

From the figure we can see the magnitude of the total displacement is less than the sum of the magnitudes of the individual displacements.

Velocity Vector

In the previous chapter we found the instantaneous velocity by calculating the derivative of the position function with respect to time. We can do the same operation in two and three dimensions, but we use vectors. The instantaneous **velocity vector** is now

$$\vec{v}(t) = \lim_{\Delta t \to 0} \frac{\vec{r}(t + \Delta t) - \vec{r}(t)}{\Delta t} = \frac{d\vec{r}}{dt}. \tag{3.2.10}$$

Let's look at the relative orientation of the position vector and velocity vector graphically. In Figure 3.2.6 we show the vectors $\vec{r}$ (t) and $\vec{r}$(t + Δt), which give the position of a particle moving along a path represented by the gray line. As Δt goes to zero, the velocity vector, given by Equation 3.2.10, becomes tangent to the path of the particle at time t.

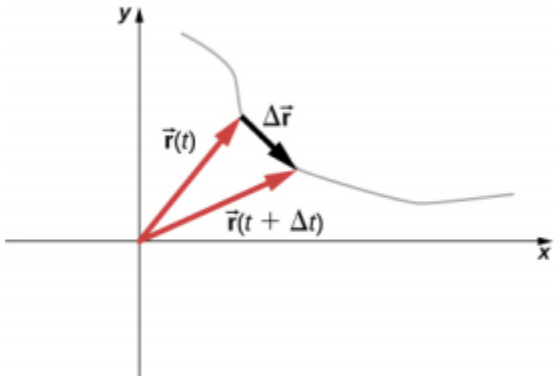


Figure 3.2.6: A particle moves along a path given by the gray line. In the limit as Δt approaches zero, the velocity vector becomes tangent to the path of the particle.

Equation 3.2.10 can also be written in terms of the components of $\vec{v}$(t). Since

$$\vec{r}(t) = x(t)\ \hat{i} + y(t)\ \hat{j} + z(t)\ \hat{k}, \tag{3.2.11}$$

we can write

$$\vec{v}(t) = v_x(t)\ \hat{i} + v_y(t)\ \hat{j} + v_z(t)\ \hat{k} \tag{3.2.12}$$

where

$$v_x(t) = \frac{dx(t)}{dt}, \quad v_y(t) = \frac{dy(t)}{dt}, \quad v_z(t) = \frac{dz(t)}{dt}. \tag{3.2.13}$$

If only the average velocity is of concern, we have the vector equivalent of the one-dimensional average velocity for two and three dimensions:

$$\vec{v}_{avg} = \frac{\vec{r}(t_2) - \vec{r}(t_1)}{t_2 - t_1}. \tag{3.2.14}$$

Example 4.3: Calculating the Velocity Vector

The position function of a particle is $\vec{r}$(t) = 2.0t^2 $\hat{i}$ + (2.0 + 3.0t) $\hat{j}$+ 5.0t $\hat{k}$ m. (a) What is the instantaneous velocity and speed at t = 2.0 s? (b) What is the average velocity between 1.0 s and 3.0 s?

Solution

Using Equation 3.2.12 and Equation 3.2.13, and taking the derivative of the position function with respect to time, we find

a.
$$v(t) = \frac{d\vec{r}(t)}{dt} = 4.0t\ \hat{i} + 3.0\ \hat{j} + 5.0\ \hat{k}\ m/s \tag{3.2.15}$$

$$\vec{v}(2.0\ s) = 8.0\ \hat{i} + 3.0\ \hat{j} + 5.0\ \hat{k}\ m/s \tag{3.2.16}$$

$$Speed\ |\vec{v}(2.0\ s)| = \sqrt{8^2 + 3^2 + 5^2} = 9.9\ m/s. \tag{3.2.17}$$

b. From Equation 3.2.14,

$$\begin{aligned}\vec{v}_{avg} &= \frac{\vec{r}(t_2) - \vec{r}(t_1)}{t_2 - t_1} = \frac{\vec{r}(3.0\ s) - \vec{r}(1.0\ s)}{3.0\ s - 1.0\ s} = \frac{(18\ \hat{i} + 11\ \hat{j} + 15\ \hat{k})m - (2\ \hat{i} + 5\ \hat{j} + 5\ \hat{k})m}{2.0\ s} \\ &= \frac{(16\ \hat{i} + 6\ \hat{j} + 10\ \hat{k})m}{2.0\ s} = 8.0\ \hat{i} + 3.0\ \hat{j} + 5.0\ \hat{k}\ m/s.\end{aligned} \tag{3.2.18}$$

Significance

We see the average velocity is the same as the instantaneous velocity at t = 2.0 s, as a result of the velocity function being linear. This need not be the case in general. In fact, most of the time, instantaneous and average velocities are not the same.

Exercise 4.1

The position function of a particle is $\vec{r}$(t) = 3.0t^3 $\hat{i}$ + 4.0 $\hat{j}$. (a) What is the instantaneous velocity at t = 3 s? (b) Is the average velocity between 2 s and 4 s equal to the instantaneous velocity at t = 3 s?

The Independence of Perpendicular Motions

When we look at the three-dimensional equations for position and velocity written in unit vector notation, Equation 3.2.2 and Equation 3.2.12, we see the components of these equations are separate and unique functions of time that do not depend on one another. Motion along the x direction has no part of its motion along the y and z directions, and similarly for the other two coordinate axes. Thus, the motion of an object in two or three dimensions can be divided into separate, independent motions along the perpendicular axes of the coordinate system in which the motion takes place.

To illustrate this concept with respect to displacement, consider a woman walking from point A to point B in a city with square blocks. The woman taking the path from A to B may walk east for so many blocks and then north (two perpendicular directions) for another set of blocks to arrive at B. How far she walks east is affected only by her motion eastward. Similarly, how far she walks north is affected only by her motion northward.

Independence of Motion

In the kinematic description of motion, we are able to treat the horizontal and vertical components of motion separately. In many cases, motion in the horizontal direction does not affect motion in the vertical direction, and vice versa.

An example illustrating the independence of vertical and horizontal motions is given by two baseballs. One baseball is dropped from rest. At the same instant, another is thrown horizontally from the same height and it follows a curved path. A stroboscope captures the positions of the balls at fixed time intervals as they fall (Figure 3.2.7).

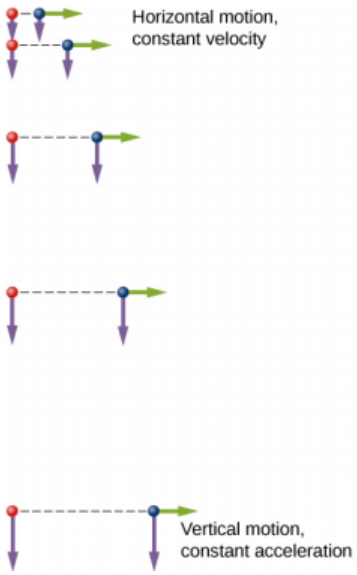


Figure 3.2.7: A diagram of the motions of two identical balls: one falls from rest and the other has an initial horizontal velocity. Each subsequent position is an equal time interval. Arrows represent the horizontal and vertical velocities at each position. The ball on the right has an initial horizontal velocity whereas the ball on the left has no horizontal velocity. Despite the difference in horizontal velocities, the vertical velocities and positions are identical for both balls, which shows the vertical and horizontal motions are independent.

It is remarkable that for each flash of the strobe, the vertical positions of the two balls are the same. This similarity implies vertical motion is independent of whether the ball is moving horizontally. (Assuming no air resistance, the vertical motion of a falling object is influenced by gravity only, not by any horizontal forces.) Careful examination of the ball thrown horizontally shows it travels the same horizontal distance between flashes. This is because there are no additional forces on the ball in the horizontal direction after it is thrown. This result means horizontal velocity is constant and is affected neither by vertical motion nor by gravity (which is vertical). Note this case is true for ideal conditions only. In the real world, air resistance affects the speed of the balls in both directions.

The two-dimensional curved path of the horizontally thrown ball is composed of two independent one-dimensional motions (horizontal and vertical). The key to analyzing such motion, called **projectile motion**, is to resolve it into motions along perpendicular directions. Resolving two-dimensional motion into perpendicular components is possible because the components are independent.

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3.3: Acceleration Vector

Learning Objectives

- Calculate the acceleration vector given the velocity function in unit vector notation.
- Describe the motion of a particle with a constant acceleration in three dimensions.
- Use the one-dimensional motion equations along perpendicular axes to solve a problem in two or three dimensions with a constant acceleration.
- Express the acceleration in unit vector notation.

Instantaneous Acceleration

In addition to obtaining the displacement and velocity vectors of an object in motion, we often want to know its **acceleration vector** at any point in time along its trajectory. This acceleration vector is the instantaneous acceleration and it can be obtained from the derivative with respect to time of the velocity function, as we have seen in a previous chapter. The only difference in two or three dimensions is that these are now vector quantities. Taking the derivative with respect to time $\vec{v}$(t), we find

$$\vec{a}(t) = \lim_{t \to 0} \frac{\vec{v}(t + \Delta t) - \vec{v}(t)}{\Delta t} = \frac{d\vec{v}(t)}{dt}. \tag{3.3.1}$$

The acceleration in terms of components is

$$\vec{a}(t) = \frac{dv_x(t)}{dt}\,\hat{i} + \frac{dv_y(t)}{dt}\,\hat{j} + \frac{dv_z(t)}{dt}\,\hat{k}. \tag{3.3.2}$$

Also, since the velocity is the derivative of the position function, we can write the acceleration in terms of the second derivative of the position function:

$$\vec{a}(t) = \frac{d^2x(t)}{dt^2}\,\hat{i} + \frac{d^2y(t)}{dt^2}\,\hat{j} + \frac{d^2z(t)}{dt^2}\,\hat{k}. \tag{3.3.3}$$

Example 4.4: Finding an Acceleration Vector

A particle has a velocity of $\vec{v}(t) = 5.0t\hat{i} + t^2\hat{j} - 2.0t^3\hat{k}\ m/s$.

a. What is the acceleration function?
b. What is the acceleration vector at t = 2.0 s? Find its magnitude and direction.

Solution

a. We take the first derivative with respect to time of the velocity function to find the acceleration. The derivative is taken component by component:

$$\vec{a}(t) = 5.0\,\hat{i} + 2.0t\,\hat{j} - 6.0t^2\,\hat{k}\ m/s^2.$$

b. Evaluating $\vec{a}(2.0\ \ s) = 5.0\hat{i} + 4.0\hat{j} - 24.0\hat{k}\ \ m/s^2$ gives us the direction in unit vector notation. The magnitude of the acceleration is

$$|\vec{a}(2.20\ s)| = \sqrt{5.0^2 + 4.0^2 + (-24.0)^2} = 24.8\ m/s^2.$$

Significance

In this example we find that acceleration has a time dependence and is changing throughout the motion. Let's consider a different velocity function for the particle.

Example 4.5: Finding a Particle Acceleration

A particle has a position function: $\vec{r}(t) = (10t - t^2)\hat{i} + 5t\hat{j} + 5t\hat{k}\ m$.

a. What is the velocity?
b. What is the acceleration?
c. Describe the motion from $t = 0\ s$.

Strategy

We can gain some insight into the problem by looking at the position function. It is linear in y and z, so we know the acceleration in these directions is zero when we take the second derivative. Also, note that the position in the x direction is zero for t = 0 s and t = 10 s.

Solution

a. Taking the derivative with respect to time of the position function, we find $\vec{v}(t) = (10 - 2t)\hat{i} + 5\hat{j} + 5\hat{k}\ m/s$. The velocity function is linear in time in the x direction and is constant in the y and z directions.
b. Taking the derivative of the velocity function, we find

$$\vec{a}(t) = -2\ \hat{i}\, m/s^2.$$

The acceleration vector is a constant in the negative x-direction.
c. The trajectory of the particle can be seen in Figure 3.3.1. Let's look in the y and z directions first. The particle's position increases steadily as a function of time with a constant velocity in these directions. In the x direction, however, the particle follows a path in positive x until t = 5 s, when it reverses direction. We know this from looking at the velocity function, which becomes zero at this time and negative thereafter. We also know this because the acceleration is negative and constant—meaning, the particle is decelerating, or accelerating in the negative direction. The particle's position reaches 25 m, where it then reverses direction and begins to accelerate in the negative x direction. The position reaches zero at t = 10 s.

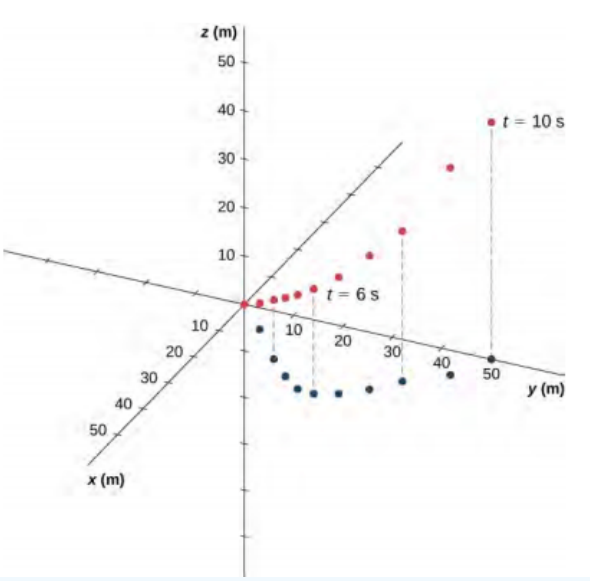


Figure 3.3.1: The particle starts at point (x, y, z) = (0, 0, 0) with position vector $\vec{r}$ = 0. The projection of the trajectory onto the xy-plane is shown. The values of y and z increase linearly as a function of time, whereas x has a turning point at t = 5 s and 25 m, when it reverses direction. At this point, the x component of the velocity becomes negative. At t = 10 s, the particle is back to 0 m in the x direction.

Exercise 4.2

Suppose the acceleration function has the form $\vec{a}$(t) = a $\hat{i}$ + b $\hat{j}$ + c $\hat{k}$ m/s^2, where a, b, and c are constants. What can be said about the functional form of the velocity function?

Constant Acceleration

Multidimensional motion with constant acceleration can be treated the same way as shown in the previous chapter for one-dimensional motion. Earlier we showed that three-dimensional motion is equivalent to three one-dimensional motions, each along an axis perpendicular to the others. To develop the relevant equations in each direction, let's consider the two-dimensional problem of a particle moving in the xy plane with constant acceleration, ignoring the z-component for the moment. The acceleration vector is

$$\vec{a} = a_{0x}\,\hat{i} + a_{0y}\,\hat{j}. \tag{3.3.4}$$

Each component of the motion has a separate set of equations similar to Equation 3.10–Equation 3.14 of the previous chapter on one-dimensional motion. We show only the equations for position and velocity in the x- and y-directions. A similar set of kinematic equations could be written for motion in the z-direction:

$$x(t) = x_0 + (v_x)_{avg}t \tag{3.3.5}$$

$$v_x(t) = v_{0x} + a_x t \tag{3.3.6}$$

$$x(t) = x_0 + v_{0x}t + \frac{1}{2}a_x t^2 \tag{3.3.7}$$

$$v_x^2(t) = v_{0x}^2 + 2a_x(x - x_0) \tag{3.3.8}$$

$$y(t) = y_0 + (v_y)_{avg}t \tag{3.3.9}$$

$$v_y(t) = v_{0y} + a_y t \tag{3.3.10}$$

$$y(t) = y_0 + v_{0y}t + \frac{1}{2}a_y t^2 \tag{3.3.11}$$

$$v_y^2(t) = v_{0y}^2 + 2a_y(y - y_0). \tag{3.3.12}$$

Here the subscript 0 denotes the initial position or velocity. Equation 3.3.5 to 3.3.12 can be substituted into Equation 4.2 and Equation 4.5 without the z-component to obtain the position vector and velocity vector as a function of time in two dimensions:

$$\vec{r}(t) = x(t)\,\hat{i} + y(t)\,\hat{j} \tag{3.3.13}$$

and

$$\vec{v}(t) = v_x(t)\,\hat{i} + v_y(t)\,\hat{j}. \tag{3.3.14}$$

The following example illustrates a practical use of the kinematic equations in two dimensions.

Example 4.6: A Skier

Figure 3.3.2 shows a skier moving with an acceleration of 2.1 m/s^2 down a slope of 15° at t = 0. With the origin of the coordinate system at the front of the lodge, her initial position and velocity are

$$\vec{r}(0) = (7.50\,\hat{i} - 50.0\,\hat{j})m$$

and

$$\vec{v}(0) = (4.1\ \hat{i} - 1.1\ \hat{j}) m/s$$

a. What are the x- and y-components of the skier's position and velocity as functions of time?
b. What are her position and velocity at t = 10.0 s?

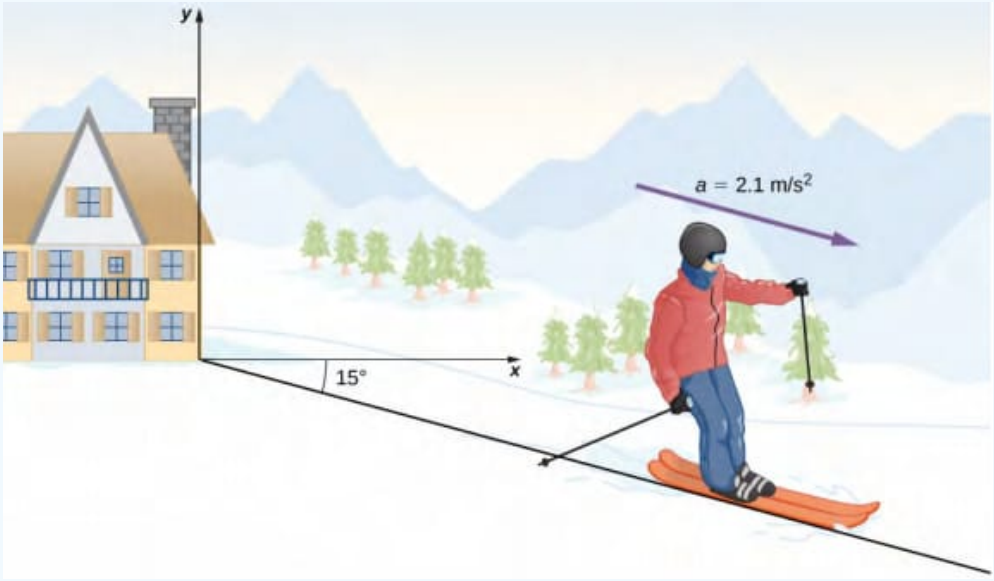


Figure 3.3.2: A skier has an acceleration of 2.1 m/s^2 down a slope of 15°. The origin of the coordinate system is at the ski lodge.

Strategy

Since we are evaluating the components of the motion equations in the x and y directions, we need to find the components of the acceleration and put them into the kinematic equations. The components of the acceleration are found by referring to the coordinate system in Figure 3.3.2. Then, by inserting the components of the initial position and velocity into the motion equations, we can solve for her position and velocity at a later time t.

Solution

a. The origin of the coordinate system is at the top of the hill with y-axis vertically upward and the x-axis horizontal. By looking at the trajectory of the skier, the x-component of the acceleration is positive and the y-component is negative. Since the angle is 15° down the slope, we find

$$a_x = (2.1\ m/s^2)\cos(15^o) = 2.0\ m/s^2 \tag{3.3.15}$$

$$a_y = (-2.1\ m/s^2)\sin(15^o) = -0.54\ m/s^2. \tag{3.3.16}$$

Inserting the initial position and velocity into Equations 3.3.6 and 3.3.7 for x, we have

$$x(t) = 75.0\ m + (4.1\ m/s)t + \frac{1}{2}(2.0\ m/s^2)t^2 \tag{3.3.17}$$

$$v_x(t) = 4.1\ m/s + (2.0\ m/s^2)t. \tag{3.3.18}$$

For y, we have

$$y(t) = -50.0.0\ m + (-1.1\ m/s)t + \frac{1}{2}(-0.54\ m/s^2)t^2 \tag{3.3.19}$$

$$v_y(t) = -1.1\ m/s + (-0.54\ m/s^2)t. \tag{3.3.20}$$

b. Now that we have the equations of motion for x and y as functions of time, we can evaluate them at t = 10.0 s:

$$x(10.0\ s) = 75.0\ m + (4.1\ m/s)(10.0\ s) + \frac{1}{2}(2.0\ m/s^2)(10.0\ s)^2 = 216.0\ m \tag{3.3.21}$$

$$v_x(10.0\ s) = 4.1\ m/s + (2.0\ m/s^2)(10.0\ s) = 24.1\ m/s \tag{3.3.22}$$

$$y(10.0) = -50.0.0\ m + (-1.1\ m/s)(10.0\ s) + \frac{1}{2}(-0.54\ m/s^2)(10.0\ s)^2 \tag{3.3.23}$$

$$v_y(10.0\ s) = -1.1\ m/s + (-0.54\ m/s^2)(10.0\ s). \tag{3.3.24}$$

The position and velocity at t = 10.0 s are, finally

$$\vec{r}(10.0\ s) = (216.0\ \hat{i} - 88.0\ \hat{j})m \tag{3.3.25}$$

$$\vec{v}(10.0\ s) = (24.1\ \hat{i} - 6.5\ \hat{j})m/s. \tag{3.3.26}$$

The magnitude of the velocity of the skier at 10.0 s is 25 m/s, which is 60 mi/h.

Significance

It is useful to know that, given the initial conditions of position, velocity, and acceleration of an object, we can find the position, velocity, and acceleration at any later time.

With Equations 3.3.1-3.3.3 we have completed the set of expressions for the position, velocity, and acceleration of an object moving in two or three dimensions. If the trajectories of the objects look something like the “Red Arrows” in the opening picture for the chapter, then the expressions for the position, velocity, and acceleration can be quite complicated. In the sections to follow we examine two special cases of motion in two and three dimensions by looking at projectile motion and circular motion.

Simulation

At this University of Colorado Boulder website, you can explore the position velocity and acceleration of a ladybug with an interactive simulation that allows you to change these parameters.

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3.4: Projectile Motion

Learning Objectives

- Use one-dimensional motion in perpendicular directions to analyze projectile motion.
- Calculate the range, time of flight, and maximum height of a projectile that is launched and impacts a flat, horizontal surface.
- Find the time of flight and impact velocity of a projectile that lands at a different height from that of launch.
- Calculate the trajectory of a projectile.

Projectile motion is the motion of an object thrown or projected into the air, subject only to acceleration as a result of gravity. The applications of projectile motion in physics and engineering are numerous. Some examples include meteors as they enter Earth's atmosphere, fireworks, and the motion of any ball in sports. Such objects are called **projectiles** and their path is called a **trajectory**. The motion of falling objects as discussed in Motion Along a Straight Line is a simple one-dimensional type of projectile motion in which there is no horizontal movement. In this section, we consider two-dimensional projectile motion, and our treatment neglects the effects of air resistance.

The most important fact to remember here is that **motions along perpendicular axes are independent** and thus can be analyzed separately. We discussed this fact in Displacement and Velocity Vectors, where we saw that vertical and horizontal motions are independent. The key to analyzing two-dimensional projectile motion is to break it into two motions: one along the horizontal axis and the other along the vertical. (This choice of axes is the most sensible because acceleration resulting from gravity is vertical; thus, there is no acceleration along the horizontal axis when air resistance is negligible.) As is customary, we call the horizontal axis the x-axis and the vertical axis the y-axis. It is not required that we use this choice of axes; it is simply convenient in the case of gravitational acceleration. In other cases we may choose a different set of axes. Figure 3.4.1 illustrates the notation for displacement, where we define $\vec{s}$ to be the total displacement, and $\vec{x}$ and $\vec{y}$ are its component vectors along the horizontal and vertical axes, respectively. The magnitudes of these vectors are s, x, and y.

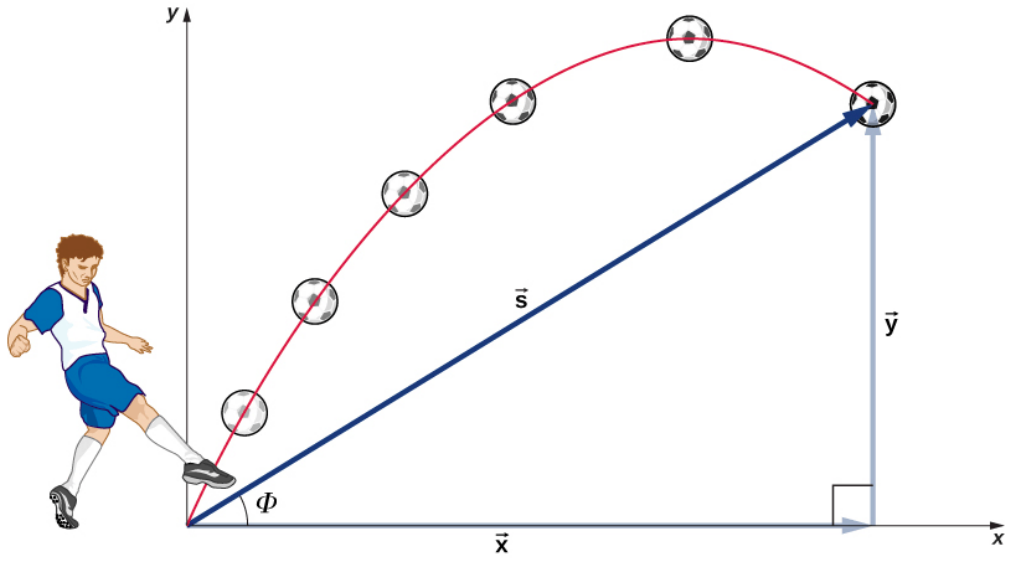


Figure 3.4.1: The total displacement s of a soccer ball at a point along its path. The vector $\vec{s}$ has components $\vec{x}$ and $\vec{y}$ along the horizontal and vertical axes. Its magnitude is s and it makes an angle ϕ with the horizontal.

To describe projectile motion completely, we must include velocity and acceleration, as well as displacement. We must find their components along the x- and y-axes. Let's assume all forces except gravity (such as air resistance and friction, for example) are negligible. Defining the positive direction to be upward, the components of acceleration are then very simple:

$$a_y = -g = -9.8\ m/s^2 (-32\ ft/s^2). \tag{3.4.1}$$

Because gravity is vertical, $a_x = 0$. If $a_x = 0$, this means the initial velocity in the x direction is equal to the final velocity in the x direction, or $v_x = v_{0x}$. With these conditions on acceleration and velocity, we can write the kinematic Equation 4.11 through Equation 4.18 for motion in a uniform gravitational field, including the rest of the kinematic equations for a constant acceleration from Motion with Constant Acceleration. The kinematic equations for motion in a uniform gravitational field become kinematic equations with $a_y = -g$, $a_x = 0$:

Horizontal Motion

$$v_{0x} = v_x, \quad x = x_0 + v_x t \tag{3.4.2}$$

Vertical Motion

$$y = y_0 + \frac{1}{2}(v_{0y} + v_y)t \tag{3.4.3}$$

$$v_y = v_{0y} - gt \tag{3.4.4}$$

$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2 \tag{3.4.5}$$

$$v_y^2 = v_{0y}^2 + 2g(y - y_0) \tag{3.4.6}$$

Using this set of equations, we can analyze projectile motion, keeping in mind some important points.

Problem-Solving Strategy: Projectile Motion

1. Resolve the motion into horizontal and vertical components along the x- and y-axes. The magnitudes of the components of displacement $\vec{s}$ along these axes are x and y. The magnitudes of the components of velocity $\vec{v}$ are $v_x = v\cos\theta$ and $v_y = v\sin\theta$, where v is the magnitude of the velocity and θ is its direction relative to the horizontal, as shown in Figure 3.4.2.
2. Treat the motion as two independent one-dimensional motions: one horizontal and the other vertical. Use the kinematic equations for horizontal and vertical motion presented earlier.
3. Solve for the unknowns in the two separate motions: one horizontal and one vertical. Note that the only common variable between the motions is time t. The problem-solving procedures here are the same as those for one-dimensional kinematics and are illustrated in the following solved examples.
4. Recombine quantities in the horizontal and vertical directions to find the total displacement $\vec{s}$ and velocity $\vec{v}$. Solve for the magnitude and direction of the displacement and velocity using

$$s = \sqrt{x^2 + y^2}. \quad \phi = \tan^{-1}\left(\frac{y}{x}\right), \quad v = \sqrt{v_x^2 + v_y^2}. \tag{3.4.7}$$

where ϕ is the direction of the displacement $\vec{s}$.

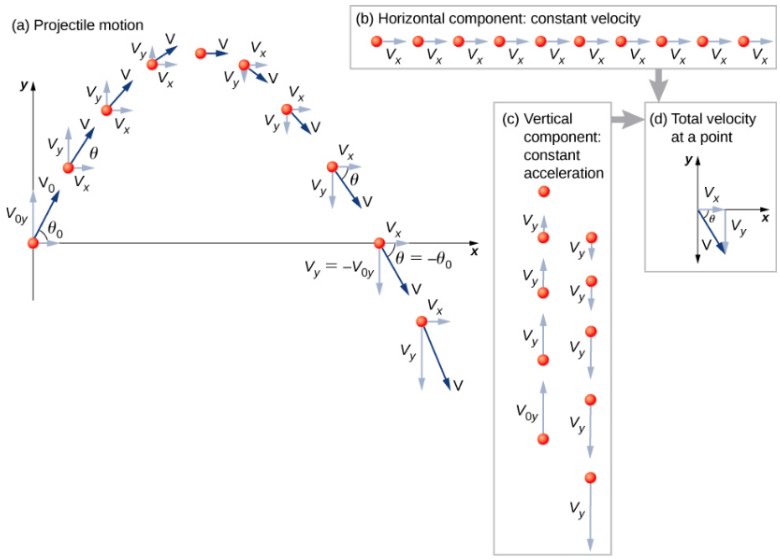


Figure 3.4.2: (a) We analyze two-dimensional projectile motion by breaking it into two independent one-dimensional motions along the vertical and horizontal axes. (b) The horizontal motion is simple, because $a_x = 0$ and v_x is a constant. (c) The velocity in the vertical direction begins to decrease as the object rises. At its highest point, the vertical velocity is zero. As the object falls toward Earth again, the vertical velocity increases again in magnitude but points in the opposite direction to the initial vertical velocity. (d) The x and y motions are recombined to give the total velocity at any given point on the trajectory.

Example 4.7: A Fireworks Projectile Explodes high and away

During a fireworks display, a shell is shot into the air with an initial speed of 70.0 m/s at an angle of 75.0° above the horizontal, as illustrated in Figure 3.4.3. The fuse is timed to ignite the shell just as it reaches its highest point above the ground. (a) Calculate the height at which the shell explodes. (b) How much time passes between the launch of the shell and the explosion? (c) What is the horizontal displacement of the shell when it explodes? (d) What is the total displacement from the point of launch to the highest point?

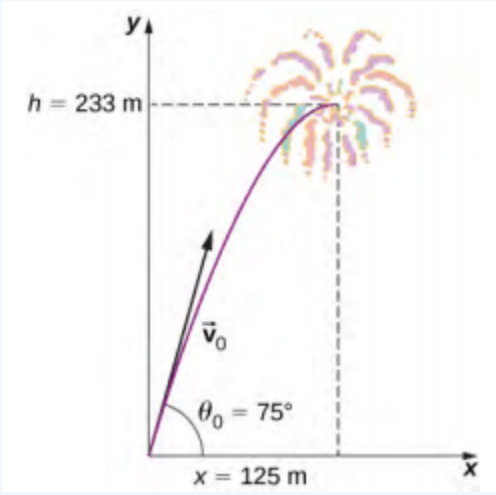


Figure 3.4.3: The trajectory of a fireworks shell. The fuse is set to explode the shell at the highest point in its trajectory, which is found to be at a height of 233 m and 125 m away horizontally.

Strategy

The motion can be broken into horizontal and vertical motions in which $a_x = 0$ and $a_y = -g$. We can then define x_0 and y_0 to be zero and solve for the desired quantities.

Solution

a. By "height" we mean the altitude or vertical position y above the starting point. The highest point in any trajectory, called the apex, is reached when $v_y = 0$. Since we know the initial and final velocities, as well as the initial position, we use the following equation to find y:

$$v_y^2 = v_{0y}^2 - 2g(y - y_0). \tag{3.4.8}$$

Because y_0 and v_y are both zero, the equation simplifies to

$$0 = v_{0y}^2 - 2gy. \tag{3.4.9}$$

Solving for y gives

$$y = \frac{v_{0y}^2}{2g}. \tag{3.4.10}$$

Now we must find v_{0y}, the component of the initial velocity in the y direction. It is given by $v_{0y} = v_0 \sin\theta_0$, where v_0 is the initial velocity of 70.0 m/s and $\theta_0 = 75°$ is the initial angle. Thus

$$v_{0y} = v_0 \sin\theta = (70.0\ m/s) \sin 75^o = 67.6\ m/s \tag{3.4.11}$$

and y is

$$y = \frac{(67.6\ m/s)^2}{2(9.80\ m/s^2)}. \tag{3.4.12}$$

Thus, we have

$$y = 233\ m. \tag{3.4.13}$$

Note that because up is positive, the initial vertical velocity is positive, as is the maximum height, but the acceleration resulting from gravity is negative. Note also that the maximum height depends only on the vertical component of the initial velocity, so that any projectile with a 67.6-m/s initial vertical component of velocity reaches a maximum height of 233 m (neglecting air resistance). The numbers in this example are reasonable for large fireworks displays, the shells of which do reach such heights before exploding. In practice, air resistance is not completely negligible, so the initial velocity would have to be somewhat larger than that given to reach the same height.

b. As in many physics problems, there is more than one way to solve for the time the projectile reaches its highest point. In this case, the easiest method is to use $v_y = v_{0y} - gt$. Because $v_y = 0$ at the apex, this equation reduces

$$0 = v_{0y} - gt \tag{3.4.14}$$

or

$$t = \frac{v_{0y}}{g} = \frac{67.6\ m/s}{9.80\ m/s^2} = 6.90\ s. \tag{3.4.15}$$

This time is also reasonable for large fireworks. If you are able to see the launch of fireworks, notice that several seconds pass before the shell explodes. Another way of finding the time is by using $y = y_0 + \frac{1}{2}(v_{0y} + v_y)t$. This is left for you as an exercise to complete.

c. Because air resistance is negligible, $a_x = 0$ and the horizontal velocity is constant, as discussed earlier. The horizontal displacement is the horizontal velocity multiplied by time as given by $x = x_0 + v_x t$, where x_0 is equal to zero. Thus,

$$x = v_x t, \tag{3.4.16}$$

where v_x is the x-component of the velocity, which is given by

$$v_x = v_0 \cos\theta = (70.0\ m/s) \cos 75^o = 18.1\ m/s. \tag{3.4.17}$$

Time t for both motions is the same, so x is

$$x = (18.1\ m/s)(6.90\ s) = 125\ m. \tag{3.4.18}$$

Horizontal motion is a constant velocity in the absence of air resistance. The horizontal displacement found here could be useful in keeping the fireworks fragments from falling on spectators. When the shell explodes, air resistance has a major effect, and many fragments land directly below.

d. The horizontal and vertical components of the displacement were just calculated, so all that is needed here is to find the magnitude and direction of the displacement at the highest point:

$$\vec{s} = 125\hat{i} + 233\hat{j} \tag{3.4.19}$$

$$|\vec{s}| = \sqrt{125^2 + 233^2} = 264\ m \tag{3.4.20}$$

$$\theta = \tan^{-1}\left(\frac{233}{125}\right) = 61.8^{o}. \tag{3.4.21}$$

Note that the angle for the displacement vector is less than the initial angle of launch. To see why this is, review Figure 3.4.1, which shows the curvature of the trajectory toward the ground level. When solving Example 4.7(a), the expression we found for y is valid for any projectile motion when air resistance is negligible. Call the maximum height y = h. Then,

$$h = \frac{v_{0y}^2}{2g}. \tag{3.4.22}$$

This equation defines the **maximum height of a projectile above its launch position** and it depends only on the vertical component of the initial velocity.

Exercise 4.3

A rock is thrown horizontally off a cliff 100.0 m high with a velocity of 15.0 m/s. (a) Define the origin of the coordinate system. (b) Which equation describes the horizontal motion? (c) Which equations describe the vertical motion? (d) What is the rock's velocity at the point of impact?

Example 4.8: Calculating projectile motion- Tennis Player

A tennis player wins a match at Arthur Ashe stadium and hits a ball into the stands at 30 m/s and at an angle 45° above the horizontal (Figure 3.4.4). On its way down, the ball is caught by a spectator 10 m above the point where the ball was hit. (a) Calculate the time it takes the tennis ball to reach the spectator. (b) What are the magnitude and direction of the ball's velocity at impact?

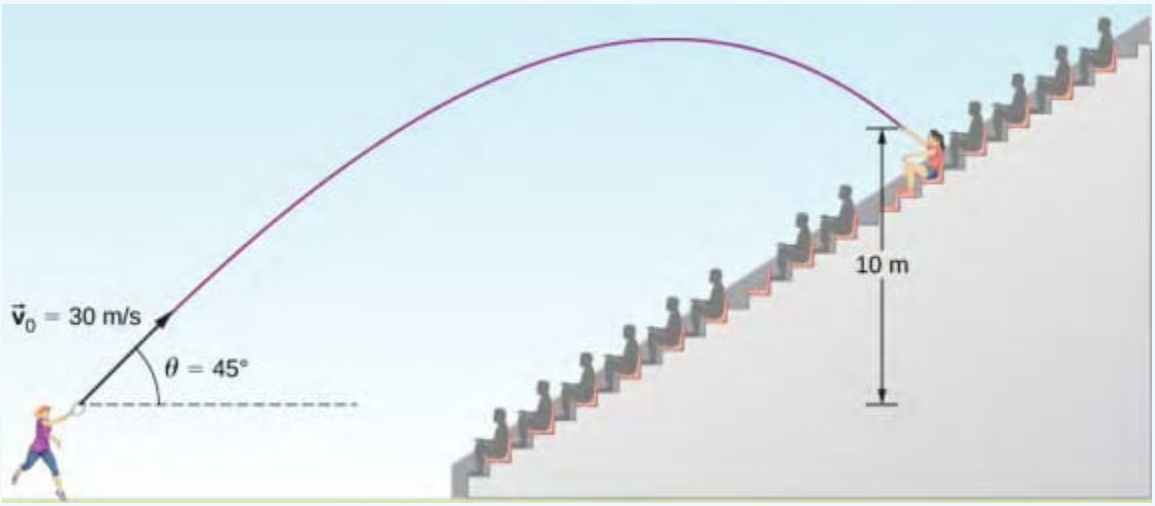


Figure 3.4.4: The trajectory of a tennis ball hit into the stands.

Strategy

Again, resolving this two-dimensional motion into two independent one-dimensional motions allows us to solve for the desired quantities. The time a projectile is in the air is governed by its vertical motion alone. Thus, we solve for t first. While the ball is rising and falling vertically, the horizontal motion continues at a constant velocity. This example asks for the final velocity. Thus, we recombine the vertical and horizontal results to obtain $\vec{v}$ at final time t, determined in the first part of the example.

Solution

a. While the ball is in the air, it rises and then falls to a final position 10.0 m higher than its starting altitude. We can find the time for this by using Equation 3.4.5:

$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2. \tag{3.4.23}$$

If we take the initial position y_0 to be zero, then the final position is y = 10 m. The initial vertical velocity is the vertical component of the initial velocity:

$$v_{0y} = v_0 \sin\theta_0 = (30.0\ m/s)\sin 45^o = 21.2\ m/s. \tag{3.4.24}$$

Substituting into Equation 3.4.5 for y gives us

$$10.0\ m = (21.2\ m/s)t - (4.90\ m/s^2)t^2. \tag{3.4.25}$$

Rearranging terms gives a quadratic equation in t:

$$(4.90\ m/s^2)t^2 - (21.2\ m/s)t + 10.0\ m = 0. \tag{3.4.26}$$

Use of the quadratic formula yields t = 3.79 s and t = 0.54 s. Since the ball is at a height of 10 m at two times during its trajectory—once on the way up and once on the way down—we take the longer solution for the time it takes the ball to reach the spectator:

$$t = 3.79\ s. \tag{3.4.27}$$

The time for projectile motion is determined completely by the vertical motion. Thus, any projectile that has an initial vertical velocity of 21.2 m/s and lands 10.0 m above its starting altitude spends 3.79 s in the air.

b. We can find the final horizontal and vertical velocities v_x and v_y with the use of the result from (a). Then, we can combine them to find the magnitude of the total velocity vector $\vec{v}$ and the angle θ it makes with the horizontal. Since v_x is constant, we can solve for it at any horizontal location. We choose the starting point because we know both the initial velocity and the initial angle. Therefore,

$$v_x = v_0 \cos\theta_0 = (30\ m/s)\cos 45^o = 21.2\ m/s. \tag{3.4.28}$$

The final vertical velocity is given by Equation 3.4.4:

$$v_y = v_{0y} - gt. \tag{3.4.29}$$

Since v_{0y} was found in part (a) to be 21.2 m/s, we have

$$v_y = 21.2\ m/s - (9.8\ m/s^2)(3.79s) = -15.9\ m/s. \tag{3.4.30}$$

The magnitude of the final velocity $\vec{v}$ is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(21.2\ m/s)^2 + (-15.9\ m/s)^2} = 26.5\ m/s. \tag{3.4.31}$$

The direction θ_v is found using the inverse tangent:

$$\theta_v = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{21.2}{-15.9}\right) = -53.1^o. \tag{3.4.32}$$

Significance

a. As mentioned earlier, the time for projectile motion is determined completely by the vertical motion. Thus, any projectile that has an initial vertical velocity of 21.2 m/s and lands 10.0 m above its starting altitude spends 3.79 s in the air.

b. The negative angle means the velocity is 53.1° below the horizontal at the point of impact. This result is consistent with the fact that the ball is impacting at a point on the other side of the apex of the trajectory and therefore has a negative y component of the velocity. The magnitude of the velocity is less than the magnitude of the initial velocity we expect since it is impacting 10.0 m above the launch elevation.

Time of Flight, Trajectory, and Range

Of interest are the time of flight, trajectory, and range for a projectile launched on a flat horizontal surface and impacting on the same surface. In this case, kinematic equations give useful expressions for these quantities, which are derived in the following sections.

Time of flight

We can solve for the time of flight of a projectile that is both launched and impacts on a flat horizontal surface by performing

some manipulations of the kinematic equations. We note the position and displacement in y must be zero at launch and at impact on an even surface. Thus, we set the displacement in y equal to zero and find

$$y - y_0 = v_{0y}t - \frac{1}{2}gt^2 = (v_0 \sin\theta_0)t - \frac{1}{2}gt^2 = 0. \tag{3.4.33}$$

Factoring, we have

$$t\left(v_0 \sin\theta_0 - \frac{gt}{2}\right) = 0. \tag{3.4.34}$$

Solving for t gives us

$$T_{tof} = \frac{2(v_0 \sin\theta_0)}{g}. \tag{3.4.35}$$

This is the **time of flight** for a projectile both launched and impacting on a flat horizontal surface. Equation 3.4.35 does not apply when the projectile lands at a different elevation than it was launched, as we saw in Example 4.8 of the tennis player hitting the ball into the stands. The other solution, t = 0, corresponds to the time at launch. The time of flight is linearly proportional to the initial velocity in the y direction and inversely proportional to g. Thus, on the Moon, where gravity is one-sixth that of Earth, a projectile launched with the same velocity as on Earth would be airborne six times as long.

Trajectory

The trajectory of a projectile can be found by eliminating the time variable t from the kinematic equations for arbitrary t and solving for y(x). We take $x_0 = y_0 = 0$ so the projectile is launched from the origin. The kinematic equation for x gives

$$x = v_{0x}t \Rightarrow t = \frac{x}{v_{0x}} = \frac{x}{v_0 \cos\theta_0}. \tag{3.4.36}$$

Substituting the expression for t into the equation for the position $y = (v_0 \sin\theta_0)t - \frac{1}{2}gt^2$ gives

$$y = (v_0 \sin\theta_0)\left(\frac{x}{v_0 \cos\theta_0}\right) - \frac{1}{2}g\left(\frac{x}{v_0 \cos\theta_0}\right)^2. \tag{3.4.37}$$

Rearranging terms, we have

$$y = (\tan\theta_0)x - \left[\frac{g}{2(v_0 \cos\theta_0)^2}\right]x^2. \tag{3.4.38}$$

This trajectory equation is of the form $y = ax + bx^2$, which is an equation of a parabola with coefficients

$$a = \tan\theta_0, \quad b = -\frac{g}{2(v_0 \cos\theta_0)^2}. \tag{3.4.39}$$

Range

From the trajectory equation we can also find the **range**, or the horizontal distance traveled by the projectile. Factoring Equation 3.4.38, we have

$$y = x\left[\tan\theta_0 - \frac{g}{2(v_0 \cos\theta_0)^2}x\right]. \tag{3.4.40}$$

The position y is zero for both the launch point and the impact point, since we are again considering only a flat horizontal surface. Setting y = 0 in this equation gives solutions x = 0, corresponding to the launch point, and

$$x = \frac{2v_0^2 \sin\theta_0 \cos\theta_0}{g}, \tag{3.4.41}$$

corresponding to the impact point. Using the trigonometric identity $2\sin\theta\cos\theta = \sin 2\theta$ and setting x = R for range, we find

$$R = \frac{v_0^2 \sin 2\theta_0}{g}. \tag{3.4.42}$$

Note particularly that Equation 3.4.42 is valid only for launch and impact on a horizontal surface. We see the range is directly proportional to the square of the initial speed v_0 and $\sin^2 \theta_0$, and it is inversely proportional to the acceleration of gravity. Thus, on the Moon, the range would be six times greater than on Earth for the same initial velocity. Furthermore, we see from the factor $\sin^2 \theta_0$ that the range is maximum at 45°. These results are shown in Figure 3.4.5. In (a) we see that the greater the initial velocity, the greater the range. In (b), we see that the range is maximum at 45°. This is true only for conditions neglecting air resistance. If air resistance is considered, the maximum angle is somewhat smaller. It is interesting that the same range is found for two initial launch angles that sum to 90°. The projectile launched with the smaller angle has a lower apex than the higher angle, but they both have the same range.

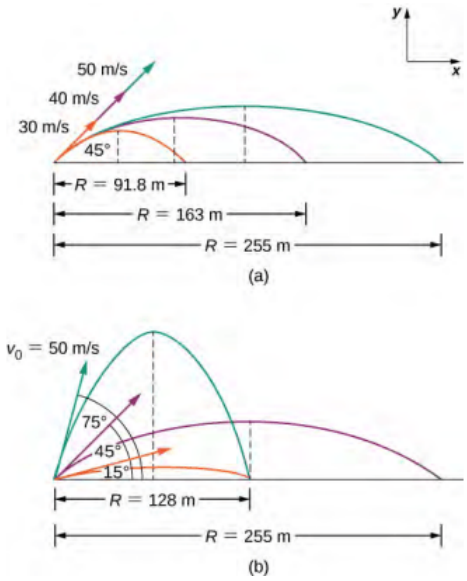


Figure 3.4.5: Trajectories of projectiles on level ground. (a) The greater the initial speed v_0, the greater the range for a given initial angle. (b) The effect of initial angle (\theta_{0}\) on the range of a projectile with a given initial speed. Note that the range is the same for initial angles of 15° and 75°, although the maximum heights of those paths are different.

Example 4.9: Comparing golf shots

A golfer finds himself in two different situations on different holes. On the second hole he is 120 m from the green and wants to hit the ball 90 m and let it run onto the green. He angles the shot low to the ground at 30° to the horizontal to let the ball roll after impact. On the fourth hole he is 90 m from the green and wants to let the ball drop with a minimum amount of rolling after impact. Here, he angles the shot at 70° to the horizontal to minimize rolling after impact. Both shots are hit and impacted on a level surface. (a) What is the initial speed of the ball at the second hole? (b) What is the initial speed of the ball at the fourth hole? (c) Write the trajectory equation for both cases. (d) Graph the trajectories.

Strategy

We see that the range equation has the initial speed and angle, so we can solve for the initial speed for both (a) and (b). When we have the initial speed, we can use this value to write the trajectory equation.

Solution

a.
$$R = \frac{v_0^2 \sin 2\theta_0}{g} \Rightarrow v_0 = \sqrt{\frac{Rg}{\sin 2\theta_0}} = \sqrt{\frac{(90.0\ m)(9.8\ m/s^2)}{\sin(2(30^o))}} = 31.9\ m/s \tag{3.4.43}$$

b.
$$R = \frac{v_0^2 \sin 2\theta_0}{g} \Rightarrow v_0 = \sqrt{\frac{Rg}{\sin 2\theta_0}} = \sqrt{\frac{(90.0\ m)(9.8\ m/s^2)}{\sin(2(70^o))}} = 37.0\ m/s \tag{3.4.44}$$

c.
$$y = x\left[\tan\theta_0 - \frac{g}{2(v_0\cos\theta_0)^2}x\right] \tag{3.4.45}$$

Second hole:
$$y = x\left[\tan 30^o - \frac{9.8\ m/s^2}{2[(31.9\ m/s)(\cos 30^o)]^2}x\right] = 0.58x - 0.0064x^2 \tag{3.4.46}$$

Fourth hole:
$$y = x\left[\tan 70^o - \frac{9.8\ m/s^2}{2[(37.0\ m/s)(\cos 70^o)]^2}x\right] = 2.75x - 0.0306x^2 \tag{3.4.47}$$

d. Using a graphing utility, we can compare the two trajectories, which are shown in Figure 3.4.6.

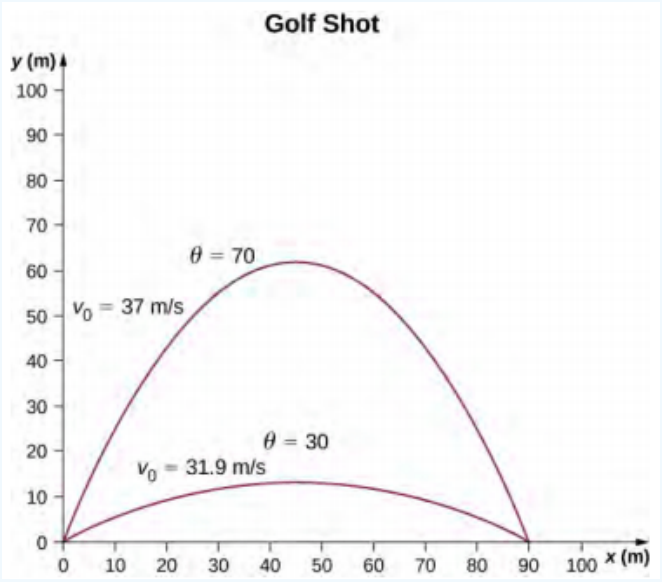


Figure 3.4.6: Two trajectories of a golf ball with a range of 90 m. The impact points of both are at the same level as the launch point.

Significance

The initial speed for the shot at 70° is greater than the initial speed of the shot at 30°. Note from Figure 3.4.6 that two projectiles launched at the same speed but at different angles have the same range if the launch angles add to 90°. The launch angles in this example add to give a number greater than 90°. Thus, the shot at 70° has to have a greater launch speed to reach 90 m, otherwise it would land at a shorter distance.

Exercise 4.4

If the two golf shots in Example 4.9 were launched at the same speed, which shot would have the greatest range?

When we speak of the range of a projectile on level ground, we assume R is very small compared with the circumference of Earth. If, however, the range is large, Earth curves away below the projectile and the acceleration resulting from gravity changes direction along the path. The range is larger than predicted by the range equation given earlier because the projectile has farther to fall than it would on level ground, as shown in Figure 3.4.7, which is based on a drawing in Newton's **Principia**. If the initial speed is great enough, the projectile goes into orbit. Earth's surface drops 5 m every 8000 m. In 1 s an object falls 5 m without air resistance. Thus, if an object is given a horizontal velocity of 8000 m/s (or 18,000 mi/hr) near Earth's surface, it will go into orbit around the planet because the surface continuously falls away from the object. This is roughly the speed of the Space Shuttle in a low Earth orbit when it was operational, or any satellite in a low Earth orbit. These and other aspects of orbital motion, such as Earth's rotation, are covered in greater depth in Gravitation.

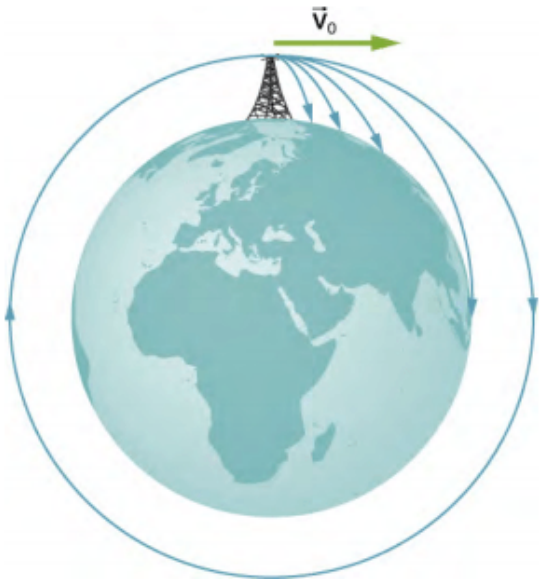


Figure 3.4.7: Projectile to satellite. In each case shown here, a projectile is launched from a very high tower to avoid air resistance. With increasing initial speed, the range increases and becomes longer than it would be on level ground because Earth curves away beneath its path. With a speed of 8000 m/s, orbit is achieved.

Simulation

At PhET Explorations: Projectile Motion, learn about projectile motion in terms of the launch angle and initial velocity.

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3.5: Uniform Circular Motion

Learning Objectives

- Solve for the centripetal acceleration of an object moving on a circular path.
- Use the equations of circular motion to find the position, velocity, and acceleration of a particle executing circular motion.
- Explain the differences between centripetal acceleration and tangential acceleration resulting from nonuniform circular motion.
- Evaluate centripetal and tangential acceleration in nonuniform circular motion, and find the total acceleration vector.

Uniform circular motion is a specific type of motion in which an object travels in a circle with a constant speed. For example, any point on a propeller spinning at a constant rate is executing uniform circular motion. Other examples are the second, minute, and hour hands of a watch. It is remarkable that points on these rotating objects are actually accelerating, although the rotation rate is a constant. To see this, we must analyze the motion in terms of vectors.

Centripetal Acceleration

In one-dimensional kinematics, objects with a constant speed have zero acceleration. However, in two- and three-dimensional kinematics, even if the speed is a constant, a particle can have acceleration if it moves along a curved trajectory such as a circle. In this case the velocity vector is changing, or $\frac{d\vec{v}}{dt} \neq 0$. This is shown in Figure $3.5.1$. As the particle moves counterclockwise in time Δt on the circular path, its position vector moves from $\vec{r}(t)$ to $\vec{r}(t+\Delta t)$. The velocity vector has constant magnitude and is tangent to the path as it changes from $\vec{v}$(t) to $\vec{v}\,(t+\Delta t)$, changing its direction only. Since the velocity vector $\vec{v}(t)$ is perpendicular to the position vector $\vec{r}$(t), the triangles formed by the position vectors and $\Delta\vec{r}$, and the velocity vectors and $\Delta\vec{v}$ are similar. Furthermore, since

$$|\vec{r}(t)| = |\vec{r}(t+\Delta t)|$$

and

$$|\vec{v}(t)| = |\vec{v}(t+\Delta t)|,$$

the two triangles are isosceles. From these facts we can make the assertion

$$\frac{\Delta v}{v} = \frac{\Delta r}{r} \tag{3.5.1}$$

or

$$\Delta v = \frac{v}{r}\Delta r. \tag{3.5.2}$$

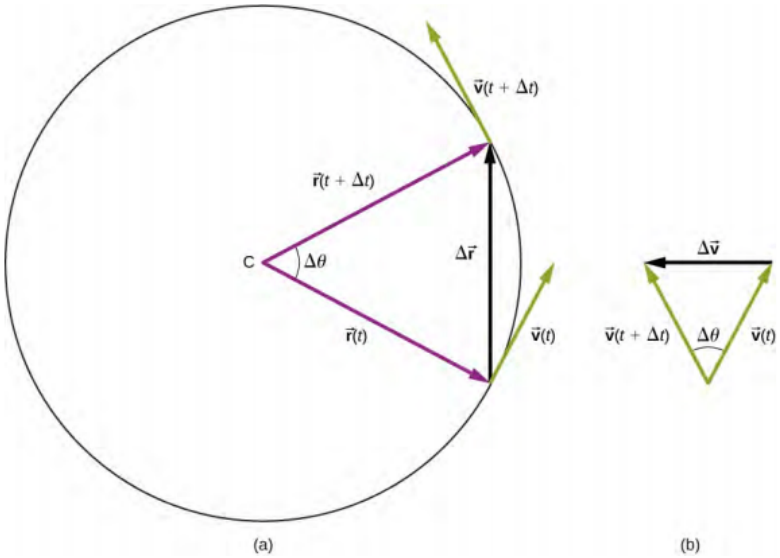


Figure 3.5.1: (a) A particle is moving in a circle at a constant speed, with position and velocity vectors at times t and $t+\Delta t$. (b) Velocity vectors forming a triangle. The two triangles in the figure are similar. The vector $\Delta\vec{v}$ points toward the center of the circle in the limit $\Delta t \to 0$.

We can find the magnitude of the acceleration from

$$a = \lim_{\Delta t \to 0}\left(\frac{\Delta v}{\Delta t}\right) = \frac{v}{r}\left(\lim_{\Delta t \to 0}\frac{\Delta r}{\Delta t}\right) = \frac{v^2}{r}. \tag{3.5.3}$$

The direction of the acceleration can also be found by noting that as Δt and therefore $\Delta\theta$ approach zero, the vector $\Delta\vec{v}$ approaches a direction perpendicular to $\vec{v}$. In the limit $\Delta t \to 0$, $\Delta\vec{v}$ is perpendicular to $\vec{v}$. Since $\vec{v}$ is tangent to the circle, the acceleration $\frac{d\vec{v}}{dt}$ points toward the center of the circle. Summarizing, a particle moving in a circle at a constant speed has an acceleration with magnitude

$$a_c = \frac{v^2}{r}. \tag{3.5.4}$$

The direction of the acceleration vector is toward the center of the circle (Figure 3.5.2). This is a radial acceleration and is called the **centripetal acceleration**, which is why we give it the subscript c. The word **centripetal** comes from the Latin words **centrum** (meaning "center") and **petere** (meaning to seek"), and thus takes the meaning "center seeking."

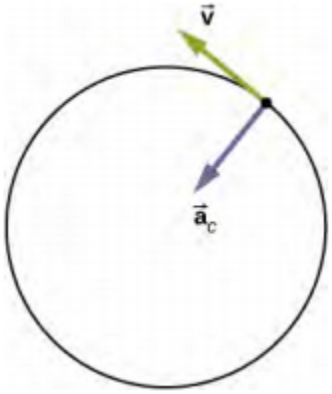


Figure 3.5.2: The centripetal acceleration vector points toward the center of the circular path of motion and is an acceleration in the radial direction. The velocity vector is also shown and is tangent to the circle.

Let's investigate some examples that illustrate the relative magnitudes of the velocity, radius, and centripetal acceleration.

Example 3.5.1: Creating an Acceleration of 1 g

A jet is flying at 134.1 m/s along a straight line and makes a turn along a circular path level with the ground. What does the radius of the circle have to be to produce a centripetal acceleration of 1 g on the pilot and jet toward the center of the circular trajectory?

Strategy

Given the speed of the jet, we can solve for the radius of the circle in the expression for the centripetal acceleration.

Solution

Set the centripetal acceleration equal to the acceleration of gravity: 9.8 m/s^2 = $\frac{v^2}{r}$.

Solving for the radius, we find

$$r = \frac{(134.1\ m/s)^2}{9.8\ m/s^2} = 1835\ m = 1.835\ km. \tag{3.5.5}$$

Significance

To create a greater acceleration than g on the pilot, the jet would either have to decrease the radius of its circular trajectory or increase its speed on its existing trajectory or both.

Exercise 4.5

A flywheel has a radius of 20.0 cm. What is the speed of a point on the edge of the flywheel if it experiences a centripetal acceleration of 900.0 cm/s^2?

Centripetal acceleration can have a wide range of values, depending on the speed and radius of curvature of the circular path. Typical centripetal accelerations are given in Table 3.5.1.

Table 3.5.1: Typical Centripetal Accelerations

Object	Centripetal Acceleration (m/s^2 or factors of g)
Earth around the Sun	5.93 x 10^{-3}
Moon around the Earth	2.73 x 10^{-3}
Satellite in geosynchronous orbit	0.233
Outer edge of a CD when playing	5.75
Jet in a barrel roll	(2-3 g)
Roller coaster	(5 g)
Electron orbiting a proton in a simple Bohr model of the atom	9.0 x 10^{22}

Equations of Motion for Uniform Circular Motion

A particle executing circular motion can be described by its position vector $\vec{r}(t)$. Figure 3.5.3 shows a particle executing circular motion in a counterclockwise direction. As the particle moves on the circle, its position vector sweeps out the angle θ with the x-axis. Vector $\vec{r}(t)$ making an angle θ with the x-axis is shown with its components along the x- and y-axes. The magnitude of the position vector is $A = |\vec{r}(t)|$ and is also the radius of the circle, so that in terms of its components,

$$\vec{r}(t) = A\cos\omega t\hat{i} + A\sin\omega t\hat{j}. \tag{3.5.6}$$

Here, ω is a constant called the **angular frequency** of the particle. The angular frequency has units of radians (rad) per second and is simply the number of radians of angular measure through which the particle passes per second. The angle θ that the position vector has at any particular time is ωt.

If T is the period of motion, or the time to complete one revolution ($2\pi\ rad$), then

$$\omega = \frac{2\pi}{T}.$$

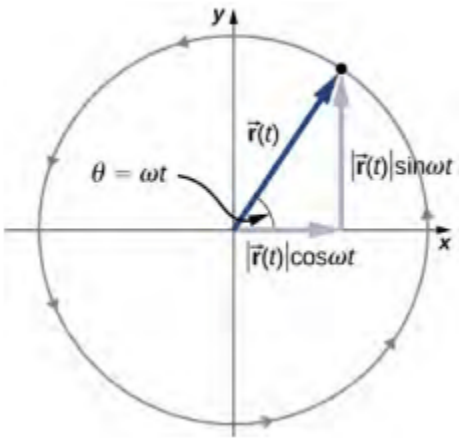


Figure 3.5.3: The position vector for a particle in circular motion with its components along the x- and y-axes. The particle moves counterclockwise. Angle θ is the angular frequency ω in radians per second multiplied by t.

Velocity and acceleration can be obtained from the position function by differentiation:

$$\vec{v}(t) = \frac{d\vec{r}(t)}{dt} = -A\omega\sin\omega t\hat{i} + A\omega\cos\omega t\hat{j}. \tag{3.5.7}$$

It can be shown from Figure 3.5.3 that the velocity vector is tangential to the circle at the location of the particle, with magnitude Aω. Similarly, the acceleration vector is found by differentiating the velocity:

$$\vec{a}(t) = \frac{d\vec{v}(t)}{dt} = -A\omega^2\cos\omega t\hat{i} - A\omega^2\sin\omega t\hat{j}. \tag{3.5.8}$$

From this equation we see that the acceleration vector has magnitude Aω^2 and is directed opposite the position vector, toward the origin, because $\vec{a}$(t) = –$\omega^2\vec{r}$(t).

Example 3.5.2: Circular Motion of a Proton

A proton has speed 5 x 10^6 m/s and is moving in a circle in the xy plane of radius r = 0.175 m. What is its position in the xy plane at time t = 2.0 x 10^{-7} s = 200 ns? At t = 0, the position of the proton is 0.175 m $\hat{i}$ and it circles counterclockwise. Sketch the trajectory.

Solution

From the given data, the proton has period and angular frequency:

$$T = \frac{2\pi r}{v} = \frac{2\pi(0.175\ m)}{5.0\times10^6\ m/s} = 2.20\times10^{-7}\ s$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{2.20\times10^{-7}\ s} = 2.856\times10^7\ rad/s.$$

The position of the particle at t = 2.0 x 10^{-7} s with A = 0.175 m is

$$\begin{aligned}\vec{r}(2.0\times10^{-7}\ s) &= A\cos\omega(2.0\times10^{-7}\ s)\hat{i} + A\sin\omega(2.0\times10^{-7}\ s)\hat{j}\ m\\ &= 0.175\cos(2.856\times10^{7}\ rad/s)(2.0\times10^{-7}\ s)\hat{i} + 0.175\sin(2.856\times10^{7}\ rad/s)(2.0\times10^{-7}\ s)\hat{j}\ m\\ &= 0.175\cos(5.712\ rad)\hat{i} + 0.175\sin(5.172\ rad)\hat{j}\ m\\ &= 0.147\hat{i} - 0.095\hat{j}\ m.\end{aligned}$$

From this result we see that the proton is located slightly below the x-axis. This is shown in Figure 3.5.4.

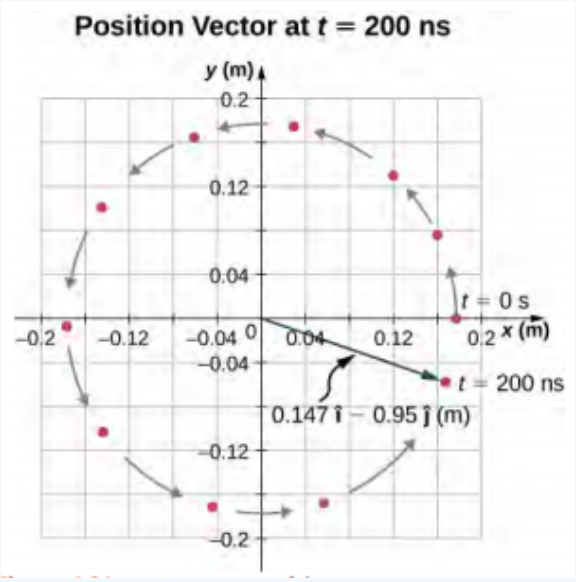


Figure 3.5.4: Position vector of the proton at $t = 2.0\times10^{-7}\ ms = 200\ ns$. The trajectory of the proton is shown. The angle through which the proton travels along the circle is 5.712 rad, which a little less than one complete revolution.

Significance

We picked the initial position of the particle to be on the x-axis. This was completely arbitrary. If a different starting position were given, we would have a different final position at t = 200 ns.

Nonuniform Circular Motion

Circular motion does not have to be at a constant speed. A particle can travel in a circle and speed up or slow down, showing an acceleration in the direction of the motion.

In uniform circular motion, the particle executing circular motion has a constant speed and the circle is at a fixed radius. If the speed of the particle is changing as well, then we introduce an additional acceleration in the direction tangential to the circle. Such accelerations occur at a point on a top that is changing its spin rate, or any accelerating rotor. In Displacement and Velocity Vectors we showed that centripetal acceleration is the time rate of change of the direction of the velocity vector. If the speed of the particle is changing, then it has a **tangential acceleration** that is the time rate of change of the magnitude of the velocity:

$$a_T = \frac{d|\vec{v}|}{dt}. \tag{3.5.9}$$

The direction of tangential acceleration is tangent to the circle whereas the direction of centripetal acceleration is radially inward toward the center of the circle. Thus, a particle in circular motion with a tangential acceleration has a **total acceleration** that is the vector sum of the centripetal and tangential accelerations:

$$\vec{a} = \vec{a}_c + \vec{a}_T. \tag{3.5.10}$$

The acceleration vectors are shown in Figure 3.5.5. Note that the two acceleration vectors $\vec{a}_c$ and $\vec{a}_T$ are perpendicular to each other, with $\vec{a}_c$ in the radial direction and $\vec{a}_T$ in the tangential direction. The total acceleration $\vec{a}$ points at an angle between $\vec{a}_c$ and $\vec{a}_T$.

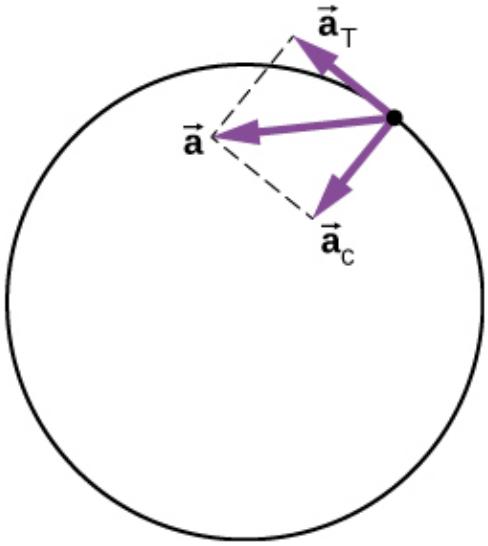


Figure 3.5.5: The centripetal acceleration points toward the center of the circle. The tangential acceleration is tangential to the circle at the particle's position. The total acceleration is the vector sum of the tangential and centripetal accelerations, which are perpendicular.

Example 3.5.3: Total Acceleration during Circular Motion

A particle moves in a circle of radius r = 2.0 m. During the time interval from t = 1.5 s to t = 4.0 s its speed varies with time according to

$$v(t) = c_1 - \frac{c_2}{t^2}, c_1 = 4.0\ m/s, c_2 = 6.0\ m \cdot s. \tag{3.5.11}$$

What is the total acceleration of the particle at t = 2.0 s?

Strategy

We are given the speed of the particle and the radius of the circle, so we can calculate centripetal acceleration easily. The direction of the centripetal acceleration is toward the center of the circle. We find the magnitude of the tangential acceleration by taking the derivative with respect to time of |v(t)| using Equation 3.5.9 and evaluating it at t = 2.0 s. We use this and the magnitude of the centripetal acceleration to find the total acceleration.

Solution

Centripetal acceleration is

$$v(2.0\ s) = \left(4.0 - \frac{6.0}{(2.0)^2}\right) m/s = 2.5\ m/s$$

$$a_c = \frac{v^2}{r} = \frac{(2.5\ m/s)^2}{2.0\ m} = 3.1\ m/s^2$$

directed toward the center of the circle. Tangential acceleration is

$$a_T = \left|\frac{d\vec{v}}{dt}\right| = \frac{2c_2}{t^3} = \frac{12.0}{(2.0)^3} m/s^2 = 1.5\ m/s^2.$$

Total acceleration is

$$|\vec{a}| = \sqrt{3.1^2 + 1.5^2} m/s^2 = 3.44\ m/s^2 \tag{3.5.12}$$

and $\theta = \tan^{-1}\left(\frac{3.1}{1.5}\right)$ = 64° from the tangent to the circle. See Figure 3.5.6.

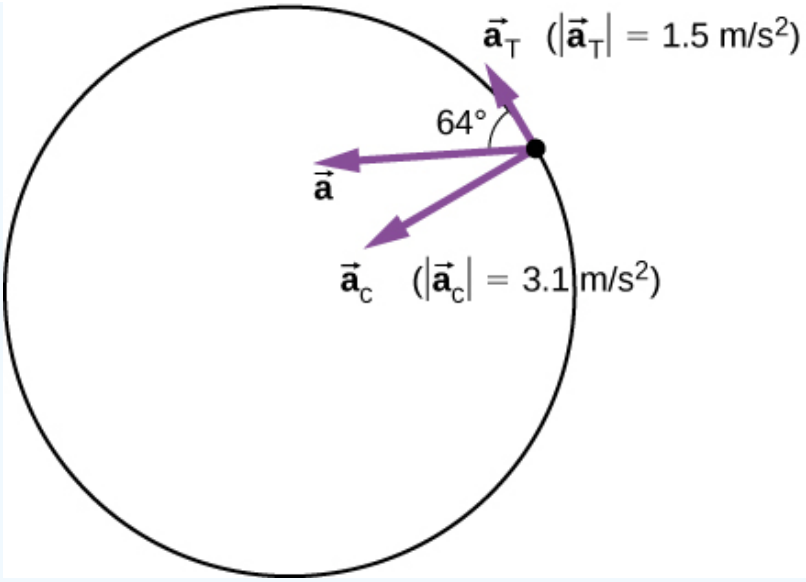


Figure 3.5.6: The tangential and centripetal acceleration vectors. The net acceleration $\vec{a}$ is the vector sum of the two accelerations.

Significance

The directions of centripetal and tangential accelerations can be described more conveniently in terms of a polar coordinate system, with unit vectors in the radial and tangential directions. This coordinate system, which is used for motion along curved paths, is discussed in detail later in the book.

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3.6: Relative Motion in One and Two Dimensions

Learning Objectives

- Explain the concept of reference frames.
- Write the position and velocity vector equations for relative motion.
- Draw the position and velocity vectors for relative motion.
- Analyze one-dimensional and two-dimensional relative motion problems using the position and velocity vector equations.

Motion does not happen in isolation. If you're riding in a train moving at 10 m/s east, this velocity is measured relative to the ground on which you're traveling. However, if another train passes you at 15 m/s east, your velocity relative to this other train is different from your velocity relative to the ground. Your velocity relative to the other train is 5 m/s west. To explore this idea further, we first need to establish some terminology.

Reference Frames

To discuss relative motion in one or more dimensions, we first introduce the concept of **reference frames**. When we say an object has a certain velocity, we must state it has a velocity with respect to a given reference frame. In most examples we have examined so far, this reference frame has been Earth. If you say a person is sitting in a train moving at 10 m/s east, then you imply the person on the train is moving relative to the surface of Earth at this velocity, and Earth is the reference frame. We can expand our view of the motion of the person on the train and say Earth is spinning in its orbit around the Sun, in which case the motion becomes more complicated. In this case, the solar system is the reference frame. In summary, all discussion of relative motion must define the reference frames involved. We now develop a method to refer to reference frames in relative motion.

Relative Motion in One Dimension

We introduce relative motion in one dimension first, because the velocity vectors simplify to having only two possible directions. Take the example of the person sitting in a train moving east. If we choose east as the positive direction and Earth as the reference frame, then we can write the velocity of the train with respect to the Earth as $\vec{v}_{TE}$ = 10 m/s $\hat{i}$ east, where the subscripts TE refer to train and Earth. Let's now say the person gets up out of /her seat and walks toward the back of the train at 2 m/s. This tells us she has a velocity relative to the reference frame of the train. Since the person is walking west, in the negative direction, we write her velocity with respect to the train as $\vec{v}_{PT}$ = –2 m/s $\hat{i}$. We can add the two velocity vectors to find the velocity of the person with respect to Earth. This relative velocity is written as

$$\vec{v}_{PE} = \vec{v}_{PT} + \vec{v}_{TE}. \tag{3.6.1}$$

Note the ordering of the subscripts for the various reference frames in Equation 3.6.1. The subscripts for the coupling reference frame, which is the train, appear consecutively in the right-hand side of the equation. Figure 3.6.1 shows the correct order of subscripts when forming the vector equation.

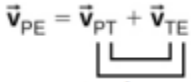


Figure 3.6.1: When constructing the vector equation, the subscripts for the coupling reference frame appear consecutively on the inside. The subscripts on the left-hand side of the equation are the same as the two outside subscripts on the right-hand side of the equation.

Adding the vectors, we find $\vec{v}_{PE}$ = 8 m/s $\hat{i}$, so the person is moving 8 m/s east with respect to Earth. Graphically, this is shown in Figure 3.6.2.



Figure 3.6.2: Velocity vectors of the train with respect to Earth, person with respect to the train, and person with respect to Earth.

Relative Velocity in Two Dimensions

We can now apply these concepts to describing motion in two dimensions. Consider a particle P and reference frames S and S′, as shown in Figure 3.6.3. The position of the origin of S′ as measured in S is $\vec{r}_{S'S}$, the position of P as measured in S′ is $\vec{r}_{PS'}$, and the position of P as measured in S is $\vec{r}_{PS}$.

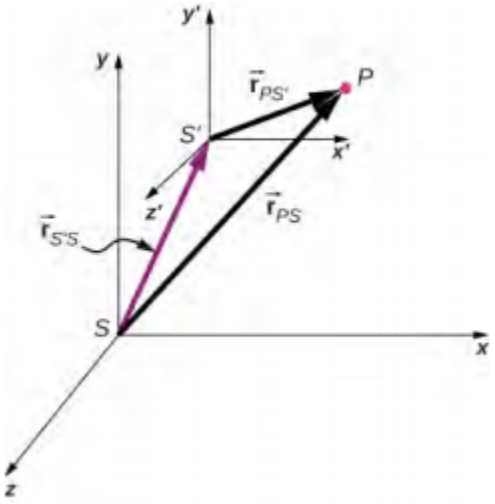


Figure 3.6.3: The positions of particle P relative to frames S and S′ are $\vec{r}_{PS}$ and $\vec{r}_{PS'}$, respectively.

From Figure 3.6.3 we see that

$$\vec{r}_{PS} = \vec{r}_{PS} + \vec{r}_{S'S}. \tag{3.6.2}$$

The relative velocities are the time derivatives of the position vectors. Therefore,

$$\vec{v}_{PS} = \vec{v}_{PS'} + \vec{v}_{S'S}. \tag{3.6.3}$$

The velocity of a particle relative to S is equal to its velocity relative to S′ plus the velocity of S′ relative to S.

We can extend Equation 3.6.3 to any number of reference frames. For particle P with velocities $\vec{v}_{PA}$, $\vec{v}_{PB}$, and $\vec{v}_{PC}$ in frames A, B, and C,

$$\vec{v}_{PC} = \vec{v}_{PA} + \vec{v}_{AB} + \vec{v}_{BC}. \tag{3.6.4}$$

We can also see how the accelerations are related as observed in two reference frames by differentiating Equation 3.6.3:

$$\vec{a}_{PS} = \vec{a}_{PS'} + \vec{a}_{S'S}. \tag{3.6.5}$$

We see that if the velocity of S′ relative to S is a constant, then $\vec{a}_{S'S}$ = 0 and

$$\vec{a}_{PS} = \vec{a}_{PS'}. \tag{3.6.6}$$

This says the acceleration of a particle is the same as measured by two observers moving at a constant velocity relative to each other.

Example 4.13: Motion of a Car Relative to a Truck

A truck is traveling south at a speed of 70 km/h toward an intersection. A car is traveling east toward the intersection at a speed of 80 km/h (Figure 3.6.4). What is the velocity of the car relative to the truck?

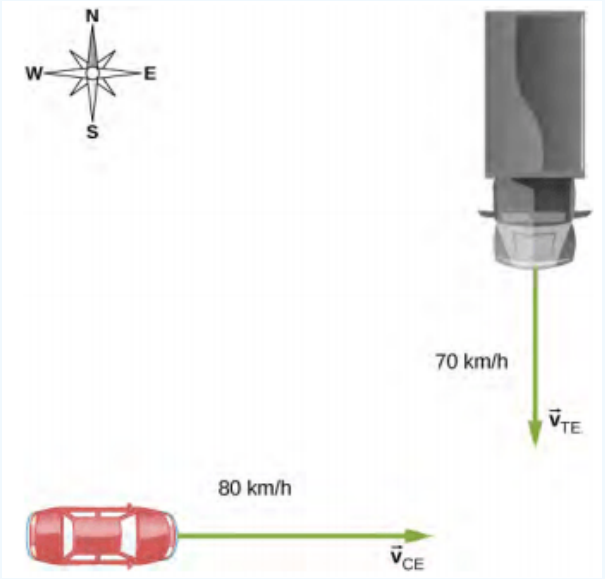


Figure 3.6.4: A car travels east toward an intersection while a truck travels south toward the same intersection.

Strategy

First, we must establish the reference frame common to both vehicles, which is Earth. Then, we write the velocities of each with respect to the reference frame of Earth, which enables us to form a vector equation that links the car, the truck, and Earth to solve for the velocity of the car with respect to the truck.

Solution

The velocity of the car with respect to Earth is $\vec{v}_{CE}$ = 80 km/h $\hat{i}$. The velocity of the truck with respect to Earth is $\vec{v}_{TE}$ = –70 km/h $\hat{j}$. Using the velocity addition rule, the relative motion equation we are seeking is

$$\vec{v}_{CT} = \vec{v}_{CE} + \vec{v}_{ET}. \tag{3.6.7}$$

Here, $\vec{v}_{CT}$ is the velocity of the car with respect to the truck, and Earth is the connecting reference frame. Since we have the velocity of the truck with respect to Earth, the negative of this vector is the velocity of Earth with respect to the truck: $\vec{v}_{ET} = -\vec{v}_{TE}$. The vector diagram of this equation is shown in Figure 3.6.5.

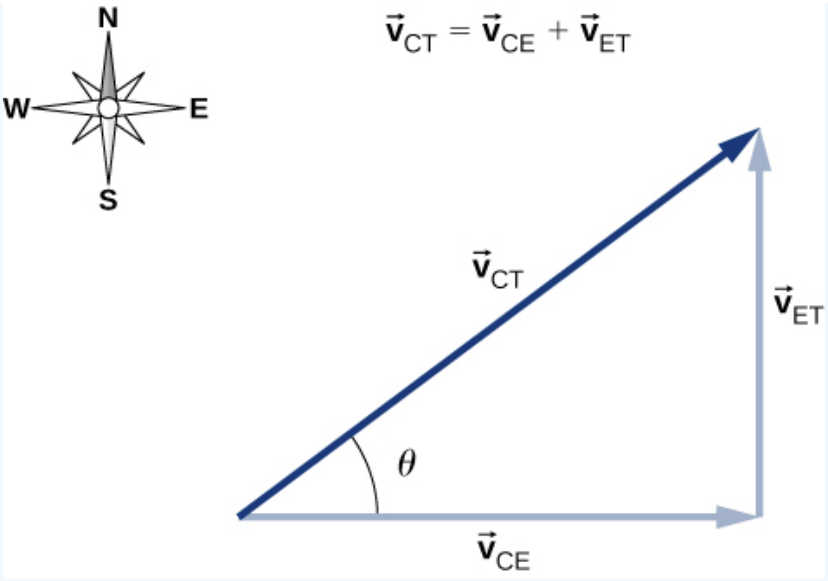


Figure 3.6.5: Vector diagram of the vector Equation 3.6.7.

We can now solve for the velocity of the car with respect to the truck:

$$|\vec{v}_{CT}| = \sqrt{(80.0\ km/h)^2 + (70.0\ km/h)^2} = 106.\ km/h$$

and

$$\theta = \tan^{-1}\left(\frac{70.0}{80.0}\right) = 41.2^o\ north\ of\ east.$$

Significance

Drawing a vector diagram showing the velocity vectors can help in understanding the relative velocity of the two objects.

Exercise 4.6

A boat heads north in still water at 4.5 m/s directly across a river that is running east at 3.0 m/s. What is the velocity of the boat with respect to Earth?

Example 4.14: Flying a Plane in a Wind

A pilot must fly his plane due north to reach his destination. The plane can fly at 300 km/h in still air. A wind is blowing out of the northeast at 90 km/h. (a) What is the speed of the plane relative to the ground? (b) In what direction must the pilot head her plane to fly due north?

Strategy

The pilot must point her plane somewhat east of north to compensate for the wind velocity. We need to construct a vector equation that contains the velocity of the plane with respect to the ground, the velocity of the plane with respect to the air, and the velocity of the air with respect to the ground. Since these last two quantities are known, we can solve for the velocity of the plane with respect to the ground. We can graph the vectors and use this diagram to evaluate the magnitude of the plane's velocity with respect to the ground. The diagram will also tell us the angle the plane's velocity makes with north with respect to the air, which is the direction the pilot must head her plane.

Solution

The vector equation is $\vec{v}_{PG} = \vec{v}_{PA} + \vec{v}_{AG}$, where P = plane, A = air, and G = ground. From the geometry in Figure 3.6.6, we can solve easily for the magnitude of the velocity of the plane with respect to the ground and the angle of the plane's heading, θ.

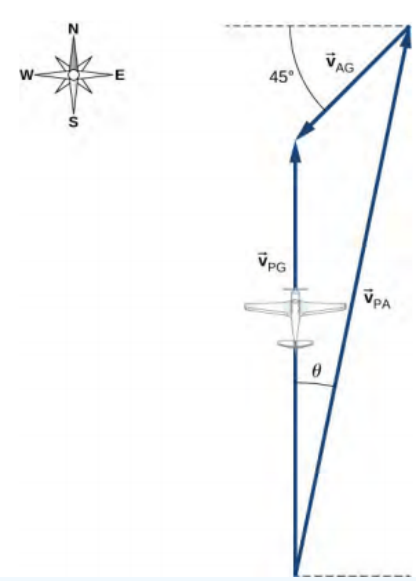


Figure 3.6.6: Vector diagram for Equation 3.6.2 showing the vectors $\vec{v}_{PA}$, $\vec{v}_{AG}$, and $\vec{v}_{PG}$.

a. Known quantities:

$$|\vec{v}_{PA}| = 300\ km/h \tag{3.6.8}$$

$$|\vec{v}_{AG}| = 90\ km/h \tag{3.6.9}$$

Substituting into the equation of motion, we obtain $|\vec{v}_{PG}|$ = 230 km/h.

b. The angle $\theta = \tan^{-1}\left(\frac{63.64}{300}\right)$ = 12° east of north.

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3.7: Motion in Two and Three Dimensions (Exercises)

Conceptual Questions

4.1 Displacement and Velocity Vectors

1. What form does the trajectory of a particle have if the distance from any point A to point B is equal to the magnitude of the displacement from A to B?
2. Give an example of a trajectory in two or three dimensions caused by independent perpendicular motions.
3. If the instantaneous velocity is zero, what can be said about the slope of the position function?

4.2 Acceleration Vector

4. If the position function of a particle is a linear function of time, what can be said about its acceleration?
5. If an object has a constant x-component of the velocity and suddenly experiences an acceleration in the y direction, does the x-component of its velocity change?
6. If an object has a constant x-component of velocity and suddenly experiences an acceleration at an angle of 70° in the x direction, does the x-component of velocity change?

4.3 Projectile Motion

7. Answer the following questions for projectile motion on level ground assuming negligible air resistance, with the initial angle being neither 0° nor 90° : (a) Is the velocity ever zero? (b) When is the velocity a minimum? A maximum? (c) Can the velocity ever be the same as the initial velocity at a time other than at t = 0? (d) Can the speed ever be the same as the initial speed at a time other than at t = 0?
8. Answer the following questions for projectile motion on level ground assuming negligible air resistance, with the initial angle being neither 0° nor 90° : (a) Is the acceleration ever zero? (b) Is the vector $\vec{v}$ ever parallel or antiparallel to the vector $\vec{a}$? (c) Is the vector $\vec{v}$ ever perpendicular to the vector $\vec{a}$? If so, where is this located?
9. A dime is placed at the edge of a table so it hangs over slightly. A quarter is slid horizontally on the table surface perpendicular to the edge and hits the dime head on. Which coin hits the ground first?

4.4 Uniform Circular Motion

10. Can centripetal acceleration change the speed of a particle undergoing circular motion?
11. Can tangential acceleration change the speed of a particle undergoing circular motion?

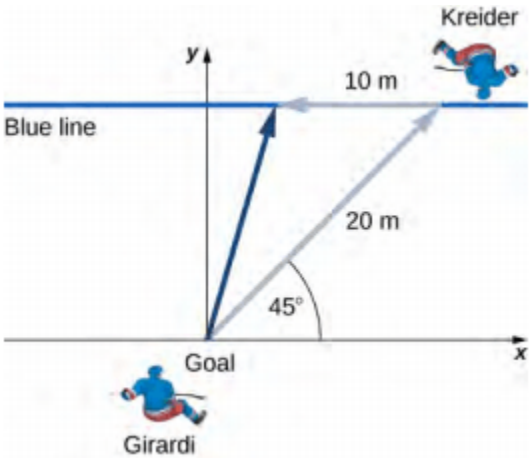
4.5 Relative Motion in One and Two Dimensions

12. What frame or frames of reference do you use instinctively when driving a car? When flying in a commercial jet?
13. A basketball player dribbling down the court usually keeps his eyes fixed on the players around him. He is moving fast. Why doesn't he need to keep his eyes on the ball?
14. If someone is riding in the back of a pickup truck and throws a softball straight backward, is it possible for the ball to fall straight down as viewed by a person standing at the side of the road? Under what condition would this occur? How would the motion of the ball appear to the person who threw it?
15. The hat of a jogger running at constant velocity falls off the back of his head. Draw a sketch showing the path of the hat in the jogger's frame of reference. Draw its path as viewed by a stationary observer. Neglect air resistance.
16. A clod of dirt falls from the bed of a moving truck. It strikes the ground directly below the end of the truck. (a) What is the direction of its velocity relative to the truck just before it hits? (b) Is this the same as the direction of its velocity relative to ground just before it hits? Explain your answers.

Problems

4.1 Displacement and Velocity Vectors

17. The coordinates of a particle in a rectangular coordinate system are (1.0, –4.0, 6.0). What is the position vector of the particle?
18. The position of a particle changes from $\vec{r}_1$ = (2.0 $\hat{i}$ + 3.0 $\hat{j}$)cm to $\vec{r}_2$ = (–4.0 $\hat{i}$ + 3.0 $\hat{j}$) cm. What is the particle's displacement?
19. The 18th hole at Pebble Beach Golf Course is a dogleg to the left of length 496.0 m. The fairway off the tee is taken to be the x direction. A golfer hits his tee shot a distance of 300.0 m, corresponding to a displacement $\Delta\vec{r}_1$ = 300.0 m $\hat{i}$, and hits his second shot 189.0 m with a displacement $\Delta\vec{r}_2$ = 172.0 m $\hat{i}$ + 80.3 m $\hat{j}$. What is the final displacement of the golf ball from the tee?
20. A bird flies straight northeast a distance of 95.0 km for 3.0 h. With the x-axis due east and the y-axis due north, what is the displacement in unit vector notation for the bird? What is the average velocity for the trip?
21. A cyclist rides 5.0 km due east, then 10.0 km 20° west of north. From this point she rides 8.0 km due west. What is the final displacement from where the cyclist started?
22. New York Rangers defenseman Daniel Girardi stands at the goal and passes a hockey puck 20 m and 45° from straight down the ice to left wing Chris Kreider waiting at the blue line. Kreider waits for Girardi to reach the blue line and passes the puck directly across the ice to him 10 m away. What is the final displacement of the puck? See the following figure.



23. The position of a particle is $\vec{r}$(t) = 4.0t^2 $\hat{i}$ – 3.0 $\hat{j}$ + 2.0t^3 $\hat{k}$ m. (a) What is the velocity of the particle at 0 s and at 1.0 s? (b) What is the average velocity between 0 s and 1.0 s?
24. Clay Matthews, a linebacker for the Green Bay Packers, can reach a speed of 10.0 m/s. At the start of a play, Matthews runs downfield at 45° with respect to the 50-yard line and covers 8.0 m in 1 s. He then runs straight down the field at 90° with respect to the 50-yard line for 12 m, with an elapsed time of 1.2 s. (a) What is Matthews' final displacement from the start of the play? (b) What is his average velocity?
25. The F-35B Lighting II is a short-takeoff and vertical landing fighter jet. If it does a vertical takeoff to 20.00-m height above the ground and then follows a flight path angled at 30° with respect to the ground for 20.00 km, what is the final displacement?

4.2 Acceleration Vector

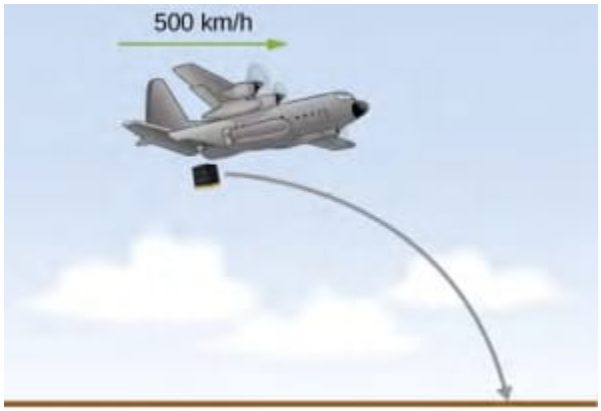
26. The position of a particle is $\vec{r}$ (t) = (3.0 ^{2} $\hat{i}$ + 5.0 $\hat{j}$ – 6.0t $\hat{k}$) m. (a) Determine its velocity and acceleration as functions of time. (b) What are its velocity and acceleration at time t = 0?
27. A particle's acceleration is (4.0 $\hat{i}$ + 3.0 $\hat{j}$)m/s^2. At t = 0, its position and velocity are zero. (a) What are the particle's position and velocity as functions of time? (b) Find the equation of the path of the particle. Draw the x- and y-axes and sketch the trajectory of the particle.
28. A boat leaves the dock at t = 0 and heads out into a lake with an acceleration of 2.0 m/s^2 $\hat{i}$. A strong wind is pushing

the boat, giving it an additional velocity of 2.0 m/s $\hat{i}$ + 1.0 m/s $\hat{j}$. (a) What is the velocity of the boat at t = 10 s? (b) What is the position of the boat at t = 10s? Draw a sketch of the boat's trajectory and position at t = 10 s, showing the x- and y-axes.

29. The position of a particle for t > 0 is given by $\vec{r}(t) = (3.0t^2\,\hat{i} - 7.0t^3\,\hat{j} - 5.0t^{-2}\,\hat{k})$ m. (a) What is the velocity as a function of time? (b) What is the acceleration as a function of time? (c) What is the particle's velocity at t = 2.0 s? (d) What is its speed at t = 1.0 s and t = 3.0 s? (e) What is the average velocity between t = 1.0 s and t = 2.0 s?
30. The acceleration of a particle is a constant. At t = 0 the velocity of the particle is (10 $\hat{i}$ + 20 $\hat{j}$)m/s. At t = 4 s the velocity is 10 $\hat{j}$ m/s. (a) What is the particle's acceleration? (b) How do the position and velocity vary with time? Assume the particle is initially at the origin.
31. A particle has a position function $\vec{r}(t) = \cos(1.0t)\,\hat{i} + \sin(1.0t)\,\hat{j} + t\,\hat{k}$, where the arguments of the cosine and sine functions are in radians. (a) What is the velocity vector? (b) What is the acceleration vector?
32. A Lockheed Martin F-35 II Lighting jet takes off from an aircraft carrier with a runway length of 90 m and a takeoff speed 70 m/s at the end of the runway. Jets are catapulted into airspace from the deck of an aircraft carrier with two sources of propulsion: the jet propulsion and the catapult. At the point of leaving the deck of the aircraft carrier, the F-35's acceleration decreases to a constant acceleration of 5.0 m/s^2 at 30° with respect to the horizontal. (a) What is the initial acceleration of the F-35 on the deck of the aircraft carrier to make it airborne? (b) Write the position and velocity of the F-35 in unit vector notation from the point it leaves the deck of the aircraft carrier. (c) At what altitude is the fighter 5.0 s after it leaves the deck of the aircraft carrier? (d) What is its velocity and speed at this time? (e) How far has it traveled horizontally?

4.3 Projectile Motion

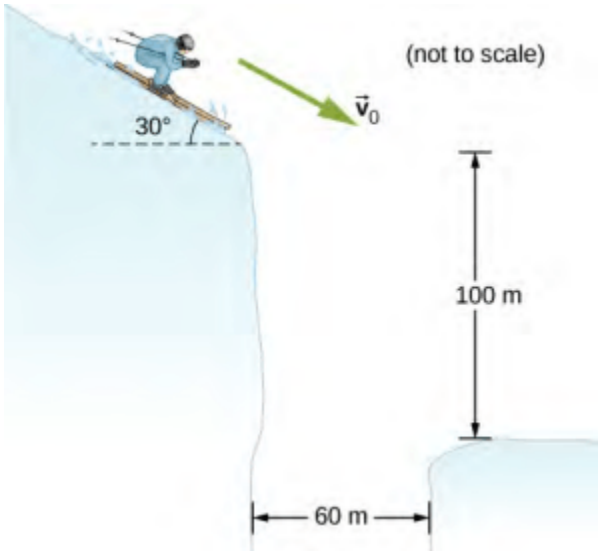
33. A bullet is shot horizontally from shoulder height (1.5 m) with and initial speed 200 m/s. (a) How much time elapses before the bullet hits the ground? (b) How far does the bullet travel horizontally?
34. A marble rolls off a tabletop 1.0 m high and hits the floor at a point 3.0 m away from the table's edge in the horizontal direction. (a) How long is the marble in the air? (b) What is the speed of the marble when it leaves the table's edge? (c) What is its speed when it hits the floor?
35. A dart is thrown horizontally at a speed of 10 m/ s at the bull's-eye of a dartboard 2.4 m away, as in the following figure. (a) How far below the intended target does the dart hit? (b) What does your answer tell you about how proficient dart players throw their darts?
36. An airplane flying horizontally with a speed of 500 km/h at a height of 800 m drops a crate of supplies (see the following figure). If the parachute fails to open, how far in front of the release point does the crate hit the ground?



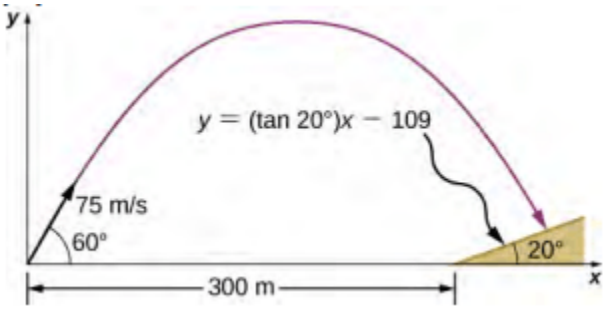
37. Suppose the airplane in the preceding problem fires a projectile horizontally in its direction of motion at a speed of 300 m/s relative to the plane. (a) How far in front of the release point does the projectile hit the ground? (b) What is its speed when it hits the ground?
38. A fastball pitcher can throw a baseball at a speed of 40 m/s (90 mi/h). (a) Assuming the pitcher can release the ball 16.7 m from home plate so the ball is moving horizontally, how long does it take the ball to reach home plate? (b) How far does the ball drop between the pitcher's hand and home plate?
39. A projectile is launched at an angle of 30° and lands 20 s later at the same height as it was launched. (a) What is the initial speed of the projectile? (b) What is the maximum altitude? (c) What is the range? (d) Calculate the displacement from the point of launch to the position on its trajectory at 15 s.
40. A basketball player shoots toward a basket 6.1 m away and 3.0 m above the floor. If the ball is released 1.8 m above the

floor at an angle of 60° above the horizontal, what must the initial speed be if it were to go through the basket?

41. At a particular instant, a hot air balloon is 100 m in the air and descending at a constant speed of 2.0 m/s. At this exact instant, a girl throws a ball horizontally, relative to herself, with an initial speed of 20 m/s. When she lands, where will she find the ball? Ignore air resistance.
42. A man on a motorcycle traveling at a uniform speed of 10 m/s throws an empty can straight upward relative to himself with an initial speed of 3.0 m/s. Find the equation of the trajectory as seen by a police officer on the side of the road. Assume the initial position of the can is the point where it is thrown. Ignore air resistance.
43. An athlete can jump a distance of 8.0 m in the broad jump. What is the maximum distance the athlete can jump on the Moon, where the gravitational acceleration is onesixth that of Earth?
44. The maximum horizontal distance a boy can throw a ball is 50 m. Assume he can throw with the same initial speed at all angles. How high does he throw the ball when he throws it straight upward?
45. A rock is thrown off a cliff at an angle of 53° with respect to the horizontal. The cliff is 100 m high. The initial speed of the rock is 30 m/s. (a) How high above the edge of the cliff does the rock rise? (b) How far has it moved horizontally when it is at maximum altitude? (c) How long after the release does it hit the ground? (d) What is the range of the rock? (e) What are the horizontal and vertical positions of the rock relative to the edge of the cliff at t = 2.0 s, t = 4.0 s, and t = 6.0 s?
46. Trying to escape his pursuers, a secret agent skis off a slope inclined at 30° below the horizontal at 60 km/h. To survive and land on the snow 100 m below, he must clear a gorge 60 m wide. Does he make it? Ignore air resistance.



47. A golfer on a fairway is 70 m away from the green, which sits below the level of the fairway by 20 m. If the golfer hits the ball at an angle of 40° with an initial speed of 20 m/s, how close to the green does she come?
48. A projectile is shot at a hill, the base of which is 300 m away. The projectile is shot at 60° above the horizontal with an initial speed of 75 m/s. The hill can be approximated by a plane sloped at 20° to the horizontal. Relative to the coordinate system shown in the following figure, the equation of this straight line is y = (tan 20°)x – 109. Where on the hill does the projectile land?



49. An astronaut on Mars kicks a soccer ball at an angle of 45° with an initial velocity of 15 m/s. If the acceleration of gravity on Mars is 3.7 m/s, (a) what is the range of the soccer kick on a flat surface? (b) What would be the range of the

same kick on the Moon, where gravity is one-sixth that of Earth?

50. Mike Powell holds the record for the long jump of 8.95 m, established in 1991. If he left the ground at an angle of 15°, what was his initial speed?
51. MIT's robot cheetah can jump over obstacles 46 cm high and has speed of 12.0 km/h. (a) If the robot launches itself at an angle of 60° at this speed, what is its maximum height? (b) What would the launch angle have to be to reach a height of 46 cm?
52. Mt. Asama, Japan, is an active volcano. In 2009, an eruption threw solid volcanic rocks that landed 1 km horizontally from the crater. If the volcanic rocks were launched at an angle of 40° with respect to the horizontal and landed 900 m below the crater, (a) what would be their initial velocity and (b) what is their time of flight?
53. Drew Brees of the New Orleans Saints can throw a football 23.0 m/s (50 mph). If he angles the throw at 10° from the horizontal, what distance does it go if it is to be caught at the same elevation as it was thrown?
54. The Lunar Roving Vehicle used in NASA's late Apollo missions reached an unofficial lunar land speed of 5.0 m/s by astronaut Eugene Cernan. If the rover was moving at this speed on a flat lunar surface and hit a small bump that projected it off the surface at an angle of 20°, how long would it be "airborne" on the Moon?
55. A soccer goal is 2.44 m high. A player kicks the ball at a distance 10 m from the goal at an angle of 25°. What is the initial speed of the soccer ball?
56. Olympus Mons on Mars is the largest volcano in the solar system, at a height of 25 km and with a radius of 312 km. If you are standing on the summit, with what initial velocity would you have to fire a projectile from a cannon horizontally to clear the volcano and land on the surface of Mars? Note that Mars has an acceleration of gravity of 3.7 m/s^2.
57. In 1999, Robbie Knievel was the first to jump the Grand Canyon on a motorcycle. At a narrow part of the canyon (69.0 m wide) and traveling 35.8 m/s off the takeoff ramp, he reached the other side. What was his launch angle?
58. You throw a baseball at an initial speed of 15.0 m/s at an angle of 30° with respect to the horizontal. What would the ball's initial speed have to be at 30° on a planet that has twice the acceleration of gravity as Earth to achieve the same range? Consider launch and impact on a horizontal surface.
59. Aaron Rogers throws a football at 20.0 m/s to his wide receiver, who is running straight down the field at 9.4 m/s. If Aaron throws the football when the wide receiver is 10.0 m in front of him, what angle does Aaron have to launch the ball at so the receiver catches it 20.0 m in front of Aaron?

4.4 Uniform Circular Motion

60. A flywheel is rotating at 30 rev/s. What is the total angle, in radians, through which a point on the flywheel rotates in 40 s?
61. A particle travels in a circle of radius 10 m at a constant speed of 20 m/s. What is the magnitude of the acceleration?
62. Cam Newton of the Carolina Panthers throws a perfect football spiral at 8.0 rev/s. The radius of a pro football is 8.5 cm at the middle of the short side. What is the centripetal acceleration of the laces on the football?
63. A fairground ride spins its occupants inside a flying saucer-shaped container. If the horizontal circular path the riders follow has an 8.00-m radius, at how many revolutions per minute are the riders subjected to a centripetal acceleration equal to that of gravity?
64. A runner taking part in the 200-m dash must run around the end of a track that has a circular arc with a radius of curvature of 30.0 m. The runner starts the race at a constant speed. If she completes the 200-m dash in 23.2 s and runs at constant speed throughout the race, what is her centripetal acceleration as she runs the curved portion of the track?
65. What is the acceleration of Venus toward the Sun, assuming a circular orbit?
66. An experimental jet rocket travels around Earth along its equator just above its surface. At what speed must the jet travel if the magnitude of its acceleration is g?
67. A fan is rotating at a constant 3600 rev/min. What is the magnitude of the acceleration of a point on one of its blades 10.0 cm from the axis of rotation?
68. A point located on the second hand of a large clock has a radial acceleration of 0.1 cm/s^2. How far is the point from the axis of rotation of the second hand?

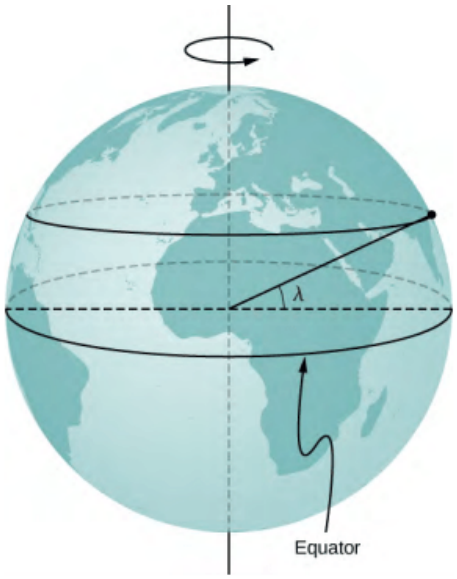
4.5 Relative Motion in One and Two Dimensions

69. The coordinate axes of the reference frame S′ remain parallel to those of S, as S′ moves away from S at a constant velocity $\vec{v}_{S'}$ = (4.0 $\hat{i}$ + 3.0 $\hat{j}$ + 5.0 $\hat{k}$) m/s. (a) If at time t = 0 the origins coincide, what is the position of the origin O′ in the S frame as a function of time? (b) How is particle position for $\vec{r}$(t) and $\vec{r}\,'$(t), as measured in S and S′, respectively, related? (c) What is the relationship between particle velocities $\vec{v}$(t) and $\vec{v}\,'$(t)? (d) How are accelerations $\vec{a}$(t) and $\vec{a}\,'$(t) related?

70. The coordinate axes of the reference frame S′ remain parallel to those of S, as S′ moves away from S at a constant velocity $\vec{v}_{S'S}$ = (1.0 $\hat{i}$ + 2.0 $\hat{j}$ + 3.0 $\hat{k}$)t m/s. (a) If at time t = 0 the origins coincide, what is the position of origin O′ in the S frame as a function of time? (b) How is particle position for $\vec{r}$(t) and $\vec{r}\,'$(t) , as measured in S and S′, respectively, related? (c) What is the relationship between particle velocities $\vec{v}$(t) and $\vec{v}\,'$(t)? (d) How are accelerations $\vec{a}$(t) and $\vec{a}\,'$(t) related?
71. The velocity of a particle in reference frame A is (2.0 $\hat{i}$ + 3.0 $\hat{j}$) m/s. The velocity of reference frame A with respect to reference frame B is 4.0 $\hat{k}$ m/s, and the velocity of reference frame B with respect to C is 2.0 $\hat{j}$ m/s. What is the velocity of the particle in reference frame C?
72. Raindrops fall vertically at 4.5 m/s relative to the earth. What does an observer in a car moving at 22.0 m/s in a straight line measure as the velocity of the raindrops?
73. A seagull can fly at a velocity of 9.00 m/s in still air. (a) If it takes the bird 20.0 min to travel 6.00 km straight into an oncoming wind, what is the velocity of the wind? (b) If the bird turns around and flies with the wind, how long will it take the bird to return 6.00 km?
74. A ship sets sail from Rotterdam, heading due north at 7.00 m/s relative to the water. The local ocean current is 1.50 m/s in a direction 40.0° north of east. What is the velocity of the ship relative to Earth?
75. A boat can be rowed at 8.0 km/h in still water. (a) How much time is required to row 1.5 km downstream in a river moving 3.0 km/h relative to the shore? (b) How much time is required for the return trip? (c) In what direction must the boat be aimed to row straight across the river? (d) Suppose the river is 0.8 km wide. What is the velocity of the boat with respect to Earth and how much time is required to get to the opposite shore? (e) Suppose, instead, the boat is aimed straight across the river. How much time is required to get across and how far downstream is the boat when it reaches the opposite shore?
76. A small plane flies at 200 km/h in still air. If the wind blows directly out of the west at 50 km/h, (a) in what direction must the pilot head her plane to move directly north across land and (b) how long does it take her to reach a point 300 km directly north of her starting point?
77. A cyclist traveling southeast along a road at 15 km/h feels a wind blowing from the southwest at 25 km/h. To a stationary observer, what are the speed and direction of the wind?
78. A river is moving east at 4 m/s. A boat starts from the dock heading 30° north of west at 7 m/s. If the river is 1800 m wide, (a) what is the velocity of the boat with respect to Earth and (b) how long does it take the boat to cross the river?

Additional Problems

79. A Formula One race car is traveling at 89.0 m/s along a straight track enters a turn on the race track with radius of curvature of 200.0 m. What centripetal acceleration must the car have to stay on the track?
80. A particle travels in a circular orbit of radius 10 m. Its speed is changing at a rate of 15.0 m/s^2 at an instant when its speed is 40.0 m/s. What is the magnitude of the acceleration of the particle?
81. The driver of a car moving at 90.0 km/h presses down on the brake as the car enters a circular curve of radius 150.0 m. If the speed of the car is decreasing at a rate of 9.0 km/h each second, what is the magnitude of the acceleration of the car at the instant its speed is 60.0 km/h?
82. A race car entering the curved part of the track at the Daytona 500 drops its speed from 85.0 m/s to 80.0 m/s in 2.0 s. If the radius of the curved part of the track is 316.0 m, calculate the total acceleration of the race car at the beginning and ending of reduction of speed.
83. An elephant is located on Earth's surface at a latitude λ. Calculate the centripetal acceleration of the elephant resulting from the rotation of Earth around its polar axis. Express your answer in terms of λ, the radius RE of Earth, and time T for one rotation of Earth. Compare your answer with g for λ = 40°.

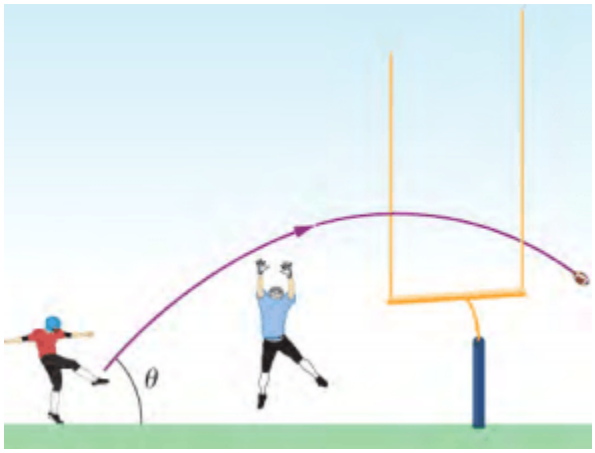


84. A proton in a synchrotron is moving in a circle of radius 1 km and increasing its speed by $v(t) = c_1 + c_2t^2$, where $c_1 = 2.0 \times 10^5$ m/s, $c_2 = 10^5$ m/s^3. (a) What is the proton's total acceleration at t = 5.0 s? (b) At what time does the expression for the velocity become unphysical?
85. A propeller blade at rest starts to rotate from t = 0 s to t = 5.0 s with a tangential acceleration of the tip of the blade at 3.00 m/s^2. The tip of the blade is 1.5 m from the axis of rotation. At t = 5.0 s, what is the total acceleration of the tip of the blade?
86. A particle is executing circular motion with a constant angular frequency of ω = 4.00 rad/s. If time t = 0 corresponds to the position of the particle being located at y = 0 m and x = 5 m, (a) what is the position of the particle at t = 10 s? (b) What is its velocity at this time? (c) What is its acceleration?
87. A particle's centripetal acceleration is $a_C = 4.0$ m/s^2 at t = 0 s where it is on the x-axis and moving counterclockwise in the xy plane. It is executing uniform circular motion about an axis at a distance of 5.0 m. What is its velocity at t = 10 s?
88. A rod 3.0 m in length is rotating at 2.0 rev/s about an axis at one end. Compare the centripetal accelerations at radii of (a) 1.0 m, (b) 2.0 m, and (c) 3.0 m.
89. A particle located initially at (1.5 $\hat{j}$ + 4.0 $\hat{k}$)m undergoes a displacement of (2.5 $\hat{i}$ + 3.2 $\hat{j}$ – 1.2 $\hat{k}$) m. What is the final position of the particle?
90. The position of a particle is given by $\vec{r}(t)$ = (50 m/s)t $\hat{i}$ – (4.9 m/s^2)t^2 $\hat{j}$. (a) What are the particle's velocity and acceleration as functions of time? (b) What are the initial conditions to produce the motion?
91. A spaceship is traveling at a constant velocity of $\vec{v}(t)$ = 250.0 $\hat{i}$ m/s when its rockets fire, giving it an acceleration of $\vec{a}$ (t) = (3.0 $\vec{i}$ + 4.0 $\hat{k}$)m/s^2. What is its velocity 5 s after the rockets fire?
92. A crossbow is aimed horizontally at a target 40 m away. The arrow hits 30 cm below the spot at which it was aimed. What is the initial velocity of the arrow?
93. A long jumper can jump a distance of 8.0 m when he takes off at an angle of 45° with respect to the horizontal. Assuming he can jump with the same initial speed at all angles, how much distance does he lose by taking off at 30°?
94. On planet Arcon, the maximum horizontal range of a projectile launched at 10 m/s is 20 m. What is the acceleration of gravity on this planet?
95. A mountain biker encounters a jump on a race course that sends him into the air at 60° to the horizontal. If he lands at a horizontal distance of 45.0 m and 20 m below his launch point, what is his initial speed?
96. Which has the greater centripetal acceleration, a car with a speed of 15.0 m/s along a circular track of radius 100.0 m or a car with a speed of 12.0 m/s along a circular track of radius 75.0 m?
97. A geosynchronous satellite orbits Earth at a distance of 42,250.0 km and has a period of 1 day. What is the centripetal acceleration of the satellite?
98. Two speedboats are traveling at the same speed relative to the water in opposite directions in a moving river. An observer on the riverbank sees the boats moving at 4.0 m/s and 5.0 m/s. (a) What is the speed of the boats relative to the river? (b) How fast is the river moving relative to the shore?

Challenge Problems

99. World's Longest Par 3. The tee of the world's longest par 3 sits atop South Africa's Hanglip Mountain at 400.0 m above the green and can only be reached by helicopter. The horizontal distance to the green is 359.0 m. Neglect air resistance and answer the following questions. (a) If a golfer launches a shot that is 40° with respect to the horizontal, what initial velocity must she give the ball? (b) What is the time to reach the green?

100. When a field goal kicker kicks a football as hard as he can at 45° to the horizontal, the ball just clears the 3-m-high crossbar of the goalposts 45.7 m away. (a) What is the maximum speed the kicker can impart to the football? (b) In addition to clearing the crossbar, the football must be high enough in the air early during its flight to clear the reach of the onrushing defensive lineman. If the lineman is 4.6 m away and has a vertical reach of 2.5 m, can he block the 45.7-m field goal attempt? (c) What if the lineman is 1.0 m away?



101. A truck is traveling east at 80 km/h. At an intersection 32 km ahead, a car is traveling north at 50 km/h. (a) How long after this moment will the vehicles be closest to each other? (b) How far apart will they be at that point?

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3.8: Motion in Two and Three Dimensions (Summary)

Key Terms

acceleration vector	instantaneous acceleration found by taking the derivative of the velocity function with respect to time in unit vector notation
angular frequency	ω, rate of change of an angle with which an object that is moving on a circular path
centripetal acceleration	component of acceleration of an object moving in a circle that is directed radially inward toward the center of the circle
displacement vector	vector from the initial position to a final position on a trajectory of a particle
position vector	vector from the origin of a chosen coordinate system to the position of a particle in two- or threedimensional space
projectile motion	motion of an object subject only to the acceleration of gravity
range	maximum horizontal distance a projectile travels
reference frame	coordinate system in which the position, velocity, and acceleration of an object at rest or moving is measured
relative velocity	velocity of an object as observed from a particular reference frame, or the velocity of one reference frame with respect to another reference frame
tangential accleration	magnitude of which is the time rate of change of speed. Its direction is tangent to the circle.
time of flight	elapsed time a projectile is in the air
total accleration	vector sum of centripetal and tangential accelerations
trajectory	path of a projectile through the air
velocity vector	vector that gives the instantaneous speed and direction of a particle; tangent to the trajectory

Key Equations

Position vector	$\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j} + z(t)\hat{k}$
Displacement vector	$\Delta\vec{r} = \vec{r}(t_2) - \vec{r}(t_1)$
Velocity vector	$\vec{v}(t) = \lim_{\Delta t \to 0} \frac{\vec{r}(t+\Delta t) - \vec{r}(t)}{\Delta t} = \frac{d\vec{r}}{dt}$
Velocity in terms of components	$\vec{v}(t) = v_x(t)\hat{i} + v_y(t)\hat{j} + v_z(t)\hat{k}$
Velocity components	$v_x(t) = \frac{dx(t)}{dt} v_y(t) = \frac{dy(t)}{dt} v_z(t) = \frac{dz(t)}{dt}$

Average velocity	$\vec{v}_{avg} = \frac{\vec{r}(t_2) - \vec{r}(t_1)}{t_2 - t_1}$
Instantaneous acceleration	$\vec{a}(t) = \lim_{\Delta t \to 0} \frac{\vec{v}(t + \Delta t) - \vec{v}(t)}{\Delta t} = \frac{d\vec{v}}{dt}$
Instantaneous acceleration, component form	$\vec{a}(t) = \frac{dv_x(t)}{dt}\hat{i} + \frac{dv_y(t)}{dt}\hat{j} + \frac{dv_z(t)}{dt}\hat{k}$
Instantaneous acceleration as second derivatives of position	$\vec{a}(t) = \frac{d^2x(t)}{dt^2}\hat{i} + \frac{d^2y(t)}{dt^2} + \frac{d^2z(t)}{dt^2}\hat{k}$
Time of flight	$T_{tof} = \frac{2(v_0 \sin\theta)}{g}$
Trajectory	$y = (\tan\theta_0)x - \left[\frac{g}{2(v_0 \cos\theta_0)^2}\right]x^2$
Range	$R = \frac{v_0^2 \sin 2\theta_0}{g}$
Centripetal accleration	$a_C = \frac{v^2}{r}$
Position vector, uniform circular motion	$\vec{r}(t) = A\cos\omega t\hat{i} + A\sin\omega t\hat{j}$
Velocity vector, uniform circular motion	$\vec{v}(t) = \frac{d\vec{r}(t)}{dt} = -A\omega\sin\omega t\hat{i} + A\omega\cos\omega t\hat{j}$
Acceleration vector, uniform circular motion	$\vec{a}(t) = \frac{d\vec{v}(t)}{dt} = -A\omega^2\cos\omega t\hat{i} - A\omega^2\sin\omega t\hat{j}$
Tangential acceleration	$a_T = \frac{d\lvert\vec{v}\rvert}{dt}$
Total acceleration	$\vec{a} = \vec{a}_C + \vec{a}_T$

Position vector in frame S is the position vector in frame S′ plus the vector from the origin of S to the origin of S′	$\vec{r}_{PS} = \vec{r}_{PS'} + \vec{r}_{S'S}$
Relative velocity equation connecting two reference frames	$\vec{v}_{PS} = \vec{v}_{PS'} + \vec{v}_{S'S}$
Relative velocity equation connecting more than two reference frames	$\vec{v}_{PC} = \vec{v}_{PA} + \vec{v}_{AB} + \vec{v}_{BC}$
Relative acceleration equation	$\vec{a}_{PS} = \vec{a}_{PS'} + \vec{a}_{S'S}$

Summary

4.1 Displacement and Velocity Vectors

- The position function $\vec{r}$(t) gives the position as a function of time of a particle moving in two or three dimensions. Graphically, it is a vector from the origin of a chosen coordinate system to the point where the particle is located at a specific time.
- The displacement vector $\Delta\vec{r}$ gives the shortest distance between any two points on the trajectory of a particle in two or three dimensions.
- Instantaneous velocity gives the speed and direction of a particle at a specific time on its trajectory in two or three dimensions, and is a vector in two and three dimensions.
- The velocity vector is tangent to the trajectory of the particle.
- Displacement $\vec{r}$(t) can be written as a vector sum of the one-dimensional displacements $\vec{x}$(t), $\vec{y}$(t), $\vec{z}$(t) along the x, y, and z directions.
- Velocity $\vec{v}$(t) can be written as a vector sum of the one-dimensional velocities v_x(t), v_y(t), v_z(t) along the x, y, and z directions.
- Motion in any given direction is independent of motion in a perpendicular direction.

4.2 Acceleration Vector

- In two and three dimensions, the acceleration vector can have an arbitrary direction and does not necessarily point along a given component of the velocity.
- The instantaneous acceleration is produced by a change in velocity taken over a very short (infinitesimal) time period. Instantaneous acceleration is a vector in two or three dimensions. It is found by taking the derivative of the velocity function with respect to time.
- In three dimensions, acceleration $\vec{a}$(t) can be written as a vector sum of the one-dimensional accelerations a_x(t), a_y(t), and a_z(t) along the x-, y-, and z-axes.
- The kinematic equations for constant acceleration can be written as the vector sum of the constant acceleration equations

in the x, y, and z directions.

4.3 Projectile Motion

- Projectile motion is the motion of an object subject only to the acceleration of gravity, where the acceleration is constant, as near the surface of Earth.
- To solve projectile motion problems, we analyze the motion of the projectile in the horizontal and vertical directions using the one-dimensional kinematic equations for x and y.
- The time of flight of a projectile launched with initial vertical velocity v_{0y} on an even surface is given by

$$T_{tof} = \frac{2(v_0 \sin\theta)}{g} \tag{3.8.23}$$

  This equation is valid only when the projectile lands at the same elevation from which it was launched.
- The maximum horizontal distance traveled by a projectile is called the range. Again, the equation for range is valid only when the projectile lands at the same elevation from which it was launched.

4.4 Uniform Circular Motion

- Uniform circular motion is motion in a circle at constant speed.
- Centripetal acceleration $\vec{a}_C$ is the acceleration a particle must have to follow a circular path. Centripetal acceleration always points toward the center of rotation and has magnitude $a_C = \frac{v^2}{r}$.
- Nonuniform circular motion occurs when there is tangential acceleration of an object executing circular motion such that the speed of the object is changing. This acceleration is called tangential acceleration $\vec{a}_T$. The magnitude of tangential acceleration is the time rate of change of the magnitude of the velocity. The tangential acceleration vector is tangential to the circle, whereas the centripetal acceleration vector points radially inward toward the center of the circle. The total acceleration is the vector sum of tangential and centripetal accelerations.
- An object executing uniform circular motion can be described with equations of motion. The position vector of the object is $\vec{r}(t) = A\cos\omega t\,\hat{i} + A\sin\omega t\,\hat{j}$, where A is the magnitude $|\vec{r}(t)|$, which is also the radius of the circle, and ω is the angular frequency.

4.5 Relative Motion in One and Two Dimensions

- When analyzing motion of an object, the reference frame in terms of position, velocity, and acceleration needs to be specified.
- Relative velocity is the velocity of an object as observed from a particular reference frame, and it varies with the choice of reference frame.
- If S and S′ are two reference frames moving relative to each other at a constant velocity, then the velocity of an object relative to S is equal to its velocity relative to S′ plus the velocity of S′ relative to S.
- If two reference frames are moving relative to each other at a constant velocity, then the accelerations of an object as observed in both reference frames are equal.

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4: PHYSICS 1- REVIEW NEWTON'S LAWS OF MOTION

4.1: Prelude to Newton's Laws of Motion

4.2: Forces

4.3: Newton's First Law

4.4: Newton's Second Law

4.5: Mass and Weight

4.6: Newton’s Third Law

4.7: Common Forces

4.8: Drawing Free-Body Diagrams

4.9: Newton's Laws of Motion (Exercises)

4.10: Newton's Laws of Motion (Summary)

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4.1: Prelude to Newton's Laws of Motion



Figure 4.1.1: The Golden Gate Bridge, one of the greatest works of modern engineering, was the longest suspension bridge in the world in the year it opened, 1937. It is still among the 10 longest suspension bridges as of this writing. In designing and building a bridge, what physics must we consider? What forces act on the bridge? What forces keep the bridge from falling? How do the towers, cables, and ground interact to maintain stability?

When you drive across a bridge, you expect it to remain stable. You also expect to speed up or slow your car in response to traffic changes. In both cases, you deal with forces. The forces on the bridge are in equilibrium, so it stays in place. In contrast, the force produced by your car engine causes a change in motion. Isaac Newton discovered the laws of motion that describe these situations.

Forces affect every moment of your life. Your body is held to Earth by force and held together by the forces of charged particles. When you open a door, walk down a street, lift your fork, or touch a baby's face, you are applying forces. Zooming in deeper, your body's atoms are held together by electrical forces, and the core of the atom, called the nucleus, is held together by the strongest force we know—strong nuclear force.

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4.2: Forces

Learning Objectives

- Distinguish between kinematics and dynamics
- Understand the definition of force
- Identify simple free-body diagrams
- Define the SI unit of force, the newton
- Describe force as a vector

The study of motion is called **kinematics**, but kinematics only describes the way objects move—their velocity and their acceleration. **Dynamics** is the study of how forces affect the motion of objects and systems. It considers the causes of motion of objects and systems of interest, where a system is anything being analyzed. The foundation of dynamics are the laws of motion stated by Isaac Newton (1642–1727). These laws provide an example of the breadth and simplicity of principles under which nature functions. They are also universal laws in that they apply to situations on Earth and in space.

Newton's laws of motion were just one part of the monumental work that has made him legendary (Figure 4.2.1). The development of Newton's laws marks the transition from the Renaissance to the modern era. Not until the advent of modern physics was it discovered that Newton's laws produce a good description of motion only when the objects are moving at speeds much less than the speed of light and when those objects are larger than the size of most molecules (about 10^{-9} m in diameter). These constraints define the realm of Newtonian mechanics. At the beginning of the twentieth century, Albert Einstein (1879–1955) developed the theory of relativity and, along with many other scientists, quantum mechanics. Quantum mechanics does not have the constraints present in Newtonian physics. All of the situations we consider in this chapter, and all those preceding the introduction of relativity in Relativity, are in the realm of Newtonian physics.

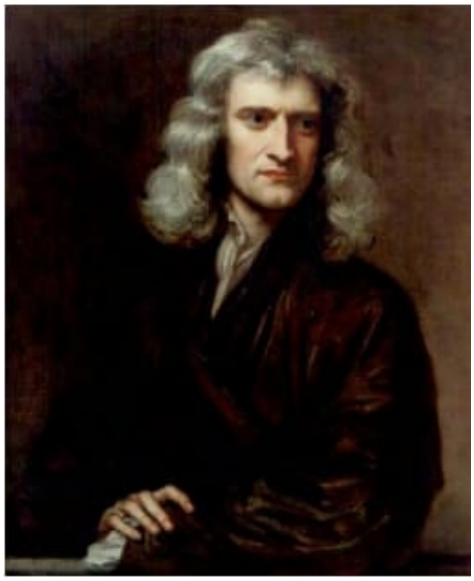


Figure 4.2.1: Isaac Newton (1642–1727) published his amazing work, Philosophiae Naturalis Principia Mathematica, in 1687. It proposed scientific laws that still apply today to describe the motion of objects (the laws of motion). Newton also discovered the law of gravity, invented calculus, and made great contributions to the theories of light and color.

Working Definition of Force

Dynamics is the study of the forces that cause objects and systems to move. To understand this, we need a working definition of force. An intuitive definition of **force**—that is, a push or a pull—is a good place to start. We know that a push or a pull has both magnitude and direction (therefore, it is a vector quantity), so we can define force as the push or pull on an object with a specific magnitude and direction. Force can be represented by vectors or expressed as a multiple of a standard force.

The push or pull on an object can vary considerably in either magnitude or direction. For example, a cannon exerts a strong force on a cannonball that is launched into the air. In contrast, Earth exerts only a tiny downward pull on a flea. Our everyday experiences also give us a good idea of how multiple forces add. If two people push in different directions on a third person, as illustrated in Figure 4.2.2, we might expect the total force to be in the direction shown. Since force is a vector, it adds just like other vectors. Forces, like other vectors, are represented by arrows and can be added using the familiar head-to-tail method or trigonometric methods. These ideas were developed in Vectors.

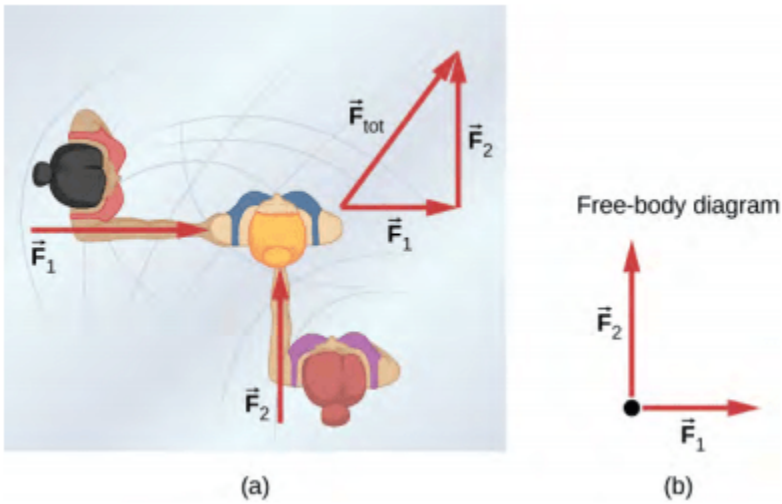


Figure 4.2.2: (a) An overhead view of two ice skaters pushing on a third skater. Forces are vectors and add like other vectors, so the total force on the third skater is in the direction shown. (b) A free-body diagram representing the forces acting on the third skater.

Figure 4.2.2(b) is our first example of a **free-body diagram**, which is a sketch showing all external forces acting on an object or system. The object or system is represented by a single isolated point (or free body), and only those forces acting on it that originate outside of the object or system—that is, **external forces**—are shown. (These forces are the only ones shown because only external forces acting on the free body affect its motion. We can ignore any internal forces within the body.) The forces are represented by vectors extending outward from the free body.

Free-body diagrams are useful in analyzing forces acting on an object or system, and are employed extensively in the study and application of Newton's laws of motion. You will see them throughout this text and in all your studies of physics. The following steps briefly explain how a free-body diagram is created; we examine this strategy in more detail in Drawing Free-Body Diagrams.

Problem-Solving Strategy: Drawing Free-Body Diagrams

1. Draw the object under consideration. If you are treating the object as a particle, represent the object as a point. Place this point at the origin of an xy-coordinate system.
2. Include all forces that act on the object, representing these forces as vectors. However, do not include the net force on the object or the forces that the object exerts on its environment.
3. Resolve all force vectors into x- and y-components.
4. Draw a separate free-body diagram for each object in the problem.

We illustrate this strategy with two examples of free-body diagrams (Figure 4.2.3). The terms used in this figure are explained in more detail later in the chapter.

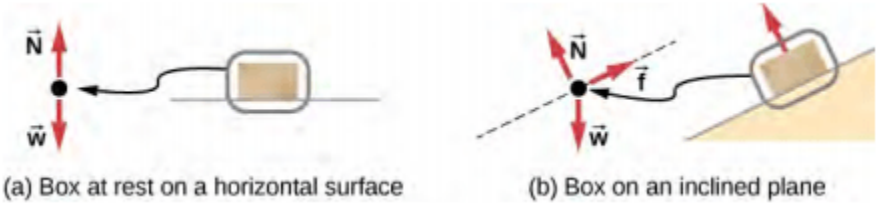


Figure 4.2.3: In these free-body diagrams, $\vec{N}$ is the normal force, $\vec{w}$ is the weight of the object, and $\vec{f}$ is the friction.

The steps given here are sufficient to guide you in this important problem-solving strategy. The final section of this chapter explains in more detail how to draw free-body diagrams when working with the ideas presented in this chapter.

Development of the Force Concept

A quantitative definition of force can be based on some standard force, just as distance is measured in units relative to a standard length. One possibility is to stretch a spring a certain fixed distance (Figure 4.2.4) and use the force it exerts to pull itself back to its relaxed shape—called a **restoring force**—as a standard. The magnitude of all other forces can be considered as multiples of this standard unit of force. Many other possibilities exist for standard forces. Some alternative definitions of force will be given later in this chapter.

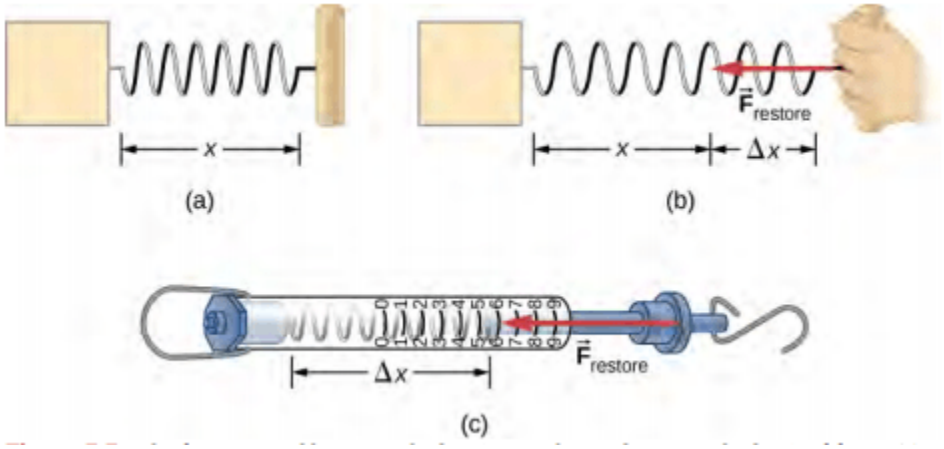


Figure 4.2.4: The force exerted by a stretched spring can be used as a standard unit of force. (a) This spring has a length x when undistorted. (b) When stretched a distance Δx , the spring exerts a restoring force $\vec{F}$ restore, which is reproducible. (c) A spring scale is one device that uses a spring to measure force. The force $\vec{F}$ restore is exerted on whatever is attached to the hook. Here, this force has a magnitude of six units of the force standard being employed.

Let's analyze force more deeply. Suppose a physics student sits at a table, working diligently on his homework (Figure 4.2.5). What external forces act on him? Can we determine the origin of these forces?

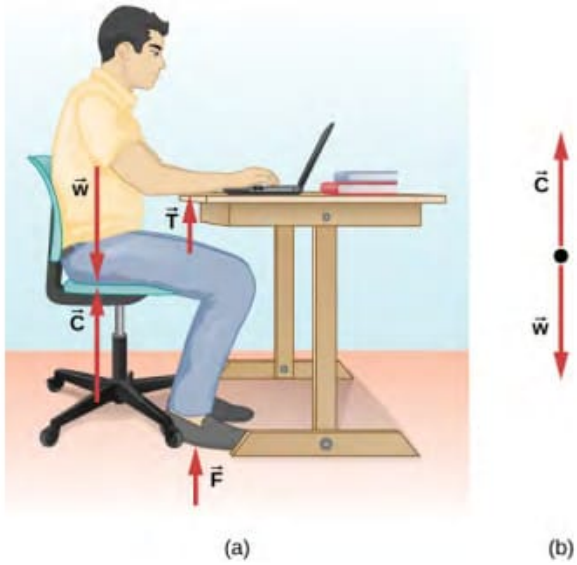


Figure 4.2.5 : (a) The forces acting on the student are due to the chair, the table, the floor, and Earth's gravitational attraction. (b) In solving a problem involving the student, we may want to consider the forces acting along the line running through his torso. A free-body diagram for this situation is shown.

In most situations, forces are grouped into two categories: **contact forces** and **field forces**. As you might guess, contact forces

are due to direct physical contact between objects. For example, the student in Figure 4.2.5 experiences the contact forces $\vec{C}$, $\vec{F}$, and $\vec{T}$, which are exerted by the chair on his posterior, the floor on his feet, and the table on his forearms, respectively. Field forces, however, act without the necessity of physical contact between objects. They depend on the presence of a "field" in the region of space surrounding the body under consideration. Since the student is in Earth's gravitational field, he feels a gravitational force $\vec{w}$; in other words, he has weight.

You can think of a field as a property of space that is detectable by the forces it exerts. Scientists think there are only four fundamental force fields in nature. These are the gravitational, electromagnetic, strong nuclear, and weak fields (we consider these four forces in nature later in this text). As noted for $\vec{w}$ in Figure 4.2.5, the gravitational field is responsible for the weight of a body. The forces of the electromagnetic field include those of static electricity and magnetism; they are also responsible for the attraction among atoms in bulk matter. Both the strong nuclear and the weak force fields are effective only over distances roughly equal to a length of scale no larger than an atomic nucleus (10^{-15} m). Their range is so small that neither field has influence in the macroscopic world of Newtonian mechanics.

Contact forces are fundamentally electromagnetic. While the elbow of the student in Figure 4.2.5 is in contact with the tabletop, the atomic charges in his skin interact electromagnetically with the charges in the surface of the table. The net (total) result is the force $\vec{T}$. Similarly, when adhesive tape sticks to a piece of paper, the atoms of the tape are intermingled with those of the paper to cause a net electromagnetic force between the two objects. However, in the context of Newtonian mechanics, the electromagnetic origin of contact forces is not an important concern.

Vector Notation for Force

As previously discussed, force is a vector; it has both magnitude and direction. The SI unit of force is called the **newton** (abbreviated N), and 1 N is the force needed to accelerate an object with a mass of 1 kg at a rate of 1 m/s^2: 1 N = 1 kg • m/s^2. An easy way to remember the size of a newton is to imagine holding a small apple; it has a weight of about 1 N.

We can thus describe a two-dimensional force in the form $\vec{F}$ = a $\hat{i}$ + b $\hat{j}$ (the unit vectors $\hat{i}$ and $\hat{j}$ indicate the direction of these forces along the x-axis and the y-axis, respectively) and a three-dimensional force in the form $\vec{F}$ = a $\hat{i}$ + b $\hat{j}$ + c $\hat{k}$. In Figure 4.2.2, let's suppose that ice skater 1, on the left side of the figure, pushes horizontally with a force of 30.0 N to the right; we represent this as $\vec{F}_1$ = 30.0 $\hat{i}$ N. Similarly, if ice skater 2 pushes with a force of 40.0 N in the positive vertical direction shown, we would write $\vec{F}_2$ = 40.0 $\hat{j}$ N. The resultant of the two forces causes a mass to accelerate—in this case, the third ice skater. This resultant is called the **net external force** $\vec{F}_{net}$ and is found by taking the vector sum of all external forces acting on an object or system (thus, we can also represent net external force as $\sum \vec{F}$):

$$\vec{F}_{net} = \sum \vec{F} = \vec{F}_1 + \vec{F}_2 + \ldots \tag{4.2.1}$$

This equation can be extended to any number of forces.

In this example, we have $\vec{F}_{net} = \sum \vec{F} = \vec{F}_1 + \vec{F}_2$ = 30.0 $\hat{i}$ + 40.0 $\hat{j}$. The hypotenuse of the triangle shown in Figure 4.2.2 is the resultant force, or net force. It is a vector. To find its magnitude (the size of the vector, without regard to direction), we use the rule given in Vectors, taking the square root of the sum of the squares of the components:

$$\vec{F}_{net} = \sqrt{(30.0\ N)^2 + (40.0\ N)^2} = 50.0\ N. \tag{4.2.2}$$

The direction is given by

$$\theta = \tan^{-1}\left(\frac{F_2}{F_1}\right) = \tan^{-1}\left(\frac{40.0}{30.0}\right) = 53.1^o, \tag{4.2.3}$$

measured from the positive x-axis, as shown in the free-body diagram in Figure 4.2.2(b).

Let's suppose the ice skaters now push the third ice skater with $\vec{F}_1$ = 3.0 $\hat{i}$ + 8.0 $\hat{j}$ N and $\vec{F}_2$ = 5.0 $\hat{i}$ + 4.0 $\hat{j}$ N. What is the resultant of these two forces? We must recognize that force is a vector; therefore, we must add using the rules for vector addition:

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 = \left(3.0\hat{i} + 8.0\hat{j}\right) + \left(5.0\hat{i} + 4.0\hat{j}\right) = 8.0\hat{i} + 12\hat{j}\ N \tag{4.2.4}$$

Exercise 5.1

Find the magnitude and direction of the net force in the ice skater example just given.

Simulation

View this interactive simulation to learn how to add vectors. Drag vectors onto a graph, change their length and angle, and sum them together. The magnitude, angle, and components of each vector can be displayed in several formats.

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4.3: Newton's First Law

Learning Objectives

- Describe Newton's first law of motion
- Recognize friction as an external force
- Define inertia
- Identify inertial reference frames
- Calculate equilibrium for a system

Experience suggests that an object at rest remains at rest if left alone and that an object in motion tends to slow down and stop unless some effort is made to keep it moving. However, Newton's first law gives a deeper explanation of this observation.

Newton's First Law of Motion

A body at rest remains at rest or, if in motion, remains in motion at constant velocity unless acted on by a net external force.

Note the repeated use of the verb "remains." We can think of this law as preserving the status quo of motion. Also note the expression "constant velocity;" this means that the object maintains a path along a straight line, since neither the magnitude nor the direction of the velocity vector changes. We can use Figure 4.3.1 to consider the two parts of Newton's first law.



Figure 4.3.1: (a) A hockey puck is shown at rest; it remains at rest until an outside force such as a hockey stick changes its state of rest; (b) a hockey puck is shown in motion; it continues in motion in a straight line until an outside force causes it to change its state of motion. Although it is slick, an ice surface provides some friction that slows the puck.

Rather than contradicting our experience, Newton's first law says that there must be a cause for any change in velocity (a change in either magnitude or direction) to occur. This cause is a net external force, which we defined earlier in the chapter. An object sliding across a table or floor slows down due to the net force of friction acting on the object. If friction disappears, will the object still slow down?

The idea of cause and effect is crucial in accurately describing what happens in various situations. For example, consider what happens to an object sliding along a rough horizontal surface. The object quickly grinds to a halt. If we spray the surface with talcum powder to make the surface smoother, the object slides farther. If we make the surface even smoother by rubbing lubricating oil on it, the object slides farther yet. Extrapolating to a frictionless surface and ignoring air resistance, we can imagine the object sliding in a straight line indefinitely. Friction is thus the cause of slowing (consistent with Newton's first law). The object would not slow down if friction were eliminated.

Consider an air hockey table (Figure 4.3.2). When the air is turned off, the puck slides only a short distance before friction slows

it to a stop. However, when the air is turned on, it creates a nearly frictionless surface, and the puck glides long distances without slowing down. Additionally, if we know enough about the friction, we can accurately predict how quickly the object slows down.



Figure 4.3.2: An air hockey table is useful in illustrating Newton's laws. When the air is off, friction quickly slows the puck; but when the air is on, it minimizes contact between the puck and the hockey table, and the puck glides far down the table.

Newton's first law is general and can be applied to anything from an object sliding on a table to a satellite in orbit to blood pumped from the heart. Experiments have verified that any change in velocity (speed or direction) must be caused by an external force. The idea of **generally applicable or universal laws** is important—it is a basic feature of all laws of physics. Identifying these laws is like recognizing patterns in nature from which further patterns can be discovered. The genius of Galileo, who first developed the idea for the first law of motion, and Newton, who clarified it, was to ask the fundamental question: "What is the cause?" Thinking in terms of cause and effect is fundamentally different from the typical ancient Greek approach, when questions such as "Why does a tiger have stripes?" would have been answered in Aristotelian fashion, such as "That is the nature of the beast." The ability to think in terms of cause and effect is the ability to make a connection between an observed behavior and the surrounding world.

Gravitation and Inertia

Regardless of the scale of an object, whether a molecule or a subatomic particle, two properties remain valid and thus of interest to physics: gravitation and inertia. Both are connected to mass. Roughly speaking, **mass** is a measure of the amount of matter in something. **Gravitation** is the attraction of one mass to another, such as the attraction between yourself and Earth that holds your feet to the floor. The magnitude of this attraction is your weight, and it is a force.

Mass is also related to **inertia**, the ability of an object to resist changes in its motion—in other words, to resist acceleration. Newton's first law is often called the **law of inertia**. As we know from experience, some objects have more inertia than others. It is more difficult to change the motion of a large boulder than that of a basketball, for example, because the boulder has more mass than the basketball. In other words, the inertia of an object is measured by its mass. The relationship between mass and weight is explored later in this chapter.

Inertial Reference Frames

Earlier, we stated Newton's first law as "A body at rest remains at rest or, if in motion, remains in motion at constant velocity unless acted on by a net external force." It can also be stated as "Every body remains in its state of uniform motion in a straight line unless it is compelled to change that state by forces acting on it." To Newton, "uniform motion in a straight line" meant constant velocity, which includes the case of zero velocity, or rest. Therefore, the first law says that the velocity of an object remains constant if the net force on it is zero.

Newton's first law is usually considered to be a statement about reference frames. It provides a method for identifying a special type of reference frame: the **inertial reference frame**. In principle, we can make the net force on a body zero. If its velocity relative to a given frame is constant, then that frame is said to be inertial. So by definition, an inertial reference frame is a reference frame in which Newton's first law is valid. Newton's first law applies to objects with constant velocity. From this fact, we can infer the following statement.

Inertial Reference Frame

A reference frame moving at constant velocity relative to an inertial frame is also inertial. A reference frame accelerating relative to an inertial frame is not inertial.

Are inertial frames common in nature? It turns out that well within experimental error, a reference frame at rest relative to the most distant, or "fixed," stars is inertial. All frames moving uniformly with respect to this fixed-star frame are also inertial. For example, a nonrotating reference frame attached to the Sun is, for all practical purposes, inertial, because its velocity relative to the fixed stars does not vary by more than one part in 10^{10}. Earth accelerates relative to the fixed stars because it rotates on its axis and revolves around the Sun; hence, a reference frame attached to its surface is not inertial. For most problems, however, such a frame serves as a sufficiently accurate approximation to an inertial frame, because the acceleration of a point on Earth's surface relative to the fixed stars is rather small (< 3.4 x 10^{-2} m/s^2). Thus, unless indicated otherwise, we consider reference frames fixed on Earth to be inertial.

Finally, no particular inertial frame is more special than any other. As far as the laws of nature are concerned, all inertial frames are equivalent. In analyzing a problem, we choose one inertial frame over another simply on the basis of convenience.

Newton's First Law and Equilibrium

Newton's first law tells us about the equilibrium of a system, which is the state in which the forces on the system are balanced. Returning to Forces and the ice skaters in Figure 5.2.2, we know that the forces $\vec{F}_1$ and $\vec{F}_2$ combine to form a resultant force, or the net external force: $\vec{F}_R = \vec{F}_{net} = \vec{F}_1 + \vec{F}_2$. To create equilibrium, we require a balancing force that will produce a net force of zero. This force must be equal in magnitude but opposite in direction to $\vec{F}_R$, which means the vector must be $-\vec{F}_R$. Referring to the ice skaters, for which we found $\vec{F}_R$ to be 30.0 $\hat{i}$ + 40.0 $\hat{j}$ N, we can determine the balancing force by simply finding $-\vec{F}_R$ = –30.0 $\hat{i}$ – 40.0 $\hat{j}$ N. See the free-body diagram in Figure 5.2.2b.

We can give Newton's first law in vector form:

$$\vec{v} = constant \; when \; \vec{F}_{net} = \vec{0} \; N. \tag{4.3.1}$$

This equation says that a net force of zero implies that the velocity $\vec{v}$ of the object is constant. (The word "constant" can indicate zero velocity.)

Newton's first law is deceptively simple. If a car is at rest, the only forces acting on the car are weight and the contact force of the pavement pushing up on the car (Figure 4.3.3). It is easy to understand that a nonzero net force is required to change the state of motion of the car. However, if the car is in motion with constant velocity, a common misconception is that the engine force propelling the car forward is larger in magnitude than the friction force that opposes forward motion. In fact, the two forces have identical magnitude.

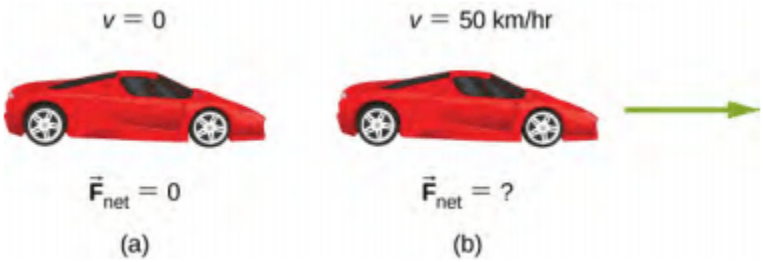


Figure 4.3.3: A car is shown (a) parked and (b) moving at constant velocity. How do Newton's laws apply to the parked car? What does the knowledge that the car is moving at constant velocity tell us about the net horizontal force on the car?

Example 5.1: When Does Newton's First Law Apply to Your Car?

Newton's laws can be applied to all physical processes involving force and motion, including something as mundane as driving a car.

a. Your car is parked outside your house. Does Newton's first law apply in this situation? Why or why not?
b. Your car moves at constant velocity down the street. Does Newton's first law apply in this situation? Why or why not?

Strategy

In (a), we are considering the first part of Newton's first law, dealing with a body at rest; in (b), we look at the second part of Newton's first law for a body in motion.

Solution

a. When your car is parked, all forces on the car must be balanced; the vector sum is 0 N. Thus, the net force is zero, and Newton's first law applies. The acceleration of the car is zero, and in this case, the velocity is also zero.
b. When your car is moving at constant velocity down the street, the net force must also be zero according to Newton's first law. The car's engine produces a forward force; friction, a force between the road and the tires of the car that opposes forward motion, has exactly the same magnitude as the engine force, producing the net force of zero. The body continues in its state of constant velocity until the net force becomes nonzero. Realize that **a net force of zero means that an object is either at rest or moving with constant velocity, that is, it is not accelerating**. What do you suppose happens when the car accelerates? We explore this idea in the next section.

Significance

As this example shows, there are two kinds of equilibrium. In (a), the car is at rest; we say it is in **static equilibrium**. In (b), the forces on the car are balanced, but the car is moving; we say that it is in **dynamic equilibrium**. (We examine this idea in more detail in Static Equilibrium and Elasticity.) Again, it is possible for two (or more) forces to act on an object yet for the object to move. In addition, a net force of zero cannot produce acceleration.

Exercise 5.2

A skydiver opens his parachute, and shortly thereafter, he is moving at constant velocity. (a) What forces are acting on him? (b) Which force is bigger?

Simulation

Engage in this simulation to predict, qualitatively, how an external force will affect the speed and direction of an object's motion. Explain the effects with the help of a free-body diagram. Use free-body diagrams to draw position, velocity, acceleration, and force graphs, and vice versa. Explain how the graphs relate to one another. Given a scenario or a graph, sketch all four graphs.

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4.4: Newton's Second Law

Learning Objectives

- Distinguish between external and internal forces
- Describe Newton's second law of motion
- Explain the dependence of acceleration on net force and mass

Newton's second law is closely related to his first law. It mathematically gives the cause-and-effect relationship between force and changes in motion. Newton's second law is quantitative and is used extensively to calculate what happens in situations involving a force. Before we can write down Newton's second law as a simple equation that gives the exact relationship of force, mass, and acceleration, we need to sharpen some ideas we mentioned earlier.

Force and Acceleration

First, what do we mean by a change in motion? The answer is that a change in motion is equivalent to a change in velocity. A change in velocity means, by definition, that there is acceleration. Newton's first law says that a net external force causes a change in motion; thus, we see that a **net external force causes nonzero acceleration**.

We defined external force in Forces as force acting on an object or system that originates outside of the object or system. Let's consider this concept further. An intuitive notion of **external** is correct—it is outside the system of interest. For example, in Figure $4.4.1a$, the system of interest is the car plus the person within it. The two forces exerted by the two students are external forces. In contrast, an internal force acts between elements of the system. Thus, the force the person in the car exerts to hang on to the steering wheel is an internal force between elements of the system of interest. Only external forces affect the motion of a system, according to Newton's first law. (The internal forces cancel each other out, as explained in the next section.) Therefore, we must define the boundaries of the system before we can determine which forces are external. Sometimes, the system is obvious, whereas at other times, identifying the boundaries of a system is more subtle. The concept of a system is fundamental to many areas of physics, as is the correct application of Newton's laws. This concept is revisited many times in the study of physics.

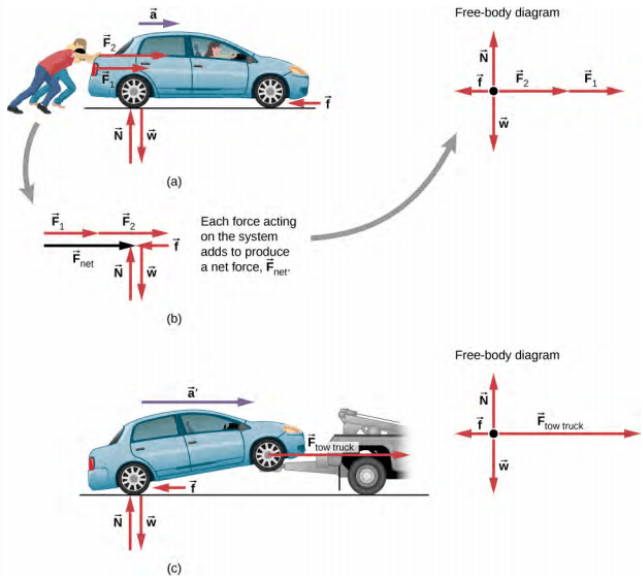


Figure $4.4.1$: Different forces exerted on the same mass produce different accelerations. (a) Two students push a stalled car. All external forces acting on the car are shown. (b) The forces acting on the car are transferred to a coordinate plane (free-body diagram) for simpler analysis. (c) The tow truck can produce greater external force on the same mass, and thus greater acceleration.

From this example, you can see that different forces exerted on the same mass produce different accelerations. In Figure $4.4.1a$, the two students push a car with a driver in it. Arrows representing all external forces are shown. The system of interest is the car

and its driver. The weight $\vec{w}$ of the system and the support of the ground $\vec{N}$ are also shown for completeness and are assumed to cancel (because there was no vertical motion and no imbalance of forces in the vertical direction to create a change in motion). The vector $\vec{f}$ represents the friction acting on the car, and it acts to the left, opposing the motion of the car. (We discuss friction in more detail in the next chapter.) In Figure $4.4.1b$, all external forces acting on the system add together to produce the net force $\vec{F}_{net}$. The free-body diagram shows all of the forces acting on the system of interest. The dot represents the center of mass of the system. Each force vector extends from this dot. Because there are two forces acting to the right, the vectors are shown collinearly. Finally, in Figure $4.4.1c$, a larger net external force produces a larger acceleration ($\vec{a}' > \vec{a}$) when the tow truck pulls the car.

It seems reasonable that acceleration would be directly proportional to and in the same direction as the net external force acting on a system. This assumption has been verified experimentally and is illustrated in Figure $4.4.1$. To obtain an equation for Newton's second law, we first write the relationship of acceleration $\vec{a}$ and net external force $\vec{F}_{net}$ as the proportionality

$$\vec{a} \propto \vec{F}_{net} \tag{4.4.1}$$

where the symbol α means "proportional to." (Recall from Forces that the net external force is the vector sum of all external forces and is sometimes indicated as $\sum \vec{F}$.) This proportionality shows what we have said in words—acceleration is directly proportional to net external force. Once the system of interest is chosen, identify the external forces and ignore the internal ones. It is a tremendous simplification to disregard the numerous internal forces acting between objects within the system, such as muscular forces within the students' bodies, let alone the myriad forces between the atoms in the objects. Still, this simplification helps us solve some complex problems.

It also seems reasonable that acceleration should be inversely proportional to the mass of the system. In other words, the larger the mass (the inertia), the smaller the acceleration produced by a given force. As illustrated in Figure $4.4.2$, the same net external force applied to a basketball produces a much smaller acceleration when it is applied to an SUV. The proportionality is written as

$$a \propto \frac{1}{m}, \tag{4.4.2}$$

where m is the mass of the system and a is the magnitude of the acceleration. Experiments have shown that acceleration is exactly inversely proportional to mass, just as it is directly proportional to net external force.

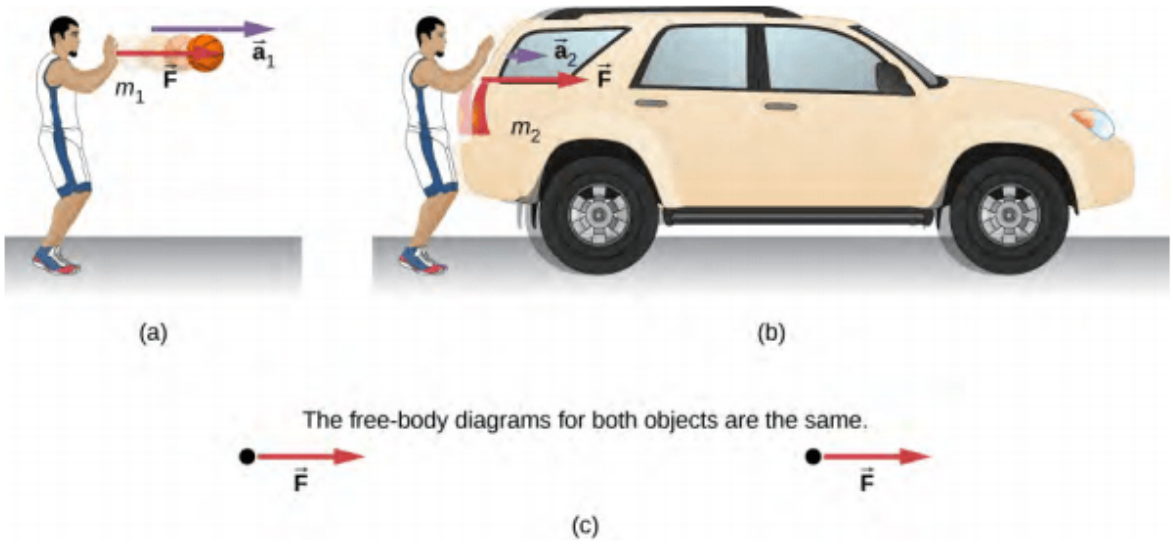


Figure $4.4.2$: The same force exerted on systems of different masses produces different accelerations. (a) A basketball player pushes on a basketball to make a pass. (Ignore the effect of gravity on the ball.) (b) The same player exerts an identical force on a stalled SUV and produces far less acceleration. (c) The free-body diagrams are identical, permitting direct comparison of the two situations. A series of patterns for free-body diagrams will emerge as you do more problems and learn how to draw them in Drawing Free-Body Diagrams.

It has been found that the acceleration of an object depends only on the net external force and the mass of the object. Combining the two proportionalities just given yields **Newton's second law**.

Newton's Second Law of Motion

The acceleration of a system is directly proportional to and in the same direction as the net external force acting on the system and is inversely proportion to its mass. In equation form, Newton's second law is

$$\vec{a} = \frac{\vec{F}_{net}}{m}, \tag{4.4.3}$$

where $\vec{a}$ is the acceleration, $\vec{F}_{net}$ is the net force, and m is the mass. This is often written in the more familiar form

$$\vec{F}_{net} = \sum \vec{F} = m\vec{a}, \tag{4.4.4}$$

but the first equation gives more insight into what Newton's second law means. When only the magnitude of force and acceleration are considered, this equation can be written in the simpler scalar form:

$$\vec{F}_{net} = ma. \tag{4.4.5}$$

The law is a cause-and-effect relationship among three quantities that is not simply based on their definitions. The validity of the second law is based on experimental verification. The free-body diagram, which you will learn to draw in Drawing Free-Body Diagrams, is the basis for writing Newton's second law.

Example 5.2: What Acceleration Can a Person Produce When Pushing a Lawn Mower?

Suppose that the net external force (push minus friction) exerted on a lawn mower is 51 N (about 11 lb.) parallel to the ground (Figure 4.4.3). The mass of the mower is 24 kg. What is its acceleration?

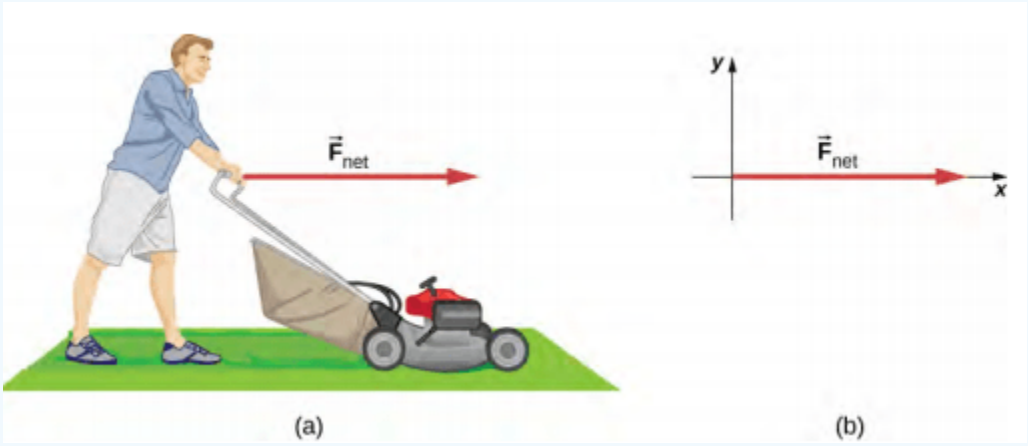


Figure 4.4.3: (a) The net force on a lawn mower is 51 N to the right. At what rate does the lawn mower accelerate to the right? (b) The free-body diagram for this problem is shown.

Strategy

This problem involves only motion in the horizontal direction; we are also given the net force, indicated by the single vector, but we can suppress the vector nature and concentrate on applying Newton's second law. Since F_{net} and m are given, the acceleration can be calculated directly from Newton's second law as $F_{net} = ma$.

Solution

The magnitude of the acceleration a is $a = \frac{F_{net}}{m}$. Entering known values gives

$$a = \frac{51\ N}{24\ kg}. \tag{4.4.6}$$

Substituting the unit of kilograms times meters per square second for newtons yields

$$a = \frac{51\ kg{\cdot}\ m/s^2}{24\ kg} = 2.1\ m/s^2. \tag{4.4.7}$$

Significance

The direction of the acceleration is the same direction as that of the net force, which is parallel to the ground. This is a result of the vector relationship expressed in Newton's second law, that is, the vector representing net force is the scalar multiple of the acceleration vector. There is no information given in this example about the individual external forces acting on the system, but we can say something about their relative magnitudes. For example, the force exerted by the person pushing the mower must be greater than the friction opposing the motion (since we know the mower moved forward), and the vertical forces must cancel because no acceleration occurs in the vertical direction (the mower is moving only horizontally). The acceleration found is small enough to be reasonable for a person pushing a mower. Such an effort would not last too long, because the person's top speed would soon be reached.

Exercise 5.3

At the time of its launch, the HMS Titanic was the most massive mobile object ever built, with a mass of 6.0 x 10^7 kg. If a force of 6 MN (6 x 10^6 N) was applied to the ship, what acceleration would it experience?

In the preceding example, we dealt with net force only for simplicity. However, several forces act on the lawn mower. The weight $\vec{w}$ (discussed in detail in Mass and Weight) pulls down on the mower, toward the center of Earth; this produces a contact force on the ground. The ground must exert an upward force on the lawn mower, known as the normal force $\vec{N}$, which we define in Common Forces. These forces are balanced and therefore do not produce vertical acceleration. In the next example, we show both of these forces. As you continue to solve problems using Newton's second law, be sure to show multiple forces.

Example 5.3: Which Force Is Bigger?

a. The car shown in Figure 4.4.4 is moving at a constant speed. Which force is bigger, $\vec{F}_{engine}$ or $\vec{F}_{friction}$? Explain.
b. The same car is now accelerating to the right. Which force is bigger, $\vec{F}_{engine}$ or $\vec{F}_{friction}$? Explain.

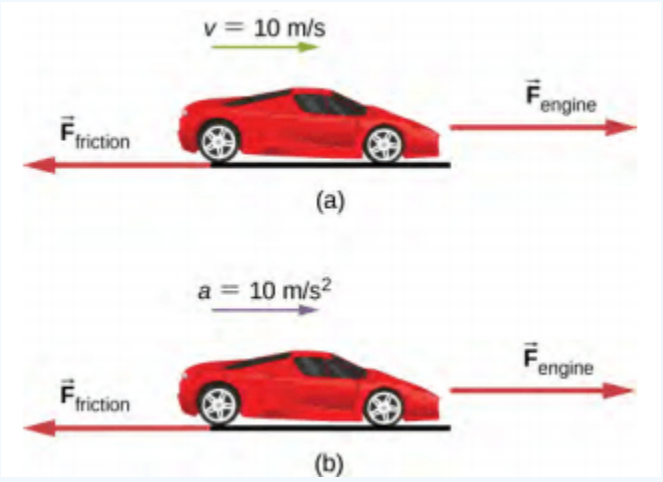


Figure 4.4.4: A car is shown (a) moving at constant speed and (b) accelerating. How do the forces acting on the car compare in each case? (a) What does the knowledge that the car is moving at constant velocity tell us about the net horizontal force on the car compared to the friction force? (b) What does the knowledge that the car is accelerating tell us about the horizontal force on the car compared to the friction force?

Strategy

We must consider Newton's first and second laws to analyze the situation. We need to decide which law applies; this, in turn, will tell us about the relationship between the forces.

Solution

a. The forces are equal. According to Newton's first law, if the net force is zero, the velocity is constant.
b. In this case, $\vec{F}_{engine}$ must be larger than $\vec{F}_{friction}$. According to Newton's second law, a net force is required to cause acceleration.

Significance

These questions may seem trivial, but they are commonly answered incorrectly. For a car or any other object to move, it must be accelerated from rest to the desired speed; this requires that the engine force be greater than the friction force. Once the car is moving at constant velocity, the net force must be zero; otherwise, the car will accelerate (gain speed). To solve problems involving Newton's laws, we must understand whether to apply Newton's first law (where $\sum \vec{F} = \vec{0}$) or Newton's second law (where $\sum \vec{F}$ is not zero). This will be apparent as you see more examples and attempt to solve problems on your own.

Example 5.4: What Rocket Thrust Accelerates This Sled?

Before manned space flights, rocket sleds were used to test aircraft, missile equipment, and physiological effects on human subjects at high speeds. They consisted of a platform that was mounted on one or two rails and propelled by several rockets.

Calculate the magnitude of force exerted by each rocket, called its thrust T, for the four-rocket propulsion system shown in Figure 4.4.5. The sled's initial acceleration is 49 m/s^2, the mass of the system is 2100 kg, and the force of friction opposing the motion is 650 N.

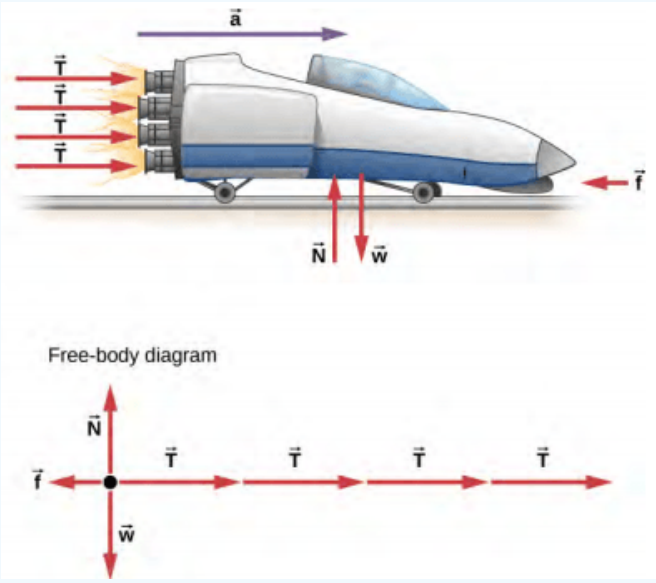


Figure 4.4.5: A sled experiences a rocket thrust that accelerates it to the right. Each rocket creates an identical thrust T. The system here is the sled, its rockets, and its rider, so none of the forces between these objects are considered. The arrow representing friction ($\vec{f}$) is drawn larger than scale.

Strategy

Although forces are acting both vertically and horizontally, we assume the vertical forces cancel because there is no vertical acceleration. This leaves us with only horizontal forces and a simpler one-dimensional problem. Directions are indicated with plus or minus signs, with right taken as the positive direction. See the free-body diagram in Figure 4.4.5.

Solution

Since acceleration, mass, and the force of friction are given, we start with Newton's second law and look for ways to find the thrust of the engines. We have defined the direction of the force and acceleration as acting "to the right," so we need to consider only the magnitudes of these quantities in the calculations. Hence we begin with

$$F_{net} = ma \tag{4.4.8}$$

where F_{net} is the net force along the horizontal direction. We can see from the figure that the engine thrusts add, whereas friction opposes the thrust. In equation form, the net external force is

$$F_{net} = 4T - f. \tag{4.4.9}$$

Substituting this into Newton's second law gives us

$$F_{net} = ma = 4T - f. \tag{4.4.10}$$

Using a little algebra, we solve for the total thrust 4T:

$$4T = ma + f. \tag{4.4.11}$$

Substituting known values yields

$$4T = ma + f = (2100\ kg)(49\ m/s^2) + 650\ N. \tag{4.4.12}$$

Therefore, the total thrust is

$$4T = 1.0 \times 10^5\ N. \tag{4.4.13}$$

Significance

The numbers are quite large, so the result might surprise you. Experiments such as this were performed in the early 1960s to test the limits of human endurance, and the setup was designed to protect human subjects in jet fighter emergency ejections. Speeds of 1000 km/h were obtained, with accelerations of 45 g's. (Recall that g, acceleration due to gravity, is 9.80 m/s^2. When we say that acceleration is 45 g's, it is 45 x 9.8 m/s^2, which is approximately 440 m/s^2.) Although living subjects are not used anymore, land speeds of 10,000 km/h have been obtained with a rocket sled.

In this example, as in the preceding one, the system of interest is obvious. We see in later examples that choosing the system of interest is crucial—and the choice is not always obvious.

Newton's second law is more than a definition; it is a relationship among acceleration, force, and mass. It can help us make predictions. Each of those physical quantities can be defined independently, so the second law tells us something basic and universal about nature.

Exercise 5.4

A 550-kg sports car collides with a 2200-kg truck, and during the collision, the net force on each vehicle is the force exerted by the other. If the magnitude of the truck's acceleration is 10 m/s^2, what is the magnitude of the sports car's acceleration?

Component Form of Newton's Second Law

We have developed Newton's second law and presented it as a vector equation in Equation 4.4.4. This vector equation can be written as three component equations:

$$\sum \vec{F}_x = m\vec{a}_x, \sum \vec{F}_y = m\vec{a}_y, \sum \vec{F}_z = m\vec{a}_z. \tag{4.4.14}$$

The second law is a description of how a body responds mechanically to its environment. The influence of the environment is the net force $\vec{F}_{net}$, the body's response is the acceleration $\vec{a}$, and the strength of the response is inversely proportional to the mass

m. The larger the mass of an object, the smaller its response (its acceleration) to the influence of the environment (a given net force). Therefore, a body's mass is a measure of its inertia, as we explained in Newton's First Law.

Example 5.5: Force on a Soccer Ball

A 0.400-kg soccer ball is kicked across the field by a player; it undergoes acceleration given by $\vec{a}$ = 3.00 $\hat{i}$ + 7.00 $\hat{j}$ m/s^2. Find (a) the resultant force acting on the ball and (b) the magnitude and direction of the resultant force.

Strategy

The vectors in $\hat{i}$ and $\hat{j}$ format, which indicate force direction along the x-axis and the y-axis, respectively, are involved, so we apply Newton's second law in vector form.

Solution

a. We apply Newton's second law:

$$\vec{F}_{net} = m\vec{a} = (0.400\ kg)\left(3.00\ \hat{i} + 7.00\ \hat{j}\ m/s^2\right) = 1.20\ \hat{i} + 2.80\ \hat{j}\ N. \tag{4.4.15}$$

b. . Magnitude and direction are found using the components of $\vec{F}_{net}$:

$$F_{net} = \sqrt{(1.20\ N)^2 + (2.80\ N)^2} = 3.05\ N\ and\ \theta = \tan^{-1}\left(\frac{2.80}{1.20}\right) = 66.8^o. \tag{4.4.16}$$

Significance

We must remember that Newton's second law is a vector equation. In (a), we are multiplying a vector by a scalar to determine the net force in vector form. While the vector form gives a compact representation of the force vector, it does not tell us how "big" it is, or where it goes, in intuitive terms. In (b), we are determining the actual size (magnitude) of this force and the direction in which it travels.

Example 5.6: Mass of a Car

Find the mass of a car if a net force of –600.0 $\hat{j}$ N produces an acceleration of –0.2 $\hat{j}$ m/s^2.

Strategy

Vector division is not defined, so $m = \frac{\vec{F}_{net}}{\vec{a}}$ cannot be performed. However, mass m is a scalar, so we can use the scalar form of Newton's second law, $m = \frac{F_{net}}{a}$.

Solution

We use m = $\frac{F_{net}}{a}$ and substitute the magnitudes of the two vectors: F_{net} = 600.0 N and a = 0.2 m/s^2. Therefore,

$$m = \frac{F_{net}}{a} = \frac{600.0\ N}{0.2\ m/s^2} = 3000\ kg.$$

Significance

Force and acceleration were given in the $\hat{i}$ and $\hat{j}$ format, but the answer, mass m, is a scalar and thus is not given in $\hat{i}$ and $\hat{j}$ form.

Example 5.7

Several Forces on a Particle A particle of mass m = 4.0 kg is acted upon by four forces of magnitudes. F_1 = 10.0 N, F_2 = 40.0 N, F_3 = 5.0 N, and F_4 = 2.0 N, with the directions as shown in the free-body diagram in Figure 4.4.6. What is the acceleration of the particle?

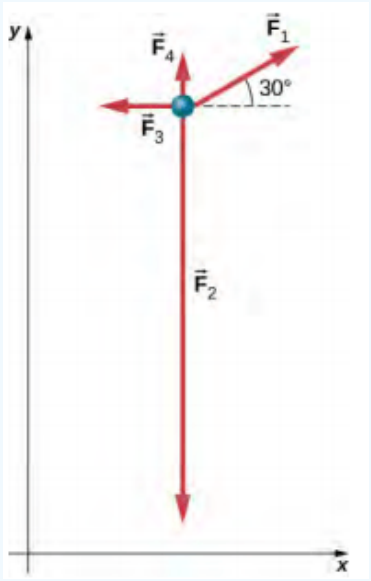


Figure 4.4.6: Four forces in the xy-plane are applied to a 4.0-kg particle.

Strategy

Because this is a two-dimensional problem, we must use a free-body diagram. First, $\vec{F}_1$ must be resolved into x- and y-components. We can then apply the second law in each direction.

Solution

We draw a free-body diagram as shown in Figure 4.4.6. Now we apply Newton's second law. We consider all vectors resolved into x- and y-components:

$$\sum F_x = ma_x \tag{4.4.17}$$

$$F_{1x} - F_{3x} = ma_x \tag{4.4.18}$$

$$F_1 \cos 30^o - F_{3x} = ma_x \tag{4.4.19}$$

$$(10.0\ N)(\cos 30^o) - 5.0\ N = (4.0\ kg)a_x \tag{4.4.20}$$

$$a_x = 0.92\ m/s^2. \tag{4.4.21}$$

Thus, the net acceleration is

$$\vec{a} = \left(0.92\ \hat{i} - 8.3\ \hat{j}\right) m/s^2, \tag{4.4.27}$$

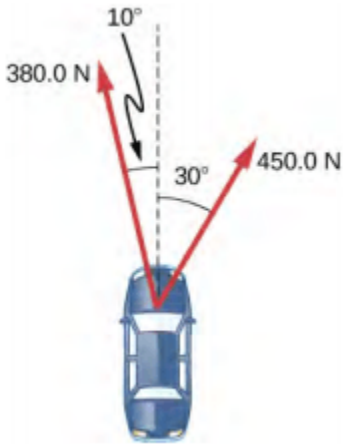
which is a vector of magnitude 8.4 m/s^2 directed at 276° to the positive x-axis.

Significance

Numerous examples in everyday life can be found that involve three or more forces acting on a single object, such as cables running from the Golden Gate Bridge or a football player being tackled by three defenders. We can see that the solution of this example is just an extension of what we have already done.

Exercise 5.5

A car has forces acting on it, as shown below. The mass of the car is 1000.0 kg. The road is slick, so friction can be ignored. (a) What is the net force on the car? (b) What is the acceleration of the car?



Newton's Second Law and Momentum

Newton actually stated his second law in terms of momentum: "The instantaneous rate at which a body's momentum changes is equal to the net force acting on the body." ("Instantaneous rate" implies that the derivative is involved.) This can be given by the vector equation

$$\vec{F}_{net} = \frac{d\vec{p}}{dt}. \tag{4.4.28}$$

This means that Newton's second law addresses the central question of motion: What causes a change in motion of an object? Momentum was described by Newton as "quantity of motion," a way of combining both the velocity of an object and its mass. We devote Linear Momentum and Collisions to the study of momentum.

For now, it is sufficient to define momentum $\vec{p}$ as the product of the mass of the object m and its velocity $\vec{v}$:

$$\vec{p} = m\vec{v}. \tag{4.4.29}$$

Since velocity is a vector, so is momentum.

It is easy to visualize momentum. A train moving at 10 m/s has more momentum than one that moves at 2 m/s. In everyday life, we speak of one sports team as "having momentum" when they score points against the opposing team.

If we substitute Equation 4.4.29 into Equation 4.4.28, we obtain

$$\vec{F}_{net} = \frac{d\vec{p}}{dt} = \frac{d(m\vec{v})}{dt}. \tag{4.4.30}$$

When m is constant, we have

$$\vec{F}_{net} = m\frac{d(\vec{v})}{dt} = m\vec{a}. \tag{4.4.31}$$

Thus, we see that the momentum form of Newton's second law reduces to the form given earlier in this section.

Simulation

Explore the forces at work when pulling a cart or pushing a refrigerator, crate, or person. Create an applied force and see how it makes objects move. Put an object on a ramp and see how it affects its motion.

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4.5: Mass and Weight

Learning Objectives

- Explain the difference between mass and weight
- Explain why falling objects on Earth are never truly in free fall
- Describe the concept of weightlessness

Mass and weight are often used interchangeably in everyday conversation. For example, our medical records often show our weight in kilograms but never in the correct units of newtons. In physics, however, there is an important distinction. Weight is the pull of Earth on an object. It depends on the distance from the center of Earth. Unlike weight, mass does not vary with location. The mass of an object is the same on Earth, in orbit, or on the surface of the Moon.

Units of Force

The equation F_{net} = ma is used to define net force in terms of mass, length, and time. As explained earlier, the SI unit of force is the newton. Since F_{net} = ma,

$$1\ N = 1\ kg{\cdot}\ m/s^2.$$

Although almost the entire world uses the newton for the unit of force, in the United States, the most familiar unit of force is the pound (lb), where 1 N = 0.225 lb. Thus, a 225-lb person weighs 1000 N.

Weight and Gravitational Force

When an object is dropped, it accelerates toward the center of Earth. Newton's second law says that a net force on an object is responsible for its acceleration. If air resistance is negligible, the net force on a falling object is the gravitational force, commonly called its **weight** $\vec{w}$, or its force due to gravity acting on an object of mass m. Weight can be denoted as a vector because it has a direction; **down** is, by definition, the direction of gravity, and hence, weight is a downward force. The magnitude of weight is denoted as w. Galileo was instrumental in showing that, in the absence of air resistance, all objects fall with the same acceleration g. Using Galileo's result and Newton's second law, we can derive an equation for weight.

Consider an object with mass m falling toward Earth. It experiences only the downward force of gravity, which is the weight $\vec{w}$. Newton's second law says that the magnitude of the net external force on an object is $\vec{F}_{net} = m\vec{a}$. We know that the acceleration of an object due to gravity is $\vec{g}$, or $\vec{a} = \vec{g}$. Substituting these into Newton's second law gives us the following equations.

Defintion: Weight

The gravitational force on a mass is its weight. We can write this in vector form, where $\vec{w}$ is weight and m is mass, as

$$\vec{w} = m\vec{g}. \tag{4.5.1}$$

In scalar form, we can write

$$w = mg. \tag{4.5.2}$$

Since g = 9.80 m/s^2 on Earth, the weight of a 1.00-kg object on Earth is 9.80 N:

$$w = mg = (1.00\ kg)(9.80 m/s^2) = 9.80\ N. \tag{4.5.3}$$

When the net external force on an object is its weight, we say that it is in **free fall**, that is, the only force acting on the object is gravity. However, when objects on Earth fall downward, they are never truly in free fall because there is always some upward

resistance force from the air acting on the object.

Acceleration due to gravity g varies slightly over the surface of Earth, so the weight of an object depends on its location and is not an intrinsic property of the object. Weight varies dramatically if we leave Earth's surface. On the Moon, for example, acceleration due to gravity is only 1.67 m/s^2. A 1.0-kg mass thus has a weight of 9.8 N on Earth and only about 1.7 N on the Moon.

The broadest definition of weight in this sense is that the weight of an object is the gravitational force on it from the nearest large body, such as Earth, the Moon, or the Sun. This is the most common and useful definition of weight in physics. It differs dramatically, however, from the definition of weight used by NASA and the popular media in relation to space travel and exploration. When they speak of "weightlessness" and "microgravity," they are referring to the phenomenon we call "free fall" in physics. We use the preceding definition of weight, force $\vec{w}$ due to gravity acting on an object of mass m, and we make careful distinctions between free fall and actual weightlessness.

Be aware that weight and mass are different physical quantities, although they are closely related. Mass is an intrinsic property of an object: It is a quantity of matter. The quantity or amount of matter of an object is determined by the numbers of atoms and molecules of various types it contains. Because these numbers do not vary, in Newtonian physics, mass does not vary; therefore, its response to an applied force does not vary. In contrast, weight is the gravitational force acting on an object, so it does vary depending on gravity. For example, a person closer to the center of Earth, at a low elevation such as New Orleans, weighs slightly more than a person who is located in the higher elevation of Denver, even though they may have the same mass.

It is tempting to equate mass to weight, because most of our examples take place on Earth, where the weight of an object varies only a little with the location of the object. In addition, it is difficult to count and identify all of the atoms and molecules in an object, so mass is rarely determined in this manner. If we consider situations in which $\vec{g}$ is a constant on Earth, we see that weight $\vec{w}$ is directly proportional to mass m, since $\vec{w} = m\vec{g}$, that is, the more massive an object is, the more it weighs. Operationally, the masses of objects are determined by comparison with the standard kilogram, as we discussed in Units and Measurement. But by comparing an object on Earth with one on the Moon, we can easily see a variation in weight but not in mass. For instance, on Earth, a 5.0-kg object weighs 49 N; on the Moon, where g is 1.67 m/s^2, the object weighs 8.4 N. However, the mass of the object is still 5.0 kg on the Moon.

Example 4.5.1: Clearing a Field

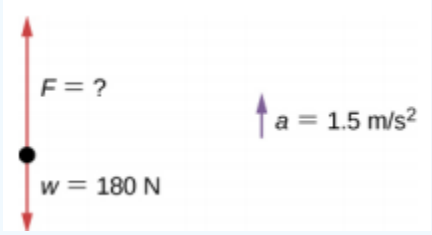
A farmer is lifting some moderately heavy rocks from a field to plant crops. He lifts a stone that weighs 40.0 lb. (about 180 N). What force does he apply if the stone accelerates at a rate of 1.5 m/s^2?

Strategy

We were given the weight of the stone, which we use in finding the net force on the stone. However, we also need to know its mass to apply Newton's second law, so we must apply the equation for weight, w = mg, to determine the mass.

Solution

No forces act in the horizontal direction, so we can concentrate on vertical forces, as shown in the following free-body diagram. We label the acceleration to the side; technically, it is not part of the free-body diagram, but it helps to remind us that the object accelerates upward (so the net force is upward).



$$w = mg$$

$$m = \frac{w}{g} = \frac{180\ N}{9.8\ m/s^2} = 18\ kg$$

$$\sum F = ma$$

$$F - w = ma$$

$$F - 180\ N = (18\ kg)(1.5\ m/s^2)$$

$$F - 180\ N = 27\ N$$

$$F = 207\ N = 210\ N \text{ to two significant figures}$$

Significance

To apply Newton's second law as the primary equation in solving a problem, we sometimes have to rely on other equations, such as the one for weight or one of the kinematic equations, to complete the solution.

Exercise 4.5.1

For 4.5.1, find the acceleration when the farmer's applied force is 230.0 N

Simulation

Can you avoid the boulder field and land safely just before your fuel runs out, as Neil Armstrong did in 1969? This version of the classic video game accurately simulates the real motion of the lunar lander, with the correct mass, thrust, fuel consumption rate, and lunar gravity. The real lunar lander is hard to control.

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4.6: Newton's Third Law

Learning Objectives

- State Newton's third law of motion
- Identify the action and reaction forces in different situations
- Apply Newton's third law to define systems and solve problems of motion

We have thus far considered force as a push or a pull; however, if you think about it, you realize that no push or pull ever occurs by itself. When you push on a wall, the wall pushes back on you. This brings us to Newton's third law.

Newton's Third Law of Motion

Whenever one body exerts a force on a second body, the first body experiences a force that is equal in magnitude and opposite in direction to the force that it exerts. Mathematically, if a body A exerts a force $\vec{F}$ on body B, then B simultaneously exerts a force $-\vec{F}$ on A, or in vector equation form,

$$\vec{F}_{AB} = -\vec{F}_{BA}. \tag{4.6.1}$$

Newton's third law represents a certain symmetry in nature: Forces always occur in pairs, and one body cannot exert a force on another without experiencing a force itself. We sometimes refer to this law loosely as "action-reaction," where the force exerted is the action and the force experienced as a consequence is the reaction. Newton's third law has practical uses in analyzing the origin of forces and understanding which forces are external to a system.

We can readily see Newton's third law at work by taking a look at how people move about. Consider a swimmer pushing off the side of a pool (Figure $4.6.1$). She pushes against the wall of the pool with her feet and accelerates in the direction opposite that of her push. The wall has exerted an equal and opposite force on the swimmer. You might think that two equal and opposite forces would cancel, but they do not **because they act on different systems**. In this case, there are two systems that we could investigate: the swimmer and the wall. If we select the swimmer to be the system of interest, as in the figure, then $F_{\text{wall on feet}}$ is an external force on this system and affects its motion. The swimmer moves in the direction of this force. In contrast, the force $F_{\text{feet on wall}}$ acts on the wall, not on our system of interest. Thus, $F_{\text{feet on wall}}$ does not directly affect the motion of the system and does not cancel $F_{\text{wall on feet}}$. The swimmer pushes in the direction opposite that in which she wishes to move. The reaction to her push is thus in the desired direction. In a free-body diagram, such as the one shown in Figure $4.6.1$, we never include both forces of an action-reaction pair; in this case, we only use $F_{\text{wall on feet}}$, not $F_{\text{feet on wall}}$.

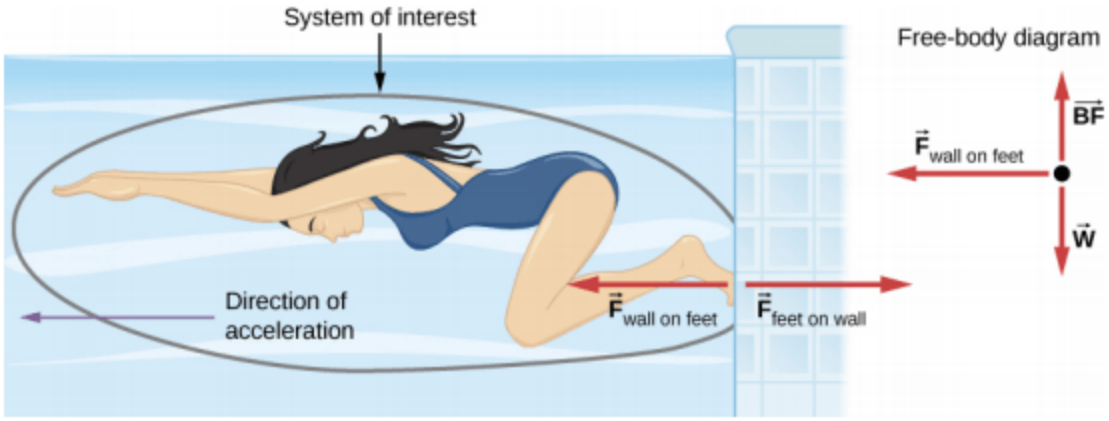


Figure 4.6.1: When the swimmer exerts a force on the wall, she accelerates in the opposite direction; in other words, the net external force on her is in the direction opposite of $F_{\text{feet on wall}}$. This opposition occurs because, in accordance with Newton's third law, the wall exerts a force $F_{\text{wall on feet}}$ on the swimmer that is equal in magnitude but in the direction opposite to the one she exerts on it. The line around the swimmer indicates the system of interest. Thus, the free-body diagram shows only $F_{\text{wall on feet}}$, w (the gravitational force), and BF, which is the buoyant force of the water supporting the swimmer's weight. The vertical forces w and BF cancel because there is no vertical acceleration.

Other examples of Newton's third law are easy to find:

- As a professor paces in front of a whiteboard, he exerts a force backward on the floor. The floor exerts a reaction force forward on the professor that causes him to accelerate forward.
- A car accelerates forward because the ground pushes forward on the drive wheels, in reaction to the drive wheels pushing backward on the ground. You can see evidence of the wheels pushing backward when tires spin on a gravel road and throw the rocks backward.
- Rockets move forward by expelling gas backward at high velocity. This means the rocket exerts a large backward force on the gas in the rocket combustion chamber; therefore, the gas exerts a large reaction force forward on the rocket. This reaction force, which pushes a body forward in response to a backward force, is called **thrust**. It is a common misconception that rockets propel themselves by pushing on the ground or on the air behind them. They actually work better in a vacuum, where they can more readily expel the exhaust gases.
- Helicopters create lift by pushing air down, thereby experiencing an upward reaction force.
- Birds and airplanes also fly by exerting force on the air in a direction opposite that of whatever force they need. For example, the wings of a bird force air downward and backward to get lift and move forward.
- An octopus propels itself in the water by ejecting water through a funnel from its body, similar to a jet ski.
- When a person pulls down on a vertical rope, the rope pulls up on the person (Figure 4.6.2).

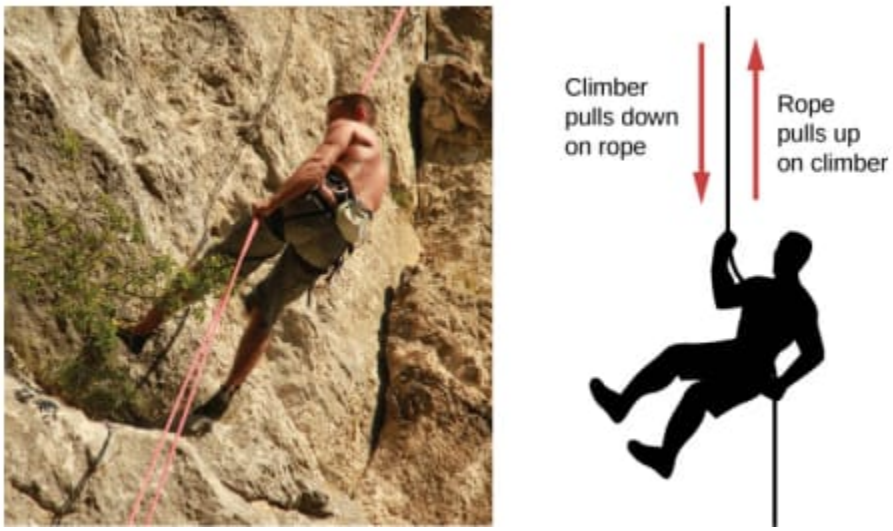


Figure 4.6.2: When the mountain climber pulls down on the rope, the rope pulls up on the mountain climber.

There are two important features of Newton's third law. First, the forces exerted (the action and reaction) are always equal in

magnitude but opposite in direction. Second, these forces are acting on different bodies or systems: A's force acts on B and B's force acts on A. In other words, the two forces are distinct forces that do not act on the same body. Thus, they do not cancel each other.

For the situation shown in Figure 5.2.5, the third law indicates that because the chair is pushing upward on the boy with force $\vec{C}$, he is pushing downward on the chair with force $-\vec{C}$. Similarly, he is pushing downward with forces $-\vec{F}$ and $-\vec{T}$ on the floor and table, respectively. Finally, since Earth pulls downward on the boy with force $\vec{w}$, he pulls upward on Earth with force $-\vec{w}$. If that student were to angrily pound the table in frustration, he would quickly learn the painful lesson (avoidable by studying Newton's laws) that the table hits back just as hard.

A person who is walking or running applies Newton's third law instinctively. For example, the runner in Figure $4.6.3$ pushes backward on the ground so that it pushes him forward.

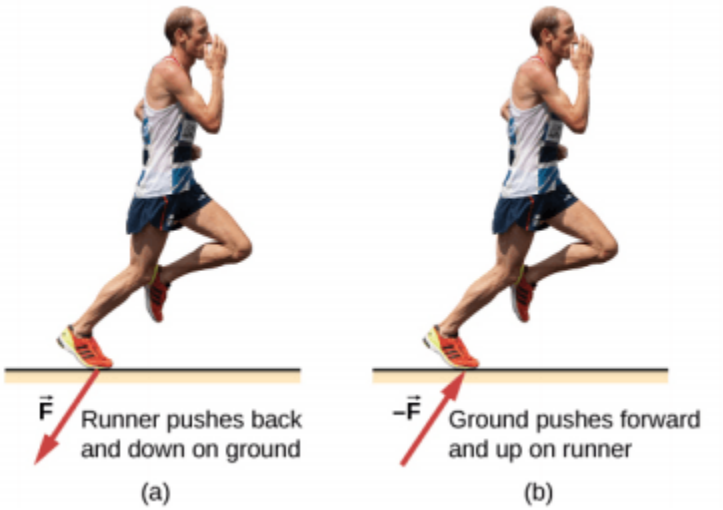


Figure $4.6.3$: The runner experiences Newton's third law. (a) A force is exerted by the runner on the ground. (b) The reaction force of the ground on the runner pushes him forward.

Example 5.9: Forces on a Stationary Object

The package in Figure $4.6.4$ is sitting on a scale. The forces on the package are $\vec{S}$, which is due to the scale, and $-\vec{w}$, which is due to Earth's gravitational field. The reaction forces that the package exerts are $-\vec{S}$ on the scale and $\vec{w}$ on Earth. Because the package is not accelerating, application of the second law yields

$$\vec{S} - \vec{w} = m\vec{a} = \vec{0}, \tag{4.6.2}$$

so

$$\vec{S} = \vec{w}. \tag{4.6.3}$$

Thus, the scale reading gives the magnitude of the package's weight. However, the scale does not measure the weight of the package; it measures the force $-\vec{S}$ on its surface. If the system is accelerating, $\vec{S}$ and $-\vec{w}$ would not be equal, as explained in Applications of Newton's Laws.

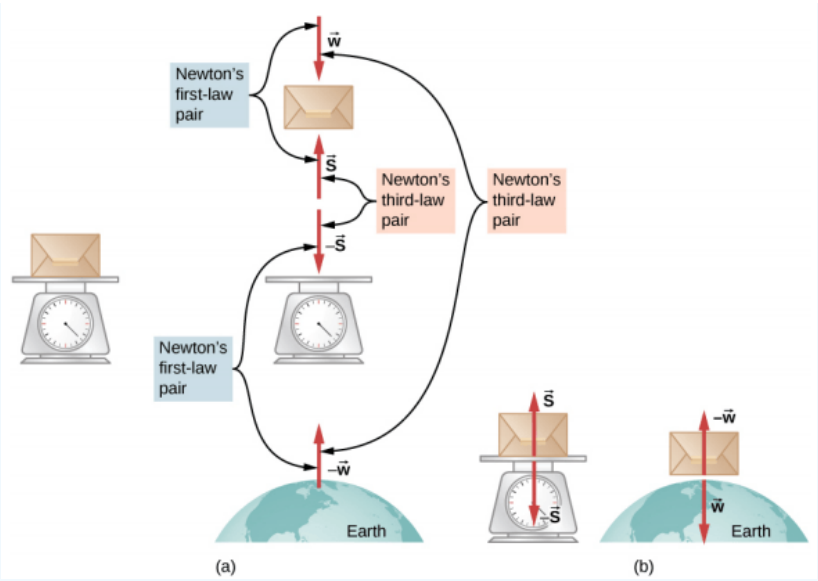


Figure 4.6.4: (a) The forces on a package sitting on a scale, along with their reaction forces. The force $\vec{w}$ is the weight of the package (the force due to Earth's gravity) and $\vec{S}$ is the force of the scale on the package. (b) Isolation of the package-scale system and the package-Earth system makes the action and reaction pairs clear.

Example 5.10: Getting Up to Speed: Choosing the Correct System

A physics professor pushes a cart of demonstration equipment to a lecture hall (Figure 4.6.5). Her mass is 65.0 kg, the cart's mass is 12.0 kg, and the equipment's mass is 7.0 kg. Calculate the acceleration produced when the professor exerts a backward force of 150 N on the floor. All forces opposing the motion, such as friction on the cart's wheels and air resistance, total 24.0 N.

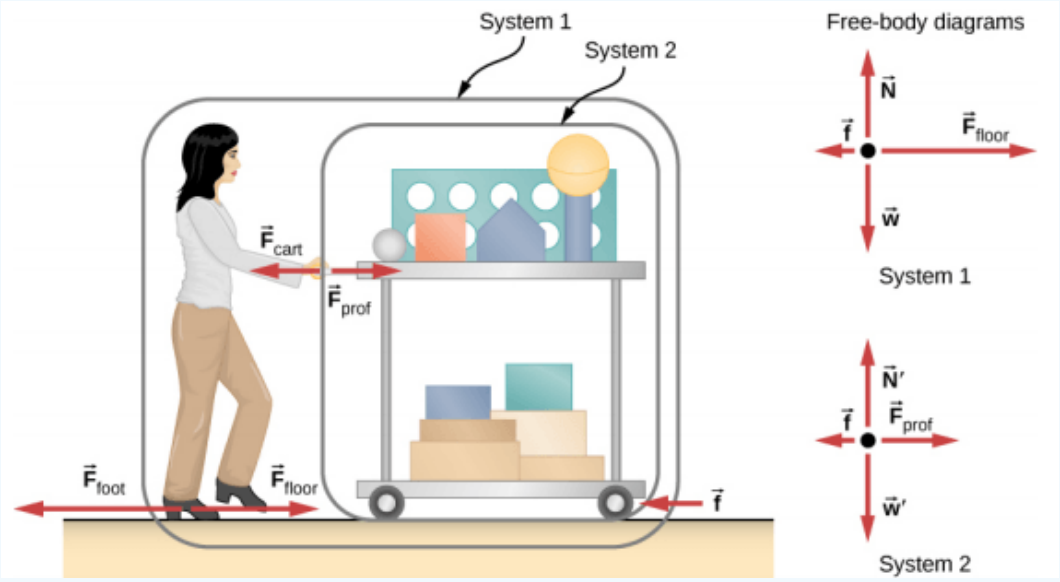


Figure 4.6.5: A professor pushes the cart with her demonstration equipment. The lengths of the arrows are proportional to the magnitudes of the forces (except for $\vec{f}$, because it is too small to drawn to scale). System 1 is appropriate for this example, because it asks for the acceleration of the entire group of objects. Only $\vec{F}_{floor}$ and $\vec{f}$ are external forces acting on System 1 along the line of motion. All other forces either cancel or act on the outside world. System 2 is chosen for the next example so that $\vec{F}_{prof}$ is an external force and enters into Newton's second law. The free-body diagrams, which serve as the basis for Newton's second law, vary with the system chosen.

Strategy

Since they accelerate as a unit, we define the system to be the professor, cart, and equipment. This is System 1 in Figure 4.6.5. The professor pushes backward with a force F_{foot} of 150 N. According to Newton's third law, the floor exerts a forward reaction force F_{floor} of 150 N on System 1. Because all motion is horizontal, we can assume there is no net force in the vertical direction. Therefore, the problem is one-dimensional along the horizontal direction. As noted, friction f opposes the motion and is thus in the opposite direction of F_{floor}. We do not include the forces F_{prof} or F_{cart} because these are internal forces, and we do not include F_{foot} because it acts on the floor, not on the system. There are no other significant forces acting on System 1. If the net external force can be found from all this information, we can use Newton's second law to find the acceleration as requested. See the free-body diagram in the figure.

Solution

Newton's second law is given by

$$a = \frac{F_{net}}{m}. \tag{4.6.4}$$

The net external force on System 1 is deduced from Figure 4.6.5 and the preceding discussion to be

$$F_{net} = F_{floor} - f = 150\ N - 24.0\ N = 126\ N. \tag{4.6.5}$$

The mass of System 1 is

$$m = (65.0 + 12.0 + 7.0)\ kg = 84\ kg. \tag{4.6.6}$$

These values of F_{net} and m produce an acceleration of

$$a = \frac{F_{net}}{m} = \frac{126\ N}{84\ kg} = 1.5\ m/s^2. \tag{4.6.7}$$

Significance

None of the forces between components of System 1, such as between the professor's hands and the cart, contribute to the net external force because they are internal to System 1. Another way to look at this is that forces between components of a system cancel because they are equal in magnitude and opposite in direction. For example, the force exerted by the professor on the cart results in an equal and opposite force back on the professor. In this case, both forces act on the same system and therefore cancel. Thus, internal forces (between components of a system) cancel. Choosing System 1 was crucial to solving this problem.

Example 5.11: Force on the Cart: Choosing a New System

Calculate the force the professor exerts on the cart in Figure 4.6.5, using data from the previous example if needed.

Strategy

If we define the system of interest as the cart plus the equipment (System 2 in Figure 4.6.5), then the net external force on System 2 is the force the professor exerts on the cart minus friction. The force she exerts on the cart, F_{prof}, is an external force acting on System 2. F_{prof} was internal to System 1, but it is external to System 2 and thus enters Newton's second law for this system.

Solution

Newton's second law can be used to find F_{prof}. We start with

$$a = \frac{F_{net}}{m}. \tag{4.6.8}$$

The magnitude of the net external force on System 2 is

$$F_{net} = F_{prof} - f. \tag{4.6.9}$$

We solve for F_{prof}, the desired quantity:

$$F_{prof} = F_{net} + f. \tag{4.6.10}$$

The value of f is given, so we must calculate net F_{net}. That can be done because both the acceleration and the mass of System 2 are known. Using Newton's second law, we see that

$$F_{net} = ma, \tag{4.6.11}$$

where the mass of System 2 is 19.0 kg (m = 12.0 kg + 7.0 kg) and its acceleration was found to be a = 1.5 m/s^2 in the previous example. Thus,

$$F_{net} = ma = (19.0\ kg)(1.5\ m/s^2) = 29\ N. \tag{4.6.12}$$

Now we can find the desired force:

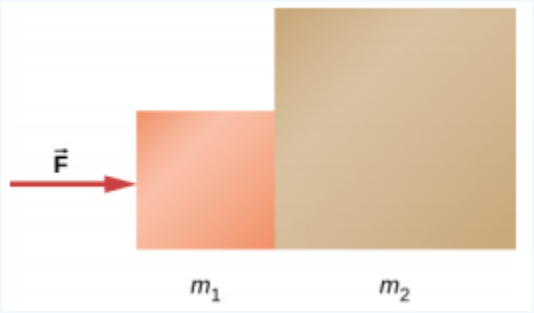
$$F_{prof} = F_{net} + f = 29\ N + 24.0\ N = 53\ N. \tag{4.6.13}$$

Significance

This force is significantly less than the 150-N force the professor exerted backward on the floor. Not all of that 150-N force is transmitted to the cart; some of it accelerates the professor. The choice of a system is an important analytical step both in solving problems and in thoroughly understanding the physics of the situation (which are not necessarily the same things).

Exercise 5.7

Two blocks are at rest and in contact on a frictionless surface as shown below, with m_1 = 2.0 kg, m_2 = 6.0 kg, and applied force 24 N. (a) Find the acceleration of the system of blocks. (b) Suppose that the blocks are later separated. What force will give the second block, with the mass of 6.0 kg, the same acceleration as the system of blocks?



Note

View this video to watch examples of action and reaction. View this video to watch examples of Newton's laws and internal and external forces.

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4.7: Common Forces

Learning Objectives

- Define normal and tension forces
- Distinguish between real and fictitious forces
- Apply Newton's laws of motion to solve problems involving a variety of forces

Forces are given many names, such as push, pull, thrust, and weight. Traditionally, forces have been grouped into several categories and given names relating to their source, how they are transmitted, or their effects. Several of these categories are discussed in this section, together with some interesting applications. Further examples of forces are discussed later in this text.

A Catalog of Forces: Normal, Tension, and Other Examples of Forces

A catalog of forces will be useful for reference as we solve various problems involving force and motion. These forces include normal force, tension, friction, and spring force.

Normal force

Weight (also called the force of gravity) is a pervasive force that acts at all times and must be counteracted to keep an object from falling. You must support the weight of a heavy object by pushing up on it when you hold it stationary, as illustrated in Figure 4.7.1(a). But how do inanimate objects like a table support the weight of a mass placed on them, such as shown in Figure 4.7.1(b)? When the bag of dog food is placed on the table, the table sags slightly under the load. This would be noticeable if the load were placed on a card table, but even a sturdy oak table deforms when a force is applied to it. Unless an object is deformed beyond its limit, it will exert a restoring force much like a deformed spring (or a trampoline or diving board). The greater the deformation, the greater the restoring force. Thus, when the load is placed on the table, the table sags until the restoring force becomes as large as the weight of the load. At this point, the net external force on the load is zero. That is the situation when the load is stationary on the table. The table sags quickly and the sag is slight, so we do not notice it. But it is similar to the sagging of a trampoline when you climb onto it.

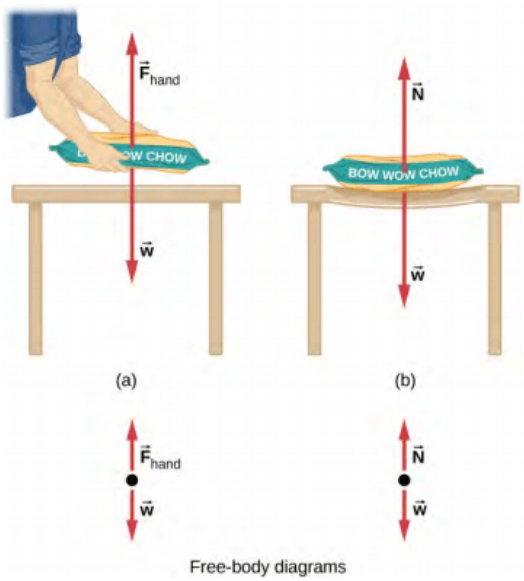


Figure 4.7.1: (a) The person holding the bag of dog food must supply an upward force $\vec{F}$ hand equal in magnitude and opposite in direction to the weight of the food $\vec{w}$ so that it doesn't drop to the ground. (b) The card table sags when the dog food is placed on it, much like a stiff trampoline. Elastic restoring forces in the table grow as it sags until they supply a force $\vec{N}$ equal in magnitude and opposite in direction to the weight of the load.

We must conclude that whatever supports a load, be it animate or not, must supply an upward force equal to the weight of the load, as we assumed in a few of the previous examples. If the force supporting the weight of an object, or a load, is perpendicular to the surface of contact between the load and its support, this force is defined as a **normal force** and here is given by the symbol $\vec{N}$. (This is not the newton unit for force, or N.) The word **normal** means perpendicular to a surface. This means that the normal force experienced by an object resting on a horizontal surface can be expressed in vector form as follows:

$$\vec{N} = -m\vec{g}.$$

In scalar form, this becomes

$$N = mg.$$

The normal force can be less than the object's weight if the object is on an incline.

Example 5.12: Weight on an Incline

Consider the skier on the slope in Figure 4.7.2. Her mass including equipment is 60.0 kg. (a) What is her acceleration if friction is negligible? (b) What is her acceleration if friction is 45.0 N?

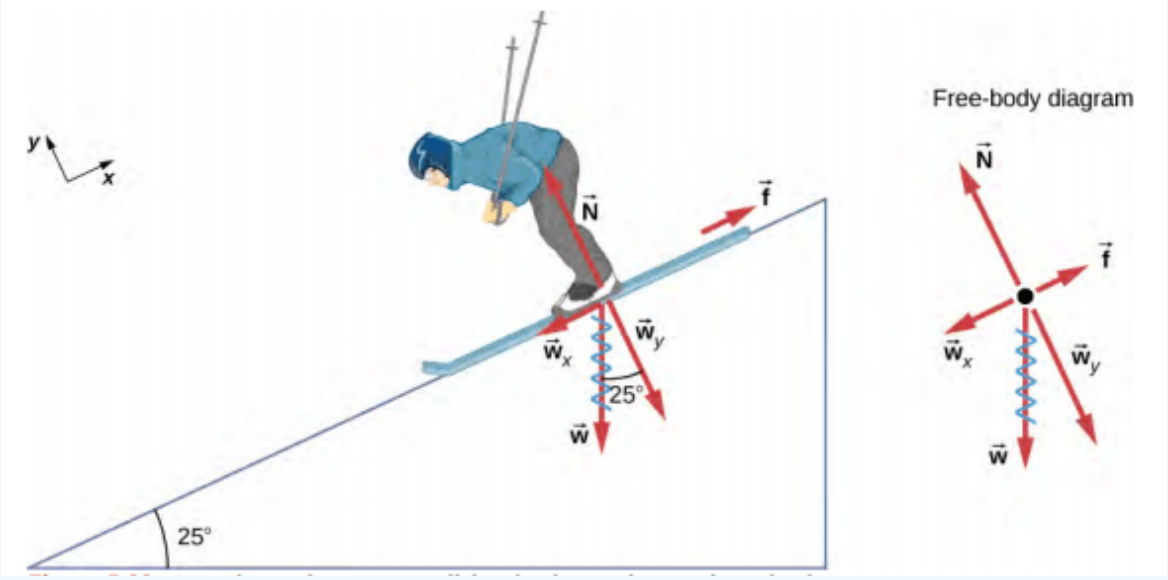


Figure 4.7.2: Since the acceleration is parallel to the slope and acting down the slope, it is most convenient to project all forces onto a coordinate system where one axis is parallel to the slope and the other is perpendicular to it (axes shown to the left of the skier). $\vec{N}$ is perpendicular to the slope and $\vec{f}$ is parallel to the slope, but $\vec{w}$ has components along both axes, namely, w_y and w_x. Here, $\vec{w}$ has a squiggly line to show that it has been replaced by these components. The force $\vec{N}$ is equal in magnitude to w_y, so there is no acceleration perpendicular to the slope, but f is less than w_x, so there is a downslope acceleration (along the axis parallel to the slope).

Strategy

This is a two-dimensional problem, since not all forces on the skier (the system of interest) are parallel. The approach we have used in two-dimensional kinematics also works well here. Choose a convenient coordinate system and project the vectors onto its axes, creating two one-dimensional problems to solve. The most convenient coordinate system for motion on an incline is one that has one coordinate parallel to the slope and one perpendicular to the slope. (Motions along mutually perpendicular axes are independent.) We use x and y for the parallel and perpendicular directions, respectively. This choice of axes simplifies this type of problem, because there is no motion perpendicular to the slope and the acceleration is downslope. Regarding the forces, friction is drawn in opposition to motion (friction always opposes forward motion) and is always parallel to the slope, w_x is drawn parallel to the slope and downslope (it causes the motion of the skier down the slope), and w_y is drawn as the component of weight perpendicular to the slope. Then, we can consider the separate problems of forces parallel to the slope and forces perpendicular to the slope.

Solution

The magnitude of the component of weight parallel to the slope is

$$w_x = w \sin 25^o = mg \sin 25^o,$$

and the magnitude of the component of the weight perpendicular to the slope is

$$w_y = w \cos 25^o = mg \cos 25^o.$$

a. Neglect friction. Since the acceleration is parallel to the slope, we need only consider forces parallel to the slope. (Forces perpendicular to the slope add to zero, since there is no acceleration in that direction.) The forces parallel to the slope are the component of the skier's weight parallel to slope w_x and friction f. Using Newton's second law, with subscripts to denote quantities parallel to the slope,

$$a_x = \frac{F_{net\ x}}{m} \tag{4.7.1}$$

where $F_{net\ x} = w_x$ - mg sin 25°, assuming no friction for this part. Therefore,

$$a_x = \frac{F_{net\ x}}{m} = \frac{mg \sin 25^o}{m} = g \sin 25^o \tag{4.7.2}$$

$$(9.80\ m/s^2)(0.4226) = 4.14\ m/s^2 \tag{4.7.3}$$

is the acceleration.

b. Include friction. We have a given value for friction, and we know its direction is parallel to the slope and it opposes motion between surfaces in contact. So the net external force is

$$F_{net\ x} = w_x - f. \tag{4.7.4}$$

Substituting this into Newton's second law, $a_x = \frac{F_{net\ x}}{m}$, gives

$$a_x = \frac{F_{net\ x}}{m} = \frac{w_x - f}{m} = \frac{mg \sin 25^o - f}{m}. \tag{4.7.5}$$

We substitute known values to obtain

$$a_x = \frac{(60.0\ kg)(9.80\ m/s^2)(0.4226) - 45.0\ N}{60.0\ kg}. \tag{4.7.6}$$

This give us

$$a_x = 3.39\ m/s^2, \tag{4.7.7}$$

which is the acceleration parallel to the incline when there is 45.0 N of opposing friction.

Significance

Since friction always opposes motion between surfaces, the acceleration is smaller when there is friction than when there is none. It is a general result that if friction on an incline is negligible, then the acceleration down the incline is a = g sin θ, regardless of mass. As discussed previously, all objects fall with the same acceleration in the absence of air resistance. Similarly, all objects, regardless of mass, slide down a frictionless incline with the same acceleration (if the angle is the same).

When an object rests on an incline that makes an angle θ with the horizontal, the force of gravity acting on the object is divided into two components: a force acting perpendicular to the plane, wy , and a force acting parallel to the plane, wx (Figure 4.7.3). The normal force $\vec{N}$ is typically equal in magnitude and opposite in direction to the perpendicular component of the weight w_y. The force acting parallel to the plane, w_x, causes the object to accelerate down the incline.

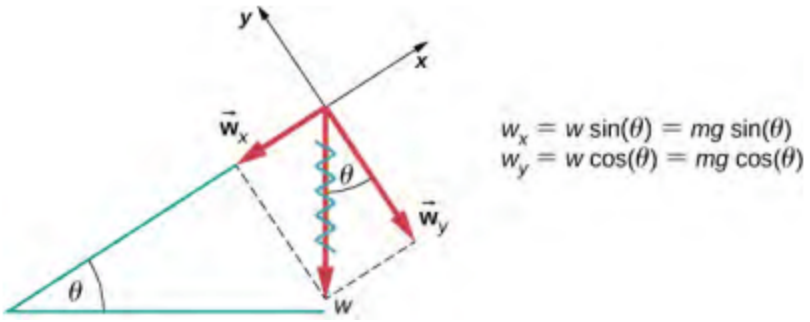


Figure 4.7.3: An object rests on an incline that makes an angle θ with the horizontal.

Be careful when resolving the weight of the object into components. If the incline is at an angle θ to the horizontal, then the magnitudes of the weight components are

$$w_x = w\sin\theta = mg\sin\theta$$

and

$$w_y = w\cos\theta = mg\cos\theta$$

We use the second equation to write the normal force experienced by an object resting on an inclined plane:

$$N = mg\cos\theta.$$

Instead of memorizing these equations, it is helpful to be able to determine them from reason. To do this, we draw the right angle formed by the three weight vectors. The angle θ of the incline is the same as the angle formed between w and w_y. Knowing this property, we can use trigonometry to determine the magnitude of the weight components:

$$\cos\theta = \frac{w_y}{w}, \quad w_y = w\cos\theta = mg\cos\theta$$

$$\sin\theta = \frac{w_x}{w}, \quad w_x = w\sin\theta = mg\sin\theta$$

Exercise 5.8

A force of 1150 N acts parallel to a ramp to push a 250-kg gun safe into a moving van. The ramp is frictionless and inclined at 17°. (a) What is the acceleration of the safe up the ramp? (b) If we consider friction in this problem, with a friction force of 120 N, what is the acceleration of the safe?

Tension

A **tension** is a force along the length of a medium; in particular, it is a pulling force that acts along a stretched flexible connector, such as a rope or cable. The word "tension" comes from a Latin word meaning "to stretch." Not coincidentally, the flexible cords that carry muscle forces to other parts of the body are called tendons. Any flexible connector, such as a string, rope, chain, wire, or cable, can only exert a pull parallel to its length; thus, a force carried by a flexible connector is a tension with a direction parallel to the connector. Tension is a pull in a connector. Consider the phrase: "You can't push a rope." Instead, tension force pulls outward along the two ends of a rope. Consider a person holding a mass on a rope, as shown in Figure 4.7.4. If the 5.00-kg mass in the figure is stationary, then its acceleration is zero and the net force is zero. The only external forces acting on the mass are its weight and the tension supplied by the rope. Thus,

$$F_{net} = T - w = 0,$$

where T and w are the magnitudes of the tension and weight, respectively, and their signs indicate direction, with up being positive. As we proved using Newton's second law, the tension equals the weight of the supported mass:

$$T = w = mg.$$

Thus, for a 5.00-kg mass (neglecting the mass of the rope), we see that

$$T = mg = (5.00\ kg)(9.80\ m/s^2) = 49.0\ N.$$

If we cut the rope and insert a spring, the spring would extend a length corresponding to a force of 49.0 N, providing a direct observation and measure of the tension force in the rope.

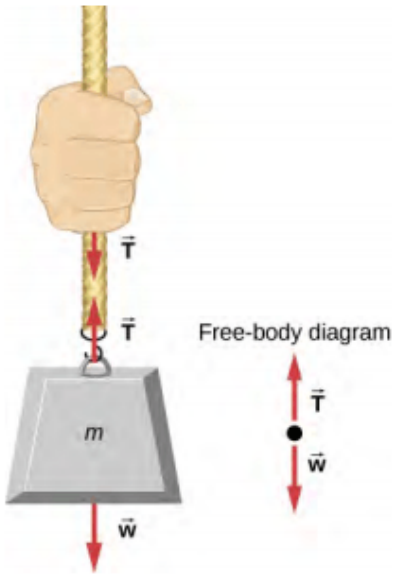


Figure 4.7.4: When a perfectly flexible connector (one requiring no force to bend it) such as this rope transmits a force $\vec{T}$, that force must be parallel to the length of the rope, as shown. By Newton's third law, the rope pulls with equal force but in opposite directions on the hand and the supported mass (neglecting the weight of the rope). The rope is the medium that carries the equal and opposite forces between the two objects. The tension anywhere in the rope between the hand and the mass is equal. Once you have determined the tension in one location, you have determined the tension at all locations along the rope.

Flexible connectors are often used to transmit forces around corners, such as in a hospital traction system, a tendon, or a bicycle brake cable. If there is no friction, the tension transmission is undiminished; only its direction changes, and it is always parallel to the flexible connector, as shown in Figure 4.7.5.

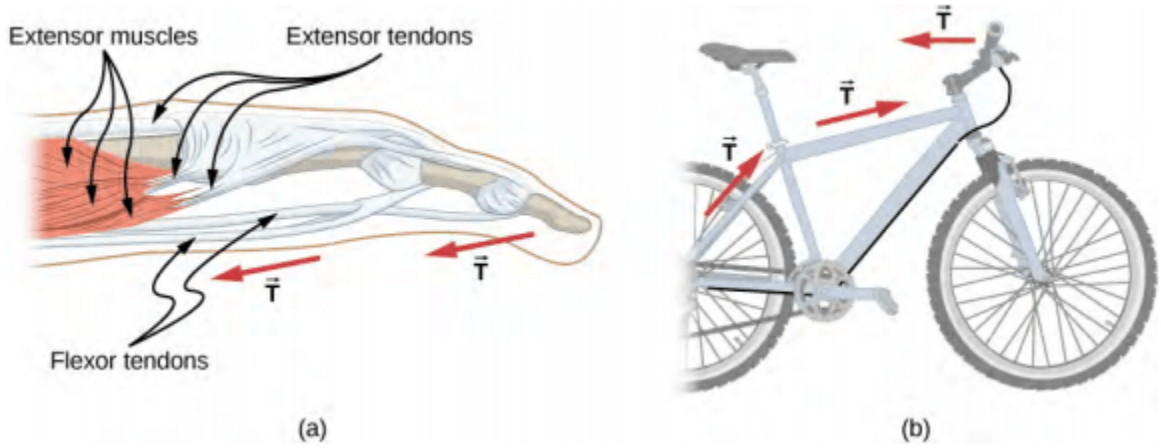


Figure 4.7.5: (a) Tendons in the finger carry force T from the muscles to other parts of the finger, usually changing the force's direction but not its magnitude (the tendons are relatively friction free). (b) The brake cable on a bicycle carries the tension T from the brake lever on the handlebars to the brake mechanism. Again, the direction but not the magnitude of T is changed.

: What is the Tension in a Tightrope?

Calculate the tension in the wire supporting the 70.0-kg tightrope walker shown in Figure 4.7.6.

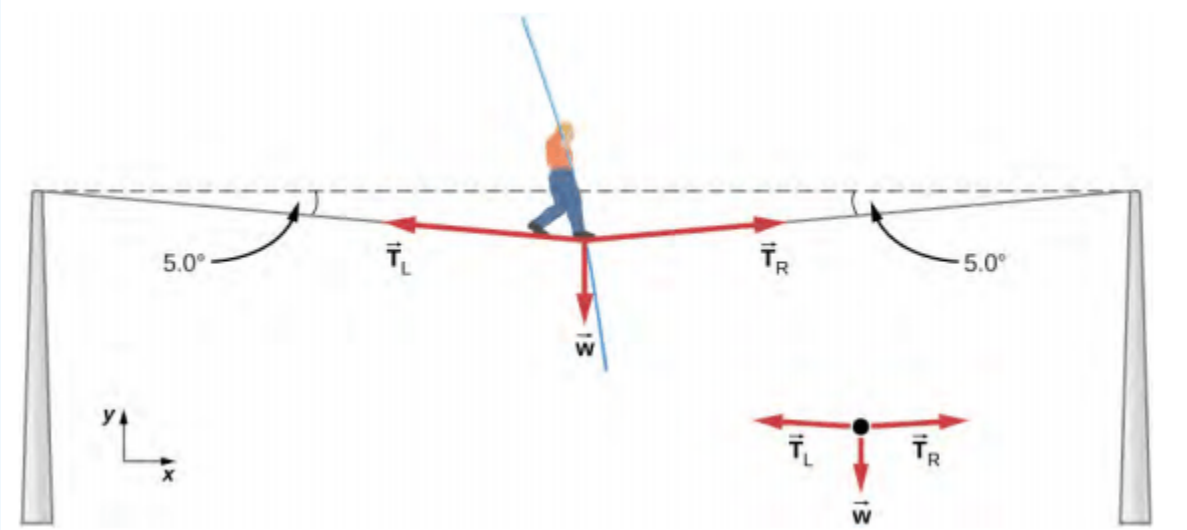


Figure 4.7.6: The weight of a tightrope walker causes a wire to sag by 5.0°. The system of interest is the point in the wire at which the tightrope walker is standing.

Strategy

As you can see in Figure 4.7.6, the wire is bent under the person's weight. Thus, the tension on either side of the person has an upward component that can support his weight. As usual, forces are vectors represented pictorially by arrows that have the same direction as the forces and lengths proportional to their magnitudes. The system is the tightrope walker, and the only external forces acting on him are his weight $\vec{w}$ and the two tensions $\vec{T}_L$ (left tension) and $\vec{T}_R$ (right tension). It is reasonable to neglect the weight of the wire. The net external force is zero, because the system is static. We can use trigonometry to find the tensions. One conclusion is possible at the outset—we can see from Figure 4.7.6(b) that the magnitudes of the tensions T_L and T_R must be equal. We know this because there is no horizontal acceleration in the rope and the only forces acting to the left and right are T_L and T_R. Thus, the magnitude of those horizontal components of the forces must be equal so that they cancel each other out.

Whenever we have two-dimensional vector problems in which no two vectors are parallel, the easiest method of solution is to pick a convenient coordinate system and project the vectors onto its axes. In this case, the best coordinate system has one horizontal axis (x) and one vertical axis (y).

Solution

First, we need to resolve the tension vectors into their horizontal and vertical components. It helps to look at a new free-body diagram showing all horizontal and vertical components of each force acting on the system (Figure 4.7.7).

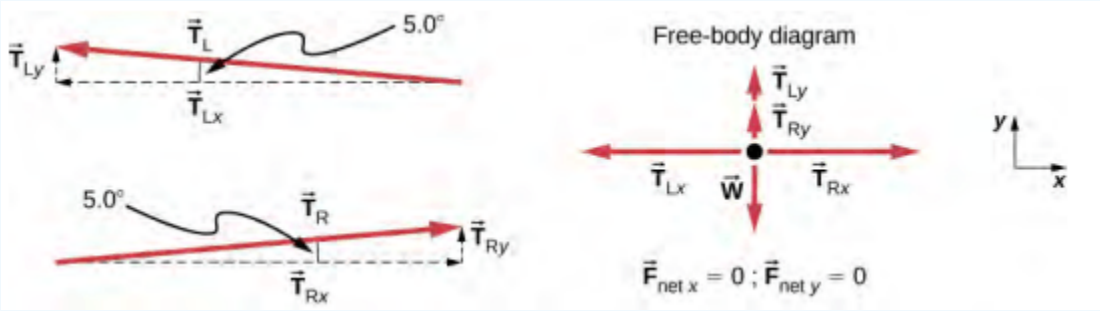


Figure 4.7.7: When the vectors are projected onto vertical and horizontal axes, their components along these axes must add to zero, since the tightrope walker is stationary. The small angle results in T being much greater than w.

Consider the horizontal components of the forces (denoted with a subscript x):

$$F_{netx} = T_{Rx} - T_{Lx}.$$

The net external horizontal force $F_{net\ x} = 0$, since the person is stationary. Thus,

$$F_{netx} = 0 = T_{Rx} - T_{Lx}.$$

$$T_{Lx} = T_{Rx}.$$

Now observe Figure 4.7.7. You can use trigonometry to determine the magnitude of T_L and T_R:

$$\cos 5.0^o = \frac{T_{Lx}}{T_L}, \quad T_{Lx} = T_L \cos 5.0^o$$

$$\cos 5.0^o = \frac{T_{Rx}}{T_R}, \quad T_{Rx} = T_R \cos 5.0^o.$$

Equating T_{Lx} and T_{Rx}:

$$T_L \cos 5.0^o = T_R \cos 5.0^o.$$

Thus,

$$T_L = T_R = T,$$

as predicted. Now, considering the vertical components (denoted by a subscript y), we can solve for T. Again, since the person is stationary, Newton's second law implies that $F_{net\ y} = 0$. Thus, as illustrated in the free-body diagram,

$$F_{nety} = T_{Ly} + T_{Ry} - w = 0.$$

We can use trigonometry to determine the relationships among T_{Ly}, T_{Ry}, and T. As we determined from the analysis in the horizontal direction, $T_L = T_R = T$:

$$\sin 5.0^o = \frac{T_{Ly}}{T_L}, \quad T_{Ly} = T_L \sin 5.0^o = T \sin 5.0^o$$

$$\sin 5.0^o = \frac{T_{Ry}}{T_R}, \quad T_{Ry} = T_R \sin 5.0^o = T \sin 5.0^o.$$

Now we can substitute the vales for T_{Ly} and T_{Ry}, into the net force equation in the vertical direction:

$$F_{nety} = T_{Ly} + T_{Ry} - w = 0$$

$$F_{nety} = 0 = T \sin 5.0^o + T \sin 5.0^o - w = 0$$

$$2T \sin 5.0^o - w = 0$$

$$2T \sin 5.0^o = w$$

and

$$T = \frac{w}{2 \sin 5.0^o} = \frac{mg}{2 \sin 5.0^o},$$

so

$$T = \frac{(70.0\ kg)(9.80\ m/s^2)}{2(0.0872)},$$

and the tension is

$$T = 3930\ N.$$

Significance

The vertical tension in the wire acts as a force that supports the weight of the tightrope walker. The tension is almost six times the 686-N weight of the tightrope walker. Since the wire is nearly horizontal, the vertical component of its tension is only a fraction of the tension in the wire. The large horizontal components are in opposite directions and cancel, so most of the tension in the wire is not used to support the weight of the tightrope walker.

If we wish to create a large tension, all we have to do is exert a force perpendicular to a taut flexible connector, as illustrated in Figure 4.7.6. As we saw in Example 5.13, the weight of the tightrope walker acts as a force perpendicular to the rope. We saw that the tension in the rope is related to the weight of the tightrope walker in the following way:

$$T = \frac{w}{2\sin\theta}.$$

We can extend this expression to describe the tension T created when a perpendicular force ($F_\perp$) is exerted at the middle of a flexible connector:

$$T = \frac{F_\perp}{2\sin\theta}.$$

The angle between the horizontal and the bent connector is represented by θ. In this case, T becomes large as θ approaches zero. Even the relatively small weight of any flexible connector will cause it to sag, since an infinite tension would result if it were horizontal (i.e., θ = 0 and sin θ = 0). For example, Figure 4.7.8 shows a situation where we wish to pull a car out of the mud when no tow truck is available. Each time the car moves forward, the chain is tightened to keep it as straight as possible. The tension in the chain is given by $T = \frac{F_\perp}{2\sin\theta}$, and since θ is small, T is large. This situation is analogous to the tightrope walker, except that the tensions shown here are those transmitted to the car and the tree rather than those acting at the point where $F_\perp$ is applied.

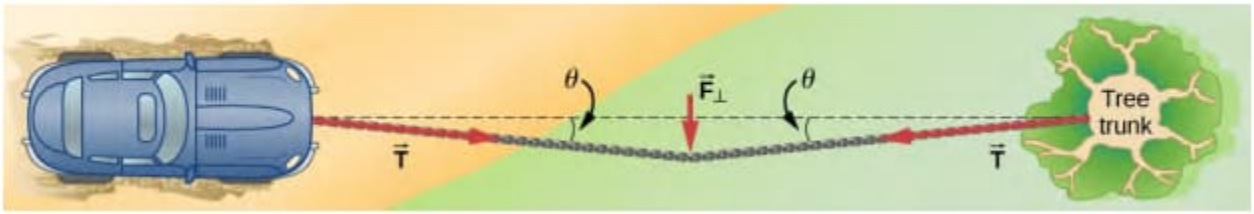


Figure 4.7.8: We can create a large tension in the chain—and potentially a big mess—by pushing on it perpendicular to its length, as shown.

Exercise 5.9

One end of a 3.0-m rope is tied to a tree; the other end is tied to a car stuck in the mud. The motorist pulls sideways on the midpoint of the rope, displacing it a distance of 0.25 m. If he exerts a force of 200.0 N under these conditions, determine the force exerted on the car.

In Applications of Newton's Laws, we extend the discussion on tension in a cable to include cases in which the angles shown are not equal.

Friction

Friction is a resistive force opposing motion or its tendency. Imagine an object at rest on a horizontal surface. The net force acting on the object must be zero, leading to equality of the weight and the normal force, which act in opposite directions. If the surface is tilted, the normal force balances the component of the weight perpendicular to the surface. If the object does not slide downward, the component of the weight parallel to the inclined plane is balanced by friction. Friction is discussed in greater detail in the next chapter.

Spring force

A spring is a special medium with a specific atomic structure that has the ability to restore its shape, if deformed. To restore its

shape, a spring exerts a restoring force that is proportional to and in the opposite direction in which it is stretched or compressed. This is the statement of a law known as Hooke's law, which has the mathematical form

$$\vec{F} = -k\vec{x}.$$

The constant of proportionality k is a measure of the spring's stiffness. The line of action of this force is parallel to the spring axis, and the sense of the force is in the opposite direction of the displacement vector (Figure $4.7.9$). The displacement must be measured from the relaxed position; x = 0 when the spring is relaxed.

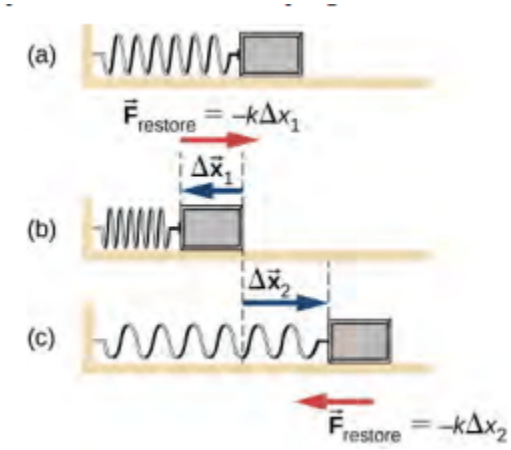


Figure $4.7.9$: A spring exerts its force proportional to a displacement, whether it is compressed or stretched. (a) The spring is in a relaxed position and exerts no force on the block. (b) The spring is compressed by displacement $\Delta\vec{x}_1$ of the object and exerts restoring force $-k\Delta\vec{x}_1$. (c) The spring is stretched by displacement $\Delta\vec{x}_1$ of the object and exerts restoring force $-k\Delta\vec{x}_2$.

Real Forces and Inertial Frames

There is another distinction among forces: Some forces are real, whereas others are not. **Real forces** have some physical origin, such as a gravitational pull. In contrast, **fictitious forces** arise simply because an observer is in an accelerating or noninertial frame of reference, such as one that rotates (like a merry-go-round) or undergoes linear acceleration (like a car slowing down). For example, if a satellite is heading due north above Earth's Northern Hemisphere, then to an observer on Earth, it will appear to experience a force to the west that has no physical origin. Instead, Earth is rotating toward the east and moves east under the satellite. In Earth's frame, this looks like a westward force on the satellite, or it can be interpreted as a violation of Newton's first law (the law of inertia). We can identify a fictitious force by asking the question, "What is the reaction force?" If we cannot name the reaction force, then the force we are considering is fictitious. In the example of the satellite, the reaction force would have to be an eastward force on Earth. Recall that an inertial frame of reference is one in which all forces are real and, equivalently, one in which Newton's laws have the simple forms given in this chapter.

Earth's rotation is slow enough that Earth is nearly an inertial frame. You ordinarily must perform precise experiments to observe fictitious forces and the slight departures from Newton's laws, such as the effect just described. On a large scale, such as for the rotation of weather systems and ocean currents, the effects can be easily observed (Figure $4.7.10$).



Figure 4.7.10: Hurricane Fran is shown heading toward the southeastern coast of the United States in September 1996. Notice the characteristic “eye” shape of the hurricane. This is a result of the Coriolis effect, which is the deflection of objects (in this case, air) when considered in a rotating frame of reference, like the spin of Earth.

The crucial factor in determining whether a frame of reference is inertial is whether it accelerates or rotates relative to a known inertial frame. Unless stated otherwise, all phenomena discussed in this text are in inertial frames.

The forces discussed in this section are real forces, but they are not the only real forces. Lift and thrust, for example, are more specialized real forces. In the long list of forces, are some more basic than others? Are some different manifestations of the same underlying force? The answer to both questions is yes, as you will see in the treatment of modern physics later in the text

Simulation

Explore forces and motion in this interactive simulation as you push household objects up and down a ramp. Lower and raise the ramp to see how the angle of inclination affects the parallel forces. Graphs show forces, energy, and work.

Simulation

Stretch and compress springs in this activity to explore the relationships among force, spring constant, and displacement. Investigate what happens when two springs are connected in series and in parallel.

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4.8: Drawing Free-Body Diagrams

Learning Objectives

- Explain the rules for drawing a free-body diagram
- Construct free-body diagrams for different situations

The first step in describing and analyzing most phenomena in physics involves the careful drawing of a free-body diagram. Free-body diagrams have been used in examples throughout this chapter. Remember that a free-body diagram must only include the external forces acting on the body of interest. Once we have drawn an accurate free-body diagram, we can apply Newton's first law if the body is in equilibrium (balanced forces; that is, $F_{net} = 0$) or Newton's second law if the body is accelerating (unbalanced force; that is, $F_{net} \neq 0$).

In Forces, we gave a brief problem-solving strategy to help you understand free-body diagrams. Here, we add some details to the strategy that will help you in constructing these diagrams.

Problem-Solving Strategy: Constructing Free-Body Diagrams

Observe the following rules when constructing a free-body diagram:

1. Draw the object under consideration; it does not have to be artistic. At first, you may want to draw a circle around the object of interest to be sure you focus on labeling the forces acting on the object. If you are treating the object as a particle (no size or shape and no rotation), represent the object as a point. We often place this point at the origin of an xy-coordinate system.
2. Include all forces that act on the object, representing these forces as vectors. Consider the types of forces described in Common Forces—normal force, friction, tension, and spring force—as well as weight and applied force. Do not include the net force on the object. With the exception of gravity, all of the forces we have discussed require direct contact with the object. However, forces that the object exerts on its environment must not be included. We never include both forces of an action-reaction pair.
3. Convert the free-body diagram into a more detailed diagram showing the x- and y-components of a given force (this is often helpful when solving a problem using Newton's first or second law). In this case, place a squiggly line through the original vector to show that it is no longer in play—it has been replaced by its x- and y-components.
4. If there are two or more objects, or bodies, in the problem, draw a separate free-body diagram for each object.

Note: If there is acceleration, we do not directly include it in the free-body diagram; however, it may help to indicate acceleration outside the free-body diagram. You can label it in a different color to indicate that it is separate from the free-body diagram.

Let's apply the problem-solving strategy in drawing a free-body diagram for a sled. In Figure $4.8.1a$, a sled is pulled by force $\vec{P}$ at an angle of 30°. In part (b), we show a free-body diagram for this situation, as described by steps 1 and 2 of the problem-solving strategy. In part (c), we show all forces in terms of their x- and y-components, in keeping with step 3.

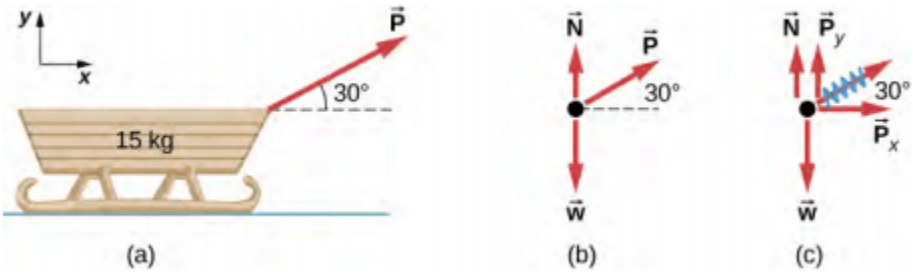


Figure $4.8.1$: (a) A moving sled is shown as (b) a free-body diagram and (c) a free-body diagram with force components.

Example 4.8.1: Two Blocks on an Inclined Plane

Construct the free-body diagram for object A and object B in Figure 4.8.1.

Strategy

We follow the four steps listed in the problem-solving strategy.

Solution

We start by creating a diagram for the first object of interest. In Figure 4.8.2*a*, object A is isolated (circled) and represented by a dot.

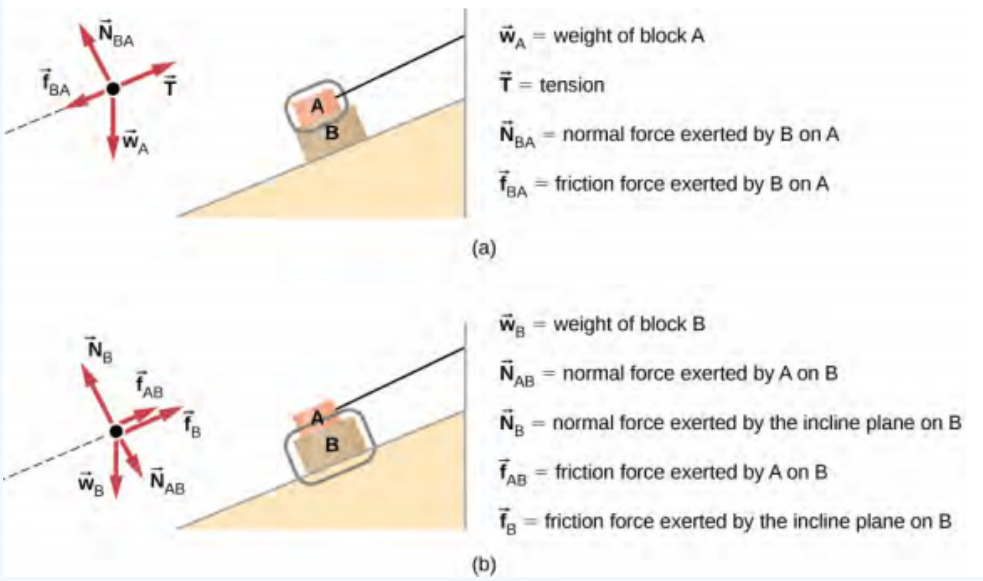


Figure 4.8.2: (a) The free-body diagram for isolated object A. (b) The free-body diagram for isolated object B. Comparing the two drawings, we see that friction acts in the opposite direction in the two figures. Because object A experiences a force that tends to pull it to the right, friction must act to the left. Because object B experiences a component of its weight that pulls it to the left, down the incline, the friction force must oppose it and act up the ramp. Friction always acts opposite the intended direction of motion.

We now include any force that acts on the body. Here, no applied force is present. The weight of the object acts as a force pointing vertically downward, and the presence of the cord indicates a force of tension pointing away from the object. Object A has one interface and hence experiences a normal force, directed away from the interface. The source of this force is object B, and this normal force is labeled accordingly. Since object B has a tendency to slide down, object A has a tendency to slide up with respect to the interface, so the friction f_{BA} is directed downward parallel to the inclined plane.

As noted in step 4 of the problem-solving strategy, we then construct the free-body diagram in Figure 5.32(b) using the same approach. Object B experiences two normal forces and two friction forces due to the presence of two contact surfaces. The interface with the inclined plane exerts external forces of N_B and f_B, and the interface with object B exerts the normal force N_{AB} and friction f_{AB}; N_{AB} is directed away from object B, and f_{AB} is opposing the tendency of the relative motion of object B with respect to object A.

Significance

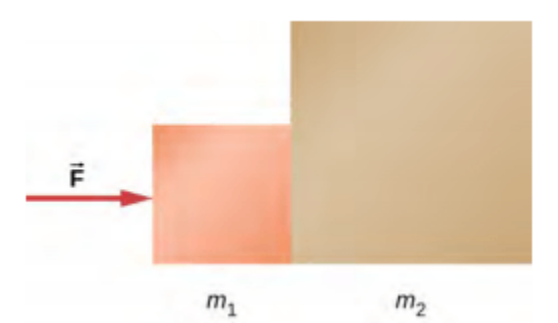
The object under consideration in each part of this problem was circled in gray. When you are first learning how to draw free-body diagrams, you will find it helpful to circle the object before deciding what forces are acting on that particular object. This focuses your attention, preventing you from considering forces that are not acting on the body

Example 4.8.2: Two Blocks in Contact

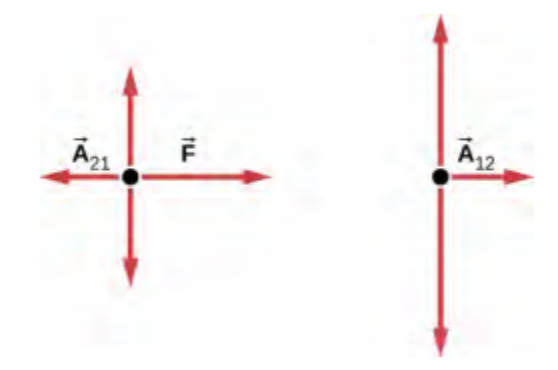
A force is applied to two blocks in contact, as shown.

Strategy

Draw a free-body diagram for each block. Be sure to consider Newton's third law at the interface where the two blocks touch.



Solution



Significance

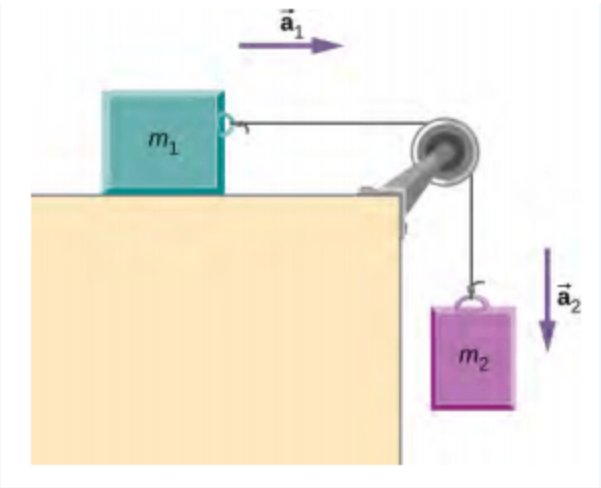
$\vec{A}_{21}$ is the action force of block 2 on block 1. $\vec{A}_{12}$ is the reaction force of block 1 on block 2. We use these free-body diagrams in Applications of Newton's Laws.

Example 4.8.3: Block on the Table (Coupled Blocks)

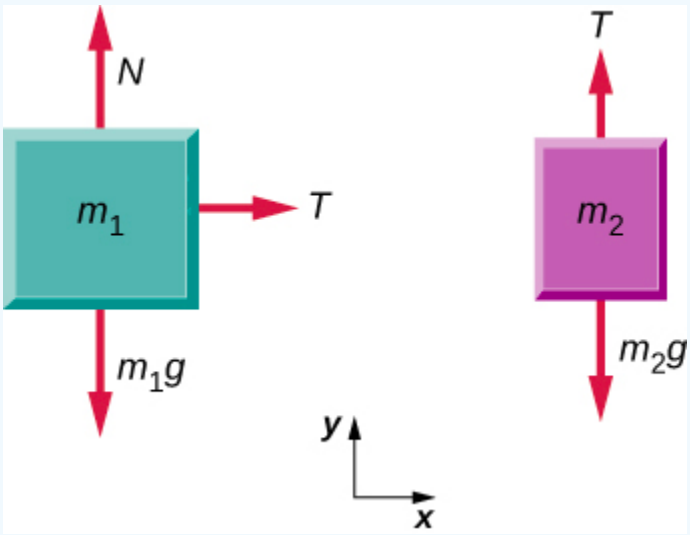
A block rests on the table, as shown. A light rope is attached to it and runs over a pulley. The other end of the rope is attached to a second block. The two blocks are said to be coupled. Block m_2 exerts a force due to its weight, which causes the system (two blocks and a string) to accelerate.

Strategy

We assume that the string has no mass so that we do not have to consider it as a separate object. Draw a free-body diagram for each block.



Solution

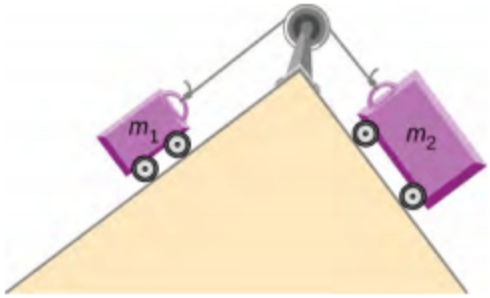


Significance

Each block accelerates (notice the labels shown for $\vec{a}_1$ and $\vec{a}_2$); however, assuming the string remains taut, they accelerate at the same rate. Thus, we have $|\vec{a}_1| = |\vec{a}_2|$. If we were to continue solving the problem, we could simply call the acceleration $\vec{a}$. Also, we use two free-body diagrams because we are usually finding tension T, which may require us to use a system of two equations in this type of problem. The tension is the same on both m_1 and m_2.

Exercise 4.8.1

a. Draw the free-body diagram for the situation shown.
b. Redraw it showing components; use x-axes parallel to the two ramps.



Simulation

View this simulation to predict, qualitatively, how an external force will affect the speed and direction of an object's motion. Explain the effects with the help of a free-body diagram. Use free-body diagrams to draw position, velocity, acceleration, and force graphs, and vice versa. Explain how the graphs relate to one another. Given a scenario or a graph, sketch all four graphs.

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4.9: Newton's Laws of Motion (Exercises)

Conceptual Questions

5.1 Forces

1. What properties do forces have that allow us to classify them as vectors?

5.2 Newton's First Law

2. Taking a frame attached to Earth as inertial, which of the following objects cannot have inertial frames attached to them, and which are inertial reference frames?
   a. A car moving at constant velocity
   b. A car that is accelerating
   c. An elevator in free fall
   d. A space capsule orbiting Earth
   e. An elevator descending uniformly
3. A woman was transporting an open box of cupcakes to a school party. The car in front of her stopped suddenly; she applied her brakes immediately. She was wearing her seat belt and suffered no physical harm (just a great deal of embarrassment), but the cupcakes flew into the dashboard and became “smushcakes.” Explain what happened.

5.3 Newton's Second Law

4. Why can we neglect forces such as those holding a body together when we apply Newton’s second law?
5. A rock is thrown straight up. At the top of the trajectory, the velocity is momentarily zero. Does this imply that the force acting on the object is zero? Explain your answer.

5.4 Mass and Weight

6. What is the relationship between weight and mass? Which is an intrinsic, unchanging property of a body?
7. How much does a 70-kg astronaut weight in space, far from any celestial body? What is her mass at this location?
8. Which of the following statements is accurate?
   a. Mass and weight are the same thing expressed in different units.
   b. If an object has no weight, it must have no mass.
   c. If the weight of an object varies, so must the mass.
   d. Mass and inertia are different concepts.
   e. Weight is always proportional to mass.
9. When you stand on Earth, your feet push against it with a force equal to your weight. Why doesn’t Earth accelerate away from you?
10. How would you give the value of $\vec{g}$ in vector form?

5.5 Newton’s Third Law

11. Identify the action and reaction forces in the following situations:
    a. Earth attracts the Moon,
    b. a boy kicks a football,
    c. a rocket accelerates upward,
    d. a car accelerates forward,
    e. a high jumper leaps, and
    f. a bullet is shot from a gun.
12. Suppose that you are holding a cup of coffee in your hand. Identify all forces on the cup and the reaction to each force.
13. (a) Why does an ordinary rifle recoil (kick backward) when fired? (b) The barrel of a recoilless rifle is open at both ends.

Describe how Newton's third law applies when one is fired. (c) Can you safely stand close behind one when it is fired?

5.6 Common Forces

14. A table is placed on a rug. Then a book is placed on the table. What does the floor exert a normal force on?
15. A particle is moving to the right. (a) Can the force on it to be acting to the left? If yes, what would happen? (b) Can that force be acting downward? If yes, why?

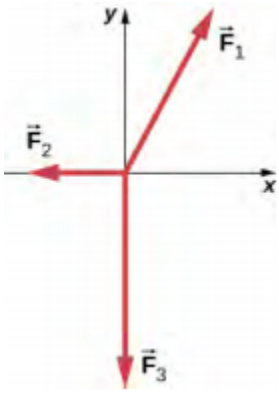
5.7 Drawing Free-Body Diagrams

16. In completing the solution for a problem involving forces, what do we do after constructing the free-body diagram? That is, what do we apply?
17. If a book is located on a table, how many forces should be shown in a free-body diagram of the book? Describe them.
18. If the book in the previous question is in free fall, how many forces should be shown in a free-body diagram of the book? Describe them.

Problems

5.1 Forces

19. Two ropes are attached to a tree, and forces of $\vec{F}_1$ = 2.0 $\hat{i}$ + 4.0 $\hat{j}$ N and $\vec{F}_2$ = 3.0 $\hat{i}$ + 6.0 $\hat{j}$ N are applied. The forces are coplanar (in the same plane). (a) What is the resultant (net force) of these two force vectors? (b) Find the magnitude and direction of this net force.
20. A telephone pole has three cables pulling as shown from above, with $\vec{F}_1$ = (300.0 $\hat{i}$ + 500.0 $\hat{j}$), $\vec{F}_2$ = −200.0 $\hat{i}$, and $\vec{F}_3$ = −800.0 $\hat{j}$. (a) Find the net force on the telephone pole in component form. (b) Find the magnitude and direction of this net force.



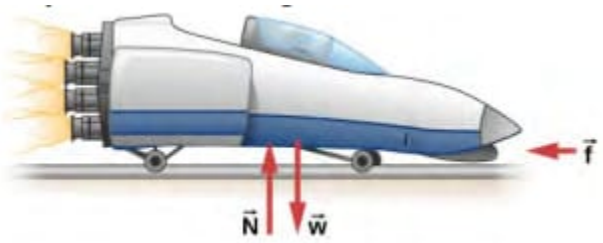
21. Two teenagers are pulling on ropes attached to a tree. The angle between the ropes is 30.0°. David pulls with a force of 400.0 N and Stephanie pulls with a force of 300.0 N. (a) Find the component form of the net force. (b) Find the magnitude of the resultant (net) force on the tree and the angle it makes with David's rope.

5.2 Newton's First Law

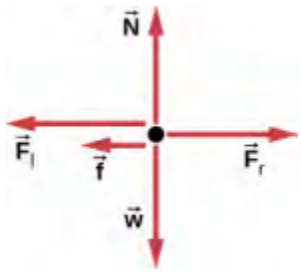
22. Two forces of $\vec{F}_1$ = 75.0 2 ($\hat{i}$ – $\hat{j}$) N and $\vec{F}_2 = \frac{150.0}{\sqrt{2}}(\hat{i} - \hat{j})$ N act on an object. Find the third force $\vec{F}_3$ that is needed to balance the first two forces.
23. While sliding a couch across a floor, Andrea and Jennifer exert forces $\vec{F}_A$ and $\vec{F}_J$ on the couch. Andrea's force is due north with a magnitude of 130.0 N and Jennifer's force is 32° east of north with a magnitude of 180.0 N. (a) Find the net force in component form. (b) Find the magnitude and direction of the net force. (c) If Andrea and Jennifer's housemates, David and Stephanie, disagree with the move and want to prevent its relocation, with what combined force $\vec{F}_{DS}$ should they push so that the couch does not move?

5.3 Newton's Second Law

24. Andrea, a 63.0-kg sprinter, starts a race with an acceleration of 4.200 m/s^2. What is the net external force on her?
25. If the sprinter from the previous problem accelerates at that rate for 20.00 m and then maintains that velocity for the remainder of a 100.00-m dash, what will her time be for the race?
26. A cleaner pushes a 4.50-kg laundry cart in such a way that the net external force on it is 60.0 N. Calculate the magnitude of his cart's acceleration.
27. Astronauts in orbit are apparently weightless. This means that a clever method of measuring the mass of astronauts is needed to monitor their mass gains or losses, and adjust their diet. One way to do this is to exert a known force on an astronaut and measure the acceleration produced. Suppose a net external force of 50.0 N is exerted, and an astronaut's acceleration is measured to be 0.893 m/s^2. (a) Calculate her mass. (b) By exerting a force on the astronaut, the vehicle in which she orbits experiences an equal and opposite force. Use this knowledge to find an equation for the acceleration of the system (astronaut and spaceship) that would be measured by a nearby observer. (c) Discuss how this would affect the measurement of the astronaut's acceleration. Propose a method by which recoil of the vehicle is avoided.
28. In Figure 5.4.3, the net external force on the 24-kg mower is given as 51 N. If the force of friction opposing the motion is 24 N, what force F (in newtons) is the person exerting on the mower? Suppose the mower is moving at 1.5 m/s when the force F is removed. How far will the mower go before stopping?
29. The rocket sled shown below decelerates at a rate of 196 m/s^2. What force is necessary to produce this deceleration? Assume that the rockets are off. The mass of the system is 2.10×10^3 kg.



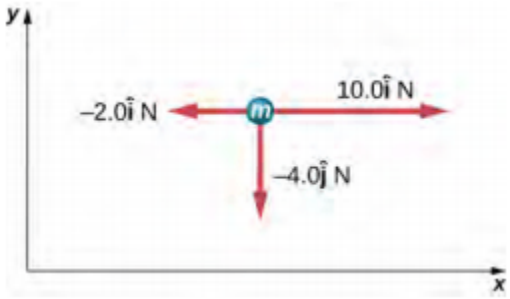
30. If the rocket sled shown in the previous problem starts with only one rocket burning, what is the magnitude of this acceleration? Assume that the mass of the system is 2.10×10^3 kg, the thrust T is 2.40×10^4 N, and the force of friction opposing the motion is 650.0 N. (b) Why is the acceleration not one-fourth of what it is with all rockets burning?
31. What is the deceleration of the rocket sled if it comes to rest in 1.10 s from a speed of 1000.0 km/h? (Such deceleration caused one test subject to black out and have temporary blindness.)
32. Suppose two children push horizontally, but in exactly opposite directions, on a third child in a wagon. The first child exerts a force of 75.0 N, the second exerts a force of 90.0 N, friction is 12.0 N, and the mass of the third child plus wagon is 23.0 kg. (a) What is the system of interest if the acceleration of the child in the wagon is to be calculated? (See the free-body diagram.) (b) Calculate the acceleration. (c) What would the acceleration be if friction were 15.0 N?



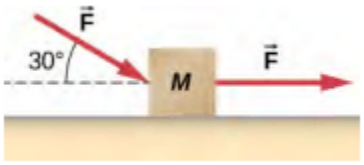
33. A powerful motorcycle can produce an acceleration of 3.50 m/s^2 while traveling at 90.0 km/h. At that speed, the forces resisting motion, including friction and air resistance, total 400.0 N. (Air resistance is analogous to air friction. It always opposes the motion of an object.) What is the magnitude of the force that motorcycle exerts backward on the ground to produce its acceleration if the mass of the motorcycle with rider is 245 kg?
34. A car with a mass of 1000.0 kg accelerates from 0 to 90.0 km/h in 10.0 s. (a) What is its acceleration? (b) What is the

net force on the car?

35. The driver in the previous problem applies the brakes when the car is moving at 90.0 km/h, and the car comes to rest after traveling 40.0 m. What is the net force on the car during its deceleration?
36. An 80.0-kg passenger in an SUV traveling at 1.00 x 10^2 km/h is wearing a seat belt. The driver slams on the brakes and the SUV stops in 45.0 m. Find the force of the seat belt on the passenger.
37. A particle of mass 2.0 kg is acted on by a single force $\vec{F}_1$ = 18 $\hat{i}$ N. (a) What is the particle's acceleration? (b) If the particle starts at rest, how far does it travel in the first 5.0 s?
38. Suppose that the particle of the previous problem also experiences forces $\vec{F}_2$ = –15 $\hat{i}$ N and $\vec{F}_3$ = 6.0 $\hat{j}$ N. What is its acceleration in this case?
39. Find the acceleration of the body of mass 5.0 kg shown below.



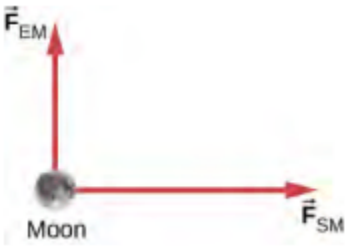
40. In the following figure, the horizontal surface on which this block slides is frictionless. If the two forces acting on it each have magnitude F = 30.0 N and M = 10.0 kg , what is the magnitude of the resulting acceleration of the block?



5.4 Mass and Weight

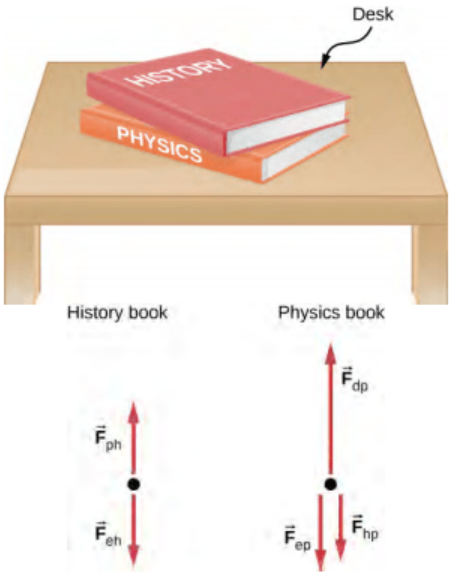
41. The weight of an astronaut plus his space suit on the Moon is only 250 N. (a) How much does the suited astronaut weigh on Earth? (b) What is the mass on the Moon? On Earth?
42. Suppose the mass of a fully loaded module in which astronauts take off from the Moon is 1.00 x 10^4 kg. The thrust of its engines is 3.00 x 10^4 N. (a) Calculate the module's magnitude of acceleration in a vertical takeoff from the Moon. (b) Could it lift off from Earth? If not, why not? If it could, calculate the magnitude of its acceleration.
43. A rocket sled accelerates at a rate of 49.0 m/s^2. Its passenger has a mass of 75.0 kg. (a) Calculate the horizontal component of the force the seat exerts against his body. Compare this with his weight using a ratio. (b) Calculate the direction and magnitude of the total force the seat exerts against his body.
44. Repeat the previous problem for a situation in which the rocket sled decelerates at a rate of 201 m/s^2. In this problem, the forces are exerted by the seat and the seat belt.
45. A body of mass 2.00 kg is pushed straight upward by a 25.0 N vertical force. What is its acceleration?
46. A car weighing 12,500 N starts from rest and accelerates to 83.0 km/h in 5.00 s. The friction force is 1350 N. Find the applied force produced by the engine.
47. A body with a mass of 10.0 kg is assumed to be in Earth's gravitational field with g = 9.80 m/s^2. What is the net force on the body if there are no other external forces acting on the object?
48. A fireman has mass m; he hears the fire alarm and slides down the pole with acceleration a (which is less than g in magnitude). (a) Write an equation giving the vertical force he must apply to the pole. (b) If his mass is 90.0 kg and he accelerates at 5.00 m/s^2, what is the magnitude of his applied force?
49. A baseball catcher is performing a stunt for a television commercial. He will catch a baseball (mass 145 g) dropped from a height of 60.0 m above his glove. His glove stops the ball in 0.0100 s. What is the force exerted by his glove on the ball?
50. When the Moon is directly overhead at sunset, the force by Earth on the Moon, F_{EM}, is essentially at 90° to the force

by the Sun on the Moon, F_{SM}, as shown below. Given that F_{EM} = 1.98 x 10^{20} N and F_{SM} = 4.36 x 10^{20} N, all other forces on the Moon are negligible, and the mass of the Moon is 7.35 x 10^{22} kg, determine the magnitude of the Moon's acceleration.



5.5 Newton's Third Law

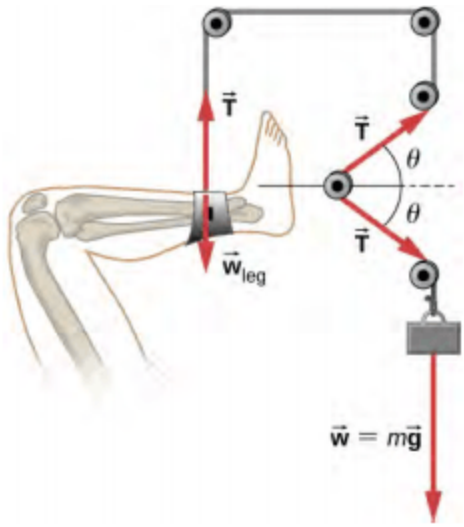
51. (a) What net external force is exerted on a 1100.0-kg artillery shell fired from a battleship if the shell is accelerated at 2.40 x 10^4 m/s^2? (b) What is the magnitude of the force exerted on the ship by the artillery shell, and why?
52. A brave but inadequate rugby player is being pushed backward by an opposing player who is exerting a force of 800.0 N on him. The mass of the losing player plus equipment is 90.0 kg, and he is accelerating backward at 1.20 m/s^2. (a) What is the force of friction between the losing player's feet and the grass? (b) What force does the winning player exert on the ground to move forward if his mass plus equipment is 110.0 kg?
53. A history book is lying on top of a physics book on a desk, as shown below; a free-body diagram is also shown. The history and physics books weigh 14 N and 18 N, respectively. Identify each force on each book with a double subscript notation (for instance, the contact force of the history book pressing against physics book can be described as $\vec{F}_{HP}$), and determine the value of each of these forces, explaining the process used.



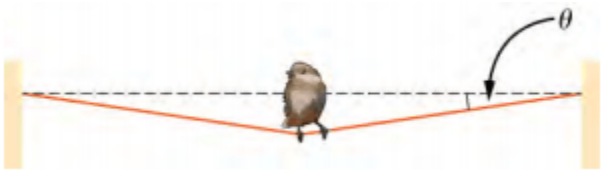
54. A truck collides with a car, and during the collision, the net force on each vehicle is essentially the force exerted by the other. Suppose the mass of the car is 550 kg, the mass of the truck is 2200 kg, and the magnitude of the truck's acceleration is 10 m/s^2. Find the magnitude of the car's acceleration.

5.6 Common Forces

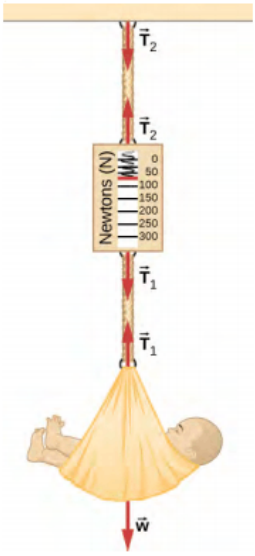
55. A leg is suspended in a traction system, as shown below. (a) Which part of the figure is used to calculate the force exerted on the foot? (b) What is the tension in the rope? Here $\vec{T}$ is the tension, $\vec{w}_{leg}$ is the weight of the leg, and $\vec{w}$ is the weight of the load that provides the tension.



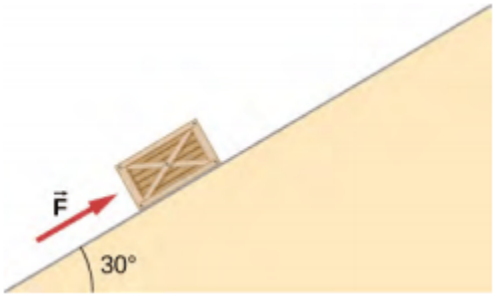
56. Suppose the shinbone in the preceding image was a femur in a traction setup for a broken bone, with pulleys and rope available. How might we be able to increase the force along the femur using the same weight?
57. A team of nine members on a tall building tug on a string attached to a large boulder on an icy surface. The boulder has a mass of 200 kg and is tugged with a force of 2350 N. (a) What is magnitude of the acceleration? (b) What force would be required to produce a constant velocity?
58. What force does a trampoline have to apply to Jennifer, a 45.0-kg gymnast, to accelerate her straight up at 7.50 m/s^2? The answer is independent of the velocity of the gymnast—she can be moving up or down or can be instantly stationary.
59. (a) Calculate the tension in a vertical strand of spider web if a spider of mass 2.00 x 10^{-5} kg hangs motionless on it. (b) Calculate the tension in a horizontal strand of spider web if the same spider sits motionless in the middle of it much like the tightrope walker in Figure 5.26. The strand sags at an angle of 12° below the horizontal. Compare this with the tension in the vertical strand (find their ratio).
60. Suppose Kevin, a 60.0-kg gymnast, climbs a rope. (a) What is the tension in the rope if he climbs at a constant speed? (b) What is the tension in the rope if he accelerates upward at a rate of 1.50 m/s^2?
61. Show that, as explained in the text, a force $F_\perp$ exerted on a flexible medium at its center and perpendicular to its length (such as on the tightrope wire in Figure 5.26) gives rise to a tension of magnitude $T = \frac{F_\perp}{2 \sin \theta}$.
62. Consider Figure 5.28. The driver attempts to get the car out of the mud by exerting a perpendicular force of 610.0 N, and the distance she pushes in the middle of the rope is 1.00 m while she stands 6.00 m away from the car on the left and 6.00 m away from the tree on the right. What is the tension T in the rope, and how do you find the answer?
63. A bird has a mass of 26 g and perches in the middle of a stretched telephone line. (a) Show that the tension in the line can be calculated using the equation $T = \frac{mg}{2 \sin \theta}$. Determine the tension when (b) θ = 5° and (c) θ = 0.5°. Assume that each half of the line is straight.



64. One end of a 30-m rope is tied to a tree; the other end is tied to a car stuck in the mud. The motorist pulls sideways on the midpoint of the rope, displacing it a distance of 2 m. If he exerts a force of 80 N under these conditions, determine the force exerted on the car.
65. Consider the baby being weighed in the following figure. (a) What is the mass of the infant and basket if a scale reading of 55 N is observed? (b) What is tension T_1 in the cord attaching the baby to the scale? (c) What is tension T_2 in the cord attaching the scale to the ceiling, if the scale has a mass of 0.500 kg? (d) Sketch the situation, indicating the system of interest used to solve each part. The masses of the cords are negligible.



66. What force must be applied to a 100.0-kg crate on a frictionless plane inclined at 30° to cause an acceleration of 2.0 m/s^2 up the plane?



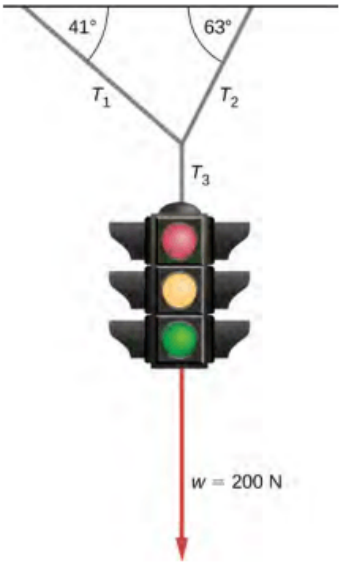
67. A 2.0-kg block is on a perfectly smooth ramp that makes an angle of 30° with the horizontal. (a) What is the block's acceleration down the ramp and the force of the ramp on the block? (b) What force applied upward along and parallel to the ramp would allow the block to move with constant velocity?

5.7 Drawing Free-Body Diagrams

68. A ball of mass m hangs at rest, suspended by a string. (a) Sketch all forces. (b) Draw the free-body diagram for the ball.
69. A car moves along a horizontal road. Draw a free-body diagram; be sure to include the friction of the road that opposes the forward motion of the car.
70. A runner pushes against the track, as shown. (a) Provide a free-body diagram showing all the forces on the runner. (**Hint**: Place all forces at the center of his body, and include his weight.) (b) Give a revised diagram showing the xy-component form.



71. The traffic light hangs from the cables as shown. Draw a free-body diagram on a coordinate plane for this situation.



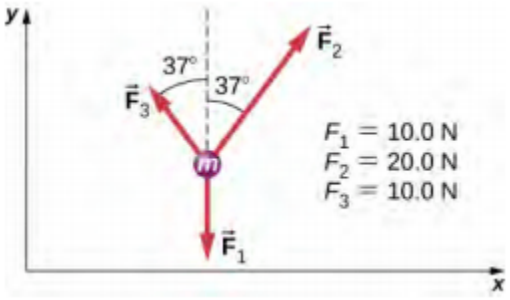
Additional Problems

72. Two small forces, $\vec{F}_1 = -2.40\ \hat{i} - 6.10t\ \hat{j}$ N and $\vec{F}_2 = 8.50\ \hat{i} - 9.70\ \hat{j}$ N, are exerted on a rogue asteroid by a pair of space tractors. (a) Find the net force. (b) What are the magnitude and direction of the net force? (c) If the mass of the asteroid is 125 kg, what acceleration does it experience (in vector form)? (d) What are the magnitude and direction of the acceleration?
73. Two forces of 25 and 45 N act on an object. Their directions differ by 70°. The resulting acceleration has magnitude of 10.0 m/s^2. What is the mass of the body?
74. A force of 1600 N acts parallel to a ramp to push a 300-kg piano into a moving van. The ramp is inclined at 20°. (a) What is the acceleration of the piano up the ramp? (b) What is the velocity of the piano when it reaches the top if the ramp is 4.0 m long and the piano starts from rest?
75. Draw a free-body diagram of a diver who has entered the water, moved downward, and is acted on by an upward force due to the water which balances the weight (that is, the diver is suspended).
76. For a swimmer who has just jumped off a diving board, assume air resistance is negligible. The swimmer has a mass of 80.0 kg and jumps off a board 10.0 m above the water. Three seconds after entering the water, her downward motion is stopped. What average upward force did the water exert on her?
77. (a) Find an equation to determine the magnitude of the net force required to stop a car of mass m, given that the initial speed of the car is v_0 and the stopping distance is x. (b) Find the magnitude of the net force if the mass of the car is

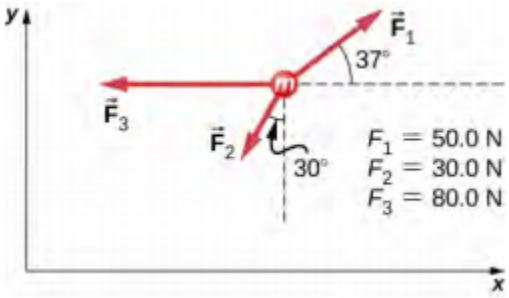
1050 kg, the initial speed is 40.0 km/h, and the stopping distance is 25.0 m.

78. A sailboat has a mass of 1.50 x 10^3 kg and is acted on by a force of 2.00 x 10^3 N toward the east, while the wind acts behind the sails with a force of 3.00 x 10^3 N in a direction 45° north of east. Find the magnitude and direction of the resulting acceleration.

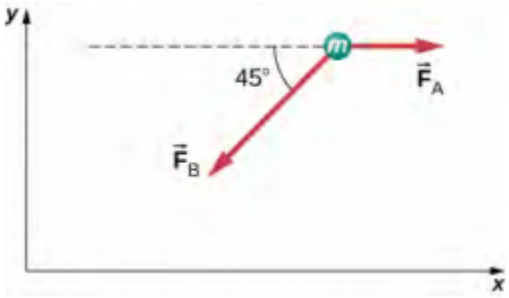
79. Find the acceleration of the body of mass 10.0 kg shown below.



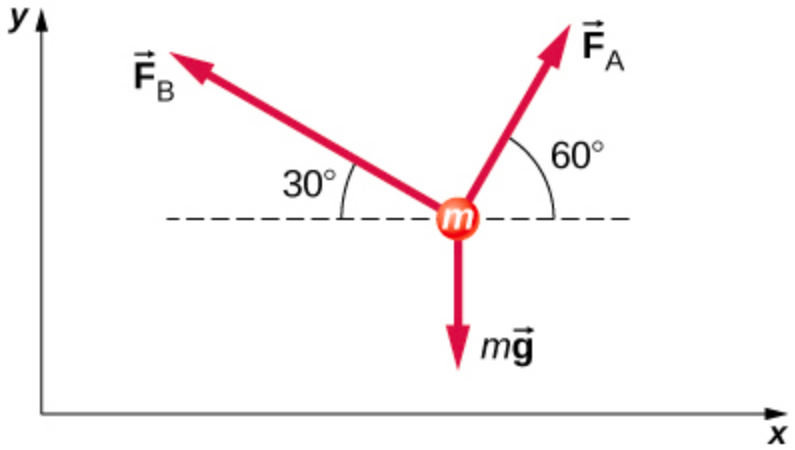
80. A body of mass 2.0 kg is moving along the x-axis with a speed of 3.0 m/s at the instant represented below. (a) What is the acceleration of the body? (b) What is the body's velocity 10.0 s later? (c) What is its displacement after 10.0 s?



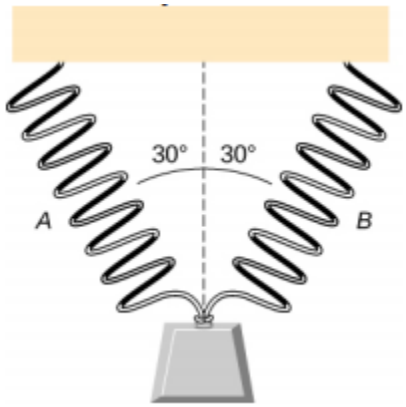
81. Force $\vec{F}_B$ has twice the magnitude of force $\vec{F}_A$. Find the direction in which the particle accelerates in this figure.



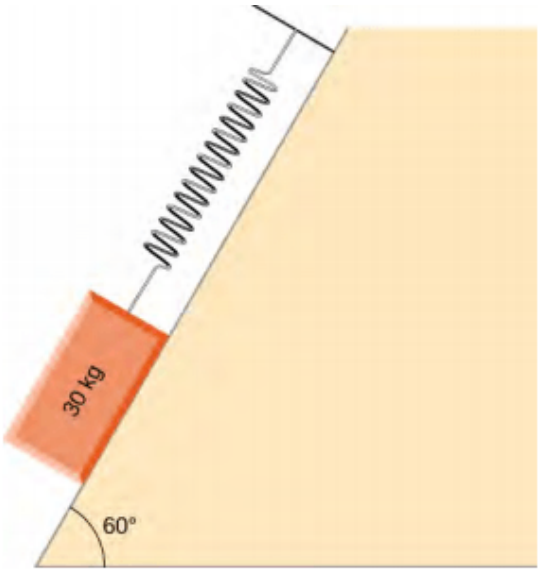
82. Shown below is a body of mass 1.0 kg under the influence of the forces $\vec{F}_A$, $\vec{F}_B$, and m$\vec{g}$. If the body accelerates to the left at 20 m/s^2, what are $\vec{F}_A$ and $\vec{F}_B$?



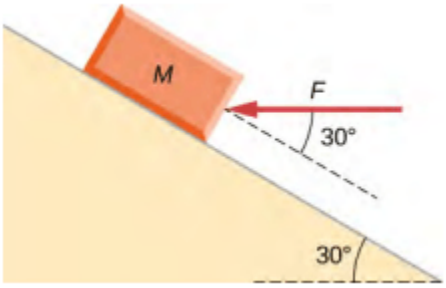
83. A force acts on a car of mass m so that the speed v of the car increases with position x as $v = kx^2$, where k is constant and all quantities are in SI units. Find the force acting on the car as a function of position.
84. A 7.0-N force parallel to an incline is applied to a 1.0-kg crate. The ramp is tilted at 20° and is frictionless. (a) What is the acceleration of the crate? (b) If all other conditions are the same but the ramp has a friction force of 1.9 N, what is the acceleration?
85. Two boxes, A and B, are at rest. Box A is on level ground, while box B rests on an inclined plane tilted at angle θ with the horizontal. (a) Write expressions for the normal force acting on each block. (b) Compare the two forces; that is, tell which one is larger or whether they are equal in magnitude. (c) If the angle of incline is 10°, which force is greater?
86. A mass of 250.0 g is suspended from a spring hanging vertically. The spring stretches 6.00 cm. How much will the spring stretch if the suspended mass is 530.0 g?
87. As shown below, two identical springs, each with the spring constant 20 N/m, support a 15.0-N weight. (a) What is the tension in spring A? (b) What is the amount of stretch of spring A from the rest position?



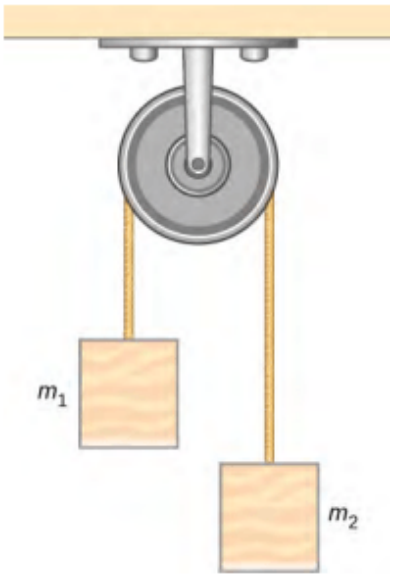
88. Shown below is a 30.0-kg block resting on a frictionless ramp inclined at 60° to the horizontal. The block is held by a spring that is stretched 5.0 cm. What is the force constant of the spring?



89. In building a house, carpenters use nails from a large box. The box is suspended from a spring twice during the day to measure the usage of nails. At the beginning of the day, the spring stretches 50 cm. At the end of the day, the spring stretches 30 cm. What fraction or percentage of the nails have been used?
90. A force is applied to a block to move it up a 30° incline. The incline is frictionless. If F = 65.0 N and M = 5.00 kg, what is the magnitude of the acceleration of the block?

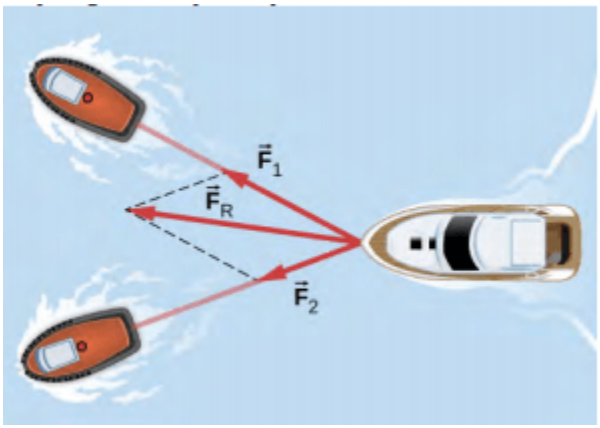


91. Two forces are applied to a 5.0-kg object, and it accelerates at a rate of 2.0 m/s^2 in the positive y-direction. If one of the forces acts in the positive x-direction with magnitude 12.0 N, find the magnitude of the other force.
92. The block on the right shown below has more mass than the block on the left ($m_2 > m_1$). Draw free-body diagrams for each block.



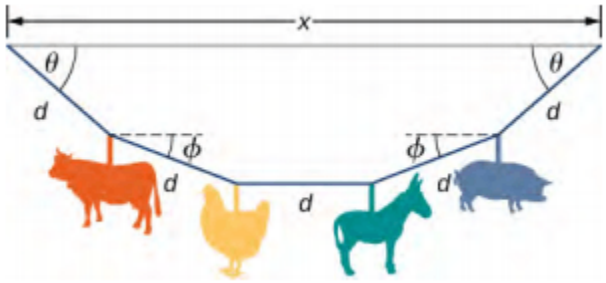
Challenge Problems

93. If two tugboats pull on a disabled vessel, as shown here in an overhead view, the disabled vessel will be pulled along the direction indicated by the result of the exerted forces. (a) Draw a free-body diagram for the vessel. Assume no friction or drag forces affect the vessel. (b) Did you include all forces in the overhead view in your free-body diagram? Why or why not?



94. A 10.0-kg object is initially moving east at 15.0 m/s. Then a force acts on it for 2.00 s, after which it moves northwest, also at 15.0 m/s. What are the magnitude and direction of the average force that acted on the object over the 2.00-s interval?
95. On June 25, 1983, shot-putter Udo Beyer of East Germany threw the 7.26-kg shot 22.22 m, which at that time was a world record. (a) If the shot was released at a height of 2.20 m with a projection angle of 45.0°, what was its initial velocity? (b) If while in Beyer's hand the shot was accelerated uniformly over a distance of 1.20 m, what was the net force on it?
96. A body of mass m moves in a horizontal direction such that at time t its position is given by $x(t) = at^4 + bt^3 + ct$, where a, b, and c are constants. (a) What is the acceleration of the body? (b) What is the time-dependent force acting on the body?
97. A body of mass m has initial velocity v_0 in the positive x-direction. It is acted on by a constant force F for time t until the velocity becomes zero; the force continues to act on the body until its velocity becomes $-v_0$ in the same amount of time. Write an expression for the total distance the body travels in terms of the variables indicated.
98. The velocities of a 3.0-kg object at t = 6.0 s and t = 8.0 s are $(3.0\,\hat{i} - 6.0\,\hat{j} + 4.0\,\hat{k})$ m/s and $(-2.0\,\hat{i} + 4.0\,\hat{k})$ m/s, respectively. If the object is moving at constant acceleration, what is the force acting on it?

99. A 120-kg astronaut is riding in a rocket sled that is sliding along an inclined plane. The sled has a horizontal component of acceleration of 5.0 m/s^2 and a downward component of 3.8 m/s^2. Calculate the magnitude of the force on the rider by the sled. (**Hint**: Remember that gravitational acceleration must be considered.)
100. Two forces are acting on a 5.0-kg object that moves with acceleration 2.0 m/s^2 in the positive y-direction. If one of the forces acts in the positive x-direction and has magnitude of 12 N, what is the magnitude of the other force?
101. Suppose that you are viewing a soccer game from a helicopter above the playing field. Two soccer players simultaneously kick a stationary soccer ball on the flat field; the soccer ball has mass 0.420 kg. The first player kicks with force 162 N at 9.0° north of west. At the same instant, the second player kicks with force 215 N at 15° east of south. Find the acceleration of the ball in $\hat{i}$ and $\hat{j}$ form.
102. A 10.0-kg mass hangs from a spring that has the spring constant 535 N/m. Find the position of the end of the spring away from its rest position. (Use g = 9.80 m/s^2.)
103. A 0.0502-kg pair of fuzzy dice is attached to the rearview mirror of a car by a short string. The car accelerates at constant rate, and the dice hang at an angle of 3.20° from the vertical because of the car's acceleration. What is the magnitude of the acceleration of the car?
104. At a circus, a donkey pulls on a sled carrying a small clown with a force given by 2.48 $\hat{i}$ + 4.33 $\hat{j}$ N . A horse pulls on the same sled, aiding the hapless donkey, with a force of 6.56 $\hat{i}$ + 5.33 $\hat{j}$ N. The mass of the sled is 575 kg. Using $\hat{i}$ and $\hat{j}$ form for the answer to each problem, find (a) the net force on the sled when the two animals act together, (b) the acceleration of the sled, and (c) the velocity after 6.50 s.
105. Hanging from the ceiling over a baby bed, well out of baby's reach, is a string with plastic shapes, as shown here. The string is taut (there is no slack), as shown by the straight segments. Each plastic shape has the same mass m, and they are equally spaced by a distance d, as shown. The angles labeled θ describe the angle formed by the end of the string and the ceiling at each end. The center length of sting is horizontal. The remaining two segments each form an angle with the horizontal, labeled ϕ. Let T_1 be the tension in the leftmost section of the string, T_2 be the tension in the section adjacent to it, and T_3 be the tension in the horizontal segment. (a) Find an equation for the tension in each section of the string in terms of the variables m, g, and θ. (b) Find the angle ϕ in terms of the angle θ. (c) If θ = 5.10°, what is the value of ϕ? (d) Find the distance x between the endpoints in terms of d and θ.



106. A bullet shot from a rifle has mass of 10.0 g and travels to the right at 350 m/s. It strikes a target, a large bag of sand, penetrating it a distance of 34.0 cm. Find the magnitude and direction of the retarding force that slows and stops the bullet.
107. An object is acted on by three simultaneous forces: $\vec{F}_1$ =(–3.00 $\hat{i}$ + 2.00 $\hat{j}$) N, $\vec{F}_2$ = (6.00 $\hat{i}$ – 4.00 $\hat{i}$) N, and $\vec{F}_3$ = (2.00 $\hat{i}$ + 5.00 $\hat{i}$) N. The object experiences acceleration of 4.23 m/s^2. (a) Find the acceleration vector in terms of m. (b) Find the mass of the object. (c) If the object begins from rest, find its speed after 5.00 s. (d) Find the components of the velocity of the object after 5.00 s.
108. In a particle accelerator, a proton has mass 1.67 x 10^{-27} kg and an initial speed of 2.00 x 10^5 m/s. It moves in a straight line, and its speed increases to 9.00 x 10^5 m/s in a distance of 10.0 cm. Assume that the acceleration is constant. Find the magnitude of the force exerted on the proton.
109. A drone is being directed across a frictionless ice-covered lake. The mass of the drone is 1.50 kg, and its velocity is 3.00 $\hat{i}$ m/s. After 10.0 s, the velocity is 9.00 $\hat{i}$ + 4.00 $\hat{j}$ m/s. If a constant force in the horizontal direction is causing this change in motion, find (a) the components of the force and (b) the magnitude of the force.

Contributors and Attributions

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4.10: Newton's Laws of Motion (Summary)

Key Terms

dynamics	study of how forces affect the motion of objects and systems
external force	force acting on an object or system that originates outside of the object or system
force	push or pull on an object with a specific magnitude and direction; can be represented by vectors or expressed as a multiple of a standard force
free fall	situation in which the only force acting on an object is gravity
free-body diagram	sketch showing all external forces acting on an object or system; the system is represented by a single isolated point, and the forces are represented by vectors extending outward from that point
Hooke's law	in a spring, a restoring force proportional to and in the opposite direction of the imposed displacement
inertia	ability of an object to resist changes in its motion
inertial reference frame	reference frame moving at constant velocity relative to an inertial frame is also inertial; a reference frame accelerating relative to an inertial frame is not inertial
law of inertia	see Newton's first law of motion
net external force	vector sum of all external forces acting on an object or system; causes a mass to accelerate
newton	SI unit of force; 1 N is the force needed to accelerate an object with a mass of 1 kg at a rate of 1 m/s^2
Newton's first law of motion	body at rest remains at rest or, if in motion, remains in motion at constant velocity unless acted on by a net external force; also known as the law of inertia
Newton's second law of motion	acceleration of a system is directly proportional to and in the same direction as the net external force acting on the system and is inversely proportional to its mass
Newton's third law of motion	whenever one body exerts a force on a second body, the first body experiences a force that is equal in magnitude and opposite in direction to the force that it exerts
normal force	force supporting the weight of an object, or a load, that is perpendicular to the surface of contact between the load and its support; the surface applies this force to an object to support the weight of the object
tension	pulling force that acts along a stretched flexible connector, such as a rope or cable
thrust	reaction force that pushes a body forward in response to a backward force
weight	force $\vec{w}$ due to gravity acting on an object of mass m

Key Equations

Net external force	$\vec{F}_{net} = \sum \vec{F} = \vec{F}_1 + \vec{F}_2 + ...$
Newton's first law	$\vec{v} = constant\ when\ \vec{F}_{net} = \vec{0}\ N$
Newton's second law, vector form	$\vec{F}_{net} = \sum \vec{F} = m\vec{a}$

Newton's second law, scalar form	$\vec{F}_{net} = ma$
Newton's second law, component form	$\sum \vec{F}_x = m\vec{a}_x, \sum \vec{F}_y = m\vec{a}_y, \sum \vec{F}_z = m\vec{a}_z$
Newton's second law, momentum form	$\vec{F}_{net} = \frac{d\vec{p}}{dt}$
Definition of weight, vector form	$\vec{w} = m\vec{g}$
Definition of weight, scalar form	$w = mg$
Newton's third law	$\vec{F}_{AB} = -\vec{F}_{BA}$
Normal force on an object resting on a horizontal surface, vector form	$\vec{N} = -m\vec{g}$
Normal force on an object resting on a horizontal surface, scalar form	$N = mg$
Normal force on an object resting on an inclined plane, scalar form	$N = mg\cos\theta$
Tension in a cable supporting an object of mass m at rest, scalar form	$T = w = mg$

Summary

5.1 Forces

- Dynamics is the study of how forces affect the motion of objects, whereas kinematics simply describes the way objects move.
- Force is a push or pull that can be defined in terms of various standards, and it is a vector that has both magnitude and direction.
- External forces are any outside forces that act on a body. A free-body diagram is a drawing of all external forces acting on a body.
- The SI unit of force is the newton (N).

5.2 Newton's First Law

- According to Newton's first law, there must be a cause for any change in velocity (a change in either magnitude or direction) to occur. This law is also known as the law of inertia.
- Friction is an external force that causes an object to slow down.
- Inertia is the tendency of an object to remain at rest or remain in motion. Inertia is related to an object's mass.
- If an object's velocity relative to a given frame is constant, then the frame is inertial. This means that for an inertial reference frame, Newton's first law is valid.
- Equilibrium is achieved when the forces on a system are balanced.
- A net force of zero means that an object is either at rest or moving with constant velocity; that is, it is not accelerating.

5.3 Newton's Second Law

- An external force acts on a system from outside the system, as opposed to internal forces, which act between components within the system.
- Newton's second law of motion says that the net external force on an object with a certain mass is directly proportional to and in the same direction as the acceleration of the object.
- Newton's second law can also describe net force as the instantaneous rate of change of momentum. Thus, a net external force causes nonzero acceleration.

5.4 Mass and Weight

- Mass is the quantity of matter in a substance.
- The weight of an object is the net force on a falling object, or its gravitational force. The object experiences acceleration due to gravity.
- Some upward resistance force from the air acts on all falling objects on Earth, so they can never truly be in free fall.
- Careful distinctions must be made between free fall and weightlessness using the definition of weight as force due to gravity acting on an object of a certain mass.

5.5 Newton's Third Law

- Newton's third law of motion represents a basic symmetry in nature, with an experienced force equal in magnitude and opposite in direction to an exerted force.
- Two equal and opposite forces do not cancel because they act on different systems.
- Action-reaction pairs include a swimmer pushing off a wall, helicopters creating lift by pushing air down, and an octopus propelling itself forward by ejecting water from its body. Rockets, airplanes, and cars are pushed forward by a thrust reaction force.
- Choosing a system is an important analytical step in understanding the physics of a problem and solving it.

5.6 Common Forces

- When an object rests on a surface, the surface applies a force to the object that supports the weight of the object. This supporting force acts perpendicular to and away from the surface. It is called a normal force.

- When an object rests on a nonaccelerating horizontal surface, the magnitude of the normal force is equal to the weight of the object.
- When an object rests on an inclined plane that makes an angle θ with the horizontal surface, the weight of the object can be resolved into components that act perpendicular and parallel to the surface of the plane.
- The pulling force that acts along a stretched flexible connector, such as a rope or cable, is called tension. When a rope supports the weight of an object at rest, the tension in the rope is equal to the weight of the object. If the object is accelerating, tension is greater than weight, and if it is decelerating, tension is less than weight.
- The force of friction is a force experienced by a moving object (or an object that has a tendency to move) parallel to the interface opposing the motion (or its tendency).
- The force developed in a spring obeys Hooke's law, according to which its magnitude is proportional to the displacement and has a sense in the opposite direction of the displacement.
- Real forces have a physical origin, whereas fictitious forces occur because the observer is in an accelerating or noninertial frame of reference.

5.7 Drawing Free-Body Diagrams

- To draw a free-body diagram, we draw the object of interest, draw all forces acting on that object, and resolve all force vectors into x- and y-components. We must draw a separate free-body diagram for each object in the problem.
- A free-body diagram is a useful means of describing and analyzing all the forces that act on a body to determine equilibrium according to Newton's first law or acceleration according to Newton's second law.

Contributors and Attributions

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5: PHYSICS 1- REVIEW APPLICATIONS OF NEWTON'S LAWS

5.1: Prelude to Applications of Newton's Laws

5.2: Solving Problems with Newton's Laws (Part 1)

5.3: Solving Problems with Newton's Laws (Part 2)

5.4: Friction (Part 1)

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5.1: Prelude to Applications of Newton's Laws



Figure 5.1.1: Stock cars racing in the Grand National Divisional race at Iowa Speedway in May, 2015. Cars often reach speeds of 200 mph (320 km/h).

Car racing has grown in popularity in recent years. As each car moves in a curved path around the turn, its wheels also spin rapidly. The wheels complete many revolutions while the car makes only part of one (a circular arc). How can we describe the velocities, accelerations, and forces involved? What force keeps a racecar from spinning out, hitting the wall bordering the track? What provides this force? Why is the track banked? We answer all of these questions in this chapter as we expand our consideration of Newton's laws of motion.

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5.2: Solving Problems with Newton's Laws (Part 1)

Learning Objectives

- Apply problem-solving techniques to solve for quantities in more complex systems of forces
- Use concepts from kinematics to solve problems using Newton's laws of motion
- Solve more complex equilibrium problems
- Solve more complex acceleration problems
- Apply calculus to more advanced dynamics problems

Success in problem solving is necessary to understand and apply physical principles. We developed a pattern of analyzing and setting up the solutions to problems involving Newton's laws in Newton's Laws of Motion; in this chapter, we continue to discuss these strategies and apply a step-by-step process.

Problem-Solving Strategies

We follow here the basics of problem solving presented earlier in this text, but we emphasize specific strategies that are useful in applying Newton's laws of motion. Once you identify the physical principles involved in the problem and determine that they include Newton's laws of motion, you can apply these steps to find a solution. These techniques also reinforce concepts that are useful in many other areas of physics. Many problem-solving strategies are stated outright in the worked examples, so the following techniques should reinforce skills you have already begun to develop.

Problem-Solving Strategy: Applying Newton's Laws of Motion

1. Identify the physical principles involved by listing the givens and the quantities to be calculated.
2. Sketch the situation, using arrows to represent all forces.
3. Determine the system of interest. The result is a free-body diagram that is essential to solving the problem.
4. Apply Newton's second law to solve the problem. If necessary, apply appropriate kinematic equations from the chapter on motion along a straight line.
5. Check the solution to see whether it is reasonable.

Let's apply this problem-solving strategy to the challenge of lifting a grand piano into a second-story apartment. Once we have determined that Newton's laws of motion are involved (if the problem involves forces), it is particularly important to draw a careful sketch of the situation. Such a sketch is shown in Figure $5.2.1a$. Then, as in Figure $5.2.1b$, we can represent all forces with arrows. Whenever sufficient information exists, it is best to label these arrows carefully and make the length and direction of each correspond to the represented force.

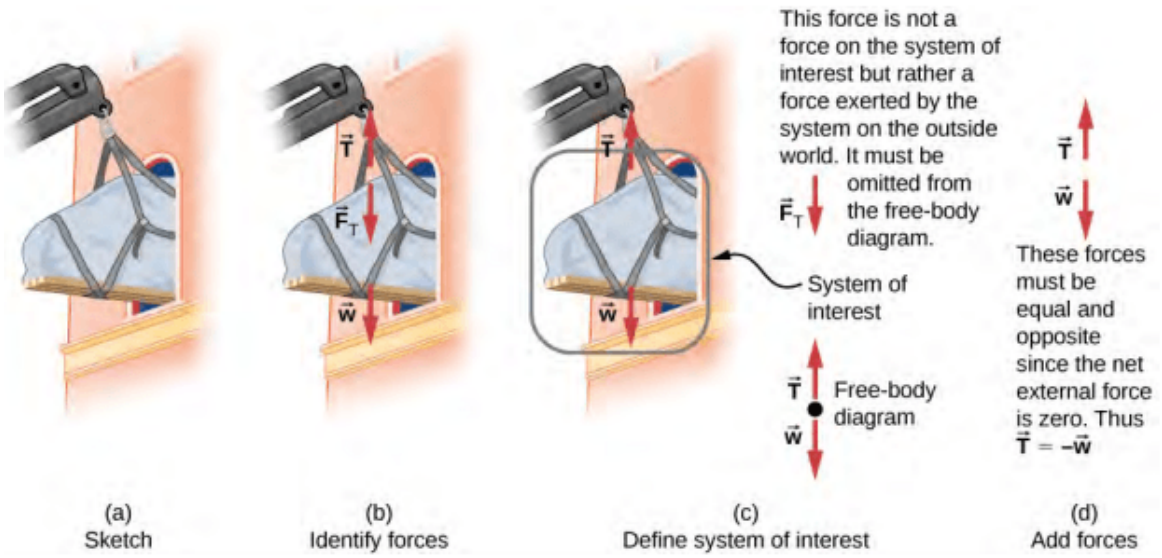


Figure 5.2.1: (a) A grand piano is being lifted to a second-story apartment. (b) Arrows are used to represent all forces: $\vec{T}$ is the tension in the rope above the piano, $\vec{F}_T$ is the force that the piano exerts on the rope, and $\vec{w}$ is the weight of the piano. All other forces, such as the nudge of a breeze, are assumed to be negligible. (c) Suppose we are given the piano's mass and asked to find the tension in the rope. We then define the system of interest as shown and draw a free-body diagram. Now $\vec{F}_T$ is no longer shown, because it is not a force acting on the system of interest; rather, $\vec{F}_T$ acts on the outside world. (d) Showing only the arrows, the head-to-tail method of addition is used. It is apparent that if the piano is stationary, $\vec{T} = -\vec{w}$.

As with most problems, we next need to identify what needs to be determined and what is known or can be inferred from the problem as stated, that is, make a list of knowns and unknowns. It is particularly crucial to identify the system of interest, since Newton's second law involves only external forces. We can then determine which forces are external and which are internal, a necessary step to employ Newton's second law. (See Figure 5.2.1*c*.) Newton's third law may be used to identify whether forces are exerted between components of a system (internal) or between the system and something outside (external). As illustrated in Newton's Laws of Motion, the system of interest depends on the question we need to answer. Only forces are shown in free-body diagrams, not acceleration or velocity. We have drawn several free-body diagrams in previous worked examples. Figure 5.2.1*c* shows a free-body diagram for the system of interest. Note that no internal forces are shown in a free-body diagram.

Once a free-body diagram is drawn, we apply Newton's second law. This is done in Figure 5.2.1*d* for a particular situation. In general, once external forces are clearly identified in free-body diagrams, it should be a straightforward task to put them into equation form and solve for the unknown, as done in all previous examples. If the problem is one-dimensional—that is, if all forces are parallel—then the forces can be handled algebraically. If the problem is two-dimensional, then it must be broken down into a pair of one-dimensional problems. We do this by projecting the force vectors onto a set of axes chosen for convenience. As seen in previous examples, the choice of axes can simplify the problem. For example, when an incline is involved, a set of axes with one axis parallel to the incline and one perpendicular to it is most convenient. It is almost always convenient to make one axis parallel to the direction of motion, if this is known. Generally, just write Newton's second law in components along the different directions. Then, you have the following equations:

$$\sum F_x = ma_x, \quad \sum F_y = ma_y. \tag{5.2.1}$$

(If, for example, the system is accelerating horizontally, then you can then set ay = 0.) We need this information to determine unknown forces acting on a system.

As always, we must check the solution. In some cases, it is easy to tell whether the solution is reasonable. For example, it is reasonable to find that friction causes an object to slide down an incline more slowly than when no friction exists. In practice, intuition develops gradually through problem solving; with experience, it becomes progressively easier to judge whether an answer is reasonable. Another way to check a solution is to check the units. If we are solving for force and end up with units of millimeters per second, then we have made a mistake.

There are many interesting applications of Newton's laws of motion, a few more of which are presented in this section. These

serve also to illustrate some further subtleties of physics and to help build problem-solving skills. We look first at problems involving particle equilibrium, which make use of Newton's first law, and then consider particle acceleration, which involves Newton's second law.

Particle Equilibrium

Recall that a particle in equilibrium is one for which the external forces are balanced. Static equilibrium involves objects at rest, and dynamic equilibrium involves objects in motion without acceleration, but it is important to remember that these conditions are relative. For example, an object may be at rest when viewed from our frame of reference, but the same object would appear to be in motion when viewed by someone moving at a constant velocity. We now make use of the knowledge attained in Newton's Laws of Motion, regarding the different types of forces and the use of free-body diagrams, to solve additional problems in particle equilibrium.

Example 6.1: Different Tensions at Different Angles

Consider the traffic light (mass of 15.0 kg) suspended from two wires as shown in Figure 5.2.2. Find the tension in each wire, neglecting the masses of the wires.

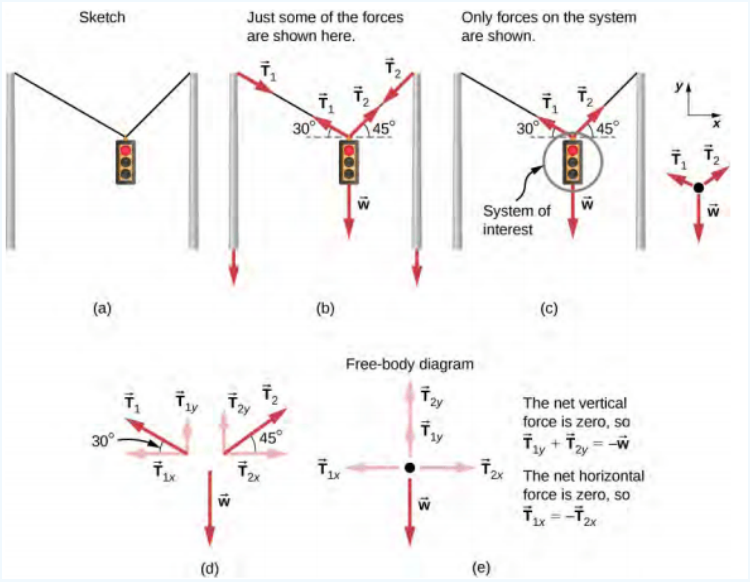


Figure 5.2.2: A traffic light is suspended from two wires. (b) Some of the forces involved. (c) Only forces acting on the system are shown here. The free-body diagram for the traffic light is also shown. (d) The forces projected onto vertical (y) and horizontal (x) axes. The horizontal components of the tensions must cancel, and the sum of the vertical components of the tensions must equal the weight of the traffic light. (e) The free-body diagram shows the vertical and horizontal forces acting on the traffic light.

Strategy

The system of interest is the traffic light, and its free-body diagram is shown in Figure 5.2.2*c*. The three forces involved are not parallel, and so they must be projected onto a coordinate system. The most convenient coordinate system has one axis vertical and one horizontal, and the vector projections on it are shown in Figure 5.2.2*d*. There are two unknowns in this problem (T_1 and T_2), so two equations are needed to find them. These two equations come from applying Newton's second law along the vertical and horizontal axes, noting that the net external force is zero along each axis because acceleration is zero.

Solution

First consider the horizontal or x-axis:

$$F_{netx} = T_{2x} - T_{1x} = 0. \tag{5.2.2}$$

Thus, as you might expect,

$$T_{1x} = T_{2x}. \tag{5.2.3}$$

This give us the following relationship:

$$T_1 \cos 30^o = T_2 \cos 45^o. \tag{5.2.4}$$

Thus,

$$T_2 = 1.225 T_1. \tag{5.2.5}$$

Note that T_1 and T_2 are not equal in this case because the angles on either side are not equal. It is reasonable that T_2 ends up being greater than T_1 because it is exerted more vertically than T_1.

Now consider the force components along the vertical or y-axis:

$$F_{nety} = T_{1y} + T_{1x} - w = 0. \tag{5.2.6}$$

This implies

$$T_{1y} + T_{2y} = w. \tag{5.2.7}$$

Substituting the expressions for the vertical components gives

$$T_1 \sin 30^o + T_2 \sin 45^o = w. \tag{5.2.8}$$

There are two unknowns in this equation, but substituting the expression for T_2 in terms of T_1 reduces this to one equation with one unknown:

$$T_1(0.500) + (1.225 T_1)(0.707) = w = mg, \tag{5.2.9}$$

which yields

$$1.366 T_1 = (15.0\ kg)(9.80\ m/s^2). \tag{5.2.10}$$

Solving this last equation gives the magnitude of T_1 to be

$$T_1 = 108\ N. \tag{5.2.11}$$

Finally, we find the magnitude of T_2 by using the relationship between them, $T_2 = 1.225\ T_1$, found above. Thus we obtain

$$T_2 = 132\ N. \tag{5.2.12}$$

Significance

Both tensions would be larger if both wires were more horizontal, and they will be equal if and only if the angles on either side are the same (as they were in the earlier example of a tightrope walker in Newton's Laws of Motion.

Example 6.2: Drag Force on a Barge

Two tugboats push on a barge at different angles (Figure 5.2.3). The first tugboat exerts a force of 2.7 x 10^5 N in the x-direction, and the second tugboat exerts a force of 3.6 x 10^5 N in the y-direction. The mass of the barge is 5.0 × 106 kg and its acceleration is observed to be 7.5 x 10^{-2} m/s^2 in the direction shown. What is the drag force of the water on the barge resisting the motion? (**Note**: Drag force is a frictional force exerted by fluids, such as air or water. The drag force opposes the motion of the object. Since the barge is flat bottomed, we can assume that the drag force is in the direction opposite of motion of the barge.)

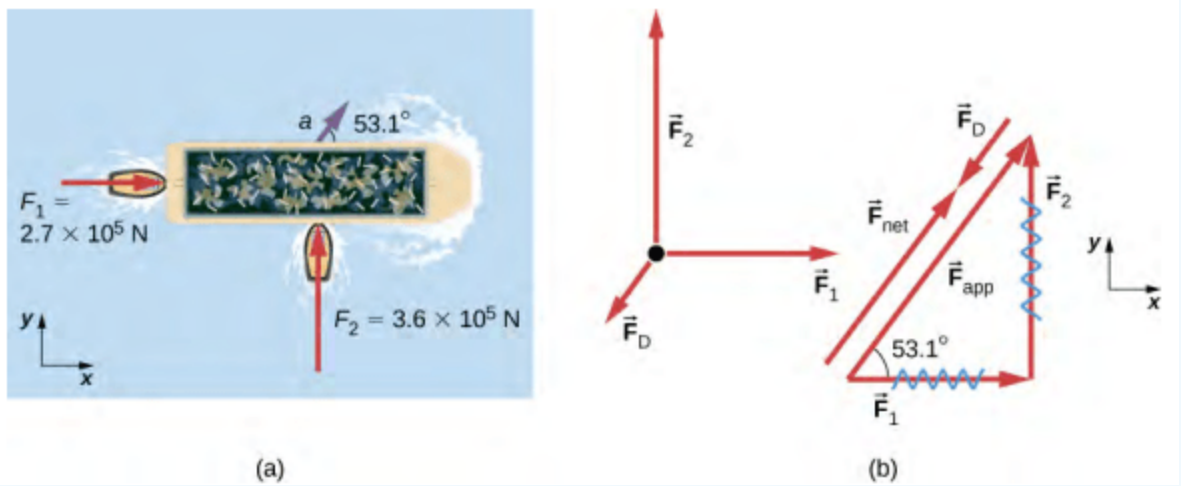


Figure 5.2.3: (a) A view from above of two tugboats pushing on a barge. (b) The free-body diagram for the ship contains only forces acting in the plane of the water. It omits the two vertical forces—the weight of the barge and the buoyant force of the water supporting it cancel and are not shown. Note that $\vec{F}_{app}$ is the total applied force of the tugboats.

Strategy

The directions and magnitudes of acceleration and the applied forces are given in Figure 5.2.3*a*. We define the total force of the tugboats on the barge as $\vec{F}_{app}$ so that

$$\vec{F}_{app} = \vec{F}_1 + \vec{F}_2. \tag{5.2.13}$$

The drag of the water $\vec{F}_D$ is in the direction opposite to the direction of motion of the boat; this force thus works against $\vec{F}_{app}$, as shown in the free-body diagram in Figure 5.2.3*b*. The system of interest here is the barge, since the forces on it are given as well as its acceleration. Because the applied forces are perpendicular, the x- and y-axes are in the same direction as $\vec{F}_1$ and $\vec{F}_2$. The problem quickly becomes a one-dimensional problem along the direction of $\vec{F}_{app}$, since friction is in the direction opposite to $\vec{F}_{app}$. Our strategy is to find the magnitude and direction of the net applied force $\vec{F}_{app}$ and then apply Newton's second law to solve for the drag force $\vec{F}_D$.

Solution

Since F_x and F_y are perpendicular, we can find the magnitude and direction of $\vec{F}_{app}$ directly. First, the resultant magnitude is given by the Pythagorean theorem:

$$\vec{F}_{app} = \sqrt{F_1^2 + F_2^2} = \sqrt{(2.7 \times 10^5\ N)^2 + (3.6 \times 10^5\ N)^2} = 4.5 \times 10^5\ N. \tag{5.2.14}$$

The angle is given by

$$\theta = \tan^{-1}\left(\frac{F_2}{F_1}\right) = \tan^{-1}\left(\frac{3.6 \times 10^5\ N}{2.7 \times 10^5\ N}\right) = 53.1^o. \tag{5.2.15}$$

From Newton's first law, we know this is the same direction as the acceleration. We also know that $\vec{F}_D$ is in the opposite direction of $\vec{F}_{app}$, since it acts to slow down the acceleration. Therefore, the net external force is in the same direction as $\vec{F}_{app}$, but its magnitude is slightly less than $\vec{F}_{app}$. The problem is now one-dimensional. From the free-body diagram, we can see that

$$F_{net} = F_{app} - F_D. \tag{5.2.16}$$

However, Newton's second law states that

$$F_{net} = ma. \tag{5.2.17}$$

Thus,

$$F_{app} - F_D = ma. \tag{5.2.18}$$

This can be solved for the magnitude of the drag force of the water F_D in terms of known quantities:

$$F_D = F_{app} - ma. \tag{5.2.19}$$

Substituting known values gives

$$F_D = (4.5 \times 10^5\ N) - (5.0 \times 10^6\ kg)(7.5 \times 10^{-2}\ m/s^2) = 7.5 \times 10^4\ N. \tag{5.2.20}$$

The direction of $\vec{F}_D$ has already been determined to be in the direction opposite to $\vec{F}_{app}$, or at an angle of 53° south of west.

Significance

The numbers used in this example are reasonable for a moderately large barge. It is certainly difficult to obtain larger accelerations with tugboats, and small speeds are desirable to avoid running the barge into the docks. Drag is relatively small for a well-designed hull at low speeds, consistent with the answer to this example, where F_D is less than 1/600th of the weight of the ship.

In Newton's Laws of Motion, we discussed the normal force, which is a contact force that acts normal to the surface so that an object does not have an acceleration perpendicular to the surface. The bathroom scale is an excellent example of a normal force acting on a body. It provides a quantitative reading of how much it must push upward to support the weight of an object. But can you predict what you would see on the dial of a bathroom scale if you stood on it during an elevator ride?

Will you see a value greater than your weight when the elevator starts up? What about when the elevator moves upward at a constant speed? Take a guess before reading the next example.

Example 6.3: What does the Bathroom Scale Read in an Elevator?

Figure 5.2.4 shows a 75.0-kg man (weight of about 165 lb.) standing on a bathroom scale in an elevator. Calculate the scale reading: (a) if the elevator accelerates upward at a rate of 1.20 m/s^2, and (b) if the elevator moves upward at a constant speed of 1 m/s.

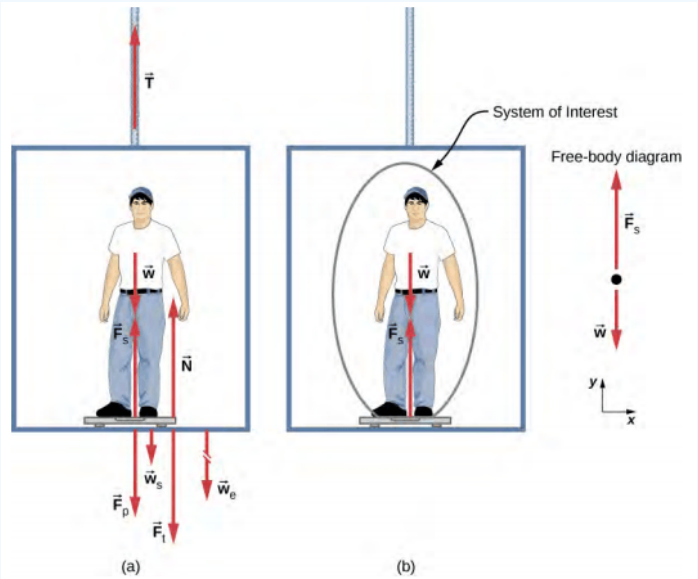


Figure 5.2.4: (a) The various forces acting when a person stands on a bathroom scale in an elevator. The arrows are approximately correct for when the elevator is accelerating upward—broken arrows represent forces too large to be drawn to scale. $\vec{T}$ is the tension in the supporting cable, $\vec{w}$ is the weight of the person, $\vec{w}_s$ is the weight of the scale, $\vec{w}_e$ is the weight of the elevator, $\vec{F}_s$ is the force of the scale on the person, $\vec{F}_p$ is the force of the person on the scale, $\vec{F}_t$ is the force of the scale on the floor of the elevator, and $\vec{N}$ is the force of the floor upward on the scale. (b) The free-body diagram shows only the external forces acting on the designated system of interest—the person—and is the diagram we use for the solution of the problem.

Strategy

If the scale at rest is accurate, its reading equals $\vec{F}_p$, the magnitude of the force the person exerts downward on it. Figure 5.2.4*a* shows the numerous forces acting on the elevator, scale, and person. It makes this one-dimensional problem look much more formidable than if the person is chosen to be the system of interest and a free-body diagram is drawn, as in Figure 5.2.4*b*. Analysis of the free-body diagram using Newton's laws can produce answers to both Figure 5.2.4*a* and (b) of this example, as well as some other questions that might arise. The only forces acting on the person are his weight $\vec{w}$ and the upward force of the scale $\vec{F}_s$. According to Newton's third law, $\vec{F}_p$ and $\vec{F}_s$ are equal in magnitude and opposite in direction, so that we need to find F_S in order to find what the scale reads. We can do this, as usual, by applying Newton's second law,

$$\vec{F}_{net} = m\vec{a}. \tag{5.2.21}$$

From the free-body diagram, we see that $\vec{F}_{net} = \vec{F}_s - \vec{w}$, so we have

$$F_s - w = ma. \tag{5.2.22}$$

Solving for F_S gives us an equation with only one unknown:

$$F_s = ma + w, \tag{5.2.23}$$

or, because w = mg, simply

$$F_s = ma + mg. \tag{5.2.24}$$

No assumptions were made about the acceleration, so this solution should be valid for a variety of accelerations in addition to those in this situation. (**Note**: We are considering the case when the elevator is accelerating upward. If the elevator is accelerating downward, Newton's second law becomes F_S – w = –ma.)

Solution

a. We have a = 1.20 m/s^2, so that

$$F_s = (75.0\ kg)(9.80\ m/s^2) + (75.0\ kg)(1.20\ m/s^2) \tag{5.2.25}$$

yielding

$$F_s = 825\ N. \tag{5.2.26}$$

b. Now, what happens when the elevator reaches a constant upward velocity? Will the scale still read more than his weight? For any constant velocity—up, down, or stationary—acceleration is zero because $a = \frac{\Delta v}{\Delta t}$ and $\Delta v = 0$. Thus,

$$F_s = ma + mg = 0 + mg \tag{5.2.27}$$

or

$$F_s = (75.0\ kg)(9.80\ m/s^2), \tag{5.2.28}$$

which gives

$$F_s = 735\ N. \tag{5.2.29}$$

Significance

The scale reading in Figure 5.2.4*a* is about 185 lb. What would the scale have read if he were stationary? Since his acceleration would be zero, the force of the scale would be equal to his weight:

$$F_{net} = ma = 0 = F_s - w \tag{5.2.30}$$

$$F_s = w = mg \tag{5.2.31}$$

$$F_s = (75.0\ kg)(9.80\ m/s^2) = 735\ N. \tag{5.2.32}$$

Thus, the scale reading in the elevator is greater than his 735-N (165-lb.) weight. This means that the scale is pushing up on the person with a force greater than his weight, as it must in order to accelerate him upward.

Clearly, the greater the acceleration of the elevator, the greater the scale reading, consistent with what you feel in rapidly accelerating versus slowly accelerating elevators. In Figure 5.2.4*b*, the scale reading is 735 N, which equals the person's weight. This is the case whenever the elevator has a constant velocity—moving up, moving down, or stationary.

Exercise 6.1

Now calculate the scale reading when the elevator accelerates downward at a rate of 1.20 m/s^2.

The solution to the previous example also applies to an elevator accelerating downward, as mentioned. When an elevator accelerates downward, a is negative, and the scale reading is **less** than the weight of the person. If a constant downward velocity is reached, the scale reading again becomes equal to the person's weight. If the elevator is in free fall and accelerating downward at g, then the scale reading is zero and the person appears to be weightless.

Example 6.4: Two Attached Blocks

Figure 5.2.5 shows a block of mass m_1 on a frictionless, horizontal surface. It is pulled by a light string that passes over a frictionless and massless pulley. The other end of the string is connected to a block of mass m_2. Find the acceleration of the blocks and the tension in the string in terms of m_1, m_2, and g.

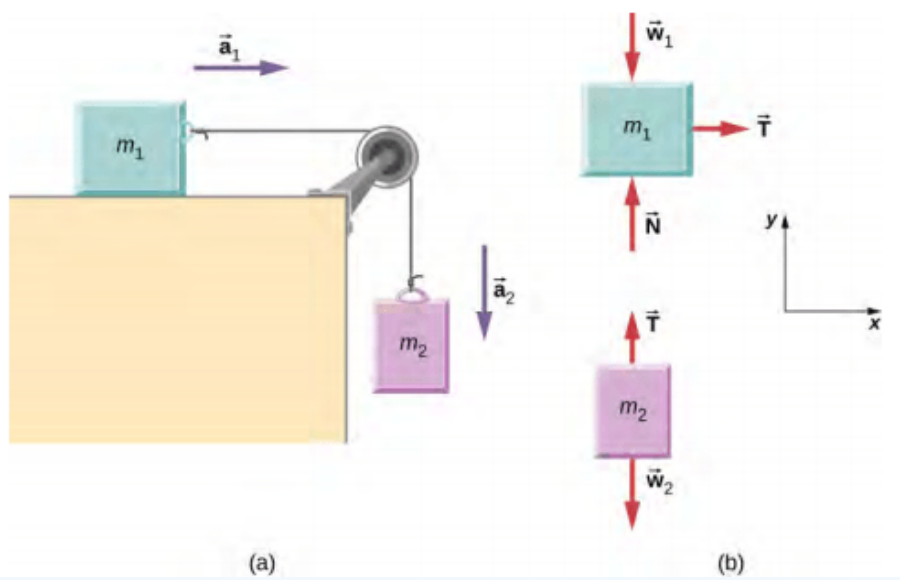


Figure 5.2.5: (a) Block 1 is connected by a light string to block 2. (b) The free-body diagrams of the blocks.

Strategy

We draw a free-body diagram for each mass separately, as shown in Figure 5.2.5. Then we analyze each one to find the required unknowns. The forces on block 1 are the gravitational force, the contact force of the surface, and the tension in the string. Block 2 is subjected to the gravitational force and the string tension. Newton's second law applies to each, so we write two vector equations:

For block 1: $\vec{T} + \vec{w}_1 + \vec{N} = m_1\vec{a}_1$

For block 2: $\vec{T} + \vec{w}_2 = m_2\vec{a}_2$.

Notice that $\vec{T}$ is the same for both blocks. Since the string and the pulley have negligible mass, and since there is no friction in the pulley, the tension is the same throughout the string. We can now write component equations for each block. All forces are either horizontal or vertical, so we can use the same horizontal/vertical coordinate system for both objects.

Solution

The component equations follow from the vector equations above. We see that block 1 has the vertical forces balanced, so we ignore them and write an equation relating the x-components. There are no horizontal forces on block 2, so only the y-equation is written. We obtain these results:

Block 1

$$\sum F_x = ma_x \tag{5.2.33}$$

$$T_x = m_1 a_{1x} \tag{5.2.34}$$

When block 1 moves to the right, block 2 travels an equal distance downward; thus, $a_{1x} = -a_{2y}$. Writing the common acceleration of the blocks as $a = a_{1x} = -a_{2y}$, we now have

$$T = m_1 a \tag{5.2.37}$$

and

$$T - m_2 g = -m_2 a. \tag{5.2.38}$$

From these two equations, we can express a and T in terms of the masses m_1 and m_2, and g:

$$a = \frac{m_2}{m_1 + m_2} g \tag{5.2.39}$$

and

$$T = \frac{m_1 m_2}{m_1 + m_2} g. \tag{5.2.40}$$

Significance

Notice that the tension in the string is less than the weight of the block hanging from the end of it. A common error in problems like this is to set $T = m_2 g$. You can see from the free-body diagram of block 2 that cannot be correct if the block is accelerating.

Check Your Understanding 6.2

Calculate the acceleration of the system, and the tension in the string, when the masses are $m_1 = 5.00$ kg and $m_2 = 3.00$ kg.

Example 6.5: Atwood Machine

A classic problem in physics, similar to the one we just solved, is that of the Atwood machine, which consists of a rope running over a pulley, with two objects of different mass attached. It is particularly useful in understanding the connection between force and motion. In Figure 5.2.6, m_1 = 2.00 kg and m_2 = 4.00 kg. Consider the pulley to be frictionless. (a) If m_2 is released, what will its acceleration be? (b) What is the tension in the string?

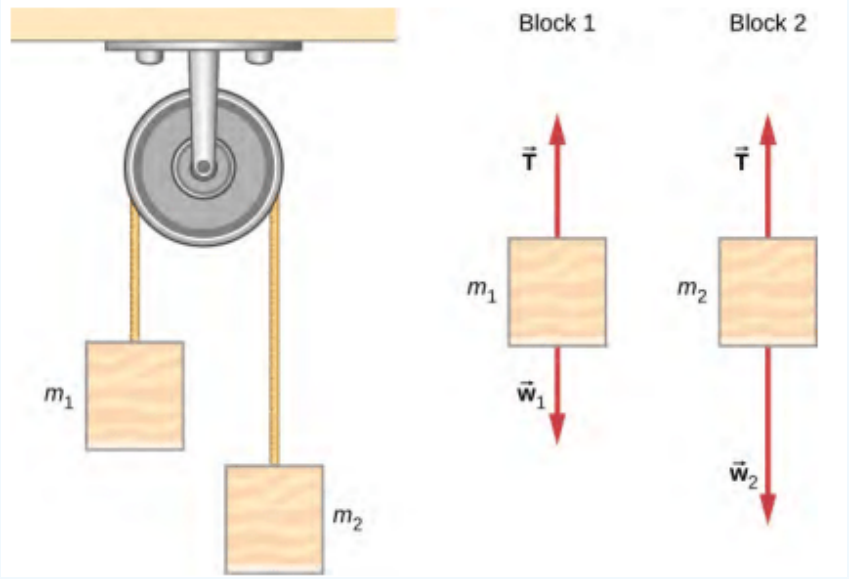


Figure 5.2.6: An Atwood machine and free-body diagrams for each of the two blocks.

Strategy

We draw a free-body diagram for each mass separately, as shown in the figure. Then we analyze each diagram to find the required unknowns. This may involve the solution of simultaneous equations. It is also important to note the similarity with the previous example. As block 2 accelerates with acceleration a_2 in the downward direction, block 1 accelerates upward with acceleration a_1. Thus, $a = a_1 = -a_2$.

Solution

a. We have

$$For\ m_1, \sum F_y = T - m_1 g = m_1 a. \quad For\ m_2, \sum F_y = T - m_2 g = -m_2 a. \tag{5.2.41}$$

(The negative sign in front of m_2 a indicates that m_2 accelerates downward; both blocks accelerate at the same rate, but in opposite directions.) Solve the two equations simultaneously (subtract them) and the result is

$$(m_2 - m_1)g = (m_1 + m_2)a. \tag{5.2.42}$$

Solving for a:

$$a = \frac{m_2 - m_1}{m_1 + m_2} g = \frac{4\ kg - 2\ kg}{4\ kg + 2\ kg}(9.8\ m/s^2) = 3.27\ m/s^2. \tag{5.2.43}$$

b. Observing the first block, we see that

$$T - m_1 g = m_1 a \tag{5.2.44}$$

$$T = m_1(g + a) = (2\ kg)(9.8\ m/s^2 + 3.27\ m/s^2) = 26.1\ N. \tag{5.2.45}$$

Significance

The result for the acceleration given in the solution can be interpreted as the ratio of the unbalanced force on the system, $(m_2 - m_1)g$, to the total mass of the system, $m_1 + m_2$. We can also use the Atwood machine to measure local gravitational field strength.

Exercise 6.3

Determine a general formula in terms of m_1, m_2 and g for calculating the tension in the string for the Atwood machine shown above.

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5.3: Solving Problems with Newton's Laws (Part 2)

Newton's Laws of Motion and Kinematics

Physics is most interesting and most powerful when applied to general situations that involve more than a narrow set of physical principles. Newton's laws of motion can also be integrated with other concepts that have been discussed previously in this text to solve problems of motion. For example, forces produce accelerations, a topic of kinematics, and hence the relevance of earlier chapters.

When approaching problems that involve various types of forces, acceleration, velocity, and/or position, listing the givens and the quantities to be calculated will allow you to identify the principles involved. Then, you can refer to the chapters that deal with a particular topic and solve the problem using strategies outlined in the text. The following worked example illustrates how the problem-solving strategy given earlier in this chapter, as well as strategies presented in other chapters, is applied to an integrated concept problem.

Example 6.6: What Force Must a Soccer Player Exert to Reach Top Speed?

A soccer player starts at rest and accelerates forward, reaching a velocity of 8.00 m/s in 2.50 s. (a) What is her average acceleration? (b) What average force does the ground exert forward on the runner so that she achieves this acceleration? The player's mass is 70.0 kg, and air resistance is negligible.

Strategy

To find the answers to this problem, we use the problem-solving strategy given earlier in this chapter. The solutions to each part of the example illustrate how to apply specific problem-solving steps. In this case, we do not need to use all of the steps. We simply identify the physical principles, and thus the knowns and unknowns; apply Newton's second law; and check to see whether the answer is reasonable.

Solution

a. We are given the initial and final velocities (zero and 8.00 m/s forward); thus, the change in velocity is Δv = 8.00 m/s . We are given the elapsed time, so Δt = 2.50 s. The unknown is acceleration, which can be found from its definition:

$$a = \frac{\Delta v}{\Delta t}. \tag{5.3.1}$$

Substituting the known values yields

$$a = \frac{8.00\ m/s}{2.50\ s} = 3.20\ m/s^2. \tag{5.3.2}$$

b. Here we are asked to find the average force the ground exerts on the runner to produce this acceleration. (Remember that we are dealing with the force or forces acting on the object of interest.) This is the reaction force to that exerted by the player backward against the ground, by Newton's third law. Neglecting air resistance, this would be equal in magnitude to the net external force on the player, since this force causes her acceleration. Since we now know the player's acceleration and are given her mass, we can use Newton's second law to find the force exerted. That is,

$$F_{net} = ma. \tag{5.3.3}$$

Substituting the known values of m and a gives

$$F_{net} = (70.0\ kg)(3.20\ m/s^2) = 224\ N. \tag{5.3.4}$$

This is a reasonable result: The acceleration is attainable for an athlete in good condition. The force is about 50 pounds, a reasonable average force.

Significance

This example illustrates how to apply problem-solving strategies to situations that include topics from different chapters. The first step is to identify the physical principles, the knowns, and the unknowns involved in the problem. The second step is to solve for the unknown, in this case using Newton's second law. Finally, we check our answer to ensure it is reasonable. These techniques for integrated concept problems will be useful in applications of physics outside of a physics course, such as in your profession, in other science disciplines, and in everyday life.

Exercise 6.4

The soccer player stops after completing the play described above, but now notices that the ball is in position to be stolen. If she now experiences a force of 126 N to attempt to steal the ball, which is 2.00 m away from her, how long will it take her to get to the ball?

Example 6.7: What Force Acts on a Model Helicopter?

A 1.50-kg model helicopter has a velocity of 5.00 $\hat{j}$ m/s at t = 0. It is accelerated at a constant rate for two seconds (2.00 s) after which it has a velocity of (6.00 $\hat{i}$ + 12.00 $\hat{j}$) m/s. What is the magnitude of the resultant force acting on the helicopter during this time interval?

Strategy

We can easily set up a coordinate system in which the x-axis ($\hat{i}$ direction) is horizontal, and the y-axis ($\hat{j}$ direction) is vertical. We know that Δt = 2.00s and Δv = (6.00 $\hat{i}$ + 12.00 $\hat{j}$ m/s) – (5.00 $\hat{j}$ m/s). From this, we can calculate the acceleration by the definition; we can then apply Newton's second law.

Solution

We have

$$a = \frac{\Delta v}{\Delta t} = \frac{(6.00\hat{i} + 12.00\hat{j}\ m/s) - (5.00\hat{j}\ m/s)}{2.00\ s} = 3.00\hat{i} + 3.50\hat{j}\ m/s^2\$\$\$\sum \vec{F} = m\vec{a} = (1.50\ kg)(3.00\hat{i} + 3.50\hat{j}\ m/s^2) = 4.5$$

The magnitude of the force is now easily found:

$$F = \sqrt{(4.50\ N)^2 + (5.25\ N)^2} = 6.91\ N. \tag{5.3.6}$$

Significance

The original problem was stated in terms of $\hat{i}$ – $\hat{j}$ vector components, so we used vector methods. Compare this example with the previous example.

Exercise 6.5

Find the direction of the resultant for the 1.50-kg model helicopter.

Example 6.8: Baggage Tractor

Figure 5.3.7(a) shows a baggage tractor pulling luggage carts from an airplane. The tractor has mass 650.0 kg, while cart A has mass 250.0 kg and cart B has mass 150.0 kg. The driving force acting for a brief period of time accelerates the system from rest and acts for 3.00 s. (a) If this driving force is given by F = (820.0t) N, find the speed after 3.00 seconds. (b) What is the horizontal force acting on the connecting cable between the tractor and cart A at this instant?

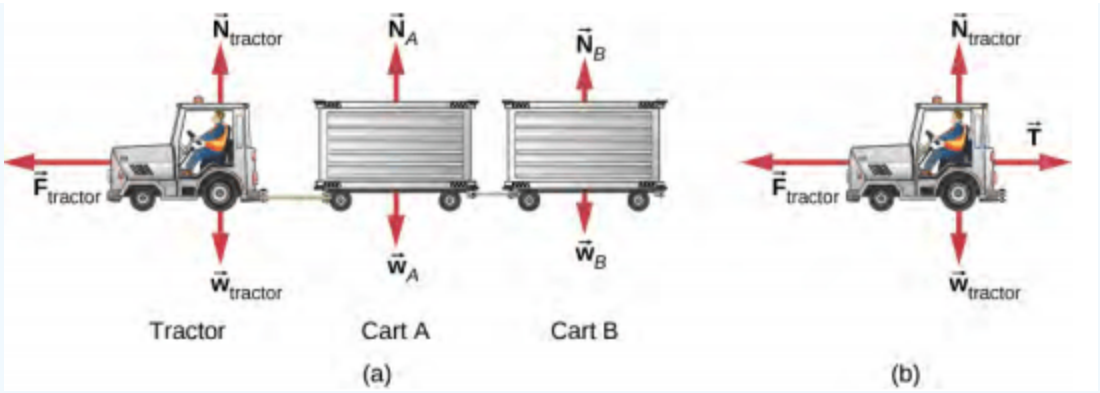


Figure 5.3.7: (a) A free-body diagram is shown, which indicates all the external forces on the system consisting of the tractor and baggage carts for carrying airline luggage. (b) A free-body diagram of the tractor only is shown isolated in order to calculate the tension in the cable to the carts.

Strategy

A free-body diagram shows the driving force of the tractor, which gives the system its acceleration. We only need to consider motion in the horizontal direction. The vertical forces balance each other and it is not necessary to consider them. For part b, we make use of a free-body diagram of the tractor alone to determine the force between it and cart A. This exposes the coupling force $\vec{T}$, which is our objective.

Solution

a. $$\sum F_x = m_{system} a_x \; and \; \sum F_x = 820.0t, \tag{5.3.7}$$

so

$$820.0t = (650.0 + 250.0 + 150.0)a \tag{5.3.8}$$

$$a = 0.7809t. \tag{5.3.9}$$

Since acceleration is a function of time, we can determine the velocity of the tractor by using a = $\frac{dv}{dt}$ with the initial condition that v_0 = 0 at t = 0. We integrate from t = 0 to t = 3:

$$\begin{aligned} dv &= adt \\ \int_0^3 dv &= \int_0^{3.00} adt = \int_0^{3.00} 0.7809tdt \\ v &= 0.3905t^2\big]_0^{3.00} = 3.51 \; m/s. \end{aligned} \tag{5.3.10}$$

b. Refer to the free-body diagram in Figure 5.3.7(b)

$$\begin{aligned} \sum F_x &= m_{tractor} a_x \\ 820.0t - T &= m_{tractor}(0.7805)t \\ (820.0)(3.00) - T &= (650.0)(0.7805)(3.00) \\ T &= 938 \; N. \end{aligned} \tag{5.3.11}$$

Significance

Since the force varies with time, we must use calculus to solve this problem. Notice how the total mass of the system was important in solving Figure 5.3.7(a), whereas only the mass of the truck (since it supplied the force) was of use in Figure 5.3.7 (b).

Recall that v = $\frac{ds}{dt}$ and a = $\frac{dv}{dt}$. If acceleration is a function of time, we can use the calculus forms developed in Motion Along a Straight Line, as shown in this example. However, sometimes acceleration is a function of displacement. In this case, we can derive an important result from these calculus relations. Solving for dt in each, we have dt = $\frac{ds}{v}$ and dt = $\frac{dv}{a}$. Now, equating these expressions, we have $\frac{ds}{v}$ = $\frac{dv}{a}$. We can rearrange this to obtain a ds = v dv.

