HAWAII COMMUNITY COLLEGE MA82X TEXTBOOK



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Licensing

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CHAPTER OVERVIEW

1: Real Numbers and Their Operations

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1.2: Adding and Subtracting Integers
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1.1: Real numbers and the Number Line

Learning Objectives

- Construct a number line and graph points on it.
- Use a number line to determine the order of real numbers.
- Determine the opposite of a real number.
- Determine the absolute value of a real number.

Definitions

A **set** is a collection of objects, typically grouped within braces { }, where each object is called an **element**. For example, {red, green, blue} is a set of colors. A **subset** is a set consisting of elements that belong to a given set. For example, {green, blue} is a subset of the color set above. A set with no elements is called the **empty set** and has its own special notation, { } or $\varnothing$.

When studying mathematics, we focus on special sets of numbers. The set of **natural (or counting) numbers**, denoted $\mathbb{N}$, is

$$\{1, 2, 3, 4, 5, \ldots\} \quad Natural\ Numbers$$

The three periods $(\ldots)$ is called an ellipsis and indicates that the numbers continue without bound. The set of **whole numbers**, denoted $\mathbb{W}$, is the set of natural numbers combined with zero.

$$\{0, 1, 2, 3, 4, 5, \ldots\} \quad Whole\ Numbers$$

The set of **integers**, denoted $\mathbb{Z}$, consists of both positive and negative whole numbers, as well as zero.

$$\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\} \quad Integers$$

Notice that the sets of natural and whole numbers are both subsets of the set of integers.

Rational numbers, denoted $\mathbb{Q}$, are defined as any number of the form $\frac{a}{b}$, where a and b are integers and b is nonzero. Decimals that repeat or terminate are rational. For example,

$$0.7 = \tfrac{7}{10} \quad \text{and} \quad 0.\overline{3} = 0.3333\cdots = \tfrac{1}{3}$$

The set of integers is a subset of the set of rational numbers because every integer can be expressed as a ratio of the integer and 1. In other words, any integer can be written over 1 and can be considered a rational number. For example,

$$5 = \tfrac{5}{1}$$

Irrational numbers are defined as any number that cannot be written as a ratio of two integers. Nonterminating decimals that do not repeat are irrational. For example,

$$\pi = 3.14159\ldots \quad \text{and} \quad \sqrt{2} = 1.41421\ldots$$

The set of **real numbers**, denoted $\mathbb{R}$, is defined as the set of all rational numbers combined with the set of all irrational numbers. Therefore, all the numbers defined so far are subsets of the set of real numbers. In summary,

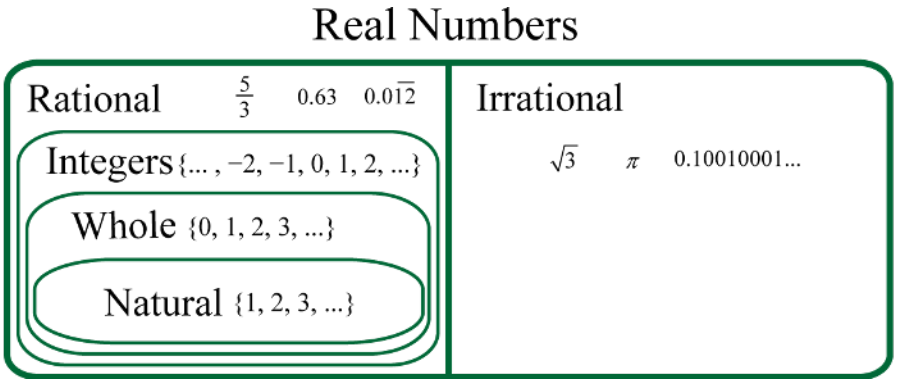


Figure 1.1.1: Real Numbers

Number Line

A **real number line**, or simply number line, allows us to visually display real numbers by associating them with unique points on a line. The real number associated with a point is called a **coordinate**. A point on the real number line that is associated with a coordinate is called its **graph**.

To construct a number line, draw a horizontal line with arrows on both ends to indicate that it continues without bound. Next, choose any point to represent the number zero; this point is called the **origin**.

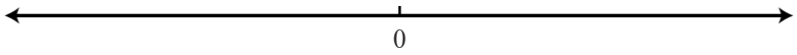


Figure 1.1.2

Mark off consistent lengths on both sides of the origin and label each tick mark to define the scale. Positive real numbers lie to the right of the origin and negative real numbers lie to the left. The number zero (0) is neither positive nor negative. Typically, each tick represents one unit.

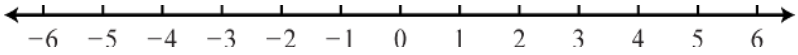


Figure 1.1.3

As illustrated below, the scale need not always be one unit. In the first number line, each tick mark represents two units. In the second, each tick mark represents $\frac{1}{7}$ of a unit.

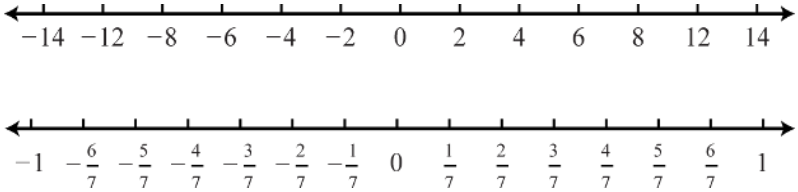


Figure 1.1.4

The graph of each real number is shown as a dot at the appropriate point on the number line. A partial graph of the set of integers $\mathbb{Z}$ follows:

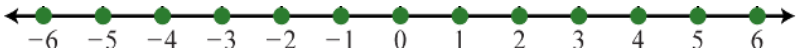


Figure 1.1.5

Example 1.1.1

Graph the following set of real numbers:

Solution

Graph the numbers on a number line with a scale where each tick mark represents $\frac{1}{2}$ of a unit.

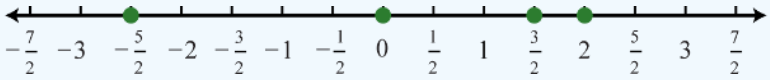


Figure 1.1.6

Ordering Real Numbers

When comparing real numbers on a number line, the larger number will always lie to the right of the smaller one. It is clear that 15 is greater than 5, but it may not be so clear to see that -1 is greater than -5 until we graph each number on a number line.

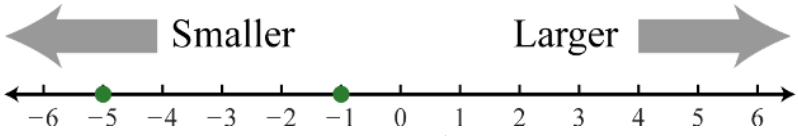


Figure 1.1.7

We use symbols to help us efficiently communicate relationships between numbers on the number line. The symbols used to describe an **equality relationship** between numbers follow:

$=$ *is equal to*
$\neq$ *is not equal to*
$\approx$ *is approximately equal to*

These symbols are used and interpreted in the following manner:

$5 = 5$ *5 is equal to 5*
$0 \neq 5$ *0 is not equal to 5*
$\pi \approx 3.14$ *pi is approximately equal to 3.14*

We next define symbols that denote an **order relationship** between real numbers.

$<$ *Less than*
$>$ *Greater than*
$\leq$ *Less than or equal to*
$\geq$ *Greater than or equal to*

These symbols allow us to compare two numbers. For example,

Since the graph of -120 is to the left of the graph of -10 on the number line, that number is less than -10. We could write an equivalent statement as follows:

Similarly, since the graph of zero is to the right of the graph of any negative number on the number line, zero is greater than any negative number.

The symbols $<$ and $>$ are used to denote **strict inequalities**, and the symbols and are used to denote **inclusive inequalities**. In some situations, more than one symbol can be correctly applied. For example, the following two statements are both true:

In addition, the "or equal to" component of an inclusive inequality allows us to correctly write the following:

The logical use of the word "or" requires that only one of the conditions need be true: the "less than" or the "equal to."

Fill in the blank with $<$, $=$, or $>$: -2 ____ -12.

Solution

Use > because the graph of -2 is to the right of the graph of -12 on a number line. Therefore, $-2 > -12$, which reads "negative two is greater than negative twelve."

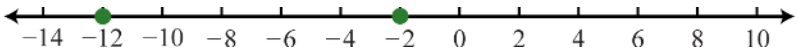


Figure 1.1.8

Answer:

$-2 > -12$

In this text, we will often point out the equivalent notation used to express mathematical quantities electronically using the standard symbols available on a keyboard. We begin with the equivalent textual notation for inequalities:

$\geq$ ">="
$\leq$ "<="
$\neq$ "!="

Many calculators, computer algebra systems, and programming languages use this notation.

Opposites

The **opposite** of any real number a is $-a$. Opposite real numbers are the same distance from the origin on a number line, but their graphs lie on opposite sides of the origin and the numbers have opposite signs.

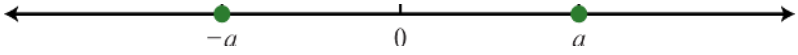


Figure 1.1.9

For example, we say that the opposite of 10 is -10.

Next, consider the opposite of a negative number. Given the integer -7, the integer the same distance from the origin and with the opposite sign is $+7$, or just 7.



Figure 1.1.10

Therefore, we say that the opposite of -7 is $-(-7)=7$. This idea leads to what is often referred to as the **double-negative property**. For any real number a,

$$-(-a)=a$$

Example 1.1.3

What is the opposite of $-\frac{3}{4}$?

Solution

Here we apply the double-negative property.

$$-(-\tfrac{3}{4})=\tfrac{3}{4}$$

Example 1.1.4

Simplify $-(-(4))$

Solution

Start with the innermost parentheses by finding the opposite of $+4$.

$$\begin{aligned}-(-(4)) &= -(-(4)) \\ &= -(-4) \\ &= 4\end{aligned}$$

Answer

4

Example 1.1.5

Simplify: $-(-(-2))$.

Solution

Apply the double-negative property starting with the innermost parentheses.

$$\begin{aligned}-(-(-2)) &= -(-(-2)) \\ &= -(2) \\ &= -2\end{aligned}$$

Answer

-2

tip

If there is an even number of consecutive negative signs, then the result is positive. If there is an odd number of consecutive negative signs, then the result is negative.

Try this!

? Exercise 1.1.1

Simplify: $-(-(-(5)))$.

Answer

-5

Procedure:

$$\begin{aligned} -(-(-(5))) &= -(-(-(5))) \\ &= -(-(-5)) \\ &= -(5) \\ &= -5 \end{aligned}$$

Video Solution:

(click to see video)

Absolute Value

The **absolute value** of a real number a, denoted $|a|$, is defined as the distance between zero (the origin) and the graph of that real number on the number line. Since it is a distance, it is always positive. For example,

$$|-4| = 4 \quad \text{and} \quad |4| = 4$$

Both 4 and -4 are four units from the origin, as illustrated below:

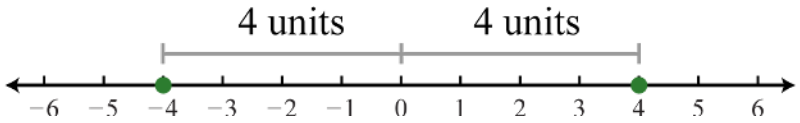


Figure 1.1.11

✓ Example 1.1.6

Simplify:

a. $|-12|$

b.$|12|$

Solution

Both -12 and 12 are twelve units from the origin on a number line. Therefore,

$$|-12| = 12 \quad \text{and} \quad |12| = 12$$

Answer

a.12 b.12

Also, it is worth noting that

$$|0| = 0$$

The absolute value can be expressed textually using the notation $\text{abs}(a)$. We often encounter negative absolute values, such as $-|3|$ or $-\text{ abs}(3)$. Notice that the negative sign is in front of the absolute value symbol. In this case, work the absolute value first and then find the opposite of the result.

Try not to confuse this with the double-negative property, which states that $-(-7) = +7$.

Simplfy: $-|-(-7)|$.

Solution

First, find the opposite of -7 inside the absolute value. Then find the opposite of the result.

$$\begin{aligned} -|-(-7)| &= -|7| \\ &= -7 \end{aligned}$$

Answer

-7

At this point, we can determine what real numbers have a particular absolute value. For example,

$$|?| = 5$$

Think of a real number whose distance to the origin is 5 units. There are two solutions: the distance to the right of the origin and the distance to the left of the origin, namely, $\{\pm 5\}$. The symbol $(\pm)$ is read "plus or minus" and indicates that there are two answers, one positive and one negative.

$$|-5| = 5 \quad \text{and} \quad |5| = 5$$

Now consider the following:

$$|?| = -5$$

Here we wish to find a value for which the distance to the origin is negative. Since negative distance is not defined, this equation has no solution. If an equation has no solution, we say the solution is the empty set: $\varnothing$.

Key Takeaways

- Any real number can be associated with a point on a line.
- Create a number line by first identifying the origin and marking off a scale appropriate for the given problem.
- Negative numbers lie to the left of the origin and positive numbers lie to the right.
- Smaller numbers always lie to the left of larger numbers on the number line.
- The opposite of a positive number is negative and the opposite of a negative number is positive.
- The absolute value of any real number is always positive because it is defined to be the distance from zero (the origin) on a number line.
- The absolute value of zero is zero.

? Exercise 1.1.2

Use set notation to list the described elements.

1. The hours on a clock.
2. The days of the week.
3. The first ten whole numbers.
4. The first ten natural numbers.
5. The first five positive even integers.
6. The first five positive odd integers.

Answer

1. $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$
3. $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$
5. $\{2, 4, 6, 8, 10\}$

Exercise 1.1.3

Determine whether the following real numbers are integers, rational, or irrational.

1. 12
2. -3
3. 4.5
4. -5
5. $0.3\overline{6}$
6. $0.\overline{3}$
7. $1.001000100001\ldots$
8. $1.00\overline{1}$
9. $e = 2.71828\ldots$
10. $\sqrt{7} = 2.645751\ldots$
11. -7
12. 3.14
13. 227
14. 1.33
15. 0
16. $8,675,309$

Answer

1: Integer, Rational

3: Rational

5: Rational

7: Irrational

9: Irrational

11: Integer, Rational

13: Rational

15: Integer, Rational

Exercise 1.1.4

True or false.

1. All integers are rational numbers.
2. All integers are whole numbers.
3. All rational numbers are whole numbers.
4. Some irrational numbers are rational.
5. All terminating decimal numbers are rational.
6. All irrational numbers are real.

Answer

1: True

3: False

5: True

Exercise 1.1.5

Choose an appropriate scale and graph the following sets of real numbers on a number line.

1. $\{-3, 0, 3\}$
2. $\{-2, 2, 4, 6, 8, 10\}$
3. $\{-2,\ -1/3,\ 2/3,\ 5/3\}$
4. $\{-5/2,\ -1/2,\ 0,\ 1/2,\ 2\}$
5. $\{-5/7,\ 0,\ 2/7,\ 1\}$
6. $\{-5,\ -2,\ -1,\ 0\}$
7. $\{-3,\ -2,\ 0,\ 2,\ 5\}$
8. $\{-2.5, -1.5, 0, 1, 2.5\}$
9. $\{0, 0.3, 0.6, 0.9, 1.2\}$
10. $\{-10, 30, 50\}$
11. $\{-6, 0, 3, 9, 12\}$
12. $\{-15, -9, 0, 9, 15\}$

Answer

1. $\{-3, 0, 3\}$

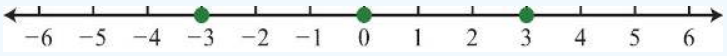


Figure 1.1.12

3. $\{-2,\ -1/3,\ 2/3,\ 5/3\}$

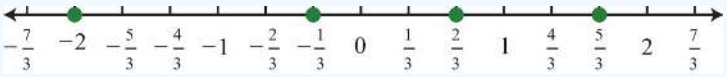


Figure 1.1.13

5. $\{-5/7,\ 0,\ 2/7,\ 1\}$

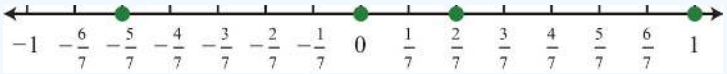


Figure 1.1.14

7. $\{-3,\ -2,\ 0,\ 2,\ 5\}$

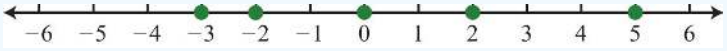


Figure 1.1.15

9. $\{0, 0.3, 0.6, 0.9, 1.2\}$

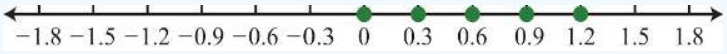


Figure 1.1.16

11. $\{-6, 0, 3, 9, 12\}$

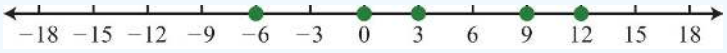


Figure 1.1.17

Exercise 1.1.6

Fill in the blank with $<, =$, or $>$.

1. -7 ___ 0
2. 30 ___ 2
3. 10 ___ -10
4. -150 ___ -75

5. -0.5 ___ -1.5
6. 0 ___ 0
7. -500 ___ 200
8. -1 ___ -200
9. -10 ___ -10
10. -40 ___ -41

Answer

1. $<$
3. $>$
5. $>$
7. $<$
9. $=$

Exercise 1.1.7

True or false.

1. $5 \neq 7$
2. $4 = 5$
3. $1 \neq 1$
4. $-5 > -10$
5. $4 \leq 4$
6. $-12 \geq 0$
7. $-10 = -10$
8. $3 > 3$
9. $-1000 < -20$
10. $0 = 0$

Answer

1. True
3. False
5. True
7. True
9. True

Exercise 1.1.8

List the numbers.

1. List three integers less than -5.
2. List three integers greater than -10.
3. List three rational numbers less than zero.
4. List three rational numbers greater than zero.
5. List three integers between -20 and -5.
6. List three rational numbers between 0 and 1.

Answer

1. $-10, -7, -6$ (answers may vary)
3. $-1, -2/3, -1/3$(answers may vary)

5. $-15, -10, -7$ (answers may vary)

Exercise 1.1.9

Translate each statement into an English sentence.

1. $10 < 20$
2. $-50 \leq -10$
3. $-4 \neq 0$
4. $30 \geq -1$
5. $0 = 0$
6. $e \approx 2.718$

Answer

1. Ten is less than twenty.
3. Negative four is not equal to zero.
5. Zero is equal to zero.

Exercise 1.1.10

Translate the following into a mathematical statement.

1. Negative seven is less than zero.
2. Twenty-four is not equal to ten.
3. Zero is greater than or equal to negative one.
4. Four is greater than or equal to negative twenty-one.
5. Negative two is equal to negative two.
6. Negative two thousand is less than negative one thousand.

Answer

1. $-7 < 0$
3. $0 \geq -1$
5. $-2 = -2$

Exercise 1.1.11

Simplify.

1. $-(-9)$
2. $-(-35)$
3. $-(10)$
4. $-(3)$
5. $-(5)$
6. $-(34)$
7. $-(-1)$
8. $-(-(-1))$
9. $-(-(1))$
10. $-(-(-3))$
11. $-(-(-(-11)))$

Answer

1. 9

3. -10

5. -5

7. 1

9. 1

11. 11

Exercise 1.1.12

Answer the following questions.

1. What is the opposite of -12
2. What is the opposite of π ?
3. What is the opposite -0.01?
4. Is the opposite of -12 smaller or larger than -11?
5. Is the opposite of 7 smaller or larger than -6?

Answer

2. $-\pi$

4. Larger

Exercise 1.1.13

Fill in the blank with $<, =$, or $>$.

1. -7 ___ $-(-8)$
2. 6 ___ $-(6)$
3. 13 ___ $-(-12)$
4. $-(-5)$ ___ $-(-2)$
5. -100 ___ $-(-(-50))$
6. 44 ___ $-(-44)$

Answer

1. $<$

3. $>$

5. $<$

Exercise 1.1.14

Simplify.

1. $|20|$
2. $|-20|$
3. $|-33|$
4. $|-0.75|$
5. $|-\frac{3}{5}|$
6. $|38|$
7. $|0|$
8. $|1|$
9. $-|12|$
10. $-|-20|$
11. $-|20|$

12. $-|-8|$
13. $-|7|$
14. $-|-316|$
15. $-(-|\frac{8}{9}|)$
16. $|-(-2)|$
17. $-|-(-3)|$
18. $-(-|5|)$
19. $-(-|-45|)$
20. $-|-(-21)|$
21. $\text{abs}(6)$
22. $\text{abs}(-7)$
23. $-\text{abs}(5)$
24. $-\text{abs}(-19)$
25. $-(-\text{abs}(9))$
26. $-\text{abs}(-(-12))$

Answer

1. 20
3. 33
5. $\frac{3}{5}$
7. 0
9. -12
11. -20
13. -7
15. $\frac{8}{9}$
17. -3
19. 45
21. 6
23. -5
25. 9

Exercise 1.1.15

Determine the unknown.

1. $|\ ?\ | = 9$
2. $|\ ?\ | = 15$
3. $|\ ?\ | = 0$
4. $|\ ?\ | = 1$
5. $|\ ?\ | = -8$
6. $|\ ?\ | = -20$
7. $|?| - 10 = -2$
8. $|?| + 5 = 14$

Answer

1. ± 9
3. 0

https://math.libretexts.org/@go/page/61343

5. $\varnothing$, No solution

7. ± 8

Exercise 1.1.16

Fill in the blank with $<, =$, or $>$.

1. $|-2|$ ____ 0
2. $|-7|$ ____ $|-10|$
3. -10 ____ $-|-2|$
4. $|-6|$ ____ $|-(-6)|$
5. $-|3|$ ____ $|-(-5)|$
6. 0 ____ $-|-(-4)|$

Answer

1. $>$

3. $<$

5. $<$

Exercise 1.1.17

Discussion Board Topics.

1. Research and discuss the history of the number zero.
2. Research and discuss the various numbering systems throughout history.
3. Research and discuss the definition and history of π.
4. Research the history of irrational numbers. Who is credited with proving that the square root of 2 is irrational and what happened to him?
5. Research and discuss the history of absolute value.
6. Discuss the “just make it positive” definition of absolute value

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1.2: Adding and Subtracting Integers

Learning Objectives

- Add and subtract signed integers.
- Translate English sentences involving addition and subtraction into mathematical statements.
- Calculate the distance between two numbers on a number line.

Addition and Subtraction $(+, -)$

Visualize adding $3+2$ on the number line by moving from zero three units to the right then another two units to the right, as illustrated below:

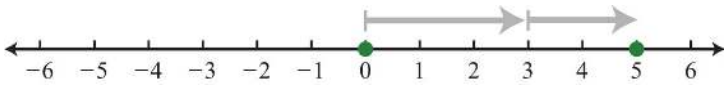


Figure 1.2.1

The illustration shows that $3+2=5$. Similarly, visualize adding two negative numbers $(-3)+(-2)$ by first moving from the origin three units to the left and then moving another two units to the left.

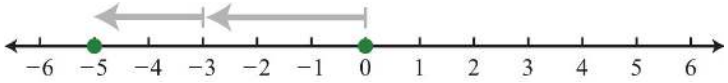


Figure 1.2.2

In this example, the illustration shows $(-3)+(-2)=-5$, which leads to the following two properties of real numbers.

$$\begin{aligned} &\textit{positive number} + \textit{positive number} = \textit{positive number} \\ &\textit{negative number} + \textit{negative number} = \textit{negative number} \end{aligned}$$

Next, we will explore addition of numbers with unlike signs. To add $3+(-7)$, first move from the origin three units to the right, then move seven units to the left as shown:

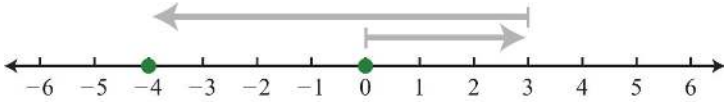


Figure 1.2.3

In this case, we can see that adding a negative number is equivalent to subtraction:

$$3+(-7)=3-7=-4$$

It is tempting to say that a positive number plus a negative number is negative, but that is not always true: $7+(-3)=7-3=4$. The result of adding numbers with unlike signs may be positive or negative. The sign of the result is the same as the sign of the number with the greatest distance from the origin. For example, the following results depend on the sign of the number 12 because it is farther from zero than 5:

$$\begin{aligned} 12+(-5)&=7 \\ -12+5&=-7 \end{aligned}$$

Example 1.2.1

Simplify: $14+(-25)$.

Solution

Here -25 is the greater distance from the origin. Therefore, the result is negative.

$$\begin{aligned} 14+(-25) &= 14-25 \\ &= -11 \end{aligned}$$

Answer

-11

Properties of Addition

Given any real numbers a, b, and c, we have the following properties of addition:

- **Additive Identity Property**:

$$a+0=0+a=a \tag{1.2.1}$$

- **Additive Inverse Property**:

$$a+(-a)=(-a)+a=0 \tag{1.2.2}$$

- **Associative Property**:

$$(a+b)+c=a+(b+c) \tag{1.2.3}$$

- **Commutative Property**:

$$a+b=b+a \tag{1.2.4}$$

Below are some examples of these properties in action.

Example 1.2.2

Simplify:

a. $5+0$

b. $10+(-10)$

Solution

a. Adding zero to any real number results in the same real number.

$$5+0=5$$

b. Adding opposites results in zero.

Answer

a. 5; b. 0

Simplify:

a. $(3+7)+4$

b. $3+(7+4)$

Solution

Parentheses group the operations that are to be performed first.

a.

$$\begin{aligned}(3+7)+4 &= 10+4\\ &= 14\end{aligned}$$

b.

$$\begin{aligned}3+(7+4) &= 3+10\\ &= 14\end{aligned}$$

These two examples both result in 14: changing the grouping of the numbers does not change the result.

$$(3+7)+4=3+(7+4)=14$$

Answer

a. 14; b. 14

At this point, we highlight the fact that addition is commutative: the order in which we add does not matter and yields the same result.

$$\begin{aligned} 2+9 &= 9+2 \\ 11 &= 11 \end{aligned}$$

On the other hand, subtraction is not commutative.

$$\begin{aligned} 2-9 &\neq 9-2 \\ -7 &\neq 7 \end{aligned}$$

We will use these properties, along with the double-negative property for real numbers, to perform more involved sequential operations. To simplify things, we will make it a general rule to first replace all sequential operations with either addition or subtraction and then perform each operation in order from left to right.

Example 1.2.4

Simplify: $4-(-10)+(-5)$.

Solution

Replace the sequential operations and then perform them from left to right.

$$\begin{aligned} 4-(-10)+(-5) &= 4+10-5 && \textit{Replace } -(-) \textit{ with addition } (+). \\ & && \textit{Replace } +(-) \textit{ with addition } (-). \\ &= 14-5 \\ &= 9 \end{aligned}$$

Answer

9

Example 1.2.5

Simplify: $-3+(-8)-(-7)$.

Solution

$$\begin{aligned} -3+(-8)-(-7) &= -3-8+7 && \textit{Replace } +(-) \textit{ with } (-). \\ & && \textit{Replace } -(-) \textit{ with } (+). \\ &= -11+7 \\ &= -4 \end{aligned}$$

Answer

-4

Example 1.2.6

Try this!

Simplify: $12-(-9)+(-6)$.

Solution

(click to see video)

Often we find the need to translate English sentences involving addition and subtraction to mathematical statements. Listed below are some key words that translate to the given operation.

Table 1.2.2

Key Words	Operation
Sum, increased by, more than, plus, added to, total	$+$
Difference, decreased by, subtracted from, less, minus	$-$

Example 1.2.7

What is the difference of 7 and -3?

Solution

The key word "difference" implies that we should subtract the numbers.

$$\begin{aligned} 7-(-3) &= 7+3 \\ &= 10 \end{aligned}$$

Answer

The difference of 7 and -3 is 10.

Example 1.2.8

What is the sum of the first five positive integers?

Solution

The initial key word to focus on is "sum"; this means that we will be adding the five numbers. The first five positive integers are $\{1, 2, 3, 4, 5\}$ Recall that 0 is neither positive nor negative.

$$1+2+3+4+5=15$$

Answer

The sum of the first five positive integers is 15.

Example 1.2.9

What is 10 subtracted from the sum of 8 and 6?

Solution

We know that subtraction is not commutative; therefore, we must take care to subtract in the correct order. First, add 8 and 6 and then subtract 10 as follows:

$$\overbrace{(8 \quad + \quad 6)}^{\textit{the sum of 8 and 6}} \qquad \overbrace{- \qquad 10}^{\textit{subtract 10 from the sum}}$$

It is important to notice that the phrase "10 subtracted from" does not translate to a mathematical statement in the order it appears. In other words, $10-(8+6)$ would be an incorrect translation and leads to an incorrect answer. After translating the sentence, perform the operations.

$$\begin{aligned} (8+6)-10 &= 14-10 \\ &= 4 \end{aligned}$$

Answer

Ten subtracted from the sum of 8 and 6 is 4.

Distance on a Number Line

One application of the absolute value is to find the distance between any two points on a number line. For real numbers a and b, the **distance formula for a number line** is given as,

$$d = |b - a|$$

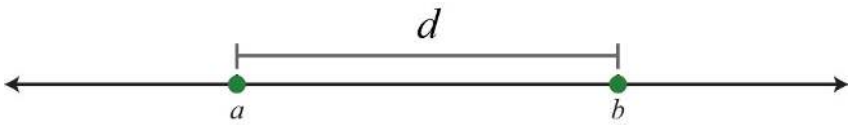


Figure 1.2.4

Example 1.2.10

Determine the distance between 2 and 7 on a number line.

Solution

On the graph we see that the distance between the two given integers is 5 units.

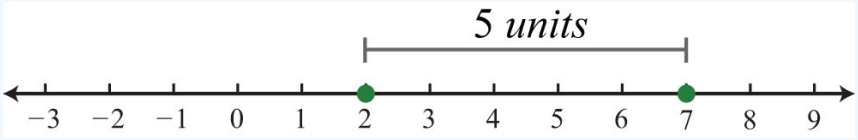


Figure 1.2.5

Using the distance formula we obtain the same result.

$$\begin{aligned} d &= |7 - 2| \\ &= |5| \\ &= 5 \end{aligned}$$

Answer

5 units

Example 1.2.11

Determine the distance between -4 and 7 on a number line.

Solution

Use the distance formula for a number line $d = |b - a|$, where $a = -4$ and $b = 7$.

$$\begin{aligned} d &= |7 - (-4)| \\ &= |7 + 4| \\ &= |11| \\ &= 11 \end{aligned}$$

Answer

11 units

It turns out that it does not matter which points are used for a and$b$; the absolute value always ensures a positive result.

Table 1.2.3

Using $a = -4$ and $b = 7$	**Using $a = 7$ and $b = -4$**
$d = \|7 - (-4)\| = \|7 + 4\| = \|11\| = 11$	$d = \|-4 - 7\| = \|-11\| = 11$

? Exercise 1.2.1

Determine the distance between -12 and -9 on the number line.

Answer

3

Video Solution

(click to see video)

Key Takeaways

- A positive number added to a positive number is positive. A negative number added to a negative number is negative.
- The sign of a positive number added to a negative number is the same as the sign of the number with the greatest distance from the origin.
- Addition is commutative and subtraction is not.
- When simplifying, it is a best practice to first replace sequential operations and then work the operations of addition and subtraction from left to right.
- The distance between any two numbers on a number line is the absolute value of their difference. In other words, given any real numbers a and b, use the formula $d = |b - a|$ to calculate the distance d between them.

? Exercise 1.2.2

Add and subtract.

1. $24 + (-18)$
2. $9 + (-11)$
3. $-31 + 5$
4. $-12 + 15$
5. $-30 + (-8)$
6. $-50 + (-25)$
7. $-7 + (-7)$
8. $-13 - (-13)$
9. $8 - 12 + 5$
10. $-3 - 7 + 4$
11. $-1 - 2 - 3 - 4$
12. $6 - (-5) + (-10) - 14$
13. $-5 + (-3) - (-7)$
14. $2 - 7 + (-9)$
15. $-30 + 20 - 8 - (-18)$
16. $10 - (-12) + (-8) - 20$
17. $5 - (-2) + (-6)$
18. $-3 + (-17) - (-13)$
19. $-10 + (-12) - (-20)$
20. $-13 + (-5) - (-25)$
21. $20 - (-4) - (-5)$
22. $17 + (-12) - (-2)$

Answer

1: 6

3: −26

5: −38

7: −14

9: 1

11: -10

13: -1

15: 0

17: 1

19: -2

21: 29

Exercise 1.2.3

Translate each sentence to a mathematical statement and then simplify.

1. Find the sum of 3, 7, and -8.
2. Find the sum of -12, -5, and 7.
3. Determine the sum of the first ten positive integers.
4. Determine the sum of the integers in the set $\{-2, -1, 0, 1, 2\}$.
5. Find the difference of 10 and 6.
6. Find the difference of 10 and -6.
7. Find the difference of -16 and -5.
8. Find the difference of -19 and 7.
9. Subtract 12 from 10.
10. Subtract -10 from -20.
11. Subtract 5 from -31.
12. Subtract -3 from 27.
13. Two less than 8.
14. Five less than -10.
15. Subtract 8 from the sum of 4 and 7.
16. Subtract -5 from the sum of 10 and -3.
17. Subtract 2 from the difference of 8 and 5.
18. Subtract 6 from the difference of -1 and 7.
19. Mandy made a $\$200$ deposit into her checking account on Tuesday. She then wrote 4 checks for $\$50.00$, $\$125.00$, $\$60.00$, and $\$45.00$. How much more than her deposit did she spend?
20. The quarterback ran the ball three times in last Sunday's football game. He gained 7 yards on one run but lost 3 yards and 8 yards on the other two. What was his total yardage running for the game?
21. The revenue for a local photographer for the month is $\$1,200$. His costs include a studio rental of $\$600$, props costing $\$105$, materials fees of $\$135$, and a make-up artist who charges $\$120$. What is his total profit for the month?
22. An airplane flying at $30,000$ feet lost $2,500$ feet in altitude and then rose $1,200$ feet. What is the new altitude of the plane?
23. The temperature was 22° at $6:00$ p.m. and dropped 26° by midnight. What was the temperature at midnight?
24. A nurse has 30 milliliters of saline solution but needs 75 milliliters of the solution. How much more does she need?
25. The width of a rectangle is 2 inches less than its length. If the length measures 16 inches, determine the width.
26. The base of a triangle is 3 feet shorter than its height. If the height measures 5 feet, find the length of the base.

Answer

1: 2

3: 55

5: 4

7: -11

9: -2

11: -36

13: 6

15: 3

17: 1

19: $80

21: $240

23: -4°

25: 14 inches

Exercise 1.2.4

Find the distance between the given numbers on a number line.

1. -3 and 12
2. 8 and -13
3. -25 and -10
4. -100 and -130
5. -7 and -20
6. 0 and -33
7. -10 and 10
8. -36 and 36
9. The coldest temperature on earth, -129°F, was recorded in 1983 at Vostok Station, Antarctica. The hottest temperature on earth, 136°F, was recorded in 1922 at Al 'Aziziyah, Libya. Calculate earth's temperature range.
10. The daily high temperature was recorded as 91°F and the low was recorded as 63°F. What was the temperature range for the day?
11. A student earned 67 points on his lowest test and 87 points on his best. Calculate his test score range.
12. On a busy day, a certain website may have $12,500$ hits. On a slow day, it may have as few as 750 hits. Calculate the range of the number of hits.

Answer

1: 15 units

3: 15 units

5: 13 units

7: 20 units

9: 265℉

11: 20 points

Exercise 1.2.5

Discussion Board Topics

1. Share an example of adding signed numbers in a real-world application.
2. Demonstrate the associative property of addition with any three real numbers.
3. Show that subtraction is not commutative

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1.3: Multiplying and Dividing Integers

Learning Objectives

- Multiply and divide signed integers.
- Translate English sentences involving multiplication and division into mathematical statements.
- Determine the prime factorization of composite numbers.
- Interpret the results of quotients involving zero.

Multiplication and Division

We begin with a review of what it means to multiply and divide signed numbers. The result of multiplying real numbers is called the **product** and the result of dividing is called the **quotient**. Recall that multiplication is equivalent to adding:

$$3 \cdot 4 = 4 + 4 + 4 = 12$$

Clearly, the product of two positive numbers is positive. Similarly, the product of a positive number and negative number can be written as shown:

We see that the product of a positive number and a negative number is negative. Next, explore the results of multiplying two negative numbers. Consider the products in the following illustration and try to identify the pattern:

$$\left.\begin{aligned} 3(-3) &= -9 \\ 2(-3) &= -6 \\ 1(-3) &= -3 \end{aligned}\right\} \quad \textit{The products increase by 3.}$$

$$0(-3) = 0 \quad \textit{Zero times any real number is zero.}$$

$$\left.\begin{aligned} (-1)(-3) &= 3 \\ (-2)(-3) &= 6 \\ (-3)(-3) &= 9 \end{aligned}\right\} \quad \textit{The pattern continues by increasing the products by 3.}$$

This shows that the product of two negative numbers is positive. To summarize,

$$\begin{aligned} \textit{positive} \times \textit{positive} &= \textit{positive} \\ \textit{positive} \times \textit{negative} &= \textit{negative} \\ \textit{negative} \times \textit{negative} &= \textit{negative} \end{aligned}$$

The rules for division are the same because division can always be rewritten as multiplication:

The rules for multiplication and division should not be confused with the fact that the sum of two negative numbers is negative.

Example 1.3.1

Simplify:

a. $(-3) + (-5)$

b. $(-3)(-5)$

Solution

Here we add and multiply the same two negative numbers.

a. The result of adding two negative numbers is negative.

$$\begin{aligned} (-3) + (-5) &= -3 - 5 \\ &= -8 \end{aligned}$$

b. The result of multiplying two negative numbers is positive.

$$(-3)(-5) = 15$$

Answer

a, -8 b.15

Given any real numbers a, b, and c, we have the following properties of multiplication:

Zero factor property:	$a \cdot 0 = 0 \cdot a = 0$
Multiplicative identity property:	$a \cdot 1 = 1 \cdot a = a$
Associative property:	$(a \cdot b) \cdot c = a \cdot (b \cdot c)$
Commutative property:	$a \cdot b = b \cdot a$

Example 1.3.2

Simplify:

a. $5 \cdot 0$

b. $10 \cdot 1$

Solution

a. Multiplying by zero results in zero.

$$5 \cdot 0 = 0$$

b. Multiplying any real number by one results in the same real number.

$$10 \cdot 1 = 10$$

Answers:

a. 0 b. 10

Example 1.3.3

Simplify:

a. $(3 \cdot 7) \cdot 2$

b. $3 \cdot (7 \cdot 2)$

Solution

a.

$$\begin{aligned}(3 \cdot 7) \cdot 2 &= 21 \cdot 2 \\ &= 42\end{aligned}$$

b.

$$\begin{aligned}3 \cdot (7 \cdot 2) &= 3 \cdot 14 \\ &= 42\end{aligned}$$

The value of each expression is 42. Changing the grouping of the numbers does not change the result.

$$(3 \cdot 7) \cdot 2 = 3 \cdot (7 \cdot 2) = 42$$

Answer

a. 42 b. 42

At this point, we highlight that multiplication is commutative: the order in which we multiply does not matter and yields the same result.

$$2 \cdot 9 = 9 \cdot 2$$
$$18 = 18$$

On the other hand, division is not commutative.

$$10 \div 5 \neq 5 \div 10$$
$$2 \neq \frac{1}{2}$$

Use these properties to perform sequential operations involving multiplication and division. When doing so, it is important to perform these operations in order from left to right.

Example 1.3.4

Simplify: $3(-2)(-5)(-1)$

Solution

Multiply two numbers at a time as follows:

$$\begin{aligned} 5(-3)(-2)(-4) &= \underbrace{5(-3)}(-2)(-4) \\ &= \underbrace{-15(-2)}(-4) \\ &= \underbrace{30(-4)} \\ &= -120 \end{aligned}$$

Figure 1.3.1

Answer

-120

Because multiplication is commutative, the order in which we multiply does not affect the final answer. When sequential operations involve multiplication and division, order does matter; hence we must work the operations from left to right to obtain a correct result.

Example 1.3.5

Simplify: $10 \div (-2)(-5)$

Solution

Perform the division first; otherwise the result will be incorrect.

Correct!

$$\begin{aligned} 10 \div (-2)(-5) &= \underbrace{10 \div (-2)}_{\textit{division first}}(-5) \\ &= -5(-5) \\ &= 25 \quad \checkmark \end{aligned}$$

Incorrect!

$$\begin{aligned} 10 \div (-2)(-5) &= 10 \div \underbrace{(-2)(-5)}_{\textit{multiplication first}} \\ &= 10 \div 10 \\ &= 1 \quad \times \end{aligned}$$

Figure 1.3.2

Notice that the order in which we multiply and divide does affect the result. Therefore, it is important to perform the operations of multiplication and division as they appear from left to right.

Answer

25

Notice that the order in which we multiply and divide does affect the final result. Therefore, it is important to perform the operations of multiplication and division as they appear from left to right.

Example 1.3.6

Simplify: $-6(3) \div (-2)(-3)$

Solution

Work the operations one at a time from left to right.

$$\begin{aligned} & -6(2) \div (-2)(-3) \\ = & -18 \div (-2)(-3) \\ = & 9(-3) \\ = & -27 \end{aligned}$$

Example 1.3.7

Try this! Simplify: $-5 \div 5 \cdot 2(-3)$

Video Solution:

(click to see video)

Within text-based applications, the symbol used for multiplication is the **asterisk** (*) and the symbol used for division is the forward slash (/).

$5 * 3$ and $14/2 = 7$

The set of **even integers** is the set of all integers that are evenly divisible by 2. We can also obtain the set of even integers by multiplying each integer by 2.

$$\{\ldots, -6, -4, -2, 0, 2, 4, 6, \ldots\} \quad \textit{Even integers}$$

The set of **odd integers** is the set of all integers that are not evenly divisible by 2.

$$\{\ldots, -5, -3, -1, 1, 3, 5, \ldots\} \quad \textit{Odd integers}$$

A **prime number** is an integer greater than 1 that is divisible only by 1 and itself. The smallest prime number is 2 and the rest are necessarily odd.

$$\{2, 3, 5, 7, 11, 13, 17, 19, 23, \ldots\} \quad \textit{Prime numbers}$$

Any integer greater than 1 that is not prime is called a **composite number** and can be written as a product of primes. When a composite number, such as 30, is written as a product, $30 = 2 \cdot 15$, we say that $2 \cdot 15$ is a **factorization** of 30 and that 2 and 15 are **factors**. Note that factors divide the number evenly. We can continue to write composite factors as products until only a product of primes remains.

$$\begin{aligned} & 30 \\ = & \ 2 \cdot 15 \\ = & \ 2 \cdot 3 \cdot 5 \end{aligned}$$

Figure 1.3.3

The **prime factorization** of 30 is $2 \cdot 3 \cdot 5$.

Example 1.3.8

Determine the prime factorization of 70.

Solution

Begin by writing 70 as a product with 2 as a factor. Then express any composite factor as a product of prime numbers.

$$\begin{aligned} 70 &= 2 \cdot 35 \\ &= 2 \cdot 5 \cdot 7 \end{aligned}$$

Since the prime factorization is unique, it does not matter how we choose to initially factor the number because the end result is the same.

$$\begin{aligned} 70 &= 7 \cdot 10 \\ &= 7 \cdot 2 \cdot 5 \\ &= 2 \cdot 5 \cdot 7 \end{aligned}$$

Answer

The prime factorization of 70 is $2 \cdot 5 \cdot 7$.

Some tests (called divisibility tests) useful for finding prime factors of composite numbers follow:

1. If the integer is even, then 2 is a factor.
2. If the sum of the digits is evenly divisible by 3, then 3 is a factor.
3. If the last digit is a 5 or 0, then 5 is a factor.

Often we find the need to translate English sentences that include multiplication and division terms to mathematical statements. Listed below are some key words that translate to the given operation.

Key Words	Operation
Product, multiplied by, of, times	* or ·
Quotient, divided by, ratio, per	/ or ÷

Example 1.3.9

Calculate the quotient of 20 and -10.

Solution

The key word “quotient” implies that we should divide.

$$20 \div (-10) = -2$$

Answer:

The quotient of 20 and -10 is -2.

Example 1.3.10

What is the product of the first three positive even integers?

Solution

The first three positive even integers are {2, 4, 6} and the key word “product” implies that we should multiply.

$$\begin{aligned} 2 \cdot 4 \cdot 6 &= 8 \cdot 6 \\ &= 48 \end{aligned}$$

Answer

The product of the first three positive even integers is 48.

Example 1.3.11

Joe is able to drive 342 miles on 18 gallons of gasoline. How many miles per gallon of gas is this?

Solution

The key word "per" indicates that we must divide the number of miles driven by the number of gallons used:

$$\frac{342 \text{ miles}}{18 \text{ gallons}} = 19 \text{ miles per gallon (mpg)}$$

Answer

Joe gets 19 miles per gallon from his vehicle.

In everyday life, we often wish to use a single value that typifies a set of values. One way to do this is to use what is called the **arithmetic mean** or **average**. To calculate an average, divide the sum of the values in the set by the number of values in that set.

Example 1.3.12

A student earns 75, 86, and 94 on his first three exams. What is the student's test average?

Solution

Add the scores and divide the sum by 3.

$$\begin{aligned} \frac{75+86+94}{3} &= \frac{255}{3} \\ &= 85 \end{aligned}$$

Answer

The student's test average is 85.

Zero and Division

Recall the relationship between multiplication and division:

$$\underbrace{12}_{\text{dividend}} \div \underbrace{6}_{\text{divisor}} = \underbrace{2}_{\text{quotient}} \quad \text{because} \quad 6 \cdot 2 = 12$$

Figure 1.3.4

In this case, the **dividend** 12 is evenly divided by the **divisor** 6 to obtain the quotient, 2. It is true in general that if we multiply the divisor by the quotient we obtain the dividend. Now consider the case where the dividend is zero and the divisor is nonzero:

$$0 \div 6 = 0 \text{ Since } 6 \cdot 0 = 0$$

This demonstrates that zero divided by any nonzero real number must be zero. Now consider a nonzero number divided by zero:

$$12 \div 0 = ? \text{ or } 0 \cdot ? = 0$$

Here any real number seems to work. For example, $0 \cdot 5 = 0$ and $0 \cdot 3 = 0$. Therefore, the quotient is uncertain or **indeterminate**.

$$0 \div 0 = \frac{0}{0} \; Is\ indeterminate.$$

In this course, we state that $0 \div 0$ is undefined.

Key Takeaways

- A positive number multiplied by a negative number is negative. A negative number multiplied by a negative number is positive.
- Multiplication is commutative and division is not.
- When simplifying, work the operations of multiplication and division in order from left to right.
- Even integers are numbers that are evenly divisible by 2 or multiples of 2, and all other integers are odd.
- A prime number is an integer greater than 1 that is divisible only by 1 and itself.

- Composite numbers are integers greater than 1 that are not prime. Composite numbers can be written uniquely as a product of primes.
- The prime factorization of a composite number is found by continuing to divide it into factors until only a product of primes remains.
- To calculate an average of a set of numbers, divide the sum of the values in the set by the number of values in the set.
- Zero divided by any nonzero number is zero. Any number divided by zero is undefined.

Exercise 1.3.1

Multiply and divide.

1. $5(-7)$
2. $-3(-8)$
3. $2(-4)(-9)$
4. $-3 \cdot 2 \cdot 5$
5. $-12(3)(0)$
6. $0(-12)(-5)$
7. $(-1)(-1)(-1)(-1)$
8. $(-1)(-1)(-1)$
9. $-100 \div 25$
10. $25 \div 5(-5)$
11. $-15(-2) \div 10(-3)$
12. $-5 \cdot 10 \div 2(-5)$
13. $(-3)(25) \div (-5)$
14. $6 * (-3)/(-9)$
15. $20/(-5) * 2$
16. $-50/2 * 5$
17. Determine the product of 11 and -3.
18. Determine the product of -7 and -22.
19. Find the product of 5 and -12.
20. Find the quotient of negative twenty-five and five.
21. Determine the quotient of -36 and 3.
22. Determine the quotient of 26 and -13.
23. Calculate the product of 3 and -8 divided by -2.
24. Calculate the product of -1 and -3 divided by 3.
25. Determine the product of the first three positive even integers.
26. Determine the product of the first three positive odd integers.

Answer

1: -35

3: 72

5: 0

7: 1

9: -4

11: -9

13: 15

15: -8

17: -33

19: -60

21: -12

23: 12

25: 48

Exercise 1.3.2

Determine the prime factorization of the following integers.

1. 105
2. 78
3. 138
4. 154
5. 165
6. 330

Answer

1: $3 \cdot 5 \cdot 7$

3: $2 \cdot 3 \cdot 23$

5: $3 \cdot 5 \cdot 11$

Exercise 1.3.3

Calculate the average of the numbers in each of the following sets.

1. $\{50, 60, 70\}$
2. $\{9, 12, 30\}$
3. $\{3, 9, 12, 30, 36\}$
4. $\{72, 84, 69, 71\}$
5. The first four positive even integers.
6. The first four positive odd integers.

Answer

1: 60

2: 18

5: 5

Exercise 1.3.4

The distance traveled D is equal to the average rate r times the time traveled t at that rate: $D = rt$. Determine the distance traveled given the rate and the time.

1. 60 miles per hour for 3 hours
2. 55 miles per hour for 3 hours
3. 15 miles per hour for 5 hours
4. 75 feet per second for 5 seconds
5. 60 kilometers per hour for 10 hours
6. 60 meters per second for 30 seconds
7. A student club ran a fund-raiser in the quad selling hot dogs. The students sold 122 hot dog meals for $3.00 each. Their costs included $50.00 for the hot dogs and buns, $25.00 for individually wrapped packages of chips, and $35.00 for the sodas. What was their profit?
8. A 230-pound man loses 4 pounds each week for 8 weeks. How much does he weigh at the end of 8 weeks?
9. Mary found that she was able to drive 264 miles on 12 gallons of gas. How many miles per gallon does her car get?

10. After filling his car with gasoline, Bill noted that his odometer reading was $45,346$ miles. After using his car for a week, he filled up his tank with 14 gallons of gas and noted that his odometer read $45,724$ miles. In that week, how many miles per gallon did Bill's car get?

Answer

1: 180 miles

3: 75 miles

5: 600 kilometers

7: $\$256.00$

9: 22 miles per gallon

11: 0

Exercise 1.3.5

Perform the operations.

1. $0 \div 9$
2. $15 \div 0$
3. $4(-7) \div 0$
4. $7(0) \div (-15)$
5. $-5(0) \div 9(0)$
6. $5 \cdot 2(-3)(-5)$
7. $-8 - 5 + (-13)$
8. $-4(-8) \div 16(-2)$
9. $50 \div (-5) \div (-10)$
10. $49 \div 7 \div (-1)$
11. $3 \cdot 4 \div 12$
12. $0 - (-8) - 12$
13. $-8 \cdot 4(-3) \div 2$
14. $0/(-3 * 8 * 5)$
15. $(-4 * 3)/(2 * (-3))$
16. $-16/(-2 * 2) * 3$
17. $-44/11 * 2$
18. $-5 * 3/(-15)$
19. $4 * 3 * 2/6$
20. $-6 * 7/(-2)$
21. During 5 consecutive winter days, the daily lows were -7°, -3°, 0°, -5°, and -10°. Calculate the average low temperature.
22. On a very cold day the temperature was recorded every 4 hours with the following results: -16°, -10°, 2°, 6°, -5°, and -13°. Determine the average temperature.
23. A student earns 9, 8, 10, 7, and 6 points on the first 5 chemistry quizzes. What is her quiz average?
24. A website tracked hits on its homepage over the Thanksgiving holiday. The number of hits for each day from Thursday to Sunday was $12,250$; $4,400$; $7,750$; and $10,200$, respectively. What was the average number of hits per day over the holiday period?

Answer

1: 0

3: Undefined

5: 0

7: -26

9: 1

11: 1

13: 48

15: 2

17: -8

19: 4

21: -5°

23: 8 points

? Exercise 1.3.1

Discussion Board Topics.

1. Demonstrate the associative property of multiplication with any three real numbers.
2. Show that division is not commutative.
3. Discuss the importance of working multiplication and division operations from left to right. Make up an example where order does matter and share the solution.
4. Discuss division involving 0. With examples, explain why the result is sometimes 0 and why it is sometimes undefined.
5. Research and discuss the fundamental theorem of arithmetic.
6. Research and discuss other divisibility tests. Provide an example for each test.
7. The arithmetic mean is one way to typify a set of values. Research other methods used to typify a set of values

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1.4: Fractions

Learning Objectives

- Reduce a fraction to lowest terms.
- Multiply and divide fractions.
- Add and subtract fractions.

Reducing

A **fraction** is a real number written as a quotient, or ratio, of two integers a and b, where $b \neq 0$.



