NBAS022 - GEOMETRY



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NBAS022 - Geometry

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1.1: Introduction to Whole Numbers

Learning Objectives

- By the end of this section, you will be able to:
- Use place value with whole numbers
- Identify multiples and and apply divisibility tests
- Find prime factorizations and least common multiples

As we begin our study of elementary algebra, we need to refresh some of our skills and vocabulary. This chapter will focus on whole numbers, integers, fractions, decimals, and real numbers. We will also begin our use of algebraic notation and vocabulary.

Use Place Value with Whole Numbers

The most basic numbers used in algebra are the numbers we use to count objects in our world: $1, 2, 3, 4$, and so on. These are called the **counting numbers**. Counting numbers are also called *natural numbers*. If we add zero to the counting numbers, we get the set of **whole numbers**.

- Counting Numbers: $1, 2, 3, \ldots$
- Whole Numbers: $0, 1, 2, 3, \ldots$

The notation "$\ldots$" is called ellipsis and means "and so on," or that the pattern continues endlessly.

We can visualize counting numbers and whole numbers on a **number line** (see Figure 1.1.1).

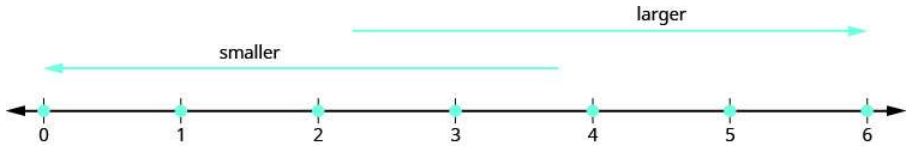


Figure 1.1.1: The numbers on the number line get larger as they go from left to right, and smaller as they go from right to left. While this number line shows only the whole numbers 0 through 6, the numbers keep going without end.

Doing the Manipulative Mathematics activity "Number Line-Part 1" will help you develop a better understanding of the counting numbers and the whole numbers.

Our **number system** is called a place value system, because the value of a digit depends on its position in a number. Figure 1.1.2 shows the **place values**. The place values are separated into groups of three, which are called periods. The periods are *ones*, *thousands*, *millions*, *billions*, *trillions*, and so on. In a written number, commas separate the periods.

Place Value														
Trillions			Billions			Millions			Thousands			Ones		
Hundred trillions	Ten trillions	Trillions	Hundred billions	Ten billions	Billions	Hundred millions	Ten millions	Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
								5	2	7	8	1	9	4

Figure 1.1.2: The number 5278194 is shown in the chart. The digit 5 is in the millions place. The digit 2 is in the hundred-thousands place. The digit 7 is in the ten-thousands place. The digit 8 is in the thousands place. The digit 1 is in the hundreds place. The digit 9 is in the tens place. The digit 4 is in the ones place.

Exercise 1.1.1

In the number 63407218, find the place value of each digit:

1. 7

2. 0
3. 1
4. 6
5. 3

Answer

Place the number in the place value chart:

Trillions			Billions			Millions			Thousands			Ones		
Hundred trillions	Ten trillions	Trillions	Hundred billions	Ten billions	Billions	Hundred millions	Ten millions	Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
							6	3	4	0	7	2	1	8

1. The 7 is in the thousands place.
2. The 0 is in the ten thousands place.
3. The 1 is in the tens place.
4. The 6 is in the ten-millions place.
5. The 3 is in the millions place.

Exercise 1.1.2

For the number 27493615 find the place value of each digit:

1. 2
2. 1
3. 4
4. 7
5. 5

Answer

1. ten millions
2. tens
3. hundred thousands
4. millions
5. ones

Exercise 1.1.3

For the number 519711641328 find the place value of each digit:

1. 9
2. 4
3. 2
4. 6
5. 7

Answer

1. billions
2. ten thousands

3. tens
4. hundred thousands
5. hundred millions

When you write a check, you write out the number in words as well as in digits. To write a number in words, write the number in each period, followed by the name of the period, without the *s* at the end. Start at the left, where the periods have the largest value. The ones period is not named. The commas separate the periods, so wherever there is a comma in the number, put a comma between the words (see Figure 1.1.3). The number 74218369is written as seventy-four million, two hundred eighteen thousand, three hundred sixty-nine.

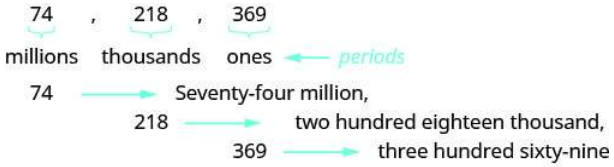


Figure 1.1.3

NAME A WHOLE NUMBER IN WORDS.

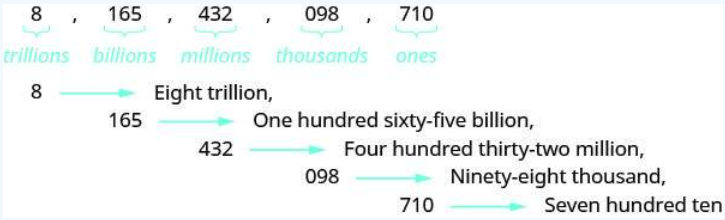
1. Start at the left and name the number in each period, followed by the period name.
2. Put commas in the number to separate the periods.
3. Do not name the ones period.

Exercise 1.1.4

Name the number 8165432098710using words.

Answer

Name the number in each period, followed by the period name.



Put the commas in to separate the periods.

So, 8165432098710is named as eight trillion, one hundred sixty-five billion, four hundred thirty-two million, ninety-eight thousand, seven hundred ten.

Exercise 1.1.5

Name the number 9,258,137,904,0619,258,137,904,061 using words.

Answer

nine trillion, two hundred fifty-eight billion, one hundred thirty-seven million, nine hundred four thousand, sixty-one

Exercise 1.1.6

Name the number 17,864,325,619,00417,864,325,619,004 using words.

Answer

seventeen trillion, eight hundred sixty-four billion, three hundred twenty-five million, six hundred nineteen thousand four

We are now going to reverse the process by writing the digits from the name of the number. To write the number in digits, we first look for the clue words that indicate the periods. It is helpful to draw three blanks for the needed periods and then fill in the blanks with the numbers, separating the periods with commas.

WRITE A WHOLE NUMBER USING DIGITS.

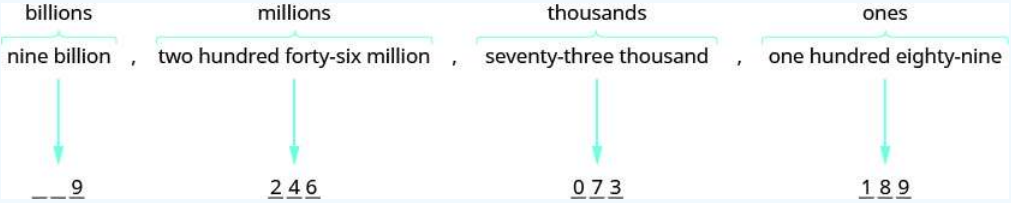
1. Identify the words that indicate periods. (Remember, the ones period is never named.)
2. Draw three blanks to indicate the number of places needed in each period. Separate the periods by commas.
3. Name the number in each period and place the digits in the correct place value position.

Exercise 1.1.7

Write *nine billion, two hundred forty-six million, seventy-three thousand, one hundred eighty-nine* as a whole number using digits.

Answer

Identify the words that indicate periods.
Except for the first period, all other periods must have three places. Draw three blanks to indicate the number of places needed in each period. Separate the periods by commas.
Then write the digits in each period.



The number is 9,246,073,189.

Exercise 1.1.8

Write the number two billion, four hundred sixty-six million, seven hundred fourteen thousand, fifty-one as a whole number using digits.

Answer

2,466,714,051

Exercise 1.1.9

Write the number eleven billion, nine hundred twenty-one million, eight hundred thirty thousand, one hundred six as a whole number using digits.

Answer

11,921,830,106

In 2013, the U.S. Census Bureau estimated the population of the state of New York as 19,651,127. We could say the population of New York was approximately 20 million. In many cases, you don't need the exact value; an approximate number is good enough.

The process of approximating a number is called **rounding**. Numbers are rounded to a specific place value, depending on how much accuracy is needed. Saying that the population of New York is approximately 20 million means that we rounded to the

millions place.

Exercise 1.1.10 How to Round Whole Numbers

Round 23,658 to the nearest hundred.

Answer

Step 1. Locate the given place value with an arrow. All digits to the left do not change.	Locate the hundreds place in 23,658.	hundredths place ↓ 23,658
Step 2. Underline the digit to the right of the given place value.	Underline the 5, which is to the right of the hundreds place.	hundredths place ↓ 23,658
Step 3. Is this digit greater than or equal to 5? Yes–add 1 to the digit in the given place value. No–do not change the digit in the given place value.	Add 1 to the 6 in the hundreds place, since 5 is greater than or equal to 5.	23,658 add 1 ↗
Step 4. Replace all digits to the right of the given place value with zeros.	Replace all digits to the right of the hundreds place with zeros.	23,700 add 1 ↗ ↖ replace with 0s So, 23,700 is rounded to the nearest hundred.

Exercise 1.1.11

Round to the nearest hundred: 17,852.

Answer

17,900

Exercise 1.1.12

Round to the nearest hundred: 468,751.

Answer

468,800

ROUND WHOLE NUMBERS.

1. Locate the given place value and mark it with an arrow. All digits to the left of the arrow do not change.
2. Underline the digit to the right of the given place value.
3. Is this digit greater than or equal to 5?
   - Yes–add 11 to the digit in the given place value.
   - No–do not change the digit in the given place value.
4. Replace all digits to the right of the given place value with zeros.

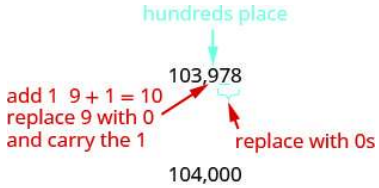
Exercise 1.1.13

Round 103,978103,978 to the nearest:

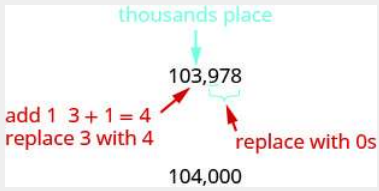
1. hundred
2. thousand
3. ten thousand

Answer

1.

Locate the hundreds place in 103,978.	
Underline the digit to the right of the hundreds place.	
Since 7 is greater than or equal to 5, add 1 to the 9. Replace all digits to the right of the hundreds place with zeros.	
	So, 104,000 is 103,978 rounded to the nearest hundred.

2.

Locate the thousands place and underline the digit to the right of the thousands place.	
Since 9 is greater than or equal to 5, add 1 to the 3. Replace all digits to the right of the hundreds place with zeros.	
	So, 104,000 is 103,978 rounded to the nearest thousand.

3.

Locate the ten thousands place and underline the digit to the right of the ten thousands place.	
Since 3 is less than 5, we leave the 0 as is, and then replace the digits to the right with zeros.	
	So, 100,000 is 103,978 rounded to the nearest ten thousand.

Exercise 1.1.14

Round 206,981 to the nearest: 1. hundred 2. thousand 3. ten thousand.

Answer

1. 207,000
2. 207,000

3. 210,000

Exercise 1.1.15

Round 784,951 to the nearest: 1. hundred 2. thousand 3. ten thousand.

Answer

1. 785,000
2. 785,000
3. 780,000

Identify Multiples and Apply Divisibility Tests

The numbers 2, 4, 6, 8, 10, and 12 are called **multiples** of 2. A multiple of 2 can be written as the **product** of a counting number and 2.

$$\underset{2\cdot 1,}{2,}\quad \underset{2\cdot 2,}{4,}\quad \underset{2\cdot 3,}{6,}\quad \underset{2\cdot 4,}{8,}\quad \underset{2\cdot 5,}{10,}\quad \underset{2\cdot 6}{12,\ldots}$$

Figure 1.1.4

Similarly, a multiple of 3 would be the product of a counting number and 3.

$$\underset{3\cdot 1,}{3,}\quad \underset{3\cdot 2,}{6,}\quad \underset{3\cdot 3,}{9,}\quad \underset{3\cdot 4,}{12,}\quad \underset{3\cdot 5,}{15,}\quad \underset{3\cdot 6}{18,\ldots}$$

Figure 1.1.5

We could find the multiples of any number by continuing this process.

Note

Doing the Manipulative Mathematics activity “Multiples” will help you develop a better understanding of multiples.

Table 1.1.1 shows the multiples of 2 through 9 for the first 12 counting numbers.

Table 1.1.1

Counting Number	1	2	3	4	5	6	7	8	9	10	11	12
Multiples of 2	2	4	6	8	10	12	14	16	18	20	22	24
Multiples of 3	3	6	9	12	15	18	21	24	27	30	33	36
Multiples of 4	4	8	12	16	20	24	28	32	36	40	44	48
Multiples of 5	5	10	15	20	25	30	35	40	45	50	55	60
Multiples of 6	6	12	18	24	30	36	42	48	54	60	66	72
Multiples of 7	7	14	21	28	35	42	49	56	63	70	77	84
Multiples of 8	8	16	24	32	40	48	56	64	72	80	88	96

Counting Number	1	2	3	4	5	6	7	8	9	10	11	12
Multiples of 9	9	18	27	36	45	54	63	72	81	90	99	108
Multiples of 10	10	20	30	40	50	60	70	80	90	100	110	120

MULTIPLE OF A NUMBER

A number is a **multiple** of n if it is the product of a counting number and n.

Another way to say that 15 is a multiple of 3 is to say that 15 is **divisible** by 3. That means that when we divide 3 into 15, we get a counting number. In fact, $15 \div 3$ is 5, so 15 is $5 \cdot 3$.

DIVISIBLE BY A NUMBER

If a number m is a multiple of n, then m is **divisible** by n

Look at the multiples of 5 in Table 1.1.1. They all end in 5 or 0. Numbers with last digit of 5 or 0 are divisible by 5. Looking for other patterns in Table 1.1.1 that shows multiples of the numbers 2 through 9, we can discover the following divisibility tests:

DIVISIBILITY TESTS

A number is divisible by:

- 2 if the last digit is 0, 2, 4, 6, or 8.
- 3 if the sum of the digits is divisible by 3.
- 5 if the last digit is 5 or 0.
- 6 if it is divisible by both 2 and 3.
- 10 if it ends with 0.

Exercise 1.1.16

Is 5625 divisible by 2? By 3? By 5? By 6? By 10?

Answer

Is 5625 divisible by 2?
Does it end in 0, 2, 4, 6, or 8? No. 5625 is not divisible by 2. (1.1.1)

Is 5625 divisible by 3?
What is the sum of the digits? $5+6+2+5=18$
Is the sum divisible by 3? Yes, 5625 is divisible by 3. (1.1.2)

Is 5625 divisible by 5 or 10?
What is the last digit? It is 5. 5625 is divisible by 5 but not by 10. (1.1.3)

Is 5625 divisible by 6?
Is it divisible by both 2 and 3? No, 5625 is not divisible by 2, so 5625 is not divisible by 6. (1.1.4)

Exercise 1.1.17

Determine whether 4,962 is divisible by 2, by 3, by 5, by 6, and by 10.

Answer

by 2, 3, and 6

Exercise 1.1.18

Determine whether 3,765 is divisible by 2, by 3, by 5, by 6, and by 10.

Answer

by 3 and 5

Find Prime Factorizations and Least Common Multiples

In mathematics, there are often several ways to talk about the same ideas. So far, we've seen that if m is a multiple of n, we can say that m is divisible by n. For example, since 72 is a multiple of 8, we say 72 is divisible by 8. Since 72 is a multiple of 9, we say 72 is divisible by 9. We can express this still another way.

Since $8 \cdot 9 = 72$, we say that 8 and 9 are **factors** of 72. When we write $72 = 8 \cdot 9$, we say we have factored 72.



Figure 1.1.6

Other ways to factor 72 are $1 \cdot 72$, $2 \cdot 36$, $3 \cdot 24$, $4 \cdot 18$ and $6 \cdot 12$. Seventy-two has many factors: 1, 2, 3, 4, 6, 8, 9, 12, 18, 36, and 72.

FACTORS

If $a \cdot b = m$, then a and b are **factors** of m.

Some numbers, like 72, have many factors. Other numbers have only two factors.

Note

Doing the Manipulative Mathematics activity "Model Multiplication and Factoring" will help you develop a better understanding of multiplication and factoring.

PRIME NUMBER AND COMPOSITE NUMBER

A **prime number** is a counting number greater than 1, whose only factors are 1 and itself.

A **composite number** is a counting number that is not prime. A composite number has factors other than 1 and itself.

Note

Doing the Manipulative Mathematics activity "Prime Numbers" will help you develop a better understanding of prime numbers.

The counting numbers from 2 to 19 are listed in Figure 1.1.7, with their factors. Make sure to agree with the "prime" or "composite" label for each!

Number	Factors	Prime or Composite?
2	1,2	Prime
3	1,3	Prime
4	1,2,4	Composite
5	1,5	Prime
6	1,2,3,6	Composite
7	1,7	Prime
8	1,2,4,8	Composite
9	1,3,9	Composite
10	1,2,5,10	Composite

Number	Factors	Prime or Composite?
11	1,11	Prime
12	1,2,3,4,6,12	Composite
13	1,13	Prime
14	1,2,7,14	Composite
15	1,3,5,15	Composite
16	1,2,4,8,16	Composite
17	1,17	Prime
18	1,2,3,6,9,18	Composite
19	1,19	Prime

Figure 1.1.7

The **prime numbers** less than 20 are 2, 3, 5, 7, 11, 13, 17, and 19. Notice that the only even prime number is 2.

A composite number can be written as a unique **product** of primes. This is called the **prime factorization** of the number. Finding the prime factorization of a composite number will be useful later in this course.

PRIME FACTORIZATION

The **prime factorization** of a number is the product of prime numbers that equals the number.

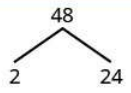
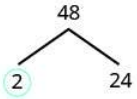
To find the prime factorization of a composite number, find any two factors of the number and use them to create two branches. If a factor is prime, that branch is complete. Circle that prime!

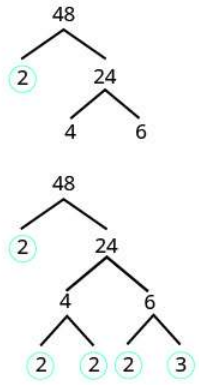
If the factor is not prime, find two factors of the number and continue the process. Once all the branches have circled primes at the end, the factorization is complete. The composite number can now be written as a product of prime numbers.

Exercise 1.1.19

Factor 48.

Answer

Step 1. Find two factors whose product is the given number. Use these numbers to create two branches.	$48 = 2 \cdot 24$	
Step 2. If a factor is prime, that branch is complete. Circle the prime.	2 is prime. Circle the prime.	

Step 3. If a factor is not prime, write it as the product of two factors and continue the process.	24 is not prime. Break it into 2 more factors. 4 and 6 are not prime. Break them each into two factors. 2 and 3 are prime, so circle them.	
Step 4. Write the composite number as the product of all the circled primes.		$48 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3$

We say $2 \cdot 2 \cdot 2 \cdot 2 \cdot 3$ is the prime factorization of 48. We generally write the primes in ascending order. Be sure to multiply the factors to verify your answer!

If we first factored 48 in a different way, for example as $6 \cdot 8$, the result would still be the same. Finish the prime factorization and verify this for yourself.

Exercise 1.1.20

Find the prime factorization of 80.

Answer

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 5$

Exercise 1.1.21

Find the prime factorization of 60.

Answer

$2 \cdot 2 \cdot 3 \cdot 5$

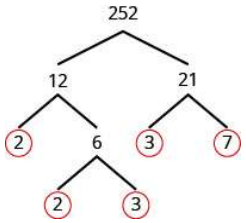
FIND THE PRIME FACTORIZATION OF A COMPOSITE NUMBER.

1. Find two factors whose product is the given number, and use these numbers to create two branches.
2. If a factor is prime, that branch is complete. Circle the prime, like a bud on the tree.
3. If a factor is not prime, write it as the product of two factors and continue the process.
4. Write the composite number as the product of all the circled primes.

Exercise 1.1.22

Find the prime factorization of 252.

Answer

Step 1. Find two factors whose product is 252. 12 and 21 are not prime. Break 12 and 21 into two more factors. Continue until all primes are factored.	
Step 2. Write 252 as the product of all the circled primes.	$252 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 7$

? Exercise 1.1.23

Find the prime factorization of 126.

Answer

$2 \cdot 3 \cdot 3 \cdot 7$

? Exercise 1.1.24

Find the prime factorization of 294.

Answer

$2 \cdot 3 \cdot 7 \cdot 7$

One of the reasons we look at multiples and primes is to use these techniques to find the **least common multiple** of two numbers. This will be useful when we add and subtract fractions with different **denominator**s. Two methods are used most often to find the least common multiple and we will look at both of them.

The first method is the Listing Multiples Method. To find the least common multiple of 12 and 18, we list the first few multiples of 12 and 18:

12: 12, 24, **36**, 48, 60, **72**, 84, 96, **108**...

18: 18, **36**, 54, **72**, 90, **108**...

Common Multiples: 36, 72, 108...

Least Common Multiple: 36

Figure 1.1.8

Notice that some numbers appear in both lists. They are the **common multiples** of 12 and 18.

We see that the first few common multiples of 12 and 18 are 36, 72, and 108. Since 36 is the smallest of the common multiples, we call it the *least common multiple*. We often use the abbreviation LCM.

LEAST COMMON MULTIPLE

The **least common multiple** (LCM) of two numbers is the smallest number that is a multiple of both numbers.

The procedure box lists the steps to take to find the LCM using the prime factors method we used above for 12 and 18.

FIND THE LEAST COMMON MULTIPLE BY LISTING MULTIPLES.

1. List several multiples of each number.
2. Look for the smallest number that appears on both lists.
3. This number is the LCM.

Exercise 1.1.25

Find the least common multiple of 15 and 20 by listing multiples.

Answer

Make lists of the first few multiples of 15 and of 20, and use them to find the least common multiple.	**15:** 15, 30, 45, **60**, 75, 90, 105, 120 **20:** 20, 40, **60**, 80, 100, 120, 140, 160
Look for the smallest number that appears in both lists.	The first number to appear on both lists is 60, so 60 is the least common multiple of 15 and 20.

Notice that 120 is in both lists, too. It is a common multiple, but it is not the *least* common multiple.

Exercise 1.1.26

Find the least common multiple by listing multiples: 9 and 12.

Answer

36

Exercise 1.1.27

Find the least common multiple by listing multiples: 18 and 24.

Answer

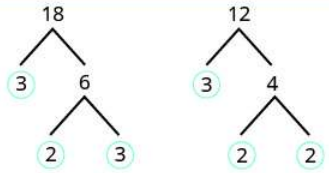
72

Our second method to find the least common multiple of two numbers is to use The Prime Factors Method. Let's find the LCM of 12 and 18 again, this time using their prime factors.

Exercise 1.1.28

Find the Least Common Multiple (LCM) of 12 and 18 using the prime factors method.

Answer

Step 1. Write each number as a product of primes.		
Step 2. List the primes of each number. Match primes vertically when possible.	List the primes of 12. List the primes of 18. Line up with the primes of 12 when possible. If not create a new column.	$12 = 2 \cdot 2 \cdot 3$ $18 = 2 \cdot \quad 3 \cdot 3$
Step 3. Bring down the number from each column.		$12 = 2 \cdot 2 \cdot 3$ $18 = 2 \cdot \quad 3 \cdot 3$ $\text{LCM} = 2 \cdot 2 \cdot 3 \cdot 3$
Step 4. Multiply the factors.		$\text{LCM} = 36$

Notice that the prime factors of $12(2 \cdot 2 \cdot 3)$ and the prime factors of $18(2 \cdot 3 \cdot 3)$ are included in the LCM $(2 \cdot 2 \cdot 3 \cdot 3)$. So 36 is the least common multiple of 12 and 18.

By matching up the common primes, each common prime factor is used only once. This way you are sure that 36 is the *least* common multiple.

Exercise 1.1.29

Find the LCM using the prime factors method: 9 and 12.

Answer

36

Exercise 1.1.30

Find the LCM using the prime factors method: 18 and 24.

Answer

72

FIND THE LEAST COMMON MULTIPLE USING THE PRIME FACTORS METHOD.

1. Write each number as a product of primes.
2. List the primes of each number. Match primes vertically when possible.
3. Bring down the columns.
4. Multiply the factors.

Exercise 1.1.31

Find the Least Common Multiple (LCM) of 24 and 36 using the prime factors method.

Answer

Find the primes of 24 and 36. Match primes vertically when possible. Bring down all columns.	$24 = 2 \cdot 2 \cdot 2 \cdot 3$ $36 = 2 \cdot 2 \cdot \quad 3 \cdot 3$ $\text{LCM} = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$
Multiply the factors.	$\text{LCM} = 72$
	The LCM of 24 and 36 is 72.

Exercise 1.1.32

Find the LCM using the prime factors method: 21 and 28.

Answer

84

Exercise 1.1.33

Find the LCM using the prime factors method: 24 and 32.

Answer

96

Note

Access this online resource for additional instruction and practice with using whole numbers. You will need to enable Java in your web browser to use the application.

- Sieve of Eratosthenes

Key Concepts

- **Place Value** as in Figure.
- **Name a Whole Number in Words**
  1. Start at the left and name the number in each period, followed by the period name.
  2. Put commas in the number to separate the periods.
  3. Do not name the ones period.
- **Write a Whole Number Using Digits**
  1. Identify the words that indicate periods. (Remember the ones period is never named.)
  2. Draw 3 blanks to indicate the number of places needed in each period. Separate the periods by commas.
  3. Name the number in each period and place the digits in the correct place value position.
- **Round Whole Numbers**
  1. Locate the given place value and mark it with an arrow. All digits to the left of the arrow do not change.
  2. Underline the digit to the right of the given place value.
  3. Is this digit greater than or equal to 5?
     - Yes—add 1 to the digit in the given place value.
     - No—do not change the digit in the given place value.
  4. Replace all digits to the right of the given place value with zeros.
- **Divisibility Tests:** A number is divisible by:
  - 2 if the last digit is 0, 2, 4, 6, or 8.
  - 3 if the sum of the digits is divisible by 3.
  - 5 if the last digit is 5 or 0.
  - 6 if it is divisible by both 2 and 3.
  - 10 if it ends with 0.
- **Find the Prime Factorization of a Composite Number**
  1. Find two factors whose product is the given number, and use these numbers to create two branches.
  2. If a factor is prime, that branch is complete. Circle the prime, like a bud on the tree.
  3. If a factor is not prime, write it as the product of two factors and continue the process.
  4. Write the composite number as the product of all the circled primes.
- **Find the Least Common Multiple by Listing Multiples**
  1. List several multiples of each number.
  2. Look for the smallest number that appears on both lists.
  3. This number is the LCM.
- **Find the Least Common Multiple Using the Prime Factors Method**
  1. Write each number as a product of primes.
  2. List the primes of each number. Match primes vertically when possible.
  3. Bring down the columns.

4. Multiply the factors.

Glossary

composite number

A composite number is a counting number that is not prime. A composite number has factors other than 1 and itself.

counting numbers

The counting numbers are the numbers 1, 2, 3, …

divisible by a number

If a number m is a multiple of n, then m is divisible by n. (If 6 is a multiple of 3, then 6 is divisible by 3.)

factors

If $a \cdot b = m$, then a and b are factors of m. Since $3 \cdot 4 = 12$, then 3 and 4 are factors of 12.

least common multiple

The least common multiple of two numbers is the smallest number that is a multiple of both numbers.

multiple of a number

A number is a multiple of n if it is the product of a counting number and n.

number line

A number line is used to visualize numbers. The numbers on the number line get larger as they go from left to right, and smaller as they go from right to left.

origin

The origin is the point labeled 0 on a number line.

prime factorization

The prime factorization of a number is the product of prime numbers that equals the number.

prime number

A prime number is a counting number greater than 1, whose only factors are 1 and itself.

whole numbers

The whole numbers are the numbers 0, 1, 2, 3, ...

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1.1E: Exercises

Practice Makes Perfect

Use Place Value with Whole Numbers

In the following exercises, find the place value of each digit in the given numbers.

Exercise 1.1E.34

51,493

1. 1
2. 4
3. 9
4. 5
5. 3

Answer

1. thousands
2. hundreds
3. tens
4. ten thousands
5. ones

Exercise 1.1E.35

87,210

1. 2
2. 8
3. 0
4. 7
5. 1

Exercise 1.1E.36

164,285

1. 5
2. 6
3. 1
4. 8
5. 2

Answer

1. ones
2. ten thousands
3. hundred thousands
4. tens
5. hundreds

Exercise 1.1E.37

395,076

1. 5
2. 3
3. 7
4. 0
5. 9

Exercise 1.1E.38

93,285,170

1. 9
2. 8
3. 7
4. 5
5. 3

Answer

1. ten millions
2. ten thousands
3. tens
4. thousands
5. millions

Exercise 1.1E.39

36,084,215

1. 8
2. 6
3. 5
4. 4
5. 3

Exercise 1.1E.40

7,284,915,860,132

1. 7
2. 4
3. 5
4. 3
5. 0

Answer

1. trillions
2. billions
3. millions
4. tens
5. thousands

Exercise 1.1E.41

2,850,361,159,433

1. 9
2. 8
3. 6
4. 4
5. 2

In the following exercises, name each number using words.

Exercise 1.1E.42

1,078

Answer

one thousand, seventy-eight

Exercise 1.1E.43

5,902

Exercise 1.1E.44

364,510

Answer

three hundred sixty-four thousand, five hundred ten

Exercise 1.1E.45

146,023

Exercise 1.1E.46

5,846,103

Answer

five million, eight hundred forty-six thousand, one hundred three

Exercise 1.1E.47

1,458,398

Exercise 1.1E.48

37,889,005

Answer

thirty-seven million, eight hundred eighty-nine thousand, five

Exercise 1.1E.49

62,008,465

In the following exercises, write each number as a whole number using digits.

Exercise 1.1E.50

four hundred twelve

Answer

412

Exercise 1.1E.51

two hundred fifty-three

Exercise 1.1E.52

thirty-five thousand, nine hundred seventy-five

Answer

35,975

Exercise 1.1E.53

sixty-one thousand, four hundred fifteen

Exercise 1.1E.54

eleven million, forty-four thousand, one hundred sixty-seven

Answer

11,044,167

Exercise 1.1E.55

eighteen million, one hundred two thousand, seven hundred eighty-three

Exercise 1.1E.56

three billion, two hundred twenty-six million, five hundred twelve thousand, seventeen

Answer

3,226,512,017

Exercise 1.1E.57

eleven billion, four hundred seventy-one million, thirty-six thousand, one hundred six

In the following, round to the indicated place value.

Exercise 1.1E.58

Round to the nearest ten.

1. 386
2. 2,931

Answer

1. 390
2. 2,930

Exercise 1.1E.59

Round to the nearest ten.

1. 792
2. 5,647

Exercise 1.1E.60

Round to the nearest hundred.

1. 13,748
2. 391,794

Answer

1. 13,700
2. 391,800

Exercise 1.1E.61

Round to the nearest hundred.

1. 28,166
2. 481,628

Exercise 1.1E.62

Round to the nearest ten.

1. 1,492
2. 1,497

Answer

1. 1,490
2. 1,500

Exercise 1.1E.63

Round to the nearest ten.

1. 2,791
2. 2,795

Exercise 1.1E.64

Round to the nearest hundred.

1. 63,994
2. 63,940

Answer

1. 64,000
2. 63,900

Exercise 1.1E.65

Round to the nearest hundred.

1. 49,584
2. 49,548

In the following exercises, round each number to the nearest ⓐ hundred, ⓑ thousand, ⓒ ten thousand.

Exercise 1.1E.66

392,546

Answer

1. 392,500
2. 393,000
3. 390,000

Exercise 1.1E.67

619,348

Exercise 1.1E.68

2,586,991

Answer

1. 2,587,000
2. 2,587,000
3. 2,590,000

Exercise 1.1E.69

4,287,965

Identify Multiples and Factors

In the following exercises, use the divisibility tests to determine whether each number is divisible by 2, 3, 5, 6, and 10.

Exercise 1.1E.70

84

Answer

divisible by 2, 3, and 6

Exercise 1.1E.71

9,696

Exercise 1.1E.72

75

Answer

divisible by 3 and 5

Exercise 1.1E.73

78

Exercise 1.1E.74

900

Answer

divisible by 2, 3, 5, 6, and 10

Exercise 1.1E.75

800

Exercise 1.1E.76

986

Answer

divisible by 2

Exercise 1.1E.77

942

Exercise 1.1E.78

350

Answer

divisible by 2, 5, and 10

Exercise 1.1E.79

550

Exercise 1.1E.80

22,335

Answer

divisible by 3 and 5

Exercise 1.1E.81

39,075

Find Prime Factorizations and Least Common Multiples

In the following exercises, find the prime factorization.

Exercise 1.1E.82

86

Answer

$2 \cdot 43$

Exercise 1.1E.83

78

Exercise 1.1E.84

132

Answer

$2 \cdot 2 \cdot 3 \cdot 11$

Exercise 1.1E.85

455

Exercise 1.1E.86

693

Answer

$3 \cdot 3 \cdot 7 \cdot 11$

Exercise 1.1E.87

400

Exercise 1.1E.88

432

Answer

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3$

Exercise 1.1E.89

627

Exercise 1.1E.90

2,160

Answer

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 5$

Exercise 1.1E.91

2,520

In the following exercises, find the least common multiple of the each pair of numbers using the multiples method.

Exercise 1.1E.92

8, 12

Answer

24

Exercise 1.1E.93

4, 3

Exercise 1.1E.94

12, 16

Answer

48

Exercise 1.1E.95

30, 40

Exercise 1.1E.96

20, 30

Answer

60

Exercise 1.1E.97

44, 55

In the following exercises, find the least common multiple of each pair of numbers using the prime factors method.

Exercise 1.1E.98

8, 12

Answer

24

Exercise 1.1E.99

12, 16

Exercise 1.1E.100

28, 40

Answer

280

Exercise 1.1E.101

84, 90

Exercise 1.1E.102

55, 88

Answer

440

Exercise 1.1E.103

60, 72

Everyday Math

Exercise 1.1E.104

Writing a Check Jorge bought a car for $24,493. He paid for the car with a check. Write the purchase price in words.

Answer

twenty-four thousand, four hundred ninety-three dollars

Exercise 1.1E.105

Writing a Check Marissa's kitchen remodeling cost $18,549. She wrote a check to the contractor. Write the amount paid in words.

Exercise 1.1E. 106

Buying a Car Jorge bought a car for $24,493. Round the price to the nearest

1. ten
2. hundred
3. thousand; and
4. ten-thousand.

Answer

1. $24,490
2. $24,500
3. $24,000
4. $20,000

Exercise 1.1E. 107

Remodeling a Kitchen Marissa's kitchen remodeling cost $18,549, Round the cost to the nearest

1. ten
2. hundred
3. thousand and
4. ten-thousand.

Exercise 1.1E. 108

Population The population of China was 1,339,724,852 on November 1, 2010. Round the population to the nearest

1. billion
2. hundred-million; and
3. million.

Answer

1. 1,000,000,000
2. 1,300,000,000
3. 1,340,000,000

Exercise 1.1E. 109

Astronomy The average distance between Earth and the sun is 149,597,888 kilometers. Round the distance to the nearest

1. hundred-million
2. ten-million; and
3. million.

Exercise 1.1E. 110

Grocery Shopping Hot dogs are sold in packages of 10, but hot dog buns come in packs of eight. What is the smallest number that makes the hot dogs and buns come out even?

Answer

40

Exercise 1.1E.111

Grocery Shopping Paper plates are sold in packages of 12 and party cups come in packs of eight. What is the smallest number that makes the plates and cups come out even?

Writing Exercises

Exercise 1.1E.112

Give an everyday example where it helps to round numbers.

Exercise 1.1E.113

If a number is divisible by 2 and by 3 why is it also divisible by 6?

Exercise 1.1E.114

What is the difference between prime numbers and composite numbers?

Answer

Answers may vary.

Exercise 1.1E.115

Explain in your own words how to find the prime factorization of a composite number, using any method you prefer.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
use place value with whole numbers.			
identify multiples and apply divisibility tests.			
find prime factorizations and least common multiples.			

ⓑ If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them. What did you do to become confident of your ability to do these things? Be specific.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is important to make sure you have a strong foundation before you move on. Who can you ask for help? Your fellow classmates and instructor are good resources. Is there a place on campus where math tutors are available? Can your study skills be improved?

…no—I don't get it! This is a warning sign and you must not ignore it. You should get help right away or you will quickly be overwhelmed. See your instructor as soon as you can to discuss your situation. Together you can come up with a plan to get you the help you need.

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1.2: Add and Subtract Integers

Learning Objectives

- Use negatives and opposites
- Simplify: expressions with absolute value
- Add integers
- Subtract integers

A more thorough introduction to the topics covered in this section can be found in the *Prealgebra* chapter, **Integers**.

Use Negatives and Opposites

Our work so far has only included the counting numbers and the whole numbers. But if you have ever experienced a temperature below zero or accidentally overdrawn your checking account, you are already familiar with negative numbers. **Negative numbers** are numbers less than 0. The **negative numbers** are to the left of zero on the number line. See Figure 1.2.1.

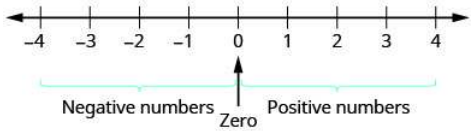


Figure 1.2.1. The number line shows the location of positive and negative numbers.

The arrows on the ends of the number line indicate that the numbers keep going forever. There is no biggest positive number, and there is no smallest negative number.

Is zero a positive or a negative number? Numbers larger than zero are positive, and numbers smaller than zero are negative. Zero is neither positive nor negative.

Consider how numbers are ordered on the number line. Going from left to right, the numbers increase in value. Going from right to left, the numbers decrease in value. See Figure 1.2.2.

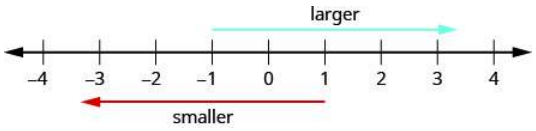


Figure 1.2.2. The numbers on a number line increase in value going from left to right and decrease in value going from right to left.

Doing the Manipulative Mathematics activity “Number Line-part 2” will help you develop a better understanding of integers.

Remember that we use the notation:

$a < b$ (read “a is less than b”) when a is to the left of b on the number line.

$a > b$ (read “a is greater than b”) when a is to the right of b on the number line.

Now we need to extend the number line which showed the whole numbers to include negative numbers, too. The numbers marked by points in Figure 1.2.3. are called the integers. The integers are the numbers $\ldots -3, -2, -1, 0, 1, 2, 3 \ldots$

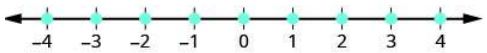


Figure 1.2.3. All the marked numbers are called *integers*.

Example 1.2.1

Order each of the following pairs of numbers, using $<$ or $>$:

1. 14 ___ 6
2. -1 ___ 9
3. -1 ___ -4
4. 2 ___ -20

Solution:

It may be helpful to refer to the number line shown.

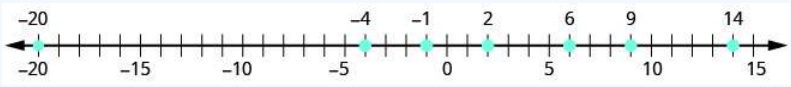


Figure 1.2.4

1. $14 \_\_\_ 6$

   $14 > 6$ 14 is to the right of 6 on the number line.

2. $-1 \_\_\_ 9$

   $-1 < 9$ -1 is to the left of 9 on the number line.

3. $-1 \_\_\_ -4$

   $-1 > -4$ -1 is to the right of -4 on the number line.

4. $2 \_\_\_ -20$

   $2 > -20$ 2 is to the right of -20 on the number line.

Try It 1.2.1

Order each of the following pairs of numbers, using $<$ or $>$:

1. $15 \_\_\_ 7$
2. $-2 \_\_\_ 5$
3. $-3 \_\_\_ -7$
4. $5 \_\_\_ -17$

Answer

1. $<$
2. $>$
3. $<$
4. $>$

Try It 1.2.2

Order each of the following pairs of numbers, using $<$ or $>$:

1. $8 \_\_\_ 13$
2. $3 \_\_\_ -4$
3. $-5 \_\_\_ -2$
4. $9 \_\_\_ -21$

Answer

1. $<$
2. $>$
3. $<$
4. $>$

You may have noticed that, on the **number line**, the negative numbers are a mirror image of the positive numbers, with zero in the middle. Because the numbers 2 and -2 are the same distance from zero, they are called **opposites**. The opposite of 2 is -2, and the opposite of -2 is 2.

OPPOSITE

The **opposite** of a number is the number that is the same distance from zero on the number line but on the opposite side of zero.

Figure 1.2.5 illustrates the definition.



Figure 1.2.5: The opposite of 3 is −3.

Sometimes in algebra the same symbol has different meanings. Just like some words in English, the specific meaning becomes clear by looking at how it is used. You have seen the symbol "−" used in three different ways.

$10-4$ Between two numbers, it indicates the operation of *subtraction*.
We read $10-4$ as "10 minus 4."

-8 In front of a number, it indicates a *negative* number.
We read -8 as "negative eight."

$-x$ In front of a variable, it indicates the *opposite*.
We read $-x$ as "the opposite of x"

$-(-2)$ Here there are two "−" signs.
The one in the parentheses tells us the number is negative 2.
The one outside the parentheses tells us to take the *opposite* of -2.
We read $-(-2)$ as "the opposite of negative two."

OPPOSITE NOTATION

$-a$ means the opposite of the number a.

The notation $-a$ is read as "the opposite of a."

Example 1.2.2

Find:

1. the opposite of 7
2. the opposite of -10
3. the opposite of $-(-6)$

Solution:

1. -7 is the same distance from 0 as 7, but on the opposite side of 0. The opposite of 7 is -7.



2. 10 is the same distance from 0 as -10, but on the opposite side of 0. The opposite of -10 is 10.



3. The opposite of $-(-6)$ is -6.



Try It 1.2.3

Find:

1. the opposite of 4
2. the opposite of -3
3. $-(-1)$

Answer

1. -4
2. 3
3. 1

Try It 1.2.4

Find:

1. the opposite of 8
2. the opposite of -5
3. $-(-5)$

Answer

1. -8
2. 5
3. 5

Our work with opposites gives us a way to define the integers. The whole numbers and their opposites are called the **integers**. The integers are the numbers $\ldots -3, -2, -1, 0, 1, 2, 3 \ldots$

INTEGERS

The whole numbers and their opposites are called the **integers**.

The integers are the numbers

$$\ldots -3, -2, -1, 0, 1, 2, 3 \ldots$$

When evaluating the opposite of a **variable**, we must be very careful. Without knowing whether the variable represents a positive or negative number, we don't know whether −x−x is positive or negative. We can see this in Example 1.2.1.

Example 1.2.3:

Evaluate

1. $-x$, when $x = 8$
2. $-x$, when $x = -8$

Solution:

1.

To evaluate when $x = 8$ means to substitute 8 for x.	
	-x
Substitute 8 for x.	–(8)
Write the opposite of 8.	-8

2.

To evaluate when $x = -8$ means to substitute -8 for $-x$.	
	-x
Substitute -8 for x.	$-(-8)$
Write the opposite of -8.	8

Try It 1.2.5

Evaluate $-n$, when

1. $n = 4$
2. $n = -4$

Answer

1. -4
2. 4

Try It 1.2.6

Evaluate $-m$, when

1. $m = 11$
2. $m = -11$

Answer

1. -11
2. 11

Simplify: Expressions with Absolute Value

We saw that numbers such as 2 and -2 are opposites because they are the same distance from 0 on the number line. They are both two units from 0. The distance between 0 and any number on the number line is called the **absolute value** of that number.

ABSOLUTE VALUE

The **absolute value** of a number is its distance from 0 on the number line.

The absolute value of a number \(n\) is written as $|n|$.

For example,

- -5 is 5 units away from 0, so $|-5| = 5$.
- 5 is 5 units away from 0, so $|5| = 5$.

Figure $1.2.6$ illustrates this idea.

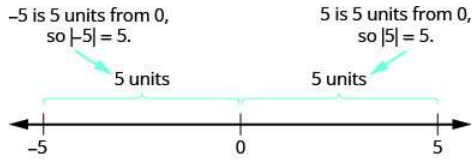


Figure $1.2.6$: The integers 5 and are 5 units away from 0.

The absolute value of a number is never negative (because distance cannot be negative). The only number with absolute value equal to zero is the number zero itself, because the distance from 0 to 0 on the number line is zero units.

PROPERTY OF ABSOLUTE VALUE

$|n| \geq 0$ for all numbers

Absolute values are always greater than or equal to zero!

Mathematicians say it more precisely, "absolute values are always non-negative." Non-negative means greater than or equal to zero.

Example 1.2.4:

Simplify:

1. $|3|$
2. $|-44|$
3. $|0|$

Solution:

The absolute value of a number is the distance between the number and zero. Distance is never negative, so the absolute value is never negative.

1. $|3| = 3$
2. $|-44| = 44$
3. $|0| = 0$

Try It 1.2.7

Simplify:

1. $|4|$
2. $|-28|$
3. $|0|$

Answer

1. 4
2. 28
3. 0

Try It 1.2.8

Simplify:

1. $|-13|$
2. $|47|$

Answer

1. 13
2. 47

In the next example, we'll order expressions with absolute values. Remember, positive numbers are always greater than negative numbers!

Example 1.2.5:

Fill in $<, >$, or $=$ for each of the following pairs of numbers:

1. $|-5|$ ____ $-|-5|$
2. 8 ____ $-|-8|$
3. -9 ____ $-|-9|$
4. $-(-16)$ ____ $-|-16|$

Solution:

1.

Simplification	$\lvert-5\rvert$	____	$-\lvert-5\rvert$
Order.	5	____	-5
	5	$>$	-5
	$\lvert-5\rvert$	$>$	$-\lvert-5\rvert$

2.

Simplification	8	____	$-\lvert-8\rvert$
Order.	8	____	-8
	8	$>$	-8

so

	$\lvert8\rvert$	$>$	$-\lvert-8\rvert$

3.

Simplification	-9	____	$-\lvert-9\rvert$
Order.	-9	____	-9
	-9	$=$	-9

so

	-9	$=$	$-\lvert-9\rvert$

4.

Simplification	$-(-16)$	____	$-\lvert16\rvert$
Order.	16	____	-16
	16	$>$	-16

so

	$-(-16)$	$>$	$-\lvert-16\rvert$

Try It 1.2.9

Fill in $<, >$, or $=$ for each of the following pairs of numbers:

1. $|-9|$ ____ $-|-9|$
2. 2 ____ $-|-2|$
3. -8 ____ $-|-8|$
4. $-(-9)$ ____ $-|-9|$

Answer

1. $>$
2. $>$
3. $<$
4. $>$

Try It 1.2.10

Fill in $<, >,$ or $=$ for each of the following pairs of numbers:

1. 7 ____ $-|-7|$
2. $-(-10)$ ____ $-|-10|$
3. $|-4|$ ____ $-|-4|$
4. -1 ____ $|-1|$

Answer

1. $>$
2. $>$
3. $>$
4. $<$

We now add absolute value bars to our list of grouping symbols. When we use the order of operations, first we simplify inside the absolute value bars as much as possible, then we take the **absolute value** of the resulting number.

GROUPING SYMBOLS

Parentheses	$()$	Braces	$\{\}$
Brackets	$[\,]$	Absolute	$\|\,\|$

In the next example, we simplify the expressions inside absolute value bars first, just like we do with parentheses.

Example 1.2.6:

Simplify: $24-|19-3(6-2)|$

Solution:

	$24-\|19-3(6-2)\|$
Work inside parentheses first: subtract 2 from 6	$24-\|19-3(4)\|$
Multiply $3(4)$	$24-\|19-12\|$
Subtract inside the absolute value bars.	$24-\|7\|$
Take the absolute value.	$24-7$
Subtract.	17

Try It 1.2.11

Simplify: $19-|11-4(3-1)|$

Answer

16

Try It 1.2.12

Simplify: $9-|8-4(7-5)|$

Answer

9

Example 1.2.7

Evaluate:

1. $|x|$ when $x = -35$
2. $|y|$ when $y = -20$
3. $-|u|$ when $u = 12$
4. $-|p|$ when $p = -14$

Solution:

1. $|x|$ when $x = -35$

	$\|x\|$
Substitute -35 for x	$\|-35\|$
Take the absolute value.	35

2. $|y|$ when $y = -20$

	$\|-y\|$
Substitute -20 for y	$\|-(-20)\|$
Simplify	$\|20\|$
Take the absolute value.	20

3. $-|u|$ when $u = 12$

	$-\|u\|$
Substitute 12 for u	$\|-12\|$
Take the absolute value.	-12

4. $-|p|$ when $p = -14$

	$-\|p\|$
Substitute -14 for p	$-\|-14\|$
Take the absolute value.	-14

Try It 1.2.13

Evaluate:

1. $|x|$ when $x = -17$
2. $|y|$ when $y = -39$
3. $-|m|$ when $m = 22$
4. $-|p|$ when $p = -11$

Answer

1. 17
2. 39
3. -22
4. -11

Try It 1.2.14

Evaluate:

1. $|y|$ when $y = -23$
2. $|-y|$ when $y = -21$
3. $-|n|$ when $n = 37$
4. $-|q|$ when $q = -49$

Answer

1. 23
2. 21
3. -37
4. -49

Add Integers

Most students are comfortable with the addition and subtraction facts for positive numbers. But doing addition or subtraction with both positive and negative numbers may be more challenging.

Doing the Manipulative Mathematics activity "Addition of Signed Numbers" will help you develop a better understanding of adding integers."

We will use two color counters to model addition and subtraction of negatives so that you can visualize the procedures instead of memorizing the rules.

We let one color (blue) represent positive. The other color (red) will represent the negatives. If we have one positive counter and one negative counter, the value of the pair is zero. They form a neutral pair. The value of this neutral pair is zero.

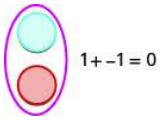
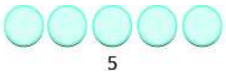


Figure 1.2.7

We will use the counters to show how to add the four addition facts using the numbers $5, -5$ and $3, -3$.

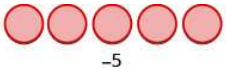
$$5+3 \quad -5+(-3) \quad -5+3 \quad 5+(-3)$$

To add $5+3$, we realize that $5+3$ means the sum of 5 and 3.

We start with 5 positives.	
And then we add 3 positives.	
We now have 8 positives. The sum of 5 and 3 is 8.	

Now we will add $-5+(-3)$. Watch for similarities to the last example $5+3=8$.

To add $-5+(-3)$, we realize this means the sum of -5 and -3.

We start with 5 negatives.	
And then we add 3 negatives.	
We now have 8 negative. The sum of -5 and -3 is -8.	

In what ways were these first two examples similar?

- The first example adds 5 positives and 3 positives—both positives.
- The second example adds 5 negatives and 3 negatives—both negatives.

In each case we got 8 — either 8 positives or 8 negatives.

When the signs were the same, the counters were all the same color, and so we added them.



Figure 1.2.7

? Example 1.2.8

Add:

1. $1+4$
2. $-1+(-4)$

Solution:

1. 

1 positive plus 4 positives is 5 positives.

2. 

1 negative plus 4 negatives is 5 negatives.

? Try It 1.2.15

Add:

1. $2+4$
2. $-2+(-4)$

Answer

1. 6
2. -6

? Try It 1.2.16

Add:

1. $2+5$
2. $-2+(-5)$

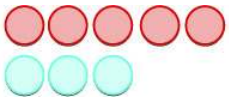
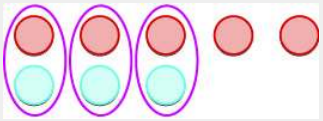
Answer

1. 7
2. -7

So what happens when the signs are different? Let's add $-5+3$. We realize this means the sum of -5 and 3. When the counters were the same color, we put them in a row. When the counters are a different color, we line them up under each other.

Table 1.2.1

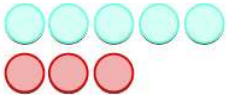
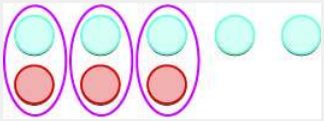
	$-5+3$ means the sum of -5 and 3.
We start with 5 negatives.	

And then we add 3 positives.	
We remove any neutral pairs.	
We have 2 negatives left.	
The sum of -5 and 3 is -2.	$-5+3=2$

Notice that there were more negatives than positives, so the result was negative.

Let's now add the last combination, $5+(-3)$.

Table 1.2.2

	$5+(-3)$ means the sum of -5 and -3.
We start with 5 positives.	
And then we add 3 negatives.	
We remove any neutral pairs.	
We have 2 positives left.	
The sum of 5 and -3 is 2.	$5+(-3)=2$

When we use counters to model addition of positive and negative integers, it is easy to see whether there are more positive or more negative counters. So we know whether the sum will be positive or negative.

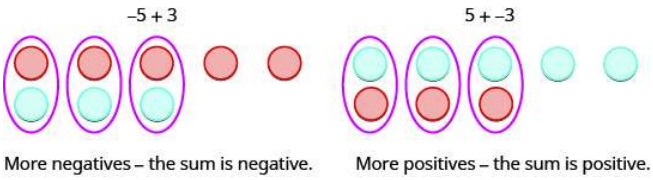


Figure 1.2.8

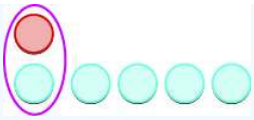
? Example 1.2.9

Add:

1. $-1+5$
2. $1+(-5)$

Solution:

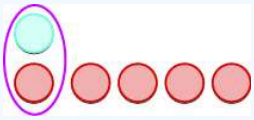
1. $-1+5$



There are more positives, so the sum is positive.

So, $-1+5=4$.

2. $1+(-5)$



There are more negatives, so the sum is negative.

So, $1+(-5)=-4$

? Try It 1.2.17

Add:

1. $-2+4$
2. $2+(-4)$

Answer

1. 2
2. -2

? Try It 1.2.18

Add:

1. $-2+5$
2. $2+(-5)$

Answer

1. 3
2. -3

Now that we have added small positive and negative integers with a model, we can visualize the model in our minds to simplify problems with any numbers.

When you need to add numbers such as $37+(-53)$, you really don't want to have to count out 37 blue counters and 53 red counters. With the model in your mind, can you visualize what you would do to solve the problem?

Picture 37 blue counters with 53 red counters lined up underneath. Since there would be more red (negative) counters than blue (positive) counters, the sum would be *negative*. How many more red counters would there be? Because $53-37=16$, there are 16 more red counters.

Therefore, the sum of $37+(-53)$ is -16.

$$37+(-53)=-16$$

Let's try another one. We'll add $-74+(-27)$. Again, imagine 74 red counters and 27 more red counters, so we'd have 101 red counters. This means the sum is -101.

$$-74+(-27)=-101$$

Let's look again at the results of adding the different combinations of $5, -5$ and $3, -3$.

ADDITION OF POSITIVE AND NEGATIVE INTEGERS

$5+3=8$	$-5+(-3)=-8$
both positive, sum positive	both negative, sum negative

When the signs are the same, the counters would be all the same color, so add them.

$-5+3=-2$	$5+(-3)=2$
different signs, more negatives, sum negative	different signs, more positives, sum positive

When the signs are different, some of the counters would make neutral pairs, so subtract to see how many are left.

Visualize the model as you simplify the expressions in the following examples.

Example 1.2.10

Simplify:

1. $19+(-47)$
2. $-14+(-36)$

Solution:

1. Since the signs are different, we subtract 19 from 47. The answer will be negative because there are more negatives than positives.

Add. $19+(-47)=-28$

2. Since the signs are same, we add. The answer will be negative because there are more negatives than positives.

Add. $-14+(-36)=-50$

Try It 1.2.19

Simplify:

1. $-31+(-19)$
2. $15+(-32)$

Answer

1. -50
2. -17

Try It 1.2.20

Simplify:

1. $-42+(-28)$
2. $25+(-61)$

Answer

1. -70
2. -36

The techniques used up to now extend to more complicated problems, like the ones we've seen before. Remember to follow the order of operations!

Example 1.2.11

Simplify:

$-5+3(-2+7)$

Solution:

	$-5+3(-2+7)$
Simplify inside the parenthesis	$-5+3(5)$
Multiply	$-5+15$
add left to right	10

Try It 1.2.21

Simplify:

$-2+5(-4+7)$

Answer

13

Try It 1.2.22

Simplify:

$-4+2(-3+5)$

Answer

0

Subtract Integers

Note

Doing the Manipulative Mathematics activity "Subtraction of Signed Numbers" will help you develop a better understanding of subtracting integers.

We will continue to use counters to model the subtraction. Remember, the blue counters represent positive numbers and the red counters represent negative numbers.

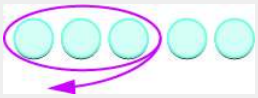
Perhaps when you were younger, you read "$5-3$" as "5 take away 3." When you use counters, you can think of subtraction the same way!

We will model the four subtraction facts using the numbers 5 and 3.

$$5-3 \quad -5-(-3)) \quad -5-3 \quad 5-(-3)$$

To subtract $5-3$, we restate the problem as "5 take away 3."

Table 1.2.3

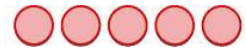
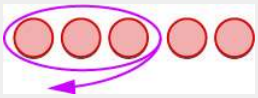
We start with 5 positives.	
We 'take away' 3 positives.	

We have 2 positives left.	
The difference of 5 and 3 is 2.	2

Now we will subtract $-5-(-3)$. Watch for similarities to the last example $5-3=2$.

To subtract $-5-(-3)$, we restate this as "-5 take away -3"

Table 1.2.4

We start with 5 negatives.	
We 'take away' 3 negatives.	
We have 2 negatives left.	
The difference of -5 and -3 is -2.	-2

Notice that these two examples are much alike: The first example, we subtract 3 positives from 5 positives and end up with 2 positives.

In the second example, we subtract 3 negatives from 5 negatives and end up with 2 negatives.

Each example used counters of only one color, and the "take away" model of subtraction was easy to apply.

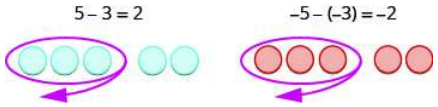


Figure 1.2.9

Example 1.2.12

Subtract:

1. $7-5$
2. $-7-(-5)$

Solution:

1. $7-5$
   2 Take 5 positives from 7 positives and get 2 positives
2. $-7-(-5)$
   -2 Take 5 negatives from 7 negatives and get 2 negatives

Try It 1.2.23

Subtract:

1. $6-4$
2. $-6-(-4)$

Answer

1. 2
2. -2

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Try It 1.2.24

Subtract:

1. $7-4$
2. $-7-(-4)$

Answer

1. 3
2. -3

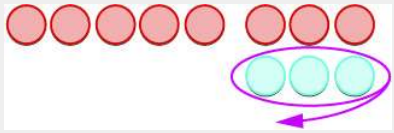
What happens when we have to subtract one positive and one negative number? We'll need to use both white and red counters as well as some **neutral pairs**. Adding a neutral pair does not change the value. It is like changing quarters to nickels—the value is the same, but it looks different.

- To subtract $-5-3$, we restate it as -5 take away 3.

We start with 5 negatives. We need to take away 3 positives, but we do not have any positives to take away.

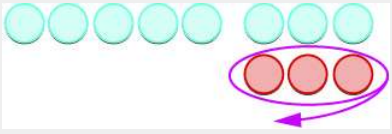
Remember, a neutral pair has value zero. If we add 0 to 5 its value is still 5. We add neutral pairs to the 5 negatives until we get 3 positives to take away.

Table 1.2.5

	$-5-3$ means -5 take away 3
We start with 5 negatives.	
We now add the neutrals needed to get 3 positives.	
We remove the 3 positives.	
We are left with 8 negatives.	
The difference of -5 and 3 is -8.	$-5-3=-8$

And now, the fourth case, $5-(-3)$. We start with 5 positives. We need to take away 3 negatives, but there are no negatives to take away. So we add neutral pairs until we have 3 negatives to take away.

Table 1.2.6

	$5-(-3)$ means 5 take away -3
We start with 5 positives.	
We now add the needed neutrals pairs.	
We remove the 3 negatives.	

We are left with 8 positives.	 8 positives
The difference of 5 and -3 is 8.	$5-(-3)=8$

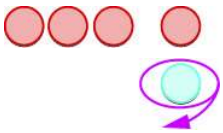
Example 1.2.13

Subtract:

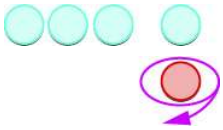
1. $-3-1$
2. $3-(-1)$

Solution:

1.

Take 1 positive from the one added neutral pair.		$-3-1$ -4

2.

Take 1 negative from the one added neutral pair.		$3-(-1)$ 4

Try It 1.2.25

Subtract:

1. $-6-4$
2. $6-(-4)$

Answer

1. -10
2. 10

Try It 1.2.26

Subtract:

1. $-7-4$
2. $7-(-4)$

Answer

1. -11
2. 11

Have you noticed that *subtraction of signed numbers can be done by adding the opposite*? In Exercise 1.2.33, $-3-1$ is the same as $-3+(-1)$ and $3-(-1)$ is the same as $3+1$. You will often see this idea, the **subtraction property**, written as follows:

SUBTRACTION PROPERTY

$$a-b=a+(-b)$$

Subtracting a number is the same as adding its opposite.

Look at these two examples.

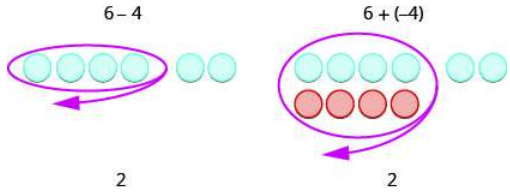


Figure 1.2.10: $6-4$ gives the same answer as $6+(-4)$.

Of course, when you have a subtraction problem that has only positive numbers, like $6-4$, you just do the subtraction. You already knew how to subtract $6-4$ long ago. But *knowing* that $6-4$ gives the same answer as $6+(-4)$ helps when you are subtracting negative numbers. Make sure that you understand how $6-4$ and $6+(-4)$ give the same results!

Example 1.2.14

Simplify:

1. $13-8$ and $13+(-8)$
2. $-17-9$ and $-17+(-9)$

Solution:

1. Subtract. $13-8$ and $13+(-8)$
   5 5
2. Subtract. $-17-9$ and $-17+(-9)$
   -26 -26

Try It 1.2.27

Simplify:

1. $21-13$ and $21+(-13)$
2. $-11-7$ and $-11+(-7)$

Answer

1. 8
2. -18

Try It 1.2.28

Simplify:

1. $15-7$ and $15+(-7)$
2. $-14-8$ and $-14+(-8)$

Answer

1. 8
2. -22

Look at what happens when we subtract a negative.

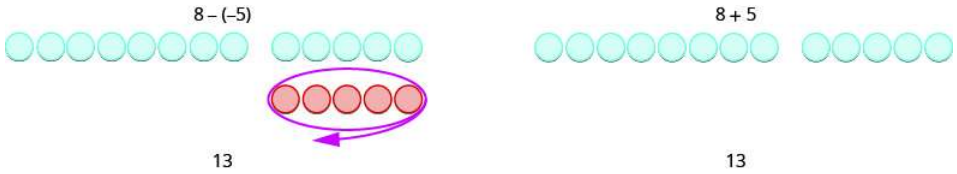


Figure 1.2.11: $8-(-5)$ gives the same answer as $8+5$

Subtracting a negative number is like adding a positive!

You will often see this written as $a-(-b)=a+b$.

Does that work for other numbers, too? Let's do the following example and see.

Example 1.2.15

Simplify:

1. $9-(-15)$ and $9+15$
2. $-7-(-4)$ and $-7+4$

Solution:

1.		$9-(-15)$	$9+15$
	Subtract	24	24
2.		$-7-(-4)$	$-7+4$
	Subtract	-3	-3

Try It 1.2.29

Simplify:

1. $6-(-13)$ and $6+13$
2. $-5-(-1)$ and $-5+1$

Answer

1. 19
2. -4

Try It 1.2.30

Simplify:

1. $4-(-19)$ and $4+19$
2. $-4-(-7)$ and $-4+7$

Answer

1. 23
2. 3

Let's look again at the results of subtracting the different combinations of $5, -5$ and $3, -3$.

SUBTRACTION OF INTEGERS

$5-3$	$-5-(-3)$
2	-2
5 positives take away 3 positives	5 negatives take away 3 negatives
2 positives	2 negatives

When there would be enough counters of the color to take away, subtract.

$-5-3$	$5-(-3)$
-8	8
5 negatives, want to take away 3 positives need neutral pairs	5 positives, want to take away 3 negatives need neutral pairs

When there would be not enough counters of the color to take away, add.

What happens when there are more than three integers? We just use the order of operations as usual.

? Example 1.2.16

Simplify:

$7-(-4-3)-9$

Solution:

	$7-(-4-3)-9$
Simplify inside the parenthesis first.	$7-(-7)-9$
Subtract left to right.	$14-9$
Subtract	5

? Try It 1.2.31

Simplify:

$8-(-3-1)-9$

Answer

3

? Try It 1.2.32

Simplify:

$12-(-9-6)-14$

Answer

12

Access these online resources for additional instruction and practice with adding and subtracting integers. You will need to enable Java in your web browser to use the applications.

- Add Colored Chip
- Subtract Colored Chip

Key Concepts

- **Addition of Positive and Negative Integers**

$5+3$	$-5+(-3)$
8	-8
both positive, sum positive	both negative, sum negative

$-5+3$	$5+(-3)$
-2	2
different signs, more negatives sum negative	different signs, more positives sum positive

- **Property of Absolute Value**:

$|n| \geq 0$ for all numbers. Absolute values are always greater than or equal to zero!

- **Subtraction of Integers**

$5-3$	$-5-(-3)$
2	-2
5 positives take away 3 positives 2 positives	5 negatives take away 3 negatives 2 negatives

$-5-3$	$5-(-3)$
-8	8
5 negatives, want to subtract 3 positives need neutral pairs	5 positives, want to subtract 3 negatives need neutral pairs

- **Subtraction Property:** Subtracting a number is the same as adding its opposite.

Glossary

absolute value

The absolute value of a number is its distance from 0 on the number line. The absolute value of a number nn is written as |n|.

integers

The whole numbers and their opposites are called the integers: $\ldots -3, -2, -1, 0, 1, 2, 3 \ldots$

opposite

The opposite of a number is the number that is the same distance from zero on the number line but on the opposite side of zero: $-a$ means the opposite of the number. The notation $-a$ is read “the opposite of a.”

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1.2E: Exercises

Practice Makes Perfect

Use Negatives and Opposites of Integers

In the following exercises, order each of the following pairs of numbers, using < or >.

Exercise 1.2E.1

1. 9___4
2. −3___6
3. −8___−2
4. 1___−10

Answer

1. >
2. <
3. <
4. >

Exercise 1.2E.2

1. −7___3
2. −10___−5
3. 2___−6
4. 8___9

In the following exercises, find the opposite of each number.

Exercise 1.2E.3

1. 2
2. −6

Answer

1. −2
2. 6

Exercise 1.2E.4

1. 9
2. −4

In the following exercises, simplify.

Exercise 1.2E.5

−(−4)

Answer

4

Exercise 1.2E.6

$-(-8)$

Exercise 1.2E.7

$-(-15)$

Answer

15

Exercise 1.2E.8

$-(-11)$

In the following exercises, evaluate.

Exercise 1.2E.9

$-c$ when

1. $c=12$
2. $c=-12$

Answer

1. -12
2. 12

1. Exercise 1.2E.10

$-d$ when

1. $d=21$
2. $d=-21$

Simplify Expressions with Absolute Value

In the following exercises, simplify.

Exercise 1.2E.11

1. $|-32|$
2. $|0|$
3. $|16|$

Answer

1. 32
2. 0
3. 16

Exercise 1.2E.12

1. $|0|$
2. $|-40|$
3. $|22|$

In the following exercises, fill in <, >, or = for each of the following pairs of numbers.

Exercise 1.2E.13

1. $-6 \_\_\_ |-6|$
2. $-|-3| \_\_\_ -3$

Answer

1. $<$
2. $=$

Exercise 1.2E.14

1. $|-5| \_\_\_ -|-5|$
2. $9 \_\_\_ -|-9|$

In the following exercises, simplify.

Exercise 1.2E.15

$-(-5)$ and $-|-5|$

Answer

$5, -5$

Exercise 1.2E.16

$-|-9|$ and $-(-9)$

Exercise 1.2E.17

$8|-7|$

Answer

56

Exercise 1.2E.18

$5|-5|$

Exercise 1.2E.19

$|15-7|-|14-6|$

Answer

0

Exercise 1.2E.20

$|17-8|-|13-4|$

Exercise 1.2E.21

$18-|2(8-3)|$

Answer

8

Exercise 1.2E.22

$18-|3(8-5)|$

In the following exercises, evaluate.

Exercise 1.2E.23

1. $-|p|$ when $p=19$
2. $-|q|$ when $q=-33$

Answer

1. -19
2. -33

Exercise 1.2E.24

1. $-|a|$ when $a=60$
2. $-|b|$ when $b=-12$

Add Integers

In the following exercises, simplify each expression.

Exercise 1.2E.25

$-21+(-59)$

Answer

-80

Exercise 1.2E.26

$-35+(-47)$

Exercise 1.2E.27

$48+(-16)$

Answer

32

Exercise 1.2E.28

$34+(-19)$

Exercise 1.2E.29

$-14+(-12)+4$

Answer

-22

Exercise 1.2E.30

$-17+(-18)+6$

Exercise 1.2E.31

$135+(-110)+83$

Answer

108

Exercise 1.2E.32

$-38+27+(-8)+12$

Exercise 1.2E.33

$19+2(-3+8)$

Answer

29

Exercise 1.2E.34

$24+3(-5+9)$

Subtract Integers

In the following exercises, simplify.

Exercise 1.2E.35

$8-2$

Answer

6

Exercise 1.2E.36

$-6-(-4)$

Exercise 1.2E.37

$-5-4$

Answer

-9

Exercise 1.2E.38

$-7-2$

Exercise 1.2E.39

$8-(-4)$

Answer

12

Exercise 1.2E.40

$7-(-3)$

Exercise 1.2E.41

1. $44-28$
2. $44+(-28)$

Answer

1. 16
2. 16

Exercise 1.2E.42

1. $35-16$
2. $35+(-16)$

Exercise 1.2E.43

1. $27-(-18)$
2. $27+18$

Answer

1. 45
2. 45

Exercise 1.2E.44

1. $46-(-37)$
2. $46+37$

In the following exercises, simplify each expression.

Exercise 1.2E.45

$15-(-12)$

Answer

27

Exercise 1.2E.46

$14-(-11)$

Exercise 1.2E.47

$48-87$

Answer

-39

Exercise 1.2E.48

$45-69$

Exercise 1.2E.49

$-17-42$

Answer

-59

Exercise 1.2E.50

$-19-46$

Exercise 1.2E.51

$-103-(-52)$

Answer

-51

Exercise 1.2E.52

$-105-(-68)$

Exercise 1.2E.53

$-45-(-54)$

Answer

9

Exercise 1.2E.54

$-58-(-67)$

Exercise 1.2E.55

$8-3-7$

Answer

-2

Exercise 1.2E.56

$9-6-5$

Exercise 1.2E.57

$-5-4+7$

Answer

-2

Exercise 1.2E.58

$-3-8+4$

Exercise 1.2E.59

$-14-(-27)+9$

Answer

22

Exercise 1.2E.60

$64+(-17)-9$

Exercise 1.2E.61

$(2-7)-(3-8)$

Answer

0

Exercise 1.2E.62

$(1-8)-(2-9)$

Exercise 1.2E.63

$-(6-8)-(2-4)$

Answer

4

Exercise 1.2E.64

$-(4-5)-(7-8)$

Exercise 1.2E.65

$25-[10-(3-12)]$

Answer

6

Exercise 1.2E.66

$32-[5-(15-20)]$

Exercise 1.2E.67

$6.3-4.3-7.2$

Answer

-5.2

Exercise 1.2E.68

$5.7-8.2-4.9$

Exercise 1.2E.69

5^2-6^2

Answer

-11

Exercise 1.2E.70

6^2-7^2

Everyday Math

Exercise 1.2E.71

Elevation The highest elevation in the United States is Mount McKinley, Alaska, at 20,320 feet above sea level. The lowest elevation is Death Valley, California, at 282 feet below sea level.

Use integers to write the elevation of:

1. Mount McKinley.
2. Death Valley.

Answer

1. 20,329
2. -282

Exercise 1.2E.72

Extreme temperatures The highest recorded temperature on Earth was 58° Celsius, recorded in the Sahara Desert in 1922. The lowest recorded temperature was 90° below 0° Celsius, recorded in Antarctica in 1983.

Use integers to write the:

1. highest recorded temperature.
2. lowest recorded temperature.

Exercise 1.2E.73

State budgets In June, 2011, the state of Pennsylvania estimated it would have a budget surplus of $540 million. That same month, Texas estimated it would have a budget deficit of $27 billion.

Use integers to write the budget of:

1. Pennsylvania.
2. Texas.

Answer

1. $540 million
2. −$27 billion

Exercise 1.2E.74

College enrollments Across the United States, community college enrollment grew by 1,400,000 students from Fall 2007 to Fall 2010. In California, community college enrollment declined by 110,171 students from Fall 2009 to Fall 2010.

Use integers to write the change in enrollment:

1. in the U.S. from Fall 2007 to Fall 2010.
2. in California from Fall 2009 to Fall 2010.

Exercise 1.2E.75

Stock Market The week of September 15, 2008 was one of the most volatile weeks ever for the US stock market. The closing numbers of the Dow Jones Industrial Average each day were:

Monday	−504
Tuesday	+142
Wednesday	−449
Thursday	+410
Friday	+369

What was the overall change for the week? Was it positive or negative?

Answer

-32

Exercise 1.2E.76

Stock Market During the week of June 22, 2009, the closing numbers of the Dow Jones Industrial Average each day were:

Monday	−201
Tuesday	−16
Wednesday	−23
Thursday	+172
Friday	−34

What was the overall change for the week? Was it positive or negative?

Writing Exercises

Exercise 1.2E.77

Give an example of a negative number from your life experience.

Answer

Answers may vary

Exercise 1.2E.78

What are the three uses of the "−" sign in algebra? Explain how they differ.

Exercise 1.2E.79

Explain why the sum of −8 and 2 is negative, but the sum of 8 and −2 is positive.

Answer

Answers may vary

Exercise 1.2E.80

Give an example from your life experience of adding two negative numbers.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
use negatives and opposites of integers.			
simplify expressions with absolute value.			
add integers.			
subtract integers.			

ⓑ What does this checklist tell you about your mastery of this section? What steps will you take to improve?

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1.3: Multiply and Divide Integers

Learning Objectives

By the end of this section, you will be able to:

- Multiply integers
- Divide integers
- Simplify expressions with integers
- Evaluate variable expressions with integers
- Translate English phrases to algebraic expressions
- Use integers in applications

A more thorough introduction to the topics covered in this section can be found in the *Prealgebra* chapter, **Integers**.

Multiply Integers

Since multiplication is mathematical shorthand for repeated addition, our model can easily be applied to show **multiplication** of **integers**. Let's look at this concrete model to see what patterns we notice. We will use the same examples that we used for addition and subtraction. Here, we will use the model just to help us discover the pattern.

We remember that $a \cdot b$ means add a, b times. Here, we are using the model just to help us discover the pattern.

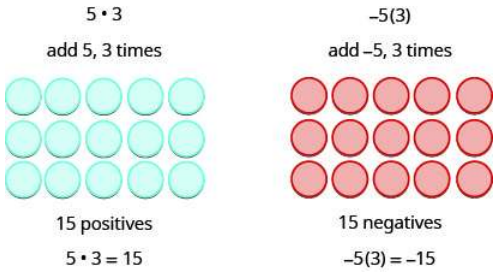


Figure 1.3.1

The next two examples are more interesting.

What does it mean to multiply 5 by -3? It means subtract $5, 3$ times. Looking at subtraction as "taking away," it means to take away $5, 3$ times. But there is nothing to take away, so we start by adding neutral pairs on the workspace. Then we take away 5 three times.

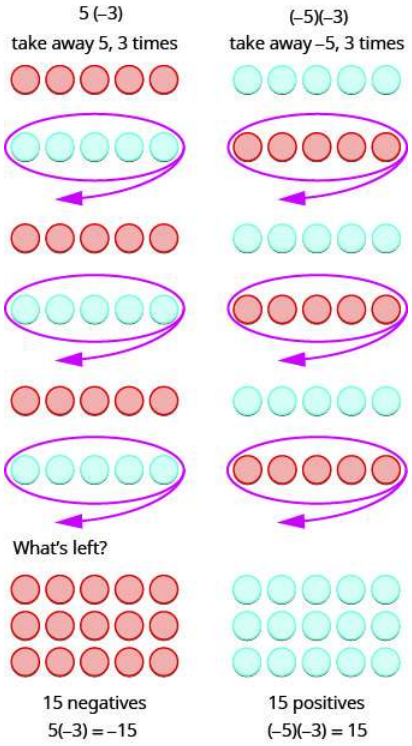


Figure 1.3.2

In summary:

$$5 \cdot 3 = 15 \qquad -5(3) = -15$$
$$5(-3) = -15 \qquad (-5)(-3) = 15$$

Notice that for multiplication of two signed numbers, when the:

- signs are the *same*, the product is *positive*.
- signs are *different*, the product is *negative*.

We'll put this all together in the chart below.

MULTIPLICATION OF SIGNED NUMBERS

For multiplication of two signed numbers:

Table 1.3.1

Same signs	Product	Example
Two positives	Positive	$7 \cdot 4 = 28$
Two negatives	Positive	$-8(-6) = 48$

Table 1.3.2

Different signs	Product	Example
Positive · negative	Negative	$7(-9) = -63$
Negative · positive	Negative	$-5 \cdot 10 = -50$

Example 1.3.1

Multiply:

1. $-9 \cdot 3$
2. $-2(-5)$
3. $4(-8)$
4. $7 \cdot 6$

Solution

1. $-9 \cdot 3 = -27$ Multiply, noting that the signs are different, so the product is negative.
2. $-2(-5) = 10$ Multiply, noting that the signs are same, so the product is positive.
3. $4(-8) = -32$ Multiply, with different signs.
4. $7 \cdot 6 = 42$ Multiply, with the same signs.

Try It 1.3.2

Multiply:

1. $-6 \cdot 8$
2. $-4(-7)$
3. $9(-7)$
4. $5 \cdot 12$

Answer

1. -48
2. 28
3. -63
4. 60

Try It 1.3.3

Multiply:

1. $-8 \cdot 7$
2. $-6(-9)$
3. $7(-4)$
4. $3 \cdot 13$

Answer

1. -56
2. 54
3. -28
4. 39

When we multiply a number by 1, the result is the same number. What happens when we multiply a number by -1? Let's multiply a positive number and then a negative number by -1 to see what we get.

Multiply.	Multiply.
$-1 \cdot 4 = -4$	$-1(-3) = 3$
Note that -4 is the opposite of 4.	Note that 3 is the opposite of -3

Each time we multiply a number by -1, we get its opposite!

MULTIPLICATION BY −1

$$-1a = -a$$

Multiplying a number by -1 gives its opposite.

Example 1.3.4

Multiply:

1. $-1 \cdot 7$
2. $-1(-11)$

Solution

1.		$-1 \cdot 7$
	Multiply, noting that the signs are different so the product is negative.	-7
		-7 is the opposite of 7.
2.		$-1(-11)$
	Multiply, noting that the signs are the same so the product is positive.	11
		11 is the opposite of -11.

Try It 1.3.5

Multiply:

1. $-1 \cdot 9$
2. $-1 \cdot (-17)$

Answer

1. -9
2. 17

Try It 1.3.6

Multiply:

1. $-1 \cdot 8$
2. $-1 \cdot (-16)$

Answer

1. -8
2. 16

Divide Integers

What about **division**? Division is the inverse operation of multiplication. So, $15 \div 3 = 5$ because $5 \cdot 3 = 15$. In words, this expression says that 15 can be divided into three groups of five each because adding five three times gives 15. Look at some examples of multiplying integers, to figure out the rules for dividing integers.

$$\begin{aligned} 5 \cdot 3 &= 15 \quad \text{so} \quad 15 \div 3 = 5 \\ -5 \cdot 3 &= -15 \quad \text{so} \quad -15 \div 3 = -5 \\ (5)(-3) &= -15 \quad \text{so} \quad -15 \div (-3) = 5 \\ (-5)(-3) &= 15 \quad \text{so} \quad 15 \div (-3) = -5 \end{aligned}$$

Division follows the same rules as multiplication!

For division of two signed numbers, when the:

- signs are the *same*, the quotient is *positive*.
- signs are *different*, the quotient is *negative*.

And remember that we can always check the answer of a division problem by multiplying.

MULTIPLICATION AND DIVISION OF SIGNED NUMBERS

For multiplication and division of two signed numbers:

- If the signs are the same, the result is positive.
- If the signs are different, the result is negative.

Table 1.3.3

Same signs	Result
Two positives	Positive
Two negatives	Positive
If the signs are the same, the result is positive.	

Table 1.3.4

Different signs	Result
Positive and negative	Negative
Negative and positive	Negative
If the signs are different, the result is negative.	

Example 1.3.7

1. $-27 \div 3$
2. $-100 \div (-4)$

Solution

1. $-27 \div 3$
   Divide, with different signs, the quotient is negative. -9
2. $-100 \div (-4)$
   Divide, with signs that are the same the quotient is negative. 25

Try It 1.3.8

Divide:

1. $-42 \div 6$
2. $-117 \div (-3)$

Answer

1. -7
2. 39

Try It 1.3.9

Divide:

1. $-63 \div 7$
2. $-115 \div (-5)$

Answer

1. -9
2. 23

Simplify Expressions with Integers

What happens when there are more than two numbers in an expression? The order of operations still applies when negatives are included. Remember My Dear Aunt Sally?

Let's try some examples. We'll simplify expressions that use all four operations with integers—addition, subtraction, multiplication, and division. Remember to follow the order of operations.

Example 1.3.10

Simplify:

$7(-2)+4(-7)-6$

Solution

	$7(-2)+4(-7)-6$
Multiply first.	$-14+(-28)-6$
Add.	$-42-6$
Subtract	-48

Try It 1.3.11

Simplify:

$8(-3)+5(-7)-4$

Answer

-63

Try It 1.3.12

Simplify:

$9(-3)+7(-8)-1$

Answer

-84

Example 1.3.13

Simplify:

1. $(-2)^4$
2. -2^4

Solution

1. $(-2)^4$

Write in expanded form.	$(-2)(-2)(-2)(-2)$
Multiply	$4(-2)(-2)$
Multiply	$-8(-2)$
Multiply	16

2. -2^4

Write in expanded form. We are asked to find the opposite of 2^4.	$-(2\cdot 2\cdot 2\cdot 2)$
Multiply	$-(4\cdot 2\cdot 2)$
Multiply	$-(8\cdot 2)$
Multiply	-16

Notice the difference in parts (1) and (2). In part (1), the exponent means to raise what is in the parentheses, the (-2) to the 4^{th} power. In part (2), the exponent means to raise just the 2 to the 4^{th} power and then take the opposite.

? Try It 1.3.14

Simplify:

1. $(-3)^4$
2. -3^4

Answer

1. 81
2. -81

? Try It 1.3.15

Simplify:

1. $(-7)^2$
2. -7^2

Answer

1. 49
2. -49

The next example reminds us to simplify inside parentheses first.

✓ Example 1.3.16

Simplify:

$12-3(9-12)$

Solution

$12-3(9-12)$	
$=12-3(-3)$	Subtract in parentheses first.
$=12-(-9)$	Multiply.
$=12+9$	Add the opposite.
$=21$	

Try It 1.3.17

Simplify:

$17-4(8-11)$

Answer

29

Try It 1.3.18

Simplify:

$16-6(7-13)$

Answer

52

Example 1.3.19

Simplify:

$8(-9)\div(-2)^3$

Solution

	$8(-9)\div(-2)^3$
Exponents first	$8(-9)\div(-8)$
Multiply.	$-72\div(-8)$
Divide	9

Try It 1.3.20

Simplify:

$12(-9)\div(-3)^3$

Answer

4

Try It 1.3.21

Simplify:

$18(-4)\div(-2)^3$

Answer

9

Example 1.3.22

Simplify:

$-30\div 2+(-3)(-7)$

Solution

	$-30 \div 2 + (-3)(-7)$
Multiply and divide left to right, so divide first.	$-15 + (-3)(-7)$
Multiply.	$-15 + 21$
Add	6

Try It 1.3.23

Simplify:

$-27 \div 3 + (-5)(-6)$

Answer

21

Try It 1.3.24

Simplify:

$-32 \div 4 + (-2)(-7)$

Answer

6

Evaluate Variable Expressions with Integers

Remember that to evaluate an expression means to substitute a number for the variable in the expression. Now we can use negative numbers as well as positive numbers.

Example 1.3.25

When $n = -5$, evaluate:

1. $n + 1$
2. $-n + 1$.

Solution

1.

	$n + 1$
Substitute -5 for n	$-5 + 1$
Simplify.	-4

2.

	$n + 1$
Substitute -5 for n	$-(-5) + 1$
Simplify.	$5 + 1$
Add.	6

Try It 1.3.26

When $n = -8$, evaluate:

1. $n + 2$
2. $-n + 2$.

Answer

1. -6
2. 10

Try It 1.3.27

When $y = -9$, evaluate:

1. $y + 8$
2. $-y + 8$.

Answer

1. -1
2. 17

Example 1.3.28

Evaluate $(x + y)^2$ when $x = -18$ and $y = 24$.

Solution

	$(x+y)^2$
Substitute -18 for x and 24 for y	$(-18+24)^2$
Add inside parentheses	$(6)^2$
Simplify.	36

Try It 1.3.29

Evaluate $(x + y)^2$ when $x = -15$ and $y = 29$.

Answer

196

Try It 1.3.30

Evaluate $(x + y)^3$ when $x = -8$ and $y = 10$.

Answer

8

Example 1.3.31

Evaluate $20 - z$ when

1. $z = 12$
2. $z = -12$

Solution

1.

	$20 - z$
Substitute 12 for z.	$20 - 12$
Subtract	8

2.

	$20 - z$
Substitute -12 for z.	$20 - (-12)$
Subtract	32

Try It 1.3.32

Evaluate $17 - k$ when

1. $k = 19$
2. $k = -19$

Answer

1. -2
2. 36

Try It 1.3.33

Evaluate $-5 - b$ when

1. $b = 14$
2. $b = -14$

Answer

1. -19
2. 9

Example 1.3.34

Evaluate:

$2x^2 + 3x + 8$ when $x = 4$.

Solution

Substitute 4 for x. Use parentheses to show multiplication.

	$2x^2 + 3x + 8$
Substitute	$2(4)^2 + 3(4) + 8$
Evaluate exponents.	$2(16) + 3(4) + 8$
Multiply.	$32 + 12 + 8$
Add.	52

Try It 1.3.35

Evaluate:

$3x^2 - 2x + 6$ when $x = -3$.

Answer

39

Try It 1.3.36

Evaluate:

$4x^2 - x - 5$ when $x = -2$.

Answer

13

Translate Phrases to Expressions with Integers

Our earlier work translating English to algebra also applies to phrases that include both positive and negative numbers.

Example 1.3.37

Translate and simplify: the sum of 8 and -12, increased by 3.

Solution

	the **sum** of 8 and -12, increased by 3
Translate.	$[8+(-12)]+3$
Simplify. Be careful not to confuse the brackets with an absolute value sign.	$(-4)+3$
Add.	-1

Try It 1.3.38

Translate and simplify: the sum of 9 and -16, increased by 4.

Answer

$(9+(-16))+4-3$

Try It 1.3.39

Translate and simplify: the sum of -8 and -12, increased by 7.

Answer

$(-8+(-12))+7-13$

When we first introduced the operation symbols, we saw that the expression may be read in several ways. They are listed in the chart below.

Table 1.3.5

$a-b$
a minus b
the difference of a and b
b subtracted from a
b less than a

Be careful to get a and b in the right order!

Example 1.3.40

Translate and then simplify

1. the difference of 13 and -21
2. subtract 24 from -19.

Solution

1.

	the **difference** of 13 and -21
Translate.	$13-(-21)$
Simplify.	34

2.

	subtract 24 **from** -19
Translate.	$-19-24$
Remember, subtract b from a means $a-b$	
Simplify.	-43

Try It 1.3.41

Translate and simplify

1. the difference of 14 and -23
2. subtract 21 from -17.

Answer

1. $14-(-23); 37$
2. $-17-21; -38$

Try It 1.3.42

Translate and simplify

1. the difference of 11 and -19
2. subtract 18 from -11.

Answer

1. $11-(-19); 30$
2. $-11-18; -29$

Once again, our prior work translating English to algebra transfers to phrases that include both multiplying and dividing integers. Remember that the key word for **multiplication** is "**product**" and for **division** is "**quotient**."

Example 1.3.43

Translate to an algebraic expression and simplify if possible: the product of -2 and 14.

Solution

	the product of -2 and 14
Translate.	$(-2)(14)$
Simplify.	-28

Try It 1.3.44

Translate to an algebraic expression and simplify if possible: the product of -5 and 12.

Answer

$-5(12); -60$

Try It 1.3.45

Translate to an algebraic expression and simplify if possible: the product of 8 and -13.

Answer

$-8(13); -104$

Example 1.3.46

Translate to an algebraic expression and simplify if possible: the quotient of -56 and -7.

Solution

$$\begin{array}{ll} & \text{the quotient of } -56 \text{ and } -7 \\ \text{Translate.} & -56 \div (-7) \\ \text{Simplify.} & 8 \end{array} \quad (1.3.1)$$

Try It 1.3.47

Translate to an algebraic expression and simplify if possible: the quotient of -63 and -9.

Answer

$-63 \div (-9); 7$

Try It 1.3.48

Translate to an algebraic expression and simplify if possible: the quotient of -72 and -9.

Answer

$-72 \div (-9); 8$

Use Integers in Applications

We'll outline a plan to solve applications. It's hard to find something if we don't know what we're looking for or what to call it! So when we solve an application, we first need to determine what the problem is asking us to find. Then we'll write a phrase that gives the information to find it. We'll translate the phrase into an expression and then simplify the expression to get the answer. Finally, we summarize the answer in a sentence to make sure it makes sense.

How to Apply a Strategy to Solve Applications with Integers

Example 1.3.49

The temperature in Urbana, Illinois one morning was 11 degrees. By mid-afternoon, the temperature had dropped to -9 degrees. What was the difference of the morning and afternoon temperatures?

Solution

Step 1. Read the problem. Make sure all the words and ideas are understood.	
Step 2. Identify what we are asked to find.	the difference of the morning and afternoon temperatures
Step 3. Write a phrase that gives the information to find it.	the difference of 11 and -9
Step 4. Translate the phrase to an expression.	$11 - (-9)$
Step 5. Simplify the expression.	20
Step 6. Write a complete sentence that answers the question.	The difference in temperatures was 20 degrees.

Try It 1.3.50

The temperature in Anchorage, Alaska one morning was 15 degrees. By mid-afternoon the temperature had dropped to 30 degrees below zero. What was the difference in the morning and afternoon temperatures?

Answer

The difference in temperatures was 45 degrees.

Try It 1.3.51

The temperature in Denver was -6 degrees at lunchtime. By sunset the temperature had dropped to -15 degrees. What was the difference in the lunchtime and sunset temperatures?

Answer

The difference in temperatures was 9 degrees.

APPLY A STRATEGY TO SOLVE APPLICATIONS WITH INTEGERS.

1. Read the problem. Make sure all the words and ideas are understood
2. Identify what we are asked to find.
3. Write a phrase that gives the information to find it.
4. Translate the phrase to an expression.
5. Simplify the expression.
6. Answer the question with a complete sentence.

Example 1.3.52

The Mustangs football team received three penalties in the third quarter. Each penalty gave them a loss of fifteen yards. What is the number of yards lost?

Solution

Step 1. Read the problem. Make sure all the words and ideas are understood.	
Step 2. Identify what we are asked to find.	the number of yards lost
Step 3. Write a phrase that gives the information to find it.	three times a 15-yard penalty
Step 4. Translate the phrase to an expression.	$3(-15)$
Step 5. Simplify the expression.	-45
Step 6. Write a complete sentence that answers the question.	The team lost 45 yards.

Try It 1.3.53

The Bears played poorly and had seven penalties in the game. Each penalty resulted in a loss of 15 yards. What is the number of yards lost due to penalties?

Answer

The Bears lost 105 yards.

? Try It 1.3.54

Bill uses the ATM on campus because it is convenient. However, each time he uses it he is charged a $2 fee. Last month he used the ATM eight times. How much was his total fee for using the ATM?

Answer

A $16 fee was deducted from his checking account.

Key Concepts

- **Multiplication and Division of Two Signed Numbers**
  - Same signs—Product is positive
  - Different signs—Product is negative
- **Strategy for Applications**
  1. Identify what you are asked to find.
  2. Write a phrase that gives the information to find it.
  3. Translate the phrase to an expression.
  4. Simplify the expression.
  5. Answer the question with a complete sentence.

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1.3E: Exercises

Practice Makes Perfect

Multiply Integers

In the following exercises, multiply.

Exercise 1.3E.55

$-4 \cdot 8$

Answer

-32

Exercise 1.3E.56

$-3 \cdot 9$

Exercise 1.3E.57

$9(-7)$

Answer

-63

Exercise 1.3E.58

$13(-5)$

Exercise 1.3E.59

$-1 \cdot 6$

Answer

-6

Exercise 1.3E.60

$-1 \cdot 3$

Exercise 1.3E.61

$-1(-14)$

Answer

14

Exercise 1.3E.62

$-1(-19)$

Divide Integers

In the following exercises, divide.

Exercise 1.3E.63

$-24 \div 6$

Answer

-4

Exercise 1.3E.64

$35 \div (-7)$

Exercise 1.3E.65

$-52 \div (-4)$

Answer

13

Exercise 1.3E.66

$-84 \div (-6)$

Exercise 1.3E.67

$-180 \div 15$

Answer

-12

Exercise 1.3E.68

$-192 \div 12$

Simplify Expressions with Integers

In the following exercises, simplify each expression.

Exercise 1.3E.69

$5(-6)+7(-2)-3$

Answer

-47

Exercise 1.3E.70

$8(-4)+5(-4)-6$

Exercise 1.3E.71

$(-2)^6$

Answer

64

Exercise 1.3E.72

$(-3)^5$

Exercise 1.3E.73

$(-4)^2$

Answer

-16

Exercise 1.3E.74

$(-6)^2$

Exercise 1.3E.75

$-3(-5)(6)$

Answer

90

Exercise 1.3E.76

$-4(-6)(3)$

Exercise 1.3E.77

$(8-11)(9-12)$

Answer

9

Exercise 1.3E.78

$(6-11)(8-13)$

Exercise 1.3E.79

$26-3(2-7)$

Answer

41

Exercise 1.3E.80

$23-2(4-6)$

Exercise 1.3E.81

$65 \div (-5) + (-28) \div (-7)$

Answer

-9

Exercise 1.3E.82

$52 \div (-4) + (-32) \div (-8)$

Exercise 1.3E.83

$9-2[3-8(-2)]$

Answer

-29

Exercise 1.3E.84

$11-3[7-4(-20)]$

Exercise 1.3E.85

$(-3)^2 - 24 \div (8-2)$

Answer

5

Exercise 1.3E.86

$(-4)^2 - 32 \div (12-4)$

Evaluate Variable Expressions with Integers

In the following exercises, evaluate each expression.

Exercise 1.3E.87

y+(−14) when

1. y=−33
2. y=30

Answer

1. −47
2. 16

Exercise 1.3E.88

x+(−21) when

1. x=−27
2. x=44

Exercise 1.3E.89

1. a+3 when a=−7
2. −a+3 when a=−7

Answer

1. −4
2. 10

Exercise 1.3E.90

1. d+(−9) when d=−8
2. −d+(−9) when d=−8

Exercise 1.3E.91

m+n when
m=−15,n=7

Answer

-8

Exercise 1.3E.92

p+q when
p=−9,q=17

Exercise 1.3E.93

r+s when r=−9,s=−7

Answer

-16

Exercise 1.3E.94

t+u when t=−6,u=−5

Exercise 1.3E.95

$(x+y)^2$ when
x=−3,y=14

Answer

121

Exercise 1.3E.96

$(y+z)^2$ when
y=−3, z=15

Exercise 1.3E.97

−2x+17 when

1. x=8
2. x=−8

Answer

1. 1
2. 33

Exercise 1.3E.98

−5y+14 when

1. y=9
2. y=−9

Exercise 1.3E.99

10−3m when

1. m=5
2. m=−5

Answer

1. −5
2. 25

Exercise 1.3E.100

18−4n when

1. n=3
2. n=−3

Exercise 1.3E.101

$2w^2-3w+7$ when
w=−2

Answer

21

Exercise 1.3E.102

$3u^2-4u+5$

Exercise 1.3E.103

9a−2b−8 when
a=−6 and b=−3

Answer

-56

Exercise 1.3E.104

7m−4n−2 when
m=−4 and n=−9

Translate English Phrases to Algebraic Expressions

In the following exercises, translate to an algebraic expression and simplify if possible.

Exercise 1.3E.105

the sum of 3 and −15, increased by 7

Answer

(3+(−15))+7;−5

Exercise 1.3E.106

the sum of −8 and −9, increased by 23

Exercise 1.3E.107

the difference of 10 and −18

Answer

10−(−18);28

Exercise 1.3E.108

subtract 11 from −25

Exercise 1.3E.109

the difference of −5 and −30

Answer

−5−(−30);25

Exercise 1.3E.110

subtract −6 from −13

Exercise 1.3E.111

the product of −3 and 15

Answer

$-3 \cdot 15; -45$

Exercise 1.3E.112

the product of −4 and 16

Exercise 1.3E.113

the quotient of −60 and −20

Answer

$-60 \div (-20)\,; 3$

Exercise 1.3E.114

the quotient of −40 and −20

Exercise 1.3E.115

the quotient of −6 and the sum of *a* and *b*

Answer

$\frac{-6}{a+b}$

Exercise 1.3E.116

the quotient of −6 and the sum of *a* and *b*

Exercise 1.3E.117

the product of −10 and the difference of p and q

Answer

−10(p−q)

Exercise 1.3E.118

the product of −13 and the difference of c and d

Use Integers in Applications

In the following exercises, solve.

Exercise 1.3E.119

Temperature On January 15, the high temperature in Anaheim, California, was 84°. That same day, the high temperature in Embarrass, Minnesota was −12°. What was the difference between the temperature in Anaheim and the temperature in Embarrass?

Answer

96°

Exercise 1.3E.120

Temperature On January 21, the high temperature in Palm Springs, California, was 89°, and the high temperature in Whitefield, New Hampshire was −31°. What was the difference between the temperature in Palm Springs and the temperature in Whitefield?

Exercise 1.3E.121

Football At the first down, the Chargers had the ball on their 25 yard line. On the next three downs, they lost 6 yards, gained 10 yards, and lost 8 yards. What was the yard line at the end of the fourth down?

Answer

21

Exercise 1.3E.122

Football At the first down, the Steelers had the ball on their 30 yard line. On the next three downs, they gained 9 yards, lost 14 yards, and lost 2 yards. What was the yard line at the end of the fourth down?

Exercise 1.3E.123

Checking Account Mayra has $124 in her checking account. She writes a check for $152. What is the new balance in her checking account?

Answer

−$28

Exercise 1.3E.124

Checking Account Selina has $165 in her checking account. She writes a check for $207. What is the new balance in her checking account?

Exercise 1.3E.125

Checking Account Diontre has a balance of −$38 in his checking account. He deposits $225 to the account. What is the new balance?

Answer

$187

Exercise 1.3E.126

Checking Account Reymonte has a balance of −$49 in his checking account. He deposits $281 to the account. What is the new balance?

Everyday Math

Exercise 1.3E.127

Stock market Javier owns 300 shares of stock in one company. On Tuesday, the stock price dropped $12 per share. What was the total effect on Javier's portfolio?

Answer

Weight loss In the first week of a diet program, eight women lost an average of 3 pounds each. What was the total weight change for the eight women?

Exercise 1.3E.128

Weight loss In the first week of a diet program, eight women lost an average of 3 pounds each. What was the total weight change for the eight women?

Writing Exercises

Exercise 1.3E.129

In your own words, state the rules for multiplying integers.

Answer

Answers may vary

Exercise 1.3E.130

In your own words, state the rules for dividing integers.

Exercise 1.3E.131

Why is $-2^4 \neq (-2)^4$?

Answer

Answers may vary

Exercise 1.3E.132

Why is $-4^3 \neq (-4)^3$?

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
multiply integers.			
divide integers.			
simplify expressions with integers.			
evaluate variable expressions with integers.			
translate English phrases to algebraic expressions.			
use integers in applications.			

ⓑ On a scale of 1–10, how would you rate your mastery of this section in light of your responses on the checklist? How can you improve this?

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1.4: Visualize Fractions

Learning Objectives

By the end of this section, you will be able to:

- Find equivalent fractions
- Simplify fractions
- Multiply fractions
- Divide fractions
- Simplify expressions written with a fraction bar
- Translate phrases to expressions with fractions

Note

A more thorough introduction to the topics covered in this section can be found in the *Prealgebra* chapter, **Fractions**.

Find Equivalent Fractions

Fractions are a way to represent parts of a whole. The fraction $\frac{1}{3}$ means that one whole has been divided into 3 equal parts and each part is one of the three equal parts. See Figure 1.4.1. The fraction $\frac{2}{3}$ represents two of three equal parts. In the fraction $\frac{2}{3}$, the 2 is called the **numerator** and the 3 is called the **denominator**.

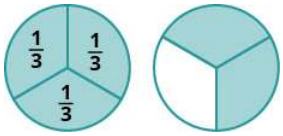


Figure 1.4.1: The circle on the left has been divided into 3 equal parts. Each part is $\frac{1}{3}$ of the 3 equal parts. In the circle on the right, $\frac{2}{3}$ of the circle is shaded (2 of the 3 equal parts).

Doing the Manipulative Mathematics activity "Model Fractions" will help you develop a better understanding of fractions, their numerators and denominators.

FRACTION

A **fraction** is written $\frac{a}{b}$, where $b \neq 0$ and

- a is the **numerator** and b is the **denominator**.

A fraction represents parts of a whole. The denominator b is the number of equal parts the whole has been divided into, and the numerator a indicates how many parts are included.

If a whole pie has been cut into 6 pieces and we eat all 6 pieces, we ate $\frac{6}{6}$ pieces, or, in other words, one whole pie.



Figure 1.4.2

So $\frac{6}{6} = 1$. This leads us to the property of one that tells us that any number, except zero, divided by itself is 1.

PROPERTY OF ONE

$$\frac{a}{a} = 1 \quad (a \neq 0) \tag{1.4.1}$$

Any number, except zero, divided by itself is one.

Note

Doing the Manipulative Mathematics activity "Fractions Equivalent to One" will help you develop a better understanding of fractions that are equivalent to one.

If a pie was cut in 6 pieces and we ate all 6, we ate $\frac{6}{6}$ pieces, or, in other words, one whole pie. If the pie was cut into 8 pieces and we ate all 8, we ate $\frac{8}{8}$ pieces, or one whole pie. We ate the same amount—one whole pie.

The fractions $\frac{6}{6}$ and $\frac{8}{8}$ have the same value, 1, and so they are called equivalent fractions. **Equivalent fractions** are fractions that have the same value.

Let's think of pizzas this time. Figure 1.4.3 shows two images: a single pizza on the left, cut into two equal pieces, and a second pizza of the same size, cut into eight pieces on the right. This is a way to show that $\frac{1}{2}$ is equivalent to $\frac{4}{8}$. In other words, they are **equivalent fractions**.

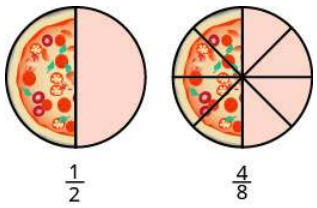


Figure 1.4.3: Since the same amount is of each pizza is shaded, we see that $\frac{1}{2}$ is equivalent to $\frac{4}{8}$. They are equivalent fractions.

EQUIVALENT FRACTIONS

Equivalent fractions are fractions that have the same value.

How can we use mathematics to change $\frac{1}{2}$ into $\frac{4}{8}$? How could we take a pizza that is cut into 2 pieces and cut it into 8 pieces? We could cut each of the 2 larger pieces into 4 smaller pieces! The whole pizza would then be cut into 88 pieces instead of just 2. Mathematically, what we've described could be written like this as $\frac{1 \cdot 4}{2 \cdot 4} = \frac{4}{8}$. See Figure 1.4.4.

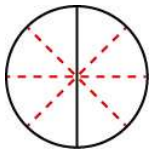


Figure 1.4.4: Cutting each half of the pizza into 4 pieces, gives us pizza cut into 8 pieces: $\frac{1 \cdot 4}{2 \cdot 4} = \frac{4}{8}$

This model leads to the following property:

EQUIVALENT FRACTIONS PROPERTY

If a, b, c are numbers where $b \neq 0, c \neq 0$, then

$$\frac{a}{b} = \frac{a \cdot c}{b \cdot c} \tag{1.4.2}$$

If we had cut the pizza differently, we could get

$$\frac{1\cdot 2}{2\cdot 2}=\frac{2}{4} \text{ so } \frac{1}{2}=\frac{2}{4}$$

$$\frac{1\cdot 3}{2\cdot 3}=\frac{3}{6} \text{ so } \frac{1}{2}=\frac{3}{6}$$

$$\frac{1\cdot 10}{2\cdot 10}=\frac{10}{20} \text{ so } \frac{1}{2}=\frac{10}{20}$$

Figure 1.4.5

So, we say $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, and $\frac{10}{20}$ are equivalent fractions.

Note

Doing the Manipulative Mathematics activity "Equivalent Fractions" will help you develop a better understanding of what it means when two fractions are equivalent.

Example 1.4.1

Find three fractions equivalent to $\frac{2}{5}$.

Solution

To find a fraction equivalent to $\frac{2}{5}$, we multiply the numerator and denominator by the same number. We can choose any number, except for zero. Let's multiply them by 2, 3, and then 5.

$$\frac{2\cdot 2}{5\cdot 2}=\frac{4}{10} \quad \frac{2\cdot 3}{5\cdot 3}=\frac{6}{15} \quad \frac{2\cdot 5}{5\cdot 5}=\frac{10}{25}$$

So, $\frac{4}{10}$, $\frac{6}{15}$, and $\frac{10}{25}$ are equivalent to $\frac{2}{5}$.