Example 6.9: Motion of a Projectile Fired Vertically

A 10.0-kg mortar shell is fired vertically upward from the ground, with an initial velocity of 50.0 m/s (see Figure 5.3.8). Determine the maximum height it will travel if atmospheric resistance is measured as $F_D = (0.0100\ v^2)$ N, where v is the speed at any instant.

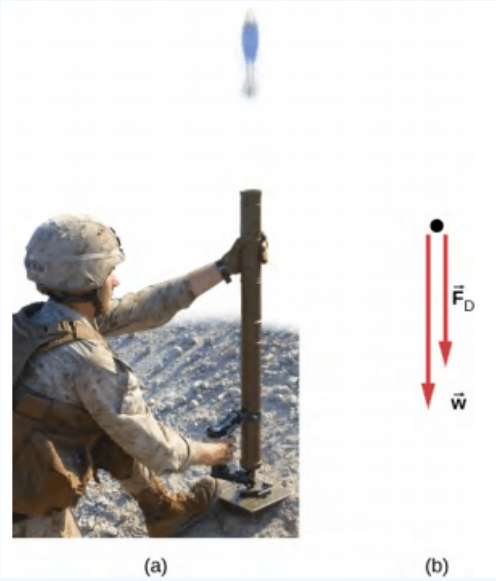


Figure 5.3.8: (a) The mortar fires a shell straight up; we consider the friction force provided by the air. (b) A free-body diagram is shown which indicates all the forces on the mortar shell.

Strategy

The known force on the mortar shell can be related to its acceleration using the equations of motion. Kinematics can then be used to relate the mortar shell's acceleration to its position.

Solution

Initially, $y_0 = 0$ and $v_0 = 50.0$ m/s. At the maximum height y = h, v = 0. The free-body diagram shows F_D to act downward, because it slows the upward motion of the mortar shell. Thus, we can write

$$\begin{aligned} \sum F_y &= ma_y \\ -F_D - w &= ma_y \\ -0.0100v^2 - 98.0 &= 10.0a \\ a &= -0.00100v^2 - 9.80. \end{aligned} \tag{5.3.12}$$

The acceleration depends on v and is therefore variable. Since a = f(v), we can relate a to v using the rearrangement described above,

$$ads = vdv. \tag{5.3.13}$$

We replace ds with dy because we are dealing with the vertical direction,

$$\begin{aligned} ady &= vdv \\ (-0.00100v^2 - 9.80)dy &= vdv. \end{aligned} \tag{5.3.14}$$

We now separate the variables (v's and dv's on one side; dy on the other):

$$\begin{aligned}\int_0^h dy &= \int_{50.0}^0 \frac{vdv}{(-0.00100v^2 - 9.80)} \\ &= -\int_{50.0}^0 \frac{vdv}{(-0.00100v^2 + 9.80)} \\ &= (-5 \times 10^3)\ln(0.00100v^2 + 9.80)\Big|_{50.0}^0.\end{aligned} \tag{5.3.15}$$

Thus, h = 114 m.

Significance

Notice the need to apply calculus since the force is not constant, which also means that acceleration is not constant. To make matters worse, the force depends on v (not t), and so we must use the trick explained prior to the example. The answer for the height indicates a lower elevation if there were air resistance. We will deal with the effects of air resistance and other drag forces in greater detail in Drag Force and Terminal Speed.

Exercise 6.6

If atmospheric resistance is neglected, find the maximum height for the mortar shell. Is calculus required for this solution?

Simulation

Explore the forces at work in this simulation when you try to push a filing cabinet. Create an applied force and see the resulting frictional force and total force acting on the cabinet. Charts show the forces, position, velocity, and acceleration vs. time. View a free-body diagram of all the forces (including gravitational and normal forces).

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5.4: Friction (Part 1)

Learning Objectives

- Describe the general characteristics of friction
- List the various types of friction
- Calculate the magnitude of static and kinetic friction, and use these in problems involving Newton's laws of motion

When a body is in motion, it has resistance because the body interacts with its surroundings. This resistance is a force of friction. Friction opposes relative motion between systems in contact but also allows us to move, a concept that becomes obvious if you try to walk on ice. Friction is a common yet complex force, and its behavior still not completely understood. Still, it is possible to understand the circumstances in which it behaves.

Static and Kinetic Friction

The basic definition of friction is relatively simple to state.

Friction

Friction is a force that opposes relative motion between systems in contact.

There are several forms of friction. One of the simpler characteristics of sliding friction is that it is parallel to the contact surfaces between systems and is always in a direction that opposes motion or attempted motion of the systems relative to each other. If two systems are in contact and moving relative to one another, then the friction between them is called kinetic friction. For example, friction slows a hockey puck sliding on ice. When objects are stationary, static friction can act between them; the static friction is usually greater than the kinetic friction between two objects.

Static and Kinetic Friction

If two systems are in contact and stationary relative to one another, then the friction between them is called **static friction**.
If two systems are in contact and moving relative to one another, then the friction between them is called **kinetic friction**.

Imagine, for example, trying to slide a heavy crate across a concrete floor—you might push very hard on the crate and not move it at all. This means that the static friction responds to what you do—it increases to be equal to and in the opposite direction of your push. If you finally push hard enough, the crate seems to slip suddenly and starts to move. Now static friction gives way to kinetic friction. Once in motion, it is easier to keep it in motion than it was to get it started, indicating that the kinetic frictional force is less than the static frictional force. If you add mass to the crate, say by placing a box on top of it, you need to push even harder to get it started and also to keep it moving. Furthermore, if you oiled the concrete you would find it easier to get the crate started and keep it going (as you might expect).

Figure 5.4.1 is a crude pictorial representation of how friction occurs at the interface between two objects. Close-up inspection of these surfaces shows them to be rough. Thus, when you push to get an object moving (in this case, a crate), you must raise the object until it can skip along with just the tips of the surface hitting, breaking off the points, or both. A considerable force can be resisted by friction with no apparent motion. The harder the surfaces are pushed together (such as if another box is placed on the crate), the more force is needed to move them. Part of the friction is due to adhesive forces between the surface molecules of the two objects, which explains the dependence of friction on the nature of the substances. For example, rubber-soled shoes slip less than those with leather soles. Adhesion varies with substances in contact and is a complicated aspect of surface physics. Once an object is moving, there are fewer points of contact (fewer molecules adhering), so less force is required to keep the object moving. At small but nonzero speeds, friction is nearly independent of speed.

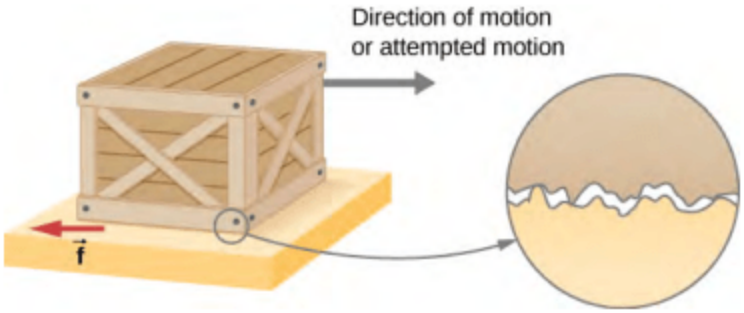


Figure 5.4.1: Frictional forces, such as $\vec{f}$, always oppose motion or attempted motion between objects in contact. Friction arises in part because of the roughness of the surfaces in contact, as seen in the expanded view. For the object to move, it must rise to where the peaks of the top surface can skip along the bottom surface. Thus, a force is required just to set the object in motion. Some of the peaks will be broken off, also requiring a force to maintain motion. Much of the friction is actually due to attractive forces between molecules making up the two objects, so that even perfectly smooth surfaces are not friction-free. (In fact, perfectly smooth, clean surfaces of similar materials would adhere, forming a bond called a "cold weld.")

The magnitude of the frictional force has two forms: one for static situations (static friction), the other for situations involving motion (kinetic friction). What follows is an approximate empirical (experimentally determined) model only. These equations for static and kinetic friction are not vector equations.

Magnitude of Static Friction

The magnitude of static friction f_s is

$$f_s \leq \mu_s N, \tag{5.4.1}$$

where μ_s is the coefficient of static friction and N is the magnitude of the normal force.

The symbol ≤ means **less than or equal to**, implying that static friction can have a maximum value of μ_sN. Static friction is a responsive force that increases to be equal and opposite to whatever force is exerted, up to its maximum limit. Once the applied force exceeds f_s (max), the object moves. Thus,

$$f_s(max) = \mu_s N. \tag{5.4.2}$$

Magnitude of Kinetic Friction

The magnitude of kinetic friction f_k is given by

$$f_k \leq \mu_k N, \tag{5.4.3}$$

where μ_k is the coefficient of kinetic friction.

A system in which $f_k = \mu_k N$ is described as a system in which friction behaves simply. The transition from static friction to kinetic friction is illustrated in Figure 5.4.2

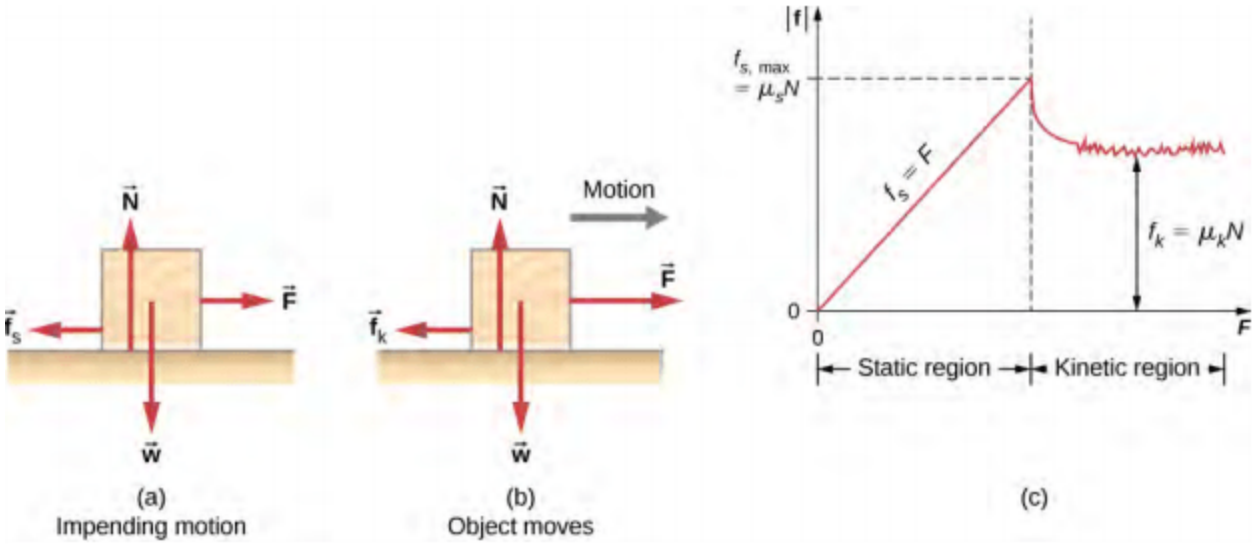


Figure 5.4.2: (a) The force of friction $\vec{f}$ between the block and the rough surface opposes the direction of the applied force $\vec{F}$. The magnitude of the static friction balances that of the applied force. This is shown in the left side of the graph in (c). (b) At some point, the magnitude of the applied force is greater than the force of kinetic friction, and the block moves to the right. This is shown in the right side of the graph. (c) The graph of the frictional force versus the applied force; note that fs (max) > f_k. This means that $\mu_s > \mu_k$

As you can see in Table 6.1, the coefficients of kinetic friction are less than their static counterparts. The approximate values of μ are stated to only one or two digits to indicate the approximate description of friction given by the preceding two equations.

Table 6.1 - Approximate Coefficients of Static and Kinetic Friction

System	Static Friction μ_s	Kinetic Friction μ_k
Rubber on dry concrete	1.0	0.7
Rubber on wet concrete	0.5-0.7	0.3-0.5
Wood on wood	0.5	0.3
Waxed wood on wet snow	0.14	0.1
Metal on wood	0.5	0.3
Steel on steel (dry)	0.6	0.3
Steel on steel (oiled)	0.05	0.03
Teflon on steel	0.04	0.04
Bone lubricated by synovial fluid	0.016	0.015
Shoes on wood	0.9	0.7
Shoes on ice	0.1	0.05
Ice on ice	0.1	0.03
Steel on ice	0.4	0.02

Equation 5.4.1 and Equation 5.4.3 include the dependence of friction on materials and the normal force. The direction of friction is always opposite that of motion, parallel to the surface between objects, and perpendicular to the normal force. For example, if the crate you try to push (with a force parallel to the floor) has a mass of 100 kg, then the normal force is equal to its weight,

$$w = mg = (100\ kg)(9.80\ m/s^2) = 980\ N, \tag{5.4.4}$$

perpendicular to the floor. If the coefficient of static friction is 0.45, you would have to exert a force parallel to the floor greater

than

$$f_s(max) = \mu_s N = (0.45)(980\ N) = 440\ N \tag{5.4.5}$$

to move the crate. Once there is motion, friction is less and the coefficient of kinetic friction might be 0.30, so that a force of only

$$f_k = \mu_k N = (0.30)(980\ N) = 290\ N \tag{5.4.6}$$

keeps it moving at a constant speed. If the floor is lubricated, both coefficients are considerably less than they would be without lubrication. Coefficient of friction is a unitless quantity with a magnitude usually between 0 and 1.0. The actual value depends on the two surfaces that are in contact.

Many people have experienced the slipperiness of walking on ice. However, many parts of the body, especially the joints, have much smaller coefficients of friction—often three or four times less than ice. A joint is formed by the ends of two bones, which are connected by thick tissues. The knee joint is formed by the lower leg bone (the tibia) and the thighbone (the femur). The hip is a ball (at the end of the femur) and socket (part of the pelvis) joint. The ends of the bones in the joint are covered by cartilage, which provides a smooth, almost-glassy surface. The joints also produce a fluid (synovial fluid) that reduces friction and wear. A damaged or arthritic joint can be replaced by an artificial joint (Figure 5.4.3). These replacements can be made of metals (stainless steel or titanium) or plastic (polyethylene), also with very small coefficients of friction.

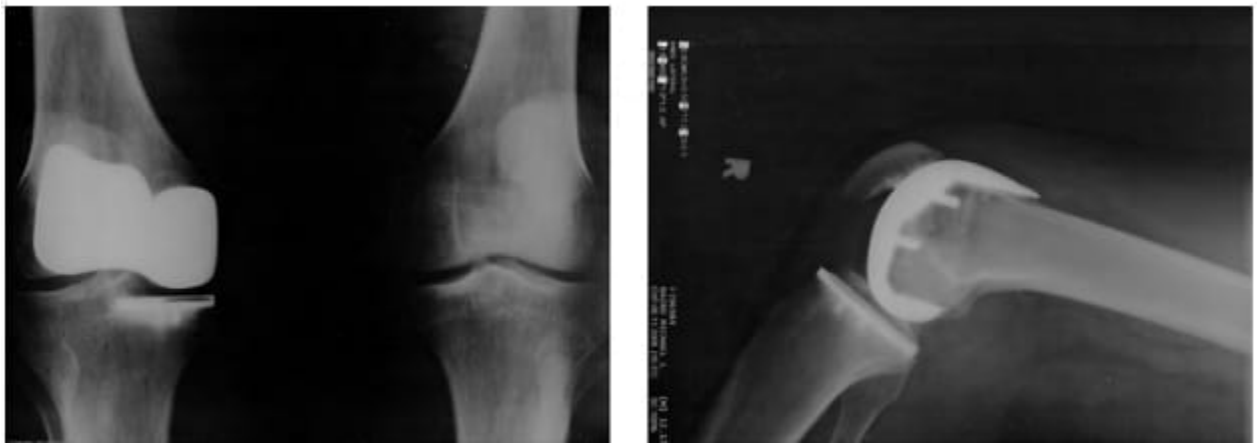


Figure 5.4.3: Artificial knee replacement is a procedure that has been performed for more than 20 years. These post-operative X-rays show a right knee joint replacement. (credit: Mike Baird)

Natural lubricants include saliva produced in our mouths to aid in the swallowing process, and the slippery mucus found between organs in the body, allowing them to move freely past each other during heartbeats, during breathing, and when a person moves. Hospitals and doctor's clinics commonly use artificial lubricants, such as gels, to reduce friction.

The equations given for static and kinetic friction are empirical laws that describe the behavior of the forces of friction. While these formulas are very useful for practical purposes, they do not have the status of mathematical statements that represent general principles (e.g., Newton's second law). In fact, there are cases for which these equations are not even good approximations. For instance, neither formula is accurate for lubricated surfaces or for two surfaces siding across each other at high speeds. Unless specified, we will not be concerned with these exceptions.

Example 6.10: Static and Kinetic Friction

A 20.0-kg crate is at rest on a floor as shown in Figure 5.4.4. The coefficient of static friction between the crate and floor is 0.700 and the coefficient of kinetic friction is 0.600. A horizontal force $\vec{P}$ is applied to the crate. Find the force of friction if (a) $\vec{P}$ = 20.0 N, (b) $\vec{P}$ = 30.0 N, (c) $\vec{P}$ = 120.0 N, and (d) $\vec{P}$ = 180.0 N.

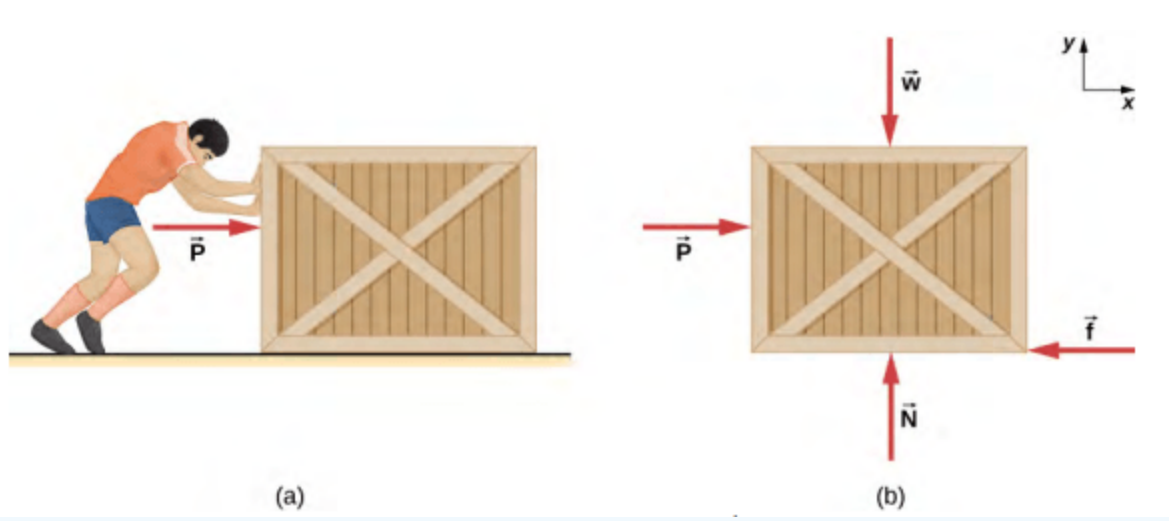


Figure 5.4.4: (a) A crate on a horizontal surface is pushed with a force $\vec{P}$. (b) The forces on the crate. Here, $\vec{f}$ may represent either the static or the kinetic frictional force.

Strategy

The free-body diagram of the crate is shown in Figure 5.4.4*b*. We apply Newton's second law in the horizontal and vertical directions, including the friction force in opposition to the direction of motion of the box.

Solution

Newton's second law gives

$$\sum F_x = ma_x \tag{5.4.7}$$

$$P - f = ma_x \tag{5.4.8}$$

Here we are using the symbol f to represent the frictional force since we have not yet determined whether the crate is subject to station friction or kinetic friction. We do this whenever we are unsure what type of friction is acting. Now the weight of the crate is

$$w = (20.0\ kg)(9.80\ m/s^2) = 196\ N, \tag{5.4.11}$$

which is also equal to N. The maximum force of static friction is therefore (0.700)(196 N) = 137 N. As long as $\vec{P}$ is less than 137 N, the force of static friction keeps the crate stationary and $f_s = \vec{P}$. Thus, (a) f_s = 20.0 N, (b) f_s = 30.0 N, and (c) f_s = 120.0 N. (d) If $\vec{P}$ = 180.0 N, the applied force is greater than the maximum force of static friction (137 N), so the crate can no longer remain at rest. Once the crate is in motion, kinetic friction acts. Then

$$f_k = \mu_k N = (0.600)(196\ N) = 118\ N, \tag{5.4.12}$$

and the acceleration is

$$a_x = \frac{\vec{P} - f_k}{m} = \frac{180.0\ N - 118\ N}{20.0\ kg} = 3.10\ m/s^2. \tag{5.4.13}$$

Significance

This example illustrates how we consider friction in a dynamics problem. Notice that static friction has a value that matches the applied force, until we reach the maximum value of static friction. Also, no motion can occur until the applied force equals the force of static friction, but the force of kinetic friction will then become smaller.

Exercise 6.7

A block of mass 1.0 kg rests on a horizontal surface. The frictional coefficients for the block and surface are μ_s = 0.50 and μ_k = 0.40. (a) What is the minimum horizontal force required to move the block? (b) What is the block's acceleration when this force is applied?

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5.5: Friction (Part 2)

Friction and the Inclined Plane

One situation where friction plays an obvious role is that of an object on a slope. It might be a crate being pushed up a ramp to a loading dock or a skateboarder coasting down a mountain, but the basic physics is the same. We usually generalize the sloping surface and call it an inclined plane but then pretend that the surface is flat. Let's look at an example of analyzing motion on an inclined plane with friction.

Example 5.5.1: Downhill Skier

A skier with a mass of 62 kg is sliding down a snowy slope at a constant velocity. Find the coefficient of kinetic friction for the skier if friction is known to be 45.0 N.

Strategy

The magnitude of kinetic friction is given as 45.0 N. Kinetic friction is related to the normal force N by $f_k = \mu_k N$; thus, we can find the coefficient of kinetic friction if we can find the normal force on the skier. The normal force is always perpendicular to the surface, and since there is no motion perpendicular to the surface, the normal force should equal the component of the skier's weight perpendicular to the slope. (See Figure 5.5.1, which repeats a figure from the chapter on Newton's laws of motion.)

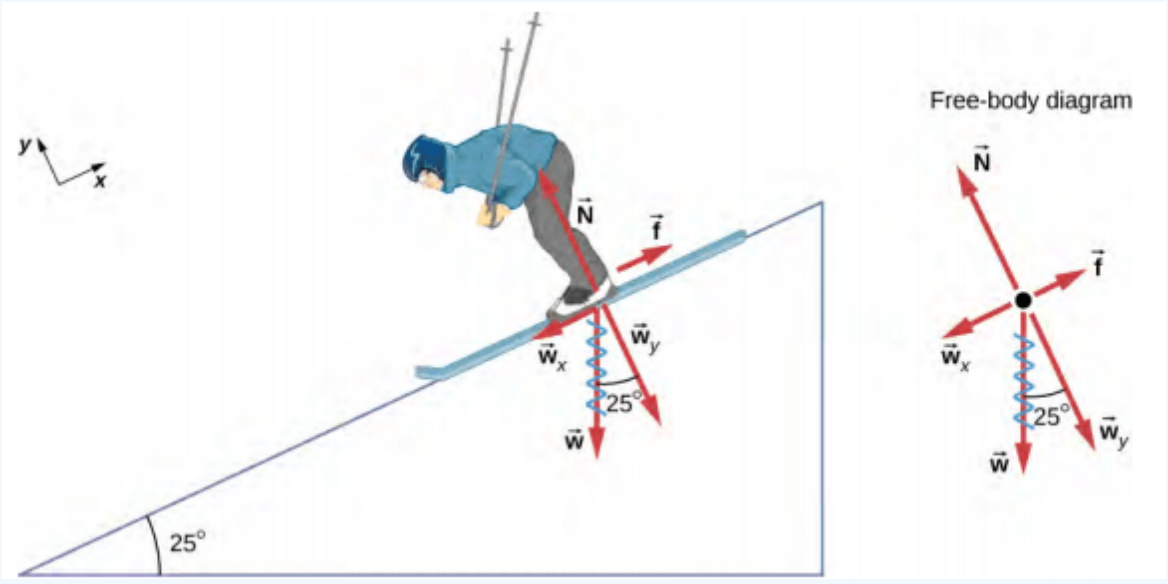


Figure 5.5.1: The motion of the skier and friction are parallel to the slope, so it is most convenient to project all forces onto a coordinate system where one axis is parallel to the slope and the other is perpendicular (axes shown to left of skier). The normal force $\vec{N}$ is perpendicular to the slope, and friction $\vec{f}$ is parallel to the slope, but the skier's weight $\vec{w}$ has components along both axes, namely $\vec{w}_y$ and $\vec{w}_x$. The normal force $\vec{N}$ is equal in magnitude to $\vec{w}_y$, so there is no motion perpendicular to the slope. However, $\vec{f}$ is less than $\vec{w}_x$ in magnitude, so there is acceleration down the slope (along the x-axis).

We have

$$N = w_y = w\cos 25^o = mg\cos 25^o. \tag{5.5.1}$$

Substituting this into our expression for kinetic friction, we obtain

$$f_k = \mu_k mg\cos 25^o, \tag{5.5.2}$$

which can now be solved for the coefficient of kinetic friction μ_k.

Solution

Solving for μ_k gives

$$\mu_k = \frac{f_k}{N} = \frac{f_k}{w\cos 25^o} = \frac{f_k}{mg\cos 25^o}. \tag{5.5.3}$$

Substituting known values on the right-hand side of the equation,

$$\mu_k = \frac{45.0\ N}{(62\ kg)(9.80\ m/s^2)(0.906)} = 0.082. \tag{5.5.4}$$

Significance

This result is a little smaller than the coefficient listed in Table 6.1 for waxed wood on snow, but it is still reasonable since values of the coefficients of friction can vary greatly. In situations like this, where an object of mass m slides down a slope that makes an angle θ with the horizontal, friction is given by $f_k = \mu_k$ mg cos θ. All objects slide down a slope with constant acceleration under these circumstances.

We have discussed that when an object rests on a horizontal surface, the normal force supporting it is equal in magnitude to its weight. Furthermore, simple friction is always proportional to the normal force. When an object is not on a horizontal surface, as with the inclined plane, we must find the force acting on the object that is directed perpendicular to the surface; it is a component of the weight.

We now derive a useful relationship for calculating coefficient of friction on an inclined plane. Notice that the result applies only for situations in which the object slides at constant speed down the ramp.

An object slides down an inclined plane at a constant velocity if the net force on the object is zero. We can use this fact to measure the coefficient of kinetic friction between two objects. As shown in Example $5.5.1$, the kinetic friction on a slope is $f_k = \mu_k$ mg cos θ. The component of the weight down the slope is equal to mg sin θ (see the free-body diagram in Figure $5.5.1$). These forces act in opposite directions, so when they have equal magnitude, the acceleration is zero. Writing these out,

$$\mu_k mg\cos\theta = mg\sin\theta. \tag{5.5.5}$$

Solving for μ_k, we find that

$$\mu_k = \frac{mg\sin\theta}{mg\cos\theta} = \tan\theta. \tag{5.5.6}$$

Put a coin on a book and tilt it until the coin slides at a constant velocity down the book. You might need to tap the book lightly to get the coin to move. Measure the angle of tilt relative to the horizontal and find μ_k. Note that the coin does not start to slide at all until an angle greater than θ is attained, since the coefficient of static friction is larger than the coefficient of kinetic friction. Think about how this may affect the value for μ_k and its uncertainty.

Atomic-Scale Explanations of Friction

The simpler aspects of friction dealt with so far are its macroscopic (large-scale) characteristics. Great strides have been made in the atomic-scale explanation of friction during the past several decades. Researchers are finding that the atomic nature of friction seems to have several fundamental characteristics. These characteristics not only explain some of the simpler aspects of friction—they also hold the potential for the development of nearly friction-free environments that could save hundreds of billions of dollars in energy which is currently being converted (unnecessarily) into heat.

Figure $5.5.2$ illustrates one macroscopic characteristic of friction that is explained by microscopic (small-scale) research. We have noted that friction is proportional to the normal force, but not to the amount of area in contact, a somewhat counterintuitive notion. When two rough surfaces are in contact, the actual contact area is a tiny fraction of the total area because only high spots touch. When a greater normal force is exerted, the actual contact area increases, and we find that the friction is proportional to this area.

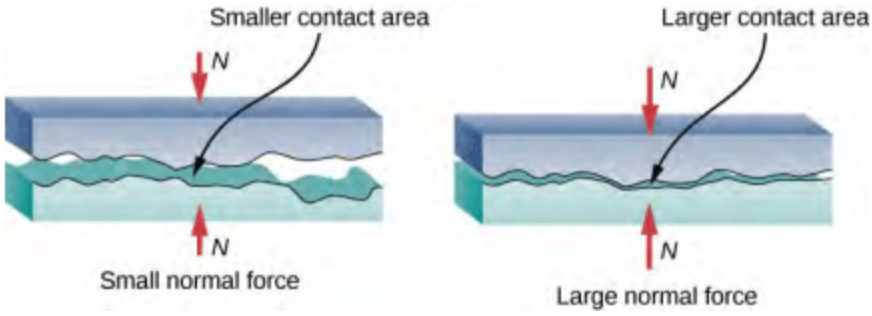


Figure 5.5.2: Two rough surfaces in contact have a much smaller area of actual contact than their total area. When the normal force is larger as a result of a larger applied force, the area of actual contact increases, as does friction.

However, the atomic-scale view promises to explain far more than the simpler features of friction. The mechanism for how heat is generated is now being determined. In other words, why do surfaces get warmer when rubbed? Essentially, atoms are linked with one another to form lattices. When surfaces rub, the surface atoms adhere and cause atomic lattices to vibrate—essentially creating sound waves that penetrate the material. The sound waves diminish with distance, and their energy is converted into heat. Chemical reactions that are related to frictional wear can also occur between atoms and molecules on the surfaces. Figure 5.5.3 shows how the tip of a probe drawn across another material is deformed by atomic-scale friction. The force needed to drag the tip can be measured and is found to be related to shear stress, which is discussed in Static Equilibrium and Elasticity. The variation in shear stress is remarkable (more than a factor of 1012) and difficult to predict theoretically, but shear stress is yielding a fundamental understanding of a large-scale phenomenon known since ancient times—friction.

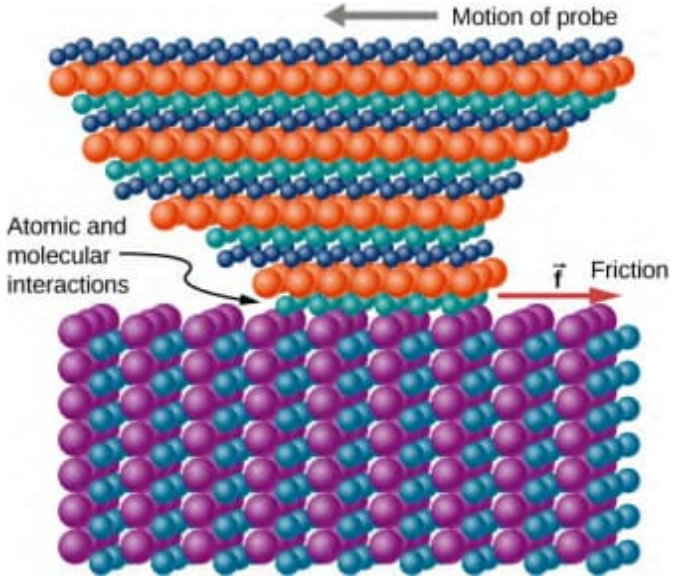


Figure 5.5.3: The tip of a probe is deformed sideways by frictional force as the probe is dragged across a surface. Measurements of how the force varies for different materials are yielding fundamental insights into the atomic nature of friction.

Simulation

Describe a model for friction on a molecular level. Describe matter in terms of molecular motion. The description should include diagrams to support the description; how the temperature affects the image; what are the differences and similarities between solid, liquid, and gas particle motion; and how the size and speed of gas molecules relate to everyday objects.

Example 5.5.2: Sliding Blocks

The two blocks of Figure 5.5.4 are attached to each other by a massless string that is wrapped around a frictionless pulley. When the bottom 4.00-kg block is pulled to the left by the constant force $\vec{P}$, the top 2.00-kg block slides across it to the right. Find the magnitude of the force necessary to move the blocks at constant speed. Assume that the coefficient of kinetic friction between all surfaces is 0.400.

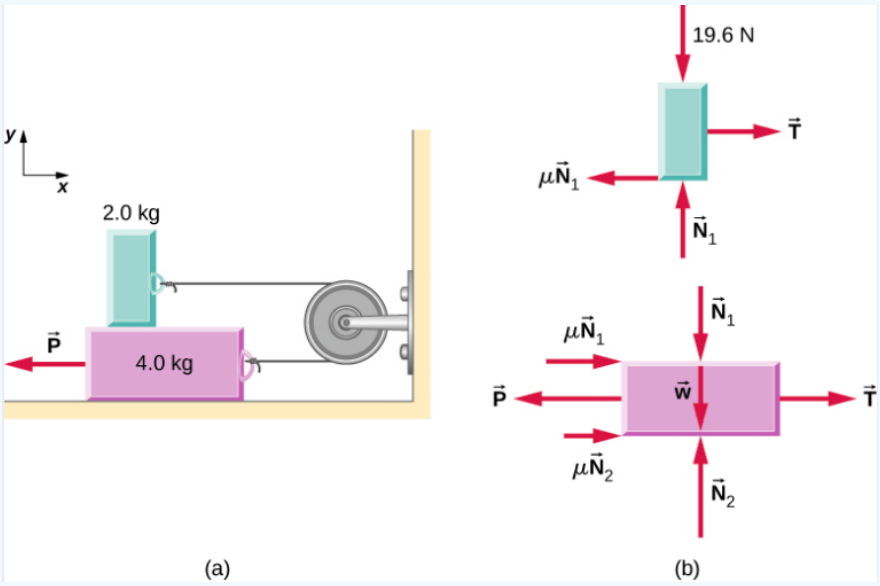


Figure 5.5.4: (a) Each block moves at constant velocity. (b) Free-body diagrams for the blocks.

Strategy

We analyze the motions of the two blocks separately. The top block is subjected to a contact force exerted by the bottom block. The components of this force are the normal force N_1 and the frictional force $-0.400\ N_1$. Other forces on the top block are the tension T in the string and the weight of the top block itself, 19.6 N. The bottom block is subjected to contact forces due to the top block and due to the floor. The first contact force has components $-N_1$ and $0.400\ N_1$, which are simply reaction forces to the contact forces that the bottom block exerts on the top block. The components of the contact force of the floor are N_2 and $0.400\ N_2$. Other forces on this block are –P, the tension T, and the weight –39.2 N. Solution Since the top block is moving horizontally to the right at constant velocity, its acceleration is zero in both the horizontal and the vertical directions. From Newton's second law,

$$\sum F_x = m_2 a_x \tag{5.5.7}$$

$$T - 0.400\ N_1 = 0 \tag{5.5.8}$$

Solving for the two unknowns, we obtain N_1 = 19.6 N and T = 0.40 N_1 = 7.84 N. The bottom block is also not accelerating, so the application of Newton's second law to this block gives

$$\sum F_x = m_2 a_x \tag{5.5.11}$$

$$T - P + 0.400\ N_1 + 0.400\ N_2 = 0 \tag{5.5.12}$$

The values of N_1 and T were found with the first set of equations. When these values are substituted into the second set of equations, we can determine N_2 and P. They are

$$N_2 = 58.8\ N\ and\ P = 39.2\ N. \tag{5.5.15}$$

Significance

Understanding what direction in which to draw the friction force is often troublesome. Notice that each friction force labeled in Figure 5.5.4 acts in the direction opposite the motion of its corresponding block.

Example 5.5.3: A Crate on an Accelerating Truck

A 50.0-kg crate rests on the bed of a truck as shown in Figure 5.5.5. The coefficients of friction between the surfaces are μ_k = 0.300 and μ_s = 0.400. Find the frictional force on the crate when the truck is accelerating forward relative to the ground at (a) 2.00 m/s^2, and (b) 5.00 m/s^2.

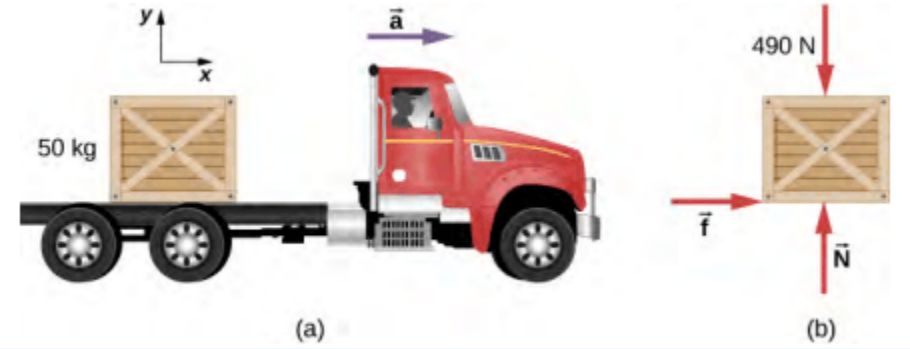


Figure 5.5.5: (a) A crate rests on the bed of the truck that is accelerating forward. (b) The free-body diagram of the crate.

Strategy

The forces on the crate are its weight and the normal and frictional forces due to contact with the truck bed. We start by assuming that the crate is not slipping. In this case, the static frictional force fs acts on the crate. Furthermore, the accelerations of the crate and the truck are equal.

Solution

a. Application of Newton's second law to the crate, using the reference frame attached to the ground, yields

$$\begin{aligned} \sum F_x &= ma_x \\ f_s &= (50.0\ kg)(2.00\ m/s^2) \\ &= 1.00 \times 10^2\ N \end{aligned} \tag{5.5.16}$$

We can now check the validity of our no-slip assumption. The maximum value of the force of static friction is

$$\mu_s N = (0.400)(4.90 \times 10^2\ N) = 196\ N, \tag{5.5.18}$$

whereas the **actual** force of static friction that acts when the truck accelerates forward at 2.00 m/s^2 is only 1.00 x 10^2 N. Thus, the assumption of no slipping is valid.

b. If the crate is to move with the truck when it accelerates at 5.0 m/s^2, the force of static friction must be

$$f_s = ma_x = (50.0\ kg)(5.00\ m/s^2) = 250\ N. \tag{5.5.19}$$

Since this exceeds the maximum of 196 N, the crate must slip. The frictional force is therefore kinetic and is

$$f_k = \mu_k N = (0.300)(4.90 \times 10^2\ N) = 147\ N. \tag{5.5.20}$$

The horizontal acceleration of the crate relative to the ground is now found from $$

$$\begin{aligned} \sum F_x &= ma_x \\ 147\ N &= (50.0\ kg)a_x, \\ so\ a_x &= 2.94\ m/s^2. \end{aligned} \tag{5.5.21}$$

\]

Significance

Relative to the ground, the truck is accelerating forward at 5.0 m/s^2 and the crate is accelerating forward at 2.94 m/s^2. Hence the crate is sliding backward relative to the bed of the truck with an acceleration 2.94 m/s^2 – 5.00 m/s^2 = –2.06 m/s^2.

Example 5.5.4: Snowboarding

Earlier, we analyzed the situation of a downhill skier moving at constant velocity to determine the coefficient of kinetic friction. Now let's do a similar analysis to determine acceleration. The snowboarder of Figure 5.5.6 glides down a slope that is inclined at θ = 13° to the horizontal. The coefficient of kinetic friction between the board and the snow is μ_k = 0.20. What is the acceleration of the snowboarder?

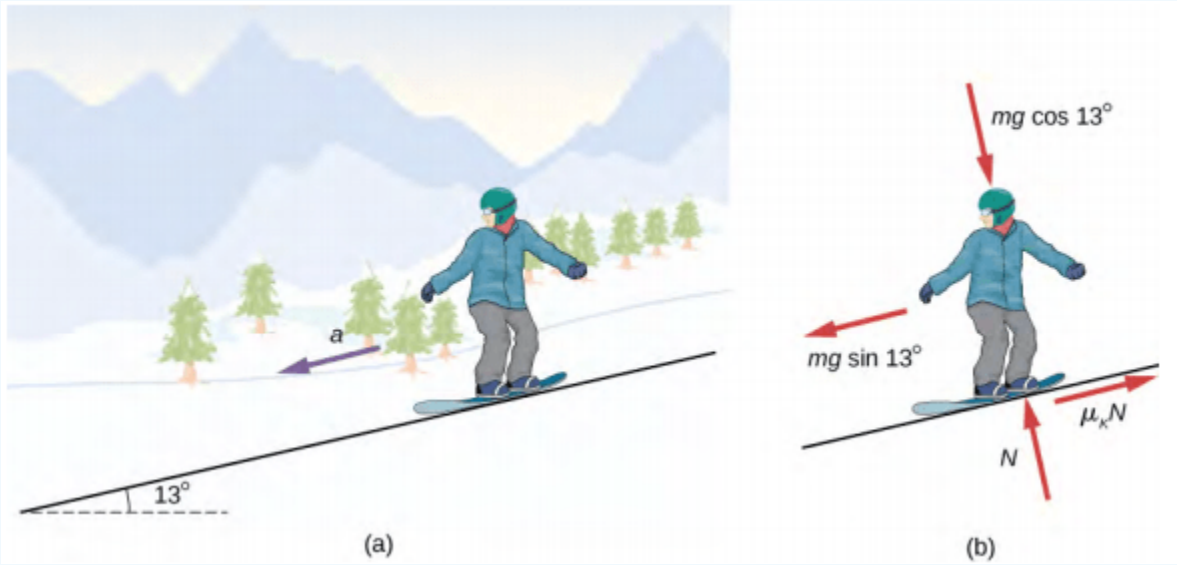


Figure 5.5.6: (a) A snowboarder glides down a slope inclined at 13° to the horizontal. (b) The free-body diagram of the snowboarder.

Strategy

The forces acting on the snowboarder are her weight and the contact force of the slope, which has a component normal to the incline and a component along the incline (force of kinetic friction). Because she moves along the slope, the most convenient reference frame for analyzing her motion is one with the x-axis along and the y-axis perpendicular to the incline. In this frame, both the normal and the frictional forces lie along coordinate axes, the components of the weight are mg sin θ along the slope and mg cos θ at right angles into the slope , and the only acceleration is along the x-axis (a_y = 0).

Solution

We can now apply Newton's second law to the snowboarder:

$$\begin{aligned} \sum F_x &= ma_x \\ mg\sin\theta - \mu_k N &= ma_x \end{aligned} \tag{5.5.22}$$

From the second equation, N = mg cos θ. Upon substituting this into the first equation, we find

$$\begin{aligned} a_x &= g(\sin\theta - \mu_k \cos\theta) \\ &= g(\sin 13^o - 0.520 \cos 13^o) = 0.29\ m/s^2. \end{aligned} \tag{5.5.24}$$

Significance

Notice from this equation that if θ is small enough or μ_k is large enough, a_x is negative, that is, the snowboarder slows down.

Exercise 5.5.4

The snowboarder is now moving down a hill with incline 10.0°. What is the skier's acceleration?

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5.6: Centripetal Force

Learning Objectives

- Explain the equation for centripetal acceleration
- Apply Newton's second law to develop the equation for centripetal force
- Use circular motion concepts in solving problems involving Newton's laws of motion

In Motion in Two and Three Dimensions, we examined the basic concepts of circular motion. An object undergoing circular motion, like one of the race cars shown at the beginning of this chapter, must be accelerating because it is changing the direction of its velocity. We proved that this centrally directed acceleration, called centripetal acceleration, is given by the formula

$$a_c = \frac{v^2}{r} \tag{5.6.1}$$

where v is the velocity of the object, directed along a tangent line to the curve at any instant. If we know the angular velocity ω, then we can use

$$a_c = r\omega^2. \tag{5.6.2}$$

Angular velocity gives the rate at which the object is turning through the curve, in units of rad/s. This acceleration acts along the radius of the curved path and is thus also referred to as a radial acceleration.

An acceleration must be produced by a force. Any force or combination of forces can cause a centripetal or radial acceleration. Just a few examples are the tension in the rope on a tether ball, the force of Earth's gravity on the Moon, friction between roller skates and a rink floor, a banked roadway's force on a car, and forces on the tube of a spinning centrifuge. Any net force causing uniform circular motion is called a **centripetal force**. The direction of a centripetal force is toward the center of curvature, the same as the direction of centripetal acceleration. According to Newton's second law of motion, net force is mass times acceleration: F_{net} = ma. For uniform circular motion, the acceleration is the centripetal acceleration: a = a_c. Thus, the magnitude of centripetal force F_c is

$$F_c = ma_c. \tag{5.6.3}$$

By substituting the expressions for centripetal acceleration a_c ($a_c = \frac{v^2}{r}; a_c = r\omega^2$), we get two expressions for the centripetal force F_c in terms of mass, velocity, angular velocity, and radius of curvature:

$$F_c = m\frac{v^2}{r}; \quad F_c = mr\omega^2. \tag{5.6.4}$$

You may use whichever expression for centripetal force is more convenient. Centripetal force $\vec{F}_c$ is always perpendicular to the path and points to the center of curvature, because $\vec{a}_c$ is perpendicular to the velocity and points to the center of curvature. Note that if you solve the first expression for r, you get

$$r = \frac{mv^2}{F_c}. \tag{5.6.5}$$

This implies that for a given mass and velocity, a large centripetal force causes a small radius of curvature—that is, a tight curve, as in Figure 5.6.1.

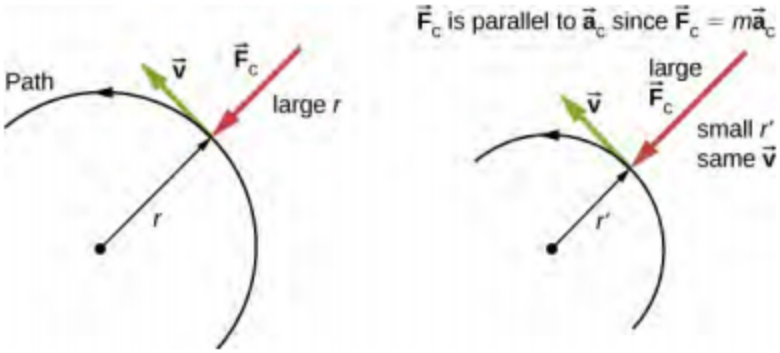


Figure 5.6.1: The frictional force supplies the centripetal force and is numerically equal to it. Centripetal force is perpendicular to velocity and causes uniform circular motion. The larger the F_c, the smaller the radius of curvature r and the sharper the curve. The second curve has the same v, but a larger F_c produces a smaller r′.

Example 5.6.1: What Coefficient of Friction Do Cars Need on a Flat Curve?

a. Calculate the centripetal force exerted on a 900.0-kg car that negotiates a 500.0-m radius curve at 25.00 m/s.
b. Assuming an unbanked curve, find the minimum static coefficient of friction between the tires and the road, static friction being the reason that keeps the car from slipping (Figure 5.6.2).

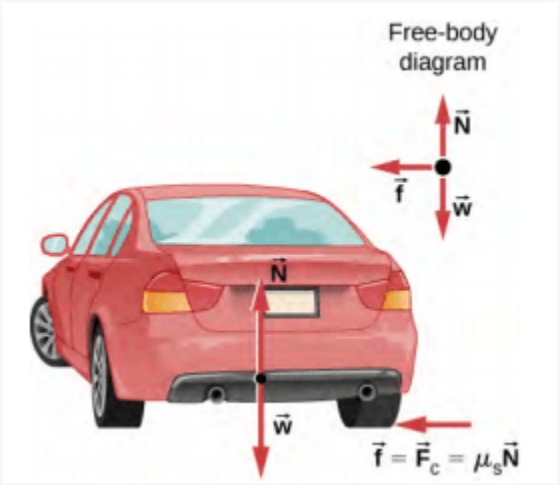


Figure 5.6.2: This car on level ground is moving away and turning to the left. The centripetal force causing the car to turn in a circular path is due to friction between the tires and the road. A minimum coefficient of friction is needed, or the car will move in a larger-radius curve and leave the roadway.

Strategy

a. We know that $F_c = m\frac{v^2}{r}$. Thus

$$F_c = m\frac{v^2}{r} = \frac{(900.0\ kg)(25.00\ m/s)^2}{(500.0\ m)} = 1125\ N. \tag{5.6.6}$$

b. Figure 5.6.2 shows the forces acting on the car on an unbanked (level ground) curve. Friction is to the left, keeping the car from slipping, and because it is the only horizontal force acting on the car, the friction is the centripetal force in this case. We know that the maximum static friction (at which the tires roll but do not slip) is μ_s N, where μ_s is the static coefficient of friction and N is the normal force. The normal force equals the car's weight on level ground, so N = mg. Thus the centripetal force in this situation is

$$F_c = f = \mu_s N = \mu_s mg. \tag{5.6.7}$$

Now we have a relationship between centripetal force and the coefficient of friction. Using the equation

$$F_c = m\frac{v^2}{r}. \tag{5.6.8}$$

we obtain

$$m\frac{v^2}{r} = \mu_s mg. \tag{5.6.9}$$

We solve this for μ_s, noting that mass cancels, and obtain

$$\mu_s = \frac{v^2}{rg}. \tag{5.6.10}$$

Substituting the knowns,

$$\mu_s = \frac{(25.00\ m/s)^2}{(500.0\ m)(9.80\ m/s^2)} = 0.13. \tag{5.6.11}$$

(Because coefficients of friction are approximate, the answer is given to only two digits.)

Significance

The coefficient of friction found in Figure $5.6.2b$ is much smaller than is typically found between tires and roads. The car still negotiates the curve if the coefficient is greater than 0.13, because static friction is a responsive force, able to assume a value less than but no more than μ_sN. A higher coefficient would also allow the car to negotiate the curve at a higher speed, but if the coefficient of friction is less, the safe speed would be less than 25 m/s. Note that mass cancels, implying that, in this example, it does not matter how heavily loaded the car is to negotiate the turn. Mass cancels because friction is assumed proportional to the normal force, which in turn is proportional to mass. If the surface of the road were banked, the normal force would be less, as discussed next.

Exercise 5.6.1

A car moving at 96.8 km/h travels around a circular curve of radius 182.9 m on a flat country road. What must be the minimum coefficient of static friction to keep the car from slipping?

Banked Curves

Let us now consider **banked curves**, where the slope of the road helps you negotiate the curve (Figure $5.6.3$). The greater the angle θ , the faster you can take the curve. Race tracks for bikes as well as cars, for example, often have steeply banked curves. In an "ideally banked curve," the angle θ is such that you can negotiate the curve at a certain speed without the aid of friction between the tires and the road. We will derive an expression for θ for an ideally banked curve and consider an example related to it.

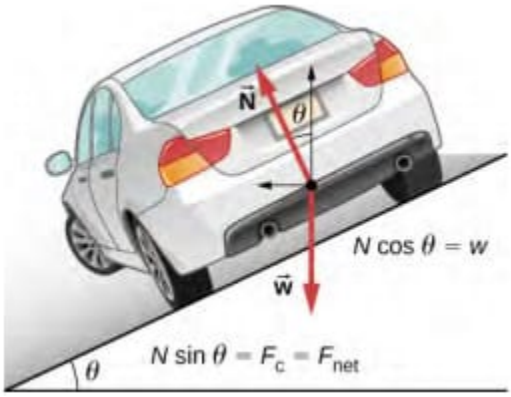


Figure $5.6.3$: The car on this banked curve is moving away and turning to the left.

For **ideal banking**, the net external force equals the horizontal centripetal force in the absence of friction. The components of the normal force N in the horizontal and vertical directions must equal the centripetal force and the weight of the car, respectively. In cases in which forces are not parallel, it is most convenient to consider components along perpendicular axes—in this case, the

vertical and horizontal directions.

Figure 5.6.3 shows a free-body diagram for a car on a frictionless banked curve. If the angle θ is ideal for the speed and radius, then the net external force equals the necessary centripetal force. The only two external forces acting on the car are its weight $\vec{w}$ and the normal force of the road $\vec{N}$. (A frictionless surface can only exert a force perpendicular to the surface—that is, a normal force.) These two forces must add to give a net external force that is horizontal toward the center of curvature and has magnitude $\frac{mv^2}{r}$. Because this is the crucial force and it is horizontal, we use a coordinate system with vertical and horizontal axes. Only the normal force has a horizontal component, so this must equal the centripetal force, that is,

$$N \sin \theta = \frac{mv^2}{r}. \tag{5.6.12}$$

Because the car does not leave the surface of the road, the net vertical force must be zero, meaning that the vertical components of the two external forces must be equal in magnitude and opposite in direction. From Figure 5.6.3, we see that the vertical component of the normal force is N cos θ, and the only other vertical force is the car's weight. These must be equal in magnitude; thus,

$$N \cos \theta = mg. \tag{5.6.13}$$

Now we can combine these two equations to eliminate N and get an expression for θ, as desired. Solving the second equation for N = $\frac{mg}{(\cos \theta)}$ and substituting this into the first yields

$$\begin{aligned} mg \frac{\sin \theta}{\cos \theta} &= \frac{mv^2}{r} \\ mg \tan \theta &= \frac{mv^2}{r} \\ \tan \theta &= \frac{v^2}{rg}. \end{aligned} \tag{5.6.14}$$

Taking the inverse tangent gives

$$\theta = \tan^{-1} \left(\frac{v^2}{rg} \right). \tag{5.6.15}$$

This expression can be understood by considering how θ depends on v and r. A large θ is obtained for a large v and a small r. That is, roads must be steeply banked for high speeds and sharp curves. Friction helps, because it allows you to take the curve at greater or lower speed than if the curve were frictionless. Note that θ does not depend on the mass of the vehicle.

Example 5.6.2: What Is the Ideal Speed to Take a Steeply Banked Tight Curve?

Curves on some test tracks and race courses, such as Daytona International Speedway in Florida, are very steeply banked. This banking, with the aid of tire friction and very stable car configurations, allows the curves to be taken at very high speed. To illustrate, calculate the speed at which a 1000-m radius curve banked at 31.0° should be driven if the road were frictionless.

Strategy

We first note that all terms in the expression for the ideal angle of a banked curve except for speed are known; thus, we need only rearrange it so that speed appears on the left-hand side and then substitute known quantities.

Solution

Starting with

$$\tan \theta = \frac{v^2}{rg}, \tag{5.6.16}$$

we get

$$v = \sqrt{rg \tan \theta}. \tag{5.6.17}$$

Noting that tan 31.0° = 0.609, we obtain

$$v = \sqrt{(100.0\ m)(9.80\ m/s^2)(0.609)} = 24.4\ m/s. \tag{5.6.18}$$

Significance

This is just about 165 km/h, consistent with a very steeply banked and rather sharp curve. Tire friction enables a vehicle to take the curve at significantly higher speeds.

Airplanes also make turns by banking. The lift force, due to the force of the air on the wing, acts at right angles to the wing. When the airplane banks, the pilot is obtaining greater lift than necessary for level flight. The vertical component of lift balances the airplane's weight, and the horizontal component accelerates the plane. The banking angle shown in Figure 5.6.4 is given by θ. We analyze the forces in the same way we treat the case of the car rounding a banked curve.

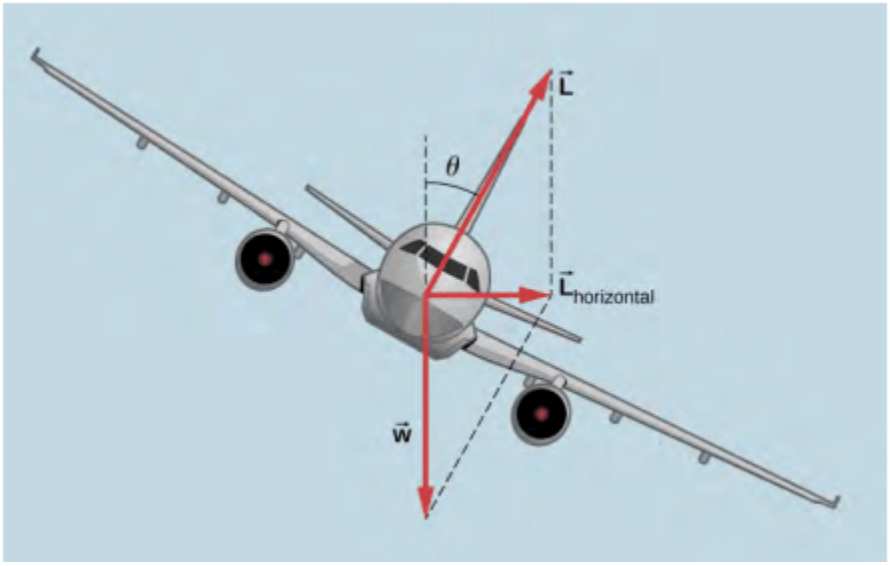


Figure 5.6.4: In a banked turn, the horizontal component of lift is unbalanced and accelerates the plane. The normal component of lift balances the plane's weight. The banking angle is given by θ. Compare the vector diagram with that shown in Figure 6.22.

Simulation

Join the ladybug in an exploration of rotational motion. Rotate the merry-go-round to change its angle or choose a constant angular velocity or angular acceleration. Explore how circular motion relates to the bug's xy-position, velocity, and acceleration using vectors or graphs.

Note

A circular motion requires a force, the so-called centripetal force, which is directed to the axis of rotation. This simplified model of a carousel demonstrates this force.

Inertial Forces and Noninertial (Accelerated) Frames: The Coriolis Force

What do taking off in a jet airplane, turning a corner in a car, riding a merry-go-round, and the circular motion of a tropical cyclone have in common? Each exhibits inertial forces—forces that merely seem to arise from motion, because the observer's frame of reference is accelerating or rotating. When taking off in a jet, most people would agree it feels as if you are being pushed back

into the seat as the airplane accelerates down the runway. Yet a physicist would say that you tend to remain stationary while the seat pushes forward on you. An even more common experience occurs when you make a tight curve in your car—say, to the right (Figure 5.6.5). You feel as if you are thrown (that is, forced) toward the left relative to the car. Again, a physicist would say that you are going in a straight line (recall Newton's first law) but the car moves to the right, not that you are experiencing a force from the left.

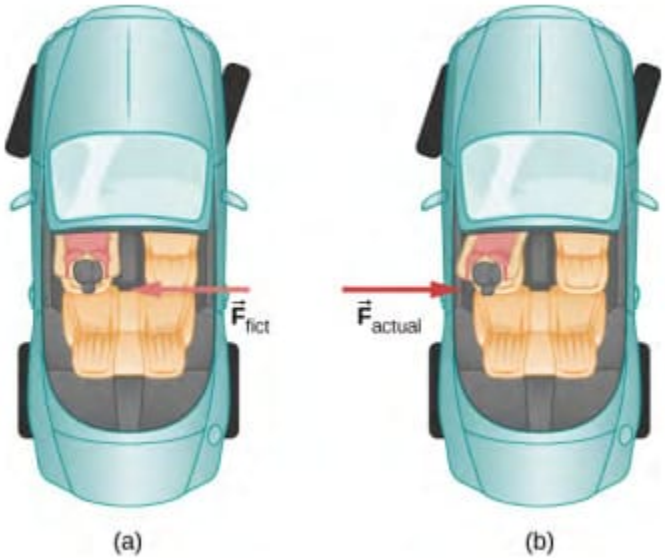


Figure 5.6.5: (a) The car driver feels herself forced to the left relative to the car when she makes a right turn. This is an inertial force arising from the use of the car as a frame of reference. (b) In Earth's frame of reference, the driver moves in a straight line, obeying Newton's first law, and the car moves to the right. There is no force to the left on the driver relative to Earth. Instead, there is a force to the right on the car to make it turn.

We can reconcile these points of view by examining the frames of reference used. Let us concentrate on people in a car. Passengers instinctively use the car as a frame of reference, whereas a physicist might use Earth. The physicist might make this choice because Earth is nearly an inertial frame of reference, in which all forces have an identifiable physical origin. In such a frame of reference, Newton's laws of motion take the form given in Newton's Laws of Motion. The car is a **noninertial frame of reference** because it is accelerated to the side. The force to the left sensed by car passengers is an **inertial force** having no physical origin (it is due purely to the inertia of the passenger, not to some physical cause such as tension, friction, or gravitation). The car, as well as the driver, is actually accelerating to the right. This inertial force is said to be an inertial force because it does not have a physical origin, such as gravity.

A physicist will choose whatever reference frame is most convenient for the situation being analyzed. There is no problem to a physicist in including inertial forces and Newton's second law, as usual, if that is more convenient, for example, on a merry-go-round or on a rotating planet. Noninertial (accelerated) frames of reference are used when it is useful to do so. Different frames of reference must be considered in discussing the motion of an astronaut in a spacecraft traveling at speeds near the speed of light, as you will appreciate in the study of the special theory of relativity.

Let us now take a mental ride on a merry-go-round—specifically, a rapidly rotating playground merry-go-round (Figure 5.6.6). You take the merry-go-round to be your frame of reference because you rotate together. When rotating in that noninertial frame of reference, you feel an inertial force that tends to throw you off; this is often referred to as a centrifugal force (not to be confused with centripetal force). Centrifugal force is a commonly used term, but it does not actually exist. You must hang on tightly to counteract your inertia (which people often refer to as centrifugal force). In Earth's frame of reference, there is no force trying to throw you off; we emphasize that centrifugal force is a fiction. You must hang on to make yourself go in a circle because otherwise you would go in a straight line, right off the merry-go-round, in keeping with Newton's first law. But the force you exert acts toward the center of the circle.

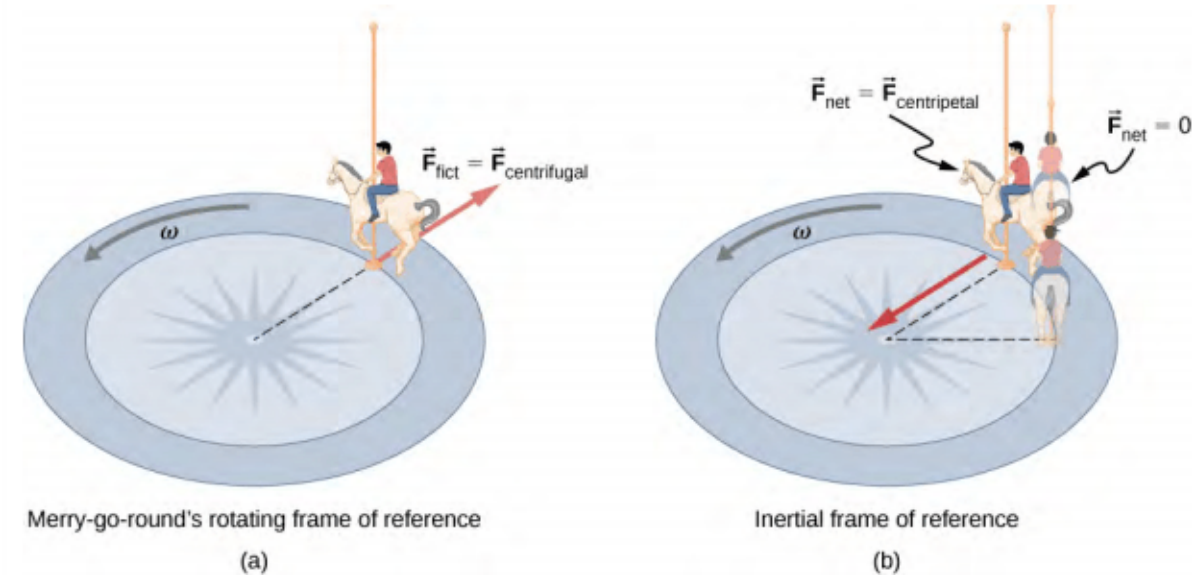


Figure 5.6.6: (a) A rider on a merry-go-round feels as if he is being thrown off. This inertial force is sometimes mistakenly called the centrifugal force in an effort to explain the rider's motion in the rotating frame of reference. (b) In an inertial frame of reference and according to Newton's laws, it is his inertia that carries him off (the unshaded rider has $F_{net} = 0$ and heads in a straight line). A force, $F_{centripetal}$, is needed to cause a circular path.

This inertial effect, carrying you away from the center of rotation if there is no centripetal force to cause circular motion, is put to good use in centrifuges (Figure 5.6.7). A centrifuge spins a sample very rapidly, as mentioned earlier in this chapter. Viewed from the rotating frame of reference, the inertial force throws particles outward, hastening their sedimentation. The greater the angular velocity, the greater the centrifugal force. But what really happens is that the inertia of the particles carries them along a line tangent to the circle while the test tube is forced in a circular path by a centripetal force.

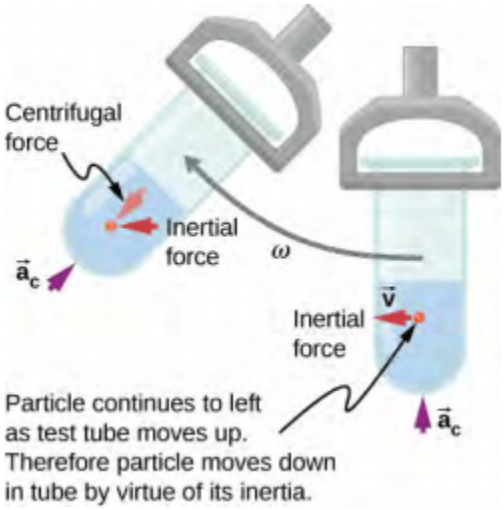


Figure 5.6.7: Centrifuges use inertia to perform their task. Particles in the fluid sediment settle out because their inertia carries them away from the center of rotation. The large angular velocity of the centrifuge quickens the sedimentation. Ultimately, the particles come into contact with the test tube walls, which then supply the centripetal force needed to make them move in a circle of constant radius.

Let us now consider what happens if something moves in a rotating frame of reference. For example, what if you slide a ball directly away from the center of the merry-go-round, as shown in Figure 5.6.8? The ball follows a straight path relative to Earth (assuming negligible friction) and a path curved to the right on the merry-go-round's surface. A person standing next to the merry-go-round sees the ball moving straight and the merry-go-round rotating underneath it. In the merry-go-round's frame of reference, we explain the apparent curve to the right by using an inertial force, called the **Coriolis force**, which causes the ball to curve to the right. The Coriolis force can be used by anyone in that frame of reference to explain why objects follow curved paths and allows us to apply Newton's laws in noninertial frames of reference.

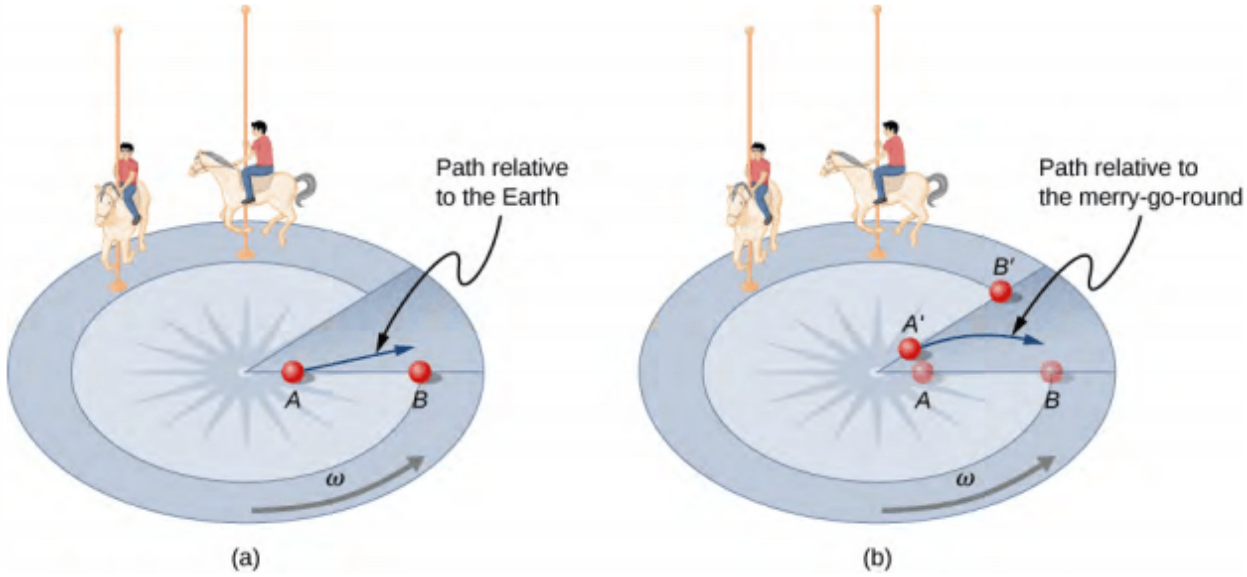


Figure 5.6.8: Looking down on the counterclockwise rotation of a merry-go-round, we see that a ball slid straight toward the edge follows a path curved to the right. The person slides the ball toward point B, starting at point A. Both points rotate to the shaded positions (A' and B') shown in the time that the ball follows the curved path in the rotating frame and a straight path in Earth's frame.

Up until now, we have considered Earth to be an inertial frame of reference with little or no worry about effects due to its rotation. Yet such effects **do** exist—in the rotation of weather systems, for example. Most consequences of Earth's rotation can be qualitatively understood by analogy with the merry-go-round. Viewed from above the North Pole, Earth rotates counterclockwise, as does the merry-go-round in Figure 5.6.8. As on the merry-go-round, any motion in Earth's Northern Hemisphere experiences a Coriolis force to the right. Just the opposite occurs in the Southern Hemisphere; there, the force is to the left. Because Earth's angular velocity is small, the Coriolis force is usually negligible, but for large-scale motions, such as wind patterns, it has substantial effects.

The Coriolis force causes hurricanes in the Northern Hemisphere to rotate in the counterclockwise direction, whereas tropical cyclones in the Southern Hemisphere rotate in the clockwise direction. (The terms hurricane, typhoon, and tropical storm are regionally specific names for cyclones, which are storm systems characterized by low pressure centers, strong winds, and heavy rains.) Figure 5.6.9 helps show how these rotations take place. Air flows toward any region of low pressure, and tropical cyclones contain particularly low pressures. Thus winds flow toward the center of a tropical cyclone or a low-pressure weather system at the surface. In the Northern Hemisphere, these inward winds are deflected to the right, as shown in the figure, producing a counterclockwise circulation at the surface for low-pressure zones of any type. Low pressure at the surface is associated with rising air, which also produces cooling and cloud formation, making low-pressure patterns quite visible from space. Conversely, wind circulation around high-pressure zones is clockwise in the Southern Hemisphere but is less visible because high pressure is associated with sinking air, producing clear skies.

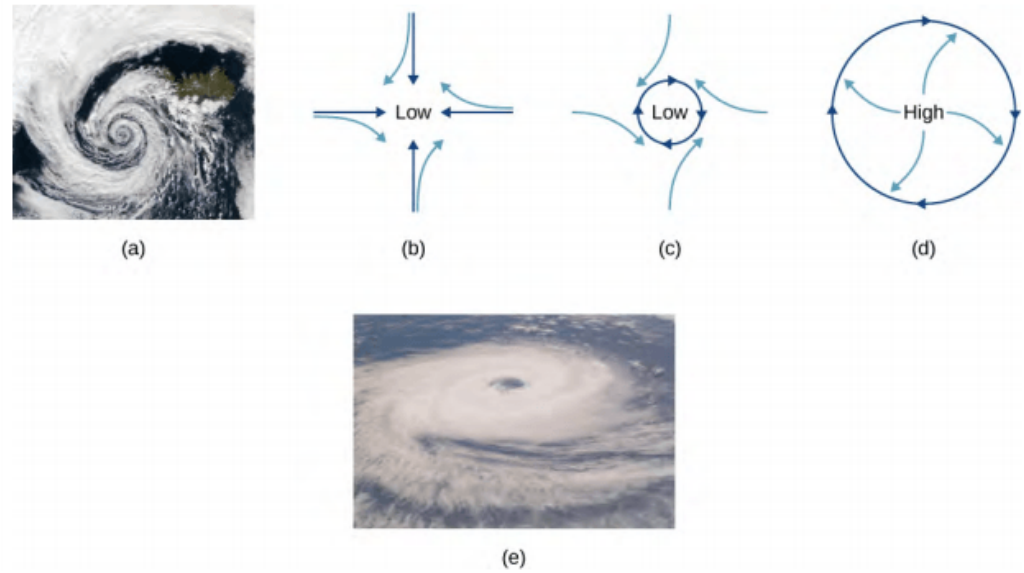


Figure 5.6.9: (a) The counterclockwise rotation of this Northern Hemisphere hurricane is a major consequence of the Coriolis force. (b) Without the Coriolis force, air would flow straight into a low-pressure zone, such as that found in tropical cyclones. (c) The Coriolis force deflects the winds to the right, producing a counterclockwise rotation. (d) Wind flowing away from a high-pressure zone is also deflected to the right, producing a clockwise rotation. (e) The opposite direction of rotation is produced by the Coriolis force in the Southern Hemisphere, leading to tropical cyclones. (credit a and credit e: modifications of work by NASA)

The rotation of tropical cyclones and the path of a ball on a merry-go-round can just as well be explained by inertia and the rotation of the system underneath. When noninertial frames are used, inertial forces, such as the Coriolis force, must be invented to explain the curved path. There is no identifiable physical source for these inertial forces. In an inertial frame, inertia explains the path, and no force is found to be without an identifiable source. Either view allows us to describe nature, but a view in an inertial frame is the simplest in the sense that all forces have origins and explanations.

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5.7: Drag Force and Terminal Speed

Learning Objectives

- Express the drag force mathematically
- Describe applications of the drag force
- Define terminal velocity
- Determine an object's terminal velocity given its mass

Another interesting force in everyday life is the force of drag on an object when it is moving in a fluid (either a gas or a liquid). You feel the drag force when you move your hand through water. You might also feel it if you move your hand during a strong wind. The faster you move your hand, the harder it is to move. You feel a smaller drag force when you tilt your hand so only the side goes through the air—you have decreased the area of your hand that faces the direction of motion.

Drag Forces

Like friction, the **drag force** always opposes the motion of an object. Unlike simple friction, the drag force is proportional to some function of the velocity of the object in that fluid. This functionality is complicated and depends upon the shape of the object, its size, its velocity, and the fluid it is in. For most large objects such as cyclists, cars, and baseballs not moving too slowly, the magnitude of the drag force F_D is proportional to the square of the speed of the object. We can write this relationship mathematically as $F_D \propto v^2$. When taking into account other factors, this relationship becomes

$$F_D = \frac{1}{2} C \rho A v^2, \tag{5.7.1}$$

where C is the drag coefficient, A is the area of the object facing the fluid, and ρ is the density of the fluid. (Recall that density is mass per unit volume.) This equation can also be written in a more generalized fashion as $F_D = bv^2$, where b is a constant equivalent to $0.5C\rho A$. We have set the exponent n for these equations as 2 because when an object is moving at high velocity through air, the magnitude of the drag force is proportional to the square of the speed. As we shall see in Fluid Mechanics, for small particles moving at low speeds in a fluid, the exponent n is equal to 1.

Definition: Drag Force

Drag force F_D is proportional to the square of the speed of the object. Mathematically,

$$F_D = \frac{1}{2} C \rho A v^2, \tag{5.7.2}$$

where C is the drag coefficient, A is the area of the object facing the fluid, and ρ is the density of the fluid.

Athletes as well as car designers seek to reduce the drag force to lower their race times (Figure $5.7.1A$). Aerodynamic shaping of an automobile can reduce the drag force and thus increase a car's gas mileage. The value of the drag coefficient C is determined empirically, usually with the use of a wind tunnel (Figure $5.7.1B$).



Figure 5.7.1: (A) From racing cars to bobsled racers, aerodynamic shaping is crucial to achieving top speeds. Bobsleds are designed for speed and are shaped like a bullet with tapered fins. (credit: "U.S. Army"/Wikimedia Commons) (B): NASA researchers test a model plane in a wind tunnel. (credit: NASA/Ames).

The drag coefficient can depend upon velocity, but we assume that it is a constant here. Table 5.7.1 lists some typical drag coefficients for a variety of objects. Notice that the drag coefficient is a dimensionless quantity. At highway speeds, over 50% of the power of a car is used to overcome air drag. The most fuel-efficient cruising speed is about 70–80 km/h (about 45–50 mi/h). For this reason, during the 1970s oil crisis in the United States, maximum speeds on highways were set at about 90 km/h (55 mi/h).

Table 5.7.1: Typical Values of Drag Coefficient C

Object	C
Airfoil	0.05
Toyota Camry	0.28
Ford Focus	0.32
Honda Civic	0.36
Ferrari Testarossa	0.37
Dodge Ram Pickup	0.43
Sphere	0.45
Hummer H2 SUV	0.64
Skydiver (feet first)	0.70
Bicycle	0.90
Skydiver (horizontal)	1.0
Circular flat plate	1.12

Substantial research is under way in the sporting world to minimize drag. The dimples on golf balls are being redesigned, as are the clothes that athletes wear. Bicycle racers and some swimmers and runners wear full bodysuits. Australian Cathy Freeman wore a full body suit in the 2000 Sydney Olympics and won a gold medal in the 400-m race. Many swimmers in the 2008 Beijing Olympics wore (Speedo) body suits; it might have made a difference in breaking many world records (Figure 5.7.2). Most elite swimmers (and cyclists) shave their body hair. Such innovations can have the effect of slicing away milliseconds in a race, sometimes making the difference between a gold and a silver medal. One consequence is that careful and precise guidelines must be continuously developed to maintain the integrity of the sport.



Figure 5.7.2: Body suits, such as this LZR Racer Suit, have been credited with aiding in many world records after their release in 2008. Smoother "skin" and more compression forces on a swimmer's body provide at least 10% less drag. (credit: NASA/Kathy Barnstorff)

Terminal Velocity

Some interesting situations connected to Newton's second law occur when considering the effects of drag forces upon a moving object. For instance, consider a skydiver falling through air under the influence of gravity. The two forces acting on him are the force of gravity and the drag force (ignoring the small buoyant force). The downward force of gravity remains constant regardless of the velocity at which the person is moving. However, as the person's velocity increases, the magnitude of the drag force increases until the magnitude of the drag force is equal to the gravitational force, thus producing a net force of zero. A zero net force means that there is no acceleration, as shown by Newton's second law. At this point, the person's velocity remains constant and we say that the person has reached his **terminal velocity** (v_T). Since F_D is proportional to the speed squared, a heavier skydiver must go faster for F_D to equal his weight. Let's see how this works out more quantitatively.

At the terminal velocity,

$$F_{net} = mg - F_D = ma = 0. \tag{5.7.3}$$

Thus,

$$mg = F_D. \tag{5.7.4}$$

Using the equation for drag force, we have

$$mg = \frac{1}{2} C\rho A v_T^2. \tag{5.7.5}$$

Solving for the velocity, we obtain

$$v_T = \sqrt{\frac{2mg}{\rho CA}}. \tag{5.7.6}$$

Assume the density of air is ρ = 1.21 kg/m^3. A 75-kg skydiver descending head first has a cross-sectional area of approximately A = 0.18 m^2 and a drag coefficient of approximately C = 0.70. We find that

$$v_T = \sqrt{\frac{2(75\ kg)(9.80\ m/s^2)}{(1.21\ kg/m^3)(0.70)(0.18\ m^2)}} = 98\ m/s = 350\ km/h. \tag{5.7.7}$$

This means a skydiver with a mass of 75 kg achieves a terminal velocity of about 350 km/h while traveling in a pike (head first) position, minimizing the area and his drag. In a spread-eagle position, that terminal velocity may decrease to about 200 km/h as

the area increases. This terminal velocity becomes much smaller after the parachute opens.

Example 5.7.1: Terminal Velocity of a Skydiver

Find the terminal velocity of an 85-kg skydiver falling in a spread-eagle position.

Strategy

At terminal velocity, $F_{net} = 0$. Thus, the drag force on the skydiver must equal the force of gravity (the person's weight). Using the equation of drag force, we find $mg = \frac{1}{2}\rho CAv^2$.

Solution

The terminal velocity v_T can be written as

$$v_T = \sqrt{\frac{2mg}{\rho CA}} = \sqrt{\frac{2(85\ kg)(9.80\ m/s^2)}{(1.21\ kg/m^3)(1.0)(0.70\ m^2)}} = 44\ m/s. \tag{5.7.8}$$

Significance

This result is consistent with the value for v_T mentioned earlier. The 75-kg skydiver going feet first had a terminal velocity of v_T = 98 m/s. He weighed less but had a smaller frontal area and so a smaller drag due to the air.

Exercise 5.7.1

Find the terminal velocity of a 50-kg skydiver falling in spread-eagle fashion.

The size of the object that is falling through air presents another interesting application of air drag. If you fall from a 5-m-high branch of a tree, you will likely get hurt—possibly fracturing a bone. However, a small squirrel does this all the time, without getting hurt. You do not reach a terminal velocity in such a short distance, but the squirrel does.

The following interesting quote on animal size and terminal velocity is from a 1928 essay by a British biologist, J. B. S. Haldane, titled "On Being the Right Size."

> "To the mouse and any smaller animal, [gravity] presents practically no dangers. You can drop a mouse down a thousand-yard mine shaft; and, on arriving at the bottom, it gets a slight shock and walks away, provided that the ground is fairly soft. A rat is killed, a man is broken, and a horse splashes. For the resistance presented to movement by the air is proportional to the surface of the moving object. Divide an animal's length, breadth, and height each by ten; its weight is reduced to a thousandth, but its surface only to a hundredth. So the resistance to falling in the case of the small animal is relatively ten times greater than the driving force."

The above quadratic dependence of air drag upon velocity does not hold if the object is very small, is going very slow, or is in a denser medium than air. Then we find that the drag force is proportional just to the velocity. This relationship is given by Stokes' law.

Stokes' law

For a spherical object falling in a medium, the drag force is

$$F_s = 6\pi r\eta v, \tag{5.7.9}$$

where r is the radius of the object, η is the viscosity of the fluid, and v is the object's velocity.

Good examples of Stokes' law are provided by microorganisms, pollen, and dust particles. Because each of these objects is so small, we find that many of these objects travel unaided only at a constant (terminal) velocity. Terminal velocities for bacteria (size

about \(1\, \mu m) can be about \(2\, \mu m/s. To move at a greater speed, many bacteria swim using flagella (organelles shaped like little tails) that are powered by little motors embedded in the cell.

Sediment in a lake can move at a greater terminal velocity (about 5 μm/s), so it can take days for it to reach the bottom of the lake after being deposited on the surface.

If we compare animals living on land with those in water, you can see how drag has influenced evolution. Fish, dolphins, and even massive whales are streamlined in shape to reduce drag forces. Birds are streamlined and migratory species that fly large distances often have particular features such as long necks. Flocks of birds fly in the shape of a spearhead as the flock forms a streamlined pattern (Figure 5.7.3). In humans, one important example of streamlining is the shape of sperm, which need to be efficient in their use of energy.



Figure 5.7.3: Geese fly in a V formation during their long migratory travels. This shape reduces drag and energy consumption for individual birds, and also allows them a better way to communicate. (credit: "Julo"/Wikimedia Commons)

In lecture demonstrations, we do measurements of the drag force on different objects. The objects are placed in a uniform airstream created by a fan. Calculate the Reynolds number and the drag coefficient.

YouTube Video

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

Video 5.7.1: Fluid Mechanics - Drag force - Flow simulation

The Calculus of Velocity-Dependent Frictional Forces

When a body slides across a surface, the frictional force on it is approximately constant and given by $\mu_k N$. Unfortunately, the frictional force on a body moving through a liquid or a gas does not behave so simply. This drag force is generally a complicated function of the body's velocity. However, for a body moving in a straight line at moderate speeds through a liquid such as water, the frictional force can often be approximated by

$$f_R = -bv, \tag{5.7.10}$$

where b is a constant whose value depends on the dimensions and shape of the body and the properties of the liquid, and v is

the velocity of the body. Two situations for which the frictional force can be represented this equation are a motorboat moving through water and a small object falling slowly through a liquid.

Let's consider the object falling through a liquid. The free-body diagram of this object with the positive direction downward is shown in Figure 5.7.4. Newton's second law in the vertical direction gives the differential equation

$$mg - bv = m\frac{dv}{dt}, \tag{5.7.11}$$

where we have written the acceleration as $\frac{dv}{dt}$. As v increases, the frictional force $-bv$ increases until it matches mg. At this point, there is no acceleration and the velocity remains constant at the terminal velocity v_T. From the previous equation,

$$mg - bv_T = 0, \tag{5.7.12}$$

so

$$v_T = \frac{mg}{b}. \tag{5.7.13}$$

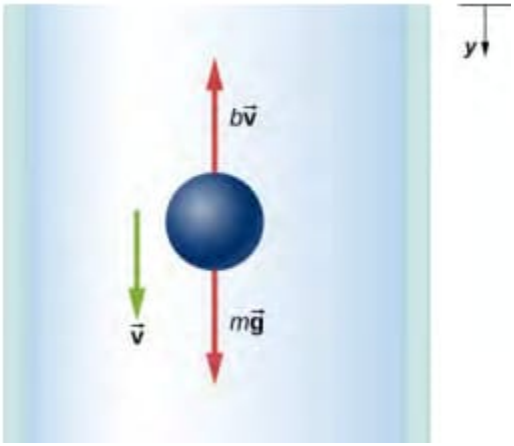


Figure 5.7.4: Free-body diagram of an object falling through a resistive medium.

We can find the object's velocity by integrating the differential equation for v. First, we rearrange terms in this equation to obtain

$$\frac{dv}{g - \left(\frac{b}{m}\right)v} = dt. \tag{5.7.14}$$

Assuming that $v = 0$ at \9t = 0\), integration of Equation 5.7.14 yields

$$\int_0^v \frac{dv'}{g - \left(\frac{b}{m}\right)v'} = \int_0^t dt', \tag{5.7.15}$$

or

$$-\frac{m}{b}\ln\left(g - \frac{b}{m}v'\right)\Big|_0^v = t'|_0^t, \tag{5.7.16}$$

where v' and t' are dummy variables of integration. With the limits given, we find

$$-\frac{m}{b}[\ln\left(g - \frac{b}{m}v\right) - \ln g] = t. \tag{5.7.17}$$

Since $\ln A - \ln B = \ln(\left(\frac{A}{B}\right)$, and $\ln(\left(\frac{A}{B}\right) = x$ implies $e^x = \frac{A}{B}$, we obtain

$$\frac{g-\left(\frac{bv}{m}\right)}{g}=e^{-\frac{bt}{m}}, \tag{5.7.18}$$

and

$$v=\frac{mg}{b}\left(1-e^{-\frac{bt}{m}}\right). \tag{5.7.19}$$

Notice that as t → ∞, v → $\frac{mg}{b}$ = v_T, which is the terminal velocity.

The position at any time may be found by integrating the equation for v. With v = $\frac{dy}{dt}$,

$$dy=\frac{mg}{b}\left(1-e^{-\frac{bt}{m}}\right)dt. \tag{5.7.20}$$

Assuming y = 0 when t = 0,

$$\int_0^y dy'=\frac{mg}{b}\int_0^t\left(1-e^{-\frac{bt}{m}}\right)dt', \tag{5.7.21}$$

which integrates to

$$y=\frac{mg}{b}t+\frac{m^2g}{b^2}\left(e^{-\frac{bt}{m}}-1\right). \tag{5.7.22}$$

Example 5.7.2: Effect of the Resistive Force on a Motorboat

A motorboat is moving across a lake at a speed v_0 when its motor suddenly freezes up and stops. The boat then slows down under the frictional force $f_R=-bv$.

a. What are the velocity and position of the boat as functions of time?
b. If the boat slows down from 4.0 to 1.0 m/s in 10 s, how far does it travel before stopping?

Solution

a. With the motor stopped, the only horizontal force on the boat is f_R = –bv, so from Newton's second law,

$$m\frac{dv}{dt}=-bv, \tag{5.7.23}$$

which we can write as

$$\frac{dv}{v}=-\frac{b}{m}dt. \tag{5.7.24}$$

Integrating this equation between the time zero when the velocity is v_0 and the time t when the velocity is v, we have

$$\int_0^v\frac{dv'}{v'}=-\frac{b}{m}\int_0^t dt'. \tag{5.7.25}$$

Thus,

$$\ln\frac{v}{v_0}=-\frac{b}{m}t, \tag{5.7.26}$$

which, since lnA = x implies e^x = A, we can write this as

$$v=v_0e^{-\frac{bt}{m}}. \tag{5.7.27}$$

Now from the definition of velocity,

$$\frac{dx}{dt}=v_0e^{-\frac{bt}{m}}, \tag{5.7.28}$$

so we have

$$dx = v_0 e^{-\frac{bt}{m}} dt. \tag{5.7.29}$$

With the initial position zero, we have

$$\int_0^x dx' = v_0 \int_0^t e^{-\frac{bt'}{m}} dt', \tag{5.7.30}$$

and

$$x = -\frac{mv_0}{b} e^{-\frac{bt}{m}} \Big|_0^t = \frac{mv_0}{b}\left(1 - e^{-\frac{bt}{m}}\right). \tag{5.7.31}$$

As time increases, $e^{-\frac{bt}{m}} \rightarrow 0$, and the position of the boat approaches a limiting value

$$x_{max} = \frac{mv_0}{b}. \tag{5.7.32}$$

Although this tells us that the boat takes an infinite amount of time to reach x_{max}, the boat effectively stops after a reasonable time. For example, at $t = \frac{10m}{b}$, we have

$$v = v_0 e^{-10} \simeq 4.5 \times 10^{-5} v_0, \tag{5.7.33}$$

whereas we also have

$$x = x_{max}\left(1 - e^{-10}\right) \simeq 0.99995 x_{max}. \tag{5.7.34}$$

Therefore, the boat's velocity and position have essentially reached their final values.

b. With $v_0 = 4.0$ m/s and $v = 1.0$ m/s, we have 1.0 m/s = (4.0 m/s) $e^{(-\frac{bt}{m})(10\ s)}$, so

$$\ln 0.25 = -\ln 4.0 = -\frac{b}{m}(10\ s), \tag{5.7.35}$$

and

$$\frac{b}{m} = \frac{1}{10} \ln 4.0\ s^{-1} = 0.14\ s^{-1}. \tag{5.7.36}$$

Now the boat's limiting position is

$$x_{max} = \frac{mv_0}{b} = \frac{4.0\ m/s}{0.14\ s^{-1}} = 29\ m. \tag{5.7.37}$$

Significance

In the both of the previous examples, we found "limiting" values. The terminal velocity is the same as the limiting velocity, which is the velocity of the falling object after a (relatively) long time has passed. Similarly, the limiting distance of the boat is the distance the boat will travel after a long amount of time has passed. Due to the properties of exponential decay, the time involved to reach either of these values is actually not too long (certainly not an infinite amount of time!) but they are quickly found by taking the limit to infinity.

Exercise 5.7.2

Suppose the resistive force of the air on a skydiver can be approximated by $f = -bv^2$. If the terminal velocity of a 100-kg skydiver is 60 m/s, what is the value of b?

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5.8: Applications of Newton's Laws (Exercises)

Conceptual Questions

6.1 Solving Problems with Newton's Laws

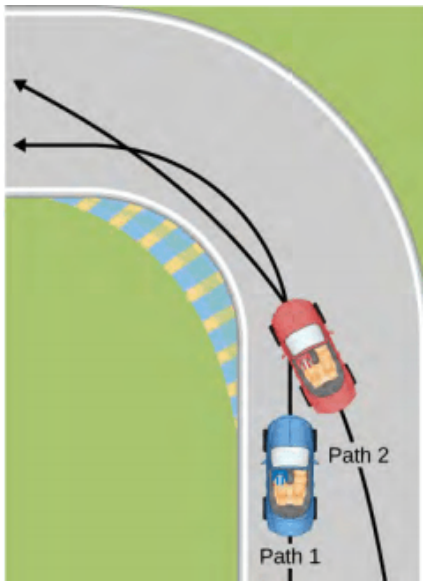
1. To simulate the apparent weightlessness of space orbit, astronauts are trained in the hold of a cargo aircraft that is accelerating downward at g. Why do they appear to be weightless, as measured by standing on a bathroom scale, in this accelerated frame of reference? Is there any difference between their apparent weightlessness in orbit and in the aircraft?

6.2 Friction

2. The glue on a piece of tape can exert forces. Can these forces be a type of simple friction? Explain, considering especially that tape can stick to vertical walls and even to ceilings.
3. When you learn to drive, you discover that you need to let up slightly on the brake pedal as you come to a stop or the car will stop with a jerk. Explain this in terms of the relationship between static and kinetic friction.
4. When you push a piece of chalk across a chalkboard, it sometimes screeches because it rapidly alternates between slipping and sticking to the board. Describe this process in more detail, in particular, explaining how it is related to the fact that kinetic friction is less than static friction. (The same slip-grab process occurs when tires screech on pavement.)
5. A physics major is cooking breakfast when she notices that the frictional force between her steel spatula and Teflon frying pan is only 0.200 N. Knowing the coefficient of kinetic friction between the two materials, she quickly calculates the normal force. What is it?

6.3 Centripetal Force

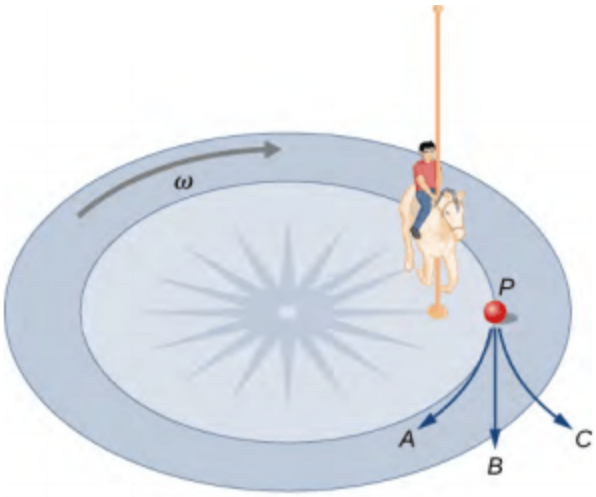
6. If you wish to reduce the stress (which is related to centripetal force) on high-speed tires, would you use large- or small-diameter tires? Explain.
7. Define centripetal force. Can any type of force (for example, tension, gravitational force, friction, and so on) be a centripetal force? Can any combination of forces be a centripetal force?
8. If centripetal force is directed toward the center, why do you feel that you are 'thrown' away from the center as a car goes around a curve? Explain.
9. Race car drivers routinely cut corners, as shown below (Path 2). Explain how this allows the curve to be taken at the greatest speed.



10. Many amusement parks have rides that make vertical loops like the one shown below. For safety, the cars are attached to the rails in such a way that they cannot fall off. If the car goes over the top at just the right speed, gravity alone will supply the centripetal force. What other force acts and what is its direction if:
    a. The car goes over the top at faster than this speed?
    b. The car goes over the top at slower than this speed?

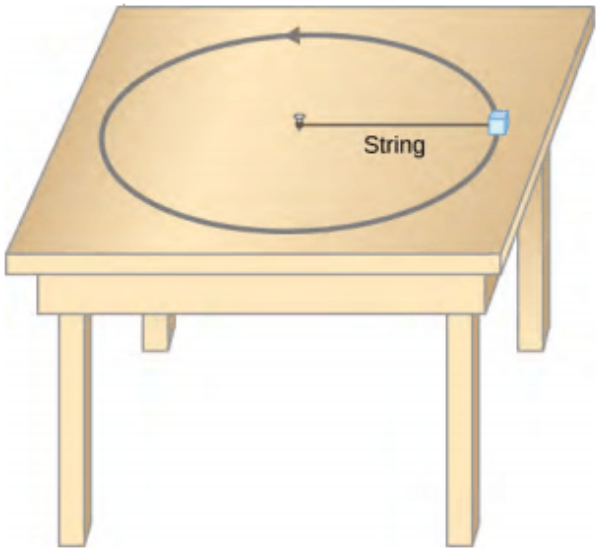


11. What causes water to be removed from clothes in a spin-dryer?
12. As a skater forms a circle, what force is responsible for making his turn? Use a free-body diagram in your answer.
13. Suppose a child is riding on a merry-go-round at a distance about halfway between its center and edge. She has a lunch box resting on wax paper, so that there is very little friction between it and the merry-go-round. Which path shown below will the lunch box take when she lets go? The lunch box leaves a trail in the dust on the merry-go-round. Is that trail straight, curved to the left, or curved to the right? Explain your answer.



Merry-go-round's rotating frame of reference

14. Do you feel yourself thrown to either side when you negotiate a curve that is ideally banked for your car's speed? What is the direction of the force exerted on you by the car seat?
15. Suppose a mass is moving in a circular path on a frictionless table as shown below. In Earth's frame of reference, there is no centrifugal force pulling the mass away from the center of rotation, yet there is a force stretching the string attaching the mass to the nail. Using concepts related to centripetal force and Newton's third law, explain what force stretches the string, identifying its physical origin.



16. When a toilet is flushed or a sink is drained, the water (and other material) begins to rotate about the drain on the way down. Assuming no initial rotation and a flow initially directly straight toward the drain, explain what causes the rotation and which direction it has in the Northern Hemisphere. (Note that this is a small effect and in most toilets the rotation is caused by directional water jets.) Would the direction of rotation reverse if water were forced up the drain?
17. A car rounds a curve and encounters a patch of ice with a very low coefficient of kinetic fiction. The car slides off the road. Describe the path of the car as it leaves the road.
18. In one amusement park ride, riders enter a large vertical barrel and stand against the wall on its horizontal floor. The barrel is spun up and the floor drops away. Riders feel as if they are pinned to the wall by a force something like the gravitational force. This is an inertial force sensed and used by the riders to explain events in the rotating frame of reference of the barrel. Explain in an inertial frame of reference (Earth is nearly one) what pins the riders to the wall, and identify all forces acting on them.

19. Two friends are having a conversation. Anna says a satellite in orbit is in free fall because the satellite keeps falling toward Earth. Tom says a satellite in orbit is not in free fall because the acceleration due to gravity is not 9.80 m/s^2. Who do you agree with and why?
20. A nonrotating frame of reference placed at the center of the Sun is very nearly an inertial one. Why is it not exactly an inertial frame?

6.4 Drag Force and Terminal Speed

21. Athletes such as swimmers and bicyclists wear body suits in competition. Formulate a list of pros and cons of such suits.
22. Two expressions were used for the drag force experienced by a moving object in a liquid. One depended upon the speed, while the other was proportional to the square of the speed. In which types of motion would each of these expressions be more applicable than the other one?
23. As cars travel, oil and gasoline leaks onto the road surface. If a light rain falls, what does this do to the control of the car? Does a heavy rain make any difference?
24. Why can a squirrel jump from a tree branch to the ground and run away undamaged, while a human could break a bone in such a fall?

Problems

6.1 Solving Problems with Newton's Laws

25. A 30.0-kg girl in a swing is pushed to one side and held at rest by a horizontal force $\vec{F}$ so that the swing ropes are 30.0° with respect to the vertical. (a) Calculate the tension in each of the two ropes supporting the swing under these conditions. (b) Calculate the magnitude of $\vec{F}$.
26. Find the tension in each of the three cables supporting the traffic light if it weighs 2.00 x 10^2 N.

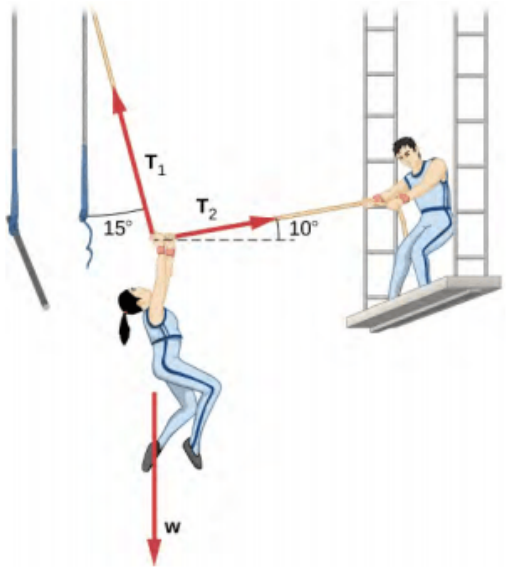


27. Three forces act on an object, considered to be a particle, which moves with constant velocity v = (3 $\hat{i}$ – 2 $\hat{j}$) m/s. Two of the forces are $\vec{F}_1$ = (3 $\hat{i}$ + 5 $\hat{j}$ – 6k $\hat{k}$) N and $\vec{F}_2$ = (4 $\hat{i}$ – 7 $\hat{j}$ + 2 $\hat{k}$) N. Find the third force.
28. A flea jumps by exerting a force of 1.20 x 10^{-5} N straight down on the ground. A breeze blowing on the flea parallel to the ground exerts a force of 0.500 x 10^{-6} N on the flea while the flea is still in contact with the ground. Find the direction and magnitude of the acceleration of the flea if its mass is 6.00 x 10^{-7} kg . Do not neglect the gravitational force.
29. Two muscles in the back of the leg pull upward on the Achilles tendon, as shown below. (These muscles are called the medial and lateral heads of the gastrocnemius muscle.) Find the magnitude and direction of the total force on the

Achilles tendon. What type of movement could be caused by this force?



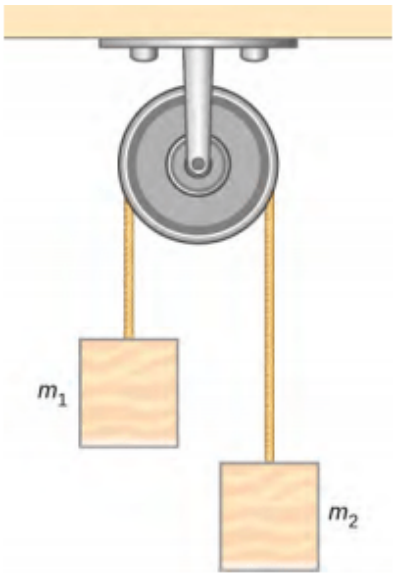
30. After a mishap, a 76.0-kg circus performer clings to a trapeze, which is being pulled to the side by another circus artist, as shown here. Calculate the tension in the two ropes if the person is momentarily motionless. Include a free-body diagram in your solution.



31. A 35.0-kg dolphin decelerates from 12.0 to 7.50 m/s in 2.30 s to join another dolphin in play. What average force was exerted to slow the first dolphin if it was moving horizontally? (The gravitational force is balanced by the buoyant force of the water.)
32. When starting a foot race, a 70.0-kg sprinter exerts an average force of 650 N backward on the ground for 0.800 s. (a) What is his final speed? (b) How far does he travel?
33. A large rocket has a mass of 2.00×10^6 kg at takeoff, and its engines produce a thrust of 3.50×10^7 N. (a) Find its initial acceleration if it takes off vertically. (b) How long does it take to reach a velocity of 120 km/h straight up, assuming constant mass and thrust?
34. A basketball player jumps straight up for a ball. To do this, he lowers his body 0.300 m and then accelerates through this distance by forcefully straightening his legs. This player leaves the floor with a vertical velocity sufficient to carry him 0.900 m above the floor. (a) Calculate his velocity when he leaves the floor. (b) Calculate his acceleration while he is straightening his legs. He goes from zero to the velocity found in (a) in a distance of 0.300 m. (c) Calculate the force

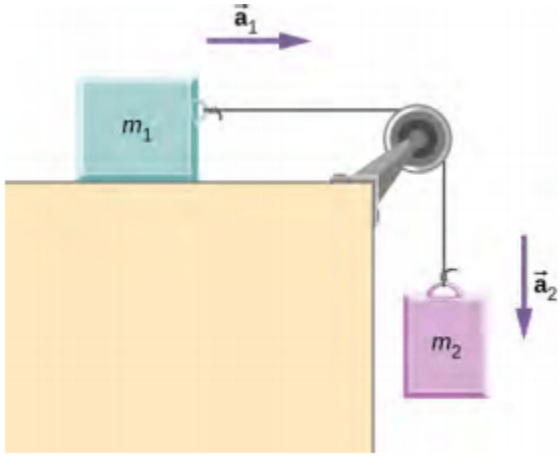
he exerts on the floor to do this, given that his mass is 110.0 kg.

35. A 2.50-kg fireworks shell is fired straight up from a mortar and reaches a height of 110.0 m. (a) Neglecting air resistance (a poor assumption, but we will make it for this example), calculate the shell's velocity when it leaves the mortar. (b) The mortar itself is a tube 0.450 m long. Calculate the average acceleration of the shell in the tube as it goes from zero to the velocity found in (a). (c) What is the average force on the shell in the mortar? Express your answer in newtons and as a ratio to the weight of the shell.
36. A 0.500-kg potato is fired at an angle of 80.0° above the horizontal from a PVC pipe used as a "potato gun" and reaches a height of 110.0 m. (a) Neglecting air resistance, calculate the potato's velocity when it leaves the gun. (b) The gun itself is a tube 0.450 m long. Calculate the average acceleration of the potato in the tube as it goes from zero to the velocity found in (a). (c) What is the average force on the potato in the gun? Express your answer in newtons and as a ratio to the weight of the potato.
37. An elevator filled with passengers has a mass of 1.70×10^3 kg. (a) The elevator accelerates upward from rest at a rate of 1.20 m/s^2 for 1.50 s. Calculate the tension in the cable supporting the elevator. (b) The elevator continues upward at constant velocity for 8.50 s. What is the tension in the cable during this time? (c) The elevator decelerates at a rate of 0.600 m/s^2 for 3.00 s. What is the tension in the cable during deceleration? (d) How high has the elevator moved above its original starting point, and what is its final velocity?
38. A 20.0-g ball hangs from the roof of a freight car by a string. When the freight car begins to move, the string makes an angle of 35.0° with the vertical. (a) What is the acceleration of the freight car? (b) What is the tension in the string?
39. A student's backpack, full of textbooks, is hung from a spring scale attached to the ceiling of an elevator. When the elevator is accelerating downward at 3.8 m/s^2, the scale reads 60 N. (a) What is the mass of the backpack? (b) What does the scale read if the elevator moves upward while slowing down at a rate 3.8 m/s^2? (c) What does the scale read if the elevator moves upward at constant velocity? (d) If the elevator had no brakes and the cable supporting it were to break loose so that the elevator could fall freely, what would the spring scale read?
40. A service elevator takes a load of garbage, mass 10.0 kg, from a floor of a skyscraper under construction, down to ground level, accelerating downward at a rate of 1.2 m/s^2. Find the magnitude of the force the garbage exerts on the floor of the service elevator?
41. A roller coaster car starts from rest at the top of a track 30.0 m long and inclined at 20.0° to the horizontal. Assume that friction can be ignored. (a) What is the acceleration of the car? (b) How much time elapses before it reaches the bottom of the track?
42. The device shown below is the Atwood's machine considered in Example 6.5. Assuming that the masses of the string and the frictionless pulley are negligible, (a) find an equation for the acceleration of the two blocks; (b) find an equation for the tension in the string; and (c) find both the acceleration and tension when block 1 has mass 2.00 kg and block 2 has mass 4.00 kg.

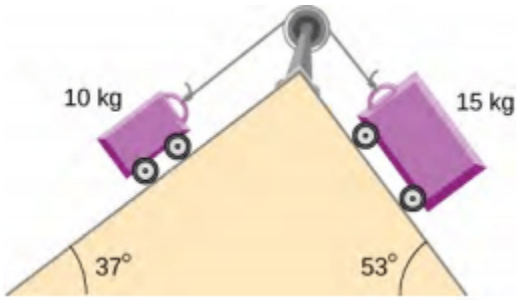


43. Two blocks are connected by a massless rope as shown below. The mass of the block on the table is 4.0 kg and the hanging mass is 1.0 kg. The table and the pulley are frictionless. (a) Find the acceleration of the system. (b) Find the tension in the rope. (c) Find the speed with which the hanging mass hits the floor if it starts from rest and is initially

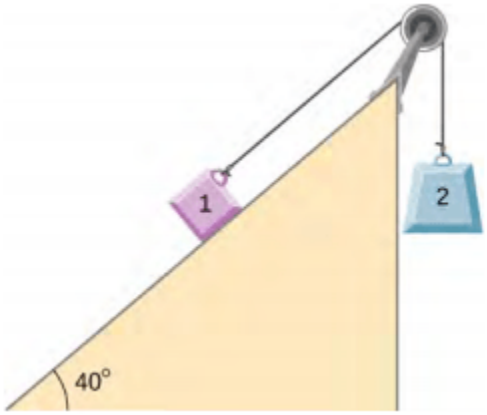
located 1.0 m from the floor.



44. Shown below are two carts connected by a cord that passes over a small frictionless pulley. Each cart rolls freely with negligible friction. Calculate the acceleration of the carts and the tension in the cord.



45. A 2.00 kg block (mass 1) and a 4.00 kg block (mass 2) are connected by a light string as shown; the inclination of the ramp is 40.0°. Friction is negligible. What is (a) the acceleration of each block and (b) the tension in the string?

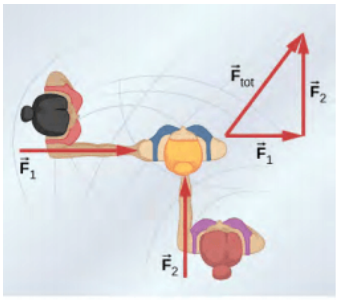


6.2 Friction

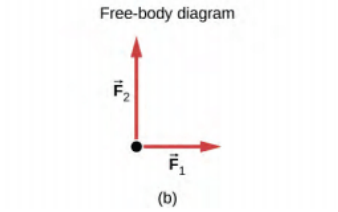
46. (a) When rebuilding his car's engine, a physics major must exert 3.00×10^2 N of force to insert a dry steel piston into a steel cylinder. What is the normal force between the piston and cylinder? (b) What force would he have to exert if the steel parts were oiled?
47. (a) What is the maximum frictional force in the knee joint of a person who supports 66.0 kg of her mass on that

knee? (b) During strenuous exercise, it is possible to exert forces to the joints that are easily 10 times greater than the weight being supported. What is the maximum force of friction under such conditions? The frictional forces in joints are relatively small in all circumstances except when the joints deteriorate, such as from injury or arthritis. Increased frictional forces can cause further damage and pain.

48. Suppose you have a 120-kg wooden crate resting on a wood floor, with coefficient of static friction 0.500 between these wood surfaces. (a) What maximum force can you exert horizontally on the crate without moving it? (b) If you continue to exert this force once the crate starts to slip, what will its acceleration then be? The coefficient of sliding friction is known to be 0.300 for this situation.
49. (a) If half of the weight of a small 1.00×10^3-kg utility truck is supported by its two drive wheels, what is the maximum acceleration it can achieve on dry concrete? (b) Will a metal cabinet lying on the wooden bed of the truck slip if it accelerates at this rate? (c) Solve both problems assuming the truck has four-wheel drive.
50. A team of eight dogs pulls a sled with waxed wood runners on wet snow (mush!). The dogs have average masses of 19.0 kg, and the loaded sled with its rider has a mass of 210 kg. (a) Calculate the acceleration of the dogs starting from rest if each dog exerts an average force of 185 N backward on the snow. (b) Calculate the force in the coupling between the dogs and the sled.
51. Consider the 65.0-kg ice skater being pushed by two others shown below. (a) Find the direction and magnitude of F_{tot}, the total force exerted on her by the others, given that the magnitudes F_1 and F_2 are 26.4 N and 18.6 N, respectively. (b) What is her initial acceleration if she is initially stationary and wearing steel-bladed skates that point in the direction of F_{tot}? (c) What is her acceleration assuming she is already moving in the direction of F_{tot}? (Remember that friction always acts in the direction opposite that of motion or attempted motion between surfaces in contact.)

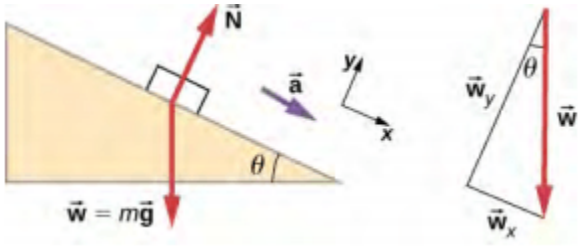


(a)

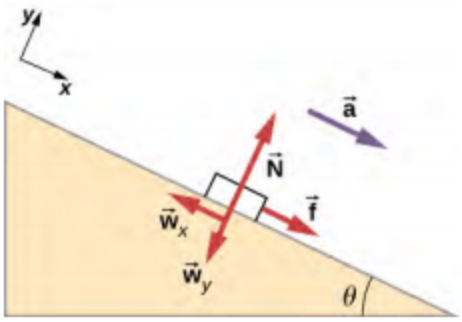


(b)

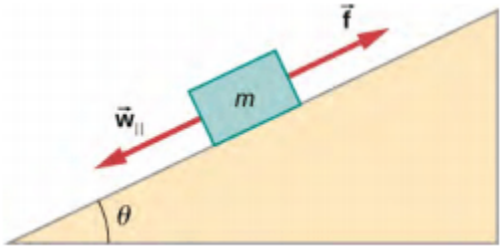
52. Show that the acceleration of any object down a frictionless incline that makes an angle θ with the horizontal is a = g sin θ. (Note that this acceleration is independent of mass.)



53. Show that the acceleration of any object down an incline where friction behaves simply (that is, where $f_k = \mu_k N$) is a = g(sin θ – μ_k cos θ). Note that the acceleration is independent of mass and reduces to the expression found in the previous problem when friction becomes negligibly small (μ_k = 0).



54. Calculate the deceleration of a snow boarder going up a 5.00° slope, assuming the coefficient of friction for waxed wood on wet snow. The result of the preceding problem may be useful, but be careful to consider the fact that the snow boarder is going uphill.
55. A machine at a post office sends packages out a chute and down a ramp to be loaded into delivery vehicles. (a) Calculate the acceleration of a box heading down a 10.0° slope, assuming the coefficient of friction for a parcel on waxed wood is 0.100. (b) Find the angle of the slope down which this box could move at a constant velocity. You can neglect air resistance in both parts.
56. If an object is to rest on an incline without slipping, then friction must equal the component of the weight of the object parallel to the incline. This requires greater and greater friction for steeper slopes. Show that the maximum angle of an incline above the horizontal for which an object will not slide down is $\theta = \tan^{-1} \mu_s$. You may use the result of the previous problem. Assume that a = 0 and that static friction has reached its maximum value.



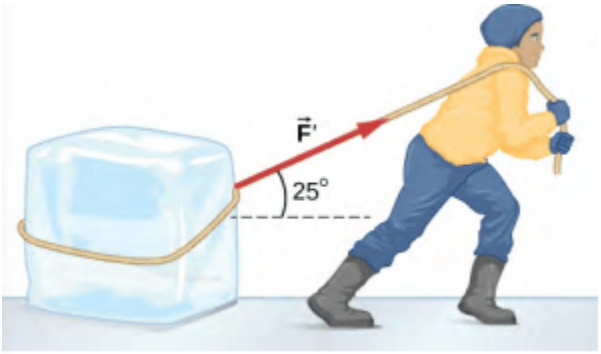
57. Calculate the maximum acceleration of a car that is heading down a 6.00° slope (one that makes an angle of 6.00° with the horizontal) under the following road conditions. You may assume that the weight of the car is evenly distributed on all four tires and that the coefficient of static friction is involved—that is, the tires are not allowed to slip during the deceleration. (Ignore rolling.) Calculate for a car: (a) On dry concrete. (b) On wet concrete. (c) On ice, assuming that μ_s = 0.100, the same as for shoes on ice.
58. Calculate the maximum acceleration of a car that is heading up a 4.00° slope (one that makes an angle of 4.00° with the horizontal) under the following road conditions. Assume that only half the weight of the car is supported by the two drive wheels and that the coefficient of static friction is involved—that is, the tires are not allowed to slip during the acceleration. (Ignore rolling.) (a) On dry concrete. (b) On wet concrete. (c) On ice, assuming that μ_s = 0.100, the same as for shoes on ice.
59. Repeat the preceding problem for a car with four-wheel drive.
60. A freight train consists of two 8.00×10^5-kg engines and 45 cars with average masses of 5.50×10^5 kg. (a) What force must each engine exert backward on the track to accelerate the train at a rate of 5.00×10^{-2} m/s^2 if the force of friction is 7.50×10^5 N, assuming the engines exert identical forces? This is not a large frictional force for such a massive system. Rolling friction for trains is small, and consequently, trains are very energy-efficient transportation systems. (b) What is the force in the coupling between the 37th and 38th cars (this is the force each exerts on the other), assuming all cars have the same mass and that friction is evenly distributed among all of the cars and engines?
61. Consider the 52.0-kg mountain climber shown below. (a) Find the tension in the rope and the force that the mountain climber must exert with her feet on the vertical rock face to remain stationary. Assume that the force is exerted parallel to her legs. Also, assume negligible force exerted by her arms. (b) What is the minimum coefficient of friction between her shoes and the cliff?



62. A contestant in a winter sporting event pushes a 45.0-kg block of ice across a frozen lake as shown below. (a) Calculate the minimum force F he must exert to get the block moving. (b) What is its acceleration once it starts to move, if that force is maintained?



63. The contestant now pulls the block of ice with a rope over his shoulder at the same angle above the horizontal as shown below. Calculate the minimum force F he must exert to get the block moving. (b) What is its acceleration once it starts to move, if that force is maintained?



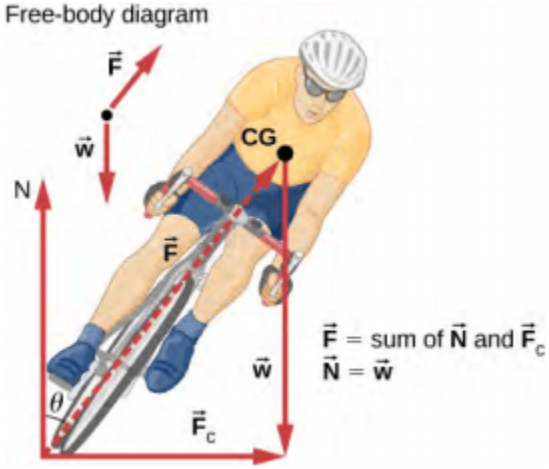
64. At a post office, a parcel that is a 20.0-kg box slides down a ramp inclined at 30.0° with the horizontal. The coefficient of kinetic friction between the box and plane is 0.0300. (a) Find the acceleration of the box. (b) Find the velocity of the box as it reaches the end of the plane, if the length of the plane is 2 m and the box starts at rest.

6.3 Centripetal Force

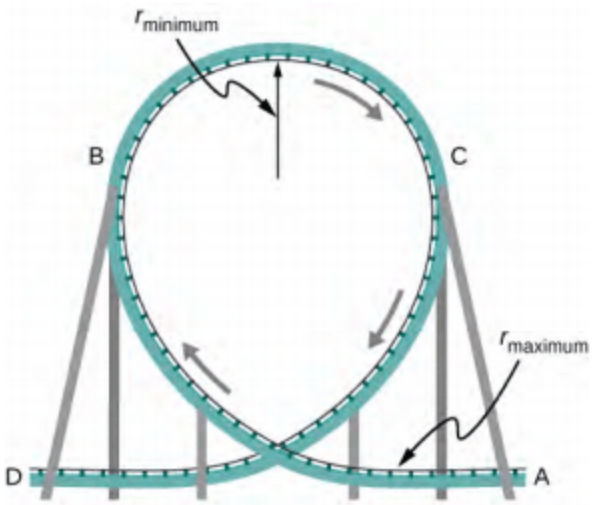
65. (a) A 22.0-kg child is riding a playground merry-go-round that is rotating at 40.0 rev/min. What centripetal force is

exerted if he is 1.25 m from its center? (b) What centripetal force is exerted if the merry-go-round rotates at 3.00 rev/min and he is 8.00 m from its center? (c) Compare each force with his weight.

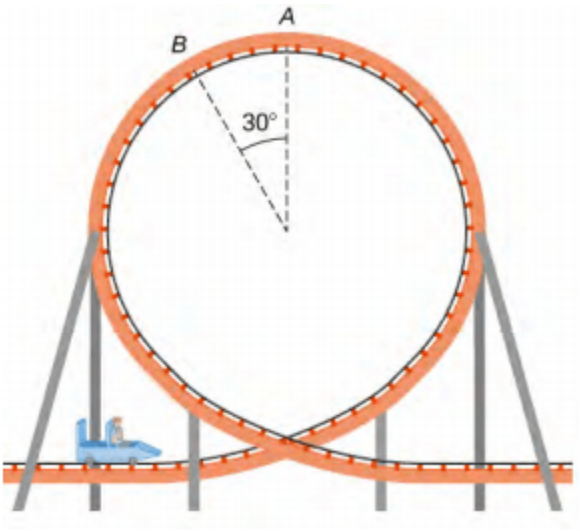
66. Calculate the centripetal force on the end of a 100-m (radius) wind turbine blade that is rotating at 0.5 rev/s. Assume the mass is 4 kg.
67. What is the ideal banking angle for a gentle turn of 1.20-km radius on a highway with a 10^5 km/h speed limit (about 65 mi/h), assuming everyone travels at the limit?
68. What is the ideal speed to take a 100.0-m-radius curve banked at a 20.0° angle?
69. (a) What is the radius of a bobsled turn banked at 75.0° and taken at 30.0 m/s, assuming it is ideally banked? (b) Calculate the centripetal acceleration. (c) Does this acceleration seem large to you?
70. Part of riding a bicycle involves leaning at the correct angle when making a turn, as seen below. To be stable, the force exerted by the ground must be on a line going through the center of gravity. The force on the bicycle wheel can be resolved into two perpendicular components—friction parallel to the road (this must supply the centripetal force) and the vertical normal force (which must equal the system's weight). (a) Show that θ (as defined as shown) is related to the speed v and radius of curvature r of the turn in the same way as for an ideally banked roadway—that is, $\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$. (b) Calculate θ for a 12.0-m/s turn of radius 30.0 m (as in a race).



71. If a car takes a banked curve at less than the ideal speed, friction is needed to keep it from sliding toward the inside of the curve (a problem on icy mountain roads). (a) Calculate the ideal speed to take a 100.0 m radius curve banked at 15.0°. (b) What is the minimum coefficient of friction needed for a frightened driver to take the same curve at 20.0 km/h?
72. Modern roller coasters have vertical loops like the one shown here. The radius of curvature is smaller at the top than on the sides so that the downward centripetal acceleration at the top will be greater than the acceleration due to gravity, keeping the passengers pressed firmly into their seats. What is the speed of the roller coaster at the top of the loop if the radius of curvature there is 15.0 m and the downward acceleration of the car is 1.50 g?



73. A child of mass 40.0 kg is in a roller coaster car that travels in a loop of radius 7.00 m. At point A the speed of the car is 10.0 m/s, and at point B, the speed is 10.5 m/s. Assume the child is not holding on and does not wear a seat belt. (a) What is the force of the car seat on the child at point A? (b) What is the force of the car seat on the child at point B? (c) What minimum speed is required to keep the child in his seat at point A?



74. In the simple Bohr model of the ground state of the hydrogen atom, the electron travels in a circular orbit around a fixed proton. The radius of the orbit is 5.28×10^{-11} m, and the speed of the electron is 2.18×10^{6} m/s. The mass of an electron is 9.11×10^{-31} kg. What is the force on the electron?
75. Railroad tracks follow a circular curve of radius 500.0 m and are banked at an angle of 5.0°. For trains of what speed are these tracks designed?
76. The CERN particle accelerator is circular with a circumference of 7.0 km. (a) What is the acceleration of the protons (m = 1.67×10^{-27} kg) that move around the accelerator at 5% of the speed of light? (The speed of light is v = 3.00×10^{8} m/s.) (b) What is the force on the protons?
77. A car rounds an unbanked curve of radius 65 m. If the coefficient of static friction between the road and car is 0.70, what is the maximum speed at which the car traverse the curve without slipping?
78. A banked highway is designed for traffic moving at 90.0 km/h. The radius of the curve is 310 m. What is the angle of banking of the highway?

6.4 Drag Force and Terminal Speed

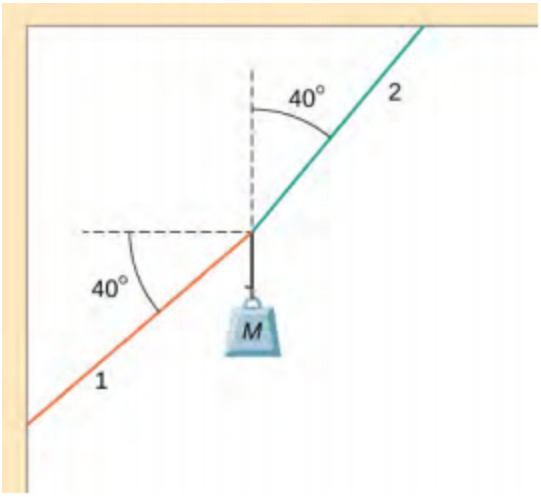
79. The terminal velocity of a person falling in air depends upon the weight and the area of the person facing the fluid. Find

the terminal velocity (in meters per second and kilometers per hour) of an 80.0-kg skydiver falling in a pike (headfirst) position with a surface area of 0.140 m^2.

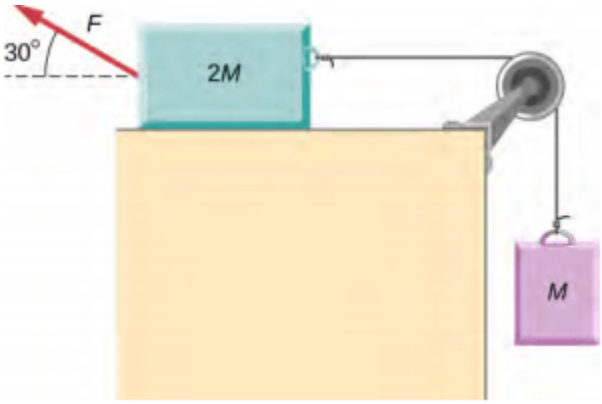
80. A 60.0-kg and a 90.0-kg skydiver jump from an airplane at an altitude of 6.00 x 10^3 m, both falling in the pike position. Make some assumption on their frontal areas and calculate their terminal velocities. How long will it take for each skydiver to reach the ground (assuming the time to reach terminal velocity is small)? Assume all values are accurate to three significant digits.
81. A 560-g squirrel with a surface area of 930 cm^2 falls from a 5.0-m tree to the ground. Estimate its terminal velocity. (Use a drag coefficient for a horizontal skydiver.) What will be the velocity of a 56-kg person hitting the ground, assuming no drag contribution in such a short distance?
82. To maintain a constant speed, the force provided by a car's engine must equal the drag force plus the force of friction of the road (the rolling resistance). (a) What are the drag forces at 70 km/h and 100 km/h for a Toyota Camry? (Drag area is 0.70 m^2) (b) What is the drag force at 70 km/h and 100 km/h for a Hummer H2? (Drag area is 2.44 m^2) Assume all values are accurate to three significant digits.
83. By what factor does the drag force on a car increase as it goes from 65 to 110 km/h?
84. Calculate the velocity a spherical rain drop would achieve falling from 5.00 km (a) in the absence of air drag (b) with air drag. Take the size across of the drop to be 4 mm, the density to be 1.00 x 10^3 kg/m^3, and the surface area to be πr^2.
85. Using Stokes' law, verify that the units for viscosity are kilograms per meter per second.
86. Find the terminal velocity of a spherical bacterium (diameter 2.00 μ_m) falling in water. You will first need to note that the drag force is equal to the weight at terminal velocity. Take the density of the bacterium to be 1.10 x 10^3 kg/m^3.
87. Stokes' law describes sedimentation of particles in liquids and can be used to measure viscosity. Particles in liquids achieve terminal velocity quickly. One can measure the time it takes for a particle to fall a certain distance and then use Stokes' law to calculate the viscosity of the liquid. Suppose a steel ball bearing (density 7.8 x 10^3 kg/m^3, diameter 3.0 mm) is dropped in a container of motor oil. It takes 12 s to fall a distance of 0.60 m. Calculate the viscosity of the oil.
88. Suppose that the resistive force of the air on a skydiver can be approximated by $f = -bv^2$. If the terminal velocity of a 50.0-kg skydiver is 60.0 m/s, what is the value of b?
89. A small diamond of mass 10.0 g drops from a swimmer's earring and falls through the water, reaching a terminal velocity of 2.0 m/s. (a) Assuming the frictional force on the diamond obeys $f = -bv$, what is b? (b) How far does the diamond fall before it reaches 90 percent of its terminal speed?

Additional Problems

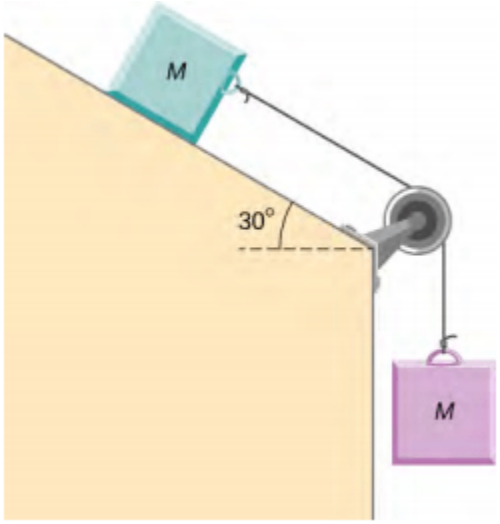
90. (a) What is the final velocity of a car originally traveling at 50.0 km/h that decelerates at a rate of 0.400 m/s^2 for 50.0 s? Assume a coefficient of friction of 1.0. (b) What is unreasonable about the result? (c) Which premise is unreasonable, or which premises are inconsistent?
91. A 75.0-kg woman stands on a bathroom scale in an elevator that accelerates from rest to 30.0 m/s in 2.00 s. (a) Calculate the scale reading in newtons and compare it with her weight. (The scale exerts an upward force on her equal to its reading.) (b) What is unreasonable about the result? (c) Which premise is unreasonable, or which premises are inconsistent?
92. (a) Calculate the minimum coefficient of friction needed for a car to negotiate an unbanked 50.0 m radius curve at 30.0 m/s. (b) What is unreasonable about the result? (c) Which premises are unreasonable or inconsistent?
93. As shown below, if M = 5.50 kg, what is the tension in string 1?



94. As shown below, if F = 60.0 N and M = 4.00 kg, what is the magnitude of the acceleration of the suspended object? All surfaces are frictionless.



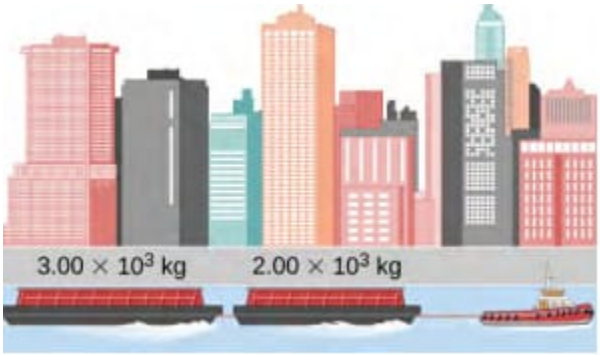
95. As shown below, if M = 6.0 kg, what is the tension in the connecting string? The pulley and all surfaces are frictionless.



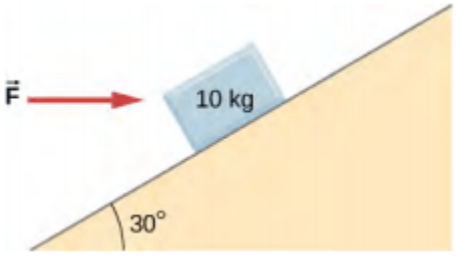
96. A small space probe is released from a spaceship. The space probe has mass 20.0 kg and contains 90.0 kg of fuel. It starts from rest in deep space, from the origin of a coordinate system based on the spaceship, and burns fuel at the rate of 3.00 kg/s. The engine provides a constant thrust of 120.0 N. (a) Write an expression for the mass of the space

probe as a function of time, between 0 and 30 seconds, assuming that the engine ignites fuel beginning at t = 0. (b) What is the velocity after 15.0 s? (c) What is the position of the space probe after 15.0 s, with initial position at the origin? (d) Write an expression for the position as a function of time, for t > 30.0 s.

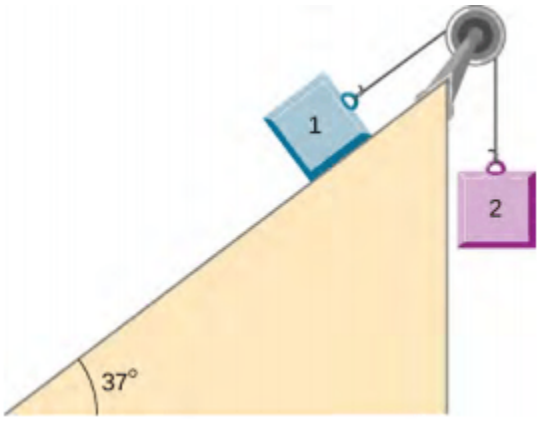
97. A half-full recycling bin has mass 3.0 kg and is pushed up a 40.0° incline with constant speed under the action of a 26-N force acting up and parallel to the incline. The incline has friction. What magnitude force must act up and parallel to the incline for the bin to move down the incline at constant velocity?
98. A child has mass 6.0 kg and slides down a 35° incline with constant speed under the action of a 34-N force acting up and parallel to the incline. What is the coefficient of kinetic friction between the child and the surface of the incline?
99. The two barges shown here are coupled by a cable of negligible mass. The mass of the front barge is 2.00 x 10^3 kg and the mass of the rear barge is 3.00 x 10^3 kg. A tugboat pulls the front barge with a horizontal force of magnitude 20.0 x 10^3 N, and the frictional forces of the water on the front and rear barges are 8.00 x 10^3 N and 10.0 x 10^3 N, respectively. Find the horizontal acceleration of the barges and the tension in the connecting cable.



100. If the order of the barges of the preceding exercise is reversed so that the tugboat pulls the 3.00 x 10^3 -kg barge with a force of 20.0 x 10^3 N, what are the acceleration of the barges and the tension in the coupling cable?
101. An object with mass m moves along the x-axis. Its position at any time is given by $x(t) = pt^3 + qt^2$ where p and q are constants. Find the net force on this object for any time t.
102. A helicopter with mass 2.35 x 10^4 kg has a position given by $\vec{r}(t) = (0.020\ t^3)\ \hat{i} + (2.2t)\ \hat{j} - (0.060\ t^2)$ |9\hat{k}\). Find the net force on the helicopter at t = 3.0 s.
103. Located at the origin, an electric car of mass m is at rest and in equilibrium. A time dependent force of $\vec{F}(t)$ is applied at time t = 0, and its components are $F_x(t) = p + nt$ and $F_y(t) = qt$ where p, q, and n are constants. Find the position $\vec{r}(t)$ and velocity $\vec{v}(t)$ as functions of time t.
104. A particle of mass m is located at the origin. It is at rest and in equilibrium. A time-dependent force of $\vec{F}(t)$ is applied at time t = 0, and its components are $F_x(t) = pt$ and $F_y(t) = n + qt$ where p, q, and n are constants. Find the position $\vec{r}(t)$ and velocity $\vec{v}(t)$ as functions of time t.
105. A 2.0-kg object has a velocity of 4.0 $\hat{i}$ m/s at t = 0. A constant resultant force of (2.0 $\hat{i}$ + 4.0 $\hat{j}$) N then acts on the object for 3.0 s. What is the magnitude of the object's velocity at the end of the 3.0-s interval?
106. A 1.5-kg mass has an acceleration of (4.0 $\hat{i}$ – 3.0 $\hat{j}$) m/s^2. Only two forces act on the mass. If one of the forces is (2.0 $\hat{i}$ – 1.4 $\hat{j}$) N, what is the magnitude of the other force?
107. A box is dropped onto a conveyor belt moving at 3.4 m/s. If the coefficient of friction between the box and the belt is 0.27, how long will it take before the box moves without slipping?
108. Shown below is a 10.0-kg block being pushed by a horizontal force $\vec{F}$ of magnitude 200.0 N. The coefficient of kinetic friction between the two surfaces is 0.50. Find the acceleration of the block.



109. As shown below, the mass of block 1 is m_1 = 4.0 kg, while the mass of block 2 is m_2 = 8.0 kg. The coefficient of friction between m_1 and the inclined surface is μ_k = 0.40. What is the acceleration of the system?



110. A student is attempting to move a 30-kg mini-fridge into her dorm room. During a moment of inattention, the mini-fridge slides down a 35 degree incline at constant speed when she applies a force of 25 N acting up and parallel to the incline. What is the coefficient of kinetic friction between the fridge and the surface of the incline?
111. A crate of mass 100.0 kg rests on a rough surface inclined at an angle of 37.0° with the horizontal. A massless rope to which a force can be applied parallel to the surface is attached to the crate and leads to the top of the incline. In its present state, the crate is just ready to slip and start to move down the plane. The coefficient of friction is 80% of that for the static case. (a) What is the coefficient of static friction? (b) What is the maximum force that can be applied upward along the plane on the rope and not move the block? (c) With a slightly greater applied force, the block will slide up the plane. Once it begins to move, what is its acceleration and what reduced force is necessary to keep it moving upward at constant speed? (d) If the block is given a slight nudge to get it started down the plane, what will be its acceleration in that direction? (e) Once the block begins to slide downward, what upward force on the rope is required to keep the block from accelerating downward?
112. A car is moving at high speed along a highway when the driver makes an emergency braking. The wheels become locked (stop rolling), and the resulting skid marks are 32.0 meters long. If the coefficient of kinetic friction between tires and road is 0.550, and the acceleration was constant during braking, how fast was the car going when the wheels became locked?
113. A crate having mass 50.0 kg falls horizontally off the back of the flatbed truck, which is traveling at 100 km/h. Find the value of the coefficient of kinetic friction between the road and crate if the crate slides 50 m on the road in coming to rest. The initial speed of the crate is the same as the truck, 100 km/h.



114. A 15-kg sled is pulled across a horizontal, snow-covered surface by a force applied to a rope at 30 degrees with the horizontal. The coefficient of kinetic friction between the sled and the snow is 0.20. (a) If the force is 33 N, what is the horizontal acceleration of the sled? (b) What must the force be in order to pull the sled at constant velocity?
115. A 30.0-g ball at the end of a string is swung in a vertical circle with a radius of 25.0 cm. The tangential velocity is 200.0 cm/s. Find the tension in the string: (a) at the top of the circle, (b) at the bottom of the circle, and (c) at a distance of 12.5 cm from the center of the circle (r = 12.5 cm).
116. A particle of mass 0.50 kg starts moves through a circular path in the xy-plane with a position given by $\vec{r}$(t) = (4.0 cos 3t) $\hat{i}$ + (4.0 sin 3t) $\hat{j}$ where r is in meters and t is in seconds. (a) Find the velocity and acceleration vectors as functions of time. (b) Show that the acceleration vector always points toward the center of the circle (and thus represents centripetal acceleration). (c) Find the centripetal force vector as a function of time.
117. A stunt cyclist rides on the interior of a cylinder 12 m in radius. The coefficient of static friction between the tires and

the wall is 0.68. Find the value of the minimum speed for the cyclist to perform the stunt.

118. When a body of mass 0.25 kg is attached to a vertical massless spring, it is extended 5.0 cm from its unstretched length of 4.0 cm. The body and spring are placed on a horizontal frictionless surface and rotated about the held end of the spring at 2.0 rev/s. How far is the spring stretched?

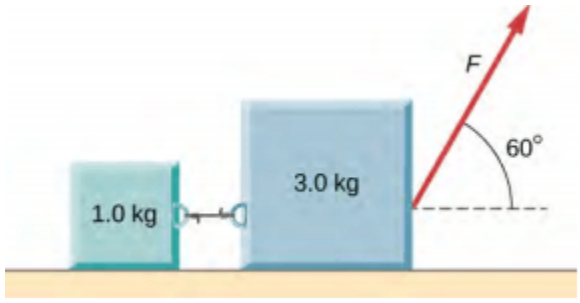
119. Railroad tracks follow a circular curve of radius 500.0 m and are banked at an angle of 5.00°. For trains of what speed are these tracks designed?

120. A plumb bob hangs from the roof of a railroad car. The car rounds a circular track of radius 300.0 m at a speed of 90.0 km/h. At what angle relative to the vertical does the plumb bob hang?

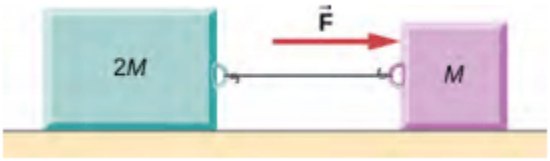
121. An airplane flies at 120.0 m/s and banks at a 30° angle. If its mass is 2.50 x 10^3 kg, (a) what is the magnitude of the lift force? (b) what is the radius of the turn?

122. The position of a particle is given by $\vec{r}(t) = A(\cos \omega t\, \hat{i} + \sin \omega t\, \hat{j})$, where ω is a constant. (a) Show that the particle moves in a circle of radius A. (b) Calculate $\frac{d\vec{r}}{dt}$ and then show that the speed of the particle is a constant $A\omega$. (c) Determine $\frac{d^2\vec{r}}{dt^2}$ and show that a is given by $a_c = r\omega^2$. (d) Calculate the centripetal force on the particle. [**Hint**: For (b) and (c), you will need to use $\left(\frac{d}{dt}\right)(\cos \omega t) = -\omega \sin \omega t$ and $\left(\frac{d}{dt}\right)(\sin \omega t) = \omega \cos \omega t$.

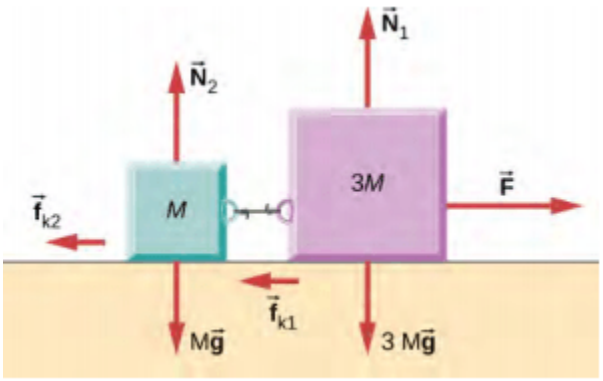
123. Two blocks connected by a string are pulled across a horizontal surface by a force applied to one of the blocks, as shown below. The coefficient of kinetic friction between the blocks and the surface is 0.25. If each block has an acceleration of 2.0 m/s^2 to the right, what is the magnitude F of the applied force?



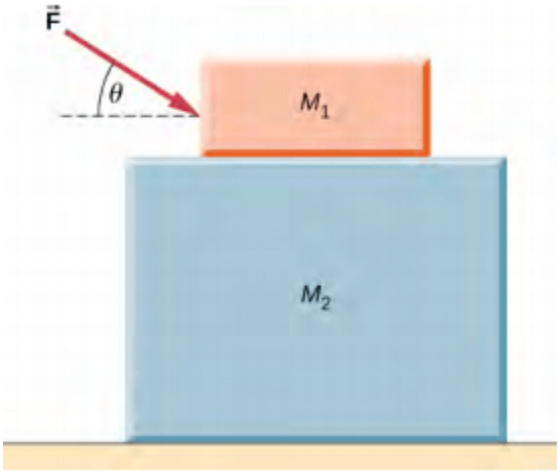
124. As shown below, the coefficient of kinetic friction between the surface and the larger block is 0.20, and the coefficient of kinetic friction between the surface and the smaller block is 0.30. If F = 10 N and M = 1.0 kg, what is the tension in the connecting string?



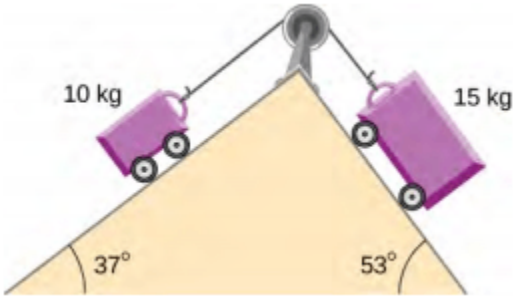
125. In the figure, the coefficient of kinetic friction between the surface and the blocks is μ_k. If M = 1.0 kg, find an expression for the magnitude of the acceleration of either block (in terms of F, μ_k, and g).



126. Two blocks are stacked as shown below, and rest on a frictionless surface. There is friction between the two blocks (coefficient of friction μ). An external force is applied to the top block at an angle θ with the horizontal. What is the maximum force F that can be applied for the two blocks to move together?

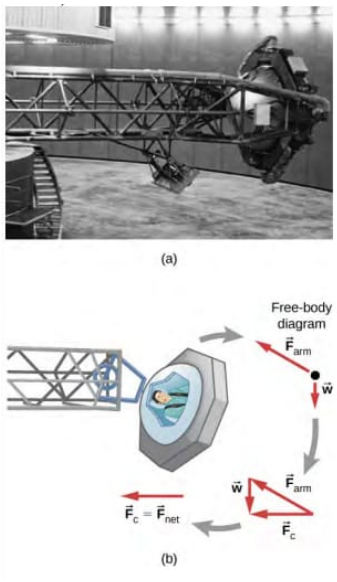


127. A box rests on the (horizontal) back of a truck. The coefficient of static friction between the box and the surface on which it rests is 0.24. What maximum distance can the truck travel (starting from rest and moving horizontally with constant acceleration) in 3.0 s without having the box slide?
128. A double-incline plane is shown below. The coefficient of friction on the left surface is 0.30, and on the right surface 0.16. Calculate the acceleration of the system.



Challenge Problems

129. In a later chapter, you will find that the weight of a particle varies with altitude such that $w = \frac{mgr_0^2}{r^2}$ where r_0 is the radius of Earth and r is the distance from Earth's center. If the particle is fired vertically with velocity v_0 from Earth's surface, determine its velocity as a function of position r. (Hint: use a $dr = v\,dv$, the rearrangement mentioned in the text.)
130. A large centrifuge, like the one shown below, is used to expose aspiring astronauts to accelerations similar to those experienced in rocket launches and atmospheric reentries. (a) At what angular velocity is the centripetal acceleration 10g if the rider is 15.0 m from the center of rotation? (b) The rider's cage hangs on a pivot at the end of the arm, allowing it to swing outward during rotation as shown in the bottom accompanying figure. At what angle θ below the horizontal will the cage hang when the centripetal acceleration is 10g? (Hint: The arm supplies centripetal force and supports the weight of the cage. Draw a free-body diagram of the forces to see what the angle θ should be.)



131. A car of mass 1000.0 kg is traveling along a level road at 100.0 km/h when its brakes are applied. Calculate the stopping distance if the coefficient of kinetic friction of the tires is 0.500. Neglect air resistance. (**Hint**: since the distance traveled is of interest rather than the time, x is the desired independent variable and not t. Use the Chain Rule to change the variable: $\frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx}$.)
132. An airplane flying at 200.0 m/s makes a turn that takes 4.0 min. What bank angle is required? What is the percentage increase in the perceived weight of the passengers?
133. A skydiver is at an altitude of 1520 m. After 10.0 seconds of free fall, he opens his parachute and finds that the air resistance, F_D, is given by the formula $F_D = -bv$, where b is a constant and v is the velocity. If b = 0.750, and the mass of the skydiver is 82.0 kg, first set up differential equations for the velocity and the position, and then find: (a) the speed of the skydiver when the parachute opens, (b) the distance fallen before the parachute opens, (c) the terminal velocity after the parachute opens (find the limiting velocity), and (d) the time the skydiver is in the air after the parachute opens.
134. In a television commercial, a small, spherical bead of mass 4.00 g is released from rest at t = 0 in a bottle of liquid shampoo. The terminal speed is observed to be 2.00 cm/s. Find (a) the value of the constant b in the equation $v = \frac{mg}{b}\left(1 - e^{-\frac{bt}{m}}\right)$, and (b) the value of the resistive force when the bead reaches terminal speed.
135. A boater and motor boat are at rest on a lake. Together, they have mass 200.0 kg. If the thrust of the motor is a constant force of 40.0 N in the direction of motion, and if the resistive force of the water is numerically equivalent to 2 times the speed v of the boat, set up and solve the differential equation to find: (a) the velocity of the boat at time t; (b) the limiting velocity (the velocity after a long time has passed).

Contributors and Attributions

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5.9: Applications of Newton's Laws (Summary)

Key Terms

banked curve	curve in a road that is sloping in a manner that helps a vehicle negotiate the curve
centripetal force	any net force causing uniform circular motion
Coriolis force	inertial force causing the apparent deflection of moving objects when viewed in a rotating frame of reference
drag force	force that always opposes the motion of an object in a fluid; unlike simple friction, the drag force is proportional to some function of the velocity of the object in that fluid
friction	force that opposes relative motion or attempts at motion between systems in contact
ideal banking	sloping of a curve in a road, where the angle of the slope allows the vehicle to negotiate the curve at a certain speed without the aid of friction between the tires and the road; the net external force on the vehicle equals the horizontal centripetal force in the absence of friction
inertial force	force that has no physical origin
kinetic friction	force that opposes the motion of two systems that are in contact and moving relative to each other
noninertial frame of reference	accelerated frame of reference
static friction	force that opposes the motion of two systems that are in contact and are not moving relative to each other
terminal velocity	constant velocity achieved by a falling object, which occurs when the weight of the object is balanced by the upward drag force

Key Equations

Magnitude of static friction	$f_s \leq \mu_s N$
Magnitude of kinetic friction	$f_k = \mu_k N$
Centripetal force	$F_c = m\frac{v^2}{r} = mr\omega^2$
Ideal angle of a banked curve	$\tan\theta = \frac{v^2}{rg}$
Drag force	$F_D = \frac{1}{2}C\rho A v^2$

Stokes' law	$F_s = 6\pi r\eta v$

Summary

6.1 Solving Problems with Newton's Laws

- Newton's laws of motion can be applied in numerous situations to solve motion problems.
- Some problems contain multiple force vectors acting in different directions on an object. Be sure to draw diagrams, resolve all force vectors into horizontal and vertical components, and draw a free-body diagram. Always analyze the direction in which an object accelerates so that you can determine whether $F_{net} = ma$ or $F_{net} = 0$.
- The normal force on an object is not always equal in magnitude to the weight of the object. If an object is accelerating vertically, the normal force is less than or greater than the weight of the object. Also, if the object is on an inclined plane, the normal force is always less than the full weight of the object.
- Some problems contain several physical quantities, such as forces, acceleration, velocity, or position. You can apply concepts from kinematics and dynamics to solve these problems.

6.2 Friction

- Friction is a contact force that opposes the motion or attempted motion between two systems. Simple friction is proportional to the normal force N supporting the two systems.
- The magnitude of static friction force between two materials stationary relative to each other is determined using the coefficient of static friction, which depends on both materials.
- The kinetic friction force between two materials moving relative to each other is determined using the coefficient of kinetic friction, which also depends on both materials and is always less than the coefficient of static friction.

6.3 Centripetal Force

- Centripetal force $\vec{F}_c$ is a "center-seeking" force that always points toward the center of rotation. It is perpendicular to linear velocity and has the magnitude

$$F_c = ma_c. \tag{5.9.7}$$

- Rotating and accelerated frames of reference are noninertial. Inertial forces, such as the Coriolis force, are needed to explain motion in such frames.

6.4 Drag Force and Terminal Speed

- Drag forces acting on an object moving in a fluid oppose the motion. For larger objects (such as a baseball) moving at a velocity in air, the drag force is determined using the drag coefficient (typical values are given in Table 6.2), the area of the object facing the fluid, and the fluid density.
- For small objects (such as a bacterium) moving in a denser medium (such as water), the drag force is given by Stokes' law.

Contributors and Attributions

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6: PHYSICS 1- REVIEW WORK AND KINETIC ENERGY

6.1: Prelude to Work and Kinetic Energy

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6.3: Kinetic Energy

6.4: Work-Energy Theorem

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6.7: Work and Kinetic Energy (Summary)

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6.1: Prelude to Work and Kinetic Energy

In this chapter, we discuss some basic physical concepts involved in every physical motion in the universe, going beyond the concepts of force and change in motion, which we discussed in Motion in Two and Three Dimensions and Newton's Laws of Motion. These concepts are work, kinetic energy, and power. We explain how these quantities are related to one another, which will lead us to a fundamental relationship called the work-energy theorem. In the next chapter, we generalize this idea to the broader principle of conservation of energy.



Figure 6.1.1: A sprinter exerts her maximum power to do as much work on herself as possible in the short time that her foot is in contact with the ground. This adds to her kinetic energy, preventing her from slowing down during the race. Pushing back hard on the track generates a reaction force that propels the sprinter forward to win at the finish. (credit: modification of work by Marie-Lan Nguyen)

The application of Newton's laws usually requires solving differential equations that relate the forces acting on an object to the accelerations they produce. Often, an analytic solution is intractable or impossible, requiring lengthy numerical solutions or simulations to get approximate results. In such situations, more general relations, like the work-energy theorem (or the conservation of energy), can still provide useful answers to many questions and require a more modest amount of mathematical calculation. In particular, you will see how the work-energy theorem is useful in relating the speeds of a particle at different points along its trajectory, to the forces acting on it, even when the trajectory is otherwise too complicated to deal with. Thus, some aspects of motion can be addressed with fewer equations and without vector decompositions.

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6.2: Work

Learning Objectives

- Represent the work done by any force
- Evaluate the work done for various forces

In physics, **work** represents a type of energy. Work is done when a force acts on something that undergoes a displacement from one position to another. Forces can vary as a function of position, and displacements can be along various paths between two points. We first define the increment of work dW done by a force $\vec{F}$ acting through an infinitesimal displacement $d\vec{r}$ as the dot product of these two vectors:

$$dW = \vec{F} \cdot d\vec{r} = |\vec{F}||d\vec{r}| \cos\theta. \tag{6.2.1}$$

Then, we can add up the contributions for infinitesimal displacements, along a path between two positions, to get the total work.

Work Done by a Force

The work done by a force is the integral of the force with respect to displacement along the path of the displacement:

$$W_{AB} = \int_{path\ AB} \vec{F} \cdot d\vec{r}. \tag{6.2.2}$$

The vectors involved in the definition of the work done by a force acting on a particle are illustrated in Figure 6.2.1.

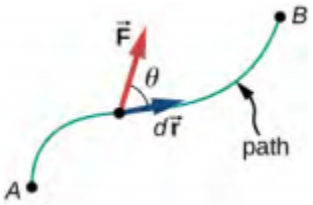


Figure 6.2.1: Vectors used to define work. The force acting on a particle and its infinitesimal displacement are shown at one point along the path between A and B. The infinitesimal work is the dot product of these two vectors; the total work is the integral of the dot product along the path.

We choose to express the dot product in terms of the magnitudes of the vectors and the cosine of the angle between them, because the meaning of the dot product for work can be put into words more directly in terms of magnitudes and angles. We could equally well have expressed the dot product in terms of the various components introduced in Vectors. In two dimensions, these were the x- and y-components in Cartesian coordinates, or the r- and φ-components in polar coordinates; in three dimensions, it was just x-, y-, and z-components. Which choice is more convenient depends on the situation. In words, you can express Equation 6.2.1 for the work done by a force acting over a displacement as a product of one component acting parallel to the other component. From the properties of vectors, it doesn't matter if you take the component of the force parallel to the displacement or the component of the displacement parallel to the force—you get the same result either way.

Recall that the magnitude of a force times the cosine of the angle the force makes with a given direction is the component of the force in the given direction. The components of a vector can be positive, negative, or zero, depending on whether the angle between the vector and the component-direction is between 0° and 90° or 90° and 180°, or is equal to 90°. As a result, the work done by a force can be positive, negative, or zero, depending on whether the force is generally in the direction of the displacement, generally opposite to the displacement, or perpendicular to the displacement. The maximum work is done by a given force when it is along the direction of the displacement ($\cos\theta = \pm 1$), and zero work is done when the force is perpendicular to the displacement ($\cos\theta = 0$).

The units of work are units of force multiplied by units of length, which in the SI system is newtons times meters, N • m. This

combination is called a joule, for historical reasons that we will mention later, and is abbreviated as J. In the English system, still used in the United States, the unit of force is the pound (lb) and the unit of distance is the foot (ft), so the unit of work is the foot-pound (ft • lb).

Work Done by Constant Forces and Contact Forces

The simplest work to evaluate is that done by a force that is constant in magnitude and direction. In this case, we can factor out the force; the remaining integral is just the total displacement, which only depends on the end points A and B, but not on the path between them:

$$W_{AB} = \vec{F} \cdot \int_A^B d\vec{r} = \vec{F} \cdot (\vec{r}_B - \vec{r}_A) = |\vec{F}||\vec{r}_B - \vec{r}_A| \cos\theta \; (constant \; force).$$

We can also see this by writing out Equation 6.2.2 in Cartesian coordinates and using the fact that the components of the force are constant:

$$\begin{aligned} W_{AB} &= \int_{path\ AB} \vec{F} \cdot d\vec{r} = \int_{path\ AB} (F_x dx + F_y dy + F_z dz) = F_x \int_A^B dx + F_y \int_A^B dy + F_z \int_A^B dz \\ &= F_x(x_B - x_A) + F_y(y_B - y_A) + F_z(z_B - z_A) = \vec{F} \cdot (\vec{r}_B - \vec{r}_A). \end{aligned}$$

Figure $6.2.2a$ shows a person exerting a constant force $\vec{F}$ along the handle of a lawn mower, which makes an angle θ with the horizontal. The horizontal displacement of the lawn mower, over which the force acts, is $\vec{d}$. The work done on the lawn mower is

$$W = \vec{F} \cdot \vec{d} = Fd\cos\theta,$$

which the figure also illustrates as the horizontal component of the force times the magnitude of the displacement.

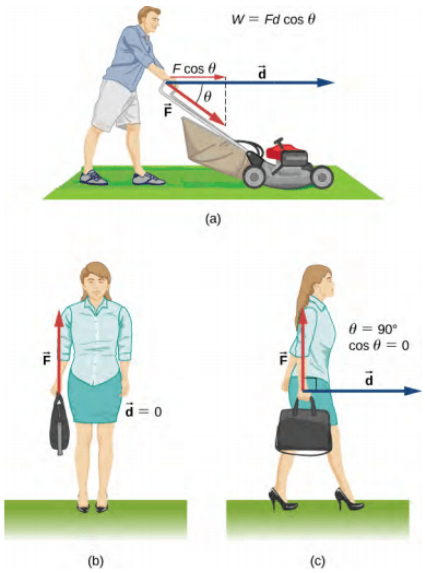


Figure 6.2.2: Work done by a constant force. (a) A person pushes a lawn mower with a constant force. The component of the force parallel to the displacement is the work done, as shown in the equation in the figure. (b) A person holds a briefcase. No work is done because the displacement is zero. (c) The person in (b) walks horizontally while holding the briefcase. No work is done because cos θ is zero.

Figure $6.2.2b$ shows a person holding a briefcase. The person must exert an upward force, equal in magnitude to the weight of the briefcase, but this force does no work, because the displacement over which it acts is zero. So why do you eventually feel tired just holding the briefcase, if you're not doing any work on it? The answer is that muscle fibers in your arm are contracting and doing work inside your arm, even though the force your muscles exert externally on the briefcase doesn't do any work on it. (Part of the force you exert could also be tension in the bones and ligaments of your arm, but other muscles in your body would be doing work to maintain the position of your arm.)

In Figure $6.2.2c$, where the person in (b) is walking horizontally with constant speed, the work done by the person on the briefcase

is still zero, but now because the angle between the force exerted and the displacement is 90° ($\vec{F}$ perpendicular to $\vec{d}$) and cos 90° = 0.

Example 6.2.1: Calculating the Work You Do to Push a Lawn Mower

How much work is done on the lawn mower by the person in Figure 6.2.2*a* if he exerts a constant force of 75.0 N at an angle 35° below the horizontal and pushes the mower 25.0 m on level ground?

Strategy

We can solve this problem by substituting the given values into the definition of work done on an object by a constant force, stated in the equation W = Fd cos θ. The force, angle, and displacement are given, so that only the work W is unknown.

Solution

The equation for the work is

$$W = Fd\cos\theta.$$

Substituting the known values gives

$$W = (75.0\ N)(25.0\ m)\cos(35.0^o) = 1.54 \times 10^3\ J.$$

Significance

Even though one and a half kilojoules may seem like a lot of work, we will see in Potential Energy and Conservation of Energy that it's only about as much work as you could do by burning one sixth of a gram of fat.

When you mow the grass, other forces act on the lawn mower besides the force you exert—namely, the contact force of the ground and the gravitational force of Earth. Let's consider the work done by these forces in general. For an object moving on a surface, the displacement d $\vec{r}$ is tangent to the surface. The part of the contact force on the object that is perpendicular to the surface is the normal force $\vec{N}$. Since the cosine of the angle between the normal and the tangent to a surface is zero, we have

$$dW_N = \vec{N}\cdot d\vec{r} = \vec{0}.$$

The normal force never does work under these circumstances. (Note that if the displacement d $\vec{r}$ did have a relative component perpendicular to the surface, the object would either leave the surface or break through it, and there would no longer be any normal contact force. However, if the object is more than a particle, and has an internal structure, the normal contact force can do work on it, for example, by displacing it or deforming its shape. This will be mentioned in the next chapter.)

The part of the contact force on the object that is parallel to the surface is friction, $\vec{f}$. For this object sliding along the surface, kinetic friction $\vec{f}_k$ is opposite to d $\vec{r}$, relative to the surface, so the work done by kinetic friction is negative. If the magnitude of $\vec{f}_k$ is constant (as it would be if all the other forces on the object were constant), then the work done by friction is

$$W_{fr} = \int_A^B \vec{f}_k\cdot d\vec{r} = -f_k\int_A^B |dr| = -f_k|l_{AB}|. \tag{6.2.3}$$

where $|l_{AB}|$ is the path length on the surface. (Note that, especially if the work done by a force is negative, people may refer to the work done against this force, where $dW_{against} = -dW_{by}$. The work done against a force may also be viewed as the work required to overcome this force, as in "How much work is required to overcome...?") The force of static friction, however, can do positive or negative work. When you walk, the force of static friction exerted by the ground on your back foot accelerates you for part of each step. If you're slowing down, the force of the ground on your front foot decelerates you. If you're driving your car at the speed limit on a straight, level stretch of highway, the negative work done by kinetic friction of air resistance is balanced by the positive work done by the static friction of the road on the drive wheels. You can pull the rug out from under an object in such a way that it slides backward relative to the rug, but forward relative to the floor. In this case, kinetic friction exerted by the rug on the object could be in the same direction as the displacement of the object, relative to the floor, and do positive work. The bottom line is that you need to analyze each particular case to determine the work done by the forces, whether positive, negative or zero.

Example 6.2.2: Moving a Couch

You decide to move your couch to a new position on your horizontal living room floor. The normal force on the couch is 1 kN and the coefficient of friction is 0.6. (a) You first push the couch 3 m parallel to a wall and then 1 m perpendicular to the wall (A to B in Figure 6.2.3). How much work is done by the frictional force? (b) You don't like the new position, so you move the couch straight back to its original position (B to A in Figure 6.2.3). What was the total work done against friction moving the couch away from its original position and back again?

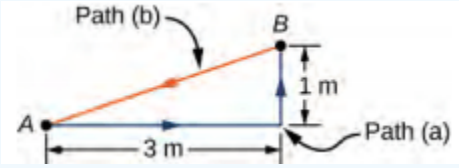


Figure 6.2.3: Top view of paths for moving a couch.

Strategy

The magnitude of the force of kinetic friction on the couch is constant, equal to the coefficient of friction times the normal force, $f_K = \mu_K N$. Therefore, the work done by it is $W_{fr} = -f_K d$, where d is the path length traversed. The segments of the paths are the sides of a right triangle, so the path lengths are easily calculated. In part (b), you can use the fact that the work done against a force is the negative of the work done by the force.

Solution

a. The work done by friction is

$$W = -(0.6)(1\ kN)(3\ m + 1\ m) = -2.4\ kJ.$$

b. The length of the path along the hypotenuse is $\sqrt{10}$ m, so the total work done against friction is

$$W = (0.6)(1\ kN)(3\ m + 1\ m + \sqrt{10}\ m) = 4.3\ kJ.$$

Significance

The total path over which the work of friction was evaluated began and ended at the same point (it was a closed path), so that the total displacement of the couch was zero. However, the total work was not zero. The reason is that forces like friction are classified as nonconservative forces, or dissipative forces, as we discuss in the next chapter.

Exercise 7.1

Can kinetic friction ever be a constant force for all paths?

The other force on the lawn mower mentioned above was Earth's gravitational force, or the weight of the mower. Near the surface of Earth, the gravitational force on an object of mass m has a constant magnitude, mg, and constant direction, vertically down. Therefore, the work done by gravity on an object is the dot product of its weight and its displacement. In many cases, it is convenient to express the dot product for gravitational work in terms of the x-, y-, and z-components of the vectors. A typical coordinate system has the x-axis horizontal and the y-axis vertically up. Then the gravitational force is $-mg\,\hat{j}$, so the work done by gravity, over any path from A to B, is

$$W_{grav,\ AB} = -mg\hat{j} \cdot (\vec{r}_B - \vec{r}_A) = -mg(y_B - y_A). \tag{6.2.4}$$

The work done by a constant force of gravity on an object depends only on the object's weight and the difference in height through which the object is displaced. Gravity does negative work on an object that moves upward ($y_B > y_A$), or, in other words, you must do positive work against gravity to lift an object upward. Alternately, gravity does positive work on an object that moves downward ($y_B < y_A$), or you do negative work against gravity to "lift" an object downward, controlling its descent so it doesn't drop to the ground. ("Lift" is used as opposed to "drop".)

Example 6.2.3: Shelving a Book

You lift an oversized library book, weighing 20 N, 1 m vertically down from a shelf, and carry it 3 m horizontally to a table (Figure 6.2.4). How much work does gravity do on the book? (b) When you're finished, you move the book in a straight line back to its original place on the shelf. What was the total work done against gravity, moving the book away from its original position on the shelf and back again?

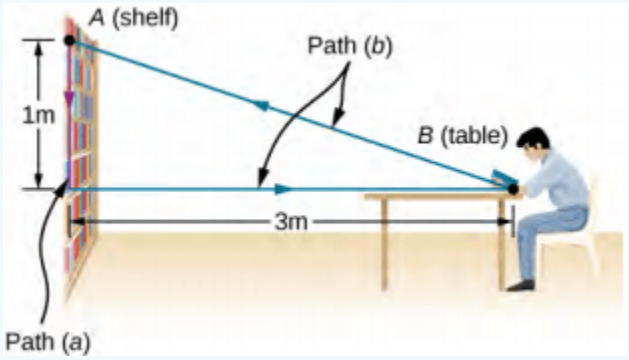


Figure 6.2.4: Side view of the paths for moving a book to and from a shelf.

Strategy

We have just seen that the work done by a constant force of gravity depends only on the weight of the object moved and the difference in height for the path taken, $W_{AB} = -mg(y_B - y_A)$. We can evaluate the difference in height to answer (a) and (b).

Solution

a. Since the book starts on the shelf and is lifted down $y_B - y_A = -1$ m, we have

$$W = -(20\ N)(-1\ m) = 20J.$$

b. There is zero difference in height for any path that begins and ends at the same place on the shelf, so W = 0.

Significance

Gravity does positive work (20 J) when the book moves down from the shelf. The gravitational force between two objects is an attractive force, which does positive work when the objects get closer together. Gravity does zero work (0 J) when the book moves horizontally from the shelf to the table and negative work (–20 J) when the book moves from the table back to the shelf. The total work done by gravity is zero [20 J + 0 J + (–20 J) = 0].

Unlike friction or other dissipative forces, described in Example 6.2.2, the total work done against gravity, over any closed path, is zero. Positive work is done against gravity on the upward parts of a closed path, but an equal amount of negative work is done against gravity on the downward parts. In other words, work done **against** gravity, lifting an object **up**, is "given back" when the object comes back down. Forces like gravity (those that do zero work over any closed path) are classified as conservative forces and play an important role in physics.

Exercise 7.2

Can Earth's gravity ever be a constant force for all paths?

Work Done by Forces that Vary

In general, forces may vary in magnitude and direction at points in space, and paths between two points may be curved. The infinitesimal work done by a variable force can be expressed in terms of the components of the force and the displacement along the path,

$$dW = F_x dx + F_y dy + F_z dz.$$

Here, the components of the force are functions of position along the path, and the displacements depend on the equations of the path. (Although we chose to illustrate dW in Cartesian coordinates, other coordinates are better suited to some situations.) Equation 6.2.2 defines the total work as a line integral, or the limit of a sum of infinitesimal amounts of work. The physical concept of work is straightforward: you calculate the work for tiny displacements and add them up. Sometimes the mathematics can seem complicated, but the following example demonstrates how cleanly they can operate.

Example 6.2.4: Work Done by a Variable Force over a Curved Path

An object moves along a parabolic path y = (0.5 m^{-1})x^2 from the origin A = (0, 0) to the point B = (2 m, 2 m) under the action of a force $\vec{F}$ = (5 N/m)y $\hat{i}$ + (10 N/m)x $\hat{j}$ (Figure 6.2.5). Calculate the work done.

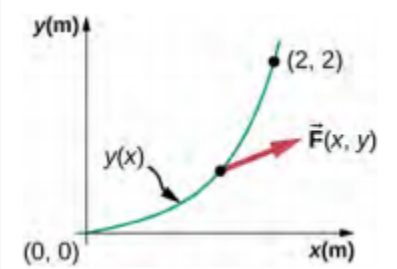


Figure 6.2.5: The parabolic path of a particle acted on by a given force.

Strategy

The components of the force are given functions of x and y. We can use the equation of the path to express y and dy in terms of x and dx; namely,

$$y = (0.5\ m^{-1})x^2 \ and \ dy = 2(0.5\ m^{-1})xdx.$$

Then, the integral for the work is just a definite integral of a function of x.

Solution

The infinitesimal element of work is

$$\begin{aligned} dW &= F_x dx + F_y dy = (5\ N/m)ydx + (10\ N/m)xdy \\ &= (5\ N/m)(0.5\ m^{-1})x^2 dx + (10\ N/m)2(0.5\ m^{-1})x^2 dx = (12.5\ N/m^2)x^2 dx. \end{aligned}$$

The integral of x^2 is $\frac{x^3}{3}$, so

$$W = \int_0^{2\ m} (12.5\ N/m^2)x^2 dx = (12.5\ N/m^2)\frac{x^3}{3}\Big|_0^{2\ m} = (12.5\ N/m^2)\left(\frac{8}{3}\right) = 33.3\ J.$$

Significance

This integral was not hard to do. You can follow the same steps, as in this example, to calculate line integrals representing work for more complicated forces and paths. In this example, everything was given in terms of x- and y-components, which are easiest to use in evaluating the work in this case. In other situations, magnitudes and angles might be easier.

Exercise 6.2.4

Find the work done by the same force in Example 6.2.4 over a cubic path, y = (0.25 m^{-2})x^3, between the same points A = (0, 0) and B = (2 m, 2 m).

You saw in Example 6.2.4 that to evaluate a line integral, you could reduce it to an integral over a single variable or parameter. Usually, there are several ways to do this, which may be more or less convenient, depending on the particular case. In Example 6.2.4, we reduced the line integral to an integral over x, but we could equally well have chosen to reduce everything to a function of y. We didn't do that because the functions in y involve the square root and fractional exponents, which may be less familiar, but for illustrative purposes, we do this now. Solving for x and dx, in terms of y, along the parabolic path, we get

$$x = \sqrt{\frac{y}{(0.5\ m^{-1})}} = \sqrt{(2\ m)y}\ and\ dx = \sqrt{(2\ m)} \times \frac{1}{2}\frac{dy}{\sqrt{y}} = \frac{dy}{\sqrt{(2\ m^{-1})y}}.$$

The components of the force, in terms of y, are

$$F_x = (5\ N/m)y\ and\ F_y = (10\ N/m)x = (10\ N/m)\sqrt{(2m)y},$$

so the infinitesimal work element becomes

$$\begin{aligned} dW = F_x dx + F_y dy &= \frac{(5\ N/m)ydy}{\sqrt{(2\ m^{-1})y}} + (10\ N/m)\sqrt{(2\ m)y}dy \\ &= (5\ N\cdot m^{-1/2})\left(\frac{1}{\sqrt{2}} + 2\sqrt{2}\right)\sqrt{y}dy = (17.7\ N\cdot m^{-1/2})y^{1/2}dy. \end{aligned}$$

The integral of $y^{1/2}$ is $\frac{2}{3}\ y^{3/2}$, so the work done from A to B is

$$W = \int_0^{2\ m} (17.7\ N\cdot m^{-1/2})y^{1/2}dy = (17.7\ N\cdot m^{-1/2})\frac{2}{3}(2\ m)^{2/3} = 33.3\ J.$$

As expected, this is exactly the same result as before.

One very important and widely applicable variable force is the force exerted by a perfectly elastic spring, which satisfies Hooke's law $\vec{F} = -k\Delta\vec{x}$, where k is the spring constant, and $\Delta\vec{x} = \vec{x} - \vec{x}_{eq}$ is the displacement from the spring's unstretched (equilibrium) position (Newton's Laws of Motion). Note that the unstretched position is only the same as the equilibrium position if no other forces are acting (or, if they are, they cancel one another). Forces between molecules, or in any system undergoing small displacements from a stable equilibrium, behave approximately like a spring force.

To calculate the work done by a spring force, we can choose the x-axis along the length of the spring, in the direction of increasing length, as in Figure 6.2.6, with the origin at the equilibrium position $x_{eq} = 0$. (Then positive x corresponds to a stretch and negative x to a compression.) With this choice of coordinates, the spring force has only an x-component, $F_x = -kx$, and the work done when x changes from x_A to x_B is

$$W_{spring,\ AB} = \int_A^B F_x dx = -k\int_A^B x dx = -k\frac{x^2}{2}\Big|_A^B = -\frac{1}{2}k\left(x_B^2 - x_A^2\right). \tag{6.2.5}$$

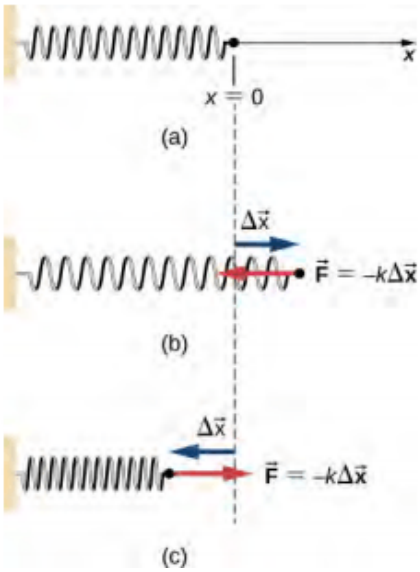


Figure 6.2.6: (a) The spring exerts no force at its equilibrium position. The spring exerts a force in the opposite direction to (b) an extension or stretch, and (c) a compression.

Notice that W_{AB} depends only on the starting and ending points, A and B, and is independent of the actual path between them, as long as it starts at A and ends at B. That is, the actual path could involve going back and forth before ending.

Another interesting thing to notice about Equation 6.2.5 is that, for this one-dimensional case, you can readily see the correspondence between the work done by a force and the area under the curve of the force versus its displacement. Recall that, in general, a one-dimensional integral is the limit of the sum of infinitesimals, f(x)dx , representing the area of strips, as shown in Figure 6.2.7. In Equation 6.2.5, since F = –kx is a straight line with slope –k, when plotted versus x, the "area" under the line is just an algebraic combination of triangular "areas," where "areas" above the x-axis are positive and those below are negative, as shown in Figure 6.2.8. The magnitude of one of these "areas" is just one-half the triangle's base, along the x-axis, times the triangle's height, along the force axis. (There are quotation marks around "area" because this base-height product has the units of work, rather than square meters.)

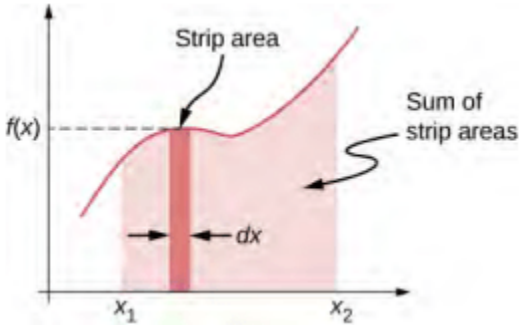


Figure 6.2.7: A curve of f(x) versus x showing the area of an infinitesimal strip, f(x)dx, and the sum of such areas, which is the integral of f(x) from x_1 to x_2.

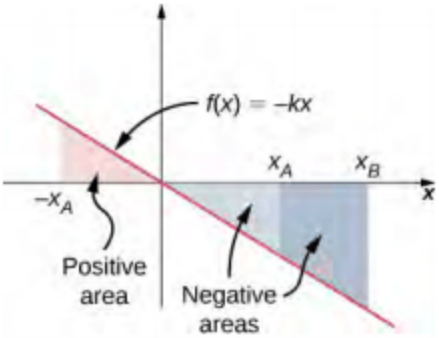


Figure 6.2.8: Curve of the spring force f(x) = –kx versus x, showing areas under the line, between x_A and x_B, for both positive and negative values of x_A. When x_A is negative, the total area under the curve for the integral in Equation 6.2.5 is the sum of positive and negative triangular areas. When x_A is positive, the total area under the curve is the difference between two negative triangles.

Example 6.2.5: Work Done by a Spring Force

A perfectly elastic spring requires 0.54 J of work to stretch 6 cm from its equilibrium position, as in Figure 6.2.6b. (a) What is its spring constant k? (b) How much work is required to stretch it an additional 6 cm?

Strategy

Work "required" means work done against the spring force, which is the negative of the work in Equation 6.2.5, that is

$$W = \frac{1}{2}k(x_B^2 - x_A^2).$$

For part (a), x_A = 0 and x_B = 6 cm; for part (b), x_B = 6 cm and x_B = 12 cm. In part (a), the work is given and you can solve for the spring constant; in part (b), you can use the value of k, from part (a), to solve for the work.

Solution

a. \[W = 0.54\; J = \frac{1}{2} k [(6\; cm)^{2} - 0],\; so\; k = 3\; N/cm \ldotp \nonumber$$
b. \[W = \frac{1}{2} (3\; N/cm) [(12\; cm)^{2} - (6\; cm)^{2}],\; so\; k = 1.62\; J \ldotp \nonumber$$

Significance

Since the work done by a spring force is independent of the path, you only needed to calculate the difference in the quantity $\frac{1}{2}kx^2$ at the end points. Notice that the work required to stretch the spring from 0 to 12 cm is four times that required to stretch it from 0 to 6 cm, because that work depends on the square of the amount of stretch from equilibrium, $\frac{1}{2}kx^2$. In this circumstance, the work to stretch the spring from 0 to 12 cm is also equal to the work for a composite path from 0 to 6 cm followed by an additional stretch from 6 cm to 12 cm. Therefore, 4W(0 cm to 6 cm) = W(0 cm to 6 cm) + W(6 cm to 12 cm) , or W(6 cm to 12 cm) = 3W(0 cm to 6 cm) , as we found above.

Exercise 6.2.5

The spring in Example 6.2.5 is compressed 6 cm from its equilibrium length. (a) Does the spring force do positive or negative work and (b) what is the magnitude?

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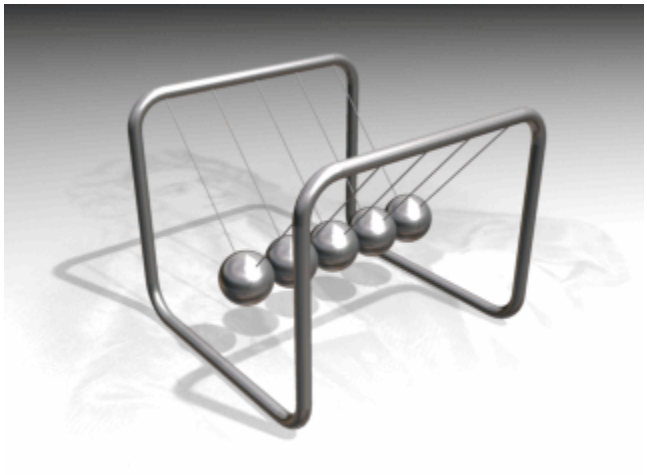
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6.3: Kinetic Energy

Learning Objectives

- Calculate the kinetic energy of a particle given its mass and its velocity or momentum
- Evaluate the kinetic energy of a body, relative to different frames of reference

It's plausible to suppose that the greater the velocity of a body, the greater effect it could have on other bodies. This does not depend on the direction of the velocity, only its magnitude. At the end of the seventeenth century, a quantity was introduced into mechanics to explain collisions between two perfectly elastic bodies, in which one body makes a head-on collision with an identical body at rest. The first body stops, and the second body moves off with the initial velocity of the first body. (If you have ever played billiards or croquet, or seen a model of Newton's Cradle, you have observed this type of collision.) The idea behind this quantity was related to the forces acting on a body and was referred to as "the energy of motion." Later on, during the eighteenth century, the name **kinetic energy** was given to energy of motion.



Newton's cradle in motion. One ball is set in motion and soon collides with the rest, conveying the energy through the rest of the balls and eventually to the last ball, which in turn is set in motion. (CC SA-BY 3.0; Dominique Toussaint).

With this history in mind, we can now state the classical definition of kinetic energy. Note that when we say "classical," we mean non-relativistic, that is, at speeds much less that the speed of light. At speeds comparable to the speed of light, the special theory of relativity requires a different expression for the kinetic energy of a particle, as discussed in Relativity. Since objects (or systems) of interest vary in complexity, we first define the kinetic energy of a particle with mass m.

Kinetic Energy

The kinetic energy of a particle is one-half the product of the particle's mass m and the square of its speed v:

$$K = \frac{1}{2}mv^2. \tag{6.3.1}$$

We then extend this definition to any system of particles by adding up the kinetic energies of all the constituent particles:

$$K = \sum \frac{1}{2}mv^2. \tag{6.3.2}$$

Note that just as we can express Newton's second law in terms of either the rate of change of momentum or mass times the rate of change of velocity, so the kinetic energy of a particle can be expressed in terms of its mass and momentum ($\vec{p}$ = m $\vec{v}$), instead of its mass and velocity. Since v = $\frac{p}{m}$, we see that

$$K = \frac{1}{2}m\left(\frac{p}{m}\right)^2 = \frac{p^2}{2m} \tag{6.3.3}$$

also expresses the kinetic energy of a single particle. Sometimes, this expression is more convenient to use than Equation 6.3.1. The units of kinetic energy are mass times the square of speed, or kg • m^2/s^2. But the units of force are mass times acceleration, kg • m/s^2, so the units of kinetic energy are also the units of force times distance, which are the units of work, or joules. You will see in the next section that work and kinetic energy have the same units, because they are different forms of the same, more general, physical property.

Example 6.3.1: Kinetic Energy of an Object

a. What is the kinetic energy of an 80-kg athlete, running at 10 m/s?
b. The Chicxulub crater in Yucatan, one of the largest existing impact craters on Earth, is thought to have been created by an asteroid, traveling at 22 km/s and releasing 4.2 x 10^{23} J of kinetic energy upon impact. What was its mass?
c. In nuclear reactors, thermal neutrons, traveling at about 2.2 km/s, play an important role. What is the kinetic energy of such a particle?

Strategy

To answer these questions, you can use the definition of kinetic energy in Equation 6.3.1. You also have to look up the mass of a neutron.

Solution

Do not forget to convert km into m to do these calculations, although, to save space, we omitted showing these conversions.

a. $$K = \frac{1}{2}(80\ kg)(10\ m/s)^2 = 4.0\ kJ.$$

b. $$m = \frac{2K}{v^2} = \frac{2(4.2 \times 10^{23}\ J)}{22\ km/s)^2} = 1.7 \times 10^{15}\ kg.$$

c. $$K = \frac{1}{2}(1.68 \times 110^{-27}\ kg)(2.2\ km/s)^2 = 4.1 \times 10^{-21}\ J.$$

Significance

In this example, we used the way mass and speed are related to kinetic energy, and we encountered a very wide range of values for the kinetic energies. Different units are commonly used for such very large and very small values. The energy of the impactor in part (b) can be compared to the explosive yield of TNT and nuclear explosions, 1 megaton = 4.18 x 10^{15} J. The Chicxulub asteroid's kinetic energy was about a hundred million megatons. At the other extreme, the energy of subatomic particle is expressed in electron-volts, 1 eV = 1.6 x 10^{-19} J. The thermal neutron in part (c) has a kinetic energy of about one fortieth of an electronvolt.

Exercise 6.3.1

a. A car and a truck are each moving with the same kinetic energy. Assume that the truck has more mass than the car. Which has the greater speed?
b. A car and a truck are each moving with the same speed. Which has the greater kinetic energy?

Because velocity is a relative quantity, you can see that the value of kinetic energy must depend on your frame of reference. You can generally choose a frame of reference that is suited to the purpose of your analysis and that simplifies your calculations. One such frame of reference is the one in which the observations of the system are made (likely an external frame). Another choice is a frame that is attached to, or moves with, the system (likely an internal frame). The equations for relative motion, discussed in Motion in Two and Three Dimensions, provide a link to calculating the kinetic energy of an object with respect to different frames of reference.

Example 6.3.2: Kinetic Energy Relative to Different Frames

A 75.0-kg person walks down the central aisle of a subway car at a speed of 1.50 m/s relative to the car, whereas the train is moving at 15.0 m/s relative to the tracks.

a. What is the person's kinetic energy relative to the car?
b. What is the person's kinetic energy relative to the tracks?
c. What is the person's kinetic energy relative to a frame moving with the person?

Strategy

Since speeds are given, we can use $\frac{1}{2}mv^2$ to calculate the person's kinetic energy. However, in part (a), the person's speed is relative to the subway car (as given); in part (b), it is relative to the tracks; and in part (c), it is zero. If we denote the car frame by C, the track frame by T, and the person by P, the relative velocities in part (b) are related by $\vec{v}_{PT} = \vec{v}_{PC} + \vec{v}_{CT}$. We can assume that the central aisle and the tracks lie along the same line, but the direction the person is walking relative to the car isn't specified, so we will give an answer for each possibility, $v_{PT} = v_{CT} \pm v_{PC}$, as shown in Figure 6.3.1.

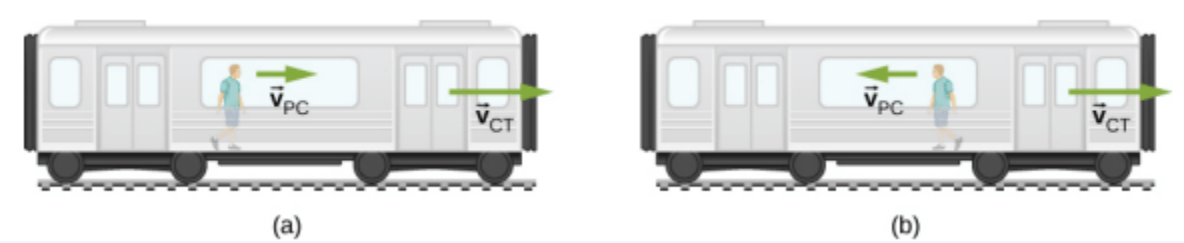


Figure 6.3.1: The possible motions of a person walking in a train are (a) toward the front of the car and (b) toward the back of the car.

Solution

a. $$K = \frac{1}{2}(75.0\ kg)(11.50\ m/s)^2 = 84.4\ J.$$

b. $$v_{PT} = (15.0 \pm 1.50)7; m/s.$$

Therefore, the two possible values for kinetic energy relative to the car are

$$K = \frac{1}{2}(75.0\ kg)(13.5\ m/s)^2 = 6.83\ kJ$$

and

$$K = \frac{1}{2}(75.0\ kg)(16.5\ m/s)^2 = 10.2\ kJ.$$

c. In a frame where $v_P = 0$, $K = 0$ as well.

Significance

You can see that the kinetic energy of an object can have very different values, depending on the frame of reference. However, the kinetic energy of an object can never be negative, since it is the product of the mass and the square of the speed, both of which are always positive or zero.

Exercise 6.3.2

You are rowing a boat parallel to the banks of a river. Your kinetic energy relative to the banks is less than your kinetic energy relative to the water. Are you rowing with or against the current?

The kinetic energy of a particle is a single quantity, but the kinetic energy of a system of particles can sometimes be divided into various types, depending on the system and its motion. For example:

- If all the particles in a system have the same velocity, the system is undergoing translational motion and has **translational**

kinetic energy.
- If an object is rotating, it could have **rotational kinetic energy**.
- If it is vibrating, it could have **vibrational kinetic energy**.

The kinetic energy of a system, relative to an internal frame of reference, may be called internal kinetic energy. The kinetic energy associated with random molecular motion may be called thermal energy. These names will be used in later chapters of the book, when appropriate. Regardless of the name, every kind of kinetic energy is the same physical quantity, representing energy associated with motion.

Example 6.3.3: Special Names for Kinetic Energy

a. A player lobs a mid-court pass with a 624-g basketball, which covers 15 m in 2 s. What is the basketball's horizontal translational kinetic energy while in flight?
b. An average molecule of air, in the basketball in part (a), has a mass of 29 u, and an average speed of 500 m/s, relative to the basketball. There are about 3 x 10^{23} molecules inside it, moving in random directions, when the ball is properly inflated. What is the average translational kinetic energy of the random motion of all the molecules inside, relative to the basketball?
c. How fast would the basketball have to travel relative to the court, as in part (a), so as to have a kinetic energy equal to the amount in part (b)?

Strategy

In part (a), first find the horizontal speed of the basketball and then use the definition of kinetic energy in terms of mass and speed, K = $\frac{1}{2}mv^2$. Then in part (b), convert unified units to kilograms and then use K = $\frac{1}{2}mv^2$ to get the average translational kinetic energy of one molecule, relative to the basketball. Then multiply by the number of molecules to get the total result. Finally, in part (c), we can substitute the amount of kinetic energy in part (b), and the mass of the basketball in part (a), into the definition K = $\frac{1}{2}mv^2$, and solve for v.

Solution

a. The horizontal speed is $\frac{(15\ m)}{(2\ s)}$, so the horizontal kinetic energy of the basketball is

$$\frac{1}{2}(0.624\ kg)(7.5\ m/s)^2 = 17.6\ J.$$

b. The average translational kinetic energy of a molecule is

$$\frac{1}{2}(29\ u)(1.66 \times 10^{-27}\ kg/u)(500\ m/s)^2 = 6.02 \times 10^{-21}\ J,$$

and the total kinetic energy of all the molecules is

$$(3 \times 10^{23})(6.02 \times 10^{-21}\ J) = 1.80\ kJ.$$

c.
$$v = \sqrt{\frac{2(1.8\ kJ)}{(0.624\ kg)}} = 76.0\ m/s.$$

Significance

In part (a), this kind of kinetic energy can be called the horizontal kinetic energy of an object (the basketball), relative to its surroundings (the court). If the basketball were spinning, all parts of it would have not just the average speed, but it would also have rotational kinetic energy. Part (b) reminds us that this kind of kinetic energy can be called internal or thermal kinetic energy. Notice that this energy is about a hundred times the energy in part (a). How to make use of thermal energy will be the subject of the chapters on thermodynamics. In part (c), since the energy in part (b) is about 100 times that in part (a), the speed should be about 10 times as big, which it is (76 compared to 7.5 m/s).

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6.4: Work-Energy Theorem

Learning Objectives

- Apply the work-energy theorem to find information about the motion of a particle, given the forces acting on it
- Use the work-energy theorem to find information about the forces acting on a particle, given information about its motion

We have discussed how to find the work done on a particle by the forces that act on it, but how is that work manifested in the motion of the particle? According to Newton's second law of motion, the sum of all the forces acting on a particle, or the net force, determines the rate of change in the momentum of the particle, or its motion. Therefore, we should consider the work done by all the forces acting on a particle, or the **net work**, to see what effect it has on the particle's motion.

Let's start by looking at the net work done on a particle as it moves over an infinitesimal displacement, which is the dot product of the net force and the displacement:

$$dW_{net} = \vec{F}_{net} \cdot d\vec{r}.$$

Newton's second law tells us that

$$\vec{F}_{net} = m\left(\frac{d\vec{v}}{dt}\right)$$

so

$$dW_{net} = m\left(\frac{d\vec{v}}{dt}\right) \cdot d\vec{r}.$$

For the mathematical functions describing the motion of a physical particle, we can rearrange the differentials dt, etc., as algebraic quantities in this expression, that is,

$$\begin{aligned} dW_{net} &= m\left(\frac{d\vec{v}}{dt}\right) \cdot d\vec{r} \\ &= m\, d\vec{v} \cdot \left(\frac{d\vec{r}}{dt}\right) \\ &= m\vec{v} \cdot d\vec{v}, \end{aligned}$$

where we substituted the velocity for the time derivative of the displacement and used the commutative property of the dot product. Since derivatives and integrals of scalars are probably more familiar to you at this point, we express the dot product in terms of Cartesian coordinates before we integrate between any two points A and B on the particle's trajectory. This gives us the net work done on the particle:

$$W_{net,\ AB} = \int_A^B (mv_x dv_x + mv_y dv_y + mv_z dv_z \tag{6.4.1}$$

$$= \frac{1}{2}m\left|v_x^2 + v_y^2 + v_z^2\right|_A^B = \left|\frac{1}{2}mv^2\right|_A^B = K_B - K_A. \tag{6.4.2}$$

In the middle step, we used the fact that the square of the velocity is the sum of the squares of its Cartesian components, and in the last step, we used the definition of the particle's kinetic energy. This important result is called the work-energy theorem.

Work-Energy Theorem

The net work done on a particle equals the change in the particle's kinetic energy:

$$W_{net} = K_B - K_A. \tag{6.4.3}$$



Figure 6.4.1: Horse pulls are common events at state fairs. The work done by the horses pulling on the load results in a change in kinetic energy of the load, ultimately going faster. (credit: "Jassen"/ Flickr)

According to this theorem, when an object slows down, its final kinetic energy is less than its initial kinetic energy, the change in its kinetic energy is negative, and so is the net work done on it. If an object speeds up, the net work done on it is positive. When calculating the net work, you must include all the forces that act on an object. If you leave out any forces that act on an object, or if you include any forces that do not act on it, you will get a wrong result.

The importance of the work-energy theorem, and the further generalizations to which it leads, is that it makes some types of calculations much simpler to accomplish than they would be by trying to solve Newton's second law. For example, in the section on Newton's Laws of Motion, we found the speed of an object sliding down a frictionless plane by solving Newton's second law for the acceleration and using kinematic equations for constant acceleration, obtaining

$$v_f^2 = v_i^2 + 2g(s_f - s_i)\sin\theta, \tag{6.4.4}$$

where s is the displacement down the plane.

We can also get this result from the work-energy theorem (Equation 6.4.3). Since only two forces are acting on the object—gravity and the normal force—and the normal force does not do any work, the net work is just the work done by gravity. This only depends on the object's weight and the difference in height, so

$$W_{net} = W_{grav} = -mg(y_f - y_i), \tag{6.4.5}$$

where y is positive up. The work-energy theorem says that this equals the change in kinetic energy:

$$-mg(y_f - y_i) = \frac{1}{2}(v_f^2 - v_i^2). \tag{6.4.6}$$

Using a right triangle, we can see that

$$(y_f - y_i) = (s_f - s - i)\sin\theta,$$

so the result for the final speed is the same.

What is gained by using the work-energy theorem? The answer is that for a frictionless plane surface, not much. However, Newton's second law is easy to solve only for this particular case, whereas the work-energy theorem gives the final speed for any shaped frictionless surface. For an arbitrary curved surface, the normal force is not constant, and Newton's second law may be difficult or impossible to solve analytically. Constant or not, for motion along a surface, the normal force never does any work, because it's perpendicular to the displacement. A calculation using the work-energy theorem avoids this difficulty and applies to

more general situations.

Problem-Solving Strategy: Work-Energy Theorem

1. Draw a free-body diagram for each force on the object.
2. Determine whether or not each force does work over the displacement in the diagram. Be sure to keep any positive or negative signs in the work done.
3. Add up the total amount of work done by each force.
4. Set this total work equal to the change in kinetic energy and solve for any unknown parameter.
5. Check your answers. If the object is traveling at a constant speed or zero acceleration, the total work done should be zero and match the change in kinetic energy. If the total work is positive, the object must have sped up or increased kinetic energy. If the total work is negative, the object must have slowed down or decreased kinetic energy

Example 6.4.1: Loop-the-Loop

The frictionless track for a toy car includes a loop-the-loop of radius R. How high, measured from the bottom of the loop, must the car be placed to start from rest on the approaching section of track and go all the way around the loop?

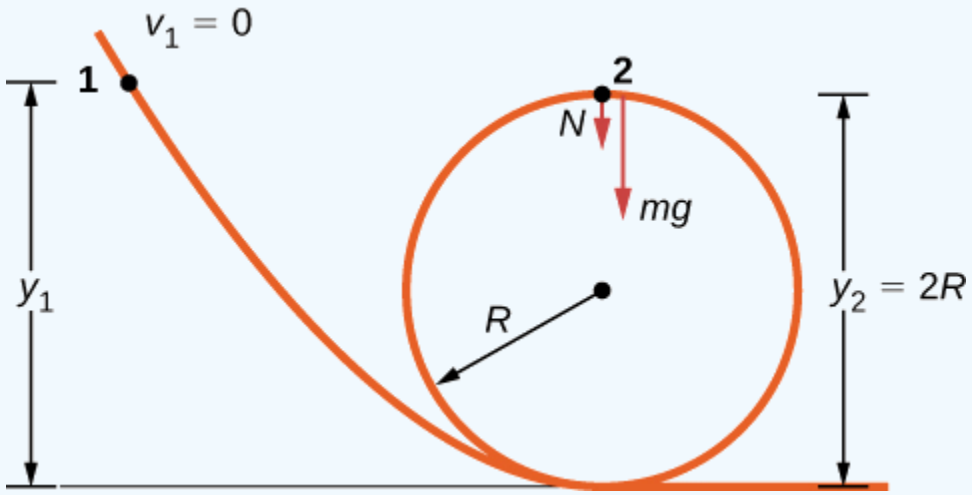


Figure 6.4.2: A frictionless track for a toy car has a loop-the-loop in it. How high must the car start so that it can go around the loop without falling off?

Strategy

The free-body diagram at the final position of the object is drawn in Figure 6.4.2. The gravitational work is the only work done over the displacement that is not zero. Since the weight points in the same direction as the net vertical displacement, the total work done by the gravitational force is positive. From the work-energy theorem, the starting height determines the speed of the car at the top of the loop,

$$mg(y_2 - y_1) = \frac{1}{2}mv_2^2,$$

where the notation is shown in the accompanying figure. At the top of the loop, the normal force and gravity are both down and the acceleration is centripetal, so

$$a_{top} = \frac{F}{m} = \frac{N + mg}{m} = \frac{v_2^2}{R}.$$

The condition for maintaining contact with the track is that there must be some normal force, however slight; that is, $N > 0$. Substituting for v_2^2 and N, we can find the condition for y_1.

Solution

Implement the steps in the strategy to arrive at the desired result:

$$N = -mg + \frac{mv_2^2}{R} = \frac{-mgR + 2mg(y_1 - 2R)}{R} > 0 \text{ or } y_1 > \frac{5R}{2}.$$

Significance

On the surface of the loop, the normal component of gravity and the normal contact force must provide the centripetal acceleration of the car going around the loop. The tangential component of gravity slows down or speeds up the car. A child would find out how high to start the car by trial and error, but now that you know the work-energy theorem, you can predict the minimum height (as well as other more useful results) from physical principles. By using the work-energy theorem, you did not have to solve a differential equation to determine the height.

Exercise 6.4.1

Suppose the radius of the loop-the-loop in Example 6.4.1 is 15 cm and the toy car starts from rest at a height of 45 cm above the bottom. What is its speed at the top of the loop?

In situations where the motion of an object is known, but the values of one or more of the forces acting on it are not known, you may be able to use the work-energy theorem to get some information about the forces. Work depends on the force and the distance over which it acts, so the information is provided via their product.

Example 6.4.2: Determining a Stopping Force

A bullet has a mass of 40 grains (2.60 g) and a muzzle velocity of 1100 ft./s (335 m/s). It can penetrate eight 1-inch pine boards, each with thickness 0.75 inches. What is the average stopping force exerted by the wood, as shown in Figure 6.4.3?

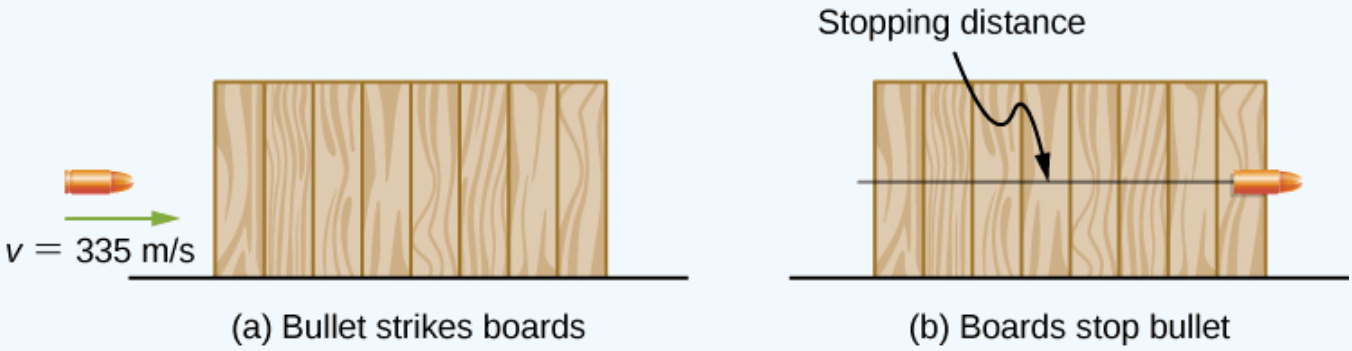


Figure 6.4.3: The boards exert a force to stop the bullet. As a result, the boards do work and the bullet loses kinetic energy

Strategy

We can assume that under the general conditions stated, the bullet loses all its kinetic energy penetrating the boards, so the work-energy theorem says its initial kinetic energy is equal to the average stopping force times the distance penetrated. The change in the bullet's kinetic energy and the net work done stopping it are both negative, so when you write out the work-energy theorem, with the net work equal to the average force times the stopping distance, that's what you get. The total thickness of eight 1-inch pine boards that the bullet penetrates is 8 x $\frac{3}{4}$ in. = 6 in. = 15.2 cm.

Solution

Applying the work-energy theorem, we get

$$W_{net} = -F_{ave}\Delta s_{stop} = -K_{initial},$$

so

$$F_{ave} = \frac{\frac{1}{2}mv^2}{\Delta s_{stop}} = \frac{\frac{1}{2}(2.66 \times 10^{-3}\ kg)(335\ m/s)^2}{0.152\ m} = 960\ N.$$

Significance

We could have used Newton's second law and kinematics in this example, but the work-energy theorem also supplies an answer to less simple situations. The penetration of a bullet, fired vertically upward into a block of wood, is discussed in one section of Asif Shakur's recent article ["Bullet-Block Science Video Puzzle." **The Physics Teacher** (January 2015) 53(1): 15-16]. If the bullet is fired dead center into the block, it loses all its kinetic energy and penetrates slightly farther than if fired off-center. The reason is that if the bullet hits off-center, it has a little kinetic energy after it stops penetrating, because the block rotates. The work-energy theorem implies that a smaller change in kinetic energy results in a smaller penetration. You will understand more of the physics in this interesting article after you finish reading Angular Momentum.

Learn more about work and energy in this PhET simulation (https://phet.colorado.edu/en/simulation/the-ramp) called "the ramp." Try changing the force pushing the box and the frictional force along the incline. The work and energy plots can be examined to note the total work done and change in kinetic energy of the box.

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6.5: Power

Learning Objectives

- Relate the work done during a time interval to the power delivered
- Find the power expended by a force acting on a moving body

The concept of work involves force and displacement; the work-energy theorem relates the net work done on a body to the difference in its kinetic energy, calculated between two points on its trajectory. None of these quantities or relations involves time explicitly, yet we know that the time available to accomplish a particular amount of work is frequently just as important to us as the amount itself. In the chapter-opening figure, several sprinters may have achieved the same velocity at the finish, and therefore did the same amount of work, but the winner of the race did it in the least amount of time.

We express the relation between work done and the time interval involved in doing it, by introducing the concept of power. Since work can vary as a function of time, we first define **average power** as the work done during a time interval, divided by the interval,

$$P_{ave} = \frac{\Delta W}{\Delta t}. \tag{6.5.1}$$

Then, we can define the **instantaneous power** (frequently referred to as just plain **power**).

Definition: Power

Power is defined as the rate of doing work, or the limit of the average power for time intervals approaching zero,

$$P = \frac{dW}{dt}. \tag{6.5.2}$$

If the power is constant over a time interval, the average power for that interval equals the instantaneous power, and the work done by the agent supplying the power is

$$W = P\Delta t. \tag{6.5.3}$$

If the power during an interval varies with time (i.e., $P(t)$), then the work done is the time integral of the power,

$$W = \int P(t)dt. \tag{6.5.4}$$

The work-energy theorem relates how work can be transformed into kinetic energy. Since there are other forms of energy as well, as we discuss in the next chapter, we can also define power as the rate of transfer of energy. Work and energy are measured in units of joules, so power is measured in units of joules per second, which has been given the SI name watts, abbreviation W: 1 J/s = 1 W. Another common unit for expressing the power capability of everyday devices is horsepower: 1 hp = 746 W.

Example 6.5.1: Pull-Up Power

An 80-kg army trainee does pull-ups on a horizontal bar (Figure 6.5.1). It takes the trainee 0.8 seconds to raise the body from a lower position to where the chin is above the bar. How much power do the trainee's muscles supply moving his body from the lower position to where the chin is above the bar? (**Hint**: Make reasonable estimates for any quantities needed.)

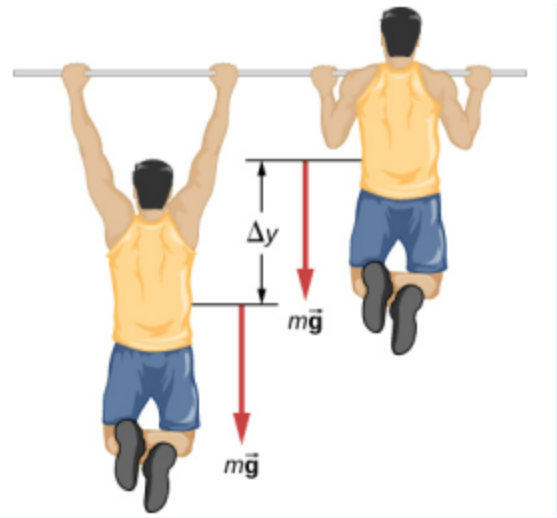


Figure 6.5.1: What is the power expended in doing ten pull-ups in ten seconds?

Strategy

The work done against gravity, going up or down a distance Δy, is mgΔy. Let's assume that Δy = 2 ft ≈ 60 cm. Also, assume that the arms comprise 10% of the body mass and are not included in the moving mass. With these assumptions, we can calculate the work done.

Solution

The result we get, applying our assumptions, is

$$P = \frac{\text{mg}(\Delta y)}{t} = \frac{0.9(80\ \text{kg})\,(9.8\ \text{m/s}^2)\,(0.60\ \text{m})}{0.8\ \text{s}} = 529\ \text{W} \tag{6.5.5}$$

Significance

This is typical for power expenditure in strenuous exercise; in everyday units, it's somewhat more than one horsepower (1 hp = 746 W).

Exercise 6.5.1

Estimate the power expended by a weightlifter raising a 150-kg barbell 2 m in 3 s.

Answer available at the end of this section.

The power involved in moving a body can also be expressed in terms of the forces acting on it. If a force $\vec{F}$ acts on a body that is displaced $d\vec{r}$ in a time dt, the power expended by the force is

$$P = \frac{dW}{dt} = \frac{\vec{F} \cdot d\vec{r}}{dt} = \vec{F} \cdot \left(\frac{d\vec{r}}{dt}\right) = \vec{F} \cdot \vec{v}, \tag{6.5.6}$$

where $\vec{v}$ is the velocity of the body. The fact that the limits implied by the derivatives exist, for the motion of a real body, justifies the rearrangement of the infinitesimals.

Example 6.5.2: Automotive Power Driving Uphill

How much power must an automobile engine expend to move a 1200-kg car up a 15% grade at 90 km/h (Figure 6.5.2)? Assume that 25% of this power is dissipated overcoming air resistance and friction.

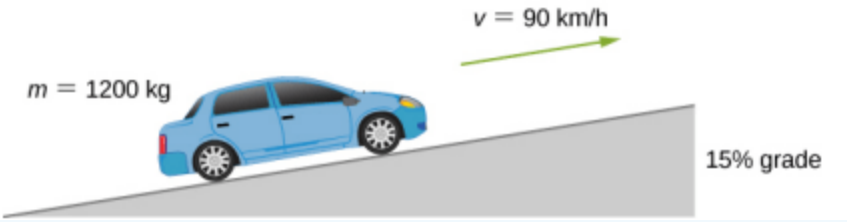


Figure 6.5.2: We want to calculate the power needed to move a car up a hill at constant speed.

Strategy

At constant velocity, there is no change in kinetic energy, so the net work done to move the car is zero. Therefore the power supplied by the engine to move the car equals the power expended against gravity and air resistance. By assumption, 75% of the power is supplied against gravity, which equals $m\vec{g} \cdot \vec{v}$ = mgv sin θ, where θ is the angle of the incline. A 15% grade means tan θ = 0.15. This reasoning allows us to solve for the power required.

Solution

Carrying out the suggested steps, we find

$$0.75P = mgv\sin(\tan^{-1} 0.15), \tag{6.5.7}$$

or

$$P = \frac{(1200 \times 9.8\ N)(\frac{90\ m}{3.6\ s})\sin(8.53^o)}{0.75} = 58\ kW, \tag{6.5.8}$$

or about 78 hp. (You should supply the steps used to convert units.)

Significance

This is a reasonable amount of power for the engine of a small to mid-size car to supply (1 hp = 0.746 kW). Note that this is only the power expended to move the car. Much of the engine's power goes elsewhere, for example, into waste heat. That's why cars need radiators. Any remaining power could be used for acceleration, or to operate the car's accessories.

Answers

Exercise 6.5.1

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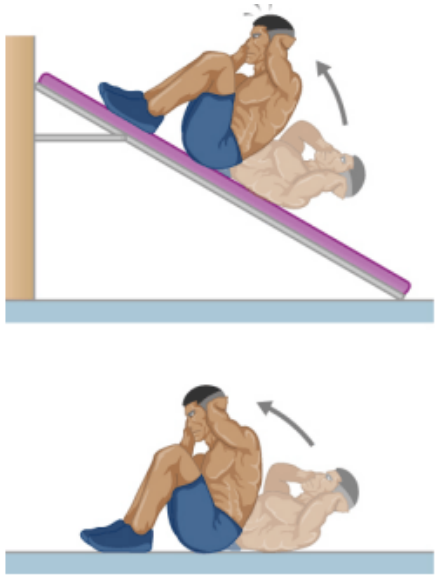
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6.6: Work and Kinetic Energy (Exercises)

Conceptual Questions

7.1 Work

1. Give an example of something we think of as work in everyday circumstances that is not work in the scientific sense. Is energy transferred or changed in form in your example? If so, explain how this is accomplished without doing work.
2. Give an example of a situation in which there is a force and a displacement, but the force does no work. Explain why it does no work.
3. Describe a situation in which a force is exerted for a long time but does no work. Explain.
4. A body moves in a circle at constant speed. Does the centripetal force that accelerates the body do any work? Explain.
5. Suppose you throw a ball upward and catch it when it returns at the same height. How much work does the gravitational force do on the ball over its entire trip?
6. Why is it more difficult to do sit-ups while on a slant board than on a horizontal surface? (See below.)



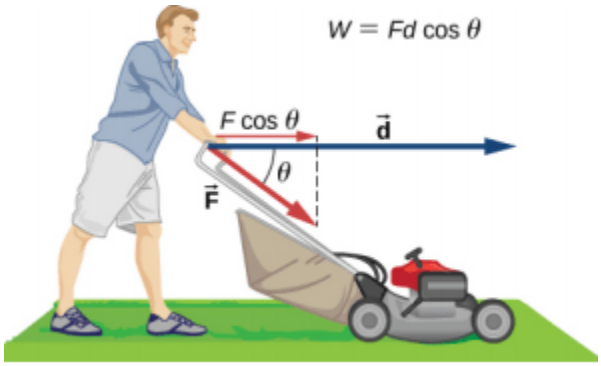
7. As a young man, Tarzan climbed up a vine to reach his tree house. As he got older, he decided to build and use a staircase instead. Since the work of the gravitational force mg is path independent, what did the King of the Apes gain in using stairs?

7.2 Kinetic Energy

8. A particle of m has a velocity of $v_x\,\hat{i} + v_y\,\hat{j} + v_z\,\hat{k}$. Is its kinetic energy given by $m(v_x^2\,\hat{i} + v_y^2\,\hat{j} + v_z^2\,\hat{k})/2$? If not, what is the correct expression?
9. One particle has mass m and a second particle has mass 2m. The second particle is moving with speed v and the first with speed 2v. How do their kinetic energies compare?
10. A person drops a pebble of mass m_1 from a height h, and it hits the floor with kinetic energy K. The person drops another pebble of mass m_2 from a height of 2h, and it hits the floor with the same kinetic energy K. How do the masses of the pebbles compare?

7.3 Work-Energy Theorem

11. Under what conditions would it lose energy?



12. Work done on a system puts energy into it. Work done by a system removes energy from it. Give an example for each statement.
13. Two marbles of masses m and 2m are dropped from a height h. Compare their kinetic energies when they reach the ground.
14. Compare the work required to accelerate a car of mass 2000 kg from 30.0 to 40.0 km/h with that required for an acceleration from 50.0 to 60.0 km/h.
15. Suppose you are jogging at constant velocity. Are you doing any work on the environment and vice versa?
16. Two forces act to double the speed of a particle, initially moving with kinetic energy of 1 J. One of the forces does 4 J of work. How much work does the other force do?

7.4 Power

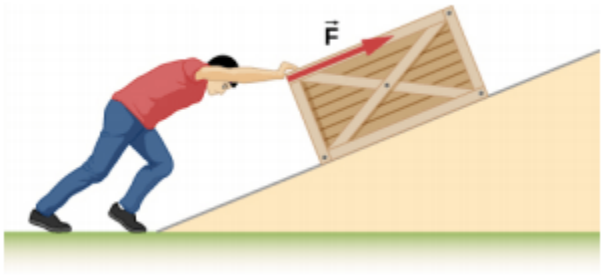
17. Most electrical appliances are rated in watts. Does this rating depend on how long the appliance is on? (When off, it is a zero-watt device.) Explain in terms of the definition of power.
18. Explain, in terms of the definition of power, why energy consumption is sometimes listed in kilowatt-hours rather than joules. What is the relationship between these two energy units?
19. A spark of static electricity, such as that you might receive from a doorknob on a cold dry day, may carry a few hundred watts of power. Explain why you are not injured by such a spark.
20. Does the work done in lifting an object depend on how fast it is lifted? Does the power expended depend on how fast it is lifted?
21. Can the power expended by a force be negative?
22. How can a 50-W light bulb use more energy than a 1000-W oven?

Problems

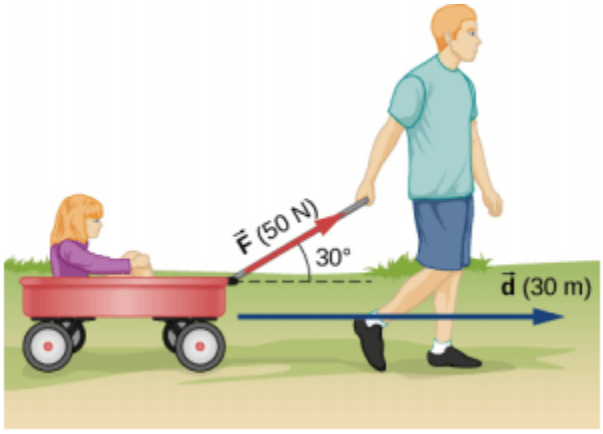
7.1 Work

23. How much work does a supermarket checkout attendant do on a can of soup he pushes 0.600 m horizontally with a force of 5.00 N?
24. A 75.0-kg person climbs stairs, gaining 2.50 m in height. Find the work done to accomplish this task.
25. (a) Calculate the work done on a 1500-kg elevator car by its cable to lift it 40.0 m at constant speed, assuming friction averages 100 N. (b) What is the work done on the lift by the gravitational force in this process? (c) What is the total work done on the lift?
26. Suppose a car travels 108 km at a speed of 30.0 m/s, and uses 2.0 gal of gasoline. Only 30% of the gasoline goes into useful work by the force that keeps the car moving at constant speed despite friction. (The energy content of gasoline is about 140 MJ/gal.) (a) What is the magnitude of the force exerted to keep the car moving at constant speed? (b) If the required force is directly proportional to speed, how many gallons will be used to drive 108 km at a speed of 28.0 m/s?
27. Calculate the work done by an 85.0-kg man who pushes a crate 4.00 m up along a ramp that makes an angle of 20.0°

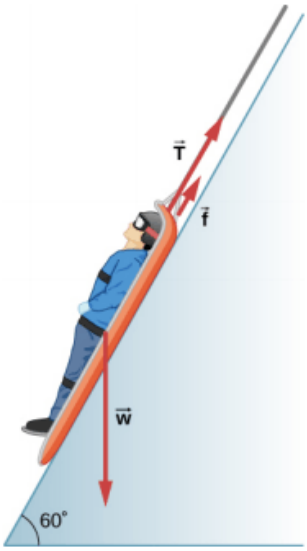
with the horizontal (see below). He exerts a force of 500 N on the crate parallel to the ramp and moves at a constant speed. Be certain to include the work he does on the crate and on his body to get up the ramp.



28. How much work is done by the boy pulling his sister 30.0 m in a wagon as shown below? Assume no friction acts on the wagon.



29. A shopper pushes a grocery cart 20.0 m at constant speed on level ground, against a 35.0 N frictional force. He pushes in a direction 25.0° below the horizontal. (a) What is the work done on the cart by friction? (b) What is the work done on the cart by the gravitational force? (c) What is the work done on the cart by the shopper? (d) Find the force the shopper exerts, using energy considerations. (e) What is the total work done on the cart?
30. Suppose the ski patrol lowers a rescue sled and victim, having a total mass of 90.0 kg, down a 60.0° slope at constant speed, as shown below. The coefficient of friction between the sled and the snow is 0.100. (a) How much work is done by friction as the sled moves 30.0 m along the hill? (b) How much work is done by the rope on the sled in this distance? (c) What is the work done by the gravitational force on the sled? (d) What is the total work done?



31. A constant 20-N force pushes a small ball in the direction of the force over a distance of 5.0 m. What is the work done by the force?
32. A toy cart is pulled a distance of 6.0 m in a straight line across the floor. The force pulling the cart has a magnitude of 20 N and is directed at 37° above the horizontal. What is the work done by this force?
33. A 5.0-kg box rests on a horizontal surface. The coefficient of kinetic friction between the box and surface is μ_K = 0.50. A horizontal force pulls the box at constant velocity for 10 cm. Find the work done by (a) the applied horizontal force, (b) the frictional force, and (c) the net force.
34. A sled plus passenger with total mass 50 kg is pulled 20 m across the snow (μk = 0.20) at constant velocity by a force directed 25° above the horizontal. Calculate (a) the work of the applied force, (b) the work of friction, and (c) the total work.
35. Suppose that the sled plus passenger of the preceding problem is pushed 20 m across the snow at constant velocity by a force directed 30° below the horizontal. Calculate (a) the work of the applied force, (b) the work of friction, and (c) the total work.
36. How much work does the force F(x) = (–2.0/x) N do on a particle as it moves from x = 2.0 m to x = 5.0 m?
37. How much work is done against the gravitational force on a 5.0-kg briefcase when it is carried from the ground floor to the roof of the Empire State Building, a vertical climb of 380 m?
38. It takes 500 J of work to compress a spring 10 cm. What is the force constant of the spring?
39. A bungee cord is essentially a very long rubber band that can stretch up to four times its unstretched length. However, its spring constant varies over its stretch [see Menz, P.G. “The Physics of Bungee Jumping.” **The Physics Teacher** (November 1993) 31: 483-487]. Take the length of the cord to be along the x-direction and define the stretch x as the length of the cord l minus its un-stretched length l_0 ; that is, $x = l - l_0$ (see below). Suppose a particular bungee cord has a spring constant, for $0 \le x \le 4.88$ m, of k_1 = 204 N/m and for 4.88 m ≤ x, of k_2 = 111 N/m. (Recall that the spring constant is the slope of the force F(x) versus its stretch x.) (a) What is the tension in the cord when the stretch is 16.7 m (the maximum desired for a given jump)? (b) How much work must be done against the elastic force of the bungee cord to stretch it 16.7 m?

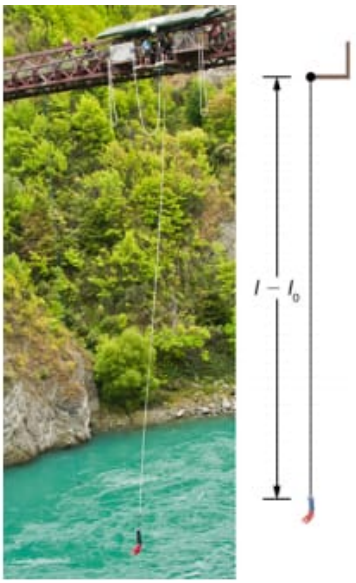


Figure 7.16 - (credit: Graeme Churchard)

40. A bungee cord exerts a nonlinear elastic force of magnitude $F(x) = k_1x + k_2x^3$, where x is the distance the cord is stretched, $k_1 = 204$ N/m and $k_2 = -0.233$ N/m^3. How much work must be done on the cord to stretch it 16.7 m?
41. Engineers desire to model the magnitude of the elastic force of a bungee cord using the equation \[F(x) = a \Bigg[\frac{x + 9\; m}{9\; m} – \left(\dfrac{9\; m}{x + 9\; m}\right)^{2} \Bigg],$$where x is the stretch of the cord along its length and a is a constant. If it takes 22.0 kJ of work to stretch the cord by 16.7 m, determine the value of the constant a.
42. A particle moving in the xy-plane is subject to a force

$$\vec{F}(x,y) = (50\ N \cdot m^2)\frac{(x\hat{i} + y\hat{j})}{(x^2 + y^2)^{3/2}}, \tag{6.6.1}$$

where x and y are in meters. Calculate the work done on the particle by this force, as it moves in a straight line from the point (3 m, 4 m) to the point (8 m, 6 m).
43. A particle moves along a curved path y(x) = (10 m){1 + cos[(0.1 m^{-1})x]}, from x = 0 to x = 10π m, subject to a tangential force of variable magnitude F(x) = (10 N)sin[(0.1 m^{-1})x]. How much work does the force do? (**Hint**: Consult a table of integrals or use a numerical integration program.)

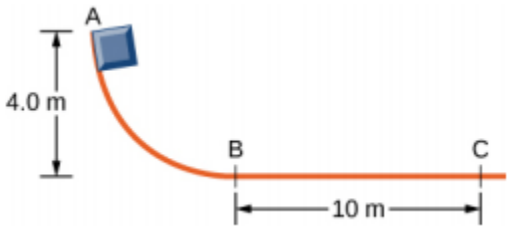
7.2 Kinetic Energy

44. Compare the kinetic energy of a 20,000-kg truck moving at 110 km/h with that of an 80.0-kg astronaut in orbit moving at 27,500 km/h.
45. (a) How fast must a 3000-kg elephant move to have the same kinetic energy as a 65.0-kg sprinter running at 10.0 m/s? (b) Discuss how the larger energies needed for the movement of larger animals would relate to metabolic rates.
46. Estimate the kinetic energy of a 90,000-ton aircraft carrier moving at a speed of at 30 knots. You will need to look up the definition of a nautical mile to use in converting the unit for speed, where 1 knot equals 1 nautical mile per hour.
47. Calculate the kinetic energies of (a) a 2000.0-kg automobile moving at 100.0 km/h; (b) an 80.-kg runner sprinting at 10. m/s; and (c) a 9.1 x 10^{-31}-kg electron moving at 2.0 x 10^7 m/s.
48. A 5.0-kg body has three times the kinetic energy of an 8.0-kg body. Calculate the ratio of the speeds of these bodies.
49. An 8.0-g bullet has a speed of 800 m/s. (a) What is its kinetic energy? (b) What is its kinetic energy if the speed is halved?

7.3 Work-Energy Theorem

50. (a) Calculate the force needed to bring a 950-kg car to rest from a speed of 90.0 km/h in a distance of 120 m (a fairly typical distance for a non-panic stop). (b) Suppose instead the car hits a concrete abutment at full speed and is brought to a stop in 2.00 m. Calculate the force exerted on the car and compare it with the force found in part (a).

51. A car's bumper is designed to withstand a 4.0-km/h (1.1-m/s) collision with an immovable object without damage to the body of the car. The bumper cushions the shock by absorbing the force over a distance. Calculate the magnitude of the average force on a bumper that collapses 0.200 m while bringing a 900-kg car to rest from an initial speed of 1.1 m/s.
52. Boxing gloves are padded to lessen the force of a blow. (a) Calculate the force exerted by a boxing glove on an opponent's face, if the glove and face compress 7.50 cm during a blow in which the 7.00-kg arm and glove are brought to rest from an initial speed of 10.0 m/s. (b) Calculate the force exerted by an identical blow in the days when no gloves were used, and the knuckles and face would compress only 2.00 cm. Assume the change in mass by removing the glove is negligible. (c) Discuss the magnitude of the force with glove on. Does it seem high enough to cause damage even though it is lower than the force with no glove?
53. Using energy considerations, calculate the average force a 60.0-kg sprinter exerts backward on the track to accelerate from 2.00 to 8.00 m/s in a distance of 25.0 m, if he encounters a headwind that exerts an average force of 30.0 N against him.
54. A 5.0-kg box has an acceleration of 2.0 m/s^2 when it is pulled by a horizontal force across a surface with μ_K = 0.50. Find the work done over a distance of 10 cm by (a) the horizontal force, (b) the frictional force, and (c) the net force. (d) What is the change in kinetic energy of the box?
55. A constant 10-N horizontal force is applied to a 20-kg cart at rest on a level floor. If friction is negligible, what is the speed of the cart when it has been pushed 8.0 m?
56. In the preceding problem, the 10-N force is applied at an angle of 45° below the horizontal. What is the speed of the cart when it has been pushed 8.0 m?
57. Compare the work required to stop a 100-kg crate sliding at 1.0 m/s and an 8.0-g bullet traveling at 500 m/s.
58. A wagon with its passenger sits at the top of a hill. The wagon is given a slight push and rolls 100 m down a 10° incline to the bottom of the hill. What is the wagon's speed when it reaches the end of the incline. Assume that the retarding force of friction is negligible.
59. An 8.0-g bullet with a speed of 800 m/s is shot into a wooden block and penetrates 20 cm before stopping. What is the average force of the wood on the bullet? Assume the block does not move.
60. A 2.0-kg block starts with a speed of 10 m/s at the bottom of a plane inclined at 37° to the horizontal. The coefficient of sliding friction between the block and plane is mu_k = 0.30. (a) Use the work-energy principle to determine how far the block slides along the plane before momentarily coming to rest. (b) After stopping, the block slides back down the plane. What is its speed when it reaches the bottom? (**Hint**: For the round trip, only the force of friction does work on the block.)
61. When a 3.0-kg block is pushed against a massless spring of force constant 4.5 x 10^3 N/m, the spring is compressed 8.0 cm. The block is released, and it slides 2.0 m (from the point at which it is released) across a horizontal surface before friction stops it. What is the coefficient of kinetic friction between the block and the surface?
62. A small block of mass 200 g starts at rest at A, slides to B where its speed is v_B = 8.0 m/s, then slides along the horizontal surface a distance 10 m before coming to rest at C. (See below.) (a) What is the work of friction along the curved surface? (b) What is the coefficient of kinetic friction along the horizontal surface?



63. A small object is placed at the top of an incline that is essentially frictionless. The object slides down the incline onto a rough horizontal surface, where it stops in 5.0 s after traveling 60 m. (a) What is the speed of the object at the bottom of the incline and its acceleration along the horizontal surface? (b) What is the height of the incline?
64. When released, a 100-g block slides down the path shown below, reaching the bottom with a speed of 4.0 m/s. How much work does the force of friction do?



65. A 0.22LR-caliber bullet like that mentioned in Example 7.10 is fired into a door made of a single thickness of 1-inch pine boards. How fast would the bullet be traveling after it penetrated through the door?
66. A sled starts from rest at the top of a snow-covered incline that makes a 22° angle with the horizontal. After sliding 75 m down the slope, its speed is 14 m/s. Use the work-energy theorem to calculate the coefficient of kinetic friction between the runners of the sled and the snowy surface.

7.4 Power

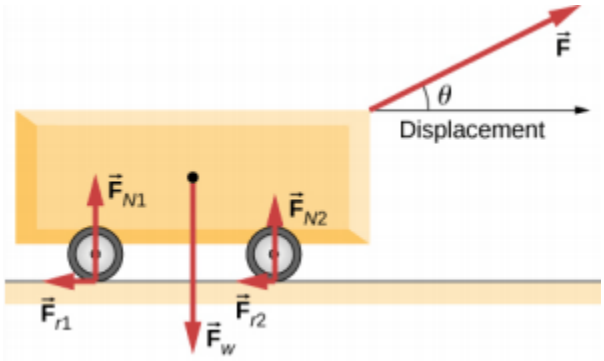
67. A person in good physical condition can put out 100 W of useful power for several hours at a stretch, perhaps by pedaling a mechanism that drives an electric generator. Neglecting any problems of generator efficiency and practical considerations such as resting time: (a) How many people would it take to run a 4.00-kW electric clothes dryer? (b) How many people would it take to replace a large electric power plant that generates 800 MW?
68. What is the cost of operating a 3.00-W electric clock for a year if the cost of electricity is $0.0900 per kW • h?
69. A large household air conditioner may consume 15.0 kW of power. What is the cost of operating this air conditioner 3.00 h per day for 30.0 d if the cost of electricity is $0.110 per kW • h?
70. (a) What is the average power consumption in watts of an appliance that uses 5.00 kW • h of energy per day? (b) How many joules of energy does this appliance consume in a year?
71. (a) What is the average useful power output of a person who does 6.00×10^6 J of useful work in 8.00 h? (b) Working at this rate, how long will it take this person to lift 2000 kg of bricks 1.50 m to a platform? (Work done to lift his body can be omitted because it is not considered useful output here.)
72. A 500-kg dragster accelerates from rest to a final speed of 110 m/s in 400 m (about a quarter of a mile) and encounters an average frictional force of 1200 N. What is its average power output in watts and horsepower if this takes 7.30 s?
73. (a) How long will it take an 850-kg car with a useful power output of 40.0 hp (1 hp equals 746 W) to reach a speed of 15.0 m/s, neglecting friction? (b) How long will this acceleration take if the car also climbs a 3.00-m high hill in the process?
74. (a) Find the useful power output of an elevator motor that lifts a 2500-kg load a height of 35.0 m in 12.0 s, if it also increases the speed from rest to 4.00 m/s. Note that the total mass of the counterbalanced system is 10,000 kg—so that only 2500 kg is raised in height, but the full 10,000 kg is accelerated. (b) What does it cost, if electricity is $0.0900 per kW • h ?
75. (a) How long would it take a 1.50×10^5-kg airplane with engines that produce 100 MW of power to reach a speed of 250 m/s and an altitude of 12.0 km if air resistance were negligible? (b) If it actually takes 900 s, what is the power? (c) Given this power, what is the average force of air resistance if the airplane takes 1200 s? (**Hint**: You must find the distance the plane travels in 1200 s assuming constant acceleration.)
76. Calculate the power output needed for a 950-kg car to climb a 2.00° slope at a constant 30.0 m/s while encountering wind resistance and friction totaling 600 N.
77. A man of mass 80 kg runs up a flight of stairs 20 m high in 10 s. (a) how much power is used to lift the man? (b) If the man's body is 25% efficient, how much power does he expend?
78. The man of the preceding problem consumes approximately 1.05×10^7 J (2500 food calories) of energy per day in maintaining a constant weight. What is the average power he produces over a day? Compare this with his power production when he runs up the stairs.
79. An electron in a television tube is accelerated uniformly from rest to a speed of 8.4×10^7 m/s over a distance of 2.5 cm. What is the power delivered to the electron at the instant that its displacement is 1.0 cm?
80. Coal is lifted out of a mine a vertical distance of 50 m by an engine that supplies 500 W to a conveyer belt. How much coal per minute can be brought to the surface? Ignore the effects of friction.
81. A girl pulls her 15-kg wagon along a flat sidewalk by applying a 10-N force at 37° to the horizontal. Assume that friction is negligible and that the wagon starts from rest. (a) How much work does the girl do on the wagon in the first 2.0 s. (b) How much instantaneous power does she exert at t = 2.0 s ?
82. A typical automobile engine has an efficiency of 25%. Suppose that the engine of a 1000-kg automobile has a

maximum power output of 140 hp. What is the maximum grade that the automobile can climb at 50 km/h if the frictional retarding force on it is 300 N?

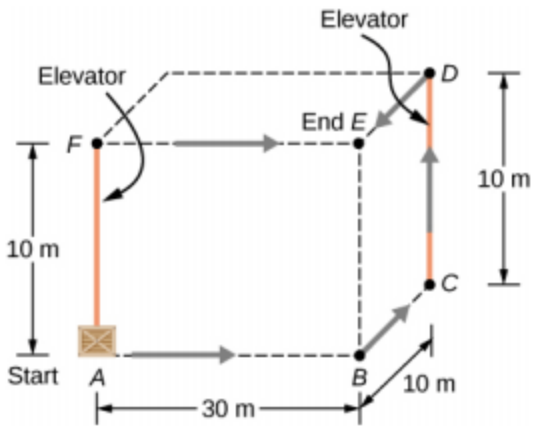
83. When jogging at 13 km/h on a level surface, a 70-kg man uses energy at a rate of approximately 850 W. Using the facts that the "human engine" is approximately 25% efficient, determine the rate at which this man uses energy when jogging up a 5.0° slope at this same speed. Assume that the frictional retarding force is the same in both cases.

Additional Problems

84. A cart is pulled a distance D on a flat, horizontal surface by a constant force F that acts at an angle θ with the horizontal direction. The other forces on the object during this time are gravity (F_w), normal forces (F_{N1}) and (F_{N2}), and rolling frictions F_{r1} and F_{r2}, as shown below. What is the work done by each force?



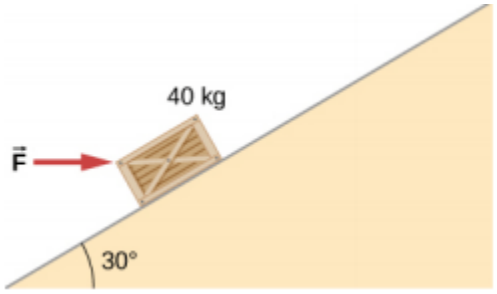
85. Consider a particle on which several forces act, one of which is known to be constant in time: $\vec{F}_1$ = (3 N) $\hat{i}$ + (4 N) $\hat{j}$. As a result, the particle moves along the x-axis from x = 0 to x = 5 m in some time interval. What is the work done by $\vec{F}_1$?
86. Consider a particle on which several forces act, one of which is known to be constant in time: $\vec{F}_1$ = (3 N) $\hat{i}$ + (4 N) $\hat{j}$. As a result, the particle moves first along the x-axis from x = 0 to x = 5 m and then parallel to the y-axis from y = 0 to y = 6 m. What is the work done by $\vec{F}_1$?
87. Consider a particle on which several forces act, one of which is known to be constant in time: $\vec{F}_1$ = (3 N) $\hat{i}$ + (4 N) $\hat{j}$. As a result, the particle moves along a straight path from a Cartesian coordinate of (0 m, 0 m) to (5 m, 6 m). What is the work done by $\vec{F}_1$?
88. Consider a particle on which a force acts that depends on the position of the particle. This force is given by $\vec{F}_1$ = (2y) $\hat{i}$ + (3x) $\hat{j}$. Find the work done by this force when the particle moves from the origin to a point 5 meters to the right on the x-axis.
89. A boy pulls a 5-kg cart with a 20-N force at an angle of 30° above the horizontal for a length of time. Over this time frame, the cart moves a distance of 12 m on the horizontal floor. (a) Find the work done on the cart by the boy. (b) What will be the work done by the boy if he pulled with the same force horizontally instead of at an angle of 30° above the horizontal over the same distance?
90. A crate of mass 200 kg is to be brought from a site on the ground floor to a third floor apartment. The workers know that they can either use the elevator first, then slide it along the third floor to the apartment, or first slide the crate to another location marked C below, and then take the elevator to the third floor and slide it on the third floor a shorter distance. The trouble is that the third floor is very rough compared to the ground floor. Given that the coefficient of kinetic friction between the crate and the ground floor is 0.100 and between the crate and the third floor surface is 0.300, find the work needed by the workers for each path shown from A to E. Assume that the force the workers need to do is just enough to slide the crate at constant velocity (zero acceleration). Note: The work by the elevator against the force of gravity is not done by the workers.



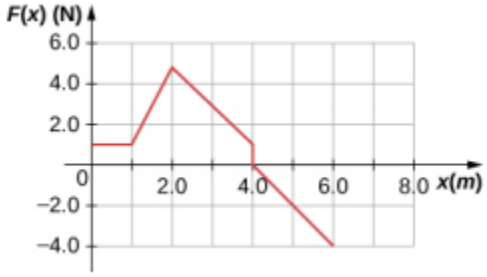
91. A hockey puck of mass 0.17 kg is shot across a rough floor with the roughness different at different places, which can be described by a position-dependent coefficient of kinetic friction. For a puck moving along the x-axis, the coefficient of kinetic friction is the following function of x, where x is in m: $\mu(x) = 0.1 + 0.05x$. Find the work done by the kinetic frictional force on the hockey puck when it has moved (a) from x = 0 to x = 2 m, and (b) from x = 2 m to x = 4 m.
92. A horizontal force of 20 N is required to keep a 5.0 kg box traveling at a constant speed up a frictionless incline for a vertical height change of 3.0 m. (a) What is the work done by gravity during this change in height? (b) What is the work done by the normal force? (c) What is the work done by the horizontal force?
93. A 7.0-kg box slides along a horizontal frictionless floor at 1.7 m/s and collides with a relatively massless spring that compresses 23 cm before the box comes to a stop. (a) How much kinetic energy does the box have before it collides with the spring? (b) Calculate the work done by the spring. (c) Determine the spring constant of the spring.
94. You are driving your car on a straight road with a coefficient of friction between the tires and the road of 0.55. A large piece of debris falls in front of your view and you immediate slam on the brakes, leaving a skid mark of 30.5 m (100-feet) long before coming to a stop. A policeman sees your car stopped on the road, looks at the skid mark, and gives you a ticket for traveling over the 13.4 m/s (30 mph) speed limit. Should you fight the speeding ticket in court?
95. A crate is being pushed across a rough floor surface. If no force is applied on the crate, the crate will slow down and come to a stop. If the crate of mass 50 kg moving at speed 8 m/s comes to rest in 10 seconds, what is the rate at which the frictional force on the crate takes energy away from the crate?
96. Suppose a horizontal force of 20 N is required to maintain a speed of 8 m/s of a 50 kg crate. (a) What is the power of this force? (b) Note that the acceleration of the crate is zero despite the fact that 20 N force acts on the crate horizontally. What happens to the energy given to the crate as a result of the work done by this 20 N force?
97. Grains from a hopper falls at a rate of 10 kg/s vertically onto a conveyor belt that is moving horizontally at a constant speed of 2 m/s. (a) What force is needed to keep the conveyor belt moving at the constant velocity? (b) What is the minimum power of the motor driving the conveyor belt?
98. A cyclist in a race must climb a 5° hill at a speed of 8 m/s. If the mass of the bike and the biker together is 80 kg, what must be the power output of the biker to achieve the goal?

Challenge Problems

99. Shown below is a 40-kg crate that is pushed at constant velocity a distance 8.0 m along a 30° incline by the horizontal force $\vec{F}$. The coefficient of kinetic friction between the crate and the incline is $\mu_k = 0.40$. Calculate the work done by (a) the applied force, (b) the frictional force, (c) the gravitational force, and (d) the net force.



100. The surface of the preceding problem is modified so that the coefficient of kinetic friction is decreased. The same horizontal force is applied to the crate, and after being pushed 8.0 m, its speed is 5.0 m/s. How much work is now done by the force of friction? Assume that the crate starts at rest.
101. The force F(x) varies with position, as shown below. Find the work done by this force on a particle as it moves from x = 1.0 m to x = 5.0 m.



102. Find the work done by the same force in Example 7.4, between the same points, A = (0, 0) and B = (2 m, 2 m) , over a circular arc of radius 2 m, centered at (0, 2 m). Evaluate the path integral using Cartesian coordinates. (**Hint**: You will probably need to consult a table of integrals.)
103. Answer the preceding problem using polar coordinates.
104. Find the work done by the same force in Example 7.4, between the same points, A = (0, 0) and B = (2 m, 2 m), over a circular arc of radius 2 m, centered at (2 m, 0). Evaluate the path integral using Cartesian coordinates. (**Hint**: You will probably need to consult a table of integrals.)
105. Answer the preceding problem using polar coordinates.
106. Constant power P is delivered to a car of mass m by its engine. Show that if air resistance can be ignored, the distance covered in a time t by the car, starting from rest, is given by $s = \left(\frac{8P}{9m}\right)^{1/2} t^{3/2}$.
107. Suppose that the air resistance a car encounters is independent of its speed. When the car travels at 15 m/ s, its engine delivers 20 hp to its wheels. (a) What is the power delivered to the wheels when the car travels at 30 m/ s? (b) How much energy does the car use in covering 10 km at 15 m/s? At 30 m/s? Assume that the engine is 25% efficient. (c) Answer the same questions if the force of air resistance is proportional to the speed of the automobile. (d) What do these results, plus your experience with gasoline consumption, tell you about air resistance?
108. Consider a linear spring, as in Figure 7.7(a), with mass M uniformly distributed along its length. The left end of the spring is fixed, but the right end, at the equilibrium position x = 0, is moving with speed v in the x-direction. What is the total kinetic energy of the spring? (**Hint**: First express the kinetic energy of an infinitesimal element of the spring dm in terms of the total mass, equilibrium length, speed of the right-hand end, and position along the spring; then integrate.)

Contributors and Attributions

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6.7: Work and Kinetic Energy (Summary)

Key Terms

average power	work done in a time interval divided by the time interval
kinetic energy	energy of motion, one-half an object's mass times the square of its speed
net work	work done by all the forces acting on an object
power	(or instantaneous power) rate of doing work
work	done when a force acts on something that undergoes a displacement from one position to another
work done by a force	integral, from the initial position to the final position, of the dot product of the force and the infinitesimal displacement along the path over which the force acts
work-energy theorem	net work done on a particle is equal to the change in its kinetic energy

Key Equations

Work done by a force over an infinitesimal displacement	$dW = \vec{F} \cdot d\vec{r} = \lvert\vec{F}\rvert\lvert d\vec{r}\rvert \cos\theta$
Work done by a force acting along a path from A to B	$W_{AB} = \int_{path\ AB} \vec{F} \cdot d\vec{r}$
Work done by a constant force of kinetic friction	$W_{fr} = -f_k \lvert l_{AB}\rvert$
Work done going from A to B by Earth's gravity, near its surface	$W_{grav,\ AB} = -mg(y_B - y_A)$
Work done going from A to B by one-dimensional spring force	$W_{spring,\ AB} = \left(\frac{1}{2}k\right)(x_B^2 - x_A^2)$
Kinetic energy of a non-relativistic particle	$K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$

Work-energy theorem	$W_{net} = K_B - K_A$
Power as rate of doing work	$P = \frac{dW}{dt}$
Power as the dot product of force and velocity	$P = \vec{F} \cdot \vec{v}$

Summary

7.1 Work

- The infinitesimal increment of work done by a force, acting over an infinitesimal displacement, is the dot product of the force and the displacement.
- The work done by a force, acting over a finite path, is the integral of the infinitesimal increments of work done along the path.
- The work done against a force is the negative of the work done by the force.
- The work done by a normal or frictional contact force must be determined in each particular case.
- The work done by the force of gravity, on an object near the surface of Earth, depends only on the weight of the object and the difference in height through which it moved.
- The work done by a spring force, acting from an initial position to a final position, depends only on the spring constant and the squares of those positions.

7.2 Kinetic Energy

- The kinetic energy of a particle is the product of one-half its mass and the square of its speed, for non-relativistic speeds.
- The kinetic energy of a system is the sum of the kinetic energies of all the particles in the system.
- Kinetic energy is relative to a frame of reference, is always positive, and is sometimes given special names for different types of motion.

7.3 Work-Energy Theorem

- Because the net force on a particle is equal to its mass times the derivative of its velocity, the integral for the net work done on the particle is equal to the change in the particle's kinetic energy. This is the work-energy theorem.
- You can use the work-energy theorem to find certain properties of a system, without having to solve the differential equation for Newton's second law.

7.4 Power

- Power is the rate of doing work; that is, the derivative of work with respect to time.
- Alternatively, the work done, during a time interval, is the integral of the power supplied over the time interval.
- The power delivered by a force, acting on a moving particle, is the dot product of the force and the particle's velocity

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7: PHYSICS 1- REVIEW POTENTIAL ENERGY AND CONSERVATION OF ENERGY

7.1: Prelude to Potential Energy and Conservation of Energy

7.2: Potential Energy of a System

7.3: Conservative and Non-Conservative Forces

7.4: Conservation of Energy

7.5: Potential Energy Diagrams and Stability

7.6: Sources of Energy

7.7: Potential Energy and Conservation of Energy (Exercises)

7.8: Potential Energy and Conservation of Energy (Summary)

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7.1: Prelude to Potential Energy and Conservation of Energy

In George Rhoads' rolling ball sculpture, the principle of conservation of energy governs the changes in the ball's kinetic energy and relates them to changes and transfers for other types of energy associated with the ball's interactions. In this chapter, we introduce the important concept of potential energy. This will enable us to formulate the law of conservation of mechanical energy and to apply it to simple systems, making solving problems easier. In the final section on sources of energy, we will consider energy transfers and the general law of conservation of energy. Throughout this book, the law of conservation of energy will be applied in increasingly more detail, as you encounter more complex and varied systems, and other forms of energy.



Figure 7.1.1: Shown here is part of a Ball Machine sculpture by George Rhoads. A ball in this contraption is lifted, rolls, falls, bounces, and collides with various objects, but throughout its travels, its kinetic energy changes in definite, predictable amounts, which depend on its position and the objects with which it interacts. (credit: modification of work by Roland Tanglao)

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7.2: Potential Energy of a System

Learning Objectives

- Relate the difference of potential energy to work done on a particle for a system without friction or air drag
- Explain the meaning of the zero of the potential energy function for a system
- Calculate and apply the gravitational potential energy for an object near Earth's surface and the elastic potential energy of a mass-spring system

In Work, we saw that the work done on an object by the constant gravitational force, near the surface of Earth, over any displacement is a function only of the difference in the positions of the end-points of the displacement. This property allows us to define a different kind of energy for the system than its kinetic energy, which is called **potential energy**. We consider various properties and types of potential energy in the following subsections.

Potential Energy Basics

In Motion in Two and Three Dimensions, we analyzed the motion of a projectile, like kicking a football in Figure 7.2.1. For this example, let's ignore friction and air resistance. As the football rises, the work done by the gravitational force on the football is negative, because the ball's displacement is positive vertically and the force due to gravity is negative vertically. We also noted that the ball slowed down until it reached its highest point in the motion, thereby decreasing the ball's kinetic energy. This loss in kinetic energy translates to a gain in gravitational potential energy of the football-Earth system.

As the football falls toward Earth, the work done on the football is now positive, because the displacement and the gravitational force both point vertically downward. The ball also speeds up, which indicates an increase in kinetic energy. Therefore, energy is converted from gravitational potential energy back into kinetic energy.

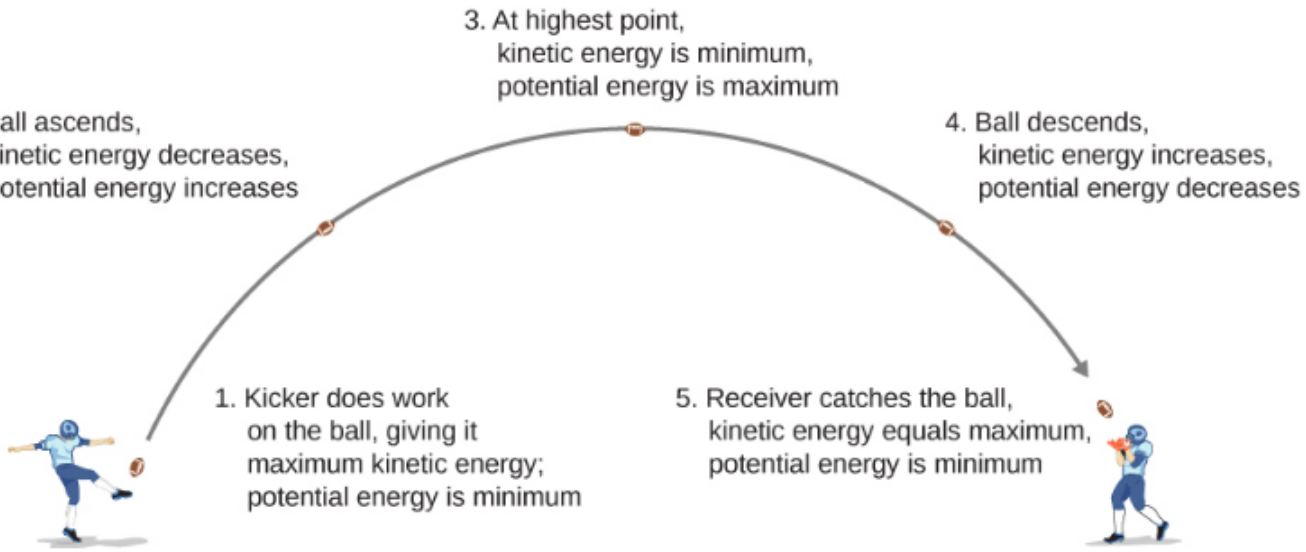


Figure 7.2.1: As a football starts its descent toward the wide receiver, gravitational potential energy is converted back into kinetic energy.

Based on this scenario, we can define the difference of potential energy from point *A* to point *B* as the negative of the work done:

$$\Delta U_{AB} = U_B - U_A = -W_{AB} \tag{7.2.1}$$

This formula explicitly states a **potential energy difference**, not just an absolute potential energy. Therefore, we need to define potential energy at a given position in such a way as to state standard values of potential energy on their own, rather than potential energy differences. We do this by rewriting the potential energy function in terms of an arbitrary constant,

$$\Delta U = U(\vec{\mathbf{r}}) - U\left(\vec{\mathbf{r}}_0\right) \tag{7.2.2}$$

The choice of the potential energy at a starting location of $\vec{r}_0$ is made out of convenience in the given problem. Most importantly, whatever choice is made should be stated and kept consistent throughout the given problem. There are some well-accepted choices of initial potential energy. For example, the lowest height in a problem is usually defined as zero potential energy, or if an object is in space, the farthest point away from the system is often defined as zero potential energy. Then, the potential energy, with respect to zero at $\vec{r}_0$, is just $U(\vec{r})$.

As long as there is no friction or air resistance, the change in kinetic energy of the football equals negative of the change in gravitational potential energy of the football. This can be generalized to any potential energy:

$$\Delta K_{AB} = -\Delta U_{AB} \tag{7.2.3}$$

Let's look at a specific example, choosing zero potential energy for gravitational potential energy at convenient points.

Example 7.2.1: Basic Properties of Potential Energy

A particle moves along the *x*-axis under the action of a force given by F = -ax^2, where a = 3 N/m^2. (a) What is the difference in its potential energy as it moves from x_A = 1 m to x_B = 2 m? (b) What is the particle's potential energy at x = 1 m with respect to a given 0.5 J of potential energy at x=0?

Strategy

(a) The difference in potential energy is the negative of the work done, as defined by Equation 7.2.1. The work is defined in the previous chapter as the dot product of the force with the distance. Since the particle is moving forward in the *x*-direction, the dot product simplifies to a multiplication ($\hat{i} \cdot \hat{i}$ = 1). To find the total work done, we need to integrate the function between the given limits. After integration, we can state the work or the change in potential energy. (b) The potential energy function, with respect to zero at x=0, is the indefinite integral encountered in part (a), with the constant of integration determined from Equation 7.2.3. Then, we substitute the *x*-value into the function of potential energy to calculate the potential energy at x = 1.

Solution

a. The work done by the given force as the particle moves from coordinate x to x + dx in one dimension is

$$dW = \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} = F dx = -ax^2 dx$$

Substituting this expression into Equation 7.2.1, we obtain

$$\Delta U = -W = \int_{x_1}^{x_2} ax^2 dx = \frac{1}{3}\left(3\ \mathrm{N/m^2}\right) x^3 \Big|_{1\mathrm{m}}^{2\mathrm{m}} = 7\ \mathrm{J}$$

b. The indefinite integral for the potential energy function in part (a) is

$$U(x) = \frac{1}{3} a x^3 + \text{ const. },$$

and we want the constant to be determined by

$$U(0) = 0.5\ J.$$

Thus, the potential energy with respect to zero at x = 0 is just

$$U(x) = \frac{1}{3} a x^3 + 0.5\ \mathrm{J}$$

Therefore, the potential energy at x = 1 m is

$$U(1\ \mathrm{m}) = \frac{1}{3}\left(3\ \mathrm{N/m^2}\right)(1\ \mathrm{m})^3 + 0.5\ \mathrm{J} = 1.5\ \mathrm{J}.$$

Significance

In this one-dimensional example, any function we can integrate, independent of path, is conservative. Notice how we applied the definition of potential energy difference to determine the potential energy function with respect to zero at a chosen point. Also notice that the potential energy, as determined in part (b), at x = 1 m is U(1 m) = 1 J and at x = 2 m is U(2 m) = 8 J; their difference is the result in part (a).

Exercise 7.2.1

In Example 7.2.1, what are the potential energies of the particle at x = 1 m and x = 2 m with respect to zero at x = 1.5 m? Verify that the difference of potential energy is still 7 J.

Systems of Several Particles

In general, a system of interest could consist of several particles. The difference in the potential energy of the system is the negative of the work done by gravitational or elastic forces, which, as we will see in the next section, are conservative forces. The potential energy difference depends only on the initial and final positions of the particles, and on some parameters that characterize the interaction (like mass for gravity or the spring constant for a Hooke's law force).

It is important to remember that potential energy is a property of the interactions between objects in a chosen system, and not just a property of each object. This is especially true for electric forces, although in the examples of potential energy we consider below, parts of the system are either so big (like Earth, compared to an object on its surface) or so small (like a massless spring), that the changes those parts undergo are negligible when included in the system.

Types of Potential Energy

For each type of interaction present in a system, you can label a corresponding type of potential energy. The total potential energy of the system is the sum of the potential energies of all the types. (This follows from the additive property of the dot product in the expression for the work done.) Let's look at some specific examples of types of potential energy discussed in Work. First, we consider each of these forces when acting separately, and then when both act together.

Gravitational Potential Energy Near Earth's Surface

The system of interest consists of our planet, Earth, and one or more particles near its surface (or bodies small enough to be considered as particles, compared to Earth). The gravitational force on each particle (or body) is just its weight *mg* near the surface of Earth, acting vertically down. According to Newton's third law, each particle exerts a force on Earth of equal magnitude but in the opposite direction. Newton's second law tells us that the magnitude of the acceleration produced by each of these forces on Earth is *mg* divided by Earth's mass. Since the ratio of the mass of any ordinary object to the mass of Earth is vanishingly small, the motion of Earth can be completely neglected. Therefore, we consider this system to be a group of single-particle systems, subject to the uniform gravitational force of Earth.

In Work, the work done on a body by Earth's uniform gravitational force, near its surface, depended on the mass of the body, the acceleration due to gravity, and the difference in height the body traversed, as given by Equation 7.2.4. By definition, this work is the negative of the difference in the gravitational potential energy, so that difference is

$$\Delta U_{\text{grav}} = -W_{\text{grav},AB} = mg\left(y_B - y_A\right). \tag{7.2.4}$$

You can see from this that the gravitational potential energy function, near Earth's surface, is

$$U(y) = mgy + \text{ const.} \tag{7.2.5}$$

You can choose the value of the constant, as described in the discussion of Equation 7.2.2; however, for solving most problems, the most convenient constant to choose is zero for when y=0, which is the lowest vertical position in the problem.

Example 7.2.2: Gravitational PotentIAL Energy of a hiker

The summit of Great Blue Hill in Milton, MA, is 147 m above its base and has an elevation above sea level of 195 m (Figure 7.2.2). (Its Native American name, *Massachusett*, was adopted by settlers for naming the Bay Colony and state near its location.) A 75-kg hiker ascends from the base to the summit. What is the gravitational potential energy of the hiker-Earth system with respect to zero gravitational potential energy at base height, when the hiker is (a) at the base of the hill, (b) at the summit, and (c) at sea level, afterward?

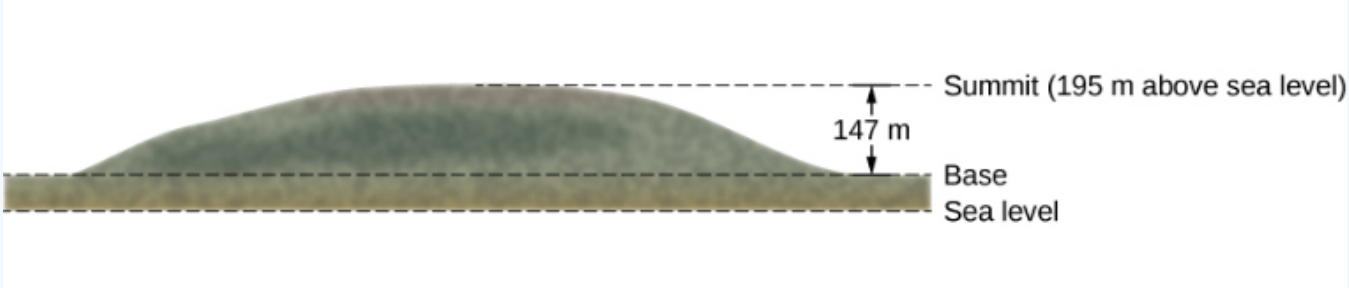


Figure 7.2.2: Sketch of the profile of Great Blue Hill, Milton, MA. The altitudes of the three levels are indicated.

Strategy

First, we need to pick an origin for the *y*-axis and then determine the value of the constant that makes the potential energy zero at the height of the base. Then, we can determine the potential energies from Equation 7.2.5, based on the relationship between the zero potential energy height and the height at which the hiker is located.

Solution

a. Let's choose the origin for the *y*-axis at base height, where we also want the zero of potential energy to be. This choice makes the constant equal to zero and

$$U(\text{ base }) = U(0) = 0$$

b. At the summit, y = 147 m, so

$$U(\text{ summit }) = U(147\text{ m}) = mgh = (75 \times 9.8\text{ N})(147\text{ m}) = 108\text{ kJ}.$$

c. At sea level, y = (147 - 195) m = -48 m, so

$$U\text{ (sea-level)} = (75 \times 9.8\text{N})(-48\text{m}) = -35.3\text{kJ}.$$

Significance

Besides illustrating the use of Equation 7.2.4 and Equation 7.2.5, the values of gravitational potential energy we found are reasonable. The gravitational potential energy is higher at the summit than at the base, and lower at sea level than at the base. Gravity does work on you on your way up, too! It does negative work and not quite as much (in magnitude), as your muscles do. But it certainly does work. Similarly, your muscles do work on your way down, as negative work. The numerical values of the potential energies depend on the choice of zero of potential energy, but the physically meaningful differences of potential energy do not. [Note that since Equation 7.2.2 is a difference, the numerical values do not depend on the origin of coordinates.]

Exercise 7.2.2

What are the values of the gravitational potential energy of the hiker at the base, summit, and sea level, with respect to a sea-level zero of potential energy?

Elastic Potential Energy

In Work, we saw that the work done by a perfectly elastic spring, in one dimension, depends only on the spring constant and the squares of the displacements from the unstretched position, as given in Equation 7.2.5. This work involves only the properties of a Hooke's law interaction and not the properties of real springs and whatever objects are attached to them. Therefore, we can define the difference of elastic potential energy for a spring force as the negative of the work done by the spring force in this equation, before we consider systems that embody this type of force. Thus,

$$\Delta U = -W_{AB} = \frac{1}{2}k\left(x_B^2 - x_A^2\right) \tag{7.2.6}$$

where the object travels from point *A* to point *B*. The potential energy function corresponding to this difference is

$$U(x) = \frac{1}{2}kx^2 + \text{ const.} \tag{7.2.7}$$

If the spring force is the only force acting, it is simplest to take the zero of potential energy at x = 0, when the spring is at its unstretched length. Then, the constant is Equation 7.2.7 is zero. (Other choices may be more convenient if other forces are acting.)

Example 7.2.3: Spring Potential Energy

A system contains a perfectly elastic spring, with an unstretched length of 20 cm and a spring constant of 4 N/cm. (a) How much elastic potential energy does the spring contribute when its length is 23 cm? (b) How much more potential energy does it contribute if its length increases to 26 cm?

Strategy

When the spring is at its unstretched length, it contributes nothing to the potential energy of the system, so we can use Equation 7.2.7 with the constant equal to zero. The value of *x* is the length minus the unstretched length. When the spring is expanded, the spring's displacement or difference between its relaxed length and stretched length should be used for the *x*-value in calculating the potential energy of the spring.

Solution

a. The displacement of the spring is x = 23 cm – 20 cm = 3 cm, so the contributed potential energy is U = $\frac{1}{2}$kx^2 = $\frac{1}{2}$(4 N/cm)(3 cm)^2 = 0.18 J.
b. When the spring's displacement is x = 26 cm – 20 cm = 6 cm, the potential energy is U = $\frac{1}{2}$kx^2 = $\frac{1}{2}$(4 N/cm)(6 cm)^2 = 0.72 J, which is a 0.54-J increase over the amount in part (a).

Significance

Calculating the elastic potential energy and potential energy differences from Equation 7.2.7 involves solving for the potential energies based on the given lengths of the spring. Since *U* depends on x^2, the potential energy for a compression (negative *x*) is the same as for an extension of equal magnitude.

Exercise 7.2.3

When the length of the spring in Example 8.2.3 changes from an initial value of 22.0 cm to a final value, the elastic potential energy it contributes changes by –0.0800J. Find the final length.

Gravitational and Elastic Potential Energy

A simple system embodying both gravitational and elastic types of potential energy is a one-dimensional, vertical mass-spring system. This consists of a massive particle (or block), hung from one end of a perfectly elastic, massless spring, the other end of which is fixed, as illustrated in Figure 7.2.3.

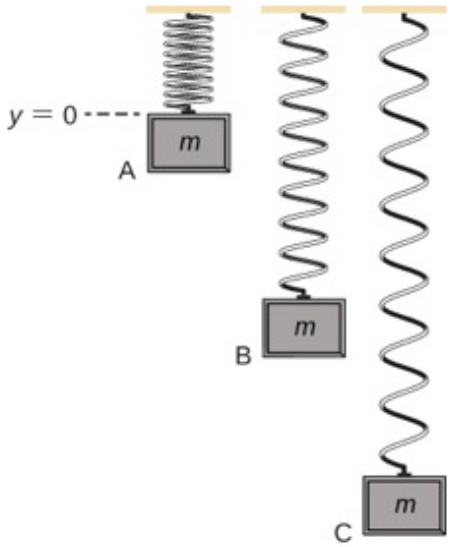


Figure 7.2.1: A vertical mass-spring system, with the positive *y*-axis pointing upward. The mass is initially at an unstretched spring length, point A. Then it is released, expanding past point B to point C, where it comes to a stop.

First, let's consider the potential energy of the system. We need to define the constant in the potential energy function of Equation 7.2.5. Often, the ground is a suitable choice for when the gravitational potential energy is zero; however, in this case, the highest point or when y = 0 is a convenient location for zero gravitational potential energy. Note that this choice is arbitrary, and the problem can be solved correctly even if another choice is picked.

We must also define the elastic potential energy of the system and the corresponding constant, as detailed in Equation 7.2.7. This is where the spring is unstretched, or at the y = 0 position.

If we consider that the total energy of the system is conserved, then the energy at point A equals point C. The block is placed just on the spring so its initial kinetic energy is zero. By the setup of the problem discussed previously, both the gravitational potential energy and elastic potential energy are equal to zero. Therefore, the initial energy of the system is zero. When the block arrives at point C, its kinetic energy is zero. However, it now has both gravitational potential energy and elastic potential energy. Therefore, we can solve for the distance y that the block travels before coming to a stop:

$$\begin{aligned} K_{\mathrm{A}} + U_A &= K_C + U_{\mathrm{C}} \\ 0 &= 0 + mgy_C + \frac{1}{2}k\left(y_C\right)^2 \\ y_{\mathrm{C}} &= \frac{-2mg}{k} \end{aligned} \tag{7.2.8}$$



Figure 7.2.4: A bungee jumper transforms gravitational potential energy at the start of the jump into elastic potential energy at the bottom of the jump.

Example 7.2.4: Potential energy of a vertical mass-spring system

A block weighing 1.2 N is hung from a spring with a spring constant of 6.0 N/m, as shown in Figure 7.2.3. (a) What is the maximum expansion of the spring, as seen at point C? (b) What is the total potential energy at point B, halfway between A and C? (c) What is the speed of the block at point B?

Strategy

In part (a) we calculate the distance y_C as discussed in the previous text. Then in part (b), we use half of the y value to calculate the potential energy at point B using equations Equation 7.2.4 and Equation 7.2.6. This energy must be equal to the kinetic energy, Equation 7.3.1, at point B since the initial energy of the system is zero. By calculating the kinetic energy at point B, we can now calculate the speed of the block at point B.

Solution

a. Since the total energy of the system is zero at point A as discussed previously, the maximum expansion of the spring is calculated to be:

$$\begin{aligned} y_C &= \tfrac{-2mg}{k} \\ y_C &= \tfrac{-2(1.2\text{ N})}{(6.0\text{ N/m})} = -0.40\text{ m} \end{aligned} \tag{7.2.9}$$

b. The position of y_B is half of the position at y_C or -0.20 m. The total potential energy at point B would therefore be:

$$\begin{aligned} U_B &= mgy_B + \left(\frac{1}{2}ky_B\right)^2 \\ U_B &= (1.2\text{ N})(-0.20\text{ m}) + \frac{1}{2}(6\text{ N/m})(-0.20\text{ m})^2 \\ U_B &= -0.12\text{ J} \end{aligned} \tag{7.2.10}$$

c. The mass of the block is the weight divided by gravity.

$$m = \frac{F_w}{g} = \frac{1.2\text{ N}}{9.8\text{ m/s}^2} = 0.12\text{ kg}$$

The kinetic energy at point B therefore is 0.12 J because the total energy is zero. Therefore, the speed of the block at point B is equal to

$$K = \frac{1}{2}mv^2$$
$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(0.12\text{ J})}{(0.12\text{ kg})}} = 1.4\text{ m/s} \tag{7.2.11}$$

Significance

Even though the potential energy due to gravity is relative to a chosen zero location, the solutions to this problem would be the same if the zero energy points were chosen at different locations.

Exercise 7.2.4

Suppose the mass in Equation 7.2.6 is doubled while keeping the all other conditions the same. Would the maximum expansion of the spring increase, decrease, or remain the same? Would the speed at point B be larger, smaller, or the same compared to the original mass?

Simulation

View this simulation to learn about conservation of energy with a skater! Build tracks, ramps and jumps for the skater and view the kinetic energy, potential energy and friction as he moves. You can also take the skater to different planets or even space!

A sample chart of a variety of energies is shown in Table 7.2.1 to give you an idea about typical energy values associated with certain events. Some of these are calculated using kinetic energy, whereas others are calculated by using quantities found in a form of potential energy that may not have been discussed at this point.

Table 7.2.1: Energy of Various Objects and Phenomena

Object/phenomenon	Energy in joules
Big Bang	10^{68}
Annual world energy use	4.0×10^{20}
Large fusion bomb (9 megaton)	3.8×10^{16}
Hiroshima-size fission bomb (10 kiloton)	4.2×10^{13}
1 barrel crude oil	5.9×10^{9}
1 ton TNT	4.2×10^{9}
1 gallon of gasoline	1.2×10^{8}
Daily adult food intake (recommended)	1.2×10^{7}
1000-kg car at 90 km/h	3.1×10^{5}
Tennis ball at 100 km/h	22
Mosquito (10^{-2} g at 0.5 m/s)	1.3×10^{-6}
Single electron in a TV tube beam	4.0×10^{-15}
Energy to break one DNA strand	10^{-19}

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7.3: Conservative and Non-Conservative Forces

Learning Objectives

- Characterize a conservative force in several different ways
- Specify mathematical conditions that must be satisfied by a conservative force and its components
- Relate the conservative force between particles of a system to the potential energy of the system
- Calculate the components of a conservative force in various cases

In Potential Energy and Conservation of Energy, any transition between kinetic and potential energy conserved the total energy of the system. This was path independent, meaning that we can start and stop at any two points in the problem, and the total energy of the system—kinetic plus potential—at these points are equal to each other. This is characteristic of a **conservative force**. We dealt with conservative forces in the preceding section, such as the gravitational force and spring force. When comparing the motion of the football in Figure 8.2.1, the total energy of the system never changes, even though the gravitational potential energy of the football increases, as the ball rises relative to ground and falls back to the initial gravitational potential energy when the football player catches the ball. **Non-conservative forces** are dissipative forces such as friction or air resistance. These forces take energy away from the system as the system progresses, energy that you can't get back. These forces are path dependent; therefore it matters where the object starts and stops.

Definition: Conservative Force

The work done by a conservative force is independent of the path; in other words, the work done by a conservative force is the same for any path connecting two points:

$$W_{AB,\ path-1} = \int_{AB,\ path-1} \vec{F}_{cons} \cdot d\vec{r} = W_{AB,\ path-2} = \int_{AB,\ path-2} \vec{F}_{cons} \cdot d\vec{r}. \tag{7.3.1}$$

The work done by a non-conservative force depends on the path taken. Equivalently, a force is conservative if the work it does around any closed path is zero:

$$W_{closed\ path} = \oint \vec{E}_{cons} \cdot d\vec{r} = 0. \tag{7.3.2}$$

In Equation 7.3.2, we use the notation of a circle in the middle of the integral sign for a line integral over a closed path, a notation found in most physics and engineering texts.] Equations 7.3.1 and 7.3.2 are equivalent because any closed path is the sum of two paths: the first going from A to B, and the second going from B to A. The work done going along a path from B to A is the negative of the work done going along the same path from A to B, where A and B are any two points on the closed path:

$$\begin{aligned} 0 = \int \vec{F}_{cons} \cdot d\vec{r} &= \int_{AB,\ path-1} \vec{F}_{cons} \cdot d\vec{r} + \int_{BA,\ path-2} \vec{F}_{cons} \cdot d\vec{r} \\ &= \int_{AB,\ path-1} \vec{F}_{cons} \cdot d\vec{r} - \int_{AB,\ path-2} \vec{F}_{cons} \cdot d\vec{r} = 0. \end{aligned} \tag{7.3.3}$$

You might ask how we go about proving whether or not a force is conservative, since the definitions involve any and all paths from A to B, or any and all closed paths, but to do the integral for the work, you have to choose a particular path. One answer is that the work done is independent of path if the infinitesimal work $\vec{F} \cdot d\vec{r}$ is an **exact differential**, the way the infinitesimal net work was equal to the exact differential of the kinetic energy, $dW_{net} = m\vec{v} \cdot d\vec{v} = d\frac{1}{2}mv^2$, when we derived the work-energy theorem in Work-Energy Theorem. There are mathematical conditions that you can use to test whether the infinitesimal work done by a force is an exact differential, and the force is conservative. These conditions only involve differentiation and are thus relatively easy to apply. In two dimensions, the condition for $\vec{F} \cdot d\vec{r}$ = $F_x dx + F_y dy$ to be an exact differential is

$$\frac{dF_x}{dy} = \frac{dF_y}{dx}. \tag{7.3.4}$$

You may recall that the work done by the force in Example 7.2.4 depended on the path. For that force,

$$F_x = (5\ N/m)y\ and\ F_y = (10\ N/m)x. \tag{7.3.5}$$

Therefore,

$$\left(\frac{dF_x}{dy}\right) = 5\ N/m \neq \left(\frac{dF_y}{dx}\right) = 10\ N/m, \tag{7.3.6}$$

which indicates it is a non-conservative force. Can you see what you could change to make it a conservative force?

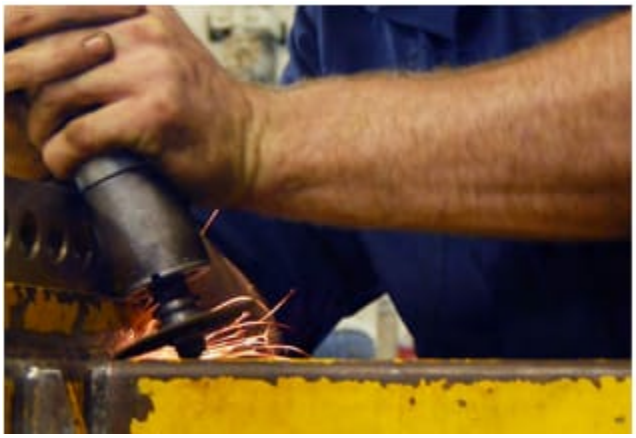


Figure 7.3.1: A grinding wheel applies a non-conservative force, because the work done depends on how many rotations the wheel makes, so it is path-dependent.

Example 7.3.1: Conservative or Not?

Which of the following two-dimensional forces are conservative and which are not? Assume a and b are constants with appropriate units:

a. $axy^3\hat{i} + ayx^3\hat{j}$,
b. $a\left[\left(\frac{y^2}{x}\right)\hat{i} + 2y\ln\left(\frac{x}{b}\right)\hat{j}\right]$,
c. $\frac{ax\hat{i}+ay\hat{j}}{x^2+y^2}$

Strategy

Apply the condition stated in Equation 7.3.4, namely, using the derivatives of the components of each force indicated. If the derivative of the y-component of the force with respect to x is equal to the derivative of the x-component of the force with respect to y, the force is a conservative force, which means the path taken for potential energy or work calculations always yields the same results.

Solution

a:

$$\frac{dF_x}{dy} = \frac{d(axy^3)}{dy} = 3axy^2$$

and

$$\frac{dF_y}{dx} = \frac{d(ayx^3)}{dx} = 3ayx^2,$$

so this force is non-conservative.

b:

$$\frac{dF_x}{dy} = \frac{d\left(\frac{ay^2}{x}\right)}{dy} = \frac{2ay}{x}$$

and

$$\frac{dF_y}{dx} = \frac{d(2ay\ln\left(\frac{x}{b}\right))}{dx} = \frac{2ay}{x},$$

so this force is conservative.

c:

$$\frac{dF_x}{dy} = \frac{d\left(\frac{ax}{(x^2+y^2)}\right)}{dy} = -\frac{ax(2y)}{(x^2+y^2)^2} = \frac{dF_y}{dx} = \frac{d\left(\frac{ay}{(x^2+y^2)}\right)}{dx}, \tag{7.3.7}$$

again conservative.

Significance

The conditions in Equation 7.3.4 are derivatives as functions of a single variable; in three dimensions, similar conditions exist that involve more derivatives.

Exercise 7.3.1

A two-dimensional, conservative force is zero on the x- and y-axes, and satisfies the condition $\left(\frac{dF_x}{dy}\right) = \left(\frac{dF_y}{dy}\right)$ = (4 N/m^3)xy. What is the magnitude of the force at the point $x = y = 1\ m$?

Before leaving this section, we note that non-conservative forces do not have potential energy associated with them because the energy is lost to the system and can't be turned into useful work later. So there is always a conservative force associated with every potential energy. We have seen that potential energy is defined in relation to the work done by conservative forces. That relation, Equation 8.2.1, involved an integral for the work; starting with the force and displacement, you integrated to get the work and the change in potential energy. However, integration is the inverse operation of differentiation; you could equally well have started with the potential energy and taken its derivative, with respect to displacement, to get the force. The infinitesimal increment of potential energy is the dot product of the force and the infinitesimal displacement,

$$dU = -\vec{F}\cdot d\vec{l} = -F_l dl. \tag{7.3.8}$$

Here, we chose to represent the displacement in an arbitrary direction by d$\vec{l}$, so as not to be restricted to any particular coordinate direction. We also expressed the dot product in terms of the magnitude of the infinitesimal displacement and the component of the force in its direction. Both these quantities are scalars, so you can divide by dl to get

$$F_l = -\frac{dU}{dl}. \tag{7.3.9}$$

This equation gives the relation between force and the potential energy associated with it. In words, the component of a conservative force, in a particular direction, equals the negative of the derivative of the corresponding potential energy, with respect to a displacement in that direction. For one-dimensional motion, say along the x-axis, Equation 7.3.9 give the entire vector force,

$$\bar{F} = F_x\hat{i} = -\frac{\partial U}{\partial x}\hat{i}. \tag{7.3.10}$$

In two dimensions,

$$\vec{F} = F_x\hat{i} + F_y\hat{j} \tag{7.3.11}$$

$$= -\left(\frac{\partial U}{\partial x}\right)\hat{i} - \left(\frac{\partial U}{\partial y}\right)\hat{j}. \tag{7.3.12}$$

From this equation, you can see why Equation 7.3.9 is the condition for the work to be an exact differential, in terms of the derivatives of the components of the force. In general, a partial derivative notation is used. If a function has many variables in it, the derivative is taken only of the variable the partial derivative specifies. The other variables are held constant. In three dimensions, you add another term for the z-component, and the result is that the force is the negative of the gradient of the potential energy. However, we won't be looking at three-dimensional examples just yet.

Example 7.3.2: Force due to a Quartic Potential Energy

The potential energy for a particle undergoing one-dimensional motion along the x-axis is

$$U(x) = \frac{1}{4}cx^4,$$

where c = 8 N/m^3. Its total energy at x = 0 is 2 J, and it is not subject to any non-conservative forces. Find (a) the positions where its kinetic energy is zero and (b) the forces at those positions.

Strategy

a. We can find the positions where K = 0, so the potential energy equals the total energy of the given system.
b. Using Equation 7.3.9, we can find the force evaluated at the positions found from the previous part, since the mechanical energy is conserved.

Solution

a. The total energy of the system of 2 J equals the quartic elastic energy as given in the problem

$$2\ J = \frac{1}{4}(8\ N/m^3)x_f^4. \tag{7.3.13}$$

Solving for x_f results in $x_f = \pm 1$ m.

b. From Equation 7.3.9,

$$F_x = -\frac{dU}{dx} = -cx^3. \tag{7.3.14}$$

Thus, evaluating the force at ±1 m , we get

$$\vec{F} = -(8\ N/m^3)(\pm 1\ m)^3\hat{i} = \pm 8\ N\hat{i}. \tag{7.3.15}$$

At both positions, the magnitude of the forces is 8 N and the directions are toward the origin, since this is the potential energy for a restoring force.

Significance

Finding the force from the potential energy is mathematically easier than finding the potential energy from the force, because differentiating a function is generally easier than integrating one.

Exercise 7.3.2

Find the forces on the particle in Example 7.3.2 when its kinetic energy is 1.0 J at $x = 0$.

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7.4: Conservation of Energy

Learning Objectives

- Formulate the principle of conservation of mechanical energy, with or without the presence of non-conservative forces
- Use the conservation of mechanical energy to calculate various properties of simple systems

In this section, we elaborate and extend the result we derived in Potential Energy of a System, where we re-wrote the work-energy theorem in terms of the change in the kinetic and potential energies of a particle. This will lead us to a discussion of the important principle of the conservation of mechanical energy. As you continue to examine other topics in physics, in later chapters of this book, you will see how this conservation law is generalized to encompass other types of energy and energy transfers. The last section of this chapter provides a preview.

The terms 'conserved quantity' and 'conservation law' have specific, scientific meanings in physics, which are different from the everyday meanings associated with the use of these words. (The same comment is also true about the scientific and everyday uses of the word 'work.') In everyday usage, you could conserve water by not using it, or by using less of it, or by re-using it. Water is composed of molecules consisting of two atoms of hydrogen and one of oxygen. Bring these atoms together to form a molecule and you create water; dissociate the atoms in such a molecule and you destroy water. However, in scientific usage, a **conserved quantity** for a system stays constant, changes by a definite amount that is transferred to other systems, and/or is converted into other forms of that quantity. A conserved quantity, in the scientific sense, can be transformed, but not strictly created or destroyed. Thus, there is no physical law of conservation of water.

Systems with a Single Particle or Object

We first consider a system with a single particle or object. Returning to our development of Equation 8.2.2, recall that we first separated all the forces acting on a particle into conservative and non-conservative types, and wrote the work done by each type of force as a separate term in the work-energy theorem. We then replaced the work done by the conservative forces by the change in the potential energy of the particle, combining it with the change in the particle's kinetic energy to get Equation 8.2.2. Now, we write this equation without the middle step and define the sum of the kinetic and potential energies, K + U = E; to be the **mechanical energy** of the particle.

Conservation of Energy

The mechanical energy E of a particle stays constant unless forces outside the system or non-conservative forces do work on it, in which case, the change in the mechanical energy is equal to the work done by the non-conservative forces:

$$W_{nc,\,AB} = \Delta(K+U)_{AB} = \Delta E_{AB}. \tag{7.4.1}$$

This statement expresses the concept of **energy conservation** for a classical particle as long as there is no non-conservative work. Recall that a classical particle is just a point mass, is nonrelativistic, and obeys Newton's laws of motion. In Relativity, we will see that conservation of energy still applies to a non-classical particle, but for that to happen, we have to make a slight adjustment to the definition of energy.

It is sometimes convenient to separate the case where the work done by non-conservative forces is zero, either because no such forces are assumed present, or, like the normal force, they do zero work when the motion is parallel to the surface. Then

$$0 = W_{nc,\,AB} = \Delta(K+U)_{AB} = \Delta E_{AB}. \tag{7.4.2}$$

In this case, the conservation of mechanical energy can be expressed as follows: The mechanical energy of a particle does not change if all the non-conservative forces that may act on it do no work. Understanding the concept of energy conservation is the important thing, not the particular equation you use to express it.

Problem-Solving Strategy: Conservation of Energy

1. Identify the body or bodies to be studied (the system). Often, in applications of the principle of mechanical energy conservation, we study more than one body at the same time.
2. Identify all forces acting on the body or bodies.
3. Determine whether each force that does work is conservative. If a non-conservative force (e.g., friction) is doing work, then mechanical energy is not conserved. The system must then be analyzed with non-conservative work, Equation 7.4.2.
4. For every force that does work, choose a reference point and determine the potential energy function for the force. The reference points for the various potential energies do not have to be at the same location.
5. Apply the principle of mechanical energy conservation by setting the sum of the kinetic energies and potential energies equal at every point of interest.

Example 8.7: Simple pendulum

A particle of mass m is hung from the ceiling by a massless string of length 1.0 m, as shown in Figure 7.4.1. The particle is released from rest, when the angle between the string and the downward vertical direction is 30°. What is its speed when it reaches the lowest point of its arc?

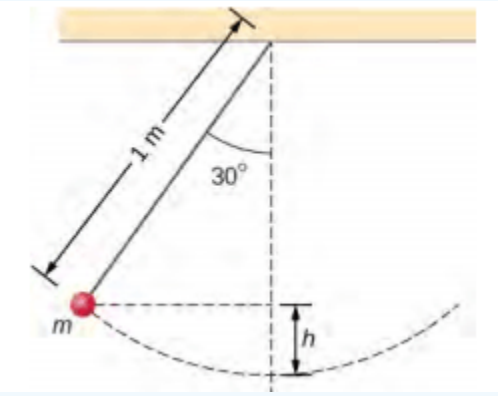


Figure 7.4.1: A particle hung from a string constitutes a simple pendulum. It is shown when released from rest, along with some distances used in analyzing the motion.

Strategy

Using our problem-solving strategy, the first step is to define that we are interested in the particle-Earth system. Second, only the gravitational force is acting on the particle, which is conservative (step 3). We neglect air resistance in the problem, and no work is done by the string tension, which is perpendicular to the arc of the motion. Therefore, the mechanical energy of the system is conserved, as represented by Equation 7.4.2, 0 = Δ(K + U). Because the particle starts from rest, the increase in the kinetic energy is just the kinetic energy at the lowest point. This increase in kinetic energy equals the decrease in the gravitational potential energy, which we can calculate from the geometry. In step 4, we choose a reference point for zero gravitational potential energy to be at the lowest vertical point the particle achieves, which is mid-swing. Lastly, in step 5, we set the sum of energies at the highest point (initial) of the swing to the lowest point (final) of the swing to ultimately solve for the final speed.

Solution

We are neglecting non-conservative forces, so we write the energy conservation formula relating the particle at the highest point (initial) and the lowest point in the swing (final) as

$$K_i + U_i = K_f + U_f. \tag{7.4.3}$$

Since the particle is released from rest, the initial kinetic energy is zero. At the lowest point, we define the gravitational potential energy to be zero. Therefore our conservation of energy formula reduces to

$$0 + mgh = \frac{1}{2}mv^2 + 0$$
$$v = \sqrt{2gh}. \tag{7.4.4}$$

The vertical height of the particle is not given directly in the problem. This can be solved for by using trigonometry and two givens: the length of the pendulum and the angle through which the particle is vertically pulled up. Looking at the diagram, the vertical dashed line is the length of the pendulum string. The vertical height is labeled h. The other partial length of the vertical string can be calculated with trigonometry. That piece is solved for by

$$\cos\theta = \frac{x}{L} = L\cos\theta. \tag{7.4.5}$$

Therefore, by looking at the two parts of the string, we can solve for the height h,

$$\begin{aligned} x + h &= L \\ L\cos\theta + h &= L \\ h &= L - L\cos\theta \\ &= L(1 - \cos\theta). \end{aligned} \tag{7.4.6}$$

We substitute this height into the previous expression solved for speed to calculate our result:

$$v = \sqrt{2gL(1-\cos\theta)} = \sqrt{2(9.8\ m/s^2)(1\ m)(1-\cos 30^o)} = 1.62\ m/s. \tag{7.4.7}$$

Significance

We found the speed directly from the conservation of mechanical energy, without having to solve the differential equation for the motion of a pendulum (see Oscillations). We can approach this problem in terms of bar graphs of total energy. Initially, the particle has all potential energy, being at the highest point, and no kinetic energy. When the particle crosses the lowest point at the bottom of the swing, the energy moves from the potential energy column to the kinetic energy column. Therefore, we can imagine a progression of this transfer as the particle moves between its highest point, lowest point of the swing, and back to the highest point (Figure 7.4.2). As the particle travels from the lowest point in the swing to the highest point on the far right hand side of the diagram, the energy bars go in reverse order from (c) to (b) to (a).

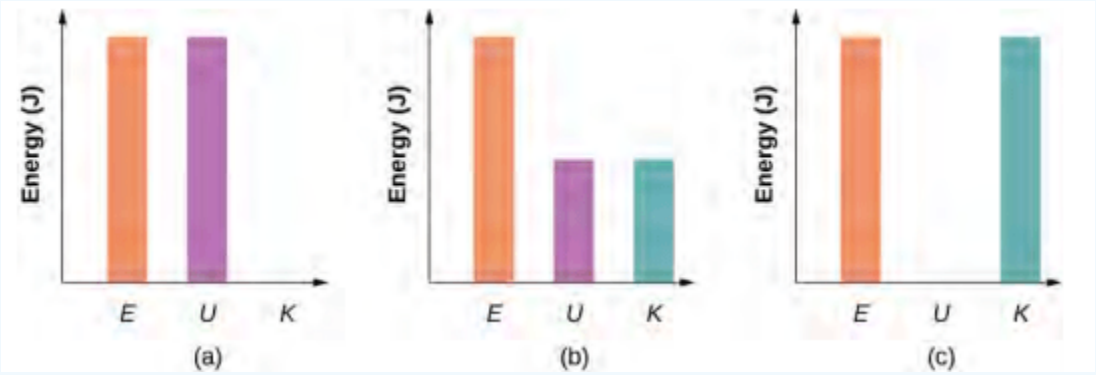


Figure 7.4.2: Bar graphs representing the total energy (E), potential energy (U), and kinetic energy (K) of the particle in different positions. (a) The total energy of the system equals the potential energy and the kinetic energy is zero, which is found at the highest point the particle reaches. (b) The particle is midway between the highest and lowest point, so the kinetic energy plus potential energy bar graphs equal the total energy. (c) The particle is at the lowest point of the swing, so the kinetic energy bar graph is the highest and equal to the total energy of the system.

Exercise 8.7

How high above the bottom of its arc is the particle in the simple pendulum above, when its speed is 0.81 m/s?

Example 8.8: Air resistance on a falling object

A helicopter is hovering at an altitude of 1 km when a panel from its underside breaks loose and plummets to the ground (Figure 7.4.3). The mass of the panel is 15 kg, and it hits the ground with a speed of 45 m/s. How much mechanical energy was dissipated by air resistance during the panel's descent?

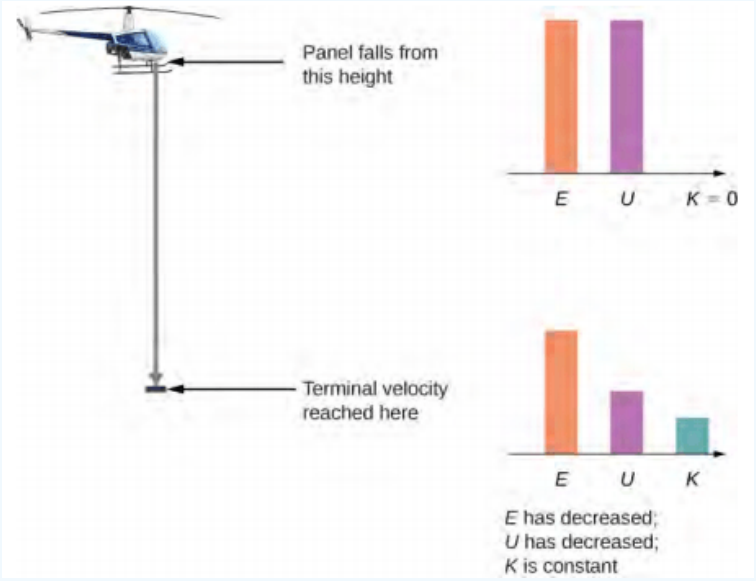


Figure 7.4.3: A helicopter loses a panel that falls until it reaches terminal velocity of 45 m/s. How much did air resistance contribute to the dissipation of energy in this problem?

Strategy

Step 1: Here only one body is being investigated.

Step 2: Gravitational force is acting on the panel, as well as air resistance, which is stated in the problem.

Step 3: Gravitational force is conservative; however, the non-conservative force of air resistance does negative work on the falling panel, so we can use the conservation of mechanical energy, in the form expressed by Equation 7.4.1, to find the energy dissipated. This energy is the magnitude of the work:

$$\Delta E_{diss} = |W_{nc,if}| = |\Delta(K+U)_{if}|. \tag{7.4.8}$$

Step 4: The initial kinetic energy, at yi = 1 km, is zero. We set the gravitational potential energy to zero at ground level out of convenience.

Step 5: The non-conservative work is set equal to the energies to solve for the work dissipated by air resistance.

Solution

The mechanical energy dissipated by air resistance is the algebraic sum of the gain in the kinetic energy and loss in potential energy. Therefore the calculation of this energy is

$$\begin{aligned} \Delta E_{diss} &= |K_f - K_i 9 U_f - U_i| \\ &= \left|\frac{1}{2}(15\ kg)(45\ m/s)^2 - 0 + 0 - (15\ kg)(9.8\ m/s^2)(1000\ m)\right| \\ &= 130\ kJ. \end{aligned} \tag{7.4.9}$$

Significance

Most of the initial mechanical energy of the panel (U_i), 147 kJ, was lost to air resistance. Notice that we were able to calculate the energy dissipated without knowing what the force of air resistance was, only that it was dissipative.

Exercise 8.8

You probably recall that, neglecting air resistance, if you throw a projectile straight up, the time it takes to reach its maximum height equals the time it takes to fall from the maximum height back to the starting height. Suppose you cannot neglect air resistance, as in Example 8.8. Is the time the projectile takes to go up (a) greater than, (b) less than, or (c) equal to the time it takes to come back down? Explain.

In these examples, we were able to use conservation of energy to calculate the speed of a particle just at particular points in its motion. But the method of analyzing particle motion, starting from energy conservation, is more powerful than that. More advanced treatments of the theory of mechanics allow you to calculate the full time dependence of a particle's motion, for a given potential energy. In fact, it is often the case that a better model for particle motion is provided by the form of its kinetic and potential energies, rather than an equation for force acting on it. (This is especially true for the quantum mechanical description of particles like electrons or atoms.)

We can illustrate some of the simplest features of this energy-based approach by considering a particle in one-dimensional motion, with potential energy U(x) and no non-conservative interactions present. Equation 7.4.1 and the definition of velocity require

$$K = \frac{1}{2}mv^2 = E - U(x) \tag{7.4.10}$$

$$v = \frac{dx}{dt} = \sqrt{\frac{2(E-U(x))}{m}}. \tag{7.4.11}$$

Separate the variables x and t and integrate, from an initial time t = 0 to an arbitrary time, to get

$$t = \int_0^t dt = \int_{x_0}^{x} \frac{dx}{\sqrt{\frac{2(E-U(x))}{m}}}. \tag{7.4.12}$$

If you can do the integral in Equation 7.4.12, then you can solve for x as a function of t.

Example 8.9: Constant Acceleration

Use the potential energy U(x) = –E $\left(\frac{x}{x_0}\right)$, for E > 0, in Equation 7.4.12 to find the position x of a particle as a function of time t.

Strategy

Since we know how the potential energy changes as a function of x, we can substitute for U(x) in Equation 7.4.12, integrate, and then solve for x. This results in an expression of x as a function of time with constants of energy E, mass m, and the initial position x_0.

Solution

Following the first two suggested steps in the above strategy,

$$t = \int_{x_0}^{x} \frac{dx}{\sqrt{\left(\frac{2E}{mx_0}\right)(x_0 - x)}} = \frac{1}{\sqrt{\left(\frac{2E}{mx_0}\right)}} \Big| -2\sqrt{(x_0 - x)} \Big|_{x_0}^{x} = \frac{-2\sqrt{(x_0 - x)}}{\sqrt{\left(\frac{2E}{mx_0}\right)}}. \tag{7.4.13}$$

Solving for the position, we obtain

$$x(t) = x_0 - \frac{1}{2}\left(\frac{E}{mx_0}\right)t^2. \tag{7.4.14}$$

Significance

The position as a function of time, for this potential, represents one-dimensional motion with constant acceleration, a = $\left(\frac{E}{mx_0}\right)$, starting at rest from position x_0. This is not so surprising, since this is a potential energy for a constant force, F = $-\frac{dU}{dx}$ = $\frac{E}{x_0}$, and a = $\frac{F}{m}$.

Exercise 8.9

What potential energy U(x) can you substitute in Equation 7.4.2 that will result in motion with constant velocity of 2 m/s for a particle of mass 1 kg and mechanical energy 1 J?

We will look at another more physically appropriate example of the use of Equation 7.4.2 after we have explored some further implications that can be drawn from the functional form of a particle's potential energy.

Systems with Several Particles or Objects

Systems generally consist of more than one particle or object. However, the conservation of mechanical energy, in one of the forms in Equation 7.4.1 or Equation 7.4.2, is a fundamental law of physics and applies to any system. You just have to include the kinetic and potential energies of all the particles, and the work done by all the non-conservative forces acting on them. Until you learn more about the dynamics of systems composed of many particles, in Linear Momentum and Collisions, Fixed-Axis Rotation, and Angular Momentum, it is better to postpone discussing the application of energy conservation to then.

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7.5: Potential Energy Diagrams and Stability

Learning Objectives

- Create and interpret graphs of potential energy
- Explain the connection between stability and potential energy

Often, you can get a good deal of useful information about the dynamical behavior of a mechanical system just by interpreting a graph of its potential energy as a function of position, called a **potential energy diagram**. This is most easily accomplished for a one-dimensional system, whose potential energy can be plotted in one two-dimensional graph—for example, U(x) versus x—on a piece of paper or a computer program. For systems whose motion is in more than one dimension, the motion needs to be studied in three-dimensional space. We will simplify our procedure for one-dimensional motion only.

First, let's look at an object, freely falling vertically, near the surface of Earth, in the absence of air resistance. The mechanical energy of the object is conserved, E = K + U, and the potential energy, with respect to zero at ground level, is U(y) = mgy, which is a straight line through the origin with slope mg . In the graph shown in Figure 7.5.1, the x-axis is the height above the ground y and the y-axis is the object's energy.

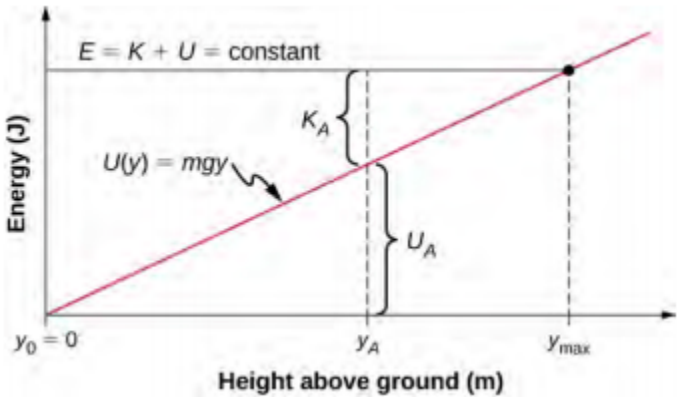


Figure 7.5.1: The potential energy graph for an object in vertical free fall, with various quantities indicated.

The line at energy E represents the constant mechanical energy of the object, whereas the kinetic and potential energies, K_A and U_A, are indicated at a particular height y_A. You can see how the total energy is divided between kinetic and potential energy as the object's height changes. Since kinetic energy can never be negative, there is a maximum potential energy and a maximum height, which an object with the given total energy cannot exceed:

$$K = E - U \geq 0, \tag{7.5.1}$$

$$U \leq E. \tag{7.5.2}$$

If we use the gravitational potential energy reference point of zero at y_0, we can rewrite the gravitational potential energy U as mgy. Solving for y results in

$$y \leq \frac{E}{mg} = y_{max}. \tag{7.5.3}$$

We note in this expression that the quantity of the total energy divided by the weight (mg) is located at the maximum height of the particle, or y_{max}. At the maximum height, the kinetic energy and the speed are zero, so if the object were initially traveling upward, its velocity would go through zero there, and y_{max}would be a turning point in the motion. At ground level, $y_0 = 0$, the potential energy is zero, and the kinetic energy and the speed are maximum:

$$U_0 = 0 = E - K_0, \tag{7.5.4}$$

$$E = K_0 = \frac{1}{2}mv_0^2, \tag{7.5.5}$$

$$v_0 = \pm\sqrt{\frac{2E}{m}}. \tag{7.5.6}$$

The maximum speed $\pm v_0$ gives the initial velocity necessary to reach y_{max}, the maximum height, and $-v_0$ represents the final velocity, after falling from y_{max}. You can read all this information, and more, from the potential energy diagram we have shown.

Consider a mass-spring system on a frictionless, stationary, horizontal surface, so that gravity and the normal contact force do no work and can be ignored (Figure 7.5.2). This is like a one-dimensional system, whose mechanical energy E is a constant and whose potential energy, with respect to zero energy at zero displacement from the spring's unstretched length, x = 0, is $U(x) = \frac{1}{2}kx^2$.

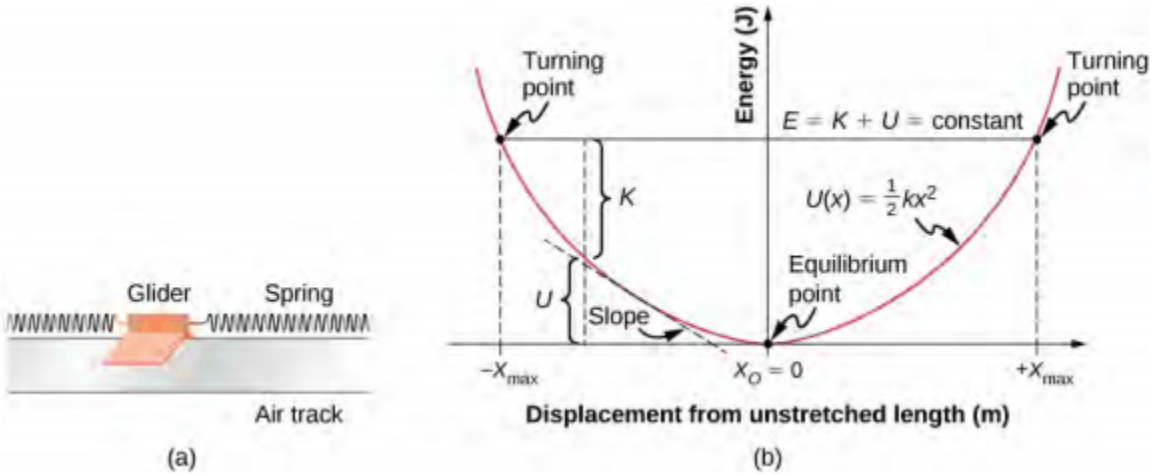


Figure 7.5.2: (a) A glider between springs on an air track is an example of a horizontal mass-spring system. (b) The potential energy diagram for this system, with various quantities indicated.

You can read off the same type of information from the potential energy diagram in this case, as in the case for the body in vertical free fall, but since the spring potential energy describes a variable force, you can learn more from this graph. As for the object in vertical free fall, you can deduce the physically allowable range of motion and the maximum values of distance and speed, from the limits on the kinetic energy, $0 \le K \le E$. Therefore, K = 0 and U = E at a **turning point**, of which there are two for the elastic spring potential energy,

$$x_{max} = \pm\sqrt{\frac{2E}{k}}. \tag{7.5.7}$$

The glider's motion is confined to the region between the turning points, $-x_{max} \le x \le x_{max}$. This is true for any (positive) value of E because the potential energy is unbounded with respect to x. For this reason, as well as the shape of the potential energy curve, U(x) is called an infinite potential well. At the bottom of the potential well, x = 0, U = 0 and the kinetic energy is a maximum, K = E, so $v_{max} = \pm\sqrt{\frac{2E}{m}}$.

However, from the slope of this potential energy curve, you can also deduce information about the force on the glider and its acceleration. We saw earlier that the negative of the slope of the potential energy is the spring force, which in this case is also the net force, and thus is proportional to the acceleration. When x = 0, the slope, the force, and the acceleration are all zero, so this is an **equilibrium point**. The negative of the slope, on either side of the equilibrium point, gives a force pointing back to the equilibrium point, F = ±kx, so the equilibrium is termed stable and the force is called a restoring force. This implies that U(x) has a relative minimum there. If the force on either side of an equilibrium point has a direction opposite from that direction of position change, the equilibrium is termed unstable, and this implies that U(x) has a relative maximum there.

Example 8.10: Quartic and Quadratic Potential Energy Diagram

The potential energy for a particle undergoing one-dimensional motion along the x-axis is $U(x) = 2(x^4 - x^2)$, where U is in joules and x is in meters. The particle is not subject to any non-conservative forces and its mechanical energy is constant at E = −0.25 J. (a) Is the motion of the particle confined to any regions on the x-axis, and if so, what are they? (b) Are there any equilibrium points, and if so, where are they and are they stable or unstable?

Strategy

First, we need to graph the potential energy as a function of x. The function is zero at the origin, becomes negative as x increases in the positive or negative directions (x^2 is larger than x^4 for x < 1), and then becomes positive at sufficiently large |x|. Your graph should look like a double potential well, with the zeros determined by solving the equation U(x) = 0, and the extremes determined by examining the first and second derivatives of U(x), as shown in Figure 7.5.3.

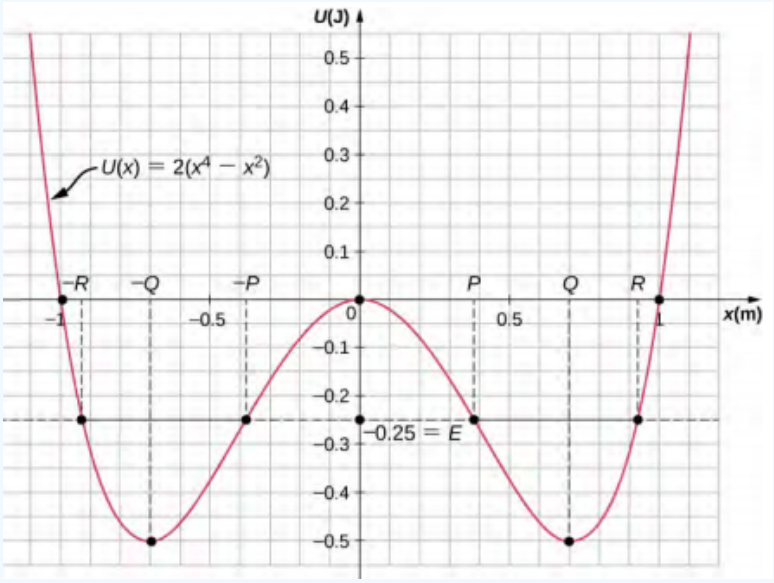


Figure 7.5.3: The potential energy graph for a one-dimensional, quartic and quadratic potential energy, with various quantities indicated.

You can find the values of (a) the allowed regions along the x-axis, for the given value of the mechanical energy, from the condition that the kinetic energy can't be negative, and (b) the equilibrium points and their stability from the properties of the force (stable for a relative minimum and unstable for a relative maximum of potential energy). You can just eyeball the graph to reach qualitative answers to the questions in this example. That, after all, is the value of potential energy diagrams.

You can see that there are two allowed regions for the motion (E > U) and three equilibrium points (slope $\frac{dU}{dx}$ = 0), of which the central one is unstable $\left(\frac{d^2U}{dx^2} < 0\right)$, and the other two are stable $\left(\frac{d^2U}{dx^2} > 0\right)$.

Solution

a. To find the allowed regions for x, we use the condition

$$K = E - U = -\frac{1}{4} - 2(x^4 - x^2) \geq 0. \tag{7.5.8}$$

If we complete the square in x 2 , this condition simplifies to $2\left(x^2 - \frac{1}{2}\right)^2 \leq \frac{1}{4}$, which we can solve to obtain

$$\frac{1}{2} - \sqrt{\frac{1}{8}} \leq x^2 \leq \frac{1}{2} + \sqrt{\frac{1}{8}}. \tag{7.5.9}$$

This represents two allowed regions, $x_p \leq x \leq x_R$ and $-x_R \leq x \leq -x_p$, where x_p = 0.38 and x_R = 0.92 (in meters).

b. To find the equilibrium points, we solve the equation

$$\frac{dU}{dx} = 8x^3 - 4x = 0 \tag{7.5.10}$$

and find x = 0 and x = ±x_Q, where $x_Q = \frac{1}{\sqrt{2}}$ = 0.707 (meters). The second derivative

$$\frac{d^2U}{dx^2} = 24x^2 - 4 \tag{7.5.11}$$

is negative at x = 0, so that position is a relative maximum and the equilibrium there is unstable. The second derivative is positive at x = ±x_Q, so these positions are relative minima and represent stable equilibria.

Significance

The particle in this example can oscillate in the allowed region about either of the two stable equilibrium points we found, but it does not have enough energy to escape from whichever potential well it happens to initially be in. The conservation of mechanical energy and the relations between kinetic energy and speed, and potential energy and force, enable you to deduce much information about the qualitative behavior of the motion of a particle, as well as some quantitative information, from a graph of its potential energy.

Exercise 8.10

Repeat Example 8.10 when the particle's mechanical energy is +0.25 J.

Before ending this section, let's practice applying the method based on the potential energy of a particle to find its position as a function of time, for the one-dimensional, mass-spring system considered earlier in this section.

Example 8.11: Sinusoidal Oscillations

Find x(t) for a particle moving with a constant mechanical energy E > 0 and a potential energy U(x) = $\frac{1}{2}kx^2$, when the particle starts from rest at time t = 0.

Strategy

We follow the same steps as we did in Example 8.9. Substitute the potential energy U into Equation 8.4.9 and factor out the constants, like m or k. Integrate the function and solve the resulting expression for position, which is now a function of time.

Solution

Substitute the potential energy in Equation 8.4.9 and integrate using an integral solver found on a web search:

\[t = \int_{x_{0}}^{x} \frac{dx}{\sqrt{\left(\dfrac{k}{m}\right) \Big[\left(\dfrac{2E}{k}\right) - x^{2} \Big]}} = \sqrt{\frac{m}{k}} \Bigg[\sin^{-1} \left(\dfrac{x}{\sqrt{\frac{2E}{k}}}\right) - \sin^{-1} \left(\frac{x_{0}}{\sqrt{\frac{2E}{k}}}\right) \Bigg] \ldotp$$From the initial conditions at t = 0, the initial kinetic energy is zero and the initial potential energy is $\frac{1}{2}kx_0^2$ = E, from which you can see that $\frac{x_0}{\sqrt{\left(\frac{2E}{k}\right)}}$ = ±1 and $\sin^{-1}(\pm)$ = ±90°. Now you can solve for x:

$$x(t) = \sqrt{\left(\frac{2E}{k}\right)} \sin\left[\left(\sqrt{\frac{k}{m}}\right) t \pm 90^o\right] = \pm\sqrt{\left(\frac{2E}{k}\right)} \cos\left[\left(\sqrt{\frac{k}{m}}\right) t\right]. \tag{7.5.12}$$

Significance

A few paragraphs earlier, we referred to this mass-spring system as an example of a harmonic oscillator. Here, we anticipate that a harmonic oscillator executes sinusoidal oscillations with a maximum displacement of $\sqrt{\left(\frac{2E}{k}\right)}$ (called the amplitude) and a rate of oscillation of $\left(\frac{1}{2\pi}\right)\sqrt{\frac{k}{m}}$ (called the frequency). Further discussions about oscillations can be found in Oscillations.

Exercise 8.11

Find x(t) for the mass-spring system in Example 8.11 if the particle starts from $x_0 = 0$ at $t = 0$. What is the particle's initial velocity?

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7.6: Sources of Energy

Learning Objectives

- Describe energy transformations and conversions in general terms
- Explain what it means for an energy source to be renewable or nonrenewable

In this section, we have studied energy. We learned that energy can take different forms and can be transferred from one form to another. You will find that energy is discussed in many everyday, as well as scientific, contexts, because it is involved in all physical processes. It will also become apparent that many situations are best understood, or most easily conceptualized, by considering energy. So far, no experimental results have contradicted the conservation of energy. In fact, whenever measurements have appeared to conflict with energy conservation, new forms of energy have been discovered or recognized in accordance with this principle.

What are some other forms of energy? Many of these are covered in later chapters (also see Figure 7.6.1), but let's detail a few here:

- Atoms and molecules inside all objects are in random motion. The internal kinetic energy from these random motions is called **thermal energy**, because it is related to the temperature of the object. Note that thermal energy can also be transferred from one place to another, not transformed or converted, by the familiar processes of conduction, convection, and radiation. In this case, the energy is known as **heat energy**.
- **Electrical energy** is a common form that is converted to many other forms and does work in a wide range of practical situations.
- Fuels, such as gasoline and food, have **chemical energy**, which is potential energy arising from their molecular structure. Chemical energy can be converted into thermal energy by reactions like oxidation. Chemical reactions can also produce electrical energy, such as in batteries. Electrical energy can, in turn, produce thermal energy and light, such as in an electric heater or a light bulb.
- Light is just one kind of electromagnetic radiation, or **radiant energy**, which also includes radio, infrared, ultraviolet, X-rays, and gamma rays. All bodies with thermal energy can radiate energy in electromagnetic waves.
- **Nuclear energy** comes from reactions and processes that convert measurable amounts of mass into energy. Nuclear energy is transformed into radiant energy in the Sun, into thermal energy in the boilers of nuclear power plants, and then into electrical energy in the generators of power plants. These and all other forms of energy can be transformed into one another and, to a certain degree, can be converted into mechanical work.

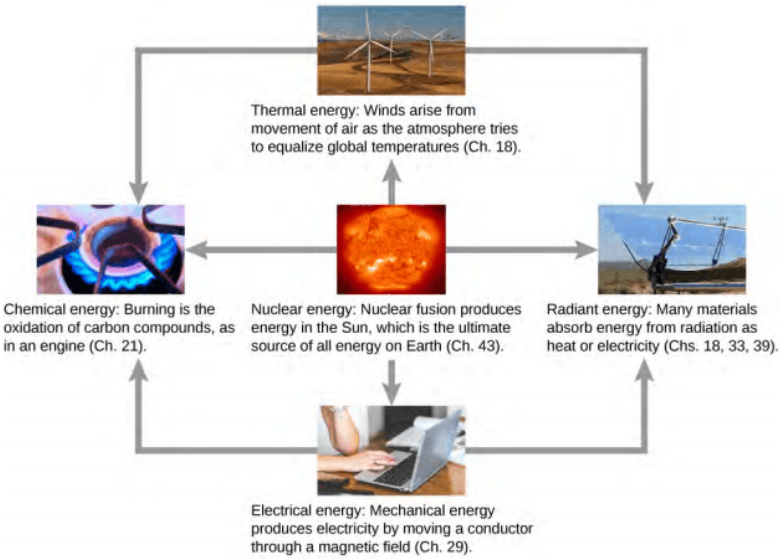


Figure 7.6.1: Energy that we use in society takes many forms, which be converted from one into another depending on the process involved. We will study many of these forms of energy in later chapters in this text. (credit "sun": EIT SOHO Consortium, ESA, NASA; credit "solar panels": "kjkolb"/Wikimedia Commons; credit "gas burner": Steven Depolo)

The transformation of energy from one form into another happens all the time. The chemical energy in food is converted into thermal energy through metabolism; light energy is converted into chemical energy through photosynthesis. Another example of energy conversion occurs in a solar cell. Sunlight impinging on a solar cell produces electricity, which can be used to run electric motors or heat water. In an example encompassing many steps, the chemical energy contained in coal is converted into thermal energy as it burns in a furnace, to transform water into steam, in a boiler. Some of the thermal energy in the steam is then converted into mechanical energy as it expands and spins a turbine, which is connected to a generator to produce electrical energy. In these examples, not all of the initial energy is converted into the forms mentioned, because some energy is always transferred to the environment.

Energy is an important element at all levels of society. We live in a very interdependent world, and access to adequate and reliable energy resources is crucial for economic growth and for maintaining the quality of our lives. The principal energy resources used in the world are shown in Figure 7.6.2. The figure distinguishes between two major types of energy sources: **renewable** and **non-renewable**, and further divides each type into a few more specific kinds. Renewable sources are energy sources that are replenished through naturally occurring, ongoing processes, on a time scale that is much shorter than the anticipated lifetime of the civilization using the source. Non-renewable sources are depleted once some of the energy they contain is extracted and converted into other kinds of energy. The natural processes by which non-renewable sources are formed typically take place over geological time scales.

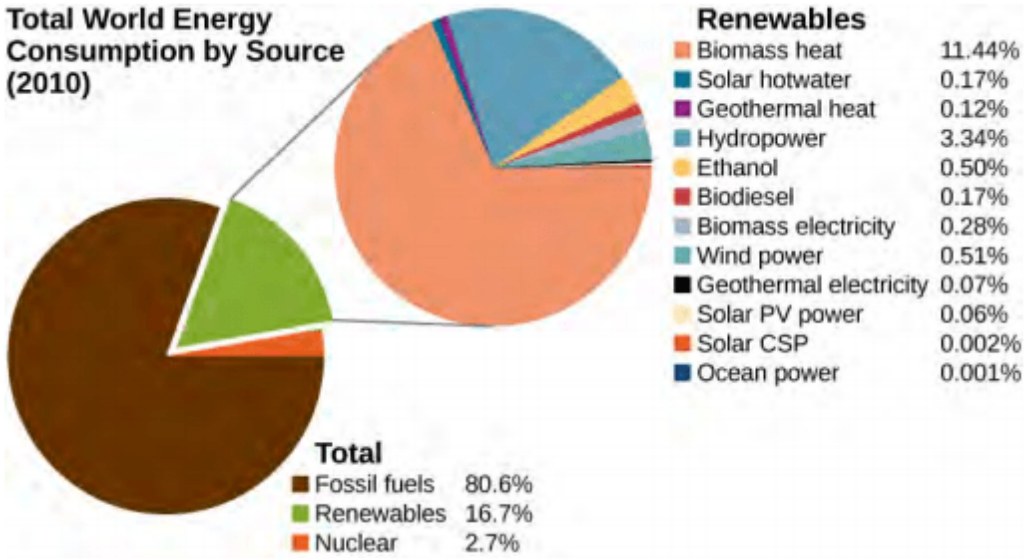


Figure 7.6.2: World energy consumption by source; the percentage of renewables is increasing, accounting for 19% in 2012.

Our most important non-renewable energy sources are fossil fuels, such as coal, petroleum, and natural gas. These account for about 81% of the world's energy consumption, as shown in the figure. Burning fossil fuels creates chemical reactions that transform potential energy, in the molecular structures of the reactants, into thermal energy and products. This thermal energy can be used to heat buildings or to operate steam-driven machinery. Internal combustion and jet engines convert some of the energy of rapidly expanding gases, released from burning gasoline, into mechanical work. Electrical power generation is mostly derived from transferring energy in expanding steam, via turbines, into mechanical work, which rotates coils of wire in magnetic fields to generate electricity. Nuclear energy is the other non-renewable source shown in Figure 7.6.2 and supplies about 3% of the world's consumption. Nuclear reactions release energy by transforming potential energy, in the structure of nuclei, into thermal energy, analogous to energy release in chemical reactions. The thermal energy obtained from nuclear reactions can be transferred and converted into other forms in the same ways that energy from fossil fuels are used.

An unfortunate byproduct of relying on energy produced from the combustion of fossil fuels is the release of carbon dioxide into the atmosphere and its contribution to global warming. Nuclear energy poses environmental problems as well, including the safety and disposal of nuclear waste. Besides these important consequences, reserves of non-renewable sources of energy are limited and, given the rapidly growing rate of world energy consumption, may not last for more than a few hundred years. Considerable effort is going on to develop and expand the use of renewable sources of energy, involving a significant percentage of the world's physicists and engineers.

Four of the renewable energy sources listed in Figure 7.6.2—those using material from plants as fuel (biomass heat, ethanol, biodiesel, and biomass electricity)—involve the same types of energy transformations and conversions as just discussed for fossil and nuclear fuels. The other major types of renewable energy sources are hydropower, wind power, geothermal power, and solar power.

Hydropower is produced by converting the gravitational potential energy of falling or flowing water into kinetic energy and then into work to run electric generators or machinery. Converting the mechanical energy in ocean surface waves and tides is in development. Wind power also converts kinetic energy into work, which can be used directly to generate electricity, operate mills, and propel sailboats.

The interior of Earth has a great deal of thermal energy, part of which is left over from its original formation (gravitational potential energy converted into thermal energy) and part of which is released from radioactive minerals (a form of natural nuclear energy). It will take a very long time for this geothermal energy to escape into space, so people generally regard it as a renewable source, when actually, it's just inexhaustible on human time scales.

The source of solar power is energy carried by the electromagnetic waves radiated by the Sun. Most of this energy is carried by visible light and infrared (heat) radiation. When suitable materials absorb electromagnetic waves, radiant energy is converted into thermal energy, which can be used to heat water, or when concentrated, to make steam and generate electricity (Figure 7.6.3). However, in another important physical process, known as the photoelectric effect, energetic radiation impinging on certain materials is directly converted into electricity. Materials that do this are called photovoltaics (PV in Figure 7.6.2). Some solar power

systems use lenses or mirrors to concentrate the Sun's rays, before converting their energy through photovoltaics, and these are qualified as CSP in Figure 7.6.2.



Figure 7.6.3: Solar cell arrays found in a sunny area converting the solar energy into stored electrical energy. (credit: Sarah Swenty)

As we finish this chapter on energy and work, it is relevant to draw some distinctions between two sometimes misunderstood terms in the area of energy use. As we mentioned earlier, the "law of conservation of energy" is a very useful principle in analyzing physical processes. It cannot be proven from basic principles but is a very good bookkeeping device, and no exceptions have ever been found. It states that the total amount of energy in an isolated system always remains constant. Related to this principle, but remarkably different from it, is the important philosophy of energy conservation. This concept has to do with seeking to decrease the amount of energy used by an individual or group through reducing activities (e.g., turning down thermostats, diving fewer kilometers) and/or increasing conversion efficiencies in the performance of a particular task, such as developing and using more efficient room heaters, cars that have greater miles-per-gallon ratings, energy-efficient compact fluorescent lights, etc.

Since energy in an isolated system is not destroyed, created, or generated, you might wonder why we need to be concerned about our energy resources, since energy is a conserved quantity. The problem is that the final result of most energy transformations is waste heat, that is, work that has been "degraded" in the energy transformation. We will discuss this idea in more detail in the chapters on thermodynamics.

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7.7: Potential Energy and Conservation of Energy (Exercises)

Conceptual Questions

8.1 Potential Energy of a System

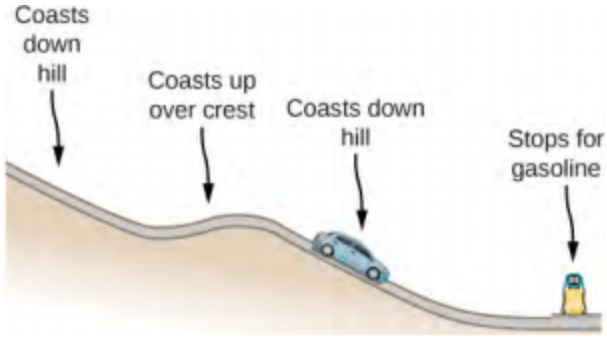
1. The kinetic energy of a system must always be positive or zero. Explain whether this is true for the potential energy of a system.
2. The force exerted by a diving board is conservative, provided the internal friction is negligible. Assuming friction is negligible, describe changes in the potential energy of a diving board as a swimmer drives from it, starting just before the swimmer steps on the board until just after his feet leave it.
3. Describe the gravitational potential energy transfers and transformations for a javelin, starting from the point at which an athlete picks up the javelin and ending when the javelin is stuck into the ground after being thrown.
4. A couple of soccer balls of equal mass are kicked off the ground at the same speed but at different angles. Soccer ball A is kicked off at an angle slightly above the horizontal, whereas ball B is kicked slightly below the vertical. How do each of the following compare for ball A and ball B? (a) The initial kinetic energy and (b) the change in gravitational potential energy from the ground to the highest point? If the energy in part (a) differs from part (b), explain why there is a difference between the two energies.
5. What is the dominant factor that affects the speed of an object that started from rest down a frictionless incline if the only work done on the object is from gravitational forces?
6. Two people observe a leaf falling from a tree. One person is standing on a ladder and the other is on the ground. If each person were to compare the energy of the leaf observed, would each person find the following to be the same or different for the leaf, from the point where it falls off the tree to when it hits the ground: (a) the kinetic energy of the leaf; (b) the change in gravitational potential energy; (c) the final gravitational potential energy?

8.2 Conservative and Non-Conservative Forces

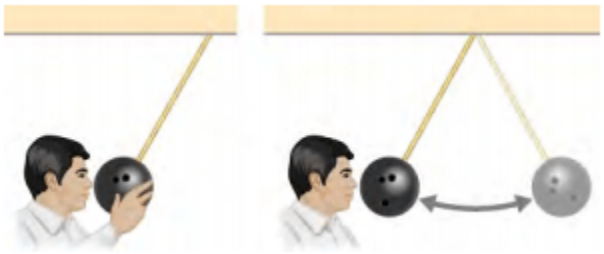
7. What is the physical meaning of a non-conservative force?
8. A bottle rocket is shot straight up in the air with a speed 30 m/s. If the air resistance is ignored, the bottle would go up to a height of approximately 46 m. However, the rocket goes up to only 35 m before returning to the ground. What happened? Explain, giving only a qualitative response.
9. An external force acts on a particle during a trip from one point to another and back to that same point. This particle is only effected by conservative forces. Does this particle's kinetic energy and potential energy change as a result of this trip?

8.3 Conservation of Energy

10. When a body slides down an inclined plane, does the work of friction depend on the body's initial speed? Answer the same question for a body sliding down a curved surface.
11. Consider the following scenario. A car for which friction is not negligible accelerates from rest down a hill, running out of gasoline after a short distance (see below). The driver lets the car coast farther down the hill, then up and over a small crest. He then coasts down that hill into a gas station, where he brakes to a stop and fills the tank with gasoline. Identify the forms of energy the car has, and how they are changed and transferred in this series of events.



12. A dropped ball bounces to one-half its original height. Discuss the energy transformations that take place.
13. " E = K + U constant is a special case of the work-energy theorem." Discuss this statement.
14. In a common physics demonstration, a bowling ball is suspended from the ceiling by a rope. The professor pulls the ball away from its equilibrium position and holds it adjacent to his nose, as shown below. He releases the ball so that it swings directly away from him. Does he get struck by the ball on its return swing? What is he trying to show in this demonstration?



15. A child jumps up and down on a bed, reaching a higher height after each bounce. Explain how the child can increase his maximum gravitational potential energy with each bounce.
16. Can a non-conservative force increase the mechanical energy of the system?
17. Neglecting air resistance, how much would I have to raise the vertical height if I wanted to double the impact speed of a falling object?
18. A box is dropped onto a spring at its equilibrium position. The spring compresses with the box attached and comes to rest. Since the spring is in the vertical position, does the change in the gravitational potential energy of the box while the spring is compressing need to be considered in this problem?

Problems

8.1 Potential Energy of a System

19. Using values from Table 8.2, how many DNA molecules could be broken by the energy carried by a single electron in the beam of an old-fashioned TV tube? (These electrons were not dangerous in themselves, but they did create dangerous X-rays. Later-model tube TVs had shielding that absorbed X-rays before they escaped and exposed viewers.)
20. If the energy in fusion bombs were used to supply the energy needs of the world, how many of the 9-megaton variety would be needed for a year's supply of energy (using data from Table 8.1)?
21. A camera weighing 10 N falls from a small drone hovering 20 m overhead and enters free fall. What is the gravitational potential energy change of the camera from the drone to the ground if you take a reference point of (a) the ground being zero gravitational potential energy? (b) The drone being zero gravitational potential energy? What is the gravitational potential energy of the camera (c) before it falls from the drone and (d) after the camera lands on the ground if the reference point of zero gravitational potential energy is taken to be a second person looking out of a building 30 m from the ground?
22. Someone drops a 50 – g pebble off of a docked cruise ship, 70.0 m from the water line. A person on a dock 3.0 m from the water line holds out a net to catch the pebble. (a) How much work is done on the pebble by gravity during the drop? (b) What is the change in the gravitational potential energy during the drop? If the gravitational potential energy is zero at the water line, what is the gravitational potential energy (c) when the pebble is dropped? (d) When it reaches

the net? What if the gravitational potential energy was 30.0 Joules at water level? (e) Find the answers to the same questions in (c) and (d).

23. A cat's crinkle ball toy of mass 15 g is thrown straight up with an initial speed of 3 m/s. Assume in this problem that air drag is negligible. (a) What is the kinetic energy of the ball as it leaves the hand? (b) How much work is done by the gravitational force during the ball's rise to its peak? (c) What is the change in the gravitational potential energy of the ball during the rise to its peak? (d) If the gravitational potential energy is taken to be zero at the point where it leaves your hand, what is the gravitational potential energy when it reaches the maximum height? (e) What if the gravitational potential energy is taken to be zero at the maximum height the ball reaches, what would the gravitational potential energy be when it leaves the hand? (f) What is the maximum height the ball reaches?

8.2 Conservative and Non-Conservative Forces

24. A force F(x) = (3.0/x) N acts on a particle as it moves along the positive x-axis. (a) How much work does the force do on the particle as it moves from x = 2.0 m to x = 5.0 m? (b) Picking a convenient reference point of the potential energy to be zero at x = ∞, find the potential energy for this force.
25. A force F(x) = ($-5.0x^2 + 7.0x$) N acts on a particle. (a) How much work does the force do on the particle as it moves from x = 2.0 m to x = 5.0 m? (b) Picking a convenient reference point of the potential energy to be zero at x = ∞, find the potential energy for this force.
26. Find the force corresponding to the potential energy $U(x) = -\frac{a}{x} + \frac{b}{x^2}$.
27. The potential energy function for either one of the two atoms in a diatomic molecule is often approximated by $U(x) = -\frac{a}{x^{12}} - \frac{b}{x^6}$ where x is the distance between the atoms. (a) At what distance of separation does the potential energy have a local minimum (not at x = ∞)? (b) What is the force on an atom at this separation? (c) How does the force vary with the separation distance?
28. A particle of mass 2.0 kg moves under the influence of the force $F(x) = \left(\frac{3}{\sqrt{x}}\right)$ N. If its speed at x = 2.0 m is v = 6.0 m/s, what is its speed at x = 7.0 m?
29. A particle of mass 2.0 kg moves under the influence of the force F(x) = ($-5x^2 + 7x$) N. If its speed at x = −4.0 m is v = 20.0 m/s, what is its speed at x = 4.0 m?
30. A crate on rollers is being pushed without frictional loss of energy across the floor of a freight car (see the following figure). The car is moving to the right with a constant speed v_0 . If the crate starts at rest relative to the freight car, then from the work-energy theorem, $Fd = \frac{mv^2}{2}$, where d, the distance the crate moves, and v, the speed of the crate, are both measured relative to the freight car. (a) To an observer at rest beside the tracks, what distance d′ is the crate pushed when it moves the distance d in the car? (b) What are the crate's initial and final speeds v_0' and v′ as measured by the observer beside the tracks? (c) Show that $Fd' = \frac{m(v')^2}{2} - \frac{m(v_0')^2}{2}$ and, consequently, that work is equal to the change in kinetic energy in both reference systems.

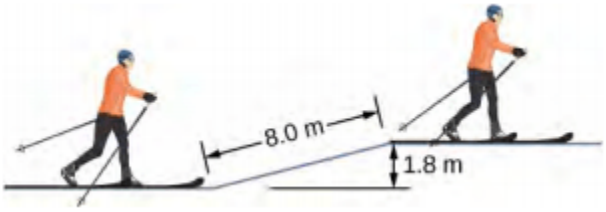


8.3 Conservation of Energy

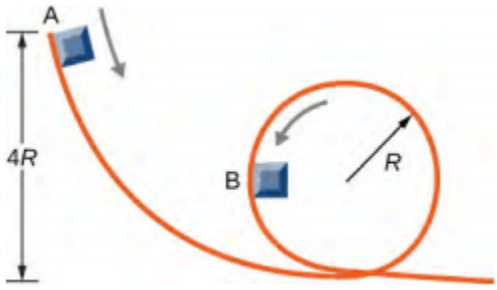
31. A boy throws a ball of mass 0.25 kg straight upward with an initial speed of 20 m/s When the ball returns to the boy, its speed is 17 m/s How much much work does air resistance do on the ball during its flight?
32. A mouse of mass 200 g falls 100 m down a vertical mine shaft and lands at the bottom with a speed of 8.0 m/s. During its fall, how much work is done on the mouse by air resistance?
33. Using energy considerations and assuming negligible air resistance, show that a rock thrown from a bridge 20.0 m above water with an initial speed of 15.0 m/s strikes the water with a speed of 24.8 m/s independent of the direction thrown. (**Hint**: show that $K_i + U_i = K_f + U_f$)
34. A 1.0-kg ball at the end of a 2.0-m string swings in a vertical plane. At its lowest point the ball is moving with a speed of

10 m/s. (a) What is its speed at the top of its path? (b) What is the tension in the string when the ball is at the bottom and at the top of its path?

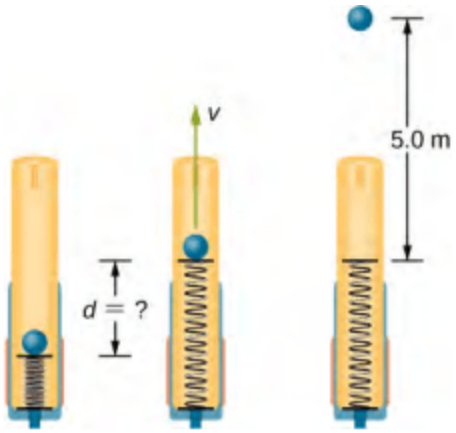
35. Ignoring details associated with friction, extra forces exerted by arm and leg muscles, and other factors, we can consider a pole vault as the conversion of an athlete's running kinetic energy to gravitational potential energy. If an athlete is to lift his body 4.8 m during a vault, what speed must he have when he plants his pole?
36. Tarzan grabs a vine hanging vertically from a tall tree when he is running at 9.0 m/s. (a) How high can he swing upward? (b) Does the length of the vine affect this height?
37. Assume that the force of a bow on an arrow behaves like the spring force. In aiming the arrow, an archer pulls the bow back 50 cm and holds it in position with a force of 150 N. If the mass of the arrow is 50 g and the "spring" is massless, what is the speed of the arrow immediately after it leaves the bow?
38. A 100 – kg man is skiing across level ground at a speed of 8.0 m/s when he comes to the small slope 1.8 m higher than ground level shown in the following figure. (a) If the skier coasts up the hill, what is his speed when he reaches the top plateau? Assume friction between the snow and skis is negligible. (b) What is his speed when he reaches the upper level if an 80 – N frictional force acts on the skis?



39. A sled of mass 70 kg starts from rest and slides down a 10° incline 80 m long. It then travels for 20 m horizontally before starting back up an 8° incline. It travels 80 m along this incline before coming to rest. What is the net work done on the sled by friction?
40. A girl on a skateboard (total mass of 40 kg) is moving at a speed of 10 m/s at the bottom of a long ramp. The ramp is inclined at 20° with respect to the horizontal. If she travels 14.2 m upward along the ramp before stopping, what is the net frictional force on her?
41. A baseball of mass 0.25 kg is hit at home plate with a speed of 40 m/s. When it lands in a seat in the left-field bleachers a horizontal distance 120 m from home plate, it is moving at 30 m/s. If the ball lands 20 m above the spot where it was hit, how much work is done on it by air resistance?
42. A small block of mass m slides without friction around the loop-the-loop apparatus shown below. (a) If the block starts from rest at A, what is its speed at B? (b) What is the force of the track on the block at B?



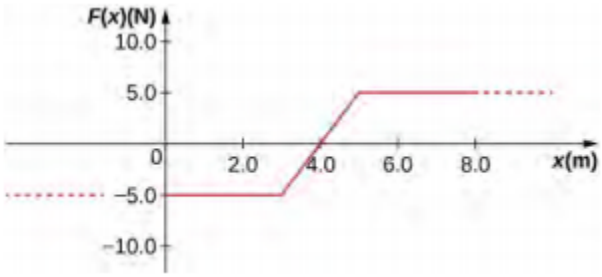
43. The massless spring of a spring gun has a force constant k = 12 N/cm. When the gun is aimed vertically, a 15-g projectile is shot to a height of 5.0 m above the end of the expanded spring. (See below.) How much was the spring compressed initially?



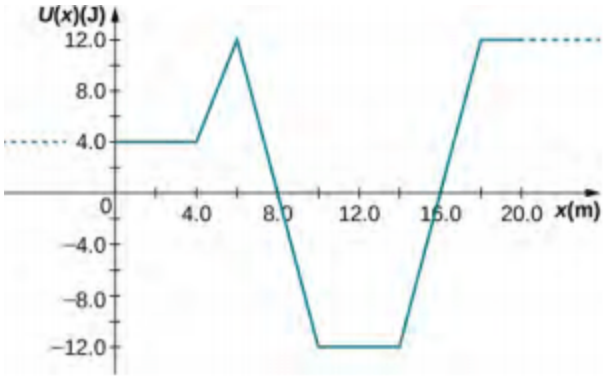
44. A small ball is tied to a string and set rotating with negligible friction in a vertical circle. If the ball moves over the top of the circle at its slowest possible speed (so that the tension in the string is negligible), what is the tension in the string at the bottom of the circle, assuming there is no additional energy added to the ball during rotation?

