

CALCULUS-BASED ELECTRICITY AND MAGNETISM (WORKBOOK)



Andrew Totah-McCarty

Chabot College

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1: STEADY CIRCUITS

1.1: Voltage

1.2: Current

1.3: Circuit Devices and Measurements

1.4: Equivalent Resistance and Circuit Analysis

1.1: Voltage

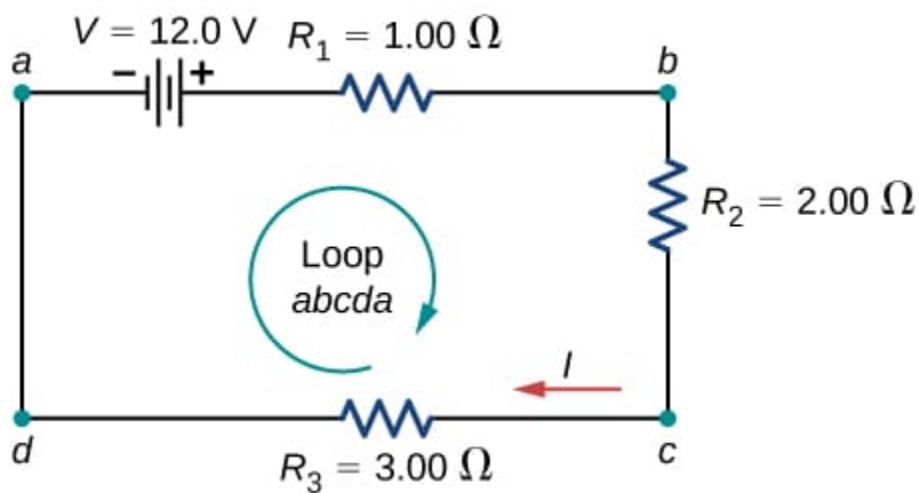
1. Voltage

Learning Objectives

- Model voltage changes across batteries, resistors, and wires qualitatively by annotating circuit schematics and graphing relative voltage changes on Voltage diagrams.
- Calculate voltage drops across different resistors along the same path without junctions when the total voltage drop in that path is known.
- Construct voltage equations for different loop paths using Kirchhoff's Voltage Rule in series and parallel circuits that include batteries, resistors, and wires.

1.1 Voltage in a Circuit

Model



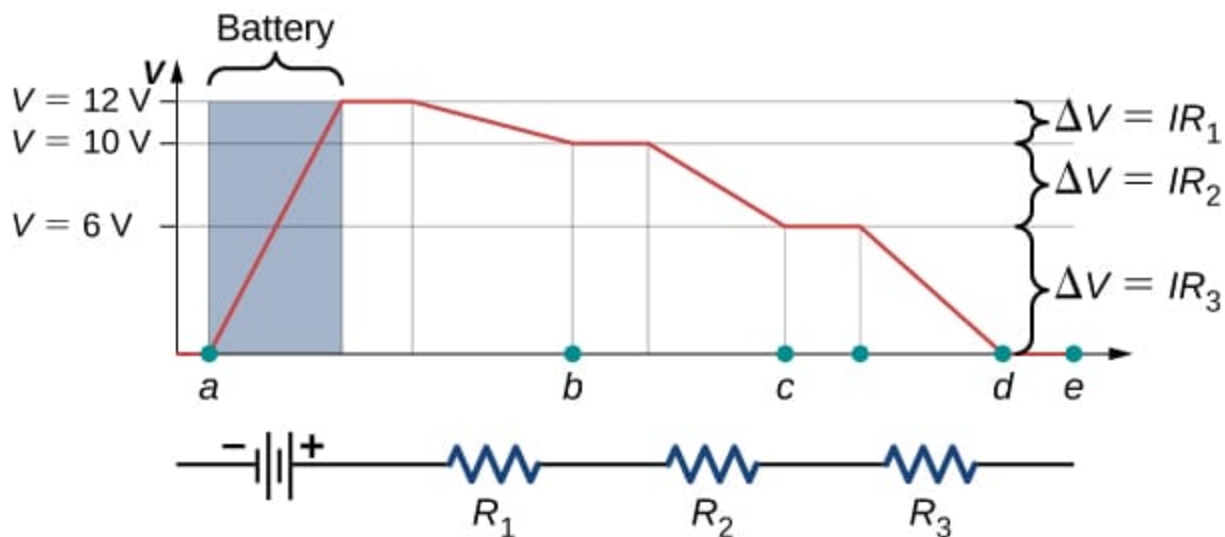


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Explore

Examine the Voltage vs. Length Diagrams (right).

1. What is the x-axis label and units? What is the y-axis label and units?
2. Look at the part of the graph with a positive slope. Does the voltage rise or fall during this part of the circuit? What circuit element (look below the graph) causes this rise or fall?
3. Look at the part of the graph with a negative slope. Does the voltage rise or fall during this part of the circuit? What circuit element (look below the graph) causes this rise or fall?
4. Use highlighters or colored pencils to highlight the horizontal parts of the graph using different colors for different voltage values.
5. What parts of the physical circuit correspond to the horizontal segments of the voltage graph? Use the same highlighter colors to color the circuit schematic to show which parts of the circuit correspond to those segments on the graph.

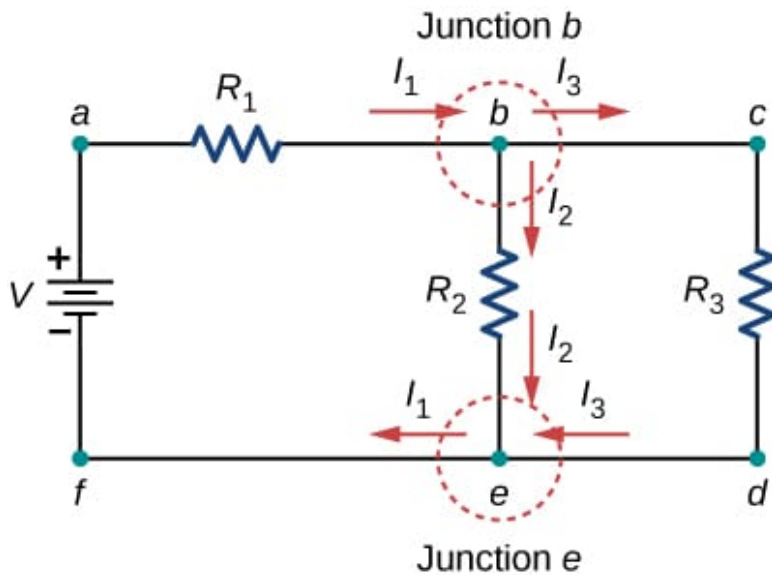
Critical Thinking & Concept Invention

1. Starting at point A, tell a story that identifies what circuit element is between each lettered point and whether the voltage increases, decreases, or does not change as it goes through that circuit element.
2. Identify patterns. How do the different components of a circuit affect the voltage in a circuit? Create an organized summary on your whiteboard to present the key ideas to the class.

Component	Effect on Voltage
Batteries	
Resistors	
Wires	

Apply

Extend your voltage patterns to the circuit below that has multiple paths.

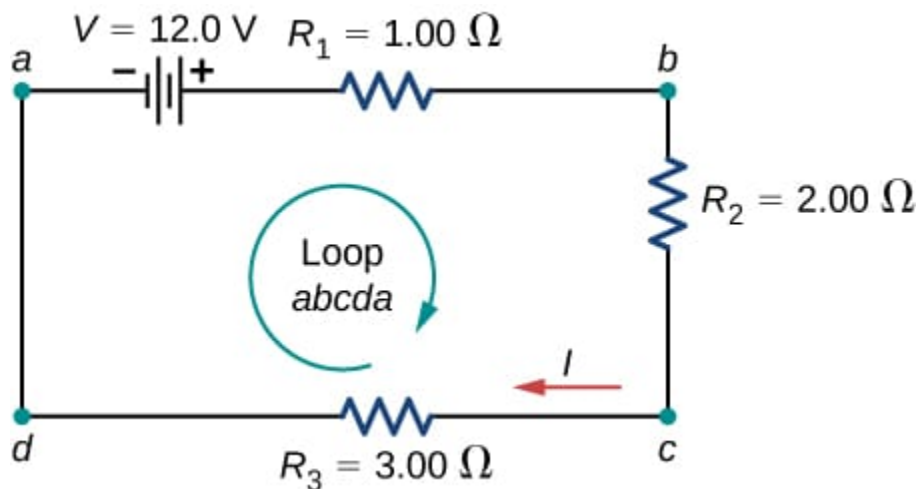


- Copy the circuit onto your whiteboard.
- Make the wires in the circuit different colors to show different values of voltage.
- Decide as a group if color changes or does not change as you pass through a junction. Make this statement on your whiteboard:

Color (Voltage) does change / does not change through junctions.

1.2 Voltage Splitting

Model



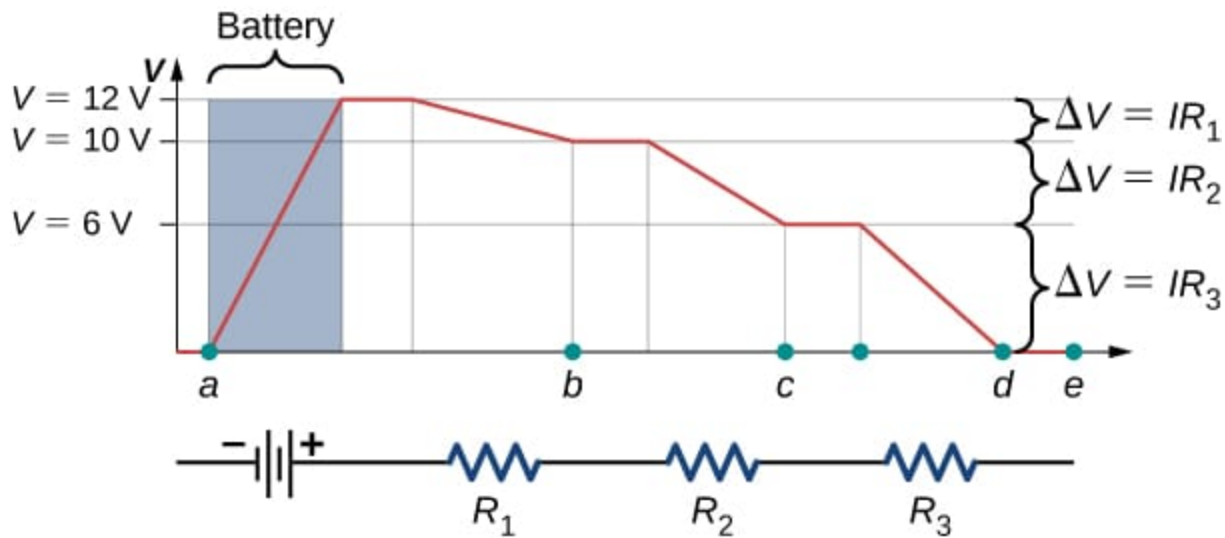


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Explore

1. Use the information from the model to fill out the table.

Resistor	Change in Voltage (ΔV)	Resistance (Ω)
R_1		
R_2		
R_3		
$R_1 + R_2 + R_3$		

Critical Thinking & Concept Invention

1. Identify patterns. As the voltage drops from the battery voltage to zero:
 - Circuit elements with more resistance have greater / lesser voltage drops.
 - A circuit element with no resistance would be:
 - a. a flat line
 - b. a vertical line
 - c. a line slanted down
 - d. a line slanted up
2. Part to Whole: Proportionally, what fraction of the total resistance is each resistor? Write an equation to calculate this for each resistor using variables first, then use the resistance values to calculate fractions below each equation.

R_1	R_2	R_3

3. Compare these fractions to the voltage drops for each resistor. How could you use the fractions to calculate the voltage drops

if they were not given on the voltage graph?

4. Generalize: Create an equation to determine the voltage drop for different resistors along the same path.

Note: This equation only holds true for resistors in series along a path without a junction.

Apply

A circuit is constructed with a 16 V battery, a $100\ \Omega$, $160\ \Omega$, and $220\ \Omega$ resistor. There are no junctions.

- Draw the circuit. Choose any location for the battery and resistors. Label them with their values.
- Draw a Kirchhoff path that will make a voltage rise for the battery and a voltage drop for the resistors.
- Use the voltage splitting equation to determine the voltage drop for each resistor.
- Draw a Voltage vs. Length diagram (the x-axis will be qualitative).

1.3 Kirchhoff Voltage Rule

Model

Circuit A

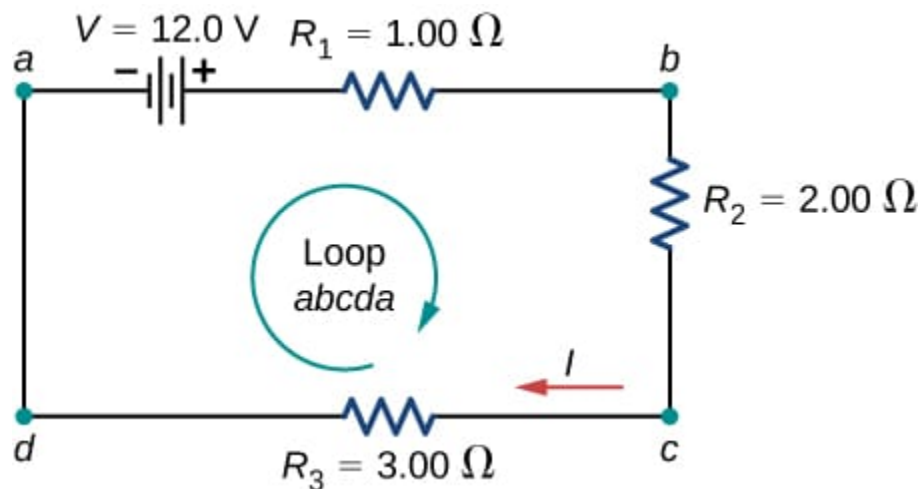


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Circuit B

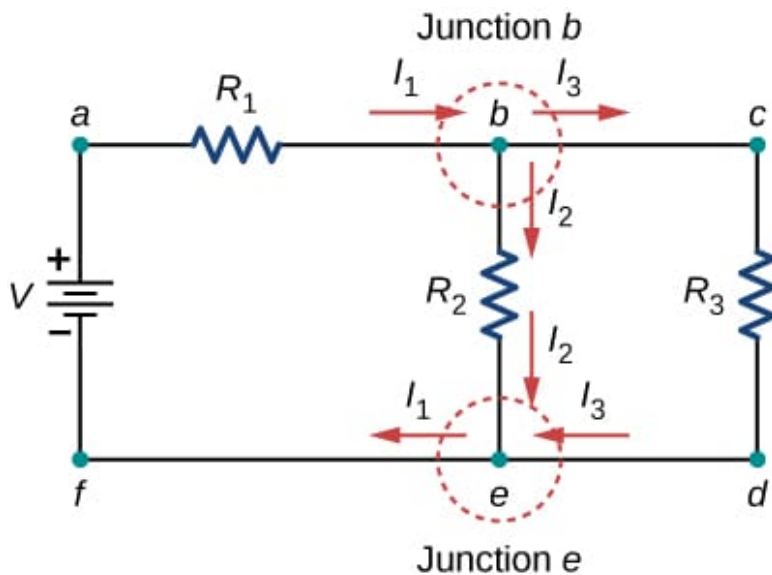


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Circuit A

$$\sum \Delta V_{abcda} = 0$$

Circuit B

$$\sum \Delta V = 0$$

$$\sum \Delta V = 0$$

Explore

1. How many circuit elements (batteries or resistors) exist along the loop $abcda$ in Circuit A?
2. Underneath the loop equation $\sum \Delta V_{abcda} = 0$, write a ΔV for each circuit element on the left side of the equation. Keep the right side equal to zero.
3. Draw two loops on Circuit B and label them based on the lettered points those loops pass through, similar to the loop on Circuit A:
 - Make the first loop go through the battery and resistor R_2 .
 - Make the second loop go through the battery and resistor R_3 .
4. Label subscripts on the two loop equations that include the lettered points that each separate path goes through. (Some letters might appear in both subscripts.)
5. How many circuit elements (batteries or resistors) exist along each of your loops in Circuit B?
6. Underneath the loop equations, write a ΔV for each circuit element on the left side of each equation. Keep the right side equal to zero.

Critical Thinking & Concept Invention

1. Kirchhoff's Voltage Rule is often stated as:

$$\sum \Delta V_{\text{closed loop}} = 0$$

Explain what this means using examples from the model and your additions to the model.

2. Explain how to generate voltage equations from a circuit using this rule.
3. Some students identify all circuit elements in the entire circuit (without drawing loops) and set the sum of all voltages equal to zero. Does this always work? Explain why or why not using examples from the model and the equations you created.

Apply

Analyze the circuit below by completing the following:

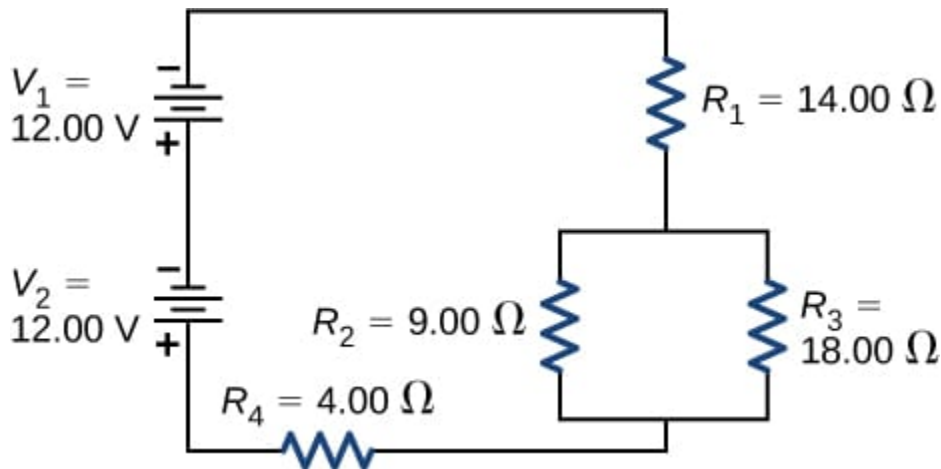


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- Draw two Kirchhoff loops and construct a voltage equation for each closed loop.
- Put (+) and (-) signs above each ΔV term in your equations.
- For each equation, fill in the battery values on a new line.
- Decide as a group whether the voltage splitting equation can be used here to solve for the voltage drops across each resistor. Explain why or why not.
- If you conclude that the voltage splitting equation can be used, determine the voltage drop across each resistor.

1.4 Multiple Batteries

Model

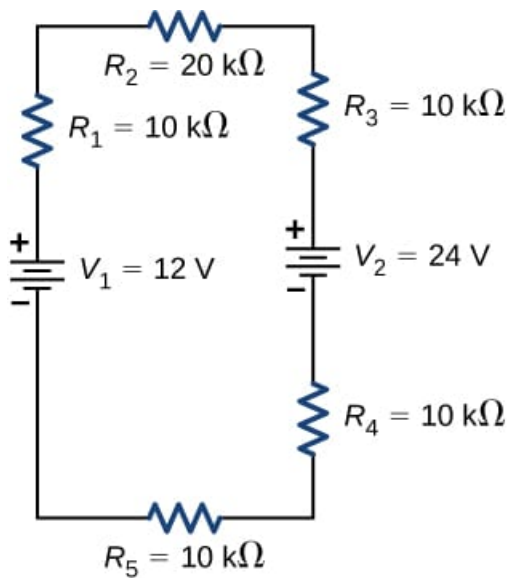


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Explore

1. Draw a loop that is counterclockwise on this circuit.
2. Examine each battery:
 - If the path is in the direction from positive to negative in the battery, put a plus sign.
 - If the path is in the direction from negative to positive, put a negative sign next to the battery.

Critical Thinking & Concept Invention

1. Analyze the circuit:
 - Create a Kirchhoff Voltage Rule equation for the circuit.
 - Add the battery voltages into that equation (include the appropriate + and - signs). What is the total voltage drop for all resistors?
 - Use the voltage splitting equation to determine the voltage drop for each resistor.
 - Construct a Voltage vs. Position graph next to the circuit schematic in the model. Show the symbols of the circuit elements underneath the x-axis and label each voltage rise or drop.
2. Summarize: How do batteries affect the voltage in a circuit? Be sure to discuss when and why voltage might increase or decrease as a result of a battery.

1.2: Current

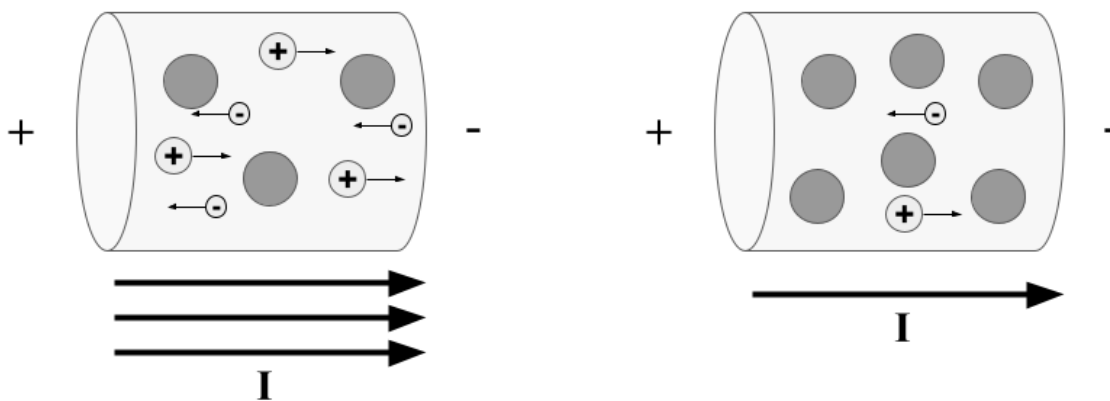
1. Current

Learning Objectives

- Qualitatively show how resistors and junctions affect current in a circuit by annotating circuits to show comparative amounts of current along different paths.
- Calculate the current, resistance, and voltage change across resistors using Ohm's Relation.
- Construct current equations for junctions using Kirchhoff's Current Rule.

1.1 Charge and Current

Model



Two different resistors are connected to the same voltage source. In this model, we zoom in to see the microscopic differences.

Explore

1. Identify the direction of each flow as from “+ to -” or from “- to +”.
 - Negative Charges
 - Positive Charges
 - Current (I)
2. Which wire (left or right) has more current passing through it?

Critical Thinking & Concept Invention

1. Which (left or right) model do you think it is easier for the charges to flow through? Why?
2. Which model (left or right) do you think has more resistance? Why do you think that?
3. Identify a Pattern:

When two resistors have the same change in voltage, the resistor with more resistance has _____ current.

Summary

The current in a circuit is based on the rate that charges flow through a circuit:

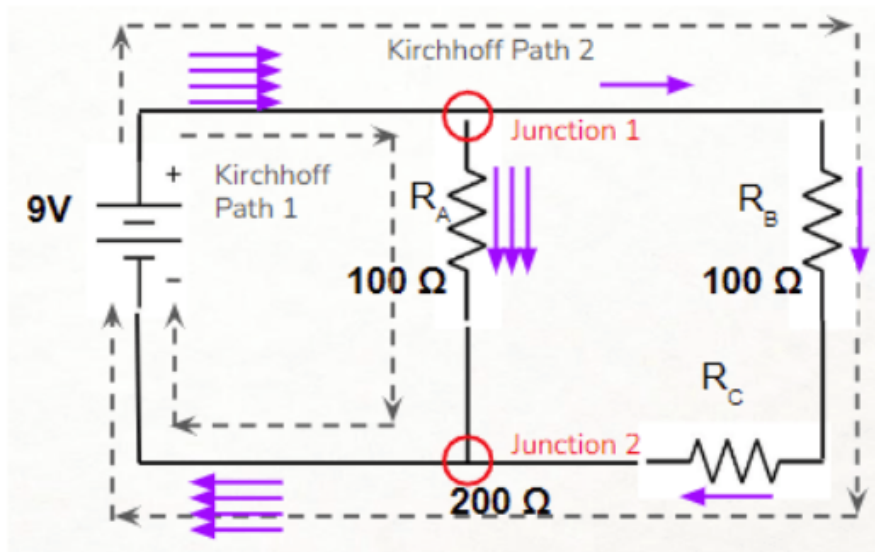
$$I \equiv \frac{dQ}{dt}$$

Typically negative charges are the ones that actually move through a circuit, but the current is defined as the direction of positive flow. This means that in most circuits, the current points in the opposite direction that the charges move.

The resistance of an object is based partly on the microscopic structure of that object (its resistivity). The geometry of the object (length and cross-sectional area) also affect the total resistance.

1.2 Kirchhoff Current Rule

Model



Explore

1. How does the amount of current (number of arrows) in R_B compare to R_C ?
2. Examine each Kirchhoff Path and identify the amount of current (based on the number of arrows) in a given circuit element on each path. Also identify the current in and out of each junction.

Path	Total Path Resistance	Number of Arrows Through Each Circuit Element
1		
2		

- Examine each junction. Identify the number of arrows (amount of current) going into and out of each junction.

Junction	Current (Number of Arrows) Going Into the Junction	Current (Number of Arrows) Coming Out of the Junction
1		
2		

Critical Thinking & Concept Invention

- Identify trends:
 - Resistors along the same path have (the same / different) amounts of current.
 - Paths with more resistance have (more / less) current than paths with less resistance.
- Construct a rule:

At a junction, the current going into the junction _____.

- Circuit Models:

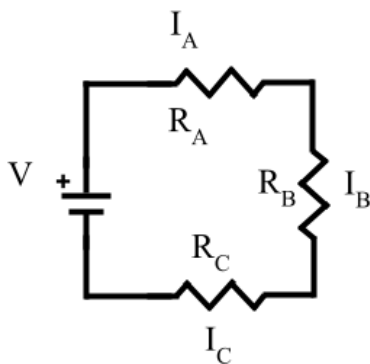
We show relative amounts of current in different parts of a circuit schematic by _____.

Apply

The circuit below has three resistors.

$$R_A > R_B \text{ and } R_B = R_C$$

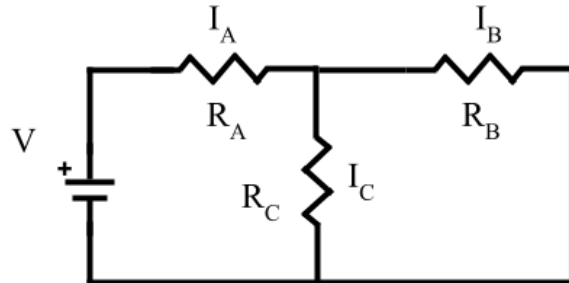
- Rank the current through each resistor (I_A , I_B , I_C) using $>$, $<$, and $=$ signs.
- Rank the voltage drop across each resistor (ΔV_A , ΔV_B , ΔV_C) using $>$, $<$, and $=$ signs.
- Draw arrows next to each resistor to show the direction of current and compare the relative amount of current going through the resistor.



The circuit below has three resistors.

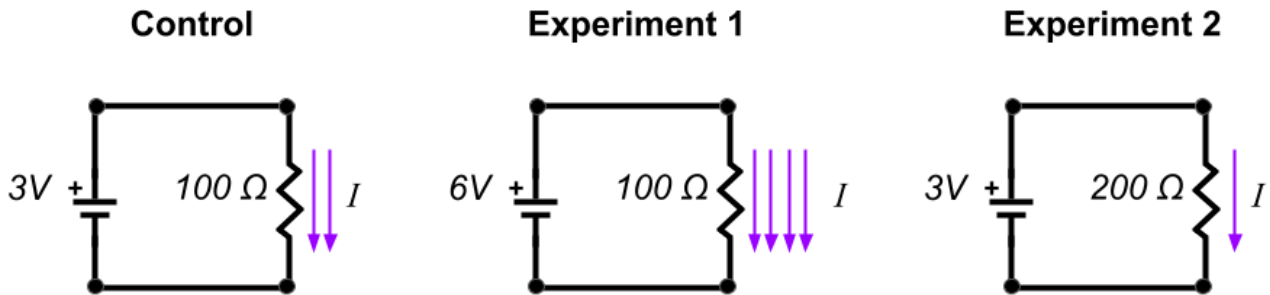
$$R_A > R_B \text{ and } R_B > R_C$$

- Rank the current through each resistor (I_A , I_B , I_C) using $>$, $<$, and $=$ signs.
- Construct a mathematical equation that relates the currents in all three resistors (I_A , I_B , I_C).
- Rank the voltage drop across each resistor (ΔV_A , ΔV_B , ΔV_C) using $>$, $<$, and $=$ signs.
- Draw arrows next to each resistor to show the direction of current and compare the relative amount of current going through the resistor.



1.3 Ohm's Relation

Model



Explore

1. Fill in the blanks to identify which quantity changed and if that caused the current to increase or decrease.

In Experiment 1, _____ increased, which caused the current to _____.

In Experiment 2, _____ increased, which caused the current to _____.

Critical Thinking & Concept Invention

1. Select the appropriate proportionality:

$$I \propto \Delta V$$

OR

$$I \propto \frac{1}{\Delta V}$$

$$I \propto R$$

OR

$$I \propto \frac{1}{R}$$

- Combining the two proportionalities together gives an equation for current. Then rearrange that same equation to solve for the change in voltage.

$$I =$$

$$\Delta V =$$

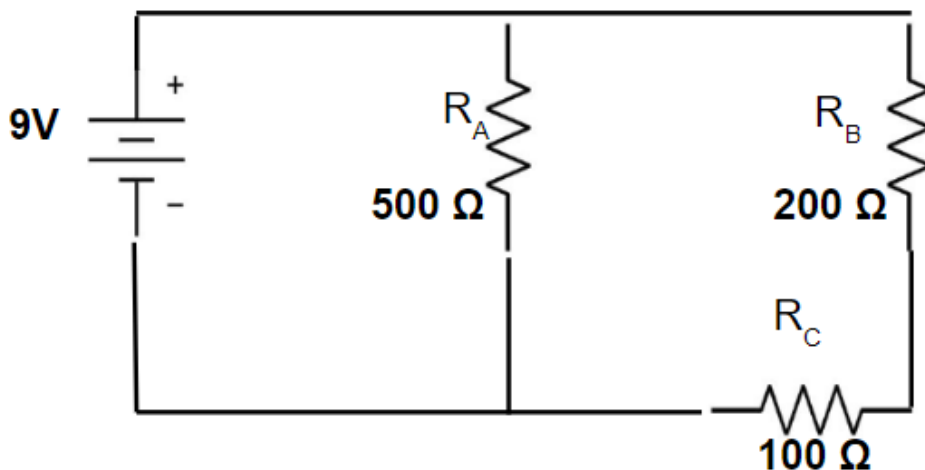
Summary

The relationship between the voltage difference, resistance, and current is called **Ohm's Relation**. It is only valid for resistors and gives the magnitude of those quantities. The sign (+/-) of the voltage difference is based on the direction of our Kirchhoff path through resistors, batteries, and other circuit elements.

Apply

Mark up and analyze the circuit below:

- Draw two Kirchhoff Paths and add arrows for current near each junction (label them) and through each resistor. Verify that your arrows align with Kirchhoff's Current Law and patterns relating current and resistance.
- Determine the Voltage Drop (ΔV) for each resistor (remember Voltage Splitting).
- Use Ohm's Relation to determine the current in each resistor (I_A , I_B , I_C).
- Determine the current through the battery (I_0) and explain how you determined this value.



1.4 Voltage Sign Conventions

Model

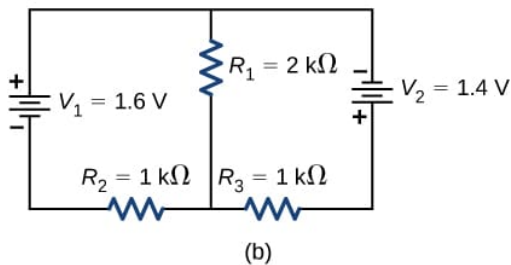
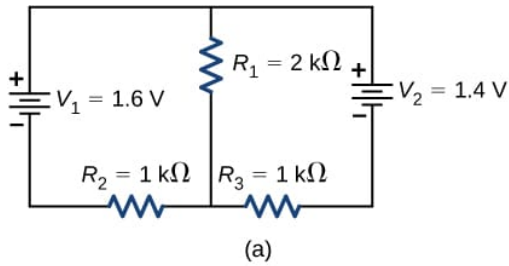


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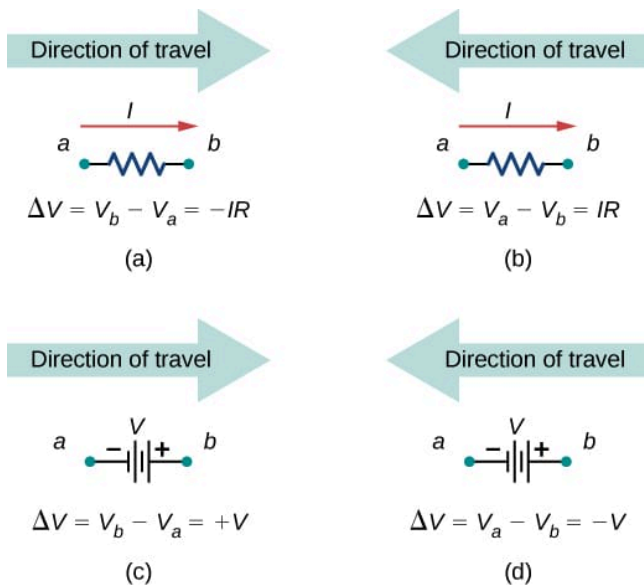


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Explore

- Quantitatively Analyze the Circuit:
 - Color code the circuits to show different wire voltages.
 - Draw two Kirchhoff Paths for each circuit.
 - Construct a set of voltage equations for each path.

- Put a (+) or (-) above the battery voltage terms in the equation using the key to the left of the circuits.
- Put a (+) or (-) above the resistor voltage terms in the equation using the key to the left of the circuits.

Critical Thinking & Concept Invention

1. Solve the Circuit Equations:
 - Substitute the battery voltages (with +/- signs) in for the battery voltage terms in the equations.
 - Substitute Ohm's Relation for the resistor voltage terms in the equations.
 - Identify the current magnitudes and directions in each circuit from your equations.
2. Annotate the circuit by drawing current arrows on each path and next to each resistor. Check that they follow the Kirchhoff Current Rule.
3. Summarize: How do you take into account the sign for resistor and battery voltage changes (ΔV 's)?

Apply

Construct Voltage and Current equations to determine the magnitude of:

V_1 , V_2 , and R_4

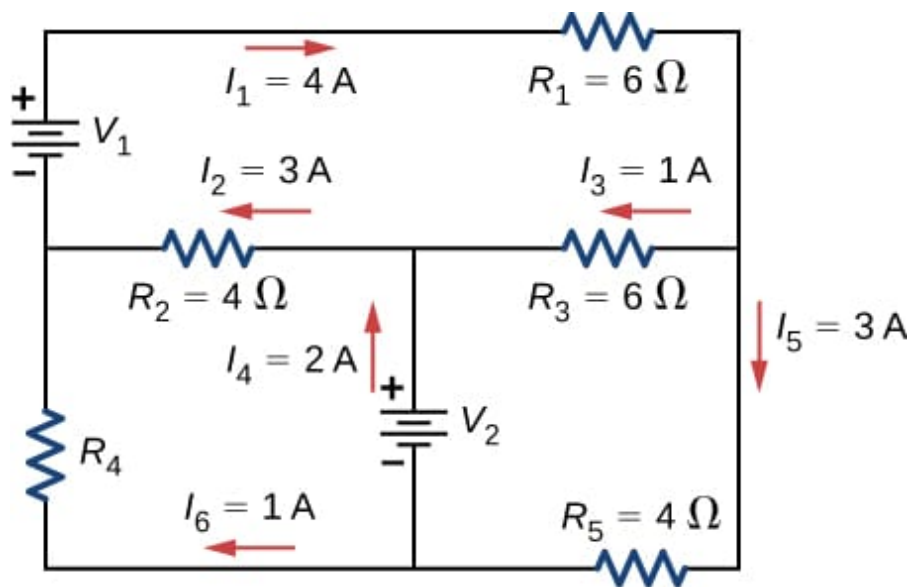


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1.3: Circuit Devices and Measurements

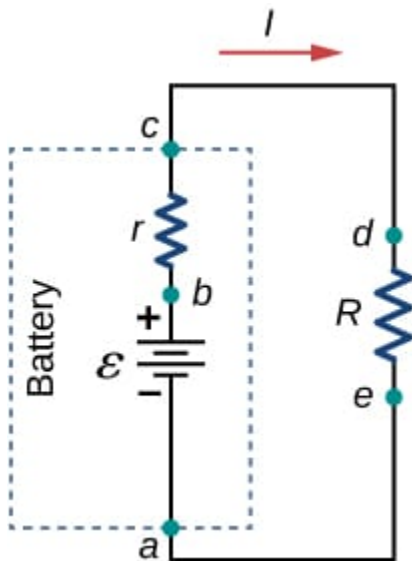
1. Circuit Devices and Measurements

Learning Objectives

- Model physical batteries as a voltage source with a small resistor.
 - Describe and calculate the effects of ideal voltmeters and ammeters on circuits based on how they are connected to those circuits.
 - Calculate the power dissipated in a resistor and describe how the current through and change in voltage across the resistor affect this quantity.
-

1.1 Batteries and Internal Resistance

Model



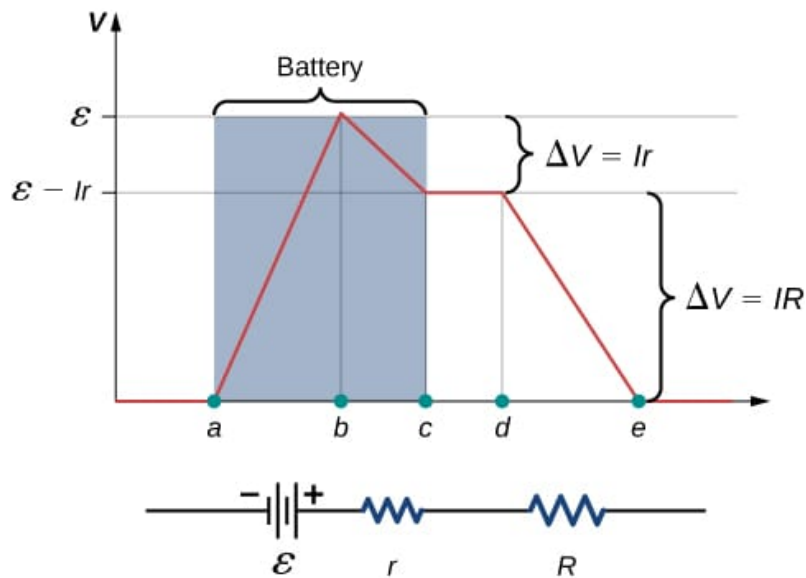


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Explore

1. Color-code the wires on the circuit and voltage diagram to show different voltage values.
2. Which letters represent the physical terminals of the battery?
3. Below the model, construct a Voltage Equation for the circuit using the Kirchhoff Voltage Rule that follows the circuit clockwise.
4. In your voltage equation, draw one box around the term(s) that represent the entire physical battery with a dashed line.

Critical Thinking & Concept Invention

1. Identify the terminal voltage of the battery as an equation.

$$\Delta V_{\text{terminal}} =$$

2. Identify a pattern:

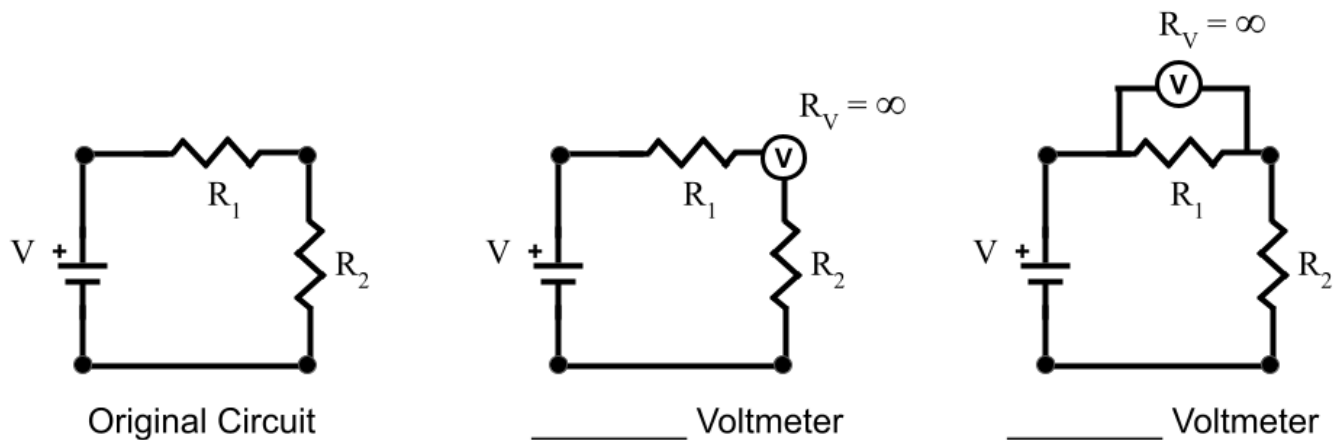
As the current through the battery increases, $\Delta V_{\text{terminal}}$...

3. If the internal resistance of the battery is $r = 5\Omega$, identify the terminal voltage of the battery for the following currents:
 - $I = 0.1A$
 - $I = 0.3A$
 - $I = 1A$
 - $I = 0A$
4. Identify the resistance of the resistor (R) for each of the currents above.

1.2 Voltage Meters

Model

A circuit with two resistors is shown below. We want to use a voltmeter to measure the voltage drop of resistor R_1 . The circle with a V represents the voltmeter device, which measures the voltage difference across itself.



Explore

- Annotate the models with the following:
 - Color code the wires to show different voltage values.
 - Set up voltage equation(s) for the different paths and current equations for any junctions. Be sure to have equations for the resistor and voltmeter in each circuit (they might be in the same equation or need their own).
 - Label the voltmeter “series” if it is in series with the resistor and “parallel” if it is in parallel with the resistor.
- According to the diagram, what is the resistance of a voltmeter?

Critical Thinking & Concept Invention

- Visually, which configuration looks like the voltage difference across the voltmeter matches a resistor in the circuit? Explain your thinking.
- Calculate the voltage difference and current through the resistor and voltmeter in each case. Use values:

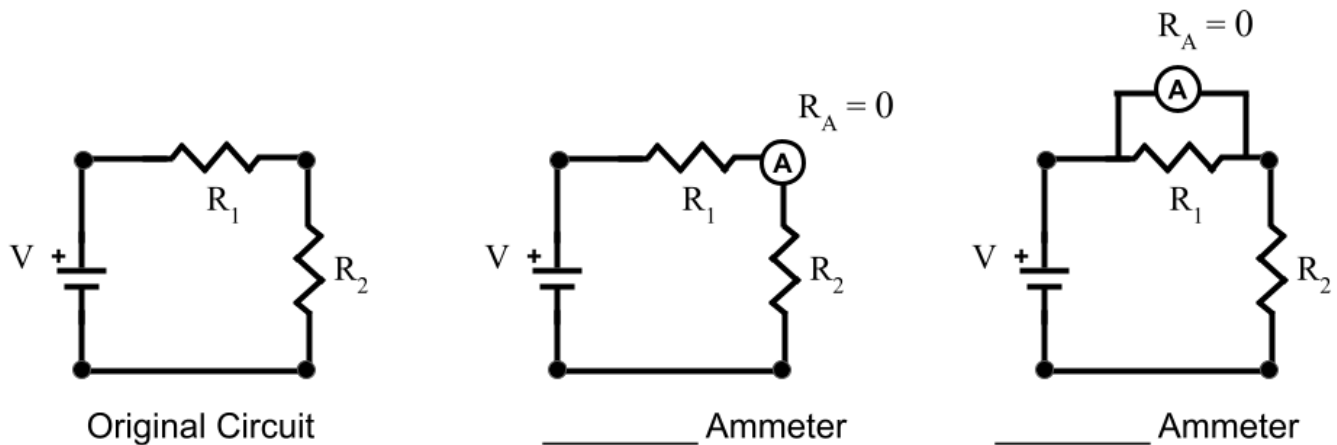
$$V = 12V, \quad R_1 = 150\Omega, \quad R_2 = 175\Omega$$

- Based on your calculations, should voltmeters be set up in series or parallel with resistors? Explain your reasoning.
- What might be problematic about setting up a voltmeter the wrong way?

1.3 Current Meters

Model

A circuit with two resistors is shown below. We want to use an ammeter to measure the current through resistor R_1 . The circle with an A represents the ammeter device, which measures the current going through itself.



Explore

- Annotate the models with the following:
 - Color code the wires to show different voltage values.
 - Set up voltage equation(s) for the different paths and current equations for any junctions. Be sure to have equations for the resistor and voltmeter in each circuit (they might be in the same equation or need their own).
 - Label the ammeter “series” if it is in series with the resistor and “parallel” if it is in parallel with the resistor.
- According to the diagram, what is the resistance of an ammeter?

Critical Thinking & Concept Invention

- Visually, which configuration looks like the current through the ammeter matches the current through a resistor in the circuit? Explain your thinking.
- Calculate the voltage difference and current through the resistor and ammeter in each case. Use values:

$$V = 12V, \quad R_1 = 150\Omega, \quad R_2 = 175\Omega$$

- Based on your calculations, should ammeters be set up in series or parallel with resistors? Explain your reasoning.
- What might be problematic about setting up an ammeter the wrong way?

Summary

The way we connect voltmeters and ammeters to a circuit can have an impact on the voltages and currents in the circuit. If connected incorrectly, the measurements will not reflect the same values in the circuit when those devices are not present and the circuit could become dangerous or destructive.

Voltmeters are connected in _____ with resistors because resistors in _____ have the same voltage drop. Voltmeters have a resistance of _____ so that they draw zero current.

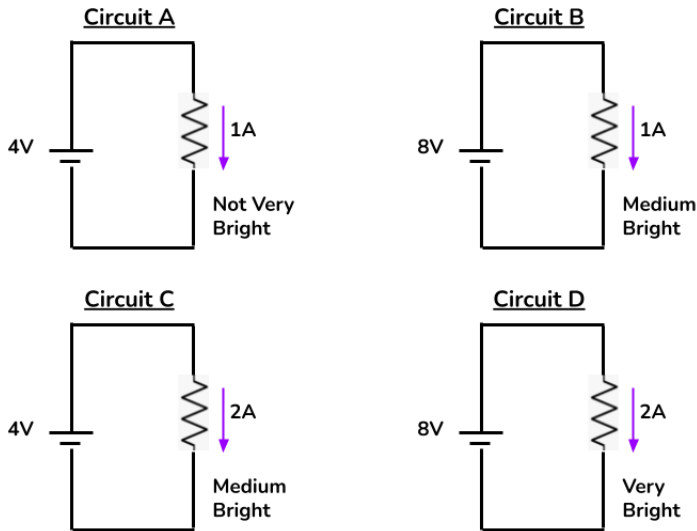
Ammeters are connected in _____ with resistors because resistors in _____ have the same current. Ammeters have a resistance of _____ so that they do not result in a voltage drop.

Memorizing these patterns can be a helpful shortcut, but the important thing is that you can use the resistances of ideal

voltmeters and ammeters to predict their effect and measured values in a circuit by treating them like resistors.

1.4 Brightness and Power

Model



Explore

1. Fill in the table to organize information about voltage, current, and light brightness.

Circuit	Voltage Drop	Current	Brightness
A			
B			
C			
D			

2. Summarize how brightness seems to relate to voltage and current.

Critical Thinking & Concept Invention

1. What we perceive as brightness is based on the power of the lightbulb. Identify the appropriate proportionalities based on the previous patterns.

$$P \propto I \text{ OR } P \propto \frac{1}{I}$$

$$P \propto \Delta V \text{ OR } P \propto \frac{1}{\Delta V}$$

2. Use your two proportional relationships to suggest an equation for power. Then use Ohm's Law to create two other versions

of the equation: one without current and one without voltage.

$$P =$$

$$P =$$

$$P =$$

3. As a group, derive the equation for power a different way:

- Use the relationship that $P = \frac{dE}{dt}$ and that charges moving through a circuit satisfy $dE = dU_{ele}$ to construct an equation for power below the energy equation.
- Group the differential quantities together. As a group, identify what this differential ratio represents and rewrite the equation below.

$$dU_{ele} = dQ \Delta V$$

$$P_{ele} =$$

$$P_{ele} =$$

Summary

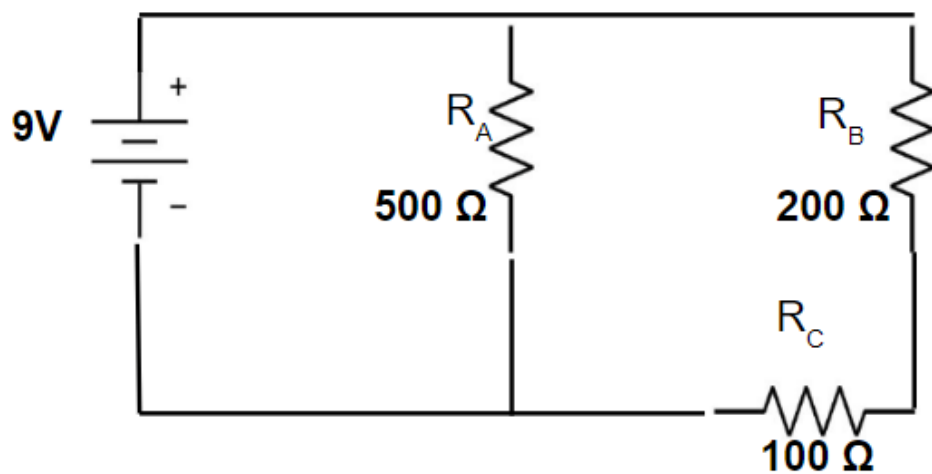
Power represents how much energy flows into or out of a system every second. Power has units of Watts (W), which are Joules per second (J/s).

In lightbulbs and resistors, power is typically negative because energy leaves the circuit. The negative sign comes from the ΔV part of the equation when using $P = I\Delta V$. It is often easier to calculate the magnitude of power (absolute value) and interpret the direction energy flows based on the context of the problem.

This is necessary if you use either of the other equations for power since they involve squared quantities.

Apply

Determine the power in each resistor and the total power from the battery (sum of all powers). Discuss which version of the power equation you want to use.



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1.4: Equivalent Resistance and Circuit Analysis

1. Equivalent Resistance

Learning Objectives

- Use Kirchhoff Rules to reduce a circuit with multiple resistors in Series into an equivalent circuit.
- Use Kirchhoff Rules to reduce a circuit with multiple resistors in Parallel into an equivalent circuit.
- Apply Equivalent Resistance rules for series and parallel resistors to reduce complex circuits into an equivalent circuit.

1.1 Equivalent Resistance for Series Circuits

Model

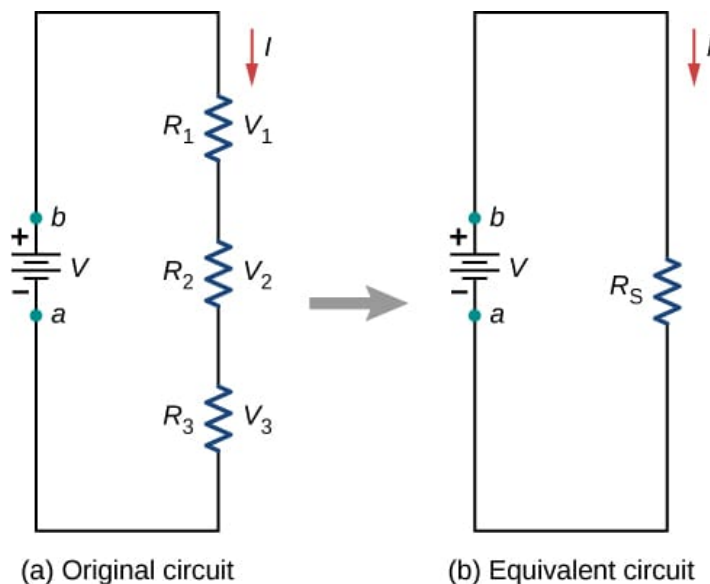


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Explore

- Color-code wires in each circuit to show the different voltage values. Be consistent with which colors mark the voltage of the positive side of the battery and the negative side of the battery.
- Construct a voltage equation for each circuit using Kirchhoff's Voltage Rule.

Critical Thinking & Concept Invention

Use both voltage equations to make comparisons between the two circuits:

- Use the battery value and Ohm's Relation to substitute in for the voltage changes in each circuit.
- Solve the equation for the current (I) in the original circuit.
- Substitute that expression for I from the original circuit into the equivalent circuit and solve for R_S .
- Interpret the result: How does the equivalent resistance of multiple resistors in series compare to the resistance of each of those resistors?

Summary

For resistors connected in series:

$$R_S = R_1 + R_2 + R_3 + \dots$$

The equivalent resistance of resistors in series is the sum of the individual resistances.

Apply

Use the concept of equivalent resistance to determine the current and voltage drop of each resistor in the circuit below.

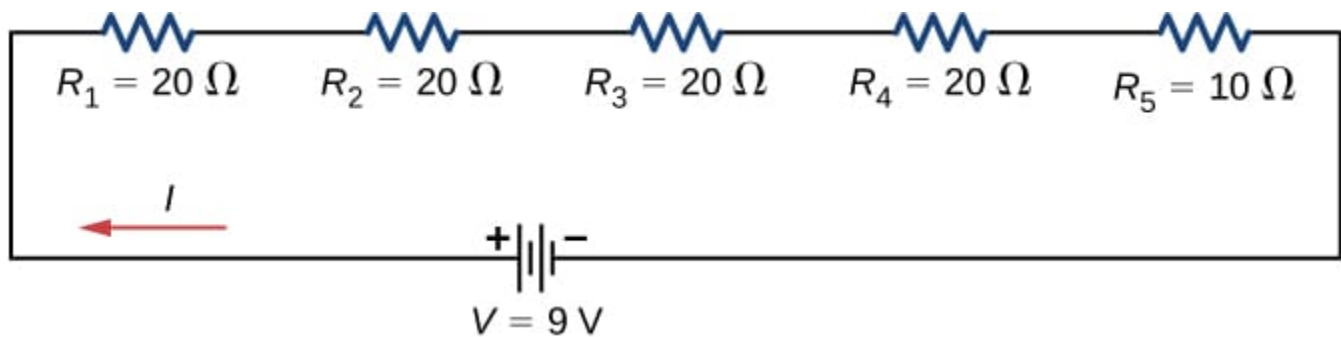


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1.2 Equivalent Resistance for Parallel Circuits

Model

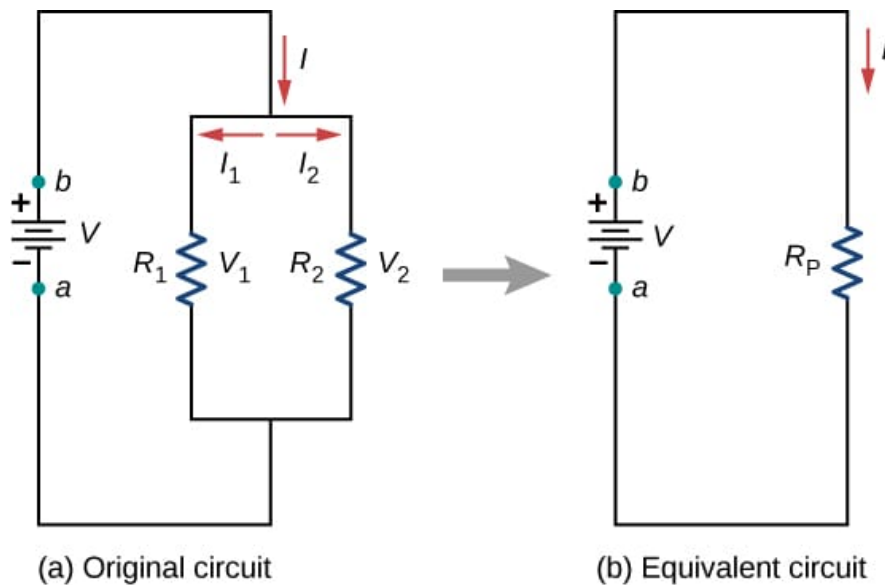


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Explore

- Color-code wires in each circuit to show the different voltage values. Be consistent with which colors mark the voltage of the positive side of the battery and the negative side of the battery.
- Construct a current equation using Kirchhoff's Current Rule to determine the current that leaves the battery (I).
- Construct a set of voltage equations for each circuit using Kirchhoff's Voltage Rule. Make sure you have an equation for each resistor in the original circuit.

Critical Thinking & Concept Invention

Use both voltage equations to make comparisons between the two circuits:

- Use the battery value and Ohm's Relation to substitute in for the voltage changes in each circuit.
- Solve each voltage equation for the currents through each resistor in the original circuit.
- Substitute those expressions for the currents into the current equation to determine the current from the battery (I).
- Substitute that expression for I from the original circuit into the equivalent circuit and solve for R_P . Cancel and simplify as appropriate.
- Interpret the result: How does the equivalent resistance of multiple resistors in parallel compare to the resistance of each of those resistors?