Figure 1.4.1

The integer above the fraction bar is called the **numerator** and the integer below is called the **denominator**. The numerator is often called the "part" and the denominator is often called the "whole." **Equivalent fractions** are two equal ratios expressed using different numerators and denominators. For example,

$$\frac{50}{100} = \frac{1}{2}$$

Fifty parts out of 100 is the same ratio as 1 part out of 2 and represents the same real number. Consider the following factorizations of 50 and 100:

$$\begin{aligned} 50 &= 2 \cdot 25 \\ 100 &= 4 \cdot 25 \end{aligned}$$

The numbers 50 and 100 share the factor 25. A shared factor is called a common factor. We can rewrite the ratio $\frac{50}{100}$ as follows:

$$\frac{50}{100} = \frac{2 \cdot 25}{4 \cdot 25}$$

Making use of the multiplicative identity property and the fact that $\frac{25}{25} = 1$, we have

$$\frac{50}{100} = \frac{2 \cdot \cancel{25}}{4 \cdot \cancel{25}} = \frac{2}{4} \cdot 1 = \frac{2}{4}$$

Figure 1.4.2

Dividing $\frac{25}{25}$ and replacing this factor with a 1 is called **canceling**. Together, these basic steps for finding equivalent fractions define the process of **reducing**. Since factors divide their product evenly, we achieve the same result by dividing both the numerator and denominator by 25 as follows:

$$\frac{50 \div 25}{100 \div 25} = \frac{2}{4}$$

Figure 1.4.3

Finding equivalent fractions where the numerator and denominator have no common factor other than 1 is called **reducing to lowest terms**. When learning how to reduce to lowest terms, it is helpful to first rewrite the numerator and denominator as a product of primes and then cancel. For example,

$$\frac{50}{100} = \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{5}} \cdot \overset{1}{\cancel{5}}}{2 \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{5}} \cdot \underset{1}{\cancel{5}}}$$

$$= \frac{1}{2}$$

Figure 1.4.4

We achieve the same result by dividing the numerator and denominator by the **greatest common factor** (GCF). The GCF is the largest number that divides both the numerator and denominator evenly. One way to find the GCF of 50 and 100 is to list all the factors of each and identify the largest number that appears in both lists. Remember, each number is also a factor of itself.

$$\begin{array}{ll} \{1, 2, 5, 10, 25, 50\} & \textit{Factors of } 50 \\ \{1, 2, 4, 5, 10, 20, 25, 50, 100\} & \textit{Factors of } 100 \end{array}$$

Common factors are listed in bold, and we see that the greatest common factor is 50. We use the following notation to indicate the GCF of two numbers: $\text{GCF}(50, 100) = 50$. After determining the GCF, reduce by dividing both the numerator and the denominator as follows:

$$\frac{50 \div 50}{100 \div 50} = \frac{1}{2}$$

Figure 1.4.5

Example 1.4.1

Reduce to lowest terms: $\frac{105}{300}$.

Solution

Rewrite the numerator and denominator as a product of primes and then cancel.

$$\frac{105}{300} = \frac{\overset{1}{\cancel{3}} \cdot \overset{1}{\cancel{5}} \cdot 7}{2 \cdot 2 \cdot \underset{1}{\cancel{3}} \cdot \underset{1}{\cancel{5}} \cdot 5}$$

$$= \frac{7}{2 \cdot 2 \cdot 5}$$

$$= \frac{7}{20}$$

Figure 1.4.6

Alternatively, we achieve the same result if we divide both the numerator and denominator by the $\text{GCF}(105, 300)$ A quick way to find the GCF of the two numbers requires us to first write each as a product of primes. The GCF is the product of all the common prime factors.

$$\left.\begin{aligned} 105 &= 3\cdot 5\cdot 7 \\ 300 &= 2\cdot 2\cdot 3\cdot 5\cdot 5 \end{aligned}\right\} \quad \text{GCF}(105, 300) = 3\cdot 5 = 15$$

Figure 1.4.7

In this case, the common prime factors are 3 and 5 and the greatest common factor of 105 and 300 is 15.

$$\frac{105}{300} = \frac{105\div 15}{300\div 15} = \frac{7}{20}$$

Figure 1.4.8

Answer:

$\frac{7}{20}$

Example 1.4.2

Try this! Reduce to lowest terms: $\frac{32}{96}$.

Video Solution:

(click to see video)

An **improper fraction** is one where the numerator is larger than the denominator. A mixed number is a number that represents the sum of a whole number and a fraction. For example, $5\frac{1}{2}$ is a mixed number that represents the sum $5+\frac{1}{2}$. Use long division to convert an improper fraction to a mixed number; the remainder is the numerator of the fractional part.

Example 1.4.3

Write $\frac{23}{5}$ as a mixed number.

Solution

Notice that 5 divides into 23 four times with a remainder of 3.

$$\begin{array}{r} 4 \\ 5\overline{)\,23} \\ -20 \\ \hline 3 \end{array}$$

Figure 1.4.9

We then can write

$$\begin{aligned} \frac{23}{5} &= 4+\frac{3}{5} \\ &= 4\frac{3}{5} \end{aligned}$$

Note that the denominator of the fractional part of the mixed number remains the same as the denominator of the original fraction.

Answer

$4\frac{3}{5}$

To convert mixed numbers to improper fractions, multiply the whole number by the denominator and then add the numerator; write this result over the original denominator.

Example 1.4.4

Write $3\frac{5}{7}$ as an improper fraction.

Solution

Obtain the numerator by multiplying 7 times 3 and then add 5.

$$\begin{aligned} 3\frac{5}{7} &= \frac{7\cdot 3+5}{7} \\ &= \frac{21+5}{7} \\ &= \frac{26}{7} \end{aligned}$$

Answer

$\frac{26}{7}$

It is important to note that converting to a mixed number is not part of the reducing process. We consider improper fractions, such as 267, to be reduced to lowest terms. In algebra it is often preferable to work with improper fractions, although in some applications, mixed numbers are more appropriate.

Example 1.4.5

Try this! Convert $10\frac{1}{2}$ to an improper fraction.

Solution

(click to see video)

Multiplying and Dividing Fractions

In this section, assume that a, b, c, and d are all nonzero integers. The product of two fractions is the fraction formed by the product of the numerators and the product of the denominators. In other words, to multiply fractions, multiply the numerators and multiply the denominators:

$$\frac{a}{b}\cdot\frac{c}{d}=\frac{ac}{bd}$$

Example 1.4.6

Multiply: $\frac{2}{3}\cdot\frac{5}{7}$

Solution

Multiply the numerators and multiply the denominators.

$$\begin{aligned} \frac{2}{3}\cdot\frac{5}{7} &= \frac{2\cdot 5}{3\cdot 7} \\ &= \frac{10}{21} \end{aligned}$$

Answer:

$\frac{10}{21}$

Example 1.4.7

Multiply: $\frac{5}{9}\left(-\frac{1}{4}\right)$

Solution

Recall that the product of a positive number and a negative number is negative.

$$\begin{aligned}\frac{5}{9}\left(-\frac{1}{4}\right) &= -\frac{5\cdot 1}{9\cdot 4}\\ &= -\frac{5}{36}\end{aligned}$$

Answer:

$-\frac{5}{36}$

Example 1.4.8

Multiply: $\frac{2}{3}\cdot 5\frac{3}{4}$

Solution

Begin by converting $5\frac{3}{4}$ to an improper fraction.

$$\begin{aligned}\frac{2}{3}\cdot 5\frac{3}{4} &= \frac{2}{3}\cdot\frac{23}{4}\\ &= \frac{\cancel{2}^{1}}{3}\cdot\frac{23}{\cancel{4}_{2}}\\ &= \frac{23}{6}\\ &= 3\frac{5}{6}\end{aligned}$$

Figure 1.4.10

In this example, we noticed that we could reduce before we multiplied the numerators and the denominators. Reducing in this way is called **cross canceling**, and can save time when multiplying fractions.

Answer

$3\frac{5}{6}$

Two real numbers whose product is 1 are called **reciprocals**. Therefore, $\frac{a}{b}$ and $\frac{b}{a}$ are reciprocals because $\frac{a}{b}\cdot\frac{b}{a}=\frac{ab}{ab}=1$. For example,

$$\frac{2}{3}\cdot\frac{3}{2}=\frac{6}{6}=1$$

Because their product is 1, $\frac{2}{3}$ and $\frac{3}{2}$ are reciprocals. Some other reciprocals are listed below:

$\frac{5}{8}$ and $\frac{8}{5}$ $\quad$ 7 and $\frac{1}{7}$ $\quad$ $-\frac{4}{5}$ and $-\frac{5}{4}$

This definition is important because dividing fractions requires that you multiply the dividend by the reciprocal of the divisor.

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$$

Example 1.4.9

Divide: $\frac{2}{3} \div \frac{5}{7}$

Solution

Multiply $\frac{2}{3}$ by the reciprocal of $\frac{5}{7}$.

$$\begin{aligned} \frac{2}{3} \div \frac{5}{7} &= \frac{2}{3} \cdot \frac{7}{5} \\ &= \frac{2 \cdot 7}{3 \cdot 5} \\ &= \frac{14}{15} \end{aligned}$$

Answer:

\(\frac{14}{15\)

You also need to be aware of other forms of notation that indicate division: / and —. For example,

$$5/(1/2) = 5 * (2/1) = (5/1) * (2/1) = 10/1 = 10$$

Or

$$\frac{\frac{7}{8}}{\frac{2}{3}} = \frac{7}{8} \div \frac{2}{3} = \frac{7}{8} \cdot \frac{3}{2} = \frac{21}{16}$$

The latter is an example of a **complex fraction**, which is a fraction whose numerator, denominator, or both are fractions.

Note

Students often ask why dividing is equivalent to multiplying by the reciprocal of the divisor. A mathematical explanation comes from the fact that the product of reciprocals is 1. If we apply the multiplicative identity property and multiply numerator and denominator by the reciprocal of the denominator, then we obtain the following:

$$\frac{\frac{7}{8}}{\frac{2}{3}} = \frac{\frac{7}{8}}{\frac{2}{3}} \cdot 1 = \frac{\frac{7}{8}}{\frac{2}{3}} \cdot \frac{\frac{3}{2}}{\frac{3}{2}} = \frac{\frac{7}{8} \cdot \frac{3}{2}}{1} = \frac{7}{8} \cdot \frac{3}{2}$$

Figure 1.4.11

Before multiplying, look for common factors to cancel; this eliminates the need to reduce the end result.

Example 1.4.10

Divide: $\frac{\frac{5}{2}}{\frac{7}{4}}$.

Solution

$$\frac{\frac{5}{2}}{\frac{7}{4}} = \frac{5}{2} \div \frac{7}{4}$$

$$= \frac{5}{2} \cdot \frac{4}{7}$$

$$= \frac{5}{\cancel{2}_{1}} \cdot \frac{\cancel{4}^{2}}{7}$$

$$= \frac{5 \cdot 2}{1 \cdot 7}$$

$$= \frac{10}{7}$$

Figure 1.4.12

Answer

$\frac{10}{7}$

When dividing by an integer, it is helpful to rewrite it as a fraction over 1.

Example 1.4.11

Divide: $\frac{2}{3} \div 6$

Solution

Rewrite 6 as $\frac{6}{1}$ and multiply by its reciprocal.

$$\frac{2}{3} \div 6 = \frac{2}{3} \div \frac{6}{1}$$
$$= \frac{2}{3} \cdot \frac{1}{6}$$
$$= \frac{\cancel{2}^{1}}{3} \cdot \frac{1}{\cancel{6}_{3}}$$
$$= \frac{1 \cdot 1}{3 \cdot 3}$$
$$= \frac{1}{9}$$

Figure 1.4.13

Answer:

$\frac{1}{9}$

Also, note that we only cancel when working with multiplication. Rewrite any division problem as a product *before* canceling.

Example 1.4.12

Try this! Divide: $5 \div 2\frac{3}{5}$

Video Solution:

(click to see video)

Adding and Subtracting Fractions

Negative fractions are indicated with the negative sign in front of the fraction bar, in the numerator, or in the denominator. All such forms are equivalent and interchangeable.

$$\frac{-3}{4} = -\frac{3}{4} = \frac{3}{-4}$$

Adding or subtracting fractions requires a **common denominator**. In this section, assume the common denominator c is a nonzero integer.

It is good practice to use positive common denominators by expressing negative fractions with negative numerators. In short, avoid negative denominators.

Subtract: $\frac{12}{15} - \frac{3}{15}$

Solution

The two fractions have a common denominator 15. Therefore, subtract the numerators and write the result over the common denominator:

$$\begin{aligned}\frac{12}{15}-\frac{3}{15} &= \frac{12-3}{15} && \textit{Subtract the numerators.}\\ &= \frac{9}{15}\\ &= \frac{9\div 3}{15\div 3} && \textit{Reduce.}\\ &= \frac{3}{5}\end{aligned}$$

Answer

$\frac{3}{5}$

Most problems that you are likely to encounter will have **unlike denominators**. In this case, first find equivalent fractions with a common denominator before adding or subtracting the numerators. One way to obtain equivalent fractions is to divide the numerator and the denominator by the same number. We now review a technique for finding equivalent fractions by multiplying the numerator and the denominator by the same number. It should be clear that $5/5$ is equal to 1 and that 1 multiplied times any number is that number:

$$\frac{1}{2}=\frac{1}{2}\cdot 1=\frac{1}{2}\cdot\frac{5}{5}=\frac{5}{10}$$

We have equivalent fractions $\frac{1}{2}=\frac{5}{10}$. Use this idea to find equivalent fractions with a common denominator to add or subtract fractions. The steps are outlined in the following example.

Example 1.4.14

Subtract: $\frac{7}{15}-\frac{3}{10}$

Solution

Step 1: Determine a common denominator. To do this, use the **least common multiple** (LCM) of the given denominators. The LCM of 15 and 10 is indicated by $\text{LCM}(15,10)$. Try to think of the smallest number that both denominators divide into evenly. List the multiples of each number:

$$\{10,\ 20,\ \mathbf{30},\ 40,\ 50,\ \mathbf{60},\ldots\} \quad \textit{Multiples of 10}$$

$$\{15,\ \mathbf{30},\ 45,\ \mathbf{60},\ 75,\ 90,\ldots\} \quad \textit{Multiples of 15}$$

Common multiples are listed in bold, and the least common multiple is 30.

$$\text{LCM}(10,15)=30$$

Step 2: Multiply the numerator and the denominator of each fraction by values that result in equivalent fractions with the determined common denominator.

$$\begin{aligned}\frac{7}{15}-\frac{3}{10} &= \frac{7\cdot 2}{15\cdot 3}-\frac{3\cdot 3}{10\cdot 3}\\ &= \frac{14}{30}-\frac{9}{30}\end{aligned}$$

Step 3: Add or subtract the numerators, write the result over the common denominator and then reduce if possible.

$$\begin{aligned}\frac{14}{30}-\frac{9}{30} &= \frac{14-9}{30}\\ &= \frac{5}{30}\\ &= \frac{5\div 5}{30\div 5}\\ &= \frac{1}{6}\end{aligned}$$

Answer:

$\frac{1}{6}$

The least common multiple of the denominators is called the **least common denominator** (LCD). Finding the LCD is often the difficult step. It is worth finding because if any common multiple other than the least is used, then there will be more steps involved when reducing.

Example 1.4.15

Add: $\frac{5}{10}+\frac{1}{18}$

Solution

First, determine that the LCM$(10,18)$ is 90 and then find equivalent fractions with 90 as the denominator.

$$\begin{aligned}\frac{5}{10}+\frac{1}{18} &= \frac{5\cdot 9}{10\cdot 9}+\frac{1\cdot 5}{18\cdot 5}\\ &= \frac{45}{90}+\frac{5}{90}\\ &= \frac{45+5}{90}\\ &= \frac{50}{90}\\ &= \frac{50\div 10}{90\div 10}\\ &= \frac{5}{9}\end{aligned}$$

Answer

$\frac{5}{9}$

Example 1.4.16

Try this! Add: $\frac{2}{30}+\frac{5}{21}$

Video Solution:

(click to see video)

Example 1.4.17

Simplify: $2\frac{1}{3}+\frac{3}{5}-\frac{1}{2}$

Solution

Begin by converting $2\frac{1}{3}$ to an improper fraction.

$$2\frac{1}{3}+\frac{3}{5}-\frac{1}{2}=\frac{7}{3}+\frac{3}{5}-\frac{1}{2}$$ *Convert to improper fractions.*

$$=\frac{7\cdot 10}{3\cdot 10}+\frac{3\cdot 6}{5\cdot 6}-\frac{1\cdot 15}{2\cdot 15}$$ $\text{LCM}(3,\ 5,\ 2)\ =\ 30$

$$=\frac{70}{30}+\frac{18}{30}-\frac{15}{30}$$ *Equivalent fractions with a common denominator*

$$=\frac{70+18-15}{30}$$

$$=\frac{73}{30}$$

$$=2\frac{13}{30}$$

Answer:

$2\frac{13}{30}$

In general, it is preferable to work with improper fractions. However, when the original problem involves mixed numbers, if appropriate, present your answers as mixed numbers. Also, mixed numbers are often preferred when working with numbers on a number line and with real-world applications.

Example 1.4.18

Try this! Subtract: $\frac{5}{7}-2\frac{1}{7}$

Video Solution:

(click to see video)

Example 1.4.19

How many $\frac{1}{2}$ inch thick paperback books can be stacked to fit on a shelf that is $1\frac{1}{2}$ feet in height?

Solution

First, determine the height of the shelf in inches. To do this, use the fact that there are 12 inches in 1 foot and multiply as follows:

$$1\frac{1}{2}\text{ ft} = 1\frac{1}{2}\text{ ft} \cdot \left(\frac{12\text{ in}}{1\text{ ft}}\right)$$
$$= \frac{3}{\cancel{2}_{1}} \cdot \frac{\cancel{12}^{6}}{1}\text{ in}$$
$$= 18\text{ in}$$

Next, determine how many notebooks will fit by dividing the height of the shelf by the thickness of each book.

$$18\text{ in} \div \frac{1}{2}\text{ in} = 18 \cdot \frac{2}{1}$$
$$= 18 \cdot 2$$
$$= 36$$

Answer

36 books can be stacked on the shelf.

Key Takeaways:

- Fractions are not unique; there are many ways to express the same ratio. Find equivalent fractions by multiplying or dividing the numerator and the denominator by the same real number.
- Equivalent fractions in lowest terms are generally preferred. It is a good practice to always reduce.
- In algebra, improper fractions are generally preferred. However, in real-life applications, mixed number equivalents are often preferred. We may present answers as improper fractions unless the original question contains mixed numbers, or it is an answer to a real-world or geometric application.
- Multiplying fractions does not require a common denominator; multiply the numerators and multiply the denominators to obtain the product. It is a best practice to cancel any common factors in the numerator and the denominator before multiplying.
- Reciprocals are rational numbers whose product is equal to 1. Given a fraction $\frac{a}{b}$, its reciprocal is $\frac{b}{a}$.
- Divide fractions by multiplying the dividend by the reciprocal of the divisor. In other words, multiply the numerator by the reciprocal of the denominator.
- Rewrite any division problem as a product *before* canceling.
- Adding or subtracting fractions requires a common denominator. When the denominators of any number of fractions are the same, simply add or subtract the numerators and write the result over the common denominator.
- Before adding or subtracting fractions, ensure that the denominators are the same by finding equivalent fractions with a common denominator. Multiply the numerator and the denominator of each fraction by the appropriate value to find the equivalent fractions.
- Typically, it is best to convert all mixed numbers to improper fractions before beginning the process of adding, subtracting, multiplying, or dividing.

? Exercise 1.4.1

Reduce each fraction to lowest terms.

1. $\frac{5}{30}$
2. $\frac{6}{24}$

3. $\frac{30}{70}$
4. $\frac{18}{27}$
5. $\frac{44}{84}$
6. $\frac{54}{90}$
7. $\frac{135}{30}$
8. $\frac{105}{300}$
9. $\frac{18}{6}$
10. $\frac{256}{16}$
11. $\frac{126}{45}$
12. $\frac{52}{234}$
13. $\frac{54}{162}$
14. $\frac{2000}{3000}$
15. $\frac{270}{360}$

Answer

1: $1/6$

3: $3/7$

5: $11/21$

7: $9/2$

9: 3

11: $14/5$

13: $1/3$

15: $3/4$

Exercise 1.4.2

Rewrite as an improper fraction.

1. $4\frac{3}{4}$
2. $2\frac{1}{2}$
3. $5\frac{7}{15}$
4. $1\frac{1}{2}$
5. $3\frac{5}{8}$
6. $1\frac{3}{4}$
7. $-2\frac{1}{2}$
8. $-1\frac{3}{4}$

Answer

1: $5/2$

3: $3/2$

5: $7/4$

7: $-7/4$

Exercise 1.4.3

Rewrite as a mixed number.

1. $\frac{15}{2}$
2. $\frac{9}{2}$
3. $\frac{40}{13}$
4. $\frac{103}{25}$
5. $\frac{73}{10}$
6. $\frac{-52}{7}$
7. $\frac{-59}{6}$

Answer

2: $4\frac{1}{2}$

4: $4\frac{3}{25}$

6: $-7\frac{3}{7}$

Exercise 1.4.4

Multiply and reduce to lowest terms.

1. $\frac{2}{3} \cdot \frac{5}{7}$
2. $\frac{1}{5} \cdot \frac{4}{8}$
3. $\frac{1}{2} \cdot \frac{1}{3}$
4. $\frac{3}{4} \cdot \frac{20}{9}$
5. $\frac{5}{7} \cdot \frac{49}{10}$
6. $\frac{2}{3} \cdot \frac{9}{12}$
7. $\frac{6}{14} \cdot \frac{21}{12}$
8. $\frac{44}{15} \cdot \frac{15}{11}$
9. $3\frac{3}{4} \cdot 2\frac{1}{3}$
10. $2\frac{7}{10} \cdot 5\frac{5}{6}$
11. $\frac{3}{11}(-\frac{5}{2})$
12. $-\frac{4}{5}(\frac{9}{5})$
13. $(-\frac{9}{5}(-\frac{3}{10})$
14. $\frac{6}{7}(-\frac{14}{3})$
15. $(-\frac{9}{12})(-\frac{4}{8})$
16. $-\frac{3}{8}(-\frac{4}{15})$
17. $\frac{1}{7} \cdot \frac{1}{2} \cdot \frac{1}{3}$
18. $\frac{3}{5} \cdot \frac{15}{21} \cdot \frac{7}{27}$
19. $\frac{2}{5} \cdot 3\frac{1}{8} \cdot \frac{4}{5}$
20. $2\frac{4}{9} \cdot \frac{2}{5} \cdot 2\frac{5}{11}$

Answer

1: $10/21$

3: $1/6$

5: $7/2$

7: $3/4$

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9: 834

11: $-15/22$

13:$27/50$

15: $3/8$

17: $1/42$

19: 1

Exercise 1.4.5

Determine the reciprocal of the following numbers.

1. $\frac{1}{2}$
2. $\frac{8}{5}$
3. $-\frac{2}{3}$
4. $-\frac{4}{3}$
5. 10
6. -4
7. $2\frac{1}{3}$
8. $1\frac{5}{8}$

Answer

1: 2

3: $-3/2$

5: $1/10$

7: $3/7$

9: $3/4$

Exercise 1.4.6

Divide and reduce to lowest terms.

1. $\frac{1}{2} \div \frac{2}{3}$
2. $\frac{5}{9} \div \frac{1}{3}$
3. $\frac{5}{8} \div \left(-\frac{4}{5}\right)$
4. $\left(-\frac{2}{5}\right) \div \frac{15}{3}$
5. $\dfrac{-\frac{6}{7}}{-\frac{6}{7}}$
6. $\dfrac{-\frac{1}{2}}{\frac{1}{4}}$
7. $\dfrac{-\frac{10}{3}}{-\frac{5}{20}}$
8. $\dfrac{\frac{2}{3}}{\frac{9}{2}}$
9. $\dfrac{\frac{30}{50}}{\frac{5}{3}}$

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10. $\frac{\frac{1}{2}}{2}$
11. $\frac{5}{\frac{2}{5}}$
12. $\frac{-6}{\frac{5}{4}}$
13. $2\frac{1}{2} \div \frac{5}{3}$
14. $4\frac{2}{3} \div 3\frac{1}{2}$
15. $5 \div 2\frac{3}{5}$
16. $4\frac{3}{5} \div 23$

Answer

1: $3/4$

3: $-25/32$

5: 1

7: $40/3$

9: $9/25$

11: $25/2$

13: $1\frac{1}{2}$

15: $1\frac{12}{13}$

? Exercise 1.4.7

Add or subtract and reduce to lowest terms.

1. $\frac{17}{20} - \frac{5}{20}$
2. $\frac{4}{9} - \frac{13}{9}$
3. $\frac{3}{5} + \frac{1}{5}$
4. $\frac{11}{15} + \frac{9}{15}$
5. $\frac{5}{7} - 2\frac{1}{7}$
6.
7. $\frac{1}{2} + \frac{1}{3}$
8. $\frac{1}{5} - \frac{1}{4}$
9. $\frac{3}{4} - \frac{5}{2}$
10. $\frac{3}{8} + \frac{7}{16}$
11. $\frac{7}{15} - \frac{3}{10}$
12. $\frac{3}{10} + \frac{2}{14}$
13. $\frac{2}{30} + \frac{5}{21}$
14. $\frac{3}{18} - \frac{1}{24}$
15. $5\frac{1}{2} + 2\frac{1}{3}$
16. $1\frac{3}{4} + 2\frac{1}{10}$
17. $\frac{1}{2} + \frac{1}{3} + \frac{1}{6}$
18. $\frac{2}{3} + \frac{3}{5} - \frac{2}{9}$
19. $\frac{7}{3} - \frac{3}{2} + \frac{2}{15}$
20. $\frac{9}{4} - \frac{3}{2} + \frac{3}{8}$
21.
22. $\frac{2}{3} - 4\frac{1}{2} + 3\frac{1}{6}$

23. $1 - \frac{6}{16} + \frac{3}{18}$
24. $3 - \frac{1}{21} - \frac{1}{15}$

Answer

1: $3/5$

3: $4/5$

5: $-1\frac{3}{7}$

7: $5/6$

9: $-7/4$

11: $1/6$

13: $32/105$

15: $7\frac{5}{6}$

17: 1

19: $29/30$

21: $2\frac{2}{3}$

23: $19/24$

Exercise 1.4.8

Perform the operations. Reduce answers to lowest terms.

1. $\frac{3}{14} \cdot \frac{7}{3} \div \frac{1}{8}$
2. $\frac{1}{2} \cdot \left(-\frac{4}{5}\right) \div \frac{14}{15}$
3. $\frac{1}{2} \div \frac{3}{4} \cdot \frac{1}{5}$
4. $-\frac{5}{9} \div \frac{5}{3} \cdot \frac{5}{2}$
5.
6.
7. $\frac{4}{5} \div 4 \cdot \frac{1}{2}$
8. $\frac{5}{3} \div 15 \cdot \frac{2}{3}$
9. What is the product of $\frac{3}{16}$ and $\frac{4}{9}$?
10. What is the product of $-\frac{24}{5}$ and $\frac{25}{8}$?
11. What is the quotient of $\frac{5}{9}$ and $\frac{25}{3}$?
12. What is the quotient of $-\frac{16}{5}$ and 32?
13. Subtract $\frac{1}{6}$ from the sum of $\frac{9}{2}$ and $\frac{2}{3}$.
14. Subtract $\frac{1}{4}$ from the sum of $\frac{3}{4}$ and $\frac{6}{5}$.
15. What is the total width when 3 boards, each with a width of $2\frac{5}{8}$ inches, are glued together?
16. The precipitation in inches for a particular 3-day weekend was published as $\frac{3}{10}$ inches on Friday, $1\frac{1}{2}$ inches on Saturday, and $\frac{3}{4}$ inches on Sunday. Calculate the total precipitation over this period.
17. A board that is $5\frac{1}{4}$ feet long is to be cut into 7 pieces of equal length. What is length of each piece?
18. How many $\frac{3}{4}$ inch thick notebooks can be stacked into a box that is 2 feet high?
19. In a mathematics class of 44 students, one-quarter of the students signed up for a special Saturday study session. How many students signed up?
20. Determine the length of fencing needed to enclose a rectangular pen with dimensions $35\frac{1}{2}$ feet by $20\frac{2}{3}$ feet.
21. Each lap around the track measures $\frac{1}{4}$ mile. How many laps are required to complete a $2\frac{1}{2}$ mile run?
22. A retiree earned a pension that consists of three-fourths of his regular monthly salary. If his regular monthly salary was $\$5,200$, then what monthly payment can the retiree expect from the pension plan?

Answer

1: 4

3: $2/15$

5: $9/28$

7: $1/10$

9: $1/12$

11: $1/15$

13: 5

15: $7\frac{7}{8}$ inches

17: $\frac{3}{4}$ feet

19: 11 students

21: 10 laps

Exercise 1.4.1

Discussion Board Topics

1. Does 0 have a reciprocal? Explain.
2. Explain the difference between the LCM and the GCF. Give an example.
3. Explain the difference between the LCM and LCD.
4. Why is it necessary to find an LCD in order to add or subtract fractions?
5. Explain how to determine which fraction is larger, $\frac{7}{16}$ or $\frac{1}{2}$.

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1.5: Review of Decimals and Percents

Skills to Develop

- Convert fractions to decimals and back.
- Perform operations with decimals.
- Round off decimals to a given place.
- Define a percent.
- Convert percents to decimals and back.
- Convert fractions to percents and back.

Decimals

In this section, we provide a brief review of the decimal system. A real number in decimal form, a **decimal** consists of a decimal point, digits (0 through 9) to the left of the decimal point representing the whole number part, and digits to the right of the decimal point representing the fractional part. The digits represent powers of 10 as shown in the set $\{\ldots, 1,000, 100, 10, 1, 1/10, 1/100, 1/1,000, \ldots\}$ according to the following diagram:

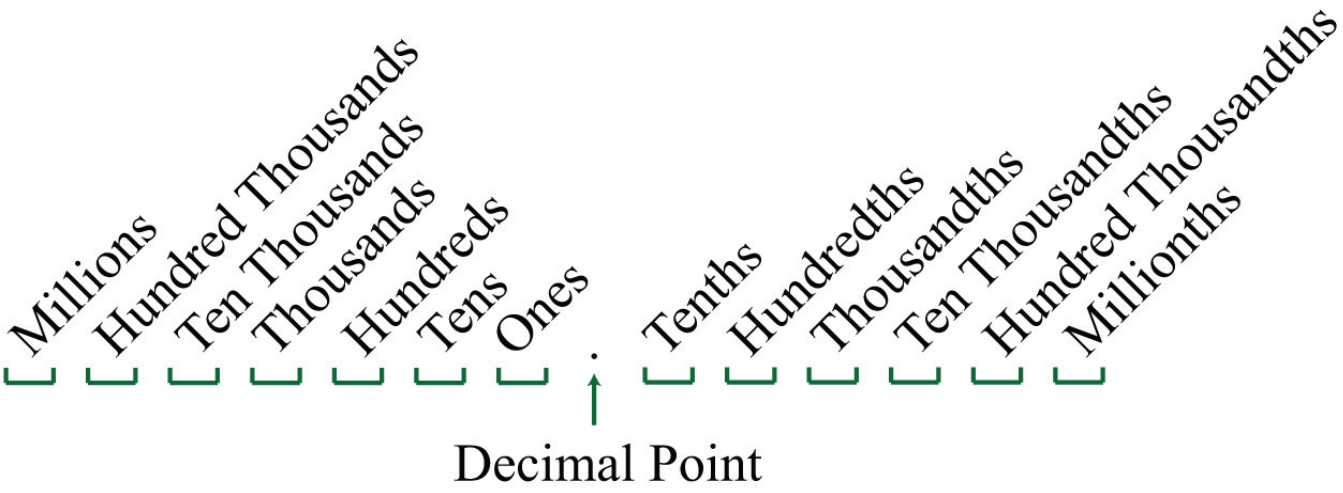


Figure 1.5.1

For example, the decimal 538.3 can be written in the following expanded form:

$$5 \cdot 100 + 3 \cdot 10 + 8 \cdot 1 + 3 \cdot \frac{1}{10}$$

After simplifying, we obtain the mixed number $538\frac{3}{10}$. Use this process to convert decimals to mixed numbers.

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1.6: Exponents and Square Roots

Learning Objectives

- Interpret exponential notation with positive integer exponents.
- Calculate the nth power of a real number.
- Calculate the exact and approximate value of the square root of a real number.

Exponential Notation and Positive Integer Exponents

If a number is repeated as a factor numerous times, then we can write the product in a more compact form using exponential notation. For example,

$$5 \cdot 5 \cdot 5 \cdot 5 = 5^4$$

The base is the factor, and the positive integer exponent indicates the number of times the base is repeated as a factor. In the above example, the base is 5 and the exponent is 4. In general, if a is the base that is repeated as a factor n times, then

$$a^n = \underbrace{a \cdot a \cdot a \cdot \ldots \cdot a}_{n \text{ factors of } a}$$

***Figure* 1.6.1**

When the exponent is 2, we call the result a square. For example,

$$3^2 = 3 \cdot 3 = 9$$

The number 3 is the base and the integer 2 is the exponent. The notation 3^2 can be read two ways: “three squared” or “3 raised to the second power.” The base can be any real number.

It is important to study the difference between the ways the last two examples are calculated. In the example $(-7)^2$, the base is -7 as indicated by the parentheses. In the example -5^2, the base is 5, not -5, so only the 5 is squared and the result remains negative. To illustrate this, write

$$-5^2 = -1 \cdot 5^2 = -1 \cdot 5 \cdot 5 = -25$$

This subtle distinction is very important because it determines the sign of the result.

The textual notation for exponents is usually denoted using the caret $(^{})$ symbol as follows:

$$8^2 = 8 \wedge 2 = 8 * 8 = 64$$
$$-5.1^2 = -5.1 \wedge 2 = -5.1 * 5.1 = -26.01$$

The square of an integer is called a perfect square. The ability to recognize perfect squares is useful in our study of algebra. The squares of the integers from 1 to 15 should be memorized. A partial list of perfect squares follows:

$$\{0, 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, \ldots\}$$

Exercise 1.6.1

Simplify

$$(-12)^2.$$

Answer

144

When the exponent is 3 we call the result a cube. For example,

$$3^3 = 3 \cdot 3 \cdot 3 = 27$$

The notation 3^3 can be read two ways: “three cubed” or “3 raised to the third power.” As before, the base can be any real number.

Note that the result of cubing a negative number is negative. The cube of an integer is called a perfect cube. The ability to recognize perfect cubes is useful in our study of algebra. The cubes of the integers from 1 to 10 should be memorized. A partial list of perfect cubes follows:

$\{0, 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000, \dots\}$

? Exercise 1.6.2

Simplify $(-2)^3$.

Answer

-8

If the exponent is greater than 3, then the notation an is read "a raised to the nth power."

$$10^6 = 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 1,000,000$$
$$(-1)^4 = (-1)(-1)(-1)(-1) = 1$$
$$\left(\frac{1}{3}\right)^5 = \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{243}$$

Notice that the result of a negative base with an even exponent is positive. The result of a negative base with an odd exponent is negative. These facts are often confused when negative numbers are involved. Study the following four examples carefully:

Table 1.6.1

The base is (-2)	The base is 2
$(-2)^4 = (-2) \cdot (-2) \cdot (-2) \cdot (-2) = +16$ $(-2)^3 = (-2) \cdot (-2) \cdot (-2) = -8$	$-2^4 = -2 \cdot 2 \cdot 2 \cdot 2 = -16$ $-2^3 = -2 \cdot 2 \cdot 2 = -8$

The parentheses indicate that the negative number is to be used as the base.

✓ Example 1.6.1

Calculate:

a. $\left(-\frac{1}{3}\right)^3$
b. $\left(-\frac{1}{3}\right)^4$

Solution:

The base is $-\frac{1}{3}$ for both problems.

a. Use the base as a factor three times.

$$\left(-\frac{1}{3}\right)^3 = \left(-\frac{1}{3}\right)\left(-\frac{1}{3}\right)\left(-\frac{1}{3}\right)$$
$$= -\frac{1}{27}$$

b. Use the base as a factor four times.

$$\left(-\frac{1}{3}\right)^4 = \left(-\frac{1}{3}\right)\left(-\frac{1}{3}\right)\left(-\frac{1}{3}\right)\left(-\frac{1}{3}\right)$$
$$= +\frac{1}{81}$$

Answer:

a. $-\frac{1}{27}$; b. $\frac{1}{81}$

Exercise 1.6.3

Simplify:

$$-10^4 \text{ and } (-10)^4.$$

Answer

$-10,000 \text{and } 10,000$

Square Root of a Real Number

Think of finding the square root of a number as the inverse of squaring a number. In other words, to determine the square root of 25 the question is, "What number squared equals 25?" Actually, there are two answers to this question, 5 and -5.

$$5^2 = 25 \quad \text{and} (-5)^2 = 25$$

When asked for the square root of a number, we implicitly mean the principal (nonnegative) square root. Therefore we have,

$$\sqrt{a^2} = a \text{, if } a \geq 0 \text{ or more generally } \sqrt{a^2} = |a|$$

As an example, $\sqrt{25} = 5$, which is read "square root of 25 equals 5." The symbol $\sqrt{}$ is called the radical sign and 25 is called the radicand. The alternative textual notation for square roots follows:

$$\sqrt{16} = textsqrt(16) = 4$$

It is also worthwhile to note that

$$\sqrt{1} = 1 \quad \text{and} \quad \sqrt{0} = 0$$

This is the case because $1^2 = 1$ and $0^2 = 0$.

Example 1.6.2

Simplify:

$$\sqrt{10,000}.$$

Solution:

$10,000 \text{is a perfect square because } 100 \cdot 100 = 10,000.$

$$\begin{aligned} \sqrt{10,000} &= \sqrt{(100)^2} \\ &= 100 \end{aligned}$$

Answer:

$$100$$

Example 1.6.3

Simplify:

$$\sqrt{\tfrac{1}{9}}.$$

Solution:

Here we notice that $\frac{1}{9}$ is a square because $\frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9}$.

$$\begin{aligned} \sqrt{\frac{1}{9}} &= \sqrt{\left(\frac{1}{3}\right)^2} \\ &= \frac{1}{3} \end{aligned}$$

Answer:

$\frac{1}{3}$

Given a and b as positive real numbers, use the following property to simplify square roots whose radicands are not squares:

$$\sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b}$$

The idea is to identify the largest square factor of the radicand and then apply the property shown above. As an example, to simplify $\sqrt{8}$ notice that 8 is not a perfect square. However, $8 = 4 \cdot 2$ and thus has a perfect square factor other than 1. Apply the property as follows:

$$\begin{aligned} \sqrt{8} &= \sqrt{4 \cdot 2} \\ &= \sqrt{4} \cdot \sqrt{2} \\ &= 2 \cdot \sqrt{2} \\ &= 2\sqrt{2} \end{aligned}$$

Here $2\sqrt{2}$ is a simplified irrational number. You are often asked to find an approximate answer rounded off to a certain decimal place. In that case, use a calculator to find the decimal approximation using either the original problem or the simplified equivalent.

$$\sqrt{8} = 2\sqrt{2} \approx 2.83$$

On a calculator, try $2.83 \wedge 2$. What do you expect? Why is the answer not what you would expect?

It is important to mention that ***the radicand must be positive***. For example, $\sqrt{-9}$ is undefined since there is no real number that when squared is negative. Try taking the square root of a negative number on your calculator. What does it say?

Note

Taking the square root of a negative number is defined later in the course.

Example 1.6.4

Simplify and give an approximate answer rounded to the nearest hundredth:

$$\sqrt{75}.$$

Solution:

The radicand 75 can be factored as $25 \cdot 3$ where the factor 25 is a perfect square.

$$\begin{aligned} \sqrt{75} &= \sqrt{25 \cdot 3} && \textit{The largest perfect square} \\ &= \sqrt{25} \cdot \sqrt{3} && \textit{factor of } 75 \textit{ is } 25. \\ &= 5 \cdot \sqrt{3} \\ &= 5\sqrt{3} && \textit{Exact answer} \\ &\approx 8.66 && \textit{Approximate answer} \end{aligned}$$

Answer:

$$\sqrt{75} \approx 8.66$$

As a check, calculate (\sqrt{75}\) and $5\sqrt{3}$ on a calculator and verify that the both results are approximately 8.66.

Example 1.6.5

Simplify:

$$\sqrt{180}.$$

Solution:

$$\begin{aligned} \sqrt{180} &= \sqrt{36 \cdot 5} \\ &= \sqrt{36} \cdot \sqrt{5} \\ &= 6 \cdot \sqrt{5} \\ &= 6\sqrt{5} \end{aligned}$$

Since the question did not ask for an approximate answer, we present the exact answer.

Answer:

$$6\sqrt{5}$$

Example 1.6.6

Simplify:

$$-5\sqrt{162}.$$

Solution:

$$\begin{aligned} -5\sqrt{162} &= -5 \cdot \sqrt{81 \cdot 2} \\ &= -5 \cdot \sqrt{81} \cdot \sqrt{2} \\ &= -5 \cdot 9 \cdot \sqrt{2} \\ &= -45 \cdot \sqrt{2} \\ &= -45\sqrt{2} \end{aligned}$$

Answer:

$$-45\sqrt{2}$$

Exercise 1.6.4

Simplify and give an approximate answer rounded to the nearest hundredth:

$$\sqrt{128}.$$

Answer

$8\sqrt{2} \approx 11.31$

A right triangle is a triangle where one of the angles measures 90°. The side opposite the right angle is the longest side, called the hypotenuse, and the other two sides are called legs. Numerous real-world applications involve this geometric figure. The Pythagorean theorem states that given any right triangle with legs measuring a and b units, the square of the measure of the hypotenuse c is equal to the sum of the squares of the measures of the legs: $a^2 + b^2 = c^2$. In other words, the hypotenuse of any right triangle is equal to the square root of the sum of the squares of its legs.

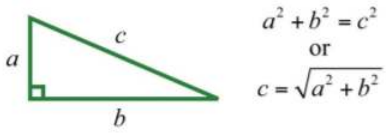


***Figure* 1.6.1**

Example 1.6.7

If the two legs of a right triangle measure 3 units and 4 units, then find the length of the hypotenuse.

Solution:

Given the lengths of the legs of a right triangle, use the formula $c = \sqrt{a^2 + b^2}$ to find the length of the hypotenuse.

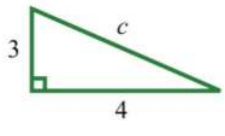


***Figure* 1.6.2**

$$\begin{aligned} c &= \sqrt{a^2+b^2} \\ c &= \sqrt{3^2+4^2} \\ &= \sqrt{9+16} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

Answer:

$$c = 5 \text{ units}$$

When finding the hypotenuse of a right triangle using the Pythagorean theorem, the radicand is not always a perfect square.

Example 1.6.8

If the two legs of a right triangle measure 2 units and 6 units, find the length of the hypotenuse.

Solution:

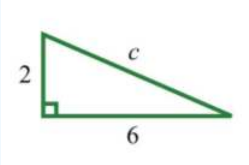


***Figure* 1.6.3**

$$\begin{aligned} c &= \sqrt{a^2+b^2} \\ &= \sqrt{2^2+6^2} \\ &= \sqrt{4+36} \\ &= \sqrt{40} \\ &= \sqrt{4 \cdot 10} \\ &= \sqrt{4} \cdot \sqrt{10} \\ &= 2 \cdot \sqrt{10} \end{aligned}$$

Answer:

$$c = 2\sqrt{10} \text{ units}$$

Key Takeaways

- When using exponential notation a^n, the base a is used as a factor n times.
- When the exponent is 2, the result is called a square. When the exponent is 3, the result is called a cube.
- Memorize the squares of the integers up to 15 and the cubes of the integers up to 10. They will be used often as you progress in your study of algebra.
- When negative numbers are involved, take care to associate the exponent with the correct base. Parentheses group a negative number raised to some power.
- A negative base raised to an even power is positive.
- A negative base raised to an odd power is negative.
- The square root of a number is a number that when squared results in the original number. The principal square root is the positive square root.
- Simplify a square root by looking for the largest perfect square factor of the radicand. Once a perfect square is found, apply the property $\sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b}$, where a and b are nonnegative, and simplify.
- Check simplified square roots by calculating approximations of the answer using both the original problem and the simplified answer on a calculator to verify that the results are the same.
- Find the length of the hypotenuse of any right triangle given the lengths of the legs using the Pythagorean theorem.

Exercise 1.6.5 Square of a Number

Simplify.

1. 10^2
2. 12^2
3. $(-9)^2$
4. -12^2
5. 11^2
6. $(-20)^2$
7. 0^2
8. 1^2
9. $-(-8)^2$
10. $-(13)^2$
11. $(\frac{1}{2})^2$
12. $(-\frac{2}{3})^2$
13. 0.5^2
14. 1.25^2
15. $(-2.6)^2$
16. $-(-5.1)^2$
17. $(2\frac{1}{3})^2$
18. $(5\frac{3}{4})^2$

Answer

1. 100
3. 81
5. 121
7. 0
9. -64
11. $\frac{1}{4}$
13. $.25$
15. 6.76
17. $5\frac{4}{9}$

Exercise 1.6.6 Square of a Number

If s is the length of the side of a square, then the area is given by $A = s^2$.

1. Determine the area of a square given that a side measures 5 inches.
2. Determine the area of a square given that a side measures 2.3 feet.
3. List all the squares of the integers 0 through 15.
4. List all the squares of the integers from -15 to 0.
5. List the squares of all the rational numbers in the set $\{0, \frac{1}{3}, \frac{2}{3}, 1, \frac{4}{3}, \frac{5}{3}, 2\}$.
6. List the squares of all the rational numbers in the set $\{0, \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}\}$.

Answer

1. 25 square inches
3. $\{0, 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225\}$
5. $\{0, \frac{1}{9}, \frac{4}{9}, 1, \frac{16}{9}, \frac{25}{9}, 4\}$

Exercise 1.6.7 Integer Exponents

Simplify.

1. 5^3
2. 2^6
3. $(-1)^4$
4. $(-3)^3$
5. -1^4
6. $(-2)^4$
7. -7^3
8. $(-7)^3$
9. $-(-3)^3$
10. $-(-10)^4$
11. $(-1)^{20}$
12. $(-1)^{21}$
13. $(-6)^3$
14. -3^4
15. 1^{100}
16. 0^{100}
17. $-(\frac{1}{2})^3$
18. $(\frac{1}{2})^6$
19. $(\frac{5}{2})^3$
20. $(-\frac{3}{4})^4$
21. List all the cubes of the integers -5 through 5.
22. List all the cubes of the integers from -10 to 0.
23. List all the cubes of the rational numbers in the set $\{-\frac{2}{3}, -\frac{1}{3}, 0, \frac{1}{3}, \frac{2}{3}\}$.
24. List all the cubes of the rational numbers in the set $\{-\frac{3}{7}, -\frac{1}{7}, 0, \frac{1}{7}, \frac{3}{7}\}$.

Answer

1. 125
3. 1
5. -1
7. -343
9. 27
11. 1
13. -216
15. 1
17. $-\frac{1}{8}$
19. $\frac{12}{58}$
21. $\{-125, -64, -27, -8, -1, 0, 1, 8, 27, 64, 125\}$
23. $\{-\frac{8}{27}, -\frac{1}{27}, 0, \frac{1}{27}, \frac{8}{27}\}$

Exercise 1.6.8 Square Root of a Number

Determine the exact answer in simplified form.

1. $\sqrt{121}$
2. $\sqrt{81}$
3. $\sqrt{100}$
4. $\sqrt{169}$
5. $-\sqrt{25}$
6. $-\sqrt{144}$
7. $\sqrt{12}$
8. $\sqrt{27}$
9. $\sqrt{45}$
10. $\sqrt{50}$
11. $\sqrt{98}$
12. $\sqrt{2000}$
13. $\sqrt{\frac{1}{4}}$
14. $\sqrt{\frac{9}{16}}$
15. $\sqrt{\frac{5}{9}}$
16. $\sqrt{\frac{8}{36}}$
17. $\sqrt{0.64}$
18. $\sqrt{0.81}$
19. $\sqrt{30^2}$
20. $\sqrt{15^2}$
21. $\sqrt{(-2)^2}$
22. $\sqrt{(-5)^2}$
23. $\sqrt{-9}$
24. $\sqrt{-16}$
25. $3\sqrt{16}$
26. $5\sqrt{18}$
27. $-2\sqrt{36}$
28. $-3\sqrt{32}$
29. $6\sqrt{200}$
30. $10\sqrt{27}$

Answer

1. 11
3. 10
5. -5
7. $2\sqrt{3}$
9. $3\sqrt{5}$
11. $7\sqrt{2}$
13. $\frac{1}{2}$
15. $5\sqrt{3}$
17. 0.8
19. 30
21. 2

23. Not real

25. 12

27. -12

29. $60\sqrt{2}$

Exercise 1.6.9 Square Root of a Number

Approximate the following to the nearest hundredth.

1. $\sqrt{2}$
2. $\sqrt{3}$
3. $\sqrt{10}$
4. $\sqrt{15}$
5. $2\sqrt{3}$
6. $5\sqrt{2}$
7. $-6\sqrt{5}$
8. $-4\sqrt{6}$
9. $\sqrt{79}$
10. $\sqrt{54}$
11. $-\sqrt{162}$
12. $-\sqrt{86}$
13. If the two legs of a right triangle measure 6 units and 8 units, then find the length of the hypotenuse.
14. If the two legs of a right triangle measure 5 units and 12 units, then find the length of the hypotenuse.
15. If the two legs of a right triangle measure 9 units and 12 units, then find the length of the hypotenuse.
16. If the two legs of a right triangle measure $\frac{3}{2}$ units and 2 units, then find the length of the hypotenuse.
17. If the two legs of a right triangle both measure 1 unit, then find the length of the hypotenuse.
18. If the two legs of a right triangle measure 1 unit and 5 units, then find the length of the hypotenuse.
19. If the two legs of a right triangle measure 2 units and 4 units, then find the length of the hypotenuse.
20. If the two legs of a right triangle measure 3 units and 9 units, then find the length of the hypotenuse.

Answer

1. 1.41

3. 3.16

5. 3.46

7. -13.42

9. 8.89

11. -12.73

13. 10 units

15. 15 units

17. $\sqrt{2}$ units

19. $2\sqrt{5}$ units

Exercise 1.6.10 Discussion Board Topics

1. Why is the result of an exponent of 2 called a square? Why is the result of an exponent of 3 called a cube?
2. Research and discuss the history of the Pythagorean theorem.
3. Research and discuss the history of the square root.
4. Discuss the importance of the principal square root.

Answer

1. Answers may vary

3. Answers may vary

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1.7: Order of Operations

Learning Objectives

- Identify and work with grouping symbols.
- Understand the order of operations.
- Simplify using the order of operations.

Grouping Symbols

In a computation where more than one operation is involved, grouping symbols help tell us which operations to perform first. The grouping symbols commonly used in algebra are

$$\begin{array}{ll} (\ ) & \textit{Parentheses} \\ [\ ] & \textit{Brackets} \\ \{\ \} & \textit{Braces} \\ - & \textit{Fraction bar} \end{array}$$

All of the above grouping symbols, as well as absolute value, have the same order of precedence. Perform operations inside the innermost grouping symbol or absolute value first.

Example 1.7.1

Simplify:

$$5-(4-12)\,.$$

Solution:

Perform the operations within the parentheses first. In this case, first subtract 12 from 4.

$$\begin{aligned} 5-(4-12) &= 5-(-8) \\ &= 5+8 \\ &= 13 \end{aligned}$$

Answer:

$$13$$

Example 1.7.2

Simplify:

$$3\{-2[-(-3-1)]\}.$$

Solution:

Answer:

-24

Example 1.7.3

Simplify:

$$\frac{5-|4-(-3)|}{|-3|-(5-7)}.$$

Solution:

$$\begin{aligned}\frac{5-|4-(-3)|}{|-3|-(5-7)} &= \frac{5-|4+3|}{|-3|-(-2)} \\ &= \frac{5-|7|}{|-3|+2} \\ &= \frac{5-7}{3+2} \\ &= \frac{-2}{5} \\ &= -\frac{2}{5}\end{aligned}$$

Answer:

$$-\tfrac{2}{5}$$

? Exercise 1.7.1

Simplify:

$$-[-3(2+3)].$$

Answer

15

Order of Operations

When several operations are to be applied within a calculation, we must follow a specific order to ensure a single correct result.

1. Perform all calculations within the innermost **parentheses** or grouping symbols.
2. Evaluate all **exponents**.
3. Perform **multiplication and division** operations from left to right.
4. Finally, perform all remaining **addition and subtraction** operations from left to right.

Note

Caution: Note that multiplication and division operations ***must*** be worked from ***left to right***.