Try It 1.4.2

Find three fractions equivalent to $\frac{3}{5}$.

Answer

$\frac{6}{10}$, $\frac{9}{15}$, $\frac{12}{20}$; answers may vary

Try It 1.4.3

Find three fractions equivalent to $\frac{4}{5}$.

Answer

$\frac{8}{10}$, $\frac{12}{15}$, $\frac{16}{20}$; answers may vary

Simplify Fractions

A fraction is considered **simplified** if there are no common factors, other than 1, in its numerator and denominator.

For example,

- $\frac{2}{3}$ is simplified because there are no common factors of 2 and 3.
- $\frac{10}{15}$ is not simplified because 5 is a common factor of 10 and 15.

SIMPLIFIED FRACTION

A fraction is considered **simplified** if there are no common factors in its numerator and denominator.

The phrase *reduce a fraction* means to simplify the fraction. We simplify, or reduce, a fraction by removing the common factors of the numerator and denominator. A fraction is not simplified until all common factors have been removed. If an expression has fractions, it is not completely simplified until the fractions are simplified.

In Exercise 1.4.4, we used the equivalent fractions property to find equivalent fractions. Now we'll use the equivalent fractions property in reverse to simplify fractions. We can rewrite the property to show both forms together.

EQUIVALENT FRACTIONS PROPERTY

If a, b, c are numbers where $b \neq 0, c \neq 0$,

$$\text{then } \frac{a}{b} = \frac{a \cdot c}{b \cdot c} \text{ and } \frac{a \cdot c}{b \cdot c} = \frac{a}{b} \tag{1.4.3}$$

Example 1.4.4

Simplify: $-\frac{32}{56}$

Solution

	$-\frac{32}{56}$
Rewrite the numerator and denominator showing the common factors.	$-\frac{4 \cdot 8}{7 \cdot 8}$
Simplify using the equivalent fractions property.	$-\frac{4}{7}$

Notice that the fraction $-\frac{4}{7}$ is simplified because there are no more common factors.

p

Try It 1.4.5

Simplify: $-\frac{42}{54}$

Answer

$-\frac{7}{9}$

Try It 1.4.6

Simplify: $-\frac{30}{54}$

Answer

$-\frac{5}{9}$

Sometimes it may not be easy to find common factors of the numerator and denominator. When this happens, a good idea is to factor the numerator and the denominator into **prime numbers**. Then divide out the common factors using the equivalent fractions property.

Example 1.4.7

Simplify: $-\frac{210}{385}$

Solution

Step 1. Rewrite the numerator and denominator to show the common factors. If needed, factor the numerator and denominator into prime numbers first.	Rewrite 210 and 385 as the product of the primes.	$-\frac{210}{385}$ $-\frac{2\cdot 3\cdot 5\cdot 7}{5\cdot 7\cdot 11}$
Step 2. Simplify using the equivalent fractions property by dividing out common factors.	Mark the common factors 5 and 7. Divide out the common factors.	$-\frac{2\cdot 3\cdot \cancel{5}\cdot \cancel{7}}{\cancel{5}\cdot \cancel{7}\cdot 11}$ $-\frac{2\cdot 3}{11}$
Step 3. Multiplify the remaining factors, if necessary.		$-\frac{6}{11}$

Try It 1.4.8

Simplify: $-\frac{69}{120}$

Answer

$-\frac{23}{40}$

Try It 1.4.9

Simplify: $-\frac{120}{192}$

Answer

$-\frac{5}{8}$

We now summarize the steps you should follow to simplify fractions.

SIMPLIFY A FRACTION.

1. Rewrite the numerator and denominator to show the common factors.
   If needed, factor the numerator and denominator into prime numbers first.
2. Simplify using the equivalent fractions property by dividing out common factors.
3. Multiply any remaining factors, if needed.

Example 1.4.10

Simplify: $\frac{5x}{5y}$

Solution

$\frac{5x}{5y}$

Rewrite showing the common factors, then divide out the common factors.	Rewrite showing the common factors, then divide out the common factors. $\frac{5x}{5y}$ $\frac{5 \cdot x}{5 \cdot y}$ Simplify. $\frac{x}{y}$
Simplify.	$\frac{x}{y}$

Try It 1.4.11

Simplify: $\frac{7x}{7y}$

Answer

$\frac{x}{y}$

Try It 1.4.12

Simplify: $\frac{3a}{3b}$

Answer

$\frac{a}{b}$

Multiply Fractions

Many people find multiplying and dividing fractions easier than adding and subtracting fractions. So we will start with fraction multiplication.

Doing the Manipulative Mathematics activity "Model Fraction Multiplication" will help you develop a better understanding of multiplying fractions.

We'll use a model to show you how to multiply two fractions and to help you remember the procedure. Let's start with $\frac{3}{4}$.



Figure 1.4.6

Now we'll take $\frac{1}{2}$ of $\frac{3}{4}$.

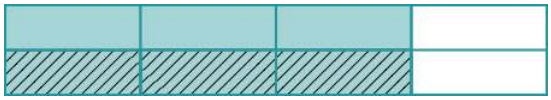


Figure 1.4.6

Notice that now, the whole is divided into 8 equal parts. So $\frac{1}{2} \cdot \frac{3}{4} = \frac{3}{8}$.

To multiply fractions, we multiply the numerators and multiply the denominators.

FRACTION MULTIPLICATION

If a, b, c and d are numbers where $b \neq 0$ and $d \neq 0$, then

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd} \tag{1.4.4}$$

To multiply fractions, multiply the numerators and multiply the denominators.

When **multiplying fractions**, the properties of positive and negative numbers still apply, of course. It is a good idea to determine the sign of the **product** as the first step. In Exercise 1.4.13, we will multiply negative and a positive, so the product will be negative.

Example 1.4.13

Multiply: $-\frac{11}{12} \cdot \frac{5}{7}$

Solution

The first step is to find the sign of the product. Since the signs are the different, the product is negative.

$$\begin{array}{ll} & -\frac{11}{12} \cdot \frac{5}{7} \\ \text{Determine the sign of the product; multiply.} & -\frac{11 \cdot 5}{12 \cdot 7} \\ \text{Are there any common factors in the numerator} & \\ \text{and the denominator? No} & -\frac{55}{84} \end{array} \tag{1.4.5}$$

Try It 1.4.14

Multiply: $-\frac{10}{28} \cdot \frac{8}{15}$

Answer

$-\frac{4}{21}$

Try It 1.4.15

Multiply: $-\frac{9}{20} \cdot \frac{5}{12}$

Answer

$-\frac{3}{16}$

When multiplying a fraction by an integer, it may be helpful to write the integer as a fraction. Any integer, a, can be written as $\frac{a}{1}$. So, for example, $3 = \frac{3}{1}$.

Example 1.4.16

Multiply: $-\frac{12}{5}(-20x)$

Solution

Determine the sign of the product. The signs are the same, so the product is positive.

	$-\frac{12}{5}(-20x)$
Write $20x$ as a fraction.	$\frac{12}{5}(\frac{20x}{1})$
Multiply.	
Rewrite 20 to show the common factor 5 and divide it out.	$\frac{12 \cdot 4 \cdot \cancel{5}x}{\cancel{5} \cdot 1}$
Simplify.	$48x$

Try It 1.4.17

Multiply: $\frac{11}{3}(-9a)$

Answer

$-33a$

Try It 1.4.18

Multiply: $\frac{13}{7}(-14b)$

Answer

$-26b$

Divide Fractions

Now that we know how to multiply fractions, we are almost ready to divide. Before we can do that, that we need some vocabulary.

The **reciprocal** of a fraction is found by inverting the fraction, placing the numerator in the denominator and the denominator in the numerator. The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$.

Notice that $\frac{2}{3} \cdot \frac{3}{2} = 1$. A number and its reciprocal multiply to 1.

To get a **product** of positive 1 when multiplying two numbers, the numbers must have the same sign. So reciprocals must have the same sign.

The reciprocal of $-\frac{10}{7}$ is $-\frac{7}{10}$, since $-\frac{10}{7}(-\frac{7}{10}) = 1$.

RECIPROCAL

The **reciprocal** of $\frac{a}{b}$ is $\frac{b}{a}$.

A number and its reciprocal multiply to one $\frac{a}{b} \cdot \frac{b}{a} = 1$

Note

Doing the Manipulative Mathematics activity “Model Fraction Division” will help you develop a better understanding of dividing fractions.

To divide fractions, we multiply the first fraction by the reciprocal of the second.

FRACTION DIVISION

If a, b, c and d are numbers where $b \neq 0, c \neq 0$ and $d \neq 0$, then

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} \tag{1.4.6}$$

To divide fractions, we multiply the first fraction by the reciprocal of the second.

We need to say $b \neq 0, c \neq 0$ and $d \neq 0$ to be sure we don't divide by zero!

Example 1.4.19

Divide: $-\frac{2}{3} \div \frac{n}{5}$

Solution

	$-\frac{2}{3} \div \frac{n}{5}$
To divide, multiply the first fraction by the reciprocal of the second.	$-\frac{2}{3} \cdot \frac{5}{n}$
Multiply.	$-\frac{10}{3n}$

(1.4.7)

Try It 1.4.20

Divide: $-\frac{3}{5} \div \frac{p}{7}$.

Answer

$-\frac{21}{5p}$

Try It 1.4.21

Divide: $-\frac{5}{8} \div \frac{q}{3}$.

Answer

$-\frac{15}{8q}$

Example 1.4.22

Find the quotient:

$-\frac{7}{18} \div (-\frac{14}{27})$

Solution

	$-\frac{7}{18} \div (-\frac{14}{27})$

To divide, multiply the first fraction by the reciprocal of the second.	$-\frac{7}{18} \cdot -\frac{27}{14}$
Determine the sign of the product, and then multiply..	$\frac{7 \cdot 27}{18 \cdot 14}$
Rewrite showing common factors.	$\frac{7 \cdot 9 \cdot 3}{9 \cdot 2 \cdot 7 \cdot 2}$
Remove common factors.	$\frac{3}{2 \cdot 2}$
Simplify.	$\frac{3}{4}$

? Try It 1.4.23

Find the quotient:

$-\frac{7}{8} \div (-\frac{14}{27})$

Answer

$\frac{4}{15}$

? Try It 1.4.24

Find the quotient:

$-\frac{7}{8} \div (-\frac{14}{27})$

Answer

$\frac{2}{3}$

There are several ways to remember which steps to take to multiply or divide fractions. One way is to repeat the call outs to yourself. If you do this each time you do an exercise, you will have the steps memorized.

- “To multiply fractions, multiply the numerators and multiply the denominators.”
- “To divide fractions, multiply the first fraction by the reciprocal of the second.”

Another way is to keep two examples in mind:

One fourth of two pizzas is one half of a pizza.



$$2 \cdot \frac{1}{4}$$

$$\frac{2}{1} \cdot \frac{1}{4}$$

$$\frac{2}{4}$$

$$\frac{1}{2}$$

There are eight quarters in $2.00.



$$2 \div \frac{1}{4}$$

$$\frac{2}{1} \div \frac{1}{4}$$

$$\frac{2}{1} \cdot \frac{4}{1}$$

$$8$$

Figure 1.4.7

The numerators or denominators of some fractions contain fractions themselves. A fraction in which the numerator or the denominator is a fraction is called a **complex fraction**.

COMPLEX FRACTION

A **complex fraction** is a fraction in which the numerator or the denominator contains a fraction.

Some examples of complex fractions are:

$$\frac{\frac{6}{7}}{3} \quad \frac{\frac{3}{4}}{\frac{5}{8}} \quad \frac{\frac{x}{2}}{\frac{5}{6}} \tag{1.4.8}$$

To simplify a complex fraction, we remember that the fraction bar means **division**. For example, the complex fraction $\frac{\frac{3}{4}}{\frac{5}{8}}$ means $\frac{3}{4} \div \frac{5}{8}$.

Example 1.4.25

Simplify: $\frac{\frac{3}{4}}{\frac{5}{8}}$

Solution

	$\frac{\frac{3}{4}}{\frac{5}{8}}$
Rewrite as division.	$\frac{3}{4} \div \frac{5}{8}$
Multiply the first fraction by the reciprocal of the second.	$\frac{3}{4} \cdot \frac{8}{5}$
Multiply.	$\frac{3 \cdot 8}{4 \cdot 5}$
Look for common factors.	$\frac{3 \cdot \cancel{4} \cdot 2}{\cancel{4} \cdot 5}$
Divide out common factors and simplify.	$\frac{6}{5}$

Try It 1.4.26

Simplify: $\frac{\frac{2}{3}}{\frac{5}{6}}$

Answer

$\frac{4}{5}$

Try It 1.4.27

Simplify: $\frac{\frac{3}{7}}{\frac{6}{11}}$

Answer

$\frac{11}{14}$

Example 1.4.28

Simplify: $\frac{\frac{x}{2}}{\frac{xy}{6}}$

Solution

	$\frac{\frac{x}{2}}{\frac{xy}{6}}$
Rewrite as division.	$\frac{x}{2} \div \frac{xy}{6}$
Multiply the first fraction by the reciprocal of the second.	$\frac{x}{2} \cdot \frac{6}{xy}$
Multiply.	$\frac{x \cdot 6}{2 \cdot xy}$
Look for common factors.	$\frac{x \cdot 3 \cdot 2}{2 \cdot x \cdot y}$
Divide out common factors and simplify.	$\frac{3}{y}$

Try It 1.4.29

Simplify: $\frac{\frac{a}{8}}{\frac{ab}{6}}$

Answer

$\frac{3}{4b}$

Try It 1.4.30

Simplify: $\dfrac{\frac{p}{2}}{\frac{pq}{8}}$

Answer

$\dfrac{4}{q}$

Simplify Expressions with a Fraction Bar

The line that separates the numerator from the denominator in a fraction is called a fraction bar. A **fraction bar** acts as grouping symbol. The order of operations then tells us to simplify the numerator and then the denominator. Then we divide.

To simplify the expression $\dfrac{5-3}{7+1}$, we first simplify the numerator and the denominator separately. Then we divide.

$$\begin{array}{l} \dfrac{5-3}{7+1} \\ \dfrac{2}{8} \\ \dfrac{1}{4} \end{array} \tag{1.4.9}$$

SIMPLIFY AN EXPRESSION WITH A FRACTION BAR.

1. Simplify the expression in the numerator. Simplify the expression in the denominator.
2. Simplify the fraction.

Example 1.4.31

Simplify: $\dfrac{4-2(3)}{2^2+2}$

Solution

	$\dfrac{4-2(3)}{2^2+2}$
Use the order of operations to simplify the numerator and the denominator.	$\dfrac{4-6}{4+2}$
Simplify the numerator and the denominator	$\dfrac{-2}{6}$
Simplify. A negative divided by a positive is negative.	$-\dfrac{1}{3}$

(1.4.10)

Try It 1.4.32

Simplify: $\dfrac{6-3(5)}{3^2+3}$

Answer

$-\dfrac{3}{4}$

Try It 1.4.33

Simplify: $\frac{4-4(6)}{3^2+3}$

Answer

$-\frac{5}{3}$

Where does the negative sign go in a fraction? Usually the negative sign is in front of the fraction, but you will sometimes see a fraction with a negative numerator, or sometimes with a negative denominator. Remember that fractions represent division. When the numerator and denominator have different signs, the quotient is negative.

$$\begin{aligned} \frac{-1}{3} &= -\frac{1}{3} \quad \frac{\text{negative}}{\text{positive}} = \text{negative} \\ \frac{1}{-3} &= -\frac{1}{3} \quad \frac{\text{positive}}{\text{negative}} = \text{negative} \end{aligned} \tag{1.4.11}$$

PLACEMENT OF NEGATIVE SIGN IN A FRACTION

For any positive numbers a and b,

$$\frac{-a}{b} = \frac{a}{-b} = -\frac{a}{b} \tag{1.4.12}$$

Example 1.4.34

Simplify: $\frac{4(-3)+6(-2)}{-3(2)-2}$

Solution

The fraction bar acts like a grouping symbol. So completely simplify the numerator and the denominator separately.

$$\begin{aligned} & \frac{4(-3)+6(-2)}{-3(2)-2} \\ \text{Multiply.} \quad & \frac{-12+(-12)}{-6-2} \\ \text{Simplify.} \quad & \frac{-24}{-8} \\ \text{Divide.} \quad & 3 \end{aligned} \tag{1.4.13}$$

Try It 1.4.35

Simplify: $\frac{8(-2)+4(-3)}{-5(2)+3}$

Answer

4

Try It 1.4.36

Simplify: $\frac{7(-1)+9(-3)}{-5(3)-2}$

Answer

2

Translate Phrases to Expressions with Fractions

Now that we have done some work with fractions, we are ready to translate phrases that would result in expressions with fractions.

The English words **quotient** and **ratio** are often used to describe fractions. Remember that "quotient" means **division**. The quotient of aa and bb is the result we get from dividing a by b, or $\frac{a}{b}$.

Example 1.4.37

Translate the English phrase into an algebraic expression: the quotient of the difference of m and n, and p.

Solution

We are looking for the *quotient of* the difference of m and n, and p. This means we want to divide the difference of m and n, and p.

$$\frac{m-n}{p} \tag{1.4.14}$$

Try It 1.4.38

Translate the English phrase into an algebraic expression: the quotient of the difference of a and b, and cd.

Answer

$\frac{a-b}{cd}$

Try It 1.4.39

Translate the English phrase into an algebraic expression: the quotient of the sum of p and q, and r.

Answer

$\frac{p+q}{r}$

Key Concepts

- **Equivalent Fractions Property:** If a, b, c are numbers where $b \neq 0, c \neq 0$, then $\frac{a}{b} = \frac{a \cdot c}{b \cdot c}$ and $\frac{a \cdot c}{b \cdot c} = \frac{a}{b}$
- **Fraction Division:** If a, b, c and d are numbers where $b \neq 0, c \neq 0$ and $d \neq 0$, then $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$. To divide fractions, multiply the first fraction by the reciprocal of the second.
- **Fraction Multiplication:** If a, b, c and d are numbers where $b \neq 0, d \neq 0$, then $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$. To multiply fractions, multiply the numerators and multiply the denominators.
- **Placement of Negative Sign in a Fraction:** For any positive numbers a and b, $\frac{-a}{a} = \frac{a}{-a} = -\frac{a}{b}$
- **Property of One:** $\frac{a}{a} = 1$; Any number, except zero, divided by itself is one.
- **Simplify a Fraction**
  1. Rewrite the numerator and denominator to show the common factors. If needed, factor the numerator and denominator into prime numbers first.
  2. Simplify using the equivalent fractions property by dividing out common factors.
  3. Multiply any remaining factors.
- **Simplify an Expression with a Fraction Bar**
  1. Simplify the expression in the numerator. Simplify the expression in the denominator.
  2. Simplify the fraction.

Glossary

complex fraction

A complex fraction is a fraction in which the numerator or the denominator contains a fraction.

denominator

The denominator is the value on the bottom part of the fraction that indicates the number of equal parts into which the whole has been divided.

equivalent fractions

Equivalent fractions are fractions that have the same value.

fraction

A fraction is written $\frac{a}{b}$, where $b \neq 0$, a is the numerator and b is the denominator. A fraction represents parts of a whole. The denominator b is the number of equal parts the whole has been divided into, and the numerator aa indicates how many parts are included.

numerator

The numerator is the value on the top part of the fraction that indicates how many parts of the whole are included.

reciprocal

The reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$. A number and its reciprocal multiply to one: $\frac{a}{b} \cdot \frac{b}{a} = 1$.

simplified fraction

A fraction is considered simplified if there are no common factors in its numerator and denominator.

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1.4E: Exercises

Practice Makes Perfect

Find Equivalent Fractions

In the following exercises, find three fractions equivalent to the given fraction. Show your work, using figures or algebra.

Exercise 1.4E.1

$\frac{3}{8}$

Answer

$\frac{6}{16}, \frac{9}{24}, \frac{12}{32}$, answers may vary

Exercise 1.4E.2

$\frac{5}{8}$

Exercise 1.4E.3

$\frac{5}{9}$

Answer

$\frac{10}{18}, \frac{15}{27}, \frac{20}{36}$, answers may vary

Exercise 1.4E.4

$\frac{1}{8}$

Simplify Fractions

In the following exercises, simplify.

Exercise 1.4E.5

$-\frac{40}{88}$

Answer

$-\frac{5}{11}$

Exercise 1.4E.6

$-\frac{63}{99}$

Exercise 1.4E.7

$-\frac{108}{63}$

Answer

$-\frac{12}{7}$

Exercise 1.4E.8

$-\frac{104}{48}$

Exercise 1.4E.9

$\frac{120}{252}$

Answer

$\frac{10}{21}$

Exercise 1.4E.10

$\frac{182}{294}$

Exercise 1.4E.11

$-\frac{3x}{12y}$

Answer

$-\frac{x}{4y}$

Exercise 1.4E.12

$-\frac{4x}{32y}$

Exercise 1.4E.13

$\frac{14x^2}{21y}$

Answer

$\frac{2x^2}{3y}$

Exercise 1.4E.14

$$\frac{24a}{32b^2}$$

Multiply Fractions

In the following exercises, multiply.

Exercise 1.4E.15

$$\frac{3}{4} \cdot \frac{9}{10}$$

Answer

$$\frac{27}{40}$$

Exercise 1.4E.16

$$\frac{4}{5} \cdot \frac{2}{7}$$

Exercise 1.4E.17

$$-\frac{2}{3} \cdot -\frac{3}{8}$$

Answer

$$\frac{1}{4}$$

Exercise 1.4E.18

$$-\frac{3}{4}\left(-\frac{4}{9}\right)$$

Exercise 1.4E.19

$$-\frac{5}{9} \cdot \frac{3}{10}$$

Answer

$$-\frac{1}{6}$$

Exercise 1.4E.20

$$-\frac{3}{8} \cdot \frac{4}{15}$$

Exercise 1.4E.21

$$\left(-\frac{14}{15}\right)\left(\frac{9}{20}\right)$$

Answer

$$-\frac{21}{50}$$

Exercise 1.4E.22

$$\left(-\frac{9}{10}\right)\left(\frac{25}{33}\right)$$

Exercise 1.4E.23

$$\left(-\frac{63}{84}\right)\left(-\frac{44}{90}\right)$$

Answer

$$\frac{11}{30}$$

Exercise 1.4E.24

$$\left(-\frac{63}{60}\right)\left(-\frac{40}{88}\right)$$

Exercise 1.4E.25

$$4 \cdot \frac{5}{11}$$

Answer

$$\frac{20}{11}$$

Exercise 1.4E.26

$$5 \cdot \frac{8}{3}$$

Exercise 1.4E.27

$$\frac{3}{7} \cdot 21n$$

Answer

9n

Exercise 1.4E.28

$$\frac{5}{6} \cdot 30m$$

Exercise 1.4E.29

$-8 \cdot \frac{17}{4}$

Answer

-34

Exercise 1.4E.30

$(-1)\left(-\frac{6}{7}\right)$

Divide Fractions

In the following exercises, divide.

Exercise 1.4E.31

$\frac{3}{4} \div \frac{2}{3}$

Answer

$\frac{9}{8}$

Exercise 1.4E.32

$\frac{4}{5} \div \frac{3}{4}$

Exercise 1.4E.33

$-\frac{7}{9} \div \left(-\frac{7}{4}\right)$

Answer

$\frac{4}{9}$

Exercise 1.4E.34

$-\frac{5}{6} \div \left(-\frac{5}{6}\right)$

Exercise 1.4E.35

$\frac{3}{4} \div \frac{x}{11}$

Answer

$\frac{33}{4x}$

Exercise 1.4E.36

$\frac{2}{5} \div \frac{y}{9}$

Exercise 1.4E.37

$\frac{5}{18} \div -\frac{15}{24}$

Answer

$-\frac{4}{9}$

Exercise 1.4E.38

$\frac{7}{18} \div \left(-\frac{14}{27}\right)$

Exercise 1.4E.39

$\frac{8u}{15} \div \frac{12v}{25}$

Answer

$\frac{10u}{9v}$

Exercise 1.4E.40

$\frac{12r}{25} \div \frac{18s}{35}$

Exercise 1.4E.41

$-5 \div \frac{1}{2}$

Answer

-10

Exercise 1.4E.42

$-3 \div \frac{1}{4}$

Exercise 1.4E.43

$\frac{3}{4} \div (-12)$

Answer

$\frac{1}{16}$

Exercise 1.4E.44

$-15 \div -\frac{5}{3}$

In the following exercises, simplify.

Exercise 1.4E.45

$$\frac{-\frac{8}{21}}{\frac{12}{35}}$$

Answer

$-\frac{10}{9}$

Exercise 1.4E.46

$$\frac{-\frac{9}{16}}{\frac{33}{40}}$$

Exercise 1.4E.47

$$\frac{-\frac{4}{5}}{2}$$

Answer

$-\frac{2}{5}$

Exercise 1.4E.48

$$\frac{5}{\frac{3}{10}}$$

Exercise 1.4E.49

$$\frac{\frac{m}{3}}{\frac{n}{2}}$$

Answer

$\frac{2m}{3n}$

Exercise 1.4E.50

$$\frac{-\frac{3}{8}}{-\frac{y}{12}}$$

Simplify Expressions Written with a Fraction Bar

In the following exercises, simplify.

Exercise 1.4E.51

$$\frac{22+3}{10}$$

Answer

$$\frac{5}{2}$$

Exercise 1.4E.52

$$\frac{19-4}{6}$$

Exercise 1.4E.53

$$\frac{48}{24-15}$$

Answer

$$\frac{16}{3}$$

Exercise 1.4E.54

$$\frac{46}{4+4}$$

Exercise 1.4E.55

$$\frac{-6+6}{8+4}$$

Answer

$$0$$

Exercise 1.4E.56

$$\frac{-6+3}{17-8}$$

Exercise 1.4E.57

$$\frac{4 \cdot 3}{6 \cdot 6}$$

Answer

$$\frac{1}{3}$$

Exercise 1.4E.58

$$\frac{6 \cdot 6}{9 \cdot 2}$$

Exercise 1.4E.59

$$\frac{4^2 - 1}{25}$$

Answer

$$\frac{3}{5}$$

Exercise 1.4E.60

$$\frac{7^2 + 1}{60}$$

Exercise 1.4E.61

$$\frac{8 \cdot 3 + 2 \cdot 9}{14 + 3}$$

Answer

$$\frac{42}{17}$$

Exercise 1.4E.62

$$\frac{9 \cdot 6 - 4 \cdot 7}{22 + 3}$$

Exercise 1.4E.63

$$\frac{5 \cdot 6 - 3 \cdot 4}{4 \cdot 5 - 2 \cdot 3}$$

Answer

$$\frac{9}{7}$$

Exercise 1.4E.64

$$\frac{8 \cdot 9 - 7 \cdot 6}{5 \cdot 6 - 9 \cdot 2}$$

Exercise 1.4E.65

$$\frac{5^2 - 3^2}{3 - 5}$$

Answer

-8

Exercise 1.4E.66

$$\frac{6^2 - 4^2}{4 - 6}$$

Exercise 1.4E.67

$$\frac{7 \cdot 4 - 2(8 - 5)}{9 \cdot 3 - 3 \cdot 5}$$

Answer

$$\frac{11}{6}$$

Exercise 1.4E.68

$$\frac{9 \cdot 7 - 3(12 - 8)}{8 \cdot 7 - 6 \cdot 6}$$

Exercise 1.4E.69

$$\frac{9(8 - 2) - 3(15 - 7)}{6(7 - 1) - 3(17 - 9)}$$

Answer

$$\frac{5}{2}$$

Exercise 1.4E.70

$$\frac{8(9 - 2) - 4(14 - 9)}{7(8 - 3) - 3(16 - 9)}$$

Translate Phrases to Expressions with Fractions

In the following exercises, translate each English phrase into an algebraic expression.

Exercise 1.4E.71

the quotient of r and the sum of s and 10

Answer

$$\frac{r}{s+10}$$

Exercise 1.4E.72

the quotient of A and the difference of 3 and B

Exercise 1.4E.73

the quotient of the difference of x and y, and -3

Answer

$$\frac{x-y}{-3}$$

Exercise 1.4E.74

the quotient of the sum of m and n, and $4q$

Everyday Math

Exercise 1.4E.75

Baking. A recipe for chocolate chip cookies calls for $\frac{3}{4}$ cup brown sugar. Imelda wants to double the recipe.

1. How much brown sugar will Imelda need? Show your calculation.
2. Measuring cups usually come in sets of $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$, and 1 cup. Draw a diagram to show two different ways that Imelda could measure the brown sugar needed to double the cookie recipe.

Answer

1. $1\frac{1}{2}$ cups
2. answers will vary

Exercise 1.4E.76

Baking. Nina is making 4 pans of fudge to serve after a music recital. For each pan, she needs $\frac{2}{3}$ cup of condensed milk.

1. How much condensed milk will Nina need? Show your calculation.
2. Measuring cups usually come in sets of $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$, and 1 cup. Draw a diagram to show two different ways that Nina could measure the condensed milk needed for 4 pans of fudge.

Exercise 1.4E.77

Portions Don purchased a bulk package of candy that weighs 5 pounds. He wants to sell the candy in little bags that hold $\frac{1}{4}$ pound. How many little bags of candy can he fill from the bulk package?

Answer

20 bags

Exercise 1.4E.78

Portions Kristen has $\frac{3}{4}$ yards of ribbon that she wants to cut into 6 equal parts to make hair ribbons for her daughter's 6 dolls. How long will each doll's hair ribbon be?

Writing Exercises

Exercise 1.4E.79

Rafael wanted to order half a medium pizza at a restaurant. The waiter told him that a medium pizza could be cut into 6 or 8 slices. Would he prefer 3 out of 6 slices or 4 out of 8 slices? Rafael replied that since he wasn't very hungry, he would prefer 3 out of 6 slices. Explain what is wrong with Rafael's reasoning.

Answer

Answers may vary

Exercise 1.4E.80

Give an example from everyday life that demonstrates how $\frac{1}{2} \cdot \frac{2}{3}$ is $\frac{1}{3}$.

Exercise 1.4E.81

Explain how you find the reciprocal of a fraction.

Answer

Answers may vary

Exercise 1.4E.82

Explain how you find the reciprocal of a negative number.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
find equivalent fractions.			
simplify fractions.			
multiply fractions.			
divide fractions.			
simplify expressions written with a fraction bar.			
translate phrases to expressions with fractions.			

ⓑ After looking at the checklist, do you think you are well prepared for the next section? Why or why not?

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1.5: Add and Subtract Fractions

Learning Objectives

By the end of this section, you will be able to:

- Add or subtract fractions with a common denominator
- Add or subtract fractions with different denominators
- Use the order of operations to simplify complex fractions
- Evaluate variable expressions with fractions

Note

A more thorough introduction to the topics covered in this section can be found in the *Prealgebra*chapter, **Fractions**.

Add or Subtract Fractions with a Common Denominator

When we multiplied fractions, we just multiplied the numerators and multiplied the denominators right straight across. To add or subtract fractions, they must have a common denominator.

FRACTION ADDITION AND SUBTRACTION

If a, b, and c are numbers where $c \neq 0$, then

$$\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c} \quad \text{and} \quad \frac{a}{c}-\frac{b}{c}=\frac{a-b}{c} \tag{1.5.1}$$

To add or subtract fractions, add or subtract the numerators and place the result over the common denominator.

Manipulative Mathematics

Doing the Manipulative Mathematics activities “Model Fraction Addition” and “Model Fraction Subtraction” will help you develop a better understanding of adding and subtracting fractions.

Example 1.5.1

Find the sum: $\frac{x}{3}+\frac{2}{3}$.

Solution

$$\begin{array}{ll} & \frac{x}{3}+\frac{2}{3} \\ \text{Add the numerators and place the sum over the common denominator} & \frac{x+2}{3} \end{array} \tag{1.5.2}$$

Try It 1.5.2

Find the sum: $\frac{x}{4}+\frac{3}{4}$.

Answer

$\frac{x+3}{4}$

Try It 1.5.3

Find the sum: $\frac{y}{8}+\frac{5}{8}$.

Answer

$\frac{y+5}{8}$

Example 1.5.4

Find the difference: $-\frac{23}{24}-\frac{13}{24}$

Solution

	$-\frac{23}{24}-\frac{13}{24}$
Subtract the numerators and place the difference over the common denominator	$\frac{-23-13}{24}$ (1.5.3)
Simplify.	$\frac{-36}{24}$
Simplify. Remember, $-\frac{a}{b}=\frac{-a}{b}$	$-\frac{3}{2}$

Try It 1.5.5

Find the difference: $-\frac{19}{28}-\frac{7}{28}$

Answer

$-\frac{26}{28}$

Try It 1.5.6

Find the difference: $-\frac{27}{32}-\frac{1}{32}$

Answer

$-\frac{7}{8}$

Example 1.5.7

Find the difference: $-\frac{10}{x}-\frac{4}{x}$

Solution

	$-\frac{10}{x}-\frac{4}{x}$	
Subtract the numerators and place the difference over the common denominator	$\frac{-14}{x}$	(1.5.4)
Rewrite with the sign in front of the fraction.	$-\frac{14}{x}$	

Try It 1.5.8

Find the difference: $-\frac{9}{x}-\frac{7}{x}$

Answer

$-\frac{16}{x}$

Try It 1.5.9

Find the difference: $-\frac{17}{a}-\frac{5}{a}$

Answer

$-\frac{22}{a}$

Now we will do an example that has both addition and subtraction.

Example 1.5.10

Simplify: $\frac{3}{8}+(-\frac{5}{8})-\frac{1}{8}$

Solution

Add and Subtract fractions — do they have a common denominator? Yes.	$\frac{3}{8}+(-\frac{5}{8})-\frac{1}{8}$	
Add and subtract the numerators and place the result over the common denominator.	$\frac{3+(-5)-1}{8}$	(1.5.5)
Simplify left to right.	$\frac{-2-1}{8}$	
Simplify.	$-\frac{3}{8}$	

Try It 1.5.11

Simplify: $\frac{2}{9}+(-\frac{4}{9})-\frac{7}{9}$

Answer

-1

Try It 1.5.12

Simplify: $\frac{2}{5}+(-\frac{4}{9})-\frac{7}{9}$

Answer

$-\frac{2}{3}$

Add or Subtract Fractions with Different Denominators

As we have seen, to add or subtract fractions, their denominators must be the same. The **least common denominator** (LCD) of two fractions is the smallest number that can be used as a common **denominator** of the fractions. The LCD of the two fractions is the least common multiple (LCM) of their denominators.

LEAST COMMON DENOMINATOR

The least common denominator (LCD) of two fractions is the least common multiple (LCM) of their denominators.

Note

Doing the Manipulative Mathematics activity "Finding the Least Common Denominator" will help you develop a better understanding of the LCD.

After we find the least common denominator of two fractions, we convert the fractions to equivalent fractions with the LCD. Putting these steps together allows us to add and subtract fractions because their denominators will be the same!

Example 1.5.13

Add: $\frac{7}{12}+\frac{5}{18}$

Solution

Step 1. Do they have a common denominator? No—rewrite each fraction with the LCD (least common denominator).	No. Find the LCD of 12, 18. Change into equivalent fractions with the LCD, 36. Do not simplify the equivalent fractions! If you do, you'll get back to the original fractions and lose the common denominator!	$12=2\cdot2\cdot3$ $18=2\cdot\quad3\cdot3$ $\text{LCD}=2\cdot2\cdot3\cdot3$ $\text{LCD}=36$ $\frac{7}{12}+\frac{5}{18}$ $\frac{7\cdot3}{12\cdot3}+\frac{5\cdot2}{18\cdot2}$ $\frac{21}{36}+\frac{10}{36}$
Step 2. Add or subtract the fractions.	Add.	$\frac{31}{36}$
Step 3. Simplify, if possible.	Because 31 is a prime number, it has no factors in common with 36. The answer is simplified.	

Try It 1.5.14

Add: $\frac{7}{12}+\frac{11}{15}$

Answer

$\frac{79}{60}$

Try It 1.5.15

Add: $\frac{13}{15}+\frac{17}{20}$

Answer

$\frac{103}{60}$

ADD OR SUBTRACT FRACTIONS.

1. Do they have a common denominator?
   - Yes—go to step 2.
   - No—rewrite each fraction with the LCD (least common denominator). Find the LCD. Change each fraction into an equivalent fraction with the LCD as its denominator.
2. Add or subtract the fractions.
3. Simplify, if possible.

When finding the equivalent fractions needed to create the common denominators, there is a quick way to find the number we need to multiply both the numerator and denominator. This method works if we found the LCD by factoring into primes.

Look at the factors of the LCD and then at each column above those factors. The "missing" factors of each denominator are the numbers we need.

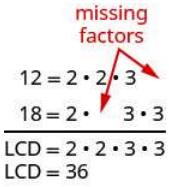


Figure: 1.5.1

In Exercise 1.5.13, the LCD, 36, has two factors of 2 and two factors of 3.

The numerator 12 has two factors of 2 but only one of 3—so it is "missing" one 3—we multiply the numerator and denominator by 3.

The numerator 18 is missing one factor of 2—so we multiply the numerator and denominator by 2.

We will apply this method as we subtract the fractions in Exercise 1.5.16.

Example 1.5.16

Subtract: $\frac{7}{15}-\frac{19}{24}$

Solution

Do the fractions have a common denominator? No, so we need to find the LCD.

Find the LCD. $\frac{7}{15}-\frac{19}{24}$ $15=3\cdot5$ $24=2\cdot2\cdot2\cdot3$ $LCD=2\cdot2\cdot2\cdot3\cdot5$ $LCD=120$	
Notice, 15 is “missing” three factors of 2 and 24 is “missing” the 5 from the factors of the LCD. So we multiply 8 in the first fraction and 5 in the second fraction to get the LCD.	
Rewrite as equivalent fractions with the LCD.	$\frac{7\cdot8}{15\cdot8}-\frac{19\cdot5}{24\cdot5}$
Simplify.	$\frac{56}{120}-\frac{95}{120}$
Subtract.	$-\frac{39}{120}$
Check to see if the answer can be simplified.	$-\frac{13\cdot3}{40\cdot3}$
Both 39 and 120 have a factor of 3.	
Simplify.	$-\frac{13}{40}$

Do not simplify the equivalent fractions! If you do, you’ll get back to the original fractions and lose the common denominator!

? Try It 1.5.17

Subtract: $\frac{13}{24}-\frac{17}{32}$

Answer

$\frac{1}{96}$

? Try It 1.5.18

Subtract: $\frac{7}{15}-\frac{19}{24}$

Answer

$\frac{75}{224}$

In the next example, one of the fractions has a variable in its numerator. Notice that we do the same steps as when both numerators are numbers.

✓ Example 1.5.19

Add: $\frac{3}{5}+\frac{x}{8}$

Solution

The fractions have different denominators.

	$\frac{3}{5}+\frac{x}{8}$

Find the LCD. $\begin{array}{l} 5= \quad 5 \\ 8=2\cdot 2\cdot 2 \\ \text{LCD}=2\cdot 2\cdot 2\cdot 5 \\ \text{LCD}=40 \end{array}$	
Rewrite as equivalent fractions with the LCD.	$\frac{3\cdot 8}{5\cdot 8}+\frac{x\cdot 5}{8\cdot 5}$
Simplify.	$\frac{24}{40}+\frac{5x}{40}$
Add.	$\frac{24+5x}{40}$

Remember, we can only add like terms: 24 and $5x$ are not like terms.

Try It 1.5.20

Add: $\frac{y}{6}+\frac{7}{9}$

Answer

$\frac{3y+14}{18}$

Try It 1.5.21

Add: $\frac{x}{6}+\frac{7}{15}$

Answer

$\frac{15x+42}{153}$

We now have all four operations for fractions. Table $1.5.1$ summarizes **fraction operations**.

Table 1.5.1

Fraction Multiplication	**Fraction Division**
$\frac{a}{b}\cdot\frac{c}{d}=\frac{ac}{bd}$ Multiply the numerators and multiply the denominators	$\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c}$ Multiply the first fraction by the reciprocal of the second.
Fraction Addition	**Fraction Subtraction**
$\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c}$ Add the numerators and place the sum over the common denominator.	$\frac{a}{c}-\frac{b}{c}=\frac{a-b}{c}$ Subtract the numerators and place the difference over the common denominator.
To multiply or divide fractions, and LCD is NOT needed. To add or subtract fractions, an LCD is needed.	

Example 1.5.22

Simplify:

1. $\frac{5x}{6}-\frac{3}{10}$
2. $\frac{5x}{6}\cdot\frac{3}{10}$.

Solution

First ask, “What is the operation?” Once we identify the operation that will determine whether we need a common denominator. Remember, we need a common denominator to add or subtract, but not to multiply or divide.

1. What is the operation? The operation is subtraction.

$$
\begin{array}{ll}
\text{Do the fractions have a common denominator? No.} & \frac{5x}{6}-\frac{3}{10} \\
\text{Rewrite each fractions as an equivalent fraction with the LCD.} & \frac{5x\cdot 5}{6\cdot 5}-\frac{3\cdot 3}{10\cdot 3} \\
 & \frac{25x}{30}-\frac{9}{30} \\
\text{Subtract the numerators and place the difference over the common denominators.} & \frac{25x-9}{30} \\
\text{Simplify, if possible. There are no common factors.} & \\
\text{The fraction is simplified.} &
\end{array}
\tag{1.5.6}
$$

2. What is the operation? Multiplication.

$$
\begin{array}{ll}
 & \frac{5x}{6}\cdot\frac{3}{10} \\
\text{To multiply fractions, multiply the numerators and multiply the denominators} & \frac{5x\cdot 3}{6\cdot 10} \\
\text{Rewrite, showing common factors.} & \frac{5x\cdot 3}{2\cdot 3\cdot 2\cdot 5} \\
\text{Simplify.} & \frac{x}{4}
\end{array}
\tag{1.5.7}
$$

? Try It 1.5.23

Simplify:

1. $\frac{3a}{4}-\frac{8}{9}$
2. $\frac{3a}{4}\cdot\frac{8}{9}$

Answer

1. $\frac{27a-32}{36}$
2. $\frac{2a}{3}$

? Try It 1.5.24

Simplify:

1. $\frac{4k}{5}-\frac{1}{6}$
2. $\frac{4k}{5}\cdot\frac{1}{6}$

Answer

1. $\frac{24k-5}{30}$
2. $\frac{2k}{15}$

Use the Order of Operations to Simplify Complex Fractions

We have seen that a complex fraction is a fraction in which the numerat or or denominator contains a fraction. The fraction bar indicates **division**. We simplified the complex fraction $\frac{\frac{3}{4}}{\frac{5}{8}}$ by dividing $\frac{3}{4}$ by $\frac{5}{8}$.

Now we'll look at complex fractions where the numerator or denominator contains an expression that can be simplified. So we first must completely simplify the numerator and denominator separately using the order of operations. Then we divide the numerator by the denominator.

Example 1.5.25: How to simplify complex fractions

Simplify: $\frac{(\frac{1}{2})^2}{4+3^2}$

Solution

Step 1. Simplify the numerator. * Remember, $(\frac{1}{2})^2$ means $\frac{1}{2}\cdot\frac{1}{2}$.	$\frac{(\frac{1}{2})^2}{4+3^2}$ $\frac{\frac{1}{4}}{4+3^2}$
Step 2. Simplify the denominator.	$\frac{\frac{1}{4}}{4+9}$ $\frac{\frac{1}{4}}{13}$
Step 3. Divide the numerator by the denominator. Simplify if possible. * Remember, $13=\frac{13}{1}$	$\frac{1}{4}\div 13$ $\frac{1}{4}\cdot\frac{1}{13}$ $\frac{1}{52}$

Try It 1.5.26

Simplify: $\frac{(\frac{1}{3})^2}{2^3+2}$

Answer

$\frac{1}{90}$

Try It 1.5.27

Simplify: $\frac{1+4^2}{(\frac{1}{4})^2}$

Answer

272

SIMPLIFY COMPLEX FRACTIONS.

1. Simplify the numerator.
2. Simplify the denominator.
3. Divide the numerator by the denominator. Simplify if possible.

Example 1.5.28

Simplify: $\dfrac{\frac{1}{2}+\frac{2}{3}}{\frac{3}{4}-\frac{1}{6}}$

Solution

	$\dfrac{(\frac{1}{2}+\frac{2}{3})}{(\frac{3}{4}-\frac{1}{6})}$	
Simplify the numerator (LCD $= 6$) and simplify the denominator (LCD $= 12$).	$\dfrac{(\frac{3}{6}+\frac{4}{6})}{(\frac{9}{12}-\frac{2}{12})}$	
Simplify.	$\dfrac{(\frac{7}{6})}{(\frac{7}{12})}$	(1.5.8)
Divide the numerator by the denominator.	$\frac{7}{6} \div \frac{7}{12}$	
Simplify.	$\frac{7}{6} \cdot \frac{12}{7}$	
Divide out common factors.	$\frac{7 \cdot 6 \cdot 2}{6 \cdot 7}$	
Simplify.	2	

Try It 1.5.29

Simplify: $\dfrac{\frac{1}{3}+\frac{1}{2}}{\frac{3}{4}-\frac{1}{3}}$

Answer

2

Try It 1.5.30

Simplify: $\dfrac{\frac{2}{3}-\frac{1}{2}}{\frac{1}{4}+\frac{1}{3}}$

Answer

$\frac{2}{7}$

Evaluate Variable Expressions with Fractions

We have evaluated expressions before, but now we can evaluate expressions with fractions. Remember, to evaluate an expression, we substitute the value of the variable into the expression and then simplify.

Example 1.5.31

Evaluate $x + \dfrac{1}{3}$ when

1. $x = -\dfrac{1}{3}$

2. $x = -\frac{3}{4}$

Solution

1. To evaluate $x + \frac{1}{3}$ when $x = -\frac{1}{3}$, substitute $-\frac{1}{3}$ for x in the expression.

	$x+\frac{1}{3}$
Substitute $-\frac{1}{3}$ for x.	$-\frac{1}{3}+\frac{1}{3}$
Simplify.	0

2. To evaluate $x + \frac{1}{3}$ when $x = -\frac{3}{4}$, substitute $-\frac{3}{4}$ for x in the expression.

	$x+\frac{1}{3}$
Substitute $-\frac{3}{4}$ for x.	$-\frac{3}{4}+\frac{1}{3}$
Rewrite as equivalent fractions with the LCD, 12.	$-\frac{3\cdot3}{4\cdot3}+\frac{1\cdot4}{3\cdot4}$
Simplify.	$-\frac{9}{12}+\frac{4}{12}$
Add.	$-\frac{5}{12}$

? Try It 1.5.32

Evaluate $x + \frac{3}{4}$ when

1. $x = -\frac{7}{4}$
2. $x = -\frac{5}{4}$

Answer

1. -1
2. $-\frac{1}{2}$

? Try It 1.5.33

Evaluate $y + \frac{1}{2}$ when

1. $y = \frac{2}{3}$
2. $y = -\frac{3}{4}$

Answer

1. $\frac{7}{6}$
2. $-\frac{1}{12}$

Example 1.5.34

Evaluate $-\frac{5}{6}-y$ when $y=-\frac{2}{3}$

Solution

	$-\frac{5}{6}-y$
Substitute $-\frac{2}{3}$ for y.	$-\frac{5}{6}-\left(-\frac{2}{3}\right)$
Rewrite as equivalent fractions with the LCD, 6.	$-\frac{5}{6}-\left(-\frac{4}{6}\right)$
Subtract.	$\frac{-5-(-4)}{6}$
Simplify.	$-\frac{1}{6}$

Try It 1.5.35

Evaluate $y+\frac{1}{2}$ when $y=\frac{2}{3}$

Answer

$-\frac{1}{4}$

Try It 1.5.36

Evaluate $y+\frac{1}{2}$ when $y=\frac{2}{3}$

Answer

$-\frac{17}{8}$

Example 1.5.37

Evaluate $2x^2y$ when $x=\frac{1}{4}$ and $y=-\frac{2}{3}$.

Solution

Substitute the values into the expression.

	$2x^2y$
Substitute $\frac{1}{4}$ for x and $-\frac{2}{3}$ for y.	$2\left(\frac{1}{4}\right)^2\left(-\frac{2}{3}\right)$
Simplify exponents first.	$2\left(\frac{1}{16}\right)\left(-\frac{2}{3}\right)$
Multiply. Divide out the common factors. Notice we write 16 as $2\cdot 2\cdot 4$ to make it easy to remove	$-\frac{\cancel{2}\cdot 1\cdot\cancel{2}}{\cancel{2}\cdot\cancel{2}\cdot 4\cdot 3}$
Simplify.	$-\frac{1}{12}$

Try It 1.5.38

Evaluate $3ab^2$ when $a=-\frac{2}{3}$ and $b=-\frac{1}{2}$.

Answer

$-\frac{1}{2}$

Try It 1.5.39

Evaluate $4c^3d$ when $c=-\frac{1}{2}$ and $d=-\frac{4}{3}$.

Answer

$\frac{2}{3}$

The next example will have only variables, no constants.

Example 1.5.40

Evaluate $\frac{p+q}{r}$ when $p=-4, q=-2$, and $r=8$.

Solution

To evaluate $\frac{p+q}{r}$ when $p=-4, q=-2$, and $r=8$, we substitute the values into the expression.

	$\frac{p+q}{r}$
Substitute –4 for p, –2 for q and 8 for r.	$\frac{-4+(-2)}{8}$
Add in the numerator first.	$\frac{-6}{8}$
Simplify.	$-\frac{3}{4}$

Try It 1.5.41

Evaluate $\frac{a+b}{c}$ when $a=-8, b=-7$, and $c=6$.

Answer

$-\frac{5}{2}$

Try It 1.5.42

Evaluate $\frac{x+y}{z}$ when $x=9, y=-18$, and $z=-6$.

Answer

$\frac{3}{2}$

Key Concepts

- **Fraction Addition and Subtraction:** If a, b, and c are numbers where $c \neq 0$, then
  $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$ and $\frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$
  To add or subtract fractions, add or subtract the numerators and place the result over the common denominator.
- **Strategy for Adding or Subtracting Fractions**
  1. Do they have a common denominator?
     Yes—go to step 2.
     No—Rewrite each fraction with the LCD (Least Common Denominator). Find the LCD. Change each fraction into an equivalent fraction with the LCD as its denominator.
  2. Add or subtract the fractions.
  3. Simplify, if possible. To multiply or divide fractions, an LCD IS NOT needed. To add or subtract fractions, an LCD IS needed.
- **Simplify Complex Fractions**
  1. Simplify the numerator.
  2. Simplify the denominator.
  3. Divide the numerator by the denominator. Simplify if possible.

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1.5E: Exercises

Practice Makes Perfect

Add and Subtract Fractions with a Common Denominator

In the following exercises, add.

Exercise 1.5E.1

$$\frac{6}{13}+\frac{5}{13}$$

Answer

$\frac{11}{13}$

Exercise 1.5E.2

$$\frac{4}{15}+\frac{7}{15}$$

Exercise 1.5E.3

$$\frac{x}{4}+\frac{3}{4}$$

Answer

$\frac{x+3}{4}$

Exercise 1.5E.4

$$\frac{8}{q}+\frac{6}{q}$$

Exercise 1.5E.5

$$-\frac{3}{16}+\left(-\frac{7}{16}\right)$$

Answer

$-\frac{5}{8}$

Exercise 1.5E.6

$$-\frac{5}{16}+\left(-\frac{9}{16}\right)$$

Exercise 1.5E.7

$$-\frac{8}{17}+\frac{15}{17}$$

Answer

$\frac{7}{17}$

Exercise 1.5E.8

$$-\frac{9}{19}+\frac{17}{19}$$

Exercise 1.5E.9

$$\frac{6}{13}+\left(-\frac{10}{13}\right)+\left(-\frac{12}{13}\right)$$

Answer

$-\frac{16}{13}$

Exercise 1.5E.10

$$\frac{5}{12}+\left(-\frac{7}{12}\right)+\left(-\frac{11}{12}\right)$$

In the following exercises, subtract.

Exercise 1.5E.11

$$\frac{11}{15}-\frac{7}{15}$$

Answer

$\frac{4}{15}$

Exercise 1.5E.12

$$\frac{9}{13}-\frac{4}{13}$$

Exercise 1.5E.13

$$\frac{11}{12}-\frac{5}{12}$$

Answer

$\frac{1}{2}$

Exercise 1.5E.14

$$\frac{7}{12}-\frac{5}{12}$$

Exercise 1.5E.15

$$\frac{19}{21}-\frac{4}{21}$$

Answer

$\frac{5}{7}$

Exercise 1.5E.16

$$\frac{17}{21} - \frac{8}{21}$$

Exercise 1.5E.17

$$\frac{5y}{8} - \frac{7}{8}$$

Answer

$\frac{5y-7}{8}$

Exercise 1.5E.18

$$\frac{11z}{13} - \frac{8}{13}$$

Exercise 1.5E.19

$$-\frac{23}{u} - \frac{15}{u}$$

Answer

$-\frac{38}{u}$

Exercise 1.5E.20

$$-\frac{29}{v} - \frac{26}{v}$$

Exercise 1.5E.21

$$-\frac{3}{5} - \left(-\frac{4}{5}\right)$$

Answer

$\frac{1}{5}$

Exercise 1.5E.22

$$-\frac{3}{7} - \left(-\frac{5}{7}\right)$$

Exercise 1.5E.23

$$-\frac{7}{9} - \left(-\frac{5}{9}\right)$$

Answer

$-\frac{2}{9}$

Exercise 1.5E.24

$$-\frac{8}{11}-\left(-\frac{5}{11}\right)$$

Mixed Practice

In the following exercises, simplify.

Exercise 1.5E.25

$$-\frac{5}{18}\cdot\frac{9}{10}$$

Answer

$-\frac{1}{4}$

Exercise 1.5E.26

$$-\frac{3}{14}\cdot\frac{7}{12}$$

Exercise 1.5E.27

$$\frac{n}{5}-\frac{4}{5}$$

Answer

$\frac{n-4}{5}$

Exercise 1.5E.28

$$\frac{6}{11}-\frac{s}{11}$$

Exercise 1.5E.29

$$-\frac{7}{24}+\frac{2}{24}$$

Answer

$-frac524$

Exercise 1.5E.30

$$-\frac{5}{18}+\frac{1}{18}$$

Exercise 1.5E.31

$$\frac{8}{15}\div\frac{12}{5}$$

Answer

$\frac{2}{9}$

Exercise 1.5E.32

$\frac{7}{12} \div \frac{9}{28}$

Add or Subtract Fractions with Different Denominators

In the following exercises, add or subtract.

Exercise 1.5E.33

$\frac{1}{2} + \frac{1}{7}$

Answer

$\frac{9}{14}$

Exercise 1.5E.34

$\frac{1}{3} + \frac{1}{8}$

Exercise 1.5E.35

$\frac{1}{3} - \left(-\frac{1}{9}\right)$

Answer

$\frac{4}{9}$

Exercise 1.5E.36

$\frac{1}{4} - \left(-\frac{1}{8}\right)$

Exercise 1.5E.37

$\frac{7}{12} + \frac{5}{12}$

Answer

$\frac{29}{24}$

Exercise 1.5E.38

$\frac{5}{12} + \frac{3}{8}$

Exercise 1.5E.39

$\frac{7}{12} - \frac{9}{16}$

Answer

$\frac{1}{48}$

Exercise 1.5E.40

$\frac{7}{16} - \frac{5}{12}$

Exercise 1.5E.41

$\frac{2}{3} - \frac{3}{8}$

Answer

$\frac{7}{24}$

Exercise 1.5E.42

$\frac{5}{6} - \frac{3}{4}$

Exercise 1.5E.43

$-\frac{11}{30} + \frac{27}{40}$

Answer

$\frac{37}{120}$

Exercise 1.5E.44

$-\frac{9}{20} + \frac{17}{30}$

Exercise 1.5E.45

$-\frac{13}{30} + \frac{25}{42}$

Answer

$\frac{17}{105}$

Exercise 1.5E.46

$-\frac{23}{30} + \frac{5}{48}$

Exercise 1.5E.47

$-\frac{39}{56} - \frac{22}{35}$

Answer

$-\frac{53}{40}$

Exercise 1.5E.48

$-\frac{33}{49} - \frac{18}{35}$

Exercise 1.5E.49

$-\frac{2}{3}-\left(-\frac{3}{4}\right)$

Answer

$\frac{1}{12}$

Exercise 1.5E.50

$-\frac{3}{4}-\left(-\frac{4}{5}\right)$

Exercise 1.5E.51

$1+\frac{7}{8}$

Answer

$\frac{15}{8}$

Exercise 1.5E.52

$1-\frac{3}{10}$

Exercise 1.5E.53

$\frac{x}{3}+\frac{1}{4}$

Answer

$\frac{4x+3}{12}$

Exercise 1.5E.54

$\frac{y}{2}+\frac{2}{3}$

Exercise 1.5E.55

$\frac{y}{4}-\frac{3}{5}$

Answer

$\frac{5y-12}{20}$

Exercise 1.5E.56

$\frac{x}{5}-\frac{1}{4}$

Mixed Practice

In the following exercises, simplify.

Exercise 1.5E.57

1. $\frac{2}{3}+\frac{1}{6}$
2. $\frac{2}{3} \div \frac{1}{6}$

Answer

1. $\frac{5}{6}$
2. 4

Exercise 1.5E.58

1. $-\frac{2}{5}-\frac{1}{8}$
2. $-\frac{2}{5} \cdot \frac{1}{8}$

Exercise 1.5E.59

1. $\frac{5n}{6} \div \frac{8}{15}$
2. $\frac{5n}{6}-\frac{8}{15}$

Answer

1. $\frac{25n}{16}$
2. $\frac{25n-16}{30}$

Exercise 1.5E.60

1. $\frac{3a}{8} \div \frac{7}{12}$
2. $\frac{3a}{8}-\frac{7}{12}$

Exercise 1.5E.61

$-\frac{3}{8} \div \left(-\frac{3}{10}\right)$

Answer

$\frac{5}{4}$

Exercise 1.5E.62

$-\frac{5}{12} \div \left(-\frac{5}{9}\right)$

Exercise 1.5E.63

$-\frac{3}{8}+\frac{5}{12}$

Answer

$\frac{1}{24}$

Exercise 1.5E.64

$-\frac{1}{8}+\frac{7}{12}$

Exercise 1.5E.65

$\frac{5}{6} - \frac{1}{9}$

Answer

$\frac{13}{18}$

Exercise 1.5E.66

$\frac{5}{9} - \frac{1}{6}$

Exercise 1.5E.67

$-\frac{7}{15} - \frac{y}{4}$

Answer

$\frac{-28-15y}{60}$

Exercise 1.5E.68

$-\frac{3}{8} - \frac{x}{11}$

Exercise 1.5E.69

$\frac{11}{12a} \cdot \frac{9a}{16}$

Answer

$\frac{33}{64}$

Exercise 1.5E.70

$\frac{10y}{13} \cdot \frac{8}{15y}$

Use the Order of Operations to Simplify Complex Fractions

In the following exercises, simplify.

Exercise 1.5E.71

$\frac{2^3+4^2}{\left(\frac{2}{3}\right)^2}$

Answer

54

Exercise 1.5E.72

$\frac{3^3-3^2}{\left(\frac{3}{4}\right)^2}$

Exercise 1.5E.73

$$\frac{\left(\frac{3}{5}\right)^2}{\left(\frac{3}{7}\right)^2}$$

Answer

$\frac{49}{25}$

Exercise 1.5E.74

$$\frac{\left(\frac{3}{4}\right)^2}{\left(\frac{5}{8}\right)^2}$$

Exercise 1.5E.75

$$\frac{2}{\frac{1}{3}+\frac{1}{5}}$$

Answer

$\frac{15}{4}$

Exercise 1.5E.76

$$\frac{5}{\frac{1}{4}+\frac{1}{3}}$$

Exercise 1.5E.77

$$\frac{\frac{7}{8}-\frac{2}{3}}{\frac{1}{2}+\frac{3}{8}}$$

Answer

$\frac{5}{21}$

Exercise 1.5E.78

$$\frac{\frac{3}{4}-\frac{3}{5}}{\frac{1}{4}+\frac{2}{5}}$$

Exercise 1.5E.79

$$\frac{1}{2}+\frac{2}{3}\cdot\frac{5}{12}$$

Answer

$\frac{7}{9}$

Exercise 1.5E.80

$$\frac{1}{3}+\frac{2}{5}\cdot\frac{3}{4}$$

Exercise 1.5E.81

$1-\frac{3}{5} \div \frac{1}{10}$

Answer

-5

Exercise 1.5E.82

$1-\frac{5}{6} \div \frac{1}{12}$

Exercise 1.5E.83

$\frac{2}{3}+\frac{1}{6}+\frac{3}{4}$

Answer

$\frac{19}{12}$

Exercise 1.5E.84

$\frac{2}{3}+\frac{1}{4}+\frac{3}{5}$

Exercise 1.5E.85

$\frac{3}{8}-\frac{1}{6}+\frac{3}{4}$

Answer

$\frac{23}{24}$

Exercise 1.5E.86

$\frac{2}{5}+\frac{5}{8}-\frac{3}{4}$

Exercise 1.5E.87

$12\left(\frac{9}{20}-\frac{4}{15}\right)$

Answer

$\frac{11}{5}$

Exercise 1.5E.88

$8\left(\frac{15}{16}-\frac{5}{6}\right)$

Exercise 1.5E.89

$\frac{\frac{5}{8}+\frac{1}{6}}{\frac{19}{24}}$

Answer

1

Exercise 1.5E.90

$\frac{\frac{1}{6}+\frac{3}{10}}{\frac{14}{30}}$

Exercise 1.5E.91

$\left(\frac{5}{9}+\frac{1}{6}\right) \div \left(\frac{2}{3}-\frac{1}{2}\right)$

Answer

$\frac{13}{3}$

Exercise 1.5E.92

$\left(\frac{3}{4}+\frac{1}{6}\right) \div \left(\frac{5}{8}-\frac{1}{3}\right)$

Evaluate Variable Expressions with Fractions

In the following exercises, evaluate.

Exercise 1.5E.93

$x+\left(-\frac{5}{6}\right)$ when

1. $x=\frac{1}{3}$
2. $x=-\frac{1}{6}$

Answer

1. $-\frac{1}{2}$
2. -1

Exercise 1.5E.94

$x+\left(-\frac{11}{12}\right)$ when

1. $x=\frac{11}{12}$
2. $x=-\frac{3}{4}$

Exercise 1.5E.95

$x-\frac{2}{5}$ when

1. $x=\frac{3}{5}$
2. $x=-\frac{3}{5}$

Answer

1. $\frac{1}{5}$
2. -1

Exercise 1.5E.96

$x - \frac{1}{3}$ when

1. $x = \frac{2}{3}$
2. $x = -\frac{2}{3}$

Exercise 1.5E.97

$\frac{7}{10} - w$ when

1. $w = \frac{1}{2}$
2. $w = -\frac{1}{2}$

Answer

1. $\frac{1}{5}$
2. $\frac{6}{5}$

Exercise 1.5E.98

$\frac{5}{12} - w$ when

1. $w = \frac{1}{4}$
2. $w = -\frac{1}{4}$

Exercise 1.5E.99

$2x^2y^3$ when $x = -\frac{2}{3}$ and $y = -\frac{1}{2}$

Answer

$-\frac{1}{9}$

Exercise 1.5E.100

$8u^2v^3$ when $u = -\frac{3}{4}$ and $v = -\frac{1}{2}$

Exercise 1.5E.101

$\frac{a+b}{a-b}$ when $a = -3, b = 8$

Answer

$-\frac{5}{11}$

Exercise 1.5E.102

$\frac{r-s}{r+s}$ when $r = 10, s = -5$

Everyday Math

Exercise 1.5E.103

Decorating Laronda is making covers for the throw pillows on her sofa. For each pillow cover, she needs $\frac{1}{2}$ yard of print fabric and \frac{3}{8}\) yard of solid fabric. What is the total amount of fabric Laronda needs for each pillow cover?

Answer

$\frac{7}{8}$ yard

Exercise 1.5E.104

Baking Vanessa is baking chocolate chip cookies and oatmeal cookies. She needs $\frac{1}{2}$ cup of sugar for the chocolate chip cookies and $\frac{1}{4}$ of sugar for the oatmeal cookies. How much sugar does she need altogether?

Writing Exercises

Exercise 1.5E.105

Why do you need a common denominator to add or subtract fractions? Explain.

Answer

Answers may vary

Exercise 1.5E.106

How do you find the LCD of 2 fractions?

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
add and subtract fractions with different denominators.			
identify and use fraction operations.			
use the order of operations to simplify complex fractions.			
evaluate variable expressions with fractions.			

ⓑ After looking at the checklist, do you think you are well-prepared for the next chapter? Why or why not?

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1.6: Decimals

Learning Objectives

By the end of this section, you will be able to:

- Name and write decimals
- Round decimals
- Add and subtract decimals
- Multiply and divide decimals
- Convert decimals, fractions, and percents

Note

A more thorough introduction to the topics covered in this section can be found in the *Prealgebra*chapter, **Decimals**.

Name and Write Decimals

Decimals are another way of writing **fraction**s whose denominators are powers of 10.

$$\begin{aligned} 0.1 &= \tfrac{1}{10} && 0.1 \text{ is "one tenth"} \\ 0.01 &= \tfrac{1}{100} && 0.01 \text{ is "one hundredth} \\ 0.001 &= \tfrac{1}{1{,}000} && 0.001 \text{ is "one thousandth} \\ 0.0001 &= \tfrac{1}{10{,}000} && 0.0001 \text{ is "one ten-thousandth"} \end{aligned} \tag{1.6.1}$$

Notice that "ten thousand" is a number larger than one, but "one ten-thousand**th**" is a number smaller than one. The "th" at the end of the name tells you that the number is smaller than one.

When we name a whole number, the name corresponds to the place value based on the powers of ten. We read 10,000 as "ten thousand" and 10,000,000 as "ten million." Likewise, the names of the **decimal** places correspond to their fraction values. Figure 1.6.1 shows the names of the **place values** to the left and right of the decimal point.

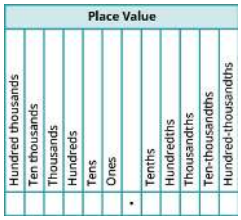


Figure 1.6.1: Place value of decimal numbers are shown to the left and right of the decimal point.

Exercise 1.6.1

Name the decimal 4.3.

Answer

Step 1. Name the number to the left of the decimal point.	4 is to the left of the decimal point.	4.3 four _____	**Step 2.** Write 'and' for the decimal point.		four and _____
Step 3. Name the 'number' part to the right of the decimal point as if it were a whole number.	3 is to the right of the decimal point.	four and three _____	**Step 4.** Name the decimal place.		four and three tenths

Exercise 1.6.2

Name the decimal 6.7.

Answer

six and seven tenths

Exercise 1.6.3

Name the decimal 5.8.

Answer

five and eight tenths

We summarize the steps needed to name a **decimal** below.

NAME A DECIMAL.

1. Name the number to the left of the decimal point.
2. Write "and" for the decimal point.
3. Name the "number" part to the right of the decimal point as if it were a whole number.
4. Name the decimal place of the last digit.

Exercise 1.6.4

Name the decimal: -15.571.

Answer

	-15.571
Name the number to the left of the decimal point.	negative fifteen _____
Write “and” for the decimal point.	negative fifteen and _____
Name the number to the right of the decimal point.	negative fifteen and five hundred seventy-one _____
The 1 is in the thousandths place.	negative fifteen and five hundred seventy-one thousandths

Exercise 1.6.5

Name the decimal: -13.461.

Answer

negative thirteen and four hundred sixty-one thousandths

Exercise 1.6.6

Name the decimal: -2.053.

Answer

negative two and fifty-three thousandths

When we write a check we write both the numerals and the name of the number. Let’s see how to write the decimal from the name.

Exercise 1.6.7: How to Write decimals

Write “fourteen and twenty-four thousandths” as a decimal.

Answer

Step 1. Look for the word ‘and’; it locates the decimal point. Place a decimal point under the word ‘and’. Translate the words before ‘and’ into the whole number and place to the left of the decimal point.		fourteen and twenty-four thousandths fourteen and twenty-four thousandths ____ . _____ 14. _____
Step 2. Mark the number of decimal places needed to the right of the decimal point by noting the place value indicated by the last word.	The last word is ‘thousandths’.	14. ____ ____ ____ tenths hundredths thousandths
Step 3. Translate the words after ‘and’ into the number to the right of the decimal point. Write the number in the spaces – putting the final digit in the last place.		14. ____ 2 4
Step 4. Fill in zeros for empty place holders as needed.	Zeros are needed in the tenths place.	14. 0 2 4 Fourteen and twenty-four thousandths is written 14.024.

Exercise 1.6.8

Write as a decimal: thirteen and sixty-eight thousandths.

Answer

13.068

Exercise 1.6.9

Write as a decimal: five and ninety-four thousandths.

Answer

5.094

We summarize the steps to writing a **decimal**.

WRITE A DECIMAL.

1. Look for the word “and”—it locates the decimal point.
   - Place a decimal point under the word “and.” Translate the words before “and” into the whole number and place it to the left of the decimal point.
   - If there is no “and,” write a “0” with a decimal point to its right.
2. Mark the number of decimal places needed to the right of the decimal point by noting the place value indicated by the last word.
3. Translate the words after “and” into the number to the right of the decimal point. Write the number in the spaces—putting the final digit in the last place.
4. Fill in zeros for place holders as needed.

Round Decimals

Rounding decimals is very much like rounding whole numbers. We will round decimals with a method based on the one we used to round whole numbers.

Exercise 1.6.10

Round 18.379 to the nearest hundredth.

Answer

Step 1. Locate the given place value and mark it with an arrow.		hundredths place ↓ 18.379	**Step 2.** Underline the digit to the right of the given place value.		hundredths place ↓ 18.379
Step 3. Is this digit greater than or equal to 5? **Yes:** Add 1 to the digit in the given place value. **No:** Do not change the digit in the given place value.	Because 9 is greater than or equal to 5, add 1 to the 7.	18.37 9 add 1 delete	**Step 4.** Rewrite the number, removing all digits to the right of the rounding digit.		18.38 18.38 is 18.379 rounded to the nearest hundredth.

Exercise 1.6.11

Round to the nearest hundredth: 1.047.

Answer

1.05

Exercise 1.6.12

Round to the nearest hundredth: 9.173.

Answer

9.17

We summarize the steps for rounding a **decimal** here.

ROUND DECIMALS.

1. Locate the given place value and mark it with an arrow.
2. Underline the digit to the right of the place value.
3. Is this digit greater than or equal to 5?
   - Yes—add 1 to the digit in the given place value.
   - No—do not change the digit in the given place value.
4. Rewrite the number, deleting all digits to the right of the rounding digit.

Exercise 1.6.13

Round 18.379 to the nearest

1. tenth
2. whole number.

Answer

Round 18.379

1. to the nearest tenth

Locate the tenths place with an arrow.	tenths place ↓ 18.379
Underline the digit to the right of the given place value.	tenths place ↓ 18.379
Because 7 is greater than or equal to 5, add 1 to the 3.	18.379 add 1 delete
Rewrite the number, deleting all digits to the right of the rounding digit.	18.4
Notice that the deleted digits were NOT replaced with zeros.	So, 18.379 rounded to the nearest tenth is 18.4.

2. to the nearest whole number

Locate the ones place with an arrow.	ones place ↓ 18.379
Underline the digit to the right of the given place value.	ones place ↓ 18.379

Since 3 is not greater than or equal to 5, do not add 1 to the 8.	18.379 do not add 1 delete
Rewrite the number, deleting all digits to the right of the rounding digit.	18
	So, 18.379 rounded to the nearest whole number is 18.

Exercise 1.6.14

Round 6.582 to the nearest

1. hundredth
2. tenth
3. whole number.

Answer

1. 6.58
2. 6.6
3. 7

Exercise 1.6.15

Round 15.2175 to the nearest

1. thousandth
2. hundredth
3. tenth.

Answer

1. 15.218
2. 15.22
3. 15.2

Add and Subtract Decimals

To add or subtract decimals, we line up the decimal points. By lining up the decimal points this way, we can add or subtract the corresponding **place values**. We then add or subtract the numbers as if they were whole numbers and then place the decimal point in the sum.

ADD OR SUBTRACT DECIMALS.

1. Write the numbers so the decimal points line up vertically.
2. Use zeros as place holders, as needed.
3. Add or subtract the numbers as if they were whole numbers. Then place the decimal point in the answer under the decimal points in the given numbers.

Exercise 1.6.16

Add: $23.5+41.38$.

Answer

Write the numbers so that the decimal points line up vertically.

$$\begin{array}{r} 23.50 \\ +41.38 \\ \hline \end{array} \tag{1.6.2}$$

Put 0 as a placeholder after the 5 in 23.5. Remember, $\frac{5}{10}=\frac{50}{100}$, so $0.5=0.50$

$$\begin{array}{r} 23.50 \\ +41.38 \\ \hline \end{array} \tag{1.6.3}$$

Add the numbers as if they were whole numbers . Then place the decimal point in the sum.

$$\begin{array}{r} 23.50 \\ +41.38 \\ \hline 64.88 \end{array} \tag{1.6.4}$$

Exercise 1.6.17

Add: $4.8+11.69$.

Answer

16.49

Exercise 1.6.18

Add: $5.123+18.47$.

Answer

23.593

Exercise 1.6.19

Subtract: $20 - 14.65$.

Answer

Write the numbers so that the decimal points line up vertically.

$$\begin{array}{r} 20 - 14.65 \\ 20. \\ -14.65 \\ \hline \end{array}$$

Remember, 20 is a whole number, so place the decimal point after the 0.

Put zeros to the right as placeholders.

$$\begin{array}{rr} 20.00 & (1.6.8) \\ -14.65 & (1.6.9) \\ \hline \end{array}$$

Write the numbers so that the decimal points line up vertically.

$$\begin{array}{r} {}^{9}\quad{}^{9} \\ 1\,\not{1}\not{0}\;\not{1}\not{0}\;\not{1}\not{0} \\ \not{2}\;\not{0}.\;\not{0}\;\not{0} \\ -14.65 \\ \hline 5.35 \end{array}$$

Exercise 1.6.20

Subtract: $10 - 9.58$.

Answer

0.42

Exercise 1.6.21

Subtract: $50 - 37.42$.

Answer

12.58

Multiply and Divide Decimals

Multiplying decimals is very much like multiplying whole numbers—we just have to determine where to place the decimal point. The procedure for multiplying decimals will make sense if we first convert them to fractions and then multiply.

So let's see what we would get as the **product** of decimals by converting them to fractions first. We will do two examples side-by-side. Look for a pattern!

Table 1.6.1

	(0.3) (0.7)	(0.2) (0.46)
Convert to fractions.	$\frac{3}{10} \cdot \frac{7}{10}$	$\frac{2}{10} \cdot \frac{46}{100}$
Multiply.	$\frac{21}{100}$	$\frac{92}{1000}$
Convert to decimals.	0.21	0.092

Notice, in the first example, we multiplied two numbers that each had one digit after the decimal point and the product had two decimal places. In the second example, we multiplied a number with one decimal place by a number with two decimal places and the product had three decimal places.

We multiply the numbers just as we do whole numbers, temporarily ignoring the decimal point. We then count the number of decimal points in the factors and that sum tells us the number of decimal places in the product.

The rules for multiplying positive and negative numbers apply to decimals, too, of course!

When *multiplying* two numbers,

- if their signs are the *same* the product is *positive*.
- if their signs are *different* the product is *negative*.

When we multiply signed decimals, first we determine the sign of the product and then multiply as if the numbers were both positive. Finally, we write the product with the appropriate sign.

MULTIPLY DECIMALS.

1. Determine the sign of the product.
2. Write in vertical format, lining up the numbers on the right. Multiply the numbers as if they were whole numbers, temporarily ignoring the decimal points.
3. Place the decimal point. The number of decimal places in the product is the sum of the number of decimal places in the factors.
4. Write the product with the appropriate sign.

Exercise 1.6.22

Multiply: $(-3.9)(4.075)$.

Answer

	$(-3.9)(4.075)$
The signs are different. The product will be negative.	
Write in vertical format, lining up the numbers on the right.	4.075 × 3.9
Multiply.	4.075 × 3.9 36675 12225 158925
Add the number of decimal places in the factors $(1+3)$. (–3.9) (4.075) 1 place 3 places Place the decimal point 4 places from the right.	4.075 × 3.9 36675 12225 15.8925 4 places
The signs are different, so the product is negative.	$(-3.9)(4.075) = -15.8925$

Exercise 1.6.23

Multiply: $-4.5(6.107)$

Answer

-27.4815

Exercise 1.6.24

Multiply: $-10.79(8.12)$.

Answer

-87.6148

In many of your other classes, especially in the sciences, you will multiply decimals by powers of 10 (10, 100, 1000, etc.). If you multiply a few products on paper, you may notice a pattern relating the number of zeros in the power of 10 to number of decimal places we move the decimal point to the right to get the product.

MULTIPLY A DECIMAL BY A POWER OF TEN.

1. Move the decimal point to the right the same number of places as the number of zeros in the power of 10.
2. Add zeros at the end of the number as needed.

Exercise 1.6.25

Multiply 5.63

1. by 10
2. by 100
3. by 1,000.

Answer

By looking at the number of zeros in the multiple of ten, we see the number of places we need to move the decimal to the right.

ⓐ

	$5.63(10)$
There is 1 zero in 10, so move the decimal point 1 place to the right.	5.63 56.3

ⓑ

	$5.63(100)$
There are 2 zeros in 100, so move the decimal point 2 places to the right.	There are 2 zeros in 100, so move the decimal point 2 places to the right. 5.63 (100) 5.63 563

ⓒ

There are 3 zeros in 1,000, so move the decimal point 3 places to the right.	5.63
A zero must be added at the end.	5,630

Exercise 1.6.26

Multiply 2.58

1. by 10
2. by 100
3. by 1,000.

Answer

1. 25.8
2. 258
3. 2,580

Exercise 1.6.27

Multiply 14.2

1. by 10
2. by 100
3. by 1,000.

Answer

1. 142
2. 1,420
3. 14,200

Just as with multiplication, **division** of decimals is very much like dividing whole numbers. We just have to figure out where the decimal point must be placed.

To divide decimals, determine what power of 10 to multiply the denominator by to make it a whole number. Then multiply the numerator by that same power of 10. Because of the equivalent fractions property, we haven't changed the value of the fraction! The effect is to move the decimal points in the numerator and denominator the same number of places to the right. For example:

$$\begin{array}{c} \frac{0.8}{0.4} \\ \frac{0.8(10)}{0.4(10)} \\ \frac{8}{4} \end{array} \tag{1.6.16}$$

We use the rules for dividing positive and negative numbers with decimals, too. When dividing signed decimals, first determine the sign of the quotient and then divide as if the numbers were both positive. Finally, write the quotient with the appropriate sign.

We review the notation and vocabulary for division:

$$\underset{\text{dividend}}{a} \div \underset{\text{divisor}}{b} = \underset{\text{quotient}}{c} \qquad \begin{array}{r} \overset{\text{quotient}}{c} \\ \underset{\text{divisor}}{b}\,\overline{)\,\underset{\text{dividend}}{a}} \end{array} \tag{1.6.17}$$

We'll write the steps to take when dividing decimals, for easy reference.

DIVIDE DECIMALS.

1. Determine the sign of the quotient.
2. Make the divisor a whole number by "moving" the decimal point all the way to the right. "Move" the decimal point in the dividend the same number of places—adding zeros as needed.
3. Divide. Place the decimal point in the quotient above the decimal point in the dividend.
4. Write the quotient with the appropriate sign.

Exercise 1.6.28

Divide: $-25.65 \div (-0.06)$.

Answer

Remember, you can "move" the decimals in the divisor and dividend because of the Equivalent Fractions Property.

	$-25.65 \div (-0.06)$
The signs are the same.	The quotient is positive.
Make the divisor a whole number by "moving" the decimal point all the way to the right.	
"Move" the decimal point in the dividend the same number of places.	$0.06\overline{)25.65}$
Divide. Place the decimal point in the quotient above the decimal point in the dividend.	$\begin{array}{r} 427.5 \\ 006\overline{)2565.0} \\ -24 \\ \hline 16 \\ -12 \\ \hline 45 \\ -42 \\ \hline 30 \\ 30 \end{array}$
Write the quotient with the appropriate sign.	$-25.65 \div (-0.06) = 427.5$

Exercise 1.6.29

Divide: $-23.492 \div (-0.04)$.

Answer

687.3

Exercise 1.6.30

Divide: $-4.11 \div (-0.12)$.

Answer

34.25

A common application of dividing whole numbers into decimals is when we want to find the price of one item that is sold as part of a multi-pack. For example, suppose a case of 24 water bottles costs $3.99. To find the price of one water bottle, we would divide $3.99 by 24. We show this division in Exercise 1.6.31. In calculations with money, we will round the answer to the nearest cent (hundredth).

Exercise 1.6.31

Divide: $3.99 \div 24$.

Answer

.99 divided by 24 is given. A long division problem is set up with 24 dividing 3.99. A table is given with directions on the left and the mathematical steps on the right. The first step reads “Place the decimal point in the quotient above the decimal point in the dividend. Divide as usual. When do we stop? Since this division involves money, we round it to the nearest cent (hundredth). To do this, we must carry the division to the thousandths place.” To the right of this, we have a long division problem set up with 24 dividing 3.990. The quotient is given as 0.166. To show the work, below 3.990 it reads 24, solid horizontal line, 159, 144, solid horizontal line, 150, 144, solid horizontal line, and finally 6. The fifth step reads “Round to the nearest cent.” To the right of this, we have $0.166 is approximately equal to $0.17 and hence >.99 divided by 24 is $0.17.">

	$3.99 \div 24$
Place the decimal point in the quotient above the decimal point in the dividend.	
Divide as usual. When do we stop? Since this division involves money, we round it to the nearest cent (hundredth.) To do this, we must carry the division to the thousandths place.	$$\begin{array}{r} 0.166 \\ 24\overline{)3.990} \\ \underline{24} \\ 159 \\ \underline{144} \\ 150 \\ \underline{144} \\ 6 \end{array}$$
Round to the nearest cent.	$0.166 \approx $0.17 $3.99 \div 2 \approx $0.17

Exercise 1.6.32

Divide: $6.99 \div 36$.

Answer

$0.19

Exercise 1.6.33

Divide: $4.99 \div 12$.

Answer

$0.42

Convert Decimals, Fractions, and Percents

We convert decimals into fractions by identifying the place value of the last (farthest right) digit. In the decimal 0.03 the 3 is in the hundredths place, so 100 is the denominator of the fraction equivalent to 0.03.

$$00.03 = \frac{3}{100} \tag{1.6.18}$$

Notice, when the number to the left of the decimal is zero, we get a fraction whose numerator is less than its denominator. Fractions like this are called **proper fractions**.

The steps to take to convert a decimal to a fraction are summarized in the procedure box.

CONVERT A DECIMAL TO A PROPER FRACTION.

1. Determine the place value of the final digit.
2. Write the fraction.
   - numerator—the “numbers” to the right of the decimal point
   - denominator—the place value corresponding to the final digit

Exercise 1.6.34

Write 0.374 as a fraction.

Answer

	0.374
Determine the place value of the final digit.	0.3 (tenths) 7 (hundredths) 4 (thousandths)
Write the fraction for 0.374: • The numerator is 374. • The denominator is 1,000.	$\frac{374}{1000}$
Simplify the fraction.	$\frac{2 \cdot 187}{2 \cdot 500}$
Divide out the common factors.	$\frac{187}{500}$ so, $0.374 = \frac{187}{500}$

Did you notice that the number of zeros in the denominator of $\frac{374}{1000}$ is the same as the number of decimal places in 0.374?

Exercise 1.6.35

Write 0.234 as a fraction.

Answer

$\frac{117}{500}$

Exercise 1.6.36

Write 0.024 as a fraction.

Answer

$\frac{3}{125}$

We've learned to convert decimals to fractions. Now we will do the reverse—convert fractions to decimals. Remember that the fraction bar means division. So $\frac{4}{5}$ can be written $4 \div 5$ or $5\overline{)4}$. This leads to the following method for converting a fraction to a decimal.

CONVERT A FRACTION TO A DECIMAL.

To convert a fraction to a decimal, divide the numerator of the fraction by the denominator of the fraction.

Exercise 1.6.37

Write $-\frac{5}{8}$ as a decimal.

Answer

Since a fraction bar means division, we begin by writing $\frac{5}{8}$ as $8\overline{)5}$. Now divide.

$$\begin{array}{r} 0.625 \\ 8\overline{)5.000} \\ \underline{48} \\ 20 \\ \underline{16} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

so, $-\frac{5}{8} = -0.625$

Exercise 1.6.38

Write $-\frac{7}{8}$ as a decimal.

Answer

−0.875

Exercise 1.6.39

Write $-\frac{3}{8}$ as a decimal.

Answer

−0.375

When we divide, we will not always get a zero remainder. Sometimes the quotient ends up with a decimal that repeats. A **repeating decimal** is a decimal in which the last digit or group of digits repeats endlessly. A bar is placed over the repeating block of digits to indicate it repeats.

REPEATING DECIMAL

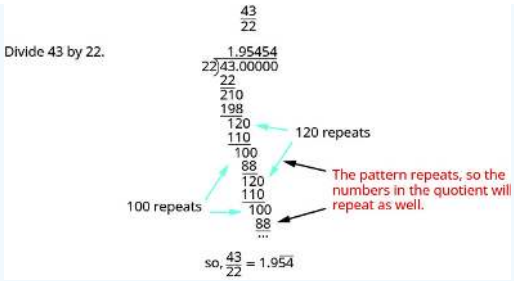
A **repeating decimal** is a decimal in which the last digit or group of digits repeats endlessly.

A bar is placed over the repeating block of digits to indicate it repeats.

Exercise 1.6.40

Write $\frac{43}{22}$ as a decimal.

Answer



Exercise 1.6.41

Write $\frac{27}{11}$ as a decimal.

Answer

$2.\overline{45}$

Exercise 1.6.42

Write $\frac{51}{22}$ as a decimal.

Answer

$2.3\overline{18}$

Sometimes we may have to simplify expressions with fractions and decimals together.

Exercise 1.6.43

Simplify: $\frac{7}{8}+6.4$.

Answer

First we must change one number so both numbers are in the same form. We can change the fraction to a decimal, or change the decimal to a fraction. Usually it is easier to change the fraction to a decimal.

		$\frac{7}{8}+6.4$
Change $\frac{7}{8}$ to a decimal.	$8\overline{)7.000}$ 64 60 56 40 40 0	
Add.		$0.875+6.4$
		7.275
		So, $\frac{7}{8}+6.4=7.275$

Exercise 1.6.44

Simplify: $\frac{3}{8}+4.9$.

Answer

5.275

Exercise 1.6.45

Simplify: $5.7+\frac{13}{20}$.

Answer

6.35

A **percent** is a ratio whose denominator is 100. Percent means per hundred. We use the percent symbol, %, to show percent.

PERCENT

A **percent** is a ratio whose denominator is 100.

Since a percent is a ratio, it can easily be expressed as a fraction. Percent means per 100, so the denominator of the fraction is 100. We then change the fraction to a decimal by dividing the numerator by the denominator.

	6%	78%	135%
Write as a ratio with denominator 100.	$\frac{6}{100}$	$\frac{78}{100}$	$\frac{135}{100}$
Change the fraction to a decimal by dividing the numerator by the denominator.	0.06	0.78	1.35

(1.6.19)

Do you see the pattern? *To convert a percent number to a decimal number, we move the decimal point two places to the left.*



Figure 1.6.2

Exercise 1.6.46

Convert each percent to a decimal:

1. 62%
2. 135%
3. 35.7%.

Answer

1.	
	62%
Move the decimal point two places to the left.	0.62
2.	
	135%
Move the decimal point two places to the left.	1.35
3.	
	5.7%
Move the decimal point two places to the left.	0.057

Exercise 1.6.47

Convert each percent to a decimal:

1. 9%
2. 87%
3. 3.9%.

Answer

1. 0.09
2. 0.87
3. 0.039

Exercise 1.6.48

Convert each percent to a decimal:

1. 3%
2. 91%
3. 8.3%.

Answer

1. 0.03
2. 0.91
3. 0.083

Converting a decimal to a percent makes sense if we remember the definition of percent and keep place value in mind.

To convert a decimal to a percent, remember that percent means per hundred. If we change the decimal to a fraction whose denominator is 100, it is easy to change that fraction to a percent.

	0.83	1.05	0.075
Write as a fraction	$\frac{83}{100}$	$1\frac{5}{100}$	$\frac{75}{1000}$
The denominator is 100.		$\frac{105}{100}$	$\frac{7.5}{100}$
Write the ratio as a percent.	83%	105%	7.5%

(1.6.20)

Recognize the pattern? *To convert a decimal to a percent, we move the decimal point two places to the right and then add the percent sign.*

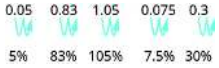


Figure 1.6.3

Exercise 1.6.49

Convert each decimal to a percent:

1. 0.51
2. 1.25
3. 0.093.

Answer

1.	
	0.51
Move the decimal point two places to the right.	51

2.	
	1.25
Move the decimal point two places to the right.	125

3.	
	0.093
Move the decimal point two places to the right.	9.3

Exercise 1.6.50

Convert each decimal to a percent:

1. 0.17
2. 1.75
3. 0.0825

Answer

1. 17%
2. 175%
3. 8.25%

Exercise 1.6.51

Convert each decimal to a percent:

1. 0.41
2. 2.25
3. 0.0925.

Answer

1. 41%
2. 225%
3. 9.25%

Key Concepts

- **Name a Decimal**
  1. Name the number to the left of the decimal point.
  2. Write ”and” for the decimal point.
  3. Name the “number” part to the right of the decimal point as if it were a whole number.
  4. Name the decimal place of the last digit.
- **Write a Decimal**
  1. Look for the word ‘and’—it locates the decimal point. Place a decimal point under the word ‘and.’ Translate the words before ‘and’ into the whole number and place it to the left of the decimal point. If there is no “and,” write a “0” with a decimal point to its right.
  2. Mark the number of decimal places needed to the right of the decimal point by noting the place value indicated by the last word.
  3. Translate the words after ‘and’ into the number to the right of the decimal point. Write the number in the spaces—putting the final digit in the last place.
  4. Fill in zeros for place holders as needed.
- **Round a Decimal**
  1. Locate the given place value and mark it with an arrow.
  2. Underline the digit to the right of the place value.
  3. Is this digit greater than or equal to 5? Yes—add 1 to the digit in the given place value. No—do not change the digit in the given place value.
  4. Rewrite the number, deleting all digits to the right of the rounding digit.
- **Add or Subtract Decimals**
  1. Write the numbers so the decimal points line up vertically.
  2. Use zeros as place holders, as needed.
  3. Add or subtract the numbers as if they were whole numbers. Then place the decimal in the answer under the decimal points in the given numbers.
- **Multiply Decimals**
  1. Determine the sign of the product.

2. Write in vertical format, lining up the numbers on the right. Multiply the numbers as if they were whole numbers, temporarily ignoring the decimal points.
3. Place the decimal point. The number of decimal places in the product is the sum of the decimal places in the factors.
4. Write the product with the appropriate sign.

- **Multiply a Decimal by a Power of Ten**
  1. Move the decimal point to the right the same number of places as the number of zeros in the power of 10.
  2. Add zeros at the end of the number as needed.
- **Divide Decimals**
  1. Determine the sign of the quotient.
  2. Make the divisor a whole number by “moving” the decimal point all the way to the right. “Move” the decimal point in the dividend the same number of places - adding zeros as needed.
  3. Divide. Place the decimal point in the quotient above the decimal point in the dividend.
  4. Write the quotient with the appropriate sign.
- **Convert a Decimal to a Proper Fraction**
  1. Determine the place value of the final digit.
  2. Write the fraction: numerator—the ‘numbers’ to the right of the decimal point; denominator—the place value corresponding to the final digit.
- **Convert a Fraction to a Decimal** Divide the numerator of the fraction by the denominator.

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1.6E: Exercises

Practice Makes Perfect

Needs to be integrated from https://cnx.org/contents/CImQfPDv@8.46:NciJ1qa0@19/1-7-Decimals

Name and Write Decimals

In the following exercises, write as a decimal.

531.

Twenty-nine and eighty-one hundredths

532.

Sixty-one and seventy-four hundredths

533.

Seven tenths

534.

Six tenths

535.

Twenty-nine thousandth

536.

Thirty-five thousandths

537.

Negative eleven and nine ten-thousandths

538.

Negative fifty-nine and two ten-thousandths

In the following exercises, name each decimal.

539.

5.5

540.

14.02

541.

8.71

542.

2.64

543.

0.002

544.

0.479

545.

−17.9−17.9

546.

−31.4−31.4

Round Decimals

In the following exercises, round each number to the nearest tenth.

547.

0.67

548.

0.49

549.

2.84

550.

4.63

In the following exercises, round each number to the nearest hundredth.

551.

0.845

552.

0.761

553.

0.299

554.

0.697

555.

4.098

556.

7.096

In the following exercises, round each number to the nearest ⓐ hundredth ⓑ tenth ⓒ whole number.

557.

5.781

558.

1.6381

559.

63.479

560.

84.28184.281

Add and Subtract Decimals

In the following exercises, add or subtract.

561.

16.92+7.5616.92+7.56

562.

248.25−91.29248.25−91.29

563.

21.76−30.9921.76−30.99

564.

38.6+13.6738.6+13.67

565.

−16.53−24.38−16.53−24.38

566.

−19.47−32.58−19.47−32.58

567.

−38.69+31.47−38.69+31.47

568.

29.83+19.7629.83+19.76

569.

72.5−10072.5−100

570.

86.2−10086.2−100

571.

15+0.7315+0.73

572.

27+0.8727+0.87

573.

91.95−(−10.462)91.95−(−10.462)

574.

94.69−(−12.678)94.69−(−12.678)

575.

55.01−3.755.01−3.7

576.

59.08−4.659.08−4.6

577.

2.51−7.42.51−7.4

578.

3.84−6.13.84−6.1

Multiply and Divide Decimals

In the following exercises, multiply.

579.

(0.24)(0.6)(0.24)(0.6)

580.

(0.81)(0.3)(0.81)(0.3)

581.

(5.9)(7.12)(5.9)(7.12)

582.

(2.3)(9.41)(2.3)(9.41)

583.

(−4.3)(2.71)(−4.3)(2.71)

584.

(−8.5)(1.69)(−8.5)(1.69)

585.

(−5.18)(−65.23)(−5.18)(−65.23)

586.

(−9.16)(−68.34)(−9.16)(−68.34)

587.

(0.06)(21.75)(0.06)(21.75)

588.

(0.08)(52.45)(0.08)(52.45)

589.

(9.24)(10)(9.24)(10)

590.

(6.531)(10)(6.531)(10)

591.

(55.2)(1000)(55.2)(1000)

592.

(99.4)(1000)(99.4)(1000)

In the following exercises, divide.

593.

4.75÷254.75÷25

594.

12.04÷4312.04÷43

595.

$117.25÷48$117.25÷48

596.

$109.24÷36$109.24÷36

597.

0.6÷0.20.6÷0.2

598.

0.8÷0.40.8÷0.4

599.

1.44÷(−0.3)1.44÷(−0.3)

600.

1.25÷(−0.5)1.25÷(−0.5)

601.

−1.75÷(−0.05)−1.75÷(−0.05)

602.

−1.15÷(−0.05)−1.15÷(−0.05)

603.

5.2÷2.55.2÷2.5

604.

6.5÷3.256.5÷3.25

605.

11÷0.5511÷0.55

606.

14÷0.3514÷0.35

Convert Decimals, Fractions and Percents

In the following exercises, write each decimal as a fraction.

607.

0.04

608.

0.19

609.

0.52

610.

0.78

611.

1.25

612.

1.35

613.

0.375

614.

0.464

615.

0.095

616.

0.085

In the following exercises, convert each fraction to a decimal.

617.

17201720

618.

13201320

619.

114114

620.

174174

621.

−31025−31025

622.

−28425−28425

623.

15111511

624.

18111811

625.

1511115111

626.

2511125111

627.

2.4+582.4+58

628.

3.9+9203.9+920

In the following exercises, convert each percent to a decimal.

629.

1%

630.

2%

631.

63%

632.

71%

633.

150%

634.

250%

635.

21.4%

636.

39.3%

637.

7.8%

638.

6.4%

In the following exercises, convert each decimal to a percent.

639.

0.01

640.

0.03

641.

1.35

642.

1.56

643.

3

644.

4

645.

0.0875

646.

0.0625

647.

2.254

648.

2.317

Everyday Math

649.

Salary Increase Danny got a raise and now makes $58,965.95 a year. Round this number to the nearest
ⓐ dollar
ⓑ thousand dollars
ⓒ ten thousand dollars.

650.

New Car Purchase Selena's new car cost $23,795.95. Round this number to the nearest
ⓐ dollar
ⓑ thousand dollars
ⓒ ten thousand dollars.

651.

Sales Tax Hyo Jin lives in San Diego. She bought a refrigerator for $1,624.99 and when the clerk calculated the sales tax it came out to exactly $142.186625. Round the sales tax to the nearest
ⓐ penny and
ⓑ dollar.

652.

Sales Tax Jennifer bought a $1,038.99 dining room set for her home in Cincinnati. She calculated the sales tax to be exactly $67.53435. Round the sales tax to the nearest
ⓐ penny and
ⓑ dollar.

653.

Paycheck Annie has two jobs. She gets paid $14.04 per hour for tutoring at City College and $8.75 per hour at a coffee shop. Last week she tutored for 8 hours and worked at the coffee shop for 15 hours.
ⓐ How much did she earn?
ⓑ If she had worked all 23 hours as a tutor instead of working both jobs, how much more would she have earned?

654.

Paycheck Jake has two jobs. He gets paid $7.95 per hour at the college cafeteria and $20.25 at the art gallery. Last week he worked 12 hours at the cafeteria and 5 hours at the art gallery.

ⓐ How much did he earn?

ⓑ If he had worked all 17 hours at the art gallery instead of working both jobs, how much more would he have earned?

Writing Exercises

655.

How does knowing about US money help you learn about decimals?

656.

Explain how you write “three and nine hundredths” as a decimal.

657.

Without solving the problem “44 is 80% of what number” think about what the solution might be. Should it be a number that is greater than 44 or less than 44? Explain your reasoning.

658.

When the Szetos sold their home, the selling price was 500% of what they had paid for the house 30 years ago. Explain what 500% means in this context.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
name and write decimals.			
round decimals.			
add and subtract decimals.			
multiply and divide decimals.			
convert decimals, fractions, and percents.			

ⓑ What does this checklist tell you about your mastery of this section? What steps will you take to improve?

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1.7: Solve Equations with Variables and Constants on Both Sides

Learning Objectives

By the end of this section, you will be able to:

- Solve an equation with constants on both sides
- Solve an equation with variables on both sides
- Solve an equation with variables and constants on both sides

Note

Before you get started, take this readiness quiz.

1. Simplify: 4y−9+9.
   If you missed this problem, review Exercise 1.10.20.

Solve Equations with Constants on Both Sides

In all the equations we have solved so far, all the variable terms were on only one side of the equation with the constants on the other side. This does not happen all the time—so now we will learn to solve equations in which the variable terms, or constant terms, or both are on both sides of the equation.

Our strategy will involve choosing one side of the equation to be the "variable side", and the other side of the equation to be the "constant side." Then, we will use the Subtraction and Addition Properties of Equality to get all the variable terms together on one side of the equation and the constant terms together on the other side.

By doing this, we will transform the equation that began with variables and constants on both sides into the form $ax = b$. We already know how to solve equations of this form by using the Division or Multiplication Properties of Equality.

Example 1.7.1

Solve: $7x + 8 = -13$.

Solution

In this equation, the variable is found only on the left side. It makes sense to call the left side the "variable" side. Therefore, the right side will be the "constant" side. We will write the labels above the equation to help us remember what goes where.

$$\overset{\text{variable}}{7x} + 8 = \overset{\text{constant}}{-13}$$

Since the left side is the "xx", or variable side, the 8 is out of place. We must "undo" adding 8 by subtracting 8, and to keep the equality we must subtract 8 from both sides.

	$\overset{\text{variable}}{7x} + 8 = \overset{\text{constant}}{-13}$
Use the Subtraction Property of Equality.	$7x + 8 - 8 = -13 - 8$
Simplify.	$7x = -21$
Now all the variables are on the left and the constant on the right. The equation looks like those you learned to solve earlier.	
Use the Division Property of Equality.	$\frac{7x}{7} = \frac{-21}{7}$
Simplify.	$x = -3$
Check:	$7x + 8 = -13$
Let x=−3.	$7(-3) + 8 \stackrel{?}{=} -13$

	$-21+8\stackrel{?}{=}-13$ $-13=-13\checkmark$	

Try It 1.7.2

Solve: $3x+4=-8$.

Answer

$x=-4$

Try It 1.7.3

Solve: $5a+3=-37$.

Answer

$a=-8$

Example 1.7.4

Solve: $8y-9=31$.

Solution

Notice, the variable is only on the left side of the equation, so we will call this side the "variable" side, and the right side will be the "constant" side. Since the left side is the "variable" side, the 9 is out of place. It is subtracted from the 8y, so to "undo" subtraction, add 9 to both sides. Remember, whatever you do to the left, you must do to the right.

		variable constant $8y-9=31$
Add 9 to both sides.		$8y-9=31$ $+9\quad+9$
Simplify.		$8y=40$
		The variable is now on one side and the constants on the other. We continue from here as we did earlier.
Divide both sides by 8.		$\frac{8y}{8}=\frac{40}{8}$
Simplify.		$y=5$
Check:	$8y-9=31$	
Let y=5.	$8\cdot5-9\stackrel{?}{=}31$	
	$40-9\stackrel{?}{=}31$	
	$31=31\checkmark$	

Try It 1.7.5

Solve: $5y-9=16$.

Answer

$y = 5$

Try It 1.7.6

Solve: $3m - 8 = 19$.

Answer

$m = 9$

Solve Equations with Variables and Constants on Both Sides

The next example will be the first to have variables and constants on both sides of the equation. It may take several steps to solve this equation, so we need a clear and organized strategy.

Example 1.7.7

Solve: $9x = 8x - 6$.

Solution

Here the variable is on both sides, but the constants only appear on the right side, so let's make the right side the "constant" side. Then the left side will be the "variable" side.

	$9x = 8x - 6$
We don't want any x's on the right, so subtract the 8x from both sides.	$9x - 8x = 8x - 8x - 6$
Simplify.	$x = -6$
We succeeded in getting the variables on one side and the constants on the other, and have obtained the solution.	
Check:	$9x = 8x - 6$
Let x=−6.	$9(-6) \stackrel{?}{=} 8(-6) - 6$
	$-54 \stackrel{?}{=} -48 - 6$
	$-54 = -54$ ✓

Try It 1.7.8

Solve: $6n = 5n - 10$.

Answer

$n = -10$

Try It 1.7.9

Solve: $-6c = -7c - 1$

Answer

$c = -1$

Example 1.7.10

Solve: $5y - 9 = 8y$

Solution

The only constant is on the left and the y's are on both sides. Let's leave the constant on the left and get the variables to the right.

	$\overset{\text{constant}}{5y} - 9 = \overset{\text{variable}}{8y}$
Subtract 5y from both sides.	$5y - 5y - 9 = 8y - 5y$
Simplify.	$-9 = 3y$
We have the y's on the right and the constants on the left. Divide both sides by 3.	$\frac{-9}{3} = \frac{3y}{3}$
Simplify.	$-3 = y$
Check:	$5y - 9 = 8y$
Let $y = -3$.	$5(-3) - 9 \stackrel{?}{=} 8(-3)$
	$-15 - 9 \stackrel{?}{=} -24$
	$-24 = -24$ ✓

Try It 1.7.11

Solve: $3p - 14 = 5p$.

Answer

$p = -7$

Try It 1.7.12

Solve: $8m + 9 = 5m$

Answer

$m = -3$

Example 1.7.13

Solve: $12x = -x + 26$

Solution

The only constant is on the right, so let the left side be the "variable" side.

	$\overset{\text{variable}}{12x} = \overset{\text{constant}}{-x + 26}$
Remove the $-x$ from the right side by adding x to both sides.	$12x + x = -x + x + 26$
Simplify.	$13x = 26$
All the x's are on the left and the constants are on the right. Divide both sides by 13.	$\frac{13x}{13} = \frac{26}{13}$

Simplify.	$x = 2$

Try It 1.7.14

Solve: $12j = -4j + 32$

Answer

$j = 2$

Try It 1.7.15

Solve: $8h = -4h + 12$

Answer

$h = 1$

Solve Equations with Variables and Constants on Both Sides

The next example will be the first to have variables and constants on both sides of the equation. It may take several steps to solve this equation, so we need a clear and organized strategy.

Example 1.7.16: How to Solve Equations with Variables and Constants on Both Sides

Solve: $7x + 5 = 6x + 2$

Solution

Step 1. Choose which side will be the "variable" side—the other side will be the "constant" side.	The variable terms are $7x$ and $6x$. Since 7 is greater than 6, we will make the left side the "x" side. The right side will be the "constant" side.	variable constant $7x + 5 = 6x + 2$
Step 2. Collect the variable terms to the "variable" side of the equation, using the addition or subtraction property of equality.	With the right side as the "constant" side, the $6x$ is out of place, so subtract $6x$ from both sides. Combine like terms. Now, the variable is only on the left side!	$7x - 6x + 5 = 6x - 6x + 2$ $x + 5 = 2$
Step 3. Collect all the constants to the other side of the equation, using the addition or subtraction property of equality.	The right side is the "constant" side, so the 5 is out of place. Subtract 5 from both sides. Simplify.	$x + 5 - 5 = 2 - 5$ $x = -3$

Step 4. Make the coefficient of the variable equal 1, using the multiplication or division property of equality.	The coefficient of x is one. The equation is solved.	
Step 5. Check.	Let $x = -3$ Simplify. Add.	**Check:** $7x + 6 = 6x + 2$ $(-3) + 5 = 6(-3) + 2$ $-21 + 5 = -18 + 2$ $-16 = -16$ ✓

? Try It 1.7.17

Solve: $12x + 8 = 6x + 2$.

Answer

$x = -1$

? Try It 1.7.18

Solve: $9y + 4 = 7y + 12$.

Answer

$y = 4$

We'll list the steps below so you can easily refer to them. But we'll call this the 'Beginning Strategy' because we'll be adding some steps later in this chapter.

BEGINNING STRATEGY FOR SOLVING EQUATIONS WITH VARIABLES AND CONSTANTS ON BOTH SIDES OF THE EQUATION.