Summary

For resistors connected in parallel:

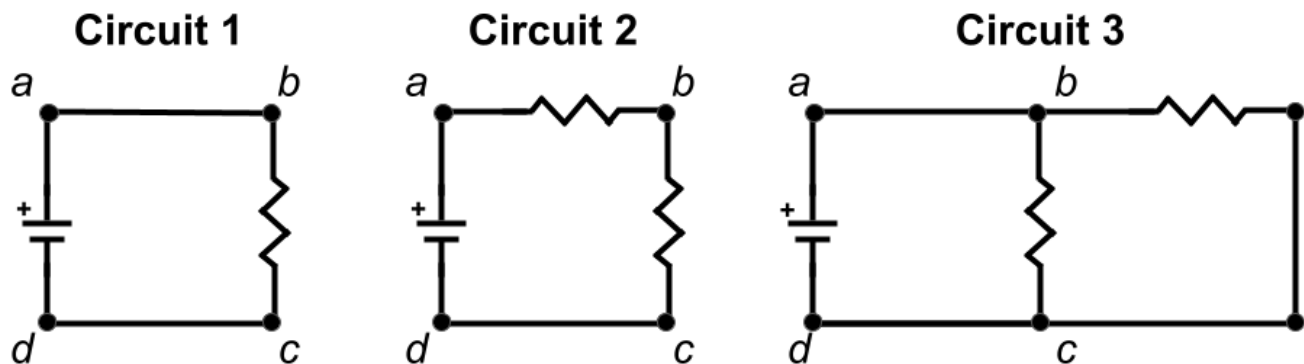
$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$$

The equivalent resistance of parallel resistors is always less than the resistance of the smallest individual resistor.

Apply

A single lightbulb with resistance R is connected to a battery with voltage V . Two other circuits are constructed with identical batteries (Voltage V) and identical resistors (each with resistance R).

1. Compare the current in each circuit (I_1, I_2, I_3) at point a using $>$, $<$, and $=$ signs.
2. Compare the voltage drop across the light bulb between points b and c in each circuit ($\Delta V_1, \Delta V_2, \Delta V_3$) using $>$, $<$, and $=$ signs.
3. Compare the brightness (power dissipated) in the light bulb between points b and c in each circuit (P_1, P_2, P_3) using $>$, $<$, and $=$ signs.



1.3 Simplifying Circuits

Model

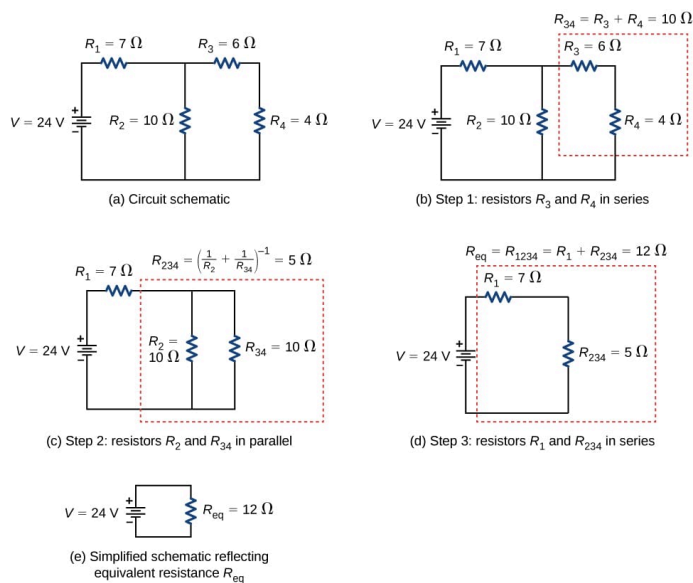


Image Credit: OpenStax University Physics Vol 2 (<https://openstax.org/details/books/university-physics-volume-2>)

Explore

1. Color-code wires in each circuit to show the different voltage values. Be consistent with which colors mark which voltages as the circuit is simplified (some colors will disappear).
2. Circle junctions in the circuits.
3. Which of the following is true in the original circuit (a)?
 - There are two resistors in parallel with the same colors (voltage) on each side of the resistor.
 - There are two resistors in series without a junction in between them.
4. Which of the following is true in the first equivalent circuit (c)?
 - There are two resistors in parallel with the same colors (voltage) on each side of the resistor.
 - There are two resistors in series without a junction in between them.
5. Which of the following is true in the second equivalent circuit (d)?
 - There are two resistors in parallel with the same colors (voltage) on each side of the resistor.
 - There are two resistors in series without a junction in between them.

Critical Thinking & Concept Invention

1. Summarize: When can you reduce a circuit's resistors in series to an equivalent resistor?
2. Summarize: When can you reduce a circuit's resistors in parallel to an equivalent resistor?
3. Verify all of the calculations used to determine the equivalent resistance.
4. Determine the current in each path and the voltage drop across each resistor.

Apply

1. Simplify the circuit below using the concept of equivalent resistance. At each stage:
 - Color-code the circuit.
 - Circle junctions.

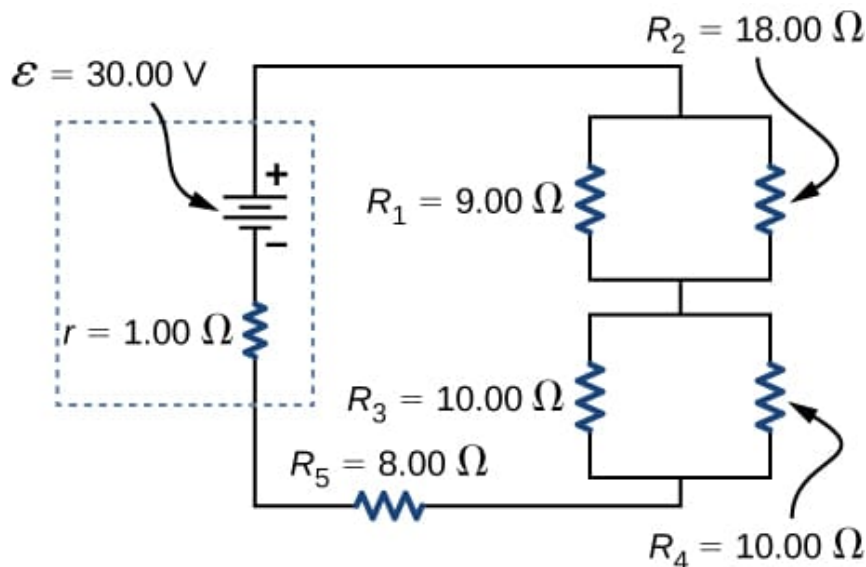


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2. Resolve the circuit below using the concept of equivalent resistance.

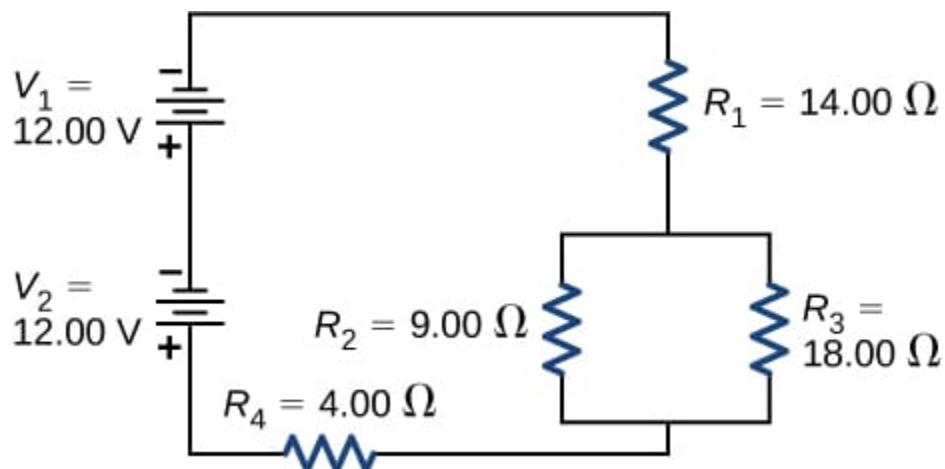


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Equivalent Resistance Reference Equations

Series Equivalent Resistance

$$R_S = \sum_i R_i$$

Parallel Equivalent Resistance

$$\frac{1}{R_P} = \sum_i \frac{1}{R_i}$$

Ohm's Relation

$$\Delta V = IR$$

Kirchhoff Current Rule

$$\sum I_{in} = \sum I_{out}$$

Kirchhoff Voltage Rule

$$\sum \Delta V_{closed\ loop} = 0$$

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2: CAPACITORS AND INDUCTORS

2.1: Capacitors

2.2: Inductors

2.3: Modeling Voltage Changes with Capacitors and Inductors

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2.1: Capacitors

2.1. Capacitors

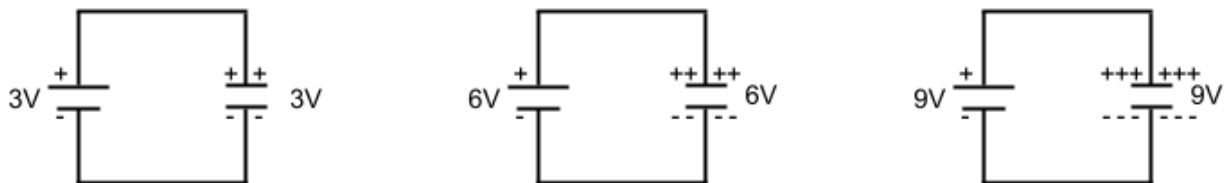
Learning Objectives

- Determine the voltage drop across a capacitor based on the charge and capacitance.
- Construct Voltage Equations for RC circuits using Kirchhoff's Voltage Rule.
- Calculate equivalent capacitance for capacitors in series and parallel.

2.1.1 Charge and Voltage

Model

The circuits below have a capacitor that has been connected to a battery for a long time. The voltage value at the battery is the terminal voltage of the battery. Each + or – sign represents $+q$ or $-q$ charge on the capacitor.



Explore

1. Choose a color for the wire connected to the negative side of the battery and call that 0V. Use that same color for all three circuits.
2. Color code the remaining wires different colors for different voltage values.
3. Complete the table to compare the three circuits.

Battery Voltage	Capacitor Voltage	Number of + on Capacitor	Net Charge on Capacitor
3V			
6V			
9V			

Critical Thinking & Concept Invention

As a group, discuss and complete the following statements:

- Given enough time, $\Delta V_{\text{battery}}$ and $\Delta V_{\text{Capacitor}}$ _____.
- As the voltage across a capacitor increases, the amount of charge on each plate _____.

$$Q \propto \Delta V_C$$

OR

$$Q \propto \frac{1}{\Delta V_C}$$

- The total net charge on both plates of a capacitor is _____.
- This means the amount of positive charge on one plate _____.

Summary

Capacitors are devices that can accumulate charge. The amount of charge a capacitor can hold is based on the electric potential difference (voltage) applied to the two plates of the capacitor.

$$Q_C = C\Delta V_C$$

The capacitance (C) of a capacitor represents the amount of charge per volt that a capacitor can accumulate and is a property of the geometric shape and materials used in the capacitor.

The unit for capacitance is Farads (F), which are equivalent to Coulombs per Volt:

$$F = \frac{C}{V}$$

We will explore those details later in the course. If we rearrange this equation to solve for the voltage across the capacitor, we can use capacitor voltage in Kirchhoff Voltage Rule equations.

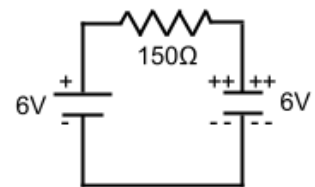
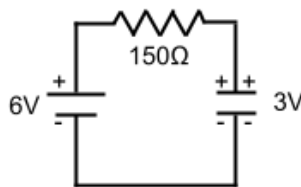
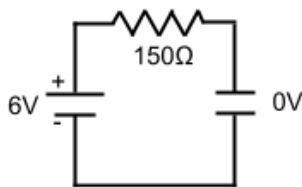
$$\Delta V_C = \frac{Q}{C}$$

The sign of this voltage difference follows the same rule as batteries: the positive side is a higher voltage than the negative side.

2.1.2 Kirchhoff Loops with Capacitors

Model

This model uses an ideal voltage source (no internal resistance) and shows the process of a capacitor charging. The first image on the left shows the circuit just as it is connected. The circuit on the right shows the final state of the circuit, after which there are no changes. Each + or – sign represents $+q$ or $-q$ charge on the capacitor.



Explore

1. Draw a Kirchhoff path in the direction that current will flow, based on the battery terminals.
2. Construct a Voltage Equation using Kirchhoff's Voltage Rule for the circuit and put a (+) or (−) sign over each ΔV term. You can use:
 - ΔV_S for the voltage source
 - ΔV_R for the voltage difference across the resistor
 - ΔV_C for the voltage difference between the two plates of the capacitor

Critical Thinking & Concept Invention

1. Do the charges that go through the resistor (as current) make a complete loop around the circuit? If not, where do they end up? Explain your thinking.
2. Use Ohm's Relation and the equation for capacitor voltage to determine the potential difference across the resistor (ΔV_R) and the current through the resistor (I) in each of the three circuit snapshots. Be careful with signs (+/−).
3. Summarize how voltages, currents, and charges relate in RC circuits:

If $Q_C = 0$, then:

$\Delta V_R = \underline{\hspace{2cm}}$ and $I_R = \underline{\hspace{2cm}}$

If $\Delta V_C = -\Delta V_S$, then:

$\Delta V_R = \underline{\hspace{2cm}}$ and $I_R = \underline{\hspace{2cm}}$

This value for I_R makes sense when a capacitor is “fully charged” because $\underline{\hspace{2cm}}$.

Summary

Capacitors are a break in the circuit where charge can accumulate. Current can flow through wires to either plate of a capacitor, but current does not actually flow through a capacitor.

The current going through wires connected to a capacitor represents the rate that charges are added to or removed from the capacitor. Negative charges flow toward or away from the plates of a capacitor until there is no longer a voltage drop between the battery terminal and the capacitor plate.

2.1.3 Capacitors in Series Circuits

Model

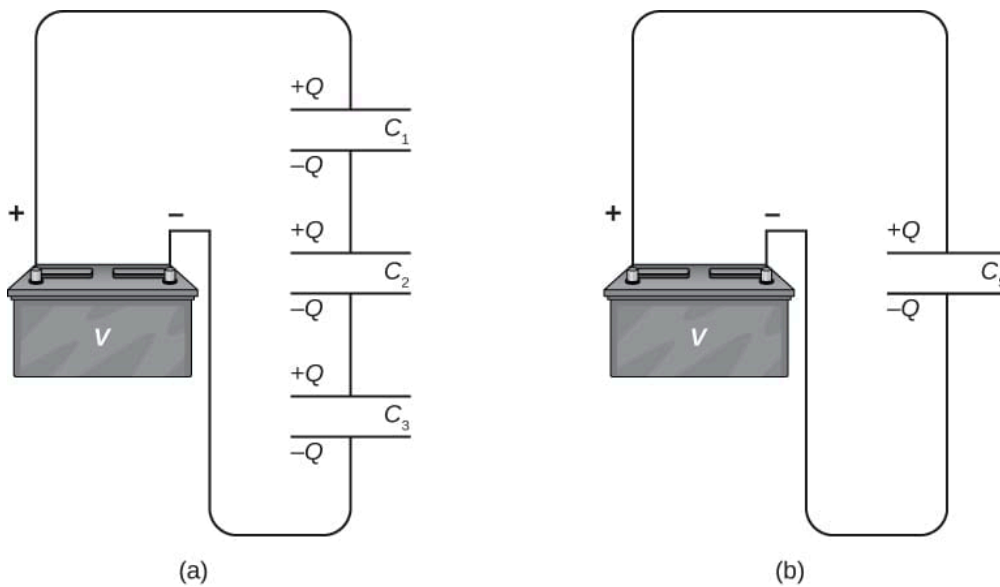


Image Credit: OpenStax University Physics Vol 2 (<https://openstax.org/details/books/university-physics-volume-2>)

Explore

1. On the left model (a), highlight each wire a different color, including the plate of the capacitor that is connected to that wire.
2. On the right model (b), highlight each wire using the same two colors connected to the positive and negative battery terminals as in model (a).
3. Circle the wires (if any) that have a net charge of zero:

$$\Sigma Q = 0$$

4. Which wires (if any) do not have a net charge of zero?

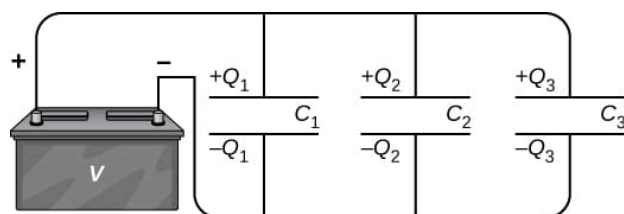
$$\Sigma Q \neq 0$$

Critical Thinking & Concept Invention

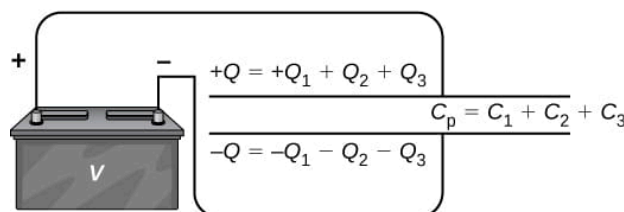
1. How does the amount of charge on each capacitor in series compare? Why is this necessary?
2. How does the charge on the series capacitors compare to the charge of the single capacitor in model (b)?
3. Compare the original series circuit (a) to the equivalent circuit (b):
 - Construct a Voltage Equation using Kirchhoff's Voltage Rule for each.
 - Plug in the appropriate $\frac{Q}{C}$ for each voltage term (note the subscripts and assume capacitances may be different).
 - Solve the system of two equations to generate an equation for C_S in terms of C_1 , C_2 , and C_3 .
 - You may review equivalent resistance relationships for guidance.

2.1.4 Capacitors in Parallel Circuits

Model



(a)



(b)

Image Credit: OpenStax University Physics Vol 2 (<https://openstax.org/details/books/university-physics-volume-2>)

Explore

1. On the top model (a), highlight each wire a different color, including the plate of the capacitor connected to that wire. Remember that color does not change at junctions.
2. On the bottom model (b), highlight each wire using the same two colors connected to the positive and negative battery terminals as in model (a).
3. Draw Kirchhoff paths in the direction current would flow from the battery. Draw enough paths so there is a path for every capacitor.
4. Circle the wires (if any) that have a net charge of zero:

$$\Sigma Q = 0$$

5. Which wires (if any) do not have a net charge of zero?

$$\Sigma Q \neq 0$$

Critical Thinking & Concept Invention

1. Which is the same for each capacitor in model (a): the charge (Q) or the voltage difference (ΔV)?
2. Compare the original parallel circuit (a) to the equivalent circuit (b):
 - Construct a set of Voltage Equations using Kirchhoff's Voltage Rule for each capacitor.
 - Plug in the appropriate $\frac{Q}{C}$ for each voltage term (note the subscripts and assume capacitances may be different).
 - Solve the system of equations to generate an equation for C_P in terms of C_1 , C_2 , and C_3 .
 - You may review equivalent resistance relationships for guidance.

Summary

The patterns for equivalent capacitance are the inverse of the patterns for equivalent resistance.

Resistance	Capacitance
$R_S =$	$C_S =$
$R_P =$	$C_P =$

Similar to equivalent resistance:

- You can only simplify a set of capacitors in series to the equivalent series capacitance if _____.
- You can only simplify a set of capacitors in parallel to the equivalent parallel capacitance if _____.

2.2: Inductors

2.2.1. Inductors

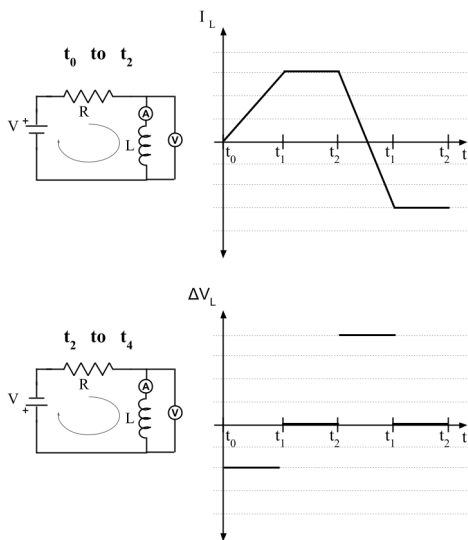
Learning Objectives

- Correlate the emf produced by an inductor (inductor voltage) to the time dependence of current through the inductor.
- Determine the sign of the emf produced by an inductor (inductor voltage) based on qualitative descriptions of current.
- Describe how inductors affect circuits over time when switches change the connections in a circuit.

2.2.1 Inductor Voltage

Model

A battery is connected to a resistor and an inductor in series. At time t_2 , the battery is flipped. The graphs are rough approximations of the current and voltage measured in the circuit.



Explore

1. Add vertical dashed lines on both graphs to divide them into sections.
2. For each section of the graphs, identify the sign (+, −, or 0) of:
 - The y-value
 - The slope (derivative)
 - The area (integral)

Critical Thinking & Concept Invention

1. How does ΔV_L seem to relate to I_L ? Which graph property (y-value, derivative, or integral) appears most connected?
2. Construct a proportionality including the correct sign:

$$\Delta V_L \propto$$

Summary

When the current through an inductor changes, an emf is induced across the inductor. This emf affects current in a way that can be modeled similarly to a potential difference (ΔV). Although the physical mechanism differs, devices such as voltmeters measure it as a potential difference, so we approximate it as a voltage.

The emf across an inductor is given by:

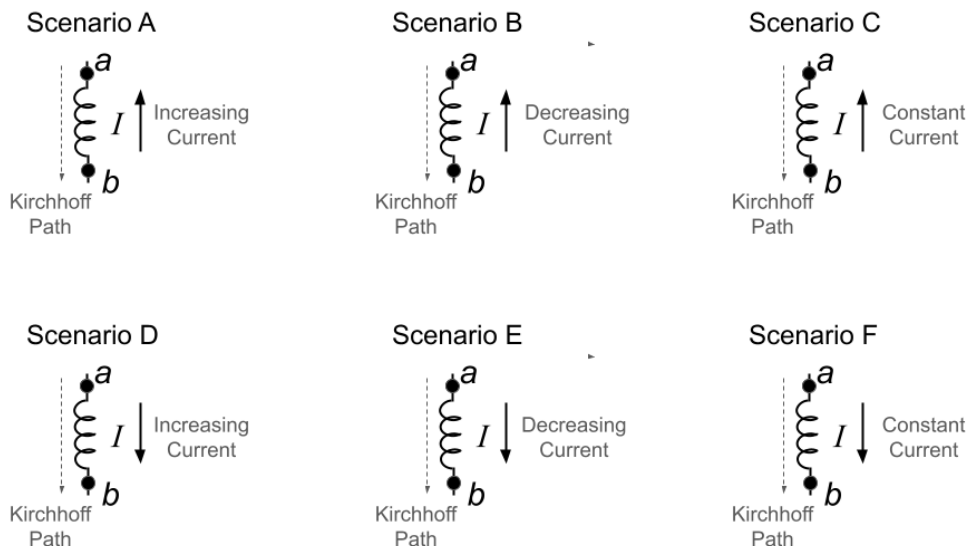
$$\Delta V_L =$$

The scalar quantity L is called the inductance of the inductor and is a physical property determined by the geometry and configuration of the wire.

The cause of this emf will be explored later when studying magnetic fields and electromagnetic induction.

2.2.2 Inductor Sign Convention

Model



Scenario	+ / - / 0 Current (<i>I</i>)	+ / - / 0 $\frac{dI}{dt}$	+ / - / 0 ΔV_L
A			
B			

C			
D			
E			
F			

Explore

1. Fill in the sign of I for each scenario based on the direction of current and the chosen Kirchhoff path direction.
2. Without overthinking, predict the sign (+, −, or 0) of the voltage difference between points b and a:

$$V_b - V_a$$

Critical Thinking & Concept Invention

1. Use the descriptions of how current is changing or remaining constant to determine the sign of:

$$\frac{dI}{dt}$$

for each scenario.

2. Determine the sign of ΔV_L for each scenario.
3. Summarize the process: How do you determine whether there is a voltage drop ($\Delta V_L < 0$), voltage rise ($\Delta V_L > 0$), or no voltage change ($\Delta V_L = 0$) across an inductor?

Summary

The sign of the voltage difference across an inductor depends on the rate of change of current,

$$\frac{dI}{dt}$$

rather than the direction of current alone.

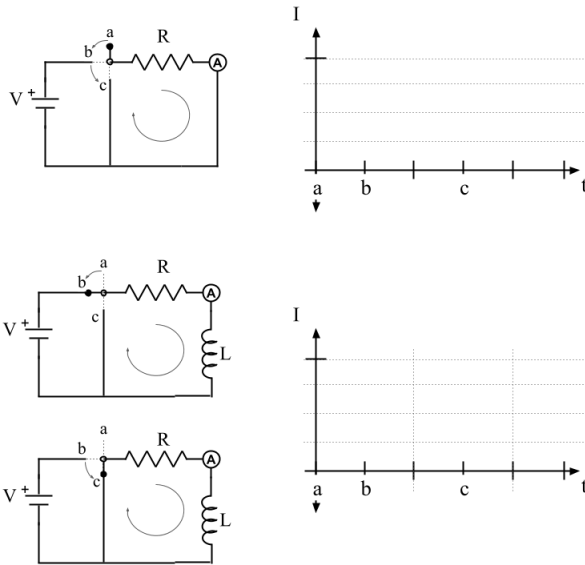
Usually we first identify the current direction and then determine whether the magnitude of the current is increasing or decreasing to determine the sign of

$$\frac{dI}{dt}.$$

2.2.3 Time-Dependent Inductive Circuits

Model

The models below show circuits containing a switch that changes how components are connected. Assume the switch remains at each labeled position until the next labeled position on the time axis.



Explore

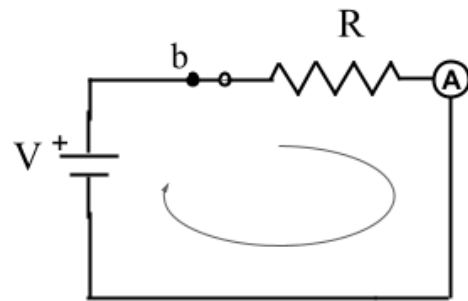
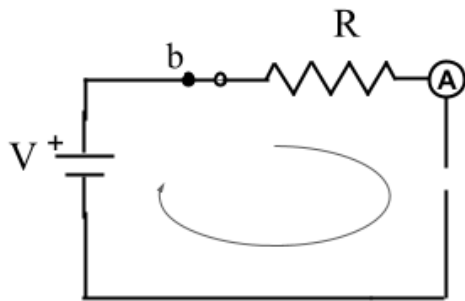
Focus only on the upper model containing the circuit without an inductor.

1. Draw an arrow next to the resistor showing the direction of current when connected to the battery.
2. Determine whether each switch movement increases or decreases current:
 - Moving the switch from a to b: current _____
 - Moving the switch from b to c: current _____
3. Identify an expression for the current through the resistor when connected to the battery using the variables shown in the model. Label the fourth dashed line on each current graph with this value.
4. Sketch the current graph for the circuit without an inductor. Consider first where the current is zero.

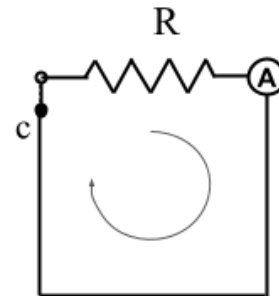
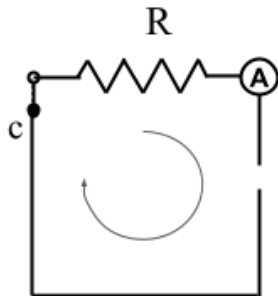
Critical Thinking & Concept Invention

Focus on the lower circuit containing the inductor.

1. Immediately after the switch moves from a to b, is $\frac{dI}{dt}$ positive, negative, or zero?
2. Immediately after the switch moves from a to b, is ΔV_L positive, negative, or zero?
3. Based on the sign of ΔV_L , draw a battery symbol whose polarity matches the inductor voltage.
4. Annotate the circuit with the switch at position b:
 - Sketch the battery symbol with correct polarity.
 - Label voltage V to match the battery voltage at the first instant.
 - Color-code wires to indicate voltage values.
 - Show how the inductor behaves like a wire after a long time.



5. Repeat the above analysis for the switch moving from b to c.



- 1.
6. Use your circuit diagrams to identify the current at moments b and c and plot points on the current graph.
7. Determine the current a long time after moments b and c and sketch those values on the graph.
8. Connect the current values smoothly and continuously from the points at b and c to their later values.
9. Summarize the effect of inductors on circuits, including when their effect is strongest and weakest.

Summary

Inductors affect circuits whenever the current changes. Switching events create large changes in current and therefore large inductive effects.

Inductors slow those changes, causing current to ramp up gradually and ramp down gradually rather than changing instantaneously.

The total area under the current-versus-time graph appears similar with and without an inductor; the current is redistributed over time.

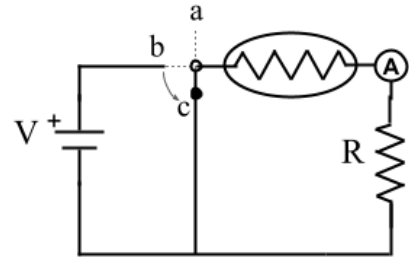
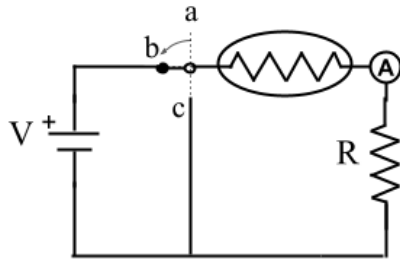
2.2.4 Comparing Current Over Time

In this activity, compare circuits containing a battery, a light bulb, and one additional circuit element connected in series. The resistor enclosed by an oval represents the light bulb.

All circuits include a switch. When the switch moves from b to c, assume it remained at position b for a long time beforehand.

Model 1

Resistor



1. For each model, the switch moves to the position shown. Describe the brightness of the light bulb:

A to B

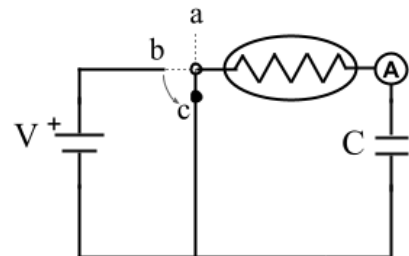
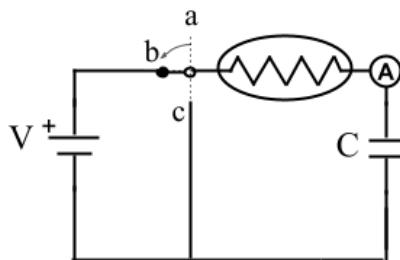
- Initially...
- Then the light...
- After a long time...

B to C

- Initially...
- Then the light...
- After a long time...

Model 2

Capacitor



2. For each model, describe the brightness of the light bulb:

A to B

- Initially...
- Then the light (during the first few τ_C)...
- After a long time...

B to C

- Initially...
 - Then the light (during the first few τ_C)...
 - After a long time...
3. For each model, draw two circuit diagrams:
- Immediately after the switch changes position.
 - A long time after the switch changes position.

Replace the resistor, capacitor, or inductor with either a battery or a wire depending on whether a voltage difference exists across that element. Include the correct polarity for any battery representation.

Model 3

Inductor

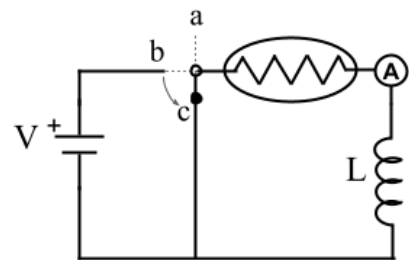
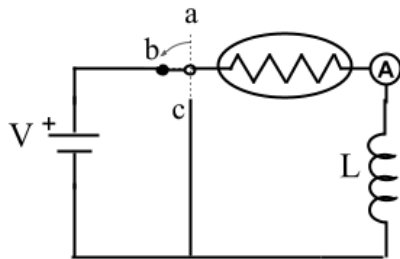


Image needed here with Alt Text:

Switched inductor circuits used to compare light bulb brightness over time.

4. For each model, describe the brightness of the light bulb:

A to B

- Initially...
- Then the light (during the first few τ_L)...
- After a long time...

B to C

- Initially...
 - Then the light (during the first few τ_L)...
 - After a long time...
5. For each model, draw two circuit diagrams:
- Immediately after the switch changes position.
 - A long time after the switch changes position.

Replace the resistor, capacitor, or inductor with either a battery or a wire depending on whether a voltage difference exists across that element. Include the correct polarity for any battery representation.

2.3: Modeling Voltage Changes with Capacitors and Inductors

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3: TIME-DEPENDENT CIRCUIT EQUATIONS

3.1: Differential Equations

3.2: RC Circuits

3.3: RL Circuits

3.4: LC Circuits

3.5: RLC Circuits

3.6: AC Circuits

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3.1: Differential Equations

1. Circuit Differential Equations and LC Circuits

Learning Objectives

- Identify patterns in derivatives of exponential functions to match them to first- and second-order differential equations.
- Identify patterns in derivatives of sine and cosine functions to match them to second-order differential equations.
- Derive equations for charge and current in LC circuits as functions of time.

1.1 Differential Equations Part 1: Exponential Functions

Model

Time and Space Models

Time	Space
$T(t) = Ae^{+\omega t} + Be^{-\omega t}$	$R(x) = Ae^{+kx} + Be^{-kx}$
(1) $\frac{dT}{dt} =$	$\frac{dR}{dx} =$
(2) Assume $B = 0$	
(3) Assume $A = 0$	
(4) $\frac{d^2T}{dt^2} =$	$\frac{d^2R}{dx^2} =$
(5) $\frac{d^2T}{dt^2} =$	$\frac{d^2R}{dx^2} =$

Explore

- Identify the following for each function:
 - The variable (symbol) that the function shows explicit dependence on.
 - What that symbol represents.
 - The coefficient that appears in front of that variable in the equation.

Function	Variable Symbol	Variable Represents	Coefficient Symbol
$T(t)$			
$R(x)$			

- Describe the structure of each equation, including what is similar and different about the A and B terms.
- Complete the model by doing the following:
 - Take the derivative of each function and write it on line (1) of the model.
 - Assume $B = 0$ and $A \neq 0$. Rewrite the derivative using the original function (T or R). Put this on line (2).
 - Assume $A = 0$ and $B \neq 0$. Rewrite the derivative using the original function (T or R). Put this on line (3).
 - Assume $A \neq 0$ and $B \neq 0$. Take the second derivative of each function. Put this on line (4).
 - Substitute the original function into the second derivative. Put this on line (5).

Critical Thinking & Concept Invention

1. What function $T(t)$ satisfies $\frac{dT}{dt} = \omega T$? Explain why this works based on derivative rules for exponential functions.
2. What function $T(t)$ satisfies $\frac{dT}{dt} = -\omega T$? Explain what is necessary to obtain the negative sign.
3. What function $T(t)$ satisfies $\frac{d^2T}{dt^2} = \omega^2 T$? Explain why this works based on derivative rules for exponential functions.
4. It is possible to obtain a similar exponential function satisfying $\frac{d^2T}{dt^2} = -\omega^2 T$. What might that function be? Consider what determines the coefficient in the second derivative.

Summary

First-order differential equations such as $\frac{d}{dt}$ and $\frac{d}{dx}$ can often be solved analytically by separating variables and integrating. Exponential functions satisfy:

$$\frac{dT}{dt} = (\pm)\omega T$$

or

$$\frac{dR}{dx} = (\pm)kR$$

The general solutions are:

$$T(t) = Ae^{+\omega t} + Be^{-\omega t}$$

$$R(x) = Ae^{+kx} + Be^{-kx}$$

The sign determines whether A or B equals zero. Constants are determined using boundary conditions. For time-based functions, ω is often related to the time constant:

$$\tau = \frac{1}{\omega}$$

or

$$\omega = \frac{1}{\tau}$$

Apply

1. A resistor, voltage source, and inductor are connected in series. The voltage source has an emf of 9 V, the resistor has a resistance of $150 \, \Omega$, and the inductor has an inductance of $30 \times 10^{-3} \, \text{H}$. After the current reaches its maximum value, the switch is changed so only the resistor and inductor remain connected.

Recall:

$$I(t) = \frac{\varepsilon}{R} (e^{-t/\tau_L})$$

- Find the current at $t = 0$.
- Determine constants A and B in $I(t) = Ae^{+\omega t} + Be^{-\omega t}$.
- Determine ω .

1.2 Differential Equations Part 2: Sine and Cosine

Model

Time	Space
$T(t) = A \cos(\omega t) + B \sin(\omega t)$	$R(x) = A \cos(kx) + B \sin(kx)$
(1) $\frac{dT}{dt} =$	$\frac{dR}{dx} =$
(2) $\frac{d^2T}{dt^2} =$	$\frac{d^2R}{dx^2} =$
(3) $\frac{d^2T}{dt^2} =$	$\frac{d^2R}{dx^2} =$

Explore

- Describe the structure of each equation, including similarities and differences between the A and B terms.
- Complete the model:
 - Take the derivative and write it on line (1).
 - Take the second derivative and write it on line (2).
 - Substitute the original function into the second derivative and write it on line (3).

Critical Thinking & Concept Invention

- Which proportionality aligns with sine and cosine derivatives?
 $\frac{dT}{dt} \propto (\pm)T$
 or
 $\frac{d^2T}{dt^2} \propto (\pm)T$
- Write the relationship as an equality, including sign and coefficient.
- Explain why the sign is positive or negative.

Summary

Sine and cosine functions satisfy:

$$\frac{d^2T}{dt^2} = -\omega^2 T$$

or

$$\frac{d^2R}{dx^2} = -k^2 R$$

General solutions:

$$T(t) = A \cos(\omega t) + B \sin(\omega t)$$

$$R(x) = A \cos(kx) + B \sin(kx)$$

The period is:

$$T = \frac{2\pi}{\omega}$$

Another equivalent form is:

$$I(t) = C \cos(\omega t + \phi)$$

Examples:

- $\phi = 0$: $C \cos(\omega t)$
- $\phi = \frac{\pi}{2}$: $C \sin(\omega t)$

Apply

1. A circuit has current:

$$I(t) = (1.2 \text{ A}) \sin(6\pi t)$$

- Find the current at $t = 0$.
- Determine ω .
- Determine the period.
- Determine the maximum current and the first time it occurs.
- Determine ϕ if written as $I(t) = (1.2 \text{ A}) \cos(6\pi t + \phi)$.

3.2: RC Circuits

3.2. Charging and Discharging Capacitors

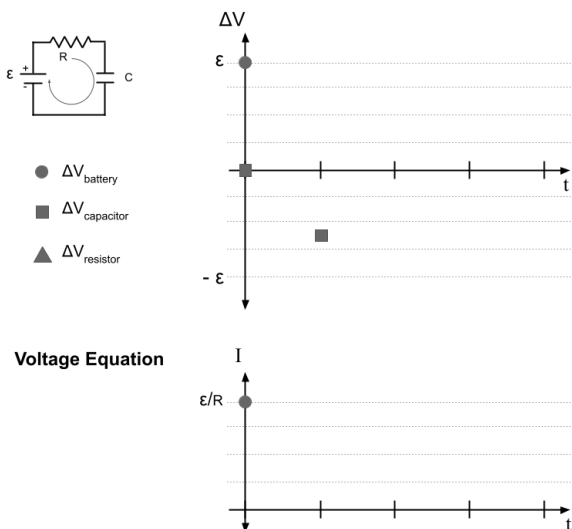
Learning Objectives

- Qualitatively describe and plot the voltage vs. time and current vs. time for elements in an RC circuit that is charging and discharging.
- Set up and solve a differential equation for charge vs. time in an RC circuit that is either charging or discharging.
- Derive equations for voltage and current as functions of time for different circuit elements in charging and discharging RC circuits.

3.2.1 Time-dependent Voltage and Current for Capacitors

Model

A capacitor is initially uncharged and connected to a resistor and a voltage source as shown in the diagram. The graph for potential difference across each circuit element is partially filled in. The origin is time t_0 and each subsequent “tick” on the x-axis represents times t_1, t_2, t_3, t_4 . The graph of current vs. time below is for the current through the resistor at those same clock-times.



Explore

1. Construct a voltage equation using the Kirchhoff Voltage Rule for this circuit and put (+) / (–) above each ΔV term.
2. Use the equation to solve for the voltage drop across the resistor (ΔV_R) at t_0 and t_1 .

At t_0 , $\Delta V_R =$

At t_1 , $\Delta V_R =$

- Plot those voltage drop values for the resistor on the graph.

Critical Thinking & Concept Invention

- How should the current through the resistor compare ($>$, $<$, $=$) at times t_0 and t_1 ?

I_0 _____ I_1

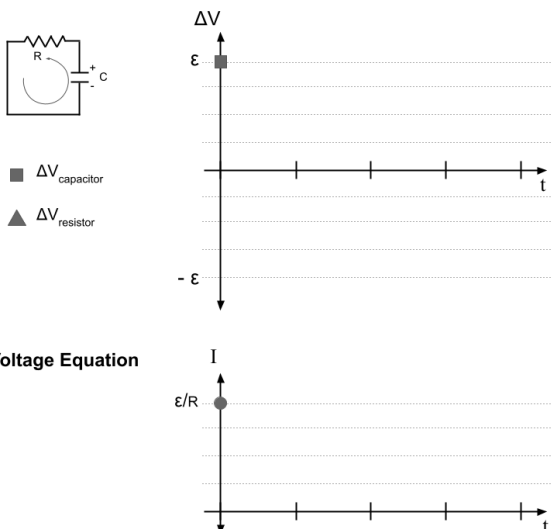
- Based on your answer to the previous question, how should the amount of charge that is added to the capacitor from t_1 to t_2 compare ($>$, $<$, $=$) to the amount of charge added to the capacitor from t_0 to t_1 ?

$\Delta Q_{1 \rightarrow 2}$ _____ $\Delta Q_{0 \rightarrow 1}$

- Based on your answer to the previous question, discuss how much the voltage drop across the capacitor should change compared to the previous two values. Plot an approximate value for the voltage drop across the capacitor for time t_2 , then plot the voltage drop for the resistor at t_2 .
- Plot the voltage difference for the voltage source, resistor, and capacitor for t_3 and t_4 .
- Use the voltage drops you plotted for the resistor to qualitatively plot the current through the resistor at each clock-time.
- Describe the way voltage changes or does not change for each circuit element while a capacitor is charging. Explain why this occurs.
- Describe the way current changes or does not change through a circuit while a capacitor is charging. Explain why this occurs.

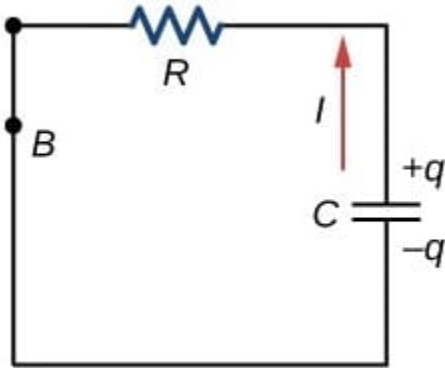
Apply

A capacitor is initially charged to a voltage of $\Delta V_C = \varepsilon$ and is connected to a resistor as shown in the diagram. Construct a voltage equation, then qualitatively plot the potential difference across the capacitor and resistor as time goes on, then sketch a curve through the points for each graph.



3.2.2 Separation of Variables: Discharging

Model



Explore

1. Draw a Kirchhoff Path in the direction of the current.
2. Construct a Voltage Equation using Kirchhoff's Voltage Rule for the circuit and put a (+) or (−) sign over each ΔV term.
3. Substitute into the equation the equations for ΔV_R and ΔV_C with appropriate sign (+/−) in front of them.

Critical Thinking & Concept Invention

1. Discuss as a group: Does the current represent the rate charge is entering or leaving the capacitor?
2. If Q represents the charge of the capacitor, which of the following should be used for I ? Circle one.

$$I = \frac{dQ}{dt}$$

OR

$$I = -\frac{dQ}{dt}$$

3. Solve the voltage equation:
 - Rewrite your voltage equation with your selection above in place of I .
 - **Separate Variables:** Perform algebra necessary to group the charge quantities (Q and dQ) on one side of the equation and the time quantities (t and/or dt) on the other side. If there is a minus sign, place it on the time side.
 - Integrate both sides (Q_0 to Q and 0 to t).
 - Perform necessary algebra and logarithmic identities to get an equation for:

$$Q =$$

Summary

The charge on a discharging capacitor is an exponential decay that depends on the initial charge (Q_0) and the values for the resistance (R) and capacitance (C) of the circuit.

The product RC is called the time constant (τ) for the capacitive circuit.

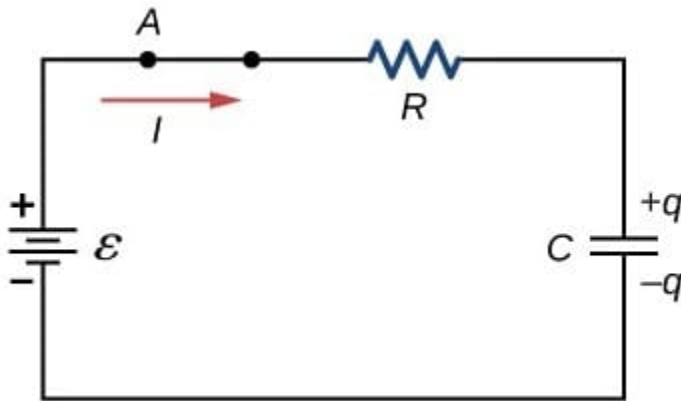
$$Q(t) =$$

$$\tau = RC$$

Graphs of charge, voltage, or current for capacitive circuits are typically scaled so that the x-axis is integer multiples of the time constant.

3.2.3 Separation of Variables: Charging

Model



Explore

1. Draw a Kirchhoff Path in the direction of the current.
2. Construct a Voltage Equation using Kirchhoff's Voltage Rule for the circuit and put a (+) or (−) sign over each ΔV term.
3. Substitute into the equation the equations for ΔV_{source} , ΔV_R , and ΔV_C with appropriate sign (+/−) in front of them.

Critical Thinking & Concept Invention

1. Discuss as a group: Does the current represent the rate charge is entering or leaving the capacitor?
2. If Q represents the charge of the capacitor, which of the following should be used for I ? Circle one.

$$I = \frac{dQ}{dt}$$

OR

$$I = -\frac{dQ}{dt}$$

3. Solve the voltage equation:

- Rewrite your voltage equation with your selection above in place of I .
- **Separate Variables:** Perform algebra necessary to group the charge quantities (Q and dQ) on one side of the equation and the time quantities (t and/or dt) on the other side. If there is a minus sign, place it on the time side.
- Integrate both sides (Q_0 to Q and 0 to t).
- Perform necessary algebra and logarithmic identities to get an equation for:

$$Q =$$

- Factor out any common factors from terms.

4. Remind each other how the current relates to charge and perform the necessary calculus operation to your $Q(t)$ equation to generate an $I(t)$ equation.

5. Generate a $\Delta V(t)$ equation for both the resistor and capacitor using your equations for $I(t)$ and $Q(t)$.

$$\Delta V_R(t) =$$

$$\Delta V_C(t) =$$

6. Put all of your $\Delta V(t)$ terms back into your original voltage equation to check your results. Does Kirchhoff's Voltage Rule still hold true?

Summary

Circuits with capacitors and inductors are not steady circuits in that current and voltages change with time throughout the circuit.

Many textbooks use lower-case variables (i , q , v) for time-dependent current, charge, and voltage. This document uses the same variable symbols with explicit time dependence shown using (t) , but both conventions are considered correct.

Kirchhoff's Voltage Rule (and an extension of it) is still true for each moment in time, even though the voltages and current might change with time.

We solve these circuits by setting up the necessary voltage and current equations, then solving the differential equation by relating current and charge.

Memorizing equations can be a bad idea when dealing with complicated circuits that vary with time. It is always best to start with the fundamental concepts:

- Kirchhoff's Voltage Rule
- Kirchhoff's Current Rule

Then generate specific equations for the circuit you are analyzing.

3.3: RL Circuits

3.3 RL Circuits

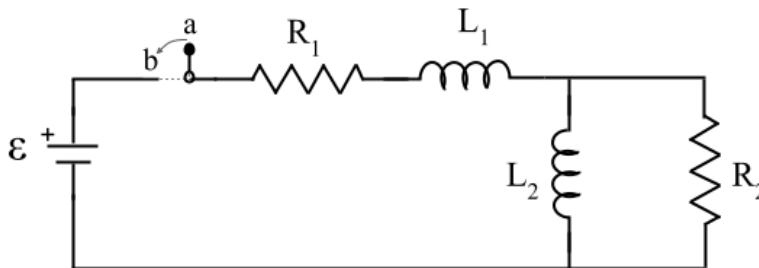
Learning Objectives

- Analyze complex circuits using Kirchhoff's Voltage Rule and the relationship for inductor voltage.
- Analyze complex circuits using Kirchhoff's Current Rule and the relationship for inductor voltage.
- Derive time-dependent equations for current in RL circuits with switches that connect and disconnect constant voltage sources.

3.3.1 RL Circuit Voltage Equations

Model

The circuit below contains an emf source, resistors, and inductors. The circuit is an open circuit until the switch is moved from position a to position b.



Explore

1. Draw a set of Kirchhoff Paths in the direction you expect the current to be in (do not worry too much).
2. Construct Kirchhoff Voltage equations.
3. Circle the circuit elements that are in series. These circuit elements have the same _____.
4. Box the circuit elements that are in parallel. These circuit elements have the same _____.

Critical Thinking & Concept Invention

1. When the switch first closes, do inductors tend to act more like batteries ($\Delta V \neq 0$) or wires ($\Delta V = 0$)?
2. When the switch is first closed, what is the current through the inductors?

$$I_{L1} =$$

$$I_{L2} =$$

- Based on that response, what is the current through each resistor?

$$I_{R1} =$$

$$I_{R2} =$$

- Use Ohm's Relation to identify the voltage drop for each resistor. Plug those values into the Voltage Equations to find the inductor voltage of each inductor. Copy the responses below:

$$\Delta V_{L1} =$$

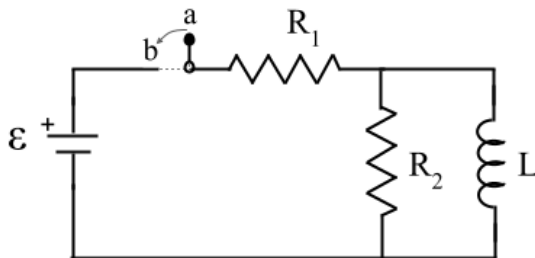
$$\Delta V_{L2} =$$

- Redraw the circuit with a battery symbol or wire in place of the inductors based on the voltage values you calculated. Color-code the circuit to show different voltage values in different wires.
- What, if anything, was surprising about this result?
- Determine the current and voltage difference for every circuit element a long time after the switch is closed.
- Redraw the circuit with a battery symbol or wire in place of the inductors based on the voltage values you calculated. Color-code the circuit to show different voltage values in different wires.

3.3.2 RL Circuit Current Equations

Model

The circuit below contains an emf source, resistors, and inductors. The circuit is an open circuit until the switch is moved from position a to position b.



Explore

- Draw a set of Kirchhoff Voltage paths and construct a Kirchhoff Voltage Rule equation for each path.
- Circle the first junction and construct a Kirchhoff Current Rule equation for that junction.
- Redraw the circuit with either a battery symbol or wire symbol in place of the inductor to represent the time:
 - Immediately as the switch is closed
 - A short time after the switch is closed
 - A long time after the switch is closed

4. At which of the times listed in the previous question is the current through the inductor zero ($I_L = 0$)?

Critical Thinking & Concept Invention

- For the moment immediately after the switch is closed:
 - Determine the currents (I_1, I_2, I_3).
 - Determine the voltage differences (ΔV 's) across each circuit element (resistors and inductors).
- After the switch has been closed for a long time:
 - Determine the currents (I_1, I_2, I_3).
 - Determine the voltage differences (ΔV 's) across each circuit element (resistors and inductors).
- Is the inductor voltage always the same as the battery voltage when a switch connects it to the battery?
- What rule should you remember about voltage/current for an inductor?

Summary

Inductors always oppose changes in current. When thinking about inductors in complex circuits with switches, it is best to rely on the current through the inductor starting out the same as it was before a switch is flipped.

From there, you might put that result into a Kirchhoff Current Rule equation or into an Ohm's Relation equation for a resistor in series with the inductor.

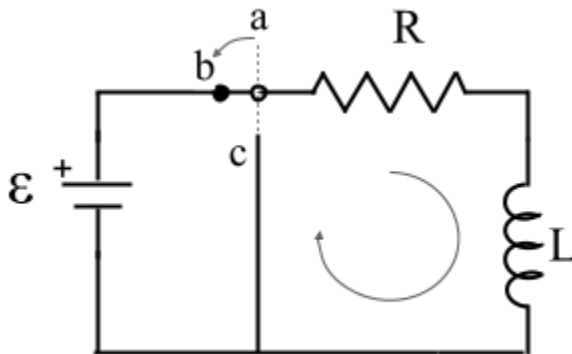
Starting out with voltage can be tricky and might be misleading in more complex circuits. While we do have a voltage equation for inductors, it is better to remember that their effects are more related to current than to voltage.

After a long time, if nothing is changing the current (steady state), the inductor behaves like a wire, which could result in current bypassing other circuit elements on parallel paths.

3.3.3 Separation of Variables: Connecting an Inductor to a Voltage Source

Model

The models below show a circuit that includes a switch that can change how circuit elements are connected. The switch is moved to position b at $t = 0s$.



Explore

1. Draw a Kirchhoff Path in the direction of the eventual current.
2. Construct a Voltage Equation using Kirchhoff's Voltage Rule for the circuit and put a (+) or (−) sign over each ΔV term.
3. Substitute into the equation the equations or variables for ΔV_{source} , ΔV_R , and ΔV_L with appropriate signs (+/−) in front of them.

Critical Thinking & Concept Invention

1. Discuss as a group: Is $\frac{dI}{dt}$ positive or negative in this context for this Kirchhoff Path?
2. What should the initial current in the circuit be?

$$I_0 =$$

3. Solve the voltage equation:
 - **Separate Variables:** Perform algebra necessary to group the current quantities (I and dI) on one side of the equation and time quantities (t and/or dt) on the other side. If there is a minus sign, place it on the time side.
 - Integrate both sides (I_0 to $I(t)$ and 0 to t).
 - Perform the necessary algebra and logarithmic identities to obtain an equation for:

$$I(t) =$$

Summary

The current through an inductor is the non-inductive current ($\frac{\mathcal{E}}{R}$) minus an exponential decay that depends on the values of the resistance (R) and inductance (L) of the circuit.

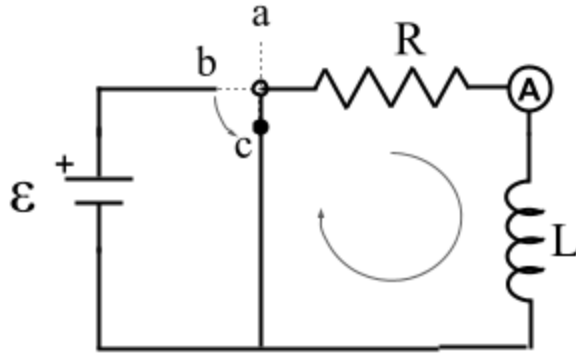
Inductive circuits also have a time constant (τ_L) that characterizes the time it takes to reach steady state.

$$\tau_L = \frac{L}{R}$$

3.3.4 Separation of Variables: Switching Out the Voltage Source

Model

The models below show a circuit that includes a switch that can change how circuit elements are connected. The switch has been in position b for a long time and then is switched to position c at $t = 0s$.



Explore

1. Draw a Kirchhoff Path in the direction of the current.
2. Construct a Voltage Equation using Kirchhoff's Voltage Rule for the circuit and put a (+) or (−) sign over each ΔV term.
3. Substitute into the equation the equations for ΔV_R and ΔV_L with appropriate signs (+/−) in front of them.

Critical Thinking & Concept Invention

1. Discuss as a group: Is $\frac{dI}{dt}$ positive or negative in this context for this Kirchhoff Path?
2. What should the initial current in the circuit be?

$$I_0 =$$

3. Solve the voltage equation:
 - **Separate Variables:** Perform algebra necessary to group the current quantities (I and dI) on one side of the equation and time quantities (t and/or dt) on the other side. If there is a minus sign, place it on the time side.
 - Integrate both sides (I_0 to $I(t)$ and 0 to t).
 - Perform the necessary algebra and logarithmic identities to obtain an equation for:

$$I(t) =$$

Summary

Use Kirchhoff's Voltage Rule together with the inductor voltage relationship to derive the time-dependent current in an RL circuit after the voltage source is removed. The resulting current decreases exponentially toward zero with a characteristic time constant determined by the circuit resistance and inductance.

3.4: LC Circuits

3.4 Circuit Differential Equations and LC Circuits

Learning Objectives

- Identify patterns in derivatives of exponential functions to match them to first- and second-order differential equations.
- Identify patterns in derivatives of sine and cosine functions to match them to second-order differential equations.
- Derive equations for charge and current in LC circuits as functions of time.