✓ Example 1.7.4

Simplify:

$$5^2-4\cdot 3\div 12\ .$$

Solution:

First, evaluate 5^2 and then perform multiplication and division as they appear from left to right.

$$\begin{aligned}5^2-4\cdot 3\div 12 &= 25-\underbrace{4\cdot 3}\div 12 \\ &= 25-\underbrace{12\div 12} \\ &= \underbrace{25}-1 \\ &= 24\end{aligned}$$

Because multiplication and division operations should be worked from left to right, it is sometimes correct to perform division before multiplication.

Example 1.7.5

Simplify:

$$2^4 - 12 \div 3 \cdot 2 + 11 \ .$$

Solution:

Begin by evaluating the exponent, $2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16$.

$$\begin{aligned} 2^4 - 12 \div 3 \cdot 2 + 11 &= 16 - \underbrace{12 \div 3} \cdot 2 + 11 \\ &= 16 - \underbrace{4 \cdot 2} + 11 \\ &= \underbrace{16 - 8} + 11 \\ &= \underbrace{8 + 11} \\ &= 19 \end{aligned}$$

Multiplying first leads to an incorrect result.

$$\begin{aligned} 2^4 - 12 \div 3 \cdot 2 + 11 &= 16 - 12 \div \underbrace{3 \cdot 2}_{Incorrect} + 11 \\ &= 16 - \underbrace{12 \div 6} + 11 \\ &= \underbrace{16 - 2} + 11 \\ &= \underbrace{14 + 11} \\ &= 25 \quad x \end{aligned}$$

Answer:

19

Example 1.7.6

Simplify:

$$-3 - 5^2 + (-7)^2 \ .$$

Solution:

Take care to correctly identify the base when squaring.

$$\begin{aligned} -3 - 5^2 + (-7)^2 &= -3 - 25 + 49 \\ &= -28 + 49 \\ &= 21 \end{aligned}$$

Answer:

21

Example 1.7.7

Simplify:

$$5 - 3[2^3 - 5 + 7(-3)] \ .$$

Solution:

It is tempting to first subtract $5 - 3$, but this will lead to an incorrect result. The order of operations requires us to simplify within the brackets first.

$$\begin{aligned} 5-3[2^3-5+7(-3)] &= 5-3[8-5-21] \\ &= 5-3[-18] \\ &= 5+54 \\ &= 59 \end{aligned}$$

Subtracting $5-3$ first leads to an incorrect result.

$$\begin{aligned} 5-3[2^3-5+7(-3)] &= \underbrace{5-3}_{Incorrect}\ [8-5-21] \\ &= 2[-18] \\ &= -36 \quad x \end{aligned}$$

Answer:

$$59$$

Example 1.7.8

Simplify:

$$-3^2-[5-(4^2-10)]\ .$$

Solution:

Perform the operations within the innermost parentheses first.

Answer:

-8

Example 1.7.9

Simplify:

$$\left(-\tfrac{2}{3}\right)^2 \div \left[\tfrac{5}{3}-\left(-\tfrac{1}{2}\right)^3\right]\ .$$

Solution:

$$\begin{aligned} \left(-\frac{2}{3}\right)^2 \div \left[\frac{5}{3}-\left(-\frac{1}{2}\right)^3\right] &= \left(-\frac{2}{3}\right)^2 \div \left[\frac{5}{3}-\left(-\frac{1}{8}\right)\right] \\ &= \left(-\frac{2}{3}\right)^2 \div \left[\frac{5}{3}+\frac{1}{8}\right] \\ &= \left(-\frac{2}{3}\right)^2 \div \left[\frac{40}{24}+\frac{3}{24}\right] \\ &= \left(-\frac{2}{3}\right)^2 \div \frac{43}{24} \\ &= \frac{4}{\cancel{9}_{3}} \cdot \frac{\cancel{24}^{8}}{43} \\ &= \frac{32}{129} \end{aligned}$$

Answer:

$\frac{32}{129}$

We are less likely to make a mistake if we work one operation at a time. Some problems may involve an absolute value, in which case we assign it the same order of precedence as parentheses.

Example 1.7.10

Simplify:

$$2-4|-4-3|+(-2)^4 \ .$$

Solution:

We begin by evaluating the absolute value and then the exponent $(-2)^4=(-2)(-2)(-2)(-2)=+16$.

Answer:

-10

Exercise 1.7.2

Simplify:

$$10\div 5\cdot 2|(-4)+|-3||+(-3)^2 \ .$$

Answer

13

Key Takeaways

- Grouping symbols indicate which operations to perform first. We usually group mathematical operations with parentheses, brackets, braces, and the fraction bar. We also group operations within absolute values. All groupings have the same order of precedence: the operations within the innermost grouping are performed first.
- When applying operations within a calculation, follow the order of operations to ensure a single correct result.
  - Address innermost parentheses or groupings first.
  - Simplify all exponents.
  - Perform multiplication and division operations from left to right.
  - Finally, perform addition and subtraction operations from left to right.
- It is important to highlight the fact that multiplication and division operations should be applied as they appear from left to right. It is a common mistake to always perform multiplication before division, which, as we have seen, in some cases produces incorrect results.

Exercise 1.7.3 Order of Operations

Simplify.

1. $-7-3\cdot 5$
2. $3+2\cdot 3$
3. $-3(2)-6^2$
4. $2(-3)^2+5(-4)$
5. $\frac{6}{3}*2$
6. $6/(3*2)$
7. $-\frac{1}{2}-\frac{3}{5}\cdot\frac{2}{3}$
8. $\frac{5}{8}\div\frac{1}{2}-\frac{5}{6}$
9. $3.2^2-6.9\div 2.3$
10. $8.2-3\div 1.2\cdot 2.1$
11. $2+3(-2)-7$
12. $8\div 2-3\cdot 2$
13. $3+6^2\div 12$
14. $5-4^2\div(-8)$
15. $-9-3\cdot 2\div 3(-2)$
16. $-2-3^2+(-2)^2$

17. $12 \div 6 \cdot 2 - 2^2$
18. $4 \cdot 3 \div 12 \cdot 2 - (-2)^2$
19. $(-5)^2 - 2(5)^2 \div 10$
20. $-3(4-7)+2$
21. $(-2+7)^2 - 10^2$
22. $10 - 7(3+2) + 7^2$
23. $-7 - 3(4 - 2 \cdot 8)$
24. $5 - 3[6 - (2+7)]$
25. $1 + 2[(-2)^3 - (-3)^2]$
26. $-3[2(7-5) \div 4 \cdot (-2) + (-3)^3]$
27. $-7^2 - [-20 - (-3)^2] - (-10)$
28. $4.7 - 3.2(4 - 1.2^3)$
29. $-5.4(6.1 - 3.1 \div 0.1) - 8.2^2$
30. $-7.3^2 + (-9.3)^2 - 37.8 \div 1.8$
31. $2 - 7(3^2 - 3 + 4 \cdot 3)$
32. $(\frac{1}{2})^2 - (-\frac{2}{3})^2$
33. $(\frac{1}{2})^3 + (-2)^3$
34. $(-\frac{1}{3})^2 - (-\frac{2}{3})^3$
35. $\frac{1}{3} - \frac{1}{2} \cdot \frac{1}{5}$
36. $\frac{5}{8} \div \frac{3}{2} \cdot \frac{14}{15}$
37. $5 \cdot \frac{2}{15} - (\frac{1}{2})^3$
38. $\frac{5}{17}(\frac{3}{5} - \frac{4}{35})$
39. $\frac{3}{16} \div (\frac{5}{12} - \frac{1}{2} + \frac{2}{3}) \cdot 4$
40. $(\frac{2}{3})^2 - (\frac{1}{2})^2$
41. $\frac{1}{2}[\frac{3}{4} \cdot (-4)^2 - 2]^2$
42. $6 \cdot [(\frac{2}{3})^2 - (\frac{1}{2})^2] \div (-2)^2$
43. $(-5)^2 + \frac{3}{2} - \frac{4}{2} + 2 \cdot 7$
44. $(-3.2 - 3.3)(8.7 - 4.7)(-4.7 + 3.9 + 2.1)$
45. $2 - [3 - (5-7)^2]3(6-32)$
46. $\frac{2+3 \cdot 6 - 4 \cdot 32}{2-32}$
47. $(2+7) \cdot 2 - \frac{23}{10} + \frac{9}{2} + 3^3$
48. $\frac{(-1-3)^2 - 1}{5 - 3 \cdot (-7+22) - 5}$
49. $(7 + 4 * (-2))/(-3 + (-2)^2)$
50. $4 + 3 * ((-3)^3 + 5^2)/6 - 2^2$

Answer

1. -22
3. -42
5. 4
7. $-\frac{9}{10}$
9. 7.24
11. -11
13. 6
15. -5
17. 0
19. 20

21. -75

23. 29

25. -33

27. -10

29. 67.22

31. -124

33. $-\frac{63}{8}$

35. $\frac{7}{30}$

37. $\frac{13}{24}$

39. $\frac{9}{7}$

41. 50

43. -17

45. $-\frac{1}{3}$

47. $\frac{5}{59}$

49. -1

Exercise 1.7.4 Order of Operations

1. Mary purchased 14 bottles of water at $\$0.75$ per bottle, 4 pounds of assorted candy at $\$3.50$ per pound, and 16 packages of microwave popcorn costing $\$0.50$ each for her party. What was her total bill?
2. Joe bought four 8-foot 2-by-4 boards for $\$24.00$. How much did he spend per linear foot?
3. Margaret bought two cases of soda at the local discount store for $\$23.52$. If each case contained 24 bottles, how much did she spend per bottle?
4. Billy earns $\$12.00$ per hour and "time and a half" for every hour he works over 40 hours a week. What is his pay for 47 hours of work this week?
5. Audry bought 4 bags of marbles each containing 15 assorted marbles. If she wishes to divide them up evenly between her 3 children, how many will each child receive?
6. Mark and Janet carpooled home from college for the Thanksgiving holiday. They shared the driving, but Mark drove twice as far as Janet. If Janet drove 135 miles, then how many miles was the entire trip?

Answer

1. $\$32.50$

3. $\$0.49$

5. 20 marbles

Exercise 1.7.5 Order of Operations with Absolute Values

Simplify.

1. $3+2|-5|$
2. $9-4|-3|$
3. $-(-|2|+|-10|)$
4. $-(|-6|-|-8|)$
5. $|-(40-|-22|)|$
6. $||-5|-|10||$
7. $-(|-8|-5)^2$

8. $(|-1|-|-2|)^2$
9. $-4+2|2^2-3^2|$
10. $-10-|4-5^2|$
11. $-|(-5)^2+4^2\div 8|$
12. $-(-3-[6-|-7|])$
13. $-2[7-(4+|-7|)]$
14. $3-7|-2-3|+4^3$
15. $7-5|6^2-5^2|+(-7)^2$
16. $(-4)^2-|-5+(-2)^3|-3^2$
17. $\frac{2}{3}-|\frac{1}{2}-(-\frac{4}{3})^2|$
18. $-30|\frac{10}{3}-\frac{1}{2}\div\frac{1}{5}|$
19. $(-4)^3-(2-|-4|)\div|-3^2+7|$
20. $[10-3(6-|-8|)]\div 4-5^2$

Answer

1. 13
3. -8
5. 18
7. -9
11. 6
13. -27
15. 8
17. 1
19. $-\frac{11}{18}$
21. -63

? Exercise 1.7.6 Order of Operations with Absolute Values

Find the distance between the given numbers on a number line.

1. $\frac{1}{2}$ and $-\frac{1}{4}$
2. $-\frac{3}{4}$ and $-\frac{2}{3}$
3. $-\frac{5}{8}$ and $-\frac{3}{4}$
4. $-\frac{7}{5}$ and $\frac{3}{7}$
5. -0.5 and 8.3
6. 10.7 and -2.8
7. $3\frac{1}{5}$ and $-2\frac{1}{3}$
8. $5\frac{3}{4}$ and 0

Answer

1. $\frac{3}{4}$ unit
3. $\frac{1}{8}$ unit
5. 8.8 units
7. $5\frac{8}{15}$ units

Exercise 1.7.7 Discussion Board Topics

1. Convert various examples in this section to equivalent expressions using text-based symbols.
2. What is PEMDAS and what is it missing?
3. Discuss the importance of proper grouping and give some examples.
4. Experiment with the order of operations on a calculator and share your results.

Answer

1. Answers may vary
3. Answers may vary

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1.8: Algebraic Expressions

The associative property of multiplication is valid for all numbers.

Associative Property of Multiplication

Let a, b, and c be any numbers. Then:

$$a \cdot (b \cdot c) = (a \cdot b) \cdot c$$

The associative property of multiplication is useful in a number of situations.

Example 1.8.1

Simplify: $-3(4y)$.

Solution

Currently, the grouping $-3(4y)$ demands that we first multiply 4 and y. However, we can use the associative property of multiplication to regroup, first multiplying -3 and 4.

$$\begin{aligned} -3(4y) &= (-3 \cdot 4)y \quad \text{The associative property of multiplication.} \\ &= -12y \quad \text{Multiply: } -3 \cdot 4 = -12 \end{aligned}$$

Thus, $-3(4y) = -12y$.

Exercise 1.8.1

Simplify: $2(3x)$.

Answer

$6x$

Let's look at another example.

Example 1.8.2

Simplify: $-2(-4xy)$.

Solution

Currently, the grouping $-2(-4xy)$ demands that we first multiply -4 and xy. However, we can use the associative property of multiplication to regroup, first multiplying -2 and -4.

$$\begin{aligned} -2(-4xy) &= (-2 \cdot (-4))xy \quad \text{The associative property of multiplication.} \\ &= 8xy \quad \text{Multiply: } -2 \cdot (-4) = 8 \end{aligned}$$

Thus, $-2(-4xy) = 8xy$.

Exercise 1.8.2

Simplify: $-3\left(-8u^2\right)$.

Answer

$24u^2$

In practice, we can move quicker if we perform the regrouping mentally, then simply write down the answer. For example:

$$-2(-4t) = 8t \quad \text{and} \quad 2\left(-5z^2\right) = -10z^2 \quad \text{and} \quad -3\left(4u^3\right) = -12u^3$$

The Distributive Property

We now discuss a property that couples addition and multiplication. Consider the expression $2 \cdot (3+5)$. The *Rules Guiding Order of Operations* require that we first simplify the expression inside the parentheses.

$$\begin{aligned} 2 \cdot (3+5) &= 2 \cdot 8 \quad \text{Add: } 3+5=8 \\ &= 16 \quad \text{Multiply: } 2 \cdot 8 = 16 \end{aligned}$$

Alternatively, we can instead distribute the 2 times each term in the parentheses. That is, we will first multiply the 3 by 2, then multiply the 5 by 2. Then we add the results.

$$\begin{aligned} 2 \cdot (3+5) &= 2 \cdot 3 + 2 \cdot 5 \quad \text{Distribute the 2.} \\ &= 6+10 \quad \text{Multiply: } 2 \cdot 3 = 6 \text{ and } 2 \cdot 5 = 10 \\ &= 16 \quad \text{Add: } 6+10=16 \end{aligned}$$

Note that both methods produce the same result, namely 16. This example demonstrates an extremely important property of numbers called the *distributive property*.

The Distributive Property

Let a, b, and c be any numbers. Then:

$$a \cdot (b+c) = a \cdot b + a \cdot c$$

That is, multiplication is distributive with respect to addition.

Example 1.8.3

Use the distributive property to expand $2(3x+7)$.

Solution

First distribute the 2 times each term in the parentheses. Then simplify.

$$\begin{aligned} 2(3x+7) &= 2(3x)+2(7) \quad \text{Use the distributive property.} \\ &= 6x+14 \quad \text{Multiply: } 2(3x)=6x \text{ and } 2(7)=14 \end{aligned}$$

Thus, $2(3x+7) = 6x+14$.

Exercise 1.8.3

Expand: $5(2y+7)$.

Answer

$10y+35$

Multiplication is also distributive with respect to subtraction.

Example 1.8.4

Use the distributive property to expand $-2(5y-6)$.

Solution

Change to addition by adding the opposite, then apply the distributive property.

$$\begin{aligned} -2(5y-6) &= -2(5y+(-6)) \quad \text{Add the opposite.} \\ &= -2(5y)+(-2)(-6) \quad \text{Use the distributive property.} \\ &= -10y+12 \quad \text{Multiply: } -2(5y)=-10y \text{ and } (-2)(-6)=12 \end{aligned}$$

Thus, $-2(5y-6) = -10y+12$

Exercise 1.8.4

Expand: $-3(2z-7)$.

Answer

$-6z+21$

Speeding Things Up a Bit

In Example 1.8.4, we changed the subtraction to addition, applied the distributive property, then several steps later we were finished. However, if you understand that subtraction is really the same as adding the opposite, and if you are willing to do a few steps in your head, you should be able to simply write down the answer immediately following the given problem.

If you look at the expression $-2(5y-6)$ from Example 1.8.4 again, only this time think "multiply -2 times $5y$, then multiply -2 times -6, then the result is immediate.

$$-2(5y-6)=-10y+12$$

Let's try this "speeding it up" technique in a couple more examples.

Example 1.8.5

Use the distributive property to expand $-3(-2x+5y-12)$.

Solution

To distribute the -3, we simply think as follows: "$-3(-2x)=6x$, $-3(5y)=-15y$, and $-3(-12)=36$." This sort of thinking allows us to write down the answer immediately without any additional steps.

$$-3(-2x+5y-12)=6x-15y+36$$

Exercise 1.8.5

Expand: $-3(-2a+3b-7)$.

Answer

$6a-9b+21$

Example 1.8.6

Use the distributive property to expand $-5(-2a-5b+8)$.

Solution

To distribute the -5, we simply think as follows: "$-5(-2a)=10a$, $-5(-5b)=25b$, and $-5(8)=-40$." This sort of thinking allows us to write down the answer immediately without any additional steps.

$$-5(-2a-5b+8)=10a+25b-40$$

Exercise 1.8.6

Expand: $-4(-x-2y-7)$.

Answer

$4x+8y+28$

Distributing a Negative Sign

Recall that negating a number is equivalent to multiplying the number by -1.

Multiplicative Property of Minus One

If a is any number, then:

$$(-1)a=-a$$

This means that if we negate an expression, it is equivalent to multiplying the expression by -1.

Example 1.8.7

Expand $-(7x-8y-10)$.

Solution

First, negating is equivalent to multiplying by -1. Then we can change subtraction to addition by "adding the opposite" and use the distributive property to finish the expansion.

$$\begin{aligned} -(7x-8y-10) &= -1(7x-8y-10) && \text{Negating is equivalent to multiplying by } -1 \\ &= -1(7x+(-8y)+(-10)) && \text{Add the opposite.} \\ &= -1(7x)+(-1)(-8y)+(-1)(-10) && \text{Distribute the } -1 \\ &= -7x+8y+10 && \text{Multiply.} \end{aligned}$$

Thus, $-(7x-8y-10) = -7x+8y+10$

Exercise 1.8.7

Expand: $-(-a-2b+11)$.

Answer

$a+2b-11$

While being mathematically precise, the technique of Example 1.8.7 can be simplified by noting that negating an expression surrounded by parentheses simply changes the sign of each term inside the parentheses to the opposite sign.

Once we understand this, we can simply "distribute the minus sign" and write:

$$-(7x-8y-10) = -7x+8y+10$$

In similar fashion,

$$-(-3a+5b-c) = 3a-5b+c$$

and,

$$-(-3x-8y+11) = 3x+8y-11$$

Combining Like Terms

We can use the distributive property to distribute a number times a sum.

$$a(b+c) = ab+ac$$

However, the distributive property can also be used in reverse, to "unmultiply" or factor an expression. Thus, we can start with the expression $ab+ac$ and "factor out" the common factor a as follows:

$$ab+ac = a(b+c)$$

You can also factor out the common factor on the right.

$$ac+bc = (a+b)c$$

We can use this latter technique to combine *like terms*.

Example 1.8.8

Simplify: $7x+5x$.

Solution

Use the distributive property to factor out the common factor x from each term, then simplify the result.

$$\begin{aligned} 7x+5x &= (7+5)x \quad \text{Factor out an } x \text{ using the distributive property.} \\ &= 12x \quad \text{Simplify: } 7+5=12 \end{aligned}$$

Thus, $7x+5x=12x$.

Exercise 1.8.8

Simplify: $3y+8y$.

Answer

$11y$

Example 1.8.9

Simplify: $-8a^2+5a^2$.

Solution

Use the distributive property to factor out the common factor a^2 from each term, then simplify the result.

$$\begin{aligned} -8a^2+5a^2 &= (-8+5)a^2 \quad \text{Factor out an } a^2 \text{ using the distributive property.} \\ &= -3a^2 \quad \text{Simplify: } -8+5=-3 \end{aligned}$$

Thus, $-8a^2+5a^2=-3a^2$.

Exercise 1.8.9

Simplify: $-5z^3+9z^3$.

Answer

$4z^3$

Examples 1.8.8 and 1.8.9 combine what are known as "like terms." Examples 1.8.8 and 1.8.9 also suggest a possible shortcut for combining like terms.

Like Terms

Two terms are called like terms if they have identical variable parts, which means that the terms must contain the same variables raised to the same exponents.

For example, $2x^2y$ and $11x^2y$ are like terms because they contain identical variables raised to the same exponents. On the other hand, $-3st^2$ and $4s^2t$ are not like terms. They contain the same variables, but the variables are not raised to the same exponents.

Consider the like terms $2x^2y$ and $11x^2y$. The numbers 2 and 11 are called the coefficients of the like terms. We can use the distributive property to combine these like terms as we did in Examples 1.8.8 and 1.8.9, factoring out the common factor x^2y.

$$\begin{aligned} 2x^2y+11x^2y &= (2+11)x^2y \\ &= 13x^2y \end{aligned}$$

However, a much quicker approach is simply to add the coefficients of the like terms, keeping the same variable part. That is, $2+11=13$, so:

$$2x^2y+11x^2y=13x^2y$$

This is the procedure we will follow from now on.

Example 1.8.10

Simplify: $-8w^2+17w^2$.

Solution

These are like terms. If we add the coefficients -8 and 17, we get 9. Thus:

$$-8w^2+17w^2=9w^2 \quad \text{Add the coefficients and repeat the variable part.}$$

Exercise 1.8.10

Simplify: $4ab-15ab$.

Answer

$-11ab$

Example 1.8.11

Simplify: $-4uv-9uv$.

Solution

These are like terms. If we add -4 and -9, we get -13. Thus:

$$-4uv-9uv=-13uv \quad \text{Add the coefficients and repeat the variable part.}$$

Exercise 1.8.11

Simplify: $-3xy-8xy$.

Answer

$-11xy$

Example 1.8.12

Simplify: $-3x^2y+2xy^2$

Solution

These are not like terms. They do not have the same variable parts. They do have the same variables, but the variables are not raised to the same exponents. Consequently, this expression is already simplified as much as possible.

$$-3x^2y+2xy^2 \quad \text{Unlike terms. Already simplified.}$$

Exercise 1.8.12

Simplify: $5ab+11bc$.

Answer

$5ab+11bc$

Sometimes we have more than just a single pair of like terms. In that case, we want to group together the like terms and combine them.

Example 1.8.13

Simplify: $-8u-4v-12u+9v$.

Solution

Use the associative and commutative property of addition to change the order and regroup, then combine line terms.

$$\begin{aligned} -8u-4v-12u+9v &= (-8u-12u)+(-4v+9v) \quad \text{Reorder and regroup.} \\ &= -20u+5v \quad \text{Combine like terms.} \end{aligned}$$

Note that $-8u-12u=-20u$ and $-4v+9v=5v$.

Alternate solution

You may skip the reordering and regrouping step if you wish, simply combining like terms mentally. That is, it is entirely possible to order your work as follows:

$$-8u-4v-12u+9v=-20u+5v \quad \text{Combine like terms.}$$

Exercise 1.8.13

Simplify: $-3z^2+4z-8z^2-9z$.

Answer

$-11z^2-5z$

In Example 1.8.13, the "Alternate solution" allows us to move more quickly and will be the technique we follow from here on, grouping and combining terms mentally.

Order of Operations

Now that we know how to combine like terms, let's tackle some more complicated expressions that require the *Rules Guiding Order of Operations*.

Rules Guiding Order of Operations

When evaluating expressions, proceed in the following order.

1. Evaluate expressions contained in grouping symbols first. If grouping symbols are nested, evaluate the expression in the innermost pair of grouping symbols first.
2. Evaluate all exponents that appear in the expression.
3. Perform all multiplications and divisions in the order that they appear in the expression, moving left to right.
4. Perform all additions and subtractions in the order that they appear in the expression, moving left to right.

Example 1.8.14

Simplify: $4(-3a+2b)-3(4a-5b)$.

Solution

Use the distributive property to distribute the 4 and the -3, then combine like terms.

$$\begin{aligned} 4(-3a+2b)-3(4a-5b) &= -12a+8b-12a+15b \quad \text{Distribute.} \\ &= -24a+23b \quad \text{Combine like terms.} \end{aligned}$$

Note that $-12a-12a=-24a$ and $8b+15b=23b$

Exercise 1.8.14

Simplify: $-2x-3(5-2x)$.

Answer

$4x-15$

Example 1.8.15

Simplify: $-2(3x-4y)-(5x-2y)$.

Solution

Use the distributive property to multiply -2 times $3x-4y$, then distribute the minus sign times each term of the expression $5x-2y$. After that, combine like terms.

$$\begin{aligned} -2(3x-4y)-(5x-2y) &= -6x+8y-5x+2y \quad \text{Distribute.} \\ &= -11x+10y \quad \text{Combine like terms.} \end{aligned}$$

Note that $-6x-5x=-11x$ and $8y+2y=10y$.

Exercise 1.8.15

Simplify: $-3(u+v)-(u-5v)$.

Answer

$-4u+2v$

Example 1.8.16

Simplify: $-2\left(x^2y-3xy^2\right)-4\left(-x^2y+3xy^2\right)$.

Solution

Use the distributive property to multiply -2 times x^2y-3xy^2 and -4 times $-x^2y+3xy^2$. After that, combine like terms.

$$\begin{aligned}-2\left(x^2y-3xy^2\right)-4\left(-x^2y+3xy^2\right) &= -2x^2y+6xy^2+4x^2y-12xy^2\\ &= 2x^2y-6xy^2\end{aligned}$$

Note that $-2x^2y+4x^2y=2x^2y$ and $6xy^2-12xy^2=-6xy^2$.

Exercise 1.8.16

Simplify: $8u^2v-3\left(u^2v+4uv^2\right)$.

Answer

$5u^2v-12uv^2$

When grouping symbols are nested, evaluate the expression inside the innermost pair of grouping symbols first.

Example 1.8.17

Simplify: $-2x-2(-2x-2[-2x-2])$.

Solution

Inside the parentheses, we have the expression $-2x-2[-2x-2]$. *The Rules Guiding Order of Operations* dictate that we should multiply first, expanding $-2[-2x-2]$ and combining like terms.

$$\begin{aligned}-2x-2(-2x-2[-2x-2]) &= -2x-2(-2x+4x+2)\\ &= -2x-2(2x+2)\end{aligned}$$

In the remaining expression, we again multiply first, expanding $-2(2x+2)$ and combining like terms.

$$\begin{aligned}&= -2x-4x-4\\ &= -6x-4\end{aligned}$$

Exercise 1.8.17

Simplify: $x-2[-x+4(x+1)]$.

Answer

$-5x-8$

Contributors

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1.E: Review Exercises and Sample Exam

Review Exercises

Exercise $1.E.1$ Real Numbers and the Number Line

Choose an appropriate scale and graph the following sets of real numbers on a number line.

1. $\{-4, 0, 4\}$
2. $\{-30, 10, 40\}$
3. $\{-12, -3, 9\}$
4. $\{-10, 8, 10\}$

Answer

1.

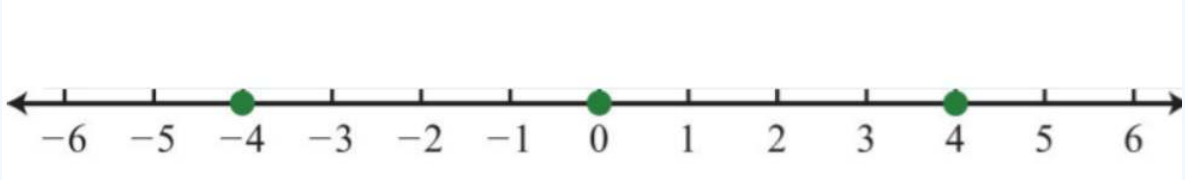


Figure 1.E.1

3.

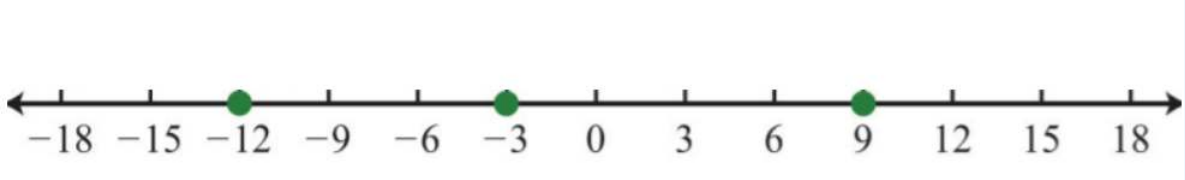


Figure 1.E.2

Exercise $1.E.2$ Real Numbers and the Number Line

Fill in the blank with $<, =,$ or $>$.

1. $0 \_\_ -9$
2. $-75 \_\_ -5$
3. $-12 \_\_ -(-3)$
4. $-(-23) \_\_ 23$
5. $|-20| \_\_ -|-30|$
6. $-|6| \_\_ -|-(-8)|$

Answer

1. $>$
3. $<$
5. $>$

Exercise $1.E.3$ Real Numbers and the Number Line

Determine the unknown.

1. $|?| = 2$
2. $|?| = 1$
3. $|?| = -7$
4. $|?| = 0$

Answer

1. ± 2

3. ø, No solution

Exercise $1.E.4$ Real Numbers and the Number Line

Translate the following into a mathematical statement.

1. Negative eight is less than or equal to zero.
2. Seventy-eight is not equal to twelve.
3. Negative nine is greater than negative ten.
4. Zero is equal to zero.

Answer

1. $-8 \leq 0$

3. $-9 > -10$

Exercise $1.E.5$ Adding and Subtracting Integers

Simplify.

1. $12 + (-7)$
2. $20 + (-32)$
3. $-23 - (-7)$
4. $-8 - (-8)$
5. $-3 - (-13) + (-1)$
6. $9 + (-15) - (-8)$
7. $(7 - 10) - 3$
8. $(-19 + 6) - 2$

Answer

1. 5

3. -16

5. 9

7. -6

Exercise $1.E.6$ Adding and Subtracting Integers

Find the distance between the given numbers on a number line.

1. -8 and 14
2. -35 and -6
3. What is 2 less than 17?

4. What is 3 less than -20?
5. Subtract 30 from the sum of 8 and 12.
6. Subtract 7 from the difference of -5 and 7.
7. An airplane flying at $22,000$ feet descended $8,500$ feet and then ascended $5,000$ feet. What is the new altitude of the airplane?
8. The width of a rectangle is 5 inches less than its length. If the length measures 22 inches, then determine the width.

Answer

1. 22 units
3. 15
5. -10
7. $18,500$ feet

Exercise $1.E.7$ Multiplying and Dividing Integers

Simplify.

1. $10 \div 5 \cdot 2$
2. $36 \div 6 \cdot 2$
3. $-6(4) \div 2(-3)$
4. $120 \div (-5)(-3)(-2)$
5. $-8(-5) \div 0$
6. $-24(0) \div 8$
7. Find the product of -6 and 9.
8. Find the quotient of -54 and -3.
9. James was able to drive 234 miles on 9 gallons of gasoline. How many miles per gallon did he get?
10. If a bus travels at an average speed of 54 miles per hour for 3 hours, then how far does the bus travel?

Answer

1. 4
3. 36
5. Undefined
7. -54
9. 26 miles per gallon

Exercise $1.E.8$ Fractions

Reduce each fraction to lowest terms.

1. $\frac{180}{300}$
2. $\frac{252}{324}$
3. Convert to a mixed number: $\frac{23}{8}$.
4. Convert to an improper fraction: $3\frac{5}{9}$.

Answer

1. $\frac{3}{5}$
3. $2\frac{7}{8}$

Exercise 1.E.9 Fractions

Simplify.

1. $\frac{3}{5}\left(-\frac{2}{7}\right)$
2. $-\frac{5}{8}\left(-\frac{1}{3}\right)$
3. $-\frac{3}{4} \div \frac{6}{7}$
4. $\frac{4}{15} \div \frac{28}{3}$
5. $4\frac{4}{5} \div 6$
6. $5 \div 8\frac{1}{3}$
7. $\frac{5}{4} \div \frac{15}{2} \cdot 6$
8. $\frac{5}{24} \div \frac{3}{2} \div \frac{5}{12}$
9. $\frac{1}{12} - \frac{1}{4}$
10. $\frac{5}{6} - \frac{3}{14}$
11. $\frac{3}{4} + \frac{2}{3} - \frac{1}{12}$
12. $\frac{3}{10} + \frac{5}{12} - \frac{1}{6}$
13. Subtract $\frac{2}{3}$ from the sum of $-\frac{1}{2}$ and $\frac{2}{9}$.
14. Subtract $\frac{5}{6}$ from the difference of $\frac{1}{3}$ and $\frac{7}{2}$.
15. If a bus travels at an average speed of 54 miles per hour for $2\frac{1}{3}$ hours, then how far does the bus travel?
16. Determine the length of fencing needed to enclose a rectangular pen with dimensions $12\frac{1}{2}$ feet by $8\frac{3}{4}$ feet.

Answer

1. $-\frac{6}{35}$
3. $-\frac{7}{8}$
5. $\frac{4}{5}$
7. 1
9. $-\frac{1}{6}$
11. $\frac{4}{3}$
13. $-\frac{17}{18}$
15. 126 miles

Exercise 1.E.10 Decimals and Percents

1. Write as a mixed number: 5.32.
2. Write as a decimal: $7\frac{3}{25}$

Answer

1. $5\frac{8}{25}$

Exercise 1.E.11 Decimals and Percents

Perform the operations.

1. $6.032 + 2.19$
2. $12.106 - 9.21$
3. 4.23×5.13
4. $9.246 \div 4.02$

Answer

1. 8.222
3. 21.6999

Exercise $1.E.12$ Decimals and Percents

Convert to a decimal.

1. 7.2%
2. $5\frac{3}{8}\%$
3. 147%
4. $27\frac{1}{2}\%$

Answer

1. 0.072
3. 1.47

Exercise $1.E.13$ Decimals and Percents

Convert to a percent.

1. 0.055
2. 1.75
3. $\frac{9}{10}$
4. $\frac{5}{6}$
5. Mary purchased 3 boxes of t-shirts for a total of $\$126$. If each box contains 24 t-shirts, then what is the cost of each t-shirt?
6. A retail outlet is offering 12% off the original $\$39.99$ price of tennis shoes. What is the price after the discount?
7. If an item costs $\$129.99$, then what is the total after adding $7\frac{1}{4}\%$ sales tax?
8. It is estimated that 8.3% of the total student population carpools to campus each day. If there are $13,000$ students, then estimate the number of students that carpool to campus.

Answer

1. 5.5%
3. 90%
5. $\$1.75$
7. $\$139.41$

Exercise $1.E.14$ Exponents and Square Roots

Simplify.

1. 8^2
2. $(-5)^2$
3. -4^2
4. $-(-3)^2$
5. $(\frac{2}{9})^2$
6. $(1\frac{2}{3})^2$
7. 3^3
8. $(-4)^3$
9. $(\frac{2}{5})^3$

10. $(-\frac{1}{6})^3$
11. $-(-2)^4$
12. $-(-1)^5$
13. $\sqrt{49}$
14. $\sqrt{225}$
15. $2\sqrt{25}$
16. $-\sqrt{121}$
17. $3\sqrt{50}$
18. $-4\sqrt{12}$
19. $4\sqrt{9}$
20. $8\sqrt{25}$
21. Calculate the area of a square with sides measuring 3 centimeters. $(A = s^2)$
22. Calculate the volume of a cube with sides measuring 3 centimeters. $(V = s^3)$
23. Determine the length of the diagonal of a square with sides measuring 3 centimeters.
24. Determine the length of the diagonal of a rectangle with dimensions 2 inches by 4 inches.

Answer

1. 64
3. -16
5. $\frac{4}{81}$
7. 27
9. $\frac{81}{25}$
11. -16
13. 7
15. 10
17. $15\sqrt{2}$
19. $\frac{2}{3}$
21. 9 square centimeters
23. $3\sqrt{2}$ centimeters

? Exercise 1.E. 15 Order of Operations

Simplify.

1. $-5(2) - 7^2$
2. $1 - 4^2 + 2(-3)^2$
3. $2 + 3(6 - 2 \cdot 4)^3$
4. $5 - 3(8 - 3 \cdot 4)^2$
5. $-2^3 + 6(3^2 - 4) + (-3)^2$
6. $5^2 - 40 \div 5(-2)^2 - (-4)$
7. $\frac{3}{4}[\frac{2}{9}(-3)^2 - 4]^2$
8. $(\frac{1}{2})^2 - \frac{3}{4} \div \frac{9}{16} - \frac{1}{3}$
9. $\frac{2-3(6-3^2)^2}{4 \cdot 5 - 5^2}$
10. $\frac{(2 \cdot 8 - 6^2)^2 - 10}{273 - (2(-5)^3 - 7)}$
11. $8 - 5|3 \cdot 4 - (-2)^4|$
12. $|14| - |-3 - 52|$

Answer

1. -59
3. -22
5. 31
7. 3
9. 5
11. -12

Exercise $1.E.16$ Order of Operations

Find the distance between the given numbers on a number line.

1. -14 and 22
2. -42 and -2
3. $\frac{7}{8}$ and $-\frac{1}{5}$
4. $-5\frac{1}{2}$ and $-1\frac{1}{4}$

Answer

1. 36 units
3. $\frac{43}{40}$ units

Sample Exam

Exercise $1.E.17$

1. List three integers greater than -10.
2. Determine the unknown(s): $|?| = 13$.
3. Fill in the blank with $<, =,$ or $: -|-100|\_9^2$.
4. Convert to a fraction: $33\frac{1}{3}\%$
5. Convert to a percent: $2\frac{3}{4}$.
6. Reduce: $\frac{75}{225}$.

Answer

1. $\{-5, 0, 5\}$(answers may vary)
3. $<$
5. 275%

Exercise $1.E.18$

Calculate the following.

1. a. $(-7)^2$; b. $-(-7)^2$; c. -7^2
2. a. $(-3)^3$; b. $-(-3)^3$; c. -3^3
3. a. $|10|$; b. $-|-10|$; c. $-|10|$

Answer

1. a. 49; b. -49; c. -49
3. a. 10; b. -10; c. -10

Exercise $1.E.19$

Simplify.

1. $-(-(-1))$
2. $\frac{2}{3}+\frac{1}{5}-\frac{3}{10}$
3. $10-(-12)+(-8)-20$
4. $-8(4)(-3)\div 2$
5. $\frac{1}{2}\cdot(-\frac{4}{5})\div\frac{14}{15}$
6. $\frac{3}{5}\cdot\frac{1}{2}-\frac{2}{3}$
7. $4\cdot 5-20\div 5\cdot 2$
8. $10-7(3-8)-5^2$
9. $3+2|-2^2-(-1)|+(-2)^2$
10. $\frac{1}{3}[5^2-(7-|-2|)+15\cdot 2\div 3]$
11. $\sqrt{116}$
12. $3\sqrt{72}$

Answer

2. $\frac{17}{30}$
4. 48
6. $-\frac{11}{30}$
8. 20
10. 10
12. $18\sqrt{2}$

Exercise $1.E.20$

1. Subtract 2 from the sum of 8 and -10.
2. Subtract 10 from the product of 8 and -10.
3. A student earns $9, 8, 10, 7$,and 8 points on the first 5 chemistry quizzes. What is her quiz average?
4. An $8\frac{3}{4}$ foot plank is cut into 5 pieces of equal length. What is the length of each piece?

Answer

2. -90
4. $1\frac{3}{4}$ feet

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CHAPTER OVERVIEW

2: Solving Linear Equations and Inequalities

2.1: Solve Equations Using the Subtraction and Addition Properties of Equality

2.1E: Exercises

2.2: Solve Equations using the Division and Multiplication Properties of Equality

2.2E: Exercises

2.3: Solve Equations with Variables and Constants on Both Sides

2.3E: Exercises

2.4: Use a General Strategy to Solve Linear Equations

2.4E: Exercises

2.5: Solve Equations with Fractions or Decimals

2.5E: Exercises

2.6: Solve a Formula for a Specific Variable

2.6E: Exercises

2.7: Solve Linear Inequalities

2.7E: Exercises

2.8: Chapter 2 Review Exercises

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2.1: Solve Equations Using the Subtraction and Addition Properties of Equality

Learning Objectives

By the end of this section, you will be able to:

- Verify a solution of an equation
- Solve equations using the Subtraction and Addition Properties of Equality
- Solve equations that require simplification
- Translate to an equation and solve
- Translate and solve applications

Quiz

Before you get started, take this readiness quiz.

1. Evaluate $x+4$ when $x=-3$.
   If you missed this problem, review Exercise 1.5.25.
2. Evaluate $15-y$ when $y=-5$.
   If you missed this problem, review Exercise 1.5.31.
3. Simplify $4(4n+1)-15n$.
   If you missed this problem, review Exercise 1.10.49.
4. Translate into algebra "5 is less than x."
   If you missed this problem, review Exercise 1.3.43.

Verify a Solution of an Equation

Solving an equation is like discovering the answer to a puzzle. The purpose in solving an equation is to find the value or values of the variable that make each side of the equation the same – so that we end up with a true statement. Any value of the variable that makes the equation true is called a solution to the equation. It is the answer to the puzzle!

Definition: Solution of an Equation

A **solution of an equation** is a value of a variable that makes a true statement when substituted into the equation.

TO DETERMINE WHETHER A NUMBER IS A Solution TO AN EQUATION

1. Substitute the number in for the variable in the equation.
2. Simplify the expressions on both sides of the equation.
3. Determine whether the resulting equation is true (the left side is equal to the right side)
   - If it is true, the number is a solution.
   - If it is not true, the number is not a solution.

Example 2.1.1

Determine whether $x=\frac{3}{2}$ is a solution of $4x-2=2x+1$.

Solution

Since a solution to an equation is a value of the variable that makes the equation true, begin by substituting the value of the solution for the variable.

	$4x-2=2x+1$
Substitute $\frac{3}{2}$ for x.	$4\left(\frac{3}{2}\right)-2\stackrel{?}{=}2\left(\frac{3}{2}\right)+1$
Multiply.	$6-2\stackrel{?}{=}3+1$
Subtract.	$4=4\checkmark$

Since $x=\frac{3}{2}$ results in a true equation (4 is in fact equal to 4), $\frac{3}{2}$ is a solution to the equation $4x-2=2x+1$.

Try It 2.1.2

Is $y = \frac{4}{3}$ a solution of $9y + 2 = 6y + 3$?

Answer

no

Try It 2.1.3

Is $y = \frac{7}{5}$ a solution of $5y + 3 = 10y - 4$?

Answer

yes

Solve Equations Using the Subtraction and Addition Properties of Equality

We are going to use a model to clarify the process of solving an equation. An envelope represents the variable – since its contents are unknown – and each counter represents one. We will set out one envelope and some counters on our workspace, as shown in Figure 2.1.1. Both sides of the workspace have the same number of counters, but some counters are "hidden" in the envelope. Can you tell how many counters are in the envelope?

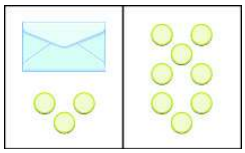


Figure 2.1.1: The illustration shows a model of an equation with one variable. On the left side of the workspace is an unknown (envelope) and three counters, while on the right side of the workspace are eight counters.

What are you thinking? What steps are you taking in your mind to figure out how many counters are in the envelope?

Perhaps you are thinking: "I need to remove the 3 counters at the bottom left to get the envelope by itself. The 3 counters on the left can be matched with 3 on the right and so I can take them away from both sides. That leaves five on the right—so there must be 5 counters in the envelope." See Figure 2.1.2 for an illustration of this process.

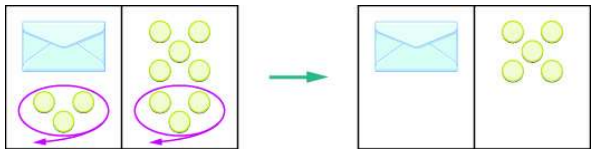


Figure 2.1.2: The illustration shows a model for solving an equation with one variable. On both sides of the workspace remove three counters, leaving only the unknown (envelope) and five counters on the right side. The unknown is equal to five counters.

What algebraic equation would match this situation? In Figure 2.1.3 each side of the workspace represents an expression and the center line takes the place of the equal sign. We will call the contents of the envelope x.

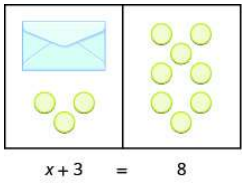


Figure 2.1.3: The illustration shows a model for the equation $x + 3 = 8$.

Let's write algebraically the steps we took to discover how many counters were in the envelope:

Table 2.1.1

	$x + 3 = 8$
First, we took away three from each side.	$x + 3 - 3 = 8 - 3$
Then we were left with five.	$x = 5$

Check:

Five in the envelope plus three more does equal eight!

$$5+3=8$$

Our model has given us an idea of what we need to do to solve one kind of equation. The goal is to isolate the variable by itself on one side of the equation. To solve equations such as these mathematically, we use the **Subtraction Property of Equality**.

SUBTRACTION PROPERTY OF EQUALITY

For any numbers a, b, and c,

$$\begin{aligned} &\text{If} \quad a=b \\ &\text{then} \quad a-c=b-c \end{aligned}$$

When you subtract the same quantity from both sides of an equation, you still have equality.

Note

Doing the Manipulative Mathematics activity "Subtraction Property of Equality" will help you develop a better understanding of how to solve equations by using the **Subtraction Property of Equality**.

Let's see how to use this property to solve an equation. Remember, the goal is to isolate the variable on one side of the equation. And we check our solutions by substituting the value into the equation to make sure we have a true statement.

Example 2.1.4

Solve: $y+37=-13$.

Solution

To get y by itself, we will undo the addition of 37 by using the Subtraction Property of Equality.

	$y+37=-13$
Subtract 37 from each side to 'undo' the addition.	$y+37=-13$ $-37 \quad -37$
Simplify.	$y=-50$
Check:	$y+37=-13$
Substitute $y=-50$	$-50+37=-13$
	$-13 \stackrel{?}{=} -13$ ✓

Since y=−50 makes y+37=−13 a true statement, we have the solution to this equation.

Try It 2.1.5

Solve: $x+19=-27$.

Answer

$x=-46$

Try It 2.1.6

Solve: $x+16=-34$.

Answer

$x=-50$

What happens when an equation has a number subtracted from the variable, as in the equation $x-5=8$? We use another property of equations to solve equations where a number is subtracted from the variable. We want to isolate the variable, so to 'undo' the subtraction we will add the number to both sides. We use the **Addition Property of Equality**.

ADDITION PROPERTY OF EQUALITY

For any numbers a, b, and c,

$$\begin{aligned} &\text{If} \quad a=b \\ &\text{then} \quad a+c=b+c \end{aligned}$$

When you add the same quantity from both sides of an equation, you still have equality.

In Exercise 2.1.4, 37 was added to the y and so we subtracted 37 to 'undo' the addition. In Exercise 2.1.7, we will need to 'undo' subtraction by using the **Addition Property of Equality**.

Example 2.1.7

Solve: $a-28=-37$.

Solution

	$a-28=-37$
Add 28 to each side to 'undo' the subtraction.	$a-28+28=-37+28$
Simplify.	$a=-9$
Check:	$a-28=-37$
Substitute $a=-9$	$-9-28=-37$
	$-37\stackrel{?}{=}-37$ ✓
	The solution to $a-28=-37$ is $a=-9$.

Try It 2.1.8

Solve: $n-61=-75$.

Answer

$n=-14$

Try It 2.1.9

Solve: $p-41=-73$.

Answer

$p=-32$

Example 2.1.10

Solve: $x-\frac{5}{8}=\frac{3}{4}$

Solution

	$x-\frac{5}{8}=\frac{3}{4}$
Use the Addition Property of Equality.	$x-\frac{5}{8}+\frac{5}{8}=\frac{3}{4}+\frac{5}{8}$
Find the LCD to add the fractions on the right.	$x=\frac{6}{8}+\frac{5}{8}$
Simplify.	$x=\frac{11}{8}$

Check:	$x-\frac{5}{8}=\frac{3}{4}$	
Substitute $x=\frac{11}{8}$	$\frac{11}{8}-\frac{5}{8}\stackrel{?}{=}\frac{3}{4}$	
Subtract.	$\frac{6}{8}\stackrel{?}{=}\frac{3}{4}$	
Simplify.	$\frac{3}{4}=\frac{3}{4}\checkmark$	
		The solution to $x-\frac{5}{8}=\frac{3}{4}$ is $x=\frac{11}{8}$.

Try It 2.1.11

Solve: $p-\frac{2}{3}=\frac{5}{6}$.

Answer

$p=\frac{9}{6}p=\frac{3}{2}$

Try It 2.1.12

Solve: $q-\frac{1}{2}=\frac{5}{6}$.

Answer

$q=\frac{4}{3}$

The next example will be an equation with decimals.

Example 2.1.13

Solve: $n-0.63=-4.2$.

Solution

		$n-0.63=-4.2$
Use the Addition Property of Equality.		$n-0.63+0.63=-4.2+0.63$
Add.		$n=-3.57$
Check:	$n=-3.57$	
Let $n=-3.57$.	$-3.57-0.63\stackrel{?}{=}-4.2$	
	$-4.2=-4.2\checkmark$	

Try It 2.1.14

Solve: $b-0.47=-2.1$.

Answer

$b=-1.63$

Try It 2.1.15

Solve: $c-0.93=-4.6$.

Answer

$c=-3.67$

Solve Equations That Require Simplification

In the previous examples, we were able to isolate the variable with just one operation. Most of the equations we encounter in algebra will take more steps to solve. Usually, we will need to simplify one or both sides of an equation before using the Subtraction or Addition Properties of Equality.

You should always simplify as much as possible before you try to isolate the variable. Remember that to simplify an expression means to do all the operations in the expression. Simplify one side of the equation at a time. Note that **simplification** is different from the process used to solve an equation in which we apply an operation to both sides.

Example 2.1.16: How to Solve Equations That Require Simplification

Solve: $9x-5-8x-6=7$.

Solution

Step 1. Simplify the expressions on each side as much as possible.	Rearrange the terms, using the Commutative Property of Addition. Combine like terms. Notice that each side is now simplified as much as possible.	$9x-5-8x-6=7$ $9x-8x-5-6=7$ $x-11=7$
Step 2. Isolate the variable.	Now isolate x. Undo subtraction by adding 11 to both sides.	$x-11+11=7+11$
Step 3. Simplify the expressions on both sides of the equation.		$x=18$
Step 4. Check the solution.		**Check:** Substitute $x=18$. $9x-5-8x-6=7$ $9(18)-5-8(18)-6\stackrel{?}{=}7$ $162-5-144-6\stackrel{?}{=}7$ $157-144-6\stackrel{?}{=}7$ $13-6\stackrel{?}{=}7$ $7=7$ ✓ The solution to $9x-5-8x-6=7$ is $x=18$.

Try It 2.1.17

Solve: $8y-4-7y-7=4$.

Answer

$y=15$

Try It 2.1.18

Solve: $6z+5-5z-4=3$.

Answer

$z=2$

Example 2.1.19

Solve: $5(n-4)-4n=-8$.

Solution

We simplify both sides of the equation as much as possible before we try to isolate the variable.

	$5(n-4)-4n=-8$
Distribute on the left.	$5n-20-4n=-8$
Use the Commutative Property to rearrange terms.	$5n-4n-20=-8$
Combine like terms.	$n-20=-8$
Each side is as simplified as possible. Next, isolate n.	
Undo subtraction by using the Addition Property of Equality.	$n-20+20=-8+20$
Add.	$n=12$
Check. Substitute $n=12$. $5(n-4)-4n=-8$ $5(12-4)-4(12)\stackrel{?}{=}-8z$ $5(8)-48\stackrel{?}{=}-8$ $40-48\stackrel{?}{=}-8$ $-8=-8$ ✓	
	The solution to $5(n-4)-4n=-8$ is $n=12$.

Try It 2.1.20

Solve: $5(p-3)-4p=-10$.

Answer

$p=5$

Try It 2.1.21

Solve: $4(q+2)-3q=-8$.

Answer

$q=-16$

Example 2.1.22

Solve: $3(2y-1)-5y=2(y+1)-2(y+3)$.