8.4 Potential Energy Diagrams and Stability

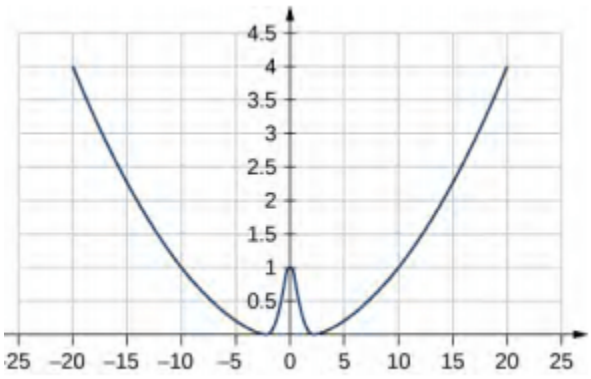
45. A mysterious constant force of 10 N acts horizontally on everything. The direction of the force is found to be always pointed toward a wall in a big hall. Find the potential energy of a particle due to this force when it is at a distance x from the wall, assuming the potential energy at the wall to be zero.
46. A single force $F(x) = -4.0x$ (in newtons) acts on a 1.0-kg body. When x = 3.5 m, the speed of the body is 4.0 m/s. What is its speed at x = 2.0 m?
47. A particle of mass 4.0 kg is constrained to move along the x-axis under a single force $F(x) = -cx^3$, where $c = 8.0\ N/m^3$. The particle's speed at A, where $x_A = 1.0$ m, is 6.0 m/s. What is its speed at B, where $x_B = -2.0$ m?
48. The force on a particle of mass 2.0 kg varies with position according to $F(x) = -3.0x^2$ (x in meters, F(x) in newtons). The particle's velocity at x = 2.0 m is 5.0 m/s. Calculate the mechanical energy of the particle using (a) the origin as the reference point and (b) x = 4.0 m as the reference point. (c) Find the particle's velocity at x = 1.0 m. Do this part of the problem for each reference point.
49. A 4.0-kg particle moving along the x-axis is acted upon by the force whose functional form appears below. The velocity of the particle at x = 0 is v = 6.0 m/s. Find the particle's speed at x = (a) 2.0 m, (b) 4.0 m, (c) 10.0 m, (d) Does the particle turn around at some point and head back toward the origin? (e) Repeat part (d) if v = 2.0 m/s at x = 0.



50. A particle of mass 0.50 kg moves along the x-axis with a potential energy whose dependence on x is shown below. (a) What is the force on the particle at x = 2.0, 5.0, 8.0, and 12 m? (b) If the total mechanical energy E of the particle is −6.0 J, what are the minimum and maximum positions of the particle? (c) What are these positions if E = 2.0 J? (d) If E = 16 J, what are the speeds of the particle at the positions listed in part (a)?



51. (a) Sketch a graph of the potential energy function U(x) = $\frac{kx^2}{2} + Ae^{-\alpha x^2}$, where k, A, and α are constants. (b) What is the force corresponding to this potential energy? (c) Suppose a particle of mass m moving with this potential energy has a velocity v_a when its position is x = a. Show that the particle does not pass through the origin unless $A \leq \frac{mv_a^2+ka^2}{2\left(1-e^{-\alpha a^2}\right)}$.

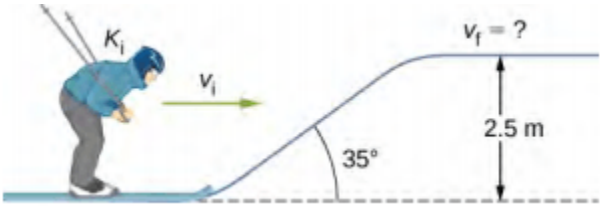


8.5 Sources of Energy

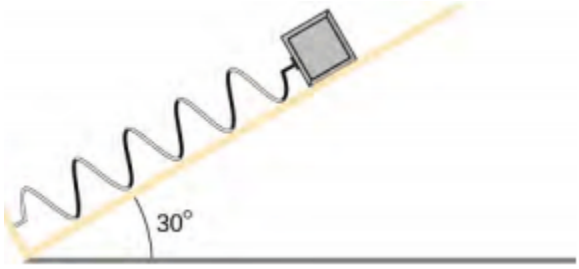
52. In the cartoon movie Pocahontas (https://openstaxcollege.org/l/21pocahontclip), Pocahontas runs to the edge of a cliff and jumps off, showcasing the fun side of her personality. (a) If she is running at 3.0 m/s before jumping off the cliff and she hits the water at the bottom of the cliff at 20.0 m/s, how high is the cliff? Assume negligible air drag in this cartoon. (b) If she jumped off the same cliff from a standstill, how fast would she be falling right before she hit the water?
53. In the reality television show “Amazing Race” (https://openstaxcollege.org/l/21amazraceclip), a contestant is firing 12-kg watermelons from a slingshot to hit targets down the field. The slingshot is pulled back 1.5 m and the watermelon is considered to be at ground level. The launch point is 0.3 m from the ground and the targets are 10 m horizontally away. Calculate the spring constant of the slingshot.
54. In the Back to the Future movies (https://openstaxcollege.org/l/21bactofutclip), a DeLorean car of mass 1230 kg travels at 88 miles per hour to venture back to the future. (a) What is the kinetic energy of the DeLorian? (b) What spring constant would be needed to stop this DeLorean in a distance of 0.1m?
55. In the Hunger Games movie (https://openstaxcollege.org/l/21HungGamesclip), Katniss Everdeen fires a 0.0200-kg arrow from ground level to pierce an apple up on a stage. The spring constant of the bow is 330 N/m and she pulls the arrow back a distance of 0.55 m. The apple on the stage is 5.00 m higher than the launching point of the arrow. At what speed does the arrow (a) leave the bow? (b) strike the apple?
56. In a “Top Fail” video (https://openstaxcollege.org/l/21topfailvideo), two women run at each other and collide by hitting exercise balls together. If each woman has a mass of 50 kg, which includes the exercise ball, and one woman runs to the right at 2.0 m/s and the other is running toward her at 1.0 m/s, (a) how much total kinetic energy is there in the system? (b) If energy is conserved after the collision and each exercise ball has a mass of 2.0 kg, how fast would the balls fly off toward the camera?
57. In a Coyote/Road Runner cartoon clip (https://openstaxcollege.org/l/21coyroadcarcl), a spring expands quickly and sends the coyote into a rock. If the spring extended 5 m and sent the coyote of mass 20 kg to a speed of 15 m/s, (a)

what is the spring constant of this spring? (b) If the coyote were sent vertically into the air with the energy given to him by the spring, how high could he go if there were no non-conservative forces?

58. In an iconic movie scene, Forrest Gump (https://openstaxcollege.org/l/21ForrGumpvid) runs around the country. If he is running at a constant speed of 3 m/s, would it take him more or less energy to run uphill or downhill and why?
59. In the movie Monty Python and the Holy Grail (https://openstaxcollege.org/l/21monpytmovcl) a cow is catapulted from the top of a castle wall over to the people down below. The gravitational potential energy is set to zero at ground level. The cow is launched from a spring of spring constant 1.1×10^4 N/m that is expanded 0.5 m from equilibrium. If the castle is 9.1 m tall and the mass of the cow is 110 kg, (a) what is the gravitational potential energy of the cow at the top of the castle? (b) What is the elastic spring energy of the cow before the catapult is released? (c) What is the speed of the cow right before it lands on the ground?
60. A 60.0-kg skier with an initial speed of 12.0 m/s coasts up a 2.50-m high rise as shown. Find her final speed at the top, given that the coefficient of friction between her skis and the snow is 0.80.



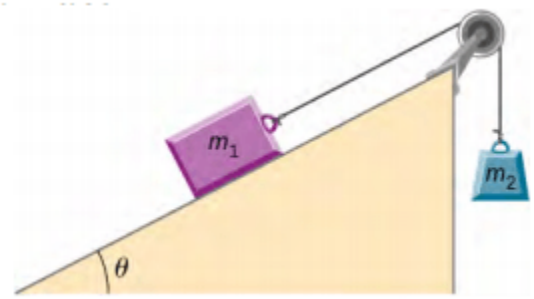
61. (a) How high a hill can a car coast up (engines disengaged) if work done by friction is negligible and its initial speed is 110 km/h? (b) If, in actuality, a 750-kg car with an initial speed of 110 km/h is observed to coast up a hill to a height 22.0 m above its starting point, how much thermal energy was generated by friction? (c) What is the average force of friction if the hill has a slope of 2.5° above the horizontal?
62. A 5.00×10^5-kg subway train is brought to a stop from a speed of 0.500 m/s in 0.400 m by a large spring bumper at the end of its track. What is the spring constant k of the spring?
63. A pogo stick has a spring with a spring constant of 2.5×10^4 N/m, which can be compressed 12.0 cm. To what maximum height from the uncompressed spring can a child jump on the stick using only the energy in the spring, if the child and stick have a total mass of 40 kg?
64. A block of mass 500 g is attached to a spring of spring constant 80 N/m (see the following figure). The other end of the spring is attached to a support while the mass rests on a rough surface with a coefficient of friction of 0.20 that is inclined at angle of 30°. The block is pushed along the surface till the spring compresses by 10 cm and is then released from rest. (a) How much potential energy was stored in the block-spring-support system when the block was just released? (b) Determine the speed of the block when it crosses the point when the spring is neither compressed nor stretched. (c) Determine the position of the block where it just comes to rest on its way up the incline.



65. A block of mass 200 g is attached at the end of a massless spring at equilibrium length of spring constant 50 N/m. The other end of the spring is attached to the ceiling and the mass is released at a height considered to be where the gravitational potential energy is zero. (a) What is the net potential energy of the block at the instant the block is at the lowest point? (b) What is the net potential energy of the block at the midpoint of its descent? (c) What is the speed of the block at the midpoint of its descent?
66. A T-shirt cannon launches a shirt at 5.00 m/s from a platform height of 3.00 m from ground level. How fast will the shirt be traveling if it is caught by someone whose hands are (a) 1.00 m from ground level? (b) 4.00 m from ground level? Neglect air drag.
67. A child (32 kg) jumps up and down on a trampoline. The trampoline exerts a spring restoring force on the child with a constant of 5000 N/m. At the highest point of the bounce, the child is 1.0 m above the level surface of the trampoline.

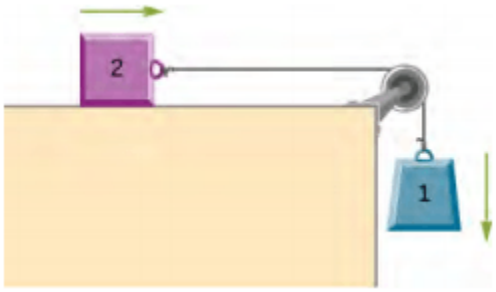
What is the compression distance of the trampoline? Neglect the bending of the legs or any transfer of energy of the child into the trampoline while jumping.

68. Shown below is a box of mass m1 that sits on a frictionless incline at an angle above the horizontal θ. This box is connected by a relatively massless string, over a frictionless pulley, and finally connected to a box at rest over the ledge, labeled m_2. If m_1 and m_2 are a height h above the ground and $m_2 >> m_1$: (a) What is the initial gravitational potential energy of the system? (b) What is the final kinetic energy of the system?

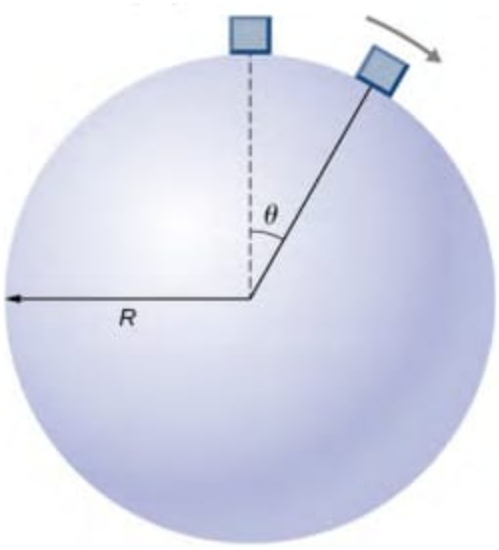


Additional Problems

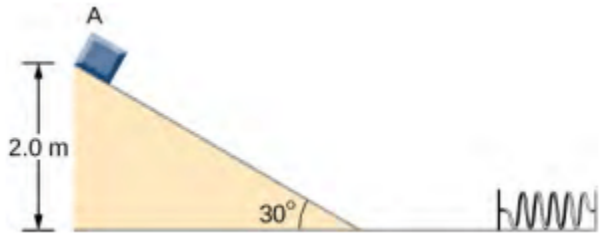
69. A massless spring with force constant k = 200 N/m hangs from the ceiling. A 2.0-kg block is attached to the free end of the spring and released. If the block falls 17 cm before starting back upwards, how much work is done by friction during its descent?
70. A particle of mass 2.0 kg moves under the influence of the force $F(x) = (-5x^2 + 7x)$ N. Suppose a frictional force also acts on the particle. If the particle's speed when it starts at x = –4.0 m is 0.0 m/s and when it arrives at x = 4.0 m is 9.0 m/s, how much work is done on it by the frictional force between x = –4.0 m and x = 4.0 m?
71. Block 2 shown below slides along a frictionless table as block 1 falls. Both blocks are attached by a frictionless pulley. Find the speed of the blocks after they have each moved 2.0 m. Assume that they start at rest and that the pulley has negligible mass. Use m_1 = 2.0 kg and m_2 = 4.0 kg.



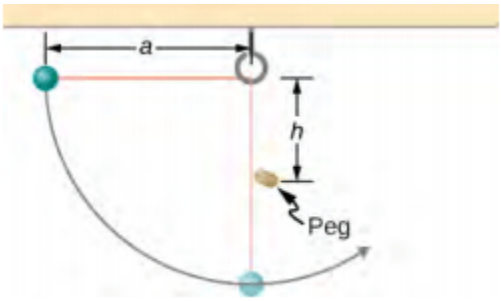
72. A body of mass m and negligible size starts from rest and slides down the surface of a frictionless solid sphere of radius R. (See below.) Prove that the body leaves the sphere when $\theta = \cos^{-1}(2/3)$.



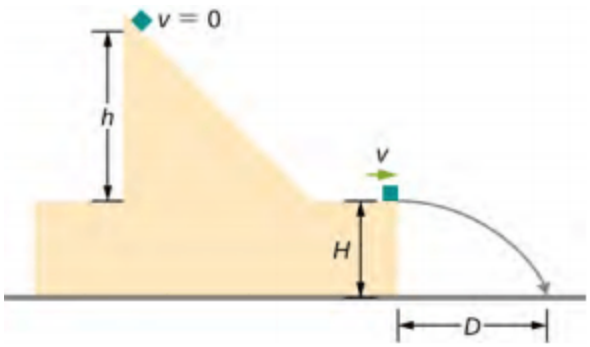
73. A mysterious force acts on all particles along a particular line and always points towards a particular point P on the line. The magnitude of the force on a particle increases as the cube of the distance from that point; that is $F \propto r^3$, if the distance from P to the position of the particle is r. Let b be the proportionality constant, and write the magnitude of the force as $F = br^3$. Find the potential energy of a particle subjected to this force when the particle is at a distance D from P, assuming the potential energy to be zero when the particle is at P.
74. An object of mass 10 kg is released at point A, slides to the bottom of the 30° incline, then collides with a horizontal massless spring, compressing it a maximum distance of 0.75 m. (See below.) The spring constant is 500 M/m, the height of the incline is 2.0 m, and the horizontal surface is frictionless. (a) What is the speed of the object at the bottom of the incline? (b) What is the work of friction on the object while it is on the incline? (c) The spring recoils and sends the object back toward the incline. What is the speed of the object when it reaches the base of the incline? (d) What vertical distance does it move back up the incline?



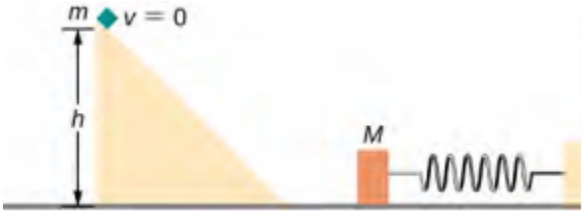
75. Shown below is a small ball of mass m attached to a string of length a. A small peg is located a distance h below the point where the string is supported. If the ball is released when the string is horizontal, show that h must be greater than 3a/5 if the ball is to swing completely around the peg.



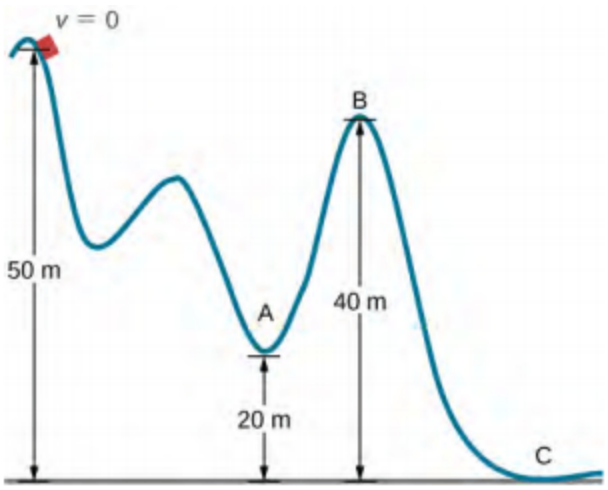
76. A block leaves a frictionless inclined surface horizontally after dropping off by a height h. Find the horizontal distance D where it will land on the floor, in terms of h, H, and g



77. A block of mass m, after sliding down a frictionless incline, strikes another block of mass M that is attached to a spring of spring constant k (see below). The blocks stick together upon impact and travel together. (a) Find the compression of the spring in terms of m, M, h, g, and k when the combination comes to rest. (b) The loss of kinetic energy as a result of the bonding of the two masses upon impact is stored in the so-called binding energy of the two masses. Calculate the binding energy.



78. A block of mass 300 g is attached to a spring of spring constant 100 N/m. The other end of the spring is attached to a support while the block rests on a smooth horizontal table and can slide freely without any friction. The block is pushed horizontally till the spring compresses by 12 cm, and then the block is released from rest. (a) How much potential energy was stored in the block-spring support system when the block was just released? (b) Determine the speed of the block when it crosses the point when the spring is neither compressed nor stretched. (c) Determine the speed of the block when it has traveled a distance of 20 cm from where it was released.
79. Consider a block of mass 0.200 kg attached to a spring of spring constant 100 N/m. The block is placed on a frictionless table, and the other end of the spring is attached to the wall so that the spring is level with the table. The block is then pushed in so that the spring is compressed by 10.0 cm. Find the speed of the block as it crosses (a) the point when the spring is not stretched, (b) 5.00 cm to the left of point in (a), and (c) 5.00 cm to the right of point in (a).
80. A skier starts from rest and slides downhill. What will be the speed of the skier if he drops by 20 meters in vertical height? Ignore any air resistance (which will, in reality, be quite a lot), and any friction between the skis and the snow.
81. Repeat the preceding problem, but this time, suppose that the work done by air resistance cannot be ignored. Let the work done by the air resistance when the skier goes from A to B along the given hilly path be –2000 J. The work done by air resistance is negative since the air resistance acts in the opposite direction to the displacement. Supposing the mass of the skier is 50 kg, what is the speed of the skier at point B?
82. Two bodies are interacting by a conservative force. Show that the mechanical energy of an isolated system consisting of two bodies interacting with a conservative force is conserved. (**Hint**: Start by using Newton's third law and the definition of work to find the work done on each body by the conservative force.)
83. In an amusement park, a car rolls in a track as shown below. Find the speed of the car at A, B, and C. Note that the work done by the rolling friction is zero since the displacement of the point at which the rolling friction acts on the tires is momentarily at rest and therefore has a zero displacement.



84. A 200-g steel ball is tied to a 2.00-m “massless” string and hung from the ceiling to make a pendulum, and then, the ball is brought to a position making a 30° angle with the vertical direction and released from rest. Ignoring the effects of the air resistance, find the speed of the ball when the string (a) is vertically down, (b) makes an angle of 20° with the vertical and (c) makes an angle of 10° with the vertical.
85. A hockey puck is shot across an ice-covered pond. Before the hockey puck was hit, the puck was at rest. After the hit, the puck has a speed of 40 m/s. The puck comes to rest after going a distance of 30 m. (a) Describe how the energy of the puck changes over time, giving the numerical values of any work or energy involved. (b) Find the magnitude of the net friction force.
86. A projectile of mass 2 kg is fired with a speed of 20 m/s at an angle of 30° with respect to the horizontal. (a) Calculate the initial total energy of the projectile given that the reference point of zero gravitational potential energy at the launch position. (b) Calculate the kinetic energy at the highest vertical position of the projectile. (c) Calculate the gravitational potential energy at the highest vertical position. (d) Calculate the maximum height that the projectile reaches. Compare this result by solving the same problem using your knowledge of projectile motion.
87. An artillery shell is fired at a target 200 m above the ground. When the shell is 100 m in the air, it has a speed of 100 m/s. What is its speed when it hits its target? Neglect air friction.
88. How much energy is lost to a dissipative drag force if a 60-kg person falls at a constant speed for 15 meters?
89. A box slides on a frictionless surface with a total energy of 50 J. It hits a spring and compresses the spring a distance of 25 cm from equilibrium. If the same box with the same initial energy slides on a rough surface, it only compresses the spring a distance of 15 cm, how much energy must have been lost by sliding on the rough surface?

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7.8: Potential Energy and Conservation of Energy (Summary)

Key Terms

conservative force	force that does work independent of path
conserved quantity	one that cannot be created or destroyed, but may be transformed between different forms of itself
energy conservation	total energy of an isolated system is constant
equilibrium point	position where the assumed conservative, net force on a particle, given by the slope of its potential energy curve, is zero
exact differential	is the total differential of a function and requires the use of partial derivatives if the function involves more than one dimension
mechanical energy	sum of the kinetic and potential energies
non-conservative force	force that does work that depends on path
non-renewable	energy source that is not renewable, but is depleted by human consumption
potential energy	function of position, energy possessed by an object relative to the system considered
potential energy diagram	graph of a particle's potential energy as a function of position
potential energy difference	negative of the work done acting between two points in space
renewable	energy source that is replenished by natural processes, over human time scales
turning point	position where the velocity of a particle, in one-dimensional motion, changes sign

Key Equations

Difference of potential energy	$\Delta U_{AB} = U_B - U_A = -W_{AB}$
Potential energy with respect to zero of potential energy at $\vec{r}$	$\vec{r}_0 \Delta U = U(\vec{r}) - U(\vec{r}_0)$
Gravitational potential energy near Earth's surface	$U(y) = mgy + const.$
Potential energy for an ideal spring	$U(x) = \frac{1}{2}kx^2 + const.$

Work done by conservative force over a closed path	$W_{closed\ path} = \oint \vec{E}_{cons} \cdot d\vec{r} = 0$
Condition for conservative force in two dimensions	$\left(\frac{dF_x}{dy}\right) = \left(\frac{dF_y}{dx}\right)$
Conservative force is the negative derivative of potential energy	$F_l = -\frac{dU}{dl}$
Conservation of energy with no non-conservative forces	$0 = W_{nc,\ AB} = \Delta(K+U)_{AB} = \Delta E_{AB}.$

Summary

8.1 Potential Energy of a System

- For a single-particle system, the difference of potential energy is the opposite of the work done by the forces acting on the particle as it moves from one position to another.
- Since only differences of potential energy are physically meaningful, the zero of the potential energy function can be chosen at a convenient location.
- The potential energies for Earth's constant gravity, near its surface, and for a Hooke's law force are linear and quadratic functions of position, respectively.

8.2 Conservative and Non-Conservative Forces

- A conservative force is one for which the work done is independent of path. Equivalently, a force is conservative if the work done over any closed path is zero.
- A non-conservative force is one for which the work done depends on the path.
- For a conservative force, the infinitesimal work is an exact differential. This implies conditions on the derivatives of the force's components.
- The component of a conservative force, in a particular direction, equals the negative of the derivative of the potential energy for that force, with respect to a displacement in that direction.

8.3 Conservation of Energy

- A conserved quantity is a physical property that stays constant regardless of the path taken.
- A form of the work-energy theorem says that the change in the mechanical energy of a particle equals the work done on it by non-conservative forces.
- If non-conservative forces do no work and there are no external forces, the mechanical energy of a particle stays constant. This is a statement of the conservation of mechanical energy and there is no change in the total mechanical energy.
- For one-dimensional particle motion, in which the mechanical energy is constant and the potential energy is known, the particle's position, as a function of time, can be found by evaluating an integral that is derived from the conservation of mechanical energy.

8.4 Potential Energy Diagrams and Stability

- Interpreting a one-dimensional potential energy diagram allows you to obtain qualitative, and some quantitative, information about the motion of a particle.
- At a turning point, the potential energy equals the mechanical energy and the kinetic energy is zero, indicating that the direction of the velocity reverses there.
- The negative of the slope of the potential energy curve, for a particle, equals the one-dimensional component of the conservative force on the particle. At an equilibrium point, the slope is zero and is a stable (unstable) equilibrium for a potential energy minimum (maximum).

8.5 Sources of Energy

- Energy can be transferred from one system to another and transformed or converted from one type into another. Some of the basic types of energy are kinetic, potential, thermal, and electromagnetic.
- Renewable energy sources are those that are replenished by ongoing natural processes, over human time scales. Examples are wind, water, geothermal, and solar power.
- Non-renewable energy sources are those that are depleted by consumption, over human time scales. Examples are fossil fuel and nuclear power.

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8: ELECTRIC CHARGES AND FIELDS

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8.2: Electric Charge
8.3: Conductors, Insulators, and Charging by Induction
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8.5: Electric Field
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8.1: Prelude to Electric Charges and Fields

Back when we were studying Newton's laws, we identified several physical phenomena as forces. We did so based on the effect they had on a physical object: Specifically, they caused the object to accelerate. Later, when we studied impulse and momentum, we expanded this idea to identify a force as any physical phenomenon that changed the momentum of an object. In either case, the result is the same: We recognize a force by the effect that it has on an object.



Figure 8.1.1: Electric charges exist all around us. They can cause objects to be repelled from each other or to be attracted to each other. (credit: modification of work by Sean McGrath)

In Gravitation, we examined the force of gravity, which acts on all objects with mass. In this chapter, we begin the study of the electric force, which acts on all objects with a property called charge. The electric force is much stronger than gravity (in most systems where both appear), but it can be a force of attraction or a force of repulsion, which leads to very different effects on objects. The electric force helps keep atoms together, so it is of fundamental importance in matter. But it also governs most everyday interactions we deal with, from chemical interactions to biological processes.

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8.2: Electric Charge

Learning Objectives

By the end of this section, you will be able to:

- Describe the concept of electric charge
- Explain qualitatively the force electric charge creates

You are certainly familiar with electronic devices that you activate with the click of a switch, from computers to cell phones to television. And you have certainly seen electricity in a flash of lightning during a heavy thunderstorm. But you have also most likely experienced electrical effects in other ways, maybe without realizing that an electric force was involved. Let's take a look at some of these activities and see what we can learn from them about electric charges and forces.

Discoveries

You have probably experienced the phenomenon of **static electricity**: When you first take clothes out of a dryer, many (not all) of them tend to stick together; for some fabrics, they can be very difficult to separate. Another example occurs if you take a woolen sweater off quickly—you can feel (and hear) the static electricity pulling on your clothes, and perhaps even your hair. If you comb your hair on a dry day and then put the comb close to a thin stream of water coming out of a faucet, you will find that the water stream bends toward (is attracted to) the comb (Figure 8.2.1).

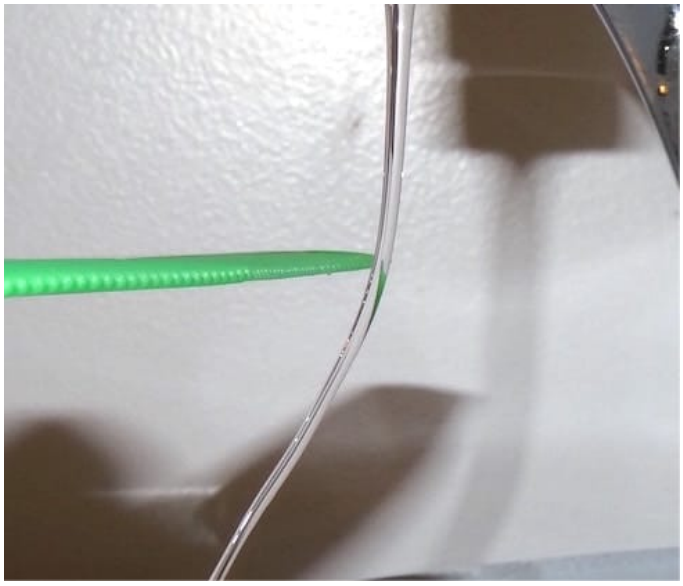


Figure 8.2.1: An electrically charged comb attracts a stream of water from a distance. Note that the water is not touching the comb. (credit: Jane Whitney)

Suppose you bring the comb close to some small strips of paper; the strips of paper are attracted to the comb and even cling to it (Figure 8.2.2). In the kitchen, quickly pull a length of plastic cling wrap off the roll; it will tend to cling to most any nonmetallic material (such as plastic, glass, or food). If you rub a balloon on a wall for a few seconds, it will stick to the wall. Probably the most annoying effect of static electricity is getting shocked by a doorknob (or a friend) after shuffling your feet on some types of carpeting.

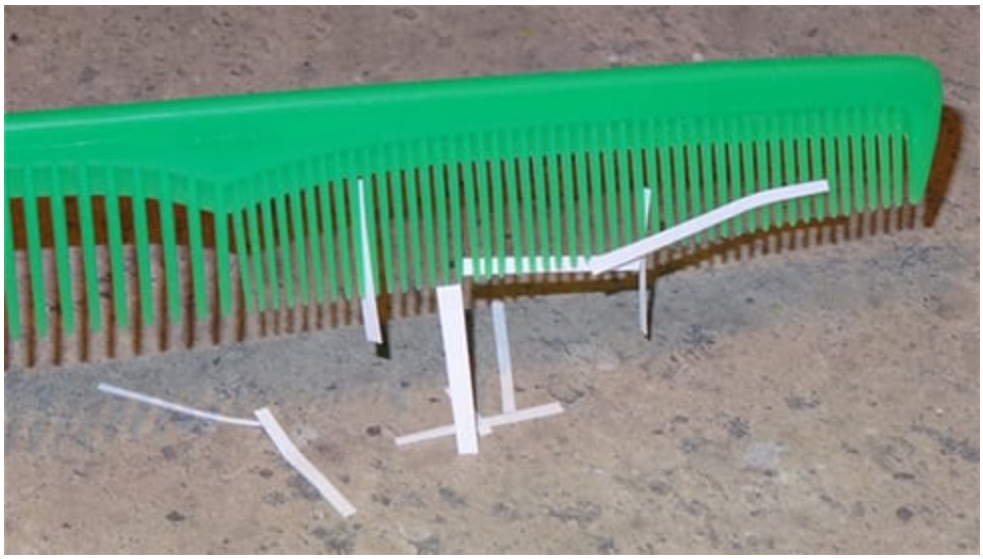


Figure 8.2.2: After being used to comb hair, this comb attracts small strips of paper from a distance, without physical contact. Investigation of this behavior helped lead to the concept of the electric force.

Many of these phenomena have been known for centuries. The ancient Greek philosopher Thales of Miletus (624–546 BCE) recorded that when amber (a hard, translucent, fossilized resin from extinct trees) was vigorously rubbed with a piece of fur, a force was created that caused the fur and the amber to be attracted to each other (Figure 8.2.3). Additionally, he found that the rubbed amber would not only attract the fur, and the fur attract the amber, but they both could affect other (nonmetallic) objects, even if not in contact with those objects (Figure 8.2.4).



Figure 8.2.3: Borneo amber is mined in Sabah, Malaysia, from shale-sandstone-mudstone veins. When a piece of amber is rubbed with a piece of fur, the amber gains more electrons, giving it a net negative charge. At the same time, the fur, having lost electrons, becomes positively charged. (credit: "Sebakoamber"/Wikimedia Commons)

The English physicist William Gilbert (1544–1603) also studied this attractive force, using various substances. He worked with amber, and, in addition, he experimented with rock crystal and various precious and semi-precious gemstones. He also experimented with several metals. He found that the metals never exhibited this force, whereas the minerals did. Moreover, although an electrified amber rod would attract a piece of fur, it would repel another electrified amber rod; similarly, two electrified pieces of fur would repel each other.

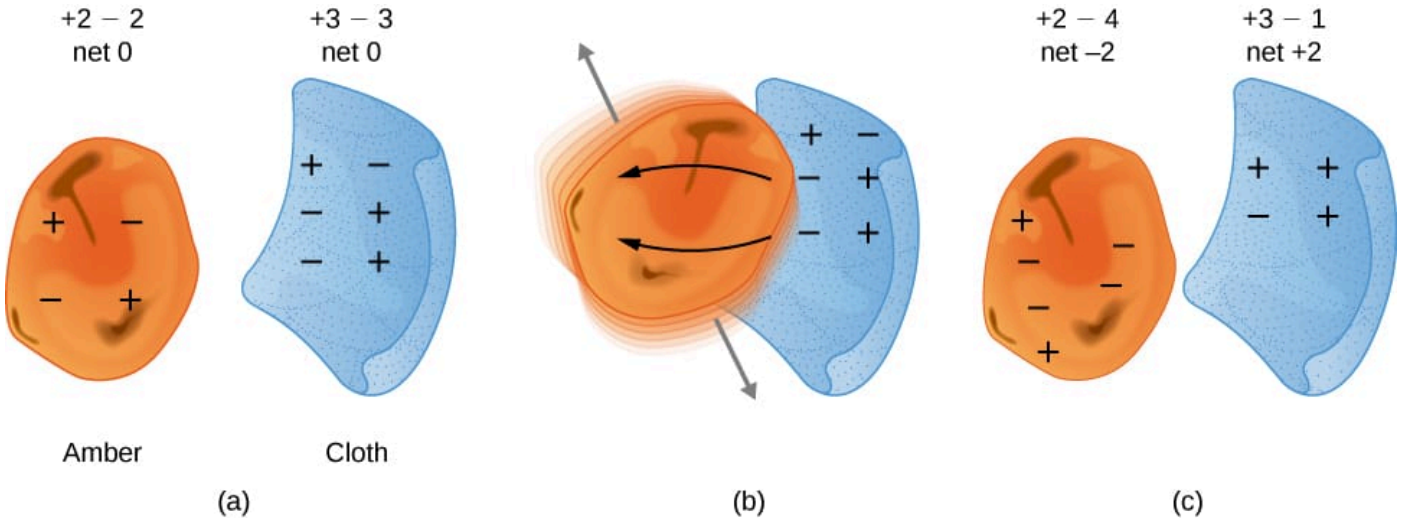


Figure 8.2.4: When materials are rubbed together, charges can be separated, particularly if one material has a greater affinity for electrons than another. (a) Both the amber and cloth are originally neutral, with equal positive and negative charges. Only a tiny fraction of the charges are involved, and only a few of them are shown here. (b) When rubbed together, some negative charge is transferred to the amber, leaving the cloth with a net positive charge. (c) When separated, the amber and cloth now have net charges, but the absolute value of the net positive and negative charges will be equal.

This suggested there were two types of an electric property; this property eventually came to be called **electric charge**. The difference between the two types of electric charge is in the directions of the electric forces that each type of charge causes: These forces are repulsive when the same type of charge exists on two interacting objects and attractive when the charges are of opposite types. The SI unit of electric charge is the **coulomb** (C), after the French physicist Charles Augustine de Coulomb (1736–1806).

The most peculiar aspect of this new force is that it does not require physical contact between the two objects in order to cause an acceleration. This is an example of a so-called “long-range” force. (Or, as James Clerk Maxwell later phrased it, “action at a distance.”) With the exception of gravity, all other forces we have discussed so far act only when the two interacting objects actually touch.

The American physicist and statesman Benjamin Franklin found that he could concentrate charge in a “Leyden jar,” which was essentially a glass jar with two sheets of metal foil, one inside and one outside, with the glass between them (Figure 8.2.4). This created a large electric force between the two foil sheets.

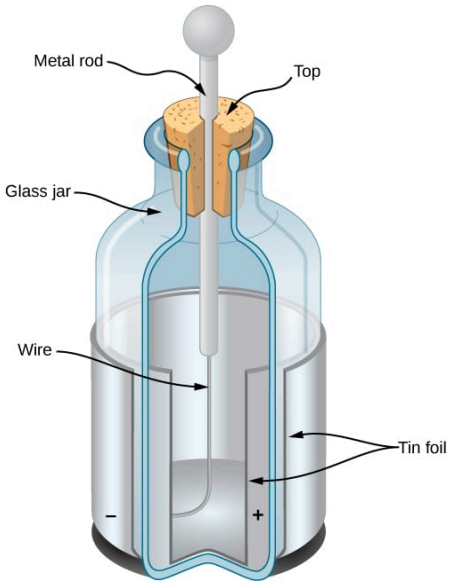


Figure 8.2.4: A Leyden jar (an early version of what is now called a capacitor) allowed experimenters to store large amounts of electric charge. Benjamin Franklin used such a jar to demonstrate that lightning behaved exactly like the electricity he got from the equipment in his laboratory.

Franklin pointed out that the observed behavior could be explained by supposing that one of the two types of charge remained motionless, while the other type of charge flowed from one piece of foil to the other. He further suggested that an excess of what he called this "electrical fluid" be called "positive electricity" and the deficiency of it be called "negative electricity." His suggestion, with some minor modifications, is the model we use today. (With the experiments that he was able to do, this was a pure guess; he had no way of actually determining the sign of the moving charge. Unfortunately, he guessed wrong; we now know that the charges that flow are the ones Franklin labeled negative, and the positive charges remain largely motionless. Fortunately, as we'll see, it makes no practical or theoretical difference which choice we make, as long as we stay consistent with our choice.)

Let's list the specific observations that we have of this **electric force**:

- The force acts without physical contact between the two objects.
- The force can be either attractive or repulsive: If two interacting objects carry the same sign of charge, the force is repulsive; if the charges are of opposite sign, the force is attractive. These interactions are referred to as **electrostatic repulsion** and **electrostatic attraction**, respectively.
- Not all objects are affected by this force.
- The magnitude of the force decreases (rapidly) with increasing separation distance between the objects.

To be more precise, we find experimentally that the magnitude of the force decreases as the square of the distance between the two interacting objects increases. Thus, for example, when the distance between two interacting objects is doubled, the force between them decreases to one fourth what it was in the original system. We can also observe that the surroundings of the charged objects affect the magnitude of the force. However, we will explore this issue in a later chapter.

Properties of Electric Charge

In addition to the existence of two types of charge, several other properties of charge have been discovered.

- **Charge is quantized.** This means that electric charge comes in discrete amounts, and there is a smallest possible amount of charge that an object can have. In the SI system, this smallest amount is $e \equiv 1.602 \times 10^{-19}$ C. No free particle can have less charge than this, and, therefore, the charge on any object—the charge on all objects—must be an integer multiple of this amount. All macroscopic, charged objects have charge because electrons have either been added or taken away from them, resulting in a net charge.
- **The magnitude of the charge is independent of the type.** Phrased another way, the smallest possible positive charge (to four significant figures) is $+1.602 \times 10^{-19}$ C, and the smallest possible negative charge is -1.602×10^{-19}; these values are exactly equal. This is simply how the laws of physics in our universe turned out.
- **Charge is conserved.** Charge can neither be created nor destroyed; it can only be transferred from place to place, from one

object to another. Frequently, we speak of two charges “canceling”; this is verbal shorthand. It means that if two objects that have equal and opposite charges are physically close to each other, then the (oppositely directed) forces they apply on some other charged object cancel, for a net force of zero. It is important that you understand that the charges on the objects by no means disappear, however. The net charge of the universe is constant.

- **Charge is conserved in closed systems.** In principle, if a negative charge disappeared from your lab bench and reappeared on the Moon, conservation of charge would still hold. However, this never happens. If the total charge you have in your local system on your lab bench is changing, there will be a measurable flow of charge into or out of the system. Again, charges can and do move around, and their effects can and do cancel, but the net charge in your local environment (if closed) is conserved. The last two items are both referred to as the **law of conservation of charge**.

The Source of Charges: The Structure of the Atom

Once it became clear that all matter was composed of particles that came to be called atoms, it also quickly became clear that the constituents of the atom included both positively charged particles and negatively charged particles. The next question was, what are the physical properties of those electrically charged particles?

The negatively charged particle was the first one to be discovered. In 1897, the English physicist J. J. Thomson was studying what was then known as *cathode rays*. Some years before, the English physicist William Crookes had shown that these “rays” were negatively charged, but his experiments were unable to tell any more than that. (The fact that they carried a negative electric charge was strong evidence that these were not rays at all, but particles.) Thomson prepared a pure beam of these particles and sent them through crossed electric and magnetic fields, and adjusted the various field strengths until the net deflection of the beam was zero. With this experiment, he was able to determine the charge-to-mass ratio of the particle. This ratio showed that the mass of the particle was much smaller than that of any other previously known particle—1837 times smaller, in fact. Eventually, this particle came to be called the **electron**.

Since the atom as a whole is electrically neutral, the next question was to determine how the positive and negative charges are distributed within the atom. Thomson himself imagined that his electrons were embedded within a sort of positively charged paste, smeared out throughout the volume of the atom. However, in 1908, the New Zealand physicist Ernest Rutherford showed that the positive charges of the atom existed within a tiny core—called a nucleus—that took up only a very tiny fraction of the overall volume of the atom, but held over 99% of the mass (see Linear Momentum and Collisions.) In addition, he showed that the negatively charged electrons perpetually orbited about this nucleus, forming a sort of electrically charged cloud that surrounds the nucleus (Figure 8.2.5). Rutherford concluded that the nucleus was constructed of small, massive particles that he named **protons**.

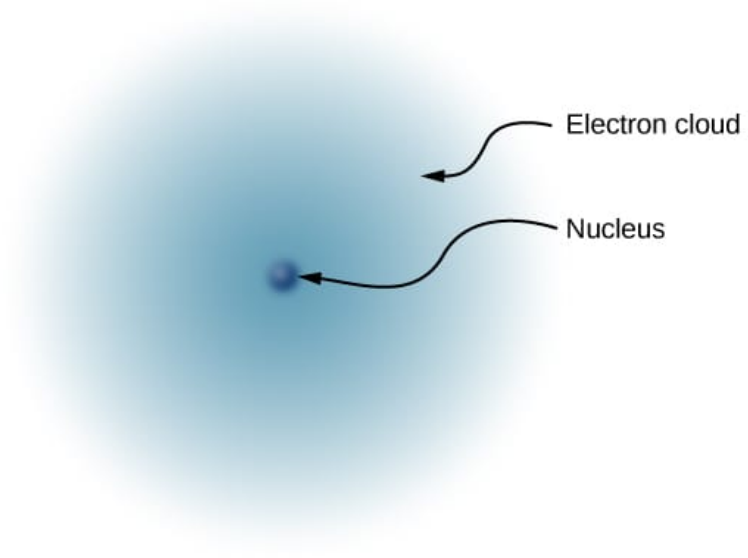


Figure 8.2.5: This simplified model of a hydrogen atom shows a positively charged nucleus (consisting, in the case of hydrogen, of a single proton), surrounded by an electron “cloud.” The charge of the electron cloud is equal (and opposite in sign) to the charge of the nucleus, but the electron does not have a definite location in space; hence, its representation here is as a cloud. Normal macroscopic amounts of matter contain immense numbers of atoms and molecules, and, hence, even greater numbers of individual negative and positive charges.

Since it was known that different atoms have different masses, and that ordinarily atoms are electrically neutral, it was natural

to suppose that different atoms have different numbers of protons in their nucleus, with an equal number of negatively charged electrons orbiting about the positively charged nucleus, thus making the atoms overall electrically neutral. However, it was soon discovered that although the lightest atom, hydrogen, did indeed have a single proton as its nucleus, the next heaviest atom—helium—has twice the number of protons (two), but *four* times the mass of hydrogen.

This mystery was resolved in 1932 by the English physicist James Chadwick, with the discovery of the **neutron**. The neutron is, essentially, an electrically neutral twin of the proton, with no electric charge, but (nearly) identical mass to the proton. The helium nucleus therefore has two neutrons along with its two protons. (Later experiments were to show that although the neutron is electrically neutral overall, it does have an internal charge *structure*. Furthermore, although the masses of the neutron and the proton are *nearly* equal, they aren't exactly equal: The neutron's mass is very slightly larger than the mass of the proton. That slight mass excess turned out to be of great importance. That, however, is a story that will have to wait until our study of modern physics in Nuclear Physics.)

Thus, in 1932, the picture of the atom was of a small, massive nucleus constructed of a combination of protons and neutrons, surrounded by a collection of electrons whose combined motion formed a sort of negatively charged "cloud" around the nucleus (Figure 8.2.6). In an electrically neutral atom, the total negative charge of the collection of electrons is equal to the total positive charge in the nucleus. The very low-mass electrons can be more or less easily removed or added to an atom, changing the net charge on the atom (though without changing its type). An atom that has had the charge altered in this way is called an **ion**. Positive ions have had electrons removed, whereas negative ions have had excess electrons added. We also use this term to describe molecules that are not electrically neutral.

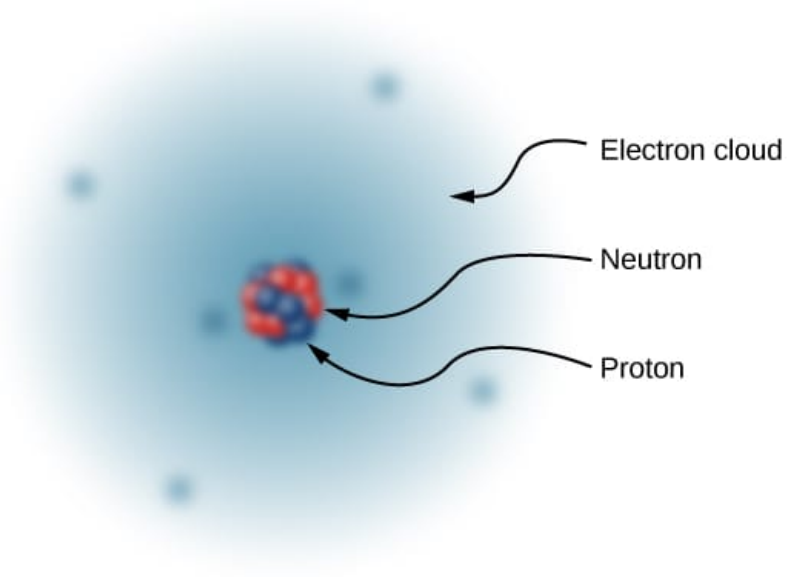


Figure 8.2.6: The nucleus of a carbon atom is composed of six protons and six neutrons. As in hydrogen, the surrounding six electrons do not have definite locations and so can be considered to be a sort of cloud surrounding the nucleus.

The story of the atom does not stop there, however. In the latter part of the twentieth century, many more subatomic particles were discovered in the nucleus of the atom: pions, neutrinos, and quarks, among others. With the exception of the photon, none of these particles are directly relevant to the study of electromagnetism, so we defer further discussion of them until the chapter on particle physics (Particle Physics and Cosmology).

A Note on Terminology

As noted previously, electric charge is a property that an object can have. This is similar to how an object can have a property that we call mass, a property that we call density, a property that we call temperature, and so on. Technically, we should always say something like, "Suppose we have a particle that carries a charge of $\mu\mathrm{C}$." However, it is very common to say instead, "Suppose we have a $\mu\mathrm{C}$ charge." Similarly, we often say something like, "Six charges are located at the vertices of a regular hexagon." A charge is not a particle; rather, it is a *property* of a particle. Nevertheless, this terminology is extremely common (and is frequently used in this book, as it is everywhere else). So, keep in the back of your mind what we really mean when we refer to a "charge."

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8.3: Conductors, Insulators, and Charging by Induction

Learning Objectives

By the end of this section, you will be able to:

- Explain what a conductor is
- Explain what an insulator is
- List the differences and similarities between conductors and insulators
- Describe the process of charging by induction

In the preceding section, we said that scientists were able to create electric charge only on nonmetallic materials and never on metals. To understand why this is the case, you have to understand more about the nature and structure of atoms. In this section, we discuss how and why electric charges do—or do not—move through materials (Figure 8.3.1). A more complete description is given in a later chapter.



Figure 8.3.1: This power adapter uses metal wires and connectors to conduct electricity from the wall socket to a laptop computer. The conducting wires allow electrons to move freely through the cables, which are shielded by rubber and plastic. These materials act as insulators that don't allow electric charge to escape outward. (credit: modification of work by "Evan-Amos"/Wikimedia Commons)

Conductors and Insulators

As discussed in the previous section, electrons surround the tiny nucleus in the form of a (comparatively) vast cloud of negative charge. However, this cloud does have a definite structure to it. Let's consider an atom of the most commonly used conductor, copper.

For reasons that will become clear in Atomic Structure, there is an outermost electron that is only loosely bound to the atom's nucleus. It can be easily dislodged; it then moves to a neighboring atom. In a large mass of copper atoms (such as a copper wire or a sheet of copper), these vast numbers of outermost electrons (one per atom) wander from atom to atom, and are the electrons that do the moving when electricity flows. These wandering, or "free," electrons are called **conduction electrons**, and copper is therefore an excellent **conductor** (of electric charge). All conducting elements have a similar arrangement of their electrons, with one or two conduction electrons. This includes most metals.

Insulators, in contrast, are made from materials that lack conduction electrons; charge flows only with great difficulty, if at all. Even if excess charge is added to an insulating material, it cannot move, remaining indefinitely in place. This is why insulating

materials exhibit the electrical attraction and repulsion forces described earlier, whereas conductors do not; any excess charge placed on a conductor would instantly flow away (due to mutual repulsion from existing charges), leaving no excess charge around to create forces. Charge cannot flow along or through an **insulator**, so its electric forces remain for long periods of time. (Charge will dissipate from an insulator, given enough time.) As it happens, amber, fur, and most semi-precious gems are insulators, as are materials like wood, glass, and plastic.

Charging by Induction

Let's examine in more detail what happens in a conductor when an electrically charged object is brought close to it. As mentioned, the conduction electrons in the conductor are able to move with nearly complete freedom. As a result, when a charged insulator (such as a positively charged glass rod) is brought close to the conductor, the (total) charge on the insulator exerts an electric force on the conduction electrons. Since the rod is positively charged, the conduction electrons (which themselves are negatively charged) are attracted, flowing toward the insulator to the near side of the conductor (Figure 8.3.2).

Now, the conductor is still overall electrically neutral; the conduction electrons have changed position, but they are still in the conducting material. However, the conductor now has a charge *distribution*; the near end (the portion of the conductor closest to the insulator) now has more negative charge than positive charge, and the reverse is true of the end farthest from the insulator. The relocation of negative charges to the near side of the conductor results in an overall positive charge in the part of the conductor farthest from the insulator. We have thus created an electric charge distribution where one did not exist before. This process is referred to as *inducing polarization*—in this case, polarizing the conductor. The resulting separation of positive and negative charge is called **polarization**, and a material, or even a molecule, that exhibits polarization is said to be polarized. A similar situation occurs with a negatively charged insulator, but the resulting polarization is in the opposite direction.

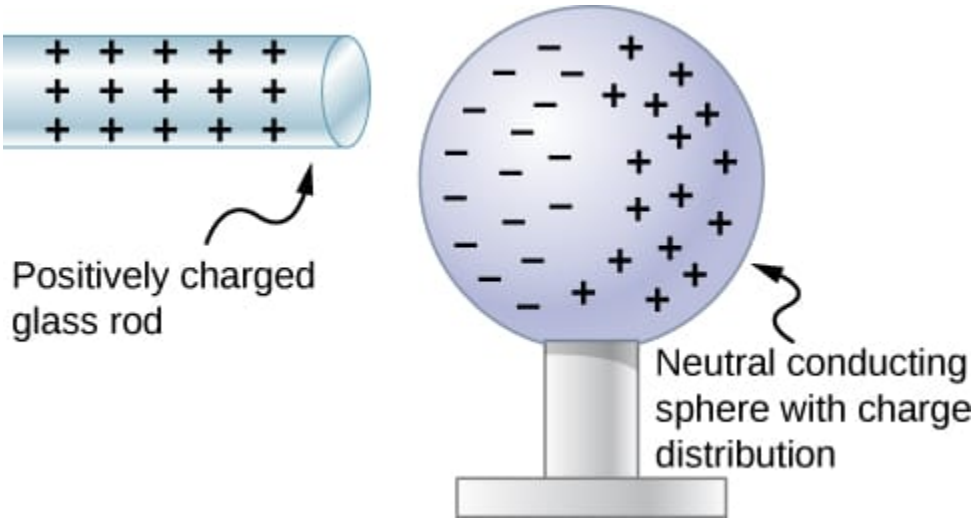


Figure 8.3.2: Induced polarization. A positively charged glass rod is brought near the left side of the conducting sphere, attracting negative charge and leaving the other side of the sphere positively charged. Although the sphere is overall still electrically neutral, it now has a charge distribution, so it can exert an electric force on other nearby charges. Furthermore, the distribution is such that it will be attracted to the glass rod.

The result is the formation of what is called an electric **dipole**, from a Latin phrase meaning "two ends." The presence of electric charges on the insulator—and the electric forces they apply to the conduction electrons—creates, or "induces," the dipole in the conductor.

Neutral objects can be attracted to any charged object. The pieces of straw attracted to polished amber are neutral, for example. If you run a plastic comb through your hair, the charged comb can pick up neutral pieces of paper. Figure 8.3.3 shows how the polarization of atoms and molecules in neutral objects results in their attraction to a charged object.

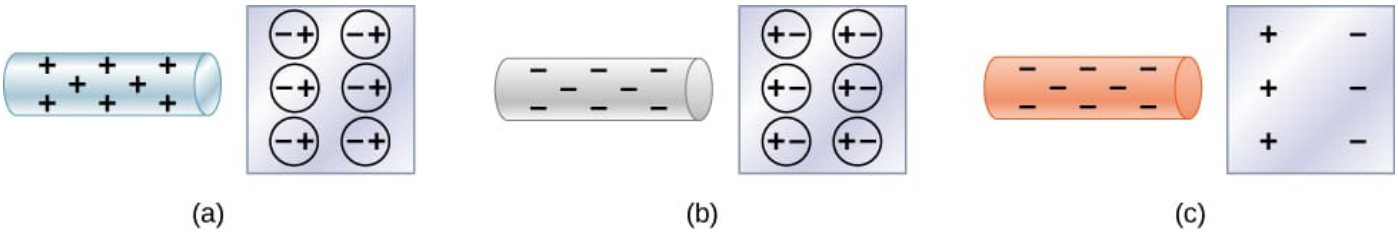


Figure 8.3.3: Both positive and negative objects attract a neutral object by polarizing its molecules. (a) A positive object brought near a neutral insulator polarizes its molecules. There is a slight shift in the distribution of the electrons orbiting the molecule, with unlike charges being brought nearer and like charges moved away. Since the electrostatic force decreases with distance, there is a net attraction. (b) A negative object produces the opposite polarization, but again attracts the neutral object. (c) The same effect occurs for a conductor; since the unlike charges are closer, there is a net attraction.

When a charged rod is brought near a neutral substance, an insulator in this case, the distribution of charge in atoms and molecules is shifted slightly. Opposite charge is attracted nearer the external charged rod, while like charge is repelled. Since the electrostatic force decreases with distance, the repulsion of like charges is weaker than the attraction of unlike charges, and so there is a net attraction. Thus, a positively charged glass rod attracts neutral pieces of paper, as will a negatively charged rubber rod. Some molecules, like water, are polar molecules. Polar molecules have a natural or inherent separation of charge, although they are neutral overall. Polar molecules are particularly affected by other charged objects and show greater polarization effects than molecules with naturally uniform charge distributions.

When the two ends of a dipole can be separated, this method of **charging by induction** may be used to create charged objects without transferring charge. In Figure 8.3.4, we see two neutral metal spheres in contact with one another but insulated from the rest of the world. A positively charged rod is brought near one of them, attracting negative charge to that side, leaving the other sphere positively charged.

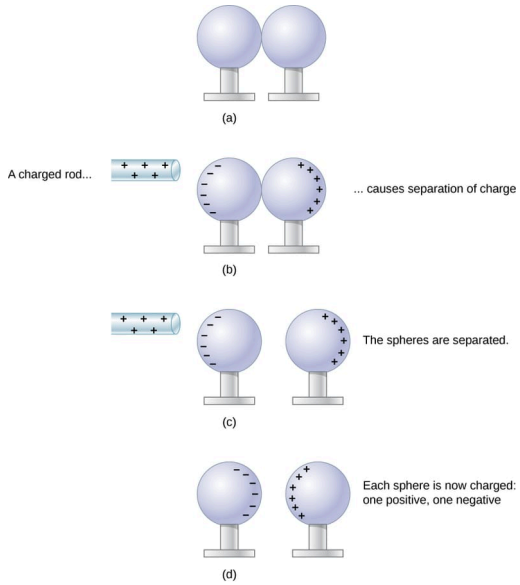


Figure 8.3.4: Charging by induction. (a) Two uncharged or neutral metal spheres are in contact with each other but insulated from the rest of the world. (b) A positively charged glass rod is brought near the sphere on the left, attracting negative charge and leaving the other sphere positively charged. (c) The spheres are separated before the rod is removed, thus separating negative and positive charges. (d) The spheres retain net charges after the inducing rod is removed—without ever having been touched by a charged object.

Another method of charging by induction is shown in Figure 8.3.5. The neutral metal sphere is polarized when a charged rod is brought near it. The sphere is then grounded, meaning that a conducting wire is run from the sphere to the ground. Since Earth is large and most of the ground is a good conductor, it can supply or accept excess charge easily. In this case, electrons are attracted to the sphere through a wire called the ground wire, because it supplies a conducting path to the ground. The ground connection is broken before the charged rod is removed, leaving the sphere with an excess charge opposite to that of the rod. Again, an opposite charge is achieved when charging by induction, and the charged rod loses none of its excess charge.

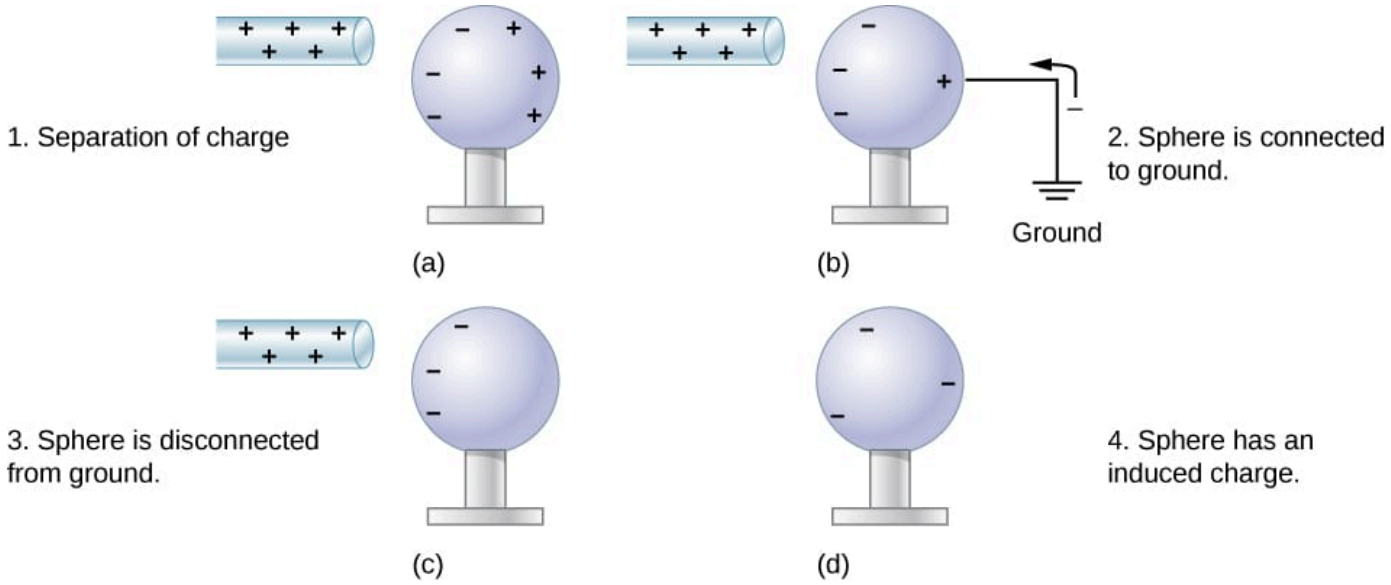


Figure 8.3.5: Charging by induction using a ground connection. (a) A positively charged rod is brought near a neutral metal sphere, polarizing it. (b) The sphere is grounded, allowing electrons to be attracted from Earth's ample supply. (c) The ground connection is broken. (d) The positive rod is removed, leaving the sphere with an induced negative charge.

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8.4: Coulomb's Law

Learning Objectives

By the end of this section, you will be able to:

- Describe the electric force, both qualitatively and quantitatively
- Calculate the force that charges exert on each other
- Determine the direction of the electric force for different source charges
- Correctly describe and apply the superposition principle for multiple source charges

Experiments with electric charges have shown that if two objects each have electric charge, then they exert an electric force on each other. The magnitude of the force is linearly proportional to the net charge on each object and inversely proportional to the square of the distance between them. (Interestingly, the force does not depend on the mass of the objects.) The direction of the force vector is along the imaginary line joining the two objects and is dictated by the signs of the charges involved.

Let

- $q_1, q_2 =$ the net electric charge of the two objects;
- $\vec{r}_{12} =$ the vector displacement from q_1 to q_2.

The electric force $\vec{F}$ on one of the charges is proportional to the magnitude of its own charge and the magnitude of the other charge, and is inversely proportional to the square of the distance between them:

$$F \propto \frac{q_1 q_2}{r_{12}^2}.$$

This proportionality becomes an equality with the introduction of a proportionality constant. For reasons that will become clear in a later chapter, the proportionality constant that we use is actually a collection of constants. (We discuss this constant shortly.)

Coulomb's Law

The magnitude of the electric force (or **Coulomb force**) between two electrically charged particles is equal to

$$|\mathbf{F}_{12}| = \frac{1}{4\pi\varepsilon_0}\frac{|q_1 q_2|}{r_{12}^2} \tag{8.4.1}$$

The unit vector r has a magnitude of 1 and points along the axis as the charges. If the charges have the same sign, the force is in the same direction as r showing a repelling force. If the charges have different signs, the force is in the opposite direction of r showing an attracting force. (Figure 8.4.1).

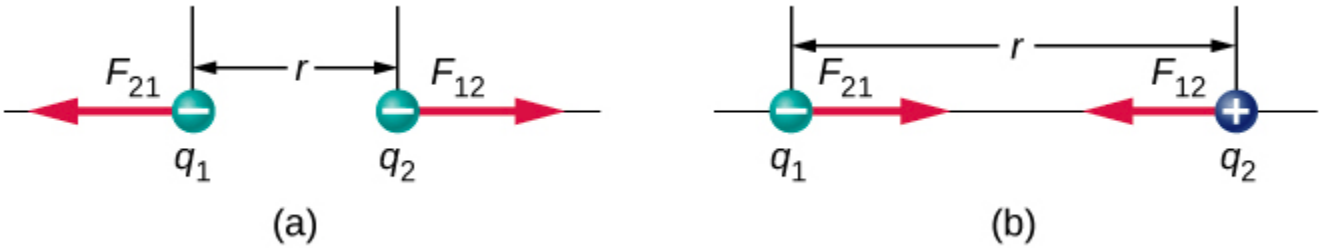


Figure 8.4.1: The electrostatic force $\vec{F}$ between point charges q_1 and q_2 separated by a distance r is given by Coulomb's law. Note that Newton's third law (every force exerted creates an equal and opposite force) applies as usual—the force on q_1 is equal in magnitude and opposite in direction to the force it exerts on q_2. (a) Like charges; (b) unlike charges.

It is important to note that the electric force is not constant; it is a function of the separation distance between the two charges. If either the test charge or the source charge (or both) move, then $\vec{r}$ changes, and therefore so does the force. An immediate consequence of this is that direct application of Newton's laws with this force can be mathematically difficult, depending on the specific problem at hand. It can (usually) be done, but we almost always look for easier methods of calculating whatever physical quantity we are interested in. (Conservation of energy is the most common choice.)

Finally, the new constant ϵ_0 in Coulomb's law is called the *permittivity of free space*, or (better) the **permittivity of vacuum**. It has a very important physical meaning that we will discuss in a later chapter; for now, it is simply an empirical proportionality constant. Its numerical value (to three significant figures) turns out to be

$$\epsilon_0 = 8.85 \times 10^{-12} \frac{\mathrm{C}^2}{\mathrm{N} \cdot \mathrm{m}^2}.$$

These units are required to give the force in Coulomb's law the correct units of newtons. Note that in Coulomb's law, the permittivity of vacuum is only part of the proportionality constant. For convenience, we often define a Coulomb's constant:

$$k_e = \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9 \frac{\mathrm{N} \cdot \mathrm{m}^2}{\mathrm{C}^2}.$$

Example 8.4.1: The Force on the Electron in Hydrogen

A hydrogen atom consists of a single proton and a single electron. The proton has a charge of $+e$ and the electron has $-e$. In the "ground state" of the atom, the electron orbits the proton at most probable distance of $5.29 \times 10^{-11}\mathrm{m}$ (Figure 8.4.2). Calculate the electric force on the electron due to the proton.

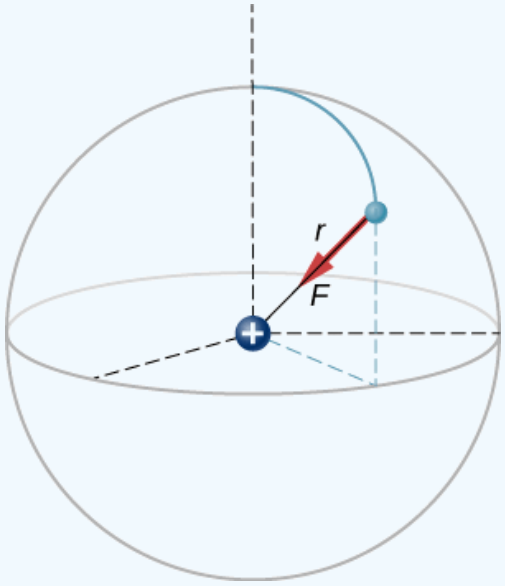


Figure 8.4.2: A schematic depiction of a hydrogen atom, showing the force on the electron. This depiction is only to enable us to calculate the force; the hydrogen atom does not really look like this.

Strategy

For the purposes of this example, we are treating the electron and proton as two point particles, each with an electric charge, and we are told the distance between them; we are asked to calculate the force on the electron. We thus use Coulomb's law (Equation 8.4.1).

Solution

Our two charges are,

$$\begin{aligned} q_1 &= +e \\ &= +1.602 \times 10^{-19}\,\text{C} \\ q_2 &= -e \\ &= -1.602 \times 10^{-19}\,\text{C} \end{aligned}$$

and the distance between them

$$r = 5.29 \times 10^{-11}\,\text{m}.$$

The magnitude of the force on the electron (Equation 8.4.1) is

$$\begin{aligned} F &= \frac{1}{4\pi\epsilon_0}\frac{|q_1 q_2|}{r_{12}^2} \\ &= \frac{1}{4\pi\left(8.85 \times 10^{-12}\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)}\frac{(1.602 \times 10^{-19}\,\text{C})^2}{(5.29 \times 10^{-11}\,\text{m})^2} \\ &= 8.25 \times 10^{-8}\,\text{N}. \end{aligned}$$

As for the direction, since the charges on the two particles are opposite, the force is attractive; the force on the electron points radially directly toward the proton, everywhere in the electron's orbit. The force is thus expressed as

$$\vec{F} = (8.25 \times 10^{-8}\,\text{N})\hat{r}.$$

Significance

This is a three-dimensional system, so the electron (and therefore the force on it) can be anywhere in an imaginary spherical shell around the proton. In this "classical" model of the hydrogen atom, the electrostatic force on the electron points in the inward centripetal direction, thus maintaining the electron's orbit. But note that the quantum mechanical model of hydrogen (discussed in Quantum Mechanics) is utterly different.

Exercise 8.4.1

What would be different if the electron also had a positive charge?

Answer available at the end of this section.

Multiple Source Charges

The analysis that we have done for two particles can be extended to an arbitrary number of particles; we simply repeat the analysis, two charges at a time. Specifically, we ask the question: Given N charges (which we refer to as source charge), what is the net electric force that they exert on some other point charge (which we call the test charge)? Note that we use these terms because we can think of the test charge being used to test the strength of the force provided by the source charges.

Like all forces that we have seen up to now, the net electric force on our test charge is simply the vector sum of each individual electric force exerted on it by each of the individual test charges. Thus, we can calculate the net force on the test charge Q by calculating the force on it from each source charge, taken one at a time, and then adding all those forces together (as vectors). This ability to simply add up individual forces in this way is referred to as the **principle of superposition**, and is one of the more important features of the electric force. In mathematical form, this becomes

$$\vec{F}(r) = \frac{1}{4\pi\epsilon_0}Q\sum_{i=1}^{N}\frac{q_i}{r_i^2}\hat{r}_i^2. \tag{8.4.2}$$

In this expression, Q represents the charge of the particle that is experiencing the electric force $\vec{F}$, and is located at $\vec{r}$ from the origin; the q_i's are the N source charges, and the vectors $\vec{r}_i = r_i\hat{r}_i$ are the displacements from the position of the ith charge to the position of Q. Each of the N unit vectors points directly from its associated source charge toward the test charge. All of this

is depicted in Figure 8.4.3. Please note that there is no physical difference between Q and q_i; the difference in labels is merely to allow clear discussion, with Q being the charge we are determining the force on.

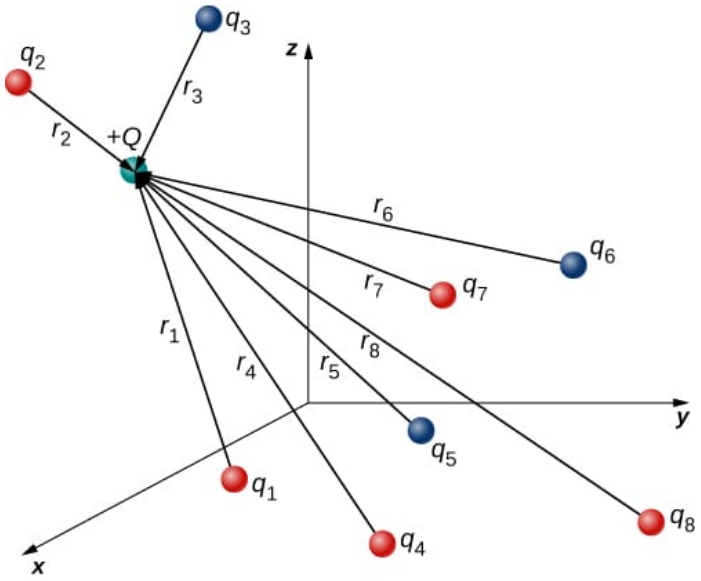


Figure 8.4.3: The eight source charges each apply a force on the single test charge Q. Each force can be calculated independently of the other seven forces. This is the essence of the superposition principle.

(Note that the force vector $\vec{F}_i$ does not necessarily point in the same direction as the unit vector $\hat{r}_i$; it may point in the opposite direction, $-\hat{r}_i$. The signs of the source charge and test charge determine the direction of the force on the test charge.)

There is a complication, however. Just as the source charges each exert a force on the test charge, so too (by Newton's third law) does the test charge exert an equal and opposite force on each of the source charges. As a consequence, each source charge would change position. However, by Equation 8.4.1, the force on the test charge is a function of position; thus, as the positions of the source charges change, the net force on the test charge necessarily changes, which changes the force, which again changes the positions. Thus, the entire mathematical analysis quickly becomes intractable. Later, we will learn techniques for handling this situation, but for now, we make the simplifying assumption that the source charges are fixed in place somehow, so that their positions are constant in time. (The test charge is allowed to move.) With this restriction in place, the analysis of charges is known as **electrostatics**, where "statics" refers to the constant (that is, static) positions of the source charges and the force is referred to as an **electrostatic force**.

Example 8.4.2: The Net Force from Two Source Charges

Three different, small charged objects are placed as shown in Figure 8.4.4. The charges q_1 and q_3 are fixed in place; q_2 is free to move. Given $q_1 = 2e,\ q_2 = -3e$, and $q_3 = -5e$, and that $d = 2.0 \times 10^{-7}$ m, what is the net force on the middle charge q_2?

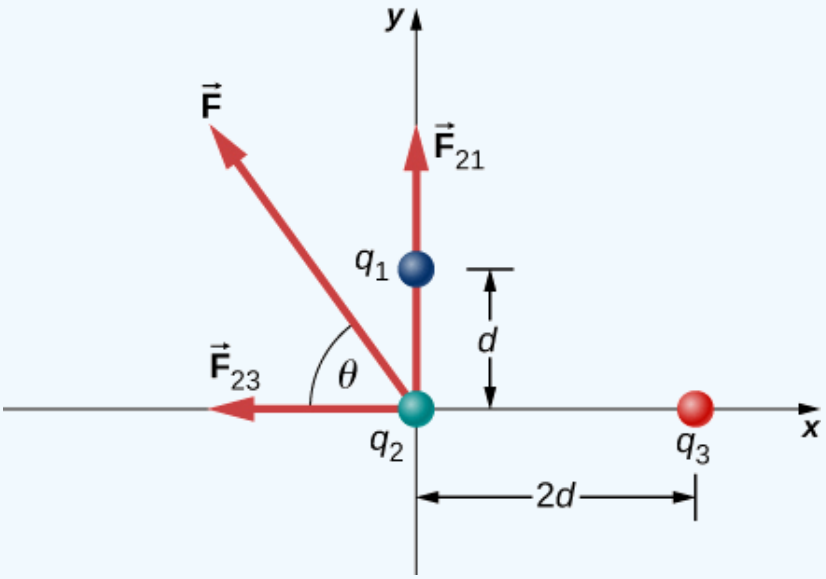


Figure 8.4.4: Source charges q_1 and q_3 each apply a force on q_2.

Strategy

We use Coulomb's law again. The way the question is phrased indicates that q_2 is our test charge, so that q_1 and q_3 are source charges. The principle of superposition says that the force on q_2 from each of the other charges is unaffected by the presence of the other charge. Therefore, we write down the force on q_2 from each and add them together as vectors.

Solution

We have two source charges q_1 and q_3 a test charge q_2, distances r_{21} and r_{23} and we are asked to find a force. This calls for Coulomb's law and superposition of forces. There are two forces:

$$\vec{F} = \vec{F}_{21} + \vec{F}_{23} = \frac{1}{4\pi\epsilon_0}\left[\frac{q_2 q_1}{r_{21}^2}\hat{j} + \left(-\frac{q_2 q_3}{r_{23}^2}\hat{i}\right)\right].$$

We cannot add these forces directly because they don't point in the same direction: $\vec{F}_{12}$ points only in the $-x$-direction, while $\vec{F}_{13}$ points only in the $+y$-direction. The net force is obtained from applying the Pythagorean theorem to its x- and y -components:

$$F = \sqrt{F_x^2 + F_y^2}$$

and

$$\begin{aligned} F_x &= -F_{23} = -\frac{1}{4\pi\epsilon_0}\frac{q_2 q_3}{r_{23}^2} \\ &= -\left(8.99\times10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{(4.806\times10^{-19}\,\text{C})(8.01\times10^{-19}\,\text{C})}{(4.00\times10^{-7}\,\text{m})^2} \\ &= -2.16\times10^{-14}\,\text{N} \end{aligned}$$

and

$$\begin{aligned} F_y &= F_{21} = \frac{1}{4\pi\epsilon_0}\frac{q_2 q_1}{r_{21}} \\ &= \left(9.99\times10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{(4.806\times10^{-19}\,\text{C})(3.204\times10^{-19}\,\text{C})}{(2.00\times10^{-7}\,\text{m})^2} \\ &= 3.46\times10^{-14}\,\text{N}. \end{aligned}$$

We find that

$$\begin{aligned} F &= \sqrt{F_x^2 + F_y^2} \\ &= 4.08 \times 10^{-14}\,\text{N} \end{aligned}$$

at an angle of

$$\begin{aligned} \phi &= \tan^{-1}\left(\frac{F_y}{F_x}\right) \\ &= \tan^{-1}\left(\frac{3.46 \times 10^{-14}\,\text{N}}{-2.16 \times 10^{-14}\,\text{N}}\right) \\ &= -58^\circ, \end{aligned}$$

that is, 58° above the $-x$-axis, as shown in the diagram.

Significance

Notice that when we substituted the numerical values of the charges, we did not include the negative sign of either q_1 or q_3. Recall that negative signs on vector quantities indicate a reversal of direction of the vector in question. But for electric forces, the direction of the force is determined by the types (signs) of both interacting charges; we determine the force directions by considering whether the signs of the two charges are the same or are opposite. If you also include negative signs from negative charges when you substitute numbers, you run the risk of mathematically reversing the direction of the force you are calculating. Thus, the safest thing to do is to calculate just the magnitude of the force, using the absolute values of the charges, and determine the directions physically.

It's also worth noting that the only new concept in this example is how to calculate the electric forces; everything else (getting the net force from its components, breaking the forces into their components, finding the direction of the net force) is the same as force problems you have done earlier.

Exercise 8.4.2

What would be different in Example $8.4.2$ if q_1 were negative rather than positive?

Answer available at the end of this section.

Answers

Exercise $8.4.1$

The force would point outward.

Exercise $8.4.2$

The net force would point 58° below the $-x$-axis.

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8.5: Electric Field

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Explain the purpose of the electric field concept
- Describe the properties of the electric field
- Calculate the field of a collection of source charges of either sign

As we showed in the preceding section, the net electric force on a test charge is the vector sum of all the electric forces acting on it, from all of the various source charges, located at their various positions. But what if we use a different test charge, one with a different magnitude, or sign, or both? Or suppose we have a dozen different test charges we wish to try at the same location? We would have to calculate the sum of the forces from scratch. Fortunately, it is possible to define a quantity, called the **electric field**, which is independent of the test charge. It only depends on the configuration of the source charges, and once found, allows us to calculate the force on any test charge.

Defining a Field

Suppose we have N source charges $q_1, q_2, q_3, ...q_N$ located at positions $\vec{r}_1, \vec{r}_2, \vec{r}_3, ...\vec{r}_N$, applying N electrostatic forces on a test charge Q. The net force on Q is

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + ... + \vec{F}_N \tag{8.5.1}$$

$$= \frac{1}{4\pi\epsilon_0}\left(\frac{Qq_1}{r_1^2}\hat{r}_1 + \frac{Qq_2}{r_2^2}\hat{r}_2 + \frac{Qq_3}{r_3^2}\hat{r}_3 + ... + \frac{Qq_N}{r_1^2}\hat{r}_N\right) \tag{8.5.2}$$

$$= Q\left[\frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{r_1^2}\hat{r}_1 + \frac{q_2}{r_2^2}\hat{r}_2 + \frac{q_3}{r_3^2}\hat{r}_3 + ... + \frac{q_N}{r_1^2}\hat{r}_N\right)\right] \tag{8.5.3}$$

We can rewrite this as

$$\vec{F} = Q\vec{E} \tag{8.5.4}$$

where

$$\vec{E} \equiv \frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{r_1^2}\hat{r}_1 + \frac{q_2}{r_2^2}\hat{r}_2 + \frac{q_3}{r_3^2}\hat{r}_3 + ... + \frac{q_N}{r_1^2}\hat{r}_N\right) \tag{8.5.5}$$

or, more compactly,

$$\vec{E}(P) \equiv \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{N}\frac{q_i}{r_i^2}\hat{r}_i. \tag{8.5.6}$$

This expression is called the electric field at position $P = P(x, y, z)$ of the N source charges. Here, P is the location of the point in space where you are calculating the field and is relative to the positions $\vec{r}_i$ of the source charges (Figure $8.5.1$). Note that we have to impose a coordinate system to solve actual problems.

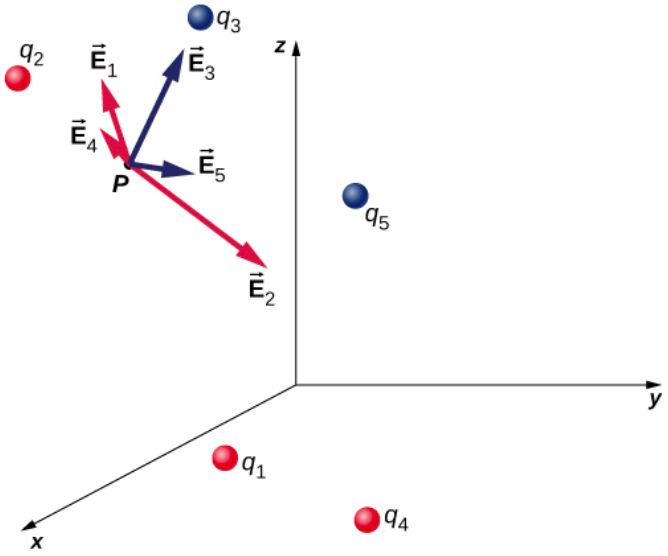


Figure 8.5.1: Each of these five source charges creates its own electric field at every point in space; shown here are the field vectors at an arbitrary point P. Like the electric force, the net electric field obeys the superposition principle.

Notice that the calculation of the electric field makes no reference to the test charge. Thus, the physically useful approach is to calculate the electric field and then use it to calculate the force on some test charge later, if needed. Different test charges experience different forces Equation 8.5.4, but it is the same electric field Equation 8.5.6. That being said, recall that there is no fundamental difference between a test charge and a source charge; these are merely convenient labels for the system of interest. Any charge produces an electric field; however, just as Earth's orbit is not affected by Earth's own gravity, a charge is not subject to a force due to the electric field it generates. Charges are only subject to forces from the electric fields of other charges.

In this respect, the electric field $\vec{E}$ of a point charge is similar to the gravitational field $\vec{g}$ of Earth; once we have calculated the gravitational field at some point in space, we can use it any time we want to calculate the resulting force on any mass we choose to place at that point. In fact, this is exactly what we do when we say the gravitational field of Earth (near Earth's surface) has a value of $9.81\ \mathrm{m/s^2}$ and then we calculate the resulting force (i.e., weight) on different masses. Also, the general expression for calculating $\vec{g}$ at arbitrary distances from the center of Earth (i.e., not just near Earth's surface) is very similar to the expression for E:

$$\vec{g} = G\frac{M}{r^2}\hat{r}$$

where G is a proportionality constant, playing the same role for $\vec{g}$ as $\dfrac{1}{4\pi\epsilon_0}$ does for $\vec{E}$. The value of $\vec{g}$ is calculated once and is then used in an endless number of problems.

To push the analogy further, notice the units of the electric field: From $F = QE$, the units of **E** are newtons per coulomb, N/C, that is, the electric field applies a force on each unit charge. Now notice the units of g: From $w = mg$ the units of g are newtons per kilogram, N/kg, that is, the gravitational field applies a force on each unit mass. We could say that the gravitational field of Earth, near Earth's surface, has a value of 9.81 N/kg.

The Meaning of "Field"

Recall from your studies of gravity that the word "field" in this context has a precise meaning. A field, in physics, is a physical quantity whose value depends on (is a function of) position, relative to the source of the field. In the case of the electric field, Equation 8.5.6 shows that the value of $\vec{E}$ (both the magnitude and the direction) depends on where in space the point P is located, measured from the locations $\vec{r}_i$ of the source charges q_i.

In addition, since the electric field is a vector quantity, the electric field is referred to as a *vector field*. (The gravitational field is also a vector field.) In contrast, a field that has only a magnitude at every point is a *scalar field*. The temperature in a room is an example of a scalar field. It is a field because the temperature, in general, is different at different locations in the room, and it is a scalar field because temperature is a scalar quantity.

Also, as you did with the gravitational field of an object with mass, you should picture the electric field of a charge-bearing object (the source charge) as a continuous, immaterial substance that surrounds the source charge, filling all of space—in principle, to $\pm\infty$ in all directions. The field exists at every physical point in space. To put it another way, the electric charge on an object alters the space around the charged object in such a way that all other electrically charged objects in space experience an electric force as a result of being in that field. The electric field, then, is the mechanism by which the electric properties of the source charge are transmitted to and through the rest of the universe. (Again, the range of the electric force is infinite.)

We will see in subsequent chapters that the speed at which electrical phenomena travel is the same as the speed of light. There is a deep connection between the electric field and light.

Superposition

Yet another experimental fact about the field is that it obeys the superposition principle. In this context, that means that we can (in principle) calculate the total electric field of many source charges by calculating the electric field of only q_1 at position **P**, then calculate the field of q_2 at **P**, while—and this is the crucial idea—ignoring the field of, and indeed even the existence of, q_1. We can repeat this process, calculating the field of each individual source charge, independently of the existence of any of the other charges. The total electric field, then, is the vector sum of all these fields. That, in essence, is what Equation $8.5.6$ says.

In the next section, we describe how to determine the shape of an electric field of a source charge distribution and how to sketch it.

The Direction of the Field

Equation $8.5.6$ enables us to determine the magnitude of the electric field, but we need the direction also. We use the convention that the direction of any electric field vector is the same as the direction of the electric force vector that the field would apply to a positive test charge placed in that field. Such a charge would be repelled by positive source charges (the force on it would point away from the positive source charge) but attracted to negative charges (the force points toward the negative source).

Direction of the Electric Field

By convention, all electric fields $\vec{E}$ point away from positive source charges and point toward negative source charges.

Phet Simulation: Electric Field of Dreams

Download the Electric Field of Dreams PhET simulation and add charges to the and see how they react to the electric field. Turn on a background electric field and adjust the direction and magnitude.

Example $8.5.1A$: The E-field of an Atom

In an ionized helium atom, the most probable distance between the nucleus and the electron is $r = 26.5 \times 10^{-12}$ m. What is the electric field due to the nucleus at the location of the electron?

Strategy

Note that although the electron is mentioned, it is not used in any calculation. The problem asks for an electric field, not a force; hence, there is only one charge involved, and the problem specifically asks for the field due to the nucleus. Thus, the electron is a red herring; only its distance matters. Also, since the distance between the two protons in the nucleus is much, much smaller than the distance of the electron from the nucleus, we can treat the two protons as a single charge +2e (Figure $8.5.2$).

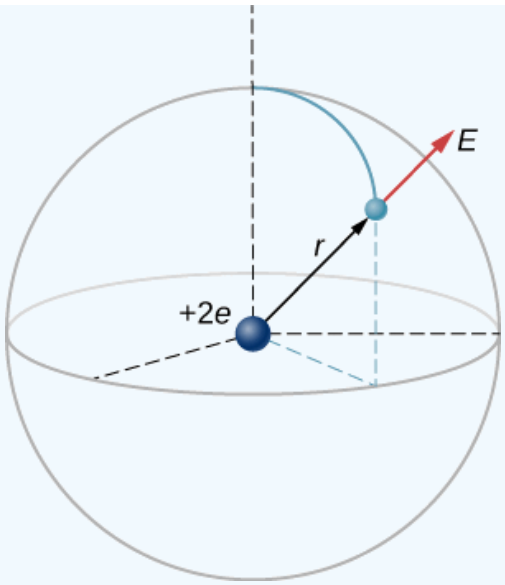


Figure 8.5.2: A schematic representation of a helium atom. Again, helium physically looks nothing like this, but this sort of diagram is helpful for calculating the electric field of the nucleus.

Solution

The electric field is calculated by

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{N}\frac{q_i}{r_i^2}\hat{r}_i.$$

Since there is only one source charge (the nucleus), this expression simplifies to

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}.$$

Here, $q = 2e = 2(1.6 \times 10^{-19}\ \text{C})$ (since there are two protons) and r is given; substituting gives

$$\begin{aligned}\vec{E} &= \frac{1}{4\pi\left(8.85 \times 10^{-12}\dfrac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)}\frac{2(1.6 \times 10^{-19}\,\text{C})}{26.5 \times 10^{-12}\,\text{m})^2}\hat{r} \\ &= 4.1 \times 10^{12}\frac{\text{N}}{\text{C}}\hat{r}.\end{aligned}$$

The direction of $\vec{E}$ is radially away from the nucleus in all directions. Why? Because a positive test charge placed in this field would accelerate radially away from the nucleus (since it is also positively charged), and again, the convention is that the direction of the electric field vector is defined in terms of the direction of the force it would apply to positive test charges.

Example 8.5.1B: The E-Field above Two Equal Charges

a. Find the electric field (magnitude and direction) a distance z above the midpoint between two equal charges $+q$ that are a distance d apart (Figure 8.5.3). Check that your result is consistent with what you'd expect when $z \gg d$.
b. The same as part (a), only this time make the right-hand charge $-q$ instead of $+q$.

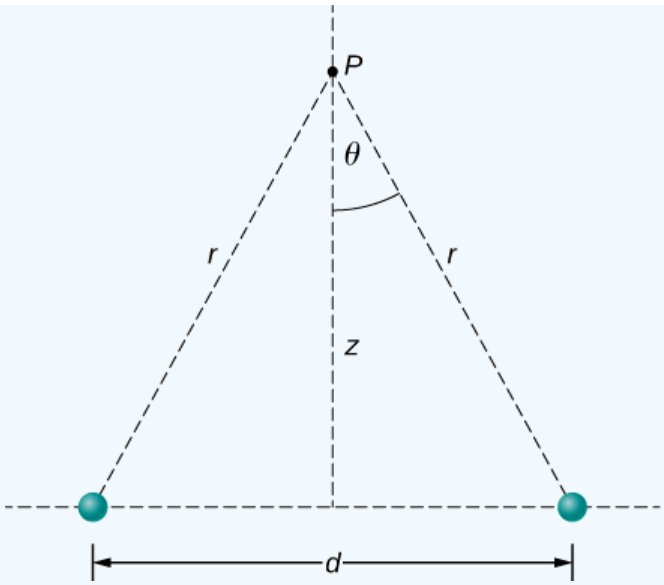


Figure 8.5.3: Finding the field of two identical source charges at the point P. Due to the symmetry, the net field at P is entirely vertical. (Notice that this is **not** true away from the midline between the charges.)

Strategy

We add the two fields as vectors, per Equation 8.5.6. Notice that the system (and therefore the field) is symmetrical about the vertical axis; as a result, the horizontal components of the field vectors cancel. This simplifies the math. Also, we take care to express our final answer in terms of only quantities that are given in the original statement of the problem: q, z, d, and constants (π, ϵ_0).

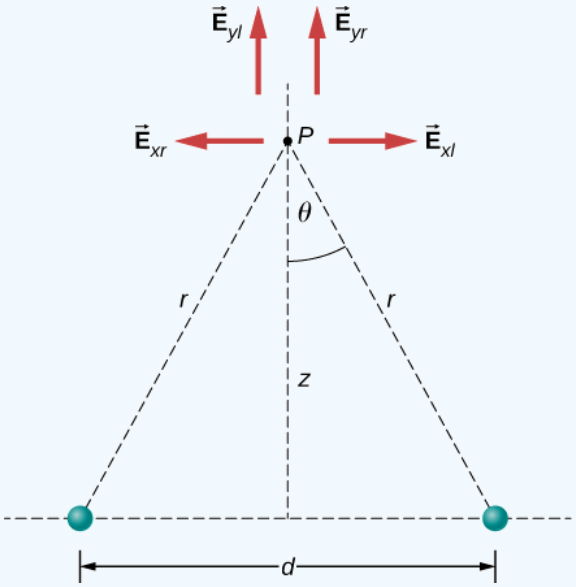


Figure 8.5.4. Note that the horizontal components of the electric fields from the two charges cancel each other out, while the vertical components add together.

Solution

a. By symmetry, the horizontal (x)-components of $\vec{E}$ cancel (Figure 8.5.4);

$$E_x = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\sin\theta - \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\sin\theta$$
$$= 0.$$

The vertical (z)-component is given by

$$E_z = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\cos\,\theta + \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\cos\,\theta$$
$$= \frac{1}{4\pi\epsilon_0}\frac{2q}{r^2}\cos\,\theta.$$

Since none of the other components survive, this is the entire electric field, and it points in the $\hat{k}$ direction. Notice that this calculation uses the principle of **superposition**; we calculate the fields of the two charges independently and then add them together.

What we want to do now is replace the quantities in this expression that we don't know (such as r), or can't easily measure (such as $\cos\theta$ with quantities that we do know, or can measure. In this case, by geometry,

$$r^2 = z^2 + \left(\frac{d}{2}\right)^2$$

and

$$\cos\,\theta = \frac{z}{R} = \frac{z}{\left[z^2 + \left(\frac{d}{2}\right)^2\right]^{1/2}}.$$

Thus, substituting,

$$\vec{E}(z) = \frac{1}{4\pi\epsilon_0}\frac{2q}{\left[z^2 + \left(\frac{d}{2}\right)^2\right]^2}\frac{z}{\left[z^2 + \left(\frac{d}{2}\right)^2\right]^{1/2}}\hat{k}.$$

Simplifying, the desired answer is

$$\vec{E}(z) = \frac{1}{4\pi\epsilon_0}\frac{2qz}{\left[z^2 + \left(\frac{d}{2}\right)^2\right]^{3/2}}\hat{k}. \tag{8.5.7}$$

b. If the source charges are equal and opposite, the vertical components cancel because

$$E_z = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\,cos\,\theta - \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\cos\,\theta = 0$$

and we get, for the horizontal component of $\vec{E}$.

$$\vec{E}(z) = \frac{1}{4\pi\epsilon_0}\frac{qd}{\left[z^2 + \left(\frac{d}{2}\right)^2\right]^{3/2}}\hat{i}. \tag{8.5.8}$$

Significance

It is a very common and very useful technique in physics to check whether your answer is reasonable by evaluating it at extreme cases. In this example, we should evaluate the field expressions for the cases $d = 0,\ z \gg d$, and $z \to \infty$, and confirm that the resulting expressions match our physical expectations. Let's do so:

Let's start with Equation 8.5.7, the field of two identical charges. From far away (i.e., $z >> d$), the two source charges should "merge" and we should then "see" the field of just one charge, of size $2q$. So, let $z \gg d$; then we can neglect d^2 in Equation 8.5.7 to obtain

$$\lim_{d\to 0}\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{2qz}{[z^2]^{3/2}}\hat{k}$$
$$= \frac{1}{4\pi\epsilon_0}\frac{2qz}{z^3}\hat{k}$$
$$= \frac{1}{4\pi\epsilon_0}\frac{2q}{z^2}\hat{k},$$

which is the correct expression for a field at a distance z away from a charge $2q$.

Next, we consider the field of equal and opposite charges, Equation 8.5.8. It can be shown (via a Taylor expansion) that for $d \ll z \ll \infty$, , this becomes

$$\vec{E}(z) = \frac{1}{4\pi\epsilon_0}\frac{qd}{z^3}\hat{i},$$

which is the field of a dipole, a system that we will study in more detail later. (Note that the units of $\vec{E}$ are still correct in this expression, since the units of d in the numerator cancel the unit of the "extra" z in the denominator.) If z is *very* large $(z \to \infty)$, then $E \to 0$, as it should; the two charges "merge" and so cancel out.

Exercise 8.5.1

What is the electric field due to a single point particle?

Answer available at the end of this section.

Phe Simulation: Electric Field Hockey

Try this simulation of electric field hockey to get the charge in the goal by placing other charges on the field.

Answers

Exercise 8.5.1

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

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8.6: Calculating Electric Fields of Charge Distributions

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Explain what a continuous source charge distribution is and how it is related to the concept of quantization of charge
- Describe line charges, surface charges, and volume charges
- Calculate the field of a continuous source charge distribution of either sign

The charge distributions we have seen so far have been discrete: made up of individual point particles. This is in contrast with a **continuous charge distribution**, which has at least one nonzero dimension. If a charge distribution is continuous rather than discrete, we can generalize the definition of the electric field. We simply divide the charge into infinitesimal pieces and treat each piece as a point charge.

Note that because charge is quantized, there is no such thing as a "truly" continuous charge distribution. However, in most practical cases, the total charge creating the field involves such a huge number of discrete charges that we can safely ignore the discrete nature of the charge and consider it to be continuous. This is exactly the kind of approximation we make when we deal with a bucket of water as a continuous fluid, rather than a collection of $\mathrm{H_2O}$ molecules.

Our first step is to define a charge density for a charge distribution along a line, across a surface, or within a volume, as shown in Figure $8.6.1$.

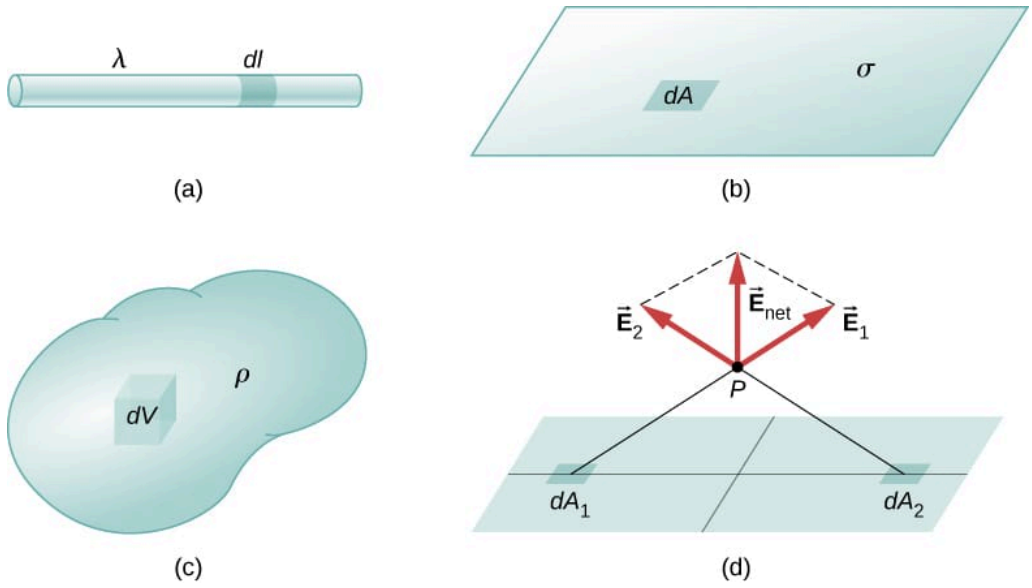


Figure $8.6.1$: The configuration of charge differential elements for a (a) line charge, (b) sheet of charge, and (c) a volume of charge. Also note that (d) some of the components of the total electric field cancel out, with the remainder resulting in a net electric field.

Definitions: Charge Densities

Definitions of charge density:

- **linear charge density:** $\lambda \equiv$ charge per unit length (Figure $8.6.1a$); units are coulombs per meter $(\mathrm{C/m})$
- **surface charge density:** $\sigma \equiv$ charge per unit area (Figure $8.6.1b$); units are coulombs per square meter $(\mathrm{C/m^2})$
- **volume charge density:** $\rho \equiv$ charge per unit volume (Figure $8.6.1c$); units are coulombs per square meter $(\mathrm{C/m^3})$

For a line charge, a surface charge, and a volume charge, the summation in the definition of an Electric field discussed previously

becomes an integral and q_i is replaced by $dq = \lambda\ dl$, $\sigma\ dA$, or $\rho\ dV$, respectively:

$$\vec{E}(P) = \underbrace{\frac{1}{4\pi\epsilon_0}\sum_{i=1}^{N}\left(\frac{q_i}{r^2}\right)\hat{r}}_{\text{Point charges}} \tag{8.6.1}$$

$$\vec{E}(P) = \underbrace{\frac{1}{4\pi\epsilon_0}\int_{\text{line}}\left(\frac{\lambda\, dl}{r^2}\right)\hat{r}}_{\text{Line charge}} \tag{8.6.2}$$

$$\vec{E}(P) = \underbrace{\frac{1}{4\pi\epsilon_0}\int_{\text{surface}}\left(\frac{\sigma\, dA}{r^2}\right)\hat{r}}_{\text{Surface charge}} \tag{8.6.3}$$

$$\vec{E}(P) = \underbrace{\frac{1}{4\pi\epsilon_0}\int_{\text{volume}}\left(\frac{\rho\, dV}{r^2}\right)\hat{r}}_{\text{Volume charge}} \tag{8.6.4}$$

The integrals in Equations 8.6.1-8.6.4 are generalizations of the expression for the field of a point charge. They implicitly include and assume the principle of superposition. The "trick" to using them is almost always in coming up with correct expressions for dl, dA, or dV, as the case may be, expressed in terms of **r**, and also expressing the charge density function appropriately. It may be constant; it might be dependent on location.

Note carefully the meaning of r in these equations: It is the distance from the charge element ($q_i,\ \lambda\ dl,\ \sigma\ dA,\ \rho\ dV$) to the location of interest, $P(x, y, z)$ (the point in space where you want to determine the field). However, don't confuse this with the meaning of $\hat{r}$; we are using it and the vector notation $\vec{E}$ to write three integrals at once. That is, Equation 8.6.2 is actually

$$E_x(P) = \frac{1}{4\pi\epsilon_0}\int_{\text{line}}\left(\frac{\lambda\, dl}{r^2}\right)_x, \tag{8.6.5}$$

$$E_y(P) = \frac{1}{4\pi\epsilon_0}\int_{\text{line}}\left(\frac{\lambda\, dl}{r^2}\right)_y, \tag{8.6.6}$$

$$E_z(P) = \frac{1}{4\pi\epsilon_0}\int_{\text{line}}\left(\frac{\lambda\, dl}{r^2}\right)_z \tag{8.6.7}$$

Example 8.6.1: Electric Field of a Line Segment

Find the electric field a distance z above the midpoint of a straight line segment of length L that carries a uniform line charge density λ.

Strategy

Since this is a continuous charge distribution, we conceptually break the wire segment into differential pieces of length dl, each of which carries a differential amount of charge

$$dq = \lambda\, dl.$$

Then, we calculate the differential field created by two symmetrically placed pieces of the wire, using the symmetry of the setup to simplify the calculation (Figure 8.6.2). Finally, we integrate this differential field expression over the length of the wire (half of it, actually, as we explain below) to obtain the complete electric field expression.

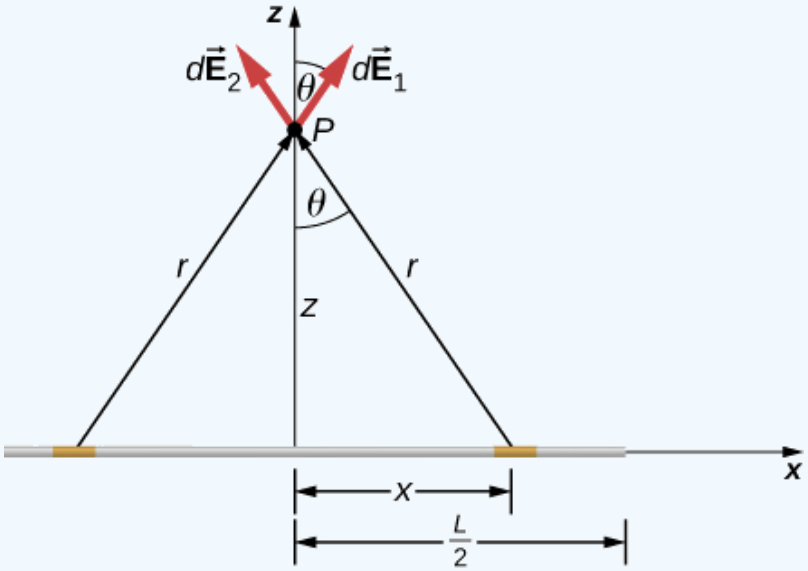


Figure 8.6.2: A uniformly charged segment of wire. The electric field at point P can be found by applying the superposition principle to symmetrically placed charge elements and integrating.

Solution

Before we jump into it, what do we expect the field to "look like" from far away? Since it is a finite line segment, from far away, it should look like a point charge. We will check the expression we get to see if it meets this expectation.

The electric field for a line charge is given by the general expression

$$\vec{E}(P) = \frac{1}{4\pi\epsilon_0}\int_{\text{line}} \frac{\lambda dl}{r^2}\hat{r}.$$

The symmetry of the situation (our choice of the two identical differential pieces of charge) implies the horizontal (x)-components of the field cancel, so that the net field points in the z-direction. Let's check this formally.

The total field $\vec{E}(P)$ is the vector sum of the fields from each of the two charge elements (call them $\vec{E}_1$ and $\vec{E}_2$, for now):

$$\begin{aligned}\vec{E}(P) &= \vec{E}_1 + \vec{E}_2 \\ &= E_{1x}\hat{i} + E_{1z}\hat{k} + E_{2x}(-\hat{i}) + E_{2z}\hat{k}.\end{aligned}$$

Because the two charge elements are identical and are the same distance away from the point P where we want to calculate the field, $E_{1x} = E_{2x}$, so those components cancel. This leaves

$$\begin{aligned}\vec{E}(P) &= E_{1z}\hat{k} + E_{2z}\hat{k} \\ &= E_1\,\cos\theta\,\hat{k} + E_2\,\cos\theta\,\hat{k}.\end{aligned}$$

These components are also equal, so we have

$$\begin{aligned}\vec{E}(P) &= \frac{1}{4\pi\epsilon_0}\int \frac{\lambda dl}{r^2}\cos\theta\,\hat{k} + \frac{1}{4\pi\epsilon_0}\int \frac{\lambda dl}{r^2}\cos\theta\,\hat{k} \\ &= \frac{1}{4\pi\epsilon_0}\int_0^{L/2} \frac{2\lambda dx}{r^2}\cos\theta\,\hat{k}\end{aligned}$$

where our differential line element dl is dx, in this example, since we are integrating along a line of charge that lies on the x -axis. (The limits of integration are 0 to $\frac{L}{2}$, not $-\frac{L}{2}$ to $+\frac{L}{2}$, because we have constructed the net field from two differential pieces of charge dq. If we integrated along the entire length, we would pick up an erroneous factor of 2.)

In principle, this is complete. However, to actually calculate this integral, we need to eliminate all the variables that are not given. In this case, both r and θ change as we integrate outward to the end of the line charge, so those are the variables to get rid of. We can do that the same way we did for the two point charges: by noticing that

$$r = (z^2 + x^2)^{1/2}$$

and

$$\cos\theta = \frac{z}{r} = \frac{z}{(z^2 + x^2)^{1/2}}.$$

Substituting, we obtain

$$\begin{aligned}\vec{E}(P) &= \frac{1}{4\pi\epsilon_0}\int_0^{L/2}\frac{2\lambda dx}{(z^2+x^2)}\frac{z}{(z^2+x^2)^{1/2}}\hat{k} \\ &= \frac{1}{4\pi\epsilon_0}\int_0^{L/2}\frac{2\lambda z}{(z^2+x^2)^{3/2}}dx\,\hat{k} \\ &= \frac{2\lambda z}{4\pi\epsilon_0}\left[\frac{x}{z^2\sqrt{z^2+x^2}}\right]_0^{L/2}\hat{k}.\end{aligned}$$

which simplifies to

$$\vec{E}(z) = \frac{1}{4\pi\epsilon_0}\frac{\lambda L}{z\sqrt{z^2+\frac{L^2}{4}}}\hat{k}. \tag{8.6.8}$$

Significance

Notice, once again, the use of symmetry to simplify the problem. This is a very common strategy for calculating electric fields. The fields of nonsymmetrical charge distributions have to be handled with multiple integrals and may need to be calculated numerically by a computer.

Exercise 8.6.1

How would the strategy used above change to calculate the electric field at a point a distance z above one end of the finite line segment?

Answer available at the end of this section.

Example 8.6.2: Electric Field of an Infinite Line of Charge

Find the electric field a distance z above the midpoint of an infinite line of charge that carries a uniform line charge density λ.

Strategy

This is exactly like the preceding example, except the limits of integration will be $-\infty$ to $+\infty$.

Solution

Again, the horizontal components cancel out, so we wind up with

$$\vec{E}(P) = \frac{1}{4\pi\epsilon_0}\int_{-\infty}^{\infty}\frac{\lambda dx}{r^2}\cos\theta\hat{k}$$

where our differential line element dl is dx, in this example, since we are integrating along a line of charge that lies on the x -axis. Again,

$$\begin{aligned}\cos\theta &= \frac{z}{r} \\ &= \frac{z}{(z^2+x^2)^{1/2}}.\end{aligned}$$

Substituting, we obtain

$$\begin{aligned}\vec{E}(P) &= \frac{1}{4\pi\epsilon_0}\int_{-\infty}^{\infty}\frac{\lambda dx}{(z^2+x^2)}\,\frac{z}{(z^2+x^2)^{1/2}}\hat{k} \\ &= \frac{1}{4\pi\epsilon_0}\int_{-\infty}^{\infty}\frac{\lambda z}{(z^2+x^2)^{3/2}}dx\,\hat{k} \\ &= \frac{1}{4\pi\epsilon_0}\left[\frac{x}{z^2\sqrt{z^2+x^2}}\right]_{-\infty}^{\infty}\hat{k}\end{aligned}$$

which simplifies to

$$\vec{E}(z) = \frac{1}{4\pi\epsilon_0}\frac{2\lambda}{z}\hat{k}.$$

Significance

Our strategy for working with continuous charge distributions also gives useful results for charges with infinite dimension.

In the case of a finite line of charge, note that for $z \gg L$, z^2 dominates the L in the denominator, so that Equation 8.6.8 simplifies to

$$\vec{E} \approx \frac{1}{4\pi\epsilon_0}\frac{\lambda L}{z^2}\hat{k}.$$

If you recall that $\lambda L = q$ the total charge on the wire, we have retrieved the expression for the field of a point charge, as expected.

In the limit $L \to \infty$ on the other hand, we get the field of an **infinite straight wire**, which is a straight wire whose length is much, much greater than either of its other dimensions, and also much, much greater than the distance at which the field is to be calculated:

$$\vec{E}(z) = \frac{1}{4\pi\epsilon_0}\frac{2\lambda}{z}\hat{k}. \tag{8.6.9}$$

An interesting artifact of this infinite limit is that we have lost the usual $1/r^2$ dependence that we are used to. This will become even more intriguing in the case of an infinite plane.

Example 8.6.3A: Electric Field due to a Ring of Charge

A ring has a uniform charge density λ, with units of coulomb per unit meter of arc. Find the electric field at a point on the axis passing through the center of the ring.

Strategy

We use the same procedure as for the charged wire. The difference here is that the charge is distributed on a circle. We divide the circle into infinitesimal elements shaped as arcs on the circle and use polar coordinates shown in Figure 8.6.3.

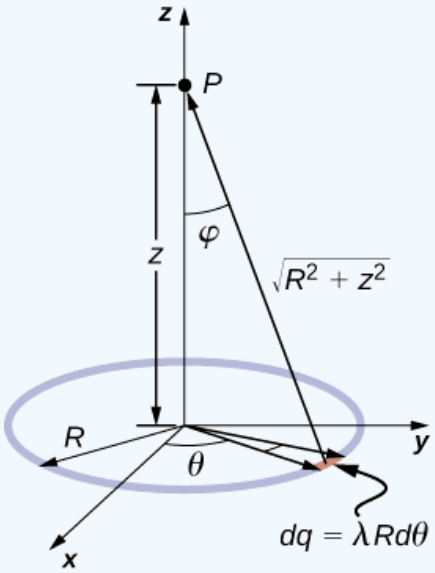


Figure 8.6.3: The system and variable for calculating the electric field due to a ring of charge.

Solution

The electric field for a line charge is given by the general expression

$$\vec{E}(P) = \frac{1}{4\pi\epsilon_0}\int_{\text{line}} \frac{\lambda dl}{r^2}\hat{r}.$$

A general element of the arc between θ and $\theta + d\theta$ is of length $R\ d\theta$ and therefore contains a charge equal to $\lambda R\ d\theta$. The element is at a distance of $r = \sqrt{z^2 + R^2}$ from P, the angle is $\cos\phi = \dfrac{z}{\sqrt{z^2 + R^2}}$ and therefore the electric field is

$$\begin{aligned}
\vec{E}(P) &= \frac{1}{4\pi\epsilon_0}\int_{\text{line}} \frac{\lambda dl}{r^2}\hat{r} = \frac{1}{4\pi\epsilon_0}\int_0^{2\pi} \frac{\lambda R d\theta}{z^2 + R^2}\frac{z}{\sqrt{z^2 + R^2}}\hat{z} \\
&= \frac{1}{4\pi\epsilon_0}\frac{\lambda R z}{(z^2 + R^2)^{3/2}}\hat{z}\int_0^{2\pi} d\theta \\
&= \frac{1}{4\pi\epsilon_0}\frac{2\pi\lambda R z}{(z^2 + R^2)^{3/2}}\hat{z} \\
&= \frac{1}{4\pi\epsilon_0}\frac{q_{tot} z}{(z^2 + R^2)^{3/2}}\hat{z}.
\end{aligned}$$

Significance

As usual, symmetry simplified this problem, in this particular case resulting in a trivial integral. Also, when we take the limit of $z \gg R$, we find that

$$\vec{E} \approx \frac{1}{4\pi\epsilon_0}\frac{q_{tot}}{z^2}\hat{z},$$

as we expect.

Example 8.6.3B: The Field of a Disk

Find the electric field of a circular thin disk of radius R and uniform charge density at a distance z above the center of the disk (Figure 8.6.4)

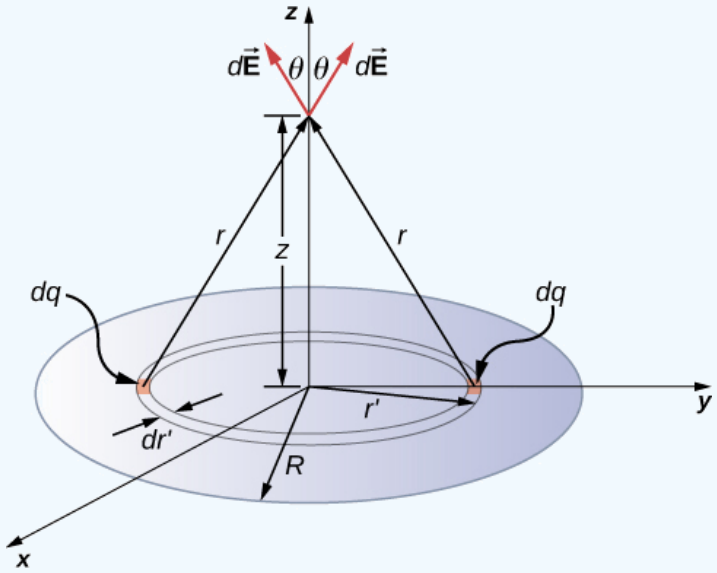


Figure 8.6.4: A uniformly charged disk. As in the line charge example, the field above the center of this disk can be calculated by taking advantage of the symmetry of the charge distribution.

Strategy

The electric field for a surface charge is given by

$$\vec{E}(P) = \frac{1}{4\pi\epsilon_0}\int_{\text{surface}} \frac{\sigma\, dA}{r^2}\hat{r}.$$

To solve surface charge problems, we break the surface into symmetrical differential "stripes" that match the shape of the surface; here, we'll use rings, as shown in the figure. Again, by symmetry, the horizontal components cancel and the field is entirely in the vertical $(\hat{k})$ direction. The vertical component of the electric field is extracted by multiplying by θ, so

$$\vec{E}(P) = \frac{1}{4\pi\epsilon_0}\int_{surface} \frac{\sigma\, dA}{r^2}\cos\theta\,\hat{k}.$$

As before, we need to rewrite the unknown factors in the integrand in terms of the given quantities. In this case,

$$dA = 2\pi r' dr'$$

$$r^2 = r'^2 + z^2$$

$$\cos\theta = \frac{z}{(r'^2+z^2)^{1/2}}.$$

(Please take note of the two different "r's" here; r is the distance from the differential ring of charge to the point P where we wish to determine the field, whereas r' is the distance from the center of the disk to the differential ring of charge.) Also, we already performed the polar angle integral in writing down dA.

Solution

Substituting all this in, we get

$$\begin{aligned}\vec{E}(P) &= \vec{E}(z)\\ &= \frac{1}{4\pi\epsilon_0}\int_0^R \frac{\sigma(2\pi r' dr')z}{(r'^2+z^2)^{3/2}}\hat{k}\\ &= \frac{1}{4\pi\epsilon_0}(2\pi\sigma z)\left(\frac{1}{z} - \frac{1}{\sqrt{R^2+z^2}}\right)\hat{k}\end{aligned}$$

or, more simply,

$$\vec{E}(z) = \frac{1}{4\pi\epsilon_0}\left(2\pi\sigma - \frac{2\pi\sigma z}{\sqrt{R^2+z^2}}\right)\hat{k}. \tag{8.6.10}$$

Significance

Again, it can be shown (via a Taylor expansion) that when $z \gg R$, this reduces to

$$\vec{E}(z) \approx \frac{1}{4\pi\epsilon_0}\frac{\sigma\pi R^2}{z^2}\hat{k},$$

which is the expression for a point charge $Q = \sigma\pi R^2$.

Exercise 8.6.3

How would the above limit change with a uniformly charged rectangle instead of a disk?

Answer available at the end of this section.

As $R \to \infty$, Equation 8.6.10 reduces to the field of an infinite plane, which is a flat sheet whose area is much, much greater than its thickness, and also much, much greater than the distance at which the field is to be calculated:

$$\vec{E} = \lim_{R\to\infty}\frac{1}{4\pi\epsilon_0}\left(2\pi\sigma - \frac{2\pi\sigma z}{\sqrt{R^2+z^2}}\right)\hat{k} \tag{8.6.11}$$

$$= \frac{\sigma}{2\epsilon_0}\hat{k}. \tag{8.6.12}$$

Note that this field is constant. This surprising result is, again, an artifact of our limit, although one that we will make use of repeatedly in the future. To understand why this happens, imagine being placed above an infinite plane of constant charge. Does the plane look any different if you vary your altitude? No—you still see the plane going off to infinity, no matter how far you are from it. It is important to note that Equation 8.6.12 is because we are above the plane. If we were below, the field would point in the $-\hat{k}$ direction.

Example 8.6.4: The Field of Two Infinite Planes

Find the electric field everywhere resulting from two infinite planes with equal but opposite charge densities (Figure 8.6.5).

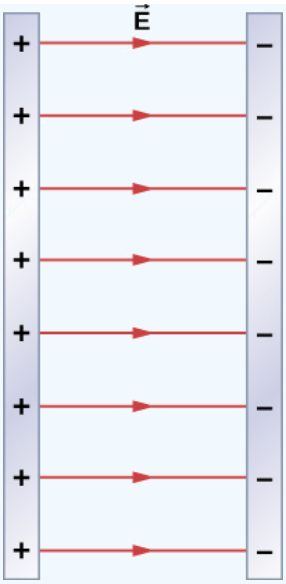


Figure 8.6.5: Two charged infinite planes. Note the direction of the electric field.

Strategy

We already know the electric field resulting from a single infinite plane, so we may use the principle of superposition to find the field from two.

Solution

The electric field points away from the positively charged plane and toward the negatively charged plane. Since the σ are equal and opposite, this means that in the region outside of the two planes, the electric fields cancel each other out to zero. However, in the region between the planes, the electric fields add, and we get

$$\vec{E} = \frac{\sigma}{\epsilon_0}\hat{i}$$

for the electric field. The $\hat{i}$ is because in the figure, the field is pointing in the $+x$-direction.

Significance

Systems that may be approximated as two infinite planes of this sort provide a useful means of creating uniform electric fields.

Exercise 8.6.1

What would the electric field look like in a system with two parallel positively charged planes with equal charge densities?

Answer available at the end of this section.

Answers

Exercise 8.6.1

We will no longer be able to take advantage of symmetry. Instead, we will need to calculate each of the two components of the electric field with their own integral.

Exercise 8.6.3

The point charge would be $Q = \sigma ab$ where a and b are the sides of the rectangle but otherwise identical.

Exercise 8.6.1

The electric field would be zero in between, and have magnitude $\frac{\sigma}{\epsilon_0}$ everywhere else.

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8.7: Electric Field Lines

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8.8: Electric Dipoles

Learning Objectives

By the end of this section, you will be able to:

- Describe a permanent dipole
- Describe an induced dipole
- Define and calculate an electric dipole moment
- Explain the physical meaning of the dipole moment

Earlier we discussed, and calculated, the electric field of a dipole: two equal and opposite charges that are "close" to each other. (In this context, "close" means that the distance d between the two charges is much, much less than the distance of the field point P, the location where you are calculating the field.) Let's now consider what happens to a dipole when it is placed in an external field $\vec{E}$. We assume that the dipole is a **permanent dipole**; it exists without the field, and does not break apart in the external field.

Rotation of a Dipole due to an Electric Field

For now, we deal with only the simplest case: The external field is uniform in space. Suppose we have the situation depicted in Figure 8.8.1, where we denote the distance between the charges as the vector $\vec{d}$, pointing from the negative charge to the positive charge.

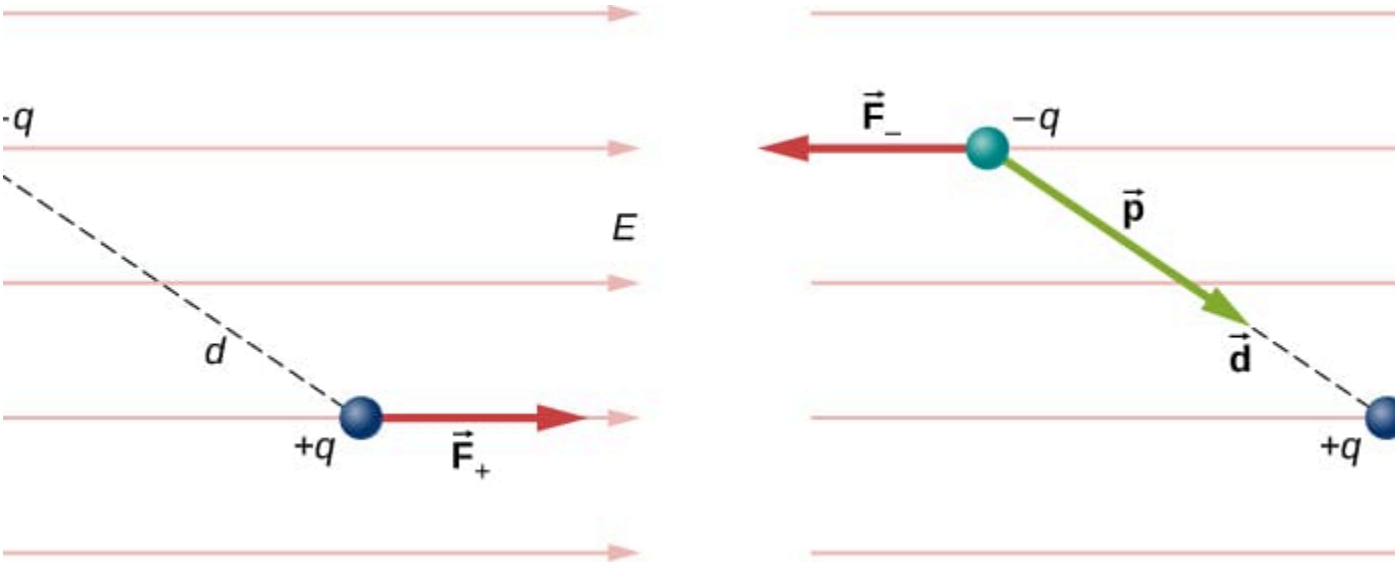


Figure 8.8.1: A dipole in an external electric field. (a) The net force on the dipole is zero, but the net torque is not. As a result, the dipole rotates, becoming aligned with the external field. (b) The dipole moment is a convenient way to characterize this effect. The $\vec{d}$ points in the same direction as $\vec{p}$.

The forces on the two charges are equal and opposite, so there is no net force on the dipole. However, there is a torque:

$$\vec{r} = \left(\frac{\vec{d}}{2} \times \vec{F}_+\right) + \left(-\frac{\vec{d}}{2} \times \vec{F}_-\right) \tag{8.8.1}$$

$$= \left[\left(\frac{\vec{d}}{2}\right) \times \left(+q\vec{E}\right) + \left(-\frac{\vec{d}}{2}\right) \times \left(-q\vec{E}\right)\right] \tag{8.8.2}$$

$$= q\vec{d} \times \vec{E}. \tag{8.8.3}$$

The quantity qd (the magnitude of each charge multiplied by the vector distance between them) is a property of the dipole; its value, as you can see, determines the torque that the dipole experiences in the external field. It is useful, therefore, to define this product as the so-called **dipole moment** of the dipole:

$$\vec{p} \equiv q\vec{d}\,.$$

We can therefore write

$$\vec{r} = \vec{p} \times \vec{E}.$$

Recall that a torque changes the angular velocity of an object, the dipole, in this case. In this situation, the effect is to rotate the dipole (that is, align the direction of $\vec{p}$) so that it is parallel to the direction of the external field.

Induced Dipoles

Neutral atoms are, by definition, electrically neutral; they have equal amounts of positive and negative charge. Furthermore, since they are spherically symmetrical, they do not have a "built-in" dipole moment the way most asymmetrical molecules do. They obtain one, however, when placed in an external electric field, because the external field causes oppositely directed forces on the positive nucleus of the atom versus the negative electrons that surround the nucleus. The result is a new charge distribution of the atom, and therefore, an **induced dipole** moment (Figure $8.8.2$).

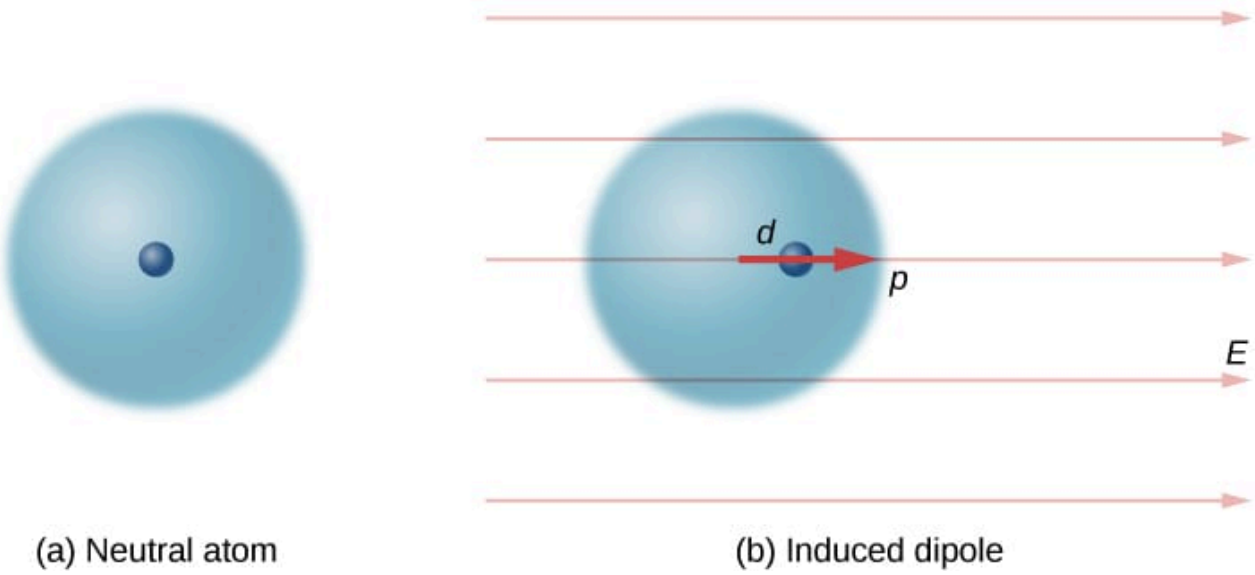


Figure $8.8.2$: A dipole is induced in a neutral atom by an external electric field. The induced dipole moment is aligned with the external field.

An important fact here is that, just as for a rotated polar molecule, the result is that the dipole moment ends up aligned parallel to the external electric field. Generally, the magnitude of an induced dipole is much smaller than that of an inherent dipole. For both kinds of dipoles, notice that once the alignment of the dipole (rotated or induced) is complete, the net effect is to decrease the total electric field

$$\vec{E}_{total} = \vec{E}_{external} + \vec{E}_{dipole}$$

in the regions outside the dipole charges (Figure $8.8.3$). By "outside" we mean further from the charges than they are from each other. This effect is crucial for capacitors, as you will see in Capacitance.

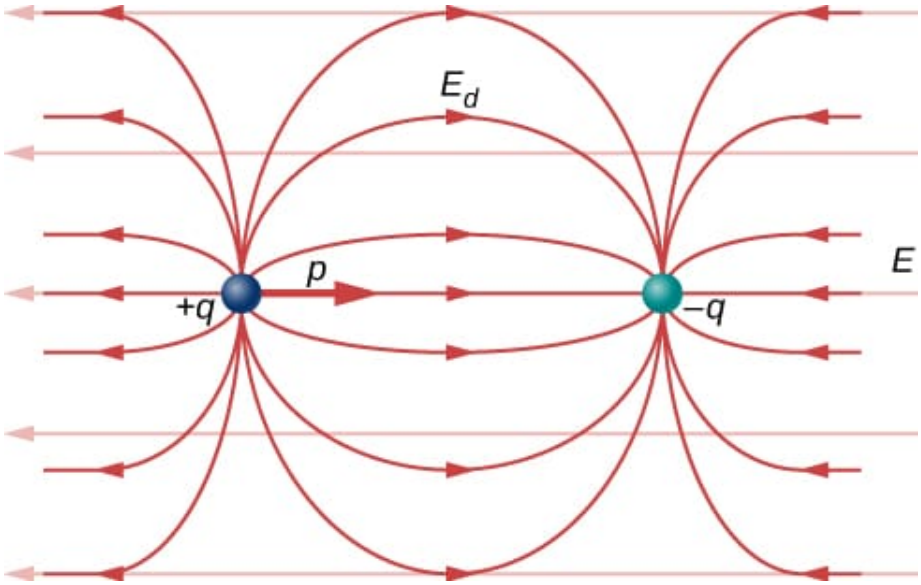


Figure 8.8.3: The net electric field is the vector sum of the field of the dipole plus the external field.

Recall that we found the electric field of a dipole. If we rewrite it in terms of the dipole moment we get:

$$\vec{E}(z) = \frac{1}{4\pi\epsilon_0}\frac{\vec{p}}{z^3}.$$

The form of this field is shown in Figure 8.8.3. Notice that along the plane perpendicular to the axis of the dipole and midway between the charges, the direction of the electric field is opposite that of the dipole and gets weaker the further from the axis one goes. Similarly, on the axis of the dipole (but outside it), the field points in the same direction as the dipole, again getting weaker the further one gets from the charges.

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8.9: Electric Charges and Fields (Summary)

Key Terms

charging by induction	process by which an electrically charged object brought near a neutral object creates a charge separation in that object
conduction electron	electron that is free to move away from its atomic orbit
conductor	material that allows electrons to move separately from their atomic orbits; object with properties that allow charges to move about freely within it
continuous charge distribution	total source charge composed of so large a number of elementary charges that it must be treated as continuous, rather than discrete
coulomb	SI unit of electric charge
Coulomb force	another term for the electrostatic force
Coulomb's law	mathematical equation calculating the electrostatic force vector between two charged particles
dipole	two equal and opposite charges that are fixed close to each other
dipole moment	property of a dipole; it characterizes the combination of distance between the opposite charges, and the magnitude of the charges
electric charge	physical property of an object that causes it to be attracted toward or repelled from another charged object; each charged object generates and is influenced by a force called an electric force
electric field	physical phenomenon created by a charge; it "transmits" a force between a two charges
electric force	noncontact force observed between electrically charged objects
electron	particle surrounding the nucleus of an atom and carrying the smallest unit of negative charge
electrostatic attraction	phenomenon of two objects with opposite charges attracting each other
electrostatic force	amount and direction of attraction or repulsion between two charged bodies; the assumption is that the source charges have no acceleration
electrostatic repulsion	phenomenon of two objects with like charges repelling each other
electrostatics	study of charged objects which are not in motion
field line	smooth, usually curved line that indicates the direction of the electric field
field line density	number of field lines per square meter passing through an imaginary area; its purpose is to indicate the field strength at different points in space
induced dipole	typically an atom, or a spherically symmetric molecule; a dipole created due to opposite forces displacing the positive and negative charges
infinite straight wire	straight wire whose length is much, much greater than either of its other dimensions, and also much, much greater than the distance at which the field is to be calculated
insulator	material that holds electrons securely within their atomic orbits
ion	atom or molecule with more or fewer electrons than protons

law of conservation of charge	net electric charge of a closed system is constant
linear charge density	amount of charge in an element of a charge distribution that is essentially one-dimensional (the width and height are much, much smaller than its length); its units are C/m
neutron	neutral particle in the nucleus of an atom, with (nearly) the same mass as a proton
permanent dipole	typically a molecule; a dipole created by the arrangement of the charged particles from which the dipole is created
permittivity of vacuum	also called the permittivity of free space, and constant describing the strength of the electric force in a vacuum
polarization	slight shifting of positive and negative charges to opposite sides of an object
principle of superposition	useful fact that we can simply add up all of the forces due to charges acting on an object
proton	particle in the nucleus of an atom and carrying a positive charge equal in magnitude to the amount of negative charge carried by an electron
static electricity	buildup of electric charge on the surface of an object; the arrangement of the charge remains constant ("static")
superposition	concept that states that the net electric field of multiple source charges is the vector sum of the field of each source charge calculated individually
surface charge density	amount of charge in an element of a two-dimensional charge distribution (the thickness is small); its units are C/m^2
volume charge density	amount of charge in an element of a three-dimensional charge distribution; its units are C/m^3

Key Equations

Coulomb's law	$\vec{F_{12}}(r) = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}^2}\hat{r_{12}}$
Superposition of electric forces	$\vec{F}(r) = \frac{1}{4\pi\varepsilon_0}Q\sum_{i=1}^{N}\frac{q_i}{r_i^2}\hat{r_i}$
Electric force due to an electric field	$\vec{F} = Q\vec{E}$
Electric field at point P	$\vec{E}(P) \equiv \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N}\frac{q_i}{r_i^2}\hat{r_i}$
Field of an infinite wire	$\vec{E}(z) = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{z}\hat{k}$
Field of an infinite plane	$\vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{k}$
Dipole moment	$\vec{p} \equiv q\vec{d}$
Torque on dipole in external E-field	$\vec{\tau} = \vec{p} \times \vec{E}$

Summary

5.2 Electric Charge

- There are only two types of charge, which we call positive and negative. Like charges repel, unlike charges attract, and the force between charges decreases with the square of the distance.

- The vast majority of positive charge in nature is carried by protons, whereas the vast majority of negative charge is carried by electrons. The electric charge of one electron is equal in magnitude and opposite in sign to the charge of one proton.
- An ion is an atom or molecule that has nonzero total charge due to having unequal numbers of electrons and protons.
- The SI unit for charge is the coulomb (C), with protons and electrons having charges of opposite sign but equal magnitude; the magnitude of this basic charge is $e \equiv 1.602 \times 10^{-19}\mathrm{C}$
- Both positive and negative charges exist in neutral objects and can be separated by bringing the two objects into physical contact; rubbing the objects together can remove electrons from the bonds in one object and place them on the other object, increasing the charge separation.
- For macroscopic objects, negatively charged means an excess of electrons and positively charged means a depletion of electrons.
- The law of conservation of charge states that the net charge of a closed system is constant.

5.3 Conductors, Insulators, and Charging by Induction

- A conductor is a substance that allows charge to flow freely through its atomic structure.
- An insulator holds charge fixed in place.
- Polarization is the separation of positive and negative charges in a neutral object. Polarized objects have their positive and negative charges concentrated in different areas, giving them a charge distribution.

5.4 Coulomb's Law

- Coulomb's law gives the magnitude of the force between point charges. It is

  $$\vec{F}_{12}(r) = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12}$$

  where q_1 and q_2 are two point charges separated by a distance **r**. This Coulomb force is extremely basic, since most charges are due to point-like particles. It is responsible for all electrostatic effects and underlies most macroscopic forces.

5.5 Electric Field

- The electric field is an alteration of space caused by the presence of an electric charge. The electric field mediates the electric force between a source charge and a test charge.
- The electric field, like the electric force, obeys the superposition principle
- The field is a vector; by definition, it points away from positive charges and toward negative charges.

5.6 Calculating Electric Fields of Charge Distributions

- A very large number of charges can be treated as a continuous charge distribution, where the calculation of the field requires integration. Common cases are:
  - one-dimensional (like a wire); uses a line charge density λ
  - two-dimensional (metal plate); uses surface charge density σ
  - three-dimensional (metal sphere); uses volume charge density ρ
- The "source charge" is a differential amount of charge dq. Calculating dq depends on the type of source charge distribution:

  $$dq = \lambda\ dl; dq = \sigma\ dA; dq = \rho\ dV.$$

- Symmetry of the charge distribution is usually key.
- Important special cases are the field of an "infinite" wire and the field of an "infinite" plane.

5.7 Electric Field Lines

- Electric field diagrams assist in visualizing the field of a source charge.
- The magnitude of the field is proportional to the field line density.
- Field vectors are everywhere tangent to field lines.

5.8 Electric Dipoles

- If a permanent dipole is placed in an external electric field, it results in a torque that aligns it with the external field.
- If a nonpolar atom (or molecule) is placed in an external field, it gains an induced dipole that is aligned with the external field.
- The net field is the vector sum of the external field plus the field of the dipole (physical or induced).
- The strength of the polarization is described by the dipole moment of the dipole, $\vec{p} = q\vec{d}$.

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8.10: Electric Charges and Fields (Exercises)

Conceptual Questions

5.2 Electric Charge

1. There are very large numbers of charged particles in most objects. Why, then, don't most objects exhibit static electricity?

2. Why do most objects tend to contain nearly equal numbers of positive and negative charges?

3. A positively charged rod attracts a small piece of cork.

(a) Can we conclude that the cork is negatively charged?

(b) The rod repels another small piece of cork. Can we conclude that this piece is positively charged?

4. Two bodies attract each other electrically. Do they both have to be charged? Answer the same question if the bodies repel one another.

5. How would you determine whether the charge on a particular rod is positive or negative?

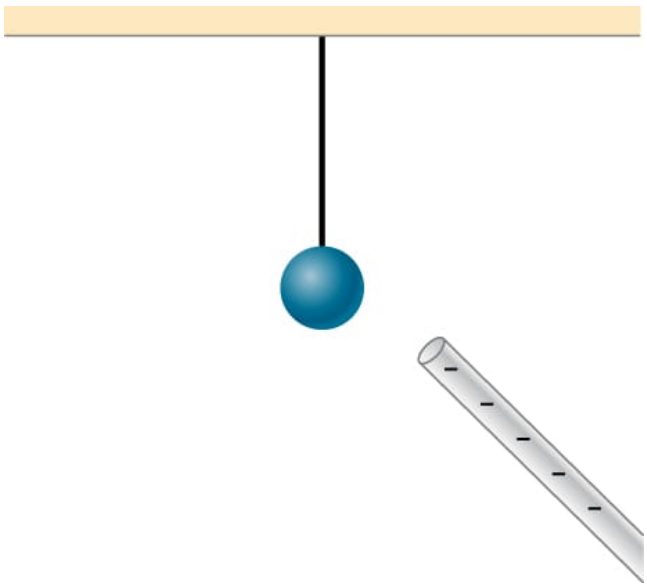
5.3 Conductors, Insulators, and Charging by Induction

6. An eccentric inventor attempts to levitate a cork ball by wrapping it with foil and placing a large negative charge on the ball and then putting a large positive charge on the ceiling of his workshop. Instead, while attempting to place a large negative charge on the ball, the foil flies off. Explain.

7. When a glass rod is rubbed with silk, it becomes positive and the silk becomes negative—yet both attract dust. Does the dust have a third type of charge that is attracted to both positive and negative? Explain.

8. Why does a car always attract dust right after it is polished? (Note that car wax and car tires are insulators.)

9. Does the uncharged conductor shown below experience a net electric force?



10. While walking on a rug, a person frequently becomes charged because of the rubbing between his shoes and the rug. This charge then causes a spark and a slight shock when the person gets close to a metal object. Why are these shocks so much more common on a dry day?

11. Compare charging by conduction to charging by induction.

12. Small pieces of tissue are attracted to a charged comb. Soon after sticking to the comb, the pieces of tissue are repelled from it. Explain.

13. Trucks that carry gasoline often have chains dangling from their undercarriages and brushing the ground. Why?

14. Why do electrostatic experiments work so poorly in humid weather?

15. Why do some clothes cling together after being removed from the clothes dryer? Does this happen if they're still damp?

16. Can induction be used to produce charge on an insulator?

17. Suppose someone tells you that rubbing quartz with cotton cloth produces a third kind of charge on the quartz. Describe what you might do to test this claim.

18. A handheld copper rod does not acquire a charge when you rub it with a cloth. Explain why.

19. Suppose you place a charge **q** near a large metal plate.

(a) If **q** is attracted to the plate, is the plate necessarily charged?

(b) If **q** is repelled by the plate, is the plate necessarily charged?

5.4 Coulomb's Law

20. Would defining the charge on an electron to be positive have any effect on Coulomb's law?

21. An atomic nucleus contains positively charged protons and uncharged neutrons. Since nuclei do stay together, what must we conclude about the forces between these nuclear particles?

22. Is the force between two fixed charges influenced by the presence of other charges?

5.5 Electric Field

23. When measuring an electric field, could we use a negative rather than a positive test charge?

24. During fair weather, the electric field due to the net charge on Earth points downward. Is Earth charged positively or negatively?

25. If the electric field at a point on the line between two charges is zero, what do you know about the charges?

26. Two charges lie along the **x**-axis. Is it true that the net electric field always vanishes at some point (other than infinity) along the **x**-axis?

5.6 Calculating Electric Fields of Charge Distributions

27. Give a plausible argument as to why the electric field outside an infinite charged sheet is constant.

28. Compare the electric fields of an infinite sheet of charge, an infinite, charged conducting plate, and infinite, oppositely charged parallel plates.

29. Describe the electric fields of an infinite charged plate and of two infinite, charged parallel plates in terms of the electric field of an infinite sheet of charge.

30. A negative charge is placed at the center of a ring of uniform positive charge. What is the motion (if any) of the charge? What if the charge were placed at a point on the axis of the ring other than the center?

5.7 Electric Field Lines

31. If a point charge is released from rest in a uniform electric field, will it follow a field line? Will it do so if the electric field is not uniform?

32. Under what conditions, if any, will the trajectory of a charged particle not follow a field line?

33. How would you experimentally distinguish an electric field from a gravitational field?

34. A representation of an electric field shows 10 field lines perpendicular to a square plate. How many field lines should pass perpendicularly through the plate to depict a field with twice the magnitude?

35. What is the ratio of the number of electric field lines leaving a charge 10q and a charge q?

5.8 Electric Dipoles

36. What are the stable orientation(s) for a dipole in an external electric field? What happens if the dipole is slightly perturbed from these orientations?

Problems

5.2 Electric Charge

37. Common static electricity involves charges ranging from nanocoulombs to microcoulombs.

(a) How many electrons are needed to form a charge of –2.00 nC?

(b) How many electrons must be removed from a neutral object to leave a net charge of **0.500μC**?

38. If 1.80×10^{20} electrons move through a pocket calculator during a full day's operation, how many coulombs of charge moved through it?

39. To start a car engine, the car battery moves 3.75×10^{21} electrons through the starter motor. How many coulombs of charge were moved?

40. A certain lightning bolt moves 40.0 C of charge. How many fundamental units of charge is this?

41. A 2.5-g copper penny is given a charge of $-2.0 \times 10^{-9} C$.

(a) How many excess electrons are on the penny?

(b) By what percent do the excess electrons change the mass of the penny?

42. A 2.5-g copper penny is given a charge of $4.0 \times 10^{-9} C$.

(a) How many electrons are removed from the penny?

(b) If no more than one electron is removed from an atom, what percent of the atoms are ionized by this charging process?

5.3 Conductors, Insulators, and Charging by Induction

43. Suppose a speck of dust in an electrostatic precipitator has 1.0000×10^{12} protons in it and has a net charge of –5.00 nC (a very large charge for a small speck). How many electrons does it have?

44. An amoeba has 1.00×10^{16} protons and a net charge of 0.300 pC.

(a) How many fewer electrons are there than protons?

(b) If you paired them up, what fraction of the protons would have no electrons?

45. A 50.0-g ball of copper has a net charge of **2.00μC**. What fraction of the copper's electrons has been removed? (Each copper atom has 29 protons, and copper has an atomic mass of 63.5.)

46. What net charge would you place on a 100-g piece of sulfur if you put an extra electron on 1 in 10^{12} of its atoms? (Sulfur has an atomic mass of 32.1 u.)

47. How many coulombs of positive charge are there in 4.00 kg of plutonium, given its atomic mass is 244 and that each plutonium atom has 94 protons?

5.4 Coulomb's Law

48. Two point particles with charges **+3μC** and **+5μC** are held in place by 3-N forces on each charge in appropriate directions. (a) Draw a free-body diagram for each particle. (b) Find the distance between the charges.

49. Two charges **+3μC** and **+12μC** are fixed 1 m apart, with the second one to the right. Find the magnitude and direction of the net force on a –2-nC charge when placed at the following locations:

(a) halfway between the two

(b) half a meter to the left of the **+3μC** charge

(c) half a meter above the **+12μC** charge in a direction perpendicular to the line joining the two fixed charges

50. In a salt crystal, the distance between adjacent sodium and chloride ions is $2.82 \times 10^{-10} m$. What is the force of attraction between the two singly charged ions?

51. Protons in an atomic nucleus are typically $10^{-15} m$ apart. What is the electric force of repulsion between nuclear protons?

52. Suppose Earth and the Moon each carried a net negative charge **–Q**. Approximate both bodies as point masses and point charges.

(a) What value of **Q** is required to balance the gravitational attraction between Earth and the Moon?

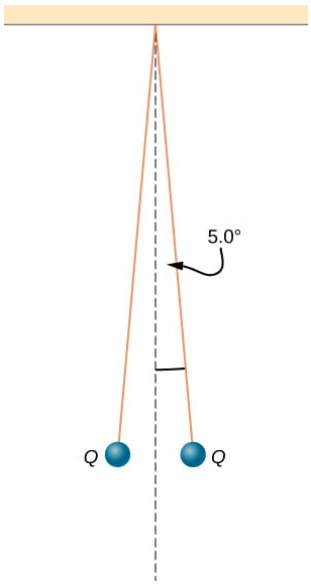
(b) Does the distance between Earth and the Moon affect your answer? Explain.

(c) How many electrons would be needed to produce this charge?

53. Point charges $q_1 = 50\mu C$ and $q_2 = -25\mu C$ are placed 1.0 m apart. What is the force on a third charge $q_3 = 20\mu C$ placed midway between q_1 and q_2?

54. Where must q_3 of the preceding problem be placed so that the net force on it is zero?

55. Two small balls, each of mass 5.0 g, are attached to silk threads 50 cm long, which are in turn tied to the same point on the ceiling, as shown below. When the balls are given the same charge **Q**, the threads hang at **5.0°** to the vertical, as shown below. What is the magnitude of **Q**? What are the signs of the two charges?

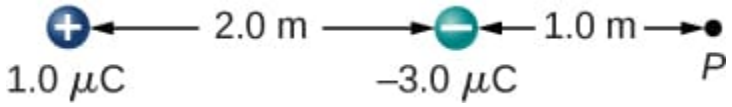


56. Point charges $Q_1 = 2.0\mu C$ and $Q_2 = 4.0\mu C$ are located at $\vec{r}_1 = (4.0\hat{i} - 2.0\hat{j} + 5.0\hat{k})m$ and $\vec{r}_2 = (8.0\hat{i} + 5.0\hat{j} - 9.0\hat{k})m$. What is the force of Q_2 on Q_1?

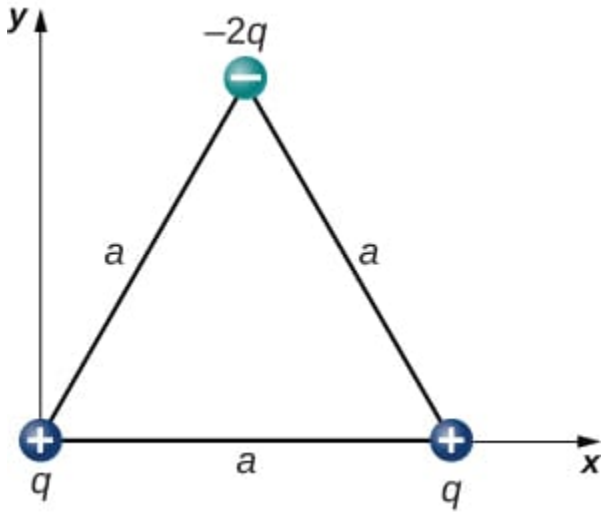
57. The net excess charge on two small spheres (small enough to be treated as point charges) is **Q**. Show that the force of repulsion between the spheres is greatest when each sphere has an excess charge **Q/2**. Assume that the distance between the spheres is so large compared with their radii that the spheres can be treated as point charges.

58. Two small, identical conducting spheres repel each other with a force of 0.050 N when they are 0.25 m apart. After a conducting wire is connected between the spheres and then removed, they repel each other with a force of 0.060 N. What is the original charge on each sphere?

59. A charge **q=2.0μC** is placed at the point **P** shown below. What is the force on **q**?



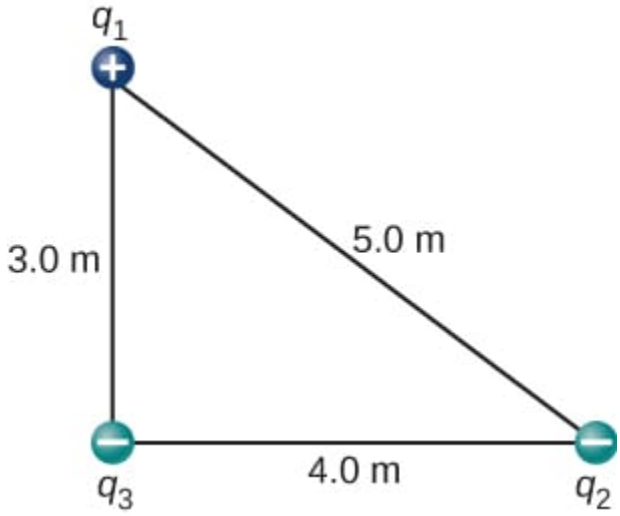
60. What is the net electric force on the charge located at the lower right-hand corner of the triangle shown here?



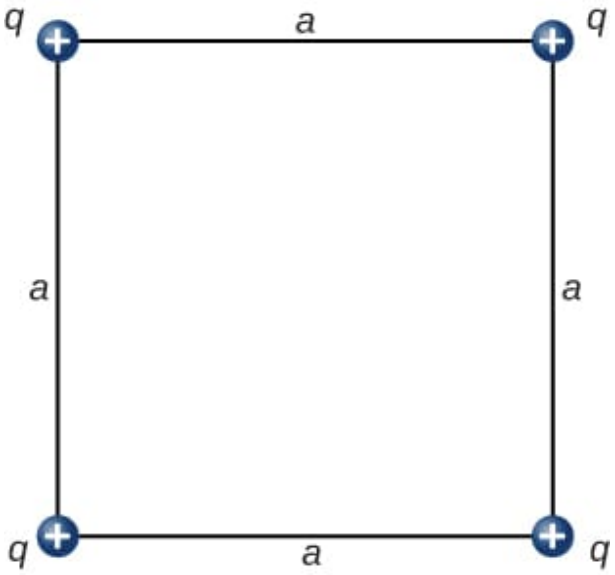
61. Two fixed particles, each of charge $5.0 \times 10^{-6}C$, are 24 cm apart. What force do they exert on a third particle of charge

$-2.5 \times 10^{-6} C$ that is 13 cm from each of them?

62. The charges $q_1 = 2.0 \times 10^{-7} C, q_2 = -4.0 \times 10^{-7} C$, and $q_3 = -1.0 \times 10^{-7} C$ are placed at the corners of the triangle shown below. What is the force on q_1?



63. What is the force on the charge q at the lower-right-hand corner of the square shown here?



64. Point charges $q_1 = 10 \mu C$ and $q_2 = -30 \mu C$ are fixed at $r_1 = (3.0\hat{i} - 4.0\hat{j}) m$ and $r_2 = (9.0\hat{i} + 6.0\hat{j}) m$. What is the force of q_2 on q_1?

5.5 Electric Field

65. A particle of charge $2.0 \times 10^{-8} C$ experiences an upward force of magnitude $4.0 \times 10^{-6} N$ when it is placed in a particular point in an electric field.

(a) What is the electric field at that point?

(b) If a charge $q = -1.0 \times 10^{-8} C$ is placed there, what is the force on it?

66. On a typical clear day, the atmospheric electric field points downward and has a magnitude of approximately 100 N/

C. Compare the gravitational and electric forces on a small dust particle of mass $2.0 \times 10^{-15} g$ that carries a single electron charge. What is the acceleration (both magnitude and direction) of the dust particle?

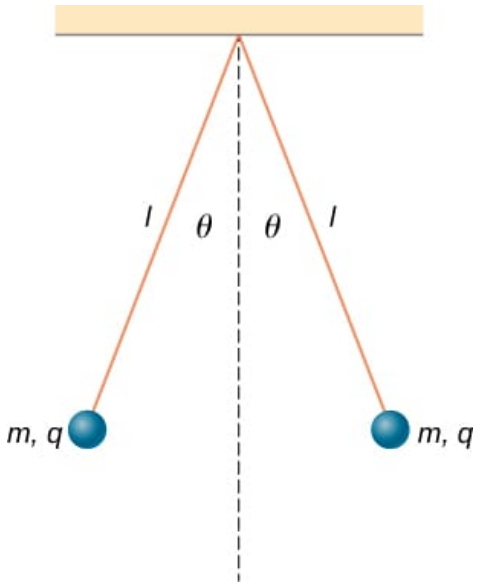
67. Consider an electron that is 10−10m10−10m from an alpha particle (q=3.2×10−19C).(q=3.2×10−19C).

(a) What is the electric field due to the alpha particle at the location of the electron?

(b) What is the electric field due to the electron at the location of the alpha particle?

(c) What is the electric force on the alpha particle? On the electron?

68. Each the balls shown below carries a charge q and has a mass m. The length of each thread is l, and at equilibrium, the balls are separated by an angle 2θ. How does θ vary with **q** and **l**? Show that θ satisfies \(\displaystyle sin(θ)^2tan(θ)=\frac{q^2}{16πε0gl^2m\).



69. What is the electric field at a point where the force on a $-2.0 \times 10^{-6} - C$ charge is $(4.0\hat{i} - 6.0\hat{j}) \times 10^{-6} N$?

70. A proton is suspended in the air by an electric field at the surface of Earth. What is the strength of this electric field?

71. The electric field in a particular thundercloud is $2.0 \times 10^{5} N/C$. What is the acceleration of an electron in this field?

72. A small piece of cork whose mass is 2.0 g is given a charge of $5.0 \times 10^{-7} C$. What electric field is needed to place the cork in equilibrium under the combined electric and gravitational forces?

73. If the electric field is **100N/C** at a distance of 50 cm from a point charge **q**, what is the value of **q**?

74. What is the electric field of a proton at the first Bohr orbit for hydrogen $(r = 5.29 \times 10^{-11} m)$? What is the force on the electron in that orbit?

75. (a) What is the electric field of an oxygen nucleus at a point that is $10^{-10} m$ from the nucleus?

(b) What is the force this electric field exerts on a second oxygen nucleus placed at that point?

76. Two point charges, $q_1 = 2.0 \times 10^{-7} C$ and $q_2 = -6.0 \times 10^{-8} C$, are held 25.0 cm apart.

(a) What is the electric field at a point 5.0 cm from the negative charge and along the line between the two charges?

(b)What is the force on an electron placed at that point?

77. Point charges $q_1 = 50 \mu C$ and $q_2 = -25 \mu C$ are placed 1.0 m apart.

(a) What is the electric field at a point midway between them?

(b) What is the force on a charge $q_3 = 20\mu C$ situated there?

78. Can you arrange the two point charges $q_1 = -2.0 \times 10^{-6}C$ and $q_2 = 4.0 \times 10^{-6}C$ along the **x**-axis so that $E = 0$ at the origin?

79. Point charges $q_1 = q_2 = 4.0 \times 10^{-6}C$ are fixed on the **x**-axis at $x = -3.0m$ and $x = 3.0m$. What charge **q** must be placed at the origin so that the electric field vanishes at **x=0,y=3.0m**?

5.6 Calculating Electric Fields of Charge Distributions

80. A thin conducting plate 1.0 m on the side is given a charge of $-2.0 \times 10^{-6}C$. An electron is placed 1.0 cm above the center of the plate. What is the acceleration of the electron?

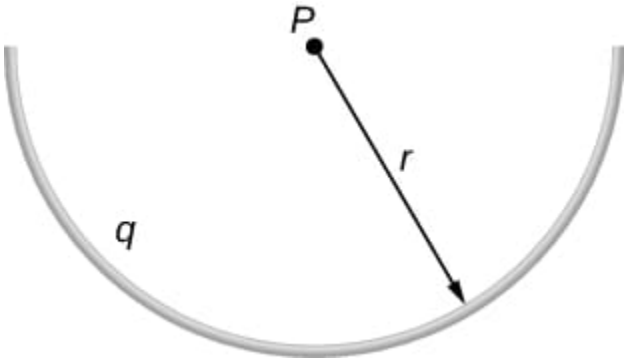
81. Calculate the magnitude and direction of the electric field 2.0 m from a long wire that is charged uniformly at $\lambda = 4.0 \times 10^{-6}C/m$.

82. Two thin conducting plates, each 25.0 cm on a side, are situated parallel to one another and 5.0 mm apart. If 10^{11} electrons are moved from one plate to the other, what is the electric field between the plates?

83. The charge per unit length on the thin rod shown below is λ. What is the electric field at the point **P**? (Hint: Solve this problem by first considering the electric field $d\vec{E}$ at **P** due to a small segment **dx** of the rod, which contains charge $dq = \lambda dx$. Then find the net field by integrating $d\vec{E}$ over the length of the rod.)



84. The charge per unit length on the thin semicircular wire shown below is **λ**. What is the electric field at the point **P**?



85. Two thin parallel conducting plates are placed 2.0 cm apart. Each plate is 2.0 cm on a side; one plate carries a net charge of **8.0μC,** and the other plate carries a net charge of **–8.0μC**. What is the charge density on the inside surface of each plate? What is the electric field between the plates?

86. A thin conducting plate 2.0 m on a side is given a total charge of **–10.0μC**.

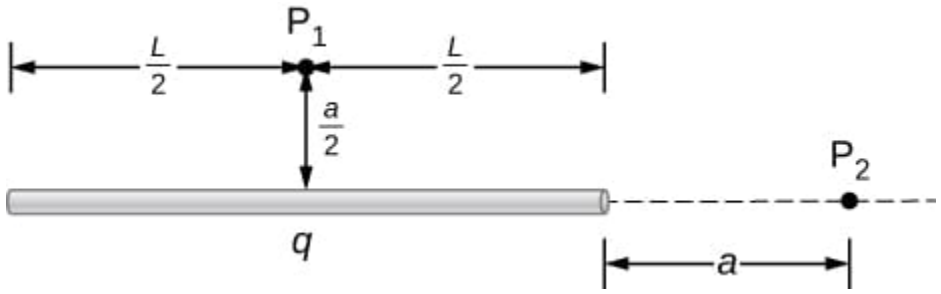
(a) What is the electric field **1.0cm** above the plate?

(b) What is the force on an electron at this point?

(c) Repeat these calculations for a point 2.0 cm above the plate.

(d) When the electron moves from 1.0 to 2,0 cm above the plate, how much work is done on it by the electric field?

87. A total charge **q** is distributed uniformly along a thin, straight rod of length **L** (see below). What is the electric field at P_1 ? At P_2?



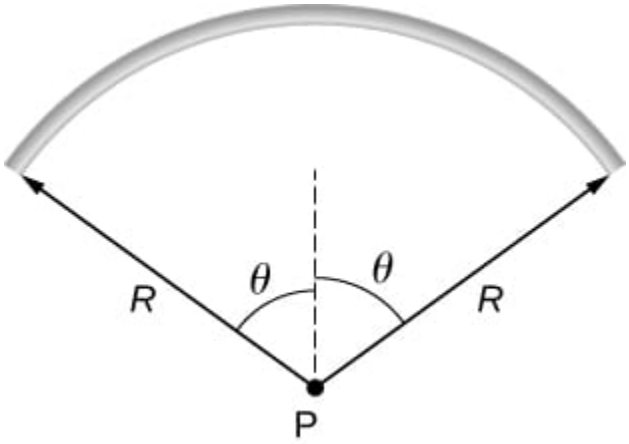
88. Charge is distributed along the entire **x**-axis with uniform density **λ**. How much work does the electric field of this charge distribution do on an electron that moves along the **y**-axis from $y = a$ to $y = b$?

89. Charge is distributed along the entire **x**-axis with uniform density λ_x and along the entire **y**-axis with uniform density λ_y . Calculate the resulting electric field at

(a) $\vec{r} = a\hat{i} + b\hat{j}$ and

(b) $\vec{r} = c\hat{k}$.

90. A rod bent into the arc of a circle subtends an angle 2θ at the center **P** of the circle (see below). If the rod is charged uniformly with a total charge **Q**, what is the electric field at **P**?.

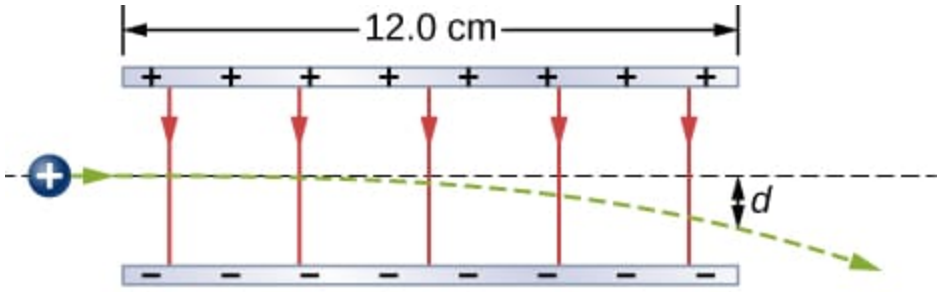


91. A proton moves in the electric field $\vec{E} = 200\hat{i}N/C$. (a) What are the force on and the acceleration of the proton? (b) Do the same calculation for an electron moving in this field.

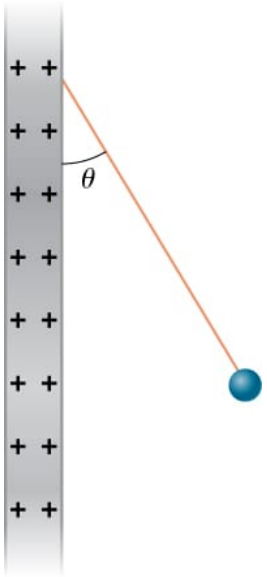
92. An electron and a proton, each starting from rest, are accelerated by the same uniform electric field of 200 N/C. Determine the distance and time for each particle to acquire a kinetic energy of $3.2 \times 10^{-16} J$.

93. A spherical water droplet of radius **25μm** carries an excess 250 electrons. What vertical electric field is needed to balance the gravitational force on the droplet at the surface of the earth?

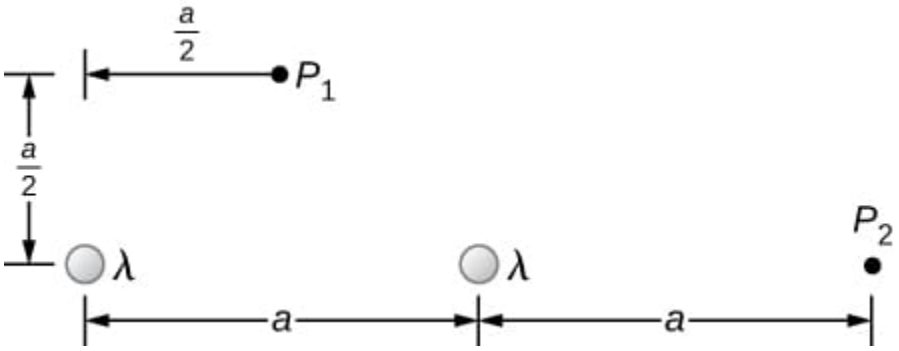
94. A proton enters the uniform electric field produced by the two charged plates shown below. The magnitude of the electric field is $4.0 \times 10^5 N/C$, and the speed of the proton when it enters is $1.5 \times 10^7 m/s$. What distance **d** has the proton been deflected downward when it leaves the plates?



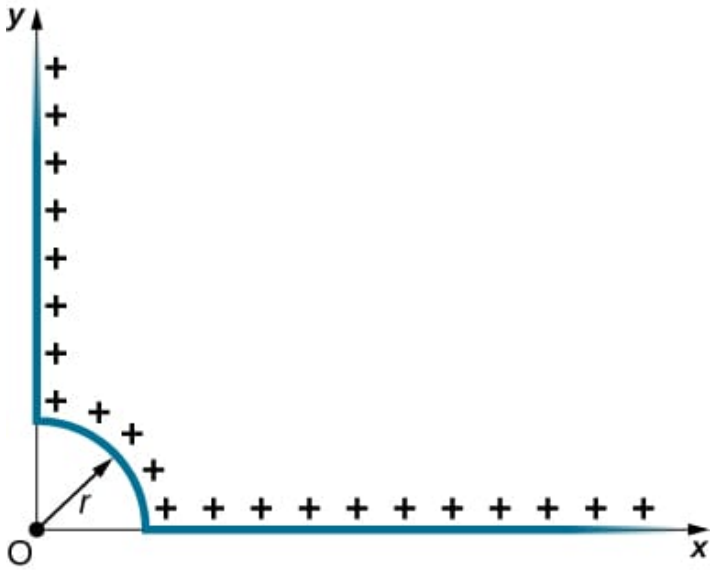
95. Shown below is a small sphere of mass 0.25 g that carries a charge of $9.0 \times 10^{-10} C$. The sphere is attached to one end of a very thin silk string 5.0 cm long. The other end of the string is attached to a large vertical conducting plate that has a charge density of $30 \times 10^{-6} C/m^2$. What is the angle that the string makes with the vertical?



96. Two infinite rods, each carrying a uniform charge density λ, are parallel to one another and perpendicular to the plane of the page. (See below.) What is the electrical field at P_1? At P_2?



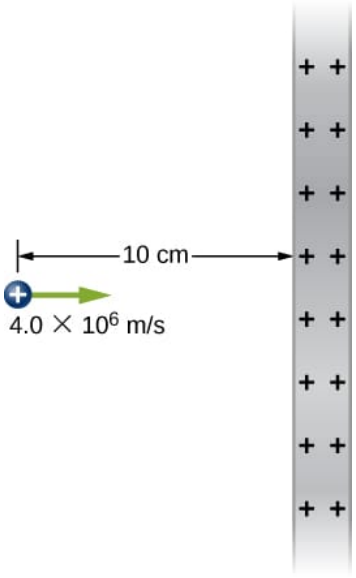
97. Positive charge is distributed with a uniform density λ along the positive **x**-axis from r to ∞, along the positive **y**-axis from r to ∞, and along a **90°** arc of a circle of radius r, as shown below. What is the electric field at **O**?



98. From a distance of 10 cm, a proton is projected with a speed of $v = 4.0 \times 10^6 m/s$ directly at a large, positively charged plate whose charge density is $\sigma = 2.0 \times 10^{-5} C/m^2$.. (See below.)

(a) Does the proton reach the plate?

(b) If not, how far from the plate does it turn around?



99. A particle of mass m and charge $-q$ moves along a straight line away from a fixed particle of charge **Q**. When the distance between the two particles is $r_0, -q$ is moving with a speed v_0.

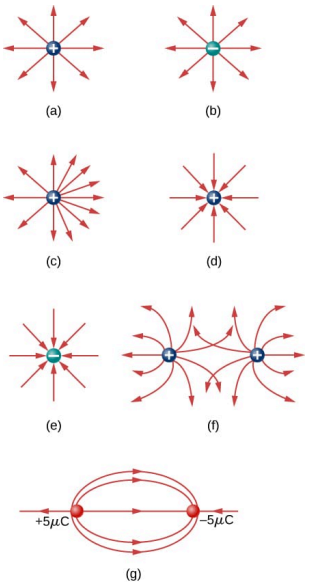
(a) Use the work-energy theorem to calculate the maximum separation of the charges.

(b) What do you have to assume about v_0 to make this calculation?

(c) What is the minimum value of v_0 such that $-q$ escapes from **Q**?

5.7 Electric Field Lines

100. Which of the following electric field lines are incorrect for point charges? Explain why.



101. In this exercise, you will practice drawing electric field lines. Make sure you represent both the magnitude and direction of the electric field adequately. Note that the number of lines into or out of charges is proportional to the charges.

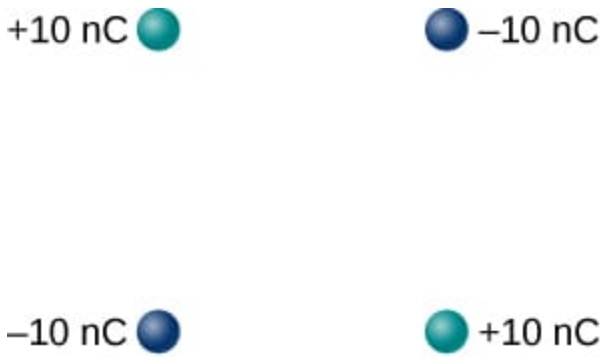
(a) Draw the electric field lines map for two charges **+20μC** and **−20μC** situated 5 cm from each other.

(b) Draw the electric field lines map for two charges **+20μC** and **+20μC** situated 5 cm from each other.

(c) Draw the electric field lines map for two charges **+20μC** and **−30μC** situated 5 cm from each other.

102. Draw the electric field for a system of three particles of charges **+1μC, +2μC** and **−3μC** fixed at the corners of an equilateral triangle of side 2 cm.

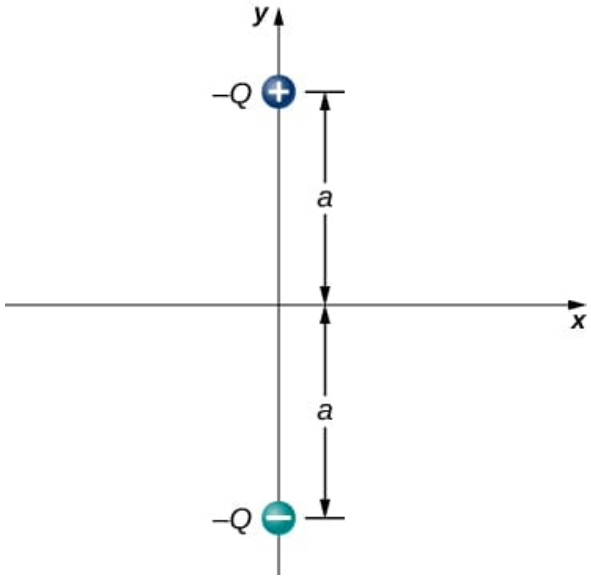
103. Two charges of equal magnitude but opposite sign make up an electric dipole. A quadrupole consists of two electric dipoles that are placed anti-parallel at two edges of a square as shown. Draw the electric field of the charge distribution.



104. Suppose the electric field of an isolated point charge decreased with distance as $1/r^{2+\delta}$ rather than as $1/r^2$. Show that it is then impossible to draw continuous field lines so that their number per unit area is proportional to **E**.

5.8 Electric Dipoles

105. Consider the equal and opposite charges shown below. (a) Show that at all points on the **x**-axis for which $|x| \gg a, E \approx Qa/2\pi\varepsilon_0 x^3$. (b) Show that at all points on the **y**-axis for which $|y| \gg a, E \approx Qa/\pi\varepsilon_0 y^3$.



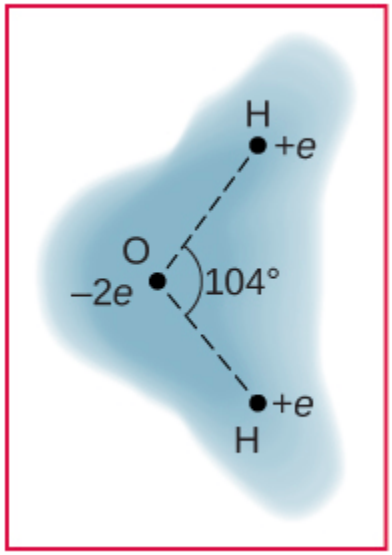
106. (a) What is the dipole moment of the configuration shown above? If **Q=4.0μC**,

(b) what is the torque on this dipole with an electric field of $4.0 \times 10^5 N/C\hat{i}$?

(c) What is the torque on this dipole with an electric field of $-4.0 \times 10^5 N/C\hat{i}$?

(d) What is the torque on this dipole with an electric field of $\pm 4.0 \times 10^5 N/C\hat{j}$?

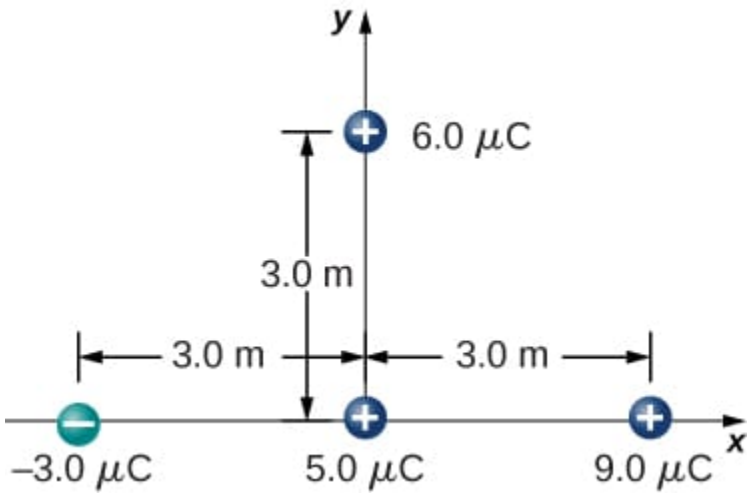
107. A water molecule consists of two hydrogen atoms bonded with one oxygen atom. The bond angle between the two hydrogen atoms is **104°** (see below). Calculate the net dipole moment of a hypothetical water molecule where the charge at the oxygen molecule is –2e and at each hydrogen atom is +e. The net dipole moment of the molecule is the vector sum of the individual dipole moment between the two O-Hs. The separation O-H is 0.9578 angstroms.



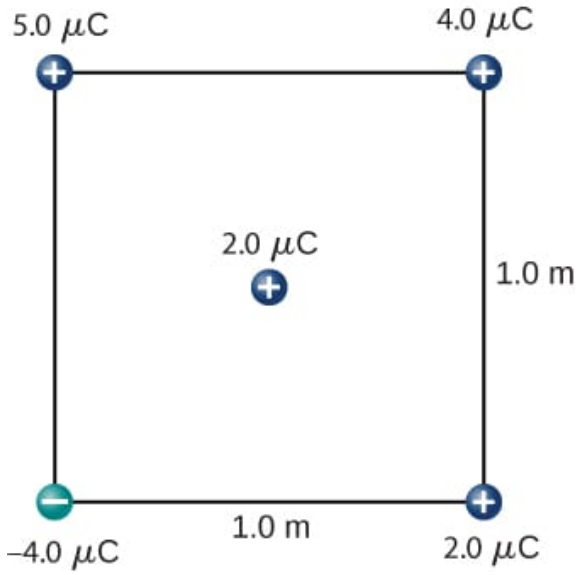
Additional Problems

108. Point charges $q_1 = 2.0\mu C$ and $q_1 = 4.0\mu C$ are located at $r_1 = (4.0\hat{i} - 2.0\hat{j} + 2.0\hat{k})m$ and $r_2 = (8.0\hat{i} + 5.0\hat{j} - 9.0\hat{k})m$. What is the force of q_2 on q_1?

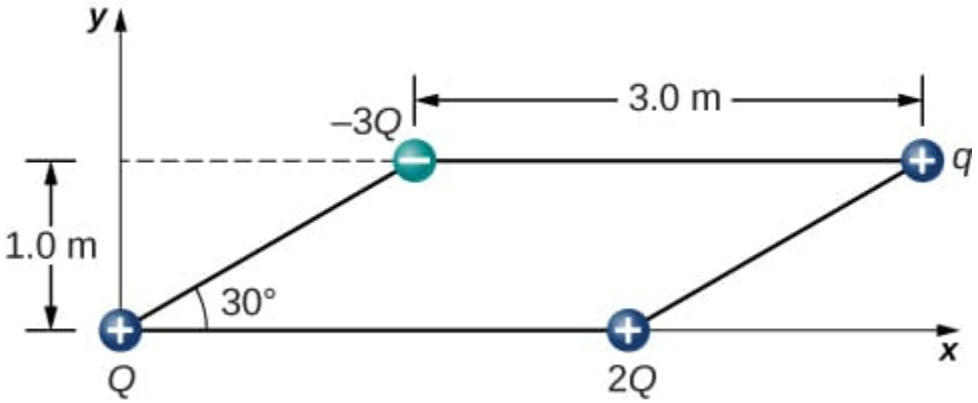
109. What is the force on the **5.0-μC** charges shown below?



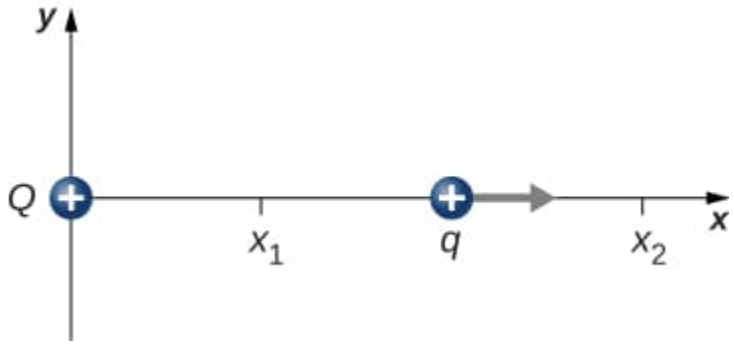
110. What is the force on the **2.0-μC** charge placed at the center of the square shown below?



111. Four charged particles are positioned at the corners of a parallelogram as shown below. If $q = 5.0\mu C$ and $Q = 8.0\mu C$, what is the net force on **q**?

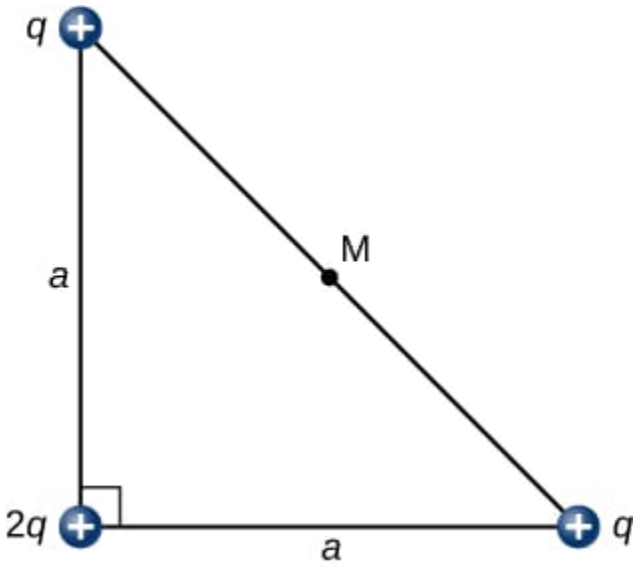


112. A charge **Q** is fixed at the origin and a second charge **q** moves along the **x**-axis, as shown below. How much work is done on **q** by the electric force when **q** moves from x_1 to x_2?

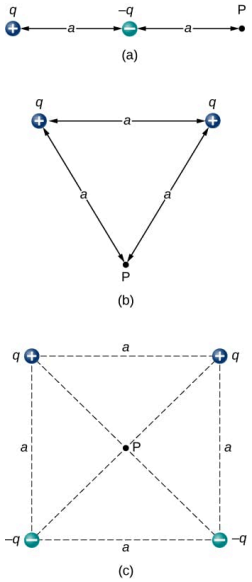


113. A charge **q=−2.0μC** is released from rest when it is 2.0 m from a fixed charge $Q = 6.0\mu C$. What is the kinetic energy of **q** when it is 1.0 m from **Q**?

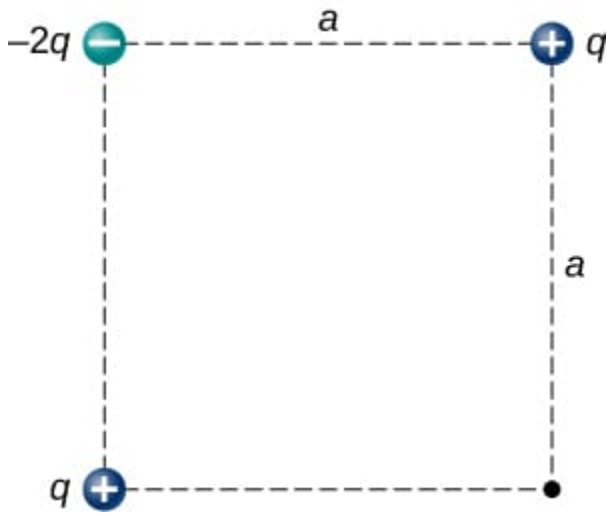
114. What is the electric field at the midpoint M of the hypotenuse of the triangle shown below?



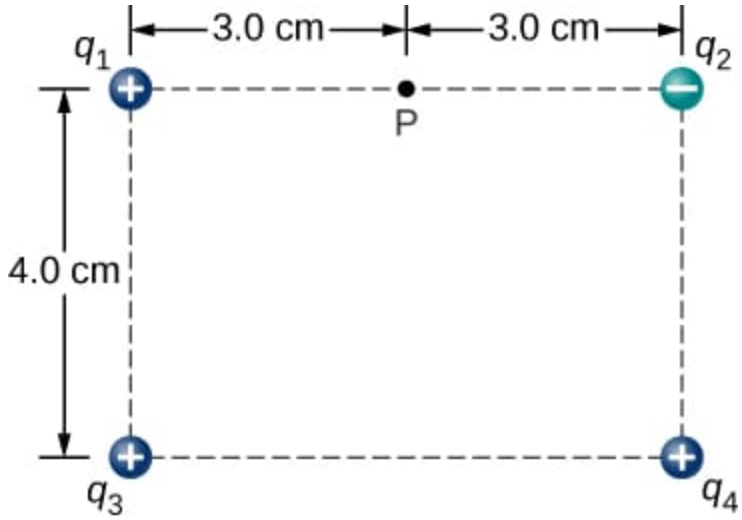
115. Find the electric field at P for the charge configurations shown below.



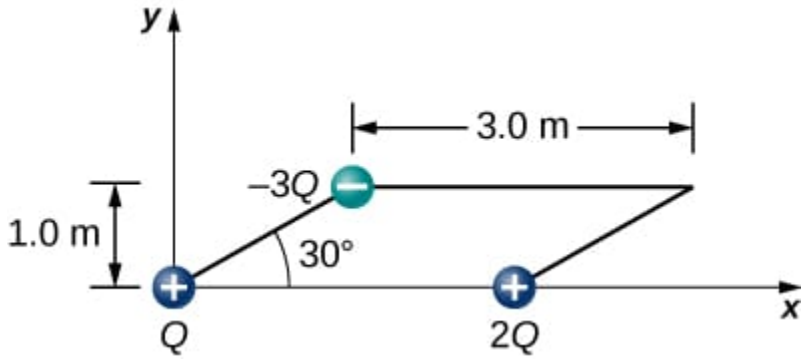
116. (a) What is the electric field at the lower-right-hand corner of the square shown below? (b) What is the force on a charge **q** placed at that point?



117. Point charges are placed at the four corners of a rectangle as shown below: $q_1 = 2.0 \times 10^{-6} C, q_2 = -2.0 \times 10^{-6} C, q_3 = 4.0 \times 10^{-6} C$, and $q_4 = 1.0 \times 10^{-6} C$. What is the electric field at **P**?



118. Three charges are positioned at the corners of a parallelogram as shown below. (a) If $Q = 8.0\mu C$, what is the electric field at the unoccupied corner? (b) What is the force on a **5.0-μC** charge placed at this corner?



119. A positive charge **q** is released from rest at the origin of a rectangular coordinate system and moves under the influence of the electric field $\vec{E} = E_0(1 + x/a)\hat{i}$. What is the kinetic energy of **q** when it passes through $x = 3a$?

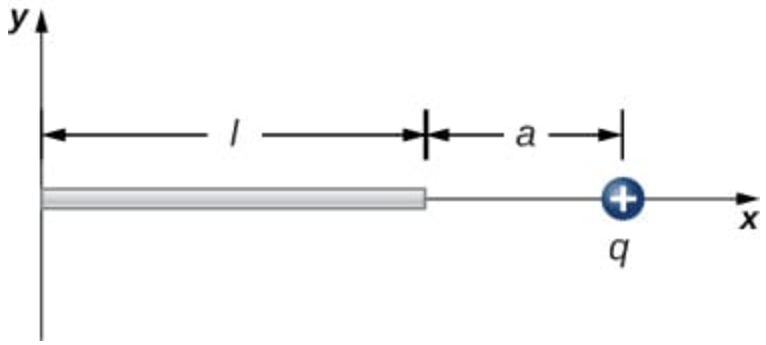
120. A particle of charge $-q$ and mass **m** is placed at the center of a uniformaly charged ring of total charge **Q** and radius **R**. The particle is displaced a small distance along the axis perpendicular to the plane of the ring and released. Assuming that the particle is constrained to move along the axis, show that the particle oscillates in simple harmonic motion with a frequency $f = \frac{1}{2\pi}\sqrt{\frac{qQ}{4\pi\varepsilon_0 mR^3}}$.

121. Charge is distributed uniformly along the entire **y**-axis with a density y_λ and along the positive **x**-axis from $x = a$ to $x = b$ with a density λ_x. What is the force between the two distributions?

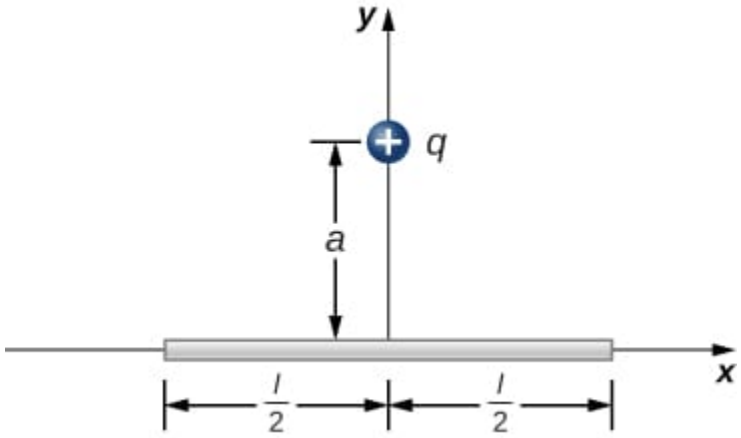
122. The circular arc shown below carries a charge per unit length $\lambda = \lambda_0 cos\theta$, where θ is measured from the **x**-axis. What is the electric field at the origin?

123. Calculate the electric field due to a uniformly charged rod of length **L**, aligned with the **x**-axis with one end at the origin; at a point **P** on the **z**-axis.

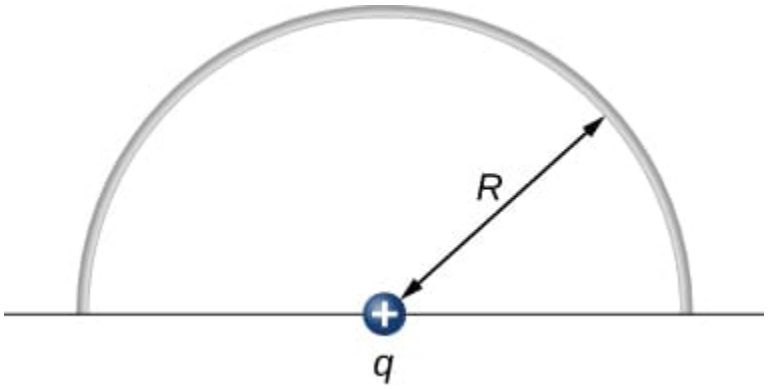
124. The charge per unit length on the thin rod shown below is λ. What is the electric force on the point charge **q**? Solve this problem by first considering the electric force $d\vec{F}$ on **q** due to a small segment dx of the rod, which contains charge λdx. Then, find the net force by integrating $d\vec{F}$ over the length of the rod.



125. The charge per unit length on the thin rod shown here is λ. What is the electric force on the point charge **q**? (See the preceding problem.)



126. The charge per unit length on the thin semicircular wire shown below is λ. What is the electric force on the point charge **q**? (See the preceding problems.)



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8.11: Electric Charges and Fields (Answer)

Check Your Understanding

5.1. The force would point outward.

5.2. The net force would point **58°** below the **–x**-axis.

5.3. $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$

5.4. We will no longer be able to take advantage of symmetry. Instead, we will need to calculate each of the two components of the electric field with their own integral.

5.5. The point charge would be $Q = \sigma ab$ where a and b are the sides of the rectangle but otherwise identical.

5.6. The electric field would be zero in between, and have magnitude $\frac{\sigma}{\varepsilon_0}$ everywhere else.

Conceptual Questions

1. There are mostly equal numbers of positive and negative charges present, making the object electrically neutral.

3. a. yes;

b. yes

5. Take an object with a known charge, either positive or negative, and bring it close to the rod. If the known charged object is positive and it is repelled from the rod, the rod is charged positive. If the positively charged object is attracted to the rod, the rod is negatively charged.

7. No, the dust is attracted to both because the dust particle molecules become polarized in the direction of the silk.

9. Yes, polarization charge is induced on the conductor so that the positive charge is nearest the charged rod, causing an attractive force.

11. Charging by conduction is charging by contact where charge is transferred to the object. Charging by induction first involves producing a polarization charge in the object and then connecting a wire to ground to allow some of the charge to leave the object, leaving the object charged.

13. This is so that any excess charge is transferred to the ground, keeping the gasoline receptacles neutral. If there is excess charge on the gasoline receptacle, a spark could ignite it.

15. The dryer charges the clothes. If they are damp, the presence of water molecules suppresses the charge.

17. There are only two types of charge, attractive and repulsive. If you bring a charged object near the quartz, only one of these two effects will happen, proving there is not a third kind of charge.

19. a. No, since a polarization charge is induced. b. Yes, since the polarization charge would produce only an attractive force.

21. The force holding the nucleus together must be greater than the electrostatic repulsive force on the protons.

23. Either sign of the test charge could be used, but the convention is to use a positive test charge.

25. The charges are of the same sign.

27. At infinity, we would expect the field to go to zero, but because the sheet is infinite in extent, this is not the case. Everywhere you are, you see an infinite plane in all directions.

29. The infinite charged plate would have $E = \frac{\sigma}{2\varepsilon_0}$ everywhere. The field would point toward the plate if it were negatively charged and point away from the plate if it were positively charged. The electric field of the parallel plates would be zero between them if they had the same charge, and **E** would be $E = \frac{\sigma}{\varepsilon_0}$ everywhere else. If the charges were opposite, the situation is reversed, zero outside the plates and $E = \frac{\sigma}{\varepsilon_0}$ between them.

31. yes; no

33. At the surface of Earth, the gravitational field is always directed in toward Earth's center. An electric field could move a charged particle in a different direction than toward the center of Earth. This would indicate an electric field is present.

35. 10

Problems

37. a. $2.00 \times 10^{-9} C(\frac{1}{1.602 \times 10^{-19}} e/C) = 1.248 \times 10^{10} electrons2$;

b. $0.500 \times 10^{-6} C(\frac{1}{1.602 \times 10^{-19}} e/C) = 3.121 \times 10^{12} electrons$

39. $\frac{3.750 \times 10^{21} e}{6.242 \times 10^{18} e/C} = -600.8C$

41. a. $2.0 \times 10^{-9} C(6.242 \times 10^{18} e/C) = 1.248 \times 10^{10} e$;

b. $9.109 \times 10^{-31} kg(1.248 \times 10^{10} e) = 1.137 \times 10^{-20} kg$, $\frac{1.137 \times 10^{-20} kg}{2.5 \times 10^{-3} kg} = 4.548 \times 10^{-18}$ or 4.545×10^{-16}

43. $5.00 \times 10^{-9} C(6.242 \times 10^{18} e/C) = 3.121 \times 10^{10} e; 3.121 \times 10^{10} e + 1.0000 \times 10^{12} e = 1.0312 \times 10^{12} e$.

45. atomic mass of copper atom times $1u = 1.055 \times 10^{-25} kg$; number of copper atoms = $4.739 \times 10^{23} atoms$; number of electrons equals 29 times number of atoms or $1.374 \times 10^{25} electrons$; $\frac{2.00 \times 10^{-6} C(6.242 \times 10^{18} e/C)}{1.374 \times 10^{25} e} = 9.083 \times 10^{-13}$ or 9.083×10^{-11}.

47. $244.00u(1.66 \times 10^{-27} kg/u) = 4.050 \times 10^{-25} kg$; $\frac{4.00kg}{4.050 \times 10^{-25} kg} = 9.877 \times 10^{24} atoms$ $9.877 \times 10^{24}(94) = 9.284 \times 10^{26} protons$ $9.284 \times 10^{26} protons; 9.284 \times 10^{26}(1.602 \times 10^{-19} C/p) = 1.487 \times 10^{8} C$

49. a. charge 1 is $3\mu C$; charge 2 is $12\mu C$, $F_{31} = 2.16 \times 10^{-4} N$ to the left,

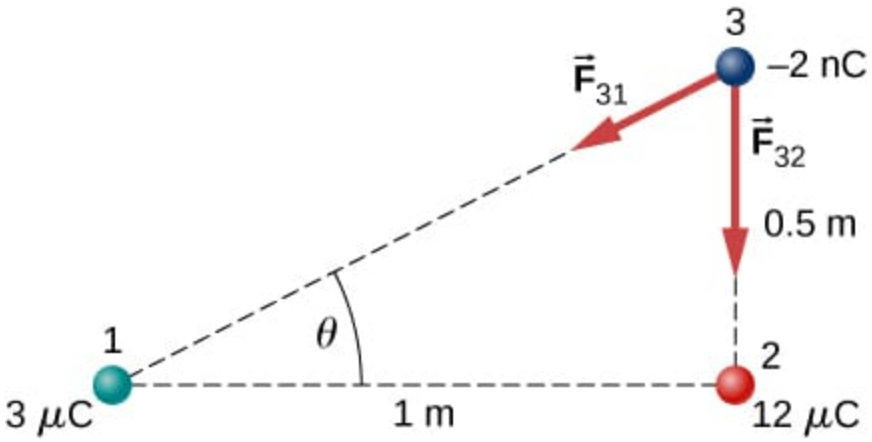
$F_{32} = 8.63 \times 10^{-4} N$ to the right,

$F_{net} = 6.47 \times 10^{-4} N$ to the right;

b. $F_{31} = 2.16 \times 10^{-4} N$ to the right,

$F_{32} = 9.59 \times 10^{-5} N$ to the right,

$F_{net} = 3.12 \times 10^{-4} N$ to the right,



c. $\vec{F}_{31x} = -2.76 \times 10^{-5} N\hat{i}$,

$\vec{F}_{31y} = -1.38 \times 10^{-5} N\hat{j}$,

$\vec{F}_{32y} = -8.63 \times 10^{-4} N\hat{j}$,

$\vec{F}_{net} = -2.76 \times 10^{-5} N\hat{i} - 8.77 \times 10^{-4} N\hat{j}$

51. $F = 230.7N$

53. $F = 53.94N$

55. The tension is $T = 0.049N$. The horizontal component of the tension is $0.0043N$

$d = 0.088m, q = 6.1 \times 10^{-8} C.$

The charges can be positive or negative, but both have to be the same sign.

57. Let the charge on one of the spheres be **rQ**, where r is a fraction between 0 and 1. In the numerator of Coulomb's law, the term involving the charges is $rQ(1-r)Q$. This is equal to $(r - r^2)Q^2$. Finding the maximum of this term gives $1 - 2r = 0 \Rightarrow r = \frac{1}{2}$

59. Define right to be the positive direction and hence left is the negative direction, then $F = -0.05N$

61. The particles form triangle of sides 13, 13, and 24 cm. The **x**-components cancel, whereas there is a contribution to the y-component from both charges 24 cm apart. The y-axis passing through the third charge bisects the 24-cm line, creating two right triangles of sides 5, 12, and 13 cm. $F_y = 2.56N$ in the negative **y**-direction since the force is attractive. The net force from both charges is $\vec{F}_{net} = -5.12N\hat{j}$

63. The diagonal is $\sqrt{2}a$ and the components of the force due to the diagonal charge has a factor $cos\theta = \frac{1}{\sqrt{2}}$; $\vec{F}_{net} = [k\frac{q^2}{a^2} + k\frac{q^2}{2a^2}\frac{1}{\sqrt{2}}]\hat{i} - [k\frac{q^2}{a^2} + k\frac{q^2}{2a^2}\frac{1}{\sqrt{2}}]\hat{j}$

65. $a. E = 2.0 \times 10^{-2}\frac{N}{C}$;

$b. F = 2.0 \times 10^{-19} N$

67. a. $E = 2.88 \times 10^{11} N/C$;

b. $E = 1.44 \times 10^{11} N/C$;

c. $F = 4.61 \times 10^{-8} N$ on alpha particl

$F = 4.61 \times 10^{-8} N$ on electron

69. $E = (-2.0\hat{i} + 3.0\hat{j})N$

71. $F = 3.204 \times 10^{-14} N$,

$a = 3.517 \times 10^{16} m/s^2$

73. $q = 2.78 \times 10^{-9} C$

75. a. $E = 1.15 \times 10^{12} N/C$;

b. $F = 1.47 \times 10^{-6} N$

77. If the q_2 is to the right of q_1, the electric field vector from both charges point to the right.

a. $E = 2.70 \times 10^{6} N/C$;

b. $F = 54.0N$

79. There is **45°** right triangle geometry. The **x**-components of the electric field at $y = 3m$ cancel. The **y**-components give $E(y = 3m) = 2.83 \times 10^{3} N/C$.

At the origin we have a a negative charge of magnitide $q = -2.83 \times 10^{-6} C$

81. $\vec{E}(z) = 3.6 \times 10^{4} N\hat{k}$

83. $dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dx}{(x+a)^2}, E = \frac{\lambda}{4\pi\varepsilon_0}[\frac{1}{l+a} - \frac{1}{a}]$

85. $\sigma = 0.02 C/m^2$ $E = 2.26 \times 10^{9} N/C$

87. At P_1 : $\vec{E}(y) = \frac{1}{4\pi\varepsilon_0}\frac{\lambda L}{y\sqrt{y^2 + \frac{L^2}{4}}}\hat{j} \Rightarrow \frac{1}{4\pi\varepsilon_0}\frac{q}{\frac{a}{2}\sqrt{(\frac{a}{2})^2 + \frac{L^2}{4}}}\hat{j} = \frac{1}{\pi\varepsilon_0}\frac{q}{a\sqrt{a^2 + L^2}}\hat{j}$

At P_2: Put the origin at the end of **L**.

$dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dx}{(x+a)^2}, \vec{E} = -\frac{q}{4\pi\varepsilon_0 l}[\frac{1}{l+a} - \frac{1}{a}]\hat{i}$

89. a. $\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda_x}{a}\hat{i} + \frac{1}{4\pi\varepsilon_0}\frac{2\lambda_y}{b}\hat{j}$;

b. $\frac{1}{4\pi\varepsilon_0}\frac{2(\lambda_x + \lambda_y)}{c}\hat{k}$

91. a. $\vec{F} = 3.2 \times 10^{-17} N\hat{i}$,

$\vec{a} = 1.92 \times 10^{10} m/s^2\hat{i}$;

b. $\vec{F} = -3.2 \times 10^{-17} N\hat{i}$,

$\vec{a} = -3.51 \times 10^{13} m/s^2\hat{i}$

93. $m = 6.5 \times 10^{-11} kg$,

$E = 1.6 \times 10^{7} N/C$

95. $E = 1.70 \times 10^{6} N/C$,

$F = 1.53 \times 10^{-3} NTcos\theta = mgTsin\theta = qE$,

$tan\theta = 0.62 \Rightarrow \theta = 32.0\ °,$

This is independent of the length of the string.

97. circular arc $dE_x(-\hat{i}) = \frac{1}{4\pi\varepsilon_0}\frac{\lambda ds}{r^2}cos\theta(-\hat{i},$

$\vec{E}_x = \frac{\lambda}{4\pi\varepsilon_0 r}(-\hat{i}),$

$dEy(-\hat{i}\hat{}) = \frac{1}{4\pi\varepsilon_0}\frac{\lambda ds}{r^2}sin\theta(-\hat{j}),$

$\vec{E}_y = \frac{\lambda}{4\pi\varepsilon_0 r}(-\hat{j});$

y-axis: $\vec{E}_x = \frac{\lambda}{4\pi\varepsilon_0 r}(-\hat{i});$

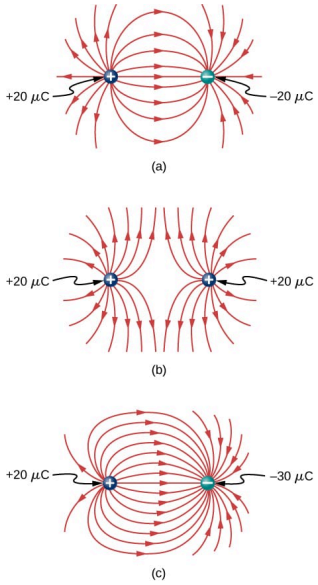
x-axis: $\vec{E}_y = \frac{\lambda}{4\pi\varepsilon_0 r}(-\hat{j}),$

$\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r}(-\hat{i}) + \frac{\lambda}{2\pi\varepsilon_0 r}(-\hat{j})$

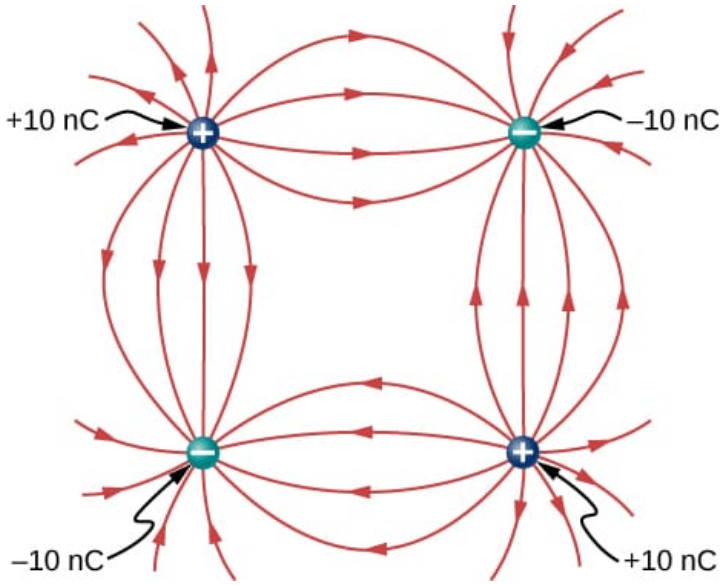
99. a. $W = \frac{1}{2}m(v^2 - v_0^2), \frac{Qq}{4\pi\varepsilon_0}(\frac{1}{r} - \frac{1}{r_0}) = \frac{1}{2}m(v^2 - v_0^2) \Rightarrow r_0 - r = \frac{4\pi\varepsilon_0}{Qq}\frac{1}{2}rr_0m(v^2 - v_0^2);$

b. $r_0 - r$ is negative; therefore, $v_0 > v, r \to \infty$, and $v \to 0 : \frac{Qq}{4\pi\varepsilon_0}(-\frac{1}{r_0}) = -\frac{1}{2}mv_0^2 \Rightarrow v_0 = \sqrt{\frac{Qq}{2\pi\varepsilon_0 mr_0}}$

101.



103.



105. $E_x = 0, E_y = \frac{1}{4\pi\varepsilon_0}[\frac{2q}{(x^2+a^2)}\frac{a}{\sqrt{(x^2+a^2)}} \Rightarrow x \gg a \Rightarrow \frac{1}{2\pi\varepsilon_0}\frac{qa}{x^3}$

$E_y = \frac{q}{4\pi\varepsilon_0}[\frac{2ya+2ya}{(y-a)^2(y+a)^2}] \Rightarrow y \gg a \Rightarrow \frac{1}{\pi\varepsilon_0}\frac{qa}{y^3}$

107. The net dipole moment of the molecule is the vector sum of the individual dipole moments between the two O-H. The separation O-H is 0.9578 angstroms:

$\vec{p} = 1.889 \times 10^{-29} Cm\hat{i}$

Additional Problems

109. $\vec{F}_{net} = [-8.99 \times 10^9 \frac{3.0\times10^{-6}(5.0\times10^{-6})}{(3.0m)^2} - 8.99 \times 10^9 \frac{9.0\times10^{-6}(5.0\times10^{-6})}{(3.0m)^2}]\hat{i}, -8.99 \times 10^9 \frac{6.0\times10^{-6}(5.0\times10^{-6})}{(3.0m)^2}\hat{j}$
$= -0.06N\hat{i} - 0.03N\hat{j}$

111. Charges **Q** and **q** form a right triangle of sides 1 m and $3+\sqrt{3}m$. Charges **2Q** and **q** form a right triangle of sides 1 m and $\sqrt{3}m$.

$F_x = 0.049N,$

$F_y = 0.09N,$

$\vec{F}_{net} = 0.049N\hat{i} + 0.09N\hat{j}$

113. W=0.054J

115. a. $\vec{E} = \frac{1}{4\pi\varepsilon_0}(\frac{q}{(2a)^2} - \frac{q}{a^2})\hat{i};$

b. $\vec{E} = \frac{\sqrt{3}}{4\pi\varepsilon_0}\frac{q}{a^2}(-\hat{j});$

c. $\vec{E} = \frac{2}{\pi\varepsilon_0}\frac{q}{a^2}\frac{1}{\sqrt{2}}(-\hat{j})$

117. $\vec{E} = 6.4 \times 10^6(\hat{i}) + 1.5 \times 10^7(\hat{j})N/C$

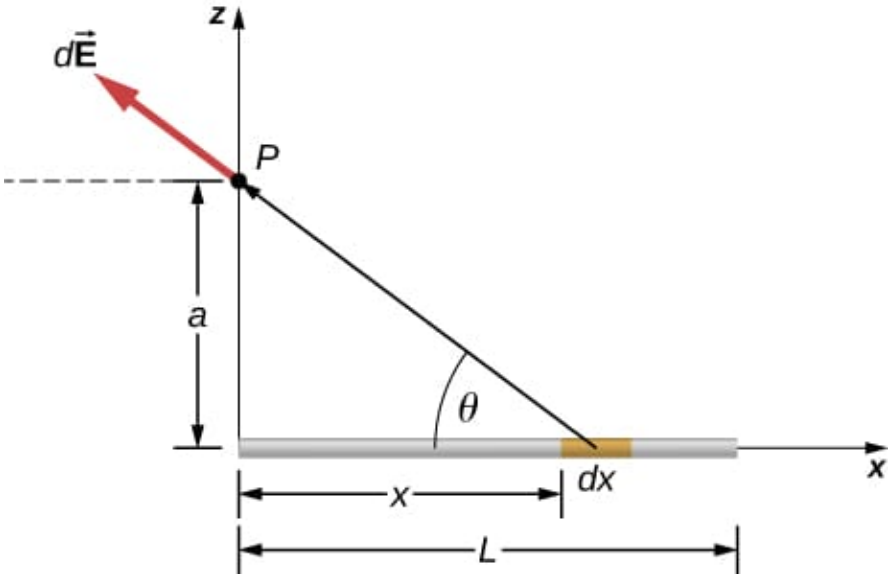
119. $F = qE_0(1+x/a)\ W = \frac{1}{2}m(v^2 - v_0^2),$

$$\frac{1}{2}mv^2 = qE_0(\frac{15a}{2})J$$

121. Electric field of wire at **x**: $\vec{E}(x) = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda_y}{x}\hat{i}$,

$$dF = \frac{\lambda_y\lambda_x}{2\pi\varepsilon_0}(lnb - lna)$$

123.



$$dEx = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dx}{(x^2+a^2)}\frac{x}{\sqrt{x^2+a^2}},$$

$$\vec{E}_x = \frac{\lambda}{4\pi\varepsilon_0}[\frac{1}{\sqrt{L^2+a^2}} - \frac{1}{a}]\hat{i},$$

$$dE_z = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dx}{(x^2+a^2)}\frac{a}{\sqrt{x^2+a^2}},$$

$$\vec{E}_z = \frac{\lambda}{4\pi\varepsilon_0 a}\frac{L}{\sqrt{L^2+a^2}}\hat{k},$$

Substituting z for a, we have:

$$\vec{E}(z) = \frac{\lambda}{4\pi\varepsilon_0}[\frac{1}{\sqrt{L^2+z^2}} - \frac{1}{z}]\hat{i} + \frac{\lambda}{4\pi\varepsilon_0 z}\frac{L}{\sqrt{L^2+z^2}}\hat{k}$$

125. There is a net force only in the **y**-direction. Let θ be the angle the vector from **dx** to **q** makes with the **x**-axis. The components along the **x**-axis cancel due to symmetry, leaving the **y**-component of the force.

$$dF_y = \frac{1}{4\pi\varepsilon_0}\frac{aq\lambda dx}{(x^2+a^2)^{3/2}},$$

$$Fy = \frac{1}{2\pi\varepsilon_0}\frac{q\lambda}{a}[\frac{l/2}{((l/2)^2+a^2)^{1/2}}]$$

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9: GAUSS'S LAW

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9.1: Prelude to Gauss's Law

Flux is a general and broadly applicable concept in physics. However, in this chapter, we concentrate on the flux of the electric field. This allows us to introduce Gauss's law, which is particularly useful for finding the electric fields of charge distributions exhibiting spatial symmetry. The main topics discussed here are

1. **Electric flux.** We define electric flux for both open and closed surfaces.
2. **Gauss's law.** We derive Gauss's law for an arbitrary charge distribution and examine the role of electric flux in Gauss's law.
3. **Calculating electric fields with Gauss's law.** The main focus of this chapter is to explain how to use Gauss's law to find the electric fields of spatially symmetrical charge distributions. We discuss the importance of choosing a Gaussian surface and provide examples involving the applications of Gauss's law.
4. **Electric fields in conductors.** Gauss's law provides useful insight into the absence of electric fields in conducting materials.



Figure 9.1.1: This chapter introduces the concept of flux, which relates a physical quantity and the area through which it is flowing. Although we introduce this concept with the electric field, the concept may be used for many other quantities, such as fluid flow. (credit: modification of work by "Alessandro"/Flickr)

So far, we have found that the electrostatic field begins and ends at point charges and that the field of a point charge varies inversely with the square of the distance from that charge. These characteristics of the electrostatic field lead to an important mathematical relationship known as Gauss's law. This law is named in honor of the extraordinary German mathematician and scientist Karl Friedrich Gauss (Figure 9.1.2). Gauss's law gives us an elegantly simple way of finding the electric field, and, as you will see, it can be much easier to use than the integration method described in the previous chapter. However, there is a catch—Gauss's law has a limitation in that, while always true, it can be readily applied only for charge distributions with certain symmetries.



Figure 9.1.2: Karl Friedrich Gauss (1777–1855) was a legendary mathematician of the nineteenth century. Although his major contributions were to the field of mathematics, he also did important work in physics and astronomy.

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9.2: Electric Flux

Learning Objectives

By the end of this section, you will be able to:

- Define the concept of flux
- Describe electric flux
- Calculate electric flux for a given situation

The concept of **flux** describes how much of something goes through a given area. More formally, it is the dot product of a vector field (in this chapter, the electric field) with an area. You may conceptualize the flux of an electric field as a measure of the number of electric field lines passing through an area (Figure $9.2.1$). The larger the area, the more field lines go through it and, hence, the greater the flux; similarly, the stronger the electric field is (represented by a greater density of lines), the greater the flux. On the other hand, if the area rotated so that the plane is aligned with the field lines, none will pass through and there will be no flux.

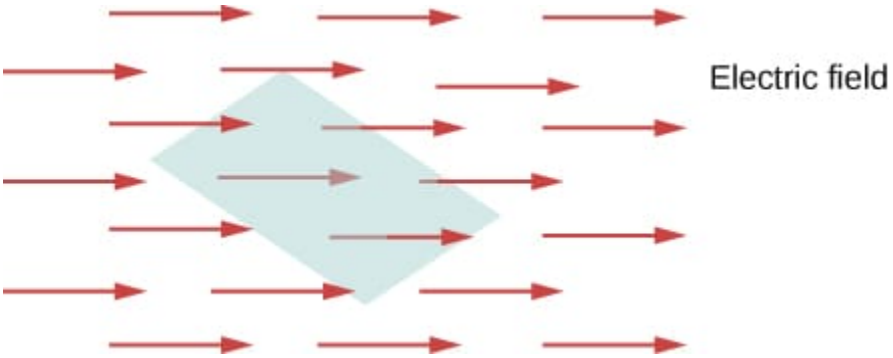


Figure $9.2.1$: The flux of an electric field through the shaded area captures information about the "number" of electric field lines passing through the area. The numerical value of the electric flux depends on the magnitudes of the electric field and the area, as well as the relative orientation of the area with respect to the direction of the electric field.

A macroscopic analogy that might help you imagine this is to put a hula hoop in a flowing river. As you change the angle of the hoop relative to the direction of the current, more or less of the flow will go through the hoop. Similarly, the amount of flow through the hoop depends on the strength of the current and the size of the hoop. Again, flux is a general concept; we can also use it to describe the amount of sunlight hitting a solar panel or the amount of energy a telescope receives from a distant star, for example.

To quantify this idea, Figure $9.2.1a$ shows a planar surface S_1 of area A_1 that is perpendicular to the uniform electric field $\vec{E} = E\hat{y}$. If N field lines pass through S_1, then we know from the definition of electric field lines (Electric Charges and Fields) that $N/A \propto E$, or $N \propto EA_1$.

The quantity EA_1 is the **electric flux** through S_1. We represent the electric flux through an open surface like S_1 by the symbol Φ. Electric flux is a scalar quantity and has an SI unit of newton-meters squared per coulomb ($\mathrm{N \cdot m^2/C}$). Notice that $N \propto EA_1$ may also be written as $N \propto \Phi$, demonstrating that *electric flux is a measure of the number of field lines crossing a surface.*

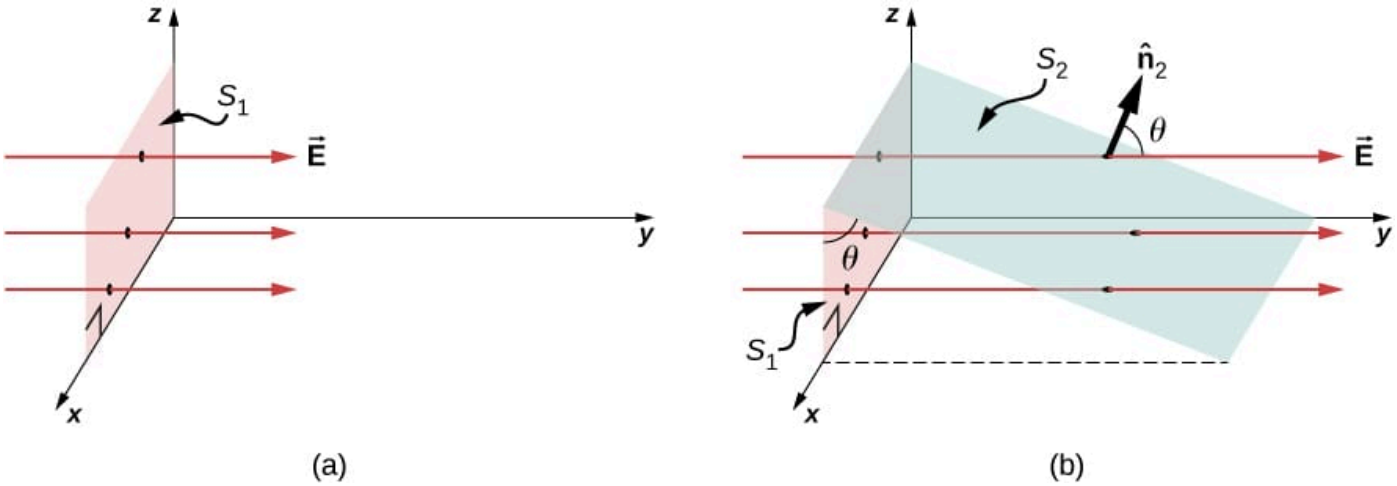


Figure 9.2.2: (a) A planar surface S_1 of area A_1 is perpendicular to the electric field $E\hat{j}$. N field lines cross surface S_1. (b) A surface S_2 of area A_2 whose projection onto the xz-plane is S_1. The same number of field lines cross each surface.

Now consider a planar surface that is not perpendicular to the field. How would we represent the electric flux? Figure $9.2.2b$ shows a surface S_2 of area A_2 that is inclined at an angle θ to the xz-plane and whose projection in that plane is S_1 (area A_1). The areas are related by $A_2\ cos\ \theta = A_1$. Because the same number of field lines crosses both S_1 and S_2, the fluxes through both surfaces must be the same. The flux through S_2 is therefore $\Phi = EA_1 = EA_2\ cos\ \theta$. Designating $\hat{n}_2$ as a unit vector normal to S_2 (see Figure $9.2.2b$), we obtain

$$\Phi = \vec{E} \cdot \hat{n}_2 A_2.$$

Note

Check out this video to observe what happens to the flux as the area changes in size and angle, or the electric field changes in strength.

Area Vector

For discussing the flux of a vector field, it is helpful to introduce an area vector $\vec{A}$. This allows us to write the last equation in a more compact form. What should the magnitude of the area vector be? What should the direction of the area vector be? What are the implications of how you answer the previous question?

The **area vector** of a flat surface of area $\vec{A}$ has the following magnitude and direction:

- Magnitude is equal to area (A)
- Direction is along the normal to the surface $(\hat{n})$; that is, perpendicular to the surface.

Since the normal to a flat surface can point in either direction from the surface, the direction of the area vector of an open surface needs to be chosen, as shown in Figure $9.2.3$.

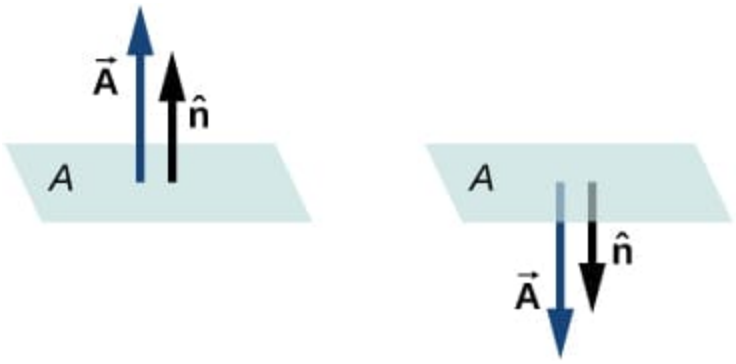


Figure 9.2.3: The direction of the area vector of an open surface needs to be chosen; it could be either of the two cases displayed here. The area vector of a part of a closed surface is defined to point from the inside of the closed space to the outside. This rule gives a unique direction.

Since $\hat{n}$ is a unit normal to a surface, it has two possible directions at every point on that surface (Figure $9.2.1a$). For an open surface, we can use either direction, as long as we are consistent over the entire surface. $9.2.1c$ of the figure shows several cases.

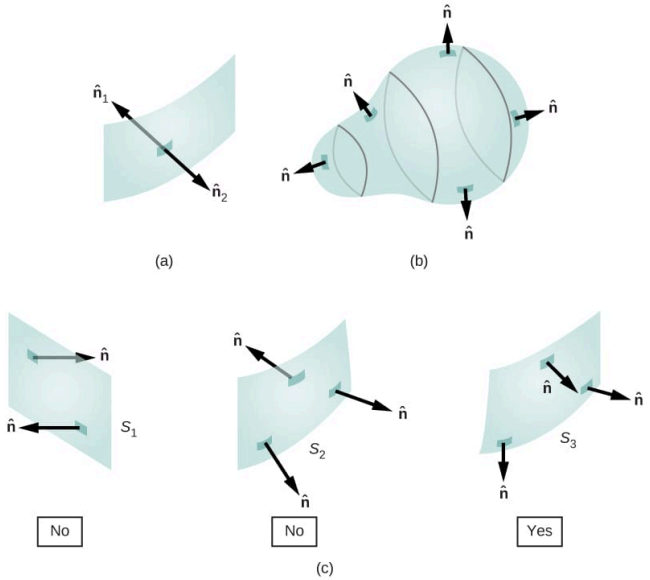


Figure 9.2.4: (a) Two potential normal vectors arise at every point on a surface. (b) The outward normal is used to calculate the flux through a closed surface. (c) Only S_3 has been given a consistent set of normal vectors that allows us to define the flux through the surface.

However, if a surface is closed, then the surface encloses a volume. In that case, the direction of the normal vector at any point on the surface points from the inside to the outside. On a *closed surface* such as that of Figure $9.2.1b$, $\hat{n}$ is chosen to be the *outward normal* at every point, to be consistent with the sign convention for electric charge.

Electric Flux

Now that we have defined the area vector of a surface, we can define the electric flux of a uniform electric field through a flat area as the scalar product of the electric field and the area vector:

$$\Phi = \vec{E} \cdot \vec{A} \quad (\text{uniform } \vec{E}, \text{flat surface}).$$

Figure $9.2.5$ shows the electric field of an oppositely charged, parallel-plate system and an imaginary box between the plates. The electric field between the plates is uniform and points from the positive plate toward the negative plate. A calculation of the flux of this field through various faces of the box shows that the net flux through the box is zero. Why does the flux cancel out here?

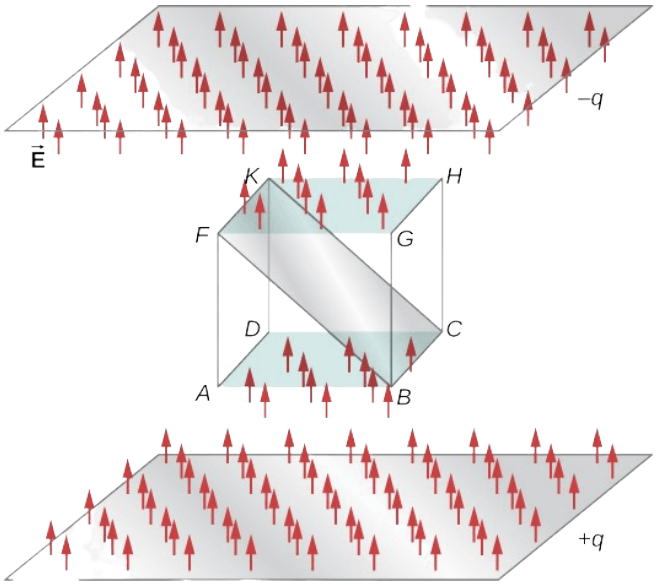


Figure 9.2.5: Electric flux through a cube, placed between two charged plates. Electric flux through the bottom face ($ABCD$) is negative, because $\vec{E}$ is in the opposite direction to the normal to the surface. The electric flux through the top face ($FGHK$) is positive, because the electric field and the normal are in the same direction. The electric flux through the other faces is zero, since the electric field is perpendicular to the normal vectors of those faces. The net electric flux through the cube is the sum of fluxes through the six faces. Here, the net flux through the cube is equal to zero. The magnitude of the flux through rectangle $BCKF$ is equal to the magnitudes of the flux through both the top and bottom faces.

The reason is that the sources of the electric field are outside the box. Therefore, if any electric field line enters the volume of the box, it must also exit somewhere on the surface because there is no charge inside for the lines to land on. Therefore, quite generally, electric flux through a closed surface is zero if there are no sources of electric field, whether positive or negative charges, inside the enclosed volume. In general, when field lines leave (or "flow out of") a closed surface, Φ is positive; when they enter (or "flow into") the surface, Φ is negative.

Any smooth, non-flat surface can be replaced by a collection of tiny, approximately flat surfaces, as shown in Figure 9.2.6. If we divide a surface S into small patches, then we notice that, as the patches become smaller, they can be approximated by flat surfaces. This is similar to the way we treat the surface of Earth as locally flat, even though we know that globally, it is approximately spherical.

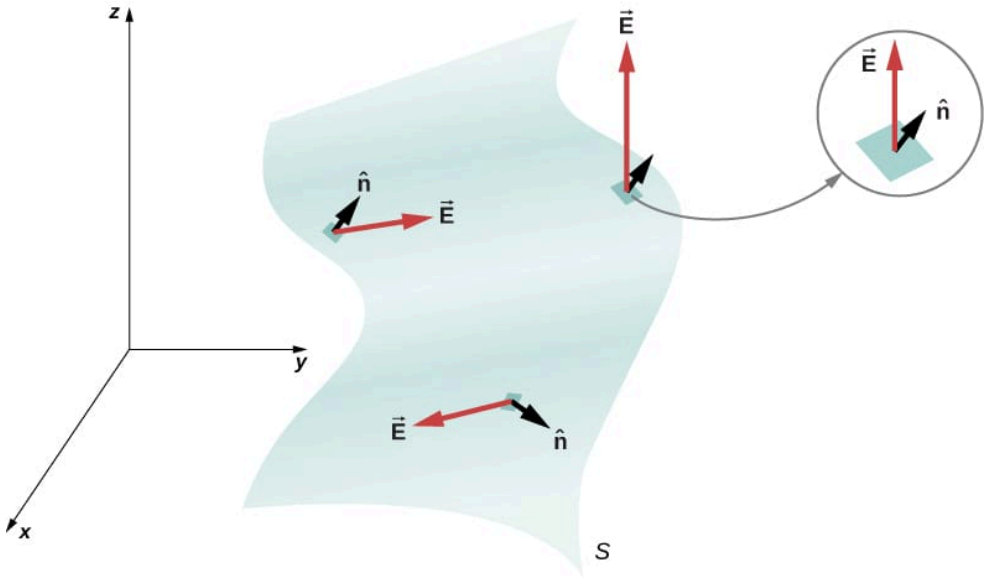


Figure 9.2.6: A surface is divided into patches to find the flux.

To keep track of the patches, we can number them from 1 through N. Now, we define the area vector for each patch as the area of

the patch pointed in the direction of the normal. Let us denote the area vector for the ith patch by $\delta\vec{A}_i$. (We have used the symbol δ to remind us that the area is of an arbitrarily small patch.) With sufficiently small patches, we may approximate the electric field over any given patch as uniform. Let us denote the average electric field at the location of the ith patch by $\vec{E}_i$.

$$\vec{E}_i = \text{average electric field over the } i\text{th patch.}$$

Therefore, we can write the electric flux Φ through the area of the ith patch as

$$\Phi_i = \vec{E}_i \cdot \delta\vec{A}_i \quad (i\text{th patch}).$$

The flux through each of the individual patches can be constructed in this manner and then added to give us an estimate of the net flux through the entire surface S, which we denote simply as Φ.

$$\Phi = \sum_{i=1}^{N} \Phi_i = \sum_{i=1}^{N} \vec{E}_i \cdot \delta\vec{A}_i \quad (N \text{ patch estimate}).$$

This estimate of the flux gets better as we decrease the size of the patches. However, when you use smaller patches, you need more of them to cover the same surface. In the limit of infinitesimally small patches, they may be considered to have area dA and unit normal $\hat{n}$. Since the elements are infinitesimal, they may be assumed to be planar, and $\vec{E}_i$ may be taken as constant over any element. Then the flux $d\Phi$ through an area dA is given by $d\Phi = \vec{E} \cdot \hat{n} dA$. It is positive when the angle between $\vec{E}_i$ and $\hat{n}$ is less than 90° and negative when the angle is greater than 90°. The net flux is the sum of the infinitesimal flux elements over the entire surface. With infinitesimally small patches, you need infinitely many patches, and the limit of the sum becomes a surface integral. With $\int_S$ representing the integral over S,

$$\Phi = \int_S \vec{E} \cdot \hat{n}\, dA = \int_S \vec{E} \cdot d\vec{A} \quad (\text{open surface}).$$

In practical terms, surface integrals are computed by taking the antiderivatives of both dimensions defining the area, with the edges of the surface in question being the bounds of the integral.

To distinguish between the flux through an open surface like that of Figure $9.2.2$ and the flux through a closed surface (one that completely bounds some volume), we represent flux through a closed surface by

$$\Phi = \oint_S \vec{E} \cdot \hat{n}\, dA = \oint_S \vec{E} \cdot d\vec{A} \quad (\text{closed surface})$$

where the circle through the integral symbol simply means that the surface is closed, and we are integrating over the entire thing. If you only integrate over a portion of a closed surface, that means you are treating a subset of it as an open surface.

Example $9.2.1$: Flux of a Uniform Electric Field

A constant electric field of magnitude E_0 points in the direction of the positive z-axis (Figure $9.2.7$). What is the electric flux through a rectangle with sides a and b in the (a) xy-plane and in the (b) xz-plane?

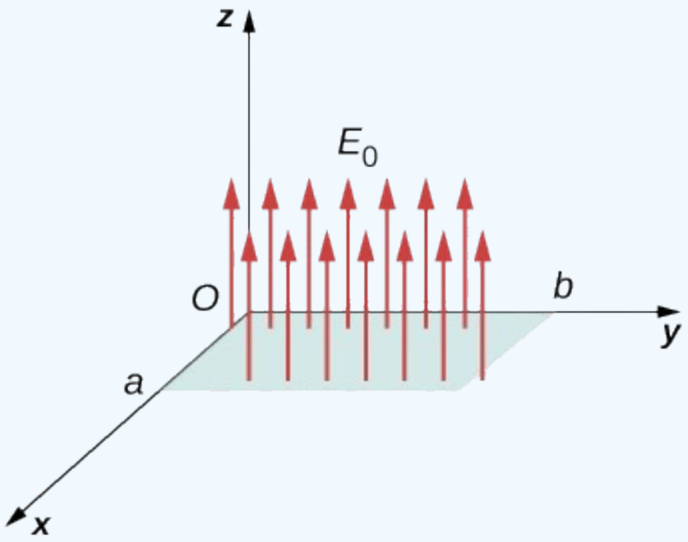


Figure 9.2.7: Calculating the flux of E_0 through a rectangular surface.

Strategy

Apply the definition of flux: $\Phi = \vec{E} \cdot \vec{A}$ (uniform $\vec{E}$), where the definition of dot product is crucial.

Solution

1. In this case, $\Phi = \vec{E}_0 \cdot \vec{A} = E_0 A = E_0 ab$.
2. Here, the direction of the area vector is either along the positive y-axis or toward the negative y-axis. Therefore, the scalar product of the electric field with the area vector is zero, giving zero flux.

Significance

The relative directions of the electric field and area can cause the flux through the area to be zero.

Flux of a Uniform Electric Field through a Closed Surface

A constant electric field of magnitude E_0 points in the direction of the positive z-axis (Figure 9.2.8). What is the net electric flux through a cube?

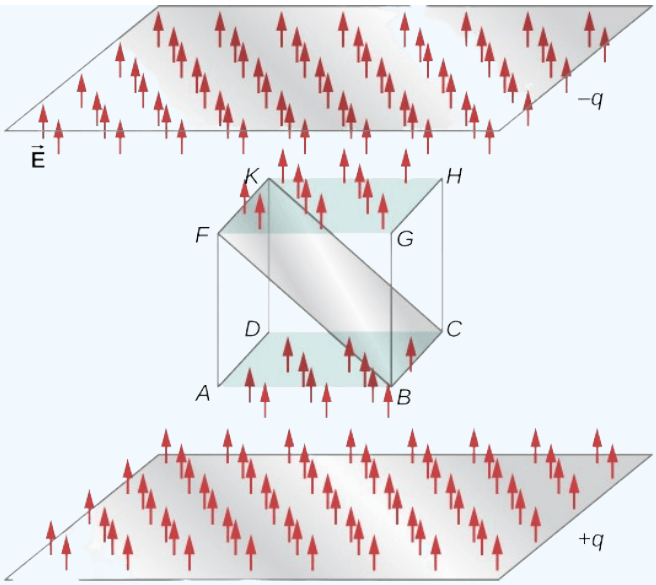


Figure 9.2.8: Calculating the flux of E_0 through a closed cubic surface.

Strategy

Apply the definition of flux: $\Phi = \vec{E} \cdot \vec{A}$ (uniform $\vec{E}$), noting that a closed surface eliminates the ambiguity in the direction of the area vector.

Solution

Through the top face of the cube $\Phi = \vec{E}_0 \cdot \vec{A} = E_0 A$.

Through the bottom face of the cube, $\Phi = \vec{E}_0 \cdot \vec{A} = -E_0 A$, because the area vector here points downward.

Along the other four sides, the direction of the area vector is perpendicular to the direction of the electric field. Therefore, the scalar product of the electric field with the area vector is zero, giving zero flux.

The net flux is $\Phi_{net} = E_0 A - E_0 A + 0 + 0 + 0 + 0 = 0$.

Significance

The net flux of a uniform electric field through a closed surface is zero.

Example 9.2.3: Electric Flux through a Plane, Integral Method

A uniform electric field $\vec{E}$ of magnitude 10 N/C is directed parallel to the yz-plane at 30° above the xy-plane, as shown in Figure 9.2.9. What is the electric flux through the plane surface of area $6.0\ \mathrm{m}^2$ located in the xz-plane? Assume that $\hat{n}$ points in the positive y-direction.

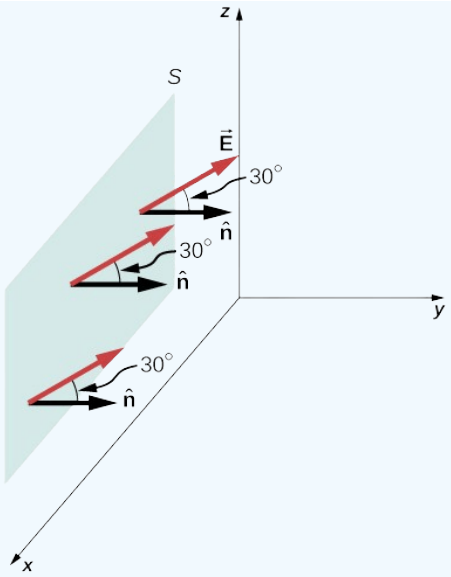


Figure 9.2.9: The electric field produces a net electric flux through the surface S.

Strategy

Apply $\Phi = \int_S \vec{E} \cdot \hat{n} dA$, where the direction and magnitude of the electric field are constant.

Solution

The angle between the uniform electric field $\vec{E}$ and the unit normal $\hat{n}$ to the planar surface is $30°$. Since both the direction and magnitude are constant, E comes outside the integral. All that is left is a surface integral over dA, which is A. Therefore, using the open-surface equation, we find that the electric flux through the surface is

$$\Phi = \int_S \vec{E} \cdot \hat{n}\, dA = EA \cos\theta$$

$$= (10\,\mathrm{N/C})(6.0\,\mathrm{m}^2)(\cos 30°) = 52\,\mathrm{N \cdot m^2/C}.$$

Significance

Again, the relative directions of the field and the area matter, and the general equation with the integral will simplify to the simple dot product of area and electric field.

Exercise 9.2.1

What angle should there be between the electric field and the surface shown in Figure 9.2.9 in the previous example so that no electric flux passes through the surface?

Answer available at the end of this section.

Example 9.2.4 : Inhomogeneous Electric Field

What is the total flux of the electric field $\vec{E} = cy^2\hat{k}$ through the rectangular surface shown in Figure 9.2.10?

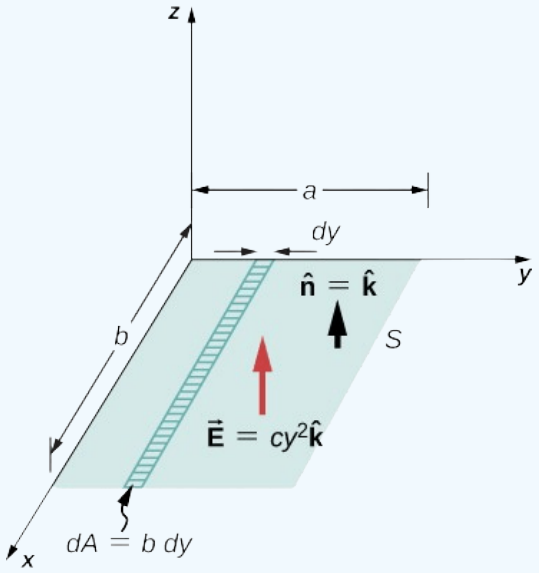


Figure 9.2.10: Since the electric field is not constant over the surface, an integration is necessary to determine the flux.

Strategy

Apply $\Phi = \int_S \vec{E} \cdot \hat{n}\ dA$. We assume that the unit normal $\hat{n}$ to the given surface points in the positive z-direction, so $\hat{n} = \hat{k}$. Since the electric field is not uniform over the surface, it is necessary to divide the surface into infinitesimal strips along which $\vec{E}$ is essentially constant. As shown in Figure 9.2.10, these strips are parallel to the x-axis, and each strip has an area $dA = b\ dy$.

Solution

From the open surface integral, we find that the net flux through the rectangular surface is

$$\Phi = \int_S \vec{E} \cdot \hat{n} dA = \int_0^a (cy^2\hat{k}) \cdot \hat{k}(b\, dy)$$
$$= cb \int_0^a y^2 dy = \frac{1}{3}a^3bc.$$

Significance

For a non-constant electric field, the integral method is required.

Exercise 9.2.2

If the electric field in Example 9.2.4 is $\vec{E} = mx\hat{k}$. what is the flux through the rectangular area?

Answer available at the end of this section.

Answers

Exercise 9.2.1

Place it so that its unit normal is perpendicular to $\vec{E}$.

Exercise 9.2.2

$mab^2/2$

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9.3: Explaining Gauss's Law

Learning Objectives

By the end of this section, you will be able to:

- State Gauss's law
- Explain the conditions under which Gauss's law may be used
- Apply Gauss's law in appropriate systems

We can now determine the electric flux through an arbitrary closed surface due to an arbitrary charge distribution. We found that if a closed surface does not have any charge inside where an electric field line can terminate, then any electric field line entering the surface at one point must necessarily exit at some other point of the surface. Therefore, if a closed surface does not have any charges inside the enclosed volume, then the electric flux through the surface is zero. Now, what happens to the electric flux if there are some charges inside the enclosed volume? Gauss's law gives a quantitative answer to this question.

To get a feel for what to expect, let's calculate the electric flux through a spherical surface around a positive point charge q, since we already know the electric field in such a situation. Recall that when we place the point charge at the origin of a coordinate system, the electric field at a point P that is at a distance r from the charge at the origin is given by

$$\vec{E}_p = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r},$$

where $\hat{r}$ is the radial vector from the charge at the origin to the point P. We can use this electric field to find the flux through the spherical surface of radius r, as shown in Figure $9.3.1$.

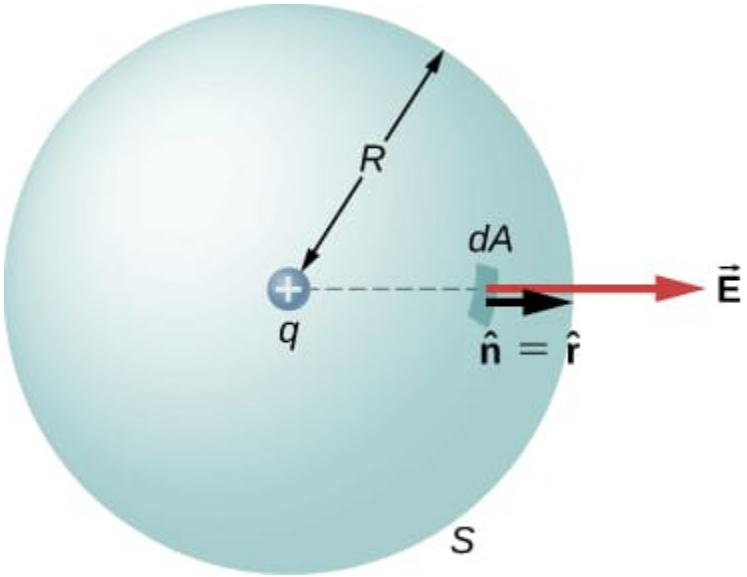


Figure $9.3.1$: A closed spherical surface surrounding a point charge q.

Then we apply $\Phi = \int_S \vec{E}\cdot\hat{n}dA$ to this system and substitute known values. On the sphere, $\hat{n}$ and $r = R$ so for an infinitesimal area dA,

$$\begin{aligned} d\Phi &= \vec{E}\cdot\hat{n}dA \\ &= \frac{1}{4\pi\epsilon_0}\frac{q}{R^2}\hat{r}\cdot\hat{r}dA \\ &= \frac{1}{4\pi\epsilon_0}\frac{q}{R^2}dA. \end{aligned}$$

We now find the net flux by integrating this flux over the surface of the sphere:

$$\Phi = \frac{1}{4\pi\epsilon_0}\frac{q}{R^2}\oint_S dA = \frac{1}{4\pi\epsilon_0}\frac{q}{R^2}(4\pi R^2) = \frac{q}{\epsilon_0}.$$

where the total surface area of the spherical surface is $4\pi R^2$. This gives the flux through the closed spherical surface at radius r as

$$\Phi = \frac{q}{\epsilon_0}.$$

A remarkable fact about this equation is that the flux is independent of the size of the spherical surface. This can be directly attributed to the fact that the electric field of a point charge decreases as $1/r^2$ with distance, which just cancels the r^2 rate of increase of the surface area.

Electric Field Lines Picture

An alternative way to see why the flux through a closed spherical surface is independent of the radius of the surface is to look at the electric field lines. Note that every field line from q that pierces the surface at radius R_1 also pierces the surface at R_2 (Figure $9.3.2$).

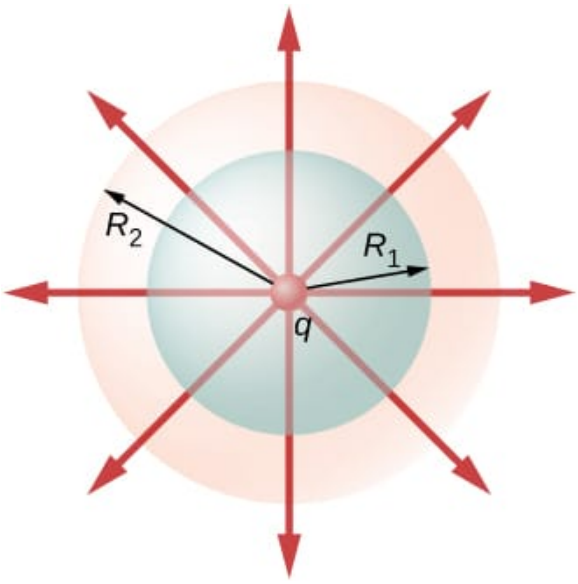


Figure $9.3.2$: Flux through spherical surfaces of radii R_1 and R_2 enclosing a charge q are equal, independent of the size of the surface, since all E-field lines that pierce one surface from the inside to outside direction also pierce the other surface in the same direction.

Therefore, the net number of electric field lines passing through the two surfaces from the inside to outside direction is equal. This net number of electric field lines, which is obtained by subtracting the number of lines in the direction from outside to inside from the number of lines in the direction from inside to outside gives a visual measure of the electric flux through the surfaces.

You can see that if no charges are included within a closed surface, then the electric flux through it must be zero. A typical field line enters the surface at dA_1 and leaves at dA_2. Every line that enters the surface must also leave that surface. Hence the net "flow" of the field lines into or out of the surface is zero (Figure $9.3.3a$). The same thing happens if charges of equal and opposite sign are included inside the closed surface, so that the total charge included is zero (Figure $9.3.3b$). A surface that includes the same amount of charge has the same number of field lines crossing it, regardless of the shape or size of the surface, as long as the surface encloses the same amount of charge (Figure $9.3.3c$).

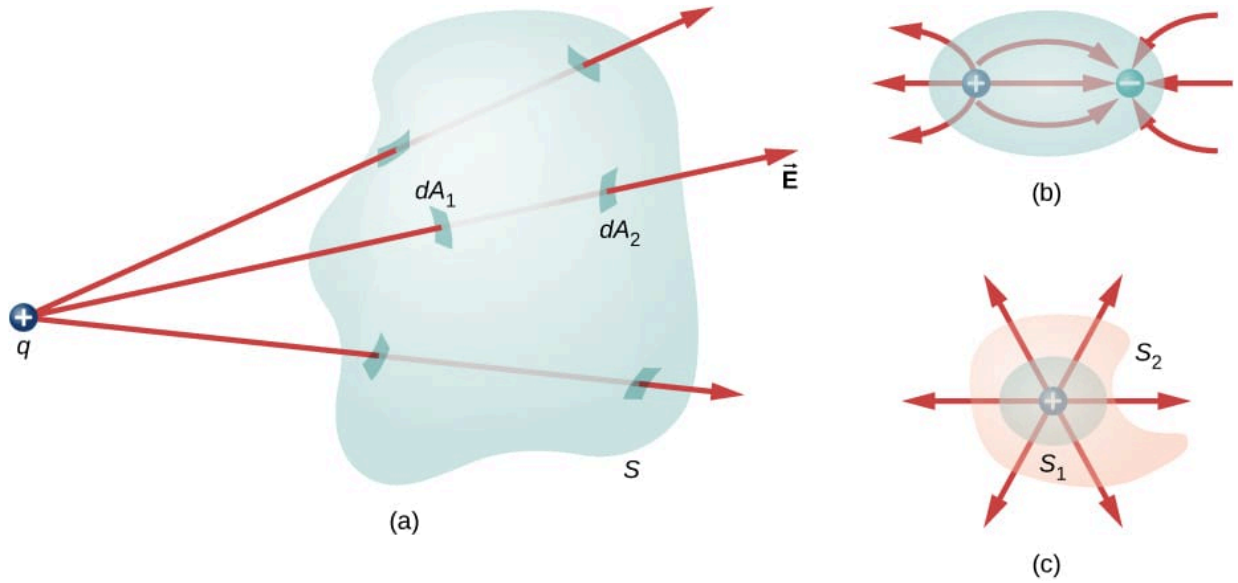


Figure 9.3.3: Understanding the flux in terms of field lines. (a) The electric flux through a closed surface due to a charge outside that surface is zero. (b) Charges are enclosed, but because the net charge included is zero, the net flux through the closed surface is also zero. (c) The shape and size of the surfaces that enclose a charge does not matter because all surfaces enclosing the same charge have the same flux.

Statement of Gauss's Law

Gauss's law generalizes this result to the case of any number of charges and any location of the charges in the space inside the closed surface. According to Gauss's law, the flux of the electric field $\vec{E}$ through any closed surface, also called a **Gaussian surface**, is equal to the net charge enclosed (q_{enc}) divided by the permittivity of free space (ϵ_0):

$$\Phi_{\text{Closed Surface}} = \frac{q_{enc}}{\epsilon_0}.$$

This equation holds for *charges of either sign,* because we define the area vector of a closed surface to point outward. If the enclosed charge is negative (Figure 9.3.4*b*), then the flux through either S or S' is negative.

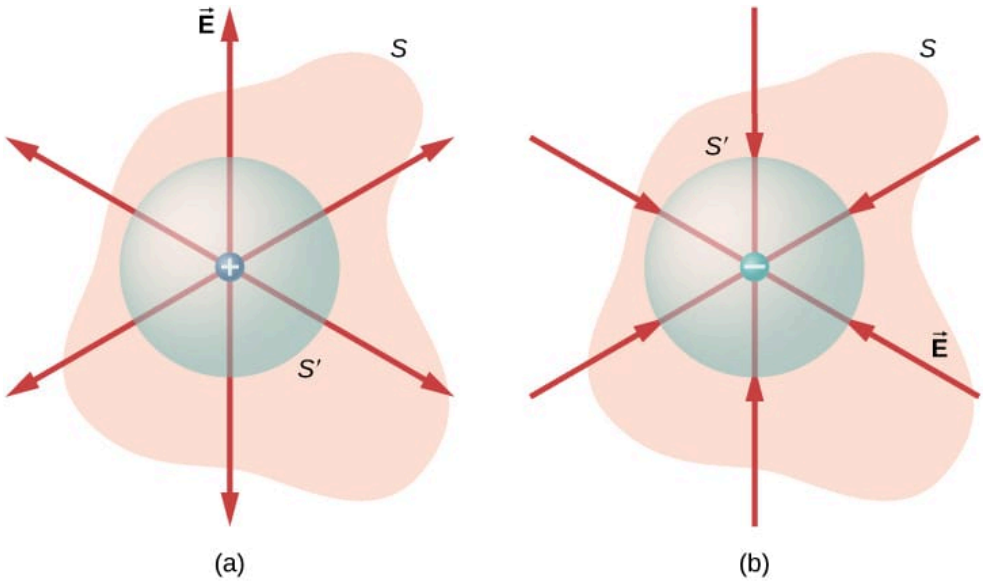


Figure 9.3.4: The electric flux through any closed surface surrounding a point charge q is given by Gauss's law. (a) Enclosed charge is positive. (b) Enclosed charge is negative.

The Gaussian surface does not need to correspond to a real, physical object; indeed, it rarely will. It is a mathematical construct that may be of any shape, provided that it is closed. However, since our goal is to integrate the flux over it, we tend to choose

shapes that are highly symmetrical.

If the charges are discrete point charges, then we just add them. If the charge is described by a continuous distribution, then we need to integrate appropriately to find the total charge that resides inside the enclosed volume. For example, the flux through the Gaussian surface S of Figure $9.3.5$ is

$$\Phi = (q_1 + q_2 + q_5)/\epsilon_0.$$

Note that q_{enc} is simply the sum of the point charges. If the charge distribution were continuous, we would need to integrate appropriately to compute the total charge within the Gaussian surface.

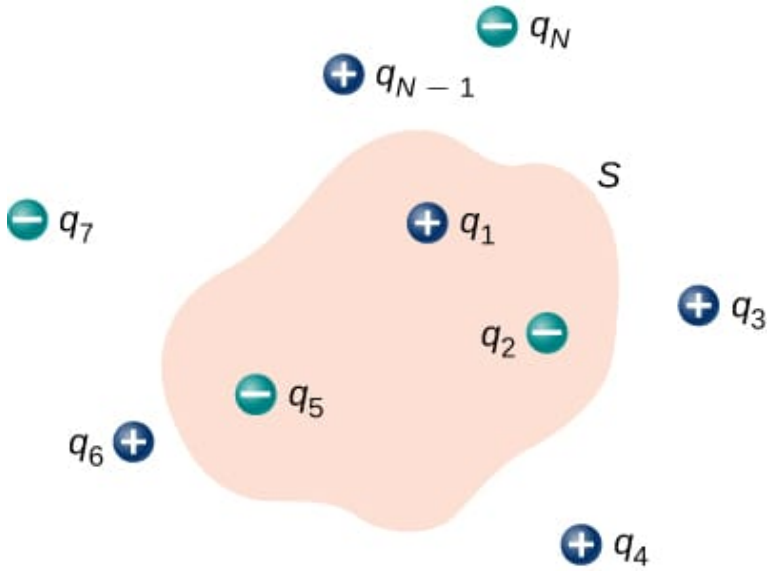


Figure $9.3.5$: The flux through the Gaussian surface shown, due to the charge distribution, is $\Phi = (q_1 + q_2 + q_5)/\epsilon_0$.

Recall that the principle of superposition holds for the electric field. Therefore, the total electric field at any point, including those on the chosen Gaussian surface, is the sum of all the electric fields present at this point. This allows us to write Gauss's law in terms of the total electric field.

Gauss's Law

The flux Φ of the electric field $\vec{E}$ through any closed surface **S** (a Gaussian surface) is equal to the net charge enclosed (q_{enc}) divided by the permittivity of free space (ϵ_0):

$$\Phi = \oint_S \vec{E} \cdot \hat{n} dA = \frac{q_{enc}}{\epsilon_0}.$$

To use Gauss's law effectively, you must have a clear understanding of what each term in the equation represents. The field $\vec{E}$ is the **total electric field** at every point on the Gaussian surface. This total field includes contributions from charges both inside and outside the Gaussian surface. However, q_{enc} is just the charge **inside** the Gaussian surface. Finally, the Gaussian surface is any closed surface in space. That surface can coincide with the actual surface of a conductor, or it can be an imaginary geometric surface. The only requirement imposed on a Gaussian surface is that it be closed (Figure $9.3.5$).



Figure 9.3.6: A **Klein bottle** partially filled with a liquid. Could the Klein bottle be used as a Gaussian surface?

Example 9.3.1: Electric Flux through Gaussian Surfaces

Calculate the electric flux through each Gaussian surface shown in Figure 9.3.7.

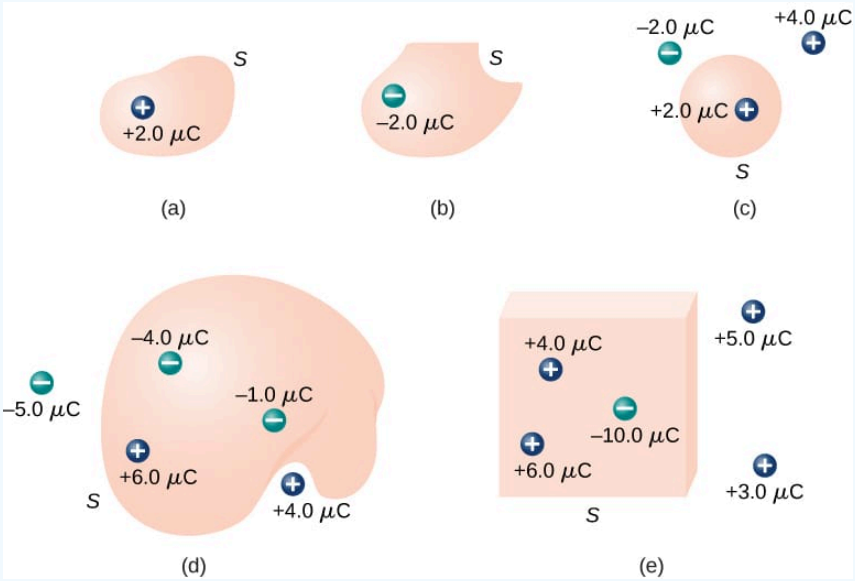


Figure 9.3.7: Various Gaussian surfaces and charges.

Strategy

From Gauss's law, the flux through each surface is given by q_{enc}/ϵ_0, where q_{enc} is the charge enclosed by that surface.

Solution

For the surfaces and charges shown, we find

a. $\Phi = \frac{2.0\ \mu C}{\epsilon_0} = 2.3 \times 10^5\ \text{N} \cdot \text{m}^2/\text{C}.$

b. $\Phi = \frac{-2.0\ \mu C}{\epsilon_0} = -2.3 \times 10^5\ \text{N} \cdot \text{m}^2/\text{C}.$

c. $\Phi = \frac{2.0\ \mu C}{\epsilon_0} = 2.3 \times 10^5\ \text{N} \cdot \text{m}^2/\text{C}.$

d. $\Phi = \frac{-4.0\ \mu C + 6.0\ \mu C - 1.0\ \mu C}{\epsilon_0} = 1.1 \times 10^5\ \text{N} \cdot \text{m}^2/\text{C}.$

e. $\Phi = \frac{4.0\ \mu C + 6.0\ \mu C - 10.0\ \mu C}{\epsilon_0} = 0.$

Significance

In the special case of a closed surface, the flux calculations become a sum of charges. In the next section, this will allow us to work with more complex systems.

Exercise 9.3.1

Calculate the electric flux through the closed cubical surface for each charge distribution shown in Figure 9.3.8.

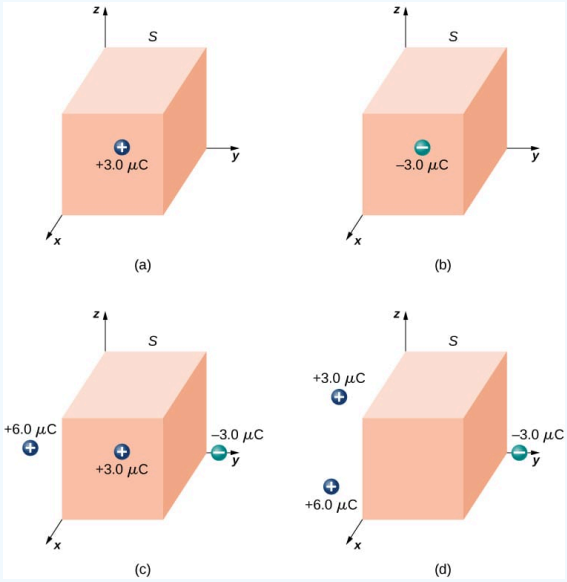


Figure 9.3.8: A cubical Gaussian surface with various charge distributions.

Answer a available at the end of this section.

Use this simulation to adjust the magnitude of the charge and the radius of the Gaussian surface around it. See how this affects the total flux and the magnitude of the electric field at the Gaussian surface.

Answers

Exercise 9.3.1

$3.4 \times 10^5\ \text{N} \cdot \text{m}^2/\text{C}$

$-3.4 \times 10^5\ \text{N} \cdot \text{m}^2/\text{C}$

$3.4 \times 10^5\ \text{N} \cdot \text{m}^2/\text{C}$

0

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9.4: Applying Gauss's Law

Learning Objectives

By the end of this section, you will be able to:

- Explain what spherical, cylindrical, and planar symmetry are
- Recognize whether or not a given system possesses one of these symmetries
- Apply Gauss's law to determine the electric field of a system with one of these symmetries

Gauss's law is very helpful in determining expressions for the electric field, even though the law is not directly about the electric field; it is about the electric flux. It turns out that in situations that have certain symmetries (spherical, cylindrical, or planar) in the charge distribution, we can deduce the electric field based on knowledge of the electric flux. In these systems, we can find a Gaussian surface S over which the electric field has constant magnitude. Furthermore, if $\vec{E}$ is parallel to $\hat{n}$ everywhere on the surface, then $\vec{E} \cdot \hat{n} = E$. (If $\vec{E}$ and $\hat{n}$ are antiparallel everywhere on the surface, $\vec{E} \cdot \hat{n} = -E$.) Gauss's law then simplifies to

$$\Phi = \oint_S \vec{E} \cdot \hat{n} dA = E \oint_S dA = EA = \frac{q_{enc}}{\epsilon_0},$$

where A is the area of the surface. Note that these symmetries lead to the transformation of the flux integral into a product of the magnitude of the electric field and an appropriate area. When you use this flux in the expression for Gauss's law, you obtain an algebraic equation that you can solve for the magnitude of the electric field, which looks like

$$E \approx \frac{q_{enc}}{\epsilon_0 \, \text{area}}.$$

The direction of the electric field at point P is obtained from the symmetry of the charge distribution and the type of charge in the distribution. Therefore, Gauss's law can be used to determine $\vec{E}$. Here is a summary of the steps we will follow:

Problem-Solving Strategy: Gauss's Law

1. *Identify the spatial symmetry of the charge distribution.* This is an important first step that allows us to choose the appropriate Gaussian surface. As examples, an isolated point charge has spherical symmetry, and an infinite line of charge has cylindrical symmetry.
2. *Choose a Gaussian surface with the same symmetry as the charge distribution and identify its consequences.* With this choice, $\vec{E} \cdot \hat{n}$ is easily determined over the Gaussian surface.
3. *Evaluate the integral $\oint_S \vec{E} \cdot \hat{n} \; dA$ over the Gaussian surface, that is, calculate the flux through the surface.* The symmetry of the Gaussian surface allows us to factor $\vec{E} \cdot \hat{n}$ outside the integral.
4. *Determine the amount of charge enclosed by the Gaussian surface.* This is an evaluation of the right-hand side of the equation representing Gauss's law. It is often necessary to perform an integration to obtain the net enclosed charge.
5. *Evaluate the electric field of the charge distribution.* The field may now be found using the results of steps 3 and 4.

Basically, there are only three types of symmetry that allow Gauss's law to be used to deduce the electric field. They are

- A charge distribution with spherical symmetry
- A charge distribution with cylindrical symmetry
- A charge distribution with planar symmetry

To exploit the symmetry, we perform the calculations in appropriate coordinate systems and use the right kind of Gaussian surface for that symmetry, applying the remaining four steps.

Charge Distribution with Spherical Symmetry

A charge distribution has **spherical symmetry** if the density of charge depends only on the distance from a point in space and not on the direction. In other words, if you rotate the system, it doesn't look different. For instance, if a sphere of radius R is uniformly charged with charge density ρ_0 then the distribution has spherical symmetry (Figure $9.4.1a$). On the other hand, if a sphere of radius R is charged so that the top half of the sphere has uniform charge density ρ_1 and the bottom half has a uniform charge density $\rho_2 \neq \rho_1$ then the sphere does not have spherical symmetry because the charge density depends on the direction (Figure $9.4.1b$). Thus, it is not the shape of the object but rather the shape of the charge distribution that determines whether or not a system has spherical symmetry.

Figure $9.4.1c$ shows a sphere with four different shells, each with its own uniform charge density. Although this is a situation where charge density in the full sphere is not uniform, the charge density function depends only on the distance from the center and not on the direction. Therefore, this charge distribution does have spherical symmetry.

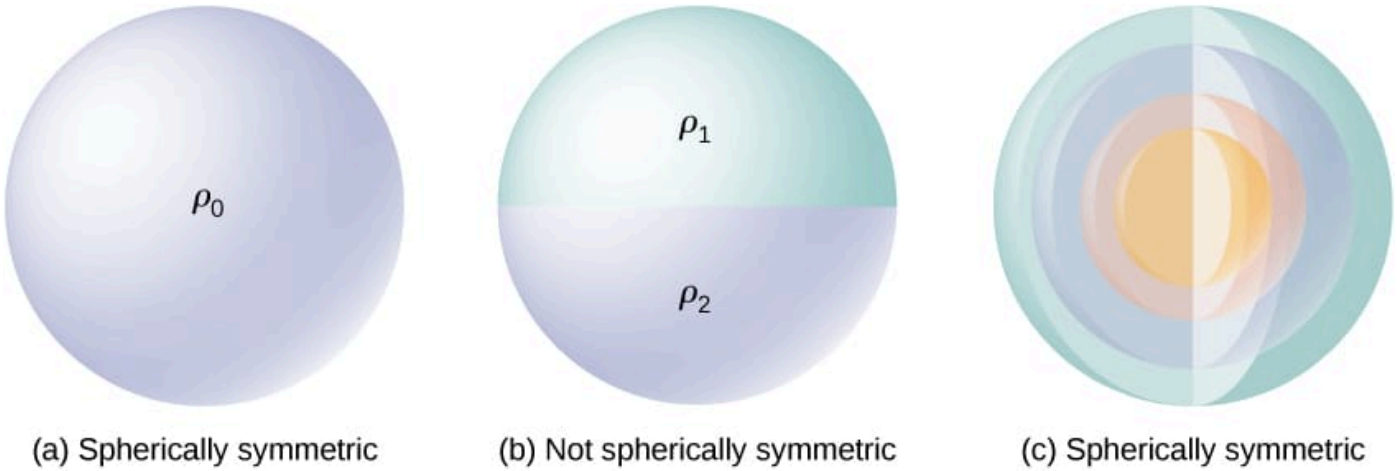


Figure $9.4.1$: Illustrations of spherically symmetrical and nonsymmetrical systems. Different shadings indicate different charge densities. Charges on spherically shaped objects do not necessarily mean the charges are distributed with spherical symmetry. The spherical symmetry occurs only when the charge density does not depend on the direction. In (a), charges are distributed uniformly in a sphere. In (b), the upper half of the sphere has a different charge density from the lower half; therefore, (b) does not have spherical symmetry. In (c), the charges are in spherical shells of different charge densities, which means that charge density is only a function of the radial distance from the center; therefore, the system has spherical symmetry.

One good way to determine whether or not your problem has spherical symmetry is to look at the charge density function in spherical coordinates, $\rho(r, \theta, \phi)$. If the charge density is only a function of r, that is $\rho = \rho(r)$, then you have spherical symmetry. If the density depends on θ or ϕ, you could change it by rotation; hence, you would not have spherical symmetry.

Consequences of symmetry

In all spherically symmetrical cases, the electric field at any point must be radially directed, because the charge and, hence, the field must be invariant under rotation. Therefore, using spherical coordinates with their origins at the center of the spherical charge distribution, we can write down the expected form of the electric field at a point P located at a distance r from the center:

$$\text{Spherical symmetry}: \ \vec{E}_P = E_P(r)\hat{r},$$

where $\hat{r}$ is the unit vector pointed in the direction from the origin to the field point P. The radial component E_p of the electric field can be positive or negative. When $E_p > 0$, the electric field at P points away from the origin, and when $E_p < 0$, the electric field at P points toward the origin.

Gaussian surface and flux calculations

We can now use this form of the electric field to obtain the flux of the electric field through the Gaussian surface. For spherical symmetry, the Gaussian surface is a closed spherical surface that has the same center as the center of the charge distribution. Thus, the direction of the area vector of an area element on the Gaussian surface at any point is parallel to the direction of the

electric field at that point, since they are both radially directed outward (Figure 9.4.2).

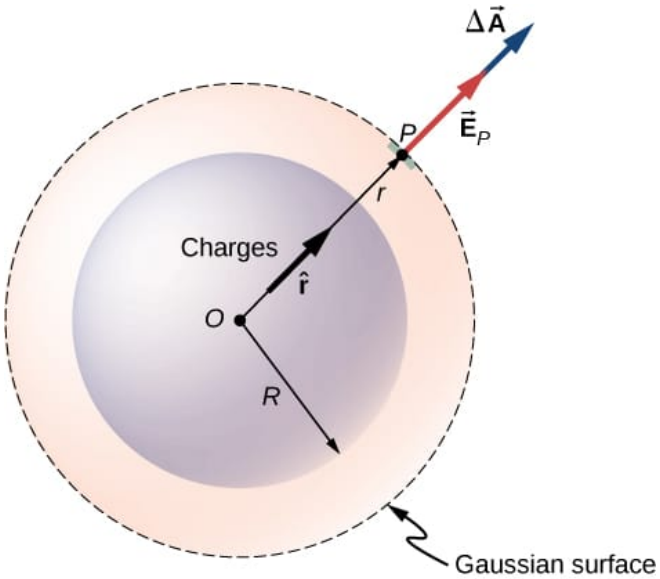


Figure 9.4.2: The electric field at any point of the spherical Gaussian surface for a spherically symmetrical charge distribution is parallel to the area element vector at that point, giving flux as the product of the magnitude of electric field and the value of the area. Note that the radius R of the charge distribution and the radius r of the Gaussian surface are different quantities.

The magnitude of the electric field $\vec{E}$ must be the same everywhere on a spherical Gaussian surface concentric with the distribution. For a spherical surface of radius r:

$$\Phi = \oint_S \vec{E}_p \cdot \hat{n} dA = E_p \oint_S dA = E_p 4\pi r^2.$$

Using Gauss's law

According to Gauss's law, the flux through a closed surface is equal to the total charge enclosed within the closed surface divided by the permittivity of vacuum ϵ_0. Let q_{enc} be the total charge enclosed inside the distance$S$ from the origin, which is the space inside the Gaussian spherical surface of radius r. This gives the following relation for Gauss's law:

$$4\pi r^2 E = \frac{q_{enc}}{\epsilon_0}.$$

Hence, the electric field at point P that is a distance r from the center of a spherically symmetrical charge distribution has the following magnitude and direction:

$$\text{Magnitude: } E(r) = \frac{1}{4\pi\epsilon_0}\frac{q_{enc}}{r^2}$$

Direction: radial from O to P or from P to O.

The direction of the field at point P depends on whether the charge in the sphere is positive or negative. For a net positive charge enclosed within the Gaussian surface, the direction is from O to P, and for a net negative charge, the direction is from P to O . This is all we need for a point charge, and you will notice that the result above is identical to that for a point charge. However, Gauss's law becomes truly useful in cases where the charge occupies a finite volume.

Computing Enclosed Charge

The more interesting case is when a spherical charge distribution occupies a volume, and asking what the electric field inside the charge distribution is thus becomes relevant. In this case, the charge enclosed depends on the distance r of the field point relative to the radius of the charge distribution R, such as that shown in Figure 9.4.3.

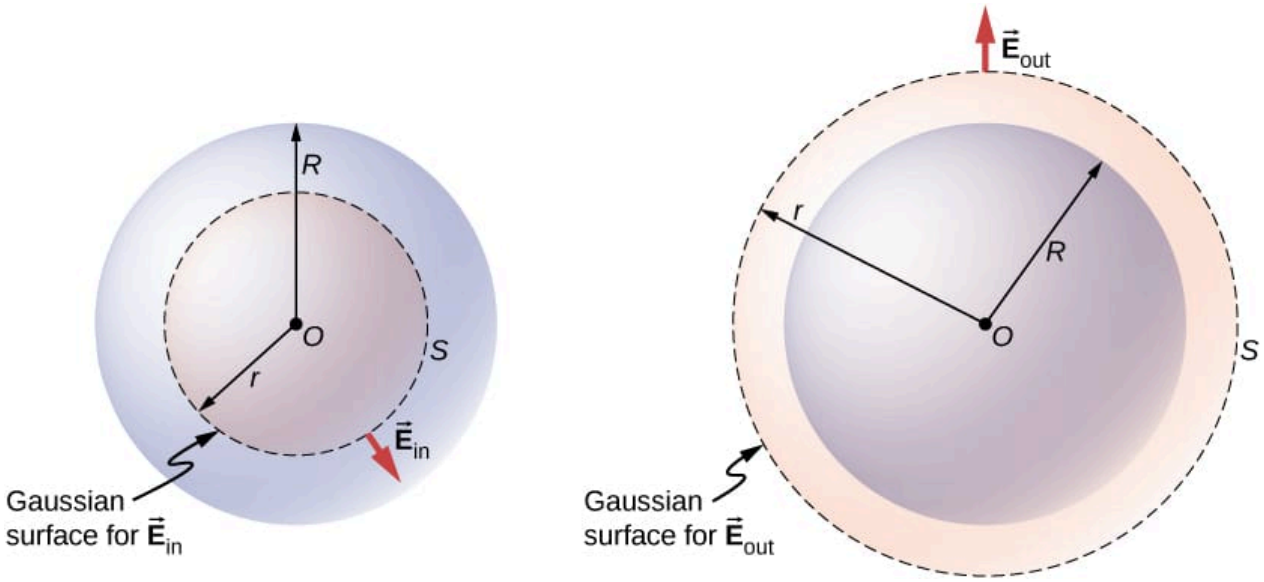


Figure 9.4.3: A spherically symmetrical charge distribution and the Gaussian surface used for finding the field (a) inside and (b) outside the distribution.

If point P is located outside the charge distribution—that is, if $r \geq R$ —then the Gaussian surface containing P encloses all charges in the sphere. In this case, q_{enc} equals the total charge in the sphere. On the other hand, if point P is within the spherical charge distribution, that is, if $r < R$, then the Gaussian surface encloses a smaller sphere than the sphere of charge distribution. In this case, q_{enc} is less than the total charge present in the sphere. Referring to Figure 9.4.3, we can write q_{enc} as

$$q_{enc} = q_{tot} \text{ (total charge) if } r \geq R$$

$$q_{enc} = q_{within\, r<R} \text{ (only charge within } r < R) \text{ if } r < R$$

The field at a point outside the charge distribution is also called $\vec{E}_{out}$, and the field at a point inside the charge distribution is called $\vec{E}_{in}$. Focusing on the two types of field points, either inside or outside the charge distribution, we can now write the magnitude of the electric field as

$$P \text{ outside sphere } E_{out} = \frac{1}{4\pi\epsilon_0}\frac{q_{tot}}{r^2}$$

$$P \text{ inside sphere } E_{in} = \frac{1}{4\pi\epsilon_0}\frac{q_{\text{within } r<R}}{r^2}.$$

Note that the electric field outside a spherically symmetrical charge distribution is identical to that of a point charge at the center that has a charge equal to the total charge of the spherical charge distribution. This is remarkable since the charges are not located at the center only. We now work out specific examples of spherical charge distributions, starting with the case of a uniformly charged sphere.

Uniformly Charged Sphere

A sphere of radius R, such as that shown in Figure 9.4.3, has a uniform volume charge density ρ_0. Find the electric field at a point outside the sphere and at a point inside the sphere.

Strategy

Apply the Gauss's law problem-solving strategy, where we have already worked out the flux calculation.

Solution

The charge enclosed by the Gaussian surface is given by

$$q_{enc} = \int \rho_0 dV = \int_0^r \rho_0 4\pi r'^2 dr' = \rho\left(\frac{4}{3}\pi r^3\right).$$

The answer for electric field amplitude can then be written down immediately for a point outside the sphere, labeled E_{out} and a point inside the sphere, labeled E_{in}.

$$E_{out} = \frac{1}{4\pi\epsilon_0}\frac{q_{tot}}{r^2},\ q_{tot} = \frac{4}{3}\pi R^3\,\rho_0,$$

$$E_{in} = \frac{q_{enc}}{4\pi\epsilon_0 r^2} = \frac{\rho_0 r}{3\epsilon_0},\ \text{since}\, q_{enc} = \frac{4}{3}\pi r^3\rho_0.$$

It is interesting to note that the magnitude of the electric field increases inside the material as you go out, since the amount of charge enclosed by the Gaussian surface increases with the volume. Specifically, the charge enclosed grows $\propto r^3$, whereas the field from each infinitesimal element of charge drops off $\propto 1/r^2$ with the net result that the electric field within the distribution increases in strength linearly with the radius. The magnitude of the electric field outside the sphere decreases as you go away from the charges, because the included charge remains the same but the distance increases. Figure $9.4.4$ displays the variation of the magnitude of the electric field with distance from the center of a uniformly charged sphere.

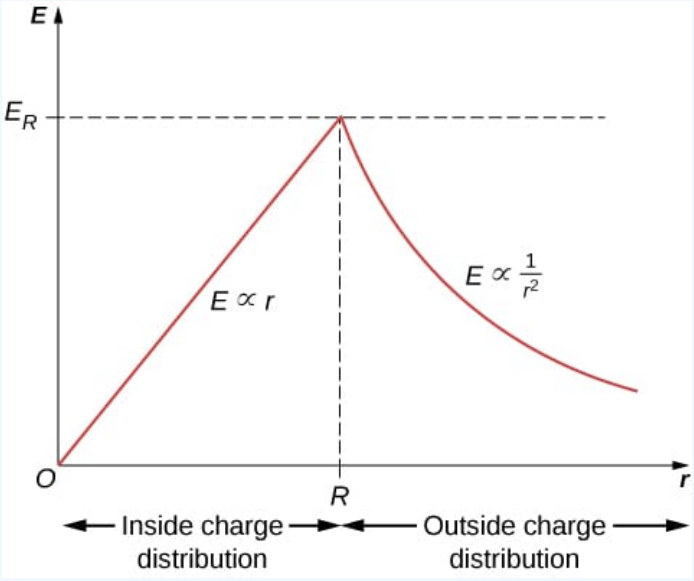


Figure $9.4.4$: Electric field of a uniformly charged, non-conducting sphere increases inside the sphere to a maximum at the surface and then decreases as $1/r^2$. Here, $E_R = \frac{\rho_0 R}{3\epsilon_0}$. The electric field is due to a spherical charge distribution of uniform charge density and total charge Q as a function of distance from the center of the distribution.

The direction of the electric field at any point P is radially outward from the origin if ρ_0 is positive, and inward (i.e., toward the center) if ρ_0 is negative. The electric field at some representative space points are displayed in Figure $9.4.5$ whose radial coordinates **r** are $r = R/2,\ r = R$, and $r = 2R$.

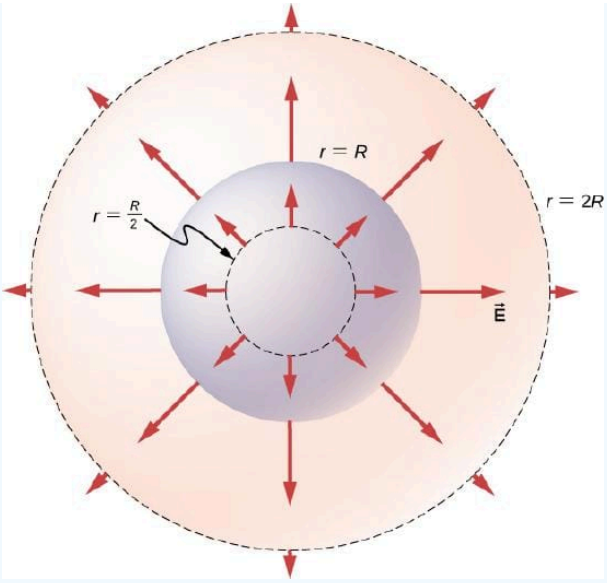


Figure 9.4.5: Electric field vectors inside and outside a uniformly charged sphere.

Significance

Notice that E_{out} has the same form as the equation of the electric field of an isolated point charge. In determining the electric field of a uniform spherical charge distribution, we can therefore assume that all of the charge inside the appropriate spherical Gaussian surface is located at the center of the distribution.

Non-Uniformly Charged Sphere

A non-conducting sphere of radius R has a non-uniform charge density that varies with the distance from its center as given by

$$\rho(r) = ar^n \, (r \leq R;\, n \geq 0),$$

where a is a constant. We require $n \geq 0$ so that the charge density is not undefined at $r = 0$. Find the electric field at a point outside the sphere and at a point inside the sphere.

Strategy

Apply the Gauss's law strategy given above, where we work out the enclosed charge integrals separately for cases inside and outside the sphere.

Solution

Since the given charge density function has only a radial dependence and no dependence on direction, we have a spherically symmetrical situation. Therefore, the magnitude of the electric field at any point is given above and the direction is radial. We just need to find the enclosed charge q_{enc}, which depends on the location of the field point.

A note about symbols: We use r' for locating charges in the charge distribution and **r** for locating the field point(s) at the Gaussian surface(s). The letter R is used for the radius of the charge distribution.

As charge density is not constant here, we need to integrate the charge density function over the volume enclosed by the Gaussian surface. Therefore, we set up the problem for charges in one spherical shell, say between r' and $r' + dr'$ as shown in Figure 9.4.6. The volume of charges in the shell of infinitesimal width is equal to the product of the area of surface $4\pi r'^2$ and the thickness dr'. Multiplying the volume with the density at this location, which is ar'^n, gives the charge in the shell:

$$dq = ar'^n 4\pi r'^2 dr'.$$

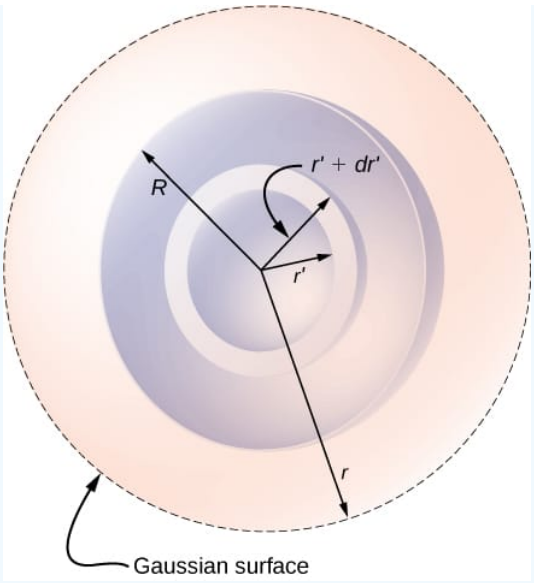


Figure 9.4.6: Spherical symmetry with non-uniform charge distribution. In this type of problem, we need four radii: R is the radius of the charge distribution, r is the radius of the Gaussian surface, r' is the inner radius of the spherical shell, and $r' + dr'$ is the outer radius of the spherical shell. The spherical shell is used to calculate the charge enclosed within the Gaussian surface. The range for r' is from 0 to r for the field at a point inside the charge distribution and from 0 to R for the field at a point outside the charge distribution. If $r > R$, then the Gaussian surface encloses more volume than the charge distribution, but the additional volume does not contribute to q_{enc}.

(a) **Field at a point outside the charge distribution.** In this case, the Gaussian surface, which contains the field point P, has a radius r that is greater than the radius R of the charge distribution, $r > R$. Therefore, all charges of the charge distribution are enclosed within the Gaussian surface. Note that the space between $r' = R$ and $r' = r$ is empty of charges and therefore does not contribute to the integral over the volume enclosed by the Gaussian surface:

$$q_{enc} = \int dq = \int_0^R ar'^n 4\pi r'^2 dr' = \frac{4\pi a}{n+3} R^{n+3}.$$

This is used in the general result for E_{out} above to obtain the electric field at a point outside the charge distribution as

$$\vec{E}_{out} = \left[\frac{aR^{n+3}}{\epsilon_0(n+3)}\right]\frac{1}{r^2}\hat{r},$$

where $\hat{r}$ is a unit vector in the direction from the origin to the field point at the Gaussian surface.

(b) **Field at a point inside the charge distribution.** The Gaussian surface is now buried inside the charge distribution, with $r < R$. Therefore, only those charges in the distribution that are within a distance r of the center of the spherical charge distribution count in r_{enc}:

$$q_{enc} = \int_0^r ar'^n 4\pi r'^2 dr' = \frac{4\pi a}{n+3} r^{n+3}.$$

Now, using the general result above for $\vec{E}_{in}$, we find the electric field at a point that is a distance r from the center and lies within the charge distribution as

$$\vec{E}_{in} = \left[\frac{a}{\epsilon_0(n+3)}\right] r^{n+1}\hat{r},$$

where the direction information is included by using the unit radial vector.

Exercise 9.4.1

Check that the electric fields for the sphere reduce to the correct values for a point charge.

Answer available at the end of this section.

Charge Distribution with Cylindrical Symmetry

A charge distribution has **cylindrical symmetry** if the charge density depends only upon the distance r from the axis of a cylinder and must not vary along the axis or with direction about the axis. In other words, if your system varies if you rotate it around the axis, or shift it along the axis, you do not have cylindrical symmetry.

Figure $9.4.7$ shows four situations in which charges are distributed in a cylinder. A uniform charge density ρ_0 in an infinite straight wire has a cylindrical symmetry, and so does an infinitely long cylinder with constant charge density ρ_0. An infinitely long cylinder that has different charge densities along its length, such as a charge density ρ_1 for $z > 0$ and $\rho_2 \neq \rho_1$ for $z < 0$, does not have a usable cylindrical symmetry for this course. Neither does a cylinder in which charge density varies with the direction, such as a charge density ρ_1 for $0 \leq \theta < \pi$ and $\rho_2 \neq \rho_1$ for $\pi \leq \theta < 2\pi$. A system with concentric cylindrical shells, each with uniform charge densities, albeit different in different shells, as in FiFigure $9.4.7d$, does have cylindrical symmetry if they are infinitely long. The infinite length requirement is due to the charge density changing along the axis of a finite cylinder. In real systems, we don't have infinite cylinders; however, if the cylindrical object is considerably longer than the radius from it that we are interested in, then the approximation of an infinite cylinder becomes useful.

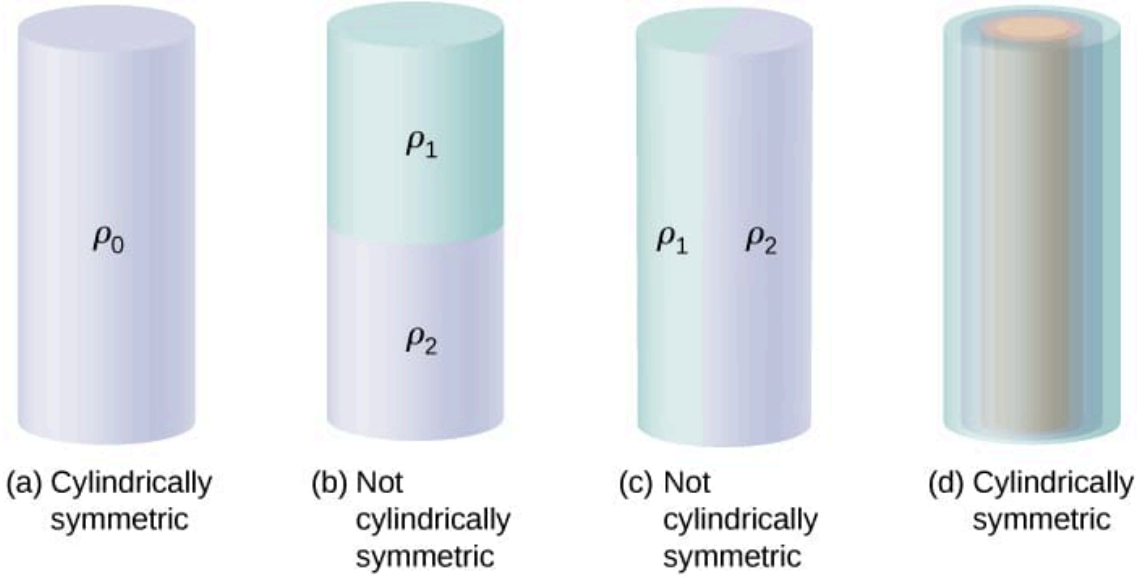


Figure $9.4.7$: To determine whether a given charge distribution has cylindrical symmetry, look at the cross-section of an "infinitely long" cylinder. If the charge density does not depend on the polar angle of the cross-section or along the axis, then you have cylindrical symmetry. (a) Charge density is constant in the cylinder; (b) upper half of the cylinder has a different charge density from the lower half; (c) left half of the cylinder has a different charge density from the right half; (d) charges are constant in different cylindrical rings, but the density does not depend on the polar angle. Cases (a) and (d) have cylindrical symmetry, whereas (b) and (c) do not.

Consequences of symmetry

In all cylindrically symmetrical cases, the electric field E_p at any point P must also display cylindrical symmetry.

Cylindrical symmetry: $\vec{E}_p = E_p(r)\hat{r}$, where r is the distance from the axis and $\hat{r}$ is a unit vector directed perpendicularly away from the axis (Figure $9.4.8$).

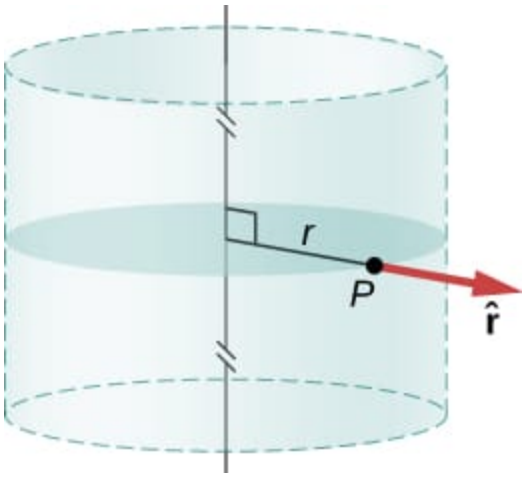


Figure 9.4.8: The electric field in a cylindrically symmetrical situation depends only on the distance from the axis. The direction of the electric field is pointed away from the axis for positive charges and toward the axis for negative charges.

Gaussian surface and flux calculation

To make use of the direction and functional dependence of the electric field, we choose a closed Gaussian surface in the shape of a cylinder with the same axis as the axis of the charge distribution. The flux through this surface of radius s and height L is easy to compute if we divide our task into two parts: (a) a flux through the flat ends and (b) a flux through the curved surface (Figure 9.4.9).

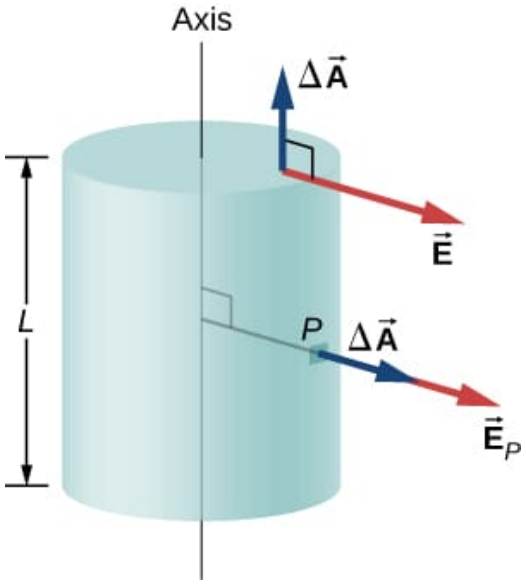


Figure 9.4.9: The Gaussian surface in the case of cylindrical symmetry. The electric field at a patch is either parallel or perpendicular to the normal to the patch of the Gaussian surface.

The electric field is perpendicular to the cylindrical side and parallel to the planar end caps of the surface. The flux through the cylindrical part is

$$\int_S \vec{E} \cdot \hat{n}\, dA = E \int_S dA = E(2\pi rL),$$

whereas the flux through the end caps is zero because $\vec{E} \cdot \hat{n} = 0$ there. Thus, the flux is

$$\int_S \vec{E} \cdot \hat{n}\, dA = E(2\pi rL) + 0 + 0 = 2\pi rLE.$$

Using Gauss's law

According to Gauss's law, the flux must equal the amount of charge within the volume enclosed by this surface, divided by the permittivity of free space. When you do the calculation for a cylinder of length L, you find that q_{enc} of Gauss's law is directly proportional to L. Let us write it as charge per unit length (λ_{enc}) times length L:

$$q_{enc} = \lambda_{enc} L.$$

Hence, Gauss's law for any cylindrically symmetrical charge distribution yields the following magnitude of the electric field a distance s away from the axis:

$$\text{Magnitude: } E(r) = \frac{\lambda_{enc}}{2\pi\epsilon_0}\frac{1}{r}.$$

The charge per unit length λ_{enc} depends on whether the field point is inside or outside the cylinder of charge distribution, just as we have seen for the spherical distribution.

Computing enclosed charge

Let R be the radius of the cylinder within which charges are distributed in a cylindrically symmetrical way. Let the field point P be at a distance s from the axis. (The side of the Gaussian surface includes the field point P.) When $r > R$ (that is, when P is outside the charge distribution), the Gaussian surface includes all the charge in the cylinder of radius R and length L. When $r < R$ (P is located inside the charge distribution), then only the charge within a cylinder of radius s and length L is enclosed by the Gaussian surface:

$\lambda_{enc} = (\text{total charge}) \text{ if } r \geq R$

$\lambda_{enc} = (\text{only charge within } t < R) \text{ if } r < R$

Uniformly Charged Cylindrical Shell

A very long non-conducting cylindrical shell of radius R has a uniform surface charge density σ_0 Find the electric field (a) at a point outside the shell and (b) at a point inside the shell.

Strategy

Apply the Gauss's law strategy given earlier, where we treat the cases inside and outside the shell separately.

Solution

a. **Electric field at a point outside the shell.** For a point outside the cylindrical shell, the Gaussian surface is the surface of a cylinder of radius $r > R$ and length L, as shown in Figure $9.4.10$. The charge enclosed by the Gaussian cylinder is equal to the charge on the cylindrical shell of length L. Therefore, λ_{enc} is given by

$$\lambda_{enc} = \frac{\sigma_0 2\pi RL}{L} = 2\pi R\sigma_0.$$

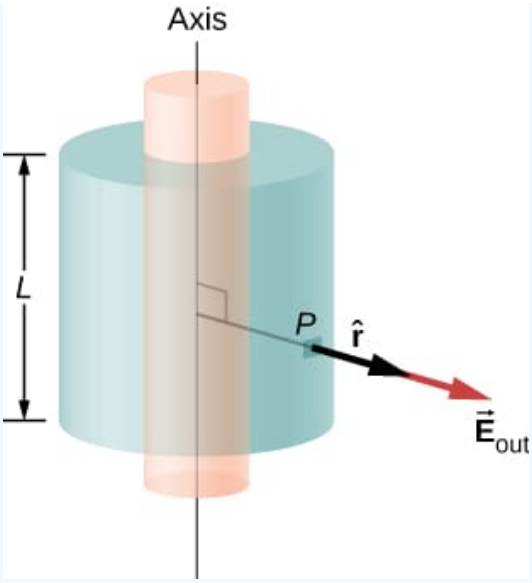


Figure 9.4.10: A Gaussian surface surrounding a cylindrical shell.

Hence, the electric field at a point P outside the shell at a distance r away from the axis is

$$\vec{E} = \frac{2\pi R\sigma_0}{2\pi\epsilon_0}\frac{1}{r}\hat{r} = \frac{R\sigma_0}{\epsilon_0}\frac{1}{r}\hat{r}\,(r > R)$$

where $\hat{r}$ is a unit vector, perpendicular to the axis and pointing away from it, as shown in the figure. The electric field at P points in the direction of $\hat{r}$ given in Figure 9.4.10 if $\sigma_0 > 0$ and in the opposite direction to $\hat{r}$ if $\sigma_0 < 0$.

b. **Electric field at a point inside the shell.** For a point inside the cylindrical shell, the Gaussian surface is a cylinder whose radius r is less than R (Figure 9.4.11). This means no charges are included inside the Gaussian surface:

$$\lambda_{enc} = 0.$$

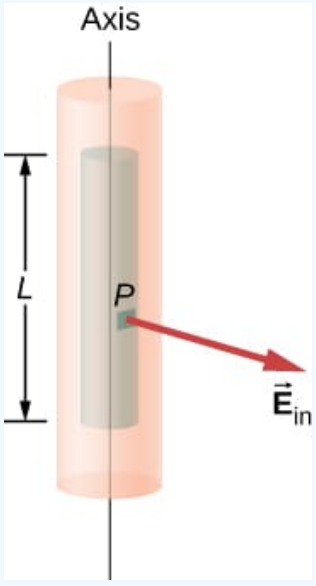


Figure 9.4.11: A Gaussian surface within a cylindrical shell.

This gives the following equation for the magnitude of the electric field E_{in} at a point whose r is less than R of the shell of charges.

\[E_{in} 2\pi rL = 0 (r

This gives us

$$E_{in} = 0 (r < R).$$

Significance

Notice that the result inside the shell is exactly what we should expect: No enclosed charge means zero electric field. Outside the shell, the result becomes identical to a wire with uniform charge $R\sigma$.

Exercise 9.4.2

A thin straight wire has a uniform linear charge density λ_0. Find the electric field at a distance d from the wire, where d is much less than the length of the wire.

Answer available at the end of this section.

Charge Distribution with Planar Symmetry

A planar symmetry of charge density is obtained when charges are uniformly spread over a large flat surface. In planar symmetry, all points in a plane parallel to the plane of charge are identical with respect to the charges.

Consequences of symmetry

We take the plane of the charge distribution to be the xy-plane and we find the electric field at a space point P with coordinates (x, y, z). Since the charge density is the same at all (x, y)-coordinates in the $z = 0$ plane, by symmetry, the electric field at P cannot depend on the x- or y-coordinates of point P, as shown in Figure 9.4.12. Therefore, the electric field at P can only depend on the distance from the plane and has a direction either toward the plane or away from the plane. That is, the electric field at P has only a nonzero z-component.

Uniform charges in xy plane: $\vec{E} = E(z)\hat{z}$ where z is the distance from the plane and $\hat{z}$ is the unit vector normal to the plane. Note that in this system, $E(z) = E(-z)$, although of course they point in opposite directions.

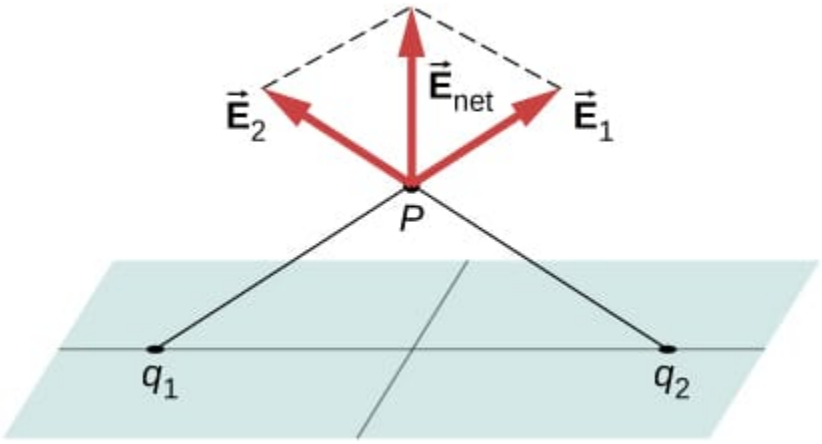


Figure 9.4.12: The components of the electric field parallel to a plane of charges cancel out the two charges located symmetrically from the field point P. Therefore, the field at any point is pointed vertically from the plane of charges. For any point P and charge q_1, we can always find a q_2 with this effect.

Gaussian surface and flux calculation

In the present case, a convenient Gaussian surface is a box, since the expected electric field points in one direction only. To keep the Gaussian box symmetrical about the plane of charges, we take it to straddle the plane of the charges, such that one face containing the field point P is taken parallel to the plane of the charges. In Figure 9.4.13, sides I and II of the Gaussian surface

(the box) that are parallel to the infinite plane have been shaded. They are the only surfaces that give rise to nonzero flux because the electric field and the area vectors of the other faces are perpendicular to each other.

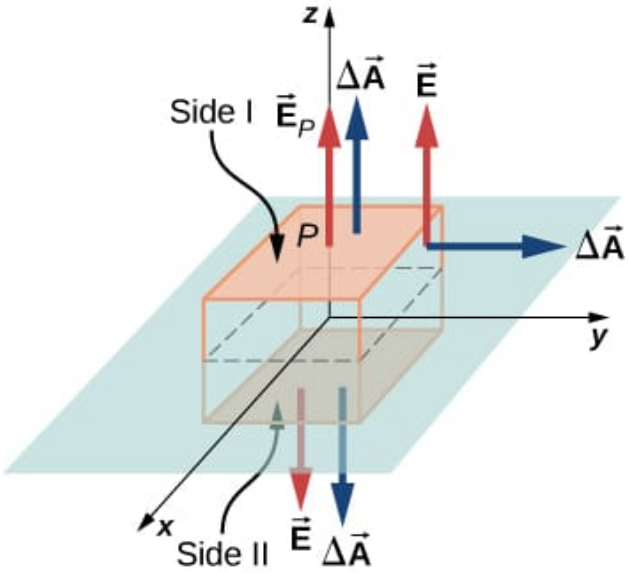


Figure $9.4.13$: A thin charged sheet and the Gaussian box for finding the electric field at the field point P. The normal to each face of the box is from inside the box to outside. On two faces of the box, the electric fields are parallel to the area vectors, and on the other four faces, the electric fields are perpendicular to the area vectors.

Let A be the area of the shaded surface on each side of the plane and E_P be the magnitude of the electric field at point P. Since sides I and II are at the same distance from the plane, the electric field has the same magnitude at points in these planes, although the directions of the electric field at these points in the two planes are opposite to each other.

Magnitude at I or II: $E(z) = E_p$.

If the charge on the plane is positive, then the direction of the electric field and the area vectors are as shown in Figure $9.4.13$. Therefore, we find for the flux of electric field through the box

$$\Phi = \int_S \vec{E}_p \cdot \hat{n} dA = E_p A + E_p A + 0 + 0 + 0 + 0 = 2E_p A$$

where the zeros are for the flux through the other sides of the box. Note that if the charge on the plane is negative, the directions of electric field and area vectors for planes I and II are opposite to each other, and we get a negative sign for the flux. According to Gauss's law, the flux must equal q_{enc}/ϵ_0. From Figure $9.4.13$, we see that the charges inside the volume enclosed by the Gaussian box reside on an area A of the xy-plane. Hence,

$$q_{enc} = \sigma_0 A.$$

Using the equations for the flux and enclosed charge in Gauss's law, we can immediately determine the electric field at a point at height z from a uniformly charged plane in the xy-plane:

$$\vec{E}_p = \frac{\sigma_0}{2\epsilon_0}\hat{n}.$$

The direction of the field depends on the sign of the charge on the plane and the side of the plane where the field point P is located. Note that above the plane, $\hat{n} = +\hat{z}$, while below the plane, $\hat{n} = -\hat{z}$.

You may be surprised to note that the electric field does not actually depend on the distance from the plane; this is an effect of the assumption that the plane is infinite. In practical terms, the result given above is still a useful approximation for finite planes near the center.

Answers

Exercise $9.4.1$

In this case, there is only $\vec{E}_{out}$. So, yes.

Exercise $9.4.2$

$\vec{E} = \frac{\lambda_0}{2\pi\epsilon_0}\frac{1}{d}\hat{r}$; This agrees with the calculation of Calculating Electric Fields of Charge Distributions where we found the electric field by integrating over the charged wire. Notice how much simpler the calculation of this electric field is with Gauss's law.

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9.5: Conductors in Electrostatic Equilibrium

Learning Objectives

By the end of this section, you will be able to:

- Describe the electric field within a conductor at equilibrium
- Describe the electric field immediately outside the surface of a charged conductor at equilibrium
- Explain why if the field is not as described in the first two objectives, the conductor is not at equilibrium

So far, we have generally been working with charges occupying a volume within an insulator. We now study what happens when free charges are placed on a conductor. Generally, in the presence of a (generally external) electric field, the free charge in a conductor redistributes and very quickly reaches electrostatic equilibrium. The resulting charge distribution and its electric field have many interesting properties, which we can investigate with the help of Gauss's law and the concept of electric potential.

The Electric Field inside a Conductor Vanishes

If an electric field is present inside a conductor, it exerts forces on the **free electrons** (also called conduction electrons), which are electrons in the material that are not bound to an atom. These free electrons then accelerate. However, moving charges by definition means nonstatic conditions, contrary to our assumption. Therefore, when electrostatic equilibrium is reached, the charge is distributed in such a way that the electric field inside the conductor vanishes.

If you place a piece of a metal near a positive charge, the free electrons in the metal are attracted to the external positive charge and migrate freely toward that region. The region the electrons move to then has an excess of electrons over the protons in the atoms and the region from where the electrons have migrated has more protons than electrons. Consequently, the metal develops a negative region near the charge and a positive region at the far end (Figure $9.5.1$). As we saw in the preceding chapter, this separation of equal magnitude and opposite type of electric charge is called **polarization**. If you remove the external charge, the electrons migrate back and neutralize the positive region.

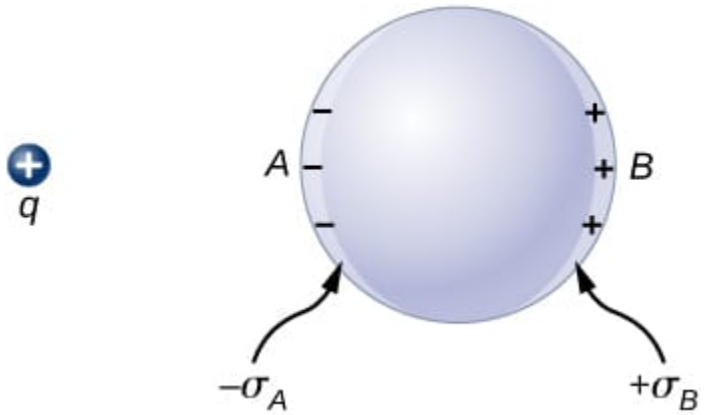


Figure $9.5.1$: Polarization of a metallic sphere by an external point charge $+q$. The near side of the metal has an opposite surface charge compared to the far side of the metal. The sphere is said to be polarized. When you remove the external charge, the polarization of the metal also disappears.

The polarization of the metal happens only in the presence of external charges. You can think of this in terms of electric fields. The external charge creates an external electric field. When the metal is placed in the region of this electric field, the electrons and protons of the metal experience electric forces due to this external electric field, but only the conduction electrons are free to move in the metal over macroscopic distances. The movement of the conduction electrons leads to the polarization, which creates an induced electric field in addition to the external electric field (Figure $9.5.2$). The net electric field is a vector sum of the fields of $+q$ and the surface charge densities $-\sigma_A$ and $+\sigma_B$. This means that the net field inside the conductor is different from the field outside the conductor.

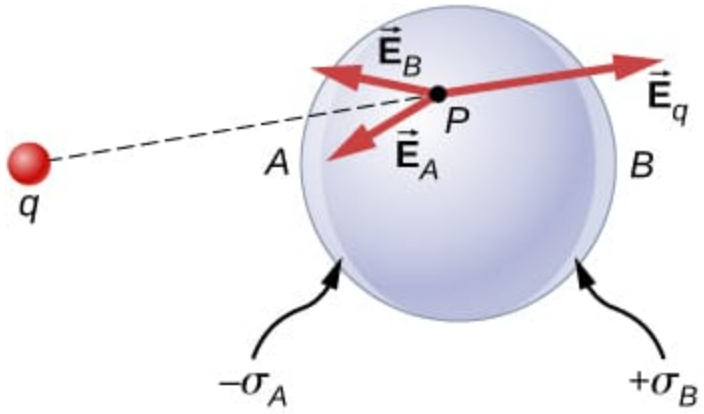


Figure 9.5.2: In the presence of an external charge q, the charges in a metal redistribute. The electric field at any point has three contributions, from $+q$ and the induced charges $-\sigma_A$ and $+\sigma_B$. Note that the surface charge distribution will not be uniform in this case.

The redistribution of charges is such that the sum of the three contributions at any point P inside the conductor is

$$\vec{E}_p = \vec{E}_q + \vec{E}_B + \vec{E}_A = \vec{0}.$$

Now, thanks to Gauss's law, we know that there is no net charge enclosed by a Gaussian surface that is solely within the volume of the conductor at equilibrium. That is, $q_{enc} = 0$ and hence

$$\vec{E}_{net} = \vec{0} \text{ (at points inside a conductor)}.$$

Charge on a Conductor

An interesting property of a conductor in static equilibrium is that extra charges on the conductor end up on the outer surface of the conductor, regardless of where they originate. Figure 9.5.3 illustrates a system in which we bring an external positive charge inside the cavity of a metal and then touch it to the inside surface. Initially, the inside surface of the cavity is negatively charged and the outside surface of the conductor is positively charged. When we touch the inside surface of the cavity, the induced charge is neutralized, leaving the outside surface and the whole metal charged with a net positive charge.

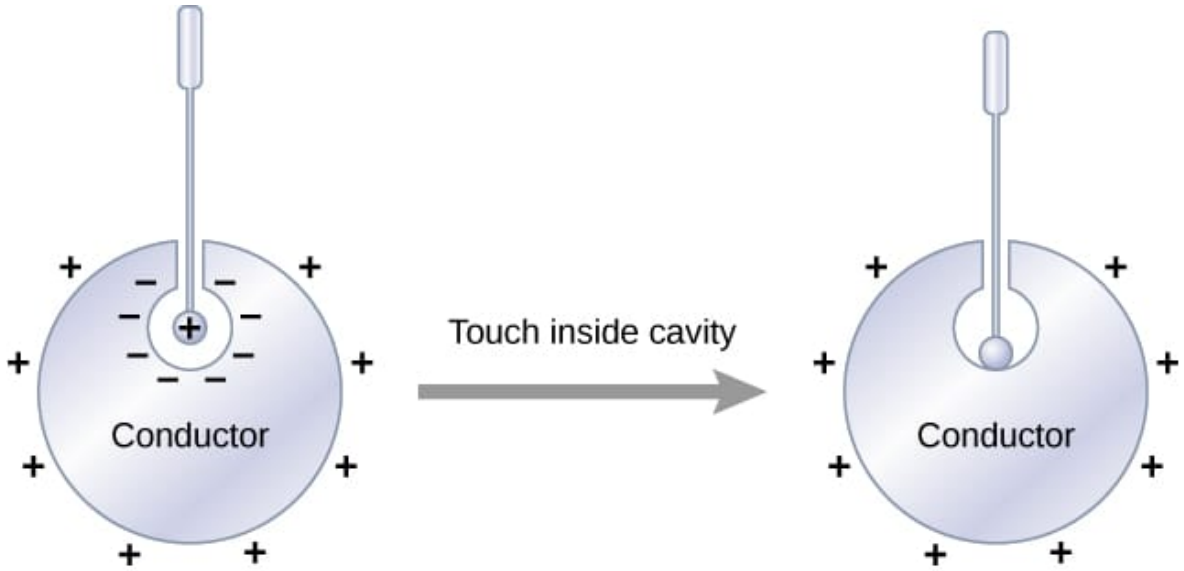


Figure 9.5.3: Electric charges on a conductor migrate to the outside surface no matter where you put them initially.

To see why this happens, note that the Gaussian surface in iFigure 9.5.4 (the dashed line) follows the contour of the actual surface of the conductor and is located an infinitesimal distance *within* it. Since $E = 0$ everywhere inside a conductor,

$$\oint \vec{E} \cdot \hat{n} dA = 0.$$

Thus, from Gauss' law, there is no net charge inside the Gaussian surface. But the Gaussian surface lies just below the actual surface of the conductor; consequently, there is no net charge inside the conductor. Any excess charge must lie on its surface.

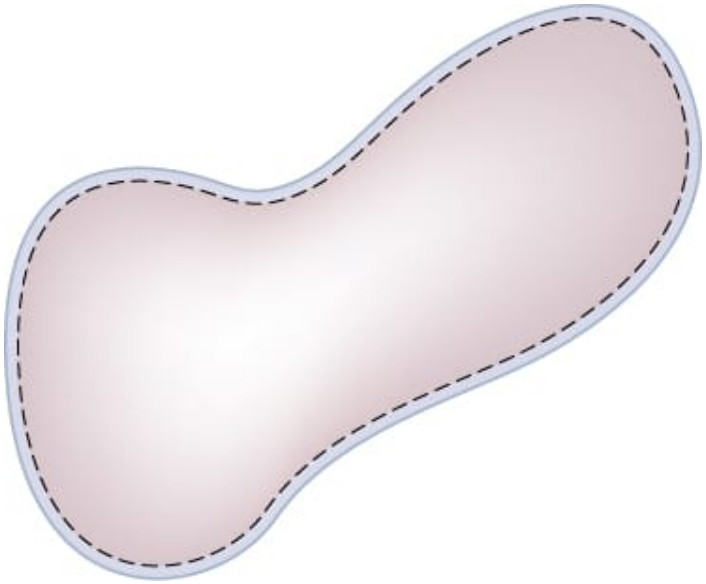


Figure 9.5.4: The dashed line represents a Gaussian surface that is just beneath the actual surface of the conductor.

This particular property of conductors is the basis for an extremely accurate method developed by Plimpton and Lawton in 1936 to verify Gauss's law and, correspondingly, Coulomb's law. A sketch of their apparatus is shown in Figure 9.5.5. Two spherical shells are connected to one another through an electrometer E, a device that can detect a very slight amount of charge flowing from one shell to the other. When switch S is thrown to the left, charge is placed on the outer shell by the battery B. Will charge flow through the electrometer to the inner shell?

No. Doing so would mean a violation of Gauss's law. Plimpton and Lawton did not detect any flow and, knowing the sensitivity of their electrometer, concluded that if the radial dependence in Coulomb's law were $1/r^{2+\delta}$, δ would be less than 2×10^{-9} [1]. More recent measurements place δ at less than 3×10^{-16} [2], a number so small that the validity of Coulomb's law seems indisputable.

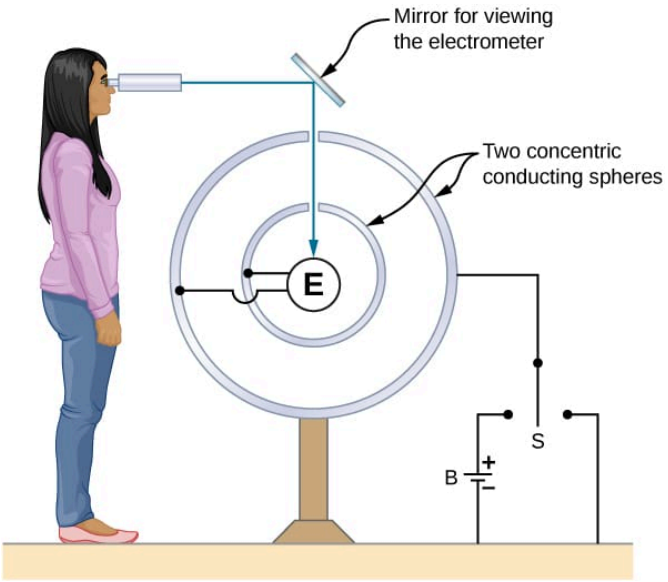


Figure 9.5.5: A representation of the apparatus used by Plimpton and Lawton. Any transfer of charge between the spheres is detected by the electrometer E.

[1]S. Plimpton and W. Lawton. 1936. “A Very Accurate Test of Coulomb’s Law of Force between Charges.” Physical Review 50, No. 11:
1066, doi:10.1103/PhysRev.50.1066

[2]E. Williams, J. Faller, and H. Hill. 1971. “New Experimental Test of Coulomb’s Law: A Laboratory Upper Limit on the Photon Rest Mass.” Physical Review Letters 26, No. 12: 721, doi:10.1103/PhysRevLett.26.721

The Electric Field at the Surface of a Conductor

If the electric field had a component parallel to the surface of a conductor, free charges on the surface would move, a situation contrary to the assumption of electrostatic equilibrium. Therefore, the electric field is always perpendicular to the surface of a conductor.

At any point just above the surface of a conductor, the surface charge density δ and the magnitude of the electric field E are related by

$$E = \frac{\sigma}{\epsilon_0}.$$

To see this, consider an infinitesimally small Gaussian cylinder that surrounds a point on the surface of the conductor, as in Figure $9.5.6$. The cylinder has one end face inside and one end face outside the surface. The height and cross-sectional area of the cylinder are δ and ΔA, respectively. The cylinder’s sides are perpendicular to the surface of the conductor, and its end faces are parallel to the surface. Because the cylinder is infinitesimally small, the charge density σ is essentially constant over the surface enclosed, so the total charge inside the Gaussian cylinder is $\sigma \Delta A$. Now E is perpendicular to the surface of the conductor outside the conductor and vanishes within it, because otherwise, the charges would accelerate, and we would not be in equilibrium. Electric flux therefore crosses only the outer end face of the Gaussian surface and may be written as $E \Delta A$ since the cylinder is assumed to be small enough that E is approximately constant over that area. From Gauss’ law,

$$E \Delta A = \frac{\sigma \Delta A}{\epsilon_0}.$$

Thus

$$E = \frac{\sigma}{\epsilon_0}.$$

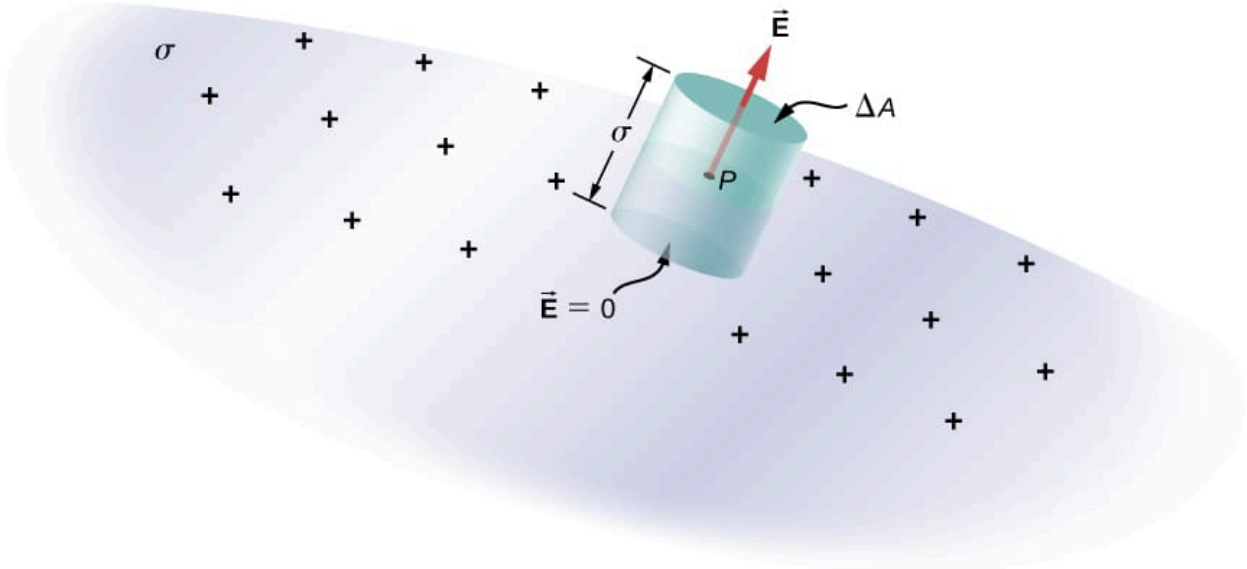


Figure $9.5.6$: An infinitesimally small cylindrical Gaussian surface surrounds point P, which is on the surface of the conductor. The field $\vec{E}$ is perpendicular to the surface of the conductor outside the conductor and vanishes within it.

Electric Field of a Conducting Plate

The infinite conducting plate in Figure $9.5.7$ has a uniform surface charge density σ. Use Gauss' law to find the electric field outside the plate. Compare this result with that previously calculated directly.

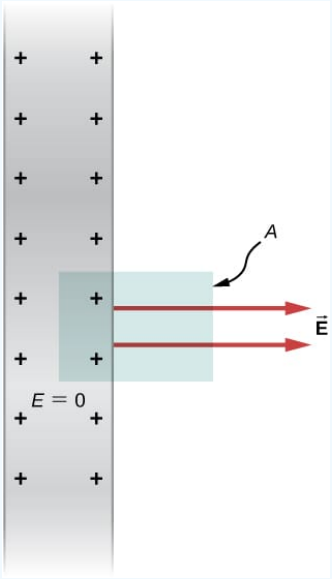


Figure $9.5.7$: A side view of an infinite conducting plate and Gaussian cylinder with cross-sectional area A.

Strategy

For this case, we use a cylindrical Gaussian surface, a side view of which is shown.

Solution

The flux calculation is similar to that for an infinite sheet of charge from the previous chapter with one major exception: The left face of the Gaussian surface is inside the conductor where $\vec{E} = \vec{0}$, so the total flux through the Gaussian surface is EA rather than $2EA$. Then from Gauss' law,

$$EA = \frac{\sigma A}{\epsilon_0}$$

and the electric field outside the plate is

$$E = \frac{\sigma}{\epsilon_0}.$$

Significance

This result is in agreement with the result from the previous section, and consistent with the rule stated above.

Electric Field between Oppositely Charged Parallel Plates

Two large conducting plates carry equal and opposite charges, with a surface charge density σ of magnitude $6.81 \times 10^{-7} \mathrm{C/m^2}$, as shown in Figure $9.5.8$. The separation between the plates is $l = 6.50 \text{ mm}$. What is the electric field between the plates?

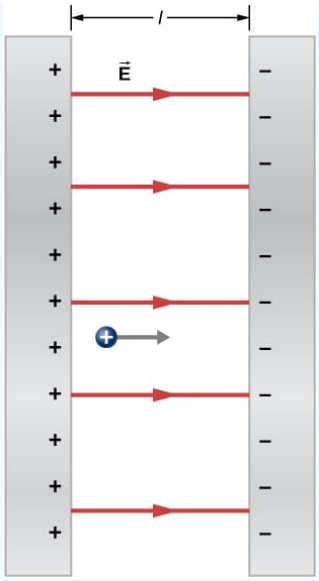


Figure 9.5.8: The electric field between oppositely charged parallel plates. A test charge is released at the positive plate.

Strategy Note that the electric field at the surface of one plate only depends on the charge on that plate. Thus, apply $E = \sigma/\epsilon_0$ with the given values.

Solution The electric field is directed from the positive to the negative plate, as shown in the figure, and its magnitude is given by

$$E = \frac{\sigma}{\epsilon_0} = \frac{6.81 \times 10^{-7}\mathrm{C/m^2}}{8.85 \times 10^{-12}\mathrm{C^2/N \cdot m^2}} = 7.69 \times 10^4\mathrm{N/C}$$

Significance

This formula is applicable to more than just a plate. Furthermore, two-plate systems will be important later.

A Conducting Sphere

The isolated conducting sphere (Figure 9.5.9) has a radius R and an excess charge q. What is the electric field both inside and outside the sphere?

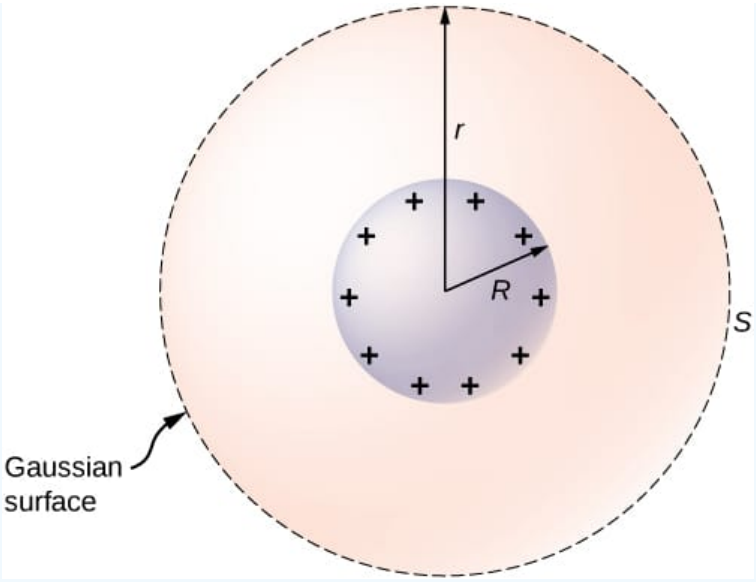


Figure 9.5.9: An isolated conducting sphere.

Strategy The sphere is isolated, so its surface change distribution and the electric field of that distribution are spherically symmetrical. We can therefore represent the field as $\vec{E} = E(r)\hat{r}$. To calculate $E(r)$, we apply Gauss's law over a closed spherical surface S of radius r that is concentric with the conducting sphere.

Solution

Since r is constant and $\hat{n} = \hat{r}$ on the sphere,

$$\oint_S \vec{E} \cdot \hat{n} dA = E(r) \oint_S dA = E(r) 4\pi r^2.$$

For $r < R$, S is within the conductor, so $q_{enc} = 0$, and Gauss's law gives

$$E(r) = 0,$$

as expected inside a conductor. If $r > R$, S encloses the conductor so $q_{enc} = q$. From Gauss's law,

$$E(r) 4\pi r^2 = \frac{q}{\epsilon_0}.$$

The electric field of the sphere may therefore be written as

$$\vec{E} = \vec{0}\,(r < R),$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}\,(r \geq R).$$

Significance

Notice that in the region $r \geq R$, the electric field due to a charge q placed on an isolated conducting sphere of radius R is identical to the electric field of a point charge q located at the center of the sphere. The difference between the charged metal and a point charge occurs only at the space points inside the conductor. For a point charge placed at the center of the sphere, the electric field is not zero at points of space occupied by the sphere, but a conductor with the same amount of charge has a zero electric field at those points (Figure 9.5.10). However, there is no distinction at the outside points in space where $r > R$, and we can replace the isolated charged spherical conductor by a point charge at its center with impunity.

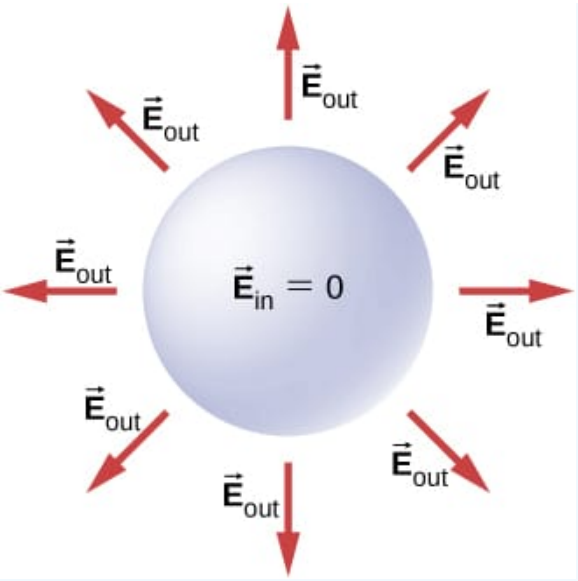


Figure 9.5.10: Electric field of a positively charged metal sphere. The electric field inside is zero, and the electric field outside is same as the electric field of a point charge at the center, although the charge on the metal sphere is at the surface.

Exercise 9.5.1

How will the system above change if there are charged objects external to the sphere?

Answer available at the end of this section.

For a conductor with a cavity, if we put a charge $+q$ inside the cavity, then the charge separation takes place in the conductor, with $-q$ amount of charge on the inside surface and a $+q$ amount of charge at the outside surface (Figure $9.5.11a$). For the same conductor with a charge $+q$ outside it, there is no excess charge on the inside surface; both the positive and negative induced charges reside on the outside surface (Figure $9.5.11b$).

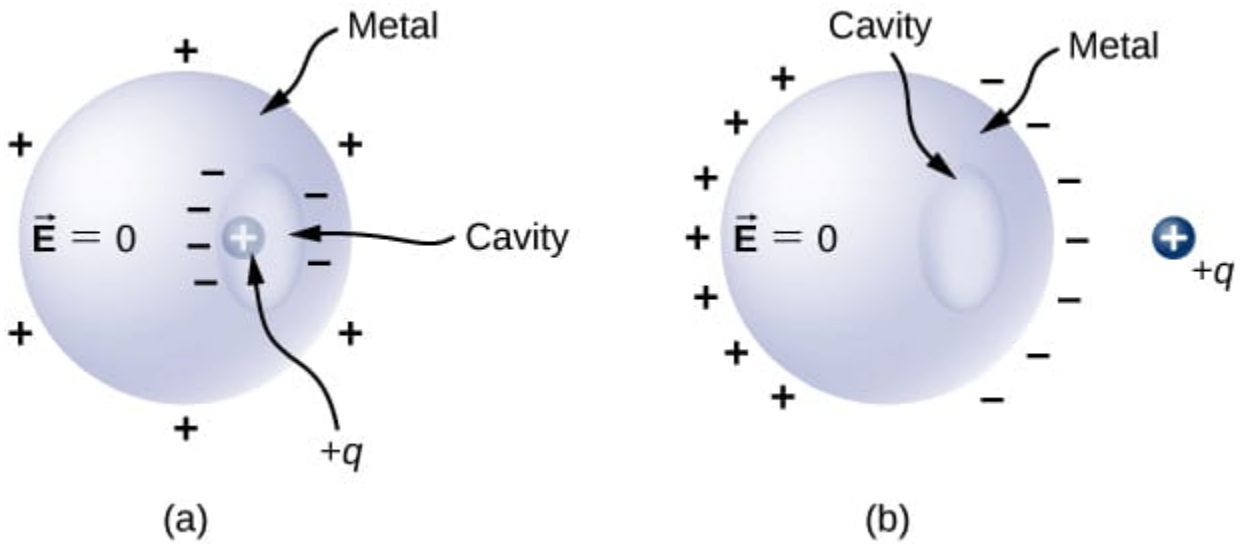


Figure 9.5.11: (a) A charge inside a cavity in a metal. The distribution of charges at the outer surface does not depend on how the charges are distributed at the inner surface, since the E-field inside the body of the metal is zero. That magnitude of the charge on the outer surface does depend on the magnitude of the charge inside, however. (b) A charge outside a conductor containing an inner cavity. The cavity remains free of charge. The polarization of charges on the conductor happens at the surface.

If a conductor has two cavities, one of them having a charge $+q_a$ inside it and the other a charge $-q_b$the polarization of the conductor results in $-q_a$on the inside surface of the cavity a, $+q_b$ on the inside surface of the cavity b, and $q_a - q_b$ on the outside

surface (Figure 9.5.12). The charges on the surfaces may not be uniformly spread out; their spread depends upon the geometry. The only rule obeyed is that when the equilibrium has been reached, the charge distribution in a conductor is such that the electric field by the charge distribution in the conductor cancels the electric field of the external charges at all space points inside the body of the conductor.

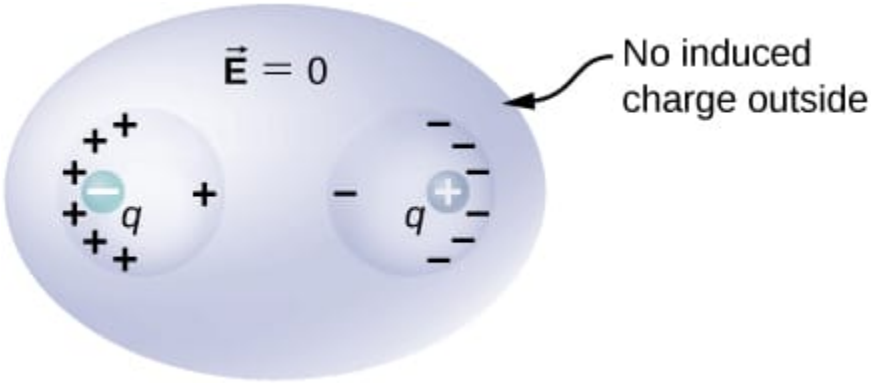


Figure 9.5.12: The charges induced by two equal and opposite charges in two separate cavities of a conductor. If the net charge on the cavity is nonzero, the external surface becomes charged to the amount of the net charge.

Answers

Exercise 9.5.1

If there are other charged objects around, then the charges on the surface of the sphere will not necessarily be spherically symmetrical; there will be more in certain direction than in other directions.

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9.6: Gauss's Law (Summary)

Key Terms

area vector	vector with magnitude equal to the area of a surface and direction perpendicular to the surface
cylindrical symmetry	system only varies with distance from the axis, not direction
electric flux	dot product of the electric field and the area through which it is passing
flux	quantity of something passing through a given area
free electrons	also called conduction electrons, these are the electrons in a conductor that are not bound to any particular atom, and hence are free to move around
Gaussian surface	any enclosed (usually imaginary) surface
planar symmetry	system only varies with distance from a plane
spherical symmetry	system only varies with the distance from the origin, not in direction

Key Equations

Definition of electric flux, for uniform electric field	$\Phi = \vec{E} \cdot \vec{A} \rightarrow EA\cos\theta$
Electric flux through an open surface	$\Phi = \int_S \vec{E} \cdot \hat{n} dA = \int_S \vec{E} \cdot d\vec{A}$
Electric flux through a closed surface	$\Phi = \oint_S \vec{E} \cdot \hat{n} dA = \oint_S \vec{E} \cdot d\vec{A}$
Gauss's law	$\Phi = \oint_S \vec{E} \cdot \hat{n} dA = \frac{q_{enc}}{\varepsilon_0}$
Gauss's Law for systems with symmetry	$\Phi = \oint_S \vec{E} \cdot \hat{n} dA = E \oint_S dA = EA = \frac{q_{enc}}{\varepsilon_0}$
The magnitude of the electric field just outside the surface of a conductor	$E = \frac{\sigma}{\varepsilon_0}$

Summary

6.2 Electric Flux

- The electric flux through a surface is proportional to the number of field lines crossing that surface. Note that this means the magnitude is proportional to the portion of the field perpendicular to the area.
- The electric flux is obtained by evaluating the surface integral

  $$\Phi = \oint_S \vec{E} \cdot \hat{n} dA = \oint_S \vec{E} \cdot d\vec{A},$$

  where the notation used here is for a closed surface S.

6.3 Explaining Gauss's Law

- Gauss's law relates the electric flux through a closed surface to the net charge within that surface,

  $$\Phi = \oint_S \vec{E} \cdot \hat{n} dA = \frac{q_{enc}}{\varepsilon_0},$$

  where q_{enc} is the total charge inside the Gaussian surface S.
- All surfaces that include the same amount of charge have the same number of field lines crossing it, regardless of the shape or size of the surface, as long as the surfaces enclose the same amount of charge.

6.4 Applying Gauss's Law

- For a charge distribution with certain spatial symmetries (spherical, cylindrical, and planar), we can find a Gaussian surface over which $\vec{E} \cdot \hat{n} = E$, where E is constant over the surface. The electric field is then determined with Gauss's law.
- For spherical symmetry, the Gaussian surface is also a sphere, and Gauss's law simplifies to $4\pi r^2 E = \frac{q_{enc}}{\varepsilon_0}$.
- For cylindrical symmetry, we use a cylindrical Gaussian surface, and find that Gauss's law simplifies to $2\pi r L E = \frac{q_{enc}}{\varepsilon_0}$.
- For planar symmetry, a convenient Gaussian surface is a box penetrating the plane, with two faces parallel to the plane and the remainder perpendicular, resulting in Gauss's law being $2AE = \frac{q_{enc}}{\varepsilon_0}$.

6.5 Conductors in Electrostatic Equilibrium

- The electric field inside a conductor vanishes.
- Any excess charge placed on a conductor resides entirely on the surface of the conductor.
- The electric field is perpendicular to the surface of a conductor everywhere on that surface.
- The magnitude of the electric field just above the surface of a conductor is given by $E = \frac{\sigma}{\varepsilon_0}$.

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9.7: Gauss's Law (Exercises)

Conceptual Questions

6.2 Electric Flux

1. Discuss how would orient a planar surface of area **A** in a uniform electric field of magnitude E_0 to obtain

(a) the maximum flux and

(b) the minimum flux through the area.

2. What are the maximum and minimum values of the flux in the preceding question?

3. The net electric flux crossing a closed surface is always zero. True or false?

4. The net electric flux crossing an open surface is never zero. True or false?

6.3 Explaining Gauss's Law

5. Two concentric spherical surfaces enclose a point charge **q**. The radius of the outer sphere is twice that of the inner one. Compare the electric fluxes crossing the two surfaces.

6. Compare the electric flux through the surface of a cube of side length a that has a charge **q** at its center to the flux through a spherical surface of radius a with a charge q at its center.

7. (a) If the electric flux through a closed surface is zero, is the electric field necessarily zero at all points on the surface?

(b) What is the net charge inside the surface?

8. Discuss how Gauss's law would be affected if the electric field of a point charge did not vary as $1/r^2$.

9. Discuss the similarities and differences between the gravitational field of a point mass **m** and the electric field of a point charge **q**.

10. Discuss whether Gauss's law can be applied to other forces, and if so, which ones.

11. Is the term $\vec{E}$ in Gauss's law the electric field produced by just the charge inside the Gaussian surface?

12. Reformulate Gauss's law by choosing the unit normal of the Gaussian surface to be the one directed inward.

6.4 Applying Gauss's Law

13. Would Gauss's law be helpful for determining the electric field of two equal but opposite charges a fixed distance apart?

14. Discuss the role that symmetry plays in the application of Gauss's law. Give examples of continuous charge distributions in which Gauss's law is useful and not useful in determining the electric field.

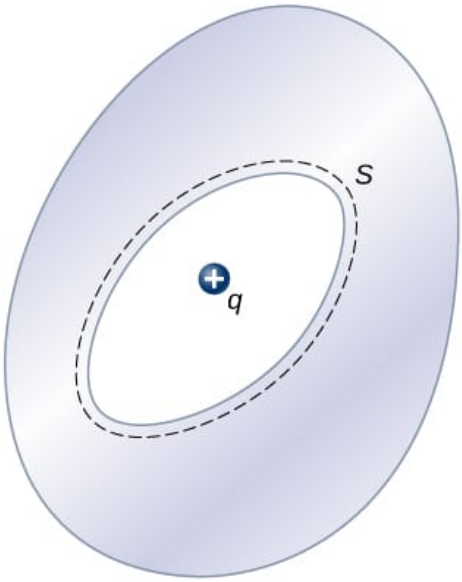
15. Discuss the restrictions on the Gaussian surface used to discuss planar symmetry. For example, is its length important? Does the cross-section have to be square? Must the end faces be on opposite sides of the sheet?

6.5 Conductors in Electrostatic Equilibrium

16. Is the electric field inside a metal always zero?

17. Under electrostatic conditions, the excess charge on a conductor resides on its surface. Does this mean that all the conduction electrons in a conductor are on the surface?

18. A charge **q** is placed in the cavity of a conductor as shown below. Will a charge outside the conductor experience an electric field due to the presence of **q**?