3.4.1 Differential Equations Part 1: Exponential Functions

Model

Time and Space Models

Time	Space
$T(t) = Ae^{+\omega t} + Be^{-\omega t}$	$R(x) = Ae^{+kx} + Be^{-kx}$
(1) $\frac{dT}{dt} =$	$\frac{dR}{dx} =$
(2) Assume $B = 0$	
(3) Assume $A = 0$	
(4) $\frac{d^2T}{dt^2} =$	$\frac{d^2R}{dx^2} =$
(5) $\frac{d^2T}{dt^2} =$	$\frac{d^2R}{dx^2} =$

Explore

1. Identify the following for each function:
 - The variable (symbol) that the function shows explicit dependence on.
 - What that symbol represents.
 - The coefficient that appears in front of that variable in the equation.

Function	Variable Symbol	Variable Represents	Coefficient Symbol
$T(t)$			
$R(x)$			

2. Describe the structure of each equation, including what is similar and different about the A and B terms.
3. Complete the model by doing the following:
 - Take the derivative of each function and write it on line (1) of the model.
 - Assume $B = 0$ and $A \neq 0$. Rewrite the derivative using the original function (T or R). Put this on line (2).
 - Assume $A = 0$ and $B \neq 0$. Rewrite the derivative using the original function (T or R). Put this on line (3).
 - Assume $A \neq 0$ and $B \neq 0$. Take the second derivative of each function. Put this on line (4).
 - Substitute the original function into the second derivative. Put this on line (5).

Critical Thinking & Concept Invention

1. What function $T(t)$ satisfies $\frac{dT}{dt} = \omega T$? Explain why this works based on derivative rules for exponential functions.
2. What function $T(t)$ satisfies $\frac{dT}{dt} = -\omega T$? Explain what is necessary to obtain the negative sign.
3. What function $T(t)$ satisfies $\frac{d^2T}{dt^2} = \omega^2 T$? Explain why this works based on derivative rules for exponential functions.
4. It is possible to obtain a similar exponential function satisfying $\frac{d^2T}{dt^2} = -\omega^2 T$. What might that function be? Consider what determines the coefficient in the second derivative.

Summary

First-order differential equations such as $\frac{d}{dt}$ and $\frac{d}{dx}$ can often be solved analytically by separating variables and integrating. Exponential functions satisfy:

$$\frac{dT}{dt} = (\pm)\omega T$$

or

$$\frac{dR}{dx} = (\pm)kR$$

The general solutions are:

$$T(t) = Ae^{+\omega t} + Be^{-\omega t}$$

$$R(x) = Ae^{+kx} + Be^{-kx}$$

The sign determines whether A or B equals zero. Constants are determined using boundary conditions. For time-based functions, ω is often related to the time constant:

$$\tau = \frac{1}{\omega}$$

or

$$\omega = \frac{1}{\tau}$$

Apply

1. A resistor, voltage source, and inductor are connected in series. The voltage source has an emf of 9 V, the resistor has a resistance of $150 \, \Omega$, and the inductor has an inductance of $30 \times 10^{-3} \, \text{H}$. After the current reaches its maximum value, the switch is changed so only the resistor and inductor remain connected.

Recall:

$$I(t) = \frac{\varepsilon}{R} (e^{-t/\tau_L})$$

- Find the current at $t = 0$.
- Determine constants A and B in $I(t) = Ae^{+\omega t} + Be^{-\omega t}$.
- Determine ω .

3.4.2 Differential Equations Part 2: Sine and Cosine

Model

Time	Space
$T(t) = A \cos(\omega t) + B \sin(\omega t)$	$R(x) = A \cos(kx) + B \sin(kx)$
(1) $\frac{dT}{dt} =$	$\frac{dR}{dx} =$
(2) $\frac{d^2T}{dt^2} =$	$\frac{d^2R}{dx^2} =$
(3) $\frac{d^2T}{dt^2} =$	$\frac{d^2R}{dx^2} =$

Explore

1. Describe the structure of each equation, including similarities and differences between the A and B terms.
2. Complete the model:
 - Take the derivative and write it on line (1).
 - Take the second derivative and write it on line (2).
 - Substitute the original function into the second derivative and write it on line (3).

Critical Thinking & Concept Invention

1. Which proportionality aligns with sine and cosine derivatives?
 $\frac{dT}{dt} \propto (\pm)T$
 or
 $\frac{d^2T}{dt^2} \propto (\pm)T$
2. Write the relationship as an equality, including sign and coefficient.
3. Explain why the sign is positive or negative.

Summary

Sine and cosine functions satisfy:

$$\frac{d^2T}{dt^2} = -\omega^2 T$$

or

$$\frac{d^2R}{dx^2} = -k^2 R$$

General solutions:

$$T(t) = A \cos(\omega t) + B \sin(\omega t)$$

$$R(x) = A \cos(kx) + B \sin(kx)$$

The period is:

$$T = \frac{2\pi}{\omega}$$

Another equivalent form is:

$$T(t) = C \cos(\omega t + \phi)$$

Examples:

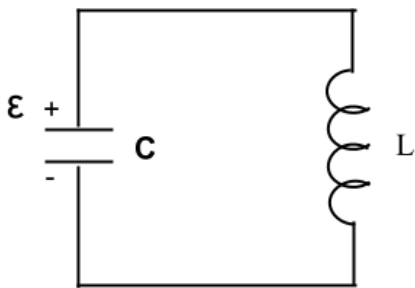
- $\phi = 0$: $C \cos(\omega t)$
- $\phi = \frac{\pi}{2}$: $C \sin(\omega t)$

Apply

1. A circuit has current:
 $I(t) = (1.2 \text{ A}) \sin(6\pi t)$
 - Find the current at $t = 0$.
 - Determine ω .
 - Determine the period.
 - Determine the maximum current and the first time it occurs.
 - Determine ϕ if written as $I(t) = (1.2 \text{ A}) \cos(6\pi t + \phi)$.

3.4.3 LC Circuits

Model



A circuit is connected as shown. At time $t = 0$, the capacitor has a voltage ε , and there is no current in the circuit.

Explore

1. What is the initial charge q_0 of the capacitor? Recall $\Delta V_C = \frac{Q}{C}$.
2. Describe what should happen when the switch is closed.
3. Draw an arrow showing the initial current direction.
4. Draw a Kirchhoff path and write the voltage equation using Kirchhoff's Voltage Rule.
5. Substitute relationships for ΔV_C and ΔV_L , but do not substitute numerical values.

Critical Thinking & Concept Invention

1. Which relationship describes the current?

$$I = + \frac{dq_c}{dt}$$

or

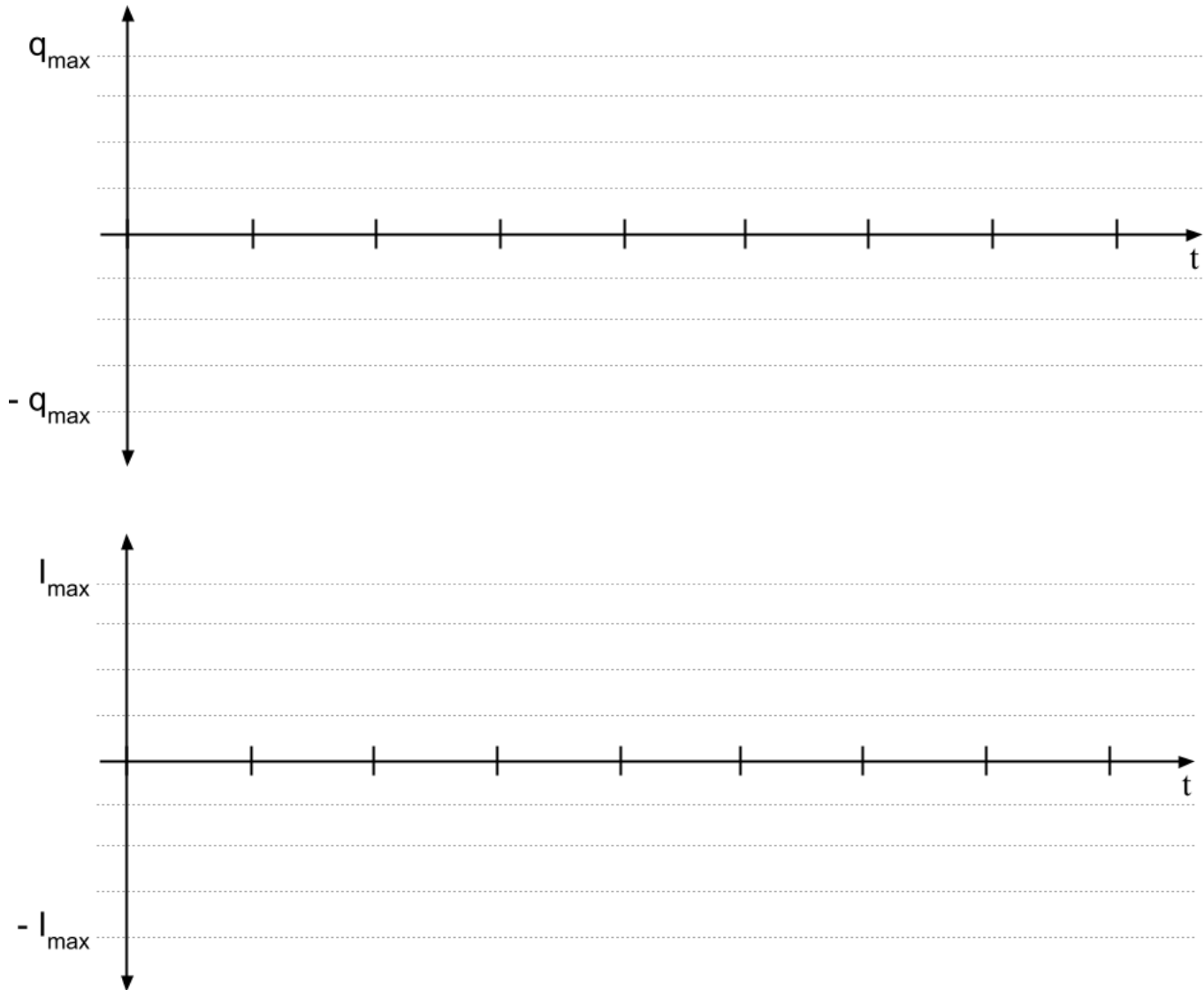
$$I = - \frac{dq_c}{dt}$$
2. Substitute your choice into the voltage equation.
3. Isolate the derivative or second derivative.
4. Which general solution satisfies the differential equation?

$$Ae^{\omega t} + Be^{-\omega t}$$

or

$$A \cos(\omega t) + B \sin(\omega t)$$

5. Determine values of A and B .
6. Write the solution for $q(t)$.
7. Use the relationship between q and I to derive $I(t)$.
8. Sketch both functions over one period.



Apply

1. In an LC circuit:
 - $L = 2.0 \times 10^{-2} \text{ H}$
 - $C = 8.0 \times 10^{-6} \text{ F}$
 - $q_0 = 1.2 \times 10^{-5} \text{ C}$

Determine:

- The angular frequency.
- The time for the capacitor to become completely discharged.
- An equation for $q(t)$.
- An equation for $I(t)$.

- The maximum current.

3.4.4 Energy in Capacitors and Inductors

Model

$$P = \frac{dE}{dt}$$

$$P_{\text{electrical}} = I\Delta V$$

Capacitors	Inductors
$P_C = I\Delta V_C$	$P_L = I\Delta V_L$
$\frac{dU_C}{dt} =$	$\frac{dU_L}{dt} =$

Explore

- How are power and energy related?
 - You can determine power by taking the _____ of energy.
 - You can determine energy by taking the _____ of power.
- Electric power is based on the product of what two electrical quantities?
- Complete the expressions for ΔV_C and ΔV_L .

Critical Thinking & Concept Invention

- Derive expressions for potential energy stored in a capacitor and an inductor.
 - Replace I with its relationship to charge q .
 - Cancel common factors where appropriate.
 - Rewrite with constants in front and integrate both sides.
 - Represent $\int dU$ as ΔU .
 - Integrate from 0 to q or 0 to I .
 - Rewrite capacitor energy using voltage instead of charge.
- What is similar about the structure of each equation?

Summary

Capacitors and inductors store and release energy, allowing them to behave like temporary emf sources. In an LC circuit, maximum capacitor charge occurs when current is zero, and maximum current occurs when capacitor charge is zero. Energy continuously transfers between the capacitor and inductor at a frequency determined by the inductance and capacitance.

3.5: RLC Circuits

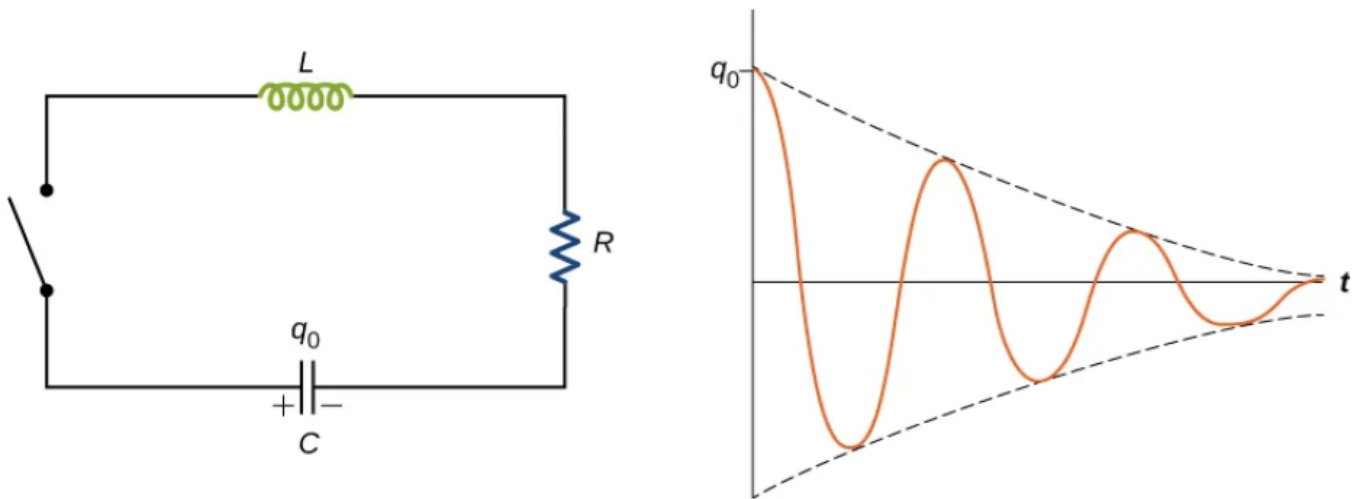
3.5 RLC Circuits

Learning Objectives

- Compare the time-dependent equation for charge in an RLC circuit to RC and LC circuits.
- Interpret different conditions for the oscillation frequency of an RLC circuit (underdamped, overdamped, critically damped).
- Identify instantaneous voltage and current from a phasor diagram for circuits with AC power supplies and resistors.

3.3.1 RLC Charge vs Time

Model



Explore

1. What circuit elements are present in this circuit? What circuit elements are not present?
2. Prior to closing the switch, what are the values for:
 - Capacitor charge and voltage:
 - $q =$
 - $\Delta V_C =$
 - Resistor current and voltage:
 - $I_0 =$
 - $\Delta V_R =$
3. After the switch is closed:
 - Draw the direction of the current when it first becomes non-zero.
 - Draw a Kirchhoff path in that direction.
 - Construct a voltage equation for that path and place (+), (-), or (0) symbols over the equation for the moment the switch is closed at $t = 0$ s.
 - Fill in the second line of the voltage equation using the general expressions for ΔV_C , ΔV_R , and ΔV_L .
 - Add a third line to the voltage equation by substituting any I terms with q terms using the appropriate derivatives. Consider whether the relationship between capacitor charge and current should initially be positive or negative.
4. How is this voltage equation different from previous ones? Why might it be harder to solve?

Critical Thinking & Concept Invention

Examine the graph of charge next to the circuit schematic. This is the solution to the differential equation for charge in this circuit.

1. Which part of the graph resembles the graph of an RL circuit?
2. Which general solution matches the RL circuit?
 - $T(t) = Ae^{+\omega t} + Be^{-\omega t}$
 - OR
 - $T(t) = C \cos(\omega' t + \phi)$
3. Which part of the graph resembles the graph of an LC circuit?
4. Which general solution matches the LC circuit?
 - $T(t) = Ae^{+\omega t} + Be^{-\omega t}$
 - OR
 - $T(t) = C \cos(\omega' t + \phi)$
5. Suppose:

$$q_{RLC}(t) = q_{RC}(t) \times q_{LC}(t)$$

What would the general structure of that function be? Interpret the initial value of charge (q_0) to decide if any of the A's or B's should be zero and what the overall coefficient should be at $t = 0$ s if $\phi = 0$.

$$q_{RLC}(t) =$$

Summary

The charge in an RLC circuit oscillates in a similar way to an LC circuit, but the oscillation dampens due to the exponential decay caused by the resistor. This gives it properties of RL and LC circuits.

The general expression for an RLC circuit is:

$$q(t) = q_0 e^{-\omega t} \cos(\omega' t + \phi) \quad (3.5.1)$$

The oscillation frequency is:

$$\omega' = \sqrt{\omega_{LC}^2 - \omega^2} \quad (3.5.2)$$

where

$$\omega_{LC} = \sqrt{\frac{1}{LC}} \quad (3.5.3)$$

The time constant for this circuit is:

$$\tau_{RLC} = 2\tau_L = \frac{2L}{R} \quad (3.5.4)$$

and

$$\omega = \frac{R}{2L} \quad (3.5.5)$$

Putting all this together:

$$q(t) = q_0 e^{-Rt/(2L)} \cos(\omega' t + \phi) \quad (3.5.6)$$

$$\omega' = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2} \quad (3.5.7)$$

3.5.2 Damped Frequency

Model

The oscillation frequency of an RLC circuit is given by:

$$\omega' = \sqrt{\omega_0^2 - \frac{1}{4}\omega_{RL}^2} \quad (3.5.8)$$

$$\omega' = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \quad (3.5.9)$$

Damped Frequency Table

ω'	$q(t)$	$I(t)$
$R = 0$		
$C = 0$		
$L = 0$		

Explore

- Which term(s) inside the square root depend on:
 - R
 - C
 - L
- Fill in the first column of the table (ω') for scenarios where $R = 0$, $C = 0$, and $L = 0$.

Critical Thinking & Concept Invention

Reducing

- Review the equation for $q(t)$ for an RLC circuit. Copy that equation with the appropriate substitutions for ω and ω' for scenarios where $R = 0$, $C = 0$, and $L = 0$ into the second column of the table.
- Review the equation for $I(t)$ for an RLC circuit. Copy that equation with the appropriate substitutions for ω and ω' for scenarios where $R = 0$, $C = 0$, and $L = 0$ into the third column of the table.
- Review those equations. Do they look familiar? Is anything different?

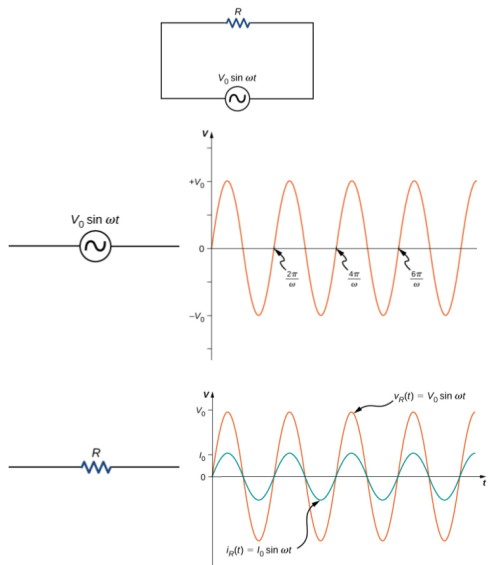
Reducing

- Under what relational values of R , C , and L is:
 - $\omega' > 0$
 - $\omega' < 0$
 - $\omega' = 0$
- Assume $\omega' < 0$. Factor out the negative from the square root and write a new equation for ω' in that scenario.

$\omega' =$

3.5.3 AC Circuit Intro

Model



The circuit below has a voltage/emf source that changes at a regular frequency ω . This source is connected to a resistor. Below the circuit are graphs of the source voltage and resistor voltage over time.

Explore

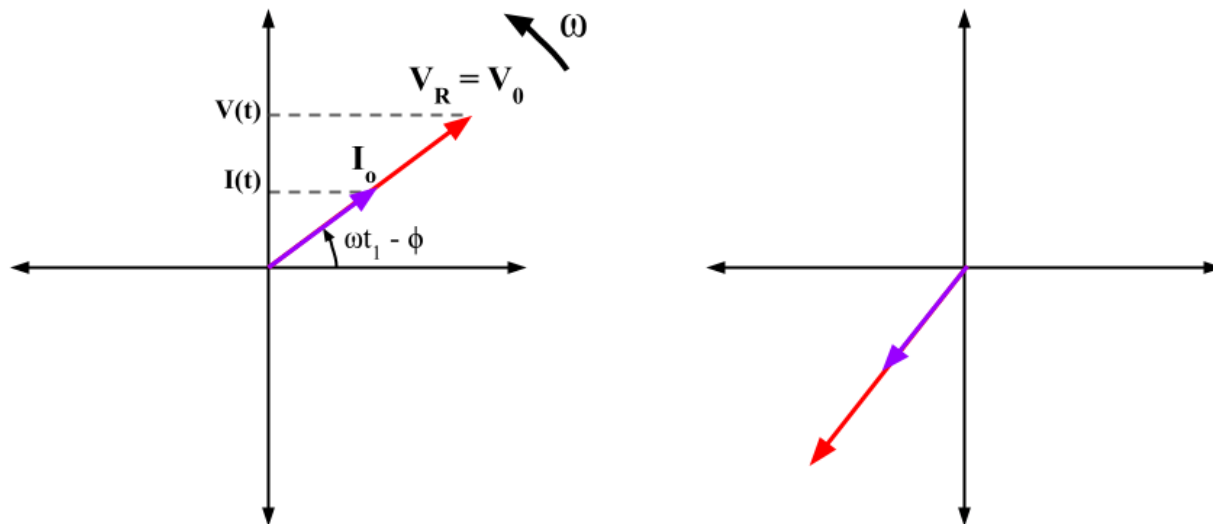
1. Draw a vertical dashed line at the first source voltage maximum and minimum that extends through both graphs.
2. Draw a horizontal dashed line from the first maximum to the second maximum and from the first minimum to the second minimum. Label this line T for period.
3. What variable is used to represent maximum voltage?

Critical Thinking & Concept Invention

1. Do the voltage maxima and minima for the source and resistor occur at the same time or at different times? If different, which circuit element reaches the maximum first?
2. Do the source and resistor have the same maximum voltage? If not, which is greater? Explain why this makes sense.
3. What is the period in terms of the frequency ω ?
4. What do you think the quantity ω represents?

3.5.4 Phasor Intro

Model



The graphs below show the voltage and current of the circuit from the previous model in a diagram called a phasor.

Explore

Focus on the diagram on the left.

1. Two vectors are shown on the diagram. What do those vectors represent?
2. What variable is listed for the magnitude of the vector itself (length of the vector)?
3. Which component (x or y) of those vectors represents the magnitude of those quantities at the snapshot in time shown (t)?
4. Are the two vectors aligned with each other or are they separated by some angle?
5. What direction do these vectors rotate (clockwise or counterclockwise)?

Critical Thinking & Concept Invention

1. Do these vectors seem to change their magnitude, direction, or both?
2. For the quantity or quantities that change with time, what is the equation for that quantity?
3. Draw a curved loop to show the angle measurement for the vectors on the second phasor (right).
4. Use what you said about how the instantaneous voltage and current are shown (that is, what component of the vector represents them) to label $V(t)$ and $I(t)$ on the second phasor diagram. Are these values positive or negative?
5. At what angles would $V(t) = 0$? What is $I(t)$ at those angles?
6. At what angle(s) would $V(t)$ be at its maximum? Would $I(t)$ be at a maximum, minimum, or neither at that angle?

Summary

The voltage drop across a resistor is given by the Ohm relation, even if the voltage and current change with time:

$$\Delta V_R = IR \quad (3.5.10)$$

$$\Delta V_R(t) = I(t)R \quad (3.5.11)$$

The resistance R is a constant, so voltage and current are proportional to each other. They are in phase, so they experience maxima and minima together and have the same period. The value of R simply scales the current amplitude from the voltage

amplitude and changes the units from volts to amps.

Phasor diagrams are a way to represent how the amplitude of voltage and current changes with time when the voltage source has a constant frequency. The vector lengths represent the maximum values, and the projection on the y-axis represents the instantaneous values, which are scaled by the sine function.

3.6: AC Circuits

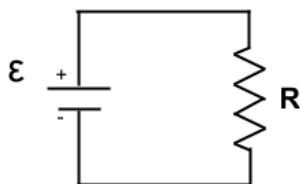
3.6 Alternating Current (AC) Circuits

Learning Objectives

- Describe how voltage peaks relate to current peaks (in phase, leading, lagging) for resistive, capacitive, and inductive circuits.
- Construct trigonometric equations for voltage and current in AC circuits.
- Derive equations for reactance and impedance in AC circuits.
- Use phasors as a model for relating voltages across different circuit elements in an AC RLC circuit.

3.6.1 Voltage and Current Relationships

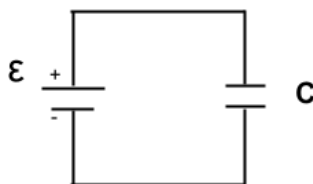
Model



$$\Sigma \Delta V = 0$$

$$\Delta V_R =$$

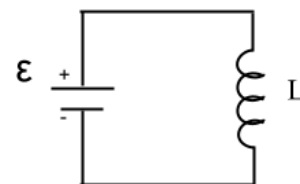
Current _____ Voltage



$$\Sigma \Delta V = 0$$

$$\Delta V_C =$$

Current _____ Voltage



$$\Sigma \Delta V = 0$$

$$\Delta V_L =$$

Current _____ Voltage

Explore

On each circuit:

- Draw a Kirchhoff Loop.
- Construct a voltage equation for that loop with the appropriate expression inserted for ΔV_R , ΔV_C , or ΔV_L .
- Write an equation that relates each ΔV to current (I), using/showing a calculus operation if necessary.
- Explain how voltage and current are related for that circuit element.

Critical Thinking & Concept Invention

- For each circuit element (resistor, capacitor, inductor), does the element experience current before voltage, voltage before current, or current and voltage at the same time? Based on your discussion, choose one of the following to fill in the line relating current and voltage under each circuit.
 - leads
 - lags
 - aligns with

- Underneath those fill-ins, explain your reasoning.

Summary

In Resistors, the voltage difference across the resistor is determined from the circuit configuration and results in a current through the resistor. Because the current is based on the voltage, the two quantities are _____ in resistors.

In Capacitors, the voltage difference across the capacitor is determined from _____ on the capacitor, which results from the current through the wires that connect to the capacitor. Because it takes time to accumulate charge, the current _____ the voltage in capacitors.

In Inductors, the voltage difference (induced emf) across the inductor is determined from _____. Because the inductor creates a voltage to oppose changes to current, the current _____ the voltage in inductors.

3.6.2 AC Voltage and Current Graphs

Model

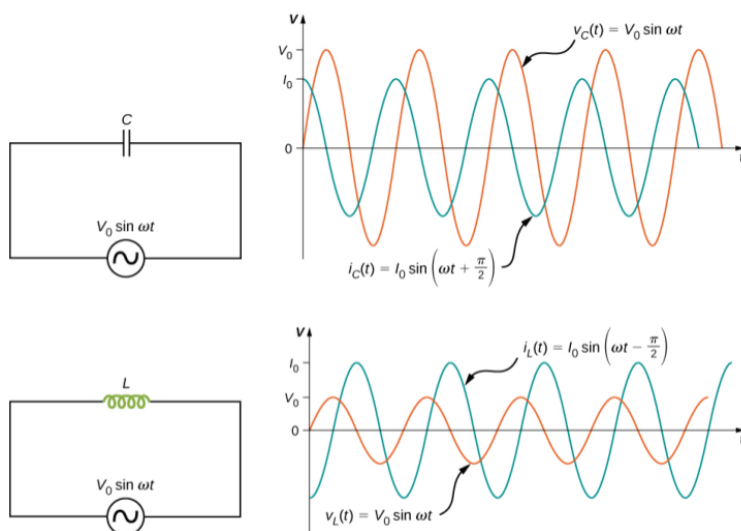


Image Credit: OpenStax University Physics Vol 2 (<https://openstax.org/details/books/university-physics-volume-2>)

Explore

- What two quantities seem to be represented on the y-axis? What is the x-axis?
- What type of function do these graphs seem to resemble?
- Add a vertical dashed line that goes through the first maximum/minimum (whichever is first) for both current and voltage on all three graphs.
- To the right of each graph, write "Voltage _____" using one of the following:
 - leads (if voltage peaks before current peaks)
 - lags (if voltage peaks after current peaks)

Critical Thinking & Concept Invention

- What is the offset between voltage and current in each of the following forms of measurement? Your answer might include fractions and/or π 's:
 - Radians

- Degrees
- Cycles

2. The general form of the equation for cyclical voltage and current is $y(t) = A \sin(\omega t + \phi)$. Explain what each constant in the equation represents and annotate the graph to visually represent each of the quantities:

- A
- ω
- ϕ

Summary

Cyclical patterns are often represented by sine or cosine functions. The general form is:

$$y(t) = A \sin(\omega t + \phi) \quad (3.6.1)$$

Since sine and cosine functions differ only based on a phase shift, we can choose one and stick with it throughout a problem (or use identities to keep the same function, even through derivatives/integrals).

$$A \cos(\omega t) = A \sin\left(\omega t + \frac{\pi}{2}\right) \quad (3.6.2)$$

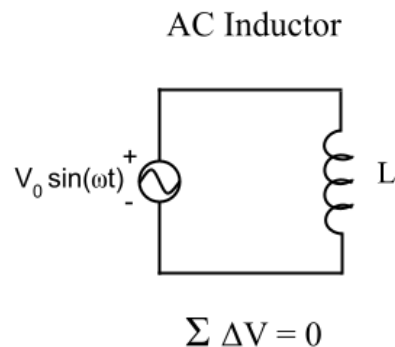
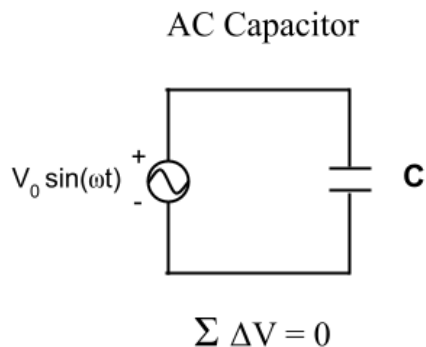
$$-A \cos(\omega t) = A \sin\left(\omega t - \frac{\pi}{2}\right) \quad (3.6.3)$$

The $\pm \frac{\pi}{2}$ represents a quarter-cycle phase shift between the functions, which is the same as a 90° shift between voltage and current.

3.6.3 Reactance

Model

The circuit below represent the same types of circuits as the previous models. In this activity, we will explore the mathematical equations that result in the equations for voltage and current for each circuit.



Explore

1. Draw a Kirchhoff Path in the direction that current will initially (once it becomes nonzero) travel. Use the $+/-$ signs on the voltage source to identify the initial voltage polarity (it will change with time).
2. Construct a voltage equation for the path you drew with the source voltage and the equations for ΔV_C and ΔV_L substituted in.

Critical Thinking & Concept Invention

1. On the next line, rewrite each equation but isolate the variable that most relates to charge (q , I , or $\frac{dI}{dt}$).
2. Perform the necessary calculus operation (derivative or integral) to transform that charge quantity into current, $I(t)$. Remember, you need to apply an operation to both sides of the equation. If needed, use the relationship between sine and cosine to represent the voltage in terms of $\sin(\omega t + \phi)$.
3. The collection of quantities in front of a trigonometric function collectively represents the amplitude of the function (I_0 or V_0 , depending on the relationship). Determine I_0 from the equation you just constructed.
4. AC Capacitors and Inductors can form an equation that resembles Ohm's Relation for resistors:

$$V_0 = I_0 X \quad (3.6.4)$$

where X_C is called Capacitive Reactance and X_L is called Inductive Reactance. Determine X_C and X_L as functions of C , L , ω .

$$X_C = \quad (3.6.5)$$

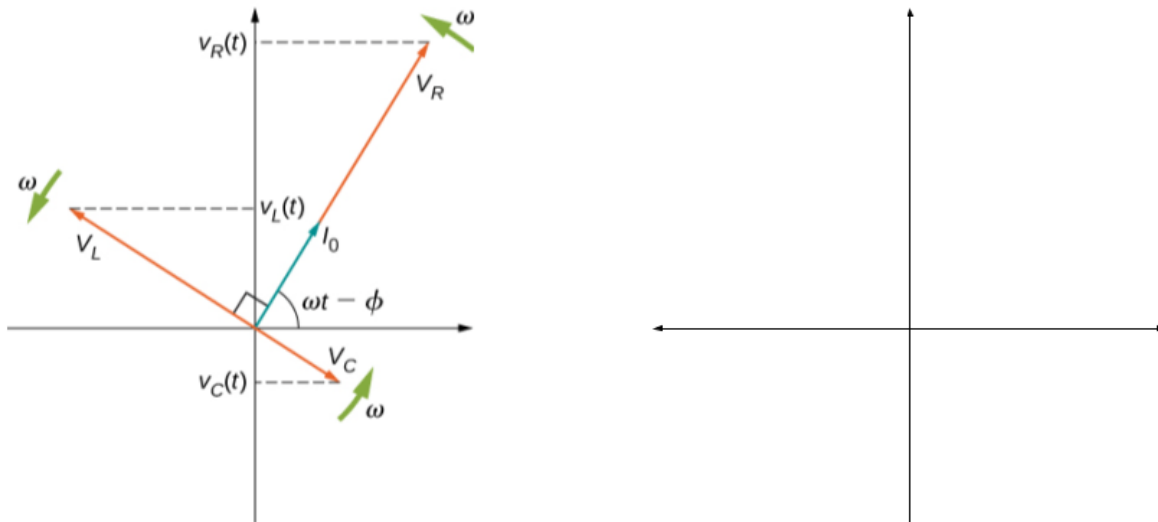
$$X_L = \quad (3.6.6)$$

Summary

Reactance is a property similar to resistance in that it represents a proportionality between current and voltage in the particular circuit element. When a combination of resistors, capacitors, and/or inductors are in series, they all have the same _____. Based on the resistance or reactance, we can then determine the _____ for each circuit element, which is often not the same.

3.6.4 AC RLC Phasor: Part 1

Model



Left Image Credit: OpenStax University Physics Vol 2 (<https://openstax.org/details/books/university-physics-volume-2>)

Explore

1. Which of the V 's is aligned with current?
2. What direction (clockwise or counterclockwise) is this model rotating?
3. Label the other V 's as either leading or lagging based on its relationship to the vector for current.

4. What is the angle between the different V vectors?
5. What is the angle (in terms of given constants and variables) between the vector I_0 and the positive x-axis?
6. The actual measured values at any moment in time are listed at the end of the dashed lines. Are these values measured on the x-axis or the y-axis?

Critical Thinking & Concept Invention

1. Based on what you said for the last Explore question and the convention that angles are measured from the positive x-axis, would the measured values come from a sine or cosine relationship with the hypotenuse?
2. Label each vector by replacing the V using Ohm's Relation and the appropriate expression for resistance or reactance.
3. Construct an equation for the magnitude of the vector addition of V_L and V_C . Consider: are these vectors aligned or opposing?
4. On the blank graph, redraw vector $\vec{V}_R$ and draw the vector sum of $\vec{V}_L$ and $\vec{V}_C$. Then add those vectors together graphically (tip to tail) and draw in the resultant. Label this V_0 .
5. Summarize how all of the vectors ($\vec{V}_R$, $\vec{V}_L$, $\vec{V}_C$, and $\vec{V}_0$) relate to the current $\vec{I}_0$ and each other.

Summary

The voltages across a source, resistor, capacitor, and inductor for alternating current is represented visually as a phasor diagram. Voltage is not really a vector, but vector mathematics can be used to represent cyclical changes. The vector length is the amplitude of each function, which represents the maximum value. The actual measured value at a moment in time is the y-value on the phase plot.

3.6.5 AC RLC Phasor: Part 2

Model

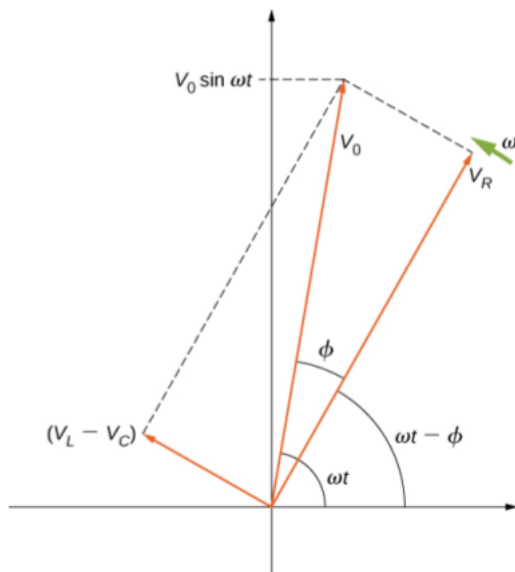


Image Credit: OpenStax University Physics Vol 2 (<https://openstax.org/details/books/university-physics-volume-2>)

Explore

1. What are the labels on each of the three vectors and which vector represents two different circuit elements?
2. If you put the three voltage vectors together as a right triangle, which would be the hypotenuse?
3. Describe what each of the following seem to represent on this diagram:
 - ϕ

◦ ωt

Critical Thinking & Concept Invention

1. At time $t = 0$ s, which V vector would align with the x-axis? What would the measured voltage of the circuit element be?
2. To the right of the model, construct an equation relating V_0 to the other vectors using what you know about right-triangle geometry.
3. Below that, construct an equation for the angle ϕ using trigonometric ratios.
4. The source voltage and the circuit current can also be related in an equation that resembles Ohm's Relation:

$$V_0 = I_0 Z \quad (3.6.7)$$

where Z is called the Impedance of the AC circuit. Replace the V 's in the equation (based on their relationship to current (I_0) and their resistance/reactance equations) to identify Z in terms of:

- R , X_L , and X_C
- R , C , L , ω

$$Z = \quad (3.6.8)$$

$$Z = \quad (3.6.9)$$

5. Identify equations for ϕ in terms of:

- R , X_L , and X_C
- R , C , L , ω

$$\phi = \quad (3.6.10)$$

$$\phi = \quad (3.6.11)$$

Summary

Impedance (Z) in a circuit is a combination of resistance and reactance of the different circuit elements. The quantity represents how the current in the circuit is affected by all of the different circuit elements.

$$Z = \quad (3.6.12)$$

The phase shift between the voltage source (V_0) and the current (which aligns with V_R) is influenced by how capacitive or inductive the circuit is:

- If the circuit is more capacitive ($V_C > V_L$), the current is shifted _____.
- If the circuit is more inductive ($V_L > V_C$), the current is _____.
- If the circuit is equally capacitive and inductive ($V_C = V_L$), the current is _____.

Apply

The output of an ac generator connected to an RLC series combination has a frequency of $f = 200$ Hz and an amplitude of $V_0 = 0.100$ V. If $R = 4.00 \, \Omega$, $L = 3.00 \times 10^{-3}$ H, and $C = 8.00 \times 10^{-4}$ F,

1. Determine the:
 - Capacitive reactance
 - Inductive reactance
 - Circuit impedance
 - Current Amplitude (I_0)
 - phase difference between current and emf of the generator (ϕ) in degrees and radians (in terms of decimal π)
2. Construct a Phasor diagram for time $t = 0$ s.

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4: ELECTRIC AND MAGNETIC FORCES

- 4.1: Charges and Electric Fields
- 4.2: Electric Field Lines and Equipotentials
- 4.3: Torque on an Electric Dipole
- 4.4: Magnetic Force on a Moving Charge
- 4.5: Magnetic Force and Torque on a Current Carrying Wire

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4.1: Charges and Electric Fields

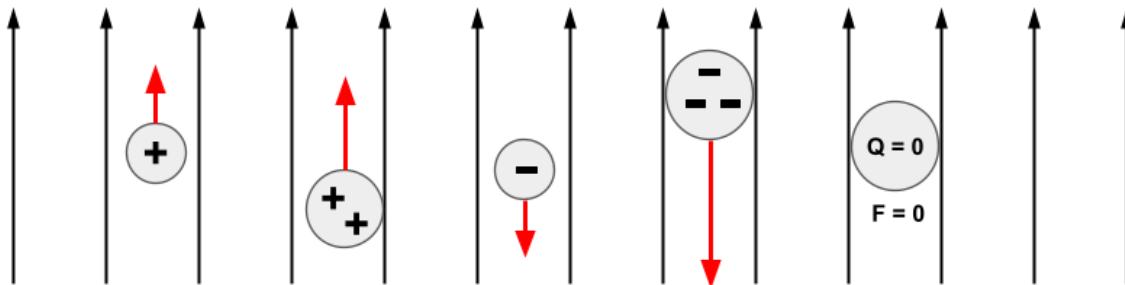
4.1 Charges and Electric Fields

Learning Objectives

- Describe how positive and negative charges are influenced by electric fields.
- Predict how objects with different charge attract or repel each other.
- Determine Electric Force vectors from on charges in an Electric field.

4.1.1 Electric Force and Electric Fields

Model



The set of parallel arrows that all point in the same direction represent a constant electric field in the direction of the arrows. The arrows attached to each charged circle represent the electric force that charge experiences from the electric field.

Explore

1. Which direction is the electric field?
2. Circle the charges that experience an upward force. What is true about all of them?
3. Box the objects that experience a downward force. What is true about all of them?
4. There should be one object left over. How is it different from the rest?

Critical Thinking & Concept Invention

1. Summarize the patterns you observed:
 - Positive (+) charges experience a force that is _____ the direction of the electric field.
 - Negative (−) charges experience a force that is _____ the direction of the electric field.
 - Objects with more charge experience _____ force.
 - Neutral objects _____.

Summary

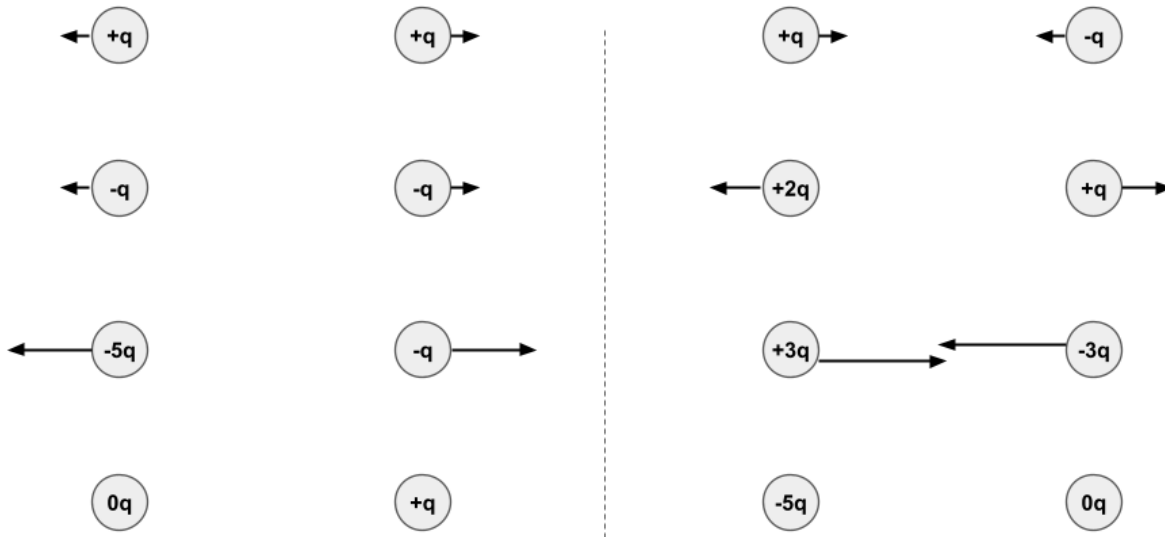
The electric force an object experiences is based on the charge (q) of an object and the strength and direction of the electric field ($\vec{E}$). This force is given by the vector equation:

$$\vec{F}_e = q\vec{E} \quad (4.1.1)$$

The negative sign that occurs when q is negative flips the direction of the force so that it opposes the direction of the electric field.

4.1.2 Interactions Between Charges

Model



Explore

- Next to each pair, write *Attracts* or *Repels* based on the direction of the forces.
- Fill in the table below to identify the number of charges in each pair and the strength of the attractive or repulsive force.

Charge Interaction Table

Object Pair	Left Charge (Q_L)	Right Charge (Q_R)	Attracts/Repels	Force Magnitude
A				
B				
C				
D				

Critical Thinking & Concept Invention

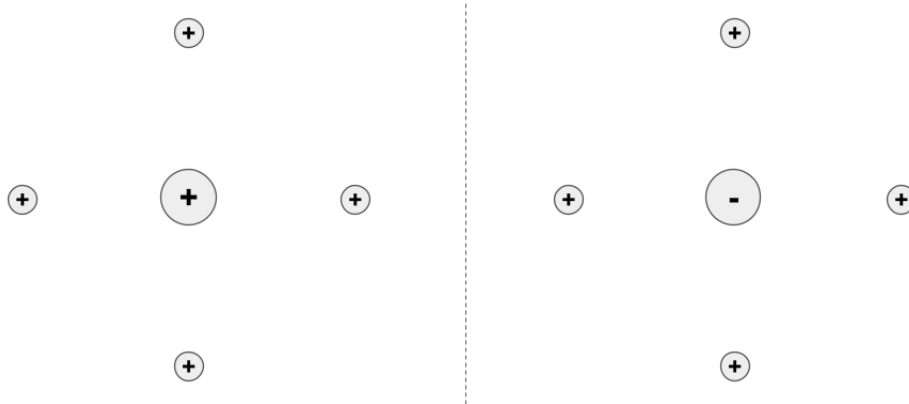
- Identify whether each of the following depends on the types of charge (+/-) or the amount of charge:
 - The direction of the electric force (attraction or repulsion) depends on _____.
 - The magnitude of the electric force depends on _____.
- Create a rule: When do objects attract and when do they repel?
- Create a rule: What determines the numerical magnitude of the electric force between two objects?

Apply

In each scenario below, the middle charge is fixed and cannot move.

- Annotate the diagram with the following:

- Next to each charge around the middle charge, write *attract* or *repel* to identify the direction of the force on that charge from the middle charge only.
- Next to each outer charge, draw an arrow to identify the direction that the electric field must point to create a force that matches your selection of attraction or repulsion.



4.1.3 Electric Force

Model

$\vec{E} = \begin{bmatrix} 2 \\ 2 \end{bmatrix} \frac{\text{N}}{\text{C}}$ everywhere

$\vec{F}_1 = q_1 \vec{E} = () \begin{bmatrix} 2 \\ 2 \end{bmatrix} \frac{\text{N}}{\text{C}} = \begin{bmatrix} \\ \\ \end{bmatrix} \text{N}$
 $\vec{F}_2 = q_2 \vec{E} = () \begin{bmatrix} 2 \\ 2 \end{bmatrix} \frac{\text{N}}{\text{C}} = \begin{bmatrix} \\ \\ \end{bmatrix} \text{N}$

Explore

- Fill in the equation to determine the force that each charge experiences, then draw a force vector for each charge.
- Is Charge 1 positive or negative? Is Charge 2 positive or negative?
 - Charge 1 is _____.
 - Charge 2 is _____.
- How does the direction of the force compare to the direction of the electric field?
 - The force on Charge 1 points _____ the direction of the electric field.
 - The force on Charge 2 points _____ the direction of the electric field.

Critical Thinking & Concept Invention

1. How does the direction of the force on positive and negative charges compare to the direction of the electric field?
 - For positive charges: _____
 - For negative charges: _____
2. Explain mathematically why the direction differs for positive and negative charges.
3. If both charges were free to move, identify the direction each charge would accelerate and explain your reasoning.

Summary

The electromagnetic force on a charged object depends on its charge q , the electric field $\vec{E}$, its velocity $\vec{v}$, and the magnetic field $\vec{B}$:

$$\vec{F}_e = q \left(\vec{E} + \vec{v} \times \vec{B} \right) \quad (4.1.2)$$

When the magnetic field is zero, this simplifies to:

$$\vec{F}_e = q\vec{E} \quad (4.1.3)$$

A negative value of q reverses the direction of the force relative to the electric field.

4.2: Electric Field Lines and Equipotentials

4.2 Field Lines and Equipotentials

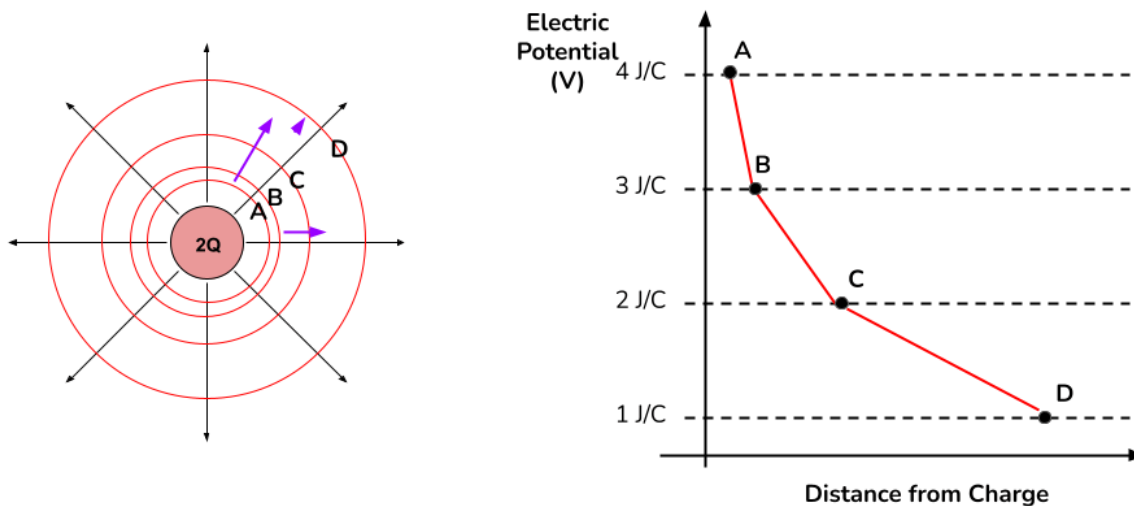
Learning Objectives

- Mathematically relate electric field and electric potential using the concept of slope.
- Qualitatively draw electric field lines from charges and interpret electric field strength from those lines.
- Use electric field lines to construct equipotential lines and vice versa.

4.2.1 Electric Potential and Electric Field

Model

The model on the left shows rings of constant electric potential (equipotential contours) and arrows that represent the electric field vector at three different points. On the right is a graph of electric potential (Voltage) versus distance from the central charge.



Explore

- Fill in the table below to identify and compare trends:

Electric Field and Potential Comparison

In the Region Between...	Electric Field High / Medium / Low	Electric Potential Close / Medium / Far	Steepness of Graph Steepest / Medium / Least Steep
A and B			
B and C			
C and D			

- What is the difference in electric potential between:
 - A and B:

- B and C:
- C and D:

Critical Thinking & Concept Invention

Identify Patterns

1. The difference in Electric Potential between each consecutive electric potential line:
 - Increases
 - Decreases
 - Is always the same.
2. When the electric field strength is High:
 - The slope on the Electric Potential vs Distance graph is (steep / not steep).
 - The electric potential lines are (far apart / close together).
3. When the electric field strength is Low:
 - The slope on the Electric Potential vs Distance graph is (steep / not steep).
 - The electric potential lines are (far apart / close together).
4. The direction of the electric field vector points ("up hill" / "downhill") on the Electric Potential vs Distance graph.

Summary

The Electric Field is a vector quantity that is equal to the spatial rate of change of electric potential and points in the direction of decreasing electric potential. This can be represented in one dimension mathematically as:

$$\vec{E}_{avg} = -\frac{\Delta V}{\Delta l} \quad (4.2.1)$$

The negative sign makes the electric field point toward decreasing potential. Rate quantities are averages when using the secant line slope. In the limit that $l \rightarrow 0$, the expression becomes a derivative:

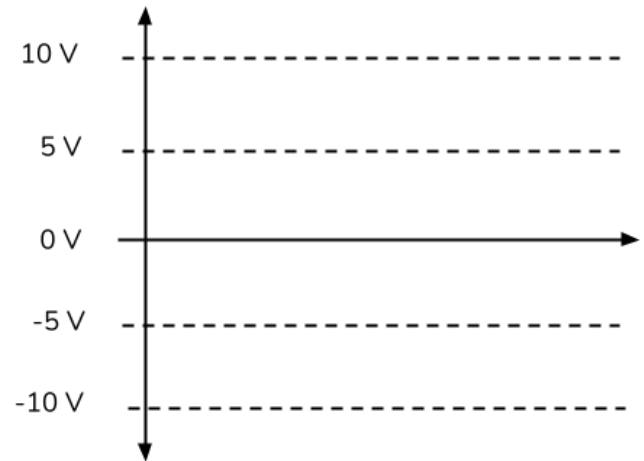
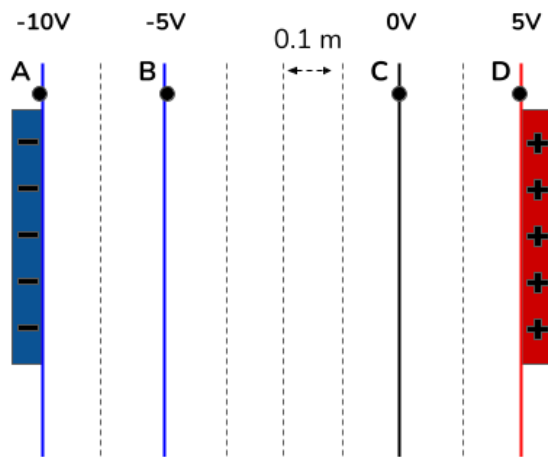
$$\vec{E} = -\frac{dV}{dl} \quad (4.2.2)$$

This relationship works for one-dimensional scenarios. Vector algebra and vector calculus are necessary for two- and three-dimensional applications. A two- or three-dimensional slope/derivative is called a gradient. The Electric Vector Field is the negative gradient of the Electric Potential Scalar Field:

$$\vec{E} = -\vec{\nabla} V \quad (4.2.3)$$

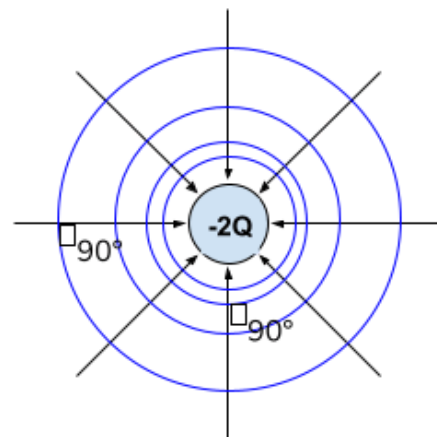
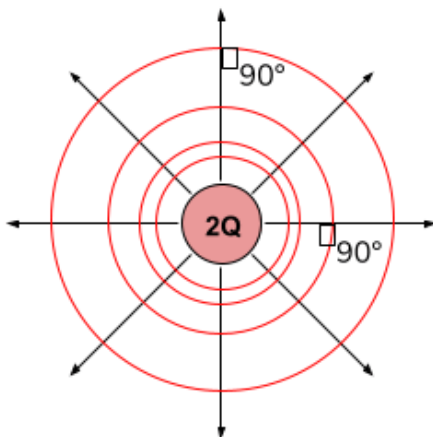
Apply

Use the electric field contours to make a graph of electric potential versus distance. Assume linear change between plotted points. Then calculate the electric field in each region and sketch electric field vectors on the diagram.



4.2.2 Equipotential Contours and Electric Field Lines

Model



Explore

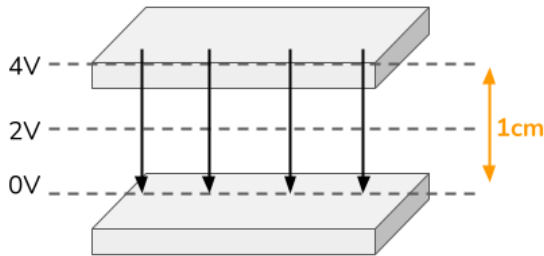
- Where are the electric potential contours (circles) closest together? Are the electric field lines close together or far apart at that location?
 - The electric potential lines are closest together _____.
 - The electric field lines are near / far at that location.
- Where are the electric potential contours (circles) farthest apart? Are the electric field lines close together or far apart at that location?
 - The electric potential lines are farthest apart _____.
 - The electric field lines are near / far at that location.
- What angle do the electric potential lines make with the electric field lines when they intersect?

Critical Thinking & Concept Invention

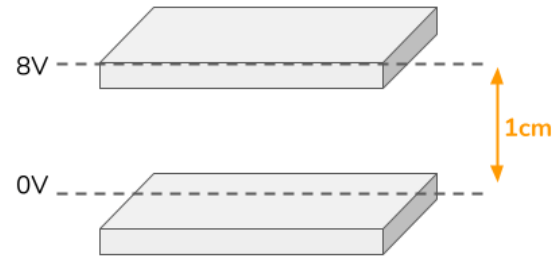
- Summarize how you could draw Electric Potential Lines from Electric Field Lines:
 - Start electric potential lines at electric field lines by making a _____ degree angle.
 - When electric field lines are close together, make the electric potential lines _____.
 - When electric field lines are far apart, make the electric potential lines _____.

Apply

- Complete the model by adding:
 - Equipotentials at 2 V spacings and Electric Field lines on Capacitor B.
 - Charges on both plates of Capacitor A and B that match the electric field lines you drew.