Solution

We simplify both sides of the equation before we isolate the variable.

	$3(2y-1)-5y=2(y+1)-2(y+3)$
Distribute on both sides.	$6y-3-5y=2y+2-2y-6$
Use the Commutative Property of Addition.	$6y-5y-3=2y-2y+2-6$
Combine like terms.	$y-3=-4$
Each side is as simplified as possible. Next, isolate y.	
Undo subtraction by using the Addition Property of Equality.	$y-3+3=-4+3$

Add.	$y = -1$
Check. Let y=−1. $3(2y-1)-5y=2(y+1)-2(y+3)$ $3(2(-1)-1)-5(-1) \stackrel{?}{=} 2(-1+1)-2(-1+3)$ $3(-2-1)+5 \stackrel{?}{=} 2(0)-2(2)$ $3(-3)+5 \stackrel{?}{=} -4$ $-9+5 \stackrel{?}{=} -4$ $-4=-4$ ✓	
	The solution to $3(2y-1)-5y=2(y+1)-2(y+3)3(2y-1)-5y=2(y+1)-2(y+$ is $y=-1$.

Try It 2.1.23

Solve: $4(2h-3)-7h=6(h-2)-6(h-1)$.

Answer

$h=6$

Try It 2.1.24

Solve: $2(5x+2)-9x=3(x-2)-3(x-4)$.

Answer

$x=2$

Translate to an Equation and Solve

To solve applications algebraically, we will begin by translating from English sentences into equations. Our first step is to look for the word (or words) that would translate to the **equals sign**. Here are some of the words that are commonly used.

Equals =

- is
- is equal to
- is the same as
- the result is
- gives
- was
- will be

The steps we use to translate a sentence into an equation are listed below.

TRANSLATE AN ENGLISH SENTENCE TO AN ALGEBRAIC EQUATION

1. Locate the “equals” word(s). Translate to an equals sign (=).
2. Translate the words to the left of the “equals” word(s) into an algebraic expression.
3. Translate the words to the right of the “equals” word(s) into an algebraic expression.

Example 2.1.25

Translate and solve: Eleven more than x is equal to 54.

Solution

Translate.	Eleven more than x is equal to 54 $x+11 = 54$
Subtract 11 from both sides.	$x+11-11 = 54-11$
Simplify.	$x = 43$
Check: Is 54 eleven more than 43? $43+11 = 54\checkmark$	

Try It 2.1.26

Translate and solve: Ten more than x is equal to 41.

Answer

$x+10=41; \quad x=31$

Try It 2.1.27

Translate and solve: Twelve less than x is equal to 51.

Answer

$y-12=51; \quad y=63$

Example 2.1.28

Translate and solve: The difference of $12t$ and $11t$ is -14.

Solution

Translate.	The difference of $12t$ and $11t$ is -14 $12t-11t = -14$
Simplify.	$t = -14$
Check: $12(-14)-11(-14) \stackrel{?}{=} -14$ $-168+154 \stackrel{?}{=} -14$ $-14 = -14\checkmark$	

Try It 2.1.29

Translate and solve: The difference of $4x$ and $3x$ is 14.

Answer

$4x-3x=14; \quad x=14$

Try It 2.1.30

Translate and solve: The difference of $7a$ and $6a$ is -8.

Answer

$7a-6a=-8; \quad a=-8$

Translate and Solve Applications

Most of the time a question that requires an algebraic solution comes out of a real life question. To begin with that question is asked in English (or the language of the person asking) and not in math symbols. Because of this, it is an important skill to be able to translate an everyday situation into algebraic language.

We will start by restating the problem in just one sentence, assign a variable, and then translate the sentence into an equation to solve. When assigning a variable, choose a letter that reminds you of what you are looking for. For example, you might use q for the number of quarters if you were solving a problem about coins.

Example 2.1.31: How to Solve Translate and Solve Applications

The MacIntyre family recycled newspapers for two months. The two months of newspapers weighed a total of 57 pounds. The second month, the newspapers weighed 28 pounds. How much did the newspapers weigh the first month?

Solution

Step 1. Read the problem. Make sure all the words and ideas are understood.	The problem is about the weight of newspapers.	
Step 2. Identify what we are asked to find.	What are we asked to find?	"How much did the newspapers weigh the 2nd month?"
Step 3. Name what we are looking for. Choose a variable to represent that quantity.	Choose a variable.	Let w = weight of the newspapers the 1st month
Step 4. Translate into an equation. It may be helpful to restate the problem in one sentence with the important information.	Restate the problem. We know the weight of the newspapers the second month is 28 pounds. Translate into an equation, using the variable w.	Weight of newspapers the 1st month plus the weight of the newspapers the 2nd month equals 57 pounds. Weight from 1st month plus 28 equals 57. $w + 28 = 57$
Step 5. Solve the equation using good algebra techniques.	Solve.	$w + 28 - 28 = 57 - 28$ $w = 29$
Step 6. Check the answer in the problem and make sure it makes sense.	Does 1st month's weight plus 2nd month's weight equal 57 pounds?	**Check:** Does 1st month's weight plus 2nd month's weight equal 57 pounds? $29 + 28 \stackrel{?}{=} 57$ $57 = 57$ ✓
Step 7. Answer the question with a complete sentence.	Write a sentence to answer "How much did the newspapers weigh the 2nd month?"	The 2nd month the newspapers weighed 29 pounds.

Try It 2.1.32

Translate into an algebraic equation and solve:

The Pappas family has two cats, Zeus and Athena. Together, they weigh 23 pounds. Zeus weighs 16 pounds. How much does Athena weigh?

Answer

7 pounds

Try It 2.1.33

Translate into an algebraic equation and solve:

Sam and Henry are roommates. Together, they have 68 books. Sam has 26 books. How many books does Henry have?

Answer

42 books

SOLVE AN APPLICATION.

1. **Read** the problem. Make sure all the words and ideas are understood.
2. **Identify** what we are looking for.
3. **Name** what we are looking for. Choose a variable to represent that quantity.
4. **Translate** into an equation. It may be helpful to restate the problem in one sentence with the important information.
5. **Solve** the equation using good algebra techniques.
6. **Check** the answer in the problem and make sure it makes sense.
7. **Answer** the question with a complete sentence.

Example 2.1.34

Randell paid $28,675 for his new car. This was $875 less than the sticker price. What was the sticker price of the car?

Solution

Step 1. Read the problem.

Step 2. Identify what we are looking for. "What was the sticker price of the car?"

Step 3. Name what we are looking for. Choose a variable to represent that quantity. Let s = the sticker price of the car.

Step 4. Translate into an equation. Restate the problem in one sentence. $28675 is $875 less than the sticker price

$28675 is $875 less than s

Step 5. Solve the equation.

$$28675 = s - 875$$
$$28675 + 875 = s - 875 + 875$$
$$29550 = s$$

Step 6. Check the answer.

Is $875 less than $29550 equal to $28675?

$$29550 - 875 \stackrel{?}{=} 28675$$
$$28675 = 28675 \checkmark$$

Step 7. Answer the question with a complete sentence. The sticker price of the car was $29550.

Try It 2.1.35

Translate into an algebraic equation and solve:

Eddie paid $19875 for his new car. This was $1025 less than the sticker price. What was the sticker price of the car?

Answer

$20900

Try It 2.1.36

Translate into an algebraic equation and solve:

The admission price for the movies during the day is $7.75. This is $3.25 less the price at night. How much does the movie cost at night?

Answer

$11.00

Key Concepts

- **To Determine Whether a Number is a Solution to an Equation**
  1. **Substitute the number in for the variable in the equation.**
  2. **Simplify the expressions on both sides of the equation.**
  3. **Determine whether the resulting statement is true.**
     - If it is true, the number is a solution.
     - If it is not true, the number is not a solution.
- **Addition Property of Equality**
  - For any numbers a, b and c, if $a = b$, then $a + c = b + c$.
- **Subtraction Property of Equality**
  - For any numbers a, b and c, if $a = b$, then $a - c = b - c$.
- **To Translate a Sentence to an Equation**
  1. Locate the “equals” word(s). Translate to an equal sign (=).
  2. Translate the words to the left of the “equals” word(s) into an algebraic expression.
  3. Translate the words to the right of the “equals” word(s) into an algebraic expression.
- **To Solve an Application**
  1. Read the problem. Make sure all the words and ideas are understood.
  2. Identify what we are looking for.
  3. Name what we are looking for. Choose a variable to represent that quantity.
  4. Translate into an equation. It may be helpful to restate the problem in one sentence with the important information.
  5. Solve the equation using good algebra techniques.
  6. Check the answer in the problem and make sure it makes sense.
  7. Answer the question with a complete sentence.

Glossary

solution of an equation

A solution of an equation is a value of a variable that makes a true statement when substituted into the equation.

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2.1E: Exercises

Practice Makes Perfect

Verify a Solution of an Equation

In the following exercises, determine whether the given value is a solution to the equation.

Exercise 2.1E.1

Is $y = \frac{5}{3}$ a solution of
$6y + 10 = 12y$?

Answer

Yes

Exercise 2.1E.2

Is $x = \frac{9}{4}$ a solution of
$4x + 9 = 8x$?

Exercise 2.1E.3

Is $u = -\frac{1}{2}$ a solution of
$8u - 1 = 6u$?

Answer

No

Exercise 2.1E.4

Is $v = -\frac{1}{3}$ a solution of
$9v - 2 = 3v$?

Solve Equations using the Subtraction and Addition Properties of Equality

In the following exercises, solve each equation using the Subtraction and Addition Properties of Equality.

Exercise 2.1E.5

$x + 24 = 35$

Answer

x = 11

Exercise 2.1E.6

$x + 17 = 22$

Exercise 2.1E.7

$y + 45 = -66$

Answer

y = -111

Exercise 2.1E.8

$y + 39 = -83$

Exercise 2.1E.9

$b + \frac{1}{4} = \frac{3}{4}$

Answer

$b = \frac{1}{2}$

Exercise 2.1E.10

$a + \frac{2}{5} = \frac{4}{5}$

Exercise 2.1E.11

$p + 2.4 = -9.3$

Answer

p = -11.7

Exercise 2.1E.12

$m + 7.9 = 11.6$

Exercise 2.1E.13

$a - 45 = 76$

Answer

a = 121

Exercise 2.1E.14

$a - 30 = 57$

Exercise 2.1E.15

$m - 18 = -200$

Answer

m = -182

Exercise 2.1E.16

$m - 12 = -12$

Exercise 2.1E.17

$x - \frac{1}{3} = 2$

Answer

$x = \frac{7}{3}$

Exercise 2.1E.18

$x - \frac{1}{5} = 4$

Exercise 2.1E.19

$y - 3.8 = 10$

Answer

y = 10.8

Exercise 2.1E.20

$y - 7.2 = 5$

Exercise 2.1E.21

$x - 165 = -420$

Answer

$x = -255$

Exercise 2.1E.22

$z - 101 = -314$

Exercise 2.1E.23

$z + 0.52 = -8.5$

Answer

$z = -9.02$

Exercise 2.1E.24

$x + 0.93 = -4.1$

Exercise 2.1E.25

$q + \frac{3}{4} = \frac{1}{2}$

Answer

$q = -\frac{1}{4}$

Exercise 2.1E.26

$p+\frac{1}{3}=\frac{5}{6}$

Exercise 2.1E.27

$p-\frac{2}{5}=\frac{2}{3}$

Answer

$p=\frac{16}{15}$

Exercise 2.1E.28

$y-\frac{3}{4}=\frac{3}{5}$

Solve Equations that Require Simplification

In the following exercises, solve each equation.

Exercise 2.1E.29

$c+31-10=46$

Answer

c = 25

Exercise 2.1E.30

$m+16-28=5$

Exercise 2.1E.31

$9x+5-8x+14=20$

Answer

x = 1

Exercise 2.1E.32

$6x+8-5x+16=32$

Exercise 2.1E.33

$-6x-11+7x-5=-16$

Answer

x = 0

Exercise 2.1E.34

$-8n-17+9n-4=-41$

Exercise 2.1E.35

$5(y-6)-4y=-6$

Answer

$y=24$

Exercise 2.1E.36

$9(y-2)-8y=-16$

Exercise 2.1E.37

$8(u+1.5)-7u=4.9$

Answer

$u=-7.1$

Exercise 2.1E.38

$5(w+2.2)-4w=9.3$

Exercise 2.1E.39

$6a-5(a-2)+9=-11$

Answer

$a=-30$

Exercise 2.1E.40

$8c-7(c-3)+4=-16$

Exercise 2.1E.41

$6(y-2)-5y=4(y+3)-4(y-1)$

Answer

y =28

Exercise 2.1E.42

$9(x-1)-8x=-3(x+5)+3(x-5)$

Exercise 2.1E.43

$3(5n-1)-14n+9=10(n-4)-6n-4(n+1)$

Answer

n = -50

Exercise 2.1E.44

$2(8m+3)-15m-4=9(m+6)-2(m-1)-7m$

Exercise 2.1E.45

$-(j+2)+2j-1=5$

Answer

j = 8

Exercise 2.1E.46

$-(k+7)+2k+8=7$

Exercise 2.1E.47

$-\left(\frac{1}{4}a-\frac{3}{4}\right)+\frac{5}{4}a=-2$

Answer

$a=-\frac{11}{4}$

Exercise 2.1E.48

$-\left(\frac{2}{3}d-\frac{1}{3}\right)+\frac{5}{3}d=-4$

Exercise 2.1E.49

$8(4x+5)-5(6x)-x$
$=53-6(x+1)+3(2x+2)$

Answer

x=13

Exercise 2.1E.50

$6(9y-1)-10(5y)-3y$
$=22-4(2y-12)+8(y-6)$

Translate to an Equation and Solve

In the following exercises, translate to an equation and then solve it.

Exercise 2.1E.51

Nine more than x is equal to 52.

Answer

$x+9=52; x=43$

Exercise 2.1E.52

The sum of x and -15 is 23.

Exercise 2.1E.53

Ten less than m is -14.

Answer

$m-10=-14; m=-4$

Exercise 2.1E.54

Three less than y is -19.

Exercise 2.1E.55

The sum of y and -30 is 40.

Answer

$y+(-30)=40; y=70$

Exercise 2.1E.56

Twelve more than p is equal to 67.

Exercise 2.1E.57

The difference of $9x$ and $8x$ is 107.

Answer

$9x-8x=107; 107$

Exercise 2.1E.58

The difference of $5c$ and $4c$ is 602.

Exercise 2.1E.59

The difference of $5c$ and $4c$ is 602

Answer

$n-\frac{1}{6}=\frac{1}{2}; \frac{2}{3}$

Exercise 2.1E.60

The difference of f and $\frac{1}{3}$ is $\frac{1}{12}$.

Exercise 2.1E.61

The sum of $-4n$ and $5n$ is -82

Answer

$-4n+5n=-82;-82$

Exercise 2.1E.62

The sum of $-9m$ and $10m$ is -95

Translate and Solve Applications

In the following exercises, translate into an equation and solve.

Exercise 2.1E.63

Distance Avril rode her bike a total of 18 miles, from home to the library and then to the beach. The distance from Avril's house to the library is 7 miles. What is the distance from the library to the beach?

Answer

11 miles

Exercise 2.1E.64

Reading Jeff read a total of 54 pages in his History and Sociology textbooks. He read 41 pages in his History textbook. How many pages did he read in his Sociology textbook?

Exercise 2.1E.65

Age Eva's daughter is 15 years younger than her son. Eva's son is 22 years old. How old is her daughter?

Answer

7 years old

Exercise 2.1E.66

Age Pablo's father is 3 years older than his mother. Pablo's mother is 42 years old. How old is his father?

Exercise 2.1E.67

Groceries For a family birthday dinner, Celeste bought a turkey that weighed 5 pounds less than the one she bought for Thanksgiving. The birthday turkey weighed 16 pounds. How much did the Thanksgiving turkey weigh?

Answer

21 pounds

Exercise 2.1E.68

Weight Allie weighs 8 pounds less than her twin sister Lorrie. Allie weighs 124 pounds. How much does Lorrie weigh?

Exercise 2.1E. 69

Health Connor's temperature was 0.7 degrees higher this morning than it had been last night. His temperature this morning was 101.2 degrees. What was his temperature last night?

Answer

100.5 degrees

Exercise 2.1E. 70

Health The nurse reported that Tricia's daughter had gained 4.2 pounds since her last checkup and now weighs 31.6 pounds. How much did Tricia's daughter weigh at her last checkup?

Exercise 2.1E. 71

Salary Ron's paycheck this week was $17.43 less than his paycheck last week. His paycheck this week was $103.76. How much was Ron's paycheck last week?

Answer

$121.19

Exercise 2.1E. 72

Textbooks Melissa's math book cost $22.85 less than her art book cost. Her math book cost $93.75. How much did her art book cost?

Everyday Math

Exercise 2.1E. 73

Construction Miguel wants to drill a hole for a $\frac{5}{8}$ inch screw. The hole should be $\frac{1}{12}$ inch smaller than the screw. Let d equal the size of the hole he should drill. Solve the equation $d = \frac{5}{8} - \frac{1}{12}$ to see what size the hole should be.

Answer

$d = \frac{13}{24}$ inch

Exercise 2.1E. 74

Baking Kelsey needs $\frac{2}{3}$ cup of sugar for the cookie recipe she wants to make. She only has $\frac{3}{8}$ cup of sugar and will borrow the rest from her ner neighbor. Let s equal the amount of sugar she will borrow. Solve the equation $\frac{3}{8} + s = \frac{2}{3}$ to find the amount of sugar she should ask to borrow.

Writing Exercises

Exercise 2.1E. 75

Is -8 a solution to the equation $3x = 16 - 5x$? How do you know?

Answer

No. Justifications will vary.

Exercise $2.1E.76$

What is the first step in your solution to the equation $10x + 2 = 4x + 26$?

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
verify a solution of an equation.			
solve equations using the subtraction and addition properties of equality.			
solve equations that require simplification.			
translate to an equation and solve.			
translate and solve applications.			

ⓑ If most of your checks were:

…confidently. Congratulations! You have achieved your goals in this section! Reflect on the study skills you used so that you can continue to use them. What did you do to become confident of your ability to do these things? Be specific!

…with some help. This must be addressed quickly as topics you do not master become potholes in your road to success. Math is sequential - every topic builds upon previous work. It is important to make sure you have a strong foundation before you move on. Who can you ask for help? Your fellow classmates and instructor are good resources. Is there a place on campus where math tutors are available? Can your study skills be improved?

…no - I don't get it! This is critical and you must not ignore it. You need to get help immediately or you will quickly be overwhelmed. See your instructor as soon as possible to discuss your situation. Together you can come up with a plan to get you the help you need.

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2.2: Solve Equations using the Division and Multiplication Properties of Equality

Learning Objectives

By the end of this section, you will be able to:

- Solve equations using the Division and Multiplication Properties of Equality
- Solve equations that require simplification
- Translate to an equation and solve
- Translate and solve applications

Note

Before you get started, take this readiness quiz.

1. Simplify: $-7\left(\frac{1}{-7}\right)$.
   If you missed this problem, review Exercise 1.6.13.
2. Evaluate $9x+2$ when $x=-3$.
   If you missed this problem, review Exercise 1.5.34.

Solve Equations Using the Division and Multiplication Properties of Equality

You may have noticed that all of the equations we have solved so far have been of the form $x+a=b$ or $x-a=b$. We were able to isolate the variable by adding or subtracting the constant term on the side of the equation with the variable. Now we will see how to solve equations that have a variable multiplied by a constant and so will require division to isolate the variable.

Let's look at our puzzle again with the envelopes and counters in Figure 2.2.1.

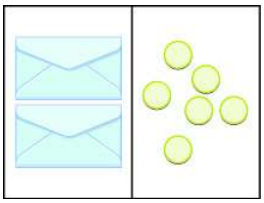


Figure 2.2.1: The illustration shows a model of an equation with one variable multiplied by a constant. On the left side of the workspace are two instances of the unknown (envelope), while on the right side of the workspace are six counters.

In the illustration there are two identical envelopes that contain the same number of counters. Remember, the left side of the workspace must equal the right side, but the counters on the left side are "hidden" in the envelopes. So how many counters are in each envelope?

How do we determine the number? We have to separate the counters on the right side into two groups of the same size to correspond with the two envelopes on the left side. The 6 counters divided into 2 equal groups gives 3 counters in each group (since $6\div 2=3$).

What equation models the situation shown in Figure 2.2.2? There are two envelopes, and each contains xx counters. Together, the two envelopes must contain a total of 6 counters.

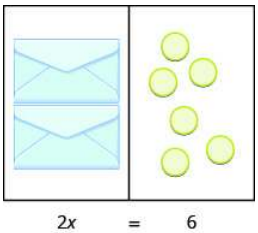


Figure 2.2.2: The illustration shows a model of the equation $2x=6$.

	$2x=6$

If we divide both sides of the equation by 2, as we did with the envelopes and counters,	$\frac{2x}{2}=\frac{6}{2}$
we get:	$x=3$

We found that each envelope contains 3 counters. Does this check? We know $2 \cdot 3 = 6$, so it works! Three counters in each of two envelopes does equal six!

This example leads to the **Division Property of Equality**.

THE DIVISION PROPERTY OF EQUALITY

For any numbers *a*, *b*, and *c*, and $c \neq 0$,

$$\begin{array}{rcl} \text{If} & a & = & b \\ \text{then} & \frac{a}{c} & = & \frac{b}{c} \end{array} \qquad (2.2.1)$$

When you divide both sides of an equation by any non-zero number, you still have equality.

Note

Doing the Manipulative Mathematics activity "**Division Property of Equality**" will help you develop a better understanding of how to solve equations by using the Division Property of Equality.

The goal in solving an equation is to 'undo' the operation on the variable. In the next example, the variable is multiplied by 5, so we will divide both sides by 5 to 'undo' the multiplication.

Example 2.2.1

Solve: $5x = -27$.

Solution

To isolate x, "undo" the multiplication by 5.		$5x=-27$
Divide to 'undo' the multiplication.		$\frac{5x}{5}=\frac{-27}{5}$
Simplify.		$x=-\frac{27}{5}$
Check:	$5x=-27$	
Substitute $-\frac{27}{5}$ for x.	$5\left(-\frac{27}{5}\right)\stackrel{?}{=}-27$	
	$-27=-27$ ✓	
		Since this is a true statement, $x=-\frac{27}{5}$ is the solution to $5x=-27$.

Try It 2.2.2

Solve: $3y = -41$.

Answer

$y=-\frac{41}{3}$

Try It 2.2.3

Solve: $4z = -55$.

Answer

$y = -\frac{55}{4}$

Consider the equation $\frac{x}{4} = 3$. We want to know what number divided by 4 gives 3. So to "undo" the division, we will need to multiply by 4. The **Multiplication Property of Equality** will allow us to do this. This property says that if we start with two equal quantities and multiply both by the same number, the results are equal.

THE MULTIPLICATION PROPERTY OF EQUALITY

For any numbers *a*, *b*, and *c*,

$$\begin{array}{lllll} \text{If} & a & = & b \\ \text{then} & ac & = & bc \end{array} \qquad (2.2.2)$$

If you multiply both sides of an equation by the same number, you still have equality.

Example 2.2.4

Solve: $\frac{y}{-7} = -14$

Solution

Here y is divided by −7. We must multiply by −7 to isolate y.

	$\frac{y}{-7} = -14$
Multiply both sides by −7.	$-7\left(\frac{y}{-7}\right) = -7(-14)$
Multiply.	$\frac{-7y}{7} = 98$
Simplify.	$y = 98$
Check: $\frac{y}{-7} = -14$	
Substitute y=98.	$\frac{98}{-7} \stackrel{?}{=} -14$
Divide.	$-14 = -14$ ✓

Try It 2.2.5

Solve: $\frac{a}{-7} = -42$

Answer

$a = 294$

Try It 2.2.6

Solve: $\frac{b}{-6} = -24$

Answer

$b = 144$

Example 2.2.7

Solve: $-n = 9$

Solution

	$-n = 9$
Remember −n is equivalent to −1n.	$-1n = 9$
Divide both sides by −1.	$\frac{-1n}{-1} = \frac{9}{-1}$
Divide.	$n = -9$

Notice that there are two other ways to solve −n=9. We can also solve this equation by multiplying both sides by −1 and also by taking the opposite of both sides.

Check:	$-n = 9$	
Substitute n=−9.	$-(-9) \stackrel{?}{=} 9$	
Simplify.	$9 = 9$ ✓	

Try It 2.2.8

Solve: $-k = 8$.

Answer

$k = -8$

Try It 2.2.9

Solve: $-g = 3$.

Answer

$g = -3$

Example 2.2.10

Solve: $\frac{3}{4}x = 12$

Solution

Since the product of a number and its reciprocal is 1, our strategy will be to isolate x by multiplying by the reciprocal of $\frac{3}{4}$.

	$\frac{3}{4}x = 12$
Multiply by the reciprocal of $\frac{3}{4}$.	$\frac{4}{3} \cdot \frac{3}{4}x = \frac{4}{3} \cdot 12$
Reciprocals multiply to 1.	$1x = \frac{4}{3} \cdot \frac{12}{1}$
Multiply.	$x = 16$

Notice that we could have divided both sides of the equation $\frac{3}{4}x = 12$ by $\frac{3}{4}$ to isolate x. While this would work, most people would find multiplying by the reciprocal easier.

Check:	$\frac{3}{4}x = 12$	

Substitute $x = 16$.	$\frac{3}{4} \cdot 16 \stackrel{?}{=} 12$	
	$12 = 12$ ✓	

Try It 2.2.11

Solve: $\frac{2}{5}n = 14$.

Answer

$n = 35$

Try It 2.2.12

Solve: $\frac{5}{6}y = 15$.

Answer

$y = 18$

In the next example, all the variable terms are on the right side of the equation. As always, our goal in solving the equation is to isolate the variable.

Example 2.2.13

Solve: $\frac{8}{15} = -\frac{4}{5}x$

Solution

	$\frac{8}{15} = -\frac{4}{5}x$
Multiply by the reciprocal of $-\frac{4}{5}$.	$\left(-\frac{5}{4}\right)\left(\frac{8}{15}\right) = \left(-\frac{5}{4}\right)\left(-\frac{4}{5}x\right)$
Reciprocals multiply to 1.	$-\frac{\cancel{5} \cdot \cancel{4} \cdot 2}{\cancel{4} \cdot 3 \cdot \cancel{5}} = 1x$
Multiply.	$-\frac{2}{3} = x$

Check:	$\frac{8}{15} = -\frac{4}{5}x$
Let $x = -\frac{2}{3}$.	$\frac{8}{15} = -\frac{4}{5}\left(-\frac{2}{3}\right)$
	$\frac{8}{15} = \frac{8}{15}$ ✓

Try It 2.2.14

Solve: $\frac{9}{25} = -\frac{4}{5}z$

Answer

$z = -\frac{9}{5}$

Try It 2.2.15

$\frac{5}{6} = -\frac{8}{3}r$

Answer

$r = -\frac{5}{16}$

Solve Equations That Require Simplification

Many equations start out more complicated than the ones we have been working with.

With these more complicated equations the first step is to simplify both sides of the equation as much as possible. This usually involves combining like terms or using the distributive property.

Example 2.2.16

Solve: $14 - 23 = 12y - 4y - 5y$.

Solution

Begin by simplifying each side of the equation.

	$14 - 23 \stackrel{?}{=} -36 + 12 + 15$
Simplify each side.	$-9 = -9$ ✓
Divide both sides by 3 to isolate y.	$14 - 23 = 12y - 4y - 5y$
Divide.	$-9 = 3y$
Check:	$14 - 23 = 12y - 4y - 5y$
Substitute $y = -3$.	$14 - 23 \stackrel{?}{=} 12(-3) - 4(-3) - 5(-3)$
	$14 - 23 \stackrel{?}{=} -36 + 12 + 15$
	$-9 = -9$ ✓

Try It 2.2.17

Solve: $18 - 27 = 15c - 9c - 3c$.

Answer

$c = -3$

Try It 2.2.18

Solve: $18 - 22 = 12x - x - 4x$.

Answer

$x = -\frac{4}{7}$

Example 2.2.19

Solve: $-4(a - 3) - 7 = 25$.

Solution

Here we will simplify each side of the equation by using the distributive property first.

	$-4(a-3)-7=25$
Distribute.	$-4a+12-7=25$
Simplify.	$-4a+5=25$
Simplify.	$-4a=20$
Divide both sides by -4 to isolate a.	$\frac{-4a}{-4}=\frac{20}{-4}$
Divide.	$a=-5$
Check:	$-4(a-3)-7=25$
Substitute $a=-5$	$-4(-5-3)-7\stackrel{?}{=}25$
	$-4(-8)-7\stackrel{?}{=}25$
	$32-7\stackrel{?}{=}25$
	$25=25$ ✓

Try It 2.2.20

Solve: $-4(q-2)-8=24$.

Answer

$q=-6$

Try It 2.2.21

Solve: $-6(r-2)-12=30$.

Answer

$r=-5$

Now we have covered all four properties of equality—subtraction, addition, division, and multiplication. We'll list them all together here for easy reference.

PROPERTIES OF EQUALITY

When you add, subtract, multiply, or divide the same quantity from both sides of an equation, you still have equality.

$$\begin{array}{ll} \textbf{Subtraction Property of Equality} & \textbf{Addition Property of Equality} \\ \text{For any real numbers a, b, and c,} & \text{For any real numbers a, b, and c,} \\ \text{if } a=b, & \text{if } a=b, \\ \text{then } a-c=b-c & \text{then } a+c=b+c \\ \textbf{Divition Property of Equality} & \textbf{Multiplication Property of Equality} \\ \text{For any real numbers a, b, and c,} & \text{For any real numbers a, b, and c,} \\ \text{if } a=b, & \text{if } a=b, \\ \text{then } a-c=b-c & \text{then } a+c=b+c \end{array} \tag{2.2.3}$$

When you add, subtract, multiply, or divide the same quantity from both sides of an equation, you still have equality.

Translate to an Equation and Solve

In the next few examples, we will translate sentences into equations and then solve the equations. You might want to review the translation table in the previous chapter.

Example 2.2.22

Translate and solve: The number 143 is the product of −11 and *y*.

Solution

Translate.	The number 143 is the product of –11 and y. $143 = -11y$
Divide by −11.	$\frac{143}{-11} = \frac{-11y}{-11}$
Simplify.	$-13 = y$
Check: $143 = -11y$ $143 \stackrel{?}{=} -11(-13)$ (2.2.4) $143 = 143\checkmark$	

Try It 2.2.23

Translate and solve: The number 132 is the product of *−12* and *y*.

Answer

132=−12y;y=−11

Try It 2.2.24

Translate and solve: The number 117 is the product of *−13* and *z*.

Answer

117=−13z;z=−9

Example 2.2.25

Translate and solve: *n* divided by 8 is −32.

Solution

Begin by translating the sentence into an equation. Translate.		*n* divided by 8 is –32. $\frac{n}{8} = -32$
Multiple both sides by 8.		$8 \cdot \frac{n}{8} = 8(-32)$
Simplify.		$n = -256$
Check:	Is nn divided by 8 equal to −32?	
Let $n = -256$.	Is −256 divided by 88 equal to −32?	
Translate.	$\frac{-256}{8} \stackrel{?}{=} -32$	

Simplify.	$-32 = -32$✓

Try It 2.2.26

Translate and solve: nn divided by 7 is equal to −21.

Answer

$\frac{n}{7} = -21; n = -147$

Try It 2.2.27

Translate and solve: n divided by 8 is equal to −56.

Answer

$\frac{n}{8} = -56; n = -448$

Example 2.2.28

Translate and solve: The quotient of yy and −4 is 68.

Solution

Begin by translating the sentence into an equation.

Translate.	The quotient of y and -4 is 68. $\frac{y}{-4} = 68$
Multiply both sides by -4.	$-4\left(\frac{y}{-4}\right) = -4(68)$
Simplify.	$y = -272$
Check:	Is the quotient of y and −4 equal to 68?
Let y=−272.	Is the quotient of −272 and −4 equal to 68?
Translate.	$\frac{-272}{-4} \stackrel{?}{=} 68$
Simplify.	$68 = 68$✓

Try It 2.2.29

Translate and solve: The quotient of q and −8 is 72.

Answer

$\frac{q}{-8} = 72; q = -576$

Try It 2.2.30

Translate and solve: The quotient of p and −9 is 81.

Answer

$\frac{p}{-9} = 81; p = -729$

Example 2.2.31

Translate and solve: Three-fourths of p is 18.

Solution

Begin by translating the sentence into an equation. Remember, “of” translates into multiplication.

Translate.	Three-fourths of p is 18. $\frac{3}{4}p = 18$
Multiply both sides by $\frac{4}{3}$.	$\frac{4}{3} \cdot \frac{3}{4}p = \frac{4}{3} \cdot 18$
Simplify.	$p = 24$
Check:	Is three-fourths of p equal to 18?
Let p = 24.	Is three-fourths of 24 equal to 18?
Translate.	$\frac{3}{4} \cdot 24 \stackrel{?}{=} 18$
Simplify.	$18 = 18$✓

Try It 2.2.32

Translate and solve: Two-fifths of f is 16.

Answer

$\frac{2}{5}f = 16; f = 40$

Try It 2.2.33

Translate and solve: Three-fourths of f is 21.

Answer

$\frac{3}{4}f = 21; f = 28$

Example 2.2.34

Translate and solve: The sum of three-eighths and x is one-half.

Solution

Begin by translating the sentence into an equation.

Translate.	The sum of three – eighths and x is $\frac{1}{2}$ $\frac{3}{8} + x = \frac{1}{2}$
Subtract $\frac{3}{8}$ from each side.	$\frac{3}{8} - \frac{3}{8} + x = \frac{1}{2} - \frac{3}{8}$
Simplify and rewrite fractions with common denominators.	$x = \frac{4}{8} - \frac{3}{8}$
Simplify.	$x = \frac{1}{8}$

Check:	Is the sum of three-eighths and x equal to one-half?
Let $x = \frac{1}{8}$.	Is the sum of three-eighths and one-eighth equal to one-half?
Translate.	$\frac{3}{8} + \frac{1}{8} \stackrel{?}{=} \frac{1}{2}$
Simplify.	$\frac{4}{8} \stackrel{?}{=} \frac{1}{2}$
Simplify.	$\frac{1}{2} = \frac{1}{2}$ ✓

Try It 2.2.35

Translate and solve: The sum of five-eighths and *x* is one-fourth.

Answer

$\frac{5}{8} + x = \frac{1}{4}; x = -\frac{3}{8}$

Try It 2.2.36

Translate and solve: The sum of three-fourths and *x* is five-sixths.

Answer

$\frac{3}{4} + x = \frac{5}{6}; x = \frac{1}{12}$

Translate and Solve Applications

To solve applications using the Division and Multiplication Properties of Equality, we will follow the same steps we used in the last section. We will restate the problem in just one sentence, assign a variable, and then translate the sentence into an equation to solve.

Example 2.2.37

Denae bought 6 pounds of grapes for $10.74. What was the cost of one pound of grapes?

Solution

What are you asked to find?	The cost of 1 pound of grapes	
Assign a variable.	Let c = the cost of one pound.	
Write a sentence that gives the information to find it.	The cost of 6 pounds is $10.74	
Translate into an equation.	$6c = 10.74$	
Solve.	$\frac{6c}{c} = \frac{10.74}{6}$ $c = 1.79$	(2.2.5)
	The grapes cost $1.79 per pound.	
Check: If one pound costs $1.79, *do* 6 pounds cost $10.74?		
$6(1.79) \stackrel{?}{=} 10.74$ $10.74 = 10.74$✓		

? Try It 2.2.38

Translate and solve:

Arianna bought a 24-pack of water bottles for $9.36. What was the cost of one water bottle?

Answer

$0.39

? Try It 2.2.39

Translate and solve:

At JB's Bowling Alley, 6 people can play on one lane for $34.98. What is the cost for each person?

Answer

$5.83

✓ Example 2.2.40

Andreas bought a used car for $12,000. Because the car was 4-years old, its price was $\frac{3}{4}$ of the original price, when the car was new. What was the original price of the car?

Solution

What are you asked to find?	The original price of the car
Assign a variable.	Let p = the original price.
Write a sentence that gives the information to find it.	\$12000 is $\frac{3}{4}$ of the original price.
Translate into an equation.	$12000 = \frac{3}{4}p$
Solve.	$\frac{3}{4}(12000) = \frac{4}{3} \cdot \frac{3}{4}p$ $16000 = p$
	The original cost of the car was \$16000.

(2.2.6)

Check: Is $\frac{3}{4}$ of \$16000 equal to \$12000

$\frac{3}{4} \cdot 16000 \stackrel{?}{=} 12000$
$12000 = 12000$✓

? Try It 2.2.41

Translate and solve:

The annual property tax on the Mehta's house is \$1,800, calculated as $\frac{15}{1000}$ of the assessed value of the house. What is the assessed value of the Mehta's house?

Answer

\$120000

? Try It 2.2.42

Translate and solve:

Stella planted 14 flats of flowers in $\frac{2}{3}$ of her garden. How many flats of flowers would she need to fill the whole garden?

Answer

21 flats

Key Concepts

- **The Division Property of Equality**—For any numbers a, b, and c, and $c \neq 0$, if $a = b$, then $\frac{a}{c} = \frac{b}{c}$.
  When you divide both sides of an equation by any non-zero number, you still have equality.
- **The Multiplication Property of Equality**—For any numbers a, b, and c, if $a = b$, then $ac = bc$.
  If you multiply both sides of an equation by the same number, you still have equality.

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2.2E: Exercises

Practice Makes Perfect

Solve Equations Using the Division and Multiplication Properties of Equality

In the following exercises, solve each equation using the Division and Multiplication Properties of Equality and check the solution.

Exercise 2.2E.1

$8x = 56$

Answer

$x = 7$

Exercise 2.2E.2

$7p = 63$

Exercise 2.2E.3

$-5c = 55$

Answer

$c = -11$

Exercise 2.2E.4

$-9x = -27$

Exercise 2.2E.5

$-809 = 15y$

Answer

$y = -\frac{809}{15}$

Exercise 2.2E.6

$-731 = 19y$

Exercise 2.2E.7

$-37p = -541$

Answer

$p = -\frac{541}{37}$

Exercise 2.2E.8

$-19m = -586$

Exercise 2.2E.9

$0.25z = 3.25$

Answer

z= 13

Exercise 2.2E.10

$0.75a = 11.25$

Exercise 2.2E.11

$-13x = 0$

Answer

$x = 0$

Exercise 2.2E.12

$24x = 0$

Exercise 2.2E.13

$\frac{x}{4} = 35$

Answer

$x = 140$

Exercise 2.2E.14

$\frac{z}{2} = 54$

Exercise 2.2E.15

$-20 = \frac{q}{-5}$

Answer

$q = 100$

Exercise 2.2E.16

$\frac{c}{-3} = -12$

Exercise 2.2E.17

$\frac{y}{9} = -16$

Answer

$y = -144$

Exercise 2.2E.18

$\frac{q}{6} = -38$

Exercise 2.2E.19

$\frac{m}{-12} = 45$

Answer

$m = -540$

Exercise 2.2E.20

$-24 = \frac{p}{-20}$

Exercise 2.2E.21

$-y = 6$

Answer

$y = -6$

Exercise 2.2E.22

$-u = 15$

Exercise 2.2E.23

$-v = -72$

Answer

$v = 72$

Exercise 2.2E.24

$-x = -39$

Exercise 2.2E.25

$\frac{2}{3}y = 48$

Answer

$y = 72$

Exercise 2.2E.26

$\frac{3}{5}r = 75$

Exercise 2.2E.27

$-\frac{5}{8}w = 40$

Answer

$w = -64$

Exercise 2.2E.28

$24 = -\frac{3}{4}x$

Exercise 2.2E.29

$-\frac{2}{5} = \frac{1}{10}a$

Answer

$a = -4$

Exercise 2.2E.30

$-\frac{1}{3}q = -\frac{5}{6}$

Exercise 2.2E.31

$-\frac{7}{10}x = -\frac{14}{3}$

Answer

$x = \frac{20}{3}$

Exercise 2.2E.32

$\frac{3}{8}y = -\frac{1}{4}$

Exercise 2.2E.33

$\frac{7}{12} = -\frac{3}{4}p$

Answer

$p = -\frac{7}{9}$

Exercise 2.2E.34

$\frac{11}{18} = -\frac{5}{6}q$

Exercise 2.2E.35

$-\frac{5}{18} = -\frac{10}{9}u$

Answer

$u = \frac{1}{4}$

Exercise 2.2E.36

$-\frac{7}{20}=-\frac{7}{4}v$

Solve Equations That Require Simplification

In the following exercises, solve each equation requiring simplification.

Exercise 2.2E.37

$100-16=4p-10p-p$

Answer

$p=-12$

Exercise 2.2E.38

$-18-7=5t-9t-6t$

Exercise 2.2E.39

$\frac{7}{8}n-\frac{3}{4}n=9+2$

Answer

$n=88$

Exercise 2.2E.40

$\frac{5}{12}q+\frac{1}{2}q=25-3$

Exercise 2.2E.41

$0.25d+0.10d=6-0.75$

Answer

d=15

Exercise 2.2E.42

$0.05p-0.01p=2+0.24$

Exercise 2.2E.43

$-10(q-4)-57=93$

Answer

$q=-11$

Exercise 2.2E.44

$-12(d-5)-29=43$

Exercise 2.2E.45

$-10(x+4)-19=85$

Answer

$x=-\frac{72}{5}$

Exercise 2.2E.46

$-15(z+9)-11=75$

Mixed Practice

In the following exercises, solve each equation.

Exercise 2.2E.47

$\frac{9}{10}x=90$

Answer

$x=100$

Exercise 2.2E.48

$\frac{5}{12}y=60$

Exercise 2.2E.49

$y+46=55$

Answer

$y=9$

Exercise 2.2E.50

$x+33=41$

Exercise 2.2E.51

$\frac{w}{-2}=99$

Answer

$w=-198$

Exercise 2.2E.52

$\frac{s}{-3}=-60$

Exercise 2.2E.53

$27 = 6a$

Answer

$a = \frac{9}{2}$

Exercise 2.2E.54

$-a = 7$

Exercise 2.2E.55

$-x = 2$

Answer

$x = -2$

Exercise 2.2E.56

$z - 16 = -59$

Exercise 2.2E.57

$m - 41 = -14$

Answer

$m = 27$

Exercise 2.2E.58

$0.04r = 52.60$

Exercise 2.2E.59

$63.90 = 0.03p$

Answer

$p = 2130$

Exercise 2.2E.60

$-15x = -120$

Exercise 2.2E.61

$84 = -12z$

Answer

$y = -7$

Exercise 2.2E.62

$19.36 = x - 0.2x$

Exercise 2.2E.63

$c - 0.3c = 35.70$

Answer

$c = 51$

Exercise 2.2E.64

$-y = -9$

Exercise 2.2E.65

$-x = -8$

Answer

$x = 8$

Translate to an Equation and Solve

In the following exercises, translate to an equation and then solve.

Exercise 2.2E.66

187 is the product of -17 and m

Exercise 2.2E.67

133 is the product of -19 and n

Answer

$133 = -19n; n = -7$

Exercise 2.2E.68

-184 is the product of 23 and p

Exercise 2.2E.69

-152 is the product of 8 and q

Answer

$-152 = 8q; q = -19$

Exercise 2.2E.70

u divided by 7 is equal to -49

Exercise 2.2E.71

r divided by 12 is equal to -48

Answer

$\frac{r}{12} = -48; r = -576$

Exercise 2.2E.72

h divided by -13 is equal to -65

Exercise 2.2E.73

j divided by -20 is equal to -80

Answer

$\frac{j}{-20} = -80; j = 1,600$

Exercise 2.2E.74

The quotient c and -19 is 38.

Exercise 2.2E.75

The quotient of b and -6 is 18

Answer

$\frac{b}{-6} = 18; b = -108$

Exercise 2.2E.76

The quotient of h and 26 is -52

Exercise 2.2E.77

The quotient k and 22 is -66

Answer

$\frac{k}{22} = -66; k = -1,452$

Exercise 2.2E.78

Five-sixths of y is 15

Exercise 2.2E.79

Three-tenths of x is 15

Answer

$\frac{3}{10}x = 15; x = 50$

Exercise 2.2E.80

Four-thirds of w is 36

Exercise 2.2E.81

Five-halves of v is 50

Answer

$\frac{5}{2}v = 50; v = 20$

Exercise 2.2E.82

The sum of nine-tenths and g is two-thirds.

Exercise 2.2E.83

The sum of two-fifths and f is one-half.

Answer

$\frac{2}{5} + f = \frac{1}{2}; f = \frac{1}{10}$

Exercise 2.2E.84

The difference of p and one-sixth is two-thirds.

Exercise 2.2E.85

The difference of q and one-eighth is three-fourths.

Answer

$q - \frac{1}{8} = \frac{3}{4}; q = \frac{7}{8}$

Translate and Solve Applications

In the following exercises, translate into an equation and solve.

Exercise 2.2E.86

Kindergarten Connie's kindergarten class has 24 children. She wants them to get into 4 equal groups. How many children will she put in each group?

Exercise 2.2E.87

Balloons Ramona bought 18 balloons for a party. She wants to make 3 equal bunches. How many balloons did she use in each bunch?

Answer

6 balloons

Exercise 2.2E.88

Tickets Mollie paid $36.25 for 5 movie tickets. What was the price of each ticket?

Exercise 2.2E.89

Shopping Serena paid $12.96 for a pack of 12 pairs of sport socks. What was the price of pair of sport socks?

Answer

$1.08

Exercise 2.2E.90

Sewing Nancy used 14 yards of fabric to make flags for one-third of the drill team. How much fabric, would Nancy need to make flags for the whole team?

Exercise 2.2E.91

MPG John's SUV gets 18 miles per gallon (mpg). This is half as many mpg as his wife's hybrid car. How many miles per gallon does the hybrid car get?

Answer

36 mpg

Exercise 2.2E.92

Height Aiden is 27 inches tall. He is $\frac{3}{8}$ as tall as his father. How tall is his father?

Exercise 2.2E.92

Real estate Bea earned $\$11,700$ commission for selling a house, calculated as $\frac{6}{100}$ of the selling price. What was the selling price of the house?

Answer

$195,000

Everyday Math

Exercise 2.2E.93

Commissions Every week Perry gets paid $\$150$ plus 12% of his total sales amount. Solve the equation $840 = 150 + 0.12(a - 1250)$ for a to find the total amount Perry must sell in order to be paid $\$840$ one week.

Exercise 2.2E.94

Stamps Travis bought $9.45 worth of 49-cent stamps and 21-cent stamps. The number of 21-cent stamps was 5 less than the number of 49-cent stamps. Solve the equation 0.49s+0.21(s−5)=9.45 for s, to find the number of 49-cent stamps Travis bought.

Answer

15 49-cent stamps

Writing Exercises

Exercise 2.2E.95

Frida started to solve the equation −3x=36 by adding 3 to both sides. Explain why Frida's method will not solve the equation.

Exercise 2.2E.96

Emiliano thinks $x = 40$ is the solution to the equation $\frac{1}{2}x = 80$. Explain why he is wrong.

Answer

Answers will vary.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
solve equations using the Division and Multiplication Properties of equality.			
solve equations that require simplification.			
translate to an equation and solve.			
translate and solve applications.			

ⓑ What does this checklist tell you about your mastery of this section? What steps will you take to improve?

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2.3: Solve Equations with Variables and Constants on Both Sides

Learning Objectives

By the end of this section, you will be able to:

- Solve an equation with constants on both sides
- Solve an equation with variables on both sides
- Solve an equation with variables and constants on both sides

Note

Before you get started, take this readiness quiz.

1. Simplify: 4y−9+9.
   If you missed this problem, review Exercise 1.10.20.

Solve Equations with Constants on Both Sides

In all the equations we have solved so far, all the variable terms were on only one side of the equation with the constants on the other side. This does not happen all the time—so now we will learn to solve equations in which the variable terms, or constant terms, or both are on both sides of the equation.

Our strategy will involve choosing one side of the equation to be the "variable side", and the other side of the equation to be the "constant side." Then, we will use the Subtraction and Addition Properties of Equality to get all the variable terms together on one side of the equation and the constant terms together on the other side.

By doing this, we will transform the equation that began with variables and constants on both sides into the form $ax = b$. We already know how to solve equations of this form by using the Division or Multiplication Properties of Equality.

Example 2.3.1

Solve: $7x + 8 = -13$.

Solution

In this equation, the variable is found only on the left side. It makes sense to call the left side the "variable" side. Therefore, the right side will be the "constant" side. We will write the labels above the equation to help us remember what goes where.

variable constant
$7x + 8 = -13$

Since the left side is the "xx", or variable side, the 8 is out of place. We must "undo" adding 8 by subtracting 8, and to keep the equality we must subtract 8 from both sides.

	variable constant $7x + 8 = -13$
Use the Subtraction Property of Equality.	$7x + 8 - 8 = -13 - 8$
Simplify.	$7x = -21$
Now all the variables are on the left and the constant on the right. The equation looks like those you learned to solve earlier.	
Use the Division Property of Equality.	$\frac{7x}{7} = \frac{-21}{7}$
Simplify.	$x = -3$
Check:	$7x + 8 = -13$
Let x=−3.	$7(-3) + 8 \stackrel{?}{=} -13$

	$-21+8\stackrel{?}{=}-13$ $-13=-13\checkmark$	

Try It 2.3.2

Solve: $3x+4=-8$.

Answer

$x=-4$

Try It 2.3.3

Solve: $5a+3=-37$.

Answer

$a=-8$

Example 2.3.4

Solve: $8y-9=31$.

Solution

Notice, the variable is only on the left side of the equation, so we will call this side the "variable" side, and the right side will be the "constant" side. Since the left side is the "variable" side, the 9 is out of place. It is subtracted from the 8y, so to "undo" subtraction, add 9 to both sides. Remember, whatever you do to the left, you must do to the right.

		variable constant $8y-9=31$
Add 9 to both sides.		$8y-9=31$ $+9\quad+9$
Simplify.		$8y=40$
		The variable is now on one side and the constants on the other. We continue from here as we did earlier.
Divide both sides by 8.		$\frac{8y}{8}=\frac{40}{8}$
Simplify.		$y=5$
Check:	$8y-9=31$	
Let y=5.	$8\cdot5-9\stackrel{?}{=}31$	
	$40-9\stackrel{?}{=}31$	
	$31=31\checkmark$	

Try It 2.3.5

Solve: $5y-9=16$.

Answer

$y = 5$

Try It 2.3.6

Solve: $3m - 8 = 19$.

Answer

$m = 9$

Solve Equations with Variables and Constants on Both Sides

The next example will be the first to have variables and constants on both sides of the equation. It may take several steps to solve this equation, so we need a clear and organized strategy.

Example 2.3.7

Solve: $9x = 8x - 6$.

Solution

Here the variable is on both sides, but the constants only appear on the right side, so let's make the right side the "constant" side. Then the left side will be the "variable" side.

	$9x = 8x - 6$
We don't want any x's on the right, so subtract the 8x from both sides.	$9x - 8x = 8x - 8x - 6$
Simplify.	$x = -6$
We succeeded in getting the variables on one side and the constants on the other, and have obtained the solution.	
Check:	$9x = 8x - 6$
Let x=−6.	$9(-6) \stackrel{?}{=} 8(-6) - 6$
	$-54 \stackrel{?}{=} -48 - 6$
	$-54 = -54$ ✓

Try It 2.3.8

Solve: $6n = 5n - 10$.

Answer

$n = -10$

Try It 2.3.9

Solve: $-6c = -7c - 1$

Answer

$c = -1$

Example 2.3.10

Solve: $5y - 9 = 8y$

Solution

The only constant is on the left and the y's are on both sides. Let's leave the constant on the left and get the variables to the right.

	constant variable $5y - 9 = 8y$
Subtract 5y from both sides.	$5y - 5y - 9 = 8y - 5y$
Simplify.	$-9 = 3y$
We have the y's on the right and the constants on the left. Divide both sides by 3.	$\frac{-9}{3} = \frac{3y}{3}$
Simplify.	$-3 = y$
Check:	$5y - 9 = 8y$
Let $y = -3$.	$5(-3) - 9 \stackrel{?}{=} 8(-3)$
	$-15 - 9 \stackrel{?}{=} -24$
	$-24 = -24$ ✓

Try It 2.3.11

Solve: $3p - 14 = 5p$.