1. Choose which side will be the "variable" side—the other side will be the "constant" side.
2. Collect the variable terms to the "variable" side of the equation, using the Addition or Subtraction Property of Equality.
3. Collect all the constants to the other side of the equation, using the Addition or Subtraction Property of Equality.
4. Make the coefficient of the variable equal 1, using the Multiplication or Division Property of Equality.
5. Check the solution by substituting it into the original equation.

In Step 1, a helpful approach is to make the "variable" side the side that has the variable with the larger coefficient. This usually makes the arithmetic easier.

✓ Example 1.7.19

Solve: $8n - 4 = -2n + 6$.

Solution

In the first step, choose the variable side by comparing the coefficients of the variables on each side.

Since $8 > -2$, make the left side the "variable" side.	variable constant $8n - 4 = -2n + 6$
We don't want variable terms on the right side—add 2n to both sides to leave only constants on the right.	$8n + 2n - 4 = -2n + 2n + 6$
Combine like terms.	$10n - 4 = 6$

We don't want any constants on the left side, so add 4 to both sides.	$10n-4+4=6+4$
Simplify.	$10n=10$
The variable term is on the left and the constant term is on the right. To get the coefficient of nn to be one, divide both sides by 10.	$\frac{10n}{10}=\frac{10}{10}$
Simplify.	$n=1$
Check:	$8n-4=-2n+6$
Let n=1.	$8\cdot 1-4\stackrel{?}{=}-2\cdot 1+6$
	$8-4\stackrel{?}{=}-2+6$
	$4=4\checkmark$

Try It 1.7.20

Solve: $8q-5=-4q+7$

Answer

$q=1$

Try It 1.7.21

Solve: $7n-3=n+3$

Answer

$n=1$

Example 1.7.22

Solve: $7a-3=13a+7$

Solution

In the first step, choose the variable side by comparing the coefficients of the variables on each side.

Since 13>7, make the right side the "variable" side and the left side the "constant" side.

	constant variable $7a-3=13a+7$
Subtract 7a from both sides to remove the variable term from the left.	$7a-7a-3=13a-7a+7$
Combine like terms.	$-3=6a+7$
Subtract 7 from both sides to remove the constant from the right.	$-3-7=6a+7-7$
Simplify.	$-10=6a$
Divide both sides by 6 to make 1 the coefficient of a.	$\frac{-10}{6}=\frac{6a}{6}$
Simplify.	$-\frac{5}{3}=a$
Check:	$7a-3=13a+7$
Let $a=-\frac{5}{3}$.	$7\left(-\frac{5}{3}\right)-3\stackrel{?}{=}13\left(-\frac{5}{3}\right)+7$

	$-\frac{35}{3}-\frac{9}{3}\stackrel{?}{=}-\frac{65}{3}+\frac{21}{3}$	
	$-\frac{54}{3}=-\frac{54}{3}\checkmark$	

Try It 1.7.23

Solve: $2a-2=6a+18$

Answer

$a=-5$

Try It 1.7.24

Solve: $4k-1=7k+17$

Answer

$k=-6$

In the last example, we could have made the left side the "variable" side, but it would have led to a negative coefficient on the variable term. (Try it!) While we could work with the negative, there is less chance of errors when working with positives. The strategy outlined above helps avoid the negatives!

To solve an equation with fractions, we just follow the steps of our strategy to get the solution!

Example 1.7.25

Solve: $\frac{4}{5}x+6=\frac{1}{4}x-2$

Solution

Since $\frac{5}{4}>\frac{1}{4}$, make the left side the "variable" side and the right side the "constant" side.

	variable constant $\frac{5}{4}x+6=\frac{1}{4}x-2$
Subtract $\frac{1}{4}x$ from both sides.	$\frac{5}{4}x-\frac{1}{4}x+6=\frac{1}{4}x-\frac{1}{4}x-2$
Combine like terms.	$x+6=-2$
Subtract 6 from both sides.	$x+6-6=-2-6$
Simplify.	$x=-8$
Check: Let $x=-8$	$\frac{5}{4}x+6 = \frac{1}{4}x-2$ $\frac{5}{4}(-8)+6 \stackrel{?}{=} \frac{1}{4}(-8)-2$ $-10+6 \stackrel{?}{=} -2-2$ $-4 = -4\checkmark$

Try It 1.7.26

Solve: $\frac{7}{8}x-12=-\frac{1}{8}x-2$

Answer

$x = 10$

Try It 1.7.27

Solve: $\frac{7}{6}x + 11 = \frac{1}{6}y + 8$

Answer

$y = -3$

We will use the same strategy to find the solution for an equation with decimals.

Example 1.7.28

Solve: $7.8x + 4 = 5.4x - 8$.

Solution

Since $7.8 > 5.4$, make the left side the "variable" side and the right side the "constant" side.

	variable side constant side $7.8x + 4 = 5.4x - 8$
Subtract 5.4x from both sides.	$7.8x - 5.4x + 4 = 5.4x - 5.4x - 8$
Combine like terms.	$2.4x + 4 = -8$
Subtract 4 from both sides.	$2.4x + 4 - 4 = -8 - 4$
Simplify.	$2.4x = -12$
Use the Division Property of Equality.	$\frac{2.4x}{2.4} = \frac{-12}{2.4}$
Simplify.	$x = -5$
Check:	$7.8x + 4 = 5.4x - 5$
Let $x = -5$	$7.8(-5) + 4 = 5.4(-5) - 8$
	$-39 + 4 \stackrel{?}{=} -27 - 8$
	$-35 = -35$ ✓

Try It 1.7.29

Solve: $2.8x + 12 = -1.4x - 9$

Answer

$x = -5$

Try It 1.7.30

Solve: $3.6y + 8 = 1.2y - 4$

Answer

$y = -5$

Key Concepts

- **Beginning Strategy for Solving an Equation with Variables and Constants on Both Sides of the Equation**
  1. Choose which side will be the “variable” side—the other side will be the “constant” side.
  2. Collect the variable terms to the “variable” side of the equation, using the Addition or Subtraction Property of Equality.
  3. Collect all the constants to the other side of the equation, using the Addition or Subtraction Property of Equality.
  4. Make the coefficient of the variable equal 1, using the Multiplication or Division Property of Equality.
  5. Check the solution by substituting it into the original equation.

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1.7E: Exercises

Practice Makes Perfect

Solve Equations with Constants on Both Sides

In the following exercises, solve the following equations with constants on both sides.

Exercise 1.7E.1

$9x - 3 = 60$

Exercise 1.7E.2

$12x - 8 = 64$

Answer

$x = 6$

Exercise 1.7E.3

$14w + 5 = 117$

Exercise 1.7E.4

$15y + 7 = 97$

Answer

$y = 6$

Exercise 1.7E.5

$2a + 8 = -28$

Exercise 1.7E.6

$3m + 9 = -15$

Answer

$m = -8$

Exercise 1.7E.7

$-62 = 8n - 6$

Exercise 1.7E.8

$-77 = 9b - 5$

Answer

$b = -8$

Exercise 1.7E.9

$35 = -13y + 9$

Exercise 1.7E.10

$60 = -21x - 24$

Answer

$x = -4$

Exercise 1.7E.11

$-12p - 9 = 9$

Exercise 1.7E.12

$-14q - 2 = 16$

Answer

$q = -\frac{9}{7}$

Solve Equations with Variables on Both Sides

In the following exercises, solve the following equations with variables on both sides.

Exercise 1.7E.13

$19z = 18z - 7$

Exercise 1.7E.14

$21k = 20k - 11$

Answer

$k = -11$

Exercise 1.7E.15

$9x + 36 = 15x$

Exercise 1.7E.16

$8x + 27 = 11x$

Answer

$x = 9$

Exercise 1.7E.17

$c = -3c - 20$

Exercise 1.7E.18

$b = -4b - 15$

Answer

$b = -3$

Exercise 1.7E.19

$9q = 44 - 2q$

Exercise 1.7E.20

$5z = 39 - 8z$

Answer

$z = 3$

Exercise 1.7E.21

$6y + \frac{1}{2} = 5y$

Exercise 1.7E.22

$4x + \frac{3}{4} = 3x$

Answer

$x = -\frac{3}{4}$

Exercise 1.7E.23

$-18a - 8 = -22a$

Exercise 1.7E.24

$-11r - 8 = -7r$

Answer

$r = -2$

Solve Equations with Variables and Constants on Both Sides

In the following exercises, solve the following equations with variables and constants on both sides.

Exercise 1.7E.25

$8x - 15 = 7x + 3$

Exercise 1.7E.26

$6x - 17 = 5x + 2$

Answer

$x = 19$

Exercise 1.7E.27

$26 + 13d = 14d + 11$

Exercise 1.7E.28

$21 + 18f = 19f + 14$

Answer

$f = 7$

Exercise 1.7E.29

$2p - 1 = 4p - 33$

Exercise 1.7E.30

$12q - 5 = 9q - 20$

Answer

$q = -5$

Exercise 1.7E.31

$4a + 5 = -a - 40$

Exercise 1.7E.32

$8c + 7 = -3c - 37$

Answer

$c = -4$

Exercise 1.7E.33

$5y - 30 = -5y + 30$

Exercise 1.7E.34

$7x - 17 = -8x + 13$

Answer

$x = 2$

Exercise 1.7E.35

$7s + 12 = 5 + 4s$

Exercise 1.7E.36

$9p+14=6+4p$

Answer

$p=-\frac{8}{5}$

Exercise 1.7E.37

$2z-6=23-z$

Exercise 1.7E.38

$3y-4=12-y$

Answer

$y=4$

Exercise 1.7E.39

$\frac{5}{3}c-3=\frac{2}{3}c-16$

Exercise 1.7E.40

$\frac{7}{4}m-7=\frac{3}{4}m-13$

Answer

$m=-6$

Exercise 1.7E.41

$8-\frac{2}{5}q=\frac{3}{5}q+6$

Exercise 1.7E.42

$11-\frac{1}{5}a=\frac{4}{5}a+4$

Answer

$a=7$

Exercise 1.7E.43

$\frac{4}{3}n+9=\frac{1}{3}n-9$

Exercise 1.7E.44

$\frac{5}{4}a+15=\frac{3}{4}a-5$

Answer

$a=-40$

Exercise 1.7E.45

$\frac{1}{4}y+7=\frac{3}{4}y-3$

Exercise 1.7E.46

$\frac{3}{5}p+2=\frac{4}{5}p-1$

Answer

$p=15$

Exercise 1.7E.47

$14n+8.25=9n+19.60$

Exercise 1.7E.48

$13z+6.45=8z+23.75$

Answer

$z=3.46$

Exercise 1.7E.49

$2.4w-100=0.8w+28$

Exercise 1.7E.50

$2.7w-80=1.2w+10$

Answer

$w=60$

Exercise 1.7E.51

$5.6r+13.1=3.5r+57.2$

Exercise 1.7E.52

$6.6x-18.9=3.4x+54.7$

Answer

$x=23$

Everyday Math

Exercise 1.7E.53

Concert tickets At a school concert the total value of tickets sold was $1506. Student tickets sold for $6 and adult tickets sold for $9. The number of adult tickets sold was 5 less than 3 times the number of student tickets. Find the number of student tickets sold, *s*, by solving the equation 6s+27s−45=1506. Add exercises text here.

Exercise $1.7E.54$

Making a fence Jovani has 150 feet of fencing to make a rectangular garden in his backyard. He wants the length to be 15 feet more than the width. Find the width, w, by solving the equation $150 = 2w + 30 + 2w$.

Answer

30 feet

Writing Exercises

Exercise $1.7E.55$

Solve the equation $\frac{6}{5}y - 8 = \frac{1}{5}y + 7$ explaining all the steps of your solution as in the examples in this section.

Exercise $1.7E.56$

Solve the equation $10x + 14 = -2x + 38$ explaining all the steps of your solution as in this section.

Answer

$x = 2$ Justifications will vary.

Exercise $1.7E.57$

When solving an equation with variables on both sides, why is it usually better to choose the side with the larger coefficient of x to be the "variable" side?

Exercise $1.7E.58$

Is $x = -2$ a solution to the equation $5 - 2x = -4x + 1$? How do you know?

Answer

Yes. Justifications will vary.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
solve an equation with constants on both sides.			
solve an equation with variables on both sides.			
solve an equation with variables and constants on both sides.			

ⓑ What does this checklist tell you about your mastery of this section? What steps will you take to improve?

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1.8: Use a General Strategy to Solve Linear Equations

Learning Objectives

By the end of this section, you will be able to:

- Solve equations using a general strategy
- Classify equations

Note

Before you get started, take this readiness quiz.

1. Simplify: $-(a-4)$.
   If you missed this problem, review Exercise 1.10.46
2. Multiply: $\frac{3}{2}(12x+20)$
   If you missed this problem, review Exercise 1.10.34.
3. Simplify: $5-2(n+1)$
   If you missed this problem, review Exercise 1.10.49.
4. Multiply: $3(7y+9)$
   If you missed this problem, review Exercise 1.10.34.
5. Multiply: $(2.5)(6.4)$
   If you missed this problem, review Exercise 1.8.19.

Solve Equations Using the General Strategy

Until now we have dealt with solving one specific form of a linear equation. It is time now to lay out one overall strategy that can be used to solve any linear equation. Some equations we solve will not require all these steps to solve, but many will.

Beginning by simplifying each side of the equation makes the remaining steps easier.

Example 1.8.1: How to Solve Linear Equations Using the General Strategy

Solve: $-6(x+3)=24$.

Solution

Step 1. Simplify each side of the equation as much as possible.	Use the Distributive Property.	$-6(x+3)=24$
	Notice that each side of the equation is simplified as much as possible.	$-6x-18=24$
Step 2. Collect all variable terms on one side of the equation.	Nothing to do – all x's are on the left side.	
Step 3. Collect constant terms on the other side of the equation.	To get constants only on the right, add 18 to each side.	$-6x-18+18=24+18$
	Simplify.	$-6x=42$

Step 4. Make the coefficient of the variable term to equal to 1.	Divide each side by –6.	$\frac{-6x}{-6} = \frac{42}{-6}$
	Simplify.	$x = -7$
Step 5. Check the solution.	Let $x = -7$	**Check:** $-6(x + 3) = 24$
	Simplify.	$-6(-7 + 3) \stackrel{?}{=} 24$
	Multiply.	$-6(-4) \stackrel{?}{=} 24$
		$24 = 24$ ✓

Try It 1.8.2

Solve: $5(x+3) = 35$

Answer

$x = 4$

Try It 1.8.3

Solve: $6(y-4) = -18$

Answer

$y = 1$

GENERAL STRATEGY FOR SOLVING LINEAR EQUATIONS.

1. **Simplify each side of the equation as much as possible.**
   Use the Distributive Property to remove any parentheses.
   Combine like terms.
2. **Collect all the variable terms on one side of the equation.**
   Use the Addition or Subtraction Property of Equality.
3. **Collect all the constant terms on the other side of the equation.**
   Use the Addition or Subtraction Property of Equality.
4. **Make the coefficient of the variable term to equal to 1.**
   Use the Multiplication or Division Property of Equality.
   State the solution to the equation.
5. **Check the solution.** Substitute the solution into the original equation to make sure the result is a true statement.

Example 1.8.4

Solve: $-(y+9) = 8$

Solution

	$-(y + 9) = 8$
Simplify each side of the equation as much as possible by distributing.	$-y - 9 = 8$
The only y term is on the left side, so all variable terms are on the left side of the equation.	
Add 9 to both sides to get all constant terms on the right side of the equation.	$-y - 9 + 9 = 8 + 9$
Simplify.	$-y = 17$

Rewrite $-y$ as $-1y$.		$-1y = 17$
Make the coefficient of the variable term to equal to 1 by dividing both sides by -1.		$\frac{-1y}{-1} = \frac{17}{-1}$
Simplify.		$y = -17$
Check:	$-(y+9) = 8$	
Let y=−17.	$-(-17+9) \stackrel{?}{=} 8$	
	$-(-8) \stackrel{?}{=} 8$	
	$8 = 8$ ✓	

Try It 1.8.5

Solve: $-(y+8) = -2$

Answer

$y = -6$

Try It 1.8.6

Solve: $-(z+4) = -12$

Answer

$z = 8$

Example 1.8.7

Solve: $5(a-3)+5 = -10$

Solution

		$5(a-3)+5 = -10$
Simplify each side of the equation as much as possible.		
Distribute.		$5a-15+5 = -10$
Combine like terms.		$5a-10 = -10$
The only a term is on the left side, so all variable terms are on one side of the equation.		
Add 10 to both sides to get all constant terms on the other side of the equation.		$5a-10+10 = -10+10$
Simplify.		$5a = 0$
Make the coefficient of the variable term to equal to 11 by dividing both sides by 55.		$\frac{5a}{5} = \frac{0}{5}$
Simplify.		$a = 0$
Check:	$5(a-3)+5 = -10$	
Let a=0.	$5(0-3)+5 \stackrel{?}{=} -10$	
	$5(-3)+5 \stackrel{?}{=} -10$	
	$-15+5 \stackrel{?}{=} -10$	

	$-10 = -10$ ✓

? Try It 1.8.8

Solve: $2(m-4)+3=-1$

Answer

$m=2$

? Try It 1.8.9

Solve:$7(n-3)-8=-15$

Answer

$n=2$

✓ Example 1.8.10

Solve: $\frac{2}{3}(6m-3)=8-m$

Solution

	$\frac{2}{3}(6m-3)=8-m$
Distribute.	$4m-2=8-m$
Add m to get the variables only to the left.	$4m+m-2=8-m+m$
Simplify.	$5m-2=8$
Add 2 to get constants only on the right.	$5m-2+2=8+2$
Simplify.	$5m=10$
Divide by 5.	$\frac{5m}{5}=\frac{10}{5}$
Simplify.	$m=2$
Check:	$\frac{2}{3}(6m-3)=8-m$
Let m=2.	$\frac{2}{3}(6\cdot 2-3)\stackrel{?}{=}8-2$
	$\frac{2}{3}(12-3)\stackrel{?}{=}6$
	$\frac{2}{3}(9)\stackrel{?}{=}6$
	$6=6$ ✓

? Try It 1.8.11

Solve: $\frac{1}{3}(6u+3)=7-u$

Answer

$u=2$

Try It 1.8.12

Solve: $\frac{2}{3}(9x-12)=8+2x$

Answer

$x=4$

Example 1.8.13

Solve: $8-2(3y+5)=0$

Solution

	$8-2(3y+5)=0$
Simplify—use the Distributive Property.	$8-6y-10=0$
Combine like terms.	$-6y-2=0$
Add 2 to both sides to collect constants on the right.	$-6y-2+2=0+2$
Simplify.	$-6y=2$
Divide both sides by -6−6.	$\frac{-6y}{-6}=\frac{2}{-6}$
Simplify.	$y=-\frac{1}{3}$
Check: Let y=−13. $8-2(3y+5)=0$ $8-2\left[3\left(-\frac{1}{3}\right)+5\right]=0$ $8-2(-1+5)\stackrel{?}{=}0$ $8-2(4)\stackrel{?}{=}0$ $8-8\stackrel{?}{=}0$ $0=0$ ✓	

Try It 1.8.14

Solve: $12-3(4j+3)=-17$

Answer

$j=\frac{5}{3}$

Try It 1.8.15

Solve: $-6-8(k-2)=-10$

Answer

$k=\frac{5}{2}$

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Example 1.8.16

Solve: $4(x-1)-2=5(2x+3)+6$

Solution

	$4(x-1)-2=5(2x+3)+6$
Distribute.	$4x-4-2=10x+15+6$
Combine like terms.	$4x-6=10x+21$
Subtract 4x to get the variables only on the right side since $10>4$.	$4x-4x-6=10x-4x+21$
Simplify.	$-6=6x+21$
Subtract 21 to get the constants on left.	$-6-21=6x+21-21$
Simplify.	$-27=6x$
Divide by 6.	$\frac{-27}{6}=\frac{6x}{6}$
Simplify.	$-\frac{9}{2}=x$
Check:	$4(x-1)-2=5(2x+3)+6$
Let $x=-\frac{9}{2}$.	$4\left(-\frac{9}{2}-1\right)-2\stackrel{?}{=}5\left[2\left(-\frac{9}{2}\right)+3\right]+6$
	$4\left(-\frac{11}{2}\right)-2\stackrel{?}{=}5(-9+3)+6$
	$-22-2\stackrel{?}{=}5(-6)+6$
	$-24\stackrel{?}{=}-30+6$
	$-24=-24$ ✓

Try It 1.8.17

Solve: $6(p-3)-7=5(4p+3)-12$

Answer

$p=-2$

Try It 1.8.18

Solve: $8(q+1)-5=3(2q-4)-1$

Answer

$q=-8$

Example 1.8.19

Solve: $10[3-8(2s-5)]=15(40-5s)$

Solution

	$10[3-8(2s-5)]=15(40-5s)$

Simplify from the innermost parentheses first.		$10[3-16s+40]=15(40-5s)$
Combine like terms in the brackets.		$10[43-16s]=15(40-5s)$
Distribute.		$430-160s=600-75s$
Add 160s to get the s's to the right.		$430-160s+160s=600-75s+160s$
Simplify.		$430=600+85s$
Subtract 600 to get the constants to the left.		$430-600=600+85s-600$
Simplify.		$-170=85s$
Divide.		$\frac{-170}{85}=\frac{85s}{85}$
Simplify.		$-2=s$
Check:	$10[3-8(2s-5)]=15(40-5s)$	
Substitute s=−2.	$10[3-8(2(-2)-5)] \stackrel{?}{=} 15(40-5(-2))$	
	$10[3-8(-4-5)] \stackrel{?}{=} 15(40+10)$	
	$10[3-8(-9)] \stackrel{?}{=} 15(50)$	
	$10[3+72] \stackrel{?}{=} 750$	
	$10[75] \stackrel{?}{=} 750$	
	$750=750$ ✓	

? Try It 1.8.20

Solve: $6[4-2(7y-1)]=8(13-8y)$.

Answer

$y=-\frac{17}{5}$

? Try It 1.8.21

Solve: $12[1-5(4z-1)]=3(24+11z)$.

Answer

$z=0$

✓ Example 1.8.22

Solve: $0.36(100n+5)=0.6(30n+15)$.

Solution

	$0.36(100n+5)=0.6(30n+15)$
Distribute.	$36n+1.8=18n+9$
Subtract 18n to get the variables to the left.	$36n-18n+1.8=18n-18n+9$
Simplify.	$18n+1.8=9$

Subtract 1.8 to get the constants to the right.		$18n + 1.8 - 1.8 = 9 - 1.8$
Simplify.		$18n = 7.2$
Divide.		$\frac{18n}{18} = \frac{7.2}{18}$
Simplify.		$n = 0.4$
Check:	$0.36(100n + 5) = 0.6(30n + 15)$	
Let n=0.4.	$0.36(100(0.4) + 5) \stackrel{?}{=} 0.6(30(0.4) + 15)$	
	$0.36(40 + 5) \stackrel{?}{=} 0.6(12 + 15)$	
	$0.36(45) \stackrel{?}{=} 0.6(27)$	
	$16.2 = 16.2$ ✓	

? Try It 1.8.23

Solve: $0.55(100n+8) = 0.6(85n+14)$.

Answer

$n = 1$

? Try It 1.8.24

Solve: $0.15(40m-120) = 0.5(60m+12)$.

Answer

$m = -1$

Classify Equations

Consider the equation we solved at the start of the last section, 7x+8=−13. The solution we found was x=−3. This means the equation 7x+8=−13 is true when we replace the variable, *x*, with the value −3. We showed this when we checked the solution x=−3 and evaluated 7x+8=−13 for x=−3.

$$7(-3) + 8 \stackrel{?}{=} -13$$
$$-21 + 8 \stackrel{?}{=} -13$$
$$-13 = -13 \checkmark$$

If we evaluate 7x+8 for a different value of *x*, the left side will not be −13.

The equation 7x+8=−13 is true when we replace the variable, *x*, with the value −3, but not true when we replace *x* with any other value. Whether or not the equation 7x+8=−13 is true depends on the value of the variable. Equations like this are called conditional equations.

All the equations we have solved so far are conditional equations.

CONDITIONAL EQUATION

An equation that is true for one or more values of the variable and false for all other values of the variable is a **conditional equation**.

Now let's consider the equation 2y+6=2(y+3). Do you recognize that the left side and the right side are equivalent? Let's see what happens when we solve for *y*.

	$2y+6=2(y+3)$
Distribute.	$2y+6=2y+6$
Subtract 2y to get the y's to one side.	$2y-2y+6=2y-2y+6$
Simplify—the y's are gone!	$6=6$

But 6=6 is true.

This means that the equation 2y+6=2(y+3) is true for any value of *y*. We say the solution to the equation is all of the real numbers. An equation that is true for any value of the variable like this is called an **identity**.

IDENTITY

An equation that is true for any value of the variable is called an **identity**.

The solution of an identity is all real numbers.

What happens when we solve the equation 5z=5z−1?

	$5z=5z-1$
Subtract 5z to get the constant alone on the right.	$5z-5z=5z-5z-1$
Simplify—the z's are gone!	$0\neq -1$

But $0 \neq -1$.

Solving the equation 5z=5z−1 led to the false statement 0=−1. The equation 5z=5z−1 will not be true for any value of z. It has no solution. An equation that has no solution, or that is false for all values of the variable, is called a **contradiction**.

CONTRADICTION

An equation that is false for all values of the variable is called a **contradiction**.

A contradiction has no solution.

Example 1.8.25

Classify the equation as a conditional equation, an identity, or a contradiction. Then state the solution.

$6(2n-1)+3=2n-8+5(2n+1)$

Solution

$6(2n-1)+3=2n-8+5(2n+1)$	
Distribute.	$12n-6+3=2n-8+10n+5$
Combine like terms.	$12n-3=12n-3$
Subtract 12n to get the nn's to one side.	$12n-12n-3=12n-12n-3$
Simplify.	$-3=-3$
This is a true statement.	The equation is an identity. The solution is all real numbers.

Try It 1.8.26

Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution:

$4+9(3x-7)=-42x-13+23(3x-2)$

Answer

identity; all real numbers

Try It 1.8.27

Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution:

$8(1-3x)+15(2x+7)=2(x+50)+4(x+3)+1$

Answer

identity; all real numbers

Example 1.8.28

Classify as a conditional equation, an identity, or a contradiction. Then state the solution.

$10+4(p-5)=0$

Solution

	$10+4(p-5)=0$
Distribute.	$10+4p-20=0$
Combine like terms.	$4p-10=0$
Add 10 to both sides.	$4p-10+10=0+10$
Simplify.	$4p=10$
Divide.	$\frac{4p}{4}=\frac{10}{4}$
Simplify.	$p=\frac{5}{2}$
The equation is true when $p=frac52$.	This is a conditional equation. The solution is $p=frac52$.

Try It 1.8.29

Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution: $11(q+3)-5=19$

Answer

conditional equation; \(q = \frac{9}{11}\

Try It 1.8.30

Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution: $6+14(k-8)=95$

Answer

conditional equation; $k = \frac{193}{14}$

Example 1.8.31

Classify the equation as a conditional equation, an identity, or a contradiction. Then state the solution.

$5m + 3(9 + 3m) = 2(7m - 11)$

Solution

	$5m + 3(9 + 3m) = 2(7m - 11)$
Distribute.	$5m + 27 + 9m = 14m - 22$
Combine like terms.	$14m + 27 = 14m - 22$
Subtract 14m from both sides.	$14m + 27 - 14m = 14m - 22 - 14m$
Simplify.	$27 \neq -22$
But $27 \neq -22$.	The equation is a contradiction. It has no solution.

Try It 1.8.32

Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution:

$12c + 5(5 + 3c) = 3(9c - 4)$

Answer

contradiction; no solution

Try It 1.8.33

Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution:

$4(7d + 18) = 13(3d - 2) - 11d$

Answer

contradiction; no solution

Type of equation	What happens when you solve it?	Solution
Conditional Equation	True for one or more values of the variables and false for all other values	One or more values
Identity	**True** for any value of the variable	All real numbers
Contradiction	**False** for all values of the variable	No solution

Key Concepts

- **General Strategy for Solving Linear Equations**
  1. Simplify each side of the equation as much as possible.
     Use the Distributive Property to remove any parentheses.
     Combine like terms.

2. Collect all the variable terms on one side of the equation.
   Use the Addition or Subtraction Property of Equality.
3. Collect all the constant terms on the other side of the equation.
   Use the Addition or Subtraction Property of Equality.
4. Make the coefficient of the variable term to equal to 1.
   Use the Multiplication or Division Property of Equality.
   State the solution to the equation.
5. Check the solution.
   Substitute the solution into the original equation.

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1.8E: Exercises

Practice Makes Perfect

Solve Equations Using the General Strategy for Solving Linear Equations

In the following exercises, solve each linear equation.

Exercise 1.8E.1

$15(y-9)=-60$

Exercise 1.8E.2

$21(y-5)=-42$

Answer

$y=3$

Exercise 1.8E.3

$-9(2n+1)=36$

Exercise 1.8E.4

$-16(3n+4)=32$

Answer

$n=-2$

Exercise 1.8E.5

$8(22+11r)=0$

Exercise 1.8E.6

$5(8+6p)=0$

Answer

$p=-\frac{4}{3}$

Exercise 1.8E.7

$-(w-12)=30$

Exercise 1.8E.8

$-(t-19)=28$

Answer

$t=-9$

Exercise 1.8E.9

$9(6a+8)+9=81$

Exercise 1.8E.10

$8(9b-4)-12=100$

Answer

$b=2$

Exercise 1.8E.11

$32+3(z+4)=41$

Exercise 1.8E.12

$21+2(m-4)=25$

Answer

$m=6$

Exercise 1.8E.13

$51+5(4-q)=56$

Exercise 1.8E.14

$-6+6(5-k)=15$

Answer

$k=\frac{3}{2}$

Exercise 1.8E.15

$2(9s-6)-62=16$

Exercise 1.8E.16

$8(6t-5)-35=-27$

Answer

$t=1$

Exercise 1.8E.17

$3(10-2x)+54=0$

Exercise 1.8E.18

$-2(11-7x)+54=4$

Answer

$x=-2$

Exercise 1.8E.19

$\frac{2}{3}(9c-3)=22$

Exercise 1.8E.20

$\frac{3}{5}(10x-5)=27$

Answer

$x=5$

Exercise 1.8E.21

$\frac{1}{5}(15c+10)=c+7$

Exercise 1.8E.22

$\frac{1}{4}(20d+12)=d+7$

Answer

$d=1$

Exercise 1.8E.23

$18-(9r+7)=-16$

Exercise 1.8E.24

$15-(3r+8)=28$

Answer

$r=-7$

Exercise 1.8E.25

$5-(n-1)=19$

Exercise 1.8E.26

$-3-(m-1)=13$

Answer

$m=-15$

Exercise 1.8E.27

$11-4(y-8)=43$

Exercise 1.8E.28

$18-2(y-3)=32$

Answer

$y=-4$

Exercise 1.8E.29

$24-8(3v+6)=0$

Exercise 1.8E.30

$35-5(2w+8)=-10$

Answer

$w=\frac{1}{2}$

Exercise 1.8E.31

$4(a-12)=3(a+5)$

Exercise 1.8E.32

$-2(a-6)=4(a-3)$

Answer

$a=4$

Exercise 1.8E.33

$2(5-u)=-3(2u+6)$

Exercise 1.8E.34

$5(8-r)=-2(2r-16)$

Answer

$r=8$

Exercise 1.8E.35

$3(4n-1)-2=8n+3$

Exercise 1.8E.36

$9(2m-3)-8=4m+7$

Answer

$m=3$

Exercise 1.8E.37

$12+2(5-3y)=-9(y-1)-2$

Exercise 1.8E.38

$-15+4(2-5y)=-7(y-4)+4$

Answer

$y=-3$

Exercise 1.8E.39

$8(x-4)-7x=14$

Exercise 1.8E.40

$5(x-4)-4x=14$

Answer

$x=34$

Exercise 1.8E.41

$5+6(3s-5)=-3+2(8s-1)$

Exercise 1.8E.42

$-12+8(x-5)=-4+3(5x-2)$

Answer

$x=-6$

Exercise 1.8E.43

$4(u-1)-8=6(3u-2)-7$

Exercise 1.8E.44

$7(2n-5)=8(4n-1)-9$

Answer

$n=-1$

Exercise 1.8E.45

$4(p-4)-(p+7)=5(p-3)$

Exercise 1.8E.46

$3(a-2)-(a+6)=4(a-1)$

Answer

$a=-4$

Exercise 1.8E.47

$-(9y+5)-(3y-7)$
$=16-(4y-2)$

Exercise 1.8E.48

$-(7m+4)-(2m-5)$
$=14-(5m-3)$

Answer

$m=-4$

Exercise 1.8E.49

$4[5-8(4c-3)]$
$=12(1-13c)-8$

Exercise 1.8E.50

$5[9-2(6d-1)]$
$=11(4-10d)-139$

Answer

$d=-3$

Exercise 1.8E.51

$3[-9+8(4h-3)]$
$=2(5-12h)-19$

Exercise 1.8E.52

$3[-14+2(15k-6)]$
$=8(3-5k)-24$

Answer

$k=\frac{3}{5}$

Exercise 1.8E.53

$5[2(m+4)+8(m-7)]$
$=2[3(5+m)-(21-3m)]$

Exercise 1.8E.54

$10[5(n+1)+4(n-1)]$
$=11[7(5+n)-(25-3n)]$

Answer

$n=-5$

Exercise 1.8E.55

$5(1.2u-4.8)=-12$

Exercise 1.8E.56

$4(2.5v-0.6)=7.6$

Answer

$v=1$

Exercise 1.8E.57

$0.25(q-6)=0.1(q+18)$

Exercise 1.8E.58

$0.2(p-6)=0.4(p+14)$

Answer

$p=-34$

Exercise 1.8E.59

$0.2(30n+50)=28$

Exercise 1.8E.60

$0.5(16m+34)=-15$

Answer

$m=-4$

Classify Equations

In the following exercises, classify each equation as a conditional equation, an identity, or a contradiction and then state the solution.

Exercise 1.8E.61

$23z+19=3(5z-9)+8z+46$

Exercise 1.8E.62

$15y+32=2(10y-7)-5y+46$

Answer

identity; all real numbers

Exercise 1.8E.63

$5(b-9)+4(3b+9)=6(4b-5)-7b+21$

Exercise 1.8E.64

$9(a-4)+3(2a+5)=7(3a-4)-6a+7$

Answer

identity; all real numbers

Exercise 1.8E.65

$18(5j-1)+29=47$

Exercise 1.8E.66

$24(3d-4)+100=52$

Answer

conditional equation; $d=\frac{2}{3}$

Exercise 1.8E.67

$22(3m-4)=8(2m+9)$

Exercise 1.8E.68

$30(2n-1)=5(10n+8)$

Answer

conditional equation; $n=7$

Exercise 1.8E.69

$7v+42=11(3v+8)-2(13v-1)$

Exercise 1.8E.70

$18u-51=9(4u+5)-6(3u-10)$

Answer

contradiction; no solution

Exercise 1.8E.71

$3(6q-9)+7(q+4)=5(6q+8)-5(q+1)$

Exercise 1.8E.72

$5(p+4)+8(2p-1)=9(3p-5)-6(p-2)$

Answer

contradiction; no solution

Exercise 1.8E.73

$12(6h-1)=8(8h+5)-4$

Exercise 1.8E.74

$9(4k-7)=11(3k+1)+4$

Answer

conditional equation; $k=26$

Exercise 1.8E.75

$45(3y-2)=9(15y-6)$

Exercise 1.8E.76

$60(2x-1)=15(8x+5)$

Answer

contradiction; no solution

Exercise 1.8E.77

$16(6n+15)=48(2n+5)$

Exercise 1.8E.78

$36(4m+5)=12(12m+15)$

Answer

identity; all real numbers

Exercise 1.8E.79

$9(14d+9)+4d=13(10d+6)+3$

Exercise 1.8E.80

$11(8c+5)-8c=2(40c+25)+5$

Answer

identity; all real numbers

Everyday Math

Exercise 1.8E.81

Fencing Micah has 44 feet of fencing to make a dog run in his yard. He wants the length to be 2.5 feet more than the width. Find the length, L, by solving the equation 2L+2(L−2.5)=44.

Exercise 1.8E.82

Coins Rhonda has $\$1.90$ in nickels and dimes. The number of dimes is one less than twice the number of nickels. Find the number of nickels, n, by solving the equation $0.05n+0.10(2n-1)=1.90$.

Answer

8 nickels

Writing Exercises

Exercise 1.8E.83

Using your own words, list the steps in the general strategy for solving linear equations.

Exercise 1.8E.84

Explain why you should simplify both sides of an equation as much as possible before collecting the variable terms to one side and the constant terms to the other side.

Answer

Answers will vary.

Exercise 1.8E.85

What is the first step you take when solving the equation $3-7(y-4)=38$? Why is this your first step?

Exercise 1.8E.86

Solve the equation $\frac{1}{4}(8x+20)=3x-4$ explaining all the steps of your solution as in the examples in this section.

Answer

Answers will vary.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objective of this section.

I can...	Confidently	With some help	No-I don't get it!
solve equations using the general strategy for solving linear equations.			
classify equations.			

ⓑ On a scale of 1-10, how would you rate your mastery of this section in light of your responses on the checklist? How can you improve this?

Glossary

conditional equation

An equation that is true for one or more values of the variable and false for all other values of the variable is a conditional equation.

contradiction

An equation that is false for all values of the variable is called a contradiction. A contradiction has no solution.

identity

An equation that is true for any value of the variable is called an identity. The solution of an identity is all real numbers.

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1.9: Solve a Formula for a Specific Variable

Learning Objectives

By the end of this section, you will be able to:

- Use the Distance, Rate, and Time formula
- Solve a formula for a specific variable

Quiz

Before you get started, take this readiness quiz.

1. Solve: $15t = 120$.
   If you missed this problem, review Exercise 2.2.1.
2. Solve: $6x + 24 = 96$.
   If you missed this problem, review Exercise 2.3.1.

Use the Distance, Rate, and Time Formula

One formula you will use often in algebra and in everyday life is the formula for distance traveled by an object moving at a constant rate. Rate is an equivalent word for "speed." The basic idea of rate may already familiar to you. Do you know what distance you travel if you drive at a steady rate of 60 miles per hour for 2 hours? (This might happen if you use your car's cruise control while driving on the highway.) If you said 120 miles, you already know how to use this formula!

DISTANCE, RATE, AND TIME

For an object moving at a uniform (constant) rate, the distance traveled, the elapsed time, and the rate are related by the formula:

$$d = rt \quad \text{where} \quad \begin{aligned} d &= \text{distance} \\ r &= \text{rate} \\ t &= \text{time} \end{aligned} \tag{1.9.1}$$

We will use the Strategy for Solving Applications that we used earlier in this chapter. When our problem requires a formula, we change Step 4. In place of writing a sentence, we write the appropriate formula. We write the revised steps here for reference.

SOLVE AN APPLICATION (WITH A FORMULA).

1. **Read** the problem. Make sure all the words and ideas are understood.
2. **Identify** what we are looking for.
3. **Name** what we are looking for. Choose a variable to represent that quantity.
4. **Translate** into an equation. Write the appropriate formula for the situation. Substitute in the given information.
5. **Solve** the equation using good algebra techniques.
6. **Check** the answer in the problem and make sure it makes sense.
7. **Answer** the question with a complete sentence.

You may want to create a mini-chart to summarize the information in the problem. See the chart in this first example.

Example 1.9.1

Jamal rides his bike at a uniform rate of 12 miles per hour for $3\frac{1}{2}$ hours. What distance has he traveled?

Solution

Step 1. Read the problem.	

Step 2. Identify what you are looking for.	distance traveled
Step 3. Name. Choose a variable to represent it.	Let d = distance.
Step 4. Translate: Write the appropriate formula.	$d = rt$
	$d = ?$ $r = 12$ mph $t = 3\frac{1}{2}$ hours
Substitute in the given information.	$d = 12 \cdot 3\frac{1}{2}$
Step 5. Solve the equation.	$d = 42$ miles
Step 6. Check	
Does 42 miles make sense?	
Jamal rides:	
12 miles in 1 hour, 24 miles in 2 hours, 36 miles in 3 hours, 42 miles in $3\frac{1}{2}$ hours is reasonable 48 miles in 4 hours.	
Step 7. Answer the question with a complete sentence.	Jamal rode 42 miles.

Try It 1.9.2

Lindsay drove for $5\frac{1}{2}$ hours at 60 miles per hour. How much distance did she travel?

Answer

330 miles

Try It 1.9.3

Trinh walked for $2\frac{1}{3}$ hours at 3 miles per hour. How far did she walk?

Answer

7 miles

Example 1.9.4

Rey is planning to drive from his house in San Diego to visit his grandmother in Sacramento, a distance of 520 miles. If he can drive at a steady rate of 65 miles per hour, how many hours will the trip take?

Solution

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	How many hours (time)
Step 3. Name. Choose a variable to represent it.	Let t = time.
	$d = 520$ miles $r = 65$ mph $t = ?$ hours

Step 4. Translate. Write the appropriate formula.	$d = rt$
Substitute in the given information.	$520 = 65t$
Step 5. Solve the equation.	$t = 8$
Step 6. Check. Substitute the numbers into the formula and make sure the result is a true statement.	
$d = rt$ $520 \stackrel{?}{=} 65 \cdot 8$ $520 = 520 \checkmark$	
Step 7. Answer the question with a complete sentence. Rey's trip will take 8 hours.	

Try It 1.9.5

Lee wants to drive from Phoenix to his brother's apartment in San Francisco, a distance of 770 miles. If he drives at a steady rate of 70 miles per hour, how many hours will the trip take?

Answer

11 hours

Try It 1.9.6

Yesenia is 168 miles from Chicago. If she needs to be in Chicago in 3 hours, at what rate does she need to drive?

Answer

56 mph

Solve a Formula for a Specific Variable

You are probably familiar with some geometry formulas. A formula is a mathematical description of the relationship between variables. Formulas are also used in the sciences, such as chemistry, physics, and biology. In medicine they are used for calculations for dispensing medicine or determining body mass index. Spreadsheet programs rely on formulas to make calculations. It is important to be familiar with formulas and be able to manipulate them easily.

In Exercise 1.9.1 and Exercise 1.9.4, we used the formula $d = rt$. This formula gives the value of d, distance, when you substitute in the values of r and t, the rate and time. But in Exercise 1.9.4, we had to find the value of t. We substituted in values of d and r and then used algebra to solve for tt. If you had to do this often, you might wonder why there is not a formula that gives the value of t when you substitute in the values of d and r. We can make a formula like this by solving the formula $d = rt$ for t.

To solve a formula for a specific variable means to isolate that variable on one side of the equals sign with a coefficient of 1. All other variables and constants are on the other side of the equals sign. To see how to solve a formula for a specific variable, we will start with the distance, rate and time formula.

Example 1.9.7

Solve the formula d=rt for t:

1. when d=520 and r=65
2. in general

Solution

We will write the solutions side-by-side to demonstrate that solving a formula in general uses the same steps as when we have numbers to substitute.

1. when d=520 and r=65		2. in general	
Write the formula.	$d = rt$	Write the formula.	$d = rt$
Substitute.	$520 = 65t$		
Divide, to isolate t.	$\frac{520}{65} = \frac{65t}{65}$	Divide, to isolate tt.	$\frac{d}{r} = \frac{rt}{t}$
Simplify.	$8 = t$	Simplify.	$\frac{d}{r} = t$

We say the formula $t = \frac{d}{r}$ is solved for t.

Try It 1.9.8

Solve the formula $d = rt$ for r:

1. when d=180 and t=4
2. in general

Answer

1. $r = 45$
2. $r = \frac{d}{t}$

Try It 1.9.9

Solve the formula $d = rt$ for r:

1. when d=780 and t=12
2. in general

Answer

1. $r = 65$
2. \(r = \frac{d}{rt\)

Example 1.9.10

Solve the formula $A = \frac{1}{2}bh$ for h:

1. when $A = 90$ and $b = 15$
2. in general

Solution

1. when $A = 90$ and $b = 15$		2. in general	
Write the formula.	$A = \frac{1}{2}bh$	Write the formula.	$A = \frac{1}{2}bh$
Substitute.	$90 = \frac{1}{2} \cdot 15 \cdot h$		
Clear the fractions.	$2 \cdot 90 = 2 \cdot \frac{1}{2}15h$	Clear the fractions.	$2 \cdot A = 2 \cdot \frac{1}{2}bh$
Simplify.	$180 = 15h$	Simplify.	$2A = bh$
Solve for h.	$12 = h$	Solve for hh.	$\frac{2A}{b} = h$

We can now find the height of a triangle, if we know the area and the base, by using the formula $h = \frac{2A}{b}$

Try It 1.9.11

Solve the formula $A = \frac{1}{2}bh$ for h:

1. when $A = 170$ and $b = 17$
2. in general

Answer

1. $h = 20$
2. $h = \frac{2A}{b}$

Try It 1.9.12

Solve the formula $A = \frac{1}{2}bh$ for h:

1. when $A = 62$ and $h = 31$
2. in general

Answer

1. $b = 4$
2. $b = \frac{2A}{h}$

The formula $I = Prt$ is used to calculate simple interest, *I*, for a principal, *P*, invested at rate, *r*, for *t* years.

Example 1.9.13

Solve the formula I=Prt to find the principal, P:

1. when I=$5,600, r=4% ,t=7years
2. in general

Solution

1. I=$5,600, r=4% ,t=7years		2. in general	
Write the formula.	$I = Prt$	Write the formula.	$I = Prt$
Substitute.	$5600 = P(0.04)(7)$		
Simplify.	$5600 = P(0.28)$	Simplify.	$I = P(rt)$
Divide, to isolate *P*.	$\frac{5600}{0.28} = \frac{P(0.28)}{0.28}$	Divide, to isolate *P*.	$\frac{I}{rt} = \frac{P(rt)}{rt}$
Simplify.	$20,000 = P$	Simplify.	$\frac{I}{rt} = P$
The principal is	$20,000		$P = \frac{I}{rt}$

Try It 1.9.14

Solve the formula I=Prt to find the principal, P:

1. when I=$2160, r=6% ,t=3 years
2. in general

Answer

1. $12000
2. $P = \frac{1}{rt}$

Try It 1.9.15

Solve the formula I=Prt to find the principal, P:

1. when I=$5400, r= 12% ,t=5 years
2. in general

Answer

1. $9000
2. $P = \frac{1}{rt}$

Later in this class, and in future algebra classes, you'll encounter equations that relate two variables, usually *x* and *y*. You might be given an equation that is solved for *y* and need to solve it for *x*, or vice versa. In the following example, we're given an equation with both *x* and *y* on the same side and we'll solve it for *y*.

Example 1.9.16

Solve the formula 3x+2y=18 for *y*:

1. when x=4
2. in general

Solution

1. when x=4		2. in general	
	$3x + 2y = 18$		$3x + 2y = 18$
Substitute.	$3(4) + 2y = 18$		
Subtract to isolate the y-term.	$12 - 12 + 2y = 18 - 12$	Subtract to isolate the y-term.	$3x - 3x + 2y = 18 - 3x$
Divide.	$\frac{2y}{2} = \frac{6}{2}$	Divide.	$\frac{2y}{2} = \frac{18}{2} - \frac{3x}{2}$
Simplify.	$y = 3$	Simplify.	$y = -\frac{3x}{2} + 9$

Try It 1.9.17

Solve the formula 3x+4y=10 for *y*:

1. when $x = \frac{14}{3}$
2. in general

Answer

1. $y = -1$
2. $y = \frac{10-3x}{4}$

Try It 1.9.18

Solve the formula 5x+2y=18 for *y*:

1. when $x = 4$

2. in general

Answer

1. $y = -1$
2. $y = \frac{18-5x}{2}$

In Exercise 1.9.7 through Exercise 1.9.18 we used the numbers in part 1 as a guide to solving in general in part 2. Now we will solve a formula in general without using numbers as a guide.

Example 1.9.19

Solve the formula P=a+b+c for a.

Solution

We will isolate aa on one side of the equation.	$P = a + b + c$
Both b and c are added to a, so we subtract them from both sides of the equation.	$P - b - c = a + b + c - b - c$
Simplify.	$P - b - c = a$ $a = P - b - c$

Try It 1.9.20

Solve the formula P=a+b+c for *b*.

Answer

b=P−a−c

Try It 1.9.21

Solve the formula P=a+b+c for *c*.

Answer

c=P−a−b

Example 1.9.22

Solve the formula 6x+5y=13 for *y*.

Solution

	$6x + 5y = 13$
Subtract 6x from both sides to isolate the term with y.	$6x - 6x + 5y = 13 - 6x$
Simplify.	$5y = 13 - 6x$
Divide by 5 to make the coefficient 1.	$\frac{5y}{5} = \frac{13-6x}{5}$
Simplify.	$y = \frac{13-6x}{5}$

The fraction is simplified. We cannot divide 13−6x by 5.

? Try It 1.9.23

Solve the formula 4x+7y=9 for *y*.

Answer

$y = \frac{9-4x}{7}$

? Try It 1.9.24

Solve the formula 5x+8y=1 for *y*.

Answer

$y = \frac{1-5x}{8}$

Key Concepts

- **To Solve an Application (with a formula)**
  1. **Read** the problem. Make sure all the words and ideas are understood.
  2. **Identify** what we are looking for.
  3. **Name** what we are looking for. Choose a variable to represent that quantity.
  4. **Translate** into an equation. Write the appropriate formula for the situation. Substitute in the given information.
  5. **Solve** the equation using good algebra techniques.
  6. **Check** the answer in the problem and make sure it makes sense.
  7. **Answer** the question with a complete sentence.
- **Distance, Rate and Time**
  For an object moving at a uniform (constant) rate, the distance traveled, the elapsed time, and the rate are related by the formula: d=rt where *d* = distance, *r* = rate, *t* = time.
- **To solve a formula for a specific variable** means to get that variable by itself with a coefficient of 1 on one side of the equation and all other variables and constants on the other side.

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1.9E: Exercises

Practice Makes Perfect

Use the Distance, Rate, and Time Formula

In the following exercises, solve.

Exercise 1.9E.1

Steve drove for $8\frac{1}{2}$ hours at 72 miles per hour. How much distance did he travel?

Exercise 1.9E.2

Socorro drove for $4\frac{5}{6}$ hours at 60 miles per hour. How much distance did she travel?

Answer

290 miles

Exercise 1.9E.3

Yuki walked for $1\frac{3}{4}$ hours at 4 miles per hour. How far did she walk?

Exercise 1.9E.4

Francie rode her bike for $2\frac{1}{2}$ hours at 12 miles per hour. How far did she ride?

Answer

30 miles

Exercise 1.9E.5

Connor wants to drive from Tucson to the Grand Canyon, a distance of 338 miles. If he drives at a steady rate of 52 miles per hour, how many hours will the trip take?

Exercise 1.9E.6

Megan is taking the bus from New York City to Montreal. The distance is 380 miles and the bus travels at a steady rate of 76 miles per hour. How long will the bus ride be?

Answer

5 hours

Exercise 1.9E.7

Aurelia is driving from Miami to Orlando at a rate of 65 miles per hour. The distance is 235 miles. To the nearest tenth of an hour, how long will the trip take?

Exercise 1.9E.8

Kareem wants to ride his bike from St. Louis to Champaign, Illinois. The distance is 180 miles. If he rides at a steady rate of 16 miles per hour, how many hours will the trip take?

Answer

11.25 hours

Exercise 1.9E.9

Javier is driving to Bangor, 240 miles away. If he needs to be in Bangor in 4 hours, at what rate does he need to drive?

Exercise 1.9E.10

Alejandra is driving to Cincinnati, 450 miles away. If she wants to be there in 6 hours, at what rate does she need to drive?

Answer

75 mph

Exercise 1.9E.11

Aisha took the train from Spokane to Seattle. The distance is 280 miles and the trip took 3.5 hours. What was the speed of the train?

Exercise 1.9E.12

Philip got a ride with a friend from Denver to Las Vegas, a distance of 750 miles. If the trip took 10 hours, how fast was the friend driving?

Answer

75 mph

Solve a Formula for a Specific Variable

In the following exercises, use the formula $d = rt$.

Exercise 1.9E.13

Solve for t

1. when $d = 350$ and $r = 70$
2. in general

Exercise 1.9E.14

Solve for t

1. when $d = 240$ and $r = 60$
2. in general

Answer

1. $t = 4$
2. $t = \frac{d}{r}$

Exercise 1.9E.15

Solve for t

1. when $d = 510$ and $r = 60$

2. in general

Exercise 1.9E.16

Solve for t

1. when $d = 175$ and $r = 50$
2. in general

Answer

1. $t = 3.5$
2. $t = \frac{d}{r}$

Exercise 1.9E.17

Solve for r

1. when $d = 204$ and $t = 3$
2. in general

Exercise 1.9E.18

Solve for r

1. when $d = 420$ and $t = 6$
2. in general

Answer

1. $r = 70$
2. $r = \frac{d}{t}$

Exercise 1.9E.19

Solve for r

1. when $d = 160$ and $t = 2.5$
2. in general

Exercise 1.9E.20

Solve for r

1. when $d = 180$ and $t = 4.5$
2. in general

Answer

1. $r = 40$
2. $r = \frac{d}{t}$

In the following exercises, use the formula $A = \frac{1}{2}bh$

Exercise 1.9E.21

Solve for b

1. when $A = 126$ and $h = 18$
2. in general

Exercise 1.9E.22

Solve for h

1. when $A = 176$ and $b = 22$
2. in general

Answer

1. $h = 16$
2. $h = \frac{2A}{b}$

Exercise 1.9E.23

Solve for h

1. when $A = 375$ and $b = 25$
2. in general

Exercise 1.9E.24

Solve for b

1. when $A = 65$ and $h = 13$
2. in general

Answer

1. $b = 10$
2. $b = \frac{2A}{h}$

In the following exercises, use the formula $I = Prt$.

Exercise 1.9E.25

Solve for the principal, P for

1. $I = \$5,480$, $r = 4\%$, $t = 7$ years
2. in general

Exercise 1.9E.26

Solve for the principal, P for

1. $I = \$3,950$, $r = 6\%$, $t = 5$ years
2. in general

Answer

1. $P = \$13,166.67$
2. $P = \frac{I}{rt}$

Exercise 1.9E.27

Solve for the time, t for

1. $I = \$2,376$, $P = \$9,000$, $r = 4.4\%$
2. in general

Exercise 1.9E.28

Solve for the time, t for

1. $I = \$624$, $P = \$6,000$, $r = 5.2\%$
2. in general

Answer

1. $t = 2$ years
2. $t = \frac{I}{Pr}$

In the following exercises, solve.

Exercise 1.9E.29

Solve the formula $2x + 3y = 12$ for y

1. when $x = 3$
2. in general

Exercise 1.9E.30

Solve the formula $5x + 2y = 10$ for y

1. when $x = 4$
2. in general

Answer

1. $y = -5$
2. $y = \frac{10-5x}{2}$

Exercise 1.9E.31

Solve the formula $3x - y = 7$ for y

1. when $x = -2$
2. in general

Exercise 1.9E.32

Solve the formula $4x + y = 5$ for y

1. when $x = -3$
2. in general

Answer

1. $y = 17$
2. $y = 5 - 4x$

Exercise 1.9E.33

Solve $a+b=90$ for b.

Exercise 1.9E.34

Solve $a+b=90$ for a

Answer

$a=90-b$

Exercise 1.9E.35

Solve $180=a+b+c$ for a

Exercise 1.9E.36

Solve $180=a+b+c$ for c

Answer

$c=180-a-b$

Exercise 1.9E.37

Solve the formula $8x+y=15$ for y

Exercise 1.9E.38

Solve the formula $9x+y=13$ for y

Answer

$y=13-9x$

Exercise 1.9E.39

Solve the formula $-4x+y=-6$ for y

Exercise 1.9E.40

Solve the formula $-5x+y=-1$ for y

Answer

$y=-1+5x$

Exercise 1.9E.41

Solve the formula $4x+3y=7$ for y

Exercise 1.9E.42

Solve the formula $3x + 2y = 11$ for y

Answer

$y = \frac{11-3x}{2}$

Exercise 1.9E.43

Solve the formula $x - y = -4$ for y

Exercise 1.9E.44

Solve the formula $x - y = -3$ for y

Answer

$y = 3 + x$

Exercise 1.9E.45

Solve the formula $P = 2L + 2W$ for L

Exercise 1.9E.46

Solve the formula $P = 2L + 2W$ for W

Answer

$W = \frac{P-2L}{2}$

Exercise 1.9E.47

Solve the formula $C = \pi d$ for d

Exercise 1.9E.48

Solve the formula $C = \pi d$ for π

Answer

$\pi = \frac{C}{d}$

Exercise 1.9E.49

Solve the formula $V = LWH$ for L

Exercise 1.9E.50

Solve the formula $V = LWH$ for H

Answer

$H = \frac{V}{LW}$

Everyday Math

Exercise 1.9E.51

Converting temperature While on a tour in Greece, Tatyana saw that the temperature was 40° Celsius. Solve for F in the formula $C = \frac{5}{9}(F - 32)$ to find the Fahrenheit temperature.

Exercise 1.9E.52

Converting temperature Yon was visiting the United States and he saw that the temperature in Seattle one day was 50ºFahrenheit. Solve for C in the formula $F = \frac{9}{5}C + 32$ to find the Celsius temperature.

Answer

10°C

Writing Exercises

Exercise 1.9E.53

Solve the equation $2x + 3y = 6$ for y

1. when $x = -3$
2. in general
3. Which solution is easier for you, 1 or 2? Why?

Exercise 1.9E.54

Solve the equation $5x - 2y = 10$ for x

1. when $y = 10$
2. in general
3. Which solution is easier for you, 1 or 2? Why?

Answer

Answers will vary.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
use the distance, rate, and time formula.			
solve a formula for a specific variable.			

ⓑ What does this checklist tell you about your mastery of this section? What steps will you take to improve?

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1.10: Solve Linear Inequalities

Learning Objectives

By the end of this section, you will be able to:

- Graph inequalities on the number line
- Solve inequalities using the Subtraction and Addition Properties of inequality
- Solve inequalities using the Division and Multiplication Properties of inequality
- Solve inequalities that require simplification
- Translate to an inequality and solve

Note

Before you get started, take this readiness quiz.

1. Translate from algebra to English: $15 > x$.
   If you missed this problem, review Exercise 1.3.1.
2. Solve: $n - 9 = -42$.
   If you missed this problem, review Exercise 2.1.7.
3. Solve: $-5p = -23$.
   If you missed this problem, review Exercise 2.2.1.
4. Solve: $3a - 12 = 7a - 20$.
   If you missed this problem, review Exercise 2.3.22.

Graph Inequalities on the Number Line

Do you remember what it means for a number to be a solution to an equation? A solution of an equation is a value of a variable that makes a true statement when substituted into the equation.

What about the solution of an inequality? What number would make the inequality $x > 3$ true? Are you thinking, 'x could be 4'? That's correct, but x could be 5 too, or 20, or even 3.001. Any number greater than 3 is a solution to the inequality $x > 3$.

We show the solutions to the inequality $x > 3$ on the number line by shading in all the numbers to the right of 3, to show that all numbers greater than 3 are solutions. Because the number 3 itself is not a solution, we put an open parenthesis at 3. The graph of $x > 3$ is shown in Figure 1.10.1. Please note that the following convention is used: light blue arrows point in the positive direction and dark blue arrows point in the negative direction.

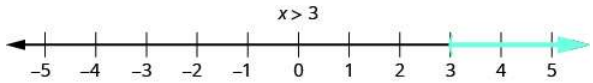


Figure 1.10.1: The inequality $x > 3$ is graphed on this number line.

The graph of the inequality $x \geq 3$ is very much like the graph of $x > 3$, but now we need to show that 3 is a solution, too. We do that by putting a bracket at $x = 3$, as shown in Figure 1.10.2

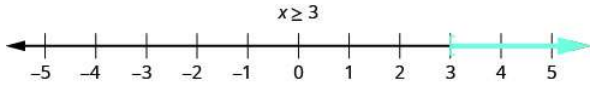


Figure 1.10.2: The inequality $x \geq 3$ is graphed on this number line.

Notice that the open parentheses symbol, (, shows that the endpoint of the inequality is not included. The open bracket symbol, [, shows that the endpoint is included.

Exercise 1.10.1

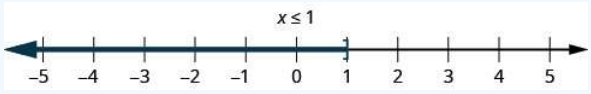
Graph on the number line:

1. $x \leq 1$
2. $x < 5$

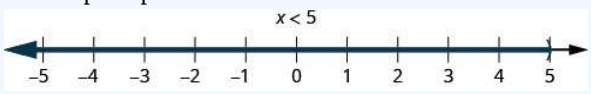
3. $x > -1$

Answer

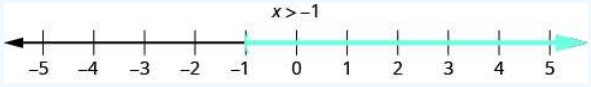
1. $x \leq 1$ This means all numbers less than or equal to 1. We shade in all the numbers on the number line to the left of 1 and put a bracket at x=1 to show that it is included.



2. $x < 5$ This means all numbers less than 5, but not including 5. We shade in all the numbers on the number line to the left of 5 and put a parenthesis at x=5 to show it is not included.



3. $x > -1$ This means all numbers greater than −1, but not including −1. We shade in all the numbers on the number line to the right of −1, then put a parenthesis at x=−1 to show it is not included.

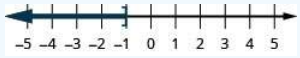


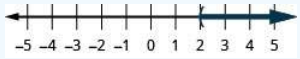
Exercise 1.10.2

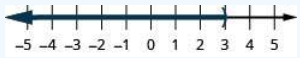
Graph on the number line:

1. $x \leq -1$
2. $x > 2$
3. $x < 3$

Answer

1. 

2. 

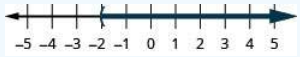
3. 

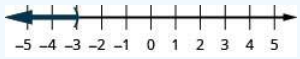
Exercise 1.10.3

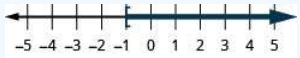
Graph on the number line:

1. $x > -2$
2. $x < -3$
3. $x \geq -1$

Answer

1. 

2. 

3. 

We can also represent inequalities using *interval notation.* As we saw above, the inequality $x > 3$ means all numbers greater than 3. There is no upper end to the solution to this inequality. In **interval notation**, we express $x > 3$ as $(3, \infty)$. The symbol ∞ is read as 'infinity'. It is not an actual number. Figure $1.10.3$shows both the number line and the interval notation.

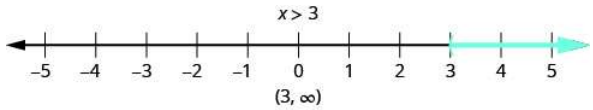


Figure 1.10.3: The inequality $x > 3$ is graphed on this number line and written in interval notation.

The inequality $x \leq 1$ means all numbers less than or equal to 1. There is no lower end to those numbers. We write $x \leq 1$ in interval notation as $(-\infty, 1]$. The symbol $-\infty$ is read as 'negative infinity'. Figure 1.10.4 shows both the number line and interval notation.

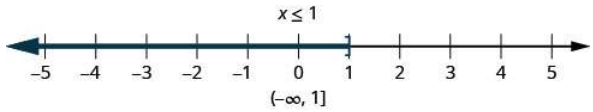


Figure 1.10.4: The inequality $x \leq 1$ is graphed on this number line and written in interval notation.

INEQUALITIES, NUMBER LINES, AND INTERVAL NOTATION



Did you notice how the parenthesis or bracket in the interval notation matches the symbol at the endpoint of the arrow? These relationships are shown in Figure 1.10.5



Figure 1.10.5: The notation for inequalities on a number line and in interval notation use similar symbols to express the endpoints of intervals.

Exercise 1.10.4

Graph on the number line and write in interval notation.

1. $x \geq -3$
2. $x < 2.5$
3. $x \leq \frac{3}{5}$

Answer

1.

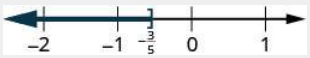
	$x \geq -3$
Shade to the right of −3, and put a bracket at −3.	–4 –3 –2 –1
Write in interval notation.	[–3, ∞)

2.

	$x < 2.5$
Shade to the left of 2.5, and put a parenthesis at 2.5.	0 1 2 2.5 3
Write in interval notation.	(–∞, 2.5)

3.

	$x \leq -\frac{3}{5}$

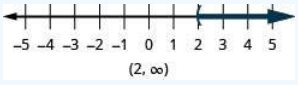
Shade to the left of $-\frac{3}{5}$, and put a bracket at $-\frac{3}{5}$.	
Write in interval notation.	$\left(-\infty, -\frac{3}{5}\right]$

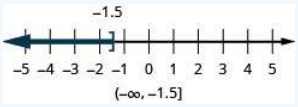
Exercise 1.10.5

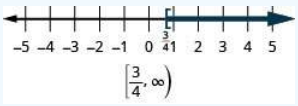
Graph on the number line and write in interval notation:

1. $x > 2$
2. $x \leq -1.5$
3. $x \geq \frac{3}{4}$

Answer

1. 

2. 

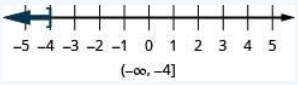
3. 

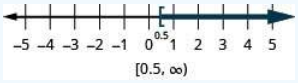
Exercise 1.10.6

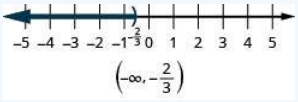
Graph on the number line and write in interval notation:

1. $x \leq -4$
2. $x \geq 0.5$
3. $x < -\frac{2}{3}$

Answer

1. 

2. 

3. 

Solve Inequalities using the Subtraction and Addition Properties of Inequality

The Subtraction and Addition Properties of Equality state that if two quantities are equal, when we add or subtract the same amount from both quantities, the results will be equal.

PROPERTIES OF EQUALITY

Subtraction Property of Equality	**Addition Property of Equality**	
For any numbers $a, b,$ and c, if $a = b$, then $a - c = b - c$.	For any numbers $a, b,$ and c if $a = b$ then $a + c = b + c$	(1.10.1)

Similar properties hold true for inequalities.

Table 1.10.1

For example, we know that −4 is less than 2.	$-4 < 2$
If we subtract 5 from both quantities, is the left side still less than the right side?	$-4-5 \stackrel{?}{<} 2-5$
We get −9 on the left and −3 on the right.	$-9 \stackrel{?}{<} -3$
And we know −9 is less than −3.	$-9 < -3$
	The inequality sign stayed the same.

Similarly we could show that the inequality also stays the same for addition.

This leads us to the Subtraction and Addition Properties of Inequality.

PROPERTIES OF INEQUALITY

Subtraction Property of Inequality	**Addition Property of Inequality**
For any numbers $a, b,$ and c,	For any numbers $a, b,$ and c
if $a < b$	if $a < b$
then $a - c < b - c$.	then $a + c < b + c$
if $a > b$	if $a > b$
then $a - c > b - c$.	then $a + c > b + c$

(1.10.2)

We use these properties to solve inequalities, taking the same steps we used to solve equations. Solving the inequality $x + 5 > 9$, the steps would look like this:

$$\begin{array}{lrcl} & x+5 & > & 9 \\ \text{Subtract 5 from both sides to isolate } x. & x+5-5 & > & 9-5 \\ & x & > & 4 \end{array} \quad (1.10.3)$$

Any number greater than 4 is a solution to this inequality.

Exercise 1.10.7

Solve the inequality $n - \frac{1}{2} \leq \frac{5}{8}$, graph the solution on the number line, and write the solution in interval notation.

Answer

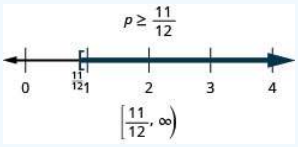
	$n - \frac{1}{2} \leq \frac{5}{8}$
Add $\frac{1}{2}$ to both sides of the inequality.	$n - \frac{1}{2} + \frac{1}{2} \leq \frac{5}{8} + \frac{1}{2}$
Simplify.	$n \leq \frac{9}{8}$
Graph the solution on the number line.	
Write the solution in interval notation.	

Exercise 1.10.8

Solve the inequality, graph the solution on the number line, and write the solution in interval notation.

$p - \frac{3}{4} \geq \frac{1}{6}$

Answer

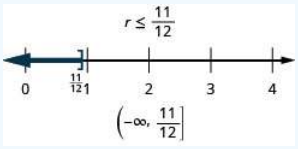


Exercise 1.10.9

Solve the inequality, graph the solution on the number line, and write the solution in interval notation.

$r - \frac{1}{3} \leq \frac{7}{12}$

Answer



Solve Inequalities using the Division and Multiplication Properties of Inequality

The Division and Multiplication Properties of Equality state that if two quantities are equal, when we divide or multiply both quantities by the same amount, the results will also be equal (provided we don't divide by 0).

PROPERTIES OF EQUALITY

Division Property of Equality	**MUltiplication Property of Equality**
For any numbers a, b, c, and $c \neq 0$	For any numbers a, b, c
if $a = b$	if $a = b$
then $\frac{a}{c} = \frac{b}{c}$	then $ac = bc$

(1.10.4)

Are there similar properties for inequalities? What happens to an inequality when we divide or multiply both sides by a constant?

Consider some numerical examples.

Table 1.10.2

	$10 < 15$		$10 < 15$
Divide both sides by 5.	$\frac{10}{5} \,?\, \frac{15}{5}$	Multiply both sides by 5.	$10(5) \,?\, 15(5)$
Simplify.	$2 \,?\, 3$		$50 \,?\, 75$
Fill in the inequality signs.	$2 < 3$		$50 < 75$

The inequality signs stayed the same.

Does the inequality stay the same when we divide or multiply by a negative number?

Table 1.10.3

	$10 < 15$		$10 < 15$

Divide both sides by -5.	$\frac{10}{-5} \; ? \; \frac{15}{-5}$	Multiply both sides by -5.	$10(-5) \; ? \; 15(-5)$
Simplify.	$-2 \; ? \; -3$		$-50 \; ? \; -75$
Fill in the inequality signs.	$-2 > -3$		$-50 > -75$

The inequality signs reversed their direction.

When we divide or multiply an inequality by a positive number, the inequality sign stays the same. When we divide or multiply an inequality by a negative number, the inequality sign reverses.

Here are the Division and Multiplication Properties of Inequality for easy reference.

DIVISION AND MULTIPLICATION PROPERTIES OF INEQUALITY

For any real numbers a,b,c

$$\begin{aligned} &\text{if } a<b \text{ and } c>0, \text{ then } && \tfrac{a}{c}<\tfrac{b}{c} \text{ and } ac<bc \\ &\text{if } a>b \text{ and } c>0, \text{ then } && \tfrac{a}{c}>\tfrac{b}{c} \text{ and } ac>bc \\ &\text{if } a<b \text{ and } c<0, \text{ then } && \tfrac{a}{c}>\tfrac{b}{c} \text{ and } ac>bc \\ &\text{if } a>b \text{ and } c<0, \text{ then } && \tfrac{a}{c}<\tfrac{b}{c} \text{ and } ac<bc \end{aligned} \tag{1.10.5}$$

When we **divide or multiply** an inequality by a:

- **positive** number, the inequality stays the **same**.
- **negative** number, the inequality **reverses**.

Exercise 1.10.10

Solve the inequality $7y < 42$, graph the solution on the number line, and write the solution in interval notation.

Answer

	$7y < 42$
Divide both sides of the inequality by 7. Since $7 > 0$, the inequality stays the same.	$\frac{7y}{7} < \frac{42}{7}$
Simplify.	$y < 6$
Graph the solution on the number line.	
Write the solution in interval notation.	$(-\infty, 6)$

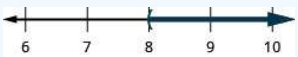
Exercise 1.10.11

Solve the inequality, graph the solution on the number line, and write the solution in interval notation.

$9c > 72$

Answer

$c > 8$



$(8, \infty)$

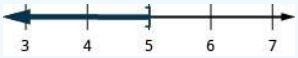
Exercise 1.10.12

Solve the inequality, graph the solution on the number line, and write the solution in interval notation.

$12d \leq 60$

Answer

$d \leq 5$



$(-\infty, 5]$

Exercise 1.10.13

Solve the inequality $-10a \geq 50$, graph the solution on the number line, and write the solution in interval notation.

Answer

	$-10a \geq 50$
Divide both sides of the inequality by −10. Since $-10 < 0$, the inequality reverses.	$\frac{-10a}{-10} \leq \frac{50}{-10}$
Simplify.	$a \leq -5$
Graph the solution on the number line.	
Write the solution in interval notation.	$(-\infty, -5]$

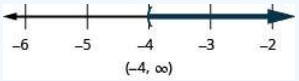
Exercise 1.10.14

Solve each inequality, graph the solution on the number line, and write the solution in interval notation.

$-8q < 32$

Answer

$q > -4$

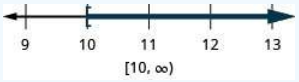


Exercise 1.10.15

Solve each inequality, graph the solution on the number line, and write the solution in interval notation.

$-7r \leq -70$

Answer



SOLVING INEQUALITIES

Sometimes when solving an inequality, the variable ends up on the right. We can rewrite the inequality in reverse to get the variable to the left.

$$x > a \text{ has the same meaning as } a < x \tag{1.10.6}$$

Think about it as "If Xavier is taller than Alex, then Alex is shorter than Xavier."

Exercise 1.10.16

Solve the inequality $-20 < \frac{4}{5}u$, graph the solution on the number line, and write the solution in interval notation.

Answer

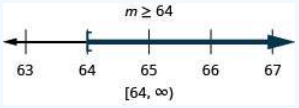
	$-20 < \frac{4}{5}u$
Multiply both sides of the inequality by $\frac{5}{4}$. Since $\frac{5}{4} > 0$, the inequality stays the same.	$\frac{5}{4}(-20) < \frac{5}{4}\left(\frac{4}{5}u\right)$
Simplify.	$-25 < u$
Rewrite the variable on the left.	$u > -25$
Graph the solution on the number line.	
Write the solution in interval notation.	$(-25, \infty)$

Exercise 1.10.17

Solve the inequality, graph the solution on the number line, and write the solution in interval notation.

$24 \leq \frac{3}{8}m$

Answer

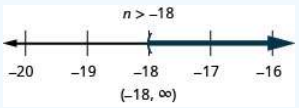


Exercise 1.10.18

Solve the inequality, graph the solution on the number line, and write the solution in interval notation.

$-24 < \frac{4}{3}n$

Answer



Exercise 1.10.19

Solve the inequality $\frac{t}{-2} \geq 8$, graph the solution on the number line, and write the solution in interval notation.

Answer

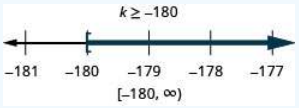
	$\frac{t}{-2} \geq 8$
Multiply both sides of the inequality by -2. Since $-2 < 0$, the inequality reverses.	$-2\left(\frac{t}{-2}\right) \leq -2(8)$
Simplify.	$t \leq -16$
Graph the solution on the number line.	
Write the solution in interval notation.	$(-\infty, -16]$

? Exercise 1.10.20

Solve the inequality, graph the solution on the number line, and write the solution in interval notation.

$\frac{k}{-12} \leq 15$

Answer

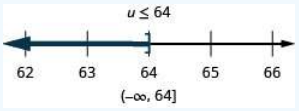


? Exercise 1.10.21

Solve the inequality, graph the solution on the number line, and write the solution in interval notation.

$\frac{u}{-4} \geq -16$

Answer



Solve Inequalities That Require Simplification

Most inequalities will take more than one step to solve. We follow the same steps we used in the general strategy for solving linear equations, but be sure to pay close attention during multiplication or division.

? Exercise 1.10.22

Solve the inequality $4m \leq 9m + 17$, graph the solution on the number line, and write the solution in interval notation.

Answer

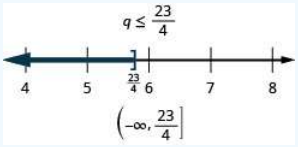
	$4m \leq 9m + 17$
Subtract 9m from both sides to collect the variables on the left.	$4m - 9m \leq 9m - 9m + 17$
Simplify.	$-5m \leq 17$
Divide both sides of the inequality by -5, and reverse the inequality.	$\frac{-5m}{-5} \geq \frac{17}{-5}$

Simplify.	$m \geq -\frac{17}{5}$
Graph the solution on the number line.	
Write the solution in interval notation.	$\left[-\frac{17}{5}, \infty\right)$

Exercise 1.10.23

Solve the inequality $3q \geq 7q - 23$, graph the solution on the number line, and write the solution in interval notation.

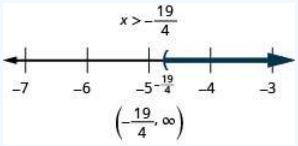
Answer



Exercise 1.10.24

Solve the inequality $6x < 10x + 19$, graph the solution on the number line, and write the solution in interval notation.

Answer



Exercise 1.10.25

Solve the inequality $8p + 3(p - 12) > 7p - 28$ graph the solution on the number line, and write the solution in interval notation.

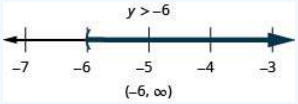
Answer

Simplify each side as much as possible.	8p+3(p−12)>7p−28
Distribute.	8p+3p−36>7p−28
Combine like terms.	11p−36>7p−28
Subtract 7p from both sides to collect the variables on the left.	11p−36−7p>7p−28−7p
Simplify.	4p−36>−28
Add 36 to both sides to collect the constants on the right.	4p−36+36>−28+36
Simplify.	4p>8
Divide both sides of the inequality by 4; the inequality stays the same.	$\frac{4p}{4} > 84$
Simplify.	$p > 2$
Graph the solution on the number line.	
Write the solution in interval notation.	$(2, \infty)$

Exercise 1.10.26

Solve the inequality $9y+2(y+6)>5y-24$, graph the solution on the number line, and write the solution in interval notation.

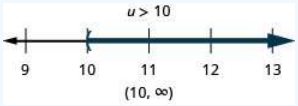
Answer



Exercise 1.10.27

Solve the inequality $6u+8(u-1)>10u+32$, graph the solution on the number line, and write the solution in interval notation.

Answer



Just like some equations are identities and some are contradictions, inequalities may be identities or contradictions, too. We recognize these forms when we are left with only constants as we solve the inequality. If the result is a true statement, we have an identity. If the result is a false statement, we have a contradiction.

Exercise 1.10.28

Solve the inequality $8x-2(5-x)<4(x+9)+6x$, graph the solution on the number line, and write the solution in interval notation.

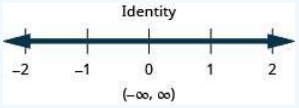
Answer

Simplify each side as much as possible.	8x−2(5−x)<4(x+9)+6x
Distribute.	8x−10+2x<4x+36+6x
Combine like terms.	10x−10<10x+36
Subtract 10x from both sides to collect the variables on the left.	10x−10−10x<10x+36−10x
Simplify.	−10<36
The xx's are gone, and we have a true statement.	The inequality is an identity. The solution is all real numbers.
Graph the solution on the number line.	
Write the solution in interval notation.	$(-\infty,\infty)$

Exercise 1.10.29

Solve the inequality $4b-3(3-b)>5(b-6)+2b$, graph the solution on the number line, and write the solution in interval notation.

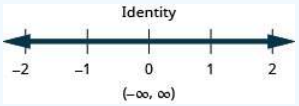
Answer



Exercise 1.10.30

Solve the inequality $9h-7(2-h)<8(h+11)+8h$, graph the solution on the number line, and write the solution in interval notation.

Answer



Exercise 1.10.31

Solve the inequality $\frac{1}{3}a-\frac{1}{8}a>\frac{5}{24}a+\frac{3}{4}$, graph the solution on the number line, and write the solution in interval notation.

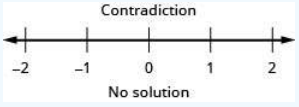
Answer

	$\frac{1}{3}a-\frac{1}{8}a>\frac{5}{24}a+\frac{3}{4}$
Multiply both sides by the LCD, 24, to clear the fractions.	$24\left(\frac{1}{3}a-\frac{1}{8}a\right)>24\left(\frac{5}{24}a+\frac{3}{4}\right)$
Simplify.	$8a-3a>5a+18$
Combine like terms.	$5a>5a+18$
Subtract 5a from both sides to collect the variables on the left.	$5a-5a>5a-5a+18$
Simplify.	$0>18$
The statement is false!	The inequality is a contradiction.
	There is no solution.
Graph the solution on the number line.	
Write the solution in interval notation.	There is no solution.

Exercise 1.10.32

Solve the inequality $\frac{1}{4}x-\frac{1}{12}x>\frac{1}{6}x+\frac{7}{8}$, graph the solution on the number line, and write the solution in interval notation.

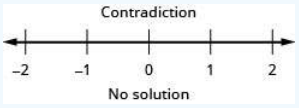
Answer



Exercise 1.10.33

Solve the inequality $\frac{2}{5}z - \frac{1}{3}z < \frac{1}{15}z - \frac{3}{5}$, graph the solution on the number line, and write the solution in interval notation.

Answer



Translate to an Inequality and Solve

To translate English sentences into inequalities, we need to recognize the phrases that indicate the inequality. Some words are easy, like ‘more than’ and ‘less than’. But others are not as obvious.

Think about the phrase ‘at least’ – what does it mean to be ‘at least 21 years old’? It means 21 or more. The phrase ‘at least’ is the same as ‘greater than or equal to’.

Table 1.10.4shows some common phrases that indicate inequalities.

Table 1.10.4

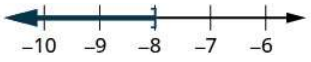
>	≥	<	≤
" data-valign="middle" class="lt-math-15134">is greater than	is greater than or equal to	is less than	is less than or equal to
" data-valign="middle" class="lt-math-15134">is more than	is at least	is smaller than	is at most
" data-valign="middle" class="lt-math-15134">is larger than	is no less than	has fewer than	is no more than
" data-valign="middle" class="lt-math-15134">exceeds	is the minimum	is lower than	is the maximum

Exercise 1.10.34

Translate and solve. Then write the solution in interval notation and graph on the number line.

Twelve times c is no more than 96.

Answer

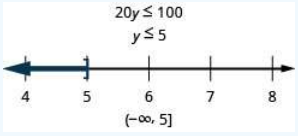
Translate.	Twelve times c is no more than 96 $12c \leq 96$
Solve—divide both sides by 12.	$\frac{12c}{12} \leq \frac{96}{12}$
Simplify.	$c \leq 8$
Write in interval notation.	$(-\infty, -8]$
Graph on the number line.	

Exercise 1.10.35

Translate and solve. Then write the solution in interval notation and graph on the number line.

Twenty times y is at most 100

Answer

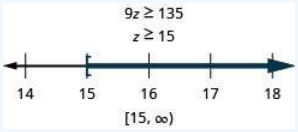


Exercise 1.10.36

Translate and solve. Then write the solution in interval notation and graph on the number line.

Nine times z is no less than 135

Answer



Exercise 1.10.37

Translate and solve. Then write the solution in interval notation and graph on the number line.

Thirty less than x is at least 45.

Answer

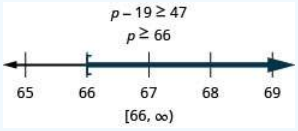
Translate.	Thirty less than x is at least 45. $x - 30 \geq 45$
Solve—add 30 to both sides.	$x - 30 + 30 \geq 45 + 30$
Simplify.	$x \geq 75$
Write in interval notation.	$[75, \infty)$
Graph on the number line.	

Exercise 1.10.38

Translate and solve. Then write the solution in interval notation and graph on the number line.

Nineteen less than p is no less than 47

Answer



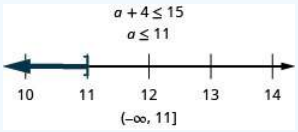
https://math.libretexts.org/@go/page/72120

Exercise 1.10.39

Translate and solve. Then write the solution in interval notation and graph on the number line.

Four more than a is at most 15.

Answer



Key Concepts

- **Subtraction Property of Inequality**
  For any numbers a, b, and c,
  if a<b then a−c<b−c and
  if a>b then a−c>b−c.
- **Addition Property of Inequality**
  For any numbers a, b, and c,
  if a<b then a+c<b+c and
  if a>b then a+c>b+c.
- **Division and Multiplication Properties of Inequality**
  For any numbers a, b, and c,
  if a<b and c>0, then ac<bc and ac>bc.
  if a>b and c>0, then ac>bc and ac>bc.
  if a<b and c<0, then ac>bc and ac>bc.
  if a>b and c<0, then ac<bc and ac<bc.
- When we **divide or multiply** an inequality by a:
  - **positive** number, the inequality stays the **same**.
  - **negative** number, the inequality **reverses**.

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1.10E: Exercises

Practice Makes Perfect

Graph Inequalities on the Number Line

In the following exercises, graph each inequality on the number line

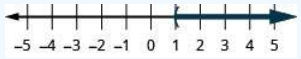
Exercise 1.10.1

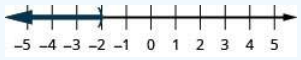
1. $x \leq 2$
2. $x > -1$
3. $x < 0$

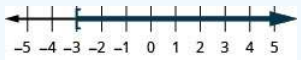
Exercise 1.10.2

1. $x > 1$
2. $x < -2$
3. $x \geq -3$

Answer

1. 

2. 

3. 

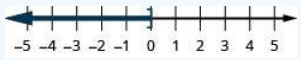
Exercise 1.10.3

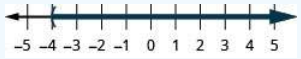
1. $x \geq -3$
2. $x < 4$
3. $x \leq -2$

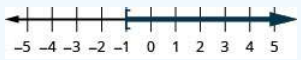
Exercise 1.10.4

1. $x \leq 0$
2. $x > -4$
3. $x \geq -1$

Answer

1. 

2. 

3. 

In the following exercises, graph each inequality on the number line and write in interval notation.

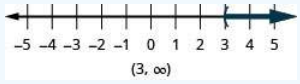
Exercise 1.10.5

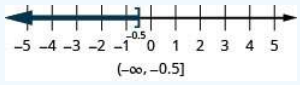
1. $x < -2$
2. $x \geq -3.5$
3. $x \leq \frac{2}{3}$

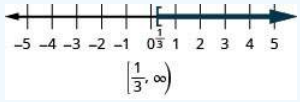
Exercise 1.10.6

1. $x > 3$
2. $x \leq -0.5$
3. $x \geq \frac{1}{3}$

Answer

1. 

2. 

3. 

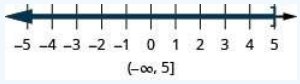
Exercise 1.10.7

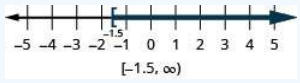
1. $x \geq -4$
2. x<2.5
3. $x > -\frac{3}{2}$

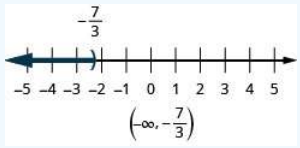
Exercise 1.10.8

1. $x \leq 5$
2. $x \geq -1.5x$
3. x<−73

Answer

1. 

2. 

3. 

Solve Inequalities using the Subtraction and Addition Properties of Inequality

In the following exercises, solve each inequality, graph the solution on the number line, and write the solution in interval notation.

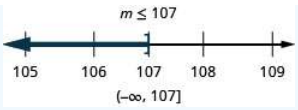
Exercise 1.10.9

$n - 11 < 33$

Exercise 1.10.10

$m - 45 \leq 62$

Answer



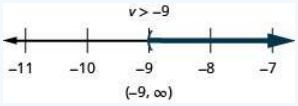
Exercise 1.10.11

$u+25>21$

Exercise 1.10.12

$v+12>3$

Answer



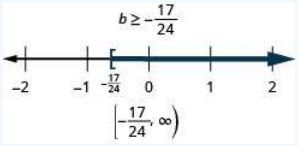
Exercise 1.10.13

$a+\frac{3}{4}\geq\frac{7}{10}$

Exercise 1.10.14

$b+\frac{7}{8}\geq\frac{1}{6}$

Answer



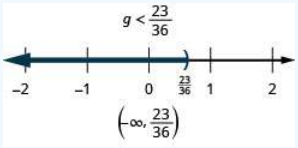
Exercise 1.10.15

$f-\frac{13}{20}<-\frac{5}{12}$

Exercise 1.10.16

$g-\frac{11}{12}<-\frac{5}{18}$

Answer



Solve Inequalities using the Division and Multiplication Properties of Inequality

In the following exercises, solve each inequality, graph the solution on the number line, and write the solution in interval notation.

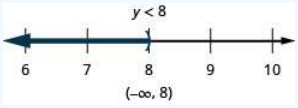
Exercise 1.10.17

$8x > 72$

Exercise 1.10.18

$6y < 48$

Answer



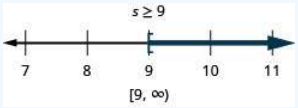
Exercise 1.10.19

$7r \leq 56$

Exercise 1.10.20

$9s \geq 81$

Answer



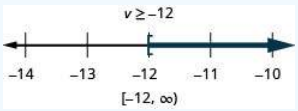
Exercise 1.10.21

$-5u \geq 65$

Exercise 1.10.22

$-8v \leq 96$

Answer



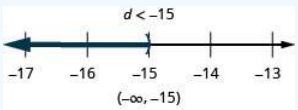
Exercise 1.10.23

$-9c < 126$

Exercise 1.10.24

$-7d > 105$

Answer



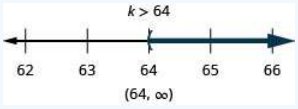
Exercise 1.10.25

$20 > \frac{2}{5}h$

Exercise 1.10.26

$40 < \frac{5}{8}k$

Answer



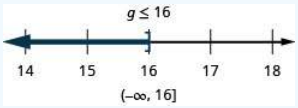
Exercise 1.10.27

$\frac{7}{6}j \geq 42$

Exercise 1.10.28

$\frac{9}{4}g \leq 36$

Answer



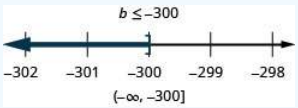
Exercise 1.10.29

$\frac{a}{-3} \leq 9$

Exercise 1.10.30

$\frac{b}{-10} \geq 30$

Answer



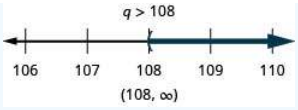
Exercise 1.10.31

$-25 < \frac{p}{-5}$

Exercise 1.10.32

$-18 > \frac{q}{-6}$

Answer



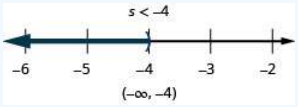
Exercise 1.10.33

$9t \geq -27$

Exercise 1.10.34

$7s < -28$

Answer



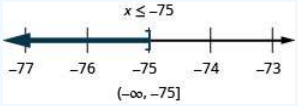
Exercise 1.10.35

$\frac{2}{3}y > -36$

Exercise 1.10.36

$\frac{3}{5}x \leq -45$

Answer



Solve Inequalities That Require Simplification

In the following exercises, solve each inequality, graph the solution on the number line, and write the solution in interval notation.

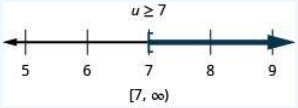
Exercise 1.10.37

$4v \geq 9v - 40$

Exercise 1.10.38

$5u \leq 8u - 21$

Answer



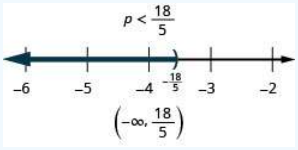
Exercise 1.10.39

$13q < 7q - 29$

Exercise 1.10.40

$9p > 14p - 18$

Answer



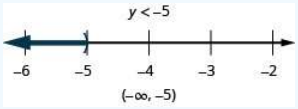
Exercise 1.10.41

$12x + 3(x + 7) > 10x - 24$

Exercise 1.10.42

$9y + 5(y + 3) < 4y - 35$

Answer



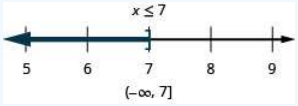
Exercise 1.10.43

$6h - 4(h - 1) \leq 7h - 11$

Exercise 1.10.44

$4k - (k - 2) \geq 7k - 26$

Answer



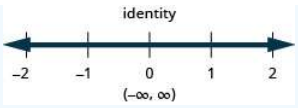
Exercise 1.10.45

$8m - 2(14 - m) \geq 7(m - 4) + 3m$

Exercise 1.10.46

$6n - 12(3 - n) \leq 9(n - 4) + 9n$

Answer



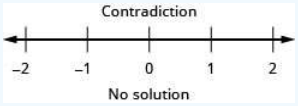
Exercise 1.10.47

$\frac{3}{4}b-\frac{1}{3}b<\frac{5}{12}b-\frac{1}{2}$

Exercise 1.10.48

$9u+5(2u-5)\geq 12(u-1)+7u$

Answer



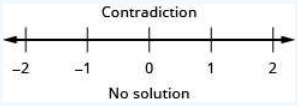
Exercise 1.10.49

$\frac{2}{3}g-\frac{1}{2}(g-14)\leq\frac{1}{6}(g+42)$

Exercise 1.10.50

$\frac{5}{6}a-\frac{1}{4}a>\frac{7}{12}a+\frac{2}{3}$

Answer



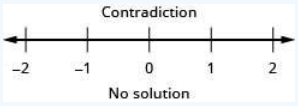
Exercise 1.10.51

$\frac{4}{5}h-\frac{2}{3}(h-9)\geq\frac{1}{15}(2h+90)$

Exercise 1.10.52

$12v+3(4v-1)\leq 19(v-2)+5v$

Answer



Mixed practice

In the following exercises, solve each inequality, graph the solution on the number line, and write the solution in interval notation.

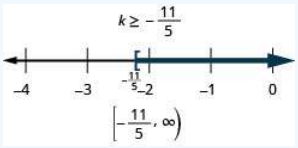
Exercise 1.10.53

$15k\leq -40$

Exercise 1.10.54

$35k \geq -77$

Answer



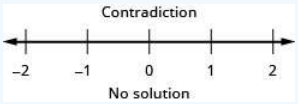
Exercise 1.10.55

$23p - 2(6 - 5p) > 3(11p - 4)$

Exercise 1.10.56

$18q - 4(10 - 3q) < 5(6q - 8)$

Answer



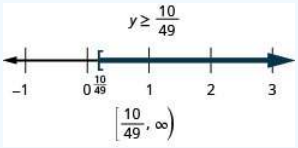
Exercise 1.10.57

$-\frac{9}{4}x \geq -\frac{5}{12}$

Exercise 1.10.58

$-\frac{21}{8}y \leq -\frac{15}{28}$

Answer



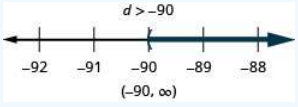
Exercise 1.10.59

$c + 34 < -99$

Exercise 1.10.60

$d + 29 > -61$

Answer



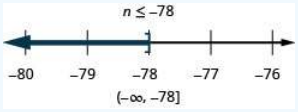
Exercise 1.10.61

$\frac{m}{18} \geq -4$

Exercise 1.10.62

$\frac{n}{13} \leq -6$

Answer



Translate to an Inequality and Solve

In the following exercises, translate and solve .Then write the solution in interval notation and graph on the number line.

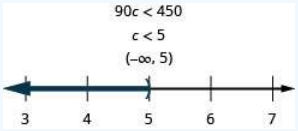
Exercise 1.10.63

Fourteen times d is greater than 56.

Exercise 1.10.64

Ninety times c is less than 450.

Answer



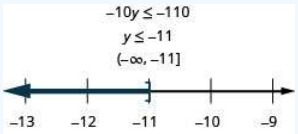
Exercise 1.10.65

Eight times z is smaller than -40

Exercise 1.10.66

Ten times y is at most -110

Answer



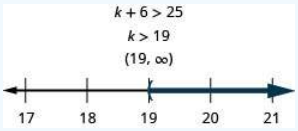
Exercise 1.10.67

Three more than h is no less than 25

Exercise 1.10.68

Six more than k exceeds 25

Answer



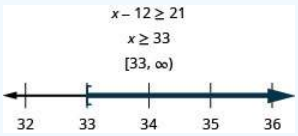
Exercise 1.10.69

Ten less than w is at least 39.

Exercise 1.10.70

Twelve less than x is no less than 21

Answer



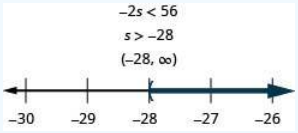
Exercise 1.10.71

Negative five times r is no more than 95.

Exercise 1.10.72

Negative two times s is lower than 56

Answer



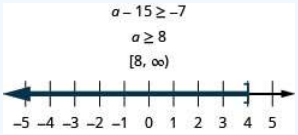
Exercise 1.10.73

Nineteen less than b is at most -22

Exercise 1.10.74

Fifteen less than a is at least -7

Answer



Everyday Math

Exercise 1.10.75

Safety A child's height, *h*, must be at least 57 inches for the child to safely ride in the front seat of a car. Write this as an inequality.

Exercise 1.10.76

Fighter pilots The maximum height, *h*, of a fighter pilot is 77 inches. Write this as an inequality.

Answer

$h \leq 77$

Exercise 1.10.77

Elevators The total weight, *w*, of an elevator's passengers can be no more than 1,200 pounds. Write this as an inequality.

Exercise 1.10.78

Shopping The number of items, *n*, a shopper can have in the express check-out lane is at most 8. Write this as an inequality.

Answer

$n \leq 8$

Writing Exercises

Exercise 1.10.79

Give an example from your life using the phrase 'at least'.

Exercise 1.10.80

Give an example from your life using the phrase 'at most'.

Answer

Answers will vary.

Exercise 1.10.81

Explain why it is necessary to reverse the inequality when solving $-5x > 10$

Exercise 1.10.82

Explain why it is necessary to reverse the inequality when solving $\frac{n}{-3} < 12$

Answer

Answers will vary.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
graph inequalities on the number line.			
solve inequalities using the Subtraction and Addition Properties of Inequality.			
solve inequalities using the Division and Multiplication Properties of Inequality.			
solve inequalities that require simplification.			
translate to an inequality and solve.			

ⓑ What does this checklist tell you about your mastery of this section? What steps will you take to improve?

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1.11: Use the Rectangular Coordinate System

Learning Objectives

By the end of this section, you will be able to:

- Plot points in a rectangular coordinate system
- Verify solutions to an equation in two variables
- Complete a table of solutions to a linear equation
- Find solutions to a linear equation in two variables

Note

Before you get started, take this readiness quiz.

1. Evaluate $x+3$ when $x=-1$.
   If you missed this problem, review Example 1.5.25.
2. Evaluate $2x-5y$ when $x=3$ and y=−2.
   If you missed this problem, review Example 1.5.28.
3. Solve for y: $40-4y=20$
   If you missed this problem, review Example 2.3.1.

Plot Points on a Rectangular Coordinate System

Just like maps use a grid system to identify locations, a grid system is used in algebra to show a relationship between two variables in a **rectangular coordinate system**. The rectangular coordinate system is also called the *xy*-plane or the 'coordinate plane'.

The horizontal number line is called the *x-axis*. The vertical number line is called the *y-axis*. The *x*-axis and the *y*-axis together form the rectangular coordinate system. These axes divide a plane into four regions, called **quadrants**. The quadrants are identified by Roman numerals, beginning on the upper right and proceeding counterclockwise. See Figure \(\PageIndex{1}\).

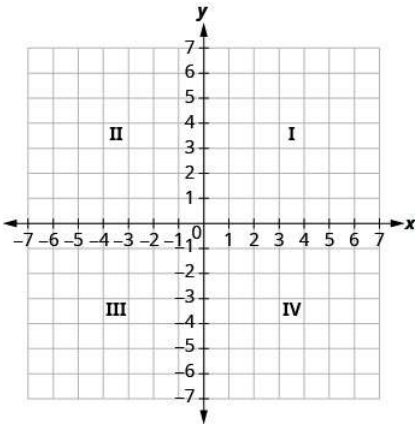


Figure 1.11.1: 'Quadrant' has the root 'quad,' which means 'four.'

In the **rectangular coordinate system**, every point is represented by an *ordered pair*. The first number in the ordered pair is the ***x*-coordinate** of the point, and the second number is the ***y*-coordinate** of the point.

ORDERED PAIR

An **ordered pair**, (x,y)(x,y), gives the coordinates of a point in a rectangular coordinate system.

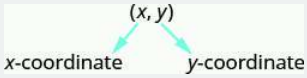


Figure 1.11.2

The first number is the *x*-coordinate.

The second number is the *y*-coordinate.

The phrase 'ordered pair' means the order is important. What is the ordered pair of the point where the axes cross? At that point both coordinates are zero, so its ordered pair is $(0, 0)$. The point $(0, 0)$ has a special name. It is called the **origin**.

THE ORIGIN

The point $(0, 0)$ is called the **origin**. It is the point where the *x*-axis and *y*-axis intersect.

We use the coordinates to locate a point on the *xy*-plane. Let's plot the point $(1, 3)$ as an example. First, locate 1 on the *x*-axis and lightly sketch a vertical line through $x = 1$. Then, locate 3 on the *y*-axis and sketch a horizontal line through $y = 3$. Now, find the point where these two lines meet — that is the point with coordinates $(1, 3)$.

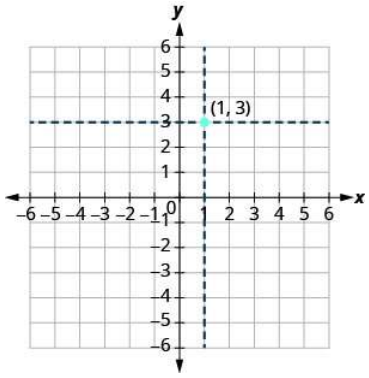


Figure 1.11.3

Notice that the vertical line through $x = 1$ and the horizontal line through $y = 3$ are not part of the graph. We just used them to help us locate the point $(1, 3)$.

Example 1.11.1

Plot each point in the rectangular coordinate system and identify the **quadrant** in which the point is located:

a. (−5,4)
b. (−3,−4)
c. (2,−3)
d. (−2,3)
e. $(3, \frac{5}{2})$

Solution

The first number of the coordinate pair is the *x*-coordinate, and the second number is the *y*-coordinate.

a. Since x=−5, the point is to the left of the *y*-axis. Also, since y=4, the point is above the *x*-axis. The point (−5,4) is in Quadrant II.
b. Since x=−3, the point is to the left of the *y*-axis. Also, since y=−4, the point is below the *x*-axis. The point (−3,−4) is in Quadrant III.
c. Since x=2, the point is to the right of the *y*-axis. Since y=−3, the point is below the *x*-axis. The point (2,−3) is in Quadrant IV.
d. Since x=−2, the point is to the left of the *y*-axis. Since y=3, the point is above the *x*-axis. The point (−2,3) is in Quadrant II.
e. Since x=3, the point is to the right of the *y*-axis. Since $y = \frac{5}{2}$, the point is above the *x*-axis. (It may be helpful to write $\frac{5}{2}$ as a mixed number or decimal.) The point $(3, \frac{5}{2})$ is in Quadrant I.

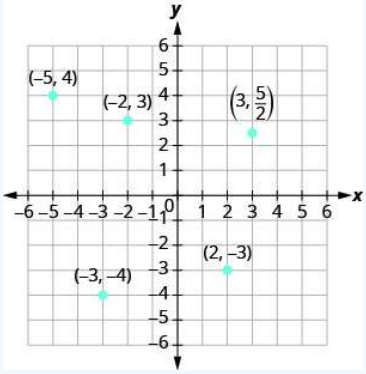


Figure 1.11.4

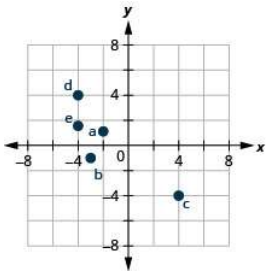
Try It 1.11.2

Plot each point in a rectangular coordinate system and identify the quadrant in which the point is located:

a. (−2,1)
b. (−3,−1)
c. (4,−4)
d. (−4,4)
e. $(-4, \frac{3}{2})$

Answer

The point a (-2,1) is in Quadrant II.
The point b (-3,-1) is in Quadrant III.
The point c (4,-4) is in Quadrant IV.
The point d (-4,4) is in Quadrant II.
The point e $(-4, \frac{3}{2})$ is in Quadrant II.



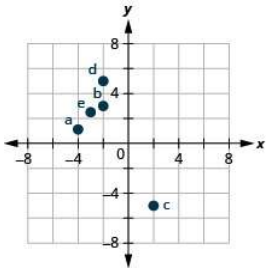
Try It 1.11.3

Plot each point in a rectangular coordinate system and identify the quadrant in which the point is located:

a. (−4,1)
b. (−2,3)
c. (2,−5)
d. (−2,5)
e. $(-3, \frac{5}{2})$

Answer

The point a (-4, 1) is in Quadrant II.
The point b (-2, 3) is in Quadrant II.
The point c (2, -5) is in Quadrant IV.
The point d (-2, 5) is in Quadrant II.
The point e $(-3, \frac{5}{2})$ is in Quadrant II.



How do the signs affect the location of the points? You may have noticed some patterns as you graphed the points in the previous example.

For the point in Figure 1.11.4 in Quadrant IV, what do you notice about the signs of the coordinates? What about the signs of the coordinates of points in the third quadrant? The second quadrant? The first quadrant?

Can you tell just by looking at the coordinates in which quadrant the point (−2,5) is located? In which quadrant is (2,−5) located?

QUADRANTS

We can summarize sign patterns of the quadrants in this way.

Quadrant I	Quadrant II	Quadrant III	Quadrant IV	
(x, y)	(x, y)	(x, y)	(x, y)	(1.11.1)
$(+, +)$	$(-, +)$	$(-, -)$	$(+, -)$	

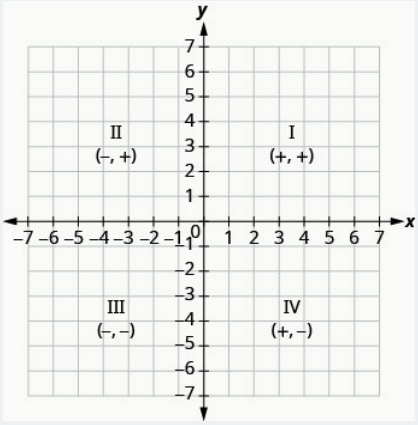


Figure 1.11.5

What if one coordinate is zero as shown in Figure 1.11.6? Where is the point (0,4) located? Where is the point (−2,0) located?