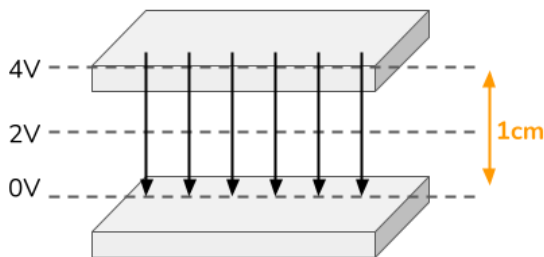


Capacitor A

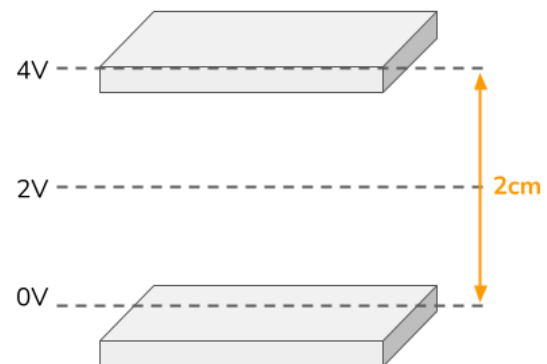


Capacitor B

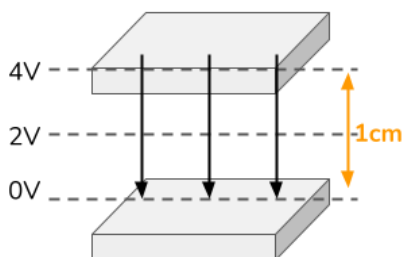
- For each of the following capacitor pairs:
 - Draw the electric field lines that match the electric potential shown.
 - Draw charges on the plates that are appropriate for the electric field lines.
 - Identify which capacitor (A or B) has the highest capacitance ($C = \frac{Q}{\Delta V}$).
 - Select the proportionality that matches how capacitance relates to the quantity that is different for Capacitor A and Capacitor B.



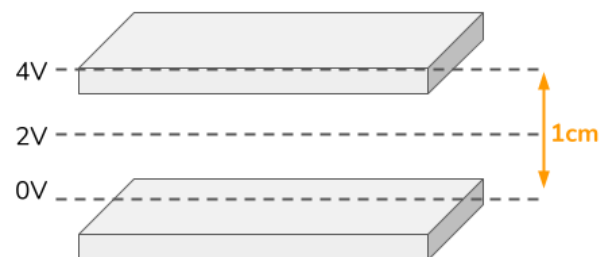
Capacitor A



Capacitor B



Capacitor A



Capacitor B

$$C \propto d \quad \text{OR} \quad C \propto \frac{1}{d} \quad (4.2.4)$$

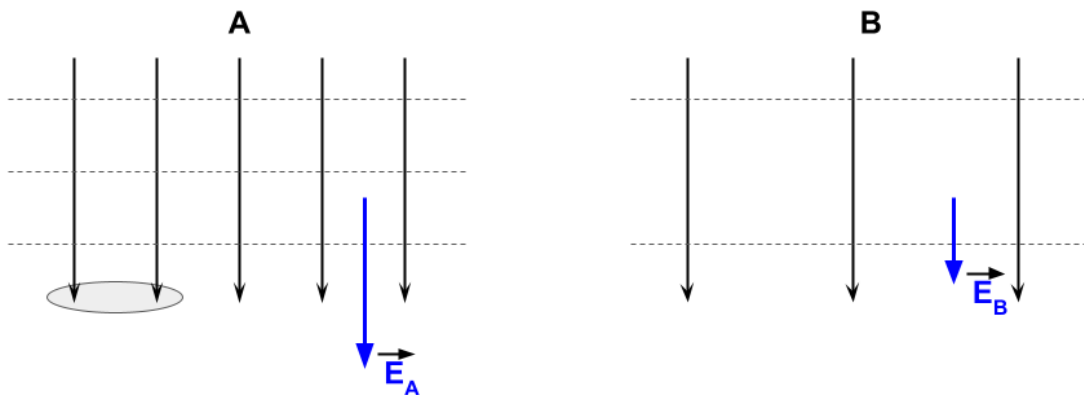
$$C \propto A \quad \text{OR} \quad C \propto \frac{1}{A} \quad (4.2.5)$$

4.2.3 Electric Field Vectors from Equipotentials

Model

----- Equipotential Line

→ Electric Field Line



Explore

1. Draw an oval on Scenario B that would have the same Electric Flux going into it as the one drawn in Model A.
2. Use the model to complete the table.

Electric Field, Equipotential, and Flux Comparison

Scenario	E Lines (close/far)	V Contours (close/far)	E-Vector Magnitude (greater/lesser)	Flux Area (greater/lesser)
A				
B				

Critical Thinking & Concept Invention

1. As a group, summarize patterns relating electric field lines, electric potential contours, and the magnitude of the electric field.

$$\vec{E} = -\frac{\Delta V}{\Delta l} \quad (4.2.6)$$

- Closer equipotential lines = _____ slope = $|\vec{E}|$
- Farther equipotential lines = _____ slope = $|\vec{E}|$

$$V_e = -\sum \vec{E} \cdot \Delta \vec{l} \quad (4.2.7)$$

- Greater Electric Field = _____ distance between Equipotentials

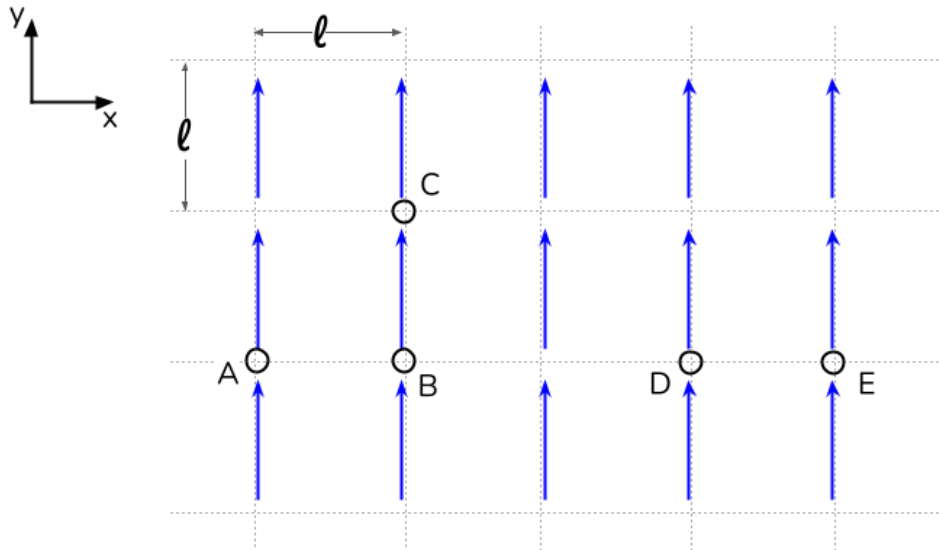
- Lesser Electric Field = _____ distance between Equipotentials

$$\Phi_E = \vec{E}_{avg} \cdot \vec{A} \quad (4.2.8)$$

- Greater Electric Field = _____ Area for same Flux
- Lesser Electric Field = _____ Area for same Flux

4.2.4 Calculating Electric Potential Differences

Model



Explore

1. Draw a path that connects letters in alphabetical order by making straight lines between consecutive letters.
2. Identify the directions of electric field and path for each segment of the overall journey.

Path Segment Analysis

	A to B	B to C	C to D	D to E
$\vec{E}$ Direction (+/- x or y)				
$\Delta \vec{l}$ Direction (+/- x or y)				
$\vec{E}$ and $\Delta \vec{l}$ are ($\parallel$ / $\perp$ / neither)				

Critical Thinking & Concept Invention

1. Identify the direction of increasing Electric Potential (V_e).
2. Label the last row " ΔV " and identify the sign (+ / - / 0) of the change in electric potential along each segment.
3. Use the relationship

$$\Delta V = -\vec{E} \cdot \Delta \vec{l} \quad (4.2.9)$$

to determine the change in potential for each segment of the path. Use column vectors to explicitly show this dot product.

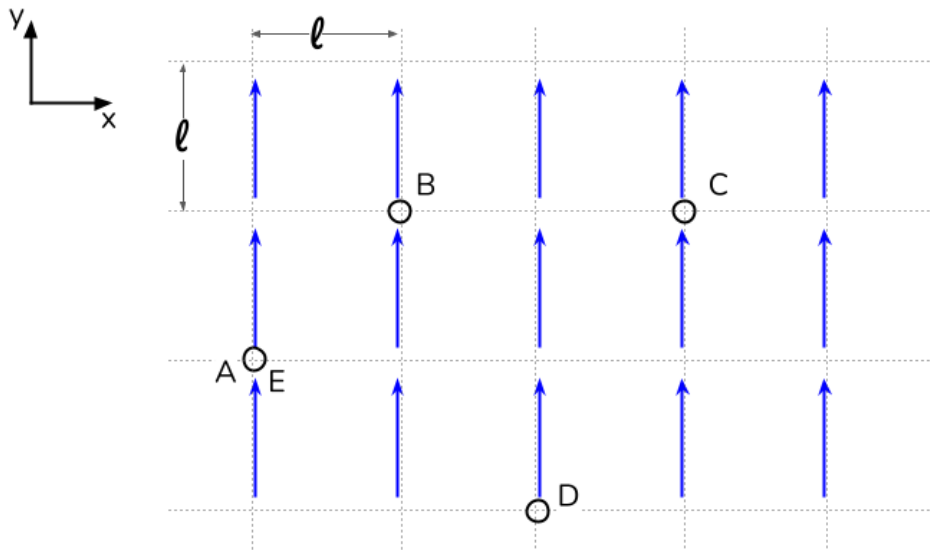
4. Find the total change in electric potential from A to E by summing each individual change:

$$\sum \vec{E} \cdot \Delta \vec{l} \quad (4.2.10)$$

Apply

1. Draw a path that connects letters in alphabetical order by making straight lines between consecutive letters. Note that A and E are the same position.
2. Along each path, identify the sign (+ / - / 0) in the change in electric potential (ΔV) along that path.
3. Determine the change in electric potential along each path. Explicitly show vectors in column vector notation to take dot products.
4. Determine the Circulation (Γ_E) of the closed path, which is the sum of each change in electric potential:

$$\Gamma_E = \sum \vec{E} \cdot \Delta \vec{l} \quad (4.2.11)$$



Summary

For constant electric fields,

$$\Delta V = -\vec{E} \cdot \Delta \vec{l} \quad (4.2.12)$$

It can sometimes be easier to break up the path into segments. The change in electric potential from one end to the other is then the sum of each segment on that path:

$$\Delta V = -\sum \vec{E} \cdot \Delta \vec{l} \quad (4.2.13)$$

If the electric field is not constant, we need to take the limit as $\Delta l \rightarrow 0$ and add up all of those segments. This becomes an integral:

$$\Delta V = -\int \vec{E} \cdot d\vec{l} \quad (4.2.14)$$

If we take this integral (or sum) over a closed loop, the result is called the Circulation of the Electric Field (Γ_E).

$$\Gamma_E \equiv \oint \vec{E} \cdot d\vec{l} \quad (4.2.15)$$

The circulation of the electric field is given by Faraday's Law which states:

$$\Gamma_E = -\frac{d\Phi_B}{dt} \quad (4.2.16)$$

In the equation above, Φ_B is magnetic flux. When magnetic fields are not changing, $\frac{d\Phi_B}{dt} = 0$ and the circulation of the electric

field is zero.

Static charges do not create or affect magnetic fields, so all electric fields that are created by charges are conservative fields, which means the circulation is zero everywhere. Until we learn about magnetism, we will assume:

$$\Gamma_E = 0 \quad (4.2.17)$$

4.3: Torque on an Electric Dipole

4.3 Electric Dipoles

Learning Objectives

- Identify vector dot products and cross products in derivations.
- Determine the torque and potential energy of an electric dipole.

Force and Torque Review

From Mechanics, we know the following relationships for force and torque:

$$\vec{a} = \frac{1}{m} \sum \vec{F} \quad (4.3.1)$$

$$\vec{\tau} = \vec{r} \times \vec{F} \quad (4.3.2)$$

$$\vec{\alpha} = \frac{1}{I} \sum \vec{\tau} \quad (4.3.3)$$

In this activity, we will explore force and torque from electric fields. Recall the electric force on a charged particle is:

$$\vec{F}_E = q\vec{E} \quad (4.3.4)$$

In this equation, the charge q is a signed quantity and a negative sign causes the force to point in the opposite direction of the electric field.

Vector Patterns

If you have a scalar relationship that is a product of two vector quantities multiplied by $\cos \theta$, that relationship follows dot product mathematics.

$$ab \cos \theta \rightarrow \vec{a} \cdot \vec{b} \quad (4.3.5)$$

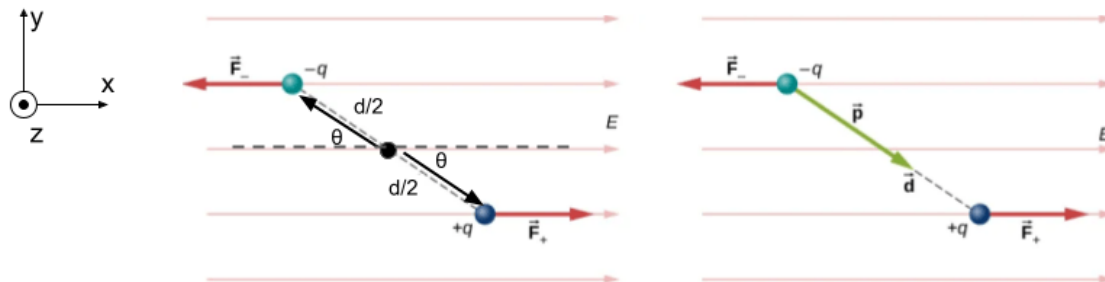
If you have a vector relationship that is a product of two vector quantities multiplied by $\sin \theta$, that relationship might follow cross product mathematics. To be sure, you need to check that the direction of the vector also follows the Right Hand Rule.

$$ab \sin \theta \rightarrow \vec{a} \times \vec{b} \quad \text{or} \quad \vec{b} \times \vec{a} \quad (4.3.6)$$

Look out for these relationships in your calculations to find vector relations between quantities.

4.3.1 Dipole Torque

Model



$$\sum \vec{\tau} = \sum (\vec{r} \times \vec{F}) \quad (4.3.7)$$

$$\vec{\tau} = \vec{\tau}_+ + \vec{\tau}_- \quad (4.3.8)$$

$$\vec{\tau}_{\text{dipole}} = \quad (4.3.9)$$

$$\sum \vec{\tau} = \sum \vec{r} \times \vec{F} \quad (4.3.10)$$

$$\vec{\tau}_+ \quad \vec{\tau}_- \quad (4.3.11)$$

$$\begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix} \times \begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix} + \begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix} \times \begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix} \quad (4.3.12)$$

$$\begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix} + \begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix} \quad (4.3.13)$$

$$\vec{\tau}_{\text{dipole}} = \begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix} \quad (4.3.14)$$

$$\vec{\tau}_{\text{dipole}} = \quad (4.3.15)$$

Explore

1. Use the equation for torque to put a variable $\vec{r}$ or $\vec{F}$ with appropriate subscripts on each column vector in the first line of the equation.
2. Examine the model to fill in key information:
 - Determine the vectors from the origin to each charge and fill in the first column vector of each cross product.
 - Determine the force of the field on each charge and fill in the second column vector of each cross product. Be careful about sign.

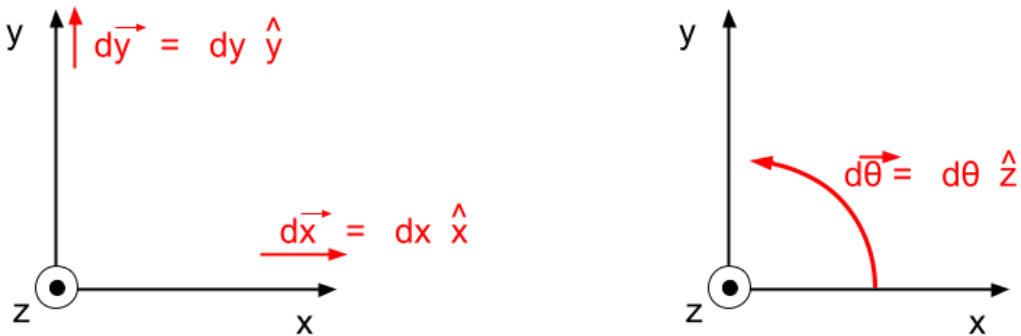
Critical Thinking & Concept Invention

- Determine the torque on the dipole:
 - Take the vector cross product to determine each torque.
 - Add the torques together to get the net torque.
 - Replace any Qd with p .
 - Look at the vector patterns and replace scalar products with vector products that align with those patterns.
 - Your answer should include the dipole moment vector $\vec{p}$.

Unit Vector Changes

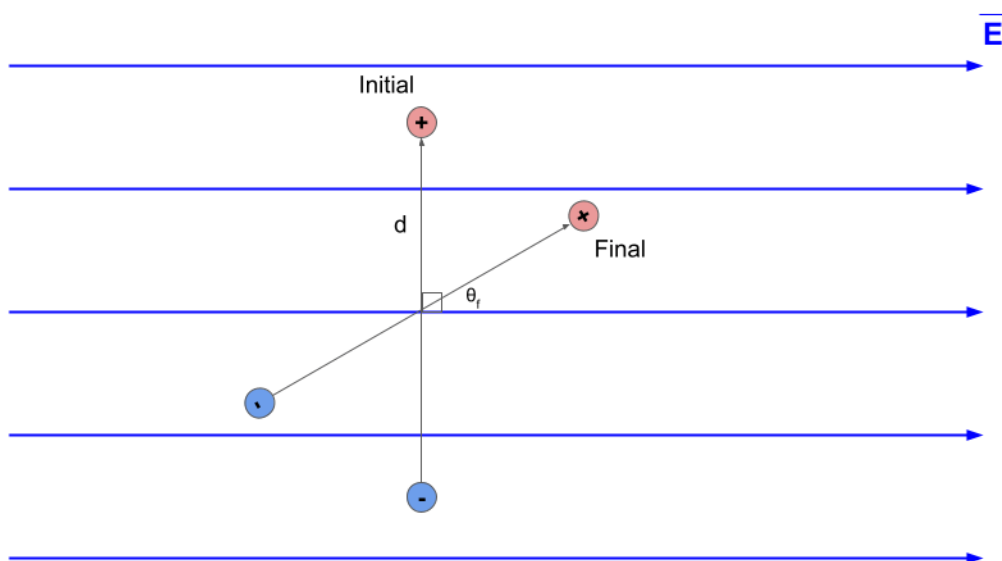
In Cartesian coordinates the direction of the coordinate unit vectors are fixed. This is why we can use Cartesian components in column vectors. In other coordinate systems, the coordinate unit vectors do not necessarily change in the direction of the coordinates themselves. This can make dot and cross products more challenging.

As an example, the direction that theta changes is a counterclockwise direction. We represent this with an axial vector in the z-direction.



1.2 Dipole Potential Energy

Model



$$\Delta U = -W$$

(4.3.16)

$$W = \int \vec{\tau} \cdot d\vec{\theta} \quad (4.3.17)$$

$$W = \int \tau \hat{\tau} \cdot d\theta \hat{\theta} \quad (4.3.18)$$

$$\Delta U = \quad (4.3.19)$$

Explore

1. Identify the direction of $\vec{\tau}$ and $d\vec{\theta}$ as coordinate unit vectors. Fill in those unit vectors in the work integral.
2. Examine the model to fill in key information:
 - What is the initial angle of the dipole? Put that value (in radians) on the lower limit of the work integral.
 - What is the final angle of the dipole? Put that value or variable on the upper limit of the work integral.

Critical Thinking & Concept Invention

1. Determine the change in potential energy:
 - Use the equation you have for torque from the previous section to fill in the $\vec{\tau}$ part of the integral.
 - Use the version with sine/cosine and include a unit vector for direction.
 - Remind each other which direction the angle θ changes on a unit circle.
 - Use the Right Hand Rule to determine the direction of that change and write in that unit vector after $d\theta$.
 - Evaluate the integral to determine the work done to the dipole as it rotates.
 - Use the relationship between work and potential energy to determine the change in potential energy of the dipole due to its rotation.
 - Examine your result. Can you turn this into a vector equation that includes the dipole moment vector $\vec{p}$?

Summary

When an electric dipole is in an external field, it can experience a torque that does work on the dipole. We can use vector concepts to identify the torque on and potential energy of an electric dipole in an electric field.

$$\vec{p} = q\vec{d} \quad (4.3.20)$$

$$\vec{\tau} = \quad (4.3.21)$$

$$U = \quad (4.3.22)$$

By convention, we identify a potential energy of zero when the dipole is perpendicular to the field. This means the potential energy can range from $+pE$ to $-pE$ with a maximum change in potential energy of $\pm 2pE$.

4.4: Magnetic Force on a Moving Charge

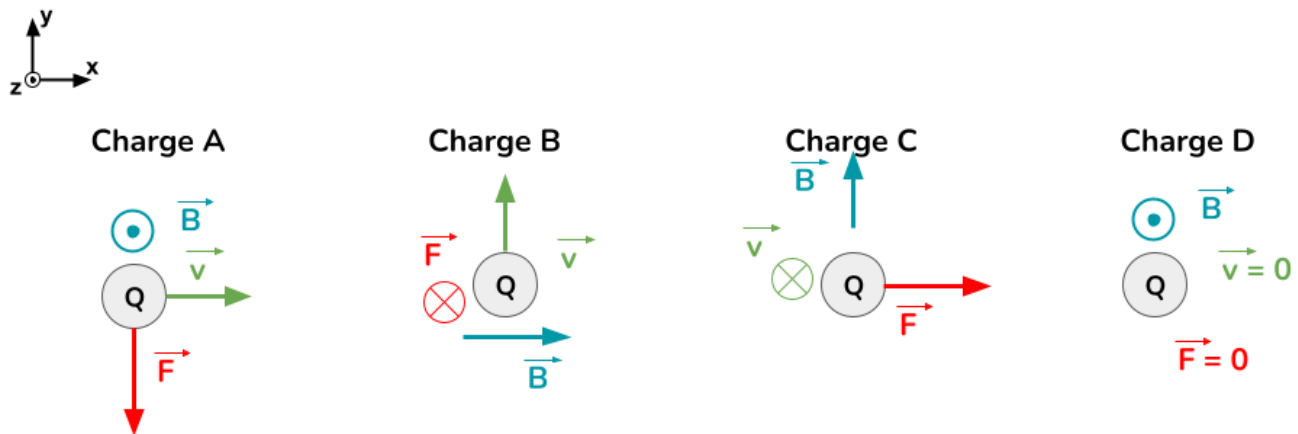
4.4 Magnetic Force

Learning Objectives

- Determine the direction of the magnetic force on moving positive and negative charges.
- Calculate the magnitude of the magnetic force on moving positive and negative charges.
- Determine the magnetic force between current-carrying wires.

4.4.1 Magnetic Force on a Moving Charge

Model



Explore

1. Use the model to identify the direction of each vector for each charge. Put zero if the vector quantity is zero.

Magnetic Force Direction Table

Charge	Velocity (x, y, z, 0)	Magnetic Field (x, y, z, 0)	Force (x, y, z, 0)
A			
B			
C			
D			

2. Do any of the vectors ever point in the same direction (zero doesn't count)?

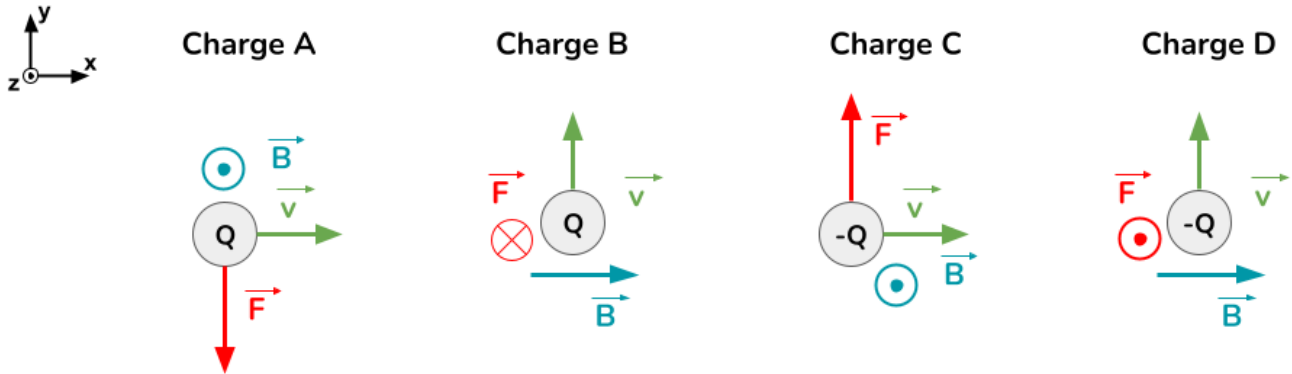
Critical Thinking & Concept Invention

1. What angle do these vectors make with each other?
2. Align your fingers (not your thumb) with the magnetic field and your thumb with the velocity vector. What part of your hand

points in the direction of the force? Check with each charge.

4.4.2 Magnetic Force: Positive and Negative Charges

Model



Explore

1. Use the model to identify the direction of each vector for each charge. Put zero if the vector quantity is zero.

Positive and Negative Charge Magnetic Force Table

Charge	Velocity (x, y, z, θ)	Magnetic Field (x, y, z, θ)	Force (x, y, z, θ)
A			
B			
C			
D			

2. Do any of the vectors ever point in the same direction (zero doesn't count)?

Critical Thinking & Concept Invention

1. Do negative and positive charges respond to magnetic fields in the same way? If not, describe the difference.

Apply

Identify the direction of the magnetic force on a negative charge that travels through the following fields.

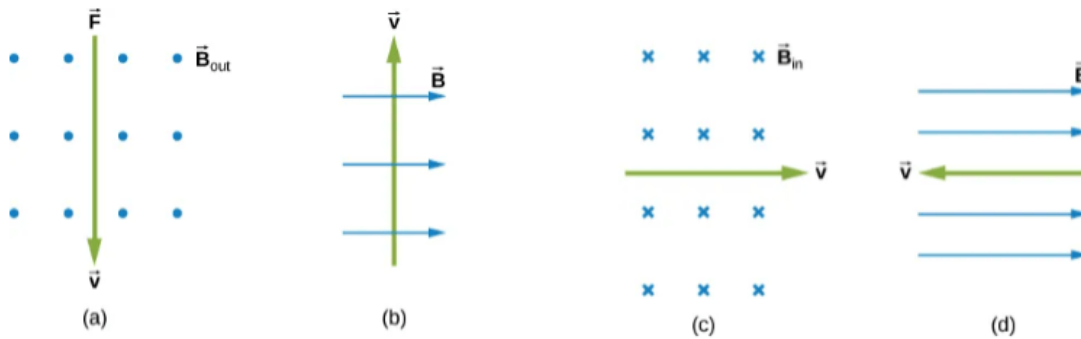
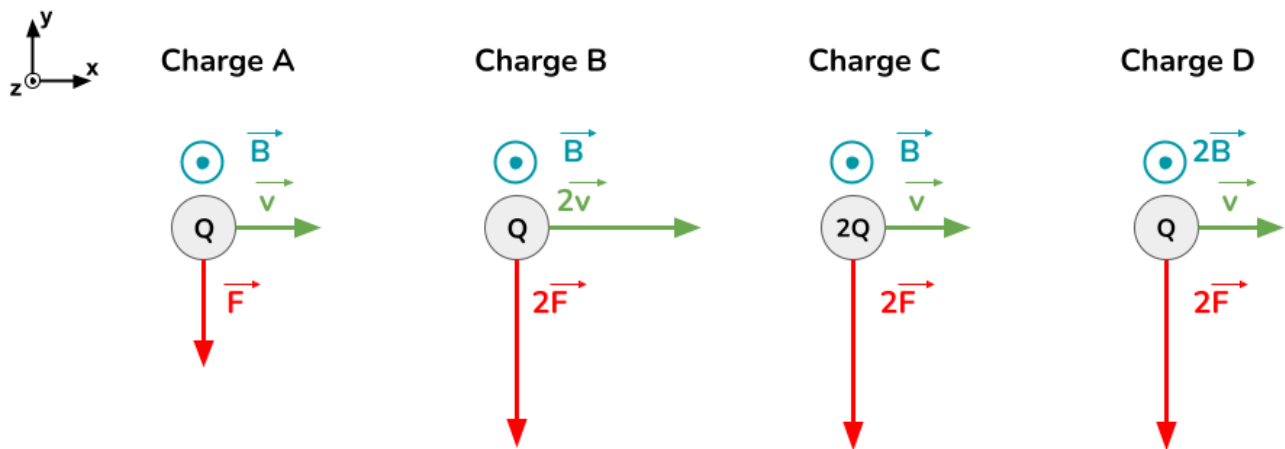


Image Credit: OpenStax University Physics Vol 2 (<https://openstax.org/details/books/university-physics-volume-2>)

4.4.3 Magnetic Force: Charge and Velocity

Model



Explore

1. How does the magnitude of the magnetic force on Charge B compare to Charge A? What else is different about Charge B that might have caused this change in force?
2. How does the magnitude of the magnetic force on Charge C compare to Charge A? What else is different about Charge C that might have caused this change in force?
3. How does the magnitude of the magnetic force on Charge D compare to Charge A? What else is different about Charge D that might have caused this change in force?

Critical Thinking & Concept Invention

1. Identify a proportionality between the magnitude of the magnetic force on charged particles and various quantities:

$$F_B \propto Q \quad \text{OR} \quad F_B \propto \frac{1}{Q} \quad (4.4.1)$$

$$F_B \propto v \quad \text{OR} \quad F_B \propto \frac{1}{v} \quad (4.4.2)$$

$$F_B \propto B \quad \text{OR} \quad F_B \propto \frac{1}{B} \quad (4.4.3)$$

2. Predict the single equation that will include these proportional relationships:

$$F_B = \quad (4.4.4)$$

3. Based on use of the Right Hand Rule, the equation is likely a cross product. Which of the following cross products aligns with the direction of the force?

$$\vec{B} \times \vec{v} \quad \text{OR} \quad \vec{v} \times \vec{B} \quad (4.4.5)$$

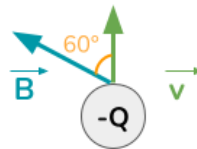
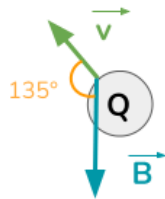
Summary

The magnetic force on a particle is based on the magnetic field, as well as the particle's charge and velocity. The force is proportional to all three and has a direction based on the Right Hand Rule, resulting in the equation:

$$\vec{F}_B = \quad (4.4.6)$$

Apply

Set up a coordinate system and use column vectors to determine the magnetic force on each particle.

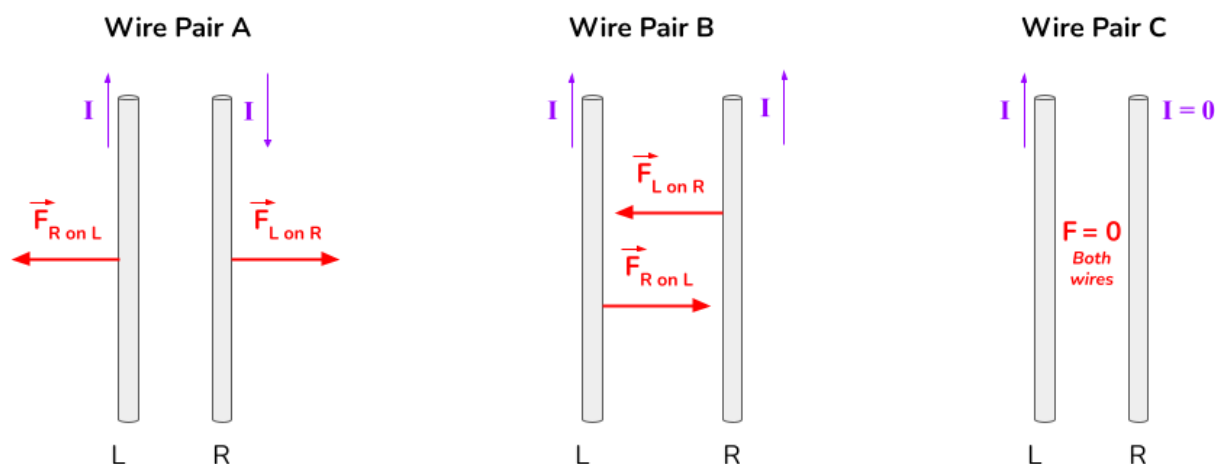


$$\begin{bmatrix} \\ \\ \end{bmatrix} \times \begin{bmatrix} \\ \\ \end{bmatrix} = \begin{bmatrix} \\ \\ \end{bmatrix} \quad (4.4.7)$$

$$\begin{bmatrix} \\ \\ \end{bmatrix} \times \begin{bmatrix} \\ \\ \end{bmatrix} = \begin{bmatrix} \\ \\ \end{bmatrix} \quad (4.4.8)$$

4.4.4 Magnetic Force Between Wires

Model



Explore

1. In Wire Pair A is the current in each wire the same direction or opposite direction? Do the forces between the wires attract or repel?
2. In Wire Pair B is the current in each wire the same direction or opposite direction? Do the forces between the wires attract or repel?
3. Describe the direction of the current(s) in Wire Pair C. Is there any force between these wires?

Critical Thinking & Concept Invention

1. Generalize to create rules:
 - When parallel wires have current in the same direction, the force between the wires . . .
 - When parallel wires have current in opposite directions, the force between the wires . . .
2. Does there need to be current in wires to experience this force? Explain.

4.5: Magnetic Force and Torque on a Current Carrying Wire

4.5 Magnetic Forces, Torque, and Energy on Current-Carrying Wires

Learning Objectives

- Determine the magnetic force between a current-carrying wire and an external magnetic field.
- Identify vector dot products and cross products in derivations.
- Determine the torque and potential energy of a magnetic dipole.

Force and Torque Review

From Mechanics, we know the following relationships for force and torque:

$$\vec{a} = \frac{1}{m} \sum \vec{F} \quad (4.5.1)$$

$$\vec{\tau} = \vec{r} \times \vec{F} \quad (4.5.2)$$

$$\vec{\alpha} = \frac{1}{I} \sum \vec{\tau} \quad (4.5.3)$$

In this activity, we will explore force and torque from electric fields. Recall that the electric force on a charged particle is:

$$\vec{F}_B = q \vec{v} \times \vec{B} \quad (4.5.4)$$

In this equation, the charge q is a signed scalar quantity.

Vector Patterns

If you have a scalar relationship that is a product of two vector quantities multiplied by $\cos \theta$, that relationship follows dot product mathematics.

$$ab \cos \theta \longrightarrow \vec{a} \cdot \vec{b} \quad (4.5.5)$$

If you have a vector relationship that is a product of two vector quantities multiplied by $\sin \theta$, that relationship might follow cross product mathematics. To be sure, you need to check that the direction of the vector also follows the Right Hand Rule.

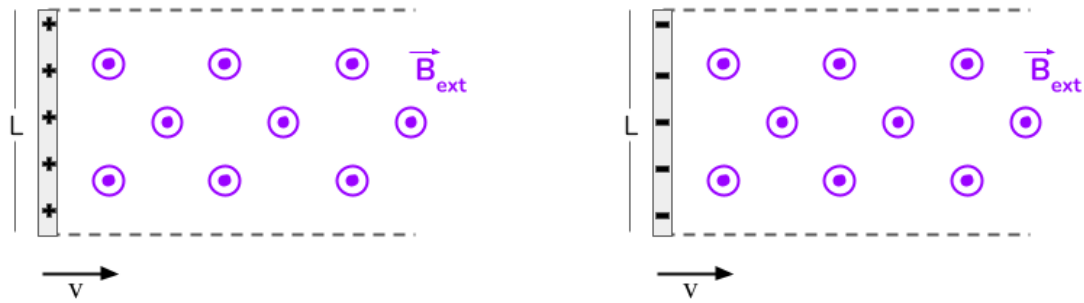
$$ab \sin \theta \longrightarrow \vec{a} \times \vec{b} \quad \text{OR} \quad \vec{b} \times \vec{a} \quad (4.5.6)$$

Look out for these relationships in your calculations to find vector relations between quantities.

4.5.1 Induced Current by Magnetic Force

Model

Below is an image of a conducting rod moving through a magnetic field with velocity $\vec{v}$. Two separate images are shown so we can make sense of the interactions of the magnetic field with the positive charges and the negative charges.



Positive charge:

$$q \begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix} \times \begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix} = \begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix} \quad (4.5.7)$$

Negative charge:

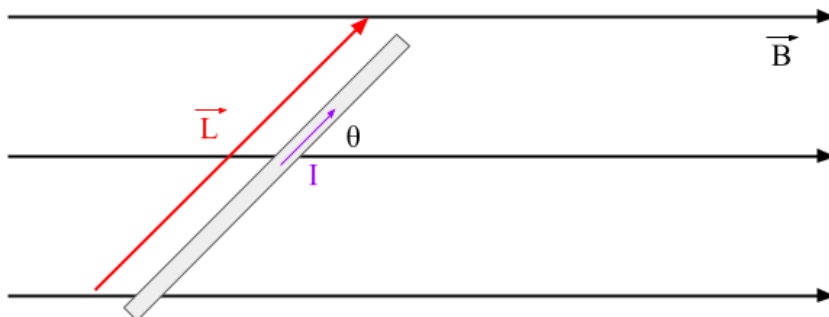
$$-q \begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix} \times \begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix} = \begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix} \quad (4.5.8)$$

Critical Thinking & Concept Invention

1. Set up a coordinate system and fill in the vectors for $\vec{v}$ and $\vec{B}$.
2. Take the cross products and put the final result into the right-most column vector. Explicitly show the sign of the charge.
3. Draw force vectors on each line of charge based on the direction of the magnetic force.
4. That force will cause the charges to move in the direction of the arrows. Put a large + and – on the ends of the rods based on which charges (+/–) accumulate where. This represents the battery/voltage terminals that would cause currents to flow according to the forces.
5. Draw an electric field vector to show the direction of the electric field that is created by this charge separation. Would this field create a force that aligns with or opposes the magnetic force? Based on your answer, will the ends continue to accumulate charge or will there eventually be an equilibrium state?

1.2 Forces on a Wire

Model



Critical Thinking & Concept Invention

1. Connect current and wire length to charge and charge velocity by rearranging the differential fraction:

$$I = \frac{dq}{dt} \quad (4.5.9)$$

$$I d\ell = \quad (4.5.10)$$

2. Use the equation for the magnetic force on a moving charge to identify a differential force on a small bit of charge moving in the wire. Construct (but do not yet solve) an integral equation for the force on the full wire.

$$\vec{F}_{B \text{ on } q} = q\vec{v} \times \vec{B} \quad (4.5.11)$$

$$d\vec{F}_{B \text{ on } q} = dq \vec{v} \times \vec{B} \quad (4.5.12)$$

$$\vec{F}_{B \text{ on wire}} = \int d\vec{F}_{B \text{ on } q} \quad (4.5.13)$$

$$\vec{F}_{B \text{ on wire}} = \int \quad (4.5.14)$$

3. The integral includes a cross product. Identify the three components of the vectors $d\vec{\ell}$ and $\vec{B}$, then take the cross product.

$$d\vec{\ell} \times \vec{B} \quad (4.5.15)$$

$$\begin{bmatrix} \\ \\ \end{bmatrix} \times \begin{bmatrix} \\ \\ \end{bmatrix} = \begin{bmatrix} \\ \\ \end{bmatrix} \quad (4.5.16)$$

4. Use the result to fill in and take the integral to determine the force of the magnetic field on the wire.

$$\vec{F}_{B \text{ on wire}} = \int \quad (4.5.17)$$

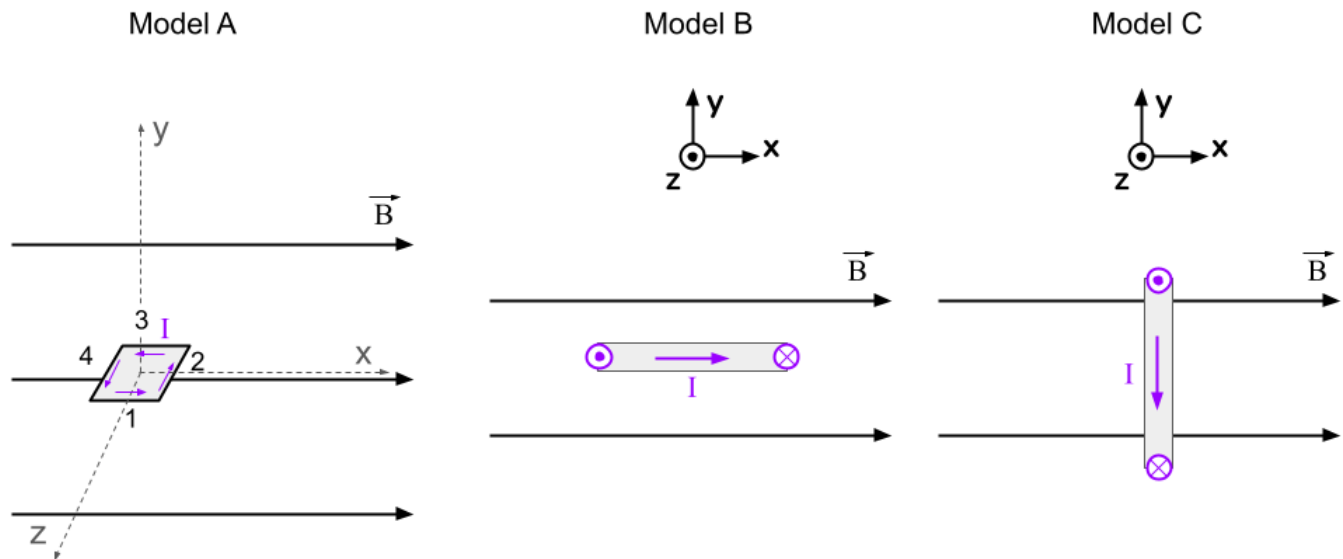
In terms of θ :

$$\vec{F}_{B \text{ on wire}} = \quad (4.5.18)$$

As a coordinate-free vector equation:

$$\vec{F}_{B \text{ on wire}} = \quad (4.5.19)$$

4.5.3 Forces on a Loop



Critical Thinking & Concept Invention

- Fill in the coordinate unit vector directions for the magnetic field and the current along the wire for each side of the square loop.

Current Loop Segment Directions

Loop Side	Current Segment	Magnetic Field
1	$\vec{L} = IL\hat{}$	$\vec{B} = B\hat{}$
2	$\vec{L} = IL\hat{}$	$\vec{B} = B\hat{}$
3	$\vec{L} = IL\hat{}$	$\vec{B} = B\hat{}$
4	$\vec{L} = IL\hat{}$	$\vec{B} = B\hat{}$

- Take the cross products to identify the magnetic force from the external magnetic field on the loop of current. Include unit vectors to show direction.

$$\vec{F}_1 = \quad (4.5.20)$$

$$\vec{F}_2 = \quad (4.5.21)$$

$$\vec{F}_3 = \quad (4.5.22)$$

$$\vec{F}_4 = \quad (4.5.23)$$

- Draw these vectors (if they are non-zero) on Model B. To help interpret, side 1 is facing you, side 4 is on the left, and side 2 is on the right.
- Determine the net force on the loop:

$$\sum \vec{F} = \quad (4.5.24)$$

- Will the loop accelerate? Will the loop have an angular acceleration? Explain your reasoning.
- Draw the forces on Model C. Will Model C have an acceleration? Will Model C have an angular acceleration? Explain your reasoning.

Summary

Loops of current (in any shape) create magnetic dipole fields that have a similar pattern to electric dipole fields.

The magnetic dipole moment is based on the area of the loop and the current in the loop:

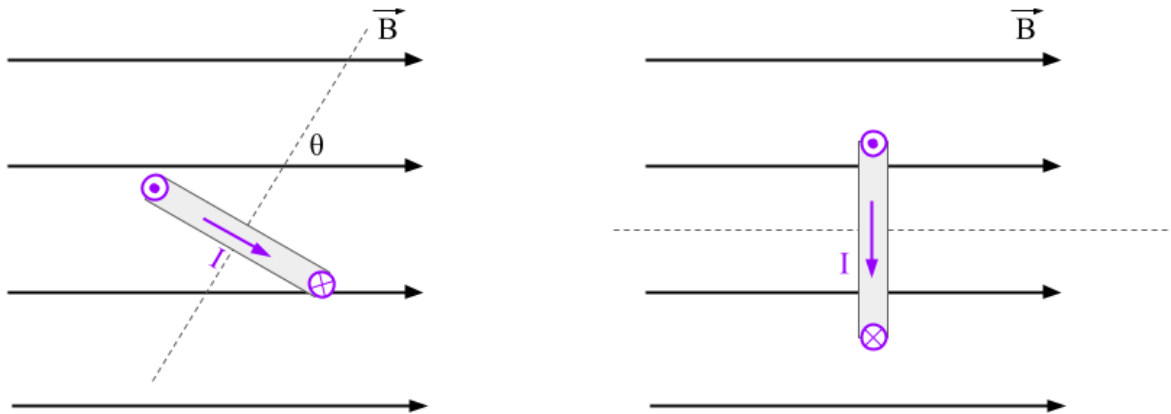
$$\vec{\mu}_B = I\vec{A} \quad (4.5.25)$$

The direction of $\vec{\mu}_B$ points in the direction of the area vector and can be found using the Right Hand Rule by curling your fingers in the direction of the current. As you find the product of I and A in your equations, replace them with μ_B to get equations based on the dipole moment of the loop.

4.5.4 Dipole Torque

Model

Below is the same side view of a loop of current as the previous cycle, when it is in two different orientations.



1. Draw the forces as vectors (arrows) on the ends of the loops, similar to how they appeared in the previous cycle.

Critical Thinking & Concept Invention

1. Examine the model to fill in key information:
 - Determine the force of the field on each current segment and fill in the second column vector of each cross product. Be careful about sign.
 - Determine the vectors from the origin to each edge of the loop where the forces are applied.
 - Draw a position vector from the origin (middle) of the loop to the two ends.
 - Identify the angle from the positive x-axis to those vectors in terms of θ . If measured clockwise, the angle will be negative.
 - A phase shift of 90° or $\frac{\pi}{2}$ shifts sine to cosine and cosine to sine. They may also pick up a sign change. Do this carefully.
 - Fill in your results in the first column vector of each cross product.
2. Determine the torques on the dipole.
 - Take the vector cross product to determine each torque.
 - Add the torques together to get the net torque.
 - Replace any IA with μ_B .
 - Look at the vector patterns and replace scalar products with vector products that align with those patterns.
 - Your answer should include the dipole moment vector $\vec{\mu}_B$.

$$\sum \vec{\tau} = \sum (\vec{r} \times \vec{F}) \quad (4.5.26)$$

$$\vec{\tau}_1 \quad \vec{\tau}_2 \quad (4.5.27)$$

$$\begin{bmatrix} \\ \\ \end{bmatrix} \times \begin{bmatrix} \\ \\ \end{bmatrix} + \begin{bmatrix} \\ \\ \end{bmatrix} \times \begin{bmatrix} \\ \\ \end{bmatrix} \quad (4.5.28)$$

In terms of θ :

$$\vec{\tau}_{\text{dipole}} = \quad (4.5.29)$$

As a coordinate-free vector equation:

$$\vec{\tau}_{\text{dipole}} = \quad (4.5.30)$$

Unit Vector Changes

In Cartesian coordinates the direction of the coordinate unit vectors are fixed. This is why we can use Cartesian components in column vectors.

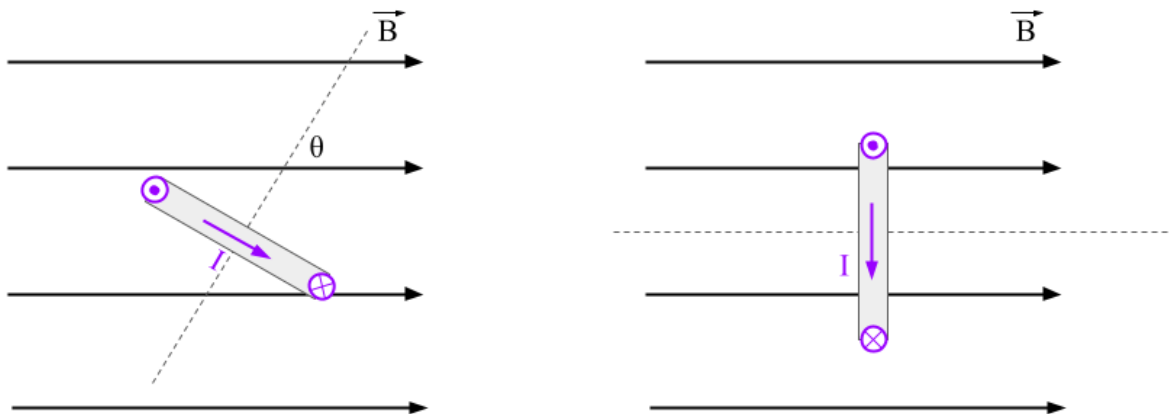
In other coordinate systems, the coordinate unit vectors do not necessarily change in the direction of the coordinates themselves. This can make dot and cross products more challenging.

As an example, the direction that θ changes is a counterclockwise direction. We represent this with an axial vector in the z-direction.

1.5 Dipole Potential Energy

Model

Below is the same side view of a current loop as in the previous cycle. You can use the results you got for torque, but use the coordinate version that has the angle θ in the equation.



Critical Thinking & Concept Invention

- Examine the model to fill in key information:
 - Draw the direction of the force on the ends of the dipole shown in each picture.
 - Identify the direction of $\vec{\tau}$ and $d\vec{\theta}$ as coordinate unit vectors.
 - Fill in those unit vectors in the Work Integral.
 - What is the initial angle of the dipole (left image)? Put that value (in radians) or variable on the lower limit of the work integral.
 - What is the final angle of the dipole (right image)? Put that value (in radians) or variable on the upper limit of the work

integral.

2. Identify the work and potential energy change of the dipole.
 - Evaluate the dot product within the integral based on the unit vectors.
 - Use the equation for torque from the previous section.
 - Evaluate the definite integral.
 - If you see IA , use $\mu_B = IA$.
3. Use the relationship between work and potential energy to determine the change in potential energy of the dipole due to its rotation.
4. Examine your result. Can you turn this into a vector equation that includes the dipole moment vector $\vec{\mu}_B$?
5. According to this equation, what is the change in potential energy as the dipole flips when the dipole vector begins at:
 - π
 - $\frac{\pi}{2}$
 - 0

$$\Delta U = -W \quad (4.5.31)$$

$$W = \int \vec{\tau} \cdot d\vec{\theta} \quad (4.5.32)$$

$$W = \int \tau \hat{\tau} \cdot d\theta \hat{\theta} \quad (4.5.33)$$

$$W = \int \quad (4.5.34)$$

In terms of θ :

$$\Delta U = \quad (4.5.35)$$

As a coordinate-free vector equation:

$$\Delta U = \quad (4.5.36)$$

6. These expressions give the change in potential energy of the dipole, which relates to the total potential energy by

$$\Delta U = U_b - U_a \quad (4.5.37)$$

Identify which term in your equation for ΔU corresponds to the generic angle (θ) and the angle ($\theta = 0$) for the final state in the model. What would U have to be to result in the equation you found?

$$U = \quad (4.5.38)$$

Summary

When a magnetic dipole is in an external magnetic field, it can experience a torque that does work on the dipole. We can use vector concepts to identify the torque on and potential energy of a magnetic dipole in a magnetic field.

$$\vec{\mu}_B = I\vec{A} \quad (4.5.39)$$

$$\vec{\tau} = \quad (4.5.40)$$

$$U = \quad (4.5.41)$$

By convention, we identify a potential energy of zero when the dipole is perpendicular to the field to simplify the equation and remove the constant. This means the potential energy can range from $+\mu_B B$ to $-\mu_B B$ with a maximum change in potential energy of $\pm 2\mu_B B$.

5: GAUSS LAW

5.1: Flux

5.2: Electric Flux from Charges

5.3: Gauss Law

5.4: Electric Potential from Gauss Law

5.5: Conductors in Electric Fields

5.6: Dielectrics in Electric Fields

5.1: Flux

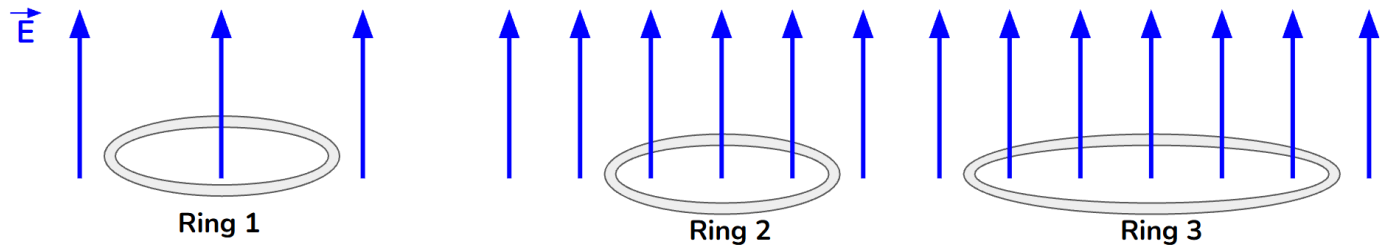
5.1 Flux

Learning Objectives

- Relate flux to electric field, area, and angle using trigonometry and vector dot products.
- Represent the area of a surface using vector concepts.

5.1.1 Electric Flux: Field and Area

Model



Explore

1. Use the model to fill in the table.

Electric Flux Comparison

Ring	Ring Size (big / small)	Electric Field Strength (strong / weak)	Number of Electric Field Lines Through the Ring
1			
2			
3			

Critical Thinking & Concept Invention

Recall that the “number of electric field lines through the ring” represents Electric Flux (Φ_E).

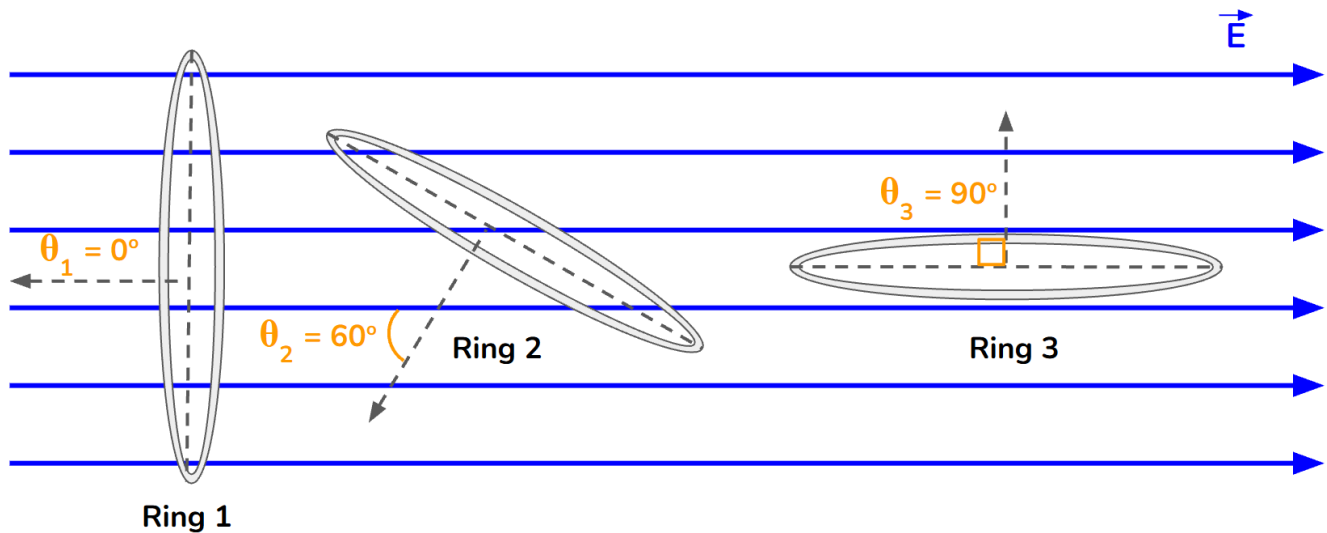
1. Describe the relationship between electric field lines and electric flux.
2. Identify two different proportional relationships for Electric Flux:

$$\Phi_E \propto \text{Area} \quad (5.1.1)$$

$$\Phi_E \propto E \quad (5.1.2)$$

5.1.2 Electric Flux: Angle

Model



Explore

1. Use the model to fill in the table.

Flux and Angle Relationship

Ring	Angle (θ) Between Electric Field and Ring Surface	$\cos(\theta)$	Number of Electric Field Lines Through the Ring
1			
2			
3			

Critical Thinking & Concept Invention

1. How does electric flux vary with θ ?
Electric flux increases when...
2. How does electric flux vary with $\cos(\theta)$?
Electric flux increases when...
3. Identify a proportional relationship for electric flux.
Reminder:

$$\cos(0^\circ) = 1 \quad (5.1.3)$$

and

$$\cos(90^\circ) = 0 \quad (5.1.4)$$

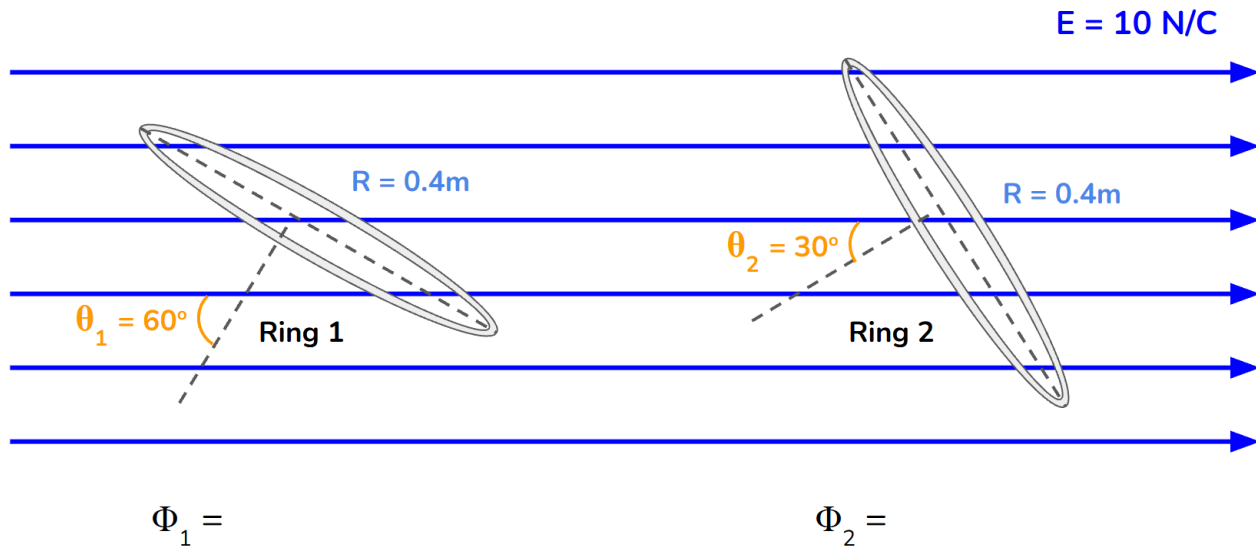
$$\Phi \propto \cos(\theta) \quad \text{or} \quad \Phi \propto \frac{1}{\cos(\theta)} \quad (5.1.5)$$

Summary

The angle between the field and the surface affects the overall flux through that surface. The greatest flux occurs when the field is perpendicular to the surface, which is parallel to the normal vector.