Answer

$p = -7$

Try It 2.3.12

Solve: $8m + 9 = 5m$

Answer

$m = -3$

Example 2.3.13

Solve: $12x = -x + 26$

Solution

The only constant is on the right, so let the left side be the "variable" side.

	variable constant $12x = -x + 26$
Remove the $-x$ from the right side by adding x to both sides.	$12x + x = -x + x + 26$
Simplify.	$13x = 26$
All the x's are on the left and the constants are on the right. Divide both sides by 13.	$\frac{13x}{13} = \frac{26}{13}$

Simplify.	$x = 2$

Try It 2.3.14

Solve: $12j = -4j + 32$

Answer

$j = 2$

Try It 2.3.15

Solve: $8h = -4h + 12$

Answer

$h = 1$

Solve Equations with Variables and Constants on Both Sides

The next example will be the first to have variables and constants on both sides of the equation. It may take several steps to solve this equation, so we need a clear and organized strategy.

Example 2.3.16: How to Solve Equations with Variables and Constants on Both Sides

Solve: $7x + 5 = 6x + 2$

Solution

Step 1. Choose which side will be the "variable" side—the other side will be the "constant" side.	The variable terms are $7x$ and $6x$. Since 7 is greater than 6, we will make the left side the "x" side. The right side will be the "constant" side.	variable constant $7x + 5 = 6x + 2$
Step 2. Collect the variable terms to the "variable" side of the equation, using the addition or subtraction property of equality.	With the right side as the "constant" side, the $6x$ is out of place, so subtract $6x$ from both sides. Combine like terms. Now, the variable is only on the left side!	$7x - 6x + 5 = 6x - 6x + 2$ $x + 5 = 2$
Step 3. Collect all the constants to the other side of the equation, using the addition or subtraction property of equality.	The right side is the "constant" side, so the 5 is out of place. Subtract 5 from both sides. Simplify.	$x + 5 - 5 = 2 - 5$ $x = -3$

Step 4. Make the coefficient of the variable equal 1, using the multiplication or division property of equality.	The coefficient of x is one. The equation is solved.	
Step 5. Check.	Let $x = -3$ Simplify. Add.	**Check:** $7x + 6 = 6x + 2$ $(-3) + 5 = 6(-3) + 2$ $-21 + 5 = -18 + 2$ $-16 = -16$ ✓

? Try It 2.3.17

Solve: $12x + 8 = 6x + 2$.

Answer

$x = -1$

? Try It 2.3.18

Solve: $9y + 4 = 7y + 12$.

Answer

$y = 4$

We'll list the steps below so you can easily refer to them. But we'll call this the 'Beginning Strategy' because we'll be adding some steps later in this chapter.

BEGINNING STRATEGY FOR SOLVING EQUATIONS WITH VARIABLES AND CONSTANTS ON BOTH SIDES OF THE EQUATION.

1. Choose which side will be the "variable" side—the other side will be the "constant" side.
2. Collect the variable terms to the "variable" side of the equation, using the Addition or Subtraction Property of Equality.
3. Collect all the constants to the other side of the equation, using the Addition or Subtraction Property of Equality.
4. Make the coefficient of the variable equal 1, using the Multiplication or Division Property of Equality.
5. Check the solution by substituting it into the original equation.

In Step 1, a helpful approach is to make the "variable" side the side that has the variable with the larger coefficient. This usually makes the arithmetic easier.

✓ Example 2.3.19

Solve: $8n - 4 = -2n + 6$.

Solution

In the first step, choose the variable side by comparing the coefficients of the variables on each side.

Since $8 > -2$, make the left side the "variable" side.	variable constant $8n - 4 = -2n + 6$
We don't want variable terms on the right side—add 2n to both sides to leave only constants on the right.	$8n + 2n - 4 = -2n + 2n + 6$
Combine like terms.	$10n - 4 = 6$

We don't want any constants on the left side, so add 4 to both sides.	$10n-4+4=6+4$
Simplify.	$10n=10$
The variable term is on the left and the constant term is on the right. To get the coefficient of nn to be one, divide both sides by 10.	$\frac{10n}{10}=\frac{10}{10}$
Simplify.	$n=1$
Check:	$8n-4=-2n+6$
Let n=1.	$8\cdot 1-4\stackrel{?}{=}-2\cdot 1+6$
	$8-4\stackrel{?}{=}-2+6$
	$4=4\checkmark$

Try It 2.3.20

Solve: $8q-5=-4q+7$

Answer

$q=1$

Try It 2.3.21

Solve: $7n-3=n+3$

Answer

$n=1$

Example 2.3.22

Solve: $7a-3=13a+7$

Solution

In the first step, choose the variable side by comparing the coefficients of the variables on each side.

Since 13>7, make the right side the "variable" side and the left side the "constant" side.

	constant variable $7a-3=13a+7$
Subtract 7a from both sides to remove the variable term from the left.	$7a-7a-3=13a-7a+7$
Combine like terms.	$-3=6a+7$
Subtract 7 from both sides to remove the constant from the right.	$-3-7=6a+7-7$
Simplify.	$-10=6a$
Divide both sides by 6 to make 1 the coefficient of a.	$\frac{-10}{6}=\frac{6a}{6}$
Simplify.	$-\frac{5}{3}=a$
Check:	$7a-3=13a+7$
Let $a=-\frac{5}{3}$.	$7\left(-\frac{5}{3}\right)-3\stackrel{?}{=}13\left(-\frac{5}{3}\right)+7$

	$-\frac{35}{3}-\frac{9}{3}\stackrel{?}{=}-\frac{65}{3}+\frac{21}{3}$	
	$-\frac{54}{3}=-\frac{54}{3}\checkmark$	

Try It 2.3.23

Solve: $2a-2=6a+18$

Answer

$a=-5$

Try It 2.3.24

Solve: $4k-1=7k+17$

Answer

$k=-6$

In the last example, we could have made the left side the "variable" side, but it would have led to a negative coefficient on the variable term. (Try it!) While we could work with the negative, there is less chance of errors when working with positives. The strategy outlined above helps avoid the negatives!

To solve an equation with fractions, we just follow the steps of our strategy to get the solution!

Example 2.3.25

Solve: $\frac{4}{5}x+6=\frac{1}{4}x-2$

Solution

Since $\frac{5}{4}>\frac{1}{4}$, make the left side the "variable" side and the right side the "constant" side.

	variable constant $\frac{5}{4}x+6=\frac{1}{4}x-2$
Subtract $\frac{1}{4}x$ from both sides.	$\frac{5}{4}x-\frac{1}{4}x+6=\frac{1}{4}x-\frac{1}{4}x-2$
Combine like terms.	$x+6=-2$
Subtract 6 from both sides.	$x+6-6=-2-6$
Simplify.	$x=-8$
Check: Let $x=-8$	$\frac{5}{4}x+6 = \frac{1}{4}x-2$ $\frac{5}{4}(-8)+6 \stackrel{?}{=} \frac{1}{4}(-8)-2$ $-10+6 \stackrel{?}{=} -2-2$ $-4 = -4\checkmark$

Try It 2.3.26

Solve: $\frac{7}{8}x-12=-\frac{1}{8}x-2$

Answer

$x = 10$

Try It 2.3.27

Solve: $\frac{7}{6}x + 11 = \frac{1}{6}y + 8$

Answer

$y = -3$

We will use the same strategy to find the solution for an equation with decimals.

Example 2.3.28

Solve: $7.8x + 4 = 5.4x - 8$.

Solution

Since $7.8 > 5.4$, make the left side the "variable" side and the right side the "constant" side.

	variable side constant side $7.8x + 4 = 5.4x - 8$
Subtract 5.4x from both sides.	$7.8x - 5.4x + 4 = 5.4x - 5.4x - 8$
Combine like terms.	$2.4x + 4 = -8$
Subtract 4 from both sides.	$2.4x + 4 - 4 = -8 - 4$
Simplify.	$2.4x = -12$
Use the Division Property of Equality.	$\frac{2.4x}{2.4} = \frac{-12}{2.4}$
Simplify.	$x = -5$
Check:	$7.8x + 4 = 5.4x - 5$
Let $x = -5$	$7.8(-5) + 4 = 5.4(-5) - 8$
	$-39 + 4 \stackrel{?}{=} -27 - 8$
	$-35 = -35$ ✓

Try It 2.3.29

Solve: $2.8x + 12 = -1.4x - 9$

Answer

$x = -5$

Try It 2.3.30

Solve: $3.6y + 8 = 1.2y - 4$

Answer

$y = -5$

Key Concepts

- **Beginning Strategy for Solving an Equation with Variables and Constants on Both Sides of the Equation**
  1. Choose which side will be the “variable” side—the other side will be the “constant” side.
  2. Collect the variable terms to the “variable” side of the equation, using the Addition or Subtraction Property of Equality.
  3. Collect all the constants to the other side of the equation, using the Addition or Subtraction Property of Equality.
  4. Make the coefficient of the variable equal 1, using the Multiplication or Division Property of Equality.
  5. Check the solution by substituting it into the original equation.

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2.3E: Exercises

Practice Makes Perfect

Solve Equations with Constants on Both Sides

In the following exercises, solve the following equations with constants on both sides.

Exercise 2.3E.1

$9x - 3 = 60$

Exercise 2.3E.2

$12x - 8 = 64$

Answer

$x = 6$

Exercise 2.3E.3

$14w + 5 = 117$

Exercise 2.3E.4

$15y + 7 = 97$

Answer

$y = 6$

Exercise 2.3E.5

$2a + 8 = -28$

Exercise 2.3E.6

$3m + 9 = -15$

Answer

$m = -8$

Exercise 2.3E.7

$-62 = 8n - 6$

Exercise 2.3E.8

$-77 = 9b - 5$

Answer

$b = -8$

Exercise 2.3E.9

$35 = -13y + 9$

Exercise 2.3E.10

$60 = -21x - 24$

Answer

$x = -4$

Exercise 2.3E.11

$-12p - 9 = 9$

Exercise 2.3E.12

$-14q - 2 = 16$

Answer

$q = -\frac{9}{7}$

Solve Equations with Variables on Both Sides

In the following exercises, solve the following equations with variables on both sides.

Exercise 2.3E.13

$19z = 18z - 7$

Exercise 2.3E.14

$21k = 20k - 11$

Answer

$k = -11$

Exercise 2.3E.15

$9x + 36 = 15x$

Exercise 2.3E.16

$8x + 27 = 11x$

Answer

$x = 9$

Exercise 2.3E.17

$c = -3c - 20$

Exercise 2.3E.18

$b = -4b - 15$

Answer

$b = -3$

Exercise 2.3E.19

$9q = 44 - 2q$

Exercise 2.3E.20

$5z = 39 - 8z$

Answer

$z = 3$

Exercise 2.3E.21

$6y + \frac{1}{2} = 5y$

Exercise 2.3E.22

$4x + \frac{3}{4} = 3x$

Answer

$x = -\frac{3}{4}$

Exercise 2.3E.23

$-18a - 8 = -22a$

Exercise 2.3E.24

$-11r - 8 = -7r$

Answer

$r = -2$

Solve Equations with Variables and Constants on Both Sides

In the following exercises, solve the following equations with variables and constants on both sides.

Exercise 2.3E.25

$8x - 15 = 7x + 3$

Exercise 2.3E.26

$6x - 17 = 5x + 2$

Answer

$x = 19$

Exercise 2.3E.27

$26 + 13d = 14d + 11$

Exercise 2.3E.28

$21 + 18f = 19f + 14$

Answer

$f = 7$

Exercise 2.3E.29

$2p - 1 = 4p - 33$

Exercise 2.3E.30

$12q - 5 = 9q - 20$

Answer

$q = -5$

Exercise 2.3E.31

$4a + 5 = -a - 40$

Exercise 2.3E.32

$8c + 7 = -3c - 37$

Answer

$c = -4$

Exercise 2.3E.33

$5y - 30 = -5y + 30$

Exercise 2.3E.34

$7x - 17 = -8x + 13$

Answer

$x = 2$

Exercise 2.3E.35

$7s + 12 = 5 + 4s$

Exercise 2.3E.36

$9p+14=6+4p$

Answer

$p=-\frac{8}{5}$

Exercise 2.3E.37

$2z-6=23-z$

Exercise 2.3E.38

$3y-4=12-y$

Answer

$y=4$

Exercise 2.3E.39

$\frac{5}{3}c-3=\frac{2}{3}c-16$

Exercise 2.3E.40

$\frac{7}{4}m-7=\frac{3}{4}m-13$

Answer

$m=-6$

Exercise 2.3E.41

$8-\frac{2}{5}q=\frac{3}{5}q+6$

Exercise 2.3E.42

$11-\frac{1}{5}a=\frac{4}{5}a+4$

Answer

$a=7$

Exercise 2.3E.43

$\frac{4}{3}n+9=\frac{1}{3}n-9$

Exercise 2.3E.44

$\frac{5}{4}a+15=\frac{3}{4}a-5$

Answer

$a=-40$

Exercise 2.3E.45

$\frac{1}{4}y+7=\frac{3}{4}y-3$

Exercise 2.3E.46

$\frac{3}{5}p+2=\frac{4}{5}p-1$

Answer

$p=15$

Exercise 2.3E.47

$14n+8.25=9n+19.60$

Exercise 2.3E.48

$13z+6.45=8z+23.75$

Answer

$z=3.46$

Exercise 2.3E.49

$2.4w-100=0.8w+28$

Exercise 2.3E.50

$2.7w-80=1.2w+10$

Answer

$w=60$

Exercise 2.3E.51

$5.6r+13.1=3.5r+57.2$

Exercise 2.3E.52

$6.6x-18.9=3.4x+54.7$

Answer

$x=23$

Everyday Math

Exercise 2.3E.53

Concert tickets At a school concert the total value of tickets sold was $1506. Student tickets sold for $6 and adult tickets sold for $9. The number of adult tickets sold was 5 less than 3 times the number of student tickets. Find the number of student tickets sold, *s*, by solving the equation 6s+27s−45=1506. Add exercises text here.

Exercise $2.3E.54$

Making a fence Jovani has 150 feet of fencing to make a rectangular garden in his backyard. He wants the length to be 15 feet more than the width. Find the width, w, by solving the equation $150 = 2w + 30 + 2w$.

Answer

30 feet

Writing Exercises

Exercise $2.3E.55$

Solve the equation $\frac{6}{5}y - 8 = \frac{1}{5}y + 7$ explaining all the steps of your solution as in the examples in this section.

Exercise $2.3E.56$

Solve the equation $10x + 14 = -2x + 38$ explaining all the steps of your solution as in this section.

Answer

$x = 2$ Justifications will vary.

Exercise $2.3E.57$

When solving an equation with variables on both sides, why is it usually better to choose the side with the larger coefficient of x to be the "variable" side?

Exercise $2.3E.58$

Is $x = -2$ a solution to the equation $5 - 2x = -4x + 1$? How do you know?

Answer

Yes. Justifications will vary.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
solve an equation with constants on both sides.			
solve an equation with variables on both sides.			
solve an equation with variables and constants on both sides.			

ⓑ What does this checklist tell you about your mastery of this section? What steps will you take to improve?

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2.4: Use a General Strategy to Solve Linear Equations

Learning Objectives

By the end of this section, you will be able to:

- Solve equations using a general strategy
- Classify equations

Note

Before you get started, take this readiness quiz.

1. Simplify: $-(a-4)$.
   If you missed this problem, review Exercise 1.10.46
2. Multiply: $\frac{3}{2}(12x+20)$
   If you missed this problem, review Exercise 1.10.34.
3. Simplify: $5-2(n+1)$
   If you missed this problem, review Exercise 1.10.49.
4. Multiply: $3(7y+9)$
   If you missed this problem, review Exercise 1.10.34.
5. Multiply: $(2.5)(6.4)$
   If you missed this problem, review Exercise 1.8.19.

Solve Equations Using the General Strategy

Until now we have dealt with solving one specific form of a linear equation. It is time now to lay out one overall strategy that can be used to solve any linear equation. Some equations we solve will not require all these steps to solve, but many will.

Beginning by simplifying each side of the equation makes the remaining steps easier.

Example 2.4.1: How to Solve Linear Equations Using the General Strategy

Solve: $-6(x+3)=24$.

Solution

Step 1. Simplify each side of the equation as much as possible.	Use the Distributive Property. Notice that each side of the equation is simplified as much as possible.	$-6(x+3)=24$ $-6x-18=24$
Step 2. Collect all variable terms on one side of the equation.	Nothing to do – all x's are on the left side.	
Step 3. Collect constant terms on the other side of the equation.	To get constants only on the right, add 18 to each side. Simplify.	$-6x-18+18=24+18$ $-6x=42$

Step 4. Make the coefficient of the variable term to equal to 1.	Divide each side by –6.	$\frac{-6x}{-6} = \frac{42}{-6}$
	Simplify.	$x = -7$
Step 5. Check the solution.	Let $x = -7$	**Check:** $-6(x + 3) = 24$
	Simplify.	$-6(-7 + 3) \stackrel{?}{=} 24$
	Multiply.	$-6(-4) \stackrel{?}{=} 24$
		$24 = 24$ ✓

Try It 2.4.2

Solve: $5(x+3) = 35$

Answer

$x = 4$

Try It 2.4.3

Solve: $6(y-4) = -18$

Answer

$y = 1$

GENERAL STRATEGY FOR SOLVING LINEAR EQUATIONS.

1. **Simplify each side of the equation as much as possible.**
   Use the Distributive Property to remove any parentheses.
   Combine like terms.
2. **Collect all the variable terms on one side of the equation.**
   Use the Addition or Subtraction Property of Equality.
3. **Collect all the constant terms on the other side of the equation.**
   Use the Addition or Subtraction Property of Equality.
4. **Make the coefficient of the variable term to equal to 1.**
   Use the Multiplication or Division Property of Equality.
   State the solution to the equation.
5. **Check the solution.** Substitute the solution into the original equation to make sure the result is a true statement.

Example 2.4.4

Solve: $-(y+9) = 8$

Solution

	$-(y + 9) = 8$
Simplify each side of the equation as much as possible by distributing.	$-y - 9 = 8$
The only y term is on the left side, so all variable terms are on the left side of the equation.	
Add 9 to both sides to get all constant terms on the right side of the equation.	$-y - 9 + 9 = 8 + 9$
Simplify.	$-y = 17$

Rewrite $-y$ as $-1y$.		$-1y = 17$
Make the coefficient of the variable term to equal to 1 by dividing both sides by -1.		$\frac{-1y}{-1} = \frac{17}{-1}$
Simplify.		$y = -17$
Check:	$-(y+9) = 8$	
Let y=−17.	$-(-17+9) \stackrel{?}{=} 8$	
	$-(-8) \stackrel{?}{=} 8$	
	$8 = 8$ ✓	

Try It 2.4.5

Solve: $-(y+8) = -2$

Answer

$y = -6$

Try It 2.4.6

Solve: $-(z+4) = -12$

Answer

$z = 8$

Example 2.4.7

Solve: $5(a-3)+5 = -10$

Solution

		$5(a-3)+5 = -10$
Simplify each side of the equation as much as possible.		
Distribute.		$5a-15+5 = -10$
Combine like terms.		$5a-10 = -10$
The only a term is on the left side, so all variable terms are on one side of the equation.		
Add 10 to both sides to get all constant terms on the other side of the equation.		$5a-10+10 = -10+10$
Simplify.		$5a = 0$
Make the coefficient of the variable term to equal to 11 by dividing both sides by 55.		$\frac{5a}{5} = \frac{0}{5}$
Simplify.		$a = 0$
Check:	$5(a-3)+5 = -10$	
Let a=0.	$5(0-3)+5 \stackrel{?}{=} -10$	
	$5(-3)+5 \stackrel{?}{=} -10$	
	$-15+5 \stackrel{?}{=} -10$	

	$-10 = -10$ ✓

? Try It 2.4.8

Solve: $2(m-4)+3=-1$

Answer

$m=2$

? Try It 2.4.9

Solve:$7(n-3)-8=-15$

Answer

$n=2$

✓ Example 2.4.10

Solve: $\frac{2}{3}(6m-3)=8-m$

Solution

	$\frac{2}{3}(6m-3)=8-m$
Distribute.	$4m-2=8-m$
Add m to get the variables only to the left.	$4m+m-2=8-m+m$
Simplify.	$5m-2=8$
Add 2 to get constants only on the right.	$5m-2+2=8+2$
Simplify.	$5m=10$
Divide by 5.	$\frac{5m}{5}=\frac{10}{5}$
Simplify.	$m=2$
Check:	$\frac{2}{3}(6m-3)=8-m$
Let m=2.	$\frac{2}{3}(6\cdot 2-3)\stackrel{?}{=}8-2$
	$\frac{2}{3}(12-3)\stackrel{?}{=}6$
	$\frac{2}{3}(9)\stackrel{?}{=}6$
	$6=6$ ✓

? Try It 2.4.11

Solve: $\frac{1}{3}(6u+3)=7-u$

Answer

$u=2$

Try It 2.4.12

Solve: $\frac{2}{3}(9x-12)=8+2x$

Answer

$x=4$

Example 2.4.13

Solve: $8-2(3y+5)=0$

Solution

	$8-2(3y+5)=0$
Simplify—use the Distributive Property.	$8-6y-10=0$
Combine like terms.	$-6y-2=0$
Add 2 to both sides to collect constants on the right.	$-6y-2+2=0+2$
Simplify.	$-6y=2$
Divide both sides by -6−6.	$\frac{-6y}{-6}=\frac{2}{-6}$
Simplify.	$y=-\frac{1}{3}$
Check: Let y=−13. $8-2(3y+5)=0$ $8-2\left[3\left(-\frac{1}{3}\right)+5\right]=0$ $8-2(-1+5)\stackrel{?}{=}0$ $8-2(4)\stackrel{?}{=}0$ $8-8\stackrel{?}{=}0$ $0=0$ ✓	

Try It 2.4.14

Solve: $12-3(4j+3)=-17$

Answer

$j=\frac{5}{3}$

Try It 2.4.15

Solve: $-6-8(k-2)=-10$

Answer

$k=\frac{5}{2}$

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Example 2.4.16

Solve: $4(x-1)-2=5(2x+3)+6$

Solution

	$4(x-1)-2=5(2x+3)+6$
Distribute.	$4x-4-2=10x+15+6$
Combine like terms.	$4x-6=10x+21$
Subtract 4x to get the variables only on the right side since $10>4$.	$4x-4x-6=10x-4x+21$
Simplify.	$-6=6x+21$
Subtract 21 to get the constants on left.	$-6-21=6x+21-21$
Simplify.	$-27=6x$
Divide by 6.	$\frac{-27}{6}=\frac{6x}{6}$
Simplify.	$-\frac{9}{2}=x$
Check:	$4(x-1)-2=5(2x+3)+6$
Let $x=-\frac{9}{2}$.	$4\left(-\frac{9}{2}-1\right)-2\stackrel{?}{=}5\left[2\left(-\frac{9}{2}\right)+3\right]+6$
	$4\left(-\frac{11}{2}\right)-2\stackrel{?}{=}5(-9+3)+6$
	$-22-2\stackrel{?}{=}5(-6)+6$
	$-24\stackrel{?}{=}-30+6$
	$-24=-24$ ✓

Try It 2.4.17

Solve: $6(p-3)-7=5(4p+3)-12$

Answer

$p=-2$

Try It 2.4.18

Solve: $8(q+1)-5=3(2q-4)-1$

Answer

$q=-8$

Example 2.4.19

Solve: $10[3-8(2s-5)]=15(40-5s)$

Solution

	$10[3-8(2s-5)]=15(40-5s)$

Simplify from the innermost parentheses first.		$10[3-16s+40]=15(40-5s)$
Combine like terms in the brackets.		$10[43-16s]=15(40-5s)$
Distribute.		$430-160s=600-75s$
Add 160s to get the s's to the right.		$430-160s+160s=600-75s+160s$
Simplify.		$430=600+85s$
Subtract 600 to get the constants to the left.		$430-600=600+85s-600$
Simplify.		$-170=85s$
Divide.		$\frac{-170}{85}=\frac{85s}{85}$
Simplify.		$-2=s$
Check:	$10[3-8(2s-5)]=15(40-5s)$	
Substitute s=−2.	$10[3-8(2(-2)-5)] \stackrel{?}{=} 15(40-5(-2))$	
	$10[3-8(-4-5)] \stackrel{?}{=} 15(40+10)$	
	$10[3-8(-9)] \stackrel{?}{=} 15(50)$	
	$10[3+72] \stackrel{?}{=} 750$	
	$10[75] \stackrel{?}{=} 750$	
	$750=750$ ✓	

? Try It 2.4.20

Solve: $6[4-2(7y-1)]=8(13-8y)$.

Answer

$y=-\frac{17}{5}$

? Try It 2.4.21

Solve: $12[1-5(4z-1)]=3(24+11z)$.

Answer

$z=0$

✓ Example 2.4.22

Solve: $0.36(100n+5)=0.6(30n+15)$.

Solution

	$0.36(100n+5)=0.6(30n+15)$
Distribute.	$36n+1.8=18n+9$
Subtract 18n to get the variables to the left.	$36n-18n+1.8=18n-18n+9$
Simplify.	$18n+1.8=9$

Subtract 1.8 to get the constants to the right.		$18n + 1.8 - 1.8 = 9 - 1.8$
Simplify.		$18n = 7.2$
Divide.		$\frac{18n}{18} = \frac{7.2}{18}$
Simplify.		$n = 0.4$
Check:	$0.36(100n + 5) = 0.6(30n + 15)$	
Let n=0.4.	$0.36(100(0.4) + 5) \stackrel{?}{=} 0.6(30(0.4) + 15)$	
	$0.36(40 + 5) \stackrel{?}{=} 0.6(12 + 15)$	
	$0.36(45) \stackrel{?}{=} 0.6(27)$	
	$16.2 = 16.2$ ✓	

? Try It 2.4.23

Solve: $0.55(100n+8) = 0.6(85n+14)$.

Answer

$n = 1$

? Try It 2.4.24

Solve: $0.15(40m - 120) = 0.5(60m + 12)$.

Answer

$m = -1$

Classify Equations

Consider the equation we solved at the start of the last section, 7x+8=−13. The solution we found was x=−3. This means the equation 7x+8=−13 is true when we replace the variable, *x*, with the value −3. We showed this when we checked the solution x=−3 and evaluated 7x+8=−13 for x=−3.

$$7(-3) + 8 \stackrel{?}{=} -13$$
$$-21 + 8 \stackrel{?}{=} -13$$
$$-13 = -13 \checkmark$$

If we evaluate 7x+8 for a different value of *x*, the left side will not be −13.

The equation 7x+8=−13 is true when we replace the variable, *x*, with the value −3, but not true when we replace *x* with any other value. Whether or not the equation 7x+8=−13 is true depends on the value of the variable. Equations like this are called conditional equations.

All the equations we have solved so far are conditional equations.

CONDITIONAL EQUATION

An equation that is true for one or more values of the variable and false for all other values of the variable is a **conditional equation**.

Now let's consider the equation 2y+6=2(y+3). Do you recognize that the left side and the right side are equivalent? Let's see what happens when we solve for *y*.

	$2y+6=2(y+3)$
Distribute.	$2y+6=2y+6$
Subtract 2y to get the y's to one side.	$2y-2y+6=2y-2y+6$
Simplify—the y's are gone!	$6=6$

But 6=6 is true.

This means that the equation 2y+6=2(y+3) is true for any value of *y*. We say the solution to the equation is all of the real numbers. An equation that is true for any value of the variable like this is called an **identity**.

IDENTITY

An equation that is true for any value of the variable is called an **identity**.

The solution of an identity is all real numbers.

What happens when we solve the equation 5z=5z−1?

	$5z=5z-1$
Subtract 5z to get the constant alone on the right.	$5z-5z=5z-5z-1$
Simplify—the z's are gone!	$0\neq -1$

But $0 \neq -1$.

Solving the equation 5z=5z−1 led to the false statement 0=−1. The equation 5z=5z−1 will not be true for any value of z. It has no solution. An equation that has no solution, or that is false for all values of the variable, is called a **contradiction**.

CONTRADICTION

An equation that is false for all values of the variable is called a **contradiction**.

A contradiction has no solution.

Example 2.4.25

Classify the equation as a conditional equation, an identity, or a contradiction. Then state the solution.

$6(2n-1)+3=2n-8+5(2n+1)$

Solution

$6(2n-1)+3=2n-8+5(2n+1)$	
Distribute.	$12n-6+3=2n-8+10n+5$
Combine like terms.	$12n-3=12n-3$
Subtract 12n to get the nn's to one side.	$12n-12n-3=12n-12n-3$
Simplify.	$-3=-3$
This is a true statement.	The equation is an identity. The solution is all real numbers.

Try It 2.4.26

Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution:

$4+9(3x-7)=-42x-13+23(3x-2)$

Answer

identity; all real numbers

Try It 2.4.27

Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution:

$8(1-3x)+15(2x+7)=2(x+50)+4(x+3)+1$

Answer

identity; all real numbers

Example 2.4.28

Classify as a conditional equation, an identity, or a contradiction. Then state the solution.

$10+4(p-5)=0$

Solution

	$10+4(p-5)=0$
Distribute.	$10+4p-20=0$
Combine like terms.	$4p-10=0$
Add 10 to both sides.	$4p-10+10=0+10$
Simplify.	$4p=10$
Divide.	$\frac{4p}{4}=\frac{10}{4}$
Simplify.	$p=\frac{5}{2}$
The equation is true when $p=frac52$.	This is a conditional equation. The solution is $p=frac52$.

Try It 2.4.29

Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution: $11(q+3)-5=19$

Answer

conditional equation; \(q = \frac{9}{11}\

Try It 2.4.30

Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution: $6+14(k-8)=95$

Answer

conditional equation; $k = \frac{193}{14}$

Example 2.4.31

Classify the equation as a conditional equation, an identity, or a contradiction. Then state the solution.

$5m + 3(9 + 3m) = 2(7m - 11)$

Solution

	$5m + 3(9 + 3m) = 2(7m - 11)$
Distribute.	$5m + 27 + 9m = 14m - 22$
Combine like terms.	$14m + 27 = 14m - 22$
Subtract 14m from both sides.	$14m + 27 - 14m = 14m - 22 - 14m$
Simplify.	$27 \neq -22$
But $27 \neq -22$.	The equation is a contradiction. It has no solution.

Try It 2.4.32

Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution:

$12c + 5(5 + 3c) = 3(9c - 4)$

Answer

contradiction; no solution

Try It 2.4.33

Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution:

$4(7d + 18) = 13(3d - 2) - 11d$

Answer

contradiction; no solution

Type of equation	What happens when you solve it?	Solution
Conditional Equation	True for one or more values of the variables and false for all other values	One or more values
Identity	**True** for any value of the variable	All real numbers
Contradiction	**False** for all values of the variable	No solution

Key Concepts

- **General Strategy for Solving Linear Equations**
  1. Simplify each side of the equation as much as possible.
     Use the Distributive Property to remove any parentheses.
     Combine like terms.

2. Collect all the variable terms on one side of the equation.
   Use the Addition or Subtraction Property of Equality.
3. Collect all the constant terms on the other side of the equation.
   Use the Addition or Subtraction Property of Equality.
4. Make the coefficient of the variable term to equal to 1.
   Use the Multiplication or Division Property of Equality.
   State the solution to the equation.
5. Check the solution.
   Substitute the solution into the original equation.

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2.4E: Exercises

Practice Makes Perfect

Solve Equations Using the General Strategy for Solving Linear Equations

In the following exercises, solve each linear equation.

Exercise 2.4E.1

$15(y-9)=-60$

Exercise 2.4E.2

$21(y-5)=-42$

Answer

$y=3$

Exercise 2.4E.3

$-9(2n+1)=36$

Exercise 2.4E.4

$-16(3n+4)=32$

Answer

$n=-2$

Exercise 2.4E.5

$8(22+11r)=0$

Exercise 2.4E.6

$5(8+6p)=0$

Answer

$p=-\frac{4}{3}$

Exercise 2.4E.7

$-(w-12)=30$

Exercise 2.4E.8

$-(t-19)=28$

Answer

$t=-9$

Exercise 2.4E.9

$9(6a+8)+9=81$

Exercise 2.4E.10

$8(9b-4)-12=100$

Answer

$b=2$

Exercise 2.4E.11

$32+3(z+4)=41$

Exercise 2.4E.12

$21+2(m-4)=25$

Answer

$m=6$

Exercise 2.4E.13

$51+5(4-q)=56$

Exercise 2.4E.14

$-6+6(5-k)=15$

Answer

$k=\frac{3}{2}$

Exercise 2.4E.15

$2(9s-6)-62=16$

Exercise 2.4E.16

$8(6t-5)-35=-27$

Answer

$t=1$

Exercise 2.4E.17

$3(10-2x)+54=0$

Exercise 2.4E.18

$-2(11-7x)+54=4$

Answer

$x=-2$

Exercise 2.4E.19

$\frac{2}{3}(9c-3)=22$

Exercise 2.4E.20

$\frac{3}{5}(10x-5)=27$

Answer

$x=5$

Exercise 2.4E.21

$\frac{1}{5}(15c+10)=c+7$

Exercise 2.4E.22

$\frac{1}{4}(20d+12)=d+7$

Answer

$d=1$

Exercise 2.4E.23

$18-(9r+7)=-16$

Exercise 2.4E.24

$15-(3r+8)=28$

Answer

$r=-7$

Exercise 2.4E.25

$5-(n-1)=19$

Exercise 2.4E.26

$-3-(m-1)=13$

Answer

$m=-15$

Exercise 2.4E.27

$11-4(y-8)=43$

Exercise 2.4E.28

$18-2(y-3)=32$

Answer

$y=-4$

Exercise 2.4E.29

$24-8(3v+6)=0$

Exercise 2.4E.30

$35-5(2w+8)=-10$

Answer

$w=\frac{1}{2}$

Exercise 2.4E.31

$4(a-12)=3(a+5)$

Exercise 2.4E.32

$-2(a-6)=4(a-3)$

Answer

$a=4$

Exercise 2.4E.33

$2(5-u)=-3(2u+6)$

Exercise 2.4E.34

$5(8-r)=-2(2r-16)$

Answer

$r=8$

Exercise 2.4E.35

$3(4n-1)-2=8n+3$

Exercise 2.4E.36

$9(2m-3)-8=4m+7$

Answer

$m=3$

Exercise 2.4E.37

$12+2(5-3y)=-9(y-1)-2$

Exercise 2.4E.38

$-15+4(2-5y)=-7(y-4)+4$

Answer

$y=-3$

Exercise 2.4E.39

$8(x-4)-7x=14$

Exercise 2.4E.40

$5(x-4)-4x=14$

Answer

$x=34$

Exercise 2.4E.41

$5+6(3s-5)=-3+2(8s-1)$

Exercise 2.4E.42

$-12+8(x-5)=-4+3(5x-2)$

Answer

$x=-6$

Exercise 2.4E.43

$4(u-1)-8=6(3u-2)-7$

Exercise 2.4E.44

$7(2n-5)=8(4n-1)-9$

Answer

$n=-1$

Exercise 2.4E.45

$4(p-4)-(p+7)=5(p-3)$

Exercise 2.4E.46

$3(a-2)-(a+6)=4(a-1)$

Answer

$a=-4$

Exercise 2.4E.47

$-(9y+5)-(3y-7)$
$=16-(4y-2)$

Exercise 2.4E.48

$-(7m+4)-(2m-5)$
$=14-(5m-3)$

Answer

$m=-4$

Exercise 2.4E.49

$4[5-8(4c-3)]$
$=12(1-13c)-8$

Exercise 2.4E.50

$5[9-2(6d-1)]$
$=11(4-10d)-139$

Answer

$d=-3$

Exercise 2.4E.51

$3[-9+8(4h-3)]$
$=2(5-12h)-19$

Exercise 2.4E.52

$3[-14+2(15k-6)]$
$=8(3-5k)-24$

Answer

$k=\frac{3}{5}$

Exercise 2.4E.53

$5[2(m+4)+8(m-7)]$
$=2[3(5+m)-(21-3m)]$

Exercise 2.4E.54

$10[5(n+1)+4(n-1)]$
$=11[7(5+n)-(25-3n)]$

Answer

$n=-5$

Exercise 2.4E.55

$5(1.2u-4.8)=-12$

Exercise 2.4E.56

$4(2.5v-0.6)=7.6$

Answer

$v=1$

Exercise 2.4E.57

$0.25(q-6)=0.1(q+18)$

Exercise 2.4E.58

$0.2(p-6)=0.4(p+14)$

Answer

$p=-34$

Exercise 2.4E.59

$0.2(30n+50)=28$

Exercise 2.4E.60

$0.5(16m+34)=-15$

Answer

$m=-4$

Classify Equations

In the following exercises, classify each equation as a conditional equation, an identity, or a contradiction and then state the solution.

Exercise 2.4E.61

$23z+19=3(5z-9)+8z+46$

Exercise 2.4E.62

$15y+32=2(10y-7)-5y+46$

Answer

identity; all real numbers

Exercise 2.4E.63

$5(b-9)+4(3b+9)=6(4b-5)-7b+21$

Exercise 2.4E.64

$9(a-4)+3(2a+5)=7(3a-4)-6a+7$

Answer

identity; all real numbers

Exercise 2.4E.65

$18(5j-1)+29=47$

Exercise 2.4E.66

$24(3d-4)+100=52$

Answer

conditional equation; $d=\frac{2}{3}$

Exercise 2.4E.67

$22(3m-4)=8(2m+9)$

Exercise 2.4E.68

$30(2n-1)=5(10n+8)$

Answer

conditional equation; $n=7$

Exercise 2.4E.69

$7v+42=11(3v+8)-2(13v-1)$

Exercise 2.4E.70

$18u-51=9(4u+5)-6(3u-10)$

Answer

contradiction; no solution

Exercise 2.4E.71

$3(6q-9)+7(q+4)=5(6q+8)-5(q+1)$

Exercise 2.4E.72

$5(p+4)+8(2p-1)=9(3p-5)-6(p-2)$

Answer

contradiction; no solution

Exercise 2.4E.73

$12(6h-1)=8(8h+5)-4$

Exercise 2.4E.74

$9(4k-7)=11(3k+1)+4$

Answer

conditional equation; $k=26$

Exercise 2.4E.75

$45(3y-2)=9(15y-6)$

Exercise 2.4E.76

$60(2x-1)=15(8x+5)$

Answer

contradiction; no solution

Exercise 2.4E.77

$16(6n+15)=48(2n+5)$

Exercise 2.4E.78

$36(4m+5)=12(12m+15)$

Answer

identity; all real numbers

Exercise 2.4E.79

$9(14d+9)+4d=13(10d+6)+3$

Exercise 2.4E.80

$11(8c+5)-8c=2(40c+25)+5$

Answer

identity; all real numbers

Everyday Math

Exercise 2.4E.81

Fencing Micah has 44 feet of fencing to make a dog run in his yard. He wants the length to be 2.5 feet more than the width. Find the length, L, by solving the equation 2L+2(L−2.5)=44.

Exercise 2.4E.82

Coins Rhonda has $\$1.90$ in nickels and dimes. The number of dimes is one less than twice the number of nickels. Find the number of nickels, n, by solving the equation $0.05n+0.10(2n-1)=1.90$.

Answer

8 nickels

Writing Exercises

Exercise 2.4E.83

Using your own words, list the steps in the general strategy for solving linear equations.

Exercise 2.4E.84

Explain why you should simplify both sides of an equation as much as possible before collecting the variable terms to one side and the constant terms to the other side.

Answer

Answers will vary.

Exercise 2.4E.85

What is the first step you take when solving the equation $3-7(y-4)=38$? Why is this your first step?

Exercise 2.4E.86

Solve the equation $\frac{1}{4}(8x+20)=3x-4$ explaining all the steps of your solution as in the examples in this section.

Answer

Answers will vary.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objective of this section.

I can...	Confidently	With some help	No-I don't get it!
solve equations using the general strategy for solving linear equations.			
classify equations.			

ⓑ On a scale of 1-10, how would you rate your mastery of this section in light of your responses on the checklist? How can you improve this?

Glossary

conditional equation

An equation that is true for one or more values of the variable and false for all other values of the variable is a conditional equation.

contradiction

An equation that is false for all values of the variable is called a contradiction. A contradiction has no solution.

identity

An equation that is true for any value of the variable is called an identity. The solution of an identity is all real numbers.

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2.5: Solve Equations with Fractions or Decimals

Learning Objectives

By the end of this section, you will be able to:

- Solve equations with fraction coefficients
- Solve equations with decimal coefficients

Note

Before you get started, take this readiness quiz.

1. Multiply: $8 \cdot 38$.
   If you missed this problem, review Exercise 1.6.16.
2. Find the LCD of $\frac{5}{6}$ and $\frac{1}{4}$.
   If you missed this problem, review Exercise 1.7.16.
3. Multiply 4.78 by 100.
   If you missed this problem, review Exercise 1.8.22.

Solve Equations with Fraction Coefficients

Let's use the general strategy for solving linear equations introduced earlier to solve the equation, $\frac{1}{8}x + \frac{1}{2} = \frac{1}{4}$.

Table 2.5.1

	$\frac{1}{8}x+\frac{1}{2}=\frac{1}{4}$
To isolate the x term, subtract $\frac{1}{2}$ from both sides.	$\frac{1}{8}x+\frac{1}{2}-\frac{1}{2}=\frac{1}{4}-\frac{1}{2}$
Simplify the left side.	$\frac{1}{8}x=\frac{1}{4}-\frac{1}{2}$
Change the constants to equivalent fractions with the LCD.	$\frac{1}{8}x=\frac{1}{4}-\frac{2}{4}$
Subtract.	$\frac{1}{8}x=-\frac{1}{4}$
Multiply both sides by the reciprocal of $\frac{1}{8}$.	$\frac{8}{1}\cdot\frac{1}{8}x=\frac{8}{1}\left(-\frac{1}{4}\right)$
Simplify.	$x=-2$

This method worked fine, but many students do not feel very confident when they see all those fractions. So, we are going to show an alternate method to solve equations with fractions. This alternate method eliminates the fractions.

We will apply the Multiplication Property of Equality and multiply both sides of an equation by the least common denominator of all the fractions in the equation. The result of this operation will be a new equation, equivalent to the first, but without fractions. This process is called "clearing" the equation of fractions.

Let's solve a similar equation, but this time use the method that eliminates the fractions.

Example 2.5.1: How to Solve Equations with Fraction Coefficients

Solve: $\frac{1}{6}y - \frac{1}{3} = \frac{5}{6}$

Solution

Step 1. Find the least common denominator of *all* the fractions in the equation.	What is the LCD of $\frac{1}{6}$, $\frac{1}{3}$, and $\frac{5}{6}$?	$\frac{1}{6}y-\frac{1}{3}=\frac{5}{6}$ LCD $=6$
Step 2. Multiply both sides of the equation by that LCD. This clears the fractions.	Multiply both sides of the equation by the LCD 6.	$6\left(\frac{1}{6}y-\frac{1}{3}\right)=6\left(\frac{5}{6}\right)$
	Use the Distributive Property.	$6\cdot\frac{1}{6}y-6\cdot\frac{1}{3}=6\cdot\frac{5}{6}$
	Simplify – and notice, no more fractions!	$y-2=5$
Step 3. Solve using the General Strategy for Solving Linear Equations.	To isolate the "y" term, add 2.	$y-2+2=5+2$
	Simplify.	$y=7$

Try It 2.5.2

Solve: $\frac{1}{4}x+\frac{1}{2}=\frac{5}{8}$

Answer

$x=\frac{1}{2}$

Try It 2.5.3

Solve: $\frac{1}{8}x+\frac{1}{2}=\frac{1}{4}$

Answer

$x=-2$

Notice in Exercise 2.5.1, once we cleared the equation of fractions, the equation was like those we solved earlier in this chapter. We changed the problem to one we already knew how to solve! We then used the General Strategy for Solving Linear Equations.

STRATEGY TO SOLVE EQUATIONS WITH FRACTION COEFFICIENTS.

1. Find the least common denominator of *all* the fractions in the equation.
2. Multiply both sides of the equation by that LCD. This clears the fractions.
3. Solve using the General Strategy for Solving Linear Equations.

Example 2.5.4

Solve: $6=\frac{1}{2}v+\frac{2}{5}v-\frac{3}{4}v$

Solution

We want to clear the fractions by multiplying both sides of the equation by the LCD of all the fractions in the equation.

Find the LCD of all fractions in the equation.	$6=\frac{1}{2}v+\frac{2}{5}v-\frac{3}{4}v$
The LCD is 20.	
Multiply both sides of the equation by 20.	$20(6)=20\cdot\left(\frac{1}{2}v+\frac{2}{5}v-\frac{3}{4}v\right)$
Distribute.	$20(6)=20\cdot\frac{1}{2}v+20\cdot\frac{2}{5}v-20\cdot\frac{3}{4}v$

Simplify—notice, no more fractions!		$120 = 10v + 8v - 15v$
Combine like terms.		$120 = 3v$
Divide by 3.		$\frac{120}{3} = \frac{3v}{3}$
Simplify.		$40 = v$
Check:	$6 = \frac{1}{2}v + \frac{2}{5}v - \frac{3}{4}v$	
Let v=40.	$6 \stackrel{?}{=} \frac{1}{2}(40) + \frac{2}{5}(40) - \frac{3}{4}(40)$	
	$6 \stackrel{?}{=} 20 + 16 - 30$	
	$6 = 6$ ✓	

Try It 2.5.5

Solve: $7 = \frac{1}{2}x + \frac{3}{4}x - \frac{2}{3}x$

Answer

$x = 12$

Try It 2.5.6

Solve: $-1 = \frac{1}{2}u + \frac{1}{4}u - \frac{2}{3}u$

Answer

$u = -12$

In the next example, we again have variables on both sides of the equation.

Example 2.5.7

Solve: $a + \frac{3}{4} = \frac{3}{8}a - \frac{1}{2}$

Solution

	$a + \frac{3}{4} = \frac{3}{8}a - \frac{1}{2}$
Find the LCD of all fractions in the equation. The LCD is 8.	
Multiply both sides by the LCD.	$8\left(a + \frac{3}{4}\right) = 8\left(\frac{3}{8}a - \frac{1}{2}\right)$
Distribute.	$8 \cdot a + 8 \cdot \frac{3}{4} = 8 \cdot \frac{3}{8}a - 8 \cdot \frac{1}{2}$
Simplify—no more fractions.	$8a + 6 = 3a - 4$
Subtract 3a3a from both sides.	$8a - 3a + 6 = 3a - 3a - 4$
Simplify.	$5a + 6 = -4$
Subtract 6 from both sides.	$5a + 6 - 6 = -4 - 6$
Simplify.	$5a = -10$

Divide by 5.		$\frac{5a}{5}=\frac{-10}{5}$
Simplify.		$a=-2$
Check:	$a+\frac{3}{4}=\frac{3}{8}a-\frac{1}{2}$	
Let a=−2.	$-2+\frac{3}{4}\stackrel{?}{=}\frac{3}{8}(-2)-\frac{1}{2}$	
	$-\frac{8}{4}+\frac{3}{4}\stackrel{?}{=}-\frac{16}{8}-\frac{4}{8}$	
	$-\frac{5}{4}=-\frac{10}{8}$	
	$-\frac{5}{4}=-\frac{5}{4}\checkmark$	

? Try It 2.5.8

Solve: $x+\frac{1}{3}=\frac{1}{6}x-\frac{1}{2}$

Answer

$x=-1$

? Try It 2.5.9

Solve: $c+\frac{3}{4}=\frac{1}{2}c-\frac{1}{4}$

Answer

$c=-2$

In the next example, we start by using the Distributive Property. This step clears the fractions right away.

✓ Example 2.5.10

Solve: $-5=\frac{1}{4}(8x+4)$

Solution

	$-5=\frac{1}{4}(8x+4)$
Distribute.	$-5=\frac{1}{4}\cdot 8x+\frac{1}{4}\cdot 4$
Simplify. Now there are no fractions.	$-5=2x+1$
Subtract 1 from both sides.	$-5-1=2x+1-1$
Simplify.	$-6=2x$
Divide by 2.	$\frac{-6}{2}=\frac{2x}{2}$
Simplify.	$-3=x$
Check:	$-5=\frac{1}{4}(8x+4)$
Let x=−3.	$-5\stackrel{?}{=}\frac{1}{2}(4(-3)+2)$

	$-5 \stackrel{?}{=} \frac{1}{2}(-12+2)$	
	$-5 \stackrel{?}{=} \frac{1}{2}(-10)$	
	$-5 = -5$ ✓	

Try It 2.5.11

Solve: $-11 = \frac{1}{2}(6p+2)$

Answer

$p = -4$

Try It 2.5.12

Solve: $8 = \frac{1}{3}(9q+6)$

Answer

$q = 2$

In the next example, even after distributing, we still have fractions to clear.

Example 2.5.13

Solve: $\frac{1}{2}(y-5) = \frac{1}{4}(y-1)$

Solution

	$\frac{1}{2}(y-5) = \frac{1}{4}(y-1)$
Distribute.	$\frac{1}{2} \cdot y - \frac{1}{2} \cdot 5 = \frac{1}{4} \cdot y - \frac{1}{4} \cdot 1$
Simplify.	$\frac{1}{2}y - \frac{5}{2} = \frac{1}{4}y - \frac{1}{4}$
Multiply by the LCD, 4.	$4\left(\frac{1}{2}y - \frac{5}{2}\right) = 4\left(\frac{1}{4}y - \frac{1}{4}\right)$
Distribute.	$4 \cdot \frac{1}{2}y - 4 \cdot \frac{5}{2} = 4 \cdot \frac{1}{4}y - 4 \cdot \frac{1}{4}$
Simplify.	$2y - 10 = y - 1$
Collect the variables to the left.	$2y - y - 10 = y - y - 1$
Simplify.	$y - 10 = -1$
Collect the constants to the right.	$y - 10 + 10 = -1 + 10$
Simplify.	$y = 9$
Check:	$\frac{1}{2}(y-5) = \frac{1}{4}(y-1)$
Let y=9.	$\frac{1}{2}(9-5) \stackrel{?}{=} \frac{1}{4}(9-1)$
Finish the check on your own.	

Try It 2.5.14

Solve: $\frac{1}{5}(n+3)=\frac{1}{4}(n+2)$

Answer

$n=2$

Try It 2.5.15

Solve: $\frac{1}{2}(m-3)=\frac{1}{4}(m-7)$

Answer

$m=-1$

Example 2.5.16

Solve: $\frac{5x-3}{4}=\frac{x}{2}$

Solution

	$\frac{5x-3}{4}=\frac{x}{2}$
Multiply by the LCD, 4.	$4\left(\frac{5x-3}{4}\right)=4\left(\frac{x}{2}\right)$
Simplify.	$5x-3=2x$
Collect the variables to the right.	$5x-5x-3=2x-5x$
Simplify.	$-3=-3x$
Divide.	$\frac{-3}{-3}=\frac{-3x}{-3}$
Simplify.	$1=x$
Check:	$\frac{5x-3}{4}=\frac{x}{2}$
Let x=1.	$\frac{5(1)-3}{4}\stackrel{?}{=}\frac{1}{2}$
	$\frac{2}{4}\stackrel{?}{=}\frac{1}{2}$
	$\frac{1}{2}=\frac{1}{2}\checkmark$

Try It 2.5.17

Solve: $\frac{4y-7}{3}=\frac{y}{6}$

Answer

$y=2$

Try It 2.5.18

Solve: $\frac{-2z-5}{4}=\frac{z}{8}$

Answer

$z = -2$

Example 2.5.19

Solve: $\frac{a}{6} + 2 = \frac{a}{4} + 3$

Solution

	$\frac{a}{6} + 2 = \frac{a}{4} + 3$
Multiply by the LCD, 12.	$12\left(\frac{a}{6} + 2\right) = 12\left(\frac{a}{4} + 3\right)$
Distribute.	$12 \cdot \frac{a}{6} + 12 \cdot 2 = 12 \cdot \frac{a}{4} + 12 \cdot 3$
Simplify.	$2a + 24 = 3a + 36$
Collect the variables to the right.	$2a - 2a + 24 = 3a - 2a + 36$
Simplify.	$24 = a + 36$
Collect the constants to the left.	$24 - 36 = a + 36 - 36$
Simplify.	$a = -12$
Check:	$\frac{a}{6} + 2 = \frac{a}{4} + 3$
Let a=−12.	$\frac{-12}{6} + 2 \stackrel{?}{=} \frac{-12}{4} + 3$
	$-2 + 2 \stackrel{?}{=} -3 + 3$
	$0 = 0$ ✓

Try It 2.5.20

Solve: $\frac{b}{10} + 2 = \frac{b}{4} + 5$

Answer

$b = -20$

Try It 2.5.21

Solve: $\frac{c}{6} + 3 = \frac{c}{3} + 4$

Answer

$c = -6$

Example 2.5.22

Solve: $\frac{4q+3}{2} + 6 = \frac{3q+5}{4}$

Solution

	$\frac{4q+3}{2} + 6 = \frac{3q+5}{4}$
Multiply by the LCD, 4.	$4\left(\frac{4q+3}{2} + 6\right) = 4\left(\frac{3q+5}{4}\right)$

Distribute.	$4\left(\frac{4q+3}{2}\right)+4\cdot 6=4\cdot\left(\frac{3q+5}{4}\right)$
Simplify.	$2(4q+3)+24=3q+5$ $8q+6+24=3q+5$ $8q+30=3q+5$
Collect the variables to the left.	$8q-3q+30=3q-3q+5$
Simplify.	$5q+30=5$
Collect the constants to the right.	$5q+30-30=5-30$
Simplify.	$5q=-25$
Divide by 5.	$\frac{5q}{5}=\frac{-25}{5}$
Simplify.	$q=-5$
Check:	$\frac{4q+3}{2}+6=\frac{3q+5}{4}$
Let q=−5.	$\frac{4(-5)+3}{2}+6\stackrel{?}{=}\frac{3(-5)+5}{4}$
Finish the check on your own.	