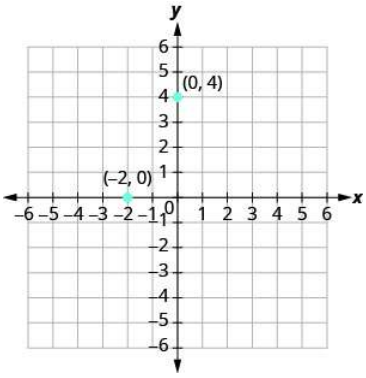


Figure 1.11.6

The point (0,4) is on the *y*-axis and the point (−2,0) is on the *x*-axis.

POINTS ON THE AXES

Points with a ***y*-coordinate** equal to 0 are on the *x*-axis, and have coordinates (a,0).

Points with an ***x*-coordinate** equal to 0 are on the *y*-axis, and have coordinates (0,b).

Example 1.11.4

Plot each point:

1. (0,5)
2. (4,0)
3. (−3,0)
4. (0,0)
5. (0,−1)

Solution

1. Since x=0, the point whose coordinates are (0,5) is on the *y*-axis.
2. Since y=0, the point whose coordinates are (4,0) is on the *x*-axis.
3. Since y=0, the point whose coordinates are (−3,0) is on the *x*-axis.
4. Since x=0 and y=0, the point whose coordinates are (0,0) is the origin.
5. Since x=0, the point whose coordinates are (0,−1) is on the *y*-axis.

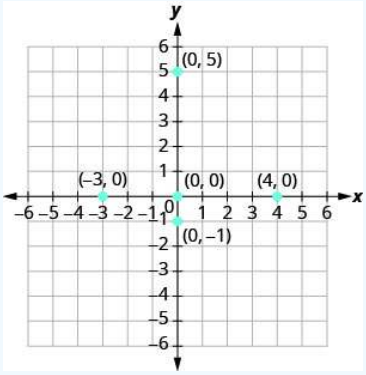


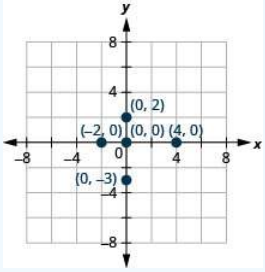
Figure 1.11.7

Try It 1.11.5

Plot each point:

1. (4,0)
2. (−2,0)
3. (0,0)
4. (0,2)
5. (0,−3).

Answer



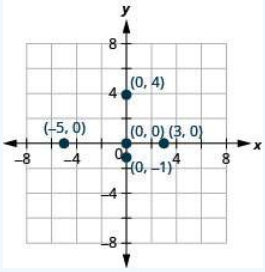
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Try It 1.11.6

Plot each point:

1. (−5,0)
2. (3,0)
3. (0,0)
4. (0,−1)
5. (0,4).

Answer



In algebra, being able to identify the coordinates of a point shown on a graph is just as important as being able to plot points. To identify the *x*-coordinate of a point on a graph, read the number on the *x*-axis directly above or below the point. To identify the *y*-coordinate of a point, read the number on the *y*-axis directly to the left or right of the point. Remember, when you write the **ordered pair** use the correct order, (x,y).

Example 1.11.7

Name the ordered pair of each point shown in the rectangular coordinate system.

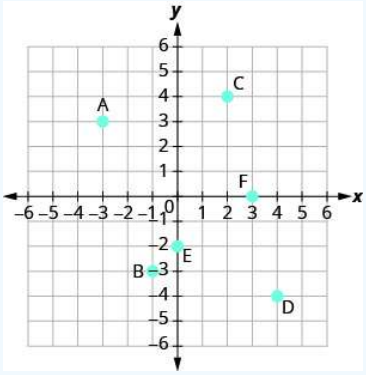


Figure 1.11.8

Solution

Point A is above −3 on the *x*-axis, so the *x*-coordinate of the point is −3.

The point is to the left of 3 on the *y*-axis, so the *y*-coordinate of the point is 3.

The coordinates of the point are (−3,3).

Point B is below −1 on the *x*-axis, so the *x*-coordinate of the point is −1.

The point is to the left of −3 on the *y*-axis, so the *y*-coordinate of the point is −3.

The coordinates of the point are (−1,−3).

Point C is above 2 on the *x*-axis, so the *x*-coordinate of the point is 2.

The point is to the right of 4 on the *y*-axis, so the *y*-coordinate of the point is 4.

The coordinates of the point are (2,4).

Point D is below 4 on the *x*-axis, so the *x*-coordinate of the point is 4.

The point is to the right of −4 on the *y*-axis, so the *y*-coordinate of the point is −4.

The coordinates of the point are (4,−4).

Point E is on the *y*-axis at y=−2. The coordinates of point E are (0,−2).

Point F is on the *x*-axis at x=3. The coordinates of point F are (3,0).

Try It 1.11.8

Name the ordered pair of each point shown in the rectangular coordinate system.

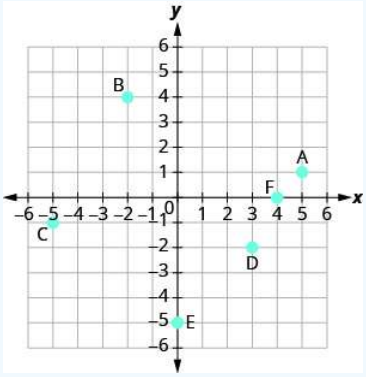


Figure 1.11.9

Answer

A: (5,1) B: (−2,4) C: (−5,−1) D: (3,−2) E: (0,−5) F: (4,0)

Try It 1.11.9

Name the ordered pair of each point shown in the rectangular coordinate system.

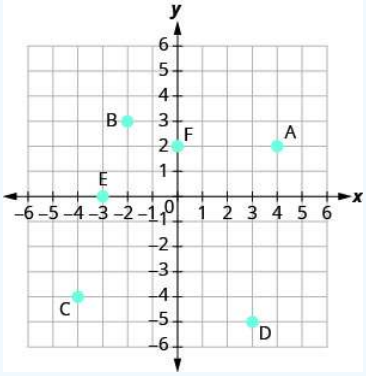


Figure 1.11.10

Answer

A: (4,2) B: (−2,3) C: (−4,−4) D: (3,−5) E: (−3,0) F: (0,2)

Verify Solutions to an Equation in Two Variables

Up to now, all the equations you have solved were equations with just one variable. In almost every case, when you solved the equation you got exactly one solution. The process of solving an equation ended with a statement like x=4. (Then, you checked the solution by substituting back into the equation.)

Here's an example of an equation in one variable, and its one solution.

$$\begin{aligned} 3x+5 &= 17 \\ 3x &= 12 \\ x &= 4 \end{aligned}$$

But equations can have more than one variable. Equations with two variables may be of the form Ax+By=C. Equations of this form are called **linear equations in two variables**.

LINEAR EQUATION

An equation of the form Ax+By=C, where A and B are not both zero, is called a **linear equation in two variables**.

Notice the word *line* in **linear**. Here is an example of a linear equation in two variables, x and y.

$$\begin{aligned} Ax+By&=C \\ x+4y&=8 \\ A=1,\ B&=4,\ C=8 \end{aligned}$$

Figure 1.11.11

The equation y=−3x+5 is also a **linear equation**. But it does not appear to be in the form Ax+By=C. We can use the Addition Property of Equality and rewrite it in Ax+By=C form.

	y	$=$	$-3x+5$
Add to both sides.	$y+3x$	$=$	$-3x+5+3x$
Simplify.	$y+3x$	$=$	5
Use the Commutative Property to put it in $Ax+By=C$ form.	$3x+y$	$=$	5

By rewriting y=−3x+5 as 3x+y=5, we can easily see that it is a linear equation in two variables because it is of the form Ax+By=C. When an equation is in the form Ax+By=C, we say it is in *standard form*.

STANDARD FORM OF LINEAR EQUATION

A linear equation is in standard form when it is written Ax+By=C.

Most people prefer to have A, B, and C be integers and $A \geq 0$ when writing a linear equation in standard form, although it is not strictly necessary.

Linear equations have infinitely many solutions. For every number that is substituted for x there is a corresponding y value. This pair of values is a *solution* to the linear equation and is represented by the ordered pair (x,y). When we substitute these values of x and y into the equation, the result is a true statement, because the value on the left side is equal to the value on the right side.

Solution OF A LINEAR EQUATION IN TWO VARIABLES

An **ordered pair** (x,y) is a **solution** of the linear equation Ax+By=C, if the equation is a true statement when the *x*- and *y*-values of the ordered pair are substituted into the equation.

Example 1.11.10

Determine which ordered pairs are solutions to the equation x+4y=8.

(a) (0,2)

(b) (2,−4)

(c) (−4,3)

Solution

Substitute the *x*- and *y*-values from each ordered pair into the equation and determine if the result is a true statement.

(a)	(b)	(c)
(0, 2)	(2, –4)	(–4, 3)
$x = 0, y = 2$	$x = 2, y = -4$	$x = -4, y = 3$
$x + 4y = 8$	$x + 4y = 8$	$x + 4y = 8$
$0 + 4 \cdot 2 \stackrel{?}{=} 8$	$2 + 4(-4) \stackrel{?}{=} 8$	$-4 + 4 \cdot 3 \stackrel{?}{=} 8$
$0 + 8 \stackrel{?}{=} 8$	$2 + (-16) \stackrel{?}{=} 8$	$-4 + 12 \stackrel{?}{=} 8$
$8 = 8$ ✓	$-14 \neq 8$	$8 = 8$ ✓
(0, 2) is a solution.	(2, –4) is not a solution.	(–4, 3) is a solution.

Try It 1.11.11

Which of the following ordered pairs are solutions to 2x+3y=6?

1. (3,0)
2. (2,0)
3. (6,−2)

Answer

1, 3

Try It 1.11.12

Which of the following ordered pairs are solutions to the equation 4x−y=8?

1. (0,8)
2. (2,0)
3. (1,−4)

Answer

2, 3

Example 1.11.13

Which of the following ordered pairs are solutions to the equation y=5x−1?

(a) (0,−1)

(b) (1,4)

(c) (−2,−7)

Solution

Substitute the *x*- and *y*-values from each **ordered pair** into the equation and determine if it results in a true statement.

(a)	(b)	(c)
(0, –1)	(1, 4)	(–2, –7)
$x = 0, y = -1$	$x = 1, y = 4$	$x = -2, y = -7$
$y = 5x - 1$	$y = 5x - 1$	$y = 5x - 1$
$-1 \stackrel{?}{=} 5(0) - 1$	$4 \stackrel{?}{=} 5(1) - 1$	$-7 \stackrel{?}{=} 5(-2) - 1$
$-1 \stackrel{?}{=} 0 - 1$	$4 \stackrel{?}{=} 5 - 1$	$-7 \stackrel{?}{=} -10 - 1$
$-1 = -1$ ✓	$4 = 4$ ✓	$-7 \neq -11$
(0, –1) is a solution.	(1, 4) is a solution.	(–2, –7) is not a solution.

Try It 1.11.14

Which of the following ordered pairs are solutions to the equation y=4x−3?

1. (0,3)
2. (1,1)
3. (−1,−1)

Answer

2

Try It 1.11.15

Which of the following ordered pairs are solutions to the equation y=−2x+6?

1. (0,6)
2. (1,4)
3. (−2,−2)

Answer

1, 2

Complete a Table of Solutions to a Linear Equation in Two Variables

In the examples above, we substituted the *x*- and *y*-values of a given ordered pair to determine whether or not it was a solution to a linear equation. But how do you find the ordered pairs if they are not given? It's easier than you might think—you can just pick a value for x and then solve the equation for y. Or, pick a value for y and then solve for x.

We'll start by looking at the solutions to the equation y = 5x−1 that we found in Example 1.11.13 We can summarize this information in a table of solutions, as shown in Table 1.11.1.

Table 1.11.1

y=5x−1		
x	**y**	**(x,y)**
0	−1	(0,−1)
1	4	(1,4)

To find a third solution, we'll let x=2 and solve for y.

	$y = 5x - 1$
Substitute $x = 2$.	$y = 5(2) - 1$
Multiply.	$y = 10 - 1$
Simplify.	$y = 9$

Figure 1.11.12

The ordered pair (2,9) is a solution to y=5x−1. We will add it to Table 1.11.2

Table 1.11.2

y=5x−1		
x	**y**	**(x,y)**
0	−1	(0,−1)
1	4	(1,4)
2	9	(2,9)

We can find more solutions to the equation by substituting in any value of x or any value of y and solving the resulting equation to get another ordered pair that is a solution. There are infinitely many solutions of this equation.

Example 1.11.16

Complete the table to find three solutions to the equation $y=4x-2$.

Table 1.11.3

$y=4x-2$		
x	**y**	**(x,y)**
0		
−1		
2		

Solution

Substitute $x=0$, $x=-1$, and $x=2$ into $y=4x-2$.

$x=0$	$x=-1$	$x=2$
$y=4x-2$	$y=4x-2$	$y=4x-2$
$y=4\cdot 0-2$	$y=4(-1)-2$	$y=4\cdot 2-2$
$y=0-2$	$y=-4-2$	$y=8-2$
$y=-2$	$y=-6$	$y=6$
$(0,-2)$	$(-1,-6)$	$(2,6)$

The results are summarized in Table 1.11.4

Table 1.11.4

$y=4x-2$		
x	**y**	**(x,y)**
0	−2	(0,−2)
−1	−6	(−1,−6)
2	6	(2,6)

Try It 1.11.17

Complete the table to find three solutions to this equation: $y=3x-1$.

Table 1.11.5

$y=3x-1$		
x	**y**	**(x,y)**
0		
−1		
2		

Answer

Table 1.11.6

$y=3x-1$

x	y	(x,y)
0	-1	(0, -1)
−1	-4	(-1, -4)
2	5	(2, 5)

Try It 1.11.18

Complete the table to find three solutions to this equation: y=6x+1.

Table 1.11.7

y=6x+1		
x	**y**	**(x,y)**
-2		

Answer

Table 1.11.8

y=6x+1		
x	**y**	**(x,y)**
0	1	(0,1)
1	7	(1,7)
−2	−11	(−2,−11)

Example 1.11.19

Complete Table 1.11.9 to find three solutions to the equation 5x−4y=20.

Table 1.11.9

5x−4y=20		
x	**y**	**(x,y)**
	0	
	5	

Solution

Substitute the given value into the equation 5x−4y=20 and solve for the other variable. Then, fill in the values in the table.

$x = 0$	$y = 0$	$y = 5$
$5x - 4y = 20$	$5x - 4y = 20$	$5x - 4y = 20$
$5 \cdot 0 - 4y = 20$	$5x - 4 \cdot 0 = 20$	$5x - 4 \cdot 5 = 20$
$0 - 4y = 20$	$5x - 0 = 20$	$5x - 20 = 20$
$-4y = 20$	$5x = 20$	$5x = 40$
$y = -5$	$x = 4$	$x = 8$
$(0, -5)$	$(4, 0)$	$(8, 5)$

The results are summarized in Table 1.11.10

Table 1.11.10

5x−4y=20		
x	**y**	**(x,y)**
0	−5	(0,−5)
4	0	(4,0)
8	5	(8,5)

Try It 1.11.20

Complete the table to find three solutions to this equation: 2x−5y=20.

Table 1.11.11

2x−5y=20		
x	**y**	**(x,y)**
-5		

Answer

Table 1.11.12

2x−5y=20		
x	**y**	**(x,y)**
0	−4	(0,−4)
10	0	(10,0)
−5	−6	(−5,−6)

Try It 1.11.21

Complete the table to find three solutions to this equation: 3x−4y=12.

Table 1.11.13

3x−4y=12		
x	**y**	**(x,y)**

-4		

Answer

Table 1.11.14

3x−4y=12		
x	**y**	**(x,y)**
0	−3	(0,−3)
4	0	(4,0)
−4	−6	(−4,−6)

Find Solutions to a Linear Equation

To find a solution to a linear equation, you really can pick *any* number you want to substitute into the equation for x or y. But since you'll need to use that number to solve for the other variable it's a good idea to choose a number that's easy to work with.

When the equation is in *y*-form, with the *y* by itself on one side of the equation, it is usually easier to choose values of x and then solve for y.

Example 1.11.22

Find three solutions to the equation y=−3x+2.

Solution

We can substitute any value we want for x or any value for y. Since the equation is in *y*-form, it will be easier to substitute in values of x. Let's pick x=0, x=1, and x=−1.

	$x=0$	$x=1$	$x=-1$
	$y=-3x+2$	$y=-3x+2$	$y=-3x+2$
Substitute the value into the equation.	$y=-3\cdot 0+2$	$y=-3\cdot 1+2$	$y=-3(-1)+2$
Simplify.	$y=0+2$	$y=-3+2$	$y=3+2$
Simplify.	$y=2$	$y=-1$	$y=5$
Write the ordered pair.	(0, 2)	(1, -1)	(-1, 5)
Check.			
y=−3x+2	y=−3x+2	y=−3x+2	
$2\stackrel{?}{=}-3\cdot 0+2$	$-1\stackrel{?}{=}-3\cdot 1+2$	$5\stackrel{?}{=}-3(-1)+2$	
$2\stackrel{?}{=}0+2$	$-1\stackrel{?}{=}-3+2$	$5\stackrel{?}{=}-3+2$	
$2=2$✓	$-1=-1$✓	$5=5$✓	

Table 1.11.15

So, (0,2), (1,−1) and (−1,5) are all solutions to y=−3x+2. We show them in Table 1.11.16

Table 1.11.16

y=−3x+2

x	y	(x,y)
0	2	(0,2)
1	−1	(1,−1)
−1	5	(−1,5)

Try It 1.11.23

Find three solutions to this equation: y=−2x+3.

Answer

Answers will vary. Answers might include (0,3), (1,1) and (−1,5).

Try It 1.11.24

Find three solutions to this equation: y=−4x+1.

Answer

Answers will vary. Answers might include (0,1), (1,−3) and (−1,5).

We have seen how using zero as one value of x makes finding the value of y easy. When an equation is in standard form, with both the x and y on the same side of the equation, it is usually easier to first find one solution when x=0 find a second solution when y=0, and then find a third solution.

Example 1.11.25

Find three solutions to the equation 3x+2y=6.

Solution

We can substitute any value we want for x or any value for y. Since the equation is in standard form, let's pick first x=0, then y=0, and then find a third point.

Table 1.11.17

$x=0$	$y=0$	$x=1$			
			$3x+2y=6$	$3x+2y=6$	$3x+2y=6$
Substitute the value into the equation.			$3(0)+2y=6$	$3x+2(0)=6$	$3(1)+2y=6$
Simplify.			$0+2y=6$	$3x+0=6$	$3+2y=6$
Solve.			$2y=6$	$3x=6$	$2y=3$
			$y=3$	$x=2$	$y=\frac{3}{2}$
Write the ordered pair.			(0, 3)	(2, 0)	$(1, \frac{3}{2})$
Check.					
3x+2y=6	3x+2y=6	3x+2y=6			
$3\cdot 0+2\cdot 3\stackrel{?}{=}6$	$3\cdot 2+2\cdot 0\stackrel{?}{=}6$	$3\cdot 1+2\cdot\frac{3}{2}\stackrel{?}{=}6$			
$0+6\stackrel{?}{=}6$	$6+0\stackrel{?}{=}6$	$3+3\stackrel{?}{=}6$			
$6=6$✓	$6=6$✓	$6=6$✓			

So (0,3), (2,0), and $(1, \frac{3}{2})$ are all solutions to the equation 3x+2y=6. We can list these three solutions in Table 1.11.18

Table 1.11.18

3x+2y=6		
x	**y**	**(x,y)**
0	3	(0,3)
2	0	(2,0)
1	$\frac{3}{2}$	$(1, \frac{3}{2})$

? Try It 1.11.26

Find three solutions to the equation 2x+3y=6.

Answer

Answers will vary. Answers might include (0,2), (3,0) and (6, −2).

? Try It 1.11.27

Find three solutions to the equation 4x+2y=8.

Answer

Answers will vary. Answers might include (0,4), (2,0) and (−1,6).

Key Concepts

- **Sign Patterns of the Quadrants**

Quadrant I	Quadrant II	Quadrant III	Quadrant IV
(x, y)	(x, y)	(x, y)	(x, y)
$(+, +)$	$(-, +)$	$(-, -)$	$(+, -)$

- **Points on the Axes**
  - On the *x*-axis, y=0. Points with a *y*-coordinate equal to 0 are on the *x*-axis, and have coordinates (a,0).
  - On the *y*-axis, x=0. Points with an *x*-coordinate equal to 0 are on the *y*-axis, and have coordinates (0,b).
- **Solution of a Linear Equation**
  - An ordered pair (x,y) is a solution of the linear equation Ax+By=C, if the equation is a true statement when the *x*- and *y*-values of the ordered pair are substituted into the equation.

Glossary

linear equation

A linear equation is of the form Ax+By=C, where A and B are not both zero, is called a linear equation in two variables.

ordered pair

An ordered pair (x,y) gives the coordinates of a point in a rectangular coordinate system.

origin

The point (0,0) is called the origin. It is the point where the *x*-axis and *y*-axis intersect.

quadrant

The *x*-axis and the *y*-axis divide a plane into four regions, called quadrants.

rectangular coordinate system

A grid system is used in algebra to show a relationship between two variables; also called the *xy*-plane or the ‘coordinate plane’.

***x*-coordinate**

The first number in an ordered pair (x,y).

***y*-coordinate**

The second number in an ordered pair (x,y).

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1.11E: Exercises

Practice Makes Perfect

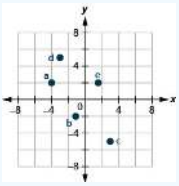
Plot Points in a Rectangular Coordinate System

In the following exercises, plot each point in a rectangular coordinate system and identify the quadrant in which the point is located.

Exercise 1.11E.1

1. (−4,2)
2. (−1,−2)
3. (3,−5)
4. (−3,5)
5. $(\frac{5}{3}, 2)$

Answer



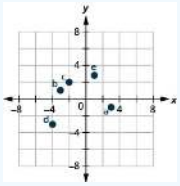
Exercise 1.11E.2

1. (−2,−3)
2. (3,−3)
3. (−4,1)
4. (4,−1)
5. $(\frac{3}{2}, 1)$

Exercise 1.11E.3

1. (3,−1)
2. (−3,1)
3. (−2,2)
4. (−4,−3)
5. $\left(1, \frac{14}{5}\right)$

Answer



Exercise 1.11E.4

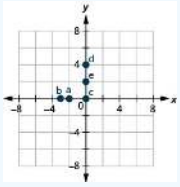
1. (−1,1)
2. (−2,−1)
3. (2,1)
4. (1,−4)
5. $\left(3, \frac{7}{2}\right)$

In the following exercises, plot each point in a rectangular coordinate system.

Exercise 1.11E.5

1. (−2,0)
2. (−3,0)
3. (0,0)
4. (0,4)
5. (0,2)

Answer



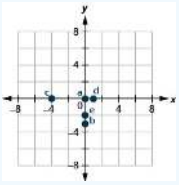
Exercise 1.11E.6

1. (0,1)
2. (0,−4)
3. (−1,0)
4. (0,0)
5. (5,0)

Exercise 1.11E.7

1. (0,0)
2. (0,−3)
3. (−4,0)
4. (1,0)
5. (0,−2)

Answer

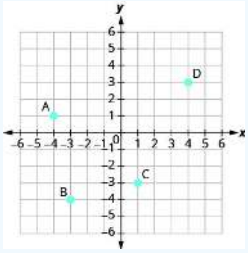


Exercise 1.11E.8

1. (−3,0)
2. (0,5)
3. (0,−2)
4. (2,0)
5. (0,0)

In the following exercises, name the ordered pair of each point shown in the rectangular coordinate system.

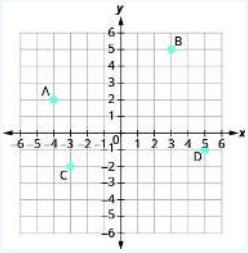
Exercise 1.11E.9



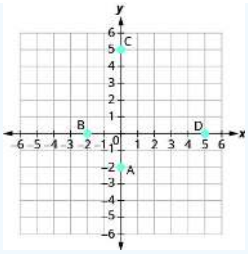
Answer

$A:(-4,1)\quad B:(-3,-4)\quad C:(1,-3)\quad D:(4,3)$

Exercise 1.11E.10



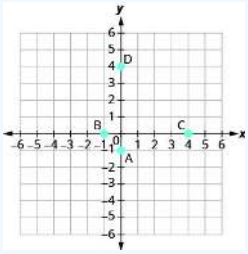
Exercise 1.11E.11



Answer

$A:(0,-2)\quad B:(-2,0)\quad C:(0,5)\quad D:(5,0)$

Exercise 1.11E.12



Verify Solutions to an Equation in Two Variables

In the following exercises, which ordered pairs are solutions to the given equations?

Exercise 1.11E.13

2x+y=6

1. (1,4)
2. (3,0)
3. (2,3)

Answer

1, 2

Exercise 1.11E.14

1. x+3y=9
2. (0,3)
3. (6,1)
4. (−3,−3)

Exercise 1.11E.15

4x−2y=8

1. (3,2)
2. (1,4)
3. (0,−4)

Answer

1, 3

Exercise 1.11E.16

3x−2y=12

1. (4,0)
2. (2,−3)
3. (1,6)

Exercise 1.11E.17

y=4x+3

1. (4,3)
2. (−1,−1)
3. $\left(\frac{1}{2},5\right)$

Answer

2, 3

Exercise 1.11E.18

y=2x−5

1. (0,−5)
2. (2,1)
3. $\left(\frac{1}{2}, -4\right)$

Exercise 1.11E.19

$y = \frac{1}{2}x - 1$

1. (2,0)
2. (−6,−4)
3. (−4,−1)

Answer

1, 2

Exercise 1.11E.20

$y = \frac{1}{3}x + 1$

1. (−3,0)
2. (9,4)
3. (−6,−1)

Complete a Table of Solutions to a Linear Equation

In the following exercises, complete the table to find solutions to each linear equation.

Exercise 1.11E.21

$y = 2x - 4$

x	y	(x,y)
0		
2		
-1		

Answer

x	y	(x,y)
0	−4	(0,−4)
2	0	(2,0)
−1	−6	(−1,−6)

Exercise 1.11E.22

$y = 3x - 1$

x	y	(x,y)
0		
2		
-1		

Exercise 1.11E.23

$y = -x + 5$

x	y	(x,y)
0		
3		
-2		

Answer

x	y	(x,y)
0	5	(0,5)
3	2	(3,2)
−2	7	(−2,7)

Exercise 1.11E.24

$y = -x + 2$

x	y	(x,y)
0		
3		
-2		

Exercise 1.11E.25

$y = \frac{1}{3}x + 1$

x	y	(x,y)
0		
3		
6		

Answer

x	y	(x,y)
0	1	(0,1)
3	2	(3,2)
6	3	(6,3)

Exercise 1.11E.26

$y = \frac{1}{2}x + 4$

x	y	(x,
0		
2		
4		

Exercise 1.11E.27

$y = -\frac{3}{2}x - 2$

x	y	(x,y)
0		
2		
-2		

Answer

x	y	(x,y)
0	−2	(0,−2)
2	−5	(2,−5)
−2	1	(−2,1)

Exercise 1.11E.28

$y = -\frac{2}{3}x - 1$

x	y	(x,y)
0		
3		
-3		

Exercise 1.11E.29

$x + 3y = 6$

x	y	(x,y)
0		
3		

	0	

Answer

x	y	(x,y)
0	2	(0,2)
3	4	(3,1)
6	0	(6,0)

Exercise 1.11E.30

x+2y=8

x	y	(x,y)
0		
4		
	0	

Exercise 1.11E.31

$2x - 5y = 10$

x	y	(x,y)
0		
10		
	0	

Answer

x	y	(x,y)
0	−2	(0,−2)
10	2	(10,2)
5	0	(5,0)

Exercise 1.11E.32

x	y	(x,y)
0		
8		
	0	

Find Solutions to a Linear Equation

In the following exercises, find three solutions to each linear equation.

Exercise 1.11E.33

$y = 5x - 8$

Answer

Answers will vary.

Exercise 1.11E.34

$y = 3x - 9$

Exercise 1.11E.35

$y = -4x + 5$

Answer

Answers will vary.

Exercise 1.11E.36

$y = -2x + 7$

Exercise 1.11E.37

$x+y=8$

Answer

Answers will vary.

Exercise 1.11E.38

$x+y=6$

Exercise 1.11E.39

$x+y=-2$

Answer

Answers will vary.

Exercise 1.11E.40

$x+y=-1$

Exercise 1.11E.41

$3x+y=5$

Answer

Answers will vary.

Exercise 1.11E.42

$2x+y=3$

Exercise 1.11E.43

$4x-y=8$

Answer

Answers will vary.

Exercise 1.11E.44

$5x-y=10$

Exercise 1.11E.45

$2x+4y=8$

Answer

Answers will vary.

Exercise 1.11E.46

$3x+2y=6$

Exercise 1.11E.47

$5x-2y=10$

Answer

Answers will vary.

Exercise 1.11E.48

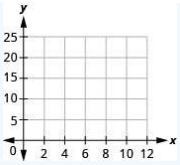
$4x-3y=12$

Everyday Math

Exercise 1.11E.49

Weight of a baby. Mackenzie recorded her baby's weight every two months. The baby's age, in months, and weight, in pounds, are listed in the table below, and shown as an ordered pair in the third column.

1. Plot the points on a coordinate plane.

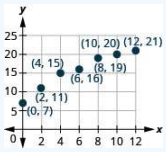


2. Why is only Quadrant I needed?

Age x	Weight y	(x,y)
0	7	(0, 7)
2	11	(2, 11)
4	15	(4, 15)
6	16	(6, 16)
8	19	(8, 19)
10	20	(10, 20)
12	21	(12, 21)

Answer

1.

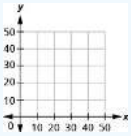


2. Age and weight are only positive.

Exercise 1.11E.50

Weight of a child. Latresha recorded her son's height and weight every year. His height, in inches, and weight, in pounds, are listed in the table below, and shown as an ordered pair in the third column.

1. Plot the points on a coordinate plane.



2. Why is only Quadrant I needed?

Height x	Weight y	(x,y)
28	22	(28, 22)
31	27	(31, 27)
33	33	(33, 33)
37	35	(37, 35)
40	41	(40, 41)
42	45	(42, 45)

Writing Exercises

Exercise 1.11E.51

Explain in words how you plot the point (4,−2) in a rectangular coordinate system?

Answer

Answers will vary.

Exercise 1.11E.52

How do you determine if an ordered pair is a solution to a given equation?

Exercise 1.11E.53

Is the point (−3,0) on the *x*-axis or *y*-axis? How do you know?

Answer

Answers will vary.

? Exercise 1.11E.54

Is the point (0,8) on the *x*-axis or *y*-axis? How do you know?

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
plot points in a rectangular coordinate system.			
identify points on a graph.			
verify solutions to an equation in two variables.			
complete a table of solutions to a linear equation.			
find solutions to a linear equation.			

ⓑ If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them. What did you do to become confident of your ability to do these things? Be specific.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math every topic builds upon previous work. It is important to make sure you have a strong foundation before you move on. Who can you ask for help? Your fellow classmates and instructor are good resources. Is there a place on campus where math tutors are available? Can your study skills be improved?

…no, I don't get it. This is a warning sign and you must not ignore it. You should get help right away or you will quickly be overwhelmed. See your instructor as soon as you can to discuss your situation. Together you can come up with a plan to get you the help you need.

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1.12: Graph Linear Equations in Two Variables

Learning Objectives

By the end of this section, you will be able to:

- Recognize the relationship between the solutions of an equation and its graph.
- Graph a linear equation by plotting points.
- Graph vertical and horizontal lines.

Note

Before you get started, take this readiness quiz.

1. Evaluate $3x+2$ when $x=-1$.
   If you missed this problem, review Exercise 1.5.34.
2. Solve $3x+2y=12$ for y in general.
   If you missed this problem, review Exercise 2.6.16.

Recognize the Relationship Between the Solutions of an Equation and its Graph

In the previous section, we found several solutions to the equation $3x+2y=6$. They are listed in Table 1.12.1. So, the ordered pairs (0,3), (2,0), and $(1, \frac{3}{2})$ are some solutions to the equation $3x+2y=6$. We can plot these solutions in the rectangular coordinate system as shown in Figure 1.12.1.

Table 1.12.1

3x+2y=6		
x	**y**	**(x,y)**
0	3	(0,3)
2	0	(2,0)
1	$\frac{3}{2}$	$(1, \frac{3}{2})$

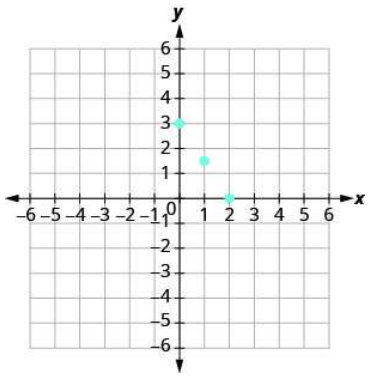


Figure 1.12.1

Notice how the points line up perfectly? We connect the points with a line to get the graph of the equation 3x+2y=6. See Figure 1.12.2 Notice the arrows on the ends of each side of the line. These arrows indicate the line continues.

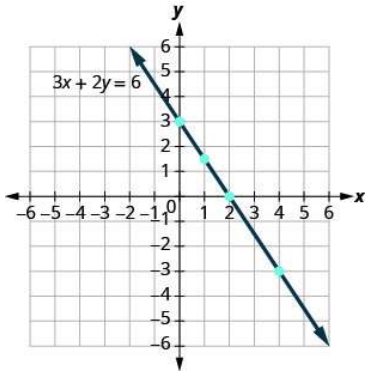


Figure 1.12.2

Every point on the line is a solution of the equation. Also, every solution of this equation is a point on this line. Points *not* on the line are not solutions.

Notice that the point whose coordinates are (−2,6) is on the line shown in Figure 1.12.3. If you substitute x=−2 and y=6 into the equation, you find that it is a solution to the equation.

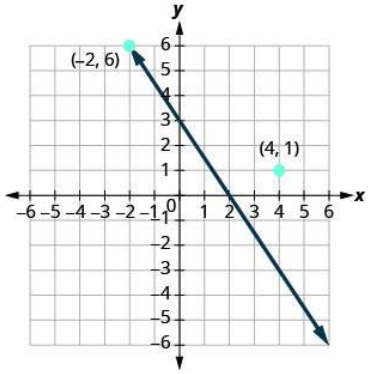


Figure 1.12.3

Test (−2, 6)

$$3x + 2y = 6$$

$$3(-2) + 2(6) = 6$$

$$-6 + 12 = 6$$

$$6 = 6 \checkmark$$

So the point (−2,6) is a solution to the equation $3x + 2y = 6$. (The phrase "the point whose coordinates are (−2,6)" is often shortened to "the point (−2,6).")

What about (4, 1) ?

$$3x + 2y = 6$$

$$3 \cdot 4 + 2 \cdot 1 = 6$$

$$12 + 2 \stackrel{?}{=} 6$$

$$14 \neq 6$$

Figure 1.12.3. This is an example of the saying, "A picture is worth a thousand words." The line shows you *all* the solutions to the equation. Every point on the line is a solution of the equation. And, every solution of this equation is on this line. This line is called the *graph* of the equation $3x + 2y = 6$.

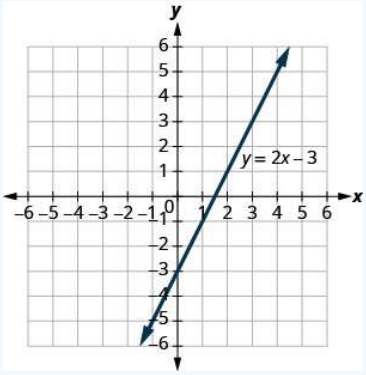
GRAPH OF A LINEAR EQUATION

The **graph of a linear equation** Ax+By=C is a line.

- Every point on the line is a solution of the equation.
- Every solution of this equation is a point on this line.

Example 1.12.1

The graph of y=2x−3 is shown.



For each ordered pair, decide:

1. Is the ordered pair a solution to the equation?
2. Is the point on the line?

A (0,−3) B (3,3) C (2,−3) D (−1,−5)

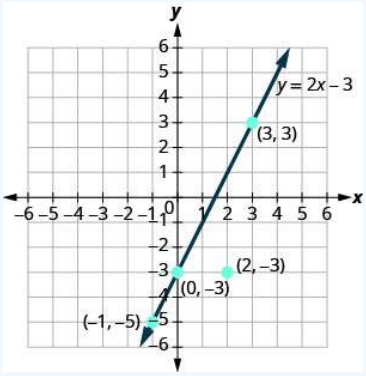
Solution

Substitute the *x*- and *y*- values into the equation to check if the ordered pair is a solution to the equation.

1.

A: (0, –3)	B: (3, 3)	C: (2, –3)	D: (–1, –5)
$y = 2x - 3$	$y = 2x - 3$	$y = 2x - 3$	$y = 2x - 3$
$-3 \stackrel{?}{=} 2(0) - 3$	$3 \stackrel{?}{=} 2(3) - 3$	$-3 \stackrel{?}{=} 2(2) - 3$	$-5 \stackrel{?}{=} 2(-1) - 3$
$-3 = -3$ ✓	$3 = 3$ ✓	$-3 \neq 1$	$-5 = -5$ ✓
(0, –3) is a solution.	(3, 3) is a solution.	(2, –3) is not a solution.	(–1, –5) is a solution.

2. Plot the points A (0,−3), B (3,3), C (2,−3), and D (−1,−5).



The points that are solutions to y=2x−3 are on the line, but the point that is not a solution is not on the line.

The points (0,−3), (3,3), and (−1,−5) are on the line y=2x−3, and the point (2,−3) is not on the line.

Try It 1.12.2

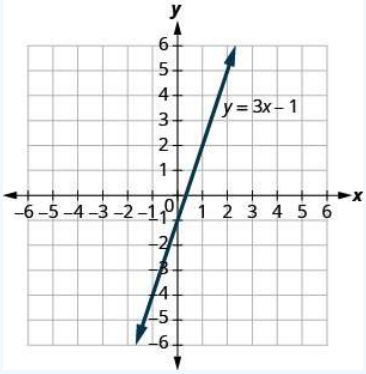
Use the graph of y=3x−1 to decide whether each ordered pair is:

- a solution to the equation.

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- on the line.

1. (0,−1)
2. (2,5)



Answer

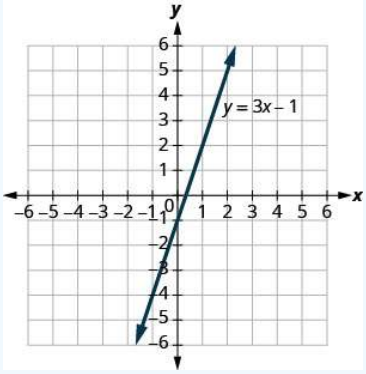
1. yes, yes
2. yes, yes

? Try It 1.12.3

Use graph of y=3x−1 to decide whether each ordered pair is:

- a solution to the equation
- on the line

1. (3,−1)
2. (−1,−4)



Answer

1. no, no
2. yes, yes

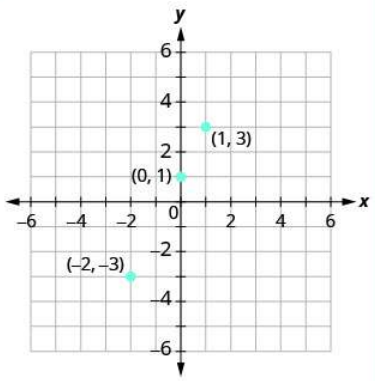
Graph a Linear Equation by Plotting Points

There are several methods that can be used to graph a linear equation. The method we used to graph 3x+2y=6 is called plotting points, or the Point–Plotting Method.

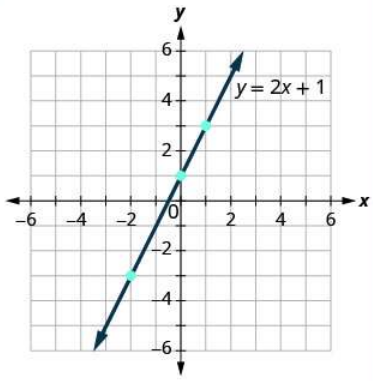
Example 1.12.4: How To Graph an Equation By Plotting Points

Graph the equation y=2x+1 by plotting points.

Solution

Step 1. Find three points whose coordinates are solutions to the equation.	You can choose any values for x or y. In this case, since y is isolated on the left side of the equation, it is easier to choose values for x.	$y = 2x + 1$ $x = 0$ $y = 2x + 1$ $y = 2 \cdot 0 + 1$ $y = 0 + 1$ $y = 1$ $x = 1$ $y = 2x + 1$ $y = 2 \cdot 1 + 1$ $y = 2 + 1$ $y = 3$ $x = -2$ $y = 2x + 1$ $y = 2(-2) + 1$ $y = -4 + 1$ $y = -3$
Organize the solutions in a table.	Put the three solutions in a table.	$y = 2x + 1$ x | y | (x, y) 0 | 1 | (0, 1) 1 | 3 | (1, 3) –2 | –3 | (–2, –3)
Step 2. Plot the points in a rectangular coordinate system. Check that the points line up. If they do not, carefully check your work!	Plot: (0, 1), (1, 3), (–2, –3). Do the points line up? Yes, the points line up.	

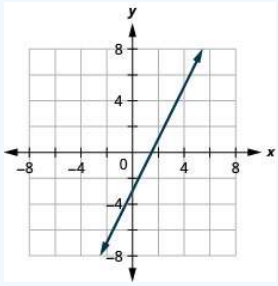
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Step 3. Draw the line through the three points. Extend the line to fill the grid and put arrows on both ends of the line.	This line is the graph of $y = 2x + 1$.	

Try It 1.12.5

Graph the equation by plotting points: y=2x−3.

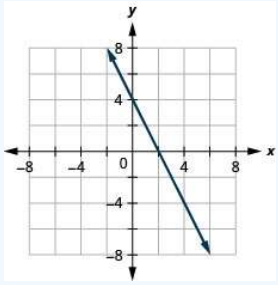
Answer



Try It 1.12.6

Graph the equation by plotting points: y=−2x+4.

Answer



The steps to take when graphing a linear equation by plotting points are summarized below.

GRAPH A LINEAR EQUATION BY PLOTTING POINTS

1. Find three points whose coordinates are solutions to the equation. Organize them in a table.
2. Plot the points in a rectangular coordinate system. Check that the points line up. If they do not, carefully check your work.
3. Draw the line through the three points. Extend the line to fill the grid and put arrows on both ends of the line.

It is true that it only takes two points to determine a line, but it is a good habit to use three points. If you only plot two points and one of them is incorrect, you can still draw a line but it will not represent the solutions to the equation. It will be the wrong line.

If you use three points, and one is incorrect, the points will not line up. This tells you something is wrong and you need to check your work. Look at the difference between part (a) and part (b) in Figure 1.12.4

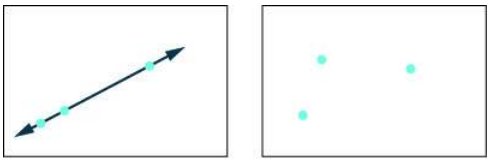


Figure 1.12.4

Let's do another example. This time, we'll show the last two steps all on one grid.

Example 1.12.7

Graph the equation y=−3x.

Solution

Find three points that are solutions to the equation. Here, again, it's easier to choose values for x. Do you see why?

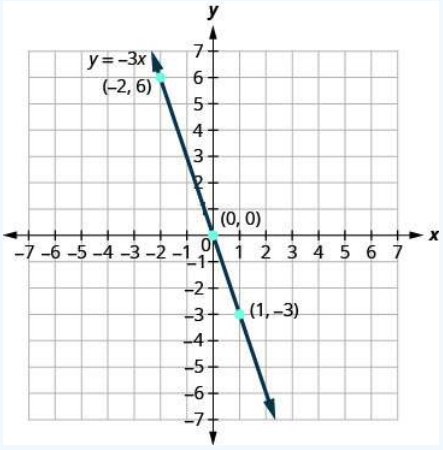
$x = 0$	$x = 1$	$x = -2$
$y = -3x$	$y = -3x$	$y = -3x$
$y = -3 \cdot 0$	$y = -3 \cdot 1$	$y = -3(-2)$
$y = 0$	$y = -3$	$y = 6$

We list the points in Table 1.12.2

Table 1.12.2

y=−3x		
x	**y**	**(x,y)**
0	0	(0,0)
1	−3	(1,−3)
−2	6	(−2,6)

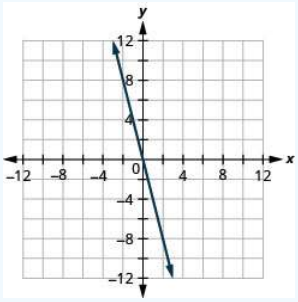
Plot the points, check that they line up, and draw the line.



Try It 1.12.8

Graph the equation by plotting points: y=−4x.

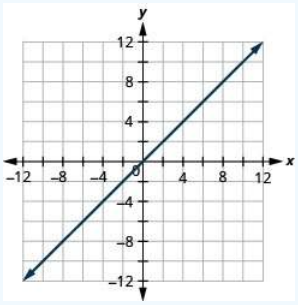
Answer



Try It 1.12.9

Graph the equation by plotting points: y=x.

Answer



When an equation includes a fraction as the coefficient of x, we can still substitute any numbers for x. But the math is easier if we make ‘good’ choices for the values of x. This way we will avoid fraction answers, which are hard to graph precisely.

Example 1.12.10

Graph the equation $y = \frac{1}{2}x + 3$.

Solution

Find three points that are solutions to the equation. Since this equation has the fraction $\frac{1}{2}$ as a coefficient of x, we will choose values of x carefully. We will use zero as one choice and multiples of 2 for the other choices. Why are multiples of 2 a good choice for values of x?

$x = 0$	$x = 2$	$x = 4$
$y = \frac{1}{2}x + 3$	$y = \frac{1}{2}x + 3$	$y = \frac{1}{2}x + 3$
$y = \frac{1}{2}(0) + 3$	$y = \frac{1}{2}(2) + 3$	$y = \frac{1}{2}(4) + 3$
$y = 0 + 3$	$y = 1 + 3$	$y = 2 + 3$
$y = 3$	$y = 4$	$y = 5$

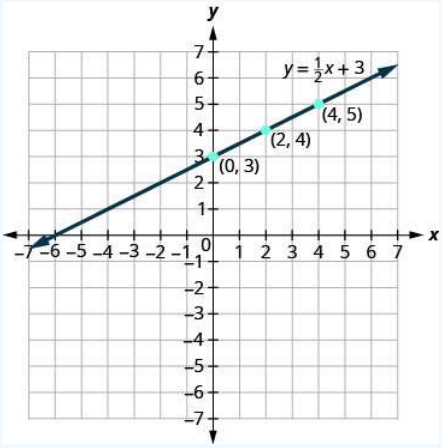
The points are shown in Table 1.12.3

Table 1.12.3

y=12x+3

x	y	(x,y)
0	3	(0,3)
2	4	(2,4)
4	5	(4,5)

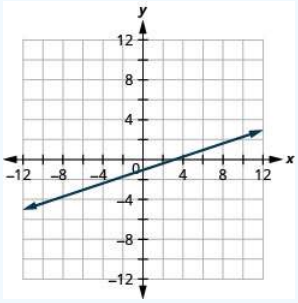
Plot the points, check that they line up, and draw the line.



Try It 1.12.11

Graph the equation $y = \frac{1}{3}x - 1$.

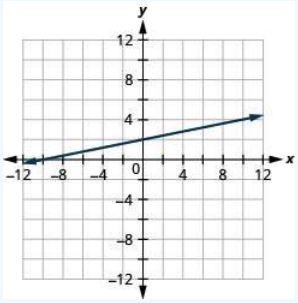
Answer



Try It 1.12.12

Graph the equation $y = \frac{1}{5}x + 2$.

Answer



So far, all the equations we graphed had y given in terms of x. Now we'll graph an equation with x and y on the same side. Let's see what happens in the equation 2x+y=3. If y=0 what is the value of x?

$$\begin{aligned} y &= 0 \\ 2x + y &= 3 \\ 2x + 0 &= 3 \\ 2x &= 3 \\ x &= \frac{3}{2} \\ &\left(\frac{3}{2}, 0\right) \end{aligned}$$

This point has a fraction for the *x*- coordinate and, while we could graph this point, it is hard to be precise graphing fractions. Remember in the example y=12x+3, we carefully chose values for x so as not to graph fractions at all. If we solve the equation 2x+y=3 for y, it will be easier to find three solutions to the equation.

$$\begin{aligned} 2x + y &= 3 \\ y &= -2x + 3 \end{aligned}$$

The solutions for x=0, x=1, and x=−1 are shown in the Table 1.12.4 The graph is shown in Figure 1.12.5

Table 1.12.4

2x+y=3		
x	**y**	**(x,y)**
0	3	(0,3)
1	1	(1,1)
−1	5	(−1,5)

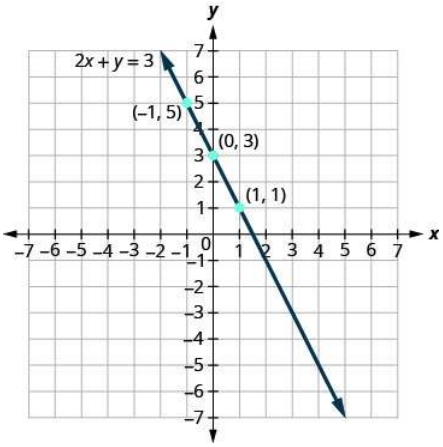


Figure 1.12.5

Can you locate the point $\left(\frac{3}{2}, 0\right)$ which we found by letting y=0, on the line?

Example 1.12.13

Graph the equation 3x+y=−1.

Solution

Find three points that are solutions to the equation.	$3x + y$	$=$	-1
First solve the equation for y.	y	$=$	$-3x - 1$

We'll let x be 0, 1, and −1 to find 3 points. The ordered pairs are shown in Table 1.12.5 Plot the points, check that they line up, and draw the line. See Figure 1.12.6

Table 1.12.5

3x+y=−1		
x	**y**	**(x,y)**
0	−1	(0,−1)
1	−4	(1,−4)
−1	2	(−1,2)

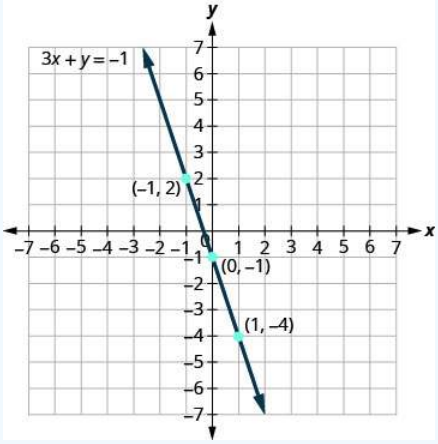
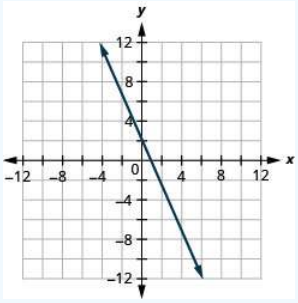


Figure 1.12.6

Try It 1.12.14

Graph the equation 2x+y=2.

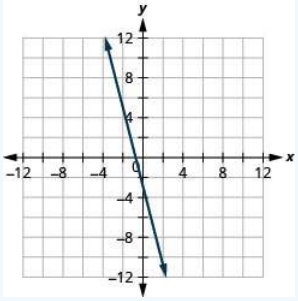
Answer



Try It 1.12.15

Graph the equation 4x+y=−3.

Answer



If you can choose any three points to graph a line, how will you know if your graph matches the one shown in the answers in the book? If the points where the graphs cross the *x*- and *y*-axis are the same, the graphs match!

The equation in Example 1.12.13was written in standard form, with both x and y on the same side. We solved that equation for y in just one step. But for other equations in standard form it is not that easy to solve for y, so we will leave them in standard form. We can still find a first point to plot by letting x=0 and solving for y. We can plot a second point by letting y=0 and then solving for x. Then we will plot a third point by using some other value for x or y.

Example 1.12.16

Graph the equation $2x-3y=6$.

Solution

Find three points that are solutions to the equation.	$2x-3y$	$=$	6
	$2x-3y$	$=$	6
First let $x=0$.	$2(0)-3y$	$=$	6
Solve for y.	$-3y$	$=$	6
	y	$=$	-2
Now let $y=0$.	$2x-3(0)$	$=$	6
Solve for x.	$2x$	$=$	6
	x	$=$	3
We need a third point. Remember, we can choose any value for x or y. We'll let $x=6$.	$2(6)-3y$	$=$	6
	$12-3y$	$=$	6
Solve for y.	$-3y$	$=$	-6
	y	$=$	2

We list the ordered pairs in Table 1.12.6. Plot the points, check that they line up, and draw the line. See Figure 1.12.7.

Table 1.12.6

2x−3y=6		
x	**yT**	**(x,y)**
0	−2	(0,−2)
3	0	(3,0)
6	2	(6,2)

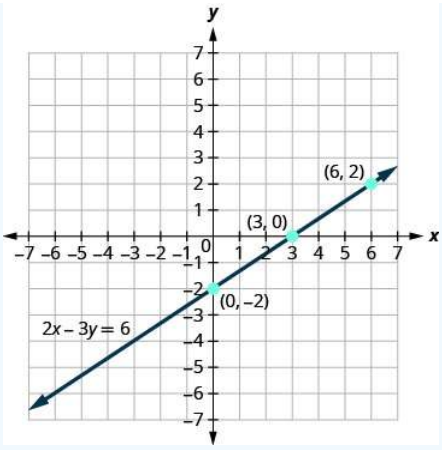
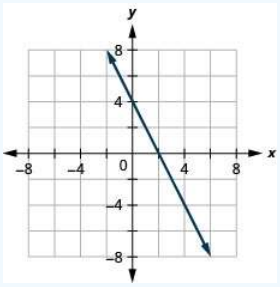


Figure 1.12.7

Try It 1.12.17

Graph the equation $4x + 2y = 8$.

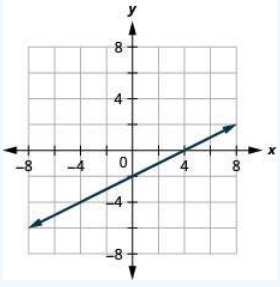
Answer



Try It 1.12.18

Graph the equation $2x - 4y = 8$.

Answer



Graph Vertical and Horizontal Lines

Can we graph an equation with only one variable? Just x and no y, or just y without an x? How will we make a table of values to get the points to plot?

Let's consider the equation x=−3. This equation has only one variable, x. The equation says that x is *always* equal to −3, so its value does not depend on y. No matter what y is, the value of x is always −3.

So to make a table of values, write −3 in for all the x values. Then choose any values for y. Since x does not depend on y, you can choose any numbers you like. But to fit the points on our coordinate graph, we'll use 1, 2, and 3 for the *y*-coordinates. See Table 1.12.7

Table 1.12.7

x=−3		
x	**y**	**(x,y)**
−3	1	(−3,1)
−3	2	(−3,2)
−3	3	(−3,3)

Plot the points from Table 1.12.7 and connect them with a straight line. Notice in Figure 1.12.8 that we have graphed a *vertical line*.

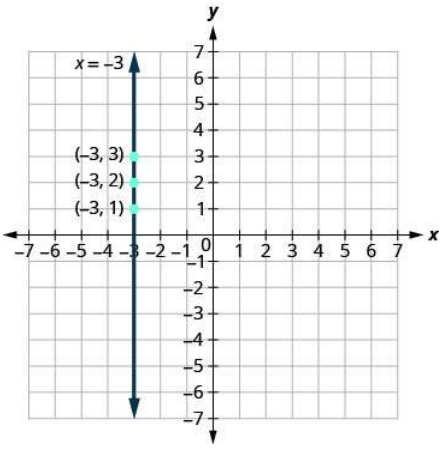


Figure 1.12.8

VERTICAL LINE

A **vertical line** is the graph of an equation of the form x=a.

The line passes through the *x*-axis at (a,0).

Example 1.12.19

Graph the equation x=2.

Solution

The equation has only one variable, x, and x is always equal to 2. We create Table 1.12.8 where x is always 2 and then put in any values for y. The graph is a vertical line passing through the *x*-axis at 2. See Figure 1.12.9

Table 1.12.8

x=2		
x	**y**	**(x,y)**
2	1	(2,1)
2	2	(2,2)
2	3	(2,3)

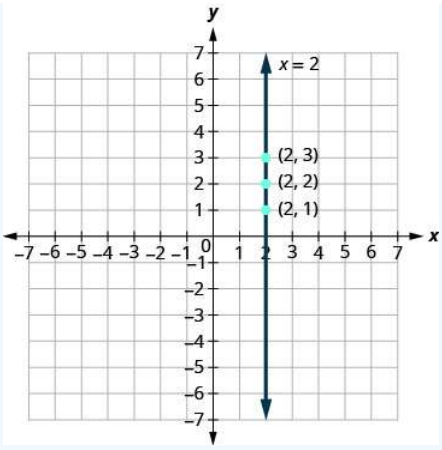
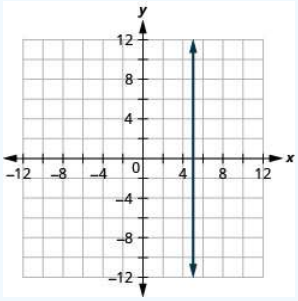


Figure 1.12.9

Try It 1.12.20

Graph the equation x=5.

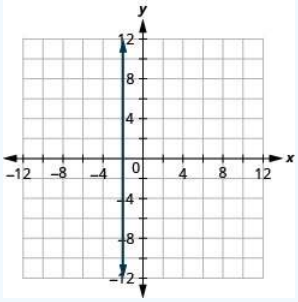
Answer



Try It 1.12.21

Graph the equation x=−2.

Answer



What if the equation has y but no x? Let's graph the equation y=4. This time the *y*- value is a constant, so in this equation, y does not depend on x. Fill in 4 for all the y's in Table 1.12.9 and then choose any values for x. We'll use 0, 2, and 4 for the *x*-coordinates.

Table 1.12.9

y=4		
x	**y**	**(x,y)**

0	4	(0,4)
2	4	(2,4)
4	4	(4,4)

The graph is a horizontal line passing through the *y*-axis at 4. See Figure 1.12.10

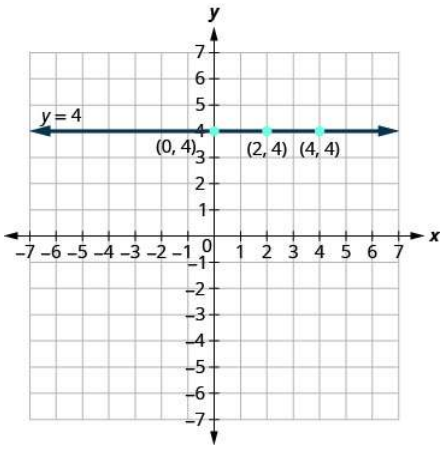


Figure 1.12.10

HORIZONTAL LINE

A **horizontal line** is the graph of an equation of the form y=b.

The line passes through the *y*-axis at (0,b).

Example 1.12.22

Graph the equation y=−1.

Solution

The equation y=−1 has only one variable, y. The value of y is constant. All the ordered pairs in Table 1.12.10have the same *y*-coordinate. The graph is a horizontal line passing through the *y*-axis at −1, as shown in Figure 1.12.11.

Table 1.12.10

y=−1		
x	**y**	**(x,y)**
	−1	(0,−1)
	−1	(3,−1)
−3	−1	(−3,−1)

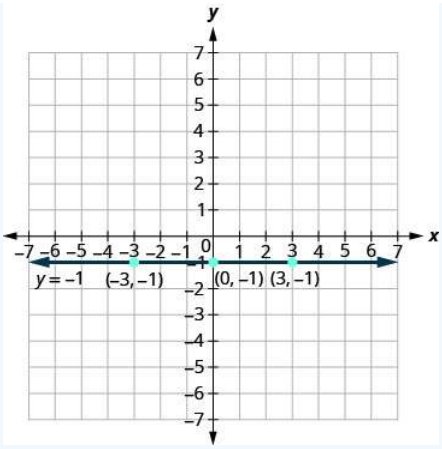
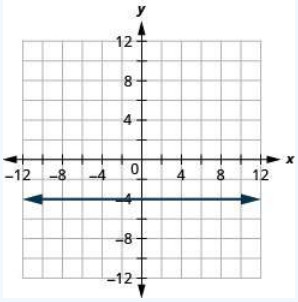


Figure 1.12.11

? Try It 1.12.23

Graph the equation y=−4.

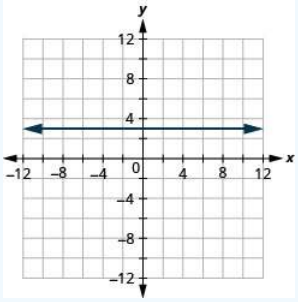
Answer



? Try It 1.12.24

Graph the equation y=3.

Answer



The equations for vertical and horizontal lines look very similar to equations like y=4x. What is the difference between the equations y=4x and y=4?

The equation y=4x has both x and y. The value of y depends on the value of x. The *y*-coordinate changes according to the value of x. The equation y=4 has only one variable. The value of y is constant. The *y*-coordinate is always 4. It does not depend on the value of x. See Table 1.12.11.

Table 1.12.11

y=4x				y=4		
x	**y**	**(x,y)**		**x**	**y**	**(x,y)**
0	0	(0,0)		0	4	(0,4)
1	4	(1,4)		1	4	(1,4)
2	8	(2,8)		2	4	(2,4)

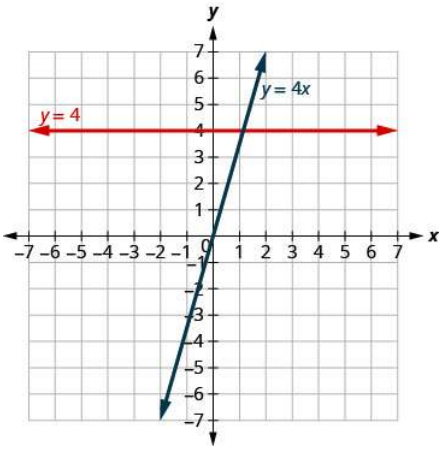


Figure 1.12.12

Notice, in Figure 1.12.12 the equation y=4x gives a slanted line, while y=4 gives a horizontal line.

Example 1.12.25

Graph y=−3x and y=−3 in the same rectangular coordinate system.

Solution

Notice that the first equation has the variable x, while the second does not. See Table 1.12.12 The two graphs are shown in Figure 1.12.13

Table 1.12.12

y=−3x				y=−3		
x	**y**	**(x,y)**		**x**	**y**	**(x,y)**
		(0,0)			−3	(0,−3)
	−3	(1,−3)			−3	(1,−3)
	−6	(2,−6)			−3	(2,−3)

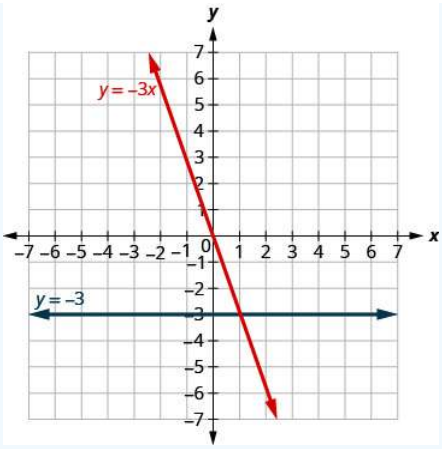
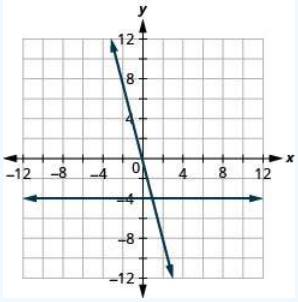


Figure 1.12.13

Try It 1.12.26

Graph y=−4x and y=−4 in the same rectangular coordinate system.

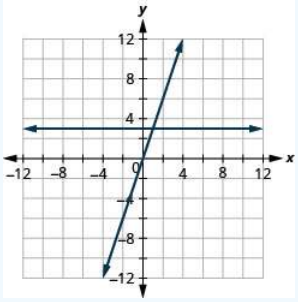
Answer



Try It 1.12.27

Graph y=3 and y=3x in the same rectangular coordinate system.

Answer



Key Concepts

- **Graph a Linear Equation by Plotting Points**
  1. Find three points whose coordinates are solutions to the equation. Organize them in a table.
  2. Plot the points in a rectangular coordinate system. Check that the points line up. If they do not, carefully check your work!
  3. Draw the line through the three points. Extend the line to fill the grid and put arrows on both ends of the line.

Glossary

graph of a linear equation

The graph of a linear equation Ax+By=C is a straight line. Every point on the line is a solution of the equation. Every solution of this equation is a point on this line.

horizontal line

A horizontal line is the graph of an equation of the form y=b. The line passes through the *y*-axis at (0,b).

vertical line

A vertical line is the graph of an equation of the form x=a. The line passes through the *x*-axis at (a,0).

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1.12E: Exercises

Practice Makes Perfect

Recognize the Relationship Between the Solutions of an Equation and its Graph

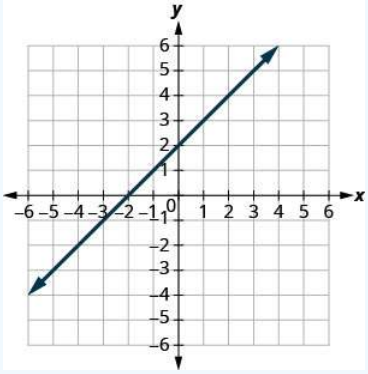
In the following exercises, for each ordered pair, decide:

1. Is the ordered pair a solution to the equation?
2. Is the point on the line?

Exercise 1.12E.1

y=x+2

1. (0,2)
2. (1,2)
3. (−1,1)
4. (−3,−1)



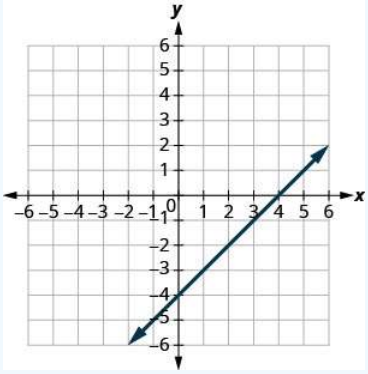
Answer

1. yes; no
2. no; no
3. yes; yes
4. yes; yes

Exercise 1.12E.2

y=x−4

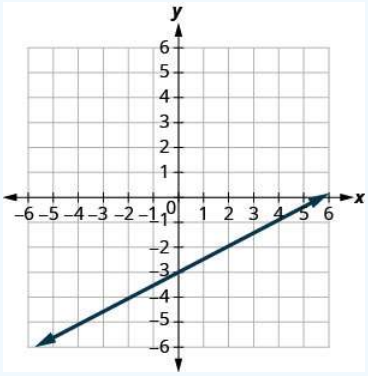
1. (0,−4)
2. (3,−1)
3. (2,2)
4. (1,−5)



Exercise $1.12E.3$

$y = \frac{1}{2}x - 3$

1. (0,−3)
2. (2,−2)
3. (−2,−4)
4. (4,1)



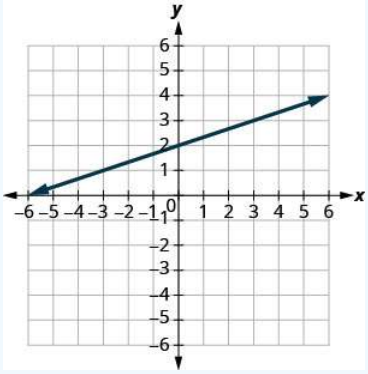
Answer

1. yes; yes
2. yes; yes
3. yes; yes
4. no; no

Exercise $1.12E.4$

$y = \frac{1}{3}x + 2$

1. (0,2)
2. (3,3)
3. (−3,2)
4. (−6,0)



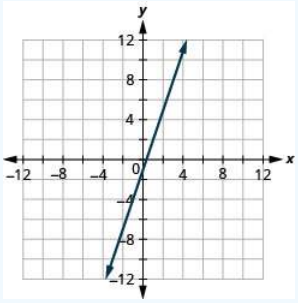
Graph a Linear Equation by Plotting Points

In the following exercises, graph by plotting points.

Exercise 1.12E.5

$y = 3x - 1$

Answer



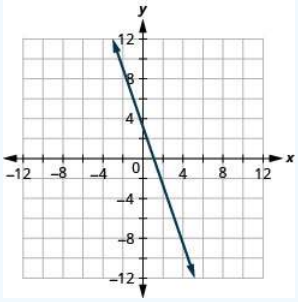
Exercise 1.12E.6

$y = 2x + 3$

Exercise 1.12E.7

$y = -2x + 2$

Answer



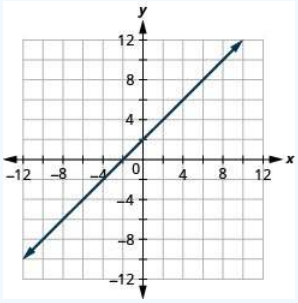
Exercise 1.12E. 8

$y = -3x + 1$

Exercise 1.12E. 9

$y = x + 2$

Answer



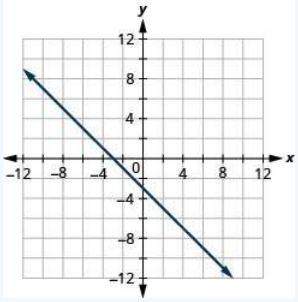
Exercise 1.12E. 10

$y = x - 3$

Exercise 1.12E. 11

$y = -x - 3$

Answer



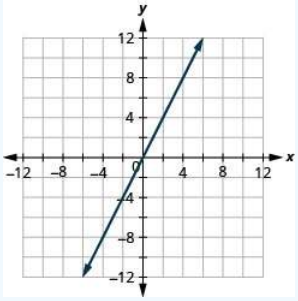
Exercise 1.12E. 12

$y = -x - 2$

Exercise 1.12E. 13

$y = 2x$

Answer



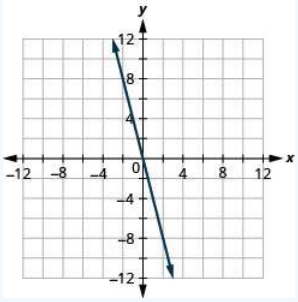
Exercise 1.12E.14

$y = 3x$

Exercise 1.12E.15

$y = -4x$

Answer



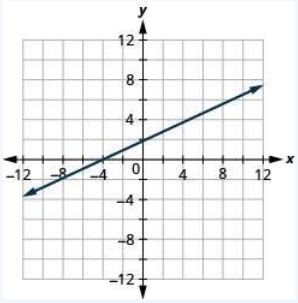
Exercise 1.12E.16

$y = -2x$

Exercise 1.12E.17

$y = \frac{1}{2}x + 2$

Answer



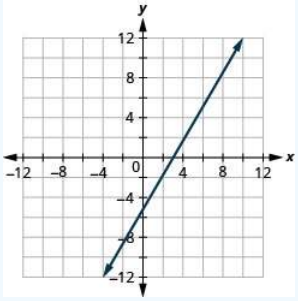
Exercise 1.12E.18

$y = \frac{1}{3}x - 1$

Exercise 1.12E.19

$y = \frac{4}{3}x - 5$

Answer



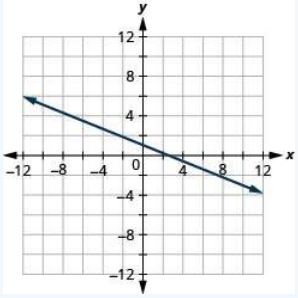
Exercise 1.12E.20

$y = \frac{3}{2}x - 3$

Exercise 1.12E.21

$y = -\frac{2}{5}x + 1$

Answer



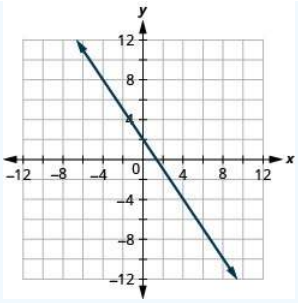
Exercise 1.12E.22

$y = -\frac{4}{5}x - 1$

Exercise 1.12E.23

$y = -\frac{3}{2}x + 2$

Answer



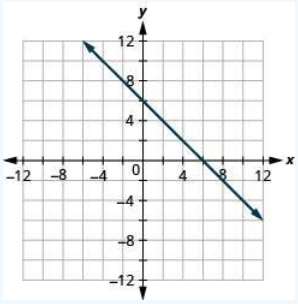
Exercise 1.12E.24

$y = -\frac{5}{3}x + 4$

Exercise 1.12E.25

$x + y = 6$

Answer



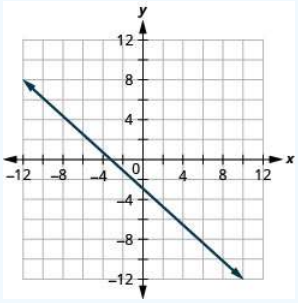
Exercise 1.12E.26

$x + y = 4$

Exercise 1.12E.27

$x + y = -3$

Answer



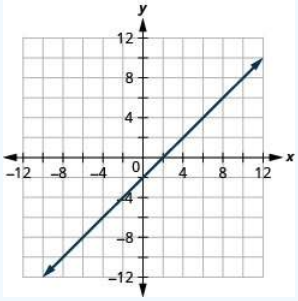
Exercise 1.12E.28

$x + y = -2$

Exercise 1.12E.29

$x - y = 2$

Answer



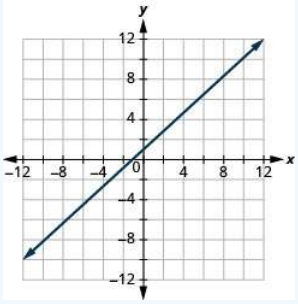
Exercise 1.12E.30

$x - y = 1$

Exercise 1.12E.31

$x - y = -1$

Answer



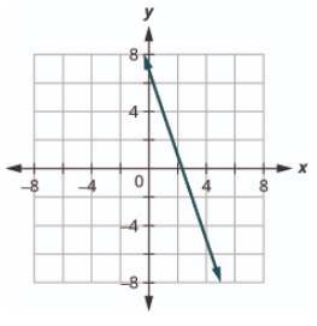
Exercise 1.12E.32

$x - y = -3$

Exercise 1.12E.33

$3x + y = 7$

Answer



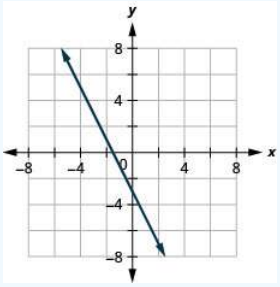
Exercise 1.12E.34

$5x + y = 6$

Exercise 1.12E.35

2x+y=−3

Answer



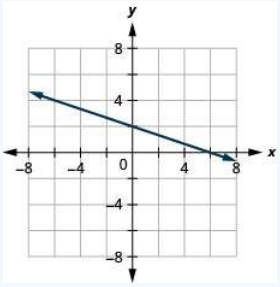
Exercise 1.12E.36

$4x + y = -5$

Exercise 1.12E.37

$\frac{1}{3}x + y = 2$

Answer



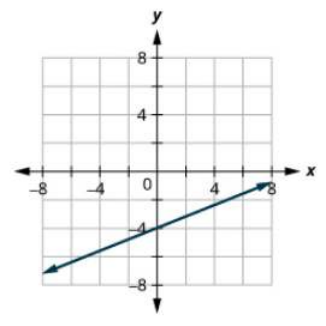
Exercise 1.12E.38

$\frac{1}{2}x + y = 3$

Exercise 1.12E.39

$\frac{2}{5}x - y = 4$

Answer



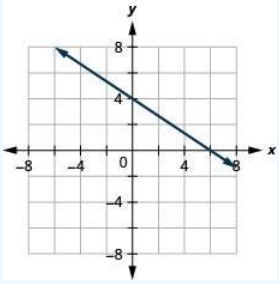
Exercise 1.12E.40

$\frac{3}{4}x - y = 6$

Exercise 1.12E.41

$2x + 3y = 12$

Answer



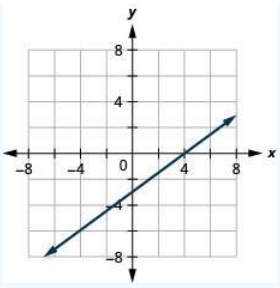
Exercise 1.12E.42

4x+2y=12

Exercise 1.12E.43

3x−4y=12

Answer



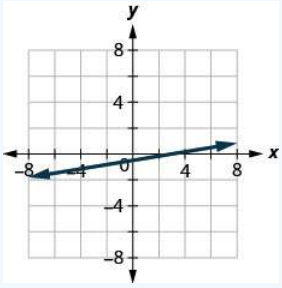
Exercise 1.12E.44

2x−5y=10

Exercise 1.12E.45

x−6y=3

Answer



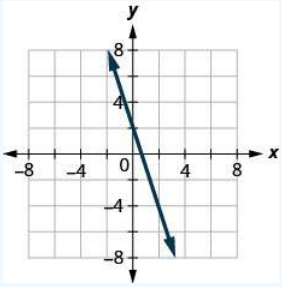
Exercise 1.12E.46

x−4y=2

Exercise 1.12E.47

5x+2y=4

Answer



Exercise 1.12E.48

3x+5y=5

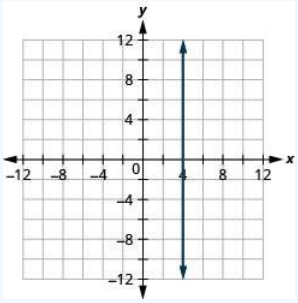
Graph Vertical and Horizontal Lines

In the following exercises, graph each equation.

Exercise 1.12E.49

x=4

Answer



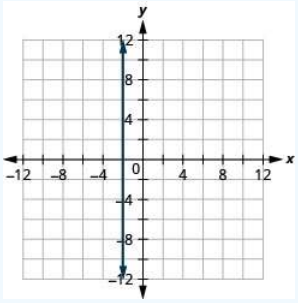
Exercise 1.12E.50

x=3

Exercise 1.12E.51

x=−2

Answer



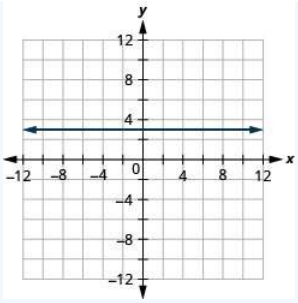
Exercise 1.12E.52

x=−5

Exercise 1.12E.53

y=3

Answer



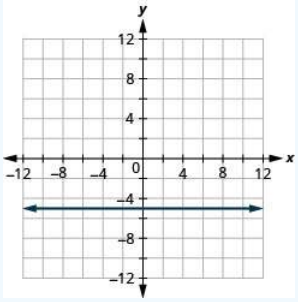
Exercise 1.12E.54

y=1

Exercise 1.12E.55

y=−5

Answer



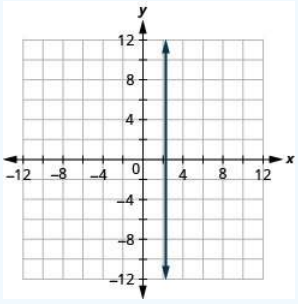
Exercise 1.12E.56

y=−2

Exercise 1.12E.57

$x = \frac{7}{3}$

Answer



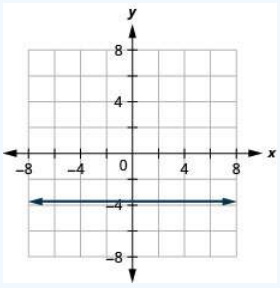
Exercise 1.12E.58

$x = \frac{5}{4}$

Exercise 1.12E.59

$y = -\frac{15}{4}$

Answer



Exercise 1.12E.60

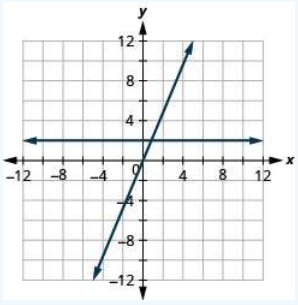
$y = -\frac{5}{3}$

In the following exercises, graph each pair of equations in the same rectangular coordinate system.

Exercise 1.12E.61

y=2x and y=2

Answer



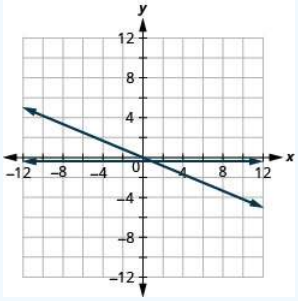
Exercise 1.12E.62

y=5x and y=5

Exercise 1.12E.63

$y = -\frac{1}{2}x$ and $y = -\frac{1}{2}$

Answer



Exercise 1.12E.64

$y = -\frac{1}{3}x$ and $y = -\frac{1}{3}$

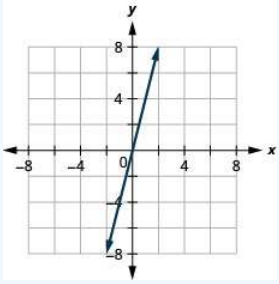
Mixed Practice

In the following exercises, graph each equation.

Exercise 1.12E.65

y=4x

Answer



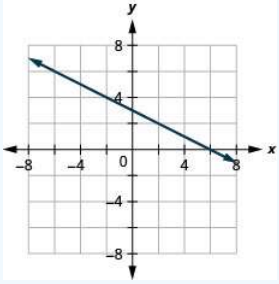
Exercise 1.12E.66

y=2x

Exercise 1.12E.67

$y = -\frac{1}{2}x + 3$

Answer



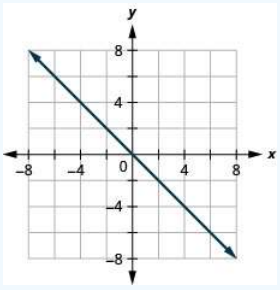
Exercise 1.12E.68

$y = \frac{1}{4}x - 2$

Exercise 1.12E.69

y=−x

Answer



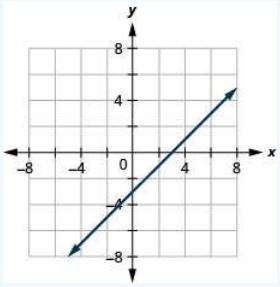
Exercise 1.12E.70

y=x

Exercise 1.12E.71

x−y=3

Answer



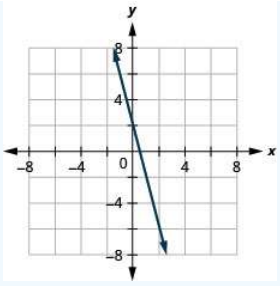
Exercise 1.12E.72

x+y=−5

Exercise 1.12E.73

4x+y=2

Answer



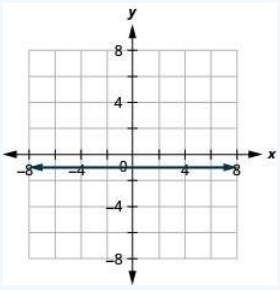
Exercise $1.12E.74$

2x+y=6

Exercise $1.12E.75$

y=−1

Answer



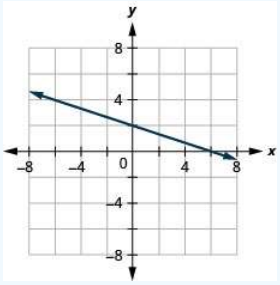
Exercise $1.12E.76$

y=5

Exercise $1.12E.77$

2x+6y=12

Answer



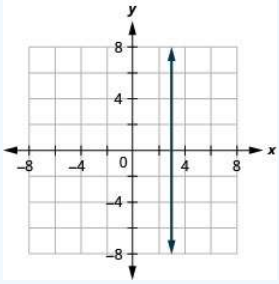
Exercise $1.12E.78$

5x+2y=10

Exercise 1.12E.79

x=3

Answer



Exercise 1.12E.80

x=−4

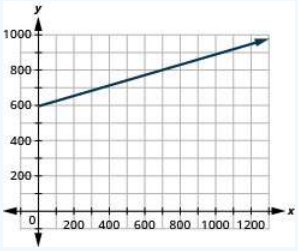
Everyday Math

Exercise 1.12E.81

Motor home cost. The Robinsons rented a motor home for one week to go on vacation. It cost them $594 plus $0.32 per mile to rent the motor home, so the linear equation y=594+0.32x gives the cost, yy, for driving xx miles. Calculate the rental cost for driving 400, 800, and 1200 miles, and then graph the line.

Answer

$722, $850, $978



Exercise 1.12E.82

Weekly earnings. At the art gallery where he works, Salvador gets paid $200 per week plus 15% of the sales he makes, so the equation y=200+0.15x gives the amount, yy, he earns for selling x dollars of artwork. Calculate the amount Salvador earns for selling $900, $1600, and $2000, and then graph the line.

Writing Exercises

Exercise 1.12E.83

Explain how you would choose three x - values to make a table to graph the line $y = \frac{1}{5}x - 2$

Answer

Answers will vary.

Exercise 1.12E. 84

What is the difference between the equations of a vertical and a horizontal line?

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
recognize the relation between the solutions of an equation and its graph.			
graph a linear equation by plotting points.			
graph vertical and horizontal lines.			

ⓑ After reviewing this checklist, what will you do to become confident for all goals?

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1.13: Graph with Intercepts

Learning Objectives

By the end of this section, you will be able to:

- Identify the x- and y- intercepts on a graph
- Find the x- and y- intercepts from an equation of a line
- Graph a line using the intercepts

Note

Before you get started, take this readiness quiz.

1. Solve: $3 \cdot 0 + 4y = -2$.
   If you missed this problem, review Exercise 2.2.13.

Identify the *x*- and *y*- Intercepts on a Graph

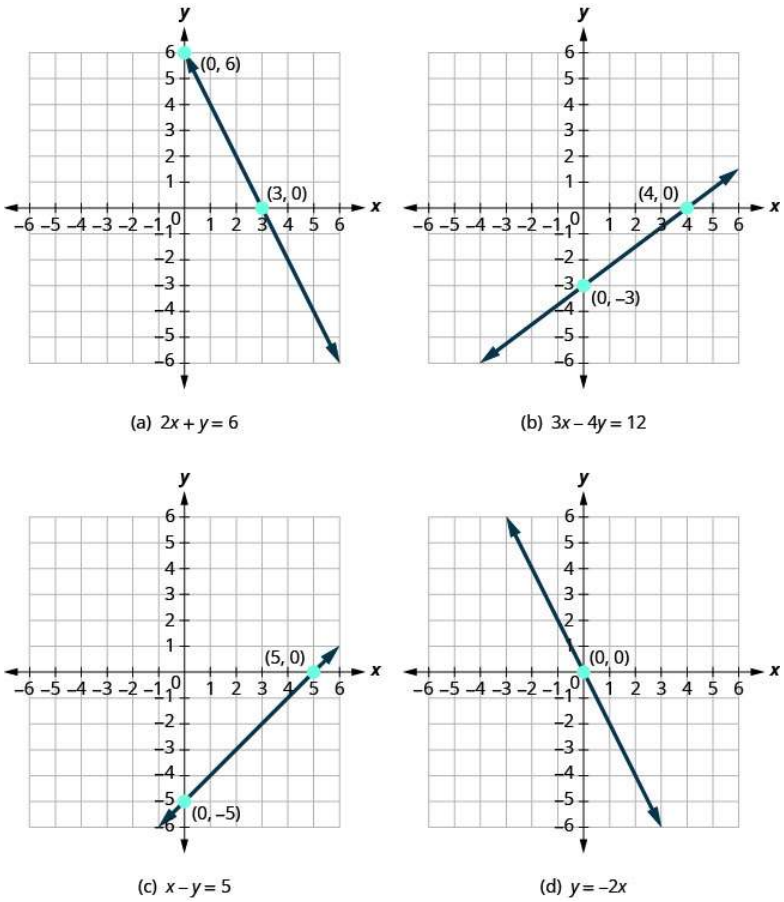
Every linear equation can be represented by a unique line that shows all the solutions of the equation. We have seen that when graphing a line by plotting points, you can use any three solutions to graph. This means that two people graphing the line might use different sets of three points.

At first glance, their two lines might not appear to be the same, since they would have different points labeled. But if all the work was done correctly, the lines should be exactly the same. One way to recognize that they are indeed the same line is to look at where the line crosses the *x*- axis and the *y*- axis. These points are called the *intercepts* of the line.

INTERCEPTS OF A LINE

The points where a line crosses the *x*- axis and the *y*- axis are called the **intercepts of a line**.

Let's look at the graphs of the lines in Figure 1.13.1.



(a) $2x + y = 6$ (b) $3x - 4y = 12$

(c) $x - y = 5$ (d) $y = -2x$

Figure 1.13.1: Examples of graphs crossing the x-negative axis.

First, notice where each of these lines crosses the x axis. See Figure 1.13.1.

Table 1.13.1

Figure	The line crosses the x-axis at:	Ordered pair of this point
Figure (a)	3	(3,0)
Figure (b)	4	(4,0)
Figure (c)	5	(5,0)
Figure (d)	0	(0,0)

Do you see a pattern?

For each row, the *y*- coordinate of the point where the line crosses the *x*- axis is zero. The point where the line crosses the *x*- axis has the form (a,0) and is called the ***x*- intercept of a line**. The *x*- intercept occurs when y is zero. Now, let's look at the points where these lines cross the *y*- axis. See Table 1.13.2

Table 1.13.2

Figure	The line crosses the x-axis at:	Ordered pair of this point
Figure (a)	6	(0,6)
Figure (b)	−3	(0,−3)
Figure (c)	−5	(0,5)
Figure (d)	0	(0,0)

What is the pattern here?

In each row, the *x*- coordinate of the point where the line crosses the *y*- axis is zero. The point where the line crosses the *y*- axis has the form (0,b) and is called the *y*- *intercept* of the line. The *y*- intercept occurs when x is zero.

X- INTERCEPT AND Y- INTERCEPT OF A LINE

The *x*- intercept is the point (a,0) where the line crosses the *x*- axis.

The *y*- intercept is the point (0,b) where the line crosses the *y*- axis.

	x	*y*
• The *x*-intercept occurs when *y* is zero.	*a*	0
• The *y*-intercept occurs when *x* is zero.	0	*b*

Figure 1.13.2

Example 1.13.1

Find the *x*- and *y*- intercepts on each graph.

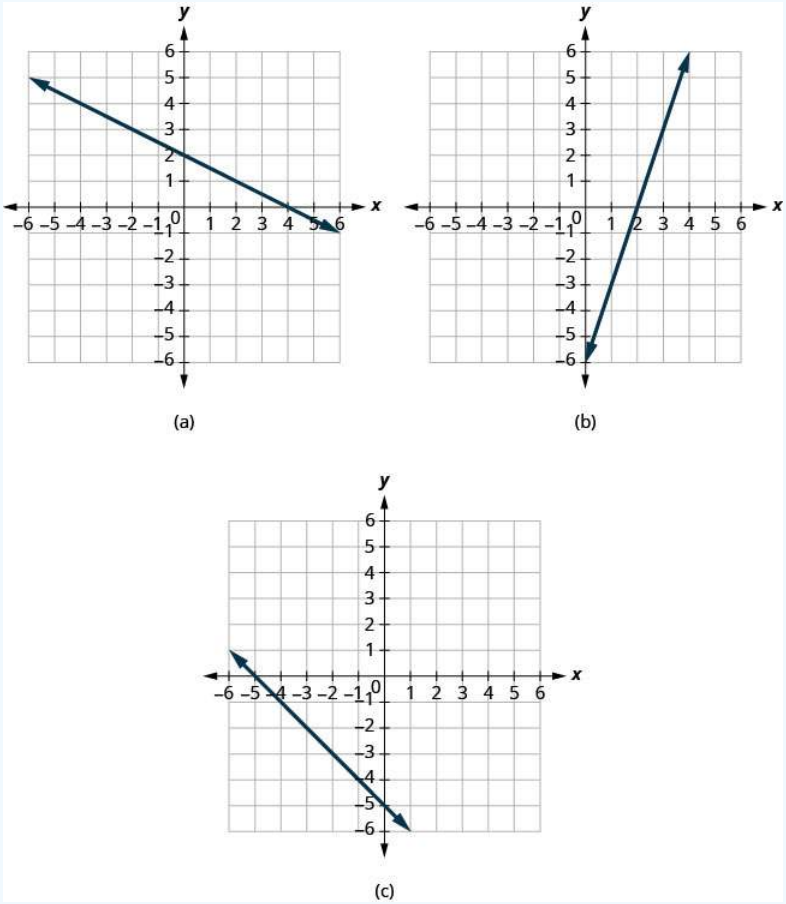


Figure 1.13.3

Solution

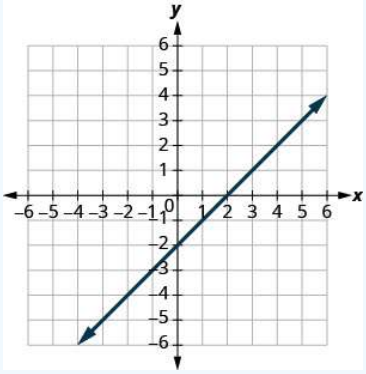
(a) The graph crosses the *x*- axis at the point (4,0). The *x*- intercept is (4,0).
The graph crosses the *y*- axis at the point (0,2). The *y*- intercept is (0,2).

(b) The graph crosses the x- axis at the point (2,0). The x- intercept is (2,0)
The graph crosses the y- axis at the point (0,−6). The y- intercept is (0,−6).

(c) The graph crosses the x- axis at the point (−5,0). The x- intercept is (−5,0).
The graph crosses the y- axis at the point (0,−5). The y- intercept is (0,−5).

Try It 1.13.2

Find the x- and y- intercepts on the graph.

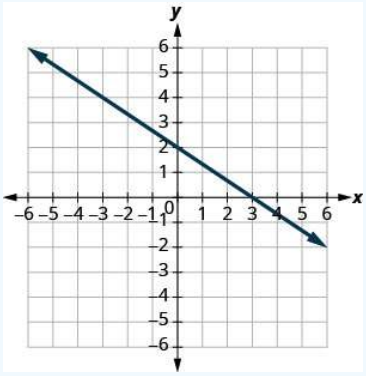


Answer

x- intercept: (2,0); y- intercept: (0,−2)

Try It 1.13.3

Find the x- and y- intercepts on the graph.



Answer

x- intercept: (3,0), y- intercept: (0,2)

Find the x- and y- Intercepts from an Equation of a Line

Recognizing that the ***x*- intercept** occurs when y is zero and that the y- intercept occurs when x is zero, gives us a method to find the intercepts of a line from its equation. To find the x- intercept, let y=0 and solve for x. To find the ***y*- intercept**, let x=0 and solve for y.

X- AND Y- INTERCEPTS FROM THE EQUATION OF A LINE

Use the equation of the line. To find:

- the *x*- intercept of the line, let y=0 and solve for x.
- the *y*- intercept of the line, let x=0 and solve for y.

Example 1.13.4

Find the intercepts of 2x+y=6.

Solution

We will let y=0 to find the *x*- intercept, and let x=0 to find the *y*- intercept. We will fill in the table, which reminds us of what we need to find.

$2x+y=6$	
x	y
	0
0	

x-intercept

y-intercept

To find the *x*- intercept, let y=0.

Table 1.13.3

	$2x+y=6$
Let $y = 0$.	$2x+0=6$
Simplify.	$2x=6$
	$x=3$
The *x*-intercept is	(3, 0)
To find the *y*-intercept, let $x = 0$.	
	$2x+y=6$
Let $x = 0$.	$2\cdot 0+y=6$
Simplify.	$0+y=6$
	$y=6$
The *y*-intercept is	(0, 6)

The intercepts are the points (3,0) and (0,6) as shown in Table 1.13.4

Table 1.13.4

2x+y=6	
x	**y**
3	0
0	6

Try It 1.13.5

Find the intercepts of 3x+y=12.

Answer

x- intercept: (4,0), *y*- intercept: (0,12)

Try It 1.13.6

Find the intercepts of x+4y=8.

Answer

x- intercept: (8,0), *y*- intercept: (0,2)

Example 1.13.7

Find the intercepts of 4x–3y=12.

Solution

To find the *x*-intercept, let $y = 0$.	
	$4x - 3y = 12$
Let $y = 0$.	$4x - 3 \cdot 0 = 12$
Simplify.	$4x - 0 = 12$
	$4x = 12$
	$x = 3$
The *x*-intercept is	(3, 0)
To find the *y*-intercept, let $x = 0$.	
	$4x - 3y = 12$
Let $x = 0$.	$4 \cdot 0 - 3y = 12$
Simplify.	$0 - 3y = 12$
	$-3y = 12$
	$y = -4$
The *y*-intercept is	(0, −4)

Table 1.13.5

The intercepts are the points (3, 0) and (0, −4) as shown in the following table.

Table 1.13.6

4x−3y=12	
x	**y**
3	0
0	−4

Try It 1.13.8

Find the intercepts of 3x–4y=12.

Answer

x- intercept: (4,0), *y*- intercept: (0,−3)

Try It 1.13.9

Find the intercepts of 2x–4y=8.

Answer

x- intercept: (4,0), *y*- intercept: (0,−2)

Graph a Line Using the Intercepts

To graph a linear equation by plotting points, you need to find three points whose coordinates are solutions to the equation. You can use the *x*- and *y*- intercepts as two of your three points. Find the intercepts, and then find a third point to ensure accuracy. Make sure the points line up—then draw the line. This method is often the quickest way to graph a line.

Example 1.13.10: How to Graph a Line Using Intercepts

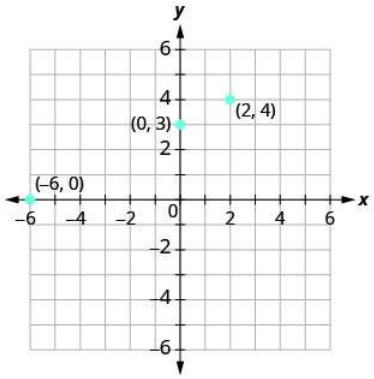
Graph –x+2y=6 using the intercepts.

Solution

Step 1. Find the *x*- and *y*-intercepts of the line. Let $y = 0$ and solve for x. Let $x = 0$ and solve for y.	Find the *x*-intercept.	Let $y = 0$ $-x + 2y = 6$ $-x + 2(0) = 6$ $-x = 6$ $x = -6$ The *x*-intercept is (–6, 0).
	Find the *y*-intercept.	Let $x = 0$ $-x + 2y = 6$ $-0 + 2y = 6$ $2y = 6$ $y = 3$ The *y*-intercept is (0, 3).
Step 2. Find another solution to the equation.	We'll use $x = 2$.	Let $x = 2$ $-x + 2y = 6$ $-2 + 2y = 6$ $2y = 8$ $y = 4$ A third point is (2, 4).

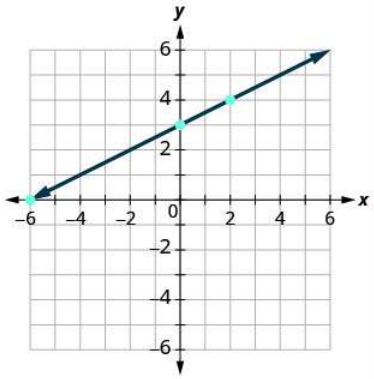
Step 3. Plot the three points. Check that the points line up.

x	y	(x, y)
–6	0	(–6, 0)
0	3	(0, 3)
2	4	(2, 4)



Step 4. Draw the line.

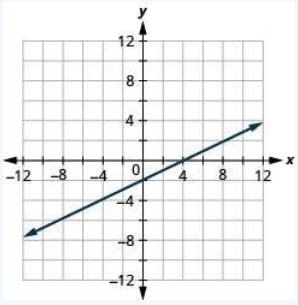
See the graph.



Try It 1.13.11

Graph x–2y=4 using the intercepts.

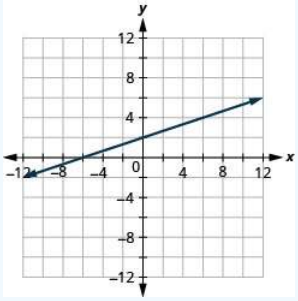
Answer



Try It 1.13.12

Graph –x+3y=6 using the intercepts.

Answer



The steps to graph a linear equation using the intercepts are summarized below.

GRAPH A LINEAR EQUATION USING THE INTERCEPTS.

1. Find the *x*- and *y*- intercepts of the line.
   - Let y=0 and solve for x
   - Let x=0 and solve for y.
2. Find a third solution to the equation.
3. Plot the three points and check that they line up.
4. Draw the line.

Example 1.13.13

Graph 4x–3y=12 using the intercepts.

Solution

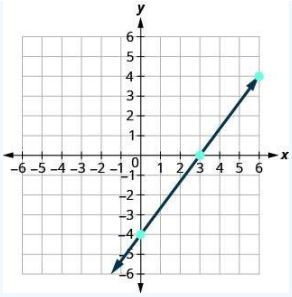
Find the intercepts and a third point.

x-intercept, let $y = 0$	*y*-intercept, let $x = 0$	third point, let $y = 4$
$4x - 3y = 12$	$4x - 3y = 12$	$4x - 3y = 12$
$4x - 3(0) = 12$	$4(0) - 3y = 12$	$4x - 3(4) = 12$
$4x = 12$	$-3y = 12$	$4x - 12 = 12$
$x = 3$	$y = -4$	$4x = 24$
		$x = 6$

We list the points in Table 1.13.7and show the graph below.

4x−3y=12		
x	**y**	**(x,y)**
3	0	(3,0)
0	−4	(0,−4)
6	4	(6,4)

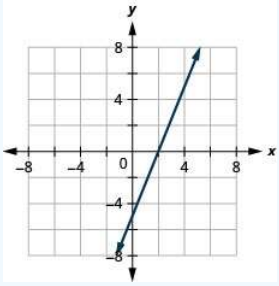
Table 1.13.7



Try It 1.13.14

Graph 5x–2y=10 using the intercepts.

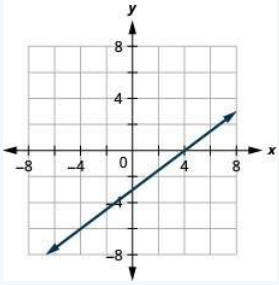
Answer



Try It 1.13.15

Graph 3x–4y=12 using the intercepts.

Answer



Example 1.13.16

Graph y=5x using the intercepts.

Solution

x-intercept	*y*-intercept
Let $y = 0$.	Let $x = 0$.
$y = 5x$	$y = 5x$
$0 = 5x$	$y = 5 \cdot 0$
$0 = x$	$y = 0$
(0, 0)	(0, 0)

This line has only one intercept. It is the point (0,0).

To ensure accuracy we need to plot three points. Since the x- and y- intercepts are the same point, we need *two* more points to graph the line.

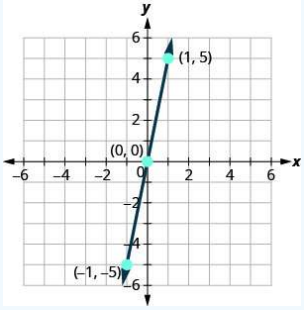
Let $x = 1$.	Let $x = -1$.
$y = 5x$	$y = 5x$
$y = 5 \cdot 1$	$y = 5(-1)$
$y = 5$	$y = -5$

See Table 1.13.8

y=5x		
x	**y**	**(x,y)**
0	0	(0,0)
1	5	(1,5)
−1	−5	(−1,−5)

Table 1.13.8

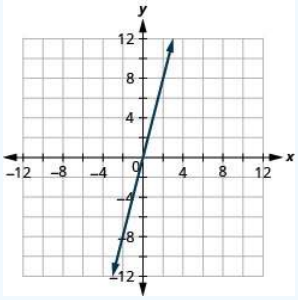
Plot the three points, check that they line up, and draw the line.



Try It 1.13.17

Graph y=4x using the intercepts.

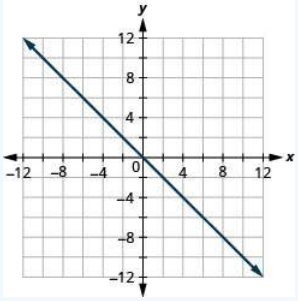
Answer



Try It 1.13.18

Graph y=−x the intercepts.

Answer



Key Concepts

- **Find the *x*- and *y*- Intercepts from the Equation of a Line**
  - Use the equation of the line to find the *x*- intercept of the line, let y=0 and solve for *x*.
  - Use the equation of the line to find the *y*- intercept of the line, let x=0 and solve for *y*.
- **Graph a Linear Equation using the Intercepts**
  1. Find the *x*- and *y*- intercepts of the line.
     Let y=0 and solve for *x*.
     Let x=0 and solve for *y*.
  2. Find a third solution to the equation.
  3. Plot the three points and then check that they line up.
  4. Draw the line.
- **Strategy for Choosing the Most Convenient Method to Graph a Line:**
  - Consider the form of the equation.
  - If it only has one variable, it is a vertical or horizontal line.
    x=a is a vertical line passing through the *x*- axis at a
    y=b is a horizontal line passing through the *y*- axis at b.
  - If *y* is isolated on one side of the equation, graph by plotting points.
  - Choose any three values for *x* and then solve for the corresponding *y*- values.
  - If the equation is of the form ax+by=c, find the intercepts. Find the *x*- and *y*- intercepts and then a third point.

Glossary

intercepts of a line

The points where a line crosses the *x*- axis and the *y*- axis are called the intercepts of the line.

***x*- intercept**

The point (a,0) where the line crosses the *x*- axis; the *x*- intercept occurs when y is zero.

***y*-intercept**

The point (0,b) where the line crosses the *y*- axis; the *y*- intercept occurs when x is zero.

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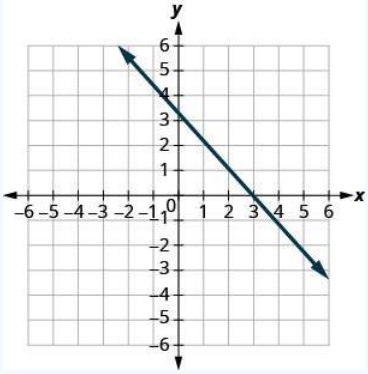
1.13E: Exercises

Practice Makes Perfect

Identify the *x*- and *y*- Intercepts on a Graph

In the following exercises, find the *x*- and *y*- intercepts on each graph.