19. The conductor in the preceding figure has an excess charge of $-5.0\mu C$. If a $2.0-\mu C$ point charge is placed in the cavity, what is the net charge on the surface of the cavity and on the outer surface of the conductor?

Problems

6.2 Electric Flux

20. A uniform electric field of magnitude $1.1\times10^4 N/C$ is perpendicular to a square sheet with sides 2.0 m long. What is the electric flux through the sheet?

21. Calculate the flux through the sheet of the previous problem if the plane of the sheet is at an angle of **60°** to the field. Find the flux for both directions of the unit normal to the sheet.

22. Find the electric flux through a rectangular area **3cm×2cm** between two parallel plates where there is a constant electric field of 30 N/C for the following orientations of the area: (a) parallel to the plates, (b) perpendicular to the plates, and (c) the normal to the area making a **30°** angle with the direction of the electric field. Note that this angle can also be given as **180°+30°**.

23. The electric flux through a square-shaped area of side 5 cm near a large charged sheet is found to be $3\times10^{-5}N\cdot m^2/C$. when the area is parallel to the plate. Find the charge density on the sheet.

24. Two large rectangular aluminum plates of area $150cm^2$ face each other with a separation of 3 mm between them. The plates are charged with equal amount of opposite charges, **±20μC**. The charges on the plates face each other. Find the flux through a circle of radius 3 cm between the plates when the normal to the circle makes an angle of **5°** with a line perpendicular to the plates. Note that this angle can also be given as **180°+5°**.

25. A square surface of area $2cm^2$ is in a space of uniform electric field of magnitude $10^3N/C$. The amount of flux through it depends on how the square is oriented relative to the direction of the electric field. Find the electric flux through the square, when the normal to it makes the following angles with electric field: (a) **30°**, (b) **90°**, and (c) **0°**. Note that these angles can also be given as **180°+θ**.

26. A vector field is pointed along the **z**-axis, \(\displaystyle \vec{v} =\frac{α}{x^2+y^2}\hat{z}.

(a) Find the flux of the vector field through a rectangle in the xy-plane between a

(b) Do the same through a rectangle in the yz-plane between a

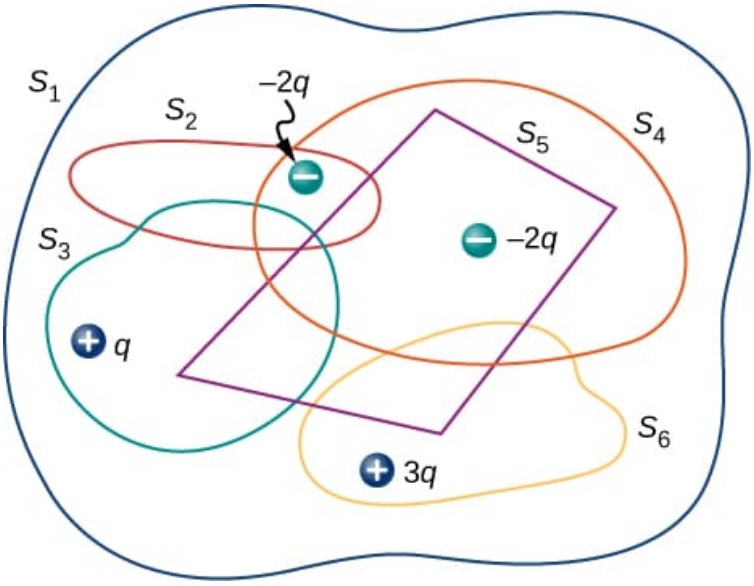
27. Consider the uniform electric field $\vec{E} = (4.0\hat{j} + 3.0\hat{k}) \times 10^3 N/C$. What is its electric flux through a circular area of radius 2.0 m that lies in the **xy**-plane?

28. Repeat the previous problem, given that the circular area is (a) in the **yz**-plane and (b) **45°** above the **xy**-plane.

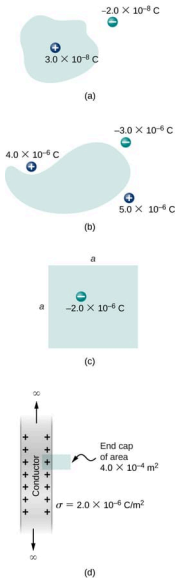
29. An infinite charged wire with charge per unit length λ lies along the central axis of a cylindrical surface of radius **r** and length **l**. What is the flux through the surface due to the electric field of the charged wire?

6.3 Explaining Gauss's Law

30. Determine the electric flux through each closed surface whose cross-section inside the surface is shown below.



31. Find the electric flux through the closed surface whose cross-sections are shown below.



32. A point charge **q** is located at the center of a cube whose sides are of length **a**. If there are no other charges in this system, what is the electric flux through one face of the cube?

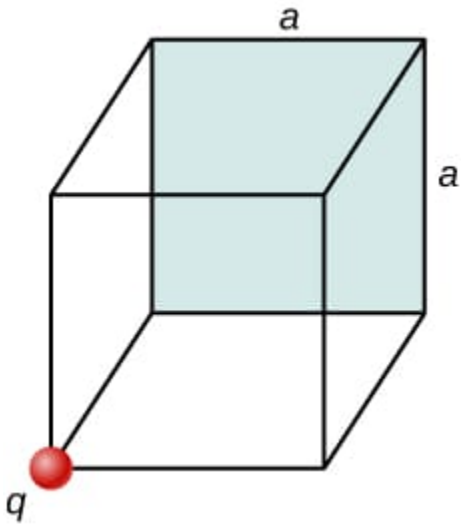
33. A point charge of $10\mu C$ is at an unspecified location inside a cube of side 2 cm. Find the net electric flux though the surfaces of the cube.

34. A net flux of $1.0 \times 10^4 N \cdot^m 2/C$ passes inward through the surface of a sphere of radius 5 cm.

(a) How much charge is inside the sphere?

(b) How precisely can we determine the location of the charge from this information?

35. A charge **q** is placed at one of the corners of a cube of side **a**, as shown below. Find the magnitude of the electric flux through the shaded face due to **q**. Assume $q > 0$.



36. The electric flux through a cubical box 8.0 cm on a side is $1.2 \times 10^3 N \cdot m^2/C$. What is the total charge enclosed by the box?

37. The electric flux through a spherical surface is $4.0 \times 10^4 N \cdot m^2/C$. What is the net charge enclosed by the surface?

38. A cube whose sides are of length **d** is placed in a uniform electric field of magnitude $E = 4.0 \times 10^3 N/C$ so that the field is perpendicular to two opposite faces of the cube. What is the net flux through the cube?

39. Repeat the previous problem, assuming that the electric field is directed along a body diagonal of the cube.

40. A total charge $5.0 \times 10^{-6} C$ is distributed uniformly throughout a cubical volume whose edges are 8.0 cm long.

(a) What is the charge density in the cube?

(b) What is the electric flux through a cube with 12.0-cm edges that is concentric with the charge distribution?

(c) Do the same calculation for cubes whose edges are 10.0 cm long and 5.0 cm long.

(d) What is the electric flux through a spherical surface of radius 3.0 cm that is also concentric with the charge distribution?

6.4 Applying Gauss's Law

41. Recall that in the example of a uniform charged sphere, $\rho_0 = Q/(\frac{4}{3}\pi R^3)$. Rewrite the answers in terms of the total charge **Q** on the sphere.

42. Suppose that the charge density of the spherical charge distribution shown in Figure 6.23 is $\rho(r) = \rho_0 r/R$ for $r \leq R$ and zero for $r > R$. Obtain expressions for the electric field both inside and outside the distribution.

43. A very long, thin wire has a uniform linear charge density of $50\mu C/m$. What is the electric field at a distance 2.0 cm from the wire?

44. A charge of $-30\mu C$ is distributed uniformly throughout a spherical volume of radius 10.0 cm. Determine the electric field due to this charge at a distance of

(a) 2.0 cm,

(b) 5.0 cm, and

(c) 20.0 cm from the center of the sphere.

45. Repeat your calculations for the preceding problem, given that the charge is distributed uniformly over the surface of a spherical conductor of radius 10.0 cm.

46. A total charge **Q** is distributed uniformly throughout a spherical shell of inner and outer radii r_1 and r_2, respectively. Show that the electric field due to the charge is

$$\vec{E} = \vec{0}\ (r \leq r_1);$$

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\left(\frac{r^3 - r_1^3}{r_2^3 - r_1^3}\right)\hat{r}\ (r_1 \leq r \leq r_2);$$

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}\ (r \geq r_2).$$

47. When a charge is placed on a metal sphere, it ends up in equilibrium at the outer surface. Use this information to determine the electric field of $+3.0\mu C$ charge put on a 5.0-cm aluminum spherical ball at the following two points in space:

(a) a point 1.0 cm from the center of the ball (an inside point) and

(b) a point 10 cm from the center of the ball (an outside point).

48. A large sheet of charge has a uniform charge density of $10\mu C/m^2$. What is the electric field due to this charge at a point just above the surface of the sheet?

49. Determine if approximate cylindrical symmetry holds for the following situations. State why or why not.

(a) A 300-cm long copper rod of radius 1 cm is charged with +500 nC of charge and we seek electric field at a point 5 cm from the center of the rod.

(b) A 10-cm long copper rod of radius 1 cm is charged with +500 nC of charge and we seek electric field at a point 5 cm from the center of the rod.

(c) A 150-cm wooden rod is glued to a 150-cm plastic rod to make a 300-cm long rod, which is then painted with a charged paint so that one obtains a uniform charge density. The radius of each rod is 1 cm, and we seek an electric field at a point that is 4 cm from the center of the rod.

(d) Same rod as (c), but we seek electric field at a point that is 500 cm from the center of the rod.

50. A long silver rod of radius 3 cm has a charge of $-5\mu C/cm$ on its surface.

(a) Find the electric field at a point 5 cm from the center of the rod (an outside point).

(b) Find the electric field at a point 2 cm from the center of the rod (an inside point).

51. The electric field at 2 cm from the center of long copper rod of radius 1 cm has a magnitude 3 N/C and directed outward from the axis of the rod.

(a) How much charge per unit length exists on the copper rod?

(b) What would be the electric flux through a cube of side 5 cm situated such that the rod passes through opposite sides of the cube perpendicularly?

52. A long copper cylindrical shell of inner radius 2 cm and outer radius 3 cm surrounds concentrically a charged long aluminum rod of radius 1 cm with a charge density of 4 pC/m. All charges on the aluminum rod reside at its surface. The inner surface of the copper shell has exactly opposite charge to that of the aluminum rod while the outer surface of the copper shell has the same charge as the aluminum rod. Find the magnitude and direction of the electric field at points that are at the following distances from the center of the aluminum rod:

(a) 0.5 cm, (b) 1.5 cm, (c) 2.5 cm, (d) 3.5 cm, and (e) 7 cm.

53. Charge is distributed uniformly with a density ρ throughout an infinitely long cylindrical volume of radius **R**. Show that the field of this charge distribution is directed radially with respect to the cylinder and that

$$E = \frac{\rho r}{2\varepsilon_0} \ (r \leq R);$$

$$E = \frac{\rho R^2}{2\varepsilon_0 r} \ (r \geq R)$$

54. Charge is distributed throughout a very long cylindrical volume of radius **R** such that the charge density increases with the distance **r** from the central axis of the cylinder according to $\rho = \alpha r$, where α is a constant. Show that the field of this charge distribution is directed radially with respect to the cylinder and that

$$E = \frac{\alpha r^2}{3\varepsilon_0} \ (r \leq R);$$

$$E = \frac{\alpha R^3}{3\varepsilon_0 r} \ (r \geq R).$$

55. The electric field 10.0 cm from the surface of a copper ball of radius 5.0 cm is directed toward the ball's center and has magnitude $4.0 \times 10^2 N/C$. How much charge is on the surface of the ball?

56. Charge is distributed throughout a spherical shell of inner radius r_1 and outer radius r_2 with a volume density given by $\rho = \rho_0 r_1/r$, where ρ_0 is a constant. Determine the electric field due to this charge as a function of **r**, the distance from the center of the shell.

57. Charge is distributed throughout a spherical volume of radius **R** with a density $\rho = \alpha r^2$, where αα is a constant. Determine the electric field due to the charge at points both inside and outside the sphere.

58. Consider a uranium nucleus to be sphere of radius $R = 7.4 \times 10^{-15} m$ with a charge of **92e** distributed uniformly throughout its volume. (a) What is the electric force exerted on an electron when it is $3.0 \times 10^{-15} m$ from the center of the nucleus? (b) What is the acceleration of the electron at this point?

59. The volume charge density of a spherical charge distribution is given by $\rho(r) = \rho_0 e^{-\alpha r}$, where ρ_0 and α are constants. What is the electric field produced by this charge distribution?

6.5 Conductors in Electrostatic Equilibrium

60. An uncharged conductor with an internal cavity is shown in the following figure. Use the closed surface S along with Gauss' law to show that when a charge **q** is placed in the cavity a total charge **–q** is induced on the inner surface of the conductor. What is the charge on the outer surface of the conductor?

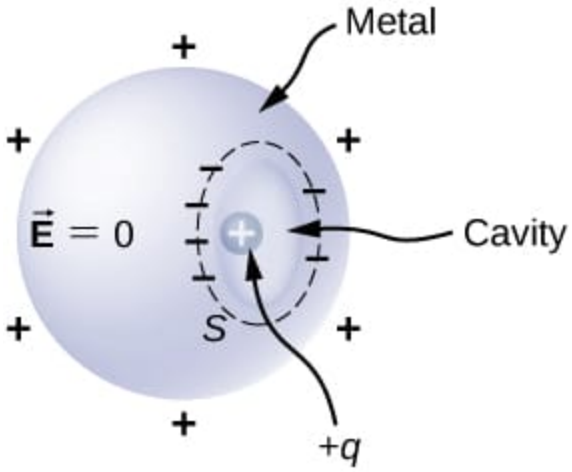
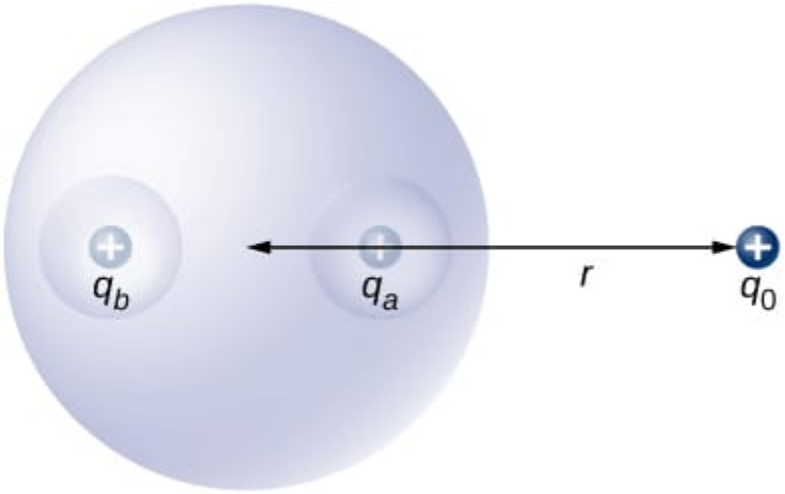


Figure 6.46: A charge inside a cavity of a metal. Charges at the outer surface do not depend on how the charges are distributed at the inner surface since E field inside the body of the metal is zero.

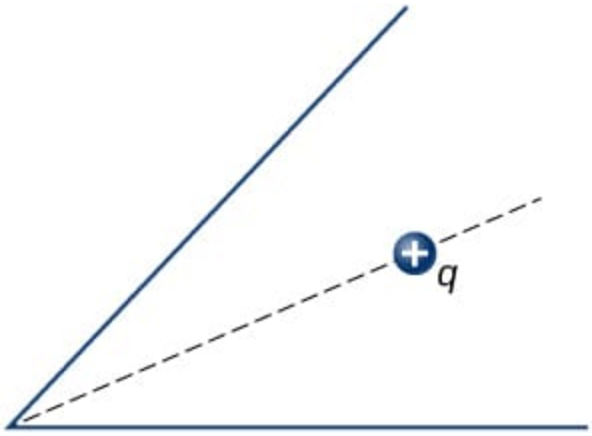
61. An uncharged spherical conductor **S** of radius **R** has two spherical cavities **A** and **B** of radii **a** and **b**, respectively as shown below. Two point charges $+q_a$ and $+q_b$ are placed at the center of the two cavities by using non-conducting supports. In addition, a point charge $+q_0$ is placed outside at a distance r from the center of the sphere.

(a) Draw approximate charge distributions in the metal although metal sphere has no net charge.

(b) Draw electric field lines. Draw enough lines to represent all distinctly different places.



62. A positive point charge is placed at the angle bisector of two uncharged plane conductors that make an angle of **45°**.See below. Draw the electric field lines.



63. A long cylinder of copper of radius 3 cm is charged so that it has a uniform charge per unit length on its surface of 3 C/m. (a) Find the electric field inside and outside the cylinder. (b) Draw electric field lines in a plane perpendicular to the rod.

64. An aluminum spherical ball of radius 4 cm is charged with **5μC** of charge. A copper spherical shell of inner radius 6 cm and outer radius 8 cm surrounds it. A total charge of **−8μC** is put on the copper shell.

(a) Find the electric field at all points in space, including points inside the aluminum and copper shell when copper shell and aluminum sphere are concentric.

(b) Find the electric field at all points in space, including points inside the aluminum and copper shell when the centers of copper shell and aluminum sphere are 1 cm apart.

65. A long cylinder of aluminum of radius **R** meters is charged so that it has a uniform charge per unit length on its surface of λ. (a) Find the electric field inside and outside the cylinder. (b) Plot electric field as a function of distance from the center of the rod.

66. At the surface of any conductor in electrostatic equilibrium, $E = \sigma/\varepsilon_0$. Show that this equation is consistent with the fact that $E = kq/r^2$ at the surface of a spherical conductor.

67. Two parallel plates 10 cm on a side are given equal and opposite charges of magnitude $5.0 \times 10^{-9} C$. The plates are 1.5 mm apart. What is the electric field at the center of the region between the plates?

68. Two parallel conducting plates, each of cross-sectional area $400 cm^2$, are 2.0 cm apart and uncharged. If 1.0×10^{12} electrons are transferred from one plate to the other, what are (a) the charge density on each plate? (b) The electric field between the plates?

69. The surface charge density on a long straight metallic pipe is σ. What is the electric field outside and inside the pipe? Assume the pipe has a diameter of **2a**.



70. A point charge $q = -5.0 \times 10^{-12} C$ is placed at the center of a spherical conducting shell of inner radius 3.5 cm and outer radius 4.0 cm. The electric field just above the surface of the conductor is directed radially outward and has magnitude 8.0 N/C.

(a) What is the charge density on the inner surface of the shell?

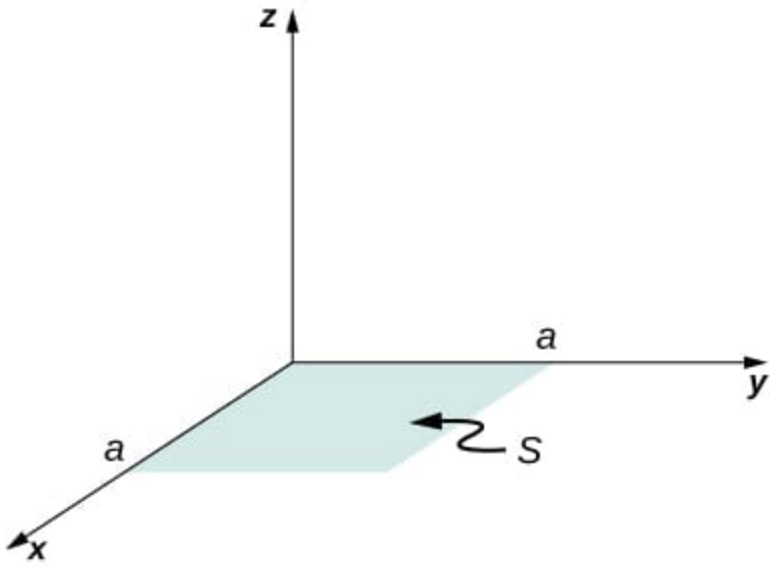
(b) What is the charge density on the outer surface of the shell?

(c) What is the net charge on the conductor?

71. A solid cylindrical conductor of radius a is surrounded by a concentric cylindrical shell of inner radius **b**. The solid cylinder and the shell carry charges **+Q** and **–Q**, respectively. Assuming that the length **L** of both conductors is much greater than a or b, determine the electric field as a function of r, the distance from the common central axis of the cylinders, for (a) \(\displaystyle rb\).

Additional Problems

72. A vector field $\vec{E}$ (not necessarily an electric field; note units) is given by $\vec{E} = 3x^2\hat{k}$. Calculate $\int_S \vec{E} \cdot \hat{n} da$, where **S** is the area shown below. Assume that $\hat{n} = \hat{k}$.

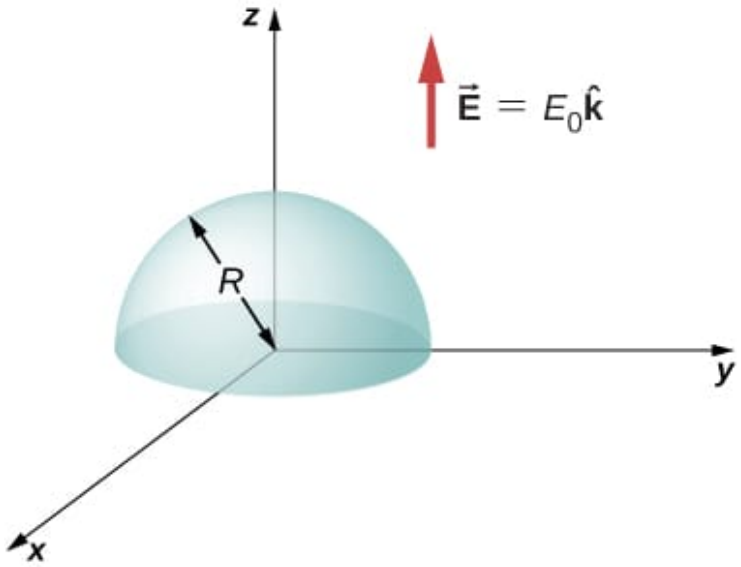


73. Repeat the preceding problem, with $\vec{E} = 2x\hat{i} + 3x^2\hat{k}$.

74. A circular area **S** is concentric with the origin, has radius a, and lies in the **yz**-plane. Calculate $\int_S \vec{E} \cdot \hat{n} dA$ for $\vec{E} = 3z^2\hat{i}$.

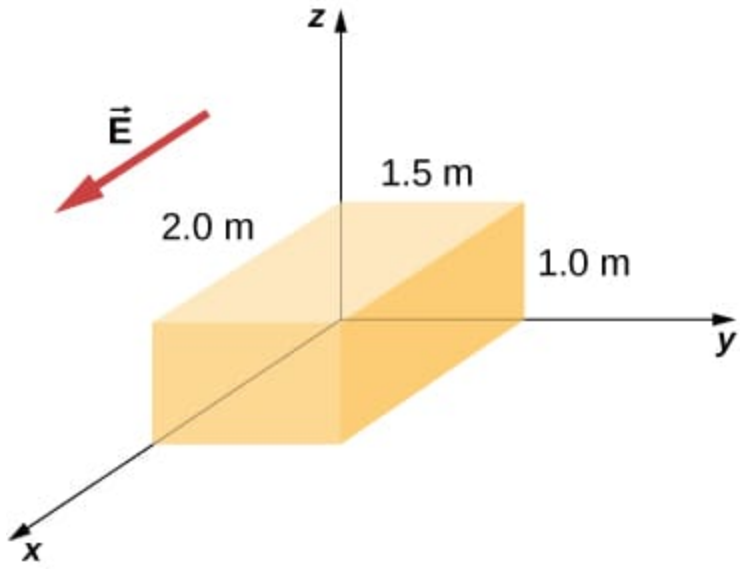
75. (a) Calculate the electric flux through the open hemispherical surface due to the electric field $\vec{E} = E_0\hat{k}$ (see below).

(b) If the hemisphere is rotated by **90°** around the **x**-axis, what is the flux through it?

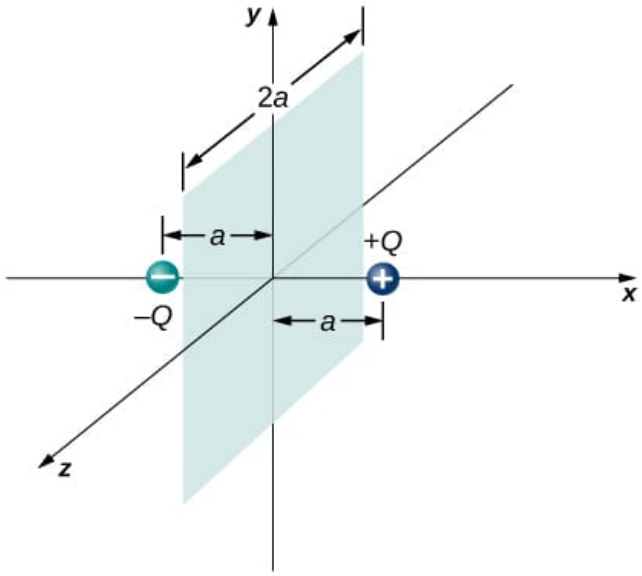


76. Suppose that the electric field of an isolated point charge were proportional to $1/r^{2+\sigma}$ rather than $1/r^2$. Determine the flux that passes through the surface of a sphere of radius **R** centered at the charge. Would Gauss's law remain valid?

77. The electric field in a region is given by $\vec{E} = a/(b + cx)\hat{i}$, where $a = 200N \cdot m/C, b = 2.0m,$ and $c = 2.0$.What is the net charge enclosed by the shaded volume shown below?



78. Two equal and opposite charges of magnitude **Q** are located on the **x-**axis at the points **+a** and **–a**, as shown below. What is the net flux due to these charges through a square surface of side **2a** that lies in the **yz**-plane and is centered at the origin? (**Hint:** Determine the flux due to each charge separately, then use the principle of superposition. You may be able to make a symmetry argument.)



79. A fellow student calculated the flux through the square for the system in the preceding problem and got 0. What went wrong?

80. A **10cm×10cm** piece of aluminum foil of 0.1 mm thickness has a charge of $20\mu C$ that spreads on both wide side surfaces evenly. You may ignore the charges on the thin sides of the edges.

(a) Find the charge density.

(b) Find the electric field 1 cm from the center, assuming approximate planar symmetry.

81. Two **10cm×10cm** pieces of aluminum foil of thickness 0.1 mm face each other with a separation of 5 mm. One of the foils has a charge of $+30\mu C$ and the other has $-30\mu C$.

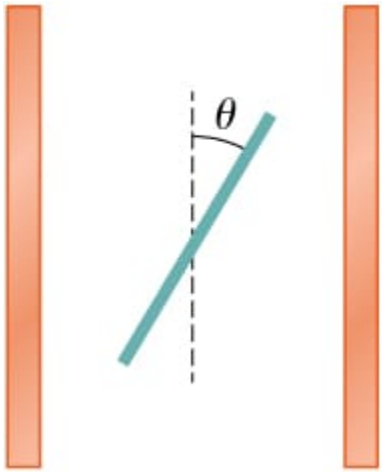
(a) Find the charge density at all surfaces, i.e., on those facing each other and those facing away.

(b) Find the electric field between the plates near the center assuming planar symmetry.

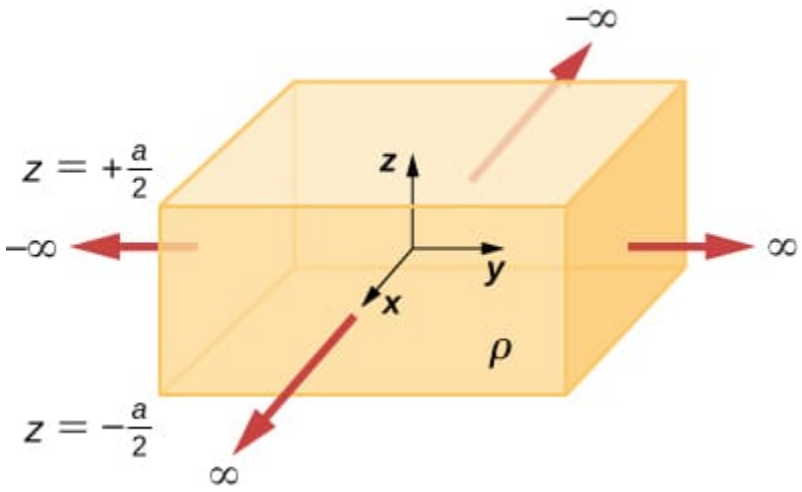
82. Two large copper plates facing each other have charge densities $\pm 4.0 C/m^2$ on the surface facing the other plate, and zero in between the plates. Find the electric flux through a **3cm×4cm** rectangular area between the plates, as shown below, for the following orientations of the area.

(a) If the area is parallel to the plates, and

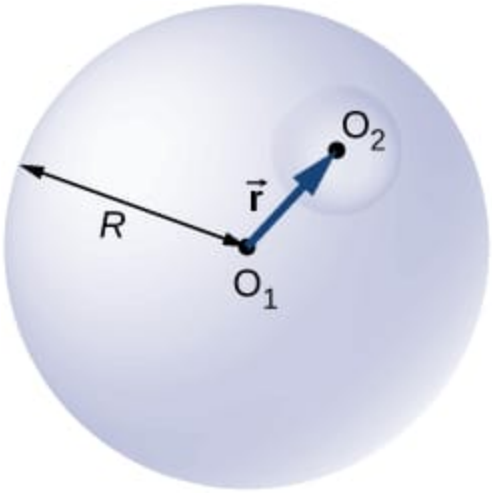
(b) if the area is tilted $\theta = 30\,°$ from the parallel direction. Note, this angle can also be $\theta = 180\,° + 30\,°$.



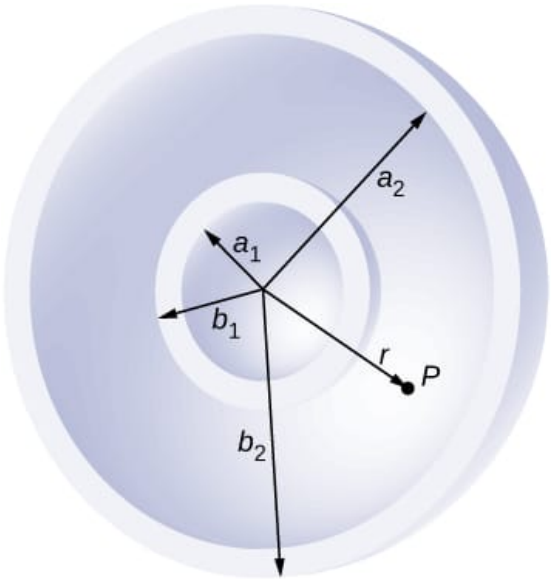
83. The infinite slab between the planes defined by $z = -a/2$ and $z = a/2$ contains a uniform volume charge density ρ(see below). What is the electric field produced by this charge distribution, both inside and outside the distribution?



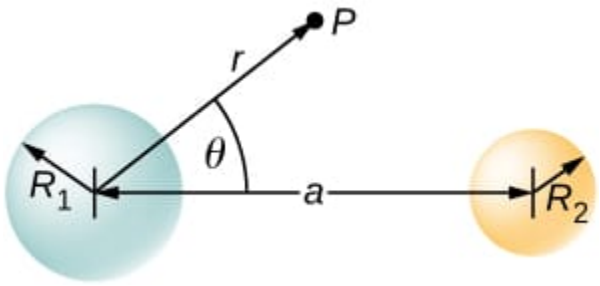
84. A total charge **Q** is distributed uniformly throughout a spherical volume that is centered at O_1 and has a radius **R**. Without disturbing the charge remaining, charge is removed from the spherical volume that is centered at O_2 (see below). Show that the electric field everywhere in the empty region is given by $\vec{E} = \dfrac{Q\vec{r}}{4\pi\varepsilon_0 R^3}$, where $\vec{r}$ is the displacement vector directed from O_1 to O_2.



85. A non-conducting spherical shell of inner radius a_1 and outer radius b_1 is uniformly charged with charged density ρ_1 inside another non-conducting spherical shell of inner radius a_2 and outer radius b_2 that is also uniformly charged with charge density ρ_2. See below. Find the electric field at space point **P** at a distance r from the common center such that (a) $r > b_2$, (b) \(\displaystyle a_2



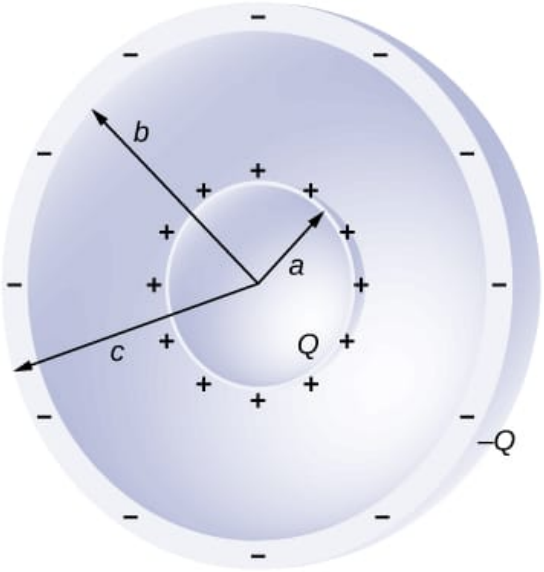
86. Two non-conducting spheres of radii R_1 and R_2 are uniformly charged with charge densities ρ_1 and ρ_2, respectively. They are separated at center-to-center distance a (see below). Find the electric field at point **P** located at a distance r from the center of sphere 1 and is in the direction θ from the line joining the two spheres assuming their charge densities are not affected by the presence of the other sphere. (**Hint:** Work one sphere at a time and use the superposition principle.)



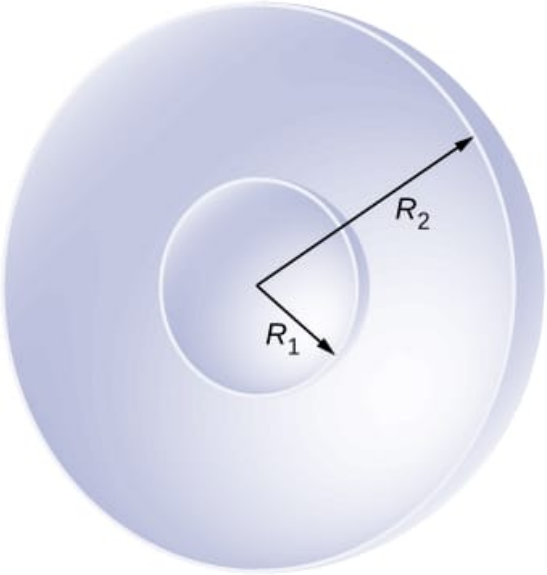
87. A disk of radius **R** is cut in a non-conducting large plate that is uniformly charged with charge density **σ** (coulomb per

square meter). See below. Find the electric field at a height h above the center of the disk. (\(\displaystyle h>>R,h<Hint: Fill the hole with $\pm\sigma$.)

88. Concentric conducting spherical shells carry charges **Q** and **–Q**, respectively (see below). The inner shell has negligible thickness. Determine the electric field for (a) \(\displaystyle rc\).



89. Shown below are two concentric conducting spherical shells of radii R_1 and R_2, each of finite thickness much less than either radius. The inner and outer shell carry net charges q_1 and q_2, respectively, where both q_1 and q_2 are positive. What is the electric field for (a) \(\displaystyle rR_2\)? (d) What is the net charge on the inner surface of the inner shell, the outer surface of the inner shell, the inner surface of the outer shell, and the outer surface of the outer shell?



90. A point charge of q=5.0×10−8Cq=5.0×10−8C is placed at the center of an uncharged spherical conducting shell of inner radius 6.0 cm and outer radius 9.0 cm. Find the electric field at (a) r=4.0cmr=4.0cm, (b) r=8.0cmr=8.0cm, and (c) r=12.0cmr=12.0cm. (d) What are the charges induced on the inner and outer surfaces of the shell?

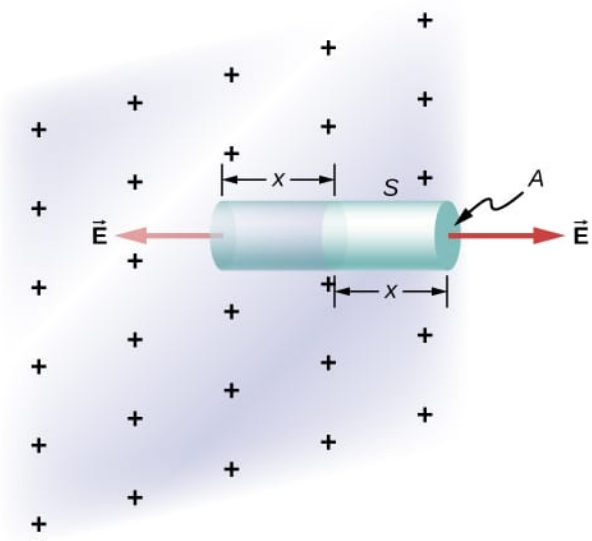
Challenge Problems

91. The Hubble Space Telescope can measure the energy flux from distant objects such as supernovae and stars. Scientists

then use this data to calculate the energy emitted by that object. Choose an interstellar object which scientists have observed the flux at the Hubble with (for example, $Vega$)[3], find the distance to that object and the size of Hubble's primary mirror, and calculate the total energy flux. (**Hint**: The Hubble intercepts only a small part of the total flux.)

92. Re-derive Gauss's law for the gravitational field, with $\vec{g}$ directed positively outward.

93. An infinite plate sheet of charge of surface charge density σ is shown below. What is the electric field at a distance **x** from the sheet? Compare the result of this calculation with that of worked out in the text.



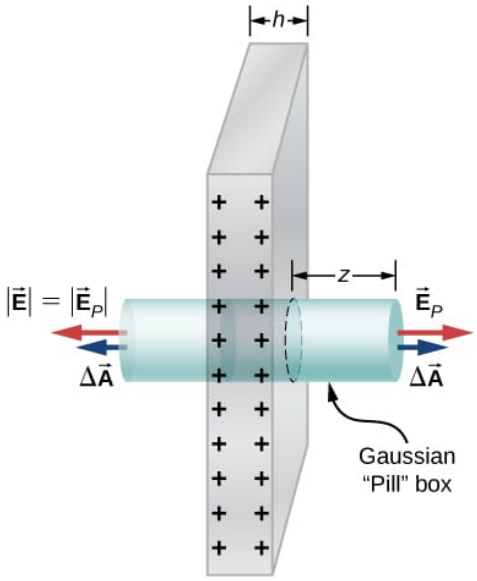
94. A spherical rubber balloon carries a total charge **Q** distributed uniformly over its surface. At $t = 0$, the radius of the balloon is **R**. The balloon is then slowly inflated until its radius reaches **2R** at the time t_0. Determine the electric field due to this charge as a function of time

(a) at the surface of the balloon,

(b) at the surface of radius **R**, and

(c) at the surface of radius **2R**. Ignore any effect on the electric field due to the material of the balloon and assume that the radius increases uniformly with time.

95. Find the electric field of a large conducting plate containing a net charge **q**. Let **A** be area of one side of the plate and **h** the thickness of the plate (see below). The charge on the metal plate will distribute mostly on the two planar sides and very little on the edges if the plate is thin.



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9.8: Gauss's Law (Answers)

Check Your Understanding

6.1. Place it so that its unit normal is perpendicular to $\vec{E}$.

6.2. $mab^2/2$

6.3 a. $3.4 \times 10^5 N \cdot m^2/C$;

b. $-3.4 \times 10^5 N \cdot m^2/C$;

c. $3.4 \times 10^5 N \cdot m^2/C$;

d. 0

6.4. In this case, there is only $\vec{E}_{out}$. So,yes.

6.5. $\vec{E} = \frac{\lambda_0}{2\pi\varepsilon_0}\frac{1}{d}\hat{r}$; This agrees with the calculation of Example 5.5 where we found the electric field by integrating over the charged wire. Notice how much simpler the calculation of this electric field is with Gauss's law.

6.6. If there are other charged objects around, then the charges on the surface of the sphere will not necessarily be spherically symmetrical; there will be more in certain direction than in other directions.

Conceptual Questions

1. a. If the planar surface is perpendicular to the electric field vector, the maximum flux would be obtained. b. If the planar surface were parallel to the electric field vector, the minimum flux would be obtained.

3. true

5. Since the electric field vector has a $\frac{1}{r^2}$ dependence, the fluxes are the same since $A = 4\pi r^2$.

7. a. no;

b. zero

9. Both fields vary as $\frac{1}{r^2}$. Because the gravitational constant is so much smaller than $\frac{1}{4\pi\varepsilon_0}$, the gravitational field is orders of magnitude weaker than the electric field.

11. No, it is produced by all charges both inside and outside the Gaussian surface.

13. No, since the situation does not have symmetry, making Gauss's law challenging to simplify.

15. Any shape of the Gaussian surface can be used. The only restriction is that the Gaussian integral must be calculable; therefore, a box or a cylinder are the most convenient geometrical shapes for the Gaussian surface.

17. yes

19. Since the electric field is zero inside a conductor, a charge of $-2.0\mu C$ is induced on the inside surface of the cavity. This will put a charge of $+2.0\mu C$ on the outside surface leaving a net charge of $-3.0\mu C$ on the surface.

Problems

21. $\Phi = \vec{E} \cdot \vec{A} \rightarrow EAcos\theta = 2.2 \times 10^4 N \cdot m^2/C$ electric field in direction of unit normal; $\Phi = \vec{E} \cdot \vec{A} \rightarrow EAcos\theta = -2.2 \times 10^4 N \cdot m^2/C$ electric field opposite to unit normal

23. $\frac{3 \times 10^{-5} N \cdot m^2/C}{(0.05m)^2} = E \Rightarrow \sigma = 2.12 \times 10^{-13} C/m^2$

25. a. $\Phi = 0.17 N \cdot m^2/C$;

b. $\Phi = 0$;

c. $\Phi = EAcos0\,^\circ = 1.0 \times 10^3 N/C(2.0 \times 10^{-4} m)^2 cos0\,^\circ = 0.20 N \cdot m^2/C$

27. $\Phi = 3.8 \times 10^4 N \cdot m^2/C$

29. $\vec{E}(z) = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{z}\hat{k}, \int \vec{E} \cdot \hat{n} dA = \frac{\lambda}{\varepsilon_0} l$

31. a. $\Phi = 3.39 \times 10^3 N \cdot m^2/C$;

b. $\Phi = 0$;

c. $\Phi = -2.25 \times 10^5 N \cdot m^2/C$;

d. $\Phi = 90.4 N \cdot m^2/C$

33. $\Phi = 1.13 \times 10^6 N \cdot m^2/C$

35. Make a cube with **q** at the center, using the cube of side a. This would take four cubes of side a to make one side of the large cube. The shaded side of the small cube would be 1/24th of the total area of the large cube; therefore, the flux through the shaded area would be $\Phi = \frac{1}{24}\frac{q}{\varepsilon_0}$.

37. $q = 3.54 \times 10^{-7} C$

39. zero, also because flux in equals flux out

41. $r > R, E = \frac{Q}{4\pi\varepsilon_0 r^2}; r < r, e = \frac{qr}{4\pi\varepsilon_0 r^3}$< p=""> </r,e=\frac{qr}{4πε_0r^3}\)<>

43. $EA = \frac{\lambda l}{\varepsilon_0} \Rightarrow E = 4.50 \times 10^7 N/C$

45. a. 0;

b. 0;

c. $\vec{E} = 6.74 \times 10^6 N/C(-\hat{r})$

47. a. 0;

b. $E = 2.70 \times 10^6 N/C$

49. a. Yes, the length of the rod is much greater than the distance to the point in question.

b. No, The length of the rod is of the same order of magnitude as the distance to the point in question.

c. Yes, the length of the rod is much greater than the distance to the point in question.

d. No. The length of the rod is of the same order of magnitude as the distance to the point in question.

51. a. $\vec{E} = \frac{R\sigma_0}{\varepsilon_0}\frac{1}{r}\hat{r} \Rightarrow \sigma_0 = 5.31 \times 10^{-11} C/m^2, \lambda = 3.33 \times 10^{-12} C/m$;

b. $\Phi = \frac{q_{enc}}{\varepsilon_0} = \frac{3.33 \times 10^{-12} C/m(0.05m)}{\varepsilon + 0} = 0.019 N \cdot m^2/C$

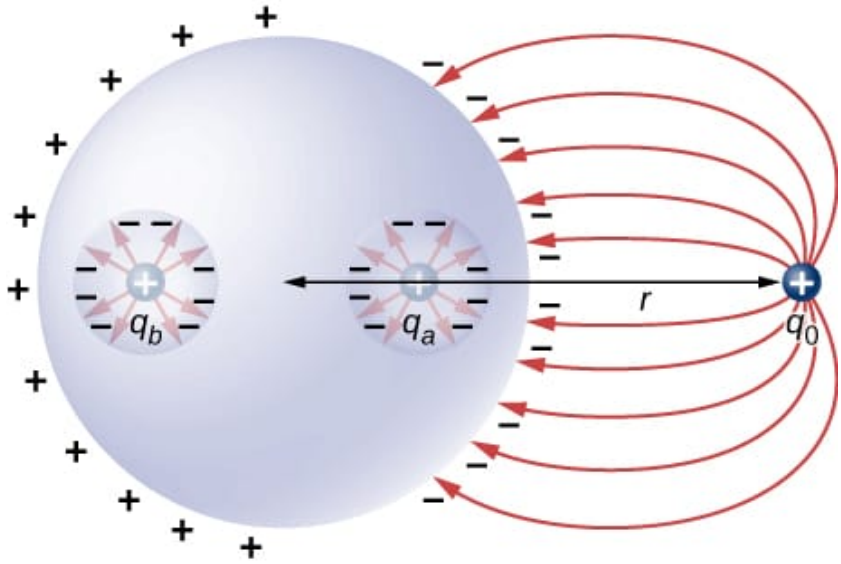
53. $E2\pi rl = \frac{\rho\pi r^2 l}{\varepsilon_0} \Rightarrow E = \frac{\rho r}{2\varepsilon_0}(r \leq R); E2\pi rl = \frac{\rho\pi R^2 l}{\varepsilon_0} \Rightarrow E = \frac{\rho R^2}{2\varepsilon_0 r}(r \geq R)$

55. $\Phi = \frac{q_{enc}}{\varepsilon_0} \Rightarrow q_{enc} = -1.0 \times 10^{-9} C$

57. $q_{enc} = \frac{4}{5}\pi\alpha r^5, E4\pi r^2 = \frac{4\pi\alpha r^5}{5\varepsilon_0} \Rightarrow E = \frac{\alpha r^3}{5\varepsilon_0}(r \leq R), q_{enc} = \frac{4}{5}\pi\alpha R^5, E4\pi r^2 = \frac{4\pi\alpha R^5}{5\varepsilon_0} \Rightarrow E = \frac{\alpha R^5}{5\varepsilon_0 r^2}(r \geq R)$

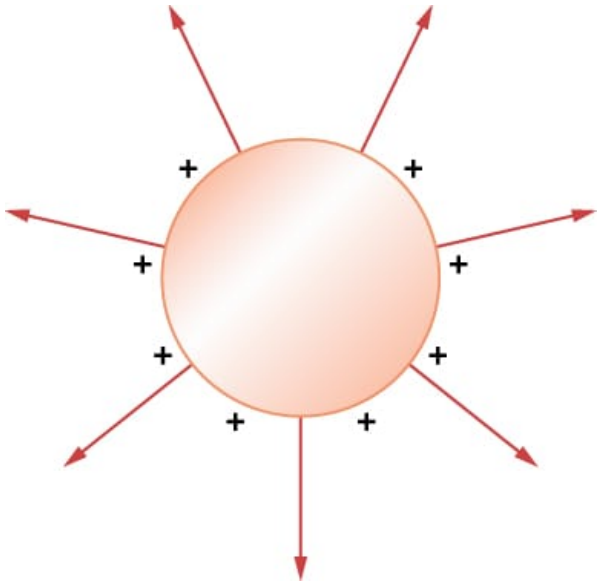
59. integrate by parts: $q_{enc} = 4\pi\rho_0[-e^{-\alpha r}(\frac{(r)^2}{\alpha} + \frac{2r}{\alpha^2} + \frac{2}{\alpha^3}) + \frac{2}{\alpha^3}] \Rightarrow E = \frac{\rho_0}{r^2\varepsilon_0}[-e^{-\alpha r}(\frac{(r)^2}{\alpha} + \frac{2r}{\alpha^2} + \frac{2}{\alpha^3}) + \frac{2}{\alpha^3}]$

61.



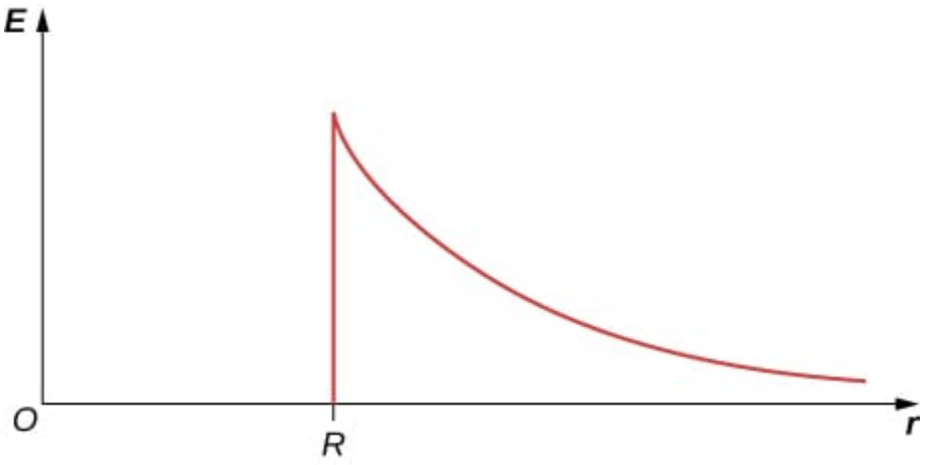
63. a. Outside: $E2\pi rl = \frac{\lambda l}{\varepsilon_0} \Rightarrow E = \frac{3.0 C/m}{2\pi\varepsilon_0 r}$; Inside $E_{in} = 0$;

b.



65. a. $E2\pi rl = \frac{\lambda l}{\varepsilon_0} \Rightarrow E = \frac{\lambda}{2\pi\varepsilon_0 r} r \geq R$ **E** inside equals **0**;

b.



67. $E = 5.65 \times 10^4 N/C$

69. $\lambda = \frac{\lambda l}{\varepsilon_0} \Rightarrow E = \frac{a\sigma}{\varepsilon_0 r} r \geq a, E = 0$ inside since q **enclosed**=**0**

71. a. $E = 0$;

b. $E2\pi rL = \frac{Q}{\varepsilon_0} \Rightarrow E = \frac{Q}{2\pi\varepsilon_0 rL}$; c. $E = 0$ since **r** would be either inside the second shell or if outside then **q** enclosed equals **0**.

Additional Problems

73. $\int \vec{E} \cdot \hat{n} dA = a^4$

75. a. $\int \vec{E} \cdot \hat{n} dA = E_0 r^2 \pi$; b. zero, since the flux through the upper half cancels the flux through the lower half of the sphere

77. $\Phi = \frac{q_{enc}}{\varepsilon_0}$; There are two contributions to the surface integral: one at the side of the rectangle at $x = 0$ and the other at the side at $x = 2.0m$; $-E(0)[1.5m^2] + E(2.0m)[1.5m^2] = \frac{q_{enc}}{\varepsilon_0} = -100Nm2/C$

where the minus sign indicates that at $x = 0$, the electric field is along positive **x** and the unit normal is along negative **x**. At $x = 2$, the unit normal and the electric field vector are in the same direction: $q_{enc} = \varepsilon_0\Phi = -8.85 \times 10^{-10}C$

79. didn't keep consistent directions for the area vectors, or the electric fields

81. a. $\sigma = 3.0 \times 10^{-3}C/m^2, +3 \times 10^{-3}C/m^2$ on one and $-3 \times 10^{-3}C/m^2$ on the other;

b. $E = 3.39 \times 10^8 N/CE = 3.39 \times 10^8 N/C$

83. Construct a Gaussian cylinder along the **z**-axis with cross-sectional area **A.**

$$|z| \geq \frac{a}{2} q_{enc} = \rho Aa, \Phi = \frac{\rho Aa}{\varepsilon_0} \Rightarrow E = \frac{\rho a}{2\varepsilon_0},$$

$$|z| \leq \frac{a}{2} q_{enc} = \rho A2z, E(2A) = \frac{\rho A2z}{\varepsilon_0} \Rightarrow E = \frac{\rho z}{\varepsilon_0}$$

85. a. $r > b_2\ E4\pi r^2 = \frac{\frac{4}{3}\pi[\rho_1(b_1^3 - a_1^3) + \rho_2(b_2^3 - a_2^3)}{\varepsilon_0} \Rightarrow E = \frac{\rho_1(b_1^3 - a_1^3) + \rho_2(b_2^3 - a_2^3)}{3\varepsilon_0 r^2}$;

b. \(\displaystyle a_2

c. \(\displaystyle b_1

d. \(\displaystyle a_1

e. 0

87. Electric field due to plate without hole: $E = \frac{\sigma}{2\varepsilon_0}$.

Electric field of just hole filled with \(\displaystyle −σE=\frac{−σ}{2ε_0}(1−\frac{z}{\sqrt{R^2+z^2}}).

Thus, $E_{net} = \frac{\sigma}{2\varepsilon_0} \frac{h}{\sqrt{R^2 + h^2}}$

89. a. $E = 0$; b. $E = \frac{q_1}{4\pi\varepsilon_0 r^2}$; c. $E = \frac{q_1 + q_2}{4\pi\varepsilon_0 r^2}$; d. 0 $q_1 - q_1, q_1 + q_2$

Challenge Problems

91. Given the referenced link, using a distance to Vega of $237 \times 10^{15} m$[4] and a diameter of 2.4 m for the primary mirror,[5] we find that at a wavelength of 555.6 nm, Vega is emitting $2.44 \times 10^{24} J/s$ at that wavelength. Note that the flux through the mirror is essentially constant.

93. The symmetry of the system forces $\vec{E}$ to be perpendicular to the sheet and constant over any plane parallel to the sheet. To calculate the electric field, we choose the cylindrical Gaussian surface shown. The cross-section area and the height of the cylinder are **A** and **2x**, respectively, and the cylinder is positioned so that it is bisected by the plane sheet. Since **E** is perpendicular to each end and parallel to the side of the cylinder, we have **EA** as the flux through each end and there is no flux through the side. The charge enclosed by the cylinder is σA, so from Gauss's law, $2EA = \frac{\sigma A}{\varepsilon_0}$, and the electric field of an infinite sheet of charge is

$E = \frac{\sigma}{2\varepsilon_0}$, in agreement with the calculation of in the text.

95. There is **Q/2** on each side of the plate since the net charge is $Q : \sigma = \frac{Q}{2A}$,

$$\oint_S \vec{E} \cdot \hat{n} dA = \frac{2\sigma \Delta A}{\varepsilon_0} \Rightarrow E_P = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 2A}$$

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10: ELECTRIC POTENTIAL

10.1: Prelude to Electric Potential
10.2: Electric Potential Energy
10.3: Electric Potential and Potential Difference
10.4: Calculations of Electric Potential
10.5: Determining Field from Potential
10.6: Equipotential Surfaces and Conductors
10.7: Applications of Electrostatics
10.8: Electric Potential (Summary)
10.9: Electric Potential (Exercises)
10.10: Electric Potential (Answer)

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10.1: Prelude to Electric Potential

In Electric Charges and Fields, we just scratched the surface (or at least rubbed it) of electrical phenomena. Two terms commonly used to describe electricity are its energy and voltage, which we show in this chapter is directly related to the potential energy in a system. We know, for example, that great amounts of electrical energy can be stored in batteries, are transmitted cross-country via currents through power lines, and may jump from clouds to explode the sap of trees. In a similar manner, at the molecular level, ions cross cell membranes and transfer information.



Figure 10.1.1: The energy released in a lightning strike is an excellent illustration of the vast quantities of energy that may be stored and released by an electric potential difference. In this chapter, we calculate just how much energy can be released in a lightning strike and how this varies with the height of the clouds from the ground. (credit: Anthony Quintano)

We also know about voltages associated with electricity. Batteries are typically a few volts, the outlets in your home frequently produce 120 volts, and power lines can be as high as hundreds of thousands of volts. But energy and voltage are not the same thing. A motorcycle battery, for example, is small and would not be very successful in replacing a much larger car battery, yet each has the same voltage. In this chapter, we examine the relationship between voltage and electrical energy, and begin to explore some of the many applications of electricity.

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10.2: Electric Potential Energy

Learning Objectives

By the end of this section, you will be able to:

- Define the work done by an electric force
- Define electric potential energy
- Apply work and potential energy in systems with electric charges

When a free positive charge q is accelerated by an electric field, it is given kinetic energy (Figure $10.2.1$). The process is analogous to an object being accelerated by a gravitational field, as if the charge were going down an electrical hill where its electric potential energy is converted into kinetic energy, although of course the sources of the forces are very different. Let us explore the work done on a charge q by the electric field in this process, so that we may develop a definition of electric potential energy.

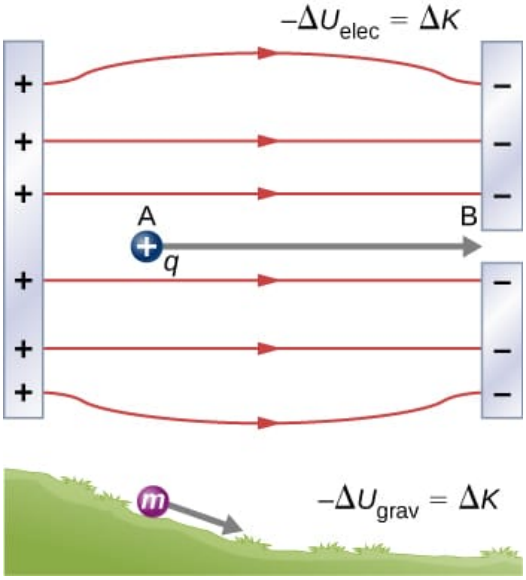


Figure $10.2.1$: A charge accelerated by an electric field is analogous to a mass going down a hill. In both cases, potential energy decreases as kinetic energy increases, $-\Delta U = \Delta K$. Work is done by a force, but since this force is conservative, we can write $W = -\Delta U$.

The electrostatic or Coulomb force is conservative, which means that the work done on q is independent of the path taken, as we will demonstrate later. This is exactly analogous to the gravitational force. When a force is conservative, it is possible to define a potential energy associated with the force. It is usually easier to work with the potential energy (because it depends only on position) than to calculate the work directly.

To show this explicitly, consider an electric charge $+q$ fixed at the origin and move another charge $+Q$ toward q in such a manner that, at each instant, the applied force $\vec{F}$ exactly balances the electric force $\vec{F}_e$ on Q (Figure $10.2.2$). The work done by the applied force $\vec{F}$ on the charge Q changes the potential energy of Q. We call this potential energy the **electrical potential energy** of Q.

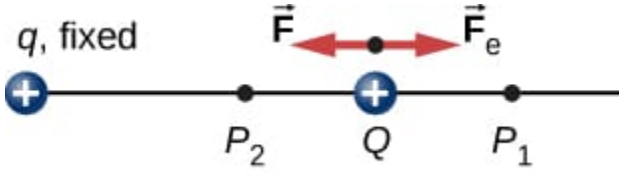


Figure $10.2.2$: Displacement of "test" charge Q in the presence of fixed "source" charge q.

The work W_{12} done by the applied force $\vec{F}$ when the particle moves from P_1 to P_2 may be calculated by

$$W_{12} = \int_{P_1}^{P_2} \vec{F} \cdot d\vec{l}\,.$$

Since the applied force $\vec{F}$ balances the electric force $\vec{F}_e$ on Q, the two forces have equal magnitude and opposite directions. Therefore, the applied force is

$$\vec{F} = -\vec{F}_e = -\frac{kqQ}{r^2}\hat{r},$$

where we have defined positive to be pointing away from the origin and r is the distance from the origin. The directions of both the displacement and the applied force in the system in Figure $10.2.2$ are parallel, and thus the work done on the system is positive.

We use the letter U to denote electric potential energy, which has units of joules (J). When a conservative force does negative work, the system gains potential energy. When a conservative force does positive work, the system loses potential energy, $\Delta U = -W$. In the system in Figure $10.2.2$, the Coulomb force acts in the opposite direction to the displacement; therefore, the work is negative. However, we have increased the potential energy in the two-charge system.

Example $10.2.1$: Kinetic Energy of a Charged Particle

A $+3.0\ \text{nC}$ charge Q is initially at rest a distance of 10 cm (r_1) from a $+5.0\ \text{nC}$ charge q fixed at the origin (Figure $10.2.3$). Naturally, the Coulomb force accelerates Q away from q, eventually reaching 15 cm (r_2).

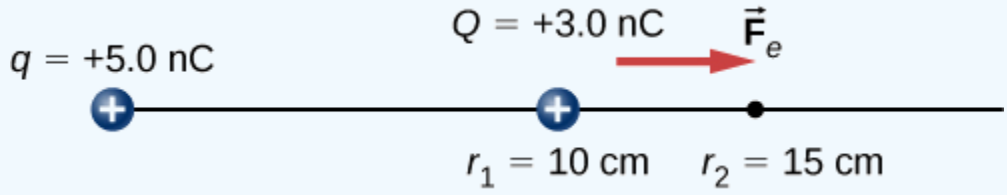


Figure $10.2.3$: The charge Q is repelled by q, thus having work done on it and gaining kinetic energy.

a. What is the work done by the electric field between r_1 and r_2?
b. How much kinetic energy does Q have at r_2?

Strategy

Calculate the work with the usual definition. Since Q started from rest, this is the same as the kinetic energy.

Solution

Integrating force over distance, we obtain

$$\begin{aligned} W_{12} &= \int_{r_1}^{r_2} \vec{F} \cdot d\vec{r} \\ &= \int_{r_1}^{r_2} \frac{kqQ}{r^2} dr \\ &= -\frac{kqQ}{r}\Big|_{r_1}^{r_2} \\ &= kqQ\left[\frac{-1}{r_2} + \frac{1}{r_1}\right] \\ &= (8.99 \times 10^9\,\text{N}\,\text{m}^2/\text{C}^2)(5.0 \times 10^{-9}\text{C})(3.0 \times 10^{-9}\text{C})\left[\frac{-1}{0.15\,\text{m}} + \frac{1}{0.10\,\text{m}}\right] \\ &= 4.5 \times 10^{-7}\,\text{J}. \end{aligned}$$

This is also the value of the kinetic energy at r_2.

Significance

Charge Q was initially at rest; the electric field of q did work on Q, so now Q has kinetic energy equal to the work done by the electric field.

Exercise 10.2.1

If Q has a mass of $4.00\ \mu\text{g}$, what is the speed of Q at r_2?

Answer available at the end of this section.

In this example, the work W done to accelerate a positive charge from rest is positive and results from a loss in U, or a negative ΔU. A value for U can be found at any point by taking one point as a reference and calculating the work needed to move a charge to the other point.

Electric Potential Energy

Work W done to accelerate a positive charge from rest is positive and results from a loss in U, or a negative ΔU. Mathematically,

$$W = -\Delta U. \tag{10.2.1}$$

Gravitational potential energy and electric potential energy are quite analogous. Potential energy accounts for work done by a conservative force and gives added insight regarding energy and energy transformation without the necessity of dealing with the force directly. It is much more common, for example, to use the concept of electric potential energy than to deal with the Coulomb force directly in real-world applications.

In polar coordinates with q at the origin and Q located at r, the displacement element vector is $\vec{dl} = \hat{r}dr$ and thus the work becomes

$$\begin{aligned} W_{12} &= kqQ\int_{r_1}^{r_2}\frac{1}{r^2}\hat{r}\cdot\hat{r}dr \\ &= \underbrace{kqQ\frac{1}{r_2}}_{\text{final point}} - \underbrace{kqQ\frac{1}{r_1}}_{\text{initial point}}. \end{aligned} \tag{10.2.2}$$

Notice that this result only depends on the endpoints and is otherwise independent of the path taken. To explore this further, compare path P_1 to P_2 with path $P_1P_3P_4P_2$ in Figure 10.2.4.

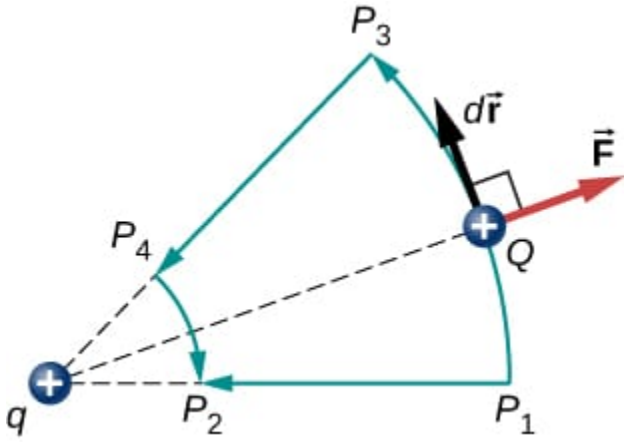


Figure 10.2.4:Two paths for displacement P_1 to P_2. The work on segments P_1P_3 and P_4P_2 are zero due to the electrical force being perpendicular to the displacement along these paths. Therefore, work on paths P_1P_2 and $P_1P_3P_4P_2$ are equal.

The segments P_1P_3 and P_4P_2 are arcs of circles centered at q. Since the force on Q points either toward or away from q, no work

is done by a force balancing the electric force, because it is perpendicular to the displacement along these arcs. Therefore, the only work done is along segment P_3P_4 which is identical to P_1P_2.

One implication of this work calculation is that if we were to go around the path $P_1P_3P_4P_2P_1$, the net work would be zero (Figure $10.2.5$). Recall that this is how we determine whether a force is conservative or not. Hence, because the electric force is related to the electric field by $\vec{F} = q\vec{E}$, the electric field is itself conservative. That is,

$$\oint \vec{E} \cdot d\vec{l} = 0.$$

Note that Q is a constant.

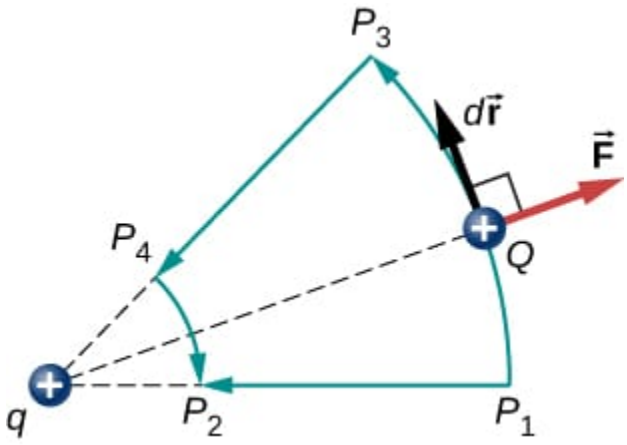


Figure $10.2.5$: A closed path in an electric field. The net work around this path is zero.

Another implication is that we may define an electric potential energy. Recall that the work done by a conservative force is also expressed as the difference in the potential energy corresponding to that force. Therefore, the work W_{ref} to bring a charge from a reference point to a point of interest may be written as

$$W_{ref} = \int_{r_{\text{ref}}}^{r} \vec{F} \cdot d\vec{l}$$

and, by Equation $10.2.1$, the difference in potential energy $(U_2 - U_1)$ of the test charge Q between the two points is

$$\Delta U = -\int_{r_{\text{ref}}}^{r} \vec{F} \cdot d\vec{l}\,.$$

Therefore, we can write a general expression for the potential energy of two point charges (in spherical coordinates):

$$\Delta U = -\int_{r_{\text{ref}}}^{r} \frac{kqQ}{r^2} dr = -\left[-\frac{kqQ}{r}\right]_{r_{\text{ref}}}^{r} = kqQ\left[\frac{1}{r} - \frac{1}{r_{\text{ref}}}\right].$$

We may take the second term to be an arbitrary constant reference level, which serves as the zero reference:

$$U(r) = k\frac{qQ}{r} - U_{\text{ref}}.$$

A convenient choice of reference that relies on our common sense is that when the two charges are infinitely far apart, there is no interaction between them. (Recall the discussion of reference potential energy in Potential Energy and Conservation of Energy.) Taking the potential energy of this state to be zero removes the term U_{ref} from the equation (just like when we say the ground is zero potential energy in a gravitational potential energy problem), and the potential energy of Q when it is separated from q by a distance r assumes the form

$$\underbrace{U(r) = k\frac{qQ}{r}}_{\text{zero reference at } r=\infty}\,.$$

This formula is symmetrical with respect to q and Q, so it is best described as the potential energy of the two-charge system.

Example 10.2.2: Potential Energy of a Charged Particle

A $+3.0\ \mathrm{nC}$ charge Q is initially at rest a distance of 10 cm (r_1) from a $+5.0\ \mathrm{nC}$ charge q fixed at the origin (Figure 10.2.6). Naturally, the Coulomb force accelerates Q away from q, eventually reaching 15 cm (r_2).

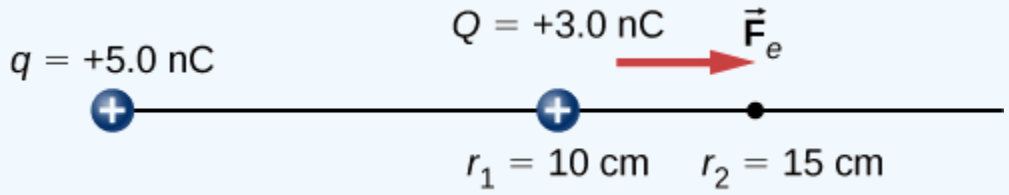


Figure 10.2.6: The charge Q is repelled by q, thus having work done on it and losing potential energy.

What is the change in the potential energy of the two-charge system from r_1 to r_2?

Strategy

Calculate the potential energy with the definition given above:

$\Delta U_{12} = -\int_{r_1}^{r_2} \vec{F} \cdot d\vec{r}$. Since Q started from rest, this is the same as the kinetic energy.

Solution

We have

$$\begin{aligned}
\Delta U_{12} &= -\int_{r_1}^{r_2} \vec{F} \cdot d\vec{r} \\
&= -\int_{r_1}^{r_2} \frac{kqQ}{r^2} dr \\
&= -\left[-\frac{kqQ}{r}\right]_{r_1}^{r_2} \\
&= kqQ\left[\frac{1}{r_2} - \frac{1}{r_1}\right] \\
&= (8.99 \times 10^9\ \mathrm{N\,m^2/C^2})(5.0 \times 10^{-9}\mathrm{C})(3.0 \times 10^{-9}\mathrm{C})\left[\frac{1}{0.15\,\mathrm{m}} - \frac{1}{0.10\,\mathrm{m}}\right] \\
&= -4.5 \times 10^{-7}\ \mathrm{J}. \qquad (10.2.3)
\end{aligned}$$

Significance

The change in the potential energy is negative, as expected, and equal in magnitude to the change in kinetic energy in this system. Recall from Example 10.2.1 that the change in kinetic energy was positive.

Exercise 10.2.2

What is the potential energy of Q relative to the zero reference at infinity at r_2 in the above example?

Answer available at the end of this section.

Due to Coulomb's law, the forces due to multiple charges on a test charge Q superimpose; they may be calculated individually and then added. This implies that the work integrals and hence the resulting potential energies exhibit the same behavior. To demonstrate this, we consider an example of assembling a system of four charges.

Example 10.2.3: Assembling Four Positive Charges

Find the amount of work an external agent must do in assembling four charges $+2.0\ \mu\text{C}$, $+3.0\ \mu\text{C}$, $+4.0\ \mu\text{C}$ and $+5.0\ \mu\text{C}$ at the vertices of a square of side 1.0 cm, starting each charge from infinity (Figure 10.2.7).

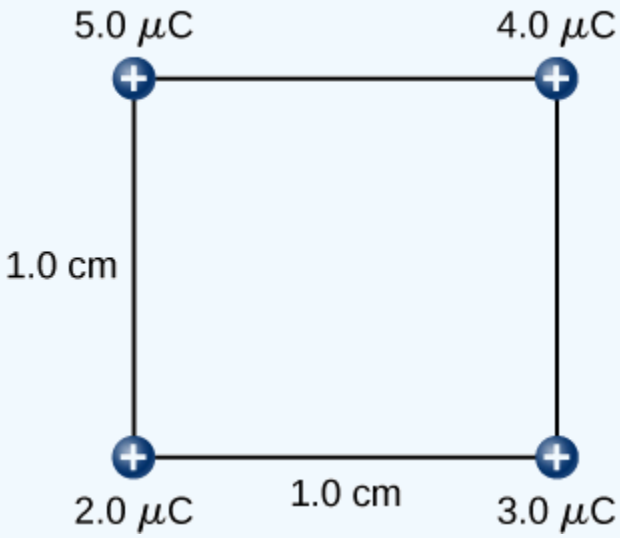


Figure 10.2.7: How much work is needed to assemble this charge configuration?

Strategy

We bring in the charges one at a time, giving them starting locations at infinity and calculating the work to bring them in from infinity to their final location. We do this in order of increasing charge.

Solution

Step 1. First bring the $+2.0\ \mu\text{C}$ charge to the origin. Since there are no other charges at a finite distance from this charge yet, no work is done in bringing it from infinity,

$$W_1 = 0.$$

Step 2. While keeping the $+2.0\ \mu\text{C}$ charge fixed at the origin, bring the $+3.0\ \mu\text{C}$ charge to $(x, y, z) = (1.0\ \text{cm},\ 0,\ 0)$ (Figure 10.2.8). Now, the applied force must do work against the force exerted by the $+2.0\ \mu\text{C}$ charge fixed at the origin. The work done equals the change in the potential energy of the $+3.0\ \mu\text{C}$ charge:

$$\begin{aligned} W_2 &= k\frac{q_1 q_2}{r_{12}} \\ &= \left(9.0 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right) \frac{(2.0 \times 10^{-6}\text{C})(3.0 \times 10^{-6}\text{C})}{1.0 \times 10^{-2}\text{m}} \\ &= 5.4\,\text{J}. \end{aligned}$$

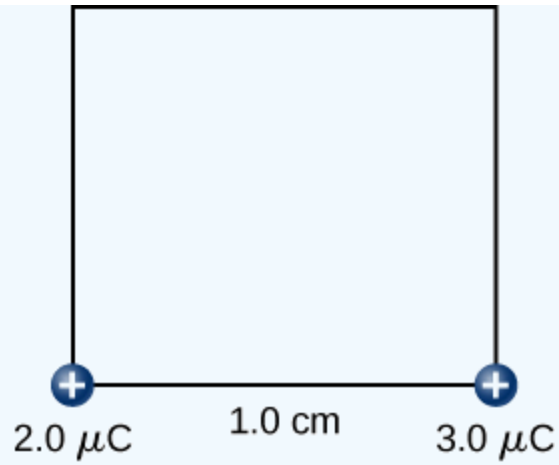


Figure 10.2.8: Step 2. Work W_2 to bring the $+3.0\ \mu\text{C}$ charge from infinity.

Step 3. While keeping the charges of $+2.0\ \mu\text{C}$ and $+3.0\ \mu\text{C}$ fixed in their places, bring in the $+4.0\ \mu\text{C}$ charge to $(x, y, z) = (1.0\ \text{cm},\ 1.0\ \text{cm},\ 0)$ (Figure 10.2.9). The work done in this step is

$$\begin{aligned} W_3 &= k\frac{q_1 q_3}{r_{13}} + k\frac{q_2 q_3}{r_{23}} \\ &= \left(9.0\times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left[\frac{(2.0\times 10^{-6}\text{C})(4.0\times 10^{-6}\text{C})}{\sqrt{2}\times 10^{-2}\text{m}} + \frac{(3.0\times 10^{-6}\text{C})(4.0\times 10^{-6}\text{C})}{1.0\times 10^{-2}\text{m}}\right] \\ &= 15.9\,\text{J}. \end{aligned}$$

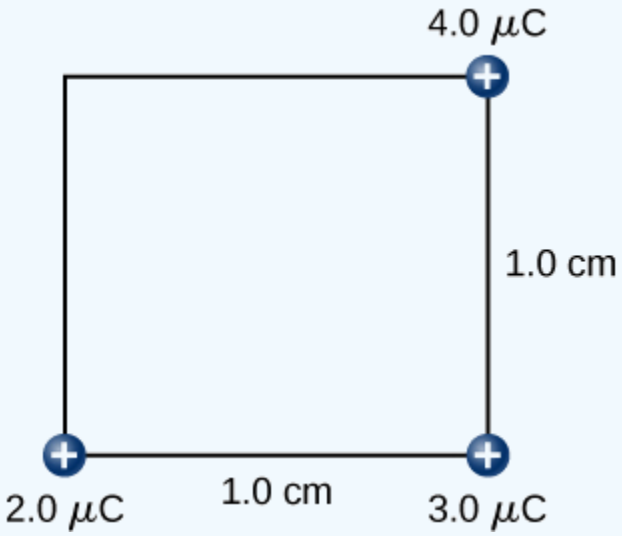


Figure 10.2.9: Step 3. The work W_3 to bring the $+4.0\ \mu\text{C}$ charge from infinity.

Step 4. Finally, while keeping the first three charges in their places, bring the $+5.0\ \mu\text{C}$ charge to $(x, y, z) = (0,\ 1.0\ \text{cm},\ 0)$ (Figure 10.2.10). The work done here is

$$\begin{aligned} W_4 &= kq_4\left[\frac{q_1}{r_{14}} + \frac{q_2}{r_{24}} + \frac{q_3}{r_{34}}\right], \\ &= \left(9.0\times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(5.0\times 10^{-6}\text{C})\left[\frac{(2.0\times 10^{-6}\text{C})}{1.0\times 10^{-2}\text{m}} + \frac{(3.0\times 10^{-6}\text{C})}{\sqrt{2}\times 10^{-2}\text{m}} + \frac{(4.0\times 10^{-6}\text{C})}{1.0\times 10^{-2}\text{m}}\right] \\ &= 36.5\,\text{J}. \end{aligned}$$

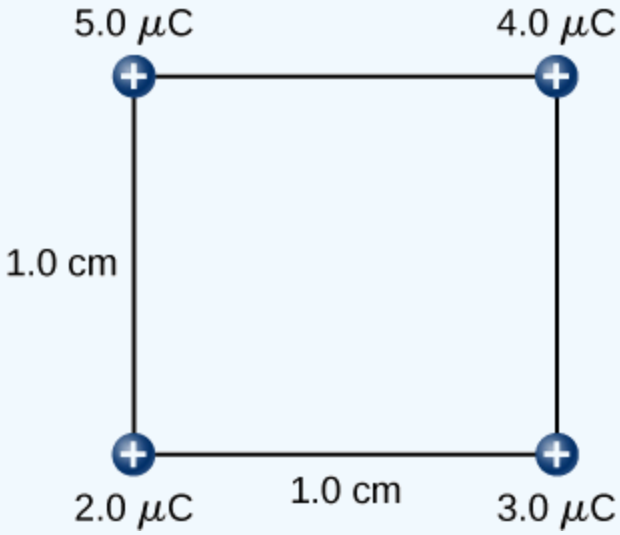


Figure 10.2.10: Step 4. The work W_4 to bring the $+5.0\ \mu\text{C}$ charge from infinity.

Hence, the total work done by the applied force in assembling the four charges is equal to the sum of the work in bringing each charge from infinity to its final position:

$$\begin{aligned} W_T &= W_1 + W_2 + W_3 + W_4 \\ &= 0 + 5.4\,\text{J} + 15.9\,\text{J} + 36.5\,\text{J} \\ &= 57.8\,\text{J}. \end{aligned}$$

Significance

The work on each charge depends only on its pairwise interactions with the other charges. No more complicated interactions need to be considered; the work on the third charge only depends on its interaction with the first and second charges, the interaction between the first and second charge does not affect the third.

Exercise 10.2.3

Is the electrical potential energy of two point charges positive or negative if the charges are of the same sign? Opposite signs? How does this relate to the work necessary to bring the charges into proximity from infinity?

Answer available at the end of this section.

Note that the electrical potential energy is positive if the two charges are of the same type, either positive or negative, and negative if the two charges are of opposite types. This makes sense if you think of the change in the potential energy ΔU as you bring the two charges closer or move them farther apart. Depending on the relative types of charges, you may have to work on the system or the system would do work on you, that is, your work is either positive or negative. If you have to do positive work on the system (actually push the charges closer), then the energy of the system should increase. If you bring two positive charges or two negative charges closer, you have to do positive work on the system, which raises their potential energy. Since potential energy is proportional to $1/r$, the potential energy goes up when r goes down between two positive or two negative charges.

On the other hand, if you bring a positive and a negative charge nearer, you have to do negative work on the system (the charges are pulling you), which means that you take energy away from the system. This reduces the potential energy. Since potential energy is negative in the case of a positive and a negative charge pair, the increase in $1/r$ makes the potential energy more negative, which is the same as a reduction in potential energy.

The result from Example 10.2.2 may be extended to systems with any arbitrary number of charges. In this case, it is most convenient to write the formula as

$$W_{12\ldots N} = \frac{k}{2}\sum_{i}^{N}\sum_{j}^{N}\frac{q_i q_j}{r_{ij}} \text{ for } i \neq j.$$

The factor of 1/2 accounts for adding each pair of charges twice.

Answers

Exercise 10.2.1

$K = \frac{1}{2}mv^2$, $v = \sqrt{2\frac{\text{K}}{\text{m}}} = \sqrt{2\frac{4.5\times10^{-7}\text{J}}{4.00\times10^{-9}\text{kg}}} = 15 \text{ m/s}.$

Exercise 10.2.2

It has kinetic energy of 4.5×10^{-7} J at point r_2 and potential energy of 9.0×10^{-7} J, which means that as Q approaches infinity, its kinetic energy totals three times the kinetic energy at r_2, since all of the potential energy gets converted to kinetic.

Exercise 10.2.3

Positive, negative, and these quantities are the same as the work you would need to do to bring the charges in from infinity

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10.3: Electric Potential and Potential Difference

Learning Objectives

By the end of this section, you will be able to:

- Define electric potential, voltage, and potential difference
- Define the electron-volt
- Calculate electric potential and potential difference from potential energy and electric field
- Describe systems in which the electron-volt is a useful unit
- Apply conservation of energy to electric systems

Recall that earlier we defined electric field to be a quantity independent of the test charge in a given system, which would nonetheless allow us to calculate the force that would result on an arbitrary test charge. (The default assumption in the absence of other information is that the test charge is positive.) We briefly defined a field for gravity, but gravity is always attractive, whereas the electric force can be either attractive or repulsive. Therefore, although potential energy is perfectly adequate in a gravitational system, it is convenient to define a quantity that allows us to calculate the work on a charge independent of the magnitude of the charge. Calculating the work directly may be difficult, since $W = \vec{F} \cdot \vec{d}$ and the direction and magnitude of $\vec{F}$ can be complex for multiple charges, for odd-shaped objects, and along arbitrary paths. But we do know that because $\vec{F} = q\vec{E}$, the work, and hence ΔU is proportional to the test charge q. To have a physical quantity that is independent of test charge, we define **electric potential** V (or simply potential, since electric is understood) to be the potential energy per unit charge:

Electric Potential

The electric potential energy per unit charge is

$$V = \frac{U}{q}. \tag{10.3.1}$$

Since U is proportional to q, the dependence on q cancels. Thus, V does not depend on q. The change in potential energy ΔU is crucial, so we are concerned with the difference in potential or potential difference ΔV between two points, where

Electric Potential Difference

The **electric potential difference** between points A and B, $V_B - V_A$ is defined to be the change in potential energy of a charge q moved from A to B, divided by the charge. Units of potential difference are joules per coulomb, given the name volt (V) after Alessandro Volta.

$$1\,\mathrm{V} = 1\,\mathrm{J/C} \tag{10.3.2}$$

The familiar term **voltage** is the common name for electric potential difference. Keep in mind that whenever a voltage is quoted, it is understood to be the potential difference between two points. For example, every battery has two terminals, and its voltage is the potential difference between them. More fundamentally, the point you choose to be zero volts is arbitrary. This is analogous to the fact that gravitational potential energy has an arbitrary zero, such as sea level or perhaps a lecture hall floor. It is worthwhile to emphasize the distinction between potential difference and electrical potential energy.

Potential Difference and Electrical Potential Energy

The relationship between potential difference (or voltage) and electrical potential energy is given by

$$\Delta V = \frac{\Delta U}{q} \tag{10.3.3}$$

or

$$\Delta U = q\Delta V. \tag{10.3.4}$$

Voltage is not the same as energy. Voltage is the energy per unit charge. Thus, a motorcycle battery and a car battery can both have the same voltage (more precisely, the same potential difference between battery terminals), yet one stores much more energy than the other because $\Delta U = q\Delta V$. The car battery can move more charge than the motorcycle battery, although both are 12-V batteries.

Example 10.3.1: Calculating Energy

You have a 12.0-V motorcycle battery that can move 5000 C of charge, and a 12.0-V car battery that can move 60,000 C of charge. How much energy does each deliver? (Assume that the numerical value of each charge is accurate to three significant figures.)

Strategy

To say we have a 12.0-V battery means that its terminals have a 12.0-V potential difference. When such a battery moves charge, it puts the charge through a potential difference of 12.0 V, and the charge is given a change in potential energy equal to $\Delta U = q\Delta V$. To find the energy output, we multiply the charge moved by the potential difference.

Solution

For the motorcycle battery, $q = 5000\ \text{C}$ and $\Delta V = 12.0\ \text{V}$. The total energy delivered by the motorcycle battery is

$$\Delta U_{cycle} = (5000\,\text{C})(12.0\,\text{V}) = (5000\,\text{C})(12.0\,\text{J/C}) = 6.00 \times 10^4\,\text{J}.$$

Similarly, for the car battery, $q = 60,000\ \text{C}$ and

$$\Delta U_{car} = (60,000\,\text{C})(12.0\,\text{V}) = 7.20 \times 10^5\,\text{J}.$$

Significance

Voltage and energy are related, but they are not the same thing. The voltages of the batteries are identical, but the energy supplied by each is quite different. A car battery has a much larger engine to start than a motorcycle. Note also that as a battery is discharged, some of its energy is used internally and its terminal voltage drops, such as when headlights dim because of a depleted car battery. The energy supplied by the battery is still calculated as in this example, but not all of the energy is available for external use.

Exercise 10.3.1

How much energy does a 1.5-V AAA battery have that can move 100 C?

Answer available at the end of this section.

Note that the energies calculated in the previous example are absolute values. The change in potential energy for the battery is negative, since it loses energy. These batteries, like many electrical systems, actually move negative charge—electrons in particular. The batteries repel electrons from their negative terminals (A) through whatever circuitry is involved and attract them to their positive terminals (B), as shown in Figure 10.3.1. The change in potential is $\Delta V = V_B - V_A = +12\ V$ and the charge q is negative, so that $\Delta U = q\Delta V$ is negative, meaning the potential energy of the battery has decreased when q has moved from A to B.

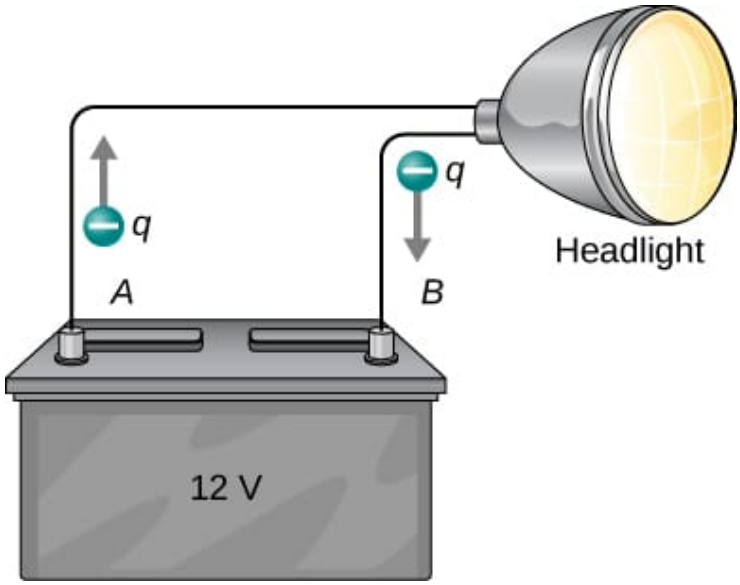


Figure 10.3.1: A battery moves negative charge from its negative terminal through a headlight to its positive terminal. Appropriate combinations of chemicals in the battery separate charges so that the negative terminal has an excess of negative charge, which is repelled by it and attracted to the excess positive charge on the other terminal. In terms of potential, the positive terminal is at a higher voltage than the negative terminal. Inside the battery, both positive and negative charges move.

Example 10.3.2: How Many Electrons Move through a Headlight Each Second?

When a 12.0-V car battery powers a single 30.0-W headlight, how many electrons pass through it each second?

Strategy

To find the number of electrons, we must first find the charge that moves in 1.00 s. The charge moved is related to voltage and energy through the equations $\Delta U = q\Delta V$. A 30.0-W lamp uses 30.0 joules per second. Since the battery loses energy, we have $\Delta U = -30\ \text{J}$ and, since the electrons are going from the negative terminal to the positive, we see that $\Delta V = +12.0\ \text{V}$.

Solution

To find the charge q moved, we solve the equation $\Delta U = q\Delta V$:

$$q = \frac{\Delta U}{\Delta V}.$$

Entering the values for ΔU and ΔV, we get

$$q = \frac{-30.0\,\text{J}}{+12.0\,\text{V}} = \frac{-30.0\,\text{J}}{+12.0\,\text{J/C}} = -2.50\,\text{C}.$$

The number of electrons n_e is the total charge divided by the charge per electron. That is,

$$n_e = \frac{-2.50\,\text{C}}{-1.60 \times 10^{-19}\text{C}/e^-} = 1.56 \times 10^{19}\,\text{electrons}.$$

Significance

This is a very large number. It is no wonder that we do not ordinarily observe individual electrons with so many being present in ordinary systems. In fact, electricity had been in use for many decades before it was determined that the moving charges in many circumstances were negative. Positive charge moving in the opposite direction of negative charge often produces identical effects; this makes it difficult to determine which is moving or whether both are moving.

Exercise 10.3.2

How many electrons would go through a 24.0-W lamp?

Answer available at the end of this section.

The Electron-Volt

The energy per electron is very small in macroscopic situations like that in the previous example—a tiny fraction of a joule. But on a submicroscopic scale, such energy per particle (electron, proton, or ion) can be of great importance. For example, even a tiny fraction of a joule can be great enough for these particles to destroy organic molecules and harm living tissue. The particle may do its damage by direct collision, or it may create harmful X-rays, which can also inflict damage. It is useful to have an energy unit related to submicroscopic effects.

Figure 10.3.2 shows a situation related to the definition of such an energy unit. An electron is accelerated between two charged metal plates, as it might be in an old-model television tube or oscilloscope. The electron gains kinetic energy that is later converted into another form—light in the television tube, for example. (Note that in terms of energy, "downhill" for the electron is "uphill" for a positive charge.) Since energy is related to voltage by $\Delta U = q\Delta V$, we can think of the joule as a coulomb-volt.

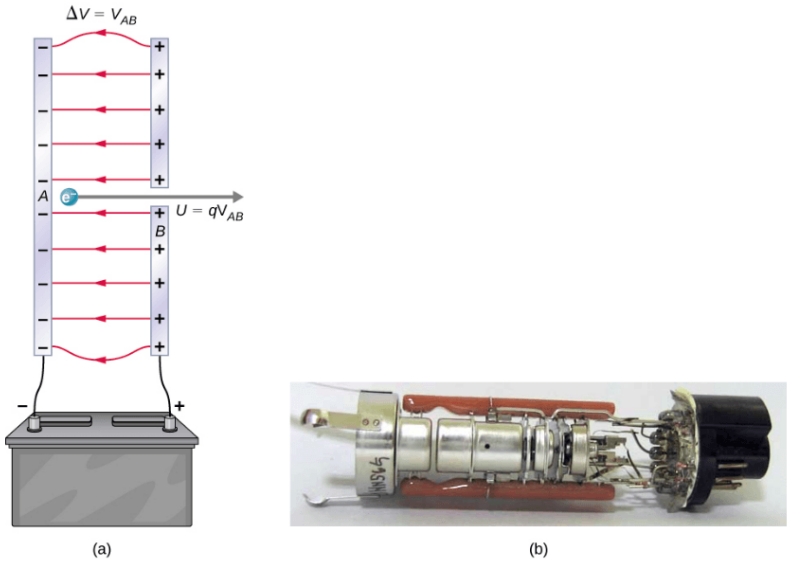


Figure 10.3.2: A typical electron gun accelerates electrons using a potential difference between two separated metal plates. By conservation of energy, the kinetic energy has to equal the change in potential energy, so $KE = qV$. The energy of the electron in electron-volts is numerically the same as the voltage between the plates. For example, a 5000-V potential difference produces 5000-eV electrons. The conceptual construct, namely two parallel plates with a hole in one, is shown in (a), while a real electron gun is shown in (b).

The Electron-Volt Unit

On the submicroscopic scale, it is more convenient to define an energy unit called the **electron-volt** (eV), which is the energy given to a fundamental charge accelerated through a potential difference of 1 V. In equation form,

$$1\,\text{eV} = (1.60 \times 10^{-19}\text{C})(1\,\text{V}) = (1.60 \times 10^{-19}\text{C})(1\,\text{J/C}) = 1.60 \times 10^{-19}\,\text{J}.$$

An electron accelerated through a potential difference of 1 V is given an energy of 1 eV. It follows that an electron accelerated through 50 V gains 50 eV. A potential difference of 100,000 V (100 kV) gives an electron an energy of 100,000 eV (100 keV), and so on. Similarly, an ion with a double positive charge accelerated through 100 V gains 200 eV of energy. These simple relationships between accelerating voltage and particle charges make the electron-volt a simple and convenient energy unit in such circumstances.

The electron-volt is commonly employed in submicroscopic processes—chemical valence energies and molecular and nuclear binding energies are among the quantities often expressed in electron-volts. For example, about 5 eV of energy is required to break up certain organic molecules. If a proton is accelerated from rest through a potential difference of 30 kV, it acquires an energy of 30 keV (30,000 eV) and can break up as many as 6000 of these molecules $(30,000\ \text{eV} \div 5\ \text{eV per molecule} = 6000\ \text{molecules})$. Nuclear decay energies are on the order of 1 MeV (1,000,000 eV) per event and can thus produce significant biological damage.

Conservation of Energy

The total energy of a system is conserved if there is no net addition (or subtraction) due to work or heat transfer. For conservative forces, such as the electrostatic force, conservation of energy states that mechanical energy is a constant.

Mechanical energy is the sum of the kinetic energy and potential energy of a system; that is, $K + U = \text{constant}$. A loss of (\U\) for a charged particle becomes an increase in its (\K\). Conservation of energy is stated in equation form as

$$K + U = \text{constant}$$

or

$$K_i + U_i = K_f + U_f$$

where i and f stand for initial and final conditions. As we have found many times before, considering energy can give us insights and facilitate problem solving.

Example 10.3.3: Electrical Potential Energy Converted into Kinetic Energy

Calculate the final speed of a free electron accelerated from rest through a potential difference of 100 V. (Assume that this numerical value is accurate to three significant figures.)

Strategy

We have a system with only conservative forces. Assuming the electron is accelerated in a vacuum, and neglecting the gravitational force (we will check on this assumption later), all of the electrical potential energy is converted into kinetic energy. We can identify the initial and final forms of energy to be

$K_i = 0$, $K_f = \frac{1}{2}mv^2$, $U_i = qV$, $U_f = 0$.

Solution

Conservation of energy states that

$$K_i + U_i = K_f + U_f.$$

Entering the forms identified above, we obtain

$$qV = \frac{mv^2}{2}.$$

We solve this for (\v\):

$$v = \sqrt{\frac{2qV}{m}}.$$

Entering values for (\q\), (\V\), and (\m\) gives

$$v = \sqrt{\frac{2(-1.60 \times 10^{-19}\text{C})(-100\,\text{J/C})}{9.11 \times 10^{-31}\text{kg}}} = 5.93 \times 10^{6}\,\text{m/s}.$$

Significance

Note that both the charge and the initial voltage are negative, as in Figure 10.3.2. From the discussion of electric charge and electric field, we know that electrostatic forces on small particles are generally very large compared with the gravitational force. The large final speed confirms that the gravitational force is indeed negligible here. The large speed also indicates how easy it is to accelerate electrons with small voltages because of their very small mass. Voltages much higher than the 100 V in this problem are typically used in electron guns. These higher voltages produce electron speeds so great that effects from special relativity must be taken into account and will be discussed elsewhere. That is why we consider a low voltage (accurately) in this example.

Exercise 10.3.3

How would this example change with a positron? A positron is identical to an electron except the charge is positive.

Answer available at the end of this section.

Voltage and Electric Field

So far, we have explored the relationship between voltage and energy. Now we want to explore the relationship between voltage and electric field. We will start with the general case for a non-uniform $\vec{E}$ field. Recall that our general formula for the potential energy of a test charge q at point P relative to reference point R is

$$U_p = -\int_R^p \vec{F} \cdot d\vec{l}\,.$$

When we substitute in the definition of electric field $(\vec{E} = \vec{F}/q)$, this becomes

$$U_p = -q\int_R^p \vec{E} \cdot d\vec{l}\,.$$

Applying our definition of potential $(V = U/q)$ to this potential energy, we find that, in general,

$$V_p = -\int_R^p \vec{E} \cdot d\vec{l}\,.$$

From our previous discussion of the potential energy of a charge in an electric field, the result is independent of the path chosen, and hence we can pick the integral path that is most convenient.

Consider the special case of a positive point charge q at the origin. To calculate the potential caused by q at a distance r from the origin relative to a reference of 0 at infinity (recall that we did the same for potential energy), let $P = r$ and $R = \infty$, with $d\vec{l} = d\vec{r} = \hat{r}dr$ and use $\vec{E} = \frac{kq}{r^2}\hat{r}$. When we evaluate the integral

$$V_p = -\int_R^p \vec{E} \cdot d\vec{l}$$

for this system, we have

$$V_r = -\int_\infty^r \frac{kq}{r^2}dr = \frac{kq}{r} - \frac{kq}{\infty} = \frac{kq}{r}.$$

This result,

$$V_r = \frac{kq}{r}$$

is the standard form of the potential of a point charge. This will be explored further in the next section.

To examine another interesting special case, suppose a uniform electric field $\vec{E}$ is produced by placing a potential difference (or voltage) ΔV across two parallel metal plates, labeled A and B (Figure 10.3.3). Examining this situation will tell us what voltage is needed to produce a certain electric field strength. It will also reveal a more fundamental relationship between electric potential

and electric field.

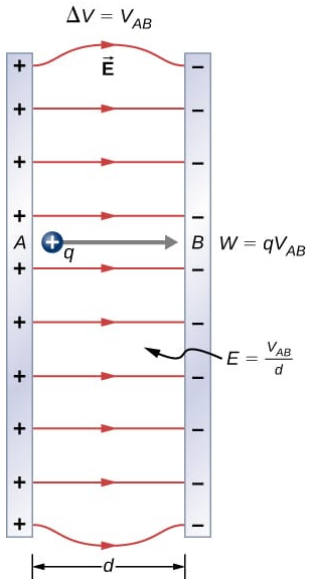


Figure $10.3.3$: The relationship between V and E for parallel conducting plates is $E = V/d$. (Note that $\Delta V = V_{AB}$ in magnitude. For a charge that is moved from plate A at higher potential to plate B at lower potential, a minus sign needs to be included as follows: $-\Delta V = V_A - V_B = V_{AB}$.)

From a physicist's point of view, either ΔV or $\vec{E}$ can be used to describe any interaction between charges. However, ΔV is a scalar quantity and has no direction, whereas $\vec{E}$ is a vector quantity, having both magnitude and direction. (Note that the magnitude of the electric field, a scalar quantity, is represented by E.) The relationship between ΔV and $\vec{E}$ is revealed by calculating the work done by the electric force in moving a charge from point A to point B. But, as noted earlier, arbitrary charge distributions require calculus. We therefore look at a uniform electric field as an interesting special case.

The work done by the electric field in Figure $10.3.3$ to move a positive charge q from A, the positive plate, higher potential, to B, the negative plate, lower potential, is

$$W = -\Delta U = -q\Delta V.$$

The potential difference between points A and B is

$$-\Delta V = -(V_B - V_A) = V_A - V_B = V_{AB}.$$

Entering this into the expression for work yields

$$W = qV_{AB}.$$

Work is $W = \vec{F} \cdot \vec{d} = Fd\cos\theta$: here $\cos\theta = 1$, since the path is parallel to the field. Thus, $W = Fd$. Since $F = qE$ we see that $W = qEd$.

Substituting this expression for work into the previous equation gives

$$qEd = qV_{AB}.$$

The charge cancels, so we obtain for the voltage between points A and B.

In uniform E-field only:

$$V_{AB} = Ed$$

$$E = \frac{V_{AB}}{d}$$

where d is the distance from A to B, or the distance between the plates in Figure $10.3.3$. Note that this equation implies that the units for electric field are volts per meter. We already know the units for electric field are newtons per coulomb; thus, the following

relation among units is valid:

$$1\,\mathrm{N/C} = 1\,\mathrm{V/m}.$$

Furthermore, we may extend this to the integral form. Substituting Equation 10.3.3 into our definition for the potential difference between points A and B we obtain

$$V_{AB} = V_B - V_A = -\int_R^B \vec{E}\cdot d\vec{l} + \int_R^A \vec{E}\cdot d\vec{l}$$

which simplifies to

$$V_B - V_A = -\int_A^B \vec{E}\cdot d\vec{l}\,.$$

As a demonstration, from this we may calculate the potential difference between two points (A and B) equidistant from a point charge q at the origin, as shown in Figure 10.3.4.

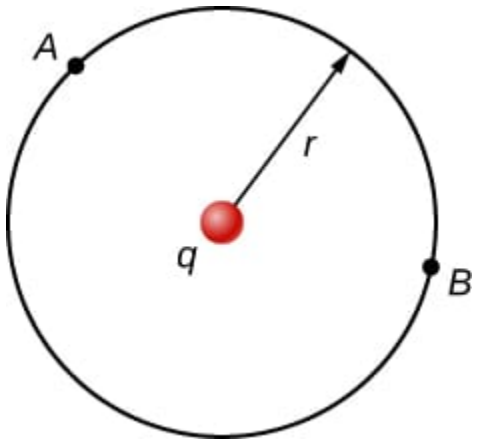


Figure 10.3.4: The arc for calculating the potential difference between two points that are equidistant from a point charge at the origin.

To do this, we integrate around an arc of the circle of constant radius r between A and B, which means we let $d\vec{l} = r\hat{\varphi}d\varphi$, while using $\vec{E} = \frac{kq}{r^2}\hat{r}$. Thus,

$$\Delta V = V_B - V_A = -\int_A^B \vec{E}\cdot d\vec{l}\,.$$

for this system becomes

$$V_B - V_A = -\int_A^B \frac{kq}{r^2}\hat{r}\cdot r\hat{\varphi}d\varphi.$$

However, $\hat{r}\cdot\hat{\varphi}$ and therefore

$$V_B - V_A = 0.$$

This result, that there is no difference in potential along a constant radius from a point charge, will come in handy when we map potentials.

Example 10.3.4A: What Is the Highest Voltage Possible between Two Plates?

Dry air can support a maximum electric field strength of about $3.0\times10^6\ \mathrm{V/m}$. Above that value, the field creates enough ionization in the air to make the air a conductor. This allows a discharge or spark that reduces the field. What, then, is the maximum voltage between two parallel conducting plates separated by 2.5 cm of dry air?

Strategy

We are given the maximum electric field E between the plates and the distance d between them. We can use the equation $V_{AB} = Ed$ to calculate the maximum voltage.

Solution

The potential difference or voltage between the plates is

$$V_{AB} = Ed.$$

Entering the given values for E and d gives

$$V_{AB} = (3.0 \times 10^6 \text{ V/m})(0.025 \text{ m}) = 7.5 \times 10^4 \text{ V}$$

or

$$V_{AB} = 75 \text{ kV}.$$

(The answer is quoted to only two digits, since the maximum field strength is approximate.)

Significance

One of the implications of this result is that it takes about 75 kV to make a spark jump across a 2.5-cm (1-in.) gap, or 150 kV for a 5-cm spark. This limits the voltages that can exist between conductors, perhaps on a power transmission line. A smaller voltage can cause a spark if there are spines on the surface, since sharp points have larger field strengths than smooth surfaces. Humid air breaks down at a lower field strength, meaning that a smaller voltage will make a spark jump through humid air. The largest voltages can be built up with static electricity on dry days (Figure 10.3.5).

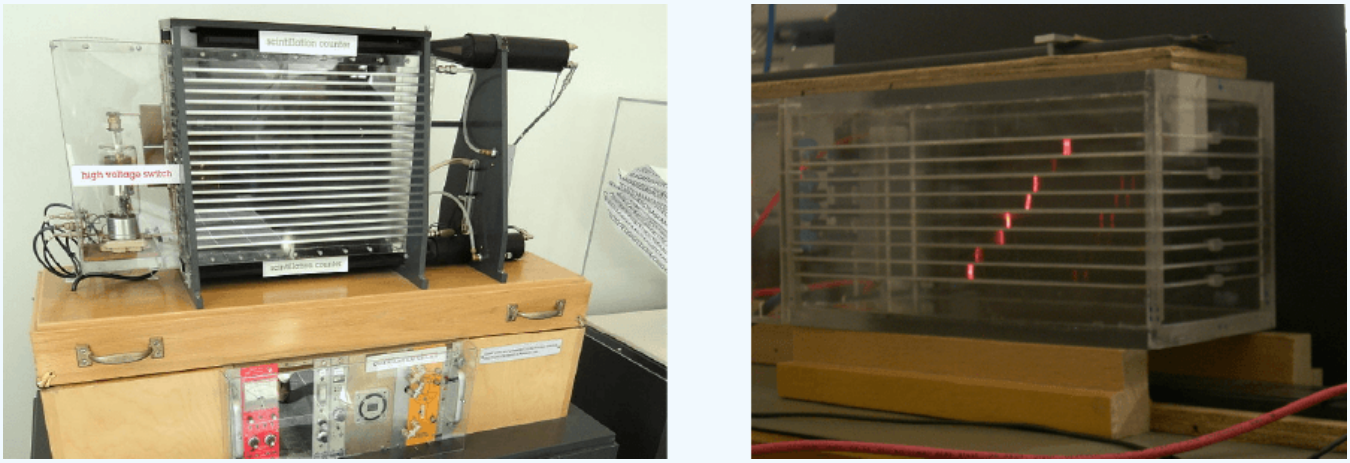


Figure 10.3.5: A spark chamber is used to trace the paths of high-energy particles. Ionization created by the particles as they pass through the gas between the plates allows a spark to jump. The sparks are perpendicular to the plates, following electric field lines between them. The potential difference between adjacent plates is not high enough to cause sparks without the ionization produced by particles from accelerator experiments (or cosmic rays). This form of detector is now archaic and no longer in use except for demonstration purposes. (credit b: modification of work by Jack Collins)

Example 10.3.1B: Field and Force inside an Electron Gun

An electron gun (Figure 10.3.2) has parallel plates separated by 4.00 cm and gives electrons 25.0 keV of energy. (a) What is the electric field strength between the plates? (b) What force would this field exert on a piece of plastic with a $0.500\ \mu\text{C}$ charge that gets between the plates?

Strategy

Since the voltage and plate separation are given, the electric field strength can be calculated directly from the expression $E = \frac{V_{AB}}{d}$. Once we know the electric field strength, we can find the force on a charge by using $\vec{F} = q\vec{E}$. Since the electric field is in only one direction, we can write this equation in terms of the magnitudes, $F = qE$.

Solution

a. The expression for the magnitude of the electric field between two uniform metal plates is

$$E = \frac{V_{AB}}{d}.$$

Since the electron is a single charge and is given 25.0 keV of energy, the potential difference must be 25.0 kV. Entering this value for V_{AB} and the plate separation of 0.0400 m, we obtain

$$E = \frac{25.0\,\text{kV}}{0.0400\,\text{m}} = 6.25 \times 10^5\,\text{V/m}.$$

b. The magnitude of the force on a charge in an electric field is obtained from the equation

$$F = qE.$$

Substituting known values gives

$$F = (0.500 \times 10^{-6}\text{C})(6.25 \times 10^5\text{V/m}) = 0.313\,\text{N}.$$

Significance Note that the units are newtons, since $1\ \text{V/m} = 1\ \text{N/C}$. Because the electric field is uniform between the plates, the force on the charge is the same no matter where the charge is located between the plates.

Example $10.3.4C$: Calculating Potential of a Point Charge

Given a point charge $q = +2.0\ \text{nC}$ at the origin, calculate the potential difference between point P_1 a distance $a = 4.0\ \text{cm}$ from q, and P_2 a distance $b = 12.0\ \text{cm}$ from q, where the two points have an angle of $\varphi = 24^\circ$ between them (Figure 10.3.6).

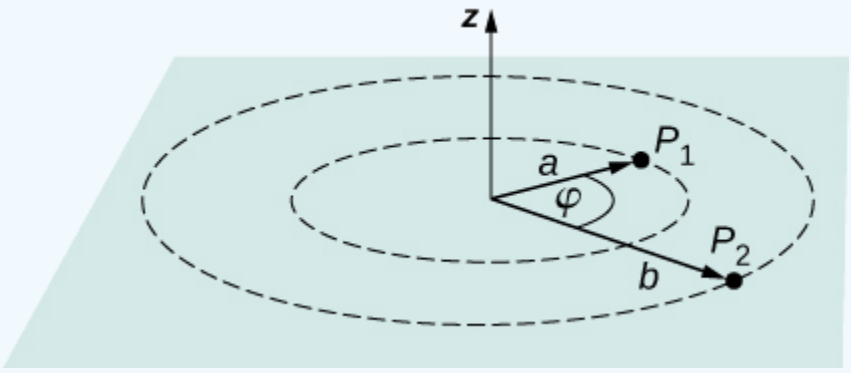


Figure 10.3.6: Find the difference in potential between P_1 and P_2.

Strategy Do this in two steps. The first step is to use $V_B - V_A = -\int_A^B \vec{E}\cdot d\vec{l}$ and let $A = a = 4.0\ \text{cm}$ and $B = b = 12.0\ \text{cm}$, with $d\vec{l} = d\vec{r} = \hat{r}dr$ and $\vec{E} = \frac{kq}{r^2}\hat{r}$. Then perform the integral. The second step is to integrate $V_B - V_A = -\int_A^B \vec{E}\cdot d\vec{l}$ around an arc of constant radius r, which means we let $d\vec{l} = r\hat{\varphi}d\varphi$ with limits $0 \le \varphi \le 24^\circ$, still using $\vec{E} = \frac{kq}{r^2}\hat{r}$.

Then add the two results together.

Solution For the first part, $V_B - V_A = -\int_A^B \vec{E}\cdot d\vec{l}$ for this system becomes $V_b - V_a = -\int_a^b \frac{kq}{r^2}\hat{r}\cdot\hat{r}dr$ which computes to

$$\Delta V = -\int_a^b \frac{kq}{r^2}dr = kq\left[\frac{1}{a} - \frac{1}{b}\right]$$

$$= (8.99 \times 10^9\ \text{N m}^2/\text{C}^2)(2.0 \times 10^{-9}\ \text{C})\left[\frac{1}{0.040\,\text{m}} - \frac{1}{0.12\,\text{m}}\right] = 300\ \text{V}.$$

For the second step, $V_B - V_A = -\int_A^B \vec{E}\cdot d\vec{l}$ becomes $\Delta V = -\int_{0^\circ}^{24^\circ} \frac{kq}{r^2}\hat{r}\cdot r\hat{\varphi}d\varphi$, but $\hat{r}\cdot\hat{\varphi} = 0$ and therefore $\Delta V = 0$. Adding the two parts together, we get 300 V.

Significance

We have demonstrated the use of the integral form of the potential difference to obtain a numerical result. Notice that, in this particular system, we could have also used the formula for the potential due to a point charge at the two points and simply taken the difference.

Exercise 10.3.4

From the examples, how does the energy of a lightning strike vary with the height of the clouds from the ground? Consider the cloud-ground system to be two parallel plates.

Answer available at the end of this section.

Before presenting problems involving electrostatics, we suggest a problem-solving strategy to follow for this topic.

Problem-Solving Strategy: Electrostatics

1. Examine the situation to determine if static electricity is involved; this may concern separated stationary charges, the forces among them, and the electric fields they create.
2. Identify the system of interest. This includes noting the number, locations, and types of charges involved.
3. Identify exactly what needs to be determined in the problem (identify the unknowns). A written list is useful. Determine whether the Coulomb force is to be considered directly—if so, it may be useful to draw a free-body diagram, using electric field lines.
4. Make a list of what is given or can be inferred from the problem as stated (identify the knowns). It is important to distinguish the Coulomb force F from the electric field E, for example.
5. Solve the appropriate equation for the quantity to be determined (the unknown) or draw the field lines as requested.
6. Examine the answer to see if it is reasonable: Does it make sense? Are units correct and the numbers involved reasonable?

Answers

Exercise 10.3.1

$\Delta U = q\Delta V = (100\ \text{C})(1.5\ \text{V}) = 150\ \text{J}$

Exercise 10.3.2

$-2.00\ C,\ n_e = 1.25 \times 10^{19}\ electrons$

Exercise 10.3.3

It would be going in the opposite direction, with no effect on the calculations as presented.

Exercise 10.3.4

Given a fixed maximum electric field strength, the potential at which a strike occurs increases with increasing height above the ground. Hence, each electron will carry more energy. Determining if there is an effect on the total number of electrons lies in the future.

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10.4: Calculations of Electric Potential

Learning Objectives

By the end of this section, you will be able to:

- Calculate the potential due to a point charge
- Calculate the potential of a system of multiple point charges
- Describe an electric dipole
- Define dipole moment
- Calculate the potential of a continuous charge distribution

Point charges, such as electrons, are among the fundamental building blocks of matter. Furthermore, spherical charge distributions (such as charge on a metal sphere) create external electric fields exactly like a point charge. The electric potential due to a point charge is, thus, a case we need to consider.

We can use calculus to find the work needed to move a test charge q from a large distance away to a distance of r from a point charge q. Noting the connection between work and potential $W = -q\Delta V$, as in the last section, we can obtain the following result.

Electric Potential V of a Point Charge

The electric potential V of a point charge is given by

$$\underbrace{V = \frac{kq}{r}}_{\text{point charge}} \tag{10.4.1}$$

where k is a constant equal to $8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$.

The potential in Equation 10.4.1 at infinity is chosen to be zero. Thus, V for a point charge decreases with distance, whereas $\vec{E}$ for a point charge decreases with distance squared:

$$E = \frac{F}{q_t} = \frac{kq}{r^2}$$

Recall that the electric potential V is a scalar and has no direction, whereas the electric field $\vec{E}$ is a vector. To find the voltage due to a combination of point charges, you add the individual voltages as numbers. To find the total electric field, you must add the individual fields as vectors, taking magnitude and direction into account. This is consistent with the fact that V is closely associated with energy, a scalar, whereas $\vec{E}$ is closely associated with force, a vector.

Example 10.4.1: What Voltage Is Produced by a Small Charge on a Metal Sphere?

Charges in static electricity are typically in the nanocoulomb (nC) to microcoulomb (μC) range. What is the voltage 5.00 cm away from the center of a 1-cm-diameter solid metal sphere that has a –3.00-nC static charge?

Strategy

As we discussed in Electric Charges and Fields, charge on a metal sphere spreads out uniformly and produces a field like that of a point charge located at its center. Thus, we can find the voltage using the equation $V = \frac{kq}{r}$.

Solution

Entering known values into the expression for the potential of a point charge (Equation 10.4.1), we obtain

$$\begin{aligned} V &= k\frac{q}{r} \\ &= (9.00 \times 10^9\,\mathrm{N \cdot m^2/C^2})\left(\frac{-3.00 \times 10^{-9}\mathrm{C}}{5.00 \times 10^{-2}\mathrm{m}}\right) \\ &= -539\,\mathrm{V}. \end{aligned}$$

Significance

The negative value for voltage means a positive charge would be attracted from a larger distance, since the potential is lower (more negative) than at larger distances. Conversely, a negative charge would be repelled, as expected.

Example 10.4.2: What Is the Excess Charge on a Van de Graaff Generator?

A demonstration **Van de Graaff generator** has a 25.0-cm-diameter metal sphere that produces a voltage of 100 kV near its surface (Figure). What excess charge resides on the sphere? (Assume that each numerical value here is shown with three significant figures.)

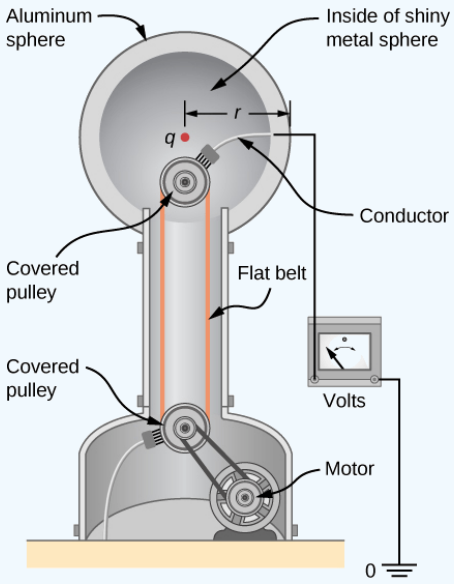


Figure 10.4.1: The voltage of this demonstration Van de Graaff generator is measured between the charged sphere and ground. Earth's potential is taken to be zero as a reference. The potential of the charged conducting sphere is the same as that of an equal point charge at its center.

Strategy

The potential on the surface is the same as that of a point charge at the center of the sphere, 12.5 cm away. (The radius of the sphere is 12.5 cm.) We can thus determine the excess charge using Equation 10.4.1

$$V = \frac{kq}{r}.$$

Solution

Solving for q and entering known values gives

$$\begin{aligned} q &= \frac{rV}{k} \\ &= \frac{(0.125\,\text{m})(100 \times 10^3\,\text{V})}{8.99 \times 10^9\,\text{N} \cdot \text{m}^2/\text{C}^2} \\ &= 1.39 \times 10^{-6}\,\text{C} \\ &= 1.39\,\mu\text{C}. \end{aligned}$$

Significance

This is a relatively small charge, but it produces a rather large voltage. We have another indication here that it is difficult to store isolated charges.

Exercise 10.4.1

What is the potential inside the metal sphere in Example 10.4.1?

Answer available at the end of this section.

The voltages in both of these examples could be measured with a meter that compares the measured potential with ground potential. Ground potential is often taken to be zero (instead of taking the potential at infinity to be zero). It is the potential difference between two points that is of importance, and very often there is a tacit assumption that some reference point, such as Earth or a very distant point, is at zero potential. As noted earlier, this is analogous to taking sea level as $h = 0$ when considering gravitational potential energy $U_g = mgh$.

Systems of Multiple Point Charges

Just as the electric field obeys a superposition principle, so does the electric potential. Consider a system consisting of N charges $q_1, q_2, ..., q_N$. What is the net electric potential V at a space point P from these charges? Each of these charges is a source charge that produces its own electric potential at point P, independent of whatever other changes may be doing. Let $V_1, V_2, ..., V_N$ be the electric potentials at P produced by the charges $q_1, q_2, ..., q_N$, respectively. Then, the net electric potential V_p at that point is equal to the sum of these individual electric potentials. You can easily show this by calculating the potential energy of a test charge when you bring the test charge from the reference point at infinity to point P:

$$V_p = V_1 + V_2 + ... + V_N = \sum_1^N V_i.$$

Note that electric potential follows the same principle of superposition as electric field and electric potential energy. To show this more explicitly, note that a test charge q_i at the point P in space has distances of $r_1, r_2, ..., r_N$ from the N charges fixed in space above, as shown in Figure 10.4.2. Using our formula for the potential of a point charge for each of these (assumed to be point) charges, we find that

$$V_p = \sum_1^N k\frac{q_i}{r_i} = k\sum_1^N \frac{q_i}{r_i}. \tag{10.4.2}$$

Therefore, the electric potential energy of the test charge is

$$U_p = q_t V_p = q_t k \sum_1^N \frac{q_i}{r_i},$$

which is the same as the work to bring the test charge into the system, as found in the first section of the chapter.

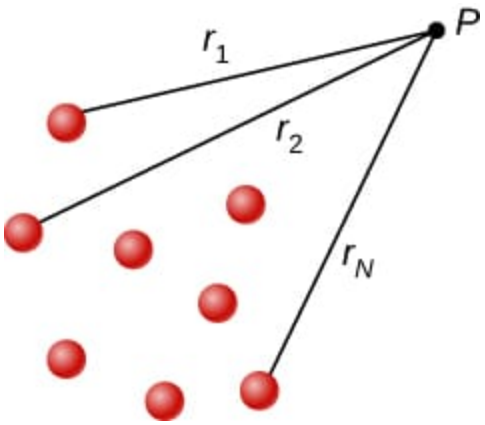


Figure 10.4.2: Notation for direct distances from charges to a space point P.

1The Electric Dipole

An **electric dipole** is a system of two equal but opposite charges a fixed distance apart. This system is used to model many real-world systems, including atomic and molecular interactions. One of these systems is the water molecule, under certain circumstances. These circumstances are met inside a microwave oven, where electric fields with alternating directions make the water molecules change orientation. This vibration is the same as heat at the molecular level.

Example 10.4.3: Electric Potential of a Dipole

Consider the dipole in Figure 10.4.3 with the charge magnitude of $q = 3.0\ \mu\text{C}$ and separation distance $d = 4.0\ \text{cm}$. What is the potential at the following locations in space? (a) (0, 0, 1.0 cm); (b) (0, 0, –5.0 cm); (c) (3.0 cm, 0, 2.0 cm).

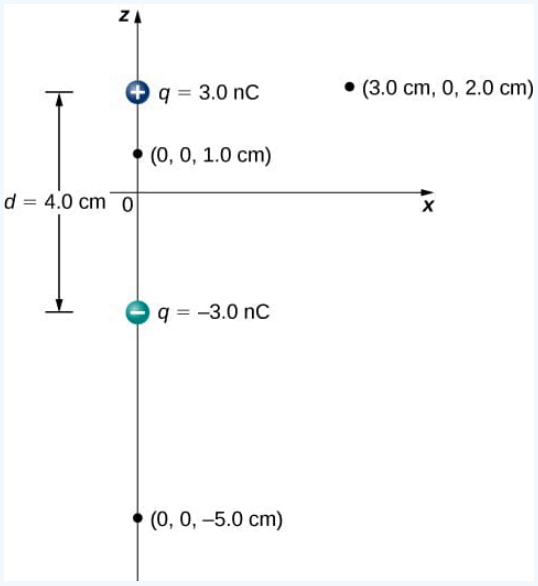


Figure 10.4.3: A general diagram of an electric dipole, and the notation for the distances from the individual charges to a point P in space.

Strategy

Apply $V_p = k\sum_1^N \frac{q_i}{r_i}$ to each of these three points.

Solution

a. $V_p = k\sum_1^N \frac{q_i}{r_i} = (9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\left(\frac{3.0\ \text{nC}}{0.010\,\text{m}} - \frac{3.0\ \text{nC}}{0.030\,\text{m}}\right) = 1.8 \times 10^3\ \text{V}$

b. $V_p = k\sum_1^N \frac{q_i}{r_i} = (9.0 \times 10^9\ \text{N} \cdot \text{m}^2/\text{C}^2)\left(\frac{3.0\ \text{nC}}{0.070\,\text{m}} - \frac{3.0\ \text{nC}}{0.030\,\text{m}}\right) = -5.1 \times 10^2\ \text{V}$

c. $V_p = k\sum_1^N \frac{q_i}{r_i} = (9.0 \times 10^9\ \text{N} \cdot \text{m}^2/\text{C}^2)\left(\frac{3.0\ \text{nC}}{0.030\,\text{m}} - \frac{3.0\ \text{nC}}{0.050\,\text{m}}\right) = 3.6 \times 10^2\ \text{V}$

Significance

Note that evaluating potential is significantly simpler than electric field, due to potential being a scalar instead of a vector.

Exercise 10.4.1

What is the potential on the x-axis? The z-axis?

Answer available at the end of this section.

Now let us consider the special case when the distance of the point P from the dipole is much greater than the distance between the charges in the dipole, $r >> d$; for example, when we are interested in the electric potential due to a polarized molecule such as a water molecule. This is not so far (infinity) that we can simply treat the potential as zero, but the distance is great enough that we can simplify our calculations relative to the previous example.

We start by noting that in Figure 10.4.4 the potential is given by

$$V_p = V_+ + V_- = k\left(\frac{q}{r_+} - \frac{q}{r_-}\right)$$

where

$$r_\pm = \sqrt{x^2 + \left(z \pm \frac{d}{2}\right)^2}.$$

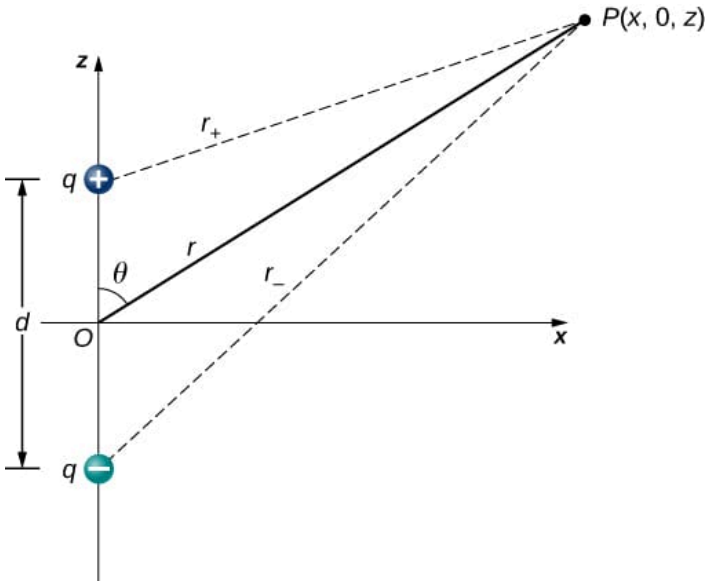


Figure 10.4.4: A general diagram of an electric dipole, and the notation for the distances from the individual charges to a point P in space.

This is still the exact formula. To take advantage of the fact that $r \gg d$, we rewrite the radii in terms of polar coordinates, with $x = r\sin\theta$ and $z = r\cos\theta$. This gives us

$$r_\pm = \sqrt{r^2\sin^2\theta + \left(r\cos\theta \pm \frac{d}{2}\right)^2}.$$

We can simplify this expression by pulling r out of the root,

$$r_{\pm} = \sqrt{\sin^2\,\theta + \left(r\,\cos\,\theta \pm \frac{d}{2}\right)^2}$$

and then multiplying out the parentheses

$$r_{\pm} = r\sqrt{\sin^2\theta + \cos^2\theta \pm \cos\theta\,\frac{d}{r} + \left(\frac{d}{2r}\right)^2} = r\sqrt{1 \pm \cos\theta\,\frac{d}{r} + \left(\frac{d}{2r}\right)^2}.$$

The last term in the root is small enough to be negligible (remember $r >> d$, and hence $(d/r)^2$ is extremely small, effectively zero to the level we will probably be measuring), leaving us with

$$r_{\pm} = r\sqrt{1 \pm \cos\theta\,\frac{d}{r}}.$$

Using the **binomial approximation** (a standard result from the mathematics of series, when a is small)

$$\frac{1}{\sqrt{1 \pm a}} \approx 1 \pm \frac{a}{2}$$

and substituting this into our formula for V_p, we get

$$V_p = k\left[\frac{q}{r}\left(1 + \frac{d\cos\theta}{2r}\right) - \frac{q}{r}\left(1 - \frac{d\cos\theta}{2r}\right)\right] = k\frac{qd\cos\theta}{r^2}.$$

This may be written more conveniently if we define a new quantity, the **electric dipole moment**,

$$\vec{p} = q\vec{d}\,,$$

where these vectors point from the negative to the positive charge. Note that this has magnitude qd. This quantity allows us to write the potential at point P due to a dipole at the origin as

$$V_p = k\frac{\vec{p}\cdot\hat{r}}{r^2}.$$

A diagram of the application of this formula is shown in Figure 10.4.5.

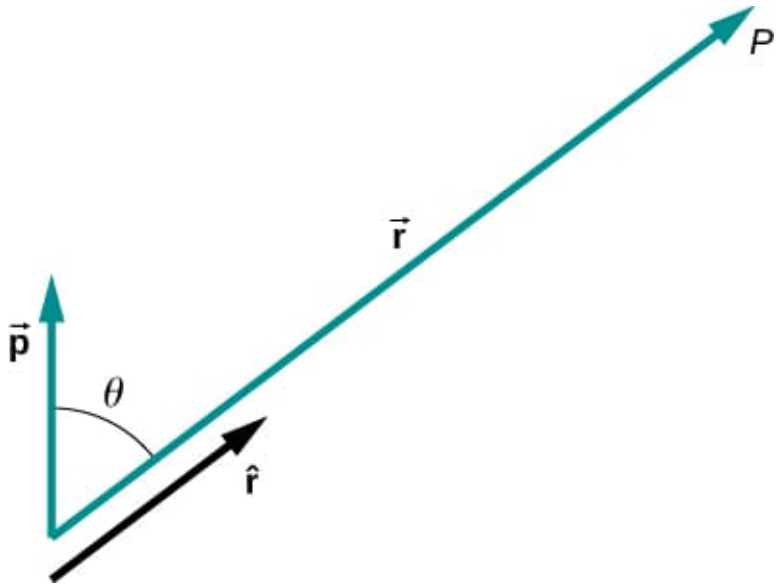


Figure 10.4.5: The geometry for the application of the potential of a dipole.

There are also higher-order moments, for quadrupoles, octupoles, and so on. You will see these in future classes.

Potential of Continuous Charge Distributions

We have been working with point charges a great deal, but what about continuous charge distributions? Recall from Equation 10.4.2 that

$$V_p = k\sum \frac{q_i}{r_i}.$$

We may treat a continuous charge distribution as a collection of infinitesimally separated individual points. This yields the integral

$$V_p = \int \frac{dq}{r}$$

for the potential at a point P. Note that r is the distance from each individual point in the charge distribution to the point P. As we saw in Electric Charges and Fields, the infinitesimal charges are given by

$$\underbrace{dq = \lambda\, dl}_{\text{one dimension}}$$

$$\underbrace{dq = \sigma\, dA}_{\text{two dimensions}}$$

$$\underbrace{dq = \rho\, dV}_{\text{three dimensions}}$$

where λ is linear charge density, σ is the charge per unit area, and ρ is the charge per unit volume.

Example 10.4.4: Potential of a Line of Charge

Find the electric potential of a uniformly charged, nonconducting wire with linear density λ (coulomb/meter) and length L at a point that lies on a line that divides the wire into two equal parts.

Strategy

To set up the problem, we choose Cartesian coordinates in such a way as to exploit the symmetry in the problem as much as possible. We place the origin at the center of the wire and orient the y-axis along the wire so that the ends of the wire are at $y = \pm L/2$. The field point P is in the xy-plane and since the choice of axes is up to us, we choose the x-axis to pass through the field point P, as shown in Figure 10.4.6.

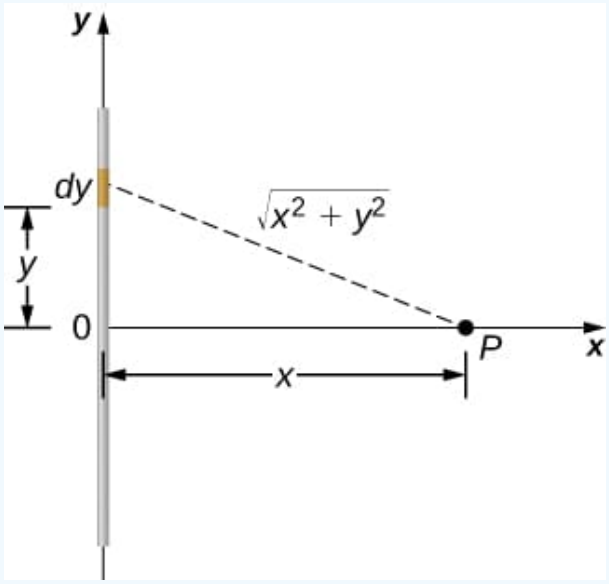


Figure 10.4.6: We want to calculate the electric potential due to a line of charge.

Solution

Consider a small element of the charge distribution between y and $y+dy$. The charge in this cell is $dq=\lambda\ dy$ and the distance from the cell to the field point P is $\sqrt{x^2+y^2}$. Therefore, the potential becomes

$$\begin{aligned} V_p &= k\int \frac{dq}{r} \\ &= k\int_{-L/2}^{L/2} \frac{\lambda\, dy}{\sqrt{x^2+y^2}} \\ &= k\lambda\left[\ln\left(y+\sqrt{y^2+x^2}\right)\right]_{-L/2}^{L/2} \\ &= k\lambda\left[\ln\left(\left(\frac{L}{2}\right)+\sqrt{\left(\frac{L}{2}\right)^2+x^2}\right)-\ln\left(\left(-\frac{L}{2}\right)+\sqrt{\left(-\frac{L}{2}\right)^2+x^2}\right)\right] \\ &= k\lambda\ln\left[\frac{L+\sqrt{L^2+4x^2}}{-L+\sqrt{L^2+4x^2}}\right]. \end{aligned}$$

Significance

Note that this was simpler than the equivalent problem for electric field, due to the use of scalar quantities. Recall that we expect the zero level of the potential to be at infinity, when we have a finite charge. To examine this, we take the limit of the above potential as x approaches infinity; in this case, the terms inside the natural log approach one, and hence the potential approaches zero in this limit. Note that we could have done this problem equivalently in cylindrical coordinates; the only effect would be to substitute r for x and z for y.

Example 10.4.5: Potential Due to a Ring of Charge

A ring has a uniform charge density λ, with units of coulomb per unit meter of arc. Find the electric potential at a point on the axis passing through the center of the ring.

Strategy

We use the same procedure as for the charged wire. The difference here is that the charge is distributed on a circle. We divide the circle into infinitesimal elements shaped as arcs on the circle and use cylindrical coordinates shown in Figure 10.4.7.

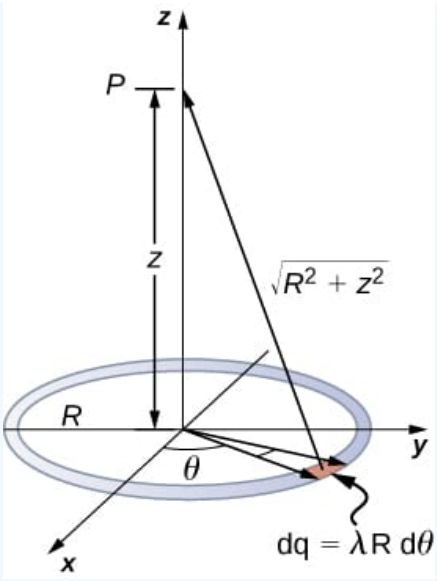


Figure 10.4.7: We want to calculate the electric potential due to a ring of charge.

Solution

A general element of the arc between θ and $\theta + d\theta$ is of length $Rd\theta$ and therefore contains a charge equal to $\lambda Rd\theta$. The element is at a distance of $\sqrt{z^2 + R^2}$ from P, and therefore the potential is

$$\begin{aligned} V_p &= k \int \frac{dq}{r} \\ &= k \int_0^{2\pi} \frac{\lambda R d\theta}{\sqrt{z^2 + R^2}} \\ &= \frac{k\lambda R}{\sqrt{z^2 + R^2}} \int_0^{2\pi} d\theta \\ &= \frac{2\pi k\lambda R}{\sqrt{z^2 + R^2}} \\ &= k \frac{q_{tot}}{\sqrt{z^2 + R^2}}. \end{aligned}$$

Significance

This result is expected because every element of the ring is at the same distance from point P. The net potential at P is that of the total charge placed at the common distance, $\sqrt{z^2 + R^2}$.

Example 10.4.6: Potential Due to a Uniform Disk of Charge

A disk of radius R has a uniform charge density σ with units of coulomb meter squared. Find the electric potential at any point on the axis passing through the center of the disk.

Strategy

We divide the disk into ring-shaped cells, and make use of the result for a ring worked out in the previous example, then integrate over **r** in addition to θ. This is shown in Figure 10.4.8.

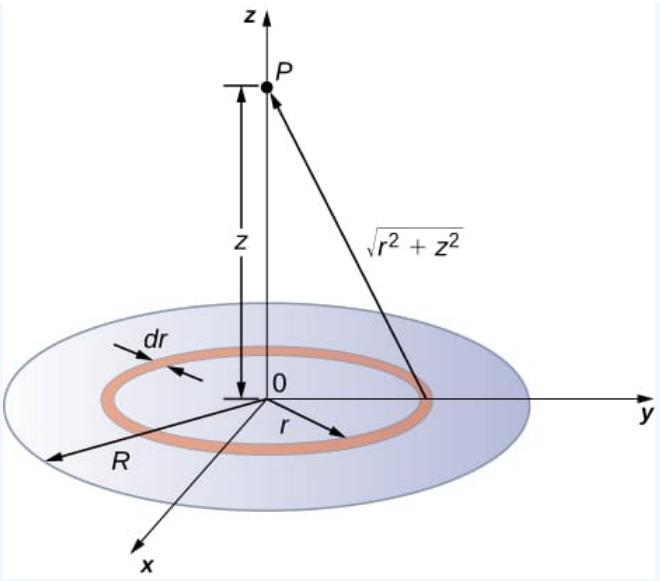


Figure 10.4.8: We want to calculate the electric potential due to a disk of charge.

Solution

An infinitesimal width cell between cylindrical coordinates r and $r+dr$ shown in Figure 10.4.8 will be a ring of charges whose electric potential dV_p at the field point has the following expression

$$dV_p = k\frac{dq}{\sqrt{z^2+r^2}}$$

where

$$dq = \sigma 2\pi r dr.$$

The superposition of potential of all the infinitesimal rings that make up the disk gives the net potential at point P. This is accomplished by integrating from $r=0$ to $r=R$:

$$\begin{aligned} V_p &= \int dV_p = k2\pi\sigma \int_0^R \frac{r\,dr}{\sqrt{z^2+r^2}}, \\ &= k2\pi\sigma(\sqrt{z^2+R^2}-\sqrt{z^2}). \end{aligned}$$

Significance

The basic procedure for a disk is to first integrate around and then over r. This has been demonstrated for uniform (constant) charge density. Often, the charge density will vary with r, and then the last integral will give different results.

Example 10.4.7: Potential Due to an Infinite Charged Wire

Find the electric potential due to an infinitely long uniformly charged wire.

Strategy

Since we have already worked out the potential of a finite wire of length L in Example 10.4.4, we might wonder if taking $L \to \infty$ in our previous result will work:

$$V_p = \lim_{L\to\infty} k\lambda \ln\left(\frac{L+\sqrt{L^2+4x^2}}{-L+\sqrt{L^2+4x^2}}\right).$$

However, this limit does not exist because the argument of the logarithm becomes $[2/0]$ as $L \to \infty$, so this way of finding V of an infinite wire does not work. The reason for this problem may be traced to the fact that the charges are not localized in some space but continue to infinity in the direction of the wire. Hence, our (unspoken) assumption that zero potential must be an infinite distance from the wire is no longer valid.

To avoid this difficulty in calculating limits, let us use the definition of potential by integrating over the electric field from the previous section, and the value of the electric field from this charge configuration from the previous chapter.

Solution

We use the integral

$$V_p = -\int_R^p \vec{E} \cdot d\vec{l}$$

where R is a finite distance from the line of charge, as shown in Figure $10.4.9$.

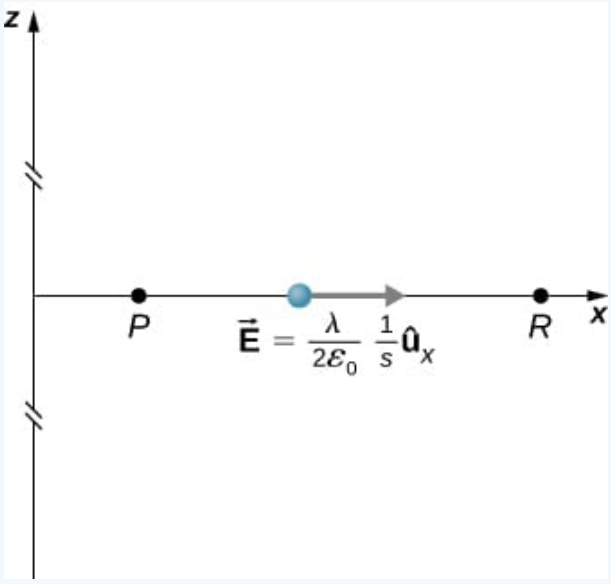


Figure $10.4.9$: Points of interest for calculating the potential of an infinite line of charge.

With this setup, we use $\vec{E}_p = 2k\lambda\frac{1}{s}\hat{s}$ and $d\vec{l} = d\vec{s}$ to obtain

$$\begin{aligned} V_p - V_R &= -\int_R^p 2k\lambda\frac{1}{s}ds \\ &= -2k\lambda \ln\frac{s_p}{s_R}. \end{aligned}$$

Now, if we define the reference potential $V_R = 0$ at $s_R = 1\ m$, this simplifies to

$$V_p = -2k\lambda\ \ln\ s_p.$$

Note that this form of the potential is quite usable; it is 0 at 1 m and is undefined at infinity, which is why we could not use the latter as a reference.

Significance

Although calculating potential directly can be quite convenient, we just found a system for which this strategy does not work well. In such cases, going back to the definition of potential in terms of the electric field may offer a way forward.

Exercise 10.4.7

What is the potential on the axis of a nonuniform ring of charge, where the charge density is $\lambda(\theta) = \lambda \cos\theta$?

Solution

It will be zero, as at all points on the axis, there are equal and opposite charges equidistant from the point of interest. Note that this distribution will, in fact, have a dipole moment.

Answers

Exercise 10.4.1

$$\begin{aligned} V &= k\frac{q}{r} \\ &= (8.99 \times 10^9\,\mathrm{N \cdot m^2/C^2})\left(\frac{-3.00 \times 10^{-9}\,\mathrm{C}}{5.00 \times 10^{-3}\,\mathrm{m}}\right) \\ &= -5390\,\mathrm{V} \end{aligned}$$

Recall that the electric field inside a conductor is zero. Hence, any path from a point on the surface to any point in the interior will have an integrand of zero when calculating the change in potential, and thus the potential in the interior of the sphere is identical to that on the surface.

Exercise 10.4.1

The x-axis the potential is zero, due to the equal and opposite charges the same distance from it. On the z-axis, we may superimpose the two potentials; we will find that for $z >> d$, again the potential goes to zero due to cancellation.

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10.5: Determining Field from Potential

Learning Objectives

By the end of this section, you will be able to:

- Explain how to calculate the electric field in a system from the given potential
- Calculate the electric field in a given direction from a given potential
- Calculate the electric field throughout space from a given potential

Recall that we were able, in certain systems, to calculate the potential by integrating over the electric field. As you may already suspect, this means that we may calculate the electric field by taking derivatives of the potential, although going from a scalar to a vector quantity introduces some interesting wrinkles. We frequently need $\vec{E}$ to calculate the force in a system; since it is often simpler to calculate the potential directly, there are systems in which it is useful to calculate V and then derive $\vec{E}$ from it.

In general, regardless of whether the electric field is uniform, it points in the direction of decreasing potential, because the force on a positive charge is in the direction of $\vec{E}$ and also in the direction of lower potential V. Furthermore, the magnitude of $\vec{E}$ equals the rate of decrease of V with distance. The faster V decreases over distance, the greater the electric field. This gives us the following result.

Relationship between Voltage and Uniform Electric Field

In equation form, the relationship between voltage and uniform electric field is

$$E = -\frac{\Delta V}{\Delta s}$$

where Δs is the distance over which the change in potential ΔV takes place. The minus sign tells us that E points in the direction of decreasing potential. The electric field is said to be the gradient (as in grade or slope) of the electric potential.

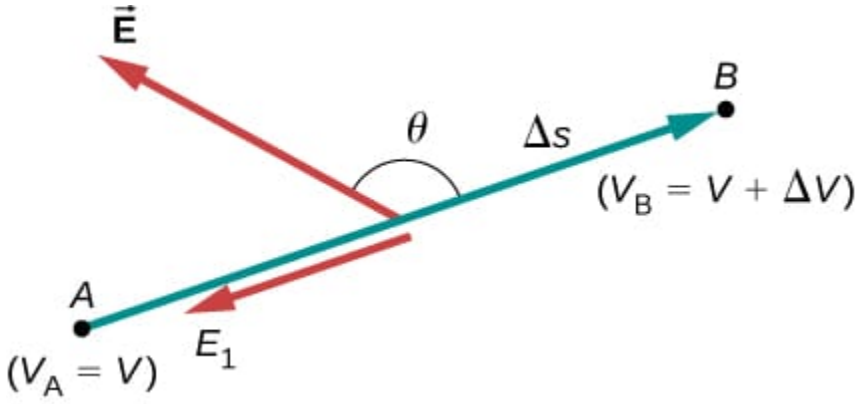


Figure $10.5.1$: The electric field component along the displacement Δs is given by $E = -\frac{\Delta V}{\Delta s}$. Note that A and B are assumed to be so close together that the field is constant along Δs.

For continually changing potentials, ΔV and Δs become infinitesimals, and we need differential calculus to determine the electric field. As shown in Figure $10.5.1$, if we treat the distance Δs as very small so that the electric field is essentially constant over it, we find that

$$E_s = -\frac{dV}{ds}.$$

Therefore, the electric field components in the Cartesian directions are given by

$$E_x = -\frac{\partial V}{\partial x},\ E_y = -\frac{\partial V}{\partial y},\ E_z = -\frac{\partial V}{\partial z}.$$

This allows us to define the “grad” or “del” vector operator, which allows us to compute the gradient in one step. In Cartesian coordinates, it takes the form

$$\vec{\nabla} = \hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}.$$

With this notation, we can calculate the electric field from the potential with

$$\vec{E} = -\vec{\nabla}V, \tag{10.5.1}$$

a process we call **calculating the gradient of the potential**.

If we have a system with either cylindrical or spherical symmetry, we only need to use the del operator in the appropriate coordinates:

$$\vec{\nabla}_{cyl} = \underbrace{\hat{r}\frac{\partial}{\partial r} + \hat{\varphi}\frac{1}{r}\frac{\partial}{\partial \varphi} + \hat{z}\frac{\partial}{\partial z}}_{\text{Cylindrical}} \tag{10.5.2}$$

$$\vec{\nabla}_{sph} = \underbrace{\hat{r}\frac{\partial}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial}{\partial \theta} + \hat{\varphi}\frac{1}{r\sin\theta}\frac{\partial}{\partial \varphi}}_{\text{Spherical}} \tag{10.5.3}$$

Example 10.5.1: Electric Field of a Point Charge

Calculate the electric field of a point charge from the potential.

Strategy

The potential is known to be $V = k\frac{q}{r}$, which has a spherical symmetry. Therefore, we use the spherical del operator (Equation 10.5.3) into Equation 10.5.1:

$$\vec{E} = -\vec{\nabla}_{sph}V.$$

Solution

Performing this calculation gives us

$$\begin{aligned}\vec{E} &= -\left(\hat{r}\frac{\partial}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial}{\partial \theta} + \hat{\varphi}\frac{1}{r\sin\theta}\frac{\partial}{\partial \varphi}\right)k\frac{q}{r} \\ &= -kq\left(\hat{r}\frac{\partial}{\partial r}\frac{1}{r} + \hat{\theta}\frac{1}{r}\frac{\partial}{\partial \theta}\frac{1}{r} + \hat{\varphi}\frac{1}{r\sin\theta}\frac{\partial}{\partial \varphi}\frac{1}{r}\right).\end{aligned}$$

This equation simplifies to

$$\vec{E} = -kq\left(\hat{r}\frac{-1}{r^2} + \hat{\theta}0 + \hat{\varphi}0\right) = k\frac{q}{r^2}\hat{r}$$

as expected.

Significance

We not only obtained the equation for the electric field of a point particle that we’ve seen before, we also have a demonstration that $\vec{E}$ points in the direction of decreasing potential, as shown in Figure 10.5.2.

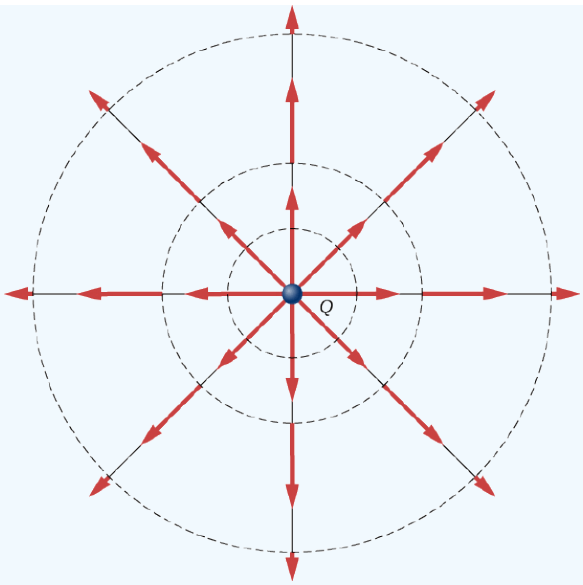


Figure 10.5.2: Electric field vectors inside and outside a uniformly charged sphere.

Example 10.5.2: Electric Field of a Ring of Charge

Use the potential found previously to calculate the electric field along the axis of a ring of charge (Figure 10.5.3).

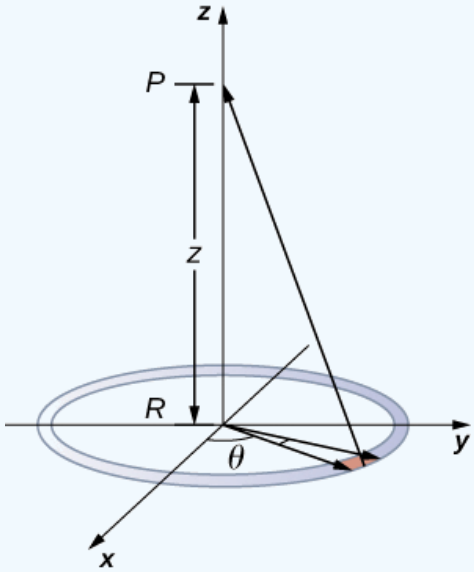


Figure 10.5.3: We want to calculate the electric field from the electric potential due to a ring charge.

Strategy

In this case, we are only interested in one dimension, the z-axis. Therefore, we use

$$E_z = -\frac{\partial V}{\partial z}$$

with the potential

$$V = k\frac{q_{tot}}{\sqrt{z^2 + R^2}}$$

found previously.

Solution

Taking the derivative of the potential yields

$$E_z = -\frac{\partial}{\partial z}\frac{kq_{tot}}{\sqrt{z^2+R^2}}$$
$$= k\frac{q_{tot}z}{(z^2+R^2)^{3/2}}.$$

Significance

Again, this matches the equation for the electric field found previously. It also demonstrates a system in which using the full del operator is not necessary.

Exercise 10.5.1

Which coordinate system would you use to calculate the electric field of a dipole?

Answer available at the end of this section.

Answers

Exercise 10.5.1

Any, but cylindrical is closest to the symmetry of a dipole.

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10.6: Equipotential Surfaces and Conductors

Learning Objectives

By the end of this section, you will be able to:

- Define equipotential surfaces and equipotential lines
- Explain the relationship between equipotential lines and electric field lines
- Map equipotential lines for one or two point charges
- Describe the potential of a conductor
- Compare and contrast equipotential lines and elevation lines on topographic maps

We can represent electric potentials (voltages) pictorially, just as we drew pictures to illustrate electric fields. This is not surprising, since the two concepts are related. Consider Figure $10.6.1$, which shows an isolated positive point charge and its electric field lines, which radiate out from a positive charge and terminate on negative charges. We use blue arrows to represent the magnitude and direction of the electric field, and we use green lines to represent places where the electric potential is constant. These are called **equipotential surfaces** in three dimensions, or **equipotential lines** in two dimensions. The term **equipotential** is also used as a noun, referring to an equipotential line or surface. The potential for a point charge is the same anywhere on an imaginary sphere of radius **r** surrounding the charge. This is true because the potential for a point charge is given by $V = kq/r$ and thus has the same value at any point that is a given distance **r** from the charge. An equipotential sphere is a circle in the two-dimensional view of Figure $10.6.1$. Because the electric field lines point radially away from the charge, they are perpendicular to the equipotential lines.

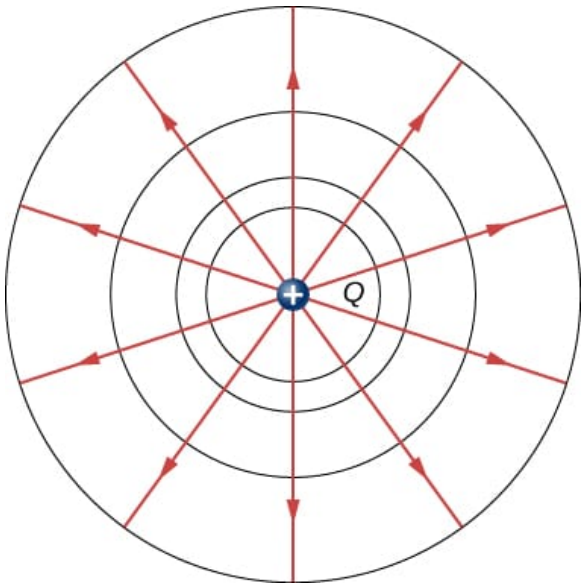


Figure $10.6.1$: An isolated point charge Q with its electric field lines in blue and equipotential lines in green. The potential is the same along each equipotential line, meaning that no work is required to move a charge anywhere along one of those lines. Work is needed to move a charge from one equipotential line to another. Equipotential lines are perpendicular to electric field lines in every case. For a three-dimensional version, explore the first media link.

It is important to note that *equipotential lines are always perpendicular to electric field lines*. No work is required to move a charge along an equipotential, since $\Delta V = 0$. Thus, the work is

$$W = -\Delta U = -q\Delta V = 0.$$

Work is zero if the direction of the force is perpendicular to the displacement. Force is in the same direction as E, so motion along an equipotential must be perpendicular to E. More precisely, work is related to the electric field by

$$W = \vec{F} \cdot \vec{d} \tag{10.6.1}$$

$$= q\vec{E} \cdot \vec{d} \tag{10.6.2}$$

$$= qEd \cos \theta$$

$$= 0.$$

Note that in Equation 10.6.1, E and F symbolize the magnitudes of the electric field and force, respectively. Neither q nor E is zero and d is also not zero. So $\cos\ \theta$ must be 0, meaning θ must be 90°. In other words, motion along an equipotential is perpendicular to **E**.

One of the rules for static electric fields and conductors is that the electric field must be perpendicular to the surface of any conductor. This implies that a *conductor is an equipotential surface in static situations*. There can be no voltage difference across the surface of a conductor, or charges will flow. One of the uses of this fact is that a conductor can be fixed at what we consider zero volts by connecting it to the earth with a good conductor—a process called **grounding**. Grounding can be a useful safety tool. For example, grounding the metal case of an electrical appliance ensures that it is at zero volts relative to Earth.

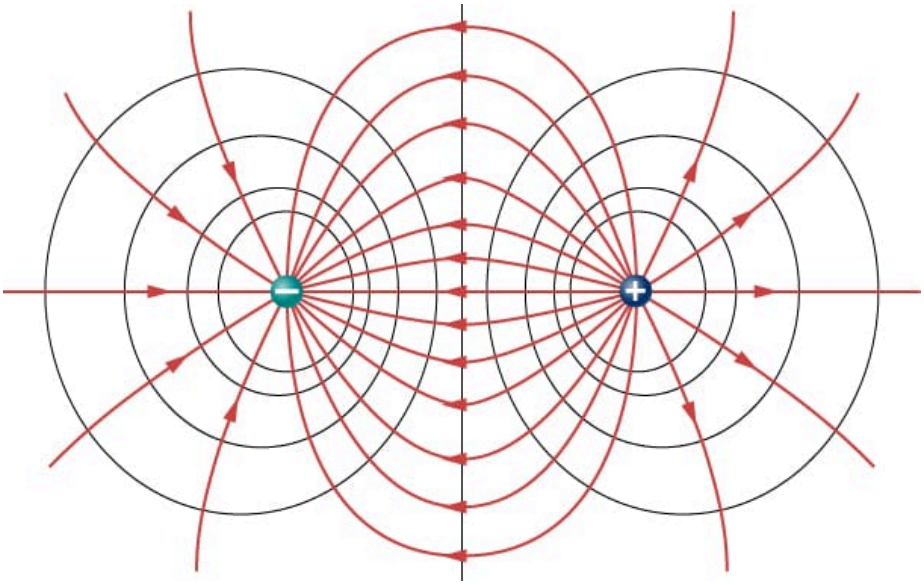


Figure 10.6.2: The electric field lines and equipotential lines for two equal but opposite charges. The equipotential lines can be drawn by making them perpendicular to the electric field lines, if those are known. Note that the potential is greatest (most positive) near the positive charge and least (most negative) near the negative charge. For a three-dimensional version, explore the first media link.

Because a conductor is an equipotential, it can replace any equipotential surface. For example, in Figure 10.6.2, a charged spherical conductor can replace the point charge, and the electric field and potential surfaces outside of it will be unchanged, confirming the contention that a spherical charge distribution is equivalent to a point charge at its center.

Figure 10.6.2 shows the electric field and equipotential lines for two equal and opposite charges. Given the electric field lines, the equipotential lines can be drawn simply by making them perpendicular to the electric field lines. Conversely, given the equipotential lines, as in Figure 10.6.2a, the electric field lines can be drawn by making them perpendicular to the equipotentials, as in Figure 10.6.2b.

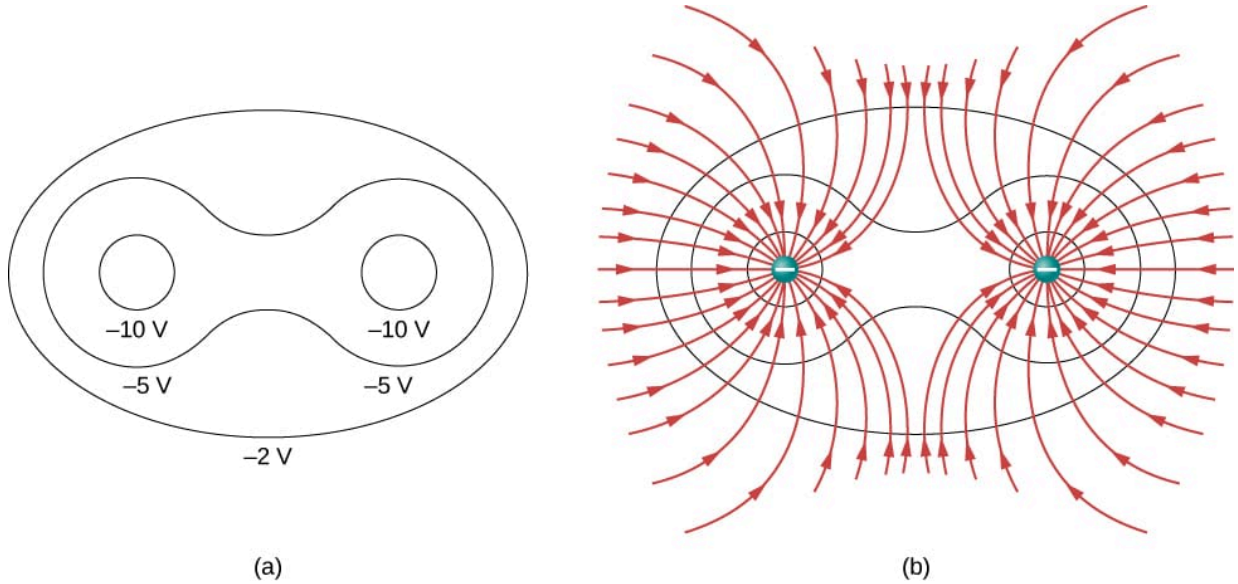


Figure 10.6.3: (a) These equipotential lines might be measured with a voltmeter in a laboratory experiment. (b) The corresponding electric field lines are found by drawing them perpendicular to the equipotentials. Note that these fields are consistent with two equal negative charges. For a three-dimensional version, play with the first media link.

To improve your intuition, we show a three-dimensional variant of the potential in a system with two opposing charges. Figure 10.6.4 displays a three-dimensional map of electric potential, where lines on the map are for equipotential surfaces. The hill is at the positive charge, and the trough is at the negative charge. The potential is zero far away from the charges. Note that the cut off at a particular potential implies that the charges are on conducting spheres with a finite radius.

Figure 10.6.4: Electric potential map of two opposite charges of equal magnitude on conducting spheres. The potential is negative near the negative charge and positive near the positive charge. This dynamic image is powered by CalcPlot3D and can be viewed here.

A two-dimensional map of the cross-sectional plane that contains both charges is shown in Figure 10.6.5. The line that is equidistant from the two opposite charges corresponds to zero potential, since at the points on the line, the positive potential from the positive charge cancels the negative potential from the negative charge. Equipotential lines in the cross-sectional plane are closed loops, which are not necessarily circles, since at each point, the net potential is the sum of the potentials from each charge.

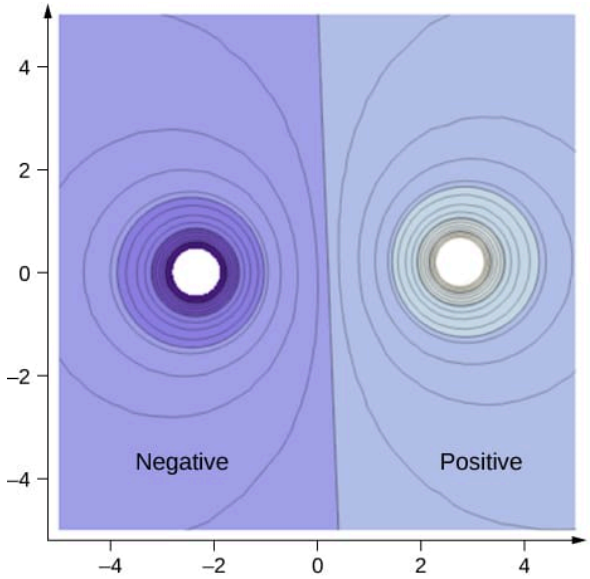


Figure 10.6.5: A cross-section of the electric potential map of two opposite charges of equal magnitude. The potential is negative near the negative charge and positive near the positive charge.

Note

View this simulation to observe and modify the equipotential surfaces and electric fields for many standard charge configurations. There's a lot to explore.

One of the most important cases is that of the familiar parallel conducting plates shown in Figure 10.6.6. Between the plates, the equipotentials are evenly spaced and parallel. The same field could be maintained by placing conducting plates at the equipotential lines at the potentials shown.

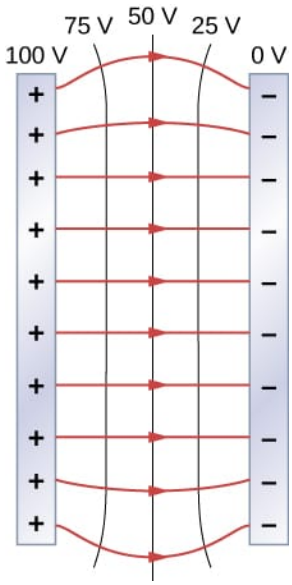


Figure 10.6.6: The electric field and equipotential lines between two metal plates. Note that the electric field is perpendicular to the equipotentials and hence normal to the plates at their surface as well as in the center of the region between them.

Consider the parallel plates Figure 10.6.6. These have equipotential lines that are parallel to the plates in the space between and evenly spaced. An example of this (with sample values) is given in Figure 10.6.6. We could draw a similar set of equipotential isolines for gravity on hills . If the hill has any extent at the same slope, the isolines along that extent would be parallel to each other. Furthermore, in regions of constant slope, the isolines would be evenly spaced. An example of real topographic lines is shown in Figure 10.6.7.

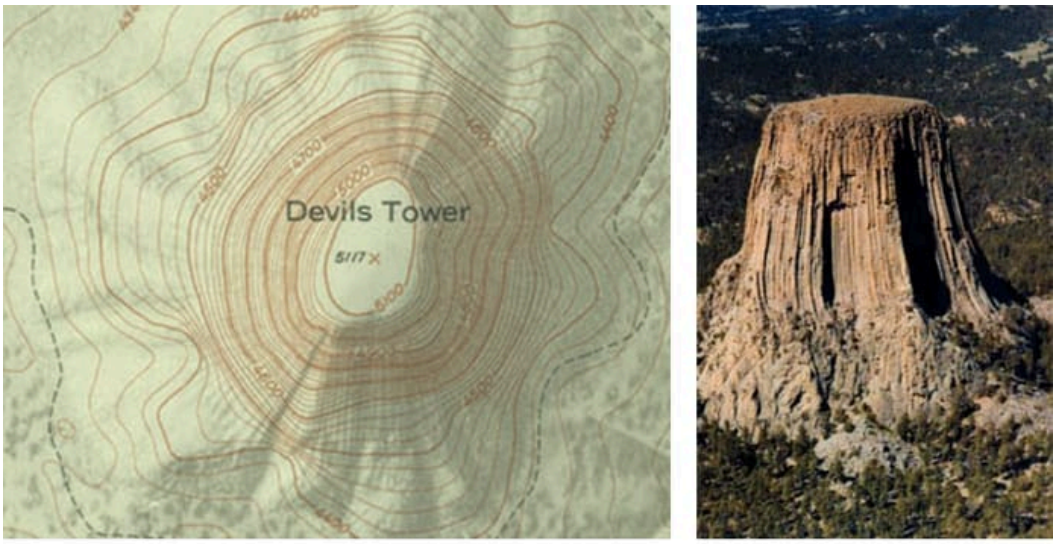


Figure 10.6.6. (a) A topographical map of Devil's Tower, Wyoming. Lines that are close together indicate very steep terrain. (b) A perspective photo of Devil's Tower shows just how steep its sides are. Notice the top of the tower has the same shape as the center of the topographical map.

Example 10.6.1: Calculating Equipotential Lines

You have seen the equipotential lines of a point charge in Figure 10.6.1. How do we calculate them? For example, if we have a $+10\text{-nC}$ charge at the origin, what are the equipotential surfaces at which the potential is (a) 100 V, (b) 50 V, (c) 20 V, and (d) 10 V?

Strategy

Set the equation for the potential of a point charge equal to a constant and solve for the remaining variable(s). Then calculate values as needed.

Solution

In $V = k\frac{q}{r}$, let V be a constant. The only remaining variable is r; hence, $r = k\frac{q}{V} = \text{constant}$. Thus, the equipotential surfaces are spheres about the origin. Their locations are:

a. $r = k\frac{q}{V} = \left(8.99 \times 10^9\ \text{N} \cdot \text{m}^2/\text{C}^2\right)\frac{(10 \times 10^{-9}\ \text{C})}{100\ \text{V}} = 0.90\ \text{m};$

b. $r = k\frac{q}{V} = \left(8.99 \times 10^9\ \text{N} \cdot \text{m}^2/\text{C}^2\right)\frac{(10 \times 10^{-9}\ \text{C})}{50\ \text{V}} = 1.8\ \text{m};$

c. $r = k\frac{q}{V} = \left(8.99 \times 10^9\ \text{N} \cdot \text{m}^2/\text{C}^2\right)\frac{(10 \times 10^{-9}\ \text{C})}{20\ \text{V}} = 4.5\ \text{m};$

d. $r = k\frac{q}{V} = \left(8.99 \times 10^9\ \text{N} \cdot \text{m}^2/\text{C}^2\right)\frac{(10 \times 10^{-9}\ \text{C})}{10\ \text{V}} = 9.0\ \text{m}.$

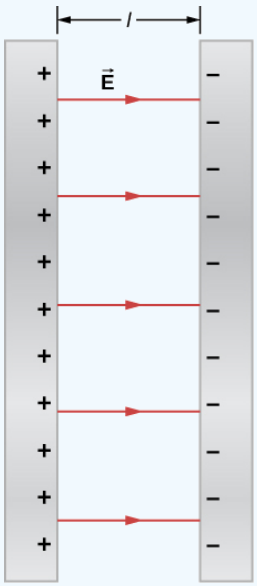
Significance

This means that equipotential surfaces around a point charge are spheres of constant radius, as shown earlier, with well-defined locations.

Example 10.6.2: Potential Difference between Oppositely Charged Parallel Plates

Two large conducting plates carry equal and opposite charges, with a surface charge density σ of magnitude $6.81 \times 10^{-7}\ \text{C}/\text{m}^2$, as shown in Figure 10.6.8. The separation between the plates is $l = 6.50\ \text{mm}$.

a. What is the electric field between the plates?
b. What is the potential difference between the plates?
c. What is the distance between equipotential planes which differ by 100 V?



Strategy

1. Since the plates are described as “large” and the distance between them is not, we will approximate each of them as an infinite plane, and apply the result from Gauss’s law in the previous chapter.
2. Use $\Delta V_{AB} = -\int_A^B \vec{E} \cdot d\vec{l}$.
3. Since the electric field is constant, find the ratio of 100 V to the total potential difference; then calculate this fraction of the distance.

Solution

a. The electric field is directed from the positive to the negative plate as shown in the figure, and its magnitude is given by

$$\begin{aligned} E &= \frac{\sigma}{\epsilon_0} \\ &= \frac{6.81 \times 10^{-7}\,\mathrm{C/m^2}}{8.85 \times 10^{-12}\,\mathrm{C^2/N \cdot m^2}} \\ &= 7.69 \times 10^4\,\mathrm{V/m}. \end{aligned}$$

b. To find the potential difference ΔV between the plates, we use a path from the negative to the positive plate that is directed against the field. The displacement vector $d\vec{l}$ and the electric field $\vec{E}$ are antiparallel so $\vec{E} \cdot d\vec{l} = -E\ dl$. The potential difference between the positive plate and the negative plate is then

$$\begin{aligned} \Delta V &= -\int E \cdot dl \\ &= E \int dl \\ &= El \\ &= (7.69 \times 10^4\,\mathrm{V/m})(6.50 \times 10^{-3}\,\mathrm{m}) \\ &= 500\,\mathrm{V} \end{aligned}$$

c. The total potential difference is 500 V, so 1/5 of the distance between the plates will be the distance between 100-V potential differences. The distance between the plates is 6.5 mm, so there will be 1.3 mm between 100-V potential differences.

Significance

You have now seen a numerical calculation of the locations of equipotentials between two charged parallel plates.

Exercise 10.6.1

What are the equipotential surfaces for an infinite line charge?

Answer available at the end of this section.

Distribution of Charges on Conductors

In Example 10.6.1 with a point charge, we found that the equipotential surfaces were in the form of spheres, with the point charge at the center. Given that a conducting sphere in electrostatic equilibrium is a spherical equipotential surface, we should expect that we could replace one of the surfaces in Example 10.6.1 with a conducting sphere and have an identical solution outside the sphere. Inside will be rather different, however.

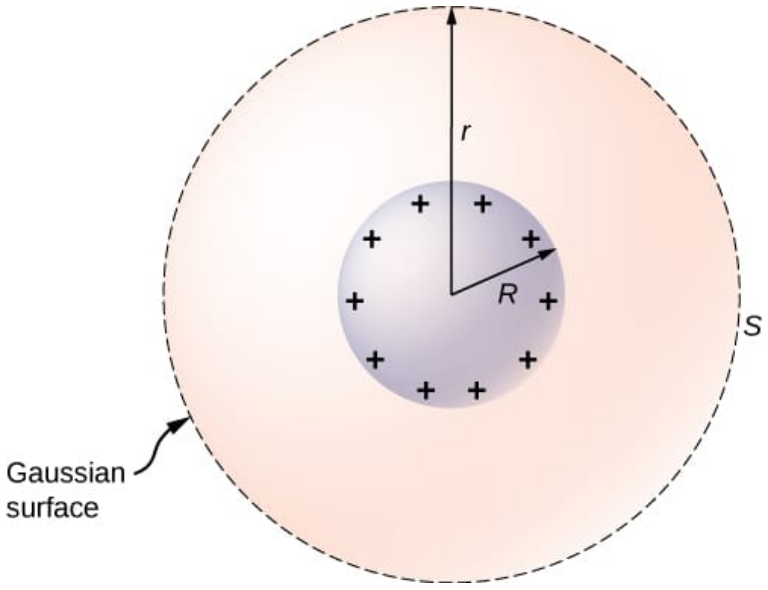


Figure 10.6.9: An isolated conducting sphere.

To investigate this, consider the isolated conducting sphere of Figure 10.6.9 that has a radius R and an excess charge q. To find the electric field both inside and outside the sphere, note that the sphere is isolated, so its surface change distribution and the electric field of that distribution are spherically symmetric. We can therefore represent the field as $\vec{E} = E(r)\hat{r}$. To calculate $E(r)$, we apply Gauss's law over a closed spherical surface S of radius r that is concentric with the conducting sphere. Since r is constant and $\hat{n} = \hat{r}$ on the sphere,

$$\oint \vec{E} \cdot \hat{n}\, da = E(r) \oint da \tag{10.6.3}$$

$$= E(r) 4\pi r^2. \tag{10.6.4}$$

For $r < R$, S is within the conductor, so recall from our previous study of Gauss's law that $q_{enc} = 0$ and Gauss's law gives $E(r) = 0$, as expected inside a conductor at equilibrium. If $r > R$, S encloses the conductor so $q_{enc} = q$. From Gauss's law,

$$E(r) 4\pi r^2 = \frac{q}{\epsilon_0}.$$

The electric field of the sphere may therefore be written as

$$E = 0\,(r < R),$$

and

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}\,(r \geq R).$$

As expected, in the region $r \geq R$, the electric field due to a charge q placed on an isolated conducting sphere of radius R is identical to the electric field of a point charge q located at the center of the sphere.

To find the electric potential inside and outside the sphere, note that for $r \geq R$, the potential must be the same as that of an isolated point charge q located at $r = 0$,

$$V(r) = \frac{1}{4\pi\epsilon_0}\frac{q}{r}(r \geq R)$$

simply due to the similarity of the electric field.

For $r < R,\ \ E = 0$, so $V(r)$ is constant in this region. Since $V(R) = q/4\pi\epsilon_0 R$,

$$V(r) = \frac{1}{4\pi\epsilon_0}\frac{q}{R}(r < R).$$

We will use this result to show that

$$\sigma_1 R_1 = \sigma_2 R_2,$$

for two conducting spheres of radii R_1 and R_2, with surface charge densities σ_1 and σ_2 respectively, that are connected by a thin wire, as shown in Figure $10.6.10$. The spheres are sufficiently separated so that each can be treated as if it were isolated (aside from the wire). Note that the connection by the wire means that this entire system must be an equipotential.

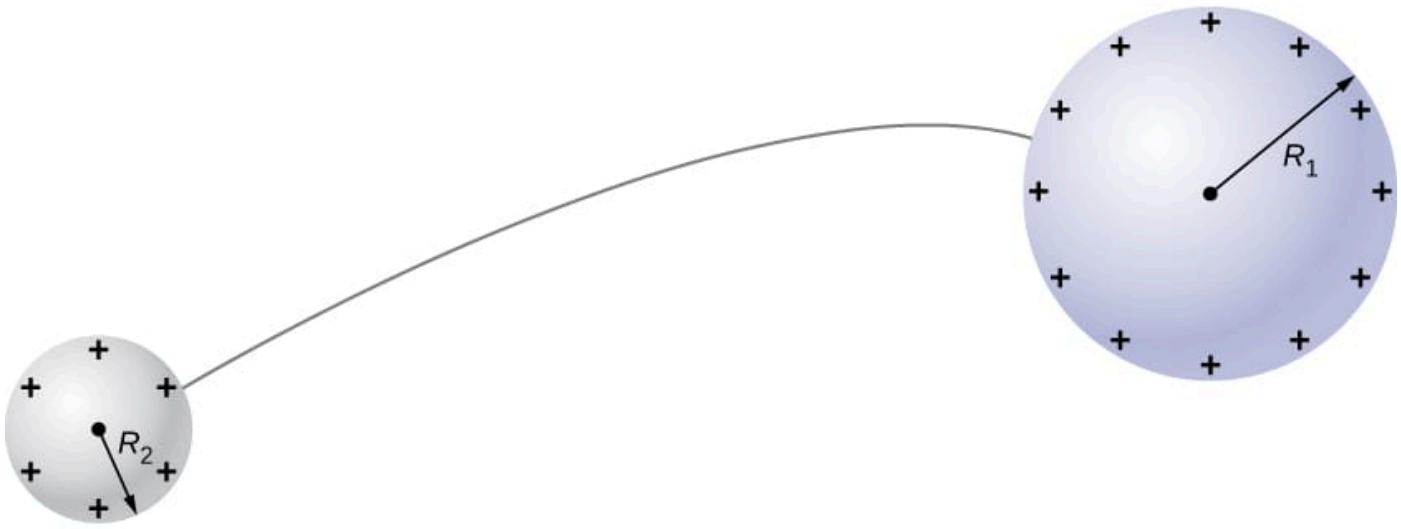


Figure $10.6.10$: Two conducting spheres are connected by a thin conducting wire.

We have just seen that the electrical potential at the surface of an isolated, charged conducting sphere of radius R is

$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{R}.$$

Now, the spheres are connected by a conductor and are therefore at the same potential; hence

$$\frac{1}{4\pi\epsilon_0}\frac{q_1}{R_1} = \frac{1}{4\pi r\epsilon_0}\frac{q_2}{R_2},$$

and

$$\frac{q_1}{R_1} = \frac{q_2}{R_2}.$$

The net charge on a conducting sphere and its surface charge density are related by $q = \sigma(4\pi R^2)$. Substituting this equation into the previous one, we find

$$\sigma_1 R_1 = \sigma_2 R_2.$$

Obviously, two spheres connected by a thin wire do not constitute a typical conductor with a variable radius of curvature. Nevertheless, this result does at least provide a qualitative idea of how charge density varies over the surface of a conductor. The equation indicates that where the radius of curvature is large (points B and D in $10.6.11$), σ and E are small.

Similarly, the charges tend to be denser where the curvature of the surface is greater, as demonstrated by the charge distribution on oddly shaped metal (Figure $10.6.11$). The surface charge density is higher at locations with a small radius of curvature than at locations with a large radius of curvature.

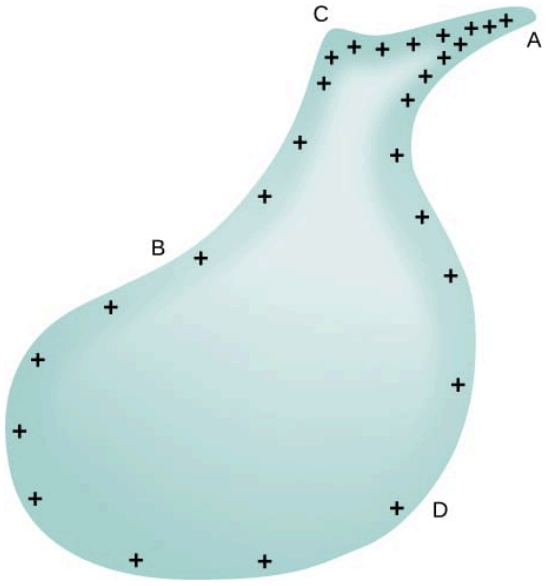


Figure $10.6.11$: The surface charge density and the electric field of a conductor are greater at regions with smaller radii of curvature.

A practical application of this phenomenon is the **lightning rod**, which is simply a grounded metal rod with a sharp end pointing upward. As positive charge accumulates in the ground due to a negatively charged cloud overhead, the electric field around the sharp point gets very large. When the field reaches a value of approximately $3.0 \times 10^6\ \mathrm{N/C}$ (the **dielectric strength** of the air), the free ions in the air are accelerated to such high energies that their collisions with air molecules actually ionize the molecules. The resulting free electrons in the air then flow through the rod to Earth, thereby neutralizing some of the positive charge. This keeps the electric field between the cloud and the ground from getting large enough to produce a lightning bolt in the region around the rod.

An important application of electric fields and equipotential lines involves the heart. The heart relies on electrical signals to maintain its rhythm. The movement of electrical signals causes the chambers of the heart to contract and relax. When a person has a heart attack, the movement of these electrical signals may be disturbed. An artificial pacemaker and a defibrillator can be used to initiate the rhythm of electrical signals. The equipotential lines around the heart, the thoracic region, and the axis of the heart are useful ways of monitoring the structure and functions of the heart. An electrocardiogram (ECG) measures the small electric signals being generated during the activity of the heart.

PheT

Play around with this simulation to move point charges around on the playing field and then view the electric field, voltages, equipotential lines, and more.

Answers

Exercise 10.6.1

infinite cylinders of constant radius, with the line charge as the axis

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10.7: Applications of Electrostatics

Learning Objectives

By the end of this section, you will be able to:

- Describe some of the many practical applications of electrostatics, including several printing technologies
- Relate these applications to Newton's second law and the electric force

The study of electrostatics has proven useful in many areas. This module covers just a few of the many applications of electrostatics.

The Van de Graaff Generator

Van de Graaff generators (or Van de Graaffs) are not only spectacular devices used to demonstrate high voltage due to static electricity—they are also used for serious research. The first was built by Robert Van de Graaff in 1931 (based on original suggestions by Lord Kelvin) for use in nuclear physics research. Figure $10.7.1$ shows a schematic of a large research version. Van de Graaffs use both smooth and pointed surfaces, and conductors and insulators to generate large static charges and, hence, large voltages.

A very large excess charge can be deposited on the sphere because it moves quickly to the outer surface. Practical limits arise because the large electric fields polarize and eventually ionize surrounding materials, creating free charges that neutralize excess charge or allow it to escape. Nevertheless, voltages of 15 million volts are well within practical limits.

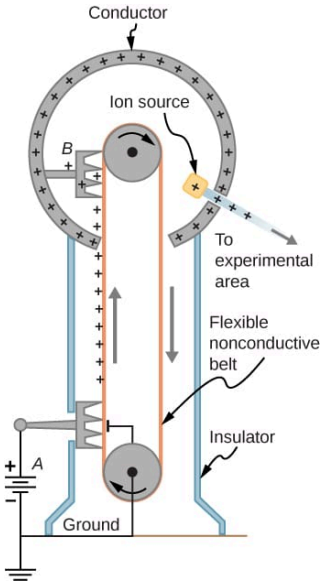


Figure $10.7.1$: Schematic of Van de Graaff generator. A battery (A) supplies excess positive charge to a pointed conductor, the points of which spray the charge onto a moving insulating belt near the bottom. The pointed conductor (B) on top in the large sphere picks up the charge. (The induced electric field at the points is so large that it removes the charge from the belt.) This can be done because the charge does not remain inside the conducting sphere but moves to its outside surface. An ion source inside the sphere produces positive ions, which are accelerated away from the positive sphere to high velocities.

Xerography

Most copy machines use an electrostatic process called xerography—a word coined from the Greek words **xeros** for dry and **graphos** for writing. The heart of the process is shown in simplified form in Figure $10.7.2$.

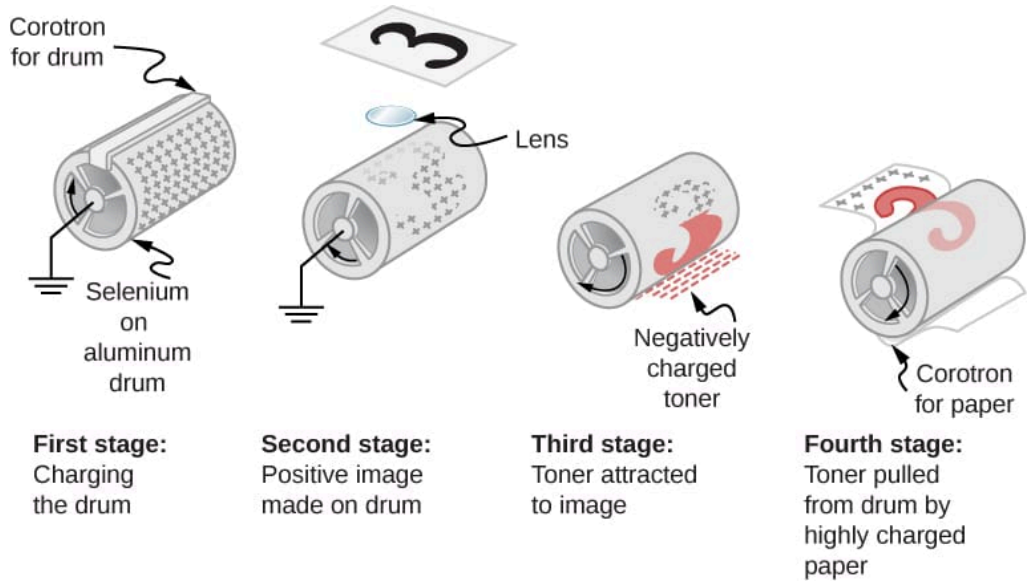


Figure 10.7.2: Xerography is a dry copying process based on electrostatics. The major steps in the process are the charging of the photoconducting drum, transfer of an image, creating a positive charge duplicate, attraction of toner to the charged parts of the drum, and transfer of toner to the paper. Not shown are heat treatment of the paper and cleansing of the drum for the next copy.

A selenium-coated aluminum drum is sprayed with positive charge from points on a device called a corotron. Selenium is a substance with an interesting property—it is a **photoconductor.** That is, selenium is an insulator when in the dark and a conductor when exposed to light.

In the first stage of the xerography process, the conducting aluminum drum is grounded so that a negative charge is induced under the thin layer of uniformly positively charged selenium. In the second stage, the surface of the drum is exposed to the image of whatever is to be copied. In locations where the image is light, the selenium becomes conducting, and the positive charge is neutralized. In dark areas, the positive charge remains, so the image has been transferred to the drum.

The third stage takes a dry black powder, called toner, and sprays it with a negative charge so that it is attracted to the positive regions of the drum. Next, a blank piece of paper is given a greater positive charge than on the drum so that it will pull the toner from the drum. Finally, the paper and electrostatically held toner are passed through heated pressure rollers, which melt and permanently adhere the toner to the fibers of the paper.

Laser Printers

Laser printers use the xerographic process to make high-quality images on paper, employing a laser to produce an image on the photoconducting drum as shown in Figure 10.7.3. In its most common application, the laser printer receives output from a computer, and it can achieve high-quality output because of the precision with which laser light can be controlled. Many laser printers do significant information processing, such as making sophisticated letters or fonts, and in the past may have contained a computer more powerful than the one giving them the raw data to be printed.

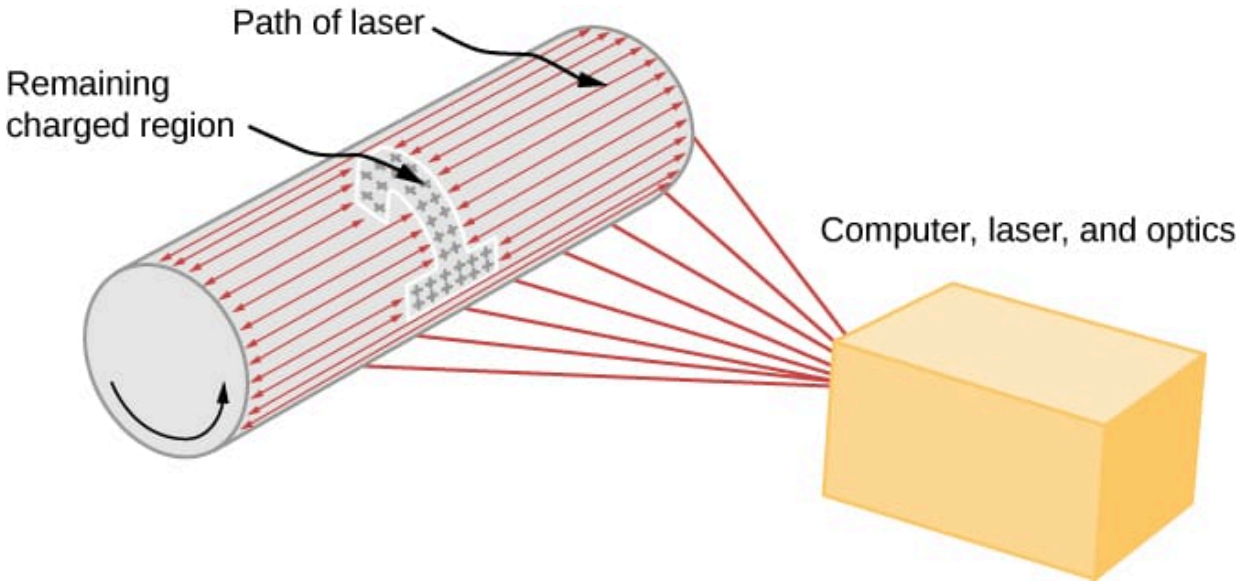


Figure 10.7.3: In a laser printer, a laser beam is scanned across a photoconducting drum, leaving a positively charged image. The other steps for charging the drum and transferring the image to paper are the same as in xerography. Laser light can be very precisely controlled, enabling laser printers to produce high-quality images.

Ink Jet Printers and Electrostatic Painting

The **ink jet printer**, commonly used to print computer-generated text and graphics, also employs electrostatics. A nozzle makes a fine spray of tiny ink droplets, which are then given an electrostatic charge (Figure 10.7.4).

Once charged, the droplets can be directed, using pairs of charged plates, with great precision to form letters and images on paper. Ink jet printers can produce color images by using a black jet and three other jets with primary colors, usually cyan, magenta, and yellow, much as a color television produces color. (This is more difficult with xerography, requiring multiple drums and toners.)

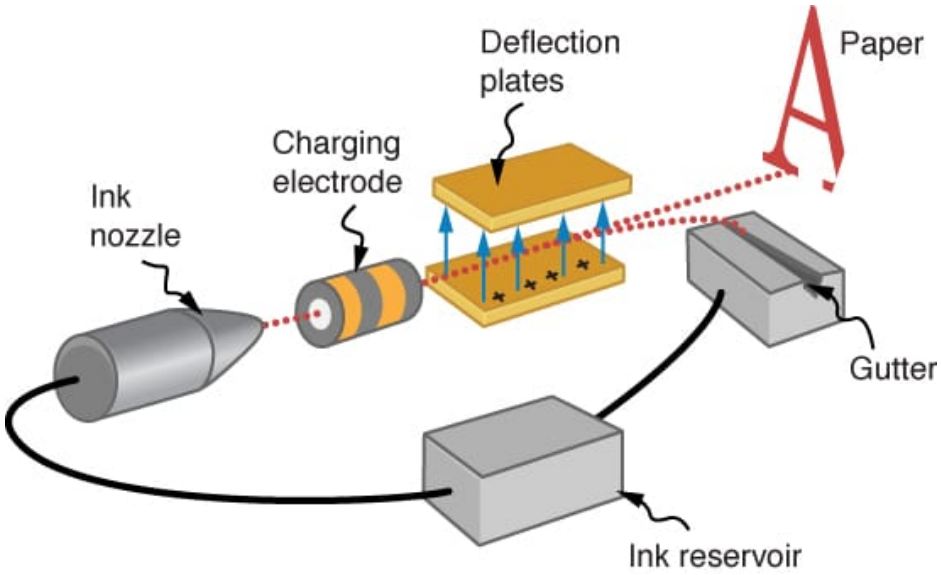


Figure 10.7.4: The nozzle of an ink-jet printer produces small ink droplets, which are sprayed with electrostatic charge. Various computer-driven devices are then used to direct the droplets to the correct positions on a page.

Electrostatic painting employs electrostatic charge to spray paint onto oddly shaped surfaces. Mutual repulsion of like charges causes the paint to fly away from its source. Surface tension forms drops, which are then attracted by unlike charges to the surface to be painted. Electrostatic painting can reach hard-to-get-to places, applying an even coat in a controlled manner. If the object

is a conductor, the electric field is perpendicular to the surface, tending to bring the drops in perpendicularly. Corners and points on conductors will receive extra paint. Felt can similarly be applied.

Smoke Precipitators and Electrostatic Air Cleaning

Another important application of electrostatics is found in air cleaners, both large and small. The electrostatic part of the process places excess (usually positive) charge on smoke, dust, pollen, and other particles in the air and then passes the air through an oppositely charged grid that attracts and retains the charged particles (Figure $10.7.5$)

Large **electrostatic precipitators** are used industrially to remove over 99% of the particles from stack gas emissions associated with the burning of coal and oil. Home precipitators, often in conjunction with the home heating and air conditioning system, are very effective in removing polluting particles, irritants, and allergens.

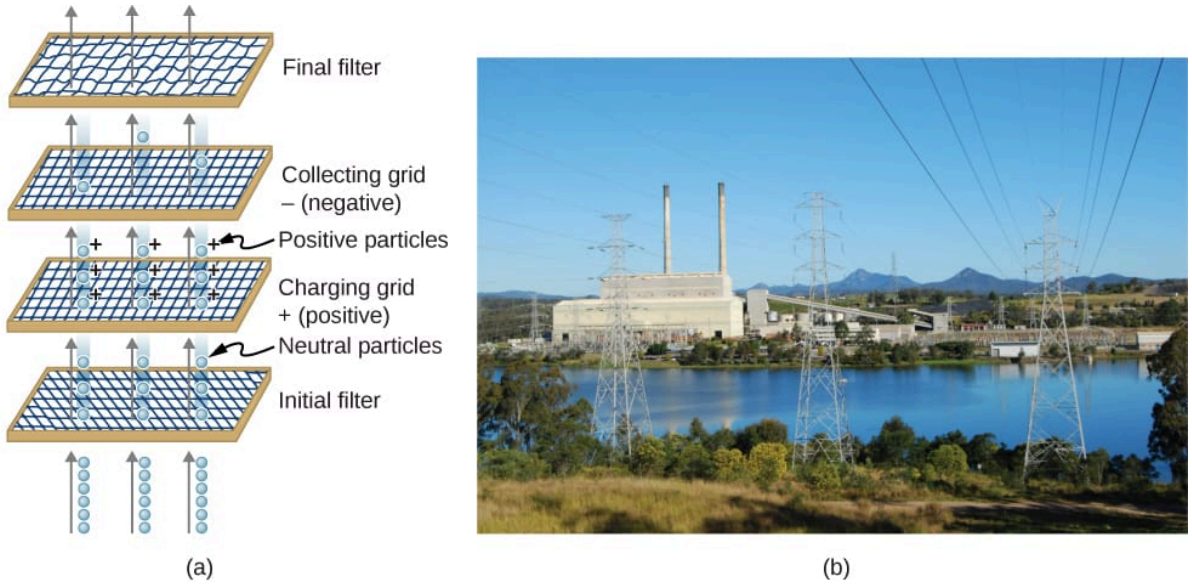


Figure $10.7.5$: (a) Schematic of an electrostatic precipitator. Air is passed through grids of opposite charge. The first grid charges airborne particles, while the second attracts and collects them. (b) The dramatic effect of electrostatic precipitators is seen by the absence of smoke from this power plant. (credit b: modification of work by “Cmdalgleish”/Wikimedia Commons)

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10.8: Electric Potential (Summary)

Key Terms

electric dipole	system of two equal but opposite charges a fixed distance apart
electric dipole moment	quantity defined as $\vec{p} = q\vec{d}$ for all dipoles, where the vector points from the negative to positive charge
electric potential	potential energy per unit charge
electric potential difference	the change in potential energy of a charge q moved between two points, divided by the charge.
electric potential energy	potential energy stored in a system of charged objects due to the charges
electron-volt	energy given to a fundamental charge accelerated through a potential difference of one volt
electrostatic precipitators	filters that apply charges to particles in the air, then attract those charges to a filter, removing them from the airstream
equipotential line	two-dimensional representation of an equipotential surface
equipotential surface	surface (usually in three dimensions) on which all points are at the same potential
grounding	process of attaching a conductor to the earth to ensure that there is no potential difference between it and Earth
ink jet printer	small ink droplets sprayed with an electric charge are controlled by electrostatic plates to create images on paper
photoconductor	substance that is an insulator until it is exposed to light, when it becomes a conductor
Van de Graaff generator	machine that produces a large amount of excess charge, used for experiments with high voltage
voltage	change in potential energy of a charge moved from one point to another, divided by the charge; units of potential difference are joules per coulomb, known as volt
xerography	dry copying process based on electrostatics

Key Equations

Potential energy of a two-charge system	$U(r) = k\frac{qQ}{r}$
Work done to assemble a system of charges	$W_{12\cdots N} = \frac{k}{2}\sum_{i}^{N}\sum_{j}^{N}\frac{q_i q_j}{r_{ij}}$ for $i \neq j$
Potential difference	$\Delta V = \frac{\Delta U}{q}$ or $\Delta U = q\Delta V$
Electric potential	$V = \frac{U}{q} = -\int_{R}^{P}\vec{E}\cdot\vec{dl}$
Potential difference between two points	$\Delta V_{AB} = V_B - V_A = -\int_{A}^{B}\vec{E}\cdot\vec{dl}$
Electric potential of a point charge	$V = \frac{kq}{r}$

Electric potential of a system of point charges	$V_P = k\sum_1^N \frac{q_i}{r_i}$
Electric dipole moment	$\vec{p} = q\vec{d}$
Electric potential due to a dipole	$V_P = k\frac{\vec{p}\cdot\hat{r}}{r^2}$
Electric potential of a continuous charge distribution	$V_P = k\int \frac{dq}{r}$
Electric field components	$E_x = -\frac{\partial V}{\partial x}, E_y = -\frac{\partial V}{\partial y}, E_z = -\frac{\partial V}{\partial z}$
Del operator in Cartesian coordinates	$\vec{\nabla} = \hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$
Electric field as gradient of potential	$\vec{E} = -\vec{\nabla}V$
Del operator in cylindrical coordinates	$\vec{\nabla} = \hat{r}\frac{\partial}{\partial r} + \hat{\varphi}\frac{1}{r}\frac{\partial}{\partial \varphi} + \hat{z}\frac{\partial}{\partial z}$
Del operator in spherical coordinates	$\vec{\nabla} = \hat{r}\frac{\partial}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial}{\partial \theta} + \hat{\varphi}\frac{1}{r\sin\theta}\frac{\partial}{\partial \varphi}$

Summary

7.2 Electric Potential Energy

- The work done to move a charge from point A to B in an electric field is path independent, and the work around a closed path is zero. Therefore, the electric field and electric force are conservative.
- We can define an electric potential energy, which between point charges is $U(r) = k\frac{qQ}{r}$, with the zero reference taken to be at infinity.
- The superposition principle holds for electric potential energy; the potential energy of a system of multiple charges is the sum of the potential energies of the individual pairs.

7.3 Electric Potential and Potential Difference

- Electric potential is potential energy per unit charge.
- The potential difference between points A and B, $V_B - V_A$, that is, the change in potential of a charge q moved from A to B, is equal to the change in potential energy divided by the charge.
- Potential difference is commonly called voltage, represented by the symbol ΔV:

$$\Delta V = \frac{\Delta U}{q} \text{ or } \Delta U = q\Delta V.$$

- An electron-volt is the energy given to a fundamental charge accelerated through a potential difference of 1 V. In equation form,

$$1 \text{ eV} = (1.60 \times 10^{-19} \text{ C})(1 \text{ V}) = (1.60 \times 10^{-19} \text{ C})(1 \text{ J/C}) = 1.60 \times 10^{-19} \text{ J}.$$

7.4 Calculations of Electric Potential

- Electric potential is a scalar whereas electric field is a vector.
- Addition of voltages as numbers gives the voltage due to a combination of point charges, allowing us to use the principle of superposition: $V_P = k\sum_1^N \frac{q_i}{r_i}$.
- An electric dipole consists of two equal and opposite charges a fixed distance apart, with a dipole moment $\vec{p} = q\vec{d}$.
- Continuous charge distributions may be calculated with $V_P = k\int \frac{dq}{r}$.

7.5 Determining Field from Potential

- Just as we may integrate over the electric field to calculate the potential, we may take the derivative of the potential to calculate the electric field.
- This may be done for individual components of the electric field, or we may calculate the entire electric field vector with the gradient operator.

7.6 Equipotential Surfaces and Conductors

- An equipotential surface is the collection of points in space that are all at the same potential. Equipotential lines are the two-dimensional representation of equipotential surfaces.
- Equipotential surfaces are always perpendicular to electric field lines.
- Conductors in static equilibrium are equipotential surfaces.
- Topographic maps may be thought of as showing gravitational equipotential lines.

7.7 Applications of Electrostatics

- Electrostatics is the study of electric fields in static equilibrium.
- In addition to research using equipment such as a Van de Graaff generator, many practical applications of electrostatics exist, including photocopiers, laser printers, ink jet printers, and electrostatic air filters.

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10.9: Electric Potential (Exercises)

Conceptual Questions

7.2: Electric Potential Energy

1. Would electric potential energy be meaningful if the electric field were not conservative?

2. Why do we need to be careful about work done **on** the system versus work done **by** the system in calculations?

3. Does the order in which we assemble a system of point charges affect the total work done?

7.3: Electric Potential and Potential Difference

4. Discuss how potential difference and electric field strength are related. Give an example.

5. What is the strength of the electric field in a region where the electric potential is constant?

6. If a proton is released from rest in an electric field, will it move in the direction of increasing or decreasing potential? Also answer this question for an electron and a neutron. Explain why.

7. Voltage is the common word for potential difference. Which term is more descriptive, voltage or potential difference?

8. If the voltage between two points is zero, can a test charge be moved between them with zero net work being done? Can this necessarily be done without exerting a force? Explain.

9. What is the relationship between voltage and energy? More precisely, what is the relationship between potential difference and electric potential energy?

10. Voltages are always measured between two points. Why?

11. How are units of volts and electron-volts related? How do they differ?

12. Can a particle move in a direction of increasing electric potential, yet have its electric potential energy decrease? Explain

7.4: Calculations of Electric Potential

13. Compare the electric dipole moments of charges $\pm Q$ separated by a distance d and charges $\pm Q/2$ separated by a distance **d/2.**

14. Would Gauss's law be helpful for determining the electric field of a dipole? Why?

15. In what region of space is the potential due to a uniformly charged sphere the same as that of a point charge? In what region does it differ from that of a point charge?

16. Can the potential of a nonuniformly charged sphere be the same as that of a point charge? Explain.

7.5: Determining Field from Potential

17. If the electric field is zero throughout a region, must the electric potential also be zero in that region?

18. Explain why knowledge of $\vec{E}(x, y, z)$ is not sufficient to determine $V(x, y, z)$. What about the other way around?

7.6: Equipotential Surfaces and Conductors

19. If two points are at the same potential, are there any electric field lines connecting them?

20. Suppose you have a map of equipotential surfaces spaced 1.0 V apart. What do the distances between the surfaces in a particular region tell you about the strength of the $\vec{E}$ in that region?

21. Is the electric potential necessarily constant over the surface of a conductor?

22. Under electrostatic conditions, the excess charge on a conductor resides on its surface. Does this mean that all of the conduction electrons in a conductor are on the surface?

23. Can a positively charged conductor be at a negative potential? Explain.

24. Can equipotential surfaces intersect?

7.7: Applications of Electrostatics

25. Why are the metal support rods for satellite network dishes generally grounded?

26. (a) Why are fish reasonably safe in an electrical storm?

(b) Why are swimmers nonetheless ordered to get out of the water in the same circumstance?

27. What are the similarities and differences between the processes in a photocopier and an electrostatic precipitator?

28. About what magnitude of potential is used to charge the drum of a photocopy machine? A web search for “xerography” may be of use.

Problems

7.2 Electric Potential Energy

29. Consider a charge $Q_1(+5.0\mu C)$ fixed at a site with another charge Q_2 (charge $+3.0\mu C$, mass $6.0\mu g$) moving in the neighboring space. (a) Evaluate the potential energy of Q_2 when it is 4.0 cm from Q_1. (b) If Q_2 starts from rest from a point 4.0 cm from Q_1, what will be its speed when it is 8.0 cm from Q_1? (**Note:** Q_1 is held fixed in its place.)

30. Two charges $Q_1(+2.00\mu C)$ and $Q_2(+2.00\mu C)$ are placed symmetrically along the **x**-axis at $x = \pm 3.00cm$. Consider a charge Q_3 of charge $+4.00\mu C$ and mass 10.0 mg moving along the **y**-axis. If Q_3 starts from rest at $y = 2.00cm$, what is its speed when it reaches $y = 4.00cm$?

31. To form a hydrogen atom, a proton is fixed at a point and an electron is brought from far away to a distance of $0.529 \times 10^{-10}m$, the average distance between proton and electron in a hydrogen atom. How much work is done?

32. (a) What is the average power output of a heart defibrillator that dissipates 400 J of energy in 10.0 ms? (b) Considering the high-power output, why doesn’t the defibrillator produce serious burns?

7.3 Electric Potential and Potential Difference

33. Find the ratio of speeds of an electron and a negative hydrogen ion (one having an extra electron) accelerated through the same voltage, assuming non-relativistic final speeds. Take the mass of the hydrogen ion to be $1.67 \times 10^{-27}kg$.

34. An evacuated tube uses an accelerating voltage of 40 kV to accelerate electrons to hit a copper plate and produce X-rays. Non-relativistically, what would be the maximum speed of these electrons?

35. Show that units of **V/m** and **N/C** for electric field strength are indeed equivalent.

36. What is the strength of the electric field between two parallel conducting plates separated by 1.00 cm and having a potential difference (voltage) between them of $1.50 \times 10^4 V$?

37. The electric field strength between two parallel conducting plates separated by 4.00 cm is $7.50 \times 10^4 V$.

(a) What is the potential difference between the plates?

(b) The plate with the lowest potential is taken to be zero volts. What is the potential 1.00 cm from that plate and 3.00 cm from the other?

38. The voltage across a membrane forming a cell wall is 80.0 mV and the membrane is 9.00 nm thick. What is the electric field strength? (The value is surprisingly large, but correct.) You may assume a uniform electric field.

39. Two parallel conducting plates are separated by 10.0 cm, and one of them is taken to be at zero volts.

(a) What is the electric field strength between them, if the potential 8.00 cm from the zero volt plate (and 2.00 cm from the other) is 450 V?

(b) What is the voltage between the plates?

40. Find the maximum potential difference between two parallel conducting plates separated by 0.500 cm of air, given the maximum sustainable electric field strength in air to be $3.0 \times 10^6 V/m$.

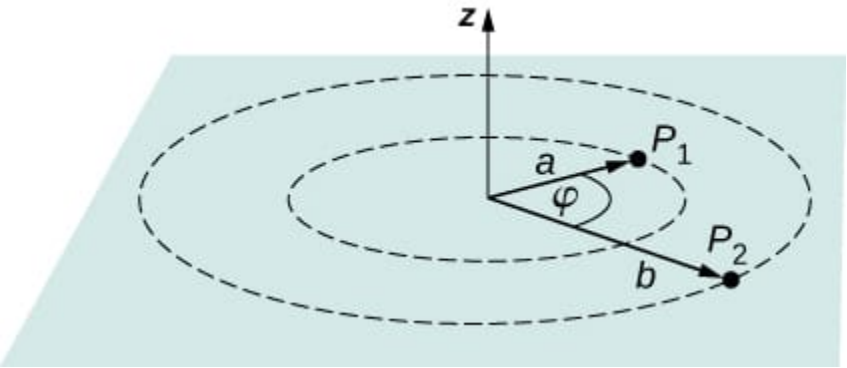
41. An electron is to be accelerated in a uniform electric field having a strength of $2.00 \times 10^6 V/m$.

(a) What energy in keV is given to the electron if it is accelerated through 0.400 m?

(b) Over what distance would it have to be accelerated to increase its energy by 50.0 GeV?

42. Use the definition of potential difference in terms of electric field to deduce the formula for potential difference between $r = r_a$ and $r = r_b$ for a point charge located at the origin. Here r is the spherical radial coordinate.

43. The electric field in a region is pointed away from the **z**-axis and the magnitude depends upon the distance s from the axis. The magnitude of the electric field is given as $E = \frac{\alpha}{s}$ where α is a constant. Find the potential difference between points P_1 and P_2, explicitly stating the path over which you conduct the integration for the line integral.



44. Singly charged gas ions are accelerated from rest through a voltage of 13.0 V. At what temperature will the average kinetic energy of gas molecules be the same as that given these ions?

7.4 Calculations of Electric Potential

45. A 0.500-cm-diameter plastic sphere, used in a static electricity demonstration, has a uniformly distributed 40.0-pC charge on its surface. What is the potential near its surface?

46. How far from a $1.00 - \mu C$ point charge is the potential 100 V? At what distance is it $2.00 \times 10^2 V$?

47. If the potential due to a point charge is $5.00 \times 10^2 V$ at a distance of 15.0 m, what are the sign and magnitude of the charge?

48. In nuclear fission, a nucleus splits roughly in half. (a) What is the potential $2.00 \times 10^{-14} m$ from a fragment that has 46 protons in it? (b) What is the potential energy in MeV of a similarly charged fragment at this distance?

49. A research Van de Graaff generator has a 2.00-m-diameter metal sphere with a charge of 5.00 mC on it. Assume the potential energy is zero at a reference point infinitely far away from the Van de Graaff.

(a) What is the potential near its surface?

(b) At what distance from its center is the potential 1.00 MV?

(c) An oxygen atom with three missing electrons is released near the Van de Graaff generator. What is its kinetic energy in MeV when the atom is at the distance found in part b?

50. An electrostatic paint sprayer has a 0.200-m-diameter metal sphere at a potential of 25.0 kV that repels paint droplets onto a grounded object.

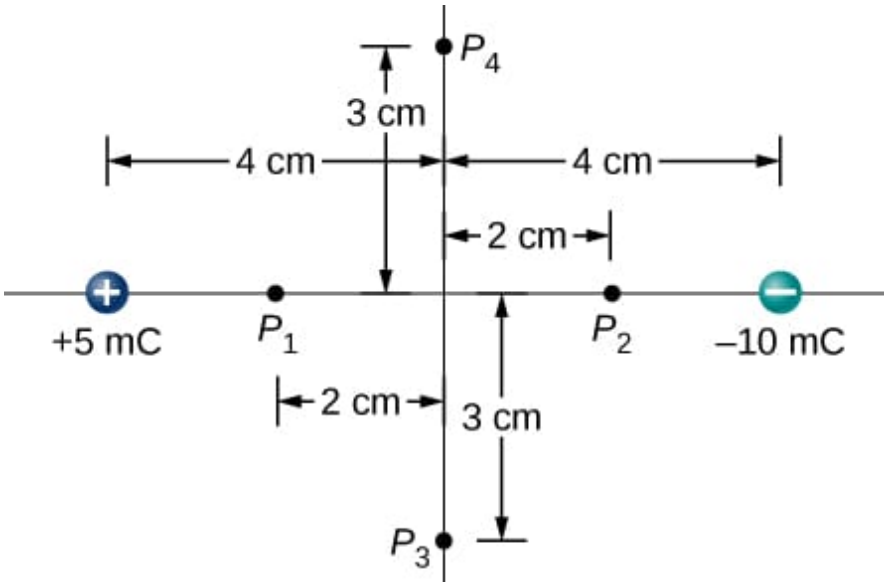
(a) What charge is on the sphere?

(b) What charge must a 0.100-mg drop of paint have to arrive at the object with a speed of 10.0 m/s?

51. (a) What is the potential between two points situated 10 cm and 20 cm from a $3.0 - \mu C$ point charge?

(b) To what location should the point at 20 cm be moved to increase this potential difference by a factor of two?

52. Find the potential at points P_1, P_2, P_3, and P_4 in the diagram due to the two given charges.



53. Two charges $-2.0 \mu C$ and $+2.0 \mu C$ are separated by 4.0 cm on the **z**-axis symmetrically about origin, with the positive one uppermost. Two space points of interest P_1 and P_2 are located 3.0 cm and 30 cm from origin at an angle **30°** with respect to the **z**-axis. Evaluate electric potentials at P_1 and P_2 in two ways:

(a) Using the exact formula for point charges, and

(b) using the approximate dipole potential formula.

54. (a) Plot the potential of a uniformly charged 1-m rod with 1 C/m charge as a function of the perpendicular distance from the center. Draw your graph from $s = 0.1m$ to $s = 1.0m$.

(b) On the same graph, plot the potential of a point charge with a 1-C charge at the origin.

(c) Which potential is stronger near the rod? (d) What happens to the difference as the distance increases? Interpret your result.

7.5 Determining Field from Potential

55. Throughout a region, equipotential surfaces are given by $z = constant$. The surfaces are equally spaced with $V = 100V$ for $z = 0.00m, V = 200V$ for $z = 0.50m, V = 300V$ for $z = 1.00m$. What is the electric field in this region?

56. In a particular region, the electric potential is given by $V = -xy^2z + 4xy$. What is the electric field in this region?

57. Calculate the electric field of an infinite line charge, throughout space.

7.6 Equipotential Surfaces and Conductors

58. Two very large metal plates are placed 2.0 cm apart, with a potential difference of 12 V between them. Consider one plate to be at 12 V, and the other at 0 V. (a) Sketch the equipotential surfaces for 0, 4, 8, and 12 V.

(b) Next sketch in some electric field lines, and confirm that they are perpendicular to the equipotential lines.

59. A very large sheet of insulating material has had an excess of electrons placed on it to a surface charge density of $-3.00nC/m^2$.

(a) As the distance from the sheet increases, does the potential increase or decrease? Can you explain why without any calculations? Does the location of your reference point matter?

(b) What is the shape of the equipotential surfaces?

(c) What is the spacing between surfaces that differ by 1.00 V?

60. A metallic sphere of radius 2.0 cm is charged with $+5.0 - \mu C$ charge, which spreads on the surface of the sphere uniformly. The metallic sphere stands on an insulated stand and is surrounded by a larger metallic spherical shell, of inner radius 5.0 cm and outer radius 6.0 cm. Now, a charge of $-5.0 - \mu C$ is placed on the inside of the spherical shell, which spreads out uniformly on the inside surface of the shell. If potential is zero at infinity, what is the potential of

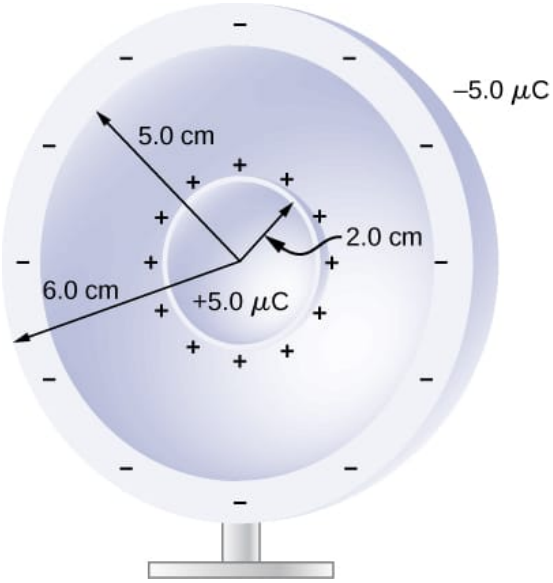
(a) the spherical shell,

(b) the sphere,

(c) the space between the two,

(d) inside the sphere, and

(e) outside the shell?



61. Two large charged plates of charge density $\pm 30 \mu C/m^2$ face each other at a separation of 5.0 mm.

(a) Find the electric potential everywhere.

(b) An electron is released from rest at the negative plate; with what speed will it strike the positive plate?

62. A long cylinder of aluminum of radius R meters is charged so that it has a uniform charge per unit length on its surface of λ.

(a) Find the electric field inside and outside the cylinder.

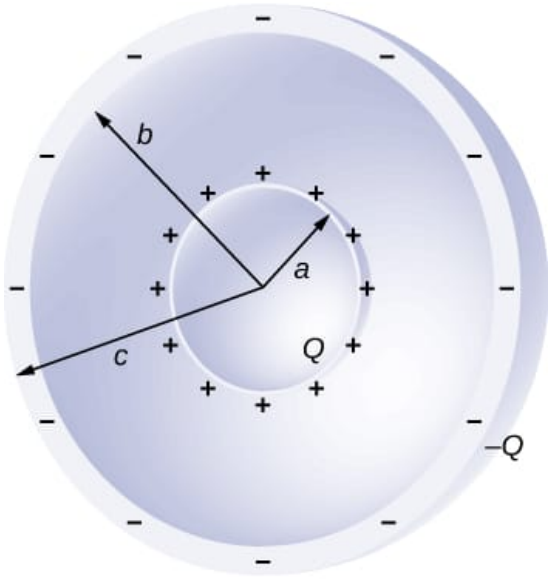
(b) Find the electric potential inside and outside the cylinder. (c) Plot electric field and electric potential as a function of distance from the center of the rod.

63. Two parallel plates 10 cm on a side are given equal and opposite charges of magnitude $5.0 \times 10^{-9} C$.The plates are 1.5 mm apart. What is the potential difference between the plates?

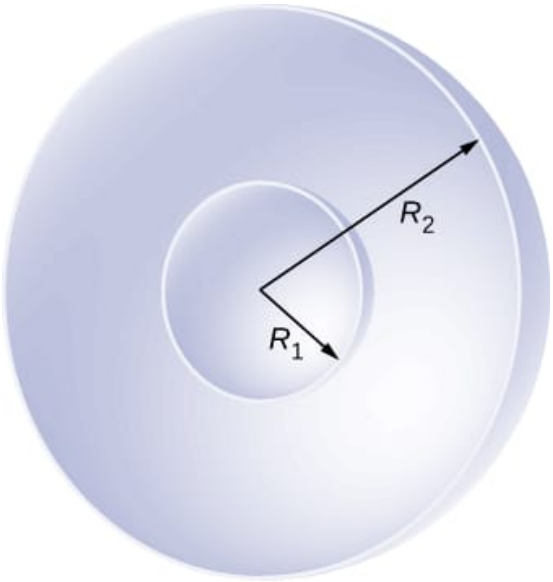
64. The surface charge density on a long straight metallic pipe is σ. What is the electric potential outside and inside the pipe? Assume the pipe has a diameter of **2a**.



65. Concentric conducting spherical shells carry charges **Q** and **–Q**, respectively. The inner shell has negligible thickness. What is the potential difference between the shells?



66. Shown below are two concentric spherical shells of negligible thicknesses and radii R_1 and R_2. The inner and outer shell carry net charges q_1 and q_2, respectively, where both q_1 and q_2 are positive. What is the electric potential in the regions (a) \(\displaystyle rR_2\)?



67. A solid cylindrical conductor of radius a is surrounded by a concentric cylindrical shell of inner radius **b**. The solid cylinder and the shell carry charges **Q** and **–Q**, respectively. Assuming that the length **L** of both conductors is much greater than **a** or **b**, what is the potential difference between the two conductors?

7.7 Applications of Electrostatics

68. (a) What is the electric field 5.00 m from the center of the terminal of a Van de Graaff with a 3.00-mC charge, noting that the field is equivalent to that of a point charge at the center of the terminal?

(b) At this distance, what force does the field exert on a $2.00 - \mu C$ charge on the Van de Graaff's belt?

69. (a) What is the direction and magnitude of an electric field that supports the weight of a free electron near the surface of Earth?

(b) Discuss what the small value for this field implies regarding the relative strength of the gravitational and electrostatic forces.

70. A simple and common technique for accelerating electrons is shown in Figure $10.9.1$, where there is a uniform electric field between two plates. Electrons are released, usually from a hot filament, near the negative plate, and there is a small hole in the positive plate that allows the electrons to continue moving.

(a) Calculate the acceleration of the electron if the field strength is $2.50 \times 10^4 N/C$.

(b) Explain why the electron will not be pulled back to the positive plate once it moves through the hole.

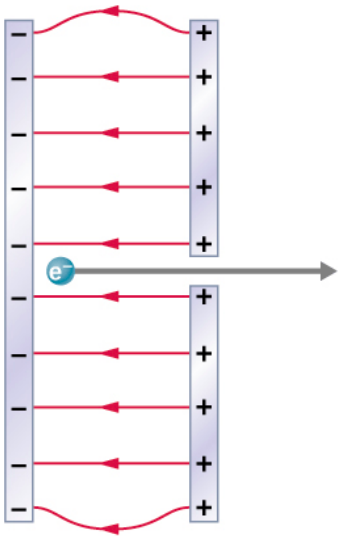


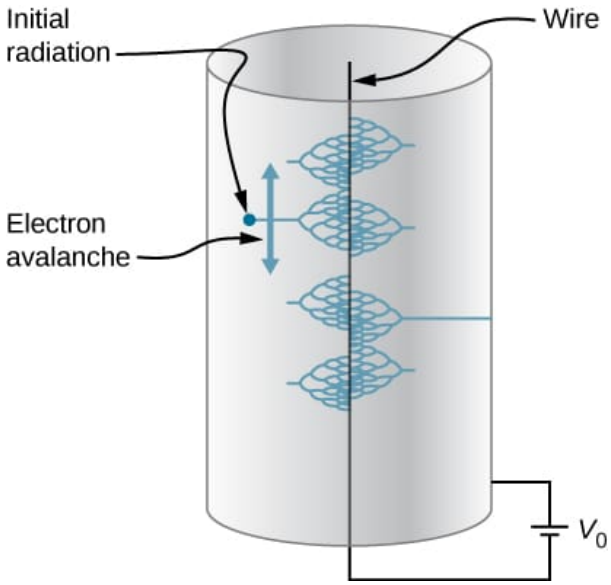
Figure $10.9.1$: Parallel conducting plates with opposite charges on them create a relatively uniform electric field used to accelerate electrons to the right. Those that go through the hole can be used to make a TV or computer screen glow or to produce X-rays.

71. In a Geiger counter, a thin metallic wire at the center of a metallic tube is kept at a high voltage with respect to the metal tube. Ionizing radiation entering the tube knocks electrons off gas molecules or sides of the tube that then accelerate towards the center wire, knocking off even more electrons. This process eventually leads to an avalanche that is detectable as a current. A particular Geiger counter has a tube of radius **R** and the inner wire of radius a is at a potential of V_0 volts with respect to the outer metal tube. Consider a point P at a distance s from the center wire and far away from the ends.

(a) Find a formula for the electric field at a point **P** inside using the infinite wire approximation.

(b) Find a formula for the electric potential at a point **P** inside.

(c) Use $V_0 = 900V, a = 3.00mm, R = 2.00cm$, and find the value of the electric field at a point 1.00 cm from the center.



72. The practical limit to an electric field in air is about $3.00 \times 10^6 N/C$. Above this strength, sparking takes place because air begins to ionize.

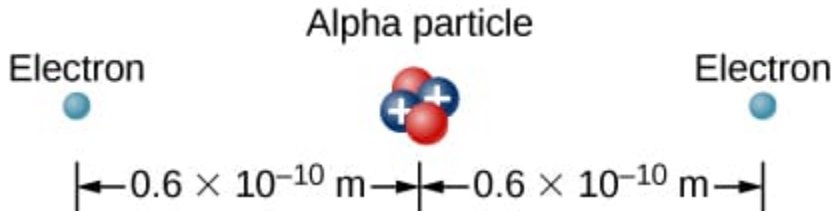
(a) At this electric field strength, how far would a proton travel before hitting the speed of light (ignore relativistic effects)?

(b) Is it practical to leave air in particle accelerators?

73. To form a helium atom, an alpha particle that contains two protons and two neutrons is fixed at one location, and two electrons are brought in from far away, one at a time. The first electron is placed at $0.600 \times 10^{-10} m$ from the alpha particle and held there while the second electron is brought to $0.600 \times 10^{-10} m$ from the alpha particle on the other side from the first electron. See the final configuration below.

(a) How much work is done in each step?

(b) What is the electrostatic energy of the alpha particle and two electrons in the final configuration?



74. Find the electrostatic energy of eight equal charges $(+3\mu C)$ each fixed at the corners of a cube of side 2 cm.

75. The probability of fusion occurring is greatly enhanced when appropriate nuclei are brought close together, but mutual Coulomb repulsion must be overcome. This can be done using the kinetic energy of high-temperature gas ions or by accelerating the nuclei toward one another.

(a) Calculate the potential energy of two singly charged nuclei separated by $1.00 \times 10^{-12} m$.

(b) At what temperature will atoms of a gas have an average kinetic energy equal to this needed electrical potential energy?

76. A bare helium nucleus has two positive charges and a mass of $6.64 \times 10^{-27} kg$.

(a) Calculate its kinetic energy in joules at 2.00 of the speed of light.

(b) What is this in electron-volts?

(c) What voltage would be needed to obtain this energy?

77. An electron enters a region between two large parallel plates made of aluminum separated by a distance of 2.0 cm and kept at a potential difference of 200 V. The electron enters through a small hole in the negative plate and moves toward the positive plate. At the time the electron is near the negative plate, its speed is $4.0 \times 10^5 m/s$. Assume the electric field between the plates to be uniform, and find the speed of electron at

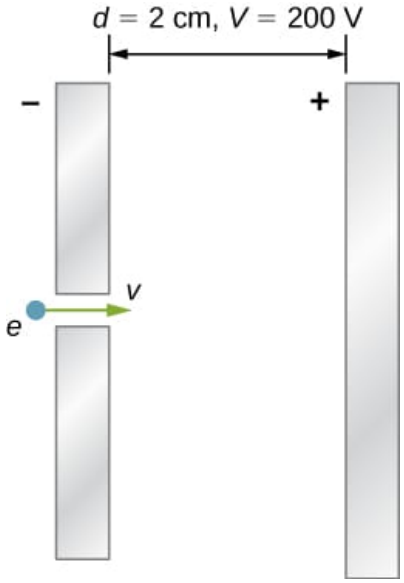
(a) 0.10 cm,

(b) 0.50 cm,

(c) 1.0 cm, and

(d) 1.5 cm from the negative plate, and

(e) immediately before it hits the positive plate.



78. How far apart are two conducting plates that have an electric field strength of $4.50 \times 10^3 V/m$ between them, if their potential difference is 15.0 kV?

79. (a) Will the electric field strength between two parallel conducting plates exceed the breakdown strength of dry air, which is $3.00 \times 10^6 V/m$, if the plates are separated by 2.00 mm and a potential difference of $5.0 \times 10^3 V$ is applied?

(b) How close together can the plates be with this applied voltage?

80. Membrane walls of living cells have surprisingly large electric fields across them due to separation of ions. What is the voltage across an 8.00-nm-thick membrane if the electric field strength across it is 5.50 MV/m? You may assume a uniform electric field.

81. A double charged ion is accelerated to an energy of 32.0 keV by the electric field between two parallel conducting plates separated by 2.00 cm. What is the electric field strength between the plates?

82. The temperature near the center of the Sun is thought to be 15 million degrees Celsius ($1.5 \times 10^7\ ^\circ C$) (or kelvin). Through what voltage must a singly charged ion be accelerated to have the same energy as the average kinetic energy of ions at this temperature?

83. A lightning bolt strikes a tree, moving 20.0 C of charge through a potential difference of $1.00 \times 1^0 2MV$.

(a) What energy was dissipated?

(b) What mass of water could be raised from **15°C** to the boiling point and then boiled by this energy?

(c) Discuss the damage that could be caused to the tree by the expansion of the boiling steam.

84. What is the potential $0.530 \times 10^{-10} m$ from a proton (the average distance between the proton and electron in a hydrogen atom)?

85. (a) A sphere has a surface uniformly charged with 1.00 C. At what distance from its center is the potential 5.00 MV? (b) What does your answer imply about the practical aspect of isolating such a large charge?

86. What are the sign and magnitude of a point charge that produces a potential of –2.00 V at a distance of 1.00 mm?

87. In one of the classic nuclear physics experiments at the beginning of the twentieth century, an alpha particle was accelerated toward a gold nucleus, and its path was substantially deflected by the Coulomb interaction. If the energy of the doubly charged alpha nucleus was 5.00 MeV, how close to the gold nucleus (79 protons) could it come before being deflected?

Additional Problems

88. A 12.0-V battery-operated bottle warmer heats 50.0 g of glass, $2.50 \times 10^2 g$ of baby formula, and $2.00 \times 10^2 g$ of aluminum from **20.0°C** to **90.0°C**.

(a) How much charge is moved by the battery?

(b) How many electrons per second flow if it takes 5.00 min to warm the formula? (Hint: Assume that the specific heat of baby formula is about the same as the specific heat of water.)

89. A battery-operated car uses a 12.0-V system. Find the charge the batteries must be able to move in order to accelerate the 750 kg car from rest to 25.0 m/s, make it climb a $2.00 \times 10^2 - m$ high hill, and finally cause it to travel at a constant 25.0 m/s while climbing with $5.00 \times 10^2 - N$ force for an hour.

90. (a) Find the voltage near a 10.0 cm diameter metal sphere that has 8.00 C of excess positive charge on it.

(b) What is unreasonable about this result?

(c) Which assumptions are responsible?

91. A uniformly charged half-ring of radius 10 cm is placed on a nonconducting table. It is found that 3.0 cm above the center of the half-ring the potential is –3.0 V with respect to zero potential at infinity. How much charge is in the half-ring?

92. A glass ring of radius 5.0 cm is painted with a charged paint such that the charge density around the ring varies continuously given by the following function of the polar angle $\theta, \lambda = (3.0 \times 10^{-6} C/m) cos^2 \theta$.. Find the potential at a point 15 cm above the center.

93. A CD disk of radius ($R = 3.0cm$) is sprayed with a charged paint so that the charge varies continually with radial distance rfrom the center in the following manner: $\sigma = -(6.0C/m)r/R$. Find the potential at a point 4 cm above the center.

94. (a) What is the final speed of an electron accelerated from rest through a voltage of 25.0 MV by a negatively charged Van de Graff terminal? (b) What is unreasonable about this result? (c) Which assumptions are responsible?

95. A large metal plate is charged uniformly to a density of $\sigma = 2.0 \times 10^{-9} C/m^2$. How far apart are the equipotential surfaces that represent a potential difference of 25 V?

96. Your friend gets really excited by the idea of making a lightning rod or maybe just a sparking toy by connecting two spheres as shown in Figure 7.39, and making R_2 so small that the electric field is greater than the dielectric strength of air, just from the usual 150 V/m electric field near the surface of the Earth. If R_1 is 10 cm, how small does R_2 need to be, and does this seem practical? (**Hint**: recall the calculation for electric field at the surface of a conductor from Gauss's Law.)

97. (a) Find $x >> L$ limit of the potential of a finite uniformly charged rod and show that it coincides with that of a point charge formula. (b) Why would you expect this result?

98. A small spherical pith ball of radius 0.50 cm is painted with a silver paint and then $-10\mu C$ of charge is placed on it. The charged pith ball is put at the center of a gold spherical shell of inner radius 2.0 cm and outer radius 2.2 cm.

(a) Find the electric potential of the gold shell with respect to zero potential at infinity.

(b) How much charge should you put on the gold shell if you want to make its potential 100 V?

99. Two parallel conducting plates, each of cross-sectional area $400cm^2$, are 2.0 cm apart and uncharged. If 1.0×10^{12} electrons are transferred from one plate to the other,

(a) what is the potential difference between the plates?

(b) What is the potential difference between the positive plate and a point 1.25 cm from it that is between the plates?

100. A point charge of $q = 5.0 \times 10^{-8}C$ is placed at the center of an uncharged spherical conducting shell of inner radius 6.0 cm and outer radius 9.0 cm. Find the electric potential at

(a) $r = 4.0cm$,

(b) $r = 8.0cm$,

(c) $r = 12.0cm$.

101. Earth has a net charge that produces an electric field of approximately 150 N/C downward at its surface.

(a) What is the magnitude and sign of the excess charge, noting the electric field of a conducting sphere is equivalent to a point charge at its center?

(b) What acceleration will the field produce on a free electron near Earth's surface?

(c) What mass object with a single extra electron will have its weight supported by this field?

102. Point charges of $25.0\mu C$ and $45.0\mu C$ are placed 0.500 m apart.

(a) At what point along the line between them is the electric field zero?

(b) What is the electric field halfway between them?

103. What can you say about two charges q_1 and q_2, if the electric field one-fourth of the way from q_1 to q_2 is zero?

104. Calculate the angular velocity ω of an electron orbiting a proton in the hydrogen atom, given the radius of the orbit is $0.530 \times 10^{-10}m$. You may assume that the proton is stationary and the centripetal force is supplied by Coulomb attraction.

105. An electron has an initial velocity of $5.00 \times 10^6 m/s$ in a uniform $2.00 \times 10^5 - N/C$ electric field. The field accelerates the electron in the direction opposite to its initial velocity.

(a) What is the direction of the electric field?

(b) How far does the electron travel before coming to rest?

(c) How long does it take the electron to come to rest?

(d) What is the electron's velocity when it returns to its starting point?

Challenge Problems

106. Three Na^+ and three Cl^- ions are placed alternately and equally spaced around a circle of radius 50 nm. Find the

electrostatic energy stored.

107. Look up (presumably online, or by dismantling an old device and making measurements) the magnitude of the potential deflection plates (and the space between them) in an ink jet printer. Then look up the speed with which the ink comes out the nozzle. Can you calculate the typical mass of an ink drop?

108. Use the electric field of a finite sphere with constant volume charge density to calculate the electric potential, throughout space. Then check your results by calculating the electric field from the potential.

109. Calculate the electric field of a dipole throughout space from the potential.

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10.10: Electric Potential (Answer)

Check Your Understanding

7.1. $K = \frac{1}{2}mv^2, v = \sqrt{2\frac{K}{m}} = \sqrt{2\frac{4.5 \times 10^{-7} J}{4.00 \times 10^{-9} kg}} = 15 m/s$

7.2. It has kinetic energy of $4.5 \times 10^{-7} J$ at point r_2 and potential energy of $9.0 \times 10^{-7} J$J, which means that as **Q** approaches infinity, its kinetic energy totals three times the kinetic energy at r_2, since all of the potential energy gets converted to kinetic.

7.3. positive, negative, and these quantities are the same as the work you would need to do to bring the charges in from infinity

7.4. $\Delta U = q\Delta V = (100C)(1.5V) = 150J$

7.5. -2.00 C, $n_e = 1.25 \times 10^{19} electrons ne$

7.6. It would be going in the opposite direction, with no effect on the calculations as presented.

7.7. Given a fixed maximum electric field strength, the potential at which a strike occurs increases with increasing height above the ground. Hence, each electron will carry more energy. Determining if there is an effect on the total number of electrons lies in the future.

7.8. $V = k\frac{q}{r} = (8.99 \times 10^9 N \cdot m^2/C^2)(\frac{-3.00 \times 10^{-9} C}{5.00 \times 10^{-3} m}) = -5390V$; recall that the electric field inside a conductor is zero. Hence, any path from a point on the surface to any point in the interior will have an integrand of zero when calculating the change in potential, and thus the potential in the interior of the sphere is identical to that on the surface.

7.9. The **x**-axis the potential is zero, due to the equal and opposite charges the same distance from it. On the **z**-axis, we may superimpose the two potentials; we will find that for $z >> d$, again the potential goes to zero due to cancellation.

7.10. It will be zero, as at all points on the axis, there are equal and opposite charges equidistant from the point of interest. Note that this distribution will, in fact, have a dipole moment.

7.11. Any, but cylindrical is closest to the symmetry of a dipole.

7.12. infinite cylinders of constant radius, with the line charge as the axis

Conceptual Questions

1. No. We can only define potential energies for conservative fields.

3. No, though certain orderings may be simpler to compute.

5. The electric field strength is zero because electric potential differences are directly related to the field strength. If the potential difference is zero, then the field strength must also be zero.

7. Potential difference is more descriptive because it indicates that it is the difference between the electric potential of two points.

9. They are very similar, but potential difference is a feature of the system; when a charge is introduced to the system, it will have a potential energy which may be calculated by multiplying the magnitude of the charge by the potential difference.

11. An electron-volt is a volt multiplied by the charge of an electron. Volts measure potential difference, electron-volts are a

unit of energy.

13. The second has 1/4 the dipole moment of the first.

15. The region outside of the sphere will have a potential indistinguishable from a point charge; the interior of the sphere will have a different potential.

17. No. It will be constant, but not necessarily zero.

19. no

21. No; it might not be at electrostatic equilibrium.

23. Yes. It depends on where the zero reference for potential is. (Though this might be unusual.)

25. So that lightning striking them goes into the ground instead of the television equipment.

27. They both make use of static electricity to stick small particles to another surface. However, the precipitator has to charge a wide variety of particles, and is not designed to make sure they land in a particular place.

Problems

29. a. $U = 3.4J$;

b. $\frac{1}{2}mv^2 = kQ_1Q_2(\frac{1}{r_i} - \frac{1}{r_f}) \rightarrow v = 750m/s$

31. $U = 4.36 \times 10^{-18}J$

33. $\frac{1}{2}m_ev_e^2 = qV, \frac{1}{2}m_Hv_H^2 = qV$,so that $\frac{m_ev_e^2}{m_Hv_H^2} = 1$ or $\frac{v_e}{v_H} = 42.8$.

35. $1V = 1J/C; 1J = 1N \cdot m \rightarrow 1V/m = 1N/C$

37. a. $V_{AB} = 3.00kV$;

b. $V_{AB} = 7.50kV$

39. a. $V_{AB} = Ed \rightarrow E = 5.63kV/m$;

b. $V_{AB} = 563V$

41. a. $\Delta K = q\Delta V$ and $V_{AB} = Ed$,so that $\Delta K = 800keV$;

b. $d = 25.0km$

43. One possibility is to stay at constant radius and go along the arc from P_1 to P_2, which will have zero potential due to the path being perpendicular to the electric field. Then integrate from a to b: $V_{ab} = \alpha ln(\frac{b}{a})$

45. $V = 144V$

47. $V = \frac{kQ}{r} \rightarrow Q = 8.33 \times 10^{-7}C$; The charge is positive because the potential is positive.

49. a. $V = 45.0MV$;

b. $V = \frac{kQ}{r} \rightarrow r = 45.0m$;

c. $\Delta U = 132MeV$

51. $V = kQ/r$; a. Relative to origin, find the potential at each point and then calculate the difference. $\Delta V = 135 \times 10^3 V$;

b. To double the potential difference, move the point from 20 cm to infinity; the potential at 20 cm is halfway between zero and that at 10 cm.

53. a. $V_{P1} = 7.4 \times 10^5 V$ and $V_{P2} = 6.9 \times 10^3 V$;

b. $V_{P1} = 6.9 \times 10^5 V$ and $V_{P2} = 6.9 \times 10^3 V$

55. The problem is describing a uniform field, so $E = 200 V/m$ in the **–z**-direction.

57. Apply $vecE = -\vec{\nabla} V$ with $\vec{\nabla} = \hat{r}\frac{\partial}{\partial r} + \hat{\varphi}\frac{1}{r}\frac{\partial}{\partial \varphi} + \hat{z}\frac{\partial}{\partial z}$ to the potential calculated earlier, $V = -2k\lambda lns : \vec{E} = 2k\lambda\frac{1}{r}\hat{r}$ as expected.

59. a. increases; the constant (negative) electric field has this effect, the reference point only matters for magnitude; b. they are planes parallel to the sheet; c. 0.006 m

61. a. from the previous chapter, the electric field has magnitude $\frac{\sigma}{\varepsilon_0}$ in the region between the plates and zero outside; defining the negatively charged plate to be at the origin and zero potential, with the positively charged plate located at **+5mm** in the **z**-direction, V=1.7×104VV=1.7×104Vso the potential is 0 for $z < 0, 1.7 \times 10^4 V(\frac{z}{5mm})$ for $0 \le z \le 5mm, 1.7 \times 10^4 V$ for $z > 5mm$;

b. $qV = \frac{1}{2}mv^2 \rightarrow v = 7.7 \times 10^7 m/s$

63. $V = 85V$

65. In the region $a \le r \le b, \vec{E} = \frac{kQ}{r^2}\hat{r}$, and **E** is zero elsewhere; hence, the potential difference is $V = kQ(\frac{1}{a} - \frac{1}{b})$.

67. From previous results $V_P - V_R = -2k\lambda ln\frac{s_P}{s_R}$., note that b is a very convenient location to define the zero level of potential: $\Delta V = -2k\frac{Q}{L}ln\frac{a}{b}$.

69. a. $F = 5.58 \times 10^{-11} N/C$;The electric field is towards the surface of Earth.

b. The coulomb force is much stronger than gravity.

71. We know from the Gauss's law chapter that the electric field for an infinite line charge is $\vec{E}_P = 2k\lambda\frac{1}{s}\hat{s}$, and from earlier in this chapter that the potential of a wire-cylinder system of this sort is $V_P = -2k\lambda ln\frac{s_P}{R}$ by integration. We are not given λ, but we are given a fixed V_0; thus, we know that $V_0 = -2k\lambda ln\frac{a}{R}$ and hence $\lambda = -\frac{V_0}{2kln(\frac{a}{R})}$. We may substitute this back in to find a. $\vec{E}_P = -\frac{V_0}{ln(\frac{a}{R})}\frac{1}{s}\hat{s}$;

b. $V_P = V_0\frac{ln(\frac{sP}{R})}{ln(\frac{a}{R})}$;

c. $4.74 \times 10^4 N/C$

73. a. $U_1 = 7.68 \times 10^{-18} J, U_2 = 5.76 \times 10^{-18} J$;

b. $U_1 + U_2 = -1.34 \times 10^{-17} J$

75. a. \(\displaystyle U=2.30×10^{–16}J;

b. $\overline{K} = \frac{3}{2}kT \rightarrow T = 1.11 \times 10^7$

77. a. $1.9 \times 10^6 m/s$;

b. $4.2 \times 10^6 m/s$;

c. $5.9 \times 10^6 m/s$;

d. $7.3 \times 10^6 m/s$;

e. $8.4 \times 10^6 m/s$

79. a. $E = 2.5 \times 10^6 V/m < 3 \times 10^6 V/m$ No, the field strength is smaller than the breakdown strength for air.

b. $d = 1.7mm$

81. $K_f = qV_{AB} = qEd \rightarrow E = 8.00 \times 10^5 V/m$

83. a. Energy=$2.00 \times 10^9 J$;

b. $Q = m(c\Delta T + L_{\nabla})\ m = 766kg$;

c. The expansion of the steam upon boiling can literally blow the tree apart.

85. a. $V = \dfrac{kQ}{r} \rightarrow r = 1.80km$;

b. A 1-C charge is a very large amount of charge; a sphere of 1.80 km is impractical.

87. The alpha particle approaches the gold nucleus until its original energy is converted to potential energy. $5.00MeV = 8.00 \times 10^{-13} J$, so $E_0 = \dfrac{qkQ}{r} \rightarrow r = 4.54 \times 10^{-14} m$

(Size of gold nucleus is about $7 \times 10^{-15} m$).

Additional Problems

89. $E_{tot} = 4.67 \times 10^7 J\ E_{tot} = qV \rightarrow q = \dfrac{E_{tot}}{V} = 3.89 \times 10^6 C$

91. $V_P = k\dfrac{q_{tot}}{\sqrt{z^2 + R^2}} \rightarrow q_{tot} = -3.5 \times 10^{-11} C$

93. $V_P = -2.2GV$

95. Recall from the previous chapter that the electric field $E_P = \dfrac{\sigma_0}{2\varepsilon_0}$ is uniform throughout space, and that for uniform fields we have $E = -\dfrac{\Delta V}{\Delta z}$ for the relation. Thus, we get $\dfrac{\sigma}{2\varepsilon_0} = \dfrac{\Delta V}{\Delta z} \rightarrow \Delta z = 0.22m$ for the distance between 25-V equipotentials.

97. a. Take the result from Example 7.13, divide both the numerator and the denominator by x, take the limit of that, and then apply a Taylor expansion to the resulting log to get: $V_P \approx k\lambda\dfrac{L}{x}$;

b. which is the result we expect, because at great distances, this should look like a point charge of $q = \lambda L$

99. a. $V = 9.0 \times 10^3 V$;

b. $-9.0 \times 10^3 V(\dfrac{1.25cm}{2.0cm}) = -5.7 \times 10^3 V$

101. a. $E = \dfrac{KQ}{r^2} \rightarrow Q = -6.76 \times 10^5 C$;

b. $F = ma = qE \rightarrow a = \frac{qE}{m} = 2.63 \times 10^{13} m/s^2 (upwards)$;

c. $F = -mg = qE \rightarrow m = \frac{-qE}{g} = 2.45 \times 10^{-18} kg$

103. If the electric field is zero ¼ from the way of q_1 and q_2, then we know from $E = k\frac{Q}{r^2}$ that $|E_1| = |E_2| \rightarrow \frac{Kq_1}{x^2} = \frac{Kq^2}{(3x)^2}$ so that $\frac{q_2}{q_1} = \frac{(3x)^2}{x^2} = 9$; the charge q_2 is 9 times larger than q_1.

105. a. The field is in the direction of the electron's initial velocity.

b. $v^2 = v_0^2 + 2ax \rightarrow x = -\frac{v_0^2}{2a} (v = 0)$. Also, $F = ma = qE \rightarrow a = \frac{qE}{m}$,$x = 3.56 \times 10^{-4} m$;

c. $v_2 = v_0 + at \rightarrow t = -\frac{v_0 m}{qE} (v = 0), \therefore t = 1.42 \times 10^{-10} s$;

d. $v = -(\frac{2qEx}{m})^{1/2} - 5.00 \times 10^6 m/s$ (opposite its initial velocity)

Challenge Problems

107. Answers will vary. This appears to be proprietary information, and ridiculously difficult to find. Speeds will be 20 m/s or less, and there are claims of 10^{-7} grams for the mass of a drop.

109. Apply $\vec{E} = -\vec{\nabla} V$ with $\vec{\nabla} = \hat{r}\frac{\partial}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial}{\partial \theta} + \hat{\varphi}\frac{1}{rsin\theta}\frac{\partial}{\partial \varphi}$ to the potential calculated earlier, $V_P = k\frac{\vec{p} \cdot \hat{r}}{r^2}$ with $\vec{p} = q\vec{d}$, and assume that the axis of the dipole is aligned with the **z-**axis of the coordinate system. Thus, the potential is $V_P = k\frac{q\vec{d} \cdot \hat{r}}{r^2} = k\frac{qdcos\theta}{r^2}$.

$\vec{E} = 2kqd(\frac{cos\theta}{r^3})\hat{r} + kqd(\frac{sin\theta}{r^3})\hat{\theta}$

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11: CAPACITANCE

11.1: Prelude to Capacitance

11.2: Capacitors and Capacitance

11.3: Capacitors in Series and in Parallel

11.4: Energy Stored in a Capacitor

11.5: Capacitor with a Dielectric

11.6: Molecular Model of a Dielectric

11.7: Capacitance (Summary)

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11.9: Capacitance (Answers)

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11.1: Prelude to Capacitance

Capacitors are important components of electrical circuits in many electronic devices, including pacemakers, cell phones, and computers. In this chapter, we study their properties, and, over the next few chapters, we examine their function in combination with other circuit elements. By themselves, capacitors are often used to store electrical energy and release it when needed; with other circuit components, capacitors often act as part of a filter that allows some electrical signals to pass while blocking others. You can see why capacitors are considered one of the fundamental components of electrical circuits.

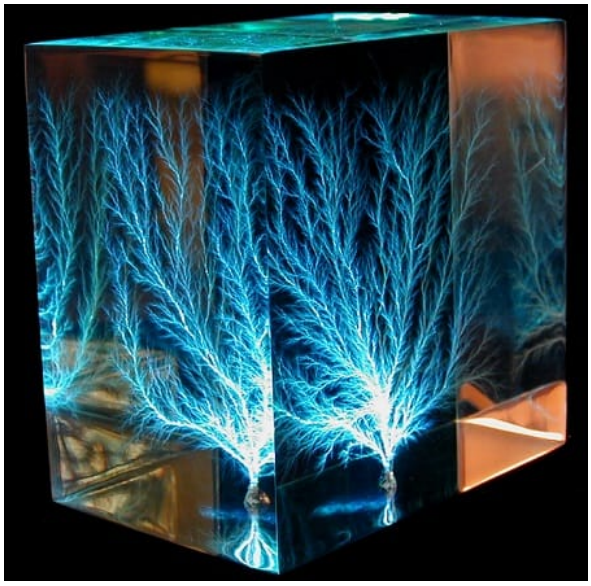


Figure 11.1.1: The tree-like branch patterns in this clear acrylic block are created by irradiating the block with an electron beam. This tree is known as a Lichtenberg figure, named for the German physicist Georg Christof Lichtenberg (1742–1799), who was the first to study these patterns. The “branches” are created by the dielectric breakdown produced by a strong electric field. (Bert Hickman).

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11.2: Capacitors and Capacitance

Learning Objectives

By the end of this section, you will be able to:

- Explain the concepts of a capacitor and its capacitance
- Describe how to evaluate the capacitance of a system of conductors

A **capacitor** is a device used to store electrical charge and electrical energy. It consists of at least two electrical conductors separated by a distance. (Note that such electrical conductors are sometimes referred to as "electrodes," but more correctly, they are "capacitor plates.") The space between capacitors may simply be a vacuum, and, in that case, a capacitor is then known as a "vacuum capacitor." However, the space is usually filled with an insulating material known as a **dielectric**. (You will learn more about dielectrics in the sections on dielectrics later in this chapter.) The amount of storage in a capacitor is determined by a property called **capacitance**, which you will learn more about a bit later in this section.

Capacitors have applications ranging from filtering static from radio reception to energy storage in heart defibrillators. Typically, commercial capacitors have two conducting parts close to one another but not touching, such as those in Figure 11.2.1. Most of the time, a dielectric is used between the two plates. When battery terminals are connected to an initially uncharged capacitor, the battery potential moves a small amount of charge of magnitude Q from the positive plate to the negative plate. The capacitor remains neutral overall, but with charges $+Q$ and $-Q$ residing on opposite plates.

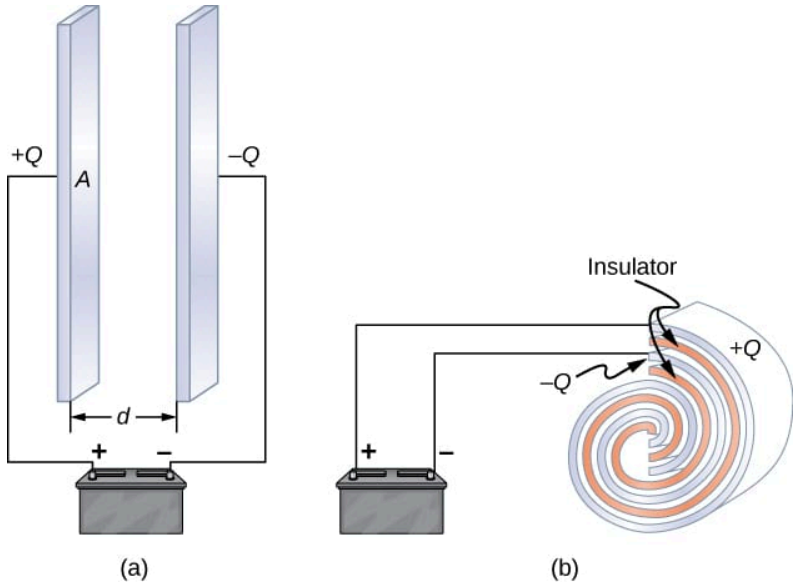


Figure 11.2.1: Both capacitors shown here were initially uncharged before being connected to a battery. They now have charges of $+Q$ and $-Q$ (respectively) on their plates. (a) A parallel-plate capacitor consists of two plates of opposite charge with area A separated by distance d. (b) A rolled capacitor has a dielectric material between its two conducting sheets (plates).

A system composed of two identical parallel-conducting plates separated by a distance is called a **parallel-plate capacitor** (Figure 11.2.2). The magnitude of the electrical field in the space between the parallel plates is $E = \sigma/\epsilon_0$, where σ denotes the surface charge density on one plate (recall that σ is the charge Q per the surface area A). Thus, the magnitude of the field is directly proportional to Q.

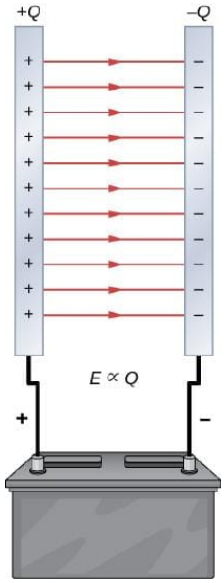


Figure 11.2.2: The charge separation in a capacitor shows that the charges remain on the surfaces of the capacitor plates. Electrical field lines in a parallel-plate capacitor begin with positive charges and end with negative charges. The magnitude of the electrical field in the space between the plates is in direct proportion to the amount of charge on the capacitor.

Capacitors with different physical characteristics (such as shape and size of their plates) store different amounts of charge for the same applied voltage V across their plates. The **capacitance** C of a capacitor is defined as the ratio of the maximum charge Q that can be stored in a capacitor to the applied voltage V across its plates. In other words, capacitance is the largest amount of charge per volt that can be stored on the device:

$$C = \frac{Q}{V} \tag{11.2.1}$$

The SI unit of capacitance is the **farad** (F), named after Michael Faraday (1791–1867). Since capacitance is the charge per unit voltage, one farad is one coulomb per one volt, or

$$1\,\mathrm{F} = \frac{1\,\mathrm{C}}{1\,\mathrm{V}}.$$

By definition, a 1.0-F capacitor is able to store 1.0 C of charge (a very large amount of charge) when the potential difference between its plates is only 1.0 V. One farad is therefore a very large capacitance. Typical capacitance values range from picofarads $(1\ \mathrm{pF} = 10{-}12\mathrm{F})$ to millifarads $(1\ \mathrm{mF} = 10^{-3}\mathrm{F})$, which also includes microfarads $(1\ \mu\mathrm{C} = 10^{-6}\mathrm{F})$. Capacitors can be produced in various shapes and sizes (Figure 11.2.3).

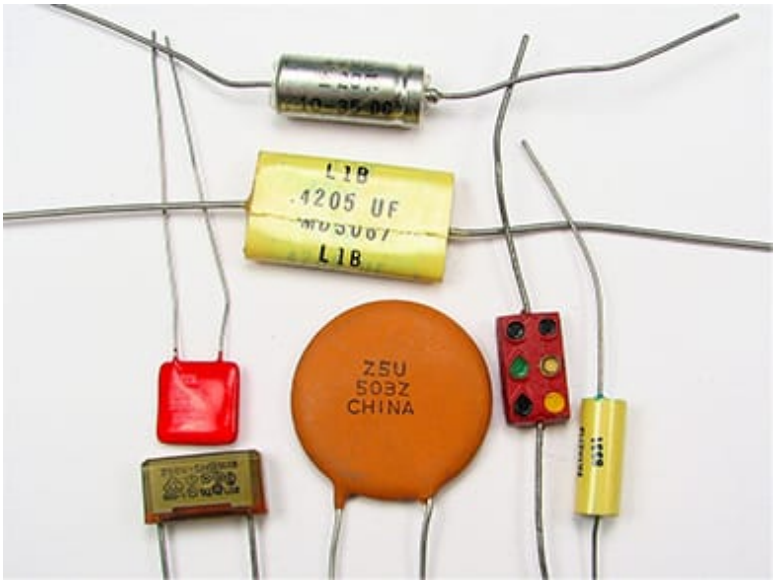


Figure 11.2.3: These are some typical capacitors used in electronic devices. A capacitor's size is not necessarily related to its capacitance value.

Calculation of Capacitance

We can calculate the capacitance of a pair of conductors with the standard approach that follows.

Problem-Solving Strategy: Calculating Capacitance

1. Assume that the capacitor has a charge Q.
2. Determine the electrical field $\vec{E}$ between the conductors. If symmetry is present in the arrangement of conductors, you may be able to use Gauss's law for this calculation.
3. Find the potential difference between the conductors from

$$V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{l}, \tag{11.2.2}$$

   where the path of integration leads from one conductor to the other. The magnitude of the potential difference is then $V = |V_B - V_A|$.
4. With V known, obtain the capacitance directly from Equation 11.2.1.

To show how this procedure works, we now calculate the capacitances of parallel-plate, spherical, and cylindrical capacitors. In all cases, we assume vacuum capacitors (empty capacitors) with no dielectric substance in the space between conductors.

Parallel-Plate Capacitor

The parallel-plate capacitor (Figure 11.2.4) has two identical conducting plates, each having a surface area A, separated by a distance d. When a voltage V is applied to the capacitor, it stores a charge Q, as shown. We can see how its capacitance may depend on A and d by considering characteristics of the Coulomb force. We know that force between the charges increases with charge values and decreases with the distance between them. We should expect that the bigger the plates are, the more charge they can store. Thus, C should be greater for a larger value of A. Similarly, the closer the plates are together, the greater the attraction of the opposite charges on them. Therefore, C should be greater for a smaller d.

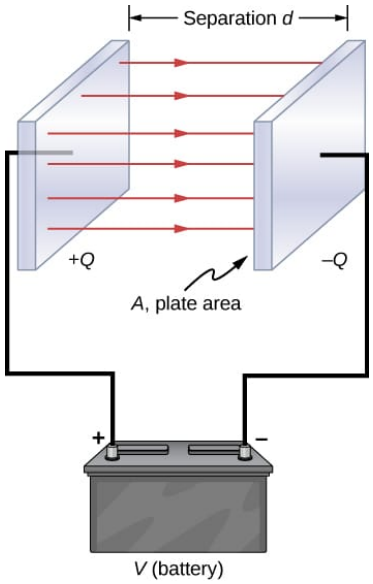


Figure 11.2.4: In a parallel-plate capacitor with plates separated by a distance \(**d**\), each plate has the same surface area \(**A**\).

We define the surface charge density σ on the plates as

$$\sigma = \frac{Q}{A}.$$

We know from previous chapters that when d is small, the electrical field between the plates is fairly uniform (ignoring edge effects) and that its magnitude is given by

$$E = \frac{\sigma}{\epsilon_0},$$

where the constant ϵ_0 is the permittivity of free space, $\epsilon_0 = 8.85 \times 10^{-12} F/m$. The SI unit of F/m is equivalent to $C^2/N \cdot m^2$. Since the electrical field $\vec{E}$ between the plates is uniform, the potential difference between the plates is

$$V = Ed = \frac{\sigma d}{\epsilon_0} = \frac{Qd}{\epsilon_0 A}.$$

Therefore Equation 11.2.1 gives the capacitance of a parallel-plate capacitor as

$$C = \frac{Q}{V} = \frac{Q}{Qd/\epsilon_0 A} = \epsilon_0 \frac{A}{d}. \tag{11.2.3}$$

Notice from this equation that capacitance is a function **only of the geometry** and what material fills the space between the plates (in this case, vacuum) of this capacitor. In fact, this is true not only for a parallel-plate capacitor, but for all capacitors: The capacitance is independent of Q or V. If the charge changes, the potential changes correspondingly so that Q/V remains constant.

Example 11.2.1A: Capacitance and Charge Stored in a Parallel-Plate Capacitor

a. What is the capacitance of an empty parallel-plate capacitor with metal plates that each have an area of $1.00\ m^2$, separated by 1.00 mm?
b. How much charge is stored in this capacitor if a voltage of $3.00 \times 10^3 V$ is applied to it?

Strategy

Finding the capacitance C is a straightforward application of Equation 11.2.3. Once we find C, we can find the charge stored by using Equation 11.2.1.

Solution

1. Entering the given values into Equation 11.2.3 yields

$$C = \epsilon_0 \frac{A}{d} = \left(8.85 \times 10^{-12} \frac{F}{m}\right) \frac{1.00\, m^2}{1.00 \times 10^{-3} m} = 8.85 \times 10^{-9} F = 8.85\, nF.$$

   This small capacitance value indicates how difficult it is to make a device with a large capacitance.
2. Inverting Equation 11.2.1 and entering the known values into this equation gives

$$Q = CV = (8.85 \times 10^{-9} F)(3.00 \times 10^{3} V) = 26.6\, \mu C.$$

Significance

This charge is only slightly greater than those found in typical static electricity applications. Since air breaks down (becomes conductive) at an electrical field strength of about 3.0 MV/m, no more charge can be stored on this capacitor by increasing the voltage.

Example 11.2.1B: A 1-F Parallel-Plate Capacitor

Suppose you wish to construct a parallel-plate capacitor with a capacitance of 1.0 F. What area must you use for each plate if the plates are separated by 1.0 mm?

Solution

Rearranging Equation 11.2.3, we obtain

$$A = \frac{Cd}{\epsilon_0} = \frac{(1.0\, F)(1.0 \times 10^{-3} m)}{8.85 \times 10^{-12} F/m} = 1.1 \times 10^{8}\, m^2.$$

Each square plate would have to be 10 km across. It used to be a common prank to ask a student to go to the laboratory stockroom and request a 1-F parallel-plate capacitor, until stockroom attendants got tired of the joke.

Exercise 11.2.1A

The capacitance of a parallel-plate capacitor is 2.0 pF. If the area of each plate is $2.4\ cm^2$, what is the plate separation?

Answer available at the end of this section.

Exercise 11.2.1B

Verify that σ/V and ϵ_0/d have the same physical units.

Spherical Capacitor

A spherical capacitor is another set of conductors whose capacitance can be easily determined (Figure 11.2.5). It consists of two concentric conducting spherical shells of radii R_1 (inner shell) and R_2 (outer shell). The shells are given equal and opposite charges $+Q$ and $-Q$, respectively. From symmetry, the electrical field between the shells is directed radially outward. We can obtain the magnitude of the field by applying Gauss's law over a spherical Gaussian surface of radius **r** concentric with the shells. The enclosed charge is $+Q$; therefore we have

$$\oint_S \vec{E} \cdot \hat{n} dA = E(4\pi r^2) = \frac{Q}{\epsilon_0}.$$

Thus, the electrical field between the conductors is

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \hat{r}.$$

We substitute this $\vec{E}$ into Equation 11.2.2 and integrate along a radial path between the shells:

$$V = \int_{R_1}^{R_2} \vec{E} \cdot d\vec{l} = \int_{R_1}^{R_2} \left(\frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \hat{r} \right) \cdot (\hat{r} dr) = \frac{Q}{4\pi\epsilon_0} \int_{R_1}^{R_2} \frac{dr}{r^2} = \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{R_1} - \frac{1}{R_2} \right).$$

In this equation, the potential difference between the plates is

$$V = -(V_2 - V_1) = V_1 - V_2.$$

We substitute this result into Equation 11.2.1 to find the capacitance of a spherical capacitor:

$$C = \frac{Q}{V} = 4\pi\epsilon_0 \frac{R_1 R_2}{R_2 - R_1}. \tag{11.2.4}$$

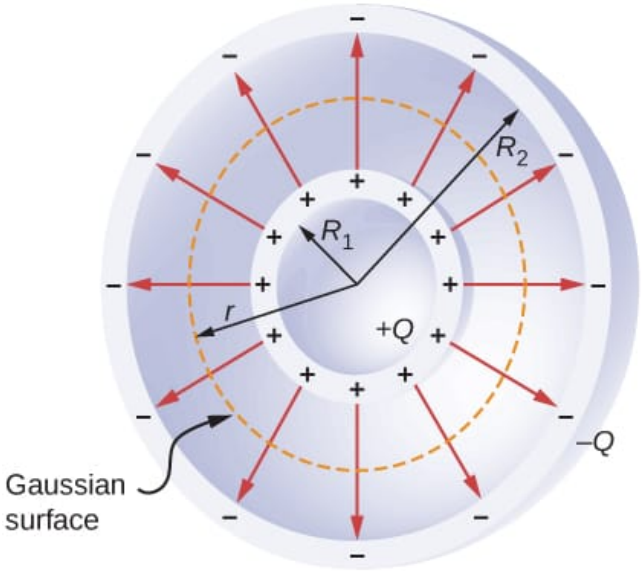


Figure 11.2.5: A spherical capacitor con**sists of two concentric conducting spheres. Note that the charges on a conductor reside on its surface.**

Example 11.2.2: Capacitance of an Isolated Sphere

Calculate the capacitance of a single isolated conducting sphere of radius R_1 and compare it with Equation 11.2.4 in the limit as $R_2 \to \infty$.

Strategy

We assume that the charge on the sphere is Q, and so we follow the four steps outlined earlier. We also assume the other conductor to be a concentric hollow sphere of infinite radius.

Solution

On the outside of an isolated conducting sphere, the electrical field is given by Equation 11.2.2. The magnitude of the potential difference between the surface of an isolated sphere and infinity is

$$
\begin{aligned}
V &= \int_{R_1}^{+\infty} \vec{E} \cdot d\vec{l} \\
&= \frac{Q}{4\pi\epsilon_0} \int_{R_1}^{+\infty} \frac{1}{r^2} \hat{r} \cdot (\hat{r}\, dr) \\
&= \frac{Q}{4\pi\epsilon_0} \int_{R_1}^{+\infty} \frac{dr}{r^2} \\
&= \frac{1}{4\pi\epsilon_0} \frac{Q}{R_1}
\end{aligned}
$$

The capacitance of an isolated sphere is therefore

$$C = \frac{Q}{V} = Q\frac{4\pi\epsilon_0 R_1}{Q} = 4\pi\epsilon_0 R_1.$$

Significance

The same result can be obtained by taking the limit of Equation 11.2.4 as $R_2 \to \infty$. A single isolated sphere is therefore equivalent to a spherical capacitor whose outer shell has an infinitely large radius.

Exercise 11.2.2

The radius of the outer sphere of a spherical capacitor is five times the radius of its inner shell. What are the dimensions of this capacitor if its capacitance is 5.00 pF?

Answer available at the end of this section.

Cylindrical Capacitor

A cylindrical capacitor consists of two concentric, conducting cylinders (Figure 11.2.6). The inner cylinder, of radius R_1, may either be a shell or be completely solid. The outer cylinder is a shell of inner radius R_2. We assume that the length of each cylinder is **l** and that the excess charges $+Q$ and $-Q$ reside on the inner and outer cylinders, respectively.

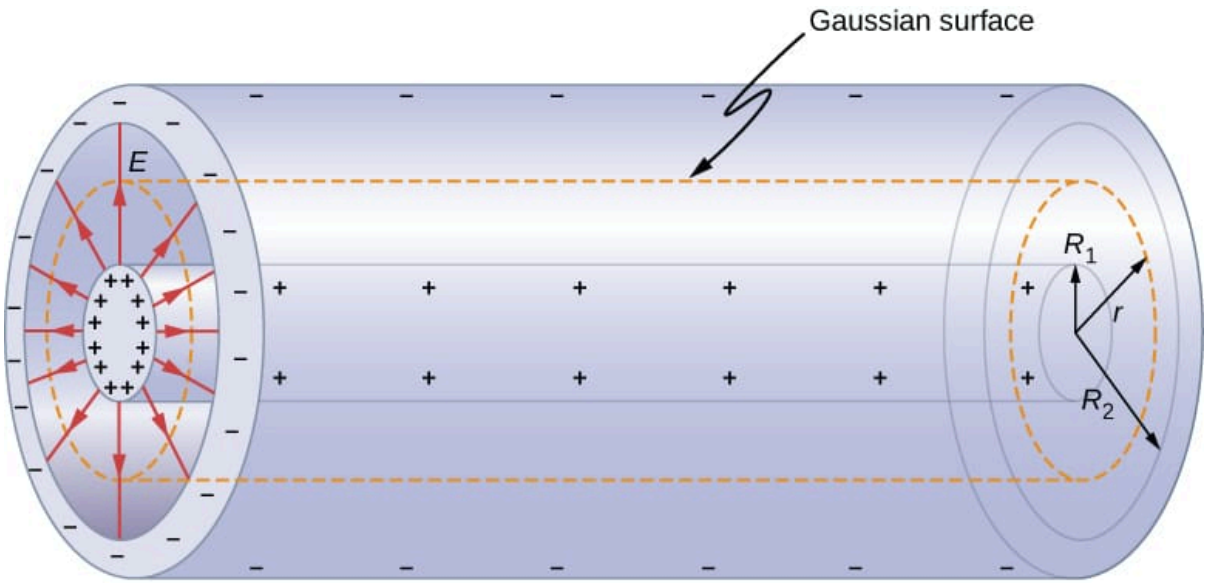


Figure 11.2.6: A cylindrical capacitor consists of two concentric, conducting cylinders. Here, the charge on the outer surface of the inner cylinder is positive (indicated by $+$) and the charge on the inner surface of the outer cylinder is negative (indicated by $-$).

With edge effects ignored, the electrical field between the conductors is directed radially outward from the common axis of the

cylinders. Using the Gaussian surface shown in Figure 11.2.6, we have

$$\oint_S \vec{E} \cdot \hat{n} dA = E(2\pi rl) = \frac{Q}{\epsilon_0}.$$

Therefore, the electrical field between the cylinders is

$$\vec{E} = \frac{1}{2\pi\epsilon_0}\frac{Q}{r\,l}\hat{r}.$$

where $\hat{r}$ is the unit radial vector along the radius of the cylinder. We can substitute into Equation 11.2.2 and find the potential difference between the cylinders:

$$V = \int_{R_1}^{R_2} \vec{E} \cdot \vec{dl}_p = \frac{Q}{2\pi\epsilon_0 l}\int_{R_1}^{R_2} \frac{1}{r}\hat{r} \cdot (\hat{r}\, dr) = \frac{Q}{2\pi\epsilon_0 l}\int_{R_1}^{R_2} \frac{dr}{r} = \frac{Q}{2\pi\epsilon_0 l}\ln\, r\Big|_{R_1}^{R_2} = \frac{Q}{2\pi\epsilon_0 l}\frac{R_2}{R_1}.$$

Thus, the capacitance of a cylindrical capacitor is

$$C = \frac{Q}{V} = \frac{2\pi\epsilon_0\, l}{\ln(R_2/R_1)}. \tag{11.2.5}$$

As in other cases, this capacitance depends only on the geometry of the conductor arrangement. An important application of Equation 11.2.5 is the determination of the capacitance per unit length of a **coaxial cable**, which is commonly used to transmit time-varying electrical signals. A coaxial cable consists of two concentric, cylindrical conductors separated by an insulating material. (Here, we assume a vacuum between the conductors, but the physics is qualitatively almost the same when the space between the conductors is filled by a dielectric.) This configuration shields the electrical signal propagating down the inner conductor from stray electrical fields external to the cable. Current flows in opposite directions in the inner and the outer conductors, with the outer conductor usually grounded. Now, from Equation 11.2.5, the capacitance per unit length of the coaxial cable is given by

$$\frac{C}{l} = \frac{2\pi\epsilon_0}{\ln(R_2/R_1)}.$$

In practical applications, it is important to select specific values of C/l. This can be accomplished with appropriate choices of radii of the conductors and of the insulating material between them.

Exercise 11.2.3

When a cylindrical capacitor is given a charge of 0.500 nC, a potential difference of 20.0 V is measured between the cylinders.

a. What is the capacitance of this system?
b. If the cylinders are 1.0 m long, what is the ratio of their radii?

Answer available at the end of this section.

Several types of practical capacitors are shown in Figure 11.2.3. Common capacitors are often made of two small pieces of metal foil separated by two small pieces of insulation (Figure 11.2.1*b*). The metal foil and insulation are encased in a protective coating, and two metal leads are used for connecting the foils to an external circuit. Some common insulating materials are mica, ceramic, paper, and Teflon™ non-stick coating.

Another popular type of capacitor is an **electrolytic capacitor**. It consists of an oxidized metal in a conducting paste. The main advantage of an electrolytic capacitor is its high capacitance relative to other common types of capacitors. For example, capacitance of one type of aluminum electrolytic capacitor can be as high as 1.0 F. However, you must be careful when using an electrolytic capacitor in a circuit, because it only functions correctly when the metal foil is at a higher potential than the conducting paste. When reverse polarization occurs, electrolytic action destroys the oxide film. This type of capacitor cannot be connected across an alternating current source, because half of the time, ac voltage would have the wrong polarity, as an alternating current reverses its polarity (see Alternating-Current Circuts on alternating-current circuits).

A **variable air capacitor** (Figure 11.2.7) has two sets of parallel plates. One set of plates is fixed (indicated as “stator”), and the other set of plates is attached to a shaft that can be rotated (indicated as “rotor”). By turning the shaft, the cross-sectional area in the overlap of the plates can be changed; therefore, the capacitance of this system can be tuned to a desired value. Capacitor tuning has applications in any type of radio transmission and in receiving radio signals from electronic devices. Any time you tune your car radio to your favorite station, think of capacitance.

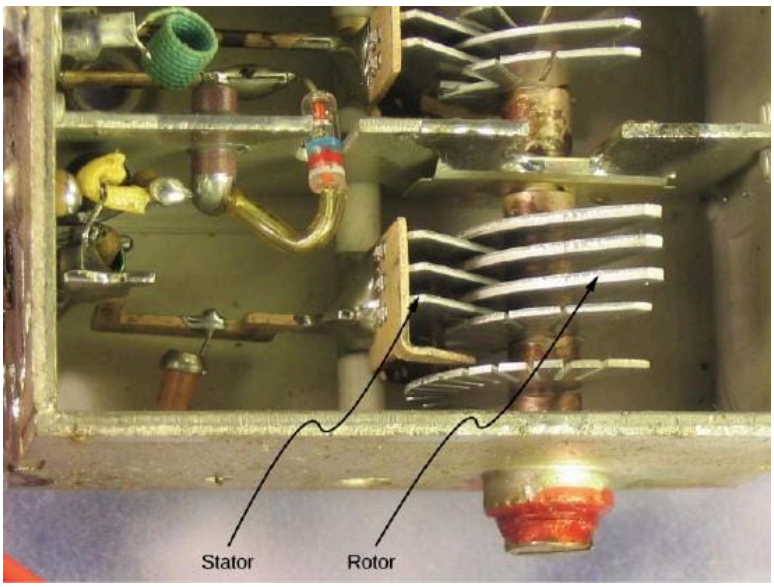


Figure 11.2.7: In a variable air capacitor, capacitance can be tuned by changing the effective area of the plates. (credit: modification of work by Robbie Sproule)

The symbols shown in Figure 11.2.8 are circuit representations of various types of capacitors. We generally use the symbol shown in Figure 11.2.8a. The symbol in Figure 11.2.8c represents a variable-capacitance capacitor. Notice the similarity of these symbols to the symmetry of a parallel-plate capacitor. An electrolytic capacitor is represented by the symbol in part Figure 11.2.8b, where the curved plate indicates the negative terminal.

Figure a shows two vertical lines. Figure b shows a vertical line to the left and another, slightly curved vertical line to the right. Figure c shows two vertical lines and an arrow cutting across them diagonally. In all figures, each line is connected to a horizontal line on the outside.

Figure 11.2.8: This shows three different circuit representations of capacitors. The symbol in (a) is the most commonly used one. The symbol in (b) represents an electrolytic capacitor. The symbol in (c) represents a variable-capacitance capacitor.

An interesting applied example of a capacitor model comes from cell biology and deals with the electrical potential in the plasma membrane of a living cell (Figure 11.2.9). **Cell membranes** separate cells from their surroundings, but allow some selected ions to pass in or out of the cell. The potential difference across a membrane is about 70 mV. The cell membrane may be 7 to 10 nm thick. Treating the cell membrane as a nano-sized capacitor, the estimate of the smallest electrical field strength across its ‘plates’ yields the value

$$E = \frac{V}{d} = \frac{70 \times 10^{-3} V}{10 \times 10^{-9} m} = 7 \times 10^{6} V/m > 3\, MV/m.$$

This magnitude of electrical field is great enough to create an electrical spark in the air.

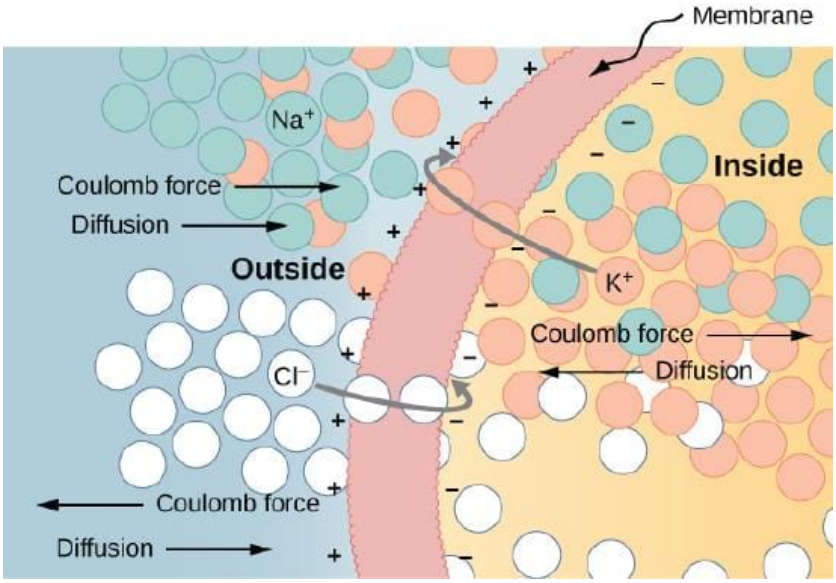


Figure 11.2.9: The semipermeable membrane of a biological cell has different concentrations of ions on its interior surface than on its exterior. Diffusion moves the K^+ (potassium) and Cl^- (chloride) ions in the directions shown, until the Coulomb force halts further transfer. In this way, the exterior of the membrane acquires a positive charge and its interior surface acquires a negative charge, creating a potential difference across the membrane. The membrane is normally impermeable to Na^+ (sodium ions).

Simulation

Visit the PhET Explorations: Capacitor Lab to explore how a capacitor works. Change the size of the plates and add a dielectric to see the effect on capacitance. Change the voltage and see charges built up on the plates. Observe the electrical field in the capacitor. Measure the voltage and the electrical field.

Answers

Exercise 11.2.1A

$1.1 \times 10^{-3} m$

Exercise 11.2.2

3.59 cm, 17.98 cm

Exercise 11.2.3

a. 25.0 pF

b. 9.2

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11.3: Capacitors in Series and in Parallel

Learning Objectives

By the end of this section, you will be able to:

- Explain how to determine the equivalent capacitance of capacitors in series and in parallel combinations
- Compute the potential difference across the plates and the charge on the plates for a capacitor in a network and determine the net capacitance of a network of capacitors

Several capacitors can be connected together to be used in a variety of applications. Multiple connections of capacitors behave as a single equivalent capacitor. The total capacitance of this equivalent single capacitor depends both on the individual capacitors and how they are connected. Capacitors can be arranged in two simple and common types of connections, known as **series** and **parallel**, for which we can easily calculate the total capacitance. These two basic combinations, series and parallel, can also be used as part of more complex connections.

The Series Combination of Capacitors

Figure $11.3.1$ illustrates a series combination of three capacitors, arranged in a row within the circuit. As for any capacitor, the capacitance of the combination is related to both charge and voltage:

$$C = \frac{Q}{V}.$$

When this series combination is connected to a battery with voltage **V**, each of the capacitors acquires an identical charge **Q**. To explain, first note that the charge on the plate connected to the positive terminal of the battery is $+Q$ and the charge on the plate connected to the negative terminal is $-Q$. Charges are then induced on the other plates so that the sum of the charges on all plates, and the sum of charges on any pair of capacitor plates, is zero. However, the potential drop $V_1 = Q/C_1$ on one capacitor may be different from the potential drop $V_2 = Q/C_2$ on another capacitor, because, generally, the capacitors may have different capacitances. The series combination of two or three capacitors resembles a single capacitor with a smaller capacitance. Generally, any number of capacitors connected in series is equivalent to one capacitor whose capacitance (called the **equivalent capacitance**) is smaller than the smallest of the capacitances in the series combination. Charge on this equivalent capacitor is the same as the charge on any capacitor in a series combination: That is, **all capacitors of a series combination have the same charge**. This occurs due to the conservation of charge in the circuit. When a charge **Q** in a series circuit is removed from a plate of the first capacitor (which we denote as $-Q$), it must be placed on a plate of the second capacitor (which we denote as $+Q$), and so on.

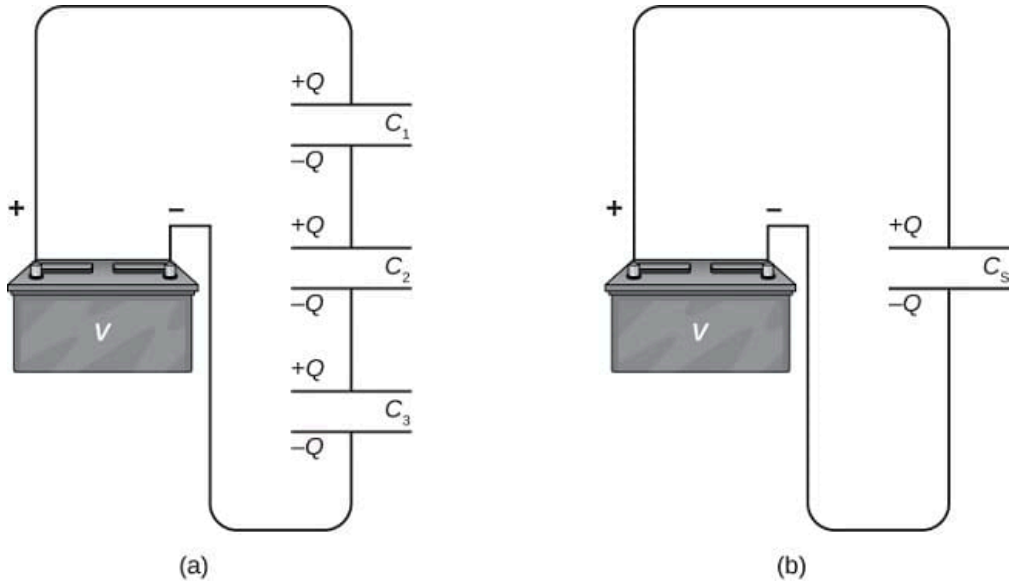


Figure 11.3.1: (a) Three capacitors are connected in series. The magnitude of the charge on each plate is Q. (b) The network of capacitors in (a) is equivalent to one capacitor that has a smaller capacitance than any of the individual capacitances in (a), and the charge on its plates is Q.

We can find an expression for the total (equivalent) capacitance by considering the voltages across the individual capacitors. The potentials across capacitors 1, 2, and 3 are, respectively, $V_1 = Q/C_1$, $V_2 = Q/C_2$, and $V_3 = Q/C_3$. These potentials must sum up to the voltage of the battery, giving the following potential balance:

$$V = V_1 + V_2 + V_3.$$

Potential V is measured across an equivalent capacitor that holds charge Q and has an equivalent capacitance C_S. Entering the expressions for V_1, V_2, and V_3, we get

$$\frac{Q}{C_S} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}.$$

Canceling the charge **Q**, we obtain an expression containing the equivalent capacitance, C_S, of three capacitors connected in series:

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}.$$

This expression can be generalized to any number of capacitors in a series network.

Series Combination

For capacitors connected in a series combination, the reciprocal of the equivalent capacitance is the sum of reciprocals of individual capacitances:

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots \tag{11.3.1}$$

Example 11.3.1: Equivalent Capacitance of a Series Network

Find the total capacitance for three capacitors connected in series, given their individual capacitances are $1.000\mu F$, $5.000\mu F$, and $8.000\mu F$.

Strategy

Because there are only three capacitors in this network, we can find the equivalent capacitance by using Equation 11.3.1 with three terms.

Solution

We enter the given capacitances into Equation 11.3.1:

$$\begin{aligned}\frac{1}{C_S} &= \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \\ &= \frac{1}{1.000\mu F} + \frac{1}{5.000\mu F} + \frac{1}{8.000\mu F} \\ &= \frac{1.325}{\mu F}.\end{aligned}$$

Now we invert this result and obtain

$$\begin{aligned}C_S &= \frac{\mu F}{1.325} \\ &= 0.755\mu F.\end{aligned}$$

Significance

Note that in a series network of capacitors, the equivalent capacitance is always less than the smallest individual capacitance in the network.

The Parallel Combination of Capacitors

A parallel combination of three capacitors, with one plate of each capacitor connected to one side of the circuit and the other plate connected to the other side, is illustrated in Figure 11.3.2*a*. Since the capacitors are connected in parallel, **they all have the same voltage V across their plates**. However, each capacitor in the parallel network may store a different charge. To find the equivalent capacitance C_p of the parallel network, we note that the total charge **Q** stored by the network is the sum of all the individual charges:

$$Q = Q_1 + Q_2 + Q_3.$$

On the left-hand side of this equation, we use the relation $Q = C_pV$, which holds for the entire network. On the right-hand side of the equation, we use the relations $Q_1 = C_1V$, $Q_2 = C_2V$, and $Q_3 = C_3V$ for the three capacitors in the network. In this way we obtain

$$C_pV = C_1V + C_2V + C_3V.$$

This equation, when simplified, is the expression for the equivalent capacitance of the parallel network of three capacitors:

$$C_p = C_1 + C_2 + C_3.$$

This expression is easily generalized to any number of capacitors connected in parallel in the network.

Parallel Combination

For capacitors connected in a parallel combination, the equivalent (net) capacitance is the sum of all individual capacitances in the network,

$$C_p = C_1 + C_2 + C_3 + \ldots \tag{11.3.2}$$

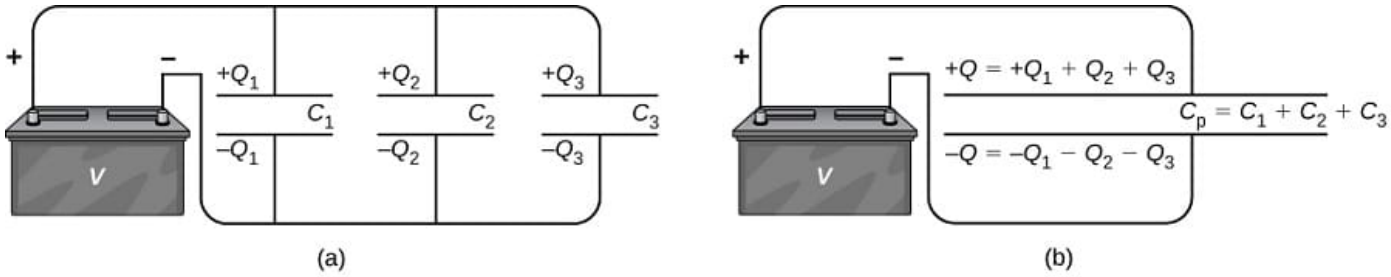


Figure 11.3.2: (a) Three capacitors are connected in parallel. Each capacitor is connected directly to the battery. (b) The charge on the equivalent capacitor is the sum of the charges on the individual capacitors.

Example 11.3.2: Equivalent Capacitance of a Parallel Network

Find the net capacitance for three capacitors connected in parallel, given their individual capacitances are $1.0\mu F$, $5.0\mu F$, and $8.0\mu F$.

Strategy

Because there are only three capacitors in this network, we can find the equivalent capacitance by using Equation 11.3.2 with three terms.

Solution

Entering the given capacitances into Equation 11.3.2 yields

$$\begin{aligned} C_p &= C_1 + C_2 + C_3 \\ &= 1.0\mu F + 5.0\mu F + 8.0\mu F \\ &= 14.0\mu F. \end{aligned}$$

Significance

Note that in a parallel network of capacitors, the equivalent capacitance is always larger than any of the individual capacitances in the network.

Capacitor networks are usually some combination of series and parallel connections, as shown in Figure 11.3.3. To find the net capacitance of such combinations, we identify parts that contain only series or only parallel connections, and find their equivalent capacitances. We repeat this process until we can determine the equivalent capacitance of the entire network. The following example illustrates this process.

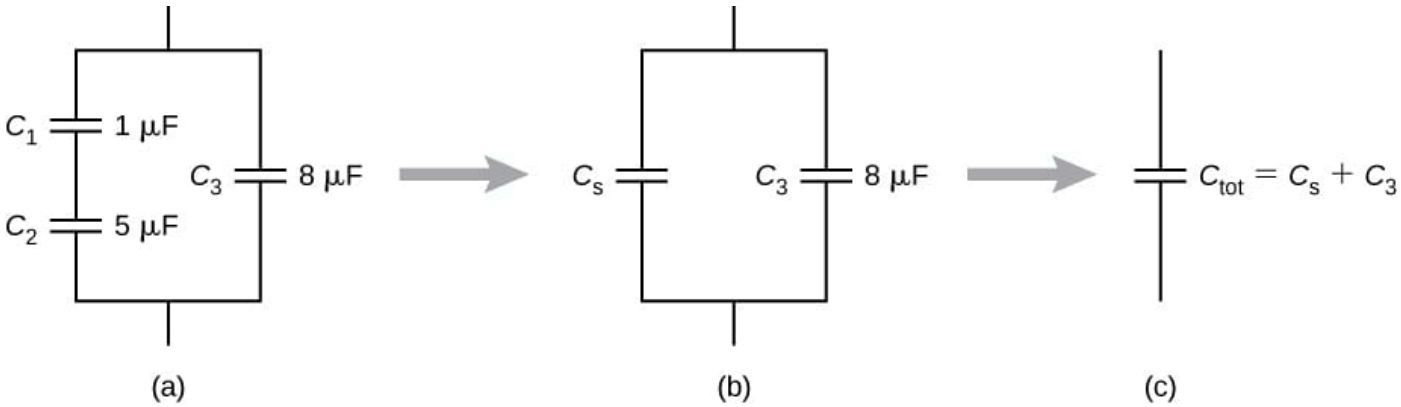


Figure 11.3.3: (a) This circuit contains both series and parallel connections of capacitors. (b) C_1 and C_2 are in series; their equivalent capacitance is C_S c) The equivalent capacitance C_S is connected in parallel with C_3. Thus, the equivalent capacitance of the entire network is the sum of C_S and C_3.

Example 11.3.3: Equivalent Capacitance of a Network

Find the total capacitance of the combination of capacitors shown in Figure 11.3.3. Assume the capacitances are known to three decimal places ($C_1 = 1.000\mu F, C_2 = 5.000\mu F, C_3 = 8.000\mu F$). Round your answer to three decimal places.

Strategy

We first identify which capacitors are in series and which are in parallel. Capacitors C_1 and C_2 are in series. Their combination, labeled C_S is in parallel with C_3.

Solution

Since C_1 and C_2 are in series, their equivalent capacitance C_S is obtained with Equation 11.3.1:

$$\begin{aligned}\frac{1}{C_S} &= \frac{1}{C_1} + \frac{1}{C_2} \\ &= \frac{1}{1.000\mu F} + \frac{1}{5.000\mu F} \\ &= \frac{1.200}{\mu F}\end{aligned}$$

Therefor

$$C_S = 0.833\mu F.$$

Capacitance C_S is connected in parallel with the third capacitance C_3, so we use Equation 11.3.2 find the equivalent capacitance **C** of the entire network:

$$\begin{aligned}C &= C_S + C_3 \\ &= 0.833\mu F + 8.000\mu F \\ &= 8.833\mu F.\end{aligned}$$

Network of Capacitors

Determine the net capacitance **C** of the capacitor combination shown in Figure 11.3.4 when the capacitances are $C_1 = 12.0\mu F, C_2 = 2.0\mu F$, and $C_3 = 4.0\mu F$. When a 12.0-V potential difference is maintained across the combination, find the charge and the voltage across each capacitor.

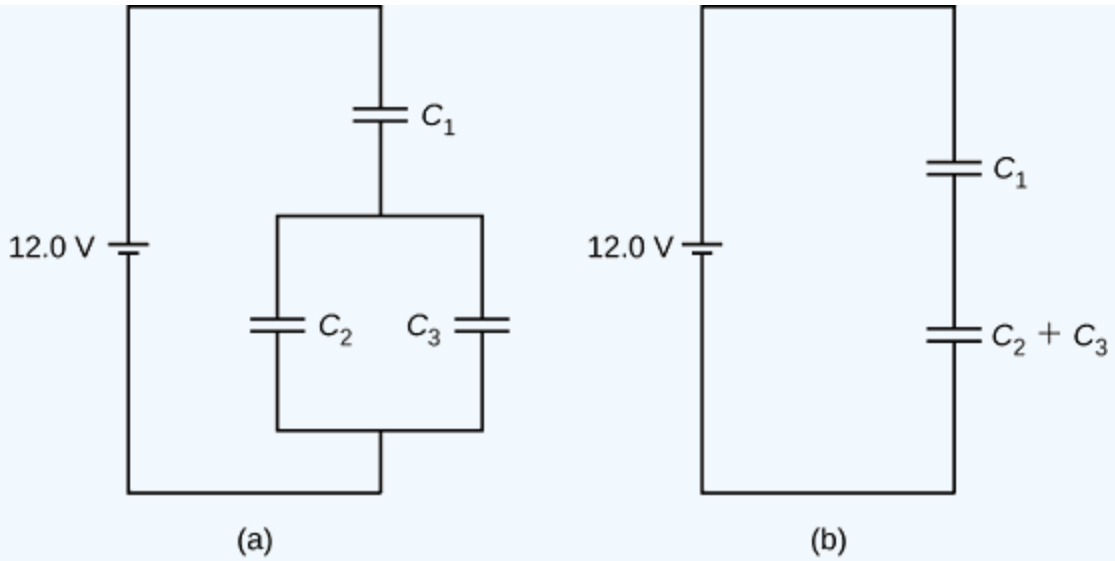


Figure 11.3.4: (a) A capacitor combination. (b) An equivalent two-capacitor combination.

Strategy We first compute the net capacitance C_{23} of the parallel connection C_2 and C_3. Then **C** is the net capacitance of the series connection C_1 and C_{23}. We use the relation $C = Q/V$ to find the charges Q_1, Q_2, and Q_3, and the voltages V_1, V_2, and V_3 across capacitors 1, 2, and 3, respectively.

Solution The equivalent capacitance for C_2 and C_3 is

$$C_{23} = C_2 + C_3 = 2.0\mu F + 4.0\mu F = 6.0\mu F.$$

The entire three-capacitor combination is equivalent to two capacitors in series,

$$\frac{1}{C} = \frac{1}{12.0\mu F} + \frac{1}{6.0\mu F} = \frac{1}{4.0\mu F} \Rightarrow C = 4.0\mu F.$$

Consider the equivalent two-capacitor combination in Figure 11.3.2*b*. Since the capacitors are in series, they have the same charge, $Q_1 = Q_{23}$. Also, the capacitors share the 12.0-V potential difference, so

$$12.0V = V_1 + V_{23} = \frac{Q_1}{C_1} + \frac{Q_{23}}{C_{23}} = \frac{Q_1}{12.0\mu F} + \frac{Q_1}{6.0\mu F} \Rightarrow Q_1 = 48.0\mu C.$$

Now the potential difference across capacitor 1 is

$$V_1 = \frac{Q_1}{C_1} = \frac{48.0\mu C}{12.0\mu F} = 4.0V.$$

Because capacitors 2 and 3 are connected in parallel, they are at the same potential difference:

$$V_2 = V_3 = 12.0V - 4.0V = 8.0V.$$

Hence, the charges on these two capacitors are, respectively,

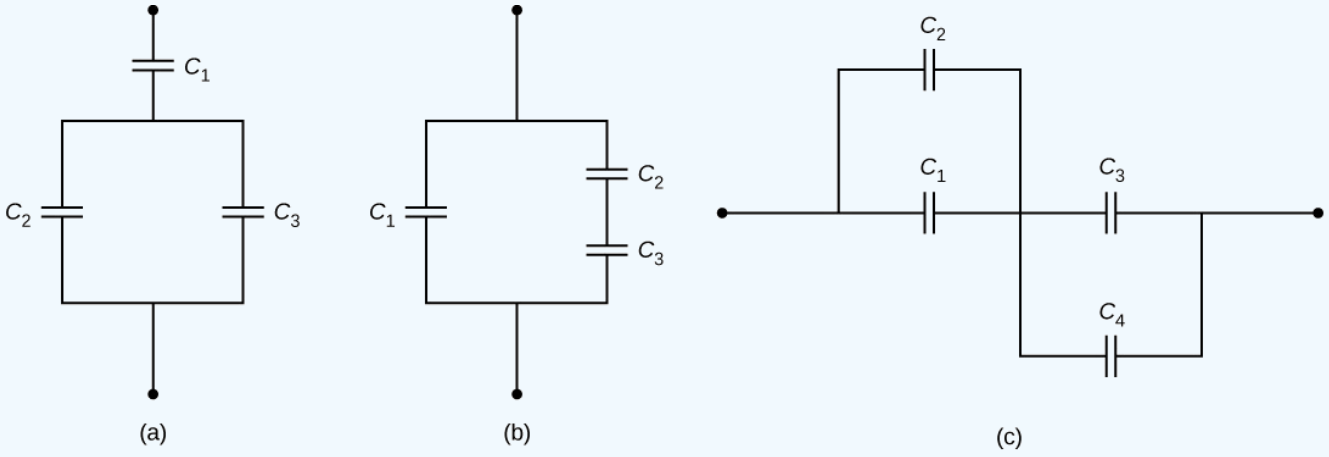
$$Q_2 = C_2V_2 = (2.0\mu F)(8.0V) = 16.0\mu C,$$

$$Q_3 = C_3V_3 = (4.0\mu F)(8.0V) = 32.0\mu C.$$

Significance As expected, the net charge on the parallel combination of C_2 and C_3 is $Q_{23} = Q_2 + Q_3 = 48.0\mu C$.

Exercise 11.3.1

Determine the net capacitance **C** of each network of capacitors shown below. Assume that $C_1 = 1.0pF, C_2 = 2.0pF, C_3 = 4.0pF$, and $C_4 = 5.0pF$. Find the charge on each capacitor, assuming there is a potential difference of 12.0 V across each network.



Answer a available at the end of this section.

Answers

Exercise 11.3.1

$C = 0.86pF, Q_1 = 10pC, Q_2 = 3.4pC, Q_3 = 6.8pC$

$C = 2.3pF, Q_1 = 12pC, Q_2 = Q_3 = 16pC$

$C = 2.3pF, Q_1 = 9.0pC, Q_2 = 18pC, Q_3 = 12pC, Q_4 = 15pC$

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11.4: Energy Stored in a Capacitor

Learning Objectives

By the end of this section, you will be able to:

- Explain how energy is stored in a capacitor
- Use energy relations to determine the energy stored in a capacitor network

Most of us have seen dramatizations of medical personnel using a defibrillator to pass an electrical current through a patient's heart to get it to beat normally. Often realistic in detail, the person applying the shock directs another person to "make it 400 joules this time." The energy delivered by the defibrillator is stored in a capacitor and can be adjusted to fit the situation. SI units of joules are often employed. Less dramatic is the use of capacitors in microelectronics to supply energy when batteries are charged (Figure 11.4.1). Capacitors are also used to supply energy for flash lamps on cameras.

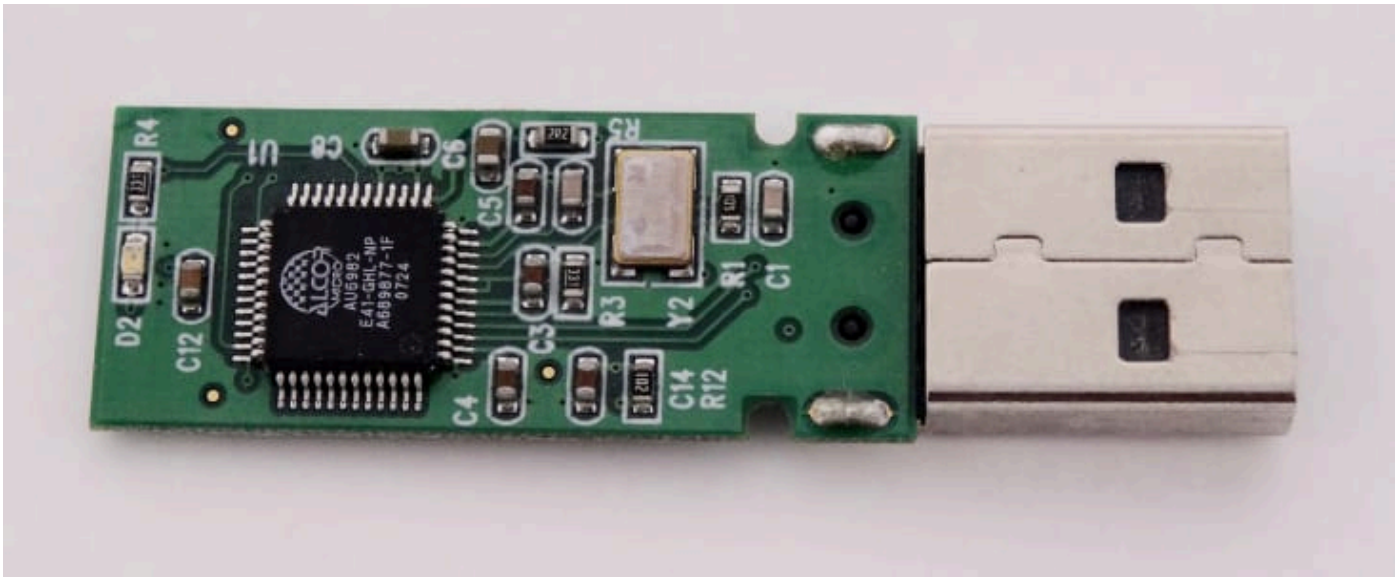


Figure 11.4.1: The capacitors on the circuit board for an electronic device follow a labeling convention that identifies each one with a code that begins with the letter "C."

The energy U_C stored in a capacitor is electrostatic potential energy and is thus related to the charge **Q** and voltage **V** between the capacitor plates. A charged capacitor stores energy in the electrical field between its plates. As the capacitor is being charged, the electrical field builds up. When a charged capacitor is disconnected from a battery, its energy remains in the field in the space between its plates.

To gain insight into how this energy may be expressed (in terms of **Q** and **V**), consider a charged, empty, parallel-plate capacitor; that is, a capacitor without a dielectric but with a vacuum between its plates. The space between its plates has a volume **Ad**, and it is filled with a uniform electrostatic field **E**. The total energy U_C of the capacitor is contained within this space. The **energy density** u_E in this space is simply U_C divided by the volume **Ad**. If we know the energy density, the energy can be found as $U_C = u_E(Ad)$. We will learn in Electromagnetic Waves (after completing the study of Maxwell's equations) that the energy density u_E in a region of free space occupied by an electrical field **E** depends only on the magnitude of the field and is

$$u_E = \frac{1}{2}\epsilon_0 E^2.$$

If we multiply the energy density by the volume between the plates, we obtain the amount of energy stored between the plates of a parallel-plate capacitor $U_C = u_E(Ad) = \frac{1}{2}\epsilon_0 E^2 Ad = \frac{1}{2}\epsilon_0 \frac{V^2}{d^2} Ad = \frac{1}{2}V^2\epsilon_0\frac{A}{d} = \frac{1}{2}V^2 C$.

In this derivation, we used the fact that the electrical field between the plates is uniform so that $E = V/d$ and $C = \epsilon_0 A/d$. Because

$C = Q/V$, we can express this result in other equivalent forms:

$$U_C = \frac{1}{2}V^2C = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV. \tag{11.4.1}$$

The expression in Equation 11.4.1 for the energy stored in a parallel-plate capacitor is generally valid for all types of capacitors. To see this, consider any uncharged capacitor (not necessarily a parallel-plate type). At some instant, we connect it across a battery, giving it a potential difference $V = q/C$ between its plates. Initially, the charge on the plates is $Q = 0$. As the capacitor is being charged, the charge gradually builds up on its plates, and after some time, it reaches the value **Q**. To move an infinitesimal charge **dq** from the negative plate to the positive plate (from a lower to a higher potential), the amount of work **dW** that must be done on **dq** is $dW = W\ dq = \frac{q}{C}dq$.

This work becomes the energy stored in the electrical field of the capacitor. In order to charge the capacitor to a charge **Q**, the total work required is

$$W = \int_0^{W(Q)} dW = \int_0^Q \frac{q}{C}dq = \frac{1}{2}\frac{Q^2}{C}.$$

Since the geometry of the capacitor has not been specified, this equation holds for any type of capacitor. The total work **W** needed to charge a capacitor is the electrical potential energy U_C stored in it, or $U_C = W$. When the charge is expressed in coulombs, potential is expressed in volts, and the capacitance is expressed in farads, this relation gives the energy in joules.

Knowing that the energy stored in a capacitor is $U_C = Q^2/(2C)$, we can now find the energy density u_E stored in a vacuum between the plates of a charged parallel-plate capacitor. We just have to divide U_C by the volume **Ad** of space between its plates and take into account that for a parallel-plate capacitor, we have $E = \sigma/\epsilon_0$ and $C = \epsilon_0 A/d$. Therefore, we obtain

$$u_E = \frac{U_C}{Ad} = \frac{1}{2}\frac{Q^2}{C}\frac{1}{Ad} = \frac{1}{2}\frac{Q^2}{\epsilon_0 A/d}\frac{1}{Ad} = \frac{1}{2}\frac{1}{\epsilon_0}\left(\frac{Q}{A}\right)^2 = \frac{\sigma^2}{2\epsilon_0} = \frac{(E\epsilon_0)^2}{2\epsilon_0} = \frac{\epsilon_0}{2}E^2$$

We see that this expression for the density of energy stored in a parallel-plate capacitor is in accordance with the general relation expressed in Equation ???. We could repeat this calculation for either a spherical capacitor or a cylindrical capacitor—or other capacitors—and in all cases, we would end up with the general relation given by Equation ???.

Energy Stored in a Capacitor

Calculate the energy stored in the capacitor network in Figure 8.3.4a when the capacitors are fully charged and when the capacitances are $C_1 = 12.0\ \mu F,\ C_2 = 2.0\ \mu F$, and $C_3 = 4.0\ \mu F$, respectively.

Strategy

We use Equation 11.4.1 to find the energy $U_1,\ U_2$, and U_3 stored in capacitors 1, 2, and 3, respectively. The total energy is the sum of all these energies.

Solution We identify $C_1 = 12.0\ \mu F$ and $V_1 = 4.0\ V,\ C_2 = 2.0\ \mu F$ and $V_2 = 8.0\ V,\ C_3 = 4.0\ \mu F$ and $V_3 = 8.0\ V$. The energies stored in these capacitors are

$$U_1 = \frac{1}{2}C_1V_1^2 = \frac{1}{2}(12.0\,\mu F)(4.0\,V)^2 = 96\,\mu J,$$

$$U_2 = \frac{1}{2}C_2V_2^2 = \frac{1}{2}(2.0\,\mu F)(8.0\,V)^2 = 64\,\mu J,$$

$$U_3 = \frac{1}{2}C_3V_3^2 = \frac{1}{2}(4.0\,\mu F)(8.0\,V)^2 = 130\,\mu J,$$

The total energy stored in this network is

$$U_C = U_1 + U_2 + U_3 = 96\,\mu J + 64\,\mu J + 130\,\mu J = 0.29\,mJ.$$

Significance

We can verify this result by calculating the energy stored in the single $4.0-\mu F$ capacitor, which is found to be equivalent to the entire network. The voltage across the network is 12.0 V. The total energy obtained in this way agrees with our previously obtained result, $U_C=\frac{1}{2}CV^2=\frac{1}{2}(4.0\ \mu F)(12.0\,V)^2=0.29\ mJ$

Exercise 11.4.1

The potential difference across a 5.0-pF capacitor is 0.40 V. (a) What is the energy stored in this capacitor? (b) The potential difference is now increased to 1.20 V. By what factor is the stored energy increased?

Answer available at the end of this section.

In a cardiac emergency, a portable electronic device known as an automated external defibrillator (AED) can be a lifesaver. A **defibrillator** (Figure 11.4.2) delivers a large charge in a short burst, or a shock, to a person's heart to correct abnormal heart rhythm (an arrhythmia). A heart attack can arise from the onset of fast, irregular beating of the heart—called cardiac or ventricular fibrillation. Applying a large shock of electrical energy can terminate the arrhythmia and allow the body's natural pacemaker to resume its normal rhythm. Today, it is common for ambulances to carry AEDs. AEDs are also found in many public places. These are designed to be used by lay persons. The device automatically diagnoses the patient's heart rhythm and then applies the shock with appropriate energy and waveform. CPR (cardiopulmonary resuscitation) is recommended in many cases before using a defibrillator.



Figure 11.4.2: Automated external defibrillators are found in many public places. These portable units provide verbal instructions for use in the important first few minutes for a person suffering a cardiac attack.

Example 11.4.2: Capacitance of a Heart Defibrillator

A heart defibrillator delivers $4.00\times10^2 J$ of energy by discharging a capacitor initially at $1.00\times10^4 V$. What is its capacitance?

Strategy

We are given U_C and **V**, and we are asked to find the capacitance **C**. We solve Equation 11.4.1 for **C** and substitute.

Solution

Solving this expression for **C** and entering the given values yields $C=2\frac{U_C}{V^2}=2\frac{4.00\times10^2 J}{(1.00\times10^4 V)^2}=8.00\ \mu F$.

Answers

Exercise 11.4.1

a. $4.0 \times 10^{-13} J$; b. 9 times

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11.5: Capacitor with a Dielectric

Learning Objectives

By the end of this section, you will be able to:

- Describe the effects a dielectric in a capacitor has on capacitance and other properties
- Calculate the capacitance of a capacitor containing a dielectric

As we discussed earlier, an insulating material placed between the plates of a capacitor is called a dielectric. Inserting a dielectric between the plates of a capacitor affects its capacitance. To see why, let's consider an experiment described in Figure 11.5.1 . Initially, a capacitor with capacitance C_0 when there is air between its plates is charged by a battery to voltage V_0. When the capacitor is fully charged, the battery is disconnected. A charge Q_0 then resides on the plates, and the potential difference between the plates is measured to be V_0. Now, suppose we insert a dielectric that **totally** fills the gap between the plates. If we monitor the voltage, we find that the voltmeter reading has dropped to a **smaller** value V. We write this new voltage value as a fraction of the original voltage V_0, with a positive number κ, $\kappa > 1$.

$$V = \frac{1}{\kappa} V_0.$$

The constant κ in this equation is called the **dielectric constant** of the material between the plates, and its value is characteristic for the material. A detailed explanation for why the dielectric reduces the voltage is given in the next section. Different materials have different dielectric constants (a table of values for typical materials is provided in the next section). Once the battery becomes disconnected, there is no path for a charge to flow to the battery from the capacitor plates. Hence, the insertion of the dielectric has no effect on the charge on the plate, which remains at a value of Q_0. Therefore, we find that the capacitance of the capacitor with a dielectric is

$$C = \frac{Q_0}{V} = \frac{Q_0}{V_0/\kappa} = \kappa \frac{Q_0}{V_0} = \kappa C_0. \tag{11.5.1}$$

This equation tells us that the **capacitance C_0 of an empty (vacuum) capacitor can be increased by a factor of κ when we insert a dielectric material to completely fill the space between its plates**. Note that Equation 11.5.1 can also be used for an empty capacitor by setting $\kappa = 1$. In other words, we can say that the dielectric constant of the vacuum is 1, which is a reference value.

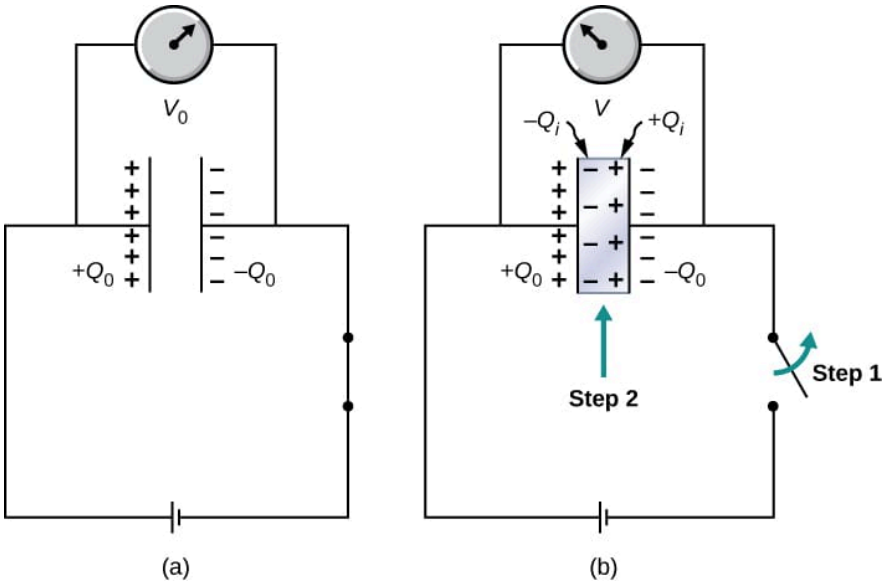


Figure 11.5.1: (a) When fully charged, a vacuum capacitor has a voltage V_0 and charge Q_0 (the charges remain on plate's inner surfaces; the schematic indicates the sign of charge on each plate). (b) In step 1, the battery is disconnected. Then, in step 2, a dielectric (that is electrically neutral) is inserted into the charged capacitor. When the voltage across the capacitor is now measured, it is found that the voltage value has decreased to $V = V_0/\kappa$. The schematic indicates the sign of the induced charge that is now present on the surfaces of the dielectric material between the plates.

The principle expressed by Equation 11.5.1 is widely used in the construction industry (Figure 11.5.2). Metal plates in an electronic stud finder act effectively as a capacitor. You place a stud finder with its flat side on the wall and move it continually in the horizontal direction. When the finder moves over a wooden stud, the capacitance of its plates changes, because wood has a different dielectric constant than a gypsum wall. This change triggers a signal in a circuit, and thus the stud is detected.

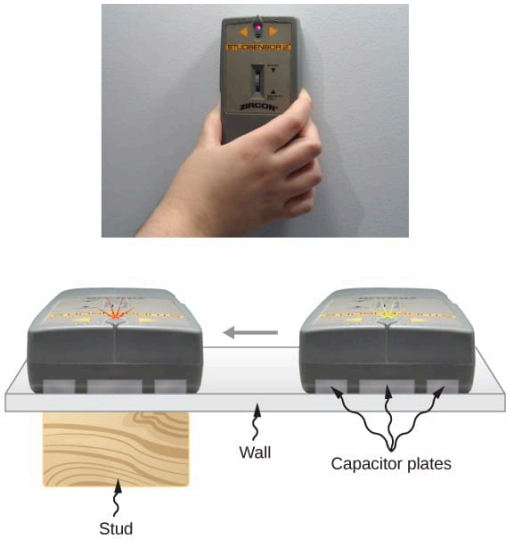


Figure 11.5.2: An electronic stud finder is used to detect wooden studs behind drywall.

The electrical energy stored by a capacitor is also affected by the presence of a dielectric. When the energy stored in an empty capacitor is U_0, the energy U stored in a capacitor with a dielectric is smaller by a factor of κ.

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{Q_0^2}{\kappa C_0} = \frac{1}{\kappa}U_0. \tag{11.5.2}$$

As a dielectric material sample is brought near an empty charged capacitor, the sample reacts to the electrical field of the charges on the capacitor plates. Just as we learned in Electric Charges and Fields on electrostatics, there will be the induced

charges on the surface of the sample; however, they are not free charges like in a conductor, because a perfect insulator does not have freely moving charges. These induced charges on the dielectric surface are of an opposite sign to the free charges on the plates of the capacitor, and so they are attracted by the free charges on the plates. Consequently, the dielectric is "pulled" into the gap, and the work to polarize the dielectric material between the plates is done at the expense of the stored electrical energy, which is reduced, in accordance with Equation 11.5.2.

Example 11.5.1: Inserting a Dielectric into an Isolated Capacitor

An empty 20.0-pF capacitor is charged to a potential difference of 40.0 V. The charging battery is then disconnected, and a piece of Teflon™ with a dielectric constant of 2.1 is inserted to completely fill the space between the capacitor plates (see Figure 11.5.1). What are the values of:

a. the capacitance,
b. the charge of the plate,
c. the potential difference between the plates, and
d. the energy stored in the capacitor with and without dielectric?

Strategy

We identify the original capacitance $C_0 = 20.0\ pF$ and the original potential difference $V_0 = 40.0\ V$ between the plates. We combine Equation 11.5.1 with other relations involving capacitance and substitute.

Solution

a. The capacitance increases to

$$C = \kappa C_0 = 2.1(20.0\,pF) = 42.0\,pF.$$

b. Without dielectric, the charge on the plates is

$$Q_0 = C_0 V_0 = (20.0\,pF)(40.0\,V) = 0.8\,nC.$$

Since the battery is disconnected before the dielectric is inserted, the plate charge is unaffected by the dielectric and remains at 0.8 nC.

c. With the dielectric, the potential difference becomes

$$V = \frac{1}{\kappa} V_0 = \frac{1}{2.1} 40.0\,V = 19.0\,V.$$

d. The stored energy without the dielectric is

$$U_0 = \frac{1}{2} C_0 V_0^2 = \frac{1}{2}(20.0\,pF)(40.0\,V)^2 = 16.0\,nJ.$$

With the dielectric inserted, we use Equation 11.5.2 to find that the stored energy decreases to

$$U = \frac{1}{\kappa} U_0 = \frac{1}{2.1} 16.0\,nJ = 7.6\,nJ.$$

Significance

Notice that the effect of a dielectric on the capacitance of a capacitor is a drastic increase of its capacitance. This effect is far more profound than a mere change in the geometry of a capacitor.

Exercise 11.5.1

When a dielectric is inserted into an isolated and charged capacitor, the stored energy decreases to 33% of its original value.

a. What is the dielectric constant?
b. How does the capacitance change?

Answer available at the end of this section.

Answers

Exercise 11.5.1

a. 3.0; b. $C = 3.0\ C_0$

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11.6: Molecular Model of a Dielectric

Learning Objectives

By the end of this section, you will be able to:

- Explain the polarization of a dielectric in a uniform electrical field
- Describe the effect of a polarized dielectric on the electrical field between capacitor plates
- Explain dielectric breakdown

We can understand the effect of a dielectric on capacitance by looking at its behavior at the molecular level. As we have seen in earlier chapters, in general, all molecules can be classified as either **polar** or **nonpolar**. There is a net separation of positive and negative charges in an isolated polar molecule, whereas there is no charge separation in an isolated nonpolar molecule (Figure $11.6.1$). In other words, polar molecules have permanent **electric-dipole moments** and nonpolar molecules do not. For example, a molecule of water is polar, and a molecule of oxygen is nonpolar. Nonpolar molecules can become polar in the presence of an external electrical field, which is called **induced polarization**.

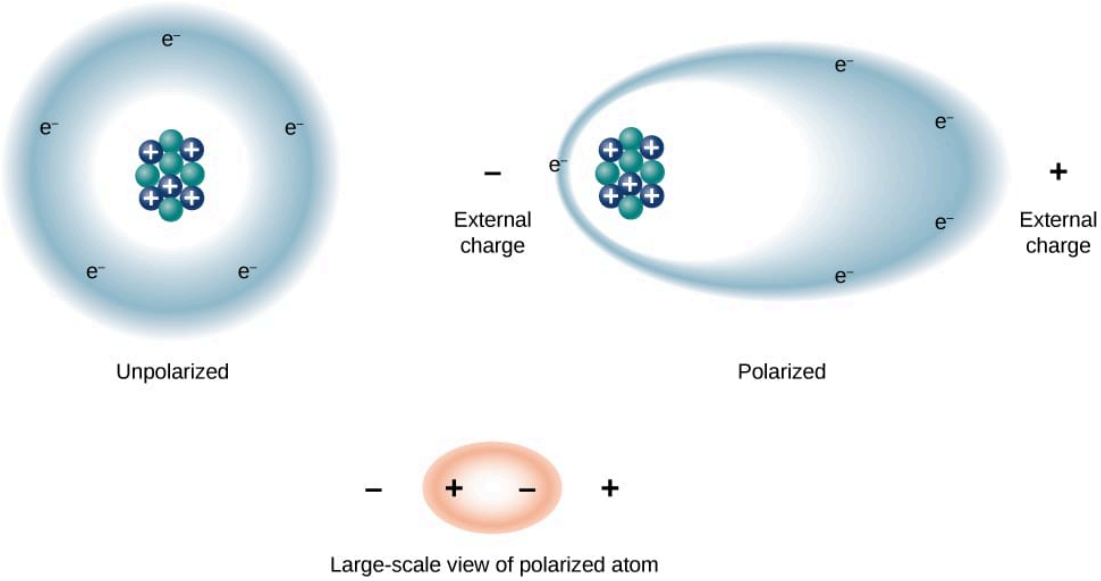


Figure $11.6.1$: The concept of polarization: In an unpolarized atom or molecule, a negatively charged electron cloud is evenly distributed around positively charged centers, whereas a polarized atom or molecule has an excess of negative charge at one side so that the other side has an excess of positive charge. However, the entire system remains electrically neutral. The charge polarization may be caused by an external electrical field. Some molecules and atoms are permanently polarized (electric dipoles) even in the absence of an external electrical field (polar molecules and atoms).

Let's first consider a dielectric composed of polar molecules. In the absence of any external electrical field, the electric dipoles are oriented randomly, as illustrated in Figure $11.6.2a$. However, if the dielectric is placed in an external electrical field $\vec{E}_0$, the polar molecules align with the external field, as shown in $11.6.2b$ of the figure. Opposite charges on adjacent dipoles within the volume of dielectric neutralize each other, so there is no net charge within the dielectric (see the dashed circles in part (b)). However, this is not the case very close to the upper and lower surfaces that border the dielectric (the region enclosed by the dashed rectangles in $11.6.2b$, where the alignment does produce a net charge. Since the external electrical field merely aligns the dipoles, the dielectric as a whole is neutral, and the surface charges induced on its opposite faces are equal and opposite. These **induced surface charges** $+Q_i$ and $-Q_i$ produce an additional electrical field $\vec{E}_i$ (an **induced electrical field**), which **opposes** the external field $\vec{E}_0$, as illustrated in part (c).

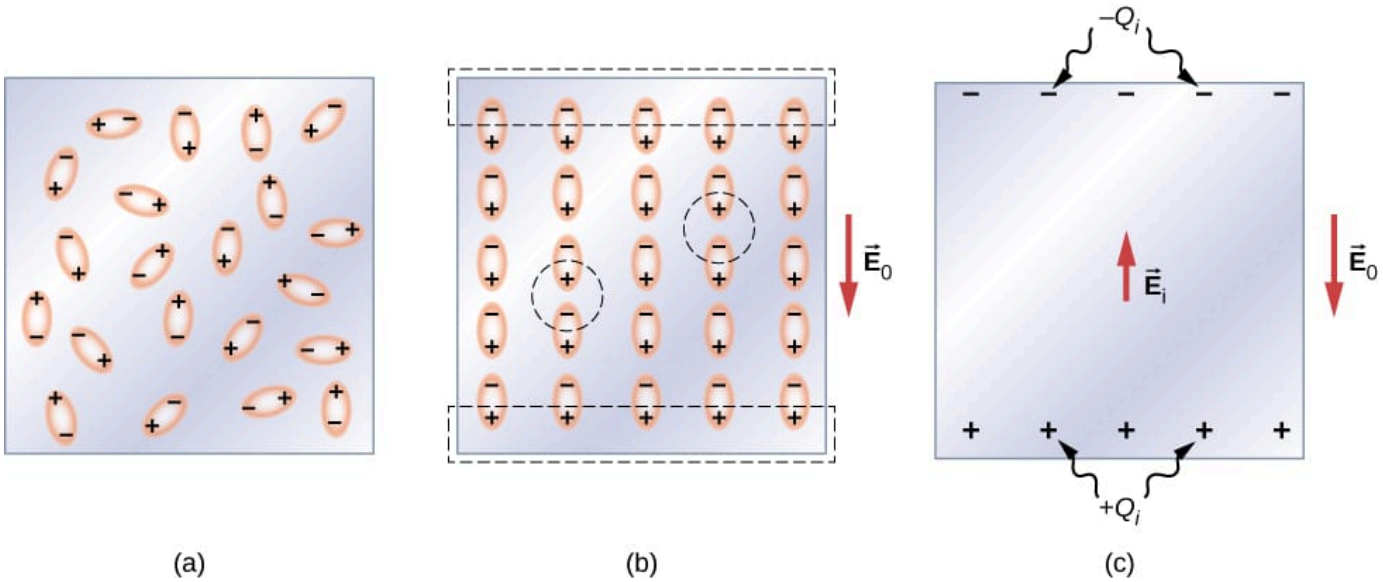


Figure 11.6.2: A dielectric with polar molecules: (a) In the absence of an external electrical field; (b) in the presence of an external electrical field $\vec{E}_0$. The dashed lines indicate the regions immediately adjacent to the capacitor plates. (c) The induced electrical field $\vec{E}_i$ inside the dielectric produced by the induced surface charge Q_i of the dielectric. Note that, in reality, the individual molecules are not perfectly aligned with an external field because of thermal fluctuations; however, the **average** alignment is along the field lines as shown.

The same effect is produced when the molecules of a dielectric are nonpolar. In this case, a nonpolar molecule acquires an induced **electric-dipole moment** because the external field $\vec{E}_0$ causes a separation between its positive and negative charges. The induced dipoles of the nonpolar molecules align with $\vec{E}_0$ in the same way as the permanent dipoles of the polar molecules are aligned (shown in part (b)). Hence, the electrical field within the dielectric is weakened regardless of whether its molecules are polar or nonpolar.

Therefore, when the region between the parallel plates of a charged capacitor, such as that shown in Figure 11.6.3a, is filled with a dielectric, within the dielectric there is an electrical field $\vec{E}_0$ due to the **free charge** Q_0 on the capacitor plates and an electrical field $\vec{E}_i$ due to the induced charge Q_i on the surfaces of the dielectric. Their vector sum gives the net electrical field $\vec{E}$ within the dielectric between the capacitor plates (shown in part (b) of the figure):

$$\vec{E} = \vec{E}_0 + \vec{E}_i. \tag{11.6.1}$$

This net field can be considered to be the field produced by an **effective charge** $Q_0 - Q_i$ on the capacitor.

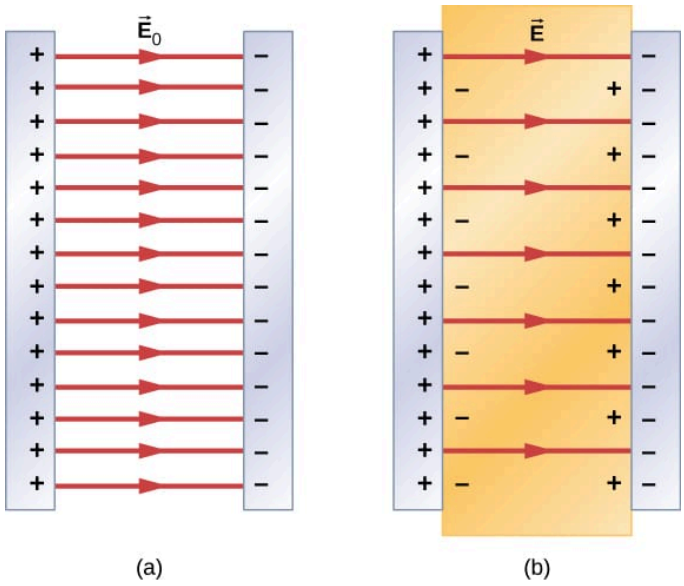


Figure 11.6.3: Electrical field: (a) In an empty capacitor, electrical field $\vec{E}_0$ (b) In a dielectric-filled capacitor, electrical field $\vec{E}$.

In most dielectrics, the net electrical field $\vec{E}$ is proportional to the field $\vec{E}_0$ produced by the free charge. In terms of these two electrical fields, the dielectric constant κ of the material is defined as

$$\kappa = \frac{E_0}{E}. \tag{11.6.2}$$

Since $\vec{E}_0$ and $\vec{E}_i$ point in opposite directions, the magnitude **E** is smaller than the magnitude $\vec{E}_0$ and therefore $\kappa > 1$. Combining Equations 11.6.1 with 11.6.2, and rearranging the terms, yields the following expression for the induced electrical field in a dielectric:

$$\vec{E}_i = \left(\frac{1}{\kappa} - 1\right)\vec{E}_0. \tag{11.6.3}$$

When the magnitude of an external electrical field becomes too large, the molecules of dielectric material start to become ionized. A molecule or an atom is ionized when one or more electrons are removed from it and become free electrons, no longer bound to the molecular or atomic structure. When this happens, the material can conduct, thereby allowing charge to move through the dielectric from one capacitor plate to the other. This phenomenon is called **dielectric breakdown**. (Figure 8.1.1 shows typical random-path patterns of electrical discharge during dielectric breakdown.) The critical value, E_c of the electrical field at which the molecules of an insulator become ionized is called the **dielectric strength** of the material. The dielectric strength imposes a limit on the voltage that can be applied for a given plate separation in a capacitor. For example, the dielectric strength of air is $E_c = 3.0\ MV/m$, so for an air-filled capacitor with a plate separation of $d = 1.00\ mm$, the limit on the potential difference that can be safely applied across its plates without causing dielectric breakdown is $V = E_c d = (3.0 \times 10^6 V/m)(1.00 \times 10^{-3} m) = 3.0\ kV$.

However, this limit becomes 60.0 kV when the same capacitor is filled with Teflon™, whose dielectric strength is about $60.0\ MV/m$. Because of this limit imposed by the dielectric strength, the amount of charge that an air-filled capacitor can store is only $Q_0 = \kappa_{air} C_0 (3.0\ kV)$ and the charge stored on the same Teflon™-filled capacitor can be as much as

$$\begin{aligned} Q &= \kappa_{teflon} C_0 (60.0\,kV) \\ &= \kappa_{teflon} \frac{Q_0}{\kappa_{air}(3.0\,kV)} (60.0\,kV) \\ &= 20 \frac{\kappa_{teflon}}{\kappa_{air}} Q_0 = 20 \frac{2.1}{1.00059} Q_0 \\ &\cong 42 Q_0. \end{aligned} \tag{11.6.4}$$

which is about 42 times greater than a charge stored on an air-filled capacitor. Typical values of dielectric constants and dielectric strengths for various materials are given in Table 11.6.1. Notice that the dielectric constant κ is exactly 1.0 for a vacuum (the empty space serves as a reference condition) and very close to 1.0 for air under normal conditions (normal pressure at room temperature). These two values are so close that, in fact, the properties of an air-filled capacitor are essentially the same as those of an empty capacitor.

Table 11.6.1: Representative Values of Dielectric Constants and Dielectric Strengths of Various Materials at Room Temperature

Material	Dielectric constant κ	Dielectric strength $E_c[\times 10^6\ V/m]$
Vacuum	1	∞
Dry air (1 atm)	1.00059	3.0
Teflon™	2.1	60 to 173
Paraffin	2.3	11
Silicon oil	2.5	10 to 15
Polystyrene	2.56	19.7
Nylon	3.4	14
Paper	3.7	16
Fused quartz	3.78	8
Glass	4 to 6	9.8 to 13.8

Material	Dielectric constant κ	Dielectric strength $E_c[\times 10^6\ V/m]$
Concrete	4.5	–
Bakelite	4.9	24
Diamond	5.5	2,000
Pyrex glass	5.6	14
Mica	6.0	118
Neoprene rubber	6.7	15.7 to 26.7
Water	80	-
Sulfuric acid	84 to 100	-
Titanium dioxide	86 to 173	–
Strontium titanate	310	8
Barium titanate	1,200 to 10,000	–
Calcium copper titanate	> 250,000	–

Not all substances listed in the table are good insulators, despite their high dielectric constants. Water, for example, consists of polar molecules and has a large dielectric constant of about 80. In a water molecule, electrons are more likely found around the oxygen nucleus than around the hydrogen nuclei. This makes the oxygen end of the molecule slightly negative and leaves the hydrogens end slightly positive, which makes the molecule easy to align along an external electrical field, and thus water has a large dielectric constant. However, the polar nature of water molecules also makes water a good solvent for many substances, which produces undesirable effects, because any concentration of free ions in water conducts electricity.

Example 11.6.1: Electrical Field and Induced Surface Charge

Suppose that the distance between the plates of the capacitor in Example 8.5.1 is 2.0 mm and the area of each plate is $4.5 \times 10^{-3} m^2$. Determine:

a. the electrical field between the plates before and after the Teflon™ is inserted, and
b. the surface charge induced on the Teflon™ surfaces.

Strategy

In part (a), we know that the voltage across the empty capacitor is $V_0 = 40\ V$, so to find the electrical fields we use the relation $V = Ed$ and Equation 11.6.3. In part (b), knowing the magnitude of the electrical field, we use the expression for the magnitude of electrical field near a charged plate $E = \sigma/\epsilon_0$, where σ is a uniform surface charge density caused by the surface charge. We use the value of free charge $Q_0 = 8.0 \times 10^{-10} C$ obtained in Example 8.5.1.

Solution

1. The electrical field E_0 between the plates of an empty capacitor is

$$E_0 = \frac{V_0}{d} = \frac{40\,V}{2.0 \times 10^{-3} m} = 2.010^4 V/m.$$

The electrical field E with the Teflon™ in place is

$$E = \frac{1}{\kappa} E_0 = \frac{1}{2.1} 2.0 \times 10^4 V/m = 9.5 \times 10^3\, V/m.$$

2. The effective charge on the capacitor is the difference between the free charge Q_0 and the induced charge Q_i. The electrical field in the Teflon™ is caused by this effective charge. Thus

$$E = \frac{1}{\epsilon_0}\sigma$$
$$= \frac{1}{\epsilon_0}\frac{Q_0 - Q_i}{A}.$$

We invert this equation to obtain Q_i, which yields

$$Q_i = Q_0 - \epsilon_0 AE$$
$$= 8.0 \times 10^{-10} C - \left(8.85 \times 10^{-12}\frac{C^2}{N \cdot m^2}\right)(4.5 \times 10^{-3} m^2)\left(9.5 \times 10^3 \frac{V}{m}\right)$$
$$= 4.2 \times 10^{-10} C = 0.42\, nC.$$

Example 11.6.2: Inserting a Dielectric into a Capacitor Connected to a Battery

When a battery of voltage V_0 is connected across an empty capacitor of capacitance C_0, the charge on its plates is Q_0, and the electrical field between its plates is E_0. A dielectric of dielectric constant κ is inserted between the plates **while the battery remains in place**, as shown in Figure 11.6.4.

a. Find the capacitance C, the voltage V across the capacitor, and the electrical field E between the plates after the dielectric is inserted.
b. Obtain an expression for the free charge Q on the plates of the filled capacitor and the induced charge Q_i on the dielectric surface in terms of the original plate charge Q_0.

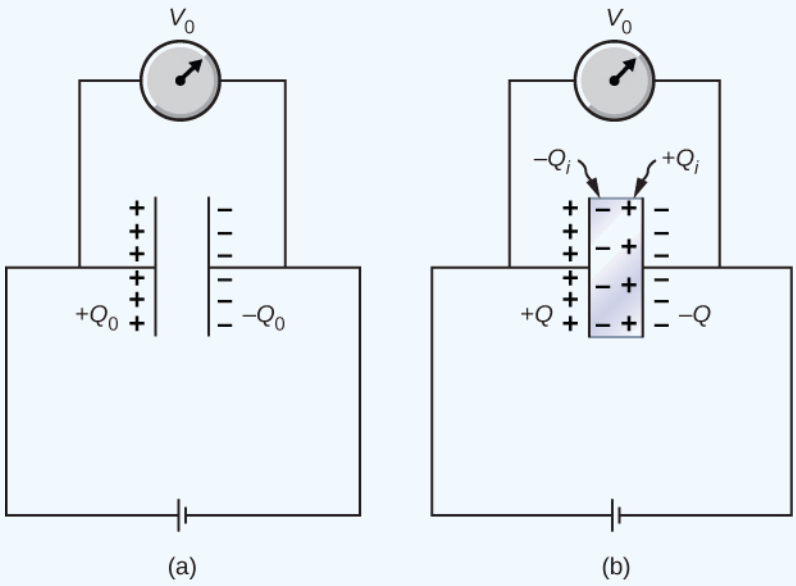


Figure 11.6.4: A dielectric is inserted into the charged capacitor while the capacitor remains connected to the battery.

Strategy

We identify the known values:V_0, C_0, E_o, κ, and Q_0. Our task is to express the unknown values in terms of these known values.