Try It 2.5.23

Solve: $\frac{3r+5}{6}+1=\frac{4r+3}{3}$

Answer

$r=1$

Try It 2.5.24

Solve: $\frac{2s+3}{2}+1=\frac{3s+2}{4}$

Answer

$s=-8$

Solve Equations with Decimal Coefficients

Some equations have decimals in them. This kind of equation will occur when we solve problems dealing with money or percentages. But decimals can also be expressed as fractions. For example, $0.3=\frac{3}{10}$ and $0.17=\frac{17}{100}$. So, with an equation with decimals, we can use the same method we used to clear fractions—multiply both sides of the equation by the least common denominator.

Example 2.5.25

Solve: $0.06x+0.02=0.25x-1.5$

Solution

Look at the decimals and think of the equivalent fractions.

$0.06=\frac{6}{100}\quad 0.02=\frac{2}{100}\quad 0.25=\frac{25}{100}\quad 1.5=1\frac{5}{10}$

Notice, the LCD is 100.

By multiplying by the LCD, we will clear the decimals from the equation.

	$0.06x + 0.02 = 0.25x - 1.5$
Multiply both side by 100.	$100(0.06x + 0.02) = 100(0.25x - 1.5)$
Distribute.	$100(0.06x) + 100(0.02) = 100(0.25x) - 100(1.5)$
Multiply, and now we have no more decimals.	$6x + 2 = 25x - 150$
Collect the variables to the right.	$6x - 6x + 2 = 25x - 6x - 150$
Simplify.	$2 = 19x - 150$
Collect the variables to the right.	$2 + 150 = 19x - 150 + 150$
Simplify.	$152 = 19x$
Divide by 19.	$\frac{152}{19} = \frac{19x}{19}$
Simplify.	$8 = x$
Check: Let x=8 $0.06(8) + 0.02 \stackrel{?}{=} 0.25(8) - 1.5$ $0.48 + 0.02 \stackrel{?}{=} 2.00 - 1.5$ $0.50 = 0.50$ ✓	

Try It 2.5.26

Solve: $0.14h + 0.12 = 0.35h - 2.4$

Answer

$h = 12$

Try It 2.5.27

Solve: $0.65k - 0.1 = 0.4k - 0.35$

Answer

$k = -1$

The next example uses an equation that is typical of the money applications in the next chapter. Notice that we distribute the decimal before we clear all the decimals.

Example 2.5.28

Solve: $0.25x + 0.05(x + 3) = 2.85$

Solution

	$0.25x + 0.05(x + 3) = 2.85$
Distribute first.	$0.25x + 0.05x + 0.15 = 2.85$
Combine like terms.	$0.30x + 0.15 = 2.85$
To clear decimals, multiply by 100.	$100(0.30x + 0.15) = 100(2.85)$
Distribute.	$30x + 15 = 285$
Subtract 15 from both sides.	$30x + 15 - 15 = 285 - 15$

Simplify.	$30x = 270$
Divide by 30.	$\frac{30x}{30} = \frac{270}{30}$
Simplify.	$x = 9$
Check it yourself by substituting x=9 into the original equation.	

Try It 2.5.29

Solve: $0.25n + 0.05(n + 5) = 2.95$

Answer

$n = 9$

Try It 2.5.30

Solve: $0.10d + 0.05(d - 5) = 2.15$

Answer

$d = 16$

Key Concepts

- **Strategy to Solve an Equation with Fraction Coefficients**
  1. Find the least common denominator of all the fractions in the equation.
  2. Multiply both sides of the equation by that LCD. This clears the fractions.
  3. Solve using the General Strategy for Solving Linear Equations.

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2.5E: Exercises

Practice Makes Perfect

Solve Equations with Fraction Coefficients

In the following exercises, solve each equation with fraction coefficients.

Exercise 2.5E.1

$\frac{1}{4}x - \frac{1}{2} = -\frac{3}{4}$

Exercise 2.5E.2

$\frac{3}{4}x - \frac{1}{2} = \frac{1}{4}$

Answer

x=1

Exercise 2.5E.3

$\frac{5}{6}y - \frac{2}{3} = -\frac{3}{2}$

Exercise 2.5E.4

$\frac{5}{6}y - \frac{1}{3} = -\frac{7}{6}$

Answer

$y = -1$

Exercise 2.5E.5

$\frac{1}{2}a + \frac{3}{8} = \frac{3}{4}$

Exercise 2.5E.6

$\frac{5}{8}b + \frac{1}{2} = -\frac{3}{4}$

Answer

$b = -2$

Exercise 2.5E.7

$2 = \frac{1}{3}x - \frac{1}{2}x + \frac{2}{3}x$

Exercise 2.5E.8

$2 = \frac{3}{5}x - \frac{1}{3}x + \frac{2}{5}x$

Answer

$x = 3$

Exercise 2.5E. 9

$\frac{1}{4}m - \frac{4}{5}m + \frac{1}{2}m = -1$

Exercise 2.5E. 10

$\frac{5}{6}n - \frac{1}{4}n - \frac{1}{2}n = -2$

Answer

$n = -24$

Exercise 2.5E. 11

$x + \frac{1}{2} = \frac{2}{3}x - \frac{1}{2}$

Exercise 2.5E. 12

$x + \frac{3}{4} = \frac{1}{2}x - \frac{5}{4}$

Answer

$x = -4$

Exercise 2.5E. 13

$\frac{1}{3}w + \frac{5}{4} = w - \frac{1}{4}$

Exercise 2.5E. 14

$\frac{3}{2}z + \frac{1}{3} = z - \frac{2}{3}$

Answer

$z = -2$

Exercise 2.5E. 15

$\frac{1}{2}x - \frac{1}{4} = \frac{1}{12}x + \frac{1}{6}$

Exercise 2.5E. 16

$\frac{1}{2}a - \frac{1}{4} = \frac{1}{6}a + \frac{1}{12}$

Answer

$a = 1$

Exercise 2.5E. 17

$\frac{1}{3}b + \frac{1}{5} = \frac{2}{5}b - \frac{3}{5}$

Exercise 2.5E.18

$\frac{1}{3}x+\frac{2}{5}=\frac{1}{5}x-\frac{2}{5}$

Answer

$x=-6$

Exercise 2.5E.19

$1=\frac{1}{6}(12x-6)$

Exercise 2.5E.20

$1=\frac{1}{5}(15x-10)$

Answer

$x=1$

Exercise 2.5E.21

$\frac{1}{4}(p-7)=\frac{1}{3}(p+5)$

Exercise 2.5E.22

$\frac{1}{5}(q+3)=\frac{1}{2}(q-3)$

Answer

$q=7$

Exercise 2.5E.23

$\frac{1}{2}(x+4)=\frac{3}{4}$

Exercise 2.5E.24

$\frac{1}{3}(x+5)=\frac{5}{6}$

Answer

$x=-\frac{5}{2}$

Exercise 2.5E.25

$\frac{5q-8}{5}=\frac{2q}{10}$

Exercise 2.5E.26

$\frac{4m+2}{6}=\frac{m}{3}$

Answer

$m=-1$

Exercise 2.5E.27

$\frac{4n+8}{4} = \frac{n}{3}$

Exercise 2.5E.28

$\frac{3p+6}{3} = \frac{p}{2}$

Answer

$p = -4$

Exercise 2.5E.29

$\frac{u}{3} - 4 = \frac{u}{2} - 3$

Exercise 2.5E.30

$\frac{v}{10} + 1 = \frac{v}{4} - 2$

Answer

$v = 20$

Exercise 2.5E.31

$\frac{c}{15} + 1 = \frac{c}{10} - 1$

Exercise 2.5E.32

$\frac{d}{6} + 3 = \frac{d}{8} + 2$

Answer

$d = -24$

Exercise 2.5E.33

$\frac{3x+4}{2} + 1 = \frac{5x+10}{8}$

Exercise 2.5E.34

$\frac{10y-2}{3} + 3 = \frac{10y+1}{9}$

Answer

$y = -1$

Exercise 2.5E.35

$\frac{7u-1}{4} - 1 = \frac{4u+8}{5}$

Exercise 2.5E.36

$\frac{3v-6}{2}+5=\frac{11v-4}{5}$

Answer

$v=4$

Solve Equations with Decimal Coefficients

In the following exercises, solve each equation with decimal coefficients.

Exercise 2.5E.37

$0.6y+3=9$

Exercise 2.5E.38

$0.4y-4=2$

Answer

$y=15$

Exercise 2.5E.39

$3.6j-2=5.2$

Exercise 2.5E.40

$2.1k+3=7.2$

Answer

$k=2$

Exercise 2.5E.41

$0.4x+0.6=0.5x-1.2$

Exercise 2.5E.42

$0.7x+0.4=0.6x+2.4$

Answer

$x=20$

Exercise 2.5E.43

$0.23x+1.47=0.37x-1.05$

Exercise 2.5E.44

$0.48x+1.56=0.58x-0.64$

Answer

$x = 22$

Exercise 2.5E.45

$0.9x - 1.25 = 0.75x + 1.75$

Exercise 2.5E.46

$1.2x - 0.91 = 0.8x + 2.29$

Answer

$x = 8$

Exercise 2.5E.47

$0.05n + 0.10(n + 8) = 2.15$

Exercise 2.5E.48

$0.05n + 0.10(n + 7) = 3.55$

Answer

$n = 19$

Exercise 2.5E.49

$0.10d + 0.25(d + 5) = 4.05$

Exercise 2.5E.50

$0.10d + 0.25(d + 7) = 5.25$

Answer

$d = 10$

Exercise 2.5E.51

$0.05(q - 5) + 0.25q = 3.05$

Exercise 2.5E.52

$0.05(q - 8) + 0.25q = 4.10$

Answer

$q = 15$

Everyday Math

Exercise 2.5E.53

Coins Taylor has $200 in dimes and pennies. The number of pennies is 2 more than the number of dimes. Solve the equation $0.10d+0.01(d+2)=2$ for d, the number of dimes.

Exercise 2.5E.54

Stamps Paula bought $22.82 worth of 49-cent stamps and 21-cent stamps. The number of 21-cent stamps was 8 less than the number of 49-cent stamps. Solve the equation $0.49s+0.21(s-8)=22.82$ for s, to find the number of 49-cent stamps Paula bought.

Answer

$s=35$

Writing Exercises

Exercise 2.5E.55

Explain how you find the least common denominator of $\frac{3}{8}$, $\frac{1}{6}$, and $\frac{2}{3}$

Exercise 2.5E.56

If an equation has several fractions, how does multiplying both sides by the LCD make it easier to solve?

Answer

Answers will vary.

Exercise 2.5E.57

If an equation has fractions only on one side, why do you have to multiply both sides of the equation by the LCD?

Exercise 2.5E.58

In the equation $0.35x+2.1=3.85$ what is the LCD? How do you know?

Answer

100. Justifications will vary.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
solve equations with fraction coefficients.			
solve equations with decimal coefficients.			

ⓑ Overall, after looking at the checklist, do you think you are well-prepared for the next section? Why or why not?

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2.6: Solve a Formula for a Specific Variable

Learning Objectives

By the end of this section, you will be able to:

- Use the Distance, Rate, and Time formula
- Solve a formula for a specific variable

Quiz

Before you get started, take this readiness quiz.

1. Solve: $15t = 120$.
   If you missed this problem, review Exercise 2.2.1.
2. Solve: $6x + 24 = 96$.
   If you missed this problem, review Exercise 2.3.1.

Use the Distance, Rate, and Time Formula

One formula you will use often in algebra and in everyday life is the formula for distance traveled by an object moving at a constant rate. Rate is an equivalent word for "speed." The basic idea of rate may already familiar to you. Do you know what distance you travel if you drive at a steady rate of 60 miles per hour for 2 hours? (This might happen if you use your car's cruise control while driving on the highway.) If you said 120 miles, you already know how to use this formula!

DISTANCE, RATE, AND TIME

For an object moving at a uniform (constant) rate, the distance traveled, the elapsed time, and the rate are related by the formula:

$$d = rt \quad \text{where} \quad \begin{aligned} d &= \text{distance} \\ r &= \text{rate} \\ t &= \text{time} \end{aligned} \tag{2.6.1}$$

We will use the Strategy for Solving Applications that we used earlier in this chapter. When our problem requires a formula, we change Step 4. In place of writing a sentence, we write the appropriate formula. We write the revised steps here for reference.

SOLVE AN APPLICATION (WITH A FORMULA).

1. **Read** the problem. Make sure all the words and ideas are understood.
2. **Identify** what we are looking for.
3. **Name** what we are looking for. Choose a variable to represent that quantity.
4. **Translate** into an equation. Write the appropriate formula for the situation. Substitute in the given information.
5. **Solve** the equation using good algebra techniques.
6. **Check** the answer in the problem and make sure it makes sense.
7. **Answer** the question with a complete sentence.

You may want to create a mini-chart to summarize the information in the problem. See the chart in this first example.

Example 2.6.1

Jamal rides his bike at a uniform rate of 12 miles per hour for $3\frac{1}{2}$ hours. What distance has he traveled?

Solution

Step 1. Read the problem.	

Step 2. Identify what you are looking for.	distance traveled
Step 3. Name. Choose a variable to represent it.	Let d = distance.
Step 4. Translate: Write the appropriate formula.	$d = rt$
	$d = ?$ $r = 12$ mph $t = 3\frac{1}{2}$ hours
Substitute in the given information.	$d = 12 \cdot 3\frac{1}{2}$
Step 5. Solve the equation.	$d = 42$ miles
Step 6. Check	
Does 42 miles make sense?	
Jamal rides:	
12 miles in 1 hour, 24 miles in 2 hours, 36 miles in 3 hours, 42 miles in $3\frac{1}{2}$ hours is reasonable 48 miles in 4 hours.	
Step 7. Answer the question with a complete sentence.	Jamal rode 42 miles.

Try It 2.6.2

Lindsay drove for $5\frac{1}{2}$ hours at 60 miles per hour. How much distance did she travel?

Answer

330 miles

Try It 2.6.3

Trinh walked for $2\frac{1}{3}$ hours at 3 miles per hour. How far did she walk?

Answer

7 miles

Example 2.6.4

Rey is planning to drive from his house in San Diego to visit his grandmother in Sacramento, a distance of 520 miles. If he can drive at a steady rate of 65 miles per hour, how many hours will the trip take?

Solution

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	How many hours (time)
Step 3. Name. Choose a variable to represent it.	Let t = time.
	$d = 520$ miles $r = 65$ mph $t = ?$ hours

Step 4. Translate. Write the appropriate formula.	$d = rt$
Substitute in the given information.	$520 = 65t$
Step 5. Solve the equation.	$t = 8$
Step 6. Check. Substitute the numbers into the formula and make sure the result is a true statement.	
$d = rt$ $520 \stackrel{?}{=} 65 \cdot 8$ $520 = 520 \checkmark$	
Step 7. Answer the question with a complete sentence. Rey's trip will take 8 hours.	

Try It 2.6.5

Lee wants to drive from Phoenix to his brother's apartment in San Francisco, a distance of 770 miles. If he drives at a steady rate of 70 miles per hour, how many hours will the trip take?

Answer

11 hours

Try It 2.6.6

Yesenia is 168 miles from Chicago. If she needs to be in Chicago in 3 hours, at what rate does she need to drive?

Answer

56 mph

Solve a Formula for a Specific Variable

You are probably familiar with some geometry formulas. A formula is a mathematical description of the relationship between variables. Formulas are also used in the sciences, such as chemistry, physics, and biology. In medicine they are used for calculations for dispensing medicine or determining body mass index. Spreadsheet programs rely on formulas to make calculations. It is important to be familiar with formulas and be able to manipulate them easily.

In Exercise 2.6.1 and Exercise 2.6.4, we used the formula $d = rt$. This formula gives the value of d, distance, when you substitute in the values of r and t, the rate and time. But in Exercise 2.6.4, we had to find the value of t. We substituted in values of d and r and then used algebra to solve for tt. If you had to do this often, you might wonder why there is not a formula that gives the value of t when you substitute in the values of d and r. We can make a formula like this by solving the formula $d = rt$ for t.

To solve a formula for a specific variable means to isolate that variable on one side of the equals sign with a coefficient of 1. All other variables and constants are on the other side of the equals sign. To see how to solve a formula for a specific variable, we will start with the distance, rate and time formula.

Example 2.6.7

Solve the formula d=rt for t:

1. when d=520 and r=65
2. in general

Solution

We will write the solutions side-by-side to demonstrate that solving a formula in general uses the same steps as when we have numbers to substitute.

1. when d=520 and r=65		2. in general	
Write the formula.	$d = rt$	Write the formula.	$d = rt$
Substitute.	$520 = 65t$		
Divide, to isolate t.	$\frac{520}{65} = \frac{65t}{65}$	Divide, to isolate tt.	$\frac{d}{r} = \frac{rt}{t}$
Simplify.	$8 = t$	Simplify.	$\frac{d}{r} = t$

We say the formula $t = \frac{d}{r}$ is solved for t.

Try It 2.6.8

Solve the formula $d = rt$ for r:

1. when d=180 and t=4
2. in general

Answer

1. $r = 45$
2. $r = \frac{d}{t}$

Try It 2.6.9

Solve the formula $d = rt$ for r:

1. when d=780 and t=12
2. in general

Answer

1. $r = 65$
2. \(r = \frac{d}{rt}\)

Example 2.6.10

Solve the formula $A = \frac{1}{2}bh$ for h:

1. when $A = 90$ and $b = 15$
2. in general

Solution

1. when $A = 90$ and $b = 15$		2. in general	
Write the formula.	$A = \frac{1}{2}bh$	Write the formula.	$A = \frac{1}{2}bh$
Substitute.	$90 = \frac{1}{2} \cdot 15 \cdot h$		
Clear the fractions.	$2 \cdot 90 = 2 \cdot \frac{1}{2}15h$	Clear the fractions.	$2 \cdot A = 2 \cdot \frac{1}{2}bh$
Simplify.	$180 = 15h$	Simplify.	$2A = bh$
Solve for h.	$12 = h$	Solve for hh.	$\frac{2A}{b} = h$

We can now find the height of a triangle, if we know the area and the base, by using the formula $h = \frac{2A}{b}$

Try It 2.6.11

Solve the formula $A = \frac{1}{2}bh$ for h:

1. when $A = 170$ and $b = 17$
2. in general

Answer

1. $h = 20$
2. $h = \frac{2A}{b}$

Try It 2.6.12

Solve the formula $A = \frac{1}{2}bh$ for h:

1. when $A = 62$ and $h = 31$
2. in general

Answer

1. $b = 4$
2. $b = \frac{2A}{h}$

The formula $I = Prt$ is used to calculate simple interest, *I*, for a principal, *P*, invested at rate, *r*, for *t* years.

Example 2.6.13

Solve the formula I=Prt to find the principal, P:

1. when I=$5,600, r=4% ,t=7years
2. in general

Solution

1. I=$5,600, r=4% ,t=7years		2. in general	
Write the formula.	$I = Prt$	Write the formula.	$I = Prt$
Substitute.	$5600 = P(0.04)(7)$		
Simplify.	$5600 = P(0.28)$	Simplify.	$I = P(rt)$
Divide, to isolate *P*.	$\frac{5600}{0.28} = \frac{P(0.28)}{0.28}$	Divide, to isolate *P*.	$\frac{I}{rt} = \frac{P(rt)}{rt}$
Simplify.	$20,000 = P$	Simplify.	$\frac{I}{rt} = P$
The principal is	$20,000		$P = \frac{I}{rt}$

Try It 2.6.14

Solve the formula I=Prt to find the principal, P:

1. when I=$2160, r=6% ,t=3 years
2. in general

Answer

1. $12000
2. $P = \frac{1}{rt}$

Try It 2.6.15

Solve the formula I=Prt to find the principal, P:

1. when I=$5400, r= 12% ,t=5 years
2. in general

Answer

1. $9000
2. $P = \frac{1}{rt}$

Later in this class, and in future algebra classes, you'll encounter equations that relate two variables, usually *x* and *y*. You might be given an equation that is solved for *y* and need to solve it for *x*, or vice versa. In the following example, we're given an equation with both *x* and *y* on the same side and we'll solve it for *y*.

Example 2.6.16

Solve the formula 3x+2y=18 for *y*:

1. when x=4
2. in general

Solution

1. when x=4		2. in general	
	$3x + 2y = 18$		$3x + 2y = 18$
Substitute.	$3(4) + 2y = 18$		
Subtract to isolate the y-term.	$12 - 12 + 2y = 18 - 12$	Subtract to isolate the y-term.	$3x - 3x + 2y = 18 - 3x$
Divide.	$\frac{2y}{2} = \frac{6}{2}$	Divide.	$\frac{2y}{2} = \frac{18}{2} - \frac{3x}{2}$
Simplify.	$y = 3$	Simplify.	$y = -\frac{3x}{2} + 9$

Try It 2.6.17

Solve the formula 3x+4y=10 for *y*:

1. when $x = \frac{14}{3}$
2. in general

Answer

1. $y = -1$
2. $y = \frac{10-3x}{4}$

Try It 2.6.18

Solve the formula 5x+2y=18 for *y*:

1. when $x = 4$

2. in general

Answer

1. $y = -1$
2. $y = \frac{18-5x}{2}$

In Exercise $2.6.7$ through Exercise $2.6.18$ we used the numbers in part 1 as a guide to solving in general in part 2. Now we will solve a formula in general without using numbers as a guide.

Example 2.6.19

Solve the formula P=a+b+c for a.

Solution

We will isolate aa on one side of the equation.	$P = a + b + c$
Both b and c are added to a, so we subtract them from both sides of the equation.	$P - b - c = a + b + c - b - c$
Simplify.	$P - b - c = a$ $a = P - b - c$

Try It 2.6.20

Solve the formula P=a+b+c for *b*.

Answer

b=P−a−c

Try It 2.6.21

Solve the formula P=a+b+c for *c*.

Answer

c=P−a−b

Example 2.6.22

Solve the formula 6x+5y=13 for *y*.

Solution

	$6x + 5y = 13$
Subtract 6x from both sides to isolate the term with y.	$6x - 6x + 5y = 13 - 6x$
Simplify.	$5y = 13 - 6x$
Divide by 5 to make the coefficient 1.	$\frac{5y}{5} = \frac{13-6x}{5}$
Simplify.	$y = \frac{13-6x}{5}$

The fraction is simplified. We cannot divide 13−6x by 5.

? Try It 2.6.23

Solve the formula 4x+7y=9 for *y*.

Answer

$y = \frac{9-4x}{7}$

? Try It 2.6.24

Solve the formula 5x+8y=1 for *y*.

Answer

$y = \frac{1-5x}{8}$

Key Concepts

- **To Solve an Application (with a formula)**
  1. **Read** the problem. Make sure all the words and ideas are understood.
  2. **Identify** what we are looking for.
  3. **Name** what we are looking for. Choose a variable to represent that quantity.
  4. **Translate** into an equation. Write the appropriate formula for the situation. Substitute in the given information.
  5. **Solve** the equation using good algebra techniques.
  6. **Check** the answer in the problem and make sure it makes sense.
  7. **Answer** the question with a complete sentence.
- **Distance, Rate and Time**
  For an object moving at a uniform (constant) rate, the distance traveled, the elapsed time, and the rate are related by the formula: d=rt where *d* = distance, *r* = rate, *t* = time.
- **To solve a formula for a specific variable** means to get that variable by itself with a coefficient of 1 on one side of the equation and all other variables and constants on the other side.

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2.6E: Exercises

Practice Makes Perfect

Use the Distance, Rate, and Time Formula

In the following exercises, solve.

Exercise 2.6E.1

Steve drove for $8\frac{1}{2}$ hours at 72 miles per hour. How much distance did he travel?

Exercise 2.6E.2

Socorro drove for $4\frac{5}{6}$ hours at 60 miles per hour. How much distance did she travel?

Answer

290 miles

Exercise 2.6E.3

Yuki walked for $1\frac{3}{4}$ hours at 4 miles per hour. How far did she walk?

Exercise 2.6E.4

Francie rode her bike for $2\frac{1}{2}$ hours at 12 miles per hour. How far did she ride?

Answer

30 miles

Exercise 2.6E.5

Connor wants to drive from Tucson to the Grand Canyon, a distance of 338 miles. If he drives at a steady rate of 52 miles per hour, how many hours will the trip take?

Exercise 2.6E.6

Megan is taking the bus from New York City to Montreal. The distance is 380 miles and the bus travels at a steady rate of 76 miles per hour. How long will the bus ride be?

Answer

5 hours

Exercise 2.6E.7

Aurelia is driving from Miami to Orlando at a rate of 65 miles per hour. The distance is 235 miles. To the nearest tenth of an hour, how long will the trip take?

Exercise 2.6E.8

Kareem wants to ride his bike from St. Louis to Champaign, Illinois. The distance is 180 miles. If he rides at a steady rate of 16 miles per hour, how many hours will the trip take?

Answer

11.25 hours

Exercise 2.6E.9

Javier is driving to Bangor, 240 miles away. If he needs to be in Bangor in 4 hours, at what rate does he need to drive?

Exercise 2.6E.10

Alejandra is driving to Cincinnati, 450 miles away. If she wants to be there in 6 hours, at what rate does she need to drive?

Answer

75 mph

Exercise 2.6E.11

Aisha took the train from Spokane to Seattle. The distance is 280 miles and the trip took 3.5 hours. What was the speed of the train?

Exercise 2.6E.12

Philip got a ride with a friend from Denver to Las Vegas, a distance of 750 miles. If the trip took 10 hours, how fast was the friend driving?

Answer

75 mph

Solve a Formula for a Specific Variable

In the following exercises, use the formula $d = rt$.

Exercise 2.6E.13

Solve for t

1. when $d = 350$ and $r = 70$
2. in general

Exercise 2.6E.14

Solve for t

1. when $d = 240$ and $r = 60$
2. in general

Answer

1. $t = 4$
2. $t = \frac{d}{r}$

Exercise 2.6E.15

Solve for t

1. when $d = 510$ and $r = 60$

2. in general

Exercise 2.6E.16

Solve for t

1. when $d = 175$ and $r = 50$
2. in general

Answer

1. $t = 3.5$
2. $t = \frac{d}{r}$

Exercise 2.6E.17

Solve for r

1. when $d = 204$ and $t = 3$
2. in general

Exercise 2.6E.18

Solve for r

1. when $d = 420$ and $t = 6$
2. in general

Answer

1. $r = 70$
2. $r = \frac{d}{t}$

Exercise 2.6E.19

Solve for r

1. when $d = 160$ and $t = 2.5$
2. in general

Exercise 2.6E.20

Solve for r

1. when $d = 180$ and $t = 4.5$
2. in general

Answer

1. $r = 40$
2. $r = \frac{d}{t}$

In the following exercises, use the formula $A = \frac{1}{2}bh$

Exercise 2.6E.21

Solve for b

1. when $A = 126$ and $h = 18$
2. in general

Exercise 2.6E.22

Solve for h

1. when $A = 176$ and $b = 22$
2. in general

Answer

1. $h = 16$
2. $h = \frac{2A}{b}$

Exercise 2.6E.23

Solve for h

1. when $A = 375$ and $b = 25$
2. in general

Exercise 2.6E.24

Solve for b

1. when $A = 65$ and $h = 13$
2. in general

Answer

1. $b = 10$
2. $b = \frac{2A}{h}$

In the following exercises, use the formula $I = Prt$.

Exercise 2.6E.25

Solve for the principal, P for

1. $I = \$5,480$, $r = 4\%$, $t = 7$ years
2. in general

Exercise 2.6E.26

Solve for the principal, P for

1. $I = \$3,950$, $r = 6\%$, $t = 5$ years
2. in general

Answer

1. $P = \$13,166.67$
2. $P = \frac{I}{rt}$

Exercise 2.6E.27

Solve for the time, t for

1. $I = \$2,376$, $P = \$9,000$, $r = 4.4\%$
2. in general

Exercise 2.6E.28

Solve for the time, t for

1. $I = \$624$, $P = \$6,000$, $r = 5.2\%$
2. in general

Answer

1. $t = 2$ years
2. $t = \frac{I}{Pr}$

In the following exercises, solve.

Exercise 2.6E.29

Solve the formula $2x + 3y = 12$ for y

1. when $x = 3$
2. in general

Exercise 2.6E.30

Solve the formula $5x + 2y = 10$ for y

1. when $x = 4$
2. in general

Answer

1. $y = -5$
2. $y = \frac{10-5x}{2}$

Exercise 2.6E.31

Solve the formula $3x - y = 7$ for y

1. when $x = -2$
2. in general

Exercise 2.6E.32

Solve the formula $4x + y = 5$ for y

1. when $x = -3$
2. in general

Answer

1. $y = 17$
2. $y = 5 - 4x$

Exercise 2.6E.33

Solve $a+b=90$ for b.

Exercise 2.6E.34

Solve $a+b=90$ for a

Answer

$a=90-b$

Exercise 2.6E.35

Solve $180=a+b+c$ for a

Exercise 2.6E.36

Solve $180=a+b+c$ for c

Answer

$c=180-a-b$

Exercise 2.6E.37

Solve the formula $8x+y=15$ for y

Exercise 2.6E.38

Solve the formula $9x+y=13$ for y

Answer

$y=13-9x$

Exercise 2.6E.39

Solve the formula $-4x+y=-6$ for y

Exercise 2.6E.40

Solve the formula $-5x+y=-1$ for y

Answer

$y=-1+5x$

Exercise 2.6E.41

Solve the formula $4x+3y=7$ for y

Exercise 2.6E.42

Solve the formula $3x+2y=11$ for y

Answer

$y=\frac{11-3x}{2}$

Exercise 2.6E.43

Solve the formula $x-y=-4$ for y

Exercise 2.6E.44

Solve the formula $x-y=-3$ for y

Answer

$y=3+x$

Exercise 2.6E.45

Solve the formula $P=2L+2W$ for L

Exercise 2.6E.46

Solve the formula $P=2L+2W$ for W

Answer

$W=\frac{P-2L}{2}$

Exercise 2.6E.47

Solve the formula $C=\pi d$ for d

Exercise 2.6E.48

Solve the formula $C=\pi d$ for π

Answer

$\pi=\frac{C}{d}$

Exercise 2.6E.49

Solve the formula $V=LWH$ for L

Exercise 2.6E.50

Solve the formula $V=LWH$ for H

Answer

$H=\frac{V}{LW}$

Everyday Math

Exercise 2.6E.51

Converting temperature While on a tour in Greece, Tatyana saw that the temperature was 40° Celsius. Solve for F in the formula $C = \frac{5}{9}(F - 32)$ to find the Fahrenheit temperature.

Exercise 2.6E.52

Converting temperature Yon was visiting the United States and he saw that the temperature in Seattle one day was 50ºFahrenheit. Solve for C in the formula $F = \frac{9}{5}C + 32$ to find the Celsius temperature.

Answer

10°C

Writing Exercises

Exercise 2.6E.53

Solve the equation $2x + 3y = 6$ for y

1. when $x = -3$
2. in general
3. Which solution is easier for you, 1 or 2? Why?

Exercise 2.6E.54

Solve the equation $5x - 2y = 10$ for x

1. when $y = 10$
2. in general
3. Which solution is easier for you, 1 or 2? Why?

Answer

Answers will vary.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
use the distance, rate, and time formula.			
solve a formula for a specific variable.			

ⓑ What does this checklist tell you about your mastery of this section? What steps will you take to improve?

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2.7: Solve Linear Inequalities

Learning Objectives

By the end of this section, you will be able to:

- Graph inequalities on the number line
- Solve inequalities using the Subtraction and Addition Properties of inequality
- Solve inequalities using the Division and Multiplication Properties of inequality
- Solve inequalities that require simplification
- Translate to an inequality and solve

Note

Before you get started, take this readiness quiz.

1. Translate from algebra to English: $15 > x$.
   If you missed this problem, review Exercise 1.3.1.
2. Solve: $n - 9 = -42$.
   If you missed this problem, review Exercise 2.1.7.
3. Solve: $-5p = -23$.
   If you missed this problem, review Exercise 2.2.1.
4. Solve: $3a - 12 = 7a - 20$.
   If you missed this problem, review Exercise 2.3.22.

Graph Inequalities on the Number Line

Do you remember what it means for a number to be a solution to an equation? A solution of an equation is a value of a variable that makes a true statement when substituted into the equation.

What about the solution of an inequality? What number would make the inequality $x > 3$ true? Are you thinking, 'x could be 4'? That's correct, but x could be 5 too, or 20, or even 3.001. Any number greater than 3 is a solution to the inequality $x > 3$.

We show the solutions to the inequality $x > 3$ on the number line by shading in all the numbers to the right of 3, to show that all numbers greater than 3 are solutions. Because the number 3 itself is not a solution, we put an open parenthesis at 3. The graph of $x > 3$ is shown in Figure 2.7.1. Please note that the following convention is used: light blue arrows point in the positive direction and dark blue arrows point in the negative direction.

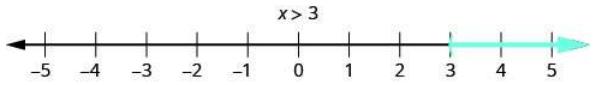


Figure 2.7.1: The inequality $x > 3$ is graphed on this number line.

The graph of the inequality $x \geq 3$ is very much like the graph of $x > 3$, but now we need to show that 3 is a solution, too. We do that by putting a bracket at $x = 3$, as shown in Figure 2.7.2.

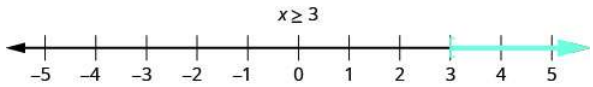


Figure 2.7.2: The inequality $x \geq 3$ is graphed on this number line.

Notice that the open parentheses symbol, (, shows that the endpoint of the inequality is not included. The open bracket symbol, [, shows that the endpoint is included.

Exercise 2.7.1

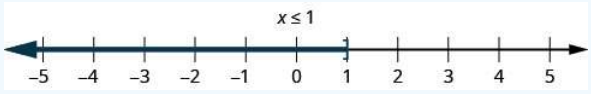
Graph on the number line:

1. $x \leq 1$
2. $x < 5$

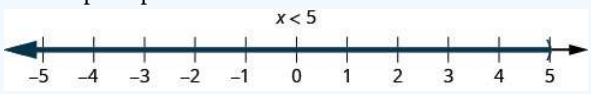
3. $x > -1$

Answer

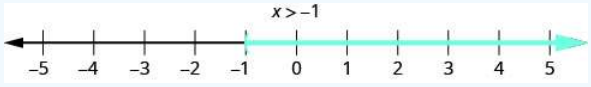
1. $x \leq 1$ This means all numbers less than or equal to 1. We shade in all the numbers on the number line to the left of 1 and put a bracket at x=1 to show that it is included.



2. $x < 5$ This means all numbers less than 5, but not including 5. We shade in all the numbers on the number line to the left of 5 and put a parenthesis at x=5 to show it is not included.



3. $x > -1$ This means all numbers greater than −1, but not including −1. We shade in all the numbers on the number line to the right of −1, then put a parenthesis at x=−1 to show it is not included.

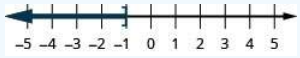


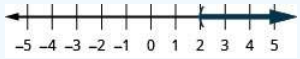
Exercise 2.7.2

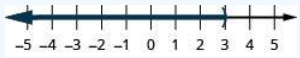
Graph on the number line:

1. $x \leq -1$
2. $x > 2$
3. $x < 3$

Answer

1. 

2. 

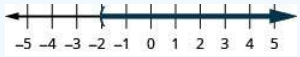
3. 

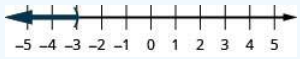
Exercise 2.7.3

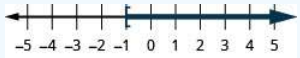
Graph on the number line:

1. $x > -2$
2. $x < -3$
3. $x \geq -1$

Answer

1. 

2. 

3. 

We can also represent inequalities using *interval notation.* As we saw above, the inequality $x > 3$ means all numbers greater than 3. There is no upper end to the solution to this inequality. In **interval notation**, we express $x > 3$ as $(3, \infty)$. The symbol ∞ is read as 'infinity'. It is not an actual number. Figure $2.7.3$ shows both the number line and the interval notation.

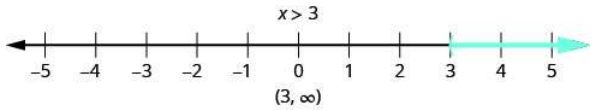


Figure 2.7.3: The inequality $x > 3$ is graphed on this number line and written in interval notation.

The inequality $x \leq 1$ means all numbers less than or equal to 1. There is no lower end to those numbers. We write $x \leq 1$ in interval notation as $(-\infty, 1]$. The symbol $-\infty$ is read as 'negative infinity'. Figure 2.7.4 shows both the number line and interval notation.

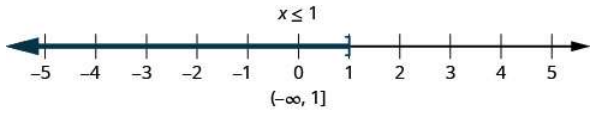


Figure 2.7.4: The inequality $x \leq 1$ is graphed on this number line and written in interval notation.

INEQUALITIES, NUMBER LINES, AND INTERVAL NOTATION



Did you notice how the parenthesis or bracket in the interval notation matches the symbol at the endpoint of the arrow? These relationships are shown in Figure 2.7.5.



Figure 2.7.5: The notation for inequalities on a number line and in interval notation use similar symbols to express the endpoints of intervals.

Exercise 2.7.4

Graph on the number line and write in interval notation.

1. $x \geq -3$
2. $x < 2.5$
3. $x \leq \frac{3}{5}$

Answer

1.

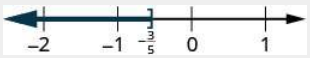
	$x \geq -3$
Shade to the right of −3, and put a bracket at −3.	
Write in interval notation.	[−3, ∞)

2.

	$x < 2.5$
Shade to the left of 2.5, and put a parenthesis at 2.5.	
Write in interval notation.	(−∞, 2.5)

3.

	$x \leq -\frac{3}{5}$

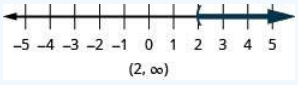
Shade to the left of $-\frac{3}{5}$, and put a bracket at $-\frac{3}{5}$.	
Write in interval notation.	$\left(-\infty, -\frac{3}{5}\right]$

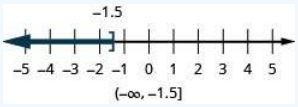
Exercise 2.7.5

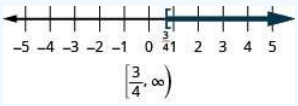
Graph on the number line and write in interval notation:

1. $x > 2$
2. $x \leq -1.5$
3. $x \geq \frac{3}{4}$

Answer

1. 

2. 

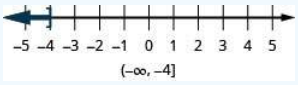
3. 

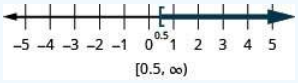
Exercise 2.7.6

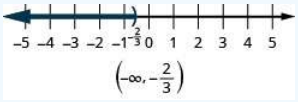
Graph on the number line and write in interval notation:

1. $x \leq -4$
2. $x \geq 0.5$
3. $x < -\frac{2}{3}$

Answer

1. 

2. 

3. 

Solve Inequalities using the Subtraction and Addition Properties of Inequality

The Subtraction and Addition Properties of Equality state that if two quantities are equal, when we add or subtract the same amount from both quantities, the results will be equal.

PROPERTIES OF EQUALITY

Subtraction Property of Equality	**Addition Property of Equality**
For any numbers $a, b,$ and c,	For any numbers $a, b,$ and c
if $a = b$,	if $a = b$
then $a - c = b - c$.	then $a + c = b + c$

(2.7.1)

Similar properties hold true for inequalities.

Table 2.7.1

For example, we know that −4 is less than 2.	$-4 < 2$
If we subtract 5 from both quantities, is the left side still less than the right side?	$-4-5 \stackrel{?}{<} 2-5$
We get −9 on the left and −3 on the right.	$-9 \stackrel{?}{<} -3$
And we know −9 is less than −3.	$-9 < -3$
	The inequality sign stayed the same.

Similarly we could show that the inequality also stays the same for addition.

This leads us to the Subtraction and Addition Properties of Inequality.

PROPERTIES OF INEQUALITY

Subtraction Property of Inequality	**Addition Property of Inequality**
For any numbers $a, b,$ and c,	For any numbers $a, b,$ and c
if $a < b$	if $a < b$
then $a - c < b - c$.	then $a + c < b + c$
if $a > b$	if $a > b$
then $a - c > b - c$.	then $a + c > b + c$

(2.7.2)

We use these properties to solve inequalities, taking the same steps we used to solve equations. Solving the inequality $x + 5 > 9$, the steps would look like this:

$$\begin{array}{lrcl} & x+5 & > & 9 \\ \text{Subtract 5 from both sides to isolate } x. & x+5-5 & > & 9-5 \\ & x & > & 4 \end{array} \tag{2.7.3}$$

Any number greater than 4 is a solution to this inequality.

Exercise 2.7.7

Solve the inequality $n - \frac{1}{2} \leq \frac{5}{8}$, graph the solution on the number line, and write the solution in interval notation.

Answer

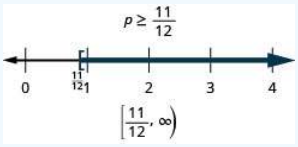
	$n - \frac{1}{2} \leq \frac{5}{8}$
Add $\frac{1}{2}$ to both sides of the inequality.	$n - \frac{1}{2} + \frac{1}{2} \leq \frac{5}{8} + \frac{1}{2}$
Simplify.	$n \leq \frac{9}{8}$
Graph the solution on the number line.	
Write the solution in interval notation.	

Exercise 2.7.8

Solve the inequality, graph the solution on the number line, and write the solution in interval notation.

$p - \frac{3}{4} \geq \frac{1}{6}$

Answer

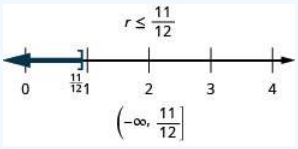


Exercise 2.7.9

Solve the inequality, graph the solution on the number line, and write the solution in interval notation.

$r - \frac{1}{3} \leq \frac{7}{12}$

Answer



Solve Inequalities using the Division and Multiplication Properties of Inequality

The Division and Multiplication Properties of Equality state that if two quantities are equal, when we divide or multiply both quantities by the same amount, the results will also be equal (provided we don't divide by 0).

PROPERTIES OF EQUALITY

Division Property of Equality	**MUltiplication Property of Equality**
For any numbers a, b, c, and $c \neq 0$	For any numbers a, b, c
if $a = b$	if $a = b$
then $\frac{a}{c} = \frac{b}{c}$	then $ac = bc$

(2.7.4)

Are there similar properties for inequalities? What happens to an inequality when we divide or multiply both sides by a constant?

Consider some numerical examples.

Table 2.7.2

	$10 < 15$		$10 < 15$
Divide both sides by 5.	$\frac{10}{5} \,?\, \frac{15}{5}$	Multiply both sides by 5.	$10(5) \,?\, 15(5)$
Simplify.	$2 \,?\, 3$		$50 \,?\, 75$
Fill in the inequality signs.	$2 < 3$		$50 < 75$

The inequality signs stayed the same.

Does the inequality stay the same when we divide or multiply by a negative number?

Table 2.7.3

	$10 < 15$		$10 < 15$

Divide both sides by -5.	$\frac{10}{-5} \; ? \; \frac{15}{-5}$	Multiply both sides by -5.	$10(-5) \; ? \; 15(-5)$
Simplify.	$-2 \; ? \; -3$		$-50 \; ? \; -75$
Fill in the inequality signs.	$-2 > -3$		$-50 > -75$

The inequality signs reversed their direction.

When we divide or multiply an inequality by a positive number, the inequality sign stays the same. When we divide or multiply an inequality by a negative number, the inequality sign reverses.

Here are the Division and Multiplication Properties of Inequality for easy reference.

DIVISION AND MULTIPLICATION PROPERTIES OF INEQUALITY

For any real numbers a,b,c

$$\begin{aligned} &\text{if } a<b \text{ and } c>0, \text{ then } && \tfrac{a}{c}<\tfrac{b}{c} \text{ and } ac<bc \\ &\text{if } a>b \text{ and } c>0, \text{ then } && \tfrac{a}{c}>\tfrac{b}{c} \text{ and } ac>bc \\ &\text{if } a<b \text{ and } c<0, \text{ then } && \tfrac{a}{c}>\tfrac{b}{c} \text{ and } ac>bc \\ &\text{if } a>b \text{ and } c<0, \text{ then } && \tfrac{a}{c}<\tfrac{b}{c} \text{ and } ac<bc \end{aligned} \tag{2.7.5}$$

When we **divide or multiply** an inequality by a:

- **positive** number, the inequality stays the **same**.
- **negative** number, the inequality **reverses**.

Exercise 2.7.10

Solve the inequality $7y < 42$, graph the solution on the number line, and write the solution in interval notation.

Answer

	$7y < 42$
Divide both sides of the inequality by 7. Since $7 > 0$, the inequality stays the same.	$\frac{7y}{7} < \frac{42}{7}$
Simplify.	$y < 6$
Graph the solution on the number line.	
Write the solution in interval notation.	$(-\infty, 6)$

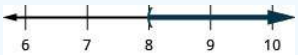
Exercise 2.7.11

Solve the inequality, graph the solution on the number line, and write the solution in interval notation.

$9c > 72$

Answer

$c > 8$



$(8, \infty)$

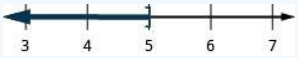
Exercise 2.7.12

Solve the inequality, graph the solution on the number line, and write the solution in interval notation.

$12d \leq 60$

Answer

$d \leq 5$



$(-\infty, 5]$

Exercise 2.7.13

Solve the inequality $-10a \geq 50$, graph the solution on the number line, and write the solution in interval notation.

Answer

	$-10a \geq 50$
Divide both sides of the inequality by -10. Since $-10 < 0$, the inequality reverses.	$\frac{-10a}{-10} \leq \frac{50}{-10}$
Simplify.	$a \leq -5$
Graph the solution on the number line.	
Write the solution in interval notation.	$(-\infty, -5]$

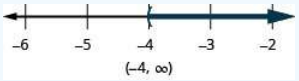
Exercise 2.7.14

Solve each inequality, graph the solution on the number line, and write the solution in interval notation.

$-8q < 32$

Answer

$q > -4$

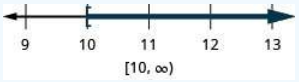


Exercise 2.7.15

Solve each inequality, graph the solution on the number line, and write the solution in interval notation.

$-7r \leq -70$

Answer



SOLVING INEQUALITIES

Sometimes when solving an inequality, the variable ends up on the right. We can rewrite the inequality in reverse to get the variable to the left.

$$x > a \text{ has the same meaning as } a < x \tag{2.7.6}$$

Think about it as "If Xavier is taller than Alex, then Alex is shorter than Xavier."

Exercise 2.7.16

Solve the inequality $-20 < \frac{4}{5}u$, graph the solution on the number line, and write the solution in interval notation.

Answer

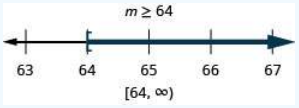
	$-20 < \frac{4}{5}u$
Multiply both sides of the inequality by $\frac{5}{4}$. Since $\frac{5}{4} > 0$, the inequality stays the same.	$\frac{5}{4}(-20) < \frac{5}{4}\left(\frac{4}{5}u\right)$
Simplify.	$-25 < u$
Rewrite the variable on the left.	$u > -25$
Graph the solution on the number line.	
Write the solution in interval notation.	$(-25, \infty)$

Exercise 2.7.17

Solve the inequality, graph the solution on the number line, and write the solution in interval notation.

$24 \leq \frac{3}{8}m$

Answer

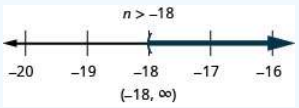


Exercise 2.7.18

Solve the inequality, graph the solution on the number line, and write the solution in interval notation.

$-24 < \frac{4}{3}n$

Answer



Exercise 2.7.19

Solve the inequality $\frac{t}{-2} \geq 8$, graph the solution on the number line, and write the solution in interval notation.

Answer

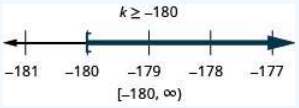
	$\frac{t}{-2} \geq 8$
Multiply both sides of the inequality by -2. Since $-2 < 0$, the inequality reverses.	$-2\left(\frac{t}{-2}\right) \leq -2(8)$
Simplify.	$t \leq -16$
Graph the solution on the number line.	
Write the solution in interval notation.	$(-\infty, -16]$

Exercise 2.7.20

Solve the inequality, graph the solution on the number line, and write the solution in interval notation.

$\frac{k}{-12} \leq 15$

Answer

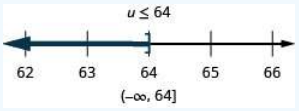


Exercise 2.7.21

Solve the inequality, graph the solution on the number line, and write the solution in interval notation.

$\frac{u}{-4} \geq -16$

Answer



Solve Inequalities That Require Simplification

Most inequalities will take more than one step to solve. We follow the same steps we used in the general strategy for solving linear equations, but be sure to pay close attention during multiplication or division.

Exercise 2.7.22

Solve the inequality $4m \leq 9m + 17$, graph the solution on the number line, and write the solution in interval notation.

Answer

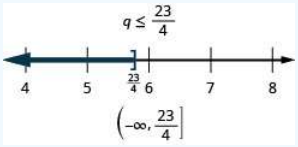
	$4m \leq 9m + 17$
Subtract 9m from both sides to collect the variables on the left.	$4m - 9m \leq 9m - 9m + 17$
Simplify.	$-5m \leq 17$
Divide both sides of the inequality by -5, and reverse the inequality.	$\frac{-5m}{-5} \geq \frac{17}{-5}$

Simplify.	$m \geq -\frac{17}{5}$
Graph the solution on the number line.	
Write the solution in interval notation.	$\left[-\frac{17}{5}, \infty\right)$

Exercise 2.7.23

Solve the inequality $3q \geq 7q - 23$, graph the solution on the number line, and write the solution in interval notation.

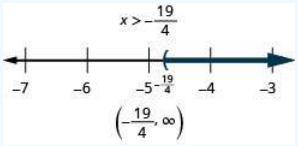
Answer



Exercise 2.7.24

Solve the inequality $6x < 10x + 19$, graph the solution on the number line, and write the solution in interval notation.

Answer



Exercise 2.7.25

Solve the inequality $8p + 3(p - 12) > 7p - 28$ graph the solution on the number line, and write the solution in interval notation.

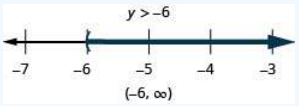
Answer

Simplify each side as much as possible.	8p+3(p−12)>7p−28
Distribute.	8p+3p−36>7p−28
Combine like terms.	11p−36>7p−28
Subtract 7p from both sides to collect the variables on the left.	11p−36−7p>7p−28−7p
Simplify.	4p−36>−28
Add 36 to both sides to collect the constants on the right.	4p−36+36>−28+36
Simplify.	4p>8
Divide both sides of the inequality by 4; the inequality stays the same.	$\frac{4p}{4} > 84$
Simplify.	$p > 2$
Graph the solution on the number line.	
Write the solution in interval notation.	$(2, \infty)$

Exercise 2.7.26

Solve the inequality $9y+2(y+6)>5y-24$, graph the solution on the number line, and write the solution in interval notation.

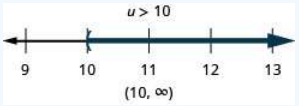
Answer



Exercise 2.7.27

Solve the inequality $6u+8(u-1)>10u+32$, graph the solution on the number line, and write the solution in interval notation.

Answer



Just like some equations are identities and some are contradictions, inequalities may be identities or contradictions, too. We recognize these forms when we are left with only constants as we solve the inequality. If the result is a true statement, we have an identity. If the result is a false statement, we have a contradiction.

Exercise 2.7.28

Solve the inequality $8x-2(5-x)<4(x+9)+6x$, graph the solution on the number line, and write the solution in interval notation.