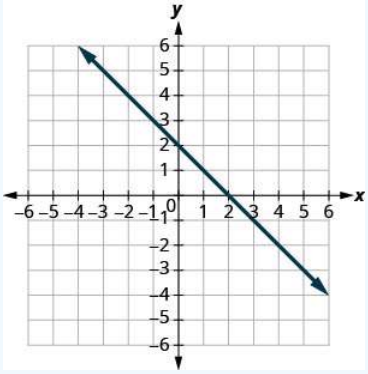
Exercise 1.13E.1



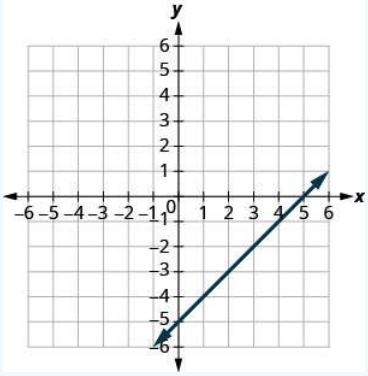
Answer

(3,0),(0,3)

Exercise 1.13E.2



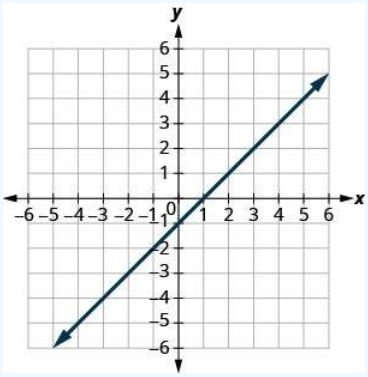
Exercise 1.13E.3



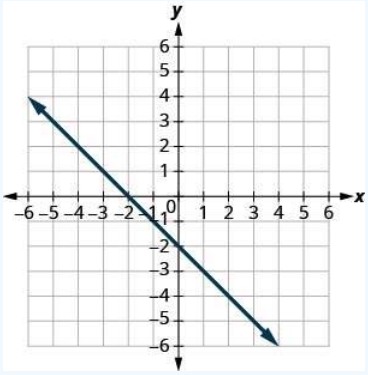
Answer

(5,0),(0,−5)

Exercise 1.13E.4



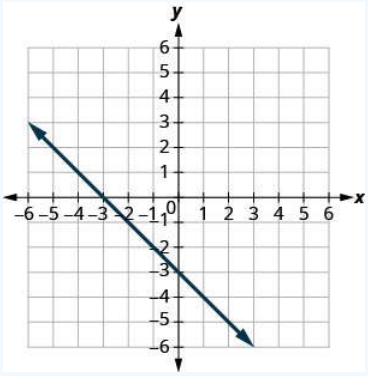
Exercise 1.13E.5



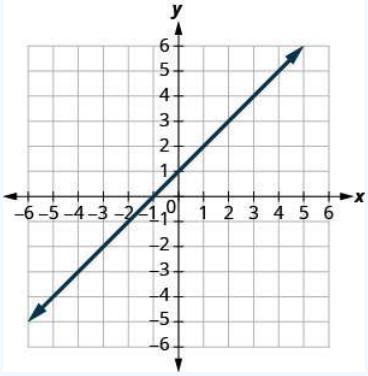
Answer

(−2,0),(0,−2)

Exercise 1.13E.6



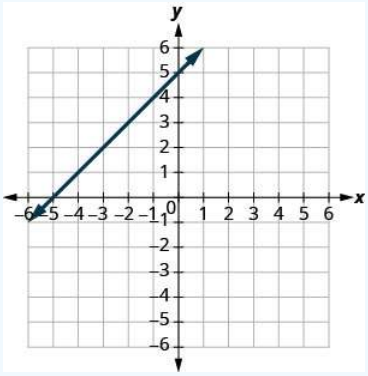
Exercise 1.13E.7



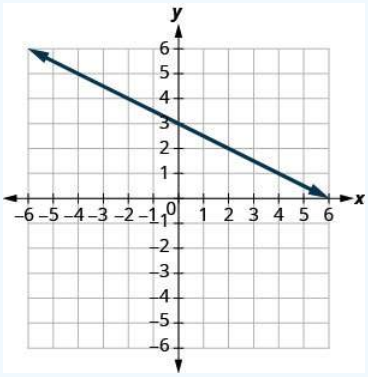
Answer

$(-1,0),(0,1)$

Exercise 1.13E.8



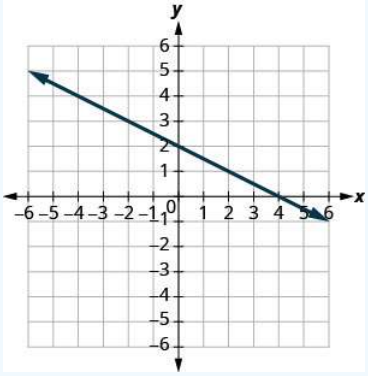
Exercise 1.13E.9



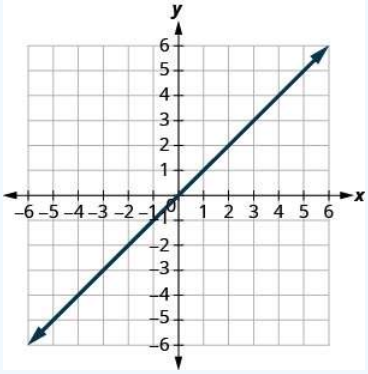
Answer

$(6,0),(0,3)$

Exercise 1.13E.10



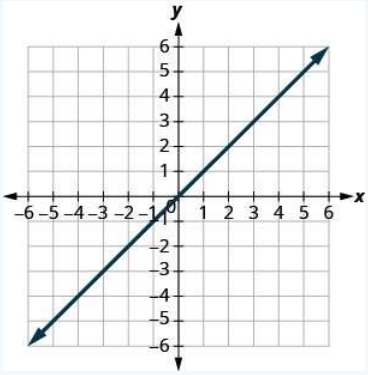
Exercise 1.13E.11



Answer

(0,0)

Exercise 1.13E.12



Find the *x*- and *y*- Intercepts from an Equation of a Line

In the following exercises, find the intercepts for each equation.

Exercise 1.13E.13

x+y=4

Answer

(4,0),(0,4)

Exercise 1.13E.14

x+y=3

Exercise 1.13E.15

x+y=−2

Answer

(−2,0),(0,−2)

Exercise 1.13E.16

x+y=−5

Exercise 1.13E.17

x–y=5

Answer

(5,0),(0,−5)

Exercise 1.13E.18

x–y=1

Exercise 1.13E.19

x–y=−3

Answer

(−3,0),(0,3)

Exercise 1.13E.20

x–y=−4

Exercise 1.13E.21

x+2y=8

Answer

(8,0),(0,4)

Exercise 1.13E.22

x+2y=10

Exercise 1.13E.23

3x+y=6

Answer

(2,0),(0,6)

Exercise 1.13E.24

3x+y=9

Exercise 1.13E.25

x–3y=12

Answer

(12,0),(0,−4)

Exercise 1.13E.26

x–2y=8

Exercise 1.13E.27

4x–y=8

Answer

(2,0),(0,−8)

Exercise 1.13E.28

5x–y=5

Exercise 1.13E.29

2x+5y=10

Answer

(5,0),(0,2)

Exercise 1.13E.30

2x+3y=6

Exercise 1.13E.31

3x–2y=12

Answer

(4,0),(0,−6)

Exercise 1.13E.32

3x–5y=30

Exercise 1.13E.33

$y = \frac{1}{3}x + 1$

Answer

(-3,0),(0,1)

Exercise 1.13E.34

$y = \frac{1}{4}x - 1$

Exercise 1.13E.35

$y = \frac{1}{5}x + 2$

Answer

(−10,0),(0,2)

Exercise 1.13E.36

$y = \frac{1}{3}x + 4$

Exercise 1.13E.37

y=3x

Answer

(0,0)

Exercise 1.13E.38

y=-2x

Exercise 1.13E.39

y=-4x

Answer

(0,0)

Exercise 1.13E. 40

y=5x

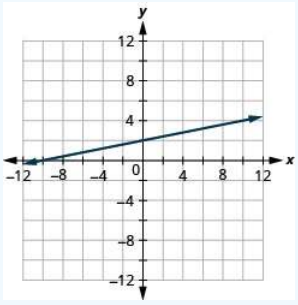
Graph a Line Using the Intercepts

In the following exercises, graph using the intercepts.

Exercise 1.13E. 41

$-x+5y=10$

Answer



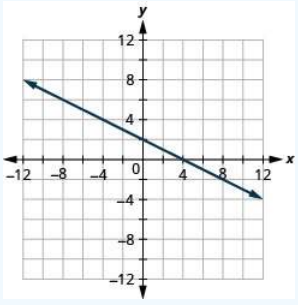
Exercise 1.13E. 42

$-x+4y=8$

Exercise 1.13E. 43

$x+2y=4$

Answer



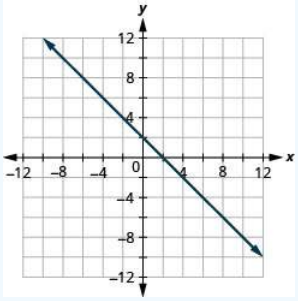
Exercise 1.13E. 44

$x+2y=6$

Exercise 1.13E. 45

$x+y=2$

Answer



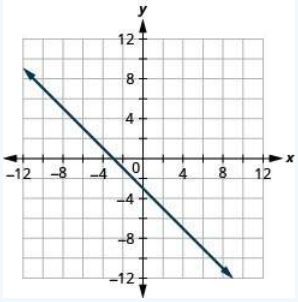
Exercise 1.13E.46

$x + y = 5$

Exercise 1.13E.47

$x + y = -3$

Answer



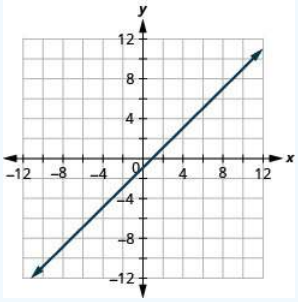
Exercise 1.13E.48

$x + y = -1$

Exercise 1.13E.49

$x - y = 1$

Answer



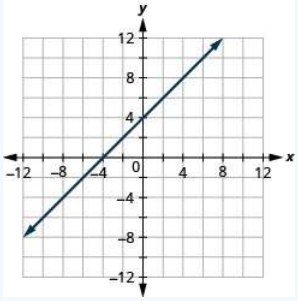
Exercise 1.13E.50

$x - y = 2$

Exercise 1.13E.51

$x - y = -4$

Answer



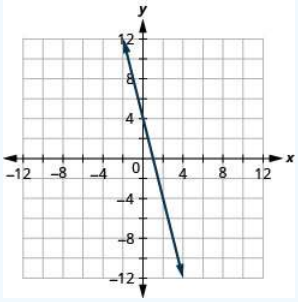
Exercise 1.13E.52

$x - y = -3$

Exercise 1.13E.53

$4x + y = 4$

Answer



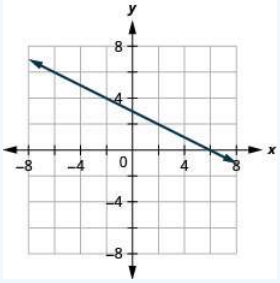
Exercise 1.13E.54

$3x + y = 3$

Exercise 1.13E.55

$2x + 4y = 12$

Answer



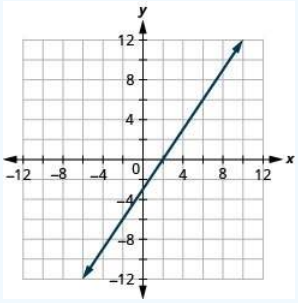
Exercise 1.13E.56

$3x + 2y = 12$

Exercise 1.13E.57

$3x - 2y = 6$

Answer



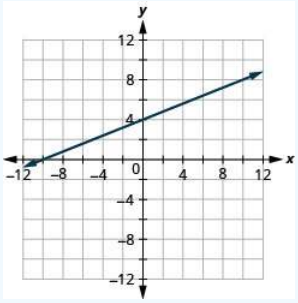
Exercise 1.13E.58

$5x - 2y = 10$

Exercise 1.13E.59

$2x - 5y = -20$

Answer



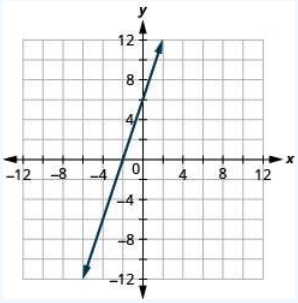
Exercise 1.13E.60

$3x - 4y = -12$

Exercise 1.13E.61

$3x - y = -6$

Answer



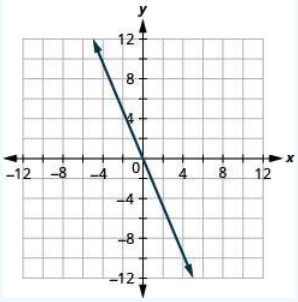
Exercise 1.13E.62

$2x - y = -8$

Exercise 1.13E.63

$y = -2x$

Answer



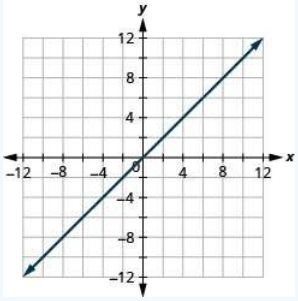
Exercise 1.13E.64

$y = -4x$

Exercise 1.13E.65

$y = x$

Answer



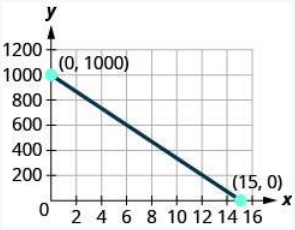
? Exercise 1.13E. 66

y=3x

Everyday Math

? Exercise 1.13E. 67

Road trip. Damien is driving from Chicago to Denver, a distance of 1000 miles. The *x*- axis on the graph below shows the time in hours since Damien left Chicago. The *y*- axis represents the distance he has left to drive.



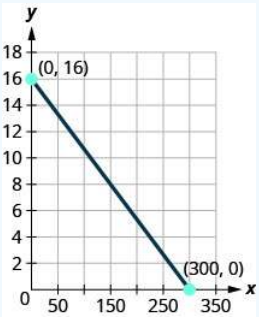
1. Find the *x*- and *y*- intercepts.
2. Explain what the *x*- and *y*- intercepts mean for Damien.

Answer

1. (0,1000),(15,0)
2. At (0,1000), he has been gone 0 hours and has 1000 miles left. At (15,0), he has been gone 15 hours and has 0 miles left to go.

? Exercise 1.13E. 68

Road trip. Ozzie filled up the gas tank of his truck and headed out on a road trip. The *x*- axis on the graph below shows the number of miles Ozzie drove since filling up. The *y*- axis represents the number of gallons of gas in the truck's gas tank.



1. Find the *x*- and *y*- intercepts.
2. Explain what the *x*- and *y*- intercepts mean for Ozzie.

Writing Exercises

Exercise 1.13E.69

How do you find the x -intercept of the graph of $3x-2y=6$?

Answer

Answers will vary.

Exercise 1.13E.70

Do you prefer to use the method of plotting points or the method using the intercepts to graph the equation 4x+y=−4? Why?

Exercise 1.13E.71

Do you prefer to use the method of plotting points or the method using the intercepts to graph the equation $y=\frac{2}{3}x-2$? Why?

Answer

Answers will vary.

Exercise 1.13E.72

Do you prefer to use the method of plotting points or the method using the intercepts to graph the equation y=6? Why?

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
identify the *x* and *y* intercepts on a graph.			
find the *x* and *y* intercepts from an equation of a line.			
graph a line using the intercepts.			

ⓑ What does this checklist tell you about your mastery of this section? What steps will you take to improve?

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1.14: Understanding the Slope of a Line

Learning Objectives

- By the end of this section, you will be able to:
- Use geoboards to model slope
- Use $m = \frac{\text{rise}}{\text{run}}$ to find the slope of a line from its graph
- Find the slope of horizontal and vertical lines
- Use the slope formula to find the slope of a line between two points
- Graph a line given a point and the slope
- Solve slope applications

Note

Before you get started, take this readiness quiz.

1. Simplify: $\frac{1-4}{8-2}$.
   If you missed this problem, review Exercise 1.6.31
2. Divide: $\frac{0}{4}, \frac{4}{0}$.
   If you missed this problem, review Exercise 1.10.16.
3. Simplify: $\frac{15}{-3}, \frac{-15}{3}, \frac{-15}{-3}$.
   If you missed this problem, review Exercise 1.6.4.

When you graph linear equations, you may notice that some lines tilt up as they go from left to right and some lines tilt down. Some lines are very steep and some lines are flatter. What determines whether a line tilts up or down or if it is steep or flat?

In mathematics, the 'tilt' of a line is called the *slope* of the line. The concept of slope has many applications in the real world. The pitch of a roof, grade of a highway, and a ramp for a wheelchair are some examples where you literally see slopes. And when you ride a bicycle, you feel the slope as you pump uphill or coast downhill.

In this section, we will explore the concept of slope.

Use Geoboards to Model Slope

A **geoboard** is a board with a grid of pegs on it. Using rubber bands on a geoboard gives us a concrete way to model lines on a coordinate grid. By stretching a rubber band between two pegs on a geoboard, we can discover how to find the slope of a line.

Doing the Manipulative Mathematics activity "Exploring Slope" will help you develop a better understanding of the slope of a line. (Graph paper can be used instead of a geoboard, if needed.)

We'll start by stretching a rubber band between two pegs as shown in Figure 1.14.1.

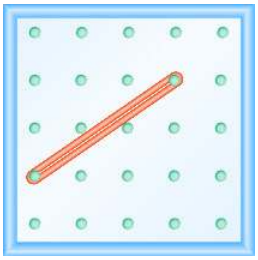


Figure 1.14.1

Doesn't it look like a line?

Now we stretch one part of the rubber band straight up from the left peg and around a third peg to make the sides of a right triangle, as shown in Figure 1.14.2

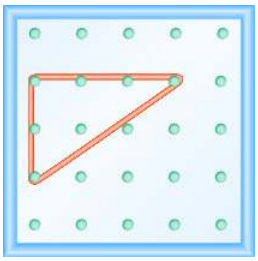


Figure 1.14.2

We carefully make a 90º angle around the third peg, so one of the newly formed lines is vertical and the other is horizontal.

To find the slope of the line, we measure the distance along the vertical and horizontal sides of the triangle. The vertical distance is called the **rise** and the horizontal distance is called the **run**, as shown in Figure 1.14.3.

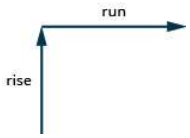


Figure 1.14.3

If our **geoboard** and rubber band look just like the one shown in Figure 1.14.4, the rise is 2. The rubber band goes up 2 units. (Each space is one unit.)

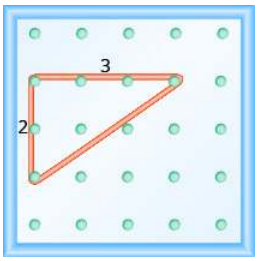


Figure 1.14.4

The rise on this geoboard is 2, as the rubber band goes up two units.

What is the run?

The rubber band goes across 3 units. The run is 3 (see Figure 1.14.4).

The slope of a line is the ratio of the rise to the run. In mathematics, it is always referred to with the letter m.

SLOPE OF A LINE

The **slope of a line** of a line is $m = \frac{\text{rise}}{\text{run}}$.

The **rise** measures the vertical change and the **run** measures the horizontal change between two points on the line.

What is the slope of the line on the geoboard in Figure 1.14.4?

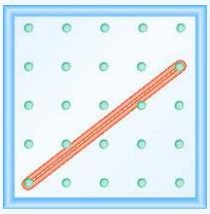
$$m = \frac{\text{rise}}{\text{run}}$$
$$m = \frac{2}{3}$$

The line has slope $\frac{2}{3}$. This means that the line rises 2 units for every 3 units of run.

When we work with geoboards, it is a good idea to get in the habit of starting at a peg on the left and connecting to a peg to the right. If the rise goes up it is positive and if it goes down it is negative. The run will go from left to right and be positive.

Example 1.14.1

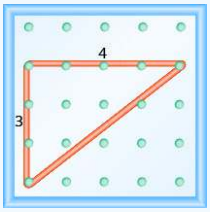
What is the slope of the line on the geoboard shown?



Solution

Use the definition of slope: $m = \frac{\text{rise}}{\text{run}}$.

Start at the left peg and count the spaces up and to the right to reach the second peg.

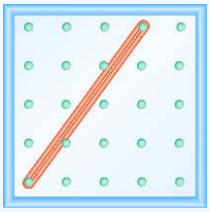


$$\begin{aligned} &\text{The rise is } 3. && m = \tfrac{3}{\text{run}} \\ &\text{The run is } 4. && m = \tfrac{3}{4} \\ & && \text{The slope is } \tfrac{3}{4}. \end{aligned} \tag{1.14.1}$$

This means that the line rises 3 units for every 4 units of run.

Try It 1.14.2

What is the slope of the line on the geoboard shown?

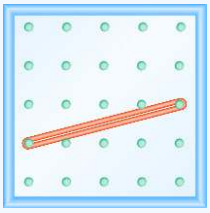


Answer

$\frac{4}{3}$

Try It 1.14.3

What is the slope of the line on the geoboard shown?

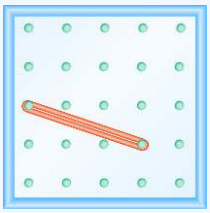


Answer

$\frac{1}{4}$

Example 1.14.4

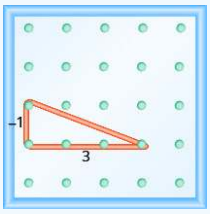
What is the slope of the line on the geoboard shown?



Solution

Use the definition of slope: $m = \frac{\text{rise}}{\text{run}}$.

Start at the left peg and count the units down and to the right to reach the second peg.

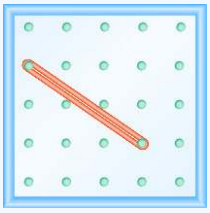


$$\begin{aligned} &\text{The rise is } -1. && m = \frac{-1}{\text{run}} \\ &\text{The run is } 3. && m = \frac{-1}{3} \\ & && m = -\frac{1}{3} \\ & && \text{The slope is } -\frac{1}{3} \end{aligned} \tag{1.14.2}$$

This means that the line drops 1 unit for every 3 units of run.

Try It 1.14.5

What is the slope of the line on the geoboard?

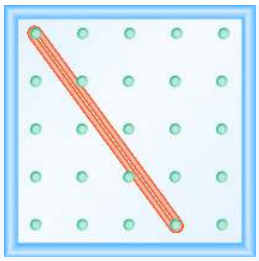


Answer

$-\frac{2}{3}$

Try It 1.14.6

What is the slope of the line on the geoboard?



Answer

$-\frac{4}{3}$

Notice that in Exercise 1.14.1 the slope is positive and in Exercise 1.14.4 the slope is negative. Do you notice any difference in the two lines shown in Figure(a) and Figure(b)?

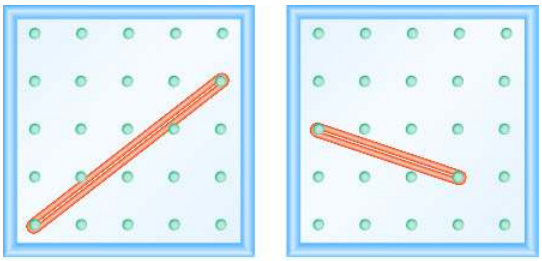


Figure 1.14.5

We 'read' a line from left to right just like we read words in English. As you read from left to right, the line in Figure(a) is going up; it has **positive slope**. The line in Figure(b) is going down; it has **negative slope**.

POSITIVE AND NEGATIVE SLOPES

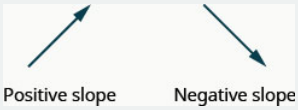


Figure 1.14.6

Example 1.14.7

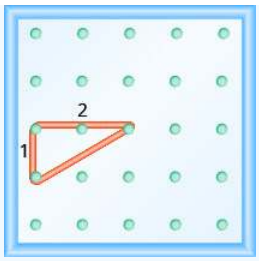
Use a geoboard to model a line with slope $\frac{1}{2}$.

Solution

To model a line on a geoboard, we need the rise and the run.

Use the slope formula.	$m = \frac{\text{rise}}{\text{run}}$
Replace m with $\frac{1}{2}$.	$\frac{1}{2} = \frac{\text{rise}}{\text{run}}$

So, the rise is 1 and the run is 2 .
Start at a peg in the lower left of the geoboard.
Stretch the rubber band up 1 unit, and then right 2 units.

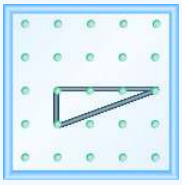


The hypotenuse of the right triangle formed by the rubber band represents a line whose slope is $\frac{1}{2}$.

Try It 1.14.8

Model the slope $m = \frac{1}{3}$. Draw a picture to show your results.

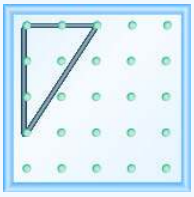
Answer



Try It 1.14.9

Model the slope $m = \frac{3}{2}$. Draw a picture to show your results.

Answer

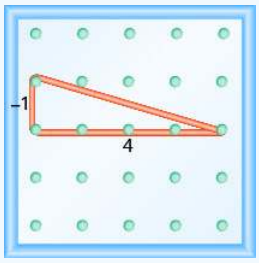


Example 1.14.10

Use a geoboard to model a line with slope $\frac{-1}{4}$.

Solution

Use the slope formula. m = $\frac{\text{rise}}{\text{run}}$.
Replace m with $\frac{-1}{4}$.
So, the rise is -1 and the run is 4.
Since the rise is negative, we choose a starting peg on the upper left that will give us room to count down.
We stretch the rubber band down 1 unit, and then right 4 units.

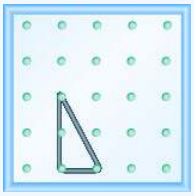


The hypotenuse of the right triangle formed by the rubber band represents a line whose slope is $\frac{-1}{4}$.

Try It 1.14.11

Model the slope $m = -2 = \frac{-2}{1}$. Draw a picture to show your results.

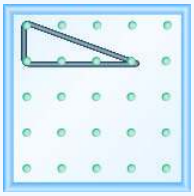
Answer



Try It 1.14.12

Model the slope $m = \frac{-1}{3}$. Draw a picture to show your results.

Answer



Use $m = \frac{\text{rise}}{\text{run}}$ to Find the Slope of a Line from its Graph

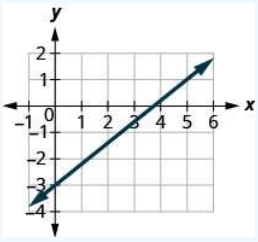
Now, we'll look at some graphs on the xy-coordinate plane and see how to find their slopes. The method will be very similar to what we just modeled on our geoboards.

To find the slope, we must count out the rise and the run. But where do we start?

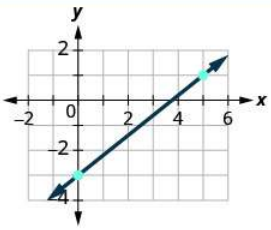
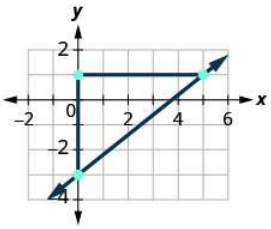
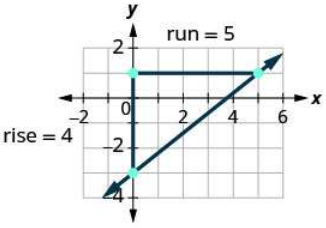
We locate two points on the line whose coordinates are integers. We then start with the point on the left and sketch a right triangle, so we can count the rise and run.

Example 1.14.13:

Find the slope of the line shown.

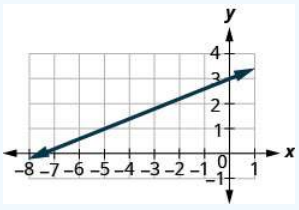


Solution

Step 1. Locate two points on the graph whose coordinates are integers.	Mark (0, –3) and (5, 1).	
Step 2. Starting with the point on the left, sketch a right triangle, going from the first point to the second point.	Starting at (0, –3), sketch a right triangle to (5, 1).	
Step 3. Count the rise and the run on the legs of the triangle.	Count the rise. Count the run.	 The rise is 4. The run is 5.
Step 4. Take the ratio of rise to run to find the slope. $m = \frac{\text{rise}}{\text{run}}$	Use the slope formula. Substitute the values of the rise and run.	$m = \frac{\text{rise}}{\text{run}}$ $m = \frac{4}{5}$ The slope of the line is $\frac{4}{5}$. This means that y increases 4 units as x increases 5 units.

Try It 1.14.14

Find the slope of the line shown.

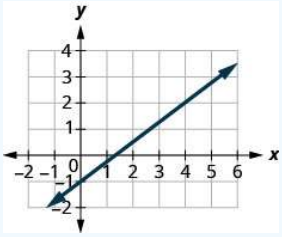


Answer

$\frac{2}{5}$

Try It 1.14.15

Find the slope of the line shown.



Answer

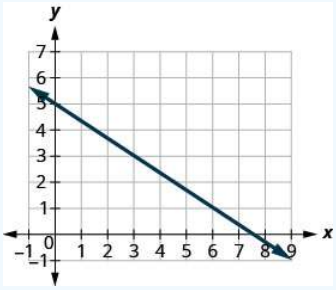
$\frac{3}{4}$

FIND THE SLOPE OF A LINE FROM ITS GRAPH

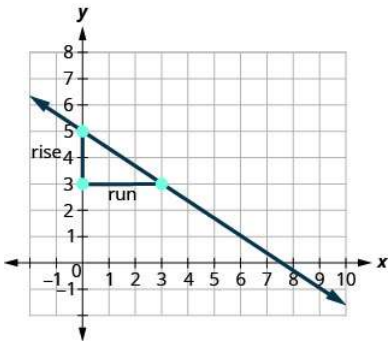
1. Locate two points on the line whose coordinates are integers.
2. Starting with the point on the left, sketch a right triangle, going from the first point to the second point.
3. Count the rise and the run on the legs of the triangle.
4. Take the ratio of rise to run to find the slope, $m = \frac{\text{rise}}{\text{run}}$.

Example 1.14.16

Find the slope of the line shown.



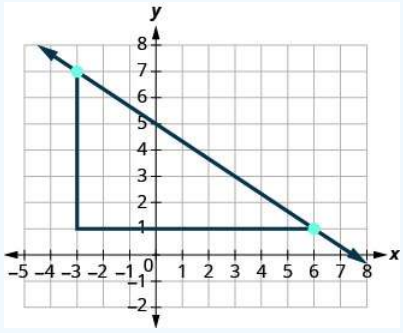
Solution

Locate two points on the graph whose coordinates are integers.	(0,5) and (3,3)
Which point is on the left?	(0,5)
Starting at (0,5), sketch a right triangle to (3,3).	
Count the rise—it is negative.	The rise is −2.
Count the run.	The run is 3.

Use the slope formula.	$m = \frac{\text{rise}}{\text{run}}$
Substitute the values of the rise and run.	$m = \frac{-2}{3}$
Simplify.	$m = -\frac{2}{3}$
	The slope of the line is $-\frac{2}{3}$.

So y increases by 3 units as x decreases by 2 units.

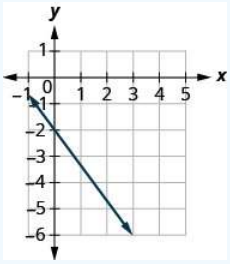
What if we used the points (−3,7) and (6,1) to find the slope of the line?



The rise would be −6 and the run would be 9. Then $m = \frac{-6}{9}$, and that simplifies to $m = -\frac{2}{3}$. Remember, it does not matter which points you use—the slope of the line is always the same.

Try It 1.14.17

Find the slope of the line shown.

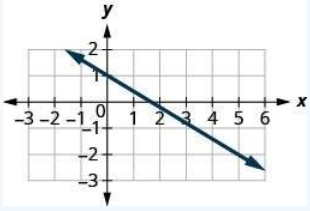


Answer

$-\frac{4}{3}$

Try It 1.14.18

Find the slope of the line shown.



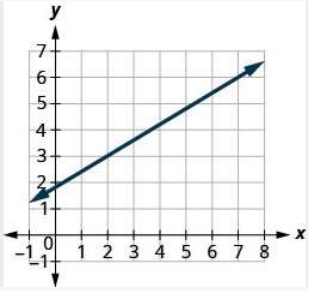
Answer

$-\frac{3}{5}$

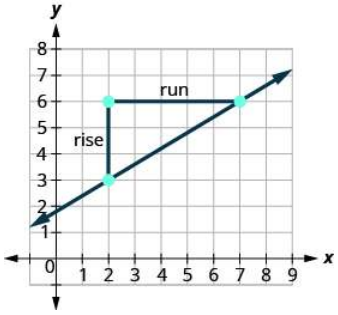
In the last two examples, the lines had *y*-intercepts with integer values, so it was convenient to use the *y*-intercept as one of the points to find the slope. In the next example, the *y*-intercept is a fraction. Instead of using that point, we'll look for two other points whose coordinates are integers. This will make the slope calculations easier.

Example 1.14.19

Find the slope of the line shown.



Solution

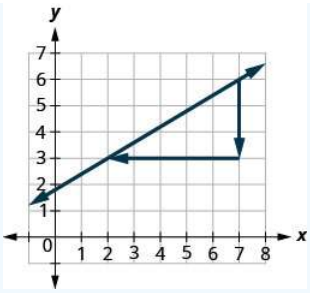
Locate two points on the graph whose coordinates are integers.	(2,3) and (7,6)
Which point is on the left?	(2,3)
Starting at (2,3), sketch a right triangle to (7,6).	
Count the rise.	The rise is 3.
Count the run.	The run is 5.
Use the slope formula.	$m = \frac{\text{rise}}{\text{run}}$
Substitute the values of the rise and run.	$m = \frac{3}{5}$
	The slope of the line is $\frac{3}{5}$.

This means that y increases 5 units as x increases 3 units.

When we used geoboards to introduce the concept of slope, we said that we would always start with the point on the left and count the rise and the run to get to the point on the right. That way the run was always positive and the rise determined whether the slope was positive or negative.

What would happen if we started with the point on the right?

Let's use the points (2,3) and (7,6) again, but now we'll start at (7,6).

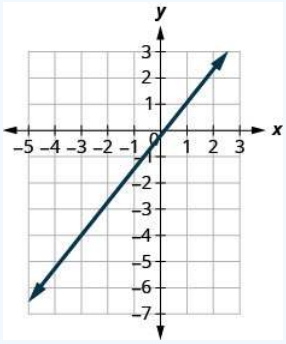


Count the rise.	The rise is -3.
Count the run. It goes from right to left, so it is negative.	The run is-5.
Use the slope formula.	$m = \frac{\text{rise}}{\text{run}}$
Substitute the values of the rise and run.	$m = \frac{-3}{-5}$
	The slope of the line is $\frac{3}{5}$

It does not matter where you start—the slope of the line is always the same.

? Try It 1.14.20

Find the slope of the line shown.

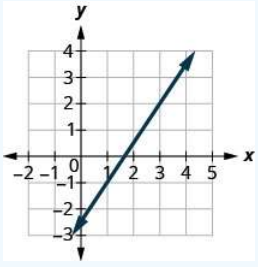


Answer

$\frac{5}{4}$

? Try It 1.14.21

Find the slope of the line shown.



Answer

$\frac{3}{2}$

Find the Slope of Horizontal and Vertical Lines

Do you remember what was special about horizontal and vertical lines? Their equations had just one variable.

Horizontal line $y = b$	**Vertical line** $x = a$
y -coordinates are the same.	x -coordinates are the same.

(1.14.3)

So how do we find the slope of the horizontal line y=4? One approach would be to graph the horizontal line, find two points on it, and count the rise and the run. Let's see what happens when we do this.

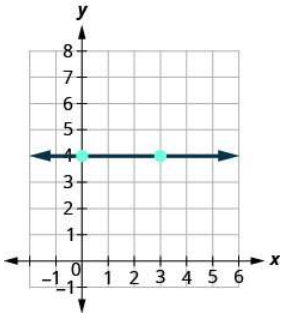


Figure 1.14.7

What is the rise?	The rise is 0.
What is the run?	The run is 3.
	$m = \frac{\text{rise}}{\text{run}}$
	$m = \frac{0}{3}$
What is the slope?	$m = 0$
	The slope of the horizontal line y = 4 is 0.

All horizontal lines have slope 0. When the *y*-coordinates are the same, the rise is 0.

SLOPE OF A HORIZONTAL LINE

The slope of a horizontal line, y=b, is 0.

The floor of your room is horizontal. Its slope is 0. If you carefully placed a ball on the floor, it would not roll away.

Now, we'll consider a vertical line, the line.

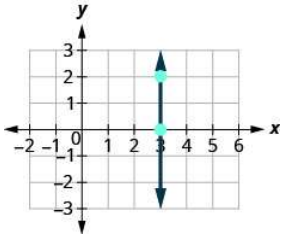


Figure 1.14.8

What is the rise?	The rise is 2.
What is the run?	The run is 0.
	$m = \frac{\text{rise}}{\text{run}}$
What is the slope?	$m = \frac{2}{0}$

But we can't divide by 0. Division by 0 is not defined. So we say that the slope of the vertical line x=3 is undefined.

The slope of any vertical line is undefined. When the *x*-coordinates of a line are all the same, the run is 0.

SLOPE OF A VERTICAL LINE

The slope of a vertical line, x=a, is undefined.

Example 1.14.22

Find the slope of each line:

(a) x=8 (b) y=−5.

Solution

(a) x=8
This is a vertical line.
Its slope is undefined.

(b) y=−5
This is a horizontal line.
It has slope 0.

Try It 1.14.23

Find the slope of the line: x=−4.

Answer

undefined

Try It 1.14.24

Find the slope of the line: y=7.

Answer

0

QUICK GUIDE TO THE SLOPES OF LINES

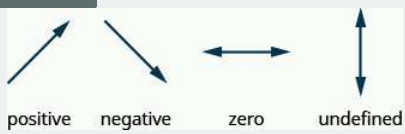


Figure 1.14.9

Remember, we ‘read’ a line from left to right, just like we read written words in English.

Use the Slope Formula to find the Slope of a Line Between Two Points

Doing the Manipulative Mathematics activity “Slope of Lines Between Two Points” will help you develop a better understanding of how to find the slope of a line between two points.

Sometimes we’ll need to find the slope of a line between two points when we don’t have a graph to count out the rise and the run. We could plot the points on grid paper, then count out the rise and the run, but as we’ll see, there is a way to find the slope without graphing. Before we get to it, we need to introduce some algebraic notation.

We have seen that an ordered pair (x,y) gives the coordinates of a point. But when we work with slopes, we use two points. How can the same symbol (x,y) be used to represent two different points? Mathematicians use subscripts to distinguish the points.

$$\begin{aligned}(x_1, y_1) &\quad \text{read } `x \text{ sub } 1, y \text{ sub } 1'\\ (x_2, y_2) &\quad \text{read } `x \text{ sub } 2, y \text{ sub } 2'\end{aligned} \tag{1.14.4}$$

The use of subscripts in math is very much like the use of last name initials in elementary school. Maybe you remember Laura C. and Laura M. in your third grade class?

We will use (x_1, y_1) to identify the first point and (x_2, y_2) to identify the second point.

If we had more than two points, we could use (x_3, y_3), (x_4, y_4), and so on.

Let's see how the rise and run relate to the coordinates of the two points by taking another look at the slope of the line between the points (2,3) and (7,6).

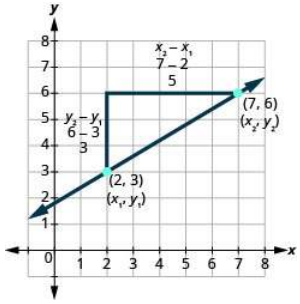


Figure 1.14.10

Since we have two points, we will use subscript notation, $\begin{pmatrix} x_1, y_1 \\ 2, 3 \end{pmatrix} \begin{pmatrix} x_2, y_2 \\ 7, 6 \end{pmatrix}$.

On the graph, we counted the rise of 3 and the run of 5.

Notice that the rise of 3 can be found by subtracting the *y*-coordinates 6 and 3.

$$3 = 6 - 3 \tag{1.14.5}$$

And the run of 5 can be found by subtracting the *x*-coordinates 7 and 2.

$$5 = 7 - 2 \tag{1.14.6}$$

We know $m = \frac{\text{rise}}{\text{run}}$. So $m = \frac{3}{5}$.

We rewrite the rise and run by putting in the coordinates $m = \frac{6-3}{7-2}$

But 6 is y_2, the *y*-coordinate of the second point and 3 is y_1, the *y*-coordinate of the first point.

So we can rewrite the slope using subscript notation. $m = \frac{y2-y1}{7-2}$

Also, 7 is x_2, the *x*-coordinate of the second point and 2 is x_1, the *x*-coordinate of the first point.

So, again, we rewrite the slope using subscript notation. $m = \frac{y2-y1}{x2-x1}$

We've shown that $m = \frac{y2-y1}{x2-x1}$ is really another version of $m = \frac{\text{rise}}{\text{run}}$. We can use this formula to find the slope of a line when we have two points on the line.

SLOPE FORMULA

The slope of the line between two points (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1} \tag{1.14.7}$$

This is the **slope formula**.

The slope is:

y of the second point minus y of the first point
over
x of the second point minus x of the first point. (1.14.8)

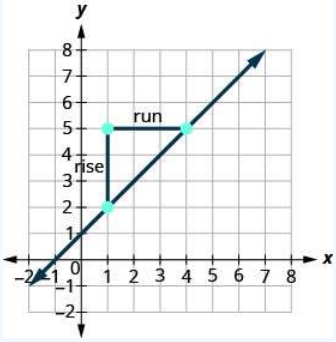
Example 1.14.25

Use the **slope formula** to find the slope of the line between the points (1,2) and (4,5).

Solution

We'll call (1,2) point #1 and (4,5) point #2.	$\begin{pmatrix} x_1, y_1 \\ 1, 2 \end{pmatrix} \begin{pmatrix} x_2, y_2 \\ 4, 5 \end{pmatrix}$
Use the slope formula.	$m = \frac{y_2 - y_1}{x_2 - x_1}$
Substitute the values.	
y of the second point minus y of the first point	$m = \frac{5-2}{x_2 - x_1}$
x of the second point minus x of the first point	$m = \frac{5-2}{4-1}$
Simplify the numerator and the denominator.	$m = \frac{3}{3}$
Simplify.	$m = 1$

Let's confirm this by counting out the slope on a graph using $m = \frac{\text{rise}}{\text{run}}$.



It doesn't matter which point you call point #1 and which one you call point #2. The slope will be the same. Try the calculation yourself.

Try It 1.14.26

Use the slope formula to find the slope of the line through the points: (8,5) and (6,3).

Answer

1

Try It 1.14.27

Use the slope formula to find the slope of the line through the points: (1,5) and (5,9).

Answer

1

Example 1.14.28

Use the slope formula to find the slope of the line through the points (−2,−3) and (−7,4).

Solution

We'll call (-2, -3) point #1 and (-7,4) point #2. $\begin{pmatrix} x_1, y_1 \\ -2, -3 \end{pmatrix} \begin{pmatrix} x_2, y_2 \\ -7, 4 \end{pmatrix}$

Use the slope formula. $m = \frac{y_2 - y_1}{x_2 - x_1}$

Substitute the values.

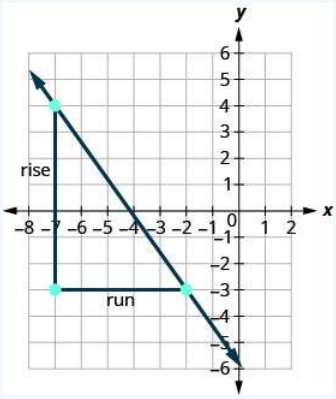
y of the second point minus y of the first point $m = \frac{4-(-3)}{x_2 - x_1}$

x of the second point minus x of the first point $m = \frac{4-(-3)}{-7-(-2)}$

Simplify the numerator and the denominator. $m = \frac{7}{-5}$

Simplify. $m = -\frac{7}{5}$

Let's verify this slope on the graph shown.



$$m = \frac{\text{rise}}{\text{run}}$$
$$m = \frac{-7}{5}$$
$$m = -\frac{7}{5}$$

Try It 1.14.29

Use the slope formula to find the slope of the line through the points: (−3,4) and (2,−1).

Answer

-1

Try It 1.14.30

Use the slope formula to find the slope of the line through the pair of points: (−2,6) and (−3,−4).

Answer

10

Graph a Line Given a Point and the Slope

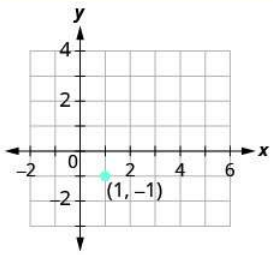
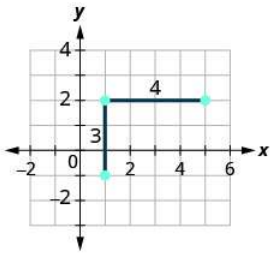
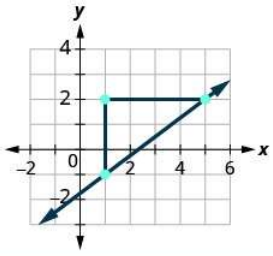
Up to now, in this chapter, we have graphed lines by plotting points, by using intercepts, and by recognizing horizontal and vertical lines.

One other method we can use to graph lines is called the **point–slope method**. We will use this method when we know one point and the slope of the line. We will start by plotting the point and then use the definition of slope to draw the graph of the line.

Example 1.14.31

Graph the line passing through the point $(1,-1)$ whose slope is $m = \frac{3}{4}$.

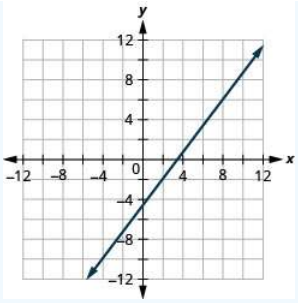
Solution

Step 1. Plot the given point.	Plot (1, –1).	
Step 2. Use the slope formula $m = \frac{\text{rise}}{\text{run}}$ to identify the rise and the run.	Identify the rise and the run.	$m = \frac{3}{4}$ $\frac{\text{rise}}{\text{run}} = \frac{3}{4}$ rise = 3 run = 4
Step 3. Starting at the given point, count out the rise and run to mark the second point.	Start at (1, –1) and count the rise and the run. Up 3 units, right 4 units.	
Step 4. Connect the points with a line.	Connect the two points with a line.	

Try It 1.14.32

Graph the line passing through the point $(2,-2)$ with the slope $m = \frac{4}{3}$.

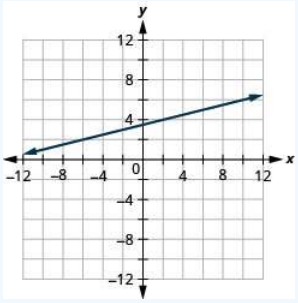
Answer



Try It 1.14.33

Graph the line passing through the point (−2,3) with the slope $m = \frac{1}{4}$.

Answer



GRAPH A LINE GIVEN A POINT AND THE SLOPE.

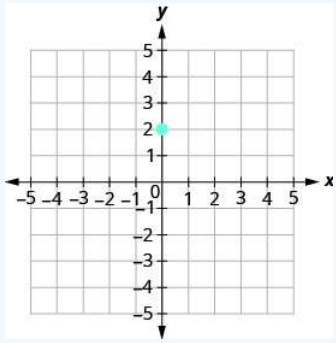
1. Plot the given point.
2. Use the slope formula $m = \frac{\text{rise}}{\text{rise}}$ to identify the rise and the run.
3. Starting at the given point, count out the rise and run to mark the second point.
4. Connect the points with a line.

Example 1.14.34

Graph the line with *y*-intercept 2 whose slope is $m = -\frac{2}{3}$.

Solution

Plot the given point, the *y*-intercept, (0,2).

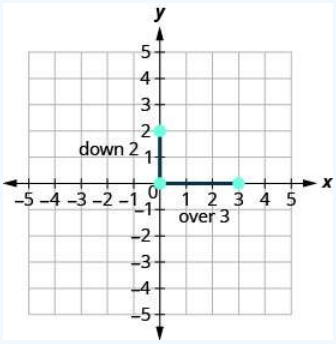


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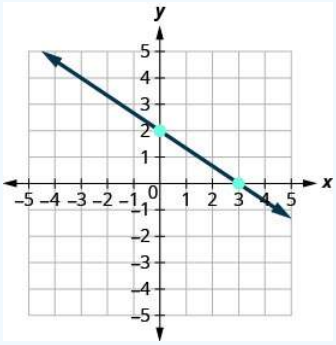
Identify the rise and the run. $m = -\frac{2}{3}$

$$\frac{\text{rise}}{\text{run}} = \frac{-2}{3}$$
$$\text{rise} = -2$$
$$\text{run} = 3$$

Count the rise and the run. Mark the second point.



Connect the two points with a line.

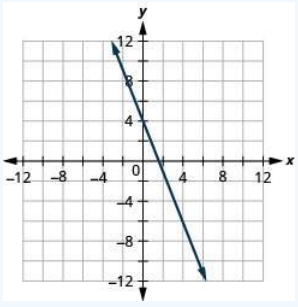


You can check your work by finding a third point. Since the slope is $m = -\frac{2}{3}$, it can be written as $m = \frac{2}{-3}$. Go back to (0,2) and count out the rise, 2, and the run, −3.

? Try It 1.14.35

Graph the line with the *y*-intercept 4 and slope $m = -\frac{5}{2}$.

Answer

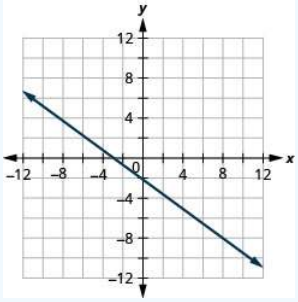


? Try It 1.14.36

Graph the line with the *x*-intercept −3 and slope $m = -\frac{3}{4}$.

Answer

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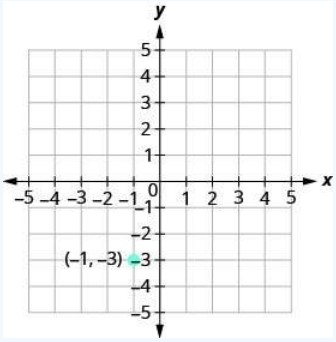


Example 1.14.37

Graph the line passing through the point $(-1,-3)$ whose slope is $m=4$.

Solution

Plot the given point.

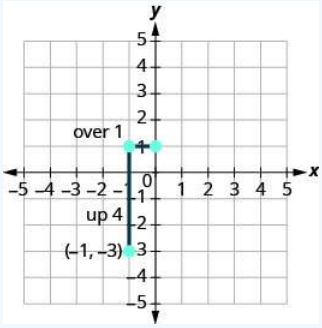


Identify the rise and the run. $m = 4$

Write 4 as a fraction. $\frac{\text{rise}}{\text{run}} = \frac{4}{1}$

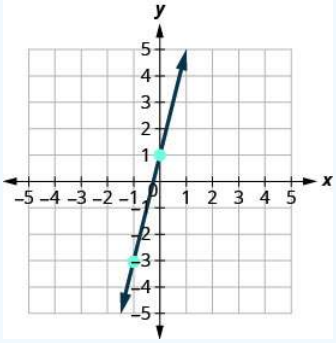
$\text{rise} = 4 \quad \text{run} = 1$

Count the rise and run and mark the second point.



Connect the two points with a line.

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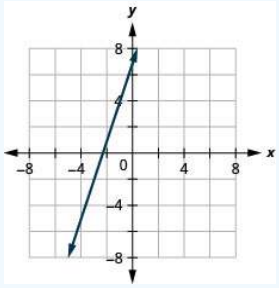


You can check your work by finding a third point. Since the slope is m=4, it can be written as $m = \frac{-4}{-1}$. Go back to (−1,−3) and count out the rise, −4, and the run, −1.

Try It 1.14.38

Graph the line with the point (−2,1) and slope m=3.

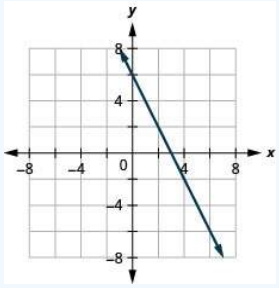
Answer



Try It 1.14.39

Graph the line with the point (4,−2) and slope m=−2.

Answer

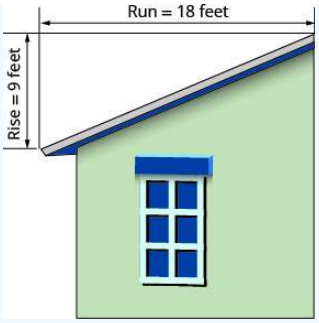


Solve Slope Applications

At the beginning of this section, we said there are many applications of slope in the real world. Let’s look at a few now.

Example 1.14.40

The ‘pitch’ of a building’s roof is the slope of the roof. Knowing the pitch is important in climates where there is heavy snowfall. If the roof is too flat, the weight of the snow may cause it to collapse. What is the slope of the roof shown?



Solution

Use the slope formula.	$m = \frac{\text{rise}}{\text{rise}}$
Substitute the values for rise and run.	$m = \frac{9}{18}$
Simplify.	$m = \frac{1}{2}$

The slope of the roof is $\frac{1}{2}$.

The roof rises 1 foot for every 2 feet of horizontal run.

Try It 1.14.41

Use Exercise 1.14.40, substituting the rise = 14 and run = 24.

Answer

$\frac{7}{12}$

Try It 1.14.42

Use Exercise 1.14.40, substituting rise = 15 and run = 36.

Answer

$\frac{5}{12}$

Example 1.14.43

Have you ever thought about the sewage pipes going from your house to the street? They must slope down $\frac{1}{4}$ inch per foot in order to drain properly. What is the required slope?



Solution

Use the slope formula.	$m = \frac{\text{rise}}{\text{run}}$
	$m = \frac{-\frac{1}{4}\text{inch}}{1\text{ foot}}$
	$m = \frac{-\frac{1}{4}\text{ inch}}{12\text{ inches}}$
Simplify.	$m = -\frac{1}{48}$
	The slope of the pipe is $-\frac{1}{48}$

The pipe drops 1 inch for every 48 inches of horizontal run.

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? Try It 1.14.44

Find the slope of a pipe that slopes down $\frac{1}{3}$ inch per foot.

Answer

$-\frac{1}{36}$

? Try It 1.14.45

Find the slope of a pipe that slopes down $\frac{3}{4}$ inch per yard.

Answer

$-\frac{1}{48}$

Access these online resources for additional instruction and practice with understanding slope of a line.

- Practice Slope with a Virtual Geoboard
- Small, Medium, and Large Virtual Geoboards
- Explore Area and Perimeter with a Geoboard

Key Concepts

- **Find the Slope of a Line from its Graph using $m = \frac{\text{rise}}{\text{run}}$**
  1. Locate two points on the line whose coordinates are integers.
  2. Starting with the point on the left, sketch a right triangle, going from the first point to the second point.
  3. Count the rise and the run on the legs of the triangle.
  4. Take the ratio of rise to run to find the slope.
- **Graph a Line Given a Point and the Slope**
  1. Plot the given point.
  2. Use the slope formula $m = \frac{\text{rise}}{\text{run}}$ to identify the rise and the run.
  3. Starting at the given point, count out the rise and run to mark the second point.
  4. Connect the points with a line.
- **Slope of a Horizontal Line**
  - The slope of a horizontal line, y=b, is 0.
- **Slope of a vertical line**
  - The slope of a vertical line, x=a, is undefined

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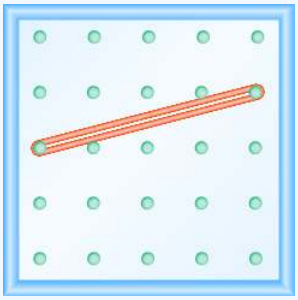
1.14E: Exercises

Practice Makes Perfect

Use Geoboards to Model Slope

In the following exercises, find the slope modeled on each geoboard.

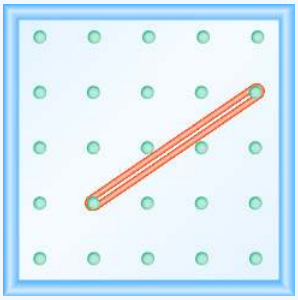
Exercise 1.14E.1



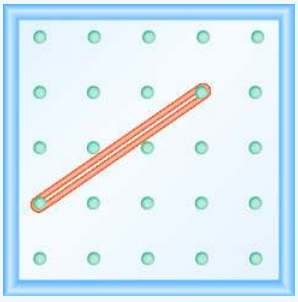
Answer

$\frac{1}{4}$

Exercise 1.14E.2



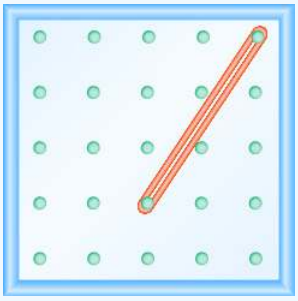
Exercise 1.14E.3



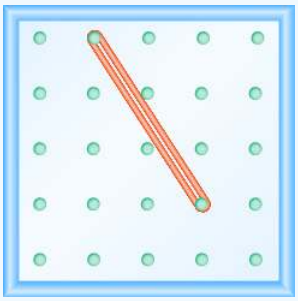
Answer

$\frac{2}{3}$

Exercise 1.14E.4



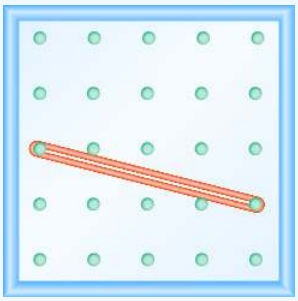
Exercise 1.14E.5



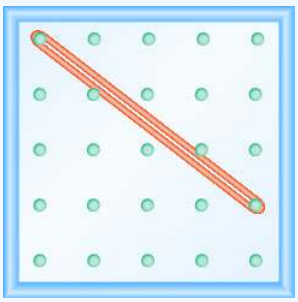
Answer

$\frac{-3}{2} = -\frac{3}{2}$

Exercise 1.14E.6



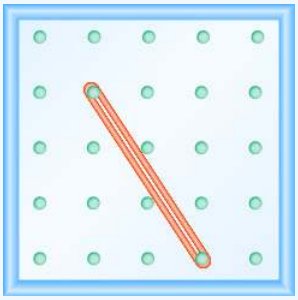
Exercise 1.14E.7



Answer

$-\frac{2}{3}$

Exercise 1.14E.8

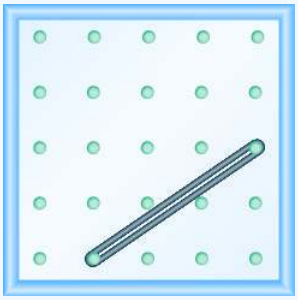


In the following exercises, model each slope. Draw a picture to show your results.

Exercise 1.14E.9

$\frac{2}{3}$

Answer



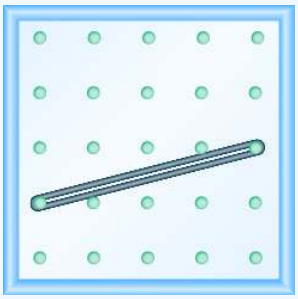
Exercise 1.14E.10

$\frac{3}{4}$

Exercise 1.14E.11

$\frac{1}{4}$

Answer



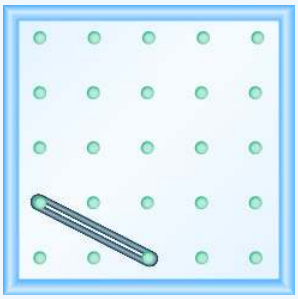
Exercise 1.14E.12

$\frac{4}{3}$

Exercise 1.14E.13

$-\frac{1}{2}$

Answer



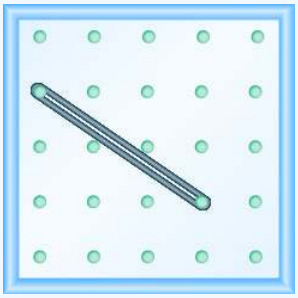
Exercise 1.14E.14

$-\frac{3}{4}$

Exercise 1.14E.15

$-\frac{2}{3}$

Answer



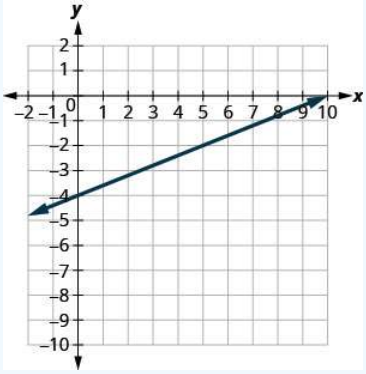
Exercise 1.14E.16

$-\frac{3}{2}$

Use $m = \frac{rise}{run}$ to find the Slope of a Line from its Graph

In the following exercises, find the slope of each line shown.

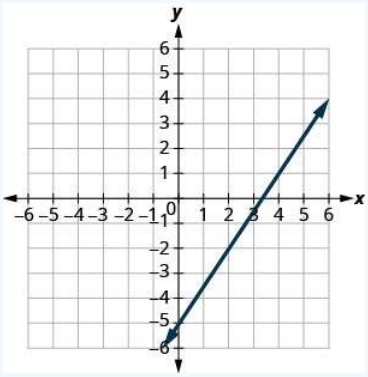
Exercise 1.14E.17



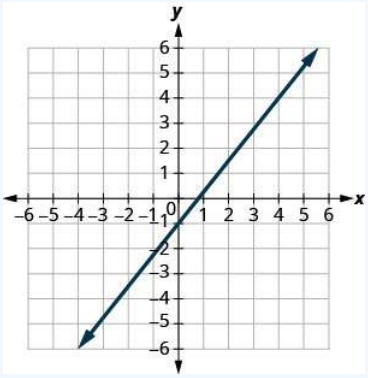
Answer

$\frac{2}{5}$

Exercise 1.14E. 18



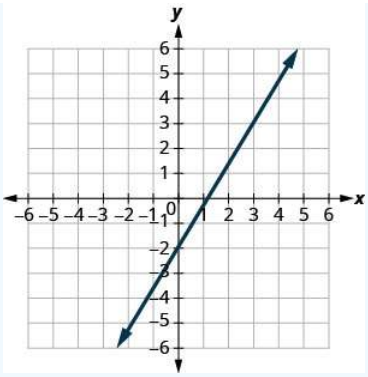
Exercise 1.14E. 19



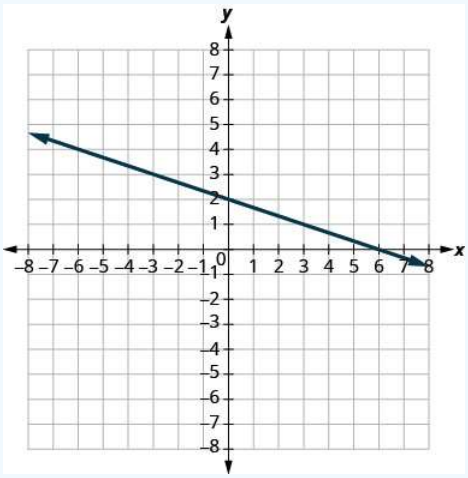
Answer

$\frac{5}{4}$

Exercise 1.14E. 20



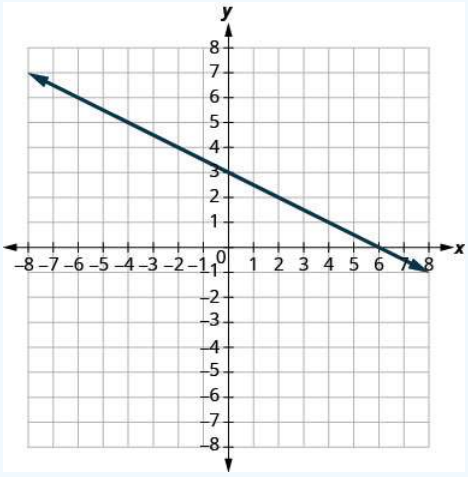
Exercise 1.14E.21



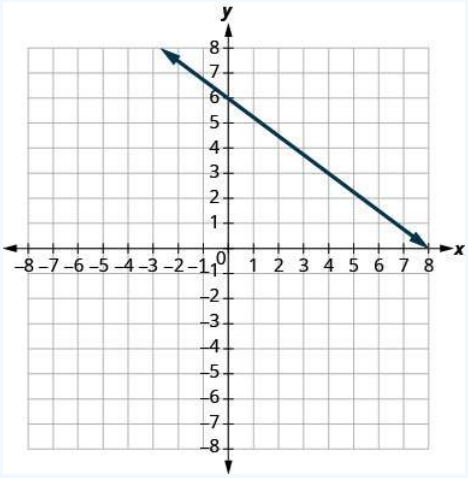
Answer

$-\frac{1}{3}$

Exercise 1.14E.22



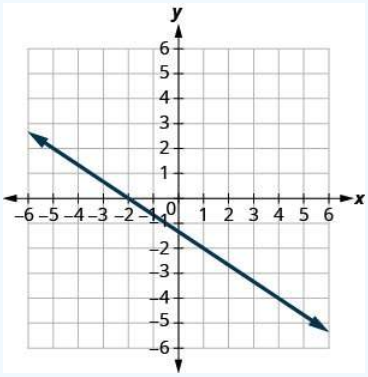
Exercise $1.14E.23$



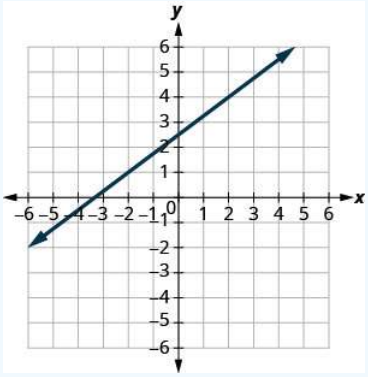
Answer

$-\frac{3}{4}$

Exercise $1.14E.24$



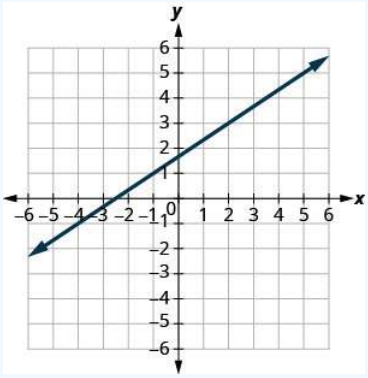
Exercise $1.14E.25$



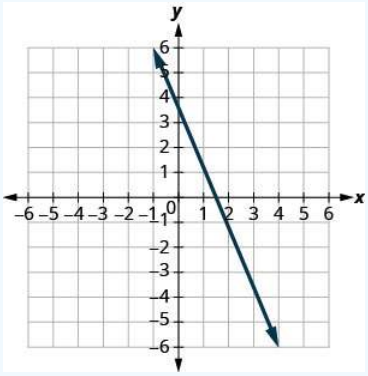
Answer

$\frac{3}{4}$

Exercise $1.14E.26$



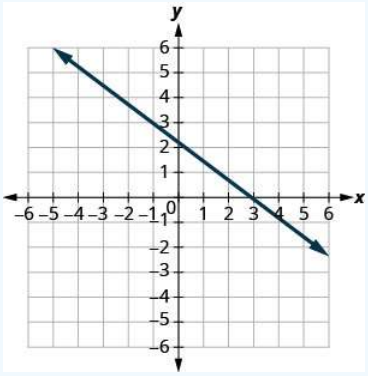
Exercise $1.14E.27$



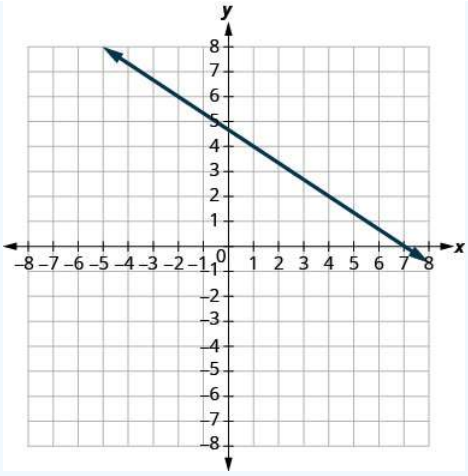
Answer

$-\frac{5}{2}$

Exercise $1.14E.28$



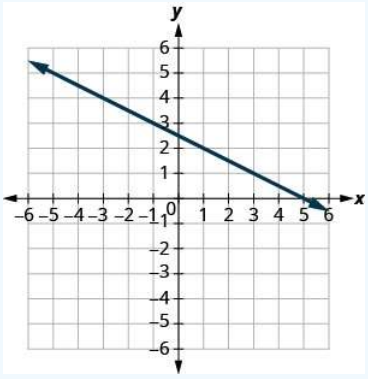
Exercise $1.14E.29$



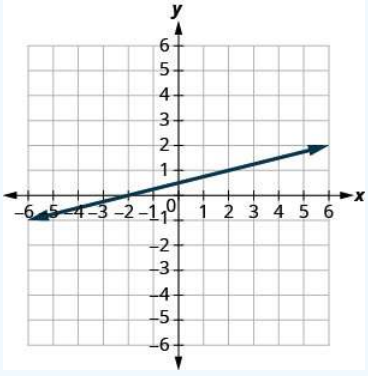
Answer

$-\frac{2}{3}$

Exercise $1.14E.30$



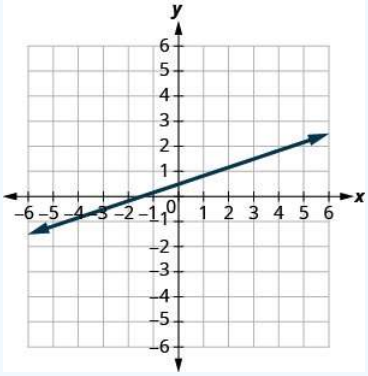
Exercise $1.14E.31$



Answer

$\frac{1}{4}$

Exercise 1.14E.32



Find the Slope of Horizontal and Vertical Lines

In the following exercises, find the slope of each line.

Exercise 1.14E.33

y=3

Answer

0

Exercise 1.14E.34

y=1

Exercise 1.14E.35

x=4

Answer

undefined

Exercise 1.14E.36

x=2

Exercise 1.14E.37

y=−2

Answer

0

Exercise 1.14E.38

y=−3

Exercise 1.14E. 39

x=−5

Answer

undefined

Exercise 1.14E. 40

x=−4

Use the Slope Formula to find the Slope of a Line between Two Points

In the following exercises, use the slope formula to find the slope of the line between each pair of points.

Exercise 1.14E. 41

(1,4),(3,9)

Answer

$\frac{5}{2}$

Exercise 1.14E. 42

(2,3),(5,7)

Exercise 1.14E. 43

(0,3),(4,6)

Answer

$\frac{3}{4}$

Exercise 1.14E. 44

(0,1),(5,4)

Exercise 1.14E. 45

(2,5),(4,0)

Answer

$-\frac{5}{2}$

Exercise 1.14E. 46

(3,6),(8,0)

Exercise 1.14E. 47

(−3,3),(4,−5)

Answer

$-\frac{8}{7}$

Exercise 1.14E.48

(−2,4),(3,−1)

Exercise 1.14E.49

(−1,−2),(2,5)

Answer

$\frac{7}{3}$

Exercise 1.14E.50

(−2,−1),(6,5)

Exercise 1.14E.51

(4,−5),(1,−2)

Answer

−1

Exercise 1.14E.52

(3,−6),(2,−2)

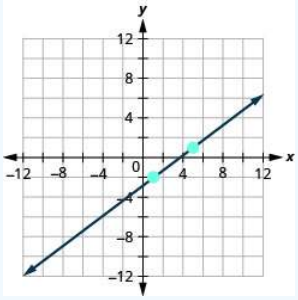
Graph a Line Given a Point and the Slope

In the following exercises, graph each line with the given point and slope.

Exercise 1.14E.53

$(1,-2); m=\frac{3}{4}$

Answer



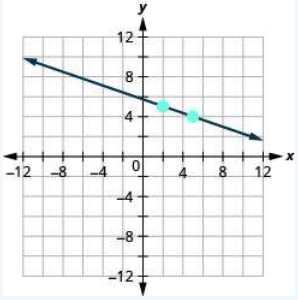
Exercise 1.14E.54

$(1,-1); m=\frac{2}{3}$

Exercise 1.14E.55

$(2, 5); m = -\frac{1}{3}$

Answer



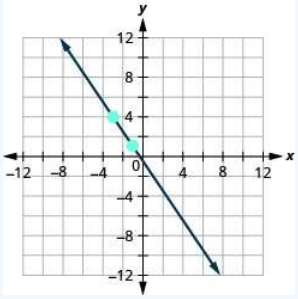
Exercise 1.14E.56

$(1, 4); m = -\frac{1}{2}$

Exercise 1.14E.57

$(-3, 4); m = -\frac{3}{2}$

Answer



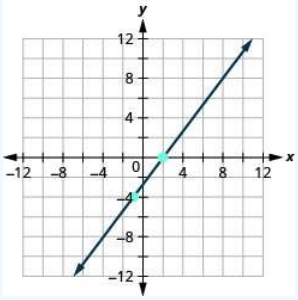
Exercise 1.14E.58

$(-2, 5); m = -\frac{5}{4}$

Exercise 1.14E.59

$(-1, -4); m = \frac{4}{3}$

Answer



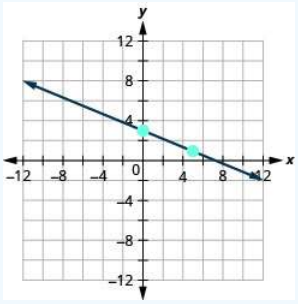
Exercise 1.14E.60

$(-3, -5); m = \frac{3}{2}$

Exercise 1.14E.61

y -intercept $3; m = -\frac{2}{5}$

Answer



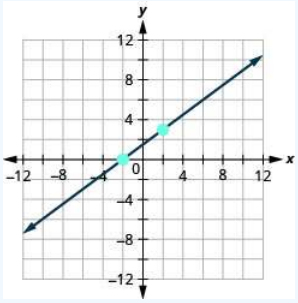
Exercise 1.14E.62

y -intercept $5; m = -\frac{4}{3}$

Exercise 1.14E.63

x -intercept $-2; m = \frac{3}{4}$

Answer



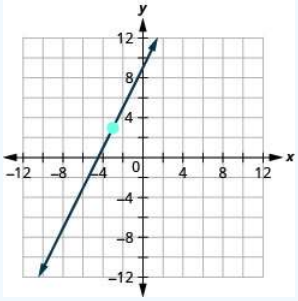
Exercise 1.14E.64

x -intercept $-1; m = \frac{1}{5}$

Exercise 1.14E.65

$(-3, 3); m = 2$

Answer



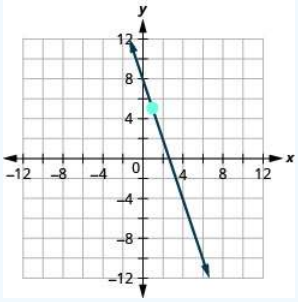
Exercise 1.14E.66

$(-4, 2); m = 4$

Exercise 1.14E.67

$(1, 5); m = -3$

Answer



Exercise 1.14E.68

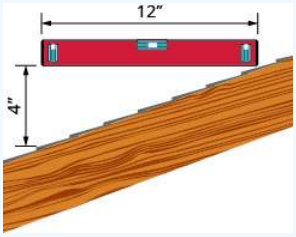
$(2, 3); m = -1$

Everyday Math

Exercise 1.14E.69

Slope of a roof. An easy way to determine the slope of a roof is to set one end of a 12 inch level on the roof surface and hold it level. Then take a tape measure or ruler and measure from the other end of the level down to the roof surface. This will give you the slope of the roof. Builders, sometimes, refer to this as pitch and state it as an "x 12 pitch" meaning $\frac{x}{12}$, where x is the measurement from the roof to the level—the rise. It is also sometimes stated as an "x-in-12 pitch".

1. What is the slope of the roof in this picture?
2. What is the pitch in construction terms?

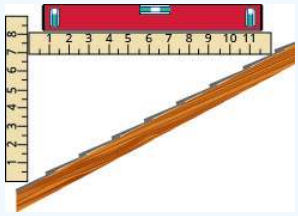


Answer

1. $\frac{1}{3}$
2. 4 12 pitch or 4-in-12 pitch

Exercise 1.14E.70

The slope of the roof shown here is measured with a 12” level and a ruler. What is the slope of this roof?



Exercise 1.14E.71

Road grade. A local road has a grade of 6%. The grade of a road is its slope expressed as a percent. Find the slope of the road as a fraction and then simplify. What rise and run would reflect this slope or grade?

Answer

$\frac{3}{50}$; rise $= 3$, run $= 50$

Exercise 1.14E.72

Highway grade. A local road rises 2 feet for every 50 feet of highway.

1. What is the slope of the highway?
2. The grade of a highway is its slope expressed as a percent. What is the grade of this highway?

Exercise 1.14E.73

Wheelchair ramp. The rules for wheelchair ramps require a maximum 1-inch rise for a 12-inch run.

1. How long must the ramp be to accommodate a 24-inch rise to the door?
2. Create a model of this ramp.

Answer

1. 288 inches (24 feet)
2. Models will vary.

Exercise 1.14E.74

Wheelchair ramp. A 1-inch rise for a 16-inch run makes it easier for the wheelchair rider to ascend a ramp.

1. How long must a ramp be to easily accommodate a 24-inch rise to the door?
2. Create a model of this ramp.

Writing Exercises

Exercise 1.14E.75

What does the sign of the slope tell you about a line?

Answer

When the slope is a positive number the line goes up from left to right. When the slope is a negative number the line goes down from left to right.

Exercise 1.14E.76

How does the graph of a line with slope $m = \frac{1}{2}$ differ from the graph of a line with slope $m = 2$?

Exercise 1.14E.77

Why is the slope of a vertical line "undefined"?

Answer

A vertical line has 0 run and since division by 0 is undefined the slope is undefined.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it
use geoboards to model slope.			
use $m = \frac{rise}{run}$ to find the slope of a line from its graph.			
find the slope of horizontal and vertical lines.			
use the slope formula to find the slope of a line between two points.			
graph a line given a point and the slope.			
solve slope applications.			

ⓑ On a scale of 1–10, how would you rate your mastery of this section in light of your responses on the checklist? How can you improve this?

Glossary

geoboard

A geoboard is a board with a grid of pegs on it.

negative slope

A negative slope of a line goes down as you read from left to right.

positive slope

A positive slope of a line goes up as you read from left to right.

rise

The rise of a line is its vertical change.

run

The run of a line is its horizontal change.

slope formula

The slope of the line between two points (x_1, y_1) and (x_2, y_2) is $m = \frac{y_2 - y_1}{x_2 - x_1}$.

slope of a line

The slope of a line is $m = \frac{\text{rise}}{\text{run}}$. The rise measures the vertical change and the run measures the horizontal change.

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1.15: Solve Systems of Equations by Graphing

Learning Objectives

By the end of this section, you will be able to:

- Determine whether an ordered pair is a solution of a system of equations
- Solve a system of linear equations by graphing
- Determine the number of solutions of linear system
- Solve applications of systems of equations by graphing

Note

Before you get started, take this readiness quiz.

1. For the equation $y = \frac{2}{3}x - 4$
   ⓐ is (6,0) a solution? ⓑ is (−3,−2) a solution?
   If you missed this problem, review Example 2.1.1.
2. Find the slope and y-intercept of the line 3x−y=12.
   If you missed this problem, review Example 4.5.7.
3. Find the x- and y-intercepts of the line 2x−3y=12.
   If you missed this problem, review Example 4.3.7.

Determine Whether an Ordered Pair is a Solution of a System of Equations

In the section on Solving Linear Equations and Inequalities we learned how to solve linear equations with one variable. Remember that the solution of an equation is a value of the variable that makes a true statement when substituted into the equation. Now we will work with **systems of linear equations**, two or more linear equations grouped together.

Definition: SYSTEM OF LINEAR EQUATIONS

When two or more linear equations are grouped together, they form a system of linear equations.

We will focus our work here on systems of two linear equations in two unknowns. Later, you may solve larger systems of equations.

An example of a system of two linear equations is shown below. We use a brace to show the two equations are grouped together to form a system of equations.

$$\begin{cases} 2x + y = 7 \\ x - 2y = 6 \end{cases} \tag{1.15.1}$$

A linear equation in two variables, like $2x + y = 7$, has an infinite number of solutions. Its graph is a line. Remember, every point on the line is a solution to the equation and every solution to the equation is a point on the line.

To solve a system of two linear equations, we want to find the values of the variables that are solutions to both equations. In other words, we are looking for the ordered pairs (x, y) that make both equations true. These are called the **solutions to a system of equations**.

Definition: SolutionS OF A SYSTEM OF EQUATIONS

Solutions of a system of equations are the values of the variables that make all the equations true. A solution of a system of two linear equations is represented by an ordered pair (x, y).

To determine if an ordered pair is a solution to a system of two equations, we substitute the values of the variables into each equation. If the ordered pair makes both equations true, it is a solution to the system.

Let's consider the system below:

$$\begin{cases} 3x-y=7 \\ x-2y=4 \end{cases} \tag{1.15.2}$$

Is the ordered pair (2,−1) a solution?

We substitute $x=2$ and $y=-1$ into both equations.

$$\begin{aligned} 3x-y &= 7 & \qquad x-2y &= 4 \\ 3(2)-(-1) &\stackrel{?}{=} 7 & \qquad 2-2(-1) &\stackrel{?}{=} 4 \\ 7 &= 7 \text{ true} & \qquad 4 &= 4 \text{ true} \end{aligned}$$

The ordered pair (2, −1) made both equations true. Therefore (2, −1) is a solution to this system.

Let's try another ordered pair. Is the ordered pair (3, 2) a solution?

We substitute $x=3$ and $y=2$ into both equations.

$$\begin{aligned} 3x-y &= 7 & \qquad x-2y &= 4 \\ 3(3)-2 &\stackrel{?}{=} 7 & \qquad 2-2(2) &\stackrel{?}{=} 4 \\ 7 &= 7 \text{ true} & \qquad -2 &= 4 \text{ false} \end{aligned}$$

The ordered pair (3, 2) made one equation true, but it made the other equation false. Since it is not a solution to **both** equations, it is not a solution to this system.

Example 1.15.1

Determine whether the ordered pair is a solution to the system: $\begin{cases} x-y=-1 \\ 2x-y=-5 \end{cases}$

1. (−2,−1)
2. (−4,−3)

Solution

1.

$$\begin{cases} x-y=-1 \\ 2x-y=-5 \end{cases}$$

We substitute $x=-2$ and $y=-1$ into both equations

$$\begin{aligned} x-y &= 1 & \qquad 2x-y &= -5 \\ -2-(-1) &= -1 & \qquad 2(-2)-(-1) &\stackrel{?}{=} -5 \\ -1 &= -1 \checkmark & \qquad -3 &= -5 \times \end{aligned}$$

(−2, −1) does not make both equations true. (−2, −1) is not a solution.

2.

We substitute $x=-4$ and $y=-3$ into both equations.

$$\begin{aligned} x-y &= -1 & \qquad 2x-y &= -5 \\ -4-(-3) &\stackrel{?}{=} -1 & \qquad 2(-4)-(-3) &\stackrel{?}{=} -5 \\ -1 &= -1 \checkmark & \qquad -5 &= -5 \checkmark \end{aligned}$$

(−4, −3) makes both equations true. (−4, −3) is a solution.

Try It 1.15.2

Determine whether the ordered pair is a solution to the system: $\begin{cases} 3x+y=0 \\ x+2y=-5 \end{cases}$

1. (1,−3)

2. (0,0)

Answer

1. yes
2. no

? Try It 1.15.3

Determine whether the ordered pair is a solution to the system: $\begin{cases} x-3y=-8 \\ -3x-y=4 \end{cases}$

1. (2,−2)
2. (−2,2)

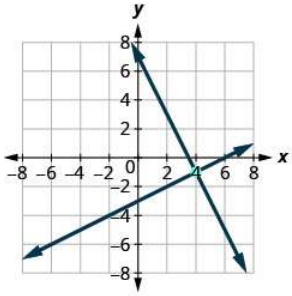
Answer

1. no
2. yes

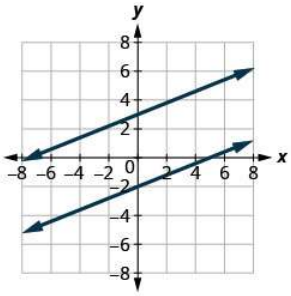
Solve a System of Linear Equations by Graphing

In this chapter we will use three methods to solve a system of linear equations. The first method we'll use is graphing. The graph of a linear equation is a line. Each point on the line is a solution to the equation. For a system of two equations, we will graph two lines. Then we can see all the points that are solutions to each equation. And, by finding what the lines have in common, we'll find the solution to the system.

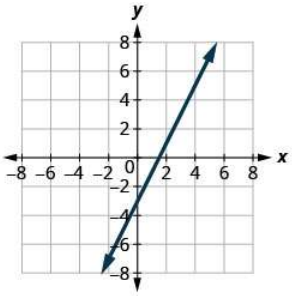
Most linear equations in one variable have one solution, but we saw that some equations, called **contradictions**, have no solutions and for other equations, called **identities**, all numbers are solutions. Similarly, when we solve a system of two linear equations represented by a graph of two lines in the same plane, there are three possible cases, as shown in Figure 1.15.1:



The lines intersect.
Intersecting lines have one point in common.
There is one solution to this system.



The lines are parallel.
Parallel lines have no points in common.
There is no solution to this system.



Both equations give the same line.
Because we have just one line, there are infinitely many solutions.

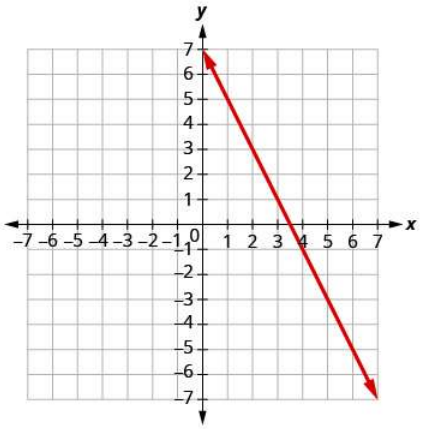
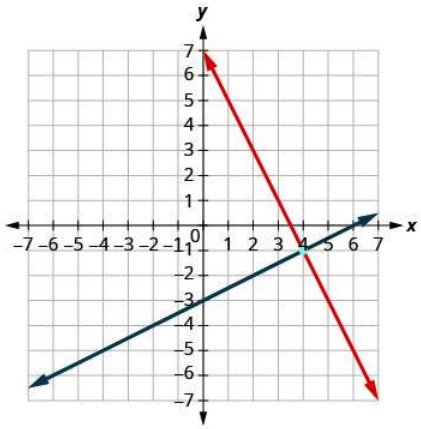
Figure 1.15.1

For the first example of solving a system of linear equations in this section and in the next two sections, we will solve the same system of two linear equations. But we'll use a different method in each section. After seeing the third method, you'll decide which method was the most convenient way to solve this system.

✔ Example 1.15.4: How to Solve a System of Linear Equations by Graphing

Solve the system by graphing: $\begin{cases} 2x+y=7 \\ x-2y=6 \end{cases}$

Solution

Step 1. Graph the first equation.	To graph the first line, write the equation in slope–intercept form. $2x + y = 7$ $y = -2x + 7$ $m = -2$ $b = 7$	$\begin{cases} 2x + y = 7 \\ x - 2y = 6 \end{cases}$ 
Step 2. Graph the second equation on the same rectangular coordinate system.	To graph the second line, use intercepts. $x - 2y = 6$ $(0, -3)$ $(6, 0)$	
Step 3. Determine whether the lines intersect, are parallel, or are the same line.	Look at the graph of the lines.	The lines intersect.
Step 4. Identify the solution to the system. If the lines intersect, identify the point of intersection. Check to make sure it is a solution to both equations. This is the solution to the system. If the lines are parallel, the system has no solution If the lines are the same, the system has an infinite number of solutions.	Since the lines intersect, find the point of intersection. Check the point in both equations.	The lines intersect at $(4, -1)$. $2x + y = 7$ $2(4) + (-1) \stackrel{?}{=} 7$ $8 - 1 \stackrel{?}{=} 7$ $7 = 7$ ✓ $x - 2y = 6$ $4 - 2(-1) \stackrel{?}{=} 6$ $6 = 6$ ✓ The solution is $(4, -1)$.

? Try It 1.15.5

Solve each system by graphing: $\begin{cases} x - 3y = -3 \\ x + y = 5 \end{cases}$

Answer

(3,2)

Try It 1.15.6

Solve each system by graphing: $\begin{cases} -x+y=1 \\ 3x+2y=12 \end{cases}$

Answer

(2,3)

The steps to use to solve a system of linear equations by graphing are shown below.

TO SOLVE A SYSTEM OF LINEAR EQUATIONS BY GRAPHING.

1. Graph the first equation.
2. Graph the second equation on the same rectangular coordinate system.
3. Determine whether the lines intersect, are parallel, or are the same line.
4. Identify the solution to the system.
   - If the lines intersect, identify the point of intersection. Check to make sure it is a solution to both equations. This is the solution to the system.
   - If the lines are parallel, the system has no solution.
   - If the lines are the same, the system has an infinite number of solutions.

Example 1.15.7

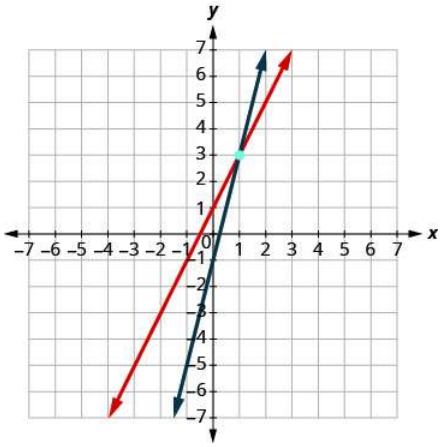
Solve the system by graphing: $\begin{cases} y=2x+1 \\ y=4x-1 \end{cases}$

Solution

Both of the equations in this system are in slope-intercept form, so we will use their slopes and y-intercepts to graph them.

$\begin{cases} y=2x+1 \\ y=4x-1 \end{cases}$

Find the slope and y-intercept of the first equation.	$y=2x+1$ $m=2$ $b=1$
Find the slope and y-intercept of the first equation.	$y=4x-1$ $m=4$ $b=-1$
Graph the two lines.	
Determine the point of intersection.	The lines intersect at (1, 3).

	
Check the solution in both equations.	$y = 2x + 1$ $\quad$ $y = 4x - 1$ $3 \stackrel{?}{=} 2 \cdot 1 + 1$ $\quad$ $3 \stackrel{?}{=} 4 \cdot 1 - 1$ $3 = 3\checkmark$ $\quad$ $3 = 3\checkmark$
	The solution is (1, 3).

? Try It 1.15.8

Solve the system by graphing: $\begin{cases} y = 2x + 2 \\ y = -x - 4 \end{cases}$

Answer

(−2,−2)

? Try It 1.15.9

Solve the system by graphing: $\begin{cases} y = 3x + 3 \\ y = -x + 7 \end{cases}$

Answer

(1,6)

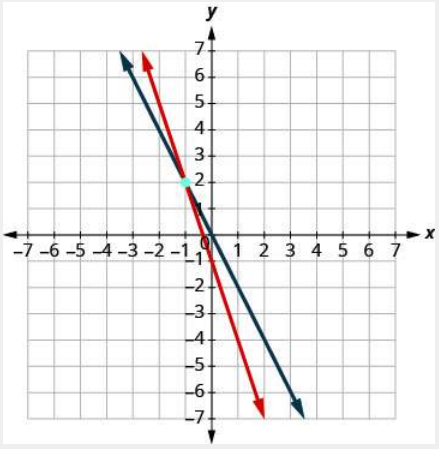
Both equations in Example 1.15.7 were given in slope–intercept form. This made it easy for us to quickly graph the lines. In the next example, we'll first re-write the equations into slope–intercept form.

✓ Example 1.15.10

Solve the system by graphing: $\begin{cases} 3x + y = -1 \\ 2x + y = 0 \end{cases}$

Solution

We'll solve both of these equations for y so that we can easily graph them using their slopes and y-intercepts. $\begin{cases} 3x + y = -1 \\ 2x + y = 0 \end{cases}$

Solve the first equation for y.	
Find the slope and y-intercept.	$3x+y=-1$ $y=-3x-1$ $m=-3$ $b=-1$
Solve the second equation for y.	$2x+y=0$ $y=-2x$
Find the slope and y-intercept.	$b=0$
Graph the lines.	
Determine the point of intersection.	The lines intersect at $(-1, 2)$.
Check the solution in both equations.	$3x+y=-1$ $\quad 3(-1)+2 \stackrel{?}{=} -1 \quad -1=-1\checkmark$ $2x+y=0$ $\quad 2(-1)+2 \stackrel{?}{=} 0 \quad 0=0\checkmark$
	The solution is $(-1, 2)$.

? Try It 1.15.11

Solve each system by graphing: $\begin{cases} -x+y=1 \\ 2x+y=10 \end{cases}$

Answer

(3,4)

? Try It 1.15.12

Solve each system by graphing: $\begin{cases} 2x+y=6 \\ x+y=1 \end{cases}$

Answer

(5,−4)

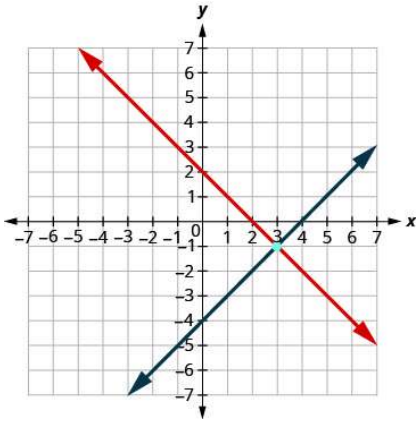
Usually when equations are given in standard form, the most convenient way to graph them is by using the intercepts. We'll do this in Example 1.15.13

Example 1.15.13

Solve the system by graphing: $\begin{cases} x+y=2 \\ x-y=4 \end{cases}$

Solution

We will find the *x*- and *y*-intercepts of both equations and use them to graph the lines.

	$x+y=2$	
To find the intercepts, let $x = 0$ and solve for y, then let $y = 0$ and solve for x.	$x+y=2 \quad x+y=2$ $0+y=2 \quad x+0=2$ $y=2 \quad x=2$	x | y 0 | 2 2 | 0
	$x-y=4$	
To find the intercepts, let $x = 0$ then let $y = 0$.	$x-y=4 \quad x-y=4$ $0-y=4 \quad x-0=4$ $-y=4 \quad x=4$ $y=-4$	x | y 0 | –4 4 | 0
Graph the line.		
Determine the point of intersection.	The lines intersect at (3, −1).	
Check the solution in both equations.	$x+y = 2 \quad x-y = 4$ $3+(-1) \stackrel{?}{=} 2 \quad 3-(-1) \stackrel{?}{=} 4$ $2 = 2\checkmark \quad 4 = 4\checkmark$ The solution is (3, −1).	

(1.15.3)

Try It 1.15.14

Solve each system by graphing: $\begin{cases} x+y=6 \\ x-y=2 \end{cases}$

Answer

(4,2)

Try It 1.15.15

Solve each system by graphing: $\begin{cases} x+y=2 \\ x-y=8 \end{cases}$

Answer

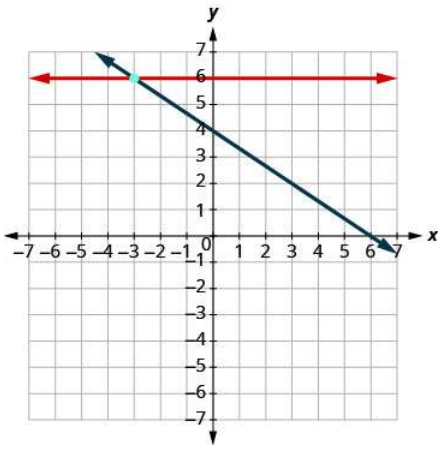
(5,−3)

Do you remember how to graph a linear equation with just one variable? It will be either a vertical or a horizontal line.

Example 1.15.16

Solve the system by graphing: $\begin{cases} y=6 \\ 2x+3y=12 \end{cases}$

Solution

	$y=6$ $2x+3y=12$
We know the first equation represents a horizontal line whose y-intercept is 6.	$y=6$
The second equation is most conveniently graphed using intercepts.	$2x+3y=12$
To find the intercepts, let $x=0$ and then $y=0$.	x | y 0 | 4 6 | 0
Graph the lines.	
Determine the point of intersection.	The lines intersect at (−3, 6).
Check the solution to both equations.	$y = 6$, $6 \stackrel{?}{=} 6$, $6 = 6\checkmark$ $2x+3y = 12$, $2(-3)+3(6) \stackrel{?}{=} 12$, $-6+18 \stackrel{?}{=} 12$, $12 = 12\checkmark$
	The solution is (−3, 6).

Try It 1.15.17

Solve each system by graphing: $\begin{cases} y = -1 \\ x + 3y = 6 \end{cases}$

Answer

$(9,-1)$

Try It 1.15.18

Solve each system by graphing: $\begin{cases} x = 4 \\ 3x - 2y = 24 \end{cases}$

Answer

$(4,-6)$

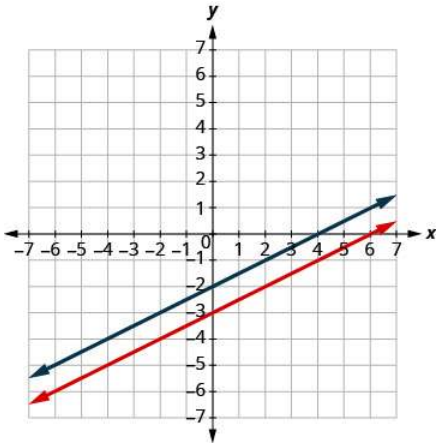
In all the systems of linear equations so far, the lines intersected and the solution was one point. In the next two examples, we'll look at a system of equations that has no solution and at a system of equations that has an infinite number of solutions.

Example 1.15.19

Solve the system by graphing: $\begin{cases} y = \frac{1}{2}x - 3 \\ x - 2y = 4 \end{cases}$

Solution

	$y = \frac{1}{2}x - 3$ $x - 2y = 4$
To graph the first equation, we will use its slope and y-intercept.	$y = \frac{1}{2}x - 3$
	$m = \frac{1}{2}$
	$b = -3$
To graph the second equation, we will use the intercepts.	$x - 2y = 4$
	x | y 0 | -2 4 | 0

Graph the lines.	
Determine the point of intersection.	The lines are parallel.
	Since no point is on both lines, there is no ordered pair that makes both equations true. There is no solution to this system.

Try It 1.15.20

Solve each system by graphing: $\begin{cases} y = -\frac{1}{4}x + 2 \\ x + 4y = -8 \end{cases}$

Answer

no solution

Try It 1.15.21

Solve each system by graphing: $\begin{cases} y = 3x - 1 \\ 6x - 2y = 6 \end{cases}$

Answer

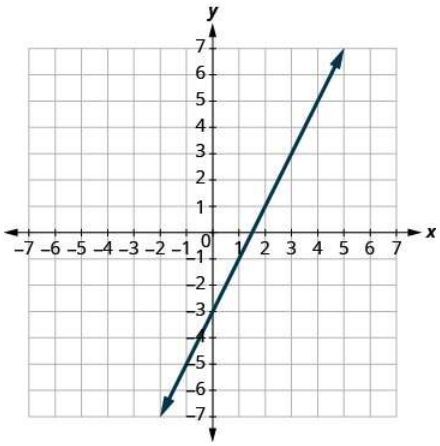
no solution

Example 1.15.22

Solve the system by graphing: $\begin{cases} y = 2x - 3 \\ -6x + 3y = -9 \end{cases}$

Solution

	$y = 2x - 3$ $-6x + 3y = -9$
Find the slope and y-intercept of the first equation.	$y = 2x - 3$ $m = 2$ $b = -3$
Find the intercepts of the second equation.	$-6x + 3y = -9$

	$\begin{array}{\|c\|c\|}\hline x & y \\ \hline 0 & -3 \\ \hline \frac{3}{2} & 0 \\ \hline \end{array}$
Graph the lines.	
Determine the point of intersection.	The lines are the same!
	Since every point on the line makes both equations true, there are infinitely many ordered pairs that make both equations true.
	There are infinitely many solutions to this system.

? Try It 1.15.23

Solve each system by graphing: $\begin{cases} y = -3x - 6 \\ 6x + 2y = -12 \end{cases}$

Answer

infinitely many solutions

? Try It 1.15.24

Solve each system by graphing: $\begin{cases} y = \frac{1}{2}x - 4 \\ 2x - 4y = 16 \end{cases}$

Answer

infinitely many solutions

If you write the second equation in Example $1.15.22$ in slope-intercept form, you may recognize that the equations have the same slope and same *y*-intercept.

When we graphed the second line in the last example, we drew it right over the first line. We say the two lines are coincident. Coincident lines have the same slope and same *y*-intercept.

COINCIDENT LINES

Coincident lines have the same slope and same *y*-intercept.

Determine the Number of Solutions of a Linear System

There will be times when we will want to know how many solutions there will be to a system of linear equations, but we might not actually have to find the solution. It will be helpful to determine this without graphing.

We have seen that two lines in the same plane must either intersect or are parallel. The systems of equations in Example 1.15.4 through Example 1.15.18all had two intersecting lines. Each system had one solution.

A system with parallel lines, like Example 1.15.19, has no solution. What happened in Example 1.15.22? The equations have **coincident lines**, and so the system had infinitely many solutions.

We'll organize these results in Figure 1.15.2below:

Graph	Number of solutions
2 intersecting lines	1
Parallel lines	None
Same line	Infinitely many

Figure 1.15.2

Parallel lines have the same slope but different *y*-intercepts. So, if we write both equations in a system of linear equations in slope–intercept form, we can see how many solutions there will be without graphing! Look at the system we solved in Example 1.15.19

$$\begin{cases} y = \frac{1}{2}x - 3 \\ x - 2y = 4 \end{cases}$$

The first line is in slope–intercept form. If we solve the second equation for y, we get

$$y = \frac{1}{2}x - 3 \qquad x - 2y = 4$$
$$x - 2y = -x + 4$$
$$y = \frac{1}{2}x - 2$$
$$m = \frac{1}{2}, b = -3 \qquad m = \frac{1}{2}, b = -2$$

The two lines have the same slope but different *y*-intercepts. They are parallel lines.

Figure 1.15.3shows how to determine the number of solutions of a linear system by looking at the slopes and intercepts.

Number of Solutions of a Linear System of Equations			
Slopes	**Intercepts**	**Type of Lines**	**Number of Solutions**
Different		Intersecting	1 point
Same	Different	Parallel	No solution
Same	Same	Coincident	Infinitely many solutions

Figure 1.15.3

Let's take one more look at our equations in Example 1.15.19that gave us parallel lines.

$$\begin{cases} y = \frac{1}{2}x - 3 \\ x - 2y = 4 \end{cases} \tag{1.15.4}$$

When both lines were in slope-intercept form we had:

$$y = \frac{1}{2}x - 3 \quad y = \frac{1}{2}x - 2 \tag{1.15.5}$$

Do you recognize that it is impossible to have a single ordered pair (x,y) that is a solution to both of those equations?

We call a system of equations like this **an inconsistent system**. It has no solution.

A system of equations that has at least one solution is called a **consistent system**.

CONSISTENT AND INCONSISTENT SYSTEMS

A **consistent system** of equations is a system of equations with at least one solution.

An **inconsistent system** of equations is a system of equations with no solution.

We also categorize the equations in a system of equations by calling the equations *independent* or *dependent*. If two equations are **independent equations**, they each have their own set of solutions. Intersecting lines and parallel lines are independent.

If two equations are dependent, all the solutions of one equation are also solutions of the other equation. When we graph two **dependent equations**, we get coincident lines.

INDEPENDENT AND DEPENDENT EQUATIONS

Two equations are **independent** if they have different solutions.

Two equations are **dependent** if all the solutions of one equation are also solutions of the other equation.

Let's sum this up by looking at the graphs of the three types of systems. See Figure 1.15.4and Figure 1.15.5

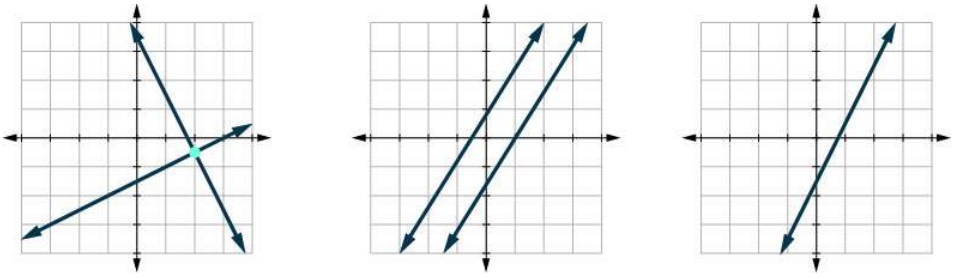


Figure 1.15.4

Lines	Intersecting	Parallel	Coincident
Number of solutions	1 point	No solution	Infinitely many
Consistent/inconsistent	Consistent	Inconsistent	Consistent
Dependent/independent	Independent	Independent	Dependent

Figure 1.15.5

Example 1.15.25

Without graphing, determine the number of solutions and then classify the system of equations: $\begin{cases} y = 3x - 1 \\ 6x - 2y = 12 \end{cases}$

Solution

We will compare the slopes and intercepts of the two lines.

$$\begin{cases} y = 3x - 1 \\ 6x - 2y = 12 \end{cases}$$

The first equation is already in slope-intercept form.

$$y = 3x - 1$$

Write the second equation inslope–intercept form.

$$\begin{aligned} 6x - 2y &= 12 \\ -2y &= -6x - 12 \\ \tfrac{-2y}{-2} &= \tfrac{-6x+12}{-2} \\ y &= 3x - 6 \end{aligned}$$

Find the slope and intercept of each line.

$$\begin{aligned} y &= 3x - 1 & y &= 3x - 6 \\ m &= 3 & m &= 3 \\ b &= -1 & b &= -6 \end{aligned}$$

Since the slopes are the same and y-intercepts are different, the lines are parallel.

A system of equations whose graphs are parallel lines has no solution and is inconsistent and independent.

Try It 1.15.26

Without graphing, determine the number of solutions and then classify the system of equations.

$$\begin{cases} y = -2x - 4 \\ 4x + 2y = 9 \end{cases}$$

Answer

no solution, inconsistent, independent

Try It 1.15.27

Without graphing, determine the number of solutions and then classify the system of equations.

$$\begin{cases} y = \frac{1}{3}x - 5 \\ x - 3y = 6 \end{cases}$$

Answer

no solution, inconsistent, independent

Example 1.15.28

Without graphing, determine the number of solutions and then classify the system of equations: $\begin{cases} 2x + y = -3 \\ x - 5y = 5 \end{cases}$

Solution

We will compare the slopes and intercepts of the two lines.

$$\begin{cases} 2x + y = -3 \\ x - 5y = 5 \end{cases}$$

Write the second equation in slope–intercept form.

$$\begin{aligned} 2x + y &= -3 \\ y &= -2x - 3 \end{aligned} \qquad \begin{aligned} x - 5y &= 5 \\ -5y &= -x + 5 \\ \tfrac{-5y}{-5} &= \tfrac{-x+5}{-5} \\ y &= \tfrac{1}{5}x - 1 \end{aligned}$$

Find the slope and intercept of each line.

$$\begin{aligned} y &= -2x - 3 \\ m &= -2 \\ b &= -3 \end{aligned} \qquad \begin{aligned} y &= \tfrac{1}{5}x - 1 \\ m &= \tfrac{1}{5} \\ b &= -1 \end{aligned}$$

Since the slopes are different, the lines intersect.