Apply

Find the flux in both rings.



5.1.3 Area Vector

Model

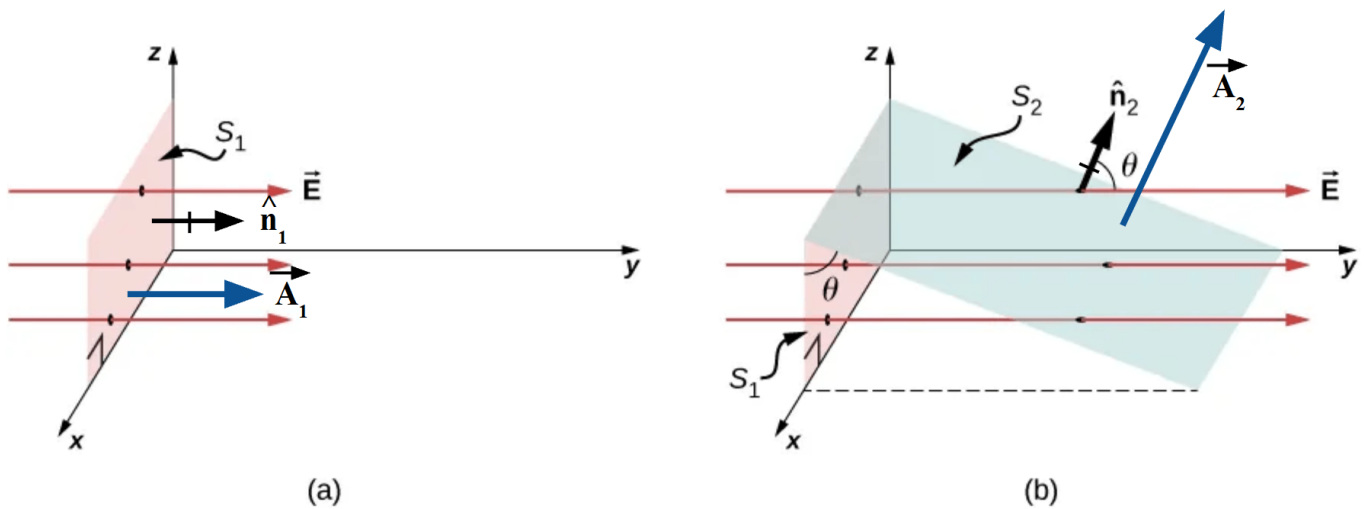


Image Credit: OpenStax University Physics Vol 2 (<https://openstax.org/details/books/university-physics-volume-2>)

Explore

Compare Scenario 1 (left) and Scenario 2 (right) by identifying which one is Greater, Lesser, or the Same.

Recall that the “number of field lines” through a surface represents the Flux (Φ).

Comparing Surface Quantities

	$ \vec{E} $	Φ_E	Area S	$\vec{A}$	Length of $\hat{n}$
1					
2					
3					

Critical Thinking & Concept Invention

1. Why do we use a vector to represent the area of a surface?
2. What determines the magnitude and direction of the vector $\vec{A}$?
3. How can the flux be the same for both surfaces if:

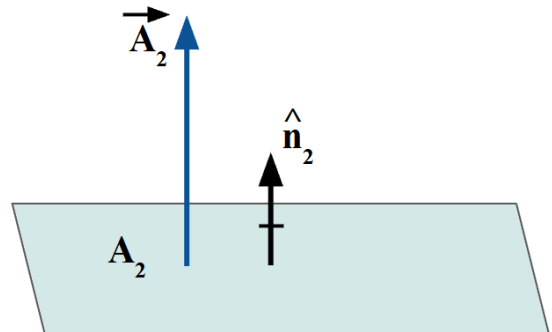
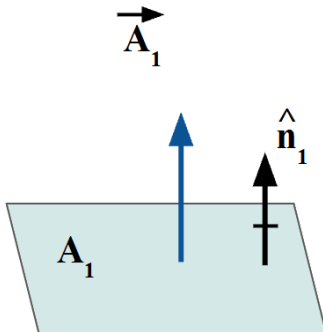
$$\Phi_E = \vec{E} \cdot \vec{A} \quad (5.1.6)$$

Summary

We can represent a surface area as a vector that incorporates the magnitude of the area with the direction of its normal vector.

The normal vector $\hat{n}$ is a unit vector. It has a magnitude of 1 and points directly outward from the surface (at a 90° angle to the surface).

$$\vec{A} = A\hat{n} \quad (5.1.7)$$



The flux calculation is a dot product between the electric field and the area vector for the identified surface. Dot products are also equal to the product of the two magnitudes and a cosine factor of the angle between the two vectors.

The surface can be real or imaginary.

$$\Phi_E = \vec{E} \cdot \vec{A} \quad (5.1.8)$$

$$\Phi_E = \vec{E} \cdot \hat{n}A \quad (5.1.9)$$

$$\Phi_E = EA \cos \theta \quad (5.1.10)$$

Apply

Find the electric flux through a rectangular area $3 \text{ cm} \times 2 \text{ cm}$ between two parallel plates where there is a constant electric field of 30 N/C for the following orientations of the area:

- Parallel to the plates.

- Perpendicular to the plates.
- With a normal vector at a 30° angle with the direction of the electric field. Note that this angle can also be given as:

$$180^\circ + 30^\circ \quad (5.1.11)$$

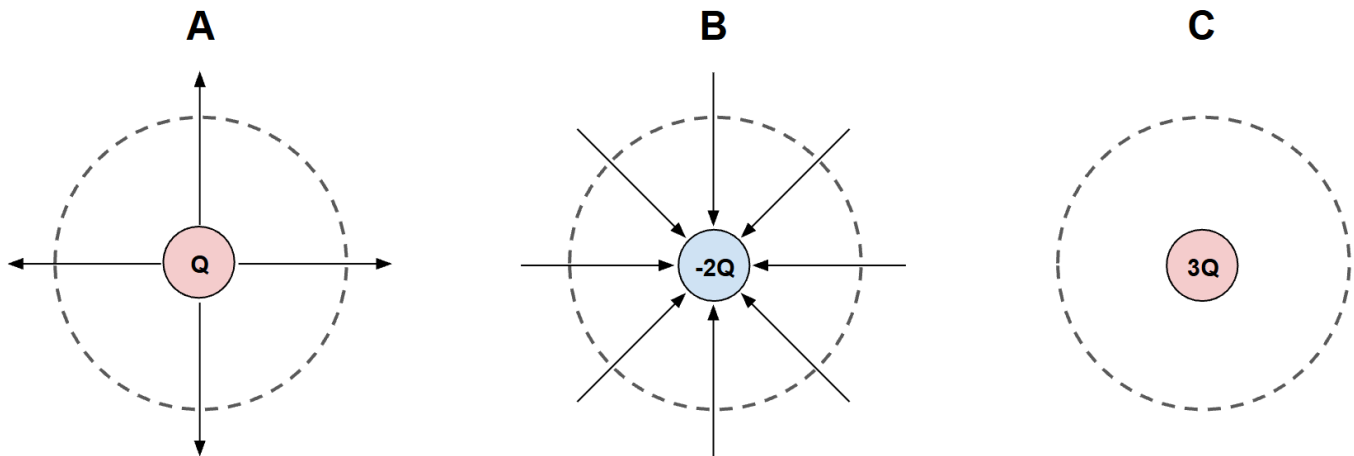
5.2: Electric Flux from Charges

Learning Objectives

- Use the concept of electric flux to predict electric field patterns around positive and negative charges.
- Identify conditions for Gaussian surfaces to reduce flux integrals to simple products.

5.2.1 Electric Fields from Charges

Model



The model below shows the electric field lines for two different charges (A and B). The electric field lines exist but are not shown for charge C.

Explore

Use the model above to fill in the table. Leave the number of lines blank for Charge C.

Electric Field Line Table

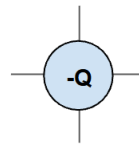
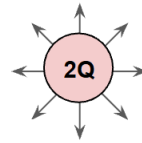
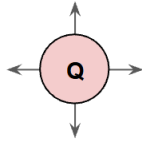
Scenario	Charge (Q)	# Lines into Charge	# Lines out of Charge
A			
B			
C			

Critical Thinking & Concept Invention

- Complete the following statements:
 - Electric field lines originate from _____ charges.
 - Electric field lines terminate on _____ charges.
- How many lines should go in or out of Charge C? Draw them, then complete the table in the Explore section.
- Generalize: What determines the amount of field lines that connect to a charged object?
- Summarize: How do you draw the electric field lines that are created by a charge?

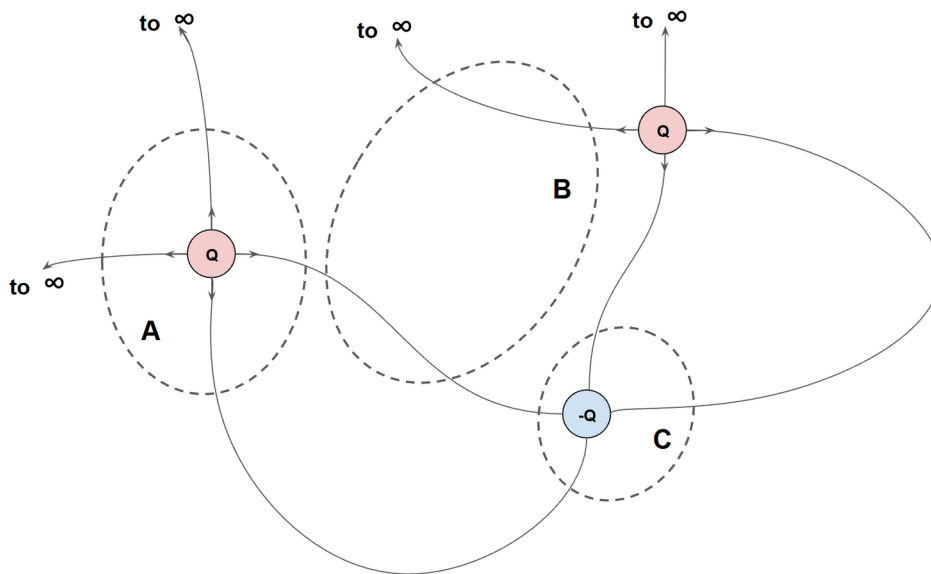
Apply

Continue the electric field lines for these charges.



5.2.2 Multiple Charges and Electric Flux

Model



Explore

- At each point where an electric field line crosses a dashed oval:
 - Put a circle on the crossing point if the field goes into the dashed oval.
 - Put an X on the crossing point if the field goes out of the dashed oval.
- What are the two options for where an electric field line can end?
- Examine your annotations to fill out the table for each closed oval.

Electric Flux Table

Oval	Charge in Oval	# Lines In	# Lines Out	Φ_E	# Lines Out – # Lines In
A					
B					
C					

Critical Thinking & Concept Invention

The difference between the number of lines that go in and the number of lines that go out of a surface is called flux (Φ). For a closed surface, lines out are considered positive and lines in are considered negative.

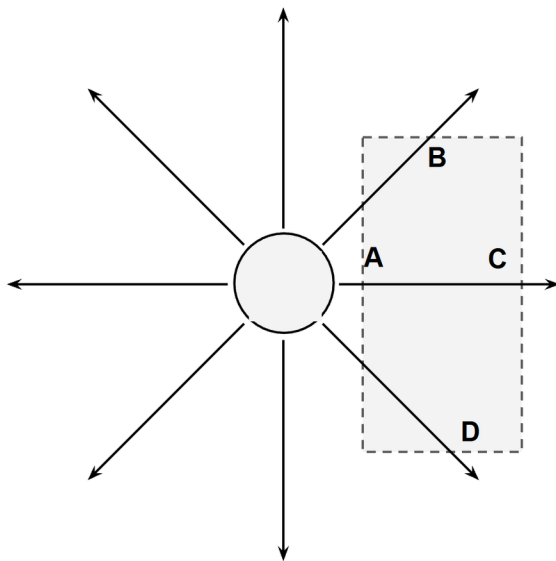
1. How does the sign of flux (+, –, or 0) relate to where the charges are?
2. How would the flux in ovals A, B, and C be different if all of the charges were doubled while keeping their sign and location?
3. Propose a proportionality that relates flux and charge:

$$\Phi_E \propto \quad (5.2.1)$$

Apply

Apply the concept of flux to the example below:

- Identify the sign of the charge (+, –, or 0) for the circle and write that inside the circle.
- Put circles on electric field lines going into the rectangle and X's on the lines as they go out of the rectangle.
- Determine the flux on each side of the rectangle (A, B, C, and D).
- Determine the total flux for the closed rectangle as a whole.
- Identify the sign of the charge (+, –, or 0) for the rectangle.



Summary

Charges create electric fields that obey Gauss' Law, which states that the electric flux for any closed surface is proportional to the charge enclosed by that surface.

$$\Phi_E \propto Q_{\text{enclosed}} \quad (5.2.2)$$

$$\Phi_E = \frac{1}{\epsilon} Q_{\text{enclosed}} \quad (5.2.3)$$

The surface can be any shape. The exact relationship has a proportionality constant (ϵ). In a vacuum (or non-polarizable medium), we replace ϵ with ϵ_0 , which is called the permittivity of free space.

For this course, we will primarily focus on electric fields in free space, so we will use:

$$\Phi_E = \frac{1}{\epsilon_0} Q_{\text{enclosed}} \quad (5.2.4)$$

$$\epsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{N \cdot m^2} \quad (5.2.5)$$

It is not always possible to determine the exact electric field at a specific location using this relationship because there can be different electric field strengths along a surface. We will often need to be creative with the type of surface we draw to take advantage of symmetries.

5.2.3 Gaussian Surfaces

Model

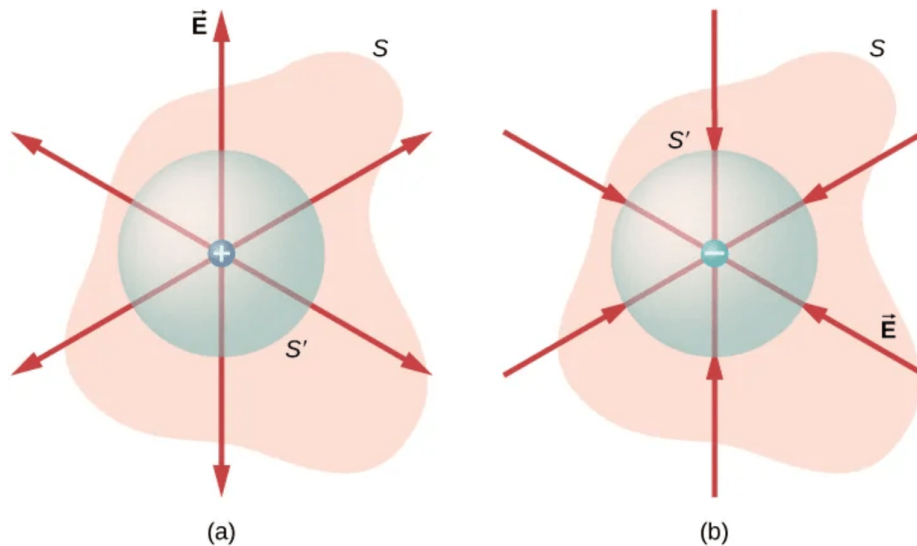


Image Credit: OpenStax University Physics Vol 2 (<https://openstax.org/details/books/university-physics-volume-2>)

Explore

1. For which enclosed surface (S or S') is the electric field at a 90° angle to the surface?
2. For which charge (+ or -) does the electric field point out of the surface?
3. For which charge (+ or -) does the electric field point into the surface?
4. Is there a surface where the electric field is uniform along the surface? If so, which one (S or S')?

Critical Thinking & Concept Invention

Electric flux is defined as:

$$\Phi_E \equiv \iint \vec{E} \cdot d\vec{A} \quad (5.2.6)$$

or

$$\Phi_E \equiv \iint \vec{E} \cdot \hat{n} dA \quad (5.2.7)$$

1. At each place where the electric field passes through the surface (S or S'), draw a normal vector for the surface.
2. For which surface (S or S') does this integral reduce to the expression below? Explain why.

$$\Phi_E = \pm \iint E dA \quad (5.2.8)$$

3. What determines the $+$ or the $-$ in front of that expression?
4. What has to be true for the expression to further reduce to:

$$\Phi_E = \pm E \iint dA \quad (5.2.9)$$

5. Evaluate the integral from the previous question.

$$\Phi_E = \quad (5.2.10)$$

6. Recall from Gauss' Law the relation between electric flux in a closed surface and the charge enclosed by that surface. Use that expression and the previous expression to solve for the electric field in terms of charge.

$$\Phi_E = \frac{Q_{\text{enc}}}{\epsilon_0} \quad (5.2.11)$$

7. Explain why we cannot solve the original flux integral for one of the surfaces (S or S').
8. Explain what needs to be true about the electric field and the Gaussian surface to make the integral expression reduce to a product of electric field magnitude and area:

$$\iint \vec{E} \cdot d\vec{A} \rightarrow \pm EA \quad (5.2.12)$$

Summary

Gauss' Law is true for all scenarios in the universe and is one of the four Maxwell Equations that describe electric and magnetic fields.

It states that the flux (Φ) for a closed surface is:

$$\Phi_E = \frac{Q_{\text{enc}}}{\epsilon_0} \quad (5.2.13)$$

Electric flux is defined by the integral:

$$\Phi_E \equiv \iint \vec{E} \cdot d\vec{A} \quad (5.2.14)$$

When we have sufficient symmetry such that the electric field is constant along a closed Gaussian surface and points perpendicular (or at a constant known angle) to that surface, the integral simplifies.

In these situations, we can use Gauss' Law to determine the electric field created by charges.

When we do not have sufficient symmetry such that the electric field is constant along a closed Gaussian surface and points perpendicular (or at a constant known angle) to that surface, we are left with a vector integral that cannot be solved if the electric field is not already known.

In these circumstances, a different method must be used to solve for the electric field created by the charges.

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5.3: Gauss Law

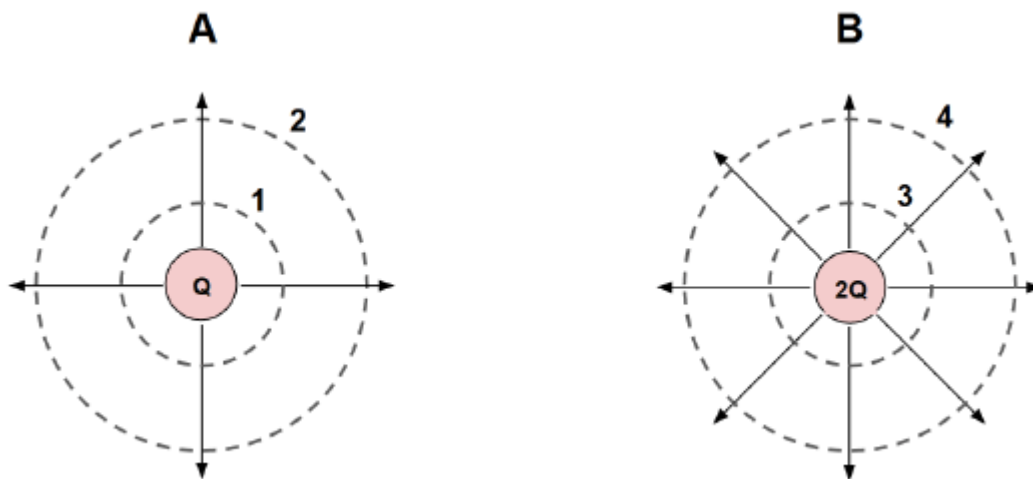
5.3 Gauss Law and Electric Field Vectors

Learning Objectives

- Use Gauss's Law and the definition of Electric Flux to determine the electric fields from point charges, lines of uniform charge density, and planes of uniform charge density.
- Identify how electric field magnitudes vary with distance from point charges, lines of charge, and planes of charge.

5.3.1 Electric Vector Field of a Point Charge

Model



The image above is a two-dimensional representation of a three-dimensional space. Imagine the dashed circles are actually spheres surrounding a charge located at the center.

Explore

1. What angle does the electric field make with the dashed Gaussian spheres?
2. On the surface of any sphere shown, is the electric field uniformly distributed, or is it stronger in some places and weaker in others? Focus on one sphere at a time.
3. For Scenario A, compare the following quantities by inserting the appropriate relation ($>$, $<$, or $=$):
 - Φ_1 ___ Φ_2
 - A_1 ___ A_2
 - E_1 ___ E_2
4. For Scenario B, compare:
 - Φ_3 ___ Φ_4
 - A_3 ___ A_4
 - E_3 ___ E_4
5. How does the flux in Scenario A compare with Scenario B?

Critical Thinking & Concept Invention

Recall the equations for Gauss's Law and the definition of Electric Flux:

$$\Phi_E = \frac{Q_{\text{enc}}}{\epsilon_0} \quad (5.3.1)$$

$$\Phi_E \equiv \iint \vec{E} \cdot d\vec{A} \quad (5.3.2)$$

1. What is the equation for the surface area of a sphere?
2. What does the integral equation for electric flux reduce to for these Gaussian surfaces? Explain why.
3. Draw radii on the model and assign unique labels.
4. Use Gauss's Law and the definition of Electric Flux to construct an equation for the electric field magnitude in terms of enclosed charge:
 - $E_1 = E_2 =$
 - $\frac{E_1}{E_2} =$
 - $E_3 = E_4 =$
 - $\frac{E_3}{E_4} =$
5. Generalize for a point charge by identifying an equation for the magnitude of the electric field as a function of distance r :

$$E_{\text{point charge}}(r) = \quad (5.3.3)$$

Summary

For a point charge, a spherical Gaussian surface results in a constant electric field magnitude everywhere on the surface. The field is perpendicular to the surface (parallel to the outward normal vector). Applying Gauss's Law yields:

$$E = \frac{Q_{\text{enc}}}{4\pi\epsilon_0 r^2} \quad (5.3.4)$$

The field strength decreases according to an inverse-square relationship because surface area increases as r^2 while total flux remains constant.

The vector form is:

$$\vec{E} = \frac{Q_{\text{enc}}}{4\pi\epsilon_0 r^2} \hat{r} \quad (5.3.5)$$

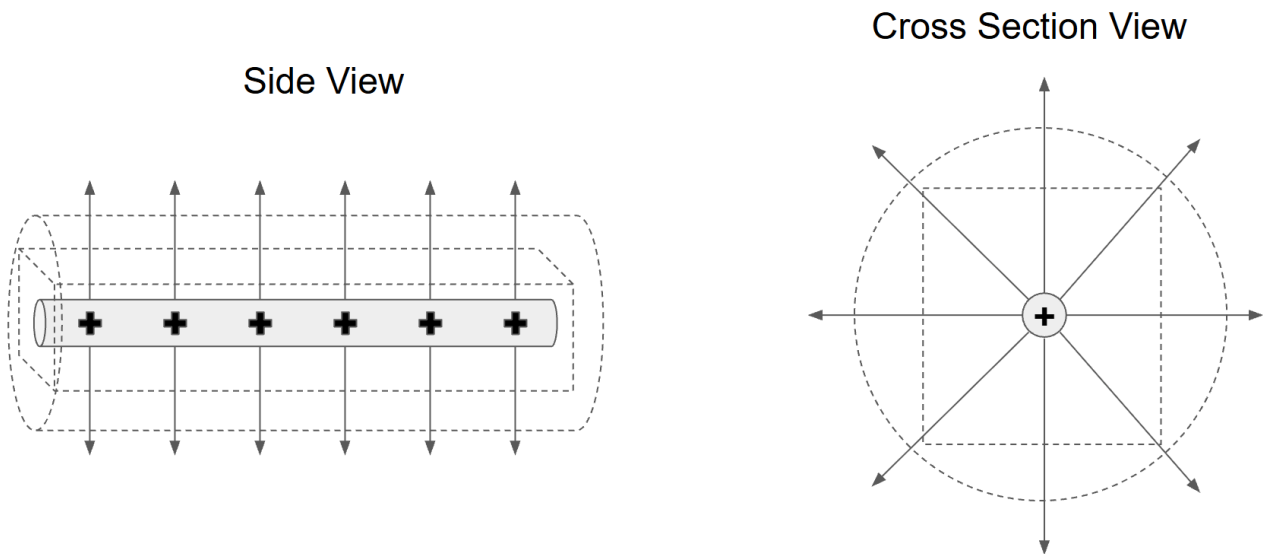
Using $\hat{r} = \frac{\vec{r}}{r}$:

$$\vec{E} = \frac{Q_{\text{enc}}}{4\pi\epsilon_0 r^3} \vec{r} \quad (5.3.6)$$

This expression is commonly called Coulomb's Law. Since all charge distributions can be represented as sums of point charges, this relationship is useful for determining electric fields when Gauss's Law cannot be applied directly.

5.3.2 Electric Field of a Line of Charge

Model



A long line of charge with length L has a uniform charge distribution. The figure shows both side and cross-sectional views using different Gaussian surfaces.

Explore

1. Do electric field lines pass through all portions of the closed Gaussian surfaces? Which portions have zero flux?
2. Label L on the side view and label a and s on the cross-sectional view.
3. On the cross-sectional view, draw the normal vectors where electric field lines intersect the surface.
4. Which surface (box, cylinder, or neither) has normal vectors parallel to the electric field everywhere?
5. Which surface has the same electric field magnitude at every point?
6. Which Gaussian surface is most appropriate for determining the electric field?

Critical Thinking & Concept Invention

1. For each Gaussian surface, identify the area contributing nonzero electric flux:
 - $A_{\text{cylinder}} =$
 - $A_{\text{rectangular prism}} =$
2. Based on the chosen surface:
 - Evaluate Gauss's Law and solve for the electric field magnitude.
 - Replace area with the surface area expression.
 - Use the linear charge density:

$$\lambda = \frac{Q}{L_{\text{enc}}} \quad (5.3.7)$$

- Apply:

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0} \quad (5.3.8)$$

3. Does the electric field decrease according to an inverse-square law? If not, what proportionality does it follow?

Summary

The electric field from a long line of charge points radially outward and decreases inversely with distance. A cylindrical Gaussian surface is appropriate, with radial distance commonly represented by s .

$$E_{\text{line of charge}} = \quad (5.3.9)$$

$$\vec{E}_{\text{line of charge}} = \quad (5.3.10)$$

This result is an approximation valid far from the ends of the charged line. Near the ends, electric flux exits through the cylinder caps, causing a slight overestimate of the field.

5.3.3 Electric Field from a Plane of Charge

Model

The model compares a two-charge configuration with a uniformly charged plane having surface charge density σ .

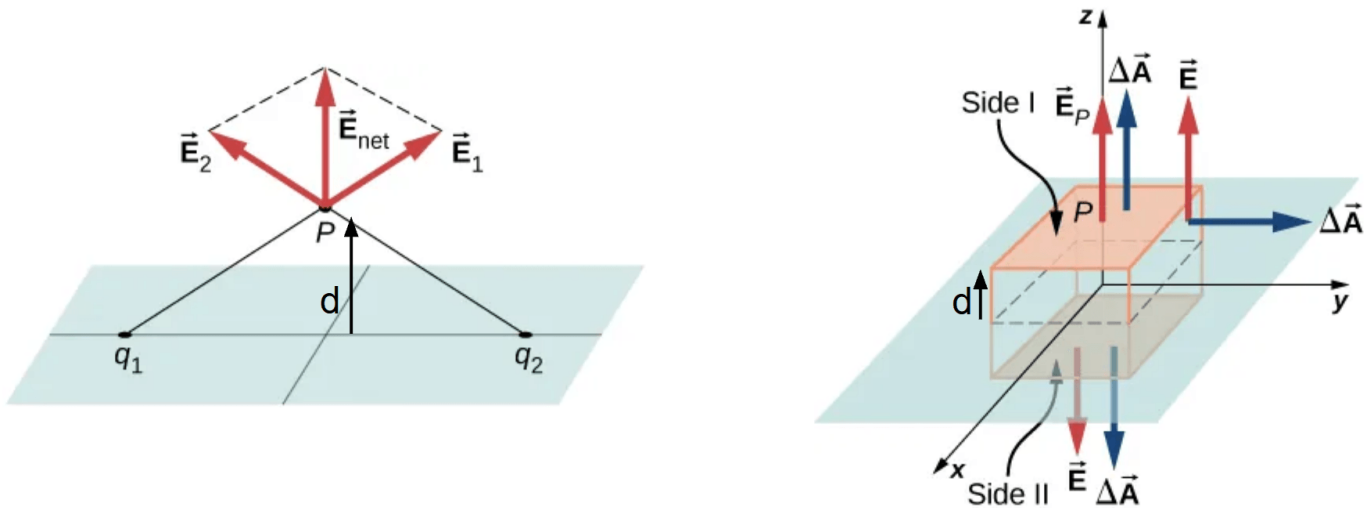


Image Credit: OpenStax University Physics Vol 2 (<https://openstax.org/details/books/university-physics-volume-2>)

Explore

Two-Charge System

- Determine the directions (+/- x , +/- y , +/- z) of:
 - $\vec{E}_1$
 - $\vec{E}_2$
 - $\vec{E}_{\text{net}}$
- What angle does the net electric field make with the rectangular surface, and why is this convenient?

Plane of Charge

- In which directions do the electric field vectors point?
- How many faces of the rectangular prism have nonzero electric flux?
- If each face has area ΔA , what total area contributes nonzero flux?

Critical Thinking & Concept Invention

1. Determine the electric field magnitude a distance d from the plane.
 - Evaluate Gauss's Law and solve for the electric field.
 - Replace area with the prism surface area.
 - Use the surface charge density:

$$\sigma = \frac{Q}{A_{\text{enc}}} \quad (5.3.11)$$

- Apply:

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0} \quad (5.3.12)$$

- Determine whether the field decreases with distance or follows another relationship.

Summary

For a large uniformly charged plane, the electric field is perpendicular to the surface and remains constant with distance from the plane.

$$E_{\text{plane}} = \quad (5.3.13)$$

$$\vec{E}_{\text{plane}} = \quad (5.3.14)$$

This approximation is valid away from the edges of the plane. Near the edges, electric flux exits through the sides of the Gaussian surface, causing slight inaccuracies. More advanced methods are required for finite planes.

Apply

A large rectangular piece of metal has a net charge Q . Determine the strength of the electric field produced by this metal near its center while ignoring edge effects.

Hint: Consider where charge distributes on a conductor and the behavior of electric fields inside and outside conductors.

- Draw the metal piece and establish coordinate axes.
- Draw the charge distribution and resulting electric field lines.
- Draw an appropriate Gaussian surface that does not need to enclose all of the charge.
- Use the same setup process as in previous sections to solve the Gauss's Law integral without explicit integration.

5.4: Electric Potential from Gauss Law

5.4 Gauss Law and Electric Potential

Learning Objectives

- Apply Gauss Law to charge distributions.
- Determine electric potential functions from electric fields using integrals.
- Use electric field and potential functions to determine charge density on a conductor.

5.4.1 Electric Field and Potential for a Parallel Plate Capacitor

Model

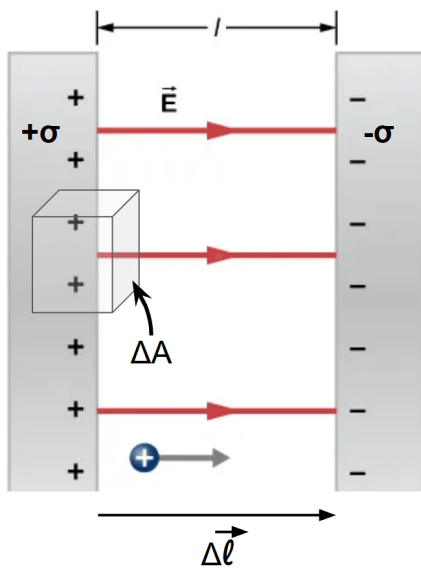


Image Credit: OpenStax University Physics Vol 2 (<https://openstax.org/details/books/university-physics-volume-2>)

Explore

1. What is the electric field inside either of the plates (they are conductors)?
2. How many faces of the Gaussian box have a non-zero electric flux? Shade those faces and draw a normal vector.
3. Compare the direction of $\vec{E}$ and $d\vec{A}$ (parallel, perpendicular, or neither).
4. Does the electric field magnitude vary with distance (for example, do the field lines converge or diverge as they leave the plate)?
5. Compare the direction of $\vec{E}$ and $d\vec{\ell}$ (parallel, perpendicular, or neither).

Critical Thinking & Concept Invention

1. Use Gauss's Law to identify the electric field magnitude created by the charge distribution.
 - Identify the enclosed charge.
 - Simplify the integral and explain how you were able to simplify it.
 - Identify the electric field in terms of σ .

- Identify the electric field in terms of capacitor charge Q if that charge is uniformly distributed over a plate with area A .

$$Q_{\text{enc}} = \quad (5.4.1)$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0} \quad (5.4.2)$$

- Determine the change in electric potential (ΔV) from the positive plate to the negative plate.
 - Simplify the integral and explain how you were able to simplify it.
 - Identify the electric potential in terms of σ .
 - Identify the electric potential in terms of capacitor charge Q if that charge is uniformly distributed over a plate with area A .
 - Using $\Delta V_C = Q/C$, identify the capacitance C .

$$\Delta V \equiv - \int \vec{E} \cdot d\vec{l} \quad (5.4.3)$$

$$\Delta V = \quad (5.4.4)$$

$$\Delta V = \quad (5.4.5)$$

$$C = \quad (5.4.6)$$

Summary

Electric potential and electric fields are related by the line integral:

$$\Delta V \equiv - \int \vec{E} \cdot d\vec{l} \quad (5.4.7)$$

Gauss's Law relates electric field to charge on a conductor:

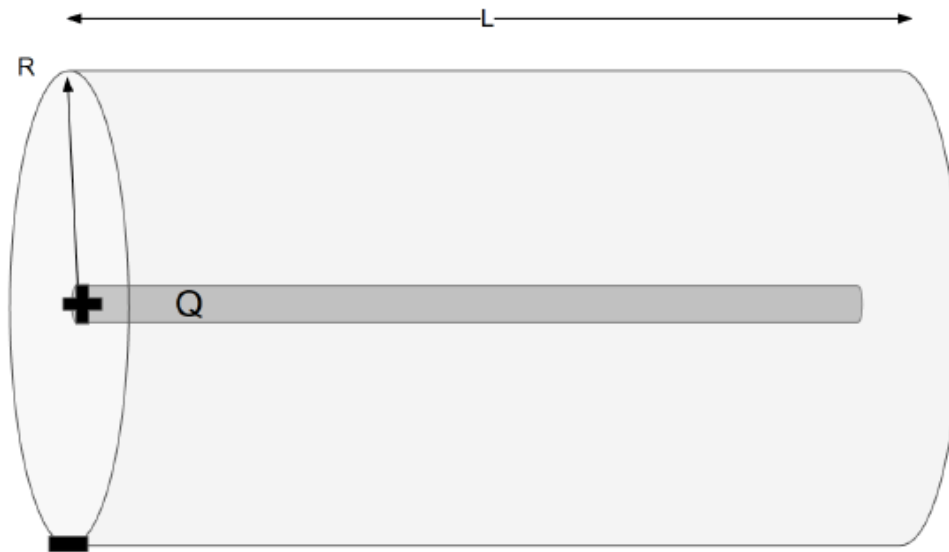
$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0} \quad (5.4.8)$$

Combining these relationships allows electric potential to be related to charge. Since capacitance is defined as the ratio of charge to voltage:

$$C \equiv \frac{Q}{\Delta V} \quad (5.4.9)$$

Apply

A cylindrical capacitor is created from a long thin wire (length L) and an outer conducting cylinder of the same length and radius R . The inner wire has charge $+Q$ and the outer cylinder has charge $-Q$. Determine the capacitance.



5.4.2 Electric Potential from Spherical Charge Distributions

Model

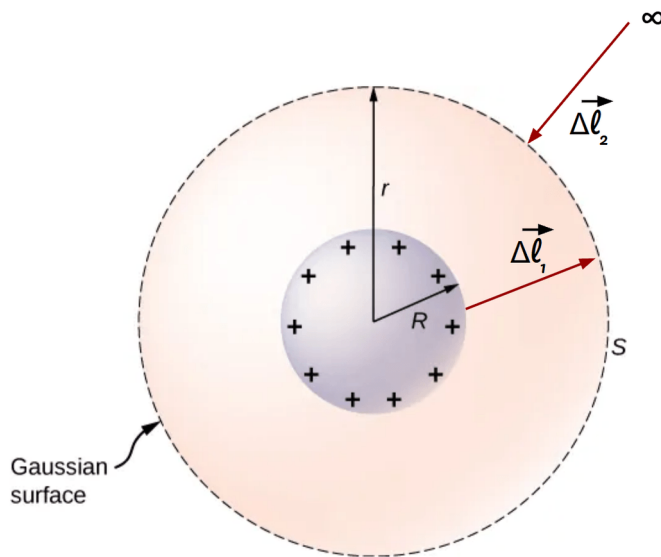


Image Credit: OpenStax University Physics Vol 2 (<https://openstax.org/details/books/university-physics-volume-2>)

Explore

1. What is the geometric shape of the charge distribution? What is the equation for its volume?
2. What is the shape of the Gaussian surface shown? What is the equation for its surface area?
3. Can Gauss's Law be reduced as shown below for this charge distribution and Gaussian surface? Explain why or why not.

$$\Phi_E \equiv \iint \vec{E} \cdot d\vec{A} \rightarrow \Phi_E = EA \quad (5.4.10)$$

Critical Thinking & Concept Invention

$$\vec{E} = \quad (5.4.11)$$

1. Use Gauss's Law to identify the electric field magnitude created by the charged sphere if its total net charge is $+Q$.

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0} \quad (5.4.12)$$

2. What coordinate system (Cartesian, cylindrical, or spherical) best aligns with the direction of this electric field?
3. Identify a vector equation for the electric field by including an appropriate unit vector.
4. Determine the change in electric potential between:
 - The radius of the charged sphere and a point on the Gaussian surface.
 - A point at infinity and a point on the Gaussian surface.

$$\Delta V \equiv - \int \vec{E} \cdot d\vec{l} \quad (5.4.13)$$

$$\Delta V \equiv - \int \vec{E} \cdot d\vec{l} \quad (5.4.14)$$

5. Use the result referenced to infinity to identify the capacitance of the charged sphere.

$$\Delta V_C = \frac{Q}{C} \quad (5.4.15)$$

$$C = \quad (5.4.16)$$

5.4.3 Electric Fields Inside and Outside of Charged Dielectrics

Model

A sphere of radius R has a total net charge Q that is uniformly distributed throughout its volume. Two Gaussian surfaces are shown: one inside the sphere and one outside the sphere.

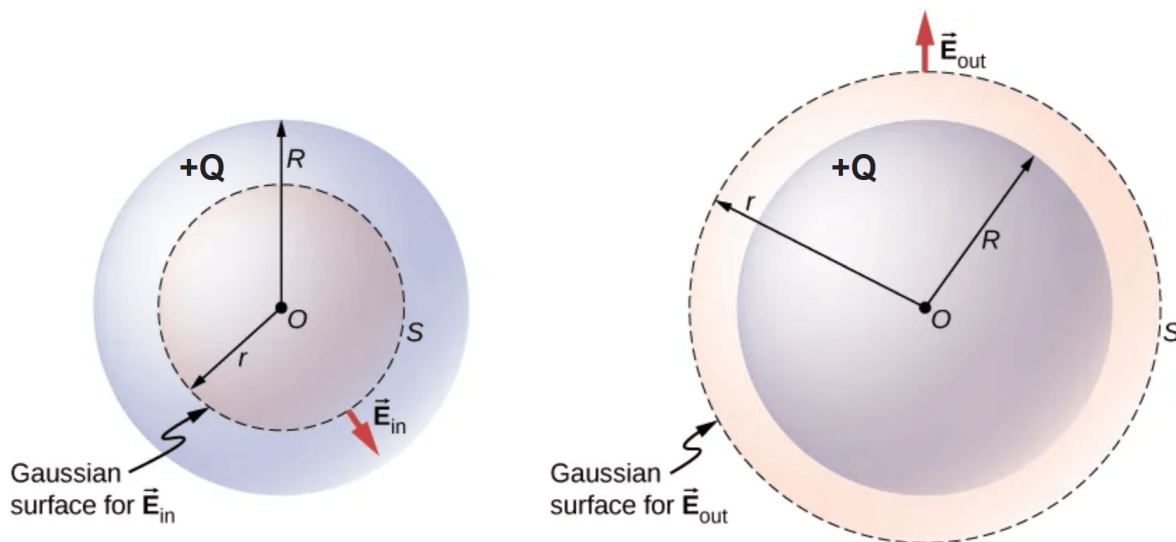


Image Credit: OpenStax University Physics Vol 2 (<https://openstax.org/details/books/university-physics-volume-2>)

Explore

1. For constant charge density, $\rho = Q/V$, where V represents volume, what is the charge density of the sphere?
2. What is the volume enclosed by each Gaussian sphere?
3. For which Gaussian surface (inside or outside) is the entire charge Q enclosed?
4. For the surface that does not enclose all of Q , what is the enclosed charge? Use the charge density relationship and geometric variables.

Critical Thinking & Concept Invention

1. Use Gauss's Law to identify the electric field magnitude created by the charge distribution.
 - Identify the enclosed charge in each case.
 - Simplify the integral and explain your reasoning.
 - Express the electric field in terms of Q .

$$Q_{\text{enc}} = \quad Q_{\text{enc}} = \quad (5.4.17)$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0} \quad (5.4.18)$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0} \quad (5.4.19)$$

2. Identify how the electric field depends on distance r :
 - Inside the sphere: $E \propto$
 - Outside the sphere: $E \propto$

Summary

Changes in electric potential relate to changes in energy for charged objects. A specific electric potential value has meaning only relative to another reference point. Choosing infinity as the zero reference defines the Coulomb potential for a point charge or uniformly charged sphere:

$$V(r) = \quad (5.4.20)$$

Apply

Suppose the spherical charge distribution has charge density

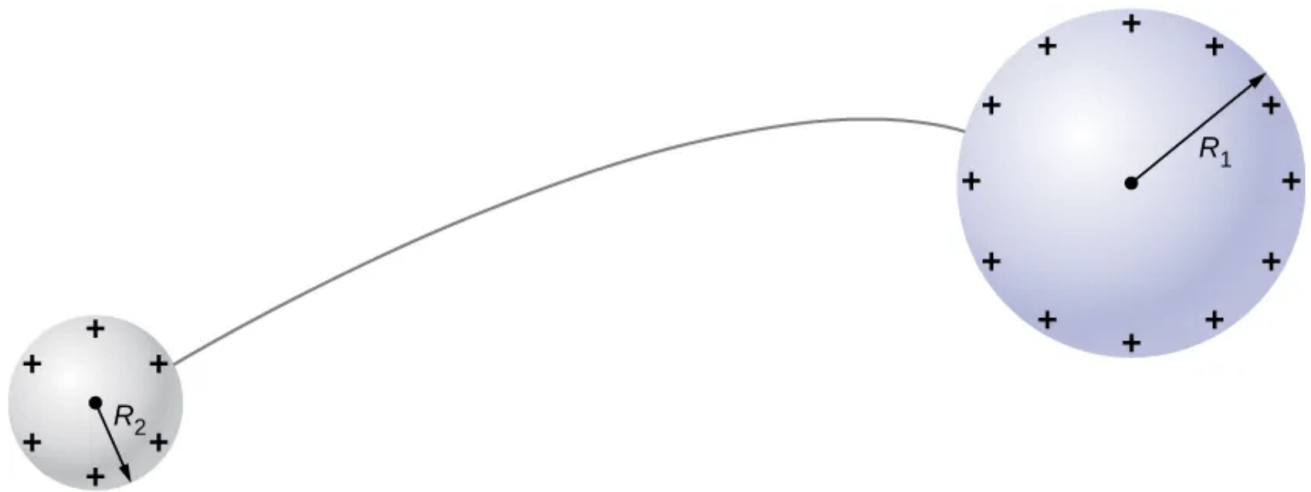
$$\rho(r) = \frac{\rho_0 r}{R} \quad (5.4.21)$$

within the object and zero for $r > R$. Find the electric field as a function of r both inside and outside the charge distribution.

5.4.4 Surface Charge Density

Model

Two conducting spheres of different sizes are connected by a wire.



Explore

1. Choose a color to represent the wire voltage in the model. Color the spheres using the same code.
2. Compare the two spheres by writing greater than, less than, or equal for:

Quantity	Sphere 1 vs. Sphere 2
Radius	
Charge	

Critical Thinking & Concept Invention

1. Because the spheres are connected by a conducting wire, how should their voltages compare? Add a voltage column to the table and indicate greater than, less than, or equal.
2. Determine how charge Q and surface charge density σ compare for each sphere.
 - Write an expression for the voltage of each sphere using the equations derived with infinity as the reference point.
 - Use the equation to identify the charge ratio $\frac{Q_1}{Q_2}$, then solve for Q_1 .
 - Using $\sigma = Q/A$, identify the surface charge density ratio $\frac{\sigma_1}{\sigma_2}$.

$$V_1 = V_2 = \quad (5.4.22)$$

$$\frac{Q_1}{Q_2} = \quad Q_1 = \quad (5.4.23)$$

$$\frac{\sigma_1}{\sigma_2} = \quad \sigma_1 = \quad (5.4.24)$$

3. How does surface charge density compare to radius of curvature for a conductor?

A greater radius of curvature results in a surface charge density _____.

5.5: Conductors in Electric Fields

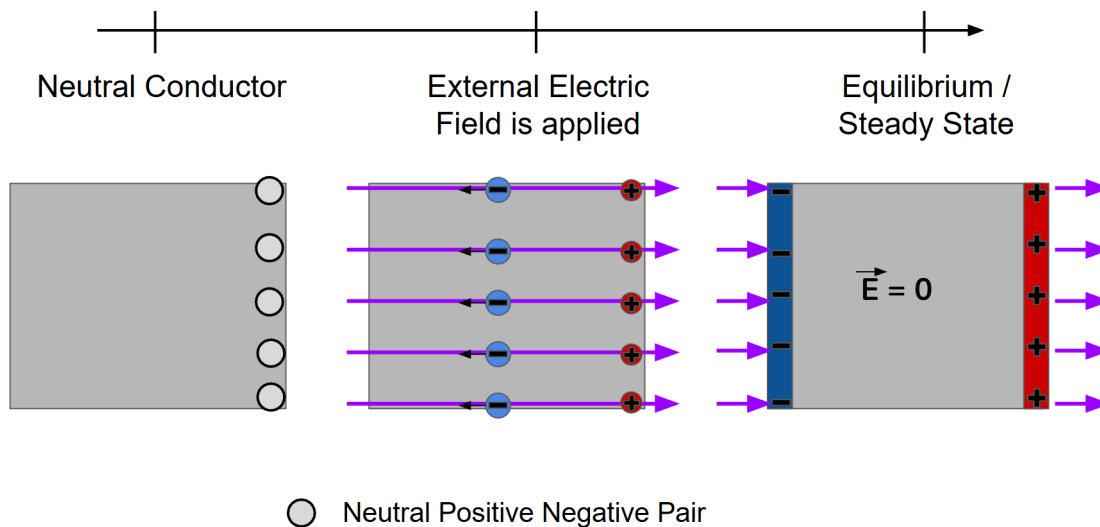
5.5 Conductors and Insulators in Electric Fields

Learning Objectives

- Predict how an electric field will influence the motion of positive and negative charges.
- Construct qualitative models to show the equilibrium state for charges in/on a conductor in the presence of a zero or nonzero external electric field.

5.5.1 Conductors in an External Electric Field

Model



Explore

1. When an electric field is applied to a conductor, what moves? What direction do they move?
2. Describe the charge distribution (i.e., where they are located) when an external electric field is applied to a conductor.
3. What is the electric field inside the middle of the conductor?

Critical Thinking & Concept Invention

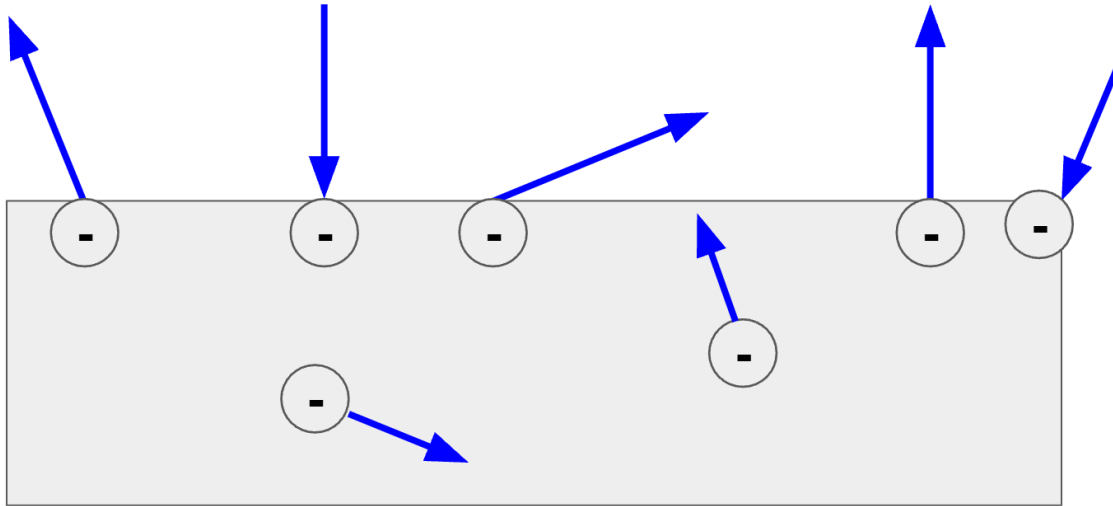
1. What direction would the electric field from the positive and negative charges point? Hint: It is not zero.
2. How is it possible that the electric field in a conductor is zero, given your answer to the previous question?

5.5.2 Surface Charges and Fields on a Conductor

Model

The box below is a conductor and the arrows represent electric field vectors at the locations shown. Recall that charges can move

freely inside a conductor but cannot leave a conductor.



Explore

1. Next to each electric field vector, draw an arrow to identify the direction of force that the charge experiences. Hint: Pay attention to the sign of the charge.
2. Forces will cause stationary charges to accelerate and therefore move. Consider what direction the charges will be pushed and draw where each charge might be a few moments later.
3. Circle any charges that were not able to move in the direction of the force on that/those charge(s).

Critical Thinking & Concept Invention

1. For the charges that do not move: Is the force parallel or perpendicular to the surface of the conductor? (One might be tricky to determine.)
2. For the charges on the surface that do move, is it the parallel or perpendicular component of the electric field, with respect to the surface, that points in the direction (or opposite) that the charge moves?
3. Summarize the patterns by choosing the correct selection:
 - A component of the electric field that is parallel to the surface of a conductor **WILL / WILL NOT** cause a charge to move.
 - A component of the electric field that is perpendicular to the surface of a conductor **WILL / WILL NOT** cause a charge to move.
4. Unless an external electric field is changing, charges will move until equilibrium, then remain still. At equilibrium, which of the following must be true:

$$E_{\parallel} = 0 \quad \text{OR} \quad E_{\perp} = 0 \quad (5.5.1)$$

5. If charges in the interior of the conductor also do not move, what must be true about the electric field inside a conductor?

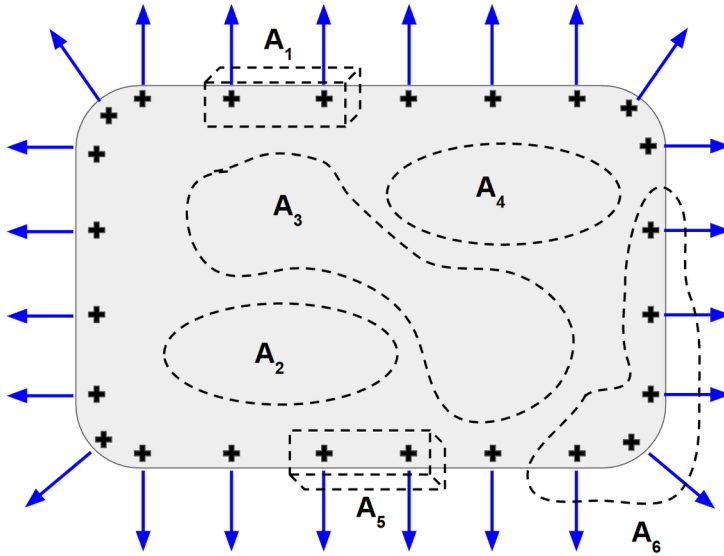
$$E_{\text{internal}} = \quad (5.5.2)$$

5.5.3 Internal Charges of a Conductor

Model

A conductor with net charges shown does not interact with any external object or fields. The arrows shown represent the electric fields created by charges in/on that conductor only. Six different closed surfaces are shown (imagine the ones that appear two-

dimensional are in fact closed three-dimensional objects of those arbitrary shapes).



Explore

1. Is the electric field zero or nonzero inside the conductor?
2. Identify which areas (if any) have:
 - Positive enclosed charge
 - Negative enclosed charge
 - Neutral or no enclosed charge
 - Positive electric flux
 - Negative electric flux
 - No electric flux

Critical Thinking & Concept Invention

Examine Areas 2, 3, and 4:

1. Is it possible that there could be a positive charge in any of those areas? Explain why or why not.
2. Is it possible that there could be a negative charge in any of those areas? Explain why or why not.
3. Construct a rule about net charge inside a conductor. Explain that rule.

5.5.4 Charges in the Cavity of a Conductor

Model

Both objects are conductors. The cavity is an empty, closed space within the conductor and the charges inside those cavities do not touch the conductor itself.

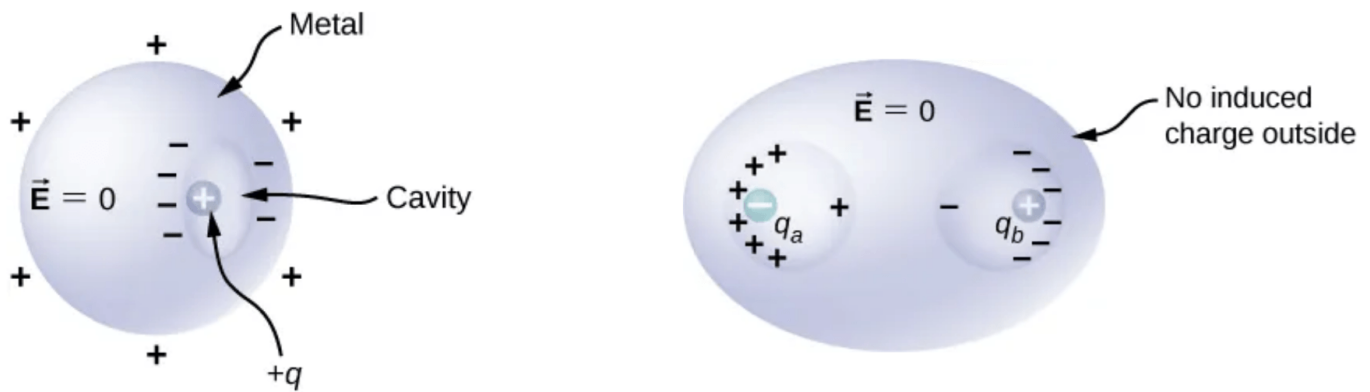


Image Credit: OpenStax University Physics Vol 2 (<https://openstax.org/details/books/university-physics-volume-2>)

Explore

1. Draw electric field lines from each charge inside the cavity. Remember that each line should originate on a positive charge and terminate on a negative charge.
2. Draw electric field lines from each charge (if present) on the surface of the conductor.
3. Identify, based on the number of lines, the flux inside each cavity for a surface that encloses the charge but not the charges along the surface of the cavity.
4. Identify, based on the number of lines, the flux outside of the full conductor in each example.