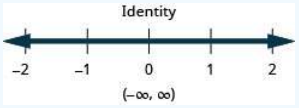
Answer

Simplify each side as much as possible.	8x−2(5−x)<4(x+9)+6x
Distribute.	8x−10+2x<4x+36+6x
Combine like terms.	10x−10<10x+36
Subtract 10x from both sides to collect the variables on the left.	10x−10−10x<10x+36−10x
Simplify.	−10<36
The xx's are gone, and we have a true statement.	The inequality is an identity. The solution is all real numbers.
Graph the solution on the number line.	
Write the solution in interval notation.	$(-\infty,\infty)$

Exercise 2.7.29

Solve the inequality $4b-3(3-b)>5(b-6)+2b$, graph the solution on the number line, and write the solution in interval notation.

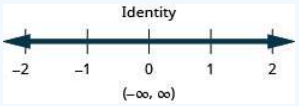
Answer



Exercise 2.7.30

Solve the inequality $9h-7(2-h)<8(h+11)+8h$, graph the solution on the number line, and write the solution in interval notation.

Answer



Exercise 2.7.31

Solve the inequality $\frac{1}{3}a-\frac{1}{8}a>\frac{5}{24}a+\frac{3}{4}$, graph the solution on the number line, and write the solution in interval notation.

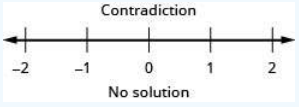
Answer

	$\frac{1}{3}a-\frac{1}{8}a>\frac{5}{24}a+\frac{3}{4}$
Multiply both sides by the LCD, 24, to clear the fractions.	$24\left(\frac{1}{3}a-\frac{1}{8}a\right)>24\left(\frac{5}{24}a+\frac{3}{4}\right)$
Simplify.	$8a-3a>5a+18$
Combine like terms.	$5a>5a+18$
Subtract 5a from both sides to collect the variables on the left.	$5a-5a>5a-5a+18$
Simplify.	$0>18$
The statement is false!	The inequality is a contradiction.
	There is no solution.
Graph the solution on the number line.	
Write the solution in interval notation.	There is no solution.

Exercise 2.7.32

Solve the inequality $\frac{1}{4}x-\frac{1}{12}x>\frac{1}{6}x+\frac{7}{8}$, graph the solution on the number line, and write the solution in interval notation.

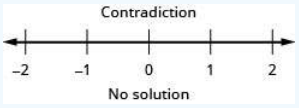
Answer



Exercise 2.7.33

Solve the inequality $\frac{2}{5}z - \frac{1}{3}z < \frac{1}{15}z - \frac{3}{5}$, graph the solution on the number line, and write the solution in interval notation.

Answer



Translate to an Inequality and Solve

To translate English sentences into inequalities, we need to recognize the phrases that indicate the inequality. Some words are easy, like ‘more than’ and ‘less than’. But others are not as obvious.

Think about the phrase ‘at least’ – what does it mean to be ‘at least 21 years old’? It means 21 or more. The phrase ‘at least’ is the same as ‘greater than or equal to’.

Table 2.7.4 shows some common phrases that indicate inequalities.

Table 2.7.4

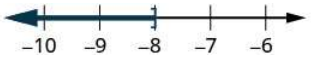
>	≥	<	≤
" data-valign="middle" class="lt-math-15134">is greater than	is greater than or equal to	is less than	is less than or equal to
" data-valign="middle" class="lt-math-15134">is more than	is at least	is smaller than	is at most
" data-valign="middle" class="lt-math-15134">is larger than	is no less than	has fewer than	is no more than
" data-valign="middle" class="lt-math-15134">exceeds	is the minimum	is lower than	is the maximum

Exercise 2.7.34

Translate and solve. Then write the solution in interval notation and graph on the number line.

Twelve times c is no more than 96.

Answer

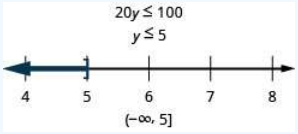
Translate.	Twelve times c is no more than 96 $12c \leq 96$
Solve—divide both sides by 12.	$\frac{12c}{12} \leq \frac{96}{12}$
Simplify.	$c \leq 8$
Write in interval notation.	$(-\infty, -8]$
Graph on the number line.	

Exercise 2.7.35

Translate and solve. Then write the solution in interval notation and graph on the number line.

Twenty times y is at most 100

Answer

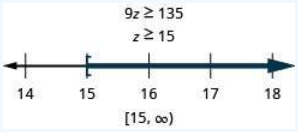


Exercise 2.7.36

Translate and solve. Then write the solution in interval notation and graph on the number line.

Nine times z is no less than 135

Answer



Exercise 2.7.37

Translate and solve. Then write the solution in interval notation and graph on the number line.

Thirty less than x is at least 45.

Answer

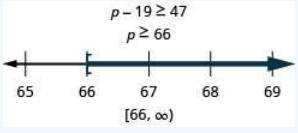
Translate.	Thirty less than x is at least 45. $x - 30 \geq 45$
Solve—add 30 to both sides.	$x - 30 + 30 \geq 45 + 30$
Simplify.	$x \geq 75$
Write in interval notation.	$[75, \infty)$
Graph on the number line.	

Exercise 2.7.38

Translate and solve. Then write the solution in interval notation and graph on the number line.

Nineteen less than p is no less than 47

Answer



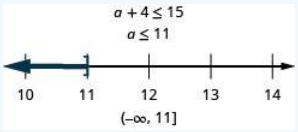
https://math.libretexts.org/@go/page/61365

Exercise 2.7.39

Translate and solve. Then write the solution in interval notation and graph on the number line.

Four more than a is at most 15.

Answer



Key Concepts

- **Subtraction Property of Inequality**
  For any numbers a, b, and c,
  if a<b then a−c<b−c and
  if a>b then a−c>b−c.
- **Addition Property of Inequality**
  For any numbers a, b, and c,
  if a<b then a+c<b+c and
  if a>b then a+c>b+c.
- **Division and Multiplication Properties of Inequality**
  For any numbers a, b, and c,
  if a<b and c>0, then ac<bc and ac>bc.
  if a>b and c>0, then ac>bc and ac>bc.
  if a<b and c<0, then ac>bc and ac>bc.
  if a>b and c<0, then ac<bc and ac<bc.
- When we **divide or multiply** an inequality by a:
  - **positive** number, the inequality stays the **same**.
  - **negative** number, the inequality **reverses**.

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2.7E: Exercises

Practice Makes Perfect

Graph Inequalities on the Number Line

In the following exercises, graph each inequality on the number line

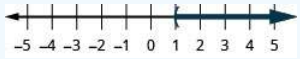
Exercise 2.7E.1

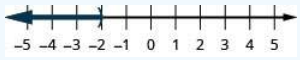
1. $x \leq 2$
2. x>−1
3. x<0

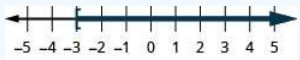
Exercise 2.7E.2

1. x>1
2. x<−2
3. $x \geq -3$

Answer

1. 

2. 

3. 

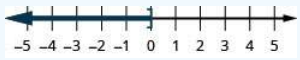
Exercise 2.7E.3

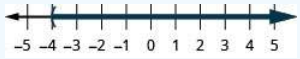
1. $x \geq -3$
2. x<4
3. $x \leq -2$

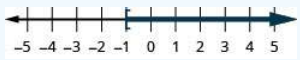
Exercise 2.7E.4

1. $x \leq 0$
2. x>−4
3. $x \geq -1$

Answer

1. 

2. 

3. 

In the following exercises, graph each inequality on the number line and write in interval notation.

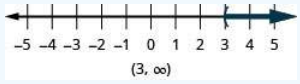
Exercise 2.7E.5

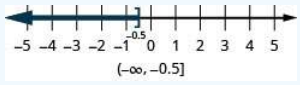
1. x<−2
2. $x \geq -3.5$
3. $x \leq \frac{2}{3}$

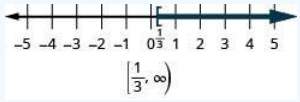
Exercise 2.7E.6

1. $x > 3$
2. $x \leq -0.5$
3. $x \geq \frac{1}{3}$

Answer

1. 

2. 

3. 

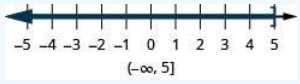
Exercise 2.7E.7

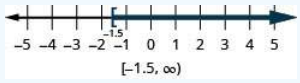
1. $x \geq -4$
2. x<2.5
3. $x > -\frac{3}{2}$

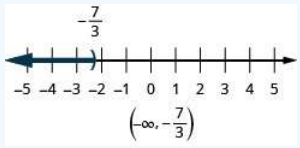
Exercise 2.7E.8

1. $x \leq 5$
2. $x \geq -1.5x$
3. x<−73

Answer

1. 

2. 

3. 

Solve Inequalities using the Subtraction and Addition Properties of Inequality

In the following exercises, solve each inequality, graph the solution on the number line, and write the solution in interval notation.

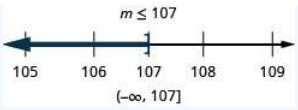
Exercise 2.7E.9

$n - 11 < 33$

Exercise 2.7E.10

$m - 45 \leq 62$

Answer



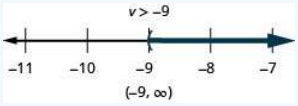
Exercise 2.7E.11

$u+25>21$

Exercise 2.7E.12

$v+12>3$

Answer



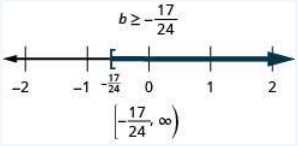
Exercise 2.7E.13

$a+\frac{3}{4}\geq\frac{7}{10}$

Exercise 2.7E.14

$b+\frac{7}{8}\geq\frac{1}{6}$

Answer



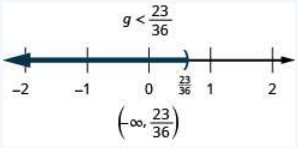
Exercise 2.7E.15

$f-\frac{13}{20}<-\frac{5}{12}$

Exercise 2.7E.16

$g-\frac{11}{12}<-\frac{5}{18}$

Answer



Solve Inequalities using the Division and Multiplication Properties of Inequality

In the following exercises, solve each inequality, graph the solution on the number line, and write the solution in interval notation.

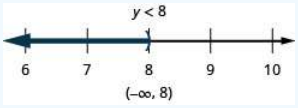
Exercise 2.7E.17

$8x > 72$

Exercise 2.7E.18

$6y < 48$

Answer



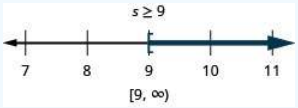
Exercise 2.7E.19

$7r \leq 56$

Exercise 2.7E.20

$9s \geq 81$

Answer



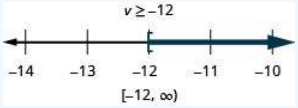
Exercise 2.7E.21

$-5u \geq 65$

Exercise 2.7E.22

$-8v \leq 96$

Answer



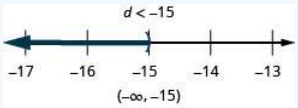
Exercise 2.7E.23

$-9c < 126$

Exercise 2.7E.24

$-7d > 105$

Answer



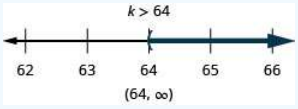
Exercise 2.7E.25

$20 > \frac{2}{5}h$

Exercise 2.7E.26

$40 < \frac{5}{8}k$

Answer



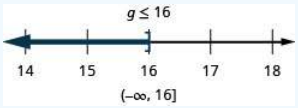
Exercise 2.7E.27

$\frac{7}{6}j \geq 42$

Exercise 2.7E.28

$\frac{9}{4}g \leq 36$

Answer



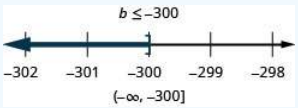
Exercise 2.7E.29

$\frac{a}{-3} \leq 9$

Exercise 2.7E.30

$\frac{b}{-10} \geq 30$

Answer



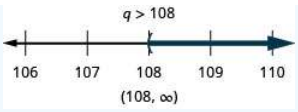
Exercise 2.7E.31

$-25 < \frac{p}{-5}$

Exercise 2.7E.32

$-18 > \frac{q}{-6}$

Answer



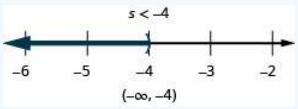
Exercise 2.7E.33

$9t \geq -27$

Exercise 2.7E.34

$7s < -28$

Answer



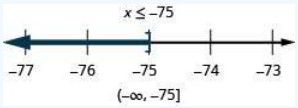
Exercise 2.7E.35

$\frac{2}{3}y > -36$

Exercise 2.7E.36

$\frac{3}{5}x \leq -45$

Answer



Solve Inequalities That Require Simplification

In the following exercises, solve each inequality, graph the solution on the number line, and write the solution in interval notation.

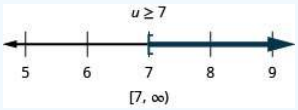
Exercise 2.7E.37

$4v \geq 9v - 40$

Exercise 2.7E.38

$5u \leq 8u - 21$

Answer



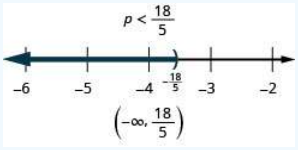
Exercise 2.7E.39

$13q < 7q - 29$

Exercise 2.7E.40

$9p > 14p - 18$

Answer



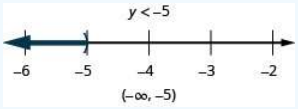
Exercise 2.7E.41

$12x + 3(x + 7) > 10x - 24$

Exercise 2.7E.42

$9y + 5(y + 3) < 4y - 35$

Answer



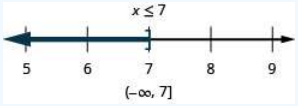
Exercise 2.7E.43

$6h - 4(h - 1) \leq 7h - 11$

Exercise 2.7E.44

$4k - (k - 2) \geq 7k - 26$

Answer



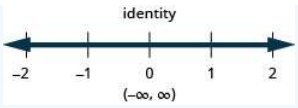
Exercise 2.7E.45

$8m - 2(14 - m) \geq 7(m - 4) + 3m$

Exercise 2.7E.46

$6n - 12(3 - n) \leq 9(n - 4) + 9n$

Answer



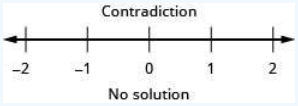
Exercise 2.7E.47

$\frac{3}{4}b-\frac{1}{3}b<\frac{5}{12}b-\frac{1}{2}$

Exercise 2.7E.48

$9u+5(2u-5)\geq 12(u-1)+7u$

Answer



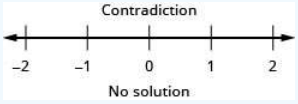
Exercise 2.7E.49

$\frac{2}{3}g-\frac{1}{2}(g-14)\leq\frac{1}{6}(g+42)$

Exercise 2.7E.50

$\frac{5}{6}a-\frac{1}{4}a>\frac{7}{12}a+\frac{2}{3}$

Answer



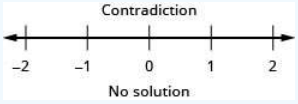
Exercise 2.7E.51

$\frac{4}{5}h-\frac{2}{3}(h-9)\geq\frac{1}{15}(2h+90)$

Exercise 2.7E.52

$12v+3(4v-1)\leq 19(v-2)+5v$

Answer



Mixed practice

In the following exercises, solve each inequality, graph the solution on the number line, and write the solution in interval notation.

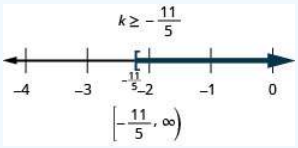
Exercise 2.7E.53

$15k\leq -40$

Exercise 2.7E.54

$35k \geq -77$

Answer



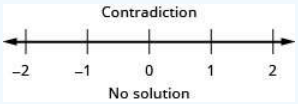
Exercise 2.7E.55

$23p - 2(6 - 5p) > 3(11p - 4)$

Exercise 2.7E.56

$18q - 4(10 - 3q) < 5(6q - 8)$

Answer



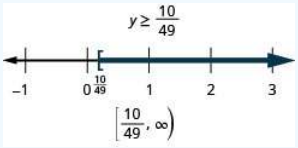
Exercise 2.7E.57

$-\frac{9}{4}x \geq -\frac{5}{12}$

Exercise 2.7E.58

$-\frac{21}{8}y \leq -\frac{15}{28}$

Answer



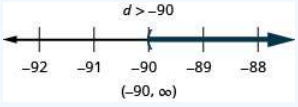
Exercise 2.7E.59

$c + 34 < -99$

Exercise 2.7E.60

$d + 29 > -61$

Answer



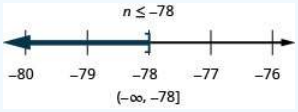
Exercise 2.7E.61

$\frac{m}{18} \geq -4$

Exercise 2.7E.62

$\frac{n}{13} \leq -6$

Answer



Translate to an Inequality and Solve

In the following exercises, translate and solve .Then write the solution in interval notation and graph on the number line.

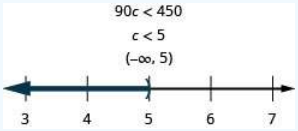
Exercise 2.7E.63

Fourteen times *d* is greater than 56.

Exercise 2.7E.64

Ninety times *c* is less than 450.

Answer



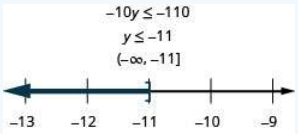
Exercise 2.7E.65

Eight times z is smaller than -40

Exercise 2.7E.66

Ten times y is at most -110

Answer



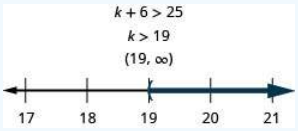
Exercise 2.7E.67

Three more than h is no less than 25

Exercise 2.7E.68

Six more than k exceeds 25

Answer



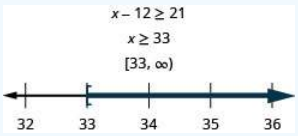
Exercise 2.7E.69

Ten less than w is at least 39.

Exercise 2.7E.70

Twelve less than x is no less than 21

Answer



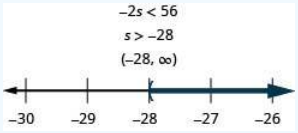
Exercise 2.7E.71

Negative five times r is no more than 95.

Exercise 2.7E.72

Negative two times s is lower than 56

Answer



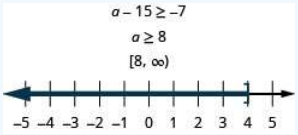
Exercise 2.7E.73

Nineteen less than b is at most -22

Exercise 2.7E.74

Fifteen less than a is at least -7

Answer



Everyday Math

Exercise 2.7E.75

Safety A child's height, *h*, must be at least 57 inches for the child to safely ride in the front seat of a car. Write this as an inequality.

Exercise 2.7E.76

Fighter pilots The maximum height, *h*, of a fighter pilot is 77 inches. Write this as an inequality.

Answer

$h \leq 77$

Exercise 2.7E.77

Elevators The total weight, *w*, of an elevator's passengers can be no more than 1,200 pounds. Write this as an inequality.

Exercise 2.7E.78

Shopping The number of items, *n*, a shopper can have in the express check-out lane is at most 8. Write this as an inequality.

Answer

$n \leq 8$

Writing Exercises

Exercise 2.7E.79

Give an example from your life using the phrase 'at least'.

Exercise 2.7E.80

Give an example from your life using the phrase 'at most'.

Answer

Answers will vary.

Exercise 2.7E.81

Explain why it is necessary to reverse the inequality when solving $-5x > 10$

Exercise 2.7E.82

Explain why it is necessary to reverse the inequality when solving $\frac{n}{-3} < 12$

Answer

Answers will vary.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
graph inequalities on the number line.			
solve inequalities using the Subtraction and Addition Properties of Inequality.			
solve inequalities using the Division and Multiplication Properties of Inequality.			
solve inequalities that require simplification.			
translate to an inequality and solve.			

ⓑ What does this checklist tell you about your mastery of this section? What steps will you take to improve?

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2.8: Chapter 2 Review Exercises

Chapter 2 Review Exercises

Solve Equations using the Subtraction and Addition Properties of Equality

Verify a Solution of an Equation

In the following exercises, determine whether each number is a solution to the equation.

Exercise 2.8.1

$10x-1=5x; x=\frac{1}{5}$

Exercise 2.8.2

$w+2=\frac{5}{8}; w=\frac{3}{8}$

Answer

no

Exercise 2.8.3

$-12n+5=8n; n=-\frac{5}{4}$

Exercise 2.8.4

$6a-3=-7a, a=\frac{3}{13}$

Answer

yes

Solve Equations using the Subtraction and Addition Properties of Equality

In the following exercises, solve each equation using the Subtraction Property of Equality.

Exercise 2.8.5

$x+7=19$

Exercise 2.8.6

$y+2=-6$

Answer

$y=-8$

Exercise 2.8.7

$a+\frac{1}{3}=\frac{5}{3}$

Exercise 2.8.8

$n+3.6=5.1$

Answer

$n=1.5$

In the following exercises, solve each equation using the Addition Property of Equality.

Exercise 2.8.9

$u-7=10$

Exercise 2.8.10

$x-9=-4$

Answer

$x=5$

Exercise 2.8.11

$c-\frac{3}{11}=\frac{9}{11}$

Exercise 2.8.12

$p-4.8=14$

Answer

$p=18.8$

In the following exercises, solve each equation.

Exercise 2.8.13

$n-12=32$

Exercise 2.8.14

$y+16=-9$

Answer

$y=-25$

Exercise 2.8.15

$f+\frac{2}{3}=4$

Exercise 2.8.16

$d-3.9=8.2$

Answer

$d = 12.1$

Solve Equations That Require Simplification

In the following exercises, solve each equation.

Exercise 2.8.17

$y + 8 - 15 = -3$

Exercise 2.8.18

$7x + 10 - 6x + 3 = 5$

Answer

$x = -8$

Exercise 2.8.19

$6(n - 1) - 5n = -14$

Exercise 2.8.20

$8(3p + 5) - 23(p - 1) = 35$

Answer

$p = -28$

Translate to an Equation and Solve

In the following exercises, translate each English sentence into an algebraic equation and then solve it.

Exercise 2.8.21

The sum of -6 and m is 25

Exercise 2.8.22

Four less than n is 13

Answer

$n - 4 = 13; n = 17$

Translate and Solve Applications

In the following exercises, translate into an algebraic equation and solve.

Exercise 2.8.23

Rochelle's daughter is 11 years old. Her son is 3 years younger. How old is her son?

Exercise 2.8.24

Tan weighs 146 pounds. Minh weighs 15 pounds more than Tan. How much does Minh weigh?

Answer

161 pounds

Exercise 2.8.25

Peter paid $9.75 to go to the movies, which was $46.25 less than he paid to go to a concert. How much did he pay for the concert?

Exercise 2.8.26

Elissa earned $\$152.84$this week, which was $\$2. .65$more than she earned last week. How much did she earn last week?

Answer

$\$131.19$

Solve Equations using the Division and Multiplication Properties of Equality

Solve Equations Using the Division and Multiplication Properties of Equality

In the following exercises, solve each equation using the division and multiplication properties of equality and check the solution.

Exercise 2.8.27

$8x = 72$

Exercise 2.8.28

$13a = -65$

Answer

$a = -5$

Exercise 2.8.29

$0.25p = 5.25$

Exercise 2.8.30

$-y = 4$

Answer

$y = -4$

Exercise 2.8.31

$\frac{n}{6} = 18$

Exercise 2.8.32

$\frac{y}{-10} = 30$

Answer

$y = -300$

Exercise 2.8.33

$36 = \frac{3}{4}x$

Exercise 2.8.34

$\frac{5}{8}u = \frac{15}{16}$

Answer

$u = \frac{3}{2}$

Exercise 2.8.35

$-18m = -72$

Exercise 2.8.36

$\frac{c}{9} = 36$

Answer

$c = 324$

Exercise 2.8.37

$0.45x = 6.75$

Exercise 2.8.38

$\frac{11}{12} = \frac{2}{3}y$

Answer

$y = \frac{11}{8}$

Solve Equations That Require Simplification

In the following exercises, solve each equation requiring simplification.

Exercise 2.8.39

$5r - 3r + 9r = 35 - 2$

Exercise 2.8.40

$24x + 8x - 11x = -7 - 14$

Answer

$x = -1$

Exercise 2.8.41

$\frac{11}{12}n - \frac{5}{6}n = 9 - 5$

Exercise 2.8.42

$-9(d - 2) - 15 = -24$

Answer

$d = 3$

Translate to an Equation and Solve

In the following exercises, translate to an equation and then solve.

Exercise 2.8.43

143 is the product of -11 and y

Exercise 2.8.44

The quotient of b and and 9 is -27

Answer

$\frac{b}{9} = -27; b = -243$

Exercise 2.8.45

The sum of q and one-fourth is one.

Exercise 2.8.46

The difference of s and one-twelfth is one fourth.

Answer

$s - \frac{1}{12} = \frac{1}{4}; s = \frac{1}{3}$

Translate and Solve Applications

In the following exercises, translate into an equation and solve.

Exercise 2.8.47

Ray paid $21 for 12 tickets at the county fair. What was the price of each ticket?

Exercise 2.8.48

Janet gets paid $\$24$ per hour. She heard that this is $\frac{3}{4}$ of what Adam is paid. How much is Adam paid per hour?

Answer

$32

Solve Equations with Variables and Constants on Both Sides

Solve an Equation with Constants on Both Sides

In the following exercises, solve the following equations with constants on both sides.

Exercise 2.8.49

$8p+7=47$

Exercise 2.8.50

$10w-5=65$

Answer

$w=7$

Exercise 2.8.51

$3x+19=-47$

Exercise 2.8.52

$32=-4-9n$

Answer

$n=-4$

Solve an Equation with Variables on Both Sides

In the following exercises, solve the following equations with variables on both sides.

Exercise 2.8.53

$7y=6y-13$

Exercise 2.8.54

$5a+21=2a$

Answer

$a=-7$

Exercise 2.8.55

$k=-6k-35$

Exercise 2.8.56

$4x - \frac{3}{8} = 3x$

Answer

$x = \frac{3}{8}$

Solve an Equation with Variables and Constants on Both Sides

In the following exercises, solve the following equations with variables and constants on both sides.

Exercise 2.8.57

$12x - 9 = 3x + 45$

Exercise 2.8.58

$5n - 20 = -7n - 80$

Answer

$n = -5$

Exercise 2.8.59

$4u + 16 = -19 - u$

Exercise 2.8.60

$\frac{5}{8}c - 4 = \frac{3}{8}c + 4$

Answer

$c = 32$

Use a General Strategy for Solving Linear Equations

Solve Equations Using the General Strategy for Solving Linear Equations

In the following exercises, solve each linear equation.

Exercise 2.8.61

$6(x + 6) = 24$

Exercise 2.8.62

$9(2p - 5) = 72$

Answer

$p = \frac{13}{2}$

Exercise 2.8.63

$-(s+4)=18$

Exercise 2.8.64

$8+3(n-9)=17$

Answer

$n=12$

Exercise 2.8.65

$23-3(y-7)=8$

Exercise 2.8.66

$\frac{1}{3}(6m+21)=m-7$

Answer

$m=-14$

Exercise 2.8.67

$4(3.5y+0.25)=365$

Exercise 2.8.68

$0.25(q-8)=0.1(q+7)$

Answer

$q=18$

Exercise 2.8.69

$8(r-2)=6(r+10)$

Exercise 2.8.70

$5+7(2-5x)=2(9x+1)$
$-(13x-57)$

Answer

$x=-1$

Exercise 2.8.71

$(9n+5)-(3n-7)$
$=20-(4n-2)$

Exercise 2.8.72

$2[-16+5(8k-6)]$
$=8(3-4k)-32$

Answer

$k=\frac{3}{4}$

Classify Equations

In the following exercises, classify each equation as a conditional equation, an identity, or a contradiction and then state the solution.

Exercise 2.8.73

$17y-3(4-2y)=11(y-1)$
$+12y-1$

Exercise 2.8.74

$9u+32=15(u-4)$
$-3(2u+21)$

Answer

contradiction; no solution

Exercise 2.8.75

$-8(7m+4)=-6(8m+9)$

Exercise 2.8.76

$21(c-1)-19(c+1)$
$=2(c-20)$

Answer

identity; all real numbers

Solve Equations with Fractions and Decimals

Solve Equations with Fraction Coefficients

In the following exercises, solve each equation with fraction coefficients.

Exercise 2.8.77

$\frac{2}{5}n-\frac{1}{10}=\frac{7}{10}$

Exercise 2.8.78

$\frac{1}{3}x+\frac{1}{5}x=8$

Answer

$x=15$

Exercise 2.8.79

$\frac{3}{4}a-\frac{1}{3}=\frac{1}{2}a-\frac{5}{6}$

Exercise 2.8.80

$\frac{1}{2}(k-3)=\frac{1}{3}(k+16)$

Answer

$k=41$

Exercise 2.8.81

$\frac{3x-2}{5}=\frac{3x+4}{8}$

Exercise 2.8.82

$\frac{5y-1}{3}+4=\frac{-8y+4}{6}$

Answer

$y=-1$

Solve Equations with Decimal Coefficients

In the following exercises, solve each equation with decimal coefficients.

Exercise 2.8.83

$0.8x-0.3=0.7x+0.2$

Exercise 2.8.84

$0.36u+2.55=0.41u+6.8$

Answer

$u=-85$

Exercise 2.8.85

$0.6p-1.9=0.78p+1.7$

Exercise 2.8.86

$0.6p-1.9=0.78p+1.7$

Answer

$d=-20$

Solve a Formula for a Specific Variable

Use the Distance, Rate, and Time Formula

In the following exercises, solve.

Exercise 2.8.87

Natalie drove for $7\frac{1}{2}$ hours at 60 miles per hour. How much distance did she travel?

Exercise 2.8.88

Mallory is taking the bus from St. Louis to Chicago. The distance is 300 miles and the bus travels at a steady rate of 60 miles per hour. How long will the bus ride be?

Answer

5 hours

Exercise 2.8.89

Aaron's friend drove him from Buffalo to Cleveland. The distance is 187 miles and the trip took 2.75 hours. How fast was Aaron's friend driving?

Exercise 2.8.90

Link rode his bike at a steady rate of 15 miles per hour for $2\frac{1}{2}$ hours. How much distance did he travel?

Answer

37.5 miles

Solve a Formula for a Specific Variable

In the following exercises, solve.

Exercise 2.8.91

Use the formula. d=rt to solve for t

1. when d=510 and r=60
2. in general

Exercise 2.8.92

Use the formula. d=rt to solve for r

1. when when d=451 and t=5.5
2. in general

Answer

1. r=82mph
2. $r = \frac{D}{t}$

Exercise 2.8.93

Use the formula $A = \frac{1}{2}bh$ to solve for b

1. when A=390 and h=26
2. in general

Exercise 2.8.94

Use the formula $A = \frac{1}{2}bh$ to solve for b

1. when A=153 and b=18
2. in general

Answer

1. $h = 17$
2. $h = \frac{2A}{b}$

Exercise 2.8.95

Use the formula I=Prt to solve for the principal, *P* for

1. I=$2,501,r=4.1%, t=5 years
2. in general

Exercise 2.8.96

Solve the formula 4x+3y=6 for *y*

1. when x=−2
2. in general

Answer

ⓐ $y = \frac{14}{3}$ ⓑ $y = \frac{6-4x}{3}$

Exercise 2.8.97

Solve $180 = a + b + c$ for c

Exercise 2.8.98

Solve the formula $V = LWH$ for H

Answer

$H = \frac{V}{LW}$

Solve Linear Inequalities

Graph Inequalities on the Number Line

In the following exercises, graph each inequality on the number line.

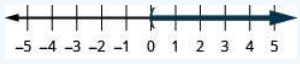
Exercise 2.8.99

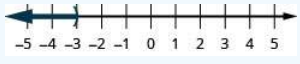
1. $x \leq 4$
2. x>−2
3. x<1

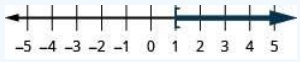
Exercise 2.8.100

1. x>0
2. x<−3
3. $x \geq -1$

Answer

1. 

2. 

3. 

In the following exercises, graph each inequality on the number line and write in interval notation.

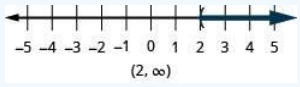
Exercise 2.8.101

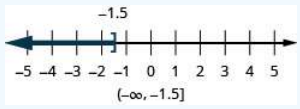
1. $x < -1$
2. $x \geq -2.5$
3. $x \leq \frac{5}{4}$

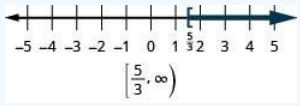
Exercise 2.8.102

1. $x > 2$
2. $x \leq -1.5$
3. $x \geq \frac{5}{3}$

Answer

1. 

2. 

3. 

Solve Inequalities using the Subtraction and Addition Properties of Inequality

In the following exercises, solve each inequality, graph the solution on the number line, and write the solution in interval notation.

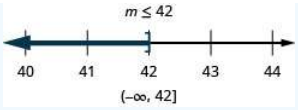
Exercise 2.8.103

$n - 12 \leq 23$

Exercise 2.8.104

$m + 14 \leq 56$

Answer



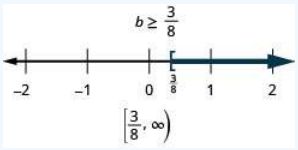
Exercise 2.8.105

$a+\frac{2}{3} \geq \frac{7}{12}$

Exercise 2.8.106

$b-\frac{7}{8} \geq -\frac{1}{2}$

Answer



Solve Inequalities using the Division and Multiplication Properties of Inequality

In the following exercises, solve each inequality, graph the solution on the number line, and write the solution in interval notation.

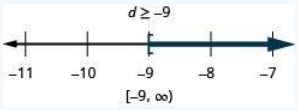
Exercise 2.8.107

$9x > 54$

Exercise 2.8.108

$-12d \leq 108$

Answer



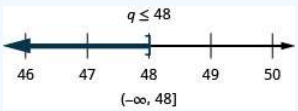
Exercise 2.8.109

$\frac{5}{2}j < -60$

Exercise 2.8.110

$\frac{q}{-2} \geq -24$

Answer



Solve Inequalities That Require Simplification

In the following exercises, solve each inequality, graph the solution on the number line, and write the solution in interval notation.

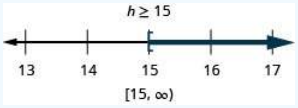
Exercise 2.8.111

$6p > 15p - 30$

Exercise 2.8.112

$9h - 7(h-1) \leq 4h - 23$

Answer



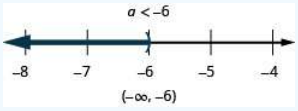
Exercise 2.8.113

$5n - 15(4-n) < 10(n-6) + 10n$

Exercise 2.8.114

$\frac{3}{8}a - \frac{1}{12}a > \frac{5}{12}a + \frac{3}{4}$

Answer



Translate to an Inequality and Solve

In the following exercises, translate and solve. Then write the solution in interval notation and graph on the number line.

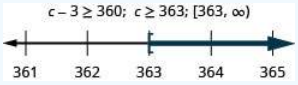
Exercise 2.8.115

Five more than z is at most 19.

Exercise 2.8.116

Three less than c is at least 360.

Answer



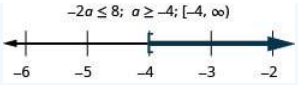
Exercise 2.8.117

Nine times n exceeds 42.

Exercise 2.8.118

Negative two times a is no more than 8.

Answer



Everyday Math

Exercise 2.8.119

Describe how you have used two topics from this chapter in your life outside of your math class during the past month.

Chapter 2 Practice Test

Exercise 2.8.1

Determine whether each number is a solution to the equation $6x - 3 = x + 20$

1. 5
2. $\frac{23}{5}$

Answer

1. no
2. yes

In the following exercises, solve each equation.

Exercise 2.8.2

$n - \frac{2}{3} = \frac{1}{4}$

Exercise 2.8.3

$\frac{9}{2}c = 144$

Answer

c=32

Exercise 2.8.4

$4y - 8 = 16$

Exercise 2.8.5

$-8x - 15 + 9x - 1 = -21$

Answer

$x = -5$

Exercise 2.8.6

$-15a = 120$

Exercise 2.8.7

$\frac{2}{3}x = 6$

Answer

$x = 9$

Exercise 2.8.8

$x - 3.8 = 8.2$

Exercise 2.8.9

$10y = -5y - 60$

Answer

$y = -4$

Exercise 2.8.10

$8n - 2 = 6n - 12$

Exercise 2.8.11

$9m - 2 - 4m - m = 42 - 8$

Answer

$m = 9$

Exercise 2.8.12

$-5(2x - 1) = 45$

Exercise 2.8.13

$-(d - 9) = 23$

Answer

$d = -14$

Exercise 2.8.14

$\frac{1}{4}(12m - 28) = 6 - 2(3m - 1)$

Exercise 2.8.15

$2(6x - 5) - 8 = -22$

Answer

$x = -\frac{1}{3}$

Exercise 2.8.16

$8(3a-5)-7(4a-3)=20-3a$

Exercise 2.8.17

$\frac{1}{4}p-\frac{1}{3}=\frac{1}{2}$

Answer

$p=\frac{10}{3}$

Exercise 2.8.18

$0.1d+0.25(d+8)=4.1$

Exercise 2.8.19

$14n-3(4n+5)=-9+2(n-8)$

Answer

contradiction; no solution

Exercise 2.8.20

$9(3u-2)-4[6-8(u-1)]=3(u-2)$

Exercise 2.8.21

Solve the formula x−2y=5 for *y*

1. when x=−3
2. in general

Answer

1. y=4
2. $y=\frac{5-x}{2}$

In the following exercises, graph on the number line and write in interval notation.

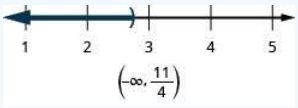
Exercise 2.8.22

$x\geq -3.5$

Exercise 2.8.23

$x<\frac{11}{4}$

Answer



https://math.libretexts.org/@go/page/61367

In the following exercises,, solve each inequality, graph the solution on the number line, and write the solution in interval notation.

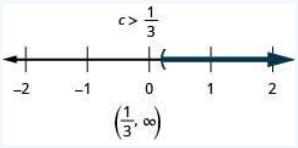
Exercise 2.8.24

$8k \geq 5k - 120$

Exercise 2.8.25

$3c - 10(c-2) < 5c + 16$

Answer



In the following exercises, translate to an equation or inequality and solve.

Exercise 2.8.26

4 less than twice *x* is 16.

Exercise 2.8.27

Fifteen more than *n* is at least 48.

Answer

$n + 15 \geq 48; n \geq 33$

Exercise 2.8.28

Samuel paid $25.82 for gas this week, which was $3.47 less than he paid last week. How much had he paid last week?

Exercise 2.8.29

Jenna bought a coat on sale for $\$120$, which was $\frac{2}{3}$ of the original price. What was the original price of the coat?

Answer

$120 = \frac{2}{3}p$; The original price was $\$180$

Exercise 2.8.30

Sean took the bus from Seattle to Boise, a distance of 506 miles. If the trip took $7\frac{2}{3}$ hours, what was the speed of the bus?

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CHAPTER OVERVIEW

3: Math Models

- 3.1: Use a Problem-Solving Strategy
  - 3.1E: Exercises
- 3.2: Solve Percent Applications
  - 3.2E: Exercises
- 3.3: Solve Mixture Applications
  - 3.3E: Exercises
- 3.4: Triangles, Rectangles, and the Pythagorean Theorem
  - 3.4E: Exercises
- 3.5: Solve Uniform Motion Applications
  - 3.5E: Exercises
- 3.6: Solve Applications with Linear Inequalities
  - 3.6E: Exercises
- 3.7: Chapter 3 Review Exercises

Thumbnail: https://www.wikihow.com/Make-a-Mathematical-Model

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3.1: Use a Problem-Solving Strategy

Learning Objectives

By the end of this section, you will be able to:

- Approach word problems with a positive attitude
- Use a problem-solving strategy for word problems
- Solve number problems

Note

Before you get started, take this readiness quiz.

1. Translate "6 less than twice x" into an algebraic expression.
   If you missed this problem, review Exercise 1.3.43.
2. Solve: $\frac{2}{3}x = 24$.
   If you missed this problem, review Exercise 2.2.10.
3. Solve: $3x + 8 = 14$.
   If you missed this problem, review Exercise 2.3.1.

Approach Word Problems with a Positive Attitude

"If you think you can… or think you can't… you're right."—Henry Ford

The world is full of word problems! Will my income qualify me to rent that apartment? How much punch do I need to make for the party? What size diamond can I afford to buy my girlfriend? Should I fly or drive to my family reunion? How much money do I need to fill the car with gas? How much tip should I leave at a restaurant? How many socks should I pack for vacation? What size turkey do I need to buy for Thanksgiving dinner, and then what time do I need to put it in the oven? If my sister and I buy our mother a present, how much does each of us pay?

Now that we can solve equations, we are ready to apply our new skills to word problems. Do you know anyone who has had negative experiences in the past with word problems? Have you ever had thoughts like the student below (Figure 3.1.1)?

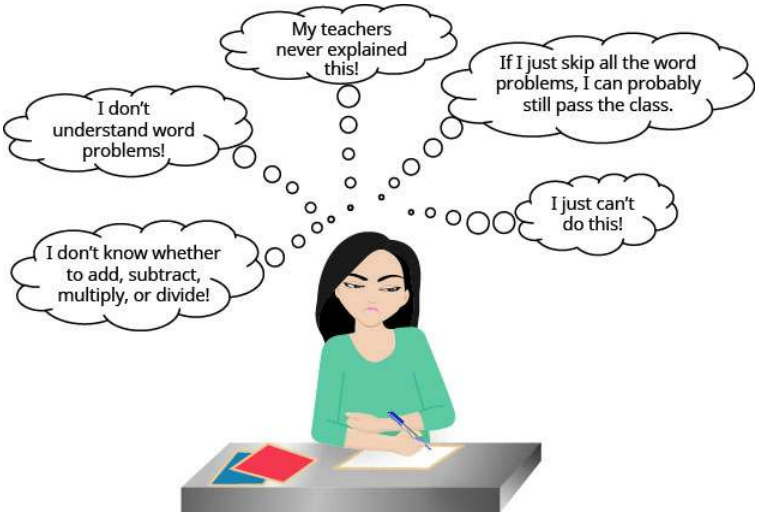


Figure 3.1.1: Negative thoughts can be barriers to success.

When we feel we have no control, and continue repeating negative thoughts, we set up barriers to success. We need to calm our fears and change our negative feelings.

Start with a fresh slate and begin to think positive thoughts. If we take control and believe we can be successful, we will be able to master word problems! Read the positive thoughts in Figure 3.1.2 and say them out loud.



Figure 3.1.2: Thinking positive thoughts is a first step towards success.

Think of something, outside of school, that you can do now but couldn't do 3 years ago. Is it driving a car? Snowboarding? Cooking a gourmet meal? Speaking a new language? Your past experiences with word problems happened when you were younger—now you're older and ready to succeed!

Use a Problem-Solving Strategy for Word Problems

We have reviewed translating English phrases into algebraic expressions, using some basic mathematical vocabulary and symbols. We have also translated English sentences into algebraic equations and solved some word problems. The word problems applied math to everyday situations. We restated the situation in one sentence, assigned a variable, and then wrote an equation to solve the problem. This method works as long as the situation is familiar and the math is not too complicated.

Now, we'll expand our strategy so we can use it to successfully solve any word problem. We'll list the strategy here, and then we'll use it to solve some problems. We summarize below an effective strategy for problem solving.

USE A PROBLEM-SOLVING STRATEGY TO SOLVE WORD PROBLEMS.

1. **Read** the problem. Make sure all the words and ideas are understood.
2. **Identify** what we are looking for.
3. **Name** what we are looking for. Choose a variable to represent that quantity.
4. **Translate** into an equation. It may be helpful to restate the problem in one sentence with all the important information. Then, translate the English sentence into an algebraic equation.
5. **Solve** the equation using good algebra techniques.
6. **Check** the answer in the problem and make sure it makes sense.
7. **Answer** the question with a complete sentence.

Example 3.1.1

Pilar bought a purse on sale for $18, which is one-half of the original price. What was the original price of the purse?

Solution

Step 1. Read the problem. Read the problem two or more times if necessary. Look up any unfamiliar words in a dictionary or on the internet.

In this problem, is it clear what is being discussed? Is every word familiar?

Let p = the original price of the purse.

Step 2. Identify what you are looking for. Did you ever go into your bedroom to get something and then forget what you were looking for? It's hard to find something if you are not sure what it is! Read the problem again and look for words that tell you what you are looking for!

In this problem, the words "what was the original price of the purse" tell us what we need to find.

Step 3. Name what we are looking for. Choose a variable to represent that quantity. We can use any letter for the variable, but choose one that makes it easy to remember what it represents.

Step 4. Translate into an equation. It may be helpful to restate the problem in one sentence with all the important information. Translate the English sentence into an algebraic equation.

Reread the problem carefully to see how the given information is related. Often, there is one sentence that gives this information, or it may help to write one sentence with all the important information. Look for clue words to help translate the sentence into algebra. Translate the sentence into an equation.

Restate the problem in one sentence with all the important information.	**18** **is** **one-half the original price.**
Translate into an equation.	$18 = \frac{1}{2} \cdot p$

Step 5. Solve the equation using good algebraic techniques. Even if you know the solution right away, using good algebraic techniques here will better prepare you to solve problems that do not have obvious answers.

Solve the equation.	$18 = \frac{1}{2}p$
Multiply both sides by 2.	$2 \cdot 18 = 2 \cdot \frac{1}{2}p$
Simplify.	$36 = p$

Step 6. Check the answer in the problem to make sure it makes sense. We solved the equation and found that $p = 36$,which means "the original price" was $\$36$.

Does $36 make sense in the problem? Yes, because 18 is one-half of 36, and the purse was on sale at half the original price.

If this were a homework exercise, our work might look like this:

Pilar bought a purse on sale for $\$18$, which is one-half the original price. What was the original price of the purse?

Step 7. Answer the question with a complete sentence. The problem asked "What was the original price of the purse?"

The answer to the question is: "The original price of the purse was $36."

	Let $p =$ the original price.
	18 is one-half the original price.
	$18 = \frac{1}{2}p$
Multiply both sides by 2.	$2 \cdot 18 = 2 \cdot \frac{1}{2}p$
Simplify.	$36 = p$
Check. Is $36 a reasonable price for a purse?	
Yes.	
Is 18 one half of 36?	
$18 \stackrel{?}{=} \frac{1}{2} \cdot 36$	
$18 = 18$✓	
	The original price of the purse was $36.

Try It 3.1.2

Joaquin bought a bookcase on sale for $120, which was two-thirds of the original price. What was the original price of the bookcase?

Answer

$180

Try It 3.1.3

Two-fifths of the songs in Mariel's playlist are country. If there are 16 country songs, what is the total number of songs in the playlist?

Answer

40

Let's try this approach with another example.

Example 3.1.4

Ginny and her classmates formed a study group. The number of girls in the study group was three more than twice the number of boys. There were 11 girls in the study group. How many boys were in the study group?

Solution

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	How many boys were in the study group?
Step 3. Name. Choose a variable to represent the number of boys.	Let $n =$ the number of boys.
Step 4. Translate. Restate the problem in one sentence with all the important information.	**The number of girls** (11) **was** **three more than twice the number of boys**
Translate into an equation.	$11 \quad = \quad 2b+3$
Step 5. Solve the equation.	$11 = 2b+3$
Subtract 3 from each side.	$11-3 = 2b+3-3$
Simplify.	$8 = 2b$
Divide each side by 2.	$\frac{8}{2} = \frac{2b}{2}$
Simplify.	$4 = b$
Step 6. Check. First, is our answer reasonable? Yes, having 4 boys in a study group seems OK. The problem says the number of girls was 3 more than twice the number of boys. If there are four boys, does that make eleven girls? Twice 4 boys is 8. Three more than 8 is 11.	
Step 7. Answer the question.	There were 4 boys in the study group.

Try It 3.1.5

Guillermo bought textbooks and notebooks at the bookstore. The number of textbooks was 3 more than twice the number of notebooks. He bought 7 textbooks. How many notebooks did he buy?

Answer

2

Try It 3.1.6

Gerry worked Sudoku puzzles and crossword puzzles this week. The number of Sudoku puzzles he completed is eight more than twice the number of crossword puzzles. He completed 22 Sudoku puzzles. How many crossword puzzles did he do?

Answer

7

Solve Number Problems

Now that we have a problem solving strategy, we will use it on several different types of word problems. The first type we will work on is “number problems.” Number problems give some clues about one or more numbers. We use these clues to write an equation. Number problems don’t usually arise on an everyday basis, but they provide a good introduction to practicing the problem solving strategy outlined above.

Example 3.1.7

The difference of a number and six is 13. Find the number.

Solution

Step 1. Read the problem. Are all the words familiar?	
Step 2. Identify what we are looking for.	the number
Step 3. Name. Choose a variable to represent the number.	Let $n =$ the number.
Step 4. Translate. Remember to look for clue words like "difference... of... and..."	
Restate the problem as one sentence.	$\underbrace{\textbf{The difference of the number and 6}}$ $\underbrace{\textbf{is}}$ $\underbrace{\textbf{13}}$
Translate into an equation.	$n - 6 \quad = \quad 13$
Step 5. Solve the equation.	$n - 6 = 13$
Simplify.	$n = 19$
Step 6. Check.	
The difference of 19 and 6 is 13. It checks!	
Step 7. Answer the question.	The number is 19.

Try It 3.1.8

The difference of a number and eight is 17. Find the number.

Answer

25

Try It 3.1.9

The difference of a number and eleven is -7. Find the number.

Answer

4

Example 3.1.10

The sum of twice a number and seven is 15. Find the number.

Solution

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	the number
Step 3. Name. Choose a variable to represent the number.	Let $n =$ the number.
Step 4. Translate.	
Restate the problem as one sentence.	The sum of twice a number and 7 is 15
Translate into an equation.	$2n + 7 = 15$
Step 5. Solve the equation.	$2n + 7 = 15$
Subtract 7 from each side and simplify.	$2n = 8$
Divide each side by 2 and simplify.	$n = 4$
Step 6. Check.	
Is the sum of twice 4 and 7 equal to 15?	
$2 \cdot 4 + 7 \stackrel{?}{=} 15$ $15 = 15\checkmark$	
Step 7. Answer the question.	The number is 4.

Did you notice that we left out some of the steps as we solved this equation? If you're not yet ready to leave out these steps, write down as many as you need.

Try It 3.1.11

The sum of four times a number and two is 14. Find the number.

Answer

3

Try It 3.1.12

The sum of three times a number and seven is 25. Find the number.

Answer

6

Some number word problems ask us to find two or more numbers. It may be tempting to name them all with different variables, but so far we have only solved equations with one variable. In order to avoid using more than one variable, we will define the numbers in terms of the same variable. Be sure to read the problem carefully to discover how all the numbers relate to each other.

Example 3.1.13

One number is five more than another. The sum of the numbers is 21. Find the numbers.

Solution

Step 1. Read the problem.		
Step 2. Identify what we are looking for.		We are looking for two numbers.
Step 3. Name. We have two numbers to name and need a name for each.		
Choose a variable to represent the first number.		Let $n = 1^{st}$ number.
What do we know about the second number?		One number is five more than another.
		$n + 5 = 2^{nd}$ number
Step 4. Translate. Restate the problem as one sentence with all the important information.		The sum of the 1st number and the 2nd number is 21.
Translate into an equation.		$1^{st}\text{ number} + 2^{nd}\text{ number} = 21$
Substitute the variable expressions.		$n + n+5 = 21$
Step 5. Solve the equation.		$n + n + 5 = 21$
Combine like terms.		$2n + 5 = 21$
Subtract 5 from both sides and simplify.		$2n = 16$
Divide by 2 and simplify.		$n = 8$ 1st number
Find the second number, too.		$n + 5$ 2nd number
		$8 + 5$
		13
Step 6. Check.		
Do these numbers check in the problem?		
Is one number 5 more than the other?	$13 \stackrel{?}{=} 8 + 5$	
Is thirteen 5 more than 8? Yes.	$13 = 13$✓	
Is the sum of the two numbers 21?	$8 + 13 \stackrel{?}{=} 21$	
	$21 = 21$✓	
Step 7. Answer the question.		The numbers are 8 and 13.

Try It 3.1.14

One number is six more than another. The sum of the numbers is twenty-four. Find the numbers.

Answer

9, 15

Try It 3.1.15

The sum of two numbers is fifty-eight. One number is four more than the other. Find the numbers.

Answer

27, 31

Example 3.1.16

The sum of two numbers is negative fourteen. One number is four less than the other. Find the numbers.