A system of equations whose graphs are intersect has 1 solution and is consistent and independent.

Try It 1.15.29

Without graphing, determine the number of solutions and then classify the system of equations.

$$\begin{cases} 3x + 2y = 2 \\ 2x + y = 1 \end{cases}$$

Answer

one solution, consistent, independent

Try It 1.15.30

Without graphing, determine the number of solutions and then classify the system of equations.

$$\begin{cases} x+4y=12 \\ -x+y=3 \end{cases}$$

Answer

one solution, consistent, independent

Example 1.15.31

Without graphing, determine the number of solutions and then classify the system of equations. $\begin{cases} 3x-2y=4 \\ y=\frac{3}{2}x-2 \end{cases}$

Solution

We will compare the slopes and intercepts of the two lines. $\begin{cases} 3x-2y &= 4 \\ y &= \frac{3}{2}x-2 \end{cases}$

Write the second equation in slope–intercept form.

$$\begin{aligned} 3x-2y &= 4 \\ -2y &= -3x+4 \\ \frac{-2y}{-2} &= \frac{-3x+4}{-2} \\ y &= \frac{3}{2}x-2 \end{aligned}$$

Find the slope and intercept of each line. $y = \frac{3}{2}x-2$

Since the equations are the same, they have the same slope and same y-intercept and so the lines are coincident.

A system of equations whose graphs are coincident lines has infinitely many solutions and is consistent and dependent.

Try It 1.15.32

Without graphing, determine the number of solutions and then classify the system of equations.

$$\begin{cases} 4x-5y=20 \\ y=\frac{4}{5}x-4 \end{cases}$$

Answer

infinitely many solutions, consistent, dependent

Try It 1.15.33

Without graphing, determine the number of solutions and then classify the system of equations.

$$\begin{cases} -2x-4y=8 \\ y=-\frac{1}{2}x-2 \end{cases}$$

Answer

infinitely many solutions, consistent, dependent

Solve Applications of Systems of Equations by Graphing

We will use the same problem solving strategy we used in Math Models to set up and solve applications of systems of linear equations. We'll modify the strategy slightly here to make it appropriate for systems of equations.

USE A PROBLEM SOLVING STRATEGY FOR SYSTEMS OF LINEAR EQUATIONS.

1. **Read** the problem. Make sure all the words and ideas are understood.
2. **Identify** what we are looking for.
3. **Name** what we are looking for. Choose variables to represent those quantities.
4. **Translate** into a system of equations.
5. **Solve** the system of equations using good algebra techniques.
6. **Check** the answer in the problem and make sure it makes sense.
7. **Answer** the question with a complete sentence.

Step 5 is where we will use the method introduced in this section. We will graph the equations and find the solution.

Example 1.15.34

Sondra is making 10 quarts of punch from fruit juice and club soda. The number of quarts of fruit juice is 4 times the number of quarts of club soda. How many quarts of fruit juice and how many quarts of club soda does Sondra need?

Solution

Step 1. Read the problem.

Step 2. Identify what we are looking for.

We are looking for the number of quarts of fruit juice and the number of quarts of club soda that Sondra will need.

Step 3. Name what we are looking for. Choose variables to represent those quantities.

Let f= number of quarts of fruit juice.
c= number of quarts of club soda

Step 4. Translate into a system of equations.

The number of quarts of fruit juice and the number of quarts of club soda is 10

$$f + c = 10$$

The number of quarts of fruit juice is four times the number of quarts of club soda

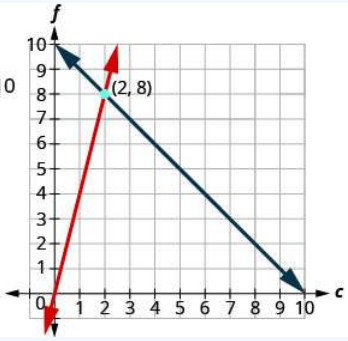
$$f = 4c$$

We now have the system. $\begin{cases} f + c = 10 \\ f = 4c \end{cases}$

Step 5. Solve the system of equations using good algebra techniques.

$f = 4c$ $\quad m = 4$ $\quad b = 0$

$f + c = 10$ $\quad f = -c + 10$ $\quad m = -1$ $\quad b = 10$



The point of intersection (2, 8) is the solution. This means Sondra needs 2 quarts of club soda and 8 quarts of fruit juice.

Step 6. Check the answer in the problem and make sure it makes sense.

Does this make sense in the problem?

Yes, the number of quarts of fruit juice, 8 is 4 times the number of quarts of club soda, 2.

Yes, 10 quarts of punch is 8 quarts of fruit juice plus 2 quarts of club soda.

Step 7. Answer the question with a complete sentence.

Sondra needs 8 quarts of fruit juice and 2 quarts of soda.

Try It 1.15.35

Manny is making 12 quarts of orange juice from concentrate and water. The number of quarts of water is 3 times the number of quarts of concentrate. How many quarts of concentrate and how many quarts of water does Manny need?

Answer

Manny needs 3 quarts juice concentrate and 9 quarts water.

Try It 1.15.36

Alisha is making an 18 ounce coffee beverage that is made from brewed coffee and milk. The number of ounces of brewed coffee is 5 times greater than the number of ounces of milk. How many ounces of coffee and how many ounces of milk does Alisha need?

Answer

Alisha needs 15 ounces of coffee and 3 ounces of milk.

Note

Access these online resources for additional instruction and practice with solving systems of equations by graphing.

- Instructional Video Solving Linear Systems by Graphing
- Instructional Video Solve by Graphing

Key Concepts

- **To solve a system of linear equations by graphing**
  1. Graph the first equation.
  2. Graph the second equation on the same rectangular coordinate system.
  3. Determine whether the lines intersect, are parallel, or are the same line.
  4. Identify the solution to the system.
     If the lines intersect, identify the point of intersection. Check to make sure it is a solution to both equations. This is the solution to the system.
     If the lines are parallel, the system has no solution.
     If the lines are the same, the system has an infinite number of solutions.
  5. Check the solution in both equations.
- Determine the number of solutions from the graph of a linear system

Graph	Number of solutions
2 intersecting lines	1
Parallel lines	None
Same line	Infinitely many

- Determine the number of solutions of a linear system by looking at the slopes and intercepts

Number of Solutions of a Linear System of Equations			
Slopes	**Intercepts**	**Type of Lines**	**Number of Solutions**
Different		Intersecting	1 point
Same	Different	Parallel	No solution
Same	Same	Coincident	Infinitely many solutions

- Determine the number of solutions and how to classify a system of equations

Lines	**Intersecting**	**Parallel**	**Coincident**
Number of solutions	1 point	No solution	Infinitely many
Consistent/inconsistent	Consistent	Inconsistent	Consistent
Dependent/independent	Independent	Independent	Dependent

- **Problem Solving Strategy for Systems of Linear Equations**
  1. **Read** the problem. Make sure all the words and ideas are understood.
  2. **Identify** what we are looking for.
  3. **Name** what we are looking for. Choose variables to represent those quantities.
  4. **Translate** into a system of equations.
  5. **Solve** the system of equations using good algebra techniques.
  6. **Check** the answer in the problem and make sure it makes sense.
  7. **Answer** the question with a complete sentence.

Glossary

coincident lines

Coincident lines are lines that have the same slope and same y-intercept.

consistent system

A consistent system of equations is a system of equations with at least one solution.

dependent equations

Two equations are dependent if all the solutions of one equation are also solutions of the other equation.

inconsistent system

An inconsistent system of equations is a system of equations with no solution.

independent equations

Two equations are independent if they have different solutions.

solutions of a system of equations

Solutions of a system of equations are the values of the variables that make all the equations true. A solution of a system of two linear equations is represented by an ordered pair (x, y).

system of linear equations

When two or more linear equations are grouped together, they form a system of linear equations.

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1.15E: Exercises

Practice Makes Perfect

Determine Whether an Ordered Pair is a Solution of a System of Equations. In the following exercises, determine if the following points are solutions to the given system of equations.

Exercise 1.15E.1

$$\begin{cases} 2x-6y=0 \\ 3x-4y=5 \end{cases}$$

1. (3,1)
2. (−3,4)

Answer

1. yes
2. no

Exercise 1.15E.2

$$\begin{cases} 7x-4y=-1 \\ -3x-2y=1 \end{cases}$$

1. $(-\frac{3}{13}, -\frac{2}{13})$
2. (1,-2)

Answer

1. yes
2. no

Exercise 1.15E.3

$$\begin{cases} 2x+y=5 \\ x+y=1 \end{cases}$$

1. (4,−3)
2. (2,0)

Answer

1. yes
2. no

Exercise 1.15E.4

$$\begin{cases} -3x+y=8 \\ -x+2y=-9 \end{cases}$$

1. (−5,−7)
2. (−5,7)

Exercise 1.15E.5

$$\begin{cases} x+y=2 \\ y=\frac{3}{4}x \end{cases}$$

1. $\left(\frac{8}{7},\frac{6}{7}\right)$
2. $\left(1,\frac{3}{4}\right)$

Answer

1. yes
2. no

Exercise 1.15E.6

$$\begin{cases} x+y=1 \\ y=\frac{2}{5}x \end{cases}$$

1. $\left(\frac{5}{7},\frac{2}{7}\right)$
2. (5,2)

Exercise 1.15E.7

$$\begin{cases} x+5y=10 \\ y=\frac{3}{5}x+1 \end{cases}$$

1. (−10,4)
2. $\left(\frac{5}{4},\frac{7}{4}\right)$

Answer

1. no
2. yes

Exercise 1.15E.8

$$\begin{cases} x+3y=9 \\ y=\frac{2}{3}x-2 \end{cases}$$

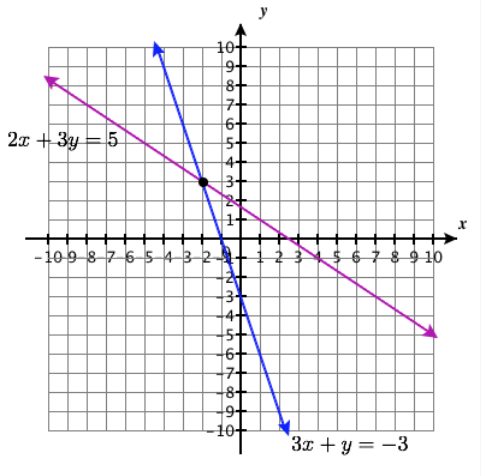
1. (−6,5)
2. $\left(5,\frac{4}{3}\right)$

Solve a System of Linear Equations by Graphing In the following exercises, solve the following systems of equations by graphing.

Exercise 1.15E.9

$$\begin{cases} 3x+y=-3 \\ 2x+3y=5 \end{cases}$$

Answer



Solution Point: $(-2, 3)$

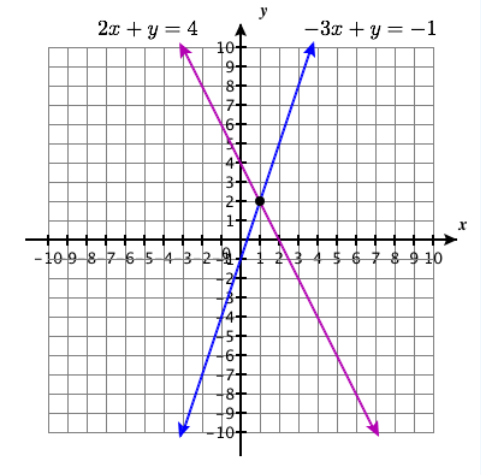
Exercise $1.15E.10$

$$\begin{cases} -x + y = 2 \\ 2x + y = -4 \end{cases}$$

Exercise $1.15E.11$

$$\begin{cases} -3x + y = -1 \\ 2x + y = 4 \end{cases}$$

Answer



Solution Point: $(1, 2)$

Exercise $1.15E.12$

$$\begin{cases} -2x + 3y = -3 \\ x + y = 4 \end{cases}$$

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Exercise 1.15E.13

$$\begin{cases} y = x + 2 \\ y = -2x + 2 \end{cases}$$

Answer

Solution Point: $(0, 2)$

Exercise 1.15E.14

$$\begin{cases} y = x - 2 \\ y = -3x + 2 \end{cases}$$

Exercise 1.15E.15

$$\begin{cases} y = \frac{3}{2}x + 1 \\ y = -\frac{1}{2}x + 5 \end{cases}$$

Answer

Solution Point: $(2, 4)$

Exercise 1.15E.16

$$\begin{cases} y = \frac{2}{3}x - 2 \\ y = -\frac{1}{3}x - 5 \end{cases}$$

Exercise 1.15E.17

$$\begin{cases} -x + y = -3 \\ 4x + 4y = 4 \end{cases}$$

Answer

Solution Point: $(2, -1)$

Exercise 1.15E.18

$$\begin{cases} x - y = 3 \\ 2x - y = 4 \end{cases}$$

Exercise 1.15E.19

$$\begin{cases} -3x + y = -1 \\ 2x + y = 4 \end{cases}$$

Answer

Solution Point: $(1, 2)$

Exercise 1.15E.20

$$\begin{cases} -3x + y = -2 \\ 4x - 2y = 6 \end{cases}$$

Exercise 1.15E.21

$$\begin{cases} x + y = 5 \\ 2x - y = 4 \end{cases}$$

Answer

Solution Point: $(3, 2)$

Exercise 1.15E.22

$$\begin{cases} x - y = 2 \\ 2x - y = 6 \end{cases}$$

Exercise 1.15E.23

$$\begin{cases} x + y = 2 \\ x - y = 0 \end{cases}$$

Answer

Solution Point: $(1, 1)$

Exercise 1.15E.24

$$\begin{cases} x + y = 6 \\ x - y = -8 \end{cases}$$

Exercise 1.15E.25

$$\begin{cases} x + y = -5 \\ x - y = 3 \end{cases}$$

Answer

Solution Point: $(-1, -4)$

Exercise 1.15E.26

$$\begin{cases} x + y = 4 \\ x - y = 0 \end{cases}$$

Exercise 1.15E.27

$$\begin{cases} x + y = -4 \\ -x + 2y = -2 \end{cases}$$

Answer

Solution Point: $(3, 3)$

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Exercise 1.15E.28

$$\begin{cases} -x+3y=3 \\ x+3y=3 \end{cases}$$

Exercise 1.15E.29

$$\begin{cases} -2x+3y=3 \\ x+3y=12 \end{cases}$$

Answer

Solution Point: $(-5, 6)$

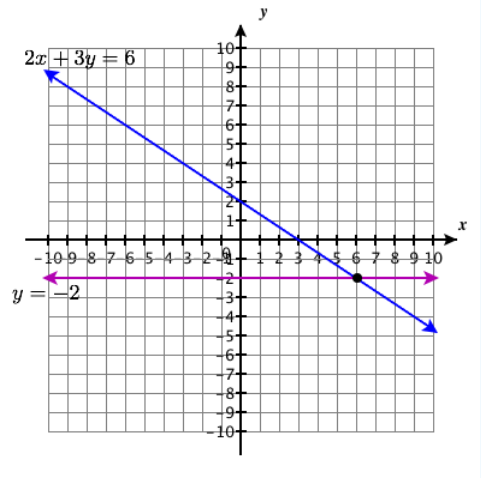
Exercise 1.15E.30

$$\begin{cases} 2x-y=4 \\ 2x+3y=12 \end{cases}$$

Exercise 1.15E.31

$$\begin{cases} 2x+3y=6 \\ y=-2 \end{cases}$$

Answer



Solution Point: $(6, -2)$

Exercise 1.15E.32

$$\begin{cases} -2x+y=2 \\ y=4 \end{cases}$$

Exercise 1.15E.33

$$\begin{cases} x-3y=-3 \\ y=2 \end{cases}$$

Answer

Solution Point: $(3, 2)$

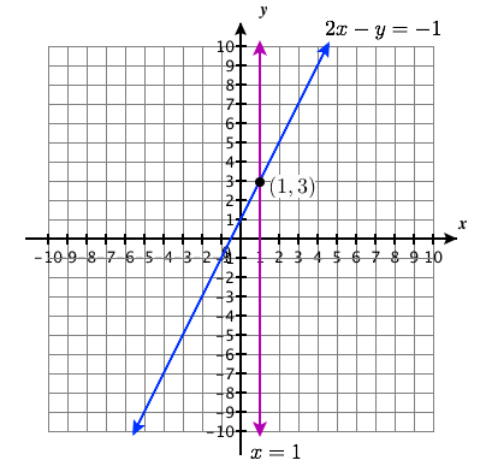
Exercise $1.15E.34$

$$\begin{cases} 2x-2y=8 \\ y=-3 \end{cases}$$

Exercise $1.15E.35$

$$\begin{cases} 2x-y=-1 \\ x=1 \end{cases}$$

Answer



Solution Point: $(1,3)$

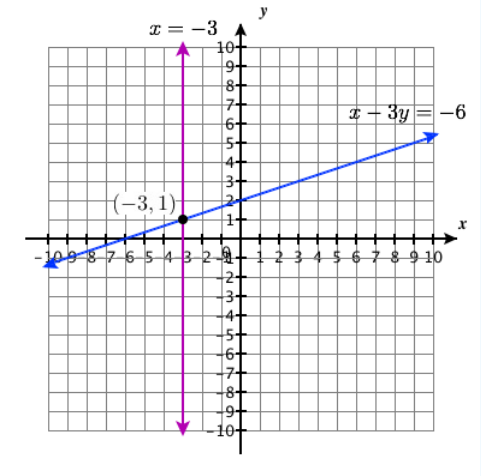
Exercise $1.15E.36$

$$\begin{cases} x+2y=2 \\ x=-2 \end{cases}$$

Exercise $1.15E.37$

$$\begin{cases} x-3y=-6 \\ x=-3 \end{cases}$$

Answer



Solution Point: $(-3, 1)$

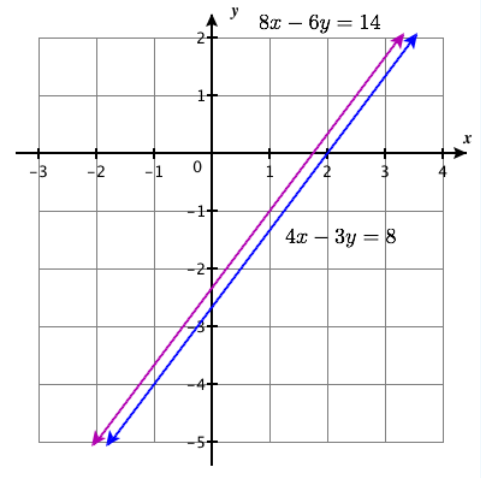
Exercise $1.15E.38$

$$\begin{cases} x + y = 4 \\ x = 1 \end{cases}$$

Exercise $1.15E.39$

$$\begin{cases} 4x - 3y = 8 \\ 8x - 6y = 14 \end{cases}$$

Answer



no solution

Exercise $1.15E.40$

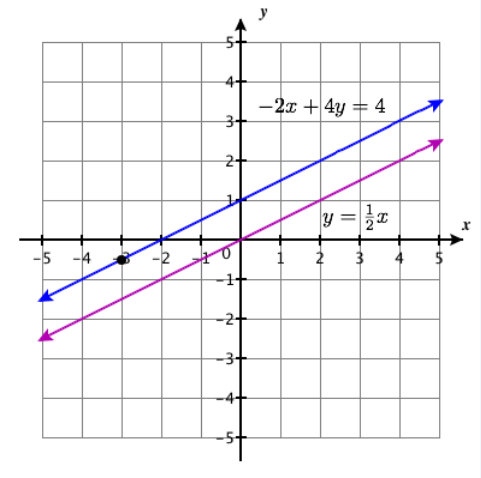
$$\begin{cases} x + 3y = 4 \\ -2x - 6y = 3 \end{cases}$$

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Exercise $1.15E.41$

$$\begin{cases} -2x+4y=4 \\ y=\frac{1}{2}x \end{cases}$$

Answer



no solution

Exercise $1.15E.42$

$$\begin{cases} 3x+5y=10 \\ y=-\frac{3}{5}x+1 \end{cases}$$

Exercise $1.15E.43$

$$\begin{cases} x=-3y+4 \\ 2x+6y=8 \end{cases}$$

Answer

no solution

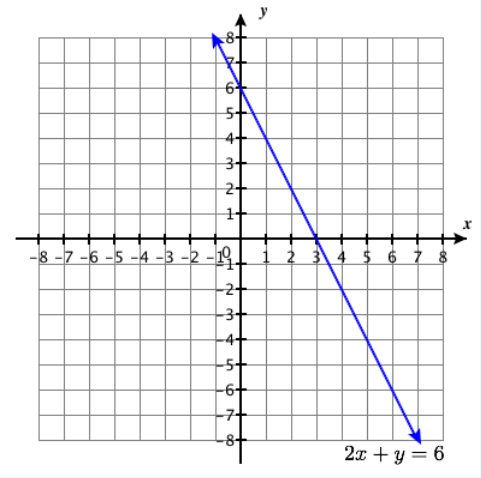
Exercise $1.15E.44$

$$\begin{cases} 4x=3y+7 \\ 8x-6y=14 \end{cases}$$

Exercise $1.15E.45$

$$\begin{cases} 2x+y=6 \\ -8x-4y=-24 \end{cases}$$

Answer



infinitely many solutions with solution set: $\{(x, y)|2x + y = 6\}$

Exercise 1.15E. 46

$$\begin{cases} 5x + 2y = 7 \\ -10x - 4y = -14 \end{cases}$$

Exercise 1.15E. 47

$$\begin{cases} x + 3y = -6 \\ 4y = -\frac{4}{3}x - 8 \end{cases}$$

Answer

infinitely many solutions with solution set: $\{(x, y)|x + 3y = -6\}$

Exercise 1.15E. 48

$$\begin{cases} -x + 2y = -6 \\ y = -\frac{1}{2}x - 1 \end{cases}$$

Exercise 1.15E. 49

$$\begin{cases} -3x + 2y = -2 \\ y = -x + 4 \end{cases}$$

Answer

Solution Point: $(2, 2)$

Exercise 1.15E. 50

$$\begin{cases} -x + 2y = -2 \\ y = -x - 1 \end{cases}$$

Determine the Number of Solutions of a Linear System Without graphing the following systems of equations, determine the number of solutions and then classify the system of equations.

Exercise 1.15E.51

$$\begin{cases} y = \frac{2}{3}x + 1 \\ -2x + 3y = 5 \end{cases}$$

Answer

0 solutions

Exercise 1.15E.52

$$\begin{cases} y = \frac{1}{3}x + 2 \\ x - 3y = 9 \end{cases}$$

Exercise 1.15E.53

$$\begin{cases} y = -2x + 1 \\ 4x + 2y = 8 \end{cases}$$

Answer

0 solutions

Exercise 1.15E.54

$$\begin{cases} y = 3x + 4 \\ 9x - 3y = 18 \end{cases}$$

Exercise 1.15E.55

$$\begin{cases} y = \frac{2}{3}x + 1 \\ 2x - 3y = 7 \end{cases}$$

Answer

0 solutions

Exercise 1.15E.56

$$\begin{cases} 3x + 4y = 12 \\ y = -3x - 1 \end{cases}$$

Exercise 1.15E.57

$$\begin{cases} 4x + 2y = 10 \\ 4x - 2y = -6 \end{cases}$$

Answer

consistent, 1 solution

Exercise 1.15E.58

$$\begin{cases} 5x+3y=4 \\ 2x-3y=5 \end{cases}$$

Exercise 1.15E.59

$$\begin{cases} y=-\frac{1}{2}x+5 \\ x+2y=10 \end{cases}$$

Answer

infinitely many solutions

Exercise 1.15E.60

$$\begin{cases} y=x+1 \\ -x+y=1 \end{cases}$$

Exercise 1.15E.61

$$\begin{cases} y=2x+3 \\ 2x-y=-3 \end{cases}$$

Answer

infinitely many solutions

Exercise 1.15E.62

$$\begin{cases} 5x-2y=10 \\ y=\frac{5}{2}x-5 \end{cases}$$

Solve Applications of Systems of Equations by Graphing In the following exercises, solve.

Exercise 1.15E.63

Molly is making strawberry infused water. For each ounce of strawberry juice, she uses three times as many ounces of water. How many ounces of strawberry juice and how many ounces of water does she need to make 64 ounces of strawberry infused water?

Answer

Molly needs 16 ounces of strawberry juice and 48 ounces of water.

Exercise 1.15E.64

Jamal is making a snack mix that contains only pretzels and nuts. For every ounce of nuts, he will use 2 ounces of pretzels. How many ounces of pretzels and how many ounces of nuts does he need to make 45 ounces of snack mix?

Exercise 1.15E.65

Enrique is making a party mix that contains raisins and nuts. For each ounce of nuts, he uses twice the amount of raisins. How many ounces of nuts and how many ounces of raisins does he need to make 24 ounces of party mix?

Answer

Enrique needs 8 ounces of nuts and 16 ounces of water.

Exercise 1.15E.66

Owen is making lemonade from concentrate. The number of quarts of water he needs is 4 times the number of quarts of concentrate. How many quarts of water and how many quarts of concentrate does Owen need to make 100 quarts of lemonade?

Everyday Math

Exercise 1.15E.67

Leo is planning his spring flower garden. He wants to plant tulip and daffodil bulbs. He will plant 6 times as many daffodil bulbs as tulip bulbs. If he wants to plant 350 bulbs, how many tulip bulbs and how many daffodil bulbs should he plant?

Answer

Leo should plant 50 tulips and 300 daffodils.

Exercise 1.15E.68

A marketing company surveys 1,200 people. They surveyed twice as many females as males. How many males and females did they survey?

Writing Exercises

Exercise 1.15E.69

In a system of linear equations, the two equations have the same slope. Describe the possible solutions to the system.

Answer

Given that it is only known that the slopes of both linear equations are the same, there are either no solutions (the graphs of the equations are parallel) or infinitely many.

Exercise 1.15E.70

In a system of linear equations, the two equations have the same intercepts. Describe the possible solutions to the system.

Self Check

After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
determine whether an ordered pair is a solution of a system of equations.			
solve a system of linear equations by graphing.			
determine the number of solutions of a linear system.			
solve applications of systems of equations by graphing.			

If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them. What did you do to become confident of your ability to do these things? Be specific.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math every topic builds upon previous work. It is important to make sure you have a strong foundation before you move on. Who can you ask for help? Your fellow classmates and instructor are good resources. Is there a place on campus where math tutors are available? Can your study skills be improved?

…no - I don't get it! This is a warning sign and you must not ignore it. You should get help right away or you will quickly be overwhelmed. See your instructor as soon as you can to discuss your situation. Together you can come up with a plan to get you the help you need.

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1.16: Solve Systems of Equations by Substitution

Learning Objectives

By the end of this section, you will be able to:

- Solve a system of equations by substitution
- Solve applications of systems of equations by substitution

Note

Before you get started, take this readiness quiz.

1. Simplify −5(3−x).
   If you missed this problem, review Example 1.10.43.
2. Simplify 4−2(n+5).
   If you missed this problem, review Example 1.10.49.
3. Solve for y. 8y−8=32−2y
   If you missed this problem, review Example 2.3.22.
4. Solve for x. 3x−9y=−3
   If you missed this problem, review Example 2.6.22.

Solving systems of linear equations by graphing is a good way to visualize the types of solutions that may result. However, there are many cases where solving a system by graphing is inconvenient or imprecise. If the graphs extend beyond the small grid with *x* and *y* both between −10 and 10, graphing the lines may be cumbersome. And if the solutions to the system are not integers, it can be hard to read their values precisely from a graph.

In this section, we will solve systems of linear equations by the substitution method.

Solve a System of Equations by Substitution

We will use the same system we used first for graphing.

$$\begin{cases} 2x+y=7 \\ x-2y=6 \end{cases}$$

We will first solve one of the equations for either *x* or *y*. We can choose either equation and solve for either variable—but we'll try to make a choice that will keep the work easy.

Then we substitute that expression into the other equation. The result is an equation with just one variable—and we know how to solve those!

After we find the value of one variable, we will substitute that value into one of the original equations and solve for the other variable. Finally, we check our solution and make sure it makes both equations true.

We'll fill in all these steps now in Example 1.16.1.

Example 1.16.1: How to Solve a System of Equations by Substitution

Solve the system by substitution. $\begin{cases} 2x+y=7 \\ x-2y=6 \end{cases}$

Solution

Step 1. Solve one of the equations for either variable.	We'll solve the first equation for y.	$2x+y=7$ $y=7-2x$
Step 2. Substitute the expression from Step 1 into the other equation.	We replace y in the second equation with the expression $7-2x$.	$x-2y=6$ $x-2(7-2x)=6$
Step 3. Solve the resulting equation.	Now we have an equation with just 1 variable. We know how to solve this!	$x-2(7-2x)=6$ $x-14+4x=6$ $5x=20$ $x=4$
Step 4. Substitute the solution in Step 3 into one of the original equations to find the other variable.	We'll use the first equation and replace x with 4.	$2x+y=7$ $2(4)+y=7$ $8+y=7$ $y=-1$
Step 5. Write the solution as an ordered pair.	The ordered pair is (x, y).	$(4,-1)$
Step 6. Check that the ordered pair is a solution to **both** original equations.	Substitute $(4,-1)$ into both equations and make sure they are both true.	$2x+y=7$ $2(4)+(-1)\stackrel{?}{=}7$ $7=7$ ✓ $x-2y=6$ $4-2(-1)\stackrel{?}{=}6$ $6=6$ ✓ Both equations are true. $(4,-1)$ is the solution to the system.

Try It 1.16.2

Solve the system by substitution. $\begin{cases} -2x+y=-11 \\ x+3y=9 \end{cases}$

Answer

(6,1)

Try It 1.16.3

Solve the system by substitution. $\begin{cases} x+3y=10 \\ 4x+y=18 \end{cases}$

Answer

(4,2)

SOLVE A SYSTEM OF EQUATIONS BY SUBSTITUTION.

1. Solve one of the equations for either variable.
2. Substitute the expression from Step 1 into the other equation.
3. Solve the resulting equation.
4. Substitute the solution in Step 3 into one of the original equations to find the other variable.

5. Write the solution as an ordered pair.
6. Check that the ordered pair is a solution to **both** original equations.

If one of the equations in the system is given in slope–intercept form, Step 1 is already done! We'll see this in Example 1.16.4

Example 1.16.4

Solve the system by substitution. $\begin{cases} x+y=-1 \\ y=x+5 \end{cases}$

Solution

The second equation is already solved for y. We will substitute the expression in place of y in the first equation.

	$x+y=-1$ $y=x+5$
The second equation is already solved for y. We will substitute into the first equation.	
Replace the y with $x+5$.	$y=x+5$ $x+y=-1$
Solve the resulting equation for x.	$x+x+5=-1$
	$2x+5=-1$
	$2x=-6$
Substitute $x=-3$ into $y=x+5$ to find y.	$x=-3$ $y=x+5$
	$y=-3+5$
The ordered pair is $(-3, 2)$.	$y=2$
Check the ordered pair in both equations: $\begin{array}{rcl\|rcl} x+y & = & -1 & y & = & x+5 \\ -3+2 & \stackrel{?}{=} & -1 & 2 & \stackrel{?}{=} & -3+5 \\ -1 & = & -1\checkmark & 2 & = & 2\checkmark \end{array}$	
	The solution is $(-3, 2)$.

Try It 1.16.5

Solve the system by substitution. $\begin{cases} x+y=6 \\ y=3x-2 \end{cases}$

Answer

(2,4)

Try It 1.16.6

Solve the system by substitution. $\begin{cases} 2x-y=1 \\ y=-3x-6 \end{cases}$

Answer

(−1,−3)

If the equations are given in standard form, we'll need to start by solving for one of the variables. In this next example, we'll solve the first equation for y.

Example 1.16.7

Solve the system by substitution. $\begin{cases} 3x+y=5 \\ 2x+4y=-10 \end{cases}$

Solution

We need to solve one equation for one variable. Then we will substitute that expression into the other equation.

Solve for y. Substitute into the other equation.	$3x+y=5$ $y=-3x+5$ $2x+4y=-10$
Replace the y with $-3x+5$.	$2x+4(-3x+5)=-10$
Solve the resulting equation for x.	$2x-12x+20=-10$
	$-10x+20=-10$ $-10x=-30$
Substitute $x=3$ into $3x+y=5$ to find y.	$x=3$ $3x+y=5$
	$3(3)+y=5$ $9+y=5$
The ordered pair is $(3,-4)$.	$y=-4$
Check the ordered pair in both equations: $3x+y=5$ $\quad$ $2x+4y=-10$ $3\cdot 3+(-4)\stackrel{?}{=}5$ $\quad$ $2\cdot 3+4(-4)\stackrel{?}{=}-10$ $9-4\stackrel{?}{=}5$ $\quad$ $6-16\stackrel{?}{=}-10$ $5=5\checkmark$ $\quad$ $-10=-10\checkmark$	
	The solution is $(3,-4)$.

Try It 1.16.8

Solve the system by substitution. $\begin{cases} 4x+y=2 \\ 3x+2y=-1 \end{cases}$

Answer

$(1,-2)$

Try It 1.16.9

Solve the system by substitution. $\begin{cases} -x+y=4 \\ 4x-y=2 \end{cases}$

Answer

$(2,6)$

In Example 1.16.7 it was easiest to solve for y in the first equation because it had a coefficient of 1. In Example 1.16.10 it will be easier to solve for x.

Example 1.16.10

Solve the system by substitution. $\begin{cases} x-2y=-2 \\ 3x+2y=34 \end{cases}$

Solution

We will solve the first equation for x and then substitute the expression into the second equation.

	$x-2y=-2$
Solve for x. Substitute into the other equation.	$x=2y-2$ $3x+2y=34$
Replace the x with $2y-2$.	$3(2y-2)+2y=34$
Solve the resulting equation for y.	$6y-6+2y=34$
Substitute $y=5$ into $x-2y=-2$ to find x.	$8y-6=34$ $8y=40$ $y=5$ $x-2y=-2$ $x-2\cdot 5=-2$ $x-10=-2$ $x=8$
The ordered pair is (8, 5).	
Check the ordered pair in both equations: $x-2y=-2$: $8-2\cdot 5 \stackrel{?}{=} -2$, $8-10 \stackrel{?}{=} -2$, $-2=-2$ ✓ $3x+2y=34$: $3\cdot 8+2\cdot 5 \stackrel{?}{=} 34$, $24+10 \stackrel{?}{=} 34$, $34=34$ ✓	
	The solution is (8, 5).

Try It 1.16.11

Solve the system by substitution. $\begin{cases} x-5y=13 \\ 4x-3y=1 \end{cases}$

Answer

(−2,−3)

Try It 1.16.12

Solve the system by substitution. $\begin{cases} x-6y=-6 \\ 2x-4y=4 \end{cases}$

Answer

(6,2)

When both equations are already solved for the same variable, it is easy to substitute!

Example 1.16.13

Solve the system by substitution. $\begin{cases} y = -2x + 5 \\ y = \frac{1}{2}x \end{cases}$

Solution

Since both equations are solved for *y*, we can substitute one into the other.

Substitute $\frac{1}{2}x$ for *y* in the first equation.	$y = \frac{1}{2}x$ $y = -2x + 5$
Replace the *y* with $\frac{1}{2}x$	$\frac{1}{2}x = -2x + 5$
Solve the resulting equation. Start by clearing the fraction.	$2\left(\frac{1}{2}x\right) = 2(-2x + 5)$
Solve for *x*.	$x = -4x + 10$
	$5x = 10$
Substitute $x = 2$ into $y = \frac{1}{2}x$ to find *y*.	$x = 2$ $y = \frac{1}{2}x$ $y = \frac{1}{2} \cdot 2$ $y = 1$
The ordered pair is (2,1).	
Check the ordered pair in both equations: $y = \frac{1}{2}x \quad y = -2x + 5$ $1 \stackrel{?}{=} \frac{1}{2} \cdot 2 \quad 1 \stackrel{?}{=} -2 \cdot 2 + 5$ $1 = 1\checkmark \quad 1 = -4 + 5$ $1 = 1\checkmark$	
	The solution is (2,1).

Try It 1.16.14

Solve the system by substitution. $\begin{cases} y = 3x - 16 \\ y = \frac{1}{3}x \end{cases}$

Answer

(6,2)

Try It 1.16.15

Solve the system by substitution. $\begin{cases} y = -x + 10 \\ y = \frac{1}{4}x \end{cases}$

Answer

(8,2)

Be very careful with the signs in the next example.

Example 1.16.16

Solve the system by substitution. $\begin{cases} 4x+2y=4 \\ 6x-y=8 \end{cases}$

Solution

We need to solve one equation for one variable. We will solve the first equation for y.

	$4x+2y=4$
Solve the first equation for y.	$2y=-4x+4$
Substitute $-2x+2$ for y in the second equation.	$y=-2x+2$ $6x-y=8$
Replace the y with $-2x+2$.	$6x-(-2x+2)=8$
Solve the equation for x.	$6x+2x-2=8$
	$8x-2=8$ $8x=10$
Substitute $x=\frac{5}{4}$ into $4x+2y=4$ to find y.	$x=\frac{5}{4}$ $4x+2y=4$ $4\left(\frac{5}{4}\right)+2y=4$ $5+2y=4$ $2y=-1$ $y=-\frac{1}{2}$
The ordered pair is $(\frac{5}{4},-\frac{1}{2})$.	
Check the ordered pair in both equations.	$\begin{aligned} 4x+2y &= 4 \\ 4(\tfrac{5}{4})+2(-\tfrac{1}{2}) &\stackrel{?}{=} 4 \\ 5-1 &\stackrel{?}{=} 4 \\ 4 &= 4\checkmark \end{aligned}$ $\begin{aligned} 6x-y &= 8 \\ 6(\tfrac{5}{4})-(-\tfrac{1}{2}) &\stackrel{?}{=} 8 \\ \tfrac{15}{4}-(-\tfrac{1}{2}) &\stackrel{?}{=} 8 \\ \tfrac{16}{2} &\stackrel{?}{=} 8 \\ 8 &= 8\checkmark \end{aligned}$
	The solution is $\left(\frac{5}{4},-\frac{1}{2}\right)$.

Try It 1.16.17

Solve the system by substitution. $\begin{cases} x-4y=-4 \\ -3x+4y=0 \end{cases}$

Answer

$(2,\frac{3}{2})$

Try It 1.16.18

Solve the system by substitution. $\begin{cases} 4x-y=0 \\ 2x-3y=5 \end{cases}$

Answer

$(-\frac{1}{2}, -2)$

In Example, it will take a little more work to solve one equation for x or y.

Example 1.16.19

Solve the system by substitution. $\begin{cases} 4x-3y=6 \\ 15y-20x=-30 \end{cases}$

Solution

We need to solve one equation for one variable. We will solve the first equation for x.

	$4x - 3y = 6$
Solve the first equation for x.	$4x = 3y + 6$
Substitute $\frac{3}{4}y+\frac{3}{2}$ for x in the second equation.	$x = \frac{3}{4}y + \frac{3}{2}$ $15y - 20x = -30$
Replace the x with $\frac{3}{4}y+\frac{3}{2}$	$15y - 20\left(\frac{3}{4}y + \frac{3}{2}\right) = -30$
Solve for y.	$15y - 15y - 30 = -30$
	$0 - 30 = -30$
	$0 = 0$

Since 0 = 0 is a true statement, the system is consistent. The equations are dependent. The graphs of these two equations would give the same line. The system has infinitely many solutions.

Try It 1.16.20

Solve the system by substitution. $\begin{cases} 2x-3y=12 \\ -12y+8x=48 \end{cases}$

Answer

infinitely many solutions

Try It 1.16.21

Solve the system by substitution. $\begin{cases} 5x+2y=12 \\ -4y-10x=-24 \end{cases}$

Answer

infinitely many solutions

Look back at the equations in Example 1.16.22 Is there any way to recognize that they are the same line?

Let's see what happens in the next example.

Example 1.16.22

Solve the system by substitution. $\begin{cases} 5x-2y=-10 \\ y=\frac{5}{2}x \end{cases}$

Solution

The second equation is already solved for y, so we can substitute for y in the first equation.

Substitute x for y in the first equation.	$y=\left(\frac{5}{2}x\right)$ $5x-2y=-10$
Replace the y with $\frac{5}{2}x$.	$5x-2\left(\frac{5}{2}x\right)=-10$
Solve for x.	$5x-5x=-10$
	$0\neq-10$

Since $0 = -10$ is a false statement the equations are inconsistent. The graphs of the two equation would be parallel lines. The system has no solutions.

Try It 1.16.23

Solve the system by substitution. $\begin{cases} 3x+2y=9 \\ y=-\frac{3}{2}x+1 \end{cases}$

Answer

no solution

Try It 1.16.24

Solve the system by substitution. $\begin{cases} 5x-3y=2 \\ y=\frac{5}{3}x-4 \end{cases}$

Answer

no solution

Solve Applications of Systems of Equations by Substitution

We'll copy here the problem solving strategy we used in the Solving Systems of Equations by Graphing section for solving systems of equations. Now that we know how to solve systems by substitution, that's what we'll do in Step 5.

HOW TO USE A PROBLEM SOLVING STRATEGY FOR SYSTEMS OF LINEAR EQUATIONS.

1. **Read** the problem. Make sure all the words and ideas are understood.
2. **Identify** what we are looking for.
3. **Name** what we are looking for. Choose variables to represent those quantities.
4. **Translate** into a system of equations.
5. **Solve** the system of equations using good algebra techniques.
6. **Check** the answer in the problem and make sure it makes sense.
7. **Answer** the question with a complete sentence.

Some people find setting up word problems with two variables easier than setting them up with just one variable. Choosing the variable names is easier when all you need to do is write down two letters. Think about this in the next example—how would you have done it with just one variable?

Example 1.16.25

The sum of two numbers is zero. One number is nine less than the other. Find the numbers.

Solution

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	We are looking for two numbers.
Step 3. Name what we are looking for.	Let n= the first number Let m= the second number
Step 4. Translate into a system of equations.	The sum of two numbers is zero.
	$n+m=0$
	One number is nine less than the other.
	$n=m-9$
The system is:	$\begin{aligned} n+m&=0\\ n&=m-9\end{aligned}$
Step 5. Solve the system of equations. We will use substitution since the second equation is solved for n.	
Substitute $m-9$ for n in the first equation.	$n=m-9$ $n+m=0$
Solve for m.	$m-9+m=0$
	$2m-9=0$
	$2m=9$
Substitute $m=\frac{9}{2}$ into the second equation and then solve for n.	$m=\frac{9}{2}$ $n=m-9$
	$m=\frac{9}{2}-9$
	$m=\frac{9}{2}-\frac{18}{2}$
	$n=-\frac{9}{2}$
Step 6. Check the answer in the problem.	Do these numbers make sense in the problem? We will leave this to you!
Step 7. Answer the question.	The numbers are $\frac{9}{2}$ and $-\frac{9}{2}$.

Try It 1.16.26

The sum of two numbers is 10. One number is 4 less than the other. Find the numbers.

Answer

The numbers are 3 and 7.

Try It 1.16.27

The sum of two number is −6. One number is 10 less than the other. Find the numbers.

Answer

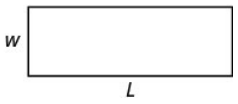
The numbers are 2 and −8.

In the Example 1.16.28 we'll use the formula for the perimeter of a rectangle, $P = 2L + 2W$.

Example 1.16.28

The perimeter of a rectangle is 88. The length is five more than twice the width. Find the length and width of the rectangle.

Solution

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	We are looking for the length and width.
Step 3. Name what we are looking for.	Let L= the length W= the width
Step 4. Translate into a system of equations.	The perimeter of a rectangle is 88.
	$2L + 2W = P$ $2L + 2W = 88$
	The length is five more than twice the width.
	$L = 2W + 5$
The system is:	$2L + 2W = 88$ $L = 2W + 5$
Step 5. Solve the system of equations. We will use substitution since the second equation is solved for L. Substitute $2W + 5$ for L in the first equation.	$L = 2W + 5$ $2L + 2W = 88$
Solve for W.	$2(2W + 5) + 2W = 88$
	$4W + 10 + 2W = 88$
	$6W + 10 = 88$
	$6W = 78$
Substitute $W = 13$ into the second equation and then solve for L.	$W = 13$ $L = 2W + 5$
	$L = 2 \cdot 13 + 5$
	$L = 31$
Step 6. Check the answer in the problem.	Does a rectangle with length 31 and width 13 have perimeter 88? Yes.
Step 7. Answer the equation.	The length is 31 and the width is 13.

Try It 1.16.29

The perimeter of a rectangle is 40. The length is 4 more than the width. Find the length and width of the rectangle.

Answer

The length is 12 and the width is 8.

Try It 1.16.30

The perimeter of a rectangle is 58. The length is 5 more than three times the width. Find the length and width of the rectangle.

Answer

The length is 23 and the width is 6.

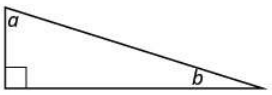
For Example 1.16.31we need to remember that the sum of the measures of the angles of a triangle is 180 degrees and that a right triangle has one 90 degree angle.

Example 1.16.31

The measure of one of the small angles of a right triangle is ten more than three times the measure of the other small angle. Find the measures of both angles.

Solution

We will draw and label a figure.

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	We are looking for the measures of the angles.
Step 3. Name what we are looking for.	Let a= the measure of the 1st angle b= the measure of the 2nd angle
Step 4. Translate into a system of equations.	The measure of one of the small angles of a right triangle is ten more than three times the measure of the other small angle.
	$a = 3b + 10$
	The sum of the measures of the angles of a triangle is 180.
	$a + b + 90 = 180$
The system is:	$a = 3b + 10$ $a + b + 90 = 180$
Step 5. Solve the system of equations. We will use substitution since the first equation is solved for a.	$a = 3b + 10$ $a + b + 90 = 180$
Substitute $3b + 10$ for a in the second equation.	$(3b + 10) + b + 90 = 180$
Solve for b.	$4b + 100 = 180$

	$4b = 80$
	$b = 20$ $a = 3b + 10$
Substitute $b = 20$ into the first equation and then solve for a.	$a = 3 \cdot 20 + 10$ $a = 70$
Step 6. Check the answer in the problem.	We will leave this to you!
Step 7. Answer the question.	The measures of the small angles are 20 and 70.

Try It 1.16.32

The measure of one of the small angles of a right triangle is 2 more than 3 times the measure of the other small angle. Find the measure of both angles.

Answer

The measure of the angles are 22 degrees and 68 degrees.

Try It 1.16.33

The measure of one of the small angles of a right triangle is 18 less than twice the measure of the other small angle. Find the measure of both angles.

Answer

The measure of the angles are 36 degrees and 54 degrees.

Example 1.16.34

Heather has been offered two options for her salary as a trainer at the gym. Option A would pay her $25,000 plus $15 for each training session. Option B would pay her $10,000 + $40 for each training session. How many training sessions would make the salary options equal?

Solution

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	We are looking for the number of training sessions that would make the pay equal.
Step 3. Name what we are looking for.	Let s= Heather's salary. n= the number of training sessions
Step 4. Translate into a system of equations.	Option A would pay her $25,000 plus $15 for each training session.
	$s = 25{,}000 + 15n$
	Option B would pay her $10,000 + $40 for each training session
	$s = 10{,}000 + 40n$
The system is:	$s = 25{,}000 + 15n$ $s = 10{,}000 + 40n$

Step 5. Solve the system of equations. We will use substitution.	$s = 25{,}000 + 15n$ $s = 10{,}000 + 40n$
Substitute $25{,}000 + 15n$ for s in the second equation.	$25{,}000 + 15n = 10{,}000 + 40n$
Solve for n.	$25{,}000 = 10{,}000 + 25n$
	$15{,}000 = 25n$
	$600 = n$
Step 6. Check the answer.	Are 600 training sessions a year reasonable? Are the two options equal when $n = 600$?
Step 7. Answer the question.	The salary options would be equal for 600 training sessions.

Try It 1.16.35

Geraldine has been offered positions by two insurance companies. The first company pays a salary of $12,000 plus a commission of $100 for each policy sold. The second pays a salary of $20,000 plus a commission of $50 for each policy sold. How many policies would need to be sold to make the total pay the same?

Answer

There would need to be 160 policies sold to make the total pay the same.

Try It 1.16.36

Kenneth currently sells suits for company A at a salary of $22,000 plus a $10 commission for each suit sold. Company B offers him a position with a salary of $28,000 plus a $4 commission for each suit sold. How many suits would Kenneth need to sell for the options to be equal?

Answer

Kenneth would need to sell 1,000 suits.

Note

Access these online resources for additional instruction and practice with solving systems of equations by substitution.

- Instructional Video-Solve Linear Systems by Substitution
- Instructional Video-Solve by Substitution

Key Concepts

- **Solve a system of equations by substitution**
  1. Solve one of the equations for either variable.
  2. Substitute the expression from Step 1 into the other equation.
  3. Solve the resulting equation.
  4. Substitute the solution in Step 3 into one of the original equations to find the other variable.
  5. Write the solution as an ordered pair.
  6. Check that the ordered pair is a solution to both original equations.

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1.16E: Exercises

Practice Makes Perfect

Solve a System of Equations by Substitution

In the following exercises, solve the systems of equations by substitution.

Exercise 1.16E.1

$$\begin{cases} 2x+y=-4 \\ 3x-2y=-6 \end{cases}$$

Answer

$(-2,0)$

Exercise 1.16E.2

$$\begin{cases} 2x+y=-2 \\ 3x-y=7 \end{cases}$$

Exercise 1.16E.3

$$\begin{cases} x-2y=-5 \\ 2x-3y=-4 \end{cases}$$

Answer

$(7,6)$

Exercise 1.16E.4

$$\begin{cases} x-3y=-9 \\ 2x+5y=4 \end{cases}$$

Exercise 1.16E.5

$$\begin{cases} 5x-2y=-6 \\ y=3x+3 \end{cases}$$

Answer

$(0,3)$

Exercise 1.16E.6

$$\begin{cases} -2x+2y=6 \\ y=-3x+1 \end{cases}$$

Exercise 1.16E.7

$$\begin{cases} 2x+3y=3 \\ y=-x+3 \end{cases}$$

Answer

$(6, -3)$

Exercise 1.16E.8

$$\begin{cases} 2x + 5y = -14 \\ y = -2x + 2 \end{cases}$$

Exercise 1.16E.9

$$\begin{cases} 2x + 5y = 1 \\ y = \frac{1}{3}x - 2 \end{cases}$$

Answer

$(3, -1)$

Exercise 1.16E.10

$$\begin{cases} 3x + 4y = 1 \\ y = -\frac{2}{5}x + 2 \end{cases}$$

Exercise 1.16E.11

$$\begin{cases} 3x - 2y = 6 \\ y = \frac{2}{3}x + 2 \end{cases}$$

Answer

$(6, 6)$

Exercise 1.16E.12

$$\begin{cases} -3x - 5y = 3 \\ y = \frac{1}{2}x - 5 \end{cases}$$

Exercise 1.16E.13

$$\begin{cases} 2x + y = 10 \\ -x + y = -5 \end{cases}$$

Answer

$(5, 0)$

Exercise 1.16E.14

$$\begin{cases} -2x + y = 10 \\ -x + 2y = 16 \end{cases}$$

Exercise 1.16E.15

$$\begin{cases} 3x+y=1 \\ -4x+y=15 \end{cases}$$

Answer

$(-2,7)$

Exercise 1.16E.16

$$\begin{cases} x+y=0 \\ 2x+3y=-4 \end{cases}$$

Exercise 1.16E.17

$$\begin{cases} x+3y=1 \\ 3x+5y=-5 \end{cases}$$

Answer

$(-5,2)$

Exercise 1.16E.18

$$\begin{cases} x+2y=-1 \\ 2x+3y=1 \end{cases}$$

Exercise 1.16E.19

$$\begin{cases} 2x+y=5 \\ x-2y=-15 \end{cases}$$

Answer

$(-1,7)$

Exercise 1.16E.20

$$\begin{cases} 4x+y=10 \\ x-2y=-20 \end{cases}$$

Exercise 1.16E.21

$$\begin{cases} y=-2x-1 \\ y=-\frac{1}{3}x+4 \end{cases}$$

Answer

$(-3,5)$

Exercise 1.16E.22

$$\begin{cases} y=x-6 \\ y=-\frac{3}{2}x+4 \end{cases}$$

Exercise 1.16E.23

$$\begin{cases} y = 2x - 8 \\ y = \frac{3}{5}x + 6 \end{cases}$$

Answer

$(10, 12)$

Exercise 1.16E.24

$$\begin{cases} y = -x - 1 \\ y = x + 7 \end{cases}$$

Exercise 1.16E.25

$$\begin{cases} 4x + 2y = 8 \\ 8x - y = 1 \end{cases}$$

Answer

$\left(\frac{1}{2}, 3\right)$

Exercise 1.16E.26

$$\begin{cases} -x - 12y = -1 \\ 2x - 8y = -6 \end{cases}$$

Exercise 1.16E.27

$$\begin{cases} 15x + 2y = 6 \\ -5x + 2y = -4 \end{cases}$$

Answer

$\left(\frac{1}{2}, -\frac{3}{4}\right)$

Exercise 1.16E.28

$$\begin{cases} 2x - 15y = 7 \\ 12x + 2y = -4 \end{cases}$$

Exercise 1.16E.29

$$\begin{cases} y = 3x \\ 6x - 2y = 0 \end{cases}$$

Answer

Infinitely many solutions. The two equations represent the same line.
The solution set is: $\{(x, y) \mid y = 3x\}$
All points that are solutions of the equation $y = 3x$ are solutions of this system.

Exercise 1.16E.30

$$\begin{cases} x = 2y \\ 4x - 8y = 0 \end{cases}$$

Exercise 1.16E.31

$$\begin{cases} 2x + 16y = 8 \\ -x - 8y = -4 \end{cases}$$

Answer

Infinitely many solutions. The two equations represent the same line.
The solution set is: $\{(x, y) \mid 2x + 16y = 8\}$
All points that are solutions of the equation $2x + 16y = 8$ are solutions of this system.

Exercise 1.16E.32

$$\begin{cases} 15x + 4y = 6 \\ -30x - 8y = -12 \end{cases}$$

Exercise 1.16E.33

$$\begin{cases} y = -4x \\ 4x + y = 1 \end{cases}$$

Answer

No solution

Exercise 1.16E.34

$$\begin{cases} y = -\frac{1}{4}x \\ x + 4y = 8 \end{cases}$$

Exercise 1.16E.35

$$\begin{cases} y = \frac{7}{8}x + 4 \\ -7x + 8y = 6 \end{cases}$$

Answer

No solution

Exercise 1.16E.36

$$\begin{cases} y = -\frac{2}{3}x + 5 \\ 2x + 3y = 11 \end{cases}$$

Solve Applications of Systems of Equations by Substitution

In the following exercises, translate to a system of equations and solve.

Exercise 1.16E.37

The sum of two numbers is 15. One number is 3 less than the other. Find the numbers.

Answer

The numbers are 6 and 9.

Exercise 1.16E.38

The sum of two numbers is 30. One number is 4 less than the other. Find the numbers.

Exercise 1.16E.39

The sum of two numbers is −26. One number is 12 less than the other. Find the numbers.

Answer

The numbers are −7 and −19.

Exercise 1.16E.40

The perimeter of a rectangle is 50. The length is 5 more than the width. Find the length and width.

Exercise 1.16E.41

The perimeter of a rectangle is 60. The length is 10 more than the width. Find the length and width.

Answer

The length is 20 and the width is 10.

Exercise 1.16E.42

The perimeter of a rectangle is 58. The length is 5 more than three times the width. Find the length and width.

Exercise 1.16E.43

The perimeter of a rectangle is 84. The length is 10 more than three times the width. Find the length and width.

Answer

The length is 34 and the width is 8.

Exercise 1.16E.44

The measure of one of the small angles of a right triangle is 14 more than 3 times the measure of the other small angle. Find the measure of both angles.

Exercise 1.16E.45

The measure of one of the small angles of a right triangle is 26 more than 3 times the measure of the other small angle. Find the measure of both angles.

Answer

The measures are 16° and 74°

Exercise 1.16E.46

The measure of one of the small angles of a right triangle is 15 less than twice the measure of the other small angle. Find the measure of both angles.

Exercise 1.16E.47

The measure of one of the small angles of a right triangle is 45 less than twice the measure of the other small angle. Find the measure of both angles.

Answer

The measures are 45° and 45°.

Exercise 1.16E.48

Maxim has been offered positions by two car dealers. The first company pays a salary of $10,000 plus a commission of $1,000 for each car sold. The second pays a salary of $20,000 plus a commission of $500 for each car sold. How many cars would need to be sold to make the total pay the same?

Exercise 1.16E.49

Jackie has been offered positions by two cable companies. The first company pays a salary of $ 14,000 plus a commission of $100 for each cable package sold. The second pays a salary of $20,000 plus a commission of $25 for each cable package sold. How many cable packages would need to be sold to make the total pay the same?

Answer

80 cable packages would need to be sold.

Exercise 1.16E.50

Amara currently sells televisions for company A at a salary of $17,000 plus a $100 commission for each television she sells. Company B offers her a position with a salary of $29,000 plus a $20 commission for each television she sells. How televisions would Amara need to sell for the options to be equal?

Exercise 1.16E.51

Mitchell currently sells stoves for company A at a salary of $12,000 plus a $150 commission for each stove he sells. Company B offers him a position with a salary of $24,000 plus a $50 commission for each stove he sells. How many stoves would Mitchell need to sell for the options to be equal?

Answer

Mitchell would need to sell 120 stoves.

Everyday Math

Exercise 1.16E.52

When Gloria spent 15 minutes on the elliptical trainer and then did circuit training for 30 minutes, her fitness app says she burned 435 calories. When she spent 30 minutes on the elliptical trainer and 40 minutes circuit training she burned 690

calories. Solve the system $\begin{cases} 15e+30c=435 \\ 30e+40c=690 \end{cases}$ for e, the number of calories she burns for each minute on the elliptical trainer, and cc, the number of calories she burns for each minute of circuit training.

Exercise 1.16E.53

Stephanie left Riverside, California, driving her motorhome north on Interstate 15 towards Salt Lake City at a speed of 56 miles per hour. Half an hour later, Tina left Riverside in her car on the same route as Stephanie, driving 70 miles per hour. Solve the system $\begin{cases} 56s=70t \\ s=t+\frac{1}{2} \end{cases}$

1. for t to find out how long it will take Tina to catch up to Stephanie.
2. what is the value of ss, the number of hours Stephanie will have driven before Tina catches up to her?

Answer

1. $t=2$ hours
2. $s=2\frac{1}{2}$ hours

Writing Exercises

Exercise 1.16E.54

Solve the system of equations
$\begin{cases} x+y=10 \\ x-y=6 \end{cases}$

1. by graphing.
2. by substitution.
3. Which method do you prefer? Why?

Exercise 1.16E.55

Solve the system of equations
$\begin{cases} 3x+y=12 \\ x=y-8 \end{cases}$ by substitution and explain all your steps in words.

Answer

Answers will vary.

Self Check

a. After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
solve a system of equations by substitution.			
solve applications of systems of equations by substitution.			

b. After reviewing this checklist, what will you do to become confident for all objectives?

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CHAPTER OVERVIEW

2: Lines, Angles, and Triangles

2.1: Lines
2.2: Angles
2.3: Angle Classifications
2.4: Parallel Lines
2.5: Triangles
2.6: Triangle CIassifications

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2.1: Lines

Geometry (from Greek words meaning earth-measure) originally developed as a means of surveying land areas, In its simplest form, it is a study of figures that can be drawn on a perfectly smooth flat surface, or **plane**. It is this **plane geometry** which we will study in this bock and which serves as a foundation for trigonometry, solid and analytic geometry, and calculus.

The simplest figures that can be drawn on a plane are the point and the line. By a line we will always mean a **straight line**. Through two distinct points one and only one (straight) line can be drawn. The line through points A and B will be denoted by $\overleftrightarrow{AB}$ (Figure 2.1.1). The arrows indicate that the line extends indefinitely in each direction, The **line segment** from A to B consists of A, B and that part of $\overleftrightarrow{AB}$ between A and B, It is denoted by AB (some textbooks use the notation $\overline{AB}$ for line segment). The ray $\overrightarrow{AB}$ is the part of $\overleftrightarrow{AB}$ which begins at A and extends indefinitely in the direction of B.

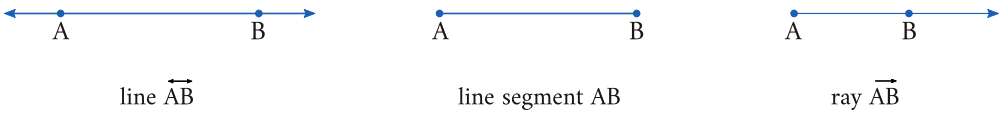


Figure 2.1.1: Line $\overleftrightarrow{AB}$, line segment $\overline{AB}$, and ray $\overrightarrow{AB}$. (CC BY-NC 4.0; Ümit Kaya via LibreTexts)

We assume everyone is familiar with the notion of length of a line segment and how it can be measured in inches, or feet, or meters, etc, The distance between two points A and B is the same as the length of AB.

Two line segments are equal if they have the same length, e.g., in Figure 2.1.2, $AB = CD$,



Figure 2.1.2: $AB = CD$. (CC BY-NC 4.0; Ümit Kaya via LibreTexts)

We often indicate two line segments are equal by marking them in the same way, e.g., in Figure 2.1.3, $AB = CD$ and $EF = GH$.



Figure 2.1.3: $AB = CD$ and $EF = GH$. (CC BY-NC 4.0; Ümit Kaya via LibreTexts)

Example 2.1.1

Find x if $AB = CD$:



Figure 2.1.E1: (CC BY-NC 4.0; Ümit Kaya via LibreTexts)

Solution

$$\begin{aligned} AB &= CD \\ 3x-6 &= x \\ 3x-x &= 6 \\ 2x &= 6 \\ x &= 3 \end{aligned}$$

Check:

AB	CD
3x - 6	x
3(3) - 6	3
9 - 6	
3	

Answer: $x = 3$.

Notice that in Example 2.1.1 we have not indicated the unit of measurement. Strictly speaking, we should specify that $AB = 3x - 6$ inches (or feet or meters) and that $BC = x$ inches. However since the answer would still be $x = 3$ we will usually omit this information to save space.

We say that B is the midpoint of AC if B is A point on AC and $AB = BC$ (Figure 2.1.4).

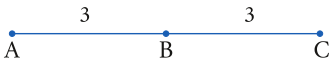


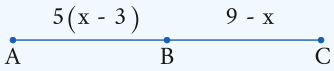
Figure 2.1.4: B is the midpoint of AC. (CC BY-NC 4.0; Ümit Kaya via LibreTexts)

Example 2.1.2

Find x and AC if B is the midpoint of AC and $AB = 5(x-3)$ and $BC = 9 - x$,

Solution

We first draw a picture to help visualize the given information:



Since B is a midpoint,

$$\begin{aligned} AB &= BC \\ 5(x-3) &= 9-x \\ 5x-15 &= 9-x \\ 5x+x &= 9+15 \\ 6x &= 24 \\ x &= 4 \end{aligned}$$

Check:

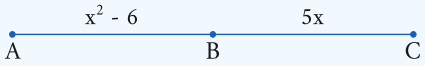
AB	BC
5(x - 3)	9 - x
5(4 - 3)	9 - 4
5(1)	5
5	

We obtain $AC = AB + BC = 5 + 5 = 10$.

Answer: $x = 4$, $AC = 10$.

Example 2.1.3

Find AB if B is the midpoint of AC:



Solution

$$\begin{aligned} AB &= BC \\ x^2-6 &= 5x \\ x^2-5x-6 &= 0 \\ (x-6)(x+1) &= 0 \end{aligned}$$

$$\begin{aligned} x-6 &= 0 \\ x &= 6 \end{aligned} \qquad \begin{aligned} x+1 &= 0 \\ x &= -1 \end{aligned}$$

If $x=6$ then $AB=x^2-6=6^2-6=36-6=30$.

If $x=-1$ then $AB=(-1)^2-6=1-6=-5$.

We reject the answer $x=-1$ and $AB=-5$ because the length of a line segment is always positive. Therefore $x=6$ and $AB=30$.

Check:

AB	=	BC
x^2 - 6		5x
6^2 - 6		5(6)
36 - 6		30
30		

Answer: $AB=30$.

Three points are collinear if they lie on the same line.



Figure 2.1.5: A, B, and C are collinear $AB=5$, $BC=3$, and $AC=8$ (CC BY-NC 4.0; Ümit Kaya via LibreTexts)

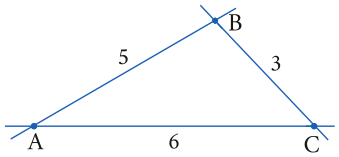


Figure 2.1.6: A, B, and C are not collinear. $AB=5$, $BC=3$, $AC=6$. (CC BY-NC 4.0; Ümit Kaya via LibreTexts)

A, B, and C are collinear if and only if $AB+BC=AC$.

Example 2.1.4

If A, B, and C are collinear and $AC=7$, find x:



Solution

$$\begin{aligned} AB+BC &= AC \\ 8-2x+x+1 &= 7 \\ 9-x &= 7 \\ 2 &= x \end{aligned}$$

Check:

AB + BC	=	AC
8 - 2x + x + 1		7
8 - 2(2) + 2 + 1		
8 - 4 + 3		
4 + 3		
7		

Answer: $x = 2$

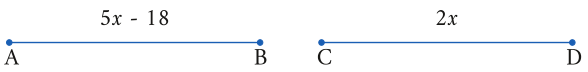
Historical Note

Geometry originated in the solution of practical problems, The architectural remains of Babylon, Egypt, and other ancient civilizations show a knowledge of simple geometric relationships, The digging of canals, erection of buildings, and the laying out of cities required computations of lengths, areas, and volumes, Surveying is said to have developed in Egypt so that tracts of land could be relocated after the annual overflow of the Nile, Geometry was also utilized by ancient civilizations in their astronomical observations and the construction of their calendars.

The Greeks transformed the practical geometry of the Babylonians and Egyptians into an organized body of knowledge. Thales (c, 636 - c. 546 B,C.), one of the "seven wise men" of antiquity, is credited with being the first to obtain geometrical results by logical reasoning, instead of just by intuition and experiment. Pythagoras (c. 582 - c. 507 B,C.) continued the work of Thales, He founded the Pythagorean school, a mystical society devoted to the unified study of philosophy, mathematics, and science, About 300 B,C., Euclid, a Greek teacher of mathematics at the university at Alexandria, wrote a systematic exposition of elementary geometry called the **Elements**, In his **Elements**, Euclid used a few simple principles, called *axioms* or *postulates*, to derive most of the mathematics known at the time, For over 2000 years, Euclid's Elements has been accepted as the standard textbook of geometry and is the basis for most other elementary texts, including this one.

Problems

1. Find x if $AB = CD$:



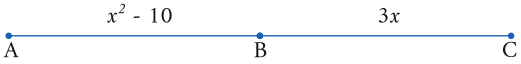
2. Find x if $AB = CD$:



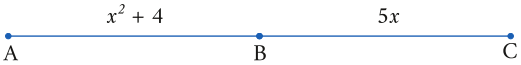
3. Find x and AC if B is the midpoint of AC and $AB = 3(x-5)$ and $BC = x+3$.

4. Find x and AC if B is the midpoint of AC and $AB = 2x+9$ and $BC = 5(x-9)$,

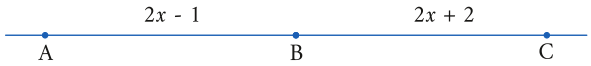
5. Find AB if B is the midpoint of AC:



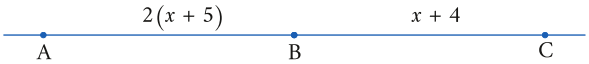
6, Find AB if B is the midpoint of AC:



7. If A, B, and C are collinear and $AC = 13$ find x:



8. If A, B, and C are collinear and $AC = 26$ find x:



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2.2: Angles

An *angle* is the figure formed by two rays with a common end point, The two rays are called the sides of the angle and the common end point is called the *vertex* of the angle, The symbol for angle is $\angle$

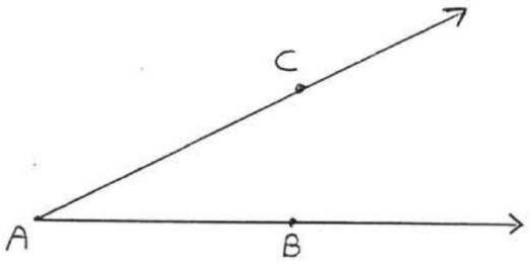


Figure 2.2.1: Angle BAC has vertex A and sides $\overrightarrow{AB}$ and $\overrightarrow{AC}$

The angle in Figure 2.2.1 has vertex A and sides AB and AC, It is denoted by $\angle BAC$ or $\angle CAB$ or simply $\angle A$. When three letters are used, the middle letter is always the vertex, In Figure 2.2.2 we would not use the notation $\angle A$ as an abbreviation for $\angle BAC$ because it could also mean $\angle CAD$ or $\angle BAD$, We could however use the simpler name $\angle x$ for $\angle BAC$ if "x" is marked in as shown,

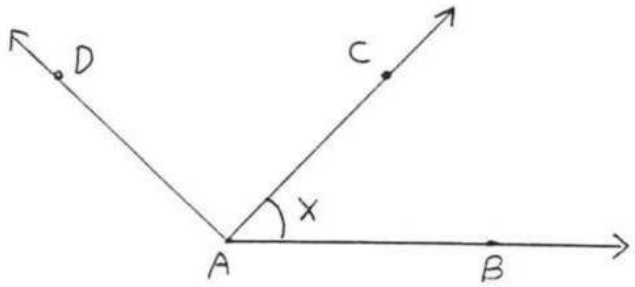


Figure 2.2.2: $\angle BAC$ may also be denoted by $\angle x$.

Angles can be measured with an instrument called a *protractor*. The unit of measurement is called a *degree* and the symbol for degree is $^\circ$.

To measure an angle, place the center of the protractor (often marked with a cross or a small circle) on the vertex of the angle, Position the protractor so that one side of the angle cuts across 0, at the beginning of the scale, and so that the other side cuts across a point further up on the scale, We use either the upper scale or the lower scale, whichever is more convenient, For example, in Figure 2.2.3, one side of $\angle BAC$ crosses 0 on the lower scale and the other side crosses 50 on the lower scale. The measure of $\angle BAC$ is therefore 50° and we write $\angle BAC = 50^\circ$.

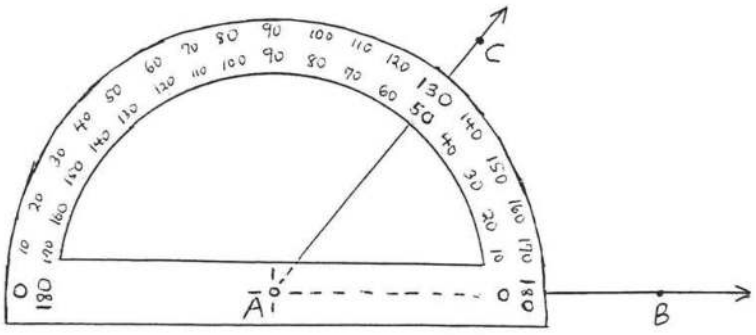


Figure 2.2.3: The protractor shows $\angle BAC = 50^\circ$

In Figure 2.2.4, side $\overrightarrow{AD}$ of $\angle DAC$ crosses 0 on the upper scale. Therefore we look on the upper scale for the point at which $\overrightarrow{AC}$ crosses and conclude that $\angle DAC = 130^\circ$.

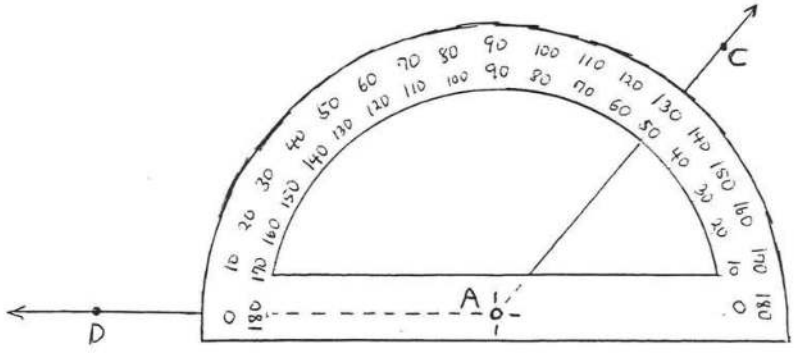


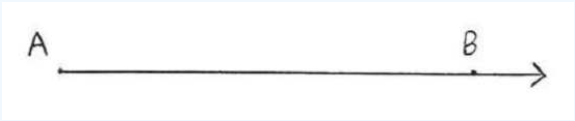
Figure 2.2.4: $\angle DAC = 130^\circ$.

Example 2.2.1

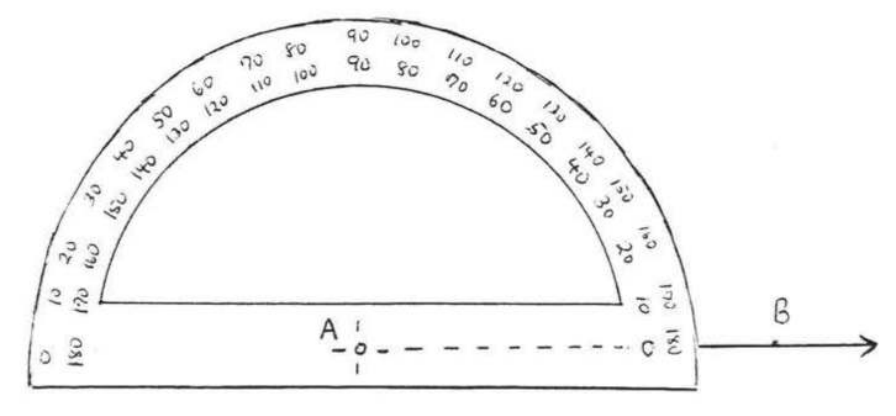
Draw an angle of 40° and label it $\angle BAC$.

Solution

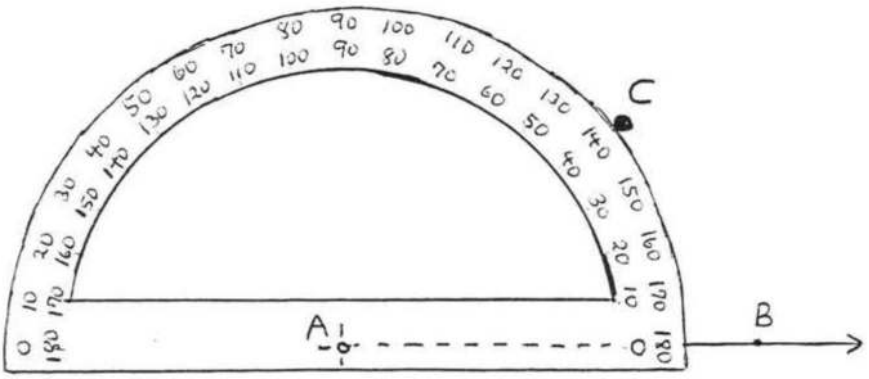
Draw ray $\overrightarrow{AB}$ using a straight edge:



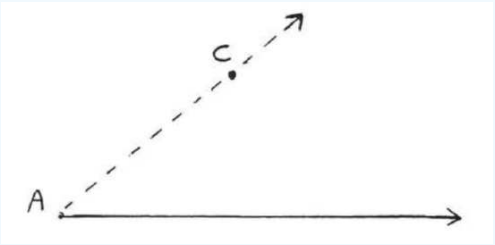
Place the protractor so that its center coincides with A and $\overrightarrow{AB}$ crosses the scale at 0:



Mark the place on the protractor corresponding to 40°. Label this point C:



Connect A with C:



Two angles are said to be equal if they have the same measure in degrees. We often indicate two angles are equal by marking them in the same way. In Figure 2.2.5, $\angle A = \angle B$.

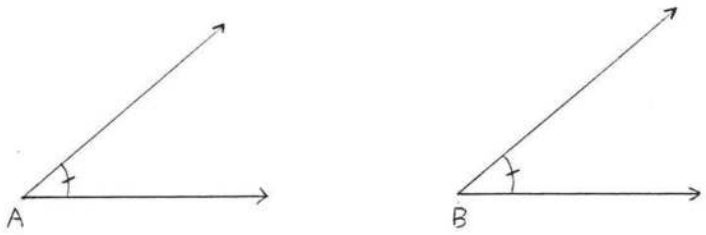


Figure 2.2.5: Equal angles.

An angle bisector is a ray which divides an angle into two equal angles. In Figure 2.2.6, $\overrightarrow{AC}$ is an angle bisector of $\angle BAD$. We also say $\overrightarrow{AC}$ bisects $\angle BAD$.

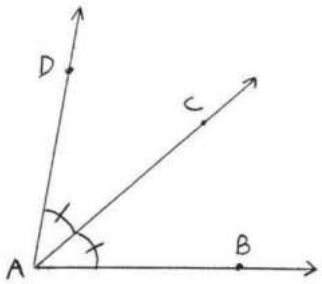
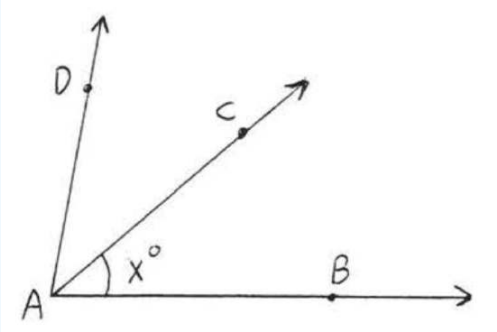


Figure 2.2.6: $\overrightarrow{AC}$ bisects $\angle BAD$.

Example 2.2.2

Find x if $\overrightarrow{AC}$ bisects $\angle BAD$ and $\angle BAD = 80°$:



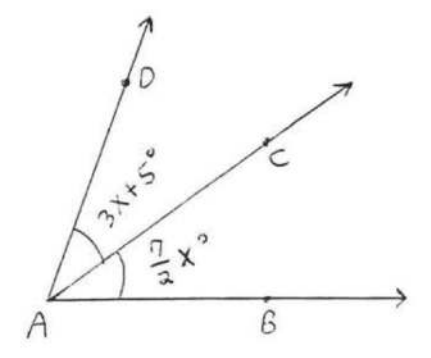
Solution

$x° = \frac{1}{2}\angle BAD = \frac{1}{2}(80°) = 40°$

Answer: $x = 40$.

Example 2.2.3

Find x if $\overrightarrow{AC}$ bisects $\angle BAD$:



Solution

$$\begin{aligned} \angle BAC &= \angle CAD \\ \frac{7}{2}x &= 3x+5 \\ (2)\frac{7}{2}x &= (2)(3x+5) \\ 7x &= 6x+10 \\ 7x-6x &= 10 \\ x &= 10 \end{aligned} \tag{2.2.1}$$

Check:

$\angle BAC = \angle CAD$

$\frac{7}{2}x^\circ = 3x + 5^\circ$

$\frac{7}{2}(10)^\circ = 3(10) + 5^\circ$

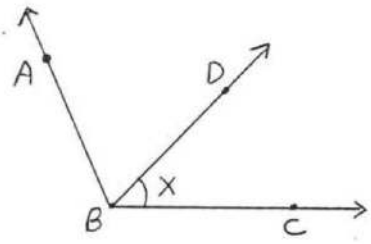
$35^\circ = 30 + 5^\circ$

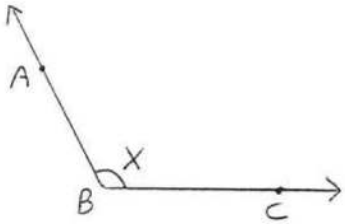
35°

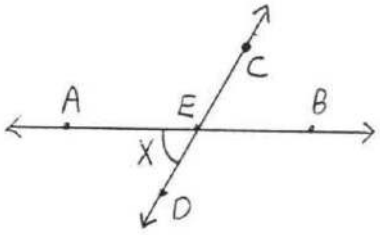
Answer: $x = 10$.

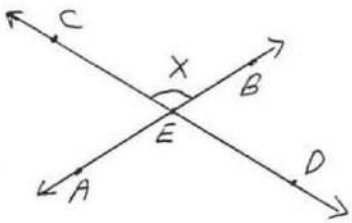
Problems

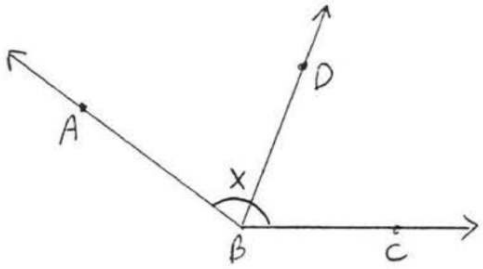
1 - 6. For each figure, give another name for $\angle x$:

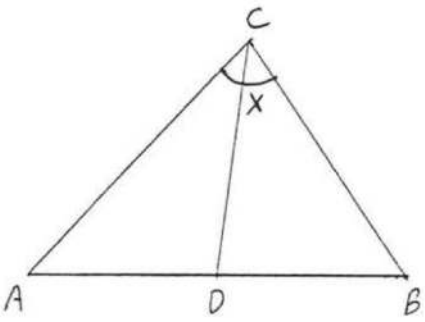
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2. 

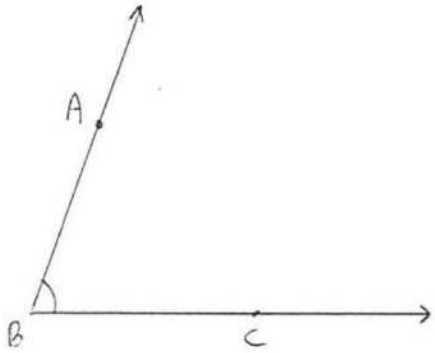
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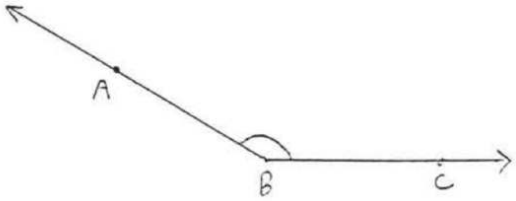
4. 

5. 

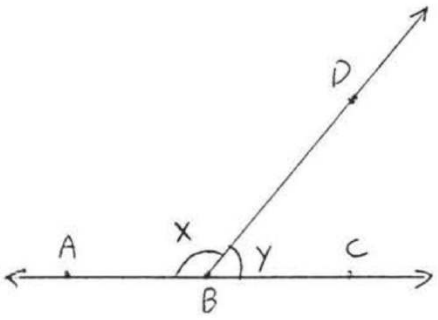
6. 

7 - 16, Measure each of the indicated angles:

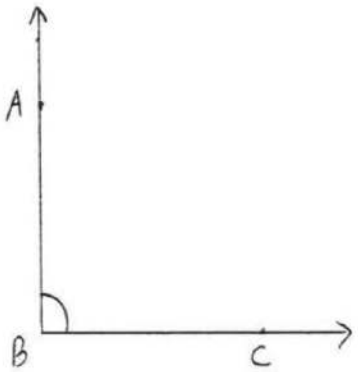
7. 

8. 

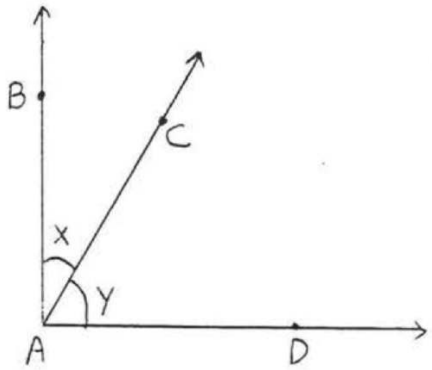
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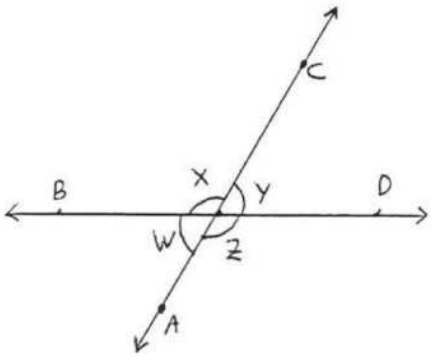
10.

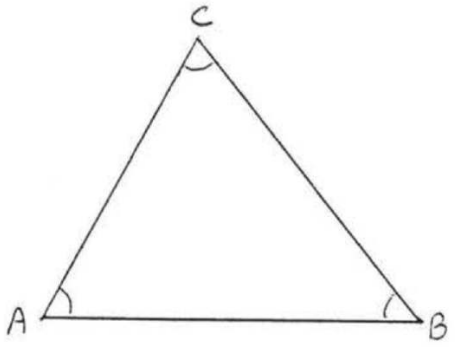


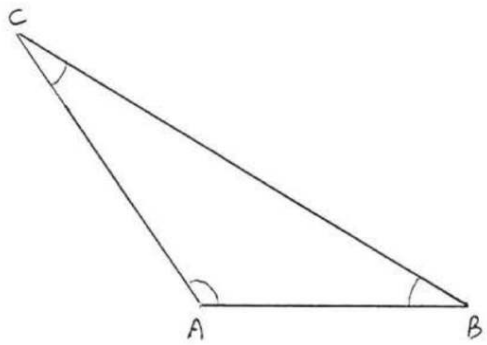
11.

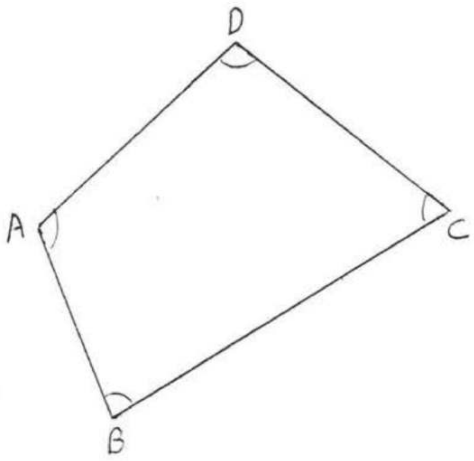


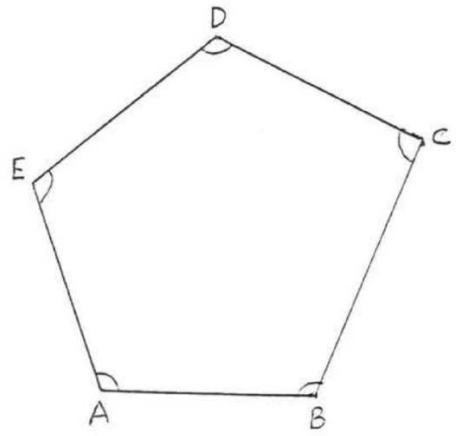
12.



13. 

14. 

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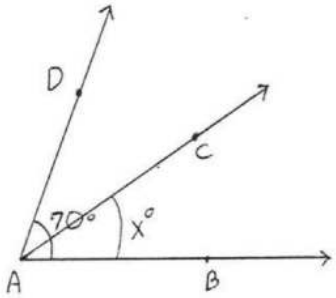
16. 

17 - 24. Draw and label each angle:

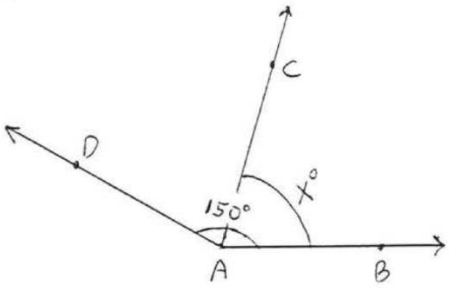
17. $\angle BAC = 30^\circ$
18. $\angle BAC = 40^\circ$
19. $\angle ABC = 45^\circ$
20. $\angle EFG = 60^\circ$
21. $\angle RST = 72^\circ$
22. $\angle XYZ = 90^\circ$
23. $\angle PQR = 135^\circ$
24. $\angle JKL = 164^\circ$

25 - 28. Find x if $\overrightarrow{AC}$ bisects $\angle BAD$:

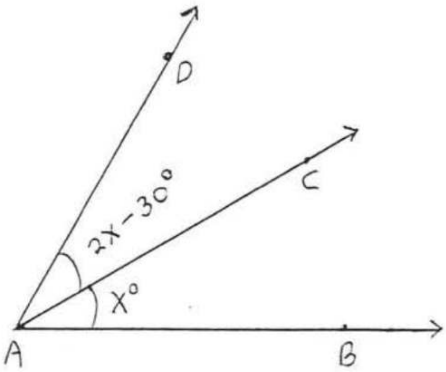
25.

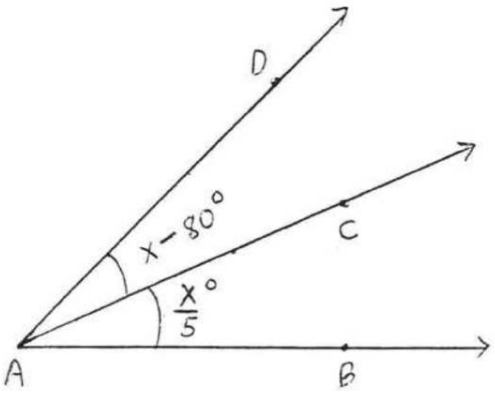


26.



27.



28. 

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2.3: Angle Classifications

Angles are classified according to their measures as follow:

- An **acute angle** is an angle whose measure is between 0° and 90°.
- A **right angle** is an angle whose measure is 90°. We often use a little square to indicate a right angle.
- An **obtuse angle** is an angle whose measure is between 90° and 180°.
- A **straight angle** is an angle whose measure is 180°. A straight angle is just a straight line with one of its points designated as the vertex.
- A **reflex angle** is an angle whose measure is greater than 180°.

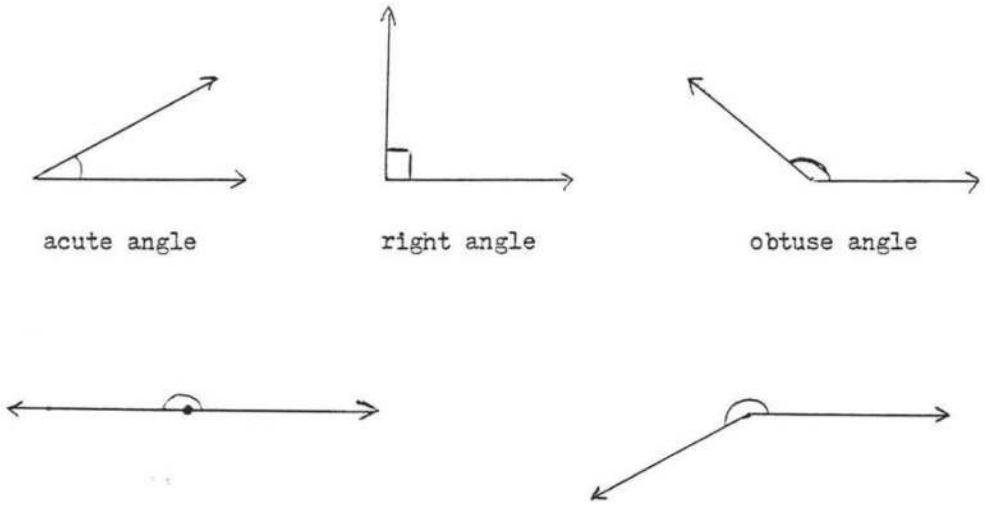


Figure 2.3.1: Angles classified according to their measures.

Notice that an angle can be measured in two ways. In Figure 2.3.2, $\angle ABC$ is a reflex of 240° or an obtuse angle of 120° depending on how it is measured. Unless otherwise indicated, we will always assume the angle has measure less than 180°.

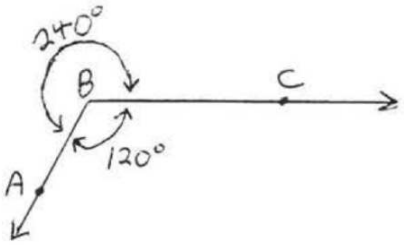


Figure 2.3.2: $\angle ABC$ can be measured in two different ways.

The lines are **perpendicular** if they meet to form right angles. In Figure 2.3.3, $\overleftrightarrow{AB}$ is perpendicular to $\overleftrightarrow{CD}$. The symbol for perpendicular is $\perp$ and we write $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$.

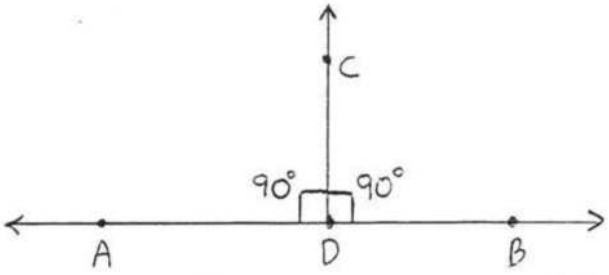


Figure 2.3.3: $\overleftrightarrow{AB}$ is perpendicular to $\overleftrightarrow{CD}$.

The **perpendicular bisector** of a line segment is a line perpendicular to the line segment at its midpoint, In Figure 2.3.4, $\overleftrightarrow{CD}$ is a perpendicular bisector of AB.

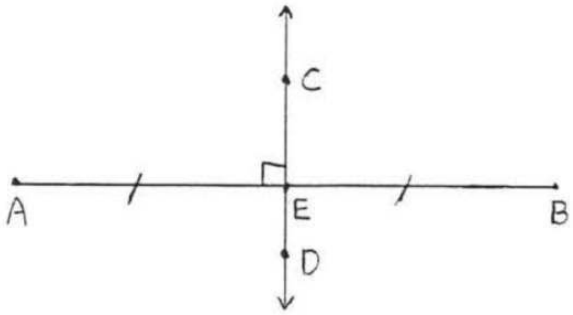


Figure 2.3.4: $\overleftrightarrow{CD}$ is a perpendicular bisector of AB.

Two angles are called complementary if the sum of their measures is 90°. Each angle is called the complement of the other. For example, angles of 60° and 30° are complementary.

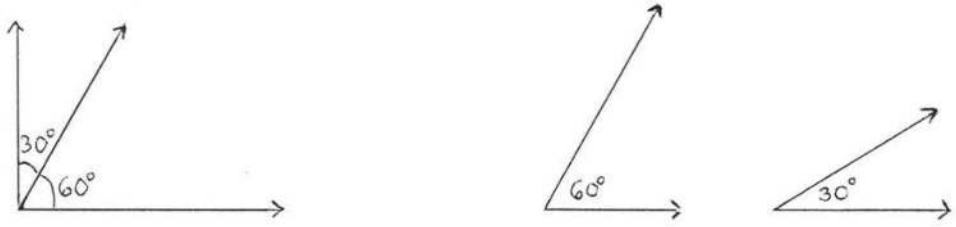


Figure 2.3.5: Complementary angles.

Example 2.3.1

Find the complement of a 40° angle.

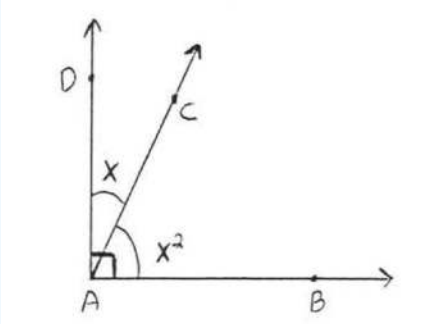
Solution

$90^\circ - 40^\circ = 50^\circ$.

Answer: 50°.

Example 2.3.2

Find x and the complementary angles:



Solution

Since $\angle BAD = 90^\circ$,

$$\begin{aligned} x^2+x &\quad 90^\circ \\ x^2+x-90 &\quad 0 \\ (x-9)(x+10) &\quad 0 \end{aligned} \tag{2.3.1}$$

$$\begin{aligned} x-9 &= 0 \qquad x+10 = 0 \\ x &= 9 \qquad\quad x = -10 \end{aligned} \tag{2.3.2}$$

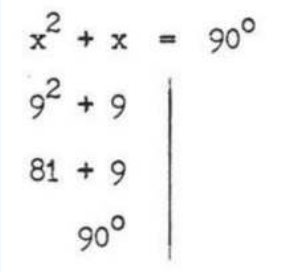
$\angle CAD = x = 90^\circ$. $\angle CAD = x = -10^\circ$.

$\angle BAC = x^2 = 9^2 = 81^\circ$.

$\angle BAC = \angle CAD = 81^\circ + 9^\circ = 90^\circ$.

We reject the answer $x = -10$ because the measure of an angle is always positive.(In trigonometry, when directed angles are introduced, angles can have negative measure. In this book, however, all angles will be thought of as having positive measure,)

Check, $x = 9$:



Answer: $x = 9$, $\angle CAD = 9^\circ$, $\angle BAC = 81^\circ$.

Two angles are called supplementary if the sum of their measures is 180°. Each angle is called the supplement of the other. For example, angle of 150° and 30° are supplementary.



Figure 2.3.6: Supplementary angles.

Example 2.3.3

Find the supplement of an angle of 40°.

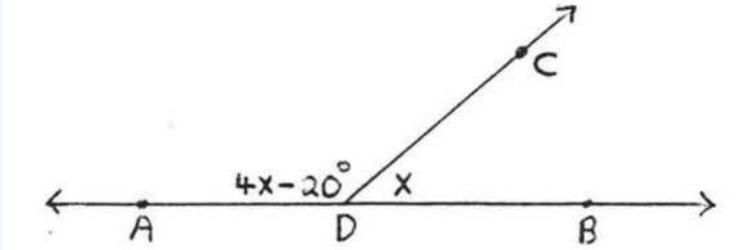
Solution

$180^\circ - 40^\circ = 140^\circ$.

Answer: 140°.

Example 2.3.4

Find x and the supplementary angles:



Solution

Since $\angle ADB = 180^\circ$,

$$\begin{aligned} 4x - 20 + x &= 180^\circ \\ 5x &= 180 + 20 \\ 5x &= 200 \\ x &= 40 \end{aligned} \tag{2.3.3}$$

$\angle ADC = 4x - 20 = 4(40) - 20 = 160 - 20 = 140^\circ$

$\angle BDC = x = 40^\circ$,

$\angle ADC + \angle BDC = 140^\circ + 40^\circ = 180^\circ$.

Check:

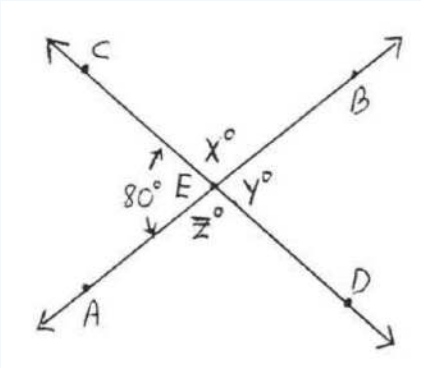
$$\begin{aligned} 4x - 20 + x &= 180^\circ \\ 4(40) - 20 + 40 & \\ 160 - 20 + 40 & \\ 140 + 40 & \\ 180^\circ & \end{aligned}$$

Answer

$x = 40, \angle ADC = 140^\circ, \angle BDC = 40^\circ$.

Example 2.3.5

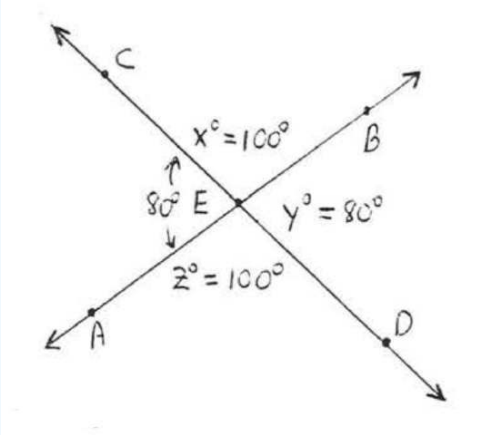
Find x, y, z:



Solution

$x^\circ = 180^\circ - 80^\circ = 100^\circ$ because x° and 80° are the measures of supplementary angles.

$$y^\circ = 180^\circ - x^\circ = 180^\circ - 100^\circ = 80^\circ.$$
$$z^\circ = 180^\circ - 80^\circ = 100^\circ. \tag{2.3.4}$$



Answer: $x = 100$, $y = 80$, $z = 100$.

When two lines intersect as in EXAMPLE E, they form two pairs of angles that are opposite to each other called vertical angles, In Figure 2.3.7, $\angle x$ and $\angle x'$ are one pair of vertical angles. $\angle y$ and $\angle y'$ a.re the other pair of vertical angles, As suggested by Example 2.3.5, $\angle x = \angle x'$ and $\angle y = \angle y'$. To see this in general, we can reason as follows: $\angle x$ is the supplement of $\angle y$ so $\angle x = 180^\circ - \angle y$. $\angle x'$ is also the supplement of $\angle y$ so $\angle x' = 180 - \angle y$. Therefore $\angle x = \angle x'$. Similarly, we can show $\angle y = \angle y'$. Therefore vertical angles are always equal.

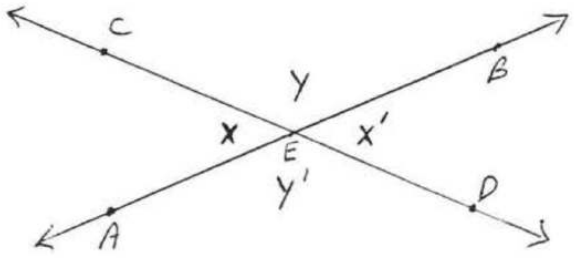
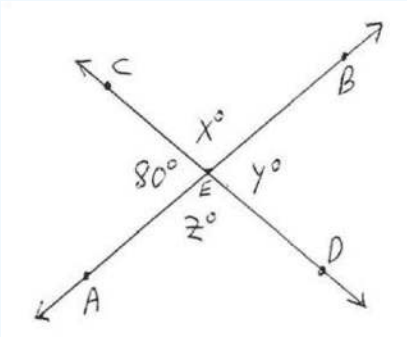


Figure 2.3.7: $\angle x$, $\angle x'$ and $\angle y$, $\angle y'$ are pairs of vertical angles.

We can now use "vertical angles are equal" in solving problems:

Example 2.3.5 (repeated)

Find x, y, and z:



Solution

$\angle x = 180^\circ - 80^\circ = 100^\circ$ because $\angle x$ is the supplement of 80°.

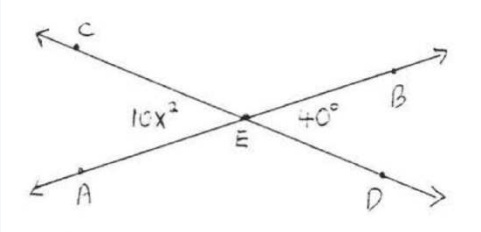
$\angle y = 80^\circ$ because vertical angles are equal.

$\angle z = \angle x = 100^\circ$ because vertical angles are equal.

Answer: $x = 100$, $y = 80$, $z = 100$.

Example 2.3.6

Find x:



Solution

Since vertical angles are equal, $10x^2 = 40^\circ$.

Method 1:

$$\begin{aligned} 10x^2 &= 40 \\ 10x^2 - 40 &= 0 \\ (10)(x^2 - 4) &= 0 \\ x^2 - 4 &= 0 \\ (x+2)(x-2) &= 0 \end{aligned}$$

$$\begin{aligned} x + 2 &= 0 \quad & x - 2 &= 0 \\ x &= -2 \quad & x &= 2 \end{aligned}$$

Method 2:

$$\begin{aligned} 10x^2 &= 40 \\ \frac{10x^2}{10} &= \frac{40}{10} \\ x^2 &= 4 \\ x &= \pm 2 \end{aligned}$$

If $x = 2$ then $\angle AEC = 10x^2 = 10(2)^2 = 10(4) = 40^\circ$.

If $x = -2$ then $\angle AEC = 10x^2 = 10(-2)^2 = 10(4) = 40^\circ$.

We accept the solution $x = -2$ even though x is negative because the value of the angle $10x^2$ is still positive.

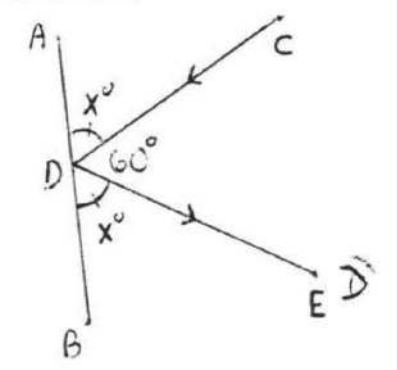
Check:



Answer: $x = 2$ or $x = -2$.

Example 2.3.7

In the diagram, AB represents a mirror, CD represents a ray of light approaching the mirror from C, and E represents the eye of a person observing the ray as it is reflected from the mirror at D. According to a law of physics, $\angle CDA$, called the angle of incidence, equals $\angle EDB$, called the angle of reflection. If $\angle CDE = 60^\circ$, how much is the angle of incidence?



Solution

Let $\x^\circ = \angle CDA = \angle EDB$.

$$\begin{aligned} x + x + 60 &= 180 \\ 2x + 60 &= 180 \\ 2x &= 120 \\ x &= 60 \end{aligned} \tag{2.3.5}$$

Answer: 60°

Note on Theorems and Postulates

The statement "vertical angles are always equal" is an example of a **theorem**. A theorem is a statement which we can prove to be true, A proof is a process of reasoning which uses statements already known to be true to show the truth of a new statement, A example of a proof is the discussion preceding the statement "vertical angles are always equal." We used facts about supplementary angles that were already known to establish the new statement, that "vertical angles are always equal."

Ideally we would like to prove all statements in mathematics which we think are true. However before we can begin proving anything we need some true statements with which to start. Such statements should be so self-evident as not to require proofs themselves, A statement of this kind, which we assume to be true without proof, is called a **postulate** or an **axiom**. An example of a postulate is the assumption that all angles can be measured in degrees. This was used without actually being stated in our proof that "vertical angles are always equal,"

Theorems, proofs, and postulates constitute the heart of mathematics and we will encounter many more of them as we continue our study of geometry.