Solution

(a) The capacitance of the filled capacitor is $C = \kappa C_0$. Since the battery is always connected to the capacitor plates, the potential difference between them does not change; hence, $V = V_0$. Because of that, the electrical field in the filled capacitor is the same as the field in the empty capacitor, so we can obtain directly that

$$E = \frac{V}{d} = \frac{V_0}{d} = E_0.$$

(b) For the filled capacitor, the free charge on the plates is

$$\begin{aligned} Q &= CV \\ &= (\kappa C_0)V_0 \\ &= \kappa(C_0V_0) \\ &= \kappa Q_0. \end{aligned}$$

The electrical field E in the filled capacitor is due to the effective charge $Q - Q_i$ (Figure $11.6.4b$). Since $E = E_0$, we have

$$\frac{Q - Q_i}{\epsilon_0 A} = \frac{Q_0}{\epsilon_0 A}.$$

Solving this equation for Q_i, we obtain for the induced charge

$$\begin{aligned} Q_i &= Q - Q_0 \\ &= \kappa Q_0 - Q_0 \\ &= (\kappa - 1)Q_0. \end{aligned}$$

Significance

Notice that for materials with dielectric constants larger than 2 (see Table $11.6.1$), the induced charge on the surface of dielectric is larger than the charge on the plates of a vacuum capacitor. The opposite is true for gasses like air whose dielectric constant is smaller than 2.

Exercise 11.6.1

Continuing with Example $11.6.2$, show that when the battery is connected across the plates the energy stored in dielectric-filled capacitor is $U = \kappa U_0$ (larger than the energy U_0 of an empty capacitor kept at the same voltage). Compare this result with the result $U = U_0/\kappa$ found previously for an isolated, charged capacitor.

Exercise 11.6.2

Repeat the calculations of previous example for the case in which the battery remains connected while the dielectric is placed in the capacitor.

Answer available at the end of this section.

Answers

Exercise $11.6.2$

a. $C_0 = 20\ pF$, $C = 42\ pF$;
b. $Q_0 = 0.8\ nC$, $Q = 1.7\ nC$:
c. $V_0 = V = 40\ V$;
d. $U_0 = 16\ nJ$, $U = 34\ nJ$

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11.7: Capacitance (Summary)

Key Terms

capacitance	amount of charge stored per unit volt
capacitor	device that stores electrical charge and electrical energy
dielectric	insulating material used to fill the space between two plates
dielectric breakdown	phenomenon that occurs when an insulator becomes a conductor in a strong electrical field
dielectric constant	factor by which capacitance increases when a dielectric is inserted between the plates of a capacitor
dielectric strength	critical electrical field strength above which molecules in insulator begin to break down and the insulator starts to conduct
energy density	energy stored in a capacitor divided by the volume between the plates
induced electric-dipole moment	dipole moment that a nonpolar molecule may acquire when it is placed in an electrical field
induced electrical field	electrical field in the dielectric due to the presence of induced charges
induced surface charges	charges that occur on a dielectric surface due to its polarization
parallel combination	components in a circuit arranged with one side of each component connected to one side of the circuit and the other sides of the components connected to the other side of the circuit
parallel-plate capacitor	system of two identical parallel conducting plates separated by a distance
series combination	components in a circuit arranged in a row one after the other in a circuit

Key Equations

Capacitance	$C = \frac{Q}{V}$
Capacitance of a parallel-plate capacitor	$C = \varepsilon_0 \frac{A}{d}$
Capacitance of a vacuum spherical capacitor	$C = 4\pi\varepsilon_0 \frac{R_1 R_2}{R_2 - R_1}$
Capacitance of a vacuum cylindrical capacitor	$C = \frac{2\pi\varepsilon_0 l}{ln(R_2/R_1)}$
Capacitance of a series combination	$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$
Capacitance of a parallel combination	$C_P = C_1 + C_2 + C_3 + \cdots$
Energy density	$u_E = \frac{1}{2}\varepsilon_0 E^2$
Energy stored in a capacitor	$U_C = \frac{1}{2}V^2 C = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV$
Capacitance of a capacitor with dielectric	$C = \kappa C_0$

Energy stored in an isolated capacitor with dielectric	$U = \frac{1}{\kappa}U_0$
Dielectric constant	$\kappa = \frac{E_0}{E}$
Induced electrical field in a dielectric	$\vec{E}_i = (\frac{1}{\kappa} - 1)\vec{E}_0$

Summary

8.2 Capacitors and Capacitance

- A capacitor is a device that stores an electrical charge and electrical energy. The amount of charge a vacuum capacitor can store depends on two major factors: the voltage applied and the capacitor's physical characteristics, such as its size and geometry.
- The capacitance of a capacitor is a parameter that tells us how much charge can be stored in the capacitor per unit potential difference between its plates. Capacitance of a system of conductors depends only on the geometry of their arrangement and physical properties of the insulating material that fills the space between the conductors. The unit of capacitance is the farad, where $1F = 1C/1V$.

8.3 Capacitors in Series and in Parallel

- When several capacitors are connected in a series combination, the reciprocal of the equivalent capacitance is the sum of the reciprocals of the individual capacitances.
- When several capacitors are connected in a parallel combination, the equivalent capacitance is the sum of the individual capacitances.
- When a network of capacitors contains a combination of series and parallel connections, we identify the series and parallel networks, and compute their equivalent capacitances step by step until the entire network becomes reduced to one equivalent capacitance.

8.4 Energy Stored in a Capacitor

- Capacitors are used to supply energy to a variety of devices, including defibrillators, microelectronics such as calculators, and flash lamps.
- The energy stored in a capacitor is the work required to charge the capacitor, beginning with no charge on its plates. The energy is stored in the electrical field in the space between the capacitor plates. It depends on the amount of electrical charge on the plates and on the potential difference between the plates.
- The energy stored in a capacitor network is the sum of the energies stored on individual capacitors in the network. It can be computed as the energy stored in the equivalent capacitor of the network.

8.5 Capacitor with a Dielectric

- The capacitance of an empty capacitor is increased by a factor of κ when the space between its plates is completely filled by a dielectric with dielectric constant κ.
- Each dielectric material has its specific dielectric constant.
- The energy stored in an empty isolated capacitor is decreased by a factor of κκ when the space between its plates is completely filled with a dielectric with dielectric constant κ.

8.6 Molecular Model of a Dielectric

- When a dielectric is inserted between the plates of a capacitor, equal and opposite surface charge is induced on the two faces of the dielectric. The induced surface charge produces an induced electrical field that opposes the field of the free charge on the capacitor plates.
- The dielectric constant of a material is the ratio of the electrical field in vacuum to the net electrical field in the material. A

capacitor filled with dielectric has a larger capacitance than an empty capacitor.
- The dielectric strength of an insulator represents a critical value of electrical field at which the molecules in an insulating material start to become ionized. When this happens, the material can conduct and dielectric breakdown is observed.

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11.8: Capacitance (Exercises)

Conceptual Questions

8.2 Capacitors and Capacitance

1. Does the capacitance of a device depend on the applied voltage? Does the capacitance of a device depend on the charge residing on it?

2. Would you place the plates of a parallel-plate capacitor closer together or farther apart to increase their capacitance?

3. The value of the capacitance is zero if the plates are not charged. True or false?

4. If the plates of a capacitor have different areas, will they acquire the same charge when the capacitor is connected across a battery?

5. Does the capacitance of a spherical capacitor depend on which sphere is charged positively or negatively?

8.3 Capacitors in Series and in Parallel

6. If you wish to store a large amount of charge in a capacitor bank, would you connect capacitors in series or in parallel? Explain.

7. What is the maximum capacitance you can get by connecting three **1.0-μF** capacitors? What is the minimum capacitance?

8.4 Energy Stored in a Capacitor

8. If you wish to store a large amount of energy in a capacitor bank, would you connect capacitors in series or parallel? Explain.

8.5 Capacitor with a Dielectric

9. Discuss what would happen if a conducting slab rather than a dielectric were inserted into the gap between the capacitor plates.

10. Discuss how the energy stored in an empty but charged capacitor changes when a dielectric is inserted if (a) the capacitor is isolated so that its charge does not change; (b) the capacitor remains connected to a battery so that the potential difference between its plates does not change.

8.6 Molecular Model of a Dielectric

11. Distinguish between dielectric strength and dielectric constant.

12. Water is a good solvent because it has a high dielectric constant. Explain.

13. Water has a high dielectric constant. Explain why it is then not used as a dielectric material in capacitors.

14. Elaborate on why molecules in a dielectric material experience net forces on them in a non-uniform electrical field but not in a uniform field.

15. Explain why the dielectric constant of a substance containing permanent molecular electric dipoles decreases with increasing temperature.

16. Give a reason why a dielectric material increases capacitance compared with what it would be with air between the plates of a capacitor. How does a dielectric material also allow a greater voltage to be applied to a capacitor? (The dielectric thus increases **C** and permits a greater **V**.)

17. Elaborate on the way in which the polar character of water molecules helps to explain water's relatively large dielectric constant.

18. Sparks will occur between the plates of an air-filled capacitor at a lower voltage when the air is humid than when it is dry. Discuss why, considering the polar character of water molecules.

Problems

8.2 Capacitors and Capacitance

19. What charge is stored in a **180.0-μF** capacitor when 120.0 V is applied to it?

20. Find the charge stored when 5.50 V is applied to an 8.00-pF capacitor.

21. Calculate the voltage applied to a **2.00-μF** capacitor when it holds **3.10μC** of charge.

22. What voltage must be applied to an 8.00-nF capacitor to store 0.160 mC of charge?

23. What capacitance is needed to store **3.00μC** of charge at a voltage of 120 V?

24. What is the capacitance of a large Van de Graaff generator's terminal, given that it stores 8.00 mC of charge at a voltage of 12.0 MV?

25. The plates of an empty parallel-plate capacitor of capacitance 5.0 pF are 2.0 mm apart. What is the area of each plate?

26. A 60.0-pF vacuum capacitor has a plate area of $0.010m^2$. What is the separation between its plates?

27. A set of parallel plates has a capacitance of **5.0μF**. How much charge must be added to the plates to increase the potential difference between them by 100 V?

28. Consider Earth to be a spherical conductor of radius 6400 km and calculate its capacitance.

29. If the capacitance per unit length of a cylindrical capacitor is 20 pF/m, what is the ratio of the radii of the two cylinders?

30. An empty parallel-plate capacitor has a capacitance of **20μF**. How much charge must leak off its plates before the voltage across them is reduced by 100 V?

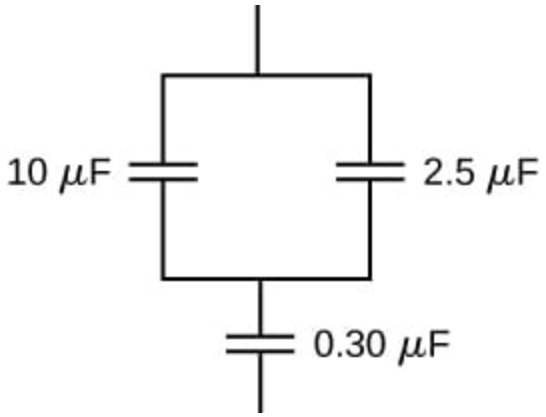
8.3 Capacitors in Series and in Parallel

31. A 4.00-pF is connected in series with an 8.00-pF capacitor and a 400-V potential difference is applied across the pair. (a) What is the charge on each capacitor?

(b) What is the voltage across each capacitor?

32. Three capacitors, with capacitances of $C_1 = 2.0\mu F, C_2 = 3.0\mu F$, and $C_3 = 6.0\mu F$, respectively, are connected in parallel. A 500-V potential difference is applied across the combination. Determine the voltage across each capacitor and the charge on each capacitor.

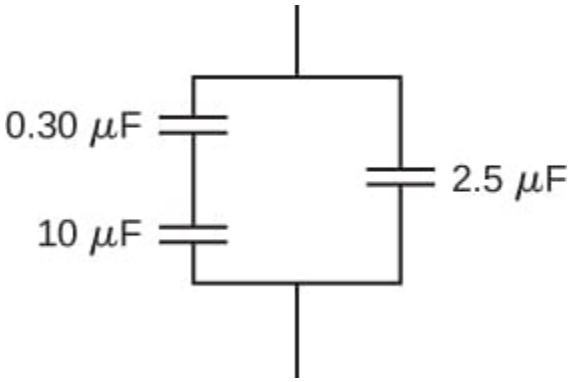
33. Find the total capacitance of this combination of series and parallel capacitors shown below.



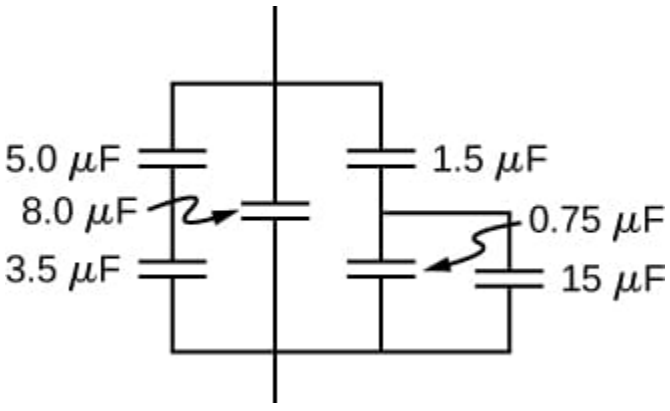
34. Suppose you need a capacitor bank with a total capacitance of 0.750 F but you have only 1.50-mF capacitors at your disposal. What is the smallest number of capacitors you could connect together to achieve your goal, and how would you connect them?

35. What total capacitances can you make by connecting a **5.00-μF** and a **8.00-μF** capacitor?

36. Find the equivalent capacitance of the combination of series and parallel capacitors shown below.



37. Find the net capacitance of the combination of series and parallel capacitors shown below.



38. A 40-pF capacitor is charged to a potential difference of 500 V. Its terminals are then connected to those of an uncharged 10-pF capacitor. Calculate:

(a) the original charge on the 40-pF capacitor;

(b) the charge on each capacitor after the connection is made; and

(c) the potential difference across the plates of each capacitor after the connection.

39. A **2.0-μF** capacitor and a **4.0-μF** capacitor are connected in series across a 1.0-kV potential. The charged capacitors are then disconnected from the source and connected to each other with terminals of like sign together. Find the charge on each capacitor and the voltage across each capacitor.

8.4 Energy Stored in a Capacitor

40. How much energy is stored in an **8.00-μF** capacitor whose plates are at a potential difference of 6.00 V?

41. A capacitor has a charge of **2.5μC** when connected to a 6.0-V battery. How much energy is stored in this capacitor?

42. How much energy is stored in the electrical field of a metal sphere of radius 2.0 m that is kept at a 10.0-V potential?

43. (a) What is the energy stored in the **10.0-μF** capacitor of a heart defibrillator charged to $9.00 \times 10^3 V$?

(b) Find the amount of the stored charge.

44. In open-heart surgery, a much smaller amount of energy will defibrillate the heart.

(a) What voltage is applied to the **8.00-μF** capacitor of a heart defibrillator that stores 40.0 J of energy?

(b) Find the amount of the stored charge.

45. A **165-μF** capacitor is used in conjunction with a dc motor. How much energy is stored in it when 119 V is applied?

46. Suppose you have a 9.00-V battery, a **2.00-μF** capacitor, and a **7.40-μF** capacitor.

(a) Find the charge and energy stored if the capacitors are connected to the battery in series.

(b) Do the same for a parallel connection.

47. An anxious physicist worries that the two metal shelves of a wood frame bookcase might obtain a high voltage if charged by static electricity, perhaps produced by friction.

(a) What is the capacitance of the empty shelves if they have area $1.00 \times 10^2 m$ and are 0.200 m apart?

(b) What is the voltage between them if opposite charges of magnitude 2.00 nC are placed on them?

(c) To show that this voltage poses a small hazard, calculate the energy stored.

(d) The actual shelves have an area 100 times smaller than these hypothetical shelves. Are his fears justified?

48. A parallel-plate capacitor is made of two square plates 25 cm on a side and 1.0 mm apart. The capacitor is connected to a 50.0-V battery. With the battery still connected, the plates are pulled apart to a separation of 2.00 mm. What are the energies stored in the capacitor before and after the plates are pulled farther apart? Why does the energy decrease even though work is done in separating the plates?

49. Suppose that the capacitance of a variable capacitor can be manually changed from 100 pF to 800 pF by turning a dial, connected to one set of plates by a shaft, from **0°** to **180°.** With the dial set at 180°180° (corresponding to **C=800pF**), the capacitor is connected to a 500-V source. After charging, the capacitor is disconnected from the source, and the dial is turned to **0°**. If friction is negligible, how much work is required to turn the dial from **180°** to **0°**?

8.5 Capacitor with a Dielectric

50. Show that for a given dielectric material, the maximum energy a parallel-plate capacitor can store is directly proportional to the volume of dielectric.

51. An air-filled capacitor is made from two flat parallel plates 1.0 mm apart. The inside area of each plate is 8.0cm28.0cm2.

(a) What is the capacitance of this set of plates?

(b) If the region between the plates is filled with a material whose dielectric constant is 6.0, what is the new capacitance?

52. A capacitor is made from two concentric spheres, one with radius 5.00 cm, the other with radius 8.00 cm.

(a) What is the capacitance of this set of conductors?

(b) If the region between the conductors is filled with a material whose dielectric constant is 6.00, what is the capacitance of the system?

53. A parallel-plate capacitor has charge of magnitude **9.00μC** on each plate and capacitance **3.00μF** when there is air between the plates. The plates are separated by 2.00 mm. With the charge on the plates kept constant, a dielectric with $\kappa = 5$ is inserted between the plates, completely filling the volume between the plates.

(a) What is the potential difference between the plates of the capacitor, before and after the dielectric has been inserted?

(b) What is the electrical field at the point midway between the plates before and after the dielectric is inserted?

54. Some cell walls in the human body have a layer of negative charge on the inside surface. Suppose that the surface charge densities are $\pm 0.50 \times 10^{-3} C/m^2$, the cell wall is $5.0 \times 10^{-9} m$ thick, and the cell wall material has a dielectric constant of **κ=5.4.**

(a) Find the magnitude of the electric field in the wall between two charge layers.

(b) Find the potential difference between the inside and the outside of the cell. Which is at higher potential?

(c) A typical cell in the human body has volume $10^{-16} m^3$. Estimate the total electrical field energy stored in the wall of a cell of this size when assuming that the cell is spherical. (**Hint:** Calculate the volume of the cell wall.)

55. A parallel-plate capacitor with only air between its plates is charged by connecting the capacitor to a battery. The capacitor is then disconnected from the battery, without any of the charge leaving the plates.

(a) A voltmeter reads 45.0 V when placed across the capacitor. When a dielectric is inserted between the plates, completely filling the space, the voltmeter reads 11.5 V. What is the dielectric constant of the material?

(b) What will the voltmeter read if the dielectric is now pulled away out so it fills only one-third of the space between the plates?

8.6 Molecular Model of a Dielectric

56. Two flat plates containing equal and opposite charges are separated by material 4.0 mm thick with a dielectric constant of 5.0. If the electrical field in the dielectric is 1.5 MV/m, what are

(a) the charge density on the capacitor plates, and

(b) the induced charge density on the surfaces of the dielectric?

57. For a Teflon™-filled, parallel-plate capacitor, the area of the plate is $50.0 cm^2$ and the spacing between the plates is 0.50 mm. If the capacitor is connected to a 200-V battery, find

(a) the free charge on the capacitor plates,

(b) the electrical field in the dielectric, and

(c) the induced charge on the dielectric surfaces.

58. Find the capacitance of a parallel-plate capacitor having plates with a surface area of $5.00 m^2$ and separated by 0.100

mm of Teflon™.

59. (a) What is the capacitance of a parallel-plate capacitor with plates of area $1.50m^2$ that are separated by 0.0200 mm of neoprene rubber?

(b) What charge does it hold when 9.00 V is applied to it?

60. Two parallel plates have equal and opposite charges. When the space between the plates is evacuated, the electrical field is $E = 3.20 \times 10^5 V/m$. When the space is filled with dielectric, the electrical field is $E = 2.50 \times 10^5 V/m$.

(a) What is the surface charge density on each surface of the dielectric?

(b) What is the dielectric constant?

61. The dielectric to be used in a parallel-plate capacitor has a dielectric constant of 3.60 and a dielectric strength of $1.60 \times 10^7 V/m$. The capacitor has to have a capacitance of 1.25 nF and must be able to withstand a maximum potential difference 5.5 kV. What is the minimum area the plates of the capacitor may have?

62. When a 360-nF air capacitor is connected to a power supply, the energy stored in the capacitor is **18.5μJ**. While the capacitor is connected to the power supply, a slab of dielectric is inserted that completely fills the space between the plates. This increases the stored energy by **23.2μJ**.

(a) What is the potential difference between the capacitor plates?

(b) What is the dielectric constant of the slab?

63. A parallel-plate capacitor has square plates that are 8.00 cm on each side and 3.80 mm apart. The space between the plates is completely filled with two square slabs of dielectric, each 8.00 cm on a side and 1.90 mm thick. One slab is Pyrex glass and the other slab is polystyrene. If the potential difference between the plates is 86.0 V, find how much electrical energy can be stored in this capacitor.

Additional Problems

64. A capacitor is made from two flat parallel plates placed 0.40 mm apart. When a charge of **0.020μC** is placed on the plates the potential difference between them is 250 V.

(a) What is the capacitance of the plates?

(b) What is the area of each plate?

(c) What is the charge on the plates when the potential difference between them is 500 V?

(d) What maximum potential difference can be applied between the plates so that the magnitude of electrical fields between the plates does not exceed 3.0 MV/m?

65. An air-filled (empty) parallel-plate capacitor is made from two square plates that are 25 cm on each side and 1.0 mm apart. The capacitor is connected to a 50-V battery and fully charged. It is then disconnected from the battery and its plates are pulled apart to a separation of 2.00 mm.

(a) What is the capacitance of this new capacitor?

(b) What is the charge on each plate?

(c) What is the electrical field between the plates?

66. Suppose that the capacitance of a variable capacitor can be manually changed from 100 to 800 pF by turning a dial connected to one set of plates by a shaft, from **0°** to **180°**. With the dial set at **180°** (corresponding to **C=800pF**), the capacitor is connected to a 500-V source. After charging, the capacitor is disconnected from the source, and the dial is turned to **0°**. (a) What is the charge on the capacitor? (b) What is the voltage across the capacitor when the dial is set to

0°?

67. Earth can be considered as a spherical capacitor with two plates, where the negative plate is the surface of Earth and the positive plate is the bottom of the ionosphere, which is located at an altitude of approximately 70 km. The potential difference between Earth's surface and the ionosphere is about 350,000 V.

(a) Calculate the capacitance of this system.

(b) Find the total charge on this capacitor.

(c) Find the energy stored in this system.

68. A **4.00-μF** capacitor and a **6.00-μF** capacitor are connected in parallel across a 600-V supply line.

(a) Find the charge on each capacitor and voltage across each.

(b) The charged capacitors are disconnected from the line and from each other. They are then reconnected to each other with terminals of unlike sign together. Find the final charge on each capacitor and the voltage across each.

69. Three capacitors having capacitances of 8.40, 8.40, and 4.20 **μF**, respectively, are connected in series across a 36.0-V potential difference.

(a) What is the charge on the **4.20-μF** capacitor?

(b) The capacitors are disconnected from the potential difference without allowing them to discharge. They are then reconnected in parallel with each other with the positively charged plates connected together. What is the voltage across each capacitor in the parallel combination?

70. A parallel-plate capacitor with capacitance **5.0μF** is charged with a 12.0-V battery, after which the battery is disconnected. Determine the minimum work required to increase the separation between the plates by a factor of 3.

71. (a) How much energy is stored in the electrical fields in the capacitors (in total) shown below?

(b) Is this energy equal to the work done by the 400-V source in charging the capacitors?

72. Three capacitors having capacitances 8.4, 8.4, and 4.2 **μF** are connected in series across a 36.0-V potential difference.

(a) What is the total energy stored in all three capacitors?

(b) The capacitors are disconnected from the potential difference without allowing them to discharge. They are then reconnected in parallel with each other with the positively charged plates connected together. What is the total energy now stored in the capacitors?

73. (a) An **8.00-μF** capacitor is connected in parallel to another capacitor, producing a total capacitance of **5.00μF**. What is the capacitance of the second capacitor?

(b) What is unreasonable about this result?

(c) Which assumptions are unreasonable or inconsistent?

74. (a) On a particular day, it takes $9.60 \times 10^3 J$ of electrical energy to start a truck's engine. Calculate the capacitance of a capacitor that could store that amount of energy at 12.0 V.

(b) What is unreasonable about this result?

(c) Which assumptions are responsible?

75. (a) A certain parallel-plate capacitor has plates of area $4.00 m^2$, separated by 0.0100 mm of nylon, and stores 0.170 C of charge. What is the applied voltage?

(b) What is unreasonable about this result?

(c) Which assumptions are responsible or inconsistent?

76. A prankster applies 450 V to an **80.0-μF** capacitor and then tosses it to an unsuspecting victim. The victim's finger is burned by the discharge of the capacitor through 0.200 g of flesh. Estimate, what is the temperature increase of the flesh? Is it reasonable to assume that no thermodynamic phase change happened?

Challenge Problems

77. A spherical capacitor is formed from two concentric spherical conducting spheres separated by vacuum. The inner sphere has radius 12.5 cm and the outer sphere has radius 14.8 cm. A potential difference of 120 V is applied to the capacitor.

(a) What is the capacitance of the capacitor?

(b) What is the magnitude of the electrical field at **r=12.6cm**, just outside the inner sphere?

(c) What is the magnitude of the electrical field at **r=14.7cm**, just inside the outer sphere?

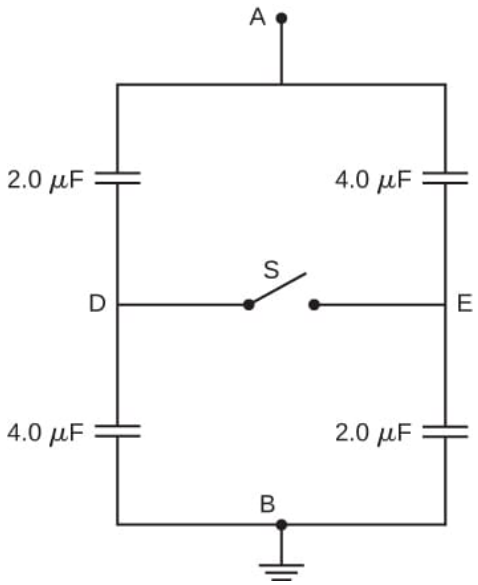
(d) For a parallel-plate capacitor the electrical field is uniform in the region between the plates, except near the edges of the plates. Is this also true for a spherical capacitor?

78. The network of capacitors shown below are all uncharged when a 300-V potential is applied between points A and B with the switch S open.

(a) What is the potential difference $V_E - V_D$?

(b) What is the potential at point E after the switch is closed?

(c) How much charge flows through the switch after it is closed?



79. Electronic flash units for cameras contain a capacitor for storing the energy used to produce the flash. In one such unit the flash lasts for 1/675 fraction of a second with an average light power output of 270 kW.

(a) If the conversion of electrical energy to light is 95% efficient (because the rest of the energy goes to thermal energy), how much energy must be stored in the capacitor for one flash?

(b) The capacitor has a potential difference between its plates of 125 V when the stored energy equals the value stored in part (a). What is the capacitance?

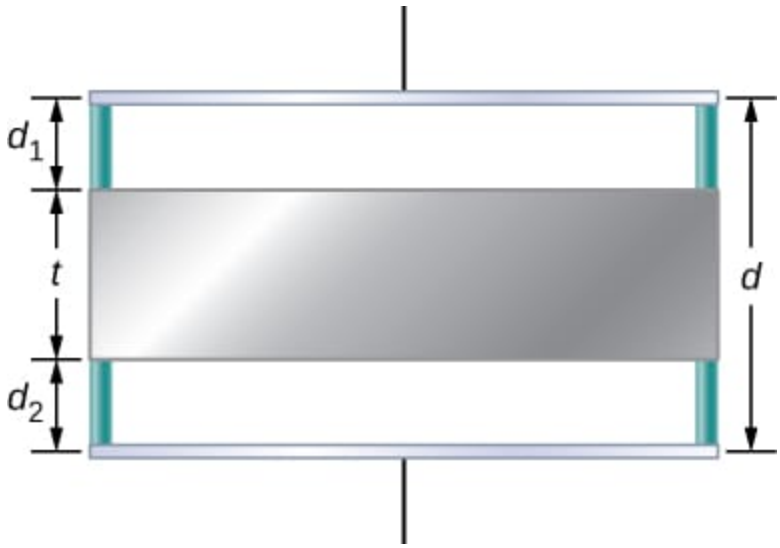
80. A spherical capacitor is formed from two concentric spherical conducting shells separated by a vacuum. The inner sphere has radius 12.5 cm and the outer sphere has radius 14.8 cm. A potential difference of 120 V is applied to the capacitor.

(a) What is the energy density at **r=12.6cm**, just outside the inner sphere?

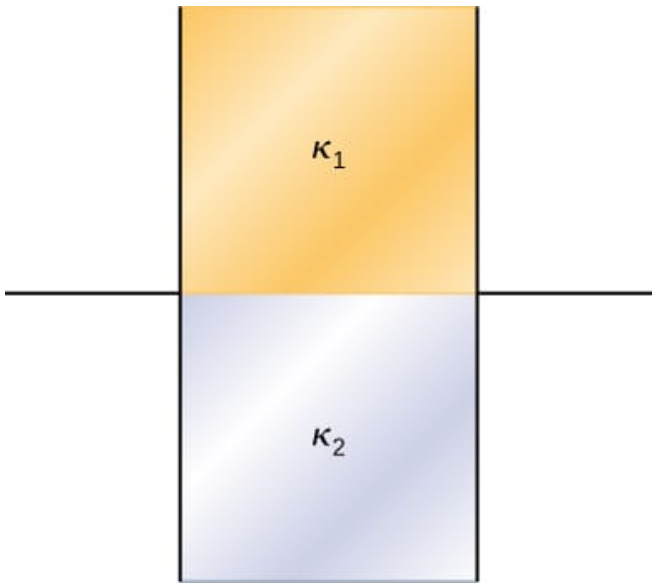
(b) What is the energy density at **r=14.7cm**, just inside the outer sphere?

(c) For the parallel-plate capacitor the energy density is uniform in the region between the plates, except near the edges of the plates. Is this also true for the spherical capacitor?

81. A metal plate of thickness t is held in place between two capacitor plates by plastic pegs, as shown below. The effect of the pegs on the capacitance is negligible. The area of each capacitor plate and the area of the top and bottom surfaces of the inserted plate are all A. What is the capacitance of this system?

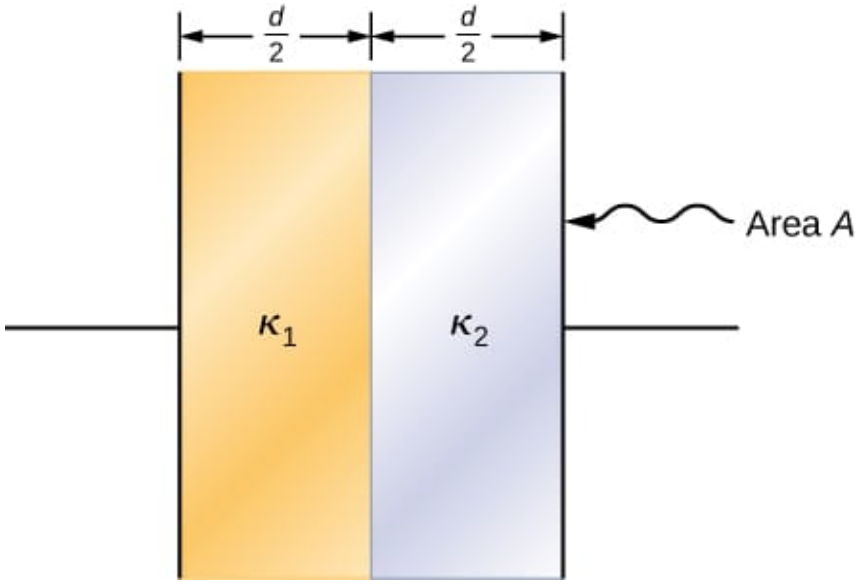


82. A parallel-plate capacitor is filled with two dielectrics, as shown below. When the plate area is A and separation between plates is d, show that the capacitance is given by $C = \varepsilon_0 \frac{A}{d} \frac{\kappa_1 + \kappa_2}{2}$.



83. A parallel-plate capacitor is filled with two dielectrics, as shown below. Show that the capacitance is given by C

$$= 2\varepsilon_0 \frac{A}{d} \frac{\kappa_1 \kappa_2}{\kappa_1 + \kappa_2}.$$



84. A capacitor has parallel plates of area $12cm^2$ separated by 2.0 mm. The space between the plates is filled with polystyrene.

(a) Find the maximum permissible voltage across the capacitor to avoid dielectric breakdown.

(b) When the voltage equals the value found in part (a), find the surface charge density on the surface of the dielectric.

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11.9: Capacitance (Answers)

Check Your Understanding

8.1. $1.1 \times 10^{-3} m$

8.3. 3.59 cm, 17.98 cm

8.4. a. 25.0 pF;

b. 9.2

8.5. a. $C = 0.86pF, Q_1 = 10pC, Q_2 = 3.4pC, Q_3 = 6.8pC$;

b. $C = 2.3pF, Q_1 = 12pC, Q_2 = Q_3 = 16pC$;

c. $C = 2.3pF, Q_1 = 9.0pC, Q_2 = 18pC, Q_3 = 12pC, Q_4 = 15pC$

8.6. a.$4.0 \times 10^{-13} J$; b. 9 times

8.7. a. 3.0; b. $C = 3.0C_0$

8.9. a. $C_0 = 20pF, C = 42pF$;

b. $Q_0 = 0.8nC, Q = 1.7nC$;

c. $V_0 = V = 40V$; d. $U_0 = 16nJ, U = 34nJ$

Conceptual Questions

1. no; yes

3. false

5. no

7. $3.0\mu F, 0.33\mu F$

9. answers may vary

11. Dielectric strength is a critical value of an electrical field above which an insulator starts to conduct; a dielectric constant is the ratio of the electrical field in vacuum to the net electrical field in a material.

13. Water is a good solvent.

15. When energy of thermal motion is large (high temperature), an electrical field must be large too in order to keep electric dipoles aligned with it.

17. answers may vary

Problems

19. 21.6 mC

21. 1.55 V

23. 25.0 nF

25. $1.1 \times 10^{-3} m^2$

27. 500 μC

29. 1:16

31. a. 1.07 nC;

b. 267 V, 133 V

33. $0.29 \mu F$

34. 500 capacitors; connected in parallel

35. $3.08 \mu F$ (series) and 13.0μ (parallel)

37. $11.4 \mu F$

39. 0.89 mC; 1.78 mC; 444 V

41. $7.5 \mu J$

43. a. 405 J; b. 90.0 mC

45. 1.15 J

47. a. $4.43 \times 10^{-9} F$;

b. 0.453 V;

c. $4.53 \times 10^{-10} J$;

d. no

49. 0.7 mJ

51. a. 7.1 pF;

b. 42 pF

53. a. before 3.00 V; after 0.600 V;

b. before 1500 V/m; after 300 V/m

55. a. 3.91;

b. 22.8 V

57. a. 37 nC;

b. 0.4 MV/m;

c. 19 nC

59. a. $4.4 \mu F$;

b. $4.0 \times 10^{-5} C$

61. $0.0135 m^2$

63. $0.185\mu J$

Additional Problems

65. a. 0.277 nF;

b. 27.7 nC;

c. 50 kV/m

67. a. 0.065 F;

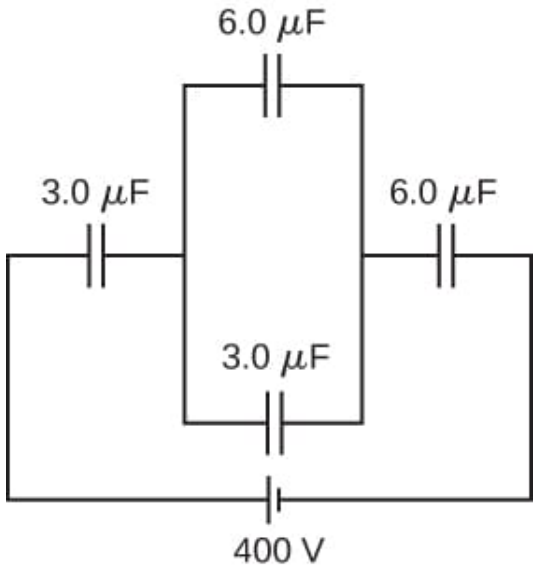
b. 23,000 C;

c. 4.0 GJ

69. a. $75.6\mu C$; b. 10.8 V

71. a. 0.13 J;

b. no, because of resistive heating in connecting wires that is always present, but the circuit schematic does not indicate resistors



73. a. $-3.00\mu F$;

b. You cannot have a negative C_2 capacitance.

c. The assumption that they were hooked up in parallel, rather than in series, is incorrect. A parallel connection always produces a greater capacitance, while here a smaller capacitance was assumed. This could only happen if the capacitors are connected in series.

75. a. 14.2 kV;

b. The voltage is unreasonably large, more than 100 times the breakdown voltage of nylon.

c. The assumed charge is unreasonably large and cannot be stored in a capacitor of these dimensions.

Challenge Problems

77. a. 89.6 pF;

b. 6.09 kV/m;

c. 4.47 kV/m;

d. no

79. a. 421 J;

b. 53.9 mF

81. $C = \varepsilon_0 A/(d_1 + d_2)$

83. proof

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12: CURRENT AND RESISTANCE

12.1: Prelude to Current and Resistance
12.2: Electrical Current
12.3: Model of Conduction in Metals
12.4: Resistivity and Resistance
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12.1: Prelude to Current and Resistance

In this chapter, we study the electrical current through a material, where the electrical current is the rate of flow of charge. We also examine a characteristic of materials known as the resistance. Resistance is a measure of how much a material impedes the flow of charge, and it will be shown that the resistance depends on temperature. In general, a good conductor, such as copper, gold, or silver, has very low resistance. Some materials, called superconductors, have zero resistance at very low temperatures.

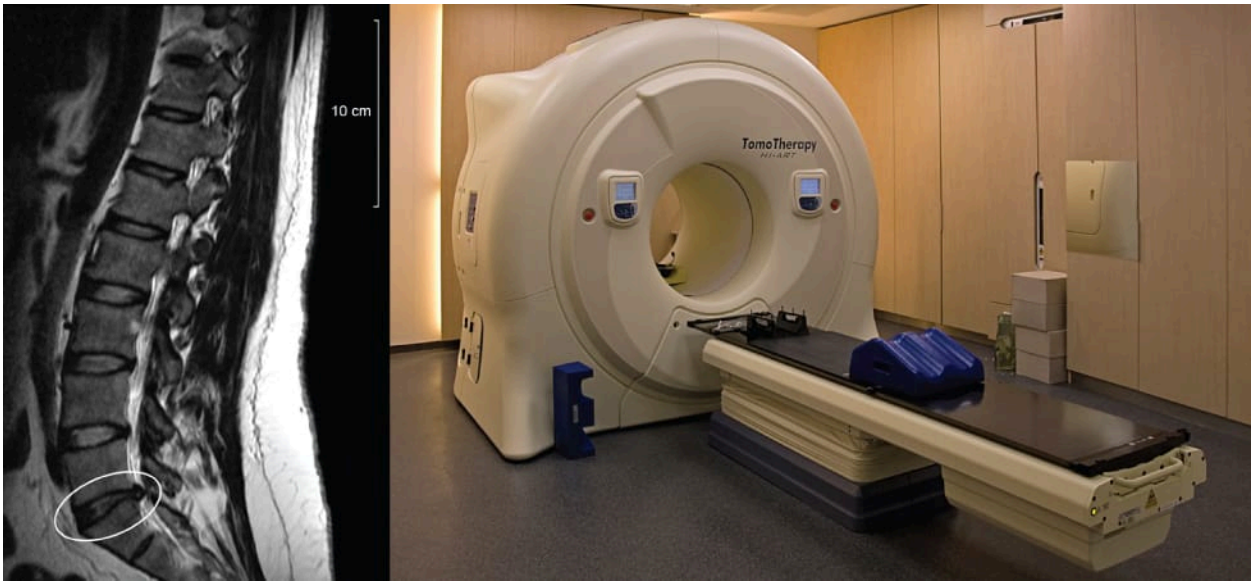


Figure 12.1.1: Magnetic resonance imaging (MRI) uses superconducting magnets and produces high-resolution images without the danger of radiation. The image on the left shows the spacing of vertebrae along a human spinal column, with the circle indicating where the vertebrae are too close due to a ruptured disc. On the right is a picture of the MRI instrument, which surrounds the patient on all sides. A large amount of electrical current is required to operate the electromagnets (credit right: modification of work by “digital cat”/Flickr).

High currents are required for the operation of electromagnets. Superconductors can be used to make electromagnets that are 10 times stronger than the strongest conventional electromagnets. These superconducting magnets are used in the construction of magnetic resonance imaging (MRI) devices that can be used to make high-resolution images of the human body. The chapter-opening picture shows an MRI image of the vertebrae of a human subject and the MRI device itself. Superconducting magnets have many other uses. For example, superconducting magnets are used in the Large Hadron Collider (LHC) to curve the path of protons in the ring.

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12.2: Electrical Current

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Describe an electrical current
- Define the unit of electrical current
- Explain the direction of current flow

Up to now, we have considered primarily static charges. When charges did move, they were accelerated in response to an electrical field created by a voltage difference. The charges lost potential energy and gained kinetic energy as they traveled through a potential difference where the electrical field did work on the charge.

Although charges do not require a material to flow through, the majority of this chapter deals with understanding the movement of charges through a material. The rate at which the charges flow past a location—that is, the amount of charge per unit time—is known as the **electrical current**. When charges flow through a medium, the current depends on the voltage applied, the material through which the charges flow, and the state of the material. Of particular interest is the motion of charges in a conducting wire. In previous chapters, charges were accelerated due to the force provided by an electrical field, losing potential energy and gaining kinetic energy. In this chapter, we discuss the situation of the force provided by an electrical field in a conductor, where charges lose kinetic energy to the material reaching a constant velocity, known as the "**drift velocity**." This is analogous to an object falling through the atmosphere and losing kinetic energy to the air, reaching a constant terminal velocity.

If you have ever taken a course in first aid or safety, you may have heard that in the event of electric shock, it is the current, not the voltage, which is the important factor on the severity of the shock and the amount of damage to the human body. Current is measured in units called amperes; you may have noticed that circuit breakers in your home and fuses in your car are rated in amps (or amperes). But what is the ampere and what does it measure?

Defining Current and the Ampere

Electrical current is defined to be the rate at which charge flows. When there is a large current present, such as that used to run a refrigerator, a large amount of charge moves through the wire in a small amount of time. If the current is small, such as that used to operate a handheld calculator, a small amount of charge moves through the circuit over a long period of time.

Electrical Current

The average electrical current I is the rate at which charge flows,

$$I_{ave} = \frac{\Delta Q}{\Delta t}, \tag{12.2.1}$$

where ΔQ is the amount of net charge passing through a given cross-sectional area in time Δt (Figure $12.2.1$). The SI unit for current is the **ampere** (A), named for the French physicist André-Marie Ampère (1775–1836). Since $I = \frac{\Delta Q}{\Delta t}$, we see that an ampere is defined as one coulomb of charge passing through a given area per second:

$$1\,\text{A} \equiv 1\frac{\text{C}}{\text{s}}.$$

The instantaneous electrical current, or simply the **electrical current**, is the time derivative of the charge that flows and is found by taking the limit of the average electrical current as $\Delta t \to 0$.

$$I = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}.$$

Most electrical appliances are rated in amperes (or amps) required for proper operation, as are fuses and circuit breakers.

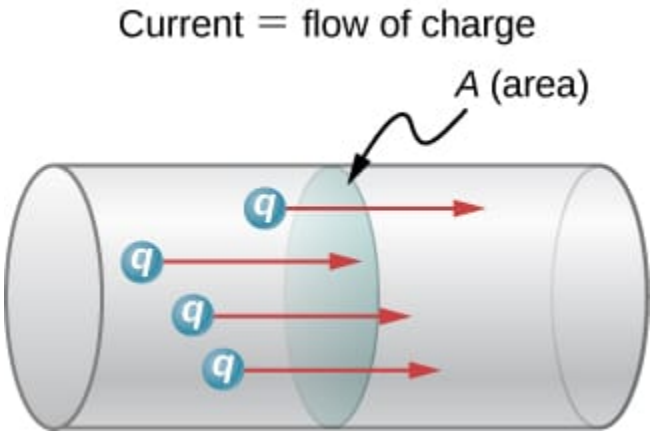


Figure 12.2.1: The rate of flow of charge is current. An ampere is the flow of one coulomb of charge through an area in one second. A current of one amp would result from 6.25×10^{18} electrons flowing through the area A each second.

Calculating the Average Current

The main purpose of a battery in a car or truck is to run the electric **starter motor**, which starts the engine. The operation of starting the vehicle requires a large current to be supplied by the battery. Once the engine starts, a device called an alternator takes over supplying the electric power required for running the vehicle and for charging the battery.

a. What is the average current involved when a truck battery sets in motion 720 C of charge in 4.00 s while starting an engine?
b. How long does it take 1.00 C of charge to flow from the battery?

Strategy

We can use the definition of the average current in Equation 12.2.1 to find the average current in part (a), since charge and time are given. For part (b), once we know the average current, we use Equation 12.2.1 to find the time required for 1.00 C of charge to flow from the battery.

Solution

a. Entering the given values for charge and time into the definition of current gives

$$\begin{aligned} I &= \frac{\Delta Q}{\Delta t} \\ &= \frac{720\,\text{C}}{4.00\,\text{s}} \\ &= 180\,\text{C/s} \\ &= 180\,\text{A}. \end{aligned}$$

b. Solving the relationship $I = \frac{\Delta Q}{\Delta t}$ for time Δt and entering the known values for charge and current gives

$$\begin{aligned} \Delta t &= \frac{\Delta Q}{I} \\ &= \frac{1.00\,\text{C}}{180\,\text{C/s}} \\ &= 5.56 \times 10^{-3}\,\text{s} \\ &= 5.56\,\text{ms}. \end{aligned}$$

Significance

a. This large value for current illustrates the fact that a large charge is moved in a small amount of time. The currents in these “starter motors” are fairly large to overcome the inertia of the engine.
b. A high current requires a short time to supply a large amount of charge. This large current is needed to supply the large amount of energy needed to start the engine.

Calculating Instantaneous Currents

Consider a charge moving through a cross-section of a wire where the charge is modeled as $Q(t) = Q_M(1 - e^{-t/\tau})$. Here, Q_M is the charge after a long period of time, as time approaches infinity, with units of coulombs, and τ is a time constant with units of seconds (Figure 12.2.2). What is the current through the wire?

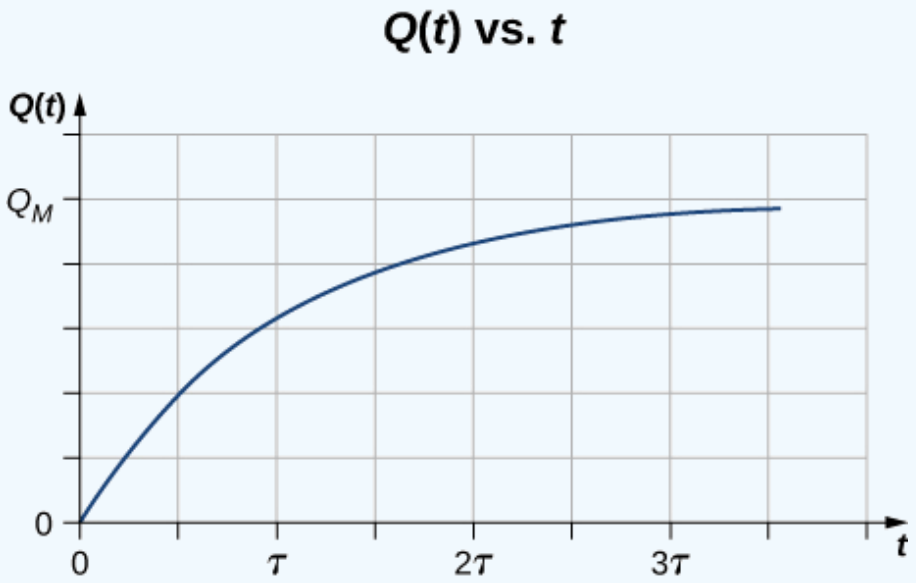


Figure 12.2.2: A graph of the charge moving through a cross-section of a wire over time.

Strategy

The current through the cross-section can be found from $I = \frac{dQ}{dt}$. Notice from the figure that the charge increases to Q_M and the derivative decreases, approaching zero, as time increases (Figure 12.2.2).

Solution

The derivative can be found using $\frac{d}{dx}e^u = e^u\frac{du}{dx}$.

$$\begin{aligned} I &= \frac{dQ}{dt} \\ &= \frac{d}{dt}\left[Q_M\left(1 - e^{-t/\tau}\right)\right] \\ &= \frac{Q_M}{\tau}e^{-t/\tau}. \end{aligned}$$

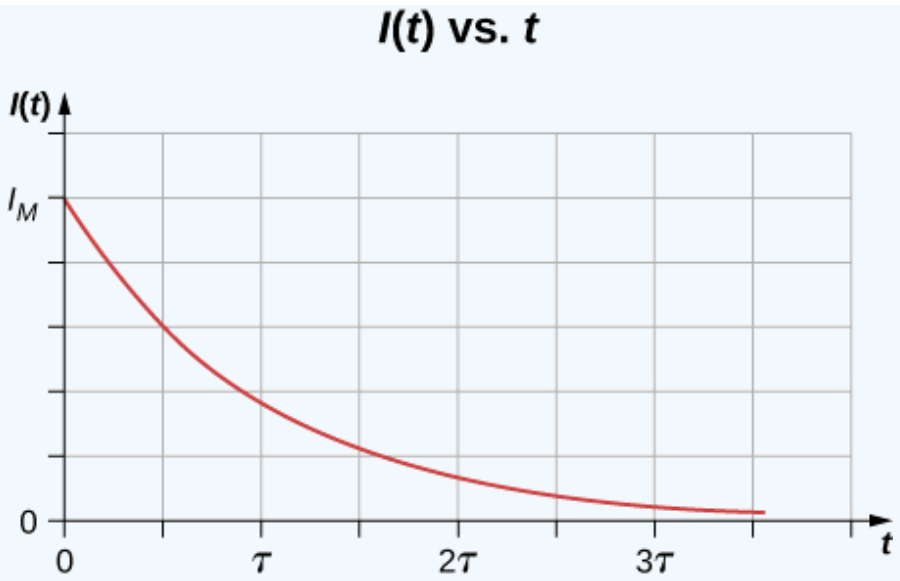


Figure 12.2.3: A graph of the current flowing through the wire over time.

Significance

The current through the wire in question decreases exponentially, as shown in Figure 12.2.3. In later chapters, it will be shown that a time-dependent current appears when a capacitor charges or discharges through a resistor. Recall that a capacitor is a device that stores charge. You will learn about the resistor in Model of Conduction in Metals.

Exercise 12.2.1A

Handheld calculators often use small solar cells to supply the energy required to complete the calculations needed to complete your next physics exam. The current needed to run your calculator can be as small as 0.30 mA. How long would it take for 1.00 C of charge to flow from the solar cells? Can solar cells be used, instead of batteries, to start traditional internal combustion engines presently used in most cars and trucks?

Answer available at the end of this section.

Exercise 12.2.1B

Circuit breakers in a home are rated in amperes, normally in a range from 10 amps to 30 amps, and are used to protect the residents from harm and their appliances from damage due to large currents. A single 15-amp circuit breaker may be used to protect several outlets in the living room, whereas a single 20-amp circuit breaker may be used to protect the refrigerator in the kitchen. What can you deduce from this about current used by the various appliances?

Answer available at the end of this section.

Current in a Circuit

In the previous paragraphs, we defined the current as the charge that flows through a cross-sectional area per unit time. In order for charge to flow through an appliance, such as the headlight shown in Figure 12.2.4, there must be a complete path (or **circuit**) from the positive terminal to the negative terminal. Consider a simple circuit of a car battery, a switch, a headlight lamp, and wires that provide a current path between the components. In order for the lamp to light, there must be a complete path for current flow. In other words, a charge must be able to leave the positive terminal of the battery, travel through the component, and back to the negative terminal of the battery. The switch is there to control the circuit. Part (a) of the figure shows the simple circuit of a car battery, a switch, a conducting path, and a headlight lamp. Also shown is the **schematic** of the circuit [part (b)]. A schematic is a graphical representation of a circuit and is very useful in visualizing the main features of a circuit. Schematics use standardized

symbols to represent the components in a circuits and solid lines to represent the wires connecting the components. The battery is shown as a series of long and short lines, representing the historic voltaic pile. The lamp is shown as a circle with a loop inside, representing the filament of an incandescent bulb. The switch is shown as two points with a conducting bar to connect the two points and the wires connecting the components are shown as solid lines. The schematic in part (c) shows the direction of current flow when the switch is closed.

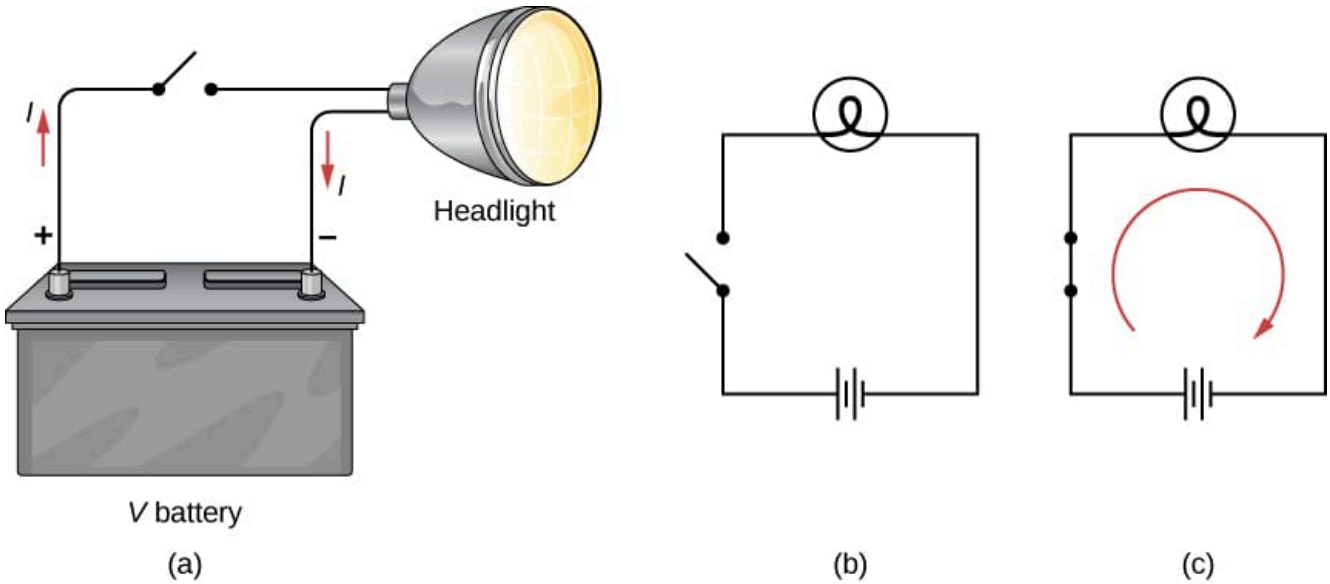


Figure 12.2.4: (a) A simple electric circuit of a headlight (lamp), a battery, and a switch. When the switch is closed, an uninterrupted path for current to flow through is supplied by conducting wires connecting a load to the terminals of a battery. (b) In this schematic, the battery is represented by parallel lines, which resemble plates in the original design of a battery. The longer lines indicate the positive terminal. The conducting wires are shown as solid lines. The switch is shown, in the open position, as two terminals with a line representing a conducting bar that can make contact between the two terminals. The lamp is represented by a circle encompassing a filament, as would be seen in an incandescent light bulb. (c) When the switch is closed, the circuit is complete and current flows from the positive terminal to the negative terminal of the battery.

When the switch is closed in Figure 12.2.4*c*, there is a complete path for charges to flow, from the positive terminal of the battery, through the switch, then through the headlight and back to the negative terminal of the battery. Note that the direction of current flow is from positive to negative. The direction of conventional current is always represented in the direction that positive charge would flow, from the positive terminal to the negative terminal.

The conventional current flows from the positive terminal to the negative terminal, but depending on the actual situation, positive charges, negative charges, or both may move. In metal wires, for example, current is carried by electrons—that is, negative charges move. In ionic solutions, such as salt water, both positive and negative charges move. This is also true in nerve cells. A Van de Graaff generator, used for nuclear research, can produce a current of pure positive charges, such as protons. In the Tevatron Accelerator at Fermilab, before it was shut down in 2011, beams of protons and antiprotons traveling in opposite directions were collided. The protons are positive and therefore their current is in the same direction as they travel. The antiprotons are negativity charged and thus their current is in the opposite direction that the actual particles travel.

A closer look at the current flowing through a wire is shown in Figure 12.2.5. The figure illustrates the movement of charged particles that compose a current. The fact that conventional current is taken to be in the direction that positive charge would flow can be traced back to American scientist and statesman Benjamin Franklin in the 1700s. Having no knowledge of the particles that make up the atom (namely the proton, electron, and neutron), Franklin believed that electrical current flowed from a material that had more of an “electrical fluid” and to a material that had less of this “electrical fluid.” He coined the term **positive** for the material that had more of this electrical fluid and **negative** for the material that lacked the electrical fluid. He surmised that current would flow from the material with more electrical fluid—the positive material—to the negative material, which has less electrical fluid. Franklin called this direction of current a positive current flow. This was pretty advanced thinking for a man who knew nothing about the atom.

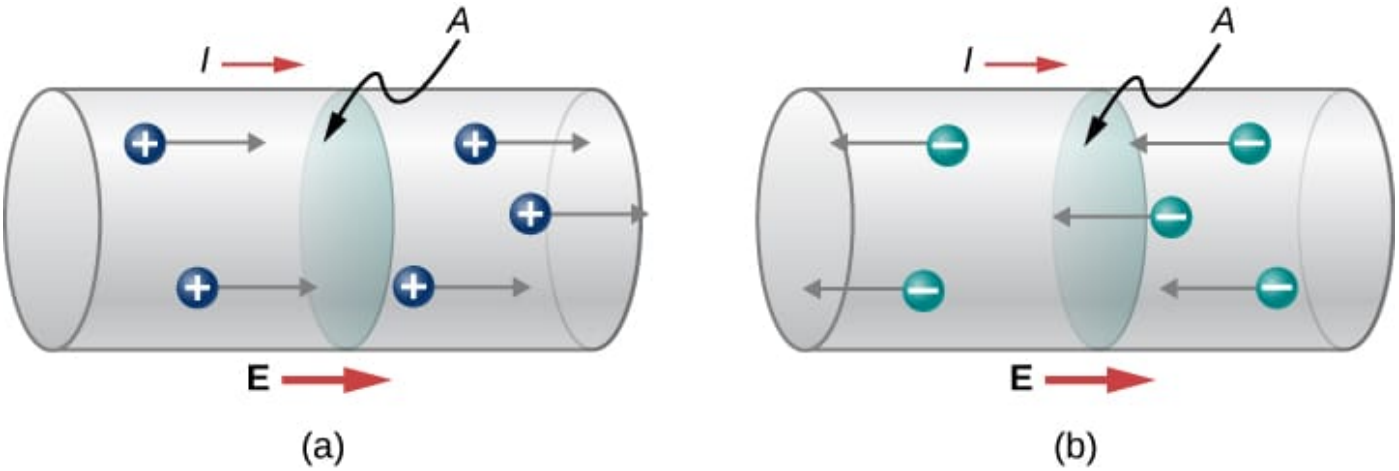


Figure 12.2.5: Current **I** is the rate at which charge moves through an area **A**, such as the cross-section of a wire. Conventional current is defined to move in the direction of the electrical field. (a) Positive charges move in the direction of the electrical field, which is the same direction as conventional current. (b) Negative charges move in the direction opposite to the electrical field. Conventional current is in the direction opposite to the movement of negative charge. The flow of electrons is sometimes referred to as electronic flow.

We now know that a material is positive if it has a greater number of protons than electrons, and it is negative if it has a greater number of electrons than protons. In a conducting metal, the current flow is due primarily to electrons flowing from the negative material to the positive material, but for historical reasons, we consider the positive current flow and the current is shown to flow from the positive terminal of the battery to the negative terminal.

It is important to realize that an electrical field is present in conductors and is responsible for producing the current (Figure 12.2.5). In previous chapters, we considered the static electrical case, where charges in a conductor quickly redistribute themselves on the surface of the conductor in order to cancel out the external electrical field and restore equilibrium. In the case of an electrical circuit, the charges are prevented from ever reaching equilibrium by an external source of electric potential, such as a battery. The energy needed to move the charge is supplied by the electric potential from the battery.

Although the electrical field is responsible for the motion of the charges in the conductor, the work done on the charges by the electrical field does not increase the kinetic energy of the charges. We will show that the electrical field is responsible for keeping the electric charges moving at a “drift velocity.”

Answers

Exercise 12.2.1*A*

The time for 1.00 C of charge to flow would be

$$\Delta t = \frac{\Delta Q}{I} = \frac{1.00\,C}{0.300 \times 10^{-3}\,\mathrm{C/s}} = 3.33 \times 10^{3}\,\mathrm{s}.$$

This is slightly less than an hour. This is quite different from the 5.55 ms for the truck battery. The calculator takes a very small amount of energy to operate, unlike the truck’s starter motor. There are several reasons that vehicles use batteries and not solar cells. Aside from the obvious fact that a light source to run the solar cells for a car or truck is not always available, the large amount of current needed to start the engine cannot easily be supplied by present-day solar cells. Solar cells can possibly be used to charge the batteries. Charging the battery requires a small amount of energy when compared to the energy required to run the engine and the other accessories such as the heater and air conditioner. Present day solar-powered cars are powered by solar panels, which may power an electric motor, instead of an internal combustion engine.

Exercise 12.2.1*B*

The total current needed by all the appliances in the living room (a few lamps, a television, and your laptop) draw less current and require less power than the refrigerator.

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12.3: Model of Conduction in Metals

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Define the drift velocity of charges moving through a metal
- Define the vector current density
- Describe the operation of an incandescent lamp

When electrons move through a conducting wire, they do not move at a constant velocity, that is, the electrons do not move in a straight line at a constant speed. Rather, they interact with and collide with atoms and other free electrons in the conductor. Thus, the electrons move in a zig-zag fashion and drift through the wire. We should also note that even though it is convenient to discuss the direction of current, current is a scalar quantity. When discussing the velocity of charges in a current, it is more appropriate to discuss the current density. We will come back to this idea at the end of this section.

Drift Velocity

Electrical signals move very rapidly. Telephone conversations carried by currents in wires cover large distances without noticeable delays. Lights come on as soon as a light switch is moved to the 'on' position. Most electrical signals carried by currents travel at speeds on the order of $10^8 \mathrm{m/s}$ a significant fraction of the speed of light. Interestingly, the individual charges that make up the current move much slower on average, typically drifting at speeds on the order of $10^{-4} \mathrm{m/s}$. How do we reconcile these two speeds, and what does it tell us about standard conductors?

The high speed of electrical signals results from the fact that the force between charges acts rapidly at a distance. Thus, when a free charge is forced into a wire, as in Figure $12.3.1$, the incoming charge pushes other charges ahead of it due to the repulsive force between like charges. These moving charges push on charges farther down the line. The density of charge in a system cannot easily be increased, so the signal is passed on rapidly. The resulting electrical shock wave moves through the system at nearly the speed of light. To be precise, this fast-moving signal, or shock wave, is a rapidly propagating change in the electrical field.

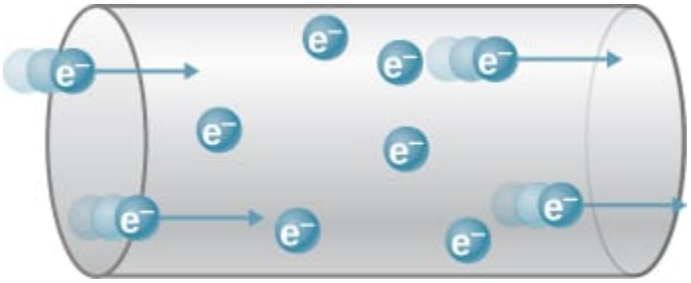


Figure $12.3.1$: When charged particles are forced into this volume of a conductor, an equal number are quickly forced to leave. The repulsion between like charges makes it difficult to increase the number of charges in a volume. Thus, as one charge enters, another leaves almost immediately, carrying the signal rapidly forward.

Good conductors have large numbers of free charges. In metals, the free charges are free electrons. (In fact, good electrical conductors are often good heat conductors too, because large numbers of free electrons can transport thermal energy as well as carry electrical current.) Figure $12.3.2$ shows how free electrons move through an ordinary conductor. The distance that an individual electron can move between collisions with atoms or other electrons is quite small. The electron paths thus appear nearly random, like the motion of atoms in a gas. But there is an electrical field in the conductor that causes the electrons to drift in the direction shown (opposite to the field, since they are negative). The **drift velocity** $\vec{v}_d$ is the average velocity of the free charges. Drift velocity is quite small, since there are so many free charges. If we have an estimate of the density of free electrons in a conductor, we can calculate the drift velocity for a given current. The larger the density, the lower the velocity required for a given current.

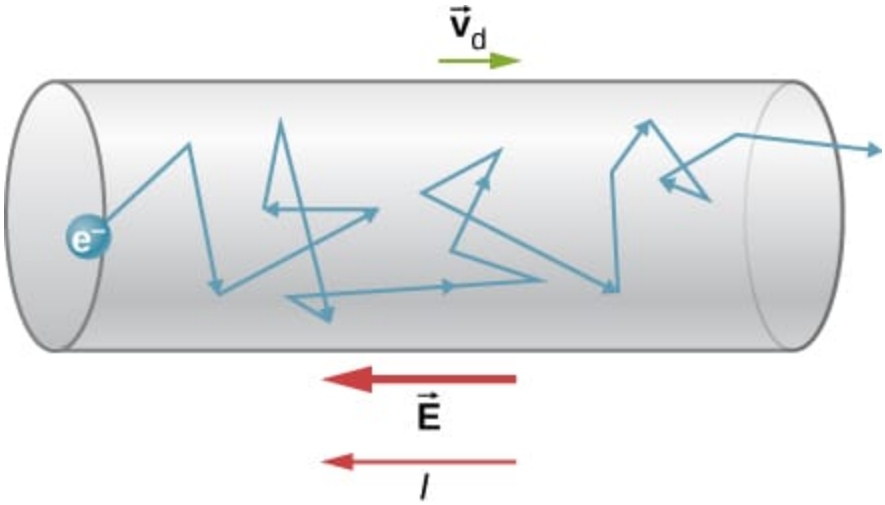


Figure 12.3.2: Free electrons moving in a conductor make many collisions with other electrons and other particles. A typical path of one electron is shown. The average velocity of the free charges is called the drift velocity $\vec{v}_d$ and for electrons, it is in the direction opposite to the electrical field. The collisions normally transfer energy to the conductor, requiring a constant supply of energy to maintain a steady current.

Free-electron collisions transfer energy to the atoms of the conductor. The electrical field does work in moving the electrons through a distance, but that work does not increase the kinetic energy (nor speed) of the electrons. The work is transferred to the conductor's atoms, often increasing temperature. Thus, a continuous power input is required to keep a current flowing. (An exception is superconductors, for reasons we shall explore in a later chapter. Superconductors can have a steady current without a continual supply of energy—a great energy savings.) For a conductor that is not a superconductor, the supply of energy can be useful, as in an incandescent light bulb filament (Figure 12.3.3). The supply of energy is necessary to increase the temperature of the tungsten filament, so that the filament glows.

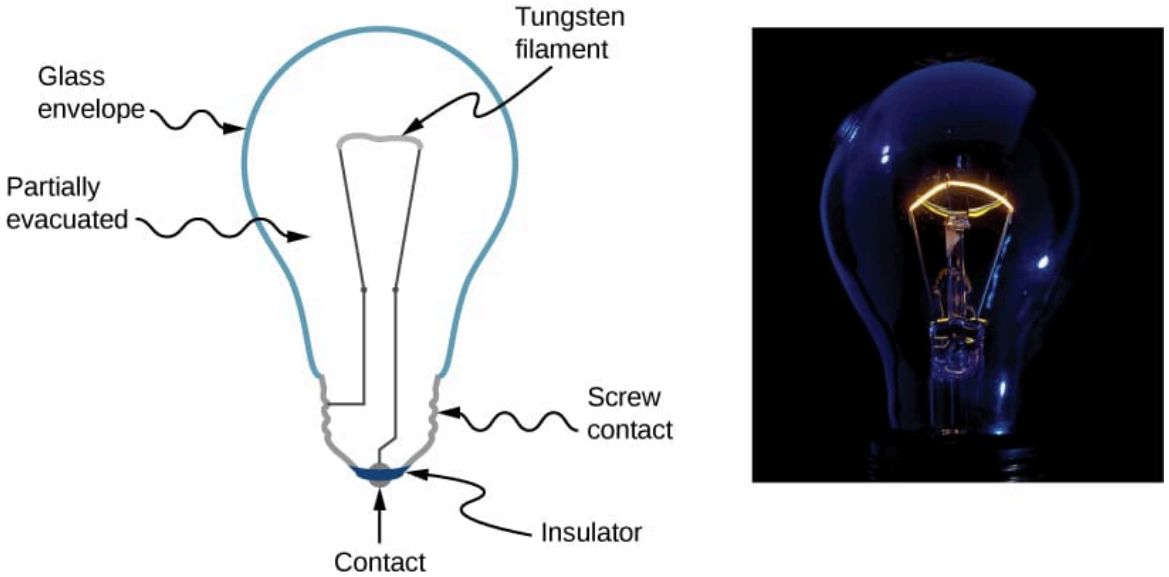


Figure 12.3.3: The incandescent lamp is a simple design. A tungsten filament is placed in a partially evacuated glass envelope. One end of the filament is attached to the screw base, which is made out of a conducting material. The second end of the filament is attached to a second contact in the base of the bulb. The two contacts are separated by an insulating material. Current flows through the filament, and the temperature of the filament becomes large enough to cause the filament to glow and produce light. However, these bulbs are not very energy efficient, as evident from the heat coming from the bulb. In the year 2012, the United States, along with many other countries, began to phase out incandescent lamps in favor of more energy-efficient lamps, such as light-emitting diode (LED) lamps and compact fluorescent lamps (CFL) (credit right: modification of work by Serge Saint).

We can obtain an expression for the relationship between current and drift velocity by considering the number of free charges in a segment of wire, as illustrated in Figure 12.3.4. The number of free charges per unit volume, or the number density of free charges, is given the symbol n where

$$n = \frac{\text{number of charges}}{\text{volume}}.$$

The value of n depends on the material. The shaded segment has a volume $Av_d\ dt$, so that the number of free charges in the volume is $nAv_d\ dt$. The charge dQ in this segment is thus $qnAv_d\ dt$, where q is the amount of charge on each carrier. (The magnitude of the charge of electrons is $q = 1.60 \times 10^{-19}$ C.) Current is charge moved per unit time; thus, if all the original charges move out of this segment in time dt, the current is

$$I = \frac{dQ}{dt} = qnAv_d.$$

Rearranging terms gives

$$v_d = \frac{I}{nqA}$$

where v_d is the drift velocity, n is the free charge density, A is the cross-sectional area of the wire, and I is the current through the wire.

The carriers of the current each have charge q and move with a drift velocity of magnitude v_d.

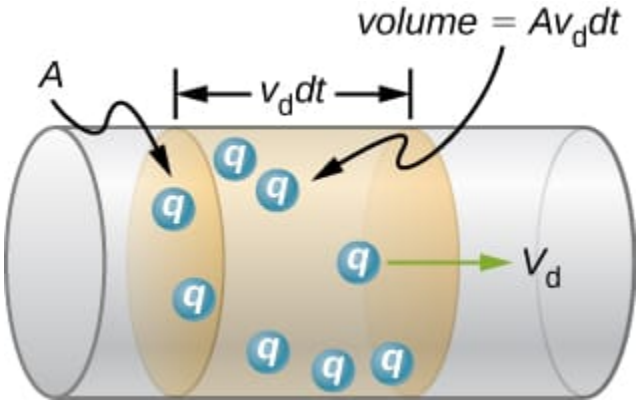


Figure 12.3.4: All the charges in the shaded volume of this wire move out in a time dt, having a drift velocity of magnitude v_d.

Note that simple drift velocity is not the entire story. The speed of an electron is sometimes much greater than its drift velocity. In addition, not all of the electrons in a conductor can move freely, and those that do move might move somewhat faster or slower than the drift velocity. So what do we mean by free electrons?

Atoms in a metallic conductor are packed in the form of a lattice structure. Some electrons are far enough away from the atomic nuclei that they do not experience the attraction of the nuclei as strongly as the inner electrons do. These are the free electrons. They are not bound to a single atom but can instead move freely among the atoms in a “sea” of electrons. When an electrical field is applied, these free electrons respond by accelerating. As they move, they collide with the atoms in the lattice and with other electrons, generating thermal energy, and the conductor gets warmer. In an insulator, the organization of the atoms and the structure do not allow for such free electrons.

As you know, electric power is usually supplied to equipment and appliances through round wires made of a conducting material (copper, aluminum, silver, or gold) that are stranded or solid. The diameter of the wire determines the current-carrying capacity—the larger the diameter, the greater the current-carrying capacity. Even though the current-carrying capacity is determined by the diameter, wire is not normally characterized by the diameter directly. Instead, wire is commonly sold in a unit known as “gauge.” Wires are manufactured by passing the material through circular forms called “drawing dies.” In order to make thinner wires, manufacturers draw the wires through multiple dies of successively thinner diameter. Historically, the gauge of the wire was related to the number of drawing processes required to manufacture the wire. For this reason, the larger the gauge, the smaller the diameter. In the United States, the American Wire Gauge (AWG) was developed to standardize the system. Household wiring commonly consists of 10-gauge (2.588-mm diameter) to 14-gauge (1.628-mm diameter) wire. A device used to measure the gauge of wire is shown in Figure 12.3.5.

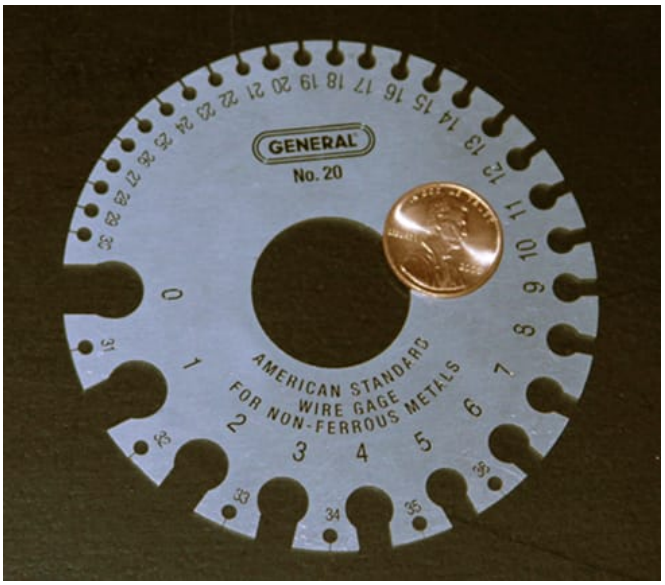


Figure 12.3.5: A device for measuring the gauge of electrical wire. As you can see, higher gauge numbers indicate thinner wires.

Example 12.3.1: Calculating Drift Velocity in a Common Wire

Calculate the drift velocity of electrons in a copper wire with a diameter of 2.053 mm (12-gauge) carrying a 20.0-A current, given that there is one free electron per copper atom. (Household wiring often contains 12-gauge copper wire, and the maximum current allowed in such wire is usually 20.0 A.) The density of copper is $8.80 \times 10^3 \text{ kg/m}^3$ and the atomic mass of copper is 63.54 g/mol.

Strategy

We can calculate the drift velocity using the equation $I = nqAv_d$. The current is $I = 20.00 \text{ A}$ and $q = 1.60 \times 10^{-19} \text{ C}$ is the charge of an electron. We can calculate the area of a cross-section of the wire using the formula $A = \pi r^2$, where r is one-half the diameter. The given diameter is 2.053 mm, so r is 1.0265 mm. We are given the density of copper, $8.80 \times 10^3 \text{ kg/m}^3$, and the atomic mass of copper is 63.54 g/mol. We can use these two quantities along with Avogadro's number, $6.02 \times 10^{23} \text{ atoms/mol}$, to determine n, the number of free electrons per cubic meter.

Solution

First, we calculate the density of free electrons in copper. There is one free electron per copper atom. Therefore, the number of free electrons is the same as the number of copper atoms per m^3. We can now find n as follows:

$$\begin{aligned} n &= \frac{1\,e^-}{\text{atom}} \times \frac{6.02 \times 10^{23}\text{atoms}}{\text{mol}} \times \frac{1\,\text{mol}}{63.54\,\text{g}} \times \frac{1000\,\text{g}}{\text{kg}} \times \frac{8.80 \times 10^3\,\text{kg}}{1\,\text{m}^3} \\ &= 8.34 \times 10^{28}\,e^-/\text{m}^3. \end{aligned}$$

The cross-sectional area of the wire is

$$\begin{aligned} A &= \pi r^2 \\ &= \pi \left(\frac{2.05 \times 10^{-3}\,\text{m}}{2} \right)^2 \\ &= 3.30 \times 10^{-6}\text{m}^2. \end{aligned}$$

Rearranging $I = nqAv_d$ to isolate drift velocity gives

$$v_d = \frac{I}{nqA}$$
$$= \frac{20.00\,\text{A}}{(8.34 \times 10^{28}/\text{m}^3)(-1.60 \times 10^{-19}\text{C})(3.30 \times 10^{-6}\text{m}^2)}$$
$$= -4.54 \times 10^{-4}\text{m/s}.$$

Significance

The minus sign indicates that the negative charges are moving in the direction opposite to conventional current. The small value for drift velocity (on the order of 10^{-4}m/s confirms that the signal moves on the order of 10^{12} times faster (about 10^{8}m/s than the charges that carry it.

Exercise 12.3.1

In Example 12.3.1, the drift velocity was calculated for a 2.053-mm diameter (12-gauge) copper wire carrying a 20-amp current. Would the drift velocity change for a 1.628-mm diameter (14-gauge) wire carrying the same 20-amp current?

Answer available at the end of this section.

Current Density

Although it is often convenient to attach a negative or positive sign to indicate the overall direction of motion of the charges, current is a scalar quantity, $I = \frac{dQ}{dt}$. It is often necessary to discuss the details of the motion of the charge, instead of discussing the overall motion of the charges. In such cases, it is necessary to discuss the current density, $\vec{J}$, a vector quantity. The **current density** is the flow of charge through an infinitesimal area, divided by the area. The current density must take into account the local magnitude and direction of the charge flow, which varies from point to point. The unit of current density is ampere per meter squared, and the direction is defined as the direction of net flow of positive charges through the area.

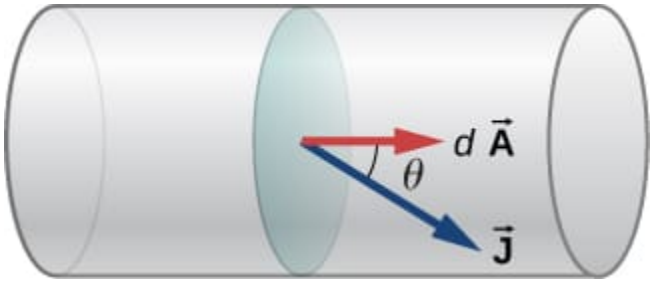


Figure 12.3.6: The current density $\vec{J}$ is defined as the current passing through an infinitesimal cross-sectional area divided by the area. The direction of the current density is the direction of the net flow of positive charges and the magnitude is equal to the current divided by the infinitesimal area.

The relationship between the current and the current density can be seen in Figure 12.3.6. The differential current flow through the area $d\vec{A}$ is found as

$$dI = \vec{J} \cdot d\vec{A} = JdA\cos\theta,$$

where θ is the angle between the area and the current density. The total current passing through area $d\vec{A}$ can be found by integrating over the area,

$$I = \iint_{area} \vec{J} \cdot d\vec{A}.$$

Consider the magnitude of the current density, which is the current divided by the area:

$$J = \frac{I}{A} = \frac{n|q|Av_d}{A} = n|q|v_d.$$

Thus, the current density is $\vec{J} = nq\vec{v}_d$. If q is positive, $\vec{v}_d$ is in the same direction as the electrical field $\vec{E}$. If q is negative, $\vec{v}_d$ is in the opposite direction of $\vec{E}$. Either way, the direction of the current density $\vec{J}$ is in the direction of the electrical field $\vec{E}$.

Example 12.3.2: Calculating the Current Density in a Wire

The current supplied to a lamp with a 100-W light bulb is 0.87 amps. The lamp is wired using a copper wire with diameter 2.588 mm (10-gauge). Find the magnitude of the current density.

Strategy

The current density is the current moving through an infinitesimal cross-sectional area divided by the area. We can calculate the magnitude of the current density using $J = \frac{I}{A}$. The current is given as 0.87 A. The cross-sectional area can be calculated to be $A = 5.26\ \text{mm}^2$.

Solution

Calculate the current density using the given current $I = 0.87\ A$ and the area, found to be $A = 5.26\ \text{mm}^2$.

$$J = \frac{I}{A} = \frac{0.87\,\text{A}}{5.26 \times 10^{-6}\,\text{m}^2} = 1.65 \times 10^5 \frac{\text{A}}{\text{m}^2}.$$

Significance

The current density in a conducting wire depends on the current through the conducting wire and the cross-sectional area of the wire. For a given current, as the diameter of the wire increases, the charge density decreases.

Exercise 12.3.2

The current density is proportional to the current and inversely proportional to the area. If the current density in a conducting wire increases, what would happen to the drift velocity of the charges in the wire?

Answer available at the end of this section.

What is the significance of the current density? The current density is proportional to the current, and the current is the number of charges that pass through a cross-sectional area per second. The charges move through the conductor, accelerated by the electric force provided by the electrical field. The electrical field is created when a voltage is applied across the conductor. In Ohm's Law, we will use this relationship between the current density and the electrical field to examine the relationship between the current through a conductor and the voltage applied.

Answers

Exercise 12.3.1

The diameter of the 14-gauge wire is smaller than the diameter of the 12-gauge wire. Since the drift velocity is inversely proportional to the cross-sectional area, the drift velocity in the 14-gauge wire is larger than the drift velocity in the 12-gauge wire carrying the same current. The number of electrons per cubic meter will remain constant.

Exercise 12.3.2

The current density in a conducting wire increases due to an increase in current. The drift velocity is inversely proportional to the current $\left(v_d = \frac{nqA}{I}\right)$, so the drift velocity would decrease.

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12.4: Resistivity and Resistance

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12.5: Ohm's Law

Learning Objectives

By the end of this section, you will be able to:

- Describe Ohm's law
- Recognize when Ohm's law applies and when it does not

We have been discussing three electrical properties so far in this chapter: current, voltage, and resistance. It turns out that many materials exhibit a simple relationship among the values for these properties, known as Ohm's law. Many other materials do not show this relationship, so despite being called Ohm's law, it is not considered a law of nature, like Newton's laws or the laws of thermodynamics. But it is very useful for calculations involving materials that do obey Ohm's law.

Description of Ohm's Law

The current that flows through most substances is directly proportional to the voltage V applied to it. The German physicist Georg Simon **Ohm** (1787–1854) was the first to demonstrate experimentally that the current in a metal wire is **directly proportional to the voltage applied**:

$$I \propto V.$$

This important relationship is the basis for **Ohm's law**. It can be viewed as a cause-and-effect relationship, with voltage the cause and current the effect. This is an empirical law, which is to say that it is an experimentally observed phenomenon, like friction. Such a linear relationship doesn't always occur. Any material, component, or device that obeys Ohm's law, where the current through the device is proportional to the voltage applied, is known as an ohmic material or **ohmic** component. Any material or component that does not obey Ohm's law is known as a **nonohmic** material or nonohmic component.

Ohm's Experiment

In a paper published in 1827, Georg Ohm described an experiment in which he measured voltage across and current through various simple electrical circuits containing various lengths of wire. A similar experiment is shown in Figure $12.5.1$. This experiment is used to observe the current through a resistor that results from an applied voltage. In this simple circuit, a resistor is connected in series with a battery. The voltage is measured with a voltmeter, which must be placed across the resistor (in parallel with the resistor). The current is measured with an ammeter, which must be in line with the resistor (in series with the resistor).

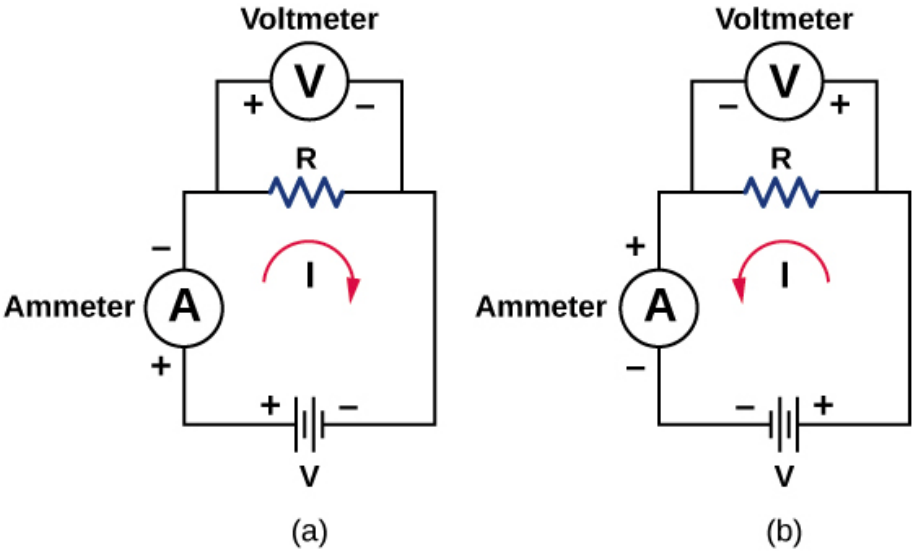


Figure 12.5.1: The experimental set-up used to determine if a resistor is an ohmic or nonohmic device. (a) When the battery is attached, the current flows in the clockwise direction and the voltmeter and ammeter have positive readings. (b) When the leads of the battery are switched, the current flows in the counterclockwise direction and the voltmeter and ammeter have negative readings.

In this updated version of Ohm's original experiment, several measurements of the current were made for several different voltages. When the battery was hooked up as in Figure 12.5.1a, the current flowed in the clockwise direction and the readings of the voltmeter and ammeter were positive. Does the behavior of the current change if the current flowed in the opposite direction? To get the current to flow in the opposite direction, the leads of the battery can be switched. When the leads of the battery were switched, the readings of the voltmeter and ammeter readings were negative because the current flowed in the opposite direction, in this case, counterclockwise. Results of a similar experiment are shown in Figure 12.5.2.

I(A)	V(V)
–2.74	–10.00
–2.25	–9.00
–2.00	–8.00
–1.75	–7.00
–1.50	–6.00
–1.49	–5.00
–1.00	–4.00
–0.51	–3.00
–0.74	–2.00
–0.49	–1.00
+0.00	+0.00
+0.49	+1.00
+0.50	+2.00
+0.99	+3.00
+0.76	+4.00
+1.01	+5.00
+1.74	+6.00
+1.75	+7.00
+2.00	+8.00
+2.49	+9.00
+2.50	+10.00

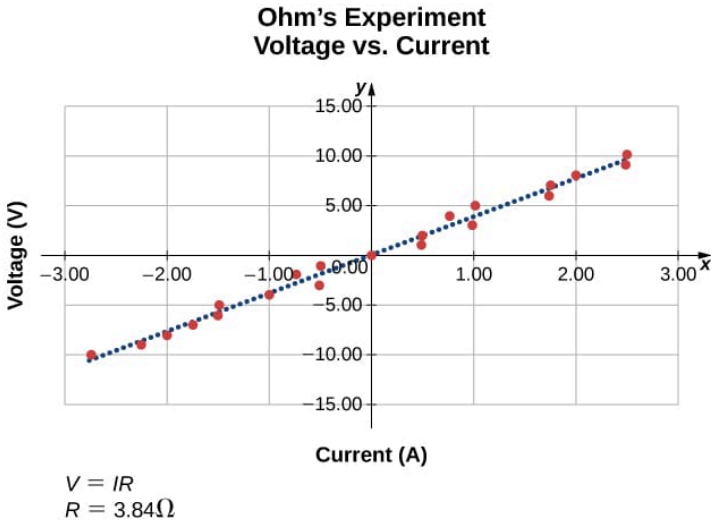


Figure 12.5.2: A resistor is placed in a circuit with a battery. The voltage applied varies from –10.00 V to +10.00 V, increased by 1.00-V increments. A plot shows values of the voltage versus the current typical of what a casual experimenter might find.

In this experiment, the voltage applied across the resistor varies from –10.00 to +10.00 V, by increments of 1.00 V. The current through the resistor and the voltage across the resistor are measured. A plot is made of the voltage versus the current, and the result is approximately linear. The slope of the line is the resistance, or the voltage divided by the current. This result is known as **Ohm's law**:

$$V = IR \tag{12.5.1}$$

where V is the voltage measured in volts across the object in question, I is the current measured through the object in amps, and R is the resistance in units of ohms. As stated previously, any device that shows a linear relationship between the voltage and the current is known as an ohmic device. A resistor is therefore an ohmic device.

Example 12.5.1: Measuring Resistance

A carbon resistor at room temperature $(20^{\circ}\mathrm{C})$ is attached to a 9.00-V battery and the current measured through the resistor is 3.00 mA.

a. What is the resistance of the resistor measured in ohms?
b. If the temperature of the resistor is increased to $60^{\circ}\mathrm{C}$ by heating the resistor, what is the current through the resistor?

Strategy

a. The resistance can be found using Ohm's law. Ohm's law states that $V = IR$, so the resistance can be found using $R = V/I$.
b. First, the resistance is temperature dependent so the new resistance after the resistor has been heated can be found using $R = R_0(1 + \alpha\Delta T)$. The current can be found using Ohm's law in the form $I = V/R$.

Solution

a. Using Ohm's law and solving for the resistance yields the resistance at room temperature:

$$R = \frac{V}{I} = \frac{9.00\,\mathrm{V}}{3.00 \times 10^{-3}\mathrm{A}} = 3.00 \times 10^{3}\,\Omega = 3.00\,\mathrm{k}\Omega$$

b. The resistance at $60^{\circ}\mathrm{C}$ can be found using $R = R_0(1 + \alpha\Delta T)$ where the temperature coefficient for carbon is $\alpha = -0.0005$.

$$R = R_0(1 + \alpha\Delta T) = 3.00 \times 10^{3}(1 - 0.0005(60^{\circ}\mathrm{C} - 20^{\circ}\mathrm{C})) = 2.94\,\mathrm{k}\Omega.$$

The current through the heated resistor is

$$I = \frac{V}{R} = \frac{9.00\,\mathrm{V}}{2.94 \times 10^{3}\,\Omega} = 3.06 \times 10^{-3}\mathrm{A} = 3.06\,\mathrm{mA}.$$

Significance

A change in temperature of $40^{\circ}\mathrm{C}$ resulted in a 2.00% change in current. This may not seem like a very great change, but changing electrical characteristics can have a strong effect on the circuits. For this reason, many electronic appliances, such as computers, contain fans to remove the heat dissipated by components in the electric circuits.

Exercise 12.5.1

The voltage supplied to your house varies as $V(t) = V_{\max}\sin(2\pi ft)$. If a resistor is connected across this voltage, will Ohm's law $V = IR$ still be valid?

Answer available at the end of this section.

Simulation: PhET

See how Ohm's law (Equation 12.5.1) relates to a simple circuit. Adjust the voltage and resistance, and see the current change according to Ohm's law. The sizes of the symbols in the equation change to match the circuit diagram.

PhET: Ohm's Law

Nonohmic devices do not exhibit a linear relationship between the voltage and the current. One such device is the semiconducting circuit element known as a **diode**. A diode is a circuit device that allows current flow in only one direction. A diagram of a simple circuit consisting of a battery, a diode, and a resistor is shown in Figure 12.5.3. Although we do not cover the

theory of the diode in this section, the diode can be tested to see if it is an ohmic or a nonohmic device.

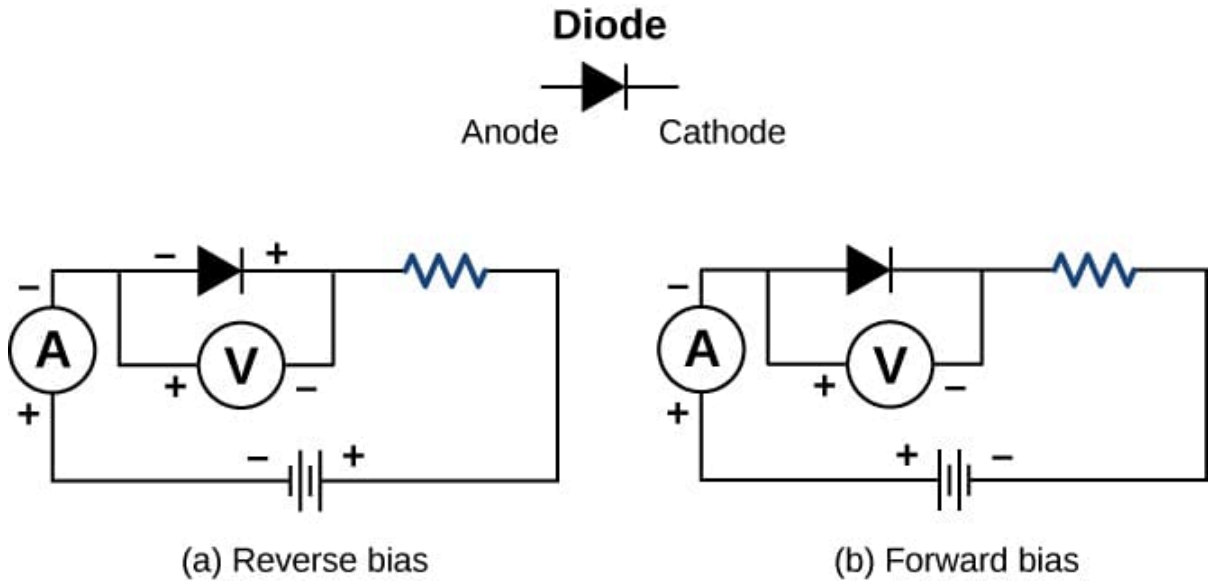


Figure 12.5.3: A diode is a semiconducting device that allows current flow only if the diode is forward biased, which means that the anode is positive and the cathode is negative.

A plot of current versus voltage is shown in Figure 12.5.4. Note that the behavior of the diode is shown as current versus voltage, whereas the resistor operation was shown as voltage versus current. A diode consists of an anode and a cathode. When the anode is at a negative potential and the cathode is at a positive potential, as shown in part (a), the diode is said to have reverse bias. With reverse bias, the diode has an extremely large resistance and there is very little current flow—essentially zero current—through the diode and the resistor. As the voltage applied to the circuit increases, the current remains essentially zero, until the voltage reaches the breakdown voltage and the diode conducts current. When the battery and the potential across the diode are reversed, making the anode positive and the cathode negative, the diode conducts and current flows through the diode if the voltage is greater than 0.7 V. The resistance of the diode is close to zero. (This is the reason for the resistor in the circuit; if it were not there, the current would become very large.) You can see from the graph in Figure 12.5.4 that the voltage and the current do not have a linear relationship. Thus, the diode is an example of a nonohmic device.

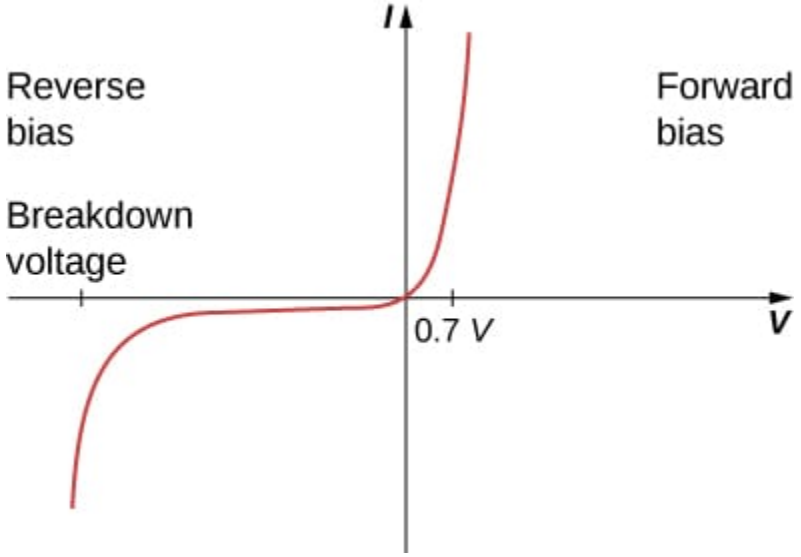


Figure 12.5.4: When the voltage across the diode is negative and small, there is very little current flow through the diode. As the voltage reaches the breakdown voltage, the diode conducts. When the voltage across the diode is positive and greater than 0.7 V (the actual voltage value depends on the diode), the diode conducts. As the voltage applied increases, the current through the diode increases, but the voltage across the diode remains approximately 0.7 V.

Ohm's law is commonly stated as $V = IR$, but originally it was stated as a microscopic view, in terms of the current density, the

conductivity, and the electrical field. This microscopic view suggests the proportionality $V \propto I$ comes from the drift velocity of the free electrons in the metal that results from an applied electrical field. As stated earlier, the current density is proportional to the applied electrical field. The reformulation of Ohm's law is credited to Gustav Kirchhoff, whose name we will see again in the next chapter.

Answers

Exercise 12.5.1

Yes, Ohm's law is still valid. At every point in time the current is equal to $I(t) = V(t)/R$, so the current is also a function of time, $I(t) = \frac{V_{\max}}{R} \sin(2\pi ft)$.

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12.6: Electrical Energy and Power

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Express electrical power in terms of the voltage and the current
- Describe the power dissipated by a resistor in an electric circuit
- Calculate the energy efficiency and cost effectiveness of appliances and equipment

In an electric circuit, electrical energy is continuously converted into other forms of energy. For example, when a current flows in a conductor, electrical energy is converted into thermal energy within the conductor. The electrical field, supplied by the voltage source, accelerates the free electrons, increasing their kinetic energy for a short time. This increased kinetic energy is converted into thermal energy through collisions with the ions of the lattice structure of the conductor. Previously, we defined power as the rate at which work is done by a force measured in watts. Power can also be defined as the rate at which energy is transferred. In this section, we discuss the time rate of energy transfer, or power, in an electric circuit.

Power in Electric Circuits

Power is associated by many people with electricity. Power transmission lines might come to mind. We also think of light bulbs in terms of their power ratings in watts. What is the expression for **electric power**?

Let us compare a 25-W bulb with a 60-W bulb (Figure $12.6.1a$). The 60-W bulb glows brighter than the 25-W bulb. Although it is not shown, a 60-W light bulb is also warmer than the 25-W bulb. The heat and light is produced by from the conversion of electrical energy. The kinetic energy lost by the electrons in collisions is converted into the internal energy of the conductor and radiation. How are voltage, current, and resistance related to electric power?

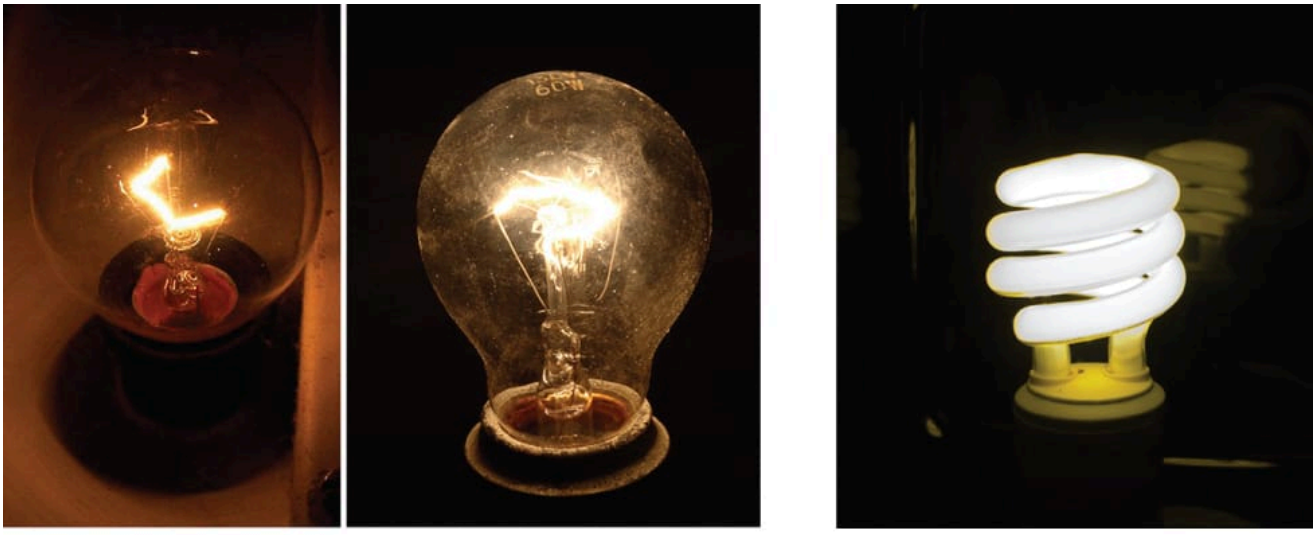


Figure $12.6.1$: (a) Pictured above are two incandescent bulbs: a 25-W bulb (left) and a 60-W bulb (right). The 60-W bulb provides a higher intensity light than the 25-W bulb. The electrical energy supplied to the light bulbs is converted into heat and light. (b) This compact fluorescent light (CFL) bulb puts out the same intensity of light as the 60-W bulb, but at 1/4 to 1/10 the input power. (credit a: modification of works by “Dickbauch”/Wikimedia Commons and Greg Westfall; credit b: modification of work by “dbgg1979”/Flickr)

To calculate electric power, consider a voltage difference existing across a material (Figure $12.6.2$). The electric potential V_1 is higher than the electric potential at V_2, and the voltage difference is negative $V = V_2 - V_1$. As discussed in Electric Potential, an electrical field exists between the two potentials, which points from the higher potential to the lower potential. Recall that the

electrical potential is defined as the potential energy per charge, $V = \Delta U/q$, and the charge ΔQ loses potential energy moving through the potential difference.

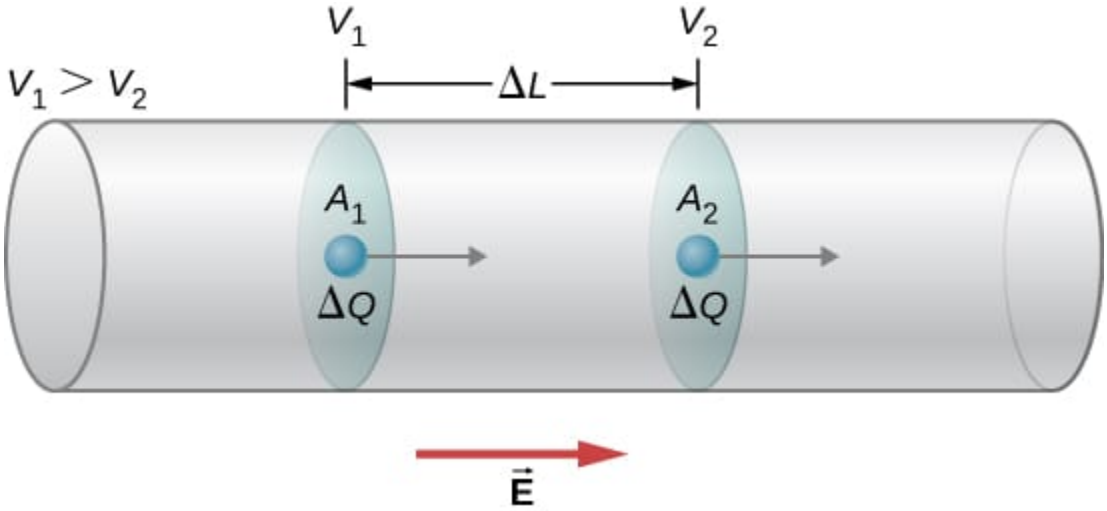


Figure 12.6.2: When there is a potential difference across a conductor, an electrical field is present that points in the direction from the higher potential to the lower potential.

If the charge is positive, the charge experiences a force due to the electrical field $\vec{F} = m\vec{a} = \Delta Q\vec{E}$. This force is necessary to keep the charge moving. This force does not act to accelerate the charge through the entire distance ΔL because of the interactions of the charge with atoms and free electrons in the material. The speed, and therefore the kinetic energy, of the charge do not increase during the entire trip across ΔL, and charge passing through area A_2 has the same drift velocity v_d as the charge that passes through area A_1. However, work is done on the charge, by the electrical field, which changes the potential energy. Since the change in the electrical potential difference is negative, the electrical field is found to be

$$E = -\frac{(V_2 - V_1)}{\Delta L} = \frac{V}{\Delta L}.$$

The work done on the charge is equal to the electric force times the length at which the force is applied,

$$W = F\Delta L = (\Delta QE)\Delta L = \left(\Delta Q\frac{V}{\Delta L}\right)\Delta L = \Delta QV = \Delta U.$$

The charge moves at a drift velocity v_d so the work done on the charge results in a loss of potential energy, but the average kinetic energy remains constant. The lost electrical potential energy appears as thermal energy in the material. On a microscopic scale, the energy transfer is due to collisions between the charge and the molecules of the material, which leads to an increase in temperature in the material. The loss of potential energy results in an increase in the temperature of the material, which is dissipated as radiation. In a resistor, it is dissipated as heat, and in a light bulb, it is dissipated as heat and light.

The power dissipated by the material as heat and light is equal to the time rate of change of the work:

$$P = \frac{\Delta U}{\Delta t} = -\frac{\Delta QV}{\Delta t} = IV$$

With a resistor, the voltage drop across the resistor is dissipated as heat. Ohm's law states that the voltage across the resistor is equal to the current times the resistance, $V = IR$. The power dissipated by the resistor is therefore

$$P = IV = I(IR) = I^2R$$

or

$$P = IV = \left(\frac{V}{R}\right)V = \frac{V^2}{R}.$$

If a resistor is connected to a battery, the power dissipated as radiant energy by the wires and the resistor is equal to

$$P = IV = I^2R = \frac{V^2}{R}.$$

The power supplied from the battery is equal to current times the voltage, $P = IV$.

Definition: Electric Power

The electric power gained or lost by any device has the form

$$P = IV.$$

The power dissipated by a resistor has the form

$$P = I^2R = \frac{V^2}{R}.$$

Different insights can be gained from the three different expressions for electric power. For example, $P = V^2/R$ implies that the lower the resistance connected to a given voltage source, the greater the power delivered. Furthermore, since voltage is squared in $P = V^2/R$, the effect of applying a higher voltage is perhaps greater than expected. Thus, when the voltage is doubled to a 25-W bulb, its power nearly quadruples to about 100 W, burning it out. If the bulb's resistance remained constant, its power would be exactly 100 W, but at the higher temperature, its resistance is higher, too.

Example 12.6.1: Calculating Power in Electric Devices

A DC winch motor is rated at 20.00 A with a voltage of 115 V. When the motor is running at its maximum power, it can lift an object with a weight of 4900.00 N a distance of 10.00 m, in 30.00 s, at a constant speed.

a. What is the power consumed by the motor?
b. What is the power used in lifting the object? Ignore air resistance.
c. Assuming that the difference in the power consumed by the motor and the power used lifting the object are dissipated as heat by the resistance of the motor, estimate the resistance of the motor.

Strategy

a. The power consumed by the motor can be found using $P = IV$.
b. The power used in lifting the object at a constant speed can be found using $P = Fv$, where the speed is the distance divided by the time. The upward force supplied by the motor is equal to the weight of the object because the acceleration is zero. (c) The resistance of the motor can be found using $P = I^2R$.

Solution

1. The power consumed by the motor is equal to $P = IV$ and the current is given as 20.00 A and the voltage is 115.00 V:
$$P = IV = (20.00\,\text{A})115.00\,\text{V} = 2300.00\,\text{W}.$$
2. The power used lifting the object is equal to $P = Fv$ where the force is equal to the weight of the object (4900 N) and the magnitude of the velocity is
$$v = \frac{10.00\,\text{m}}{30.00\,\text{s}} = 0.33\frac{\text{m}}{\text{s}}$$
$$P = Fv = (4900\,\text{N})0.33\,\text{m/s} = 1633.33\,\text{W}.$$
3. The difference in the power equals $2300.00\ \text{W} - 1633.33\ \text{W} = 666.67\ \text{W}$ and the resistance can be found using $P = I^2R$:
$$R = \frac{P}{I^2} = \frac{666.67\,\text{W}}{(20.00\,\text{A})^2} = 1.67\,\Omega.$$

Significance

The resistance of the motor is quite small. The resistance of the motor is due to many windings of copper wire. The power dissipated by the motor can be significant since the thermal power dissipated by the motor is proportional to the square of the current $(P = I^2R)$.

Exercise 12.6.1

Electric motors have a reasonably high efficiency. A 100-hp motor can have an efficiency of 90% and a 1-hp motor can have an efficiency of 80%. Why is it important to use high-performance motors?

Answer available at the end of this section.

A fuse (Figure 12.6.3) is a device that protects a circuit from currents that are too high. A fuse is basically a short piece of wire between two contacts. As we have seen, when a current is running through a conductor, the kinetic energy of the charge carriers is converted into thermal energy in the conductor. The piece of wire in the fuse is under tension and has a low melting point. The wire is designed to heat up and break at the rated current. The fuse is destroyed and must be replaced, but it protects the rest of the circuit. Fuses act quickly, but there is a small time delay while the wire heats up and breaks.



Figure 12.6.3: A fuse consists of a piece of wire between two contacts. When a current passes through the wire that is greater than the rated current, the wire melts, breaking the connection. Pictured is a "blown" fuse where the wire broke protecting a circuit (credit: modification of work by "Shardayyy"/Flickr).

Circuit breakers are also rated for a maximum current, and open to protect the circuit, but can be reset. Circuit breakers react much faster. The operation of circuit breakers is not within the scope of this chapter and will be discussed in later chapters. Another method of protecting equipment and people is the ground fault circuit interrupter (GFCI), which is common in bathrooms and kitchens. The GFCI outlets respond very quickly to changes in current. These outlets open when there is a change in magnetic field produced by current-carrying conductors, which is also beyond the scope of this chapter and is covered in a later chapter.

The Cost of Electricity

The more electric appliances you use and the longer they are left on, the higher your electric bill. This familiar fact is based on the relationship between energy and power. You pay for the energy used. Since $P = \frac{dE}{dt}$, we see that

$$E = \int Pdt$$

is the energy used by a device using power P for a time interval t. If power is delivered at a constant rate, then the energy can be found by $E = Pt$. For example, the more light bulbs burning, the greater P used; the longer they are on, the greater t is.

The energy unit on electric bills is the kilowatt-hour $(\mathrm{kW} \cdot \mathrm{h})$, consistent with the relationship $E = Pt$. It is easy to estimate the

cost of operating electrical appliances if you have some idea of their power consumption rate in watts or kilowatts, the time they are on in hours, and the cost per kilowatt-hour for your electric utility. Kilowatt-hours, like all other specialized energy units such as food calories, can be converted into joules. You can prove to yourself that $1\ \text{kW} \cdot \text{h} = 3.6 \times 10^6\ \text{J}$.

The electrical energy (E) used can be reduced either by reducing the time of use or by reducing the power consumption of that appliance or fixture. This not only reduces the cost but also results in a reduced impact on the environment. Improvements to lighting are some of the fastest ways to reduce the electrical energy used in a home or business. About 20% of a home's use of energy goes to lighting, and the number for commercial establishments is closer to 40%. Fluorescent lights are about four times more efficient than incandescent lights—this is true for both the long tubes and the compact fluorescent lights (CFLs), e.g., Figure $12.6.1b$. Thus, a 60-W incandescent bulb can be replaced by a 15-W CFL, which has the same brightness and color. CFLs have a bent tube inside a globe or a spiral-shaped tube, all connected to a standard screw-in base that fits standard incandescent light sockets. (Original problems with color, flicker, shape, and high initial investment for CFLs have been addressed in recent years.)

The heat transfer from these CFLs is less, and they last up to 10 times longer than incandescent bulbs. The significance of an investment in such bulbs is addressed in the next example. New white LED lights (which are clusters of small LED bulbs) are even more efficient (twice that of CFLs) and last five times longer than CFLs.

Example 12.6.2: Calculating the Cost Effectiveness of LED Bulb

The typical replacement for a 100-W incandescent bulb is a 20-W LED bulb. The 20-W LED bulb can provide the same amount of light output as the 100-W incandescent light bulb. What is the cost savings for using the LED bulb in place of the incandescent bulb for one year, assuming \$0.10 per kilowatt-hour is the average energy rate charged by the power company? Assume that the bulb is turned on for three hours a day.

Strategy

a. Calculate the energy used during the year for each bulb, using $E = Pt$.
b. Multiply the energy by the cost.

Solution

a. Calculate the power for each bulb.

$$E_{\text{Incandescent}} = Pt = 100\,\text{W}\left(\frac{1\,\text{kW}}{1000\,\text{W}}\right)\left(\frac{3\,\text{h}}{\text{day}}\right)(365\,\text{days}) = 109.5\,\text{kW} \cdot \text{h}$$

$$E_{\text{LED}} = Pt = 20\,\text{W}\left(\frac{1\,\text{kW}}{1000\,\text{W}}\right)\left(\frac{3\,\text{h}}{\text{day}}\right)(365\,\text{days}) = 21.9\,\text{kW} \cdot \text{h}$$

b. Calculate the cost for each.

$$\text{cost}_{\text{Incandescent}} = 109.5\,\text{kW} \cdot \text{h}\left(\frac{\$0.10}{\text{kW} \cdot \text{h}}\right) = \$10.95$$

$$\text{cost}_{\text{LED}} = 21.90\,\text{kW} \cdot \text{h}\left(\frac{\$0.10}{\text{kW} \cdot \text{h}}\right) = \$2.19$$

Significance

A LED bulb uses 80% less energy than the incandescent bulb, saving \$8.76 over the incandescent bulb for one year. The LED bulb can cost \$20.00 and the 100-W incandescent bulb can cost \$0.75, which should be calculated into the computation. A typical lifespan of an incandescent bulb is 1200 hours and is 50,000 hours for the LED bulb. The incandescent bulb would last 1.08 years at 3 hours a day and the LED bulb would last 45.66 years. The initial cost of the LED bulb is high, but the cost to the home owner will be \$0.69 for the incandescent bulbs versus \$0.44 for the LED bulbs per year. (Note that the LED bulbs are coming down in price.) The cost savings per year is approximately \$8.50, and that is just for one bulb.

Exercise 12.6.2

Is the efficiency of the various light bulbs the only consideration when comparing the various light bulbs?

Answer available at the end of this section.

Changing light bulbs from incandescent bulbs to CFL or LED bulbs is a simple way to reduce energy consumption in homes and commercial sites. CFL bulbs operate with a much different mechanism than do incandescent lights. The mechanism is complex and beyond the scope of this chapter, but here is a very general description of the mechanism. CFL bulbs contain argon and mercury vapor housed within a spiral-shaped tube. The CFL bulbs use a "ballast" that increases the voltage used by the CFL bulb. The ballast produce an electrical current, which passes through the gas mixture and excites the gas molecules. The excited gas molecules produce ultraviolet (UV) light, which in turn stimulates the fluorescent coating on the inside of the tube. This coating fluoresces in the visible spectrum, emitting visible light. Traditional fluorescent tubes and CFL bulbs had a short time delay of up to a few seconds while the mixture was being "warmed up" and the molecules reached an excited state. It should be noted that these bulbs do contain mercury, which is poisonous, but if the bulb is broken, the mercury is never released. Even if the bulb is broken, the mercury tends to remain in the fluorescent coating. The amount is also quite small and the advantage of the energy saving may outweigh the disadvantage of using mercury.

The CFL light bulbs are being replaced with LED light bulbs, where LED stands for "light-emitting diode." The diode was briefly discussed as a nonohmic device, made of semiconducting material, which essentially permits current flow in one direction. LEDs are a special type of diode made of semiconducting materials infused with impurities in combinations and concentrations that enable the extra energy from the movement of the electrons during electrical excitation to be converted into visible light. Semiconducting devices will be explained in greater detail in Condensed Matter Physics.

Commercial LEDs are quickly becoming the standard for commercial and residential lighting, replacing incandescent and CFL bulbs. They are designed for the visible spectrum and are constructed from gallium doped with arsenic and phosphorous atoms. The color emitted from an LED depends on the materials used in the semiconductor and the current. In the early years of LED development, small LEDs found on circuit boards were red, green, and yellow, but LED light bulbs can now be programmed to produce millions of colors of light as well as many different hues of white light.

Comparison of Incandescent, CFL, and LED Light Bulbs

The energy savings can be significant when replacing an incandescent light bulb or a CFL light bulb with an LED light. Light bulbs are rated by the amount of power that the bulb consumes, and the amount of light output is measured in lumens. The lumen (lm) is the SI -derived unit of luminous flux and is a measure of the total quantity of visible light emitted by a source. A 60-W incandescent light bulb can be replaced with a 13- to 15-W CFL bulb or a 6- to 8-W LED bulb, all three of which have a light output of approximately 800 lm. A table of light output for some commonly used light bulbs appears in Table 12.6.1.

The life spans of the three types of bulbs are significantly different. An LED bulb has a life span of 50,000 hours, whereas the CFL has a lifespan of 8000 hours and the incandescent lasts a mere 1200 hours. The LED bulb is the most durable, easily withstanding rough treatment such as jarring and bumping. The incandescent light bulb has little tolerance to the same treatment since the filament and glass can easily break. The CFL bulb is also less durable than the LED bulb because of its glass construction. The amount of heat emitted is 3.4 btu/h for the 8-W LED bulb, 85 btu/h for the 60-W incandescent bulb, and 30 btu/h for the CFL bulb. As mentioned earlier, a major drawback of the CFL bulb is that it contains mercury, a neurotoxin, and must be disposed of as hazardous waste. From these data, it is easy to understand why the LED light bulb is quickly becoming the standard in lighting.

Table 12.6.1: Light Output of LED, Incandescent, and CFL Light Bulbs

Light Output (lumens)	LED Light Bulbs (watts)	Incandescent Light Bulbs (watts)	CFL Light Bulbs (watts)
450	4–5	40	9–13
800	6–8	60	13–15
1100	9–13	75	18–25
1600	16–20	100	23–30
2600	25–28	150	30–55

Summary of Relationships

In this chapter, we have discussed relationships between voltages, current, resistance, and power. Figure 12.6.4 shows a summary

of the relationships between these measurable quantities for ohmic devices. (Recall that ohmic devices follow Ohm's law $V = IR$.) For example, if you need to calculate the power, use the pink section, which shows that $P = VI$, $P = \frac{V^2}{R}$, and $P = I^2R$.

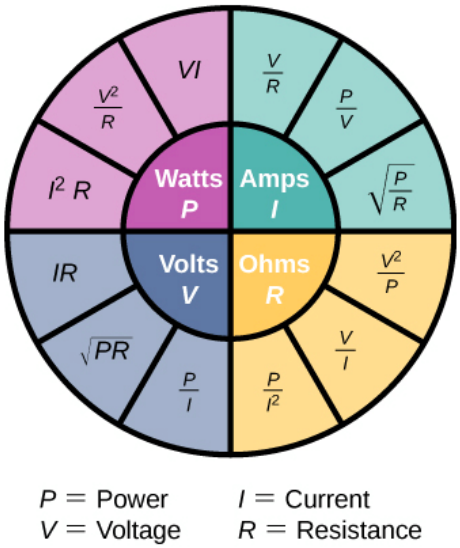


Figure 12.6.4: This circle shows a summary of the equations for the relationships between power, current, voltage, and resistance.

Which equation you use depends on what values you are given, or you measure. For example if you are given the current and the resistance, use $P = I^2R$. Although all the possible combinations may seem overwhelming, don't forget that they all are combinations of just two equations, Ohm's law $(V = IR)$ and power $(P = IV)$.

Answers

Exercise 12.6.1

Even though electric motors are highly efficient 10–20% of the power consumed is wasted, not being used for doing useful work. Most of the 10–20% of the power lost is transferred into heat dissipated by the copper wires used to make the coils of the motor. This heat adds to the heat of the environment and adds to the demand on power plants providing the power. The demand on the power plant can lead to increased greenhouse gases, particularly if the power plant uses coal or gas as fuel.

Exercise 12.6.2

No, the efficiency is a very important consideration of the light bulbs, but there are many other considerations. As mentioned above, the cost of the bulbs and the life span of the bulbs are important considerations. For example, CFL bulbs contain mercury, a neurotoxin, and must be disposed of as hazardous waste. When replacing incandescent bulbs that are being controlled by a dimmer switch with LED, the dimmer switch may need to be replaced. The dimmer switches for LED lights are comparably priced to the incandescent light switches, but this is an initial cost which should be considered. The spectrum of light should also be considered, but there is a broad range of color temperatures available, so you should be able to find one that fits your needs. None of these considerations mentioned are meant to discourage the use of LED or CFL light bulbs, but they are considerations.

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12.7: Superconductors

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Describe the phenomenon of superconductivity
- List applications of superconductivity

Touch the power supply of your laptop computer or some other device. It probably feels slightly warm. That heat is an unwanted byproduct of the process of converting household electric power into a current that can be used by your device. Although electric power is reasonably efficient, other losses are associated with it. As discussed in the section on power and energy, transmission of electric power produces I^2R line losses. These line losses exist whether the power is generated from conventional power plants (using coal, oil, or gas), nuclear plants, solar plants, hydroelectric plants, or wind farms. These losses can be reduced, but not eliminated, by transmitting using a higher voltage. It would be wonderful if these line losses could be eliminated, but that would require transmission lines that have zero resistance. In a world that has a global interest in not wasting energy, the reduction or elimination of this unwanted thermal energy would be a significant achievement. Is this possible?

The Resistance of Mercury

In 1911, Heike **Kamerlingh Onnes** of Leiden University, a Dutch physicist, was looking at the temperature dependence of the resistance of the element mercury. He cooled the sample of mercury and noticed the familiar behavior of a linear dependence of resistance on temperature; as the temperature decreased, the resistance decreased. Kamerlingh Onnes continued to cool the sample of mercury, using liquid helium. As the temperature approached $4.2\ K\ (-269.2^oC)$, the resistance abruptly went to zero (Figure $12.7.1$). This temperature is known as the **critical temperature** T_c for mercury. The sample of mercury entered into a phase where the resistance was absolutely zero. This phenomenon is known as **superconductivity**. (**Note:** If you connect the leads of a three-digit ohmmeter across a conductor, the reading commonly shows up as $0.00\ \Omega$. The resistance of the conductor is not actually zero, it is less than $0.01\ \Omega$.) There are various methods to measure very small resistances, such as the four-point method, but an ohmmeter is not an acceptable method to use for testing resistance in superconductivity.

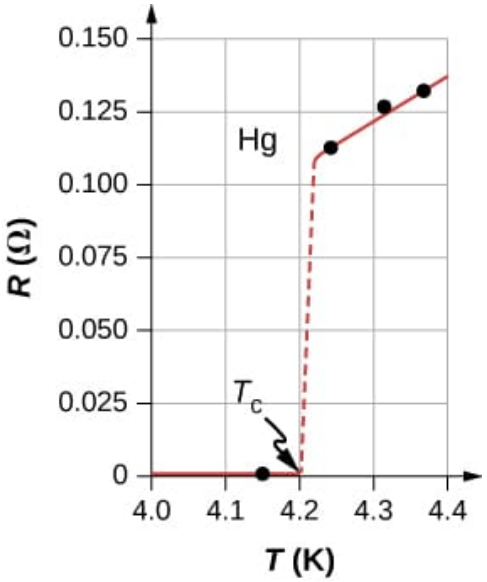


Figure $12.7.1$: The resistance of a sample of mercury is zero at very low temperatures—it is a superconductor up to the temperature of about 4.2 K. Above that critical temperature, its resistance makes a sudden jump and then increases nearly linearly with temperature.

Other Superconducting Materials

As research continued, several other materials were found to enter a superconducting phase, when the temperature reached near absolute zero. In 1941, an alloy of niobium-nitride was found that could become superconducting at $T_c = 16\ K\ (-257^oC)$ and in 1953, vanadium-silicon was found to become superconductive at $T_c = 17.5\ K\ (-255.7^oC)$. The temperatures for the transition into superconductivity were slowly creeping higher. Strangely, many materials that make good conductors, such as copper, silver, and gold, do not exhibit superconductivity. Imagine the energy savings if transmission lines for electric power-generating stations could be made to be superconducting at temperatures near room temperature! A resistance of zero ohms means no I^2R losses and a great boost to reducing energy consumption. The problem is that $T_c = 17.5\ K$ is still very cold and in the range of liquid helium temperatures. At this temperature, it is not cost effective to transmit electrical energy because of the cooling requirements.

A large jump was seen in 1986, when a team of researchers, headed by Dr. Ching Wu Chu of Houston University, fabricated a brittle, ceramic compound with a transition temperature of $T_c = 92\ K\ (-181^oC)$. The ceramic material, composed of yttrium barium copper oxide (YBCO), was an insulator at room temperature. Although this temperature still seems quite cold, it is near the boiling point of liquid nitrogen, a liquid commonly used in refrigeration. You may have noticed refrigerated trucks traveling down the highway labeled as “Liquid Nitrogen Cooled.”

YBCO ceramic is a material that could be useful for transmitting electrical energy because the cost saving of reducing the I^2R losses are larger than the cost of cooling the superconducting cable, making it financially feasible. There were and are many engineering problems to overcome. For example, unlike traditional electrical cables, which are flexible and have a decent tensile strength, ceramics are brittle and would break rather than stretch under pressure. Processes that are rather simple with traditional cables, such as making connections, become difficult when working with ceramics. The problems are difficult and complex, and material scientists and engineers are coming up with innovative solutions.

An interesting consequence of the resistance going to zero is that once a current is established in a superconductor, it persists without an applied voltage source. Current loops in a superconductor have been set up and the current loops have been observed to persist for years without decaying.

Zero resistance is not the only interesting phenomenon that occurs as the materials reach their transition temperatures. A second effect is the exclusion of magnetic fields. This is known as the **Meissner effect** (Figure 12.7.2). A light, permanent magnet placed over a superconducting sample will levitate in a stable position above the superconductor. High-speed trains have been developed that levitate on strong superconducting magnets, eliminating the friction normally experienced between the train and the tracks. In Japan, the Yamanashi Maglev test line opened on April 3, 1997. In April 2015, the MLX01 test vehicle attained a speed of 374 mph (603 km/h).

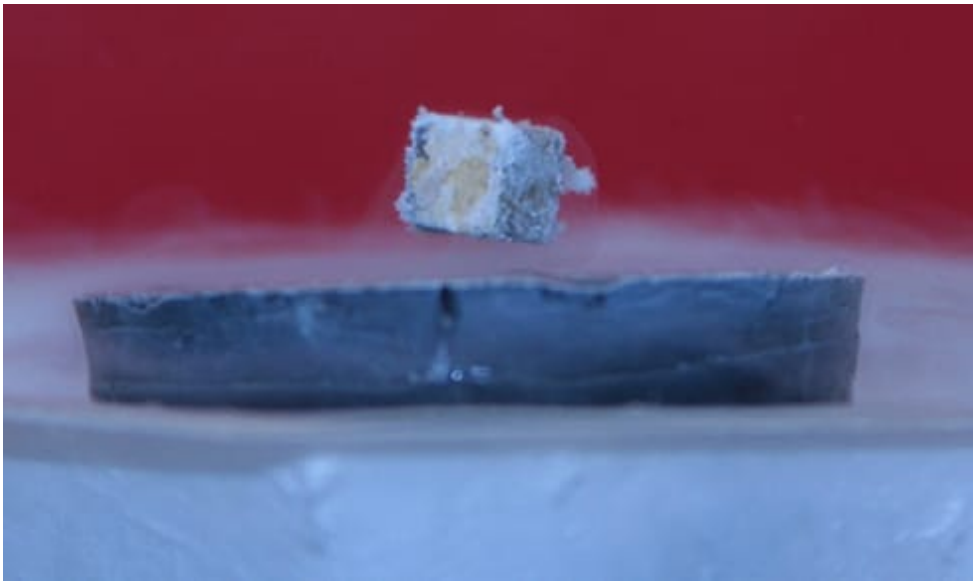


Figure 12.7.1: A small, strong magnet levitates over a superconductor cooled to liquid nitrogen temperature. The magnet levitates because the superconductor excludes magnetic fields.

Table 12.7.1 shows a select list of elements, compounds, and high-temperature superconductors, along with the critical

temperatures for which they become superconducting. Each section is sorted from the highest critical temperature to the lowest. Also listed is the critical magnetic field for some of the materials. This is the strength of the magnetic field that destroys superconductivity. Finally, the type of the superconductor is listed.

There are two types of superconductors. There are 30 pure metals that exhibit zero resistivity below their critical temperature and exhibit the Meissner effect, the property of excluding magnetic fields from the interior of the superconductor while the superconductor is at a temperature below the critical temperature. These metals are called Type I superconductors. The superconductivity exists only below their critical temperatures and below a critical magnetic field strength. Type I superconductors are well described by the BCS theory (described next). Type I superconductors have limited practical applications because the strength of the critical magnetic field needed to destroy the superconductivity is quite low.

Type II superconductors are found to have much higher critical magnetic fields and therefore can carry much higher current densities while remaining in the superconducting state. A collection of various ceramics containing barium-copper-oxide have much higher critical temperatures for the transition into a superconducting state. Superconducting materials that belong to this subcategory of the Type II superconductors are often categorized as high-temperature superconductors.

Introduction to BCS Theory

Type I superconductors, along with some Type II superconductors can be modeled using the **BCS theory**, proposed by John **Bardeen**, Leon **Cooper**, and Robert **Schrieffer**. Although the theory is beyond the scope of this chapter, a short summary of the theory is provided here. (More detail is provided in Condensed Matter Physics.) The theory considers pairs of electrons and how they are coupled together through lattice-vibration interactions. Through the interactions with the crystalline lattice, electrons near the Fermi energy level feel a small attractive force and form pairs (**Cooper pairs**), and the coupling is known as a phonon interaction. Single electrons are fermions, which are particles that obey the Pauli exclusion principle. The Pauli exclusion principle in quantum mechanics states that two identical fermions (particles with half-integer spin) cannot occupy the same quantum state simultaneously. Each electron has four quantum numbers $(n,\ \ell,\ m_\ell,\ m_s)$. The principal quantum number (n) describes the energy of the electron, the orbital angular momentum quantum number (ℓ) indicates the most probable distance from the nucleus, the magnetic quantum number m_ℓ describes the energy levels in the subshell, and the electron spin quantum number m_s describes the orientation of the spin of the electron, either up or down. As the material enters a superconducting state, pairs of electrons act more like bosons, which can condense into the same energy level and need not obey the Pauli exclusion principle. The electron pairs have a slightly lower energy and leave an energy gap above them on the order of 0.001 eV. This energy gap inhibits collision interactions that lead to ordinary resistivity. When the material is below the critical temperature, the thermal energy is less than the band gap and the material exhibits zero resistivity.

Table 12.7.1*: Superconductor Critical Temperatures*

Material	Symbol or Formula	Critical Temperature $T_c(K)$	Critical Magnetic Field $H_c(T)$	Type
Elements				
Lead	Pb	7.19	0.08	I
Lanthanum	La	(α) 4.90 - (β) 6.30		I
Tantalum	Ta	4.48	0.09	I
Mercury	Hg	(α) 4.15 - (β) 3.95	0.04	I
Tin	Sn	3.72	0.03	I
Indium	In	3.40	0.03	I
Thallium	Tl	2.39	0.03	I
Rhenium	Re	2.40	0.03	I
Thorium	Th	1.37	0.013	I
Protactinium	Pa	1.40		I
Aluminum	Al	1.20	0.01	I

Material	Symbol or Formula	Critical Temperature $T_c(K)$	Critical Magnetic Field $H_c(T)$	Type
Gallium	Ga	1.10	0.005	I
Zinc	Zn	0.86	0.014	I
Titanium	Ti	0.39	0.01	I
Uranium	U	(α) 0.68 - (β) 1.80		I
Cadmium	Cd	11.4	4.00	I
Compounds				
Niobium-germanium	Nb_3Ge	23.20	37.00	II
Niobium-tin	Nb_3Sn	18.30	30.00	II
Niobium-nitrite	NbN	16.00		II
Niobium-titanium	NbTi	10.00	15.00	II
High-Temperature Oxides				
	$HgBa_2CaCu_2O_8$	134.00		II
	$Ti_2Ba_2Ca_2Cu_3O_{10}$	125.00		II
	$YBa_2Cu_3O_7$	92.00	120.00	II

Applications of Superconductors

Superconductors can be used to make superconducting magnets. These magnets are 10 times stronger than the strongest electromagnets. These magnets are currently in use in magnetic resonance imaging (MRI), which produces high-quality images of the body interior without dangerous radiation.

Another interesting application of superconductivity is the **SQUID** (superconducting quantum interference device). A SQUID is a very sensitive magnetometer used to measure extremely subtle magnetic fields. The operation of the SQUID is based on superconducting loops containing Josephson junctions. A **Josephson junction** is the result of a theoretical prediction made by B. D. Josephson in an article published in 1962. In the article, Josephson described how a supercurrent can flow between two pieces of superconductor separated by a thin layer of insulator. This phenomenon is now called the **Josephson effect**. The SQUID consists of a superconducting current loop containing two Josephson junctions, as shown in Figure 12.7.3. When the loop is placed in even a very weak magnetic field, there is an interference effect that depends on the strength of the magnetic field.

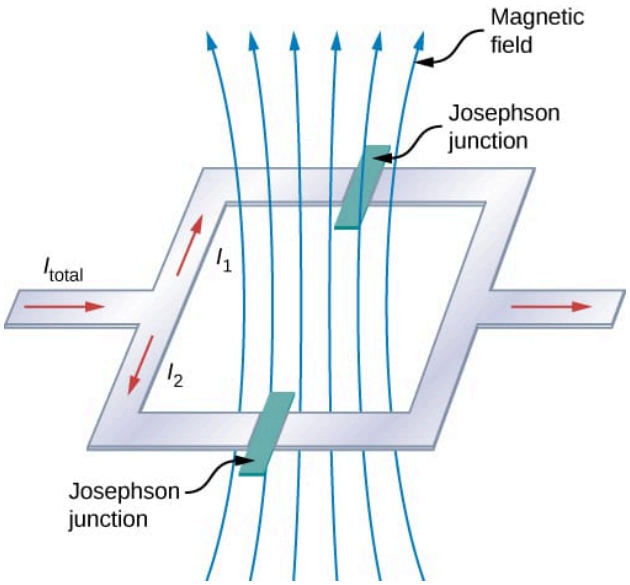


Figure 12.7.3: The SQUID (superconducting quantum interference device) uses a superconducting current loop and two Josephson junctions to detect magnetic fields as low as 10^{-14} (Earth's magnet field is on the order of $0.3 \times 10^{-5} T$).

Superconductivity is a fascinating and useful phenomenon. At critical temperatures near the boiling point of liquid nitrogen, superconductivity has special applications in MRIs, particle accelerators, and high-speed trains. Will we reach a state where we can have materials enter the superconducting phase at near room temperatures? It seems a long way off, but if scientists in 1911 were asked if we would reach liquid-nitrogen temperatures with a ceramic, they might have thought it implausible.

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12.8: Current and Resistance (Summary)

Key Terms

ampere (amp)	SI unit for current; $1\ \text{A} = 1\ \text{C/s}$
circuit	complete path that an electrical current travels along
conventional current	current that flows through a circuit from the positive terminal of a battery through the circuit to the negative terminal of the battery
critical temperature	temperature at which a material reaches superconductivity
current density	flow of charge through a cross-sectional area divided by the area
diode	nonohmic circuit device that allows current flow in only one direction
drift velocity	velocity of a charge as it moves nearly randomly through a conductor, experiencing multiple collisions, averaged over a length of a conductor, whose magnitude is the length of conductor traveled divided by the time it takes for the charges to travel the length
electrical conductivity	measure of a material's ability to conduct or transmit electricity
electrical current	rate at which charge flows, $I = \frac{dQ}{dt}$
electrical power	time rate of change of energy in an electric circuit
Josephson junction	junction of two pieces of superconducting material separated by a thin layer of insulating material, which can carry a supercurrent
Meissner effect	phenomenon that occurs in a superconducting material where all magnetic fields are expelled
nonohmic	type of a material for which Ohm's law is not valid
ohm	(Ω) unit of electrical resistance, $1\ \Omega = 1\ \text{V/A}$
ohmic	type of a material for which Ohm's law is valid, that is, the voltage drop across the device is equal to the current times the resistance
Ohm's law	empirical relation stating that the current I is proportional to the potential difference V; it is often written as $V = IR$, where R is the resistance
resistance	electric property that impedes current; for ohmic materials, it is the ratio of voltage to current, $R = V/I$
resistivity	intrinsic property of a material, independent of its shape or size, directly proportional to the resistance, denoted by ρ
schematic	graphical representation of a circuit using standardized symbols for components and solid lines for the wire connecting the components
SQUID	(Superconducting Quantum Interference Device) device that is a very sensitive magnetometer, used to measure extremely subtle magnetic fields
superconductivity	phenomenon that occurs in some materials where the resistance goes to exactly zero and all magnetic fields are expelled, which occurs dramatically at some low critical temperature (T_C)

Key Equations

Average electrical current	$I_{ave} = \frac{\Delta Q}{\Delta t}$

Definition of an ampere	$1A = 1C/s$
Electrical current	$I = \frac{dQ}{dt}$
Drift velocity	$v_d = \frac{I}{nqA}$
Current density	$I = \iint_{area} \vec{J} \cdot d\vec{A}$
Resistivity	$\rho = \frac{E}{J}$
Common expression of Ohm's law	$V = IR$
Resistivity as a function of temperature	$\rho = \rho_0[1 + \alpha(T - T_0)]$
Definition of resistance	$R \equiv \frac{V}{I}$
Resistance of a cylinder of material	$R = \rho\frac{L}{A}$
Temperature dependence of resistance	$R = R_0(1 + \alpha\Delta T)$
Electric power	$P = IV$
Power dissipated by a resistor	$P = I^2R = \frac{V^2}{R}$

Summary

9.2 Electrical Current

- The average electrical current I_{ave} is the rate at which charge flows, given by $I_{ave} = \frac{\Delta Q}{\Delta t}$, where ΔQ is the amount of charge passing through an area in time Δt.
- The instantaneous electrical current, or simply the current I, is the rate at which charge flows. Taking the limit as the change in time approaches zero, we have $I = \frac{dQ}{dt}$, where $\frac{dQ}{dt}$ is the time derivative of the charge.
- The direction of conventional current is taken as the direction in which positive charge moves. In a simple direct-current (DC) circuit, this will be from the positive terminal of the battery to the negative terminal.
- The SI unit for current is the ampere, or simply the amp (A), where $1\ \mathrm{A} = 1\mathrm{C}/\ mathrms$.
- Current consists of the flow of free charges, such as electrons, protons, and ions.

9.3 Model of Conduction in Metals

- The current through a conductor depends mainly on the motion of free electrons.
- When an electrical field is applied to a conductor, the free electrons in a conductor do not move through a conductor at a constant speed and direction; instead, the motion is almost random due to collisions with atoms and other free electrons.
- Even though the electrons move in a nearly random fashion, when an electrical field is applied to the conductor, the overall velocity of the electrons can be defined in terms of a drift velocity.
- The current density is a vector quantity defined as the current through an infinitesimal area divided by the area.
- The current can be found from the current density, $I = \iint_{area} \vec{J} \cdot d\vec{A}$.
- An incandescent light bulb is a filament of wire enclosed in a glass bulb that is partially evacuated. Current runs through the filament, where the electrical energy is converted to light and heat.

9.4 Resistivity and Resistance

- Resistance has units of ohms (Ω), related to volts and amperes by $1\Omega = 1V/A$.
- The resistance R of a cylinder of length L and cross-sectional area A is $R = \frac{\rho L}{A}$, where ρ is the resistivity of the material.

- Values of ρ in Table 9.1 show that materials fall into three groups—conductors, semiconductors, and insulators.
- Temperature affects resistivity; for relatively small temperature changes ΔT, resistivity is $\rho = \rho_0(1 + \alpha\Delta T)$, where ρ_0 is the original resistivity and α is the temperature coefficient of resistivity.
- The resistance R of an object also varies with temperature: $R = R_0(1 + \alpha\Delta T)$, where R_0 is the original resistance, and R is the resistance after the temperature change.

9.5 Ohm's Law

- Ohm's law is an empirical relationship for current, voltage, and resistance for some common types of circuit elements, including resistors. It does not apply to other devices, such as diodes.
- One statement of Ohm's law gives the relationship among current I, voltage V, and resistance R in a simple circuit as $V = IR$.
- Another statement of Ohm's law, on a microscopic level, is $J = \sigma E$.

9.6 Electrical Energy and Power

- Electric power is the rate at which electric energy is supplied to a circuit or consumed by a load.
- Power dissipated by a resistor depends on the square of the current through the resistor and is equal to $P = I^2R = \dfrac{V^2}{R}$.
- The SI unit for electric power is the watt and the SI unit for electric energy is the joule. Another common unit for electric energy, used by power companies, is the kilowatt-hour (kW · h).
- The total energy used over a time interval can be found by $E = \int Pdt$.

9.7 Superconductors

- Superconductivity is a phenomenon that occurs in some materials when cooled to very low critical temperatures, resulting in a resistance of exactly zero and the expulsion of all magnetic fields.
- Materials that are normally good conductors (such as copper, gold, and silver) do not experience superconductivity.
- Superconductivity was first observed in mercury by Heike Kamerlingh Onnes in 1911. In 1986, Dr. Ching Wu Chu of Houston University fabricated a brittle, ceramic compound with a critical temperature close to the temperature of liquid nitrogen.
- Superconductivity can be used in the manufacture of superconducting magnets for use in MRIs and high-speed, levitated trains.

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12.9: Current and Resistance (Exercises)

Conceptual Questions

9.2 Electrical Current

1. Can a wire carry a current and still be neutral—that is, have a total charge of zero? Explain.

2. Car batteries are rated in ampere-hours (**A · h**). To what physical quantity do ampere-hours correspond (voltage, current, charge, energy, power,...)?

3. When working with high-power electric circuits, it is advised that whenever possible, you work “one-handed” or “keep one hand in your pocket.” Why is this a sensible suggestion?

9.3 Model of Conduction in Metals

4. Incandescent light bulbs are being replaced with more efficient LED and CFL light bulbs. Is there any obvious evidence that incandescent light bulbs might not be that energy efficient? Is energy converted into anything but visible light?

5. It was stated that the motion of an electron appears nearly random when an electrical field is applied to the conductor. What makes the motion nearly random and differentiates it from the random motion of molecules in a gas?

6. Electric circuits are sometimes explained using a conceptual model of water flowing through a pipe. In this conceptual model, the voltage source is represented as a pump that pumps water through pipes and the pipes connect components in the circuit. Is a conceptual model of water flowing through a pipe an adequate representation of the circuit? How are electrons and wires similar to water molecules and pipes? How are they different?

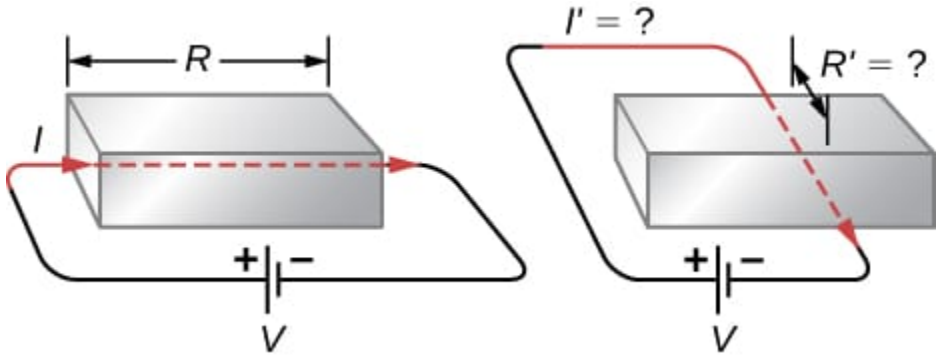
7. An incandescent light bulb is partially evacuated. Why do you suppose that is?

9.4 Resistivity and Resistance

8. The IR drop across a resistor means that there is a change in potential or voltage across the resistor. Is there any change in current as it passes through a resistor? Explain.

9. Do impurities in semiconducting materials listed in Table 9.1 supply free charges? (Hint: Examine the range of resistivity for each and determine whether the pure semiconductor has the higher or lower conductivity.)

10. Does the resistance of an object depend on the path current takes through it? Consider, for example, a rectangular bar—is its resistance the same along its length as across its width?

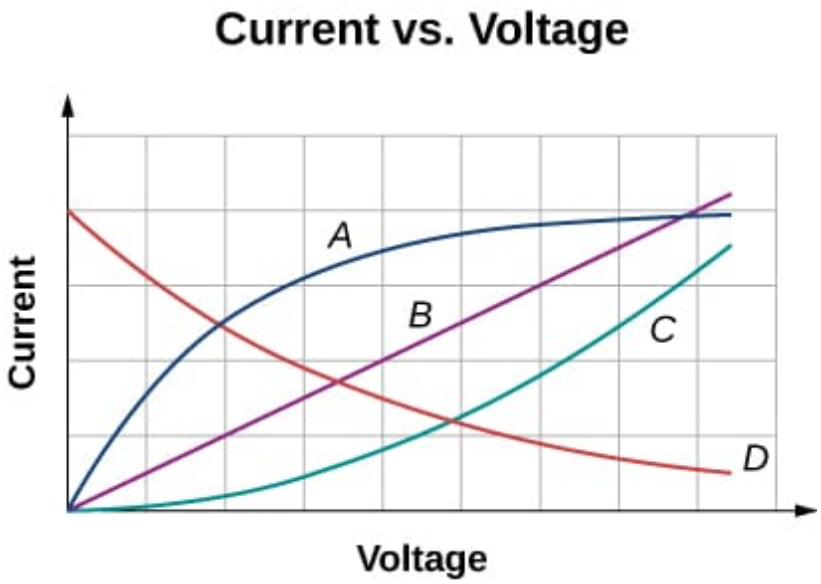


11. If aluminum and copper wires of the same length have the same resistance, which has the larger diameter? Why?

9.5 Ohm's Law

12. In Determining Field from Potential, resistance was defined as $R \equiv \frac{V}{I}$. In this section, we presented Ohm's law, which is commonly expressed as **V=IR.** The equations look exactly alike. What is the difference between Ohm's law and the definition of resistance?

13. Shown below are the results of an experiment where four devices were connected across a variable voltage source. The voltage is increased and the current is measured. Which device, if any, is an ohmic device?



14. The current I is measured through a sample of an ohmic material as a voltage V is applied. (a) What is the current when the voltage is doubled to 2V (assume the change in temperature of the material is negligible)? (b) What is the voltage applied if the current measured is 0.2I (assume the change in temperature of the material is negligible)? What will happen to the current in the material if the voltage remains constant, but the temperature of the material increases significantly?

9.6 Electrical Energy and Power

15. Common household appliances are rated at 110 V, but power companies deliver voltage in the kilovolt range and then step the voltage down using transformers to 110 V to be used in homes. You will learn in later chapters that transformers consist of many turns of wire, which warm up as current flows through them, wasting some of the energy that is given off as heat. This sounds inefficient. Why do the power companies transport electric power using this method?

16. Your electric bill gives your consumption in units of kilowatt-hour (kW · h). Does this unit represent the amount of charge, current, voltage, power, or energy you buy?

17. Resistors are commonly rated at $\frac{1}{8}W, \frac{1}{4}W, \frac{1}{2}W$, 1 W and 2 W for use in electrical circuits. If a current of $I = 2.00\ \text{A}$ is accidentally passed through a $R = 1.00\ \Omega$ resistor rated at 1 W, what would be the most probable outcome? Is there anything that can be done to prevent such an accident?

18. An immersion heater is a small appliance used to heat a cup of water for tea by passing current through a resistor. If the voltage applied to the appliance is doubled, will the time required to heat the water change? By how much? Is this a good idea?

9.7 Superconductors

19. What requirement for superconductivity makes current superconducting devices expensive to operate?

20. Name two applications for superconductivity listed in this section and explain how superconductivity is used in the

application. Can you think of a use for superconductivity that is not listed?

Problems

9.2 Electrical Current

21. A Van de Graaff generator is one of the original particle accelerators and can be used to accelerate charged particles like protons or electrons. You may have seen it used to make human hair stand on end or produce large sparks. One application of the Van de Graaff generator is to create X-rays by bombarding a hard metal target with the beam. Consider a beam of protons at 1.00 keV and a current of 5.00 mA produced by the generator.

(a) What is the speed of the protons?

(b) How many protons are produced each second?

22. A cathode ray tube (CRT) is a device that produces a focused beam of electrons in a vacuum. The electrons strike a phosphor-coated glass screen at the end of the tube, which produces a bright spot of light. The position of the bright spot of light on the screen can be adjusted by deflecting the electrons with electrical fields, magnetic fields, or both. Although the CRT tube was once commonly found in televisions, computer displays, and oscilloscopes, newer appliances use a liquid crystal display (LCD) or plasma screen. You still may come across a CRT in your study of science. Consider a CRT with an electron beam average current of 25.00μA25.00μA. How many electrons strike the screen every minute?

23. How many electrons flow through a point in a wire in 3.00 s if there is a constant current of **I=4.00A**?

24. A conductor carries a current that is decreasing exponentially with time. The current is modeled as $I = I_0 e^{-t/\tau}$, where $I_0 = 3.00A$ is the current at time **t=0.00s** and **τ=0.50s** is the time constant. How much charge flows through the conductor between **t=0.00s** and **t=3τ**?

25. The quantity of charge through a conductor is modeled as $Q = 4.00\frac{C}{s^4}t^4 - 1.00\frac{C}{s}t + 6.00mC$. What is the current at time **t=3.00s**?

26. The current through a conductor is modeled as $I(t) = I_m sin(2\pi[60Hz]t)$. Write an equation for the charge as a function of time.

27. The charge on a capacitor in a circuit is modeled as $Q(t) = Q_{max} cos(\omega t + \phi)$. What is the current through the circuit as a function of time?

9.3 Model of Conduction in Metals

28. An aluminum wire 1.628 mm in diameter (14-gauge) carries a current of 3.00 amps.

(a) What is the absolute value of the charge density in the wire?

(b) What is the drift velocity of the electrons?

(c) What would be the drift velocity if the same gauge copper were used instead of aluminum? The density of copper is $8.96 g/cm^3$ and the density of aluminum is $2.70 g/cm^3$. The molar mass of aluminum is 26.98 g/mol and the molar mass of copper is 63.5 g/mol. Assume each atom of metal contributes one free electron.

29. The current of an electron beam has a measured current of **I=50.00μA** with a radius of 1.00 mm. What is the magnitude of the current density of the beam?

30. A high-energy proton accelerator produces a proton beam with a radius of **r=0.90mm**. The beam current is **I=9.00μA** and is constant. The charge density of the beam is $n = 6.00 \times 10^{11}$ protons per cubic meter.

(a) What is the current density of the beam?

(b) What is the drift velocity of the beam?

(c) How much time does it take for 1.00×10^{10} protons to be emitted by the accelerator?

31. Consider a wire of a circular cross-section with a radius of **R=3.00mm**. The magnitude of the current density is modeled as $J = cr^2 = 5.00 \times 10^6 \frac{A}{m^4} r^2$. What is the current through the inner section of the wire from the center to **r=0.5R**?

32. A cylindrical wire has a current density from the center of the wire's cross section as $J(r) = Cr^2$ where r is in meters, J is in amps per square meter, and $C = 10^3$ A/m^4. This current density continues to the end of the wire at a radius of 1.0 mm. Calculate the current just outside of this wire.

33. The current supplied to an air conditioner unit is 4.00 amps. The air conditioner is wired using a 10-gauge (diameter 2.588 mm) wire. The charge density is $n = 8.48 \times 10^{28} \frac{electrons}{m^3}$. Find the magnitude of

(a) current density and

(b) the drift velocity.

9.4 Resistivity and Resistance

34. What current flows through the bulb of a 3.00-V flashlight when its hot resistance is **3.60Ω**?

35. Calculate the effective resistance of a pocket calculator that has a 1.35-V battery and through which 0.200 mA flows.

36. How many volts are supplied to operate an indicator light on a DVD player that has a resistance of **140Ω**, given that 25.0 mA passes through it?

37. What is the resistance of a 20.0-m-long piece of 12-gauge copper wire having a 2.053-mm diameter?

38. The diameter of 0-gauge copper wire is 8.252 mm. Find the resistance of a 1.00-km length of such wire used for power transmission.

39. If the 0.100-mm-diameter tungsten filament in a light bulb is to have a resistance of **0.200Ω** at **20.0°C**, how long should it be?

40. A lead rod has a length of 30.00 cm and a resistance of **5.00μΩ**. What is the radius of the rod?

41. Find the ratio of the diameter of aluminum to copper wire, if they have the same resistance per unit length (as they might in household wiring).

42. What current flows through a 2.54-cm-diameter rod of pure silicon that is 20.0 cm long, when 1.00×10^3 is applied to it? (Such a rod may be used to make nuclear-particle detectors, for example.)

43. (a) To what temperature must you raise a copper wire, originally at **20.0°C**, to double its resistance, neglecting any changes in dimensions? (b) Does this happen in household wiring under ordinary circumstances?

44. A resistor made of nichrome wire is used in an application where its resistance cannot change more than 1.00% from its value at **20.0°C**. Over what temperature range can it be used?

45. Of what material is a resistor made if its resistance is 40.0% greater at **100.0°C** than at **20.0°C**?

46. An electronic device designed to operate at any temperature in the range from **–10.0°C** to **55.0°C** contains pure carbon resistors. By what factor does their resistance increase over this range?**47.**

(a) Of what material is a wire made, if it is 25.0 m long with a diameter of 0.100 mm and has a resistance of **77.7Ω** at **20.0°C**? (b) What is its resistance at **150.0°C**?

48. Assuming a constant temperature coefficient of resistivity, what is the maximum percent decrease in the resistance of a constantan wire starting at **20.0°C**?

49. A copper wire has a resistance of **0.500Ω** at **20.0°C**, and an iron wire has a resistance of **0.525Ω** at the same

temperature. At what temperature are their resistances equal?

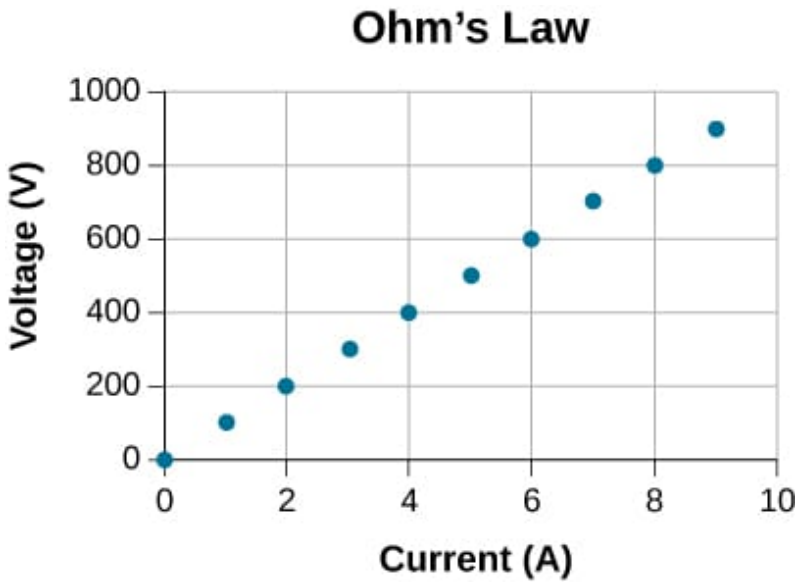
9.5 Ohm's Law

50. A **2.2-kΩ** resistor is connected across a D cell battery (1.5 V). What is the current through the resistor?

51. A resistor rated at **250kΩ** is connected across two D cell batteries (each 1.50 V) in series, with a total voltage of 3.00 V. The manufacturer advertises that their resistors are within 5% of the rated value. What are the possible minimum current and maximum current through the resistor?

52. A resistor is connected in series with a power supply of 20.00 V. The current measure is 0.50 A. What is the resistance of the resistor?

53. A resistor is placed in a circuit with an adjustable voltage source. The voltage across and the current through the resistor and the measurements are shown below. Estimate the resistance of the resistor.



54. The following table show the measurements of a current through and the voltage across a sample of material. Plot the data, and assuming the object is an ohmic device, estimate the resistance.

Table: Measurements of current through and the voltage across a sample of material

I(A)	V(V)
0	3
2	23
4	39
6	58
8	77
10	100
12	119
14	142
16	162

9.6 Electrical Energy and Power

55. A **20.00-V** battery is used to supply current to a **10-kΩ** resistor. Assume the voltage drop across any wires used for connections is negligible.

(a) What is the current through the resistor?

(b) What is the power dissipated by the resistor?

(c) What is the power input from the battery, assuming all the electrical power is dissipated by the resistor?

(d) What happens to the energy dissipated by the resistor?

56. What is the maximum voltage that can be applied to a **20-kΩ** resistor rated at $\frac{1}{4}W$?

57. A heater is being designed that uses a coil of 14-gauge nichrome wire to generate 300 W using a voltage of **V=110V.** How long should the engineer make the wire?

58. An alternative to CFL bulbs and incandescent bulbs are light-emitting diode (LED) bulbs. A 100-W incandescent bulb can be replaced by a 16-W LED bulb. Both produce 1600 lumens of light. Assuming the cost of electricity is $0.10 per kilowatt-hour, how much does it cost to run the bulb for one year if it runs for four hours a day?

59. The power dissipated by a resistor with a resistance of **R=100Ω** is **P=2.0W**. What are the current through and the voltage drop across the resistor?

60. Running late to catch a plane, a driver accidentally leaves the headlights on after parking the car in the airport parking lot. During takeoff, the driver realizes the mistake. Having just replaced the battery, the driver knows that the battery is a 12-V automobile battery, rated at 100 **A · h**. The driver, knowing there is nothing that can be done, estimates how long the lights will shine, assuming there are two 12-V headlights, each rated at 40 W. What did the driver conclude?

61. A physics student has a single-occupancy dorm room. The student has a small refrigerator that runs with a current of 3.00 A and a voltage of 110 V, a lamp that contains a 100-W bulb, an overhead light with a 60-W bulb, and various other small devices adding up to 3.00 W.

(a) Assuming the power plant that supplies 110 V electricity to the dorm is 10 km away and the two aluminum transmission cables use 0-gauge wire with a diameter of 8.252 mm, estimate the percentage of the total power supplied by the power company that is lost in the transmission.

(b) What would be the result is the power company delivered the electric power at 110 kV?

62. A 0.50-W, **220-Ω** resistor carries the maximum current possible without damaging the resistor. If the current were reduced to half the value, what would be the power consumed?

9.7 Superconductors

63. Consider a power plant is located 60 km away from a residential area uses 0-gauge $(A = 42.40mm^2)$ wire of copper to transmit power at a current of **I=100.00A**. How much more power is dissipated in the copper wires than it would be in superconducting wires?

64. A wire is drawn through a die, stretching it to four times its original length. By what factor does its resistance increase?

65. Digital medical thermometers determine temperature by measuring the resistance of a semiconductor device called a thermistor (which has **α=−0.06/°C**) when it is at the same temperature as the patient. What is a patient's temperature if the thermistor's resistance at that temperature is 82.0% of its value at **37°C** (normal body temperature)?

66. Electrical power generators are sometimes "load tested" by passing current through a large vat of water. A similar method can be used to test the heat output of a resistor. A **R=30Ω** resistor is connected to a 9.0-V battery and the resistor leads are waterproofed and the resistor is placed in 1.0 kg of room temperature water (**T=20°C**). Current runs through the resistor for 20 minutes. Assuming all the electrical energy dissipated by the resistor is converted to heat, what is the final

temperature of the water?

67. A 12-guage gold wire has a length of 1 meter.

(a) What would be the length of a silver 12-gauge wire with the same resistance?

(b) What are their respective resistances at the temperature of boiling water?

68. What is the change in temperature required to decrease the resistance for a carbon resistor by 10%?

Additional Problems

69. A coaxial cable consists of an inner conductor with radius $r_i = 0.25cm$ and an outer radius of $r_o = 0.5cm$ and has a length of 10 meters. Plastic, with a resistivity of $\rho = 2.00 \times 10^{13}\Omega \cdot m$, separates the two conductors. What is the resistance of the cable?

70. A 10.00-meter long wire cable that is made of copper has a resistance of 0.051 ohms.

(a) What is the weight if the wire was made of copper?

(b) What is the weight of a 10.00-meter-long wire of the same gauge made of aluminum?

(c)What is the resistance of the aluminum wire? The density of copper is $8960kg/m^3$ and the density of aluminum is $2760kg/m^3$.

71. A nichrome rod that is 3.00 mm long with a cross-sectional area of $1.00mm^2$ is used for a digital thermometer.

(a) What is the resistance at room temperature?

(b) What is the resistance at body temperature?

72. The temperature in Philadelphia, PA can vary between **68.00°F** and **100.00°F** in one summer day. By what percentage will an aluminum wire's resistance change during the day?

73. When 100.0 V is applied across a 5-gauge (diameter 4.621 mm) wire that is 10 m long, the magnitude of the current density is $2.0 \times 10^8 A/m^2$. What is the resistivity of the wire?

74. A wire with a resistance of **5.0Ω** is drawn out through a die so that its new length is twice times its original length. Find the resistance of the longer wire. You may assume that the resistivity and density of the material are unchanged.

75. What is the resistivity of a wire of 5-gauge wire ($A = 16.8 \times 10^{-6}m^2$), 5.00 m length, and **5.10mΩ** resistance?

76. Coils are often used in electrical and electronic circuits. Consider a coil which is formed by winding 1000 turns of insulated 20-gauge copper wire (area $0.52mm^2$) in a single layer on a cylindrical non-conducting core of radius 2.0 mm. What is the resistance of the coil? Neglect the thickness of the insulation.

77. Currents of approximately 0.06 A can be potentially fatal. Currents in that range can make the heart fibrillate (beat in an uncontrolled manner). The resistance of a dry human body can be approximately **100kΩ**.

(a) What voltage can cause 0.2 A through a dry human body?

(b) When a human body is wet, the resistance can fall to **100Ω**. What voltage can cause harm to a wet body?

78. A 20.00-ohm, 5.00-watt resistor is placed in series with a power supply.

(a) What is the maximum voltage that can be applied to the resistor without harming the resistor?

(b) What would be the current through the resistor?

79. A battery with an emf of 24.00 V delivers a constant current of 2.00 mA to an appliance. How much work does the

battery do in three minutes?

80. A 12.00-V battery has an internal resistance of a tenth of an ohm.

(a) What is the current if the battery terminals are momentarily shorted together?

(b) What is the terminal voltage if the battery delivers 0.25 amps to a circuit?

Challenge Problems

81. A 10-gauge copper wire has a cross-sectional area $A = 5.26mm^2$ and carries a current of **I=5.00A**. The density of copper is $\rho = 89.50g/cm^3$. One mole of copper atoms ($6.02 \times 10^{23} atoms$) has a mass of approximately 63.50 g. What is the magnitude of the drift velocity of the electrons, assuming that each copper atom contributes one free electron to the current?

82. The current through a 12-gauge wire is given as **I(t)=(5.00A)sin(2π60Hzt)**. What is the current density at time 15.00 ms?

83. A particle accelerator produces a beam with a radius of 1.25 mm with a current of 2.00 mA. Each proton has a kinetic energy of 10.00 MeV.

(a) What is the velocity of the protons?

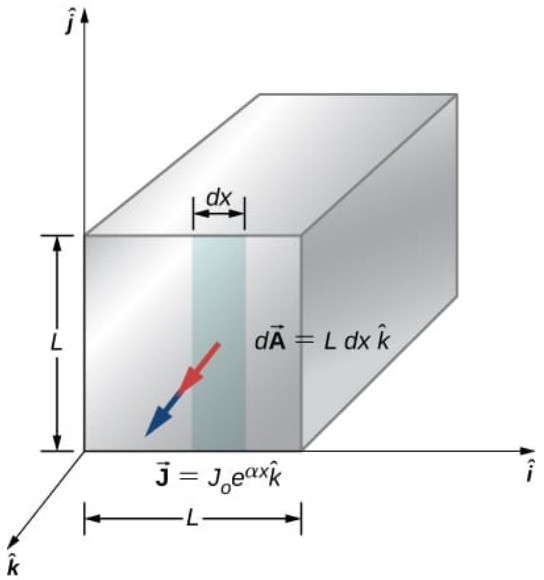
(b) What is the number (n) of protons per unit volume?

(c) How many electrons pass a cross sectional area each second?

84. In this chapter, most examples and problems involved direct current (DC). DC circuits have the current flowing in one direction, from positive to negative. When the current was changing, it was changed linearly from $I = -I_{max}$ to $I = +I_{max}$ and the voltage changed linearly from $V = -V_{max}$ to $V = +V_{max}$, where $V_{max} = I_{max}R$. Suppose a voltage source is placed in series with a resistor of **R=10Ω** that supplied a current that alternated as a sine wave, for example, $I(t) = (3.00A)sin(\frac{2\pi}{4.00s}t)$. (a) What would a graph of the voltage drop across the resistor **V(t)** versus time look like? (b) What would a plot of **V(t)** versus **I(t)** for one period look like? (Hint: If you are not sure, try plotting **V(t)** versus **I(t)** using a spreadsheet.)

85. A current of **I=25A** is drawn from a 100-V battery for 30 seconds. By how much is the chemical energy reduced?

86. Consider a square rod of material with sides of length **L=3.00cm** with a current density of $J = J_0 e^{\alpha x}\hat{k} = (0.35\frac{A}{m^2})^{e(2.1\times10^{-3}m^{-1})x}\hat{k}$ as shown below. Find the current that passes through the face of



87. A resistor of an unknown resistance is placed in an insulated container filled with 0.75 kg of water. A voltage source is connected in series with the resistor and a current of 1.2 amps flows through the resistor for 10 minutes. During this time, the temperature of the water is measured and the temperature change during this time is **ΔT=10.00°C**.

(a) What is the resistance of the resistor?

(b) What is the voltage supplied by the power supply?

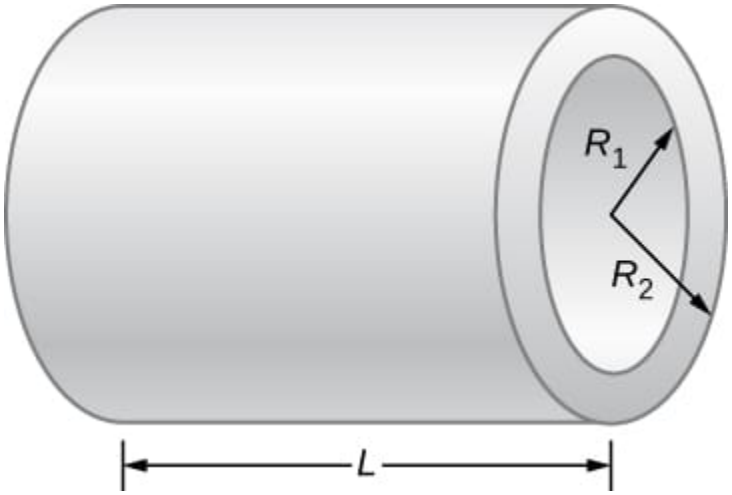
88. The charge that flows through a point in a wire as a function of time is modeled as $q(t) = q_0 e^{-t/T} = 10.0Ce^{-t/5s}$.

(a) What is the initial current through the wire at time **t=0.00s**?

(b) Find the current at time $t = \frac{1}{2}T$.

(c) At what time t will the current be reduced by one-half $I = \frac{1}{2}I_0$?

89. Consider a resistor made from a hollow cylinder of carbon as shown below. The inner radius of the cylinder is Ri=0.20mmRi=0.20mm and the outer radius is $R_0 = 0.30mm$. The length of the resistor is **L=0.90mm**. The resistivity of the carbon is $\rho = 3.5 \times 10^{-5}\Omega \cdot m$. (a) Prove that the resistance perpendicular from the axis is $R = \frac{\rho}{2\pi L} ln(\frac{R_0}{R_i})$. (b) What is the resistance?



90. What is the current through a cylindrical wire of radius **R=0.1mm** if the current density is $J = \frac{J_0}{R}r$, where $J_0 = 32000\frac{A}{m^2}$?

91. A student uses a 100.00-W, 115.00-V radiant heater to heat the student's dorm room, during the hours between sunset and sunrise, 6:00 p.m. to 7:00 a.m.

(a) What current does the heater operate at?

(b) How many electrons move through the heater?

(c) What is the resistance of the heater?

(d) How much heat was added to the dorm room?

92. A 12-V car battery is used to power a 20.00-W, 12.00-V lamp during the physics club camping trip/star party. The cable to the lamp is 2.00 meters long, 14-gauge copper wire with a charge density of $n = 9.50 \times 10^{28}m^{-3}$.

(a) What is the current draw by the lamp?

(b) How long would it take an electron to get from the battery to the lamp?

93. A physics student uses a 115.00-V immersion heater to heat 400.00 grams (almost two cups) of water for herbal tea. During the two minutes it takes the water to heat, the physics student becomes bored and decides to figure out the

resistance of the heater. The student starts with the assumption that the water is initially at the temperature of the room $T_i = 25.00\ °C$ and reaches $T_f = 100.00\ °C$. The specific heat of the water is $c = 4180 \frac{J}{kg}$. What is the resistance of the heater?

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12.10: Current and Resistance (Answers)

Check Your Understanding

9.1. The time for 1.00 C of charge to flow would be $\Delta t = \frac{\Delta Q}{I} = \frac{1.00C}{0.300 \times 10^{-3} C/s} = 3.33 \times 10^{3} s$, slightly less than an hour. This is quite different from the 5.55 ms for the truck battery. The calculator takes a very small amount of energy to operate, unlike the truck's starter motor. There are several reasons that vehicles use batteries and not solar cells. Aside from the obvious fact that a light source to run the solar cells for a car or truck is not always available, the large amount of current needed to start the engine cannot easily be supplied by present-day solar cells. Solar cells can possibly be used to charge the batteries. Charging the battery requires a small amount of energy when compared to the energy required to run the engine and the other accessories such as the heater and air conditioner. Present day solar-powered cars are powered by solar panels, which may power an electric motor, instead of an internal combustion engine.

9.2. The total current needed by all the appliances in the living room (a few lamps, a television, and your laptop) draw less current and require less power than the refrigerator.

9.3. The diameter of the 14-gauge wire is smaller than the diameter of the 12-gauge wire. Since the drift velocity is inversely proportional to the cross-sectional area, the drift velocity in the 14-gauge wire is larger than the drift velocity in the 12-gauge wire carrying the same current. The number of electrons per cubic meter will remain constant.

9.4. The current density in a conducting wire increases due to an increase in current. The drift velocity is inversely proportional to the current ($v_d = \frac{I}{nqA}$), so the drift velocity would decrease.

9.5. Silver, gold, and aluminum are all used for making wires. All four materials have a high conductivity, silver having the highest. All four can easily be drawn into wires and have a high tensile strength, though not as high as copper. The obvious disadvantage of gold and silver is the cost, but silver and gold wires are used for special applications, such as speaker wires. Gold does not oxidize, making better connections between components. Aluminum wires do have their drawbacks. Aluminum has a higher resistivity than copper, so a larger diameter is needed to match the resistance per length of copper wires, but aluminum is cheaper than copper, so this is not a major drawback. Aluminum wires do not have as high of a ductility and tensile strength as copper, but the ductility and tensile strength is within acceptable levels. There are a few concerns that must be addressed in using aluminum and care must be used when making connections. Aluminum has a higher rate of thermal expansion than copper, which can lead to loose connections and a possible fire hazard. The oxidation of aluminum does not conduct and can cause problems. Special techniques must be used when using aluminum wires and components, such as electrical outlets, must be designed to accept aluminum wires.

9.6. The foil pattern stretches as the backing stretches, and the foil tracks become longer and thinner. Since the resistance is calculated as $R = \rho \frac{L}{A}$, the resistance increases as the foil tracks are stretched. When the temperature changes, so does the resistivity of the foil tracks, changing the resistance. One way to combat this is to use two strain gauges, one used as a reference and the other used to measure the strain. The two strain gauges are kept at a constant temperature

9.7. The longer the length, the smaller the resistance. The greater the resistivity, the higher the resistance. The larger the difference between the outer radius and the inner radius, that is, the greater the ratio between the two, the greater the resistance. If you are attempting to maximize the resistance, the choice of the values for these variables will depend on the application. For example, if the cable must be flexible, the choice of materials may be limited.

9.8. Yes, Ohm's law is still valid. At every point in time the current is equal to I(t)=V(t)/RI(t)=V(t)/R, so the current is also a function of time, $I(t) = \frac{V_{max}}{R} sin(2\pi ft)$.

9.9. Even though electric motors are highly efficient 10–20% of the power consumed is wasted, not being used for doing useful work. Most of the 10–20% of the power lost is transferred into heat dissipated by the copper wires used to make the coils of the motor. This heat adds to the heat of the environment and adds to the demand on power plants providing the power. The demand on the power plant can lead to increased greenhouse gases, particularly if the power plant uses coal or

gas as fuel.

9.10. No, the efficiency is a very important consideration of the light bulbs, but there are many other considerations. As mentioned above, the cost of the bulbs and the life span of the bulbs are important considerations. For example, CFL bulbs contain mercury, a neurotoxin, and must be disposed of as hazardous waste. When replacing incandescent bulbs that are being controlled by a dimmer switch with LED, the dimmer switch may need to be replaced. The dimmer switches for LED lights are comparably priced to the incandescent light switches, but this is an initial cost which should be considered. The spectrum of light should also be considered, but there is a broad range of color temperatures available, so you should be able to find one that fits your needs. None of these considerations mentioned are meant to discourage the use of LED or CFL light bulbs, but they are considerations.

Conceptual Questions

1. If a wire is carrying a current, charges enter the wire from the voltage source's positive terminal and leave at the negative terminal, so the total charge remains zero while the current flows through it.

3. Using one hand will reduce the possibility of "completing the circuit" and having current run through your body, especially current running through your heart.

5. Even though the electrons collide with atoms and other electrons in the wire, they travel from the negative terminal to the positive terminal, so they drift in one direction. Gas molecules travel in completely random directions.

7. In the early years of light bulbs, the bulbs are partially evacuated to reduce the amount of heat conducted through the air to the glass envelope. Dissipating the heat would cool the filament, increasing the amount of energy needed to produce light from the filament. It also protects the glass from the heat produced from the hot filament. If the glass heats, it expands, and as it cools, it contacts. This expansion and contraction could cause the glass to become brittle and crack, reducing the life of the bulbs. Many bulbs are now partially filled with an inert gas. It is also useful to remove the oxygen to reduce the possibility of the filament actually burning. When the original filaments were replaced with more efficient tungsten filaments, atoms from the tungsten would evaporate off the filament at such high temperatures. The atoms collide with the atoms of the inert gas and land back on the filament.

9. In carbon, resistivity increases with the amount of impurities, meaning fewer free charges. In silicon and germanium, impurities decrease resistivity, meaning more free electrons.

11. Copper has a lower resistivity than aluminum, so if length is the same, copper must have the smaller diameter.

13. Device B shows a linear relationship and the device is ohmic.

15. Although the conductors have a low resistance, the lines from the power company can be kilometers long. Using a high voltage reduces the current that is required to supply the power demand and that reduces line losses.

17. The resistor would overheat, possibly to the point of causing the resistor to burn. Fuses are commonly added to circuits to prevent such accidents.

19. Very low temperatures necessitate refrigeration. Some materials require liquid nitrogen to cool them below their critical temperatures. Other materials may need liquid helium, which is even more costly.

Problems

21. a. $v = 4.38 \times 10^5 \frac{m}{s}$;

b. $\Delta q = 5.00 \times 10^{-3} C$,no. of protons=$3.13 \times 10^{16}$

23. $I = \frac{\Delta Q}{\Delta t}, \Delta Q = 12.00 C$, no. of electrons=$7.46 \times 10^{19}$

25. $I(t) = 0.016 \frac{C}{s^4} t^3 - 0.001 \frac{C}{s}$ $I(3.00s) = 0.431 A$

27. $I(t) = -I_{max} sin(\omega t + \phi)$

29. $|J| = 15.92 A/m^2$

31. $I = 3.98 \times 10^{-5} A$

33. a. $|J| = 7.60 \times 10^5 \frac{A}{m^2}$;

b. $v_d = 5.60 \times 10^{-5} \frac{m}{s}$

35. $R = 6.750 k\Omega$

37. $R = 0.10\Omega$

39. $R = \rho \frac{L}{A}$; $L = 3cm$

41. $\frac{R_{Al}/L_{Al}}{R_{Cu}/L_{Cu}} = \frac{\rho_{Al} \frac{1}{\pi(\frac{D_{Al}}{2})^2}}{\rho_{Cu} \frac{1}{\pi(\frac{D_{Cu}}{2})^2}} = \frac{\rho_{Al}}{\rho_{Cu}} (\frac{D_{Cu}}{D_{Al}})^2 = 1, \frac{D_{Al}}{D_{Cu}} = \sqrt{\frac{\rho_{AI}}{\rho_{Cu}}}$

43. a. $R = R_0(1 + \alpha \Delta T), 2 = 1 + \alpha \Delta T, \Delta T = 256.4\ ^\circ C, T = 276.4\ ^\circ C$;

b. Under normal conditions, no it should not occur.

45. $R = R_0(1 + \alpha \Delta T)$ $\alpha = 0.006\ ^\circ C^{-1}$, iron

47. a. $R = \rho \frac{L}{A}, \rho = 2.44 \times 10^-8 \Omega \cdot m$, gold; $R = \rho \frac{L}{A}(1 + \alpha \Delta T)$

b. $R = 2.44 \times 10^{-8} \Omega \cdot m (\frac{25m}{\pi(\frac{0.100 \times 10^{-3} m}{2})^2})(1 + 0.0034\ ^\circ C^{-1}(150\ ^\circ C - 20\ ^\circ C)) R = 112\Omega$

49. $R_{Fe} = 0.525\Omega, R_{Cu} = 0.500\Omega, \alpha_{Fe} = 0.0065\ ^\circ C^{-1}, \alpha_{Cu} = 0.0039\ ^\circ C^{-1}, R_{Fe} = R_{Cu}, R_{0Fe}(1 + \alpha_{Fe}(T - T_0)) = R_{0Cu}(1 + \alpha_{Cu}(T - T_0)), \frac{R_{0Fe}}{R_{0Cu}}(1 + \alpha_{Fe}(T - T_0)) = 1 + \alpha_{Cu}(T - T_0), T = 2.91\ ^\circ C$

51. $R_{min} = 2.375 \times 10^5 \Omega, I_{min} = 12.63 \mu A$

$R_{max} = 2.625 \times 10^5 \Omega, I_{max} = 11.43 \mu A$

53. $R = 100\Omega$

55. a. $I = 2mA$;

b. $P = 0.04mW$;

c. $P = 0.04mW$;

d. It is converted into heat.

57. $P = \frac{V^2}{R}, R = 40\Omega, A = 2.08 mm^2, \rho = 100 \times 10^{-8} \Omega \cdot m, R = \rho \frac{L}{A}, L = 83m$

59. $I = 0.14A, V = 14V$

61. a. $I \approx 3.00A + \frac{100W}{110V} + \frac{60W}{110V} + \frac{3.00W}{110V} = 4.48A$

$P = 493W$

$R = 9.91\Omega,$

$P_{loss} = 200.W$

b. $P = 493W$

$I = 0.0045A$

$R = 9.91\Omega$

$P_{loss} = 201\mu W$

63. $R_{copper} = 23.77\Omega\ P = 2.377 \times 10^5 W$

65. $R = R_0(1 + \alpha(T - T_0)$

$0.82R_0 = R_0(1 + \alpha(T - T_0)), 0.82 = 1 - 0.06(T - 37\ ^\circ C), T = 40\ ^\circ C$

67. a. $R_{Au} = R_{Ag}, \rho_{Au}\frac{L_{Au}}{A_{Au}} = \rho_{Ag}\frac{L_{Ag}}{A_{Ag}}, L_{Ag} = 1.53m;$

b. $R_{Au,20^\circ C} = 0.0074\Omega, R_{Au,100^\circ C} = 0.0094\Omega, R_{Ag,100^\circ C} = 0.0096\Omega$

Additional Problems

69. $dR = \frac{\rho}{2\pi rL}dr$

$R = \frac{\rho}{2\pi L}ln\frac{r_o}{r_i}$

$R = 2.21 \times 10^{11}\Omega$

71. a. $R_0 = 0.003\Omega;$

b. $T_c = 37.0\ ^\circ C\ R = 0.00302\Omega$

73. $\rho = 5.00 \times 10^{-8}\Omega \cdot m$

75. $\rho = 1.71 \times 10^{-8}\Omega \cdot m$

77. $a. V = 6000V;$

b. $V = 60V$

79. $P = \frac{W}{t}, W = 8.64J$

Challenge Problems

81. $V = 7.09cm^3\ n = 8.49 \times 10^{28}\frac{electrons}{m^2}\ v_d = 7.00 \times 10^{-5}\frac{m}{s}$

83. a. $v = 4.38 \times 10^7 m/s;$

b. $v = 5.81 \times 10^{13}\ \frac{protons}{m^3};$

c. 1.25 $\frac{electrons}{m^3}$

85. $E = 75kJ$

87. a. $P = 52W\ R = 36\Omega$;

b. $V = 43.54V$

89. a. $R = \frac{\rho}{2\pi L} ln(\frac{R_0}{R_i})$;

b. $R = 2.5m\Omega$

91. a. $I = 0.870A$;

b. #electrons $= 2.54 \times 10^{23}$

c. $R = 132\Omega$;

d. $q = 4.68 \times 10^6 J$

93. $P = 1045W, P = \frac{V^2}{R}, R = 12.27\Omega$

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13: DIRECT-CURRENT CIRCUITS

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13.1: Prelude to Direct-Current Circuits

In the preceding few chapters, we discussed electric components, including capacitors, resistors, and diodes. In this chapter, we use these electric components in circuits. A circuit is a collection of electrical components connected to accomplish a specific task. Figure 13.1.1 shows an amplifier circuit, which takes a small-amplitude signal and amplifies it to power the speakers in earbuds. Although the circuit looks complex, it actually consists of a set of series, parallel, and series-parallel circuits. The second section of this chapter covers the analysis of series and parallel circuits that consist of resistors. Later in this chapter, we introduce the basic equations and techniques to analyze any circuit, including those that are not reducible through simplifying parallel and series elements. But first, we need to understand how to power a circuit.

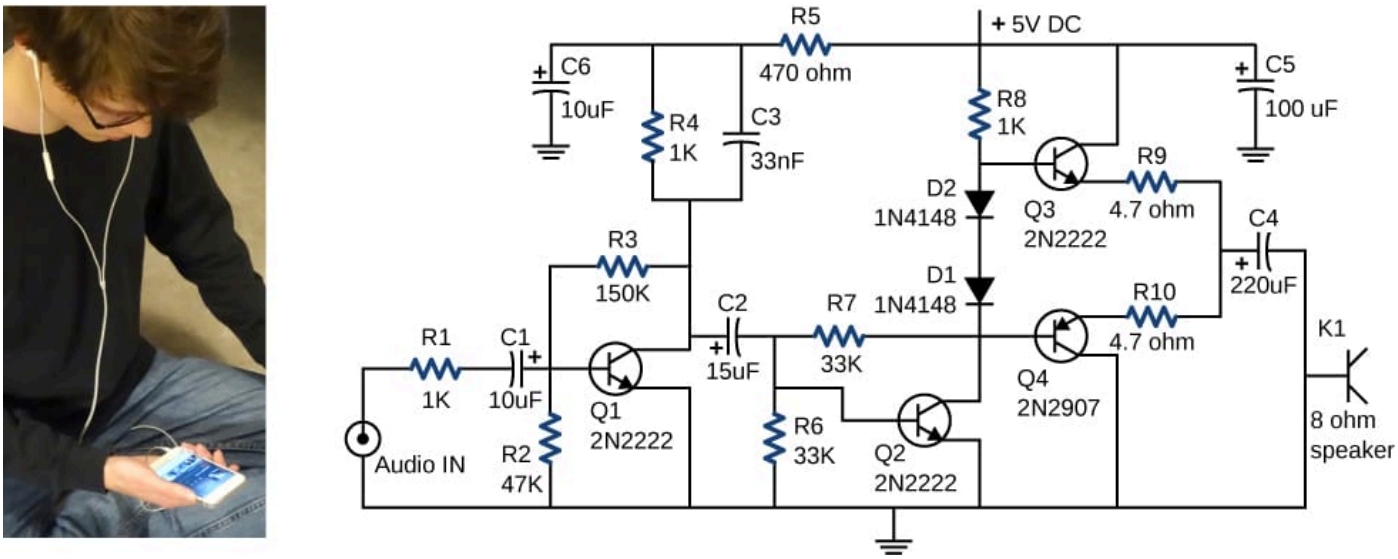


Figure 13.1.1: This circuit shown is used to amplify small signals and power the earbud speakers attached to a cellular phone. This circuit's components include resistors, capacitors, and diodes, all of which have been covered in previous chapters, as well as transistors, which are semi-conducting devices covered in Condensed Matter Physics. Circuits using similar components are found in all types of equipment and appliances you encounter in everyday life, such as alarm clocks, televisions, computers, and refrigerators. (credit: Jane Whitney)

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13.2: Electromotive Force

Learning Objectives

By the end of the section, you will be able to:

- Describe the electromotive force (emf) and the internal resistance of a battery
- Explain the basic operation of a battery

If you forget to turn off your car lights, they slowly dim as the battery runs down. Why don't they suddenly blink off when the battery's energy is gone? Their gradual dimming implies that the battery output voltage decreases as the battery is depleted. The reason for the decrease in output voltage for depleted batteries is that all voltage sources have two fundamental parts—a source of electrical energy and an internal resistance. In this section, we examine the energy source and the internal resistance.

Introduction to Electromotive Force

Voltage has many sources, a few of which are shown in Figure 13.2.1. All such devices create a **potential difference** and can supply current if connected to a circuit. A special type of potential difference is known as **electromotive force (emf)**. The emf is not a force at all, but the term 'electromotive force' is used for historical reasons. It was coined by Alessandro Volta in the 1800s, when he invented the first battery, also known as the **voltaic pile**. Because the electromotive force is not a force, it is common to refer to these sources simply as sources of emf (pronounced as the letters "ee-em-eff"), instead of sources of electromotive force.

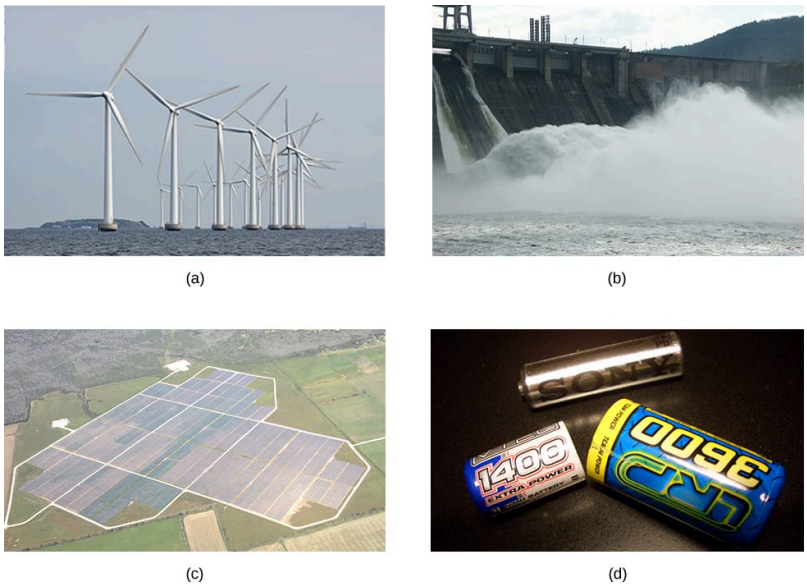


Figure 13.2.1: A variety of voltage sources. (a) The Brazos Wind Farm in Fluvanna, Texas; (b) the Krasnoyarsk Dam in Russia; (c) a solar farm; (d) a group of nickel metal hydride batteries. The voltage output of each device depends on its construction and load. The voltage output equals emf only if there is no load. (credit a: modification of work by "Leaflet"/Wikimedia Commons; credit b: modification of work by Alex Polezhaev; credit c: modification of work by US Department of Energy; credit d: modification of work by Tiaa Monto)

If the electromotive force is not a force at all, then what is the emf and what is a source of emf? To answer these questions, consider a simple circuit of a 12-V lamp attached to a 12-V battery, as shown in Figure 13.2.2. The **battery** can be modeled as a two-terminal device that keeps one terminal at a higher electric potential than the second terminal. The higher electric potential is sometimes called the positive terminal and is labeled with a plus sign. The lower-potential terminal is sometimes called the negative terminal and labeled with a minus sign. This is the source of the emf.

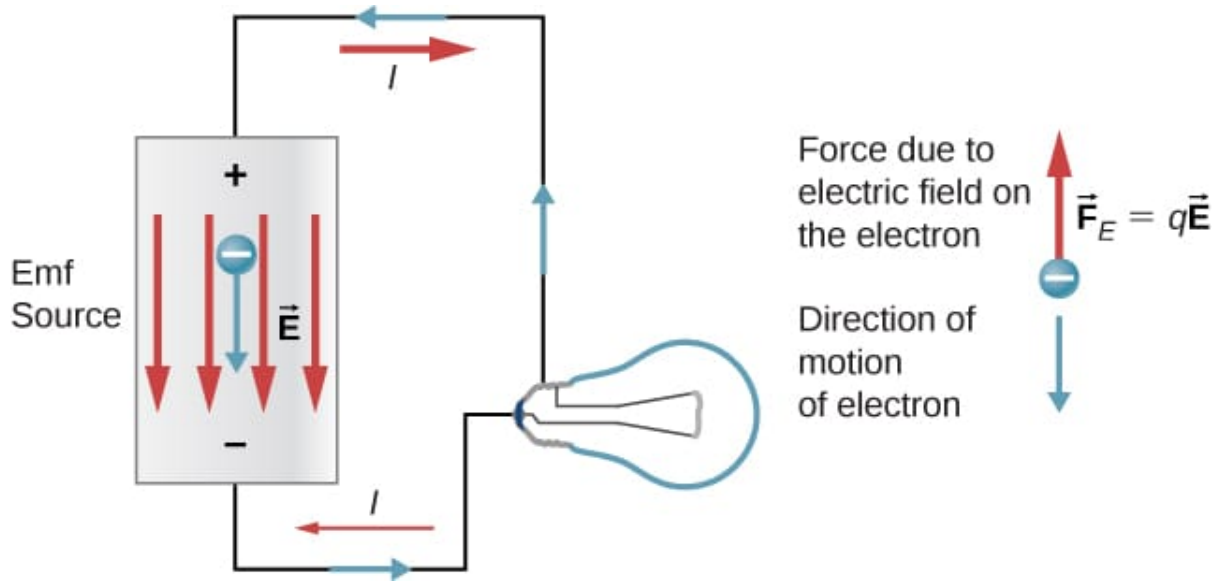


Figure 13.2.2: A source of emf maintains one terminal at a higher electric potential than the other terminal, acting as a source of current in a circuit.

When the emf source is not connected to the lamp, there is no net flow of charge within the emf source. Once the battery is connected to the lamp, charges flow from one terminal of the battery, through the lamp (causing the lamp to light), and back to the other terminal of the battery. If we consider positive (conventional) current flow, positive charges leave the positive terminal, travel through the lamp, and enter the negative terminal.

Positive current flow is useful for most of the circuit analysis in this chapter, but in metallic wires and resistors, electrons contribute the most to current, flowing in the opposite direction of positive current flow. Therefore, it is more realistic to consider the movement of electrons for the analysis of the circuit in Figure 13.2.2. The electrons leave the negative terminal, travel through the lamp, and return to the positive terminal. In order for the emf source to maintain the potential difference between the two terminals, negative charges (electrons) must be moved from the positive terminal to the negative terminal. The emf source acts as a charge pump, moving negative charges from the positive terminal to the negative terminal to maintain the potential difference. This increases the potential energy of the charges and, therefore, the electric potential of the charges.

The force on the negative charge from the electric field is in the opposite direction of the electric field, as shown in Figure 13.2.2. In order for the negative charges to be moved to the negative terminal, work must be done on the negative charges. This requires energy, which comes from chemical reactions in the battery. The potential is kept high on the positive terminal and low on the negative terminal to maintain the potential difference between the two terminals. The emf is equal to the work done on the charge per unit charge $\left(\varepsilon = \frac{dW}{dq}\right)$ when there is no current flowing. Since the unit for work is the joule and the unit for charge is the coulomb, the unit for emf is the volt $(1\ \mathrm{V} = 1\ \mathrm{J/C})$.

The **terminal voltage** V_{terminal} of a battery is voltage measured across the terminals of the battery when there is no load connected to the terminal. An ideal battery is an emf source that maintains a constant terminal voltage, independent of the current between the two terminals. An ideal battery has no internal resistance, and the terminal voltage is equal to the emf of the battery. In the next section, we will show that a real battery does have internal resistance and the terminal voltage is always less than the emf of the battery.

The Origin of Battery Potential

The combination of chemicals and the makeup of the terminals in a battery determine its emf. The lead acid battery used in cars and other vehicles is one of the most common combinations of chemicals. Figure 13.2.3 shows a single cell (one of six) of this battery. The cathode (positive) terminal of the cell is connected to a lead oxide plate, whereas the anode (negative) terminal is connected to a lead plate. Both plates are immersed in sulfuric acid, the electrolyte for the system.

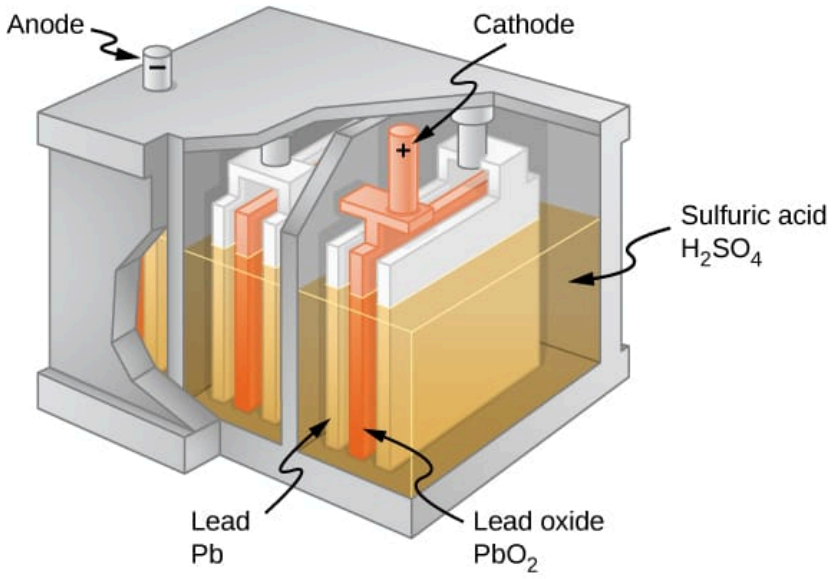


Figure 13.2.3: Chemical reactions in a lead-acid cell separate charge, sending negative charge to the anode, which is connected to the lead plates. The lead oxide plates are connected to the positive or cathode terminal of the cell. Sulfuric acid conducts the charge, as well as participates in the chemical reaction.

Knowing a little about how the chemicals in a lead-acid battery interact helps in understanding the potential created by the battery. Figure 13.2.4 shows the result of a single chemical reaction. Two electrons are placed on the anode, making it negative, provided that the cathode supplies two electrons. This leaves the cathode positively charged, because it has lost two electrons. In short, a separation of charge has been driven by a chemical reaction.

Note that the reaction does not take place unless there is a complete circuit to allow two electrons to be supplied to the cathode. Under many circumstances, these electrons come from the anode, flow through a resistance, and return to the cathode. Note also that since the chemical reactions involve substances with resistance, it is not possible to create the emf without an internal resistance.

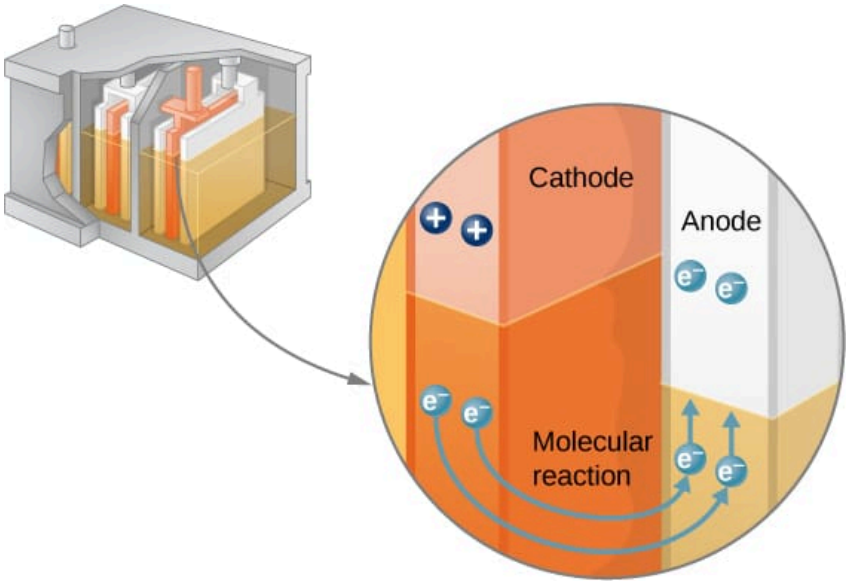


Figure 13.2.4: In a lead-acid battery, two electrons are forced onto the anode of a cell, and two electrons are removed from the cathode of the cell. The chemical reaction in a lead-acid battery places two electrons on the anode and removes two from the cathode. It requires a closed circuit to proceed, since the two electrons must be supplied to the cathode.

Internal Resistance and Terminal Voltage

The amount of resistance to the flow of current within the voltage source is called the **internal resistance**. The internal resistance r of a battery can behave in complex ways. It generally increases as a battery is depleted, due to the oxidation of the plates or the reduction of the acidity of the electrolyte. However, internal resistance may also depend on the magnitude and direction of the current through a voltage source, its temperature, and even its history. The internal resistance of rechargeable nickel-cadmium cells, for example, depends on how many times and how deeply they have been depleted. A simple model for a battery consists of an idealized emf source ε and an internal resistance r (Figure $13.2.5$).

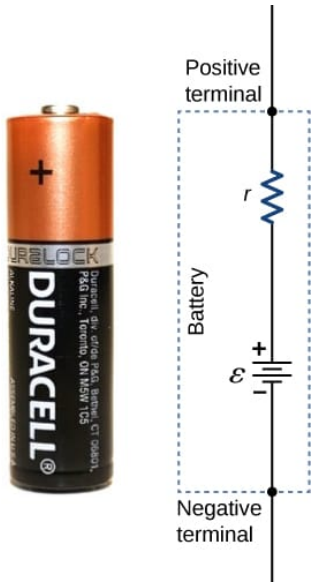


Figure $13.2.5$: A battery can be modeled as an idealized emf (ε) with an internal resistance (r). The terminal voltage of the battery is $V_{terminal} = \varepsilon - Ir$.

Suppose an external resistor, known as the load resistance R, is connected to a voltage source such as a battery, as in Figure $13.2.6$. The figure shows a model of a battery with an emf ε, an internal resistance r, and a load resistor R connected across its terminals. Using conventional current flow, positive charges leave the positive terminal of the battery, travel through the resistor, and return to the negative terminal of the battery. The terminal voltage of the battery depends on the emf, the internal resistance, and the current, and is equal to

Note

$$V_{\text{terminal}} = \varepsilon - Ir$$

For a given emf and internal resistance, the terminal voltage decreases as the current increases due to the potential drop Ir of the internal resistance.

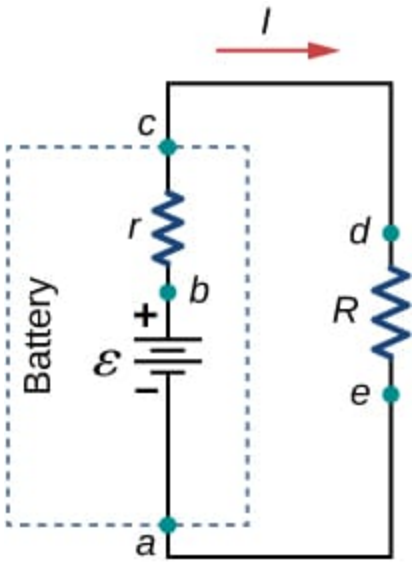


Figure 13.2.6: Schematic of a voltage source and its load resistor R. Since the internal resistance r is in series with the load, it can significantly affect the terminal voltage and the current delivered to the load.

A graph of the potential difference across each element of the circuit is shown in Figure 13.2.7. A current I runs through the circuit, and the potential drop across the internal resistor is equal to Ir. The terminal voltage is equal to $\varepsilon - Ir$, which is equal to the **potential drop** across the load resistor $IR = \varepsilon - Ir$. As with potential energy, it is the change in voltage that is important. When the term "voltage" is used, we assume that it is actually the change in the potential, or ΔV. However, Δ is often omitted for convenience.

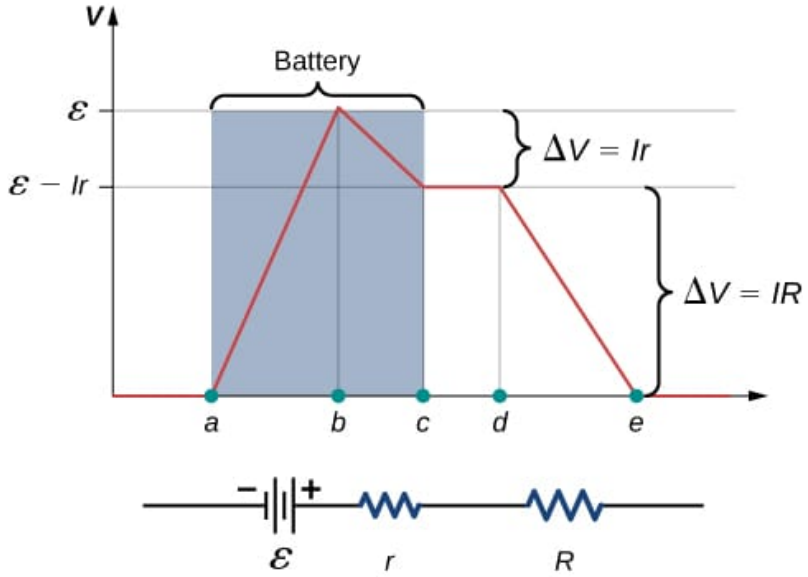


Figure 13.2.7: A graph of the voltage through the circuit of a battery and a load resistance. The electric potential increases the emf of the battery due to the chemical reactions doing work on the charges. There is a decrease in the electric potential in the battery due to the internal resistance. The potential decreases due to the internal resistance $-Ir$, making the terminal voltage of the battery equal to $(\varepsilon - Ir)$. The voltage then decreases by (IR). The current is equal to $I = \frac{\varepsilon}{r+R}$.

The current through the load resistor is $I = \frac{\varepsilon}{r+R}$. We see from this expression that the smaller the internal resistance r, the greater the current the voltage source supplies to its load R. As batteries are depleted, r increases. If r becomes a significant fraction of the load resistance, then the current is significantly reduced, as the following example illustrates.

Example 13.2.1: Analyzing a Circuit with a Battery and a Load

A given battery has a 12.00-V emf and an internal resistance of $0.100\ \Omega$. (a) Calculate its terminal voltage when connected to a $10.00\text{-}\Omega$ load. (b) What is the terminal voltage when connected to a $0.500\text{-}\Omega$ load? (c) What power does the $0.500\text{-}\Omega$ load dissipate? (d) If the internal resistance grows to $0.500\ \Omega$, find the current, terminal voltage, and power dissipated by a $0.500\text{-}\Omega$ load.

Strategy

The analysis above gave an expression for current when internal resistance is taken into account. Once the current is found, the terminal voltage can be calculated by using the equation $V_{\text{terminal}} = \varepsilon - Ir$. Once current is found, we can also find the power dissipated by the resistor.

Solution

1. Entering the given values for the emf, load resistance, and internal resistance into the expression above yields
$$I = \frac{\varepsilon}{R + r} = \frac{12.00\,\text{V}}{10.10\,\Omega} = 1.188\,\text{A}.$$
Enter the known values into the equation$V_{\text{terminal}} = \varepsilon - Ir$ to get the terminal voltage:
$$V_{\text{terminal}} = \varepsilon - Ir = 12.00\,\text{V} - (1.188\,\text{A})(0.100\,\Omega) = 11.90\,\text{V}.$$
The terminal voltage here is only slightly lower than the emf, implying that the current drawn by this light load is not significant.
2. Similarly, with $R_{\text{load}} = 0.500\ \Omega$, the current is
$$I = \frac{\varepsilon}{R + r} = \frac{12.00\,\text{V}}{0.600\,\Omega} = 20.00\,\text{A}.$$
The terminal voltage is now
$$V_{\text{terminal}} = \varepsilon - Ir = 12.00\,\text{V} - (20.00\,\text{A})(0.100\,\Omega) = 10.00\,\text{V}.$$
The terminal voltage exhibits a more significant reduction compared with emf, implying $0.500\ \Omega$ is a heavy load for this battery. A "heavy load" signifies a larger draw of current from the source but not a larger resistance.
3. The power dissipated by the $0.500\ \Omega$ load can be found using the formula $P = I^2R$. Entering the known values gives
$$P = I^2R = (20.0\,\text{A})^2(0.500\,\Omega) = 2.00 \times 10^2\,\text{W}.$$
Note that this power can also be obtained using the expression $\frac{V^2}{R}$ or IV, where V is the terminal voltage (10.0 V in this case).
4. Here, the internal resistance has increased, perhaps due to the depletion of the battery, to the point where it is as great as the load resistance. As before, we first find the current by entering the known values into the expression, yielding
$$I = \frac{\varepsilon}{R + r} = \frac{12.00\,\text{V}}{1.00\,\Omega} = 12.00\,\text{A}.$$
Now the terminal voltage is
$$V_{\text{terminal}} = \varepsilon - Ir = 12.00\,\text{V} - (12.00\,\text{A})(0.500\,\Omega) = 6.00\,\text{V},$$
and the power dissipated by the load is
$$P = I^2R = (12.00\,\text{A})^2(0.500\,\Omega) = 72.00\,\text{W}.$$
We see that the increased internal resistance has significantly decreased the terminal voltage, current, and power delivered to a load.

Significance

The internal resistance of a battery can increase for many reasons. For example, the internal resistance of a rechargeable battery increases as the number of times the battery is recharged increases. The increased internal resistance may have two effects on the battery. First, the terminal voltage will decrease. Second, the battery may overheat due to the increased power dissipated by the internal resistance.

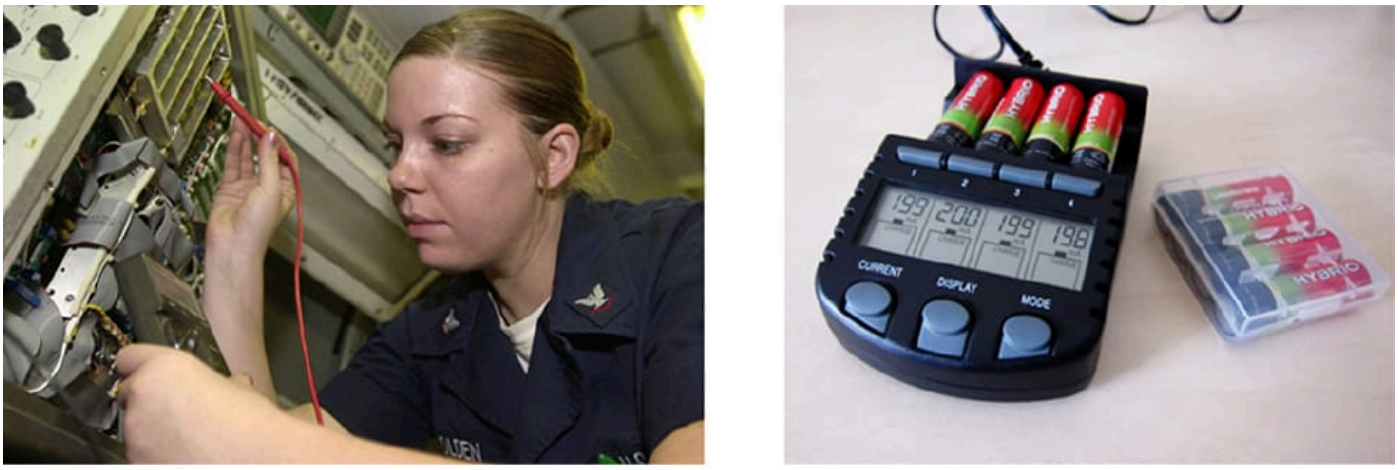
Exercise 13.2.1

If you place a wire directly across the two terminals of a battery, effectively shorting out the terminals, the battery will begin to get hot. Why do you suppose this happens?

Solution available at the end of this section.

Battery Testers

Battery testers, such as those in Figure 13.2.8, use small load resistors to intentionally draw current to determine whether the terminal potential drops below an acceptable level. Although it is difficult to measure the internal resistance of a battery, battery testers can provide a measurement of the internal resistance of the battery. If internal resistance is high, the battery is weak, as evidenced by its low terminal voltage.



(a) (b)

Figure 13.2.8: Battery testers measure terminal voltage under a load to determine the condition of a battery. (a) A US Navy electronics technician uses a battery tester to test large batteries aboard the aircraft carrier USS *Nimitz*. The battery tester she uses has a small resistance that can dissipate large amounts of power. (b) The small device shown is used on small batteries and has a digital display to indicate the acceptability of the terminal voltage. (credit a: modification of work by Jason A. Johnston; credit b: modification of work by Keith Williamson)

Some batteries can be recharged by passing a current through them in the direction opposite to the current they supply to an appliance. This is done routinely in cars and in batteries for small electrical appliances and electronic devices (Figure 13.2.9). The voltage output of the battery charger must be greater than the emf of the battery to reverse the current through it. This causes the terminal voltage of the battery to be greater than the emf, since $V = \epsilon - Ir$ and I is now negative.

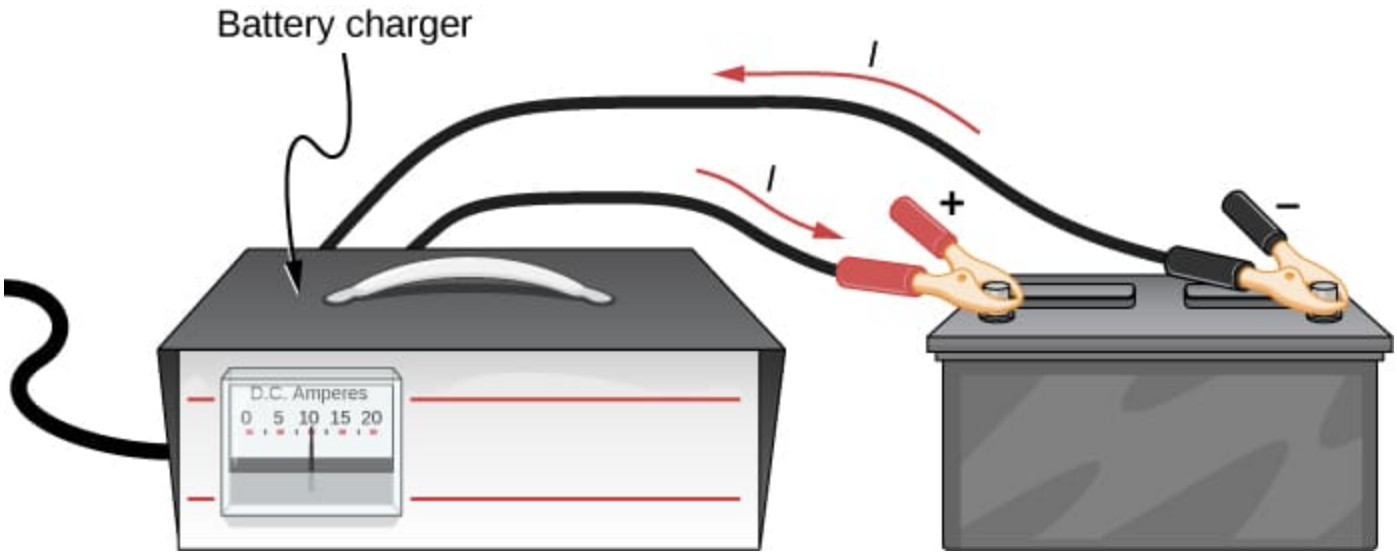


Figure 13.2.9: A car battery charger reverses the normal direction of current through a battery, reversing its chemical reaction and replenishing its chemical potential.

It is important to understand the consequences of the internal resistance of emf sources, such as batteries and solar cells, but often, the analysis of circuits is done with the terminal voltage of the battery, as we have done in the previous sections. The terminal voltage is referred to as simply as V, dropping the subscript "terminal." This is because the internal resistance of the battery is difficult to measure directly and can change over time.

Answers

Exercise 13.2.1

If a wire is connected across the terminals, the load resistance is close to zero, or at least considerably less than the internal resistance of the battery. Since the internal resistance is small, the current through the circuit will be large, $I = \frac{\varepsilon}{R+r} = \frac{\varepsilon}{0+r} = \frac{\varepsilon}{r}$. The large current causes a high power to be dissipated by the internal resistance $(P = I^2 r)$. The power is dissipated as heat.

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13.3: Resistors in Series and Parallel

Learning Objectives

By the end of the section, you will be able to:

- Define the term equivalent resistance
- Calculate the equivalent resistance of resistors connected in series
- Calculate the equivalent resistance of resistors connected in parallel

In Current and Resistance, we described the term 'resistance' and explained the basic design of a resistor. Basically, a resistor limits the flow of charge in a circuit and is an ohmic device where $V = IR$. Most circuits have more than one resistor. If several resistors are connected together and connected to a battery, the current supplied by the battery depends on the **equivalent resistance** of the circuit.

The equivalent resistance of a combination of resistors depends on both their individual values and how they are connected. The simplest combinations of resistors are series and parallel connections (Figure 13.3.1). In a series circuit, the output current of the first resistor flows into the input of the second resistor; therefore, the current is the same in each resistor. In a parallel circuit, all of the resistor leads on one side of the resistors are connected together and all the leads on the other side are connected together. In the case of a parallel configuration, each resistor has the same potential drop across it, and the currents through each resistor may be different, depending on the resistor. The sum of the individual currents equals the current that flows into the parallel connections.

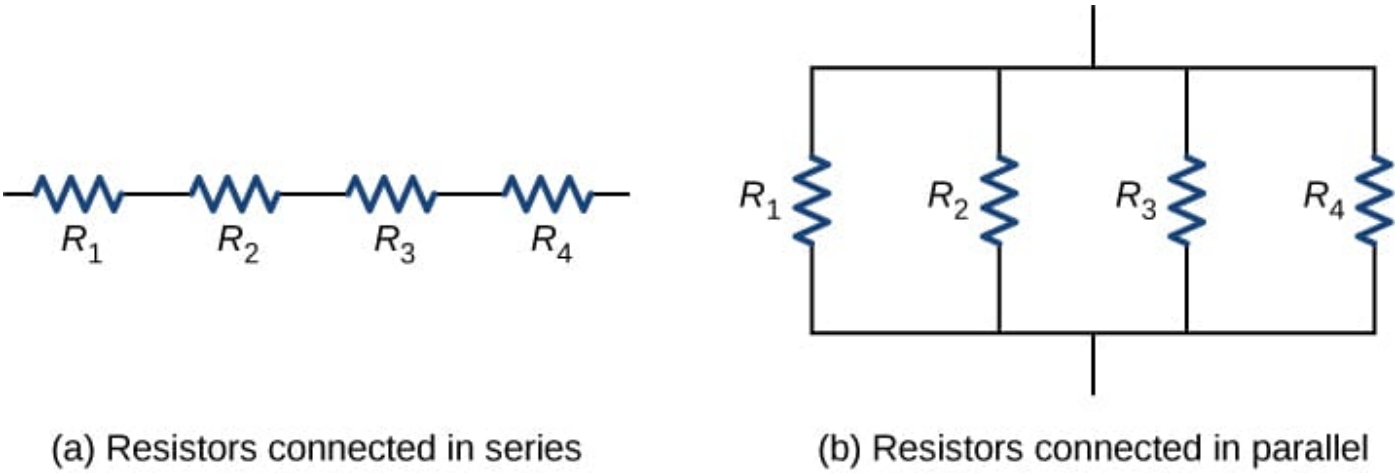


Figure 13.3.1: (a) For a series connection of resistors, the current is the same in each resistor. (b) For a parallel connection of resistors, the voltage is the same across each resistor.

Resistors in Series

Resistors are said to be in series whenever the current flows through the resistors sequentially. Consider Figure 13.3.2, which shows three resistors in series with an applied voltage equal to V_{ab}. Since there is only one path for the charges to flow through, the current is the same through each resistor. The equivalent resistance of a set of resistors in a series connection is equal to the algebraic sum of the individual resistances.

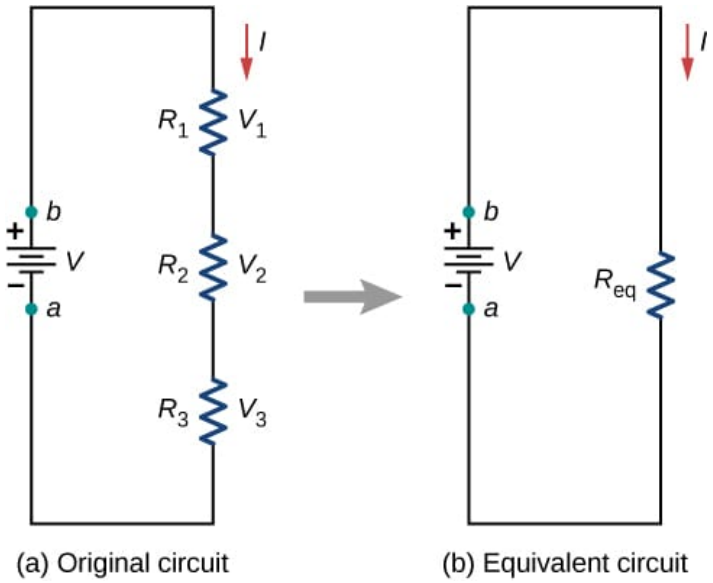


Figure 13.3.2: (a) Three resistors connected in series to a voltage source. (b) The original circuit is reduced to an equivalent resistance and a voltage source.

In Figure 13.3.2, the current coming from the voltage source flows through each resistor, so the current through each resistor is the same. The current through the circuit depends on the voltage supplied by the voltage source and the resistance of the resistors. For each resistor, a potential drop occurs that is equal to the loss of electric potential energy as a current travels through each resistor. According to Ohm's law, the potential drop V across a resistor when a current flows through it is calculated using the equation $V = IR$, where I is the current in amps (A) and R is the resistance in ohms (Ω). Since energy is conserved, and the voltage is equal to the potential energy per charge, the sum of the voltage applied to the circuit by the source and the potential drops across the individual resistors around a loop should be equal to zero:

$$\sum_{i=1}^{N} V_i = 0.$$

This equation is often referred to as Kirchhoff's loop law, which we will look at in more detail later in this chapter. For Figure 13.3.2 , the sum of the potential drop of each resistor and the voltage supplied by the voltage source should equal zero:

$$\begin{aligned} V - V_1 - V_2 - V_3 &= 0, \\ V &= V_1 + V_2 + V_3, \\ &= IR_1 + IR_2 + IR_3, \end{aligned}$$

Solving for I

$$\begin{aligned} I &= \frac{V}{R_1 + R_2 + R_3} \\ &= \frac{V}{R_S}. \end{aligned}$$

Since the current through each component is the same, the equality can be simplified to an equivalent resistance (R_S), which is just the sum of the resistances of the individual resistors.

Equivalent Resistance in Series Circuits

Any number of resistors can be connected in series. If N resistors are connected in series, the **equivalent resistance** is

$$R_S = R_1 + R_2 + R_3 + \ldots + R_{N-1} + R_N = \sum_{i=1}^{N} R_i. \tag{13.3.1}$$

One result of components connected in a series circuit is that if something happens to one component, it affects all the other components. For example, if several lamps are connected in series and one bulb burns out, all the other lamps go dark.

Example 13.3.1: Equivalent Resistance, Current, and Power in a Series Circuit

A battery with a terminal voltage of 9 V is connected to a circuit consisting of four $20\ \Omega$ and one $10\ \Omega$ resistors all in series (Figure 13.3.3). Assume the battery has negligible internal resistance.

a. Calculate the equivalent resistance of the circuit.
b. Calculate the current through each resistor.
c. Calculate the potential drop across each resistor.
d. Determine the total power dissipated by the resistors and the power supplied by the battery.

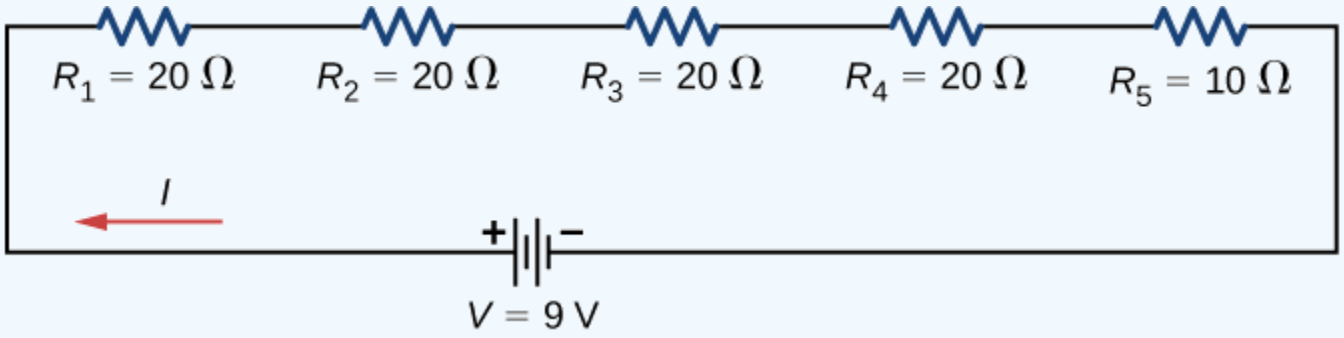


Figure 13.3.3: A simple series circuit with five resistors.

Strategy

In a series circuit, the equivalent resistance is the algebraic sum of the resistances. The current through the circuit can be found from Ohm's law and is equal to the voltage divided by the equivalent resistance. The potential drop across each resistor can be found using Ohm's law. The power dissipated by each resistor can be found using $P = I^2R$, and the total power dissipated by the resistors is equal to the sum of the power dissipated by each resistor. The power supplied by the battery can be found using $P = I\varepsilon$.

Solution

1. The equivalent resistance is the algebraic sum of the resistances (Equation 13.3.1):
$$\begin{aligned} R_S &= R_1 + R_2 + R_3 + R_4 + R_5 \\ &= 20\,\Omega + 20\,\Omega + 20\,\Omega + 20\,\Omega + 10\,\Omega = 90\,\Omega. \end{aligned}$$
2. The current through the circuit is the same for each resistor in a series circuit and is equal to the applied voltage divided by the equivalent resistance:
$$I = \frac{V}{R_S} = \frac{9\,\text{V}}{90\,\Omega} = 0.1\,\text{A}.$$
Note that the sum of the potential drops across each resistor is equal to the voltage supplied by the battery.
3. The power dissipated by a resistor is equal to $P = I^2R$, and the power supplied by the battery is equal to $P = I\varepsilon$.
$$\begin{aligned} P_1 = P_2 = P_3 = P_4 &= (0.1\,\text{A})^2(20\,\Omega) = 0.2\,\text{W}, \\ P_5 &= (0.1\,\text{A})^2(10\,\Omega) = 0.1\,\text{W}, \\ P_{\text{dissipated}} &= 0.2\,\text{W} + 0.2\,\text{W} + 0.2\,\text{W} + 0.2\,\text{W} + 0.1\,\text{W} = 0.9\,\text{W}, \\ P_{\text{source}} &= I\varepsilon = (0.1\,\text{A})(9\,\text{V}) = 0.9\,\text{W}. \end{aligned}$$

Significance

There are several reasons why we would use multiple resistors instead of just one resistor with a resistance equal to the equivalent resistance of the circuit. Perhaps a resistor of the required size is not available, or we need to dissipate the heat generated, or we want to minimize the cost of resistors. Each resistor may cost a few cents to a few dollars, but when multiplied by thousands of units, the cost saving may be appreciable.

Exercise 13.3.1

Some strings of miniature holiday lights are made to short out when a bulb burns out. The device that causes the short is called a shunt, which allows current to flow around the open circuit. A "short" is like putting a piece of wire across the component. The bulbs are usually grouped in series of nine bulbs. If too many bulbs burn out, the shunts eventually open. What causes this?

Answer available at the end of this section.

Let's briefly summarize the major features of resistors in series:

1. Series resistances add together to get the equivalent resistance (Equation 13.3.1):

$$R_S = R_1 + R_2 + R_3 + \ldots + R_{N-1} + R_N = \sum_{i=1}^{N} R_i.$$

2. The same current flows through each resistor in series.
3. Individual resistors in series do not get the total source voltage, but divide it. The total potential drop across a series configuration of resistors is equal to the sum of the potential drops across each resistor.

Resistors in Parallel

Figure 13.3.4 shows resistors in parallel, wired to a voltage source. Resistors are in parallel when one end of all the resistors are connected by a continuous wire of negligible resistance and the other end of all the resistors are also connected to one another through a continuous wire of negligible resistance. The potential drop across each resistor is the same. Current through each resistor can be found using Ohm's law $I = V/R$, where the voltage is constant across each resistor. For example, an automobile's headlights, radio, and other systems are wired in parallel, so that each subsystem utilizes the full voltage of the source and can operate completely independently. The same is true of the wiring in your house or any building.

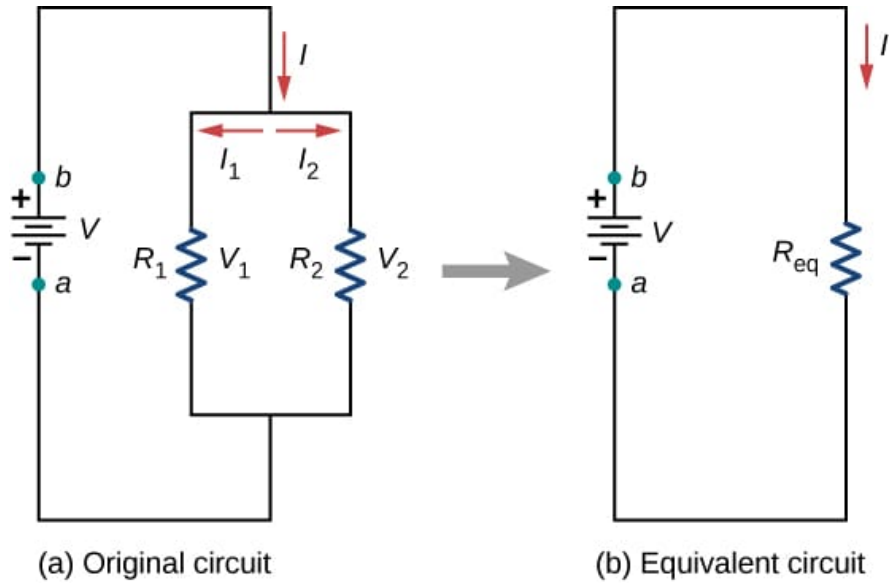


Figure 13.3.4: Two resistors connected in parallel to a voltage source. (b) The original circuit is reduced to an equivalent resistance and a voltage source.

The current flowing from the voltage source in Figure 13.3.4 depends on the voltage supplied by the voltage source and the

equivalent resistance of the circuit. In this case, the current flows from the voltage source and enters a junction, or node, where the circuit splits flowing through resistors R_1 and R_2. As the charges flow from the battery, some go through resistor R_1 and some flow through resistor R_2. The sum of the currents flowing into a junction must be equal to the sum of the currents flowing out of the junction:

$$\sum I_{in} = \sum I_{out}.$$

This equation is referred to as **Kirchhoff's junction rule** and will be discussed in detail in the next section. In Figure 13.3.4, the junction rule gives $I = I_1 + I_2$. There are two loops in this circuit, which leads to the equations $V = I_1 R_1$ and $I_1 R_1 = I_2 R_2$. Note the voltage across the resistors in parallel are the same ($V = V_1 = V_2$) and the current is additive:

$$\begin{aligned} I &= I_1 + I_2 \\ &= \frac{V_1}{R_1} + \frac{V_2}{R_2} \\ &= \frac{V}{R_1} + \frac{V}{R_2} \\ &= V\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{V}{R_P} \end{aligned}$$

Solving for the R_P

$$R_P = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}.$$

Equivalent Resistance in Parallel Circuits

Generalizing to any number of N resistors, the equivalent resistance R_P of a parallel connection is related to the individual resistances by

$$R_P = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ... + \frac{1}{R_{N-1}} + \frac{1}{R_N}\right)^{-1} = \left(\sum_{i=1}^{N} \frac{1}{R_i}\right)^{-1}. \tag{13.3.2}$$

This relationship results in an equivalent resistance R_P that is less than the smallest of the individual resistances. When resistors are connected in parallel, more current flows from the source than would flow for any of them individually, so the total resistance is lower.

Example 13.3.2: Analysis of a Parallel Circuit

Three resistors $R_1 = 1.00\ \Omega$, $R_2 = 2.00\ \Omega$, and $R_3 = 2.00\ \Omega$, are connected in parallel. The parallel connection is attached to a $V = 3.00\ \text{V}$ voltage source.

a. What is the equivalent resistance?
b. Find the current supplied by the source to the parallel circuit.
c. Calculate the currents in each resistor and show that these add together to equal the current output of the source.
d. Calculate the power dissipated by each resistor.
e. Find the power output of the source and show that it equals the total power dissipated by the resistors.

Strategy

(a) The total resistance for a parallel combination of resistors is found using Equation 13.3.2. (Note that in these calculations, each intermediate answer is shown with an extra digit.)

(b) The current supplied by the source can be found from Ohm's law, substituting R_P for the total resistance $I = \frac{V}{R_P}$.

(c) The individual currents are easily calculated from Ohm's law $\left(I_i = \frac{V_i}{R_i}\right)$, since each resistor gets the full voltage. The total current is the sum of the individual currents:

$$I = \sum_i I_i.$$

(d) The power dissipated by each resistor can be found using any of the equations relating power to current, voltage, and resistance, since all three are known. Let us use $P_i = V^2/R_i$, since each resistor gets full voltage.

(e) The total power can also be calculated in several ways, use $P = IV$.

Solution

1. The total resistance for a parallel combination of resistors is found using Equation 13.3.2. Entering known values gives

$$R_P = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1} = \left(\frac{1}{1.00\,\Omega} + \frac{1}{2.00\,\Omega} + \frac{1}{2.00\,\Omega}\right)^{-1} = 0.50\,\Omega.$$

   The total resistance with the correct number of significant digits is $R_{eq} = 0.50\ \Omega$. As predicted, R_P is less than the smallest individual resistance.
2. The total current can be found from Ohm's law, substituting R_P for the total resistance. This gives

$$I = \frac{V}{R_P} = \frac{3.00\,\text{V}}{0.50\,\Omega} = 6.00\,\text{A}.$$

   Current I for each device is much larger than for the same devices connected in series (see the previous example). A circuit with parallel connections has a smaller total resistance than the resistors connected in series.
3. The individual currents are easily calculated from Ohm's law, since each resistor gets the full voltage. Thus,

$$I_1 = \frac{V}{R_1} = \frac{3.00\,\text{V}}{1.00\,\Omega} = 3.00\,\text{A}.$$

   Similarly,

$$I_2 = \frac{V}{R_2} = \frac{3.00\,\text{V}}{2.00\,\Omega} = 1.50\,\text{A}$$

   and

$$I_3 = \frac{V}{R_3} = \frac{3.00\,\text{V}}{2.00\,\Omega} = 1.50\,\text{A}.$$

   The total current is the sum of the individual currents:

$$I_1 + I_2 + I_3 = 6.00\,\text{A}.$$

4. The power dissipated by each resistor can be found using any of the equations relating power to current, voltage, and resistance, since all three are known. Let us use $P = V^2/R$, since each resistor gets full voltage. Thus,

$$P_1 = \frac{V^2}{R_1} = \frac{(3.00\,\text{V})^2}{1.00\,\Omega} = 9.00\,\text{W}.$$

   Similarly,

$$P_2 = \frac{V^2}{R_2} = \frac{(3.00\,\text{V})^2}{2.00\,\Omega} = 4.50\,\text{W}.$$

   and

$$P_3 = \frac{V^2}{R_3} = \frac{(3.00\,\text{V})^2}{2.00\,\Omega} = 4.50\,\text{W}.$$

5. The total power can also be calculated in several ways. Choosing $P = IV$ and entering the total current yields

$$P = IV = (6.00\,\text{A})(3.00\,\text{V}) = 18.00\,\text{W}.$$

Significance

Total power dissipated by the resistors is also 18.00 W:

$$P_1 + P_2 + P_3 = 9.00\,\text{W} + 4.50\,\text{W} + 4.50\,\text{W} = 18.00\,\text{W}.$$

Notice that the total power dissipated by the resistors equals the power supplied by the source.

Exercise 13.3.2A

Consider the same potential difference $(V = 3.00\ \text{V})$ applied to the same three resistors connected in series. Would the equivalent resistance of the series circuit be higher, lower, or equal to the three resistor in parallel? Would the current through the series circuit be higher, lower, or equal to the current provided by the same voltage applied to the parallel circuit? How would the power dissipated by the resistor in series compare to the power dissipated by the resistors in parallel?

Solution available at the end of this section.

Exercise 13.3.2B

How would you use a river and two waterfalls to model a parallel configuration of two resistors? How does this analogy break down?

Solution available at the end of this section.

Let us summarize the major features of resistors in parallel:

1. Equivalent resistance is found from Equation 13.3.2 and is smaller than any individual resistance in the combination.
2. The potential drop across each resistor in parallel is the same.
3. Parallel resistors do not each get the total current; they divide it. The current entering a parallel combination of resistors is equal to the sum of the current through each resistor in parallel.

In this chapter, we introduced the equivalent resistance of resistors connected in series and resistors connected in parallel. You may recall from the Section on Capacitance, we introduced the equivalent capacitance of capacitors connected in series and parallel. Circuits often contain both capacitors and resistors. Table 13.3.1 summarizes the equations used for the equivalent resistance and equivalent capacitance for series and parallel connections.

Table 13.3.1: Summary for Equivalent Resistance and Capacitance in Series and Parallel Combinations

	Series combination
Equivalent capacitance	$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$
Equivalent resistance	$R_S = R_1 + R_2 + R_3 + \ldots = \sum_{i=1}^{N} R_i$

Combinations of Series and Parallel

More complex connections of resistors are often just combinations of series and parallel connections. Such combinations are common, especially when wire resistance is considered. In that case, wire resistance is in series with other resistances that are in parallel.

Combinations of series and parallel can be reduced to a single equivalent resistance using the technique illustrated in Figure 13.3.5. Various parts can be identified as either series or parallel connections, reduced to their equivalent resistances, and then further reduced until a single equivalent resistance is left. The process is more time consuming than difficult. Here, we note the equivalent resistance as R_{eq}.

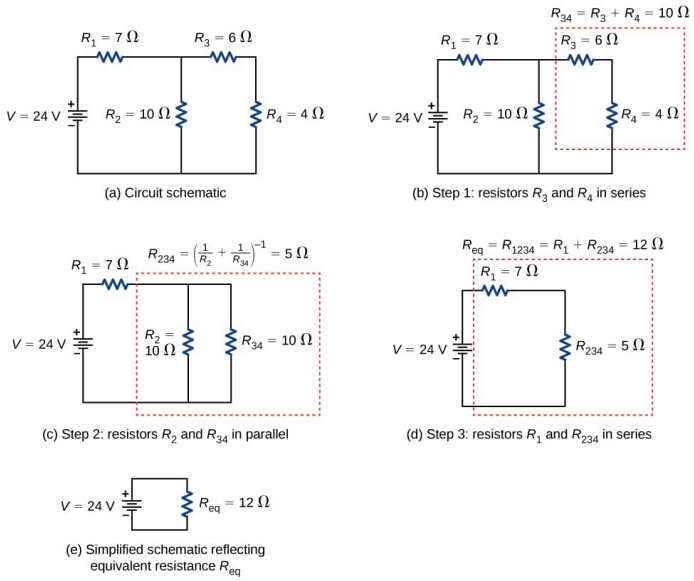


Figure 13.3.5: (a) The original circuit of four resistors. (b) Step 1: The resistors R_3 and R_4 are in series and the equivalent resistance is $R_{34} = 10\ \Omega$ (c) Step 2: The reduced circuit shows resistors R_2 and R_{34} are in parallel, with an equivalent resistance of $R_{234} = 5\ \Omega$. (d) Step 3: The reduced circuit shows that R_1 and R_{234} are in series with an equivalent resistance of $R_{1234} = 12\ \Omega$ which is the equivalent resistance R_{eq}. (e) The reduced circuit with a voltage source of $V = 24\ \text{V}$ with an equivalent resistance of $R_{eq} = 12\ \Omega$. This results in a current of $I = 2\ \text{A}$ from the voltage source.

Notice that resistors R_3 and R_4 are in series. They can be combined into a single equivalent resistance. One method of keeping track of the process is to include the resistors as subscripts. Here the equivalent resistance of R_3 and R_4 is

$$R_{34} = R_3 + R_4 = 6\,\Omega + 4\,\Omega = 10\,\Omega.$$

The circuit now reduces to three resistors, shown in Figure 13.3.5*c*. Redrawing, we now see that resistors R_2 and R_{34} constitute a parallel circuit. Those two resistors can be reduced to an equivalent resistance:

$$R_{234} = \left(\frac{1}{R_2} + \frac{1}{R_{34}}\right)^{-1} = \left(\frac{1}{10\,\Omega} + \frac{1}{10\,\Omega}\right)^{-1} = 5\,\Omega.$$

This step of the process reduces the circuit to two resistors, shown in in Figure 13.3.5*d*. Here, the circuit reduces to two resistors, which in this case are in series. These two resistors can be reduced to an equivalent resistance, which is the equivalent resistance of the circuit:

$$R_{eq} = R_{1234} = R_1 + R_{234} = 7\,\Omega + 5\Omega = 12\,\Omega.$$

The main goal of this circuit analysis is reached, and the circuit is now reduced to a single resistor and single voltage source.

Now we can analyze the circuit. The current provided by the voltage source is $I = \frac{V}{R_{eq}} = \frac{24\,\text{V}}{12\,\Omega} = 2\ \text{A}$. This current runs through resistor R_1 and is designated as I_1. The potential drop across R_1 can be found using Ohm's law:

$$V_1 = I_1 R_1 = (2\,\text{A})(7\,\Omega) = 14\,\text{V}.$$

Looking at Figure 13.3.5*c*, this leaves $24\ \text{V} - 14\ \text{V} = 10\ \text{V}$ to be dropped across the parallel combination of R_2 and R_{34}. The current through R_2 can be found using Ohm's law:

$$I_2 = \frac{V_2}{R_2} = \frac{10\,\text{V}}{10\,\Omega} = 1\,\text{A}.$$

The resistors R_3 and R_4 are in series so the currents I_3 and I_4 are equal to

$$I_3 = I_4 = I - I_2 = 2\,\text{A} - 1\,\text{A} = 1\,\text{A}.$$

Using Ohm's law, we can find the potential drop across the last two resistors. The potential drops are $V_3 = I_3 R_3 = 6\ \text{V}$ and $V_4 = I_4 R_4 = 4\ \text{V}$. The final analysis is to look at the power supplied by the voltage source and the power dissipated by the resistors.

The power dissipated by the resistors is

$$\begin{aligned} P_1 &= I_1^2 R_1 = (2\,\text{A})^2(7\,\Omega) = 28\,\text{W}, \\ P_2 &= I_2^2 R_2 = (1\,\text{A})^2(10\,\Omega) = 10\,\text{W}, \\ P_3 &= I_3^2 R_3 = (1\,\text{A})^2(6\,\Omega) = 6\,\text{W}, \\ P_4 &= I_4^2 R_4 = (1\,\text{A})^2(4\,\Omega) = 4\,\text{W}, \\ P_{dissipated} &= P_1 + P_2 + P_3 + P_4 = 48\,\text{W}. \end{aligned}$$

The total energy is constant in any process. Therefore, the power supplied by the voltage source is

$$\begin{aligned} P_s &= IV \\ &= (2\,\text{A})(24\,\text{V}) = 48\,\text{W} \end{aligned}$$

Analyzing the power supplied to the circuit and the power dissipated by the resistors is a good check for the validity of the analysis; they should be equal.

Example 13.3.3: Combining Series and Parallel Circuits

Figure 13.3.6 shows resistors wired in a combination of series and parallel. We can consider R_1 to be the resistance of wires leading to R_2 and R_3.

a. Find the equivalent resistance of the circuit.
b. What is the potential drop V_1 across resistor R_1?
c. Find the current I_2 through resistor R_2.
d. What power is dissipated by R_2?

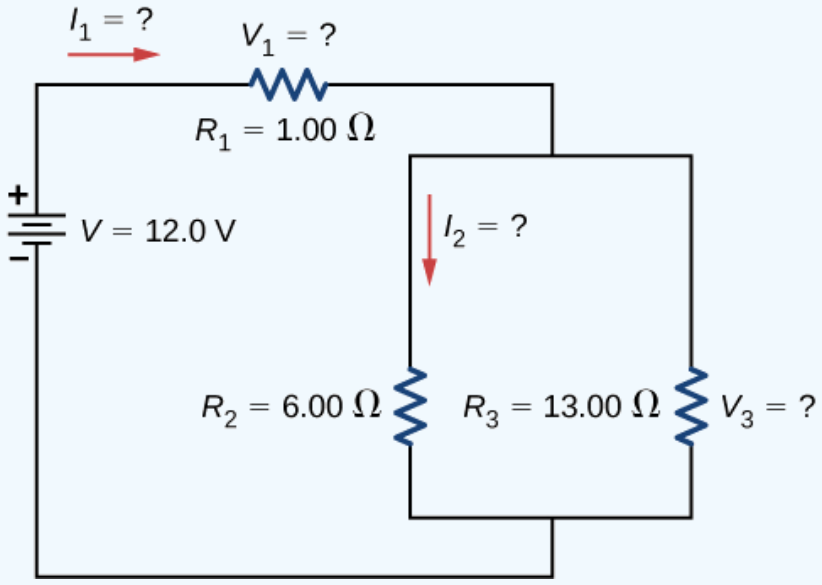


Figure 13.3.6: These three resistors are connected to a voltage source so that R_2 and R_3 are in parallel with one another and that combination is in series with R_1.

Strategy

(a) To find the equivalent resistance, first find the equivalent resistance of the parallel connection of R_2 and R_3. Then use this result to find the equivalent resistance of the series connection with R_1.

(b) The current through R_1 can be found using Ohm's law and the voltage applied. The current through R_1 is equal to the current from the battery. The potential drop V_1 across the resistor R_1 (which represents the resistance in the connecting wires) can be found using Ohm's law.

(c) The current through R_2 can be found using Ohm's law $I_2 = \frac{V_2}{R_2}$. The voltage across R_2 can be found using $V_2 = V - V_1$.

(d) Using Ohm's law $(V_2 = I_2 R_2)$, the power dissipated by the resistor can also be found using $P_2 = I_2^2 R_2 = \frac{V_2^2}{R_2}$.

Solution

1. To find the equivalent resistance of the circuit, notice that the parallel connection of R_2 and R_3 is in series with R_1, so the equivalent resistance is
$$R_{eq} = R_1 + \left(\frac{1}{R_2} + \frac{1}{R_3}\right)^{-1} = 1.00\,\Omega + \left(\frac{1}{6.00\,\Omega} + \frac{1}{13.00\,\Omega}\right)^{-1} = 5.10\,\Omega.$$
The total resistance of this combination is intermediate between the pure series and pure parallel values ($20.0\ \Omega$ and $0.804\ \Omega$, respectively).
2. The current through R_1 is equal to the current supplied by the battery:
$$I_1 = I = \frac{V}{R_{eq}} = \frac{12.0\,\text{V}}{5.10\,\Omega} = 2.35\,\text{A}.$$
The voltage across R_1 is
$$V_1 = I_1 R_1 = (2.35\,\text{A})(1\,\Omega) = 2.35\,\text{V}.$$
The voltage applied to R_2 and R_3 is less than the voltage supplied by the battery by an amount V_1. When wire resistance is large, it can significantly affect the operation of the devices represented by R_2 and R_3.
3. To find the current through R_2, we must first find the voltage applied to it. The voltage across the two resistors in parallel is the same:
$$V_2 = V_3 = V - V_1 = 12.0\,\text{V} - 2.35\,\text{V} = 9.65\,\text{V}.$$
Now we can find the current I_2 through resistance R_2 using Ohm's law:
$$I_2 = \frac{V_2}{R_2} = \frac{9.65\,\text{V}}{6.00\,\Omega} = 1.61\,\text{A}.$$
The current is less than the 2.00 A that flowed through R_2 when it was connected in parallel to the battery in the previous parallel circuit example.
4. The power dissipated by R_2 is given by
$$P_2 = I_2^2 R_2 = (1.61\,\text{A})^2(6.00\,\Omega) = 15.5\,\text{W}.$$

Significance

The analysis of complex circuits can often be simplified by reducing the circuit to a voltage source and an equivalent resistance. Even if the entire circuit cannot be reduced to a single voltage source and a single equivalent resistance, portions of the circuit may be reduced, greatly simplifying the analysis.

Exercise 13.3.3

Consider the electrical circuits in your home. Give at least two examples of circuits that must use a combination of series and parallel circuits to operate efficiently.

Solution available at the end of this section.

Practical Implications

One implication of this last example is that resistance in wires reduces the current and power delivered to a resistor. If wire resistance is relatively large, as in a worn (or a very long) extension cord, then this loss can be significant. If a large current is drawn, the IR drop in the wires can also be significant and may become apparent from the heat generated in the cord.

For example, when you are rummaging in the refrigerator and the motor comes on, the refrigerator light dims momentarily. Similarly, you can see the passenger compartment light dim when you start the engine of your car (although this may be due to

resistance inside the battery itself).

What is happening in these high-current situations is illustrated in Figure 13.3.7. The device represented by R_3 has a very low resistance, so when it is switched on, a large current flows. This increased current causes a larger IR drop in the wires represented by R_1, reducing the voltage across the light bulb (which is R_2), which then dims noticeably.

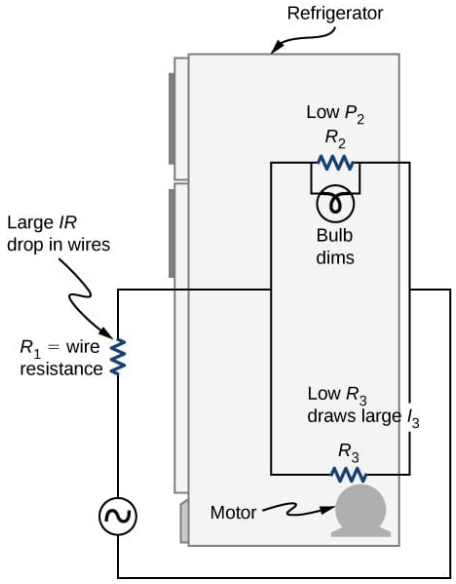


Figure 13.3.7: Why do lights dim when a large appliance is switched on? The answer is that the large current the appliance motor draws causes a significant IR drop in the wires and reduces the voltage across the light.

Problem-Solving Strategy: Series and Parallel Resistors

1. Draw a clear circuit diagram, labeling all resistors and voltage sources. This step includes a list of the known values for the problem, since they are labeled in your circuit diagram.
2. Identify exactly what needs to be determined in the problem (identify the unknowns). A written list is useful.
3. Determine whether resistors are in series, parallel, or a combination of both series and parallel. Examine the circuit diagram to make this assessment. Resistors are in series if the same current must pass sequentially through them.
4. Use the appropriate list of major features for series or parallel connections to solve for the unknowns. There is one list for series and another for parallel.
5. Check to see whether the answers are reasonable and consistent.

Example 13.3.4: Combining Series and Parallel Circuits

Two resistors connected in series $(R_1,\ R_2)$ are connected to two resistors that are connected in parallel $(R_3,\ R_4)$. The series-parallel combination is connected to a battery. Each resistor has a resistance of 10.00 Ohms. The wires connecting the resistors and battery have negligible resistance. A current of 2.00 Amps runs through resistor R_1. What is the voltage supplied by the voltage source?

Strategy

Use the steps in the preceding problem-solving strategy to find the solution for this example.

Solution

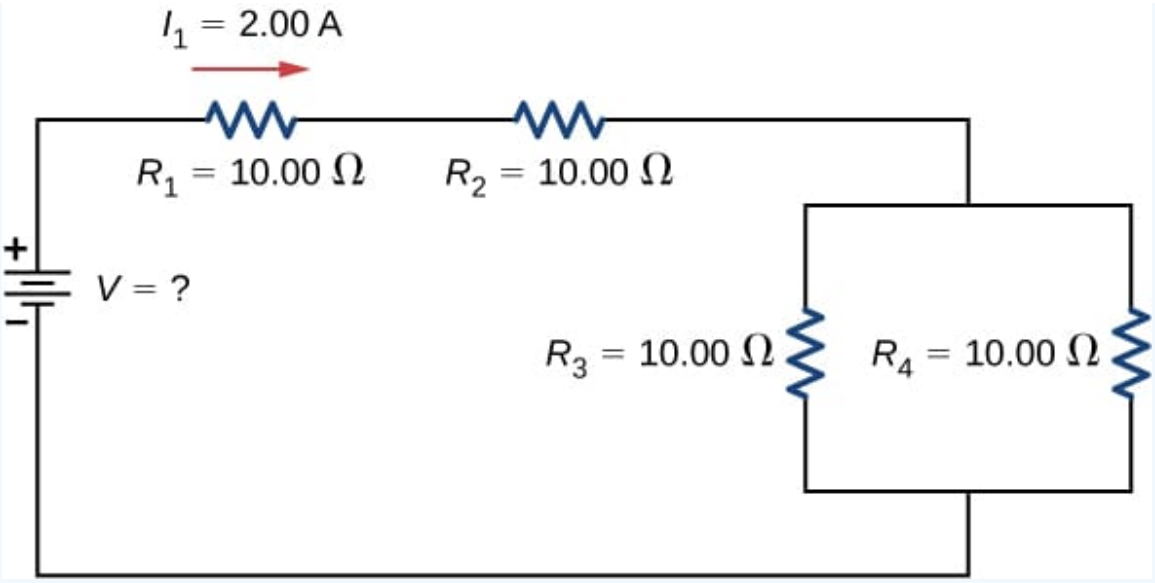


Figure 13.3.8: To find the unknown voltage, we must first find the equivalent resistance of the circuit.

1. Draw a clear circuit diagram (Figure 13.3.8).
2. The unknown is the voltage of the battery. In order to find the voltage supplied by the battery, the equivalent resistance must be found.
3. In this circuit, we already know that the resistors R_1 and R_2 are in series and the resistors R_3 and R_4 are in parallel. The equivalent resistance of the parallel configuration of the resistors R_3 and R_4 is in series with the series configuration of resistors R_1 and R_2.
4. The voltage supplied by the battery can be found by multiplying the current from the battery and the equivalent resistance of the circuit. The current from the battery is equal to the current through R_1 and is equal to 2.00 A. We need to find the equivalent resistance by reducing the circuit. To reduce the circuit, first consider the two resistors in parallel. The equivalent resistance is

$$R_{34} = \left(\frac{1}{10.00\,\Omega} + \frac{1}{10.00\,\Omega}\right)^{-1} = 5.00\,\Omega.$$

   This parallel combination is in series with the other two resistors, so the equivalent resistance of the circuit is $R_{eq} = R_1 + R_2 + R_{34} = 25.00\ \Omega$. The voltage supplied by the battery is therefore $V = IR_{eq} = 2.00\ \text{A}(25.00\ \Omega) = 50.00\ \text{V}$.
5. One way to check the consistency of your results is to calculate the power supplied by the battery and the power dissipated by the resistors. The power supplied by the battery is $P_{batt} = IV = 100.00\ \text{W}$.

Since they are in series, the current through R_2 equals the current through R_1. Since $R_3 = R_4$, the current through each will be 1.00 Amps. The power dissipated by the resistors is equal to the sum of the power dissipated by each resistor:

$$\begin{aligned} P &= I_1^2 R_1 + I_2^2 R_2 + I_3^2 R_3 + I_4^2 R_4 \\ &= 40.00\,\text{W} + 40.00\,\text{W} + 10.00\,\text{W} + 10.00\,\text{W} = 100.\,\text{W}. \end{aligned}$$

Since the power dissipated by the resistors equals the power supplied by the battery, our solution seems consistent.

Significance

If a problem has a combination of series and parallel, as in this example, it can be reduced in steps by using the preceding problem-solving strategy and by considering individual groups of series or parallel connections. When finding R_{eq} for a parallel connection, the reciprocal must be taken with care. In addition, units and numerical results must be reasonable. Equivalent series resistance should be greater, whereas equivalent parallel resistance should be smaller, for example. Power should be greater for the same devices in parallel compared with series, and so on.

Answers

Exercise 13.3.1

The equivalent resistance of nine bulbs connected in series is $9R$. The current is $I = V/9R$. If one bulb burns out, the equivalent resistance is $8R$, and the voltage does not change, but the current increases ($I = V/8R$). As more bulbs burn out, the current becomes even higher. Eventually, the current becomes too high, burning out the shunt.

Exercise $13.3.2A$

The equivalent of the series circuit would be $R_{eq} = 1.00\ \Omega + 2.00\ \Omega + 2.00\ \Omega = 5.00\ \Omega$, which is higher than the equivalent resistance of the parallel circuit $R_{eq} = 0.50\ \Omega$. The equivalent resistor of any number of resistors is always higher than the equivalent resistance of the same resistors connected in parallel. The current through the series circuit would be $I = \frac{3.00\ \text{V}}{5.00\ \Omega} = 0.60\ \text{A}$, which is lower than the sum of the currents through each resistor in the parallel circuit, $I = 6.00\ \text{A}$. This is not surprising since the equivalent resistance of the series circuit is higher. The current through a series connection of any number of resistors will always be lower than the current into a parallel connection of the same resistors, since the equivalent resistance of the series circuit will be higher than the parallel circuit. The power dissipated by the resistors in series would be $P = 1.800\ \text{W}$, which is lower than the power dissipated in the parallel circuit $P = 18.00\ \text{W}$.

Exercise $13.3.2B$

A river, flowing horizontally at a constant rate, splits in two and flows over two waterfalls. The water molecules are analogous to the electrons in the parallel circuits. The number of water molecules that flow in the river and falls must be equal to the number of molecules that flow over each waterfall, just like sum of the current through each resistor must be equal to the current flowing into the parallel circuit. The water molecules in the river have energy due to their motion and height. The potential energy of the water molecules in the river is constant due to their equal heights. This is analogous to the constant change in voltage across a parallel circuit. Voltage is the potential energy across each resistor.

The analogy quickly breaks down when considering the energy. In the waterfall, the potential energy is converted into kinetic energy of the water molecules. In the case of electrons flowing through a resistor, the potential drop is converted into heat and light, not into the kinetic energy of the electrons.

Exercise $13.3.3$

All the overhead lighting circuits are in parallel and connected to the main supply line, so when one bulb burns out, all the overhead lighting does not go dark. Each overhead light will have at least one switch in series with the light, so you can turn it on and off.

A refrigerator has a compressor and a light that goes on when the door opens. There is usually only one cord for the refrigerator to plug into the wall. The circuit containing the compressor and the circuit containing the lighting circuit are in parallel, but there is a switch in series with the light. A thermostat controls a switch that is in series with the compressor to control the temperature of the refrigerator.

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13.4: Kirchhoff's Rules

Learning Objectives

By the end of the section, you will be able to:

- State Kirchhoff's junction rule
- State Kirchhoff's loop rule
- Analyze complex circuits using Kirchhoff's rules

We have just seen that some circuits may be analyzed by reducing a circuit to a single voltage source and an equivalent resistance. Many complex circuits cannot be analyzed with the series-parallel techniques developed in the preceding sections. In this section, we elaborate on the use of Kirchhoff's rules to analyze more complex circuits. For example, the circuit in Figure 13.4.1 is known as a **multi-loop circuit**, which consists of junctions. A junction, also known as a node, is a connection of three or more wires. In this circuit, the previous methods cannot be used, because not all the resistors are in clear series or parallel configurations that can be reduced. Give it a try. The resistors R_1 and R_2 are in series and can be reduced to an equivalent resistance. The same is true of resistors R_4 and R_5. But what do you do then?

Even though this circuit cannot be analyzed using the methods already learned, two circuit analysis rules can be used to analyze any circuit, simple or complex. The rules are known as **Kirchhoff's rules**, after their inventor Gustav **Kirchhoff** (1824–1887).

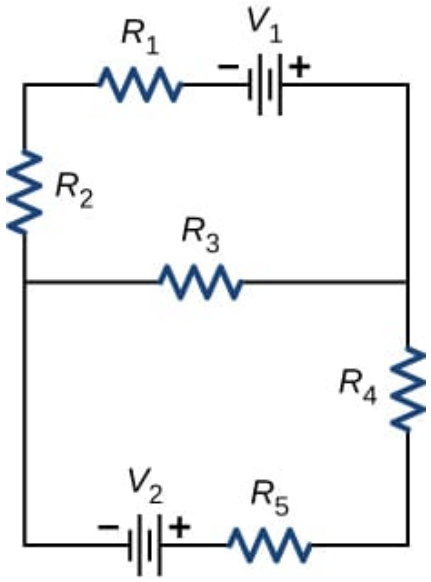


Figure 13.4.1: This circuit cannot be reduced to a combination of series and parallel connections. However, we can use Kirchhoff's rules to analyze it.

Kirchhoff's Rules: The Junction Rule and the Loop Rule

- Kirchhoff's first rule—the junction rule. The sum of all currents entering a junction must equal the sum of all currents leaving the junction:

$$\sum I_{in} = \sum I_{out}.$$

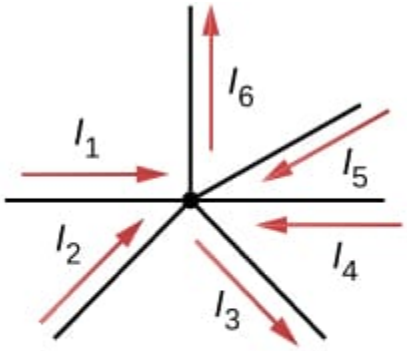
- Kirchhoff's second rule—the loop rule. The algebraic sum of changes in potential around any closed circuit path (loop) must be zero:

$$\sum V = 0.$$

We now provide explanations of these two rules, followed by problem-solving hints for applying them and a worked example that uses them.

Kirchhoff's First Rule

Kirchhoff's first rule (the **junction rule**) applies to the charge entering and leaving a junction (Figure 13.4.2). As stated earlier, a junction, or node, is a connection of three or more wires. Current is the flow of charge, and charge is conserved; thus, whatever charge flows into the junction must flow out.



$$\sum I_{\text{in}} = \sum I_{\text{out}}$$
$$I_1 + I_2 + I_4 + I_5 = I_3 + I_6$$

Figure 13.4.2: Charge must be conserved, so the sum of currents into a junction must be equal to the sum of currents out of the junction.

Although it is an over-simplification, an analogy can be made with water pipes connected in a plumbing junction. If the wires in Figure 13.4.2 were replaced by water pipes, and the water was assumed to be incompressible, the volume of water flowing into the junction must equal the volume of water flowing out of the junction.

Kirchhoff's Second Rule

Kirchhoff's second rule (the **loop rule**) applies to potential differences. The loop rule is stated in terms of potential **V** rather than potential energy, but the two are related since $U = qV$. In a closed loop, whatever energy is supplied by a voltage source, the energy must be transferred into other forms by the devices in the loop, since there are no other ways in which energy can be transferred into or out of the circuit. Kirchhoff's loop rule states that the algebraic sum of potential differences, including voltage supplied by the voltage sources and resistive elements, in any loop must be equal to zero. For example, consider a simple loop with no junctions, as in Figure 13.4.3.

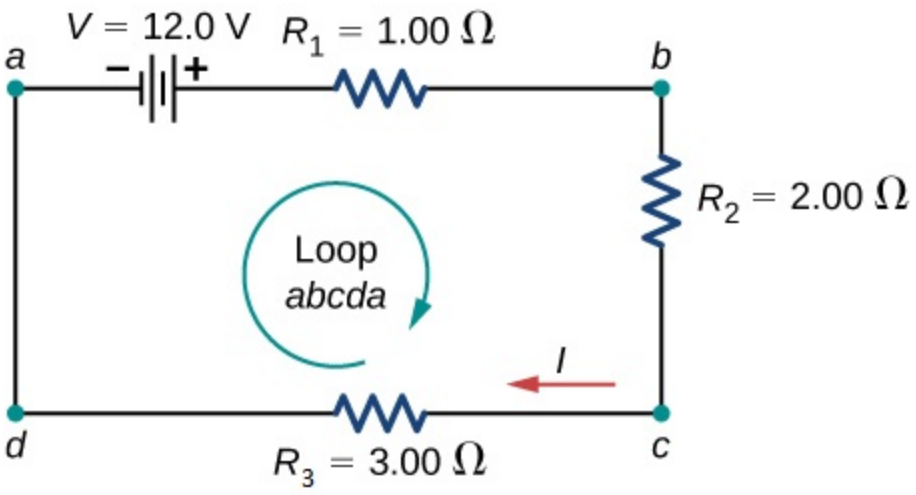


Figure 13.4.3: A simple loop with no junctions. Kirchhoff's loop rule states that the algebraic sum of the voltage differences is equal to zero.

The circuit consists of a voltage source and three external load resistors. The labels **a**, **b**, **c**, and **d** serve as references, and have no other significance. The usefulness of these labels will become apparent soon. The loop is designated as Loop **abcda**, and the labels help keep track of the voltage differences as we travel around the circuit. Start at point **a** and travel to point **b**. The voltage of the voltage source is added to the equation and the potential drop of the resistor R_1 is subtracted. From point **b** to **c**, the potential drop across R_2 is subtracted. From **c** to **d**, the potential drop across R_3 is subtracted. From points **d** to **a**, nothing is done because there are no components.

Figure 13.4.4 shows a graph of the voltage as we travel around the loop. Voltage increases as we cross the battery, whereas voltage decreases as we travel across a resistor. The **potential drop**, or change in the electric potential, is equal to the current through the resistor times the resistance of the resistor. Since the wires have negligible resistance, the voltage remains constant as we cross the wires connecting the components.

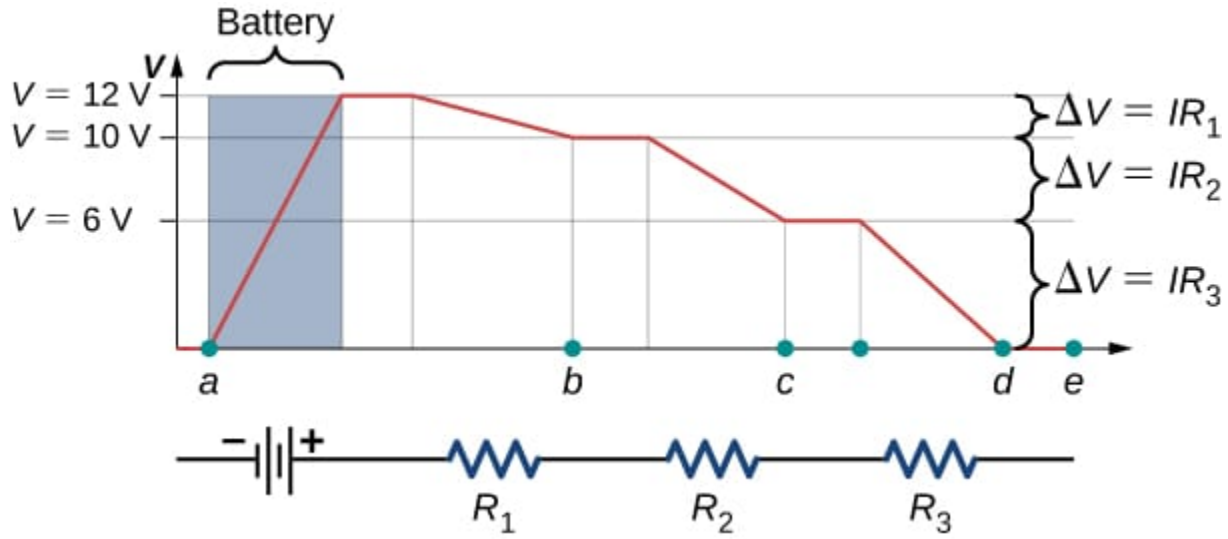


Figure 13.4.4: A voltage graph as we travel around the circuit. The voltage increases as we cross the battery and decreases as we cross each resistor. Since the resistance of the wire is quite small, we assume that the voltage remains constant as we cross the wires connecting the components.

Then Kirchhoff's loop rule states

$$V - IR_1 - IR_2 - IR_3 = 0.$$

The loop equation can be used to find the current through the loop:

$$I = \frac{V}{R_1 + R_2 + R_3} = \frac{12.00\,V}{1.00\,\Omega + 2.00\,\Omega + 3.00\,\Omega} = 2.00\,A.$$

This loop could have been analyzed using the previous methods, but we will demonstrate the power of Kirchhoff's method in the next section.

Applying Kirchhoff's Rules

By applying Kirchhoff's rules, we generate a set of linear equations that allow us to find the unknown values in circuits. These may be currents, voltages, or resistances. Each time a rule is applied, it produces an equation. If there are as many independent equations as unknowns, then the problem can be solved.

Using Kirchhoff's method of analysis requires several steps, as listed in the following procedure.

Problem-Solving Strategy: Kirchhoff's Rules

1. Label points in the circuit diagram using lowercase letters *a*, *b*, *c*, ... These labels simply help with orientation.
2. Locate the junctions in the circuit. The junctions are points where three or more wires connect. Label each junction with the currents and directions into and out of it. Make sure at least one current points into the junction and at least one current points out of the junction.
3. Choose the loops in the circuit. Every component must be contained in at least one loop, but a component may be contained in more than one loop.
4. Apply the junction rule. Again, some junctions should not be included in the analysis. You need only use enough nodes to include every current.
5. Apply the loop rule. Use the map in Figure 13.4.5.

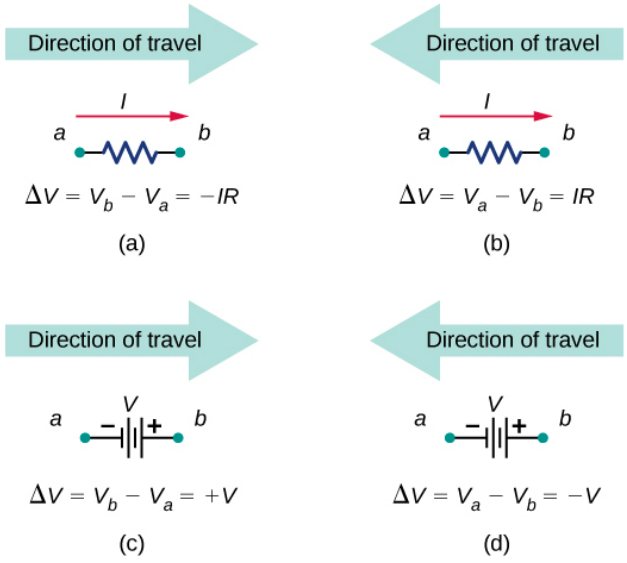


Figure 13.4.5: Each of these resistors and voltage sources is traversed from *a* to *b*. (a) When moving across a resistor in the same direction as the current flow, subtract the potential drop. (b) When moving across a resistor in the opposite direction as the current flow, add the potential drop. (c) When moving across a voltage source from the negative terminal to the positive terminal, add the potential drop. (d) When moving across a voltage source from the positive terminal to the negative terminal, subtract the potential drop.

Let's examine some steps in this procedure more closely. When locating the junctions in the circuit, do not be concerned about the direction of the currents. If the direction of current flow is not obvious, choosing any direction is sufficient as long as at least one current points into the junction and at least one current points out of the junction. If the arrow is in the opposite direction of the conventional current flow, the result for the current in question will be negative but the answer will still be correct.

The number of nodes depends on the circuit. Each current should be included in a node and thus included in at least one junction equation. Do not include nodes that are not linearly independent, meaning nodes that contain the same information.

Consider Figure 13.4.6. There are two junctions in this circuit: Junction *b* and Junction *e*. Points *a*, *c*, *d*, and *f* are not junctions, because a junction must have three or more connections. The equation for Junction *b* is $I_1 = I_2 + I_3$, and the equation for Junction *e* is $I_2 + I_3 = I_1$. These are equivalent equations, so it is necessary to keep only one of them.

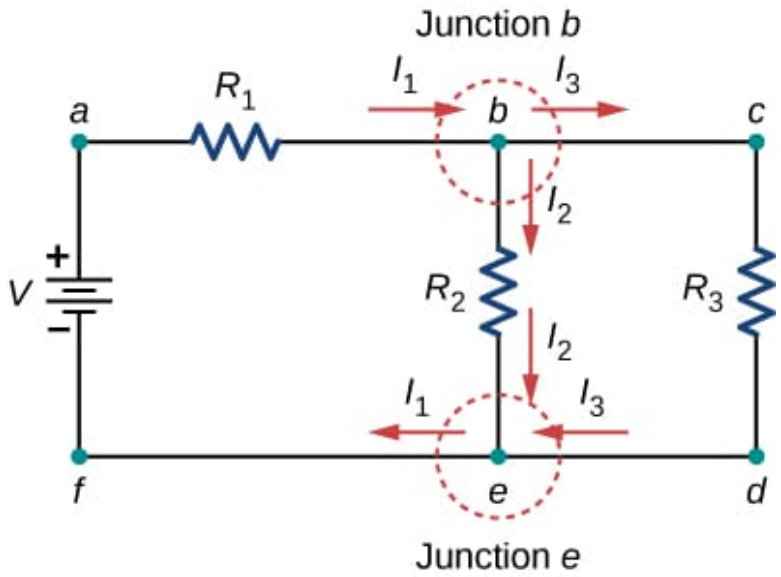


Figure 13.4.6: At first glance, this circuit contains two junctions, Junction *b* and Junction *e*, but only one should be considered because their junction equations are equivalent.

When choosing the loops in the circuit, you need enough loops so that each component is covered once, without repeating loops. Figure 13.4.7 shows four choices for loops to solve a sample circuit; choices (a), (b), and (c) have a sufficient amount of loops to solve the circuit completely. Option (d) reflects more loops than necessary to solve the circuit.

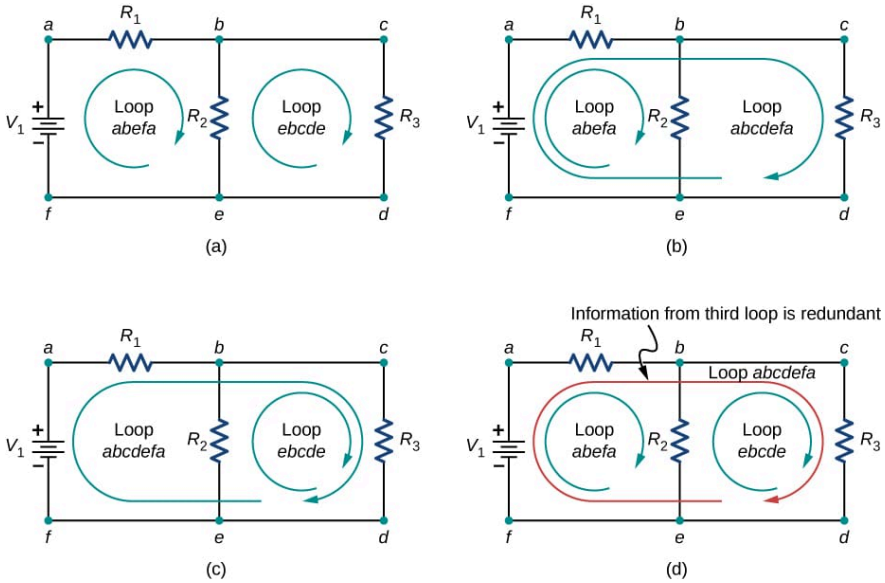


Figure 13.4.7: Panels (a)–(c) are sufficient for the analysis of the circuit. In each case, the two loops shown contain all the circuit elements necessary to solve the circuit completely. Panel (d) shows three loops used, which is more than necessary. Any two loops in the system will contain all information needed to solve the circuit. Adding the third loop provides redundant information.

Consider the circuit in Figure 13.4.8*a*. Let us analyze this circuit to find the current through each resistor. First, label the circuit as shown in part (b).

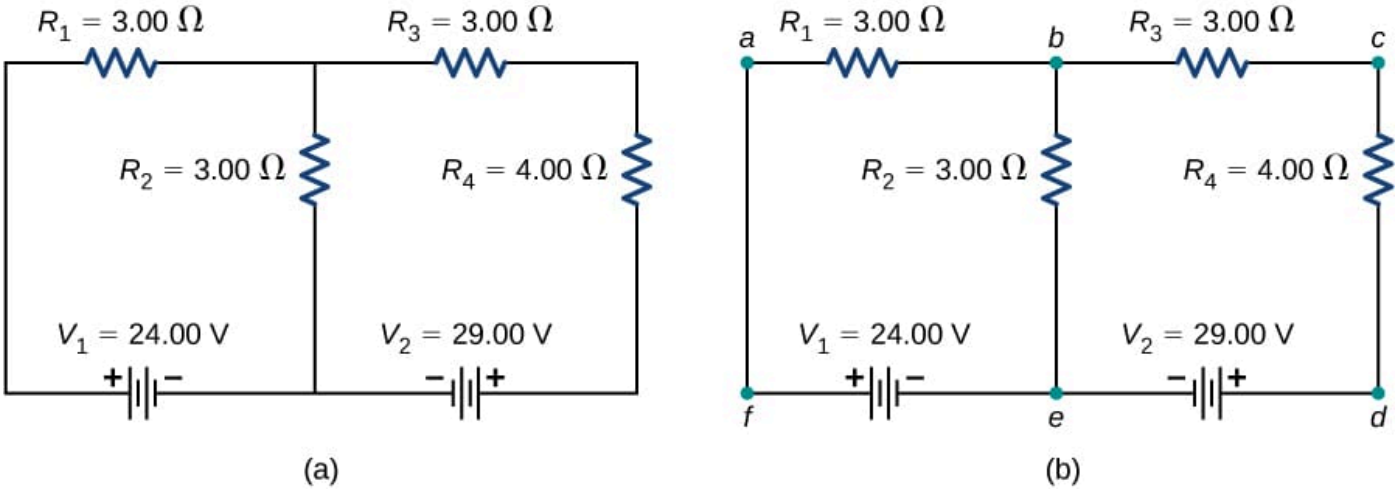


Figure 13.4.8: (a) A multi-loop circuit. (b) Label the circuit to help with orientation.

Next, determine the junctions. In this circuit, points *b* and *e* each have three wires connected, making them junctions. Start to apply Kirchhoff's junction rule $(\sum I_{in} = \sum I_{out})$ by drawing arrows representing the currents and labeling each arrow, as shown in Figure 13.4.9. Junction *b* shows that $I_1 = I_2 + I_3$ and Junction *e* shows that $I_2 + I_3 = I_1$. Since Junction *e* gives the same information of Junction *b*, it can be disregarded. This circuit has three unknowns, so we need three linearly independent equations to analyze it.

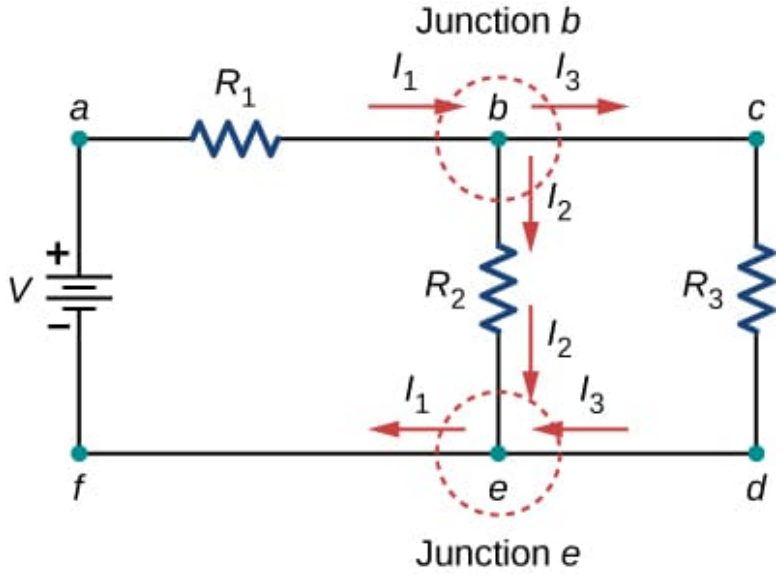


Figure 13.4.9: (a) This circuit has two junctions, labeled *b* and *e*, but only node *b* is used in the analysis. (b) Labeled arrows represent the currents into and out of the junctions.

Next we need to choose the loops. In Figure 13.4.10, Loop *abefa* includes the voltage source V_1 and resistors R_1 and R_2. The loop starts at point *a*, then travels through points *b*, *e*, and *f*, and then back to point *a*. The second loop, Loop *ebcde*, starts at point *e* and includes resistors R_2 and R_3, and the voltage source V_2.

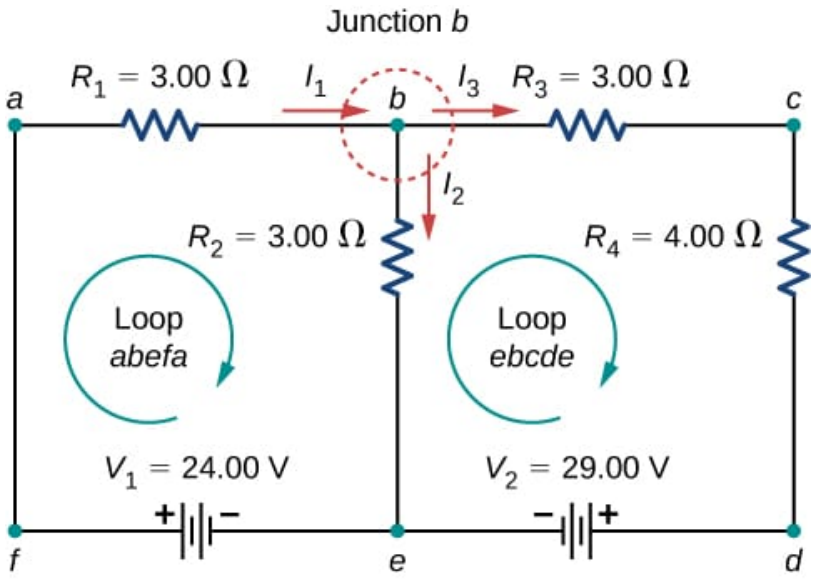


Figure 13.4.10: Choose the loops in the circuit.

Now we can apply Kirchhoff's loop rule, using the map in Figure 13.4.5. Starting at point *a* and moving to point *b*, the resistor R_1 is crossed in the same direction as the current flow I_1, so the potential drop I_1R_1 is subtracted. Moving from point *b* to point ***e***, the resistor R_2 is crossed in the same direction as the current flow I_2 so the potential drop I_2R_2 is subtracted. Moving from point *e* to point *f*, the voltage source V_1 is crossed from the negative terminal to the positive terminal, so V_1 is added. There are no components between points *f* and *a*. The sum of the voltage differences must equal zero:

$$\text{Loop } abefa: \ -I_1R_1 - I_2R_2 + V_1 = 0 \text{ or } V_1 = I_1R_1 + I_2R_2.$$

Finally, we check loop *ebcde*. We start at point *e* and move to point *b*, crossing R_2 in the opposite direction as the current flow I_2. The potential drop I_2R_2 is added. Next, we cross R_3 and R_4 in the same direction as the current flow I_3 and subtract the potential drops I_3R_3 and I_3R_4. Note that the current is the same through resistors R_3 and R_4, because they are connected in series. Finally, the voltage source is crossed from the positive terminal to the negative terminal, and the voltage source V_2 is subtracted. The sum of these voltage differences equals zero and yields the loop equation

$$\text{Loop } ebcde: \ I_2R_2 - I_3(R_3 + R_4) - V_2 = 0.$$

We now have three equations, which we can solve for the three unknowns.

$$\text{Junction } b: \ I_1 - I_2 - I_3 = 0. \tag{13.4.1}$$

$$\text{Loop } abefa: \ I_1R_1 + I_2R_2 = V_1. \tag{13.4.2}$$

$$\text{Loop } ebcde: \ I_2R_2 - I_3(R_3 + R_4) = V_2. \tag{13.4.3}$$

To solve the three equations for the three unknown currents, start by eliminating current I_2. First add Equation 13.4.1 times R_2 to Equation 13.4.2. The result is Equation 13.4.4:

$$(R_1 + R_2)I_1 - R_2I_3 = V_1.$$

$$6\,\Omega I_1 - 3\Omega I_3 = 24\,\text{V}. \tag{13.4.4}$$

Next, subtract Equation 13.4.3 from Equation 13.4.2. The result is Equation 13.4.5:

$$I_1R_1 + I_3(R_3 + R_4) = V_1 - V_2.$$

$$3\Omega I_1 + 7\Omega I_3 = -5\,\text{V}. \tag{13.4.5}$$

We can solve Equations 13.4.4 and 13.4.5 for current I_1. Adding seven times Equation 13.4.4 and three times Equation 13.4.5 results in $51\ \Omega I_1 = 153$ V, or $I_1 = 3.00$ A. Using Equation 13.4.4 results in $I_3 = -2.00$ A. Finally, Equation 13.4.1 yields $I_2 = I_1 - I_3 = 5.00$ A. One way to check that the solutions are consistent is to check the power supplied by the voltage sources and the power

dissipated by the resistors:

$$P_{\text{in}} = I_1 V_1 + I_3 V_2 = 130\,\text{W},$$

$$P_{\text{out}} = I_1^2 R_1 + I_2^2 R_2 + I_3^2 R_3 + I_3^2 R_4 = 130\,\text{W}.$$

Note that the solution for the current I_3 is negative. This is the correct answer, but suggests that the arrow originally drawn in the junction analysis is the direction opposite of conventional current flow. The power supplied by the second voltage source is 58 W and not –58 W.

Example 13.4.1: Calculating Current by Using Kirchhoff's Rules

Find the currents flowing in the circuit in Figure 13.4.11.

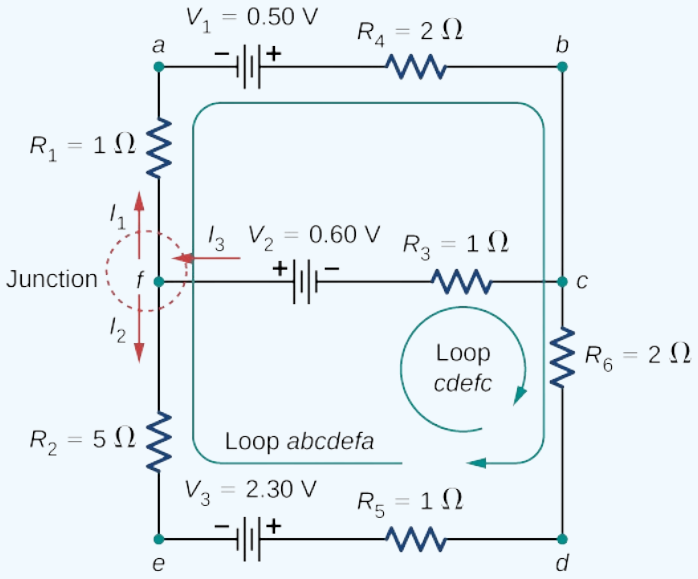


Figure 13.4.11: This circuit is combination of series and parallel configurations of resistors and voltage sources. This circuit cannot be analyzed using the techniques discussed in Electromotive Force but can be analyzed using Kirchhoff's rules.

Strategy

This circuit is sufficiently complex that the currents cannot be found using Ohm's law and the series-parallel techniques—it is necessary to use Kirchhoff's rules. Currents have been labeled I_1, I_2, and I_3 in the figure, and assumptions have been made about their directions. Locations on the diagram have been labeled with letters *a* through *h*. In the solution, we apply the junction and loop rules, seeking three independent equations to allow us to solve for the three unknown currents.

Solution

Applying the junction and loop rules yields the following three equations. We have three unknowns, so three equations are required.

$$\text{Junction } c: \ I_1 + I_2 = I_3.$$

$$\text{Loop } abcdefa: \ I_1(R_1 + R_4) - I_2(R_2 + R_5 + R_6) = V_1 - V_3.$$

$$\text{Loop } cdefc: \ I_2(R_2 + R_5 + R_6) + I_3 R_3 = V_2 + V_3.$$

Simplify the equations by placing the unknowns on one side of the equations.

$$\text{Junction } c: \ I_1 + I_2 - I_3 = 0.$$

$$\text{Loop } abcdefa: \ I_1(3\Omega) - I_2(8\Omega) = 0.5\,\text{V} - 2.30\,\text{V}.$$

$$\text{Loop } cdefc: \; I_2(8\Omega) + I_3(1\Omega) = 0.6\,\text{V} + 2.30\,\text{V}.$$

Simplify the equations. The first loop equation can be simplified by dividing both sides by 3.00. The second loop equation can be simplified by dividing both sides by 6.00.

$$\text{Junction } c: \; I_1 + I_2 - I_3 = 0.$$

$$\text{Loop } abcdefa: \; I_1(3\Omega) - I_2(8\Omega) = -1.8\,\text{V}.$$

$$\text{Loop } cdefc: \; I_2(8\Omega) + I_3(1\Omega) = 2.90\,\text{V}.$$

The results are

$$I_1 = 0.20\,\text{A},\; I_2 = 0.30\,\text{A},\; I_3 = 0.50\,\text{A}.$$

Significance

A method to check the calculations is to compute the power dissipated by the resistors and the power supplied by the voltage sources:

$$P_{R_1} = I_1^2 R_1 = 0.04\,\text{W}.$$

$$P_{R_2} = I_2^2 R_2 = 0.45\,\text{W}.$$

$$P_{R_3} = I_3^2 R_3 = 0.25\,\text{W}.$$

$$P_{R_4} = I_1^2 R_4 = 0.08\,\text{W}.$$

$$P_{R_5} = I_2^2 R_5 = 0.09\,\text{W}.$$

$$P_{R_6} = I_2^2 R_1 = 0.18\,\text{W}.$$

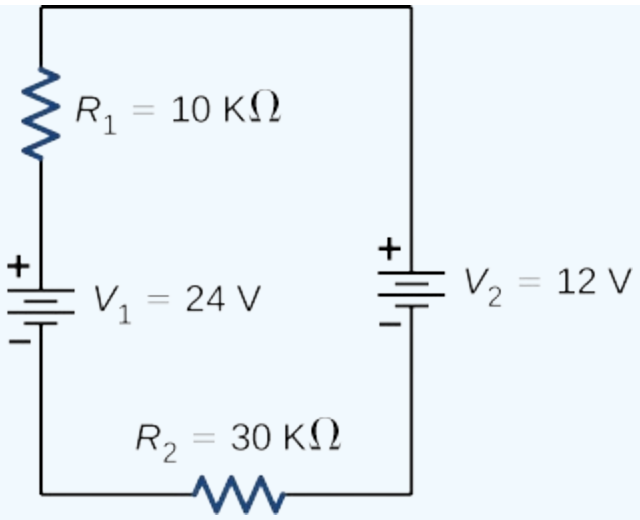
$$P_{\text{dissipated}} = 1.09\,\text{W}.$$

$$P_{\text{source}} = I_1 V_1 + I_2 V_3 + I_3 V_2 = 0.10\,\text{W} + 0.69\,\text{W} + 0.30\,\text{W} = 1.09\,\text{W}.$$

The power supplied equals the power dissipated by the resistors.

Exercise 13.4.1

In considering the following schematic and the power supplied and consumed by a circuit, will a voltage source always provide power to the circuit, or can a voltage source consume power?



Answer available at the end of this section.

Example 13.4.2: Calculating Current by Using Kirchhoff's Rules

Find the current flowing in the circuit in Figure 13.4.12.

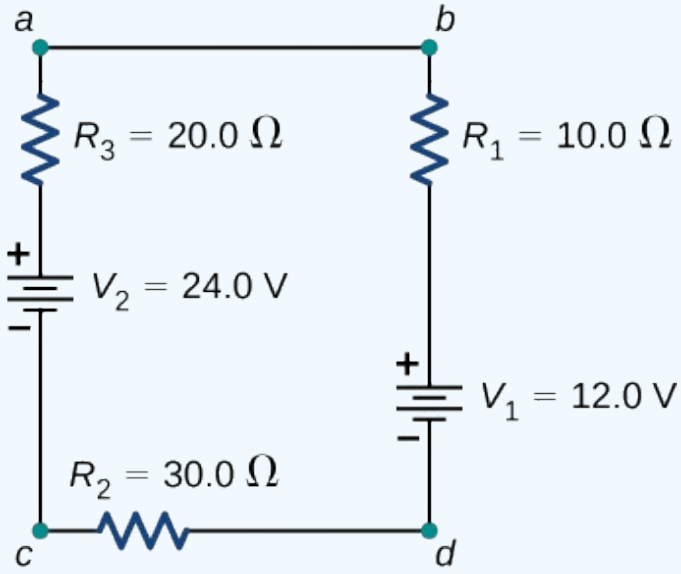


Figure 13.4.12: This circuit consists of three resistors and two batteries connected in series. Note that the batteries are connected with opposite polarities.

Strategy

This circuit can be analyzed using Kirchhoff's rules. There is only one loop and no nodes. Choose the direction of current flow. For this example, we will use the clockwise direction from point *a* to point *b*. Consider Loop *abcda* and use Figure 13.4.5 to write the loop equation. Note that according to Figure 13.4.5, battery V_1 will be added and battery V_2 will be subtracted.

Solution

Applying the junction rule yields the following three equations. We have one unknown, so one equation is required:

$$Loop\, abcda: \ -IR_1 - V_1 - IR_2 + V_2 - IR_3 = 0.$$

Simplify the equations by placing the unknowns on one side of the equations. Use the values given in the figure.

$$I(R_1 + R_2 + R_3) = V_2 - V_1.$$

$$I = \frac{V_2 - V_1}{R_1 + R_2 + R_3} = \frac{24\,\text{V} - 12\,\text{V}}{10.0\,\Omega + 30.0\,\Omega + 10.0\,\Omega} = 0.20\,\text{A}.$$

Significance

The power dissipated or consumed by the circuit equals the power supplied to the circuit, but notice that the current in the battery V_1 is flowing through the battery from the positive terminal to the negative terminal and consumes power.

$$P_{R_1} = I^2 R_1 = 0.40\,\text{W}$$

$$P_{R_2} = I^2 R_2 = 1.20\,\text{W}$$

$$P_{R_3} = I^2 R_3 = 0.80\,\text{W}$$

$$P_{V_1} = IV_1 = 2.40\,\text{W}$$

$$P_{dissipated} = 4.80\,\text{W}$$

$$P_{source} = IV_2 = 4.80\,\text{W}$$

The power supplied equals the power dissipated by the resistors and consumed by the battery V_1.

Exercise 13.4.2

When using Kirchhoff's laws, you need to decide which loops to use and the direction of current flow through each loop. In analyzing the circuit in Example 13.4.2, the direction of current flow was chosen to be clockwise, from point *a* to point *b*. How would the results change if the direction of the current was chosen to be counterclockwise, from point *b* to point *a*?

Answer available at the end of this section.

Multiple Voltage Sources

Many devices require more than one battery. Multiple voltage sources, such as batteries, can be connected in series configurations, parallel configurations, or a combination of the two.

In series, the positive terminal of one battery is connected to the negative terminal of another battery. Any number of voltage sources, including batteries, can be connected in series. Two batteries connected in series are shown in Figure 13.4.13. Using Kirchhoff's loop rule for the circuit in part (b) gives the result

$$\varepsilon_1 - Ir_1 + \varepsilon_2 - Ir_2 - IR = 0,$$

$$[(\varepsilon_1 + \varepsilon_2) - I(r_1 + r_2)] - IR = 0.$$

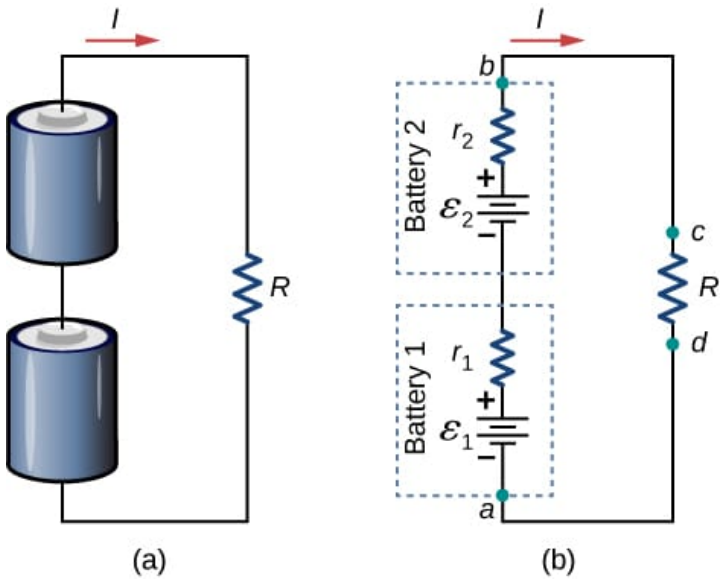


Figure 13.4.13: (a) Two batteries connected in series with a load resistor. (b) The circuit diagram of the two batteries and the load resistor, with each battery modeled as an idealized emf source and an internal resistance.

When voltage sources are in series, their internal resistances can be added together and their emfs can be added together to get the total values. Series connections of voltage sources are common—for example, in flashlights, toys, and other appliances. Usually, the cells are in series in order to produce a larger total emf. In Figure 13.4.13, the terminal voltage is

$$\begin{aligned} V_{\text{terminal}} &= (\varepsilon_1 - Ir_1) + (\varepsilon_2 - Ir_2) \\ &= [(\varepsilon_1 + \varepsilon_2) - I(r_1 + r_2) - I(r_1 + r_2)] \\ &= (\varepsilon_1 + \varepsilon_2) + Ir_{\text{eq}}. \end{aligned}$$

Note that the same current I is found in each battery because they are connected in series. The disadvantage of series connections of cells is that their internal resistances are additive.

Batteries are connected in series to increase the voltage supplied to the circuit. For instance, an LED flashlight may have two AAA cell batteries, each with a terminal voltage of 1.5 V, to provide 3.0 V to the flashlight.

Terminal Voltage for Batteries in Series

Any number of batteries can be connected in series. For *N* batteries in series, the terminal voltage is equal to

$$\begin{aligned} V_{\text{terminal}} &= (\varepsilon_1 + \varepsilon_2 + \ldots + \varepsilon_{N-1} + \varepsilon_N) - I(r_1 + r_2 + \ldots + r_{N-1} + r_N) \\ &= \sum_{i=1}^{N} \varepsilon_i - Ir_{\text{eq}} \end{aligned}$$

where the equivalent resistance is

$$r_{\text{eq}} = \sum_{i=1}^{N} r_i.$$

When a load is placed across voltage sources in series, as in Figure 13.4.14, we can find the current:

$$\begin{aligned} (\varepsilon_1 - Ir_1) + (\varepsilon_2 - Ir_2) &= IR, \\ Ir_1 + Ir_2 + IR &= \varepsilon_1 + \varepsilon_2, \\ I &= \frac{\varepsilon_1 + \varepsilon_2}{r_1 + r_2 + R}. \end{aligned}$$

As expected, the internal resistances increase the equivalent resistance.

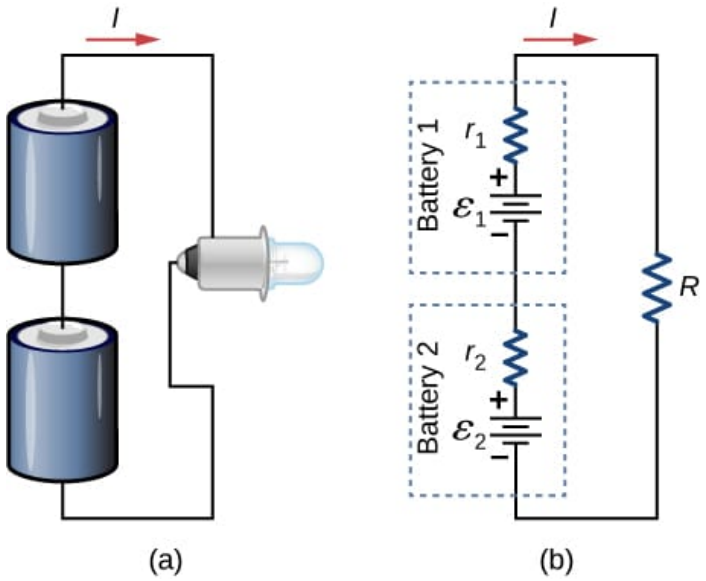


Figure 13.4.14: Two batteries connect in series to an LED bulb, as found in a flashlight.

Voltage sources, such as batteries, can also be connected in parallel. Figure 13.4.15 shows two batteries with identical emfs in parallel and connected to a load resistance. When the batteries are connect in parallel, the positive terminals are connected together and the negative terminals are connected together, and the load resistance is connected to the positive and negative terminals. Normally, voltage sources in parallel have identical emfs. In this simple case, since the voltage sources are in parallel, the total emf is the same as the individual emfs of each battery.

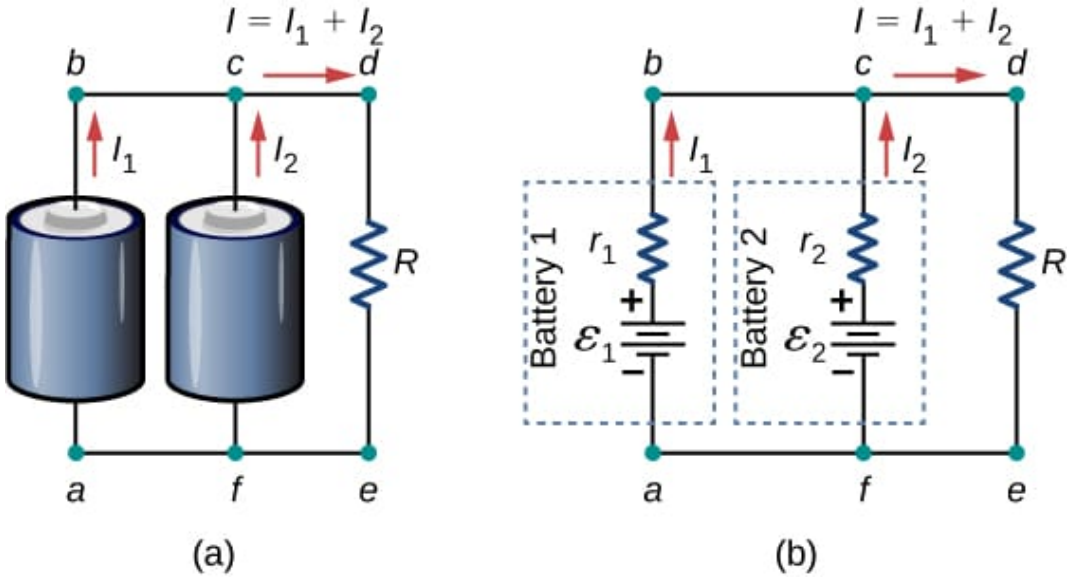


Figure 13.4.15: (a) Two batteries connect in parallel to a load resistor. (b) The circuit diagram shows the shows battery as an emf source and an internal resistor. The two emf sources have identical emfs (each labeled by ϵ) connected in parallel that produce the same emf.

Consider the Kirchhoff analysis of the circuit in Figure 13.4.15*b*. There are two loops and a node at point *b* and $\varepsilon = \varepsilon_1 = \varepsilon_2$.

Node *b*: $I_1 + I_2 - I = 0$.

- Loop *abcfa*: $\varepsilon_2 - I_1 r_1 + I_2 r_2 - \varepsilon = 0,\ \ I_1 r_1 = I_2 r_2$.
- Loop *fcdef*: $\varepsilon_2 - I_2 r_2 - IR = 0,\ \varepsilon - I_2 r_2 - IR = 0$.

Solving for the current through the load resistor results in $I = \frac{\varepsilon}{r_{eq}+R}$, where $r_{eq} = \left(\frac{1}{r_1} + \frac{1}{r_2}\right)^{-1}$. The terminal voltage is equal to the

potential drop across the load resistor $IR = \left(\frac{\varepsilon}{r_{eq}+R}\right)$.

The parallel connection reduces the internal resistance and thus can produce a larger current.

Terminal Voltage for Batteries in Parallel

Any number of batteries can be connected in parallel. For N batteries in parallel, the terminal voltage is equal to

$$V_{\text{terminal}} = \epsilon - I\left(\frac{1}{r_1} + \frac{1}{r_2} + ... + \frac{1}{r_{N-1}} + \frac{1}{r_N}\right)^{-1} = \epsilon - Ir_{\text{eq}} \tag{13.4.6}$$

where the equivalent resistance is

$$r_{\text{eq}} = \left(\sum_{i=1}^{N} \frac{1}{r_i}\right)^{-1}.$$

As an example, some diesel trucks use two 12-V batteries in parallel; they produce a total emf of 12 V but can deliver the larger current needed to start a diesel engine.

In summary, the terminal voltage of batteries in series is equal to the sum of the individual emfs minus the sum of the internal resistances times the current. When batteries are connected in parallel, they usually have equal emfs and the terminal voltage is equal to the emf minus the equivalent internal resistance times the current, where the equivalent internal resistance is smaller than the individual internal resistances. Batteries are connected in series to increase the terminal voltage to the load. Batteries are connected in parallel to increase the current to the load.

Solar Cell Arrays

Another example dealing with multiple voltage sources is that of combinations of solar cells--wired in both series and parallel combinations to yield a desired voltage and current. Photovoltaic generation, which is the conversion of sunlight directly into electricity, is based upon the photoelectric effect. The photoelectric effect is beyond the scope of this chapter and is covered in Photons and Matter Waves, but in general, photons hitting the surface of a solar cell create an electric current in the cell.

Most solar cells are made from pure silicon. Most single cells have a voltage output of about 0.5 V, while the current output is a function of the amount of sunlight falling on the cell (the incident solar radiation known as the insolation). Under bright noon sunlight, a current per unit area of about $100\ \text{mA/cm}^2$ of cell surface area is produced by typical single-crystal cells.

Individual solar cells are connected electrically in modules to meet electrical energy needs. They can be wired together in series or in parallel--connected like the batteries discussed earlier. A solar-cell array or module usually consists of between 36 and 72 cells, with a power output of 50 W to 140 W.

Solar cells, like batteries, provide a direct current (dc) voltage. Current from a dc voltage source is unidirectional. Most household appliances need an alternating current (ac) voltage.

Answers

Exercise 13.4.1

The circuit can be analyzed using Kirchhoff's loop rule. The first voltage source supplies power: $P_{\text{in}} = IV_1 = 7.20\ \text{mW}$. The second voltage source consumes power: $P_{\text{out}} = IV_2 + I^2R_1 + I^2R_2 = 7.2\ \text{mW}$.

Exercise 13.4.2

The current calculated would be equal to $I = -0.20\ \text{A}$ instead of $I = 0.20\ \text{A}$. The sum of the power dissipated and the power consumed would still equal the power supplied.

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13.5: Electrical Measuring Instruments

Learning Objectives

By the end of the section, you will be able to:

- Describe how to connect a voltmeter in a circuit to measure voltage
- Describe how to connect an ammeter in a circuit to measure current
- Describe the use of an ohmmeter

Ohm's law and Kirchhoff's method are useful to analyze and design electrical circuits, providing you with the voltages across, the current through, and the resistance of the components that compose the circuit. To measure these parameters require instruments, and these instruments are described in this section.

DC Voltmeters and Ammeters

Whereas voltmeter**s** measure voltage, ammeter**s** measure current. Some of the meters in automobile dashboards, digital cameras, cell phones, and tuner-amplifiers are actually voltmeters or ammeters (Figure 13.5.1). The internal construction of the simplest of these meters and how they are connected to the system they monitor give further insight into applications of series and parallel connections.



Figure 13.5.1: The fuel and temperature gauges (far right and far left, respectively) in this 1996 Volkswagen are voltmeters that register the voltage output of "sender" units. These units are proportional to the amount of gasoline in the tank and to the engine temperature. (credit: Christian Giersing)

Measuring Current with an Ammeter

To measure the current through a device or component, the ammeter is placed in series with the device or component. A series connection is used because objects in series have the same current passing through them. (See Figure 13.5.2, where the ammeter is represented by the symbol A.)

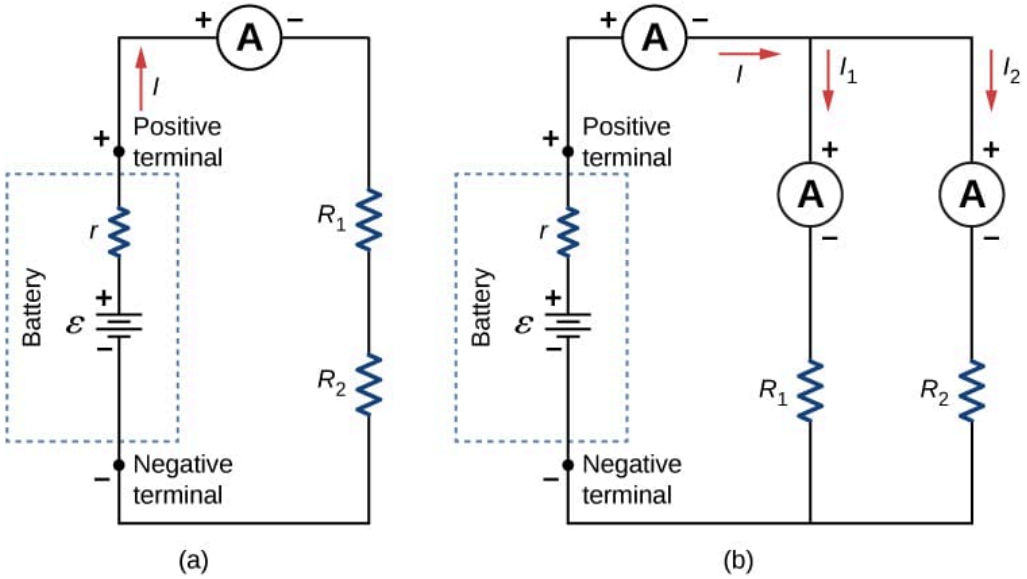


Figure 13.5.2: (a) When an ammeter is used to measure the current through two resistors connected in series to a battery, a single ammeter is placed in series with the two resistors because the current is the same through the two resistors in series. (b) When two resistors are connected in parallel with a battery, three meters, or three separate ammeter readings, are necessary to measure the current from the battery and through each resistor. The ammeter is connected in series with the component in question.

Ammeters need to have a very low resistance, a fraction of a milliohm. If the resistance is not negligible, placing the ammeter in the circuit would change the equivalent resistance of the circuit and modify the current that is being measured. Since the current in the circuit travels through the meter, ammeters normally contain a fuse to protect the meter from damage from currents which are too high.

Measuring Voltage with a Voltmeter

A voltmeter is connected in parallel with whatever device it is measuring. A parallel connection is used because objects in parallel experience the same potential difference. (See Figure 13.5.3, where the voltmeter is represented by the symbol V.)

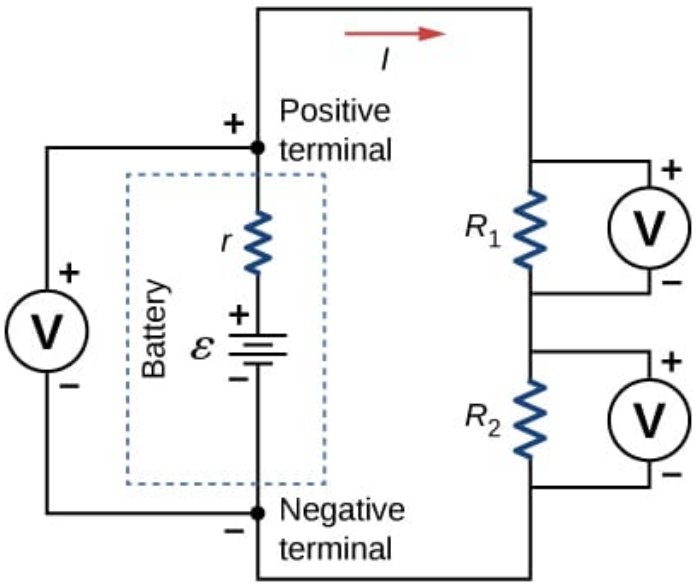


Figure 13.5.3: To measure potential differences in this series circuit, the voltmeter (V) is placed in parallel with the voltage source or either of the resistors. Note that terminal voltage is measured between the positive terminal and the negative terminal of the battery or voltage source. It is not possible to connect a voltmeter directly across the emf without including the internal resistance **r** of the battery.

Since voltmeters are connected in parallel, the voltmeter must have a very large resistance. Digital voltmeters convert the analog

voltage into a digital value to display on a digital readout (Figure 13.5.4). Inexpensive voltmeters have resistances on the order of $R_M = 10\ M\Omega$, whereas high-precision voltmeters have resistances on the order of $R_M = 10\ G\Omega$. The value of the resistance may vary, depending on which scale is used on the meter.

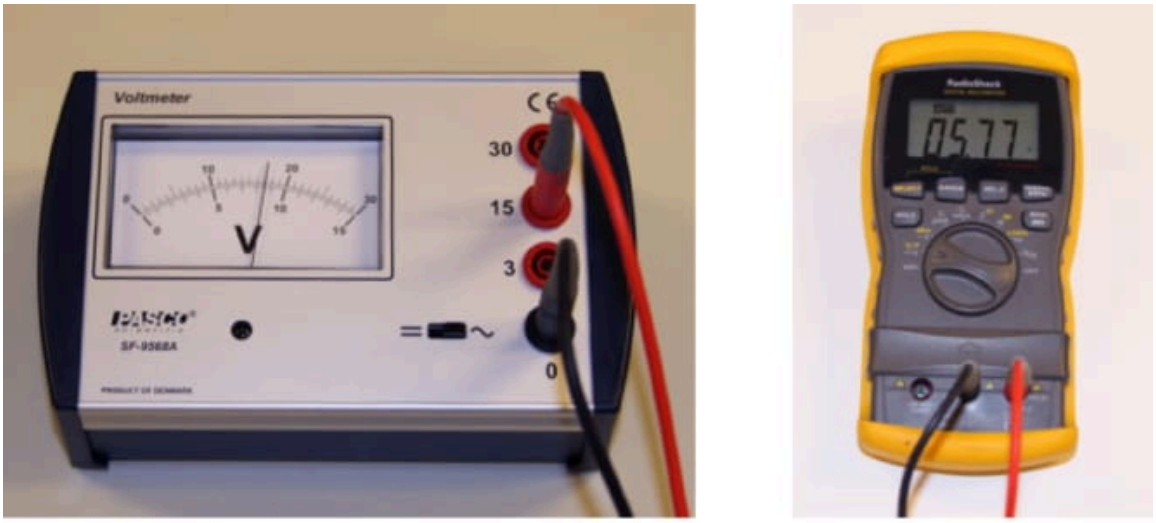


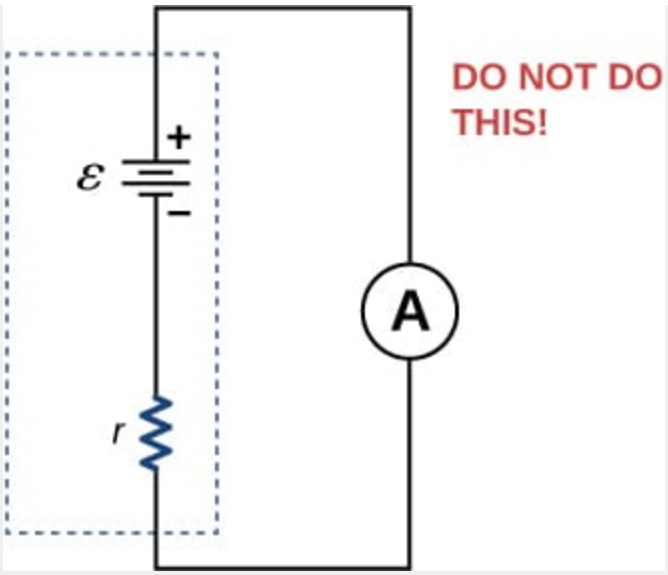
Figure 13.5.4: (a) An analog voltmeter uses a galvanometer to measure the voltage. (b) Digital meters use an analog-to-digital converter to measure the voltage. (credit a and credit b: Joseph J. Trout)

Analog and Digital Meters

You may encounter two types of meters in the physics lab: analog and digital. The term 'analog' refers to signals or information represented by a continuously variable physical quantity, such as voltage or current. An analog meter uses a galvanometer, which is essentially a coil of wire with a small resistance, in a magnetic field, with a pointer attached that points to a scale. Current flows through the coil, causing the coil to rotate. To use the galvanometer as an ammeter, a small resistance is placed in parallel with the coil. For a voltmeter, a large resistance is placed in series with the coil. A digital meter uses a component called an analog-to-digital (A to D) converter and expresses the current or voltage as a series of the digits 0 and 1, which are used to run a digital display. Most analog meters have been replaced by digital meters.

Check Your Understanding

Digital meters are able to detect smaller currents than analog meters employing galvanometers. How does this explain their ability to measure voltage and current more accurately than analog meters?



[Hide Solution]

Since digital meters require less current than analog meters, they alter the circuit less than analog meters. Their resistance as a voltmeter can be far greater than an analog meter, and their resistance as an ammeter can be far less than an analog meter. Consult Figure $13.5.3$ and Figure $13.5.2$ and their discussion in the text

Note

In this virtual lab simulation, you may construct circuits with resistors, voltage sources, ammeters and voltmeters to test your knowledge of circuit design.

Ohmmeters

An ohmmeter is an instrument used to measure the resistance of a component or device. The operation of the ohmmeter is based on Ohm's law. Traditional ohmmeters contained an internal voltage source (such as a battery) that would be connected across the component to be tested, producing a current through the component. A galvanometer was then used to measure the current and the resistance was deduced using Ohm's law. Modern digital meters use a constant current source to pass current through the component, and the voltage difference across the component is measured. In either case, the resistance is measured using Ohm's law $(R = V/I)$ where the voltage is known and the current is measured, or the current is known and the voltage is measured.

The component of interest should be isolated from the circuit; otherwise, you will be measuring the equivalent resistance of the circuit. An ohmmeter should never be connected to a "live" circuit, one with a voltage source connected to it and current running through it. Doing so can damage the meter.

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13.6: RC Circuits

Learning Objectives

By the end of the section, you will be able to:

- Describe the charging process of a capacitor
- Describe the discharging process of a capacitor
- List some applications of RC circuits

When you use a flash camera, it takes a few seconds to charge the capacitor that powers the flash. The light flash discharges the capacitor in a tiny fraction of a second. Why does charging take longer than discharging? This question and several other phenomena that involve charging and discharging capacitors are discussed in this module.

Circuits with Resistance and Capacitance

An **RC circuit** is a circuit containing resistance and capacitance. As presented in Capacitance, the capacitor is an electrical component that stores electric charge, storing energy in an electric field.

Figure $13.6.1a$ shows a simple **RC** circuit that employs a dc (direct current) voltage source ε, a resistor R, a capacitor C, and a two-position switch. The circuit allows the capacitor to be charged or discharged, depending on the position of the switch. When the switch is moved to position \(**A\)**, the capacitor charges, resulting in the circuit in Figure $13.6.1b$. When the switch is moved to position **B**, the capacitor discharges through the resistor.

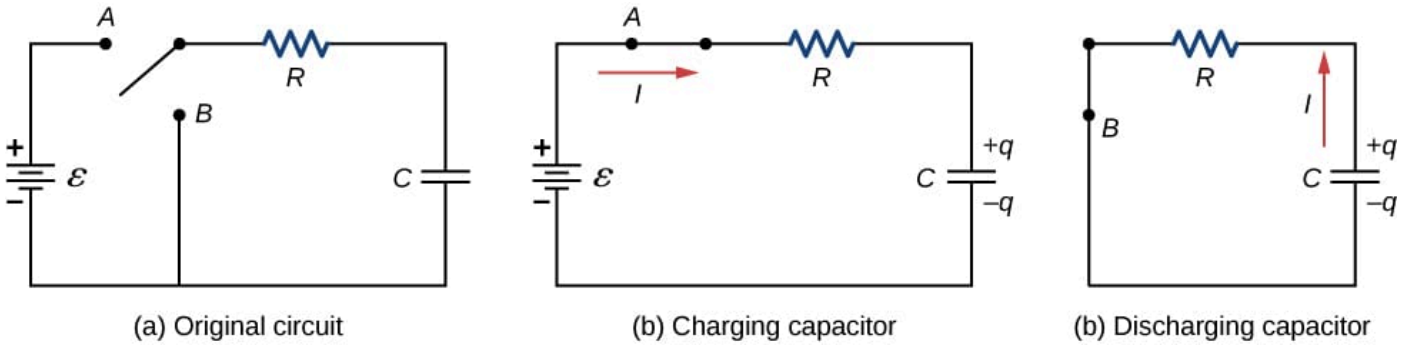


Figure $13.6.1$: (a) An **RC** circuit with a two-pole switch that can be used to charge and discharge a capacitor. (b) When the switch is moved to position **A**, the circuit reduces to a simple series connection of the voltage source, the resistor, the capacitor, and the switch. (c) When the switch is moved to position **B**, the circuit reduces to a simple series connection of the resistor, the capacitor, and the switch. The voltage source is removed from the circuit.

Charging a Capacitor

We can use Kirchhoff's loop rule to understand the charging of the capacitor. This results in the equation $\epsilon - V_R - V_C = 0$. This equation can be used to model the charge as a function of time as the capacitor charges. Capacitance is defined as $C = q/V$, so the voltage across the capacitor is $V_C = \frac{q}{C}$. Using Ohm's law, the potential drop across the resistor is $V_R = IR$, and the current is defined as $I = dq/dt$.

$$\epsilon - V_R - V_C = 0,$$

$$\epsilon - IR - \frac{q}{C} = 0,$$

$$\epsilon - R\frac{dq}{dt} - \frac{q}{C} = 0.$$

This differential equation can be integrated to find an equation for the charge on the capacitor as a function of time.

$$\epsilon - R\frac{dq}{dt} - \frac{q}{C} = 0.$$

$$\frac{dq}{dt} = \frac{\epsilon C - q}{RC},$$

$$\int_0^q \frac{dq}{\epsilon C - q} = \frac{1}{RC}\int_0^t dt.$$

Let $u = \epsilon C - q$, then $du = -dq$. The result is

$$-\int_0^q \frac{du}{u} = \frac{1}{RC}\int_0^t dt,$$

$$\ln\left(\frac{\epsilon C - q}{\epsilon C}\right) = -\frac{1}{RC}t.$$

$$\frac{\epsilon C - q}{\epsilon C} = e^{-t/RC}.$$

Simplifying results in an equation for the charge on the charging capacitor as a function of time:

$$q(t) = C\epsilon\left(1 - e^{-\frac{t}{RC}}\right) = Q\left(1 - e^{-\frac{t}{\tau}}\right).$$

A graph of the charge on the capacitor versus time is shown in Figure $13.6.2a$. First note that as time approaches infinity, the exponential goes to zero, so the charge approaches the maximum charge $Q = C\epsilon$ and has units of coulombs. The units of **RC** are seconds, units of time. This quantity is known as the **time constant**:

$$\tau = RC.$$

At time $t = \tau = RC$, the charge equal to $1 - e^{-1} = 1 - 0.368 = 0.632$ of the maximum charge $Q = C\epsilon$. Notice that the time rate change of the charge is the slope at a point of the charge versus time plot. The slope of the graph is large at time $t - 0.0\ s$ and approaches zero as time increases.

As the charge on the capacitor increases, the current through the resistor decreases, as shown in Figure $13.6.2b$. The current through the resistor can be found by taking the time derivative of the charge.

$$\begin{aligned} I(t) &= \frac{dq}{dt} \\ &= \frac{d}{dt}\left[C\epsilon\left(1 - e^{-\frac{t}{RC}}\right)\right], \\ &= C\epsilon\left(\frac{1}{RC}\right)e^{-\frac{t}{RC}} \\ &= \frac{\epsilon}{R}e^{-\frac{t}{RC}} \\ &= I_0 e^{-\frac{t}{RC}}, \end{aligned}$$

$$I(t) = I_0 e^{-t/\tau}.$$

At time $t = 0.0\ s$, the current through the resistor is $I_0 = \frac{\epsilon}{R}$. As time approaches infinity, the current approaches zero. At time $t = \tau$, the current through the resistor is $I(t = \tau) = I_0 e^{-1} = 0.368 I_0$.

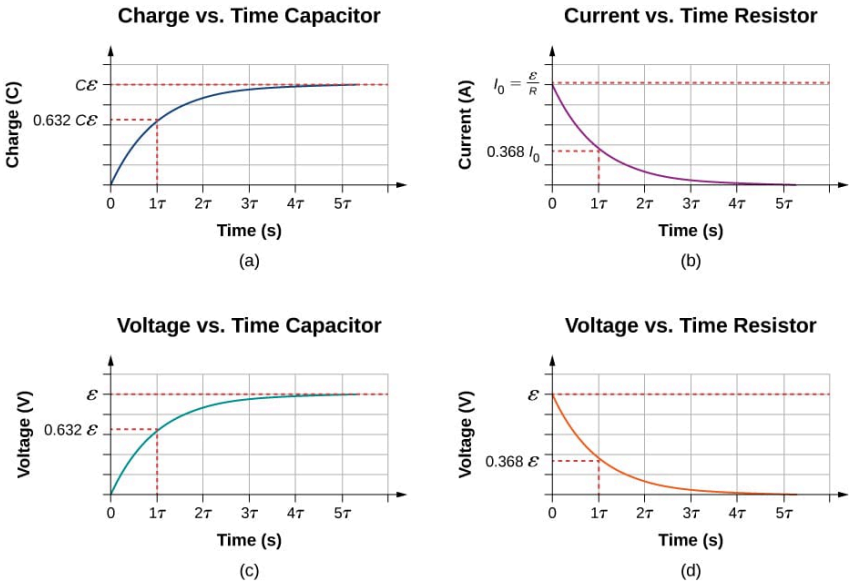


Figure 13.6.2: (a) Charge on the capacitor versus time as the capacitor charges. (b) Current through the resistor versus time. (c) Voltage difference across the capacitor. (d) Voltage difference across the resistor.

Figures $13.6.2c$ and Figure $13.6.2d$ show the voltage differences across the capacitor and the resistor, respectively. As the charge on the capacitor increases, the current decreases, as does the voltage difference across the resistor $V_R(t) = (I_0R)e^{-t/\tau} = \epsilon e^{-t/\tau}$. The voltage difference across the capacitor increases as $V_C(t) = \epsilon(1 - e^{-t/\tau})$.

Discharging a Capacitor

When the switch in Figure $13.6.3a$ is moved to position **B**, the circuit reduces to the circuit in part (c), and the charged capacitor is allowed to discharge through the resistor. A graph of the charge on the capacitor as a function of time is shown in Figure $13.6.3a$. Using Kirchhoff's loop rule to analyze the circuit as the capacitor discharges results in the equation $-V_R - V_C = 0$, which simplifies to $IR + \frac{q}{C} = 0$. Using the definition of current $\frac{dq}{dt}R = -\frac{q}{C}$ and integrating the loop equation yields an equation for the charge on the capacitor as a function of time:

$$q(t) = Qe^{-t/\tau}.$$

Here, **Q** is the initial charge on the capacitor and $\tau = RC$ is the time constant of the circuit. As shown in the graph, the charge decreases exponentially from the initial charge, approaching zero as time approaches infinity.

The current as a function of time can be found by taking the time derivative of the charge:

$$I(t) = -\frac{Q}{RC}e^{-t/\tau}.$$

The negative sign shows that the current flows in the opposite direction of the current found when the capacitor is charging. Figure $13.6.3b$ shows an example of a plot of charge versus time and current versus time. A plot of the voltage difference across the capacitor and the voltage difference across the resistor as a function of time are shown in Figures $13.6.3c$ and $13.6.3d$. Note that the magnitudes of the charge, current, and voltage all decrease exponentially, approaching zero as time increases.

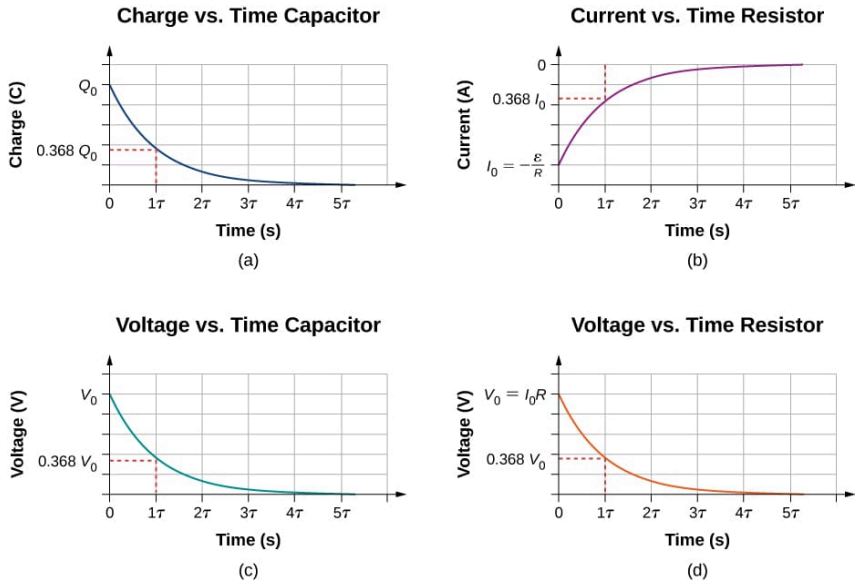
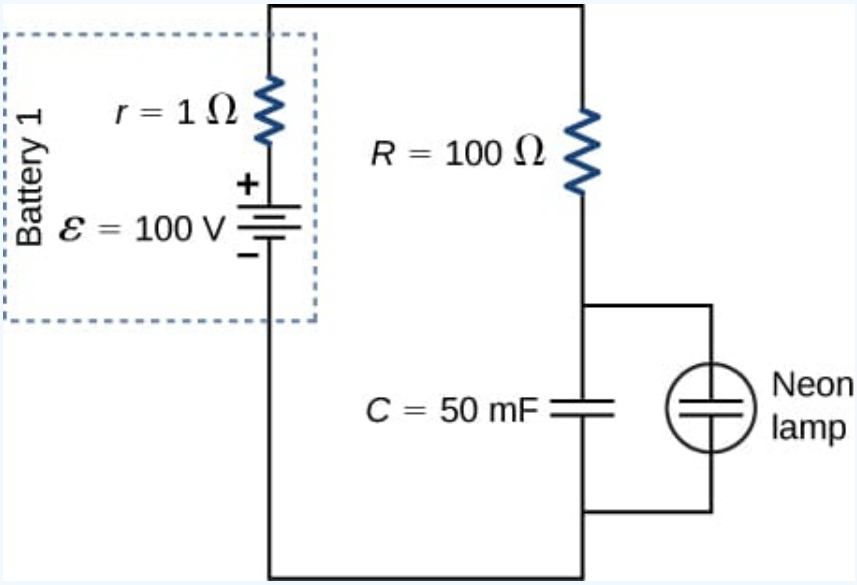


Figure 13.6.3: (a) Charge on the capacitor versus time as the capacitor discharges. (b) Current through the resistor versus time. (c) Voltage difference across the capacitor. (d) Voltage difference across the resistor.

Now we can explain why the **flash camera** mentioned at the beginning of this section takes so much longer to charge than discharge: The resistance while charging is significantly greater than while discharging. The internal resistance of the battery accounts for most of the resistance while charging. As the battery ages, the increasing internal resistance makes the charging process even slower.

Example 13.6.2: The Relaxation Oscillator

One application of an **RC** circuit is the relaxation oscillator, as shown below. The relaxation oscillator consists of a voltage source, a resistor, a capacitor, and a neon lamp. The neon lamp acts like an open circuit (infinite resistance) until the potential difference across the neon lamp reaches a specific voltage. At that voltage, the lamp acts like a short circuit (zero resistance), and the capacitor discharges through the neon lamp and produces light. In the relaxation oscillator shown, the voltage source charges the capacitor until the voltage across the capacitor is 80 V. When this happens, the neon in the lamp breaks down and allows the capacitor to discharge through the lamp, producing a bright flash. After the capacitor fully discharges through the neon lamp, it begins to charge again, and the process repeats. Assuming that the time it takes the capacitor to discharge is negligible, what is the time interval between flashes?



Strategy

The time period can be found from considering the equation $V_C(t) = \epsilon(1 - e^{=t/\tau})$. where $\tau = (R + r)C$.

Solution

The neon lamp flashes when the voltage across the capacitor reaches 80 V. The **RC** time constant is equal to $\tau = (R + r) = (101\ \Omega)(50 \times 10^{-3} F) = 5.05\ s$. We can solve the voltage equation for the time it takes the capacitor to reach 80 V:

$$V_C(t) = \epsilon(1 - e^{-t/\tau}),$$

$$e^{-t/\tau} = 1 - \frac{V_C(t)}{\epsilon},$$

$$ln(e^{-t/\tau}) = ln\left(1 - \frac{V_C(t)}{\epsilon}\right),$$

$$t = -\tau ln\left(1 - \frac{V_C(t)}{\epsilon}\right) = -5.05\,s \cdot ln\left(1 - \frac{80\,V}{100\,V}\right) = 8.13\,s.$$

Significance

One application of the relaxation oscillator is for controlling indicator lights that flash at a frequency determined by the values for **R** and **C**. In this example, the neon lamp will flash every 8.13 seconds, a frequency of $f = \frac{1}{T} = \frac{1}{8.13\,s} = 0.55\ Hz$. The relaxation oscillator has many other practical uses. It is often used in electronic circuits, where the neon lamp is replaced by a transistor or a device known as a tunnel diode. The description of the transistor and tunnel diode is beyond the scope of this chapter, but you can think of them as voltage controlled switches. They are normally open switches, but when the right voltage is applied, the switch closes and conducts. The "switch" can be used to turn on another circuit, turn on a light, or run a small motor. A relaxation oscillator can be used to make the turn signals of your car blink or your cell phone to vibrate.

RC circuits have many applications. They can be used effectively as timers for applications such as intermittent windshield wipers, pace makers, and strobe lights. Some models of intermittent windshield wipers use a variable resistor to adjust the interval between sweeps of the wiper. Increasing the resistance increases the **RC** time constant, which increases the time between the operation of the wipers.

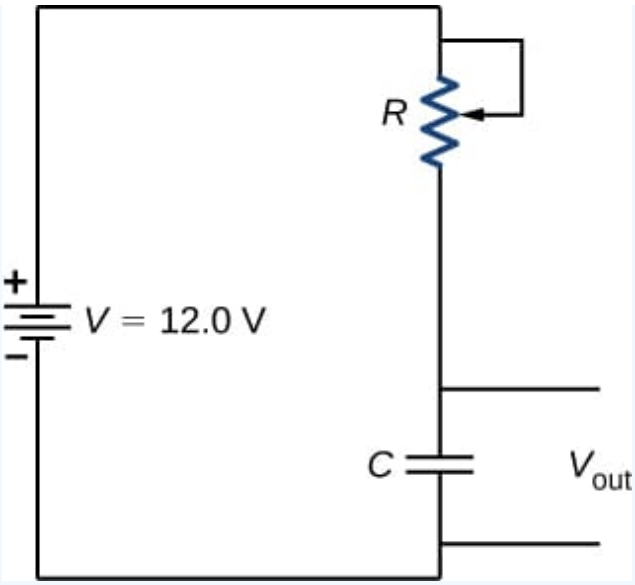
Another application is the **pacemaker**. The heart rate is normally controlled by electrical signals, which cause the muscles of the heart to contract and pump blood. When the heart rhythm is abnormal (the heartbeat is too high or too low), pace makers can be used to correct this abnormality. Pacemakers have sensors that detect body motion and breathing to increase the heart rate during physical activities, thus meeting the increased need for blood and oxygen, and an **RC** timing circuit can be used to control the time between voltage signals to the heart.

Looking ahead to the study of ac circuits (Alternating-Current Circuits), ac voltages vary as sine functions with specific frequencies. Periodic variations in voltage, or electric signals, are often recorded by scientists. These voltage signals could come from music recorded by a microphone or atmospheric data collected by radar. Occasionally, these signals can contain unwanted frequencies known as "noise." **RC** filters can be used to filter out the unwanted frequencies.

In the study of electronics, a popular device known as a 555 timer provides timed voltage pulses. The time between pulses is controlled by an **RC** circuit. These are just a few of the countless applications of **RC** circuits.

Example 13.6.2: Intermittent Windshield Wipers

A relaxation oscillator is used to control a pair of windshield wipers. The relaxation oscillator consists of a 10.00-mF capacitor and a $10.00\ k\Omega$ variable resistor known as a rheostat. A knob connected to the variable resistor allows the resistance to be adjusted from $0.00\ \Omega$ to $10.00\ k\Omega$. The output of the capacitor is used to control a voltage-controlled switch. The switch is normally open, but when the output voltage reaches 10.00 V, the switch closes, energizing an electric motor and discharging the capacitor. The motor causes the windshield wipers to sweep once across the windshield and the capacitor begins to charge again. To what resistance should the rheostat be adjusted for the period of the wiper blades be 10.00 seconds?



Strategy

The resistance considers the equation $V_{out}(t) = V(1 - e^{-t/\tau})$, where $\tau = RC$. The capacitance, output voltage, and voltage of the battery are given. We need to solve this equation for the resistance.

Solution

The output voltage will be 10.00 V and the voltage of the battery is 12.00 V. The capacitance is given as 10.00 mF. Solving for the resistance yields

$$V_{out}(t) = V(1 - e^{-t/\tau})$$

$$e^{-t/RC} = 1 - \frac{V_{out}(t)}{V},$$

$$ln(e^{-t/RC}) = ln\left(1 - \frac{V_{out}(t)}{V}\right),$$

$$-\frac{t}{RC} = ln\left(1 - \frac{V_{out}(t)}{V}\right),$$

$$R = \frac{-t}{C\,ln\left(1 - \frac{V_C(t)}{V}\right)} = \frac{-10.00\,s}{10 \times 10^{-3} F\,ln\left(1 - \frac{10\,V}{12\,V}\right)} = 558.11\,\Omega.$$

Significance

Increasing the resistance increases the time delay between operations of the windshield wipers. When the resistance is zero, the windshield wipers run continuously. At the maximum resistance, the period of the operation of the wipers is:

$$t = -RCln\left(1 - \frac{V_{out}(t)}{V}\right) = -(10 \times 10^{-3} F)(10 \times 10^{3}\,\Omega)ln\left(1 - \frac{10\,V}{12\,V}\right) = 179.18\,s = 2.98\,min.$$

The **RC** circuit has thousands of uses and is a very important circuit to study. Not only can it be used to time circuits, it can also be used to filter out unwanted frequencies in a circuit and used in power supplies, like the one for your computer, to help turn ac voltage to dc voltage.