Critical Thinking & Concept Invention

1. How do the cavity fluxes and whole-conductor fluxes compare?
2. In the cavity, is the surface charge distributed evenly or unevenly? If uneven, where is there more concentrated charge?
3. On the surface of the conductor as a whole, is the surface charge distributed evenly or unevenly? If uneven, where is there more concentrated charge?
4. If we cannot see the cavity charges, can we tell how much net charge there is inside the conductor based on the surface charge of the conductor? Explain.
5. If we cannot see the cavity charges, can we tell how or where the charge inside the conductor is distributed spatially based on the surface charge of the conductor? Explain.

Apply

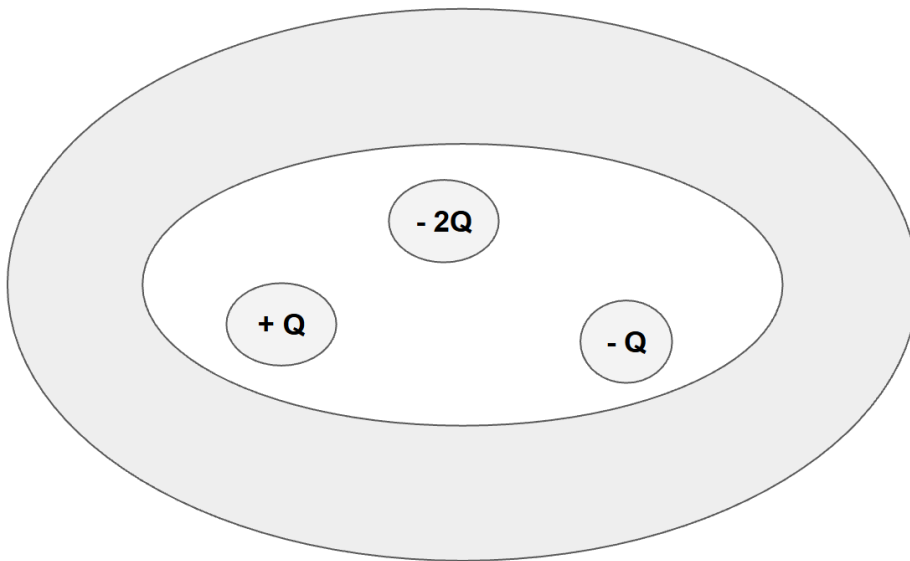
Three charges are present within the cavity of a conductor.

- Draw electric field lines inside the cavity, inside the conductor, and outside of the conductor. If the electric field is zero in any of these regions, write:

$$\vec{E} = 0 \quad (5.5.3)$$

- Draw charges to show how they will be distributed on both surfaces of the conductor (cavity and outer surface) and inside the conductor. If there is zero net charge in/on any of these regions, write:

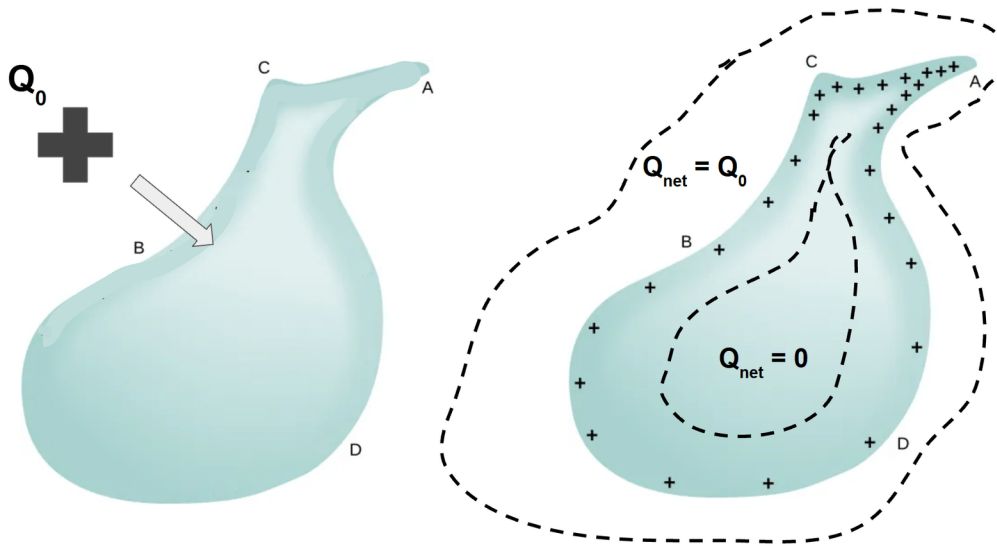
$$Q = 0 \quad (5.5.4)$$



5.5.5 Surface Charge Distribution on a Conductor

Model

A neutral conductor is shown on the left. Positive charge is added and the right image shows the distribution of charge at equilibrium. The small positive signs on the right represent a fraction of the charge added (large positive sign in the left image). The dashed lines represent theoretical closed surfaces for the purpose of electric flux models.



Explore

1. How much charge (variable, not number) is added to the conductor?
2. How much charge ends up within the inner dashed surface?
3. How much charge ends up within the outer dashed surface?
4. Is the charge distributed evenly or unevenly in/on the conductor? If uneven, describe how the charge is distributed.

Critical Thinking & Concept Invention

1. Draw an electric field line from each positive charge. Recall the rules established for how those lines should be drawn (e.g., inside/outside the conductor and angle to the surface).
2. How much of the total charge is:
 - Inside the conductor?
 - On the surface of the conductor?
3. Construct a rule:
 - Surface charge density is greater (i.e., charges are closer together) _____.
4. Identify the electric flux (in terms of given variables) for both dashed surfaces.

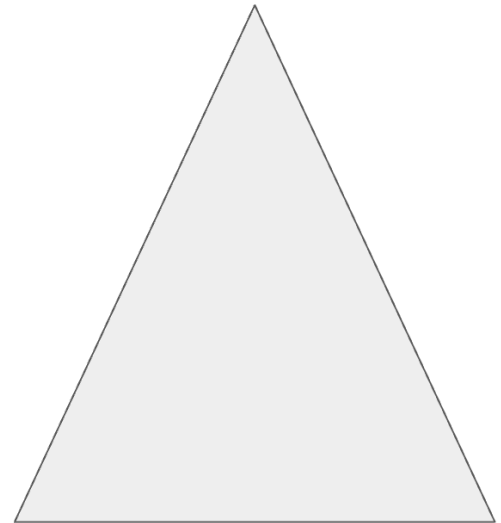
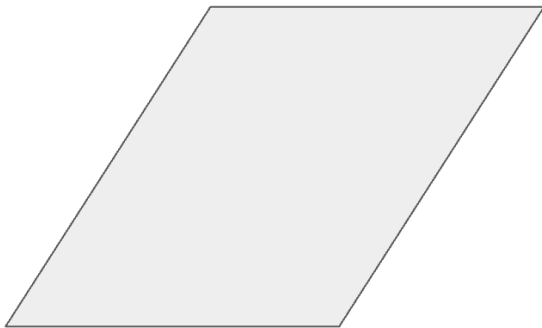
Apply

The two shapes below are conductors. Both conductors have a total charge of $-Q$.

- Draw charges (+ or – signs) to show the general distribution of charges.
- Draw electric field lines or write

$$\vec{E} = 0 \quad (5.5.5)$$

to represent the electric field inside and outside of the conductors.



Lesson Summary: Patterns and Rules for Conductors

Electric fields cause charges (generally electrons) to move. Once equilibrium is reached:

Inside the Conductor

- The net charge is: _____
- Because of this, the electric field is: _____

On the Surface of the Conductor

- If the conductor is neutral, the net surface charge is: _____
- If the conductor is charged, the net surface charge is: _____
- The distribution of charge is even if: _____
- The density of charge is greater at locations: _____

- The orientation of the electric field is: _____

5.6: Dielectrics in Electric Fields

5.6 Dielectrics

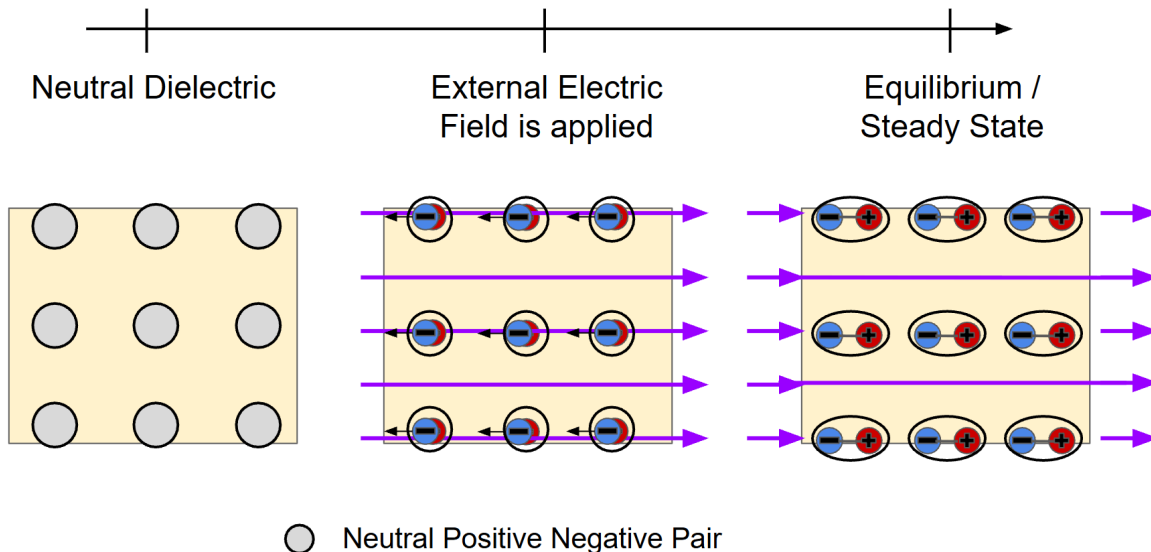
Learning Objectives

- Construct qualitative models to show the equilibrium state for charges in an insulator (dielectric) in the presence of a nonzero external electric field.
- Qualitatively model the electric fields created by the polarization of dielectrics due to external electric fields or charges.

5.6.1 Insulators (Dielectrics) in an Electric Field

Model

The model below shows the state of an insulator (dielectric) over time. Starting in the second panel, a constant electric field is applied.



Explore

1. When an electric field is applied to an insulator, what objects move? What direction do they move?
2. Are the charges able to move as far from their positive charges as in a conductor?
3. Describe the charge distribution (i.e., where they are located) when an external electric field is applied to an insulator.
4. Is the electric field zero inside an insulator? If not, is the electric field stronger or weaker than the applied electric field?

Critical Thinking & Concept Invention

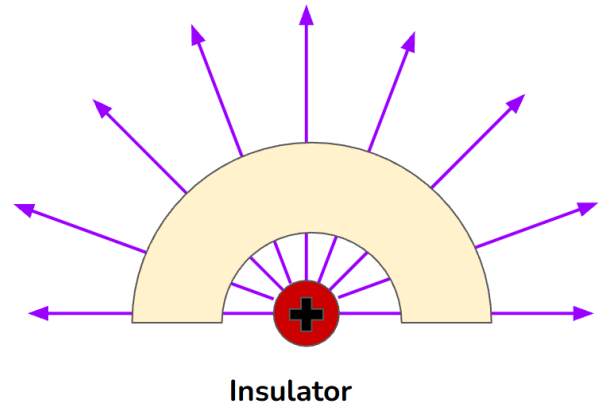
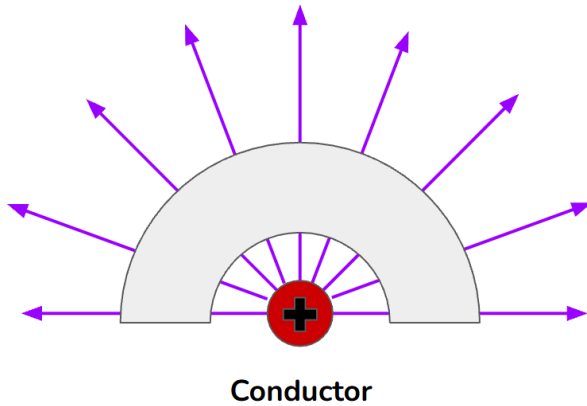
1. For each attached pair of charges, what direction would the electric field they create point? How does that compare to the applied field?
2. Why does the electric field get weaker inside an insulator?
3. What is different about how charges respond to electric fields in an insulator, compared to a conductor?

Apply

Complete the models with the following:

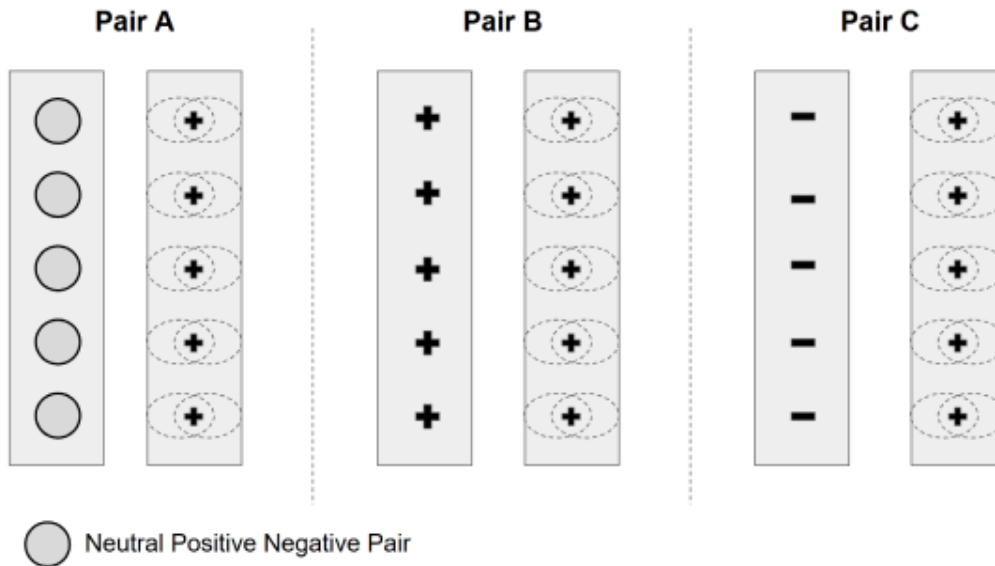
- Draw the equilibrium charge distribution on the conductor and dielectric (insulator).
- Draw the electric field lines (if any) inside the conductor and insulator. If the electric field is zero, write:

$$\vec{E} = 0 \quad (5.6.1)$$



5.6.2 Polarized Neutral Objects

Model



Explore

For each tape pair:

1. Draw the direction of the electric field vectors that are created by the charges shown on the left tape in the pair. If the field is zero, write:

$$\vec{E} = 0 \quad (5.6.2)$$

- Based on the direction of the electric field (if present), the electrons will move off of the positive charge. Identify which direction (right or left) the electrons will move and mark up the model to look like one of the provided options.



Critical Thinking & Concept Invention

- The newly separated charges create their own field. In each pair, draw (using a different color if possible) the electric field on the left side of the right tape that is created by the charge nearest to that edge of the tape. If the field is zero, write:

$$\vec{E} = 0 \quad (5.6.3)$$

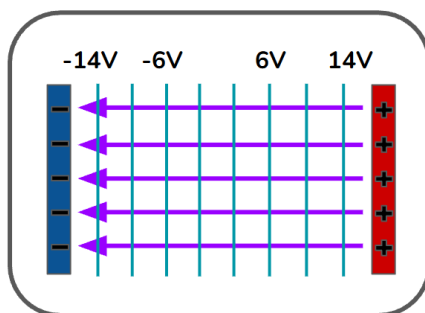
- Now draw the force on each charge of the left tape that is caused by the field from the right tape that you just drew. If the force is zero, write:

$$\vec{F} = 0 \quad (5.6.4)$$

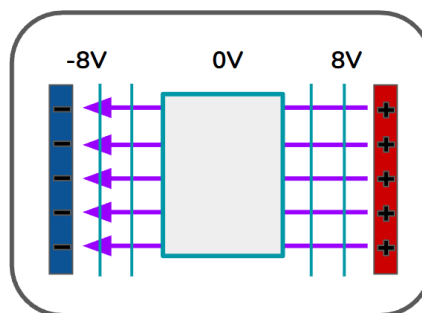
- Below each pair, write *Attracts*, *Repels*, or *No Interaction* to describe the direction of these forces.
- Summarize what you learned about charged-object and neutral-object interactions.

5.6.5 Electric Potential in Conductors and Dielectrics

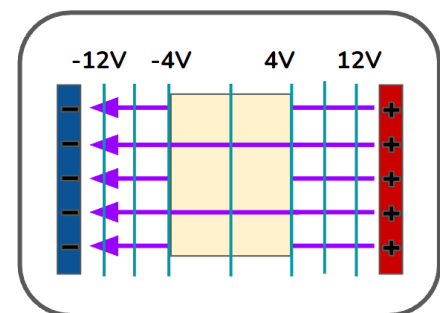
Model



Vacuum



Conductor



Insulator

Explore

- In a vacuum, are the equipotential lines (vertical lines) evenly spaced or does the spacing change between potential lines?
- How much does the voltage change between equipotential lines in this model?
- Are there equipotential lines inside the conductor? What about the insulator? If so, is the spacing the same or different than outside of that material?

Critical Thinking & Concept Invention

- Do different parts of a conductor have different electric potential values (voltages)?
- Do different parts of an insulator have different electric potential values (voltages)?
- How does the presence of a conductor affect the total number of potential lines between the charged plates? How does this affect the total difference in electric potential (positive minus negative) between the plates?
- How does the presence of an insulator affect the total number of potential lines between the charged plates? How does this affect the total difference in electric potential (positive minus negative) between the plates?

5. If you wanted to reduce the voltage between two plates as much as possible, would you add a conductor or an insulator between them? Why?

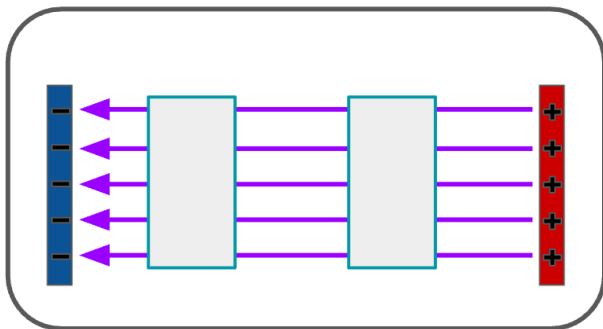
Apply

Complete the models with the following:

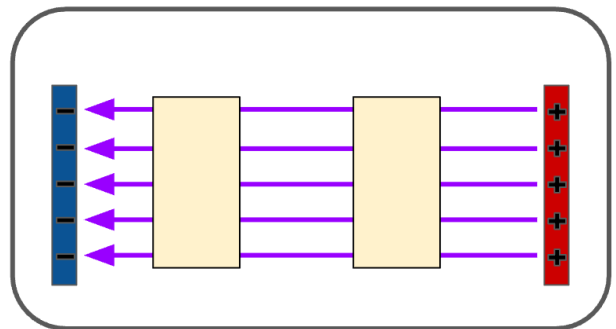
- Draw the equilibrium charge distribution on the conductors and dielectrics (insulators).
- Draw the electric field lines (if any) inside the conductors and insulators. If the electric field is zero, write:

$$\vec{E} = 0 \quad (5.6.5)$$

- Draw equipotential lines on both models. Make sure the spacing of the lines matches the established relationship between equipotential lines and electric field lines.



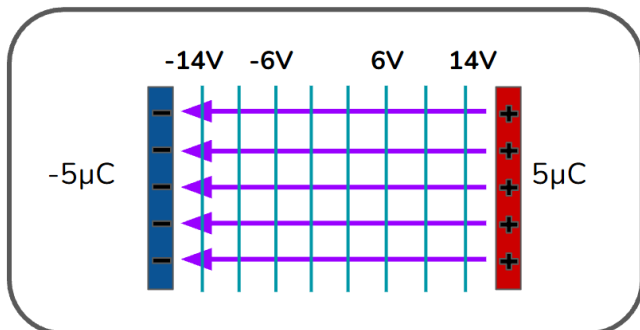
Conductor



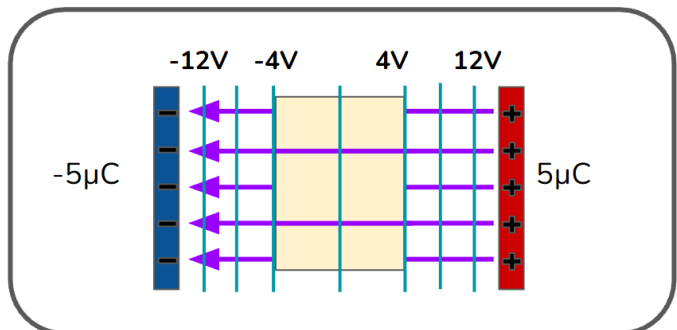
Insulator

5.6.6 Capacitance and Dielectrics

Model



Capacitor A
Vacuum between
plates



Capacitor B
Dielectric (Insulator)
is added

Explore

1. Fill in the table with information from the model.

Capacitance Comparison

Capacitor	Number of Charges on Each Plate	ΔV	Capacitance ($C = \frac{Q}{\Delta V}$)
A			
B			

Critical Thinking & Concept Invention

1. For plates with the same charge, when dielectrics are added to a capacitor, do they increase or decrease the potential difference between the plates?
2. In order to get the same voltage as a capacitor without a dielectric, would you need to increase or decrease the amount of charge on each plate?
3. Summarize: Why do dielectrics affect the capacitance of a capacitor?

Summary

If a dielectric is added between the plates of a capacitor with an initial capacitance of C_0 , the new capacitance is:

$$C = \kappa C_0 \quad (5.6.6)$$

The dielectric constant (κ) is a material property of the substance that is inserted between the capacitor plates.

Electricity does not flow through the dielectric (it is an insulator). This substance affects the electric properties of the capacitor and circuit as a whole by affecting the electric fields.

6: AMPERE AND FARADAY LAWS

6.1: Ampere's Law

6.2: Solenoids and Toroids

6.3: Faraday's Law of Induction

6.4: Inductance and Transformers

6.5: Electric Motors, Displacement Current, and EM Waves

6.1: Ampere's Law

6.1 Ampere's Law

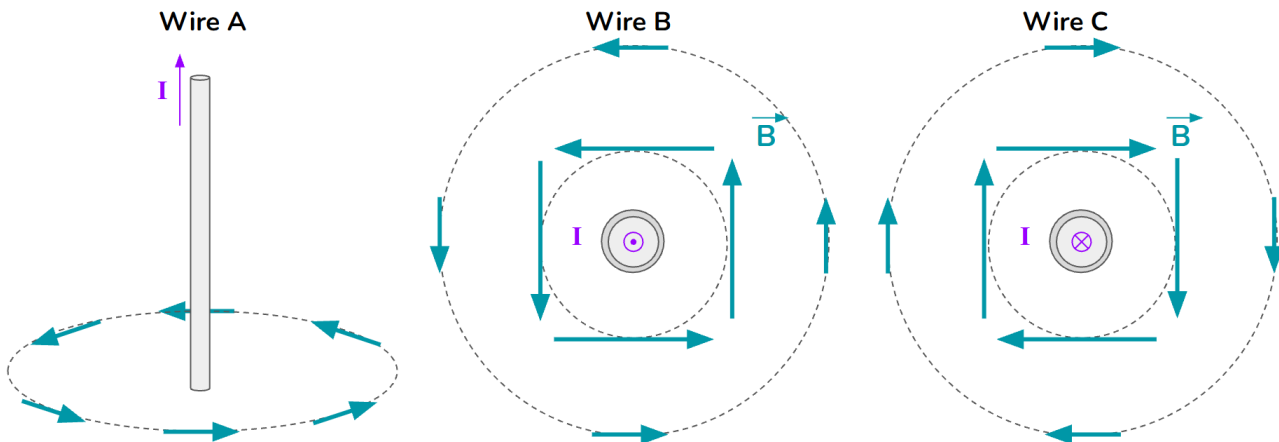
Learning Objectives

- Identify the direction of magnetic fields from currents.
- Determine the magnitude of a magnetic field from a wire or wire segment.
- Use the concept of superposition to determine the total magnetic field vector at a point from multiple current sources.

6.1.1 Magnetic Field Direction

Model

-  Out of the page/screen
 Into the page/screen



Explore

1. Describe the direction of the current (I) and magnetic field (B) created by the current for each wire.

Wire	Current Direction	Magnetic Field Direction
A		
B		
C		

2. Examine wires B and C. Does the magnetic field become stronger, weaker, or remain the same at points farther from the wire?

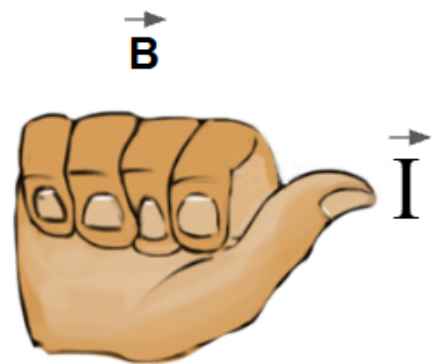
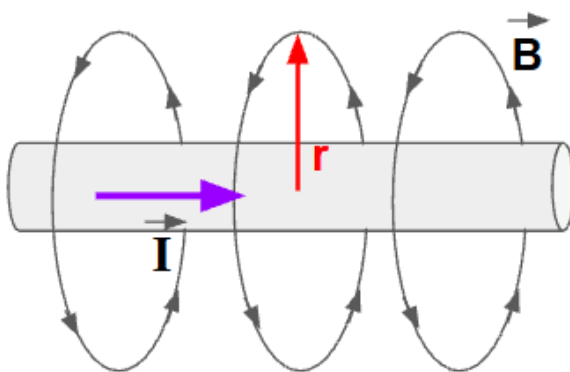
Critical Thinking & Concept Invention

1. Align your four fingers (not your thumb) with the curved magnetic field lines using both hands. Which hand places your thumb in the direction of the current?
2. Generalize the relationship between magnetic field and distance r from the wire.

$$B \propto r \quad \text{OR} \quad B \propto \frac{1}{r} \quad (6.1.1)$$

Summary

Currents (I) produce magnetic fields (B), and those fields can interact with other objects to produce forces. Magnetic field lines form continuous loops around currents with no beginning or end. The direction of the magnetic field is determined using the right-hand rule. Fingers “curl” in direction of magnetic field B and thumb points in direction of current I



Right Hand

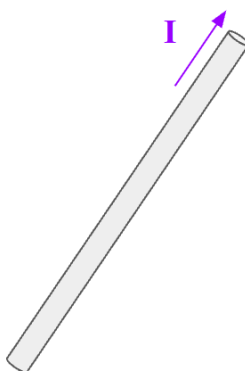
Apply

Sketch the magnetic field for each of the following wires at two different radii.

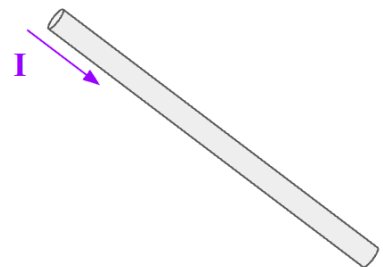
Wire D



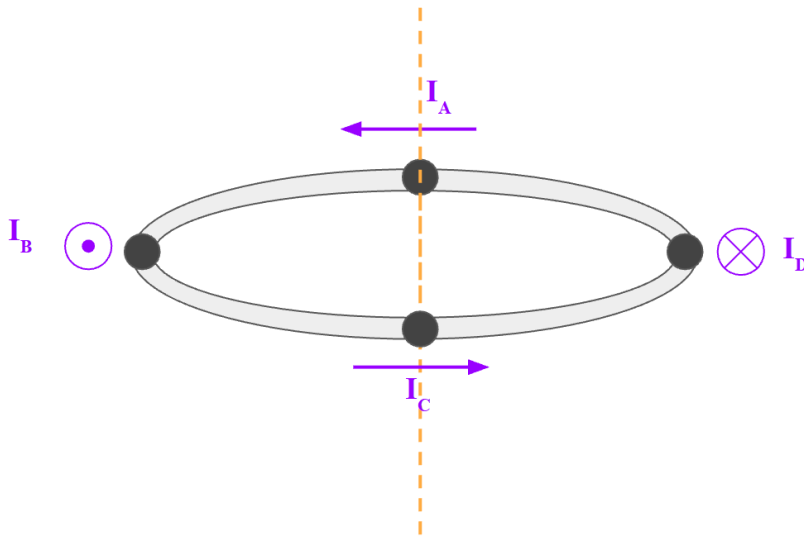
Wire E



Wire F

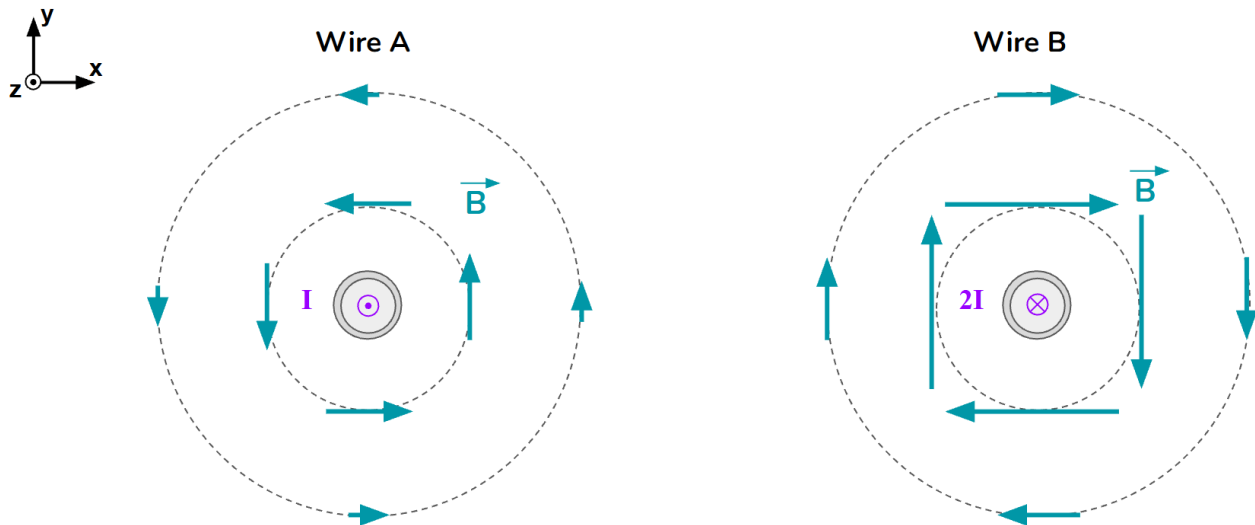


Sketch the magnetic field for a current-carrying loop



6.1.2 Magnetic Field and Current

Model



Explore

1. For wire A, is the magnetic field stronger at points closer to the wire (inner circle) or farther away (outer circle)?
2. For wire B, is the magnetic field stronger at points closer to the wire (inner circle) or farther away (outer circle)?
3. Compare the magnetic field vectors at the same distance for wires A and B. Which wire produces the greater magnetic field?
4. What difference between wires A and B might account for the difference in magnetic field?

Critical Thinking & Concept Invention

1. Describe how magnetic field magnitude depends on current and distance.
 - When distance from the wire increases, the magnitude of the magnetic field...

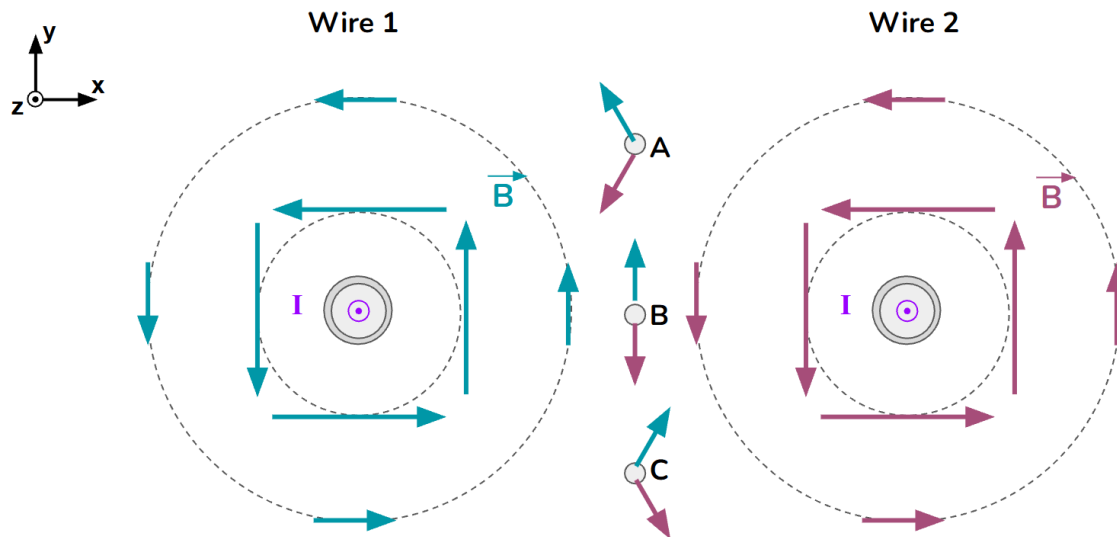
- When current increases, the magnitude of the magnetic field...
2. Choose the proportional relationships:

$$B \propto r \quad \text{OR} \quad B \propto \frac{1}{r} \quad (6.1.2)$$

$$B \propto I \quad \text{OR} \quad B \propto \frac{1}{I} \quad (6.1.3)$$

6.1.3 Magnetic Field Superposition

Model



Explore

1. Identify whether the magnetic field from each wire has positive, negative, or zero x - and y -components at each labeled point.

Point	Wire 1 B_x	Wire 2 B_x	Total B_x	Wire 1 B_y	Wire 2 B_y	Total B_y
A						
B						
C						

Critical Thinking & Concept Invention

1. Which point (A, B, or C) has a total magnetic field of zero?
2. What direction does the total magnetic field point at each point where the magnetic field is nonzero?
3. When multiple currents are present, how is the total magnetic field determined?

Summary

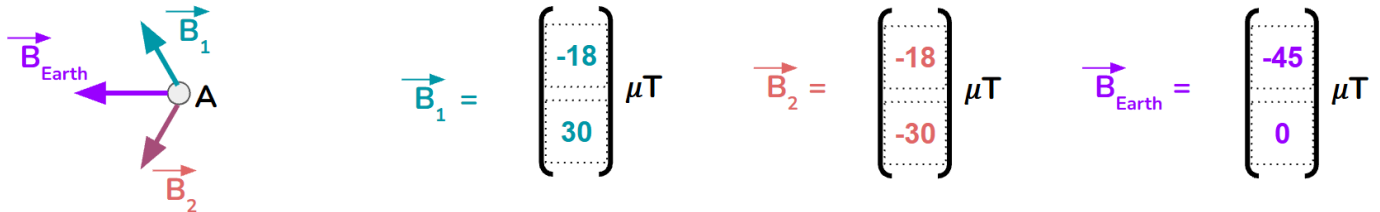
Magnetic fields are vector fields. Every moving charge produces a magnetic field, and the total magnetic field at any point is the vector sum of all individual magnetic fields.

$$\vec{B} = \sum \vec{B}_i \quad (6.1.4)$$

Vector addition is essential because direction must be considered when combining contributions from multiple sources. Sums and integrals are used to account for all contributing magnetic fields.

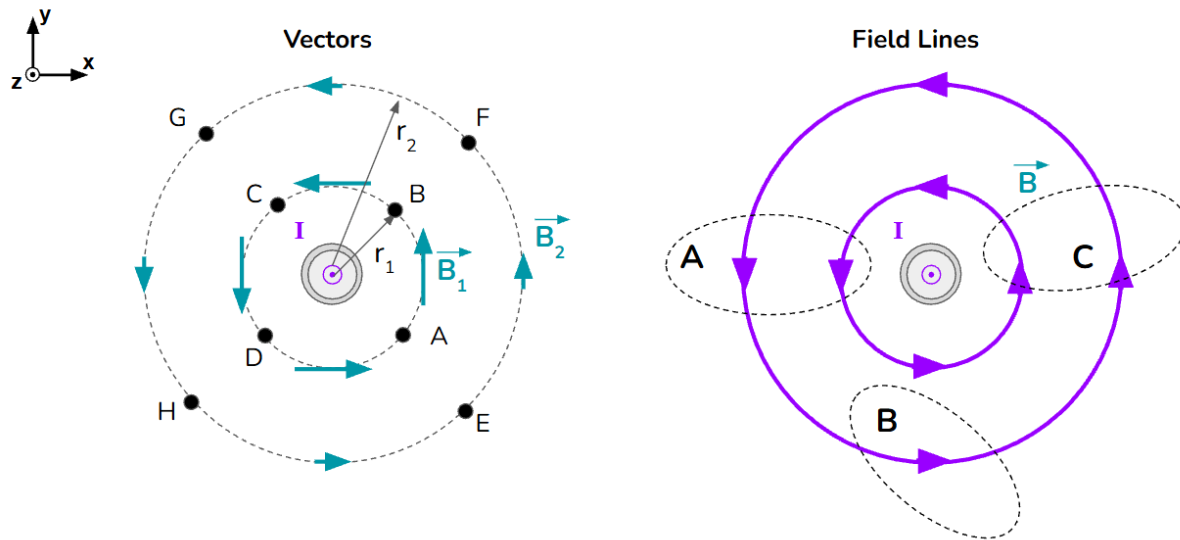
Apply

The fields from the previous two wires are shown at Point A along with Earth's magnetic field. Determine the total magnetic field graphically and numerically using column vectors.



6.1.4 Magnetic Flux and Circulation

Model



Explore

Focus on the Field Vectors to Explore Magnetic Circulation

- Imagine a path from A to B to C to D and back to A following the dashed line.
 - What is the total path length?
 - $\Delta l_1 =$
 - Along the path, how does the direction of $\vec{B}$ compare to the path direction ($\parallel$, $\perp$, or neither)?
- Imagine a path from E to F to G to H and back to E.
 - What is the total path length?
 - $\Delta l_2 =$

- How does $\vec{B}$ compare to the path direction?
- Compare field magnitudes: B_1 ___ B_2 .
- Compare path lengths: Δl_1 ___ Δl_2 .

Focus on the Field Lines to Explore Magnetic Flux

3. Circle every location where a magnetic field line enters the dashed surfaces.
4. Mark an X wherever a magnetic field line exits the dashed surfaces.
5. Identify the magnetic flux through each surface.

$$\Phi_A = \quad \Phi_B = \quad \Phi_C = \quad (6.1.5)$$

Critical Thinking & Concept Invention

Magnetic circulation is defined as:

$$\Gamma_B \equiv \oint \vec{B} \cdot d\vec{l} \quad (6.1.6)$$

1. Show how this integral can be simplified. Discuss the dot product and whether constants can be moved outside the integral.
2. Identify the circulation for both paths.

$$\Gamma_{B1} = \quad \Gamma_{B2} = \quad (6.1.7)$$

3. If the current I increases, what variable in the equation changes?
4. Propose the relationship between magnetic circulation and electric current.

$$\Gamma_B \propto I \quad \text{OR} \quad \Gamma_B \propto \frac{1}{I} \quad (6.1.8)$$

5. Solve each equation for the magnetic field.

$$B_1 = \quad B_2 = \quad (6.1.9)$$

Magnetic flux is defined as:

$$\Phi_B \equiv \iint \vec{B} \cdot d\vec{A} \quad (6.1.10)$$

6. Can you create a closed surface with nonzero magnetic flux? If so, draw one.
7. Since magnetic field lines are closed loops around currents, how does this affect magnetic flux through a closed surface?

Summary

The magnetic flux through a closed surface is always zero because magnetic field lines have no beginning or end.

$$\Phi_B = 0 \quad (6.1.11)$$

The magnetic circulation around a closed loop is proportional to the enclosed current:

$$\Gamma_B = \mu_0 I + \dots \text{something else} \quad (6.1.12)$$

Magnetic flux may be nonzero through open surfaces. The circulation equation is Ampère's Law, which will later be refined. These equations describe how currents create magnetic fields. We cannot always solve the circulation integrals easily, so we will need other techniques for certain currents when there is not sufficient symmetry.

Apply

Determine the magnetic circulation (Γ_B) for each of the three Ampère loops. The positive direction is defined by the right-hand

rule with fingers following the indicated loop direction.

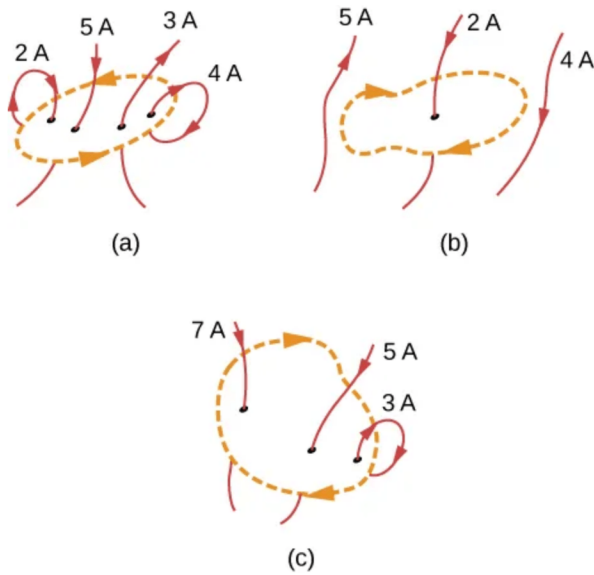
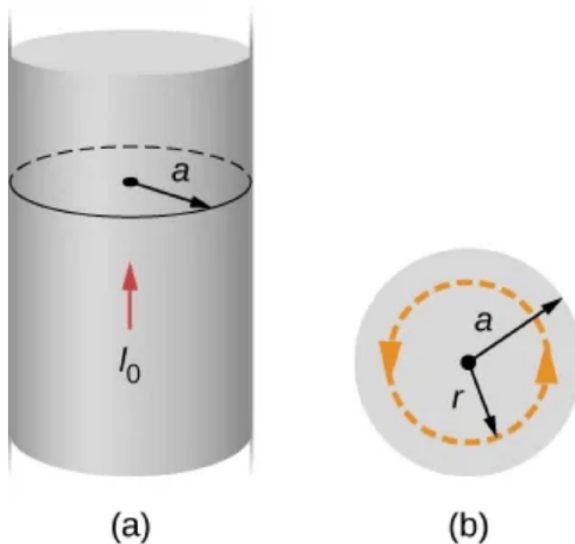


Image Credit: OpenStax University Physics Vol 2 (<https://openstax.org/details/books/university-physics-volume-2>)

6.1.5 Magnetic Field of Thick Wire

Model

$$I_0 = J A_{\text{total}} \text{ and } I_{\text{enclosed}} = J A_{\text{enclosed}}$$



Explore

1. Use the right-hand rule to identify the expected magnetic field direction along the dashed Ampère loop. Draw arrows showing the direction.
2. Should the magnetic field remain constant along the loop? If not, where is it strongest?

3. Identify the current density J from the problem variables and write it next to the model.
4. How does the radius of the Ampère loop affect the amount of enclosed current?
5. What is the maximum enclosed current for a circular Ampère loop in this example?

Critical Thinking & Concept Invention

1. Use Ampère's Law for loops smaller than the wire radius ($r < a$).
 - Write the circulation equation.
 - Evaluate the dot products and move constants outside the integral.
 - Evaluate the integral.
 - Solve for the magnetic field.

$$\Gamma_B = \mu_0 I_{\text{enc}} \quad (6.1.13)$$

$$B = \quad (6.1.14)$$

2. Use Ampère's Law for loops larger than the wire radius ($r > a$).
 - Write the circulation equation.
 - Evaluate the dot products and simplify.
 - Evaluate the integral.
 - Solve for the magnetic field.

$$\Gamma_B = \mu_0 I_{\text{enc}} \quad (6.1.15)$$

$$B = \quad (6.1.16)$$

3. Verify that both equations agree at $r = a$.

6.2: Solenoids and Toroids

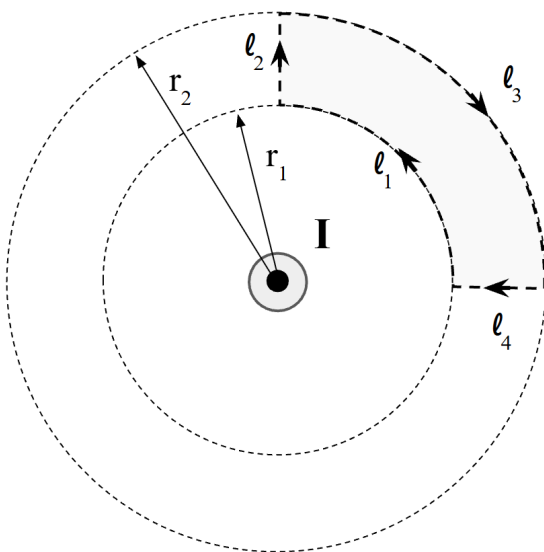
Ampère's Law - Solenoids and Toroids

Learning Objectives

- Use Ampère's Law to determine magnetic fields from currents using integrals when needed.
- Use Ampère's Law to calculate the magnetic field in regions of space around a solenoid.
- Use Ampère's Law to calculate the magnetic field in regions of space around a toroid.

6.2.1 Ampère Loops Near a Straight Wire

Model



Explore

1. Identify the general direction you expect the magnetic field to be along each path of the loop using the Right Hand Rule. Draw arrows to show this direction.
2. What is the enclosed current in the dashed Ampère loop?
3. Fill out the table to compare the path and magnetic field.

Segment	$\vec{B}$ ($\perp$ / $\parallel$ / neither)	$\vec{B} \cdot \Delta \vec{l}$ (+, -, or 0)
Δl_1		
Δl_2		
Δl_3		
Δl_4		

Critical Thinking & Concept Invention

- Use Ampère's Law to explore the magnetic fields along this loop.
 - Write a circulation equation for the loop as a sum of integrals for each segment, including any that are zero.
 - Take the dot products and move any constants outside the integral.
 - Evaluate each integral.
 - Separately solve for each nonzero magnetic field using appropriate subscripts.
 - In each equation, treat the other magnetic field and radius as constants and examine how magnetic field depends on radius.

$$\Gamma_B = \mu_0 I_{\text{enc}} \quad (6.2.1)$$

$$B = \quad (6.2.2)$$

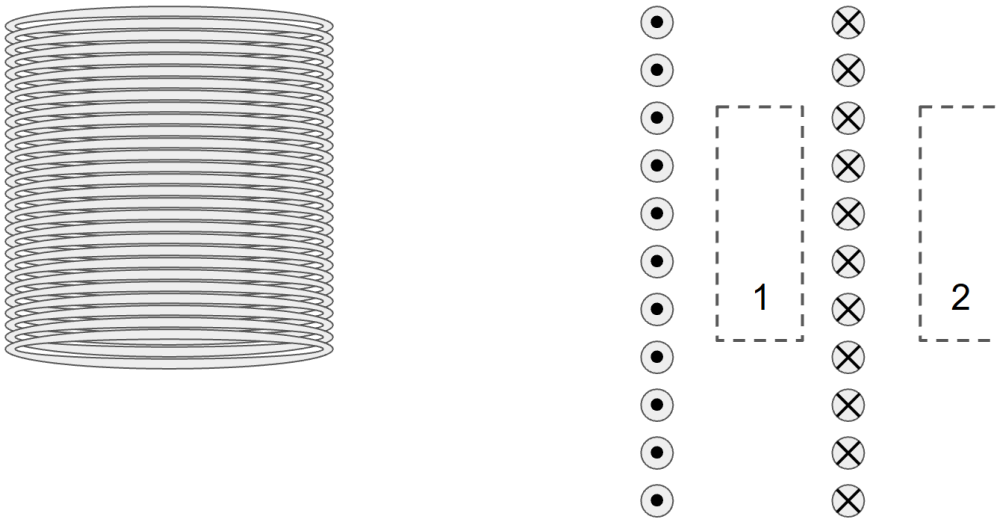
$$B = \quad (6.2.3)$$

$$B \propto r \quad \text{OR} \quad B \propto \frac{1}{r} \quad (6.2.4)$$

- Why was it useful to choose an Ampère loop that did not enclose current?

6.2.2 Ampère Loops Inside and Outside of a Solenoid

Model



Explore

- Identify the general direction you expect the magnetic field to be along each loop using the Right Hand Rule.
- Identify the enclosed current in each Ampère loop.

$$I_1 = \quad I_2 = \quad (6.2.5)$$

- What should the magnetic circulation be in each loop?

$$\Gamma_1 = \quad \Gamma_2 = \quad (6.2.6)$$

Critical Thinking & Concept Invention

1. Use Ampère's Law to explore the magnetic field along Loop 1.
 - Write the circulation equation as a sum of integrals for each segment.
 - Take the dot products and move constants outside the integral.
 - Evaluate each integral.
 - Solve for one of the nonzero magnetic fields using appropriate subscripts.

$$\Gamma_B = \mu_0 I_{\text{enc}} \quad (6.2.7)$$

$$B = \quad (6.2.8)$$

2. What must be true about the magnetic field at different places within the solenoid to make this equation true? Explain.
3. Use Ampère's Law to explore the magnetic field along Loop 2.
 - Write the circulation equation as a sum of integrals for each segment.
 - Take the dot products and move constants outside the integral.
 - Evaluate each integral.
 - Solve for one of the nonzero magnetic fields using appropriate subscripts.

$$\Gamma_B = \mu_0 I_{\text{enc}} \quad (6.2.9)$$

$$B = \quad (6.2.10)$$

4. If the farthest side of Loop 2 is placed very far away, what should the magnetic field be along that far edge?
5. Based on your previous response, what is the magnitude of the magnetic field on the near edge, which can be anywhere outside the solenoid? Explain your reasoning.

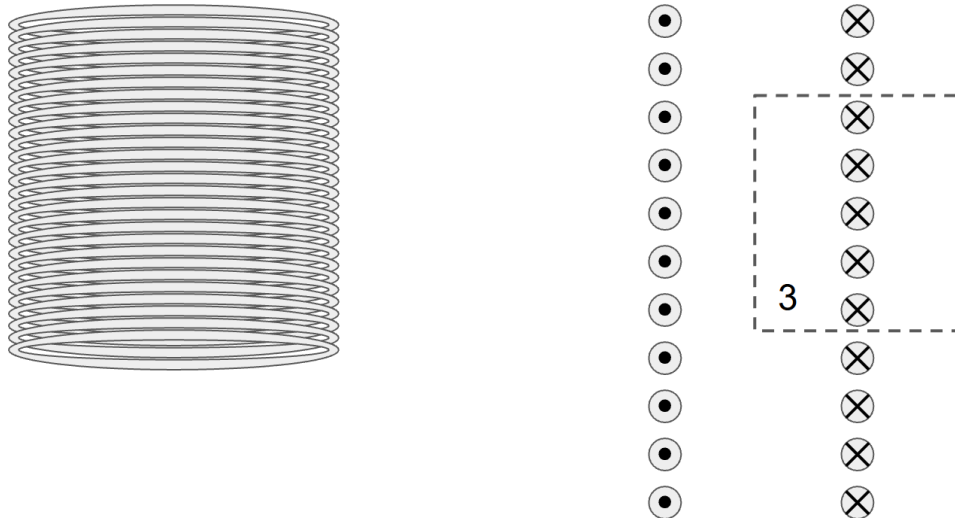
Summary

In a solenoid:

- The magnetic field inside is _____.
 - The magnetic field outside is _____.
-

6.2.3 Ampère Loop Enclosing a Solenoid

Model



Explore

1. Identify the general direction you expect the magnetic field to be along each path of the loop using the Right Hand Rule. Draw arrows to show this direction.
2. What is the enclosed current in the dashed Ampère loop? How would this change if the loop height were greater or smaller?
3. Label each segment as Δl_1 , Δl_2 , and so on.
4. Fill out the table comparing the path and magnetic field.

Segment	$\vec{B}$ ($\perp$ / $\parallel$ / neither)	$\vec{B} \cdot \Delta \vec{l}$ (+, -, or 0)
Δl_1		
Δl_2		
Δl_3		
Δl_4		

Critical Thinking & Concept Invention

1. Use Ampère's Law to explore the magnetic field along Loop 1.
 - Write the circulation equation as a sum of integrals for each segment.
 - Take the dot products and move constants outside the integral.
 - Evaluate each integral.
 - Solve for one of the nonzero magnetic fields.

$$\Gamma_B = \mu_0 I_{\text{enc}} \quad (6.2.11)$$

$$B = \quad (6.2.12)$$

2. Generalize your equation.

- For a different-sized loop, replace I_{enc} with a variable using N for the number of coils.
- Coil density is defined as $n = N/L$. Construct a new equation for the magnetic field of a solenoid in terms of n .

$$B = \quad (6.2.13)$$

$$B = \quad (6.2.14)$$

1.4 Toroid

Model

A toroid is a solenoid bent into a donut shape. The toroid shown carries current I , has N loops, an inner radius a , and an outer radius b .

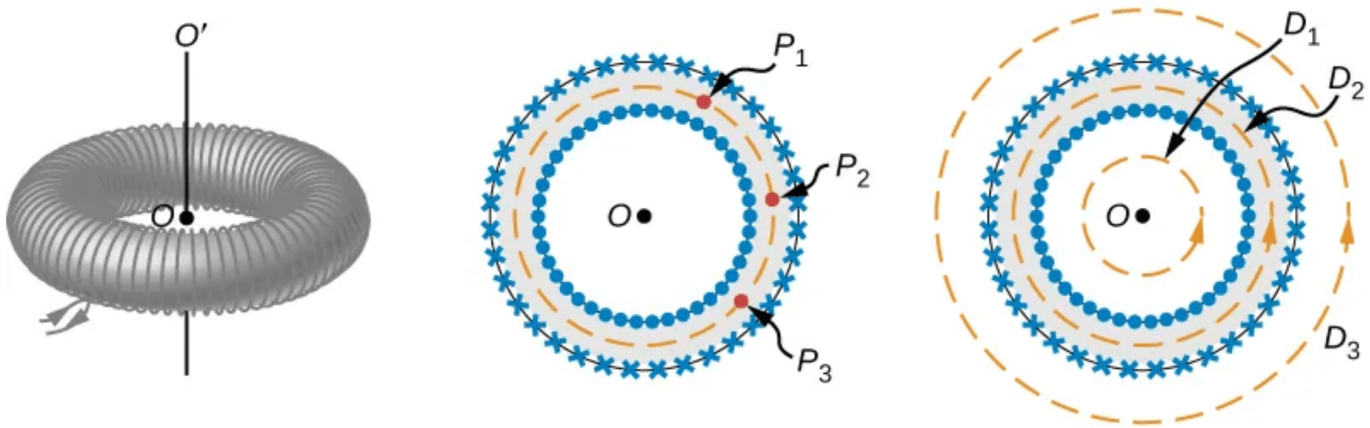


Image Credit: OpenStax University Physics Vol 2 (<https://openstax.org/details/books/university-physics-volume-2>)

Explore

Examine the middle image:

1. Compare the magnetic field magnitudes at points P_1 , P_2 , and P_3 . Circle the equality or rank them using greater-than symbols.
2. Draw magnetic field vectors at each point based on their relative magnitudes. Use the Right Hand Rule to determine direction.
3. How does the direction of the magnetic field compare at each point?

Examine the image on the right:

$$\Delta l_1 = \Delta l_2 = \Delta l_3 \quad \text{OR} \quad >> \quad (6.2.15)$$

4. How much current is enclosed in each Ampère loop?

$$I_1 = \quad I_2 = \quad I_3 = \quad (6.2.16)$$

5. Draw a radius vector to each Ampère loop and label those vectors appropriately.
6. Compare the total lengths of the three Ampère paths. Circle the equality or rank them.
7. Compare the direction of the magnetic field to the path direction for each loop.

$$\vec{B} \perp d\vec{l} \quad \text{OR} \quad \vec{B} \parallel d\vec{l} \quad \text{OR} \quad \text{neither} \quad (6.2.17)$$

Critical Thinking & Concept Invention

1. Explore the magnetic fields along paths D_1 and D_3 .
 - Write an Ampère's Law equation for each path.
 - Simplify the dot product.
 - Move constants outside the integral.
 - Solve for B_1 and B_3 .

$$\Gamma_1 = \mu_0 I_1 \quad \Gamma_3 = \mu_0 I_3 \quad (6.2.18)$$

$$B_1 = \quad B_3 = \quad (6.2.19)$$

2. What can you conclude about the magnetic field outside the toroid? Explain how you know this is true.
3. Explore the magnetic field along path D_2 .
 - Write an Ampère's Law equation for this path.
 - Simplify the dot product.
 - Move constants outside the integral.
 - Solve for B_2 .

$$\Gamma_2 = \mu_0 I_2 \quad (6.2.20)$$

$$B_2 = \quad (6.2.21)$$

4. Why is the result different for D_2 than for D_1 and D_3 ?
5. Is the magnetic field constant throughout a toroid as it is in a solenoid? If not, where is the field strongest? Explain how the equation demonstrates this.