Problems

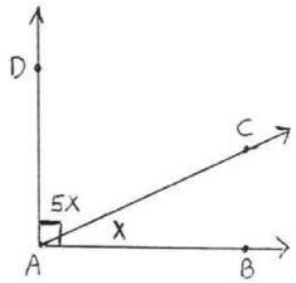
1. Find the complement of an angle of

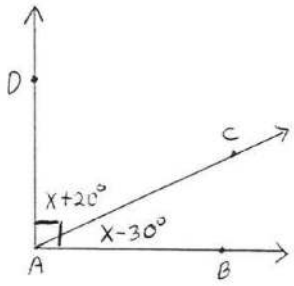
a. 37°
b. 45°
c. 53°
d. 60°

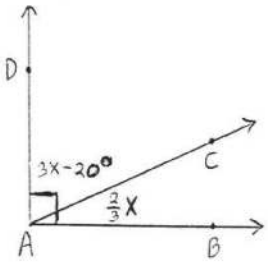
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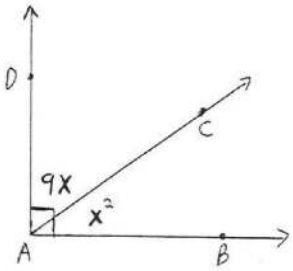
a. 30°
b. 40°
c. 50°
d. 81°

3 - 6. Find x and the complementary angles:

3. 

4. 

5. 

6. 

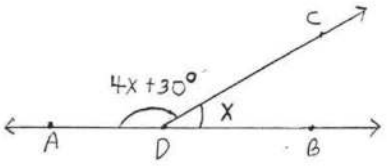
7. Find the supplement of an angle of

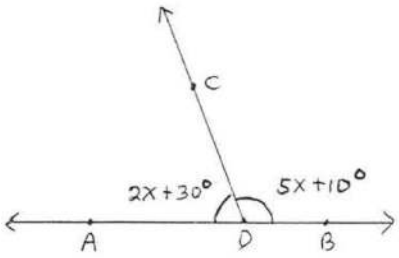
a. 30°
b. 37°
c. 90°
d. 120°

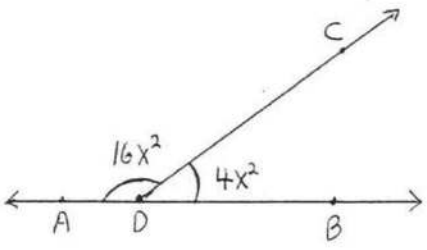
8. Find the supplement of an angle of

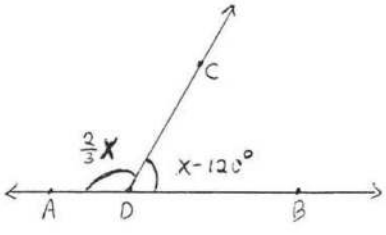
a. 45°
b. 52°
c. 85°
d. 135°

9 - 14. Find x and the supplementary angles:

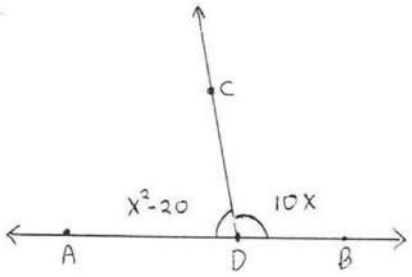
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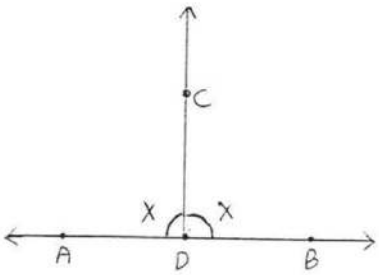
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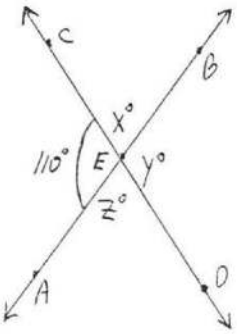


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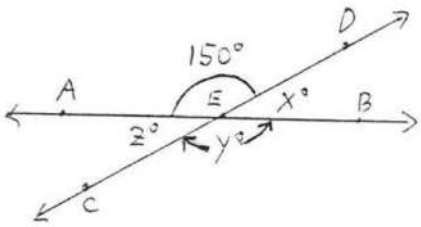


15 - 22. Find x, y, and z:

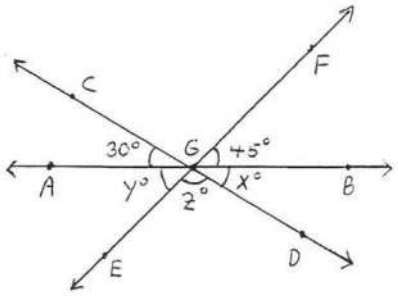
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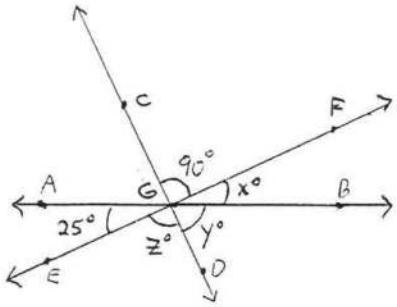
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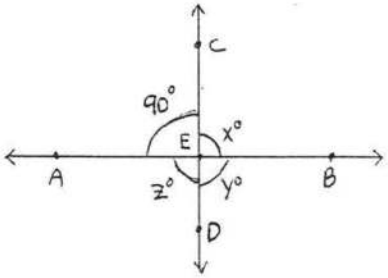
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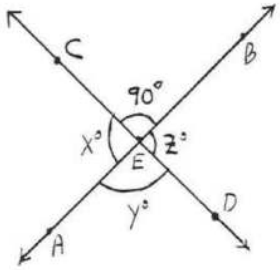
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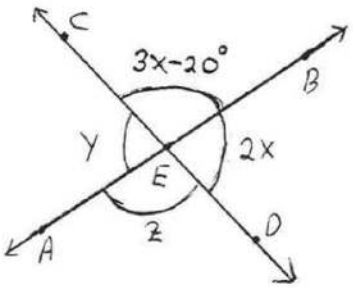
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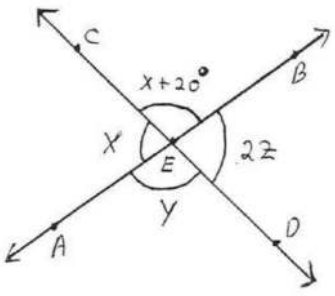
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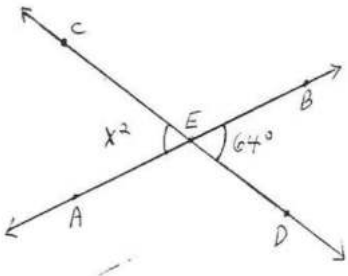
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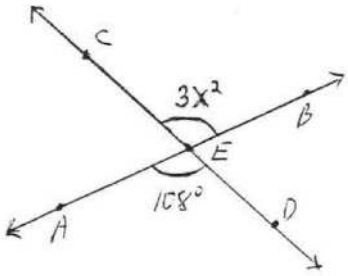
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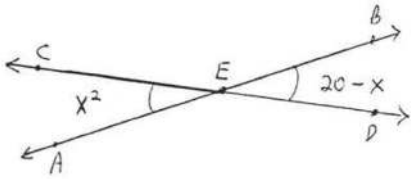
23 - 26. Find x:



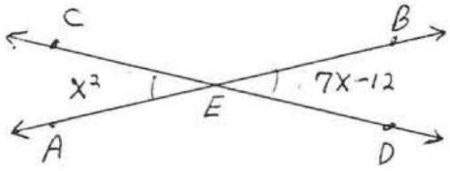
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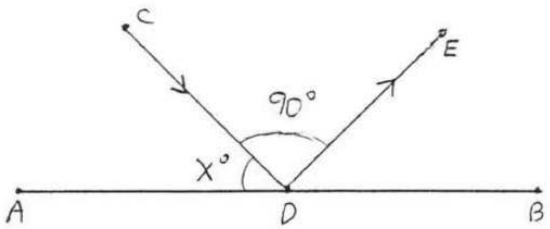


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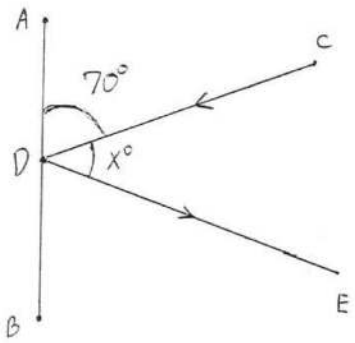


26.

27. Find the angle of incidence, $\angle CDA$:



28. Find x if the angle of incidence is 40°:



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2.4: Parallel Lines

Two lines are **parallel** if they do not meet, no matter how far they are extended. The symbol for parallel is $||$. In Figure 2.4.1, $\overleftrightarrow{AB}$ $||$ $\overleftrightarrow{CD}$. The arrow marks are used to indicate the lines are parallel.

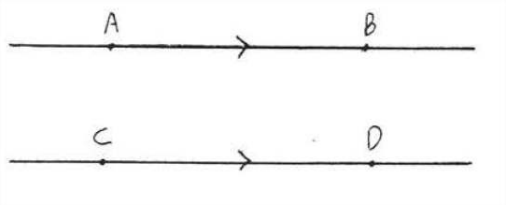


Figure 2.4.1: $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are parallel.They do not meet no matter how far they are extended.

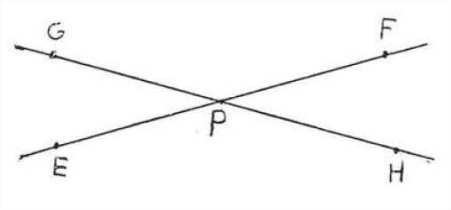


Figure 2.4.1: $\overleftrightarrow{EF}$ and $\overleftrightarrow{GH}$ are not parallel. They meet at point P.

We make the following assumption about parallel lines, called the **parallel postulate.**

Theorem 2.4.1: Parallel Postulate

The probabilities assigned to events by a distribution function on a sample space are given by

Through a point not on a given line one and only one line can be drawn parallel to the given line. So in Figure 2.4.3, there is exactly one line that can be drawn through C that is parallel to $\overleftrightarrow{AB}$.

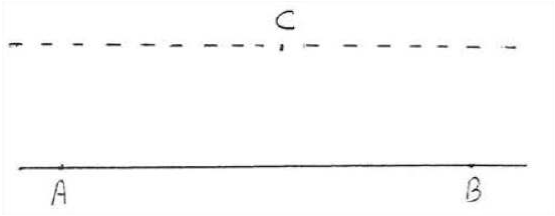


Figure 2.4.3: There is exactly one line that can be drawn through C parallel to $\overrightarrow{AB}$.

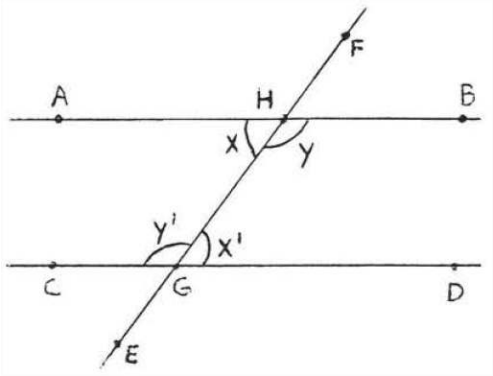


Figure 2.4.4: $\overleftrightarrow{EF}$ is a transversal.

A **transversal** is a line that intersects two other lines at two distinct points. In Figure 2.4.4, $\overleftrightarrow{EF}$ is a transversal. $\angle x$ and $\angle x'$ are called **alternate interior angles** of lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$. The word "alternate," here, means that the angles are on different sides of the transversal, one angle formed with $\overleftrightarrow{AB}$ and the other formed with $\overleftrightarrow{CD}$. The word "interior" means that they are between the two lines. Notice that they form the letter "Z." (Figure 2.4.5). $\angle y$ and $\angle y'$ are also alternate interior angles. They also form a "Z" though It is stretched out and backwards. Viewed from the side, the letter "Z" may also look like an "N."

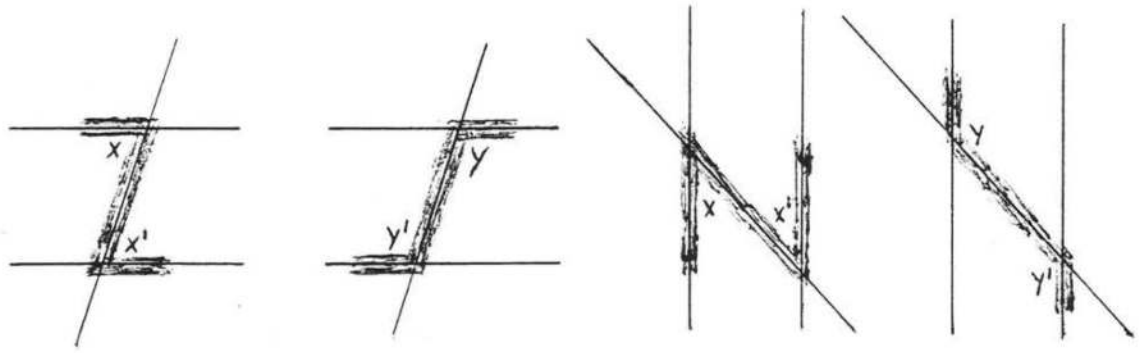


Figure 2.4.5: Alternate interior form the letters "Z" or "N". The letter may be stretched out or backwards.

Alternate interior angles are important because of the following theorem:

Theorem 2.4.1 The "Z" Theorem

If two lines are parallel then their alternate interior angles are equal, If the alternate interior angles of two lines are equal then the lines must 'oe parallel,

In Figure 2.4.6, $\overleftrightarrow{AB}$ must be parallel to $\overleftrightarrow{CD}$ because the alternate interior angles are both 30°. Notice that the other pair of alternate interior angles, $\angle y$ and $\angle y'$, are also equal. They are both 150°. In Figure 2.4.7, the lines are not parallel and none of the alternate interior angles are equal.

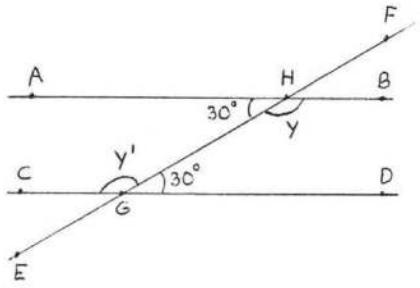


Figure 2.4.6: The lines are parallel and their alternate interior angles are equal.

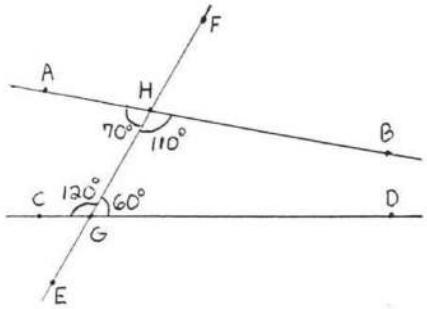
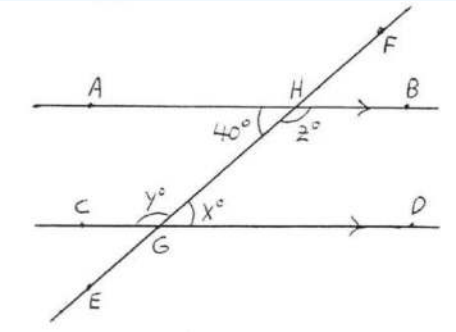


Figure 2.4.7: The lines are not parallel and their alternate interior angles are not equal.

The Proof of Theorem 2.4.1 is complicated and will be deferred to the appendix.

Example 2.4.1

Find x, y and z:



Solution

$\overleftrightarrow{AB} || \overleftrightarrow{CD}$ since the arrows indicate parallel lines. $x^\circ = 40^\circ$ because alternate interior angles of parallel lines are equal. $y^\circ = z^\circ = 180^\circ - 40^\circ = 140^\circ$.

Answer: $x = 40, y = 140, z = 140$.

Corresponding angles of two lines are two angles which are on the same side of the two lines and the same side of the transversal, In Figure 2.4.8, $\angle w$ and $\angle w'$ are corresponding angles of lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$. They form the letter "F." $\angle x$ and $\angle x'$, $\angle y$ and $\angle y'$, and $\angle z$ and $\angle z'$ are other pairs of corresponding angles of $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$. They all form the letter "F", though it might be a backwards or upside down "F" (Figure 2.4.9).

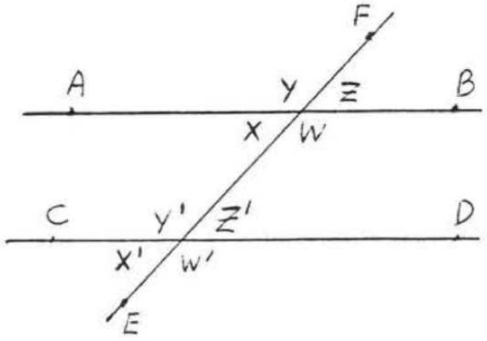


Figure 2.4.8: Four pairs of corresponding angles are illustrated.

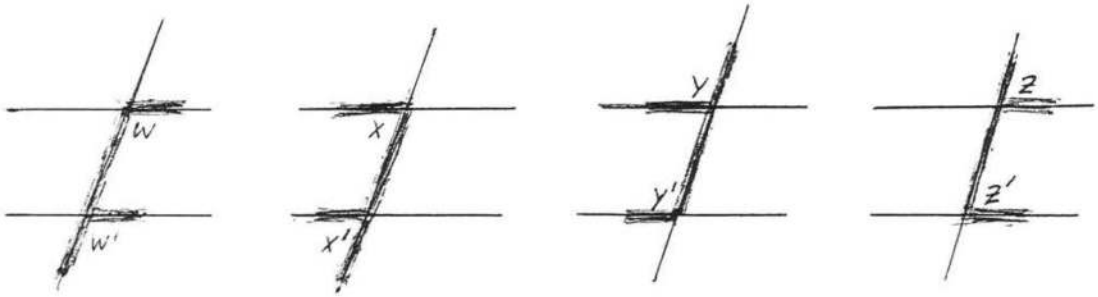


Figure 2.4.9: Corresponding angles form the letter "F," though it may be a backwards or upside down "F."

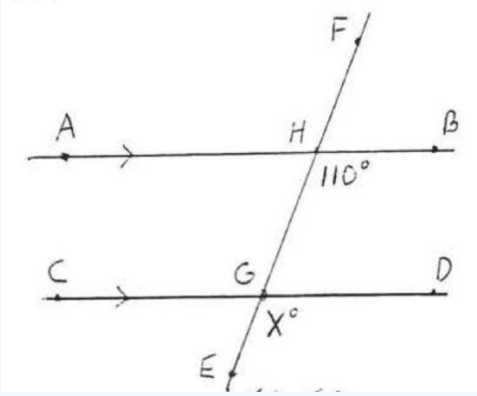
Corresponding angles are important because of the following theorem:

Theorem 2.4.2: The "F" Theorem

If two lines are parallel then their corresponding angles are equal. If the corresponding angles of two lines are equal then the lines must be parallel.

Example 2.4.2

Find x:



Solution

The arrow indicate $\overleftrightarrow{AB} || \overleftrightarrow{CD}$. Therefore $x^\circ = 110^\circ$ because x° and 110° are the measures of corresponding angles of the parallel lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$.

Answer: $x = 110$.

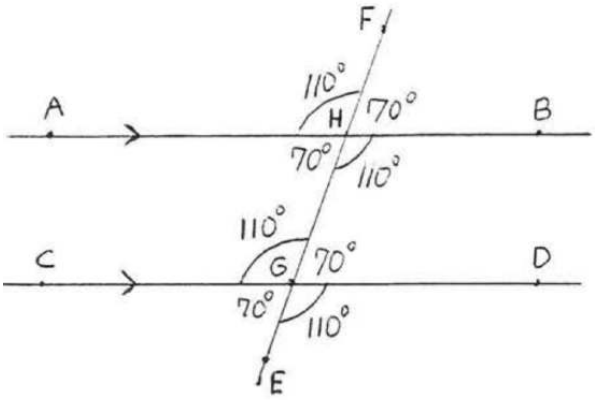


Figure $PageIndex10$: Each pair of corresponding angles is equal.

Notice that we can now find all the other angles in Example $PageIndex2$. Each one is either supplementary to one of the 110° angles or forms equal vertical angles with one of them (Figure $PageIndex10$). Therefore **all** the corresponding angles are equal, Also each pair of alternate interior angles is equal. It is not hard to see that if just one pair of corresponding angles or one pair of alternate interior angles are equal then so are all other pairs of corresponding and alternate interior angles.

Proof of Theorem 2.4.2: The corresponding angles will be equal if the alternate interior angles are equal and vice versa. Therefore Theorem 2.4.2 follows directly from Therorem 2.4.1.

In Figure 2.4.11, $\angle x$ and $\angle x'$ are called **interior angles on the same side of the transversal**.(In some textbooks, interior angles on the same sdie of the transversal are called cointerior angles.) $\angle y$ and $\angle y'$ are also interior angles on the same side of the transversal, Notice that each pair of angles forms the letter "C." Compare Figure 2.4.11 with Figure 10 and also with Example 2.4.1, The following theorem is then apparent:

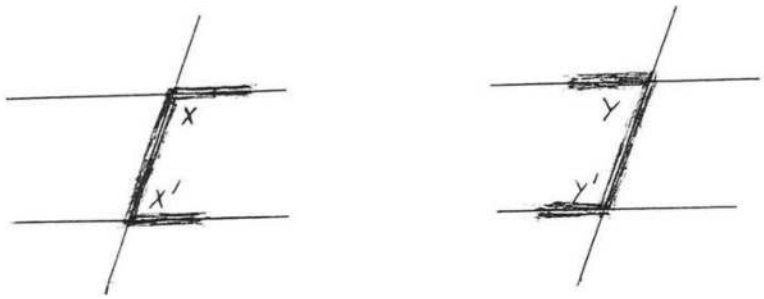


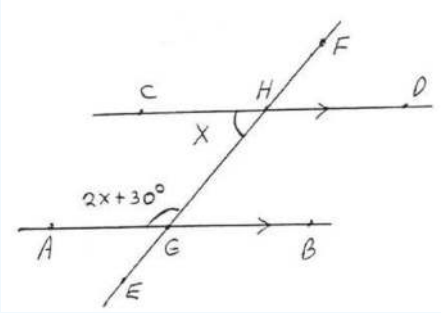
Figure 2.4.11: Interior angles on the same side of the transversal form the letter "C". It may also be a backwards "C."

Theorem 2.4.3:The "C" Theorem

If two lines are parallel then the interior angles on the same side of the transversal are supplementary (they add uP to $180°$). If the interior angles of two lines on the same side of the transversal are supplementary then the lines must be parallel.

Example 2.4.3

Find x and the marked angles:



Solution

The lines are parallel so by Theorem $2.4.3$ the two labelled angles must be supplementary.

$$\begin{aligned} x+2x+30 &= 180 \\ 3x+30 &= 180 \\ 3x &= 180-30 \\ 3x &= 150 \\ x &= 50 \end{aligned} \tag{2.4.1}$$

$\angle CHG = x = 50°$

$\angle AGH = 2x+30 = 2(50)+30 = 100+30 = 130°$.

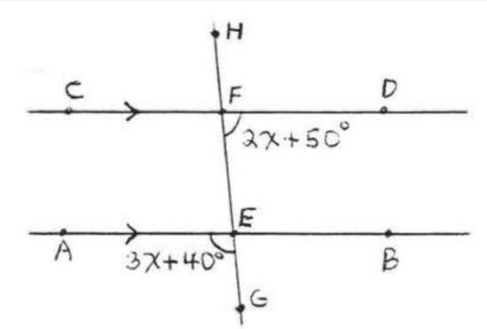
Check:

$$\begin{aligned} x + 2x + 3x &= 180° \\ 50 + 2(50) + 30 & \\ 50 + 130 & \\ 180 & \end{aligned}$$

Answer: $x = 50$, $\angle CHG = 50°$, $\angle AGHa = 130°$.

Example 2.4.4

Find x and the marked angles:



Solution

$\angle BEF = 3x + 40^\circ$ because vertical angles are equal. $\angle BEF$ and $\angle DFE$ are interior angles on the same side of the transversal, and therefore are supplementary because the lines are parallel.

$$\begin{aligned} 3x + 40 + 2x + 50 &= 180 \\ 5x + 90 &= 180 \\ 5x &= 180 - 90 \\ 5x &= 90 \\ x &= 18 \end{aligned} \tag{2.4.2}$$

$\angle AEC = 3x + 40 = 3(18) + 40 = 54 + 40 = 94^\circ$

$\angle DFE = 2x + 50 = 2(18) + 50 = 36 + 50 = 86^\circ$

Check:

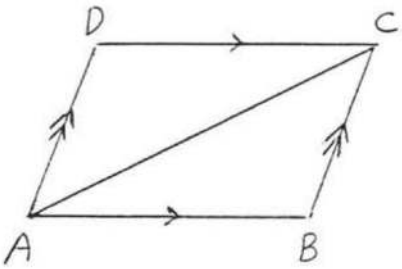
$$\begin{aligned} 3x + 40 + 2x + 50 &= 180 \\ 3(18) + 40 + 2(18) + 50 & \\ 54 + 40 + 36 + 50 & \\ 94 + 86 & \\ 180 & \end{aligned}$$

Answer: $x = 18$, $\angle AEG = 94^\circ$, $\angle DFE = 86^\circ$.

Example 2.4.5

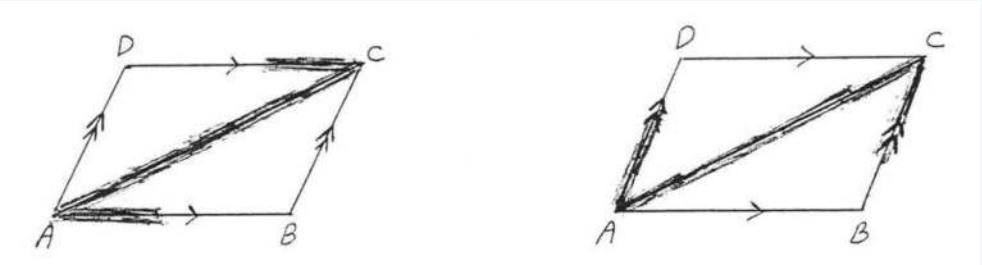
List all pairs of alternate interior angles in the diagram, (The single arrow indicates $\overleftrightarrow{AB}$ is parallel to $\overleftrightarrow{CD}$ and the double arrow indicates $\overleftrightarrow{AD}$ is parallel to $\overleftrightarrow{BC}$.

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Solution

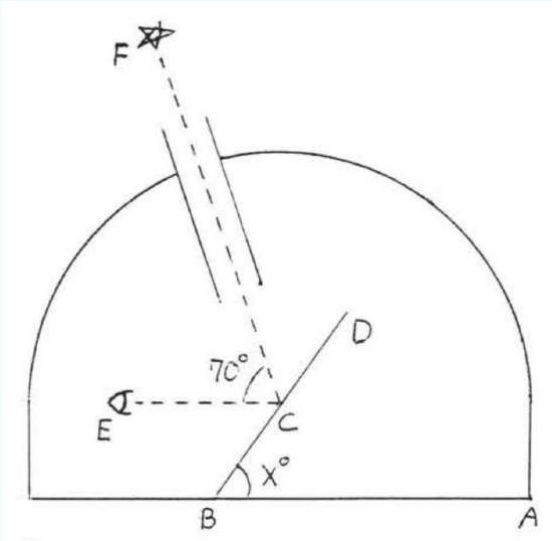
We see if a letter Z or N can be formed using the line segments in the diagram (Figure 2.4.12),



Answer: $\angle DCA$ and $\angle CAB$ are alternate interior angles of lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$. $\angle DAC$ and $\angle ACB$ are alternate interior angles of lines $\overleftrightarrow{AD}$ and $\overleftrightarrow{BC}$

Example 2.4.6

A telescope is pointed at a star 70° above the horizon, What angle x° must the mirror BD make with the horizontal so that the star can be seen in the eyepiece E?



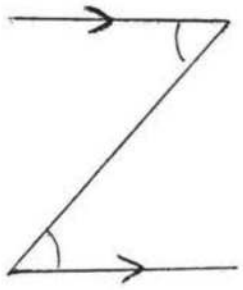
Solution

$x^\circ = \angle BCE$ because they are alternate interior angles of parallel lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CE}$. $\angle DCF = \angle BCE = x^\circ$ because the angle of incidence is equal to the angle of reflection. Therefore

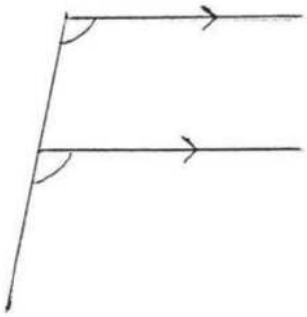
$$\begin{aligned} x+70+x &= 180 \\ 2x+70 &= 180 \\ 2x &= 110 \\ x &= 55 \end{aligned} \tag{2.4.3}$$

Answer: 55°

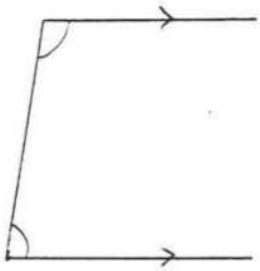
SUMMARY



Alternate interior angles of paralle lines are equal. They form the letter "Z."



Corresponding angles of parallel lines are equal. They form the letter "F."



Interior angles on the same sides of the transversal of parallel lines are supplementary. They form the letter "G."

Historical Note

The parallel postulate given earlier in this section is the equivalent of the fifth postulate of Euclid's Elements. Euclid was correct in assuming it as a postulate rather than trying to prove it as a theorem, However this did not become clear to the mathematical world until the nineteenth century, 2200 years later, In the interim, scores of prominent mathematicians attempted unsuccessfully to give a satisfactory proof of the parallel postulate. They felt that it was not as self-evident as a postulate should be, and that it required some formal justification,

In 1826, N, I, Lobachevsky, a Russian mathematician, presented a system of geometry based on the assumption that through a given point more than one straight line can be drawn parallel to a given line (Figure 2.4.13). In 1854, the German mathematician Georg Bernhard Riemann proFosed a system of geometriJ in which there are no parallel lines at all, A gecmetry in which the parallel postulate has been replaced by some other postulate is called a non-Euclidean geometry. The existence of these geometries shows that the parallel postulate need not necessarily be true. Indeed Einstein used the geometry of Riemann as the basis for his theory of relativity.

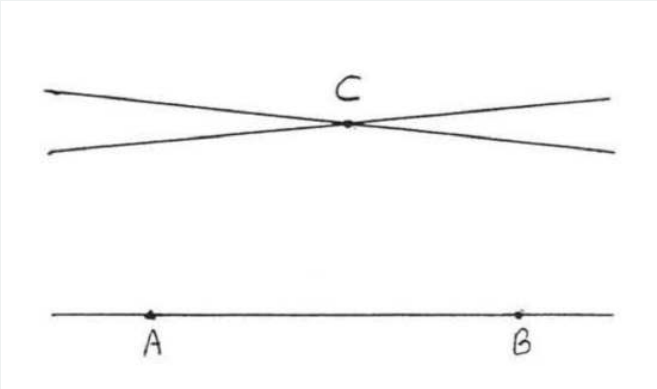


Figure $PageIndex13$: In the geometry of Lobachevsky, more than one line can be drawn through C parallel to AB.

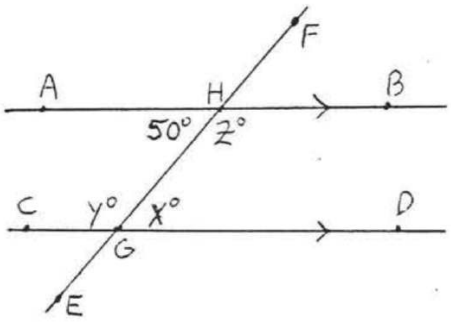
Of course our original parallel postulate makes the most sense for ordinary applications, and we use it throughout this book, However, for applications where great distances are involved, such as in astronomy, it may well be that a non-Euclidean geometry gives a better approximation of physical reality.

Problems

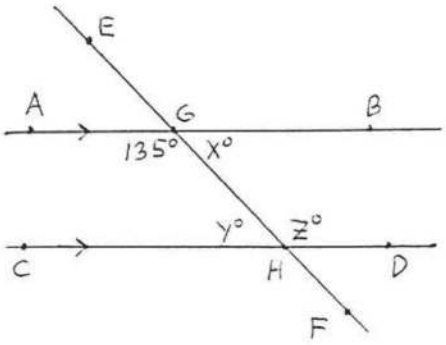
For each of the following, state the theorem(s) used in obtaining your answer (for example, "alternate interior angles of parallel lines are equal"). Lines marked with the same arrow are assumed to be parallel,

1 - 2. Find x, y, and z:

1.

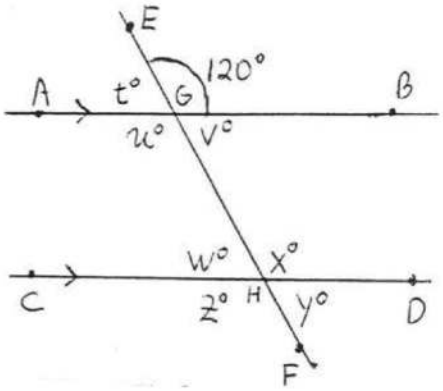


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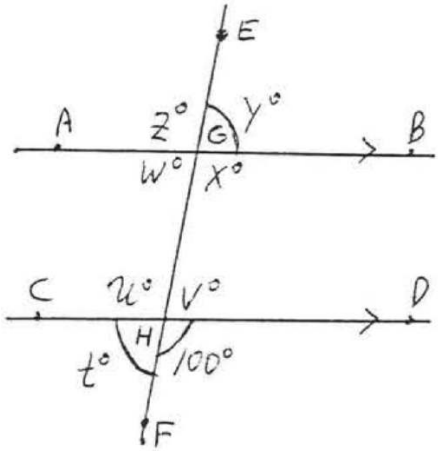


3 - 4. Find t, u, v, w, x, y, and z:

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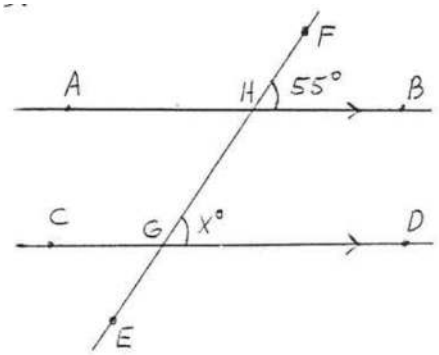


3.

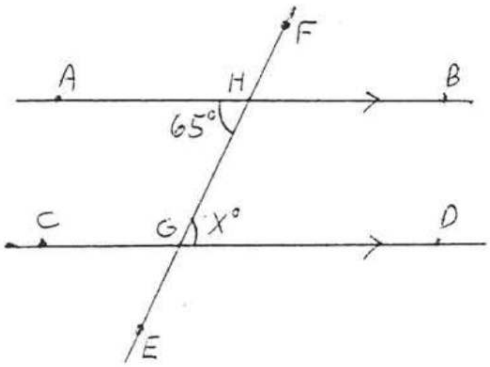


4.

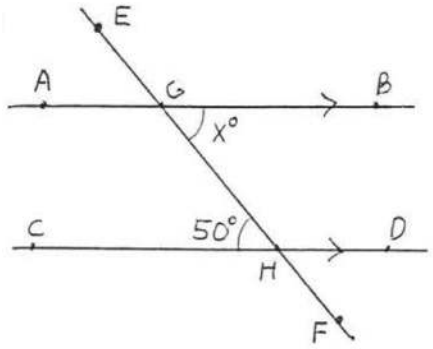
5 - 10. Find x:



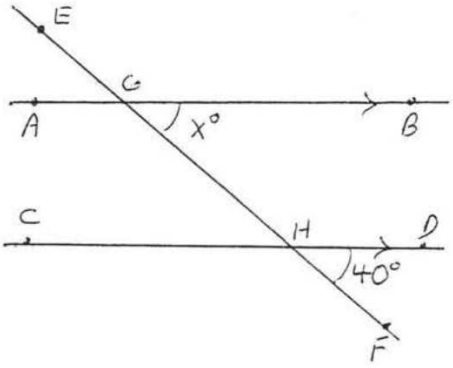
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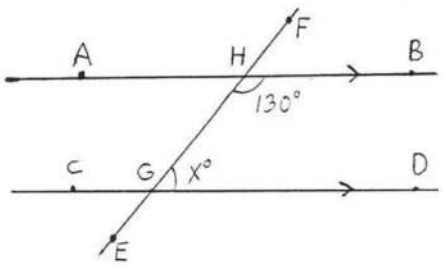
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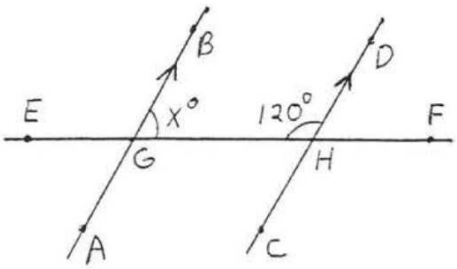
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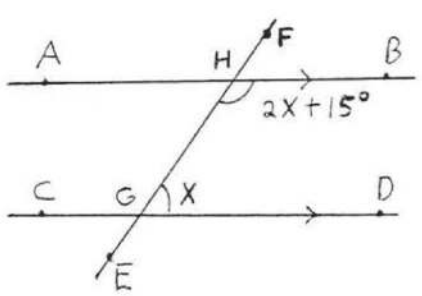


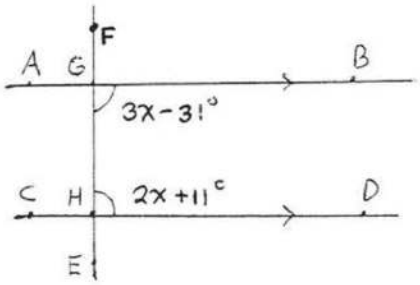
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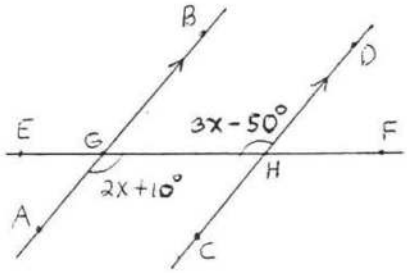


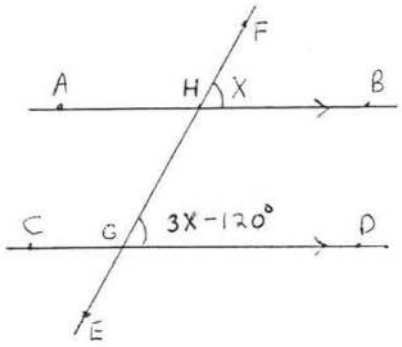
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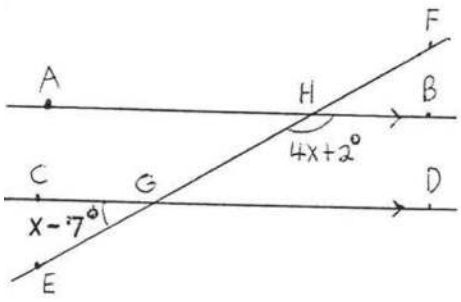
11 - 18. Find x and the marked angles:

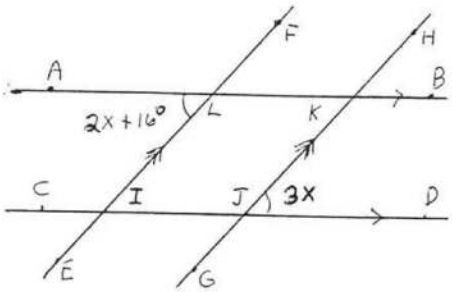
11. 

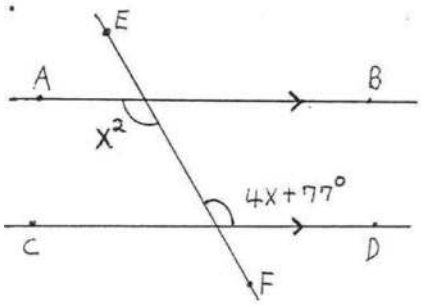
12. 

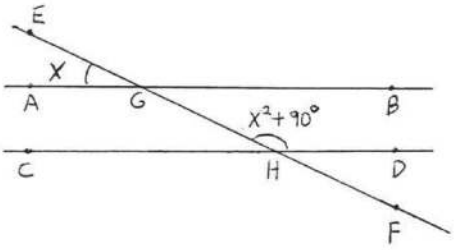
13. 

14. 

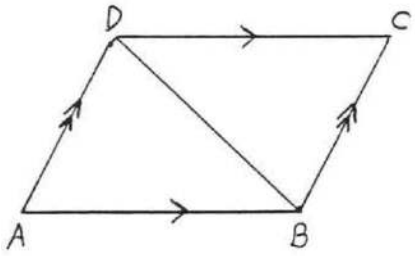
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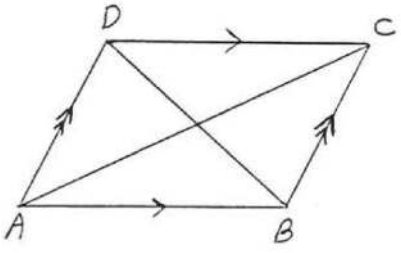
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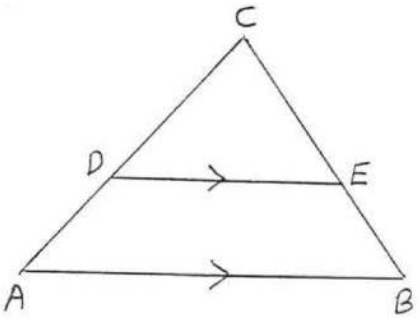
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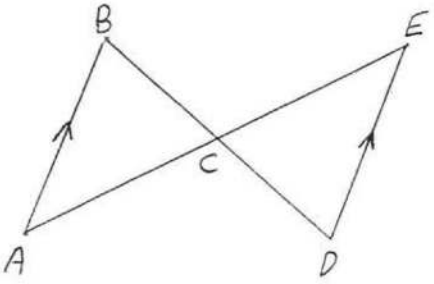
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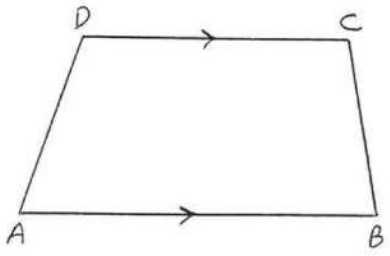
19 - 26. For each of the following, list all pairs of alternate interior angles and corresponding angles, If there are none, then list all pairs of interior angles on the same side of the transversal. Indicate the parallel lines which form each pair of angles.

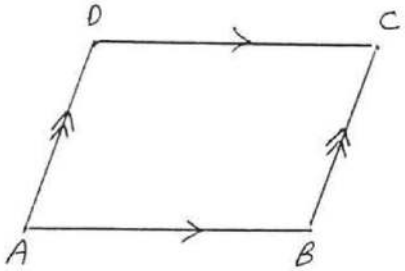
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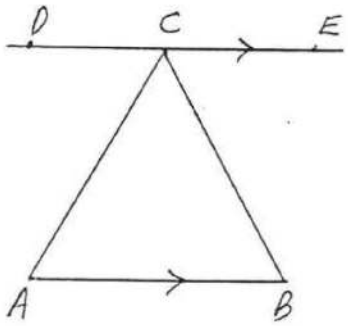
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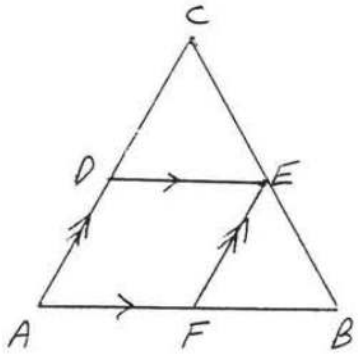
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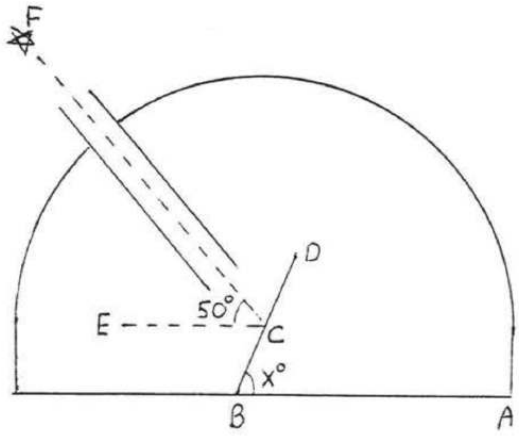
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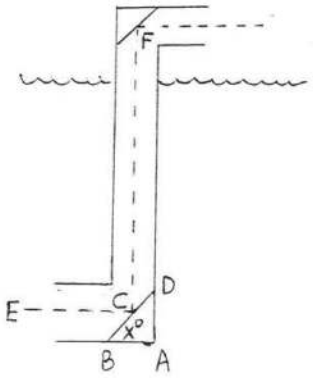
25. 

26. 

27. A telescope is pointed at a star 50° above the horizon. What angle x° must the mirror BD make wiht the horizontal so that the star can be seen in the eyepiece E?



28. A periscope is used by sailors in a submarine to see objects on the surface of the water, If $\angle ECF = 90^\circ$, what angle x° does the mirror BD make with the horizontal?



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2.5: Triangles

A **triangle** is formed when three straight line segments bound a portion of the plane. The line segments are called the **sides of the triangle**. A point where two sides meet is called a **vertex of the triangle**, and the angle formed is called an **angle of the triangle.** The symbol for triangle is $\triangle$.

The triangle in Figure 2.5.1 is denoted by $\triangle ABC$ (or $\triangle BCA$ or $\triangle CAB$, etc.).

- Its sides are AB, AC, and BC.
- Its vertices are A, B, and C.
- Its angles are $\angle A$, $\angle B$, and $\angle C$.

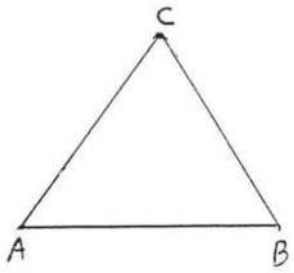


Figure 2.5.1: Triangle ABC.

The triangle is the most important figure in plane geometry, This is because figures with more than three sides can always be divided into triangles (Figure 2.5.2). If we know the properties of a triangle, we can extend this knowledge to the study of other figures as well.

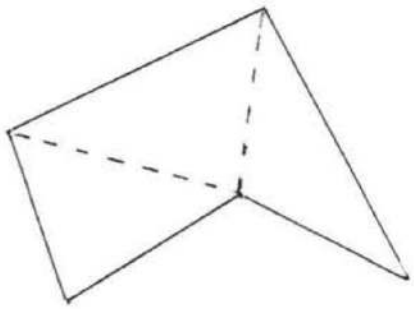


Figure 2.5.2: A closed figure formed by more than three straight lines can be divided into triangles.

A fundamental property of triangles is the following:

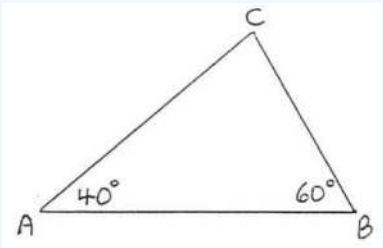
Theorem 2.5.1

The sum of the angles of a triangle is 180°.

In $\triangle ABC$ of Figure 2.5.1, $\angle A + \angle B + \angle C = 180^\circ$.

Example 2.5.1

Find $\angle C$:



Solution

$$\begin{aligned} \angle A+\angle B+\angle C &= 180^\circ \\ 40^\circ+60^\circ+\angle C &= 180^\circ \\ 100^\circ+\angle C &= 180^\circ \\ \angle C &= 180^\circ-100^\circ \\ \angle C &= 80^\circ \end{aligned}$$

Answer: $\angle C = 80^\circ$

Proof of Theorem 2.5.1: Through C draw DE parallel to AB (see Figure 2.5.3). Note that we are using the parallel postulate here, $\angle 1 = \angle A$ and $\angle 3 = \angle B$ because they are alternate interior angles of parallel lines, Therefore $\angle A+\angle B+\angle C = \angle 1+\angle 3+\angle 2 = 180^\circ$.

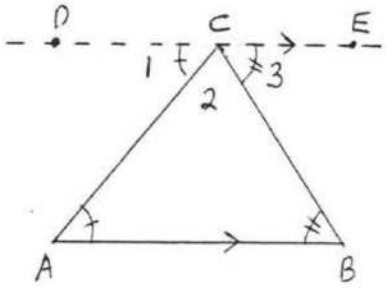
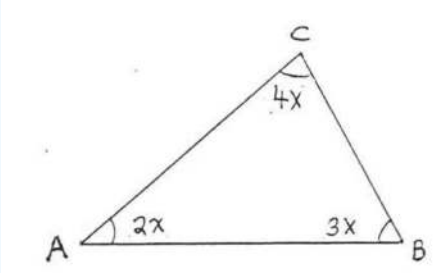


Figure 2.5.3: Through C draw DE parallel to AB.

We may verify Theorem 2.5.1 by measuring the angles of a triangle with a protractor and taking the sum, However no measuring instrument is perfectly accurate, It is reasonable to expect an answer such as 179°, 182°, 180.5°, etc. The purpose of our mathematical proof is to assure us that the sum of the angles of every triangle must be exactly \(180^{\circ}\).

Example 2.5.2

Find x:



Solution

$$\begin{aligned} \angle A+\angle B+\angle C &= 180^\circ \\ 2x+3x+4x &= 180 \\ 9x &= 180 \\ x &= 20 \end{aligned}$$

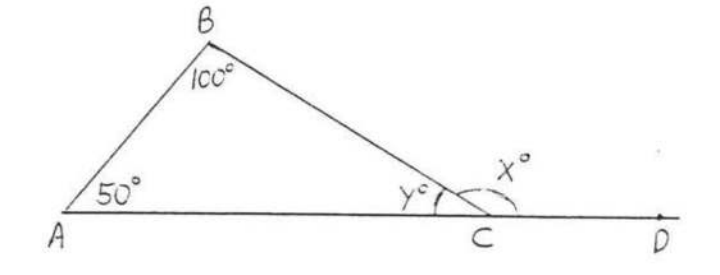
Check:

$$\begin{aligned} \angle A + \angle B + \angle C &= 180^\circ \\ 2x + 3x + 4x & \\ 2(20) + 3(20) + 4(20) & \\ 40^\circ + 60^\circ + 80^\circ & \\ 180^\circ & \end{aligned}$$

Answer: $x = 20$.

Example 2.5.3

Find y and x:



Solution

$$\begin{aligned} 50 + 100 + y &= 180 \\ 150 + y &= 180 \\ y &= 180 - 150 \\ y &= 30 \end{aligned} \tag{2.5.1}$$

$$x = 180 - 30 = 150$$

Answer: $y = 30$, $x = 150$.

In Figure 2.5.4, $\angle x$ is called an **exterior angle** of $\triangle ABC$, $\angle A$, $\angle B$, and $\angle y$ are called the **interior angles** of $\triangle ABC$. $\angle A$ and $\angle B$ are said to be the interior angles **remote** from the exterior angle $\angle x$.

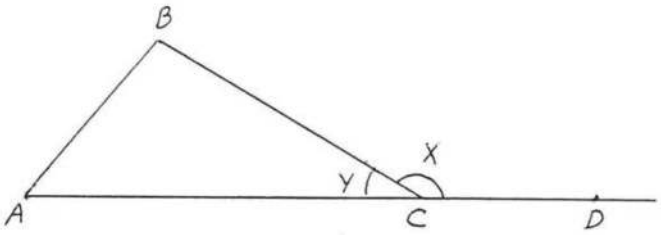


Figure 2.5.4: $\angle x$ is an exterior angle of $\triangle ABC$.

The results of Example 2.5.3 suggest the following theorem.

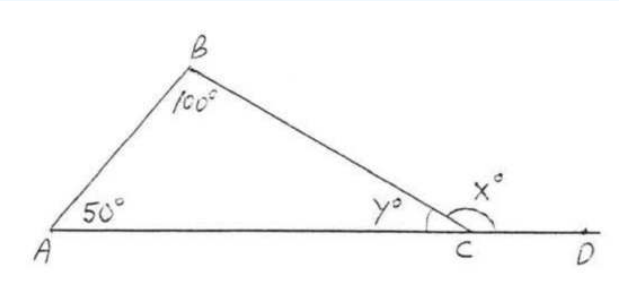
Theorem 2.5.2

An exterior angle is equal to the sum of the two remote interior angles,

In Figure 2.5.4, $\angle x = \angle A + \angle B$.

Example 2.5.3 (repeated)

Find x:



Solution

Using Theorem 2.5.2, $x^\circ = 100^\circ + 50^\circ = 150^\circ$.

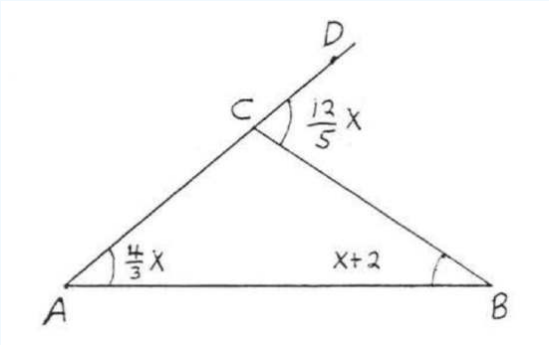
Answer: $x = 150$.

Proof of Theorem 2.5.2: We present this proof in **double-column form**, with statements in the left column and the reason for each statement in the right column. The last statement is the theorem we wish to prove.

Statements	Reasons
1. $\angle A + \angle B + \angle y = 180^\circ$	1. The sum of the angles of a triangle is 180°.
2. $\angle A + \angle B = 180^\circ - \angle y$	2. Subtract $\angle y$ from both sides of the equation, statement 1.
3. $\angle x = 180^\circ - \angle y$.	3. $\angle x$ and $\angle y$ are supplementary.
4. $\angle x = \angle A + \angle B$.	4. Both $\angle x$ (statement 3) and $\angle A + \angle B$ (statement 2) equal $180^\circ - \angle y$.

Example 2.5.4

Find x:



Solution

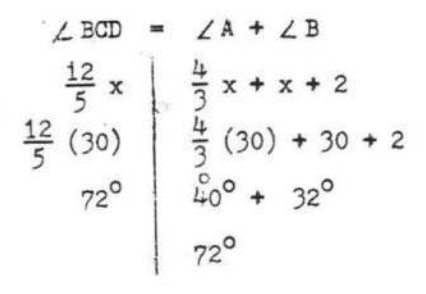
$\angle BCD$ is an exterior angle with remote interior angles $\angle A$ and $\angle B$. By Theorem 2.5.2,

$$\begin{aligned} \angle BCD &= \angle A + \angle B \\ \frac{12}{5}x &= \frac{4}{3}x + x + 2 \end{aligned} \tag{2.5.2}$$

The least common denominator (1, c, d) is 15.

$$
\begin{aligned}
(15)\frac{12}{5}x &= (15)\frac{4}{3}x+(15)x+(15)(2)\\
36x &= 20x+15x+30\\
36x &= 35x+30\\
36x-35x &= 30\\
x &= 30
\end{aligned}
$$

Check:



Answer: $x = 30$.

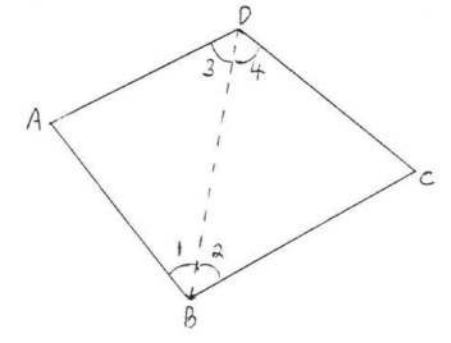
Our work on the sum of the angles of a triangle can easily be extended to other figures:

Example 2.5.5

Find the sum of the angles of a quadrilateral (four sided figure),

Solution

Divide the quadrilateral into two triangles as illustrated,



$$
\begin{aligned}
\angle A+\angle B+\angle C+\angle D &= \angle A+\angle 1+\angle 3+\angle 2+\angle 4+\angle C\\
&= 180^\circ+180^\circ\\
&= 360^\circ
\end{aligned}
$$

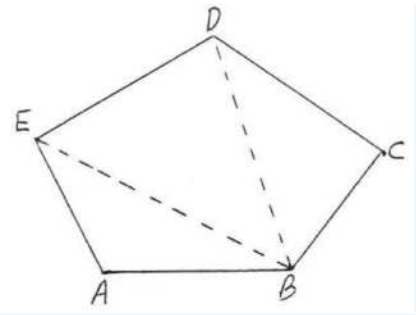
Answer: 360°.

Example 2.5.6

Find the sum of the angles of a pentagon (five-sided figure).

Solution

Divide the pentagon into three triangles as illustrated, The sum is equal to the sum of the angles of the three triangle = $(3)(180^\circ) = 540^\circ$.



Answer: 540°.

There is one more simple principle which we will derive from Theorem 2.5.1, Consider the two triangles in Figure 2.5.5.

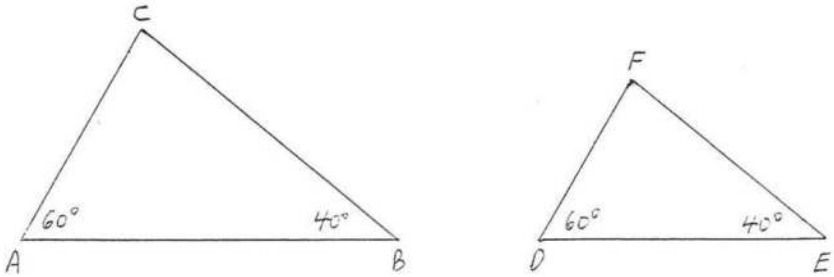


Figure 2.5.5: Each triangle has an angle of 60° and 40°.

We are given that $\angle A = \angle D = 60^\circ$ and $\angle B = \angle E = 40^\circ$. A short calculation shows that we must also have $\angle C = \angle F = 80^\circ$. This suggests the following theorem:

Theorem 2.5.3

If two angles of one triangle are equal respectively to two angles of another triangle, then their remaining angles are also equal.

In Figure 2.5.6, if $\angle A = \angle D$ and $\angle B = \angle E$ then $\angle C = \angle F$.

Proof

$\angle C = 180^\circ - (\angle A + \angle B) = 180^\circ - (\angle D + \angle E) = \angle F$.

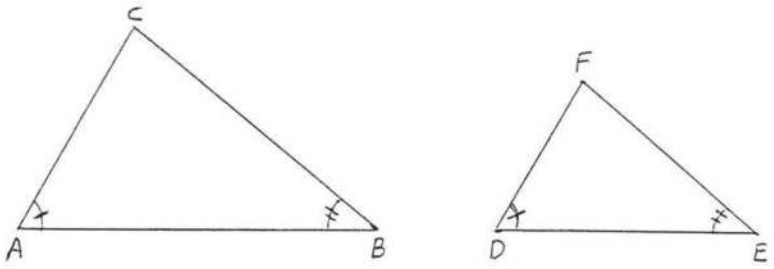


Figure 2.5.6. $\angle A = \angle D$ and $\angle B = \angle E$.

Historical Note

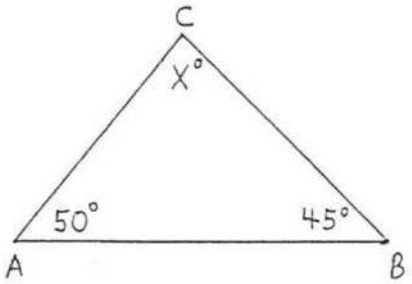
Our Theorem 2.5.1, which states that the sum of the angles of a triangle is 180°, is one of the most important consequences of the parallel postulate, Therefore, one way of testing the truth of the parallel postulate (see the **Historical Note** in Section 1.4) is to test the truth of Theorem 2.5.1, This was actually tried by the German mathematician, astronomer, and physicist, Karl Friedrich Gauss (1777 - 1855). (This is the same Gauss whose name is used as a unit of measurement in the theory of magnetism), Gauss measured the sum of the angles of the triangle formed by three mountain peaks in Germany, He found the

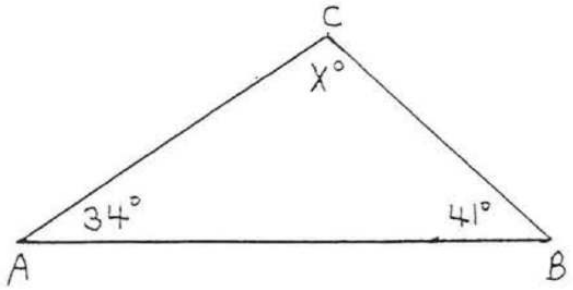
sum of the angles to be 14.85 seconds more than 180° (60 seconds 1 minute, 60 minutes = 1 degree). However this small excess could have been due to experimental error, so the sum might actually have been 180°.

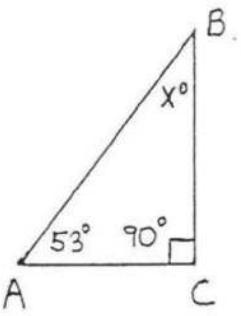
Aside from experimental error, there is another difficulty involved in verifying the angle sum theorem. According to the non-Euclidean geometry of Lobachevsky, the sum of the angles of a triangle is always **less** than 180°. In the non-Euclidean geometry of Riemann, the sum of the angles is always more than 180°, However in both cases the difference from 180° is insignificant unless the triangle is very large, Neither theory tells us exactly how large such a triangle should be, Even if we measured the angles of a very large triangle, like one formed by three stars, and found the sum to be indistinguishable from 180°, we could only say that the angle sum theorem and parallel postulate are apparently true for these large distances, These distances still might be too small to enable us to determine whichgeometric system best describes the universe as a whole,

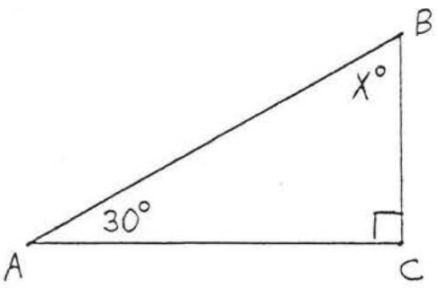
Problems

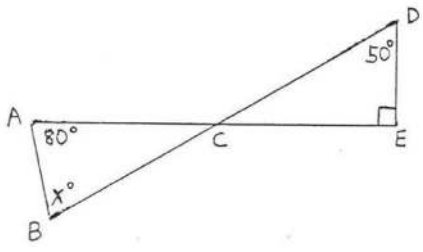
1 - 12. Find x and all the missing angles of each triangle:

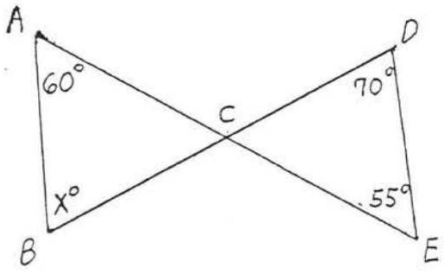
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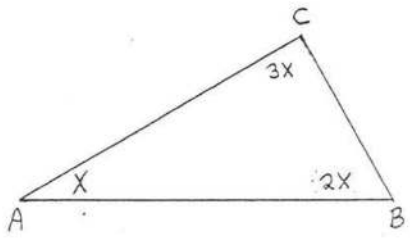
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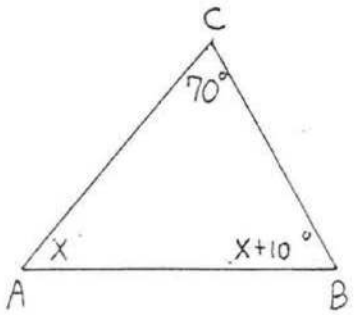
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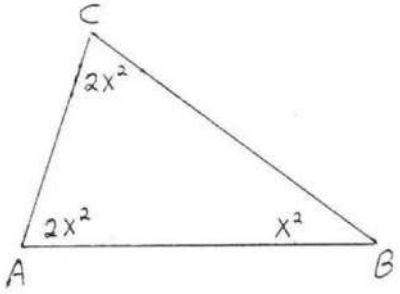
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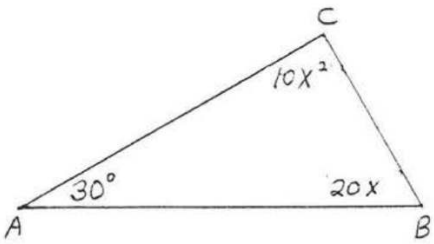
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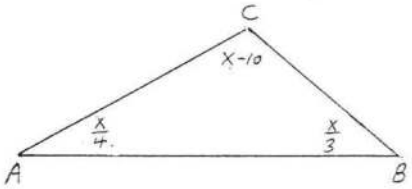
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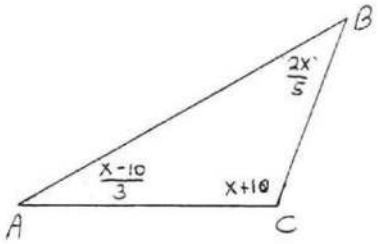
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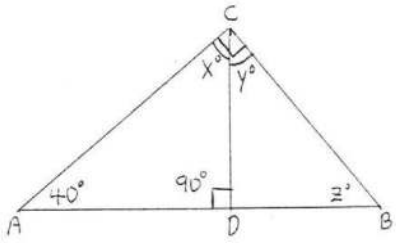
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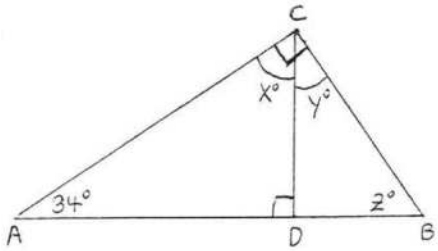
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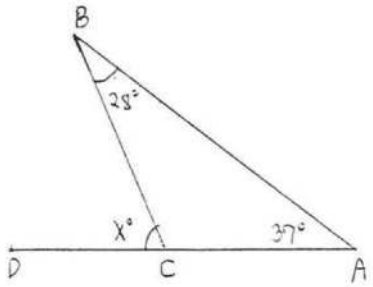
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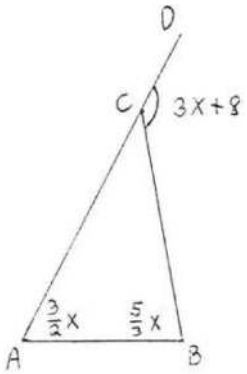
13 - 14. Find x, y, and z:

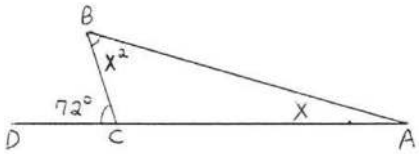
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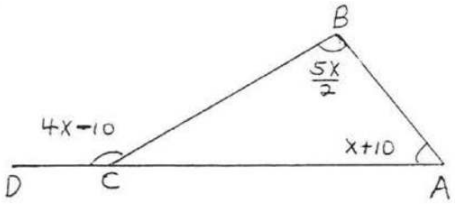
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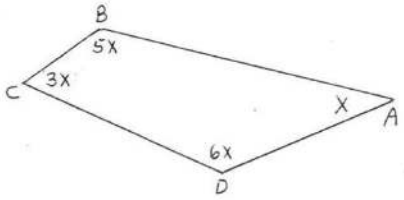
15 - 20. Find x:

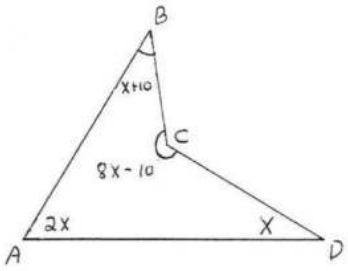
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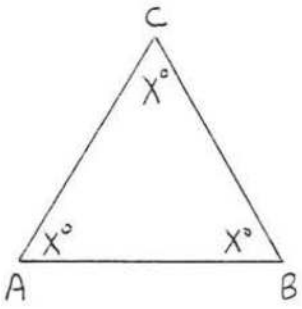
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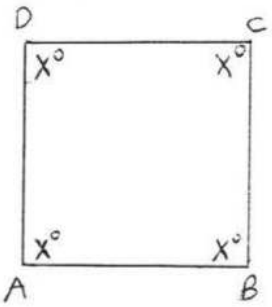
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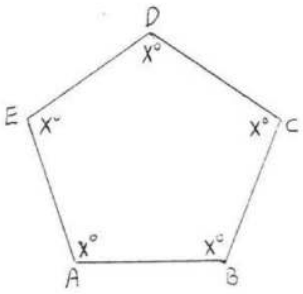
21. Find the sum of the angles of a hexagon (6-sided figure).

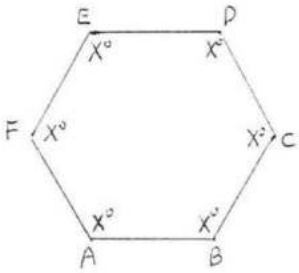
22. Find the sum of the angles of an octagon (8-sided figure).

23 - 26. Find x:

23. 

24. 

25. 

26. 

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2.6: Triangle Classifications

Triangles may be classified according to the relative lengths of their sides:

- An **equilateral** triangle has three equal sides,
- An **isosceles** triangle has two equal sides.
- A **scalene** triangle has no equal sides.

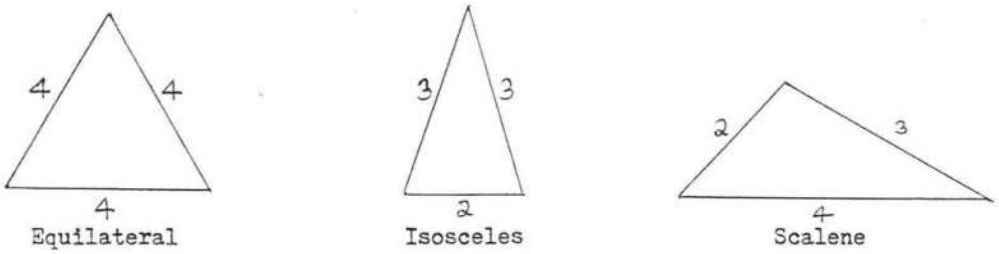


Figure 2.6.1: Triangles classified according to their sides,

Triangles may also be classified according to the measure of their angles:

- An **acute** triangle is a triangle with three acute angles.
- An **obtuse** triangle is a triangle with one obtuse angle.
- An **equiangular** triangle is a triangle with three equal angles,

Each angle of an equiangular triangle must be 60°, We will show in section 2, 5 that equiangular triangle are the same as equilateral triangles,

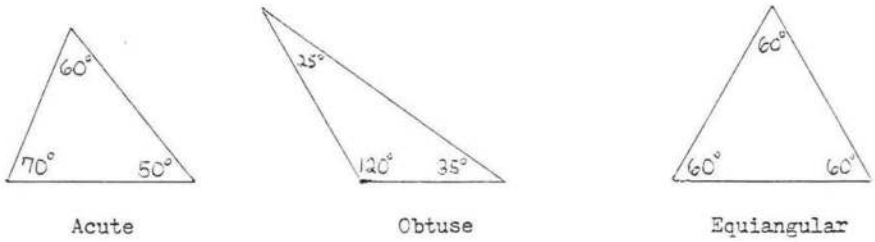


Figure 2.6.2: Triangles classified according to their angles,

A **right** triangle is a triangle with one right angle, The sides of the right angle are called the **legs** of the triangle and the remaining side is called the **hypotenuse**,

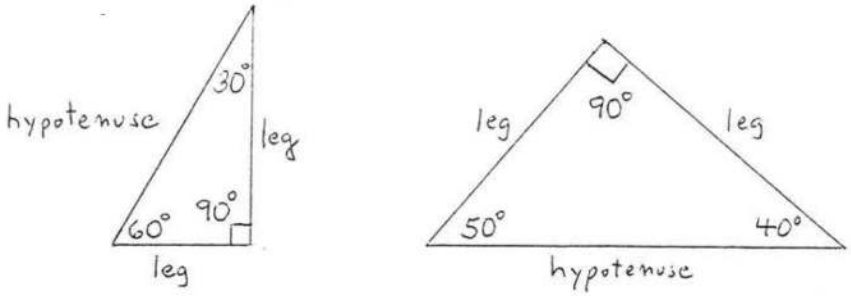
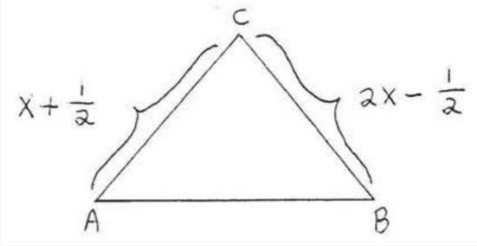


Figure 2.6.3: Right triangles.

Example 2.6.1

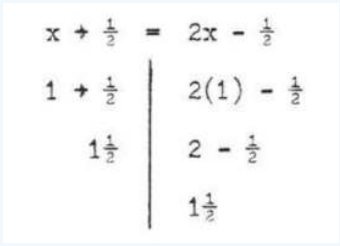
Find x if $\triangle ABC$ is isosceles with $AC = BC$:



Solution

$$\begin{aligned} x+\frac{1}{2} &= 2x-\frac{1}{2} \\ (2)(x+\frac{1}{2}) &= (2)(2x-\frac{1}{2}) \\ 2x+2(\frac{1}{2}) &= (2)(2x)-(2)(\frac{1}{2}) \\ 2x+1 &= 4x-1 \\ 1+1 &= 4x-2x \\ 2 &= 2x \\ 1 &= x \end{aligned} \tag{2.6.1}$$

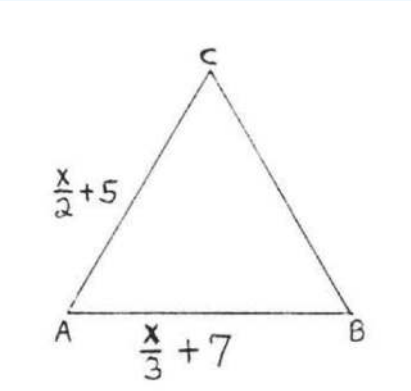
Check:



Answer: $x=1$.

Example 2.6.2

$\triangle ABC$ is equilateral. Find x:



Solution

$$\begin{aligned} \frac{x}{2}+5 &= \frac{x}{3}+7 \\ (6)(\frac{x}{2}+5) &= (6)(\frac{x}{3}+7) \\ (6)(\frac{x}{2}+(6)(5) &= (6)(\frac{x}{3}+(6)(7) \\ 3x+30 &= 2x+42 \\ 3x-2x &= 42-30 \\ x &= 12 \end{aligned} \tag{2.6.2}$$

Check:

$$\begin{array}{l|l} \frac{x}{2}+5 = \frac{x}{3}+7 & \\ \frac{12}{2}+5 & \frac{12}{3}+7 \\ 6+5 & 4+7 \\ 11 & 11 \end{array}$$

Answer: $x = 12$.

An **altitude** of a triangle is a line segment from a vertex perpendicular to the opposite·side, In Figure 4, CD and GH are altitudes, Note that altitude GH lies outside $\triangle EFG$ and side EF must be extended to meet it.

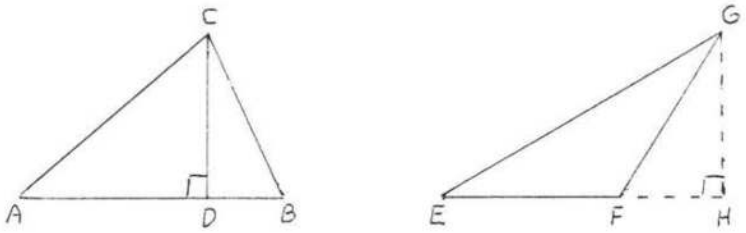


Figure 2.6.4. CD and GH are altitudes.

A **median** of a triangle is a line segment from a vertex to the midpoint of the opposite side, In Figure 5, CD is a median,

An **angle bisector** is a ray which divides an angle into two eaual angles. In Figure 2.6.6, $\overrightarrow{CD}$ is an angle bisector.

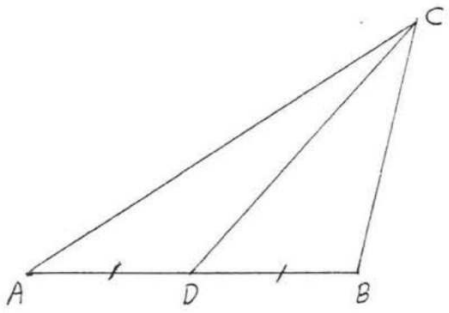


Figure 2.6.5: CD is a median.

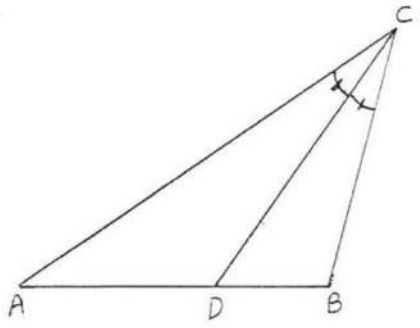
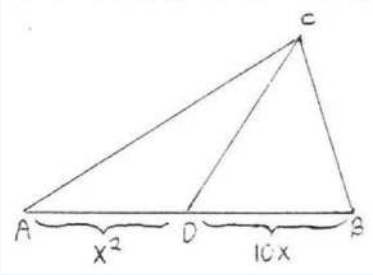


Figure 2.6.6. $\overrightarrow{CD}$ is an angle bisector of $\angle ACB$.

Example 2.6.3

Find AB if CD is a median:



Solution

$$\begin{aligned} AD &= DB \\ x^2 &= 10x \\ x^2 - 10x &= 0 \\ (x)(x-10) &= 0 \\ x = 0 \quad \text{or} \quad x - 10 &= 0 \\ x &= 10 \end{aligned} \tag{2.6.3}$$

Check, $x = 0$:

AD	=	DB
x^2		10x
0^2		10(0)
0		0

Check, $x = 10$:

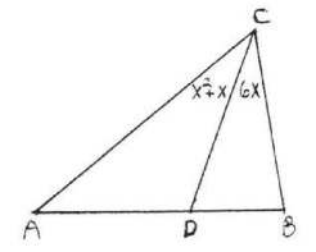
AD	=	DB
x^2		10x
10^2		10(10)
100		100

We reject the answer $x=0$ because the length of a line segment must be greater than 0, Therefore $AB=AD+DB=100+100=200$.

Answer: $AB=200$.

Example 2.6.4

Find $\angle ACB$ if $\overrightarrow{CD}$ is an angle bisector:



Solution

$$\begin{aligned} \anlge ACD &= \angle BCD \\ x^2+x &= 6x \\ x^2+x-6x &= 0 \\ x^2-5x &= 0 \\ (x)(x-5) &= 0 \end{aligned} \qquad (2.6.4)$$

$$x = 0 \quad \text{or} \quad x-5 = 0$$
$$x = 5$$

Check, $x=0$:

$\angle ACD$	$=$	$\angle BCD$
x^2+x		$6x$
0^2+0		$6(0)$
0		0

Check, $x=5$:

$\angle ACD$	$=$	$\angle BCD$
x^2+x		$6x$
5^2+5		$6(5)$
30		30

We reject the answer $x=0$ because the measures of $\angle ACD$ and $\angle BCD$ must be greater than 0°. Therefore $\angle ACB=\angle ACD+\angle BCD=30^\circ+30^\circ=60^\circ$.

Answer: $\angle ACB=60^\circ$.

The **perimeter** of a triangle is the sum of the lengths of the sides. The perimeter of $\triangle ABC$ in Figure 2.6.7 is $3+4+5=12$.

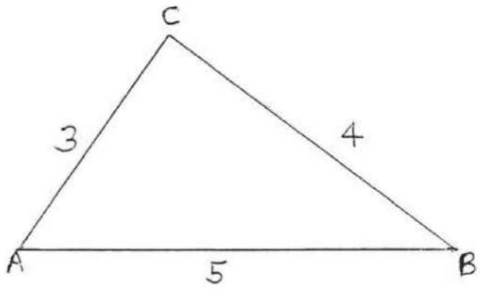


Figure 2.6.7. The perimeter of $\triangle ABC$ is 12.

Theorem 2.6.1

The sum of any two sides of a triangle is greater than the remaining side.

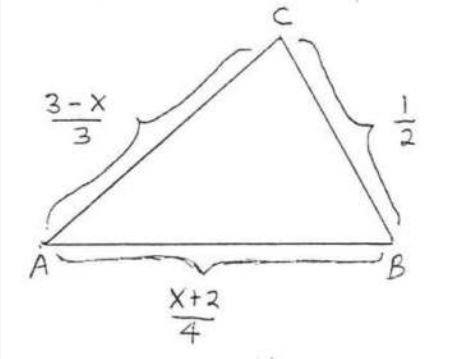
For example, in Figure 2.6.7, $AC + BC = 3 + 4 > AB = 5$.

Proof

This follows from the postulate that the shortest distance between two noints is along a straight line, For example, in Figure 2.6.7, the length AB (a straight line segment) must be less than the combined lengths of AC and CB (not on a straight line from A to B),

Example 2.6.5

Find the perimeter of the triangle in terms of x, Then find the perimeter if $x = 1$:



Solution

$$\begin{aligned} &= \\ &= \\ &= \\ &= \\ &= \end{aligned} \qquad (2.6.5)$$

If $x = 1$, $\dfrac{24 - x}{12} = \dfrac{24 - 1}{12} = \dfrac{23}{12}$.

Check:

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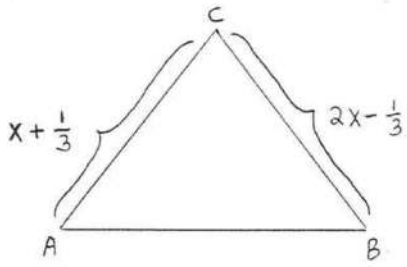
$$\frac{3-x}{3}+\frac{x+2}{4}+\frac{1}{2}=\frac{24-x}{12}$$
$$\frac{3-1}{3}+\frac{1+2}{4}+\frac{1}{2}\quad\Big|\quad\frac{24-1}{12}$$
$$\frac{2}{3}+\frac{3}{4}+\frac{1}{2}\quad\Big|\quad\frac{23}{12}$$
$$\frac{8}{12}+\frac{9}{12}+\frac{6}{12}$$
$$\frac{23}{12}$$

Answer: $\frac{24-x}{12}$, $\frac{23}{12}$.

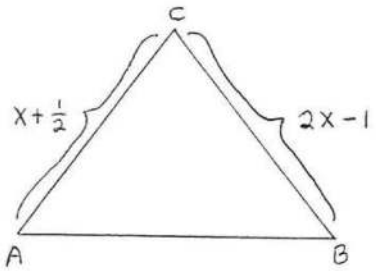
Problems

1 - 2. Find x if $\triangle ABC$ is isosceles with $AC = BC$:

1.

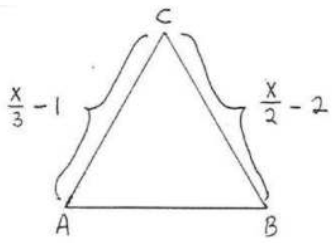


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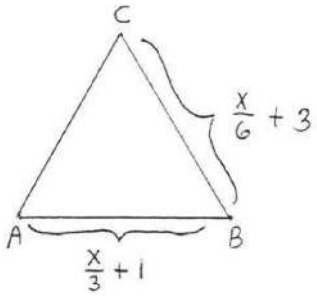


3 - 4. Find x if $\triangle ABC$ is equilateral:

3.

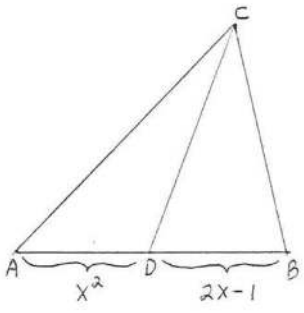


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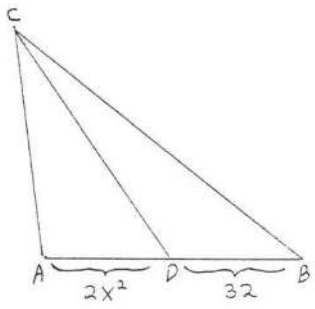


5 - 6. Find AB if CD is a median:

5.

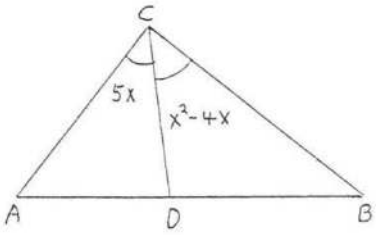


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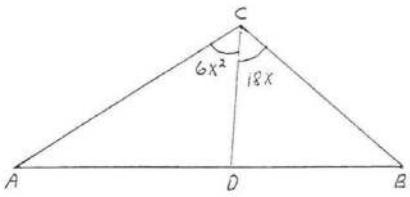


7 - 8. Find $\angle ACB$ if $\overrightarrow{CD}$ is an angle bisector:

7.

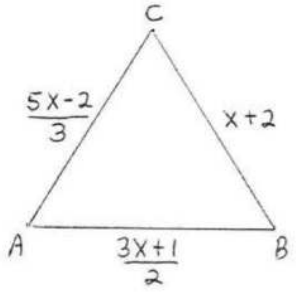


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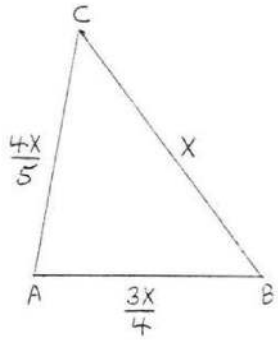


9 - 10. Find the perimeter of the triangle in terms of x, Then find the perimeter if $x = 4$:

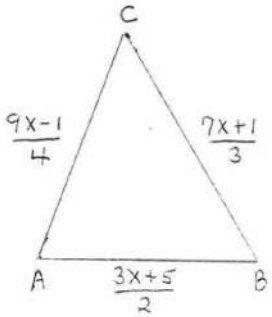
9.



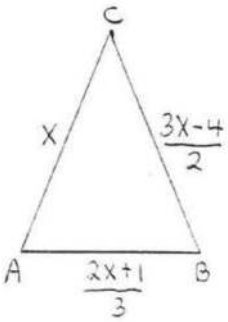
10.



11. Find x if the perimeter of $\triangle ABC$ is 33.



12. Find x if the perimeter of $\triangle ABC$ is 11.



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CHAPTER OVERVIEW

3: Congruent Triangles

3.1: The Congruence Statement
3.2: The SAS Theorem
3.3: The ASA and AAS Theorems
3.4: Proving Lines and Angles Equal
3.5: Isosceles Triangles
3.6: The SSS Theorem
3.7: The Hyp-Leg Theorem and Other Cases

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3.1: The Congruence Statement

Two triangles are said to be congruent if one can be placed over the other so that they coincide (fit together). This means that congruent triangles are exact copies of each other and when fitted together the sides and angles which coincide, called corresponding sides and angles, are equal.

In Figure 3.1.1, $\triangle ABC$ is congruent to $\triangle DEF$. The symbol for congruence is $\cong$ and we write $\triangle ABC \cong \triangle DEF$. $\angle A$ corresponds to $\angle D$, $\angle B$ corresponds to $\angle E$, and $\angle C$ corresponds to $\angle F$. Side AB corresponds to DE, BC corresponds to EF, and AC corresponds to DF.

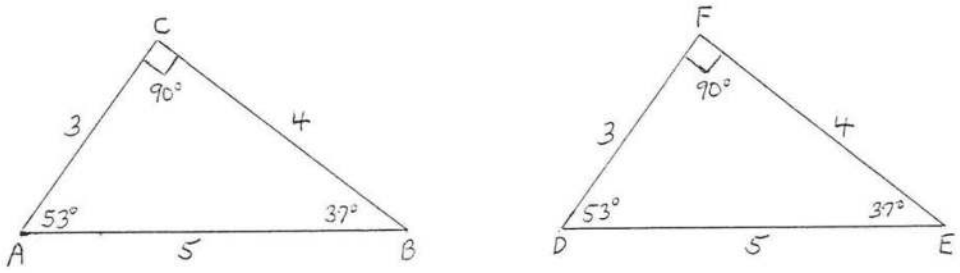


Figure 3.1.1: $\triangle ABC$ is congruent to $\triangle DEF$.

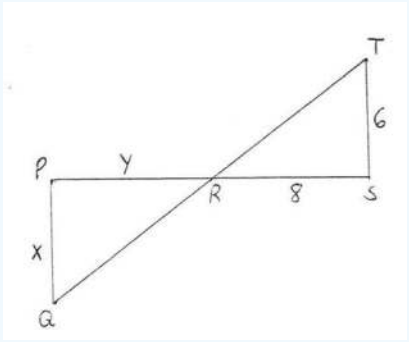
In this book the congruence statement $\triangle ABC \cong \triangle DEF$ will always be written so that corresponding vertices appear in the same order, For the triangles in Figure 3.1.1, we might also write $\triangle BAC \cong \triangle EDF$ or $\triangle ACB \cong \triangle DFE$ but never for example $\triangle ABC \cong \triangle EDF$ nor $\triangle ACB \cong \triangle DEF$. (Be warned that not all textbooks follow this practice, Many authors wil write the letters without regard to the order. If that is the case then we cannot tell which parts correspond from the congruence statement)

Therefore we can always tell which parts correspond just from the congruence statement. For example, given that $\triangle ABC \cong \triangle DEF$, side AB corresponds to side DE because each consists of the first two letters, AC corresponds to DF because each consists of the first and last letters, BC corresponds to EF because each consists of the last two letters.

Example 3.1.1

If $\triangle PQR \cong \triangle STR$

1. list the corresponding angles and sides;
2. find x and y.



Solution

(1)

$\underline{\triangle PQR}$		$\underline{\triangle STR}$	
$\angle P$	$=$	$\angle S$	(first letter of each triangle in congruence statement)
$\angle Q$	$=$	$\angle T$	(second letter)
$\angle PRQ$	$=$	$\angle SRT$	(third letter. We don't write "$\angle R = \angle R$" since each $\angle R$ is different)
PQ	$=$	ST	(first two letters)
PR	$=$	SR	(firsst and last letters)
QR	$=$	TR	(last two letters)

(2)

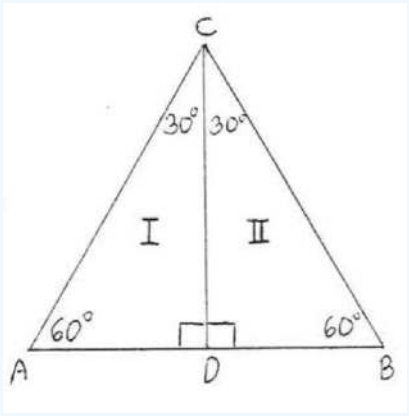
$x = PQ = ST = 6$.

$y = PR = SR = 8$.

Answer (2): $x = 6, y = 8$.

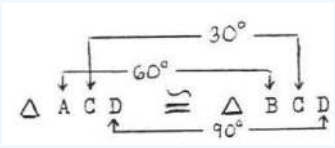
Example 3.1.2

Assuming $\triangle I \cong \triangle II$, write a congruence statement for $\triangle I$ and $\triangle II$:



Solution

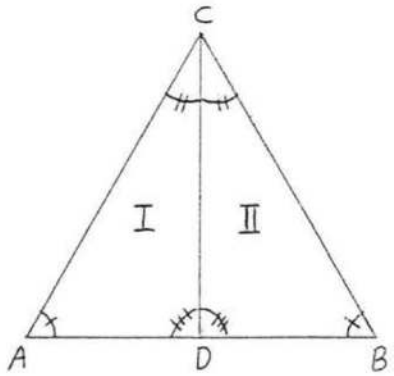
$\triangle I$		$\triangle II$	
$\angle A$	$=$	$\angle B$	(both $= 60°$)
$\angle ACD$	$=$	$\angle BCD$	(both $= 30°$)
$\angle ADC$	$=$	$\angle BDC$	(both $= 90°$)

Therefore 

Answer: $\triangle ACD \cong \triangle BCD$.

Example 3.1.3

Assuming $\triangle I \cong \triangle II$, write a congruence statement for $\triangle I$ and $\triangle II$:



Solution

The angles that are marked the same way are assumed to be equal.

$$\begin{array}{rcll} \underline{\triangle I} & & \underline{\triangle II} & \\ \angle A & = & \angle B & \text{(both marked with one stroke)} \\ \angle ACD & = & \angle BCD & \text{(both marked with two strokes)} \\ \angle ADC & = & \angle BDC & \text{(both marked with three strokes)} \end{array}$$

The relationships are the same as in Example 3.1.2.

Answer: $\triangle ACD \cong \triangle BCD$.

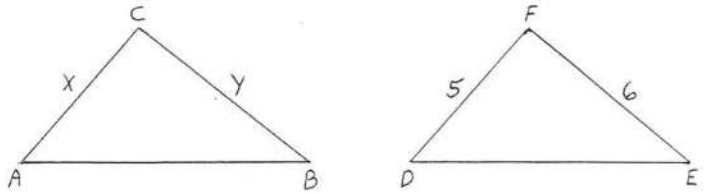
Problems

1 - 4. For each pair of congruent triangles

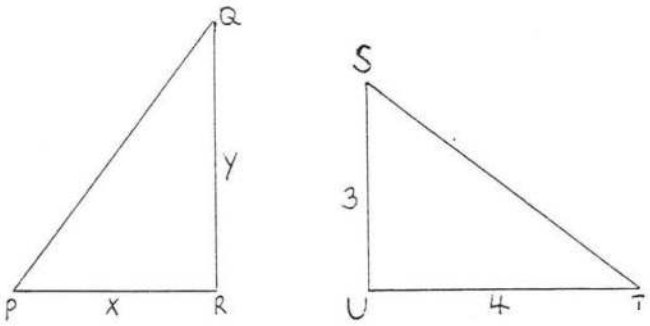
(1) list the corresponding sides and angles;

(2) find x and y.

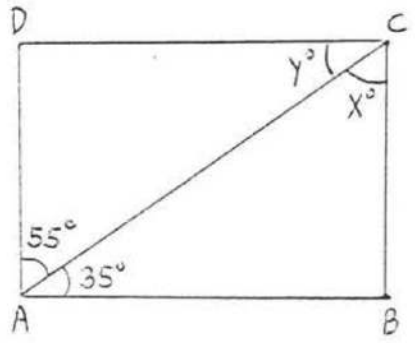
1. $\triangle ABC \cong \triangle DEF$.



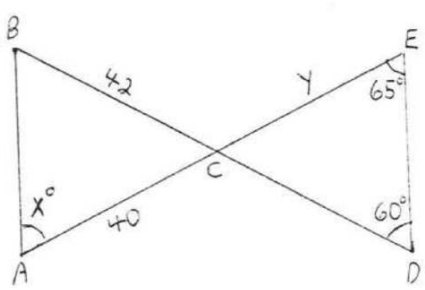
2. $\triangle PQR \cong \triangle STU$.



3. $\triangle ABC \cong \triangle CDA$.

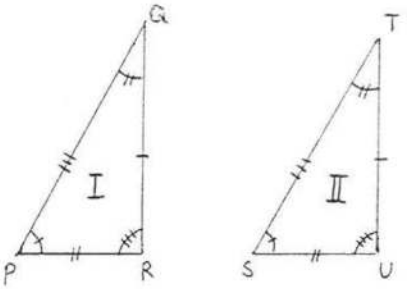


4. $\triangle ABC \cong \triangle EDC$.

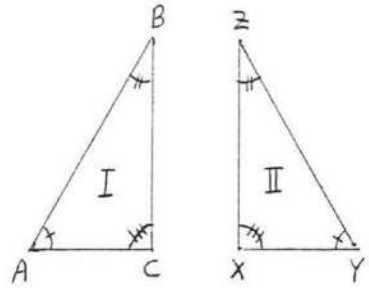


5 - 10. Write a congruence statement for each of the following. Assume the triangles are congruent and that angles or sides marked in the same way are equal.

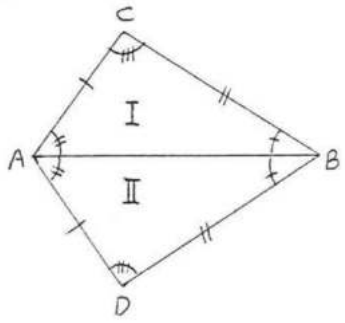
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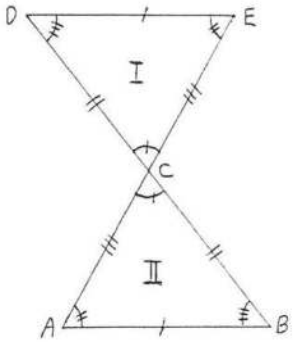
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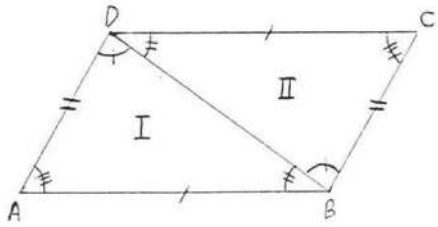


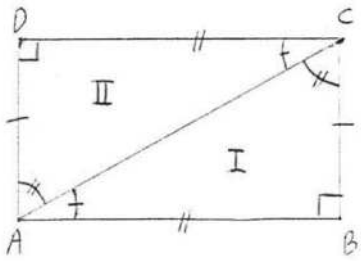
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3.2: The SAS Theorem

We have said that two triangles are congruent if all their correspond ing sides and angles are equal, However in some cases, it is possible to conclude that two triangles are congruent, with only partial information about their sides and angles.

Suppose we are told that $\triangle ABC$ has $\angle A = 53^\circ$, $AB = 5$ inches, and $AC = 3$ inches. Let us attenpt to sketch $\triangle ABC$. We first draw an angle of 53° with a protractor and label it $\angle A$. Using a ruler, we find the point 5 inches from the vertex on one side of the angle and label it B, On the other side of the angle, we find the point 3 inches from the vertex and label it C, See Figure 3.2.1, There is now only one way for us to complete our sketch of $\triangle ABC$, and that is to connect points B and C with a line segment, We could now measure BC, $\angle B$, and $\angle C$ to find the remaining parts of the triangle.

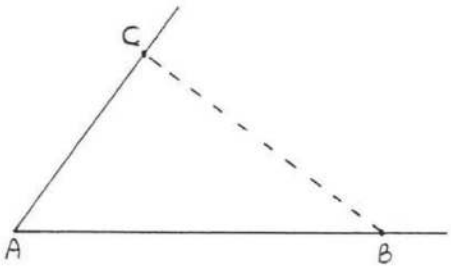


Figure 3.2.1: Sketching $\triangle ABC$.

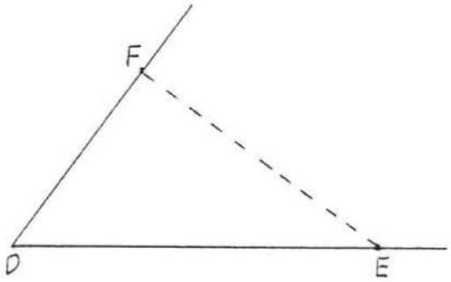


Figure 3.2.2: Sketching $\triangle DEF$.

Suppose now $\triangle DEF$ were another triangle, with $\angle D = 53^\circ$, $DE = 5$ inches, and $DF = 3$ inches. We could sketch $\triangle DEF$ just as we did $\triangle ABC$, and then measure EF, $\angle E$, and $\angle F$ (Figure 3.2.2). It is clear that we must have $BC = EF$, $\angle B = \angle E$, and $\angle C = \angle F$ because both triangles were drawn in exactly the same way. Therefore $\triangle ABC \cong \triangle DEF$.

- In $\triangle ABC$, we say that $\angle A$ is the angle included between sides AB and AC.
- In $\triangle DEF$, we say that $\angle D$ is the angle included between sides DE and DF.

Our discussion suggests the following theorem:

Theorem 3.2.1 (SAS or Side-Angle-Side Theorem)

Two triangles are congruent if two sides and the included angle of one are equal respectively to two sides and the included angle of the other,

In Figure 3.2.1 and 3.2.2, $\triangle ABC \cong \triangle DEF$ because AB, AC, and $\angle A$ are equal respectively to DE, DF and $\angle D$.

We sometimes abbreviate Theorem 3.2.1 by simply writing $SAS = SAS$.

Example 3.2.1

In $\triangle PQR$ name the angle included between sides

1. PQ and QR,
2. PQ and PR,
3. PR and QR,

Solution

Note that the included angle is named by the letter that is common to both sides, For (1), the letter "Q" is common to PQ and QR and so $\angle Q$ is included between sides PQ and QR. Similarly for (2) and (3).

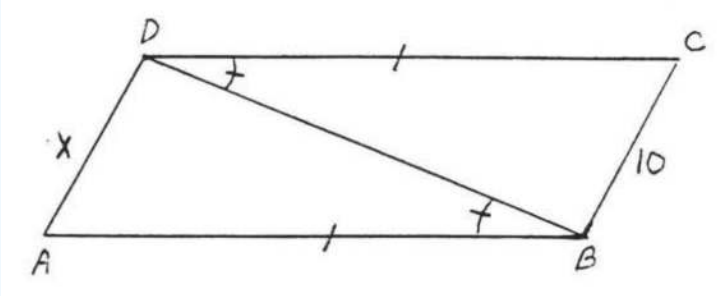
Answer: (1) $\angle Q$, (2) $\angle P$, (3) $\angle R$.

Example 3.2.2

For the two triangles in the diagram

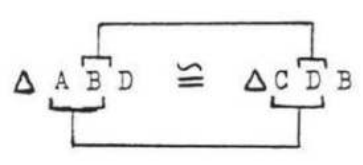
1. list two sides and an included angle of each triangle that are respectively equal, using the infonnation given in the diagram,
2. write the congruence statement,

and (3) find x by identifying a pair of corresponding sides of the congruent triangles.



Solution

(1) The angles and sides that are marked the same way in the diagram are assumed to be equal, So $\angle B$ in $\triangle ABD$ is equal to $\angle D$ in $\triangle BCD$. Therefore, "B" corresponds to "D." We also have $AB = CD$. Therefore "A" must corresponds to "C". Thus, if the triangles are congruent, the correspondence must be



Finally, BD (the same as DB) is a side common to both triangles, Summaryzing,

	$\underline{\triangle ABD}$		$\underline{\triangle CDB}$	
Side	AB	$=$	CD	(marked = in diagram)
Included Angle	$\angle B$	$=$	$\angle D$	(marked = in diagram)
Side	BD	$=$	DB	(common side)

(2) $\triangle ABD \cong \triangle CDB$ because of the SAS Theorem ($SAS = SAS$).

(3) $x = AD = CB = 10$ because AD and CB are corresponding sides (first and third letters in the congruence statement) a.~d corresponding sides of congruent triangles are equal.

Answer:

(1) AB, $\angle B$, BD of $\triangle ABD = CD$, $\angle D$, DB of $\triangle CDB$.

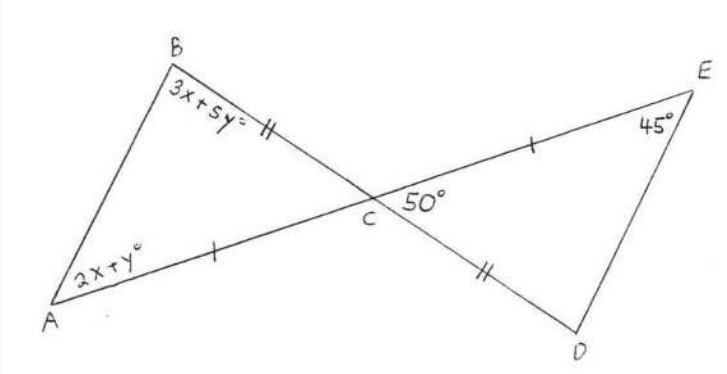
(2) $\triangle ABD \cong \triangle CDB$.

(3) $x = AD = CB = 10$.

Example 3.2.3

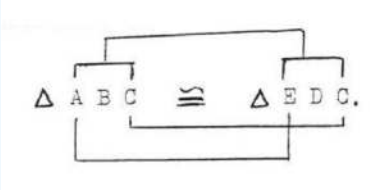
For the two triangles in the diagram

1. list two sides and an included angle of each triangle that are respectively equal, using the information given in the diagram.
2. write the congruence statement, and
3. find x and y by identifying a pair of corresponding sides of the congruent triangles.



Solution

(1) $AC = CE$ and $BC = CD$ because they are mar!,;:ed the same way. We also know that $\angle ACB = \angle ECD = 50°$ because vertical angles are equal. Therefore "C" in $\triangle ABC$ corresponds to "C" in $\triangle CDE$. Since $AC = CE$, we must have that "A" in $\triangle ABC$ corresponds to "E" in $\triangle CDE$. Thus, if the triangles are congruent, the correspondence must be

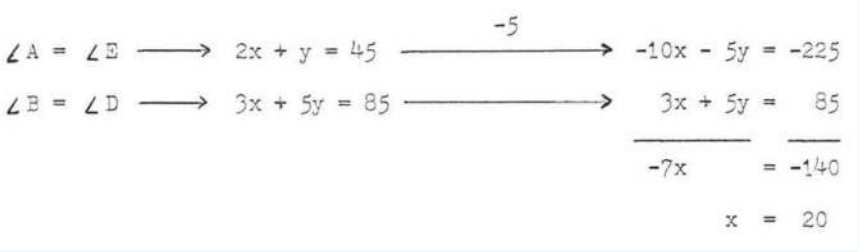


We summarize:

	$\underline{\triangle ABC}$		$\underline{\triangle EDC}$	
Side	AC	$=$	EC	(marked = in diagram)
Included Angle	$\angle ACB$	$=$	$\angle ECD$	(vertical angles are =)
Side	BC	$=$	DC	(marked = in diagram)

(2) $\triangle ABC \cong \triangle EDC$ because of the SAS theorem. ($SAS = SAS$)

(3) $\angle A = \angle E$ and $\angle B = \angle D$ because they are corres:9onding angles of the congruent triangles. $\angle D = 85°$ because the sum of the angles of $\triangle EDC$ must be $180°$. ($\angle D = 180° - (50° + 45°) = 180° - 95° = 85°$). We obtain a system of two equations in the two unknowns x and y:



Substituting for x in the first original equation,

$$\begin{aligned} 2x+y &= 45 \\ 2(20)+y &= 45 \\ 40+y &= 45 \\ y &= 45-40 \\ y &= 5 \end{aligned} \tag{3.2.1}$$

Check:

$\angle A$	$\angle E$
$2x + y$	45°
$2(20) + 5$	
$40 + 5$	
45°	

$\angle B$	$\angle D$
$3x + 5y$	85°
$3(20) + 5(5)$	
$60 + 25$	
85°	

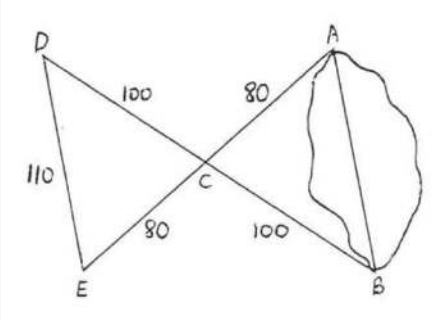
Answer:

1. AC, $\angle ACB$, BC of $\triangle ABC$ = EC, $\angle ECD$, DC of $\triangle EDC$.
2. $\triangle ABC \cong \triangle EDC$.
3. $x = 20, y = 5$.