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13.7: Household Wiring and Electrical Safety

Learning Objectives

By the end of the section, you will be able to:

- List the basic concepts involved in house wiring
- Define the terms thermal hazard and shock hazard
- Describe the effects of electrical shock on human physiology and their relationship to the amount of current through the body
- Explain the function of fuses and circuit breakers

Electricity presents two known hazards: thermal and shock. A **thermal hazard** is one in which an excessive electric current causes undesired thermal effects, such as starting a fire in the wall of a house. A **shock hazard** occurs when an electric current passes through a person. Shocks range in severity from painful, but otherwise harmless, to heart-stopping lethality. In this section, we consider these hazards and the various factors affecting them in a quantitative manner. We also examine systems and devices for preventing electrical hazards.

Thermal Hazards

Electric power causes undesired heating effects whenever electric energy is converted into thermal energy at a rate faster than it can be safely dissipated. A classic example of this is the **short circuit**, a low-resistance path between terminals of a voltage source. An example of a short circuit is shown in Figure 13.7.1. A toaster is plugged into a common household electrical outlet. Insulation on wires leading to an appliance has worn through, allowing the two wires to come into contact, or "short." As a result, thermal energy can quickly raise the temperature of surrounding materials, melting the insulation and perhaps causing a fire.

The circuit diagram shows a symbol that consists of a sine wave enclosed in a circle. This symbol represents an alternating current (ac) voltage source. In an ac voltage source, the voltage oscillates between a positive and negative maximum amplitude. Up to now, we have been considering direct current (dc) voltage sources, but many of the same concepts are applicable to ac circuits.

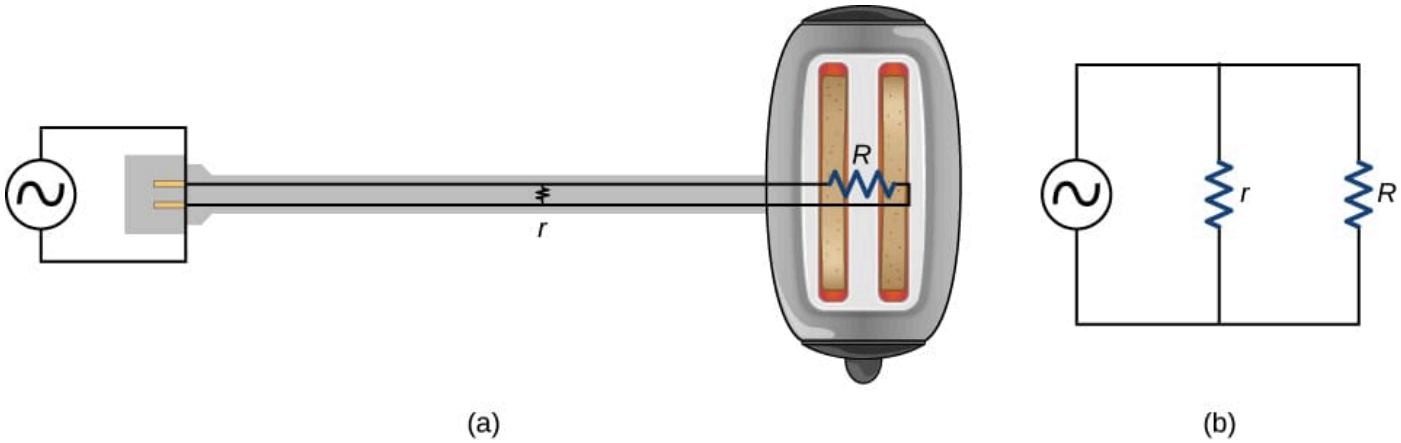


Figure 13.7.1: A short circuit is an undesired low-resistance path across a voltage source. (a) Worn insulation on the wires of a toaster allow them to come into contact with a low resistance **r**. Since $P = V^2/\tau$, thermal power is created so rapidly that the cord melts or burns. (b) A schematic of the short circuit.

Another serious thermal hazard occurs when wires supplying power to an appliance are overloaded. Electrical wires and appliances are often rated for the maximum current they can safely handle. The term "overloaded" refers to a condition where the current exceeds the rated maximum current. As current flows through a wire, the power dissipated in the supply wires is $P = I^2 R_W$, where R_W is the resistance of the wires and **I** is the current flowing through the wires. If either **I** or R_W is too large, the wires overheat. Fuses and circuit breakers are used to limit excessive currents.

Shock Hazards

Electric shock is the physiological reaction or injury caused by an external electric current passing through the body. The effect of an electric shock can be negative or positive. When a current with a magnitude above 300 mA passes through the heart, death may occur. Most electrical shock fatalities occur because a current causes ventricular fibrillation, a massively irregular and often fatal, beating of the heart. On the other hand, a heart attack victim, whose heart is in fibrillation, can be saved by an electric shock from a defibrillator.

The effects of an undesirable electric shock can vary in severity: a slight sensation at the point of contact, pain, loss of voluntary muscle control, difficulty breathing, heart fibrillation, and possibly death. The loss of voluntary muscle control can cause the victim to not be able to let go of the source of the current.

The major factors upon which the severity of the effects of electrical shock depend are

1. The amount of current **I**
2. The path taken by the current
3. The duration of the shock
4. The frequency **f** of the current $(f = 0$ for dc)

Our bodies are relatively good electric conductors due to the body's water content. A dangerous condition occurs when the body is in contact with a voltage source and "ground." The term "ground" refers to a large sink or source of electrons, for example, the earth (thus, the name). When there is a direct path to ground, large currents will pass through the parts of the body with the lowest resistance and a direct path to ground. A safety precaution used by many professions is the wearing of insulated shoes. Insulated shoes prohibit a pathway to ground for electrons through the feet by providing a large resistance. Whenever working with high-power tools, or any electric circuit, ensure that you do not provide a pathway for current flow (especially across the heart). A common safety precaution is to work with one hand, reducing the possibility of providing a current path through the heart.

Very small currents pass harmlessly and unfelt through the body. This happens to you regularly without your knowledge. The threshold of sensation is only 1 mA and, although unpleasant, shocks are apparently harmless for currents less than 5 mA. A great number of safety rules take the 5-mA value for the maximum allowed shock. At 5–30 mA and above, the current can stimulate sustained muscular contractions, much as regular nerve impulses do (Figure 13.7.2). Very large currents (above 300 mA) cause the heart and diaphragm of the lung to contract for the duration of the shock. Both the heart and respiration stop. Both often return to normal following the shock.

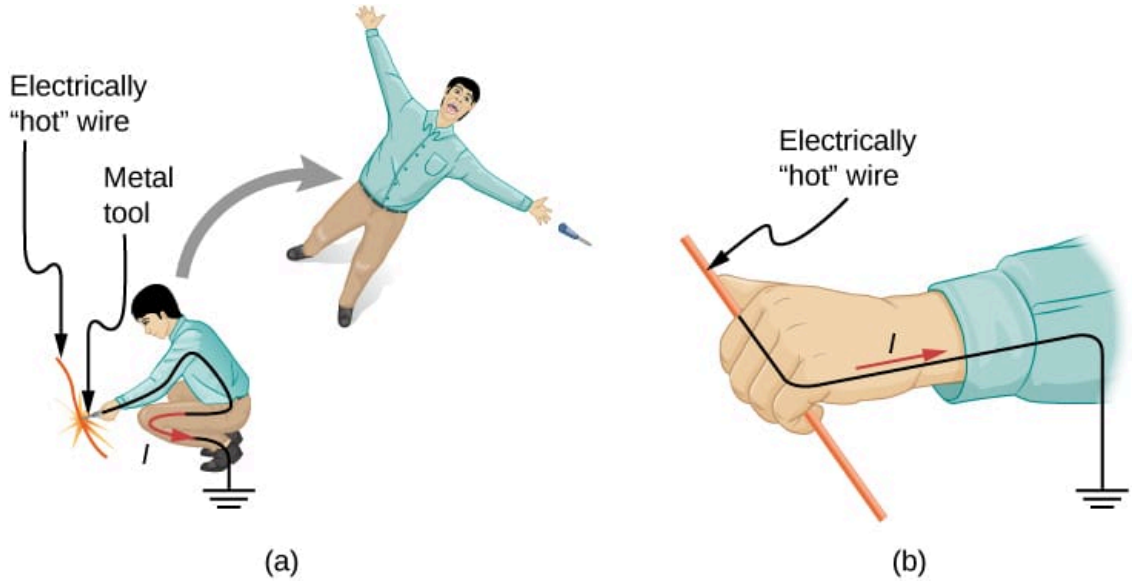


Figure 13.7.2: An electric current can cause muscular contractions with varying effects. (a) The victim is "thrown" backward by involuntary muscle contractions that extend the legs and torso. (b) The victim can't let go of the wire that is stimulating all the muscles in the hand. Those that close the fingers are stronger than those that open them.

Current is the major factor determining shock severity. A larger voltage is more hazardous, but since $I = V/R$, the severity of the

shock depends on the combination of voltage and resistance. For example, a person with dry skin has a resistance of about $200\ k\Omega$. If he comes into contact with 120-V ac, a current

$$I = (120\,V)(200\,k\Omega) = 0.6\,mA$$

passes harmlessly through him. The same person soaking wet may have a resistance of $10.0\ k\Omega$ and the same 120 V will produce a current of 12 mA—above the "can't let go" threshold and potentially dangerous.

Electrical Safety: Systems and Devices

Figure $13.7.3(a)$ shows the schematic for a simple ac circuit with no safety features. This is not how power is distributed in practice. Modern household and industrial wiring requires the **three-wire system**, shown schematically in part (b), which has several safety features, with live, neutral, and ground wires. First is the familiar circuit breaker (or fuse) to prevent thermal overload. Second is a protective case around the appliance, such as a toaster or refrigerator. The case's safety feature is that it prevents a person from touching exposed wires and coming into electrical contact with the circuit, helping prevent shocks.

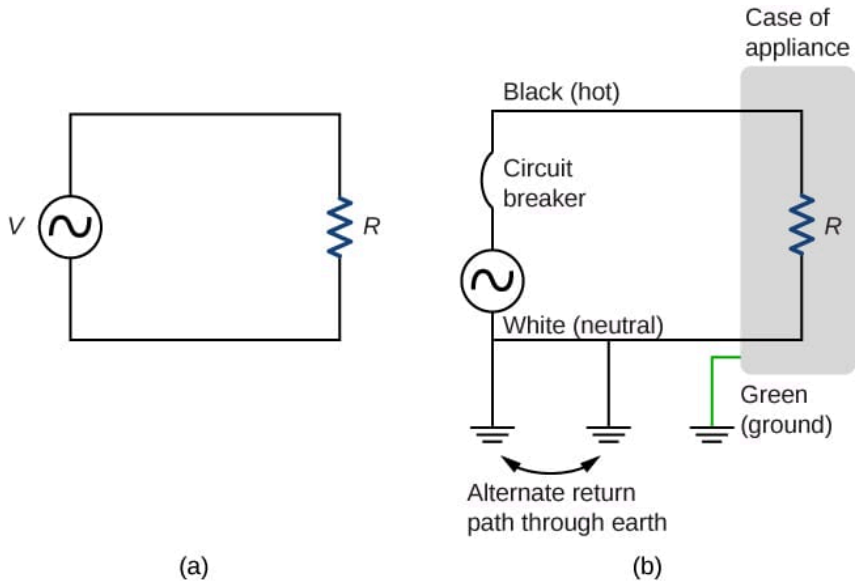


Figure $13.7.3$: (a) Schematic of a simple ac circuit with a voltage source and a single appliance represented by the resistance **R**. There are no safety features in this circuit. (b) The three-wire system connects the neutral wire to ground at the voltage source and user location, forcing it to be at zero volts and supplying an alternative return path for the current through ground. Also grounded to zero volts is the case of the appliance. A circuit breaker or fuse protects against thermal overload and is in series on the active (live/hot) wire.

There are three connections to ground shown in $13.7.3(b)$. Recall that a ground connection is a low-resistance path directly to ground. The two ground connections on the neutral wire force it to be at zero volts relative to ground, giving the wire its name. This wire is therefore safe to touch even if its insulation, usually white, is missing. The neutral wire is the return path for the current to follow to complete the circuit. Furthermore, the two ground connections supply an alternative path through ground (a good conductor) to complete the circuit. The ground connection closest to the power source could be at the generating plant, whereas the other is at the user's location. The third ground is to the case of the appliance, through the green ground wire, forcing the case, too, to be at zero volts. The live or hot wire (hereafter referred to as "live/hot") supplies voltage and current to operate the appliance. Figure $13.7.4$ shows a more pictorial version of how the three-wire system is connected through a three-prong plug to an appliance.

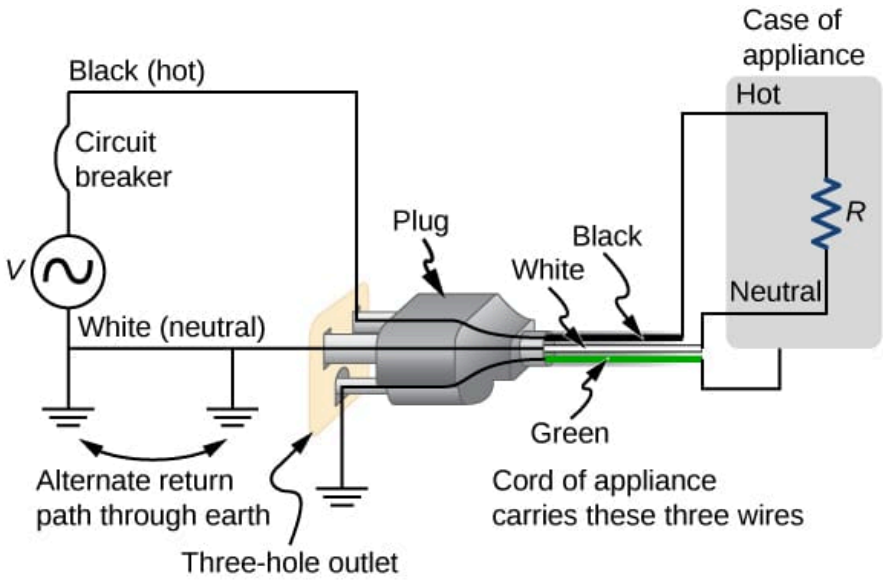


Figure 13.7.4: The standard three-prong plug can only be inserted in one way, to ensure proper function of the three-wire system.

Insulating plastic is color-coded to identify live/hot, neutral, and ground wires, but these codes vary around the world. It is essential to determine the color code in your region. Striped coatings are sometimes used for the benefit of those who are colorblind.

Grounding the case solves more than one problem. The simplest problem is worn insulation on the live/hot wire that allows it to contact the case, as shown in Figure 13.7.5. Lacking a ground connection, a severe shock is possible. This is particularly dangerous in the kitchen, where a good connection to ground is available through water on the floor or a water faucet. With the ground connection intact, the circuit breaker will trip, forcing repair of the appliance.

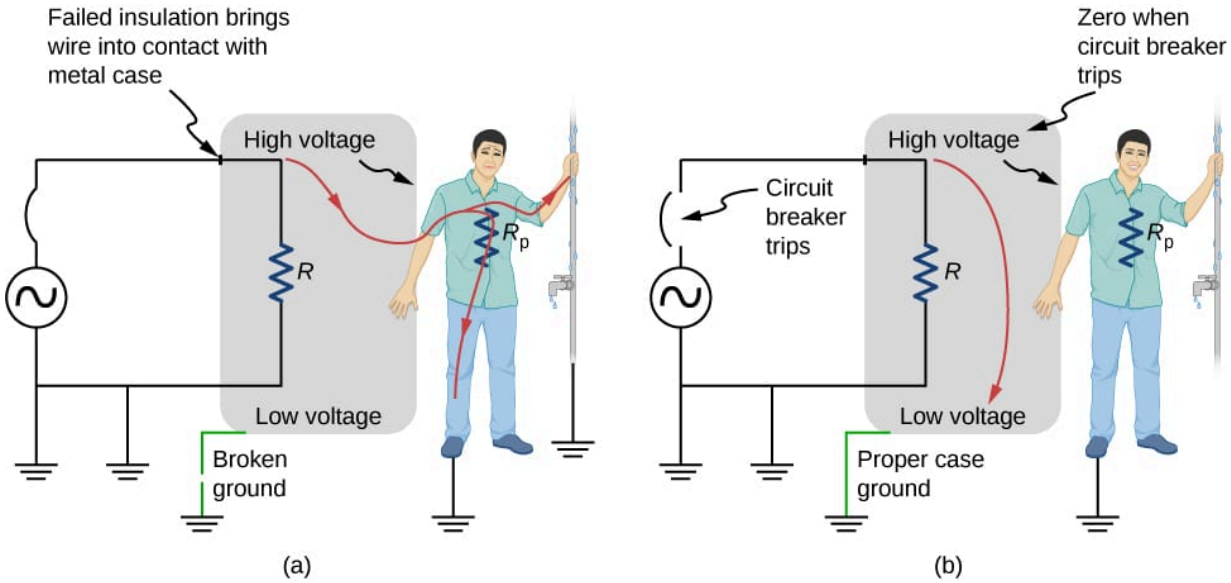


Figure 13.7.5: Worn insulation allows the live/hot wire to come into direct contact with the metal case of this appliance. (a) The ground connection being broken, the person is severely shocked. The appliance may operate normally in this situation. (b) With a proper ground, the circuit breaker trips, forcing repair of the appliance.

A **ground fault circuit interrupter** (GFCI) is a safety device found in updated kitchen and bathroom wiring that works based on electromagnetic induction. GFCIs compare the currents in the live/hot and neutral wires. When live/hot and neutral currents are not equal, it is almost always because current in the neutral is less than in the live/hot wire. Then some of the current, called a leakage current, is returning to the voltage source by a path other than through the neutral wire. It is assumed that this path presents a hazard. GFCIs are usually set to interrupt the circuit if the leakage current is greater than 5 mA, the accepted maximum harmless shock. Even if the leakage current goes safely to ground through an intact ground wire, the GFCI will trip, forcing repair

of the leakage.

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13.8: Direct-Current Circuits (Summary)

Key Terms

ammeter	instrument that measures current
electromotive force (emf)	energy produced per unit charge, drawn from a source that produces an electrical current
equivalent resistance	resistance of a combination of resistors; it can be thought of as the resistance of a single resistor that can replace a combination of resistors in a series and/or parallel circuit
internal resistance	amount of resistance to the flow of current within the voltage source
junction rule	sum of all currents entering a junction must equal the sum of all currents leaving the junction
Kirchhoff's rules	set of two rules governing current and changes in potential in an electric circuit
loop rule	algebraic sum of changes in potential around any closed circuit path (loop) must be zero
potential difference	difference in electric potential between two points in an electric circuit, measured in volts
potential drop	loss of electric potential energy as a current travels across a resistor, wire, or other component
RC circuit	circuit that contains both a resistor and a capacitor
shock hazard	hazard in which an electric current passes through a person
terminal voltage	potential difference measured across the terminals of a source when there is no load attached
thermal hazard	hazard in which an excessive electric current causes undesired thermal effects
three-wire system	wiring system used at present for safety reasons, with live, neutral, and ground wires
voltmeter	instrument that measures voltage

Key Equations

Terminal voltage of a single voltage source	$V_{terminal} = \varepsilon - Ir_{eq}$
Equivalent resistance of a series circuit	$R_{eq} = R_1 + R_2 + R_3 + \cdots + R_{N-1} + R_N = \sum_{i=1}^{N} R_i$
Equivalent resistance of a parallel circuit	$R_{eq} = (\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N})^{-1} = (\sum_{i=1}^{N} \frac{1}{R_i})^{-1}$
Junction rule	$\sum I_{in} = \sum I_{out}$
Loop rule	$\sum V = 0$
Terminal voltage of N voltage sources in series	$V_{terminal} = \sum_{i=1}^{N} \varepsilon_i - I\sum_{i=1}^{N} r_i = \sum_{i=1}^{N} \varepsilon_i - Ir_{eq}$
Terminal voltage of N voltage sources in parallel	$V_{terminal} = \varepsilon - I\sum_{i=1}^{N} (\frac{1}{r_i})^{-1} = \varepsilon - Ir_{eq}$
Charge on a charging capacitor	$q(t) = C\varepsilon(1 - e^{-\frac{t}{RC}}) = Q(1 - e^{-\frac{t}{\tau}})$
Time constant	$\tau = RC$
Current during charging of a capacitor	$I = \frac{\varepsilon}{R} e^{-\frac{t}{RC}} = I_o e^{-\frac{t}{RC}}$
Charge on a discharging capacitor	$q(t) = Qe^{-\frac{t}{\tau}}$

Current during discharging of a capacitor	$I(t) = -\frac{Q}{RC}e^{-\frac{t}{\tau}}$

Summary

10.2 Electromotive Force

- All voltage sources have two fundamental parts: a source of electrical energy that has a characteristic electromotive force (emf), and an internal resistance r. The emf is the work done per charge to keep the potential difference of a source constant. The emf is equal to the potential difference across the terminals when no current is flowing. The internal resistance **r** of a voltage source affects the output voltage when a current flows.
- The voltage output of a device is called its terminal voltage $V_{terminal}$ and is given by $V_{terminal} = \varepsilon - Ir$, where I is the electric current and is positive when flowing away from the positive terminal of the voltage source and r is the internal resistance.

10.3 Resistors in Series and Parallel

- The equivalent resistance of an electrical circuit with resistors wired in a series is the sum of the individual resistances:

$$R_s = R_1 + R_2 + R_3 + \cdots = \sum_{i=1}^{N} R_i.$$

- Each resistor in a series circuit has the same amount of current flowing through it.
- The potential drop, or power dissipation, across each individual resistor in a series is different, and their combined total is the power source input.
- The equivalent resistance of an electrical circuit with resistors wired in parallel is less than the lowest resistance of any of the components and can be determined using the formula

$$R_{eq} = (\tfrac{1}{R_1} + \tfrac{1}{R_2} + \tfrac{1}{R_3} + \cdots)^{-1} = (\textstyle\sum_{i=1}^{N} \tfrac{1}{R_i})^{-1}.$$

- Each resistor in a parallel circuit has the same full voltage of the source applied to it.
- The current flowing through each resistor in a parallel circuit is different, depending on the resistance.
- If a more complex connection of resistors is a combination of series and parallel, it can be reduced to a single equivalent resistance by identifying its various parts as series or parallel, reducing each to its equivalent, and continuing until a single resistance is eventually reached.

10.4 Kirchhoff's Rules

- Kirchhoff's rules can be used to analyze any circuit, simple or complex. The simpler series and parallel connection rules are special cases of Kirchhoff's rules.
- Kirchhoff's first rule, also known as the junction rule, applies to the charge to a junction. Current is the flow of charge; thus, whatever charge flows into the junction must flow out.
- Kirchhoff's second rule, also known as the loop rule, states that the voltage drop around a loop is zero.
- When calculating potential and current using Kirchhoff's rules, a set of conventions must be followed for determining the correct signs of various terms.
- When multiple voltage sources are in series, their internal resistances add together and their emfs add together to get the total values.
- When multiple voltage sources are in parallel, their internal resistances combine to an equivalent resistance that is less than the individual resistance and provides a higher current than a single cell.
- Solar cells can be wired in series or parallel to provide increased voltage or current, respectively.

10.5 Electrical Measuring Instruments

- Voltmeters measure voltage, and ammeters measure current. Analog meters are based on the combination of a resistor and a galvanometer, a device that gives an analog reading of current or voltage. Digital meters are based on analog-to-digital converters and provide a discrete or digital measurement of the current or voltage.
- A voltmeter is placed in parallel with the voltage source to receive full voltage and must have a large resistance to limit its effect on the circuit.

- An ammeter is placed in series to get the full current flowing through a branch and must have a small resistance to limit its effect on the circuit.
- Standard voltmeters and ammeters alter the circuit they are connected to and are thus limited in accuracy.
- Ohmmeters are used to measure resistance. The component in which the resistance is to be measured should be isolated (removed) from the circuit.

10.6 RC Circuits

- An **RC** circuit is one that has both a resistor and a capacitor.
- The time constant τ for an **RC** circuit is $\tau = RC$.
- When an initially uncharged ($q = 0$ at $t = 0$) capacitor in series with a resistor is charged by a dc voltage source, the capacitor asymptotically approaches the maximum charge.
- As the charge on the capacitor increases, the current exponentially decreases from the initial current: $I_0 = \varepsilon/R$.
- If a capacitor with an initial charge **Q** is discharged through a resistor starting at $t = 0$, then its charge decreases exponentially. The current flows in the opposite direction, compared to when it charges, and the magnitude of the charge decreases with time.

10.7 Household Wiring and Electrical Safety

- The two types of electric hazards are thermal (excessive power) and shock (current through a person). Electrical safety systems and devices are employed to prevent thermal and shock hazards.
- Shock severity is determined by current, path, duration, and ac frequency.
- Circuit breakers and fuses interrupt excessive currents to prevent thermal hazards.
- The three-wire system guards against thermal and shock hazards, utilizing live/hot, neutral, and ground wires, and grounding the neutral wire and case of the appliance.
- A ground fault circuit interrupter (GFCI) prevents shock by detecting the loss of current to unintentional paths.

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13.9: Direct-Current Circuits (Exercise)

Conceptual Questions

10.2 Electromotive Force

1. What effect will the internal resistance of a rechargeable battery have on the energy being used to recharge the battery?

2. A battery with an internal resistance of r and an emf of 10.00 V is connected to a load resistor **R=r**. As the battery ages, the internal resistance triples. How much is the current through the load resistor reduced?

3. Show that the power dissipated by the load resistor is maximum when the resistance of the load resistor is equal to the internal resistance of the battery.

10.3 Resistors in Series and Parallel

4. A voltage occurs across an open switch. What is the power dissipated by the open switch?

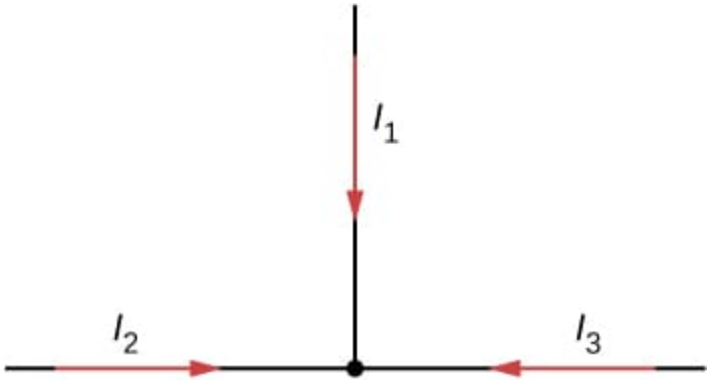
5. The severity of a shock depends on the magnitude of the current through your body. Would you prefer to be in series or in parallel with a resistance, such as the heating element of a toaster, if you were shocked by it? Explain.

6. Suppose you are doing a physics lab that asks you to put a resistor into a circuit, but all the resistors supplied have a larger resistance than the requested value. How would you connect the available resistances to attempt to get the smaller value asked for?

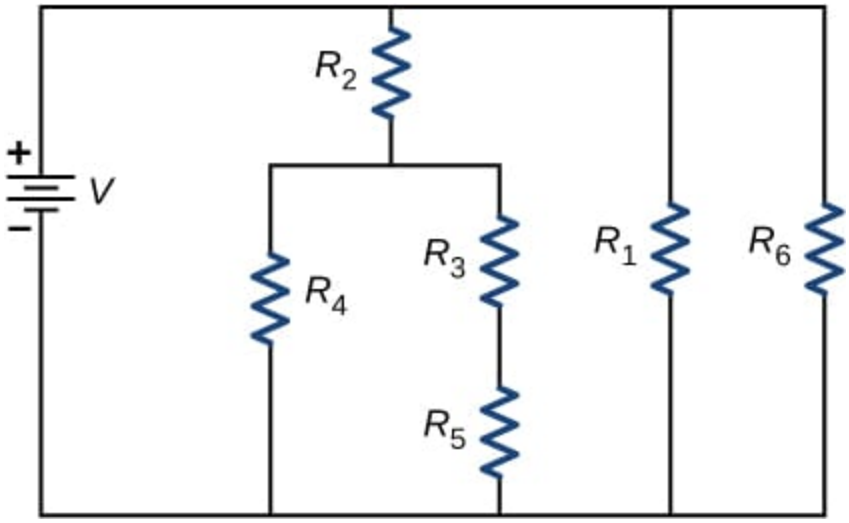
7. Some light bulbs have three power settings (not including zero), obtained from multiple filaments that are individually switched and wired in parallel. What is the minimum number of filaments needed for three power settings?

10.4 Kirchhoff's Rules

8. Can all of the currents going into the junction shown below be positive? Explain.



9. Consider the circuit shown below. Does the analysis of the circuit require Kirchhoff's method, or can it be redrawn to simplify the circuit? If it is a circuit of series and parallel connections, what is the equivalent resistance?



10. Do batteries in a circuit always supply power to a circuit, or can they absorb power in a circuit? Give an example.

11. What are the advantages and disadvantages of connecting batteries in series? In parallel?

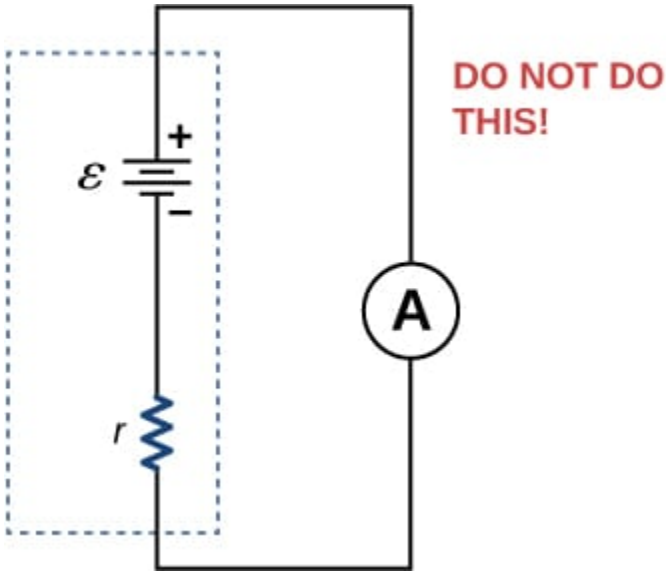
12. Semi-tractor trucks use four large 12-V batteries. The starter system requires 24 V, while normal operation of the truck's other electrical components utilizes 12 V. How could the four batteries be connected to produce 24 V? To produce 12 V? Why is 24 V better than 12 V for starting the truck's engine (a very heavy load)?

10.5 Electrical Measuring Instruments

13. What would happen if you placed a voltmeter in series with a component to be tested?

14. What is the basic operation of an ohmmeter as it measures a resistor?

15. Why should you not connect an ammeter directly across a voltage source as shown below?



10.6 RC Circuits

16. A battery, switch, capacitor, and lamp are connected in series. Describe what happens to the lamp when the switch is

closed.

17. When making an ECG measurement, it is important to measure voltage variations over small time intervals. The time is limited by the RC constant of the circuit—it is not possible to measure time variations shorter than RC. How would you manipulate R and C in the circuit to allow the necessary measurements?

10.6 Household Wiring and Electrical Safety

18. Why isn't a short circuit necessarily a shock hazard?

19. We are often advised to not flick electric switches with wet hands, dry your hand first. We are also advised to never throw water on an electric fire. Why?

Problems

10.2 Electromotive Force

20. A car battery with a 12-V emf and an internal resistance of **0.050Ω** is being charged with a current of 60 A. Note that in this process, the battery is being charged.

(a) What is the potential difference across its terminals?

(b) At what rate is thermal energy being dissipated in the battery?

(c) At what rate is electric energy being converted into chemical energy?

21. The label on a battery-powered radio recommends the use of rechargeable nickel-cadmium cells (nicads), although they have a 1.25-V emf, whereas alkaline cells have a 1.58-V emf. The radio has a **3.20Ω** resistance.

(a) Draw a circuit diagram of the radio and its batteries. Now, calculate the power delivered to the radio

(b) when using nicad cells, each having an internal resistance of **0.0400Ω**, and

(c) when using alkaline cells, each having an internal resistance of **0.200Ω**.

(d) Does this difference seem significant, considering that the radio's effective resistance is lowered when its volume is turned up?

22. An automobile starter motor has an equivalent resistance of **0.0500Ω** and is supplied by a 12.0-V battery with a **0.0100-Ω** internal resistance.

(a) What is the current to the motor?

(b) What voltage is applied to it?

(c) What power is supplied to the motor?

(d) Repeat these calculations for when the battery connections are corroded and add **0.0900Ω** to the circuit. (Significant problems are caused by even small amounts of unwanted resistance in low-voltage, high-current applications.)

23. (a) What is the internal resistance of a voltage source if its terminal potential drops by 2.00 V when the current supplied increases by 5.00 A?

(b) Can the emf of the voltage source be found with the information supplied?

24. A person with body resistance between his hands of **10.0kΩ** accidentally grasps the terminals of a 20.0-kV power supply. (Do NOT do this!)

(a) Draw a circuit diagram to represent the situation.

(b) If the internal resistance of the power supply is **2000Ω**, what is the current through his body?

(c) What is the power dissipated in his body?

(d) If the power supply is to be made safe by increasing its internal resistance, what should the internal resistance be for the maximum current in this situation to be 1.00 mA or less?

(e) Will this modification compromise the effectiveness of the power supply for driving low-resistance devices? Explain your reasoning.

25. A 12.0-V emf automobile battery has a terminal voltage of 16.0 V when being charged by a current of 10.0 A.

(a) What is the battery's internal resistance?

(b) What power is dissipated inside the battery?

(c) At what rate (in **°C/min**) will its temperature increase if its mass is 20.0 kg and it has a specific heat of **0.300kcal/kg · °C**, assuming no heat escapes?

10.3 Resistors in Series and Parallel

26. (a) What is the resistance of a $1.00 \times 10^2 - \Omega$, a $2.50 - k\Omega$, and a $4.00 - k\Omega$ resistor connected in series?

(b) In parallel?

27. What are the largest and smallest resistances you can obtain by connecting a **36.0-Ω**, a **50.0-Ω**, and a **700-Ω** resistor together?

28. An 1800-W toaster, a 1400-W speaker, and a 75-W lamp are plugged into the same outlet in a 15-A fuse and 120-V circuit. (The three devices are in parallel when plugged into the same socket.)

(a) What current is drawn by each device?

(b) Will this combination blow the 15-A fuse?

29. Your car's 30.0-W headlight and 2.40-kW starter are ordinarily connected in parallel in a 12.0-V system. What power would one headlight and the starter consume if connected in series to a 12.0-V battery? (Neglect any other resistance in the circuit and any change in resistance in the two devices.)

30. (a) Given a 48.0-V battery and **24.0-Ω** and **96.0-Ω**9 resistors, find the current and power for each when connected in series.

(b) Repeat when the resistances are in parallel.

31. Referring to the example combining series and parallel circuits and Figure 10.16, calculate I_3 in the following two different ways:

(a) from the known values of I and I_2;

(b) using Ohm's law for R_3. In both parts, explicitly show how you follow the steps in the Figure 10.17.

32. Referring to Figure 10.16,

(a) Calculate P_3 and note how it compares with P_3 found in the first two example problems in this module.

(b) Find the total power supplied by the source and compare it with the sum of the powers dissipated by the resistors.

33. Refer to Figure 10.17 and the discussion of lights dimming when a heavy appliance comes on.

(a) Given the voltage source is 120 V, the wire resistance is **0.800Ω**, and the bulb is nominally 75.0 W, what power will the bulb dissipate if a total of 15.0 A passes through the wires when the motor comes on? Assume negligible change in bulb resistance.

(b) What power is consumed by the motor?

34. Show that if two resistors R_1 and R_2 are combined and one is much greater than the other ($R_1 \gg R_2$),

(a) their series resistance is very nearly equal to the greater resistance R_1 and

(b) their parallel resistance is very nearly equal to the smaller resistance R_2.

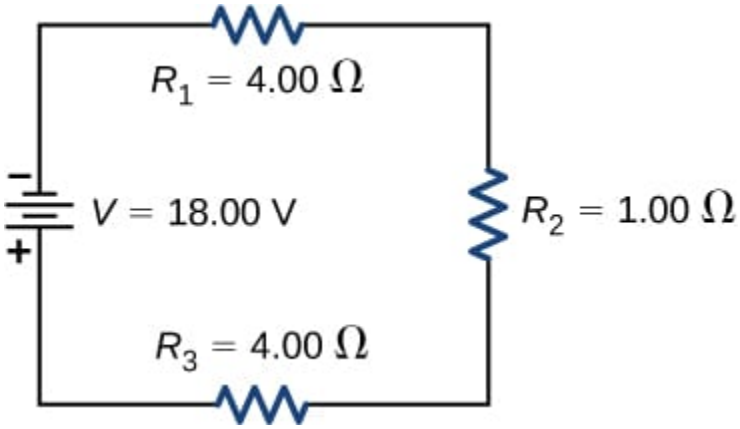
35. Consider the circuit shown below. The terminal voltage of the battery is **V=18.00V**.

(a) Find the equivalent resistance of the circuit.

(b) Find the current through each resistor.

(c) Find the potential drop across each resistor.

(d) Find the power dissipated by each resistor. (e) Find the power supplied by the battery.

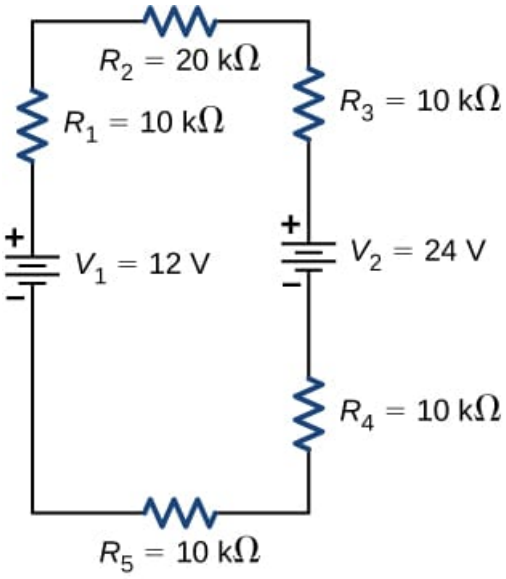


10.4 Kirchhoff's Rules

36. Consider the circuit shown below.

(a) Find the voltage across each resistor.

(b)What is the power supplied to the circuit and the power dissipated or consumed by the circuit?



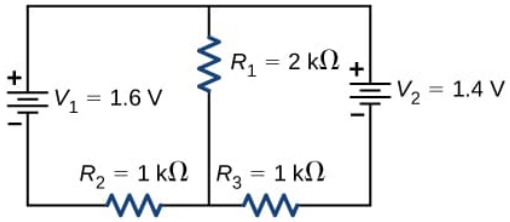
37. Consider the circuits shown below.

(a) What is the current through each resistor in part (a)?

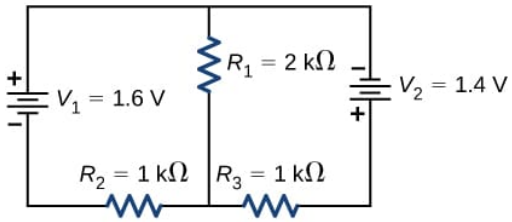
(b) What is the current through each resistor in part (b)?

(c) What is the power dissipated or consumed by each circuit?

(d) What is the power supplied to each circuit?

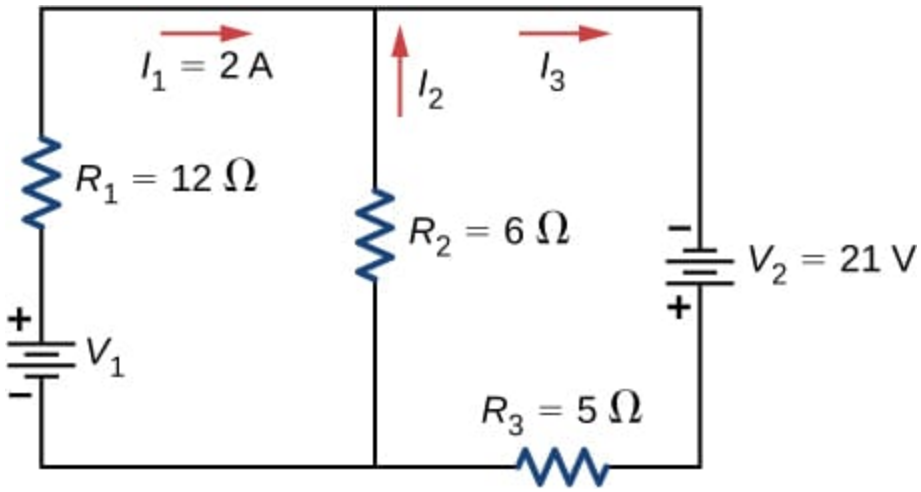


(a)

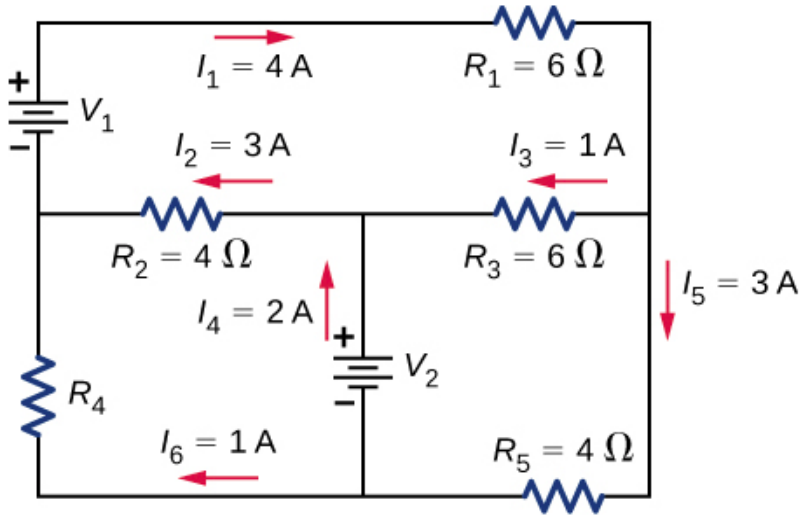


(b)

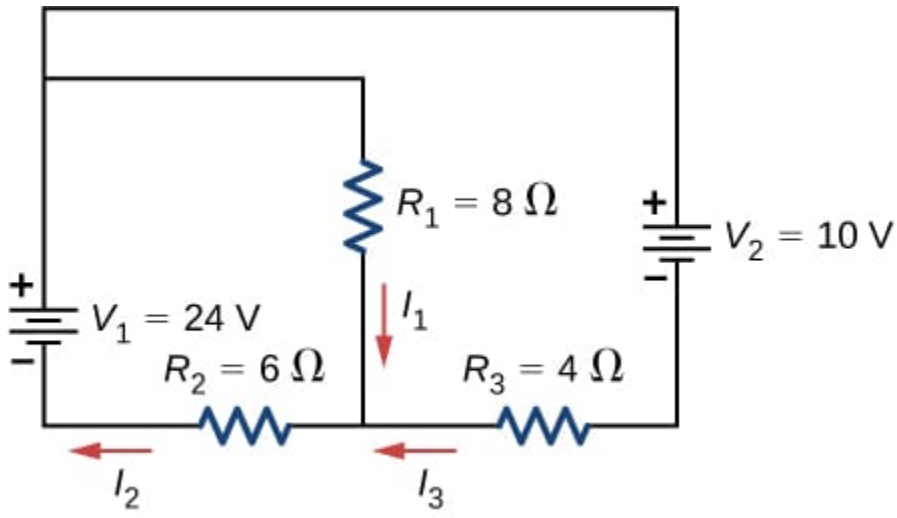
38. Consider the circuit shown below. Find V_1, I_2, and I_3.



39. Consider the circuit shown below. Find V_1, V_2, and R_4.



40. Consider the circuit shown below. Find I_1, I_2, and I_3.

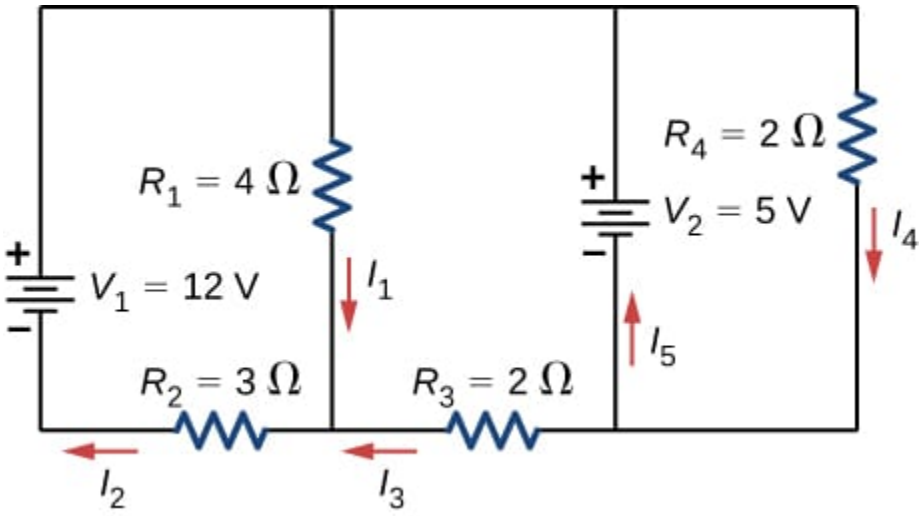


41. Consider the circuit shown below.

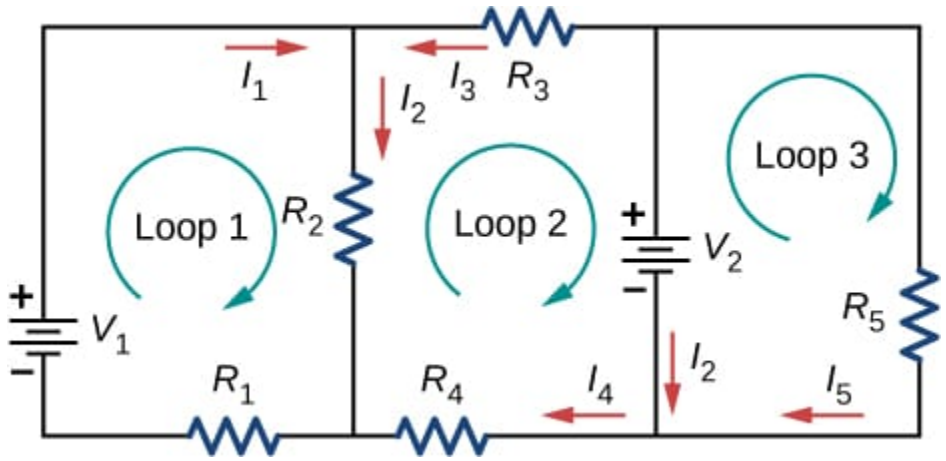
(a) Find I_1, I_2, I_3, I_4, and I_5.

(b) Find the power supplied by the voltage sources.

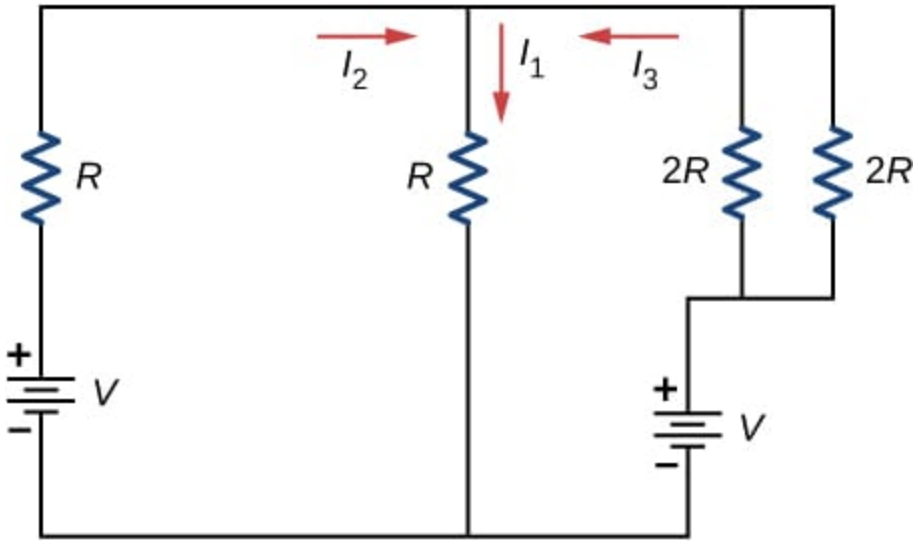
(c) Find the power dissipated by the resistors



42. Consider the circuit shown below. Write the three loop equations for the loops shown.



43. Consider the circuit shown below. Write equations for the three currents in terms of **R** and **V**.



44. Consider the circuit shown in the preceding problem. Write equations for the power supplied by the voltage sources and the power dissipated by the resistors in terms of R and V.

45. A child's electronic toy is supplied by three 1.58-V alkaline cells having internal resistances of **0.0200Ω** in series with a 1.53-V carbon-zinc dry cell having a **0.100-Ω** internal resistance. The load resistance is **10.0Ω**.

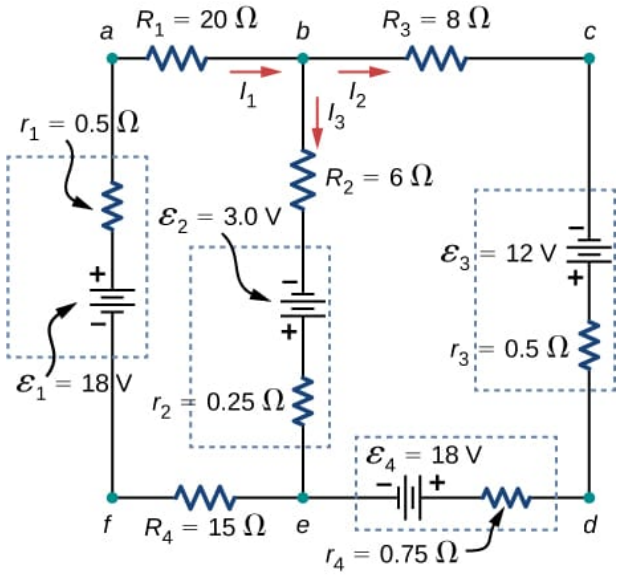
(a) Draw a circuit diagram of the toy and its batteries.

(b) What current flows?

(c) How much power is supplied to the load?

(d) What is the internal resistance of the dry cell if it goes bad, resulting in only 0.500 W being supplied to the load?

46. Apply the junction rule to Junction b shown below. Is any new information gained by applying the junction rule at e?



47. Apply the loop rule to Loop afedcba in the preceding problem.

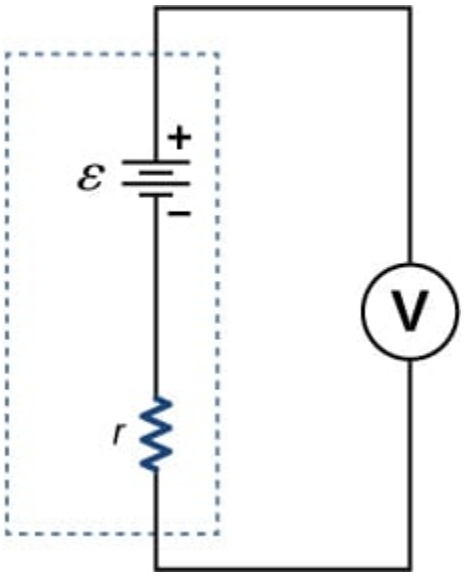
10.5 Electrical Measuring Instruments

48. Suppose you measure the terminal voltage of a 1.585-V alkaline cell having an internal resistance of **0.100Ω** by placing a **1.00-kΩ** voltmeter across its terminals (see below).

(a) What current flows?

(b) Find the terminal voltage.

(c) To see how close the measured terminal voltage is to the emf, calculate their ratio.



10.6 RC Circuits

49. The timing device in an automobile's intermittent wiper system is based on an RC time constant and utilizes a **0.500-μF** capacitor and a variable resistor. Over what range must R be made to vary to achieve time constants from 2.00 to 15.0 s?

50. A heart pacemaker fires 72 times a minute, each time a 25.0-nF capacitor is charged (by a battery in series with a resistor) to 0.632 of its full voltage. What is the value of the resistance?

51. The duration of a photographic flash is related to an RC time constant, which is **0.100μs** for a certain camera.

(a) If the resistance of the flash lamp is **0.0400Ω** during discharge, what is the size of the capacitor supplying its energy?

(b) What is the time constant for charging the capacitor, if the charging resistance is **800kΩ**?

52. A 2.00- and a **7.50-μF** capacitor can be connected in series or parallel, as can a 25.0- and a **100-kΩ** resistor. Calculate the four RC time constants possible from connecting the resulting capacitance and resistance in series.

53. A **500-Ω** resistor, an uncharged **1.50-μF** capacitor, and a 6.16-V emf are connected in series.

(a) What is the initial current?

(b) What is the RC time constant?

(c) What is the current after one time constant? (d) What is the voltage on the capacitor after one time constant?

54. A heart defibrillator being used on a patient has an RC time constant of 10.0 ms due to the resistance of the patient and the capacitance of the defibrillator.

(a) If the defibrillator has a capacitance of **8.00μF,** what is the resistance of the path through the patient? (You may neglect the capacitance of the patient and the resistance of the defibrillator.)

(b) If the initial voltage is 12.0 kV, how long does it take to decline to $6.00 \times 10^2 V$?

55. An ECG monitor must have an RC time constant less than $1.00 \times 10^2 \mu s$ to be able to measure variations in voltage over small time intervals.

(a) If the resistance of the circuit (due mostly to that of the patient's chest) is **1.00kΩ**, what is the maximum capacitance of the circuit?

(b) Would it be difficult in practice to limit the capacitance to less than the value found in (a)?

56. Using the exact exponential treatment, determine how much time is required to charge an initially uncharged 100-pF capacitor through a **75.0-MΩ** resistor to **90.0%** of its final voltage.

57. If you wish to take a picture of a bullet traveling at 500 m/s, then a very brief flash of light produced by an RC discharge through a flash tube can limit blurring. Assuming 1.00 mm of motion during one RC constant is acceptable, and given that the flash is driven by a **600-μF** capacitor, what is the resistance in the flash tube?

10.7 Household Wiring and Electrical Safety

58. (a) How much power is dissipated in a short circuit of 240-V ac through a resistance of **0.250Ω**? (b) What current flows?

59. What voltage is involved in a 1.44-kW short circuit through a **0.100-Ω** resistance?

60. Find the current through a person and identify the likely effect on her if she touches a 120-V ac source:

(a) if she is standing on a rubber mat and offers a total resistance of **300kΩ**;

(b) if she is standing barefoot on wet grass and has a resistance of only **4000kΩ**.

61. While taking a bath, a person touches the metal case of a radio. The path through the person to the drainpipe and ground has a resistance of **4000Ω**. What is the smallest voltage on the case of the radio that could cause ventricular fibrillation?

62. A man foolishly tries to fish a burning piece of bread from a toaster with a metal butter knife and comes into contact with 120-V ac. He does not even feel it since, luckily, he is wearing rubber-soled shoes. What is the minimum resistance of the path the current follows through the person?

63. (a) During surgery, a current as small as **20.0μA** applied directly to the heart may cause ventricular fibrillation. If the resistance of the exposed heart is **300Ω**, what is the smallest voltage that poses this danger?

(b) Does your answer imply that special electrical safety precautions are needed?

64. (a) What is the resistance of a 220-V ac short circuit that generates a peak power of 96.8 kW?

(b) What would the average power be if the voltage were 120 V ac?

65. A heart defibrillator passes 10.0 A through a patient's torso for 5.00 ms in an attempt to restore normal beating.

(a) How much charge passed?

(b) What voltage was applied if 500 J of energy was dissipated?

(c) What was the path's resistance? (d) Find the temperature increase caused in the 8.00 kg of affected tissue.

66. A short circuit in a 120-V appliance cord has a **0.500-Ω** resistance. Calculate the temperature rise of the 2.00 g of surrounding materials, assuming their specific heat capacity is **0.200cal/g·°C** and that it takes 0.0500 s for a circuit breaker to interrupt the current. Is this likely to be damaging?

Additional Problems

67. A circuit contains a D cell battery, a switch, a **20-Ω** resistor, and four 20-mF capacitors connected in series.

(a) What is the equivalent capacitance of the circuit?

(b) What is the RC time constant?

(c) How long before the current decreases to **50%** of the initial value once the switch is closed?

68. A circuit contains a D-cell battery, a switch, a **20-Ω** resistor, and three 20-mF capacitors. The capacitors are connected in parallel, and the parallel connection of capacitors are connected in series with the switch, the resistor and the battery.

(a) What is the equivalent capacitance of the circuit?

(b) What is the RC time constant?

(c) How long before the current decreases to **50%** of the initial value once the switch is closed?

69. Consider the circuit below. The battery has an emf of **ε=30.00V** and an internal resistance of **r=1.00Ω**.

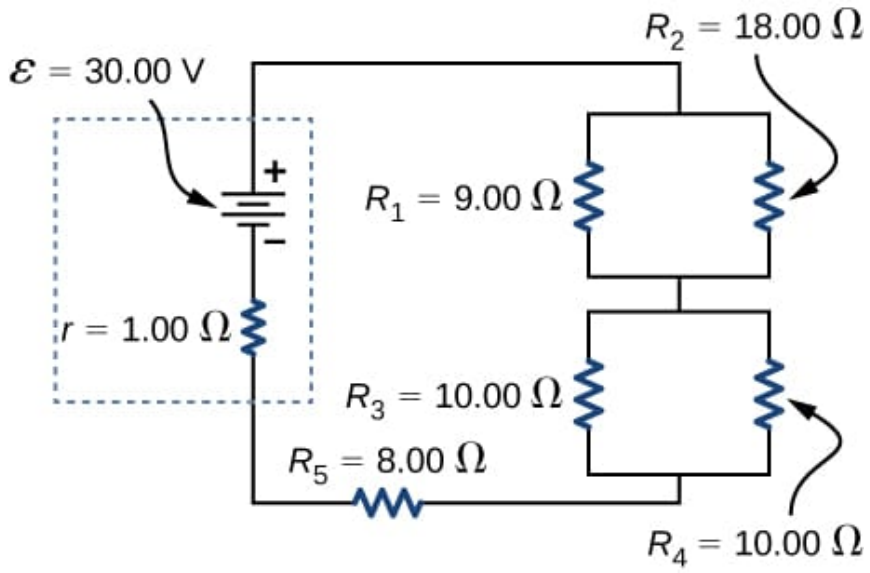
(a) Find the equivalent resistance of the circuit and the current out of the battery.

(b) Find the current through each resistor.

(c) Find the potential drop across each resistor.

(d) Find the power dissipated by each resistor.

(e) Find the total power supplied by the batteries.



70. A homemade capacitor is constructed of 2 sheets of aluminum foil with an area of 2.00 square meters, separated by paper, 0.05 mm thick, of the same area and a dielectric constant of 3.7. The homemade capacitor is connected in series with a **100.00-Ω** resistor, a switch, and a 6.00-V voltage source.

(a) What is the RC time constant of the circuit?

(b) What is the initial current through the circuit, when the switch is closed? (c) How long does it take the current to reach one third of its initial value?

71. A student makes a homemade resistor from a graphite pencil 5.00 cm long, where the graphite is 0.05 mm in diameter. The resistivity of the graphite is $\rho = 1.38 \times 10^{-5}\Omega/m$. The homemade resistor is place in series with a switch, a 10.00-mF capacitor and a 0.50-V power source.

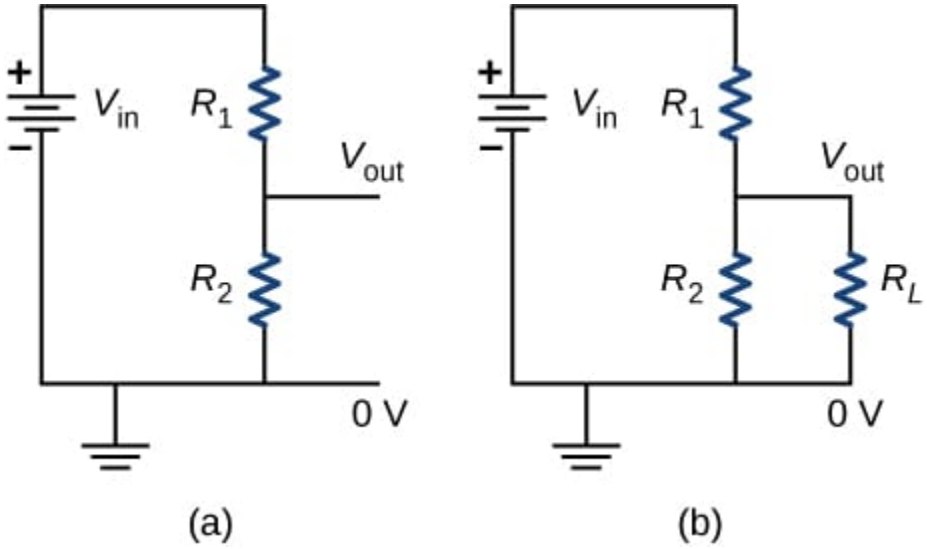
(a) What is the RC time constant of the circuit?

(b) What is the potential drop across the pencil 1.00 s after the switch is closed?

72. The rather simple circuit shown below is known as a voltage divider. The symbol consisting of three horizontal lines is represents "ground" and can be defined as the point where the potential is zero. The voltage divider is widely used in circuits and a single voltage source can be used to provide reduced voltage to a load resistor as shown in the second part of the figure. (a) What is the output voltage V_{out} of circuit

(a) in terms of R_1, R_2, and V_{in}?

(b) What is the output voltage V_{out} of circuit (b) in terms of R_1, R_2, R_L, and V_{in}?



73. Three **300-Ω** resistors are connect in series with an AAA battery with a rating of 3 AmpHours. (a) How long can the battery supply the resistors with power? (b) If the resistors are connected in parallel, how long can the battery last?

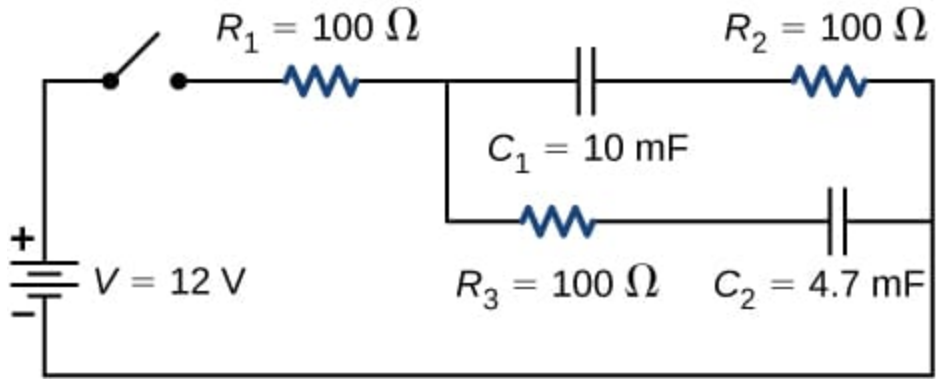
74. Consider a circuit that consists of a real battery with an emf εε and an internal resistance of r connected to a variable resistor R.

(a) In order for the terminal voltage of the battery to be equal to the emf of the battery, what should the resistance of the variable resistor be adjusted to?

(b) In order to get the maximum current from the battery, what should the resistance of the variable resistor be adjusted to?

(c) In order for the maximum power output of the battery to be reached, what should the resistance of the variable resistor be set to?

75. Consider the circuit shown below. What is the energy stored in each capacitor after the switch has been closed for a very long time?



76. Consider a circuit consisting of a battery with an emf εε and an internal resistance of r connected in series with a resistorR and a capacitor C. Show that the total energy supplied by the battery while charging the battery is equal to $\varepsilon^2 C$.

77. Consider the circuit shown below. The terminal voltages of the batteries are shown.

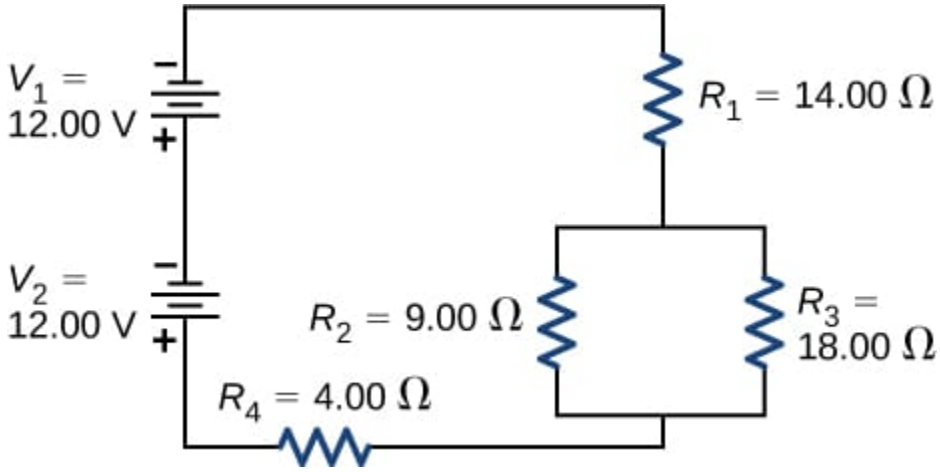
(a) Find the equivalent resistance of the circuit and the current out of the battery.

(b) Find the current through each resistor.

(c) Find the potential drop across each resistor.

(d) Find the power dissipated by each resistor.

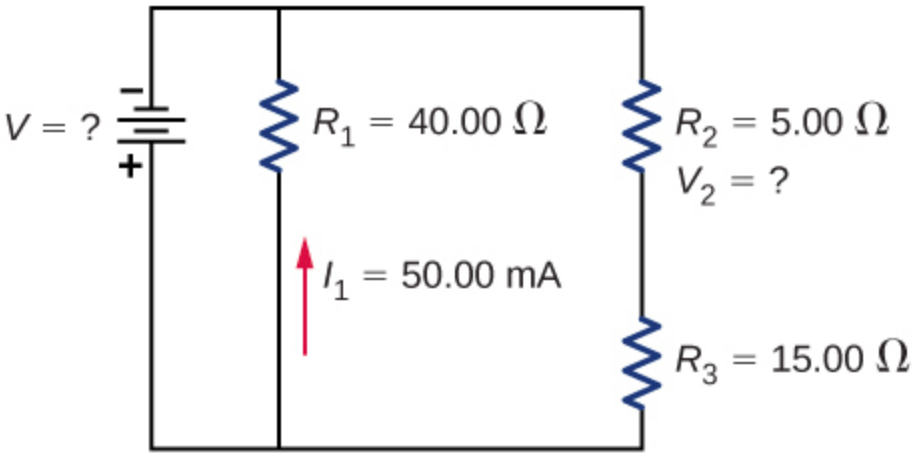
(e) Find the total power supplied by the batteries.



78. Consider the circuit shown below.

(a) What is the terminal voltage of the battery?

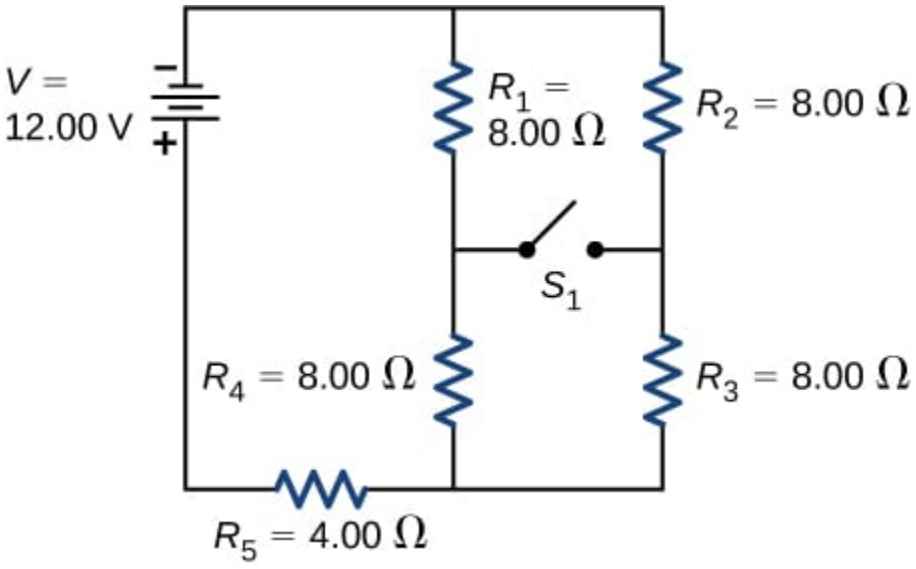
(b) What is the potential drop across resistor R_2?



79. Consider the circuit shown below.

(a)Determine the equivalent resistance and the current from the battery with switch S_1open.

(b) Determine the equivalent resistance and the current from the battery with switch S_1 closed.



80. Two resistors, one having a resistance of **145Ω**, are connected in parallel to produce a total resistance of **150Ω**.

(a) What is the value of the second resistance?

(b) What is unreasonable about this result?

(c) Which assumptions are unreasonable or inconsistent?

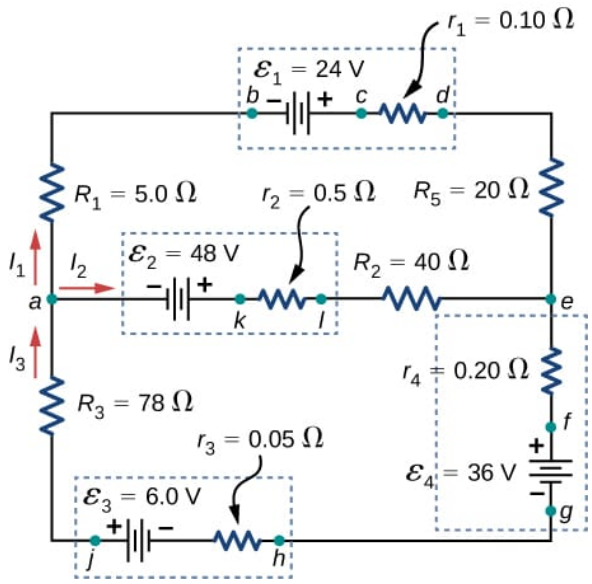
81. Two resistors, one having a resistance of **900kΩ**, are connected in series to produce a total resistance of **0.500MΩ**.

(a) What is the value of the second resistance?

(b) What is unreasonable about this result?

(c) Which assumptions are unreasonable or inconsistent?

82. Apply the junction rule at point a shown below.



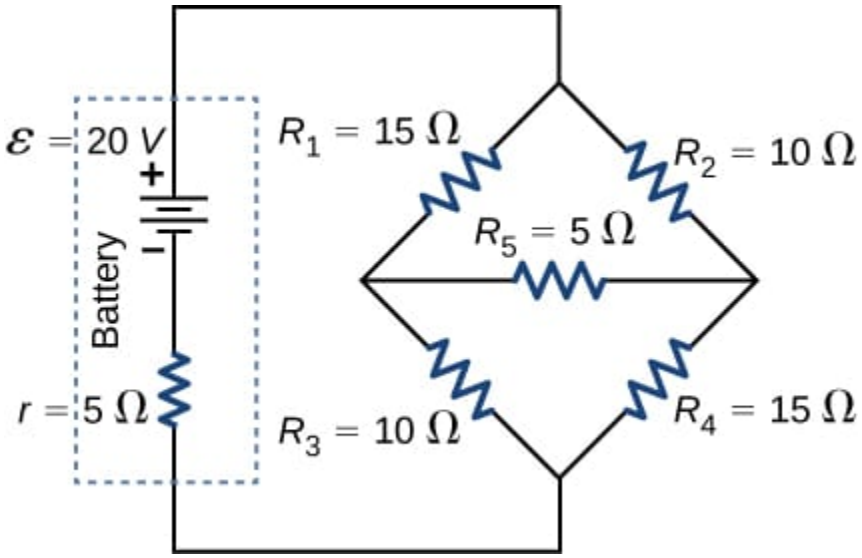
83. Apply the loop rule to Loop akledcba in the preceding problem.

84. Find the currents flowing in the circuit in the preceding problem. Explicitly show how you follow the steps in the Problem-Solving Strategy: Series and Parallel Resistors.

85. Consider the circuit shown below.

(a) Find the current through each resistor.

(b) Check the calculations by analyzing the power in the circuit.



86. A flashing lamp in a Christmas earring is based on an RC discharge of a capacitor through its resistance. The effective duration of the flash is 0.250 s, during which it produces an average 0.500 W from an average 3.00 V.

(a) What energy does it dissipate?

(b) How much charge moves through the lamp?

(c) Find the capacitance.

(d) What is the resistance of the lamp? (Since average values are given for some quantities, the shape of the pulse profile is not needed.)

87. A **160-μF** capacitor charged to 450 V is discharged through a **31.2-kΩ** resistor.

(a) Find the time constant.

(b) Calculate the temperature increase of the resistor, given that its mass is 2.50 g and its specific heat is **1.67kJ/kg · °C**, noting that most of the thermal energy is retained in the short time of the discharge.

(c) Calculate the new resistance, assuming it is pure carbon.

(d) Does this change in resistance seem significant?

Challenge Problems

88. Some camera flashes use flash tubes that require a high voltage. They obtain a high voltage by charging capacitors in parallel and then internally changing the connections of the capacitors to place them in series. Consider a circuit that uses four AAA batteries connected in series to charge six 10-mF capacitors through an equivalent resistance of **100Ω**. The connections are then switched internally to place the capacitors in series. The capacitors discharge through a lamp with a resistance of **100Ω**.

(a) What is the RC time constant and the initial current out of the batteries while they are connected in parallel?

(b) How long does it take for the capacitors to charge to **90%** of the terminal voltages of the batteries?

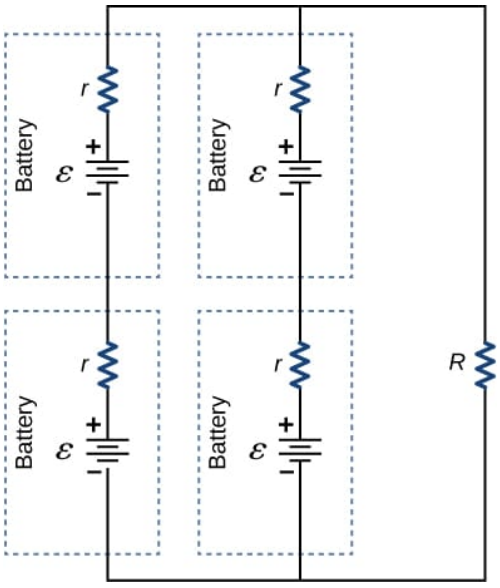
(c) What is the RC time constant and the initial current of the capacitors connected in series assuming it discharges at 90%90% of full charge?

(d) How long does it take the current to decrease to **10%** of the initial value?

89. Consider the circuit shown below. Each battery has an emf of 1.50 V and an internal resistance of **1.00Ω**.

(a) What is the current through the external resistor, which has a resistance of 10.00 ohms?

(b) What is the terminal voltage of each battery?

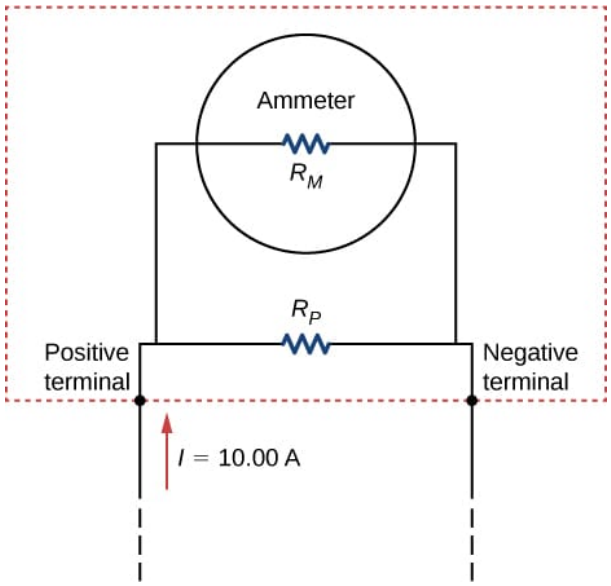


90. Analog meters use a galvanometer, which essentially consists of a coil of wire with a small resistance and a pointer with a scale attached. When current runs through the coil, the pointer turns; the amount the pointer turns is proportional to the amount of current running through the coil. Galvanometers can be used to make an ammeter if a resistor is placed in parallel with the galvanometer. Consider a galvanometer that has a resistance of **25.00Ω** and gives a full scale reading when a **50-μA** current runs through it. The galvanometer is to be used to make an ammeter that has a full scale reading of 10.00 A, as shown below. Recall that an ammeter is connected in series with the circuit of interest, so all 10 A must run through the meter.

(a) What is the current through the parallel resistor in the meter?

(b) What is the voltage across the parallel resistor?

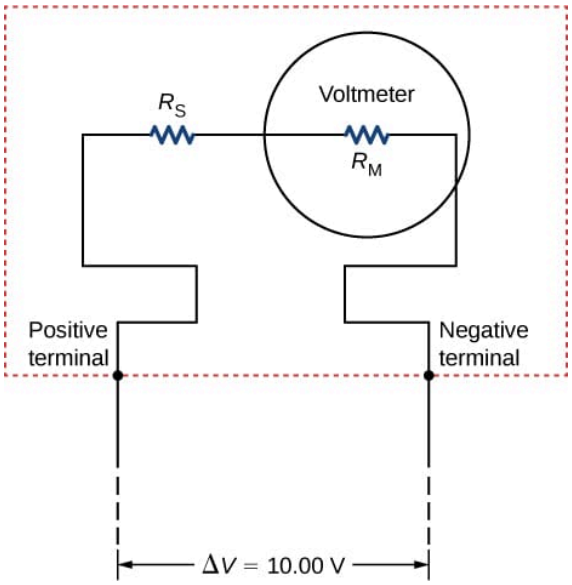
(c) What is the resistance of the parallel resistor?



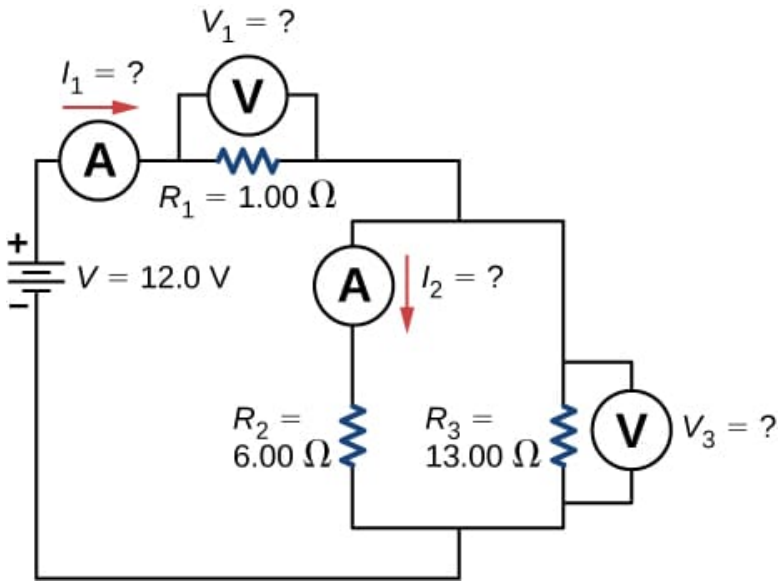
91. Analog meters use a galvanometer, which essentially consists of a coil of wire with a small resistance and a pointer with a scale attached. When current runs through the coil, the point turns; the amount the pointer turns is proportional to the amount of current running through the coil. Galvanometers can be used to make a voltmeter if a resistor is placed in series with the galvanometer. Consider a galvanometer that has a resistance of **25.00Ω** and gives a full scale reading when a **50-μA** current runs through it. The galvanometer is to be used to make an voltmeter that has a full scale reading of 10.00 V, as shown below. Recall that a voltmeter is connected in parallel with the component of interest, so the meter must have a high resistance or it will change the current running through the component.

(a) What is the potential drop across the series resistor in the meter?

(b) What is the resistance of the parallel resistor?



92. Consider the circuit shown below. Find I_1, V_1, I_2, and V_3.

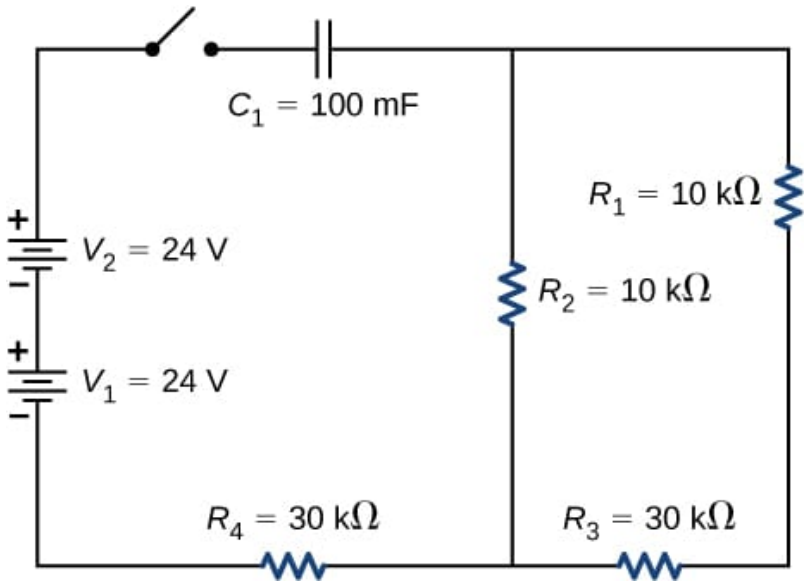


93. Consider the circuit below.

(a) What is the RC time constant of the circuit?

(b) What is the initial current in the circuit once the switch is closed?

(c) How much time passes between the instant the switch is closed and the time the current has reached half of the initial current?



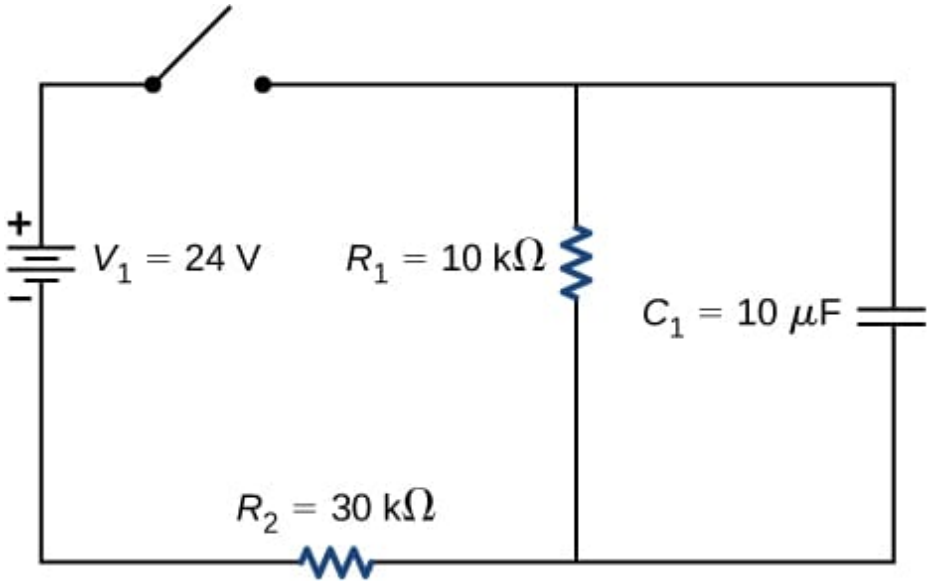
94. Consider the circuit below.

(a) What is the initial current through resistor R_2 when the switch is closed?

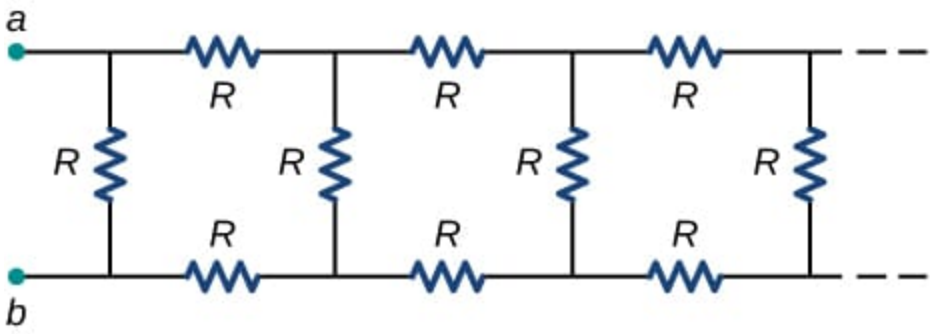
(b) What is the current through resistor R_2 when the capacitor is fully charged, long after the switch is closed?

(c) What happens if the switch is opened after it has been closed for some time?

(d) If the switch has been closed for a time period long enough for the capacitor to become fully charged, and then the switch is opened, how long before the current through resistor R_1 reaches half of its initial value?



95. Consider the infinitely long chain of resistors shown below. What is the resistance between terminals **a** and **b**?



96. Consider the circuit below. The capacitor has a capacitance of 10 mF. The switch is closed and after a long time the capacitor is fully charged.

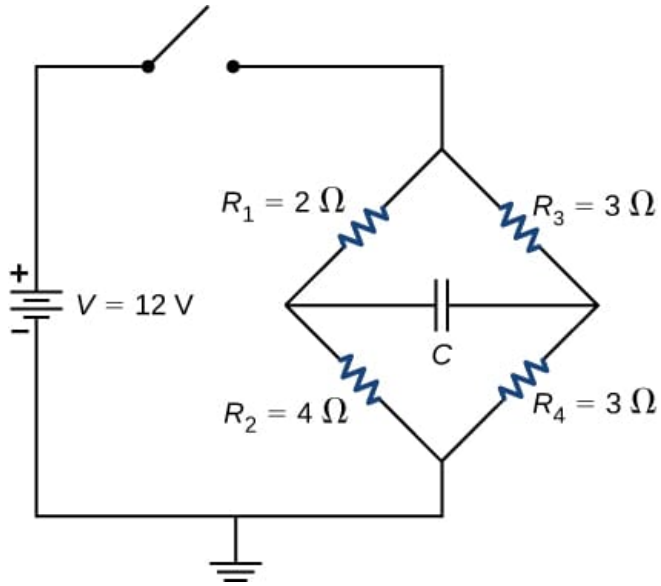
(a) What is the current through each resistor a long time after the switch is closed?

(b) What is the voltage across each resistor a long time after the switch is closed?

(c) What is the voltage across the capacitor a long time after the switch is closed?

(d) What is the charge on the capacitor a long time after the switch is closed?

(e) The switch is then opened. The capacitor discharges through the resistors. How long from the time before the current drops to one fifth of the initial value?



97. A 120-V immersion heater consists of a coil of wire that is placed in a cup to boil the water. The heater can boil one cup of **20.00°C** water in 180.00 seconds. You buy one to use in your dorm room, but you are worried that you will overload the circuit and trip the 15.00-A, 120-V circuit breaker, which supplies your dorm room. In your dorm room, you have four 100.00-W incandescent lamps and a 1500.00-W space heater.

(a) What is the power rating of the immersion heater?

(b) Will it trip the breaker when everything is turned on?

(c) If it you replace the incandescent bulbs with 18.00-W LED, will the breaker trip when everything is turned on?

98. Find the resistance that must be placed in series with a **25.0-Ω** galvanometer having a **50.0-μA** sensitivity (the same as the one discussed in the text) to allow it to be used as a voltmeter with a 3000-V full-scale reading. Include a circuit

diagram with your solution.

99. Find the resistance that must be placed in parallel with a **60.0-Ω** galvanometer having a 1.00-mA sensitivity (the same as the one discussed in the text) to allow it to be used as an ammeter with a 25.0-A full-scale reading. Include a circuit diagram with your solution.

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13.10: Direct-Current Circuits (Answers)

Check Your Understanding

10.1. If a wire is connected across the terminals, the load resistance is close to zero, or at least considerably less than the internal resistance of the battery. Since the internal resistance is small, the current through the circuit will be large, $I = \frac{\varepsilon}{R+r} = \frac{\varepsilon}{0+r} = \frac{\varepsilon}{r}$.The large current causes a high power to be dissipated by the internal resistance $(P = I^2 r)$. The power is dissipated as heat.

10.2. The equivalent resistance of nine bulbs connected in series is **9R**. The current is $I = V/9R$. If one bulb burns out, the equivalent resistance is **8R**, and the voltage does not change, but the current increases $(I = V/8R)$. As more bulbs burn out, the current becomes even higher. Eventually, the current becomes too high, burning out the shunt.

10.3. The equivalent of the series circuit would be $R_{eq} = 1.00\Omega + 2.00\Omega + 2.00\Omega = 5.00\Omega$, which is higher than the equivalent resistance of the parallel circuit $R_{eq} = 0.50\Omega$. The equivalent resistor of any number of resistors is always higher than the equivalent resistance of the same resistors connected in parallel. The current through for the series circuit would be $I = \frac{3.00V}{5.00\Omega} = 0.60A$, which is lower than the sum of the currents through each resistor in the parallel circuit, $I = 6.00A$. This is not surprising since the equivalent resistance of the series circuit is higher. The current through a series connection of any number of resistors will always be lower than the current into a parallel connection of the same resistors, since the equivalent resistance of the series circuit will be higher than the parallel circuit. The power dissipated by the resistors in series would be $P = 1.80W$, which is lower than the power dissipated in the parallel circuit $P = 18.00W$.

10.4. A river, flowing horizontally at a constant rate, splits in two and flows over two waterfalls. The water molecules are analogous to the electrons in the parallel circuits. The number of water molecules that flow in the river and falls must be equal to the number of molecules that flow over each waterfall, just like sum of the current through each resistor must be equal to the current flowing into the parallel circuit. The water molecules in the river have energy due to their motion and height. The potential energy of the water molecules in the river is constant due to their equal heights. This is analogous to the constant change in voltage across a parallel circuit. Voltage is the potential energy across each resistor.

The analogy quickly breaks down when considering the energy. In the waterfall, the potential energy is converted into kinetic energy of the water molecules. In the case of electrons flowing through a resistor, the potential drop is converted into heat and light, not into the kinetic energy of the electrons.

10.5.

1. All the overhead lighting circuits are in parallel and connected to the main supply line, so when one bulb burns out, all the overhead lighting does not go dark. Each overhead light will have at least one switch in series with the light, so you can turn it on and off.

2. A refrigerator has a compressor and a light that goes on when the door opens. There is usually only one cord for the refrigerator to plug into the wall. The circuit containing the compressor and the circuit containing the lighting circuit are in parallel, but there is a switch in series with the light. A thermostat controls a switch that is in series with the compressor to control the temperature of the refrigerator.

10.6. The circuit can be analyzed using Kirchhoff's loop rule. The first voltage source supplies power: $P_{in} = IV_1 = 7.20mW$. The second voltage source consumes power: $P_{out} = IV_2 + I^2R_1 + I^2R_2 = 7.2mW$.

10.7. The current calculated would be equal to $I = -0.20A$ instead of $I = 0.20A$. The sum of the power dissipated and the power consumed would still equal the power supplied.

10.8. Since digital meters require less current than analog meters, they alter the circuit less than analog meters. Their resistance as a voltmeter can be far greater than an analog meter, and their resistance as an ammeter can be far less than an analog meter. Consult Figure 10.36 and Figure 10.35 and their discussion in the text.

Conceptual Questions

1. Some of the energy being used to recharge the battery will be dissipated as heat by the internal resistance.

3. $P = I^2R = (\frac{\varepsilon}{r+R})^2 R = \varepsilon^2 R(r+R)^{-2}, \frac{dP}{dR} = \varepsilon^2[(r+R)^{-2} - 2R(r+R)^{-3}] = 0\ [\frac{(r+R)-2R}{(r+R)^3}] = 0, r = R$

5. It would probably be better to be in series because the current will be less than if it were in parallel.

7. two filaments, a low resistance and a high resistance, connected in parallel

9. It can be redrawn.

$$R_{eq} = [\frac{1}{R_6} + \frac{1}{R_1} + \frac{1}{R_2 + (\frac{1}{R_4} + \frac{1}{R_3+R+5})^{-1}}]^{-1}.$$

11. In series the voltages add, but so do the internal resistances, because the internal resistances are in series. In parallel, the terminal voltage is the same, but the equivalent internal resistance is smaller than the smallest individual internal resistance and a higher current can be provided.

13. The voltmeter would put a large resistance in series with the circuit, significantly changing the circuit. It would probably give a reading, but it would be meaningless.

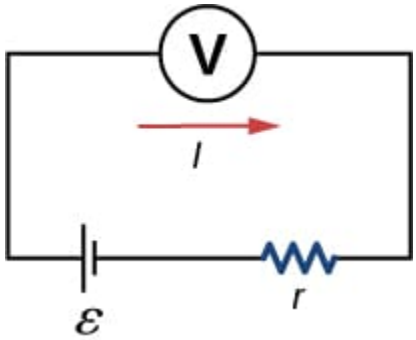
15. The ammeter has a small resistance; therefore, a large current will be produced and could damage the meter and/or overheat the battery.

17. The time constant can be shortened by using a smaller resistor and/or a smaller capacitor. Care should be taken when reducing the resistance because the initial current will increase as the resistance decreases.

19. Not only might water drip into the switch and cause a shock, but also the resistance of your body is lower when you are wet.

Problems

21. a.



b. 0.476W;

c. 0.691 W;

d. As R_L is lowered, the power difference decreases; therefore, at higher volumes, there is no significant difference.

23. a. 0.400Ω;

b. No, there is only one independent equation, so only r can be found.

25. a. 0.400Ω;

b. 40.0 W;

c. $0.0956\ ^\circ C/min$

27. largest, 786Ω, smallest, 20.32Ω

29. 29.6 W

31. a. 0.74 A;

b. 0.742 A

33. a. 60.8 W;

b. 3.18 kW

35. a. $R_s = 9.00\Omega$;

b.$I_1 = I_2 = I_3 = 2.00A$;

c. $V_1 = 8.00V, V_2 = 2.00V, V_3 = 8.00V$;

d. $P_1 = 16.00W, P_2 = 4.00W, P_3 = 16.00W$;

e. $P = 36.00W$

37. a. $I_1 = 0.6mA, I_2 = 0.4mA, I_3 = 0.2m$;

b. $I_1 = 0.04mA, I_2 = 1.52mA, I_3 = -1.48mA$;

c. $P_{out} = 0.92mW, P_{out} = 4.50mW$;

d. $P_{in} = 0.92mW, P_{in} = 4.50mW$

39. $V_1 = 42V, V_2 = 6V, R_4 = 18\Omega$

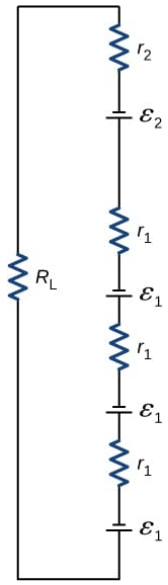
41. a. $I_1 = 1.5A, I_2 = 2A, I_3 = 0.5A, I_4 = 2.5A, I_5 = 2A$;

b. $P_{in} = I_2V_1 + I_5V_5 = 34W$;

c. $P_{out} = I_1^2R_1 + I_2^2R_2 + I_3^2R_3 + I_4^2R_4 = 34W$

43. $I_1 = \frac{2}{3}\frac{V}{R}, I_2 = \frac{1}{3}\frac{V}{R}, I_3 = \frac{1}{3}\frac{V}{R}$

45. a.



b. 0.617 A;

c. 3.81 W;

d. 18.0Ω

47. $I_1 r_1 - \varepsilon_1 + I_1 R_4 + \varepsilon_4 + I_2 r_4 + I_4 r_3 - \varepsilon_3 + I_2 R_3 + I_1 R_1 = 0$

49. 4.00 to $30.0 M\Omega$

51. a. $2.50 \mu F$;

b. 2.00 s

53. a. 12.3 mA;

b. $7.50 \times 10^{-4} s$;

c. 4.53 mA;

d. 3.89 V

55. a. $1.00 \times 10^{-7} F$;

b. No, in practice it would not be difficult to limit the capacitance to less than 100 nF, since typical capacitors range from fractions of a picofarad (pF) to milifarad (mF).

57. $3.33 \times 10^{-3} \Omega$

59. 12.0 V

61. 400 V

63. a. 6.00 mV; .

b. It would not be necessary to take extra precautions regarding the power coming from the wall. However, it is possible to generate voltages of approximately this value from static charge built up on gloves, for instance, so some precautions are necessary.

65. a. $5.00 \times 10^{-2} C$;

b. 10.0 kV;

c. $1.00k\Omega$;

d. $1.79 \times 10^{-2}\ ^{\circ}C$

Additional Problems

67. a. $C_{eq} = 5.00mF$;

b. $\tau = 0.1s$;

c. 0.069 s

69. a. $R_{eq} = 20.00\Omega$;

b. $I_r = 1.50A, I_1 = 1.00A, I_2 = 0.50A, I_3 = 0.75A, I_4 = 0.75A, I_5 = 1.50A$;

c. $V_r = 1.50V, V_1 = 9.00V, V_2 = 9.00V, V_3 = 7.50V, V_4 = 7.50V, V_5 = 12.00V$;

d. $P_r = 2.25W, P_1 = 9.00W, P_2 = 4.50W, P_3 = 5.625W, P_4 = 5.625W, P_5 = 18.00W$;

e. $P = 45.00W$

71. a. $\tau = \left(1.38 \times 10^{-5}\ \Omega\text{m}\left(\frac{5.00 \times 10^{-2}\ \text{m}}{3.14\left(\frac{0.05 \times 10^{-3}}{2}\right)^2}\right)\right) 10 \times 10^{-3}\ \text{F} = 3.52\ \text{s}$;

b. $V = 0.017A(e^{-\frac{1.00\ s}{3.52\ s}})351.59\ \Omega = 0.122\ V$

73. a. $t = \frac{3A \cdot h}{\frac{1.5V}{900\Omega}} = 1800h$;

b. $t = \frac{3A \cdot h}{\frac{1.5V}{100\Omega}} = 200h$

75. $U_1 = C_1V_1^2 = 0.72J, U_2 = C_2V_2^2 = 0.338J$

77. a. $R_{eq} = 24.00\Omega$;

b. $I_1 = 1.00A, I_2 = 0.67A, I_3 = 0.33A, I_4 = 1.00A$;

c. $V_1 = 14.00V, V_2 = 6.00V, V_3 = 6.00V, V_4 = 4.00V$;

d. $P_1 = 14.00W, P_2 = 4.04W, P_3 = 1.96W, P_4 = 4.00W$;

e. $P = 24.00W$

79. a. $R_{eq} = 12.00\Omega, I = 1.00A$;

b. $R_{eq} = 12.00\Omega, I = 1.00A$

81. a. $-400k\Omega$;

b. You cannot have negative resistance.

c. The assumption that \(\displaystyle R_{eq}

83. $E_2 - I_2r_2 - I_2R_2 + I_1R_5 + I_1r_1 - E_1 + I_1R_1 = 0$

85. a. $I = 1.17A, I_1 = 0.50A, I_2 = 0.67A, I_3 = 0.67A, I_4 = 0.50A, I_5 = 0.17A$;

b. $P_{output} = 23.4W, P_{input} = 23.4W$

87. a. 4.99 s;

b. $3.87\ ^\circ C$;

c. $3.11 \times 10^4 \Omega$;

d. No, this change does not seem significant. It probably would not be noticed.

Challenge Problems

89. a. 0.273 A; b. $V_T = 1.36V$

91. a. $V_s = V - I_M R_M = 9.99875V$;

b. $R_S = \dfrac{V_P}{I_M} = 199.975k\Omega$

93. a. $\tau = 3800s$;

b. 1.26 A;

c. $t = 2633.96s$

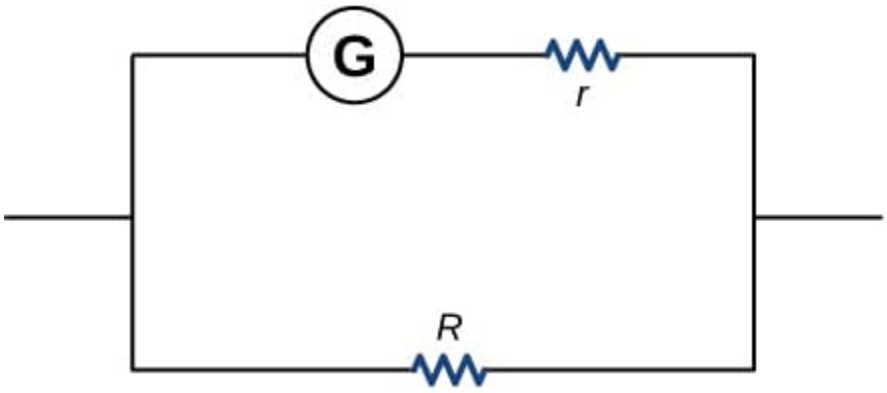
95. $R_{eq} = (\sqrt{3} - 1)R$

97. a. $P_{imheater} = \dfrac{1cup(\frac{0.000237m^3}{cup})(\frac{1000kg}{m^3})(4186\frac{J}{kg°C})(100°C - 20°C)}{180.00s} \approx 441W$,

b. $I = \dfrac{441W}{120V} + 4(\dfrac{100W}{120V}) + \dfrac{1500W}{120V} = 19.51A$; Yes, the breaker will trip.

c. $I = \dfrac{441W}{120V} + 4(\dfrac{18W}{120V}) + \dfrac{1500W}{120V} = 13.47$; No, the breaker will not trip.

99. $2.40 \times 10^{-3} \Omega$



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14: MAGNETIC FORCES AND FIELDS

14.1: Prelude to Magnetic Forces and Fields

14.2: Magnetism and Its Historical Discoveries

14.3: Magnetic Fields and Lines

14.4: Motion of a Charged Particle in a Magnetic Field

14.5: Magnetic Force on a Current-Carrying Conductor

14.6: Force and Torque on a Current Loop

14.7: The Hall Effect

14.8: Applications of Magnetic Forces and Fields

14.9: Magnetic Forces and Fields (Summary)

14.10: Magnetic Forces and Fields (Exercise)

14.11: Magnetic Forces and Fields (Answers)

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14.1: Prelude to Magnetic Forces and Fields

For the past few chapters, we have been studying electrostatic forces and fields, which are caused by electric charges at rest. These electric fields can move other free charges, such as producing a current in a circuit; however, the electrostatic forces and fields themselves come from other static charges. In this chapter, we see that when an electric charge moves, it generates other forces and fields. These additional forces and fields are what we commonly call magnetism.



Figure 14.1.1: An industrial electromagnet is capable of lifting thousands of pounds of metallic waste. (credit: modification of work by "BedfordAl"/Flickr)

Before we examine the origins of magnetism, we first describe what it is and how magnetic fields behave. Once we are more familiar with magnetic effects, we can explain how they arise from the behavior of atoms and molecules, and how magnetism is related to electricity. The connection between electricity and magnetism is fascinating from a theoretical point of view, but it is also immensely practical, as shown by an industrial electromagnet that can lift thousands of pounds of metal.

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14.2: Magnetism and Its Historical Discoveries

Learning Objectives

By the end of this section, you will be able to:

- Explain attraction and repulsion by magnets
- Describe the historical and contemporary applications of magnetism

Magnetism has been known since the time of the ancient Greeks, but it has always been a bit mysterious. You can see electricity in the flash of a lightning bolt, but when a compass needle points to magnetic north, you can't see any force causing it to rotate. People learned about magnetic properties gradually, over many years, before several physicists of the nineteenth century connected magnetism with electricity. In this section, we review the basic ideas of magnetism and describe how they fit into the picture of a magnetic field.

Brief History of Magnetism

Magnets are commonly found in everyday objects, such as toys, hangers, elevators, doorbells, and computer devices. Experimentation on these magnets shows that all magnets have two poles: One is labeled north (N) and the other is labeled south (S). Magnetic poles repel if they are alike (both N or both S), they attract if they are opposite (one N and the other S), and both poles of a magnet attract unmagnetized pieces of iron. An important point to note here is that you cannot isolate an individual magnetic pole. Every piece of a magnet, no matter how small, which contains a north pole must also contain a south pole.

Note

Visit this website for an interactive demonstration of magnetic north and south poles.

An example of a magnet is a **compass needle**. It is simply a thin bar magnet suspended at its center, so it is free to rotate in a horizontal plane. Earth itself also acts like a very large bar magnet, with its south-seeking pole near the geographic North Pole (Figure 14.2.1). The north pole of a compass is attracted toward Earth's geographic North Pole because the magnetic pole that is near the geographic North Pole is actually a south magnetic pole. Confusion arises because the geographic term "North Pole" has come to be used (incorrectly) for the magnetic pole that is near the North Pole. Thus, "**north magnetic pole**" is actually a misnomer—it should be called the **south magnetic pole**. [Note that the orientation of Earth's magnetic field is not permanent but changes ("flips") after long time intervals. Eventually, Earth's north magnetic pole may be located near its geographic North Pole.]

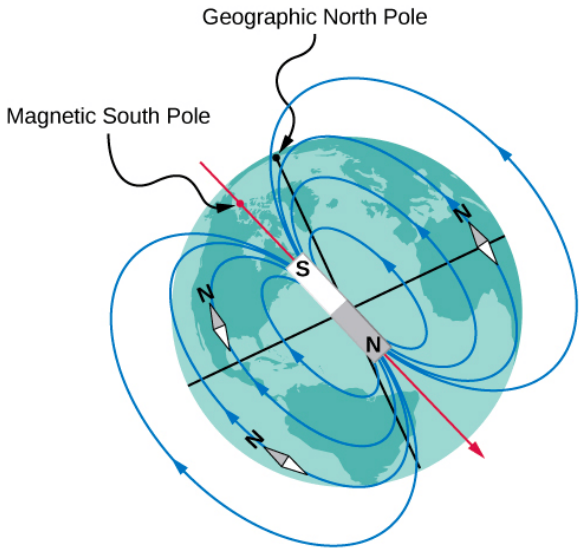


Figure 14.2.1: The north pole of a compass needle points toward the south pole of a magnet, which is how today's magnetic field is oriented from inside Earth. It also points toward Earth's geographic North Pole because the geographic North Pole is near the magnetic south pole.

Back in 1819, the Danish physicist Hans **Oersted** was performing a lecture demonstration for some students and noticed that a compass needle moved whenever current flowed in a nearby wire. Further investigation of this phenomenon convinced Oersted that an electric current could somehow cause a magnetic force. He reported this finding to an 1820 meeting of the French Academy of Science.

Soon after this report, Oersted's investigations were repeated and expanded upon by other scientists. Among those whose work was especially important were Jean-Baptiste **Biot** and Felix **Savart**, who investigated the forces exerted on magnets by currents; André Marie **Ampère**, who studied the forces exerted by one current on another; François **Arago**, who found that iron could be magnetized by a current; and Humphry **Davy**, who discovered that a magnet exerts a force on a wire carrying an electric current. Within 10 years of Oersted's discovery, Michael **Faraday** found that the relative motion of a magnet and a metallic wire induced current in the wire. This finding showed not only that a current has a magnetic effect, but that a magnet can generate electric current. You will see later that the names of Biot, Savart, Ampère, and Faraday are linked to some of the fundamental laws of electromagnetism.

The evidence from these various experiments led Ampère to propose that electric current is the source of all magnetic phenomena. To explain permanent magnets, he suggested that matter contains microscopic current loops that are somehow aligned when a material is magnetized. Today, we know that permanent magnets are actually created by the alignment of spinning electrons, a situation quite similar to that proposed by Ampère. This model of permanent magnets was developed by Ampère almost a century before the atomic nature of matter was understood. (For a full quantum mechanical treatment of magnetic spins, see Quantum Mechanics and Atomic Structure.)

Contemporary Applications of Magnetism

Today, magnetism plays many important roles in our lives. Physicists' understanding of magnetism has enabled the development of technologies that affect both individuals and society. The electronic tablet in your purse or backpack, for example, wouldn't have been possible without the applications of magnetism and electricity on a small scale (Figure 14.2.2). Weak changes in a magnetic field in a thin film of iron and chromium were discovered to bring about much larger changes in resistance, called **giant magnetoresistance**. Information can then be recorded magnetically based on the direction in which the iron layer is magnetized. As a result of the discovery of giant magnetoresistance and its applications to digital storage, the 2007 Nobel Prize in Physics was awarded to Albert Fert from France and Peter Grunberg from Germany.

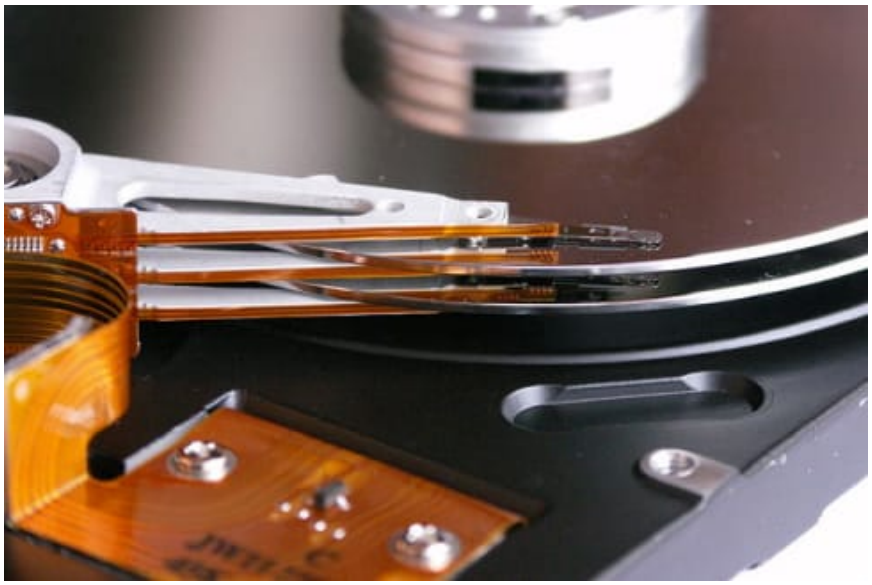


Figure 14.2.2: Engineering technology like computer storage would not be possible without a deep understanding of magnetism. (credit: Klaus Eifert)

All electric motors—with uses as diverse as powering refrigerators, starting cars, and moving elevators—contain magnets. Generators, whether producing hydroelectric power or running bicycle lights, use magnetic fields. Recycling facilities employ magnets to separate iron from other refuse. Research into using magnetic containment of fusion as a future energy source has been continuing for several years. Magnetic resonance imaging (MRI) has become an important diagnostic tool in the field of medicine, and the use of magnetism to explore brain activity is a subject of contemporary research and development. The list of applications also includes computer hard drives, tape recording, detection of inhaled asbestos, and levitation of high-speed trains. Magnetism is involved in the structure of atomic energy levels, as well as the motion of cosmic rays and charged particles trapped in the Van Allen belts around Earth. Once again, we see that all these disparate phenomena are linked by a small number of underlying physical principles.

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14.3: Magnetic Fields and Lines

Learning Objectives

By the end of this section, you will be able to:

- Define the magnetic field based on a moving charge experiencing a force
- Apply the right-hand rule to determine the direction of a magnetic force based on the motion of a charge in a magnetic field
- Sketch magnetic field lines to understand which way the magnetic field points and how strong it is in a region of space

We have outlined the properties of magnets, described how they behave, and listed some of the applications of magnetic properties. Even though there are no such things as isolated magnetic charges, we can still define the attraction and repulsion of magnets as based on a field. In this section, we define the magnetic field, determine its direction based on the right-hand rule, and discuss how to draw magnetic field lines.

Defining the Magnetic Field

A magnetic field is defined by the force that a charged particle experiences moving in this field, after we account for the gravitational and any additional electric forces possible on the charge. The magnitude of this force is proportional to the amount of charge **q**, the speed of the charged particle **v**, and the magnitude of the applied magnetic field. The direction of this force is perpendicular to both the direction of the moving charged particle and the direction of the applied magnetic field. Based on these observations, we define the magnetic field strength **B** based on the **magnetic force** $\vec{F}$ on a charge **q** moving at velocity as the cross product of the velocity and magnetic field, that is,

$$\vec{F} = q\vec{v} \times \vec{B}. \tag{14.3.1}$$

In fact, this is how we define the magnetic field $\vec{B}$ - in terms of the force on a charged particle moving in a magnetic field. The magnitude of the force is determined from the definition of the cross product as it relates to the magnitudes of each of the vectors. In other words, the magnitude of the force satisfies

$$F = qv\,B\sin\,\theta \tag{14.3.2}$$

where **θ** is the angle between the velocity and the magnetic field.

The SI unit for magnetic field strength B is called the tesla (T) after the eccentric, but brilliant inventor Nikola Tesla (1856–1943), where

$$1\,T = \frac{1\,N}{A \cdot m}.$$

A smaller unit, called the **gauss** (G) is sometimes used, where

$$1\,G = 10^{-4}\,T$$

The strongest permanent magnets have fields near 2 T; superconducting electromagnets may attain 10 T or more. Earth's magnetic field on its surface is only about $5 \times 10^{-5}\ T$ or $0.5\ G$.

Problem-Solving Strategy: Direction of the Magnetic Field by the Right-Hand Rule

The direction of the magnetic force $\vec{F}$ is perpendicular to the plane formed by $\vec{v}$ and $\vec{B}$ as determined by the **right-hand rule-1** (or RHR-1), which is illustrated in Figure 14.3.1.

1. Orient your right hand so that your fingers curl in the plane defined by the velocity and magnetic field vectors.
2. Using your right hand, sweep from the velocity toward the magnetic field with your fingers through the smallest angle possible.
3. The magnetic force is directed where your thumb is pointing.
4. If the charge was negative, reverse the direction found by these steps.

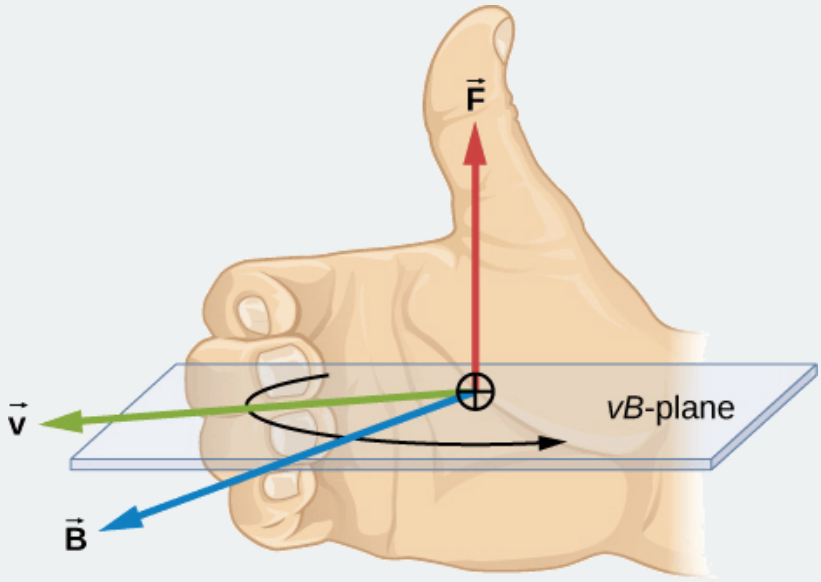


Figure 14.3.1: Magnetic fields exert forces on moving charges. The direction of the magnetic force on a moving charge is perpendicular to the plane formed by b$\vec{v}$ and $\vec{B}$ and follows the right-hand rule-1 (RHR-1) as shown. The magnitude of the force is proportional to $q,\ v,\ B$, and the sine of the angle between $\vec{v}$ and $\vec{B}$.

Note

Visit this website for additional practice with the direction of magnetic fields.

There is no magnetic force on static charges. However, there is a magnetic force on charges moving at an angle to a magnetic field. When charges are stationary, their electric fields do not affect magnets. However, when charges move, they produce magnetic fields that exert forces on other magnets. When there is relative motion, a connection between electric and magnetic forces emerges - each affects the other.

Example 14.3.1: An Alpha-Particle Moving in a Magnetic Field

An alpha-particle $(q = 3.2 \times 10^{-19}C)$ moves through a uniform magnetic field whose magnitude is 1.5 T. The field is directly parallel to the positive **z**-axis of the rectangular coordinate system of Figure 14.3.2. What is the magnetic force on the alpha-particle when it is moving (a) in the positive **x**-direction with a speed of $5.0 \times 10^4 m/s$? (b) in the negative **y**-direction with a speed of $5.0 \times 10^4 m/s$? (c) in the positive **z**-direction with a speed of $5.0 \times 10^4 m/s$? (d) with a velocity $\vec{v} = \left(2.0\hat{i} - 3.0\hat{j} + 1.0\hat{k}\right) \times 10^4 m/s$?

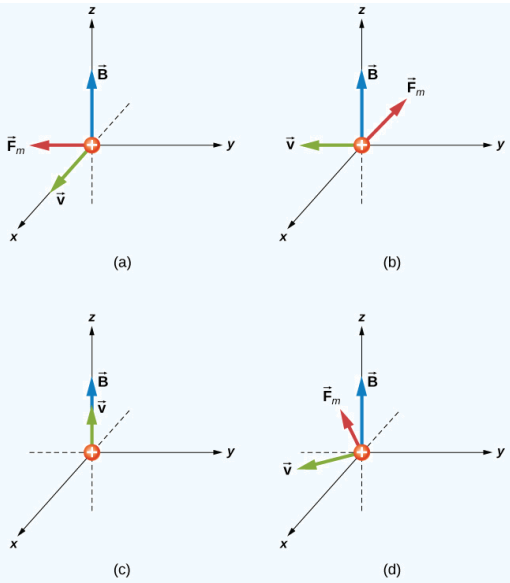


Figure 14.3.2: The magnetic forces on an alpha-particle moving in a uniform magnetic field. The field is the same in each drawing, but the velocity is different.

Strategy

We are given the charge, its velocity, and the magnetic field strength and direction. We can thus use the equation $\vec{F} = q\vec{v} \times \vec{B}$ or $F = qv\ Bsin\ \theta$ to calculate the force. The direction of the force is determined by RHR-1.

Solution

1. First, to determine the direction, start with your fingers pointing in the positive **x**-direction. Sweep your fingers upward in the direction of magnetic field. Your thumb should point in the negative **y**-direction. This should match the mathematical answer. To calculate the force, we use the given charge, velocity, and magnetic field and the definition of the magnetic force in cross-product form to calculate:

$$\vec{F} = q\vec{v} \times \vec{B} = (3.2 \times 10^{-19} C)(5.0 \times 10^{4} m/s\,\hat{i}) \times (1.5\,T\,\hat{k}) = -2.4 \times 10^{-14} N\,\hat{j}$$

2. First, to determine the directionality, start with your fingers pointing in the negative **y**-direction. Sweep your fingers upward in the direction of magnetic field as in the previous problem. Your thumb should be open in the negative **x**-direction. This should match the mathematical answer. To calculate the force, we use the given charge, velocity, and magnetic field and the definition of the magnetic force in cross-product form to calculate:

$$\vec{F} = q\vec{v} \times \vec{B} = (3.2 \times 10^{-19} C)(-5.0 \times 10^{4} m/s\,\hat{i}) \times (1.5\,T\,\hat{k}) = -2.4 \times 10^{-14} N\,\hat{i}$$

   An alternative approach is to use Equation 14.3.2 to find the magnitude of the force. This applies for both parts (a) and (b). Since the velocity is perpendicular to the magnetic field, the angle between them is 90 degrees. Therefore, the magnitude of the force is:

$$F = qv\,B \sin\,\theta = (3.2 \times 10^{-19} C)(5.0 \times 10^{4} m/s)(1.5\,T)sin(90^{o})) = 2.4 \times 10^{-14} N.$$

3. Since the velocity and magnetic field are parallel to each other, there is no orientation of your hand that will result in a force direction. Therefore, the force on this moving charge is zero. This is confirmed by the cross product. When you cross two vectors pointing in the same direction, the result is equal to zero.
4. First, to determine the direction, your fingers could point in any orientation; however, you must sweep your fingers upward in the direction of the magnetic field. As you rotate your hand, notice that the thumb can point in any **x**- or **y**-direction possible, but not in the **z**-direction. This should match the mathematical answer. To calculate the force, we use the given charge, velocity, and magnetic field and the definition of the magnetic force in cross-product form to calculate:

$$\vec{F} = q\vec{v} \times \vec{B} = (3.2 \times 10^{-19} C)((2.0\hat{i} - 3.0\hat{j} + 1.0\hat{k}) \times 10^{4} m/s) \times (1.5\,T\hat{k})$$

$$(-14.4\hat{i} - 9.6\hat{j}) \times 10^{-15} N.$$

   This solution can be rewritten in terms of a magnitude and angle in the **xy**-plane:

$$|\vec{F}| = \sqrt{F_x^2 + F_y^2} = \sqrt{(-14.4)^2 + (-9.6)^2} \times 10^{-15} N = 1.7 \times 10^{-14} N$$

$$\theta = tan^{-1}\left(\frac{F_y}{F_x}\right) = tan^{-1}\left(\frac{-9.6 \times 10^{-15} N}{-14.4 \times 10^{-15} N}\right) = 34^o.$$

The magnitude of the force can also be calculated using Equation 14.3.2. The velocity in this question, however, has three components. The **z**-component of the velocity can be neglected, because it is parallel to the magnetic field and therefore generates no force. The magnitude of the velocity is calculated from the **x**- and **y**-components. The angle between the velocity in the **xy**-plane and the magnetic field in the **z**-plane is 90 degrees. Therefore, the force is calculated to be:

$$|\vec{v}| = \sqrt{(2)^2 + (-3)^2} \times 10^4 \frac{m}{s} = 3.6 \times 10^4 \frac{m}{s}$$

$$F = qv\,B sin\,\theta = (3.2 \times 10^{-19} C)(3.6 \times 10^4 m/s)(1.5\,T) sin(90^o) = 1.7 \times 10^{-14} N$$

This is the same magnitude of force calculated by unit vectors.

Significance

The cross product in this formula results in a third vector that must be perpendicular to the other two. Other physical quantities, such as angular momentum, also have three vectors that are related by the cross product. Note that typical force values in magnetic force problems are much larger than the gravitational force. Therefore, for an isolated charge, the magnetic force is the dominant force governing the charge's motion.

Exercise 14.3.1

Repeat the previous problem with the magnetic field in the **x**-direction rather than in the **z**-direction. Check your answers with RHR-1.

Answer a available at the end of this section.

Representing Magnetic Fields

The representation of magnetic fields by **magnetic field lines** is very useful in visualizing the strength and direction of the magnetic field. As shown in Figure 14.3.3, each of these lines forms a closed loop, even if not shown by the constraints of the space available for the figure. The field lines emerge from the north pole (N), loop around to the south pole (S), and continue through the bar magnet back to the north pole.

Magnetic field lines have several hard-and-fast rules:

1. The direction of the magnetic field is tangent to the field line at any point in space. A small compass will point in the direction of the field line.
2. The strength of the field is proportional to the closeness of the lines. It is exactly proportional to the number of lines per unit area perpendicular to the lines (called the areal density).
3. Magnetic field lines can never cross, meaning that the field is unique at any point in space.
4. Magnetic field lines are continuous, forming closed loops without a beginning or end. They are directed from the north pole to the south pole.

The last property is related to the fact that the north and south poles cannot be separated. It is a distinct difference from electric field lines, which generally begin on positive charges and end on negative charges or at infinity. If isolated magnetic charges (referred to as **magnetic monopoles**) existed, then magnetic field lines would begin and end on them.

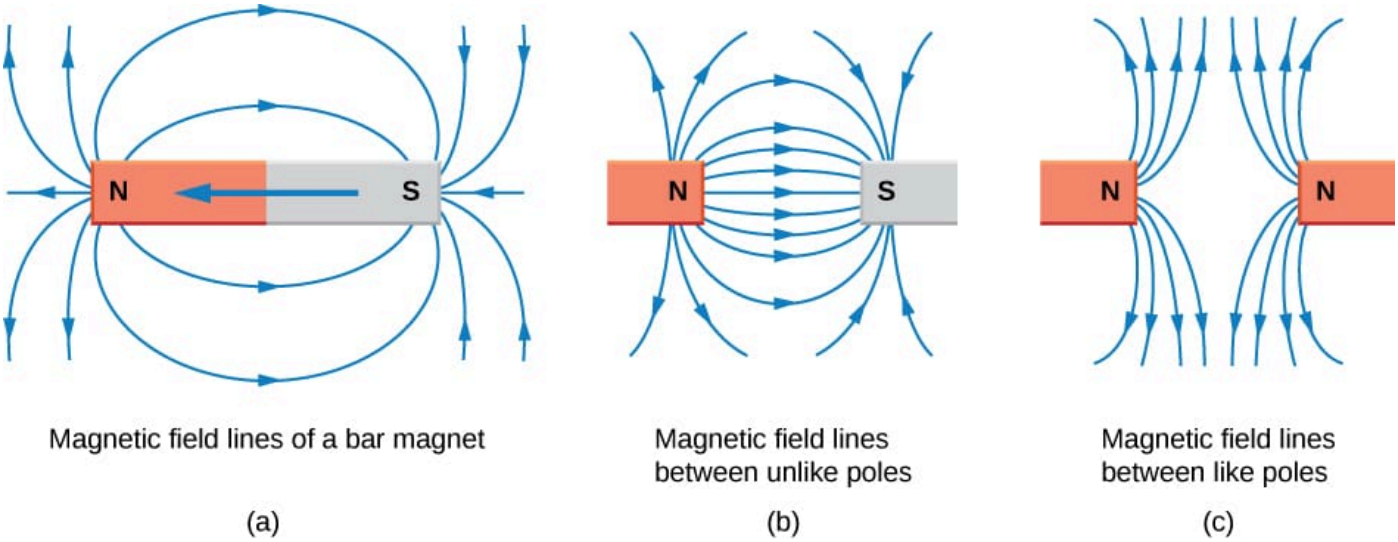


Figure 14.3.3: Magnetic field lines are defined to have the direction in which a small compass points when placed at a location in the field. The strength of the field is proportional to the closeness (or density) of the lines. If the interior of the magnet could be probed, the field lines would be found to form continuous, closed loops. To fit in a reasonable space, some of these drawings may not show the closing of the loops; however, if enough space were provided, the loops would be closed.

Answers

Exercise 14.3.1

0 N

$2.4 \times 10^{-14}\hat{k}N$

$2.4 \times 10^{-14}\hat{j}N$

$7.2\hat{j} + 2.2\hat{k}) \times 10^{-15}N$

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14.4: Motion of a Charged Particle in a Magnetic Field

Learning Objectives

By the end of this section, you will be able to:

- Explain how a charged particle in an external magnetic field undergoes circular motion
- Describe how to determine the radius of the circular motion of a charged particle in a magnetic field

A charged particle experiences a force when moving through a magnetic field. What happens if this field is uniform over the motion of the charged particle? What path does the particle follow? In this section, we discuss the circular motion of the charged particle as well as other motion that results from a charged particle entering a magnetic field.

The simplest case occurs when a charged particle moves perpendicular to a uniform **B**-field (Figure 14.4.1). If the field is in a vacuum, the magnetic field is the dominant factor determining the motion. Since the magnetic force is perpendicular to the direction of travel, a charged particle follows a curved path in a magnetic field. The particle continues to follow this curved path until it forms a complete circle. Another way to look at this is that the magnetic force is always perpendicular to velocity, so that it does no work on the charged particle. The particle's kinetic energy and speed thus remain constant. The direction of motion is affected but not the speed.

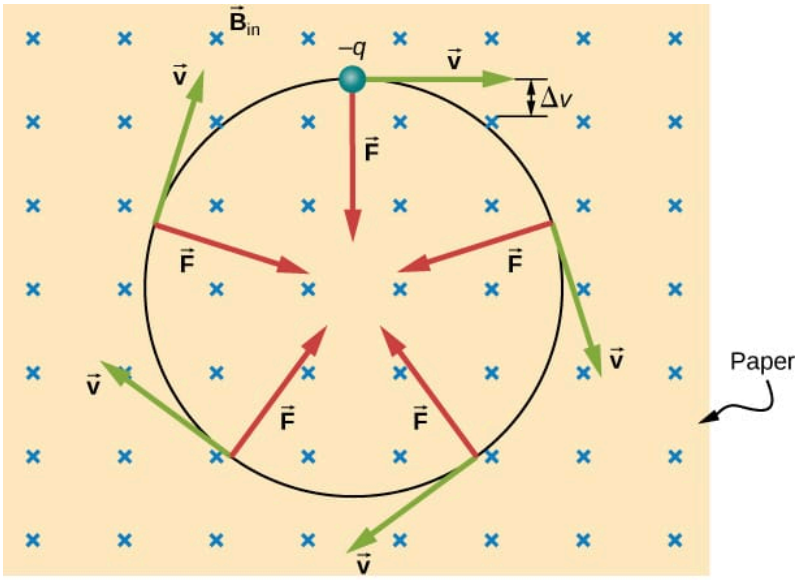


Figure 14.4.1: A negatively charged particle moves in the plane of the paper in a region where the magnetic field is perpendicular to the paper (represented by the small X's - like the tails of arrows). The magnetic force is perpendicular to the velocity, so velocity changes in direction but not magnitude. The result is uniform circular motion. (Note that because the charge is negative, the force is opposite in direction to the prediction of the right-hand rule.)

In this situation, the magnetic force supplies the centripetal force $F_C = \frac{mv^2}{r}$. Noting that the velocity is perpendicular to the magnetic field, the magnitude of the magnetic force is reduced to $F = qvB$. Because the magnetic force **F** supplies the centripetal force F_C, we have

$$qvB = \frac{mv^2}{r}.$$

Solving for **r** yields

$$r = \frac{mv}{qB}. \tag{14.4.1}$$

Here, **r** is the radius of curvature of the path of a charged particle with mass **m** and charge **q**, moving at a speed **v** that is perpendicular to a magnetic field of strength **B**. The time for the charged particle to go around the circular path is defined as the period, which is the same as the distance traveled (the circumference) divided by the speed. Based on this and Equation, we can derive the period of motion as

$$T = \frac{2\pi r}{v} = \frac{2\pi}{v}\frac{mv}{qB} = \frac{2\pi m}{qB}. \tag{14.4.2}$$

If the velocity is not perpendicular to the magnetic field, then we can compare each component of the velocity separately with the magnetic field. The component of the velocity perpendicular to the magnetic field produces a magnetic force perpendicular to both this velocity and the field:

$$v_{perp} = v \sin\theta \tag{14.4.3}$$

$$v_{para} = v \cos\theta. \tag{14.4.4}$$

where θ is the angle between **v** and **B**. The component parallel to the magnetic field creates constant motion along the same direction as the magnetic field, also shown in Equation. The parallel motion determines the **pitch p** of the helix, which is the distance between adjacent turns. This distance equals the parallel component of the velocity times the period:

$$p = v_{para}T. \tag{14.4.5}$$

The result is a **helical motion**, as shown in the following figure.

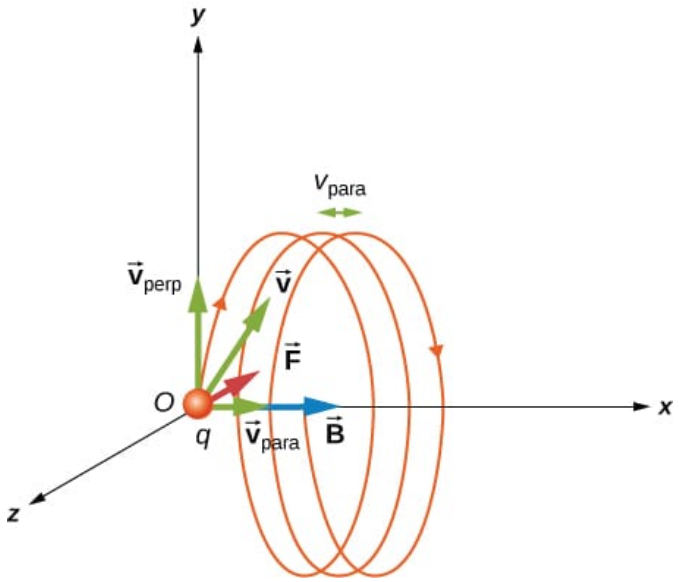


Figure 14.4.2: A charged particle moving with a velocity not in the same direction as the magnetic field. The velocity component perpendicular to the magnetic field creates circular motion, whereas the component of the velocity parallel to the field moves the particle along a straight line. The pitch is the horizontal distance between two consecutive circles. The resulting motion is helical.

While the charged particle travels in a helical path, it may enter a region where the magnetic field is not uniform. In particular, suppose a particle travels from a region of strong magnetic field to a region of weaker field, then back to a region of stronger field. The particle may reflect back before entering the stronger magnetic field region. This is similar to a wave on a string traveling from a very light, thin string to a hard wall and reflecting backward. If the reflection happens at both ends, the particle is trapped in a so-called magnetic bottle.

Trapped particles in magnetic fields are found in the Van Allen radiation belts around Earth, which are part of Earth's magnetic field. These belts were discovered by James Van Allen while trying to measure the flux of cosmic rays on Earth (high-energy particles that come from outside the solar system) to see whether this was similar to the flux measured on Earth. Van Allen found that due to the contribution of particles trapped in Earth's magnetic field, the flux was much higher on Earth than in outer space. Aurorae, like the famous aurora borealis (northern lights) in the Northern Hemisphere (Figure 14.4.3), are beautiful displays of light emitted as ions recombine with electrons entering the atmosphere as they spiral along magnetic field lines. (The ions are primarily oxygen and nitrogen atoms that are initially ionized by collisions with energetic particles in Earth's atmosphere.) Aurorae have also been observed on other planets, such as Jupiter and Saturn.

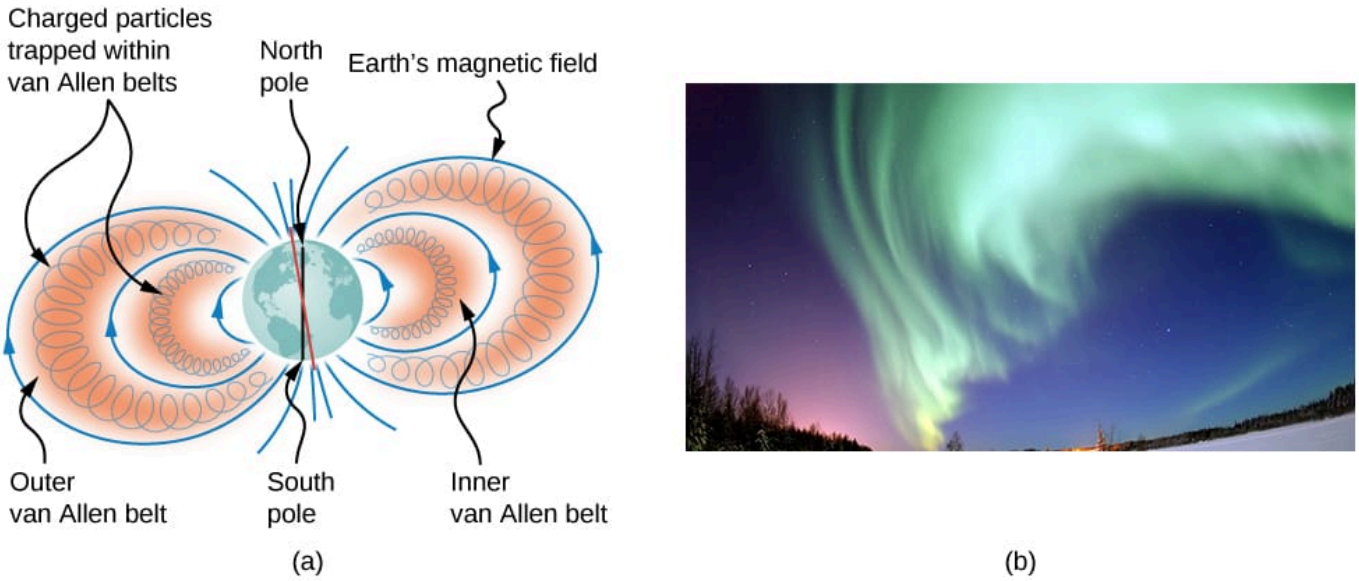


Figure 14.4.3: (a) The Van Allen radiation belts around Earth trap ions produced by cosmic rays striking Earth's atmosphere. (b) The magnificent spectacle of the aurora borealis, or northern lights, glows in the northern sky above Bear Lake near Eielson Air Force Base, Alaska. Shaped by Earth's magnetic field, this light is produced by glowing molecules and ions of oxygen and nitrogen. (credit b: modification of work by USAF Senior Airman Joshua Strang)

Example 14.4.1: Beam Deflector

A research group is investigating short-lived radioactive isotopes. They need to design a way to transport alpha-particles (helium nuclei) from where they are made to a place where they will collide with another material to form an isotope. The beam of alpha-particles $(m = 6.64 \times 10^{-27} kg,\ q = 3.2 \times 10^{-19} C)$ bends through a 90-degree region with a uniform magnetic field of 0.050 T (Figure 14.4.4). (a) In what direction should the magnetic field be applied? (b) How much time does it take the alpha-particles to traverse the uniform magnetic field region?

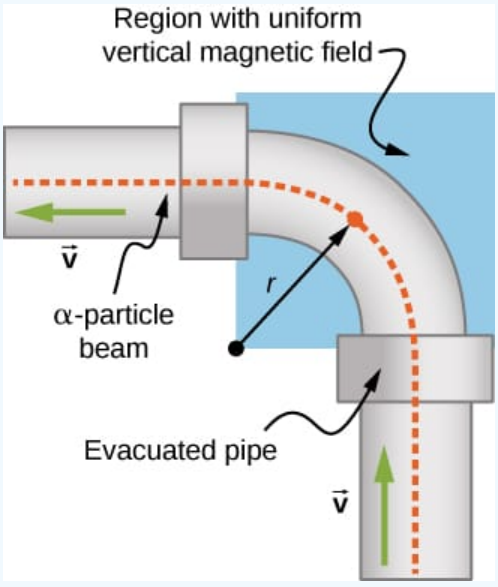


Figure 14.4.4: Top view of the beam deflector setup.

Strategy

1. The direction of the magnetic field is shown by the RHR-1. Your fingers point in the direction of **v**, and your thumb needs to point in the direction of the force, to the left. Therefore, since the alpha-particles are positively charged, the magnetic field must point down.

2. The period of the alpha-particle going around the circle is

$$T = \frac{2\pi m}{qB}.$$

Because the particle is only going around a quarter of a circle, we can take 0.25 times the period to find the time it takes to go around this path.

Solution

1. Let's start by focusing on the alpha-particle entering the field near the bottom of the picture. First, point your thumb up the page. In order for your palm to open to the left where the centripetal force (and hence the magnetic force) points, your fingers need to change orientation until they point into the page. This is the direction of the applied magnetic field.
2. The period of the charged particle going around a circle is calculated by using the given mass, charge, and magnetic field in the problem. This works out to be

$$T = \frac{2\pi m}{qB} = \frac{2\pi(6.64 \times 10^{-27} kg)}{(3.2 \times 10^{-19} C)(0.050\, T)} = 2.6 \times 10^{-6} s.$$

However, for the given problem, the alpha-particle goes around a quarter of the circle, so the time it takes would be

$$t = 0.25 \times 2.61 \times 10^{-6} s = 6.5 \times 10^{-7} s.$$

Significance

This time may be quick enough to get to the material we would like to bombard, depending on how short-lived the radioactive isotope is and continues to emit alpha-particles. If we could increase the magnetic field applied in the region, this would shorten the time even more. The path the particles need to take could be shortened, but this may not be economical given the experimental setup.

Exercise 14.4.1

A uniform magnetic field of magnitude 1.5 T is directed horizontally from west to east. (a) What is the magnetic force on a proton at the instant when it is moving vertically downward in the field with a speed of $4 \times 10^7\ m/s$? (b) Compare this force with the weight **w** of a proton.

Solution

a. $9.6 \times 10^{-12} N$ toward the south;

b. $\frac{w}{F_m} = 1.7 \times 10^{-15}$

Example 14.4.2: Helical Motion in a Magnetic Field

A proton enters a uniform magnetic field of $1.0 \times 10^{-4} T$ with a speed of $5 \times 10^5\ m/s$. At what angle must the magnetic field be from the velocity so that the pitch of the resulting helical motion is equal to the radius of the helix?

Strategy

The pitch of the motion relates to the parallel velocity times the period of the circular motion, whereas the radius relates to the perpendicular velocity component. After setting the radius and the pitch equal to each other, solve for the angle between the magnetic field and velocity or θ.

Solution

The pitch is given by Equation 14.4.5, the period is given by Equation 14.4.2, and the radius of circular motion is given by Equation 14.4.1. Note that the velocity in the radius equation is related to only the perpendicular velocity, which is where the circular motion occurs. Therefore, we substitute the sine component of the overall velocity into the radius equation to equate the pitch and radius

$$p = r$$

$$v_{\parallel} T = \frac{mv}{qB}$$

$$v\,cos\,\theta \frac{2\pi m}{qB} = \frac{mv\,sin\,\theta}{qB}$$

$$2\pi = tan\,\theta$$

$$\theta = 81.0^{o}.$$

Significance

If this angle were 0°, only parallel velocity would occur and the helix would not form, because there would be no circular motion in the perpendicular plane. If this angle were 90° only circular motion would occur and there would be no movement of the circles perpendicular to the motion. That is what creates the helical motion.

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14.5: Magnetic Force on a Current-Carrying Conductor

Learning Objectives

By the end of this section, you will be able to:

- Determine the direction in which a current-carrying wire experiences a force in an external magnetic field
- Calculate the force on a current-carrying wire in an external magnetic field

Moving charges experience a force in a magnetic field. If these moving charges are in a wire—that is, if the wire is carrying a current—the wire should also experience a force. However, before we discuss the force exerted on a current by a magnetic field, we first examine the magnetic field generated by an electric current. We are studying two separate effects here that interact closely: A current-carrying wire generates a magnetic field and the magnetic field exerts a force on the current-carrying wire.

Magnetic Fields Produced by Electrical Currents

When discussing historical discoveries in magnetism, we mentioned Oersted's finding that a wire carrying an electrical current caused a nearby compass to deflect. A connection was established that electrical currents produce magnetic fields. (This connection between electricity and magnetism is discussed in more detail in Sources of Magnetic Fields.)

The compass needle near the wire experiences a force that aligns the needle tangent to a circle around the wire. Therefore, a current-carrying wire produces circular loops of magnetic field. To determine the direction of the magnetic field generated from a wire, we use a second right-hand rule. In RHR-2, your thumb points in the direction of the current while your fingers wrap around the wire, pointing in the direction of the magnetic field produced (Figure 14.5.1). If the magnetic field were coming at you or out of the page, we represent this with a dot. If the magnetic field were going into the page, we represent this with an

These symbols come from considering a vector arrow: An arrow pointed toward you, from your perspective, would look like a dot or the tip of an arrow. An arrow pointed away from you, from your perspective, would look like a cross or an A composite sketch of the magnetic circles is shown in Figure 14.5.1, where the field strength is shown to decrease as you get farther from the wire by loops that are farther separated.

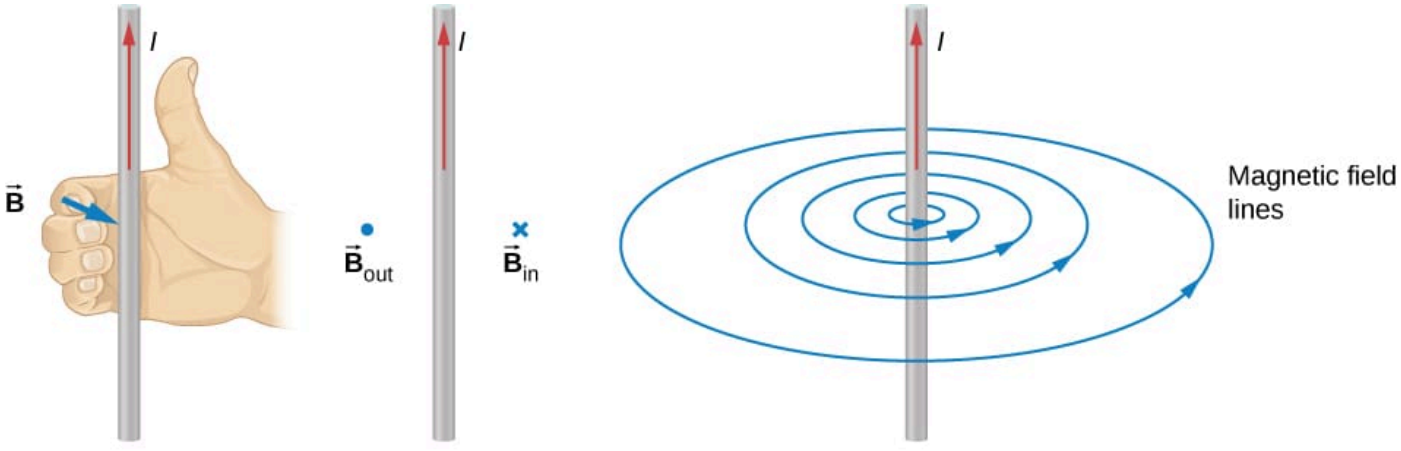


Figure 14.5.1: (a) When the wire is in the plane of the paper, the field is perpendicular to the paper. Note the symbols used for the field pointing inward (like the tail of an arrow) and the field pointing outward (like the tip of an arrow). (b) A long and straight wire creates a field with magnetic field lines forming circular loops.

Calculating the Magnetic Force

Electric current is an ordered movement of charge. A current-carrying wire in a magnetic field must therefore experience a force due to the field. To investigate this force, let's consider the infinitesimal section of wire as shown in Figure 14.5.3. The length and cross-sectional area of the section are **dl** and **A**, respectively, so its volume is $V = A \cdot dl$. The wire is formed from material that contains **n** charge carriers per unit volume, so the number of charge carriers in the section is $nA \cdot dl$. If the charge carriers move with drift velocity $\vec{v}_d$ the current **I** in the wire is (from Current and Resistance)

$$I = neAv_d.$$

The magnetic force on any single charge carrier is $e\vec{v}_d \times \vec{B}$, so the total magnetic force $d\vec{F}$ on the $nA \cdot dl$ charge carriers in the section of wire is

$$d\vec{F} = (nA \cdot dl)e\vec{v}_d \times \vec{B}.$$

We can define **dl** to be a vector of length **dl** pointing along $\vec{v}_d$, which allows us to rewrite this equation as

$$d\vec{F} = neAv_d d\vec{l} \times \vec{B},$$

or

$$d\vec{F} = Id\vec{l} \times \vec{B}. \tag{14.5.1}$$

This is the magnetic force on the section of wire. Note that it is actually the net force exerted by the field on the charge carriers themselves. The direction of this force is given by RHR-1, where you point your fingers in the direction of the current and curl them toward the field. Your thumb then points in the direction of the force.

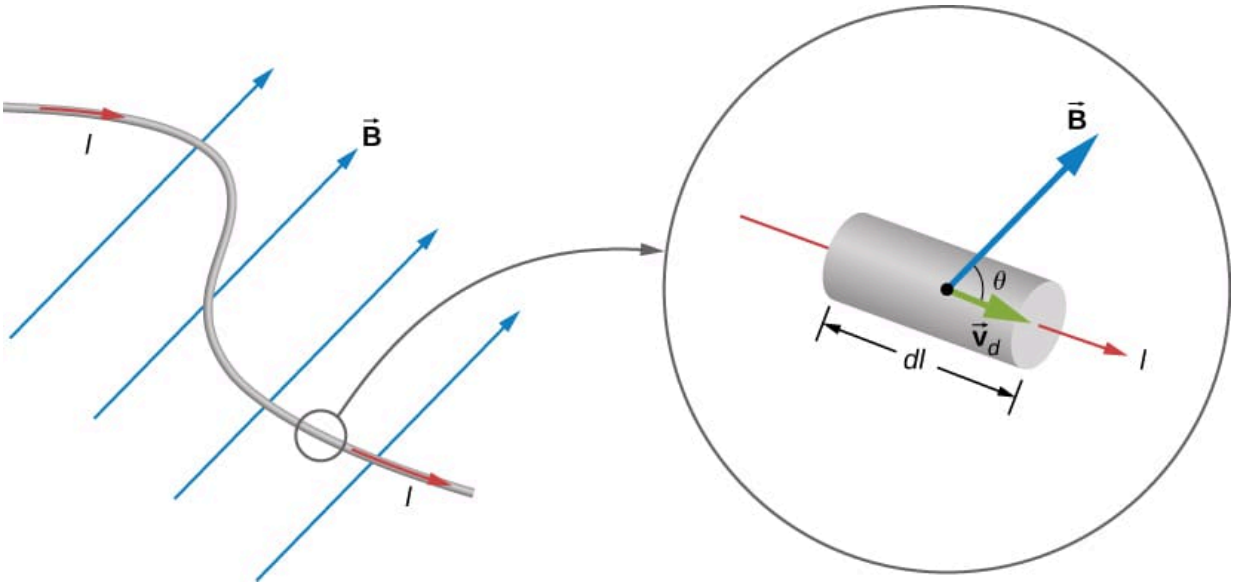


Figure 14.5.2: An infinitesimal section of current-carrying wire in a magnetic field.

To determine the magnetic force $\vec{F}$ on a wire of arbitrary length and shape, we must integrate Equation 14.5.1 over the entire wire. If the wire section happens to be straight and **B** is uniform, the equation differentials become absolute quantities, giving us

$$\vec{F} = I\vec{l} \times \vec{B}.$$

This is the force on a straight, current-carrying wire in a uniform magnetic field.

Example 14.5.1: Balancing the Gravitational and Magnetic Forces on a Current-Carrying Wire

A wire of length 50 cm and mass 10 g is suspended in a horizontal plane by a pair of flexible leads (Figure 14.5.3). The wire is then subjected to a constant magnetic field of magnitude 0.50 T, which is directed as shown. What are the magnitude and direction of the current in the wire needed to remove the tension in the supporting leads?

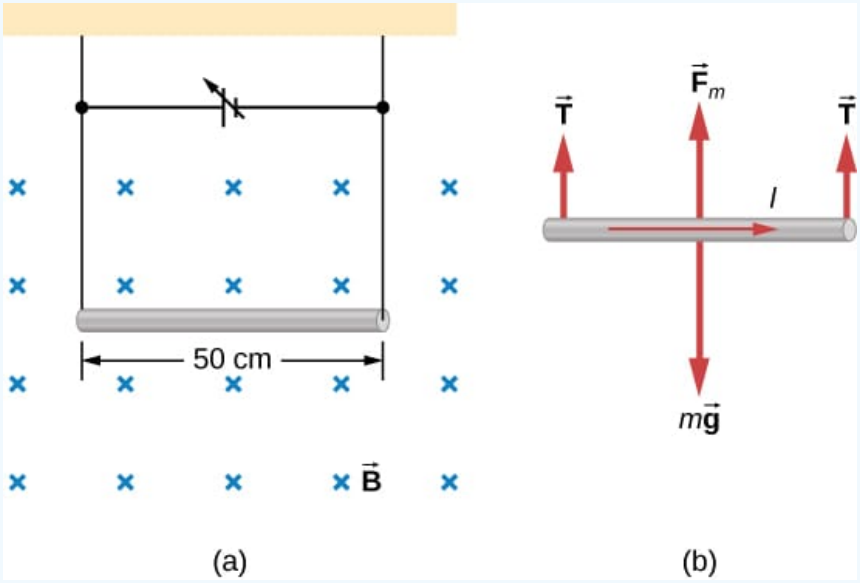


Figure 14.5.3: (a) A wire suspended in a magnetic field. (b) The free-body diagram for the wire.

Strategy

From the free-body diagram in the figure, the tensions in the supporting leads go to zero when the gravitational and magnetic forces balance each other. Using the RHR-1, we find that the magnetic force points up. We can then determine the current **I** by equating the two forces.

Solution

Equate the two forces of weight and magnetic force on the wire:

$$mg = IlB.$$

Thus,

$$I = \frac{mg}{lB} = \frac{(0.010\,kg}{9.8\,m/s^2)}(0.50\,m)(0.50\,T) = 0.39\,A.$$

Significance

This large magnetic field creates a significant force on a length of wire to counteract the weight of the wire.

Example 14.5.2: Calculating Magnetic Force on a Current-Carrying Wire

A long, rigid wire lying along the **y**-axis carries a 5.0-A current flowing in the positive **y**-direction. (a) If a constant magnetic field of magnitude 0.30 T is directed along the positive **x**-axis, what is the magnetic force per unit length on the wire? (b) If a constant magnetic field of 0.30 T is directed 30 degrees from the +**x**-axis towards the +**y**-axis, what is the magnetic force per unit length on the wire?

Strategy

The magnetic force on a current-carrying wire in a magnetic field is given by $\vec{F} = I\vec{l} \times \vec{B}$. For part a, since the current and magnetic field are perpendicular in this problem, we can simplify the formula to give us the magnitude and find the direction through the RHR-1. The angle **θ** is 90 degrees, which means $sin\ \theta = 1$. Also, the length can be divided over to the left-hand side to find the force per unit length. For part b, the current times length is written in unit vector notation, as well as the magnetic field. After the cross product is taken, the directionality is evident by the resulting unit vector.

Solution

1. We start with the general formula for the magnetic force on a wire. We are looking for the force per unit length, so we divide by the length to bring it to the left-hand side. We also set $sin\ \theta$. The solution therefore is

$$F = IlB\,sin\,\theta$$
$$\frac{F}{l} = (5.0\,A)(0.30\,T)$$
$$\frac{F}{l} = 1.5\,N/m.$$

   Directionality: Point your fingers in the positive **y**-direction and curl your fingers in the positive **x**-direction. Your thumb will point in the $-\vec{k}$ direction. Therefore, with directionality, the solution is

$$\frac{\vec{F}}{l} = -1.5\vec{k}\,N/m.$$

2. The current times length and the magnetic field are written in unit vector notation. Then, we take the cross product to find the force:

$$\vec{F} = I\vec{l} \times \vec{B} = (5.0A)l\hat{j}\times(0.30T\,cos(30^o)\hat{i}$$
$$\vec{F}/l = -1.30\hat{k}\,N/m.$$

Significance

This large magnetic field creates a significant force on a small length of wire. As the angle of the magnetic field becomes more closely aligned to the current in the wire, there is less of a force on it, as seen from comparing parts a and b.

Exercise 14.5.1

A straight, flexible length of copper wire is immersed in a magnetic field that is directed into the page. (a) If the wire's current runs in the +**x**-direction, which way will the wire bend? (b) Which way will the wire bend if the current runs in the –**x**-direction?

Solution

a. bends upward; b. bends downward

Example 14.5.3: Force on a Circular Wire

A circular current loop of radius **R** carrying a current **I** is placed in the **xy**-plane. A constant uniform magnetic field cuts through the loop parallel to the **y**-axis (Figure 14.5.4). Find the magnetic force on the upper half of the loop, the lower half of the loop, and the total force on the loop.

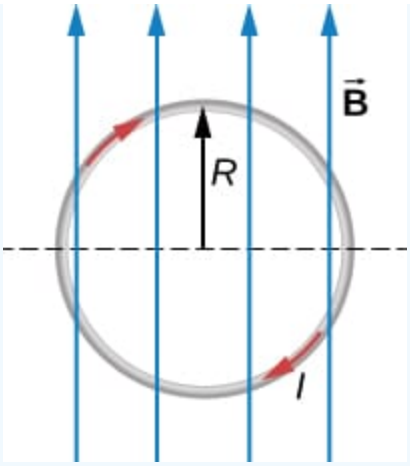


Figure 14.5.4: A loop of wire carrying a current in a magnetic field.

Strategy

The magnetic force on the upper loop should be written in terms of the differential force acting on each segment of the loop. If we integrate over each differential piece, we solve for the overall force on that section of the loop. The force on the lower loop is found in a similar manner, and the total force is the addition of these two forces.

Solution

A differential force on an arbitrary piece of wire located on the upper ring is:

$$dF = IB\,sin\,\theta\,dl,$$

where θ is the angle between the magnetic field direction (+**y**) and the segment of wire. A differential segment is located at the same radius, so using an arc-length formula, we have:

$$dl = Rd\theta$$

$$dF = IBR\,sin\,\theta\,d\theta.$$

In order to find the force on a segment, we integrate over the upper half of the circle, from 0 to π. This results in:

$$F = IBR\int_0^{\pi} sin\,\theta\,d\theta = IBR(-cos\pi + cos0) = 2IBR.$$

The lower half of the loop is integrated from π to zero, giving us:

$$F = IBR\int_{\pi}^{0} sin\,\theta\,d\theta = IBR(-cos0 + cos\pi) = -2IBR.$$

The net force is the sum of these forces, which is zero.

Significance

The total force on any closed loop in a uniform magnetic field is zero. Even though each piece of the loop has a force acting on it, the net force on the system is zero. (Note that there is a net torque on the loop, which we consider in the next section.)

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14.6: Force and Torque on a Current Loop

Learning Objectives

By the end of this section, you will be able to:

- Evaluate the net force on a current loop in an external magnetic field
- Evaluate the net torque on a current loop in an external magnetic field
- Define the magnetic dipole moment of a current loop

Motors are the most common application of magnetic force on current-carrying wires. Motors contain loops of wire in a magnetic field. When current is passed through the loops, the magnetic field exerts torque on the loops, which rotates a shaft. Electrical energy is converted into mechanical work in the process. Once the loop's surface area is aligned with the magnetic field, the direction of current is reversed, so there is a continual torque on the loop (Figure 14.6.1). This reversal of the current is done with commutators and brushes. The commutator is set to reverse the current flow at set points to keep continual motion in the motor. A basic commutator has three contact areas to avoid and dead spots where the loop would have zero instantaneous torque at that point. The brushes press against the commutator, creating electrical contact between parts of the commutator during the spinning motion.

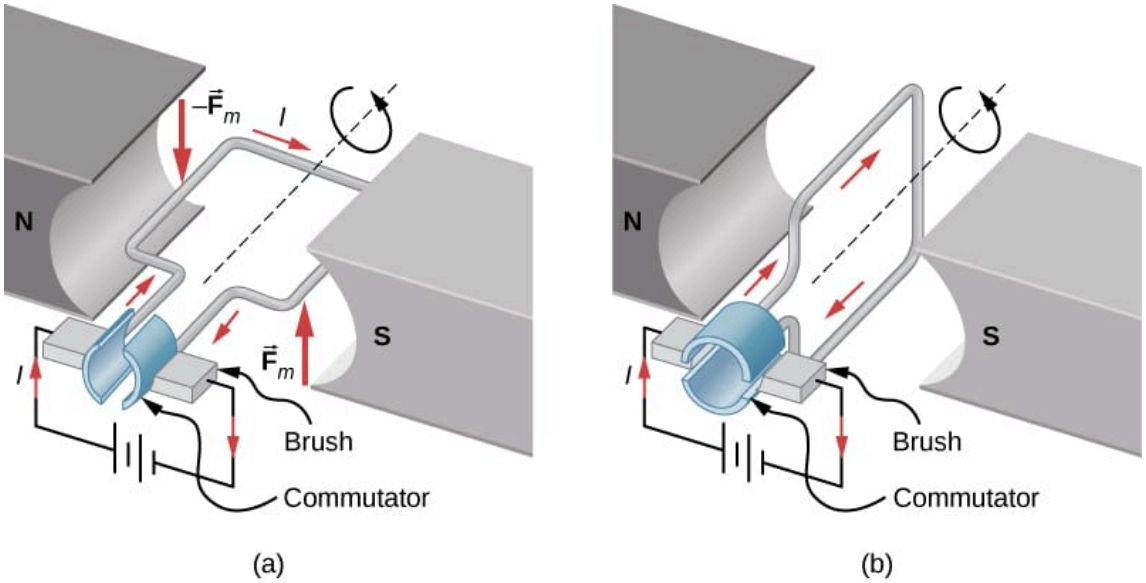


Figure 14.6.1: A simplified version of a dc electric motor. (a) The rectangular wire loop is placed in a magnetic field. The forces on the wires closest to the magnetic poles (N and S) are opposite in direction as determined by the right-hand rule-1. Therefore, the loop has a net torque and rotates to the position shown in (b). (b) The brushes now touch the commutator segments so that no current flows through the loop. No torque acts on the loop, but the loop continues to spin from the initial velocity given to it in part (a). By the time the loop flips over, current flows through the wires again but now in the opposite direction, and the process repeats as in part (a). This causes continual rotation of the loop.

In a uniform magnetic field, a current-carrying loop of wire, such as a loop in a motor, experiences both forces and torques on the loop. Figure 14.6.1 shows a rectangular loop of wire that carries a current **I** and has sides of lengths **a** and **b**. The loop is in a uniform magnetic field: $\vec{B} = B\hat{j}$. The magnetic force on a straight current-carrying wire of length **l** is given by $I\vec{l} \times \vec{B}$. To find the net force on the loop, we have to apply this equation to each of the four sides. The force on side 1 is

$$\vec{F}_1 = IaB\sin(90^o - \theta)\hat{i} = IaB\cos\theta\hat{i}$$

where the direction has been determined with the RHR-1. The current in side 3 flows in the opposite direction to that of side 1, so

$$\vec{F}_3 = -IaB\sin(90^o + \theta)\hat{i} = -IaB\cos\theta\hat{i}$$

The currents in sides 2 and 4 are perpendicular to $\vec{B}$ and the forces on these sides are

$$\vec{F}_2 = IbB\hat{k}$$

$$\vec{F}_4 = -IbB\hat{k}.$$

We can now find the net force on the loop:

$$\sum \vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = 0.$$

Although this result $(\sum F = 0)$ has been obtained for a rectangular loop, it is far more general and holds for current-carrying loops of arbitrary shapes; that is, there is no net force on a current loop in a uniform magnetic field.

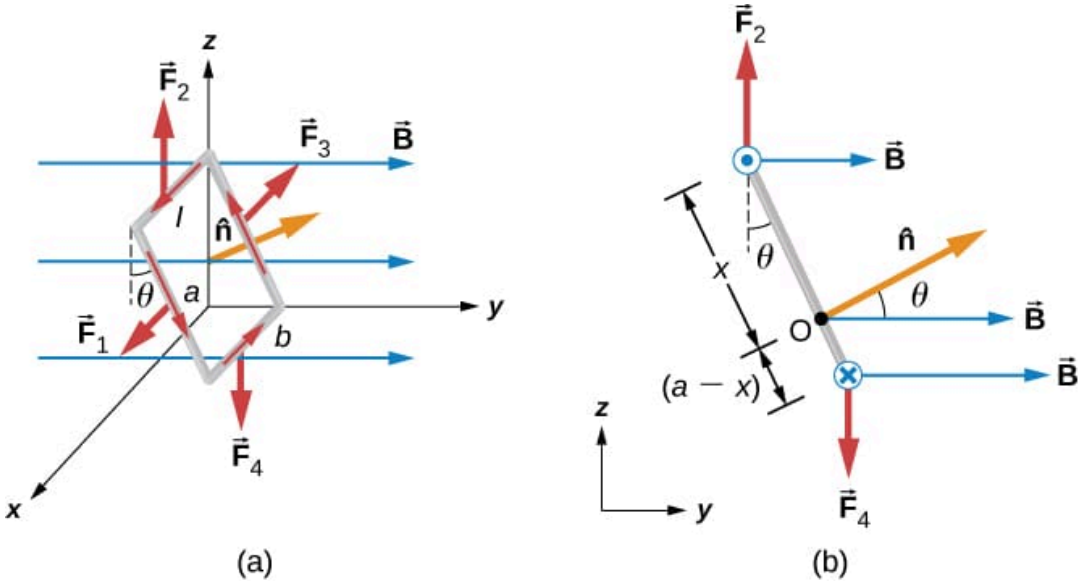


Figure 14.6.2: (a) A rectangular current loop in a uniform magnetic field is subjected to a net torque but not a net force. (b) A side view of the coil.

To find the net torque on the current loop shown in Figure $14.6.2a$, we first consider F_1 and F_3. Since they have the same line of action and are equal and opposite, the sum of their torques about any axis is zero (see Fixed-Axis Rotation). Thus, if there is any torque on the loop, it must be furnished by F_2 and F_4. Let's calculate the torques around the axis that passes through point **O** of Figure $14.6.2b$ (a side view of the coil) and is perpendicular to the plane of the page. The point **O** is a distance **x** from side 2 and a distance $(a - x)$ from side 4 of the loop. The moment arms of F_2 and F_4 are $x\ sin\ \theta$ and $(a - x)\ sin\ \theta$, respectively, so the net torque on the loop is

$$\sum \vec{\tau} = \vec{\tau}_1 + \vec{\tau}_2 + \vec{\tau}_3 + \vec{\tau}_4 = F_2 x\, sin\, \theta\hat{i} - F_4(a - x)\, sin\, (\theta)\hat{i}$$

$$-IbBx\, sin\, \theta\hat{i} - IbB(a - x) sin\, \theta\hat{i}.$$

This simplifies to

$$\vec{\tau} = -IAB\, sin\, \theta\hat{i}$$

where $A = ab$ is the area of the loop.

Notice that this torque is independent of **x**; it is therefore independent of where point **O** is located in the plane of the current loop. Consequently, the loop experiences the same torque from the magnetic field about any axis in the plane of the loop and parallel to the **x**-axis.

A closed-current loop is commonly referred to as a **magnetic dipole** and the term **IA** is known as its **magnetic dipole moment** μ**.** Actually, the magnetic dipole moment is a vector that is defined as

$$\vec{\mu} = IA\hat{n}$$

where $\hat{n}$ is a unit vector directed perpendicular to the plane of the loop (see Figure 14.6.2). The direction of $\hat{n}$ is obtained with the RHR-2—if you curl the fingers of your right hand in the direction of current flow in the loop, then your thumb points along $\hat{n}$. If the loop contains **N** turns of wire, then its magnetic dipole moment is given by

$$\vec{\mu} = NIA\hat{n}.$$

In terms of the magnetic dipole moment, the torque on a current loop due to a uniform magnetic field can be written simply as

$$\vec{\tau} = \vec{\mu} \times \vec{B}.$$

This equation holds for a current loop in a two-dimensional plane of arbitrary shape.

Using a calculation analogous to that found in Capacitance for an electric dipole, the potential energy of a magnetic dipole is

$$U = -\vec{\mu} \cdot \vec{B}.$$

Example 14.6.1: Forces and Torques on Current-Carrying Loops

A circular current loop of radius 2.0 cm carries a current of 2.0 mA. (a) What is the magnitude of its magnetic dipole moment? (b) If the dipole is oriented at 30 degrees to a uniform magnetic field of magnitude 0.50 T, what is the magnitude of the torque it experiences and what is its potential energy?

Strategy

The dipole moment is defined by the current times the area of the loop. The area of the loop can be calculated from the area of the circle. The torque on the loop and potential energy are calculated from identifying the magnetic moment, magnetic field, and angle oriented in the field.

Solution

1. The magnetic moment **μ** is calculated by the current times the area of the loop or πr^2.
$$\mu = IA = (2.0 \times 10^{-3} A)(\pi(0.02\,m)^2) = 2.5 \times 10^{-6} A \cdot m^2$$
2. The torque and potential energy are calculated by identifying the magnetic moment, magnetic field, and the angle between these two vectors. The calculations of these quantities are:
$$\tau = \vec{\mu} \times \vec{B} = \mu B \sin\theta = (2.5 \times 10^{-6} A \cdot m^2)(0.50T)sin(30^o) = 6.3 \times 10^{-7} N \cdot m$$
$$U = -\vec{\mu} \cdot \vec{B} = -\mu B cos\theta = -(2.5 \times 10^{-6} A \cdot m^2)(0.50T)cos(30^o) = -1.1 \times 10^{-6} J.$$

Significance

The concept of magnetic moment at the atomic level is discussed in the next chapter. The concept of aligning the magnetic moment with the magnetic field is the functionality of devices like magnetic motors, whereby switching the external magnetic field results in a constant spinning of the loop as it tries to align with the field to minimize its potential energy.

Exercise 14.6.1

In what orientation would a magnetic dipole have to be to produce (a) a maximum torque in a magnetic field? (b) A maximum energy of the dipole?

Solution

a. aligned or anti-aligned; b. perpendicular

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14.7: The Hall Effect

Learning Objectives

By the end of this section, you will be able to:

- Explain a scenario where the magnetic and electric fields are crossed and their forces balance each other as a charged particle moves through a velocity selector
- Compare how charge carriers move in a conductive material and explain how this relates to the Hall effect

In 1879, E.H. Hall devised an experiment that can be used to identify the sign of the predominant charge carriers in a conducting material. From a historical perspective, this experiment was the first to demonstrate that the charge carriers in most metals are negative.

Visit this website to find more information about the Hall effect.

We investigate the **Hall effect** by studying the motion of the free electrons along a metallic strip of width **l** in a constant magnetic field (Figure 14.7.1). The electrons are moving from left to right, so the magnetic force they experience pushes them to the bottom edge of the strip. This leaves an excess of positive charge at the top edge of the strip, resulting in an electric field **E** directed from top to bottom. The charge concentration at both edges builds up until the electric force on the electrons in one direction is balanced by the magnetic force on them in the opposite direction. Equilibrium is reached when:

$$eE = ev_dB \tag{14.7.1}$$

where **e** is the magnitude of the electron charge, v_d is the drift speed of the electrons, and **E** is the magnitude of the electric field created by the separated charge. Solving this for the drift speed results in

$$v_d = \frac{E}{B}. \tag{14.7.2}$$

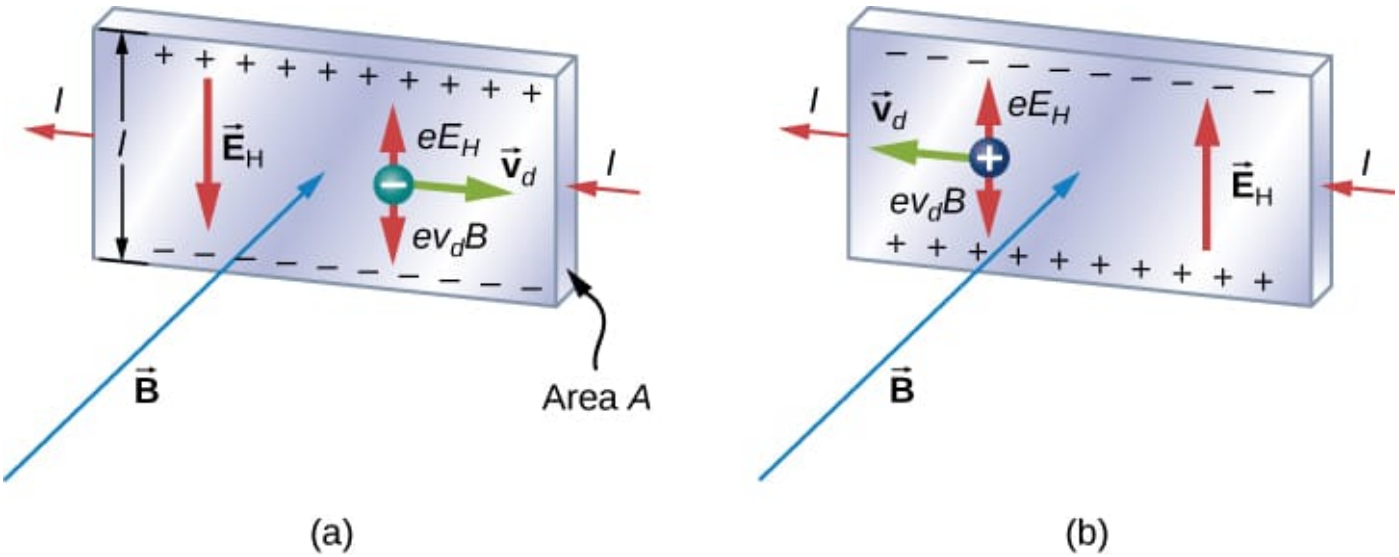


Figure 14.7.1: In the Hall effect, a potential difference between the top and bottom edges of the metal strip is produced when moving charge carriers are deflected by the magnetic field. (a) Hall effect for negative charge carriers; (b) Hall effect for positive charge carriers.

A scenario where the electric and magnetic fields are perpendicular to one another is called a crossed-field situation. If these fields produce equal and opposite forces on a charged particle with the velocity that equates the forces, these particles are able to pass through an apparatus, called a **velocity selector**, undeflected. This velocity is represented in Equation 14.7.3. Any other velocity of a charged particle sent into the same fields would be deflected by the magnetic force or electric force.

Going back to the Hall effect, if the current in the strip is **I**, then from Current and Resistance, we know that

$$I = nev_dA \tag{14.7.3}$$

where **n** is the number of charge carriers per volume and **A** is the cross-sectional area of the strip. Combining the equations for v_d and **I** results in

$$I = ne\left(\frac{E}{B}\right)A. \tag{14.7.4}$$

The field **E** is related to the potential difference **V** between the edges of the strip by

$$E = \frac{V}{l}. \tag{14.7.5}$$

The quantity V is called the **Hall potential** and can be measured with a voltmeter. Finally, combining the equations for **I** and **E** gives us

$$V = \frac{IBl}{neA} \tag{14.7.6}$$

where the upper edge of the strip in Figure 14.7.1 is positive with respect to the lower edge.

We can also combine Equation 14.7.1 and Equation 14.7.5 to get an expression for the Hall voltage in terms of the magnetic field:

$$V = Blv_d.$$

What if the charge carriers are positive, as in Figure 14.7.1? For the same current **I**, the magnitude of **V** is still given by Equation 14.7.6. However, the upper edge is now negative with respect to the lower edge. Therefore, by simply measuring the sign of **V**, we can determine the sign of the majority charge carriers in a metal.

Hall potential measurements show that electrons are the dominant charge carriers in most metals. However, Hall potentials indicate that for a few metals, such as tungsten, beryllium, and many semiconductors, the majority of charge carriers are positive. It turns out that conduction by positive charge is caused by the migration of missing electron sites (called holes) on ions. Conduction by holes is studied later in Condensed Matter Physics.

The Hall effect can be used to measure magnetic fields. If a material with a known density of charge carriers **n** is placed in a magnetic field and **V** is measured, then the field can be determined from Equation ???. In research laboratories where the fields of electromagnets used for precise measurements have to be extremely steady, a “Hall probe” is commonly used as part of an electronic circuit that regulates the field.

Example 14.7.1: Velocity Selector

An electron beam enters a crossed-field velocity selector with magnetic and electric fields of 2.0 mT and $6.0 \times 10^3\ N/C$, respectively. (a) What must the velocity of the electron beam be to traverse the crossed fields undeflected? If the electric field is turned off, (b) what is the acceleration of the electron beam and (c) what is the radius of the circular motion that results?

Strategy

The electron beam is not deflected by either of the magnetic or electric fields if these forces are balanced. Based on these balanced forces, we calculate the velocity of the beam. Without the electric field, only the magnetic force is used in Newton’s second law to find the acceleration. Lastly, the radius of the path is based on the resulting circular motion from the magnetic force.

Solution

1. The velocity of the unperturbed beam of electrons with crossed fields is calculated by Equation 14.7.2:

$$v_d = \frac{E}{B} = \frac{6 \times 10^3 N/C}{2 \times 10^{-3} T} = 3 \times 10^6 m/s.$$

2. The acceleration is calculated from the net force from the magnetic field, equal to mass times acceleration. The magnitude of the acceleration is:

$$ma = qvB$$

$$a = \frac{qvB}{m} = \frac{(1.6 \times 10^{-19} C)(3 \times 10^6 m/s)(2 \times 10^{-3} T)}{0.1 \times 10^{-31} kg} = 1.1 \times 10^{15} m/s^2.$$

3. The radius of the path comes from a balance of the circular and magnetic forces, or Equation 14.7.2:

$$r = \frac{mv}{qB} = \frac{(9.1 \times 10^{-31} kg)(3 \times 10^6 m/s)}{(1.6 \times 10^{-19} C)(2 \times 10^{-3} T)} = 8.5 \times 10^{-3} m.$$

Significance

If electrons in the beam had velocities above or below the answer in part (a), those electrons would have a stronger net force exerted by either the magnetic or electric field. Therefore, only those electrons at this specific velocity would make it through.

The Hall Potential in a Silver Ribbon

Figure 14.7.2 shows a silver ribbon whose cross section is 1.0 cm by 0.20 cm. The ribbon carries a current of 100 A from left to right, and it lies in a uniform magnetic field of magnitude 1.5 T. Using a density value of $n = 5.9 \times 10^{28}$ electrons per cubic meter for silver, find the Hall potential between the edges of the ribbon.

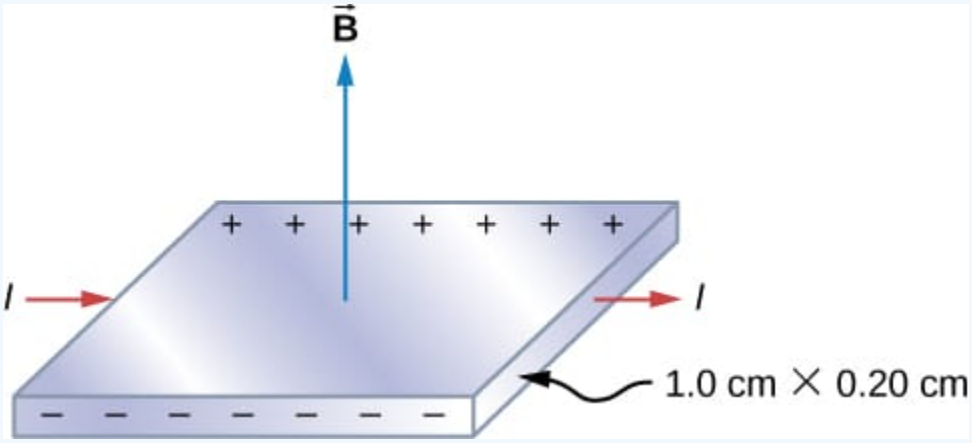


Figure 14.7.2: Finding the Hall potential in a silver ribbon in a magnetic field is shown.

Strategy

Since the majority of charge carriers are electrons, the polarity of the Hall voltage is that indicated in the figure. The value of the Hall voltage is calculated using Equation 14.7.6.

Solution

When calculating the Hall voltage, we need to know the current through the material, the magnetic field, the length, the number of charge carriers, and the area. Since all of these are given, the Hall voltage is calculated as:

$$\begin{aligned} v &= \frac{IBl}{neA} \\ &= \frac{(100\,A)(1.5\,T)(1.0 \times 10^{-2} m)}{(5.9 \times 10^{28}/m^3)(1.6 \times 10^{-19} C)(2.0 \times 10^{-5} m^2)} \\ &= 7.9 \times 10^{-6} V. \end{aligned}$$

Significance

As in this example, the Hall potential is generally very small, and careful experimentation with sensitive equipment is required for its measurement.

Exercise 14.7.1

A Hall probe consists of a copper strip, $n = 8.5 \times 10^{28}$ electrons per cubic meter, which is 2.0 cm wide and 0.10 cm thick. What is the magnetic field when $\mathbf{I}$ = 50 A and the Hall potential is

a. $4.0\ \mu V$ and
b. $6.0\ \mu V$?

Answer a available at the end of this section.

Answers

Exercise 14.7.1

1.1 T
1.6 T

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14.8: Applications of Magnetic Forces and Fields

Learning Objectives

By the end of this section, you will be able to:

- Explain how a mass spectrometer works to separate charges
- Explain how a cyclotron works

Being able to manipulate and sort charged particles allows deeper experimentation to understand what matter is made of. We first look at a mass spectrometer to see how we can separate ions by their charge-to-mass ratio. Then we discuss cyclotrons as a method to accelerate charges to very high energies.

Mass Spectrometer

The mass spectrometer is a device that separates ions according to their charge-to-mass ratios. One particular version, the Bainbridge mass spectrometer, is illustrated in Figure $14.8.1$. Ions produced at a source are first sent through a velocity selector, where the magnetic force is equally balanced with the electric force. These ions all emerge with the same speed $v = E/B$ since any ion with a different velocity is deflected preferentially by either the electric or magnetic force, and ultimately blocked from the next stage. They then enter a uniform magnetic field B_0 where they travel in a circular path whose radius **R** is given by Equation 11.4.2, $r = \frac{mv}{qB}$. The radius is measured by a particle detector located as shown in the figure.

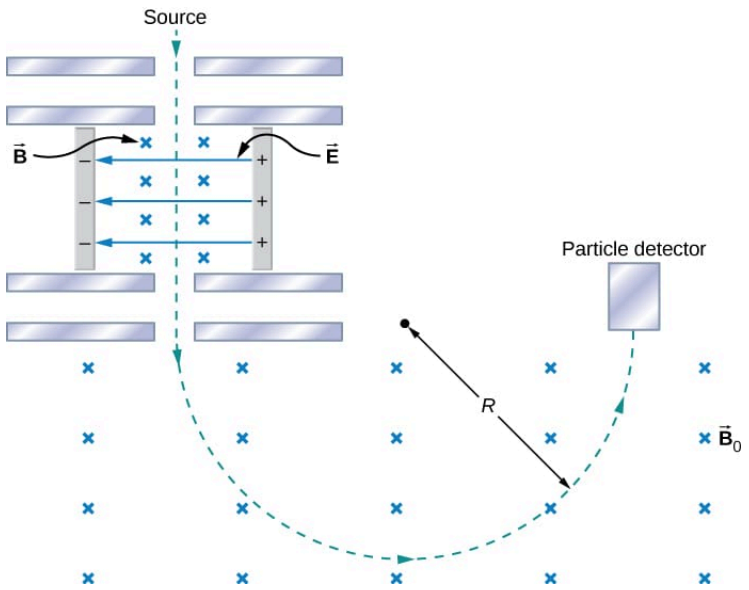


Figure $14.8.1$: A schematic of the Bainbridge mass spectrometer, showing charged particles leaving a source, followed by a velocity selector where the electric and magnetic forces are balanced, followed by a region of uniform magnetic field where the particle is ultimately detected.

The relationship between the charge-to-mass ratio **q/m** and the radius **R** is determined by combining Equation 11.4.2 and Equation 11.7.2:

$$\frac{q}{m} = \frac{E}{BB_0R}.$$

Since most ions are singly charged $(q = 1.6 \times 10^{-10}C)$, measured values of **R** can be used with this equation to determine the mass of ions. With modern instruments, masses can be determined to one part in 10^8.

An interesting use of a spectrometer is as part of a system for detecting very small leaks in a research apparatus. In low-

temperature physics laboratories, a device known as a dilution refrigerator uses a mixture of He-3, He-4, and other cryogens to reach temperatures well below 1 K. The performance of the refrigerator is severely hampered if even a minute leak between its various components occurs. Consequently, before it is cooled down to the desired temperature, the refrigerator is subjected to a leak test. A small quantity of gaseous helium is injected into one of its compartments, while an adjacent, but supposedly isolated, compartment is connected to a high-vacuum pump to which a mass spectrometer is attached. A heated filament ionizes any helium atoms evacuated by the pump. The detection of these ions by the spectrometer then indicates a leak between the two compartments of the dilution refrigerator.

In conjunction with gas chromatography, mass spectrometers are used widely to identify unknown substances. While the gas chromatography portion breaks down the substance, the mass spectrometer separates the resulting ionized molecules. This technique is used with fire debris to ascertain the cause, in law enforcement to identify illegal drugs, in security to identify explosives, and in many medicinal applications.

Cyclotron

The cyclotron was developed by E.O. Lawrence to accelerate charged particles (usually protons, deuterons, or alpha-particles) to large kinetic energies. These particles are then used for nuclear-collision experiments to produce radioactive isotopes. A cyclotron is illustrated in Figure 14.8.2. The particles move between two flat, semi-cylindrical metallic containers D1 and D2, called **dees**. The dees are enclosed in a larger metal container, and the apparatus is placed between the poles of an electromagnet that provides a uniform magnetic field. Air is removed from the large container so that the particles neither lose energy nor are deflected because of collisions with air molecules. The dees are connected to a high-frequency voltage source that provides an alternating electric field in the small region between them. Because the dees are made of metal, their interiors are shielded from the electric field.

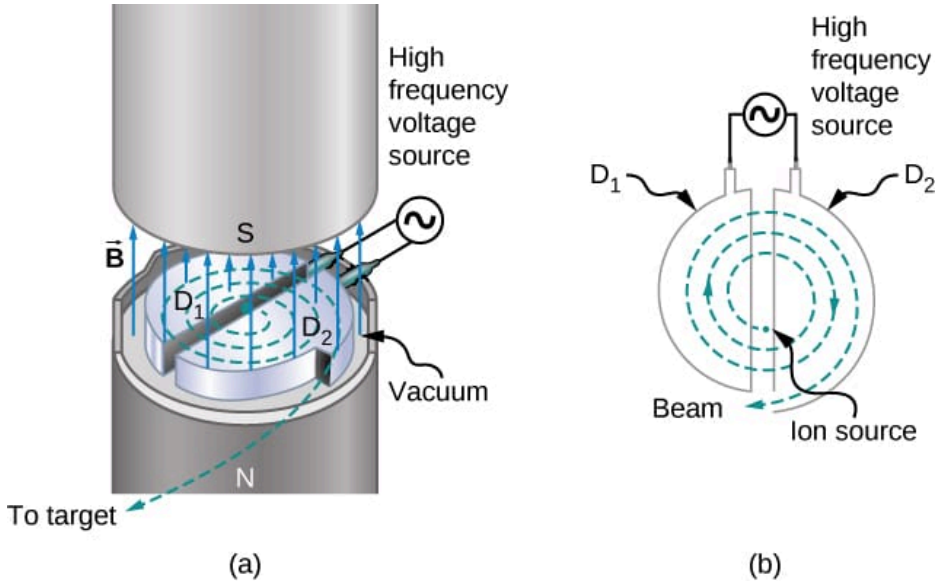


Figure 14.8.2: The inside of a cyclotron. A uniform magnetic field is applied as circulating protons travel through the dees, gaining energy as they traverse through the gap between the dees.

Suppose a positively charged particle is injected into the gap between the dees when D2 is at a positive potential relative to D1. The particle is then accelerated across the gap and enters D1 after gaining kinetic energy **qV**, where **V** is the average potential difference the particle experiences between the dees. When the particle is inside D1, only the uniform magnetic field $\vec{B}$ of the electromagnet acts on it, so the particle moves in a circle of radius

$$r = \frac{mv}{qB} \tag{14.8.1}$$

with a period of

$$T = \frac{2\pi m}{qB}. \tag{14.8.2}$$

The period of the alternating voltage course is set at **T**, so while the particle is inside D1, moving along its semicircular orbit in a time **T**/2, the polarity of the dees is reversed. When the particle reenters the gap, D1 is positive with respect to D2, and the particle is again accelerated across the gap, thereby gaining a kinetic energy **qV**. The particle then enters D2, circulates in a slightly larger circle, and emerges from D2 after spending a time **T**/2 in this dee. This process repeats until the orbit of the particle reaches the boundary of the dees. At that point, the particle (actually, a beam of particles) is extracted from the cyclotron and used for some experimental purpose.

The operation of the cyclotron depends on the fact that, in a uniform magnetic field, a particle's orbital period is independent of its radius and its kinetic energy. Consequently, the period of the alternating voltage source need only be set at the one value given by Equation 14.8.2. With that setting, the electric field accelerates particles every time they are between the dees.

If the maximum orbital radius in the cyclotron is **R**, then from Equation 14.8.1, the maximum speed of a circulating particle of mass **m** and charge **q** is

$$v_{max} = \frac{qBR}{m}.$$

Thus, its kinetic energy when ejected from the cyclotron is

$$\frac{1}{2}mv_{max}^2 = \frac{q^2B^2R^2}{2m}.$$

The maximum kinetic energy attainable with this type of cyclotron is approximately 30 MeV. Above this energy, relativistic effects become important, which causes the orbital period to increase with the radius. Up to energies of several hundred MeV, the relativistic effects can be compensated for by making the magnetic field gradually increase with the radius of the orbit. However, for higher energies, much more elaborate methods must be used to accelerate particles.

Particles are accelerated to very high energies with either linear accelerators or synchrotrons. The linear accelerator accelerates particles continuously with the electric field of an electromagnetic wave that travels down a long evacuated tube. The Stanford Linear Accelerator (SLAC) is about 3.3 km long and accelerates electrons and positrons (positively charged electrons) to energies of 50 GeV. The synchrotron is constructed so that its bending magnetic field increases with particle speed in such a way that the particles stay in an orbit of fixed radius. The world's highest-energy synchrotron is located at CERN, which is on the Swiss-French border near Geneva. CERN has been of recent interest with the verified discovery of the Higgs Boson (see Particle Physics and Cosmology). This synchrotron can accelerate beams of approximately 10^{13} protons to energies of about 10^3 GeV.

Example 14.8.1: Accelerating Alpha-Particles in a Cyclotron

A cyclotron used to accelerate alpha-particles $(m = 6.64 \times 10^{-27}kg,\ q = 3.2 \times 10^{-19}C)$ has a radius of 0.50 m and a magnetic field of 1.8 T. (a) What is the period of revolution of the alpha-particles? (b) What is their maximum kinetic energy?

Strategy

1. The period of revolution is approximately the distance traveled in a circle divided by the speed. Identifying that the magnetic force applied is the centripetal force, we can derive the period formula.
2. The kinetic energy can be found from the maximum speed of the beam, corresponding to the maximum radius within the cyclotron.

Solution

1. By identifying the mass, charge, and magnetic field in the problem, we can calculate the period:

$$T = \frac{2\pi m}{qB} = \frac{2\pi(6.64 \times 10^{-27}kg)}{(3.2 \times 10^{-19}C)(1.8T)} = 7.3 \times 10^{-8}s.$$

2. By identifying the charge, magnetic field, radius of path, and the mass, we can calculate the maximum kinetic energy:

$$\frac{1}{2}mv_{max}^2 = \frac{q^2B^2R^2}{2m} = \frac{(3.2 \times 10^{-19}C)^2(1.8T)^2(0.50m)^2}{2(6.65 \times 10^{-27}kg)} = 6.2 \times 10^{-12}J = 39\,MeV.$$

Exercise 14.8.1

A cyclotron is to be designed to accelerate protons to kinetic energies of 20 MeV using a magnetic field of 2.0 T. What is the required radius of the cyclotron?

Solution

0.32 m

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14.9: Magnetic Forces and Fields (Summary)

Key Terms

cosmic rays	comprised of particles that originate mainly from outside the solar system and reach Earth
cyclotron	device used to accelerate charged particles to large kinetic energies
dees	large metal containers used in cyclotrons that serve contain a stream of charged particles as their speed is increased
gauss	G, unit of the magnetic field strength; $1G = 10^{-4}T$
Hall effect	creation of voltage across a current-carrying conductor by a magnetic field
helical motion	superposition of circular motion with a straight-line motion that is followed by a charged particle moving in a region of magnetic field at an angle to the field
magnetic dipole	closed-current loop
magnetic dipole moment	term **IA** of the magnetic dipole, also called μ
magnetic field lines	continuous curves that show the direction of a magnetic field; these lines point in the same direction as a compass points, toward the magnetic south pole of a bar magnet
magnetic force	force applied to a charged particle moving through a magnetic field
mass spectrometer	device that separates ions according to their charge-to-mass ratios
motor (dc)	loop of wire in a magnetic field; when current is passed through the loops, the magnetic field exerts torque on the loops, which rotates a shaft; electrical energy is converted into mechanical work in the process
north magnetic pole	currently where a compass points to north, near the geographic North Pole; this is the effective south pole of a bar magnet but has flipped between the effective north and south poles of a bar magnet multiple times over the age of Earth
right-hand rule-1	using your right hand to determine the direction of either the magnetic force, velocity of a charged particle, or magnetic field
south magnetic pole	currently where a compass points to the south, near the geographic South Pole; this is the effective north pole of a bar magnet but has flipped just like the north magnetic pole
tesla	SI unit for magnetic field: $1T = 1N/A - m$
velocity selector	apparatus where the crossed electric and magnetic fields produce equal and opposite forces on a charged particle moving with a specific velocity; this particle moves through the velocity selector not affected by either field while particles moving with different velocities are deflected by the apparatus

Key Equations

Force on a charge in a magnetic field	$\vec{F} = q\vec{v} \times \vec{B}$
Magnitude of magnetic force	$F = qvBsin\theta$
Radius of a particle's path in a magnetic field	$r = \frac{mv}{qB}$

Period of a particle's motion in a magnetic field	$T = \frac{2\pi m}{qB}$
Force on a current-carrying wire in a uniform magnetic field	$\vec{F} = I\vec{l} \times \vec{B}$
Magnetic dipole moment	$\vec{\mu} = NIA\hat{n}$
Torque on a current loop	$\vec{\tau} = \vec{\mu} \times \vec{B}$
Energy of a magnetic dipole	$U = -\vec{\mu} \cdot \vec{B}$
Drift velocity in crossed electric and magnetic fields	$v_d = \frac{E}{B}$
Hall potential	$V = \frac{IBl}{neA}$
Hall potential in terms of drift velocity	$V = Blv_d$
Charge-to-mass ratio in a mass spectrometer	$\frac{q}{m} = \frac{E}{BB_0R}$
Maximum speed of a particle in a cyclotron	$v_{max} = \frac{qBR}{m}$

Summary

11.2 Magnetism and Its Historical Discoveries

- Magnets have two types of magnetic poles, called the north magnetic pole and the south magnetic pole. North magnetic poles are those that are attracted toward Earth's geographic North Pole.
- Like poles repel and unlike poles attract.
- Discoveries of how magnets respond to currents by Oersted and others created a framework that led to the invention of modern electronic devices, electric motors, and magnetic imaging technology.

11.3 Magnetic Fields and Lines

- Charges moving across a magnetic field experience a force determined by $\vec{F} = q\vec{v} \times \vec{B}$. The force is perpendicular to the plane formed by $\vec{v}$ and $\vec{B}$.
- The direction of the force on a moving charge is given by the right hand rule 1 (RHR-1): Sweep your fingers in a velocity, magnetic field plane. Start by pointing them in the direction of velocity and sweep towards the magnetic field. Your thumb points in the direction of the magnetic force for positive charges.
- Magnetic fields can be pictorially represented by magnetic field lines, which have the following properties:

    1. The field is tangent to the magnetic field line.
    2. Field strength is proportional to the line density.
    3. Field lines cannot cross.
    4. Field lines form continuous, closed loops.

- Magnetic poles always occur in pairs of north and south—it is not possible to isolate north and south poles.

11.4 Motion of a Charged Particle in a Magnetic Field

- A magnetic force can supply centripetal force and cause a charged particle to move in a circular path of radius $r = \frac{mv}{qB}$.
- The period of circular motion for a charged particle moving in a magnetic field perpendicular to the plane of motion is $T = \frac{2\pi m}{qB}$.
- Helical motion results if the velocity of the charged particle has a component parallel to the magnetic field as well as a

component perpendicular to the magnetic field.

11.5 Magnetic Force on a Current-Carrying Conductor

- An electrical current produces a magnetic field around the wire.
- The directionality of the magnetic field produced is determined by the right hand rule-2, where your thumb points in the direction of the current and your fingers wrap around the wire in the direction of the magnetic field.
- The magnetic force on current-carrying conductors is given by $\vec{F} = I\vec{l} \times \vec{B}$ where I is the current and l is the length of a wire in a uniform magnetic field B.

11.6 Force and Torque on a Current Loop

- The net force on a current-carrying loop of any plane shape in a uniform magnetic field is zero.
- The net torque τ on a current-carrying loop of any shape in a uniform magnetic field is calculated using $\tau = \vec{\mu} \times \vec{B}$ where $\vec{\mu}$ is the magnetic dipole moment and $\vec{B}$ is the magnetic field strength.
- The magnetic dipole moment μ is the product of the number of turns of wire **N,** the current in the loop **I,** and the area of the loop **A** or $\vec{\mu} = NIA\hat{n}$.

11.7 The Hall Effect

- Perpendicular electric and magnetic fields exert equal and opposite forces for a specific velocity of entering particles, thereby acting as a velocity selector. The velocity that passes through undeflected is calculated by $v = \frac{E}{B}$.
- The Hall effect can be used to measure the sign of the majority of charge carriers for metals. It can also be used to measure a magnetic field.

11.8 Applications of Magnetic Forces and Fields

- A mass spectrometer is a device that separates ions according to their charge-to-mass ratios by first sending them through a velocity selector, then a uniform magnetic field.
- Cyclotrons are used to accelerate charged particles to large kinetic energies through applied electric and magnetic fields.

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14.10: Magnetic Forces and Fields (Exercise)

Conceptual Questions

11.3 Magnetic Fields and Lines

1. Discuss the similarities and differences between the electrical force on a charge and the magnetic force on a charge.

2. (a) Is it possible for the magnetic force on a charge moving in a magnetic field to be zero?

(b) Is it possible for the electric force on a charge moving in an electric field to be zero?

(c) Is it possible for the resultant of the electric and magnetic forces on a charge moving simultaneously through both fields to be zero?

11.4 Motion of a Charged Particle in a Magnetic Field

3. At a given instant, an electron and a proton are moving with the same velocity in a constant magnetic field. Compare the magnetic forces on these particles. Compare their accelerations.

4. Does increasing the magnitude of a uniform magnetic field through which a charge is traveling necessarily mean increasing the magnetic force on the charge? Does changing the direction of the field necessarily mean a change in the force on the charge?

5. An electron passes through a magnetic field without being deflected. What do you conclude about the magnetic field?

6. If a charged particle moves in a straight line, can you conclude that there is no magnetic field present?

7. How could you determine which pole of an electromagnet is north and which pole is south?

11.5 Magnetic Force on a Current-Carrying Conductor

8. Describe the error that results from accidently using your left rather than your right hand when determining the direction of a magnetic force.

9. Considering the magnetic force law, are the velocity and magnetic field always perpendicular? Are the force and velocity always perpendicular? What about the force and magnetic field?

10. Why can a nearby magnet distort a cathode ray tube television picture?

11. A magnetic field exerts a force on the moving electrons in a current carrying wire. What exerts the force on a wire?

12. There are regions where the magnetic field of earth is almost perpendicular to the surface of Earth. What difficulty does this cause in the use of a compass?

11.7 The Hall Effect

13. Hall potentials are much larger for poor conductors than for good conductors. Why?

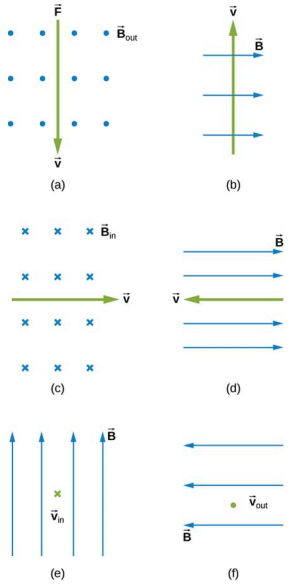
11.8 Applications of Magnetic Forces and Fields

14. Describe the primary function of the electric field and the magnetic field in a cyclotron.

Problems

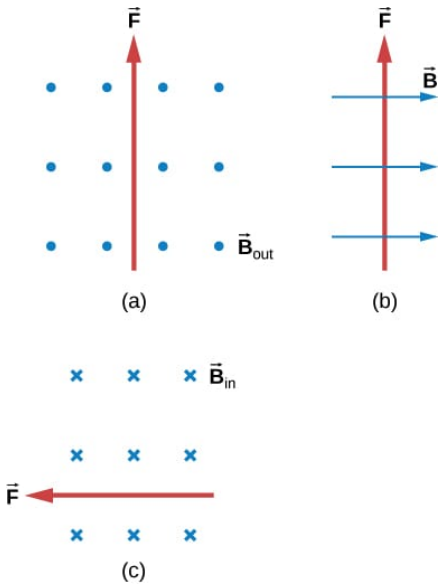
11.3 Magnetic Fields and Lines

15. What is the direction of the magnetic force on a positive charge that moves as shown in each of the six cases?



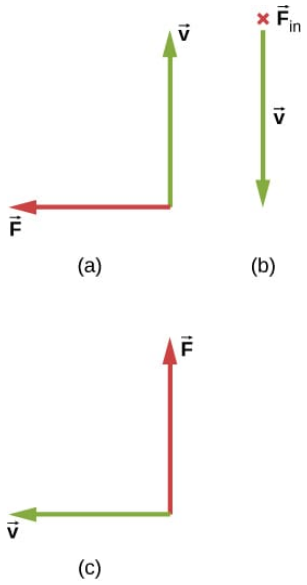
16. Repeat previous exercise for a negative charge.

17. What is the direction of the velocity of a negative charge that experiences the magnetic force shown in each of the three cases, assuming it moves perpendicular to B?



18. Repeat previous exercise for a positive charge.

19. What is the direction of the magnetic field that produces the magnetic force on a positive charge as shown in each of the three cases, assuming $\vec{\mathbf{B}}$ is perpendicular to $\vec{\mathbf{v}}$?



20. Repeat previous exercise for a negative charge.

21. (a) Aircraft sometimes acquire small static charges. Suppose a supersonic jet has a 0.500-μC charge and flies due west at a speed of 660. m/s over Earth's south magnetic pole, where the $8.00 \times 10^{-5} - T$ magnetic field points straight up. What are the direction and the magnitude of the magnetic force on the plane?

(b) Discuss whether the value obtained in part (a) implies this is a significant or negligible effect.

22. (a) A cosmic ray proton moving toward Earth at $5.00 \times 10^{7} m/s$ experiences a magnetic force of $1.70 \times 10^{-16} N$. What is the strength of the magnetic field if there is a 45° angle between it and the proton's velocity?

(b) Is the value obtained in part a. consistent with the known strength of Earth's magnetic field on its surface? Discuss.

23. An electron moving at $4.00 \times 10^{3} m/s$ in a 1.25-T magnetic field experiences a magnetic force of $1.40 \times 10^{-16} N$. What angle does the velocity of the electron make with the magnetic field? There are two answers.

24. (a) A physicist performing a sensitive measurement wants to limit the magnetic force on a moving charge in her equipment to less than $1.00 \times 10^{-12} N$. What is the greatest the charge can be if it moves at a maximum speed of 30.0 m/s in Earth's field?

(b) Discuss whether it would be difficult to limit the charge to less than the value found in (a) by comparing it with typical static electricity and noting that static is often absent.

11.4 Motion of a Charged Particle in a Magnetic Field

25. A cosmic-ray electron moves at $7.5 \times 10^{6} m/s$ perpendicular to Earth's magnetic field at an altitude where the field strength is $1.0 \times 10^{-5} T$. What is the radius of the circular path the electron follows?

26. (a) Viewers of Star Trek have heard of an antimatter drive on the Starship Enterprise. One possibility for such a futuristic energy source is to store antimatter charged particles in a vacuum chamber, circulating in a magnetic field, and then extract them as needed. Antimatter annihilates normal matter, producing pure energy. What strength magnetic field is needed to hold antiprotons, moving at $5.0 \times 10^{7} m/s$ in a circular path 2.00 m in radius? Antiprotons have the same mass as protons but the opposite (negative) charge.

(b) Is this field strength obtainable with today's technology or is it a futuristic possibility?

27. (a) An oxygen-16 ion with a mass of $2.66 \times 10^{-26} kg$ travels at $5.0 \times 10^{6} m/s$ perpendicular to a 1.20-T magnetic field, which makes it move in a circular arc with a 0.231-m radius. What positive charge is on the ion?

(b) What is the ratio of this charge to the charge of an electron? (c) Discuss why the ratio found in (b) should be an integer.

28. An electron in a TV CRT moves with a speed of $6.0 \times 10^{7} m/s$, in a direction perpendicular to Earth's field, which has a strength of $5.0 \times 10^{-5} T$. (a) What strength electric field must be applied perpendicular to the Earth's field to make the electron moves in a straight line? (b) If this is done between plates separated by 1.00 cm, what is the voltage applied? (Note that TVs are usually surrounded by a ferromagnetic material to shield against external magnetic fields and avoid the need for such a correction.)

29. (a) At what speed will a proton move in a circular path of the same radius as the electron in the previous exercise?

(b) What would the radius of the path be if the proton had the same speed as the electron?

(c) What would the radius be if the proton had the same kinetic energy as the electron?

(d) The same momentum?

30. (a) What voltage will accelerate electrons to a speed of $6.00 \times 10^{-7} m/s$? (b) Find the radius of curvature of the path of a proton accelerated through this potential in a 0.500-T field and compare this with the radius of curvature of an electron accelerated through the same potential.

31. An alpha-particle ($m = 6.64 \times 10^{-27} kg, q = 3.2 \times 10^{-19} C$) travels in a circular path of radius 25 cm in a uniform magnetic field of magnitude 1.5 T.

(a) What is the speed of the particle?

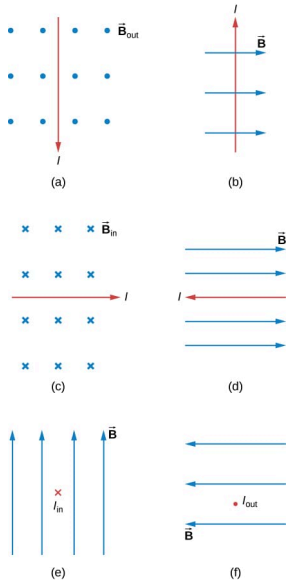
(b) What is the kinetic energy in electron-volts?

(c) Through what potential difference must the particle be accelerated in order to give it this kinetic energy?

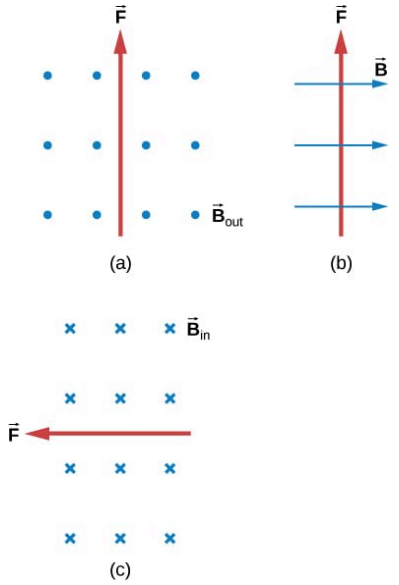
32. A particle of charge **q** and mass **m** is accelerated from rest through a potential difference **V**, after which it encounters a uniform magnetic field **B**. If the particle moves in a plane perpendicular to **B**, what is the radius of its circular orbit?

11.5 Magnetic Force on a Current-Carrying Conductor

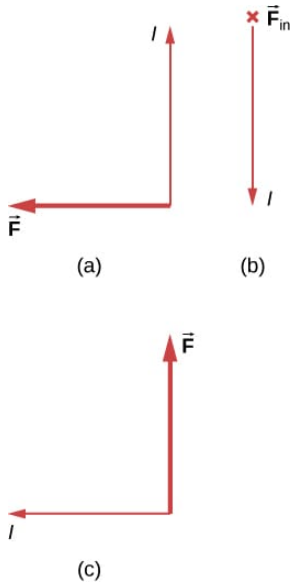
33. What is the direction of the magnetic force on the current in each of the six cases?



34. What is the direction of a current that experiences the magnetic force shown in each of the three cases, assuming the current runs perpendicular to $\vec{\mathbf{B}}$?



35. What is the direction of the magnetic field that produces the magnetic force shown on the currents in each of the three cases, assuming $\vec{\mathbf{B}}$ is perpendicular to **I**?



36. (a) What is the force per meter on a lightning bolt at the equator that carries 20,000 A perpendicular to Earth's $3.0 \times 10^{-5} T$ field? (b) What is the direction of the force if the current is straight up and Earth's field direction is due north, parallel to the ground?

37. (a) A dc power line for a light-rail system carries 1000 A at an angle of 30.0° to Earth's $5.0 \times 10^{-5} T$ field. What is the force on a 100-m section of this line?

(b) Discuss practical concerns this presents, if any.

38. A wire carrying a 30.0-A current passes between the poles of a strong magnet that is perpendicular to its field and experiences a 2.16-N force on the 4.00 cm of wire in the field. What is the average field strength?

11.6 Force and Torque on a Current Loop

39. (a) By how many percent is the torque of a motor decreased if its permanent magnets lose 5.0% of their strength?

(b) How many percent would the current need to be increased to return the torque to original values?

40. (a) What is the maximum torque on a 150-turn square loop of wire 18.0 cm on a side that carries a 50.0-A current in a 1.60-T field?

(b) What is the torque when **θ** is 10.9°?

41. Find the current through a loop needed to create a maximum torque of **9.0N·m**. The loop has 50 square turns that are 15.0 cm on a side and is in a uniform 0.800-T magnetic field.

42. Calculate the magnetic field strength needed on a 200-turn square loop 20.0 cm on a side to create a maximum torque of 300 N · m if the loop is carrying 25.0 A.

43. Since the equation for torque on a current-carrying loop is **τ = NIAB** sin **θ**, the units of N · m must equal units of $A \cdot m^2 T$. Verify this.

44. (a) At what angle θ is the torque on a current loop 90.0% of maximum?

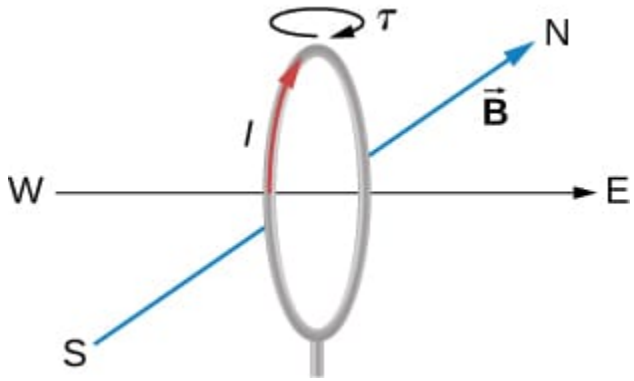
(b) 50.0% of maximum?

(c) 10.0% of maximum?

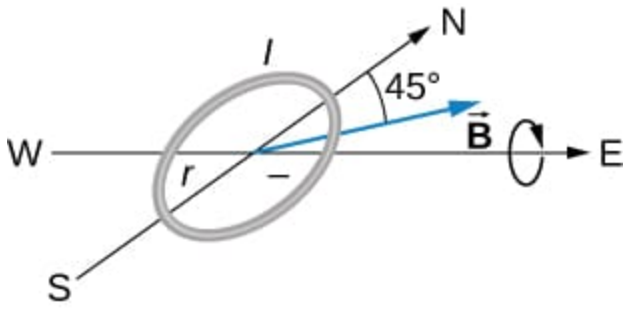
45. A proton has a magnetic field due to its spin. The field is similar to that created by a circular current loop $0.65 \times 10^{-15} m$ in radius with a current of $1.05 \times 10^{4} A$. Find the maximum torque on a proton in a 2.50-T field. (This is a significant torque on a small particle.)

46. (a) A 200-turn circular loop of radius 50.0 cm is vertical, with its axis on an east-west line. A current of 100 A circulates clockwise in the loop when viewed from the east. Earth's field here is due north, parallel to the ground, with a strength of $3.0 \times 10^{-5} T$. What are the direction and magnitude of the torque on the loop?

(b) Does this device have any practical applications as a motor?



47. Repeat the previous problem, but with the loop lying flat on the ground with its current circulating counterclockwise (when viewed from above) in a location where Earth's field is north, but at an angle 45.0° below the horizontal and with a strength of $6.0 \times 10^{-5} T$.



11.7 The Hall Effect

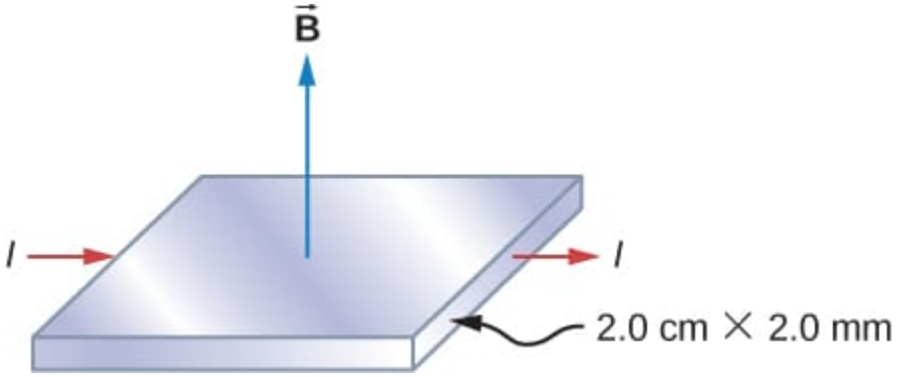
48. A strip of copper is placed in a uniform magnetic field of magnitude 2.5 T. The Hall electric field is measured to be $1.5 \times 10^{-3} V/m$.

(a) What is the drift speed of the conduction electrons?

(b) Assuming that $n = 8.0 \times 10^{28}$electrons per cubic meter and that the cross-sectional area of the strip is $5.0 \times 10^{-6} m^2$, calculate the current in the strip.

(c) What is the Hall coefficient 1/nq?

49. The cross-sectional dimensions of the copper strip shown are 2.0 cm by 2.0 mm. The strip carries a current of 100 A, and it is placed in a magnetic field of magnitude B = 1.5 T. What are the value and polarity of the Hall potential in the copper strip?



50. The magnitudes of the electric and magnetic fields in a velocity selector are $1.8 \times 10^5 V/m$ and 0.080 T, respectively.

(a) What speed must a proton have to pass through the selector?

(b) Also calculate the speeds required for an alpha-particle and a singly ionized ${}_sO^{16}$ atom to pass through the selector.

51. A charged particle moves through a velocity selector at constant velocity. In the selector, $E = 1.0 \times 10^4 N/C$ and **B** = 0.250 T. When the electric field is turned off, the charged particle travels in a circular path of radius 3.33 mm. Determine the charge-to-mass ratio of the particle.

52. A Hall probe gives a reading of **1.5μV** for a current of 2 A when it is placed in a magnetic field of 1 T. What is the magnetic field in a region where the reading is **2μV** for 1.7 A of current?

11.8 Applications of Magnetic Forces and Fields

53. A physicist is designing a cyclotron to accelerate protons to one-tenth the speed of light. The magnetic field will have a

strength of 1.5 T. Determine

(a) the rotational period of the circulating protons and

(b) the maximum radius of the protons' orbit.

54. The strengths of the fields in the velocity selector of a Bainbridge mass spectrometer are **B = 0.500 T** and $E = 1.2 \times 10^5 V/m$, and the strength of the magnetic field that separates the ions is $B_o = 0.750T$. A stream of singly charged Li ions is found to bend in a circular arc of radius 2.32 cm. What is the mass of the Li ions?

55. The magnetic field in a cyclotron is 1.25 T, and the maximum orbital radius of the circulating protons is 0.40 m.

(a) What is the kinetic energy of the protons when they are ejected from the cyclotron?

(b) What is this energy in MeV?

(c) Through what potential difference would a proton have to be accelerated to acquire this kinetic energy?

(d) What is the period of the voltage source used to accelerate the protons?

(e) Repeat the calculations for alpha-particles.

56. A mass spectrometer is being used to separate common oxygen-16 from the much rarer oxygen-18, taken from a sample of old glacial ice. (The relative abundance of these oxygen isotopes is related to climatic temperature at the time the ice was deposited.) The ratio of the masses of these two ions is 16 to 18, the mass of oxygen-16 is $2.66 \times 10^{-26} kg$, and they are singly charged and travel at $5.00 \times 10^6 m/s$ in a 1.20-T magnetic field. What is the separation between their paths when they hit a target after traversing a semicircle?

57. (a) Triply charged uranium-235 and uranium-238 ions are being separated in a mass spectrometer. (The much rarer uranium-235 is used as reactor fuel.) The masses of the ions are $3.90 \times 10^{-25} kg$ and $3.95 \times 10^{-25} kg$,respectively, and they travel at $3.0 \times 10^5 m/s$ in a 0.250-T field. What is the separation between their paths when they hit a target after traversing a semicircle? (b) Discuss whether this distance between their paths seems to be big enough to be practical in the separation of uranium-235 from uranium-238.

Additional Problems

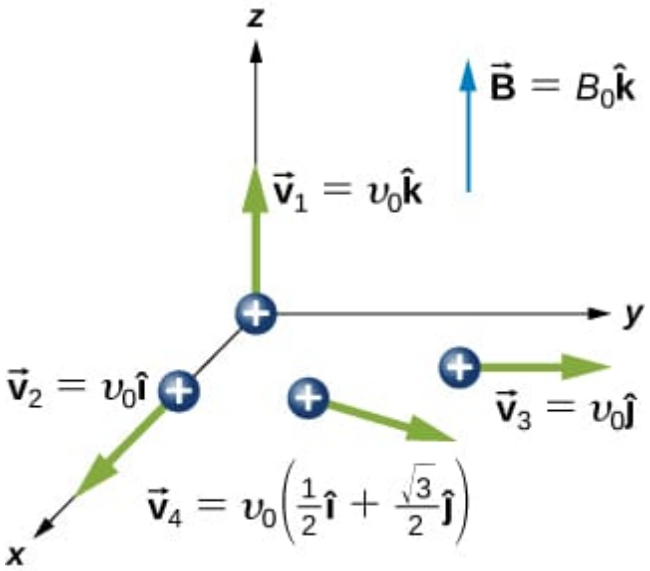
58. Calculate the magnetic force on a hypothetical particle of charge $1.0 \times 10^{-19} C$ moving with a velocity of $6.0 \times 10^4 \hat{i} m/s$ in a magnetic field of $1.2\hat{k}T$.

59. Repeat the previous problem with a new magnetic field of $(0.4\hat{i} + 1.2\hat{k})T$.

60. An electron is projected into a uniform magnetic field $(0.5\hat{i} + 0.8\hat{k})T$ with a velocity of $(3.0\hat{i} + 4.0\hat{j}) \times 10^6 m/s$. What is the magnetic force on the electron?

61. The mass and charge of a water droplet are $1.0 \times 10^{-4} g$ and $2.0 \times 10^{-8} C$, respectively. If the droplet is given an initial horizontal velocity of $5.0 \times 10^5 \hat{i} m/s$, what magnetic field will keep it moving in this direction? Why must gravity be considered here?

62. Four different proton velocities are given. For each case, determine the magnetic force on the proton in terms of e, v_0, and B_0.



63. An electron of kinetic energy 2000 eV passes between parallel plates that are 1.0 cm apart and kept at a potential difference of 300 V. What is the strength of the uniform magnetic field B that will allow the electron to travel undeflected through the plates? Assume E and B are perpendicular.

64. An alpha-particle $(m = 6.64 \times 10^{-27}kg, q = 3.2 \times 10^{-19}C)$ moving with a velocity $\vec{v} = (2.0\hat{i} - 4.0\hat{k}) \times 10^6 m/s$ enters a region where $\vec{E} = (5.0\hat{i} - 2.0\hat{j}) \times 10^4 V/m$ and $\vec{B} = (1.0\hat{i} + 4.0\hat{k}) \times 10^{-2}T$. What is the initial force on it?

65. An electron moving with a velocity $\vec{v} = (4.0\hat{i}\hat{} + 3.0\hat{j}\hat{} + 2.0\hat{k}) \times 10^6 m/s$ enters a region where there is a uniform electric field and a uniform magnetic field. The magnetic field is given by $\vec{B} = (1.0\hat{i} - 2.0\hat{j} + 4.0\hat{k}) \times 10^{-2}T$. If the electron travels through a region without being deflected, what is the electric field?

66. At a particular instant, an electron is traveling west to east with a kinetic energy of 10 keV. Earth's magnetic field has a horizontal component of $1.8 \times 10^{-5}T$ north and a vertical component of $5.0 \times 10^{-5}T$ down. (a) What is the path of the electron? (b) What is the radius of curvature of the path?

67. What is the (a) path of a proton and (b) the magnetic force on the proton that is traveling west to east with a kinetic energy of 10 keV in Earth's magnetic field that has a horizontal component of 1.8 x 10^{-5} T north and a vertical component of 5.0 x 10^{-5} T down?

68. What magnetic field is required in order to confine a proton moving with a speed of $4.0 \times 10^6 m/s$ to a circular orbit of radius 10 cm?

69. An electron and a proton move with the same speed in a plane perpendicular to a uniform magnetic field. Compare the radii and periods of their orbits.

70. A proton and an alpha-particle have the same kinetic energy and both move in a plane perpendicular to a uniform magnetic field. Compare the periods of their orbits.

71. A singly charged ion takes $2.0 \times 10^{-3}s$ to complete eight revolutions in a uniform magnetic field of magnitude $2.0 \times 10^{-2}T$. What is the mass of the ion?

72. A particle moving downward at a speed of $6.0 \times 10^6 m/s$ enters a uniform magnetic field that is horizontal and directed from east to west.

(a) If the particle is deflected initially to the north in a circular arc, is its charge positive or negative?

(b) If B = 0.25 T and the charge-to-mass ratio (q/m) of the particle is $4.0 \times 10^7 C/kg$, what is the radius of the path?

(c) What is the speed of the particle after it has moved in the field for $1.0 \times 10^{-5}s$? for 2.0 s?

73. A proton, deuteron, and an alpha-particle are all accelerated through the same potential difference. They then enter the same magnetic field, moving perpendicular to it. Compute the ratios of the radii of their circular paths. Assume that $m_d = 2m_p$ and $m_\alpha = 4m_p$.

74. A singly charged ion is moving in a uniform magnetic field of $7.5 \times 10^{-2}T$ completes 10 revolutions in $3.47 \times 10^{-4}s$. Identify the ion.

75. Two particles have the same linear momentum, but particle A has four times the charge of particle B. If both particles move in a plane perpendicular to a uniform magnetic field, what is the ratio R_A/R_B of the radii of their circular orbits?

76. A uniform magnetic field of magnitude B is directed parallel to the z-axis. A proton enters the field with a velocity $\vec{v} = (4\hat{j} + 3\hat{k}) \times 10^6 m/s$ and travels in a helical path with a radius of 5.0 cm.

(a) What is the value of B?

(b) What is the time required for one trip around the helix?

(c) Where is the proton $5.0 \times 10^{-7}s$ after entering the field?

77. An electron moving at $5.0 \times 10^6 m/s$ enters a magnetic field that makes a 75° angle with the x-axis of magnitude 0.20 T. Calculate the

(a) pitch and

(b) radius of the trajectory.

78. (a) A 0.750-m-long section of cable carrying current to a car starter motor makes an angle of 60° with Earth's $5.5 \times 10^{-5}T$ field. What is the current when the wire experiences a force of $7.0 \times 10^{-3}N$?

(b) If you run the wire between the poles of a strong horseshoe magnet, subjecting 5.00 cm of it to a 1.75-T field, what force is exerted on this segment of wire?

79. (a) What is the angle between a wire carrying an 8.00-A current and the 1.20-T field it is in if 50.0 cm of the wire experiences a magnetic force of 2.40 N?

(b) What is the force on the wire if it is rotated to make an angle of 90° with the field?

80. A 1.0-m-long segment of wire lies along the x-axis and carries a current of 2.0 A in the positive x-direction. Around the wire is the magnetic field of $(3.0\hat{i} \times 4.0\hat{k}) \times 10^{-3}T$. Find the magnetic force on this segment.

81. A 5.0-m section of a long, straight wire carries a current of 10 A while in a uniform magnetic field of magnitude $8.0 \times 10^{-3}T$. Calculate the magnitude of the force on the section if the angle between the field and the direction of the current is

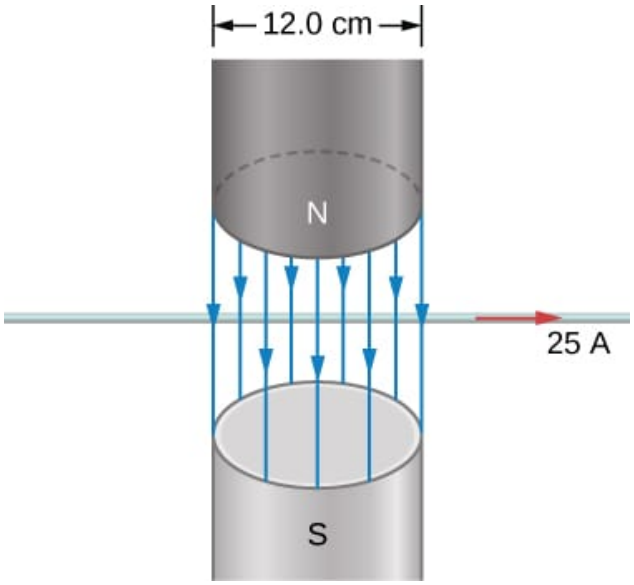
(a) 45°;

(b) 90°;

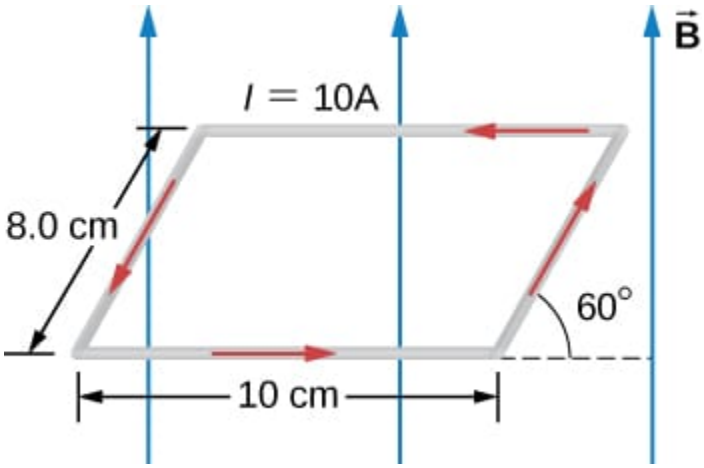
(c) 0°; or

(d) 180°.

82. An electromagnet produces a magnetic field of magnitude 1.5 T throughout a cylindrical region of radius 6.0 cm. A straight wire carrying a current of 25 A passes through the field as shown in the accompanying figure. What is the magnetic force on the wire?



83. The current loop shown in the accompanying figure lies in the plane of the page, as does the magnetic field. Determine the net force and the net torque on the loop if **I = 10 A** and **B = 1.5 T.**



84. A circular coil of radius 5.0 cm is wound with five turns and carries a current of 5.0 A. If the coil is placed in a uniform magnetic field of strength 5.0 T, what is the maximum torque on it?

85. A circular coil of wire of radius 5.0 cm has 20 turns and carries a current of 2.0 A. The coil lies in a magnetic field of magnitude 0.50 T that is directed parallel to the plane of the coil.

(a) What is the magnetic dipole moment of the coil?

(b) What is the torque on the coil?

86. A current-carrying coil in a magnetic field experiences a torque that is 75% of the maximum possible torque. What is the angle between the magnetic field and the normal to the plane of the coil?

87. A 4.0-cm by 6.0-cm rectangular current loop carries a current of 10 A. What is the magnetic dipole moment of the loop?

88. A circular coil with 200 turns has a radius of 2.0 cm.

(a) What current through the coil results in a magnetic dipole moment of 3.0 Am2?

(b) What is the maximum torque that the coil will experience in a uniform field of strength $5.0 \times 10^{-2} T$?

(c) If the angle between μ and B is 45°, what is the magnitude of the torque on the coil?

(d) What is the magnetic potential energy of coil for this orientation?

89. The current through a circular wire loop of radius 10 cm is 5.0 A.

(a) Calculate the magnetic dipole moment of the loop.

(b) What is the torque on the loop if it is in a uniform 0.20-T magnetic field such that **μ** and B are directed at **30°** to each other?

(c) For this position, what is the potential energy of the dipole?

90. A wire of length 1.0 m is wound into a single-turn planar loop. The loop carries a current of 5.0 A, and it is placed in a uniform magnetic field of strength 0.25 T.

(a) What is the maximum torque that the loop will experience if it is square?

(b) If it is circular?

(c) At what angle relative to B would the normal to the circular coil have to be oriented so that the torque on it would be the same as the maximum torque on the square coil?

91. Consider an electron rotating in a circular orbit of radius r. Show that the magnitudes of the magnetic dipole moment μ and the angular momentum L of the electron are related by:

$$\frac{\mu}{L} = \frac{e}{2m}.$$

92. The Hall effect is to be used to find the sign of charge carriers in a semiconductor sample. The probe is placed between the poles of a magnet so that magnetic field is pointed up. A current is passed through a rectangular sample placed horizontally. As current is passed through the sample in the east direction, the north side of the sample is found to be at a higher potential than the south side. Decide if the number density of charge carriers is positively or negatively charged.

93. The density of charge carriers for copper is 8.47×10^{28} electrons per cubic meter. What will be the Hall voltage reading from a probe made up of **3cm×2 cm×1 cm(L×W×T)** copper plate when a current of **1.5 A** is passed through it in a magnetic field of **2.5 T** perpendicular to the **3cm×2 cm.**

94. The Hall effect is to be used to find the density of charge carriers in an unknown material. A Hall voltage 40 **μV** for 3-A current is observed in a 3-T magnetic field for a rectangular sample with length 2 cm, width 1.5 cm, and height 0.4 cm. Determine the density of the charge carriers.

95. Show that the Hall voltage across wires made of the same material, carrying identical currents, and subjected to the same magnetic field is inversely proportional to their diameters. (Hint: Consider how drift velocity depends on wire diameter.)

96. A velocity selector in a mass spectrometer uses a 0.100-T magnetic field.

(a) What electric field strength is needed to select a speed of $4.0 \times 10^6 m/s$?

(b) What is the voltage between the plates if they are separated by 1.00 cm?

97. Find the radius of curvature of the path of a 25.0-MeV proton moving perpendicularly to the 1.20-T field of a cyclotron.

98. Unreasonable results To construct a non-mechanical water meter, a 0.500-T magnetic field is placed across the supply water pipe to a home and the Hall voltage is recorded.

(a) Find the flow rate through a 3.00-cm-diameter pipe if the Hall voltage is 60.0 mV.

(b) What would the Hall voltage be for the same flow rate through a 10.0-cm-diameter pipe with the same field applied?

99. Unreasonable results A charged particle having mass $6.64 \times 10^{-27} kg$ (that of a helium atom) moving at 8.70×105m/s perpendicular to a 1.50-T magnetic field travels in a circular path of radius 16.0 mm.

(a) What is the charge of the particle?

(b) What is unreasonable about this result?

(c) Which assumptions are responsible?

100. Unreasonable results An inventor wants to generate 120-V power by moving a 1.00-m-long wire perpendicular to Earth's $5.00 \times 10^{-5} T$ field.

(a) Find the speed with which the wire must move.

(b) What is unreasonable about this result? (c) Which assumption is responsible?

101. Unreasonable results Frustrated by the small Hall voltage obtained in blood flow measurements, a medical physicist decides to increase the applied magnetic field strength to get a 0.500-V output for blood moving at 30.0 cm/s in a 1.50-cm-diameter vessel.

(a) What magnetic field strength is needed?

(b) What is unreasonable about this result?

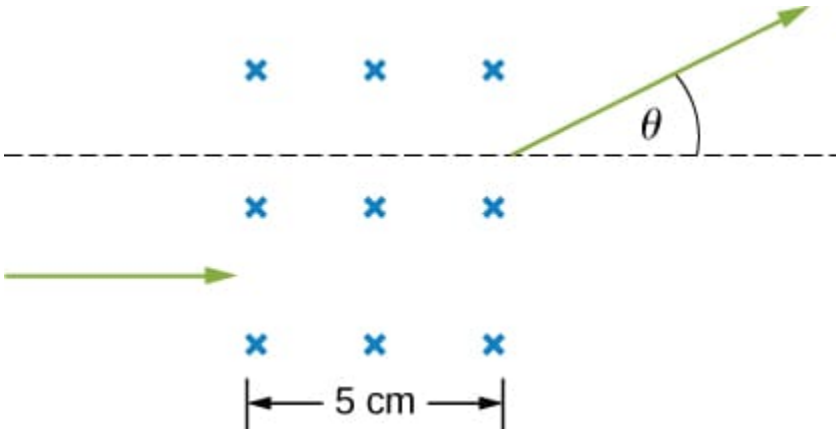
(c) Which premise is responsible?

Challenge Problems

102. A particle of charge **+q** and mass **m** moves with velocity $\hat{v_0}$ pointed in the **+y**-direction as it crosses the **x**-axis at **x = R** at a particular time. There is a negative charge **–Q** fixed at the origin, and there exists a uniform magnetic field $\hat{B_0}$ pointed in the **+z**-direction. It is found that the particle describes a circle of radius **R** about **–Q.** Find $\hat{B_0}$ in terms of the given quantities.

103. A proton of speed $v = 6 \times 10^5 m/s$ enters a region of uniform magnetic field of **B = 0.5 T** at an angle of $q = 30$ ° to the magnetic field. In the region of magnetic field proton describes a helical path with radius R and pitch p (distance between loops). Find **R** and **p**.

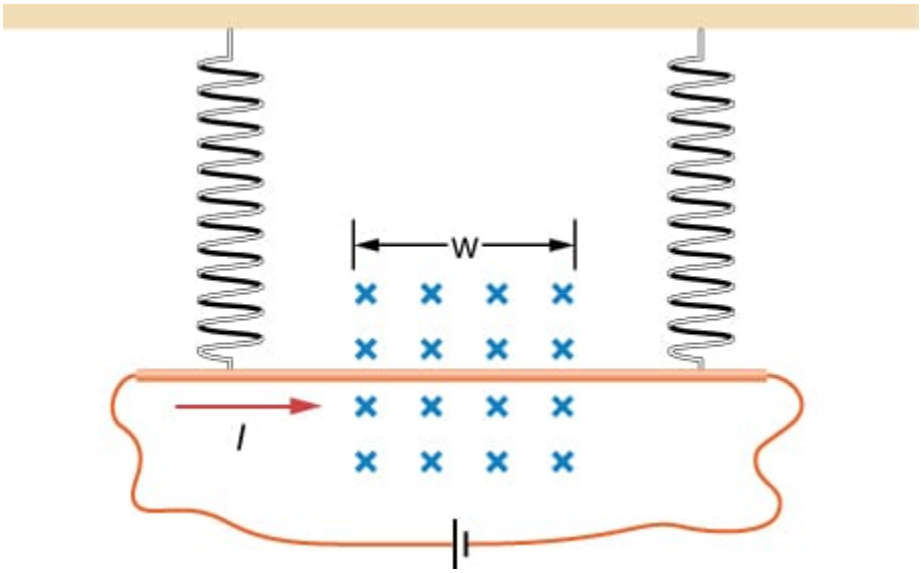
104. A particle's path is bent when it passes through a region of non-zero magnetic field although its speed remains unchanged. This is very useful for "beam steering" in particle accelerators. Consider a proton of speed $4 \times 10^6 m/s$ entering a region of uniform magnetic field 0.2 T over a 5-cm-wide region. Magnetic field is perpendicular to the velocity of the particle. By how much angle will the path of the proton be bent? (Hint: The particle comes out tangent to a circle.)



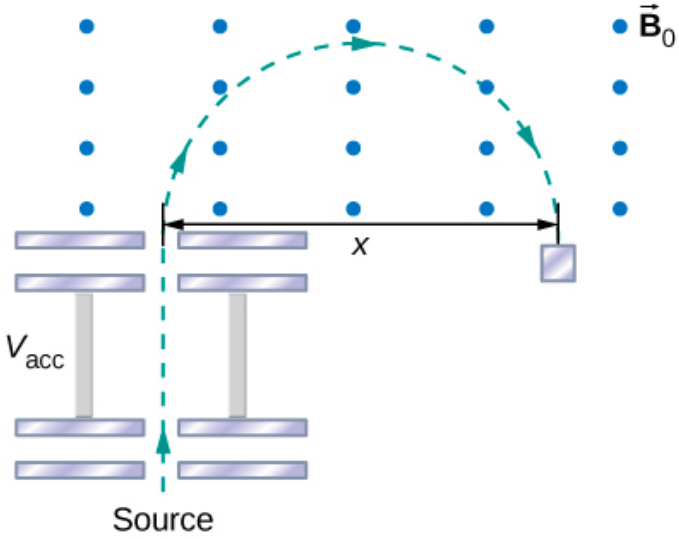
105. In a region a non-uniform magnetic field exists such that $B_x = 0, B_y = 0$, and $B_z = ax$, where a is a constant. At some time **t**, a wire of length **L** is carrying a current **I** is located along the **x-axis** from origin to **x = L**. Find the magnetic force on

the wire at this instant in time.

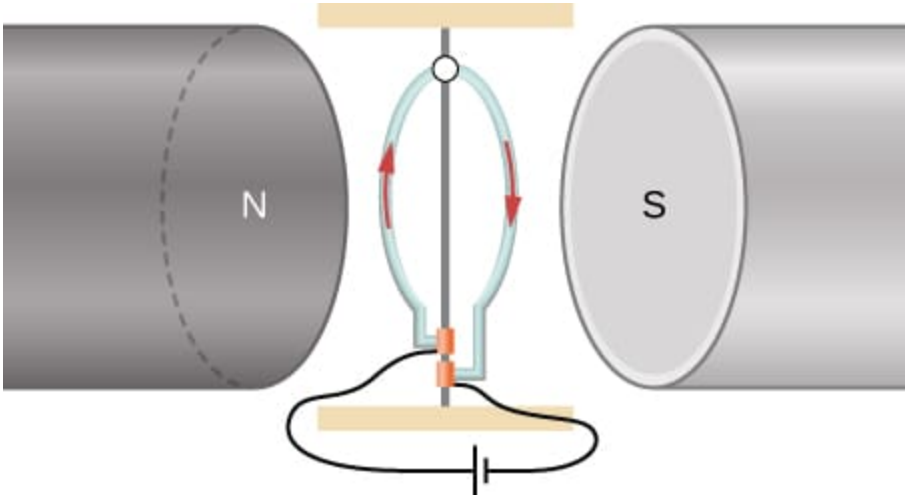
106. A copper rod of mass **m** and length **L** is hung from the ceiling using two springs of spring constant **k**. A uniform magnetic field of magnitude B_0 pointing perpendicular to the rod and spring (coming out of the page in the figure) exists in a region of space covering a length **w** of the copper rod. The ends of the rod are then connected by flexible copper wire across the terminals of a battery of voltage **V**. Determine the change in the length of the springs when a current I runs through the copper rod in the direction shown in figure. (Ignore any force by the flexible wire.)



107. The accompanied figure shows an arrangement for measuring mass of ions by an instrument called the mass spectrometer. An ion of mass **m** and charge **+q** is produced essentially at rest in source **S**, a chamber in which a gas discharge is taking place. The ion is accelerated by a potential difference V_{acc} and allowed to enter a region of constant magnetic field $\vec{B}_0$. In the uniform magnetic field region, the ion moves in a semicircular path striking a photographic plate at a distance x from the entry point. Derive a formula for mass m in terms of B_0, q, V_{acc}, and x.



108. A wire is made into a circular shape of radius **R** and pivoted along a central support. The two ends of the wire are touching a brush that is connected to a dc power source. The structure is between the poles of a magnet such that we can assume there is a uniform magnetic field on the wire. In terms of a coordinate system with origin at the center of the ring, magnetic field is $B_x = B_0, B_y = B_z = 0$, and the ring rotates about the **z-axis**. Find the torque on the ring when it is not in the xz-plane.



109. A long-rigid wire lies along the **x-axis** and carries a current of **2.5 A** in the positive x-direction. Around the wire is the magnetic field $\vec{B} = 2.0\hat{i} + 5.0x^2\hat{j}$, with **x** in meters and **B** in millitesla. Calculate the magnetic force on the segment of wire between **x = 2.0 m** and **x = 4.0 m**.

110. A circular loop of wire of area 10 cm^2 carries a current of 25 A. At a particular instant, the loop lies in the **xy**-plane and is subjected to a magnetic field $\vec{B} = (2.0\hat{i} + 6.0\hat{j} + 8.0\hat{k}) \times 10^{-3}T$. As viewed from above the **xy**-plane, the current is circulating clockwise.

(a) What is the magnetic dipole moment of the current loop?

(b) At this instant, what is the magnetic torque on the loop?

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14.11: Magnetic Forces and Fields (Answers)

Check Your Understanding

11.1. a. 0 N;

b. $2.4 \times 10^{-14}\hat{k}N$;

c. $2.4 \times 10^{-14}\hat{j}N$;

d. $(7.2\hat{j} + 2.2\hat{k}) \times 10^{-15}N$

11.2. a. $9.6 \times 10^{-12}N$ toward the south;

b. $\frac{w}{Fm} = 1.7 \times 10^{-15}$

11.3. a. bends upward;

b. bends downward

11.4. a. aligned or anti-aligned;

b. perpendicular

11.5. a. 1.1 T;

b. 1.6 T

11.6. 0.32 m

Conceptual Questions

1. Both are field dependent. Electrical force is dependent on charge, whereas magnetic force is dependent on current or rate of charge flow.

3. The magnitude of the proton and electron magnetic forces are the same since they have the same amount of charge. The direction of these forces however are opposite of each other. The accelerations are opposite in direction and the electron has a larger acceleration than the proton due to its smaller mass.

5. The magnetic field must point parallel or anti-parallel to the velocity.

7. A compass points toward the north pole of an electromagnet.

9. Velocity and magnetic field can be set together in any direction. If there is a force, the velocity is perpendicular to it. The magnetic field is also perpendicular to the force if it exists.

11. A force on a wire is exerted by an external magnetic field created by a wire or another magnet.

13. Poor conductors have a lower charge carrier density, n, which, based on the Hall effect formula, relates to a higher Hall potential. Good conductors have a higher charge carrier density, thereby a lower Hall potential.

Problems

15. a. left;

b. into the page;

c. up the page;

d. no force;

e. right;

f. down

17. a. right;

b. into the page;

c. down

19. a. into the page;

b. left;

c. out of the page

21. a. $2.64 \times 10^{-8} N$;

b. The force is very small, so this implies that the effect of static charges on airplanes is negligible.

23. $10.1\ °; 169.9\ °$

25. 4.27 m

27. a. $4.80 \times 10^{-19} C$;

b. 3;

c. This ratio must be an integer because charges must be integer numbers of the basic charge of an electron. There are no free charges with values less than this basic charge, and all charges are integer multiples of this basic charge.

29. a. $4.09 \times 10^{3} m/s$;

b. $7.83 \times 10^{3} m;$

c. $1.75 \times 10^{5} m/s$, then, $1.83 \times 10^{2} m$;

d. $4.27 m$

31. a. $1.8 \times 10^{7} m/s$;

b. $6.8 \times 10^{6} eV$;

c. $3.4 \times 10^{6} V$

33. a. left;

b. into the page;

c. up;

d. no force;

e. right;

f. down

35. a. into the page;

b. left;

c. out of the page

37. a. 2.50 N;

b. This means that the light-rail power lines must be attached in order not to be moved by the force caused by Earth's magnetic field.

39. a. $\tau = NIAB$, so τ decreases by 5.00% if **B** decreases by 5.00%;

b. 5.26% increase

41. 10.0 A

43. $A \cdot m^2 \cdot T = A \cdot m^2 . \frac{N}{A \cdot m} = N \cdot m$

45. $3.48 \times 10^{-26} N \cdot m$

47. $0.666 N \cdot m$

49. $5.8 \times 10^{-7} V$

51. $4.8 \times 10^{7} C/kg$

53. a. $4.4 \times 10^{-8} s$;

b. 0.21 m

55. a. $1.92 \times 10^{-12} J$;

b. 12 MeV;

c. 12 MV;

d. $5.2 \times 10^{-8} s$;

e. $1.92 \times 10^{-12} J$, 12 MeV, 12 V, $10.4 \times 10^{-8} s$

57. a. $2.50 \times 10^{-} 2m$;

b. Yes, this distance between their paths is clearly big enough to separate the U-235 from the U-238, since it is a distance of 2.5 cm.

Additional Problems

59. $-7.2 \times 10^{-15} N \hat{j}$

61. $9.8 \times 10^{-5} \hat{j} T$; the magnetic and gravitational forces must balance to maintain dynamic equilibrium

63. $1.13 \times 10^{-3} T$

65. $1.6\hat{i} - 1.4\hat{j} - 1.1\hat{k}) \times 10^{5} V/m$

67. a. circular motion in a north, down plane;

b. $(1.61\hat{j} - 0.58\hat{k}) \times 10^{-14} N$

69. The proton has more mass than the electron; therefore, its radius and period will be larger.

71. $1.3 \times 10^{-25} kg$

73. 1:0.707:1

75. 1/4

77. a. $2.3 \times 10^{-4} m$;

b. $1.37 \times 10^{-4} T$

79. a. $30.0\ °$;

b. 4.80 N

81. a. 0.283 N;

b. 0.4 N;

c. 0 N;

d. 0 N

83. 0 N and 0.012 Nm

85. a. $0.31 Am^2$;

b. 0.16 Nm

87. $0.024 Am^2$

89. a. $0.16 Am^2$;

b. 0.016 Nm;

c. 0.028 J

91. (Proof)

93. $4.65 \times 10^{-7} V$

95. Since $E = Blv$, where the width is twice the radius, $I = 2r, I = 2r, I = nqAv_d, v_d = \frac{I}{nqA} = \frac{I}{nq\pi r^2}$ so $E = B \times 2r \times \frac{I}{nq\pi r^2} = \frac{2IB}{nq\pi r} \propto \frac{1}{r} \propto \frac{1}{d}$. The Hall voltage is inversely proportional to the diameter of the wire.

97. $6.92 \times 10^7 m/s$; 0.602 m

99. a. $2.4 \times 10^{-19} C$;

b. not an integer multiple of e;

c. need to assume all charges have multiples of e, could be other forces not accounted for

101. a. B = 5 T;

b. very large magnet;

c. applying such a large voltage

Challenge Problems

103. $R = (mvsin\theta)/qB; p = (\frac{2\pi m}{eB})vcos\theta$

105. $IaL^2/2$

107. $m = \frac{qB_0^2}{8V_{acc}}x^2$

109. 0.01 N

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15: SOURCES OF MAGNETIC FIELDS

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15.1: Prelude to Sources of Magnetic Fields

In the preceding chapter, we saw that a moving charged particle produces a magnetic field. This connection between electricity and magnetism is exploited in electromagnetic devices, such as a computer hard drive. In fact, it is the underlying principle behind most of the technology in modern society, including telephones, television, computers, and the internet.



Figure 15.1.1: An external hard drive attached to a computer works by magnetically encoding information that can be stored or retrieved quickly. A key idea in the development of digital devices is the ability to produce and use magnetic fields in this way. (credit: modification of work by “Miss Karen”/Flickr)

In this chapter, we examine how magnetic fields are created by arbitrary distributions of electric current, using the Biot-Savart law. Then we look at how current-carrying wires create magnetic fields and deduce the forces that arise between two current-carrying wires due to these magnetic fields. We also study the torques produced by the magnetic fields of current loops. We then generalize these results to an important law of electromagnetism, called Ampère’s law.

We examine some devices that produce magnetic fields from currents in geometries based on loops, known as solenoids and toroids. Finally, we look at how materials behave in magnetic fields and categorize materials based on their responses to magnetic fields.

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15.2: The Biot-Savart Law

Learning Objectives

By the end of this section, you will be able to:

- Explain how to derive a magnetic field from an arbitrary current in a line segment
- Calculate magnetic field from the Biot-Savart law in specific geometries, such as a current in a line and a current in a circular arc

We have seen that mass produces a gravitational field and also interacts with that field. Charge produces an electric field and also interacts with that field. Since moving charge (that is, current) interacts with a magnetic field, we might expect that it also creates that field—and it does.

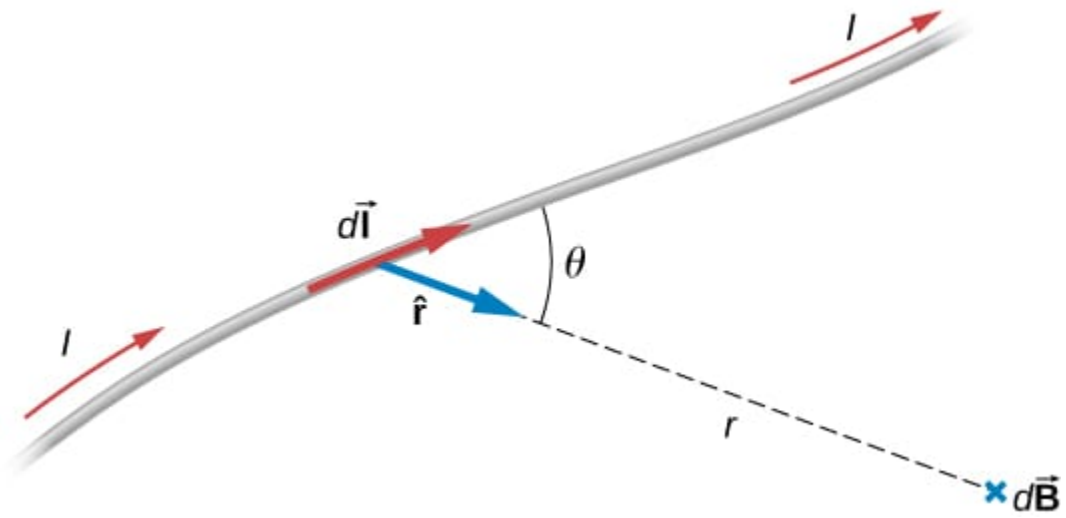


Figure 15.2.1: A current element $Id\vec{l}$ produces a magnetic field at point P given by the Biot-Savart law (Equation 15.2.4).

The equation used to calculate the magnetic field produced by a current is known as the Biot-Savart law. It is an empirical law named in honor of two scientists who investigated the interaction between a straight, current-carrying wire and a permanent magnet. This law enables us to calculate the magnitude and direction of the magnetic field produced by a current in a wire. The **Biot-Savart law** states that at any point P (Figure 15.2.1), the magnetic field $d\vec{B}$ due to an element $d\vec{l}$ of a current-carrying wire is given by

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l} \times \hat{r}}{r^2}. \tag{15.2.1}$$

The constant μ_0 is known as the **permeability of free space** and is exactly

$$\mu_0 = 4\pi \times 10^{-7}\, T \cdot m/A \tag{15.2.2}$$

in the SI system. The infinitesimal wire segment $d\vec{l}$ is in the same direction as the current I (assumed positive), r is the distance from $d\vec{l}$ to P and $\hat{r}$ is a unit vector that points from $d\vec{l}$ to P, as shown in Figure 15.2.1. The direction of $d\vec{B}$ is determined by applying the right-hand rule to the vector product $d\vec{l} \times \hat{r}$. The magnitude of $d\vec{B}$ is

$$dB = \frac{\mu_0}{4\pi}\frac{I\, dl\, \sin\theta}{r^2} \tag{15.2.3}$$

where θ is the angle between $d\vec{l}$ and $\hat{r}$. Notice that if $\theta = 0$, then $d\vec{B} = \vec{0}$. The field produced by a current element $Id\vec{l}$ has no component parallel to $d\vec{l}$.

The magnetic field due to a finite length of current-carrying wire is found by integrating Equation 15.2.2 along the wire, giving us the usual form of the Biot-Savart law.

Biot-Savart law

The magnetic field $\vec{B}$ due to an element $d\vec{l}$ of a current-carrying wire is given by

$$\vec{B} = \frac{\mu_0}{4\pi} \int_{wire} \frac{I\, d\vec{l} \times \hat{r}}{r^2}. \tag{15.2.4}$$

Since this is a vector integral, contributions from different current elements may not point in the same direction. Consequently, the integral is often difficult to evaluate, even for fairly simple geometries. The following strategy may be helpful.

Problem-Solving Strategy: Solving Biot-Savart Problems

To solve Biot-Savart law problems, the following steps are helpful:

1. Identify that the Biot-Savart law is the chosen method to solve the given problem. If there is symmetry in the problem comparing $\vec{B}$ and $d\vec{l}$, Ampère's law may be the preferred method to solve the question.
2. Draw the current element length $d\vec{l}$ and the unit vector $\hat{r}$ noting that $d\vec{l}$ points in the direction of the current and $\hat{r}$ points from the current element toward the point where the field is desired.
3. Calculate the cross product $d\vec{l} \times \hat{r}$.The resultant vector gives the direction of the magnetic field according to the Biot-Savart law.
4. Use Equation 15.2.4 and substitute all given quantities into the expression to solve for the magnetic field. Note all variables that remain constant over the entire length of the wire may be factored out of the integration.
5. Use the right-hand rule to verify the direction of the magnetic field produced from the current or to write down the direction of the magnetic field if only the magnitude was solved for in the previous part.

Calculating Magnetic Fields of Short Current Segments

A short wire of length 1.0 cm carries a current of 2.0 A in the vertical direction (Figure 15.2.2). The rest of the wire is shielded so it does not add to the magnetic field produced by the wire. Calculate the magnetic field at point **P**, which is 1 meter from the wire in the **x**-direction.

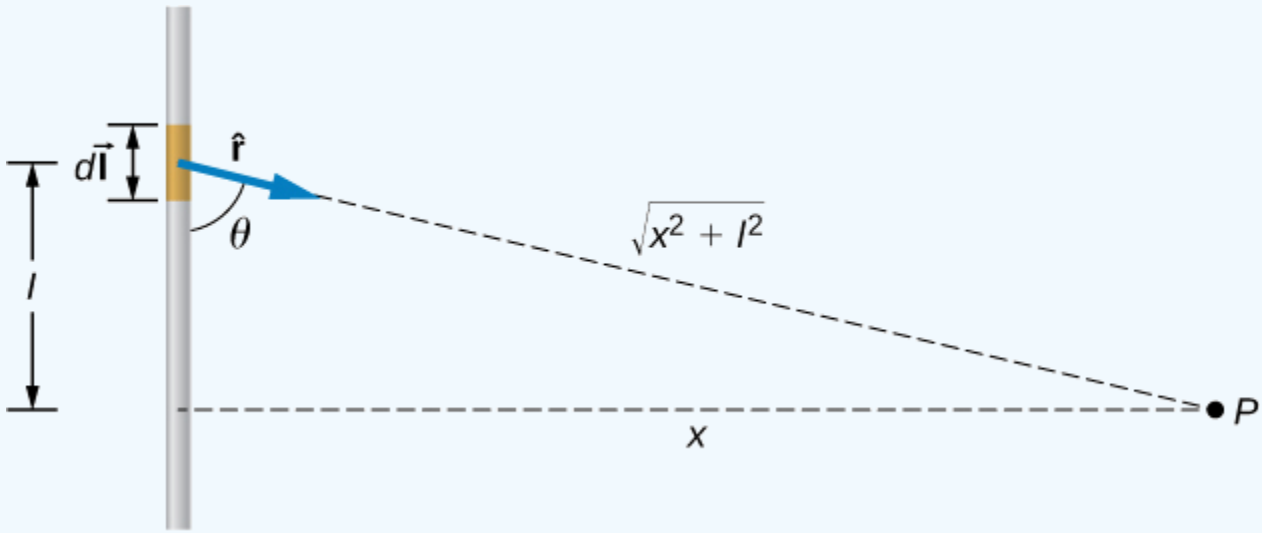


Figure 15.2.2: A small line segment carries a current I in the vertical direction. What is the magnetic field at a distance **x** from the segment?

Strategy

We can determine the magnetic field at point P using the Biot-Savart law. Since the current segment is much smaller than the distance **x**, we can drop the integral from the expression. The integration is converted back into a summation, but only for small dl, which we now write as Δl. Another way to think about it is that each of the radius values is nearly the same, no matter where the current element is on the line segment, if Δl is small compared to **x**. The angle θ is calculated using a tangent function. Using the numbers given, we can calculate the magnetic field at P.

Solution

The angle between $\Delta\vec{l}$ and $\hat{r}$ is calculated from trigonometry, knowing the distances **l** and **x** from the problem:

$$\theta = \tan^{-1}\left(\frac{1\,m}{0.01\,m}\right) = 89.4^{o}.$$

The magnetic field at point P is calculated by the Biot-Savart law (Equation 15.2.3):

$$\begin{aligned} B &= \frac{\mu_0}{4\pi}\frac{I\Delta l\,\sin\,\theta}{r^2} \\ &= (1\times 10^{-7}T\cdot m/A)\left(\frac{2\,A(0.01\,m)\,sin\,(89.4^{o})}{(1\,m)^2}\right) \\ &= 2.0\times 10^{-9}T. \end{aligned}$$

From the right-hand rule and the Biot-Savart law, the field is directed into the page.

Significance

This approximation is only good if the length of the line segment is very small compared to the distance from the current element to the point. If not, the integral form of the Biot-Savart law must be used over the entire line segment to calculate the magnetic field.

Exercise 15.2.1

Using Example 15.2.1, at what distance would **P** have to be to measure a magnetic field half of the given answer?

Solution

1.41 meters

Example 15.2.1: Calculating Magnetic Field of a Circular Arc of Wire

A wire carries a current **I** in a circular arc with radius **R** swept through an arbitrary angle θ (Figure 15.2.3). Calculate the magnetic field at the center of this arc at point **P**.

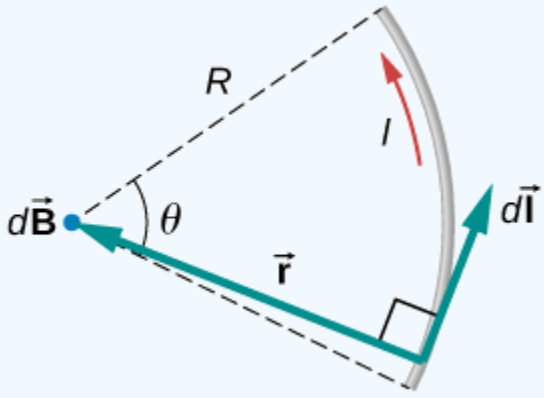


Figure 15.2.3: A wire segment carrying a current **I**. The path $d\vec{l}$ and radial direction $\hat{r}$ are indicated.

Strategy

We can determine the magnetic field at point **P** using the Biot-Savart law. The radial and path length directions are always at a right angle, so the cross product turns into multiplication. We also know that the distance along the path **dl** is related to the radius times the angle θ (in radians). Then we can pull all constants out of the integration and solve for the magnetic field.

Solution

The Biot-Savart law starts with the following equation:

$$\vec{B} = \frac{\mu_0}{4\pi} \int_{wire} \frac{I\vec{dl} \times \hat{r}}{r^2}.$$

As we integrate along the arc, all the contributions to the magnetic field are in the same direction (out of the page), so we can work with the magnitude of the field. The cross product turns into multiplication because the path dl and the radial direction are perpendicular. We can also substitute the arc length formula, $dl = r\ d\theta$:

$$B = \frac{\mu_0}{4\pi} \int_{wire} \frac{Ir\, d\theta}{r^2}.$$

The current and radius can be pulled out of the integral because they are the same regardless of where we are on the path. This leaves only the integral over the angle,

$$B = \frac{\mu_0 I}{4\pi r} \int_{wire} d\theta.$$

The angle varies on the wire from 0 to θ; hence, the result is

$$B = \frac{\mu_0 I\theta}{4\pi r}.$$

Significance

The direction of the magnetic field at point P is determined by the right-hand rule, as shown in the previous chapter. If there are other wires in the diagram along with the arc, and you are asked to find the net magnetic field, find each contribution from a wire or arc and add the results by superposition of vectors. Make sure to pay attention to the direction of each contribution. Also note that in a symmetric situation, like a straight or circular wire, contributions from opposite sides of point P cancel each other.

Exercise 15.2.2

The wire loop forms a full circle of radius **R** and current **I**. What is the magnitude of the magnetic field at the center?

Solution

$\frac{\mu_0 I}{2R}$

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15.3: Magnetic Field due to a Thin Straight Wire

Learning Objectives

By the end of this section, you will be able to:

- Explain how the Biot-Savart law is used to determine the magnetic field due to a thin, straight wire.
- Determine the dependence of the magnetic field from a thin, straight wire based on the distance from it and the current flowing in the wire.
- Sketch the magnetic field created from a thin, straight wire by using the second right-hand rule.

How much current is needed to produce a significant magnetic field, perhaps as strong as Earth's field? Surveyors will tell you that overhead electric power lines create magnetic fields that interfere with their compass readings. Indeed, when Oersted discovered in 1820 that a current in a wire affected a compass needle, he was not dealing with extremely large currents. How does the shape of wires carrying current affect the shape of the magnetic field created? We noted in Chapter 28 that a current loop created a magnetic field similar to that of a bar magnet, but what about a straight wire? We can use the Biot-Savart law to answer all of these questions, including determining the magnetic field of a long straight wire.

Figure 15.3.1 shows a section of an infinitely long, straight wire that carries a current **I**. What is the magnetic field at a point **P**, located a distance **R** from the wire?

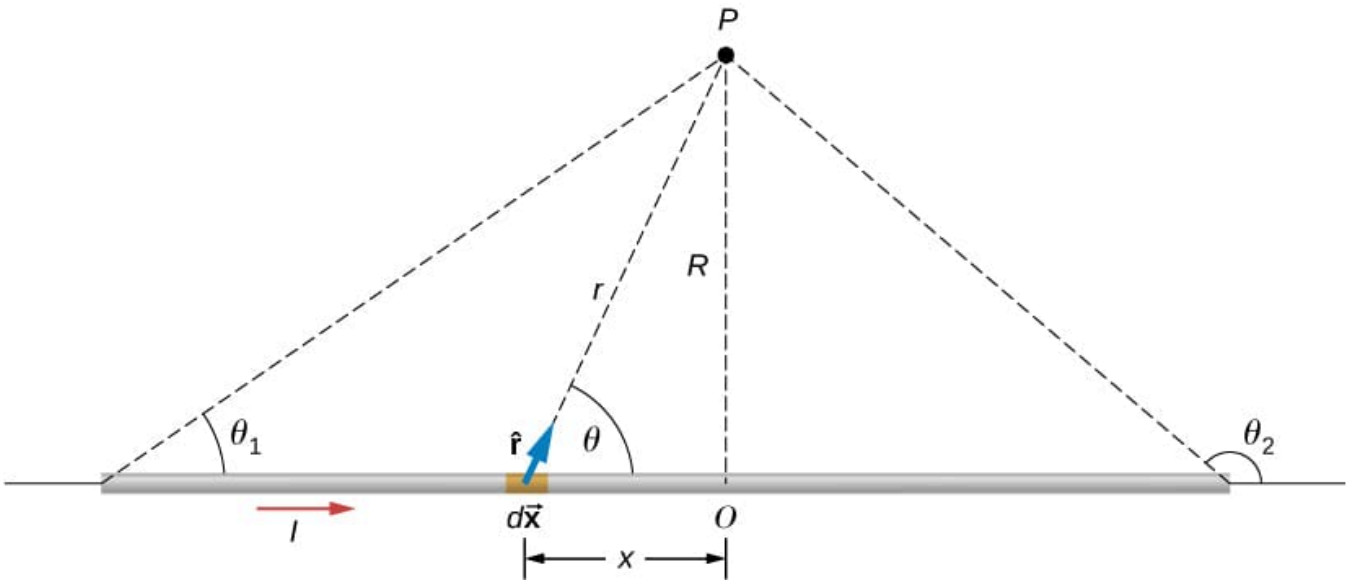


Figure 15.3.1: A section of a thin, straight current-carrying wire. The independent variable θ has the limits θ_1 and θ_2.

Let's begin by considering the magnetic field due to the current element $I\ d\vec{x}$ located at the position **x**. Using the right-hand rule 1 from the previous chapter, $d\vec{x} \times \hat{r}$ points out of the page for any element along the wire. At point P, therefore, the magnetic fields due to all current elements have the same direction. This means that we can calculate the net field there by evaluating the scalar sum of the contributions of the elements. With

$$|d\vec{x} \times \hat{r}| = (dx)(1)\ \sin\ \theta$$

we have from the **Biot-Savart law**

$$B = \frac{\mu_0}{4\pi} \int_{wire} \frac{I\ \sin\ \theta\, dx}{r^2}. \tag{15.3.1}$$

The wire is symmetrical about point O, so we can set the limits of the integration from zero to infinity and double the answer,

rather than integrate from negative infinity to positive infinity. Based on the picture and trigonometry, we can write expressions for r and $\sin\ \theta$ in terms of **x** and **R**, namely:

$$r = \sqrt{x^2 + R^2}$$

$$\sin\ \theta = \frac{R}{\sqrt{x^2 + R^2}}.$$

Substituting these expressions into Equation 15.3.1, the magnetic field integration becomes

$$B = \frac{\mu_0 I}{2\pi} \int_0^\infty \frac{R\,dx}{(x^2 + R^2)^{3/2}}.$$

Evaluating the integral yields

$$B = \frac{\mu_0 I}{2\pi R} \left[\frac{x}{(x^2 + R^2)^{1/2}} \right]_0^\infty.$$

Substituting the limits gives us the solution

$$\boxed{B = \frac{\mu_0 I}{2\pi R}.}$$

The magnetic field lines of the infinite wire are circular and centered at the wire (Figure 15.3.2), and they are identical in every plane perpendicular to the wire. Since the field decreases with distance from the wire, the spacing of the field lines must increase correspondingly with distance. The direction of this magnetic field may be found with a second form of the **right-hand rule** (Figure 15.3.2). If you hold the wire with your right hand so that your thumb points along the current, then your fingers wrap around the wire in the same sense as $\vec{B}$.

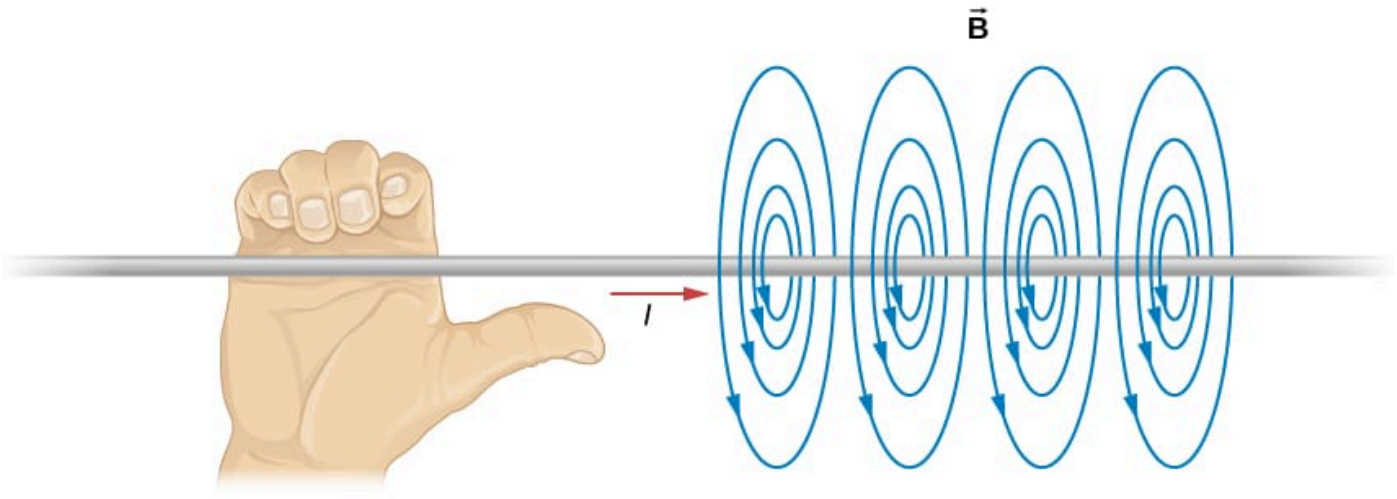


Figure 15.3.2:. Some magnetic field lines of an infinite wire. The direction of B can be found with a form of the right-hand rule.

The direction of the field lines can be observed experimentally by placing several small compass needles on a circle near the wire, as illustrated in Figure 15.3.3a. When there is no current in the wire, the needles align with Earth's magnetic field. However, when a large current is sent through the wire, the compass needles all point tangent to the circle. Iron filings sprinkled on a horizontal surface also delineate the field lines, as shown in Figure 15.3.3b.

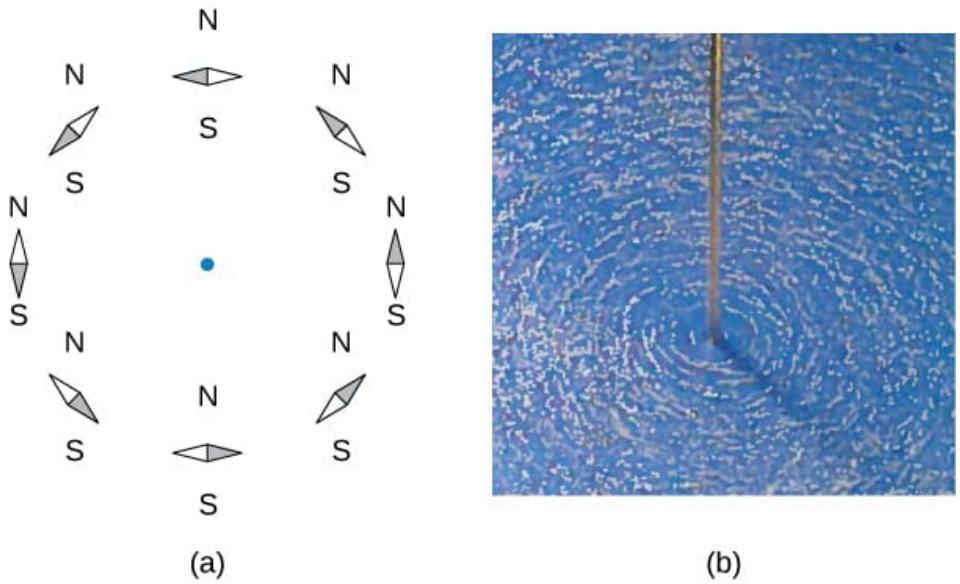


Figure 15.3.3: The shape of the magnetic field lines of a long wire can be seen using (a) small compass needles and (b) iron filings.

Example 15.3.1: Calculating Magnetic Field Due to Three Wires

Three wires sit at the corners of a square, all carrying currents of 2 amps into the page as shown in Figure 15.3.4. Calculate the magnitude of the magnetic field at the other corner of the square, point **P**, if the length of each side of the square is 1 cm.

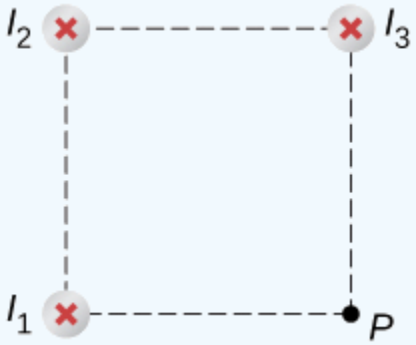


Figure 15.3.4: Three wires have current flowing into the page. The magnetic field is determined at the fourth corner of the square.

Strategy

The magnetic field due to each wire at the desired point is calculated. The diagonal distance is calculated using the Pythagorean theorem. Next, the direction of each magnetic field's contribution is determined by drawing a circle centered at the point of the wire and out toward the desired point. The direction of the magnetic field contribution from that wire is tangential to the curve. Lastly, working with these vectors, the resultant is calculated.

Solution

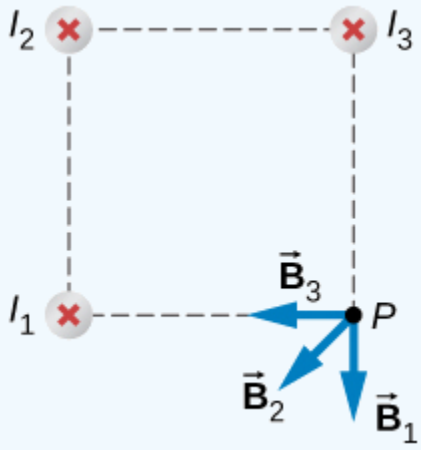
Wires 1 and 3 both have the same magnitude of magnetic field contribution at point **P**:

$$B_1 = B_3 = \frac{\mu_0 I}{2\pi R} = \frac{(4\pi \times 10^{-7} T \cdot m/A)(2\,A)}{2\pi(0.01\,m)} = 4 \times 10^{-5} T.$$

Wire 2 has a longer distance and a magnetic field contribution at point **P** of:

$$B_2 = \frac{\mu_0 I}{2\pi R} = \frac{(4\pi \times 10^{-7} T \cdot m/A)(2\,A)}{2\pi(0.01414\,m)} = 3 \times 10^{-5} T.$$

The vectors for each of these magnetic field contributions are shown.



The magnetic field in the **x**-direction has contributions from wire 3 and the **x**-component of wire 2:

$$B_{net\,x} = -4 \times 10^{-5}T - 2.83 \times 10^{-5}T\,\cos(45^o) = -6 \times 10^{-5}T.$$

The **y**-component is similarly the contributions from wire 1 and the **y**-component of wire 2:

$$B_{net\,y} = -4 \times 10^{-5}T - 2.83 \times 10^{-5}T\,\sin(45^o) = -6 \times 10^{-5}T.$$

Therefore, the net magnetic field is the resultant of these two components:

$$\begin{aligned} B_{net} &= \sqrt{B_{net\,x}^2 + B_{net\,y}} \\ &= \sqrt{(-6 \times 10^{-5}T)^2 + (-6 \times 10^{-5}T)^2} \\ &= 8.48 \times 10^{-5}T. \end{aligned}$$

Significance

The geometry in this problem results in the magnetic field contributions in the **x**- and **y**-directions having the same magnitude. This is not necessarily the case if the currents were different values or if the wires were located in different positions. Regardless of the numerical results, working on the components of the vectors will yield the resulting magnetic field at the point in need.

Exercise 15.3.1

Using Example 15.3.1, keeping the currents the same in wires 1 and 3, what should the current be in wire 2 to counteract the magnetic fields from wires 1 and 3 so that there is no net magnetic field at point P?

Solution

4 amps flowing out of the page

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15.4: Magnetic Force between Two Parallel Currents

Learning Objectives

By the end of this section, you will be able to:

- Explain how parallel wires carrying currents can attract or repel each other
- Define the ampere and describe how it is related to current-carrying wires
- Calculate the force of attraction or repulsion between two current-carrying wires

You might expect that two current-carrying wires generate significant forces between them, since ordinary currents produce magnetic fields and these fields exert significant forces on ordinary currents. But you might not expect that the force between wires is used to define the ampere. It might also surprise you to learn that this force has something to do with why large circuit breakers burn up when they attempt to interrupt large currents.

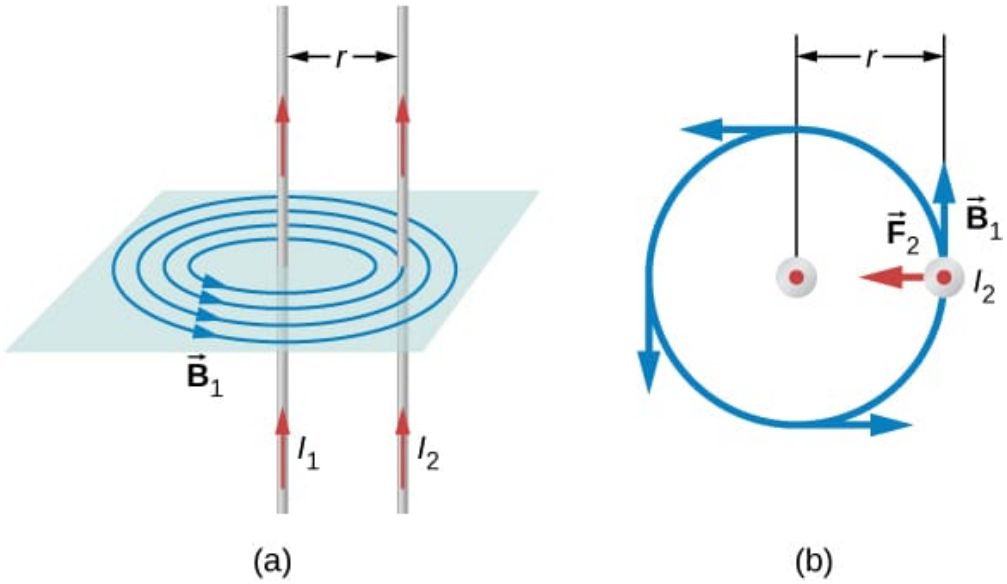


Figure 15.4.1: (a) The magnetic field produced by a long straight conductor is perpendicular to a parallel conductor, as indicated by right-hand rule (RHR)-2. (b) A view from above of the two wires shown in (a), with one magnetic field line shown for wire 1. RHR-1 shows that the force between the parallel conductors is attractive when the currents are in the same direction. A similar analysis shows that the force is repulsive between currents in opposite directions.

The force between two long, straight, and parallel conductors separated by a distance **r** can be found by applying what we have developed in the preceding sections. Figure 15.4.1 shows the wires, their currents, the field created by one wire, and the consequent force the other wire experiences from the created field. Let us consider the field produced by wire 1 and the force it exerts on wire 2 (call the force F_2). The field due to I_1 at a distance **r** is

$$B_1 = \frac{\mu_0 I_1}{2\pi r}$$

This field is uniform from the wire 1 and perpendicular to it, so the force F_2 it exerts on a length **l** of wire 2 is given by $F = IlB\ sin\ \theta$ with $sin\ \theta = 1$:

$$F_2 = I_2 l B_1. \tag{15.4.1}$$

The forces on the wires are equal in magnitude, so we just write **F** for the magnitude of F_2 (Note that $\vec{F}_1 = -\vec{F}_2$.) Since the wires are very long, it is convenient to think in terms of **F/l**, the force per unit length. Substituting the expression for B_1 into Equation 15.4.1 and rearranging terms gives

Note

$$\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}. \tag{15.4.2}$$

The ratio **F/l** is the force per unit length between two parallel currents I_1 and I_2 separated by a distance **r**. The force is attractive if the currents are in the same direction and repulsive if they are in opposite directions.

This force is responsible for the **pinch effect** in electric arcs and other plasmas. The force exists whether the currents are in wires or not. It is only apparent if the overall charge density is zero; otherwise, the Coulomb repulsion overwhelms the magnetic attraction. In an electric arc, where charges are moving parallel to one another, an attractive force squeezes currents into a smaller tube. In large circuit breakers, such as those used in neighborhood power distribution systems, the pinch effect can concentrate an arc between plates of a switch trying to break a large current, burn holes, and even ignite the equipment. Another example of the pinch effect is found in the solar plasma, where jets of ionized material, such as solar flares, are shaped by magnetic forces.

The definition of the **ampere** is based on the force between current-carrying wires. Note that for long, parallel wires separated by 1 meter with each carrying 1 ampere, the force per meter is

$$\frac{F}{l} = \frac{(4\pi \times 10^{-7} T \cdot m/A)(1\ A)^2}{(2\pi)(1\ m)} = 2 \times 10^{-7}\ N/m.$$

Since μ_0 is exactly $4\pi \times 10^{-7}\ T \cdot m/A$ by definition, and because $1\ T = 1\ N/(A \cdot m)$, the force per meter is exactly $2 \times 10^{-7}\ N/m$. This is the basis of the definition of the ampere.

Infinite-length wires are impractical, so in practice, a current balance is constructed with coils of wire separated by a few centimeters. Force is measured to determine current. This also provides us with a method for measuring the coulomb. We measure the charge that flows for a current of one ampere in one second. That is, $1\ C = 1\ A \cdot s$. For both the ampere and the coulomb, the method of measuring force between conductors is the most accurate in practice.

Example 15.4.1: Calculating Forces on Wires

Two wires, both carrying current out of the page, have a current of magnitude 5.0 mA. The first wire is located at (0.0 cm, 3.0 cm) while the other wire is located at (4.0 cm, 0.0 cm) as shown in Figure 15.4.2. What is the magnetic force per unit length of the first wire on the second and the second wire on the first?

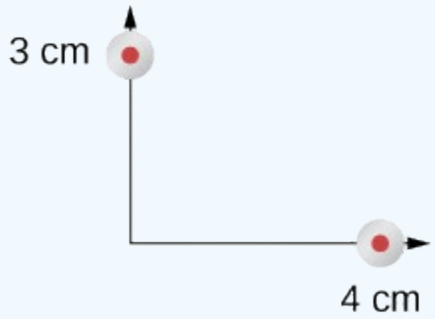


Figure 15.4.2: Two current-carrying wires at given locations with currents out of the page.

Strategy

Each wire produces a magnetic field felt by the other wire. The distance along the hypotenuse of the triangle between the wires is the radial distance used in the calculation to determine the force per unit length. Since both wires have currents flowing in the same direction, the direction of the force is toward each other.

Solution

The distance between the wires results from finding the hypotenuse of a triangle:

$$r = \sqrt{(3.0\,cm)^2 + (4.0\,cm)^2} = 5.0\,cm.$$

The force per unit length can then be calculated using the known currents in the wires:

$$\frac{F}{l} = \frac{(4\pi \times 10^{-7} T \cdot m/A)(5 \times 10^{-3} A)^2}{(2\pi)(5 \times 10^{-2} m)} = 1 \times 10^{-10}\, N/m.$$

The force from the first wire pulls the second wire. The angle between the radius and the **x**-axis is

$$\theta = tan^{-1}\left(\frac{3\,cm}{4\,cm}\right) = 36.9^o.$$

The unit vector for this is calculated by

$$cos(36.9^o)\hat{i} - sin(36.9^o)\hat{j} = 0.8\hat{i} - 0.6\hat{j}.$$

Therefore, the force per unit length from wire one on wire 2 is

$$\frac{\vec{F}}{l} = (1 \times 10^{-10}\, N/m) \times (0.8\hat{i} - 0.6\hat{j}) = (8 \times 10^{-11}\hat{i} - 6 \times 10^{-11}\hat{j})\, N/m.$$

The force per unit length from wire 2 on wire 1 is the negative of the previous answer:

$$\frac{\vec{F}}{l} = (-8 \times 10^{-11}\hat{i} + 6 \times 10^{-11}\hat{j}) N/m.$$

Significance

These wires produced magnetic fields of equal magnitude but opposite directions at each other's locations. Whether the fields are identical or not, the forces that the wires exert on each other are always equal in magnitude and opposite in direction (Newton's third law).

Exercise 15.4.1

Two wires, both carrying current out of the page, have a current of magnitude 2.0 mA and 3.0 mA, respectively. The first wire is located at (0.0 cm, 5.0 cm) while the other wire is located at (12.0 cm, 0.0 cm). What is the magnitude of the magnetic force per unit length of the first wire on the second and the second wire on the first?

Answer available at the end of this section.

Answers

Exercise 15.4.1

Both have a force per unit length of $9.23 \times 10^{-12}\ N/m$

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15.5: Magnetic Field of a Current Loop

Learning Objectives

By the end of this section, you will be able to:

- Explain how the Biot-Savart law is used to determine the magnetic field due to a current in a loop of wire at a point along a line perpendicular to the plane of the loop.
- Determine the magnetic field of an arc of current.

The circular loop of Figure 15.5.1 has a radius **R**, carries a current **I**, and lies in the **xz**-plane. What is the magnetic field due to the current at an arbitrary point **P** along the axis of the loop?

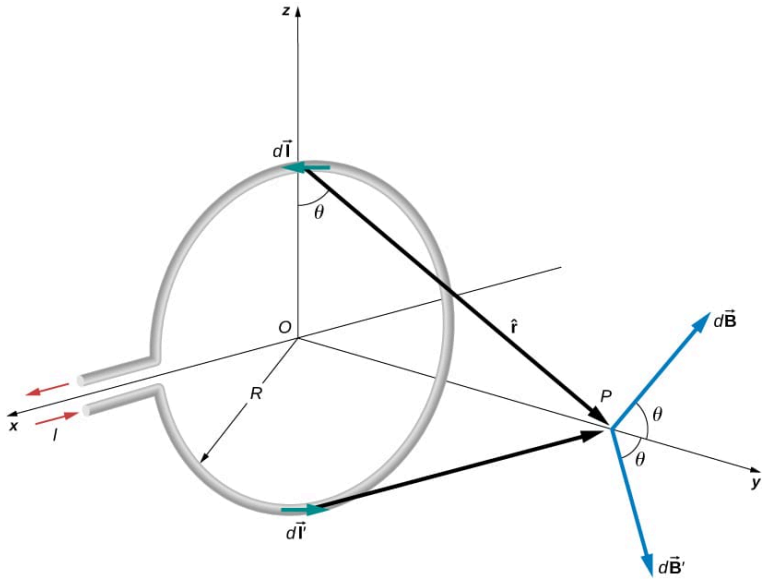


Figure 15.5.1: Determining the magnetic field at point **P** along the axis of a current-carrying loop of wire.

We can use the Biot-Savart law to find the magnetic field due to a current. We first consider arbitrary segments on opposite sides of the loop to qualitatively show by the vector results that the net magnetic field direction is along the central axis from the loop. From there, we can use the Biot-Savart law to derive the expression for magnetic field.

Let **P** be a distance **y** from the center of the loop. From the right-hand rule, the magnetic field $d\vec{B}$ at **P**, produced by the current element $I\ d\vec{l}$ is directed at an angle θ above the **y**-axis as shown. Since $d\vec{l}$ is parallel along the **x**-axis and $\hat{r}$ is in the **yz**-plane, the two vectors are perpendicular, so we have

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl\,sin\,\pi/2}{r^2} = \frac{\mu_0}{4\pi}\frac{I\,dl}{y^2+R^2} \tag{15.5.1}$$

where we have used $r^2 = y^2 + R^2$.

Now consider the magnetic field $d\vec{B}'$ due to the current element $I\ d\vec{l}\,'$, which is directly opposite $I\ d\vec{l}$ on the loop. The magnitude of $d\vec{B}'$ is also given by Equation 15.5.1, but it is directed at an angle below the **y**-axis. The components of $d\vec{B}$ and $d\vec{B}'$ perpendicular to the **y**-axis therefore cancel, and in calculating the net magnetic field, only the components along the **y**-axis need to be considered. The components perpendicular to the axis of the loop sum to zero in pairs. Hence at point **P**:

$$\vec{B} = \hat{j}\int_{loop} dB\,cos\,\theta = \hat{j}\frac{\mu_0 I}{4\pi}\int_{loop}\frac{cos\,\theta\,dl}{y^2+R^2}. \tag{15.5.2}$$

For all elements $d\vec{l}$ on the wire, **y**, **R**, and θ are constant and are related by

$$cos\,\theta = \frac{R}{\sqrt{y^2+R^2}}.$$

Now from Equation 15.5.2, the magnetic field at **P** is

$$\vec{B} = \hat{j}\frac{\mu_0 IR}{4\pi(y^2+R^2)^{3/2}}\int_{loop} dl = \frac{\mu_0 IR^2}{2(y^2+R^2)^{3/2}}\hat{j} \tag{15.5.3}$$

where we have used $\int_{loop} dl = 2\pi R$. As discussed in the previous chapter, the closed current loop is a magnetic dipole of moment $\vec{\mu} = I\ A\hat{n}$. For this example, $A = \pi R^2$ and $\hat{n} = \hat{j}$, so the magnetic field at **P** can also be written as

$$\vec{B} = \frac{\mu_0 \mu\hat{j}}{2\pi(y^2+R^2)^{3/2}}. \tag{15.5.4}$$

By setting $y = 0$ in Equation 15.5.4, we obtain the magnetic field at the center of the loop:

Note

$$\vec{B} = \frac{\mu_0 I}{2R}\hat{j}. \tag{15.5.5}$$

This equation becomes $B = \mu_0 nI/(2R)$ for a flat coil of **n** loops per length. It can also be expressed as

$$\vec{B} = \frac{\mu_0\vec{\mu}}{2\pi R^3}. \tag{15.5.6}$$

If we consider $y >> R$ in Equation 15.5.4, the expression reduces to an expression known as the magnetic field from a dipole:

$$\vec{B} = \frac{\mu_0\vec{\mu}}{2\pi y^3}. \tag{15.5.7}$$

The calculation of the magnetic field due to the circular current loop at points off-axis requires rather complex mathematics, so we'll just look at the results. The magnetic field lines are shaped as shown in Figure 15.5.2. Notice that one field line follows the axis of the loop. This is the field line we just found. Also, very close to the wire, the field lines are almost circular, like the lines of a long straight wire.

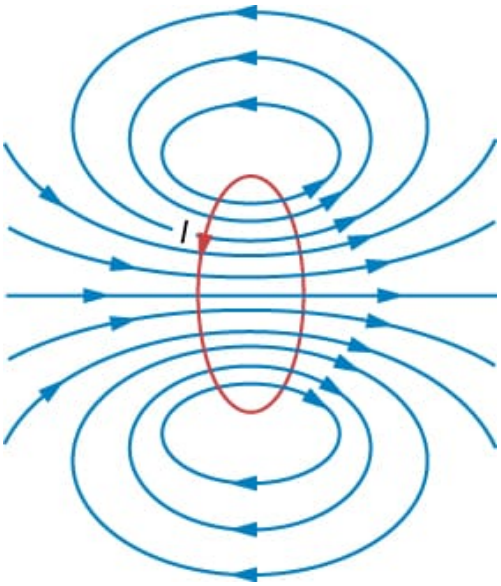


Figure 15.5.2: Sketch of the magnetic field lines of a circular current loop.

Magnetic Field between Two Loops

Two loops of wire carry the same current of 10 mA, but flow in opposite directions as seen in Figure 15.5.3. One loop is measured to have a radius of $R = 50\ cm$ while the other loop has a radius of $2R = 100\ cm$. The distance from the first loop to the point where the magnetic field is measured is 0.25 m, and the distance from that point to the second loop is 0.75 m. What is the magnitude of the net magnetic field at point **P**?

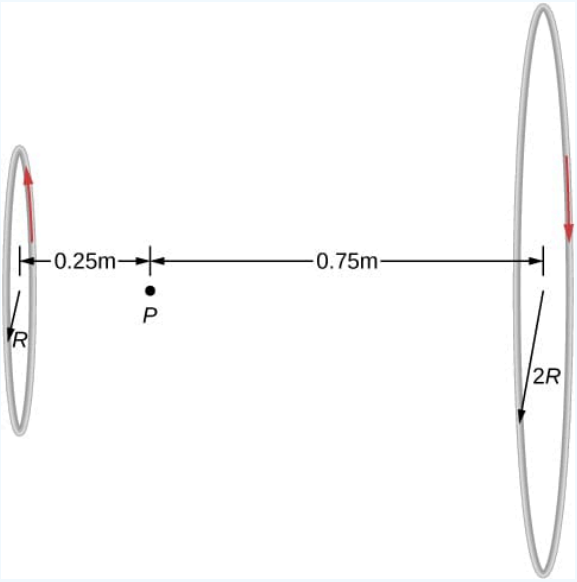


Figure 15.5.3: Two loops of different radii have the same current but flowing in opposite directions. The magnetic field at point **P** is measured to be zero.

Strategy

The magnetic field at point **P** has been determined in Equation 15.5.3. Since the currents are flowing in opposite directions, the net magnetic field is the difference between the two fields generated by the coils. Using the given quantities in the problem, the net magnetic field is then calculated.

Solution

Solving for the net magnetic field using Equation 15.5.3 and the given quantities in the problem yields

$$B = \frac{\mu_0 I R_1^2}{2(y_1^2 + R_1^2)^{3/2}} - \frac{\mu_0 I R_2^2}{2(y_2^2 + R_2^2)^{3/2}}$$

$$B = \frac{(4\pi \times 10^{-7} T \cdot m/A)(0.010\,A)(0.5\,m)^2}{2((0.25\,m)^2 + (0.5\,m)^2)^{3/2}} - \frac{(4\pi \times 10^{-7} T \cdot m/A)(0.010\,A)(1.0\,m)^2}{2((0.75\,m)^2 + (1.0\,m)^2)^{3/2}}$$

$B = 5.77 \times 10^{-9} T$ to the right.

Significance

Helmholtz coils typically have loops with equal radii with current flowing in the same direction to have a strong uniform field at the midpoint between the loops. A similar application of the magnetic field distribution created by Helmholtz coils is found in a magnetic bottle that can temporarily trap charged particles. See Magnetic Forces and Fields for a discussion on this.

Exercise 15.5.1

Using Example 15.5.1, at what distance would you have to move the first coil to have zero measurable magnetic field at point **P**?

Solution available at the end of this section.

Answers

Exercise 15.5.1

0.608 meters

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15.6: Ampère's Law

Learning Objectives

By the end of this section, you will be able to:

- Explain how Ampère's law relates the magnetic field produced by a current to the value of the current
- Calculate the magnetic field from a long straight wire, either thin or thick, by Ampère's law

A fundamental property of a static magnetic field is that, unlike an electrostatic field, it is not conservative. A conservative field is one that does the same amount of work on a particle moving between two different points regardless of the path chosen. Magnetic fields do not have such a property. Instead, there is a relationship between the magnetic field and its source, electric current. It is expressed in terms of the line integral of $\vec{B}$ and is known as **Ampère's law**. This law can also be derived directly from the Biot-Savart law. We now consider that derivation for the special case of an infinite, straight wire.

Figure $15.6.1$ shows an arbitrary plane perpendicular to an infinite, straight wire whose current **I** is directed out of the page. The magnetic field lines are circles directed counterclockwise and centered on the wire. To begin, let's consider $\oint \vec{B} \cdot d\vec{l}$ over the closed paths **M** and **N**. Notice that one path (**M**) encloses the wire, whereas the other (**N**) does not. Since the field lines are circular, $\vec{B} \cdot d\vec{l}$ is the product of **B** and the projection of **dl** onto the circle passing through $d\vec{l}$. If the radius of this particular circle is **r**, the projection is $rd\theta$, and

$$\vec{B} \cdot d\vec{l} = Br\, d\theta.$$

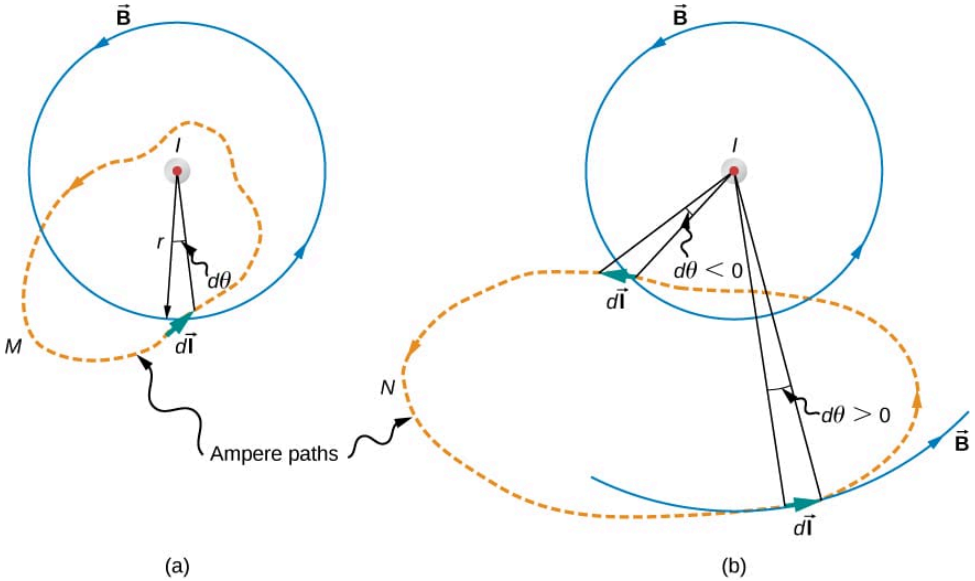


Figure $15.6.1$: The current I of a long, straight wire is directed out of the page. The integral $\oint d\theta$ equals 2π and 0, respectively, for paths M and N.

With $\vec{B}$ given by Equation 12.4.1,

$$\oint \vec{B} \cdot d\vec{l} = \oint \left(\frac{\mu_0 I}{2\pi r}\right) r\, d\theta = \frac{\mu_0 I}{2\pi} \oint d\theta.$$

For path **M**, which circulates around the wire, $\oint_M d\theta = 2\pi$ and

$$\oint_M \vec{B} \cdot d\vec{l} = \mu_0 I.$$

Path **N**, on the other hand, circulates through both positive (counterclockwise) and negative (clockwise) $d\theta$ (see Figure $15.6.1$), and since it is closed, $\oint_N d\theta = 0$. Thus for path **N**,

$$\oint_N \vec{B} \cdot d\vec{l} = 0.$$

The extension of this result to the general case is Ampère's law.

Ampere's Law

Over an arbitrary closed path,

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

where **I** is the total current passing through any open surface **S** whose perimeter is the path of integration. Only currents inside the path of integration need be considered.

To determine whether a specific current **I** is positive or negative, curl the fingers of your right hand in the direction of the path of integration, as shown in Figure $15.6.1$. If **I** passes through **S** in the same direction as your extended thumb, **I** is positive; if **I** passes through **S** in the direction opposite to your extended thumb, it is negative.

Problem-Solving Strategy: Ampère's Law

To calculate the magnetic field created from current in wire(s), use the following steps:

1. Identify the symmetry of the current in the wire(s). If there is no symmetry, use the Biot-Savart law to determine the magnetic field.
2. Determine the direction of the magnetic field created by the wire(s) by right-hand rule 2.
3. Chose a path loop where the magnetic field is either constant or zero.
4. Calculate the current inside the loop.
5. Calculate the line integral $\oint \vec{B} \cdot d\vec{l}$ around the closed loop.
6. Equate $\oint \vec{B} \cdot d\vec{l}$ with $\mu_0 I_{enc}$ with $\mu_0 I_{enc}$ and solve for $\vec{B}$.

Using Ampère's Law to Calculate the Magnetic Field Due to a Wire

Use Ampère's law to calculate the magnetic field due to a steady current **I** in an infinitely long, thin, straight wire as shown in Figure $15.6.2$.

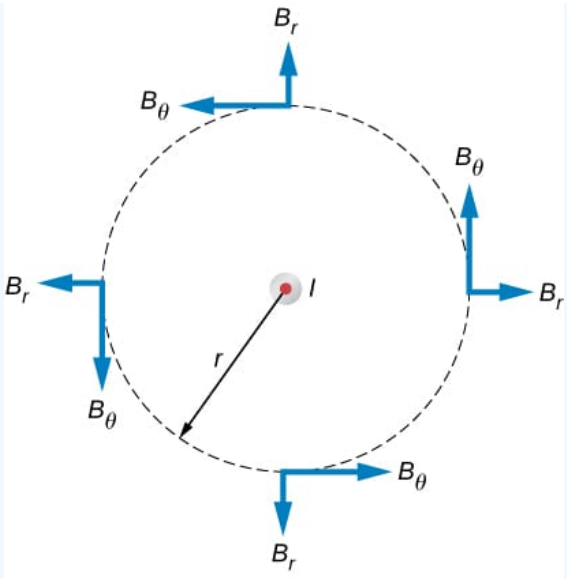


Figure 15.6.2: The possible components of the magnetic field **B** due to a current **I**, which is directed out of the page. The radial component is zero because the angle between the magnetic field and the path is at a right angle.

Strategy

Consider an arbitrary plane perpendicular to the wire, with the current directed out of the page. The possible magnetic field components in this plane, B_r and B_θ are shown at arbitrary points on a circle of radius **r** centered on the wire. Since the field is cylindrically symmetric, neither B_r nor B_θ varies with the position on this circle. Also from symmetry, the radial lines, if they exist, must be directed either all inward or all outward from the wire. This means, however, that there must be a net magnetic flux across an arbitrary cylinder concentric with the wire. The radial component of the magnetic field must be zero because $\vec{B}_r \cdot d\vec{l} = 0$. Therefore, we can apply Ampère's law to the circular path as shown.

Solution

Over this path $\vec{B}$ is constant and parallel to $d\vec{l}$, so

$$\oint \vec{B} \cdot d\vec{l} = B_\theta \oint dl = B_\theta(2\pi r).$$

Thus Ampère's law reduces to

$$B_\theta(2\pi r) = \mu_0 I.$$

Finally, since B_θ is the only component of $\vec{B}$, we can drop the subscript and write

$$B = \frac{\mu_0 I}{2\pi r}.$$

This agrees with the Biot-Savart calculation above.

Significance

Ampère's law works well if you have a path to integrate over which $\vec{B} \cdot d\vec{l}$ has results that are easy to simplify. For the infinite wire, this works easily with a path that is circular around the wire so that the magnetic field factors out of the integration. If the path dependence looks complicated, you can always go back to the Biot-Savart law and use that to find the magnetic field.

Example 15.6.2: Calculating the Magnetic Field of a Thick Wire with Ampère's Law

The radius of the long, straight wire of Figure 15.6.3 is **a**, and the wire carries a current I_0 that is distributed uniformly over its cross-section. Find the magnetic field both inside and outside the wire.

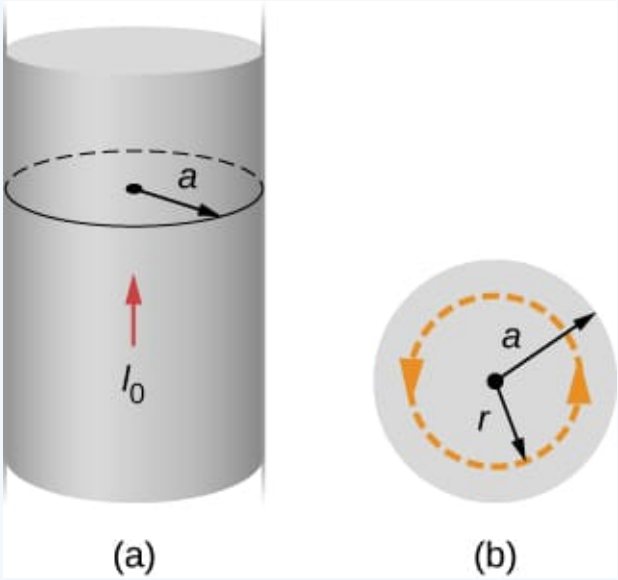


Figure 15.6.3: (a) A model of a current-carrying wire of radius **a** and current I_0. (b) A cross-section of the same wire showing the radius **a** and the Ampère's loop of radius **r**.

Strategy

This problem has the same geometry as Example 15.6.1, but the enclosed current changes as we move the integration path from outside the wire to inside the wire, where it doesn't capture the entire current enclosed (see Figure 15.6.3).

Solution

For any circular path of radius **r** that is centered on the wire,

$$\oint \vec{B} \cdot d\vec{l} = \oint Bdl = B \oint dl = B(2\pi r).$$

From Ampère's law, this equals the total current passing through any surface bounded by the path of integration.

Consider first a circular path that is inside the wire $(r \leq a)$ such as that shown in part (a) of Figure 15.6.3. We need the current **I** passing through the area enclosed by the path. It's equal to the current density **J** times the area enclosed. Since the current is uniform, the current density inside the path equals the current density in the whole wire, which is $I_0/\pi a^2$. Therefore the current **I** passing through the area enclosed by the path is

$$I = \frac{\pi r^2}{\pi a^2} I_0 = \frac{r^2}{a^2} I_0.$$

We can consider this ratio because the current density **J** is constant over the area of the wire. Therefore, the current density of a part of the wire is equal to the current density in the whole area. Using Ampère's law, we obtain

$$B(2\pi r) = \mu_0 \left(\frac{r^2}{a^2} \right) I_0,$$

and the magnetic field inside the wire is

$$B = \frac{\mu_0 I_0}{2\pi} \frac{r}{a^2} (r \leq a).$$

Outside the wire, the situation is identical to that of the infinite thin wire of the previous example; that is,

$$B = \frac{\mu_0 I_0}{2\pi r} (r \geq a).$$

The variation of **B** with **r** is shown in Figure 15.6.4.

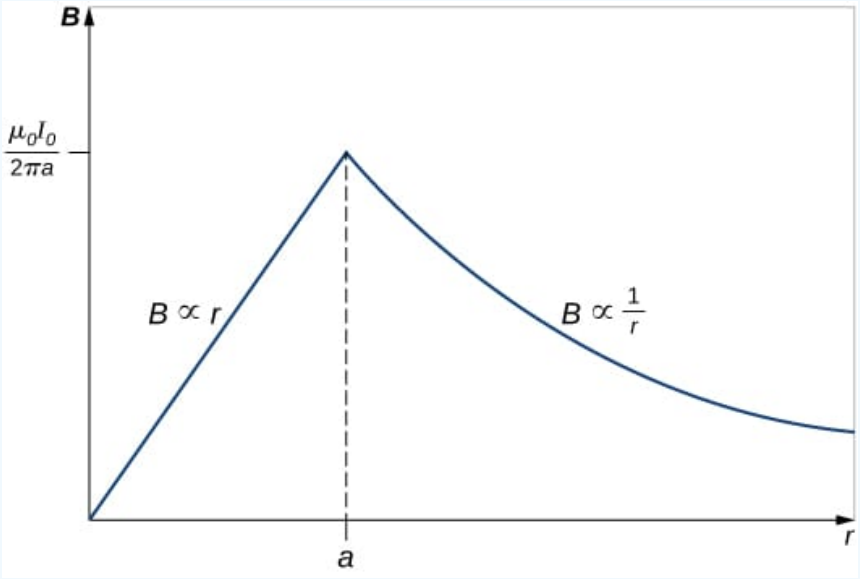


Figure 15.6.4: Variation of the magnetic field produced by a current I_0 in a long, straight wire of radius **a**.

Significance

The results show that as the radial distance increases inside the thick wire, the magnetic field increases from zero to a familiar value of the magnetic field of a thin wire. Outside the wire, the field drops off regardless of whether it was a thick or thin wire.

This result is similar to how Gauss's law for electrical charges behaves inside a uniform charge distribution, except that Gauss's law for electrical charges has a uniform volume distribution of charge, whereas Ampère's law here has a uniform area of current distribution. Also, the drop-off outside the thick wire is similar to how an electric field drops off outside of a linear charge distribution, since the two cases have the same geometry and neither case depends on the configuration of charges or currents once the loop is outside the distribution.

Using Ampère's Law with Arbitrary Paths

Use Ampère's law to evaluate $\oint \vec{B} \cdot d\vec{l}$ for the current configurations and paths in Figure 15.6.5.

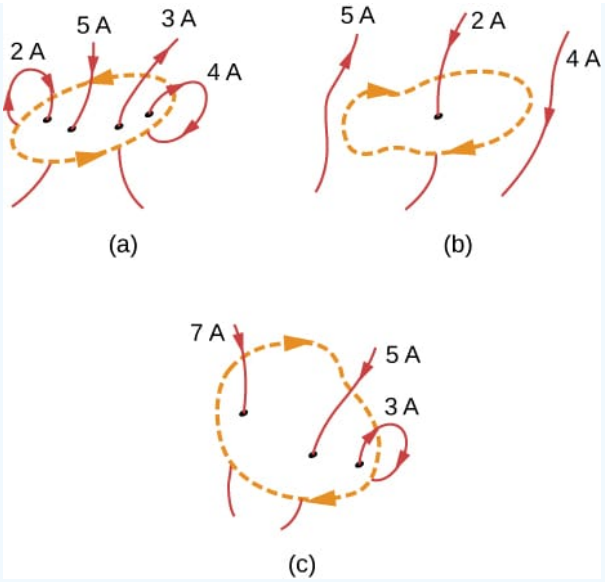


Figure 15.6.5: Current configurations and paths for Example 15.6.3.

Strategy

Ampère's law states that $\oint \vec{B} \cdot d\vec{l} = \mu_0 I$ where **I** is the total current passing through the enclosed loop. The quickest way to evaluate the integral is to calculate $\mu_0 I$ by finding the net current through the loop. Positive currents flow with your right-hand thumb if your fingers wrap around in the direction of the loop. This will tell us the sign of the answer.

Solution

(a) The current going downward through the loop equals the current going out of the loop, so the net current is zero. Thus, $\oint \vec{B} \cdot d\vec{l} = 0$.

(b) The only current to consider in this problem is 2A because it is the only current inside the loop. The right-hand rule shows us the current going downward through the loop is in the positive direction. Therefore, the answer is $\oint \vec{B} \cdot d\vec{l} = \mu_0(2\ A) = 2.51 \times 10^{-6} T \cdot m$.

(c) The right-hand rule shows us the current going downward through the loop is in the positive direction. There are $7A + 5A = 12A$ of current going downward and –3 A going upward. Therefore, the total current is 9 A and $\oint \vec{B} \cdot d\vec{l} = \mu_0(9\ A) = 5.65 \times 10^{-6} T \cdot m$.

Significance

If the currents all wrapped around so that the same current went into the loop and out of the loop, the net current would be zero and no magnetic field would be present. This is why wires are very close to each other in an electrical cord. The currents flowing toward a device and away from a device in a wire equal zero total current flow through an Ampère loop around these wires. Therefore, no stray magnetic fields can be present from cords carrying current.

Exercise 15.6.1

Consider using Ampère's law to calculate the magnetic fields of a finite straight wire and of a circular loop of wire. Why is it not useful for these calculations?

Answer available at the end of this section.

Answers

Exercise 15.6.1

In these cases the integrals around the Ampèrian loop are very difficult because there is no symmetry, so this method would not be useful.

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15.7: Solenoids and Toroids

Learning Objectives

By the end of this section, you will be able to:

- Establish a relationship for how the magnetic field of a solenoid varies with distance and current by using both the Biot-Savart law and Ampère's law
- Establish a relationship for how the magnetic field of a toroid varies with distance and current by using Ampère's law

Two of the most common and useful electromagnetic devices are called solenoids and toroids. In one form or another, they are part of numerous instruments, both large and small. In this section, we examine the magnetic field typical of these devices.

Solenoids

A long wire wound in the form of a helical coil is known as a **solenoid**. Solenoids are commonly used in experimental research requiring magnetic fields. A solenoid is generally easy to wind, and near its center, its magnetic field is quite uniform and directly proportional to the current in the wire.

Figure 15.7.1 shows a solenoid consisting of **N** turns of wire tightly wound over a length **L**. A current **I** is flowing along the wire of the solenoid. The number of turns per unit length is **N/L**; therefore, the number of turns in an infinitesimal length **dy** are (**N/L**)**dy** turns. This produces a current

$$dI = \frac{NI}{L}dy. \tag{15.7.1}$$

We first calculate the magnetic field at the point **P** of Figure 15.7.1. This point is on the central axis of the solenoid. We are basically cutting the solenoid into thin slices that are **dy** thick and treating each as a current loop. Thus, **dI** is the current through each slice. The magnetic field $d\vec{B}$ due to the current **dI** in **dy** can be found with the help of Equation 12.5.3 and Equation 15.7.1:

$$d\vec{B} = \frac{\mu_0 R^2 dI}{2(y^2+R^2)^{3/2}}\hat{j} = \left(\frac{\mu_0 I R^2 N}{2L}\hat{j}\right)\frac{dy}{(y^2+R^2)^{3/2}} \tag{15.7.2}$$

where we used Equation 15.7.1 to replace **dI**. The resultant field at **P** is found by integrating $d\vec{B}$ along the entire length of the solenoid. It's easiest to evaluate this integral by changing the independent variable from **y** to θ. From inspection of Figure 15.7.1, we have:

$$sin\,\theta = \frac{y}{\sqrt{y^2+R^2}}. \tag{15.7.3}$$

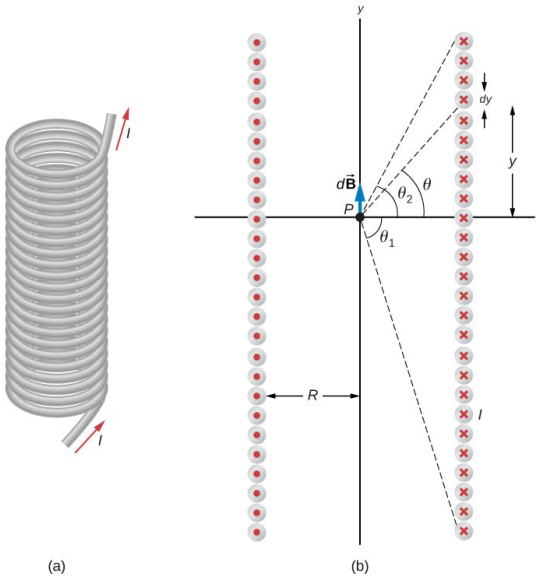


Figure 15.7.1: (a) A solenoid is a long wire wound in the shape of a helix. (b) The magnetic field at the point **P** on the axis of the solenoid is the net field due to all of the current loops.

Taking the differential of both sides of this equation, we obtain

$$cos\,\theta\,d\theta = \left[-\frac{y^2}{(y^2+R^2)^{3/2}} + \frac{1}{\sqrt{y^2+R^2}}\right] dy$$

$$= \frac{R^2 dy}{(y^2+R^2)^{3/2}}.$$

When this is substituted into the equation for $d\vec{B}$, we have

$$\vec{B} = \frac{\mu_0 IN}{2L}\hat{j}\int_{\theta_1}^{\theta_2} cos\,\theta\,d\theta = \frac{\mu_0 IN}{2L}(sin\,\theta_2 - sin\,\theta_1)\hat{j}, \tag{15.7.4}$$

which is the magnetic field along the central axis of a finite solenoid.

Of special interest is the infinitely long solenoid, for which $L \to \infty$. From a practical point of view, the infinite solenoid is one whose length is much larger than its radius ($L >> R$). In this case, $\theta_1 = \frac{-\pi}{2}$ and $\theta_2 = \frac{\pi}{2}$. Then from Equation 15.7.4, the magnetic field along the central axis of an infinite solenoid is

$$\vec{B} = \frac{\mu_0 IN}{2L}\hat{j}[sin(\pi/2) - sin(-\pi/2)] = \frac{\mu_0 IN}{L}\hat{j}$$

or

$$\vec{B} = \mu_0 nI\hat{j}, \tag{15.7.5}$$

where **n** is the number of turns per unit length. You can find the direction of $\vec{B}$ with a right-hand rule: Curl your fingers in the direction of the current, and your thumb points along the magnetic field in the interior of the solenoid.

We now use these properties, along with Ampère's law, to calculate the magnitude of the magnetic field at any location inside the infinite solenoid. Consider the closed path of Figure 15.7.2. Along segment 1, $\vec{B}$ is uniform and parallel to the path. Along segments 2 and 4, $\vec{B}$ is perpendicular to part of the path and vanishes over the rest of it. Therefore, segments 2 and 4 do not contribute to the line integral in Ampère's law. Along segment 3, $\vec{B} = 0$ because the magnetic field is zero outside the solenoid. If you consider an Ampère's law loop outside of the solenoid, the current flows in opposite directions on different segments of wire. Therefore, there is no enclosed current and no magnetic field according to Ampère's law. Thus, there is no contribution to the line integral from segment 3. As a result, we find

$$\oint \vec{B} \cdot d\vec{l} = \int_1 \vec{B} \cdot d\vec{l} = Bl. \tag{15.7.6}$$

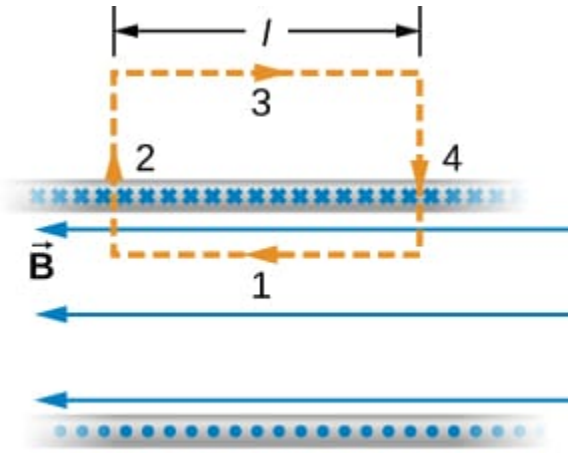


Figure 15.7.2: The path of integration used in Ampère's law to evaluate the magnetic field of an infinite solenoid.

The solenoid has **n** turns per unit length, so the current that passes through the surface enclosed by the path is **nlI**. Therefore, from Ampère's law,

$$Bl = \mu_0 nlI$$

and

Note

$$B = \mu_0 nI \tag{15.7.7}$$

within the solenoid. This agrees with what we found earlier for **B** on the central axis of the solenoid. Here, however, the location of segment 1 is arbitrary, so we have found that this equation gives the magnetic field everywhere inside the infinite solenoid.

Outside the solenoid, one can draw an Ampère's law loop around the entire solenoid. This would enclose current flowing in both directions. Therefore, the net current inside the loop is zero. According to Ampère's law, if the net current is zero, the magnetic field must be zero. Therefore, for locations outside of the solenoid's radius, the magnetic field is zero.

When a patient undergoes a **magnetic resonance imaging** (MRI) scan, the person lies down on a table that is moved into the center of a large solenoid that can generate very large magnetic fields. The solenoid is capable of these high fields from high currents flowing through superconducting wires. The large magnetic field is used to change the spin of protons in the patient's body. The time it takes for the spins to align or relax (return to original orientation) is a signature of different tissues that can be analyzed to see if the structures of the tissues is normal (Figure 15.7.3).



Figure 15.7.3: . In an MRI machine, a large magnetic field is generated by the cylindrical solenoid surrounding the patient. (credit: Liz West)

Example 15.7.1: Magnetic Field Inside a SOlenoid

A solenoid has 300 turns wound around a cylinder of diameter 1.20 cm and length 14.0 cm. If the current through the coils is 0.410 A, what is the magnitude of the magnetic field inside and near the middle of the solenoid?

Strategy

We are given the number of turns and the length of the solenoid so we can find the number of turns per unit length. Therefore, the magnetic field inside and near the middle of the solenoid is given by Equation 15.7.7. Outside the solenoid, the magnetic field is zero.

Solution

The number of turns per unit length is

$$n = \frac{300\,turns}{0.140\,m} = 2.14 \times 10^3\,turns/m.$$

The magnetic field produced inside the solenoid is

$$B = \mu_0 nI = (4\pi \times 10^{-7} T \cdot m/A)(2.14 \times 10^3 turns/m)(0.410\,A)$$

$$B = 1.10 \times 10^{-3} T.$$

Significance

This solution is valid only if the length of the solenoid is reasonably large compared with its diameter. This example is a case where this is valid.

Exercise 15.7.1

What is the ratio of the magnetic field produced from using a finite formula over the infinite approximation for an angle θ of (a) 85°? (b) 89°? The solenoid has 1000 turns in 50 cm with a current of 1.0 A flowing through the coils

Solution available at the end of this section.

Toroids

A toroid is a donut-shaped coil closely wound with one continuous wire, as illustrated in part (a) of Figure 15.7.4. If the toroid has **N** windings and the current in the wire is **I**, what is the magnetic field both inside and outside the toroid?

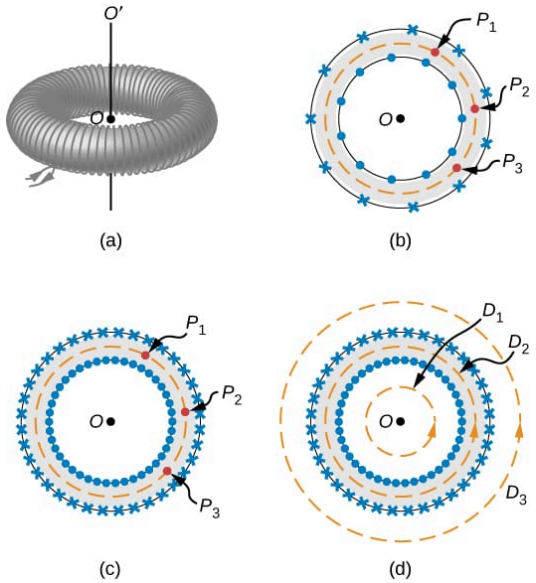


Figure 15.7.4: (a) A toroid is a coil wound into a donut-shaped object. (b) A loosely wound toroid does not have cylindrical symmetry. (c) In a tightly wound toroid, cylindrical symmetry is a very good approximation. (d) Several paths of integration for Ampère's law.

We begin by assuming cylindrical symmetry around the axis **OO'**. Actually, this assumption is not precisely correct, for as part (b) of Figure 15.7.4 shows, the view of the toroidal coil varies from point to point (for example, $P_1,\ P_2$ and P_3) on a circular path centered around **OO'**. However, if the toroid is tightly wound, all points on the circle become essentially equivalent [part (c) of Figure 15.7.4], and cylindrical symmetry is an accurate approximation.

With this symmetry, the magnetic field must be tangent to and constant in magnitude along any circular path centered on **OO'**. This allows us to write for each of the paths $D_1,\ D_2$ and D_3 shown in part (d) of Figure 15.7.4,

$$\oint \vec{B} \cdot d\vec{l} = B(2\pi r). \tag{15.7.8}$$

Ampère's law relates this integral to the net current passing through any surface bounded by the path of integration. For a path that is external to the toroid, either no current passes through the enclosing surface (path D_1), or the current passing through the surface in one direction is exactly balanced by the current passing through it in the opposite direction (path D_3). In either case, there is no net current passing through the surface, so

$$\oint B(2\pi r) = 0$$

and

$$B = 0\,(outside\,the\,toroid). \tag{15.7.9}$$

The turns of a toroid form a helix, rather than circular loops. As a result, there is a small field external to the coil; however, the derivation above holds if the coils were circular.

For a circular path within the toroid (path D_2), the current in the wire cuts the surface **N** times, resulting in a net current **NI** through the surface. We now find with Ampère's law,

$$B(2\pi r) = \mu_0 NI$$

and

Note

$$B = \frac{\mu_0 NI}{2\pi r} \, (within \, the \, toroid). \tag{15.7.10}$$

The magnetic field is directed in the counterclockwise direction for the windings shown. When the current in the coils is reversed, the direction of the magnetic field also reverses.

The magnetic field inside a toroid is not uniform, as it varies inversely with the distance **r** from the axis **OO'**. However, if the central radius **R** (the radius midway between the inner and outer radii of the toroid) is much larger than the cross-sectional diameter of the coils **r**, the variation is fairly small, and the magnitude of the magnetic field may be calculated by Equation 15.7.10 where $r = R$.

Answers

Exercise 15.7.1

a. 1.00382; b. 1.00015

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15.8: Magnetism in Matter

Learning Objectives

By the end of this section, you will be able to:

- Classify magnetic materials as paramagnetic, diamagnetic, or ferromagnetic, based on their response to a magnetic field
- Sketch how magnetic dipoles align with the magnetic field in each type of substance
- Define hysteresis and magnetic susceptibility, which determines the type of magnetic material

Why are certain materials magnetic and others not? And why do certain substances become magnetized by a field, whereas others are unaffected? To answer such questions, we need an understanding of magnetism on a microscopic level.

Within an atom, every electron travels in an orbit and spins on an internal axis. Both types of motion produce current loops and therefore magnetic dipoles. For a particular atom, the net magnetic dipole moment is the vector sum of the magnetic dipole moments. Values of μ for several types of atoms are given in Table $15.8.1$. Notice that some atoms have a zero net dipole moment and that the magnitudes of the nonvanishing moments are typically $10^{23}\ A \cdot m^2$.

Table $15.8.1$*: Magnetic Moments of Some Atoms*

Atom	Magnetic Moment $(10^{-24}\ A \cdot m^2)$
H	9.27
He	0
Li	9.27
O	13.9
Na	9.27
S	13.9

A handful of matter has approximately 10^{26} atoms and ions, each with its magnetic dipole moment. If no external magnetic field is present, the magnetic dipoles are randomly oriented—as many are pointed up as down, as many are pointed east as west, and so on. Consequently, the net magnetic dipole moment of the sample is zero. However, if the sample is placed in a magnetic field, these dipoles tend to align with the field, and this alignment determines how the sample responds to the field. On the basis of this response, a material is said to be either paramagnetic, ferromagnetic, or diamagnetic.

In a **paramagnetic material**, only a small fraction (roughly one-third) of the magnetic dipoles are aligned with the applied field. Since each dipole produces its own magnetic field, this alignment contributes an extra magnetic field, which enhances the applied field. When a **ferromagnetic material** is placed in a magnetic field, its magnetic dipoles also become aligned; furthermore, they become locked together so that a permanent magnetization results, even when the field is turned off or reversed. This permanent magnetization happens in ferromagnetic materials but not paramagnetic materials. **Diamagnetic materials** are composed of atoms that have no net magnetic dipole moment. However, when a diamagnetic material is placed in a magnetic field, a magnetic dipole moment is directed opposite to the applied field and therefore produces a magnetic field that opposes the applied field. We now consider each type of material in greater detail.

Paramagnetic Materials

For simplicity, we assume our sample is a long, cylindrical piece that completely fills the interior of a long, tightly wound solenoid. When there is no current in the solenoid, the magnetic dipoles in the sample are randomly oriented and produce no net magnetic field. With a solenoid current, the magnetic field due to the solenoid exerts a torque on the dipoles that tends to align them with the field. In competition with the aligning torque are thermal collisions that tend to randomize the orientations of the dipoles. The relative importance of these two competing processes can be estimated by comparing the energies involved. The energy

difference between a magnetic dipole aligned with and against a magnetic field is $U_B = 2\mu B$. If $\mu = 9.3 \times 10^{-24} A \cdot m^2$ (the value of atomic hydrogen) and **B** = 1.0 T, then

$$U_B = 1.9 \times 10^{-23} J.$$

At a room temperature of $27^o C$ the thermal energy per atom is

$$U_T \approx kT = (1.38 \times 10^{-23} J/K)(300\, K) = 4.1 \times 10^{-21} J,$$

which is about 220 times greater than U_B. Clearly, energy exchanges in thermal collisions can seriously interfere with the alignment of the magnetic dipoles. As a result, only a small fraction of the dipoles is aligned at any instant.

The four sketches of Figure 15.8.1 furnish a simple model of this alignment process. In part (a), before the field of the solenoid (not shown) containing the paramagnetic sample is applied, the magnetic dipoles are randomly oriented and there is no net magnetic dipole moment associated with the material. With the introduction of the field, a partial alignment of the dipoles takes place, as depicted in part (b). The component of the net magnetic dipole moment that is perpendicular to the field vanishes. We may then represent the sample by part (c), which shows a collection of magnetic dipoles completely aligned with the field. By treating these dipoles as current loops, we can picture the dipole alignment as equivalent to a current around the surface of the material, as in part (d). This fictitious surface current produces its own magnetic field, which enhances the field of the solenoid.

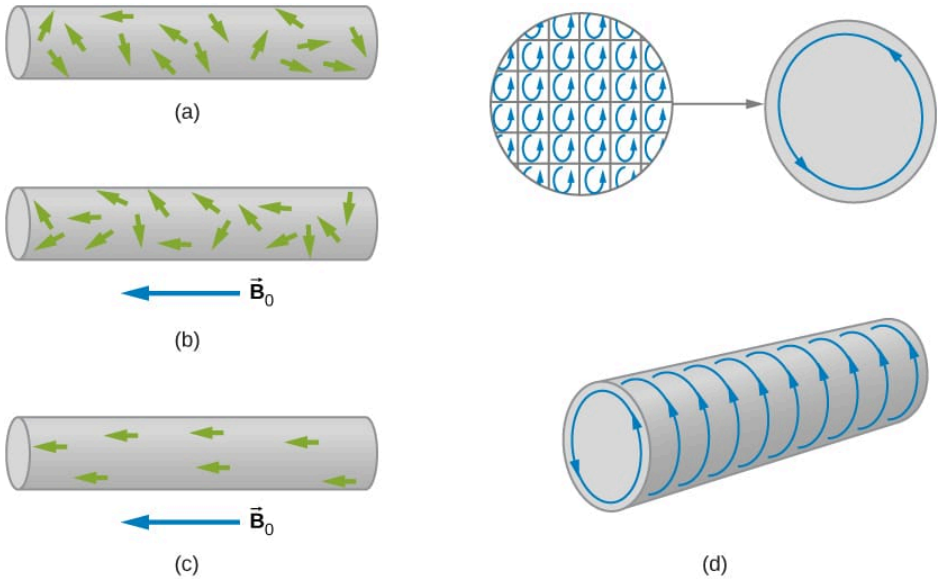


Figure 15.8.1: The alignment process in a paramagnetic material filling a solenoid (not shown). (a) Without an applied field, the magnetic dipoles are randomly oriented. (b) With a field, partial alignment occurs. (c) An equivalent representation of part (b). (d) The internal currents cancel, leaving an effective surface current that produces a magnetic field similar to that of a finite solenoid.

We can express the total magnetic field $\vec{B}$ in the material as

$$\vec{B} = \vec{B}_0 + \vec{B}_m, \tag{15.8.1}$$

where $\vec{B}_0$ is the field due to the current I_0 in the solenoid and $\vec{B}_m$ is the field due to the surface current I_m around the sample. Now $\vec{B}_m$ is usually proportional to $\vec{B}_0$ a fact we express by

$$\vec{B}_m = \chi \vec{B}_0, \tag{15.8.2}$$

where χ is a dimensionless quantity called the magnetic susceptibility. Values of χ for some paramagnetic materials are given in Table 15.8.2. Since the alignment of magnetic dipoles is so weak, χ is very small for paramagnetic materials. By combining Equation 15.8.1 and Equation 15.8.2, we obtain:

$$\vec{B} = \vec{B}_0 + \chi \vec{B}_0 = (1 + \chi)\vec{B}_0. \tag{15.8.3}$$

For a sample within an infinite solenoid, this becomes

$$B = (1 + \chi)\mu_0 nI. \tag{15.8.4}$$

This expression tells us that the insertion of a paramagnetic material into a solenoid increases the field by a factor of $(1 + \chi)$. However, since χ is so small, the field isn't enhanced very much.

The quantity

$$\mu = (1 + \chi)\mu_0. \tag{15.8.5}$$

is called the **magnetic permeability of a material**. In terms of μ, Equation 15.8.4 can be written as

$$B = \mu nI \tag{15.8.6}$$

for the filled solenoid.

*Table 15.8.2: Magnetic Susceptibilities*Note: Unless otherwise specified, values given are for room temperature.*

Paramagnetic Materials	χ	Diamagnetic Materials	χ
Aluminum	2.2×10^{-5}	Bismuth	-1.7×10^{-5}
Calcium	1.4×10^{-5}	Carbon (diamond)	-2.2×10^{-5}
Chromium	3.1×10^{-4}	Copper	-9.7×10^{-6}
Magnesium	1.2×10^{-5}	Lead	-1.8×10^{-5}
Oxygen gas (1 atm)	1.8×10^{-6}	Mercury	-2.8×10^{-5}
Oxygen liquid (90 K)	3.5×10^{-3}	Hydrogen gas (1 atm)	-2.2×10^{-9}
Tungsten	6.8×10^{-5}	Nitrogen gas (1 atm)	-6.7×10^{-9}
Air (1 atm)	3.6×10^{-7}	Water	-9.1×10^{-6}

Diamagnetic Materials

A magnetic field always induces a magnetic dipole in an atom. This induced dipole points opposite to the applied field, so its magnetic field is also directed opposite to the applied field. In paramagnetic and ferromagnetic materials, the induced magnetic dipole is masked by much stronger permanent magnetic dipoles of the atoms. However, in diamagnetic materials, whose atoms have no permanent magnetic dipole moments, the effect of the induced dipole is observable.

We can now describe the magnetic effects of diamagnetic materials with the same model developed for paramagnetic materials. In this case, however, the fictitious surface current flows opposite to the solenoid current, and the magnetic susceptibility χ is negative. Values of χ for some diamagnetic materials are also given in Table 15.8.2.

> Water is a common diamagnetic material. Animals are mostly composed of water. Experiments have been performed on frogs and mice in diverging magnetic fields. The water molecules are repelled from the applied magnetic field against gravity until the animal reaches an equilibrium. The result is that the animal is levitated by the magnetic field.

Ferromagnetic Materials

Common magnets are made of a ferromagnetic material such as iron or one of its alloys. Experiments reveal that a ferromagnetic material consists of tiny regions known as **magnetic domains**. Their volumes typically range from 10^{-12} to $10^{-8}m^3$, and they contain about 10^{17} to 10^{21} atoms. Within a domain, the magnetic dipoles are rigidly aligned in the same direction by coupling among the atoms. This coupling, which is due to quantum mechanical effects, is so strong that even thermal agitation at room temperature cannot break it. The result is that each domain has a net dipole moment. Some materials have weaker coupling and are ferromagnetic only at lower temperatures.

If the domains in a ferromagnetic sample are randomly oriented, as shown in Figure $15.8.1a$, the sample has no net magnetic dipole moment and is said to be unmagnetized. Suppose that we fill the volume of a solenoid with an unmagnetized ferromagnetic sample. When the magnetic field $\vec{B}_0$ of the solenoid is turned on, the dipole moments of the domains rotate so that they align somewhat with the field, as depicted in Figure $15.8.1b$. In addition, the aligned domains tend to increase in size at the expense of unaligned ones. The net effect of these two processes is the creation of a net magnetic dipole moment for the ferromagnet that is directed along the applied magnetic field. This net magnetic dipole moment is much larger than that of a paramagnetic sample, and the domains, with their large numbers of atoms, do not become misaligned by thermal agitation. Consequently, the field due to the alignment of the domains is quite large.

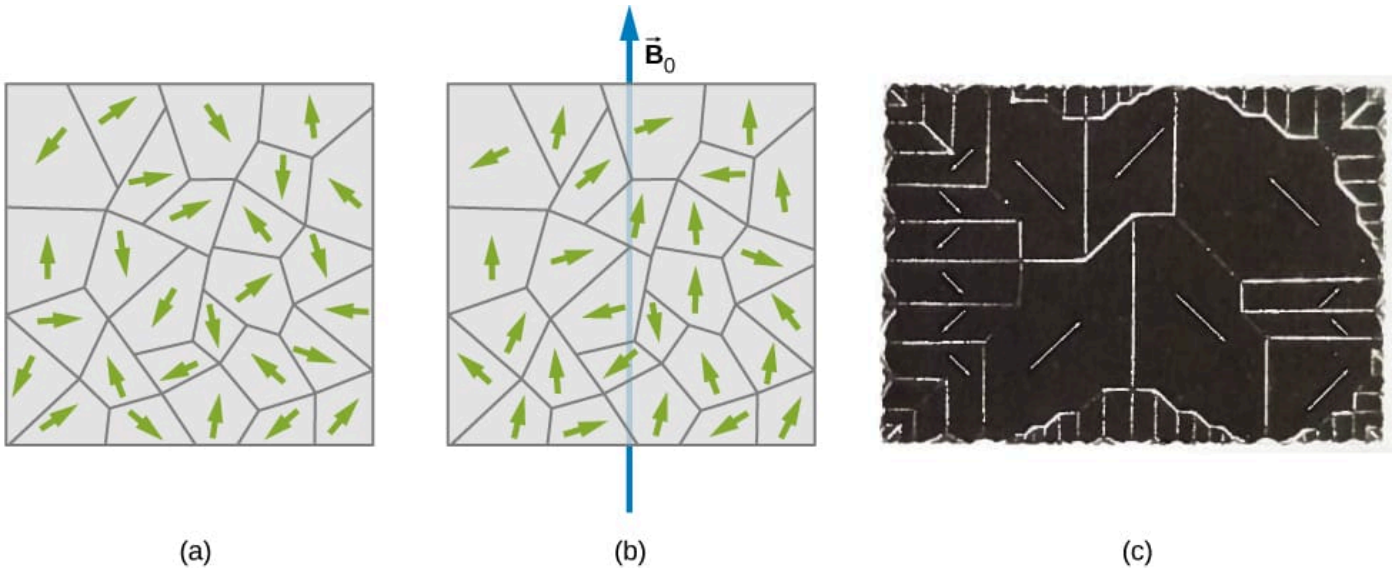


Figure $15.8.2$: (a) Domains are randomly oriented in an unmagnetized ferromagnetic sample such as iron. The arrows represent the orientations of the magnetic dipoles within the domains. (b) In an applied magnetic field, the domains align somewhat with the field. (c) The domains of a single crystal of nickel. The white lines show the boundaries of the domains. These lines are produced by iron oxide powder sprinkled on the crystal.