6.3: Faraday's Law of Induction

Faraday's Law of Induction

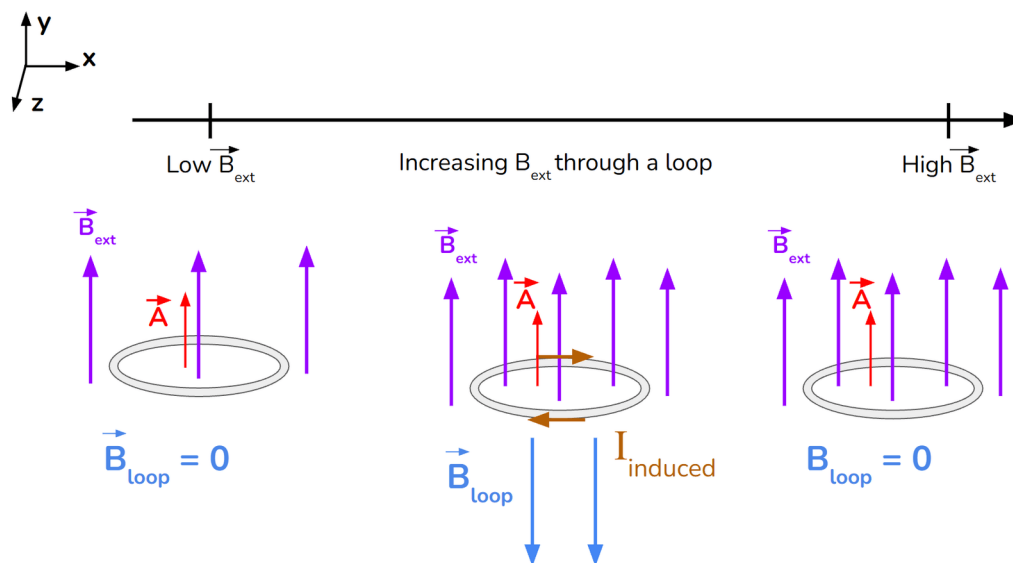
Learning Objectives

- Identify changing magnetic flux through loops of wire.
- Determine the direction of an induced current when there is a changing magnetic flux.
- Determine the electromotive force (emf) and induced current caused by a changing magnetic flux.

6.3.1 Induced emf

Model

The loop of wire initially has a low magnetic field that is constant. It then increases at a steady rate. At the end, the magnetic field is higher than it started, but is once again constant.



Explore

1. What direction (+/- x , +/- y , or +/- z) does the area vector point?
2. What direction (+/- x , +/- y , or +/- z) does the external magnetic field point?
3. Which flux variable (B , A , or $\cos(\theta)$) changed from the beginning to the end of the timeline? Did it increase or decrease?
4. What direction (+/- x , +/- y , or +/- z) does the magnetic field produced by the loop point?
5. Align your thumb with the area vector using the Right Hand Rule. Does the induced current point in the same direction or the opposite direction as your curved fingers?

Critical Thinking & Concept Invention

1. Is the flux due to the external magnetic field positive or negative? Recall:

$$\Phi_B = \vec{B} \cdot \vec{A} \quad (6.3.1)$$

2. In the middle panel, is the rate of change of the external magnetic flux,

$$\frac{d\Phi_{B,\text{ext}}}{dt}, \quad (6.3.2)$$

positive or negative?

3. Is the flux due to the loop's magnetic field positive or negative?

4. Is the sign of $\Phi_{B,\text{loop}}$ the same as or opposite to the sign of $\frac{d\Phi_{B,\text{ext}}}{dt}$?

Summary

When the magnetic flux through a loop changes, an electromotive force (emf) is created in such a way that an induced current opposes the change in magnetic flux.

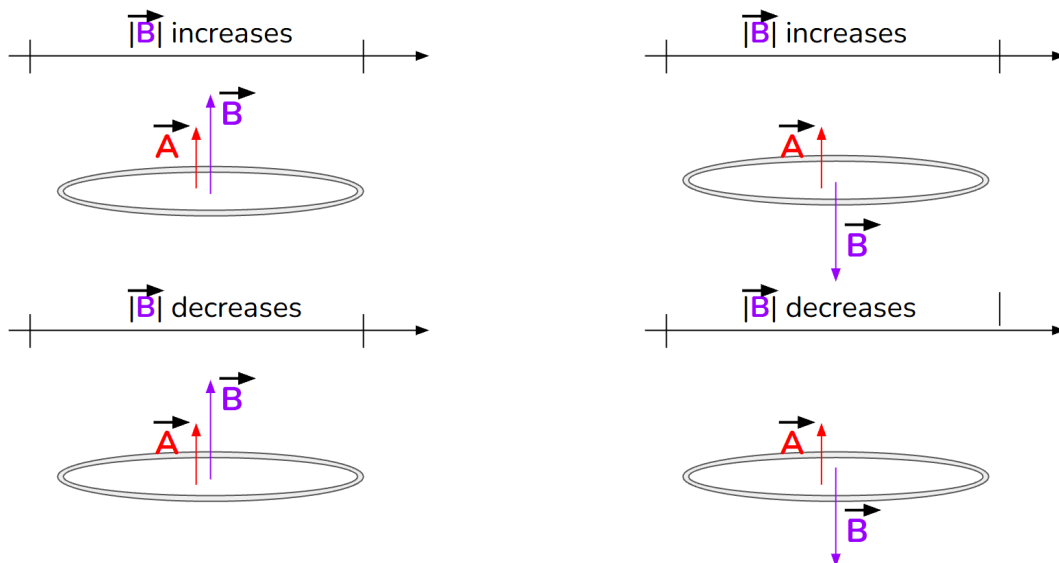
$$\varepsilon = -\frac{d\Phi_B}{dt} \quad (6.3.3)$$

The emf (electromotive force) behaves similarly to a voltage throughout the loop. Although it is not the same as electric potential, it has units of volts and affects currents in the same way. The emf causes an induced current to flow through the loop as though an equivalent electric potential were present.

- When the emf is positive, the induced current points in the same direction as your fingers when your thumb aligns with the area vector.
- When the emf is negative, the induced current points opposite the direction of your fingers when your thumb aligns with the area vector.

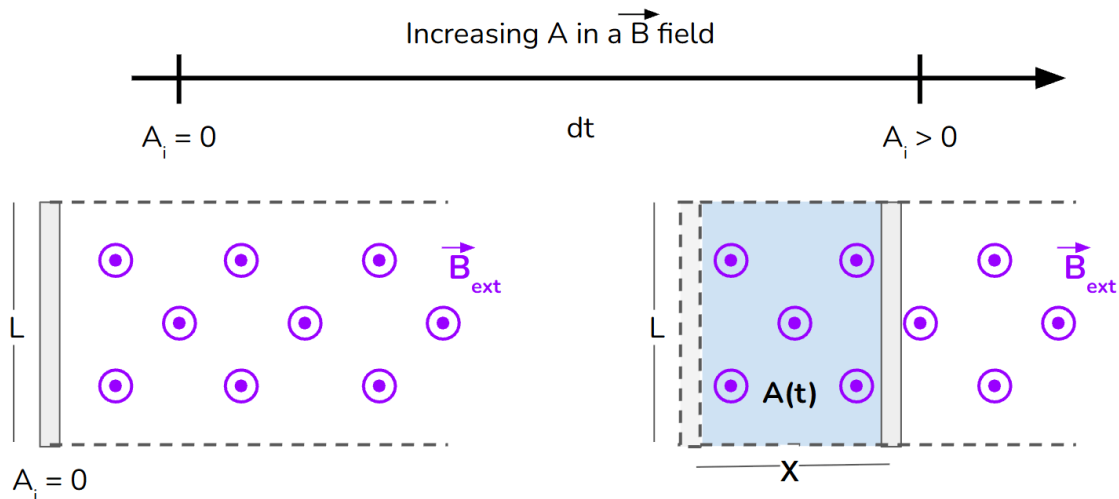
Apply

For each case, identify the sign (+ or −) of the magnetic flux Φ_B and the rate of change of magnetic flux $\frac{d\Phi_B}{dt}$. Then determine the direction of the induced current.



6.3.2 Motional emf

Model



$A =$

$A =$

$\Phi_B =$

$\frac{d\Phi_B}{dt} =$

$\varepsilon =$

Explore

1. What is the angle between the magnetic field and the line normal to the area? How does that affect the dot product $\vec{B} \cdot \vec{A}$?
2. What is the area A in terms of the variables shown in the right portion of the model? Write this expression on the model.
3. Which flux indicator (A , B , or $\cos(\theta)$) changes with time on this timeline? Rewrite the equation for magnetic flux to indicate the time dependence of that quantity.

Critical Thinking & Concept Invention

1. Take the derivative of the magnetic flux and write your result below the model. If needed, move all constants outside the derivative.
2. If a differential quotient appears, explain what that quantity represents and replace it with a single variable.
3. Determine the induced emf ε from this result and record it below the model.
4. Use the Right Hand Rule to determine the direction of the induced current. Draw an arrow on the model representing this current.
5. Based on the current direction, place positive and negative charges on the rod to indicate the polarity of the induced emf. These charges should result from the current rather than cause it.

Apply

1. Determine the direction of the induced current in each loop, or write $I_{\text{induced}} = 0$ if no current is induced. Draw arrows around each loop indicating the current direction.

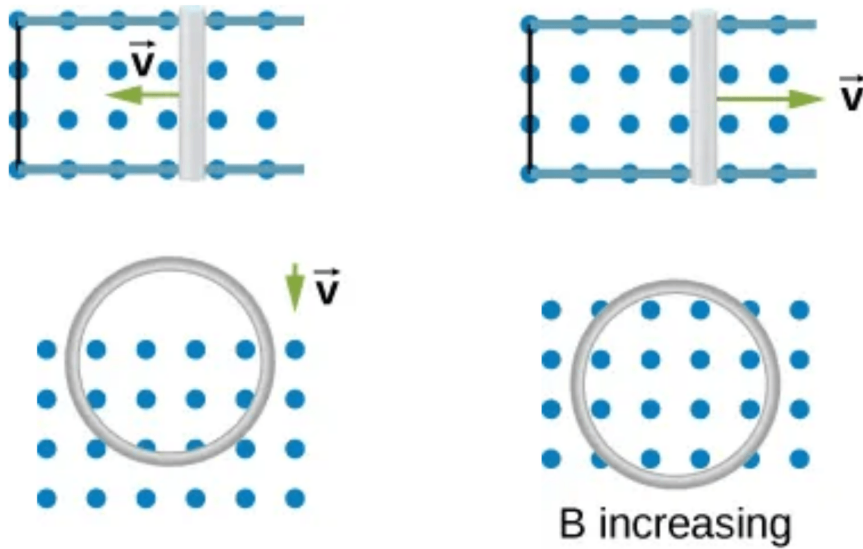


Image Credit: OpenStax University Physics Vol 2 (<https://openstax.org/details/books/university-physics-volume-2>)

1. Determine the emf in the circuit using the given variables.

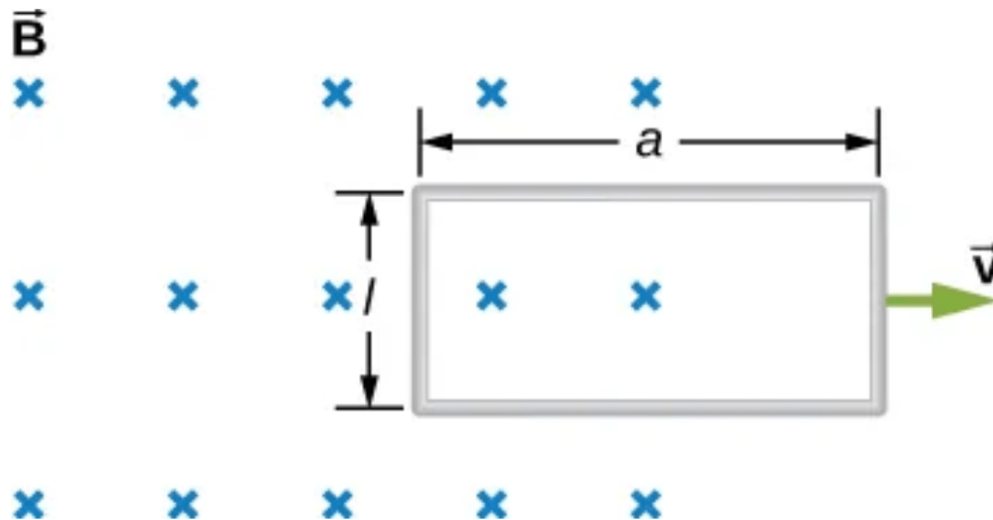
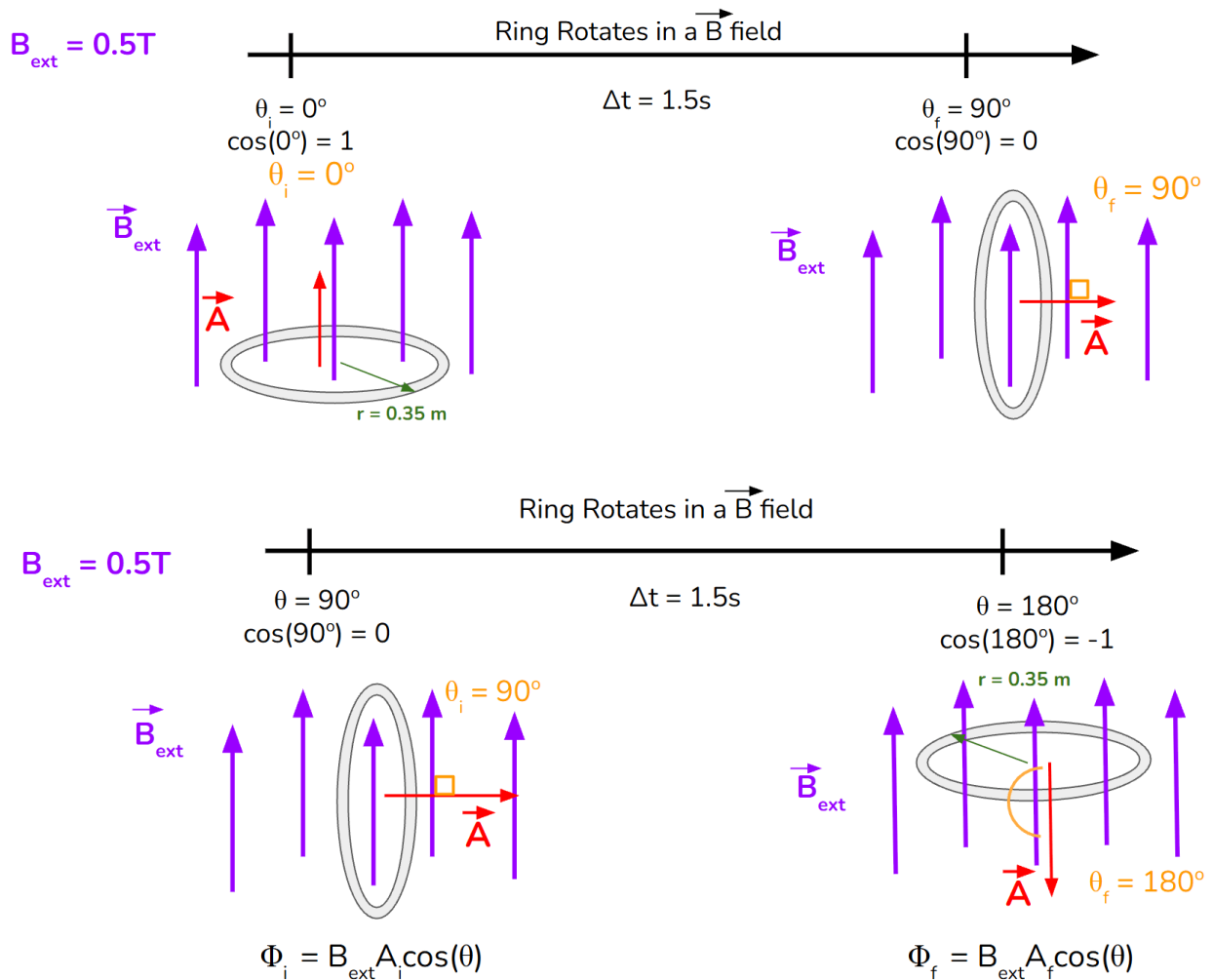


Image Credit: OpenStax University Physics Vol 2 (<https://openstax.org/details/books/university-physics-volume-2>)

6.3.3 Rotational emf

Model



Explore

- Examine the first timeline.
 - Which flux indicator (A , B , or $\cos(\theta)$) changed? Identify its initial and final values.
 - Did that quantity increase or decrease?
 - Which flux indicators remain constant? Identify their values for this situation.
- Examine the second timeline.
 - Which flux indicator (A , B , or $\cos(\theta)$) changed? Identify its initial and final values.
 - Did that quantity increase or decrease?
 - Which flux indicators remain constant? Identify their values for this situation.
- What aspects are the same between the two timelines, and what aspects are different?

Critical Thinking & Concept Invention

$$\Phi_B \quad (6.3.4)$$

$$\varepsilon \quad (6.3.5)$$

1. Do you expect $\frac{d\Phi_B}{dt}$ to be positive or negative on these timelines? Explain your reasoning.
2. Based on your answer above, do you expect the induced emf to be positive or negative? Explain.
3. Magnetic flux is defined as:

$$\Phi_B = BA \cos(\theta) \quad (6.3.6)$$

Take the time derivative of this expression, considering which variables remain constant in these models.

$$\frac{d\Phi_B}{dt} = \quad (6.3.7)$$

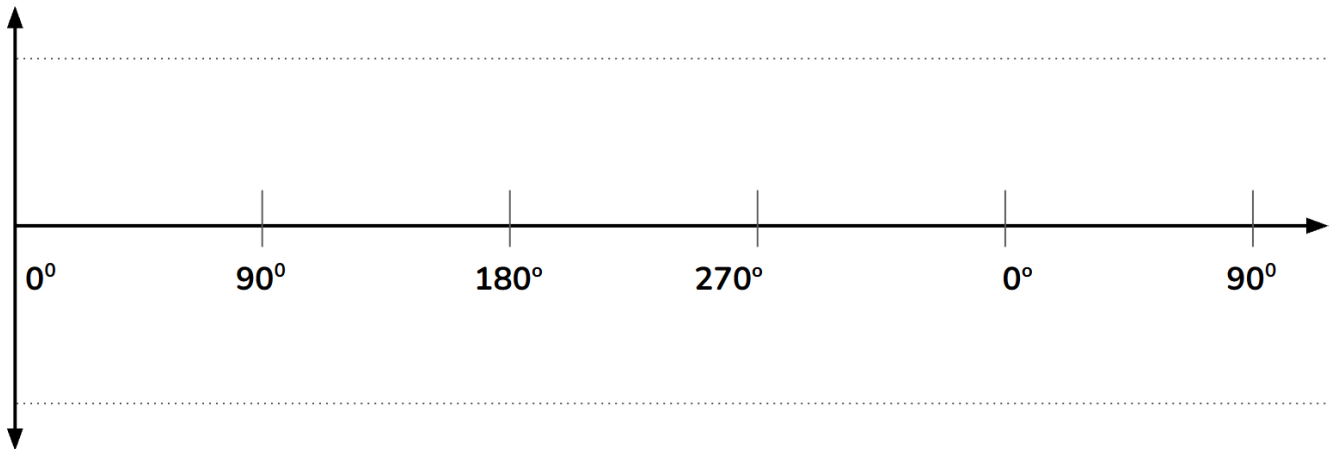
4. During which intervals is this function positive? During which intervals is it negative?
5. Recall:

$$\varepsilon = -\frac{d\Phi_B}{dt} \quad (6.3.8)$$

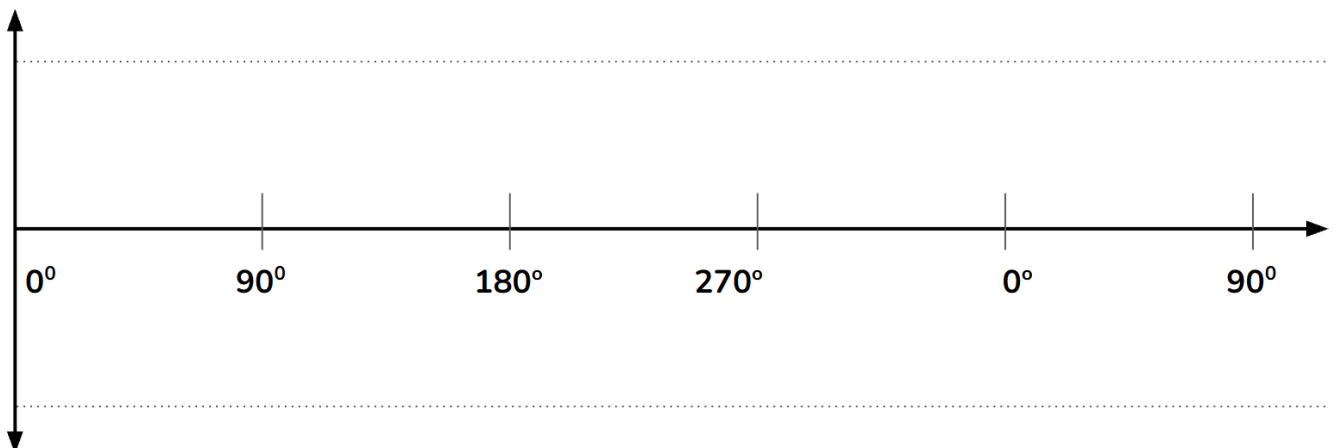
Verify the sign you predicted for the emf on each timeline. Does it agree with your expression for $\frac{d\Phi_B}{dt}$?

6. Graph both Φ_B and ε on the provided templates. Use four tick marks to represent one complete period of the function.

Flux Φ_B



EMF ε



1. If the angular speed ω is constant so that

$$\theta = \omega t, \quad (6.3.9)$$

rewrite the equations for magnetic flux and emf using this substitution.

$$\Phi_B = \quad (6.3.10)$$

$$\varepsilon = \quad (6.3.11)$$

6.4: Inductance and Transformers

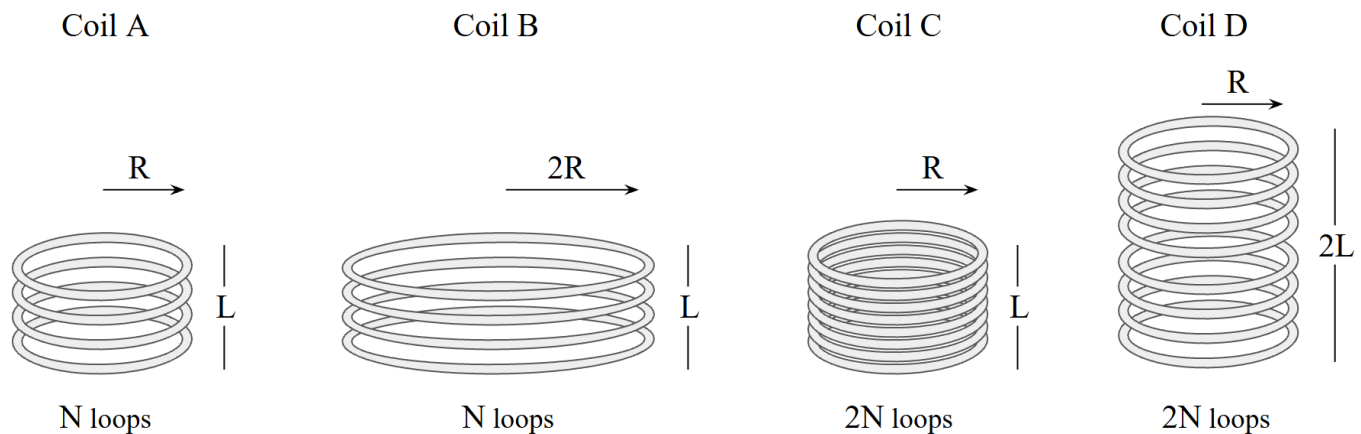
Inductance and Transformers

Learning Objectives

- Determine the self-inductance of a coil and relate emf to current through the concept of self-inductance.
- Determine the mutual inductance of two coils and relate emf to current through the concept of mutual inductance.
- Derive step-up and step-down transformer equations.

6.4.1 Self Inductance

Model



Explore

1. Rank the four coils from greatest to least for the following properties. If any are equal, place an equal sign between them.
 - Length
 - Number of Rings
 - Magnetic Field Strength
 - Total Area
2. Write an expression (and simplify if possible) for the magnetic field created by each coil using the given values or variables.

$$B_A = \quad B_B = \quad B_C = \quad B_D = \quad (6.4.1)$$

Critical Thinking & Concept Invention

1. Rank the magnetic flux through the four coils from greatest to least.
2. Analyze the coils.
 - Write an expression (and simplify if possible) for the magnetic flux through each coil based on the magnetic fields they create.
 - As a group, identify what variable in the magnetic flux expression might vary with time depending on the circuit. Rewrite that variable with explicit time dependence (for example, $V(t)$) and make it the last quantity in the expression.
 - Identify the induced emf in each circuit. Recall:

$$\varepsilon = -\frac{d\Phi}{dt} \quad (6.4.2)$$

- Recall that for inductors:

$$\Delta V_L = -L \frac{dI}{dt} \quad (6.4.3)$$

- Circle the constants in front of $\frac{dI}{dt}$.
- Based on your expression for ε , identify L for each coil.

$$\Phi_A = \quad \Phi_B = \quad \Phi_C = \quad \Phi_D = \quad (6.4.4)$$

$$\Phi_A = \quad \Phi_B = \quad \Phi_C = \quad \Phi_D = \quad (6.4.5)$$

$$\varepsilon_A = \quad \varepsilon_B = \quad \varepsilon_C = \quad \varepsilon_D = \quad (6.4.6)$$

$$L_A = \quad L_B = \quad L_C = \quad L_D = \quad (6.4.7)$$

Summary

Faraday's Law is the fundamental law upon which Kirchhoff's Voltage Rule is based. It explains all voltages and emfs in a circuit.

$$\Gamma_E = -\frac{d\Phi_B}{dt} \quad (6.4.8)$$

$$\oint \vec{E} \cdot d\vec{l} + \frac{d\Phi_B}{dt} = 0 \quad (6.4.9)$$

$$\oint \vec{E} \cdot d\vec{l} = -\sum \Delta V \quad (6.4.10)$$

$$-\sum \Delta V + L \frac{dI}{dt} = 0 \quad (6.4.11)$$

$$\sum \Delta V - L \frac{dI}{dt} = 0 \quad (6.4.12)$$

The $\sum \Delta V$ terms represent changes to electric potential and include voltages from resistors, capacitors, batteries, and other voltage sources. These voltages arise from electric fields created by charges.

Previously, the relationship

$$\Delta V_L = -L \frac{dI}{dt} \quad (6.4.13)$$

was introduced. This quantity is not actually electric potential because it is produced by changing magnetic fields rather than charges, but it behaves similarly in circuit analysis. The "voltage" associated with an inductor is identified through Faraday's Law.

Apply

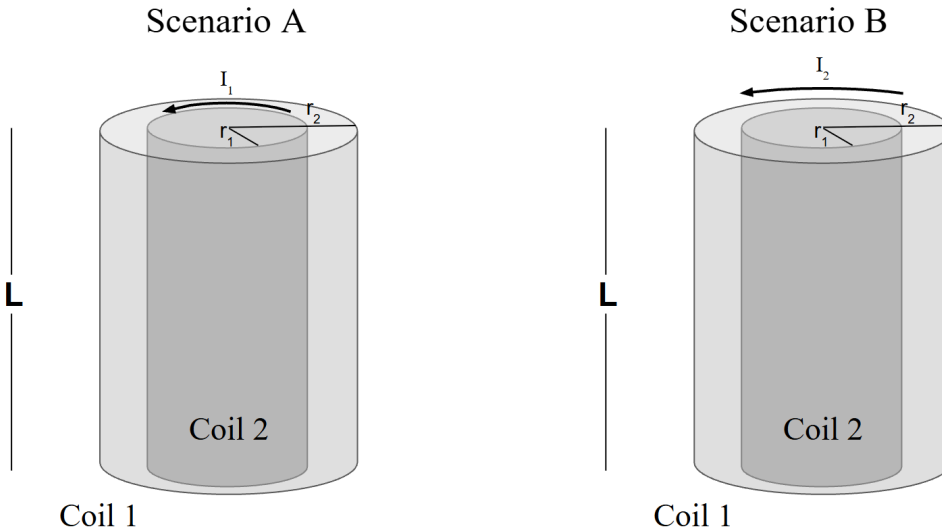
A solenoid 50 cm long is wound with 500 turns of wire. The cross-sectional area of the coil is 2.0 cm².

- Determine the self-inductance of the solenoid.
- Draw a picture and label the quantities.

6.4.2 Mutual Inductance

Model

Two coils are aligned on their common axis ("coaxial cable"). Each coil has the same total number of loops. Scenarios A and B have the same current, but differ in which coil carries the current.



Explore

- Compare the two coils using $>$, $<$, or $=$.
 - A_1 --- A_2
 - L_1 --- L_2
 - n_1 --- n_2
- Compare the magnetic field and flux for each scenario.

$$B_1 \quad B_2 \quad (6.4.14)$$

- Compare the flux in each solenoid for Scenario A.

$$\Phi_1 \quad \Phi_2 \quad (6.4.15)$$

- Compare the flux in each solenoid for Scenario B.

$$\Phi_1 \quad \Phi_2 \quad (6.4.16)$$

Critical Thinking & Concept Invention

- Compare the effects of one coil on another.
 - Identify the flux in each coil caused by the magnetic field of the other coil.
 - Rewrite that expression as a relationship to the current in the other coil.
 - When two coils affect each other, their relationship is $\Phi_1 = M_1 I_2(t)$. Identify M_1 and M_2 .
 - Compare M_1 and M_2 using $>$, $<$, or $=$.
 - Calculate the emf induced in each coil by the other coil. Express your answers in terms of M . If variables are identical, subscripts may be omitted.

$$\Phi_1 = \quad \Phi_2 = \quad (6.4.17)$$

$$\Phi_1 = I_2(t) \quad \Phi_2 = I_1(t) \quad (6.4.18)$$

$$M_1 = \quad M_2 = \quad (6.4.19)$$

$$M_1 \quad M_2 \quad (6.4.20)$$

$$\varepsilon_1 = \quad \varepsilon_2 = \quad (6.4.21)$$

Apply

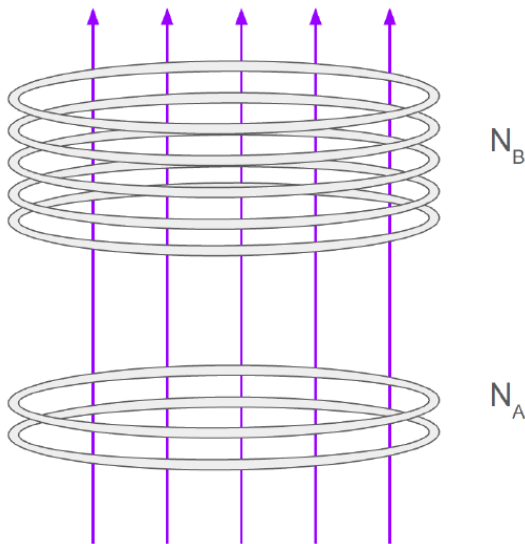
A solenoid with N_1 turns has length L_1 and radius R_1 , while a second smaller solenoid with N_2 turns has length L_2 and radius R_2 . The smaller solenoid is placed completely inside the larger solenoid so that their long axes coincide.

- Draw a picture and label the quantities.
- Determine the mutual inductance of the two solenoids.

6.4.3 Transformers

Model

Two solenoids with different loop densities are shown. Both have the same length and are assumed to be close enough (or interwoven) to neglect edge effects. The lower solenoid (A) carries current I_A , producing the magnetic field shown. As illustrated, $N_A < N_B$, although the drawing is not necessarily to scale.



Explore

1. Compare the magnetic field in each.

$$B_A = B_B \quad \text{OR} \quad > \quad (6.4.22)$$

2. Compare the magnetic flux per ring in each.

$$\frac{\Phi_1}{N_A} = \frac{\Phi_2}{N_B} \quad \text{OR} \quad > \quad (6.4.23)$$

3. Compare the total magnetic flux through each.

$$\Phi_A = \Phi_B \quad \text{OR} \quad > \quad (6.4.24)$$

Critical Thinking & Concept Invention

1. Compare quantities for each solenoid. Write your derivation in the space provided.
 - Construct an equation relating the total magnetic flux in each solenoid.
 - Take the time derivative of that relationship, then compare the emf in each.
 - Identify the circumstances under which $\varepsilon_A > \varepsilon_B$.
 - Identify the circumstances under which $\varepsilon_A < \varepsilon_B$.

Apply

A transformer is made from a solenoidal coil with 26 turns wound tightly around another coil with 330 turns. The inner solenoid is 20.0 cm long and has a diameter of 2.5 cm.

- Determine the mutual inductance of this transformer.
- If the 330-turn solenoid has a voltage peak of 120 V, determine the induced emf in the other coil.
- Determine the required rate of change of current, $\frac{dI}{dt}$, in the 330-turn solenoid that would create these voltages.

6.5: Electric Motors, Displacement Current, and EM Waves

Electric Motors, Displacement Current, and EM Waves

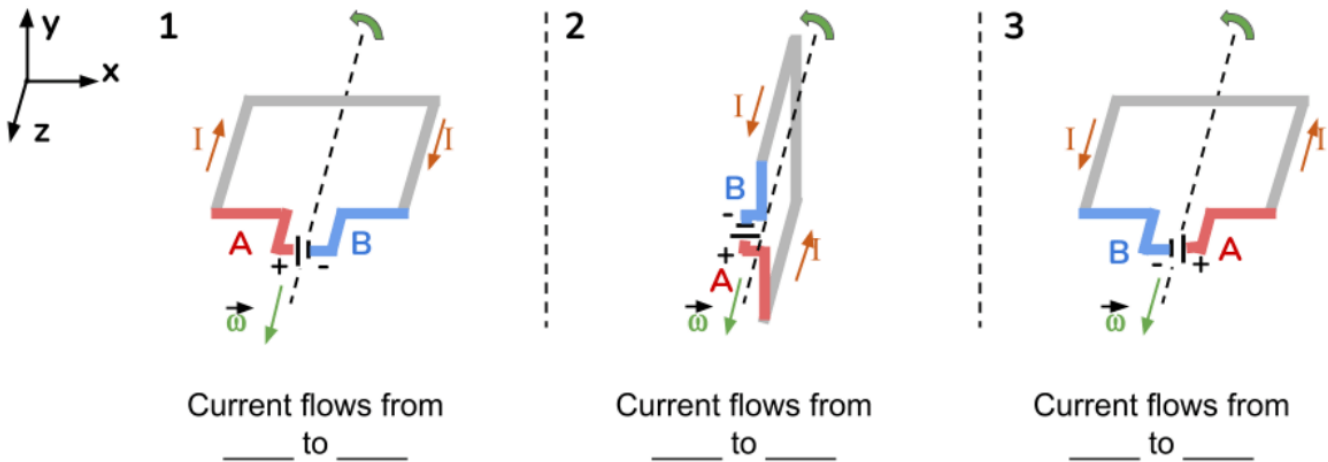
Learning Objectives

- Determine the torque from an external magnetic field on a current loop.
- Identify magnetic dipole moment and how it relates to torque and potential energy.
- Model how electric motors convert electrical energy to rotational energy.

6.5.1 Rotating Current Loop in Magnetic Field

Model

Current runs through a square loop that is located between magnets that produce a constant magnetic field directed to the right.



Explore

- Identify the direction (A to B or B to A) that current flows in each panel of this rotation.
- Draw the magnetic moment $\vec{\mu}$ on each panel using the Right Hand Rule.
- Identify the coordinate direction ($+/ - \hat{x}$, $+/ - \hat{y}$, or $+/ - \hat{z}$) of:

	1	2	3
$\vec{\mu}$			
$\vec{B}$			
$\vec{\omega}$			

Critical Thinking & Concept Invention

- Recall that torque is the cross product of the magnetic moment and the magnetic field:

$$\vec{\tau} = \vec{\mu} \times \vec{B} \quad (6.5.1)$$

Draw the torque vector on each panel. If the torque is zero, write:

$$\vec{\tau} = 0 \quad (6.5.2)$$

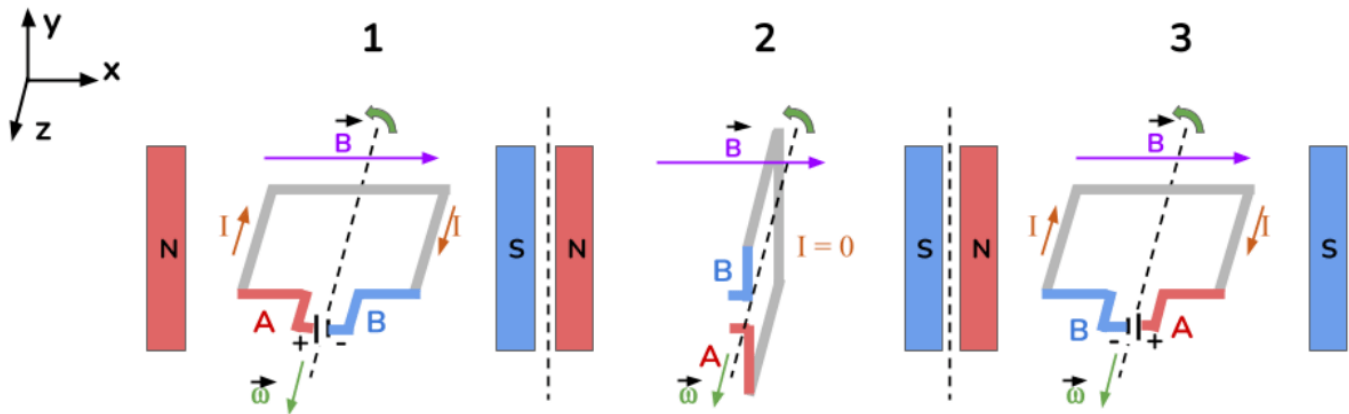
2. Identify the coordinate direction (+/- $\hat{x}$, +/- $\hat{y}$, or +/- $\hat{z}$) of:

	1	2	3
$\vec{\tau}$			
$\frac{d\vec{\omega}}{dt}$			

3. Identify whether the loop's rotation is speeding up, slowing down, or neither at each moment. Explain your reasoning.
4. Is this an effective motor? Explain why or why not.

6.5.2 Commutator

Model



Explore

- Identify the direction (A to B or B to A) that current flows in each panel. If there is no current, write "No Current."
- Draw the magnetic moment $\vec{\mu}$ on each panel using the Right Hand Rule.
- Explain what is different about this scenario from the previous one.

Critical Thinking & Concept Invention

1. Recall:

$$\vec{\tau} = \vec{\mu} \times \vec{B} \quad (6.5.3)$$

Draw the torque vector on each panel. If the torque is zero, write:

$$\vec{\tau} = 0 \quad (6.5.4)$$

2. Identify the coordinate direction (+/- $\hat{x}$, +/- $\hat{y}$, or +/- $\hat{z}$) of:

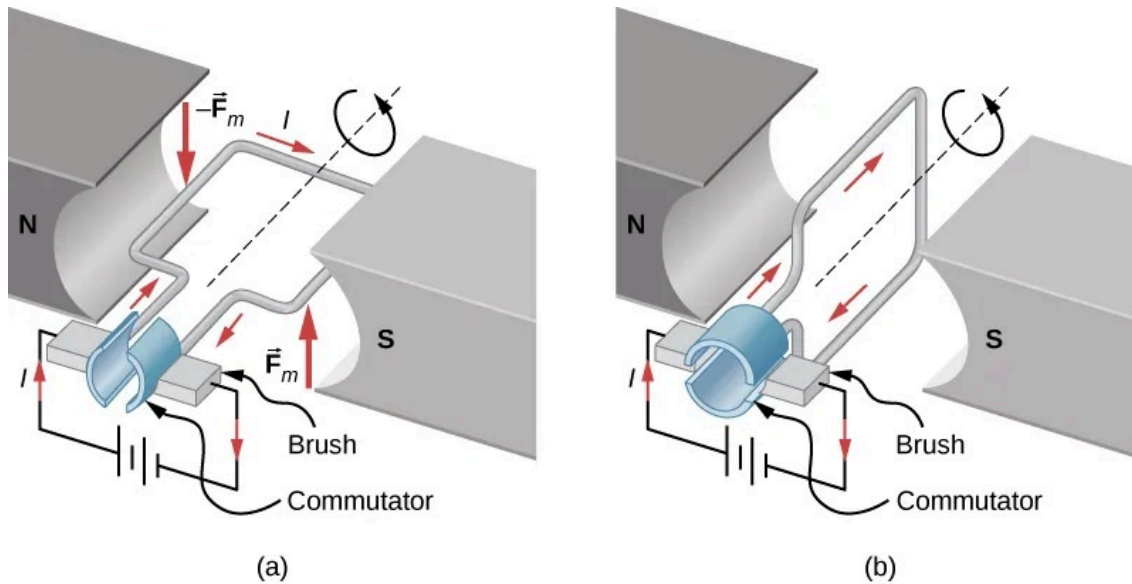
	1	2	3
--	---	---	---

$\vec{\tau}$			
$\frac{d\vec{\omega}}{dt}$			

- Identify whether the loop's rotation is speeding up, slowing down, or neither at each moment. Explain your reasoning.
- Is this an effective motor? Explain why or why not.
- Explain what must happen in a motor to continuously increase the rotation rate.

Summary

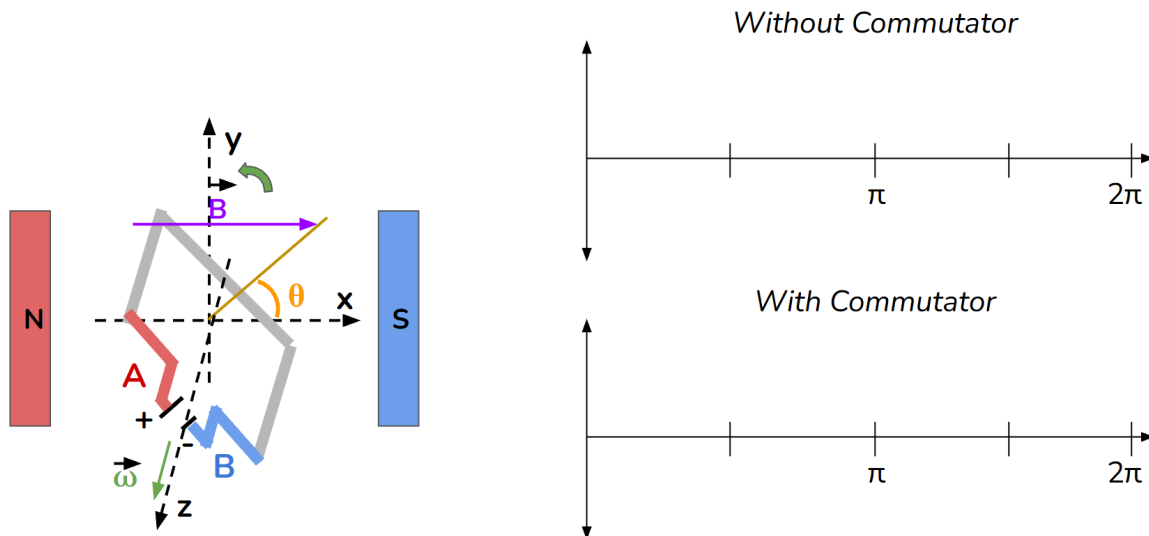
In a DC motor, the current switches direction through the circuit every half-rotation. This flips the magnetic moment vector and ensures that the torque remains in the same direction as the angular velocity, continuously increasing the rotational speed.



Apply

A square loop with current I and side length L rotates in a constant magnetic field of strength B . At a particular instant, the loop has an area vector in the x - y plane at an angle θ with respect to the x -axis.

- Identify the torque vector at this instant.
- Plot the torque on the loop as a function of θ from $\theta = 0$ to $\theta = \pi$, both with and without a commutator.



6.5.3 Displacement Current

Model

The model shows a capacitor with current flowing into and out of the plates. Two Ampere surfaces share the same closed boundary C . The model on the right is incomplete.

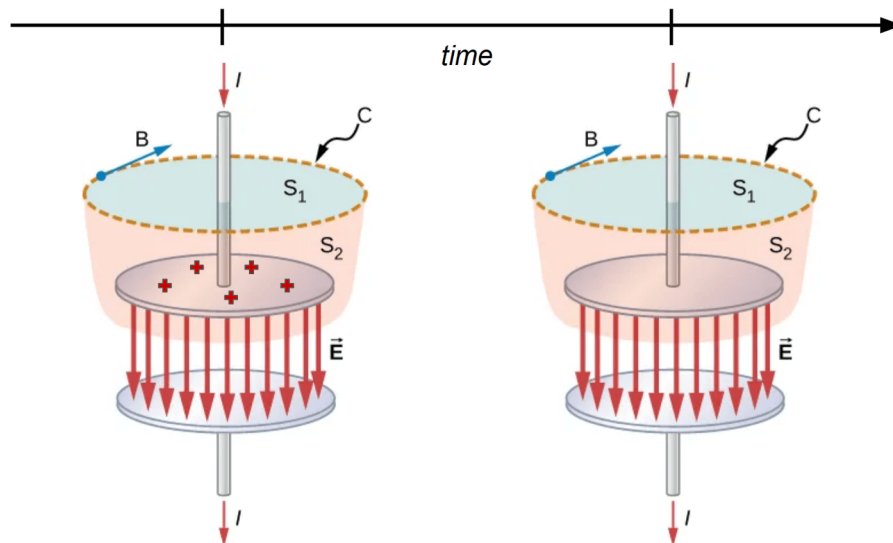


Image Credit: Modification of figure from OpenStax University Physics Vol 2 (<https://openstax.org/details/books/university-physics-volume-2>)

Explore

- Describe each shape.
- Identify the enclosed current through each surface.
 - $I_1 =$

- $I_2 =$
- 3. Identify the electric flux through each surface.
 - $\Phi_{E1} =$
 - $\Phi_{E2} =$
- 4. If current flows as shown, will there be more or fewer positive charges on the top plate over time? Draw charges accordingly.
- 5. Based on your answer, describe how the electric field magnitude and electric flux change:
 - The electric field and electric flux through S_1 will...
 - The electric field and electric flux through S_2 will...

Critical Thinking & Concept Invention

1. Ampère's Law states that magnetic circulation depends on enclosed current. Which surface suggests that the magnetic field should be zero along curve C ?
2. Using the other surface, determine the magnetic field along curve C :

$$\Gamma_B = \mu_0 I_{\text{enc}} \quad (6.5.5)$$

3. For the surface with zero enclosed current, which quantity was changing: Γ or Φ ? Include the appropriate electric or magnetic subscript.
4. Maxwell proposed a new magnetic circulation equation.
 - Identify the displacement current I_d .
 - Write the complete magnetic circulation equation.
 - Verify that it works for both surfaces by identifying I_{enc} and I_d for each.

$$\Gamma_B = \mu_0 (I_{\text{enc}} + I_d) \quad (6.5.6)$$

$$I_d = \quad (6.5.7)$$

$$\Gamma_B = \quad (6.5.8)$$

Summary

The equation developed above is known as the Maxwell–Ampère Law, and the displacement current is Maxwell's correction to Ampère's Law.

$$I_d \equiv \quad (6.5.9)$$

$$\Gamma_B = \mu_0 (I_{\text{enc}} + I_d) \quad (6.5.10)$$

$$\Gamma_B = \quad (6.5.11)$$

Displacement current is not an actual flow of charge. Instead, it is a changing electric flux that produces the same magnetic field that an ordinary current would produce. The electric constant ϵ_0 is included and often cancels a corresponding factor appearing in electric flux.

Apply

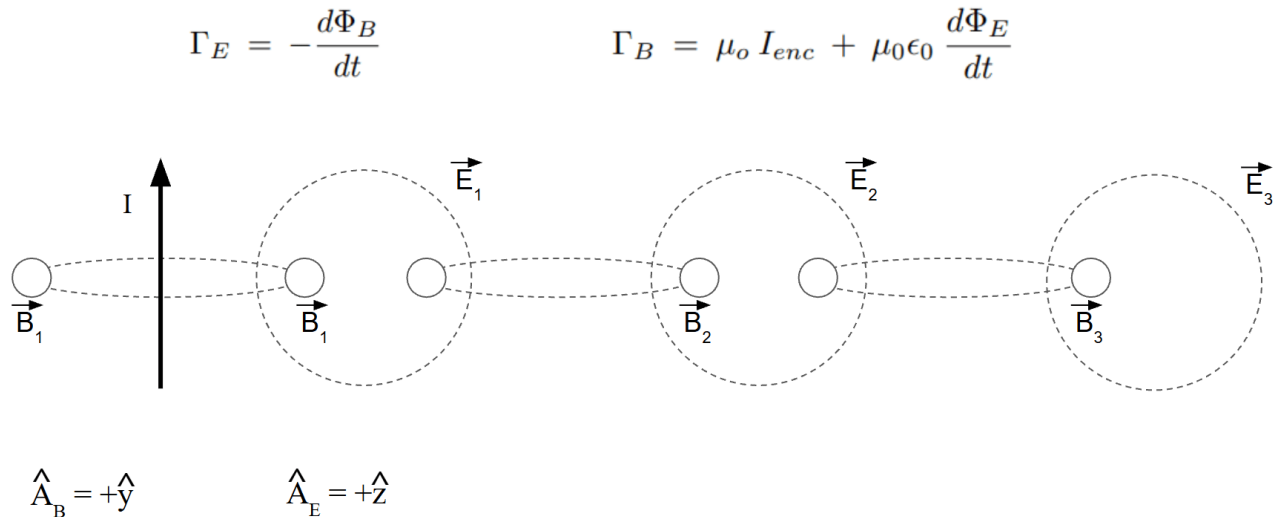
The potential difference $V(t)$ between parallel plates increases at a rate of 10^7 V/s. The plates are separated by 1.00 cm and have an area of 0.200 m^2 .

- Determine the electric field as a function of time, assuming very large plates.
- Determine the electric flux as a function of time.
- Determine the displacement current.

6.5.4 Electromagnetic Waves

Model

A current varies periodically with time, such as: $I(t) = I_0 \sin(\omega t)$. At the instant shown, the current is directed upward and increasing.



Explore

1. Identify the direction of the magnetic field created by the current by filling in the into-page or out-of-page symbols on the Ampere loop.
2. A circle labeled $\vec{E}$ intersects one magnetic field point. Determine:
 - Does the magnetic flux point into or out of the page?
 - Is the magnetic flux increasing or decreasing?
3. Determine the direction of the electric field circulation around the circle and draw arrows.

Critical Thinking & Concept Invention

1. Verify that all annotations on the left diagram are consistent.
2. Copy the electric field circulation onto the leftmost electric field circle on the right diagram.
3. Determine the direction of electric flux through the Ampere loop intersected by the electric field.
4. Assume the curling electric field is increasing. Draw the displacement current and the magnetic field it creates.
5. Assume the new magnetic field is decreasing. Draw the electric field circulation produced by the changing magnetic flux.
6. Assume the new electric field is decreasing. Draw the magnetic field circulation produced by the changing electric flux.
7. Summarize each step and explain how a changing current creates changing magnetic and electric fields at different points in space.

Summary

Gauss's Law and Ampère's Law show that electric and magnetic fields originate primarily from charges and currents. Faraday's Law and Maxwell's correction to Ampère's Law demonstrate that changing electric and magnetic fluxes can also generate fields. This allows changes in one field to create changes in the other indefinitely. The graphic below is for a field where the flux was always increasing:

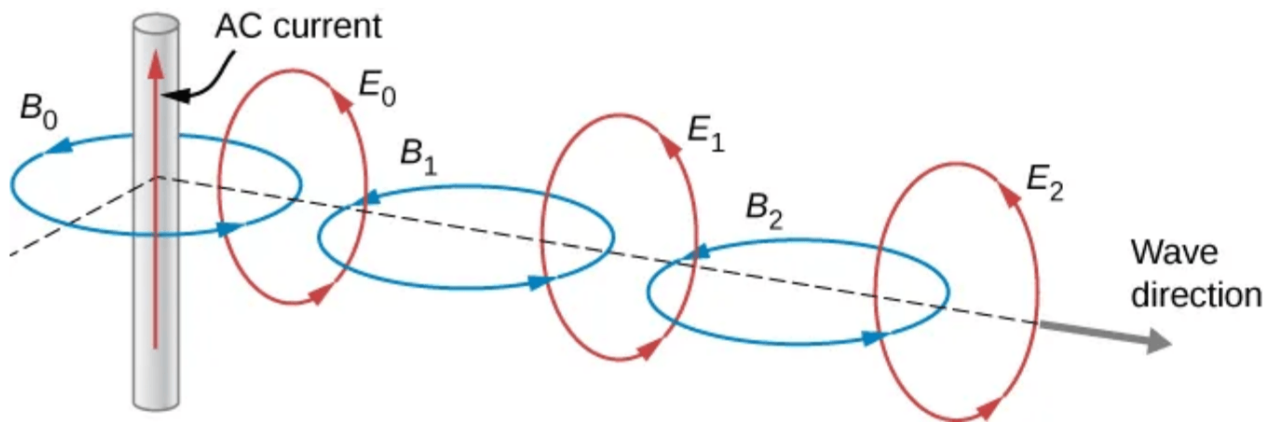
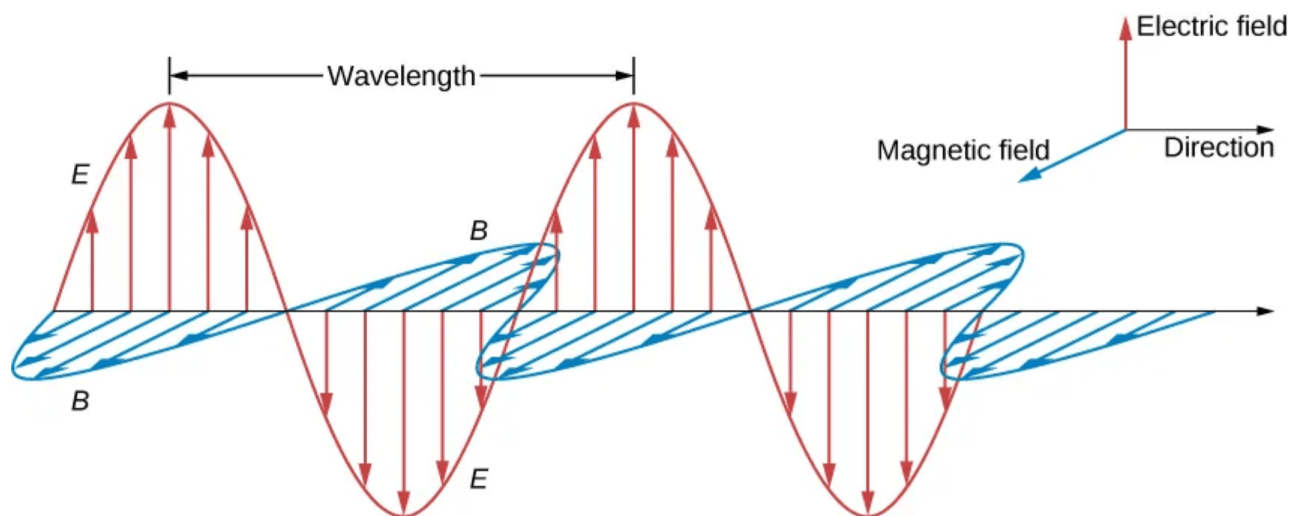


Image Credit: OpenStax University Physics Vol 2 (<https://openstax.org/details/books/university-physics-volume-2>)

Light is an electromagnetic wave. A periodic source changes one field, which changes its associated flux, producing the other field. This sequence propagates away from the source. The electric and magnetic fields remain offset by 90° because each is generated by changes in the other.



Solving **Maxwell's Wquations** gives:

$$\frac{\partial^2 E}{\partial x^2} = \epsilon_0 \mu_0 \frac{\partial^2 E}{\partial t^2} \quad (6.5.12)$$

$$\frac{\partial^2 B}{\partial z^2} = \epsilon_0 \mu_0 \frac{\partial^2 B}{\partial t^2} \quad (6.5.13)$$

Equations of this form (a partial second spatial derivative = a partial second time derivative) are wave equations. The coefficient equals the inverse square of the propagation speed (wave speed):

$$\epsilon_0 \mu_0 = \frac{1}{v^2} \quad (6.5.14)$$

$$v = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \quad (6.5.15)$$

Because both ϵ_0 and μ_0 are constants in vacuum, the speed of light is also constant:

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = 3 \times 10^8 \text{ m/s} \quad (6.5.16)$$

In other materials, the speed changes because ϵ and μ differ from their vacuum values.

7: COULOMB AND BIOT-SAVART APPROACH

7.1: Coulomb Fields from Point Charges

7.2: Coulomb Fields from Charge Distributions

7.3: Coulomb Potential

7.4: Biot-Savart Law

7.5: Electric Dipole Fields

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7.1: Coulomb Fields from Point Charges

Coulomb Fields from Point Charges

Learning Objectives

- Determine electric field vectors and force vectors from point charges using Coulomb's Law in vector form.
- Use the concept of superposition to determine the total electric field at a point from two or more electric charges.
- Use the concept of a separation vector to determine a vector Coulomb field when the source charge is not at the coordinate origin.