Example 3.2.4

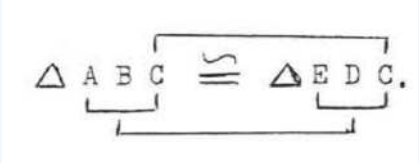
The following procedure was used to measure the d.istance AB across a pond: From a point C, AC and BC were measured and found to be 80 and 100 feet respectively. Then AC was extended to E so that $AC = CE$ and BC was extended to D so that $BC = CD$. Finally, DE we found to be 110 feet.

1. Write the congruence statement.
2. Give a reason for (1).
3. Find AB.



Solution

(1) $\angle ACB = \angle ECD$ because vertical angles are equal. Therefore the "C's" correspond, $AC = EC$ so A must correspond to E. We have



(2) $SAS = SAS$. Sides AC, BC, and included angle C of ABC are equal respectively to EC, DC, and included angle C of $\angle EDC$.

(3) $AB = ED$ ecause they are corresponding sides of congruent triangles, Since $ED = 110$, $AB = 110$.

Answer

(1) $\triangle ABC \cong \triangle EDC$.

(2) $SAS = SAS$: AC, $\angle C$, BC of $\triangle ABC = EC$, $\angle C$, DC of $\triangle EDC$.

(3) $AB = 110 feet$.

Historical Note

The SAS Theorem is Proposition 4 in Euclid's **Elements**, Both our discussion and Suclit's proof of the SAS Theoremimplicitly use the following principle: If a geometric construction is repeated in a different location (or what amounts to the same thing is "moved" to a different location) then the size and shape of the figure remain the same, There is evidence that Euclid used this principle reluctantly, and many mathematicians have since questioned its use in formal proofs, They feel that it makes too strong an assumption about the nature of physical space and is an inferior form of geometric reasoning. Bertrand Russell (1872 - 1970), for example, has suggested that we would be better off assuming the SAS Theorem as a **postulate**, This is in fact done in a system of axioms for Euclidean geometry devised by David Hilbert (1862 - 1943), a system that has gained much favor with modern mathematicians. Hilbert was the leading exponent of the "formalist school," which sought to discover exactly what assumptions underlie each branch of mathematics and to remove all logical ambiguities, Hilbert's system, however, is too formal for an introductory course in geometry,

Problems

1 - 4. For each of the following (1) draw the triangle with the two sides and the included angle and (2) measure the remaining side and angles:

1. $AB = 2$ inches, $AC = 1$ inch, $\angle A = 60^\circ$.

2. $DE = 2$ inches, $DF = 1$ inch, $\angle D = 60^\circ$.

3. $AB = 2$ inches, $AC = 3$ inches, $\angle A = 40^\circ$.

4. $DE = 2$ inches, $DF = 3$ inches, $\angle D = 40^\circ$.

5 - 8. Name the angle included between sides

5. AB and BC in $\triangle ABC$.

6. XY and YZ in $\triangle XYZ$.

7. DE and DF in $\triangle DEF$.

8. RS and TS in $\triangle RST$.

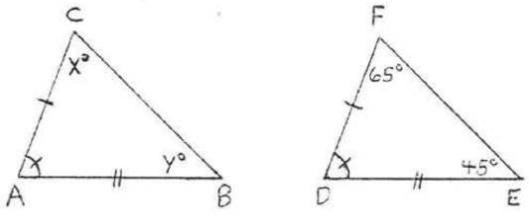
9 - 22. For each of the following.

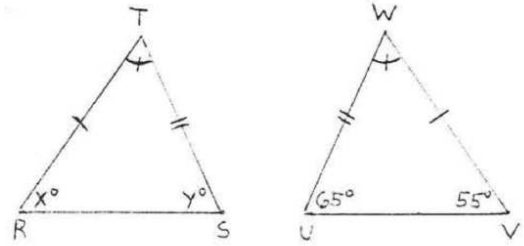
(1) list two sides and an included angle of each triangle that are respectively equal, using the information given in the diagram,

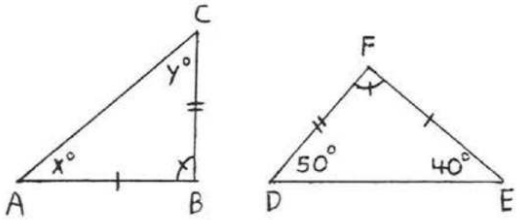
(2) write the congruence statement,

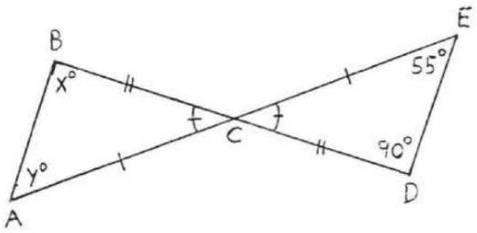
(3) find x, or x and y.

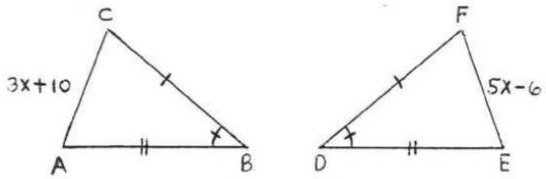
Assume that angles or sides marked in the same way are eQual.

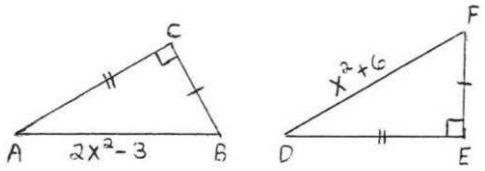
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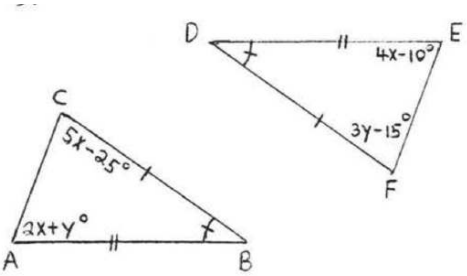
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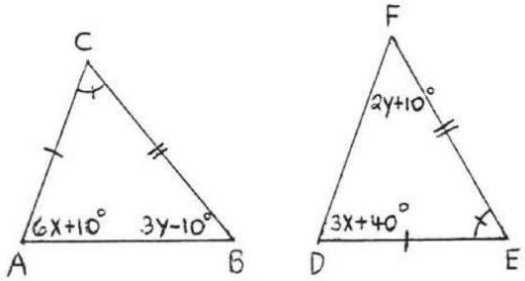
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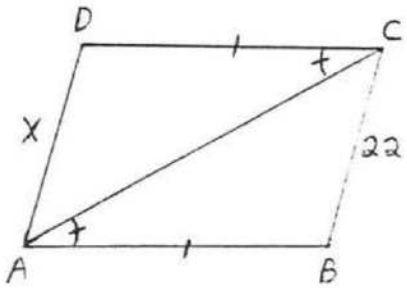
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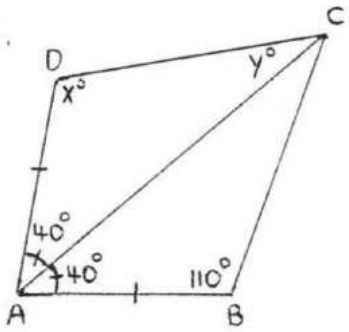
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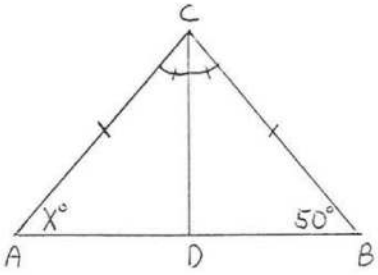
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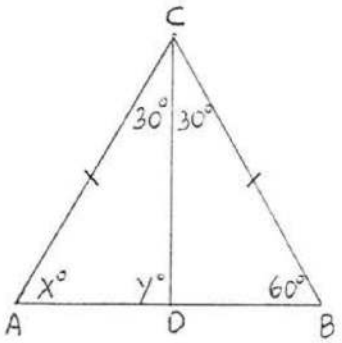
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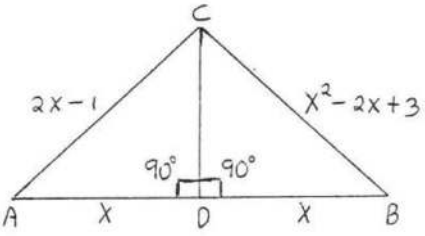
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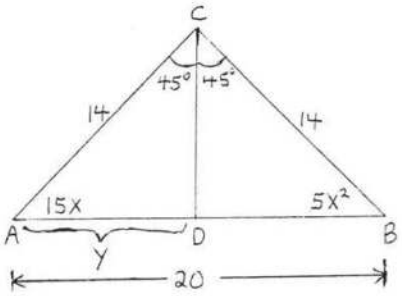
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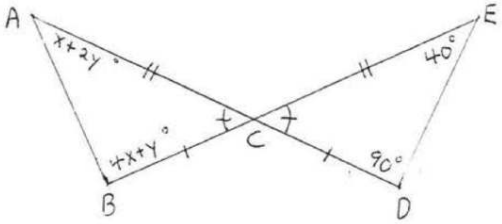
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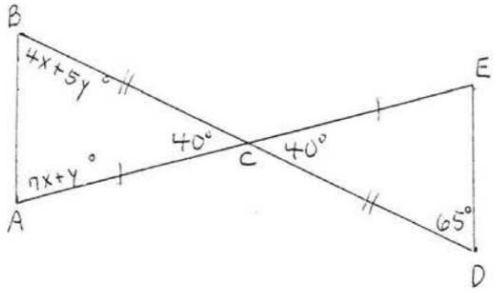
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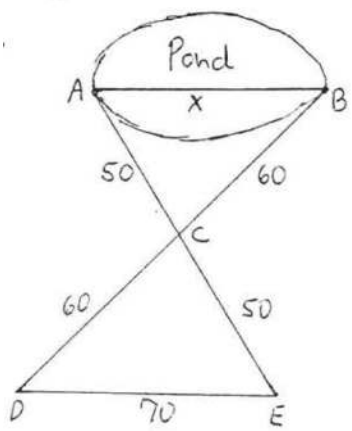
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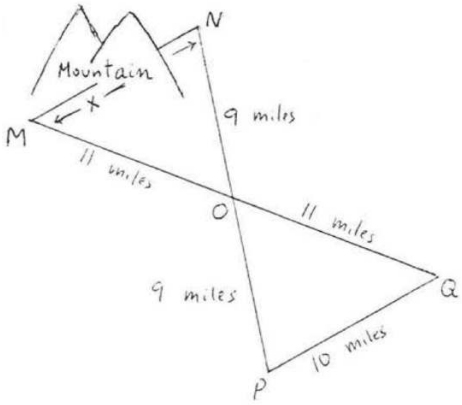
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3.3: The ASA and AAS Theorems

In this section we will consider two more cases where it is possible to conclude that triangles are congruent with only partial information about their sides and angles,

Suppose we are told that $\triangle ABC$ has $\angle A = 30°, \angle B = 40°$, and $AB =$ 2 inches. Let us attempt to sketch $\triangle ABC$. We first draw a line segment of 2 inches and label it AB, With a protractor we draw an angle of $30°$ at A and an angle of $40°$ at B (Figure 3.3.1). We extend the lines forming $\angle A$ and $\angle B$ until they meet at C. We could now measure AC, BC, and $\angle C$ to find the remaining parts of the triangle.

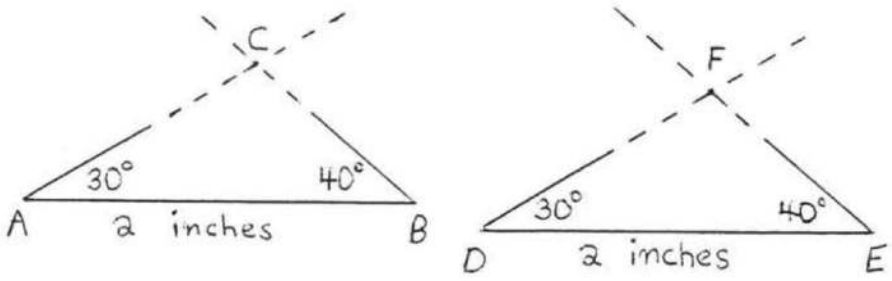


Figure 3.3.1: (left) Sketching $\triangle ABC$ and (right) Sketching $\triangle DEF$.

Let $\triangle DEF$ be another triangle, with $\angle D = 30°, \angle E = 40°$, and $DE =$ 2 inches. We could sketch $\triangle DEF$ just as we did $\triangle ABC$, and then measure DF, EF, and $\angle F$ (Figure 3.3.2). It is clear that we must have $AC = DF$, $BC = EF$, and $\angle C = \angle F$, because both triangles were drawn in exactly the same way, Therefore $\triangle ABC \cong \triangle DEF$.

In $\triangle ABC$ we say that AB is the side **included** between $\angle A$ and $\angle B$. In $\triangle DEF$ we would say that DE is the side included between $\angle D$ and $\angle E$.

Our discussion suggests the following theorem:

Theorem 3.3.1: ASA or Angle-Side-Angle Theorem

Two triangles are congruent if two angles and an included side of one are equal respectively to two angles and an included side of the other.

In Figure 3.3.1 and 3.3.2, $\triangle ABC \cong \triangle DEF$ because $\angle A, \angle B$, and AB are equal respectively to $\angle D, \angle E$, and DE.

We sometimes abbreviate Theorem 3.3.1 by simply writing $ASA = ASA$.

Example 3.3.1

In $\triangle PQR$, name the side included between

1. $\angle P$ and $\angle Q$.
2. $\angle P$ and $\angle R$.
3. $\angle Q$ and $\angle R$.

Solution

Note that the included side is named by the two letters representing each of the angles. Therefore, for (1), the side included between $\angle P$ and $\angle Q$ is named by the letters P and Q -- that is, side PQ. Similarly for (2) and (3).

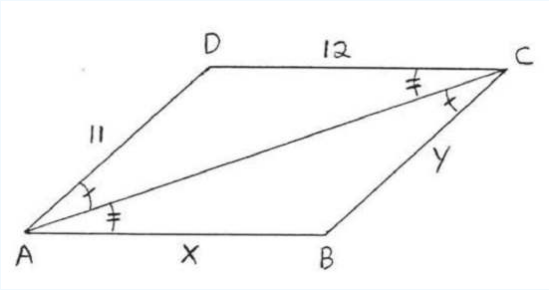
Answer: (1) PQ, (2) PR, (3) QR.

Example 3.3.2

For the two triangles in the diagram

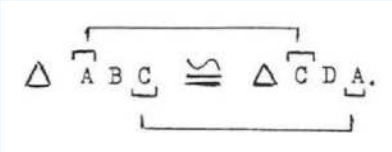
1. write the congruence statement,

2. give a reason for (1),
3. find x and y.



Solution

(1) From the diagram $\angle A$ in $\triangle ABC$ is equal to $\angle C$ in $\triangle ADC$. Therefore, "A" corresponds to "C". Also $\angle C$ in $\triangle ABC$ is equal to $\angle A$ in $\triangle ADC$. So "C" corresponds to "A". We have



(2) $\angle A, \angle C$, and included side AC of $\triangle ABC$ are equal respectively to $\angle C$, $\angle A$, and included side CA of $\triangle CDA$. ($AC = CA$ because they are just different names for the identical line segment, We sometimes say $AC = CA$ because of **identity**.) Therefore $\triangle ABC \cong \triangle CDA$ because of the ASA Theorem ($ASA = ASA$).

Summary:

	$\underline{\triangle ABC}$		$\underline{\triangle CDA}$	
Angle	$\angle BAC$	$=$	$\angle DCA$	(marked = in diagram)
Included Side	AC	$=$	CA	(identity)
Angle	$\angle BCA$	$=$	$\angle DAC$	(marked = in diagram)

(3) $AB = CD$ and $BC = DA$ because they are corresponding sides of the congruent triangles. Therefore $x = AB = CD = 12$ and $y = BC = DA = 11$.

Answer:

(1) $\triangle ABC \cong \triangle CDA$.

(2). $ASA = ASA$: $\angle A, AC, \angle C$ of $\triangle ABC = \angle C, CA, \angle A$ of $\triangle CDA$.

(3) $x = 12, y = 11$.

Let us now consider $\triangle ABC$ and $\triangle DEF$ in Figure 3.3.3. $\angle A$ and $\angle B$

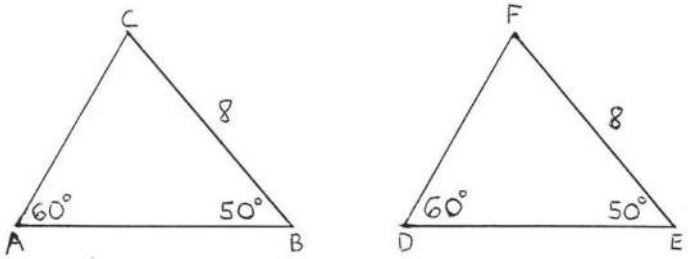


Figure 3.3.3. Two angles and a:i unincluded side of $\triangle ABC$ are equal respectively to two angles and an unincluded side of $\triangle DEF$.

of $\triangle ABC$ are equal respectively to $\angle D$ and $\angle E$ of $\triangle DEF$, yet we have no information about the sides included between these angles, AB and DE, Instead we know that the unincluded side BC is equal to the corresponding unincluded side EF. Therefore,

as things stand, we cannot use $ASA = ASA$ to conclude that the triangles are congruent, However we may show $\angle C$ equals $\angle F$ as in Theorem 3.3.3, section 1.5 $(\angle C = 180^\circ - (60^\circ + 50^\circ) = 180^\circ - 110^\circ = 70^\circ$ and $\angle F = 180^\circ - (60^\circ + 50^\circ) = 180^\circ - 110^\circ = 70^\circ)$. Then we can apply the ASA Theorem to angles Band C and their included side BC and the corresponding angles E and F with included side EF. These remarks lead us to the following theorem:

Theorem 3.3.2 (AAS or Angle-Angle-Side Theorem)

Two triangles are congruent if two angles and an unincluded side of one triangle are equal respectively to two angles and the corresponding unincluded side of the other triangle ($AAS = AAS$).

In Figure 3.3.4, if $\angle A = \angle D$, $\angle B = \angle E$ and $BC = EF$ then $\triangle ABC \cong \triangle DEF$.

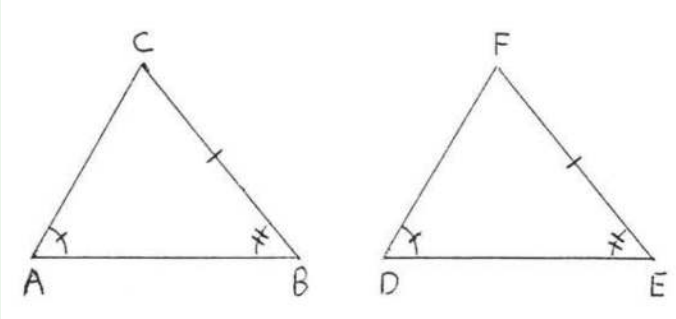


Figure 3.3.4. These two triangles are congruent by $AAS = AAS$.

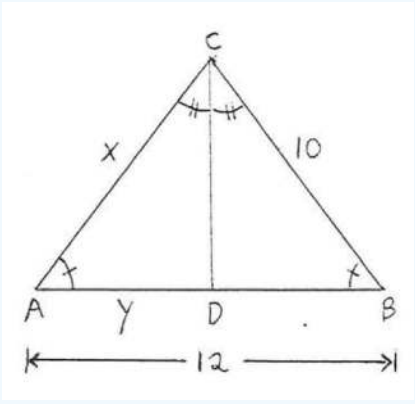
Proof

$\angle C = 180^\circ - (\angle A + \angle B) = 180^\circ - (\angle D + \angle E) = \angle F$. The triangles are then congruent by $ASA = ASA$ applied to $\angle B$. $\angle C$ and BC of $\angle ABC$ and $\angle E$, $\angle F$ and EF of $\triangle DEF$.

Example 3.3.3

For two triangles in the diagram

1. write the congruence statement,
2. give a reason for (1),
3. find x and y.



Solution

(1) $\triangle ACD \cong \triangle BCD$.

(2) $AAS = AAS$ since $\angle A$, $\angle C$ and unincluded side CD of $\angle ACD$ are equal respectively to $\angle B$, $\angle C$ and unincluded side CD of $\triangle BCD$.

	$\triangle ACD$		$\triangle BCD$	
Angle	$\angle A$	$=$	$\angle B$	(marked = in diagram)
Angle	$\angle ACD$	$=$	$\angle BCD$	(marked = in diagram)
Unincluded Side	CD	$=$	CD	(identity)

(3) $AC = BC$ and $AD = BD$ since they are corresponding sides of the congruent triangles. Therefore $x = AC = BC = 10$ and $y = AD = BD$. Since $AB = AD + BD = y + y = 2y = 12$, we must have $y = 6$.

Answer

(1) $\triangle ACD \cong \triangle BCD$

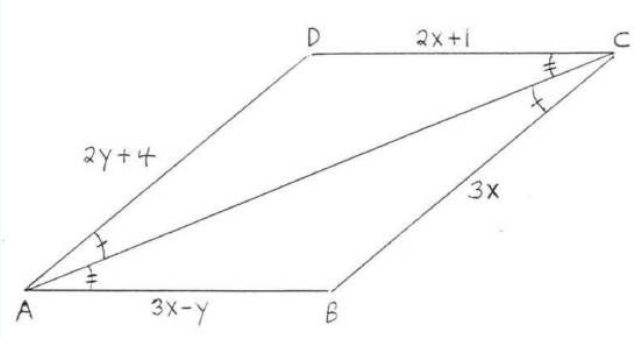
(2) $AAS = AAS$: $\angle A, \angle C, CD$ of $\triangle ACD = \angle B, \angle C, CD$ of $\triangle BCD$.

(3) $x = 10$, $y = 6$.

Example 3.3.4

For the two triangles in the diagram

i. write the congruence statement,
ii. give a reason for (1),
iii. find x and y.



Solution

Part (1) and part (2) are identical to Example 3.3.2.

(3):

$$\begin{aligned} AB &= CD \\ 3x - y &= 2x + 1 \\ 3x - 2x - y &= 1 \\ x - y &= 1 \end{aligned} \quad \text{and} \quad \begin{aligned} BC &= DA \\ 3x &= 2y + 4 \\ 3x - 2y &= 4 \end{aligned}$$

We solve these equations simultaneously for x and y:

$$\begin{aligned} x - y &= 1 \xrightarrow{\times -2} -2x + 2y = -2 \\ 3x - 2y &= 4 \longrightarrow \quad 3x - 2y = 4 \\ \hline x &= 2 \end{aligned} \qquad \begin{aligned} x - y &= 1 \\ 2 - y &= 1 \\ -y &= 1 - 2 \\ -y &= -1 \\ y &= 1 \end{aligned}$$

Check:

AB = CD	
$3x - y$	$2x + 1$
$3(2) - 1$	$2(2) + 1$
$6 - 1$	$4 + 1$
5	5

BC = DA	
$3x$	$2y + 4$
$3(2)$	$2(1) + 4$
6	$2 + 4$
	6

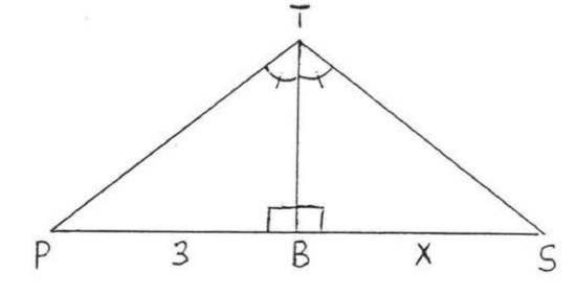
Answer:

(1) and (2) same as Example 3.3.2.

(3) $x = 2$, $y = 1$.

Example 3.3.5

From the top of a tower Ton the shore, a ship Sis sighted at sea, A point P along the coast is also sighted from T so that $\angle PTB = \angle STB$. If the distance from P to the base of the tower B is 3 miles, how far is the ship from point Bon the shore?



Solution

$\triangle PTB \cong \triangle STB$ by $ASA = ASA$. Therefore $x = SB = FB = 3$.

Answer: 3 miles

Historical Note

The method of finding the distance of ships at sea described in Example 3.3.5 has been attributed to the Greek philosopher Thales (c. 600 B.C.). We know from various authors that the ASA Theorem has been used to measure distances since ancient times, There is a story that one of Napoleon's officers used the ASA Theorem to measure the width of a river his army had to cross, (see Problem 25 below.)

Problems

1 - 4. For each of the following (1) draw the triangle with the two angles and the included side and (2) measure the remaining sides and angle,

1. $\triangle ABC$ with $\angle A = 40^\circ$, $\angle B = 50^\circ$, and $AB = 3$ inches,

2. $\triangle DEF$ with $\angle D = 40^\circ$, $\angle E = 50^\circ$, and $DE = 3$ inches,

3. $\triangle ABC$ with $\angle A = 50^\circ$, $\angle B = 40^\circ$, and $AB = 3$ inches,

4. $\triangle DEF$ with $\angle D = 50^\circ$, $\angle E = 40^\circ$, and $DE = 3$ inches.

5 - 8. Name the side included between the angles:

5. $\angle A$ and $\angle B$ in $\triangle ABC$.

6. $\angle X$ and $\angle Y$ in $\triangle XYZ$.

7. $\angle D$ and $\angle F$ in $\triangle DEF$.

8. $\angle S$ and $\angle T$ in $\triangle RST$.

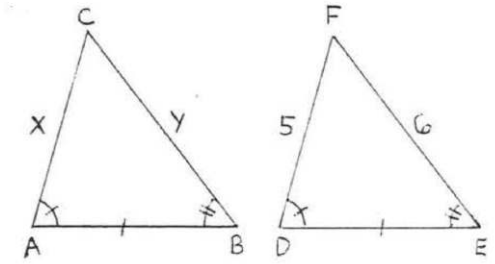
9 - 22. For each of the following

(1) write a congruence statement for the two triangles,

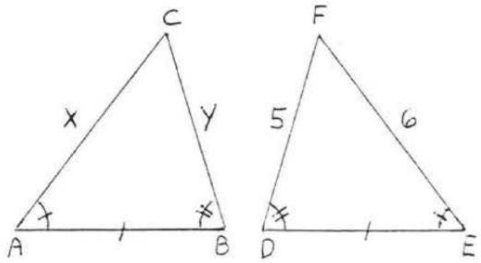
(2) give a reason for (1) (SAS, ASA, or AAS Theorems),

(3) find x, or x and y.

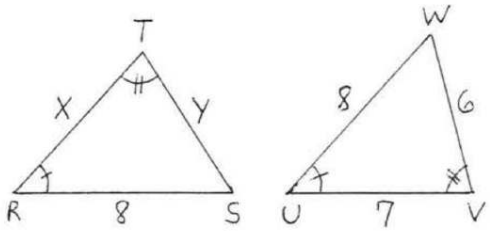
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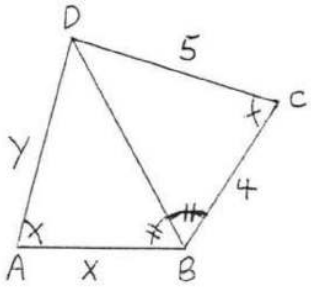
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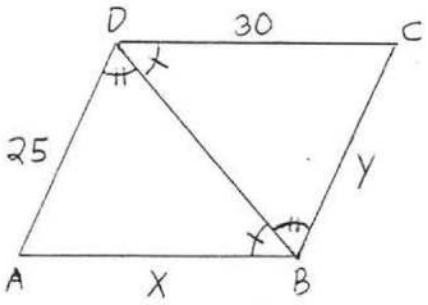
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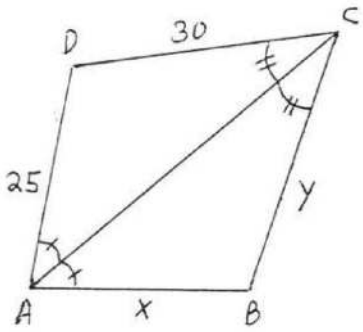
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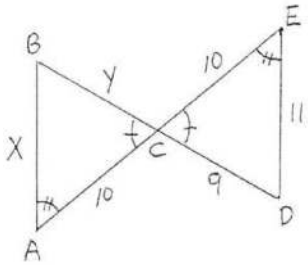
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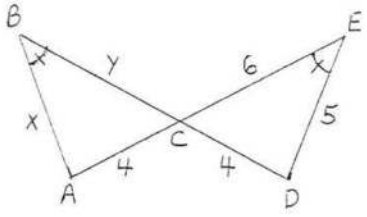
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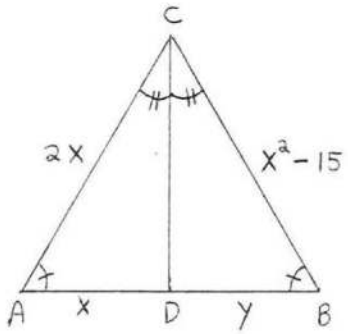


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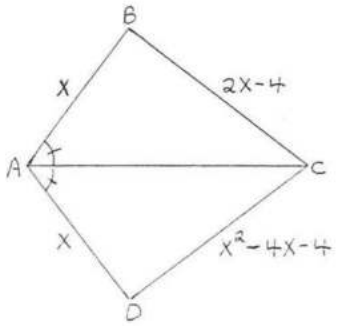


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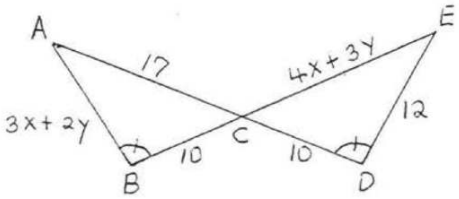




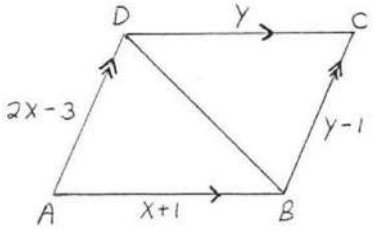
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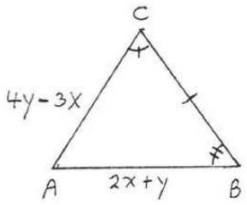
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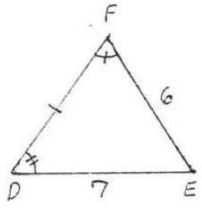
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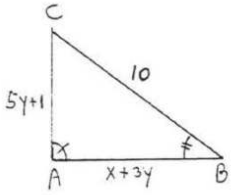


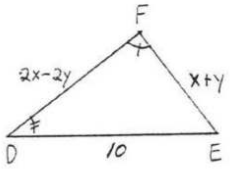
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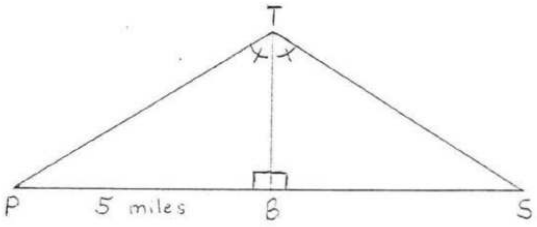




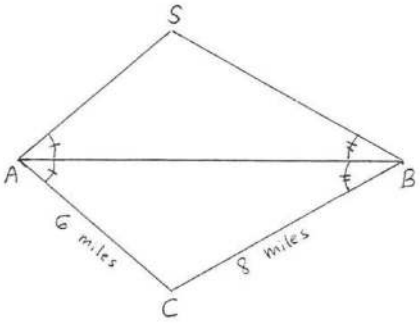
22.

23 - 26. For each of the following, include the congruence statement and the reason as part of your answer:

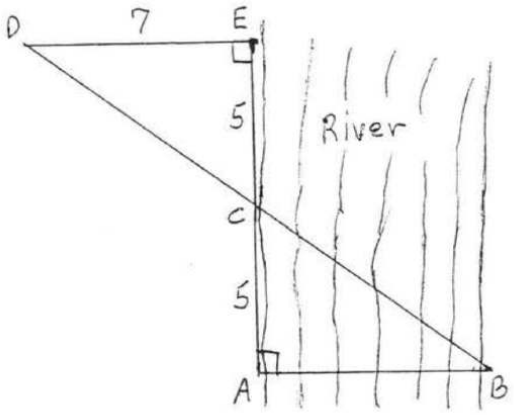
23. In the diagram how far is the ship S from the point P on the coast?



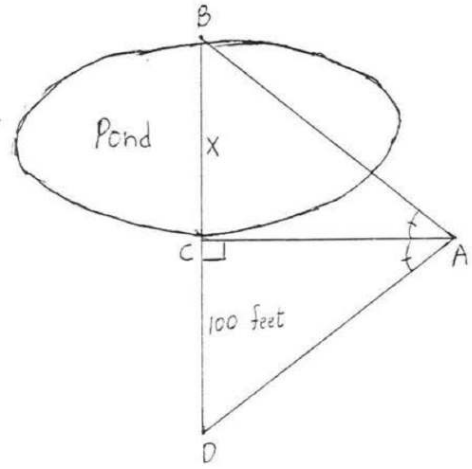
24. Ship S is observed from points A and B along the coast. Triangle ABC is then constructed and measured as in the diagram, How far is the ship from point A?



25. Find the distance AB across a river if $AC = CD = 5$ and $DE = 7$ as in the diagram.



26. that is the distance across the pond?



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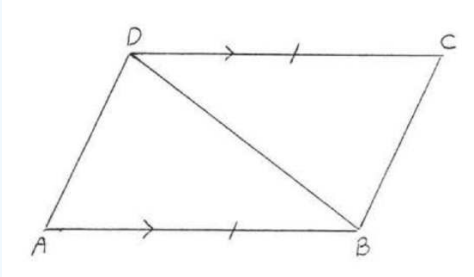
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3.4: Proving Lines and Angles Equal

We can prove lines and angles equal if we can show they are corresponding parts of congruent triangles, We find it convenient to present these proofs in **double-column form** with statements in the left columnand the reason for each statement in the right.

Example 3.4.1

Given $AB||CD$ and $AB = CD$ prove $AD = BC$



Solution

Statements	Reasons
1. $AB = CD$.	1. Given.
2. $\angle ABD = \angle CDB$.	2. Alternate interior angles of parallel lines $(AB||CD)$ are equal.
3. $BD = DB$.	3. Identity.
4. $\triangle ABD \cong \triangle CDB$	4. $SAS = SAS$: AB, $\angle B$, $BD$of $\triangle ABD = CD$, $\angle D$, DB of $\triangle CDB$.
5. $AD = BC$	5. Corresponding sides of congruent triangles are equal,

Explanation: Each of the first three statements says that a side or angle of $\triangle ABD$ is equal to the corresponding side or angle of $\triangle CDB$, To arrive at these statements, we should first write the congruence statement using the methods of the previous sections. We then select three pairs of corresponding sides or angles which are equal because of one of the following reasons:

Reasons Lines Are Equal

1. Given. This means we are asked to assume the lines are equal at the beginning of the exercise, For example, the problem will state "given $AB = CD$" or AB and CD will be marked the same way in the diagram.
2. Identity. This means the identical line segment appears in both triangles, For example, BD and DB represent the same line segment, Of course the length of a line segment is equal to itself.

Reasons Angles Are Equal

1. Given.
2. Identity.
3. Alternate interior angles of parallel lines are equal. To apply this reason we must be given that the lines are parallel.
4. Corresponding angles of parallel lines equal.
5. Vertical angles are equal.

These are not the only possible reasons but they are all that we will use at first.

We should also select the three pairs of equal sides or angles so that one of the reasons $SAS = SAS$, $ASA = ASA$, or $AAS = AAS$ can be used to justify the congruence statement in statement 4, In sections 2.6 and 2.7, we will give some additional reasons for two triangles to be congruent.

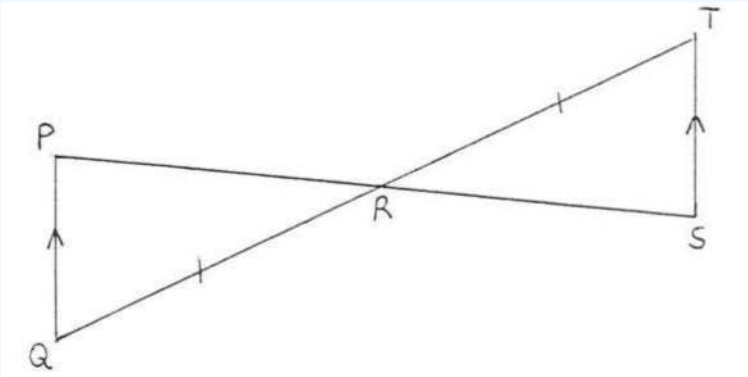
Statement 5 is the one we wish to prove, The reason is that corresponding sides (or angles) of congruent triangles are equal. Wecan use this reason here because the triangles have already been proven congruent in statement 4,

One final comment, Notice how the solution of Example $3.4.1$ conforms with our original definition of proof, Each new statement is shown to be true by using previous statements and reasons which have already been established.

Let us give another example:

Example 3.4.2

Given $QP||ST$ and $QR = TR$ prove $PR = SR$.

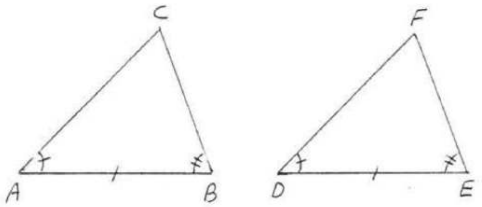


Solution

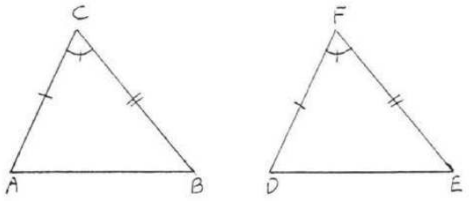
Statements	Reasons
1. $QR = TR$	1. Given.
2. $\angle Q = \angle T$.	2. Alternate interior angles of parallel lines ($QP||ST$) are equal.
3. $\angle PRQ = \angle SRT$.	3. Vertical angles are equal.
4. $\triangle PQR \cong \triangle STR$.	4. $ASA = ASA$: $\angle Q, QR, \angle R$ of $\triangle PQR = \angle T, TR, \angle R$ of $\triangle STR$.
5. $PR = SR$.	5. Corresponding sides of congruent triangles are equal,

Problems

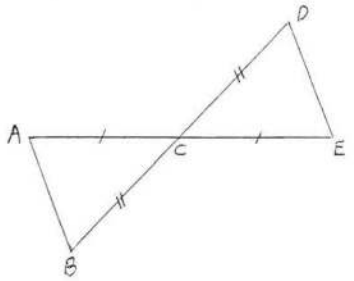
1. Given $\angle A = \angle D$, $\angle B = \angle E$, $AB = DE$. Prove $AC = DF$.



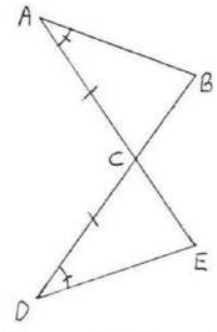
2. Given $AC = DF$, $BC = EF$, $\angle C = \angle F$. Prove $AB = DE$.



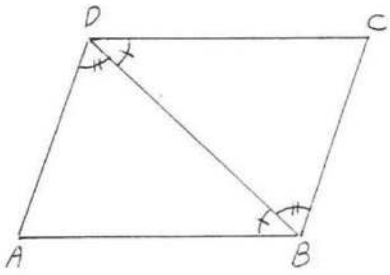
3. Given $AC = EC$ and $BC = DC$. Prove $AB = ED$.



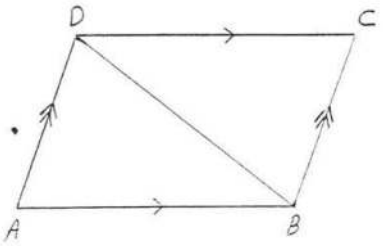
4. Given $AC = DC$, $\angle A = \angle D$. Prove $BC = EC$.



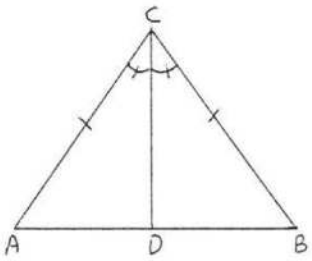
5. Given $\angle ABD = \angle CDB$ and $\angle ADB = \angle CBD$. Prove $AB = CD$.



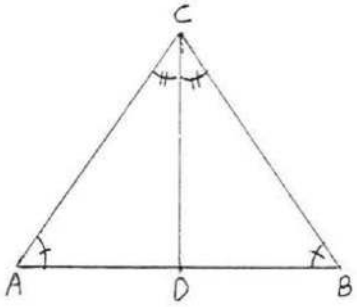
6. Given $AB||CD$ and $AD||CB$. Prove $AB = CD$.



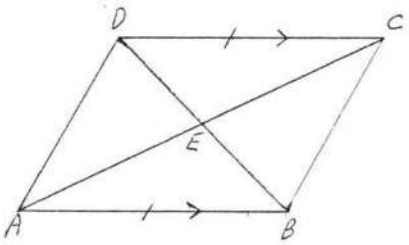
7. Given $AC = BC$ and $\angle ACD = \angle BCD$. Prove $\angle A = \angle B$.



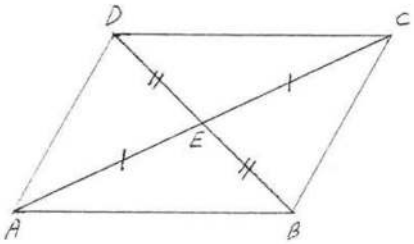
8. Given $\angle A = \angle B$, $\angle ACD = \angle BCD$. Prove $AC = BC$.



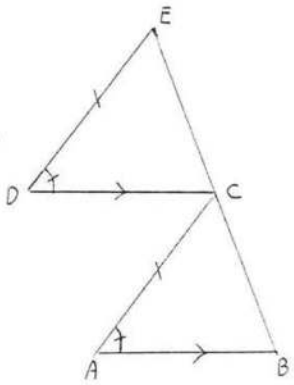
9. Given $AB||CD$ and $AB = CD$. Prove $AE = CE$. (Hint: Show $\triangle ABE \cong \triangle CDE$)



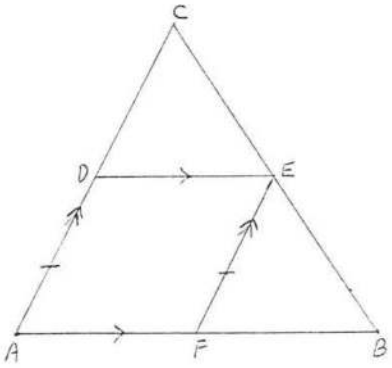
10. Given $AE = CE$ and $BE = DE$. Prove $\angle BAC = \angle CDB$.



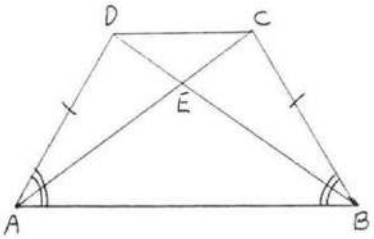
11. Given $\angle A = \angle D$, $AC = DE$, $AB||DC$. Prove $BC = CE$.



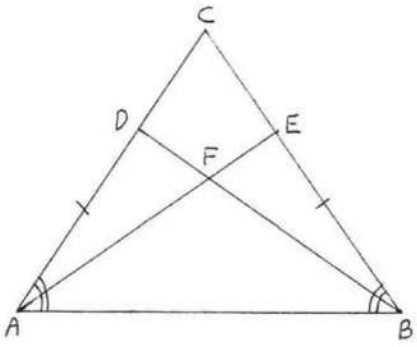
12. Given $AB||DE$, $AC||FE$ and $DC = FE$. Prove $BE = EC$.



13. Given $AD = BC$ and $\angle BAD = \angle ABC$. Prove $AC = BD$. (Hint: Show $\triangle ABD \cong \triangle BAC$)



14. Given $AD = BE$, $\angle BAC = \angle ABC$. Prove $AE = BD$.



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3.5: Isosceles Triangles

In Section 1.6, we defined a triangle to be isosceles if two of its sides are equal. Figure 3.5.1 shows an isosceles triangle $\triangle ABC$ with $AC = BC$. In $\triangle ABC$ we say that $\angle A$ is opposite side BC and $\angle B$ is opposite side AC.

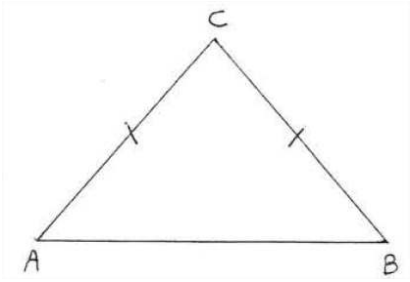


Figure 3.5.1: $\triangle ABC$ is isosceles with AC = BC.

The most important fact about isosceles triangles is the following:

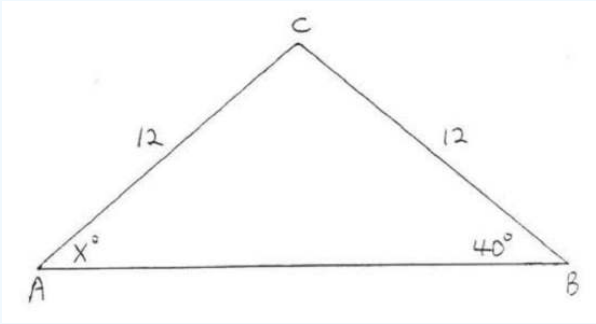
Theorem 3.5.1

If two sides of a triangle are equal the angles opposite these sides are equal.

Theorem 3.5.1 means that if $AC = BC$ in $\triangle ABC$ then $\angle A = \angle B$.

Example 3.5.1

Find x:



Solution

$AC = BC$ so $\angle A = \angle B$. Therefore, $x = 40$.

Answer: $x = 40$.

In $\triangle ABC$ if $AC = BC$ then side AB is called the **base** of the triangle and $\angle A$ and $\angle B$ are called the base angles. Therefore Theorem 3.5.1 is sometimes stated in the following way: "The base angles of an isosceles triangle are equal,"

Proof of Theorem 3.5.1: Draw CD, the angle bisector of $\angle ACB$ (Figure 3.5.2). The rest of the proof will be presented in double-column form. We have given that $AC = BC$ and $\angle ACD = \angle BCD$. We must prove $\angle A = \angle B$.

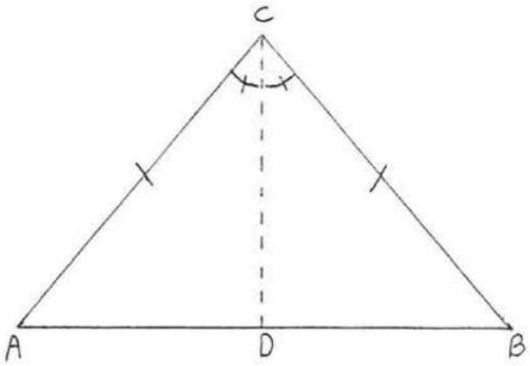
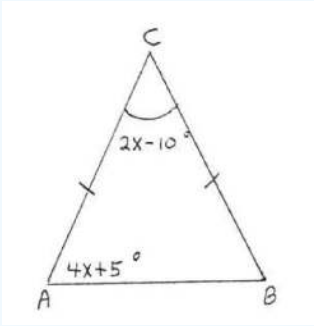


Figure 3.5.2: Draw CD, the angle bisector of $\angle ACB$.

Statements	Reasons
1. $AC = BC$.	1. Given, $\triangle ABC$ is isosceles.
2. $\angle ACD = \angle BCD$.	2. Given, CD is the angle bisector of $\angle ACB$.
3. $CD = CD$.	3. Identity.
4. $\triangle ACD \cong \triangle BCD$.	4. $SAS = SAS$: $AC, \angle C, CD$ of $\triangle ACD = BC, \angle C, CD$ of $\triangle BCD$.
5. $\angle A = \angle B$.	5. Corresponding angles of congruent trianglesare equal.

Example 3.5.2

Find $x, \angle A, \angle B$and $\angle C$:



Solution

$\angle B = \angle A = 4x + 5^\circ$ by Theorem 3.5.1. We have

$$\begin{aligned} \angle A + \angle B + \angle C &= 180^\circ \\ 4x + 5 + 4x + 5 + 2x - 10 &= 180 \\ 10x &= 180 \\ x &= 18 \end{aligned} \tag{3.5.1}$$

$\angle A = \angle B = 4x + 5^\circ = 4(18) + 5^\circ = 72 + 5^\circ = 77^\circ$.

$\angle C = 2x - 10^\circ = 2(18) - 10^\circ = 36 - 10^\circ = 26^\circ$.

Check

$$\begin{aligned} \angle A + \angle B + \angle C &= 180^\circ \\ 4x + 5 + 4x + 5 + 2x - 10 & \\ 4(18) + 5 + 4(18) + 5 + 2(18) - 10 & \\ 77^\circ + 77^\circ + 26^\circ & \\ 180^\circ & \end{aligned}$$

Answer

$x = 18$, $\angle A = 77^\circ$, $\angle B = 77^\circ$, $\angle C = 26^\circ$.

In Theorem 3.5.1 we assumed $AC = BC$ and proved $\angle A = \angle B$. We will now assume $\angle A = \angle B$ and prove $AC = BC$. '1ihen the assumption and conclusion of a statement are interchanged the result is called the converse of the original statement.

Theorem 3.5.2: The Converse of Theorem 3.5.1)

If two angles of a triangle are equal the sides opposite these angles are equal.

If Figure 4, if $\angle A = \angle B$ then $AC = BC$.

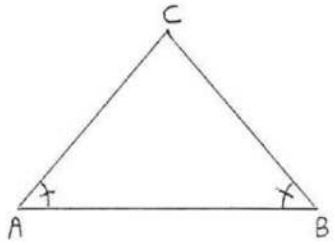
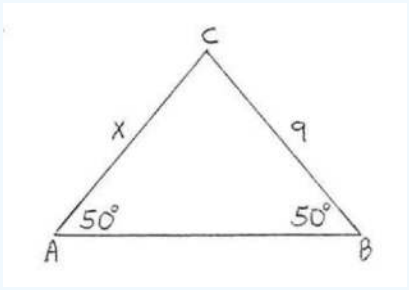


Figure 3.5.4. $\angle A = \angle B$

Example 3.5.3

Find x



Solution

$\angle A = \angle B$ so $x = AC = BC = 9$ by Theorem 3.5.2.

Answer

$x = 9$.

Proof of Theorem 3.5.2: Draw CD the angle bisector of $\angle ACB$ (Figure 3.5.5). We have $\angle ACD = \angle BCD$ and $\angle A = \angle B$. We must prove $AC = BC$.

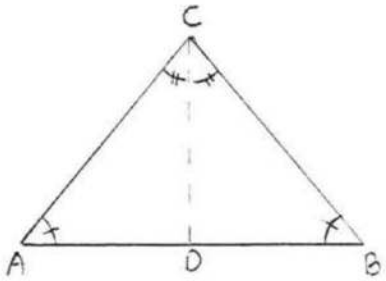


Figure 3.5.5. Draw CD, the angle bisector of $\angle ACB$.

Statements	Reasons
1. $\angle A = \angle B$.	1. Given.
2. $\angle ACD = \angle BCD$.	2. Given.
3. $CD = CD$.	3. Identity.
4. $\triangle ACD \cong \triangle BCD$.	4. $AAS = AAS$: $\angle A$, $\angle C$, CD of $\triangle ACD = \angle B$, $\angle C$, CD of $triangle BCD$.
5. $AC = BC$.	5. Corresponding sides of congruent triangles are equal

The following two theorems are **corollaries** (immediate consequences) of the two preceding theorems:

Theorem 3.5.3

An equilateral triangle is equiangular.

In Figure 3.5.7, if $AB = AC = BC$ then $\angle A = \angle B = \angle C$.

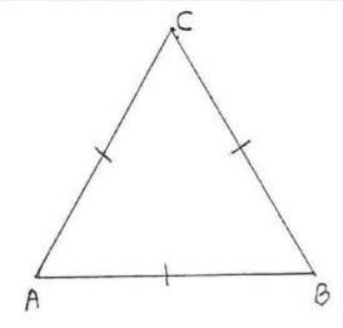


Figure 3.5.7: $\angle ABC$ is equilateral.

Proof

$AC = BC$ so by Theorem 3.5.1 $\angle B = \angle C$. Therefore $\angle A = \angle B = \angle C$.

Since the sum of the angle is 180° we must have in fact that $\angle A = \angle B = \angle C = 60^\circ$.

Theorem 3.5.4: The Converse of Theorem 3.5.3)

An equiangular triangle is equilateral.

In Figure 3.5.8, if $\angle A = \angle B = \angle C$ then $AB = AC = BC$.

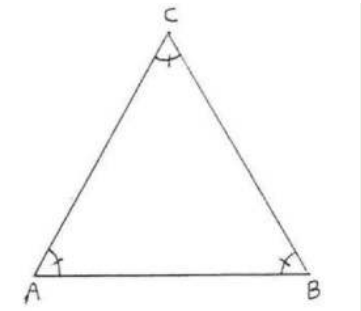


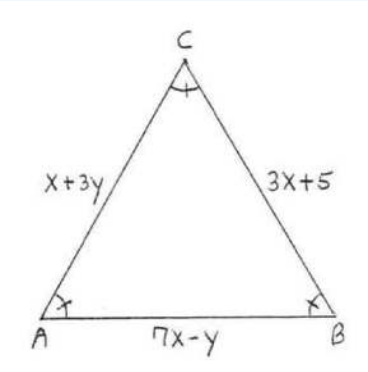
Figure 3.5.8. $\angle ABC$ is equiangular.

Proof

$\angle A = \angle B$ so by Theorem 3.5.2, $AC = BC$, $\angle B = \angle C$ by Theorem 3.5.2, $AB = AC$. Therefore $AB = AC = BC$.

Example 3.5.4

Find x, y and AC:



Solution

$\triangle ABC$ is equiangular and so by Theorem 3.5.4 is equilateral.

Therefore
$$\begin{aligned} AC &= AB \\ x+3y &= 7x-y \\ x-7x+3y+y &= 0 \\ -6x+4y &= 0 \end{aligned}$$
and
$$\begin{aligned} AB &= BC \\ 7x-y &= 3x+5 \\ 7x-3xy-y &= 5 \\ 4x-y &= 5 \end{aligned}$$

We have a system of two equations in two unknowns to solve:

$$\begin{aligned} -6x+4y &= 0 \\ 4x-y &= 5 \end{aligned} \xrightarrow{4} \begin{aligned} -6x+4y &= 0 \\ 16x-4y &= 20 \\ \hline 10x &= 20 \\ x &= 2 \end{aligned}$$

$$\begin{aligned} 4x-y &= 5 \\ 4(2)-y &= 5 \\ 8-y &= 5 \\ -y &= 5-8 \\ -y &= -3 \\ y &= 3 \end{aligned}$$

Check:

AC	=	AB
x + 3y		7x - y
2 + 3(3)		7(2) - 3
2 + 9		14 - 3
11		11

AB	=	BC
7x - y		3x + 5
7(2) - 3		3(2) + 5
14 - 3		6 + 5
11		11

Answer: $x = 2$, $y = 3$, $AC = 11$.

Historical Note

Theorem 3.5.1, the isosceles triangle theorem, is believed to have first been proven by Thales (c. 600 B,C,) - it is Proposition 5 in Euclid's **Elements**. Euclid's proof is more complicated than ours because he did not want to assume the existence of an angle bisector, Euclid's proof goes as follows:

Given $\triangle ABC$ with $AC = BC$ (as in Figure 3.5.1 at the beginning of this section), extend CA to D and CB to E so that $AD = BE$ (Figure 3.5.9). Then $\triangle DCB \cong \triangle ECA$ by $SAS = SAS$. The corresponding sides and angles of the congruent triangles are equal, so $DB = EA$, $\angle 3 = \angle 4$ and $\angle 1 + \angle 5 = \angle 2 + \angle 6$. Now $\triangle ADB \cong \triangle BEA$ by $SAS = SAS$. This gives $\angle 5 = \angle 6$ and finally $\angle 1 = \angle 2$.

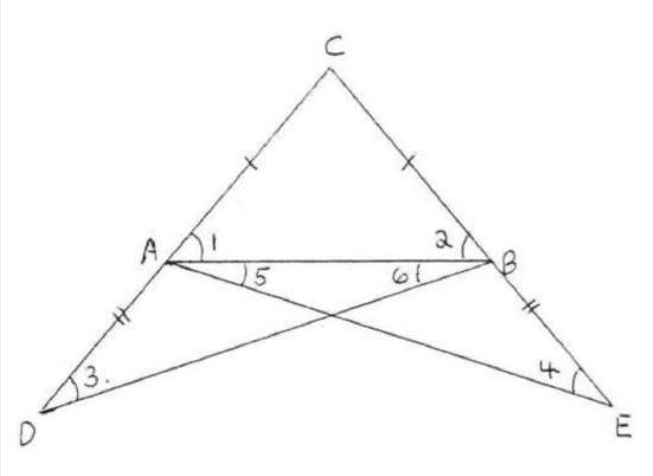


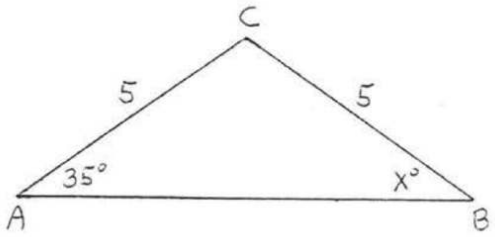
Figure 3.5.9: The "bridge of fools".

This complicated proof discouraged many students from further study in geometry during the long period when the **Elements** was the standard text, Figure 3.5.9 resembles a bridge which in the Middle Ages became known as the "bridge of fools," This was supposedly because a fool could not hope to cross this bridge and would abandon geometry at this point.

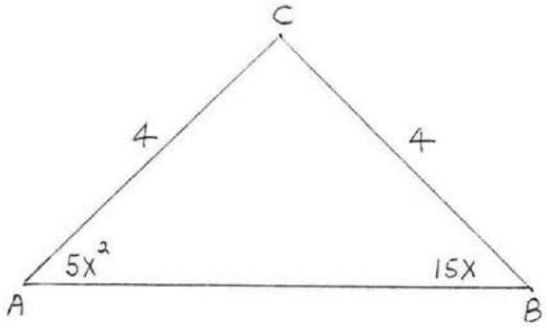
Problems

For each of the following state the theorem(s) used in obtaining your answer.

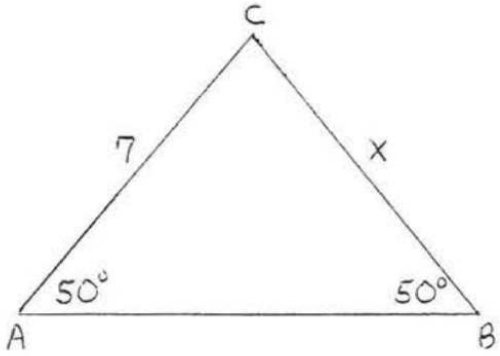
1. Find x:



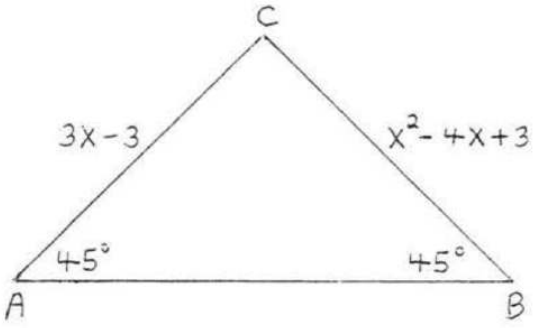
2. Find x, $\angle A$, and $\angle B$:



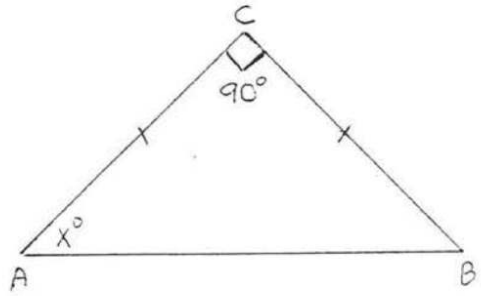
3. Find x:



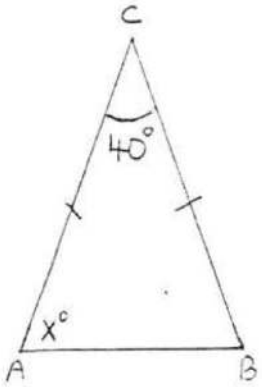
4. Find x, AC, and BC:



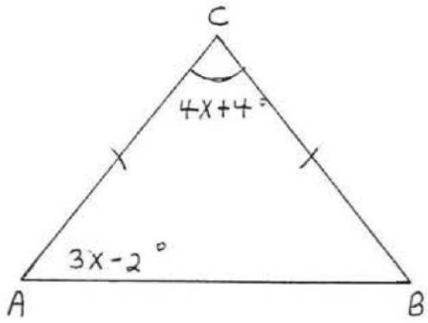
5. Find x:



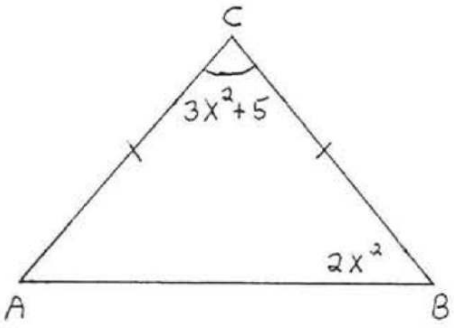
6. Find x:



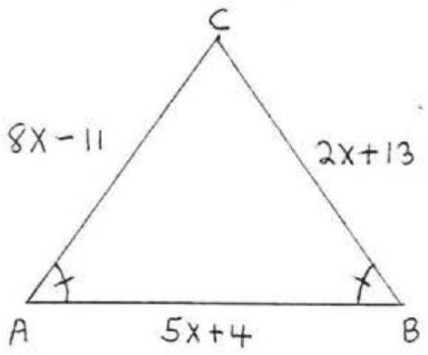
7. Find x, $\angle A$, $\angle B$, and $\angle C$:



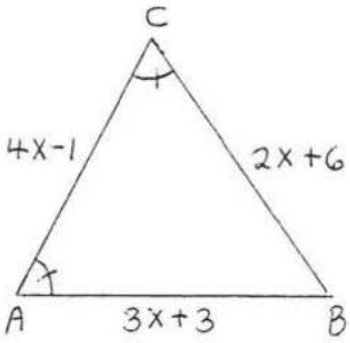
8. Find x, $\angle A$, $\angle B$, and $\angle C$:



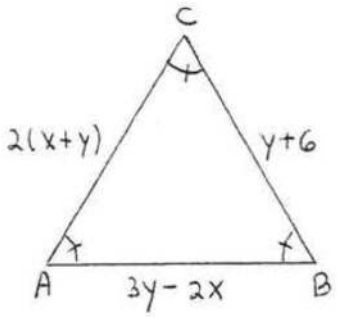
9. Find x, AB, AC, and BC:



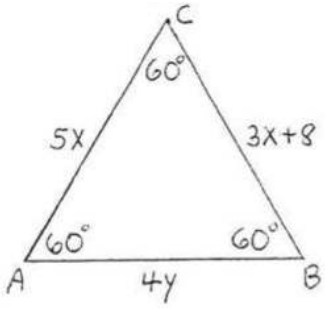
10. Find x, AB, AC, and BC:



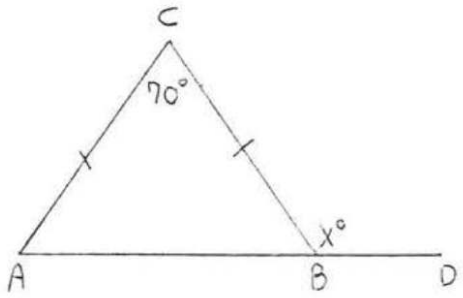
11. Find x, y, and AC:



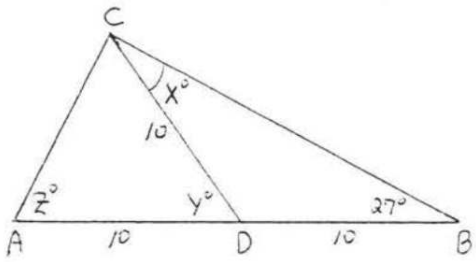
12. Find x, y, and AC:



13. Find x:



14. Find x, y, and z:



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3.6: The SSS Theorem

We now consider the case where the side of two triangles are known to be of the same length.

Theorem 3.6.1: Side-Side-Side (SSS) Theorem

Two triangles are congruent if three sides of one are equal respectively to three sides of the other ($SSS = SSS$).

Theorem 3.6.1 is demonstrated in Figure 3.6.1: if $a = d, b = e,$ and $c = f$ then $\triangle ABC \cong \triangle DEF$

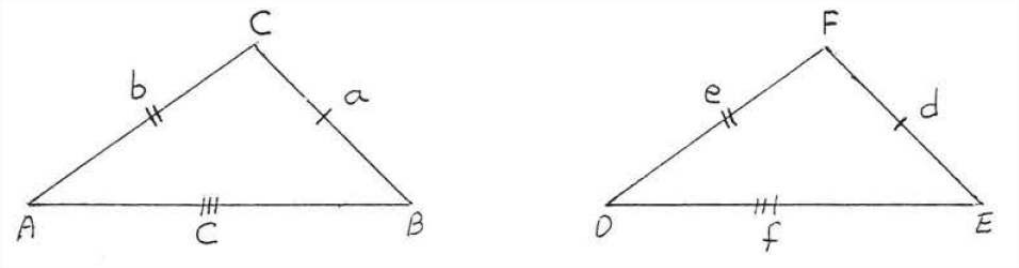
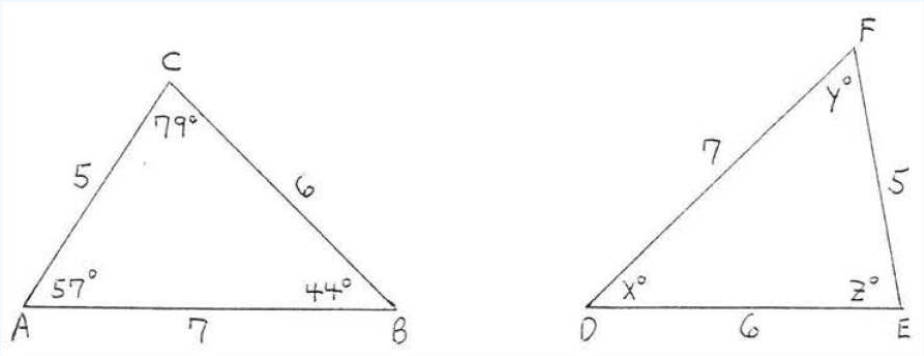


Figure 3.6.1: $\triangle ABC \cong \triangle DEF$ because $SSS = SSS$.

Example 3.6.1

Find x, y, z:



Solution

$AB = 7 = DF$. Therefore $\angle C$, the angle opposite AB must correspond to $\angle E$, the angle opposite DF. In the same way $\angle A$ corresponds to $\angle F$ and $\angle B$ corresponds to $\angle D$. We have $\triangle ABC \cong \triangle FDE$ by $SSS = SSS$, so

$x^\circ = \angle D = \angle B = 44^\circ$

$y^\circ = \angle F = \angle A = 57^\circ$

$z^\circ = \angle E = \angle C = 79^\circ$

Answer: $x = 44, y = 57, z = 79$

Proof of Theorem 3.6.1

In Figure 3.6.1, place $\triangle ABC$ and $\triangle DEF$ so that their longest sides coincide, in this case AB and DE. This can be done because $AB = c = r = DE$. Now draw CF, forming angles $1, 2, 3,$ and 4 (Figure 3.6.2). The rest of the proof will be presented in double-column form:

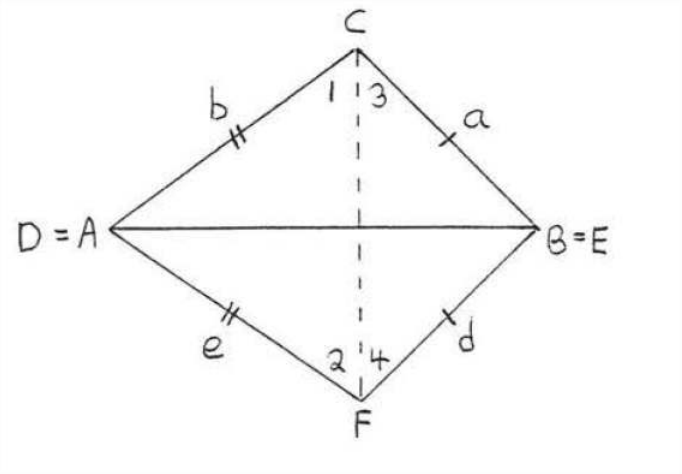
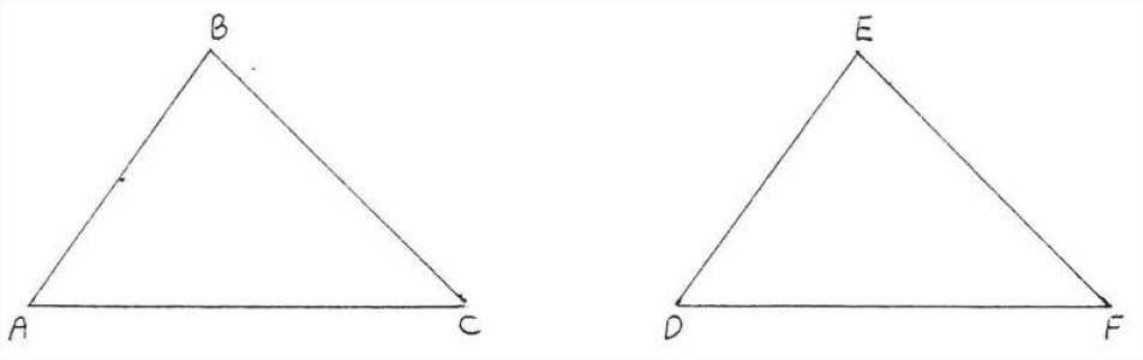


Figure 3.6.2: Place $\triangle ABC$ and $\triangle DEF$ so that AB and DE coincide and draw CF.

Statement	Reasons
1. $\angle 1 = \angle 2$.	1. The base angles of isosceles triangle CAF are equal (Theorem 3.6.1, section 2.5).
2. $\angle 3 = \angle 4$.	2. The base angles of isosceles triangle CBF are equal.
3. $\angle C = \angle F$.	3. $\angle C = \angle 1 + \angle 3 = \angle 2 + \angle 4 = \angle F$.
4. $AC = DF$.	4. Given, $AC = b = e = DF$.
5. $BC = EF$.	5. Given, $BC = a = d = EF$.
6. $\triangle ABC \cong \triangle DEF$.	6. $SAS = SAS$: $AC, \angle C, BC$ of $\triangle ABC = DF, \angle F, EF$ of $\triangle DEF$.

Example 3.6.2

Given $AB = DE, BC = EF,$ and $AC = DF$. Prove $\angle C = \angle F$



Solution

Statements	Reasons
1. $AB = DE$.	1. Given.
2. $BC = EF$.	2. Given.
3. $AC = DF$.	3. Given.

4. $\triangle ABC \cong \triangle DEF$.	4. $SSS = SSS$: AB, BC, AC of $\triangle ABC = DE$, EF, DF of $\triangle DEF$.
5. $\angle C = \angle F$.	5. Corresponding angles of congruent triangles are equal.

Application: Triangular Bracing

The SSS Theorem is the basis of an important principle of construction engineering called **triangular bracing**. Imagine the line segments in Figure 3.6.3 to be beans of wood or steel joined at the endpoints by nails or screws. If pressure is applied to one of the sides, $ABCD$ will collapse and look like $A'B'C'D'$.

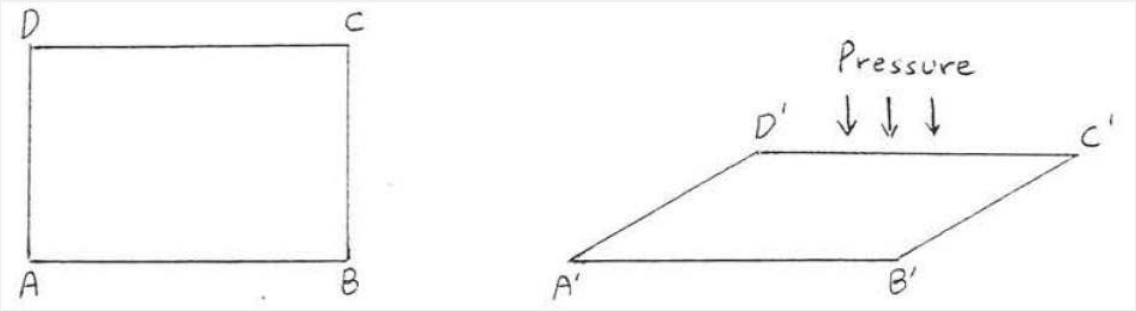


Figure 3.6.3: $ABCD$ collapses into $A'B'C'D'$, when pressure is applied.

Now suppose points A and C are joined by a new beam, called a **brace** (Figure 3.6.4). The structure will not collapse as long as the beans remain unbroken and joined together. It is impossible to deform $ABCD$ into any other shape $A'B'C'D'$ because if $AB = A'B'$, $BC = B'C'$, and $AC = A'C'$ then $\triangle ABC$ would be congruent to $\triangle A'B'C'$ by $SSS = SSS$.

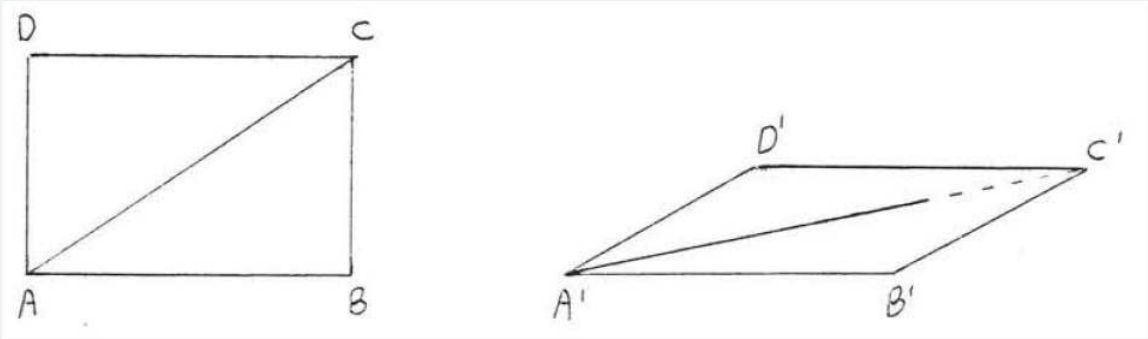


Figure 3.6.4: $ABCD$ cannot collapse into $A'B'C'D'$ as long as the beams remain unbroken and Jotned together.

We sometimes say that a triangle is a **rigid** figure; once the sides of a triangle are fixed the angles cannot be changed. Thus in Figure 3.6.4, the shape of $\triangle ABC$ cannot be changed as long as the lengths of its sides remain the same.

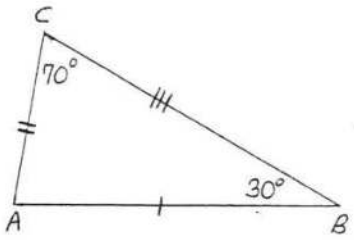
Problems

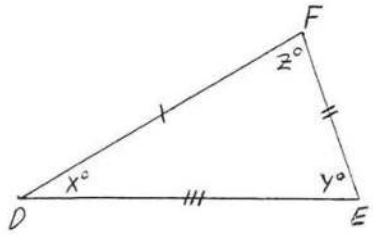
1 - 8. For each of the following (1) write the congruence statement,

(2) given the reason for (1) (SAS, ASA, AAS, or SSS Theorem), and

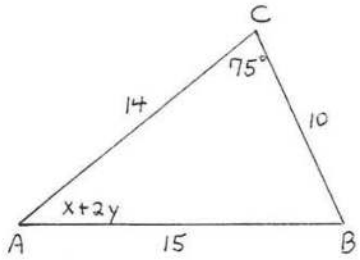
(3) find x, or x and y, or x, y, and z.

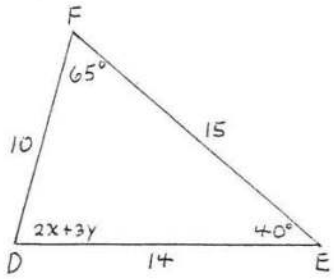
1.


C
70°
30°
A
B

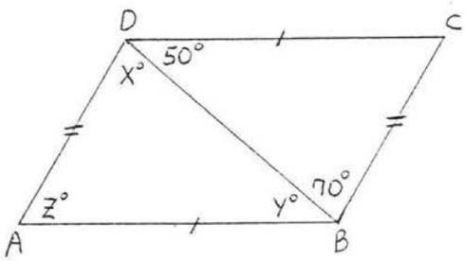

F
z°
x°
y°
D
E

2.

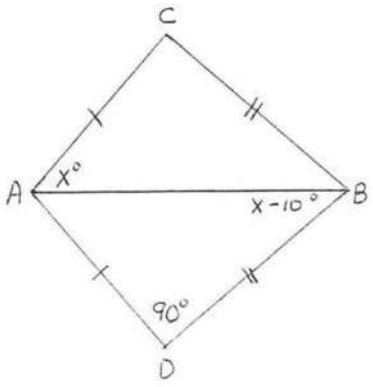

C
75°
14
10
x+2y
A
15
B


F
65°
10
15
2x+3y
40°
D
14
E

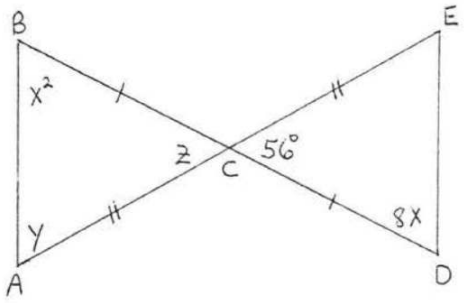
3.


D
C
50°
x°
z°
y°
70°
A
B

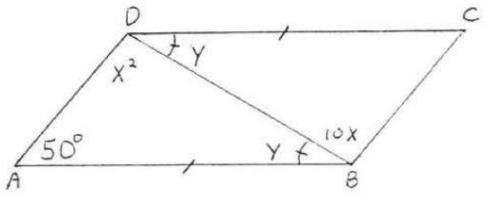
4.


C
A
x°
B
x-10°
90°
D

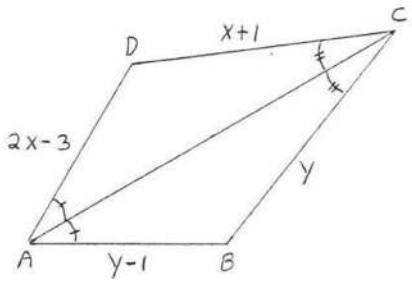
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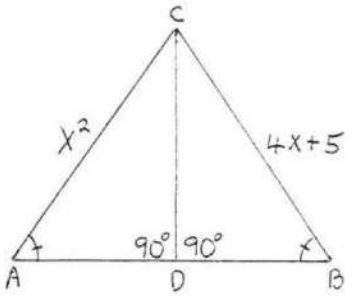
6.



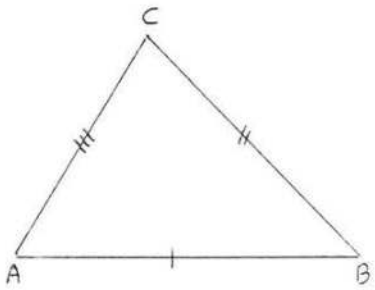
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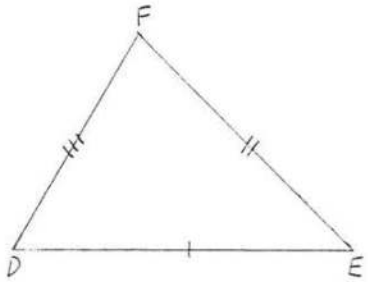


8.

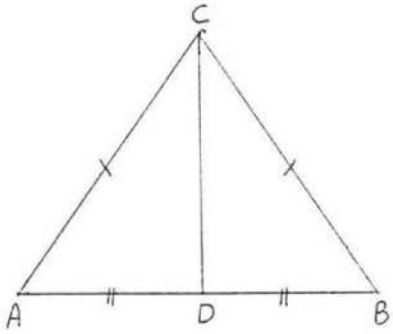


9. Given $AB = DE$, $BC = EF$, and $AC = DF$. Prove $\angle A = \angle D$.

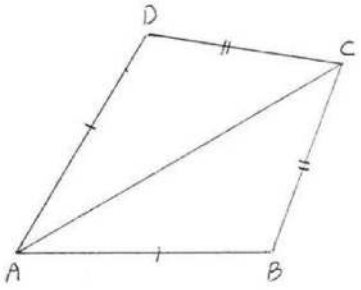




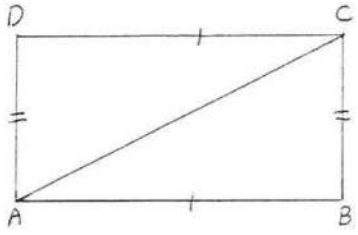
10. Given $AC = BC$. $AD = BD$. Prove $\angle ADC = \angle BDC$.



11. Given $AB = AD$, $BC = DC$. Prove $\angle BAC = \angle CAD$.

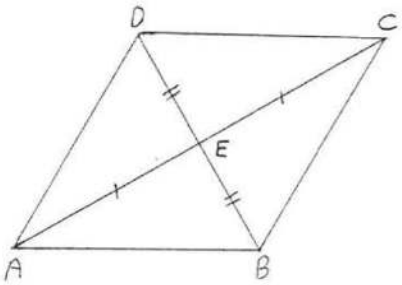


12. Given $AB = CD$, $BC = DA$. Prove $\angle BAC = \angle DCA$.

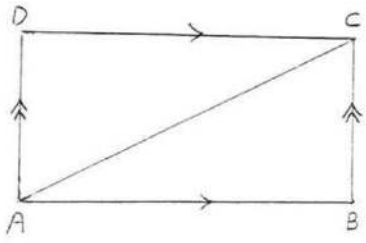


13. Given $AE = CE$, $BE = ED$. Prove $AB = CD$.

https://math.libretexts.org/@go/page/72147



14. Given $AB||CD$, $AD||BC$. Prove $AB = CD$.



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3.7: The Hyp-Leg Theorem and Other Cases

We give one more reason for two triangles to be congruent. Note that the following reason applies to **right triangles** only:

Theorem 3.7.1: Hypotenuse-Leg Theorem

Two right triangles are congruent if the hypotenuse and a leg of one triangle are respectively equal to the hypotenuse and a leg of the other triangle (Hyp-Leg = Hyp-Leg)

In Figure 3.7.1, if $AB = DE$ and $BC = BF$ then $\triangle ABC \cong \triangle DEF$.

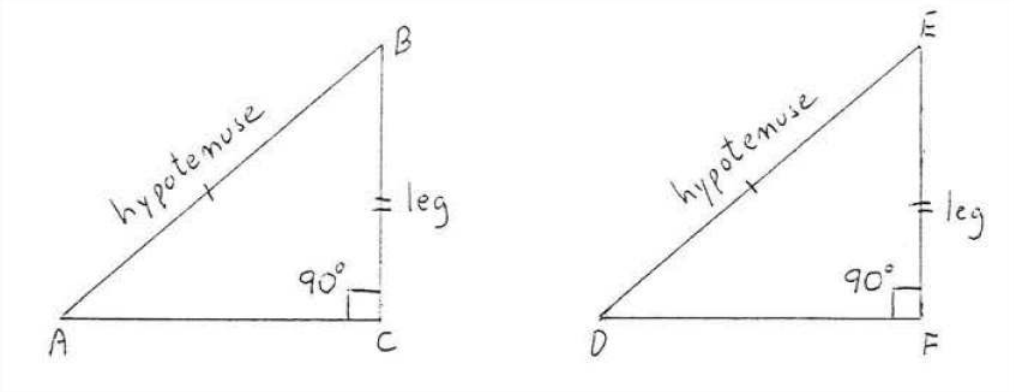
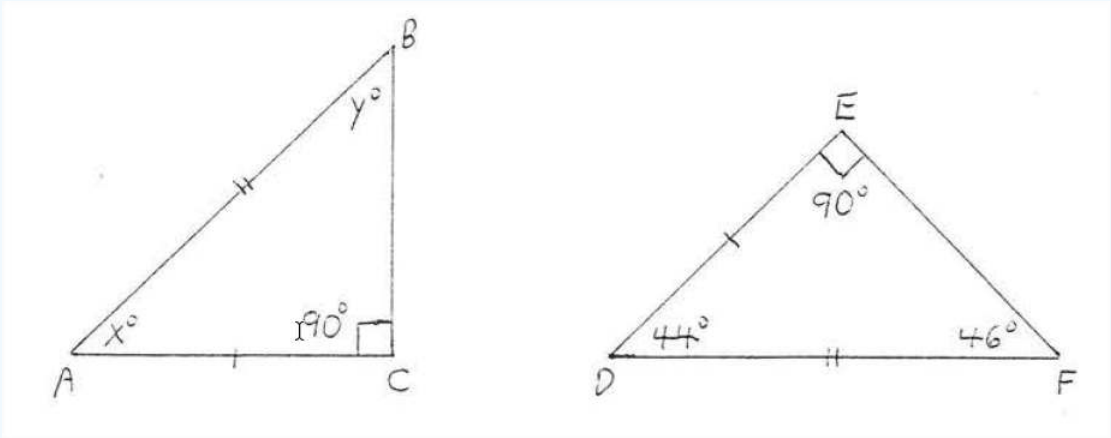


Figure 3.7.1: $\triangle ABC \cong \triangle DEF$ because Hyp-Leg = Hyp-Leg.

Example 3.7.1

Find x and y :



Solution

The hypotenuse of $\triangle ABC = AB =$ hypotenuse of $\triangle DBF = DF$ and a leg of $\triangle ABC = AC =$ a leg of $\triangle DEF = DE$. Therefore $\triangle ABC \cong \triangle DFE$ by Hyp-Leg = Hyp-Leg. So $x^\circ = \angle A = \angle D = 44^\circ$ and $y^\circ = \angle B = \angle F = 46^\circ$

Answer

$x = 44, y = 46$

Proof of Theorem 3.7.1: In Figure 3.7.1, place $\triangle DEF$ so that BC and EF coincide (see Figure 3.7.2). Then $\angle ACD = 180^\circ$ so AD is a straight line segment. $\triangle ABD$ is isosceles with $AB = DE$. Therefore $\angle A = \angle D$ because they are the base angles of isosceles triangle ABD (Theorem 3.7.1, section 2.5). Then $\triangle ABC \cong \triangle DEF$ by $AAS = AAS$.

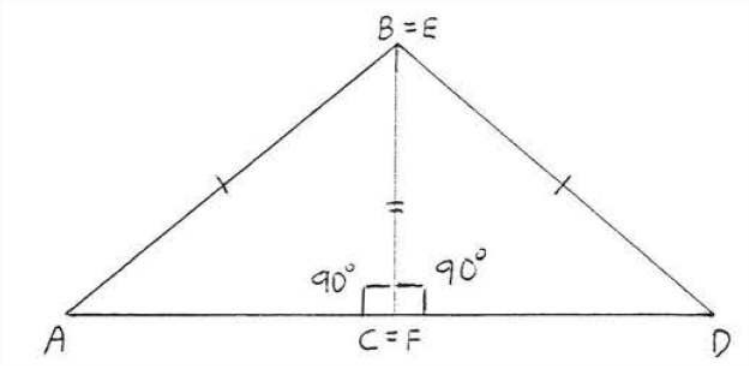
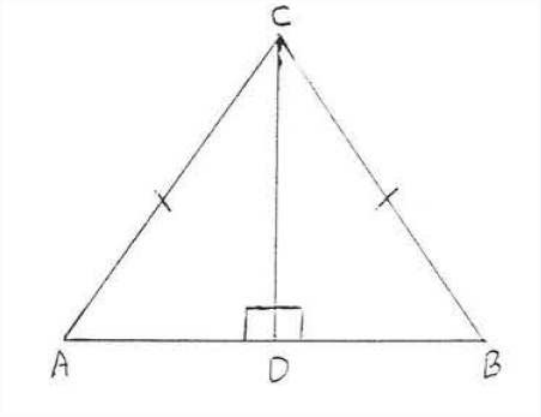


Figure 3.7.2 Place $\triangle DEF$ so that BC and EF coincide.

Example 3.7.2

Given $AC = BC, CD \perp AB$. Prove $AD = BD$.



Solution

Statements	Reasons
1. $AC = BC$.	1. Given.
2. $CD = CD$.	2. Identity.
3. $\angle ADC = \angle BDC = 90°$.	3. Given $CD \perp AB$.
4. $\triangle ACD \cong \triangle BCD$.	4. Hyp-Leg = Hyp-Leg: Hyp AC, Leg CD of $\triangle ACD =$ Hyp BD, Leg CD of $\triangle BCD$.
5. $AD = BD$.	5. Corresponding sides of congruent triangles are equal.

At this point the student might be ready to conclude that two triangles are congruent whenever any three corresponding sides or angles are equal. However this is not true in the following two cases:

1. There may be two triangles that are not congruent but have two equal sides and an equal unincluded angle (SSA = SSA).

In Figure 3.7.3, $AC = DF, BC = BF,$ and $\angle A = \angle D$ but none of the other angles or sides are equal.

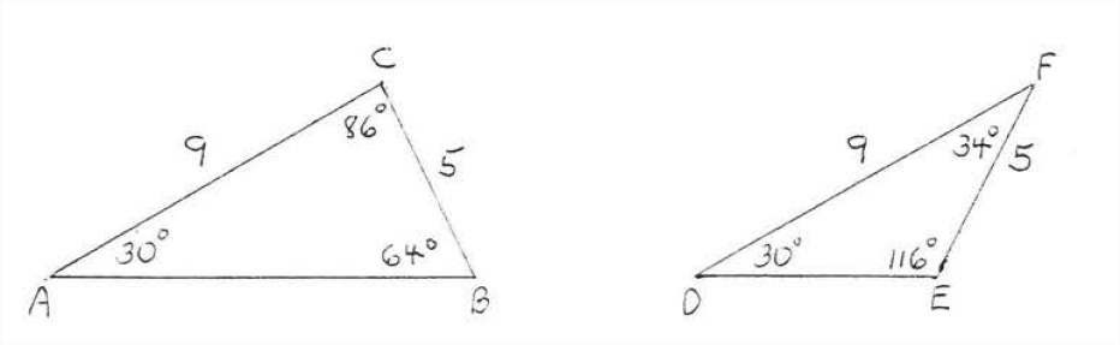


Figure 3.7.3: These two triangles satisfy $SSA = SSA$ but are not congruent.

2. There are many triangles that are not congruent but have the same three angles $(AAA = AAA)$

In Figure 3.7.4, the corresponding angles are equal but the corresponding sides are not.

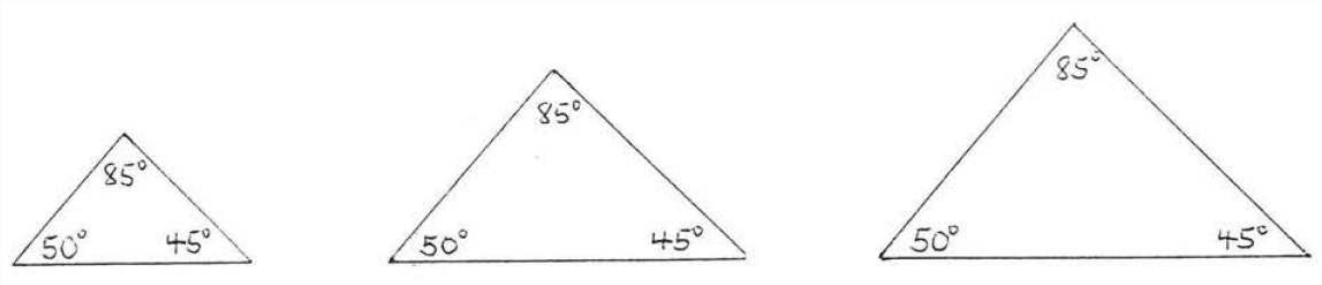
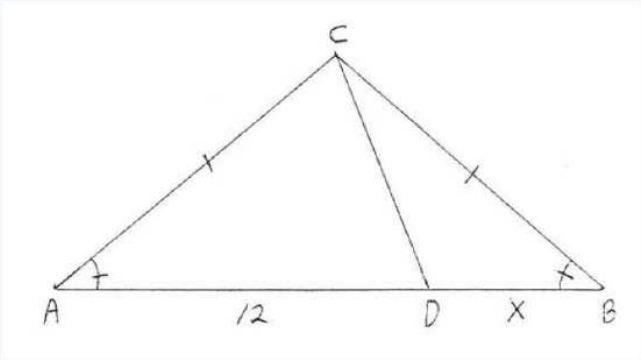


Figure 3.7.4. These triangles satisfy $AAA = AAA$ but are not congruent.

If $AAA = AAA$ the triangles are sald to be similar. Similar triangles are discussed in Chapter IV.

Example 3.7.3

Determine if the triangles are congruent. If so write the congruence statement and find x.



Solution

Solution: From the diagram $AC = BC, CD = CD$, and $\angle A = \angle B$ These are the only pairs of sides and angles which can be proven equal. $\angle A$ is not included between sides AC and CD and $\angle B$ is not included between sides BC and CD. Therefore we have only $SSA = SSA$. We cannot conclude the triangles are congruent and we cannot find x.

Answer: The triangles cannot be proven congruent.

Summary

Valid Reasons for Congruence	Invalid Reasons for Congruence
$SAS = SAS$	$SSA = SSA$
$ASA = ASA$	$AAA = AAA$

$AAS = AAS$	
$SSS = SSS$	
Hyp-Leg = Hyp-Leg (right triangles only)	

Problems

1 - 16. For each of the following determine if the triangles are congruent. If so

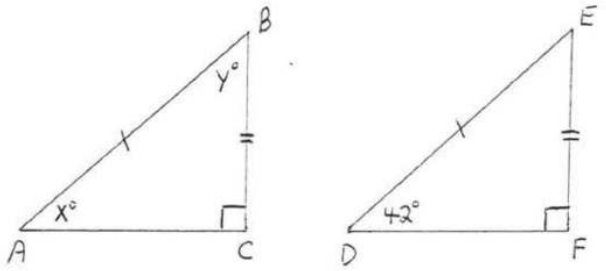
(1) write the congruence statement,

(2) give the reason for (1) ,

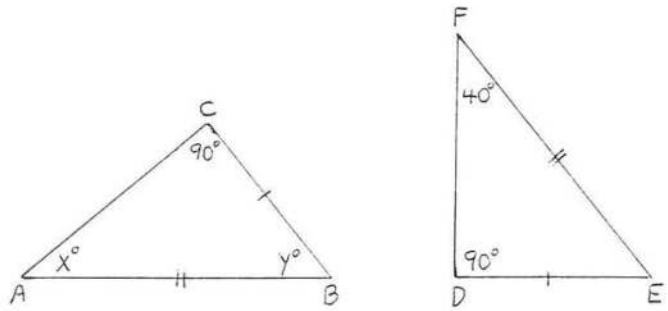
(3) find x, or x and y, or x, y, and z,

Otherwise write "the triangles cannot be proven congruent."

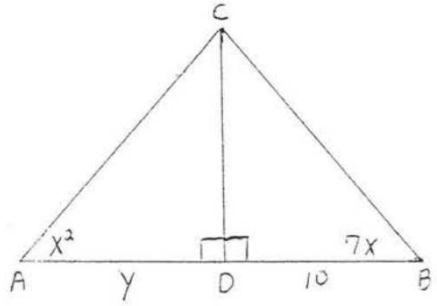
1.



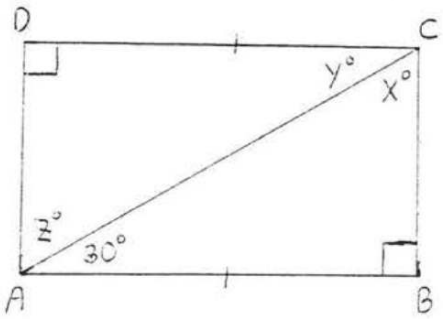
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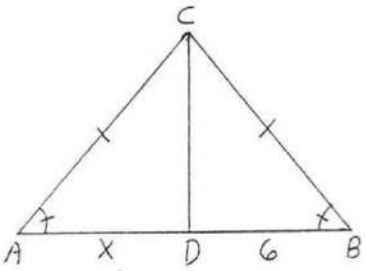
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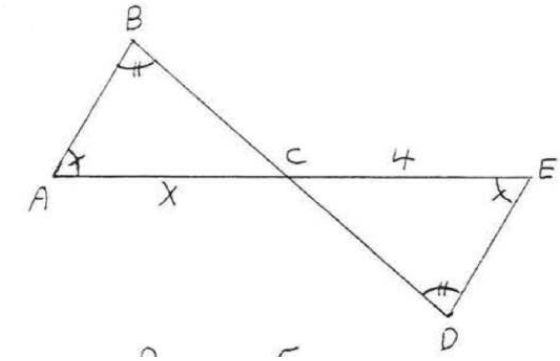
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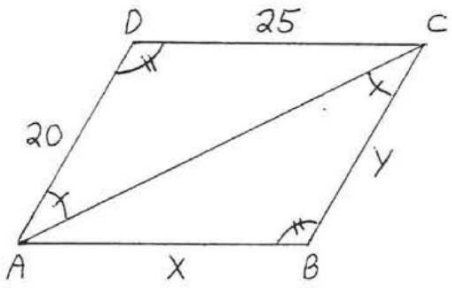
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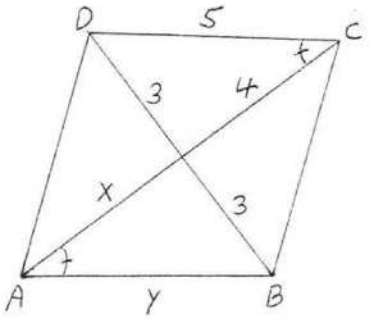
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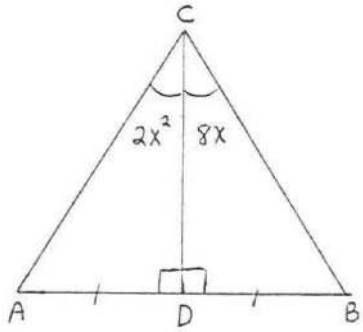
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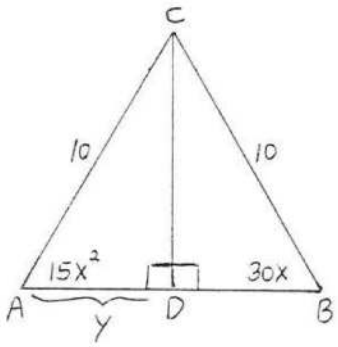
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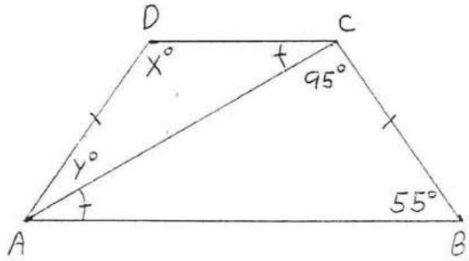
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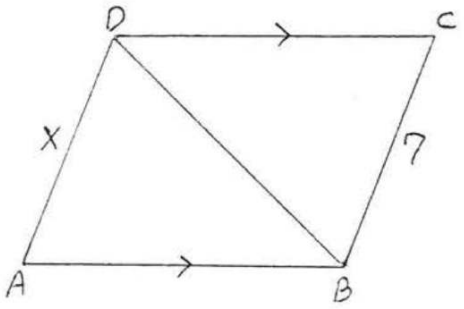
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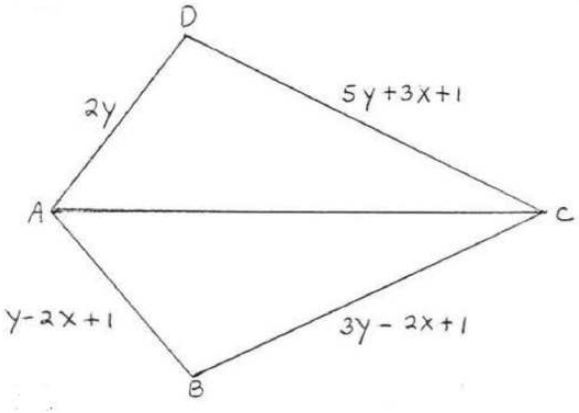
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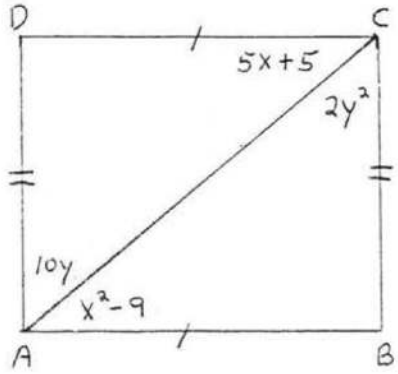
12.


D
C
X
7
A
B

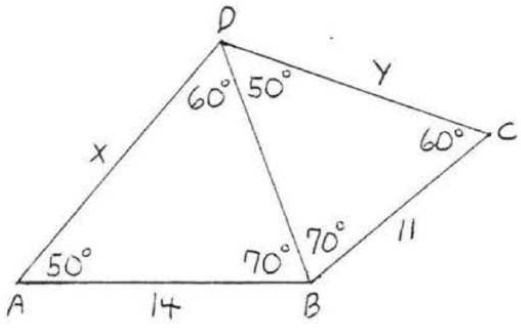
13.


D
2y
5y+3x+1
A
C
y-2x+1
3y - 2x+1
B

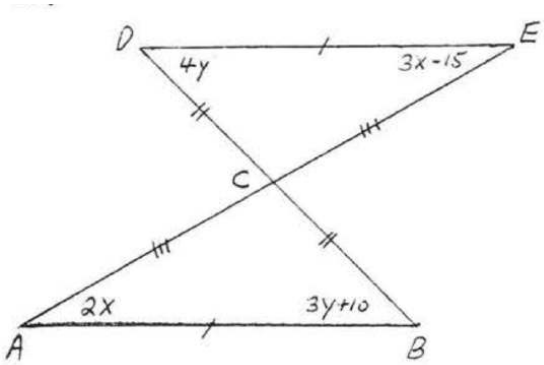
14.


D
C
5x+5
2y²
10y
x²-9
A
B

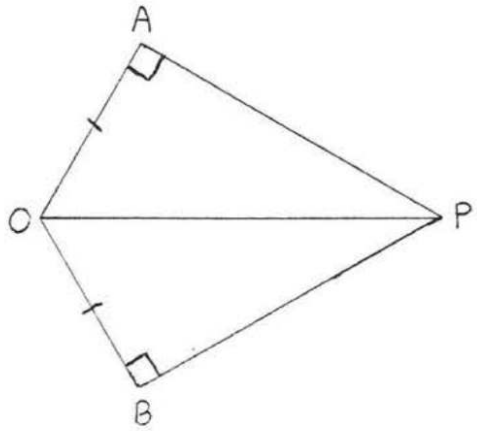
15.


D
60°
50°
Y
X
60°
C
50°
70°
70°
11
A
14
B

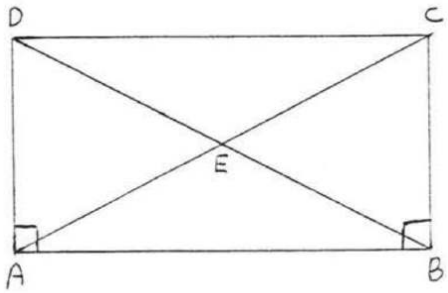
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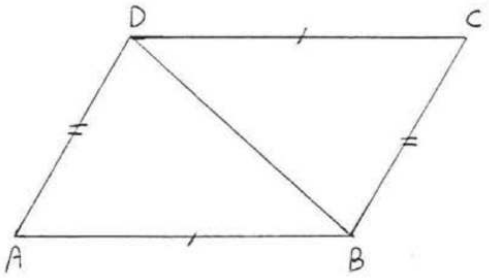
17. Given $CA = CB$ and $\angle A = \angle B = 90°$. Prove $AP = BP$.



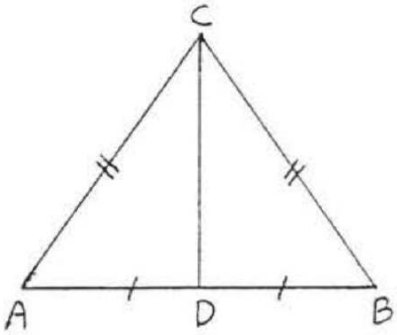
18. Given $AC = BD$ and $\angle A = \angle B = 90°$. Prove $AD = BC$. (Hint: Show $\triangle ABC \cong \triangle BAD$)



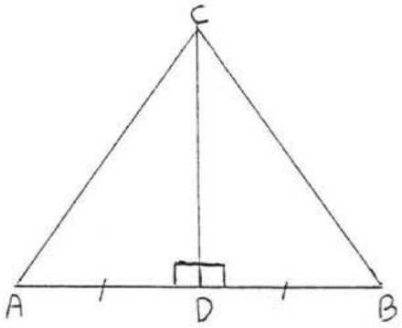
19. Given $AB = CD$ and $AD = CB$. Prove $\angle A = \angle C$.



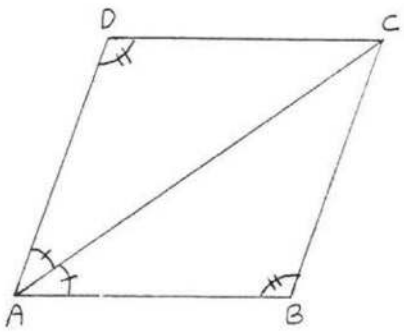
20. Given $AC = BC$ and $AD = BD$. Prove $\angle A = \angle B$.



21. Given $AD = BD$ and $AB \perp CD$. Prove $\angle A = \angle B$.



22. Given $\angle BAC = \angle DAC$ and $\angle B = \angle D$. Prove $AB = AD$.



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