Besides iron, only four elements contain the magnetic domains needed to exhibit ferromagnetic behavior: cobalt, nickel, gadolinium, and dysprosium. Many alloys of these elements are also ferromagnetic. Ferromagnetic materials can be described using Equation $15.8.3$ through Equation $15.8.6$, the paramagnetic equations. However, the value of χ for ferromagnetic material is usually on the order of 10^3 to 10^4, and it also depends on the history of the magnetic field to which the material has been subject. A typical plot of **B** (the total field in the material) versus B_0 (the applied field) for an initially unmagnetized piece of iron is shown in Figure $15.8.2c$. Some sample numbers are (1) for $B_0 = 1.0 \times 10^{-4} T$, $B = 0.60\ T$, and $\chi = ({}^{0.60}/_{1.0\times10^{-4}}) - 1 \approx 6.0 \times 10^3$; for (2) for $B_0 = 6.0 \times 10^{-4} T$, $B = 1.5\ T$, and $\chi = ({}^{1.5}/_{6.0\times10^{-4}}) - 1 \approx 2.5 \times 10^3$.

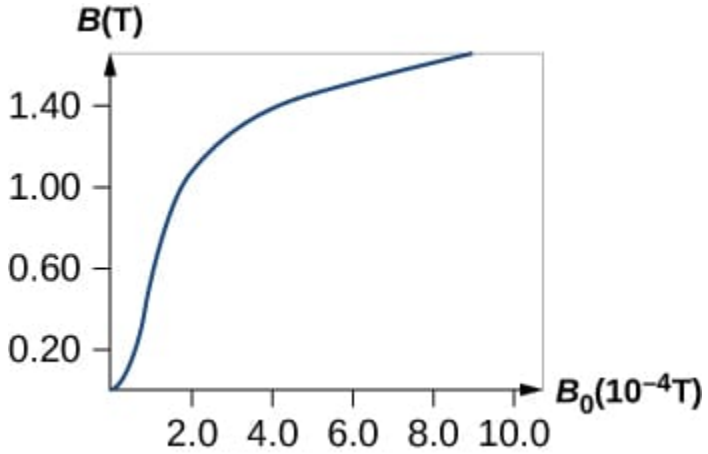


Figure $15.8.3$: (a) The magnetic field B in annealed iron as a function of the applied field B_0.

When B_0 is varied over a range of positive and negative values, **B** is found to behave as shown in Figure $15.8.3$. Note that the same

B_0 (corresponding to the same current in the solenoid) can produce different values of **B** in the material. The magnetic field **B** produced in a ferromagnetic material by an applied field B_0 depends on the magnetic history of the material. This effect is called **hysteresis**, and the curve of Figure $15.8.4$ is called a hysteresis loop. Notice that **B** does not disappear when $B_0 = 0$ (i.e., when the current in the solenoid is turned off). The iron stays magnetized, which means that it has become a permanent magnet.

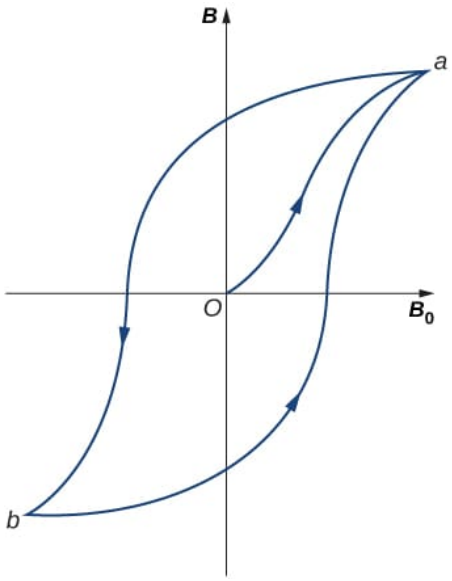


Figure $15.8.4$: A typical hysteresis loop for a ferromagnet. When the material is first magnetized, it follows a curve from 0 to **a**. When B_0 is reversed, it takes the path shown from **a** to **b**. If B_0 is reversed again, the material follows the curve from **b** to **a**.

Like the paramagnetic sample of Figure $15.8.2$, the partial alignment of the domains in a ferromagnet is equivalent to a current flowing around the surface. A bar magnet can therefore be pictured as a tightly wound solenoid with a large current circulating through its coils (the surface current). You can see in Figure $15.8.5$ that this model fits quite well. The fields of the bar magnet and the finite solenoid are strikingly similar. The figure also shows how the poles of the bar magnet are identified. To form closed loops, the field lines outside the magnet leave the north (N) pole and enter the south (S) pole, whereas inside the magnet, they leave S and enter N.

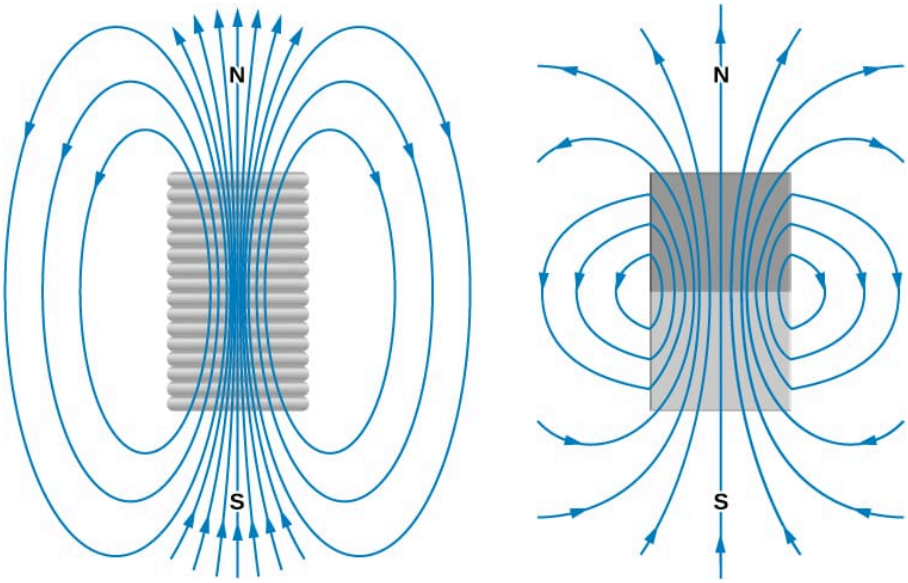


Figure $15.8.5$: Comparison of the magnetic fields of a finite solenoid and a bar magnet.

Ferromagnetic materials are found in computer hard disk drives and permanent data storage devices (Figure $15.8.6$). A material used in your hard disk drives is called a spin valve, which has alternating layers of ferromagnetic (aligning with the external magnetic field) and antiferromagnetic (each atom is aligned opposite to the next) metals. It was observed that a significant change in resistance was discovered based on whether an applied magnetic field was on the spin valve or not. This large change

in resistance creates a quick and consistent way for recording or reading information by an applied current.



Figure 15.8.6: The inside of a hard disk drive. The silver disk contains the information, whereas the thin stylus on top of the disk reads and writes information to the disk.

Example 15.8.1: Iron Core in a Coil

A long coil is tightly wound around an iron cylinder whose magnetization curve is shown in Figure 15.8.3. (a) If $n = 20$ turns per centimeter, what is the applied field B_0 when $I_0 = 0.20\ A$? (b) What is the net magnetic field for this same current? (c) What is the magnetic susceptibility in this case?

Strategy

(a) The magnetic field of a solenoid is calculated using $\vec{B} = \mu_0 n I \hat{j}$. (b) The graph is read to determine the net magnetic field for this same current. (c) The magnetic susceptibility is calculated using Equation 15.8.4 .

Solution

1. The applied field B_0 of the coil is
$$B_0 = \mu_0 n I_0 = (4\pi \times 10^{-7} T \cdot m/A)(2000/m)(0.20\ A)$$
$$B_0 = 5.0 \times 10^{-4} T.$$
2. From inspection of the magnetization curve of Figure 15.8.3, we see that, for this value of B_0, $B = 1.4\ T$. Notice that the internal field of the aligned atoms is much larger than the externally applied field.
3. The magnetic susceptibility is calculated to be
$$\chi = \frac{B}{B_0} - 1 = \frac{1.4\,T}{5.0 \times 10^{-4} T} - 1 = 2.8 \times 10^3.$$

Significance

Ferromagnetic materials have susceptibilities in the range of 10^3 which compares well to our results here. Paramagnetic materials have fractional susceptibilities, so their applied field of the coil is much greater than the magnetic field generated by the material.

Exercise 15.8.1

Repeat the calculations from the previous example for $I_0 = 0.040\ A$.

Answer available at the end of this section.

Answers

Exercise 15.8.1

a. $1.0 \times 10^{-4} T$; b. 0.60 T; c. 6.0×10^{3}

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15.9: Sources of Magnetic Fields (Summary)

Key Terms

Ampère's law	physical law that states that the line integral of the magnetic field around an electric current is proportional to the current
Biot-Savart law	an equation giving the magnetic field at a point produced by a current-carrying wire
diamagnetic materials	their magnetic dipoles align oppositely to an applied magnetic field; when the field is removed, the material is unmagnetized
ferromagnetic materials	contain groups of dipoles, called domains, that align with the applied magnetic field; when this field is removed, the material is still magnetized
hysteresis	property of ferromagnets that is seen when a material's magnetic field is examined versus the applied magnetic field; a loop is created resulting from sweeping the applied field forward and reverse
magnetic domains	groups of magnetic dipoles that are all aligned in the same direction and are coupled together quantum mechanically
magnetic susceptibility	ratio of the magnetic field in the material over the applied field at that time; positive susceptibilities are either paramagnetic or ferromagnetic (aligned with the field) and negative susceptibilities are diamagnetic (aligned oppositely with the field)
paramagnetic materials	their magnetic dipoles align partially in the same direction as the applied magnetic field; when this field is removed, the material is unmagnetized
permeability of free space	μ_0, measure of the ability of a material, in this case free space, to support a magnetic field
solenoid	thin wire wound into a coil that produces a magnetic field when an electric current is passed through it
toroid	donut-shaped coil closely wound around that is one continuous wire

Key Equations

Permeability of free space	$\mu_0 = 4\pi \times 10^{-7} T \cdot m/A$
Contribution to magnetic field from a current element	$dB = \frac{\mu_0}{4\pi}\frac{Idlsin\theta}{r^2}$
Biot–Savart law	$\vec{B} = \frac{\mu_0}{4\pi}\int_{wire}\frac{Id\vec{l}\times\hat{r}}{r^2}$
Magnetic field due to a long straight wire	$B = \frac{\mu_0 I}{2\pi R}$
Force between two parallel currents	$\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}$
Magnetic field of a current loop	$B = \frac{\mu_0 I}{2R}$ (at center of loop)
Ampère's law	$\oint \vec{B}\cdot d\vec{l} = \mu_0 I$
Magnetic field strength inside a solenoid	$B = \mu_0 nI$
Magnetic field strength inside a toroid	$B = \frac{\mu_o NI}{2\pi r}$
Magnetic permeability	$\mu = (1+\chi)\mu_0$

Magnetic field of a solenoid filled with paramagnetic material	$B = \mu nI$

Summary

12.2 The Biot-Savart Law

- The magnetic field created by a current-carrying wire is found by the Biot-Savart law.
- The current element $Id\vec{l}$ produces a magnetic field a distance r away.

12.3 Magnetic Field Due to a Thin Straight Wire

- The strength of the magnetic field created by current in a long straight wire is given by $B = \frac{\mu_0 I}{2\pi R}$ (long straight wire) where **I** is the current, **R** is the shortest distance to the wire, and the constant $\mu_0 = 4\pi \times 10^{-7} T \cdot m/s$ is the permeability of free space.
- The direction of the magnetic field created by a long straight wire is given by right-hand rule 2 (RHR-2): Point the thumb of the right hand in the direction of current, and the fingers curl in the direction of the magnetic field loops created by it.

12.4 Magnetic Force between Two Parallel Currents

- The force between two parallel currents I_1 and I_2, separated by a distance r, has a magnitude per unit length given by $\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}$.
- The force is attractive if the currents are in the same direction, repulsive if they are in opposite directions.

12.5 Magnetic Field of a Current Loop

- The magnetic field strength at the center of a circular loop is given by $B = \frac{\mu_0 I}{2R}$ **(at center of loop),** where R is the radius of the loop. RHR-2 gives the direction of the field about the loop.

12.6 Ampère's Law

- The magnetic field created by current following any path is the sum (or integral) of the fields due to segments along the path (magnitude and direction as for a straight wire), resulting in a general relationship between current and field known as Ampère's law.
- Ampère's law can be used to determine the magnetic field from a thin wire or thick wire by a geometrically convenient path of integration. The results are consistent with the Biot-Savart law.

12.7 Solenoids and Toroids

- The magnetic field strength inside a solenoid is

  $$B = \mu_0 nI \text{ (inside a solenoid)}$$

  where **n** is the number of loops per unit length of the solenoid. The field inside is very uniform in magnitude and direction.

- The magnetic field strength inside a toroid is

  $$B = \frac{\mu_o NI}{2\pi r} \text{ (within the toroid)}$$

  where **N** is the number of windings. The field inside a toroid is not uniform and varies with the distance as **1/r.**

12.8 Magnetism in Matter

- Materials are classified as paramagnetic, diamagnetic, or ferromagnetic, depending on how they behave in an applied magnetic field.
- Paramagnetic materials have partial alignment of their magnetic dipoles with an applied magnetic field. This is a positive magnetic susceptibility. Only a surface current remains, creating a solenoid-like magnetic field.
- Diamagnetic materials exhibit induced dipoles opposite to an applied magnetic field. This is a negative magnetic susceptibility.
- Ferromagnetic materials have groups of dipoles, called domains, which align with the applied magnetic field. However, when the field is removed, the ferromagnetic material remains magnetized, unlike paramagnetic materials. This magnetization of the material versus the applied field effect is called hysteresis.

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15.10: Sources of Magnetic Fields (Exercise)

Conceptual Questions

12.2 The Biot-Savart Law

1. For calculating magnetic fields, what are the advantages and disadvantages of the Biot-Savart law?

2. Describe the magnetic field due to the current in two wires connected to the two terminals of a source of emf and twisted tightly around each other.

3. How can you decide if a wire is infinite?

4. Identical currents are carried in two circular loops; however, one loop has twice the diameter as the other loop. Compare the magnetic fields created by the loops at the center of each loop.

12.3 Magnetic Field Due to a Thin Straight Wire

5. How would you orient two long, straight, current-carrying wires so that there is no net magnetic force between them? (**Hint:** What orientation would lead to one wire not experiencing a magnetic field from the other?)

12.4 Magnetic Force between Two Parallel Currents

6. Compare and contrast the electric field of an infinite line of charge and the magnetic field of an infinite line of current.

7. Is $\vec{B}$ constant in magnitude for points that lie on a magnetic field line?

12.5 Magnetic Field of a Current Loop

8. Is the magnetic field of a current loop uniform?

9. What happens to the length of a suspended spring when a current passes through it?

10. Two concentric circular wires with different diameters carry currents in the same direction. Describe the force on the inner wire.

12.6 Ampère's Law

11. Is Ampère's law valid for all closed paths? Why isn't it normally useful for calculating a magnetic field?

12.7 Solenoids and Toroids

12. Is the magnetic field inside a toroid completely uniform? Almost uniform?

13. Explain why $\vec{B} = 0$ inside a long, hollow copper pipe that is carrying an electric current parallel to the axis. Is $\vec{B} = 0$ outside the pipe?

12.8 Magnetism in Matter

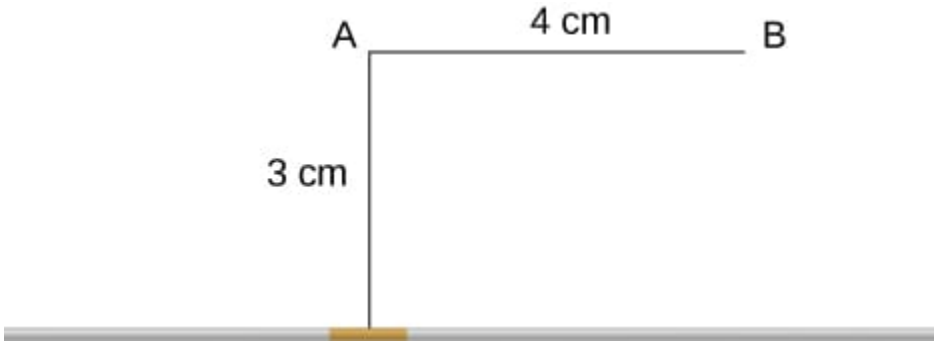
14. A diamagnetic material is brought close to a permanent magnet. What happens to the material?

15. If you cut a bar magnet into two pieces, will you end up with one magnet with an isolated north pole and another magnet with an isolated south pole? Explain your answer.

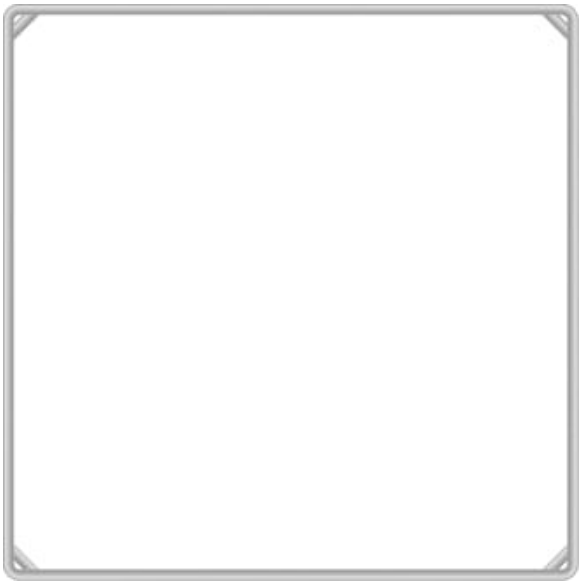
Problems

12.2 The Biot-Savart Law

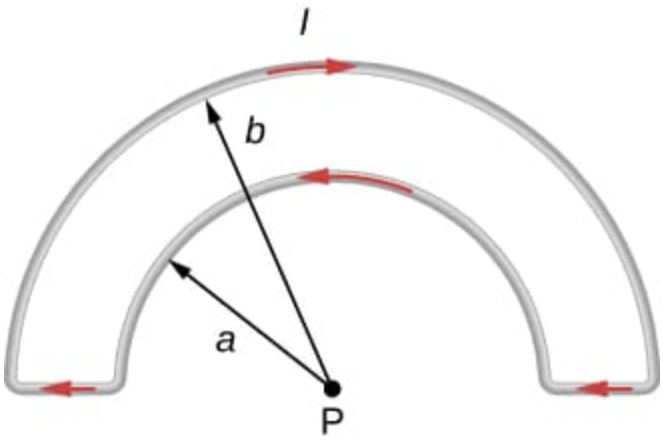
16. A 10-A current flows through the wire shown. What is the magnitude of the magnetic field due to a 0.5-mm segment of wire as measured at (a) point A and (b) point B?



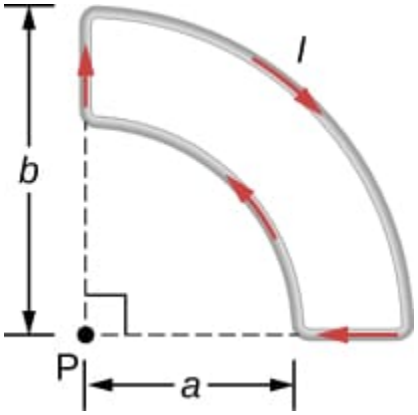
17. Ten amps flow through a square loop where each side is 20 cm in length. At each corner of the loop is a 0.01-cm segment that connects the longer wires as shown. Calculate the magnitude of the magnetic field at the center of the loop.



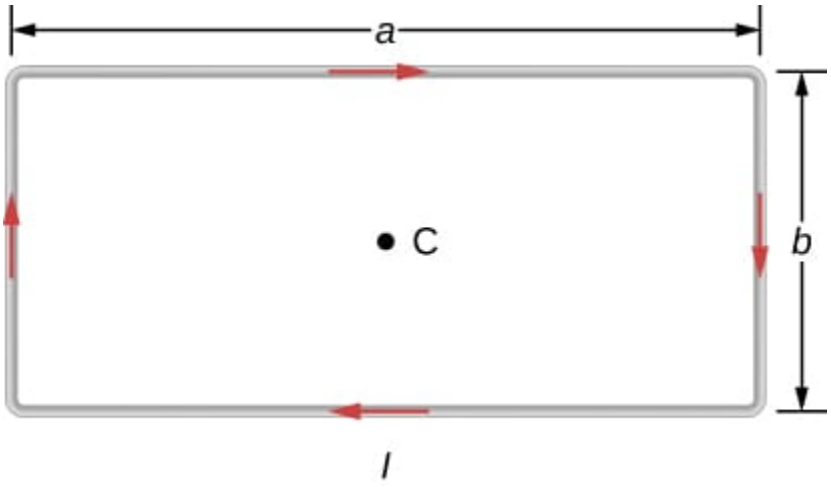
18. What is the magnetic field at P due to the current I in the wire shown?



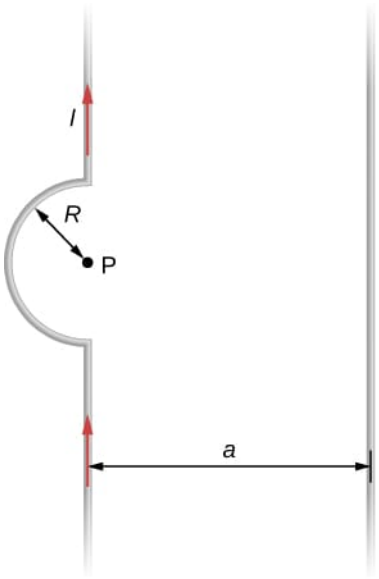
19. The accompanying figure shows a current loop consisting of two concentric circular arcs and two perpendicular radial lines. Determine the magnetic field at point P.



20. Find the magnetic field at the center C of the rectangular loop of wire shown in the accompanying figure.



21. Two long wires, one of which has a semicircular bend of radius R, are positioned as shown in the accompanying figure. If both wires carry a current **I**, how far apart must their parallel sections be so that the net magnetic field at P is zero? Does the current in the straight wire flow up or down?



12.3 Magnetic Field Due to a Thin Straight Wire

22. A typical current in a lightning bolt is 10^4 A. Estimate the magnetic field 1 m from the bolt.

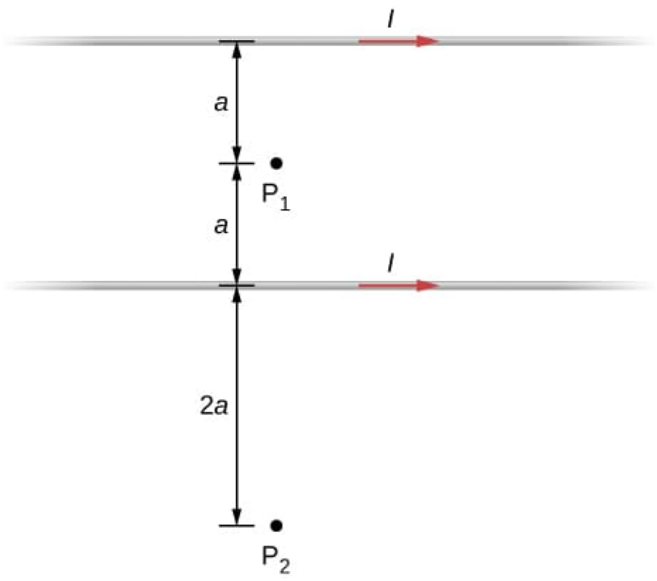
23. The magnitude of the magnetic field 50 cm from a long, thin, straight wire is **8.0μT**. What is the current through the long wire?

24. A transmission line strung 7.0 m above the ground carries a current of 500 A. What is the magnetic field on the ground directly below the wire? Compare your answer with the magnetic field of Earth.

25. A long, straight, horizontal wire carries a left-to-right current of 20 A. If the wire is placed in a uniform magnetic field of magnitude $4.0 \times 10^{-5} T$ that is directed vertically downward, what is the resultant magnitude of the magnetic field 20 cm above the wire? 20 cm below the wire?

26. The two long, parallel wires shown in the accompanying figure carry currents in the same direction. If $I_1 = 10A$ and $I_2 = 20A$, what is the magnetic field at point P?

27. The accompanying figure shows two long, straight, horizontal wires that are parallel and a distance 2a apart. If both wires carry current I in the same direction, (a) what is the magnetic field at P_1? (b) P_2?



28. Repeat the calculations of the preceding problem with the direction of the current in the lower wire reversed.

29. Consider the area between the wires of the preceding problem. At what distance from the top wire is the net magnetic field a minimum? Assume that the currents are equal and flow in opposite directions.

12.4 Magnetic Force between Two Parallel Currents

30. Two long, straight wires are parallel and 25 cm apart.

(a) If each wire carries a current of 50 A in the same direction, what is the magnetic force per meter exerted on each wire?

(b) Does the force pull the wires together or push them apart?

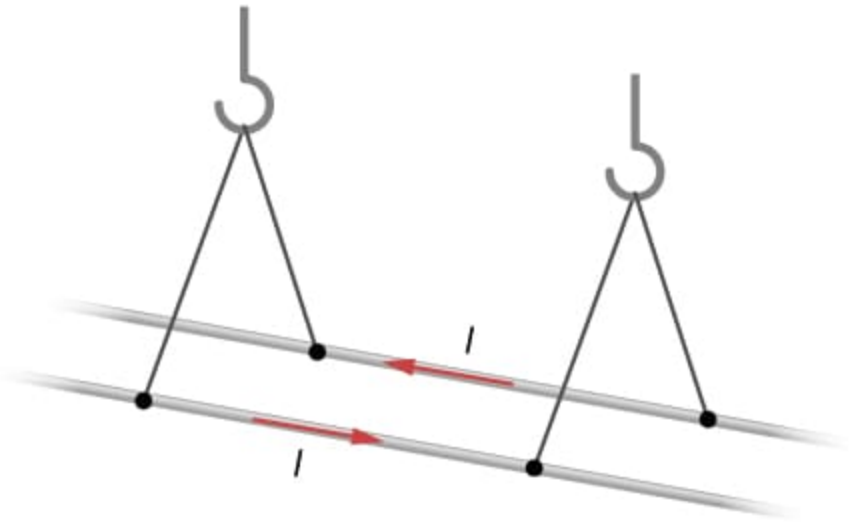
(c) What happens if the currents flow in opposite directions?

31. Two long, straight wires are parallel and 10 cm apart. One carries a current of 2.0 A, the other a current of 5.0 A.

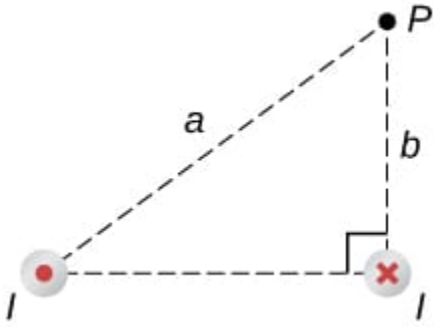
(a) If the two currents flow in opposite directions, what is the magnitude and direction of the force per unit length of one wire on the other?

(b) What is the magnitude and direction of the force per unit length if the currents flow in the same direction?

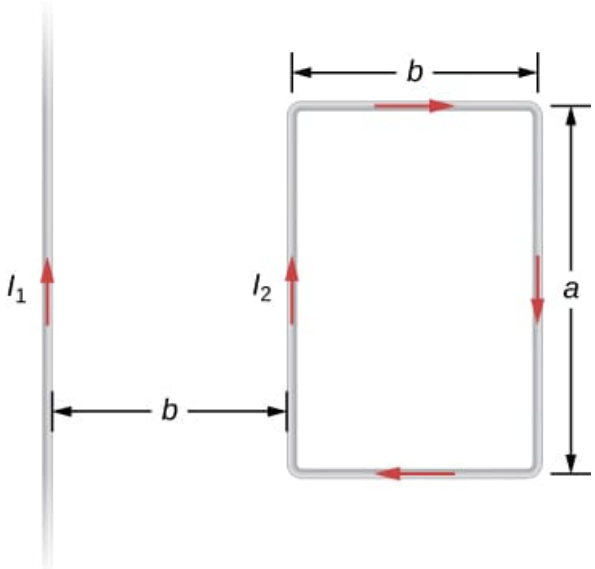
32. Two long, parallel wires are hung by cords of length 5.0 cm, as shown in the accompanying figure. Each wire has a mass per unit length of 30 g/m, and they carry the same current in opposite directions. What is the current if the cords hang at **6.0°** with respect to the vertical?



33. A circuit with current **I** has two long parallel wire sections that carry current in opposite directions. Find magnetic field at a point **P** near these wires that is a distance a from one wire and b from the other wire as shown in the figure.



34. The infinite, straight wire shown in the accompanying figure carries a current I_1. The rectangular loop, whose long sides are parallel to the wire, carries a current I_2. What are the magnitude and direction of the force on the rectangular loop due to the magnetic field of the wire?



12.5 Magnetic Field of a Current Loop

35. When the current through a circular loop is 6.0 A, the magnetic field at its center is $2.0 \times 10^{-4} T$. What is the radius of the loop?

36. How many turns must be wound on a flat, circular coil of radius 20 cm in order to produce a magnetic field of magnitude $4.0 \times 10^{-5} T$ at the center of the coil when the current through it is 0.85 A?

37. A flat, circular loop has 20 turns. The radius of the loop is 10.0 cm and the current through the wire is 0.50 A. Determine the magnitude of the magnetic field at the center of the loop.

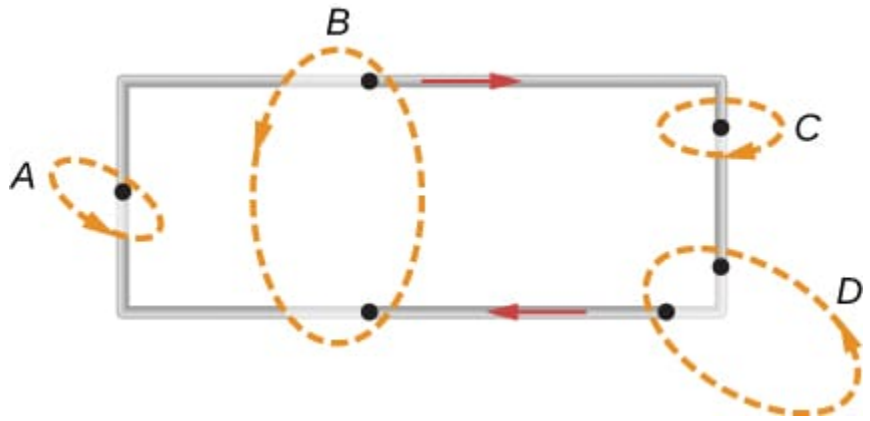
38. A circular loop of radius **R** carries a current **I**. At what distance along the axis of the loop is the magnetic field one-half its value at the center of the loop?

39. Two flat, circular coils, each with a radius **R** and wound with **N** turns, are mounted along the same axis so that they are parallel a distance **d** apart. What is the magnetic field at the midpoint of the common axis if a current **I** flows in the same direction through each coil?

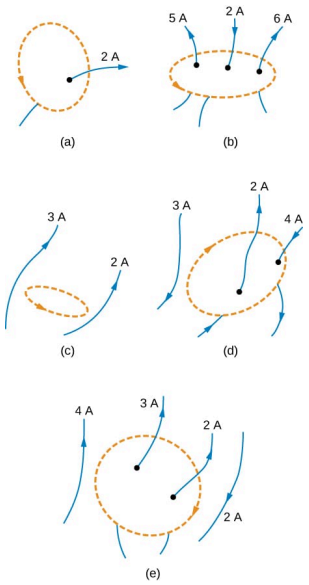
40. For the coils in the preceding problem, what is the magnetic field at the center of either coil?

12.6 Ampère's Law

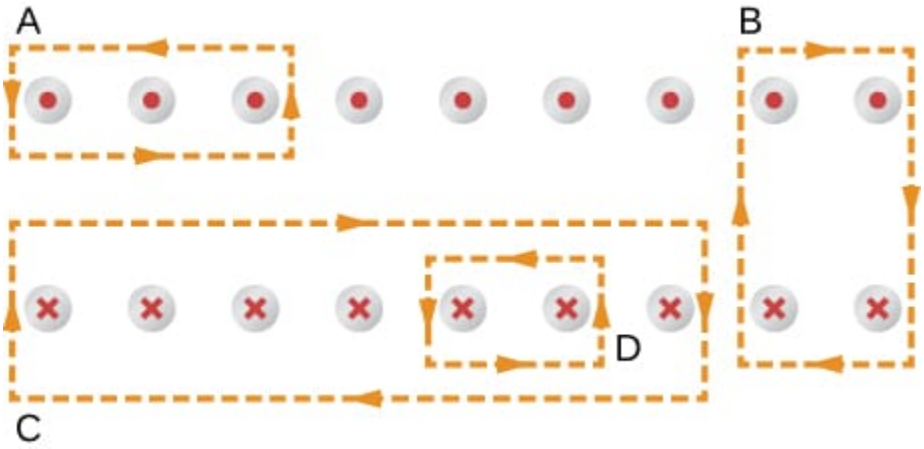
41. A current **I** flows around the rectangular loop shown in the accompanying figure. Evaluate $\oint \vec{B} \cdot \vec{dl}$ for the paths **A, B,C**, and **D.**



42. Evaluate $\oint \vec{B} \cdot \vec{dl}$ for each of the cases shown in the accompanying figure.



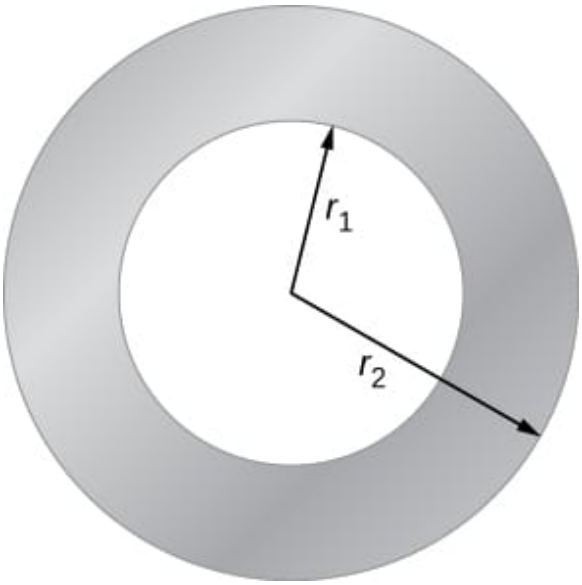
43. The coil whose lengthwise cross section is shown in the accompanying figure carries a current I and has N evenly spaced turns distributed along the length l. Evaluate $\oint \vec{B} \cdot \vec{dl}$ for the paths indicated.



44. A superconducting wire of diameter 0.25 cm carries a current of 1000 A. What is the magnetic field just outside the wire?

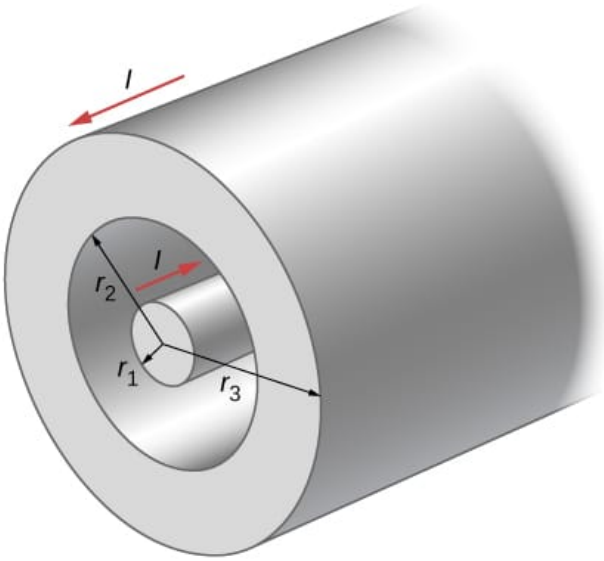
45. A long, straight wire of radius R carries a current I that is distributed uniformly over the cross-section of the wire. At what distance from the axis of the wire is the magnitude of the magnetic field a maximum?

46. The accompanying figure shows a cross-section of a long, hollow, cylindrical conductor of inner radius $r_1 = 3.0cm$ and outer radius $r_2 = 5.0cm$. A 50-A current distributed uniformly over the cross-section flows into the page. Calculate the magnetic field at $r = 2.0cm, r = 4.0cm,$ and $r = 6.0cm$.



47. A long, solid, cylindrical conductor of radius 3.0 cm carries a current of 50 A distributed uniformly over its cross-section. Plot the magnetic field as a function of the radial distance r from the center of the conductor.

48. A portion of a long, cylindrical **coaxial cable** is shown in the accompanying figure. A current I flows down the center conductor, and this current is returned in the outer conductor. Determine the magnetic field in the regions (a) $r \leq r_1$, (b) $r_2 \geq r \geq r_1$, (c) $r_3 \geq r \geq r_2$, and (d) $r \geq r_3$. Assume that the current is distributed uniformly over the cross sections of the two parts of the cable.



12.7 Solenoids and Toroids

49. A solenoid is wound with 2000 turns per meter. When the current is 5.2 A, what is the magnetic field within the solenoid?

50. A solenoid has 12 turns per centimeter. What current will produce a magnetic field of $2.0 \times 10^{-2} T$ within the solenoid?

51. If a current is 2.0 A, how many turns per centimeter must be wound on a solenoid in order to produce a magnetic field of $2.0 \times 10^{-3} T$ within it?

52. A solenoid is 40 cm long, has a diameter of 3.0 cm, and is wound with 500 turns. If the current through the windings is

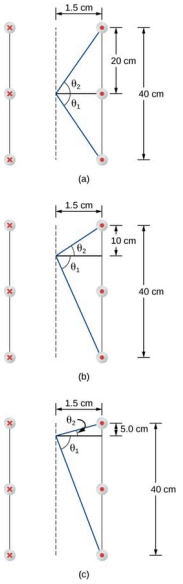
4.0 A, what is the magnetic field at a point on the axis of the solenoid that is

(a) at the center of the solenoid,

(b) 10.0 cm from one end of the solenoid, and

(c) 5.0 cm from one end of the solenoid?

(d) Compare these answers with the infinite-solenoid case.



53. Determine the magnetic field on the central axis at the opening of a semi-infinite solenoid. (That is, take the opening to be at **x=0** and the other end to be at $x = \infty$)

54. By how much is the approximation $B = \mu_0 nI$ in error at the center of a solenoid that is 15.0 cm long, has a diameter of 4.0 cm, is wrapped with **n** turns per meter, and carries a current **I**?

55. A solenoid with 25 turns per centimeter carries a current I. An electron moves within the solenoid in a circle that has a radius of 2.0 cm and is perpendicular to the axis of the solenoid. If the speed of the electron is $2.0 \times 10^5 m/s$, what is I?

56. A toroid has 250 turns of wire and carries a current of 20 A. Its inner and outer radii are 8.0 and 9.0 cm. What are the values of its magnetic field at **r=8.1, 8.5,** and **8.9cm**?

57. A toroid with a square cross section 3.0 cm × 3.0 cm has an inner radius of 25.0 cm. It is wound with 500 turns of wire, and it carries a current of 2.0 A. What is the strength of the magnetic field at the center of the square cross section?

12.8 Magnetism in Matter

58. The magnetic field in the core of an air-filled solenoid is 1.50 T. By how much will this magnetic field decrease if the air is pumped out of the core while the current is held constant?

59. A solenoid has a ferromagnetic core, **n = 1000** turns per meter, and **I = 5.0 A**. If B inside the solenoid is 2.0 T, what is **χ** for the core material?

60. A 20-A current flows through a solenoid with 2000 turns per meter. What is the magnetic field inside the solenoid if its core is (a) a vacuum and (b) filled with liquid oxygen at 90 K?

61. The magnetic dipole moment of the iron atom is about $2.1 \times 10^{-23} A \cdot m^2$.

(a) Calculate the maximum magnetic dipole moment of a domain consisting of 10^{19} iron atoms.

(b) What current would have to flow through a single circular loop of wire of diameter 1.0 cm to produce this magnetic dipole moment?

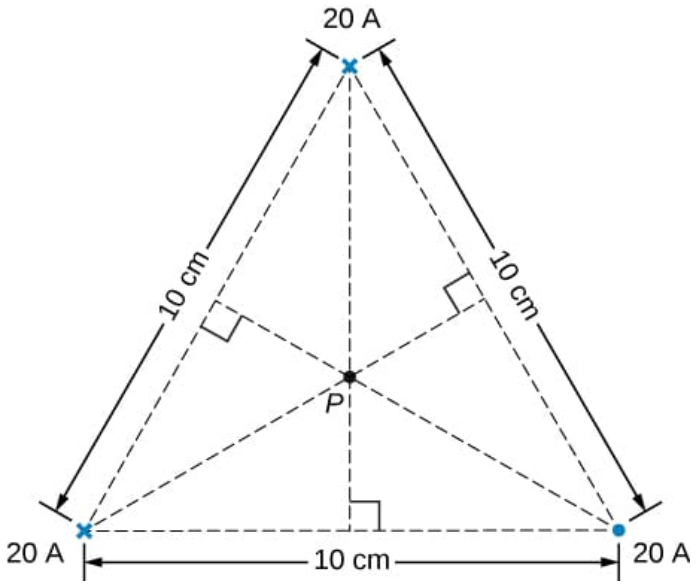
62. Suppose you wish to produce a 1.2-T magnetic field in a toroid with an iron core for which $\chi = 4.0 \times 10^3$. The toroid has a mean radius of 15 cm and is wound with 500 turns. What current is required?

63. A current of 1.5 A flows through the windings of a large, thin toroid with 200 turns per meter and a radius of 1 meter. If the toroid is filled with iron for which $\chi = 3.0 \times 10^3$, what is the magnetic field within it?

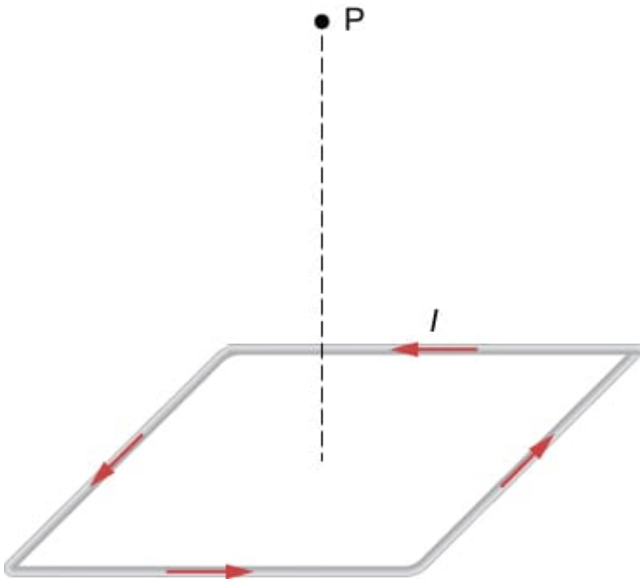
64. A solenoid with an iron core is 25 cm long and is wrapped with 100 turns of wire. When the current through the solenoid is 10 A, the magnetic field inside it is 2.0 T. For this current, what is the permeability of the iron? If the current is turned off and then restored to 10 A, will the magnetic field necessarily return to 2.0 T?

Additional Problems

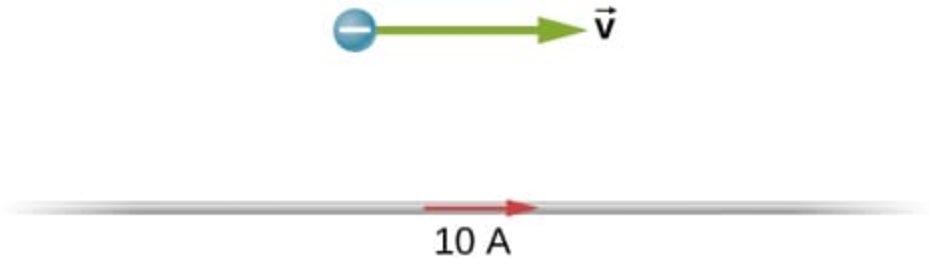
65. Three long, straight, parallel wires, all carrying 20 A, are positioned as shown in the accompanying figure. What is the magnitude of the magnetic field at the point **P**?



66. A current **I** flows around a wire bent into the shape of a square of side a. What is the magnetic field at the point P that is a distance **z** above the center of the square (see the accompanying figure)?



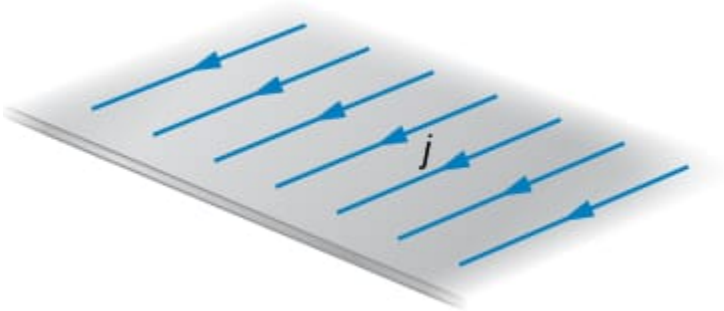
67. The accompanying figure shows a long, straight wire carrying a current of 10 A. What is the magnetic force on an electron at the instant it is 20 cm from the wire, traveling parallel to the wire with a speed of $2.0 \times 10^5 m/s$? Describe qualitatively the subsequent motion of the electron.



68. Current flows along a thin, infinite sheet as shown in the accompanying figure. The current per unit length along the sheet is J in amperes per meter.

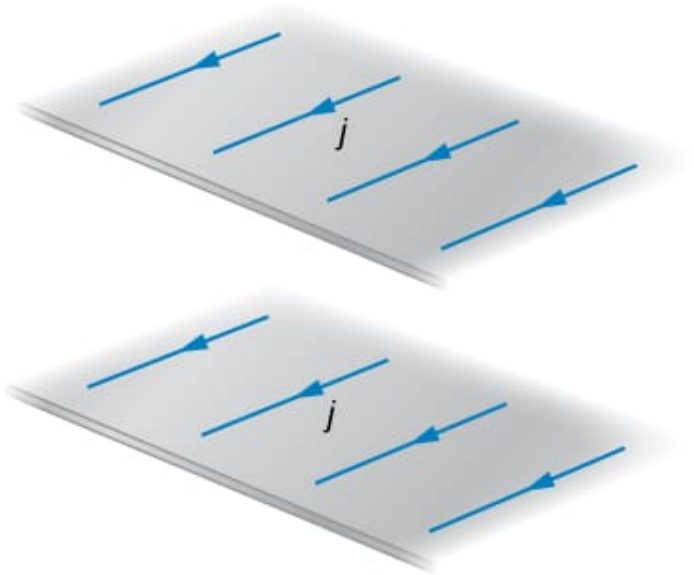
(a) Use the Biot-Savart law to show that $B = \mu_0 J/2$ on either side of the sheet. What is the direction of $\vec{B}$ on each side?

(b) Now use Ampère's law to calculate the field.

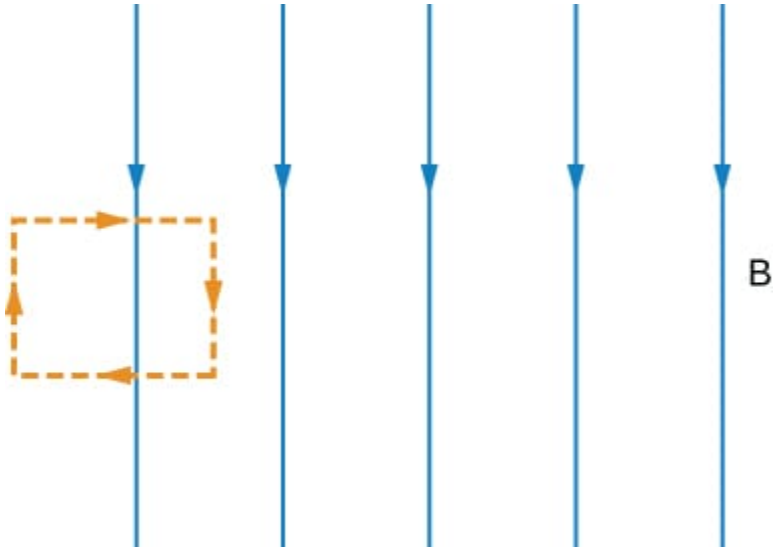


69. (a) Use the result of the previous problem to calculate the magnetic field between, above, and below the pair of infinite sheets shown in the accompanying figure.

(b) Repeat your calculations if the direction of the current in the lower sheet is reversed.



70. We often assume that the magnetic field is uniform in a region and zero everywhere else. Show that in reality it is impossible for a magnetic field to drop abruptly to zero, as illustrated in the accompanying figure. (**Hint**: Apply Ampère's law over the path shown.)



71. How is the fractional change in the strength of the magnetic field across the face of the toroid related to the fractional change in the radial distance from the axis of the toroid?

72. Show that the expression for the magnetic field of a toroid reduces to that for the field of an infinite solenoid in the limit that the central radius goes to infinity.

73. A toroid with an inner radius of 20 cm and an outer radius of 22 cm is tightly wound with one layer of wire that has a diameter of 0.25 mm.

(a) How many turns are there on the toroid?

(b) If the current through the toroid windings is 2.0 A, what is the strength of the magnetic field at the center of the toroid?

74. A wire element has $vecdl, I\vec{d}l = JAdl = Jdv$, where A and dv are the cross-sectional area and volume of the element, respectively. Use this, the Biot-Savart law, and $J = nev$ to show that the magnetic field of a moving point charge q is given

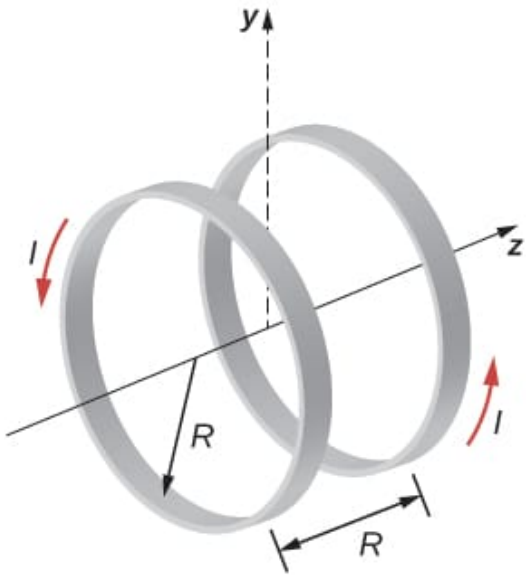
by:

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{qv \times \hat{r}}{r^2}.$$

75. A reasonably uniform magnetic field over a limited region of space can be produced with the Helmholtz coil, which consists of two parallel coils centered on the same axis. The coils are connected so that they carry the same current I. Each coil has N turns and radius R, which is also the distance between the coils.

(a) Find the magnetic field at any point on the z-axis shown in the accompanying figure.

(b) Show that dB/dz and d^2B_dz2 are both zero at **z = 0.** (These vanishing derivatives demonstrate that the magnetic field varies only slightly near **z = 0**.)



76. A charge of **4.0μC** is distributed uniformly around a thin ring of insulating material. The ring has a radius of 0.20 m and rotates at $2.0 \times 10^4 rev/min$ around the axis that passes through its center and is perpendicular to the plane of the ring. What is the magnetic field at the center of the ring?

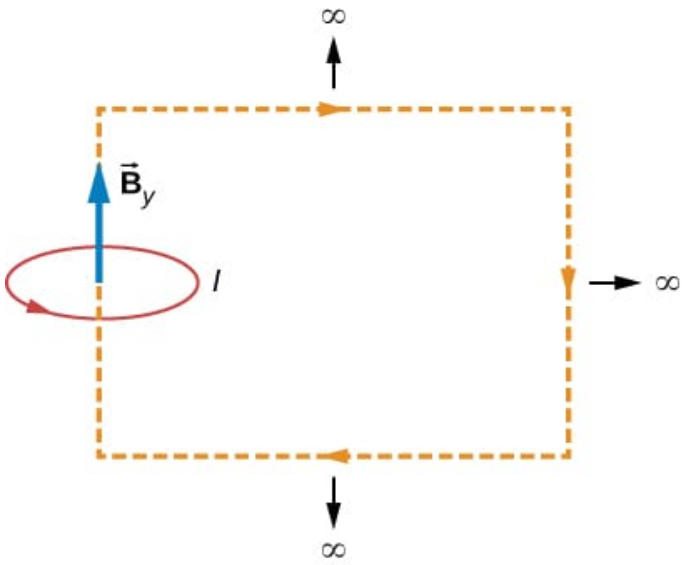
77. A thin, nonconducting disk of radius **R** is free to rotate around the axis that passes through its center and is perpendicular to the face of the disk. The disk is charged uniformly with a total charge **q**. If the disk rotates at a constant angular velocity **ω**, what is the magnetic field at its center?

78. Consider the disk in the previous problem. Calculate the magnetic field at a point on its central axis that is a distance **y** above the disk.

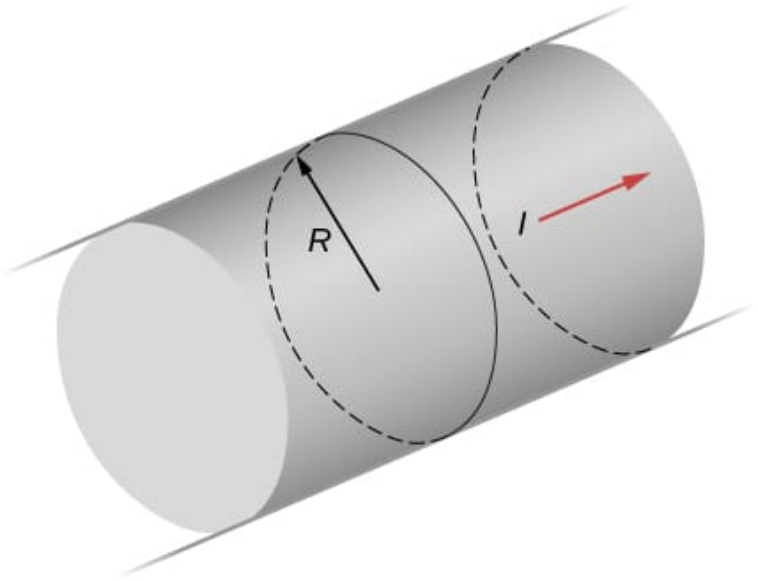
79. Consider the axial magnetic field $B_y = \mu_0 IR^2/2(y^2 + R^2)^{3/2}$ of the circular current loop shown below.

(a) Evaluate $\int_{-a}^{a} B_y dy$. Also so show that $\lim_{a\to\infty}\int_{-a}^{a} B_y dy = \mu_0 I$.

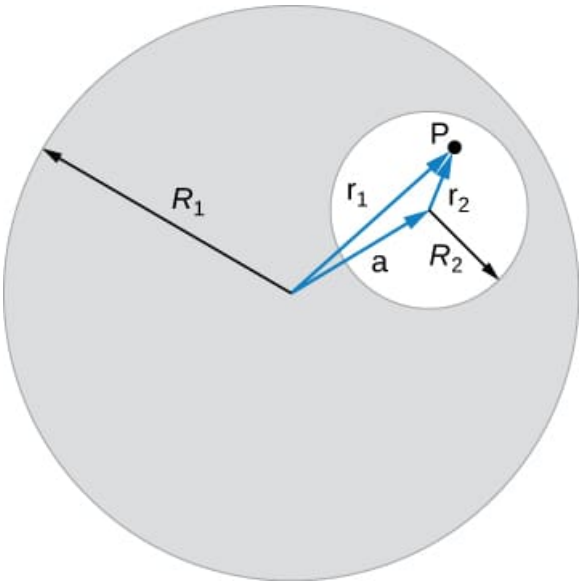
(b) Can you deduce this limit without evaluating the integral? (**Hint:**See the accompanying figure.)



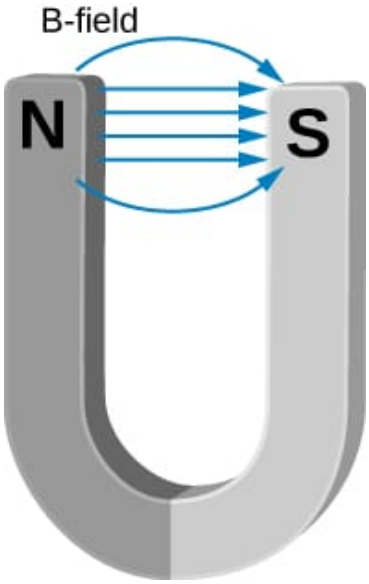
80. The current density in the long, cylindrical wire shown in the accompanying figure varies with distance r from the center of the wire according to $J = cr$, where c is a constant. (a) What is the current through the wire? (b) What is the magnetic field produced by this current for $r \leq R$? For $r \geq R$?



81. A long, straight, cylindrical conductor contains a cylindrical cavity whose axis is displaced by a from the axis of the conductor, as shown in the accompanying figure. The current density in the conductor is given by $\hat{J} = J_0\hat{k}$, where J_0 is a constant and $\hat{k}$ is along the axis of the conductor. Calculate the magnetic field at an arbitrary point P in the cavity by superimposing the field of a solid cylindrical conductor with radius R_1 and current density $\vec{J}$ onto the field of a solid cylindrical conductor with radius R_2 and current density $-\vec{J}$. Then use the fact that the appropriate azimuthal unit vectors can be expressed as $\hat{\theta}_1 = \hat{k} \times \hat{r_1}$ and $\hat{\theta}_2 = \hat{k} \times \hat{r_2}$ to show that everywhere inside the cavity the magnetic field is given by the constant $\vec{B} = \frac{1}{2}\mu_0 J_0 k \times a$, where $a = r_1 - r_2$ and $r_1 = r_1\hat{r_1}$ is the position of P relative to the center of the conductor and $2_{=}r_2\vec{r_2}$ is the position of **P** relative to the center of the cavity.

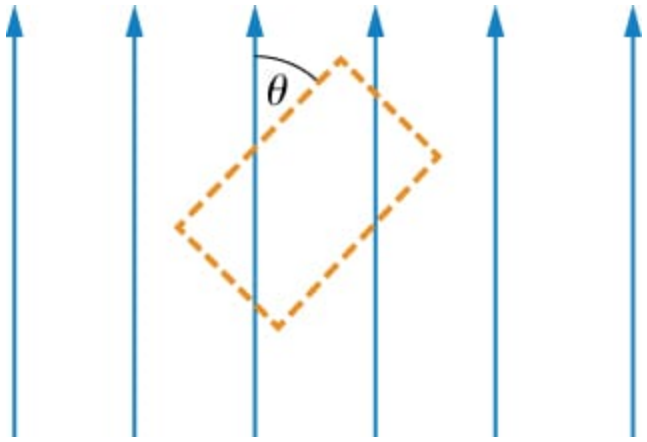


82. Between the two ends of a horseshoe magnet the field is uniform as shown in the diagram. As you move out to outside edges, the field bends. Show by Ampère's law that the field must bend and thereby the field weakens due to these bends.



83. Show that the magnetic field of a thin wire and that of a current loop are zero if you are infinitely far away.

84. An Ampère loop is chosen as shown by dashed lines for a parallel constant magnetic field as shown by solid arrows. Calculate $\vec{B} \cdot \vec{dl}$ for each side of the loop then find the entire $\oint \vec{B} \cdot \vec{dl}$. Can you think of an Ampère loop that would make the problem easier? Do those results match these?



85. A very long, thick cylindrical wire of radius R carries a current density J that varies across its cross-section. The magnitude of the current density at a point a distance **r** from the center of the wire is given by $J = J_0 \frac{r}{R}$, where J_0 is a constant. Find the magnetic field

(a) at a point outside the wire and

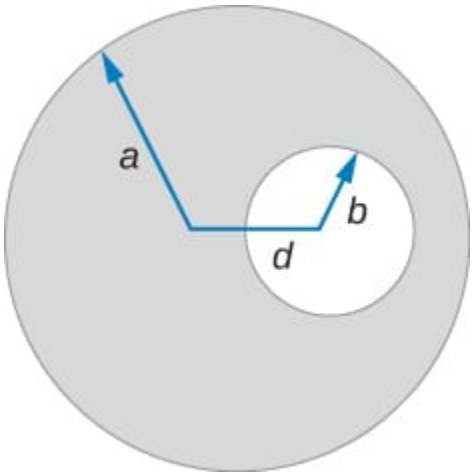
(b) at a point inside the wire. Write your answer in terms of the net current I through the wire.

86. A very long, cylindrical wire of radius a has a circular hole of radius **b** in it at a distance d from the center. The wire carries a uniform current of magnitude **I** through it. The direction of the current in the figure is out of the paper. Find the magnetic field

(a) at a point at the edge of the hole closest to the center of the thick wire,

(b) at an arbitrary point inside the hole, and

(c) at an arbitrary point outside the wire. (Hint: Think of the hole as a sum of two wires carrying current in the opposite directions.)



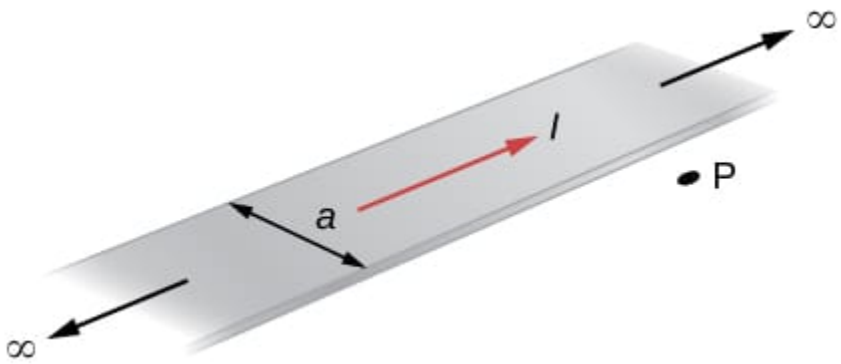
87. Magnetic field inside a torus. Consider a torus of rectangular cross-section with inner radius a and outer radius b. N turns of an insulated thin wire are wound evenly on the torus tightly all around the torus and connected to a battery producing a steady current I in the wire. Assume that the current on the top and bottom surfaces in the figure is radial, and the current on the inner and outer radii surfaces is vertical. Find the magnetic field inside the torus as a function of radial distance **r** from the axis.

88. Two long coaxial copper tubes, each of length **L**, are connected to a battery of voltage **V**. The inner tube has inner radius **a** and outer radius **b**, and the outer tube has inner radius c and outer radius **d**. The tubes are then disconnected from

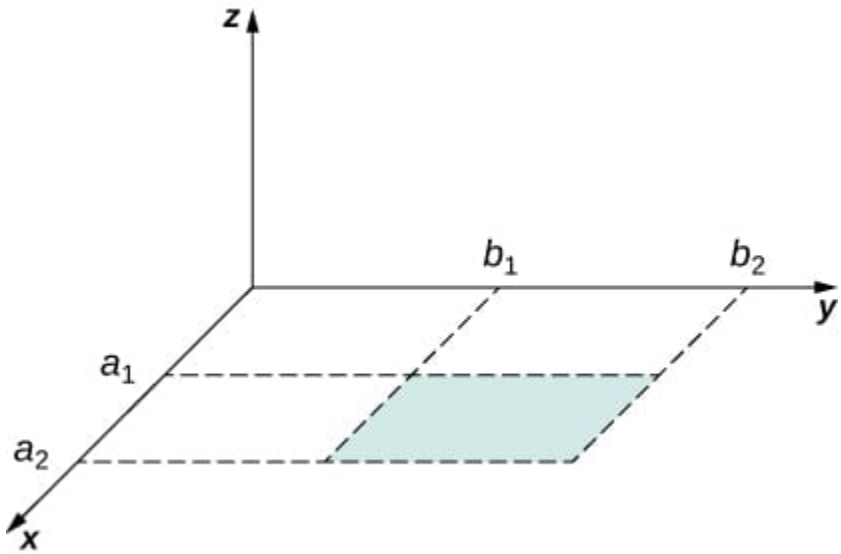
the battery and rotated in the same direction at angular speed of **ω** radians per second about their common axis. Find the magnetic field (a) at a point inside the space enclosed by the inner tube **r, and (b) at a point between the tubes b, and (c) at a point outside the tubes r>d. (Hint: Think of copper tubes as a capacitor and find the charge density based on the voltage applied, Q=VC, \(\displaystyle C=\frac{2πε_0L}{ln(c/b)}.)**

Challenge Problems

89. The accompanying figure shows a flat, infinitely long sheet of width a that carries a current I uniformly distributed across it. Find the magnetic field at the point P, which is in the plane of the sheet and at a distance x from one edge. Test your result for the limit a → 0.



90. A hypothetical current flowing in the z-direction creates the field $vecB = C[(x/y^2)\hat{i} + (1/y)\hat{j}]$ **in the rectangular region of the xy-plane shown in the accompanying figure. Use Ampère's law to find the current through the rectangle.**



91. A nonconducting hard rubber circular disk of radius R is painted with a uniform surface charge density σ. It is rotated about its axis with angular speed ω. (a) Find the magnetic field produced at a point on the axis a distance h meters from the center of the disk. (b) Find the numerical value of magnitude of the magnetic field when $\sigma = 1C/m^2, R = 20cm, h = 2cm$**, and** $\omega = 400rad/sec$**, and compare it with the magnitude of magnetic field of Earth, which is about 1/2 Gauss.**

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15.11: Sources of Magnetic Fields (Answers)

Check Your Understanding

12.1. 1.41 meters

12.2. $\frac{\mu_0 I}{2R}$

12.3. 4 amps flowing out of the page

12.4. Both have a force per unit length of $9.23 \times 10^{-12} N/m$

12.5. 0.608 meters

12.6. In these cases the integrals around the Ampèrian loop are very difficult because there is no symmetry, so this method would not be useful.

12.7. a. 1.00382;

b. 1.00015

12.8. a. $1.0 \times 10^{-4} T$;

b. 0.60 T;

c. 6.0×10^3

Conceptual Questions

1. Biot-Savart law's advantage is that it works with any magnetic field produced by a current loop. The disadvantage is that it can take a long time.

3. If you were to go to the start of a line segment and calculate the angle θ to be approximately $0\ ^\circ$, the wire can be considered infinite. This judgment is based also on the precision you need in the result.

5. You would make sure the currents flow perpendicular to one another.

7. A magnetic field line gives the direction of the magnetic field at any point in space. The density of magnetic field lines indicates the strength of the magnetic field.

9. The spring reduces in length since each coil with have a north pole-produced magnetic field next to a south pole of the next coil.

11. Ampère's law is valid for all closed paths, but it is not useful for calculating fields when the magnetic field produced lacks symmetry that can be exploited by a suitable choice of path.

13. If there is no current inside the loop, there is no magnetic field (see Ampère's law). Outside the pipe, there may be an enclosed current through the copper pipe, so the magnetic field may not be zero outside the pipe.

15. The bar magnet will then become two magnets, each with their own north and south poles. There are no magnetic monopoles or single pole magnets.

Problems

17. $5.66 \times 10^{-5} T$

19. $B = \frac{\mu_o I}{8}(\frac{1}{a} - \frac{1}{b})$ out of the page

21. $a = \frac{2R}{\pi}$; the current in the wire to the right must flow up the page.

23. 20 A

25. Both answers have the magnitude of magnetic field of $4.5 \times 10^{-5} T$.

27. At P1, the net magnetic field is zero. At P2, $B = \frac{3\mu_o I}{8\pi a}$ into the page.

29. The magnetic field is at a minimum at distance **a** from the top wire, or half-way between the wires.

31. a. $F/l = 8 \times 10^{-6}$ N/m away from the other wire;

b. $F/l = 8 \times 10^{-6}$ N/m toward the other wire

33. $B = \frac{\mu_o I a}{2\pi b^2}$ into the page

35. 0.019 m

37. $6.28 \times 10^{-5} T$

39. $B = \frac{\mu_o I R^2}{\left((\frac{d}{2})^2 + R^2\right)^{3/2}}$

41. a. $\mu 0 I$;

b. 0;

c. $\mu 0 I$;

d. 0

43. a. $3\mu_0 I$;

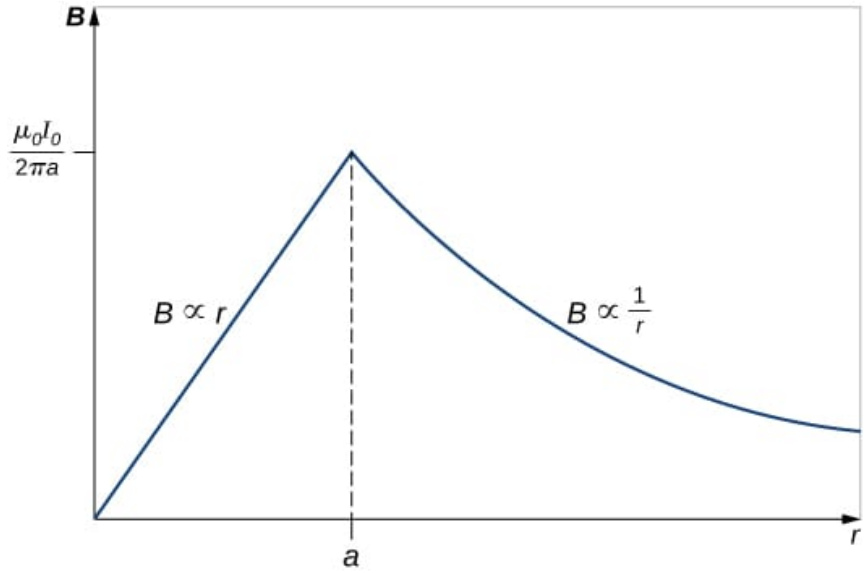
b. 0;

c. $7\mu_0 I$;

d. $-2\mu_0 I$

45. at the radius **R**

47.



49. $B = 1.3 \times 10^{-2} T$

51. roughly eight turns per cm

53. $B = \frac{1}{2}\mu_0 nI$

55. 0.0181 A

57. 0.0008 T

59. 317.31

61. $2.1 \times 10^{-4} A \cdot m^2$ $2.7A$

63. 0.18 T

Additional Problems

65. $B = 6.93 \times 10^{-5} T$

67. $3.2 \times 10^{-19} N$ in an arc away from the wire

69. a. above and below $B = \mu_0 j$, in the middle $B = 0$;

b. above and below $B = 0$, in the middle $B = \mu_0 j$

71. $\frac{dB}{B} = -\frac{dr}{r}$

73. a. 52778 turns;

b. 0.10 T

75. $B_1(x) = \frac{\mu_0 I R^2}{2(R^2 + z^2)^{3/2}}$

77. $B = \frac{\mu_0 \sigma \omega}{2} R$

79. derivation

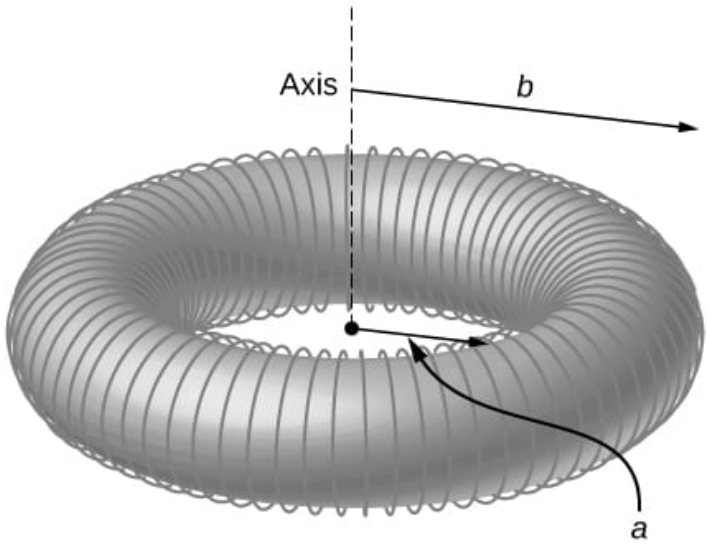
81. derivation

83. As the radial distance goes to infinity, the magnetic fields of each of these formulae go to zero.

85. a. $B = \frac{\mu_0 I}{2\pi r}$;

b. $B = \frac{\mu_0 J_0 r^2}{3R}$

87. $B(r) = \mu_0 NI/2\pi r$



Challenge Problems

89. $B = \frac{\mu_0 I}{2\pi x}$.

91. a. $B = \frac{\mu_0 \sigma \omega}{2}[\frac{2h^2 + R^2}{\sqrt{R^2 + h^0}}]$;

b. $B = 4.09 \times 10^{-5} T$, 82% of Earth's magnetic field

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16: ELECTROMAGNETIC INDUCTION

16.1: Prelude to Electromagnetic Induction
16.2: Faraday’s Law
16.3: Lenz's Law
16.4: Motional Emf
16.5: Induced Electric Fields
16.6: Eddy Currents
16.7: Electric Generators and Back Emf
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16.1: Prelude to Electromagnetic Induction

We have been considering electric fields created by fixed charge distributions and magnetic fields produced by constant currents, but electromagnetic phenomena are not restricted to these stationary situations. Most of the interesting applications of electromagnetism are, in fact, time-dependent. To investigate some of these applications, we now remove the time-independent assumption that we have been making and allow the fields to vary with time. In this and the next several chapters, you will see a wonderful symmetry in the behavior exhibited by time-varying electric and magnetic fields. Mathematically, this symmetry is expressed by an additional term in Ampère's law and by another key equation of electromagnetism called Faraday's law. We also discuss how moving a wire through a magnetic field produces an emf or voltage. Lastly, we describe applications of these principles, such as the card reader shown above.

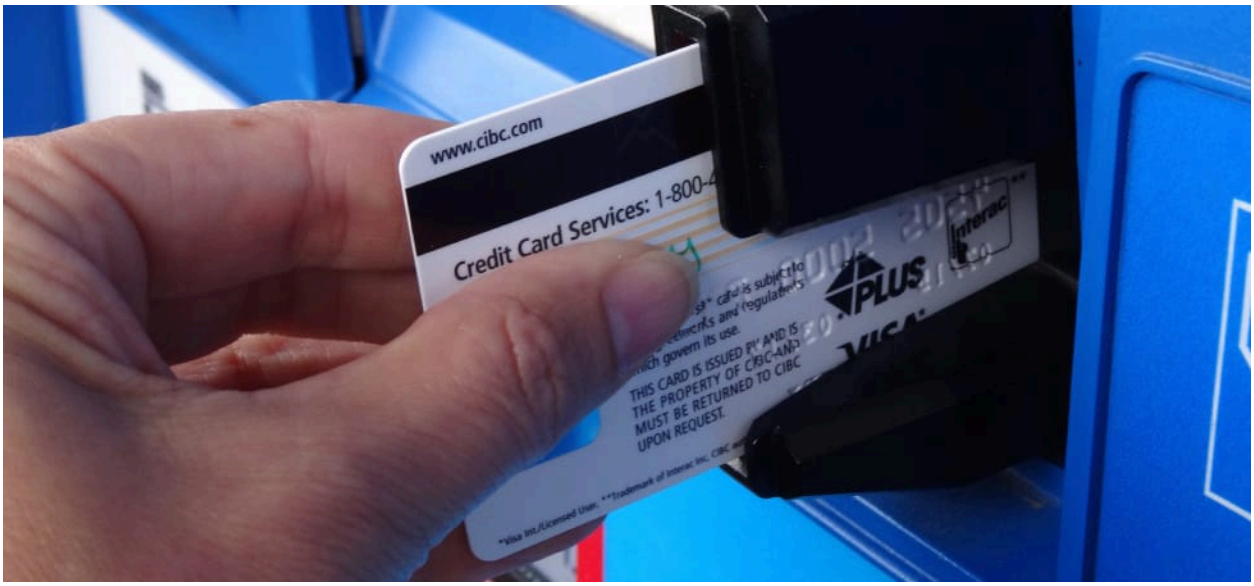


Figure 16.1.1: The black strip found on the back of credit cards and driver's licenses is a very thin layer of magnetic material with information stored on it. Reading and writing the information on the credit card is done with a swiping motion. The physical reason why this is necessary is called electromagnetic induction and is discussed in this chapter.

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16.2: Faraday's Law

Learning Objectives

By the end of this section, you will be able to:

- Determine the magnetic flux through a surface, knowing the strength of the magnetic field, the surface area, and the angle between the normal to the surface and the magnetic field
- Use Faraday's law to determine the magnitude of induced emf in a closed loop due to changing magnetic flux through the loop

The first productive experiments concerning the effects of time-varying magnetic fields were performed by Michael **Faraday** in 1831. One of his early experiments is represented in Figure 16.2.1. An **emf** is induced when the magnetic field in the coil is changed by pushing a bar magnet into or out of the coil. Emfs of opposite signs are produced by motion in opposite directions, and the directions of emfs are also reversed by reversing poles. The same results are produced if the coil is moved rather than the magnet—it is the relative motion that is important. The faster the motion, the greater the emf, and there is no emf when the magnet is stationary relative to the coil.

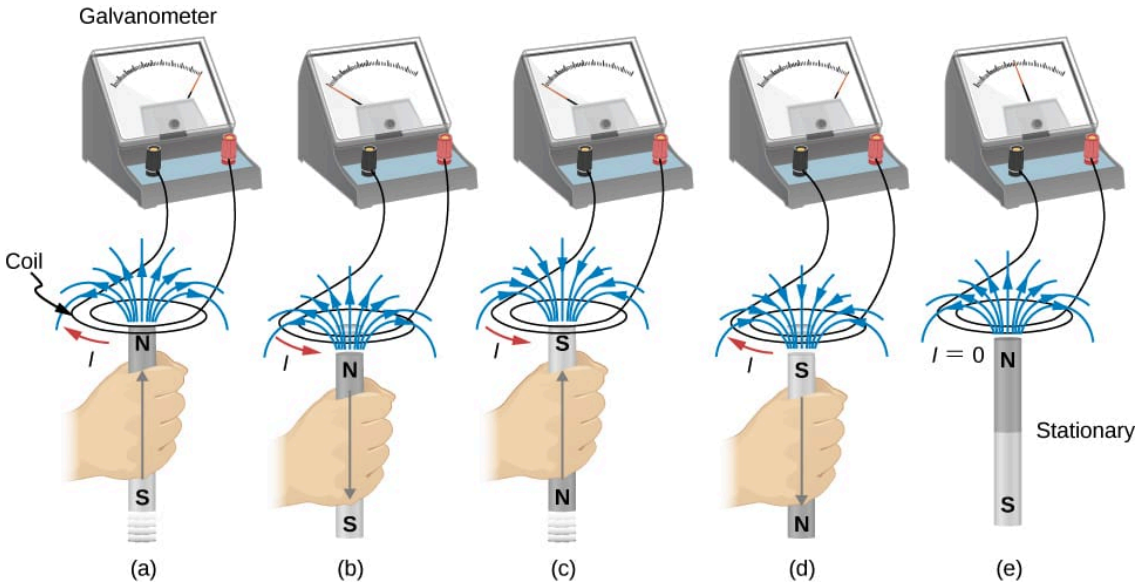


Figure 16.2.1: Movement of a magnet relative to a coil produces emfs as shown (a–d). The same emfs are produced if the coil is moved relative to the magnet. This short-lived emf is only present during the motion. The greater the speed, the greater the magnitude of the emf, and the emf is zero when there is no motion, as shown in (e).

Faraday also discovered that a similar effect can be produced using two circuits—a changing current in one circuit induces a current in a second, nearby circuit. For example, when the switch is closed in circuit 1 of Figure 16.2.1a, the ammeter needle of circuit 2 momentarily deflects, indicating that a short-lived current surge has been induced in that circuit. The ammeter needle quickly returns to its original position, where it remains. However, if the switch of circuit 1 is now suddenly opened, another short-lived current surge in the direction opposite from before is observed in circuit 2.

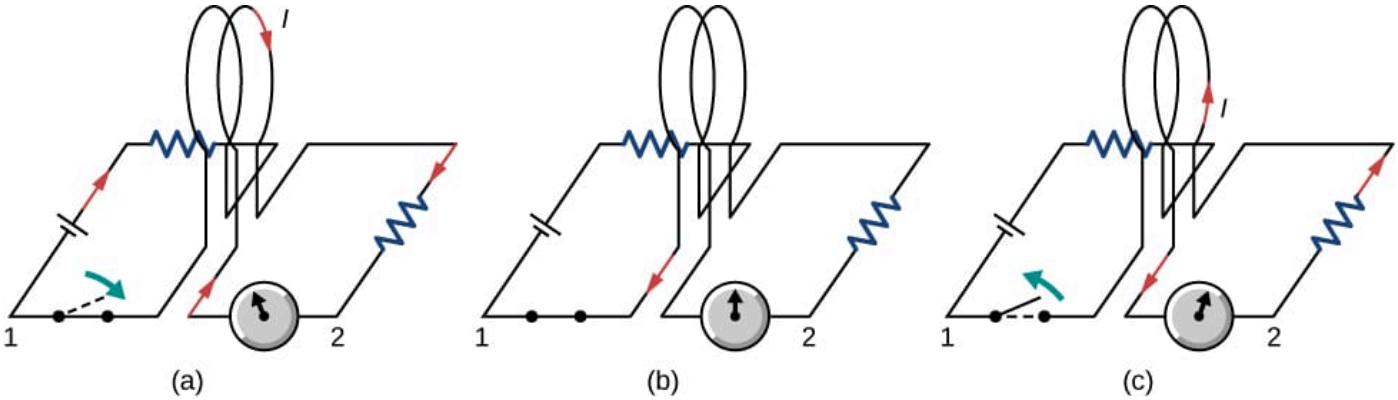


Figure 16.2.2: (a) Closing the switch of circuit 1 produces a short-lived current surge in circuit 2. (b) If the switch remains closed, no current is observed in circuit 2. (c) Opening the switch again produces a short-lived current in circuit 2 but in the opposite direction from before.

Faraday realized that in both experiments, a current flowed in the circuit containing the ammeter only when the magnetic field in the region occupied by that circuit was **changing**. As the magnet of the figure was moved, the strength of its magnetic field at the loop changed; and when the current in circuit 1 was turned on or off, the strength of its magnetic field at circuit 2 changed. Faraday was eventually able to interpret these and all other experiments involving magnetic fields that vary with time in terms of the following law.

Faraday's Law

The emf ϵ induced is the negative change in the magnetic flux Φ_m per unit time. Any change in the magnetic field or change in orientation of the area of the coil with respect to the magnetic field induces a voltage (emf).

The magnetic flux is a measurement of the amount of magnetic field lines through a given surface area, as seen in Figure 16.2.3. This definition is similar to the electric flux studied earlier. This means that if we have

$$\Phi_m = \int_S \vec{B} \cdot \hat{n} dA,$$

then the **induced emf** or the voltage generated by a conductor or coil moving in a magnetic field is

$$\epsilon = -\frac{d}{dt} \int_S \vec{B} \cdot \hat{n} dA = -\frac{d\Phi_m}{dt}.$$

The negative sign describes the direction in which the induced emf drives current around a circuit. However, that direction is most easily determined with a rule known as Lenz's law, which we will discuss shortly.

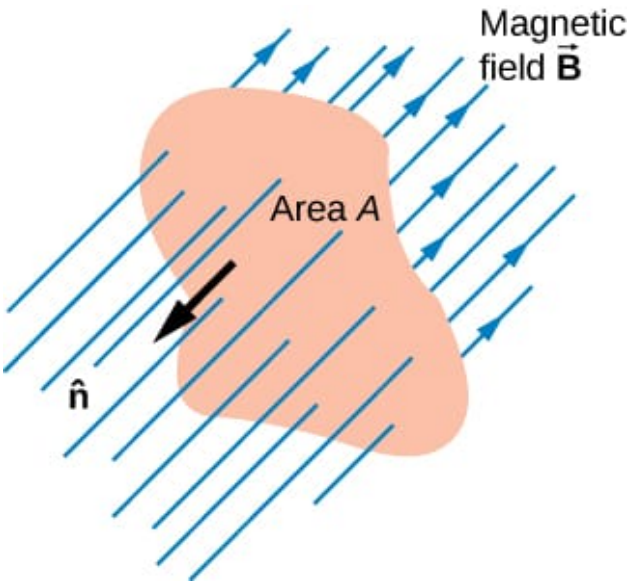


Figure 16.2.3: The magnetic flux is the amount of magnetic field lines cutting through a surface area **A** defined by the unit area vector $\hat{n}$. If the angle between the unit area $\hat{n}$ and magnetic field vector $\vec{B}$ are parallel or antiparallel, as shown in the diagram, the magnetic flux is the highest possible value given the values of area and magnetic field.

16.2.1a depicts a circuit and an arbitrary surface **S** that it bounds. Notice that **S** is an **open surface**. It can be shown that **any** open surface bounded by the circuit in question can be used to evaluate Φ_m. For example, Φ_m is the same for the various surfaces $S_1,\ S_2, \ldots$ of part (b) of the figure.

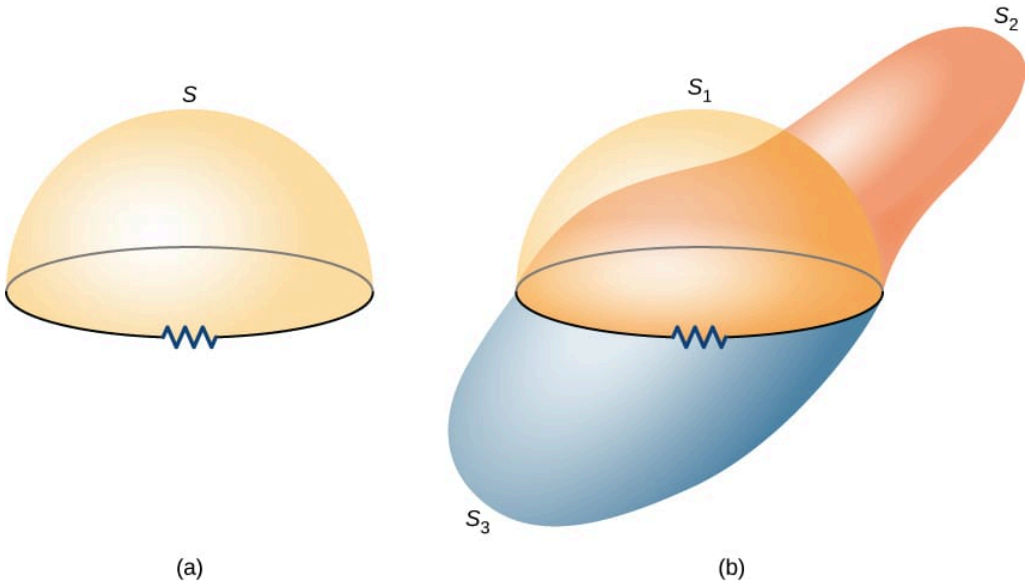


Figure 16.2.4: (a) A circuit bounding an arbitrary open surface **S**. The planar area bounded by the circuit is not part of **S**. (b) Three arbitrary open surfaces bounded by the same circuit. The value of Φ_m is the same for all these surfaces.

The SI unit for magnetic flux is the weber (Wb),

$$1\,Wb = 1\,T \cdot m^2.$$

Occasionally, the magnetic field unit is expressed as webers per square meter (Wb/m^2) instead of teslas, based on this definition. In many practical applications, the circuit of interest consists of a number **N** of tightly wound turns (Figure 16.2.5). Each turn experiences the same magnetic flux. Therefore, the net magnetic flux through the circuits is **N** times the flux through one turn, and Faraday's law is written as

$$\epsilon = -\frac{d}{dt}(N\Phi_m) = -N\frac{d\Phi_m}{dt}.$$

A Square Coil in a Changing Magnetic Field

The square coil of Figure $16.2.1$ has sides $l = 0.25\ m$ long and is tightly wound with $N = 200$ turns of wire. The resistance of the coil is $R = 5.0\ \Omega$ The coil is placed in a spatially uniform magnetic field that is directed perpendicular to the face of the coil and whose magnitude is decreasing at a rate $dB/dt = -0.040\ T/s$. (a) What is the magnitude of the emf induced in the coil? (b) What is the magnitude of the current circulating through the coil?

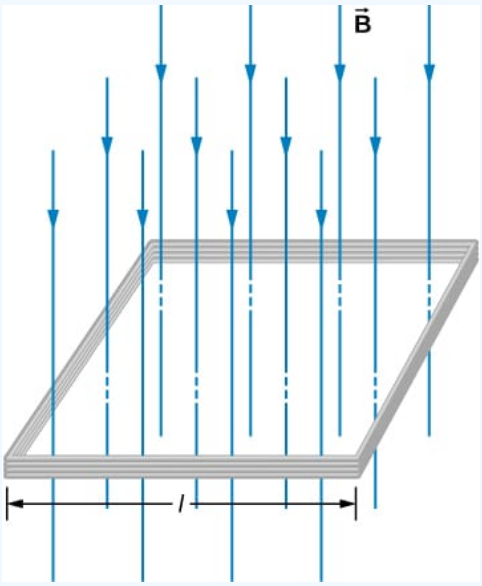


Figure $16.2.5$: A square coil with **N** turns of wire with uniform magnetic field $\vec{B}$ directed in the downward direction, perpendicular to the coil.

Strategy

The area vector, or $\hat{n}$ direction, is perpendicular to area covering the loop. We will choose this to be pointing downward so that $\vec{B}$ is parallel to $\hat{n}$ and that the flux turns into multiplication of magnetic field times area. The area of the loop is not changing in time, so it can be factored out of the time derivative, leaving the magnetic field as the only quantity varying in time. Lastly, we can apply Ohm's law once we know the induced emf to find the current in the loop.

Solution

1. The flux through one turn is

$$\Phi_m = BA = Bt^2,$$

so we can calculate the magnitude of the emf from Faraday's law. The sign of the emf will be discussed in the next section, on Lenz's law:

$$|\epsilon| = \left|-N\frac{d\Phi_m}{dt}\right| = Nl^2\frac{dB}{dt}$$
$$= (200)(0.25\,m)^2(0.040\,T/s) = 0.50\,V.$$

- The magnitude of the current induced in the coil is

$$I = \frac{\epsilon}{R} = \frac{0.50\,V}{5.0\,\Omega} = 0.10\,A.$$

Significance

If the area of the loop were changing in time, we would not be able to pull it out of the time derivative. Since the loop is a closed path, the result of this current would be a small amount of heating of the wires until the magnetic field stops changing. This may increase the area of the loop slightly as the wires are heated.

Exercise 16.2.1

A closely wound coil has a radius of 4.0 cm, 50 turns, and a total resistance of $40\ \Omega$. At what rate must a magnetic field perpendicular to the face of the coil change in order to produce Joule heating in the coil at a rate of 2.0 mW?

Solution

1.1 T/s

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16.3: Lenz's Law

Learning Objectives

By the end of this section, you will be able to:

- Use Lenz's law to determine the direction of induced emf whenever a magnetic flux changes
- Use Faraday's law with Lenz's law to determine the induced emf in a coil and in a solenoid

The direction in which the induced emf drives current around a wire loop can be found through the negative sign. However, it is usually easier to determine this direction with Lenz's law, named in honor of its discoverer, Heinrich Lenz (1804–1865). (Faraday also discovered this law, independently of Lenz.) We state Lenz's law as follows:

Lenz's Law

The direction of the induced emf drives current around a wire loop to always **oppose** the change in magnetic flux that causes the emf.

Lenz's law can also be considered in terms of conservation of energy. If pushing a magnet into a coil causes current, the energy in that current must have come from somewhere. If the induced current causes a magnetic field opposing the increase in field of the magnet we pushed in, then the situation is clear. We pushed a magnet against a field and did work on the system, and that showed up as current. If it were not the case that the induced field opposes the change in the flux, the magnet would be pulled in produce a current without anything having done work. Electric potential energy would have been created, violating the conservation of energy.

To determine an induced emf ϵ, you first calculate the magnetic flux Φ_m and then obtain $d\Phi_m/dt$. The magnitude of ϵ is given by

$$\epsilon = \left| \frac{d\Phi_m}{dt} \right| .$$

Finally, you can apply Lenz's law to determine the sense of ϵ. This will be developed through examples that illustrate the following problem-solving strategy.

Problem-Solving Strategy: Lenz's Law

To use Lenz's law to determine the directions of induced magnetic fields, currents, and emfs:

- Make a sketch of the situation for use in visualizing and recording directions.
- Determine the direction of the applied magnetic field $\vec{B}$.
- Determine whether its magnetic flux is increasing or decreasing.
- Now determine the direction of the induced magnetic field $\vec{B}$. The induced magnetic field tries to reinforce a magnetic flux that is decreasing or opposes a magnetic flux that is increasing. Therefore, the induced magnetic field adds or subtracts to the applied magnetic field, depending on the change in magnetic flux.
- Use **right-hand rule 2** (RHR-2; see Magnetic Forces and Fields) to determine the direction of the induced current **I** that is responsible for the induced magnetic field $\vec{B}$.
- The direction (or polarity) of the induced emf can now drive a conventional current in this direction.

Let's apply Lenz's law to the system of Figure $16.3.1a$. We designate the "front" of the closed conducting loop as the region containing the approaching bar magnet, and the "back" of the loop as the other region. As the north pole of the magnet moves toward the loop, the flux through the loop due to the field of the magnet increases because the strength of field lines directed from the front to the back of the loop is increasing. A current is therefore induced in the loop. By Lenz's law, the direction of the induced current must be such that its own magnetic field is directed in a way to **oppose** the changing flux caused by the field

of the approaching magnet. Hence, the induced current circulates so that its magnetic field lines through the loop are directed from the back to the front of the loop. By RHR-2, place your thumb pointing against the magnetic field lines, which is toward the bar magnet. Your fingers wrap in a counterclockwise direction as viewed from the bar magnet. Alternatively, we can determine the direction of the induced current by treating the current loop as an electromagnet that **opposes** the approach of the north pole of the bar magnet. This occurs when the induced current flows as shown, for then the face of the loop nearer the approaching magnet is also a north pole.

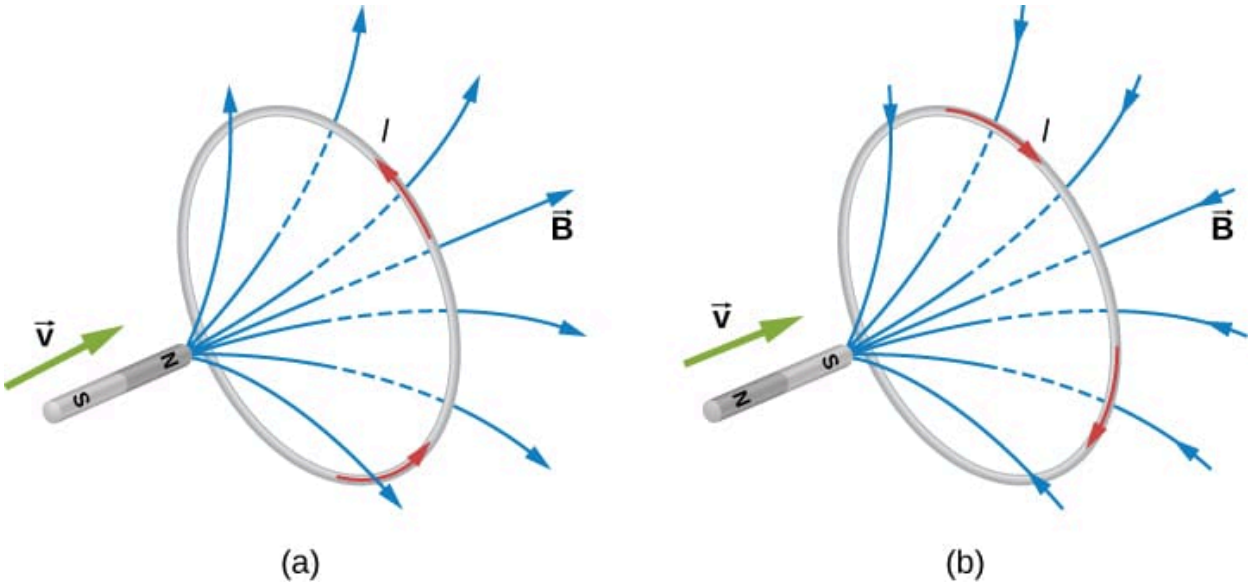


Figure 16.3.1: The change in magnetic flux caused by the approaching magnet induces a current in the loop. (a) An approaching north pole induces a counterclockwise current with respect to the bar magnet. (b) An approaching south pole induces a clockwise current with respect to the bar magnet.

Part (b) of the figure shows the south pole of a magnet moving toward a conducting loop. In this case, the flux through the loop due to the field of the magnet increases because the number of field lines directed from the back to the front of the loop is increasing. To oppose this change, a current is induced in the loop whose field lines through the loop are directed from the front to the back. Equivalently, we can say that the current flows in a direction so that the face of the loop nearer the approaching magnet is a south pole, which then repels the approaching south pole of the magnet. By RHR-2, your thumb points away from the bar magnet. Your fingers wrap in a clockwise fashion, which is the direction of the induced current.

Another example illustrating the use of Lenz's law is shown in Figure 16.3.2. When the switch is opened, the decrease in current through the solenoid causes a decrease in magnetic flux through its coils, which induces an emf in the solenoid. This emf must oppose the change (the termination of the current) causing it. Consequently, the induced emf has the polarity shown and drives in the direction of the original current. This may generate an arc across the terminals of the switch as it is opened.

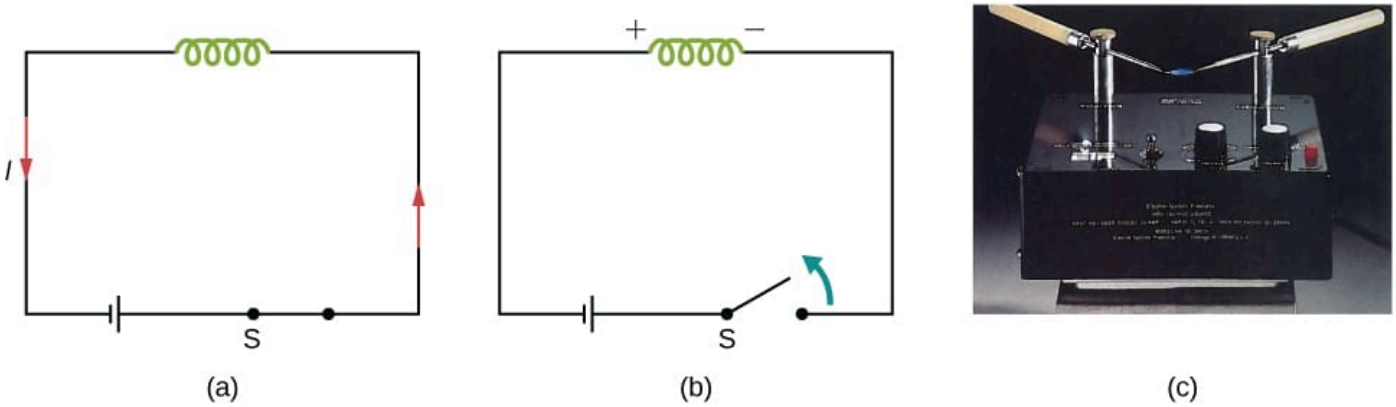


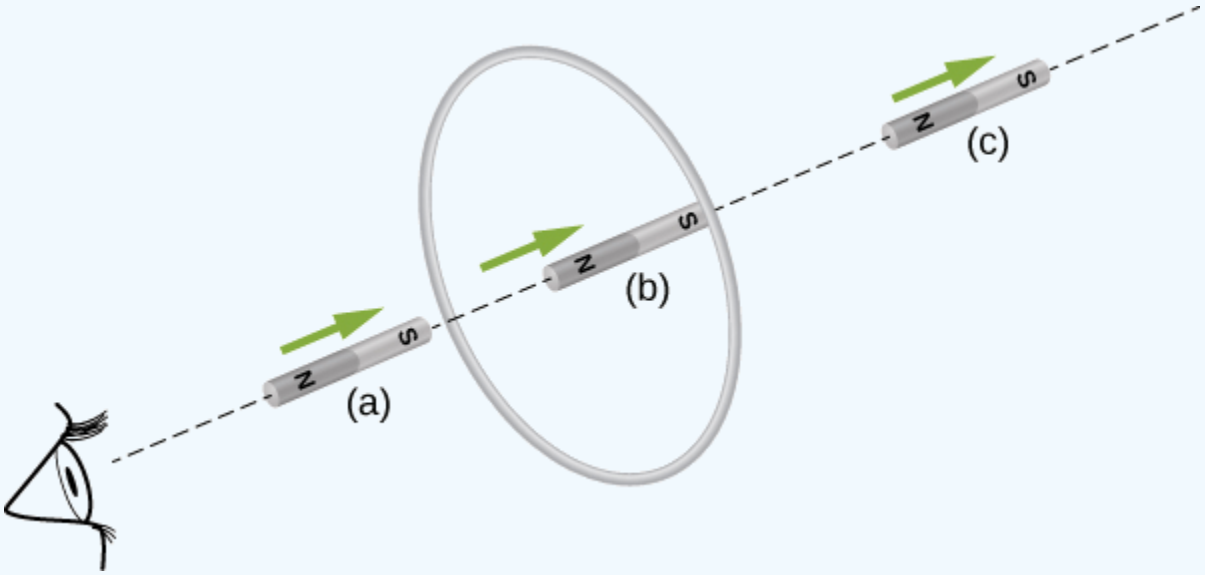
Figure 16.3.2: (a) A solenoid connected to a source of emf. (b) Opening switch S terminates the current, which in turn induces an emf in the solenoid. (c) A potential difference between the ends of the sharply pointed rods is produced by inducing an emf in a coil. This potential difference is large enough to produce an arc between the sharp points.

Exercise $16.3.1A$

Find the direction of the induced current in the wire loop shown below as the magnet enters, passes through, and leaves the loop.

Solution

To the observer shown, the current flows clockwise as the magnet approaches, decreases to zero when the magnet is centered in the plane of the coil, and then flows counterclockwise as the magnet leaves the coil.



Exercise $16.3.1B$

Verify the directions of the induced currents in Figure 13.2.2.

Example $16.3.1A$: A Circular Coil in a Changing Magnetic Field

A magnetic field $\vec{B}$ is directed outward perpendicular to the plane of a circular coil of radius $r = 0.50\ m$ (Figure $16.3.3$). The field is cylindrically symmetrical with respect to the center of the coil, and its magnitude decays exponentially according to $B = (1.5T)e^{(5.0s^{-1})t}$, where **B** is in teslas and **t** is in seconds.

a. Calculate the emf induced in the coil at the times $t_1 = 0$, $t_2 = 5.0 \times 10^{-2}s$, and $t_3 = 1.0\ s$.
b. Determine the current in the coil at these three times if its resistance is $10\ \Omega$.

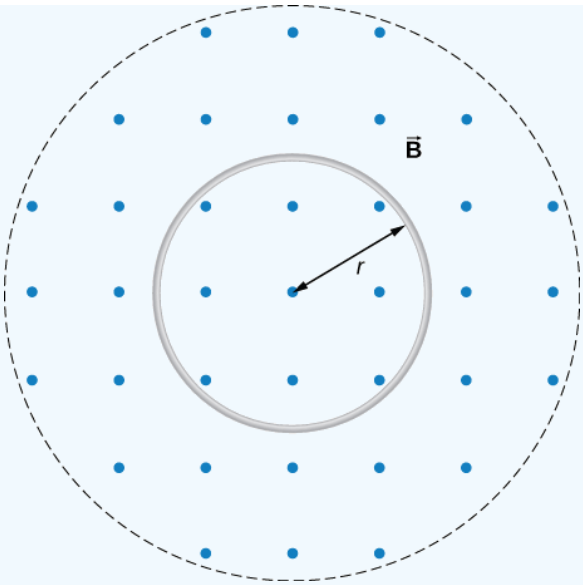


Figure 16.3.3: A circular coil in a decreasing magnetic field.

Strategy

Since the magnetic field is perpendicular to the plane of the coil and constant over each spot in the coil, the dot product of the magnetic field $\vec{B}$ and normal to the area unit vector $\hat{n}$ turns into a multiplication. The magnetic field can be pulled out of the integration, leaving the flux as the product of the magnetic field times area. We need to take the time derivative of the exponential function to calculate the emf using Faraday's law. Then we use Ohm's law to calculate the current.

Solution

1. Since $\vec{B}$ is perpendicular to the plane of the coil, the magnetic flux is given by

$$\Phi_m = B\pi r^2 = (1.5e^{-5.0t}T)\pi(0.50\,m)^2$$
$$= 1.2e^{-(5.0s^{-1})t}Wb.$$

From Faraday's law, the magnitude of the induced emf is

$$\epsilon = \left|\frac{d\Phi_m}{dt}\right| = \left|\frac{d}{dt}(1.2e^{-(5.0s^{-1})t}Wb)\right| = 6.0e^{-(5.0s^{-1})t}V.$$

Since $\vec{B}$ is directed out of the page and is decreasing, the induced current must flow counterclockwise when viewed from above so that the magnetic field it produces through the coil also points out of the page. For all three times, the sense of ε is counterclockwise; its magnitudes are

$$\epsilon(t_1) = 6.0V;\ \epsilon(t_2) = 4.7\,V;\ \epsilon(t_3) = 0040\,V.$$

2. From Ohm's law, the respective currents are

$$I(t_1) = \frac{\epsilon(t_1)}{R} = \frac{6.0\,V}{10\,\Omega} = 0.60\,A;$$
$$I(t_2) = \frac{4.7\,V}{10\,\Omega} = 0.47\,A;$$

and

$$I(t_3) = \frac{0.040\,V}{10\,\Omega} = 4.0\times10^{-3}\,A.$$

Significance

An emf voltage is created by a changing magnetic flux over time. If we know how the magnetic field varies with time over a constant area, we can take its time derivative to calculate the induced emf.

Example $16.3.1B$: Changing Magnetic Field Inside a Solenoid

The current through the windings of a solenoid with $n = 2000$ turns per meter is changing at a rate $dI/dt = 3.0\ A/s$. (See Sources of Magnetic Fields for a discussion of solenoids.) The solenoid is 50-cm long and has a cross-sectional diameter of 3.0 cm. A small coil consisting of $N = 20$ closely wound turns wrapped in a circle of diameter 1.0 cm is placed in the middle of the solenoid such that the plane of the coil is perpendicular to the central axis of the solenoid. Assuming that the infinite-solenoid approximation is valid at the location of the small coil, determine the magnitude of the emf induced in the coil.

Strategy

The magnetic field in the middle of the solenoid is a uniform value of $\mu_0 nI$. This field is producing a maximum magnetic flux through the coil as it is directed along the length of the solenoid. Therefore, the magnetic flux through the coil is the product of the solenoid's magnetic field times the area of the coil. Faraday's law involves a time derivative of the magnetic flux. The only quantity varying in time is the current, the rest can be pulled out of the time derivative. Lastly, we include the number of turns in the coil to determine the induced emf in the coil.

Solution

Since the field of the solenoid is given by $B = \mu_0 nI$, the flux through each turn of the small coil is

$$\Phi_m = \mu_0 nI\left(\frac{\pi d^2}{4}\right),$$

where d is the diameter of the coil. Now from Faraday's law, the magnitude of the emf induced in the coil is

$$\begin{aligned}\epsilon &= \left|N\frac{d\Phi_m}{dt}\right| \\ &= \left|N\mu_0 n\frac{\pi d^2}{4}\frac{dI}{dt}\right| \\ &= 20(4\pi\times 10^{-7} T\cdot m/s)(2000\, m^{-1})\frac{\pi(0.010\, m)^2}{4}(3.0\, A/s) \\ &= 1.2\times 10^{-5}\, V.\end{aligned}$$

Significance

When the current is turned on in a vertical solenoid, as shown in Figure $16.3.4$, the ring has an induced emf from the solenoid's changing magnetic flux that opposes the change. The result is that the ring is fired vertically into the air.

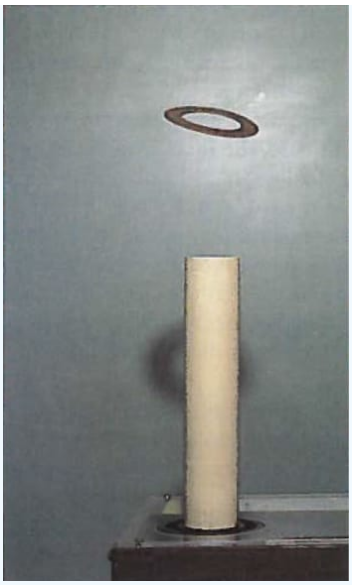


Figure 16.3.4: The jumping ring. When a current is turned on in the vertical solenoid, a current is induced in the metal ring. The stray field produced by the solenoid causes the ring to jump off the solenoid.

Note

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

A demonstration of the jumping ring from MIT.

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16.4: Motional Emf

Learning Objectives

By the end of this section, you will be able to:

- Determine the magnitude of an induced emf in a wire moving at a constant speed through a magnetic field
- Discuss examples that use motional emf, such as a rail gun and a tethered satellite

Magnetic flux depends on three factors: the strength of the magnetic field, the area through which the field lines pass, and the orientation of the field with the surface area. If any of these quantities varies, a corresponding variation in magnetic flux occurs. So far, we've only considered flux changes due to a changing field. Now we look at another possibility: a changing area through which the field lines pass including a change in the orientation of the area.

Two examples of this type of flux change are represented in Figure 16.4.1. In part (a), the flux through the rectangular loop increases as it moves into the magnetic field, and in part (b), the flux through the rotating coil varies with the angle θ.

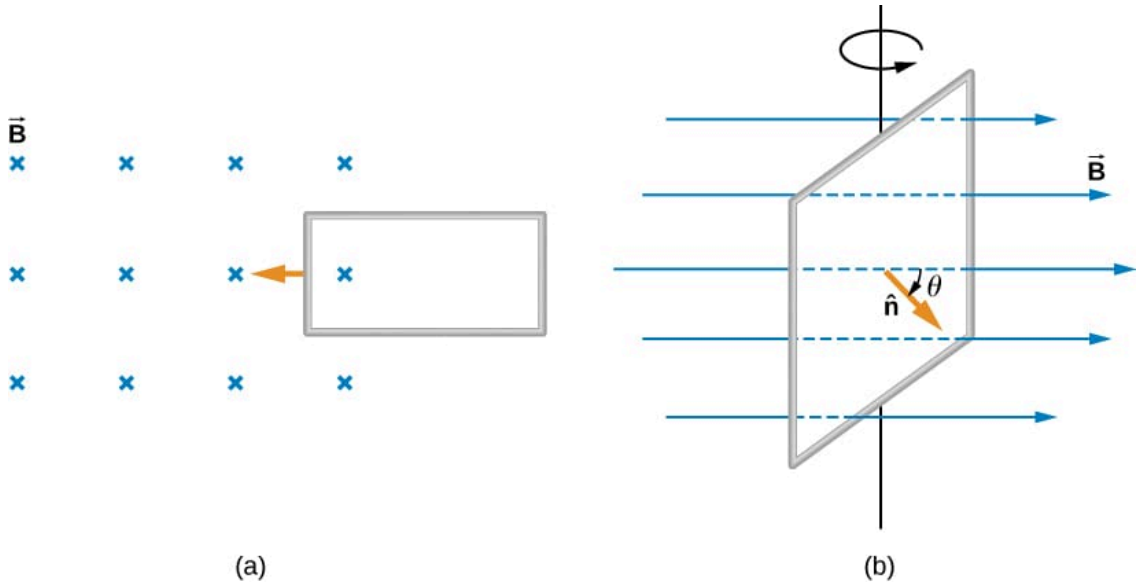


Figure 16.4.1: (a) Magnetic flux changes as a loop moves into a magnetic field; (b) magnetic flux changes as a loop rotates in a magnetic field.

It's interesting to note that what we perceive as the cause of a particular flux change actually depends on the frame of reference we choose. For example, if you are at rest relative to the moving coils of Figure 16.4.1*b*, you would see the flux vary because of a changing magnetic field—in part (a), the field moves from left to right in your reference frame, and in part (b), the field is rotating. It is often possible to describe a flux change through a coil that is moving in one particular reference frame in terms of a changing magnetic field in a second frame, where the coil is stationary. However, reference-frame questions related to magnetic flux are beyond the level of this textbook. We'll avoid such complexities by always working in a frame at rest relative to the laboratory and explain flux variations as due to either a changing field or a changing area.

Now let's look at a conducting rod pulled in a circuit, changing magnetic flux. The area enclosed by the circuit 'MNOP' of Figure 16.4.2 is **lx** and is perpendicular to the magnetic field, so we can simplify the integration of $\Phi_{\mathrm{m}} = \int_S \vec{\mathbf{B}} \cdot \hat{\mathbf{n}} dA$ into a multiplication of magnetic field and area. The magnetic flux through the open surface is therefore

$$\Phi_m = B\ l\ x.$$

Since B and l are constant and the velocity of the rod is $v = dx/dt$, we can now restate Faraday's law, Equation 13.2.2, for the magnitude of the emf in terms of the moving conducting rod as

$$\epsilon = \frac{d\Phi_m}{dt} = Bl\frac{dx}{dt} = Blv.$$

The current induced in the circuit is the emf divided by the resistance or

$$I = \frac{Blv}{R}.$$

Furthermore, the direction of the induced emf satisfies Lenz's law, as you can verify by inspection of the figure.

This calculation of motionally induced emf is not restricted to a rod moving on conducting rails. With $\vec{F} = q\vec{v} \times \vec{B}$ as the starting point, it can be shown that $\epsilon = -d\Phi_m/dt$ holds for any change in flux caused by the motion of a conductor. We saw in Faraday's Law that the emf induced by a time-varying magnetic field obeys this same relationship, which is Faraday's law. Thus Faraday's law **holds for all flux changes**, whether they are produced by a changing magnetic field, by motion, or by a combination of the two.

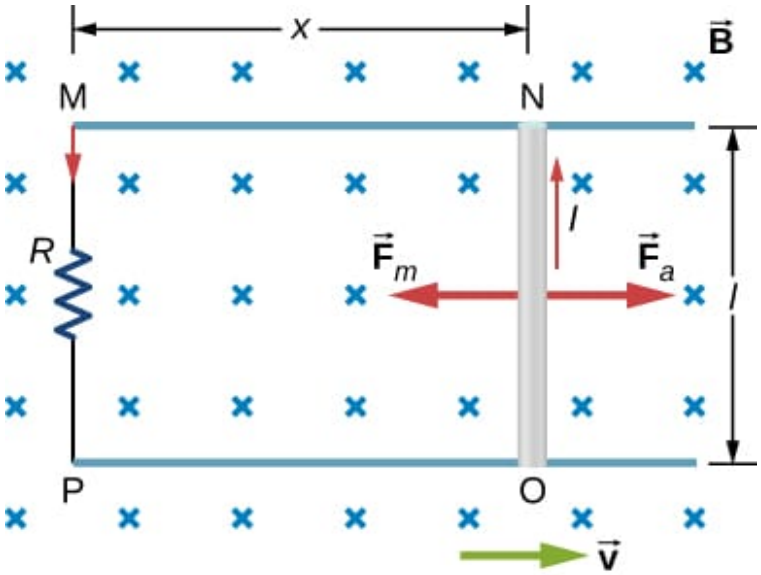


Figure 16.4.2: A conducting rod is pushed to the right at constant velocity. The resulting change in the magnetic flux induces a current in the circuit.

From an energy perspective, $\vec{F}_a$ produces power $F_a v$, and the resistor dissipates power I^2R. Since the rod is moving at constant velocity, the applied force F_a must balance the magnetic force $F_m = IlB$ on the rod when it is carrying the induced current **I**. Thus the power produced is

$$F_a v = IlBv = \frac{Blv}{R} \cdot lBv = \frac{l^2B^2v^2}{R}.$$

The power dissipated is

$$P = I^2R = \left(\frac{Blv}{R}\right)^2 R = \frac{l^2B^2v^2}{R}.$$

In satisfying the principle of energy conservation, the produced and dissipated powers are equal.

This principle can be seen in the operation of a rail gun. A **rail gun** is an electromagnetic projectile launcher that uses an apparatus similar to Figure 16.4.2 and is shown in schematic form in Figure 16.4.3. The conducting rod is replaced with a projectile or weapon to be fired. So far, we've only heard about how motion causes an emf. In a rail gun, the optimal shutting off/ramping down of a magnetic field decreases the flux in between the rails, causing a current to flow in the rod (armature) that holds the projectile. This current through the armature experiences a magnetic force and is propelled forward. Rail guns, however, are not used widely in the military due to the high cost of production and high currents: Nearly one million amps is required to produce enough energy for a rail gun to be an effective weapon.

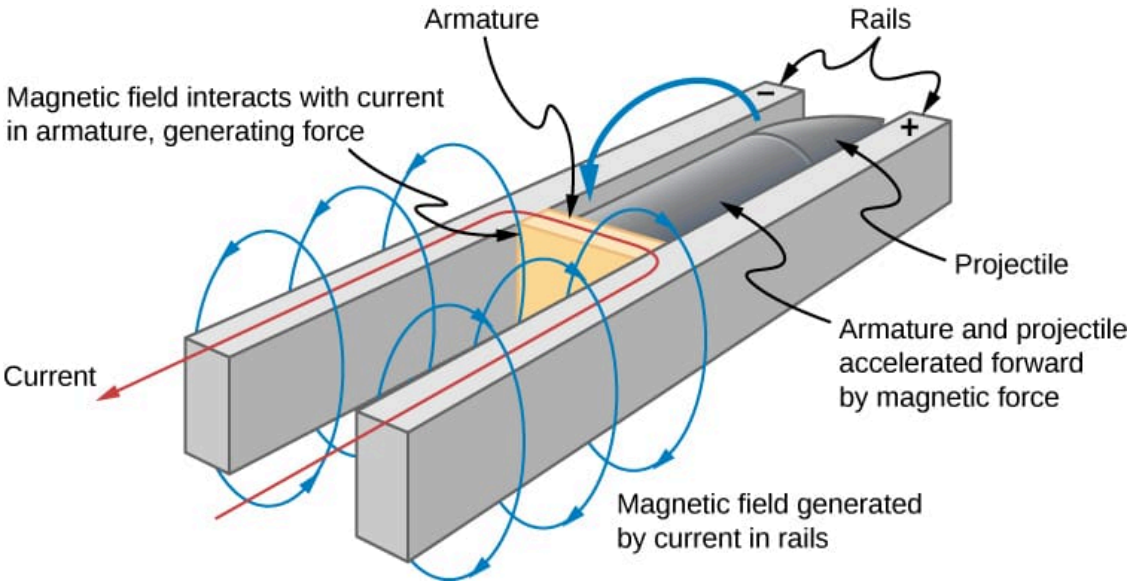


Figure 16.4.3: Current through two rails drives a conductive projectile forward by the magnetic force created.

We can calculate a motionally induced emf with Faraday's law **even when an actual closed circuit is not present**. We simply imagine an enclosed area whose boundary includes the moving conductor, calculate Φ_m, and then find the emf from Faraday's law. For example, we can let the moving rod of Figure 16.4.4 be one side of the imaginary rectangular area represented by the dashed lines. The area of the rectangle is **lx**, so the magnetic flux through it is $\Phi_m = Blx$. Differentiating this equation, we obtain

$$\frac{d\Phi_m}{dt} = Bl\frac{dx}{dt} = Blv,$$

which is identical to the potential difference between the ends of the rod that we determined earlier.

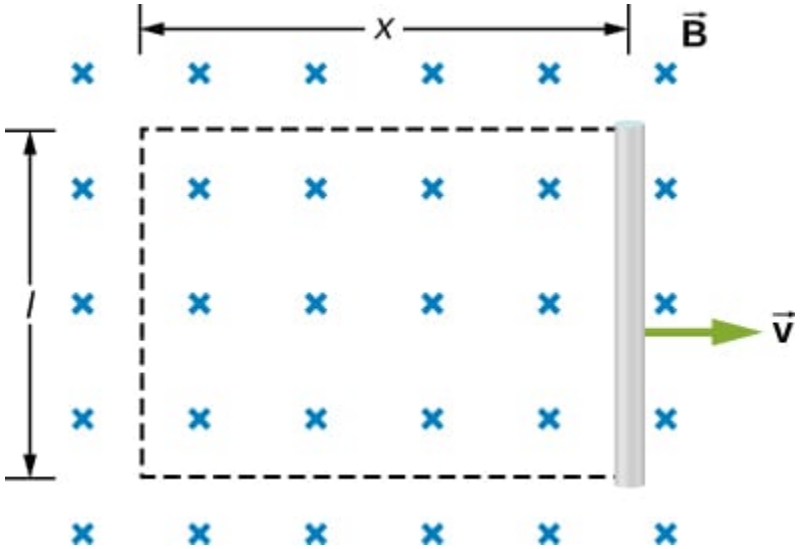


Figure 16.4.4: With the imaginary rectangle shown, we can use Faraday's law to calculate the induced emf in the moving rod.

Motional emfs in Earth's weak magnetic field are not ordinarily very large, or we would notice voltage along metal rods, such as a screwdriver, during ordinary motions. For example, a simple calculation of the motional emf of a 1.0-m rod moving at 3.0 m/s perpendicular to the Earth's field gives

$$emf = Blv = (5.0 \times 10^{-5} T)(1.0\,m)(3.0\,m/s) = 150\,\mu V.$$

This small value is consistent with experience. There is a spectacular exception, however. In 1992 and 1996, attempts were made with the space shuttle to create large motional emfs. The tethered satellite was to be let out on a 20-km length of wire, as shown in Figure 16.4.5, to create a 5-kV emf by moving at orbital speed through Earth's field. This emf could be used to convert some of

the shuttle's kinetic and potential energy into electrical energy if a complete circuit could be made. To complete the circuit, the stationary ionosphere was to supply a return path through which current could flow. (The ionosphere is the rarefied and partially ionized atmosphere at orbital altitudes. It conducts because of the ionization. The ionosphere serves the same function as the stationary rails and connecting resistor in Figure 16.4.3, without which there would not be a complete circuit.) Drag on the current in the cable due to the magnetic force $F = IlBsin\ \theta$ does the work that reduces the shuttle's kinetic and potential energy, and allows it to be converted into electrical energy. Both tests were unsuccessful. In the first, the cable hung up and could only be extended a couple of hundred meters; in the second, the cable broke when almost fully extended. Example 16.4.1 indicates feasibility in principle.

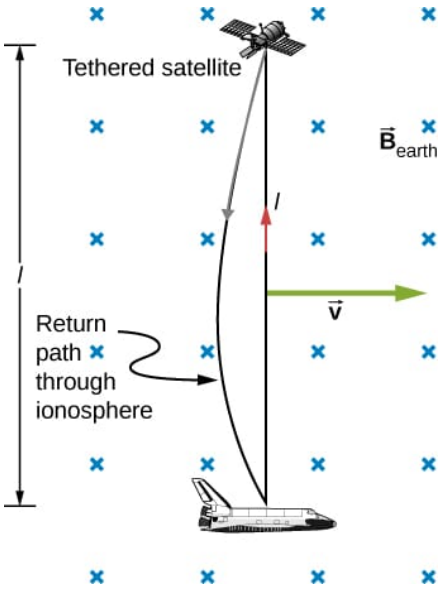


Figure 16.4.5: Motional emf as electrical power conversion for the space shuttle was the motivation for the tethered satellite experiment. A 5-kV emf was predicted to be induced in the 20-km tether while moving at orbital speed in Earth's magnetic field. The circuit is completed by a return path through the stationary ionosphere.

Example 16.4.1: Calculating the Large Motional Emf of an Object in Orbit

Calculate the motional emf induced along a 20.0-km conductor moving at an orbital speed of 7.80 km/s perpendicular to Earth's $5.00 \times 10^{-5} T$ magnetic field.

Strategy

This is a great example of using the equation motional $\epsilon = B\ l\ v$.

Solution

Entering the given values into $\epsilon = Blv$ gives

$$\begin{aligned} \epsilon &= Blv \\ &= (5.00 \times 10^{-5} T)(2.00 \times 10^{4}\, m)(7.80 \times 10^{3}\, m/s) \\ &= 7.80 \times 10^{3}\, V. \end{aligned}$$

Significance

The value obtained is greater than the 5-kV measured voltage for the shuttle experiment, since the actual orbital motion of the tether is not perpendicular to Earth's field. The 7.80-kV value is the maximum emf obtained when $\theta = 90^{o}$ and so $sin\ \theta = 1$.

Example 16.4.2: A Metal Rod Rotating in a Magnetic Field

Part (a) of Figure 16.4.6 shows a metal rod **OS** that is rotating in a horizontal plane around point **O**. The rod slides along a wire that forms a circular arc **PST** of radius r. The system is in a constant magnetic field $\vec{B}$ that is directed out of the page.

a. If you rotate the rod at a constant angular velocity ω, what is the current I in the closed loop **OPSO**? Assume that the resistor R furnishes all of the resistance in the closed loop.
b. Calculate the work per unit time that you do while rotating the rod and show that it is equal to the power dissipated in the resistor.

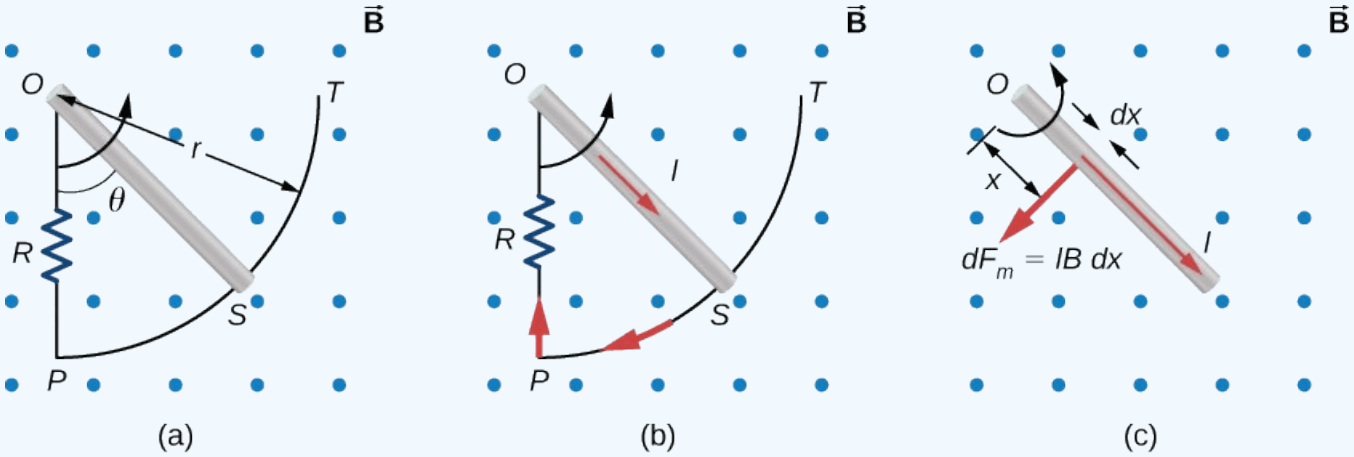


Figure 16.4.6: (a) The end of a rotating metal rod slides along a circular wire in a horizontal plane. (b) The induced current in the rod. (c) The magnetic force on an infinitesimal current segment.

Strategy

The magnetic flux is the magnetic field times the area of the quarter circle or $A = r^2\theta/2$. When finding the emf through Faraday's law, all variables are constant in time but θ, with $\omega = d\theta/dt$. To calculate the work per unit time, we know this is related to the torque times the angular velocity. The torque is calculated by knowing the force on a rod and integrating it over the length of the rod.

Solution

1. From geometry, the area of the loop **OPSO** is $A = \frac{r^2\theta}{2}$. Hence, the magnetic flux through the loop is

$$\Phi_m = BA = B\frac{r^2\theta}{2}.$$

Differentiating with respect to time and using $\omega = d\theta/dt$, we have

$$\epsilon = \left|\frac{d\Phi_m}{dt}\right| = \frac{Br^2\omega}{2}.$$

When divided by the resistance **R** of the loop, this yields for the magnitude of the induced current

$$I = \frac{\epsilon}{R} = \frac{Br^2\omega}{2R}.$$

As θ increases, so does the flux through the loop due to $\vec{B}$. To counteract this increase, the magnetic field due to the induced current must be directed into the page in the region enclosed by the loop. Therefore, as part (b) of Figure 16.4.6 illustrates, the current circulates clockwise.

2. You rotate the rod by exerting a torque on it. Since the rod rotates at constant angular velocity, this torque is equal and opposite to the torque exerted on the current in the rod by the original magnetic field. The magnetic force on the infinitesimal segment of length **dx** shown in part (c) of Figure 16.4.6 is $dF_m = IBdx$, so the magnetic torque on this segment is

$$d\tau_m = x \cdot dF_m = IBxdx.$$

The net magnetic torque on the rod is then

$$\tau_m = \int_0^r d\tau_m = IB\int_0^r x\,dx = \frac{1}{2}IBr^2.$$

The torque τ that you exert on the rod is equal and opposite to τ_m, and the work that you do when the rod rotates through an angle $d\theta$ is $dW = rd\theta$. Hence, the work per unit time that you do on the rod is

$$\frac{dW}{dt} = \tau\frac{d\theta}{dt} = \frac{1}{2}IBr^2\frac{d\theta}{dt} = \frac{1}{2}\left(\frac{Br^2\omega}{2R}\right)Br^2\omega = \frac{B^2r^4\omega^2}{4R},$$

where we have substituted for **I**. The power dissipated in the resister is $P = IR^2$, which can be written as

$$P = \left(\frac{Br^2\omega}{2R}\right)^2 R = \frac{B^2r^4\omega^2}{4R}.$$

Therefore, we see that

$$P = \frac{dW}{dt}.$$

Hence, the power dissipated in the resistor is equal to the work per unit time done in rotating the rod.

Significance

An alternative way of looking at the induced emf from Faraday's law is to integrate in space instead of time. The solution, however, would be the same. The motional emf is

$$|\epsilon| = \int B\ v\ dl.$$

The velocity can be written as the angular velocity times the radius and the differential length written as dr. Therefore,

$$\epsilon| = B\int vdr = B\omega\int_0^l rdr = \frac{1}{2}B\omega l^2,$$

which is the same solution as before.

Example 16.4.3: A Rectangular Coil Rotating in a Magnetic Field

A rectangular coil of area **A** and **N** turns is placed in a uniform magnetic field $\vec{B} = B\hat{j}$, as shown in Figure 16.4.7. The coil is rotated about the **z**-axis through its center at a constant angular velocity ω. Obtain an expression for the induced emf in the coil.

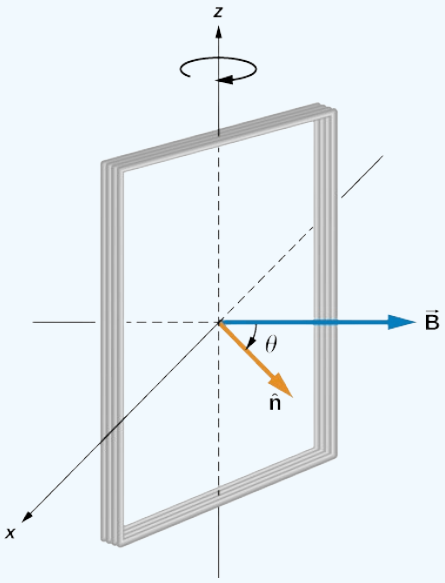


Figure 16.4.7: A rectangular coil rotating in a uniform magnetic field.

Strategy

According to the diagram, the angle between the perpendicular to the surface ($\hat{n}$) and the magnetic field ($\vec{B}$) is θ. The dot product of $B \cdot \hat{n}$ simplifies to only the $\cos\ \theta$ component of the magnetic field, namely where the magnetic field projects onto the unit area vector $\hat{n}$. The magnitude of the magnetic field and the area of the loop are fixed over time, which makes the integration simplify quickly. The induced emf is written out using Faraday's law.

Solution

When the coil is in a position such that its normal vector $\hat{n}$ makes an angle θ with the magnetic field $\vec{B}$ the magnetic flux through a single turn of the coil is

$$\Phi_m = \int_S \vec{B} \cdot \hat{n} dA = BA \cos\ \theta.$$

From Faraday's law, the emf induced in the coil is

$$\epsilon = -N\frac{d\Phi_m}{dt} = NBA \sin\ \theta \frac{d\theta}{dt}.$$

The constant angular velocity is $\omega = d\theta/dt$. The angle θ represents the time evolution of the angular velocity or ωt. This is changes the function to time space rather than θ. The induced emf therefore varies sinusoidally with time according to

$$\epsilon = \epsilon_0 \sin\ \omega t,$$

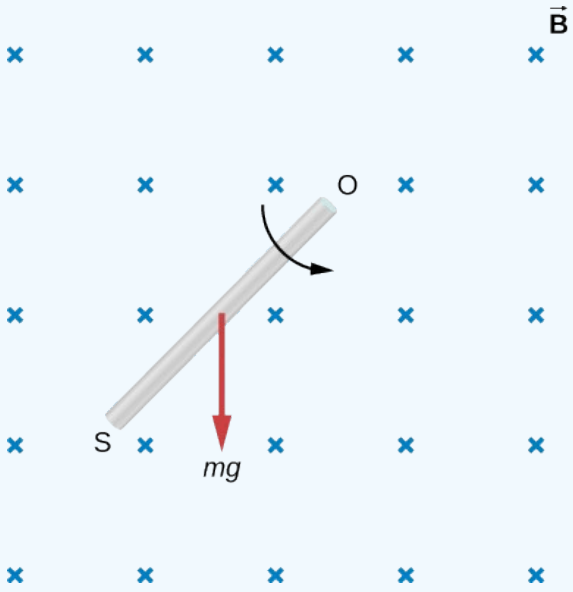
where $\epsilon_0 = NBA\omega$.

Significance

If the magnetic field strength or area of the loop were also changing over time, these variables wouldn't be able to be pulled out of the time derivative to simply the solution as shown. This example is the basis for an electric generator, as we will give a full discussion in Applications of Newton's Law.

Exercise 16.4.1

Shown below is a rod of length **l** that is rotated counterclockwise around the axis through **O** by the torque due to $m\vec{g}$. Assuming that the rod is in a uniform magnetic field $\vec{B}$, what is the emf induced between the ends of the rod when its angular velocity is ω? Which end of the rod is at a higher potential?



Answer available at the end of this section.

Exercise 16.4.2

A rod of length 10 cm moves at a speed of 10 m/s perpendicularly through a 1.5-T magnetic field. What is the potential difference between the ends of the rod?

Answer available at the end of this section.

Answers

Exercise 16.4.1

$\epsilon = Bl^2\omega/2$, with **O** at a higher potential than **S**

Exercise 16.4.2

1.5 V

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16.5: Induced Electric Fields

Learning Objectives

By the end of this section, you will be able to:

- Connect the relationship between an induced emf from Faraday's law to an electric field, thereby showing that a changing magnetic flux creates an electric field
- Solve for the electric field based on a changing magnetic flux in time

The fact that emfs are induced in circuits implies that work is being done on the conduction electrons in the wires. What can possibly be the source of this work? We know that it's neither a battery nor a magnetic field, for a battery does not have to be present in a circuit where current is induced, and magnetic fields never do work on moving charges. The answer is that the source of the work is an electric field $\vec{E}$ that is induced in the wires. The work done by $\vec{E}$ in moving a unit charge completely around a circuit is the induced emf ε; that is,

$$\epsilon = \oint \vec{E} \cdot d\vec{l},$$

where $\oint$ represents the line integral around the circuit. Faraday's law can be written in terms of the induced electric field as

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_m}{dt}.$$

There is an important distinction between the electric field induced by a changing magnetic field and the electrostatic field produced by a fixed charge distribution. Specifically, the induced electric field is nonconservative because it does net work in moving a charge over a closed path, whereas the electrostatic field is conservative and does no net work over a closed path. Hence, electric potential can be associated with the electrostatic field, but not with the induced field. The following equations represent the distinction between the two types of electric field:

$$\underbrace{\oint \vec{E} \cdot d\vec{l} \neq 0}_{\text{Induced Electric Field}}$$

$$\underbrace{\oint \vec{E} \cdot d\vec{l} = 0}_{\text{Electrostatic Electric Fields}}.$$

Our results can be summarized by combining these equations:

$$\epsilon = \oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_m}{dt}. \tag{16.5.1}$$

Example 16.5.1: Induced Electric Field in a Circular Coil

What is the induced electric field in the circular coil of Example 13.3.1A (and Figure 13.3.3) at the three times indicated?

Strategy

Using cylindrical symmetry, the electric field integral simplifies into the electric field times the circumference of a circle. Since we already know the induced emf, we can connect these two expressions by Faraday's law to solve for the induced electric field.

Solution

The induced electric field in the coil is constant in magnitude over the cylindrical surface, similar to how Ampere's law problems with cylinders are solved. Since $\vec{E}$ is tangent to the coil,

$$\oint \vec{E} \cdot d\vec{l} = \oint E dl = 2\pi r E.$$

When combined with Equation 16.5.1, this gives

$$E = \frac{\epsilon}{2\pi r}.$$

The direction of ϵ is counterclockwise, and $\vec{E}$ circulates in the same direction around the coil. The values of **E** are

$$E(t_1) = \frac{6.0\,V}{2\pi\,(0.50\,m)} = 1.9\,V/m;$$

$$E(t_2) = \frac{4.7\,V}{2\pi\,(0.50\,m)} = 1.5\,V/m;$$

$$E(t_3) = \frac{0.040\,V}{2\pi\,(0.50\,m)} = 0.013\,V/m;$$

Significance

When the magnetic flux through a circuit changes, a nonconservative electric field is induced, which drives current through the circuit. But what happens if $dB/dt \neq 0$ in free space where there isn't a conducting path? The answer is that this case can be treated **as if a conducting path were present**; that is, nonconservative electric fields are induced wherever $dB/dt \neq 0$ whether or not there is a conducting path present.

These nonconservative electric fields always satisfy Equation 16.5.1. For example, if the circular coil were removed, an electric field **in free space** at $r = 0.50\ m$ would still be directed counterclockwise, and its magnitude would still be 1.9 V/m at $t = 0$. 1.5 V/m at $t = 5.0 \times 10^{-2} s$, etc. The existence of induced electric fields is certainly **not** restricted to wires in circuits.

Example 16.5.2: Electric Field Induced by the Changing Magnetic Field of a Solenoid

Figure 16.5.1a shows a long solenoid with radius R and n turns per unit length; its current decreases with time according to $I = I_0 e^{-\alpha t}$. What is the magnitude of the induced electric field at a point a distance **r** from the central axis of the solenoid (a) when $r > R$ and (b) when $r < R$ [Figure 16.5.1b]. (c) What is the direction of the induced field at both locations? Assume that the infinite-solenoid approximation is valid throughout the regions of interest.

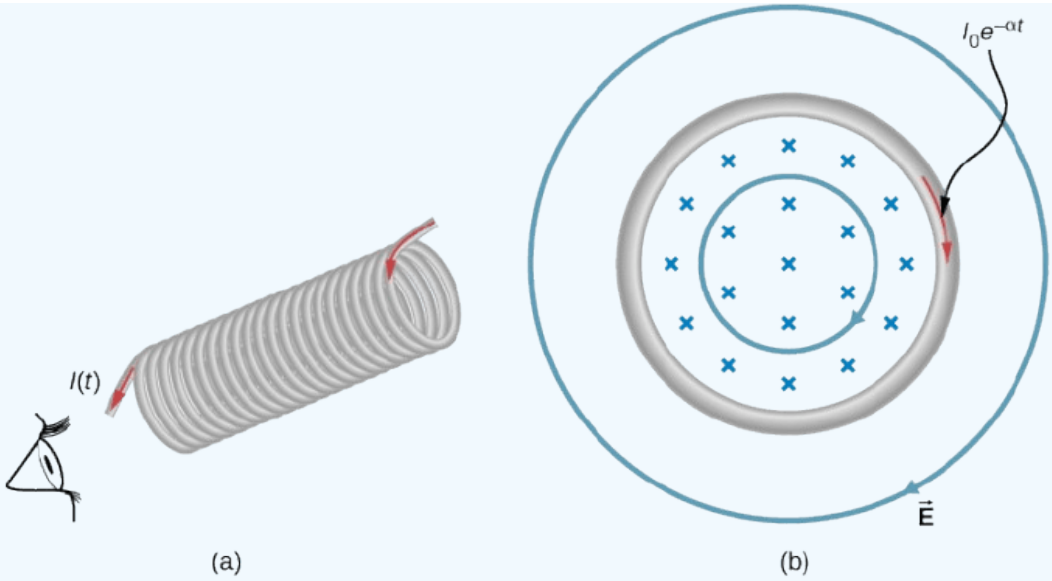


Figure 16.5.1: (a) The current in a long solenoid is decreasing exponentially. (b) A cross-sectional view of the solenoid from its left end. The cross-section shown is near the middle of the solenoid. An electric field is induced both inside and outside the solenoid.

Strategy

Using the formula for the magnetic field inside an infinite solenoid and Faraday's law, we calculate the induced emf. Since we have cylindrical symmetry, the electric field integral reduces to the electric field times the circumference of the integration path. Then we solve for the electric field.

Solution

a. The magnetic field is confined to the interior of the solenoid where

$$B = \mu_0 nI = \mu_0 nI_0 e^{-\alpha t}.$$

Thus, the magnetic flux through a circular path whose radius r is greater than R, the solenoid radius, is

$$\Phi_m = BA = \mu_0 nI_0 \pi R^2 e^{-\alpha t}.$$

The induced field $\vec{E}$ is tangent to this path, and because of the cylindrical symmetry of the system, its magnitude is constant on the path. Hence, we have

$$\left| \oint \vec{E} \cdot d\vec{l} \right| = \left| \frac{d\Phi_m}{dt} \right|,$$

$$E(2\pi r) = \left| \frac{d}{dt}(\mu_0 nI_0 \pi R^2 e^{-\alpha t}) \right| = \alpha \mu_0 nI_0 \pi R^2 e^{-\alpha t},$$

$$E = \frac{\alpha \mu_0 nI_0 R^2}{2r} e^{-\alpha t} \, (r > R).$$

b. For a path of radius **r** inside the solenoid, $\Phi_m = B\pi r^2$, so

$$E(2\pi r) = \left| \frac{d}{dt}(\mu_0 nI_0 \pi r^2 e^{-\alpha t}) \right| = \alpha \mu_0 nI_0 \pi r^2 e^{-\alpha t},$$

and the induced field is

$$E = \frac{\alpha \mu_0 nI_0 r}{2} e^{-\alpha t} \, (r < R).$$

c. The magnetic field points into the page as shown in part (b) and is decreasing. If either of the circular paths were occupied by conducting rings, the currents induced in them would circulate as shown, in conformity with Lenz's law. The induced electric field must be so directed as well.

Significance

In part (b), note that $|\vec{E}|$ increases with r inside and decreases as $1/r$ outside the solenoid, as shown in Figure $16.5.2$.

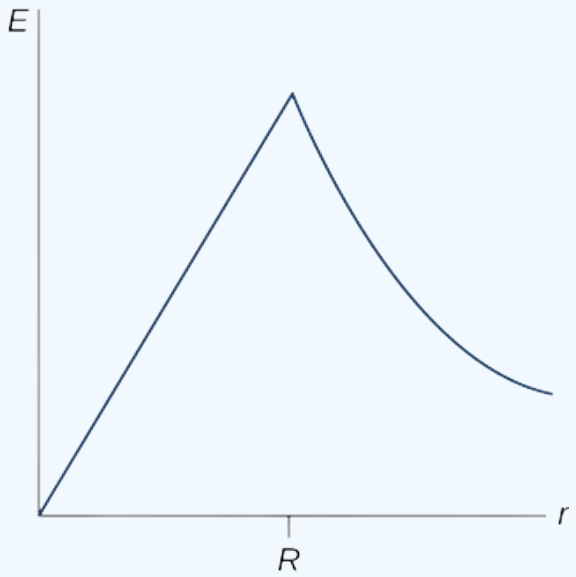


Figure $16.5.2$: The electric field vs. distance **r**. When $r < R$, the electric field rises linearly, whereas when $r > R$, the electric field falls of proportional to 1/**r**.

Exercise $16.5.1$

Suppose that the coil of Example 13.3.1A is a square rather than circular. Can Equation $16.5.1$ be used to calculate (a) the induced emf and (b) the induced electric field?

Answer available at the end of this section.

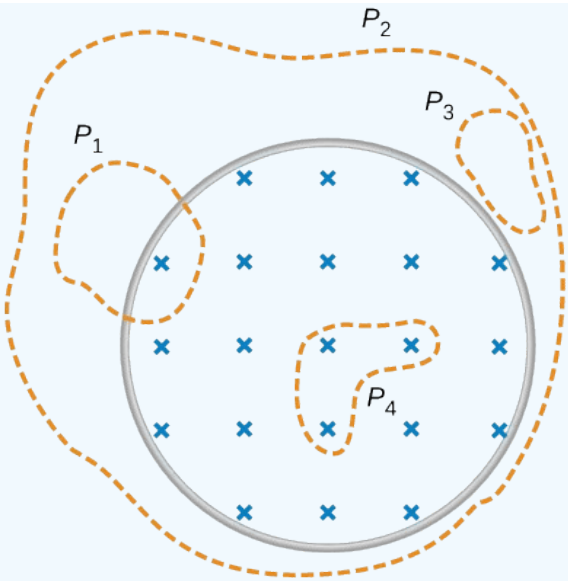
Exercise $16.5.2$

What is the magnitude of the induced electric field in Example $16.5.2$ at $t = 0$ if $r = 6.0\ cm$, $R = 2.0\ cm$, $n = 2000$ turns per meter, $I_0 = 2.0\ A$, and $\alpha = 200\ s^{-1}$?

Answer available at the end of this section.

Exercise $16.5.3$

The magnetic field shown below is confined to the cylindrical region shown and is changing with time. Identify those paths for which $\epsilon = \oint \vec{E} \cdot d\vec{l} \neq 0$.

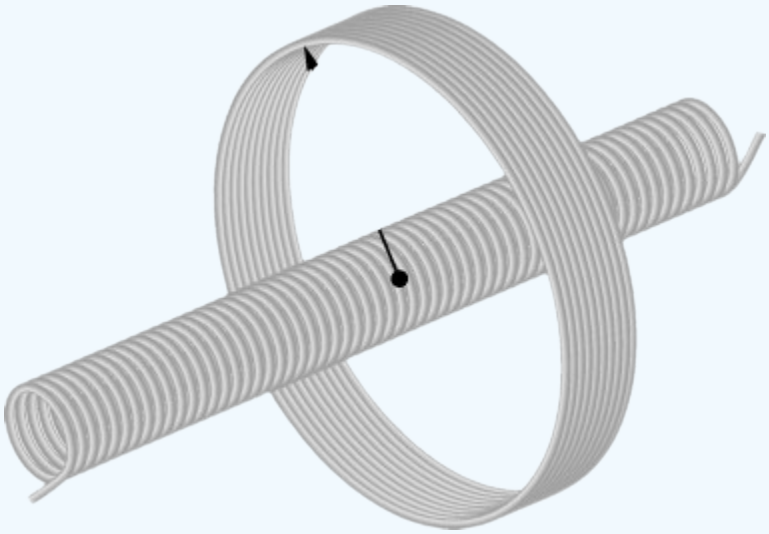


Answer available at the end of this section.

Exercise 16.5.1

A long solenoid of cross-sectional area $5.0\ cm^2$ is wound with 25 turns of wire per centimeter. It is placed in the middle of a closely wrapped coil of 10 turns and radius 25 cm, as shown below.

a. What is the emf induced in the coil when the current through the solenoid is decreasing at a rate $dI/dt = -0.20\ A/s$?
b. What is the electric field induced in the coil?



Answer available at the end of this section.

Answers

Exercise 16.5.1

a. yes; b. Yes; however there is a lack of symmetry between the electric field and coil, making $\oint \vec{E} \cdot d\vec{l}$ a more complicated relationship that can't be simplified as shown in the example.

Exercise 16.5.2

$3.4 \times 10^{-3}\ V/m$

Exercise 16.5.3

$P_1,\ P_2,\ P_4$

Exercise 16.5.1

a. $3.1 \times 10^{-6} V$; b. $2.0 \times 10^{-7}\ V/m$

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16.6: Eddy Currents

Learning Objectives

By the end of this section, you will be able to:

- Explain how eddy currents are created in metals
- Describe situations where eddy currents are beneficial and where they are not helpful

As discussed two sections earlier, a motional emf is induced when a conductor moves in a magnetic field or when a magnetic field moves relative to a conductor. If motional emf can cause a current in the conductor, we refer to that current as an **eddy current**.

Magnetic Damping

Eddy currents can produce significant drag, called **magnetic damping**, on the motion involved. Consider the apparatus shown in Figure 16.6.1, which swings a pendulum bob between the poles of a strong magnet. (This is another favorite physics demonstration.) If the bob is metal, significant drag acts on the bob as it enters and leaves the field, quickly damping the motion. If, however, the bob is a slotted metal plate, as shown in part (b) of the figure, the magnet produces a much smaller effect. There is no discernible effect on a bob made of an insulator. Why does drag occur in both directions, and are there any uses for magnetic drag?

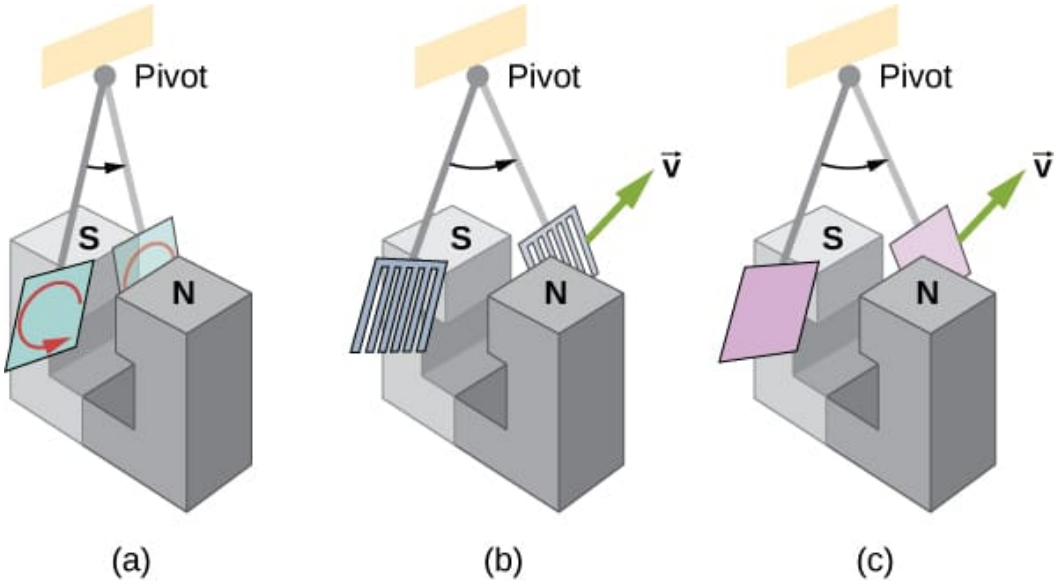


Figure 16.6.1: A common physics demonstration device for exploring eddy currents and magnetic damping. (a) The motion of a metal pendulum bob swinging between the poles of a magnet is quickly damped by the action of eddy currents. (b) There is little effect on the motion of a slotted metal bob, implying that eddy currents are made less effective. (c) There is also no magnetic damping on a nonconducting bob, since the eddy currents are extremely small.

Figure 16.6.2 shows what happens to the metal plate as it enters and leaves the magnetic field. In both cases, it experiences a force opposing its motion. As it enters from the left, flux increases, setting up an eddy current (Faraday's law) in the counterclockwise direction (Lenz's law), as shown. Only the right-hand side of the current loop is in the field, so an unopposed force acts on it to the left (RHR-1). When the metal plate is completely inside the field, there is no eddy current if the field is uniform, since the flux remains constant in this region. But when the plate leaves the field on the right, flux decreases, causing an eddy current in the clockwise direction that, again, experiences a force to the left, further slowing the motion. A similar analysis of what happens when the plate swings from the right toward the left shows that its motion is also damped when entering and leaving the field.

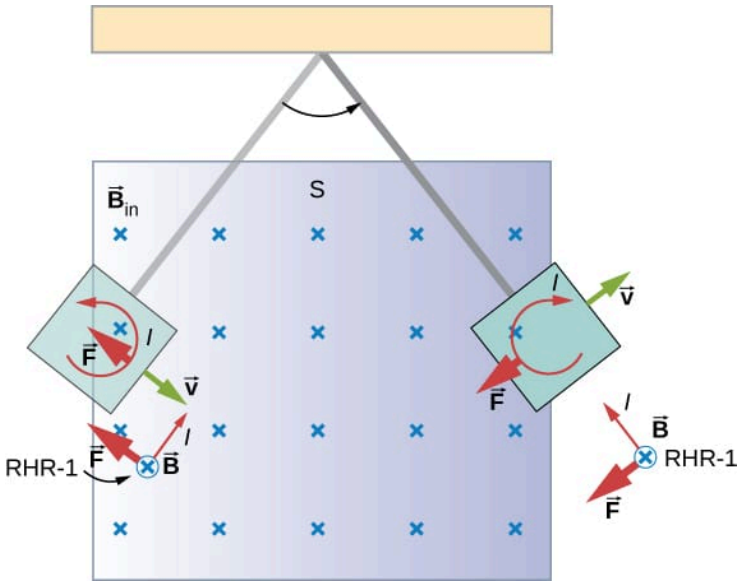


Figure 16.6.2: A more detailed look at the conducting plate passing between the poles of a magnet. As it enters and leaves the field, the change in flux produces an eddy current. Magnetic force on the current loop opposes the motion. There is no current and no magnetic drag when the plate is completely inside the uniform field.

When a slotted metal plate enters the field (Figure 16.6.3), an emf is induced by the change in flux, but it is less effective because the slots limit the size of the current loops. Moreover, adjacent loops have currents in opposite directions, and their effects cancel. When an insulating material is used, the eddy current is extremely small, so magnetic damping on insulators is negligible. If eddy currents are to be avoided in conductors, then they must be slotted or constructed of thin layers of conducting material separated by insulating sheets.

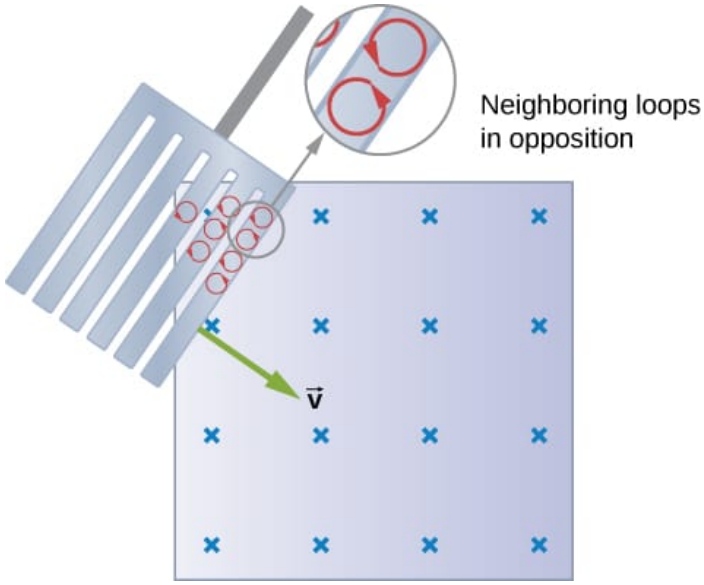


Figure 16.6.3: Eddy currents induced in a slotted metal plate entering a magnetic field form small loops, and the forces on them tend to cancel, thereby making magnetic drag almost zero.

Applications of Magnetic Damping

One use of magnetic damping is found in sensitive laboratory balances. To have maximum sensitivity and accuracy, the balance must be as friction-free as possible. But if it is friction-free, then it will oscillate for a very long time. Magnetic damping is a simple and ideal solution. With magnetic damping, drag is proportional to speed and becomes zero at zero velocity. Thus, the oscillations are quickly damped, after which the damping force disappears, allowing the balance to be very sensitive (Figure 16.6.4). In most balances, magnetic damping is accomplished with a conducting disc that rotates in a fixed field.

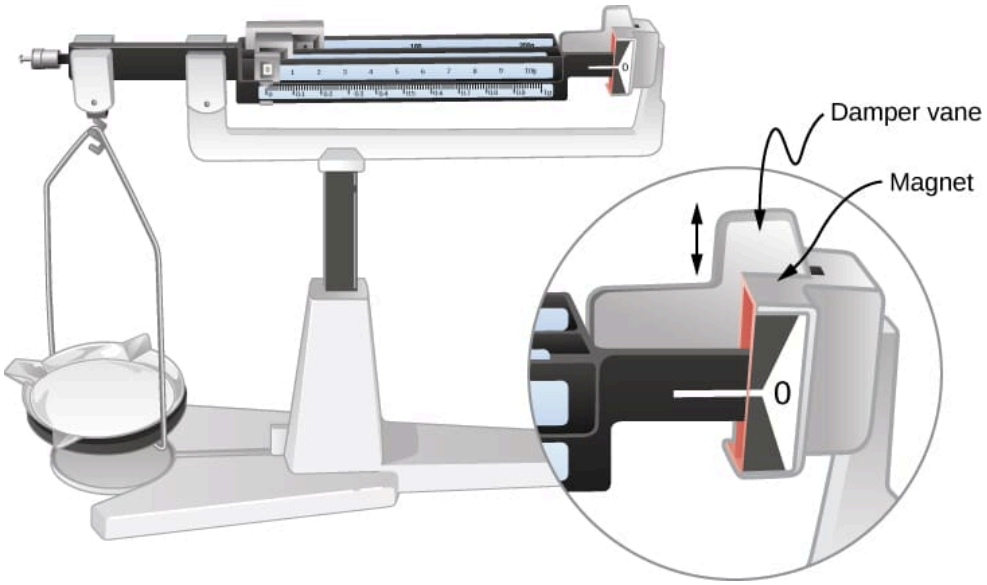


Figure 16.6.4: Magnetic damping of this sensitive balance slows its oscillations. Since Faraday's law of induction gives the greatest effect for the most rapid change, damping is greatest for large oscillations and goes to zero as the motion stops.

Since eddy currents and magnetic damping occur only in conductors, recycling centers can use magnets to separate metals from other materials. Trash is dumped in batches down a ramp, beneath which lies a powerful magnet. Conductors in the trash are slowed by magnetic damping while nonmetals in the trash move on, separating from the metals (Figure 16.6.5). This works for all metals, not just ferromagnetic ones. A magnet can separate out the ferromagnetic materials alone by acting on stationary trash.

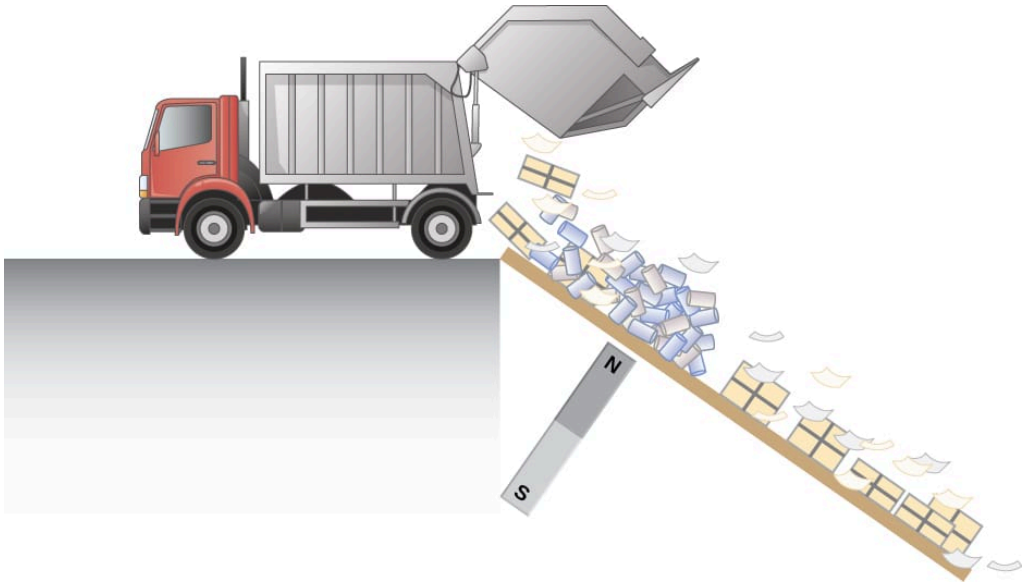


Figure 16.6.5: Metals can be separated from other trash by magnetic drag. Eddy currents and magnetic drag are created in the metals sent down this ramp by the powerful magnet beneath it. Nonmetals move on.

Other major applications of eddy currents appear in **metal detectors** and **braking systems** in trains and roller coasters. Portable metal detectors (Figure 16.6.6) consist of a primary coil carrying an alternating current and a secondary coil in which a current is induced. An eddy current is induced in a piece of metal close to the detector, causing a change in the induced current within the secondary coil. This can trigger some sort of signal, such as a shrill noise.



Figure 16.6.6: A soldier in Iraq uses a metal detector to search for explosives and weapons. (credit: U.S. Army)

Braking using eddy currents is safer because factors such as rain do not affect the braking and the braking is smoother. However, eddy currents cannot bring the motion to a complete stop, since the braking force produced decreases as speed is reduced. Thus, speed can be reduced from say 20 m/s to 5 m/s, but another form of braking is needed to completely stop the vehicle. Generally, powerful rare-earth magnets such as neodymium magnets are used in roller coasters. Figure 16.6.7 shows rows of magnets in such an application. The vehicle has metal fins (normally containing copper) that pass through the magnetic field, slowing the vehicle down in much the same way as with the pendulum bob shown in Figure 16.6.1.



Figure 16.6.7: The rows of rare-earth magnets (protruding horizontally) are used for magnetic braking in roller coasters. (credit: Stefan Scheer)

Induction cooktops have electromagnets under their surface. The magnetic field is varied rapidly, producing eddy currents in the base of the pot, causing the pot and its contents to increase in temperature. Induction cooktops have high efficiencies and good response times but the base of the pot needs to be conductors, such as iron or steel, for induction to work.

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16.7: Electric Generators and Back Emf

Learning Objectives

By the end of this section, you will be able to:

- Explain how an electric generator works
- Determine the induced emf in a loop at any time interval, rotating at a constant rate in a magnetic field
- Show that rotating coils have an induced emf; in motors this is called back emf because it opposes the emf input to the motor

A variety of important phenomena and devices can be understood with Faraday's law. In this section, we examine two of these.

Electric Generators

Electric generators induce an emf by rotating a coil in a magnetic field, as briefly discussed in Motional Emf. We now explore generators in more detail. Consider the following example.

Calculating the Emf Induced in a Generator Coil

The generator coil shown in Figure $16.7.1$ is rotated through one-fourth of a revolution (from $\theta = 0^o$ to $\theta = 90^o$) in 15.0 ms. The 200-turn circular coil has a 5.00-cm radius and is in a uniform 0.80-T magnetic field. What is the emf induced?

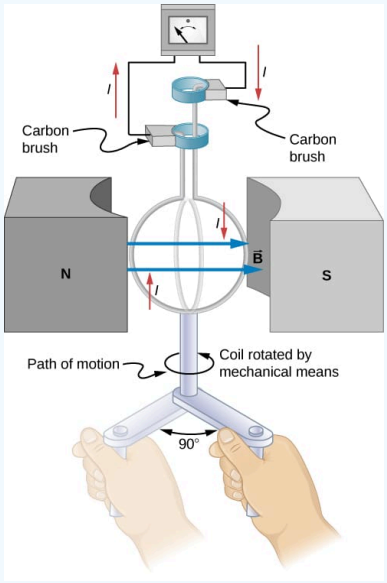


Figure $16.7.1$: When this generator coil is rotated through one-fourth of a revolution, the magnetic flux Φ_m changes from its maximum to zero, inducing an emf.

Strategy

Faraday's law of induction is used to find the emf induced:

$$\epsilon = -N\frac{d\Phi_m}{dt}.$$

We recognize this situation as the same one in Example 13.4.3. According to the diagram, the projection of the surface normal vector $\hat{n}$ to the magnetic field is initially $cos\ \theta$ and this is inserted by the definition of the dot product. The magnitude of the magnetic field and area of the loop are fixed over time, which makes the integration simplify quickly. The induced emf is written out using Faraday's law:

$$\epsilon = NBA\, sin\, \theta \frac{d\theta}{dt}.$$

Solution We are given that $N = 200,\ B = 0.80\ T$, $\theta = 90^o$, $d\theta = 90^o = \pi/2$, and $dt = 15.0\ ms$. The area of the loop is

$$A = \pi r^2 = (3.14)(0.0500\, m)^2 = 7.85 \times 10^{-3}\, m^2.$$

Entering this value gives

$$\epsilon = (200)(0.80\, T)(7.85 \times 10^{-3}\, m^2) sin(90^o) \frac{\pi/2}{15.0 \times 10^{-3} s} = 131\, V.$$

Significance

This is a practical average value, similar to the 120 V used in household power.

The emf calculated in Example 16.7.1 is the average over one-fourth of a revolution. What is the emf at any given instant? It varies with the angle between the magnetic field and a perpendicular to the coil. We can get an expression for emf as a function of time by considering the motional emf on a rotating rectangular coil of width **w** and height **l** in a uniform magnetic field, as illustrated in Figure 16.7.2.

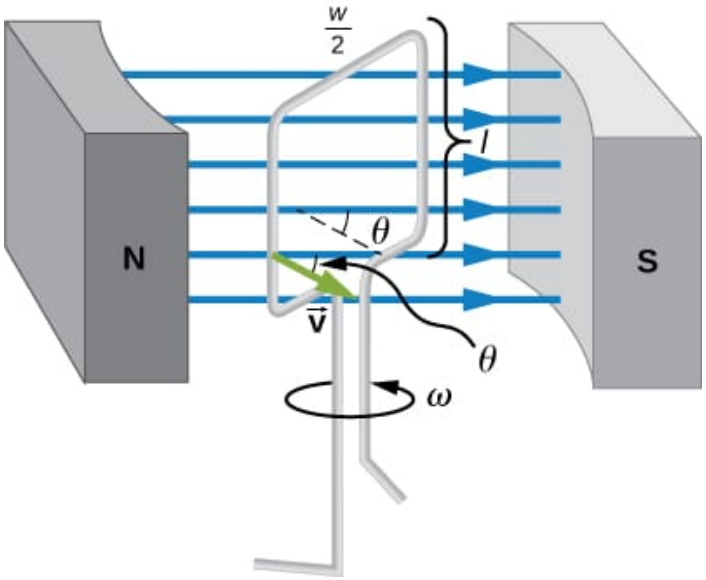


Figure 16.7.2: A generator with a single rectangular coil rotated at constant angular velocity in a uniform magnetic field produces an emf that varies sinusoidally in time. Note the generator is similar to a motor, except the shaft is rotated to produce a current rather than the other way around.

Charges in the wires of the loop experience the magnetic force, because they are moving in a magnetic field. Charges in the vertical wires experience forces parallel to the wire, causing currents. But those in the top and bottom segments feel a force perpendicular to the wire, which does not cause a current. We can thus find the induced emf by considering only the side wires. Motional emf is given to be $\epsilon = Blv$, where the velocity **v** is perpendicular to the magnetic field **B**. Here the velocity is at an angle θ with **B**, so that its component perpendicular to **B** is **v** sin θ (see Figure 16.7.2). Thus, in this case, the emf induced on each side is $\epsilon = Blv\ sin\ \theta$ and they are in the same direction. The total emf around the loop is then

$$\epsilon = 2Blv\, sin\, \theta.$$

This expression is valid, but it does not give emf as a function of time. To find the time dependence of emf, we assume the coil

rotates at a constant angular velocity ω. The angle θ is related to angular velocity by $\theta = \omega t$, so that

$$\epsilon = 2Blv\, sin(\omega t).$$

Now, linear velocity **v** is related to angular velocity ω by $v = r\omega$. Here, $r = \omega/2$, so that $v = (\omega/2)\omega$, and

$$\epsilon = 2Bl\frac{\omega}{2}\omega\, sin\, \omega t = (l\omega)Bw\, sin\, \omega t.$$

Noting that the area of the loop is $A = l\omega$, and allowing for **N** loops, we find that

$$\epsilon = NBAw\, sin\, (\omega t).$$

This is the emf induced in a generator coil of **N** turns and area **A** rotating at a constant angular velocity ω in a uniform magnetic field **B**. This can also be expressed as

$$\epsilon = \epsilon_0\, sin\, \omega t,$$

where

$$\epsilon_0 = NAB\omega$$

is the peak emf, since the maximum value of $sin(\omega t) = 1$. Note that the frequency of the oscillation is $f = \omega/2\pi$ and the period is $T = 1/f = 2\pi/\omega$. Figure $16.7.3$ shows a graph of emf as a function of time, and it now seems reasonable that ac voltage is sinusoidal.

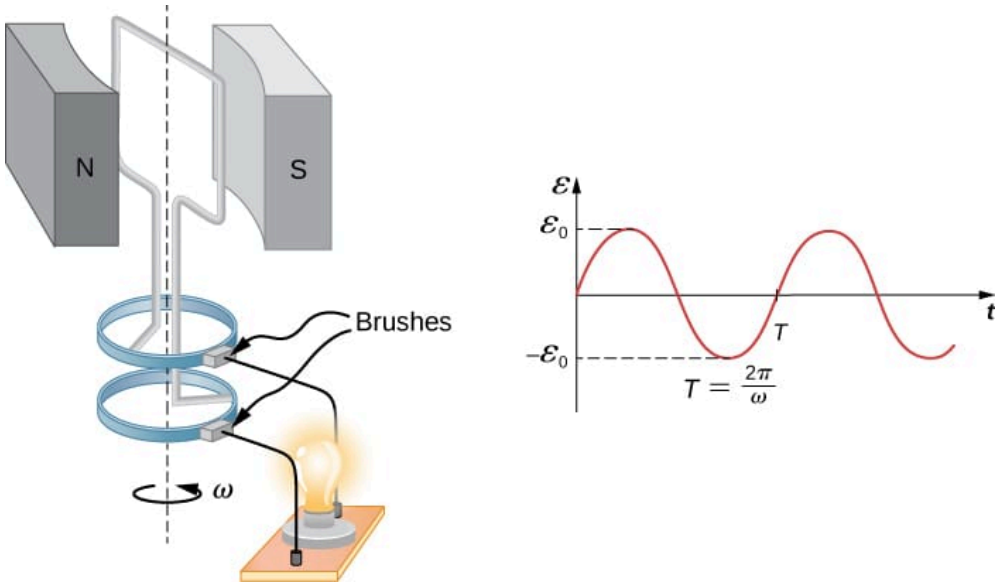


Figure $16.7.3$: The emf of a generator is sent to a light bulb with the system of rings and brushes shown. The graph gives the emf of the generator as a function of time, where ϵ_0 is the peak emf. The period is $T = 1/f = 2\pi/\omega$, where **f** is the frequency.

The fact that the peak emf is $\epsilon_0 = NBA\omega$ makes good sense. The greater the number of coils, the larger their area, and the stronger the field, the greater the output voltage. It is interesting that the faster the generator is spun (greater), the greater the emf. This is noticeable on bicycle generators—at least the cheaper varieties.

Figure $16.7.4$ shows a scheme by which a generator can be made to produce pulsed dc. More elaborate arrangements of multiple coils and split rings can produce smoother dc, although electronic rather than mechanical means are usually used to make ripple-free dc.

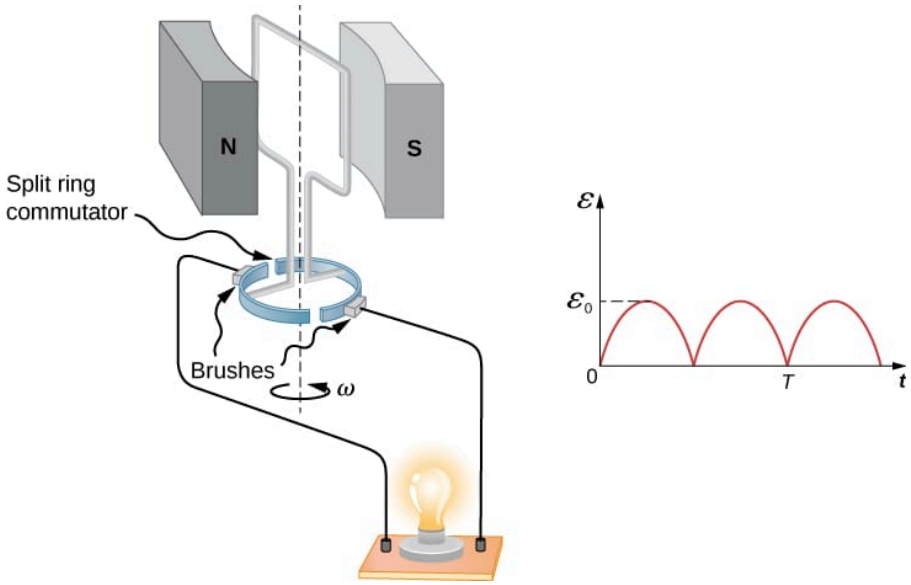


Figure 16.7.4: Split rings, called commutators, produce a pulsed dc emf output in this configuration.

In real life, electric generators look a lot different from the figures in this section, but the principles are the same. The source of mechanical energy that turns the coil can be falling water (hydropower), steam produced by the burning of fossil fuels, or the kinetic energy of wind. Figure 16.7.5 shows a cutaway view of a steam turbine; steam moves over the blades connected to the shaft, which rotates the coil within the generator. The generation of electrical energy from mechanical energy is the basic principle of all power that is sent through our electrical grids to our homes.

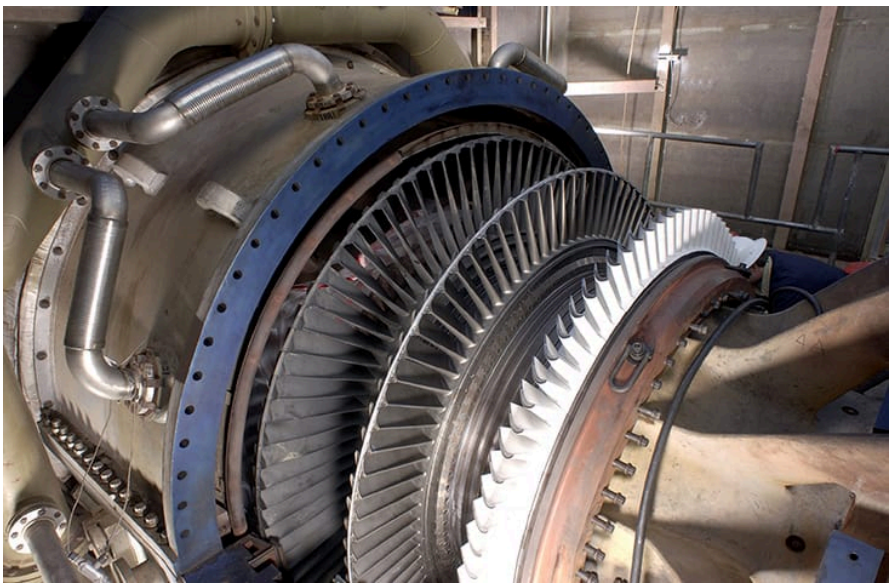


Figure 16.7.5: Steam turbine/generator. The steam produced by burning coal impacts the turbine blades, turning the shaft, which is connected to the generator.

Generators illustrated in this section look very much like the motors illustrated previously. This is not coincidental. In fact, a motor becomes a generator when its shaft rotates. Certain early automobiles used their starter motor as a generator. In the next section, we further explore the action of a motor as a generator.

Back Emf

Generators convert mechanical energy into electrical energy, whereas motors convert electrical energy into mechanical energy. Thus, it is not surprising that motors and generators have the same general construction. A motor works by sending a current through a loop of wire located in a magnetic field. As a result, the magnetic field exerts torque on the loop. This rotates a shaft,

thereby extracting mechanical work out of the electrical current sent in initially. (Refer to Force and Torque on a Current Loop for a discussion on motors that will help you understand more about them before proceeding.)

When the coil of a motor is turned, magnetic flux changes through the coil, and an emf (consistent with Faraday's law) is induced. The motor thus acts as a generator whenever its coil rotates. This happens whether the shaft is turned by an external input, like a belt drive, or by the action of the motor itself. That is, when a motor is doing work and its shaft is turning, an emf is generated. Lenz's law tells us the emf opposes any change, so that the input emf that powers the motor is opposed by the motor's self-generated emf, called the **back emf** of the motor (Figure 16.7.6).

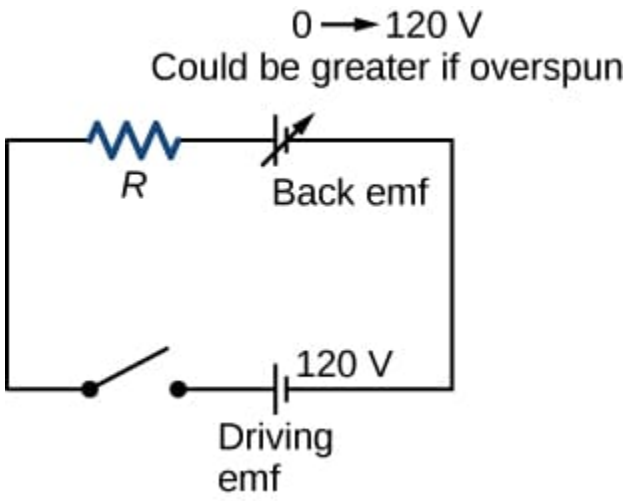


Figure 16.7.6: The coil of a dc motor is represented as a resistor in this schematic. The back emf is represented as a variable emf that opposes the emf driving the motor. Back emf is zero when the motor is not turning and increases proportionally to the motor's angular velocity.

The generator output of a motor is the difference between the supply voltage and the back emf. The back emf is zero when the motor is first turned on, meaning that the coil receives the full driving voltage and the motor draws maximum current when it is on but not turning. As the motor turns faster, the back emf grows, always opposing the driving emf, and reduces both the voltage across the coil and the amount of current it draws. This effect is noticeable in many common situations. When a vacuum cleaner, refrigerator, or washing machine is first turned on, lights in the same circuit dim briefly due to the **IR** drop produced in feeder lines by the large current drawn by the motor.

When a motor first comes on, it draws more current than when it runs at its normal operating speed. When a mechanical load is placed on the motor, like an electric wheelchair going up a hill, the motor slows, the back emf drops, more current flows, and more work can be done. If the motor runs at too low a speed, the larger current can overheat it (via resistive power in the coil, $P = I^2R$), perhaps even burning it out. On the other hand, if there is no mechanical load on the motor, it increases its angular velocity until the back emf is nearly equal to the driving emf. Then the motor uses only enough energy to overcome friction.

Eddy currents in iron cores of motors can cause troublesome energy losses. These are usually minimized by constructing the cores out of thin, electrically insulated sheets of iron. The magnetic properties of the core are hardly affected by the lamination of the insulating sheet, while the resistive heating is reduced considerably. Consider, for example, the motor coils represented in Figure 16.7.6. The coils have an equivalent resistance of $0.400\ \Omega$ and are driven by an emf of 48.0 V. Shortly after being turned on, they draw a current

$$I = V/R = (48.0\,V)/(0.400\,\Omega) = 120\,A$$

and thus dissipate $P = I^2R = 5.76\ kW$ of energy as heat transfer. Under normal operating conditions for this motor, suppose the back emf is 40.0 V. Then at operating speed, the total voltage across the coils is 8.0 V (48.0 V minus the 40.0 V back emf), and the current drawn is

$$I = V/R = (8.0\,V)/(0.400\,\Omega) = 20\,A.$$

Under normal load, then, the power dissipated is $P = IV = (20\ A)(8.0\ V) = 160\ W$. This does not cause a problem for this motor, whereas the former 5.76 kW would burn out the coils if sustained.

A Series-Wound Motor in Operation

The total resistance $(R_f + R_a)$ of a series-wound dc motor is $2.0\ \Omega$ (Figure $16.7.7$). When connected to a 120-V source (ϵ_S), the motor draws 10 A while running at constant angular velocity. (a) What is the back emf induced in the rotating coil, ϵ_i? (b) What is the mechanical power output of the motor? (c) How much power is dissipated in the resistance of the coils? (d) What is the power output of the 120-V source? (e) Suppose the load on the motor increases, causing it to slow down to the point where it draws 20 A. Answer parts (a) through (d) for this situation.

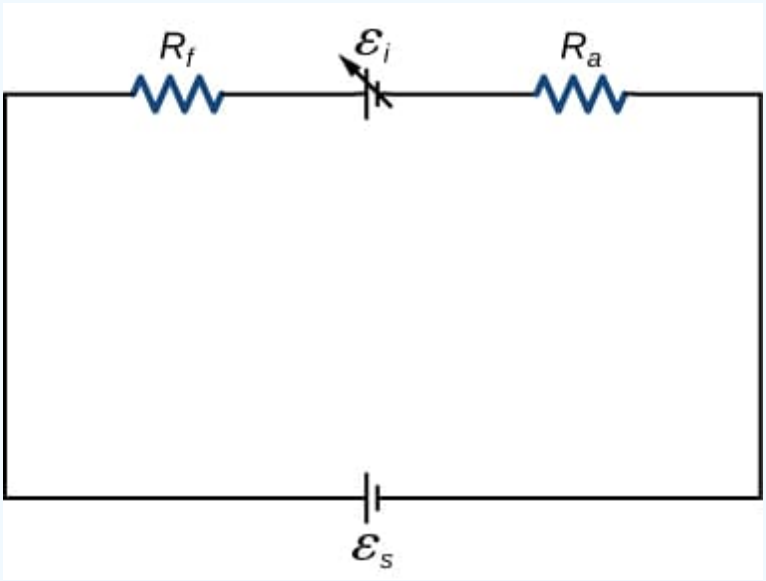


Figure $16.7.7$: Circuit representation of a series-wound direct current motor.

Strategy

The back emf is calculated based on the difference between the supplied voltage and the loss from the current through the resistance. The power from each device is calculated from one of the power formulas based on the given information.

Solution

1. The back emf is
$$\epsilon_i = \epsilon_S - I(R_f + RE_a) = 120\,V - (10\,A)(2.0\,\Omega) = 100\,V.$$
2. Since the potential across the armature is 100 V when the current through it is 10 A, the power output of the motor is
$$P_m = \epsilon_i I = (100\,V)(10\,A) = 1.0 \times 10^3\,W.$$
3. A 10-A current flows through coils whose combined resistance is $2.0\ \Omega$, so the power dissipated in the coils is
$$P_R = I^2R = (10\,A)^2(2.0\,\Omega) = 2.0 \times 10^2\,W.$$
4. Since 10 A is drawn from the 120-V source, its power output is
$$P_S = \epsilon_S I = (120\,V)(10\,A) = 1.2 \times 10^3\,W.$$
5. Repeating the same calculations with $I = 20\ A$, we find
$$\epsilon_i = 80\,V,\ P_m = 1.6 \times 10^3\,W,\ P_R = 8.0 \times 10^2\,W,\ and\ P_s = 2.4 \times 10^3\,W.$$
The motor is turning more slowly in this case, so its power output and the power of the source are larger.

Significance Notice that we have an energy balance in part (d):
$$1.2 \times 10^3\,W = 1.0 \times 10^3\,W + 2.0 \times 10^2\,W.$$

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16.8: Applications of Electromagnetic Induction

Learning Objectives

By the end of this section, you will be able to:

- Explain how computer hard drives and graphic tablets operate using magnetic induction
- Explain how hybrid/electric vehicles and transcranial magnetic stimulation use magnetic induction to their advantage

Modern society has numerous applications of Faraday's law of induction, as we will explore in this chapter and others. At this juncture, let us mention several that involve recording information using magnetic fields.

Some computer **hard drives** apply the principle of magnetic induction. Recorded data are made on a coated, spinning disk. Historically, reading these data was made to work on the principle of induction. However, most input information today is carried in digital rather than analog form—a series of 0s or 1s are written upon the spinning hard drive. Therefore, most hard drive readout devices do not work on the principle of induction, but use a technique known as **giant magnetoresistance**. Giant magnetoresistance is the effect of a large change of electrical resistance induced by an applied magnetic field to thin films of alternating ferromagnetic and nonmagnetic layers. This is one of the first large successes of nanotechnology.

Graphics tablets, or **tablet computers** where a specially designed pen is used to draw digital images, also applies induction principles. The tablets discussed here are labeled as passive tablets, since there are other designs that use either a battery-operated pen or optical signals to write with. The passive tablets are different than the touch tablets and phones many of us use regularly, but may still be found when signing your signature at a cash register. Underneath the screen, shown in Figure 16.8.1, are tiny wires running across the length and width of the screen. The pen has a tiny magnetic field coming from the tip. As the tip brushes across the screen, a changing magnetic field is felt in the wires which translates into an induced emf that is converted into the line you just drew.



Figure 16.8.1: A tablet with a specially designed pen to write with is another application of magnetic induction.

Another application of induction is the magnetic stripe on the back of your personal **credit card** as used at the grocery store or the ATM machine. This works on the same principle as the audio or video tape, in which a playback head reads personal information from your card.

Video

Check out this video to see how flashlights can use magnetic induction.

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

A magnet moves by your mechanical work through a wire. The induced current charges a capacitor that stores the charge that will light the lightbulb even while you are not doing this mechanical work.

Electric and **hybrid vehicles** also take advantage of electromagnetic induction. One limiting factor that inhibits widespread acceptance of 100% electric vehicles is that the lifetime of the battery is not as long as the time you get to drive on a full tank of gas. To increase the amount of charge in the battery during driving, the motor can act as a generator whenever the car is braking, taking advantage of the back emf produced. This extra emf can be newly acquired stored energy in the car's battery, prolonging the life of the battery.

Another contemporary area of research in which electromagnetic induction is being successfully implemented is **transcranial magnetic stimulation (TMS)**. A host of disorders, including depression and hallucinations, can be traced to irregular localized electrical activity in the brain. In transcranial magnetic stimulation, a rapidly varying and very localized magnetic field is placed close to certain sites identified in the brain. The usage of TMS as a diagnostic technique is well established.

Video

Check out this Youtube video to see how rock-and-roll instruments like electric guitars use electromagnetic induction to get those strong beats.

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16.9: Electromagnetic Induction (Summary)

Key Terms

back emf	emf generated by a running motor, because it consists of a coil turning in a magnetic field; it opposes the voltage powering the motor
eddy current	current loop in a conductor caused by motional emf
electric generator	device for converting mechanical work into electric energy; it induces an emf by rotating a coil in a magnetic field
Faraday's law	induced emf is created in a closed loop due to a change in magnetic flux through the loop
induced electric field	created based on the changing magnetic flux with time
induced emf	short-lived voltage generated by a conductor or coil moving in a magnetic field
Lenz's law	direction of an induced emf opposes the change in magnetic flux that produced it; this is the negative sign in Faraday's law
magnetic damping	drag produced by eddy currents
magnetic flux	measurement of the amount of magnetic field lines through a given area
motionally induced emf	voltage produced by the movement of a conducting wire in a magnetic field
peak emf	maximum emf produced by a generator

Key Equations

Magnetic flux	$\Phi_m = \int_S \vec{B} \cdot \hat{n} dA$
Faraday's law	$\varepsilon = -N\frac{d\Phi_m}{dt}$
Motionally induced emf	$\varepsilon = Blv$
Motional emf around a circuit	$\varepsilon = \oint \vec{E} \cdot \vec{dl} = -\frac{d\Phi_m}{dt}$
Emf produced by an electric generator	$\varepsilon = NBA\omega sin(\omega t)$

Summary

13.2 Faraday's Law

- The magnetic flux through an enclosed area is defined as the amount of field lines cutting through a surface area A defined by the unit area vector.
- The units for magnetic flux are webers, where $1Wb = 1T \cdot m^2$.
- The induced emf in a closed loop due to a change in magnetic flux through the loop is known as Faraday's law. If there is no change in magnetic flux, no induced emf is created.

13.3 Lenz's Law

- We can use Lenz's law to determine the directions of induced magnetic fields, currents, and emfs.
- The direction of an induced emf always opposes the change in magnetic flux that causes the emf, a result known as Lenz's law.

13.4 Motional Emf

- The relationship between an induced emf εε in a wire moving at a constant speed **v** through a magnetic field **B** is given by $\varepsilon = Blv$.
- An induced emf from Faraday's law is created from a motional emf that opposes the change in flux.

13.5 Induced Electric Fields

- A changing magnetic flux induces an electric field.
- Both the changing magnetic flux and the induced electric field are related to the induced emf from Faraday's law.

13.6 Eddy Currents

- Current loops induced in moving conductors are called eddy currents. They can create significant drag, called magnetic damping.
- Manipulation of eddy currents has resulted in applications such as metal detectors, braking in trains or roller coasters, and induction cooktops.

13.7 Electric Generators and Back Emf

- An electric generator rotates a coil in a magnetic field, inducing an emf given as a function of time by $\varepsilon = NBA\omega sin(\omega t)$ where **A** is the area of an **N**-turn coil rotated at a constant angular velocity ω in a uniform magnetic field $\vec{B}$.
- The peak emf of a generator is $\varepsilon_0 = NBA\omega$.
- Any rotating coil produces an induced emf. In motors, this is called back emf because it opposes the emf input to the motor.

13.8 Applications of Electromagnetic Induction

- Hard drives utilize magnetic induction to read/write information.
- Other applications of magnetic induction can be found in graphics tablets, electric and hybrid vehicles, and in transcranial magnetic stimulation.

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16.10: Electromagnetic Induction (Exercises)

Conceptual Questions

13.2 Faraday's Law

1. A stationary coil is in a magnetic field that is changing with time. Does the emf induced in the coil depend on the actual values of the magnetic field?

2. In Faraday's experiments, what would be the advantage of using coils with many turns?

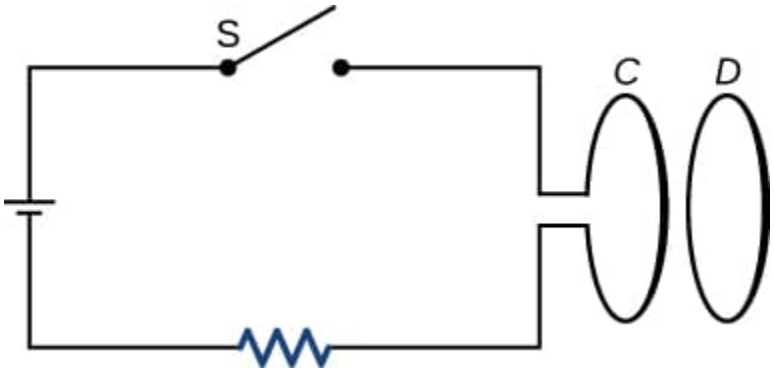
3. A copper ring and a wooden ring of the same dimensions are placed in magnetic fields so that there is the same change in magnetic flux through them. Compare the induced electric fields and currents in the rings.

4. Discuss the factors determining the induced emf in a closed loop of wire.

5. (a) Does the induced emf in a circuit depend on the resistance of the circuit?

(b) Does the induced current depend on the resistance of the circuit?

6. How would changing the radius of loop **D** shown below affect its emf, assuming **C** and **D** are much closer together compared to their radii?



7. Can there be an induced emf in a circuit at an instant when the magnetic flux through the circuit is zero?

8. Does the induced emf always act to decrease the magnetic flux through a circuit?

9. How would you position a flat loop of wire in a changing magnetic field so that there is no induced emf in the loop?

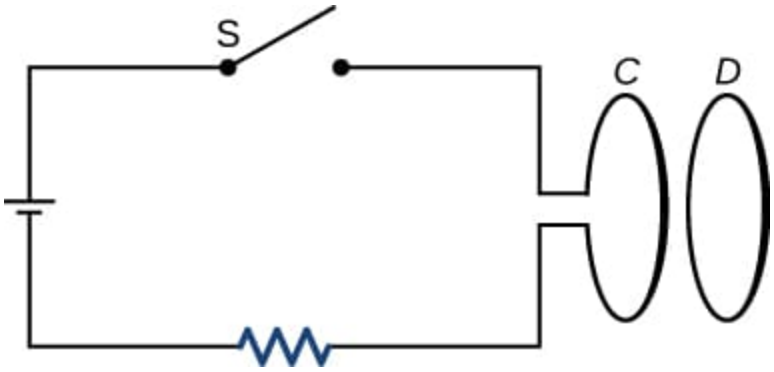
10. The normal to the plane of a single-turn conducting loop is directed at an angle **θ** to a spatially uniform magnetic field $vecB$. It has a fixed area and orientation relative to the magnetic field. Show that the emf induced in the loop is given by $\varepsilon = (dB/dt)(Acos\theta)$,where A is the area of the loop.

13.3 Lenz's Law

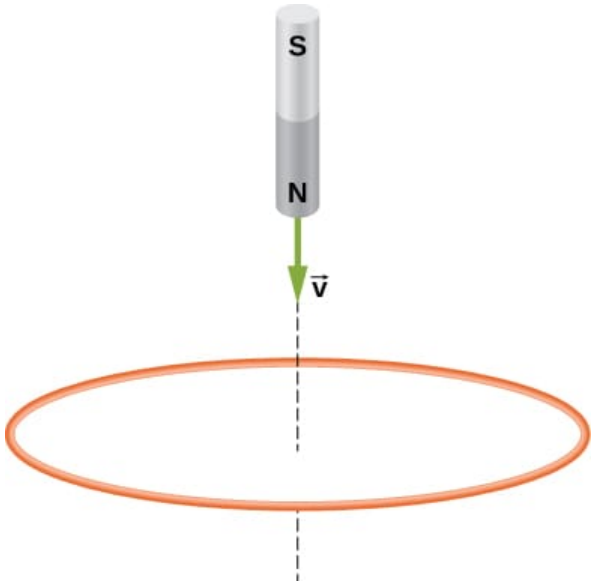
11. The circular conducting loops shown in the accompanying figure are parallel, perpendicular to the plane of the page, and coaxial.

(a) When the switch S is closed, what is the direction of the current induced in **D**?

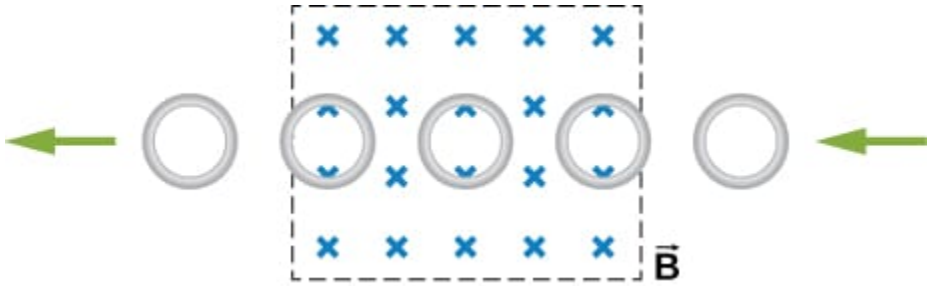
(b) When the switch is opened, what is the direction of the current induced in loop **D**?



12. The north pole of a magnet is moved toward a copper loop, as shown below. If you are looking at the loop from above the magnet, will you say the induced current is circulating clockwise or counterclockwise?

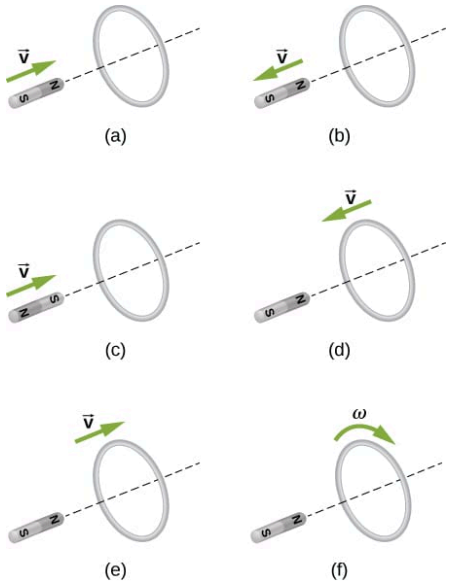


13. The accompanying figure shows a conducting ring at various positions as it moves through a magnetic field. What is the sense of the induced emf for each of those positions?



14. Show that **ε** and $d\Phi_m/dt$ have the same units.

15. State the direction of the induced current for each case shown below, observing from the side of the magnet.



13.4 Motional Emf

16. A bar magnet falls under the influence of gravity along the axis of a long copper tube. If air resistance is negligible, will there be a force to oppose the descent of the magnet? If so, will the magnet reach a terminal velocity?

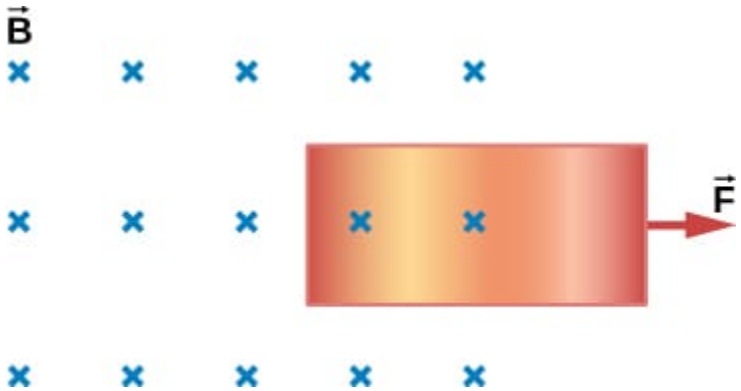
17. Around the geographic North Pole (or magnetic South Pole), Earth's magnetic field is almost vertical. If an airplane is flying northward in this region, which side of the wing is positively charged and which is negatively charged?

18. A wire loop moves translationally (no rotation) in a uniform magnetic field. Is there an emf induced in the loop?

13.5 Induced Electric Fields

19. Is the work required to accelerate a rod from rest to a speed v in a magnetic field greater than the final kinetic energy of the rod? Why?

20. The copper sheet shown below is partially in a magnetic field. When it is pulled to the right, a resisting force pulls it to the left. Explain. What happen if the sheet is pushed to the left?



13.6 Eddy Currents

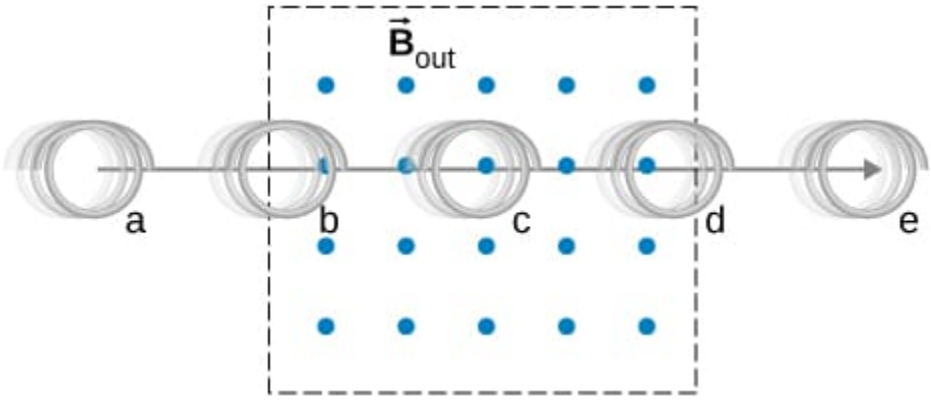
21. A conducting sheet lies in a plane perpendicular to a magnetic field $\vec{B}$ that is below the sheet. If $\vec{B}$ oscillates at a high frequency and the conductor is made of a material of low resistivity, the region above the sheet is effectively shielded from $\vec{B}$. Explain why. Will the conductor shield this region from static magnetic fields?

22. Electromagnetic braking can be achieved by applying a strong magnetic field to a spinning metal disk attached to a shaft.

(a) How can a magnetic field slow the spinning of a disk?

(b) Would the brakes work if the disk was made of plastic instead of metal?

23. A coil is moved through a magnetic field as shown below. The field is uniform inside the rectangle and zero outside. What is the direction of the induced current and what is the direction of the magnetic force on the coil at each position shown?



Problems

13.2 Faraday's Law

24. A 50-turn coil has a diameter of 15 cm. The coil is placed in a spatially uniform magnetic field of magnitude 0.50 T so that the face of the coil and the magnetic field are perpendicular. Find the magnitude of the emf induced in the coil if the magnetic field is reduced to zero uniformly in

(a) 0.10 s,

(b) 1.0 s, and

(c) 60 s.

25. Repeat your calculations of the preceding problem's time of 0.1 s with the plane of the coil making an angle of

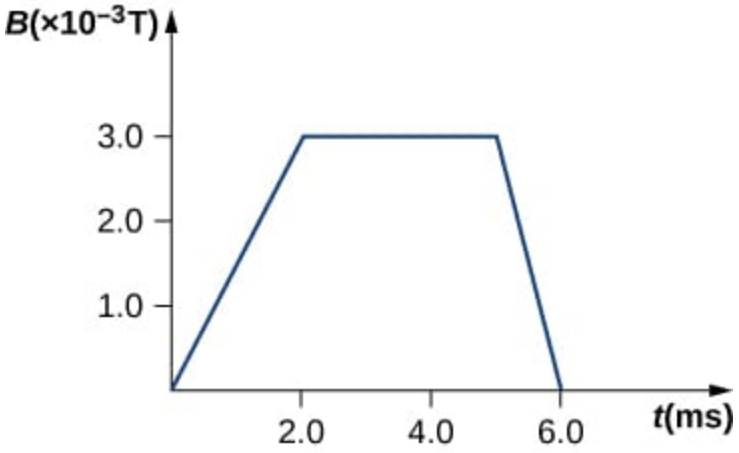
(a) **30°**,

(b) **60°,** and

(c) **90°** with the magnetic field.

26. A square loop whose sides are 6.0-cm long is made with copper wire of radius 1.0 mm. If a magnetic field perpendicular to the loop is changing at a rate of 5.0 mT/s, what is the current in the loop?

27. The magnetic field through a circular loop of radius 10.0 cm varies with time as shown below. The field is perpendicular to the loop. Plot the magnitude of the induced emf in the loop as a function of time.



28. The accompanying figure shows a single-turn rectangular coil that has a resistance of 2.0Ω.2.0Ω. The magnetic field at all points inside the coil varies according to $B = B_0 e^{-\alpha t}$, where $B_0 = 0.25T$ and **α=200Hz**. What is the current induced in the coil at

(a) **t=0.001s**,

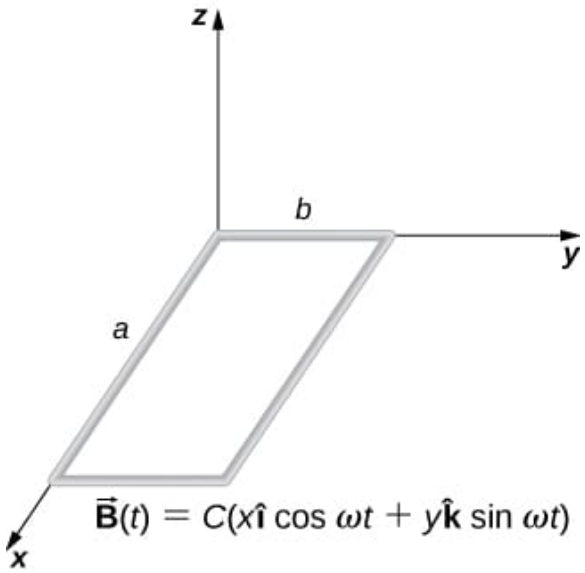
(b) **0.002 s**,

(c) **2.0 s**?

29. How would the answers to the preceding problem change if the coil consisted of 20 closely spaced turns?

30. A long solenoid with **n=10** turns per centimeter has a cross-sectional area of $5.0cm^2$ and carries a current of 0.25 A. A coil with five turns encircles the solenoid. When the current through the solenoid is turned off, it decreases to zero in 0.050 s. What is the average emf induced in the coil?

31. A rectangular wire loop with length a and width b lies in the xy-plane, as shown below. Within the loop there is a time-dependent magnetic field given by $\vec{B}(t) = C((xcos\omega t)\hat{i} + (ysin\omega t)\hat{k})$, with $\vec{B}(t)$ in tesla. Determine the emf induced in the loop as a function of time.



32. The magnetic field perpendicular to a single wire loop of diameter 10.0 cm decreases from 0.50 T to zero. The wire is made of copper and has a diameter of 2.0 mm and length 1.0 cm. How much charge moves through the wire while the field is changing?

13.3 Lenz's Law

33. A single-turn circular loop of wire of radius 50 mm lies in a plane perpendicular to a spatially uniform magnetic field. During a 0.10-s time interval, the magnitude of the field increases uniformly from 200 to 300 mT.

(a) Determine the emf induced in the loop.

(b) If the magnetic field is directed out of the page, what is the direction of the current induced in the loop?

34. When a magnetic field is first turned on, the flux through a 20-turn loop varies with time according to $\Phi_m = 5.0t^2 - 2.0t$, where Φ_m is in milliwebers, t is in seconds, and the loop is in the plane of the page with the unit normal pointing outward.

(a) What is the emf induced in the loop as a function of time? What is the direction of the induced current at

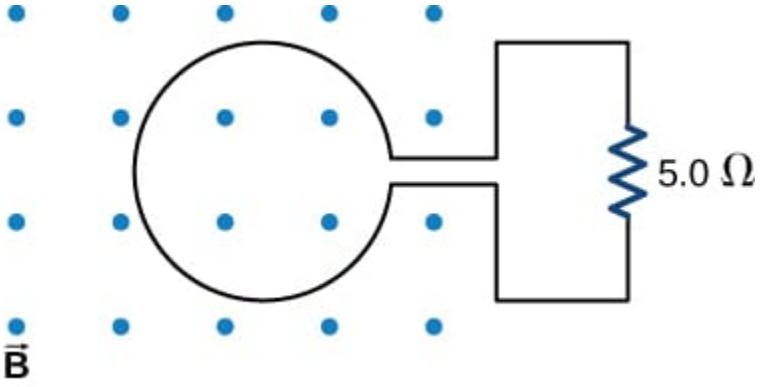
(b) t = 0,

(c) 0.10,

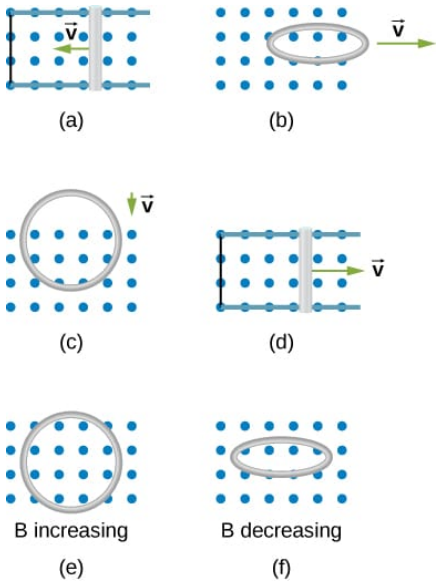
(d) 1.0, and

(e) 2.0 s?

35. The magnetic flux through the loop shown in the accompanying figure varies with time according to $\Phi_m = 2.00e^{-3t}sin(120\pi t)$,where Φ_m is in milliwebers. What are the direction and magnitude of the current through the **5.00-Ω** resistor at (a) **t=0**t=0; (b) $t = 2.17 \times 10^{-2}s$, and (c) **t=3.00s**?



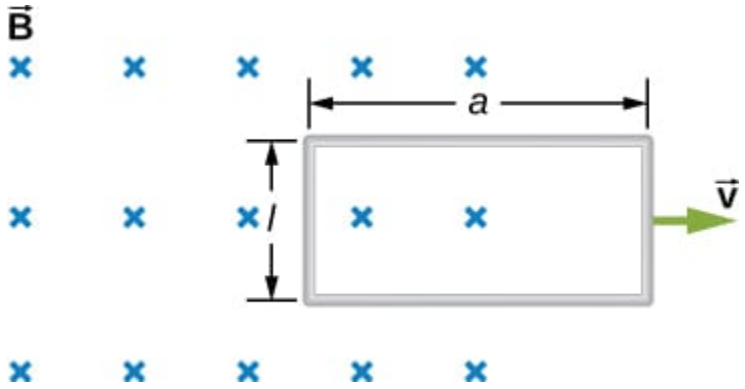
36. Use Lenz's law to determine the direction of induced current in each case.



13.4 Motional Emf

37. An automobile with a radio antenna 1.0 m long travels at 100.0 km/h in a location where the Earth's horizontal magnetic field is $5.5 \times 10^{-5} T$. What is the maximum possible emf induced in the antenna due to this motion?

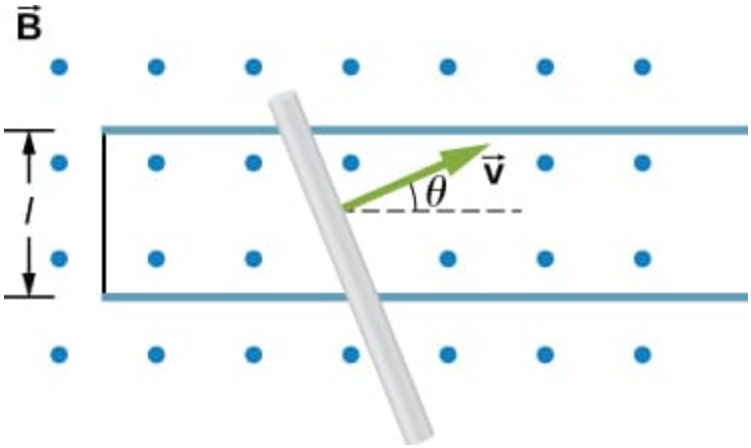
38. The rectangular loop of N turns shown below moves to the right with a constant velocity $\vec{v}$ while leaving the poles of a large electromagnet. (a) Assuming that the magnetic field is uniform between the pole faces and negligible elsewhere, determine the induced emf in the loop. (b) What is the source of work that produces this emf?



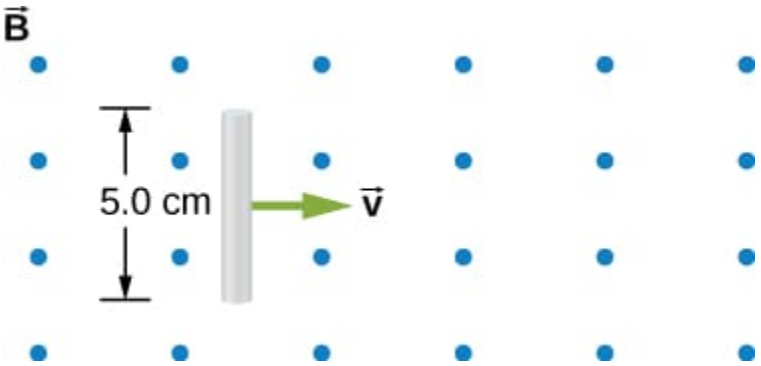
39. Suppose the magnetic field of the preceding problem oscillates with time according to $B = B_0 \sin \omega t$. What then is the emf induced in the loop when its trailing side is a distance d from the right edge of the magnetic field region?

40. A coil of 1000 turns encloses an area of$25 cm^2$. It is rotated in 0.010 s from a position where its plane is perpendicular to Earth's magnetic field to one where its plane is parallel to the field. If the strength of the field is $6.0 \times 10^{-5} T$, what is the average emf induced in the coil?

41. In the circuit shown in the accompanying figure, the rod slides along the conducting rails at a constant velocity $\vec{v}$. The velocity is in the same plane as the rails and directed at an angle θ to them. A uniform magnetic field $\vec{B}$ is directed out of the page. What is the emf induced in the rod?



42. The rod shown in the accompanying figure is moving through a uniform magnetic field of strength $B = 0.50T$ with a constant velocity of magnitude $v = 8.0m/s$.. What is the potential difference between the ends of the rod? Which end of the rod is at a higher potential?



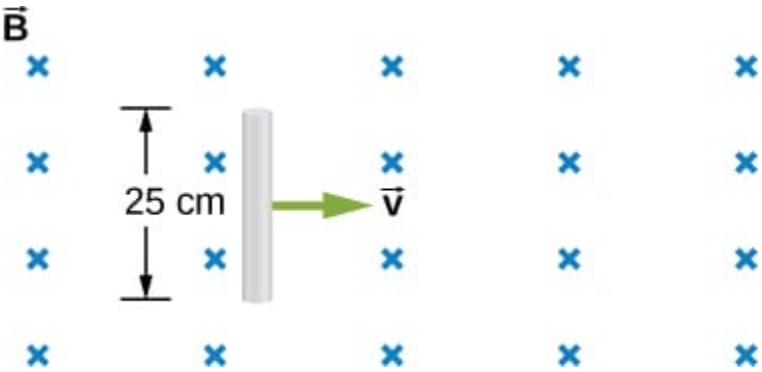
43. A 25-cm rod moves at 5.0 m/s in a plane perpendicular to a magnetic field of strength 0.25 T. The rod, velocity vector, and magnetic field vector are mutually perpendicular, as indicated in the accompanying figure. Calculate

(a) the magnetic force on an electron in the rod,

(b) the electric field in the rod, and

(c) the potential difference between the ends of the rod.

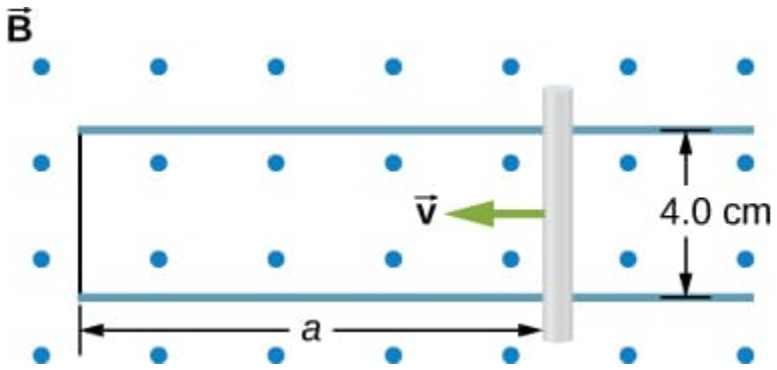
(d) What is the speed of the rod if the potential difference is 1.0 V?



44. In the accompanying figure, the rails, connecting end piece, and rod all have a resistance per unit length of **2.0Ω/cm**. The rod moves to the left at **v=3.0m/s**. If **B=0.75T** everywhere in the region, what is the current in the circuit

(a) when **a=8.0cm**?

(b) when **a=5.0cm**? Specify also the sense of the current flow.



45. The rod shown below moves to the right on essentially zero-resistance rails at a speed of **v=3.0m/**s. If **B=0.75T** everywhere in the region, what is the current through the **5.0-Ω** resistor? Does the current circulate clockwise or counterclockwise?

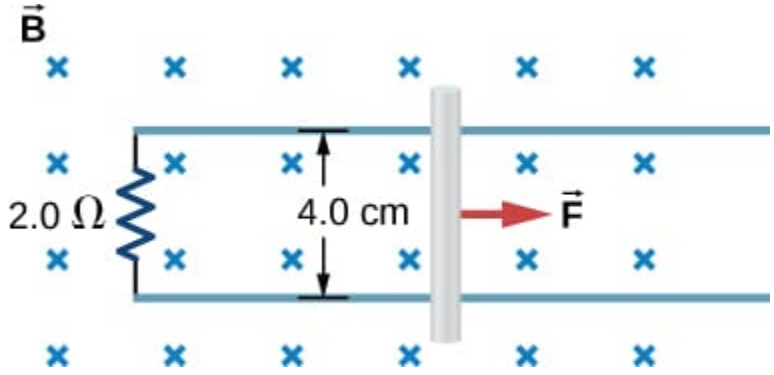
46. Shown below is a conducting rod that slides along metal rails. The apparatus is in a uniform magnetic field of strength 0.25 T, which is directly into the page. The rod is pulled to the right at a constant speed of 5.0 m/s by a force $\vec{F}$. The only significant resistance in the circuit comes from the **2.0-Ω** resistor shown.

(a) What is the emf induced in the circuit?

(b) What is the induced current? Does it circulate clockwise or counter clockwise?

(c) What is the magnitude of $\vec{F}$?

(d) What are the power output of $\vec{F}$ and the power dissipated in the resistor?



13.5 Induced Electric Fields

47. Calculate the induced electric field in a 50-turn coil with a diameter of 15 cm that is placed in a spatially uniform magnetic field of magnitude 0.50 T so that the face of the coil and the magnetic field are perpendicular. This magnetic field is reduced to zero in 0.10 seconds. Assume that the magnetic field is cylindrically symmetric with respect to the central axis of the coil.

48. The magnetic field through a circular loop of radius 10.0 cm varies with time as shown in the accompanying figure. The field is perpendicular to the loop. Assuming cylindrical symmetry with respect to the central axis of the loop, plot the induced electric field in the loop as a function of time.

49. The current I through a long solenoid with n turns per meter and radius R is changing with time as given by dI/dt. Calculate the induced electric field as a function of distance r from the central axis of the solenoid.

50. Calculate the electric field induced both inside and outside the solenoid of the preceding problem if $I = I_0 sin\omega t$..

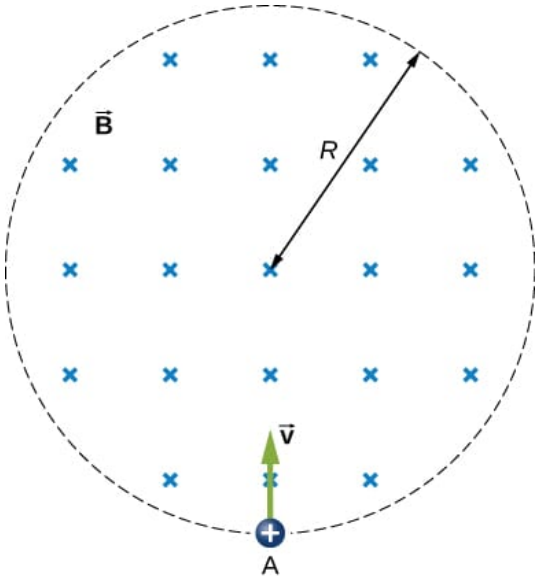
51. Over a region of radius R, there is a spatially uniform magnetic field $\vec{B}$. (See below.) At **t=0**, **B=1.0T**, after which it decreases at a constant rate to zero in 30 s.

(a) What is the electric field in the regions where $r \leq R$ and $r \geq R$ during that 30-s interval?

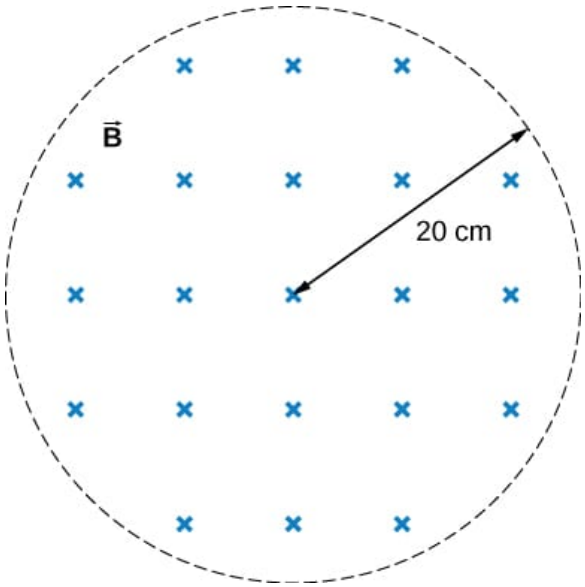
(b) Assume that **R=10.0cm**. How much work is done by the electric field on a proton that is carried once clock wise around a circular path of radius 5.0 cm?

(c) How much work is done by the electric field on a proton that is carried once counterclockwise around a circular path of any radius $r \geq R$?

(d) At the instant when **B=0.50T**, a proton enters the magnetic field at A, moving a velocity $\vec{v}(v = 5.0 \times 10^6 m/s)$ as shown. What are the electric and magnetic forces on the proton at that instant?



52. The magnetic field at all points within the cylindrical region whose cross-section is indicated in the accompanying figure starts at 1.0 T and decreases uniformly to zero in 20 s. What is the electric field (both magnitude and direction) as a function of **r**, the distance from the geometric center of the region?



53. The current in a long solenoid of radius 3 cm is varied with time at a rate of 2 A/s. A circular loop of wire of radius 5 cm and resistance **2Ω** surrounds the solenoid. Find the electrical current induced in the loop.

54. The current in a long solenoid of radius 3 cm and 20 turns/cm is varied with time at a rate of 2 A/s. Find the electric field at a distance of 4 cm from the center of the solenoid.

13.7 Electric Generators and Back Emf

55. Design a current loop that, when rotated in a uniform magnetic field of strength 0.10 T, will produce an emf $\varepsilon = \varepsilon_0 sin\omega t$, , where $\varepsilon_0 = 110V$ and $\omega = 120\pi rad/s$.

56. A flat, square coil of 20 turns that has sides of length 15.0 cm is rotating in a magnetic field of strength 0.050 T. If the maximum emf produced in the coil is 30.0 mV, what is the angular velocity of the coil?

57. A 50-turn rectangular coil with dimensions **0.15m×0.40m** rotates in a uniform magnetic field of magnitude 0.75 T at 3600 rev/min.

(a) Determine the emf induced in the coil as a function of time.

(b) If the coil is connected to a **1000-Ω** resistor, what is the power as a function of time required to keep the coil turning at 3600 rpm?

(c) Answer part (b) if the coil is connected to a **2000-Ω** resistor.

58. The square armature coil of an alternating current generator has 200 turns and is 20.0 cm on side. When it rotates at 3600 rpm, its peak output voltage is 120 V.

(a) What is the frequency of the output voltage?

(b) What is the strength of the magnetic field in which the coil is turning?

59. A flip coil is a relatively simple device used to measure a magnetic field. It consists of a circular coil of N turns wound with fine conducting wire. The coil is attached to a ballistic galvanometer, a device that measures the total charge that passes through it. The coil is placed in a magnetic field $\vec{B}$ such that its face is perpendicular to the field. It is then flipped through 180°,180°, and the total chargeQ that flows through the galvanometer is measured.

(a) If the total resistance of the coil and galvanometer is R, what is the relationship between B and Q? Because the coil is very small, you can assume that$\vec{B}$ is uniform over it.

(b) How can you determine whether or not the magnetic field is perpendicular to the face of the coil?

60. The flip coil of the preceding problem has a radius of 3.0 cm and is wound with 40 turns of copper wire. The total resistance of the coil and ballistic galvanometer is **0.20Ω**. When the coil is flipped through **180°** in a magnetic field $\vec{B}$, a change of 0.090 C flows through the ballistic galvanometer.

(a) Assuming that $\vec{B}$ and the face of the coil are initially perpendicular, what is the magnetic field?

(b) If the coil is flipped through **90°,** what is the reading of the galvanometer?

61. A 120-V, series-wound motor has a field resistance of 80 **Ω** and an armature resistance of 10 **Ω**. When it is operating at full speed, a back emf of 75 V is generated.

(a) What is the initial current drawn by the motor? When the motor is operating at full speed, where are

(b) the current drawn by the motor,

(c) the power output of the source,

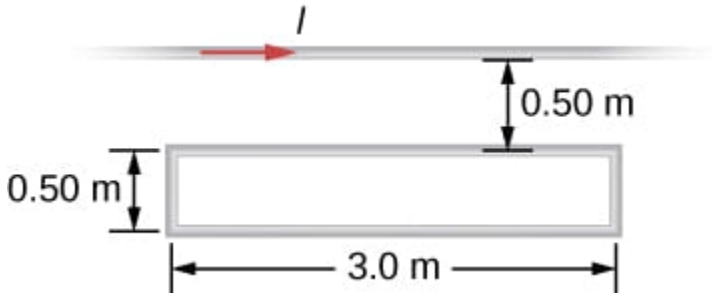
(d) the power output of the motor, and

(e) the power dissipated in the two resistances?

62. A small series-wound dc motor is operated from a 12-V car battery. Under a normal load, the motor draws 4.0 A, and when the armature is clamped so that it cannot turn, the motor draws 24 A. What is the back emf when the motor is operating normally?

Additional Problems

63. Shown in the following figure is a long, straight wire and a single-turn rectangular loop, both of which lie in the plane of the page. The wire is parallel to the long sides of the loop and is 0.50 m away from the closer side. At an instant when the emf induced in the loop is 2.0 V, what is the time rate of change of the current in the wire?



64. A metal bar of mass 500 g slides outward at a constant speed of 1.5 cm/s over two parallel rails separated by a distance of 30 cm which are part of a U-shaped conductor. There is a uniform magnetic field of magnitude 2 T pointing out of the page over the entire area. The railings and metal bar have an equivalent resistance of **150Ω**.

(a) Determine the induced current, both magnitude and direction.

(b) Find the direction of the induced current if the magnetic field is pointing into the page.

(c) Find the direction of the induced current if the magnetic field is pointed into the page and the bar moves inwards.

65. A current is induced in a circular loop of radius 1.5 cm between two poles of a horseshoe electromagnet when the current in the electromagnet is varied. The magnetic field in the area of the loop is perpendicular to the area and has a uniform magnitude. If the rate of change of magnetic field is 10 T/s, find the magnitude and direction of the induced current if resistance of the loop is **25Ω**.

66. A metal bar of length 25 cm is placed perpendicular to a uniform magnetic field of strength 3 T.

(a) Determine the induced emf between the ends of the rod when it is not moving.

(b) Determine the emf when the rod is moving perpendicular to its length and magnetic field with a speed of 50 cm/s.

67. A coil with 50 turns and area 10 cm^2 is oriented with its plane perpendicular to a 0.75-T magnetic field. If the coil is flipped over (rotated through **180°**) in 0.20 s, what is the average emf induced in it?

68. A 2-turn planer loop of flexible wire is placed inside a long solenoid of n turns per meter that carries a constant current I_0. The areaA of the loop is changed by pulling on its sides while ensuring that the plane of the loop always remains perpendicular to the axis of the solenoid. If **n=500turns** per meter, $I_0 = 20A$, and $A = 20cm^2$, what is the emf induced in the loop when **dA/dt=100**?

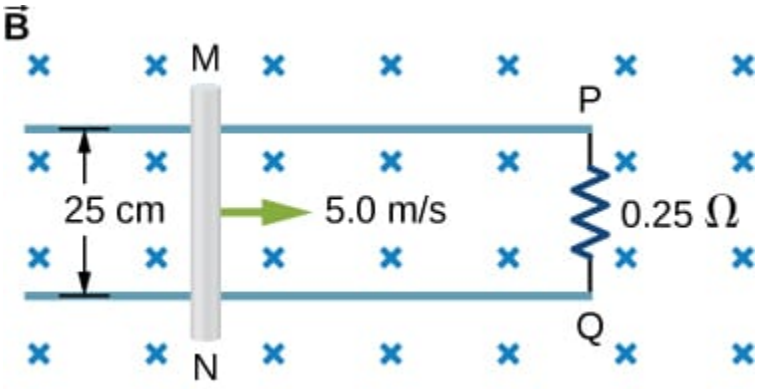
69. The conducting rod shown in the accompanying figure moves along parallel metal rails that are 25-cm apart. The system is in a uniform magnetic field of strength 0.75 T, which is directed into the page. The resistances of the rod and the rails are negligible, but the section PQ has a resistance of **0.25Ω**.

(a) What is the emf (including its sense) induced in the rod when it is moving to the right with a speed of 5.0 m/s?

(b) What force is required to keep the rod moving at this speed?

(c) What is the rate at which work is done by this force?

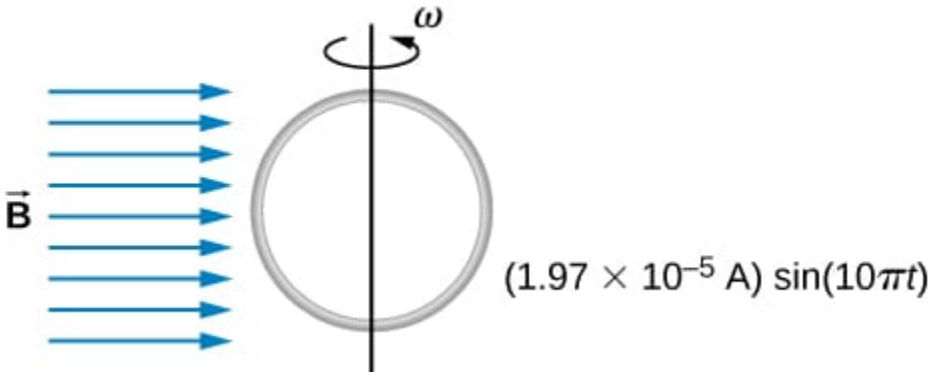
(d) What is the power dissipated in the resistor?



70. A circular loop of wire of radius 10 cm is mounted on a vertical shaft and rotated at a frequency of 5 cycles per second in a region of uniform magnetic field of 2 Gauss perpendicular to the axis of rotation.

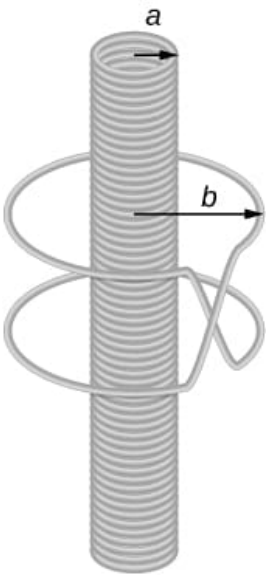
(a) Find an expression for the time-dependent flux through the ring.

(b) Determine the time-dependent current through the ring if it has a resistance of 10 Ω.



71. The magnetic field between the poles of a horseshoe electromagnet is uniform and has a cylindrical symmetry about an axis from the middle of the South Pole to the middle of the North Pole. The magnitude of the magnetic field changes as a rate of dB/dt due to the changing current through the electromagnet. Determine the electric field at a distance r from the center.

72. A long solenoid of radius a with n turns per unit length is carrying a time-dependent current $I(t) = I_0 sin(\omega t)$, where I_0 and **ω** are constants. The solenoid is surrounded by a wire of resistance **R** that has two circular loops of radius **b** with **b>a** (see the following figure). Find the magnitude and direction of current induced in the outer loops at time **t=0**.



73. A 120-V, series-wound dc motor draws 0.50 A from its power source when operating at full speed, and it draws 2.0 A when it starts. The resistance of the armature coils is **10Ω**.

(a) What is the resistance of the field coils?

(b) What is the back emf of the motor when it is running at full speed?

(c) The motor operates at a different speed and draws 1.0 A from the source. What is the back emf in this case?

74. The armature and field coils of a series-wound motor have a total resistance of **3.0Ω**. When connected to a 120-V source and running at normal speed, the motor draws 4.0 A.

(a) How large is the back emf?

(b) What current will the motor draw just after it is turned on? Can you suggest a way to avoid this large initial current?

Challenge Problems

75. A copper wire of length L is fashioned into a circular coil with **N** turns. When the magnetic field through the coil changes with time, for what value of N is the induced emf a maximum?

76. A 0.50-kg copper sheet drops through a uniform horizontal magnetic field of 1.5 T, and it reaches a terminal velocity of 2.0 m/s.

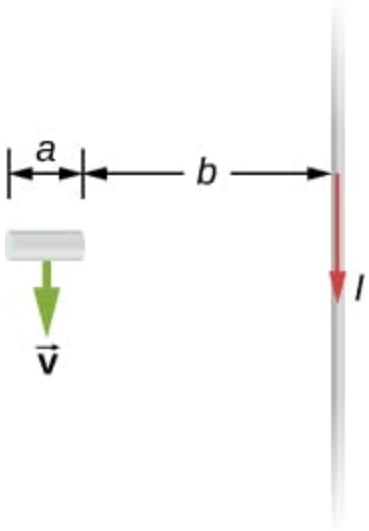
(a) What is the net magnetic force on the sheet after it reaches terminal velocity?

(b) Describe the mechanism responsible for this force.

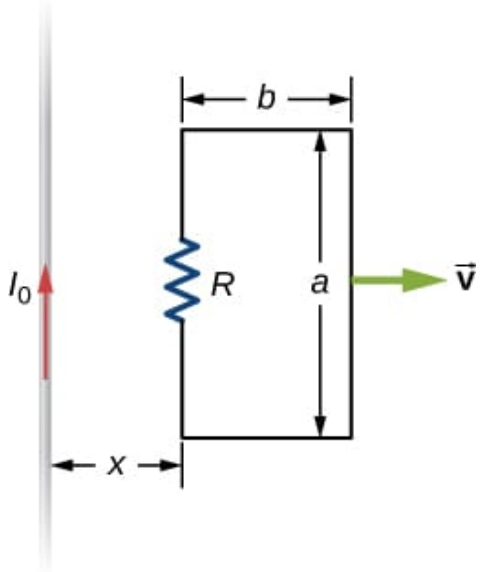
(c) How much power is dissipated as Joule heating while the sheet moves at terminal velocity?

77. A circular copper disk of radius 7.5 cm rotates at 2400 rpm around the axis through its center and perpendicular to its face. The disk is in a uniform magnetic field $\vec{B}$ of strength 1.2 T that is directed along the axis. What is the potential difference between the rim and the axis of the disk?

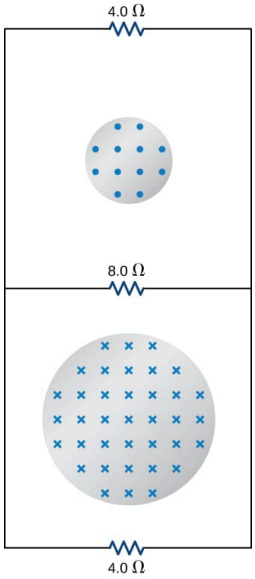
78. A short rod of length a moves with its velocity $\vec{v}$ parallel to an infinite wire carrying a current **I** (see below). If the end of the rod nearer the wire is a distance b from the wire, what is the emf induced in the rod?



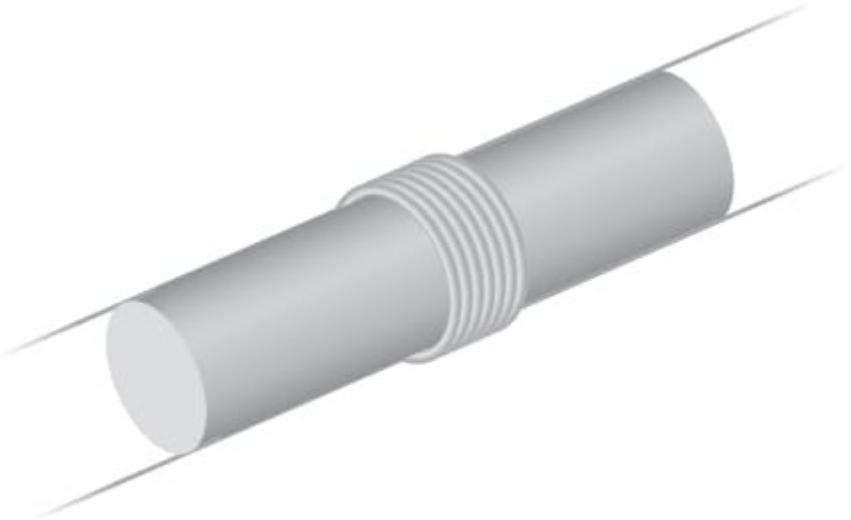
79. A rectangular circuit containing a resistance R is pulled at a constant velocity $\vec{v}$ away from a long, straight wire carrying a current I_0 (see below). Derive an equation that gives the current induced in the circuit as a function of the distance x between the near side of the circuit and the wire.



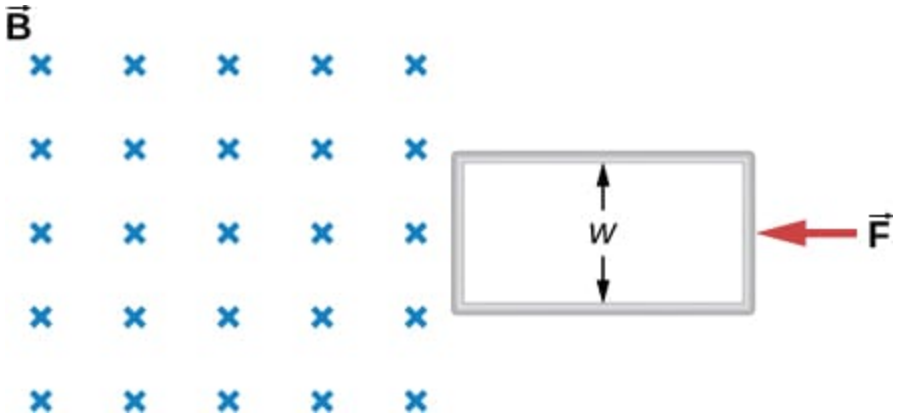
80. Two infinite solenoids cross the plane of the circuit as shown below. The radii of the solenoids are 0.10 and 0.20 m, respectively, and the current in each solenoid is changing such that **dB/dt=50.0T/s.** What are the currents in the resistors of the circuit?



81. An eight-turn coil is tightly wrapped around the outside of the long solenoid as shown below. The radius of the solenoid is 2.0 cm and it has 10 turns per centimeter. The current through the solenoid increases according to $I = I_0(1 - e^{-\alpha t})$, where $I_0 = 4.0A$ and $\alpha = 2.0 \times 10^{-2}s^{-1}$. What is the emf induced in the coil when (a) $t = 0$, (b) $t = 1.0 \times 10^2 s$, and (c) $t \to \infty$?



82. Shown below is a long rectangular loop of width **w**, length **l**, mass **m**, and resistance **R**. The loop starts from rest at the edge of a uniform magnetic field $\vec{B}$ and is pushed into the field by a constant force $\vec{F}$. Calculate the speed of the loop as a function of time.



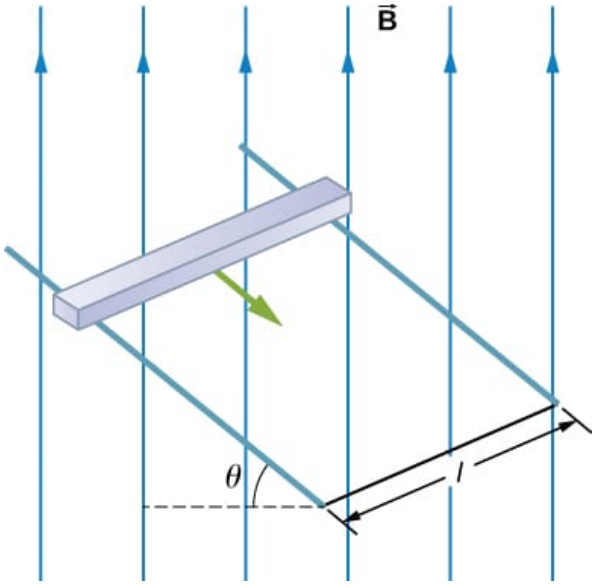
83. A square bar of mass **m** and resistance **R** is sliding without friction down very long, parallel conducting rails of negligible resistance (see below). The two rails are a distance l apart and are connected to each other at the bottom of the incline by a zero-resistance wire. The rails are inclined at an angle **θ**, and there is a uniform vertical magnetic field $\vec{B}$ throughout the region.

(a) Show that the bar acquires a terminal velocity given by $v = \frac{mgRsin\theta}{B^2l^2cos^2\theta}$.

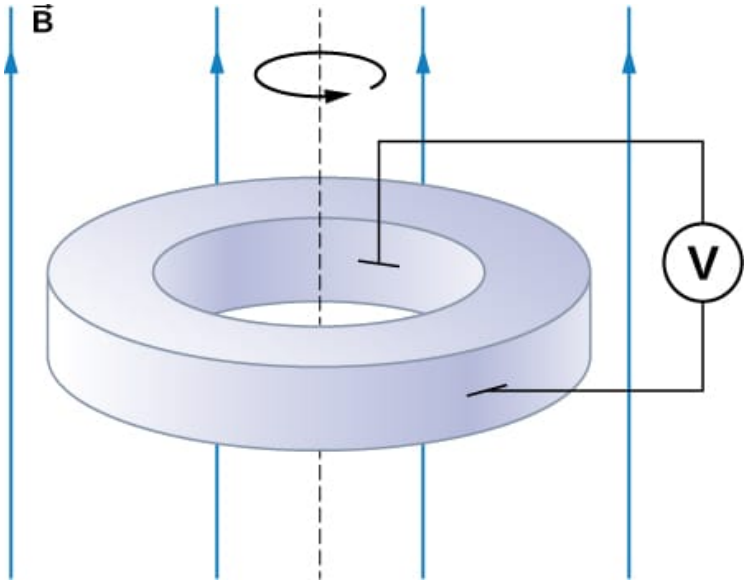
(b) Calculate the work per unit time done by the force of gravity.

(c) Compare this with the power dissipated in the Joule heating of the bar.

(d) What would happen if $\vec{B}$ were reversed?



84. The accompanying figure shows a metal disk of inner radius r_1 and other radius r_2 rotating at an angular velocity $\vec{\omega}$ while in a uniform magnetic field directed parallel to the rotational axis. The brush leads of a voltmeter are connected to the dark's inner and outer surfaces as shown. What is the reading of the voltmeter?



85. A long solenoid with 10 turns per centimeter is placed inside a copper ring such that both objects have the same central axis. The radius of the ring is 10.0 cm, and the radius of the solenoid is 5.0 cm.

(a) What is the emf induced in the ring when the current I through the solenoid is 5.0 A and changing at a rate of 100 A/s?

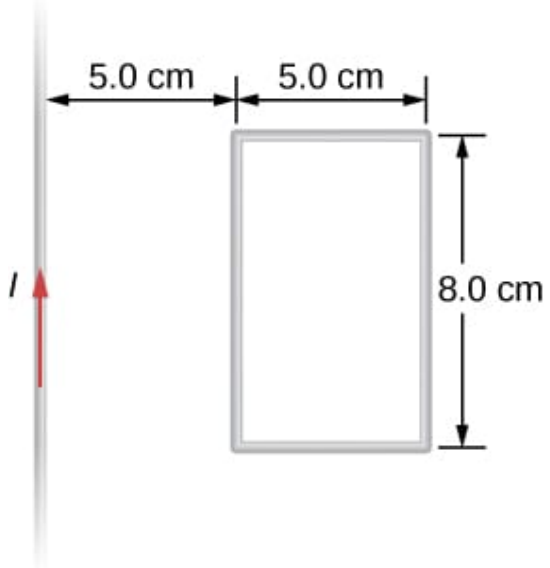
(b) What is the emf induced in the ring when **I=2.0A** and **dI/dt=100A/s**?

(c) What is the electric field inside the ring for these two cases?

(d) Suppose the ring is moved so that its central axis and the central axis of the solenoid are still parallel but no longer coincide. (You should assume that the solenoid is still inside the ring.) Now what is the emf induced in the ring?

(e) Can you calculate the electric field in the ring as you did in part (c)?

86. The current in the long, straight wire shown in the accompanying figure is given by $I = I_0 sin\omega t$, where $I_0 = 15A$ and $\omega = 120\pi rad/s$. What is the current induced in the rectangular loop at (a) **t=0** and (b) $t = 2.1 \times 10^{-3}s$? The resistance of the loop is **2.0Ω**.



87. A 500-turn coil with a $0.250 - m^2$ area is spun in Earth's $5.00 \times 10^{-5}T$ magnetic field, producing a 12.0-kV maximum emf.

(a) At what angular velocity must the coil be spun?

(b) What is unreasonable about this result?

(c) Which assumption or premise is responsible?

88. A circular loop of wire of radius 10 cm is mounted on a vertical shaft and rotated at a frequency of 5 cycles per second in a region of uniform magnetic field of $2 \times 10^{-4}T$ perpendicular to the axis of rotation.

(a) Find an expression for the time-dependent flux through the ring

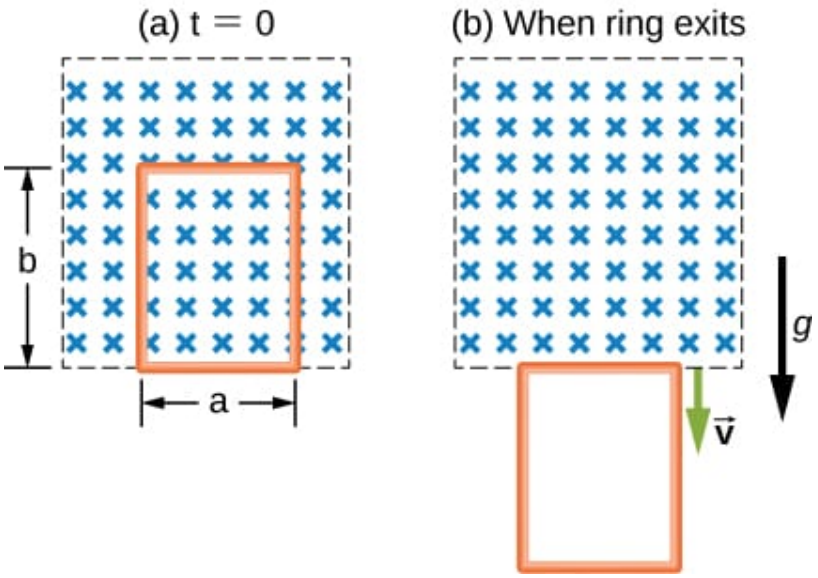
(b) Determine the time-dependent current through the ring if it has a resistance of **10Ω**.

89. A long solenoid of radius aa with nn turns per unit length is carrying a time-dependent current $I(t) = I_0 sin\omega t$ where I_0 and ωω are constants. The solenoid is surrounded by a wire of resistance R that has two circular loops of radius **b** with **b>a**. Find the magnitude and direction of current induced in the outer loops at time **t=0**.

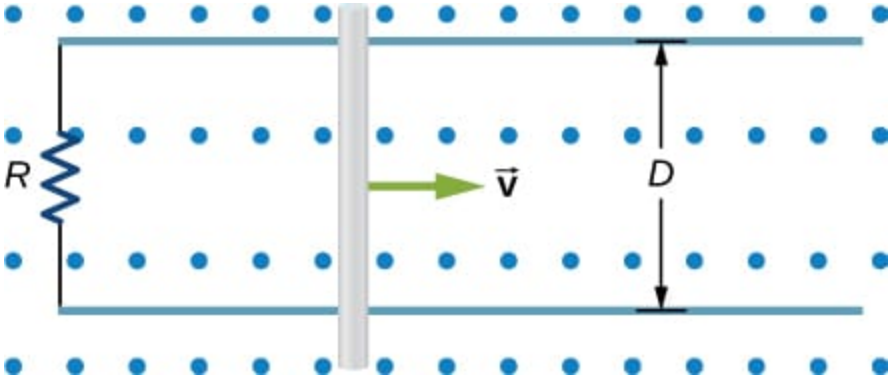
90. A rectangular copper loop of mass 100 g and resistance **0.2Ω** is in a region of uniform magnetic field that is perpendicular to the area enclosed by the ring and horizontal to Earth's surface (see below). The loop is let go from rest when it is at the edge of the nonzero magnetic field region.

(a) Find an expression for the speed when the loop just exits the region of uniform magnetic field.

(b) If it was let go at t=0t=0, what is the time when it exits the region of magnetic field for the following values: $a = 25cm, b = 50cm, B = 3T, g = 9.8m/s^2$? Assume that the magnetic field of the induced current is negligible compared to 3 T.



91. A metal bar of mass **m** slides without friction over two rails a distance **D** apart in the region that has a uniform magnetic field of magnitude B_0 and direction perpendicular to the rails (see below). The two rails are connected at one end to a resistor whose resistance is much larger than the resistance of the rails and the bar. The bar is given an initial speed of v_0 . It is found to slow down. How far does the bar go before coming to rest? Assume that the magnetic field of the induced current is negligible compared to B_0.



92. A time-dependent uniform magnetic field of magnitude **B(t)** is confined in a cylindrical region of radius **R**. A conducting rod of length **2D** is placed in the region, as shown below. Show that the emf between the ends of the rod is given by $\frac{dB}{dt} D\sqrt{R^2 - D^2}$. (**Hint**: To find the emf between the ends, we need to integrate the electric field from one end to the other. To find the electric field, use Faraday's law as "Ampère's law for **E**.")

Samuel J. Ling (Truman State University), Jeff Sanny (Loyola Marymount University), and Bill Moebs with many contributing authors. This work is licensed by OpenStax University Physics under a Creative Commons Attribution License (by 4.0).

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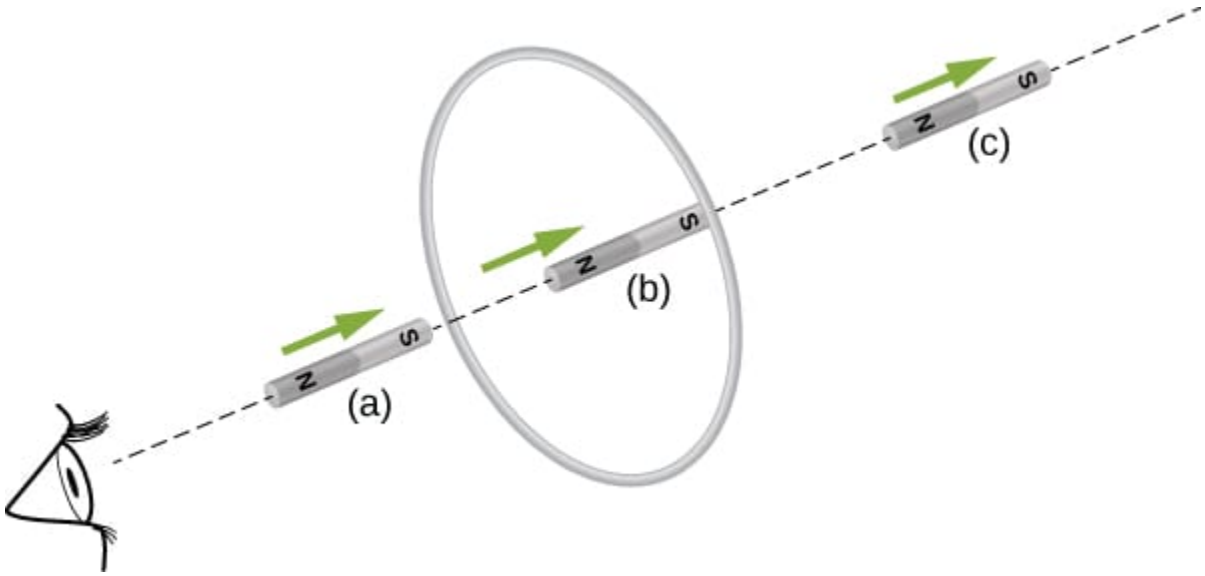
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16.11: Electromagnetic Induction (Answers)

Check Your Understanding

13.1. 1.1 T/s

13.2. To the observer shown, the current flows clockwise as the magnet approaches, decreases to zero when the magnet is centered in the plane of the coil, and then flows counterclockwise as the magnet leaves the coil.



13.4. $\varepsilon = Bl^2\omega/2$, with **O** at a higher potential than **S**

13.5. 1.5 V

13.6. a. yes;

b. Yes; however there is a lack of symmetry between the electric field and coil, making $\oint \vec{E} \cdot d\vec{l}$ a more complicated relationship that can't be simplified as shown in the example.

13.7. $3.4 \times 10^{-3} V/m$

13.8. $P_1, P)2, P_4$

13.9. a. $3.1 \times 10^{-6} V$;

b. $2.0 \times 10^{-7} V/m$

Conceptual Questions

1. The emf depends on the rate of change of the magnetic field.

3. Both have the same induced electric fields; however, the copper ring has a much higher induced emf because it conducts electricity better than the wooden ring.

5. a. no; b. yes

7. As long as the magnetic flux is changing from positive to negative or negative to positive, there could be an induced emf.

9. Position the loop so that the field lines run perpendicular to the area vector or parallel to the surface.

11. a. CW as viewed from the circuit; b. CCW as viewed from the circuit

13. As the loop enters, the induced emf creates a CCW current while as the loop leaves the induced emf creates a CW current. While the loop is fully inside the magnetic field, there is no flux change and therefore no induced current.

15. a. CCW viewed from the magnet;

b. CW viewed from the magnet;

c. CW viewed from the magnet;

d. CCW viewed from the magnet;

e. CW viewed from the magnet;

f. no current

17. Positive charges on the wings would be to the west, or to the left of the pilot while negative charges would be pulled east or to the right of the pilot. Thus, the left hand tips of the wings would be positive and the right hand tips would be negative.

19. The work is greater than the kinetic energy because it takes energy to counteract the induced emf.

21. The conducting sheet is shielded from the changing magnetic fields by creating an induced emf. This induced emf creates an induced magnetic field that opposes any changes in magnetic fields from the field underneath. Therefore, there is no net magnetic field in the region above this sheet. If the field were due to a static magnetic field, no induced emf will be created since you need a changing magnetic flux to induce an emf. Therefore, this static magnetic field will not be shielded.

23. a. zero induced current, zero force; b. clockwise induced current, force is to the left; c. zero induced current, zero force; d. counterclockwise induced current, force is to the left; e. zero induced current, zero force.

Problems

25. a. 3.8 V;

b. 2.2 V;

c. 0 V

27. $B = 1.5t, 0 \leq t < 2.0ms, B = 3.0mT, 2.0ms \leq t \leq 5.0ms,$

\(\displaystyle B=−3.0t+18mT,5.0ms

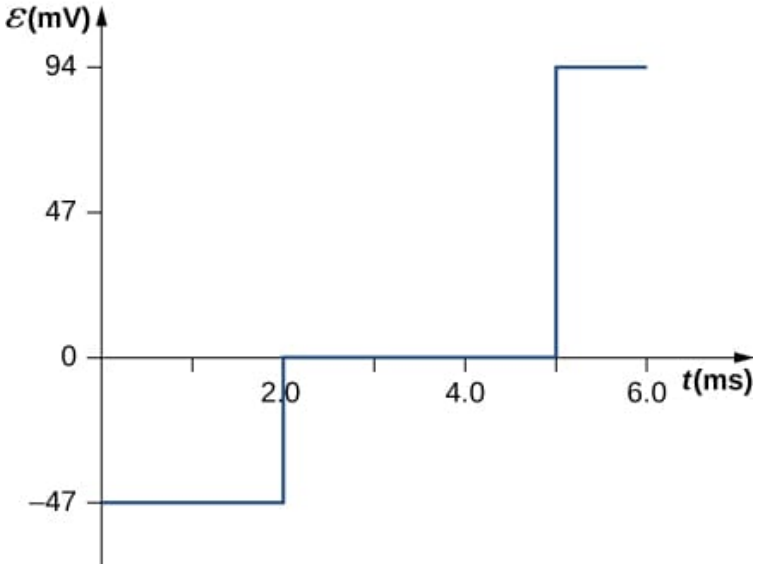
$$\varepsilon = -\frac{d\Phi m}{dt} = -\frac{d(BA)}{dt} = -A\frac{dB}{dt},$$

$$\varepsilon = -\pi(0.100m)^2(1.5T/s)$$

$$= -47mV(0 \leq t < 2.0ms),$$

$$\varepsilon = \pi(0.100m)^2(0) = 0(2.0ms \leq t \leq 5.0ms),$$

\(\displaystyle ε=−π(0.100m)^2(−3.0T/s)=94mV(5.0ms



29. Each answer is 20 times the previously given answers.

31. $\hat{n} = \hat{k}, d\Phi_m = Cysin(\omega t)dxdy,$

$$\Phi_m = \frac{Cab^2 sin(\omega t)}{2},$$

$$\varepsilon = -\frac{Cab^2 \omega cos(\omega t)}{2}.$$

33. a. $7.8 \times 10^{-3} V$;

b. CCW from the same view as the magnetic field

35. a. 150 A downward through the resistor;

b. 232 A upward through the resistor;

c. 0.093 A downward through the resistor

37. 0.0015 V

39. $\varepsilon = -B_0 ld\omega \cos(\Omega t)\mathrm{ld} + B_0 \sin(\Omega t)\mathrm{lv}$

41. $\varepsilon = Blvcos\theta$

43. a. $2 \times 10^{-19} T$;

b. 1.25 V/m;

c. 0.3125 V;

d. 16 m/s

45. 0.018 A, CW as seen in the diagram

47. 9.375 V/m

49. Inside, $B = \mu_0 nI, \oint \vec{E} \cdot d\vec{l} = (\pi r^2)\mu_0 n\frac{dI}{dt}$, so, $E = \frac{\mu_0 nr}{2} \cdot \frac{dI}{dt}$ (inside). Outside, $E(2\pi r) = \pi R^2 \mu_0 n\frac{dI}{dt}$, so, $E = \frac{\mu_0 nR^2}{2r \cdot \frac{dI}{dt}}$

(outside)

51. a. $E_{inside} = \frac{r}{2}\frac{dB}{dt}, E_{outside} = \frac{r^2}{2R}\frac{dB}{dt}$;

b. $W = 4.19 \times 10^{-23} J$;

c. 0 J;

d. $F_{mag} = 4 \times 10^{-13} N, F_{elec} = 2.7 \times 10^{-22} N$

53. $7.1 \mu A$

55. Three turns with an area of $1 m^2$

57. a. $\omega = 120\pi rad/s, \varepsilon = 850 sin 120\pi t V$;

b. $P = 720 sin^2 120\pi t W$;;

c. $P = 360 sin^2 120\pi t W$

59. a. **B** is proportional to **Q**;

b. If the coin turns easily, the magnetic field is perpendicular. If the coin is at an equilibrium position, it is parallel.

61. a. 1.33 A;

b. 0.50 A;

c. 60 W;

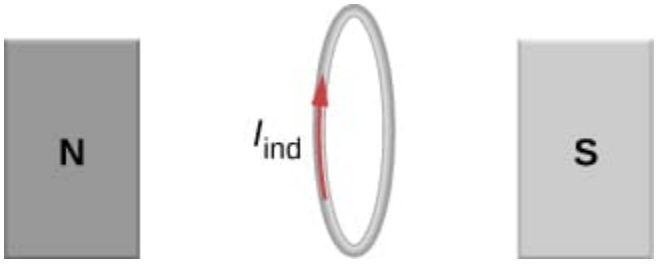
d. 22.5 W;

e. 2.5W

Additional Problems

63. 4.8×10^6 A/s

65. $2.83 \times 10^{-4} A$, the direction as follows for increasing magnetic field:



67. 0.375 V

69. a. 0.94 V;

b. 0.70 N;

c. 3.52 J/s;

d. 3.52 W

71. $(\frac{dB}{dt})\frac{A}{2\pi r}$

73. a. $R_f + R_a = \frac{120V}{2.0A} = 60\Omega$, so $R_f = 50\Omega$;

b. $I = \frac{\varepsilon_s - \varepsilon_i}{R_f + R_a}, \Rightarrow \varepsilon_i = 90V$;

c. $\varepsilon_i = 60V$

Challenge Problems

75. **N** is a maximum number of turns allowed.

77. 5.3 V

79. $\Phi = \frac{\mu_0 I_0 a}{2\pi} ln(1 + \frac{b}{x})$, so $I = \frac{\mu_0 I_0 abv}{2\pi Rx(x+b)} \varepsilon = \frac{\mu_0 I_0 abv}{2\pi x(x+b)}$

81. a. $1.01 \times 10^{-6} V$;

b. $1.37 \times 10^{-7} V$;

c. 0 V

83. a. $v = \frac{mgRsin\theta}{B^2 l^2 cos^2\theta}$;

b. $mgvsin\theta$;

c. $mc\Delta T$;

d. current would reverse direction but bar would still slide at the same speed

85. a. $B = \mu_0 nI, \Phi_m = BA = \mu_0 nIA$,

$\varepsilon = 9.9 \times 10^{-4} V$;

b. $9.9 \times 10^{-4} V$;

c. $\oint \vec{E} \cdot d\vec{l} = \varepsilon, \Rightarrow E = 1.6 \times 10^{-3} V/m$

d. $9.9 \times 10^{-4} V$;

e. no, because there is no cylindrical symmetry

87. a. $1.92 \times 10^6 rad/s = 1.83 \times 10^7 rpm$;

b. This angular velocity is unreasonably high, higher than can be obtained for any mechanical system.

c. The assumption that a voltage as great as 12.0 kV could be obtained is unreasonable.

89. $\frac{2\mu_0 \pi a^2 I_0 n\omega}{R}$

91. $\frac{mRv_o}{B^2 D^2}$

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17: INDUCTANCE

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17.1: Prelude to Inductance

In Electromagnetic Induction, we discussed how a time-varying magnetic flux induces an emf in a circuit. In many of our calculations, this flux was due to an applied time-dependent magnetic field. The reverse of this phenomenon also occurs: The current flowing in a circuit produces its own magnetic field.



Figure 17.1.1: A smartphone charging mat contains a coil that receives alternating current, or current that is constantly increasing and decreasing. The varying current induces an emf in the smartphone, which charges its battery. Note that the black box containing the electrical plug also contains a transformer (discussed in Alternating-Current Circuits) that modifies the current from the outlet to suit the needs of the smartphone. (credit: modification of work by "LG"/Flickr)

In the chapter on Electric Charges and Fields, we saw that induction is the process by which an emf is induced by changing electric flux and separation of a dipole. So far, we have discussed some examples of induction, although some of these applications are more effective than others. The smartphone charging mat in the chapter opener photo also works by induction. Is there a useful physical quantity related to how "effective" a given device is? The answer is yes, and that physical quantity is inductance. In this chapter, we look at the applications of inductance in electronic devices and how inductors are used in circuits

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17.2: Mutual Inductance

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Correlate two nearby circuits that carry time-varying currents with the emf induced in each circuit
- Describe examples in which mutual inductance may or may not be desirable

Inductance is the property of a device that tells us how effectively it induces an emf in another device. In other words, it is a physical quantity that expresses the effectiveness of a given device.

When two circuits carrying time-varying currents are close to one another, the magnetic flux through each circuit varies because of the changing current **I** in the other circuit. Consequently, an emf is induced in each circuit by the changing current in the other. This type of emf is therefore called a **mutually induced emf**, and the phenomenon that occurs is known as **mutual inductance (M)**. As an example, let's consider two tightly wound coils (Figure 17.2.1). Coils 1 and 2 have N_1 and N_2 turns and carry currents I_1 and I_2 respectively. The flux through a single turn of coil 2 produced by the magnetic field of the current in coil 1 is Φ_{12}, whereas the flux through a single turn of coil 1 due to the magnetic field of I_2 is Φ_{12}.

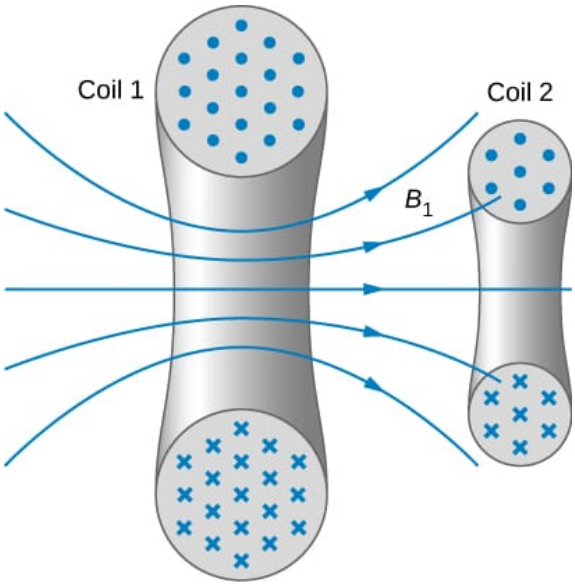


Figure 17.2.1: Some of the magnetic field lines produced by the current in coil 1 pass through coil 2.

The mutual inductance M_{21} of coil 2 with respect to coil 1 is the ratio of the flux through the N_2 turns of coil 2 produced by the magnetic field of the current in coil 1, divided by that current, that is,

$$M_{21} = \frac{N_2 \Phi_{21}}{I_1}. \tag{17.2.1}$$

Similarly, the mutual inductance of coil 1 with respect to coil 2 is

$$M_{12} = \frac{N_1 \Phi_{12}}{I_2}. \tag{17.2.2}$$

Like capacitance, mutual inductance is a geometric quantity. It depends on the shapes and relative positions of the two coils, and it is independent of the currents in the coils. The SI unit for mutual inductance **M** is called the **henry (H)** in honor of Joseph **Henry** (1799–1878), an American scientist who discovered induced emf independently of Faraday. Thus, we have $1\ H = 1\ V \cdot s/A$. From Equations 17.2.1 and 17.2.2, we can show that $M_{21} = M_{12}$, so we usually drop the subscripts associated with mutual inductance and write

$$M = \frac{N_2\Phi_{21}}{I_1} = \frac{N_1\Phi_{12}}{I_2}. \tag{17.2.3}$$

The emf developed in either coil is found by combining Faraday's law and the definition of mutual inductance. Since $N_2\Phi_{21}$ is the total flux through coil 2 due to I_1, we obtain

$$\epsilon_2 = -\frac{d}{dt}(N_2\Phi_{21}) \tag{17.2.4}$$

$$= -\frac{d}{dt}(MI_1) \tag{17.2.5}$$

$$= -M\frac{dI_1}{dt} \tag{17.2.6}$$

where we have used the fact that M is a time-independent constant because the geometry is time-independent. Similarly, we have

$$\epsilon_1 = -M\frac{dI_2}{dt}. \tag{17.2.7}$$

In Equation 17.2.7, we can see the significance of the earlier description of mutual inductance (M) as a geometric quantity. The value of M neatly encapsulates the physical properties of circuit elements and allows us to separate the physical layout of the circuit from the dynamic quantities, such as the emf and the current. Equation 17.2.7 defines the mutual inductance in terms of properties in the circuit, whereas the previous definition of mutual inductance in Equation 17.2.1 is defined in terms of the magnetic flux experienced, regardless of circuit elements. You should be careful when using Equations 17.2.6 and 17.2.6 because ϵ_1 and ϵ_2 do not necessarily represent the total emfs in the respective coils. Each coil can also have an emf induced in it because of its **self-inductance** (self-inductance will be discussed in more detail in a later section).

A large mutual inductance **M** may or may not be desirable. We want a transformer to have a large mutual inductance. But an appliance, such as an electric clothes dryer, can induce a dangerous emf on its metal case if the mutual inductance between its coils and the case is large. One way to reduce mutual inductance is to counter-wind coils to cancel the magnetic field produced (Figure 17.2.2).

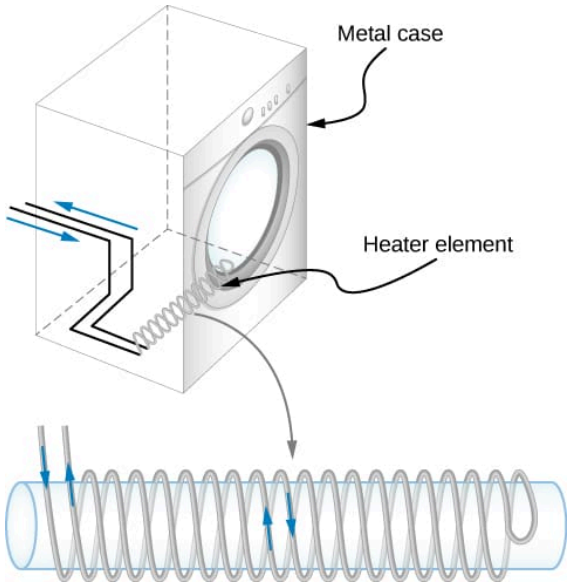


Figure 17.2.2: The heating coils of an electric clothes dryer can be counter-wound so that their magnetic fields cancel one another, greatly reducing the mutual inductance with the case of the dryer.

Digital signal processing is another example in which mutual inductance is reduced by counter-winding coils. The rapid on/off emf representing 1s and 0s in a digital circuit creates a complex time-dependent magnetic field. An emf can be generated in neighboring conductors. If that conductor is also carrying a digital signal, the induced emf may be large enough to switch 1s and 0s, with consequences ranging from inconvenient to disastrous.

Example 17.2.1: Mutual Inductance

Figure 17.2.3 shows a coil of N_2 turns and radius R_2 surrounding a long solenoid of length l_1, radius R_1, and N_1 turns.

a. What is the mutual inductance of the two coils?
b. If $N_1 = 500\ turns,\ N_2 = 10\ turns,\ R_1 = 3.10\ cm,\ l_1 = 75.0\ cm$, and the current in the solenoid is changing at a rate of 200 A/s, what is the emf induced in the surrounding coil?

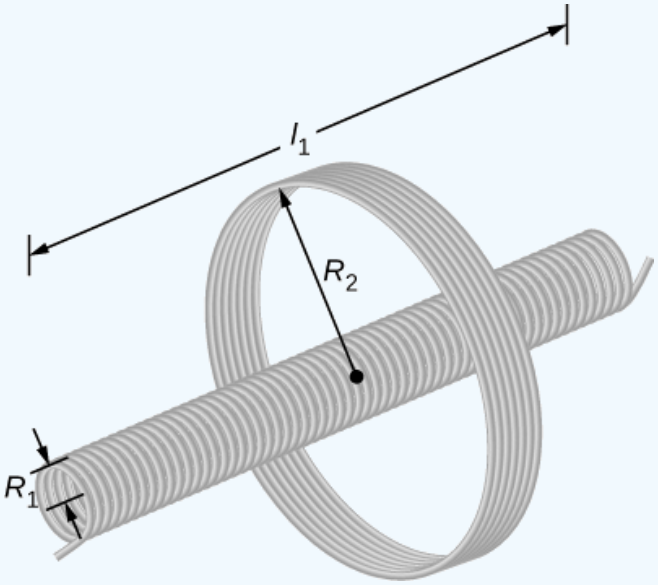


Figure 17.2.3: A solenoid surrounded by a coil.

Strategy

There is no magnetic field outside the solenoid, and the field inside has magnitude $B_1 = \mu_0(N_1/l_1)I_1$ and is directed parallel to the solenoid's axis. We can use this magnetic field to find the magnetic flux through the surrounding coil and then use this flux to calculate the mutual inductance for part (a), using Equation 17.2.3. We solve part (b) by calculating the mutual inductance from the given quantities and using Equation 17.2.6 to calculate the induced emf.

Solution

1. The magnetic flux Φ_{21} through the surrounding coil is

$$\begin{aligned}\Phi_{21} &= B_1 \pi R_1^2 \\ &= \frac{\mu_0 N_1 I_1}{l_1} \pi R_1^2.\end{aligned}$$

Now from Equation 17.2.3, the mutual inductance is

$$\begin{aligned}M &= \frac{N_2 \Phi_{21}}{I_1} \\ &= \left(\frac{N_2}{I_1}\right)\left(\frac{\mu_0 N_1 I_1}{l_1}\right)\pi R_1^2 \\ &= \frac{\mu_0 N_1 N_2 \pi R_1^2}{l_1}.\end{aligned}$$

2. Using the previous expression and the given values, the mutual inductance is

$$\begin{aligned}M &= \frac{(4\pi \times 10^{-7}\, T \cdot m/A)(500)(10)\pi(0.0310\, m)^2}{0.750\, m} \\ &= 2.53 \times 10^{-5}\, H.\end{aligned}$$

Thus, from Equation 17.2.6, the emf induced in the surrounding coil is

$$\begin{aligned}\epsilon_2 &= -M\frac{dI_1}{dt} \\ &= -(2.53\times10^{-5}H)(200\,A/s) \\ &= -5.06\times10^{-3}V.\end{aligned}$$

Significance

Notice that **M** in part (a) is independent of the radius R_2 of the surrounding coil because the solenoid's magnetic field is confined to its interior. In principle, we can also calculate **M** by finding the magnetic flux through the solenoid produced by the current in the surrounding coil. This approach is much more difficult because Φ_{12} is so complicated. However, since $M_{12} = M_{21}$, we do know the result of this calculation.

Exercise 17.2.1

A current $I(t) = (5.0\ A)\ sin\ ((120\pi\ rad/s)t)$ flows through the solenoid of part (b) of Example 17.2.1. What is the maximum emf induced in the surrounding coil?

Solution

$4.77\times10^{-2}\ V$

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17.3: Self-Inductance and Inductors

Learning Objectives

By the end of this section, you will be able to:

- Correlate the rate of change of current to the induced emf created by that current in the same circuit
- Derive the self-inductance for a cylindrical solenoid
- Derive the self-inductance for a rectangular toroid

Mutual inductance arises when a current in one circuit produces a changing magnetic field that induces an emf in another circuit. But can the magnetic field affect the current in the original circuit that produced the field? The answer is yes, and this is the phenomenon called **self-inductance**.

Inductors

Figure 17.3.1 shows some of the magnetic field lines due to the current in a circular loop of wire. If the current is constant, the magnetic flux through the loop is also constant. However, if the current **I** were to vary with time—say, immediately after switch S is closed—then the magnetic flux Φ_m would correspondingly change. Then Faraday's law tells us that an emf ϵ would be induced in the circuit, where

$$\epsilon = -\frac{d\Phi_m}{dt}. \tag{17.3.1}$$

Since the magnetic field due to a current-carrying wire is directly proportional to the current, the flux due to this field is also proportional to the current; that is,

$$\Phi_m \propto I. \tag{17.3.2}$$

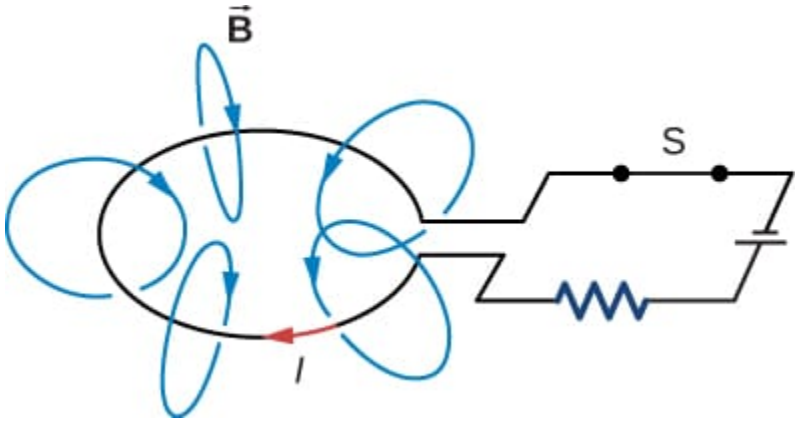


Figure 17.3.1: A magnetic field is produced by the current **I** in the loop. If **I** were to vary with time, the magnetic flux through the loop would also vary and an emf would be induced in the loop.

This can also be written as

$$\Phi_m = LI \tag{17.3.3}$$

where the constant of proportionality **L** is known as the **self-inductance** of the wire loop. If the loop has **N** turns, this equation becomes

$$\boxed{N\Phi_m = LI} \tag{17.3.4}$$

By convention, the positive sense of the normal to the loop is related to the current by the right-hand rule, so in Figure 17.3.1, the normal points downward. With this convention, Φ_m is positive in Equation 17.3.4, so **L always has a positive value**.

For a loop with **N** turns, $\epsilon = -Nd\Phi_m/dt$, so the induced emf may be written in terms of the self-inductance as

$$\boxed{\epsilon = -L\frac{dI}{dt}.} \tag{17.3.5}$$

When using this equation to determine **L**, it is easiest to ignore the signs of ϵ and dI/dt, and calculate **L** as

$$L = \frac{|\epsilon|}{|dI/dt|}.$$

Since self-inductance is associated with the magnetic field produced by a current, any configuration of conductors possesses self-inductance. For example, besides the wire loop, a long, straight wire has self-inductance, as does a coaxial cable. A coaxial cable is most commonly used by the cable television industry and may also be found connecting to your cable modem. Coaxial cables are used due to their ability to transmit electrical signals with minimal distortions. Coaxial cables have two long cylindrical conductors that possess current and a self-inductance that may have undesirable effects.



Figure 17.3.2: Symbol used to represent an inductor in a circuit.

A circuit element used to provide self-inductance is known as an **inductor**. It is represented by the symbol shown in Figure 17.3.2 , which resembles a coil of wire, the basic form of the inductor. Figure 17.3.3 shows several types of inductors commonly used in circuits.

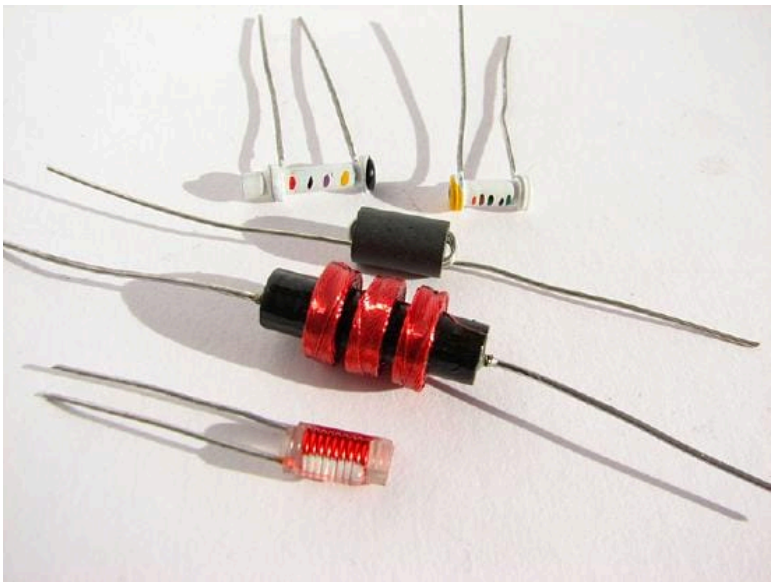


Figure 17.3.3: A variety of inductors. Whether they are encapsulated like the top three shown or wound around in a coil like the bottom-most one, each is simply a relatively long coil of wire. (credit: Windell Oskay)

In accordance with Lenz's law, the negative sign in Equation 17.3.5 indicates that the induced emf across an inductor always has a polarity that **opposes** the change in the current. For example, if the current flowing from **A** to **B** in Figure 17.3.4a were increasing, the induced emf (represented by the imaginary battery) would have the polarity shown in order to oppose the increase. If the current from **A** to **B** were decreasing, then the induced emf would have the opposite polarity, again to oppose the change in current (Figure 17.3.4b). Finally, if the current through the inductor were constant, no emf would be induced in the coil.

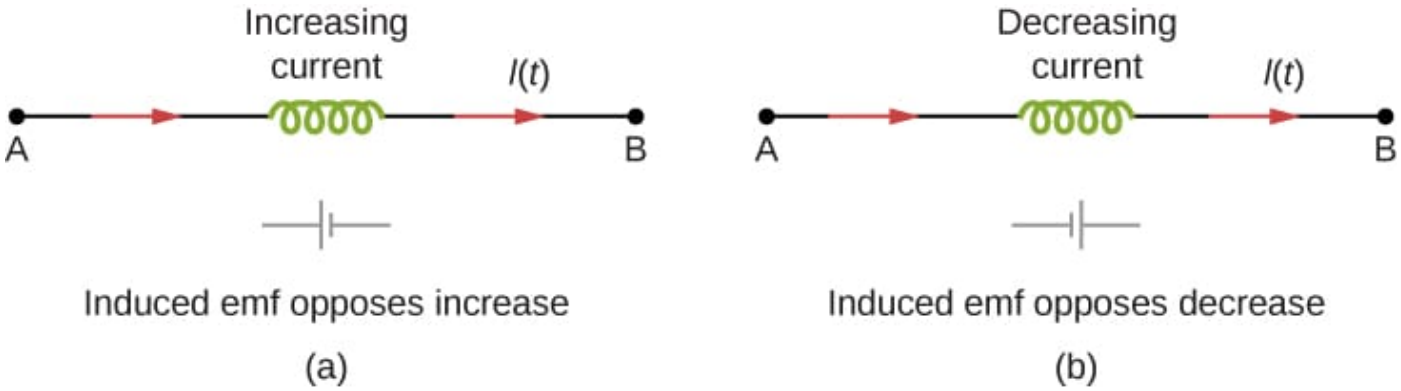


Figure 17.3.4: The induced emf across an inductor always acts to oppose the change in the current. This can be visualized as an imaginary battery causing current to flow to oppose the change in (a) and reinforce the change in (b).

One common application of inductance is to allow **traffic signals** to sense when vehicles are waiting at a street intersection. An electrical circuit with an inductor is placed in the road underneath the location where a waiting car will stop. The body of the car increases the inductance and the circuit changes, sending a signal to the traffic lights to change colors. Similarly, **metal detectors** used for airport security employ the same technique. A coil or inductor in the metal detector frame acts as both a transmitter and a receiver. The pulsed signal from the transmitter coil induces a signal in the receiver. The self-inductance of the circuit is affected by any metal object in the path (Figure 17.3.5). Metal detectors can be adjusted for sensitivity and can also sense the presence of metal on a person.



Figure 17.3.5: The familiar security gate at an airport not only detects metals, but can also indicate their approximate height above the floor. (credit: "Alexbuirds"/Wikimedia Commons)

Large induced voltages are found in **camera flashes**. Camera flashes use a battery, two inductors that function as a transformer, and a switching system or **oscillator** to induce large voltages. Recall from Oscillations on oscillations that "oscillation" is defined as the fluctuation of a quantity, or repeated regular fluctuations of a quantity, between two extreme values around an average value. Also recall (from Electromagnetic Induction on electromagnetic induction) that we need a changing magnetic field, brought about by a changing current, to induce a voltage in another coil. The oscillator system does this many times as the battery voltage is boosted to over 1000 volts. (You may hear the high-pitched whine from the transformer as the capacitor is being charged.) A capacitor stores the high voltage for later use in powering the flash.

Example 17.3.1:Self-Inductance of a Coil

An induced emf of 2.0 V is measured across a coil of 50 closely wound turns while the current through it increases uniformly from 0.0 to 5.0 A in 0.10 s. (a) What is the self-inductance of the coil? (b) With the current at 5.0 A, what is the flux through each turn of the coil?

Strategy

Both parts of this problem give all the information needed to solve for the self-inductance in part (a) or the flux through each turn of the coil in part (b). The equations needed are Equation 17.3.5 for part (a) and Equation 17.3.4 for part (b).

Solution

1. Ignoring the negative sign and using magnitudes, we have, from Equation 17.3.5,
$$L = \frac{\epsilon}{dI/dt} = \frac{2.0\,V}{5.0\,A/0.10\,s} = 4.0 \times 10^{-2} H.$$
2. From Equation 17.3.4, the flux is given in terms of the current by $\Phi_m = LI/N$, so
$$\Phi_m = \frac{(4.0 \times 10^{-2} H)(5.0\,A)}{50\,turns)} = 4.0 \times 10^{-3} Wb.$$

Significance

The self-inductance and flux calculated in parts (a) and (b) are typical values for coils found in contemporary devices. If the current is not changing over time, the flux is not changing in time, so no emf is induced.

Exercise 17.3.1

Current flows through the inductor in Figure 17.3.4 from **B** to **A** instead of from **A** to **B** as shown. Is the current increasing or decreasing in order to produce the emf given in diagram (a)? In diagram (b)?

Answer available at the end of this section.

Exercise 17.3.2

A changing current induces an emf of 10 V across a 0.25-H inductor. What is the rate at which the current is changing?

Answer available at the end of this section.

A good approach for calculating the self-inductance of an inductor consists of the following steps:

Problem-Solving Strategy: Self-Inductance

1. Assume a current **I** is flowing through the inductor.
2. Determine the magnetic field $\vec{B}$ produced by the current. If there is appropriate symmetry, you may be able to do this with Ampère's law.
3. Obtain the magnetic flux, Φ_m.
4. With the flux known, the self-inductance can be found from Equation 17.3.4, $L = N\Phi_m/I$.

To demonstrate this procedure, we now calculate the self-inductances of two inductors.

Cylindrical Solenoid

Consider a long, cylindrical solenoid with length **l**, cross-sectional area **A**, and **N** turns of wire. We assume that the length of the solenoid is so much larger than its diameter that we can take the magnetic field to be $B = \mu_0 nI$ throughout the interior of the solenoid, that is, we ignore end effects in the solenoid. With a current **I** flowing through the coils, the magnetic field produced within the solenoid is

$$B = \mu_0 \left(\frac{N}{l}\right) I, \tag{17.3.6}$$

so the magnetic flux through one turn is

$$\Phi_m = BA = \frac{\mu_0 NA}{l} I. \tag{17.3.7}$$

Using Equation 17.3.4, we find for the self-inductance of the solenoid,

Note

$$L_{solenoid} = \frac{N\Phi_m}{I} = \frac{\mu_0 N^2 A}{l}. \tag{17.3.8}$$

If $n = N/l$ is the number of turns per unit length of the solenoid, we may write Equation 17.3.8 as

$$L = \mu_0 \left(\frac{N}{l}\right)^2 Al = \mu_0 n^2 Al = \mu_0 n^2 (V), \tag{17.3.9}$$

where $V = Al$ is the volume of the solenoid. Notice that **the self-inductance of a long solenoid depends only on its physical properties** (such as the number of turns of wire per unit length and the volume), and not on the magnetic field or the current. This is true for inductors in general.

Rectangular Toroid

A toroid with a rectangular cross-section is shown in Figure 17.3.6. The inner and outer radii of the toroid are R_1 and R_2, and h is the height of the toroid. Applying Ampère's law in the same manner as we did in Example 13.5.2 for a toroid with a circular cross-section, we find the magnetic field inside a rectangular toroid is also given by

$$B = \frac{\mu_0 NI}{2\pi r},$$

where **r** is the distance from the central axis of the toroid. Because the field changes within the toroid, we must calculate the flux by integrating over the toroid's cross-section. Using the infinitesimal cross-sectional area element $da = h\ dr$ shown in Figure 17.3.6, we obtain

$$\Phi_m = \int B\, da = \int_{R_1}^{R_2} \left(\frac{\mu_0 NI}{2\pi r}\right)(h dr) = \frac{\mu_0 NhI}{2\pi} ln \frac{R_2}{R_1}. \tag{17.3.10}$$

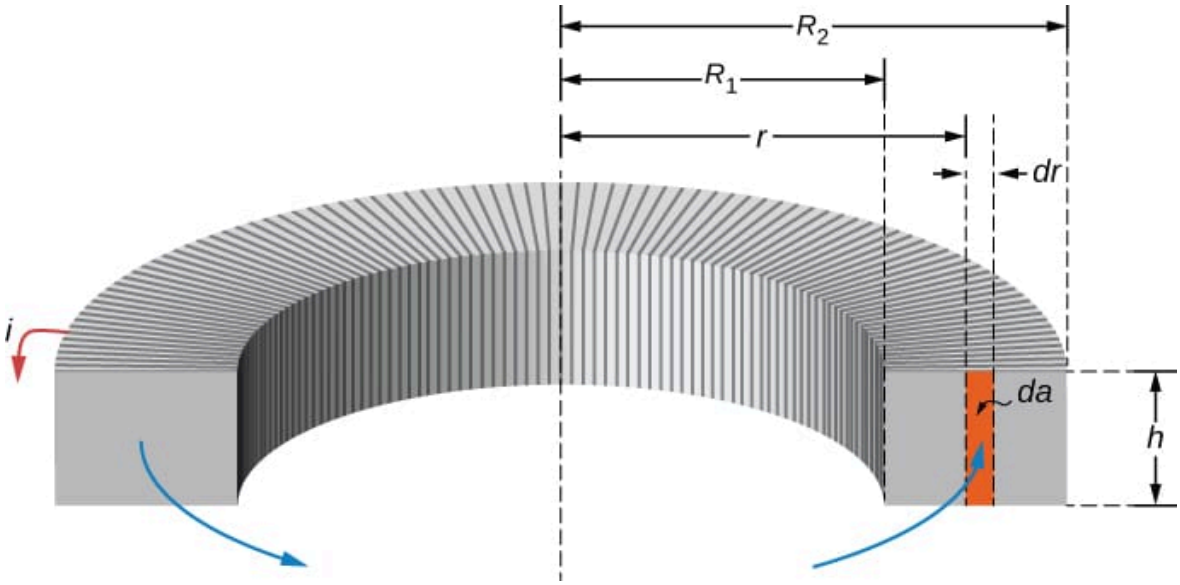


Figure 17.3.6: Calculating the self-inductance of a rectangular toroid.

Now from Equation 17.3.10, we obtain for the self-inductance of a rectangular toroid

Note

$$L = \frac{N\Phi_m}{I} = \frac{\mu_0 N^2 h}{2\pi} ln \frac{R_2}{R_1}. \tag{17.3.11}$$

As expected, the self-inductance is a constant determined by only the physical properties of the toroid.

Exercise 17.3.3

(a) Calculate the self-inductance of a solenoid that is tightly wound with wire of diameter 0.10 cm, has a cross-sectional area of $0.90\ cm^2$, and is 40 cm long. (b) If the current through the solenoid decreases uniformly from 10 to 0 A in 0.10 s, what is the emf induced between the ends of the solenoid?

Answer available at the end of this section.

Exercise 17.3.4

(a) What is the magnetic flux through one turn of a solenoid of self-inductance $8.0 \times 10^{-5} H$ when a current of 3.0 A flows through it? Assume that the solenoid has 1000 turns and is wound from wire of diameter 1.0 mm. (b) What is the cross-sectional area of the solenoid?

Answer available at the end of this section.

Answers

Exercise 17.3.1

a. decreasing; b. increasing; Since the current flows in the opposite direction of the diagram, in order to get a positive emf on the left-hand side of diagram (a), we need to decrease the current to the left, which creates a reinforced emf where the positive end is on the left-hand side. To get a positive emf on the right-hand side of diagram (b), we need to increase the current to the left, which creates a reinforced emf where the positive end is on the right-hand side.

Exercise 17.3.2

40 A/s

Exercise 17.3.3

a. $4.5 \times 10^{-5} H$; b. $4.5 \times 10^{-3} V$

Exercise 17.3.4

a. $2.4 \times 10^{-7} Wb$; b. $6.4 \times 10^{-5} m^2$

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17.4: Energy in a Magnetic Field

Learning Objectives

By the end of this section, you will be able to:

- Explain how energy can be stored in a magnetic field
- Derive the equation for energy stored in a coaxial cable given the magnetic energy density

The energy of a capacitor is stored in the electric field between its plates. Similarly, an inductor has the capability to store energy, but in its magnetic field. This energy can be found by integrating the **magnetic energy density**,

$$u_m = \frac{B^2}{2\mu_0}$$

over the appropriate volume. To understand where this formula comes from, let's consider the long, cylindrical solenoid of the previous section. Again using the infinite solenoid approximation, we can assume that the magnetic field is essentially constant and given by $B = \mu_0 nI$ everywhere inside the solenoid. Thus, the energy stored in a solenoid or the magnetic energy density times volume is equivalent to

$$U = u_m(V) = \frac{(\mu_0 nI)^2}{2\mu_0}(Al) = \frac{1}{2}(\mu_0 n^2 Al)I^2. \tag{17.4.1}$$

With the substitution of Equation 14.3.12, this becomes

$$\boxed{U = \frac{1}{2}LI^2.}$$

Although derived for a special case, this equation gives the energy stored in the magnetic field of **any** inductor. We can see this by considering an arbitrary inductor through which a changing current is passing. At any instant, the magnitude of the induced emf is $\epsilon = Ldi/dt$, where i is the induced current at that instance. Therefore, the power absorbed by the inductor is

$$P = \epsilon i = L\frac{di}{dt}i.$$

The total energy stored in the magnetic field when the current increases from **0** to **I** in a time interval from **0** to **t** can be determined by integrating this expression:

$$U = \int_0^t P dt' = \int_0^t L\frac{di}{dt'}i dt' = L\int_0^I i di = \frac{1}{2}LI^2. \tag{17.4.2}$$

Example 17.4.1: Self-Inductance of a Coaxial Cable

Figure 17.4.1 shows two long, concentric cylindrical shells of radii R_1 and R_2. As discussed in Capacitance on capacitance, this configuration is a simplified representation of a coaxial cable. The capacitance per unit length of the cable has already been calculated. Now (a) determine the magnetic energy stored per unit length of the coaxial cable and (b) use this result to find the self-inductance per unit length of the cable.

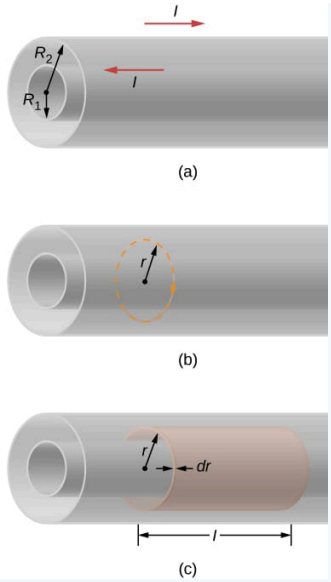


Figure 17.4.1: (a) A coaxial cable is represented here by two hollow, concentric cylindrical conductors along which electric current flows in opposite directions. (b) The magnetic field between the conductors can be found by applying Ampère's law to the dashed path. (c) The cylindrical shell is used to find the magnetic energy stored in a length **l** of the cable.

Strategy

The magnetic field both inside and outside the coaxial cable is determined by Ampère's law. Based on this magnetic field, we can use Equation 17.4.2 to calculate the energy density of the magnetic field. The magnetic energy is calculated by an integral of the magnetic energy density times the differential volume over the cylindrical shell. After the integration is carried out, we have a closed-form solution for part (a). The self-inductance per unit length is determined based on this result and Equation 17.4.2.

Solution

1. We determine the magnetic field between the conductors by applying Ampère's law to the dashed circular path shown in Figure 17.4.1*b*. Because of the cylindrical symmetry, $\vec{B}$ is constant along the path, and

$$\oint \vec{B} \cdot d\vec{l} = B(2\pi r) = \mu_0 I.$$

This gives us

$$B = \frac{\mu_0 I}{2\pi r}.$$

In the region outside the cable, a similar application of Ampère's law shows that $B = 0$, since no net current crosses the area bounded by a circular path where $r > R_2$. This argument also holds when $r < R_1$; that is, in the region within the inner cylinder. All the magnetic energy of the cable is therefore stored between the two conductors. Since the energy density of the magnetic field is

$$u_m = \frac{B^2}{2\mu_0}$$

the energy stored in a cylindrical shell of inner radius **r**, outer radius $r + dr$ and length **l** (see part (c) of the figure) is

$$u_m = \frac{\mu_0 I^2}{8\pi^2 r^2}.$$

Thus, the total energy of the magnetic field in a length **l** of the cable is

$$U = \int_{R_1}^{R_2} dU = \int_{R_1}^{R_2} \frac{\mu_0 I^2}{8\pi^2 r^2}(2\pi r l)dr = \frac{\mu_0 I^2 l}{4\pi} ln \frac{R_2}{R_1},$$

and the energy per unit length is $(\mu_0 I^2/4\pi) ln(R_2/R_1)$.

2. From Equation 17.4.2,

$$U = \frac{1}{2}LI^2,$$

where **L** is the self-inductance of a length **l** of the coaxial cable. Equating the previous two equations, we find that the self-inductance per unit length of the cable is

$$\frac{L}{l} = \frac{\mu_0}{2\pi}\ln\frac{R_2}{R_1}.$$

Significance

The inductance per unit length depends only on the inner and outer radii as seen in the result. To increase the inductance, we could either increase the outer radius (R_2) or decrease the inner radius (R_1). In the limit as the two radii become equal, the inductance goes to zero. In this limit, there is no coaxial cable. Also, the magnetic energy per unit length from part (a) is proportional to the square of the current.

Exercise 17.4.1

How much energy is stored in the inductor of Example 14.3.1 after the current reaches its maximum value?

Solution available at the end of this section.

Answers

Exercise 17.4.1

0.50 J

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17.5: RL Circuits

Learning Objectives

By the end of this section, you will be able to:

- Analyze circuits that have an inductor and resistor in series
- Describe how current and voltage exponentially grow or decay based on the initial conditions

A circuit with resistance and self-inductance is known as an **RL circuit**. Figure $17.5.1a$ shows an **RL** circuit consisting of a resistor, an inductor, a constant source of emf, and switches S_1 and S_2. When S_1 is closed, the circuit is equivalent to a single-loop circuit consisting of a resistor and an inductor connected across a source of emf (Figure $17.5.1b$). When S_1 is opened and S_2 is closed, the circuit becomes a single-loop circuit with only a resistor and an inductor (Figure $17.5.1c$).

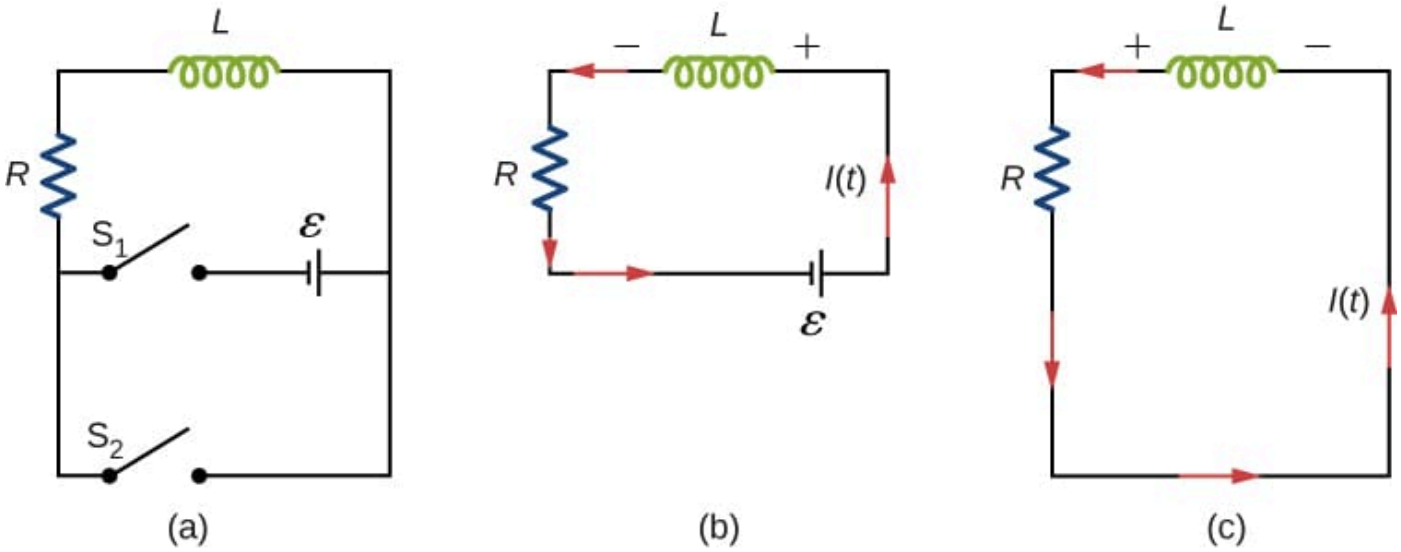


Figure $17.5.1$: (a) An RL circuit with switches S_1 and S_2. (b) The equivalent circuit with S_1 closed and S_2 open. (c) The equivalent circuit after S_1 is opened and S_2 is closed.

We first consider the **RL** circuit of Figure $17.5.1b$. Once S_1 is closed and S_2 is open, the source of emf produces a current in the circuit. If there were no self-inductance in the circuit, the current would rise immediately to a steady value of ϵ/R. However, from Faraday's law, the increasing current produces an emf $V_L = -L(dI/dt)$ across the inductor. In accordance with Lenz's law, the induced emf counteracts the increase in the current and is directed as shown in the figure. As a result, $I(t)$ starts at zero and increases asymptotically to its final value.

Applying Kirchhoff's loop rule to this circuit, we obtain

$$\epsilon - L\frac{dI}{dt} - IR = 0, \tag{17.5.1}$$

which is a first-order differential equation for $I(t)$. Notice its similarity to the equation for a capacitor and resistor in series (see **RC** Circuits). Similarly, the solution to Equation $17.5.1$ can be found by making substitutions in the equations relating the capacitor to the inductor. This gives

$$I(t) = \frac{\epsilon}{R}(1 - e^{-Rt/L}) \tag{17.5.2}$$

$$= \frac{\epsilon}{R}(1 - e^{-t/\tau_L}), \tag{17.5.3}$$

where

$$\tau_L = L/R \tag{17.5.4}$$

is the **inductive time constant** of the circuit.

The current $I(t)$ is plotted in Figure $17.5.2a$. It starts at zero, and as $t \to \infty$, $I(t)$ approaches ϵ/R asymptotically. The induced emf $V_L(t)$ is directly proportional to dI/dt, or the slope of the curve. Hence, while at its greatest immediately after the switches are thrown, the induced emf decreases to zero with time as the current approaches its final value of ϵ/R. The circuit then becomes equivalent to a resistor connected across a source of emf.

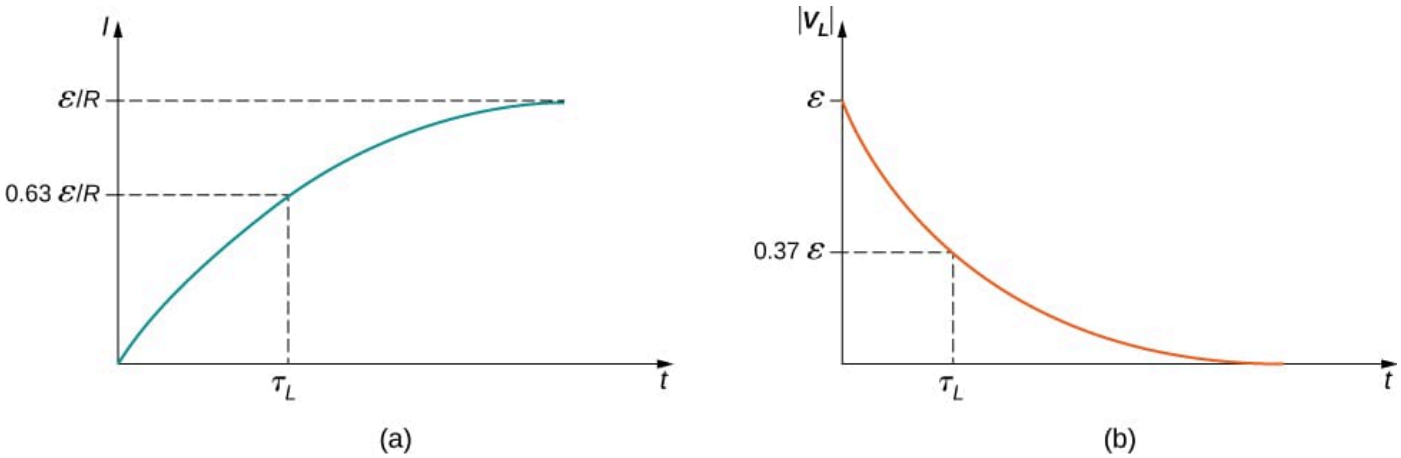


Figure $17.5.2$: Time variation of (a) the electric current and (b) the magnitude of the induced voltage across the coil in the circuit of Figure $17.5.1b$.

The energy stored in the magnetic field of an inductor is

$$U_L = \frac{1}{2}LI^2.$$

Thus, as the current approaches the maximum current ϵ/R, the stored energy in the inductor increases from zero and asymptotically approaches a maximum of $L(\epsilon/R)^2/2$.

The time constant τ_L tells us how rapidly the current increases to its final value. At $t = \tau_L$, the current in the circuit is, from Equation 17.5.3,

$$I(\tau_L) = \frac{\epsilon}{R}(1 - e^{-1}) = 0.63\frac{\epsilon}{R},$$

which is 63% of the final ϵ/R. The smaller the inductive time constant $\tau_L = L/R$, the more rapidly the current approaches ϵ/R.

We can find the time dependence of the induced voltage across the inductor in this circuit by using $V_L(t) = -L(dI/dt)$ and Equation 17.5.3:

$$V_L(t) = -L\frac{dI}{dt} = -\epsilon e^{-t/\tau_L}.$$

The magnitude of this function is plotted in Figure $17.5.2b$. The greatest value of $L(dI/dt)$ is ϵ; it occurs when dI/dt is greatest, which is immediately after S_1 is closed and S_2 is opened. In the approach to steady state, dI/dt decreases to zero. As a result, the voltage across the inductor also vanishes as $t \to \infty$.

The time constant τ_L also tells us how quickly the induced voltage decays. At $t = \tau_L$ the magnitude of the induced voltage is

$$|V_L(\tau_L)| = \epsilon e^{-1} = 0.37\epsilon = 0.37V(0).$$

The voltage across the inductor therefore drops to about 37% of its initial value after one time constant. The shorter the time constant τ_L, the more rapidly the voltage decreases.

After enough time has elapsed so that the current has essentially reached its final value, the positions of the switches in Figure $17.5.1a$ are reversed, giving us the circuit in part (c). At $t = 0$, the current in the circuit is $I(0) = \epsilon/R$. With Kirchhoff's loop rule, we

obtain

$$IR + L\frac{dI}{dt} = 0.$$

The solution to this equation is similar to the solution of the equation for a discharging capacitor, with similar substitutions. The current at time **t** is then

$$I(t) = \frac{\epsilon}{R}e^{-t/\tau_L}.$$

The current starts at $I(0) = \epsilon/R$ and decreases with time as the energy stored in the inductor is depleted (Figure $17.5.3$).

The time dependence of the voltage across the inductor can be determined from $V_L = -L(dI/dt)$:

$$V_L(0) = \epsilon e^{-t/\tau_L}. \tag{17.5.5}$$

This voltage is initially $V_L(0) = \epsilon$, and it decays to zero like the current. The energy stored in the magnetic field of the inductor, $LI^2/2$, also decreases exponentially with time, as it is dissipated by Joule heating in the resistance of the circuit.

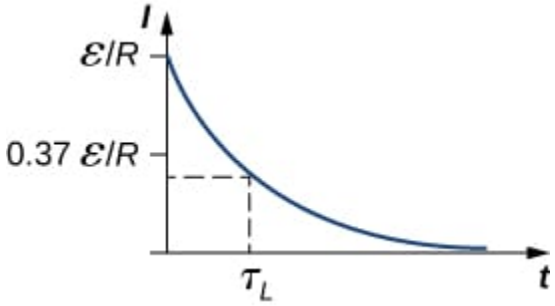


Figure $17.5.3$: Time variation of electric current in the *RL* circuit of Figure $17.5.1c$. The induced voltage across the coil also decays exponentially.

Example 17.5.1: An RL Circuit with a Source of emf

In the circuit of Figure $17.5.1a$, let $\epsilon = 2.0\ V,\ \ R = 4.0\ \Omega$, and $L = 4.0\ H$. With S_1 closed and S_2 open (Figure $17.5.1b$), (a) what is the time constant of the circuit? (b) What are the current in the circuit and the magnitude of the induced emf across the inductor at $t = 0$, at $t = 2.0\tau_L$, and as $t \to \infty$?

Strategy

The time constant for an inductor and resistor in a series circuit is calculated using Equation $17.5.4$. The current through and voltage across the inductor are calculated by the scenarios detailed from Equation $17.5.3$ and Equation $17.5.5$.

Solution

1. The inductive time constant is

$$\tau_L = \frac{L}{R} = \frac{4.0\,H}{4.0\,\Omega} = 1.0\,s.$$

2. The current in the circuit of Figure $17.5.1b$ increases according to Equation $17.5.3$:

$$I(t) = \frac{\epsilon}{R}(1 - e^{-t/\tau_L}).$$

At $t = 0$,

$$(1 - e^{-t/\tau_L}) = (1 - 1) = 0;\ so\ I(0) = 0.$$

At $t = 2.0\tau_L$ and $t \to \infty$, we have, respectively,

$$I(2.0\tau_L) = \frac{\epsilon}{R}(1 - e^{-2.0}) = (0.50\,A)(0.86) = 0.43\,A$$

and

$$I(\infty) = \frac{\epsilon}{R} = 0.50\,A.$$

From Equation 17.5.5, the magnitude of the induced emf decays as

$$|V_L(t)| = \epsilon e^{-t/\tau_L}.$$

At $t = 0$, $t = 2.0\tau_L$, and as $t \to \infty$, we obtain

$$|V_L(0)| = \epsilon = V,$$
$$|V_L(2.0\tau_L)| = (2.0\,V)e^{-2.0} = 0.27\,V$$

and

$$|V_L(\infty)| = 0.$$

Significance

If the time of the measurement were much larger than the time constant, we would not see the decay or growth of the voltage across the inductor or resistor. The circuit would quickly reach the asymptotic values for both of these (Figure 17.5.4).

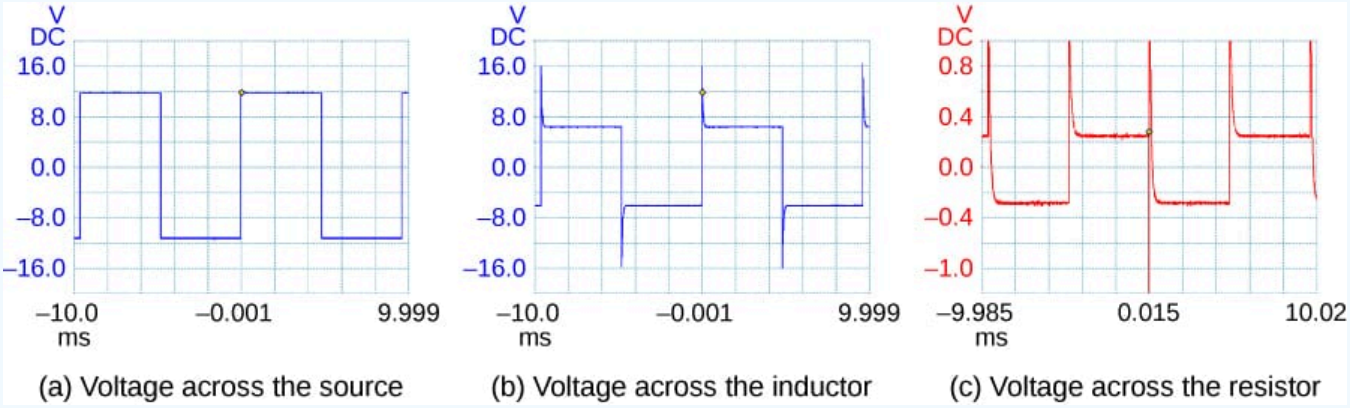


Figure 17.5.4: A generator in an **RL** circuit produces a square-pulse output in which the voltage oscillates between zero and some set value. These oscilloscope traces show (a) the voltage across the source; (b) the voltage across the inductor; (c) the voltage across the resistor.

Example 17.5.1: An RL Circuit without a Source of emf

After the current in the **RL** circuit of Example 17.5.1 has reached its final value, the positions of the switches are reversed so that the circuit becomes the one shown in Figure 17.5.1c.

a. How long does it take the current to drop to half its initial value?
b. How long does it take before the energy stored in the inductor is reduced to 1.0% of its maximum value?

Strategy

The current in the inductor will now decrease as the resistor dissipates this energy. Therefore, the current falls as an exponential decay. We can also use that same relationship as a substitution for the energy in an inductor formula to find how the energy decreases at different time intervals.

Solution

1. With the switches reversed, the current decreases according to

$$I(t) = \frac{\epsilon}{R}e^{-t/\tau_L} = I(0)e^{-t/\tau_L}.$$

At a time **t** when the current is one-half its initial value, we have

$$I(t) = 0.50I(0)\ so\, e^{-t/\tau_L} = 0.50,$$

and

$$t = -[\ln(0.50)]\tau_L = 0.69(1.0\,s) = 0.69\,s$$

where we have used the inductive time constant found in Example 17.5.1.

2. The energy stored in the inductor is given by

$$U_L(t) = \frac{1}{2}L[I(t)]^2 = \frac{1}{2}L\left(\frac{\epsilon}{R}e^{-t/\tau_L}\right)^2 = \frac{L\epsilon^2}{2R^2}e^{-2t/\tau_L}.$$

If the energy drops to 1.0% of its initial value at a time **t**, we have

$$U_L(t) = (0.010)U_L(0)\ or\ \frac{L\epsilon^2}{2R^2}e^{-2t/\tau_L} = (0.010)\frac{L\epsilon^2}{2R^2}.$$

Upon canceling terms and taking the natural logarithm of both sides, we obtain

$$-\frac{2t}{\tau_L} = \ln(0.010),$$

so

$$t = -\frac{1}{2}\tau_L \ln(0.010).$$

Since $\tau_L = 1.0\ s$, the time it takes for the energy stored in the inductor to decrease to 1.0% of its initial value is

$$t = -\frac{1}{2}(1.0\,s)\ln(0.010) = 2.3\,s.$$

Significance

This calculation only works if the circuit is at maximum current in situation (b) prior to this new situation. Otherwise, we start with a lower initial current, which will decay by the same relationship.

Exercise 17.5.1

Verify that **RC** and **L/R** have the dimensions of time

Exercise 17.5.2

a. If the current in the circuit of in Figure $17.5.1b$ increases to 90% of its final value after 5.0 s, what is the inductive time constant?
b. If $R = 20\ \Omega$, what is the value of the self-inductance?
c. If the $R = 20\ \Omega$ resistor is replaced with a $R = 100\ \Omega$ resistor, what is the time taken for the current to reach 90% of its final value?

Answer available at the end of this section.

Exercise 17.5.3

For the circuit of in Figure $17.5.1b$, show that when steady state is reached, the difference in the total energies produced by the battery and dissipated in the resistor is equal to the energy stored in the magnetic field of the coil

Answers

Exercise 17.5.2

a. 2.2 s; b. 43 H; c. 1.0 s

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17.6: Oscillations in an LC Circuit

Learning Objectives

By the end of this section, you will be able to:

- Explain why charge or current oscillates between a capacitor and inductor, respectively, when wired in series
- Describe the relationship between the charge and current oscillating between a capacitor and inductor wired in series

It is worth noting that both capacitors and inductors store energy, in their electric and magnetic fields, respectively. A circuit containing both an inductor (**L**) and a capacitor (**C**) can oscillate without a source of emf by shifting the energy stored in the circuit between the electric and magnetic fields. Thus, the concepts we develop in this section are directly applicable to the exchange of energy between the electric and magnetic fields in electromagnetic waves, or light. We start with an idealized circuit of zero resistance that contains an inductor and a capacitor, an **LC circuit**.

An **LC** circuit is shown in Figure $17.6.1$. If the capacitor contains a charge q_0 before the switch is closed, then all the energy of the circuit is initially stored in the electric field of the capacitor (Figure $17.6.1a$). This energy is

$$U_C = \frac{1}{2}\frac{q_0^2}{C}.$$

When the switch is closed, the capacitor begins to discharge, producing a current in the circuit. The current, in turn, creates a magnetic field in the inductor. The net effect of this process is a transfer of energy from the capacitor, with its diminishing electric field, to the inductor, with its increasing magnetic field.

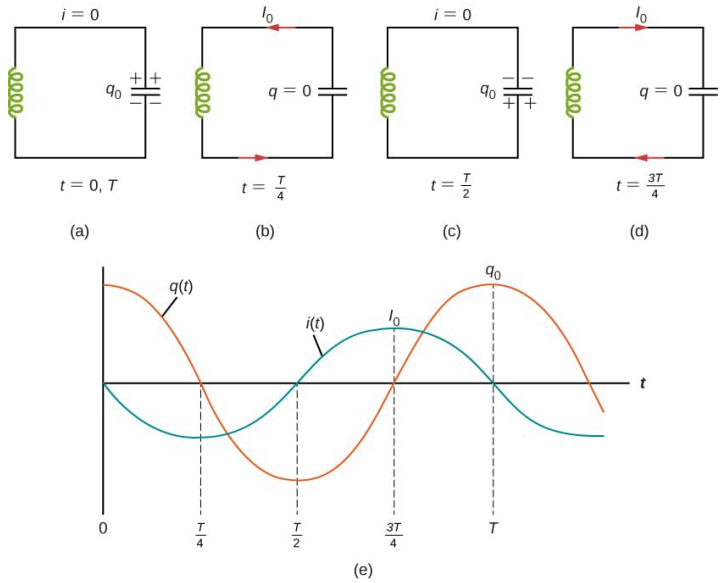


Figure $17.6.1$: (a–d) The oscillation of charge storage with changing directions of current in an **LC** circuit. (e) The graphs show the distribution of charge and current between the capacitor and inductor.

In Figure $17.6.1b$, the capacitor is completely discharged and all the energy is stored in the magnetic field of the inductor. At this instant, the current is at its maximum value I_0 and the energy in the inductor is

$$U_L = \frac{1}{2}LI_0^2.$$

Since there is no resistance in the circuit, no energy is lost through Joule heating; thus, the maximum energy stored in the capacitor is equal to the maximum energy stored at a later time in the inductor:

$$\frac{1}{2}\frac{q_0^2}{C} = \frac{1}{2}LI_0^2.$$

At an arbitrary time when the capacitor charge is **q(t)** and the current is **i(t)**, the total energy **U** in the circuit is given by

$$\frac{q^2(t)}{2C} + \frac{Li^2}{2}.$$

Because there is no energy dissipation,

$$U = \frac{1}{2}\frac{q^2}{C} + \frac{1}{2}Li^2 = \frac{1}{2}\frac{q_0^2}{C} = \frac{1}{2}LI_0^2.$$

After reaching its maximum I_0, the current **i(t)** continues to transport charge between the capacitor plates, thereby recharging the capacitor. Since the inductor resists a change in current, current continues to flow, even though the capacitor is discharged. This continued current causes the capacitor to charge with opposite polarity. The electric field of the capacitor increases while the magnetic field of the inductor diminishes, and the overall effect is a transfer of energy from the inductor **back** to the capacitor. From the law of energy conservation, the maximum charge that the capacitor re-acquires is q_0. However, as Figure $17.6.1c$ shows, the capacitor plates are charged **opposite** to what they were initially.

When fully charged, the capacitor once again transfers its energy to the inductor until it is again completely discharged, as shown in Figure $17.6.1d$. Then, in the last part of this cyclic process, energy flows back to the capacitor, and the initial state of the circuit is restored.

We have followed the circuit through one complete cycle. Its electromagnetic oscillations are analogous to the mechanical oscillations of a mass at the end of a spring. In this latter case, energy is transferred back and forth between the mass, which has kinetic energy $mv^2/2$, and the spring, which has potential energy $kx^2/2$. With the absence of friction in the mass-spring system, the oscillations would continue indefinitely. Similarly, the oscillations of an **LC** circuit with no resistance would continue forever if undisturbed; however, this ideal zero-resistance **LC** circuit is not practical, and any **LC** circuit will have at least a small resistance, which will radiate and lose energy over time.

The frequency of the oscillations in a resistance-free **LC** circuit may be found by analogy with the mass-spring system. For the circuit, $i(t) = dq(t)/dt$, the total electromagnetic energy **U** is

$$U = \frac{1}{2}Li^2 + \frac{1}{2}\frac{q^2}{C}.$$

For the mass-spring system, $v(t) = dx(t)/dt$, the total mechanical energy **E** is

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2.$$

The equivalence of the two systems is clear. To go from the mechanical to the electromagnetic system, we simply replace **m** by **L**, **v** by **i**, **k** by 1/**C**, and **x** by **q**. Now **x(t)** is given by

$$x(t) = A\,cos(\omega t + \phi)$$

where $\omega = \sqrt{k/m}$. Hence, the charge on the capacitor in an **LC** circuit is given by

$$q(t) = q_0\,cos(\omega t + \phi) \tag{17.6.1}$$

where the angular frequency of the oscillations in the circuit is

$$\omega = \sqrt{\frac{1}{LC}}. \tag{17.6.2}$$

Finally, the current in the **LC** circuit is found by taking the time derivative of **q(t)**:

$$i(t) = \frac{dq(t)}{dt} = -\omega q_0\,sin(\omega t + \phi).$$

The time variations of **q** and **I** are shown in Figure $17.6.1e$ for $\phi = 0$.

An **LC** Circuit

In an **LC** circuit, the self-inductance is 2.0×10^{-2} H and the capacitance is 8.0×10^{-6} F. At $t = 0$ all of the energy is stored in the capacitor, which has charge 1.2×10^{-5} C. (a) What is the angular frequency of the oscillations in the circuit? (b) What is the maximum current flowing through circuit? (c) How long does it take the capacitor to become completely discharged? (d) Find an equation that represents **q(t)**.

Strategy

The angular frequency of the **LC** circuit is given by Equation 17.6.2. To find the maximum current, the maximum energy in the capacitor is set equal to the maximum energy in the inductor. The time for the capacitor to become discharged if it is initially charged is a quarter of the period of the cycle, so if we calculate the period of the oscillation, we can find out what a quarter of that is to find this time. Lastly, knowing the initial charge and angular frequency, we can set up a cosine equation to find **q**(**t**).

Solution

1. From Equation 17.6.2, the angular frequency of the oscillations is
$$\omega = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(2.0 \times 10^{-2}\, H)(8.0 \times 10^{-6}\, F)}} = 2.5 \times 10^{3}\, rad/s.$$
2. The current is at its maximum I_0 when all the energy is stored in the inductor. From the law of energy conservation,
$$\frac{1}{2} L I_0^2 = \frac{1}{2} \frac{q_0^2}{C},$$
so
$$I_0 = \sqrt{\frac{1}{LC}}\, q_0 = (2.5 \times 10^{3}\, rad/s)(1.2 \times 10^{-5} C) = 3.0 \times 10^{-2} A.$$
This result can also be found by an analogy to simple harmonic motion, where current and charge are the velocity and position of an oscillator.
3. The capacitor becomes completely discharged in one-fourth of a cycle, or during a time **T**/4, where **T** is the period of the oscillations. Since
$$T = \frac{2\pi}{\omega} = \frac{2\pi}{2.5 \times 10^{3}\, rad/s} = 2.5 \times 10^{-3} s,$$
the time taken for the capacitor to become fully discharged is $(2.5 \times 10^{-3} s)/4 = 6.3 \times 10^{-4} s$.
4. The capacitor is completely charged at $t = 0$, so $q(0) = q_0$. Using 17.6.1, we obtain
$$q(0) = q_0 = q_0\, cos\, \phi.$$
Thus, $\phi = 0$, and
$$q(t) = (1.2 \times 10^{-5} C) cos(2.5 \times 10^{3} t).$$

Significance

The energy relationship set up in part (b) is not the only way we can equate energies. At most times, some energy is stored in the capacitor and some energy is stored in the inductor. We can put both terms on each side of the equation. By examining the circuit only when there is no charge on the capacitor or no current in the inductor, we simplify the energy equation.

Exercise 17.6.1

The angular frequency of the oscillations in an **LC** circuit is 2.0×10^{3} rad/s. (a) If $L = 0.10\ H$, what is **C**? (b) Suppose that at $t = 0$ all the energy is stored in the inductor. What is the value of ϕ? (c) A second identical capacitor is connected in parallel with the original capacitor. What is the angular frequency of this circuit?

Solution

a. $2.5\ \mu F$; b. $\pi/2$ rad or $3\pi/2$ rad; c. 1.4×10^3 rad/s

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17.7: RLC Series Circuits

Learning Objectives

By the end of this section, you will be able to:

- Determine the angular frequency of oscillation for a resistor, inductor, capacitor (**RLC**) series circuit
- Relate the **RLC** circuit to a damped spring oscillation

When the switch is closed in the **RLC** circuit of Figure $17.7.1a$, the capacitor begins to discharge and electromagnetic energy is dissipated by the resistor at a rate i^2R. With **U** given by Equation 14.4.2, we have

$$\frac{dU}{dt} = \frac{q}{C}\frac{dq}{dt} + Li\frac{di}{dt} = -i^2R$$

where **i** and **q** are time-dependent functions. This reduces to

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = 0. \tag{17.7.1}$$

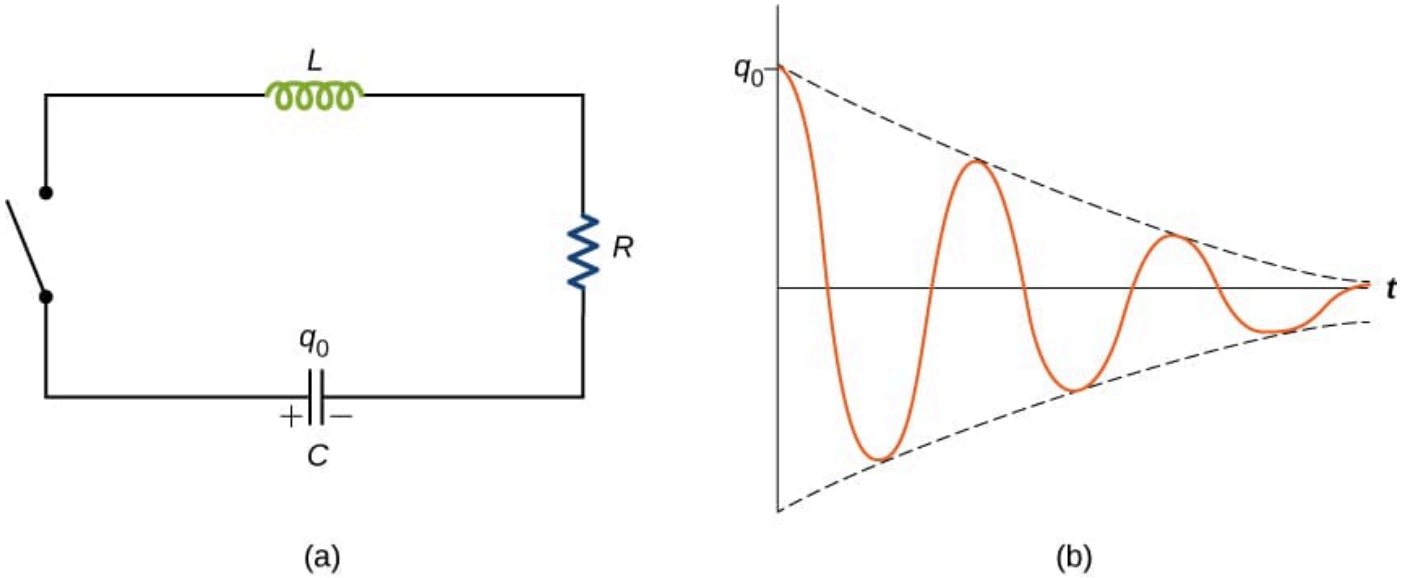


Figure $17.7.1$: (a) An **RLC** circuit. Electromagnetic oscillations begin when the switch is closed. The capacitor is fully charged initially. (b) Damped oscillations of the capacitor charge are shown in this curve of charge versus time, or **q** versus **t**. The capacitor contains a charge q_0 before the switch is closed.

This equation is analogous to

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0,$$

which is the equation of motion for a **damped mass-spring system** (you first encountered this equation in Oscillations). As we saw in that chapter, it can be shown that the solution to this differential equation takes three forms, depending on whether the angular frequency of the undamped spring is greater than, equal to, or less than **b**/2**m**. Therefore, the result can be underdamped $(\sqrt{k/m} > b/2m)$, critically damped $(\sqrt{k/m} = b/2m)$, or overdamped $(\sqrt{k/m} < b/2m)$. By analogy, the solution **q**(**t**) to the **RLC** differential equation has the same feature. Here we look only at the case of under-damping. By replacing **m** by **L**, **b** by **R**, **k** by 1/**C**, and **x** by **q** in Equation 17.7.1, and assuming $\sqrt{1/LC} > R/2L$, we obtain

Note

$$q(t) = q_0 e^{-Rt/2L} cos(\omega' t + \phi) \tag{17.7.2}$$

where the angular frequency of the oscillations is given by

Note

$$\omega' = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2} \tag{17.7.3}$$

This underdamped solution is shown in Figure $17.7.1b$. Notice that the amplitude of the oscillations decreases as energy is dissipated in the resistor. Equation $17.7.2$ can be confirmed experimentally by measuring the voltage across the capacitor as a function of time. This voltage, multiplied by the capacitance of the capacitor, then gives **q**(**t**).

Note

Try an interactive circuit construction kit that allows you to graph current and voltage as a function of time. You can add inductors and capacitors to work with any combination of **R**, **L**, and **C** circuits with both dc and ac sources.

Note

Try out a circuit-based java applet website that has many problems with both dc and ac sources that will help you practice circuit problems.

Exercise 17.7.1

In an **RLC** circuit, $L = 5.0\ mH$, $C = 6.0\ \mu F$, and $R = 200\ \Omega$. (a) Is the circuit underdamped, critically damped, or overdamped? (b) If the circuit starts oscillating with a charge of $3.0 \times 10^{-3} C$ on the capacitor, how much energy has been dissipated in the resistor by the time the oscillations cease?

Answer available at the end of this section.

Answer available at the end of this section.

Answers

Exercise $17.7.1$

a. overdamped; b. 0.75 J

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17.8: Inductance (Summary)

Key Terms

henry (H)	unit of inductance, $1H = 1\Omega \cdot s$; it is also expressed as a volt second per ampere
inductance	property of a device that tells how effectively it induces an emf in another device
inductive time constant	denoted by τ, the characteristic time given by quantity L/R of a particular series RL circuit
inductor	part of an electrical circuit to provide self-inductance, which is symbolized by a coil of wire
LC circuit	circuit composed of an ac source, inductor, and capacitor
magnetic energy density	energy stored per volume in a magnetic field
mutual inductance	geometric quantity that expresses how effective two devices are at inducing emfs in one another
RLC circuit	circuit with an ac source, resistor, inductor, and capacitor all in series.
self-inductance	effect of the device inducing emf in itself

Key Equations

Mutual inductance by flux	$M = \frac{N_2\Phi_2}{1_I} = \frac{N_1\Phi_{12}}{I_2}$
Mutual inductance in circuits	$\varepsilon_1 = -M\frac{dI_2}{dt}$
Self-inductance in terms of magnetic flux	$N\Phi_m = LI$
Self-inductance in terms of emf	$\varepsilon = -L\frac{dI}{dt}$
Self-inductance of a solenoid	$L_{solenoid} = \frac{\mu_0 N^2 A}{l}$
Self-inductance of a toroid	$L_{toroid} = \frac{\mu_0 N^2 h}{2\pi} ln\frac{R_2}{R_1}.$
Energy stored in an inductor	$U = \frac{1}{2}LI^2$
Current as a function of time for a **RL** circuit	$I(t) = \frac{\varepsilon}{R}(1 - e^{-t/\tau_L})$
Time constant for a **RL** circuitτ	$\tau_L = L/R$
Charge oscillation in **LC** circuits	$q(t) = q_0 cos(\omega t + \phi)$
Angular frequency in **LC** circuits	$\omega = \sqrt{\frac{1}{LC}}$
Current oscillations in **LC** circuits	$i(t) = -\omega q_0 sin(\omega t + \phi)$
Charge as a function of time in **RLC** circuit	$q(t) = q_0 e^{-Rt/2L} cos(\omega' t + \phi)$
Angular frequency in **RLC** circuit	$\omega' = \sqrt{\frac{1}{LC} - (\frac{R}{2L})^2}$

Summary

14.2 Mutual Inductance

- Inductance is the property of a device that expresses how effectively it induces an emf in another device.
- Mutual inductance is the effect of two devices inducing emfs in each other.
- A change in current dI_1/dt in one circuit induces an emf (ε_2) in the second:

$$\varepsilon_2 = -M\frac{dI_1}{dt},$$

where **M** is defined to be the mutual inductance between the two circuits and the minus sign is due to Lenz's law.

- Symmetrically, a change in current dI_2/dt through the second circuit induces an emf (ε_1) in the first:

$$\varepsilon_1 = -M\frac{dI_2}{dt},$$

where **M** is the same mutual inductance as in the reverse process.

14.3 Self-Inductance and Inductors

- Current changes in a device induce an emf in the device itself, called self-inductance,

$$\varepsilon = -L\frac{dI}{dt},$$

where **L** is the self-inductance of the inductor and dI/dt is the rate of change of current through it. The minus sign indicates that emf opposes the change in current, as required by Lenz's law. The unit of self-inductance and inductance is the henry (H), where $1H = 1\Omega \cdot s$.

- The self-inductance of a solenoid is

$$L = \frac{\mu_0 N^2 A}{l},$$

where **N** is its number of turns in the solenoid, **A** is its cross-sectional area, **l** is its length, and $\mu_0 = 4\pi \times 10^{-7} T \cdot m/A$ is the permeability of free space.

- The self-inductance of a toroid is

$$L = \frac{\mu_0 N^2 h}{2\pi} ln\frac{R_2}{R_1},$$

where **N** is its number of turns in the toroid, R_1 and R_2 are the inner and outer radii of the toroid, **h** is the height of the toroid, and $\mu_0 = 4\pi \times 10^{-7} T \cdot m/A$ is the permeability of free space.

14.4 Energy in a Magnetic Field

- The energy stored in an inductor **U** is

$$U = \frac{1}{2}LI^2.$$

- The self-inductance per unit length of coaxial cable is

$$\frac{L}{l} = \frac{\mu_0}{2\pi} ln\frac{R_2}{R_1}.$$

14.5 RL Circuits

- When a series connection of a resistor and an inductor—an **RL** circuit—is connected to a voltage source, the time variation of the current is

  $I(t) = \frac{\varepsilon}{R}(1 - e^{-Rt/L}) = \frac{\varepsilon}{R}(1 - e^{-t/\tau_L})$ (turning on),

  where the initial current is $I_0 = \varepsilon/R$..
- The characteristic time constant τ is $\tau_L = L/R$, where **L** is the inductance and **R** is the resistance.
- In the first time constant τ, the current rises from zero to $0.632I_0$, and to 0.632 of the remainder in every subsequent time interval τ.
- When the inductor is shorted through a resistor, current decreases as

  $I(t) = \frac{\varepsilon}{R}e^{-t/\tau_L}$ (turning off).

  Current falls to $0.368I_0$ in the first time interval τ, and to 0.368 of the remainder toward zero in each subsequent time τ.

14.6 Oscillations in an LC Circuit

- The energy transferred in an oscillatory manner between the capacitor and inductor in an **LC** circuit occurs at an angular frequency $\omega = \sqrt{\frac{1}{LC}}$.
- The charge and current in the circuit are given by

$$q(t) = q_0 cos(\omega t + \phi),$$

$$i(t) = -\omega q_0 sin(\omega t + \phi).$$

14.7 RLC Series Circuits

- The underdamped solution for the capacitor charge in an **RLC** circuit is

$$q(t) = q_0 e^{-Rt/2L} cos(\omega' t + \phi).$$

- The angular frequency given in the underdamped solution for the **RLC** circuit is

$$\omega' = \sqrt{\frac{1}{LC} - (\frac{R}{2L})^2}.$$

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17.9: Inductance (Exercise)

Conceptual Questions

14.2 Mutual Inductance

1. Show that $N\Phi_m/I$ and $\varepsilon/(dI/dt)$, which are both expressions for self-inductance, have the same units.

2. A 10-H inductor carries a current of 20 A. Describe how a 50-V emf can be induced across it.

3. The ignition circuit of an automobile is powered by a 12-V battery. How are we able to generate large voltages with this power source?

4. When the current through a large inductor is interrupted with a switch, an arc appears across the open terminals of the switch. Explain.

14.3 Self-Inductance and Inductors

5. Does self-inductance depend on the value of the magnetic flux? Does it depend on the current through the wire? Correlate your answers with the equation $N\Phi_m = LI$.

6. Would the self-inductance of a 1.0 m long, tightly wound solenoid differ from the self-inductance per meter of an infinite, but otherwise identical, solenoid?

7. Discuss how you might determine the self-inductance per unit length of a long, straight wire.

8. The self-inductance of a coil is zero if there is no current passing through the windings. True or false?

9. How does the self-inductance per unit length near the center of a solenoid (away from the ends) compare with its value near the end of the solenoid?

14.4 Energy in a Magnetic Field

10. Show that $LI^2/2$ has units of energy.

14.5 RL Circuits

11. Use Lenz's law to explain why the initial current in the RL circuit of Figure 14.12(b) is zero.

12. When the current in the **RL** circuit of Figure 14.12(b) reaches its final value ε/R, what is the voltage across the inductor? Across the resistor?

13. Does the time required for the current in an **RL** circuit to reach any fraction of its steady-state value depend on the emf of the battery?

14. An inductor is connected across the terminals of a battery. Does the current that eventually flows through the inductor depend on the internal resistance of the battery? Does the time required for the current to reach its final value depend on this resistance?

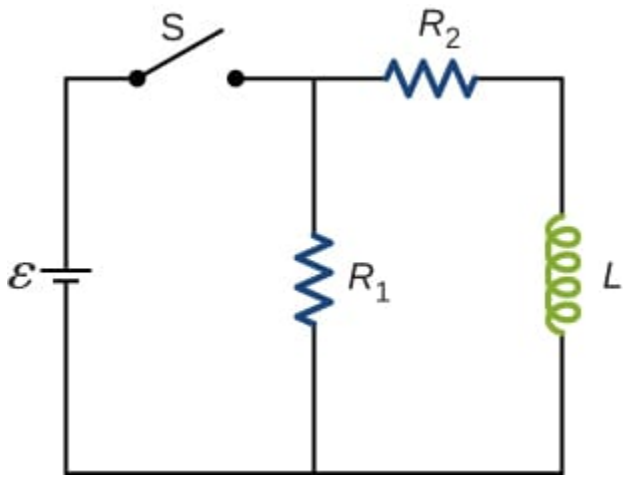
15. At what time is the voltage across the inductor of the **RL** circuit of Figure 14.12(b) a maximum?