Review

Using Gauss's Law, we previously found an equation for the electric field from a point charge:

$$E_{\text{point charge}} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \quad (7.1.1)$$

and

$$\vec{E}_{\text{point charge}} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^3} \vec{r} \quad (7.1.2)$$

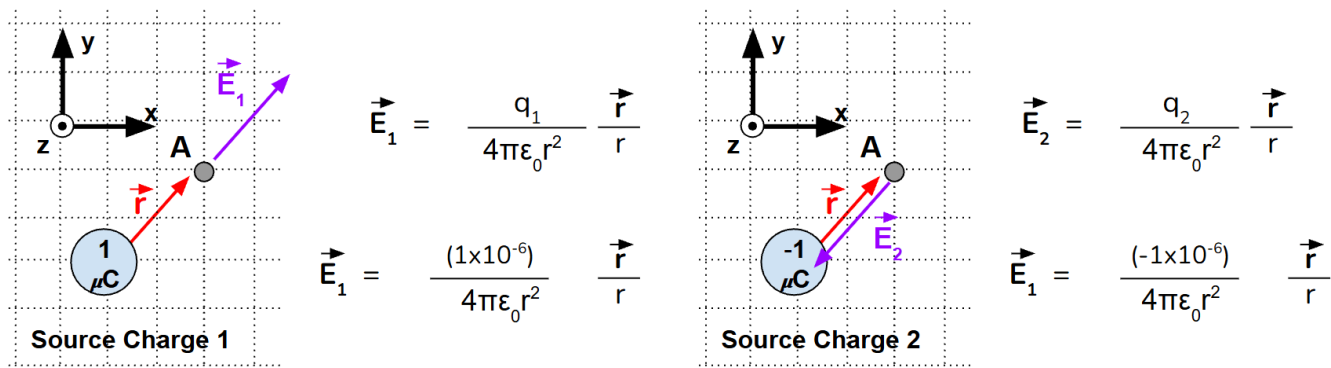
The quantity

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9, \quad (7.1.3)$$

but the variables are often left in symbolic form instead of substituting numerical values.

7.1.1 Electric Field Vectors

Model



Grid spacing are 1 mm (10^{-3}m)

Explore

- For each charge, does the separation vector $\vec{r}$ point from the source charge to the point or from the point to the source charge?
 - For the positive source charge, $\vec{r}$ points _____.
 - For the negative source charge, $\vec{r}$ points _____.
- For each source charge, does the electric field vector $\vec{E}$ point in the same direction as $\vec{r}$ or in the opposite direction?
 - For the positive source charge, $\vec{E}$ points _____.
 - For the negative source charge, $\vec{E}$ points _____.
- What is the only difference in the equation for each electric field vector?

Critical Thinking & Concept Invention

- How does the type (positive or negative) of source charge affect the direction of the electric field it creates?
 - The electric field points toward / away from positive charges.
 - The electric field points toward / away from negative charges.
- Use the equation for the electric field vector to explain your response above. Identify which part(s) of the equation determine the direction.

Summary

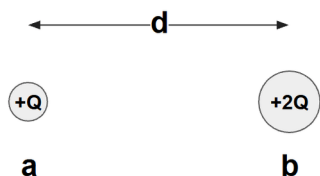
The electric field from a point charge is often called Coulomb's Law:

$$\vec{E}_{\text{point charge}} = \frac{Q}{4\pi\epsilon_0 r^3} \vec{r} \quad (7.1.4)$$

The vector $\vec{r}$ points from the source charge to the point where the electric field is evaluated. If the source charge is positive, the electric field points radially outward. If the source charge is negative, the negative sign reverses the direction so that the electric field points radially inward.

7.1.2 Force Between Charges

Model



$$\vec{F}_{b \text{ on } a} = q_a \vec{E}_b \quad \vec{F}_{a \text{ on } b} = q_b \vec{E}_a$$

$$\vec{E}_a = \frac{q_a}{4\pi\epsilon_0 r^3} \vec{r}_a$$

$$\vec{E}_b = \frac{q_b}{4\pi\epsilon_0 r^3} \vec{r}_b$$

$$\vec{E}_a = \frac{(\quad)}{4\pi\epsilon_0 (\quad)^3} \begin{pmatrix} \\ \\ \end{pmatrix}$$

$$\vec{E}_b = \frac{(\quad)}{4\pi\epsilon_0 (\quad)^3} \begin{pmatrix} \\ \\ \end{pmatrix}$$

Explore

- When identifying the force of charge b on charge a :
 - The charge comes from ____.
 - The field comes from ____.
- When identifying the force of charge a on charge b :
 - The charge comes from ____.
 - The field comes from ____.
- When identifying electric fields:
 - The field created by a uses the charge from ____.
 - The field created by b uses the charge from ____.

Critical Thinking & Concept Invention

- Complete the electric field vector equations in the model:
 - Fill in the appropriate source charge.
 - Use the correct scalar distance.
 - Construct the column vector from source to observation point.
 - Simplify into unit-vector notation.
- Use the fields to determine the forces:

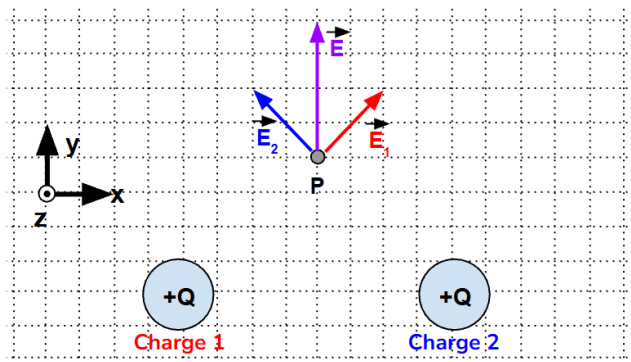
$$\vec{F}_{a \text{ on } b} = q_b \vec{E}_a \quad (7.1.5)$$

$$\vec{F}_{b \text{ on } a} = q_a \vec{E}_b \quad (7.1.6)$$

- Compare the two forces. What is the same and what is different?
- How can the forces be the same if the charges are different?

7.1.3 Electric Field Superposition

Model



$$\begin{matrix} \vec{E}_1 & + & \vec{E}_2 & = & \vec{E} \\ \text{red arrow} & & \text{blue arrow} & & \text{purple arrow} \end{matrix}$$

$$\begin{bmatrix} E_{1,x} \\ E_{1,y} \end{bmatrix} + \begin{bmatrix} E_{2,x} \\ E_{2,y} \end{bmatrix} = \begin{bmatrix} E_x \\ E_y \end{bmatrix}$$

Explore

- What direction is the field from each charge?
 - The field from Charge 1 points ____.
 - The field from Charge 2 points ____.
 - The total electric field points ____.

2. How does the total electric field relate to the individual fields?

$$\vec{E} = \quad (7.1.7)$$

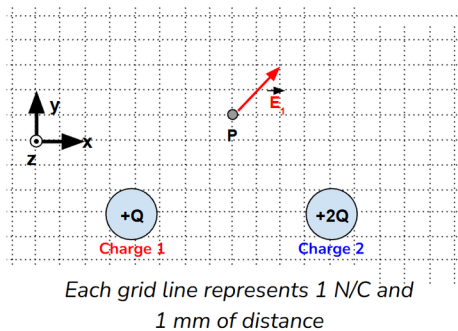
- Graphically, determine $\vec{E}$ by _____.
- Using column vectors, determine $\vec{E}$ by _____.

Critical Thinking & Concept Invention

1. How do you determine the electric field when multiple charges are present?
2. How can the total field point in a different direction from any individual field?
3. Why must vectors be used when determining the total electric field?

Apply

Complete the template to calculate the total electric field at Point P . Your answer may be expressed in terms of Q , π , and ϵ_0 . Use the provided scaling for $\vec{E}_1$ to draw $\vec{E}_2$ and illustrate the vector addition graphically.



$$\vec{E}_1 + \vec{E}_2 = \vec{E}$$

Graphical
Vector
Addition

$$\vec{E}_1 = \frac{(\quad)}{4\pi\epsilon_0(\quad)^3} \begin{pmatrix} \quad \\ \quad \end{pmatrix}$$

$$\vec{E}_2 = \frac{(\quad)}{4\pi\epsilon_0(\quad)^3} \begin{pmatrix} \quad \\ \quad \end{pmatrix}$$

$$\begin{matrix} \vec{E}_1 & + & \vec{E}_2 & = & \vec{E} \\ \begin{pmatrix} \quad \\ \quad \end{pmatrix} & + & \begin{pmatrix} \quad \\ \quad \end{pmatrix} & = & \begin{pmatrix} \quad \\ \quad \end{pmatrix} \end{matrix} \quad \frac{\text{N}}{\text{C}}$$

Column
Vector
Addition

Summary

The total electric field is the vector sum of the fields created by each individual charge. This is known as the principle of superposition.

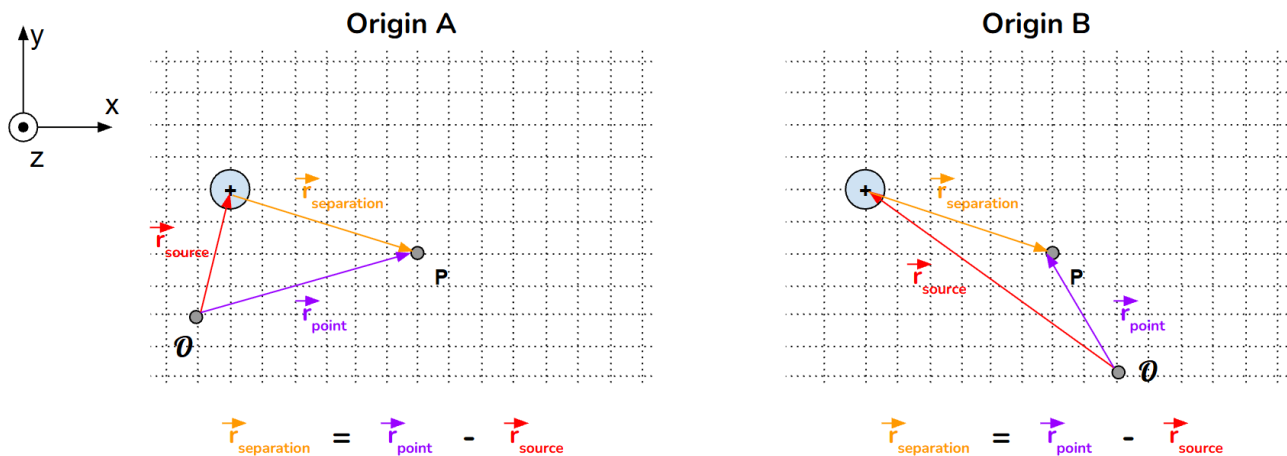
$$\vec{E} = \sum_i \vec{E}_i \quad (7.1.8)$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \cdots \quad (7.1.9)$$

Because electric fields have direction, vector addition must be used when determining the total field.

7.1.4 Separation Vectors

Model



Explore

1. Which vector or vectors differ for Origin A and Origin B?
2. Which vector or vectors remain the same for both origins?
3. Is the equation relating the three vectors affected by the choice of origin?

Critical Thinking & Concept Invention

1. Since the choice of origin does not affect the electric field vector $\vec{E}$, which vector should be used in the Coulomb field equation?
2. Suppose a charge is placed on a grid without an origin provided. List the steps needed to determine the separation vector used to calculate the electric field at a specified point.
3. If multiple charges contribute to the electric field:
 - Should the separation vector be the same or different for each charge?
 - Should the coordinate origin remain the same or change? Explain.

Summary

It is often convenient to choose a coordinate origin that differs from the source charge location. The vector used in Coulomb's Law is always the separation vector that points from the source charge to the observation point.

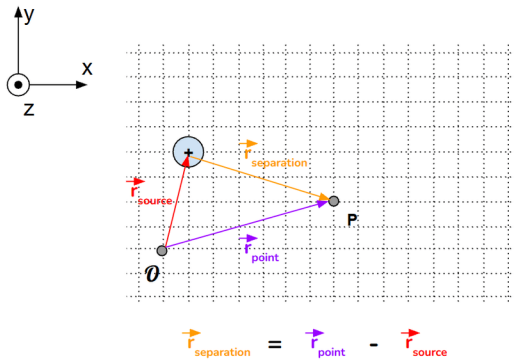
This concept becomes especially important when evaluating electric fields from many charges or continuous charge distributions, where a common coordinate system is maintained while each source has its own separation vector.

Apply

The positive charge shown has charge

$$q = 2 \mu\text{C}. \quad (7.1.10)$$

The spacing between grid lines is 1 mm. Calculate the separation vector from the indicated origin and determine the electric field vector at Point P.



$$\vec{r}_{\text{separation}} = \vec{r}_{\text{point}} - \vec{r}_{\text{source}}$$

$$\begin{pmatrix} \\ \\ \end{pmatrix} = \begin{pmatrix} \\ \\ \end{pmatrix} - \begin{pmatrix} \\ \\ \end{pmatrix}$$

$\vec{r}_{\text{separation}} =$ _____

$$\vec{E} = \frac{q}{4\pi\epsilon_0 r^3} \vec{r} = \frac{()}{4\pi\epsilon_0 ()^3} \begin{pmatrix} \\ \\ \end{pmatrix}$$

$$\vec{E} = \begin{pmatrix} \\ \\ \end{pmatrix} \frac{N}{C}$$

7.2: Coulomb Fields from Charge Distributions

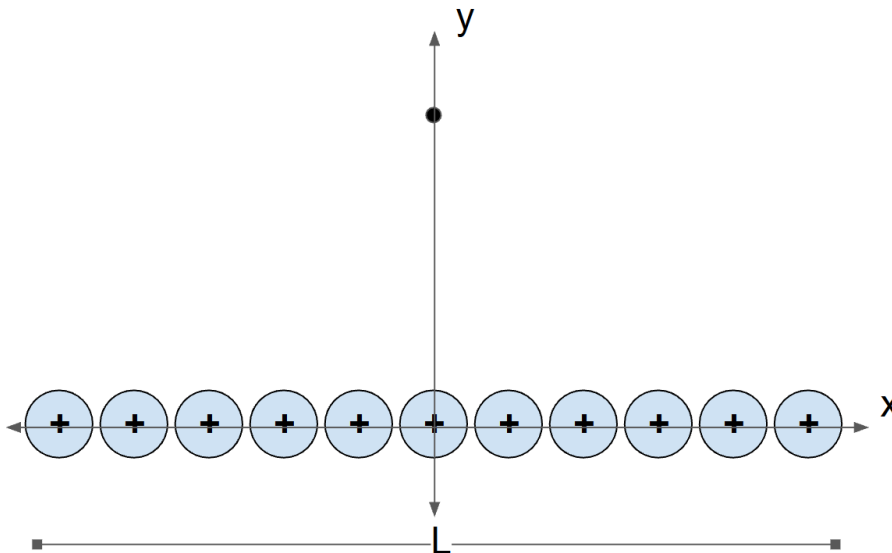
Coulomb Fields from Charge Distributions

Learning Objectives

- Adapt the **Coulomb Approach** to discrete charge distributions using summation notation.
- Characterize differential charges (dq) in terms of geometric distributions in one, two, and three dimensions.
- Use the **Coulomb Approach** in integral form to determine the electric field from continuous charge distributions using single integrals.

7.2.1 Discrete Charge Distributions

Model



Explore

1. How many charges are shown? If they all have the same charge and the total charge of the line is Q , what would the charge of each circle (ΔQ) be?
2. Label the charges in some way (for example, assign numbers or letters).
3. Draw a separation vector from each charge to the dot on the y-axis that represents the point of interest. Give each vector the same label as its corresponding charge.

Critical Thinking & Concept Invention

1. Without calculating, discuss how you would find the total electric field from the distribution.
2. Attempt to write this method using summation notation:

$$\vec{E} = \sum \quad (7.2.1)$$

3. We often take limits to turn Δ 's into d 's and summations into integrals. Try this with your equation:

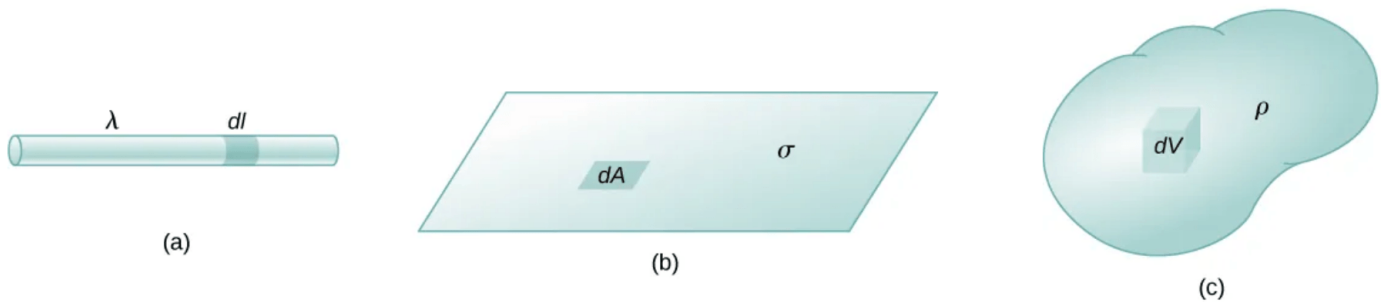
$$\vec{E} = \int \quad (7.2.2)$$

Summary

When dealing with continuous charge distributions, we break the shape into differential charge elements with charge dq that can be treated as point charges. We then add all of their electric fields using integration.

This process requires identifying the differential charge dq as a function of spatial variables and often determining both the separation vector $\vec{r}$ and its magnitude r as functions of the integration variable.

- One-dimensional shapes: $dq = \lambda \, dl$
- Two-dimensional shapes: $dq = \sigma \, dA$
- Three-dimensional shapes: $dq = \rho \, dV$



The spatial quantities dl , dA , and dV must also be related to the appropriate coordinates such as x , y , z , r , or θ .

Any charge distribution may be viewed as a collection of point charges. The **Coulomb Approach** therefore becomes

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r^3} \vec{r} \quad (7.2.3)$$

Apply

Identify the charge density of a half-circle conducting wire with total charge q .

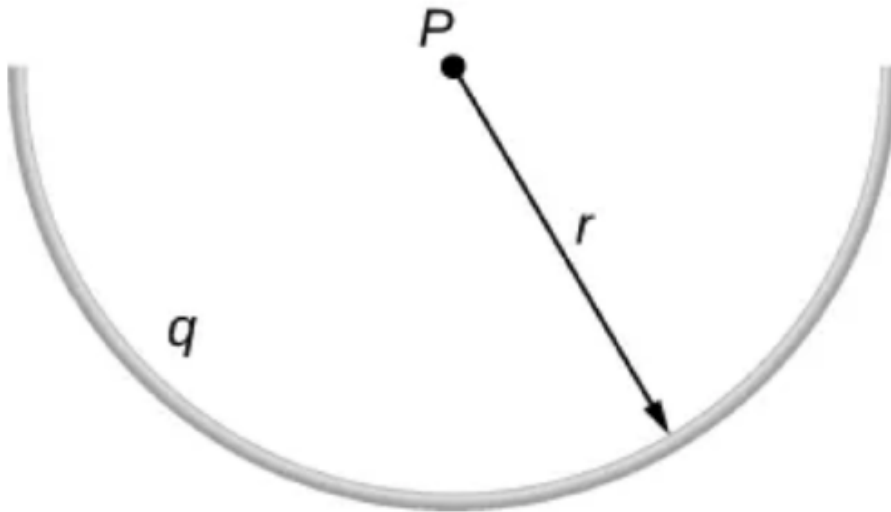
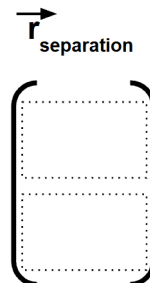
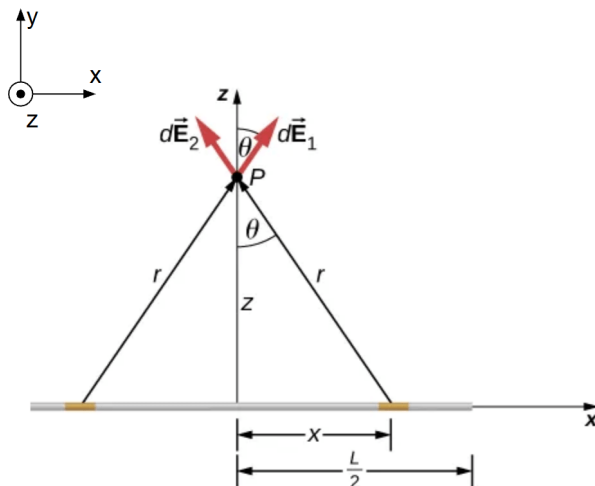


Image Credit: OpenStax University Physics Vol 2 (<https://openstax.org/details/books/university-physics-volume-2>)

7.2.2 Electric Field from a Line of Charge

Model

A line of charge with constant charge density has total charge Q . We are interested in the electric field at a point above the center



$$r_{\text{sep}}^3 =$$

$$dq =$$

of the line.

Image Credit: Adaptation of image from OpenStax University Physics Vol 2 (<https://openstax.org/details/books/university-physics-volume-2>)

Explore

1. Since dl is a differential length on the object, what variable represents dl in this model?
2. For constant linear charge density, $\lambda = Q/L$. What is λ in this scenario?
3. Write a vector expression for the separation vector using coordinate vectors:

$$\vec{r}_{\text{sep}} = \vec{r}_{\text{point}} - \vec{r}_{\text{source}} \quad (7.2.4)$$

- Identify the x - and y -components of $\vec{r}$ and write them in column-vector form.
- Determine $|\vec{r}|$. Does it change as the differential source charge moves along the line? If so, which variable changes?

Critical Thinking & Concept Invention

$$dq = \lambda dl \quad (7.2.5)$$

- Determine r_{sep}^3 .
- Identify dq . Recall that for one-dimensional objects:
- Determine the limits of integration for the entire line.
- Use Coulomb's Law to determine the electric field a distance z above the center of the line:

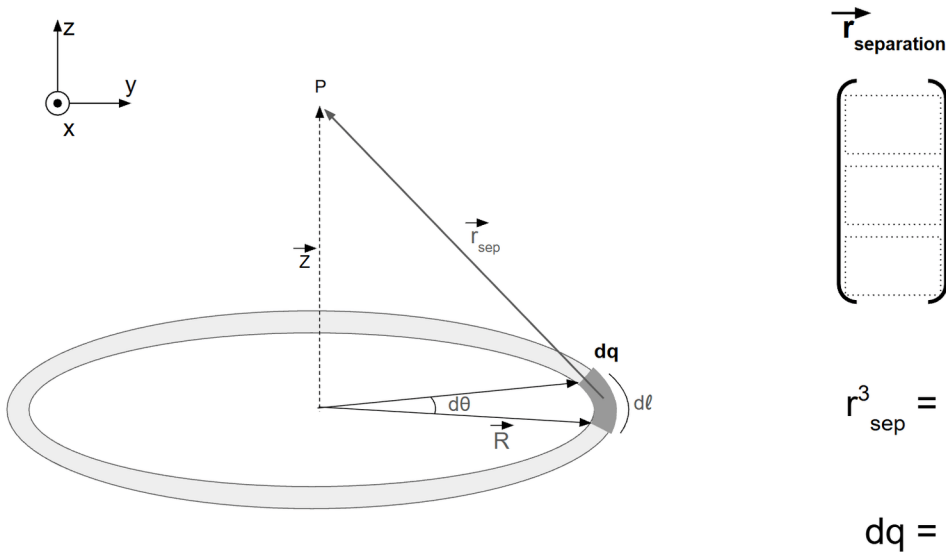
$$\vec{E} = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r^3} \vec{r} \quad (7.2.6)$$

- Repeat the process for an infinitely long line of charge.

7.2.3 Electric Field from a Ring of Charge

Model

A charged ring has total charge Q . The electric field is evaluated directly above its center.



Explore

- Express the differential arc length dl in terms of variables shown in the model.
- For constant linear charge density, determine $\lambda = Q/L$.
- Write the separation vector using the labeled vectors.
- Identify cylindrical coordinates (R, θ, z) and determine the x - and y -components of $\vec{R}$.
- Determine $|\vec{r}_{\text{sep}}|$. Does it vary around the ring?
- Evaluate:

$$\int_0^{2\pi} \cos \theta d\theta \quad (7.2.7)$$

$$\int_0^{2\pi} \sin \theta \, d\theta \quad (7.2.8)$$

Critical Thinking & Concept Invention

1. Determine the components of the separation vector.
2. Determine r_{sep}^3 .
3. Determine dq using $dq = \lambda \, dl$.
4. Determine the limits of integration for the full ring.
5. Use Coulomb's Law to determine the electric field:

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r^3} \vec{r} \quad (7.2.9)$$

Apply

Determine the electric field at point P from a half-circle conducting wire with total charge q .

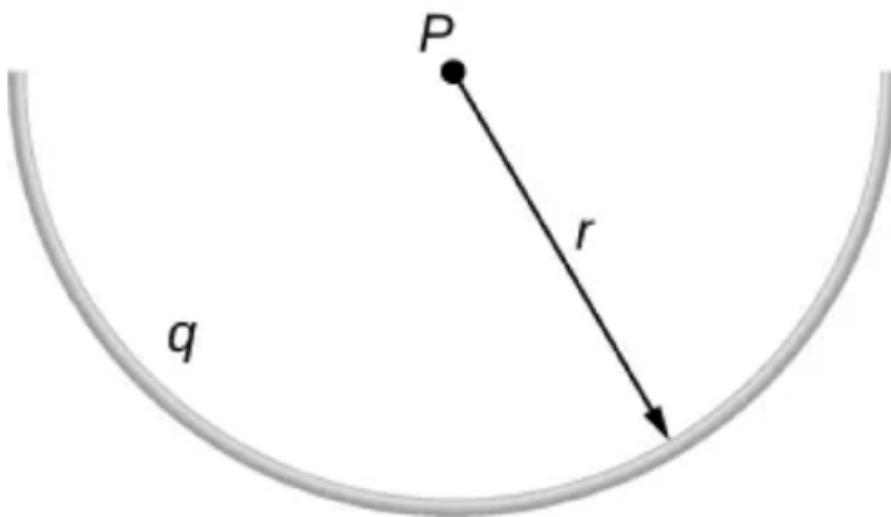
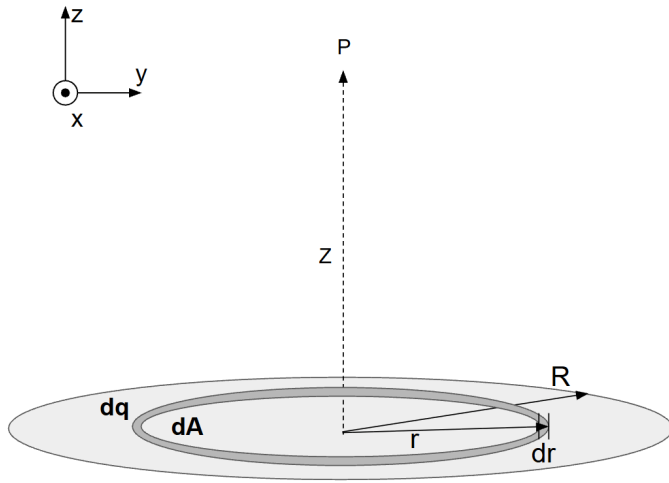


Image Credit: OpenStax University Physics Vol 2 (<https://openstax.org/details/books/university-physics-volume-2>)

7.2.4 Electric Field from a Charged Disk

Model

A charged disk has total charge Q . The electric field is evaluated directly above its center.



$$d\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{dq \vec{z}}{(r^2 + z^2)^{3/2}}$$

$$dq = \sigma dA$$

$$\sigma =$$

$$dA =$$

$$dq =$$

Explore

$$\sigma = \frac{Q}{A} \quad (7.2.10)$$

1. Identify the shape of the overall object and the shape of the differential charge element.
2. For constant surface charge density, determine:
3. What variable represents the distance from the center to the differential source charge?
4. Determine the total surface area.
5. Will every differential charge element have the same area? Explain.
6. Determine $|\vec{z}|$. Does it vary with position on the disk?

Critical Thinking & Concept Invention

$$dq = \sigma dA \quad (7.2.11)$$

1. Write the separation vector from the differential ring element.
2. Determine r_{sep}^3 .
3. Determine dq using:
4. Determine the limits of integration.
5. Starting with the charged-ring result and replacing $Q_{\text{ring}} \rightarrow dq$, determine the electric field of the disk:

$$\vec{E}_{\text{ring}} \quad (7.2.12)$$

$$\vec{E}_{\text{disk}} \quad (7.2.13)$$

Summary

When symmetry exists, higher-dimensional charge distributions can often be represented as sums of lower-dimensional objects. This allows the electric field to be determined using single integrals.

In general, the Coulomb field for a volume charge distribution is

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \iiint \frac{\rho dV}{r^3} \vec{r} \quad (7.2.14)$$

Apply

A spherical conducting shell of radius R carries total charge Q and therefore has constant surface charge density.

- Draw an appropriate diagram.
- Determine the electric field a distance r from the center when $r > R$.
- Use the Coulomb field integral rather than Gauss's Law.

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r^3} \vec{r} \quad (7.2.15)$$

7.3: Coulomb Potential

Coulomb Potential

Learning Objectives

- Adapt the Coulomb Potential to discrete charge distributions using summation notation.
- Use the integral form of the Coulomb Potential to determine the electric potential from continuous charge distributions using single integrals.

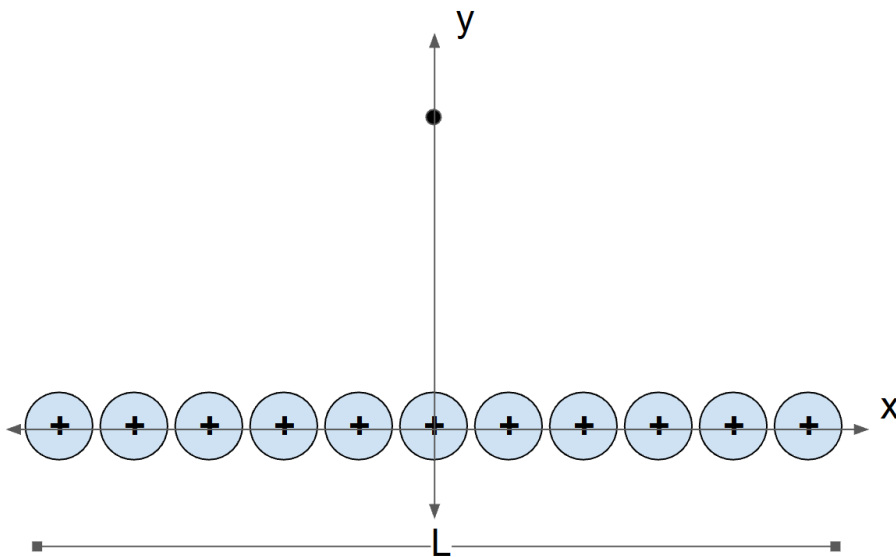
Review

Using the relationship between electric fields and electric potential, we previously found an equation for the electric potential from a point charge when our zero-point reference was at infinity:

$$V_{\text{point charge}} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r} \quad (7.3.1)$$

7.3.1 Electric Potential of a Discrete Charge Distribution

Model



Explore

1. How many charges are shown? If they all have the same charge and the total charge of the line is Q , what would the charge of each circle be (ΔQ)?
2. Label the charges in some way (for example, assign numbers or letters).
3. Draw a separation distance (scalar, not vector) from each charge to the point on the y -axis that represents the point of interest. Give each distance the same subscript label as its corresponding charge.

Critical Thinking & Concept Invention

1. Without performing calculations, discuss how you would determine the total electric potential from the distribution.
2. Express this method using summation notation:

$$V = \sum \quad (7.3.2)$$

3. Taking the limit that converts Δ quantities into differentials and summations into integrals, rewrite the expression:

$$V = \int \quad (7.3.3)$$

Apply

Determine the electric potential at point P for the collection of charges shown. Express the result in summation notation using the equation developed for a discrete collection of charges.

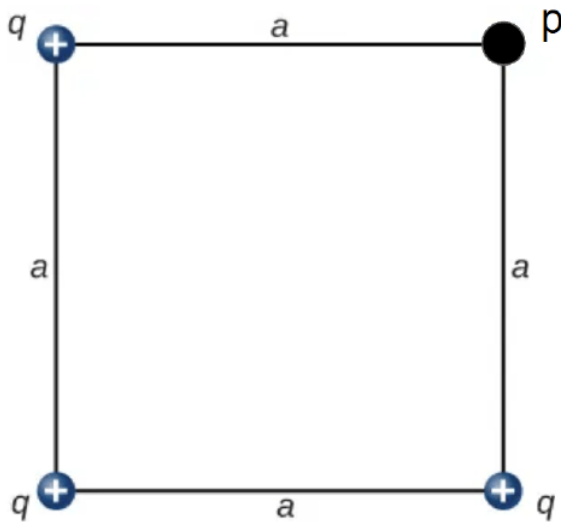


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Summary

Just as any charge distribution can be viewed as a collection of point charges when calculating electric fields, the same approach applies to electric potential. This leads to the Coulomb Potential integral:

$$V = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r} \quad (7.3.4)$$

Differential charge elements may be identified from charge densities such as λ , σ , and ρ . For a general volume distribution:

$$V = \frac{1}{4\pi\epsilon_0} \iiint \frac{\rho dV}{r} \quad (7.3.5)$$

Electric potential calculations are often simpler than electric field calculations because they involve scalar quantities rather than vectors. Once the potential is known, the electric field can be obtained from:

$$\vec{E} = -\vec{\nabla} V \quad (7.3.6)$$

7.3.2 Electric Potential of a Continuous Linear Charge Distribution

Model

A line of charge of length L with constant charge density has total charge Q . The electric potential is evaluated at a point directly above the center of the line.

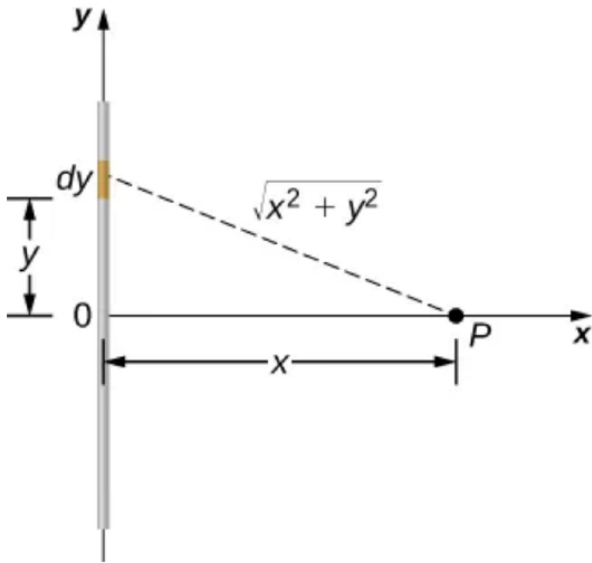


Image Credit: OpenStax University Physics Vol 2 (<https://openstax.org/details/books/university-physics-volume-2>)

Explore

1. Since dl is a differential length along the object, identify which variable represents dl in this model.
2. Determine the linear charge density λ assuming it is constant.
3. Write a scalar expression for the separation distance using the coordinates and variables shown. Determine whether this distance depends on the position of the differential charge element.
4. Identify the coordinates corresponding to:
 - The top of the rod.
 - The middle of the rod.
 - The bottom of the rod.

Critical Thinking & Concept Invention

1. Determine the differential charge element dq . Recall that for a one-dimensional object with constant charge density:

$$dq = \lambda dl \quad (7.3.7)$$

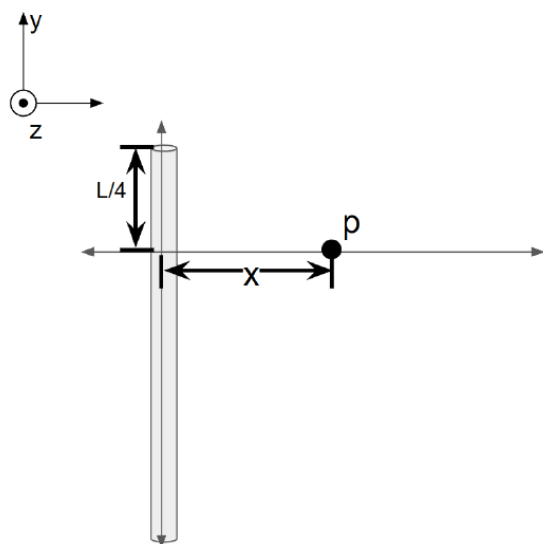
2. Determine the limits of integration needed to integrate over the entire line.
3. Use the Coulomb Potential equation to determine the electric potential a distance x from the center of the line. Constants may be factored outside the integral.

$$V = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r} \quad (7.3.8)$$

$$V = \int d \quad (7.3.9)$$

Apply

Determine the electric potential at a point located a distance $L/4$ below the end of the charged line shown in the accompanying figure.



7.3.3 Electric Potential of a Charged Ring

Model

A ring has total charge Q . The electric potential is evaluated directly above the center of the ring.

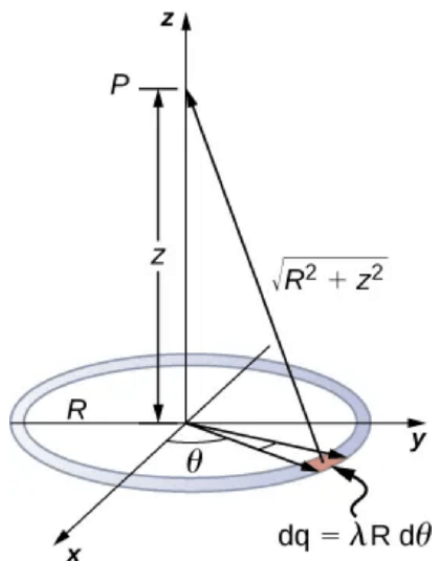


Image Credit: OpenStax University Physics Vol 2 (<https://openstax.org/details/books/university-physics-volume-2>)

Explore

1. For a one-dimensional object, express the differential length dl in terms of the variables provided.
2. For constant linear charge density, determine:

$$\lambda = \frac{Q}{L} \quad (7.3.10)$$

3. Determine dq for the ring and verify that it equals $\lambda \, dl$.
4. Write a scalar expression for the separation distance between the differential charge element and the observation point. Determine whether this distance depends on the location of the differential element.

Critical Thinking & Concept Invention

1. Determine the limits of integration needed to integrate around the complete ring.
2. Use the Coulomb Potential equation to determine the electric potential a distance z above the center of the ring. Constants may be factored outside the integral.

$$V = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r} \quad (7.3.11)$$

$$V = \int d \quad (7.3.12)$$

7.3.4 Electric Potential of a Charged Disk

Model

A disk has total charge Q . The electric potential is evaluated directly above the center of the disk.

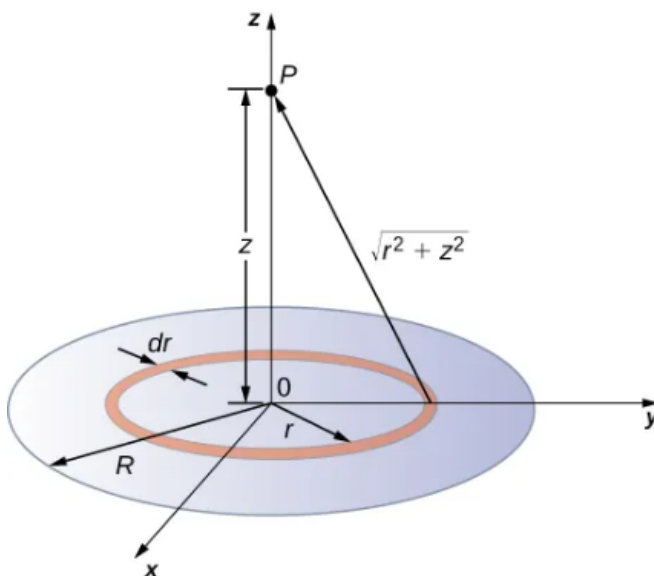


Image Credit: OpenStax University Physics Vol 2 (<https://openstax.org/details/books/university-physics-volume-2>)

Explore

1. For a two-dimensional object, determine the differential area dA of the ring-shaped element in terms of the variables provided.

2. For constant surface charge density, determine:

$$\sigma = \frac{Q}{A} \quad (7.3.13)$$

3. Write a scalar expression for the separation distance between the differential area element and the observation point. Determine whether this distance depends on position.

Critical Thinking & Concept Invention

The electric potential for a charged ring has already been determined. Use that result with the substitution $Q_{\text{ring}} \rightarrow dq$.

1. Determine dq for the disk and identify whether it depends on the location of the differential element.
2. Determine the limits of integration required to integrate over the entire disk.
3. Use the Coulomb Potential equation to determine the electric potential a distance z above the center of the disk. Constants may be factored outside the integral.

$$V = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r} \quad (7.3.14)$$

$$V = \int d \quad (7.3.15)$$

7.4: Biot-Savart Law

Biot-Savart Law

Learning Objectives

- Calculate magnetic fields from currents in straight lines using the Biot-Savart Law.
 - Calculate magnetic fields from currents in circular loops using the Biot-Savart Law.
 - Use superposition to determine magnetic fields from simple current shapes.
-

7.4.1 Biot-Savart Law

Model

The equations below represent the forces and fields from point charges when relativistic effects are not significant.

Forces

$$\vec{F}_E = q\vec{E} \quad (7.4.1)$$

$$\vec{F}_B = q\vec{v} \times \vec{B} \quad (7.4.2)$$

Fields

$$\vec{E} = k_E \frac{q\vec{r}}{r^3} \quad (7.4.3)$$

$$\vec{B} = k_B \frac{q\vec{v} \times \vec{r}}{r^3} \quad (7.4.4)$$

$$k_E = \frac{1}{4\pi\epsilon_0} \quad (7.4.5)$$

$$k_B = \frac{\mu_0}{4\pi} \quad (7.4.6)$$

Explore

1. Highlight variables or numbers that appear the same in the equations for electric fields, electric forces, magnetic fields, magnetic forces, and their constants.
2. Circle the variables that are specific to a type of field (for example, E or E subscripts). Also circle the variable that differs in the constants on the last row.
3. There should be something left over that is neither circled nor highlighted. What is it? Is it in the electric or magnetic equations?

Critical Thinking & Concept Invention

1. We know the magnetic field relates to current, but I does not appear directly in these equations. How does I relate to q ?
2. If $\vec{v} = \frac{d\vec{l}}{dt}$, how does $dq \vec{v}$ relate to I ? Hint: Write out the relationship between I and q , then manipulate the differential quantities.

3. If the equation in the model represents the magnetic field from a single moving charge, how can the total magnetic field from many moving charges be determined?

Summary

We typically have currents consisting of many moving charges. The equation can be modified using the substitutions:

$$q \rightarrow dq \quad (7.4.7)$$

$$I = \frac{dq}{dt} \quad (7.4.8)$$

$$dq \vec{v} = I d\vec{l} \quad (7.4.9)$$

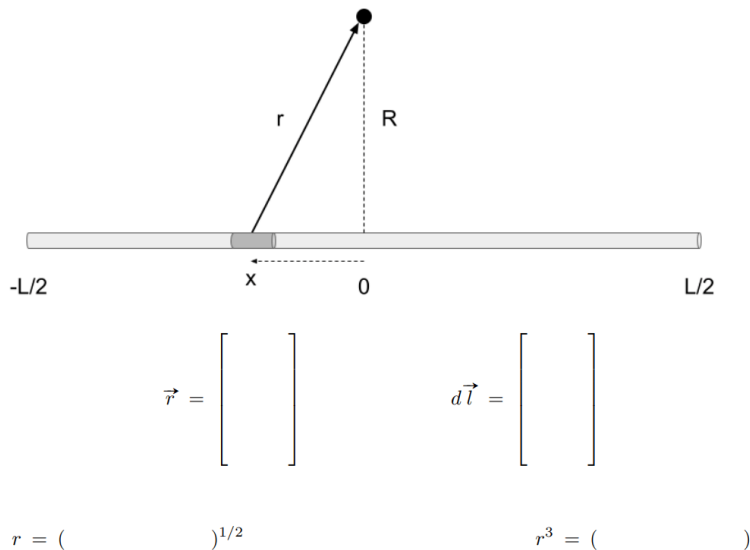
leading to the Biot-Savart Law:

$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{I d\vec{l} \times \vec{r}}{r^3} \quad (7.4.10)$$

7.4.2 Magnetic Field from a Straight Wire

Model

The model shows a wire of length L carrying current to the right. The goal is to determine the magnetic field a distance R from the midpoint.



Explore

1. Identify a coordinate system so positive and negative directions align with the wire.
2. Fill in the three components of $\vec{r}$, paying attention to sign.
3. Draw an arrow representing current flowing to the right.
4. Determine the three components of $d\vec{l}$, remembering it points in the current direction.
5. Use geometry to determine expressions for r and r^3 .

Critical Thinking & Concept Invention

- Determine the magnetic field at point P :
 - Evaluate $d\vec{l} \times \vec{r}$.
 - Write the complete Biot-Savart integral with limits.
 - Factor constants outside the integral where appropriate.
 - Match the resulting integral to a known solution and substitute constants.
- Repeat the calculation for an infinitely long wire by changing the limits of integration.

Summary

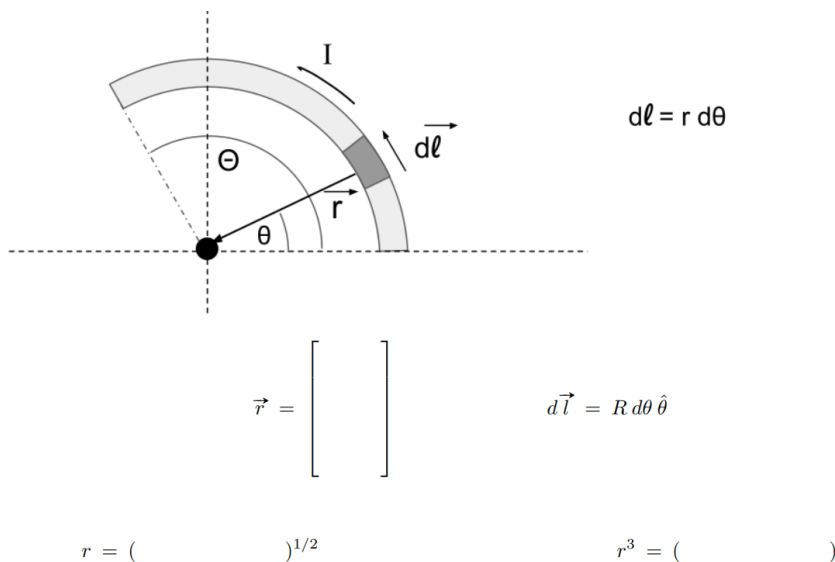
For an infinite wire every point effectively behaves as the midpoint, yielding

$$\vec{B}_{\text{wire}} = \frac{\mu_0 I}{2\pi R} \quad (7.4.11)$$

where R is the perpendicular distance from the wire. This approximation is valid whenever R is much smaller than the wire length and the observation point is not near an endpoint.

7.4.3 Magnetic Field from a Curved Wire

Model



Explore

- Draw a Cartesian coordinate system consistent with the cylindrical basis vectors.
- Fill in the components of $\vec{r}$.
- Determine expressions for r and r^3 .
- Determine whether $\vec{r}$ points in the $+\hat{r}$ or $-\hat{r}$ direction.
- Circle quantities that vary as the differential element moves around the loop.

Critical Thinking & Concept Invention

- Evaluate:

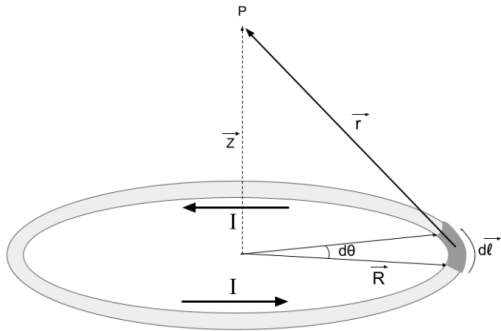
$$\hat{r} \times \hat{\theta} \quad (7.4.12)$$

$$\hat{\theta} \times \hat{r} \quad (7.4.13)$$

2. Determine $d\vec{l} \times \vec{r}$.
3. Use the Biot-Savart Law to determine the magnetic field by setting up, evaluating, and simplifying the integral.

7.4.4 Magnetic Field from a Current Loop on the Central Axis

Model



$$\vec{r} = \begin{bmatrix} \\ \\ \end{bmatrix} \quad \hat{\theta} = \begin{bmatrix} -\sin\theta \\ \cos\theta \\ 0 \end{bmatrix} \quad d\vec{l} = R d\theta \hat{\theta} = \begin{bmatrix} \\ \\ \end{bmatrix}$$

$$r = ()^{1/2} \quad r^3 = ()$$

Explore

1. Determine the components of $\vec{r}$.
2. Determine the components of $d\vec{l}$.
3. Determine expressions for r and r^3 .
4. Identify quantities that vary as the differential element moves around the loop.

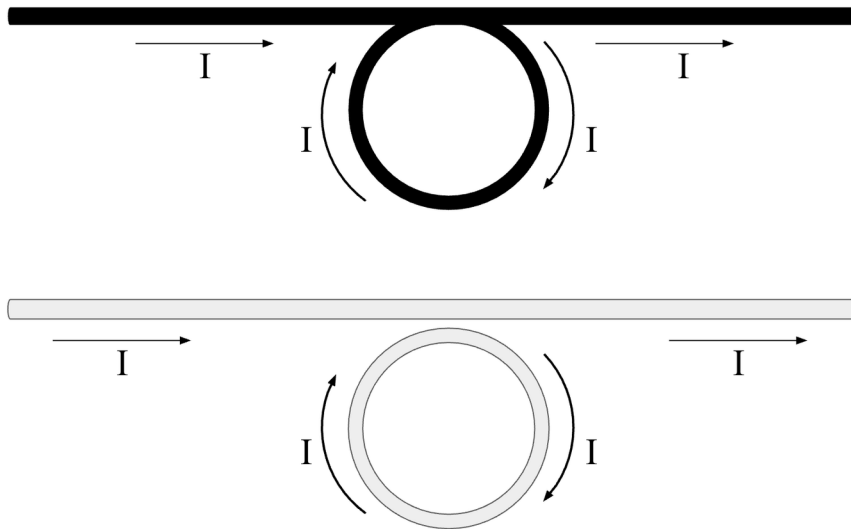
Critical Thinking & Concept Invention

1. Evaluate $d\vec{l} \times \vec{r}$.
2. Set up and evaluate the Biot-Savart integral for the magnetic field.
3. Use the result to determine the magnetic field at $z = 0$ and compare with the center-of-loop expression.
4. Explain why the calculation would become more difficult if the observation point were not on the central axis.

7.4.5 Magnetic Field Superposition

Model

The top image shows a single infinite wire carrying current I . The bottom image shows two separate wires, each carrying current I , with identical dimensions.



Explore

1. Describe how the two wire setups are similar.
2. Use the Right Hand Rule to determine the magnetic field direction at the center for the single wire.
3. Determine the magnetic field direction at the center due to each wire in the lower figure.
4. Determine whether the two magnetic fields point in the same or opposite directions and decide whether

$$B_{\text{total}} = B_{\text{line}} + B_{\text{ring}} \quad (7.4.14)$$

or

$$B_{\text{total}} = |B_{\text{line}} - B_{\text{ring}}| \quad (7.4.15)$$

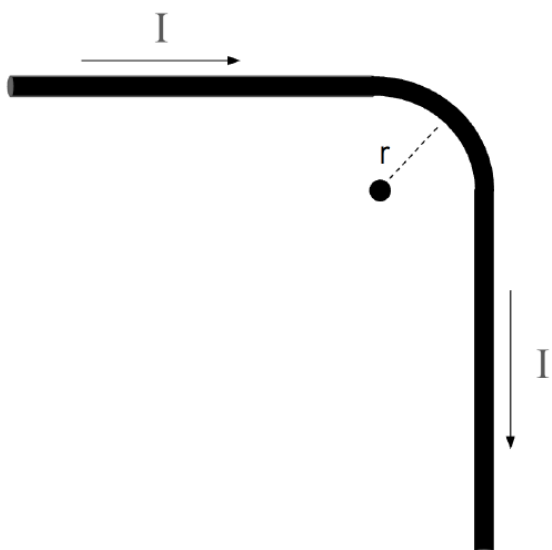
applies.

Critical Thinking & Concept Invention

1. Determine the magnetic field from the combined wire configuration:
 - Identify the field from each wire.
 - Determine the total field using vector addition.
2. Summarize how magnetic fields from complex current geometries can be found using superposition.

Apply

Calculate the magnetic field created by the wire shown by breaking it into separate sections and summing the magnetic field contribution from each piece. You may treat straight portions as infinite or assign finite lengths as appropriate.



7.5: Electric Dipole Fields

Learning Objectives

- Apply Coulomb Field and Coulomb Potential equations to dipoles.

1.3 Dipole Field Part 1

Model

Image needed here with Alt Text:

Dipole geometry for calculating electric field and potential on perpendicular axis

$$\vec{r}_L = \vec{r}_p - \vec{r}_{+Q} \quad (7.5.1)$$

$$\vec{r}_R = \vec{r}_p - \vec{r}_{-Q} \quad (7.5.2)$$

$$\vec{E}_{\text{total}} = \vec{E}_+ + \vec{E}_- \quad (7.5.3)$$

$$V_{\text{total}} = V_+ + V_- \quad (7.5.4)$$

Explore

1. Mark up the model to show the x and y component of each $\vec{r}$.
2. Examine the model to fill in key information:
 - Fill in the vectors for the position of the point and the position of each charge.
 - Determine the separation vectors from each source charge to the point of interest.
 - Use the column vectors to determine the magnitude of each separation distance and record this to the third power.
3. Draw the electric field pattern you expect from these charges.

Critical Thinking & Concept Invention

1. Determine the electric field at the point of interest:
 - Use the Coulomb field equation to determine the field from each charge.
 - Use the principle of superposition to determine the total field.
2. Determine the electric potential at the point of interest:
 - Use the Coulomb potential equation to determine the potential from each charge.
 - Use the principle of superposition to determine the total electric potential.

Summary

A positive and negative charge pair that is net neutral but has a separation of the positive and negative charge is called a dipole. The product of the charge magnitude (q) and the distance between the charges (d) is called the dipole moment:

$$\vec{p} \equiv q\vec{d} \quad (7.5.5)$$

The dipole moment is a vector that points from the negative charge to the positive charge.

The electric field and potential of a dipole at a distance r from the center of the dipole along an axis perpendicular to the dipole moment is:

$$\vec{E}_P = \quad (7.5.6)$$

$$V_P = \quad (7.5.7)$$

1.4 Dipole Field Part 2

Model

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Dipole geometry for calculating electric field and potential on parallel axis

$$\vec{r}_+ = \vec{r}_p - \vec{r}_{+Q} \quad (7.5.8)$$

$$\vec{r}_- = \vec{r}_p - \vec{r}_{-Q} \quad (7.5.9)$$

$$\vec{E}_{\text{total}} = \vec{E}_+ + \vec{E}_- \quad (7.5.10)$$

$$V_{\text{total}} = V_+ + V_- \quad (7.5.11)$$

Explore

- Mark up the model by drawing:
 - $\vec{y}$ from the origin to the point on the y-axis.
 - $\vec{r}_+$ from the positive charge to the point on the y-axis.
 - $\vec{r}_-$ from the negative charge to the point on the y-axis.
- Examine the model to fill in key information:
 - Determine the vectors from each source charge to the point of interest.
 - Fill in the column vectors in terms of $\vec{y}$.
 - Use the column vectors to determine the magnitude of each separation distance and record this to the third power.

Critical Thinking & Concept Invention

- Determine the electric field at the point of interest:
 - Use the Coulomb field equation to determine the field from each charge.
 - Use the principle of superposition to determine the total field.
- Determine the electric potential at the point of interest:
 - Use the Coulomb potential equation to determine the potential from each charge.
 - Use the principle of superposition to determine the total electric potential.

Summary

The electric field and potential of a dipole at a distance r from the center of the dipole along an axis parallel to the dipole moment is:

$$\vec{E}_P = \quad (7.5.12)$$

$$V_P = \quad (7.5.13)$$

As the distance from the center of the dipole gets much larger than the distance between the charges, this expression reduces to:

$$\vec{E}_P = \quad (7.5.14)$$

$$V_P = \quad (7.5.15)$$

Dipole Field Summary

Dipoles create a complicated electric field. In general, the field lines go from the positive charge to the negative charge.

A dipole will experience a torque in a field that causes it to rotate until its field lines generally point opposite the external field lines.

The full equation for the dipole electric field and electric potential is shown below. It is best represented in spherical coordinates, but there is a coordinate-free version as well.

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{p}{r^3} (2 \cos \theta \hat{r} + \sin \theta \hat{\theta}) \quad (7.5.16)$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{3\vec{r}(\vec{r} \cdot \vec{p}) - r^2 \vec{p}}{r^5} \quad (7.5.17)$$

$$V_{\text{dipole}} = \frac{1}{4\pi\epsilon_0} \frac{p \cos \theta}{r^2} \quad (7.5.18)$$

$$V_{\text{dipole}} = \frac{1}{4\pi\epsilon_0} \frac{\vec{r} \cdot \vec{p}}{r^3} \quad (7.5.19)$$

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