16. In the simple **RL** circuit of Figure 14.12(b), can the emf induced across the inductor ever be greater than the emf of the battery used to produce the current?

17. If the emf of the battery of Figure 14.12(b) is reduced by a factor of 2, by how much does the steady-state energy stored in the magnetic field of the inductor change?

18. A steady current flows through a circuit with a large inductive time constant. When a switch in the circuit is opened, a large spark occurs across the terminals of the switch. Explain.

19. Describe how the currents through R_1 and R_2 shown below vary with time after switch S is closed.



20. Discuss possible practical applications of **RL** circuits.

14.6 Oscillations in an LC Circuit

21. Do Kirchhoff's rules apply to circuits that contain inductors and capacitors?

22. Can a circuit element have both capacitance and inductance?

23. In an LC circuit, what determines the frequency and the amplitude of the energy oscillations in either the inductor or capacitor?

14.7 RLC Series Circuits

24. When a wire is connected between the two ends of a solenoid, the resulting circuit can oscillate like an **RLC** circuit. Describe what causes the capacitance in this circuit.

25. Describe what effect the resistance of the connecting wires has on an oscillating **LC** circuit.

26. Suppose you wanted to design an **LC** circuit with a frequency of 0.01 Hz. What problems might you encounter?

27. A radio receiver uses an **RLC** circuit to pick out particular frequencies to listen to in your house or car without hearing other unwanted frequencies. How would someone design such a circuit?

Problems

14.2 Mutual Inductance

28. When the current in one coil changes at a rate of 5.6 A/s, an emf of $6.3 \times 10^{-3} V$. is induced in a second, nearby coil. What is the mutual inductance of the two coils?

29. An **emf** of $9.7 \times 10^{-3} V$ is induced in a coil while the current in a nearby coil is decreasing at a rate of 2.7 A/s. What is the mutual inductance of the two coils?

30. Two coils close to each other have a mutual inductance of 32 mH. If the current in one coil decays according to $I = I_0 e^{-\alpha t}$, where $I_0 = 5.0A$ and $\alpha = 2.0 \times 10^3 s^{-1}$, what is the emf induced in the second coil immediately after the current starts to decay? At $t = 1.0 \times 10^{-3} s$?

31. A coil of 40 turns is wrapped around a long solenoid of cross-sectional area $7.5 \times 10^{-3} m^2$. The solenoid is 0.50 m long and has 500 turns.

(a) What is the mutual inductance of this system?

(b) The outer coil is replaced by a coil of 40 turns whose radius is three times that of the solenoid. What is the mutual inductance of this configuration?

32. A 600-turn solenoid is 0.55 m long and 4.2 cm in diameter. Inside the solenoid, a small (1.1cm×1.4cm), single-turn rectangular coil is fixed in place with its face perpendicular to the long axis of the solenoid. What is the mutual inductance of this system?

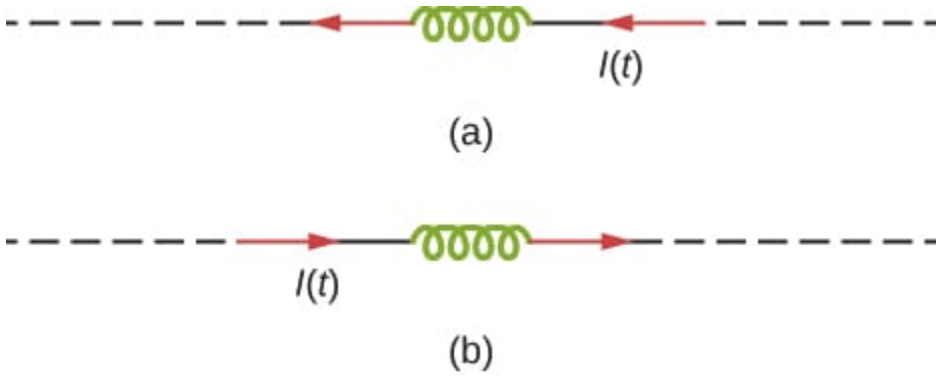
33. A toroidal coil has a mean radius of 16 cm and a cross-sectional area of $0.25 cm^2$; it is wound uniformly with 1000 turns. A second toroidal coil of 750 turns is wound uniformly over the first coil. Ignoring the variation of the magnetic field within a toroid, determine the mutual inductance of the two coils.

34. A solenoid of N_1 turns has length l_1 and radius R_1, and a second smaller solenoid of N_2 turns has length l_2 and radius R_2. The smaller solenoid is placed completely inside the larger solenoid so that their long axes coincide. What is the mutual inductance of the two solenoids?

14.3 Self-Inductance and Inductors

35. An **emf** of 0.40 V is induced across a coil when the current through it changes uniformly from 0.10 to 0.60 A in 0.30 s. What is the self-inductance of the coil?

36. The current shown in part (a) below is increasing, whereas that shown in part (b) is decreasing. In each case, determine which end of the inductor is at the higher potential.



37. What is the rate at which the current though a 0.30-H coil is changing if an emf of 0.12 V is induced across the coil?

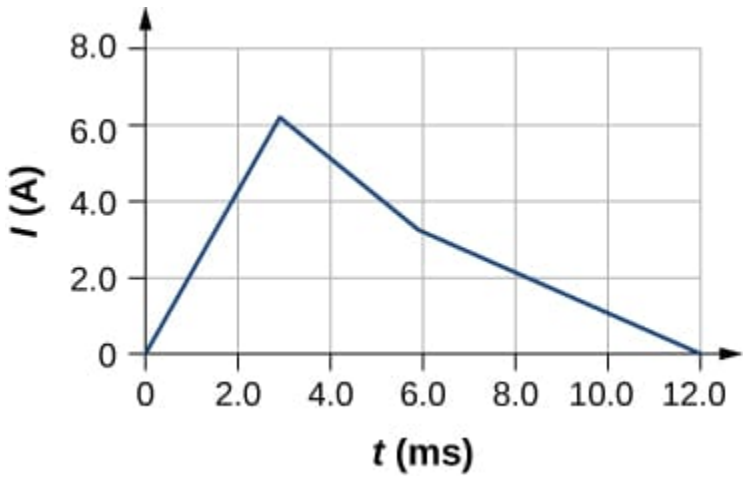
38. When a camera uses a flash, a fully charged capacitor discharges through an inductor. In what time must the 0.100-A current through a 2.00-mH inductor be switched on or off to induce a 500-V emf?

39. A coil with a self-inductance of 2.0 H carries a current that varies with time according to $I(t) = (2.0A) sin 120\pi t$. Find an expression for the emf induced in the coil.

40. A solenoid 50 cm long is wound with 500 turns of wire. The cross-sectional area of the coil is $2.0 cm^2$ What is the self-inductance of the solenoid?

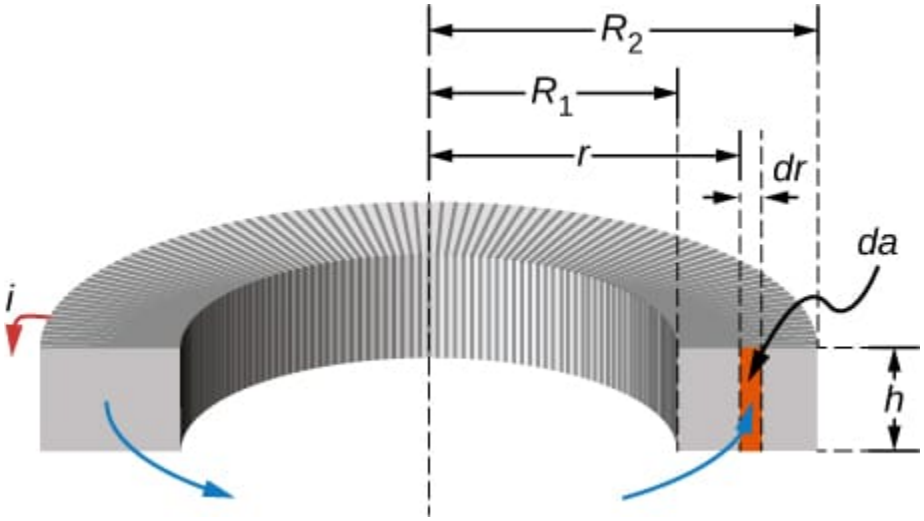
41. A coil with a self-inductance of 3.0 H carries a current that decreases at a uniform rate $dI/dt = -0.050 A/s$. What is the **emf** induced in the coil? Describe the polarity of the induced **emf.**

42. The current $I(t)$ through a 5.0-mH inductor varies with time, as shown below. The resistance of the inductor is **5.0Ω**. Calculate the voltage across the inductor at $t = 2.0ms, t = 4.0ms,$ and $t = 8.0ms$.



43. A long, cylindrical solenoid with 100 turns per centimeter has a radius of 1.5 cm. (a) Neglecting end effects, what is the self-inductance per unit length of the solenoid? (b) If the current through the solenoid changes at the rate 5.0 A/s, what is the emf induced per unit length?

44. Suppose that a rectangular toroid has 2000 windings and a self-inductance of 0.040 H. If h=0.10mh=0.10m, what is the ratio of its outer radius to its inner radius?



45. What is the self-inductance per meter of a coaxial cable whose inner radius is 0.50 mm and whose outer radius is 4.00 mm?

14.4 Energy in a Magnetic Field

46. At the instant a current of 0.20 A is flowing through a coil of wire, the energy stored in its magnetic field is $6.0 \times 10^{-3} J$. What is the self-inductance of the coil?

47. Suppose that a rectangular toroid has 2000 windings and a self-inductance of 0.040 H. If $h = 0.10m$, what is the current flowing through a rectangular toroid when the energy in its magnetic field is $2.0 \times 10^{-6} J$?

48. Solenoid **A** is tightly wound while solenoid **B** has windings that are evenly spaced with a gap equal to the diameter of the wire. The solenoids are otherwise identical. Determine the ratio of the energies stored per unit length of these solenoids when the same current flows through each.

49. A 10-H inductor carries a current of 20 A. How much ice at **0°C** could be melted by the energy stored in the magnetic field of the inductor? (Hint: Use the value $L_f = 334 J/g$ for ice.)

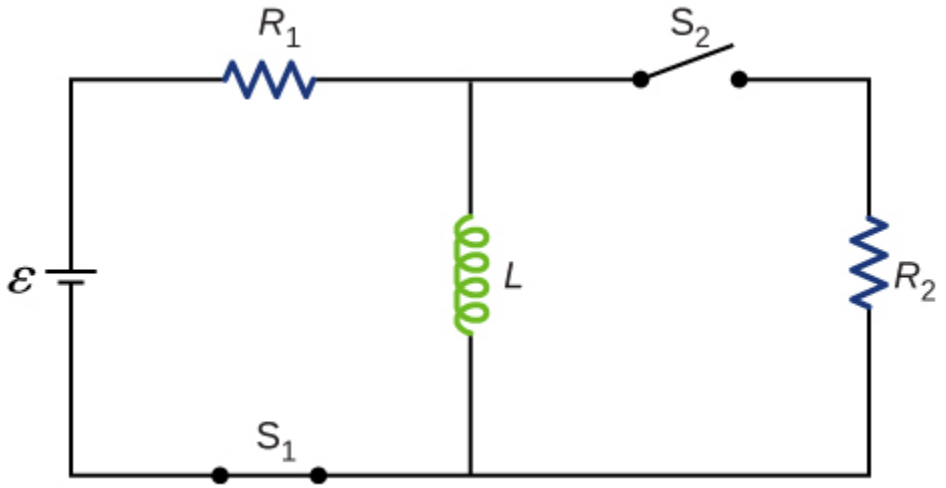
50. A coil with a self-inductance of 3.0 H and a resistance of **100Ω** carries a steady current of 2.0 A. (a) What is the energy stored in the magnetic field of the coil? (b) What is the energy per second dissipated in the resistance of the coil?

51. A current of 1.2 A is flowing in a coaxial cable whose outer radius is five times its inner radius. What is the magnetic field energy stored in a 3.0-m length of the cable?

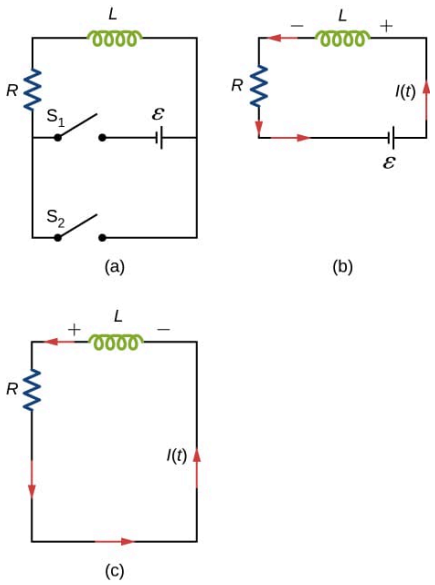
14.5 RL Circuits

52. In Figure 14.12, $\varepsilon = 12V, L = 20mH$, and $R = 5.0\Omega$. Determine (a) the time constant of the circuit, (b) the initial current through the resistor, (c) the final current through the resistor, (d) the current through the resistor when $t = 2\tau_L$, and (e) the voltages across the inductor and the resistor when $t = 2\tau_L$.

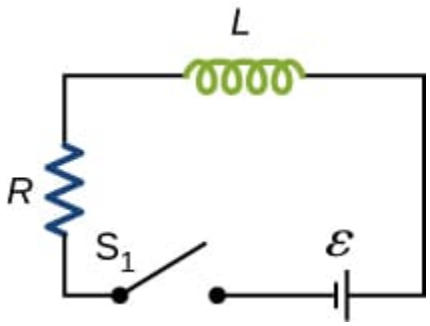
53. For the circuit shown below, $\varepsilon = 20V, L = 4.0mH$, and $R = 5.0\Omega$. After steady state is reached with S_1 closed and S_2 open, S_2 is closed and immediately thereafter (at $t = 0$) S_1 is opened. Determine (a) the current through L at $t = 0$, (b) the current through L at $t = 4.0 \times 10^{-4} s$, and (c) the voltages across L and R at $t = 4.0 \times 10^{-4} s$. $R_1 = R_2 = R$.



54. The current in the **RL** circuit shown here increases to **40%** of its steady-state value in 2.0 s. What is the time constant of the circuit?



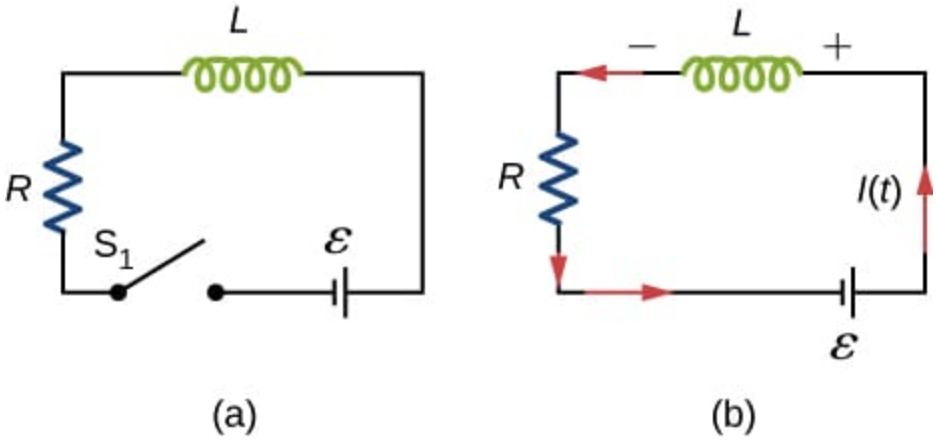
55. How long after switch S_1 is thrown does it take the current in the circuit shown to reach half its maximum value? Express your answer in terms of the time constant of the circuit.



56. Examine the circuit shown below in part (a). Determine dI/dt at the instant after the switch is thrown in the circuit of

(a), thereby producing the circuit of

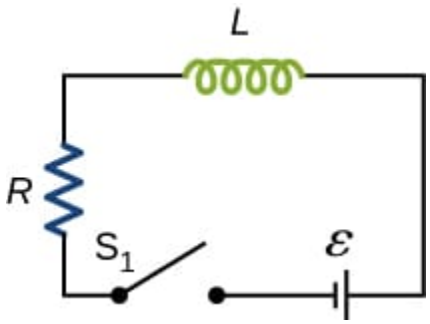
(b). Show that if I were to continue to increase at this initial rate, it would reach its maximum ε/R in one time constant.



57. The current in the **RL** circuit shown below reaches half its maximum value in 1.75 ms after the switch S_1 is thrown. Determine

(a) the time constant of the circuit and

(b) the resistance of the circuit if $L = 250mH$.

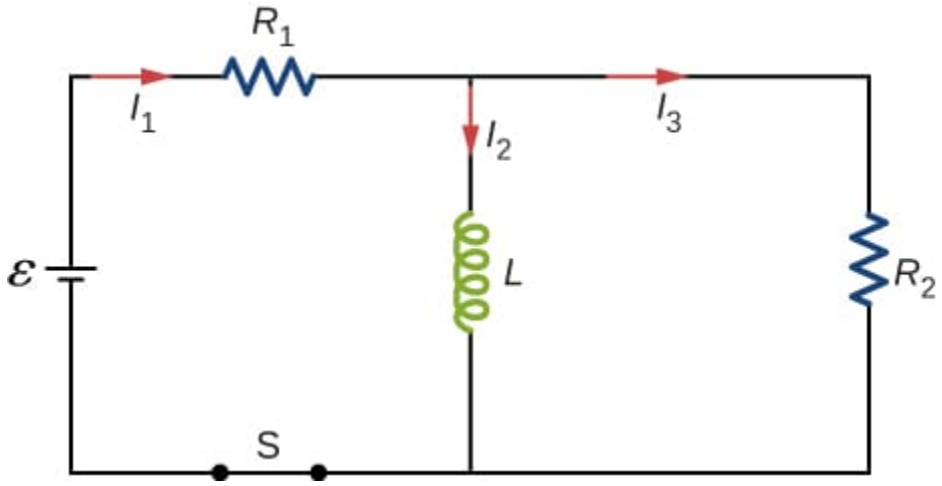


58. Consider the circuit shown below. Find I_1, I_2, and I_3 when

(a) the switch S is first closed,

(b) after the currents have reached steady-state values, and

(c) at the instant the switch is reopened (after being closed for a long time).



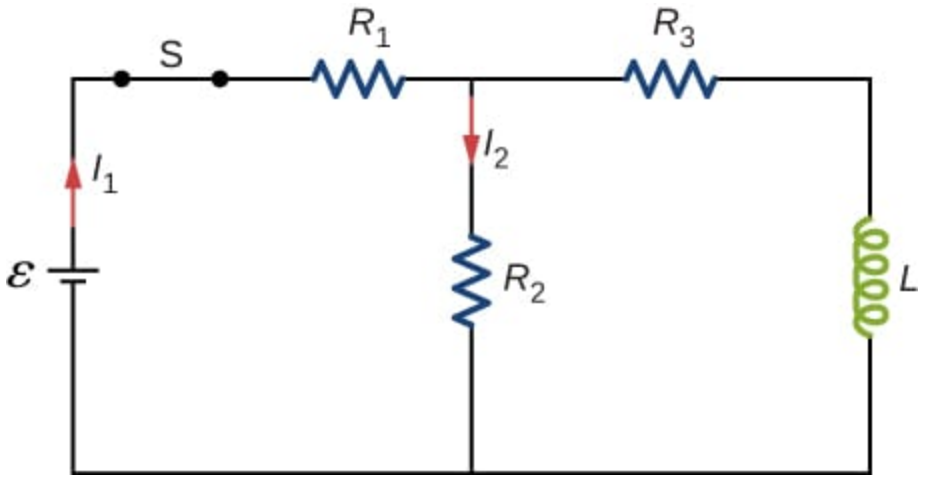
59. For the circuit shown below, $\varepsilon = 50V, R_1 = 10\Omega, R_2 = R_3 = 19.4\Omega$ and $L = 2.0mH$. Find the values of I_1 and I_2

(a) immediately after switch S is closed,

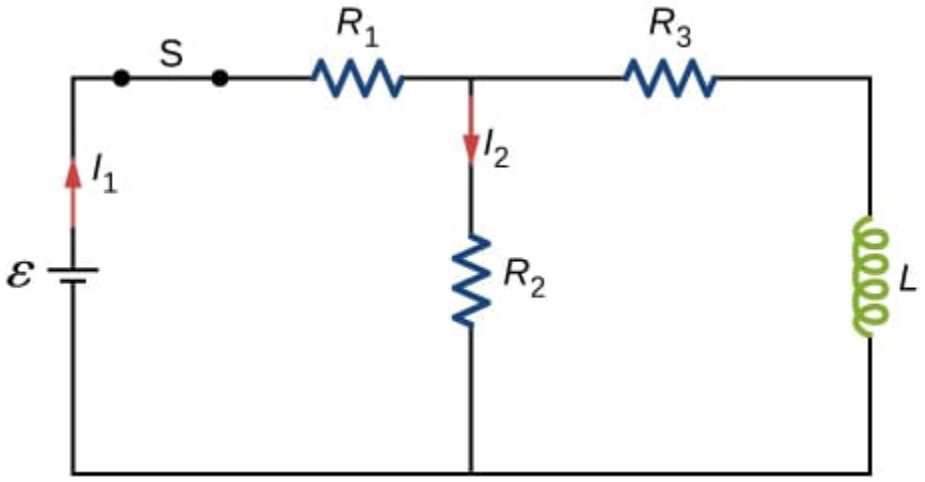
(b) a long time after S is closed,

(c) immediately after S is reopened, and

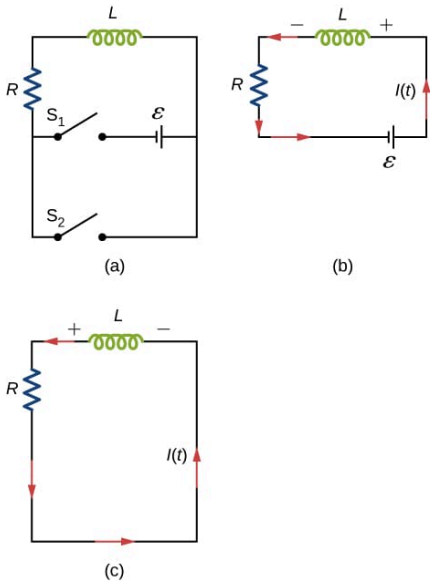
(d) a long time after S is reopened.



60. For the circuit shown below, find the current through the inductor $2.0 \times 10^{-5} s$ after the switch is reopened.



61. Show that for the circuit shown below, the initial energy stored in the inductor, $LI^2(0)/2$, is equal to the total energy eventually dissipated in the resistor, $\int_0^\infty I^2(t)Rdt$.



14.6 Oscillations in an LC Circuit

62. A 5000-pF capacitor is charged to 100 V and then quickly connected to an 80-mH inductor. Determine

(a) the maximum energy stored in the magnetic field of the inductor,

(b) the peak value of the current, and

(c) the frequency of oscillation of the circuit.

63. The self-inductance and capacitance of an **LC** circuit are 0.20 mH and 5.0 pF. What is the angular frequency at which the circuit oscillates?

64. What is the self-inductance of an **LC** circuit that oscillates at 60 Hz when the capacitance is $10\mu F$?

65. In an oscillating **LC** circuit, the maximum charge on the capacitor is $2.0 \times 10^{-6}C$ and the maximum current through the inductor is 8.0 mA.

(a) What is the period of the oscillations?

(b) How much time elapses between an instant when the capacitor is uncharged and the next instant when it is fully charged?

66. The self-inductance and capacitance of an oscillating **LC** circuit are $L = 20mH$ and $C = 1.0\mu F$, respectively.

(a) What is the frequency of the oscillations?

(b) If the maximum potential difference between the plates of the capacitor is 50 V, what is the maximum current in the circuit?

67. In an oscillating **LC** circuit, the maximum charge on the capacitor is q_m. Determine the charge on the capacitor and the current through the inductor when energy is shared equally between the electric and magnetic fields. Express your answer in terms of $q_m, L,$ and C.

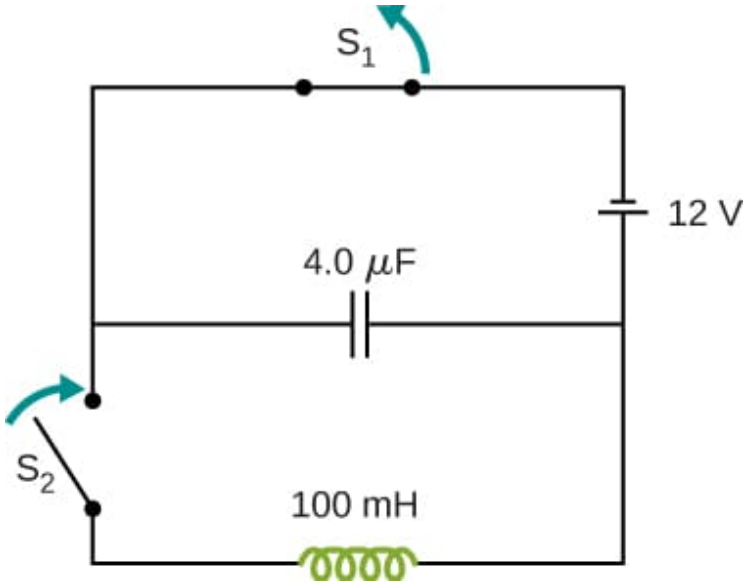
68. In the circuit shown below, S_1 is opened and S_2 is closed simultaneously. Determine

(a) the frequency of the resulting oscillations,

(b) the maximum charge on the capacitor,

(c) the maximum current through the inductor, and

(d) the electromagnetic energy of the oscillating circuit.



69. An **LC** circuit in an AM tuner (in a car stereo) uses a coil with an inductance of 2.5 mH and a variable capacitor. If the natural frequency of the circuit is to be adjustable over the range 540 to 1600 kHz (the AM broadcast band), what range of capacitance is required?

14.7 RLC Series Circuits

70. In an oscillating **RLC** circuit, $R = 5.0\Omega, L = 5.0mH$, and $C = 500\mu F$. What is the angular frequency of the oscillations?

71. In an oscillating **RLC** circuit with $L = 10mH, C = 1.5\mu F$, and $R = 2.0\Omega$, how much time elapses before the amplitude of the oscillations drops to half its initial value?

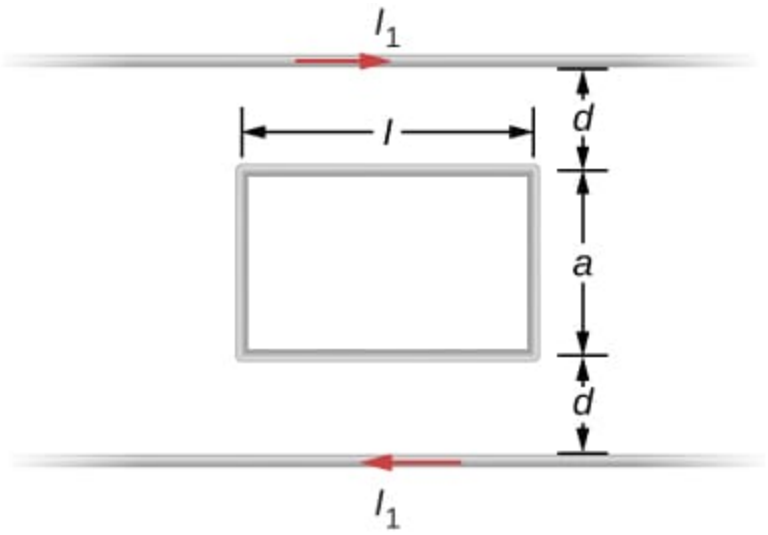
72. What resistance **R** must be connected in series with a 200-mH inductor of the resulting **RLC** oscillating circuit is to decay to **50%** of its initial value of charge in 50 cycles? To **0.10%** of its initial value in 50 cycles?

Additional Problems

73. Show that the self-inductance per unit length of an infinite, straight, thin wire is infinite.

74. Two long, parallel wires carry equal currents in opposite directions. The radius of each wire is a, and the distance between the centers of the wires is d. Show that if the magnetic flux within the wires themselves can be ignored, the self-inductance of a length l of such a pair of wires is $L = \frac{\mu_0 l}{\pi} ln \frac{d-a}{a}$. (Hint: Calculate the magnetic flux through a rectangle of length l between the wires and then use $L = N\Phi/I$.)

75. A small, rectangular single loop of wire with dimensions l, and a is placed, as shown below, in the plane of a much larger, rectangular single loop of wire. The two short sides of the larger loop are so far from the smaller loop that their magnetic fields over the smaller fields over the smaller loop can be ignored. What is the mutual inductance of the two loops?



76. Suppose that a cylindrical solenoid is wrapped around a core of iron whose magnetic susceptibility is x. Using Equation 14.9, show that the self-inductance of the solenoid is given by $L = \frac{(1+x)\mu_0 N^2 A}{l}$, where **I** is its length, **A** its cross-sectional area, and **N** its total number of turns.

77. The solenoid of the preceding problem is wrapped around an iron core whose magnetic susceptibility is 4.0×10^3.

(a) If a current of 2.0 A flows through the solenoid, what is the magnetic field in the iron core?

(b) What is the effective surface current formed by the aligned atomic current loops in the iron core?

(c) What is the self-inductance of the filled solenoid?

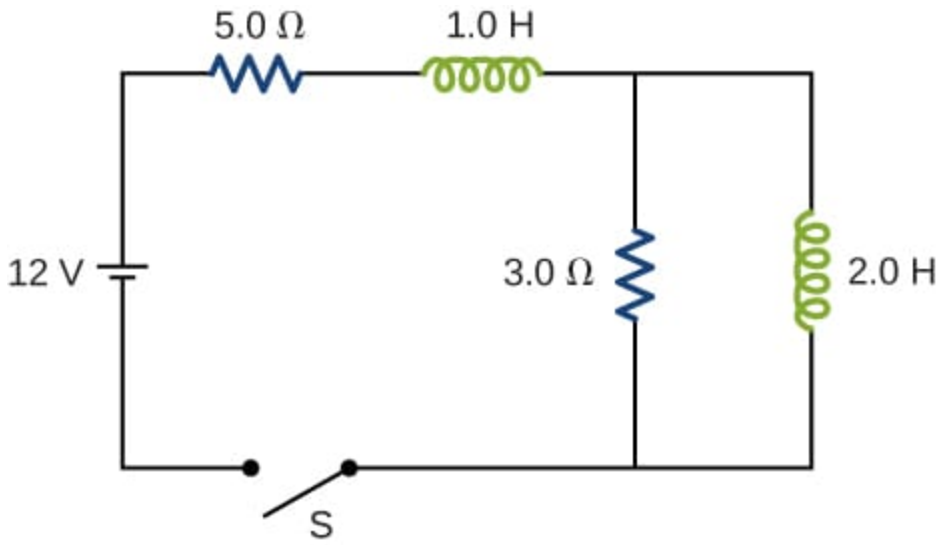
78. A rectangular toroid with inner radius $R_1 = 7.0cm$, outer radius $R_2 = 9.0cm$, height $h = 3.0$, and $N = 3000$ turns is filled with an iron core of magnetic susceptibility 5.2×10^3.

(a) What is the self-inductance of the toroid?

(b) If the current through the toroid is 2.0 A, what is the magnetic field at the center of the core?

(c) For this same 2.0-A current, what is the effective surface current formed by the aligned atomic current loops in the iron core?

79. The switch S of the circuit shown below is closed at $t = 0$. Determine (a) the initial current through the battery and (b) the steady-state current through the battery.



80. In an oscillating **RLC** circuit, $R = 7.0\Omega, L = 10mH$, and $C = 3.0\mu F$. Initially, the capacitor has a charge of $8.0\mu C$ and the current is zero. Calculate the charge on the capacitor

(a) five cycles later and

(b) 50 cycles later.

81. A 25.0-H inductor has 100 A of current turned off in 1.00 ms.

(a) What voltage is induced to oppose this?

(b) What is unreasonable about this result?

(c) Which assumption or premise is responsible?

Challenge Problems

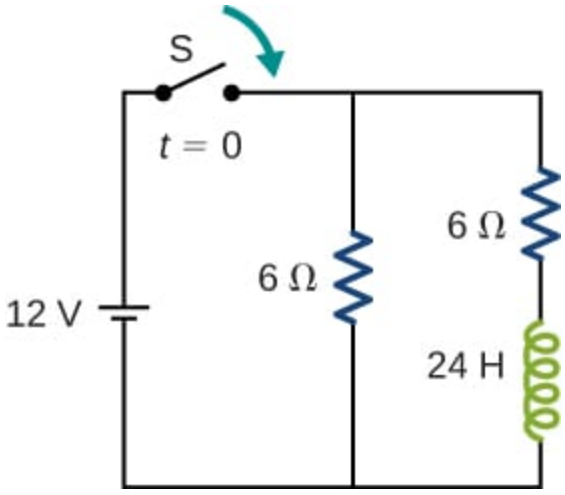
82. A coaxial cable has an inner conductor of radius a, and outer thin cylindrical shell of radius b. A current **I** flows in the inner conductor and returns in the outer conductor. The self-inductance of the structure will depend on how the current in the inner cylinder tends to be distributed. Investigate the following two extreme cases.

(a) Let current in the inner conductor be distributed only on the surface and find the self-inductance.

(b) Let current in the inner cylinder be distributed uniformly over its cross-section and find the self-inductance. Compare with your results in (a).

83. In a damped oscillating circuit the energy is dissipated in the resistor. The **Q**-factor is a measure of the persistence of the oscillator against the dissipative loss. (a) Prove that for a lightly damped circuit the energy, **U**, in the circuit decreases according to the following equation. $\frac{dU}{dt} = -2\beta U$, where $\beta = \frac{R}{2L}$. (b) Using the definition of the **Q**-factor as energy divided by the loss over the next cycle, prove that **Q**-factor of a lightly damped oscillator as defined in this problem is $Q \equiv \frac{U_{begin}}{\Delta U_{one\ cycle}}$ $= \frac{1}{2\pi R}\sqrt{\frac{L}{C}}$. (Hint: For (b), to obtain **Q**, divide **E** at the beginning of one cycle by the change ΔE over the next cycle.)

84. The switch in the circuit shown below is closed at $t = 0s$. Find currents through (a) R_1, (b) R_2, and (c) the battery as function of time.



85. A square loop of side 2 cm is placed 1 cm from a long wire carrying a current that varies with time at a constant rate of 3 A/s as shown below.

(a) Use Ampère's law and find the magnetic field as a function of time from the current in the wire.

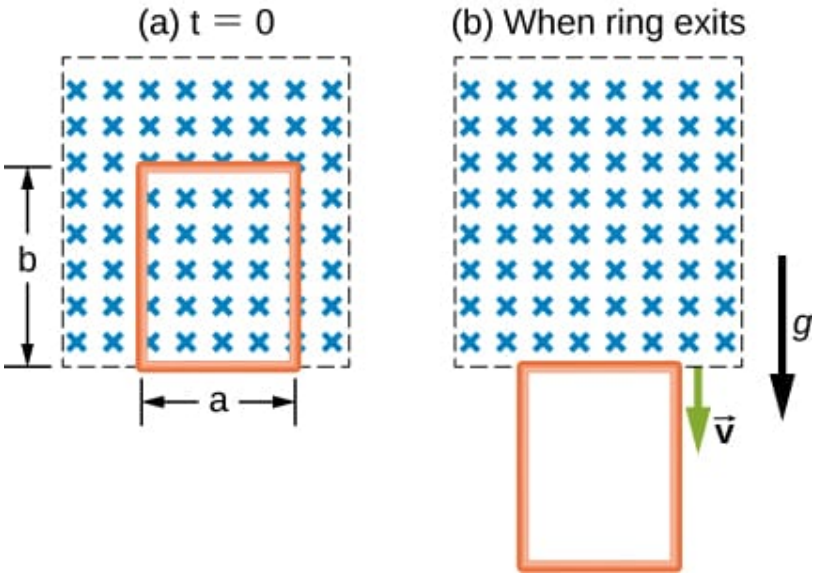
(b) Determine the magnetic flux through the loop.

(c) If the loop has a resistance of **3Ω**, how much induced current flows in the loop?

86. A rectangular copper ring, of mass 100 g and resistance **0.2Ω**, is in a region of uniform magnetic field that is perpendicular to the area enclosed by the ring and horizontal to Earth's surface. The ring is let go from rest when it is at the edge of the nonzero magnetic field region (see below).

(a) Find its speed when the ring just exits the region of uniform magnetic field.

(b) If it was let go at t=0t=0, what is the time when it exits the region of magnetic field for the following values: $a = 25cm, b = 50cm, B = 3T$, and $g = 9.8m/s^2$? Assume the magnetic field of the induced current is negligible compared to 3 T.



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17.10: Inductance (Answers)

Check Your Understanding

14.1. $4.77 \times 10^{-2} V$

14.2. a. decreasing;

b. increasing; Since the current flows in the opposite direction of the diagram, in order to get a positive emf on the left-hand side of diagram (a), we need to decrease the current to the left, which creates a reinforced emf where the positive end is on the left-hand side. To get a positive emf on the right-hand side of diagram (b), we need to increase the current to the left, which creates a reinforced emf where the positive end is on the right-hand side.

14.3. 40 A/s

14.4. a. $4.5 \times 10^{-5} H$;

b. $4.5 \times 10^{-3} V$

14.5. a. $2.4 \times 10^{-7} Wb$;

b. $6.4 \times 10^{-5} m^2$

14.6. 0.50 J

14.8. a. 2.2 s;

b. 43 H;

c. 1.0 s

14.10. a. $2.5 \mu F$;

b. $\pi/2 rad$ or $3\pi/2 rad$;

c. 1.4×10^3rad/s

14.11. a. overdamped;

b. 0.75 J

Conceptual Questions

1. $\frac{Wb}{A} = \frac{T \cdot m^2}{A} = \frac{V \cdot s}{A} = \frac{V}{A/s}$

3. The induced current from the 12-V battery goes through an inductor, generating a large voltage.

5. Self-inductance is proportional to the magnetic flux and inversely proportional to the current. However, since the magnetic flux depends on the current I, these effects cancel out. This means that the self-inductance does not depend on the current. If the emf is induced across an element, it does depend on how the current changes with time.

7. Consider the ends of a wire a part of an **RL** circuit and determine the self-inductance from this circuit.

9. The magnetic field will flare out at the end of the solenoid so there is less flux through the last turn than through the middle of the solenoid.

11. As current flows through the inductor, there is a back current by Lenz's law that is created to keep the net current at zero amps, the initial current.

13. no

15. At $t = 0$, or when the switch is first thrown.

17. 1/4

19. Initially, $I_{R1} = \frac{\varepsilon}{R_1}$ and $I_{R2} = 0$, and after a long time has passed, $I_{R1} = \frac{\varepsilon}{R_1}$ and $I_{R2} = \frac{\varepsilon}{R_2}$.

21. yes

23. The amplitude of energy oscillations depend on the initial energy of the system. The frequency in a **LC** circuit depends on the values of inductance and capacitance.

25. This creates an **RLC** circuit that dissipates energy, causing oscillations to decrease in amplitude slowly or quickly depending on the value of resistance.

27. You would have to pick out a resistance that is small enough so that only one station at a time is picked up, but big enough so that the tuner doesn't have to be set at exactly the correct frequency. The inductance or capacitance would have to be varied to tune into the station however practically speaking, variable capacitors are a lot easier to build in a circuit.

Problems

29. $M = 3.6 \times 10^{-3} H$

31. a. $3.8 \times 10^{-4} H$;

b. $3.8 \times 10^{-4} H$

33. $M_{21} = 2.3 \times 10^{-5} H$

35. 0.24 H

37. 0.4 A/s

39. $\varepsilon = 480\pi sin(120\pi t - \pi/2) V$

41. 0.15 V. This is the same polarity as the emf driving the current.

43. a. 0.089 H/m;

b. 0.44 V/m

45. $\frac{L}{l} = 4.16 \times 10^{-7} H/m$

47. 0.01 A

49. 6.0 g

51. $U_m = 7.0 \times 10^{-7} J$

53. a. 4.0 A;

b. 2.4 A;

c. on R: $V = 12V$; on L: $V = 7.9V$

55. 0.69τ

57. a. 2.52 ms;

b. 99.2Ω

59. a. $I_1 = I_2 = 1.7A$;

b. $I_1 = 2.73A, I_2 = 1.36A$;

c. $I_1 = 0, I_2 = 0.54$;

d. $I_1 = I_2 = 0$

61. proof

63. $\omega = 3.2 \times 10^7 rad/s$

65. a. $7.9 \times 10^{-4} s$;

b. $4.0 \times 10^{-4} s$

67. $q = \frac{qm}{\sqrt{2}}, I = \frac{q_m}{\sqrt{2LC}}$

69. $C = \frac{1}{4\pi^2 f^2 L}$

$f_1 = 540Hz; C_1 = 3.5 \times 10^{-11} F$

$f_2 =; 1600Hz; C_2 = 4.0 \times 10^{-12} F$

71. 6.9 ms

Additional Problems

73. proof

Outside, $B = \frac{\mu_0 I}{2\pi r}$ Inside, $B = \frac{\mu_0 Ir}{2\pi a^2}$

$U = \frac{\mu_0 I^2 l}{4\pi}(\frac{1}{4} + ln\frac{R}{a})$

So, $\frac{2U}{I^2} = \frac{\mu_0 l}{P2\pi}(\frac{1}{4} + ln\frac{R}{a})$ and $L = \infty$

75. $M = \frac{\mu_0 l}{\pi} ln\frac{d+a}{d}$

77. a. 100 T;

b. 2 A;

c. 0.50 H

79. a. 0 A;

b. 2.4 A

81. $a. 2.50 \times 10^6 V$;

(b) The voltage is so extremely high that arcing would occur and the current would not be reduced so rapidly.

(c) It is not reasonable to shut off such a large current in such a large inductor in such an extremely short time.

Challenge Problems

83. proof

85. a. $\frac{dB}{dt} = 6 \times 10^{-6} T/s$;

b. $\Phi = \frac{\mu_0 aI}{2\pi} ln(\frac{a+b}{b})$;

c. 4.0 nA

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18: ALTERNATING-CURRENT CIRCUITS

18.1: Prelude to Alternating-Current Circuits

18.2: AC Sources

18.3: Simple AC Circuits

18.4: RLC Series Circuits with AC

18.5: Power in an AC Circuit

18.6: Resonance in an AC Circuit

18.7: Transformers

18.8: Alternating-Current Circuits (Summary)

18.9: Alternating-Current Circuits (Exercise)

18.10: Alternating-Current Circuits (Answers)

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18.1: Prelude to Alternating-Current Circuits

Electric power is delivered to our homes by alternating current (ac) through high-voltage transmission lines. As explained in Transformers, transformers can then change the amplitude of the alternating potential difference to a more useful form. This lets us transmit power at very high voltages, minimizing resistive heating losses in the lines, and then furnish that power to homes at lower, safer voltages. Because constant potential differences are unaffected by transformers, this capability is more difficult to achieve with direct-current transmission.



Figure 18.1.1: The current we draw into our houses is an alternating current (ac). Power lines transmit ac to our neighborhoods, where local power stations and transformers distribute it to our homes. In this chapter, we discuss how a transformer works and how it allows us to transmit power at very high voltages and minimal heating losses across the lines.

In this chapter, we use Kirchhoff's laws to analyze four simple circuits in which ac flows. We have discussed the use of the resistor, capacitor, and inductor in circuits with batteries. These components are also part of ac circuits. However, because ac is required, the constant source of emf supplied by a battery is replaced by an ac voltage source, which produces an oscillating emf.

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18.2: AC Sources

Learning Objectives

By the end of the section, you will be able to:

- Explain the differences between direct current (dc) and alternating current (ac)
- Define characteristic features of alternating current and voltage, such as the amplitude or peak and the frequency

Most examples dealt with so far in this book, particularly those using batteries, have constant-voltage sources. Thus, once the current is established, it is constant. **Direct current (dc)** is the flow of electric charge in only one direction. It is the steady state of a constant-voltage circuit.

Most well-known applications, however, use a time-varying voltage source. **Alternating current (ac)** is the flow of electric charge that periodically reverses direction. An ac is produced by an alternating emf, which is generated in a power plant, as described in Induced Electric Fields. If the ac source varies periodically, particularly sinusoidally, the circuit is known as an ac circuit. Examples include the commercial and residential power that serves so many of our needs.

The ac voltages and frequencies commonly used in businesses and homes vary around the world. In a typical house, the potential difference between the two sides of an electrical outlet alternates sinusoidally with a frequency of 60 or 50 Hz and an amplitude of 170 or 311 V, depending on whether you live in the United States or Europe, respectively. Most people know the potential difference for electrical outlets is 120 V or 220 V in the US or Europe, but as explained later in the chapter, these voltages are not the peak values given here but rather are related to the common voltages we see in our electrical outlets. Figure 18.2.1 shows graphs of voltage and current versus time for typical dc and ac power in the United States.

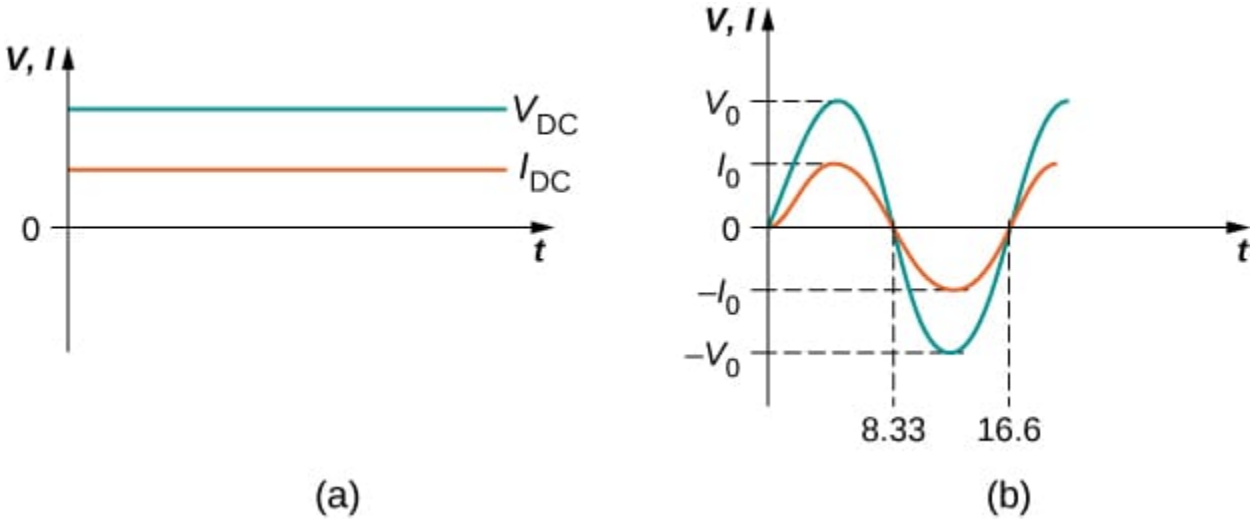


Figure 18.2.1: (a) The dc voltage and current are constant in time, once the current is established. (b) The voltage and current versus time are quite different for ac power. In this example, which shows 60-Hz ac power and time t in milliseconds, voltage and current are sinusoidal and are in phase for a simple resistance circuit. The frequencies and peak voltages of ac sources differ greatly.

Suppose we hook up a resistor to an ac voltage source and determine how the voltage and current vary in time across the resistor. Figure 18.2.2 shows a schematic of a simple circuit with an ac voltage source. The voltage fluctuates sinusoidally with time at a fixed frequency, as shown, on either the battery terminals or the resistor. Therefore, the **ac voltage**, or the "voltage at a plug," can be given by

$$v(t) = V_0 \sin \omega t,$$

where

- v is the voltage at time t,
- V_0 is the peak voltage, and
- ω is the angular frequency in radians per second.

For a typical house in the United States, $V_0 = 156\ V$ and $\omega = 120\pi\ rad/s$, whereas in Europe, $V_0 = 311\ V$ and $\omega = 100\pi\ rad/s$.

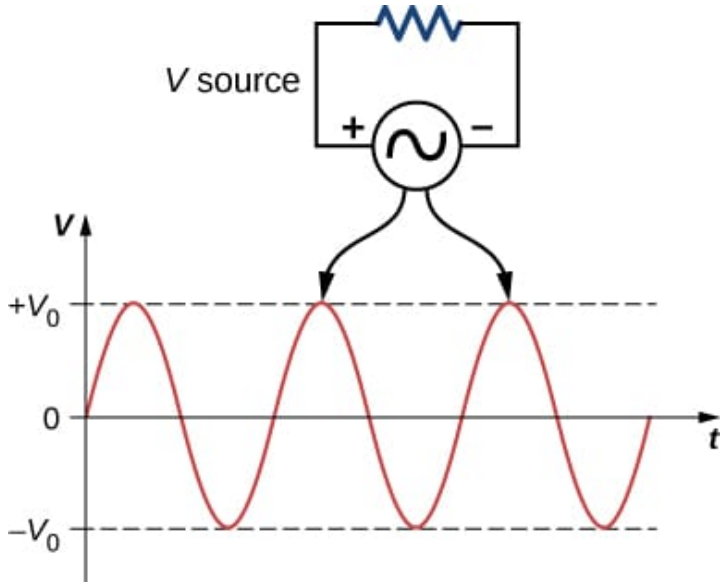


Figure 18.2.2: The potential difference **V** between the terminals of an ac voltage source fluctuates, so the source and the resistor have ac sine waves on top of each other. The mathematical expression for **v** is given by $v = V_0\ sin\ \omega t$.

For this simple resistance circuit, $I = V/R$, so the **ac current**, meaning the current that fluctuates sinusoidally with time at a fixed frequency, is

$$i(t) = I_0 \sin \omega t,$$

where

- $i(t)$ is the current at time t and
- I_0 is the peak current and is equal to V_0/R.

For this example, the voltage and current are said to be in phase, meaning that their sinusoidal functional forms have peaks, troughs, and nodes in the same place. They oscillate in sync with each other, as shown in Figure 18.2.1*b*. In these equations, and throughout this chapter, we use lowercase letters (such as i) to indicate instantaneous values and capital letters (such as I) to indicate maximum, or peak, values.

Current in the resistor alternates back and forth just like the driving voltage, since $I = V/R$. If the resistor is a fluorescent light bulb, for example, it brightens and dims 120 times per second as the current repeatedly goes through zero. A 120-Hz flicker is too rapid for your eyes to detect, but if you wave your hand back and forth between your face and a fluorescent light, you will see the stroboscopic effect of ac.

Exercise 18.2.1

If a European ac voltage source is considered, what is the time difference between the zero crossings on an ac voltage-versus-time graph?

Solution

10 ms

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18.3: Simple AC Circuits

Learning Objectives

By the end of the section, you will be able to:

- Interpret phasor diagrams and apply them to ac circuits with resistors, capacitors, and inductors
- Define the reactance for a resistor, capacitor, and inductor to help understand how current in the circuit behaves compared to each of these devices

In this section, we study simple models of ac voltage sources connected to three circuit components: (1) a resistor, (2) a capacitor, and (3) an inductor. The power furnished by an ac voltage source has an emf given by

$$v(t) = V_0 \sin \omega t,$$

as shown in Figure 18.3.1. This sine function assumes we start recording the voltage when it is $v = 0\ V$ at a time of $t = 0\ s$. A phase constant may be involved that shifts the function when we start measuring voltages, similar to the phase constant in the waves we studied in Waves. However, because we are free to choose when we start examining the voltage, we can ignore this phase constant for now. We can measure this voltage across the circuit components using one of two methods: (1) a quantitative approach based on our knowledge of circuits, or (2) a graphical approach that is explained in the coming sections.

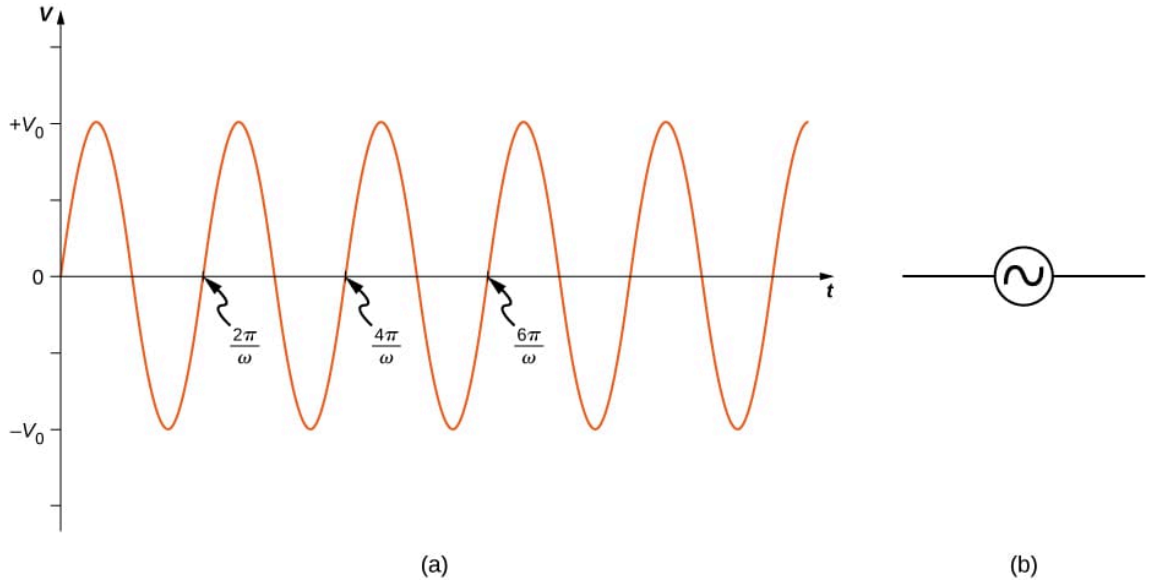


Figure 18.3.1: (a) The output $v(t) = V_0\ sin\ \omega t$ of an ac generator. (b) Symbol used to represent an ac voltage source in a circuit diagram.

Resistor

First, consider a **resistor** connected across an ac voltage source. From Kirchhoff's loop rule, the instantaneous voltage across the resistor of Figure 18.3.2a is

$$v_R(t) = V_0 \sin \omega t$$

and the instantaneous current through the resistor is

$$i_R(t) = \frac{v_R(t)}{R} = \frac{V_0}{R} \sin \omega t = I_0 \sin \omega t.$$

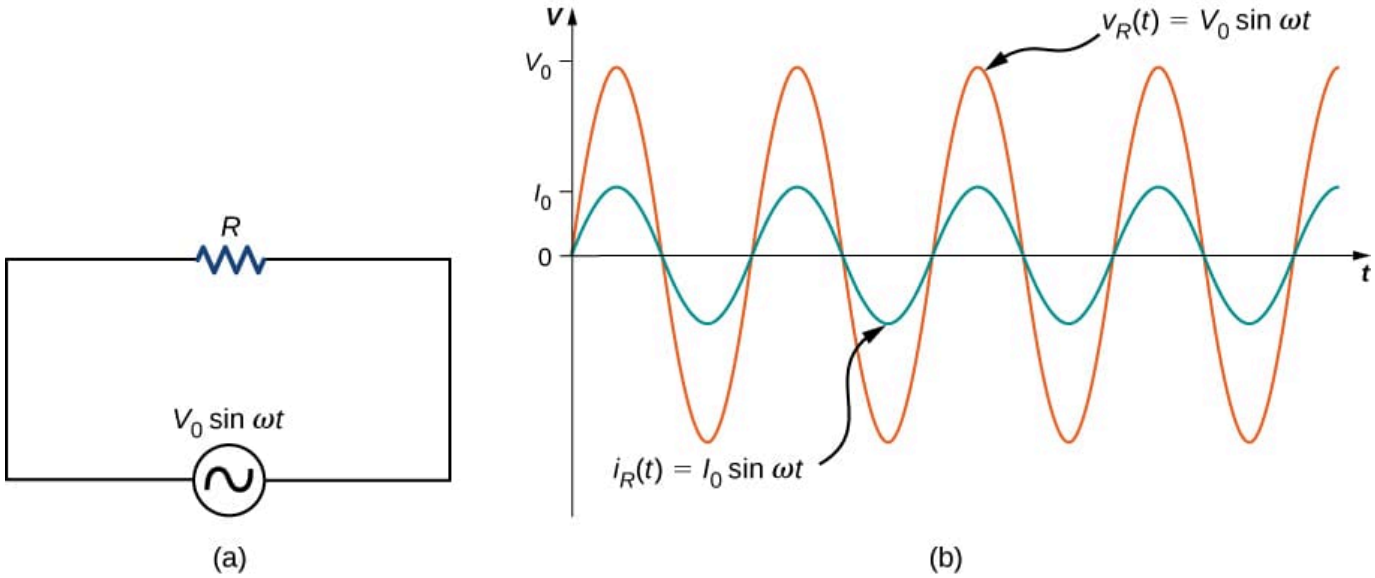


Figure 18.3.2: (a) A resistor connected across an ac voltage source. (b) The current $i_R(t)$ through the resistor and the voltage $v_R(t)$ across the resistor. The two quantities are in phase.

Here, $I_0 = V_0/R$ is the amplitude of the time-varying current. Plots of $i_R(t)$ and $v_R(t)$ are shown in Figure 18.3.2*b*. Both curves reach their maxima and minima at the same times, that is, the current through and the voltage across the resistor are in phase.

Graphical representations of the phase relationships between current and voltage are often useful in the analysis of ac circuits. Such representations are called **phasor diagrams**. The phasor diagram for $i_R(t)$ is shown in Figure 18.3.3*a*, with the current on the vertical axis. The arrow (or phasor) is rotating counterclockwise at a constant angular frequency ω, so we are viewing it at one instant in time. If the length of the arrow corresponds to the current amplitude I_0, the projection of the rotating arrow onto the vertical axis is $i_R(t) = I_0 \ sin \ \omega t$, which is the instantaneous current.

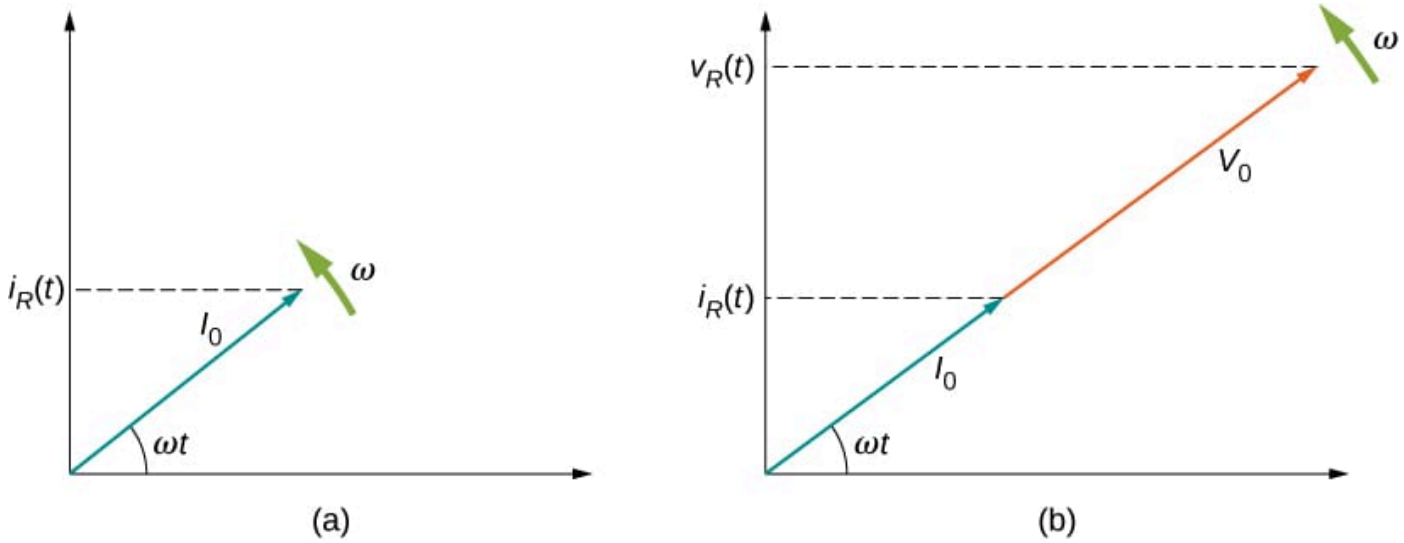


Figure 18.3.3: (a) The phasor diagram representing the current through the resistor of Figure 18.3.2 (b) The phasor diagram representing both $i_R(t)$ and $v_R(t)$.

The vertical axis on a phasor diagram could be either the voltage or the current, depending on the phasor that is being examined. In addition, several quantities can be depicted on the same phasor diagram. For example, both the current $i_R(t)$ and the voltage $v_R(t)$ are shown in the diagram of Figure 18.3.3*b*. Since they have the same frequency and are in phase, their phasors point in the same direction and rotate together. The relative lengths of the two phasors are arbitrary because they represent different quantities; however, the ratio of the lengths of the two phasors can be represented by the resistance, since one is a voltage phasor and the other is a current phasor.

Capacitor

Now let's consider a **capacitor** connected across an ac voltage source. From Kirchhoff's loop rule, the instantaneous voltage across the capacitor of Figure $18.3.4a$ is

$$v_C(t) = V_0 \, sin \, \omega t.$$

Recall that the charge in a capacitor is given by $Q = CV$. This is true at any time measured in the ac cycle of voltage. Consequently, the instantaneous charge on the capacitor is

$$q(t) = Cv_C(t) = CV_0 \, sin \, \omega t.$$

Since the current in the circuit is the rate at which charge enters (or leaves) the capacitor,

$$i_C(t) = \frac{dq(t)}{dt} = \omega CV_0 \, cos \, \omega t = I_0 \, cos \, \omega t,$$

where $I_0 = \omega CV_0$ is the current amplitude. Using the trigonometric relationship $cos \ \omega t = sin(\omega t + \pi/2)$, we may express the instantaneous current as

$$i_C(t) = I_0 \, sin \left(\omega t + \frac{\pi}{2}\right).$$

Dividing V_0 by I_0, we obtain an equation that looks similar to Ohm's law:

$$\frac{V_0}{I_0} = \frac{1}{\omega C} = X_C. \tag{18.3.1}$$

The quantity X_C is analogous to resistance in a dc circuit in the sense that both quantities are a ratio of a voltage to a current. As a result, they have the same unit, the ohm. Keep in mind, however, that a capacitor stores and discharges electric energy, whereas a resistor dissipates it. The quantity X_C is known as the capacitive reactance of the capacitor, or the opposition of a capacitor to a change in current. It depends inversely on the frequency of the ac source—high frequency leads to low capacitive reactance.

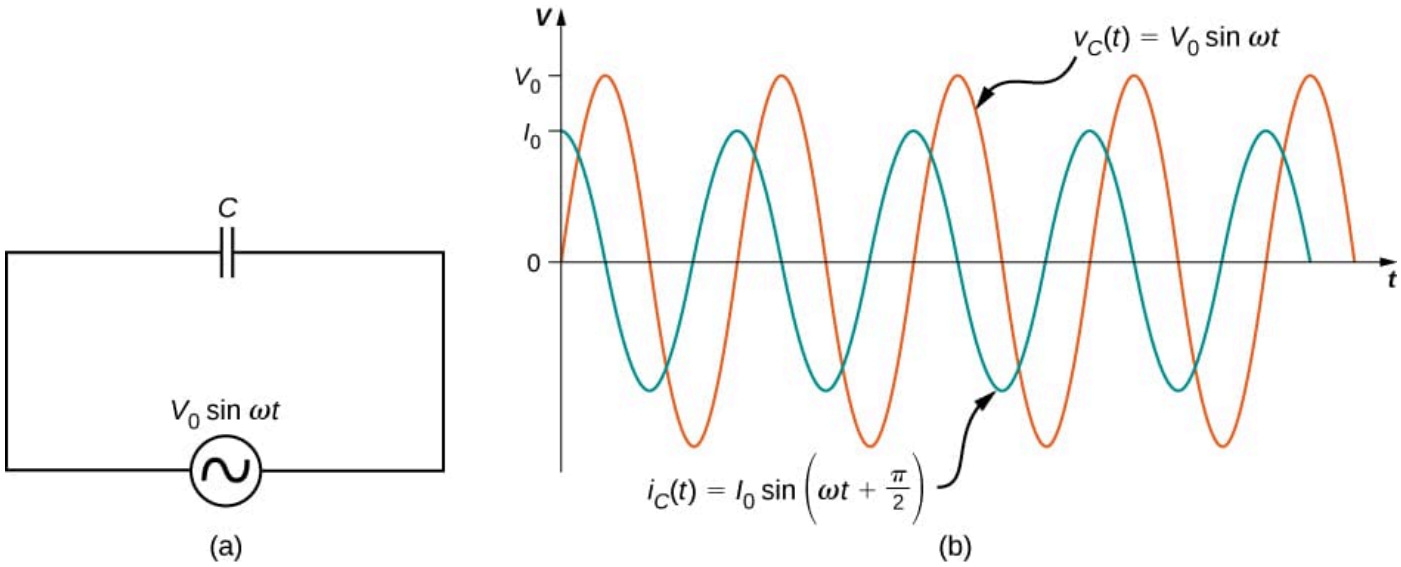


Figure $18.3.4$: (a) A capacitor connected across an ac generator. (b) The current $i_C(t)$ through the capacitor and the voltage $v_C(t)$ across the capacitor. Notice that $i_C(t)$ leads $v_C(t)$ by $\pi/2$ rad.

A comparison of the expressions for $v_C(t)$ and $i_C(t)$ shows that there is a phase difference of $\pi/2$ rad between them. When these two quantities are plotted together, the current peaks a quarter cycle (or $\pi/2$ rad) ahead of the voltage, as illustrated in Figure $18.3.4b$. The current through a capacitor leads the voltage across a capacitor by $\pi/2$ rad, or a quarter of a cycle.

The corresponding phasor diagram is shown in Figure $18.3.5$. Here, the relationship between $i_C(t)$ and $v_C(t)$ is represented by having their phasors rotate at the same angular frequency, with the current phasor leading by $\pi/2$ rad.

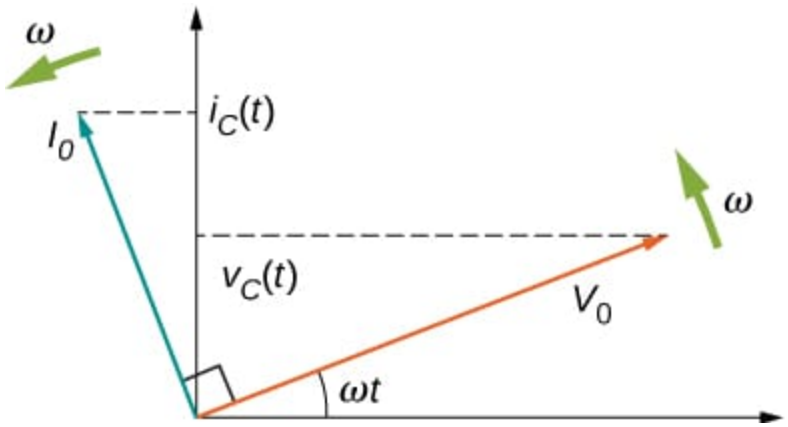


Figure 18.3.5: The current phasor leads the voltage phasor by $\pi/2$ rad as they both rotate with the same angular frequency.

To this point, we have exclusively been using peak values of the current or voltage in our discussion, namely, I_0 and V_0. However, if we average out the values of current or voltage, these values are zero. Therefore, we often use a second convention called the root mean square value, or rms value, in discussions of current and voltage. The rms operates in reverse of the terminology. First, you square the function, next, you take the mean, and then, you find the square root. As a result, the rms values of current and voltage are not zero. Appliances and devices are commonly quoted with rms values for their operations, rather than peak values. We indicate rms values with a subscript attached to a capital letter (such as I_{rms}).

Although a capacitor is basically an open circuit, an **rms current**, or the root mean square of the current, appears in a circuit with an ac voltage applied to a capacitor. Consider that

Note

$$I_{rms} = \frac{I_0}{\sqrt{2}},$$

where I_0 is the peak current in an ac system. The **rms voltage**, or the root mean square of the voltage, is

Note

$$V_{rms} = \frac{V_0}{\sqrt{2}},$$

where V_0 is the peak voltage in an ac system. The rms current appears because the voltage is continually reversing, charging, and discharging the capacitor. If the frequency goes to zero, which would be a dc voltage, X_C tends to infinity, and the current is zero once the capacitor is charged. At very high frequencies, the capacitor's reactance tends to zero—it has a negligible reactance and does not impede the current (it acts like a simple wire).

Inductor

Lastly, let's consider an **inductor** connected to an ac voltage source. From Kirchhoff's loop rule, the voltage across the inductor **L** of Figure 18.3.6*a* is

$$v_L(t) = V_0 \, sin \, \omega t. \tag{18.3.2}$$

The emf across an inductor is equal to $\epsilon = -L(di_L/dt)$; however, the potential difference across the inductor is $v_L(t) = Ldi_L(t)/dt$, because if we consider that the voltage around the loop must equal zero, the voltage gained from the ac source must dissipate through the inductor. Therefore, connecting this with the ac voltage source, we have

$$\frac{di_L(t)}{dt} = \frac{V_0}{L} \, sin \, \omega t.$$

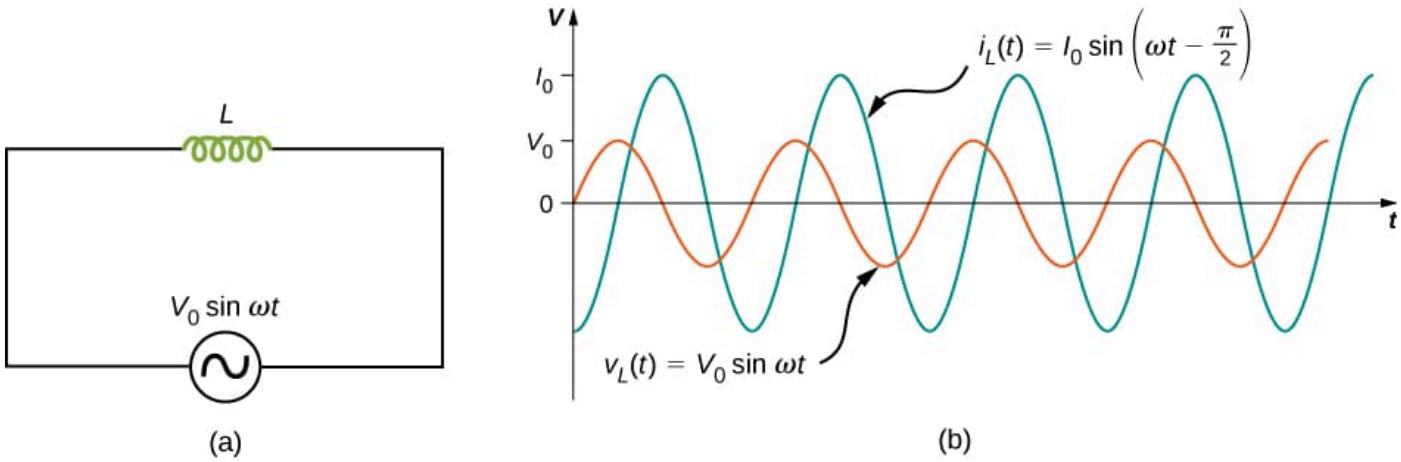


Figure 18.3.6: (a) An inductor connected across an ac generator. (b) The current $i_L(t)$ through the inductor and the voltage $v_L(t)$ across the inductor. Here $i_L(t)$ lags $v_L(t)$ by $\pi/2$ rad.

The current $i_L(t)$ is found by integrating this equation. Since the circuit does not contain a source of constant emf, there is no steady current in the circuit. Hence, we can set the constant of integration, which represents the steady current in the circuit, equal to zero, and we have

$$i_L(t) = -\frac{V_0}{\omega L} cos\, \omega t = \frac{V_0}{\omega L} sin\left(\omega t - \frac{\pi}{2}\right) = I_0\, sin\left(\omega t - \frac{\pi}{2}\right), \tag{18.3.3}$$

where $I_0 = V_0/\omega L$. The relationship between V_0 and I_0 may also be written in a form analogous to Ohm's law:

Note

$$\frac{V_0}{I_0} = \omega L = X_L. \tag{18.3.4}$$

The quantity X_L is known as the **inductive reactance** of the inductor, or the opposition of an inductor to a change in current; its unit is also the ohm. Note that X_L varies directly as the frequency of the ac source—high frequency causes high inductive reactance.

A phase difference of $\pi/2$ rad occurs between the current through and the voltage across the inductor. From Equation 18.3.2 and Equation 18.3.3, the current through an inductor lags the potential difference across an inductor by $\pi/2$ rad, or a quarter of a cycle. The phasor diagram for this case is shown in Figure 18.3.7.

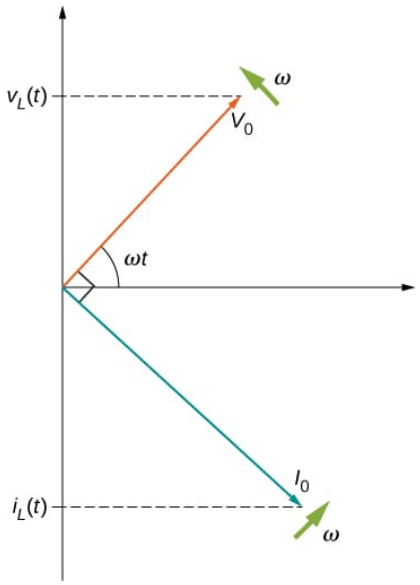


Figure 18.3.7: The current phasor lags the voltage phasor by $\pi/2$ rad as they both rotate with the same angular frequency.

Note

An animation from the University of New South Wales AC Circuits illustrates some of the concepts we discuss in this chapter. They also include wave and phasor diagrams that evolve over time so that you can get a better picture of how each changes over time.

Example 18.3.1: Simple AC CIrcuits

An ac generator produces an emf of amplitude 10 V at a frequency $f = 60\ Hz$. Determine the voltages across and the currents through the circuit elements when the generator is connected to (a) a $100\ \Omega$ resistor, (b) a $10\ \mu F$ capacitor, and (c) a 15-mH inductor.

Strategy

The entire AC voltage across each device is the same as the source voltage. We can find the currents by finding the reactance **X** of each device and solving for the peak current using $I_0 = V_0/X$.

Solution

The voltage across the terminals of the source is

$$v(t) = V_0\, sin\, \omega t = (10\, V) sin\, 120\pi t,$$

where $\omega = 2\pi f = 120\pi\ rad/s$ is the angular frequency. Since **v**(**t**) is also the voltage across each of the elements, we have

$$v(t) = v_R(t) = v_C(t) = v_L(t) = (10\, V) sin\, 120\pi t.$$

a. When $R = 100\ \Omega$, the amplitude of the current through the resistor is

$$I_0 = V_0/R = 10\, V/100\, \Omega = 0.10\, A,$$

so

$$i_R(t) = (0.10\, A) sin\, 120\pi t.$$

b. From Equation 18.3.1, the capacitive reactance is

$$X_C = \frac{1}{\omega C} = \frac{1}{(120\pi\, rad/s)(10 \times 10^{-6} F)} = 265\,\Omega,$$

so the maximum value of the current is

$$I_0 = \frac{V_0}{X_C} = \frac{10\,V}{265\,\Omega} = 3.8 \times 10^{-2}\,A$$

and the instantaneous current is given by

$$i_C(t) = (3.8 \times 10^{-2}\,A)\,sin\left(120\pi t + \frac{\pi}{2}\right).$$

c. From Equation 18.3.4, the inductive reactance is

$$X_L = \omega L = (120\pi\, rad/s)(15 \times 10^{-3} H) = 5.7\,\Omega.$$

The maximum current is therefore

$$I_0 = \frac{10\,V}{5.7\,\Omega} = 1.8\,A$$

and the instantaneous current is

$$i_L(t) = (1.8\,A)\,sin\left(120\pi t - \frac{\pi}{2}\right).$$

Significance

Although the voltage across each device is the same, the peak current has different values, depending on the reactance. The reactance for each device depends on the values of resistance, capacitance, or inductance.

Exercise 18.3.1

Repeat Example 18.3.1 for an ac source of amplitude 20 V and frequency 100 Hz.

Answer available at the end of this section.

Answers

Exercise 18.3.1

a. $(20\ V)\ sin\ 200\pi t$, $(0.20\ A)\ sin\ 200\pi t$;
b. $(20\ V)\ sin\ 200\pi t$, $(0.13\ A)\ sin\ (200\pi t + \pi/2)$;
c. $(20\ V)\ sin\ 200\pi t$, $(2.1\ A)\ sin\ (200\pi t - \pi/2)$

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18.4: RLC Series Circuits with AC

Learning Objectives

By the end of the section, you will be able to:

- Describe how the current varies in a resistor, a capacitor, and an inductor while in series with an ac power source
- Use phasors to understand the phase angle of a resistor, capacitor, and inductor ac circuit and to understand what that phase angle means
- Calculate the impedance of a circuit

The ac circuit shown in Figure 18.4.1, called an **RLC** series circuit, is a series combination of a resistor, capacitor, and inductor connected across an ac source. It produces an emf of

$$v(t) = V_0 \sin \omega t.$$

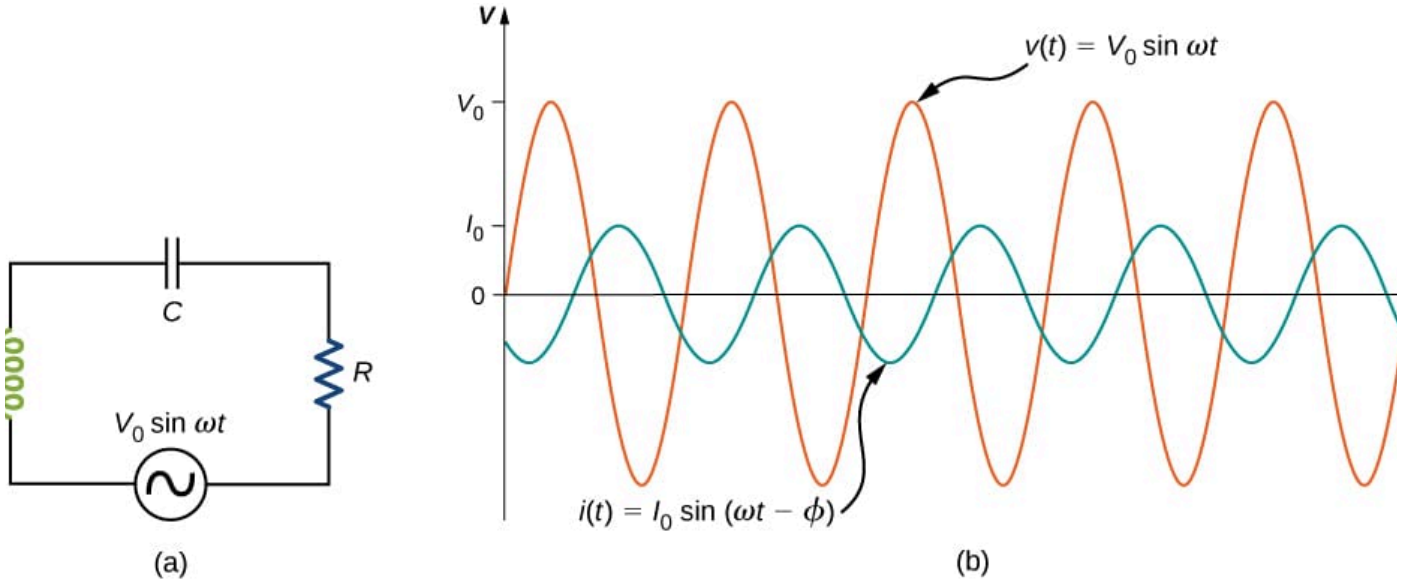


Figure 18.4.1: (a) An RLC series circuit. (b) A comparison of the generator output voltage and the current. The value of the phase difference ϕ depends on the values of R, C, and L.

Since the elements are in series, the same current flows through each element at all points in time. The relative phase between the current and the emf is not obvious when all three elements are present. Consequently, we represent the current by the general expression

$$i(t) = I_0 \sin(\omega t - \phi),$$

where I_0 is the current amplitude and ϕ is the phase angle between the current and the applied voltage. The phase angle is thus the amount by which the voltage and current are out of phase with each other in a circuit. Our task is to find I_0 and ϕ.

A phasor diagram involving $i(t)$, $v_R(t)$, $v_C(t)$, and $v_L(t)$ is helpful for analyzing the circuit. As shown in Figure 18.4.2, the phasor representing $v_R(t)$ points in the same direction as the phasor for $i(t)$; its amplitude is $V_R = I_0 R$. The $v_C(t)$ phasor lags the $\mathbf{i}(\mathbf{t})$ phasor by $\pi/2$ rad and has the amplitude $V_C = I_0 X_C$. The phasor for $v_L(t)$ leads the $\mathbf{i}(\mathbf{t})$ phasor by $\pi/2$ rad and has the amplitude $V_L = I_0 X_L$.

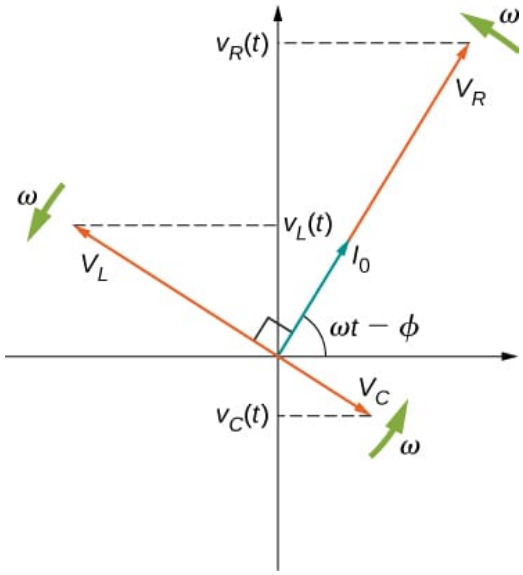


Figure 18.4.1.

At any instant, the voltage across the **RLC** combination is $v_R(t) + v_L(t) + v_C(t) = v(t)$, the emf of the source. Since a component of a sum of vectors is the sum of the components of the individual vectors—for example, $(A + B)_y = A_y + B_y$ - the projection of the vector sum of phasors onto the vertical axis is the sum of the vertical projections of the individual phasors. Hence, if we add vectorially the phasors representing $v_R(t),\ v_L(t)$, and $v_C(t)$ and then find the projection of the resultant onto the vertical axis, we obtain

$$v_R(t) + v_L(t) + v_C(t) = v(t) = V_0 \, sin \, \omega t.$$

The vector sum of the phasors is shown in Figure 18.4.3. The resultant phasor has an amplitudeV_0 and is directed at an angle ϕ with respect to the $v_R(t)$, or **i**(**t**), phasor. The projection of this resultant phasor onto the vertical axis is $v(t) = V_0 \ sin \ \omega t$. We can easily determine the unknown quantities I_0 and ϕ from the geometry of the phasor diagram. For the phase angle,

$$\phi = tan^{-1} \frac{V_L - V_C}{V_R} = tan^{-1} \frac{I_0 X_L - I_0 X_C}{I_0 R},$$

and after cancellation of I_0, this becomes

$$\phi = tan^{-1} \frac{X_L - X_C}{R}. \tag{18.4.1}$$

Furthermore, from the Pythagorean theorem,

$$V_0 = \sqrt{V_R^2 + (V_L - V_C)^2} = \sqrt{(I_0 R)^2 + (I_0 X_L - I_) X_C)^2} = I_0 \sqrt{R^2 + (X_L - X_C)^2}.$$

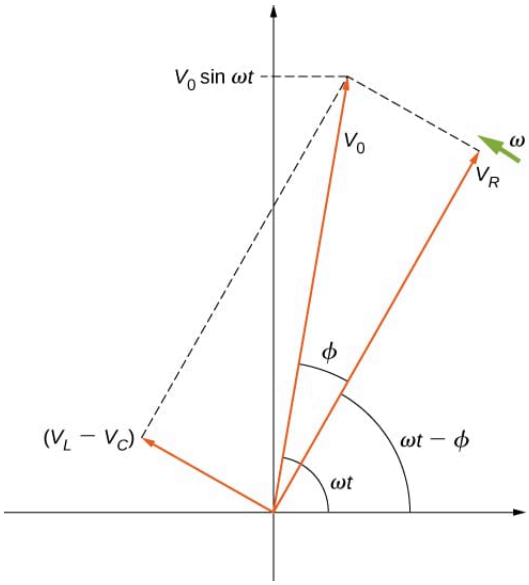


Figure 18.4.3: The resultant of the phasors for $v_L(t)$, $v_C(t)$, and $v_R(t)$ is equal to the phasor for $v_R(t) = V_0 \ sin \ \omega t$. The **i(t)** phasor (not shown) is aligned with the $v_R(t)$ phasor.

The current amplitude is therefore the ac version of Ohm's law:

$$I_0 = \frac{V_0}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V_0}{Z}, \tag{18.4.2}$$

where

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \tag{18.4.3}$$

is known as the impedance of the circuit. Its unit is the ohm, and it is the ac analog to resistance in a dc circuit, which measures the combined effect of resistance, capacitive reactance, and inductive reactance (Figure 18.4.4).



Figure 18.4.4: Power capacitors are used to balance the impedance of the effective inductance in transmission lines.

The **RLC** circuit is analogous to the wheel of a car driven over a corrugated road (Figure 18.4.5). The regularly spaced bumps in the road drive the wheel up and down; in the same way, a voltage source increases and decreases. The shock absorber acts like the resistance of the **RLC** circuit, damping and limiting the amplitude of the oscillation. Energy within the wheel system goes back and forth between kinetic and potential energy stored in the car spring, analogous to the shift between a maximum current,

with energy stored in an inductor, and no current, with energy stored in the electric field of a capacitor. The amplitude of the wheel's motion is at a maximum if the bumps in the road are hit at the resonant frequency, which we describe in more detail in Resonance in an AC Circuit.

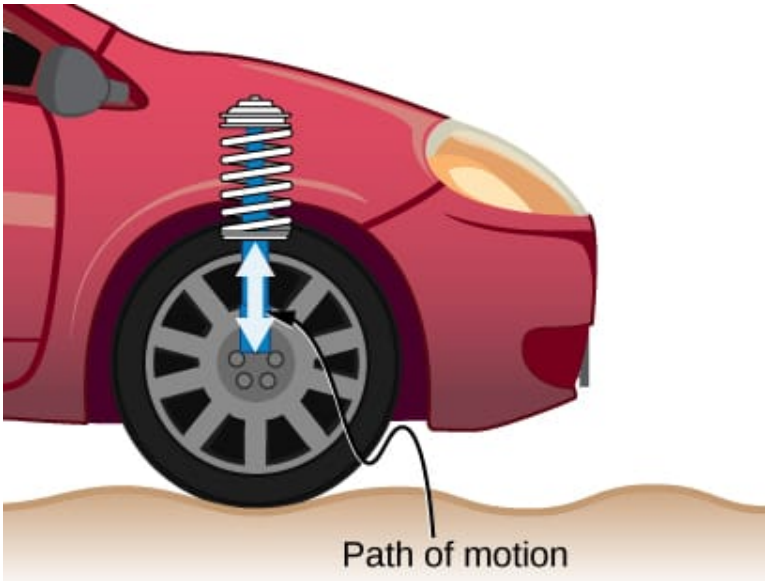


Figure 18.4.5: On a car, the shock absorber damps motion and dissipates energy. This is much like the resistance in an RLC circuit. The mass and spring determine the resonant frequency.

Problem-Solving Strategy: AC Circuits

To analyze an ac circuit containing resistors, capacitors, and inductors, it is helpful to think of each device's reactance and find the equivalent reactance using the rules we used for equivalent resistance in the past. Phasors are a great method to determine whether the emf of the circuit has positive or negative phase (namely, leads or lags other values). A mnemonic device of "ELI the ICE man" is sometimes used to remember that the emf (E) leads the current (I) in an inductor (L) and the current (I) leads the emf (E) in a capacitor (C).

Use the following steps to determine the emf of the circuit by phasors:

1. Draw the phasors for voltage across each device: resistor, capacitor, and inductor, including the phase angle in the circuit.
2. If there is both a capacitor and an inductor, find the net voltage from these two phasors, since they are antiparallel.
3. Find the equivalent phasor from the phasor in step 2 and the resistor's phasor using trigonometry or components of the phasors. The equivalent phasor found is the emf of the circuit.

Example 18.4.1: An RLC Series Circuit

The output of an ac generator connected to an **RLC** series combination has a frequency of 200 Hz and an amplitude of 0.100 V. If $R = 4.00\ \Omega,\ L = 3.00 \times 10^{-3} H$, and $C = 8.00 \times 10^{-4} F$, what are (a) the capacitive reactance, (b) the inductive reactance, (c) the impedance, (d) the current amplitude, and (e) the phase difference between the current and the emf of the generator?

Strategy

The reactances and impedance in (a)–(c) are found by substitutions into Equation 15.3.8, Equation 15.3.14, and Equation 18.4.2, respectively. The current amplitude is calculated from the peak voltage and the impedance. The phase difference between the current and the emf is calculated by the inverse tangent of the difference between the reactances divided by the resistance.

Solution

1. From Equation 15.3.8, the capacitive reactance is
$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi(200\,Hz)(8.00\times10^{-4}F)} = 0.995\,\Omega.$$
2. From Equation 15.3.14, the inductive reactance is
$$X_L = \omega L = 2\pi(200\,Hz)(3.00\times10^{-3}H) = 3.77\,\Omega.$$
3. Substituting the values of **R**, X_C, and X_L into Equation 18.4.2, we obtain for the impedance
$$Z = \sqrt{(4.00\,)^2 + (3.77\,\Omega - 0.995\,\Omega)^2} = 4.87\,\Omega.$$
4. The current amplitude is
$$I_0 = \frac{V_0}{Z} = \frac{0.100\,V}{4.87\,\Omega} = 2.05\times10^{-2}A.$$
5. From Equation 18.4.1, the phase difference between the current and the emf is
$$\phi = tan^{-1}\frac{X_L - X_C}{R} = tan^{-1}\frac{2.77\,\Omega}{4.00\,\Omega} = 0.607\,rad.$$

Significance

The phase angle is positive because the reactance of the inductor is larger than the reactance of the capacitor.

Exercise 18.4.1

Find the voltages across the resistor, the capacitor, and the inductor in the circuit of Figure 18.4.1 using $v(t) = V_0\ sin\ \omega t$ as the output of the ac generator.

Solution

$v_R = (V_0R/Z)\ sin\ (\omega t - \phi)$; $v_C = (V_0X_C/Z)\ sin\ (\omega t - \phi + \pi 2/) = -(V_0X_C/Z)\ cos\ (\omega t - \phi)$;

$v_L = (V_0X_L/Z)\ sin\ (\omega t - \phi - \pi/2) = (V_0X_L/Z)\ cos\ (\omega t - \phi)$

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18.5: Power in an AC Circuit

Learning Objectives

By the end of the section, you will be able to:

- Describe how average power from an ac circuit can be written in terms of peak current and voltage and of rms current and voltage
- Determine the relationship between the phase angle of the current and voltage and the average power, known as the power factor

A circuit element dissipates or produces power according to $P = IV$, where **I** is the current through the element and V is the voltage across it. Since the current and the voltage both depend on time in an ac circuit, the instantaneous power $p(t) = i(t)v(t)$ is also time dependent. A plot of $p(t)$ for various circuit elements is shown in Figure 18.5.1. For a resistor, $i(t)$ and $v(t)$ are in phase and therefore always have the same sign. For a capacitor or inductor, the relative signs of $i(t)$ and $v(t)$ vary over a cycle due to their phase differences. Consequently, $p(t)$ is positive at some times and negative at others, indicating that capacitive and inductive elements produce power at some instants and absorb it at others.

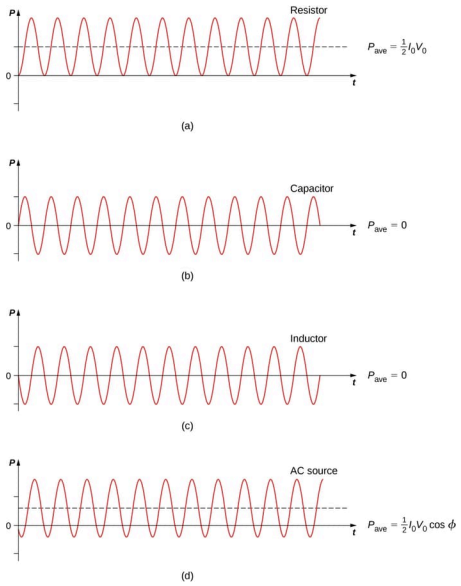


Figure 18.5.1: Graph of instantaneous power for various circuit elements. (a) For the resistor, $P_{ave} = I_0 V_0/2$, whereas for (b) the capacitor and (c) the inductor, $P_{ave} = 0$. (d) For the source, $P_{ave} = I_0 V_0 (cos\ \phi)/2$, which may be positive, negative, or zero, depending on ϕ.

Because instantaneous power varies in both magnitude and sign over a cycle, it seldom has any practical importance. What we're almost always concerned with is the power averaged over time, which we refer to as the **average power**. It is defined by the time average of the instantaneous power over one cycle:

$$P_{ave} = \frac{1}{T} \int_0^T p(t) dt, \tag{18.5.1}$$

where $T = 2\pi/\omega$ is the period of the oscillations. With the substitutions $v(t) = V_0\ \sin\ \omega t$ and $i(t) = I_0\ \sin\ (\omega t - \phi)$, Equation 18.5.1 becomes

$$P_{ave} = \frac{I_0 V_0}{T} \int_0^T \sin(\omega t - \phi)\ \sin\ \omega t\, dt.$$

Using the trigonometric difference identity

$$\sin(A-B) = \sin A \cos B - \sin B \cos A$$

we obtain

$$P_{ave} = \frac{I_0 V_0 \cos\phi}{T}\int_0^T \sin^2\omega t\, dt - \frac{I_0 V_0 \sin\phi}{T}\int_0^T \sin\omega t \cos\omega t\, dt.$$

Evaluation of these two integrals yields

$$\frac{1}{T}\int_0^T \sin^2\omega t\, dt = \frac{1}{2}$$

and

$$\frac{1}{T}\int_0^T \sin\omega t \cos\omega t\, dt = 0.$$

Hence, the average power associated with a circuit element is given by

$$\boxed{P_{\text{ave}} = \frac{1}{2} I_0 V_0 \cos\phi.} \tag{18.5.2}$$

In engineering applications, $\cos\phi$ is known as the **power factor**, which is the amount by which the power delivered in the circuit is less than the theoretical maximum of the circuit due to voltage and current being out of phase. For a resistor, $\phi = 0$, so the average power dissipated is

$$P_{ave} = \frac{1}{2} I_0 V_0. \tag{18.5.3}$$

A comparison of $p(t)$ and P_{ave} is shown in Figure 18.5.1d. To make Equation 18.5.3 look like its dc counterpart, we use the rms values I_{rms} and V_{rms} of the current and the voltage. By definition, these are

$$I_{rms} = \sqrt{i_{ave}^2}$$

and

$$V_{rms} = \sqrt{v_{ave}^2},$$

where

$$t_{ave}^2 = \frac{1}{T}\int_0^T i^2(t)dt$$

and

$$v_{ave}^2 = \frac{1}{T}\int_0^T v^2(t)dt.$$

With $i(t) = I_0 \sin(\omega t - \phi)$ and $v(t) = V_0 \sin\omega t$, we obtain

$$I_{rms} = \frac{1}{\sqrt{2}} I_0$$

and

$$V_{rms} = \frac{1}{\sqrt{2}} V_0.$$

We may then write for the average power dissipated by a resistor,

$$P_{\text{ave}} = \frac{1}{2} I_0 V_0 = I_{\text{rms}} V_{\text{rms}} = I_{\text{rms}}^2 R. \tag{18.5.4}$$

This equation further emphasizes why the rms value is chosen in discussion rather than peak values. Both Equations 18.5.2 and 18.5.4 are correct for average power, but the rms values in the formula give a cleaner representation, so the extra factor of 1/2 is not necessary.

Alternating voltages and currents are usually described in terms of their rms values. For example, the 110 V from a household outlet is an rms value. The amplitude of this source is $110\sqrt{2}\ V = 156\ V$. Because most ac meters are calibrated in terms of rms values, a typical ac voltmeter placed across a household outlet will read 110 V.

For a capacitor and an inductor, $\phi = \pi/2$ and $-\pi/2\ rad$, respectively. Since $\cos\ \pi/2 = cos(-\pi/2) = 0$, we find from Equation 18.5.2 that the average power dissipated by either of these elements is $P_{ave} = 0$. Capacitors and inductors absorb energy from the circuit during one half-cycle and then discharge it back to the circuit during the other half-cycle. This behavior is illustrated in the plots of Figures $18.5.1b$ and $18.5.1c$ which show $p(t)$ oscillating sinusoidally about zero.

The phase angle for an ac generator may have any value. If $cos\ \phi > 0$, the generator produces power; if $cos\ \phi < 0$, it absorbs power. In terms of rms values, the average power of an ac generator is written as

$$P_{ave} = I_{rms} V_{rms}\ \cos\ \phi.$$

For the generator in an **RLC** circuit,

$$tan\,\phi = \frac{X_L - X_C}{R}$$

and

$$cos\,\phi = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{R}{Z}.$$

Hence the average power of the generator is

$$P_{ave} = I_{rms} V_{rms}\ \cos\ \phi = \frac{V_{rms}}{Z} V_{rms} \frac{R}{Z} = \frac{V_{rms}^2 R}{Z^2}. \tag{18.5.5}$$

This can also be written as

$$P_{ave} = I_{rms}^2 R,$$

which designates that the power produced by the generator is dissipated in the resistor. As we can see, Ohm's law for the rms ac is found by dividing the rms voltage by the impedance.

Example 18.5.1: Power Output of a Generator

An ac generator whose emf is given by

$$v(t) = (4.00\,V)\ \sin\left[(1.00 \times 10^4\ rad/s)t\right]$$

is connected to an **RLC** circuit for which $L = 2.00 \times 10^{-3} H,\ C = 4.00 \times 10^{-6} F$, and $R = 5.00\ \Omega$.

a. What is the rms voltage across the generator?
b. What is the impedance of the circuit?
c. What is the average power output of the generator?

Strategy

The rms voltage is the amplitude of the voltage times $1/\sqrt{2}$. The impedance of the circuit involves the resistance and the reactances of the capacitor and the inductor. The average power is calculated by Equation 18.5.5 because we have the impedance of the circuit Z, the rms voltage V_{rms}, and the resistance R.

Solution

1. Since $V_0 = 4.00\ V$, the rms voltage across the generator is
$$V_{rms} = \frac{1}{\sqrt{2}}(4.00\,V) = 2.83\,V.$$
2. The impedance of the circuit is
$$\begin{aligned} Z &= \sqrt{r^2 + (x_l - x_c)^2} \\ &= \sqrt{(5.00\,\Omega)^2 + \left[(1.00\times10^4\,rad/s)(2.00\times10^{-3}\,H) - \frac{1}{(1.00\times10^4\,rad/s)(4.00\times10^{-6}\,F)}\right]^2} \\ &= 7.07\,\Omega. \end{aligned}$$
3. From Equation 18.5.5, the average power transferred to the circuit is
$$P_{ave} = \frac{V_{rms}^2 R}{Z^2} = \frac{(2.83\,V)^2(5.00\,\Omega)}{(7.07\,\Omega)^2} = 0.801\,W.$$

Significance

If the resistance is much larger than the reactance of the capacitor or inductor, the average power is a dc circuit equation of $P = V^2/R$, where **V** replaces the rms voltage.

Exercise 18.5.1A

An ac voltmeter attached across the terminals of a 45-Hz ac generator reads 7.07 V. Write an expression for the emf of the generator.

Answer available at the end of this section.

Exercise 18.5.1B

Show that the rms voltages across a resistor, a capacitor, and an inductor in an ac circuit where the rms current is I_{rms} are given by $I_{rms}R$, $I_{rms}X_C$, and $I_{rms}X_L$, respectively. Determine these values for the components of the **RLC** circuit of Equation 18.5.2.

Answer available at the end of this section.

Answers

Exercise 18.5.1A

$v(t) = (10.0\ V)\ \sin\ 90\pi t$

Exercise 18.5.1B

2.00 V; 10.01 V; 8.01 V

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18.6: Resonance in an AC Circuit

Learning Objectives

By the end of the section, you will be able to:

- Determine the peak ac resonant angular frequency for a RLC circuit
- Explain the width of the average power versus angular frequency curve and its significance using terms like bandwidth and quality factor

In the **RLC** series circuit of Figure 15.4.1, the current amplitude is, from Equation 15.4.7,

$$I_0 = \frac{V_0}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}. \tag{18.6.1}$$

If we can vary the frequency of the ac generator while keeping the amplitude of its output voltage constant, then the current changes accordingly. A plot of I_0 versus ω is shown in Figure 18.6.1.

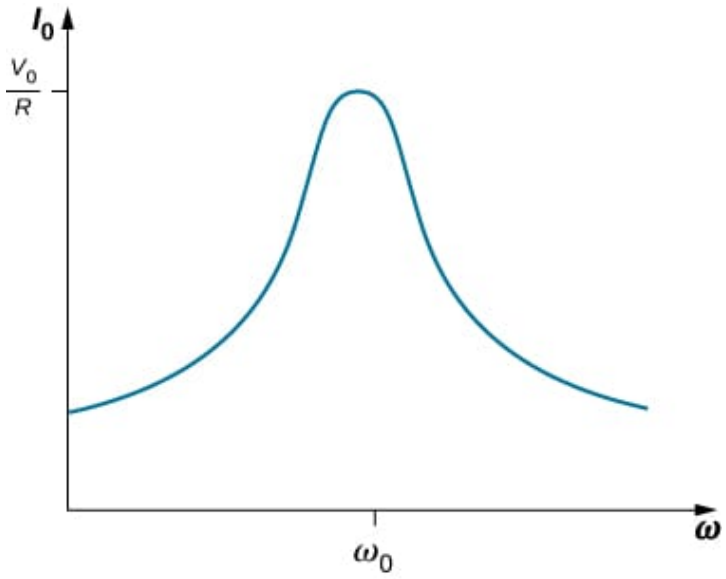


Figure 18.6.1: At an RLC circuit's resonant frequency, $\omega_0 = \sqrt{1/LC}$, the current amplitude is at its maximum value.

In Oscillations, we encountered a similar graph where the amplitude of a damped harmonic oscillator was plotted against the angular frequency of a sinusoidal driving force (see Forced Oscillations). This similarity is more than just a coincidence, as shown earlier by the application of Kirchhoff's loop rule to the circuit of Figure 15.4.1. This yields

$$L\frac{di}{dt} + iR + \frac{q}{C} = V_0 \sin \omega t, \tag{18.6.2}$$

or

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = V_0 \sin \omega t,$$

where we substituted **dq**(t)/**dt** for **i**(t). A comparison of Equation 18.6.2 and, from Oscillations, Damped Oscillations for damped harmonic motion clearly demonstrates that the driven **RLC** series circuit is the electrical analog of the driven damped harmonic oscillator.

The **resonant frequency** f_0 of the **RLC** circuit is the frequency at which the amplitude of the current is a maximum and the circuit

would oscillate if not driven by a voltage source. By inspection, this corresponds to the angular frequency $\omega_0 = 2\pi f_0$ at which the impedance **Z** in Equation 18.6.1 is a minimum, or when

$$\omega_0 L = \frac{1}{\omega_0 C} \tag{18.6.3}$$

and

$$\omega_0 = \sqrt{\frac{1}{LC}}. \tag{18.6.4}$$

This is the resonant angular frequency of the circuit. Substituting ω_0 into Equation 15.4.5, Equation 15.4.7, and Equation 15.4.8, we find that at resonance,

$$\phi = tan^{-1}(0) = 0,\ I_0 = V_0/R,\ and\ Z = R.$$

Therefore, at resonance, an **RLC** circuit is purely resistive, with the applied emf and current in phase.

What happens to the power at resonance? Equation 15.5.18 tells us how the average power transferred from an ac generator to the **RLC** combination varies with frequency. In addition, P_{ave} reaches a maximum when Z, which depends on the frequency, is a minimum, that is, when $X_L = X_C$ and $Z = R$. Thus, at resonance, the average power output of the source in an **RLC** series circuit is a maximum. From Equation 15.5.18, this maximum is V_{rms}^2/R.

Figure 18.6.2 is a typical plot of P_{ave} versus ω in the region of maximum power output. The **bandwidth** $\Delta\omega$ of the resonance peak is defined as the range of angular frequencies ω over which the average power P_{ave} is greater than one-half the maximum value of P_{ave}. The sharpness of the peak is described by a dimensionless quantity known as the **quality factor Q** of the circuit. By definition,

$$Q = \frac{\omega_0}{\Delta\omega}, \tag{18.6.5}$$

where ω_0 is the resonant angular frequency. A high **Q** indicates a sharp resonance peak. We can give **Q** in terms of the circuit parameters as

$$Q = \frac{\omega_0 L}{R}. \tag{18.6.6}$$

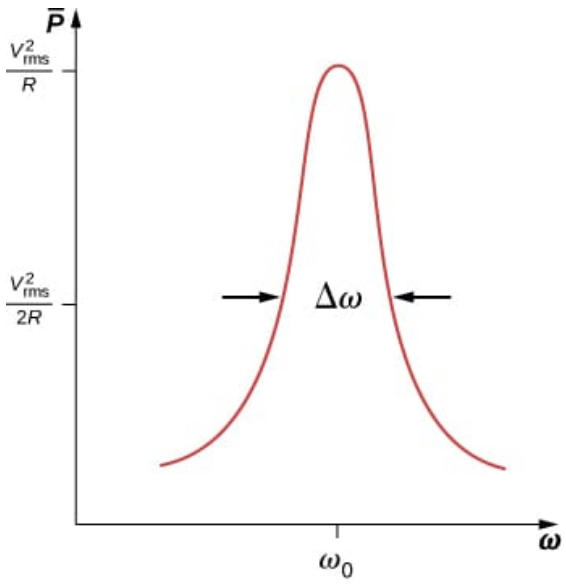


Figure 18.6.2: Like the current, the average power transferred from an ac generator to an **RLC** circuit peaks at the resonant frequency.

Resonant circuits are commonly used to pass or reject selected frequency ranges. This is done by adjusting the value of one of the elements and hence "tuning" the circuit to a particular resonant frequency. For example, in radios, the receiver is tuned to the desired station by adjusting the resonant frequency of its circuitry to match the frequency of the station. If the tuning circuit has

a high **Q**, it will have a small bandwidth, so signals from other stations at frequencies even slightly different from the resonant frequency encounter a high impedance and are not passed by the circuit. Cell phones work in a similar fashion, communicating with signals of around 1 GHz that are tuned by an inductor-capacitor circuit. One of the most common applications of capacitors is their use in ac-timing circuits, based on attaining a resonant frequency. A metal detector also uses a shift in resonance frequency in detecting metals (Figure 18.6.3).

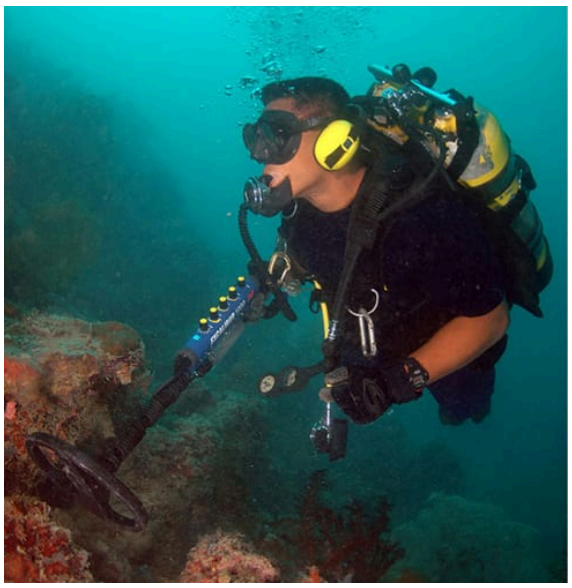


Figure 18.6.3: When a metal detector comes near a piece of metal, the self-inductance of one of its coils changes. This causes a shift in the resonant frequency of a circuit containing the coil. That shift is detected by the circuitry and transmitted to the diver by means of the headphones.

Example 18.6.1: Resonance in an RLC Series Circuit

a. What is the resonant frequency of the circuit of Example 15.3.1?
b. If the ac generator is set to this frequency without changing the amplitude of the output voltage, what is the amplitude of the current?

Strategy

The resonant frequency for a **RLC** circuit is calculated from Equation 18.6.4, which comes from a balance between the reactances of the capacitor and the inductor. Since the circuit is at resonance, the impedance is equal to the resistor. Then, the peak current is calculated by the voltage divided by the resistance.

Solution

1. The resonant frequency is found from Equation 18.6.4:

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$
$$= \frac{1}{2\pi}\sqrt{\frac{1}{(3.00\times10^{-3}H)(8.00\times10^{-4}F)}}$$
$$= 1.03\times10^{2}\,Hz.$$

2. At resonance, the impedance of the circuit is purely resistive, and the current amplitude is

$$I_0 = \frac{0.100\,V}{4.00\,\Omega} = 2.50\times10^{-2}A.$$

Significance

If the circuit were not set to the resonant frequency, we would need the impedance of the entire circuit to calculate the current.

Example 18.6.2: Power Transfer in an RLC Series Circuit at Resonance

a. What is the resonant angular frequency of an **RLC** circuit with $R = 0.200\ \Omega,\ L = 4.00 \times 10^{-3} H$, and $C = 2.00 \times 10^{-6} F$?
b. If an ac source of constant amplitude 4.00 V is set to this frequency, what is the average power transferred to the circuit?
c. Determine **Q** and the bandwidth of this circuit.

Strategy

The resonant angular frequency is calculated from Equation 18.6.4. The average power is calculated from the rms voltage and the resistance in the circuit. The quality factor is calculated from Equation 18.6.6 and by knowing the resonant frequency. The bandwidth is calculated from Equation 18.6.5 and by knowing the quality factor.

Solution

1. The resonant angular frequency is

$$\begin{aligned}\omega_0 &= \sqrt{\frac{1}{LC}} \\ &= \sqrt{\frac{1}{(4.00 \times 10^{-3} H)(2.00 \times 10^{-6} F)}} \\ &= 1.12 \times 10^4\, rad/s.\end{aligned}$$

2. At this frequency, the average power transferred to the circuit is a maximum. It is

$$P_{ave} = \frac{V_{rms}^2}{R} = \frac{[(1/\sqrt{2})(4.00\, V)]^2}{0.200\, \Omega} = 40.0\, W.$$

3. The quality factor of the circuit is

$$Q = \frac{\omega_0 L}{R} = \frac{(1.12 \times 10^4\, rad/s)(4.00 \times 10^{-3} H)}{0.200\, \Omega} = 224.$$

We then find for the bandwidth

$$\Delta\omega = \frac{\omega_0}{Q} = \frac{1.12 \times 10^4\, rad/s}{224} = 50.0\, rad/s.$$

Significance

If a narrower bandwidth is desired, a lower resistance or higher inductance would help. However, a lower resistance increases the power transferred to the circuit, which may not be desirable, depending on the maximum power that could possibly be transferred.

Exercise 18.6.1

In the circuit of Figure 15.4.1, $L = 2.0 \times 10^{-3} H,\ C = 5.0 \times 10^{-4} F$, and $R = 40\ \Omega$.

a. What is the resonant frequency?
b. What is the impedance of the circuit at resonance?
c. If the voltage amplitude is 10 V, what is **i(t)** at resonance?
d. The frequency of the AC generator is now changed to 200 Hz. Calculate the phase difference between the current and the emf of the generator.

Answer available at the end of this section.

Exercise 18.6.2

What happens to the resonant frequency of an **RLC** series circuit when the following quantities are increased by a factor of 4: (a) the capacitance, (b) the self-inductance, and (c) the resistance?

Answer available at the end of this section.

Exercise 18.6.3

The resonant angular frequency of an **RLC** series circuit is $4.0 \times 10^2\ rad/s$. An ac source operating at this frequency transfers an average power of $2.0 \times 10^{-2} W$ to the circuit. The resistance of the circuit is $0.50\ \Omega$. Write an expression for the emf of the source.

Answer available at the end of this section.

Answers

Exercise 18.6.1

a. 160 Hz; b. 40Ω; c. $(0.25A)\ \sin\ 10^3 t$; d. 0.023 rad

Exercise 18.6.2

a. halved; b. halved; c. same

Exercise 18.6.3

$v(t) = (0.14\ V)\ sin\ (4.0 \times 10^2 t)$

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18.7: Transformers

Learning Objectives

By the end of the section, you will be able to:

- Explain why power plants transmit electricity at high voltages and low currents and how they do this
- Develop relationships among current, voltage, and the number of windings in step-up and step-down transformers

Although ac electric power is produced at relatively low voltages, it is sent through transmission lines at very high voltages (as high as 500 kV). The same power can be transmitted at different voltages because power is the product $I_{rms}V_{rms}$. (For simplicity, we ignore the phase factor $\cos\ \phi$.) A particular power requirement can therefore be met with a low voltage and a high current or with a high voltage and a low current. The advantage of the high-voltage/low-current choice is that it results in lower I_{rms}^2R ohmic losses in the transmission lines, which can be significant in lines that are many kilometers long (Figure 18.7.1).

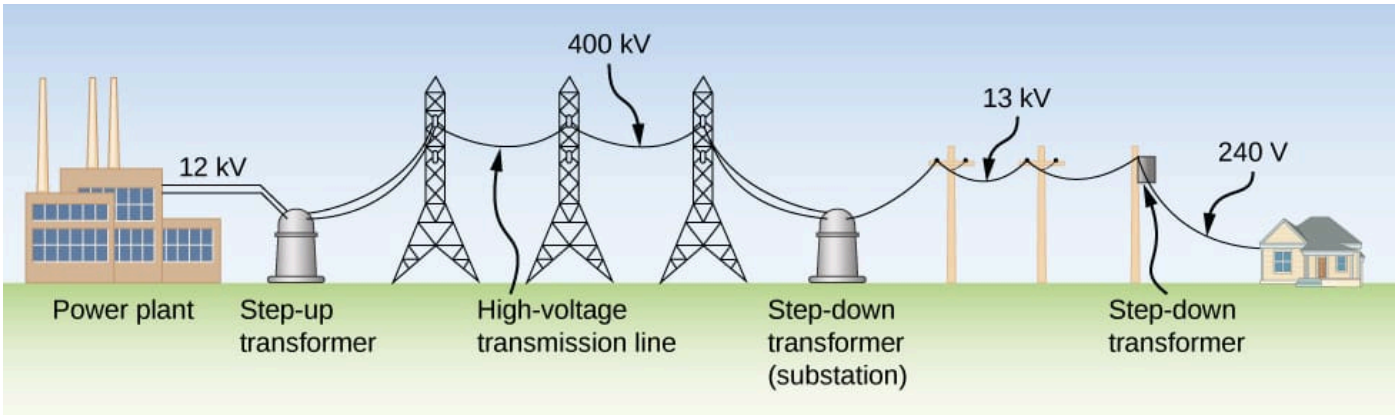


Figure 18.7.1: The rms voltage from a power plant eventually needs to be stepped down from 12 kV to 240 V so that it can be safely introduced into a home. A high-voltage transmission line allows a low current to be transmitted via a substation over long distances.

Typically, the alternating emfs produced at power plants are “stepped up” to very high voltages before being transmitted through power lines; then, they must be “stepped down” to relatively safe values (110 or 220 V rms) before they are introduced into homes. The device that transforms voltages from one value to another using induction is the **transformer** (Figure 18.7.2).

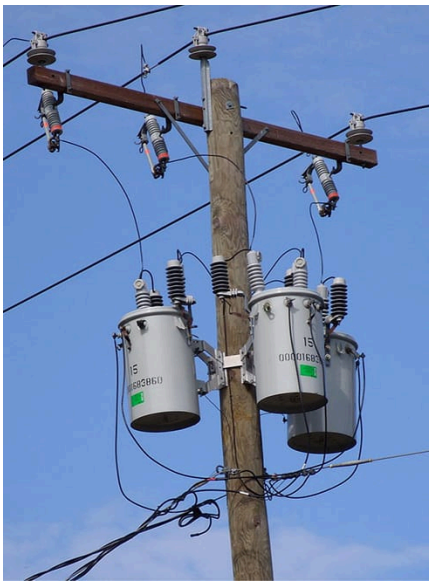


Figure 18.7.2: Transformers are used to step down the high voltages in transmission lines to the 110 to 220 V used in homes. (credit: modification of work by "Fortyseven"/Flickr)

As Figure 18.7.3 illustrates, a transformer basically consists of two separated coils, or windings, wrapped around a soft iron core. The primary winding has N_p loops, or turns, and is connected to an alternating voltage $v_p(t)$. The secondary winding has N_s turns and is connected to a load resistor R_s. We assume the ideal case for which all magnetic field lines are confined to the core so that the same magnetic flux permeates each turn of both the primary and the secondary windings. We also neglect energy losses to magnetic hysteresis, to ohmic heating in the windings, and to ohmic heating of the induced eddy currents in the core. A good transformer can have losses as low as 1% of the transmitted power, so this is not a bad assumption.

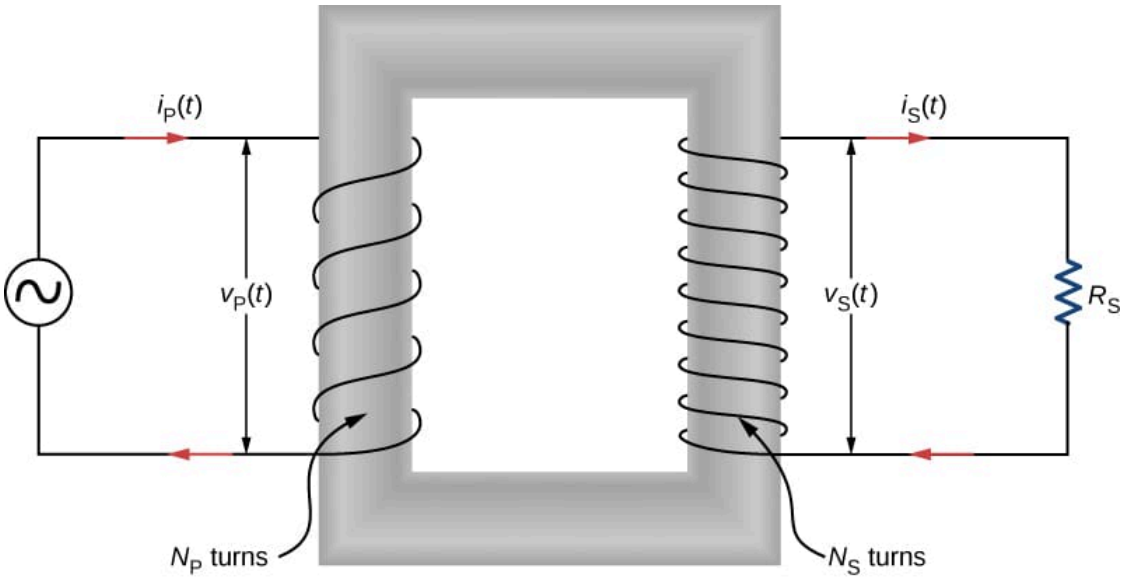


Figure 18.7.3: A step-up transformer (more turns in the secondary winding than in the primary winding). The two windings are wrapped around a soft iron core.

To analyze the transformer circuit, we first consider the primary winding. The input voltage $v_p(t)$ is equal to the potential difference induced across the primary winding. From Faraday's law, the induced potential difference is $-N_p(d\Phi/dt)$, where Φ is the flux through one turn of the primary winding. Thus,

$$v_p(t) = -N_p \frac{d\Phi}{dt}.$$

Similarly, the output voltage $v_s(t)$ delivered to the load resistor must equal the potential difference induced across the secondary

winding. Since the transformer is ideal, the flux through every turn of the secondary winding is also Φ and

$$v_s(t) = -N_s \frac{d\Phi}{dt}.$$

Combining the last two equations, we have

$$v_s(t) = \frac{N_s}{N_p} v_p(t). \tag{18.7.1}$$

Hence, with appropriate values for N_s and N_p, the input voltage $v_p(t)$ may be "stepped up" $(N_s > N_p)$ or "stepped down" $(N_s < N_p)$ to $v_s(t)$, the output voltage. This is often abbreviated as the **transformer equation**,

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}, \tag{18.7.2}$$

which shows that the ratio of the secondary to primary voltages in a transformer equals the ratio of the number of turns in their windings. For a **step-up transformer**, which increases voltage and decreases current, this ratio is greater than one; for a **step-down transformer**, which decreases voltage and increases current, this ratio is less than one.

From the law of energy conservation, the power introduced at any instant by $v_p(t)$ to the primary winding must be equal to the power dissipated in the resistor of the secondary circuit; thus,

$$i_p(t) v_p(t) = i_s(t) v_s(t).$$

When combined with Equation 18.7.2, this gives

$$i_s(t) = \frac{N_p}{N_s} i_p(t). \tag{18.7.3}$$

If the voltage is stepped up, the current is stepped down, and vice versa.

Finally, we can use $i_s(t) = v_s(t)/R_s$, along with Equation 18.7.1 and Equation 18.7.3, to obtain

$$v_p(t) = i_p \left[\left(\frac{N_p}{N_s} \right)^2 R_s \right],$$

which tells us that the input voltage $v_p(t)$ "sees" not a resistance R_s but rather a resistance

$$R_p = \left(\frac{N_p}{N_s} \right)^2 R_s.$$

Our analysis has been based on instantaneous values of voltage and current. However, the resulting equations are not limited to instantaneous values; they hold also for maximum and rms values.

Example 18.7.1: A Step-Down Transformer

A transformer on a utility pole steps the rms voltage down from 12 kV to 240 V.

a. What is the ratio of the number of secondary turns to the number of primary turns?
b. If the input current to the transformer is 2.0 A, what is the output current?
c. Determine the power loss in the transmission line if the total resistance of the transmission line is $200\ \Omega$.
d. What would the power loss have been if the transmission line was at 240 V the entire length of the line, rather than providing voltage at 12 kV? What does this say about transmission lines?

Strategy

The number of turns related to the voltages is found from Equation 18.7.1. The output current is calculated using Equation 18.7.3.

Solution

a. Using Equation 18.7.1 with rms values V_p and V_s we have

$$\frac{N_s}{N_p} = \frac{240\,V}{12 \times 10^3 V} = \frac{1}{50},$$

so the primary winding has 50 times the number of turns in the secondary winding.

b. From Equation 18.7.3, the output rms current I_s is found using the transformer equation with current

$$I_S = \frac{N_p}{N_S} I_p \tag{18.7.4}$$

such that

$$I_s = \frac{N_p}{N_s} I_p = (50)(2.0\,A) = 100\,A.$$

c. The power loss in the transmission line is calculated to be

$$P_{loss} = I_p^2 R = (2.0\,A)^2(200\,\Omega) = 800\,W.$$

d. If there were no transformer, the power would have to be sent at 240 V to work for these houses, and the power loss would be

$$P_{loss} = I_s^2 R = (100\,A)^2(200\,\Omega) = 2 \times 10^6\,W.$$

Therefore, when power needs to be transmitted, we want to avoid power loss. Thus, lines are sent with high voltages and low currents and adjusted with a transformer before power is sent into homes.

Significance

This application of a step-down transformer allows a home that uses 240-V outlets to have 100 A available to draw upon. This can power many devices in the home.

Exercise 18.7.1

A transformer steps the line voltage down from 110 to 9.0 V so that a current of 0.50 A can be delivered to a doorbell.

a. What is the ratio of the number of turns in the primary and secondary windings?
b. What is the current in the primary winding?
c. What is the resistance seen by the 110-V source?

Answer a available at the end of this section.

Answers

Exercise 18.7.1

12:1

0.042 A

$2.6 \times 10^3\ \Omega$

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18.8: Alternating-Current Circuits (Summary)

Key Terms

ac current	current that fluctuates sinusoidally with time at a fixed frequency
ac voltage	voltage that fluctuates sinusoidally with time at a fixed frequency
alternating current (ac)	flow of electric charge that periodically reverses direction
average power	time average of the instantaneous power over one cycle
bandwidth	range of angular frequencies over which the average power is greater than one-half the maximum value of the average power
capacitive reactance	opposition of a capacitor to a change in current
direct current (dc)	flow of electric charge in only one direction
impedance	ac analog to resistance in a dc circuit, which measures the combined effect of resistance, capacitive reactance, and inductive reactance
inductive reactance	opposition of an inductor to a change in current
phase angle	amount by which the voltage and current are out of phase with each other in a circuit
power factor	amount by which the power delivered in the circuit is less than the theoretical maximum of the circuit due to voltage and current being out of phase
quality factor	dimensionless quantity that describes the sharpness of the peak of the bandwidth; a high quality factor is a sharp or narrow resonance peak
resonant frequency	frequency at which the amplitude of the current is a maximum and the circuit would oscillate if not driven by a voltage source
rms current	root mean square of the current
rms voltage	root mean square of the voltage
step-down transformer	transformer that decreases voltage and increases current
step-up transformer	transformer that increases voltage and decreases current
transformer	device that transforms voltages from one value to another using induction
transformer equation	equation showing that the ratio of the secondary to primary voltages in a transformer equals the ratio of the number of turns in their windings

Key Equations

AC voltage	$v = V_0 sin\omega t$
AC current	$i = I_0 sin\omega t$
capacitive reactance	$\frac{V_0}{I_0} = \frac{1}{\omega C} = X_C$

rms voltage	$V_{rms} = \frac{V_0}{\sqrt{2}}$
rms current	$I_{rms} = \frac{I_0}{\sqrt{2}}$
inductive reactance	$\frac{V_0}{I_0} = \omega L = X_L$
Phase angle of an RLC series circuit	$\phi = tan^{-1}\frac{X_L - X_C}{R}$
AC version of Ohm's law	$I_0 = \frac{V_0}{Z}$
Impedance of an RLC series circuit	$Z = \sqrt{R^2 + (X_L - X_C)^2}$
Average power associated with a circuit element	$P_{ave} = \frac{1}{2} I_0 V_0 cos\phi$
Average power dissipated by a resistor	$P_{ave} = \frac{1}{2} I_0 V_0 = I_{rms} V_{rms} = I_{rms}^2 R$
Resonant angular frequency of a circuit	$\omega_0 = \sqrt{\frac{1}{LC}}$
Quality factor of a circuit	$Q = \frac{\omega_0}{\Delta\omega}$
Quality factor of a circuit in terms of the circuit parameters	$Q = \frac{\omega_0 L}{R}$
Transformer equation with voltage	$\frac{V_S}{V_P} = \frac{N_S}{N_P}$
Transformer equation with current	$I_S = \frac{N_P}{N_S} I_P$

Summary

15.2 AC Sources

- Direct current (dc) refers to systems in which the source voltage is constant.
- Alternating current (ac) refers to systems in which the source voltage varies periodically, particularly sinusoidally.
- The voltage source of an ac system puts out a voltage that is calculated from the time, the peak voltage, and the angular frequency.
- In a simple circuit, the current is found by dividing the voltage by the resistance. An ac current is calculated using the peak current (determined by dividing the peak voltage by the resistance), the angular frequency, and the time.

15.3 Simple AC Circuits

- For resistors, the current through and the voltage across are in phase.
- For capacitors, we find that when a sinusoidal voltage is applied to a capacitor, the voltage follows the current by one-fourth of a cycle. Since a capacitor can stop current when fully charged, it limits current and offers another form of ac resistance, called capacitive reactance, which has units of ohms.
- For inductors in ac circuits, we find that when a sinusoidal voltage is applied to an inductor, the voltage leads the current by one-fourth of a cycle.
- The opposition of an inductor to a change in current is expressed as a type of ac reactance. This inductive reactance, which has units of ohms, varies with the frequency of the ac source.

15.4 RLC Series Circuits with AC

- An **RLC** series circuit is a resistor, capacitor, and inductor series combination across an ac source.
- The same current flows through each element of an **RLC** series circuit at all points in time.

- The counterpart of resistance in a dc circuit is impedance, which measures the combined effect of resistors, capacitors, and inductors. The maximum current is defined by the ac version of Ohm's law.
- Impedance has units of ohms and is found using the resistance, the capacitive reactance, and the inductive reactance.

15.5 Power in an AC Circuit

- The average ac power is found by multiplying the rms values of current and voltage.
- Ohm's law for the rms ac is found by dividing the rms voltage by the impedance.
- In an ac circuit, there is a phase angle between the source voltage and the current, which can be found by dividing the resistance by the impedance.
- The average power delivered to an **RLC** circuit is affected by the phase angle.
- The power factor ranges from –1 to 1.

15.6 Resonance in an AC Circuit

- At the resonant frequency, inductive reactance equals capacitive reactance.
- The average power versus angular frequency plot for a **RLC** circuit has a peak located at the resonant frequency; the sharpness or width of the peak is known as the bandwidth.
- The bandwidth is related to a dimensionless quantity called the quality factor. A high quality factor value is a sharp or narrow peak.

15.7 Transformers

- Power plants transmit high voltages at low currents to achieve lower ohmic losses in their many kilometers of transmission lines.
- Transformers use induction to transform voltages from one value to another.
- For a transformer, the voltages across the primary and secondary coils, or windings, are related by the transformer equation.
- The currents in the primary and secondary windings are related by the number of primary and secondary loops, or turns, in the windings of the transformer.
- A step-up transformer increases voltage and decreases current, whereas a step-down transformer decreases voltage and increases current.

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18.9: Alternating-Current Circuits (Exercise)

Conceptual Questions

15.2 AC Sources

1. What is the relationship between frequency and angular frequency?

15.3 Simple AC Circuits

2. Explain why at high frequencies a capacitor acts as an ac short, whereas an inductor acts as an open circuit.

15.4 RLC Series Circuits with AC

3. In an **RLC** series circuit, can the voltage measured across the capacitor be greater than the voltage of the source? Answer the same question for the voltage across the inductor.

15.5 Power in an AC Circuit

4. For what value of the phase angle ϕ between the voltage output of an ac source and the current is the average power output of the source a maximum?

5. Discuss the differences between average power and instantaneous power.

6. The average ac current delivered to a circuit is zero. Despite this, power is dissipated in the circuit. Explain.

7. Can the instantaneous power output of an ac source ever be negative? Can the average power output be negative?

8. The power rating of a resistor used in ac circuits refers to the maximum average power dissipated in the resistor. How does this compare with the maximum instantaneous power dissipated in the resistor?

15.7 Transformers

9. Why do transmission lines operate at very high voltages while household circuits operate at fairly small voltages?

10. How can you distinguish the primary winding from the secondary winding in a step-up transformer?

11. Battery packs in some electronic devices are charged using an adapter connected to a wall socket. Speculate as to the purpose of the adapter.

12. Will a transformer work if the input is a dc voltage?

13. Why are the primary and secondary coils of a transformer wrapped around the same closed loop of iron?

Problems

15.2 AC Sources

14. Write an expression for the output voltage of an ac source that has an amplitude of 12 V and a frequency of 200 Hz.

15.3 Simple AC Circuits

15. Calculate the reactance of a **5.0-μF** capacitor at

(a) 60 Hz,

(b) 600 Hz, and

(c) 6000 Hz.

16. What is the capacitance of a capacitor whose reactance is **10Ω** at 60 Hz?

17. Calculate the reactance of a 5.0-mH inductor at

(a) 60 Hz,

(b) 600 Hz, and

(c) 6000 Hz.

18. What is the self-inductance of a coil whose reactance is **10Ω** at 60 Hz?

19. At what frequency is the reactance of a **20-μF** capacitor equal to that of a 10-mH inductor?

20. At 1000 Hz, the reactance of a 5.0-mH inductor is equal to the reactance of a particular capacitor. What is the capacitance of the capacitor?

21. A **50-Ω** resistor is connected across the **emf** $v(t) = (160V)sin(120\pi t)$. Write an expression for the current through the resistor.

22. A **25-μF** capacitor is connected to an emf given by $v(t) = (160V)sin(120\pi t)$.

(a) What is the reactance of the capacitor?

(b) Write an expression for the current output of the source.

23. A 100-mH inductor is connected across the emf of the preceding problem.

(a) What is the reactance of the inductor?

(b) Write an expression for the current through the inductor.

15.4 RLC Series Circuits with AC

24. What is the impedance of a series combination of a **50-Ω** resistor, a 5.0-μF5.0-μF capacitor, and a 10-μF10-μF capacitor at a frequency of 2.0 kHz?

25. A resistor and capacitor are connected in series across an ac generator. The emf of the generator is given by $v(t) = V0cos\omega t$, where $V0 = 120V, \omega = 120\pi rad/s, R = 400\Omega$, and $C = 4.0\mu F$.

(a) What is the impedance of the circuit?

(b) What is the amplitude of the current through the resistor?

(c) Write an expression for the current through the resistor.

(d) Write expressions representing the voltages across the resistor and across the capacitor.

26. A resistor and inductor are connected in series across an ac generator. The emf of the generator is given by $v(t) = V0cos\omega t$, where $V0 = 120V$ and $\omega = 120\pi rad/s$; also, $R = 400\Omega$ and $L = 1.5H$.

(a) What is the impedance of the circuit?

(b) What is the amplitude of the current through the resistor?

(c) Write an expression for the current through the resistor.

(d) Write expressions representing the voltages across the resistor and across the inductor.

27. In an **RLC** series circuit, the voltage amplitude and frequency of the source are 100 V and 500 Hz, respectively, an $R = 500\Omega, L = 0.20H$, and $C = 2.0\mu F$.

(a) What is the impedance of the circuit?

(b) What is the amplitude of the current from the source?

(c) If the emf of the source is given by $v(t) = (100V)sin1000\pi t$, how does the current vary with time?

(d) Repeat the calculations with i changed to **0.20μF**.

28. An **RLC** series circuit with $R = 600\Omega, L = 30mH$, and $C = 0.050\mu F$ is driven by an ac source whose frequency and voltage amplitude are 500 Hz and 50 V, respectively.

(a) What is the impedance of the circuit?

(b) What is the amplitude of the current in the circuit?

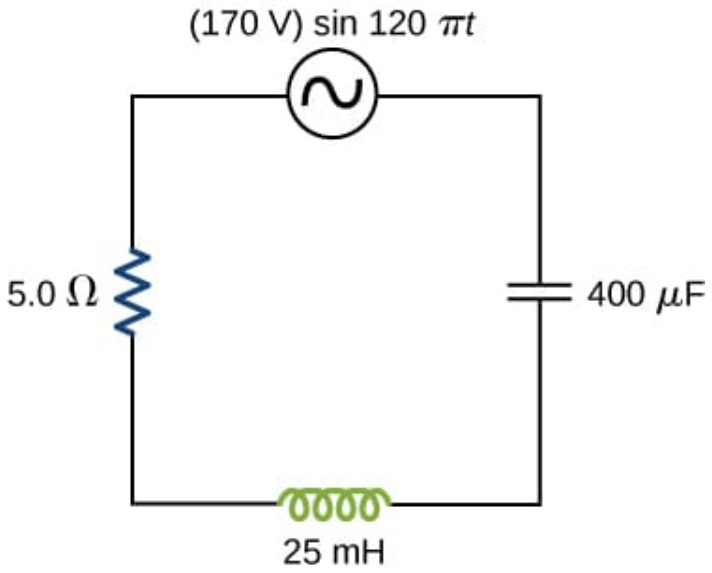
(c) What is the phase angle between the emf of the source and the current?

29. For the circuit shown below, what are

(a) the total impedance and

(b) the phase angle between the current and the emf?

(c) Write an expression for $i(t)$.



15.5 Power in an AC Circuit

30. The emf of an ac source is given by $v(t) = V0sin\omega t$, where $V0 = 100V$ and $\omega = 200\pi rad/s$.. Calculate the average power

output of the source if it is connected across

(a) a **20-μF** capacitor,

(b) a 20-mH inductor, and

(c) a **50-Ω** resistor.

31. Calculate the rms currents for an ac source is given by $v(t) = V_0 \sin \omega t$, where $V_0 = 100V$ and $\omega = 200\pi \ rad/s$ when connected across

(a) a **20-μF** capacitor,

(b) a 20-mH inductor, and

(c) a **50-Ω** resistor.

32. A 40-mH inductor is connected to a 60-Hz AC source whose voltage amplitude is 50 V. If an AC voltmeter is placed across the inductor, what does it read?

33. For an **RLC** series circuit, the voltage amplitude and frequency of the source are 100 V and 500 Hz, respectively; **R=500Ω**; and **L=0.20H**. Find the average power dissipated in the resistor for the following values for the capacitance:

(a) **C=2.0μF** and

(b) **C=0.20μF.**

34. An ac source of voltage amplitude 10 V delivers electric energy at a rate of 0.80 W when its current output is 2.5 A. What is the phase angle **ϕ** between the emf and the current?

35. An **RLC** series circuit has an impedance of **60Ω** and a power factor of 0.50, with the voltage lagging the current. (a) Should a capacitor or an inductor be placed in series with the elements to raise the power factor of the circuit? (b) What is the value of the reactance across the inductor that will raise the power factor to unity?

15.6 Resonance in an AC Circuit

36. (a) Calculate the resonant angular frequency of an **RLC** series circuit for which $R = 20\Omega, L = 75mH$, and **C=4.0μF**. (b) If **R** is changed to **300Ω**, what happens to the resonant angular frequency?

37. The resonant frequency of an **RLC** series circuit is $2.0 \times 10^3 Hz$. If the self-inductance in the circuit is 5.0 mH, what is the capacitance in the circuit?

38. (a) What is the resonant frequency of an **RLC** series circuit with **R=20Ω**, **L=2.0mH**, and **C=4.0μF**?

(b) What is the impedance of the circuit at resonance?

39. For an **RLC** series circuit, **R=100Ω**, **L=150mH**, and **C=0.25μF**.

(a) If an ac source of variable frequency is connected to the circuit, at what frequency is maximum power dissipated in the resistor?

(b) What is the quality factor of the circuit?

40. An ac source of voltage amplitude 100 V and variable frequency **f** drives an **RLC** series circuit with **R=10Ω**, **L=2.0mH**, and **C=25μF.**

(a) Plot the current through the resistor as a function of the frequency f.

(b) Use the plot to determine the resonant frequency of the circuit.

41. (a) What is the resonant frequency of a resistor, capacitor, and inductor connected in series if **R=100Ω**, **L=2.0H,** and **C=5.0μF**?

(b) If this combination is connected to a 100-V source operating at the constant frequency, what is the power output of the source?

(c) What is the **Q** of the circuit?

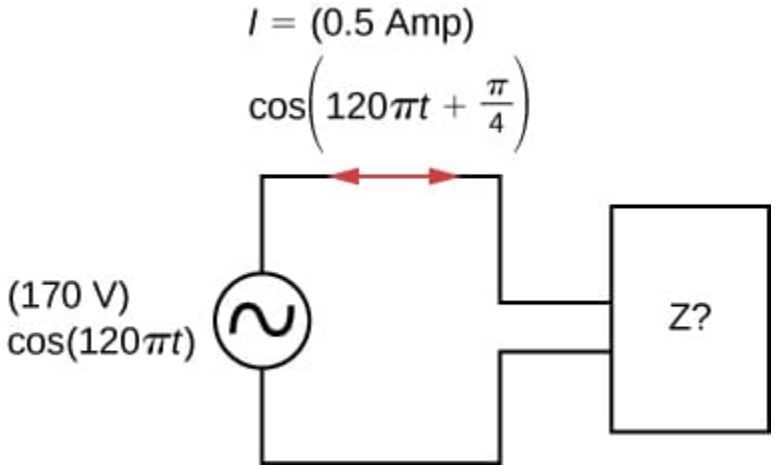
(d) What is the bandwidth of the circuit?

42. Suppose a coil has a self-inductance of 20.0 H and a resistance of **200Ω**. What

(a) capacitance and

(b) resistance must be connected in series with the coil to produce a circuit that has a resonant frequency of 100 Hz and a **Q** of 10?

43. An ac generator is connected to a device whose internal circuits are not known. We only know current and voltage outside the device, as shown below. Based on the information given, what can you infer about the electrical nature of the device and its power usage?



15.7 Transformers

44. A step-up transformer is designed so that the output of its secondary winding is 2000 V (rms) when the primary winding is connected to a 110-V (rms) line voltage.

(a) If there are 100 turns in the primary winding, how many turns are there in the secondary winding?

(b) If a resistor connected across the secondary winding draws an rms current of 0.75 A, what is the current in the primary winding?

45. A step-up transformer connected to a 110-V line is used to supply a hydrogen-gas discharge tube with 5.0 kV (rms). The tube dissipates 75 W of power.

(a) What is the ratio of the number of turns in the secondary winding to the number of turns in the primary winding?

(b) What are the rms currents in the primary and secondary windings?

(c) What is the effective resistance seen by the 110-V source?

46. An ac source of emf delivers 5.0 mW of power at an rms current of 2.0 mA when it is connected to the primary coil of a transformer. The rms voltage across the secondary coil is 20 V.

(a) What are the voltage across the primary coil and the current through the secondary coil?

(b) What is the ratio of secondary to primary turns for the transformer?

47. A transformer is used to step down 110 V from a wall socket to 9.0 V for a radio.

(a) If the primary winding has 500 turns, how many turns does the secondary winding have?

(b) If the radio operates at a current of 500 mA, what is the current through the primary winding?

48. A transformer is used to supply a 12-V model train with power from a 110-V wall plug. The train operates at 50 W of power.

(a) What is the rms current in the secondary coil of the transformer?

(b) What is the rms current in the primary coil?

(c) What is the ratio of the number of primary to secondary turns?

(d) What is the resistance of the train?

(e) What is the resistance seen by the 110-V source?

Additional Problems

49. The emf of an dc source is given by $v(t) = V_0 sin\omega t$, where $V_0 = 100V$ and $\omega = 200\pi rad/s$. Find an expression that represents the output current of the source if it is connected across

(a) a **20-μF** capacitor,

(b) a 20-mH inductor, and

(c) a **50-Ω** resistor.

50. A **700-pF** capacitor is connected across an ac source with a voltage amplitude of 160 V and a frequency of 20 kHz.

(a) Determine the capacitive reactance of the capacitor and the amplitude of the output current of the source.

(b) If the frequency is changed to 60 Hz while keeping the voltage amplitude at 160 V, what are the capacitive reactance and the current amplitude?

51. A 20-mH inductor is connected across an AC source with a variable frequency and a constant-voltage amplitude of 9.0 V.

(a) Determine the reactance of the circuit and the maximum current through the inductor when the frequency is set at 20 kHz.

(b) Do the same calculations for a frequency of 60 Hz.

52. A **30-μF** capacitor is connected across a 60-Hz ac source whose voltage amplitude is 50 V.

(a) What is the maximum charge on the capacitor?

(b) What is the maximum current into the capacitor?

(c) What is the phase relationship between the capacitor charge and the current in the circuit?

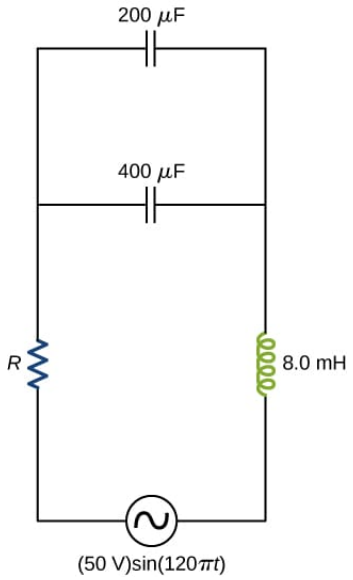
53. A 7.0-mH inductor is connected across a 60-Hz ac source whose voltage amplitude is 50 V.

(a) What is the maximum current through the inductor?

(b) What is the phase relationship between the current through and the potential difference across the inductor?

54. What is the impedance of an **RLC** series circuit at the resonant frequency?

55. What is the resistance **R** in the circuit shown below if the amplitude of the ac through the inductor is 4.24 A?



56. An ac source of voltage amplitude 100 V and frequency 1.0 kHz drives an **RLC** series circuit with **R=20Ω**, **L=4.0mH**, and **C=50μF**.

(a) Determine the rms current through the circuit.

(b) What are the rms voltages across the three elements?

(c) What is the phase angle between the emf and the current?

(d) What is the power output of the source?

(e) What is the power dissipated in the resistor?

57. In an RLC series circuit, $R = 200\Omega, L = 1.0H, C = 50\mu F, V_0 = 120V$, and $f = 50Hz$. What is the power output of the source?

58. A power plant generator produces 100 A at 15 kV (rms). A transformer is used to step up the transmission line voltage to 150 kV (rms).

(a) What is rms current in the transmission line?

(b) If the resistance per unit length of the line is $8.6 \times 10^{-8}\Omega/m$, what is the power loss per meter in the line?

(c) What would the power loss per meter be if the line voltage were 15 kV (rms)?

59. Consider a power plant located 25 km outside a town delivering 50 MW of power to the town. The transmission lines are made of aluminum cables with a $7cm^2$ cross-sectional area. Find the loss of power in the transmission lines if it is transmitted at

(a) 200 kV (rms) and

(b) 120 V (rms).

60. Neon signs require 12-kV for their operation. A transformer is to be used to change the voltage from 220-V (rms) ac to

12-kV (rms) ac.

(a) What must the ratio be of turns in the secondary winding to the turns in the primary winding?

(b) What is the maximum rms current the neon lamps can draw if the fuse in the primary winding goes off at 0.5 A?

(c) How much power is used by the neon sign when it is drawing the maximum current allowed by the fuse in the primary winding?

Challenge Problems

61. The 335-kV ac electricity from a power transmission line is fed into the primary winding of a transformer. The ratio of the number of turns in the secondary winding to the number in the primary winding is $N_s/N_p = 1000$.

(a) What voltage is induced in the secondary winding?

(b) What is unreasonable about this result?

(c) Which assumption or premise is responsible?

62. A **1.5-kΩ** resistor and 30-mH inductor are connected in series, as shown below, across a 120-V (rms) ac power source oscillating at 60-Hz frequency.

(a) Find the current in the circuit.

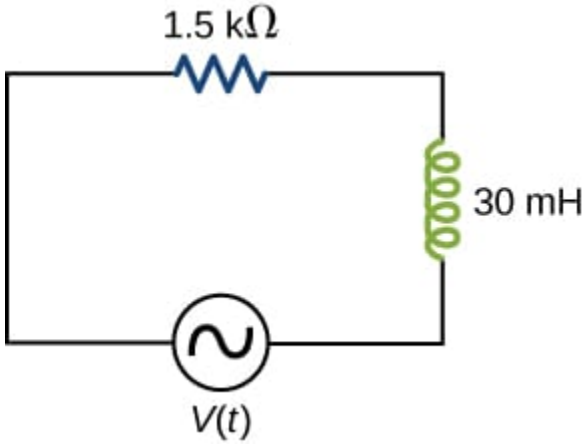
(b) Find the voltage drops across the resistor and inductor.

(c) Find the impedance of the circuit.

(d) Find the power dissipated in the resistor.

(e) Find the power dissipated in the inductor.

(f) Find the power produced by the source.



63. A **20-Ω** resistor, **50-μF** capacitor, and 30-mH inductor are connected in series with an ac source of amplitude 10 V and frequency 125 Hz.

(a) What is the impedance of the circuit?

(b) What is the amplitude of the current in the circuit?

(c) What is the phase constant of the current? Is it leading or lagging the source voltage?

(d) Write voltage drops across the resistor, capacitor, and inductor and the source voltage as a function of time.

(e) What is the power factor of the circuit? (f) How much energy is used by the resistor in 2.5 s?

64. A **200-Ω** resistor, **150-μF** capacitor, and 2.5-H inductor are connected in series with an ac source of amplitude 10 V and variable angular frequency ω.

(a) What is the value of the resonance frequency ω_R?

(b) What is the amplitude of the current if $\omega = \omega_R$?

(c) What is the phase constant of the current when $\omega = \omega_R$? Is it leading or lagging the source voltage, or is it in phase?

(d) Write an equation for the voltage drop across the resistor as a function of time when $\omega = \omega_R$.

(e) What is the power factor of the circuit when $\omega = \omega_R$?

(f) How much energy is used up by the resistor in 2.5 s when $\omega = \omega_R$?

65. Find the reactances of the following capacitors and inductors in ac circuits with the given frequencies in each case:

(a) 2-mH inductor with a frequency 60-Hz of the ac circuit;

(b) 2-mH inductor with a frequency 600-Hz of the ac circuit;

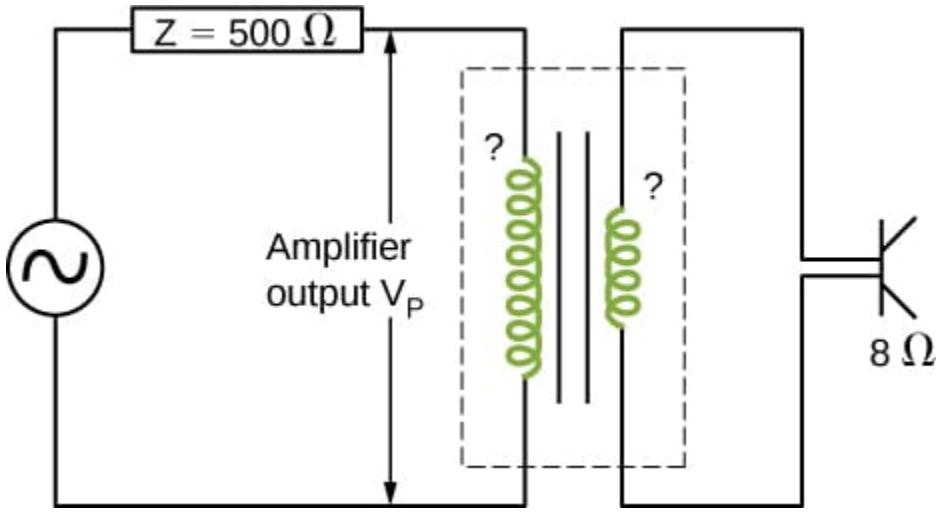
(c) 20-mH inductor with a frequency 6-Hz of the ac circuit;

(d) 20-mH inductor with a frequency 60-Hz of the ac circuit;

(e) 2-mF capacitor with a frequency 60-Hz of the ac circuit; and

(f) 2-mF capacitor with a frequency 600-Hz of the AC circuit.

66. An output impedance of an audio amplifier has an impedance of **500Ω** and has a mismatch with a low-impedance **8-Ω** loudspeaker. You are asked to insert an appropriate transformer to match the impedances. What turns ratio will you use, and why? Use the simplified circuit shown below.



67. Show that the SI unit for capacitive reactance is the ohm. Show that the SI unit for inductive reactance is also the ohm.

68. A coil with a self-inductance of 16 mH and a resistance of **6.0Ω** is connected to an ac source whose frequency can be varied. At what frequency will the voltage across the coil lead the current through the coil by **45°?**

69. An **RLC** series circuit consists of a **50-Ω** resistor, a **200-μF** capacitor, and a 120-mH inductor whose coil has a resistance of **20Ω**. The source for the circuit has an rms emf of 240 V at a frequency of 60 Hz. Calculate the rms voltages across the

(a) resistor,

(b) capacitor, and

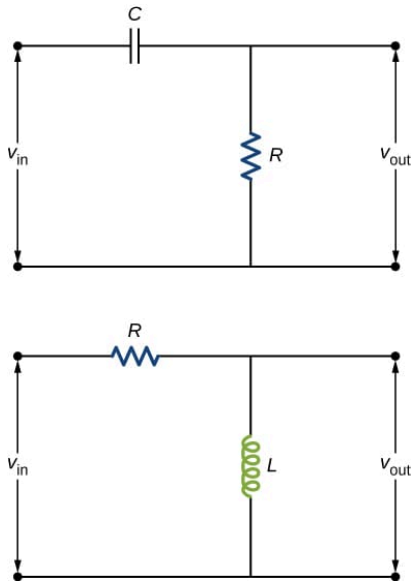
(c) inductor.

70. An **RLC** series circuit consists of a **10-Ω** resistor, an **8.0-μF** capacitor, and a 50-mH inductor. A 110-V (rms) source of variable frequency is connected across the combination. What is the power output of the source when its frequency is set to one-half the resonant frequency of the circuit?

71. Shown below are two circuits that act as crude high-pass filters. The input voltage to the circuits is v_{in}, and the output voltage is vout.vout.

(a) Show that for the capacitor circuit, $\frac{v_{out}}{v_{in}} = \frac{1}{\sqrt{1+1/\omega^2R^2C^2}}$, and for the inductor circuit, $\frac{v_{out}}{v_{in}} = \frac{\omega L}{\sqrt{R^2+\omega^2L^2}}$.

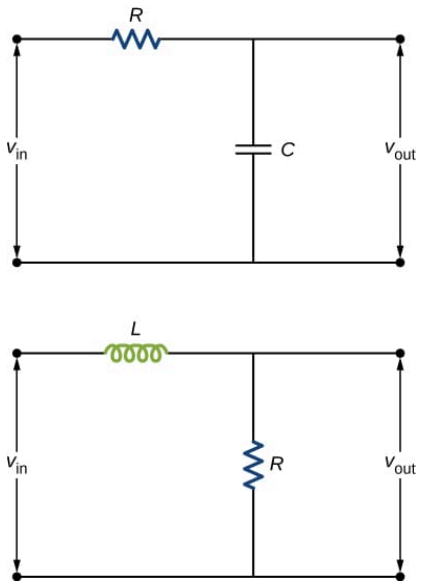
(b) Show that for high frequencies, $v_{out} \approx v_{in}$, but for low frequencies, $v_{out} \approx 0$.



72. The two circuits shown below act as crude low-pass filters. The input voltage to the circuits is v_{in}, and the output voltage is v_{out}.

(a) Show that for the capacitor circuit, $\frac{v_{out}}{v_{in}} = \frac{1}{\sqrt{1+\omega^2R^2C^2}}$, and for the inductor circuit, $\frac{v_{out}}{v_{in}} = \frac{R}{\sqrt{R^2+\omega^2L^2}}$.

(b) Show that for low frequencies, $v_{out} \approx v_{in}$, but for high frequencies, $v_{out} \approx 0$.



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18.10: Alternating-Current Circuits (Answers)

Check Your Understanding

15.1. 10 ms

15.2. a. $(20V)sin200\pi t, (0.20A)sin200\pi t$;

b. $(20V)sin200\pi t, (0.13A)sin(200\pi t + \pi/2)$;

c. $(20V)sin200\pi t, (2.1A)sin(200\pi t - \pi/2)$

15.3. $v_R = (V_0R/Z)sin(\omega t - \phi); v_C = (V_0X_C/Z)sin(\omega t - \phi + \pi/2) = -(V_0X_C/Z)cos(\omega t - \phi); v_L = (V_0X_L/Z)sin(\omega t - \phi - \pi/2) = (V_0X_L/Z)cos(\omega t - \phi)$

15.4. $v(t) = (10.0V)sin90\pi t$

15.5. 2.00 V; 10.01 V; 8.01 V

15.6. a. 160 Hz;

b. 40Ω;

c. $(0.25A)sin10^3t$;

d. 0.023 rad

15.7. a. halved;

b. halved;

c. same

15.8. $v(t) = (0.14V)sin(4.0 \times 10^2t)$

15.9. a. 12:1;

b. 0.042 A;

c. $2.6 \times 10^3\Omega$

Conceptual Questions

1. Angular frequency is 2π times frequency.

3. yes for both

5. The instantaneous power is the power at a given instant. The average power is the power averaged over a cycle or number of cycles.

7. The instantaneous power can be negative, but the power output can't be negative.

9. There is less thermal loss if the transmission lines operate at low currents and high voltages.

11. The adapter has a step-down transformer to have a lower voltage and possibly higher current at which the device can operate.

13. so each loop can experience the same changing magnetic flux

Problems

15. a. 530Ω;

b. 53Ω;

c. 5.3Ω

17. a. 1.9Ω;

b. 19Ω;

c. 190Ω

19. 360 Hz

21. $i(t) = (3.2A)sin(120\pi t)$

23. a. 38Ω;

b. $i(t) = (4.24A)sin(120\pi t - \pi/2)$

25. a. 770Ω;

b. 0.16 A;

c. $I = (0.16A)cos(120\pi t)$;

d. $v_R = 120cos(120\pi t); v_C = 120cos(120\pi t - \pi/2)$

27. a. 690Ω;

b. 0.15 A;

c. $I = (0.15A)sin(1000\pi t - 0.753)$;

d. 1100Ω, 0.092 A, $I = (0.092A)sin(1000\pi t + 1.09)$

29. a. 5.7Ω;

b. $29\ °$;

c. $I = (30.A)cos(120\pi t)$

31. a. 0.89 A;

b. 5.6A;

c. 1.4 A

33. a. 5.3 W;

b. 2.1 W

35. a. inductor;

b. $X_L = 52\Omega$

37. $1.3 \times 10^{-7} F$

39. a. 820 Hz;

b. 7.8

41. a. 50 Hz;

b. 50 W;

c. 13;

d. 25 rad/s

43. The reactance of the capacitor is larger than the reactance of the inductor because the current leads the voltage. The power usage is 30 W.

45. a. 45:1;

b. 0.68 A, 0.015 A;

c. 160Ω

47. a. 41 turns;

b. 40.9 mA

Additional Problems

49. a. $i(t) = (1.26A)sin(200\pi t + \pi/2)$;

b. $i(t) = (12.6A)sin(200\pi t - \pi/2)$;

c. $i(t) = (2A)sin(200\pi t)$

51. a. $2.5 \times 10^{3}\Omega, 3.6 \times 10^{-3}A$;

b. $7.5\Omega, 1.2A$

53. a. 19 A;

b. inductor leads by $90\,^{\circ}$

55. 11.7Ω

57. 36 W

59. a. $5.9 \times 10^{4} W$;

b. $1.64 \times 10^{11} W$

Challenge Problems

61. a. 335 MV;

b. the result is way too high, well beyond the breakdown voltage of air over reasonable distances;

c. the input voltage is too high

63. a. 20Ω;

b. 0.5 A;

c. $5.4\ °$, lagging;

d. $V_R = (9.96V)cos(250\pi t\ + 5.4\ °), V_C = (12.7V)cos(250\pi t\ + 5.4\ °-90\ °), V_L = (11.8V)cos(250\pi t\ + 5.4\ °+90\ °), V_{source} = (10.0V)cos(250\pi t)$;;

e. 0.995;

f. 6.25 J

65. a. 0.75Ω;

b. 7.5Ω;

c. 0.75Ω;

d. 7.5Ω;

e. 1.3Ω;

f. 0.13Ω

67. The units as written for inductive reactance Equation 15.8 are $\frac{rad}{s}$H. Radians can be ignored in unit analysis. The Henry can be defined as $H = \frac{V \cdot s}{A} = \Omega \cdot s$. Combining these together results in a unit of Ω for reactance.

69. a. 156 V;

b. 42 V;

c. 154 V

71. a. $\frac{v_{out}}{v_{in}} = \frac{1}{\sqrt{1 + 1/\omega^2 R^2 C^2}}$ and \(\displaystyle \frac{v_{out}}{v_{in}}=\frac{ωL}{\sqrt{R^2+ω^2L^2}};

b. $v_{out} \approx v_{in}$ and $v_{out} \approx 0$

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19: ELECTROMAGNETIC WAVES

19.1: Prelude to Electromagnetic Waves
19.2: Maxwell's Equations and Electromagnetic Waves
19.3: Plane Electromagnetic Waves
19.4: Energy Carried by Electromagnetic Waves
19.5: Momentum and Radiation Pressure
19.6: The Electromagnetic Spectrum
19.7: Electromagnetic Waves (Summary)
19.8: Electromagnetic Waves (Exercises)
19.9: Electromagnetic Waves (Answer)

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19.1: Prelude to Electromagnetic Waves

Our view of objects in the sky at night, the warm radiance of sunshine, the sting of sunburn, our cell phone conversations, and the X-rays revealing a broken bone—all are brought to us by electromagnetic waves. It would be hard to overstate the practical importance of electromagnetic waves, through their role in vision, through countless technological applications, and through their ability to transport the energy from the Sun through space to sustain life and almost all of its activities on Earth.



Figure 19.1.16: The pressure from sunlight predicted by Maxwell's equations helped produce the tail of Comet McNaught. (credit: modification of work by Sebastian Deiries—ESO)

Theory predicted the general phenomenon of electromagnetic waves before anyone realized that light is a form of an electromagnetic wave. In the mid-nineteenth century, James Clerk Maxwell formulated a single theory combining all the electric and magnetic effects known at that time. Maxwell's equations, summarizing this theory, predicted the existence of electromagnetic waves that travel at the speed of light. His theory also predicted how these waves behave, and how they carry both energy and momentum. The tails of comets, such as Comet McNaught in Figure 16.1, provide a spectacular example. Energy carried by light from the Sun warms the comet to release dust and gas. The momentum carried by the light exerts a weak force that shapes the dust into a tail of the kind seen here. The flux of particles emitted by the Sun, called the solar wind, typically produces an additional, second tail, as described in detail in this chapter.

In this chapter, we explain Maxwell's theory and show how it leads to his prediction of electromagnetic waves. We use his theory to examine what electromagnetic waves are, how they are produced, and how they transport energy and momentum. We conclude by summarizing some of the many practical applications of electromagnetic waves.

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19.2: Maxwell's Equations and Electromagnetic Waves

Learning Objectives

By the end of this section, you will be able to:

- Explain Maxwell's correction of Ampère's law by including the displacement current
- State and apply Maxwell's equations in integral form
- Describe how the symmetry between changing electric and changing magnetic fields explains Maxwell's prediction of electromagnetic waves
- Describe how Hertz confirmed Maxwell's prediction of electromagnetic waves

James Clerk **Maxwell** (1831–1879) was one of the major contributors to physics in the nineteenth century (Figure 19.2.1). Although he died young, he made major contributions to the development of the kinetic theory of gases, to the understanding of color vision, and to the nature of Saturn's rings. He is probably best known for having combined existing knowledge of the laws of electricity and of magnetism with insights of his own into a complete overarching electromagnetic theory, represented by **Maxwell's equations**.



Figure 19.2.1: James Clerk Maxwell, a nineteenth-century physicist, developed a theory that explained the relationship between electricity and magnetism, and correctly predicted that visible light consists of electromagnetic waves.

Maxwell's Correction to the Laws of Electricity and Magnetism

The four basic laws of electricity and magnetism had been discovered experimentally through the work of physicists such as Oersted, Coulomb, Gauss, and Faraday. Maxwell discovered logical inconsistencies in these earlier results and identified the incompleteness of Ampère's law as their cause.

Recall that according to Ampère's law, the integral of the magnetic field around a closed loop **C** is proportional to the current **I** passing through any surface whose boundary is loop **C** itself:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I. \tag{19.2.1}$$

There are infinitely many surfaces that can be attached to any loop, and Ampère's law stated in Equation 19.2.1 is independent of the choice of surface.

Consider the set-up in Figure 19.2.2. A source of emf is abruptly connected across a parallel-plate capacitor so that a time-dependent current **I** develops in the wire. Suppose we apply Ampère's law to loop **C** shown at a time before the capacitor is fully charged, so that $I \neq 0$. Surface S_1 gives a nonzero value for the enclosed current **I**, whereas surface S_2 gives zero for the enclosed current because no current passes through it:

$$\underbrace{\oint_C \vec{B} \cdot d\vec{s} = \mu_0 I}_{\text{if surface } S_1 \text{is used}}$$

$$\underbrace{= 0}_{\text{if surface } S_2 \text{is used}}$$

Clearly, Ampère's law in its usual form does not work here. This may not be surprising, because Ampère's law as applied in earlier chapters required a steady current, whereas the current in this experiment is changing with time and is not steady at all.

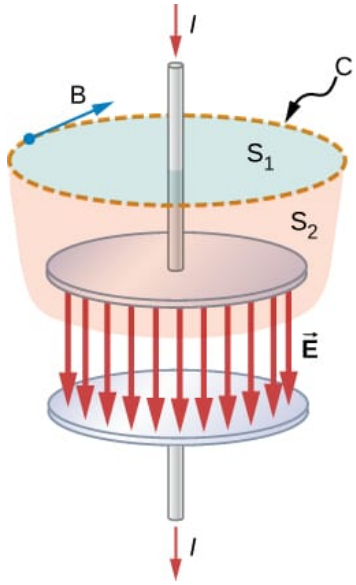


Figure 19.2.2: The currents through surface S_1 and surface S_2 are unequal, despite having the same boundary loop **C**.

How can Ampère's law be modified so that it works in all situations? Maxwell suggested including an additional contribution, called the displacement current I_d, to the real current **I**,

$$\boxed{\oint_S \vec{B} \cdot d\vec{s} = \mu_0 (I + I_d)} \tag{19.2.2}$$

where the displacement current is defined to be

$$\boxed{I_d = \epsilon_0 \frac{d\Phi_E}{dt}.} \tag{19.2.3}$$

Here ϵ_0 is the **permittivity of free space** and Φ_E is the electric flux, defined as

$$\Phi_E = \iint_{Surface\ S} \vec{E} \cdot d\vec{A}.$$

The **displacement current** is analogous to a real current in Ampère's law, entering into Ampère's law in the same way. It is produced, however, by a changing electric field. It accounts for a changing electric field producing a magnetic field, just as a real current does, but the displacement current can produce a magnetic field even where no real current is present. When this extra term is included, the modified Ampère's law equation becomes

$$\oint_C \vec{B} \cdot d\vec{s} = \mu_0 I + \epsilon_0 \mu_0 \frac{d\Phi_E}{dt}$$

and is independent of the surface **S** through which the current **I** is measured.

We can now examine this modified version of Ampère's law to confirm that it holds independent of whether the surface S_1 or the surface S_2 in Figure 19.2.2 is chosen. The electric field $\vec{E}$ corresponding to the flux Φ_E in Equation 19.2.3 is between the capacitor plates. Therefore, the $\vec{E}$ field and the displacement current through the surface S_1 are both zero, and Equation 19.2.2 takes the form

$$\oint_C \vec{B} \cdot d\vec{s} = \mu_0 I. \tag{19.2.4}$$

We must now show that for surface S_2, through which no actual current flows, the displacement current leads to the same value $\mu_0 I$ for the right side of the Ampère's law equation. For surface S_2 the equation becomes

$$\oint_C \vec{B} \cdot d\vec{s} = \mu_0 \frac{d}{dt}\left[\epsilon_0 \iint_{Surface\ S_2} \vec{E} \cdot d\vec{A}\right].$$

Gauss's law for electric charge requires a closed surface and cannot ordinarily be applied to a surface like S_1 alone or S_2 alone. But the two surfaces S_1 and S_2 form a closed surface in Figure 19.2.2 and can be used in Gauss's law. Because the electric field is zero on S_1, the flux contribution through S_1 is zero. This gives us

$$\oint_{Surface\ S_1+S_2} \vec{E} \cdot d\vec{A} = \iint_{Surface\ S_1} \vec{E} \cdot d\vec{A} + \iint_{Surface\ S_2} \vec{E} \cdot d\vec{A} \tag{19.2.5}$$

$$= 0 + \iint_{Surface\ S_2} \vec{E} \cdot d\vec{A} \tag{19.2.6}$$

$$= \iint_{Surface\ S_2} \vec{E} \cdot d\vec{A}. \tag{19.2.7}$$

Therefore, we can replace the integral over S_2 in Equation 19.2.4 with the closed Gaussian surface $S_1 + S_2$ and apply Gauss's law to obtain

$$\oint_{S_1} \vec{B} \cdot d\vec{s} = \mu_0 \frac{dQ_{in}}{dt} = \mu_0 I.$$

Thus, the modified Ampère's law equation is the same using surface S_2, where the right-hand side results from the displacement current, as it is for the surface S_1, where the contribution comes from the actual flow of electric charge.

Displacement current in a charging capacitor

A parallel-plate capacitor with capacitance **C** whose plates have area **A** and separation distance **d** is connected to a resistor **R** and a battery of voltage **V**. The current starts to flow at $t = 0$.

a. Find the displacement current between the capacitor plates at time **t**.
b. From the properties of the capacitor, find the corresponding real current $I = \frac{dQ}{dt}$, and compare the answer to the expected current in the wires of the corresponding **RC** circuit.

Strategy

We can use the equations from the analysis of an **RC** circuit (Alternating-Current Circuits) plus Maxwell's version of Ampère's law.

Solution

a. The voltage between the plates at time **t** is given by

$$V_C = \frac{1}{C}Q(t) = V_0\left(1 - e^{-t/RC}\right).$$

Let the **z**-axis point from the positive plate to the negative plate. Then the **z**-component of the electric field between the plates as a function of time **t** is

$$E_z(t) = \frac{V_0}{d}\left(1 - e^{-t/RC}\right).$$

Therefore, the z-component of the displacement current I_d between the plates is

$$I_d(t) = \epsilon_0 A \frac{\partial E_z(t)}{\partial t} = \epsilon_0 A \frac{V_0}{d} \times \frac{1}{RC} e^{-t/RC} = \frac{V_0}{R} e^{-t/RC},$$

where we have used $C = \epsilon_0 \frac{A}{d}$ for the capacitance.

b. From the expression for V_C the charge on the capacitor is

$$Q(t) = CV_C = CV_0\left(1 - e^{-t/RC}\right).$$

The current into the capacitor after the circuit is closed, is therefore

$$I = \frac{dQ}{dt} = \frac{V_0}{R} e^{-t/RC}.$$

This current is the same as I_d found in (a).

Maxwell's Equations

With the correction for the displacement current, Maxwell's equations take the form

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\epsilon_0} \text{ (Gauss's law)} \tag{19.2.8}$$

$$\oint \vec{B} \cdot d\vec{A} = 0 \text{ (Gauss's law for magnetism)} \tag{19.2.9}$$

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_m}{dt} \text{ (Faraday's law)} \tag{19.2.10}$$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I + \epsilon_0 I + \epsilon_0 \mu_0 \frac{d\Phi_E}{dt} \text{(Ampere-Maxwell law).} \tag{19.2.11}$$

Once the fields have been calculated using these four equations, the **Lorentz force equation**

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

gives the force that the fields exert on a particle with charge **q** moving with velocity $\vec{v}$. The Lorentz force equation combines the force of the electric field and of the magnetic field on the moving charge. The magnetic and electric forces have been examined in earlier modules. These four Maxwell's equations are, respectively:

Maxwell's Equations

1. **Gauss's law**

The electric flux through any closed surface is equal to the electric charge Q_{in} enclosed by the surface. Gauss's law (Equation 19.2.8) describes the relation between an electric charge and the electric field it produces. This is often pictured in terms of electric field lines originating from positive charges and terminating on negative charges, and indicating the direction of the electric field at each point in space.

2. **Gauss's law for magnetism**

The magnetic field flux through any closed surface is zero (Equation 19.2.9). This is equivalent to the statement that magnetic field lines are continuous, having no beginning or end. Any magnetic field line entering the region enclosed by the surface must also leave it. No magnetic monopoles, where magnetic field lines would terminate, are known to exist (see section on Magnetic Fields and Lines).

3. **Faraday's law**

A changing magnetic field induces an electromotive force (emf) and, hence, an electric field. The direction of the emf opposes the change. Equation 19.2.10 is Faraday's law of induction and includes Lenz's law. The electric field from a changing magnetic field has field lines that form closed loops, without any beginning or end.

4. **Ampère-Maxwell law**

Magnetic fields are generated by moving charges or by changing electric fields. This fourth of Maxwell's equations, Equation 19.2.11, encompasses Ampère's law and adds another source of magnetic fields, namely changing electric fields.

Maxwell's equations and the Lorentz force law together encompass all the laws of electricity and magnetism. The symmetry that Maxwell introduced into his mathematical framework may not be immediately apparent. Faraday's law describes how changing magnetic fields produce electric fields. The displacement current introduced by Maxwell results instead from a changing electric field and accounts for a changing electric field producing a magnetic field. The equations for the effects of both changing electric fields and changing magnetic fields differ in form only where the absence of magnetic monopoles leads to missing terms. This symmetry between the effects of changing magnetic and electric fields is essential in explaining the nature of electromagnetic waves.

Later application of Einstein's theory of relativity to Maxwell's complete and symmetric theory showed that electric and magnetic forces are not separate but are different manifestations of the same thing—the electromagnetic force. The electromagnetic force and weak nuclear force are similarly unified as the electroweak force. This unification of forces has been one motivation for attempts to unify all of the four basic forces in nature—the gravitational, electrical, strong, and weak nuclear forces (see Particle Physics and Cosmology).

The Mechanism of Electromagnetic Wave Propagation

To see how the symmetry introduced by Maxwell accounts for the existence of combined electric and magnetic waves that propagate through space, imagine a time-varying magnetic field $\vec{B}_0(t)$ produced by the high-frequency alternating current seen in Figure 19.2.3. We represent $\vec{B}_0(t)$ in the diagram by one of its field lines. From Faraday's law, the changing magnetic field through a surface induces a time-varying electric field $\vec{E}_0(t)$ at the boundary of that surface. The displacement current source for the electric field, like the Faraday's law source for the magnetic field, produces only closed loops of field lines, because of the mathematical symmetry involved in the equations for the induced electric and induced magnetic fields. A field line representation of $\vec{E}_0(t)$ is shown. In turn, the changing electric field $\vec{E}_0(t)$ creates a magnetic field $\vec{B}_1(t)$ according to the modified Ampère's law. This changing field induces $\vec{E}_1(t)$ which induces $\vec{B}_2(t)$ and so on. We then have a self-continuing process that leads to the creation of time-varying electric and magnetic fields in regions farther and farther away from **O**. This process may be visualized as the propagation of an electromagnetic wave through space.

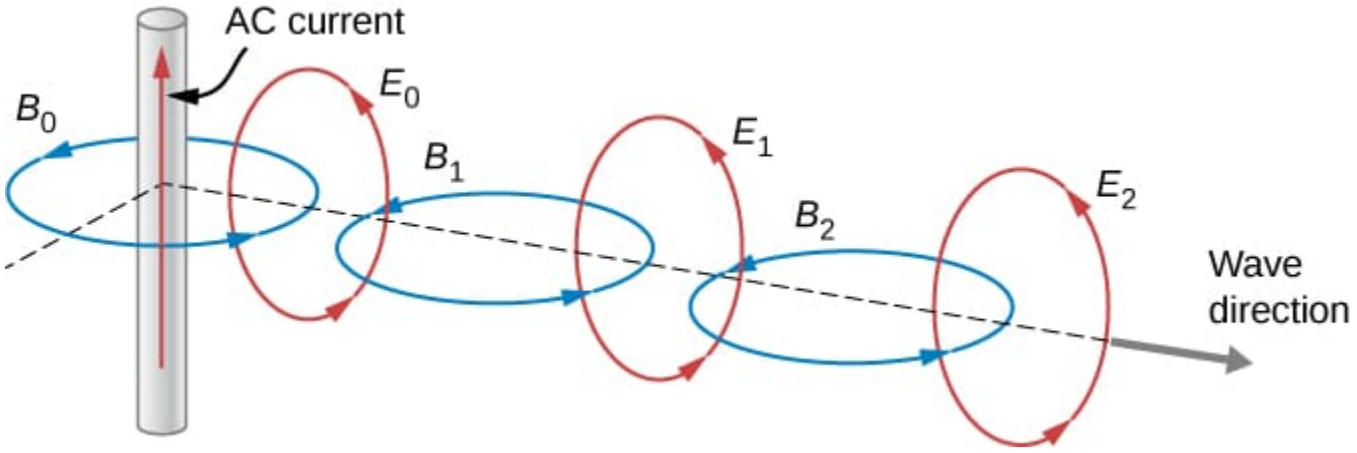


Figure 19.2.3: How changing $\vec{E}$ and $\vec{B}$ fields propagate through space.

In the next section, we show in more precise mathematical terms how Maxwell's equations lead to the prediction of electromagnetic waves that can travel through space without a material medium, implying a speed of electromagnetic waves

equal to the speed of light.

Prior to Maxwell's work, experiments had already indicated that light was a wave phenomenon, although the nature of the waves was yet unknown. In 1801, Thomas Young (1773–1829) showed that when a light beam was separated by two narrow slits and then recombined, a pattern made up of bright and dark fringes was formed on a screen. Young explained this behavior by assuming that light was composed of waves that added constructively at some points and destructively at others (see Interference). Subsequently, Jean Foucault (1819–1868), with measurements of the speed of light in various media, and Augustin Fresnel (1788–1827), with detailed experiments involving interference and diffraction of light, provided further conclusive evidence that light was a wave. So, light was known to be a wave, and Maxwell had predicted the existence of electromagnetic waves that traveled at the speed of light. The conclusion seemed inescapable: Light must be a form of electromagnetic radiation. But Maxwell's theory showed that other wavelengths and frequencies than those of light were possible for electromagnetic waves. He showed that electromagnetic radiation with the same fundamental properties as visible light should exist at any frequency. It remained for others to test, and confirm, this prediction.

Exercise 19.2.1

When the emf across a capacitor is turned on and the capacitor is allowed to charge, when does the magnetic field induced by the displacement current have the greatest magnitude?

Solution

It is greatest immediately after the current is switched on. The displacement current and the magnetic field from it are proportional to the rate of change of electric field between the plates, which is greatest when the plates first begin to charge.

Hertz's Observations

The German physicist Heinrich Hertz (1857–1894) was the first to generate and detect certain types of electromagnetic waves in the laboratory. Starting in 1887, he performed a series of experiments that not only confirmed the existence of electromagnetic waves but also verified that they travel at the speed of light.

Hertz used an alternating-current **RLC** (resistor-inductor-capacitor) circuit that resonates at a known frequency $f_0 = \frac{1}{2\pi\sqrt{LC}}$ and connected it to a loop of wire, as shown in Figure 19.2.4. High voltages induced across the gap in the loop produced sparks that were visible evidence of the current in the circuit and helped generate electromagnetic waves.

Across the laboratory, Hertz placed another loop attached to another **RLC** circuit, which could be tuned (as the dial on a radio) to the same resonant frequency as the first and could thus be made to receive electromagnetic waves. This loop also had a gap across which sparks were generated, giving solid evidence that electromagnetic waves had been received.

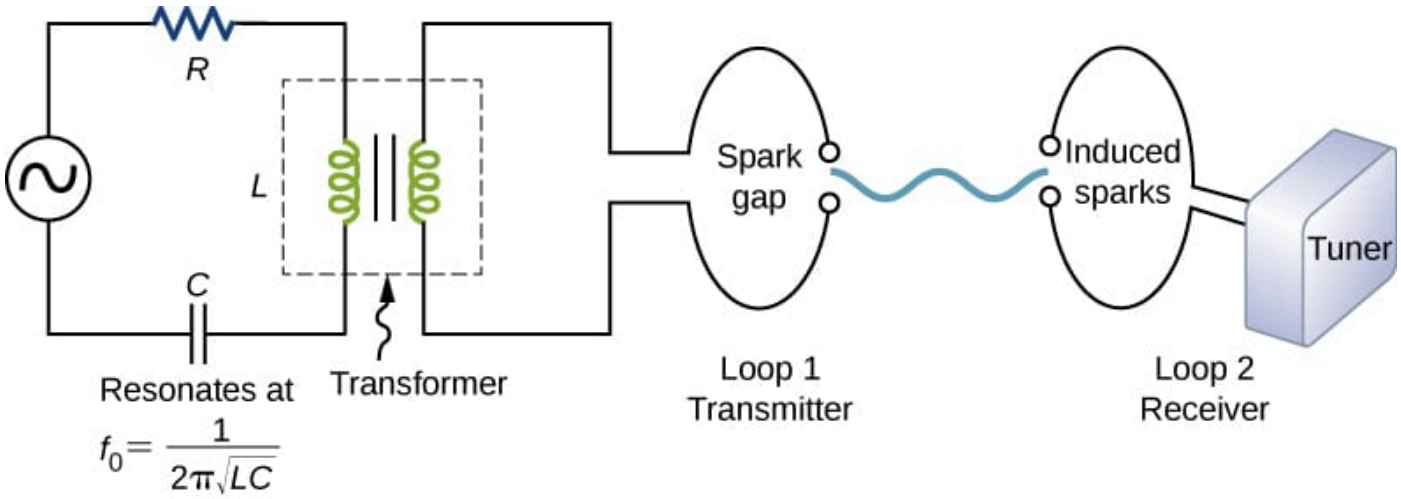


Figure 19.2.4: The apparatus used by Hertz in 1887 to generate and detect electromagnetic waves.

Hertz also studied the reflection, refraction, and interference patterns of the electromagnetic waves he generated, confirming their wave character. He was able to determine the wavelengths from the interference patterns, and knowing their frequencies, he could calculate the propagation speed using the equation $v = f\lambda$, where **v** is the speed of a wave, **f** is its frequency, and λ is its wavelength. Hertz was thus able to prove that electromagnetic waves travel at the speed of light. The SI unit for frequency, the hertz $(1\ Hz = 1\ cycle/second)$, is named in his honor.

Exercise 19.2.2

Could a purely electric field propagate as a wave through a vacuum without a magnetic field? Justify your answer.

Solution

No. The changing electric field according to the modified version of Ampère's law would necessarily induce a changing magnetic field.

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19.3: Plane Electromagnetic Waves

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Describe how Maxwell's equations predict the relative directions of the electric fields and magnetic fields, and the direction of propagation of plane electromagnetic waves
- Explain how Maxwell's equations predict that the speed of propagation of electromagnetic waves in free space is exactly the speed of light
- Calculate the relative magnitude of the electric and magnetic fields in an electromagnetic plane wave
- Describe how electromagnetic waves are produced and detected

Mechanical waves travel through a medium such as a string, water, or air. Perhaps the most significant prediction of Maxwell's equations is the existence of combined electric and magnetic (or electromagnetic) fields that propagate through space as electromagnetic waves. Because Maxwell's equations hold in free space, the predicted electromagnetic waves, unlike mechanical waves, do not require a medium for their propagation.

A general treatment of the physics of electromagnetic waves is beyond the scope of this textbook. We can, however, investigate the special case of an electromagnetic wave that propagates through free space along the **x**-axis of a given coordinate system.

Electromagnetic Waves in One Direction

An electromagnetic wave consists of an electric field, defined as usual in terms of the force per charge on a stationary charge, and a magnetic field, defined in terms of the force per charge on a moving charge. The electromagnetic field is assumed to be a function of only the **x**-coordinate and time. The **y**-component of the electric field is then written as $E_y(x,t)$, the **z**-component of the magnetic field as $B_z(x,t)$, etc. Because we are assuming free space, there are no free charges or currents, so we can set $Q_{in} = 0$ and $I = 0$ in Maxwell's equations.

The transverse nature of electromagnetic waves

We examine first what Gauss's law for electric fields implies about the relative directions of the electric field and the propagation direction in an electromagnetic wave. Assume the Gaussian surface to be the surface of a rectangular box whose cross-section is a square of side **l** and whose third side has length Δx, as shown in Figure 19.3.1. Because the electric field is a function only of **x** and **t**, the **y**-component of the electric field is the same on both the top (labeled Side 2) and bottom (labeled Side 1) of the box, so that these two contributions to the flux cancel. The corresponding argument also holds for the net flux from the **z**-component of the electric field through Sides 3 and 4. Any net flux through the surface therefore comes entirely from the **x**-component of the electric field. Because the electric field has no **y**- or **z**-dependence, $E_x(x,t)$ is constant over the face of the box with area **A** and has a possibly different value $E_x(x+\Delta x,t)$ that is constant over the opposite face of the box.

Applying Gauss's law gives

$$\text{Net flux} = -E_x(x,t)A + E_x(x+\Delta x,t)A = \frac{Q_{in}}{\epsilon_0} \tag{19.3.1}$$

where $A = l \times l$ is the area of the front and back faces of the rectangular surface. But the charge enclosed is $Q_{in} = 0$, so this component's net flux is also zero, and Equation 19.3.1 implies $E_x(x,t) = E_x(x+\Delta x,t)$ for any Δx. Therefore, if there is an **x**-component of the electric field, it cannot vary with **x**. A uniform field of that kind would merely be superposed artificially on the traveling wave, for example, by having a pair of parallel-charged plates. Such a component $E_x(x,t)$ would not be part of an electromagnetic wave propagating along the **x**-axis; so $E_x(x,t) = 0$ for this wave. Therefore, the only nonzero components of the electric field are $E_y(x,t)$ and $E_z(x,t)$ perpendicular to the direction of propagation of the wave.

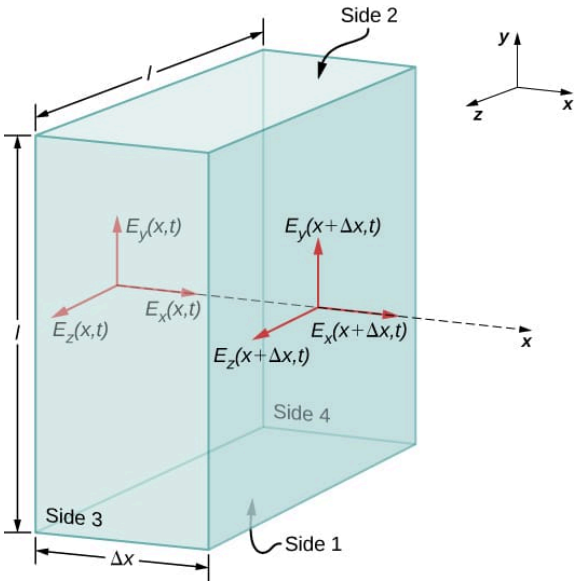


Figure 19.3.1: The surface of a rectangular box of dimensions $l \times l \times \Delta x$ is our Gaussian surface. The electric field shown is from an electromagnetic wave propagating along the x-axis.

A similar argument holds by substituting **E** for **B** and using Gauss's law for magnetism instead of Gauss's law for electric fields. This shows that the **B** field is also perpendicular to the direction of propagation of the wave. The electromagnetic wave is therefore a transverse wave, with its oscillating electric and magnetic fields perpendicular to its direction of propagation.

The speed of propagation of electromagnetic waves

We can next apply Maxwell's equations to the description given in connection with Figure 16.2.3 in the previous section to obtain an equation for the **E** field from the changing **B** field, and for the **B** field from a changing **E** field. We then combine the two equations to show how the changing **E** and **B** fields propagate through space at a speed precisely equal to the speed of light.

First, we apply Faraday's law over Side 3 of the Gaussian surface, using the path shown in Figure 19.3.2. Because $E_x(x,t) = 0$, we have

$$\oint \vec{E} \cdot d\vec{s} = -E_y(x,t)l + E_y(x+\Delta x,t)l.$$

Assuming Δx is small and approximating $E_y(x+\Delta x,t)$ by

$$E_y(x+\Delta x,t) = E_y(x,t) + \frac{\partial E_y(x,t)}{\partial x}\Delta x,$$

we obtain

$$\oint \vec{E} \cdot d\vec{s} = \frac{\partial E_y(x,t)}{\partial x}(l\Delta x).$$

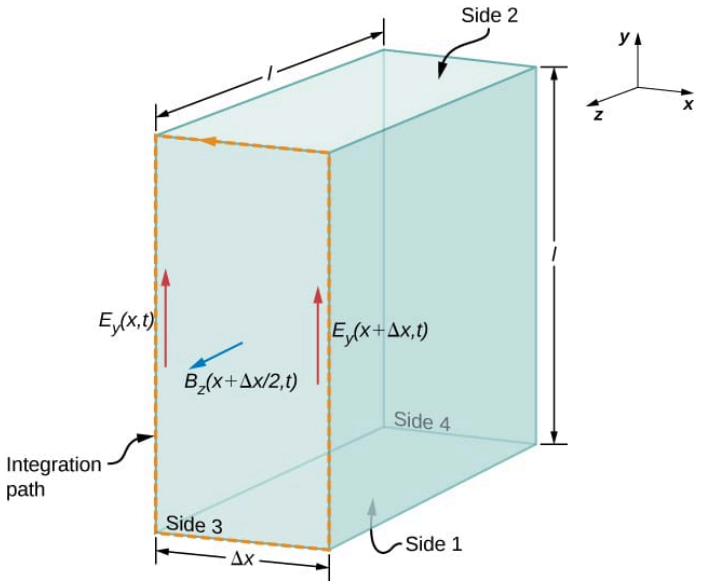


Figure 19.3.2: We apply Faraday's law to the front of the rectangle by evaluating $\oint \vec{E} \cdot d\vec{s}$ along the rectangular edge of Side 3 in the direction indicated, taking the B field crossing the face to be approximately its value in the middle of the area traversed.

Because Δx is small, the magnetic flux through the face can be approximated by its value in the center of the area traversed, namely $B_z\left(x+\frac{\Delta x}{2},t\right)$. The flux of the **B** field through Face 3 is then the **B** field times the area,

$$\oint_S \vec{B} \cdot \vec{n} dA = B_z\left(x+\frac{\Delta x}{2},t\right)(l\Delta x). \tag{19.3.2}$$

From Faraday's law,

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d}{dt}\int_S \vec{B} \cdot \vec{n} dA. \tag{19.3.3}$$

Therefore, from Equations 19.3.1 and 19.3.2,

$$\frac{\partial E_y(x,t)}{\partial x}(l\Delta x) = -\frac{\partial}{\partial t}\left[B_z\left(x+\frac{\Delta x}{2},t\right)\right](l\Delta x).$$

Canceling $l\Delta x$ and taking the limit as $\Delta x = 0$, we are left with

$$\frac{\partial E_y(x,t)}{\partial x} = -\frac{\partial B_z(x,t)}{\partial t}. \tag{19.3.4}$$

We could have applied Faraday's law instead to the top surface (numbered 2) in Figure 19.3.2, to obtain the resulting equation

$$\frac{\partial B_z(x,t)}{\partial t} = -\frac{\partial E_y(x,t)}{\partial x}. \tag{19.3.5}$$

This is the equation describing the spatially dependent **E** field produced by the time-dependent **B** field.

Next we apply the Ampère-Maxwell law (with $I = 0$) over the same two faces (Surface 3 and then Surface 2) of the rectangular box of Figure 19.3.2. Applying Equation 16.2.16,

$$\oint \vec{B} \cdot d\vec{s} = \mu_0\epsilon_0(d/dt)\int_S \vec{E} \cdot n\, da$$

to Surface 3, and then to Surface 2, yields the two equations

$$\frac{\partial E_y(x,t)}{\partial x} = -\epsilon_0\mu_0\frac{\partial E_z(x,t)}{\partial t}, \tag{19.3.6}$$

and

$$\frac{\partial B_z(x,t)}{\partial x} = -\epsilon_0\mu_0\frac{\partial E_y(x,t)}{\partial t}. \tag{19.3.7}$$

These equations describe the spatially dependent **B** field produced by the time-dependent **E** field.

We next combine the equations showing the changing **B** field producing an **E** field with the equation showing the changing **E** field producing a **B** field. Taking the derivative of Equation 19.3.4 with respect to **x** and using Equation 19.3.13 gives

$$\frac{\partial^2 E_y}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial E_y}{\partial x}\right) = -\frac{\partial}{\partial x}\left(\frac{\partial B_z}{\partial t}\right) = -\frac{\partial}{\partial t}\left(\frac{\partial B_z}{\partial x}\right) = \frac{\partial}{\partial t}\left(\epsilon_0\mu_0\frac{\partial E_y}{\partial t}\right)$$

or

$$\frac{\partial^2 E_y}{\partial x^2} = \epsilon_0\mu_0\frac{\partial^2 E_y}{\partial t^2}$$

This is the form taken by the general wave equation for our plane wave. Because the equations describe a wave traveling at some as-yet-unspecified speed **c**, we can assume the field components are each functions of **x** – **ct** for the wave traveling in the +**x**-direction, that is,

$$E_y(x,t) = f(\xi)\, where\, \xi = x - ct. \tag{19.3.8}$$

It is left as a mathematical exercise to show, using the chain rule for differentiation, that Equations 19.3.5 and 19.3.6 imply

$$1 = \epsilon_0\mu_0 c^2.$$

The speed of the electromagnetic wave in free space is therefore given in terms of the permeability and the permittivity of free space by

$$c = \frac{1}{\sqrt{\epsilon_0\mu_0}}. \tag{19.3.9}$$

We could just as easily have assumed an electromagnetic wave with field components $E_z(x,t)$ and $B_y(x,t)$. The same type of analysis with Equation 19.3.12 and 19.3.11 would also show that the speed of an electromagnetic wave is $c = 1/\sqrt{\epsilon_0\mu_0}$.

The physics of traveling electromagnetic fields was worked out by Maxwell in 1873. He showed in a more general way than our derivation that electromagnetic waves always travel in free space with a speed given by Equation 19.3.6. If we evaluate the speed $c = \frac{1}{\sqrt{\epsilon_0\mu_0}}$, we find that

$$c = \frac{1}{\sqrt{\left(8.85\times10^{-12}\frac{C^2}{N\cdot m^2}\right)\left(4\pi\times10^{-7}\frac{T\cdot m}{A}\right)}} = 3.00\times10^8 m/s,$$

which is the speed of light. Imagine the excitement that Maxwell must have felt when he discovered this equation! He had found a fundamental connection between two seemingly unrelated phenomena: electromagnetic fields and light.

Exercise 19.3.1

The wave equation was obtained by (1) finding the **E** field produced by the changing **B** field, (2) finding the **B** field produced by the changing **E** field, and combining the two results. Which of Maxwell's equations was the basis of step (1) and which of step (2)?

Answer (step 1) available at the end of this section.

How the E and B Fields Are Related

So far, we have seen that the rates of change of different components of the **E** and **B** fields are related, that the electromagnetic wave is transverse, and that the wave propagates at speed **c**. We next show what Maxwell's equations imply about the ratio of the **E** and **B** field magnitudes and the relative directions of the **E** and **B** fields.

We now consider solutions to Equation 19.3.4 in the form of plane waves for the electric field:

$$E_y(x,t) = E_0 \cos(kx - \omega t). \tag{19.3.10}$$

We have arbitrarily taken the wave to be traveling in the **+x**-direction and chosen its phase so that the maximum field strength occurs at the origin at time $t = 0$. We are justified in considering only sines and cosines in this way, and generalizing the results, because Fourier's theorem implies we can express any wave, including even square step functions, as a superposition of sines and cosines.

At any one specific point in space, the **E** field oscillates sinusoidally at angular frequency ω between $+E_0$ and $-E_0$ and similarly, the **B** field oscillates between $+B_0$ and $-B_0$. The amplitude of the wave is the maximum value of $E_y(x,t)$. The period of oscillation **T** is the time required for a complete oscillation. The frequency **f** is the number of complete oscillations per unit of time, and is related to the angular frequency ω by $\omega = 2\pi f$. The wavelength λ is the distance covered by one complete cycle of the wave, and the wavenumber **k** is the number of wavelengths that fit into a distance of 2π in the units being used. These quantities are related in the same way as for a mechanical wave:

$$\omega = 2\pi f,\ f = \frac{1}{T},\ k = \frac{2\pi}{\lambda},\ and\ c = f\lambda = \omega/k.$$

Given that the solution of E_y has the form shown in Equation ???, we need to determine the B field that accompanies it. From Equation 19.3.11, the magnetic field component B_z must obey

$$\frac{\partial B_z}{\partial t} = -\frac{\partial E_y}{\partial x}$$

$$\frac{\partial B_z}{\partial t} = -\frac{\partial}{\partial x} E_0 \cos(kx - \omega t) = kE_0\, sin\,(kx - \omega t). \tag{19.3.11}$$

Because the solution for the **B**-field pattern of the wave propagates in the **+x**-direction at the same speed **c** as the **E**-field pattern, it must be a function of $k(x - ct) = kx - \omega t$. Thus, we conclude from Equation 19.3.8 that B_z is

$$B_z(x,t) = \frac{k}{\omega} E_0 \cos(kx - \omega t) = \frac{1}{c} E_0 \cos(kx - \omega t).$$

These results may be written as

$$E_y(x,t) = E_0 \cos(kx - \omega t)$$

$$B_z(x,t) = B_0 \cos(kx - \omega t) \tag{19.3.12}$$

$$\frac{E_y}{B_z} = \frac{E_0}{B_0} = c. \tag{19.3.13}$$

Therefore, the peaks of the **E** and **B** fields coincide, as do the troughs of the wave, and at each point, the **E** and **B** fields are in the same ratio equal to the speed of light **c**. The plane wave has the form shown in Figure 19.3.3.

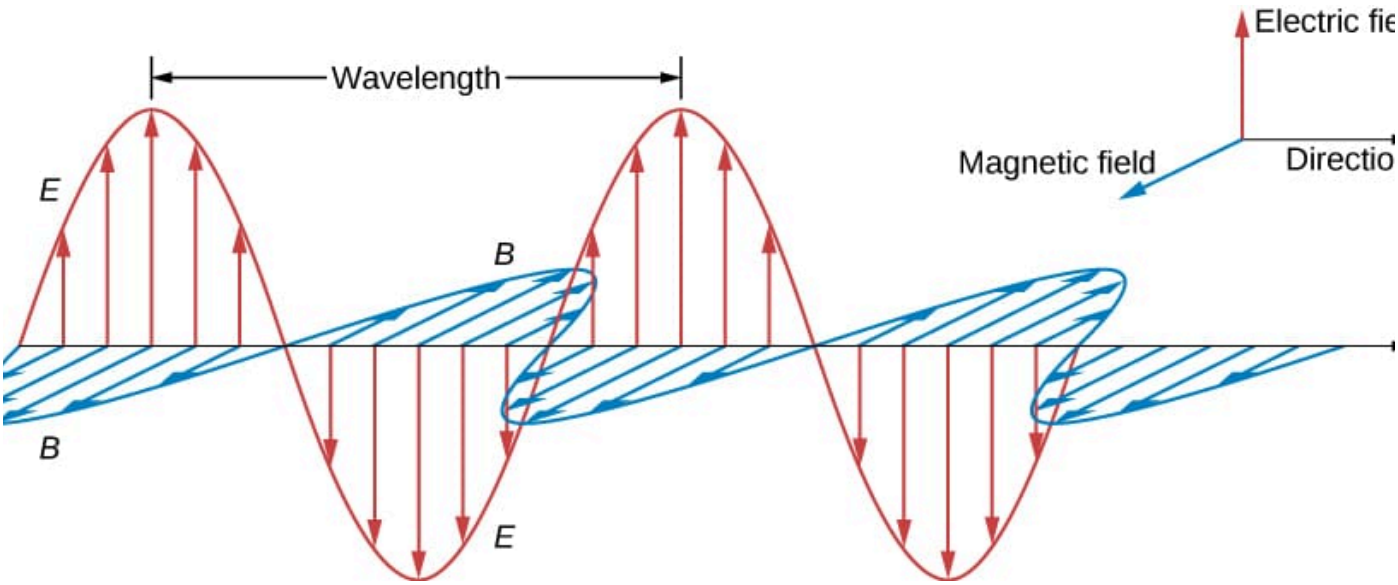


Figure 19.3.3: The plane wave solution of Maxwell's equations has the B field directly proportional to the E field at each point, with the relative directions shown.

Example 19.3.1: Calculating B-Field Strength in an Electromagnetic Wave

What is the maximum strength of the **B** field in an electromagnetic wave that has a maximum **E**-field strength of 1000 V/m?

Strategy

To find the **B**-field strength, we rearrange Equation 19.3.10 to solve for B, yielding

$$B = \frac{E}{c}.$$

Solution We are given **E**, and **c** is the speed of light. Entering these into the expression for **B** yields

$$B = \frac{1000\,V/m}{3.00 \times 10^8\,m/s} = 3.33 \times 10^{-6} T.$$

Significance

The **B**-field strength is less than a tenth of Earth's admittedly weak magnetic field. This means that a relatively strong electric field of 1000 V/m is accompanied by a relatively weak magnetic field.

Changing electric fields create relatively weak magnetic fields. The combined electric and magnetic fields can be detected in electromagnetic waves, however, by taking advantage of the phenomenon of resonance, as Hertz did. A system with the same natural frequency as the electromagnetic wave can be made to oscillate. All radio and TV receivers use this principle to pick up and then amplify weak electromagnetic waves, while rejecting all others not at their resonant frequency.

Exercise 19.3.2

What conclusions did our analysis of Maxwell's equations lead to about these properties of a plane electromagnetic wave:

a. the relative directions of wave propagation, of the **E** field, and of **B** field,
b. the speed of travel of the wave and how the speed depends on frequency, and
c. the relative magnitudes of the **E** and **B** fields.

Answer a available at the end of this section.

Production and Detection of Electromagnetic Waves

A steady electric current produces a magnetic field that is constant in time and which does not propagate as a wave. Accelerating charges, however, produce electromagnetic waves. An electric charge oscillating up and down, or an alternating current or flow of charge in a conductor, emit radiation at the frequencies of their oscillations. The electromagnetic field of a **dipole antenna** is shown in Figure 19.3.4. The positive and negative charges on the two conductors are made to reverse at the desired frequency by the output of a transmitter as the power source. The continually changing current accelerates charge in the antenna, and this results in an oscillating electric field a distance away from the antenna. The changing electric fields produce changing magnetic fields that in turn produce changing electric fields, which thereby propagate as electromagnetic waves. The frequency of this radiation is the same as the frequency of the ac source that is accelerating the electrons in the antenna. The two conducting elements of the dipole antenna are commonly straight wires. The total length of the two wires is typically about one-half of the desired wavelength (hence, the alternative name **half-wave antenna**), because this allows standing waves to be set up and enhances the effectiveness of the radiation.

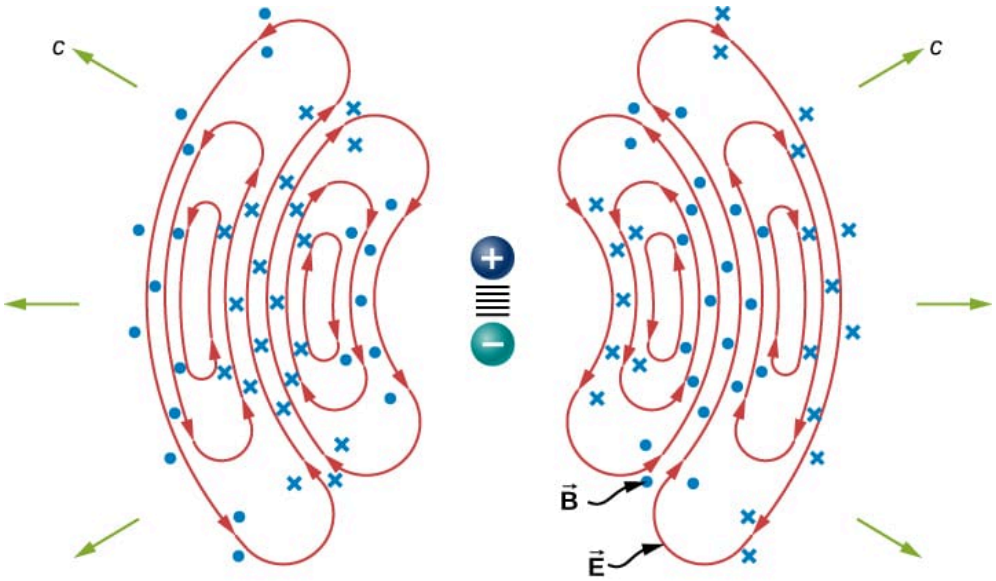


Figure 19.3.4: The oscillatory motion of the charges in a dipole antenna produces electromagnetic radiation.

The electric field lines in one plane are shown. The magnetic field is perpendicular to this plane. This radiation field has cylindrical symmetry around the axis of the dipole. Field lines near the dipole are not shown. The pattern is not at all uniform in all directions. The strongest signal is in directions perpendicular to the axis of the antenna, which would be horizontal if the antenna is mounted vertically. There is zero intensity along the axis of the antenna. The fields detected far from the antenna are from the changing electric and magnetic fields inducing each other and traveling as electromagnetic waves. Far from the antenna, the wave fronts, or surfaces of equal phase for the electromagnetic wave, are almost spherical. Even farther from the antenna, the radiation propagates like electromagnetic plane waves.

The electromagnetic waves carry energy away from their source, similar to a sound wave carrying energy away from a standing wave on a guitar string. An antenna for receiving electromagnetic signals works in reverse. Incoming electromagnetic waves induce oscillating currents in the antenna, each at its own frequency. The radio receiver includes a tuner circuit, whose resonant frequency can be adjusted. The tuner responds strongly to tshe desired frequency but not others, allowing the user to tune to the desired broadcast. Electrical components amplify the signal formed by the moving electrons. The signal is then converted into an audio and/or video format.

Note

Use this simulation to broadcast radio waves. Wiggle the transmitter electron manually or have it oscillate automatically. Display the field as a curve or vectors. The strip chart shows the electron positions at the transmitter and at the receiver.

Answers

Exercise 19.3.1

Faraday's law

the Ampère-Maxwell law

Exercise 19.3.2

The directions of wave propagation, of the **E** field, and of **B** field are all mutually perpendicular.

The speed of the electromagnetic wave is the speed of light $c = 1/\sqrt{\epsilon_0 \mu_0}$ independent of frequency.

The ratio of electric and magnetic field amplitudes is $E/B = c$.

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19.4: Energy Carried by Electromagnetic Waves

Learning Objectives

By the end of this section, you will be able to:

- Express the time-averaged energy density of electromagnetic waves in terms of their electric and magnetic field amplitudes
- Calculate the Poynting vector and the energy intensity of electromagnetic waves
- Explain how the energy of an electromagnetic wave depends on its amplitude, whereas the energy of a photon is proportional to its frequency

Anyone who has used a microwave oven knows there is energy in electromagnetic waves. Sometimes this energy is obvious, such as in the warmth of the summer Sun. Other times, it is subtle, such as the unfelt energy of gamma rays, which can destroy living cells.

Electromagnetic waves bring energy into a system by virtue of their electric and magnetic fields. These fields can exert forces and move charges in the system and, thus, do work on them. However, there is energy in an electromagnetic wave itself, whether it is absorbed or not. Once created, the fields carry energy away from a source. If some energy is later absorbed, the field strengths are diminished and anything left travels on.

Clearly, the larger the strength of the electric and magnetic fields, the more work they can do and the greater the energy the electromagnetic wave carries. In electromagnetic waves, the amplitude is the maximum field strength of the electric and magnetic fields (Figure 19.4.1). The wave energy is determined by the wave amplitude.

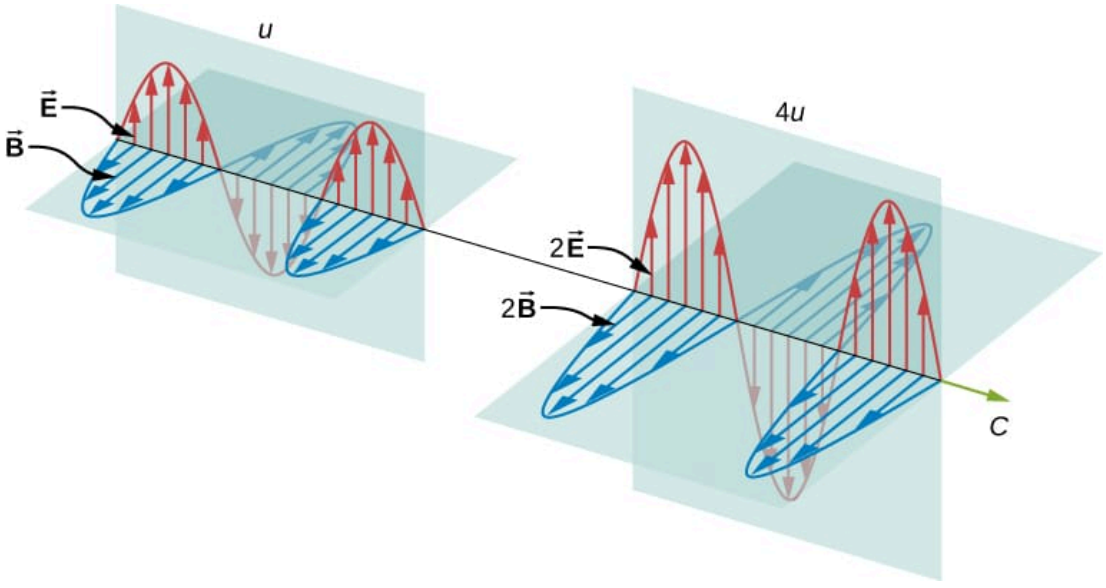


Figure 19.4.1: Energy carried by a wave depends on its amplitude. With electromagnetic waves, doubling the **E** fields and **B** fields quadruples the energy density **u** and the energy flux **uc**.

For a plane wave traveling in the direction of the positive **x**-axis with the phase of the wave chosen so that the wave maximum is at the origin at $t = 0$, the electric and magnetic fields obey the equations

$$E_y(x,t) = E_0 \cos(kx - \omega t)$$

$$B_x(x,t) = B_0 \cos(kx - \omega t).$$

The energy in any part of the electromagnetic wave is the sum of the energies of the electric and magnetic fields. This energy per

unit volume, or energy density **u**, is the sum of the energy density from the electric field and the energy density from the magnetic field. Expressions for both field energy densities were discussed earlier (u_E in Capacitance and u_B in Inductance). Combining these the contributions, we obtain

$$u(x,t) = u_E + u_B = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0}B^2.$$

The expression $E = cB = \frac{1}{\sqrt{\epsilon_0\mu_0}}B$ then shows that the magnetic energy density u_B and electric energy density u_E are equal, despite the fact that changing electric fields generally produce only small magnetic fields. The equality of the electric and magnetic energy densities leads to

$$u(x,t) = \epsilon_0 E^2 = \frac{B^2}{\mu_0}. \tag{19.4.1}$$

The energy density moves with the electric and magnetic fields in a similar manner to the waves themselves.

We can find the rate of transport of energy by considering a small time interval Δt. As shown in Figure 19.4.2, the energy contained in a cylinder of length $c\Delta t$ and cross-sectional area **A** passes through the cross-sectional plane in the interval Δt.

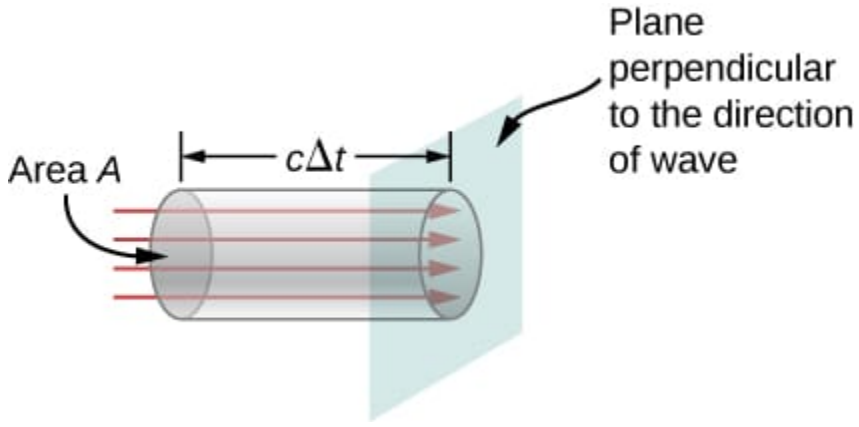


Figure 19.4.2: The energy $uAc\Delta t$ contained in the electric and magnetic fields of the electromagnetic wave in the volume $Ac\Delta t$ passes through the area A in time Δt.

The energy passing through area A in time Δt is

$$u \times volume = uAc\Delta t.$$

The energy per unit area per unit time passing through a plane perpendicular to the wave, called the **energy flux** and denoted by S, can be calculated by dividing the energy by the area A and the time interval Δt.

$$S = \frac{\text{Energy passing area } A \text{ in time } \Delta t}{A\Delta t} = uc = \epsilon_0 cE^2 = \frac{1}{\mu_0}EB.$$

More generally, the flux of energy through any surface also depends on the orientation of the surface. To take the direction into account, we introduce a vector $\vec{S}$, called the **Poynting vector**, with the following definition:

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}.$$

The cross-product of $\vec{E}$ and $\vec{B}$ points in the direction perpendicular to both vectors. To confirm that the direction of $\vec{S}$ is that of wave propagation, and not its negative, return to Figure 16.3.2. Note that Lenz's and Faraday's laws imply that when the magnetic field shown is increasing in time, the electric field is greater at x than at $x + \Delta x$. The electric field is decreasing with increasing x at the given time and location. The proportionality between electric and magnetic fields requires the electric field to increase in time along with the magnetic field. This is possible only if the wave is propagating to the right in the diagram, in which case, the relative orientations show that $\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$ is specifically in the direction of propagation of the electromagnetic wave.

The energy flux at any place also varies in time, as can be seen by substituting u from Equation 16.3.19 into Equation 19.4.1.

$$S(x,t) = c\epsilon_0 E_0^2 \cos^2(kx - \omega t) \tag{19.4.2}$$

Because the frequency of visible light is very high, of the order of $10^{14}\ Hz$, the energy flux for visible light through any area is an extremely rapidly varying quantity. Most measuring devices, including our eyes, detect only an average over many cycles. The time average of the energy flux is the **intensity** I of the electromagnetic wave and is the power per unit area. It can be expressed by averaging the cosine function in Equation 19.4.2 over one complete cycle, which is the same as time-averaging over many cycles (here, T is one period):

$$I = S_{avg} = c\epsilon_0 E_0^2 \frac{1}{T}\int_0^T \cos^2\left(2\pi\frac{t}{T}\right) dt. \tag{19.4.3}$$

We can either evaluate the integral, or else note that because the sine and cosine differ merely in phase, the average over a complete cycle for $cos^2\ (\xi)$ is the same as for $sin^2\ (\xi)$, to obtain

$$\langle\cos^2\xi\rangle = \frac{1}{2}[\langle\cos^2\xi\rangle + \langle\sin^2\xi\rangle] = \frac{1}{2}\langle 1\rangle = \frac{1}{2}.$$

where the angle brackets $\langle ...\rangle$ stand for the time-averaging operation. The intensity of light moving at speed c in vacuum is then found to be

$$I = S_{avg} = \frac{1}{2}c\epsilon_0 E_0^2 \tag{19.4.4}$$

in terms of the maximum electric field strength E_0, which is also the electric field amplitude. Algebraic manipulation produces the relationship

$$I = \frac{cB_0^2}{2\mu_0} \tag{19.4.5}$$

where B_0 is the magnetic field amplitude, which is the same as the maximum magnetic field strength. One more expression for I_{avg} in terms of both electric and magnetic field strengths is useful. Substituting the fact that $cB_0 = E_0$, the previous expression becomes

$$I = \frac{E_0 B_0}{2\mu_0}. \tag{19.4.6}$$

We can use whichever of the three preceding equations is most convenient, because the three equations are really just different versions of the same result: The energy in a wave is related to amplitude squared. Furthermore, because these equations are based on the assumption that the electromagnetic waves are sinusoidal, the peak intensity is twice the average intensity; that is, $I_0 = 2I$.

Example 19.4.1: A Laser Beam

The beam from a small laboratory laser typically has an intensity of about $1.0 \times 10^{-3} W/m^2$. Assuming that the beam is composed of plane waves, calculate the amplitudes of the electric and magnetic fields in the beam.

Strategy

Use the equation expressing intensity in terms of electric field to calculate the electric field from the intensity.

Solution

From Equation 19.4.4, the intensity of the laser beam is

$$I = \frac{1}{2}c\epsilon_0 E_0^2.$$

The amplitude of the electric field is therefore

$$E_0 = \sqrt{\frac{2}{c\epsilon_0} I}$$
$$= \sqrt{\frac{2}{(3.00 \times 10^8 m/s)(8.85 \times 10^{-12} F/m)} (1.0 \times 10^{-3} W/m^2)}$$
$$= 0.87\, V/m.$$

The amplitude of the magnetic field can be obtained from:

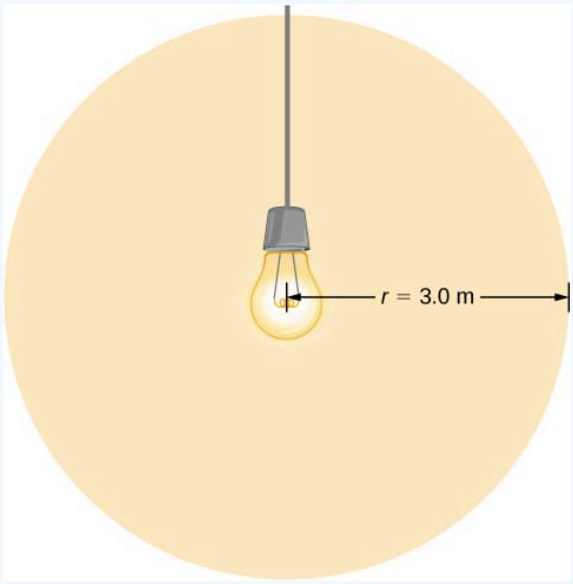
$$B_0 = \frac{E_0}{c} = 2.9 \times 10^{-9}\, T.$$

Light Bulb Fields

A light bulb emits 5.00 W of power as visible light. What are the average electric and magnetic fields from the light at a distance of 3.0 m?

Strategy

Assume the bulb's power output **P** is distributed uniformly over a sphere of radius 3.0 m to calculate the intensity, and from it, the electric field.



Solution

The power radiated as visible light is then

$I = \frac{P}{4\pi r^2} = \frac{c\epsilon_0 E_0^2}{2}$,

$E_0 = \sqrt{2\frac{P}{4\pi r^2 c\epsilon_0}} = \sqrt{2\frac{5.00\ W}{4\pi(3.0\ m)^2(3.00\times10^8\ m/s)(8.85\times10^{-12}C^2/N\cdot m^2)}} = 5.77\ N/C$,

$B_0 = E_0/c = 1.92 \times 10^{-8}\ T$.

Significance

The intensity **I** falls off as the distance squared if the radiation is dispersed uniformly in all directions.

Radio Range

A 60-kW radio transmitter on Earth sends its signal to a satellite 100 km away (Figure 19.4.3). At what distance in the same direction would the signal have the same maximum field strength if the transmitter's output power were increased to 90 kW?

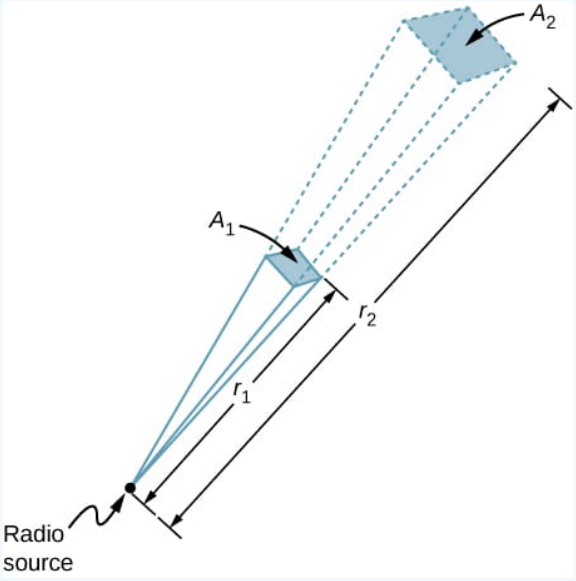


Figure 19.4.3: In three dimensions, a signal spreads over a solid angle as it travels outward from its source.

Strategy

The area over which the power in a particular direction is dispersed increases as distance squared, as illustrated in Figure 19.4.3. Change the power output **P** by a factor of (90 kW/60 kW) and change the area by the same factor to keep $I = \frac{P}{A} = \frac{c\epsilon_0 E_0^2}{2}$ the same. Then use the proportion of area **A** in the diagram to distance squared to find the distance that produces the calculated change in area.

Solution

Using the proportionality of the areas to the squares of the distances, and solving, we obtain from the diagram

$$\frac{r_2^2}{r_1^2} = \frac{A_2}{A_1} = \frac{90\,W}{60\,W},$$

$$r_2 = \sqrt{\frac{90}{60}}(100\,km)$$

$$= 122\,km.$$

Significance

The range of a radio signal is the maximum distance between the transmitter and receiver that allows for normal operation. In the absence of complications such as reflections from obstacles, the intensity follows an inverse square law, and doubling the range would require multiplying the power by four.

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19.5: Momentum and Radiation Pressure

Learning Objectives

By the end of this section, you will be able to:

- Describe the relationship of the radiation pressure and the energy density of an electromagnetic wave
- Explain how the radiation pressure of light, while small, can produce observable astronomical effects

Material objects consist of charged particles. An electromagnetic wave incident on the object exerts forces on the charged particles, in accordance with the Lorentz force. These forces do work on the particles of the object, increasing its energy, as discussed in the previous section. The energy that sunlight carries is a familiar part of every warm sunny day. A much less familiar feature of electromagnetic radiation is the extremely weak pressure that electromagnetic radiation produces by exerting a force in the direction of the wave. This force occurs because electromagnetic waves contain and transport momentum.

To understand the direction of the force for a very specific case, consider a plane electromagnetic wave incident on a metal in which electron motion, as part of a current, is damped by the resistance of the metal, so that the average electron motion is in phase with the force causing it. This is comparable to an object moving against friction and stopping as soon as the force pushing it stops (Figure 19.5.1). When the electric field is in the direction of the positive **y**-axis, electrons move in the negative **y**-direction, with the magnetic field in the direction of the positive **z**-axis. By applying the right-hand rule, and accounting for the negative charge of the electron, we can see that the force on the electron from the magnetic field is in the direction of the positive **x**-axis, which is the direction of wave propagation. When the $\vec{E}$ field reverses, the $\vec{B}$ field does too, and the force is again in the same direction. Maxwell's equations together with the Lorentz force equation imply the existence of radiation pressure much more generally than this specific example, however.

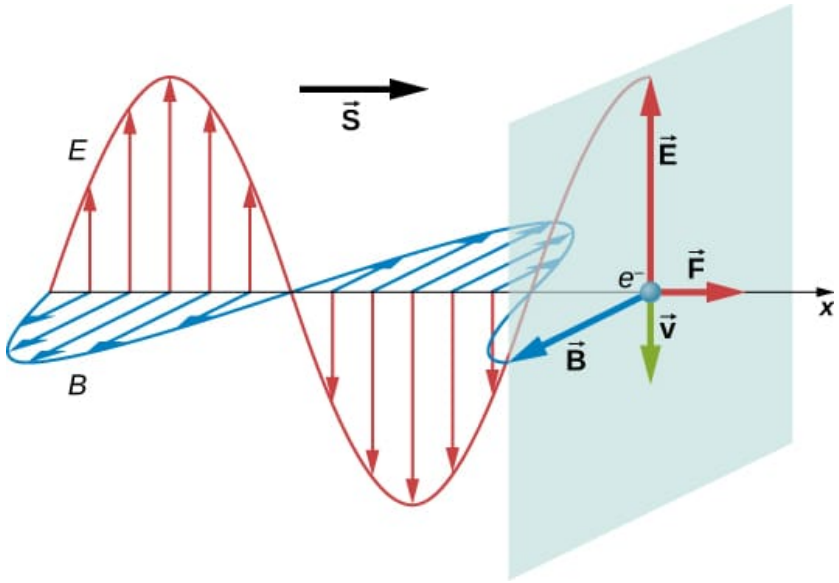


Figure 19.5.1: Electric and magnetic fields of an electromagnetic wave can combine to produce a force in the direction of propagation, as illustrated for the special case of electrons whose motion is highly damped by the resistance of a metal.

Maxwell predicted that an electromagnetic wave carries momentum. An object absorbing an electromagnetic wave would experience a force in the direction of propagation of the wave. The force corresponds to radiation pressure exerted on the object by the wave. The force would be twice as great if the radiation were reflected rather than absorbed.

Maxwell's prediction was confirmed in 1903 by Nichols and Hull by precisely measuring radiation pressures with a torsion balance. The schematic arrangement is shown in Figure 19.5.2. The mirrors suspended from a fiber were housed inside a glass container. Nichols and Hull were able to obtain a small measurable deflection of the mirrors from shining light on one of them. From the measured deflection, they could calculate the unbalanced force on the mirror, and obtained agreement with the predicted value

of the force.

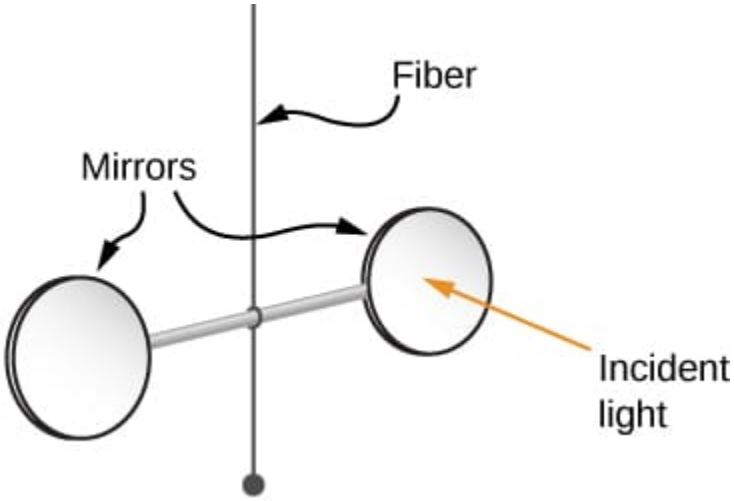


Figure 19.5.2: Simplified diagram of the central part of the apparatus Nichols and Hull used to precisely measure radiation pressure and confirm Maxwell's prediction.

The **radiation pressure** p_{rad} applied by an electromagnetic wave on a perfectly absorbing surface turns out to be equal to the energy density of the wave:

$$\underbrace{p_{rad} = u}_{\text{Perfect absorber}} \quad . \tag{19.5.1}$$

If the material is perfectly reflecting, such as a metal surface, and if the incidence is along the normal to the surface, then the pressure exerted is twice as much because the momentum direction reverses upon reflection:

$$\underbrace{p_{rad} = 2u}_{\text{Perfect reflector}} \quad . \tag{19.5.2}$$

We can confirm that the units are right:

$$[u] = \frac{J}{m^3} = \frac{N \cdot m}{m^3} = \frac{N}{m^2} = units\, of\, pressure.$$

Equations 19.5.1 and 19.5.2 give the **instantaneous** pressure, but because the energy density oscillates rapidly, we are usually interested in the time-averaged radiation pressure, which can be written in terms of intensity:

$$p = \langle p_{rad} \rangle = \begin{cases} I/c & \text{Perfect absorber} \\ 2I/c & \text{Perfect reflector} \end{cases} \tag{19.5.3}$$

Radiation pressure plays a role in explaining many observed astronomical phenomena, including the appearance of **comets**. Comets are basically chunks of icy material in which frozen gases and particles of rock and dust are embedded. When a comet approaches the Sun, it warms up and its surface begins to evaporate. The **coma** of the comet is the hazy area around it from the gases and dust. Some of the gases and dust form tails when they leave the comet. Notice in Figure 19.5.3 that a comet has **two** tails. The **ion tail** (or **gas tail**) is composed mainly of ionized gases. These ions interact electromagnetically with the solar wind, which is a continuous stream of charged particles emitted by the Sun. The force of the solar wind on the ionized gases is strong enough that the ion tail almost always points directly away from the Sun. The second tail is composed of dust particles. Because the **dust tail** is electrically neutral, it does not interact with the solar wind. However, this tail is affected by the radiation pressure produced by the light from the Sun. Although quite small, this pressure is strong enough to cause the dust tail to be displaced from the path of the comet.

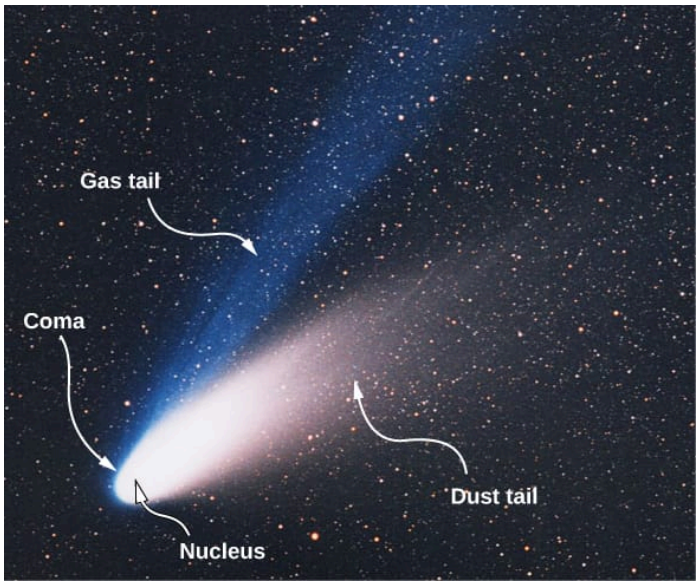


Figure 19.5.3: Evaporation of material being warmed by the Sun forms two tails, as shown in this photo of Comet Ison. (credit: modification of work by E. Slawik—ESO)

Example 19.5.1: Halley's Comet

On February 9, 1986, Comet Halley was at its closest point to the Sun, about $9.0 \times 10^{10} m$ from the center of the Sun. The average power output of the Sun is $3.8 \times 10^{26}\ W$.

a. Calculate the radiation pressure on the comet at this point in its orbit. Assume that the comet reflects all the incident light.
b. Suppose that a 10-kg chunk of material of cross-sectional area $4.0 \times 10^{-2} m^2$ breaks loose from the comet. Calculate the force on this chunk due to the solar radiation. Compare this force with the gravitational force of the Sun.

Strategy

Calculate the intensity of solar radiation at the given distance from the Sun and use that to calculate the radiation pressure. From the pressure and area, calculate the force.

Solution

a. The intensity of the solar radiation is the average solar power per unit area. Hence, at $9.0 \times 10^{10} m$ from the center of the Sun, we have

$$\begin{aligned} I &= S_{avg} \\ &= \frac{3.8 \times 10^{26}\, W}{4\pi (9.0 \times 10^{10}\, m)^2} \\ &= 3.7 \times 10^{3}\, W/m^2. \end{aligned}$$

Assuming the comet reflects all the incident radiation, we obtain from Equation 19.5.3

$$\begin{aligned} p &= \frac{2I}{c} \\ &= \frac{2(3.7 \times 10^{3}\, W/m^2)}{3.00 \times 10^{8}\, m/s} \\ &= 2.5 \times 10^{-5}\, N/m^2. \end{aligned}$$

b. The force on the chunk due to the radiation is

$$\begin{aligned} F &= pA \\ &= (2.5 \times 10^{-5} N/m^2)(4.0 \times 10^{-2} m^2) \\ &= 1.0 \times 10^{-6}\, N, \end{aligned}$$

whereas the gravitational force of the Sun is

$$\begin{aligned} F_g &= \frac{GMm}{r^2} \\ &= \frac{(6.67 \times 10^{-11}\, N \cdot m^2/kg^2)(2.0 \times 10^{30} kg)(10\, kg)}{(9.0 \times 10^{10} m)^2} \\ &= 0.16\, N. \end{aligned}$$

Significance

The gravitational force of the Sun on the chunk is therefore much greater than the force of the radiation.

After Maxwell showed that light carried momentum as well as energy, a novel idea eventually emerged, initially only as science fiction. Perhaps a spacecraft with a large reflecting **light sail** could use radiation pressure for propulsion. Such a vehicle would not have to carry fuel. It would experience a constant but small force from solar radiation, instead of the short bursts from rocket propulsion. It would accelerate slowly, but by being accelerated continuously, it would eventually reach great speeds. A spacecraft with small total mass and a sail with a large area would be necessary to obtain a usable acceleration.

When the space program began in the 1960s, the idea started to receive serious attention from NASA. The most recent development in light propelled spacecraft has come from a citizen-funded group, the Planetary Society. It is currently testing the use of light sails to propel a small vehicle built from **CubeSats**, tiny satellites that NASA places in orbit for various research projects during space launches intended mainly for other purposes.

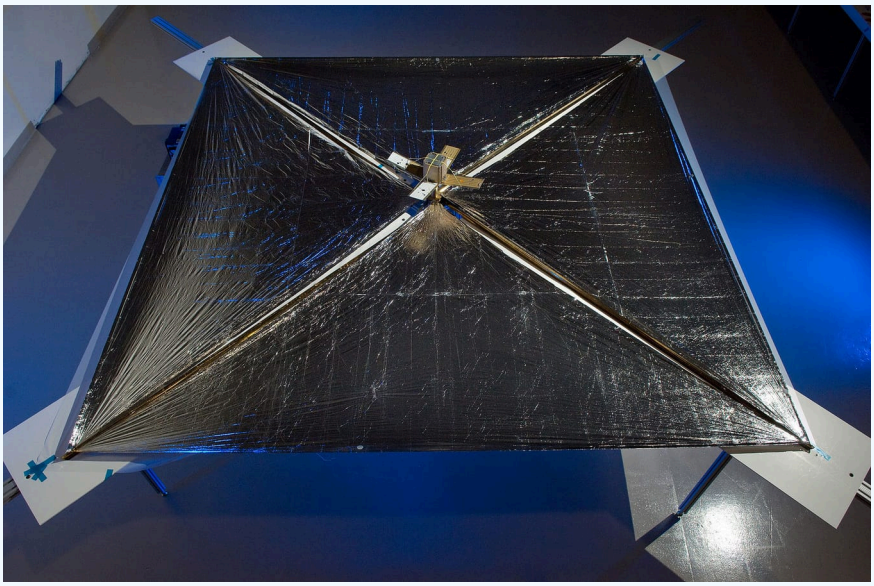
The **LightSail** spacecraft shown below (Figure 19.5.4) consists of three **CubeSats** bundled together. It has a total mass of only about 5 kg and is about the size as a loaf of bread. Its sails are made of very thin Mylar and open after launch to have a surface area of $32\ m^2$.



Figure 19.5.3: Two small CubeSat satellites deployed from the International Space Station in May, 2016. The solar sails open out when the CubeSats are far enough away from the Station.

Example 19.5.2: LightSail Acceleration

The first **LightSail** spacecraft was launched in 2015 to test the sail deployment system. It was placed in low-earth orbit in 2015 by hitching a ride on an Atlas 5 rocket launched for an unrelated mission. The test was successful, but the low-earth orbit allowed too much drag on the spacecraft to accelerate it by sunlight. Eventually, it burned in the atmosphere, as expected. The next Planetary Society's **LightSail** solar sailing spacecraft is scheduled for 2018.



The Lightsail is based on the on NASA's NanoSail-D project. (Public domain; NASA).

LightSail Acceleration

The intensity of energy from sunlight at a distance of 1 AU from the Sun is $1370\ W/m^2$. The **LightSail** spacecraft has sails with total area of $32\ m^2$ and a total mass of 5.0 kg. Calculate the maximum acceleration LightSail spacecraft could achieve from radiation pressure when it is about 1 AU from the Sun.

Strategy

The maximum acceleration can be expected when the sail is opened directly facing the Sun. Use the light intensity to calculate the radiation pressure and from it, the force on the sails. Then use Newton's second law to calculate the acceleration.

Solution

The radiation pressure is

$$F = pA = 2uA = \frac{2I}{c}A = \frac{2(1370\,W/m^2)(32\,m^2)}{(3.00 \times 10^8 m/s)} = 2.92 \times 10^{-4} N.$$

The resulting acceleration is

$$a = \frac{F}{m} = \frac{2.92 \times 10^{-4} N}{5.0\,kg} = 5.8 \times 10^{-5} m/s^2.$$

Significance

If this small acceleration continued for a year, the craft would attain a speed of 1829 m/s, or 6600 km/h.

Exercise 19.5.1

How would the speed and acceleration of a radiation-propelled spacecraft be affected as it moved farther from the Sun on an interplanetary space flight?

Solution

Its acceleration would decrease because the radiation force is proportional to the intensity of light from the Sun, which decreases with distance. Its speed, however, would not change except for the effects of gravity from the Sun and planets.

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19.6: The Electromagnetic Spectrum

Learning Objectives

By the end of this section, you will be able to:

- Explain how electromagnetic waves are divided into different ranges, depending on wavelength and corresponding frequency
- Describe how electromagnetic waves in different categories are produced
- Describe some of the many practical everyday applications of electromagnetic waves

Electromagnetic waves have a vast range of practical everyday applications that includes such diverse uses as communication by cell phone and radio broadcasting, WiFi, cooking, vision, medical imaging, and treating cancer. In this module, we discuss how electromagnetic waves are classified into categories such as radio, infrared, ultraviolet, and so on. We also summarize some of the main applications for each range.

The different categories of electromagnetic waves differ in their wavelength range, or equivalently, in their corresponding frequency ranges. Their properties change smoothly from one frequency range to the next, with different applications in each range. A brief overview of the production and utilization of electromagnetic waves is found in Table 19.6.1.

Table 19.6.1: Electromagnetic Waves

Type of wave	Production	Applications	Issues
Radio	Accelerating charges	Communications, Remote controls, MRI	Requires control for band use
Microwaves	Accelerating charges and thermal agitation	Communications, Ovens, Radar, Cell phone use	
Infrared	Thermal agitation and electronic transitions	Thermal imaging, Heating	Absorbed by atmosphere, Greenhouse effect
Visible light	Thermal agitation and electronic transitions	Photosynthesis, Human vision	
Ultraviolet	Thermal agitation and electronic transitions	Sterilization, Vitamin D production	Ozone depletion, Cancer causing
X-rays	Inner electronic transitions and fast collisions	Security, Medical diagnosis, Cancer therapy	Cancer causing
Gamma rays	Nuclear decay	Nuclear medicine, Security, Medical diagnosis, Cancer therapy	Cancer causing, Radiation damage

The relationship $c = f\lambda$ between frequency **f** and wavelength λ applies to all waves and ensures that greater frequency means smaller wavelength. Figure 19.6.2 shows how the various types of electromagnetic waves are categorized according to their wavelengths and frequencies - that is, it shows the electromagnetic spectrum.

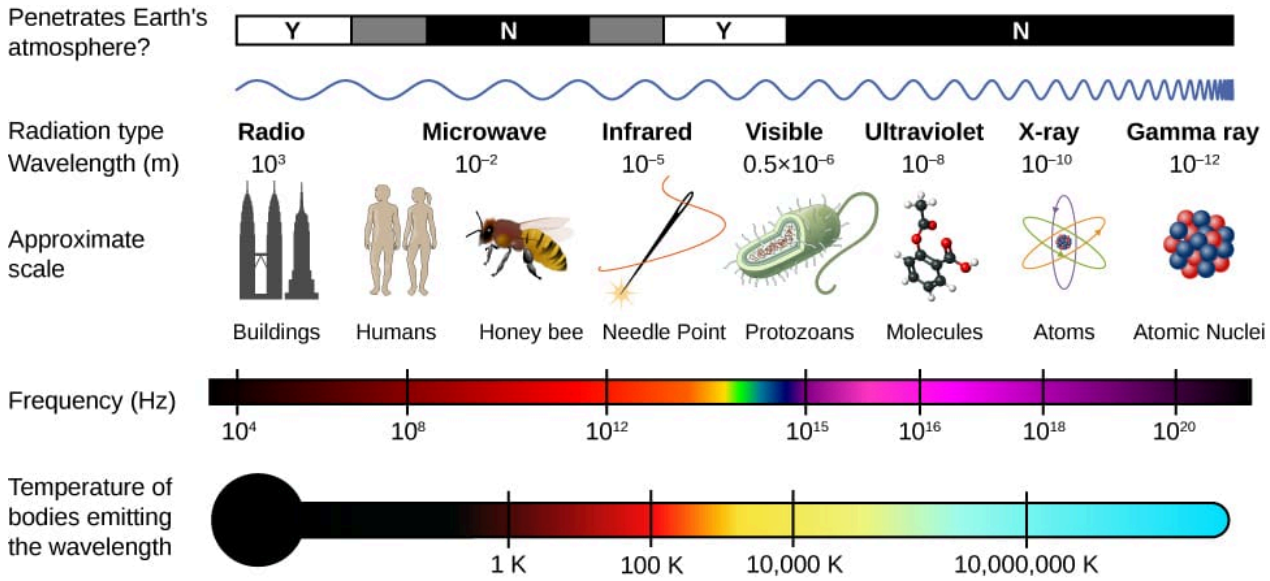


Figure 19.6.1: The electromagnetic spectrum, showing the major categories of electromagnetic waves.

Radio Waves

The term **radio waves** refers to electromagnetic radiation with wavelengths greater than about 0.1 m. Radio waves are commonly used for audio communications (i.e., for radios), but the term is used for electromagnetic waves in this range regardless of their application. Radio waves typically result from an alternating current in the wires of a broadcast antenna. They cover a very broad wavelength range and are divided into many subranges, including microwaves, electromagnetic waves used for AM and FM radio, cellular telephones, and TV signals.

There is no lowest frequency of radio waves, but ELF waves, or “extremely low frequency” are among the lowest frequencies commonly encountered, from 3 Hz to 3 kHz. The accelerating charge in the ac currents of electrical power lines produce electromagnetic waves in this range. ELF waves are able to penetrate sea water, which strongly absorbs electromagnetic waves of higher frequency, and therefore are useful for submarine communications.

In order to use an electromagnetic wave to transmit information, the amplitude, frequency, or phase of the wave is **modulated**, or varied in a controlled way that encodes the intended information into the wave. In AM radio transmission, the amplitude of the wave is modulated to mimic the vibrations of the sound being conveyed. Fourier’s theorem implies that the modulated AM wave amounts to a superposition of waves covering some narrow frequency range. Each AM station is assigned a specific carrier frequency that, by international agreement, is allowed to vary by $\pm 5\ kHz$. In FM radio transmission, the frequency of the wave is modulated to carry this information, as illustrated in Figure 19.6.2, and the frequency of each station is allowed to use 100 kHz on each side of its carrier frequency. The electromagnetic wave produces a current in a receiving antenna, and the radio or television processes the signal to produce the sound and any image. The higher the frequency of the radio wave used to carry the data, the greater the detailed variation of the wave that can be carried by modulating it over each time unit, and the more data that can be transmitted per unit of time. The assigned frequencies for AM broadcasting are 540 to 1600 kHz, and for FM are 88 MHz to108 MHz.

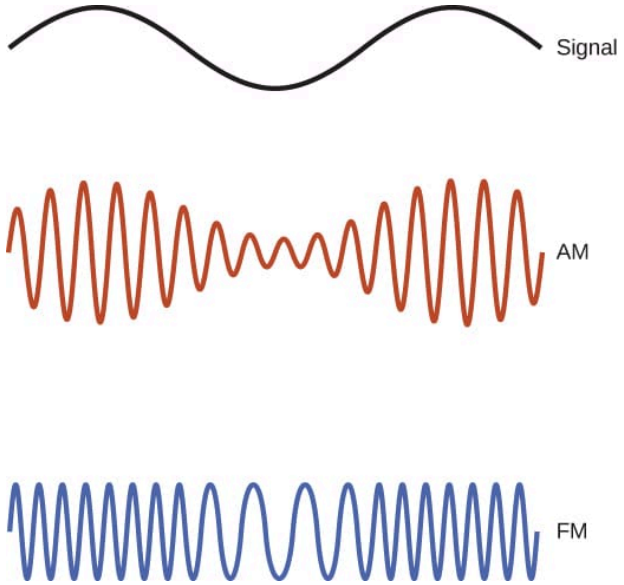


Figure 19.6.2: Electromagnetic waves are used to carry communications signals by varying the wave's amplitude (AM), its frequency (FM), or its phase.

Cell phone conversations, and **television** voice and video images are commonly transmitted as digital data, by converting the signal into a sequence of binary ones and zeros. This allows clearer data transmission when the signal is weak, and allows using computer algorithms to compress the digital data to transmit more data in each frequency range. Computer data as well is transmitted as a sequence of binary ones and zeros, each one or zero constituting one bit of data.

Microwaves

Microwaves are the highest-frequency electromagnetic waves that can be produced by currents in macroscopic circuits and devices. Microwave frequencies range from about $10^9 Hz$ to nearly $10^{12} Hz$. Their high frequencies correspond to short wavelengths compared with other radio waves—hence the name "microwave." Microwaves also occur naturally as the cosmic background radiation left over from the origin of the universe. Along with other ranges of electromagnetic waves, they are part of the radiation that any object above absolute zero emits and absorbs because of **thermal agitation**, that is, from the thermal motion of its atoms and molecules.

Most satellite-transmitted information is carried on **microwaves**. **Radar** is a common application of microwaves. By detecting and timing microwave echoes, radar systems can determine the distance to objects as diverse as clouds, aircraft, or even the surface of Venus.

Microwaves of 2.45 GHz are commonly used in microwave ovens. The electrons in a water molecule tend to remain closer to the oxygen nucleus than the hydrogen nuclei (Figure 19.6.3). This creates two separated centers of equal and opposite charges, giving the molecule a dipole moment. The oscillating electric field of the microwaves inside the oven exerts a torque that tends to align each molecule first in one direction and then in the other, with the motion of each molecule coupled to others around it. This pumps energy into the continual thermal motion of the water to heat the food. The plate under the food contains no water, and remains relatively unheated.

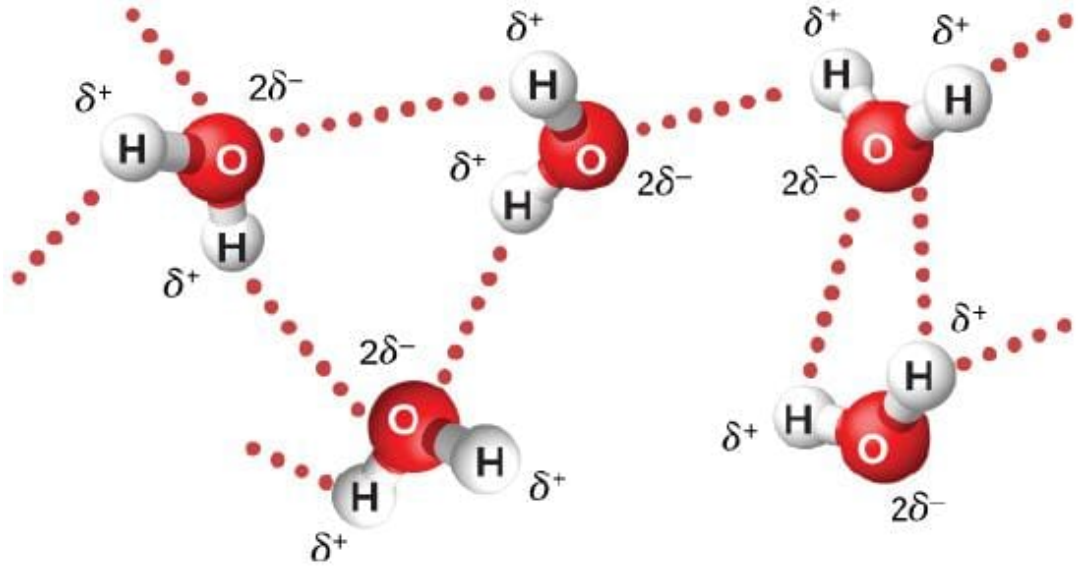


Figure 19.6.3: The oscillating electric field in a microwave oven exerts a torque on water molecules because of their dipole moment, and the torque reverses direction 4.90×10^9 times per second. Interactions between the molecules distributes the energy being pumped into them. The δ^+ and δ^- denote the charge distribution on the molecules.

The microwaves in a microwave oven reflect off the walls of the oven, so that the superposition of waves produces standing waves, similar to the standing waves of a vibrating guitar or violin string (Normal Modes of a Standing Sound Wave). A rotating fan acts as a stirrer by reflecting the microwaves in different directions, and food turntables, help spread out the hot spots.

Example 19.6.1: Why Microwave Ovens Heat Unevenly

How far apart are the hotspots in a 2.45-GHz microwave oven?

Strategy

Consider the waves along one direction in the oven, being reflected at the opposite wall from where they are generated.

Solution

The antinodes, where maximum intensity occurs, are half the wavelength apart, with separation

$$d = \frac{1}{2}\lambda \tag{19.6.1}$$

$$= \frac{1}{2}\frac{c}{f} \tag{19.6.2}$$

$$= \frac{3.00 \times 10^8 m/s}{2(2.45 \times 10^9\, Hz)} \tag{19.6.3}$$

$$= 6.02\, cm. \tag{19.6.4}$$

Significance

The distance between the hot spots in a microwave oven are determined by the wavelength of the microwaves.

A cell phone has a radio receiver and a weak radio transmitter, both of which can quickly tune to hundreds of specifically assigned microwave frequencies. The low intensity of the transmitted signal gives it an intentionally limited range. A ground-based system links the phone to only to the broadcast tower assigned to the specific small area, or cell, and smoothly transitions its connection to the next cell when the signal reception there is the stronger one. This enables a cell phone to be used while changing location.

Microwaves also provide the **WiFi** that enables owners of cell phones, laptop computers, and similar devices to connect wirelessly

to the Internet at home and at coffee shops and airports. A wireless WiFi router is a device that exchanges data over the Internet through the cable or another connection, and uses microwaves to exchange the data wirelessly with devices such as cell phones and computers. The term WiFi itself refers to the standards followed in modulating and analyzing the microwaves so that wireless routers and devices from different manufacturers work compatibly with one another. The computer data in each direction consist of sequences of binary zeros and ones, each corresponding to a binary bit. The microwaves are in the range of 2.4 GHz to 5.0 GHz range.

Other wireless technologies also use microwaves to provide everyday communications between devices. **Bluetooth** developed alongside WiFi as a standard for radio communication in the 2.4-GHz range between nearby devices, for example, to link to headphones and audio earpieces to devices such as radios, or a driver's cell phone to a hands-free device to allow answering phone calls without fumbling directly with the cell phone.

Microwaves find use also in radio tagging, using RFID (radio frequency identification) technology. Examples are RFID tags attached to store merchandize, transponder for toll booths use attached to the windshield of a car, or even a chip embedded into a pet's skin. The device responds to a microwave signal by emitting a signal of its own with encoded information, allowing stores to quickly ring up items at their cash registers, drivers to charge tolls to their account without stopping, and lost pets to be reunited with their owners. NFC (near field communication) works similarly, except it is much shorter range. Its mechanism of interaction is the induced magnetic field at microwave frequencies between two coils. Cell phones that have NFC capability and the right software can supply information for purchases using the cell phone instead of a physical credit card. The very short range of the data transfer is a desired security feature in this case.

Infrared Radiation

The boundary between the microwave and infrared regions of the electromagnetic spectrum is not well defined (Figure 19.6.1). **Infrared radiation** is generally produced by thermal motion, and the vibration and rotation of atoms and molecules. Electronic transitions in atoms and molecules can also produce **infrared radiation**. About half of the solar energy arriving at Earth is in the infrared region, with most of the rest in the visible part of the spectrum. About 23% of the solar energy is absorbed in the atmosphere, about 48% is absorbed at Earth's surface, and about 29% is reflected back into space.

The range of infrared frequencies extends up to the lower limit of visible light, just below red. In fact, infrared means "below red." Water molecules rotate and vibrate particularly well at infrared frequencies. Reconnaissance satellites can detect buildings, vehicles, and even individual humans by their infrared emissions, whose power radiation is proportional to the fourth power of the absolute temperature. More mundanely, we use infrared lamps, including those called **quartz heaters**, to preferentially warm us because we absorb infrared better than our surroundings.

The familiar handheld "remotes" for changing channels and settings on television sets often transmit their signal by modulating an infrared beam. If you try to use a TV remote without the infrared emitter being in direct line of sight with the infrared detector, you may find the television not responding. Some remotes use Bluetooth instead and reduce this annoyance.

Visible Light

Visible light is the narrow segment of the electromagnetic spectrum between about 400 nm and about 750 nm to which the normal human eye responds. Visible light is produced by vibrations and rotations of atoms and molecules, as well as by electronic transitions within atoms and molecules. The receivers or detectors of light largely utilize electronic transitions.

Red light has the lowest frequencies and longest wavelengths, whereas violet has the highest frequencies and shortest wavelengths (Figure 19.6.4). Blackbody radiation from the Sun peaks in the visible part of the spectrum but is more intense in the red than in the violet, making the sun yellowish in appearance.

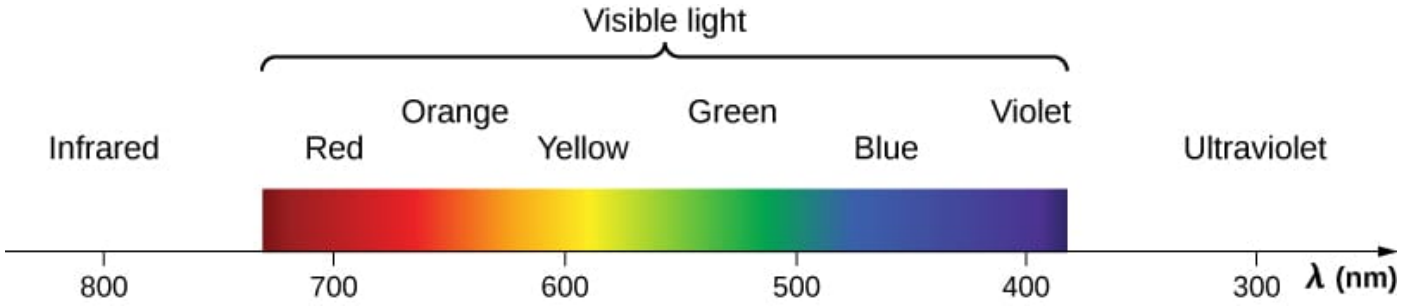


Figure 19.6.4. A small part of the electromagnetic spectrum that includes its visible components. The divisions between infrared, visible, and ultraviolet are not perfectly distinct, nor are those between the seven rainbow colors.

Living things - plants and animals - have evolved to utilize and respond to parts of the electromagnetic spectrum in which they are embedded. We enjoy the beauty of nature through visible light. Plants are more selective. Photosynthesis uses parts of the visible spectrum to make sugars.

Ultraviolet Radiation

Ultraviolet means “above violet.” The electromagnetic frequencies of **ultraviolet radiation (UV)** extend upward from violet, the highest-frequency visible light. The highest-frequency ultraviolet overlaps with the lowest-frequency X-rays. The wavelengths of ultraviolet extend from 400 nm down to about 10 nm at its highest frequencies. Ultraviolet is produced by atomic and molecular motions and electronic transitions.

UV radiation from the Sun is broadly subdivided into three wavelength ranges: UV-A (320–400 nm) is the lowest frequency, then UV-B (290–320 nm) and UV-C (220–290 nm). Most UV-B and all UV-C are absorbed by ozone (O_3) molecules in the upper atmosphere. Consequently, 99% of the solar UV radiation reaching Earth’s surface is UV-A.

Sunburn is caused by large exposures to UV-B and UV-C, and repeated exposure can increase the likelihood of skin cancer. The tanning response is a defense mechanism in which the body produces pigments in inert skin layers to reduce exposure of the living cells below.

As examined in a later chapter, the shorter the wavelength of light, the greater the energy change of an atom or molecule that absorbs the light in an electronic transition. This makes short-wavelength ultraviolet light damaging to living cells. It also explains why ultraviolet radiation is better able than visible light to cause some materials to glow, or **fluoresce**.

Besides the adverse effects of ultraviolet radiation, there are also benefits of exposure in nature and uses in technology. Vitamin D production in the skin results from exposure to UV-B radiation, generally from sunlight. Several studies suggest vitamin D deficiency is associated with the development of a range of cancers (prostate, breast, colon), as well as osteoporosis. Low-intensity ultraviolet has applications such as providing the energy to cause certain dyes to fluoresce and emit visible light, for example, in printed money to display hidden watermarks as counterfeit protection.

X-Rays

X-rays have wavelengths from about $10^{-8}m$ to $10^{-12}m$. They have shorter wavelengths, and higher frequencies, than ultraviolet, so that the energy they transfer at an atomic level is greater. As a result, X-rays have adverse effects on living cells similar to those of ultraviolet radiation, but they are more penetrating. Cancer and genetic defects can be induced by X-rays. Because of their effect on rapidly dividing cells, X-rays can also be used to treat and even cure cancer.

The widest use of X-rays is for imaging objects that are opaque to visible light, such as the human body or aircraft parts. In humans, the risk of cell damage is weighed carefully against the benefit of the diagnostic information obtained.

Gamma Rays

Soon after nuclear radioactivity was first detected in 1896, it was found that at least three distinct types of radiation were being emitted, and these were designated as alpha, beta, and gamma rays. The most penetrating nuclear radiation, the **gamma ray** γ

-ray) was later found to be an extremely high-frequency electromagnetic wave.

The lower end of the γ-ray frequency range overlaps the upper end of the X-ray range. Gamma rays have characteristics identical to X-rays of the same frequency—they differ only in source. The name “gamma rays” is generally used for electromagnetic radiation emitted by a nucleus, while X-rays are generally produced by bombarding a target with energetic electrons in an X-ray tube. At higher frequencies, γ-rays are more penetrating and more damaging to living tissue. They have many of the same uses as X-rays, including cancer therapy. Gamma radiation from radioactive materials is used in nuclear medicine.

Use this simulation to explore how light interacts with molecules in our atmosphere.

- Explore how light interacts with molecules in our atmosphere.
- Identify that absorption of light depends on the molecule and the type of light.
- Relate the energy of the light to the resulting motion.
- Identify that energy increases from microwave to ultraviolet.
- Predict the motion of a molecule based on the type of light it absorbs.

Exercise 19.6.1

How do the electromagnetic waves for the different kinds of electromagnetic radiation differ?

Answer available at the end of this section.

Answers

Exercise 19.6.1

They fall into different ranges of wavelength, and therefore also different corresponding ranges of frequency.

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19.7: Electromagnetic Waves (Summary)

Key Terms

displacement current	extra term in Maxwell's equations that is analogous to a real current but accounts for a changing electric field producing a magnetic field, even when the real current is present
gamma ray (γ ray)	extremely high frequency electromagnetic radiation emitted by the nucleus of an atom, either from natural nuclear decay or induced nuclear processes in nuclear reactors and weapons; the lower end of the γ -ray frequency range overlaps the upper end of the X-ray range, but γ rays can have the highest frequency of any electromagnetic radiation
infrared radiation	region of the electromagnetic spectrum with a frequency range that extends from just below the red region of the visible light spectrum up to the microwave region, or from $0.74\mu m$ to $300\mu m$
Maxwell's equations	set of four equations that comprise a complete, overarching theory of electromagnetism
microwaves	electromagnetic waves with wavelengths in the range from 1 mm to 1 m; they can be produced by currents in macroscopic circuits and devices
Poynting vector	vector equal to the cross product of the electric-and magnetic fields, that describes the flow of electromagnetic energy through a surface
radar	common application of microwaves; radar can determine the distance to objects as diverse as clouds and aircraft, as well as determine the speed of a car or the intensity of a rainstorm
radiation pressure	force divided by area applied by an electromagnetic wave on a surface
radio waves	electromagnetic waves with wavelengths in the range from 1 mm to 100 km; they are produced by currents in wires and circuits and by astronomical phenomena
thermal agitation	thermal motion of atoms and molecules in any object at a temperature above absolute zero, which causes them to emit and absorb radiation
ultraviolet radiation	electromagnetic radiation in the range extending upward in frequency from violet light and overlapping with the lowest X-ray frequencies, with wavelengths from 400 nm down to about 10 nm
visible light	narrow segment of the electromagnetic spectrum to which the normal human eye responds, from about 400 to 750 nm
X-ray	invisible, penetrating form of very high frequency electromagnetic radiation, overlapping both the ultraviolet range and the γ-ray range

Key Equations

Displacement current	$I_d = \varepsilon_0 \frac{d\Phi_E}{dt}$
Gauss's law	$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$
Gauss's law for magnetism	$\oint \vec{B} \cdot d\vec{A} = 0$
Faraday's law	$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_m}{dt}$
Ampère-Maxwell law	$\oint \vec{B} \cdot d\vec{s} = \mu_0 I + \varepsilon_0 \mu_0 \frac{d\Phi_E}{dt}$

Wave equation for plane EM wave	$\frac{\partial^2 E_y}{\partial x^2} = \varepsilon_0 \mu_0 \frac{\partial^2 E_y}{\partial t^2}$
Speed of EM waves	$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$
Ratio of E field to B field in electromagnetic wave	$c = \frac{E}{B}$
Energy flux (Poynting) vector	$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$
Average intensity of an electromagnetic wave	$I = S_{avg} = \frac{c\varepsilon_0 E_0^2}{2} = \frac{cB_0^2}{2\mu_0} = \frac{E_0 B_0}{2\mu_0}$
Radiation pressure	$p = \begin{cases} I/c & Perfect absorber \\ 2I/c & Perfect reflector \end{cases}$

Summary

16.2: Maxwell's Equations and Electromagnetic Waves

James Clerk Maxwell (1831–1879) was one of the major contributors to physics in the nineteenth century. Although he died young, he made major contributions to the development of the kinetic theory of gases, to the understanding of color vision, and to the nature of Saturn's rings. He is best known for having combined existing knowledge of the laws of electricity and of magnetism with insights of his own into a complete overarching electromagnetic theory, represented by Maxwell's equations.

- Maxwell's prediction of electromagnetic waves resulted from his formulation of a complete and symmetric theory of electricity and magnetism, known as Maxwell's equations.
- The four Maxwell's equations together with the Lorentz force law encompass the major laws of electricity and magnetism. The first of these is Gauss's law for electricity; the second is Gauss's law for magnetism; the third is Faraday's law of induction (including Lenz's law); and the fourth is Ampère's law in a symmetric formulation that adds another source of magnetism, namely changing electric fields.
- The symmetry introduced between electric and magnetic fields through Maxwell's displacement current explains the mechanism of electromagnetic wave propagation, in which changing magnetic fields produce changing electric fields and vice versa.
- Although light was already known to be a wave, the nature of the wave was not understood before Maxwell. Maxwell's equations also predicted electromagnetic waves with wavelengths and frequencies outside the range of light. These theoretical predictions were first confirmed experimentally by Heinrich Hertz.

16.3: Plane Electromagnetic Waves

Mechanical waves travel through a medium such as a string, water, or air. Perhaps the most significant prediction of Maxwell's equations is the existence of combined electric and magnetic (or electromagnetic) fields that propagate through space as electromagnetic waves. Because Maxwell's equations hold in free space, the predicted electromagnetic waves, unlike mechanical waves, do not require a medium for their propagation.

- Maxwell's equations predict that the directions of the electric and magnetic fields of the wave, and the wave's direction of propagation, are all mutually perpendicular. The electromagnetic wave is a transverse wave.
- The strengths of the electric and magnetic parts of the wave are related by $c = E/B$, which implies that the magnetic field **B** is very weak relative to the electric field **E**.
- Accelerating charges create electromagnetic waves (for example, an oscillating current in a wire produces electromagnetic waves with the same frequency as the oscillation).

16.4: Energy Carried by Electromagnetic Waves

- The energy carried by any wave is proportional to its amplitude squared. For electromagnetic waves, this means intensity can be expressed as

$$I = \frac{c\varepsilon_0 E_0^2}{2}$$

where I is the average intensity in W/m^2 and E_0 is the maximum electric field strength of a continuous sinusoidal wave. This can also be expressed in terms of the maximum magnetic field strength B_0 as

$$I = \frac{cB_0^2}{2\mu_0}$$

and in terms of both electric and magnetic fields as

$$I = \frac{E_0 B_0}{2\mu_0}.$$

The three expressions for I_{avg} are all equivalent.

16.5: Momentum and Radiation Pressure

- Electromagnetic waves carry momentum and exert radiation pressure.
- The radiation pressure of an electromagnetic wave is directly proportional to its energy density.
- The pressure is equal to twice the electromagnetic energy intensity if the wave is reflected and equal to the incident energy intensity if the wave is absorbed.

16.6: The Electromagnetic Spectrum

- The relationship among the speed of propagation, wavelength, and frequency for any wave is given by $v = f\lambda$, so that for electromagnetic waves, $c = f\lambda$, where f is the frequency, λ is the wavelength, and c is the speed of light.
- The electromagnetic spectrum is separated into many categories and subcategories, based on the frequency and wavelength, source, and uses of the electromagnetic waves.

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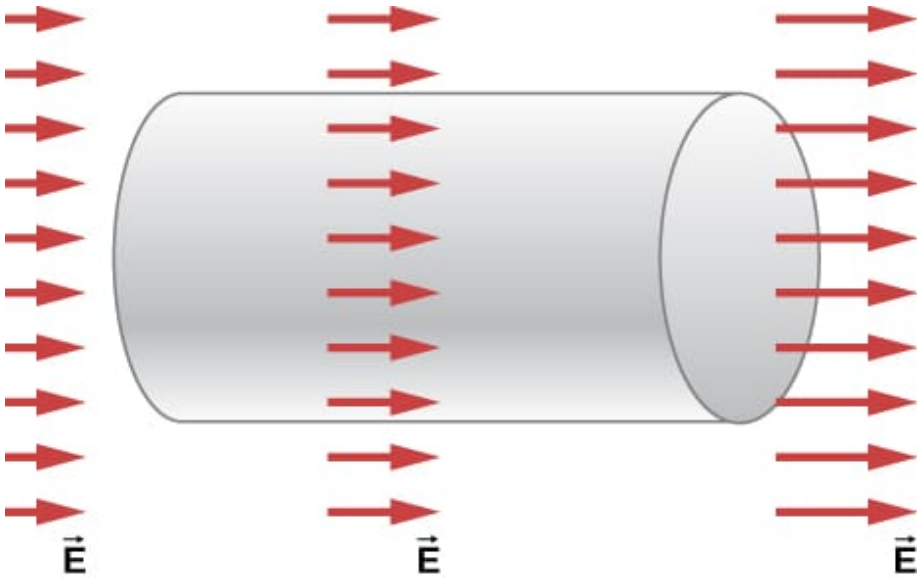
19.8: Electromagnetic Waves (Exercises)

Conceptual Questions

16.2 Maxwell's Equations and Electromagnetic Waves

1. Explain how the displacement current maintains the continuity of current in a circuit containing a capacitor.

2. Describe the field lines of the induced magnetic field along the edge of the imaginary horizontal cylinder shown below if the cylinder is in a spatially uniform electric field that is horizontal, pointing to the right, and increasing in magnitude.

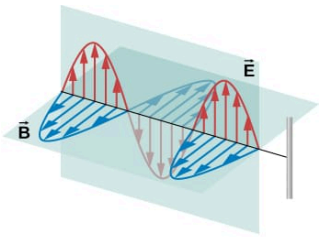


3. Why is it much easier to demonstrate in a student lab that a changing magnetic field induces an electric field than it is to demonstrate that a changing electric field produces a magnetic field?

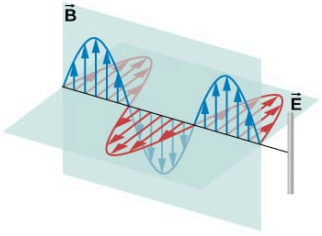
16.3 Plane Electromagnetic Waves

4. If the electric field of an electromagnetic wave is oscillating along the z-axis and the magnetic field is oscillating along the x-axis, in what possible direction is the wave traveling?

5. In which situation shown below will the electromagnetic wave be more successful in inducing a current in the wire? Explain.

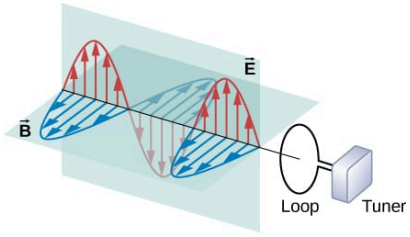


(a)

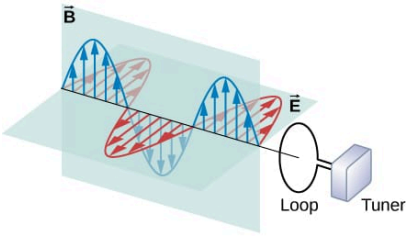


(b)

6. In which situation shown below will the electromagnetic wave be more successful in inducing a current in the loop? Explain.



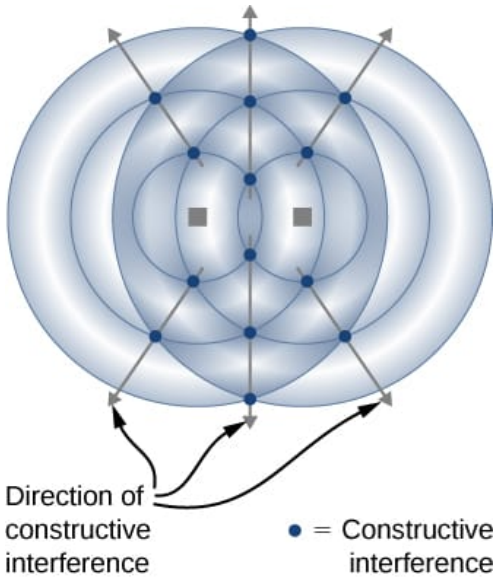
(a)



(b)

7. Under what conditions might wires in a circuit where the current flows in only one direction emit electromagnetic waves?

8. Shown below is the interference pattern of two radio antennas broadcasting the same signal. Explain how this is analogous to the interference pattern for sound produced by two speakers. Could this be used to make a directional antenna system that broadcasts preferentially in certain directions? Explain.



16.4 Energy Carried by Electromagnetic Waves

9. When you stand outdoors in the sunlight, why can you feel the energy that the sunlight carries, but not the momentum it carries?

10. How does the intensity of an electromagnetic wave depend on its electric field? How does it depend on its magnetic field?

11. What is the physical significance of the Poynting vector?

12. A 2.0-mW helium-neon laser transmits a continuous beam of red light of cross-sectional area $0.25cm^2$. If the beam does not diverge appreciably, how would its rms electric field vary with distance from the laser? Explain.

16.5 Momentum and Radiation Pressure

13. Why is the radiation pressure of an electromagnetic wave on a perfectly reflecting surface twice as large as the pressure on a perfectly absorbing surface?

14. Why did the early Hubble Telescope photos of Comet Ison approaching Earth show it to have merely a fuzzy coma around it, and not the pronounced double tail that developed later (see below)?



(credit: ESA, Hubble)

15. (a) If the electric field and magnetic field in a sinusoidal plane wave were interchanged, in which direction relative to before would the energy propagate?

(b) What if the electric and the magnetic fields were both changed to their negatives?

16.6 The Electromagnetic Spectrum

16. Compare the speed, wavelength, and frequency of radio waves and X-rays traveling in a vacuum.

17. Accelerating electric charge emits electromagnetic radiation. How does this apply in each case: (a) radio waves, (b) infrared radiation.

18. Compare and contrast the meaning of the prefix “micro” in the names of SI units in the term microwaves.

19. Part of the light passing through the air is scattered in all directions by the molecules comprising the atmosphere. The wavelengths of visible light are larger than molecular sizes, and the scattering is strongest for wavelengths of light closest to sizes of molecules.

(a) Which of the main colors of light is scattered the most?

(b) Explain why this would give the sky its familiar background color at midday.

20. When a bowl of soup is removed from a microwave oven, the soup is found to be steaming hot, whereas the bowl is only warm to the touch. Discuss the temperature changes that have occurred in terms of energy transfer.

21. Certain orientations of a broadcast television antenna give better reception than others for a particular station. Explain.

22. What property of light corresponds to loudness in sound?

23. Is the visible region a major portion of the electromagnetic spectrum?

24. Can the human body detect electromagnetic radiation that is outside the visible region of the spectrum?

25. Radio waves normally have their **E** and **B** fields in specific directions, whereas visible light usually has its **E** and **B** fields in random and rapidly changing directions that are perpendicular to each other and to the propagation direction. Can you explain why?

26. Give an example of resonance in the reception of electromagnetic waves.

27. Illustrate that the size of details of an object that can be detected with electromagnetic waves is related to their wavelength, by comparing details observable with two different types (for example, radar and visible light).

28. In which part of the electromagnetic spectrum are each of these waves:

(a) $f = 10.0kHz$,

(b) $f = \lambda = 750nm$,

(c) $f = 1.25 \times 10^8 Hz$,

(d) $0.30nm$

29. In what range of electromagnetic radiation are the electromagnetic waves emitted by power lines in a country that uses 50-Hz ac current?

30. If a microwave oven could be modified to merely tune the waves generated to be in the infrared range instead of using microwaves, how would this affect the uneven heating of the oven?

31. A leaky microwave oven in a home can sometimes cause interference with the homeowner's WiFi system. Why?

32. When a television news anchor in a studio speaks to a reporter in a distant country, there is sometimes a noticeable lag between when the anchor speaks in the studio and when the remote reporter hears it and replies. Explain what causes this delay.

Problems

16.2 Maxwell's Equations and Electromagnetic Waves

33. Show that the magnetic field at a distance **r** from the axis of two circular parallel plates, produced by placing charge **Q(t)** on the plates is

$$B_{ind} = \frac{\mu_0}{2\pi r}\frac{dQ(t)}{dt}$$

34. Express the displacement current in a capacitor in terms of the capacitance and the rate of change of the voltage across the capacitor.

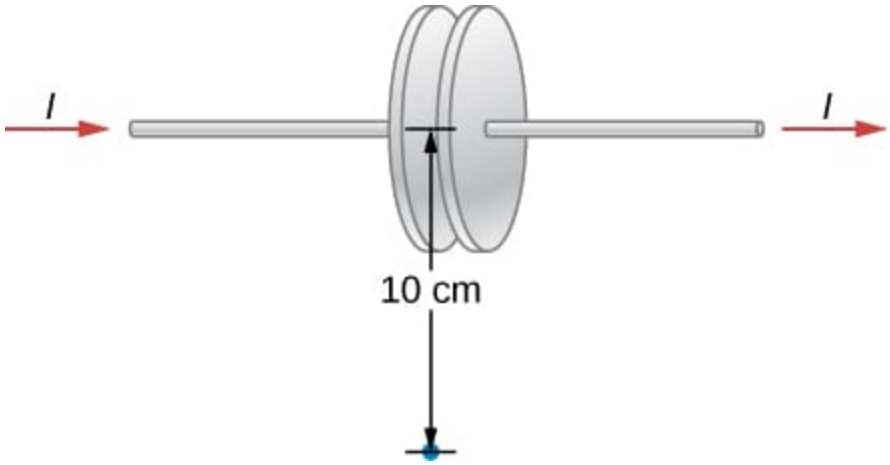
35. A potential difference $V(t) = V_0 sin\omega t$ is maintained across a parallel-plate capacitor with capacitance **C** consisting of two circular parallel plates. A thin wire with resistance **R** connects the centers of the two plates, allowing charge to leak between plates while they are charging.

(a) Obtain expressions for the leakage current $I_{res}(t)$ in the thin wire. Use these results to obtain an expression for the current $I_{real}(t)$ in the wires connected to the capacitor.

(b) Find the displacement current in the space between the plates from the changing electric field between the plates.

(c) Compare $I_{real}(t)$ with the sum of the displacement current $I_d(t)$ and resistor current $I_{res}(t)$ between the plates, and explain why the relationship you observe would be expected.

36. Suppose the parallel-plate capacitor shown below is accumulating charge at a rate of 0.010 C/s. What is the induced magnetic field at a distance of 10 cm from the capacitator?



37. The potential difference **V(t)** between parallel plates shown above is instantaneously increasing at a rate of $10^7 V/s$. What is the displacement current between the plates if the separation of the plates is 1.00 cm and they have an area of $0.200 m^2$?

38. A parallel-plate capacitor has a plate area of $A = 0.250 m^2$ and a separation of 0.0100 m. What must be must be the angular frequency ω for a voltage $V(t) = V_0 sin\omega t$ with $V_0 = 100V$ to produce a maximum displacement induced current of 1.00 A between the plates?

39. The voltage across a parallel-plate capacitor with area $A = 800 cm^2$ and separation $d = 2mm$ varies sinusoidally as $V = (15mV)cos(150t)$, where **t** is in seconds. Find the displacement current between the plates.

40. The voltage across a parallel-plate capacitor with area **A** and separation **d** varies with time **t** as $V = at^2$, where a is a constant. Find the displacement current between the plates.

16.3 Plane Electromagnetic Waves

41. If the Sun suddenly turned off, we would not know it until its light stopped coming. How long would that be, given that the Sun is $1.496 \times 10^{11} m$ away?

42. What is the maximum electric field strength in an electromagnetic wave that has a maximum magnetic field strength of $5.00 \times 10^{-4} T$ (about 10 times Earth's magnetic field)?

43. An electromagnetic wave has a frequency of 12 MHz. What is its wavelength in vacuum?

44. If electric and magnetic field strengths vary sinusoidally in time at frequency 1.00 GHz, being zero at $t = 0$, then $E = E_0 sin2\pi ft$ and $B = B_0 sin2\pi ft$.

(a) When are the field strengths next equal to zero?

(b) When do they reach their most negative value? (c) How much time is needed for them to complete one cycle?

45. The electric field of an electromagnetic wave traveling in vacuum is described by the following wave function:

$$\vec{E} = (5.00 V/m)cos[kx - (6.00 \times 10^9 s^{-1})t + 0.40]\hat{j}$$

where **k** is the wavenumber in rad/m, **x** is in m, **t** is in s.

Find the following quantities:

(a) amplitude

(b) frequency

(c) wavelength

(d) the direction of the travel of the wave

(e) the associated magnetic field wave

46. A plane electromagnetic wave of frequency 20 GHz moves in the positive y-axis direction such that its electric field is pointed along the z-axis. The amplitude of the electric field is 10 V/m. The start of time is chosen so that at $t = 0$, the electric field has a value 10 V/m at the origin.

(a) Write the wave function that will describe the electric field wave.

(b) Find the wave function that will describe the associated magnetic field wave.

47. The following represents an electromagnetic wave traveling in the direction of the positive **y**-axis:

$$E_x = 0; E_y = E_0 cos(kx - \omega t); E_z = 0$$

$$B_x = 0; B_y = 0; B_z = B_0 cos(kx - \omega t)$$

The wave is passing through a wide tube of circular cross-section of radius **R** whose axis is along the **y**-axis. Find the expression for the displacement current through the tube.

16.4 Energy Carried by Electromagnetic Waves

48. While outdoors on a sunny day, a student holds a large convex lens of radius 4.0 cm above a sheet of paper to produce a bright spot on the paper that is 1.0 cm in radius, rather than a sharp focus. By what factor is the electric field in the bright spot of light related to the electric field of sunlight leaving the side of the lens facing the paper?

49. A plane electromagnetic wave travels northward. At one instant, its electric field has a magnitude of 6.0 V/m and points eastward. What are the magnitude and direction of the magnetic field at this instant?

50. The electric field of an electromagnetic wave is given by

$$E = (6.0 \times 10^{-3} V/m) sin[2\pi(\frac{x}{18m} - \frac{t}{6.0 \times 10^{-8} s})]\hat{j}.$$

Write the equations for the associated magnetic field and Poynting vector.

51. A radio station broadcasts at a frequency of 760 kHz. At a receiver some distance from the antenna, the maximum magnetic field of the electromagnetic wave detected is $2.15 \times 10^{-11} T$.

(a) What is the maximum electric field?

(b) What is the wavelength of the electromagnetic wave?

52. The filament in a clear incandescent light bulb radiates visible light at a power of 5.00 W. Model the glass part of the bulb as a sphere of radius $r_0 = 3.00 cm$ and calculate the amount of electromagnetic energy from visible light inside the bulb.

53. At what distance does a 100-W lightbulb produce the same intensity of light as a 75-W lightbulb produces 10 m away? (Assume both have the same efficiency for converting electrical energy in the circuit into emitted electromagnetic energy.)

54. An incandescent light bulb emits only 2.6 W of its power as visible light. What is the rms electric field of the emitted light at a distance of 3.0 m from the bulb?

55. A 150-W lightbulb emits 5% of its energy as electromagnetic radiation. What is the magnitude of the average Poynting vector 10 m from the bulb?

56. A small helium-neon laser has a power output of 2.5 mW. What is the electromagnetic energy in a 1.0-m length of the beam?

57. At the top of Earth's atmosphere, the time-averaged Poynting vector associated with sunlight has a magnitude of about $1.4kW/m^2$.

(a) What are the maximum values of the electric and magnetic fields for a wave of this intensity?

(b) What is the total power radiated by the sun? Assume that the Earth is $1.5 \times 10^{11}m$ from the Sun and that sunlight is composed of electromagnetic plane waves.

58. The magnetic field of a plane electromagnetic wave moving along the z axis is given by

$\vec{B} = B_0(coskz + \omega t)\hat{j}$, where $B_0 = 5.00 \times 10^{-10}T$ and $k = 3.14 \times 10^{-2}m^{-1}$.

(a) Write an expression for the electric field associated with the wave.

(b) What are the frequency and the wavelength of the wave?

(c) What is its average Poynting vector?

59. What is the intensity of an electromagnetic wave with a peak electric field strength of 125 V/m?

60. Assume the helium-neon lasers commonly used in student physics laboratories have power outputs of 0.500 mW.

(a) If such a laser beam is projected onto a circular spot 1.00 mm in diameter, what is its intensity?

(b) Find the peak magnetic field strength.

(c) Find the peak electric field strength.

61. An AM radio transmitter broadcasts 50.0 kW of power uniformly in all directions. (a) Assuming all of the radio waves that strike the ground are completely absorbed, and that there is no absorption by the atmosphere or other objects, what is the intensity 30.0 km away? (**Hint:** Half the power will be spread over the area of a hemisphere.) (b) What is the maximum electric field strength at this distance?

62. Suppose the maximum safe intensity of microwaves for human exposure is taken to be $1.00W/m^2$.

(a) If a radar unit leaks 10.0 W of microwaves (other than those sent by its antenna) uniformly in all directions, how far away must you be to be exposed to an intensity considered to be safe? Assume that the power spreads uniformly over the area of a sphere with no complications from absorption or reflection.

(b) What is the maximum electric field strength at the safe intensity? (Note that early radar units leaked more than modern ones do. This caused identifiable health problems, such as cataracts, for people who worked near them.)

63. A 2.50-m-diameter university communications satellite dish receives TV signals that have a maximum electric field strength (for one channel) of **7.50μV/m** (see below). (a) What is the intensity of this wave? (b) What is the power received by the antenna? (c) If the orbiting satellite broadcasts uniformly over an area of $1.50 \times 10^{13}m^2$ (a large fraction of North America), how much power does it radiate?



64. Lasers can be constructed that produce an extremely high intensity electromagnetic wave for a brief time—called pulsed lasers. They are used to initiate nuclear fusion, for example. Such a laser may produce an electromagnetic wave with a maximum electric field strength of $1.00 \times 10^{11} V/m$ for a time of 1.00 ns.

(a) What is the maximum magnetic field strength in the wave?

(b) What is the intensity of the beam?

(c) What energy does it deliver on an $1.00 - mm^2$ area?

16.5 Momentum and Radiation Pressure

65. A 150-W lightbulb emits 5% of its energy as electromagnetic radiation. What is the radiation pressure on an absorbing sphere of radius 10 m that surrounds the bulb?

66. What pressure does light emitted uniformly in all directions from a 100-W incandescent light bulb exert on a mirror at a distance of 3.0 m, if 2.6 W of the power is emitted as visible light?

67. A microscopic spherical dust particle of radius **2μm** and mass **10μg** is moving in outer space at a constant speed of 30 cm/sec. A wave of light strikes it from the opposite direction of its motion and gets absorbed. Assuming the particle decelerates uniformly to zero speed in one second, what is the average electric field amplitude in the light?

68. A Styrofoam spherical ball of radius 2 mm and mass **20μg** is to be suspended by the radiation pressure in a vacuum tube in a lab. How much intensity will be required if the light is completely absorbed the ball?

69. Suppose that $\vec{S}_{avg}$ for sunlight at a point on the surface of Earth is $900 W/m^2$.

(a) If sunlight falls perpendicularly on a kite with a reflecting surface of area 0.75m20.75m2, what is the average force on the kite due to radiation pressure?

(b) How is your answer affected if the kite material is black and absorbs all sunlight?

70. Sunlight reaches the ground with an intensity of about $1.0 kW/m^2$. A sunbather has a body surface area of 0.8m20.8m2 facing the sun while reclining on a beach chair on a clear day.

(a) how much energy from direct sunlight reaches the sunbather's skin per second?

(b) What pressure does the sunlight exert if it is absorbed?

71. Suppose a spherical particle of mass m and radius **R** in space absorbs light of intensity **I** for time **t**.

(a) How much work does the radiation pressure do to accelerate the particle from rest in the given time it absorbs the light?

(b) How much energy carried by the electromagnetic waves is absorbed by the particle over this time based on the radiant energy incident on the particle?

16.6 The Electromagnetic Spectrum

72. How many helium atoms, each with a radius of about 31 pm, must be placed end to end to have a length equal to one wavelength of 470 nm blue light?

73. If you wish to detect details of the size of atoms (about 0.2 nm) with electromagnetic radiation, it must have a wavelength of about this size.

(a) What is its frequency?

(b) What type of electromagnetic radiation might this be?

74. Find the frequency range of visible light, given that it encompasses wavelengths from 380 to 760 nm.

75. (a) Calculate the wavelength range for AM radio given its frequency range is 540 to 1600 kHz.

(b) Do the same for the FM frequency range of 88.0 to 108 MHz.

76. Radio station WWVB, operated by the National Institute of Standards and Technology (NIST) from Fort Collins, Colorado, at a low frequency of 60 kHz, broadcasts a time synchronization signal whose range covers the entire continental US. The timing of the synchronization signal is controlled by a set of atomic clocks to an accuracy of $1 \times 10^{-12} s$, and repeats every 1 minute. The signal is used for devices, such as radio-controlled watches, that automatically synchronize with it at preset local times. WWVB's long wavelength signal tends to propagate close to the ground.

(a) Calculate the wavelength of the radio waves from WWVB.

(b) Estimate the error that the travel time of the signal causes in synchronizing a radio controlled watch in Norfolk, Virginia, which is 1570 mi (2527 km) from Fort Collins, Colorado.

77. An outdoor WiFi unit for a picnic area has a 100-mW output and a range of about 30 m. What output power would reduce its range to 12 m for use with the same devices as before? Assume there are no obstacles in the way and that microwaves into the ground are simply absorbed.

78. The prefix "mega" (M) and "kilo" (k), when referring to amounts of computer data, refer to factors of 1024 or 210210rather than 1000 for the prefix **kilo**, and $1024^2 = 2^{20}$ rather than 1,000,000 for the prefix **Mega** (M). If a wireless (WiFi) router transfers 150 Mbps of data, how many bits per second is that in decimal arithmetic?

79. A computer user finds that his wireless router transmits data at a rate of 75 Mbps (megabits per second). Compare the average time to transmit one bit of data with the time difference between the wifi signal reaching an observer's cell phone directly and by bouncing back to the observer from a wall 8.00 m past the observer.

80. (a) The ideal size (most efficient) for a broadcast antenna with one end on the ground is one-fourth the wavelength (**λ/4**) of the electromagnetic radiation being sent out. If a new radio station has such an antenna that is 50.0 m high, what frequency does it broadcast most efficiently? Is this in the AM or FM band?

(b) Discuss the analogy of the fundamental resonant mode of an air column closed at one end to the resonance of currents on an antenna that is one-fourth their wavelength.

81. What are the wavelengths of (a) X-rays of frequency $2.0 \times 10^{17} Hz$?

(b) Yellow light of frequency $5.1 \times 10^{14} Hz$?

(c) Gamma rays of frequency $1.0 \times 10^{23} Hz$?

82. For red light of **λ=660nm**, what are f, ω and k?

83. A radio transmitter broadcasts plane electromagnetic waves whose maximum electric field at a particular location is $1.55 \times 10^{-3} V/m$. What is the maximum magnitude of the oscillating magnetic field at that location? How does it compare with Earth's magnetic field?

84. (a) Two microwave frequencies authorized for use in microwave ovens are: 915 and 2450 MHz. Calculate the wavelength of each.

(b) Which frequency would produce smaller hot spots in foods due to interference effects?

85. During normal beating, the heart creates a maximum 4.00-mV potential across 0.300 m of a person's chest, creating a 1.00-Hz electromagnetic wave.

(a) What is the maximum electric field strength created?

(b) What is the corresponding maximum magnetic field strength in the electromagnetic wave?

(c) What is the wavelength of the electromagnetic wave?

86. Distances in space are often quoted in units of light-years, the distance light travels in 1 year.

(a) How many meters is a light-year?

(b) How many meters is it to Andromeda, the nearest large galaxy, given that it is 2.54×10^6 ly away?

(c) The most distant galaxy yet discovered is 13.4×10^9 ly away. How far is this in meters?

87. A certain 60.0-Hz ac power line radiates an electromagnetic wave having a maximum electric field strength of 13.0 kV/m.

(a) What is the wavelength of this very-low-frequency electromagnetic wave?

(b) What type of electromagnetic radiation is this wave

(c) What is its maximum magnetic field strength?

88. (a) What is the frequency of the 193-nm ultraviolet radiation used in laser eye surgery? (b) Assuming the accuracy with which this electromagnetic radiation can ablate (reshape) the cornea is directly proportional to wavelength, how much more accurate can this UV radiation be than the shortest visible wavelength of light?

Additional Problems

89. In a region of space, the electric field is pointed along the x-axis, but its magnitude changes as described by

$$E_x = (10N/C)sin(20x - 500t)$$

$$E_y = E_z = 0$$

where **t** is in nanoseconds and **x** is in cm. Find the displacement current through a circle of radius 3 cm in the $x = 0$ plane at $t = 0$.

90. A microwave oven uses electromagnetic waves of frequency $f = 2.45 \times 10^9 Hz$ to heat foods. The waves reflect from the inside walls of the oven to produce an interference pattern of standing waves whose antinodes are hot spots that can leave observable pit marks in some foods. The pit marks are measured to be 6.0 cm apart. Use the method employed by Heinrich Hertz to calculate the speed of electromagnetic waves this implies.

Use the Appendix D for the next two exercises

91. Galileo proposed measuring the speed of light by uncovering a lantern and having an assistant a known distance away uncover his lantern when he saw the light from Galileo's lantern, and timing the delay. How far away must the assistant be for the delay to equal the human reaction time of about 0.25 s?

92. Show that the wave equation in one dimension

$$\frac{\partial^2 f}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}$$

is satisfied by any doubly differentiable function of either the form $f(x - vt)$ or $f(x + vt)$.

93. On its highest power setting, a microwave oven increases the temperature of 0.400 kg of spaghetti by **45.0°C** in 120 s.

(a) What was the rate of energy absorption by the spaghetti, given that its specific heat is $3.76 \times 10^3 J/kg \cdot °C$? Assume the spaghetti is perfectly absorbing.

(b) Find the average intensity of the microwaves, given that they are absorbed over a circular area 20.0 cm in diameter.

(c) What is the peak electric field strength of the microwave?

(d) What is its peak magnetic field strength?

94. A certain microwave oven projects 1.00 kW of microwaves onto a 30-cm-by-40-cm area.

(a) What is its intensity in W/m^2?

(b) Calculate the maximum electric field strength E_0 in these waves.

(c) What is the maximum magnetic field strength B_0 ?

95. Electromagnetic radiation from a 5.00-mW laser is concentrated on a $1.00 - mm^2$ area.

(a) What is the intensity in W/m^2?

(b) Suppose a 2.00-nC electric charge is in the beam. What is the maximum electric force it experiences?

(c) If the electric charge moves at 400 m/s, what maximum magnetic force can it feel?

96. A 200-turn flat coil of wire 30.0 cm in diameter acts as an antenna for FM radio at a frequency of 100 MHz. The magnetic field of the incoming electromagnetic wave is perpendicular to the coil and has a maximum strength of $1.00 \times 10^{-12} T$.

(a) What power is incident on the coil?

(b) What average emf is induced in the coil over one-fourth of a cycle?

(c) If the radio receiver has an inductance of **2.50μH**, what capacitance must it have to resonate at 100 MHz?

97. Suppose a source of electromagnetic waves radiates uniformly in all directions in empty space where there are no absorption or interference effects.

(a) Show that the intensity is inversely proportional to r^2, the distance from the source squared.

(b) Show that the magnitudes of the electric and magnetic fields are inversely proportional to r.

98. A radio station broadcasts its radio waves with a power of 50,000 W. What would be the intensity of this signal if it is received on a planet orbiting Proxima Centuri, the closest star to our Sun, at 4.243 ly away?

99. The Poynting vector describes a flow of energy whenever electric and magnetic fields are present. Consider a long cylindrical wire of radius **r** with a current **I** in the wire, with resistance **R** and voltage **V**. From the expressions for the electric field along the wire and the magnetic field around the wire, obtain the magnitude and direction of the Poynting vector at the

surface. Show that it accounts for an energy flow into the wire from the fields around it that accounts for the Ohmic heating of the wire.

100. The Sun's energy strikes Earth at an intensity of $1.37 kW/m^2$. Assume as a model approximation that all of the light is absorbed. (Actually, about 30% of the light intensity is reflected out into space.)

(a) Calculate the total force that the Sun's radiation exerts on Earth.

(b) Compare this to the force of gravity between the Sun and Earth.

Note: Earth's mass is $5.972 \times 10^{24} kg$.

101. If a **Lightsail** spacecraft were sent on a Mars mission, by what fraction would its propulsion force be reduced when it reached Mars?

102. Lunar astronauts placed a reflector on the Moon's surface, off which a laser beam is periodically reflected. The distance to the Moon is calculated from the round-trip time.

(a) To what accuracy in meters can the distance to the Moon be determined, if this time can be measured to 0.100 ns?

(b) What percent accuracy is this, given the average distance to the Moon is 384,400 km?

103. Radar is used to determine distances to various objects by measuring the round-trip time for an echo from the object.

(a) How far away is the planet Venus if the echo time is 1000 s?

(b) What is the echo time for a car 75.0 m from a highway police radar unit?

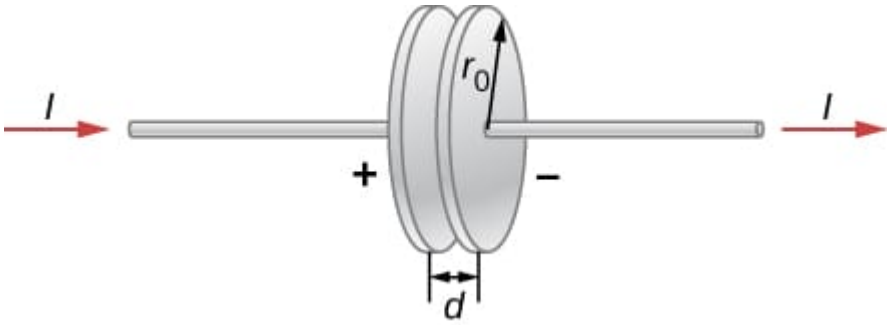
(c) How accurately (in nanoseconds) must you be able to measure the echo time to an airplane 12.0 km away to determine its distance within 10.0 m?

104. Calculate the ratio of the highest to lowest frequencies of electromagnetic waves the eye can see, given the wavelength range of visible light is from 380 to 760 nm. (Note that the ratio of highest to lowest frequencies the ear can hear is 1000.)

105. How does the wavelength of radio waves for an AM radio station broadcasting at 1030 KHz compare with the wavelength of the lowest audible sound waves (of 20 Hz). The speed of sound in air at **20°C** is about 343 m/s.

Challenge Problems

106. A parallel-plate capacitor with plate separation **d** is connected to a source of emf that places a time-dependent voltage **V(t)** across its circular plates of radius r_0 and area $A = \pi r_0^2$ (see below).



(a) Write an expression for the time rate of change of energy inside the capacitor in terms of **V(t)** and **dV(t)/dt**.

(b) Assuming that **V(t)** is increasing with time, identify the directions of the electric field lines inside the capacitor and of the magnetic field lines at the edge of the region between the plates, and then the direction of the Poynting vector

$\vec{S}$ at this location.

(c) Obtain expressions for the time dependence of **E(t)**, for **B(t)** from the displacement current, and for the magnitude of the Poynting vector at the edge of the region between the plates.

(d) From $\vec{S}$, obtain an expression in terms of **V(t)** and **dV(t)/dt** for the rate at which electromagnetic field energy enters the region between the plates.

(e) Compare the results of parts (a) and (d) and explain the relationship between them.

107. A particle of cosmic dust has a density $\rho = 2.0 g/cm^3$.

(a) Assuming the dust particles are spherical and light absorbing, and are at the same distance as Earth from the Sun, determine the particle size for which radiation pressure from sunlight is equal to the Sun's force of gravity on the dust particle.

(b) Explain how the forces compare if the particle radius is smaller.

(c) Explain what this implies about the sizes of dust particle likely to be present in the inner solar system compared with outside the Oort cloud.

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19.9: Electromagnetic Waves (Answer)

Check Your Understanding

16.1. It is greatest immediately after the current is switched on. The displacement current and the magnetic field from it are proportional to the rate of change of electric field between the plates, which is greatest when the plates first begin to charge.

16.2. No. The changing electric field according to the modified version of Ampère's law would necessarily induce a changing magnetic field.

16.3. (1) Faraday's law, (2) the Ampère-Maxwell law

16.4. a. The directions of wave propagation, of the **E** field, and of **B** field are all mutually perpendicular.

b. The speed of the electromagnetic wave is the speed of light $c = 1/\sqrt{\varepsilon_0 \mu_0}$ independent of frequency.

c. The ratio of electric and magnetic field amplitudes is $E/B = c$.

16.5. Its acceleration would decrease because the radiation force is proportional to the intensity of light from the Sun, which decreases with distance. Its speed, however, would not change except for the effects of gravity from the Sun and planets.

16.6. They fall into different ranges of wavelength, and therefore also different corresponding ranges of frequency.

Conceptual Questions

1. The current into the capacitor to change the electric field between the plates is equal to the displacement current between the plates.

3. The first demonstration requires simply observing the current produced in a wire that experiences a changing magnetic field. The second demonstration requires moving electric charge from one location to another, and therefore involves electric currents that generate a changing electric field. The magnetic fields from these currents are not easily separated from the magnetic field that the displacement current produces.

5. in (a), because the electric field is parallel to the wire, accelerating the electrons

7. A steady current in a dc circuit will not produce electromagnetic waves. If the magnitude of the current varies while remaining in the same direction, the wires will emit electromagnetic waves, for example, if the current is turned on or off.

9. The amount of energy (about $100 W/m^2$) is can quickly produce a considerable change in temperature, but the light pressure (about $3.00 \times 10^{-7} N/m^2$) is much too small to notice.

11. It has the magnitude of the energy flux and points in the direction of wave propagation. It gives the direction of energy flow and the amount of energy per area transported per second.

13. The force on a surface acting over time Δt is the momentum that the force would impart to the object. The momentum change of the light is doubled if the light is reflected back compared with when it is absorbed, so the force acting on the object is twice as great.

15. a. According to the right hand rule, the direction of energy propagation would reverse.

b. This would leave the vector $\vec{S}$, and therefore the propagation direction, the same.

17. a. Radio waves are generally produced by alternating current in a wire or an oscillating electric field between two plates;

b. Infrared radiation is commonly produced by heated bodies whose atoms and the charges in them vibrate at about the right frequency.

19. a. blue;

b. Light of longer wavelengths than blue passes through the air with less scattering, whereas more of the blue light is scattered in different directions in the sky to give it is blue color.

21. A typical antenna has a stronger response when the wires forming it are orientated parallel to the electric field of the radio wave.

23. No, it is very narrow and just a small portion of the overall electromagnetic spectrum.

25. Visible light is typically produced by changes of energies of electrons in randomly oriented atoms and molecules. Radio waves are typically emitted by an ac current flowing along a wire, that has fixed orientation and produces electric fields pointed in particular directions.

27. Radar can observe objects the size of an airplane and uses radio waves of about 0.5 cm in wavelength. Visible light can be used to view single biological cells and has wavelengths of about $10^{-7}m$.

29. ELF radio waves

31. The frequency of 2.45 GHz of a microwave oven is close to the specific frequencies in the 2.4 GHz band used for WiFi.

Problems

33. $B_{ind} = \frac{\mu_0}{P2\pi r} I_{ind} = \frac{\mu_0}{2\pi r}\varepsilon_0 \frac{\partial \Phi_E}{\partial t} = \frac{\mu_0}{2\pi r}\varepsilon_0 (A\frac{\partial E}{\partial t}) = \frac{\mu_0}{2\pi r}\varepsilon_0 A(\frac{1}{d}\frac{dV(t)}{dt}) = \frac{\mu_0}{2\pi r}[\frac{\varepsilon_0 A}{d}][\frac{1}{C}\frac{dQ(t)}{dt}] = \frac{\mu_0}{2\pi r}\frac{dQ(t)}{dt}$ because $C = \frac{\varepsilon_0 A}{d}$

35. a. $I_{res} = \frac{V_0 sin\omega t}{R}$;

b. $I_d = CV_0 \omega cos\omega t$;

c. $I_{real} = _{Ires} + \frac{dQ}{dt} = \frac{V_0 sin\omega t}{R} + CV_0 \frac{d}{dt} sin\omega t = \frac{V_0 sin\omega t}{R} + CV_0 \omega cos\omega t$; which is the sum of I_{res} and I_{real}, consistent with how the displacement current maintaining the continuity of current.

37. $1.77 \times 10^{-3} A$

39. $I_d = (7.97 \times 10^{-10} A) sin(150t)$

41. 499 s

43. 25 m

45. a. 5.00 V/m;

b. $9.55 \times 10^8 Hz$;

c. 31.4 cm;

d. toward the +**x**-axis;

e. $B = (1.67 \times 10^{-8} T) cos[kx - (6 \times 10^9 s^{-1})t + 0.40]\hat{k}$

47. $I_d = \pi \varepsilon_0 \omega R^2 E_0 sin(kx - \omega t)$

49. The magnetic field is downward, and it has magnitude $2.00 \times 10^{-8} T$.

51. a. $6.45 \times 10^{-3} V/m$;

b. 394 m

53. 11.5 m

55. $5.97 \times 10^{-3} W/m^2$

57. a. $E_0 = 1027 V/m, B_0 = 3.42 \times 10^{-6} T$;

b. $3.96 \times 10^{26} W$

59. $20.8 W/m^2$

61. a. $4.42 \times 10^{-6} W/m^2$;

b. $5.77 \times 10^{-2} V/m$

63. a. $7.47 \times 10^{-14} W/m^2$;

b. $3.66 \times 10 - ^{13} W$;

c. 1.12 W

65. $1.99 \times 10^{-11} N/m^2$

67. $F = ma = (p)(\pi r^2), p = \dfrac{ma}{\pi r^2} = \dfrac{\varepsilon_0}{2E_0^2}$

$$E_0 = \sqrt{\frac{2ma}{\varepsilon_0 \pi r^2}} = \sqrt{\frac{2(10^{-8} kg)(0.30 m/s^2)}{(8.854 \times 10^{-12} C^2/N \cdot m^2)(\pi)(2 \times 10^{-6} m)^2}}$$

$E_0 = 7.34 \times 10^6 V/m$

69. a. $4.50 \times 10^{-6} N$;

b. it is reduced to half the pressure, $2.25 \times 10^{-6} N$

71. a. $W = \dfrac{1}{2} \dfrac{\pi^2 r^4}{mc^2} I^2 t^2$;

b. $E = \pi r^2 It$

73. a. $1.5 \times 10^{18} Hz$;

b. X-rays

75. a. The wavelength range is 187 m to 556 m.

b. The wavelength range is 2.78 m to 3.41 m.

77. $P' = (\dfrac{12m}{30m})^2 (100 mW) = 16 mW$

79. time for 1 bit = 1.27×10^{-8} s, difference in travel time is 5.34×10^{-8} s

81. a. $1.5 \times 10^{-9} m$;

b. $5.9 \times 10^{-7} m$;

c. $3.0 \times 10^{-15} m$

83. $5.17 \times 10^{-12}T$, the non-oscillating geomagnetic field of 25–65 μT is much larger

85. a. $1.33 \times 10^{-2}V/m$;

b. $4.34 \times 10^{-11}T$;

c. $3.00 \times 10^{8}m$

87. a. $5.00 \times 10^{6}m$;

b. radio wave;

c. $4.33 \times 10^{-5}T$

Additional Problems

89. $I_d = (10N/C)(8.845 \times 10^{-12}C^2/N \cdot m^2)\pi(0.03m)^2(5000) = 1.25 \times 10^{-5}mA$

91. $3.75 \times 10^{7}km$, which is much greater than Earth's circumference

93. a. 564 W;

b. $1.80 \times 10^{4}W/m^2$;

c. $3.68 \times 10^{3}V/m$;

d. $1.23 \times 10^{-5}T$

95. a. $5.00 \times 10^{3}W/m^2$;

b. $3.88 \times 10^{-6}N$;

c. $5.18 \times 10^{-12}N$

97. a. $I = \dfrac{P}{A} = \dfrac{P}{4\pi r^2} \propto \dfrac{1}{r^2}$;

b. $I \propto E_0^2, B_0^2 \Rightarrow E_0^2, B_0^2 \propto \dfrac{1}{r^2} \Rightarrow E_0, B_0 \propto \dfrac{1}{r}$

99. Power into the wire= $\int \vec{S} \cdot d\vec{A} = (\dfrac{1}{\mu_0}EB)(2\pi rL) = \dfrac{1}{\mu_0}(\dfrac{V}{L})(\dfrac{\mu_0 i}{2\pi r})(2\pi rL) = iV = i^2R$

101. 0.431

103. a. $1.5 \times 10^{11}m$;

b. $5.0 \times 10^{-7}s$;

c. 33 ns

105. $sound: \lambda_{sound} = \dfrac{v_s}{f} = \dfrac{343m/s}{20.0Hz} = 17.2m$

$radio: \lambda_{radio} = \dfrac{c}{f} = \dfrac{3.00 \times 10^{8}m/s}{1030 \times 10^{3}Hz} = 291m; or 17.1\lambda_{sound}$

Challenge Problems

107. a. $0.29\mu m$;

b. The radiation pressure is greater than the Sun's gravity if the particle size is smaller, because the gravitational force varies as the radius cubed while the radiation pressure varies as the radius squared.

c. The radiation force outward implies that particles smaller than this are less likely to be near the Sun than outside the range of the Sun's radiation pressure.

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GLOSSARY

A

absolute temperature scale | scale, such as Kelvin, with a zero point that is absolute zero [OpenStax]

absolute zero | temperature at which the average kinetic energy of molecules is zero [OpenStax]

AC current | current that fluctuates sinusoidally with time at a fixed frequency [OpenStax]

AC voltage | voltage that fluctuates sinusoidally with time at a fixed frequency [OpenStax]

adiabatic process | process during which no heat is transferred to or from the system [OpenStax]

alternating current (ac) | flow of electric charge that periodically reverses direction [OpenStax]

ammeter | instrument that measures current [OpenStax]

ampere (amp) | SI unit for current; $1A = 1C/s$ [OpenStax]

Ampère's law | physical law that states that the line integral of the magnetic field around an electric current is proportional to the current [OpenStax]

area vector | vector with magnitude equal to the area of a surface and direction perpendicular to the surface [OpenStax]

average power | time average of the instantaneous power over one cycle [OpenStax]

Avogadro's number | N_A, the number of molecules in one mole of a substance; $N_A = 6.02 \times 10^{23}$ particles/mole [OpenStax]

B

back emf | emf generated by a running motor, because it consists of a coil turning in a magnetic field; it opposes the voltage powering the motor [OpenStax]

bandwidth | range of angular frequencies over which the average power is greater than one-half the maximum value of the average power [OpenStax]

Biot-Savart law | an equation giving the magnetic field at a point produced by a current-carrying wire [OpenStax]

Boltzmann constant | k_B, a physical constant that relates energy to temperature and appears in the ideal gas law; $k_B = 1.38 \times 10^{-23} J/K$ [OpenStax]

boundary | imagined walls that separate the system and its surroundings [OpenStax]

C

calorie (cal) | energy needed to change the temperature of 1.00 g of water by 1.00°C [OpenStax]

calorimeter | container that prevents heat transfer in or out [OpenStax]

calorimetry | study of heat transfer inside a container impervious to heat [OpenStax]

capacitance | amount of charge stored per unit volt [OpenStax]

capacitive reactance | opposition of a capacitor to a change in current [OpenStax]

capacitor | device that stores electrical charge and electrical energy [OpenStax]

Carnot cycle | cycle that consists of two isotherms at the temperatures of two reservoirs and two adiabatic processes connecting the isotherms [OpenStax]

Carnot engine | Carnot heat engine, refrigerator, or heat pump that operates on a Carnot cycle [OpenStax]

Carnot principle | principle governing the efficiency or performance of a heat device operating on a Carnot cycle: any reversible heat device working between two reservoirs must have the same efficiency or performance coefficient, greater than that of an irreversible heat device operating between the same two reservoirs [OpenStax]

Celsius scale | temperature scale in which the freezing point of water is 0°C and the boiling point of water is 100°C [OpenStax]

charging by induction | process by which an electrically charged object brought near a neutral object creates a charge separation in that object [OpenStax]

circuit | complete path that an electrical current travels along [OpenStax]

Clausius statement of the second law of thermodynamics | heat never flows spontaneously from a colder object to a hotter object [OpenStax]

closed system | system that is mechanically and thermally isolated from its environment [OpenStax]

coefficient of linear expansion | (α) material property that gives the change in length, per unit length, per $1-°C$ change in temperature; a constant used in the calculation of linear expansion; the coefficient of linear expansion depends to some degree on the temperature of the material [OpenStax]

coefficient of performance | measure of effectiveness of a refrigerator or heat pump [OpenStax]

coefficient of volume expansion | (β) similar to αα but gives the change in volume, per unit volume, per $1-°C$ change in temperature [OpenStax]

cold reservoir | sink of heat used by a heat engine [OpenStax]

conduction | heat transfer through stationary matter by physical contact [OpenStax]

conduction electron | electron that is free to move away from its atomic orbit [OpenStax]

conductor | material that allows electrons to move separately from their atomic orbits; object with properties that allow charges to move about freely within it [OpenStax]

continuous charge distribution | total source charge composed of so large a number of elementary charges that it must be treated as continuous, rather than discrete [OpenStax]

convection | heat transfer by the macroscopic movement of fluid [OpenStax]

conventional current | current that flows through a circuit from the positive terminal of a battery through the circuit to the negative terminal of the battery [OpenStax]

cosmic rays | comprised of particles that originate mainly from outside the solar system and reach Earth [OpenStax]

coulomb | SI unit of electric charge [OpenStax]

Coulomb force | another term for the electrostatic force [OpenStax]

Coulomb's law | mathematical equation calculating the electrostatic force vector between two charged particles [OpenStax]

critical point | for a given substance, the combination of temperature and pressure above which the liquid and gas phases are indistinguishable [OpenStax]

critical pressure | pressure at the critical point [OpenStax]

critical temperature | temperature at the critical point [OpenStax]

critical temperature | T_c at which the isotherm has a point with zero slope [OpenStax]

critical temperature | temperature at which a material reaches superconductivity [OpenStax]

current density | flow of charge through a cross-sectional area divided by the area [OpenStax]

cyclic process | process in which the state of the system at the end is same as the state at the beginning [OpenStax]

cyclotron | device used to accelerate charged particles to large kinetic energies [OpenStax]

cylindrical symmetry | system only varies with distance from the axis, not direction [OpenStax]

D

Dalton's law of partial pressures | physical law that states that the total pressure of a gas is the sum of partial pressures of the component gases [OpenStax]

dees | large metal containers used in cyclotrons that serve contain a stream of charged particles as their speed is increased [OpenStax]

degree Celsius | (°C) unit on the Celsius temperature scale [OpenStax]

degree Fahrenheit | (°F) unit on the Fahrenheit temperature scale [OpenStax]

degree of freedom

| independent kind of motion possessing energy, such as the kinetic energy of motion in one of the three orthogonal spatial directions [OpenStax]

diamagnetic materials | their magnetic dipoles align oppositely to an applied magnetic field; when the field is removed, the material is unmagnetized [OpenStax]

dielectric | insulating material used to fill the space between two plates [OpenStax]

dielectric breakdown | phenomenon that occurs when an insulator becomes a conductor in a strong electrical field [OpenStax]

dielectric constant | factor by which capacitance increases when a dielectric is inserted between the plates of a capacitor [OpenStax]

dielectric strength | critical electrical field strength above which molecules in insulator begin to break down and the insulator starts to conduct [OpenStax]

diode | nonohmic circuit device that allows current flow in only one direction [OpenStax]

dipole | two equal and opposite charges that are fixed close to each other [OpenStax]

dipole moment | property of a dipole; it characterizes the combination of distance between the opposite charges, and the magnitude of the charges [OpenStax]

direct current (dc) | flow of electric charge in only one direction [OpenStax]

disorder | measure of order in a system; the greater the disorder is, the higher the entropy [OpenStax]

displacement current | extra term in Maxwell's equations that is analogous to a real current but accounts for a changing electric field producing a magnetic field, even when the real current is present [OpenStax]

drift velocity | velocity of a charge as it moves nearly randomly through a conductor, experiencing multiple collisions, averaged over a length of a conductor, whose magnitude is the length of conductor traveled divided by the time it takes for the charges to travel the length [OpenStax]

E

eddy current | current loop in a conductor caused by motional emf [OpenStax]

efficiency (e) | output work from the engine over the input heat to the engine from the hot reservoir [OpenStax]

electric charge | physical property of an object that causes it to be attracted toward or repelled from another charged object; each charged object generates and is influenced by a force called an electric force [OpenStax]

electric dipole | system of two equal but opposite charges a fixed distance apart [OpenStax]

electric dipole moment | quantity defined as $\{vecp = q\vec{d}$ for all dipoles, where the vector points from the negative to positive charge [OpenStax]

electric field | physical phenomenon created by a charge; it "transmits" a force between a two charges [OpenStax]

electric flux | dot product of the electric field and the area through which it is passing [OpenStax]

electric force | noncontact force observed between electrically charged objects [OpenStax]

electric generator | device for converting mechanical work into electric energy; it induces an emf by rotating a coil in a magnetic field [OpenStax]

electric potential | potential energy per unit charge [OpenStax]

electric potential difference | the change in potential energy of a charge q moved between two points, divided by the charge. [OpenStax]

electric potential energy | potential energy stored in a system of charged objects due to the charges [OpenStax]

electrical conductivity | measure of a material's ability to conduct or transmit electricity [OpenStax]

electrical current | rate at which charge flows, $I = \frac{dQ}{dt}$ [OpenStax]

electrical power | time rate of change of energy in an electric circuit [OpenStax]

electromotive force (emf) | energy produced per unit charge, drawn from a source that produces an electrical current [OpenStax]

electron | particle surrounding the nucleus of an atom and carrying the smallest unit of negative charge [OpenStax]

electron-volt | energy given to a fundamental charge accelerated through a potential difference of one volt [OpenStax]

electrostatic attraction | phenomenon of two objects with opposite charges attracting each other [OpenStax]

electrostatic force | amount and direction of attraction or repulsion between two charged bodies; the assumption is that the source charges have no acceleration [OpenStax]

electrostatic precipitators | filters that apply charges to particles in the air, then attract those charges to a filter, removing them from the airstream [OpenStax]

electrostatic repulsion | phenomenon of two objects with like charges repelling each other [OpenStax]

electrostatics | study of charged objects which are not in motion [OpenStax]

emissivity | measure of how well an object radiates [OpenStax]

energy density | energy stored in a capacitor divided by the volume between the plates [OpenStax]

entropy | state function of the system that changes when heat is transferred between the system and the environment [OpenStax]

entropy statement of the second law of thermodynamics | entropy of a closed system or the entire universe never decreases [OpenStax]

environment | outside of the system being studied [OpenStax]

equation of state | describes properties of matter under given physical conditions [OpenStax]

equilibrium | thermal balance established between two objects or parts within a system [OpenStax]

equipartition theorem | theorem that the energy of a classical thermodynamic system is shared equally among its degrees of freedom [OpenStax]

equipotential line | two-dimensional representation of an equipotential surface [OpenStax]

equipotential surface | surface (usually in three dimensions) on which all points are at the same potential [OpenStax]

equivalent resistance | resistance of a combination of resistors; it can be thought of as the resistance of a single resistor that can replace a combination of resistors in a series and/or parallel circuit [OpenStax]

extensive variable | variable that is proportional to the amount of matter in the system [OpenStax]

F

Fahrenheit scale | temperature scale in which the freezing point of water is 32°F and the boiling point of water is 212°F [OpenStax]

Faraday's law | induced emf is created in a closed loop due to a change in magnetic flux through the loop [OpenStax]

ferromagnetic materials | contain groups of dipoles, called domains, that align with the applied magnetic field; when this field is removed, the material is still magnetized [OpenStax]

field line | smooth, usually curved line that indicates the direction of the electric field [OpenStax]

field line density | number of field lines per square meter passing through an imaginary area; its purpose is to indicate the field strength at different points in space [OpenStax]

first law of thermodynamics | the change in internal energy for any transition between two equilibrium states is $\Delta E_{int} = Q - W$ [OpenStax]

flux | quantity of something passing through a given area [OpenStax]

free electrons | also called conduction electrons, these are the electrons in a conductor that are not bound to any particular atom, and hence are free to move around [OpenStax]

G

gamma ray (γ ray) | extremely high frequency electromagnetic radiation emitted by the nucleus of an atom, either from natural nuclear decay or induced nuclear processes in nuclear reactors and weapons; the lower end of the γ -ray frequency range overlaps the upper end of the X-ray range, but γ rays can have the highest frequency of any electromagnetic radiation [OpenStax]

gauss | G, unit of the magnetic field strength; $1G = 10^{-4}T$ [OpenStax]

Gaussian surface | any enclosed (usually imaginary) surface [OpenStax]

greenhouse effect | warming of the earth that is due to gases such as carbon dioxide and methane that absorb infrared radiation from Earth's surface and reradiate it in all directions, thus sending some of it back toward Earth [OpenStax]

grounding | process of attaching a conductor to the earth to ensure that there is no potential difference between it and Earth [OpenStax]

H

Hall effect | creation of voltage across a current-carrying conductor by a magnetic field [OpenStax]

heat | energy transferred solely due to a temperature difference [OpenStax]

heat engine | device that converts heat into work [OpenStax]

heat of fusion | energy per unit mass required to change a substance from the solid phase to the liquid phase, or released when the substance changes from liquid to solid [OpenStax]

heat of sublimation | energy per unit mass required to change a substance from the solid phase to the vapor phase [OpenStax]

heat of vaporization | energy per unit mass required to change a substance from the liquid phase to the vapor phase [OpenStax]

heat pump | device that delivers heat to a hot reservoir [OpenStax]

heat transfer | movement of energy from one place or material to another as a result of a difference in temperature [OpenStax]

helical motion | superposition of circular motion with a straight-line motion that is followed by a charged particle moving in a region of magnetic field at an angle to the field [OpenStax]

henry (H) | unit of inductance, $1H = 1\Omega \cdot s$; it is also expressed as a volt second per ampere [OpenStax]

hot reservoir | source of heat used by a heat engine [OpenStax]

hysteresis | property of ferromagnets that is seen when a material's magnetic field is examined versus the applied magnetic field; a loop is created resulting from sweeping the applied field forward and reverse [OpenStax]

I

ideal gas | gas at the limit of low density and high temperature [OpenStax]

ideal gas law | physical law that relates the pressure and volume of a gas, far from liquefaction, to the number of gas molecules or number of

moles of gas and the temperature of the gas [OpenStax]

impedance | ac analog to resistance in a dc circuit, which measures the combined effect of resistance, capacitive reactance, and inductive reactance [OpenStax]

induced dipole | typically an atom, or a spherically symmetric molecule; a dipole created due to opposite forces displacing the positive and negative charges [OpenStax]

induced electric field | created based on the changing magnetic flux with time [OpenStax]

induced electric-dipole moment | dipole moment that a nonpolar molecule may acquire when it is placed in an electrical field [OpenStax]

induced electrical field | electrical field in the dielectric due to the presence of induced charges [OpenStax]

induced emf | short-lived voltage generated by a conductor or coil moving in a magnetic field [OpenStax]

induced surface charges | charges that occur on a dielectric surface due to its polarization [OpenStax]

inductance | property of a device that tells how effectively it induces an emf in another device [OpenStax]

inductive reactance | opposition of an inductor to a change in current [OpenStax]

inductive time constant | denoted by τ, the characteristic time given by quantity L/R of a particular series RL circuit [OpenStax]

inductor | part of an electrical circuit to provide self-inductance, which is symbolized by a coil of wire [OpenStax]

infinite straight wire | straight wire whose length is much, much greater than either of its other dimensions, and also much, much greater than the distance at which the field is to be calculated [OpenStax]

infrared radiation | region of the electromagnetic spectrum with a frequency range that extends from just below the red region of the visible light spectrum up to the microwave region, or from $0.74\mu m$ to $300\mu m$ [OpenStax]

ink jet printer | small ink droplets sprayed with an electric charge are controlled by electrostatic plates to create images on paper [OpenStax]

insulator | material that holds electrons securely within their atomic orbits [OpenStax]

intensive variable | variable that is independent of the amount of matter in the system [OpenStax]

internal energy | average of the total mechanical energy of all the molecules or entities in the system [OpenStax]

internal energy | sum of the mechanical energies of all of the molecules in it [OpenStax]

internal resistance | amount of resistance to the flow of current within the voltage source [OpenStax]

ion | atom or molecule with more or fewer electrons than protons [OpenStax]

irreversibility | phenomenon associated with a natural process [OpenStax]

irreversible process | process in which neither the system nor its environment can be restored to their original states at the same time [OpenStax]

isentropic | reversible adiabatic process where the process is frictionless and no heat is transferred [OpenStax]

isobaric process | process during which the system's pressure does not change [OpenStax]

isochoric process | process during which the system's volume does not change [OpenStax]

isothermal process | process during which the system's temperature remains constant [OpenStax]

J

Josephson junction | junction of two pieces of superconducting material separated by a thin layer of insulating material, which can carry a supercurrent [OpenStax]

junction rule | sum of all currents entering a junction must equal the sum of all currents leaving the junction [OpenStax]

K

Kelvin scale (K) | temperature scale in which 0 K is the lowest possible temperature, representing absolute zero [OpenStax]

Kelvin statement of the second law of thermodynamics | it is impossible to convert the heat from a single source into work without any other effect [OpenStax]

kilocalorie (kcal) | energy needed to change the temperature of 1.00 kg of water between 14.5°C and 15.5°C [OpenStax]

kinetic theory of gases | theory that derives the macroscopic properties of gases from the motion of the molecules they consist of [OpenStax]

Kirchhoff's rules | set of two rules governing current and changes in potential in an electric circuit [OpenStax]

L

latent heat coefficient | general term for the heats of fusion, vaporization, and sublimation [OpenStax]

law of conservation of charge | net electric charge of a closed system is constant [OpenStax]

LC circuit | circuit composed of an ac source, inductor, and capacitor [OpenStax]

Lenz's law | direction of an induced emf opposes the change in magnetic flux that produced it; this is the negative sign in Faraday's law [OpenStax]

linear charge density | amount of charge in an element of a charge distribution that is essentially one-dimensional (the width and height are much, much smaller than its length); its units are C/m [OpenStax]

loop rule | algebraic sum of changes in potential around any closed circuit path (loop) must be zero [OpenStax]

M

magnetic damping | drag produced by eddy currents [OpenStax]

magnetic dipole | closed-current loop [OpenStax]

magnetic dipole moment | term IA of the magnetic dipole, also called μ [OpenStax]

magnetic domains | groups of magnetic dipoles that are all aligned in the same direction and are coupled together quantum mechanically [OpenStax]

magnetic energy density | energy stored per volume in a magnetic field [OpenStax]

magnetic field lines | continuous curves that show the direction of a magnetic field; these lines point in the same direction as a compass points, toward the magnetic south pole of a bar magnet [OpenStax]

magnetic flux | measurement of the amount of magnetic field lines through a given area [OpenStax]

magnetic force | force applied to a charged particle moving through a magnetic field [OpenStax]

magnetic susceptibility | ratio of the magnetic field in the material over the applied field at that time; positive susceptibilities are either paramagnetic or ferromagnetic (aligned with the field) and negative susceptibilities are diamagnetic (aligned oppositely with the field) [OpenStax]

mass spectrometer | device that separates ions according to their charge-to-mass ratios [OpenStax]

Maxwell-Boltzmann distribution | function that can be integrated to give the probability of finding ideal gas molecules with speeds in the range between the limits of integration [OpenStax]

Maxwell's equations | set of four equations that comprise a complete, overarching theory of electromagnetism [OpenStax]

mean free path | average distance between collisions of a particle [OpenStax]

mean free time | average time between collisions of a particle [OpenStax]

mechanical equivalent of heat | work needed to produce the same effects as heat transfer [OpenStax]

Meissner effect | phenomenon that occurs in a superconducting material where all magnetic fields are expelled [OpenStax]

microwaves | electromagnetic waves with wavelengths in the range from 1 mm to 1 m; they can be produced by currents in macroscopic circuits and devices [OpenStax]

molar heat capacity at constant pressure | quantifies the ratio of the amount of heat added removed to the temperature while measuring at constant pressure [OpenStax]

molar heat capacity at constant volume | quantifies the ratio of the amount of heat added removed to the temperature while measuring at constant volume [OpenStax]

mole | quantity of a substance whose mass (in grams) is equal to its molecular mass [OpenStax]

most probable speed | speed near which the speeds of most molecules are found, the peak of the speed distribution function [OpenStax]

motionally induced emf | voltage produced by the movement of a conducting wire in a magnetic field [OpenStax]

motor (dc) | loop of wire in a magnetic field; when current is passed through the loops, the magnetic field exerts torque on the loops, which rotates a shaft; electrical energy is converted into mechanical work in the process [OpenStax]

mutual inductance | geometric quantity that expresses how effective two devices are at inducing emfs in one another [OpenStax]

N

net rate of heat transfer by radiation | $P_{net} = \sigma e A(T_2^4 - T_1^4)$ [OpenStax]

neutron | neutral particle in the nucleus of an atom, with (nearly) the same mass as a proton [OpenStax]

nonohmic | type of a material for which Ohm's law is not valid [OpenStax]

north magnetic pole | currently where a compass points to north, near the geographic North Pole; this is the effective south pole of a bar

magnet but has flipped between the effective north and south poles of a bar magnet multiple times over the age of Earth [OpenStax]

O

ohm | (Ω) unit of electrical resistance, $1\Omega = 1V/A$ [OpenStax]

Ohm's law | empirical relation stating that the current I is proportional to the potential difference V; it is often written as $V = IR$, where R is the resistance [OpenStax]

ohmic | type of a material for which Ohm's law is valid, that is, the voltage drop across the device is equal to the current times the resistance [OpenStax]

open system | system that can exchange energy and/or matter with its surroundings [OpenStax]

P

parallel combination | components in a circuit arranged with one side of each component connected to one side of the circuit and the other sides of the components connected to the other side of the circuit [OpenStax]

parallel-plate capacitor | system of two identical parallel conducting plates separated by a distance [OpenStax]

paramagnetic materials | their magnetic dipoles align partially in the same direction as the applied magnetic field; when this field is removed, the material is unmagnetized [OpenStax]

partial pressure | pressure a gas would create if it occupied the total volume of space available [OpenStax]

peak emf | maximum emf produced by a generator [OpenStax]

peak speed | same as "most probable speed" [OpenStax]

perfect engine | engine that can convert heat into work with 100%100% efficiency [OpenStax]

perfect refrigerator (heat pump) | refrigerator (heat pump) that can remove (dump) heat without any input of work [OpenStax]

permanent dipole | typically a molecule; a dipole created by the arrangement of the charged particles from which the dipole is created [OpenStax]

permeability of free space | μ_0, measure of the ability of a material, in this case free space, to support a magnetic field [OpenStax]

permittivity of vacuum | also called the permittivity of free space, and constant describing the strength of the electric force in a vacuum [OpenStax]

phase angle | amount by which the voltage and current are out of phase with each other in a circuit [OpenStax]

phase diagram | graph of pressure vs. temperature of a particular substance, showing at which pressures and temperatures the phases of the substance occur [OpenStax]

photoconductor | substance that is an insulator until it is exposed to light, when it becomes a conductor [OpenStax]

planar symmetry | system only varies with distance from a plane [OpenStax]

polarization | slight shifting of positive and negative charges to opposite sides of an object [OpenStax]

potential difference | difference in electric potential between two points in an electric circuit, measured in volts [OpenStax]

potential drop | loss of electric potential energy as a current travels across a resistor, wire, or other component [OpenStax]

power factor | amount by which the power delivered in the circuit is less than the theoretical maximum of the circuit due to voltage and current being out of phase [OpenStax]

Poynting vector | vector equal to the cross product of the electric-and magnetic fields, that describes the flow of electromagnetic energy through a surface [OpenStax]

principle of superposition | useful fact that we can simply add up all of the forces due to charges acting on an object [OpenStax]

proton | particle in the nucleus of an atom and carrying a positive charge equal in magnitude to the amount of negative charge carried by an electron [OpenStax]

pV diagram | graph of pressure vs. volume [OpenStax]

Q

quality factor | dimensionless quantity that describes the sharpness of the peak of the bandwidth; a high quality factor is a sharp or narrow resonance peak [OpenStax]

quasi-static process | evolution of a system that goes so slowly that the system involved is always in thermodynamic equilibrium [OpenStax]

R

radar | common application of microwaves; radar can determine the distance to objects as diverse as clouds and aircraft, as well as determine the speed of a car or the intensity of a rainstorm [OpenStax]

radiation | energy transferred by electromagnetic waves directly as a result of a temperature difference [OpenStax]

radiation pressure | force divided by area applied by an electromagnetic wave on a surface [OpenStax]

radio waves | electromagnetic waves with wavelengths in the range from 1 mm to 100 km; they are produced by currents in wires and circuits and by astronomical phenomena [OpenStax]

rate of conductive heat transfer | rate of heat transfer from one material to another [OpenStax]

RC circuit | circuit that contains both a resistor and a capacitor [OpenStax]

refrigerator | device that removes heat from a cold reservoir [OpenStax]

resistance | electric property that impedes current; for ohmic materials, it is the ratio of voltage to current, $R = V/I$ [OpenStax]

resistivity | intrinsic property of a material, independent of its shape or size, directly proportional to the resistance, denoted by ρ [OpenStax]

resonant frequency | frequency at which the amplitude of the current is a maximum and the circuit would oscillate if not driven by a voltage source [OpenStax]

reversible process | process that can be reverted to restore both the system and its environment back to their original states together [OpenStax]

reversible process | process in which both the system and the external environment theoretically can be returned to their original states [OpenStax]

right-hand rule-1 | using your right hand to determine the direction of either the magnetic force, velocity of a charged particle, or magnetic field [OpenStax]

RL circuit | A circuit with resistance and self-inductance [OpenStax]

RLC circuit | circuit with an ac source, resistor, inductor, and capacitor all in series. [OpenStax]

rms current | root mean square of the current [OpenStax]

rms voltage | root mean square of the voltage [OpenStax]

root-mean-square (rms) speed | square root of the average of the square (of a quantity) [OpenStax]

S

schematic | graphical representation of a circuit using standardized symbols for components and solid lines for the wire connecting the components [OpenStax]

self-inductance | effect of the device inducing emf in itself [OpenStax]

series combination | components in a circuit arranged in a row one after the other in a circuit [OpenStax]

shock hazard | hazard in which an electric current passes through a person [OpenStax]

solenoid | thin wire wound into a coil that produces a magnetic field when an electric current is passed through it [OpenStax]

south magnetic pole | currently where a compass points to the south, near the geographic South Pole; this is the effective north pole of a bar magnet but has flipped just like the north magnetic pole [OpenStax]

specific heat | amount of heat necessary to change the temperature of 1.00 kg of a substance by 1.00°C; also called “specific heat capacity” [OpenStax]

spherical symmetry | system only varies with the distance from the origin, not in direction [OpenStax]

SQUID | (Superconducting Quantum Interference Device) device that is a very sensitive magnetometer, used to measure extremely subtle magnetic fields [OpenStax]

static electricity | buildup of electric charge on the surface of an object; the arrangement of the charge remains constant (“static”) [OpenStax]

Stefan-Boltzmann law of radiation | $P = \sigma A e T^4$, where $\sigma = 5.67 \times 10^{-8} J/s \cdot m^2 \cdot K^4$ is the Stefan-Boltzmann constant, A is the surface area of the object, T is the absolute temperature, and e is the emissivity [OpenStax]

step-down transformer | transformer that decreases voltage and increases current [OpenStax]

step-up transformer | transformer that increases voltage and decreases current [OpenStax]

sublimation | phase change from solid to gas [OpenStax]

superconductivity | phenomenon that occurs in some materials where the resistance goes to exactly zero and all magnetic fields are expelled, which occurs dramatically at some low critical temperature (T_C) [OpenStax]

supercritical | condition of a fluid being at such a high temperature and pressure that the liquid phase cannot exist [OpenStax]

superposition | concept that states that the net electric field of multiple source charges is the vector sum of the field of each source charge calculated individually [OpenStax]

surface charge density | amount of charge in an element of a two-dimensional charge distribution (the thickness is small); its units are C/m^2 [OpenStax]

surroundings | environment that interacts with an open system [OpenStax]

T

temperature | quantity measured by a thermometer, which reflects the mechanical energy of molecules in a system [OpenStax]

terminal voltage | potential difference measured across the terminals of a source when there is no load attached [OpenStax]

tesla | SI unit for magnetic field: $1T = 1N/A - m$ [OpenStax]

thermal agitation | thermal motion of atoms and molecules in any object at a temperature above absolute zero, which causes them to emit and absorb radiation [OpenStax]

thermal conductivity | property of a material describing its ability to conduct heat [OpenStax]

thermal equilibrium | condition in which heat no longer flows between two objects that are in contact; the two objects have the same temperature [OpenStax]

thermal expansion | change in size or volume of an object with change in temperature [OpenStax]

thermal hazard | hazard in which an excessive electric current causes undesired thermal effects [OpenStax]

thermal stress | stress caused by thermal expansion or contraction [OpenStax]

thermodynamic process | manner in which a state of a system can change from initial state to final state [OpenStax]

thermodynamic system | object and focus of thermodynamic study [OpenStax]

third law of thermodynamics | absolute zero temperature cannot be reached through any finite number of cooling steps [OpenStax]

three-wire system | wiring system used at present for safety reasons, with live, neutral, and ground wires [OpenStax]

toroid | donut-shaped coil closely wound around that is one continuous wire [OpenStax]

transformer | device that transforms voltages from one value to another using induction [OpenStax]

transformer equation | equation showing that the ratio of the secondary to primary voltages in a transformer equals the ratio of the number of turns in their windings [OpenStax]

triple point | pressure and temperature at which a substance exists in equilibrium as a solid, liquid, and gas [OpenStax]

U

ultraviolet radiation | electromagnetic radiation in the range extending upward in frequency from violet light and overlapping with the lowest X-ray frequencies, with wavelengths from 400 nm down to about 10 nm [OpenStax]

universal gas constant | R, the constant that appears in the ideal gas law expressed in terms of moles, given by $R = N_A k_B$ [OpenStax]

V

Van de Graaff generator | machine that produces a large amount of excess charge, used for experiments with high voltage [OpenStax]

van der Waals equation of state | equation, typically approximate, which relates the pressure and volume of a gas to the number of gas molecules or number of moles of gas and the temperature of the gas [OpenStax]

vapor | gas at a temperature below the boiling temperature [OpenStax]

vapor pressure | pressure at which a gas coexists with its solid or liquid phase [OpenStax]

vapor pressure | partial pressure of a vapor at which it is in equilibrium with the liquid (or solid, in the case of sublimation) phase of the same substance [OpenStax]

velocity selector | apparatus where the crossed electric and magnetic fields produce equal and opposite forces on a charged particle moving with a specific velocity; this particle moves through the velocity selector not affected by either field while particles moving with different velocities are deflected by the apparatus [OpenStax]

visible light | narrow segment of the electromagnetic spectrum to which the normal human eye responds, from about 400 to 750 nm [OpenStax]

voltage | change in potential energy of a charge moved from one point to another, divided by the charge; units of potential difference are joules per coulomb, known as volt [OpenStax]

voltmeter | instrument that measures voltage [OpenStax]

volume charge density | amount of charge in an element of a three-dimensional charge distribution; its units are C/m^3 [OpenStax]

X

X-ray | invisible, penetrating form of very high frequency electromagnetic radiation, overlapping both the ultraviolet range and the γ-ray range [OpenStax]

xerography | dry copying process based on electrostatics [OpenStax]

Z

zeroth law of thermodynamics | law that states that if two objects are in thermal equilibrium, and a third object is in thermal equilibrium with one of those objects, it is also in thermal equilibrium with the other object [OpenStax]

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