Solution

Step 1. Read the problem.		
Step 2. Identify what we are looking for.		We are looking for two numbers.
Step 3. Name.		
Choose a variable.		Let $n = 1^{st}$ number.
One number is 4 less than the other.		$n - 4 = 2^{nd}$ number
Step 4. Translate.		
Write as one sentence.		The sum of the 2 numbers is negative 14.
Translate into an equation.		1st number + 2nd number is negative fourteen
Step 5. Solve the equation.		$n + n - 4 = -14$
Combine like terms.		$n + n - 4 = -14$
Add 4 to each side and simplify.		$2n - 4 = -14$
Simplify.		$2n = -10$
		$n = -5$ 1st number
		$n - 4$ 2nd number
		$-5 - 4$
		-9
Step 6. Check.		
Is -9 four less than -5?	$-5 - 4 \stackrel{?}{=} -9$	
	$-9 = -9$✓	
Is their sum -14?	$-5 + (-9) \stackrel{?}{=} -14$	
	$-14 = -14$✓	
Step 7. Answer the question.		The numbers are -5 and -9.

Try It 3.1.17

The sum of two numbers is negative twenty-three. One number is seven less than the other. Find the numbers.

Answer

-15, -8

Try It 3.1.18

The sum of two numbers is -18. One number is 40 more than the other. Find the numbers.

Answer

-29, 11

Example 3.1.19

One number is ten more than twice another. Their sum is one. Find the numbers.

Solution

Step 1. Read the problem.		
Step 2. Identify what you are looking for.		We are looking for two numbers.
Step 3. Name.		
Choose a variable.		Let $x = 1^{st}$ number.
One number is 10 more than twice another.		$2x + 10 = 2^{nd}$ number
Step 4. Translate.		
Restate as one sentence.		Their sum is one.
		The sum of the two numbers is 1.
Translate into an equation.		$x + 2x + 10 = 1$
Step 5. Solve the equation.		
Combine like terms.		$x + 2x + 10 = 1$
Subtract 10 from each side.		$3x + 10 = 1$
Divide each side by 3.		$3x = -9$
		$x = -3$ 1st number
		$2x + 10$ 2nd number
		$2(-3) + 10$
		4
Step 6. Check.		
Is ten more than twice −3 equal to 4?	$2(-3) + 10 \stackrel{?}{=} 4$	
	$-6 + 10 \stackrel{?}{=} 4$	
	$4 = 4$✓	

Is their sum 1?	$-3+4\stackrel{?}{=}1$	
	$1=1\checkmark$	
Step 7. Answer the question.		The numbers are −3 and 4.

Try It 3.1.20

One number is eight more than twice another. Their sum is negative four. Find the numbers.

Answer

$-4,\ 0$

Try It 3.1.21

One number is three more than three times another. Their sum is -5. Find the numbers.

Answer

$-3,\ -2$

Some number problems involve consecutive integers. *Consecutive integers* are integers that immediately follow each other. Examples of consecutive integers are:

$$\begin{aligned} &1, 2, 3, 4 \\ &-10, -9, -8, -7 \\ &150, 151, 152, 153 \end{aligned} \tag{3.1.1}$$

Notice that each number is one more than the number preceding it. So if we define the first integer as n, the next consecutive integer is $n+1$. The one after that is one more than $n+1$, so it is $n+1+1$, which is $n+2$.

$$\begin{aligned} &n && 1^{\text{st}} \text{ integer} \\ &n+1 && 2^{\text{nd}} \text{ consecutive integer} \\ &n+2 && 3^{\text{rd}} \text{ consecutive integer } \ldots \text{ etc.} \end{aligned} \tag{3.1.2}$$

Example 3.1.22

The sum of two consecutive integers is 47. Find the numbers.

Solution

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	two consecutive integers
Step 3. Name each number.	Let $n = 1^{st}$ integer.
	$n+1 =$ next consecutive integer
Step 4. Translate.	
Restate as one sentence.	The sum of the integers is 47.
Translate into an equation.	$n + n + 1 = 47$
Step 5. Solve the equation.	$n + n + 1 = 47$
Combine like terms.	$2n + 1 = 47$
Subtract 1 from each side.	$2n = 46$

Divide each side by 2.	$n = 23$ 1st integer
	$n + 1$ next consecutive integer
	$23 + 1$
	24
Step 6. Check.	
$23 + 24 \stackrel{?}{=} 47$ $47 = 47$✓	
Step 7. Answer the question.	The two consecutive integers are 23 and 24.

Try It 3.1.23

The sum of two consecutive integers is 95. Find the numbers.

Answer

47, 48

Try It 3.1.24

The sum of two consecutive integers is −31. Find the numbers.

Answer

-16, -15

Example 3.1.25

Find three consecutive integers whose sum is −42.

Solution

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	three consecutive integers
Step 3. Name each of the three numbers.	Let $n = 1^{st}$ integer.
	$n + 1 = 2^{nd}$ consecutive integer
	$n + 2 = 3^{rd}$ consecutive integer
Step 4. Translate.	
Restate as one sentence.	The sum of the three integers is -42.
Translate into an equation.	$n + n + 1 + n + 2 = -42$
Step 5. Solve the equation.	$n + n + 1 + n + 2 = -42$
Combine like terms.	$3n + 3 = -42$
Subtract 3 from each side.	$3n = -45$
Divide each side by 3.	$n = -15$ 1st integer
	$n + 1$ 2nd integer

	$-15+1$
	-14
	$n+2$ 3rd integer
	$-15+2$
	-13
Step 6. Check.	
$-13+(-14)+(-15) \stackrel{?}{=} -42$ $-42 = -42\checkmark$	
Step 7. Answer the question.	The three consecutive integers are −13, −14, and −15.

Try It 3.1.26

Find three consecutive integers whose sum is −96.

Answer

-33, -32, -31

Try It 3.1.27

Find three consecutive integers whose sum is −36.

Answer

-13, -12, -11

Now that we have worked with consecutive integers, we will expand our work to include consecutive even integers and consecutive odd integers. *Consecutive even integers* are even integers that immediately follow one another. Examples of consecutive even integers are:

$$\begin{aligned} &18, 20, 22 \\ &64, 66, 68 \\ &-12, -10, -8 \end{aligned} \tag{3.1.3}$$

Notice each integer is 2 more than the number preceding it. If we call the first one n, then the next one is $n+2$. The next one would be $n+2+2$ or $n+4$.

$$\begin{aligned} n \quad & 1^{\text{st}} \text{ even integer} \\ n+2 \quad & 2^{\text{nd}} \text{ consecutive even integer} \\ n+4 \quad & 3^{\text{rd}} \text{ consecutive even integer } \ldots \text{ etc.} \end{aligned} \tag{3.1.4}$$

Consecutive odd integers are odd integers that immediately follow one another. Consider the consecutive odd integers 77, 79, and 81.

$$\begin{aligned} &77, 79, 81 \\ &n, n+2, n+4 \end{aligned} \tag{3.1.5}$$

$$\begin{aligned} n \quad & 1^{\text{st}} \text{ odd integer} \\ n+2 \quad & 2^{\text{nd}} \text{ consecutive odd integer} \\ n+4 \quad & 3^{\text{rd}} \text{ consecutive odd integer } \ldots \text{ etc.} \end{aligned} \tag{3.1.6}$$

Does it seem strange to add 2 (an even number) to get from one odd integer to the next? Do you get an odd number or an even number when we add 2 to 3? to 11? to 47?

Whether the problem asks for consecutive even numbers or odd numbers, you don't have to do anything different. The pattern is still the same—to get from one odd or one even integer to the next, add 2.

Example 3.1.28

Find three consecutive even integers whose sum is 84.

Solution

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	three consecutive even integers
Step 3. Name the integers.	Let $n = 1^{st}$ even integers.
	$n+2 = 2^{nd}$ consecutive even integer
	$n+4 = 3^{rd}$ consecutive even integer
Step 4. Translate.	
Restate as one sentence.	The sum of the three even integers is 84.
Translate into an equation.	$n+n+2+n+4=84$
Step 5. Solve the equation.	
Combine like terms.	$n+n+2+n+4=84$
Subtract 6 from each side.	$3n+6=84$
Divide each side by 3.	$3n=78$
	$n=26$ 1st integer
	$n+2$ 2nd integer
	$26+2$
	28
	$n+4$ 3rd integer
	$26+4$
	30
Step 6. Check.	
$26+28+30 \stackrel{?}{=} 84$	
$84=84$✓	
Step 7. Answer the question.	The three consecutive integers are 26, 28, and 30.

(3.1.7)

Try It 3.1.29

Find three consecutive even integers whose sum is 102.

Answer

32, 34, 36

Try It 3.1.30

Find three consecutive even integers whose sum is −24.

Answer

$-10, -8, -6$

Example 3.1.31

A married couple together earns $110,000 a year. The wife earns $16,000 less than twice what her husband earns. What does the husband earn?

Solution

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	How much does the husband earn?
Step 3. Name.	
Choose a variable to represent the amount the husband earns.	Let $h =$ the amount the husband earns.
The wife earns $\$16,000$ less than twice that.	$2h - 16,000$ the amount the wife earns.
Step 4. Translate.	Together the husband and wife earn $\$110,000$.
Restate the problem in one sentence with all the important information.	The amount the husband earns plus the amount the wife earns is $110,000
Translate into an equation.	$h + 2h - 16,000 = 110,000$
Step 5. Solve the equation.	$h + 2h - 16,000 = 110,000$
Combine like terms.	$3h - 16,000 = 110,000$
Add $16,000$ to both sides and simplify.	$3h = 126,000$
Divide each side by 3.	$h = 42,000$
	$\$42,000$ amount husband earns
	$2h - 16,000$ amount wife earns
	$2(42,000) - 16,000$
	$84,000 - 16,000$
	$68,000$
Step 6. Check.	
If the wife earns $\$68,000$ and the husband earns $\$42,000$ is the total $\$110,000$? Yes!	
Step 7. Answer the question.	The husband earns $\$42,000$ a year.

Try It 3.1.32

According to the National Automobile Dealers Association, the average cost of a car in 2014 was $28,500. This was $1,500 less than 6 times the cost in 1975. What was the average cost of a car in 1975?

Answer

$5000

? Try It 3.1.33

U.S. Census data shows that the median price of new home in the United States in November 2014 was $280,900. This was $10,700 more than 14 times the price in November 1964. What was the median price of a new home in November 1964?

Answer

$19300

Key Concepts

- **Problem-Solving Strategy**
  1. **Read** the problem. Make sure all the words and ideas are understood.
  2. **Identify** what we are looking for.
  3. **Name** what we are looking for. Choose a variable to represent that quantity.
  4. **Translate** into an equation. It may be helpful to restate the problem in one sentence with all the important information. Then, translate the English sentence into an algebra equation.
  5. **Solve** the equation using good algebra techniques.
  6. **Check** the answer in the problem and make sure it makes sense.
  7. **Answer** the question with a complete sentence.
- **Consecutive Integers**
  Consecutive integers are integers that immediately follow each other.

$$\begin{array}{ll} n & 1^{\text{st}} \text{ integer} \\ n+1 & 2^{\text{nd}} \text{ consecutive integer} \\ n+2 & 3^{\text{rd}} \text{ consecutive integer } \ldots \text{ etc.} \end{array} \tag{3.1.8}$$

  Consecutive even integers are even integers that immediately follow one another.

$$\begin{array}{ll} n & 1^{\text{st}} \text{ integer} \\ n+2 & 2^{\text{nd}} \text{ consecutive even integer} \\ n+4 & 3^{\text{rd}} \text{ consecutive even integer } \ldots \text{ etc.} \end{array} \tag{3.1.9}$$

  Consecutive odd integers are odd integers that immediately follow one another.

$$\begin{array}{ll} n & 1^{\text{st}} \text{ integer} \\ n+2 & 2^{\text{nd}} \text{ consecutive odd integer} \\ n+4 & 3^{\text{rd}} \text{ consecutive odd integer } \ldots \text{ etc.} \end{array} \tag{3.1.10}$$

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3.1E: Exercises

Practice Makes Perfect

Use the Approach Word Problems with a Positive Attitude

In the following exercises, prepare the lists described.

Exercise 3.1E.1

List five positive thoughts you can say to yourself that will help you approach word problems with a positive attitude. You may want to copy them on a sheet of paper and put it in the front of your notebook, where you can read them often.

Answer

Answers will vary

Exercise 3.1E.2

List five negative thoughts that you have said to yourself in the past that will hinder your progress on word problems. You may want to write each one on a small piece of paper and rip it up to symbolically destroy the negative thoughts.

Use a Problem-Solving Strategy for Word Problems

In the following exercises, solve using the problem solving strategy for word problems. Remember to write a complete sentence to answer each question.

Exercise 3.1E.3

Two-thirds of the children in the fourth-grade class are girls. If there are 20 girls, what is the total number of children in the class?

Answer

30

Exercise 3.1E.4

Three-fifths of the members of the school choir are women. If there are 24 women, what is the total number of choir members?

Exercise 3.1E.5

Zachary has 25 country music CDs, which is one-fifth of his CD collection. How many CDs does Zachary have?

Answer

125

Exercise 3.1E.6

One-fourth of the candies in a bag of M&M's are red. If there are 23 red candies, how many candies are in the bag?

Exercise 3.1E.7

There are 16 girls in a school club. The number of girls is four more than twice the number of boys. Find the number of boys.

Answer

6

Exercise 3.1E.8

There are 18 Cub Scouts in Pack 645. The number of scouts is three more than five times the number of adult leaders. Find the number of adult leaders.

Exercise 3.1E.9

Huong is organizing paperback and hardback books for her club's used book sale. The number of paperbacks is 12 less than three times the number of hardbacks. Huong had 162 paperbacks. How many hardback books were there?

Answer

58

Exercise 3.1E.10

Jeff is lining up children's and adult bicycles at the bike shop where he works. The number of children's bicycles is nine less than three times the number of adult bicycles. There are 42 adult bicycles. How many children's bicycles are there?

Exercise 3.1E.11

Philip pays $1,620 in rent every month. This amount is $120 more than twice what his brother Paul pays for rent. How much does Paul pay for rent?

Answer

$750

Exercise 3.1E.12

Marc just bought an SUV for $54,000. This is $7,400 less than twice what his wife paid for her car last year. How much did his wife pay for her car?

Exercise 3.1E.13

Laurie has $46,000 invested in stocks and bonds. The amount invested in stocks is $8,000 less than four times the amount invested in bonds. How much does Laurie have invested in bonds?

Answer

$13,500

Exercise 3.1E.14

Erica earned a total of $50,450 last year from her two jobs. The amount she earned from her job at the store was $1,250 more than three times the amount she earned from her job at the college. How much did she earn from her job at the college?

Solve Number Problems

In the following exercises, solve each number word problem.

Exercise 3.1E.15

The sum of a number and eight is 12. Find the number.

Answer

4

Exercise 3.1E.16

The sum of a number and nine is 17. Find the number.

Exercise 3.1E.17

The difference of a number and 12 is three. Find the number.

Answer

15

Exercise 3.1E.18

The difference of a number and eight is four. Find the number.

Exercise 3.1E.19

The sum of three times a number and eight is 23. Find the number.

Answer

5

Exercise 3.1E.20

The sum of twice a number and six is 14. Find the number.

Exercise 3.1E.21

The difference of twice a number and seven is 17. Find the number.

Answer

12

Exercise 3.1E.22

The difference of four times a number and seven is 21. Find the number.

Exercise 3.1E.23

Three times the sum of a number and nine is 12. Find the number.

Answer

-5

Exercise 3.1E.24

Six times the sum of a number and eight is 30. Find the number.

Exercise 3.1E.25

One number is six more than the other. Their sum is 42. Find the numbers.

Answer

18, 24

Exercise 3.1E.26

One number is five more than the other. Their sum is 33. Find the numbers.

Exercise 3.1E.27

The sum of two numbers is 20. One number is four less than the other. Find the numbers.

Answer

8, 12

Exercise 3.1E.28

The sum of two numbers is 27. One number is seven less than the other. Find the numbers.

Exercise 3.1E.29

The sum of two numbers is −45. One number is nine more than the other. Find the numbers.

Answer

−18,−27

Exercise 3.1E.30

The sum of two numbers is −61. One number is 35 more than the other. Find the numbers.

Exercise 3.1E.31

The sum of two numbers is −316. One number is 94 less than the other. Find the numbers.

Answer

−111,−205

Exercise 3.1E.32

The sum of two numbers is −284. One number is 62 less than the other. Find the numbers.

Exercise 3.1E.33

One number is 14 less than another. If their sum is increased by seven, the result is 85. Find the numbers.

Answer

32, 46

Exercise 3.1E.34

One number is 11 less than another. If their sum is increased by eight, the result is 71. Find the numbers.

Exercise 3.1E.35

One number is five more than another. If their sum is increased by nine, the result is 60. Find the numbers.

Answer

23, 28

Exercise 3.1E.36

One number is eight more than another. If their sum is increased by 17, the result is 95. Find the numbers.

Exercise 3.1E.37

One number is one more than twice another. Their sum is −5. Find the numbers.

Answer

−2,−3

Exercise 3.1E.38

One number is six more than five times another. Their sum is six. Find the numbers.

Exercise 3.1E.39

The sum of two numbers is 14. One number is two less than three times the other. Find the numbers.

Answer

4, 10

Exercise 3.1E.40

The sum of two numbers is zero. One number is nine less than twice the other. Find the numbers.

Exercise 3.1E.41

The sum of two consecutive integers is 77. Find the integers.

Answer

38, 39

Exercise 3.1E.42

The sum of two consecutive integers is 89. Find the integers.

Exercise 3.1E.43

The sum of two consecutive integers is −23. Find the integers.

Answer

−11,−12

Exercise 3.1E.44

The sum of two consecutive integers is −37. Find the integers.

Exercise 3.1E.45

The sum of three consecutive integers is 78. Find the integers.

Answer

25, 26, 27

Exercise 3.1E.46

The sum of three consecutive integers is 60. Find the integers.

Exercise 3.1E.47

Find three consecutive integers whose sum is −36.

Answer

−11,−12,−13

Exercise 3.1E.48

Find three consecutive integers whose sum is −3.

Exercise 3.1E.49

Find three consecutive even integers whose sum is 258.

Answer

84, 86, 88

Exercise 3.1E.50

Find three consecutive even integers whose sum is 222.

Exercise 3.1E.51

Find three consecutive odd integers whose sum is 171.

Answer

55, 57, 59

Exercise 3.1E.52

Find three consecutive odd integers whose sum is 291.

Exercise 3.1E.53

Find three consecutive even integers whose sum is −36.

Answer

−10,−12,−14

Exercise 3.1E.54

Find three consecutive even integers whose sum is −84.

Exercise 3.1E.55

Find three consecutive odd integers whose sum is −213.

Answer

−69,−71,−73

Exercise 3.1E.56

Find three consecutive odd integers whose sum is −267.

Everyday Math

Exercise 3.1E.57

Sale Price Patty paid $35 for a purse on sale for $10 off the original price. What was the original price of the purse?

Answer

$45

Exercise 3.1E.58

Sale Price Travis bought a pair of boots on sale for $25 off the original price. He paid $60 for the boots. What was the original price of the boots?

Exercise 3.1E.59

Buying in Bulk Minh spent $6.25 on five sticker books to give his nephews. Find the cost of each sticker book.

Answer

$1.25

Exercise 3.1E.60

Buying in Bulk Alicia bought a package of eight peaches for $3.20. Find the cost of each peach.

Exercise 3.1E.61

Price before Sales Tax Tom paid $1,166.40 for a new refrigerator, including $86.40 tax. What was the price of the refrigerator?

Answer

$1080

Exercise 3.1E.62

Price before Sales Tax Kenji paid $2,279 for a new living room set, including $129 tax. What was the price of the living room set?

Writing Exercises

Exercise 3.1E.63

What has been your past experience solving word problems?

Answer

answers will vary

Exercise 3.1E.64

When you start to solve a word problem, how do you decide what to let the variable represent?

Exercise 3.1E.65

What are consecutive odd integers? Name three consecutive odd integers between 50 and 60.

Answer

Consecutive odd integers are odd numbers that immediately follow each other. An example of three consecutive odd integers between 50 and 60 would be 51, 53, and 55.

Exercise 3.1E.66

What are consecutive even integers? Name three consecutive even integers between −50 and −40.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
approach word problems with a positive attitude.			
use a problem solving strategy for word problems.			
solve number problems.			

ⓑ If most of your checks were:

…confidently. Congratulations! You have achieved your goals in this section! Reflect on the study skills you used so that you can continue to use them. What did you do to become confident of your ability to do these things? Be specific!

…with some help. This must be addressed quickly as topics you do not master become potholes in your road to success. Math is sequential—every topic builds upon previous work. It is important to make sure you have a strong foundation before you move on. Who can you ask for help? Your fellow classmates and instructor are good resources. Is there a place on campus where math tutors are available? Can your study skills be improved?

…no—I don't get it! This is critical and you must not ignore it. You need to get help immediately or you will quickly be overwhelmed. See your instructor as soon as possible to discuss your situation. Together you can come up with a plan to get you the help you need.

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3.2: Solve Percent Applications

Learning Objectives

By the end of this section, you will be able to:

- Translate and solve basic percent equations
- Solve percent applications
- Find percent increase and percent decrease
- Solve simple interest applications
- Solve applications with discount or mark-up

Be prepared

Before you get started, take this readiness quiz.

1. Convert 4.5% to a decimal.
   If you missed this problem, review [link].
2. Convert 0.6 to a percent.
   If you missed this problem, review [link].
3. Round 0.875 to the nearest hundredth.
   If you missed this problem, review [link].
4. Multiply (4.5)(2.38).
   If you missed this problem, review [link].
5. Solve 3.5=0.7n.
   If you missed this problem, review [link].
6. Subtract 50−37.45.
   If you missed this problem, review [link].

Translate and Solve Basic Percent Equations

We will solve percent equations using the methods we used to solve equations with fractions or decimals. Without the tools of algebra, the best method available to solve percent problems was by setting them up as proportions. Now as an algebra student, you can just translate English sentences into algebraic equations and then solve the equations.

We can use any letter you like as a variable, but it is a good idea to choose a letter that will remind us of what you are looking for. We must be sure to change the given percent to a decimal when we put it in the equation.

Example 3.2.1

Translate and solve: What number is 35% of 90?

Solution

	What number is 35% of 90?
Translate into algebra. Let n= the number.	$n = 0.35 \cdot 90$
Remember "of" means multiply, "is" means equals.	
Multiply.	$n = 31.5$
	31.5 is 35% of 90

Try It 3.2.2

Translate and solve:

What number is 45% of 80?

Answer

36

Try It 3.2.3

Translate and solve:

What number is 55% of 60?

Answer

33

We must be very careful when we translate the words in the next example. The unknown quantity will not be isolated at first, like it was in Example. We will again use direct translation to write the equation.

Example 3.2.4

Translate and solve: 6.5% of what number is $1.17?

Solution

	6.5% of what number is $1.17?
Translate. Let n= the number.	$0.065 \cdot n = 1.17$
Multiply.	$0.065n = 1.17$
Divide both sides by 0.065 and simplify.	$n = 18$
	6.5% of $18 is $1.17

Try It 3.2.5

Translate and solve:

7.5% of what number is $1.95?

Answer

$26

Try It 3.2.6

Translate and solve:

8.5% of what number is $3.06?

Answer

$36

In the next example, we are looking for the percent.

Example 3.2.7

Translate and solve: 144 is what percent of 96?

Solution

	144 is what percent of 96?
Translate into algebra. Let p= the percent.	$144 = p \cdot 96$
Multiply.	$144 = 96p$
Divide by 96 and simplify.	$1.5 = p$
Convert to percent.	$150\% = p$
	144 is 150% of 96

Note that we are asked to find percent, so we must have our final result in percent form.

Try It 3.2.8

Translate and solve:

110 is what percent of 88?

Answer

125%

Try It 3.2.9

Translate and solve:

126 is what percent of 72?

Answer

175%

Solve Applications of Percent

Many applications of **percent**—such as tips, sales tax, discounts, and interest—occur in our daily lives. To solve these applications we'll translate to a basic percent equation, just like those we solved in previous examples. Once we translate the sentence into a percent equation, we know how to solve it.

We will restate the problem solving strategy we used earlier for easy reference.

USE A PROBLEM-SOLVING STRATEGY TO SOLVE AN APPLICATION.

1. **Read** the problem. Make sure all the words and ideas are understood.
2. **Identify** what we are looking for.
3. **Name** what we are looking for. Choose a variable to represent that quantity.
4. **Translate** into an equation. It may be helpful to restate the problem in one sentence with all the important information. Then, translate the English sentence into an algebraic equation.
5. **Solve** the equation using good algebra techniques.
6. **Check** the answer in the problem and make sure it makes sense.
7. **Answer** the question with a complete sentence.

Now that we have the strategy to refer to, and have practiced solving basic percent equations, we are ready to solve percent applications. Be sure to ask yourself if your final answer makes sense—since many of the applications will involve everyday situations, you can rely on your own experience.

Example 3.2.10

Dezohn and his girlfriend enjoyed a nice dinner at a restaurant and his bill was $68.50. He wants to leave an 18% tip. If the tip will be 18% of the total bill, how much tip should he leave?

Solution

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	the amount of tip should Dezohn leave
Step 3. Name what we are looking for.	
Choose a variable to represent it.	Let t = amount of tip.
Step 4. Translate into an equation.	The tip is 18% of the total bill.
Write a sentence that gives the information to find it.	The tip is 18% of $68.50
Translate the sentence into an equation.	$t = 0.18 \cdot 68.50$
Step 5. Solve the equation. Multiply.	$t = 12.33$
Step 6. Check. Does this make sense?	
Yes, 20% is close to 18%, and 20% of $70 is $14.	
Step 7. Answer the question with a complete sentence.	Dezohn should leave a tip of $12.33.

Notice that we used t to represent the unknown tip.

Try It 3.2.11

Cierra and her sister enjoyed a dinner in a restaurant and the bill was $81.50. If she wants to leave 18% of the total bill as her tip, how much should she leave?

Answer

$14.67

Try It 3.2.12

Kimngoc had lunch at her favorite restaurant. She wants to leave 15% of the total bill as her tip. If her bill was $14.40, how much will she leave for the tip?

Answer

$2.16

Example 3.2.13

The label on Masao's breakfast cereal said that one serving of cereal provides 85 milligrams (mg) of potassium, which is 2% of the recommended daily amount. What is the total recommended daily amount of potassium?

Solution

Step 1. Read the problem.	

Step 2. Identify what we are looking for.	the total amount of potassium that is recommended
Step 3. Name what we are looking for.	
Choose a variable to represent it.	Let a= total amount of potassium.
Step 4. Translate. Write a sentence that gives the information to find it.	85 mg is 2% of the total amount
Translate into an equation.	$85 = 0.02 \cdot a$
Step 5. Solve the equation.	$4{,}250 = a$
Step 6. Check. Does this make sense?	
Yes, 2% is a small percent and 85 is a small part of 4,250.	
Step 7. Answer the question with a complete sentence.	The amount of potassium that is recommended is 4,250 mg.

Try It 3.2.14

One serving of wheat square cereal has seven grams of fiber, which is 28% of the recommended daily amount. What is the total recommended daily amount of fiber?

Answer

25 grams

Try It 3.2.15

One serving of rice cereal has 190 mg of sodium, which is 8% of the recommended daily amount. What is the total recommended daily amount of sodium?

Answer

2,375 mg

Example 3.2.16

Mitzi received some gourmet brownies as a gift. The wrapper said each brownie was 480 calories, and had 240 calories of fat. What percent of the total calories in each brownie comes from fat?

Solution

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	the percent of the total calories from fat
Step 3. Name what we are looking for.	
Choose a variable to represent it.	Let p= percent of fat.
Step 4. Translate. Write a sentence that gives the information to find it.	What percent of 480 is 240?
Translate into an equation.	$p \cdot 480 = 240$
Step 5. Solve the equation.	$480\,p = 240$
Divide by 480.	$p = 0.5$
Put in a percent form.	$p = 50\%$

Step 6. Check. Does this make sense?	
Yes, 240 is half of 480, so 50% makes sense.	
Step 7. Answer the question with a complete sentence.	Of the total calories in each brownie, 50% is fat.

Try It 3.2.17

Solve. Round to the nearest whole percent.

Veronica is planning to make muffins from a mix. The package says each muffin will be 230 calories and 60 calories will be from fat. What percent of the total calories is from fat?

Answer

26%

Try It 3.2.18

Solve. Round to the nearest whole percent.

The mix Ricardo plans to use to make brownies says that each brownie will be 190 calories, and 76 calories are from fat. What percent of the total calories are from fat?

Answer

40%

Find Percent Increase and Percent Decrease

People in the media often talk about how much an amount has increased or decreased over a certain period of time. They usually express this increase or decrease as a percent.

To find the **percent increase**, first we find the amount of increase, the difference of the new amount and the original amount. Then we find what percent the amount of increase is of the original amount.

FIND THE PERCENT INCREASE.

1. Find the amount of increase.

   $$\text{new amount} - \text{original amount} = \text{increase}$$
2. Find the percent increase.

   The increase is what percent of the original amount?

Example 3.2.19

In 2011, the California governor proposed raising community college fees from $26 a unit to $36 a unit. Find the percent increase. (Round to the nearest tenth of a percent.)

Solution

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	the percent increase
Step 3. Name what we are looking for.	
Choose a variable to represent it.	Let p = the percent.

Step 4. Translate. Write a sentence that gives the information to find it.	
First find the amount of increase.	new amount − original amount = increase
	$36-26=10$
Find the percent.	Increase is what percent of the original amount?
	10 is what percent of 26?
Translate into an equation.	$10 = p \cdot 26$
Step 5. Solve the equation.	$10 = 26p$
Divide by 26.	$0.3846\ldots = p$
Change to percent form; round to the nearest tenth.	$38.5\% = p$
Step 6. Check. Does this make sense?	
Yes, 38.5% is close to $\frac{1}{3}$, and 10 is close to $\frac{1}{3}$ of 26.	
Step 7. Answer the question with a complete sentence.	The new fees represent a 38.5% increase over the old fees.

Try It 3.2.20

Find the percent increase. (Round to the nearest tenth of a percent.)

In 2011, the IRS increased the deductible mileage cost to 55.5 cents from 51 cents.

Answer

8.8%

Try It 3.2.21

Find the percent increase.

In 1995, the standard bus fare in Chicago was $1.50. In 2008, the standard bus fare was $2.25.

Answer

50%

Finding the **percent decrease** is very similar to finding the percent increase, but now the amount of decrease is the difference of the original amount and the new amount. Then we find what percent the amount of decrease is of the original amount.

FIND THE PERCENT DECREASE.

1. Find the amount of decrease.

   $$\text{original amount} - \text{new amount} = \text{decrease}$$
2. Find the percent decrease.

   The decrease is what percent of the original amount?

Example 3.2.22

The average price of a gallon of gas in one city in June 2014 was $3.71. The average price in that city in July was $3.64. Find the percent decrease.

Solution

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	the percent decrease
Step 3. Name what we are looking for.	
Choose a variable to represent that quantity.	Let p= the percent decrease.
Step 4. Translate. Write a sentence that gives the information to find it.	
First find the amount of decrease.	3.71−3.64=0.07
Find the percent.	Decrease is what percent of the original amount?
	0.07 is what percent of 3.71?
Translate into an equation.	$0.07 = p \cdot 3.71$
Step 5. Solve the equation.	$0.07 = 3.71\,p$
Divide by 3.71.	$0.0188\ldots = p$
Change to percent form; round to the nearest tenth.	$1.9\% = p$
Step 6. Check. Does this make sense?	
Yes, if the original price was $4, a 2% decrease would be 8 cents.	
Step 7. Answer the question with a complete sentence.	The price of gas decreased 1.9%.

Try It 3.2.23

Find the percent decrease. (Round to the nearest tenth of a percent.)

The population of North Dakota was about 672,000 in 2010. The population is projected to be about 630,000 in 2020.

Answer

6.3%

Try It 3.2.24

Find the percent decrease.

Last year, Sheila's salary was $42,000. Because of furlough days, this year, her salary was $37,800.

Answer

10%

Solve Simple Interest Applications

Do you know that banks pay you to keep your money? The money a customer puts in the bank is called the **principal**, P, and the money the bank pays the customer is called the **interest**. The interest is computed as a certain percent of the principal; called the

rate of interest, *r*. We usually express rate of interest as a percent per year, and we calculate it by using the decimal equivalent of the percent. The variable *t*, (for *time*) represents the number of years the money is in the account.

To find the interest we use the simple interest formula, I=Prt.

SIMPLE INTEREST

If an amount of money, *P*, called the principal, is invested for a period of *t* years at an annual interest rate *r*, the amount of interest, *I*, earned is

$$I = Prt \quad \text{where} \quad \begin{aligned} I &= \text{interest} \\ P &= \text{principle} \\ r &= \text{rate} \\ t &= \text{time} \end{aligned} \tag{3.2.1}$$

Interest earned according to this formula is called **simple interest**.

Interest may also be calculated another way, called compound interest. This type of interest will be covered in later math classes.

The formula we use to calculate simple interest is I=Prt. To use the formula, we substitute in the values the problem gives us for the variables, and then solve for the unknown variable. It may be helpful to organize the information in a chart.

Example 3.2.25

Nathaly deposited $12,500 in her bank account where it will earn 4% interest. How much interest will Nathaly earn in 5 years?

$$\begin{aligned} I &= ? \\ P &= \$12,500 \\ r &= 4\% \\ t &= 5 \text{ years} \end{aligned}$$

Solution

Step 1. Read the problem.

Step 2. Identify what we are looking for. — the amount of interest earned

Step 3. Name what we are looking for. Choose a variable to represent that quantity — Let I = the amount of interest.

Step 4. Translate into an equation.

Write the formula. — $I = Prt$

Substitute in the given information. — $I = (12500)(.04)(5)$

Step 5. Solve the equation. — $I = 2500$

Step 6. Check: Does this make sense?

Is $2,500 is a reasonable interest on $12,500? Yes.

Step 7. Answer the question with a complete sentence. — The interest is $2500

Try It 3.2.26

Areli invested a principal of $950 in her bank account with interest rate 3%. How much interest did she earn in 5 years?

Answer

$142.50

Try It 3.2.27

Susana invested a principal of $36,000 in her bank account with interest rate 6.5%. How much interest did she earn in 3 years?

Answer

$7020

There may be times when we know the amount of interest earned on a given principal over a certain length of time, but we don't know the rate. To find the rate, we use the simple interest formula, substitute in the given values for the principal and time, and then solve for the rate.

Example 3.2.28

Loren loaned his brother $3,000 to help him buy a car. In 4 years his brother paid him back the $3,000 plus $660 in interest. What was the rate of interest?

$$\begin{aligned} I &= \$660 \\ P &= \$3000 \\ r &= ? \\ t &= 4 \text{ years} \end{aligned} \tag{3.2.2}$$

Solution

Step 1. Read the problem.

Step 2. Identify what we are looking for.	the rate of interest
Step 3. Name what we are looking for. Choose a variable to represent that quantity	Let r = the rate of interest.
Step 4. Translate into an equation.	
Write the formula.	$I = Prt$
Substitute in the given information.	$660 = (3000)r(4)$
Step 5. Solve the equation.	
	$660 = (12000)r$
Divide.	$0.055 = r$
Change to percent form.	$5.5\% = r$

Step 6. Check: Does this make sense?

$I = Prt$

$660 \stackrel{?}{=} (3000)(0.055)(4)$

$660 = 660$✓

Step 7. Answer the question with a complete sentence.	The rate of interest was 5.5%

Notice that in this example, Loren's brother paid Loren interest, just like a bank would have paid interest if Loren invested his money there.

? Try It 3.2.29

Jim loaned his sister $5,000 to help her buy a house. In 3 years, she paid him the $5,000, plus $900 interest. What was the rate of interest?

Answer

6%

? Try It 3.2.30

Hang borrowed $7,500 from her parents to pay her tuition. In 5 years, she paid them $1,500 interest in addition to the $7,500 she borrowed. What was the rate of interest?

Answer

4%

✓ Example 3.2.31

Eduardo noticed that his new car loan papers stated that with a 7.5% interest rate, he would pay $6,596.25 in interest over 5 years. How much did he borrow to pay for his car?

Solution

Step 1. Read the problem.

Step 2. Identify what we are looking for. Choose a variable to represent that quantity.	the amount borrowed (the principal)
Step 3. Name what we are looking for. Choose a variable to represent that quantity	Let P = principal borrowed.
Step 4. Translate into an equation.	
Write the formula.	$I = Prt$
Substitute in the given information.	$6596.25 = P(0.075)(5)$
Step 5. Solve the equation.	
	$6596.25 = 0.375P$
Divide.	$17590 = P$

Step 6. Check: Does this make sense?

$I = Prt$

$6596.25 \stackrel{?}{=} (17590)(0.075)(5)$

$6596.25 = 6596.25$✓

Step 7. Answer the question with a complete sentence. The principal is $17590

? Try It 3.2.32

Sean's new car loan statement said he would pay $4,866.25 in interest from an interest rate of 8.5% over 5 years. How much did he borrow to buy his new car?

Answer

$11,450

? Try It 3.2.33

In 5 years, Gloria's bank account earned $2,400 interest at 5%. How much had she deposited in the account?

Answer

$9,600

Solve Applications with Discount or Mark-up

Applications of discount are very common in retail settings. When you buy an item on sale, the original price has been discounted by some dollar amount. The **discount rate**, usually given as a percent, is used to determine the amount of the discount. To determine the **amount of discount**, we multiply the discount rate by the original price.

We summarize the discount model in the box below.

DISCOUNT

$$\begin{aligned} \text{amount of discount} &= \text{discount rate} \times \text{original price} \\ \text{sale price} &= \text{original price - amount of discount} \end{aligned} \tag{3.2.3}$$

Keep in mind that the sale price should always be less than the original price.

Example 3.2.34

Elise bought a dress that was discounted 35% off of the original price of $140. What was (a) the amount of discount and (b) the sale price of the dress?

Solution

(a) Original price = $140
Discount rate = 35%
Discount = ?

Step 1. Read the problem.

Step 2. Identify what we are looking for. — the amount of discount

Step 3. Name what we are looking for.
Choose a variable to represent that quantity. — Let d = the amount of discount.

Step 4. Translate into an equation. Write a sentence that gives the information to find it.
Translate into an equation — $d = 0.35(140)$

Step 5. Solve the equation. — $d = 49$

Step 6. Check: Does this make sense?

Is a $49 discount reasonable for a $140 dress?
Yes.

Step 7. Write a complete sentence to answer the question. — The amount of discount was $49

(b) Read the problem again.

Step 1. Identify what we are looking for.	the sale price of the dress
Step 2. Name what we are looking for.	
Choose a variable to represent that quantity.	Let s= the sale price.
Step 3. Translate into an equation.	
Write a sentence that gives the information to find it.	The sale price is the $140 minus the $49 discount
Translate into an equation.	$s = 140 - 49$
Step 4. Solve the equation.	$s = 91$
Step 5. Check. Does this make sense?	
Is the sale price less than the original price?	

Yes, $91 is less than $140.	
Step 6. Answer the question with a complete sentence.	The sale price of the dress was $91.

Try It 3.2.35

Find (a) the amount of discount and (b) the sale price:

Sergio bought a belt that was discounted 40% from an original price of $29.

Answer

(a) $11.60, (b) $17.40

Try It 3.2.36

Find (a) the amount of discount and (b) the sale price:

Oscar bought a barbecue that was discounted 65% from an original price of $395.

Answer

(a) $256.75, (b) $138.25

There may be times when we know the original price and the sale price, and we want to know the **discount rate**. To find the discount rate, first we will find the amount of discount and then use it to compute the rate as a percent of the original price. Example 3.2.37 will show this case.

Example 3.2.37

Jeannette bought a swimsuit at a sale price of $13.95. The original price of the swimsuit was $31. Find the (a) amount of discount and (b) discount rate.

Solution

(a) Original price = $31
Discount rate = ?
Discount = $13.95

Step 1. Read the problem.

Step 2. Identify what we are looking for. — the amount of discount

Step 3. Name what we are looking for.
Choose a variable to represent that quantity. — Let d = the amount of discount.

Step 4. Translate into an equation. Write a sentence that gives the information to find it. — The discount is the difference between the original price and the sale price.

Translate into an equation — $d = 31 - 13.95$

Step 5. Solve the equation. — $d = 17.05$

Step 6. Check: Does this make sense?

Is 17.05 less than 31? Yes.

Step 7. Answer the question with a complete sentence. — The amount of discount was $17.05

(b) Read the problem again.

Step 1. Identify what we are looking for.	the discount rate
Step 2. Name what we are looking for.	
Choose a variable to represent it.	Let r = the discount rate.
Step 3. Translate into an equation.	
Write a sentence that gives the information to find it.	The discount of $17.05 is what percent of $31 ?
Translate into an equation.	$17.05 = r \cdot 31$
Step 4. Solve the equation.	$17.05 = 31r$
Divide both sides by 31.	$0.55 = r$
Change to percent form.	$r = 55\%$
Step 5. Check. Does this make sense?	
Is $17.05 equal to 55% of $31?	
$17.05 \stackrel{?}{=} 0.55(31)$	
$17.05 = 17.05$✓	
Step 6. Answer the question with a complete sentence.	The rate of discount was 55%.

Try It 3.2.38

Find

a. the amount of discount and
b. the discount rate.

Lena bought a kitchen table at the sale price of $375.20. The original price of the table was $560.

Answer

a. $184.80
b. 33%

Try It 3.2.39

Find

a. the amount of discount and
b. the discount rate.

Nick bought a multi-room air conditioner at a sale price of $340. The original price of the air conditioner was $400.

Answer

a. $60
b. 15%

Applications of mark-up are very common in retail settings. The price a retailer pays for an item is called the **original cost**. The retailer then adds a **mark-up** to the original cost to get the **list price**, the price he sells the item for. The mark-up is usually calculated as a percent of the original cost. To determine the amount of mark-up, multiply the mark-up rate by the original cost.

We summarize the mark-up model in the box below.

MARK-UP

$$\begin{aligned} \text{amount of mark-up} &= \text{mark-up rate} \times \text{original cost} \\ \text{list price} &= \text{original cost} + \text{amount of mark up} \end{aligned} \tag{3.2.4}$$

Keep in mind that the list price should always be more than the original cost.

Example 3.2.40

Adam's art gallery bought a photograph at original cost $250. Adam marked the price up 40%. Find the

a. amount of mark-up and
b. the list price of the photograph.

Solution

a.

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	the amount of mark-up
Step 3. Name what we are looking for.	
Choose a variable to represent it.	Let m = the amount of markup.
Step 4. Translate into an equation.	
Write a sentence that gives the information to find it.	The mark-up is 40% of the $250 original cost

Translate into an equation.	$m = 0.40 \cdot 250$
Step 5. Solve the equation.	$m = 100$
Step 6. Check. Does this make sense?	
Yes, 40% is less than one-half and 100 is less than half of 250.	
Step 7. Answer the question with a complete sentence.	The mark-up on the photograph was $100.

b.

Step 1. Read the problem again.	
Step 2. Identify what we are looking for.	the list price
Step 3. Name what we are looking for.	
Choose a variable to represent it.	Let p= the list price.
Step 4. Translate into an equation.	
Write a sentence that gives the information to find it.	The list price is original cost plus the mark-up
Translate into an equation.	$p = 250 + 100$
Step 5. Solve the equation.	$p = 350$
Step 6. Check. Does this make sense?	
Is the list price more than the net price? Is $350 more than $250? Yes.	
Step 7. Answer the question with a complete sentence.	The list price of the photograph was $350.

Try It 3.2.41

Find

a. the amount of mark-up and
b. the list price.

Jim's music store bought a guitar at original cost $1,200. Jim marked the price up 50%.

Answer

a. $600
b. $1,800

Try It 3.2.42

Find

a. the amount of mark-up and
b. the list price.

The Auto Resale Store bought Pablo's Toyota for $8,500. They marked the price up 35%.

Answer

a. $2,975
b. $11,475

Key Concepts

- **Percent Increase** To find the percent increase:
  1. Find the amount of increase. increase=new amount−originalamount
  2. Find the percent increase. Increase is what percent of the original amount?
- **Percent Decrease** To find the percent decrease:
  1. Find the amount of decrease. decrease=original amount−newamount
  2. Find the percent decrease. Decrease is what percent of the original amount?
- **Simple Interest** If an amount of money, *P*, called the principal, is invested for a period of *t* years at an annual interest rate *r*, the amount of interest, *I*, earned is

$$I = Prt \quad \text{where} \quad \begin{aligned} I &= \text{interest} \\ P &= \text{principle} \\ r &= \text{rate} \\ t &= \text{time} \end{aligned} \tag{3.2.5}$$

- **Discount**
  - amount of discount is discount rate × original price
  - sale price is original price – discount
- **Mark-up**
  - amount of mark-up is mark-up rate × original cost
  - list price is original cost + mark up

Glossary

amount of discount

The amount of discount is the amount resulting when a discount rate is multiplied by the original price of an item.

discount rate

The discount rate is the percent used to determine the amount of a discount, common in retail settings.

interest

Interest is the money that a bank pays its customers for keeping their money in the bank.

list price

The list price is the price a retailer sells an item for.

mark-up

A mark-up is a percentage of the original cost used to increase the price of an item.

original cost

The original cost in a retail setting, is the price that a retailer pays for an item.

principal

The principal is the original amount of money invested or borrowed for a period of time at a specific interest rate.

rate of interest

The rate of interest is a percent of the principal, usually expressed as a percent per year.

simple interest

Simple interest is the interest earned according to the formula I=Prt.

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3.2E: Exercises

Practice Makes Perfect

Translate and Solve Basic Percent Equations

In the following exercises, translate and solve.

Exercise 3.2E.1

What number is 45% of 120?

Answer

54

Exercise 3.2E.2

What number is 65% of 100?

Exercise 3.2E.3

What number is 24% of 112?

Answer

26.88

Exercise 3.2E.4

What number is 36% of 124?

Exercise 3.2E.5

250% of 65 is what number?

Answer

162.5

Exercise 3.2E.6

150% of 90 is what number?

Exercise 3.2E.7

800% of 2250 is what number?

Answer

18,000

Exercise 3.2E.8

600% of 1740 is what number?

Exercise 3.2E.9

28 is 25% of what number?

Answer

112

Exercise 3.2E.10

36 is 25% of what number?

Exercise 3.2E.11

81 is 75% of what number?

Answer

108

Exercise 3.2E.12

93 is 75% of what number?

Exercise 3.2E.13

8.2% of what number is $2.87?

Answer

$35

Exercise 3.2E.14

6.4% of what number is $2.88?

Exercise 3.2E.15

11.5% of what number is $108.10?

Answer

$940

Exercise 3.2E.16

12.3% of what number is $92.25?

Exercise 3.2E.17

What percent of 260 is 78?

Answer

30%

Exercise 3.2E.18

What percent of 215 is 86?

Exercise 3.2E.19

What percent of 1500 is 540?

Answer

36%

Exercise 3.2E.20

What percent of 1800 is 846?

Exercise 3.2E.21

30 is what percent of 20?

Answer

150%

Exercise 3.2E.22

50 is what percent of 40?

Exercise 3.2E.23

840 is what percent of 480?

Answer

175%

Exercise 3.2E.24

790 is what percent of 395?

Solve Percent Applications

In the following exercises, solve.

Exercise 3.2E.25

Geneva treated her parents to dinner at their favorite restaurant. The bill was $74.25. Geneva wants to leave 16% of the total bill as a tip. How much should the tip be?

Answer

$11.88

Exercise 3.2E.26

When Hiro and his co-workers had lunch at a restaurant near their work, the bill was $90.50. They want to leave 18% of the total bill as a tip. How much should the tip be?

Exercise 3.2E.27

Trong has 12% of each paycheck automatically deposited to his savings account. His last paycheck was $2165. How much money was deposited to Trong's savings account?

Answer

$259.80

Exercise 3.2E.28

Cherise deposits 8% of each paycheck into her retirement account. Her last paycheck was $1,485. How much did Cherise deposit into her retirement account?

Exercise 3.2E.29

One serving of oatmeal has eight grams of fiber, which is 33% of the recommended daily amount. What is the total recommended daily amount of fiber?

Answer

24.2 g

Exercise 3.2E.30

One serving of trail mix has 67 grams of carbohydrates, which is 22% of the recommended daily amount. What is the total recommended daily amount of carbohydrates?

Exercise 3.2E.31

A bacon cheeseburger at a popular fast food restaurant contains 2070 milligrams (mg) of sodium, which is 86% of the recommended daily amount. What is the total recommended daily amount of sodium?

Answer

2407 mg

Exercise 3.2E.32

A grilled chicken salad at a popular fast food restaurant contains 650 milligrams (mg) of sodium, which is 27% of the recommended daily amount. What is the total recommended daily amount of sodium?

Exercise 3.2E.33

After 3 months on a diet, Lisa had lost 12% of her original weight. She lost 21 pounds. What was Lisa's original weight?

Answer

175 lb.

Exercise 3.2E.34

Tricia got a 6% raise on her weekly salary. The raise was $30 per week. What was her original salary?

Exercise 3.2E.35

Yuki bought a dress on sale for $72. The sale price was 60% of the original price. What was the original price of the dress?

Answer

$120

Exercise 3.2E.36

Kim bought a pair of shoes on sale for $40.50. The sale price was 45% of the original price. What was the original price of the shoes?

Exercise 3.2E.37

Tim left a $9 tip for a $50 restaurant bill. What percent tip did he leave?

Answer

18%

Exercise 3.2E.38

Rashid left a $15 tip for a $75 restaurant bill. What percent tip did he leave?

Exercise 3.2E.39

The nutrition fact sheet at a fast food restaurant says the fish sandwich has 380 calories, and 171 calories are from fat. What percent of the total calories is from fat?

Answer

45%

Exercise 3.2E.40

The nutrition fact sheet at a fast food restaurant says a small portion of chicken nuggets has 190 calories, and 114 calories are from fat. What percent of the total calories is from fat?

Exercise 3.2E.41

Emma gets paid $3,000 per month. She pays $750 a month for rent. What percent of her monthly pay goes to rent?

Answer

25%

Exercise 3.2E.42

Dimple gets paid $3,200 per month. She pays $960 a month for rent. What percent of her monthly pay goes to rent?

Find Percent Increase and Percent Decrease

In the following exercises, solve.

Exercise 3.2E.43

Tamanika got a raise in her hourly pay, from $15.50 to $17.36. Find the percent increase.

Answer

12%

Exercise 3.2E.44

Ayodele got a raise in her hourly pay, from $24.50 to $25.48. Find the percent increase.

Exercise 3.2E.45

Annual student fees at the University of California rose from about $4,000 in 2000 to about $12,000 in 2010. Find the percent increase.

Answer

200%

Exercise 3.2E.46

The price of a share of one stock rose from $12.50 to $50. Find the percent increase.

Exercise 3.2E.47

According to *Time* magazine annual global seafood consumption rose from 22 pounds per person in the 1960s to 38 pounds per person in 2011. Find the percent increase. (Round to the nearest tenth of a percent.)

Answer

72.7%

Exercise 3.2E.48

In one month, the median home price in the Northeast rose from $225,400 to $241,500. Find the percent increase. (Round to the nearest tenth of a percent.)

Exercise 3.2E.49

A grocery store reduced the price of a loaf of bread from $2.80 to $2.73. Find the percent decrease.

Answer

2.5%

Exercise 3.2E.50

The price of a share of one stock fell from $8.75 to $8.54. Find the percent decrease.

Exercise 3.2E.51

Hernando's salary was $49,500 last year. This year his salary was cut to $44,055. Find the percent decrease.

Answer

11%

Exercise 3.2E.52

In 10 years, the population of Detroit fell from 950,000 to about 712,500. Find the percent decrease.

Exercise 3.2E.53

In 1 month, the median home price in the West fell from $203,400 to $192,300. Find the percent decrease. (Round to the nearest tenth of a percent.)

Answer

5.5%

Exercise 3.2E.54

Sales of video games and consoles fell from $1,150 million to $1,030 million in 1 year. Find the percent decrease. (Round to the nearest tenth of a percent.)

Solve Simple Interest Applications

In the following exercises, solve.

Exercise 3.2E.55

Casey deposited $1,450 in a bank account with interest rate 4%. How much interest was earned in two years?

Answer

$116

Exercise 3.2E.56

Terrence deposited $5,720 in a bank account with interest rate 6%. How much interest was earned in 4 years?

Exercise 3.2E.57

Robin deposited $31,000 in a bank account with interest rate 5.2%. How much interest was earned in 3 years?

Answer

$4,836

Exercise 3.2E.58

Carleen deposited $16,400 in a bank account with interest rate 3.9%. How much interest was earned in 8 years?

Exercise 3.2E.59

Hilaria borrowed $8,000 from her grandfather to pay for college. Five years later, she paid him back the $8,000, plus $1,200 interest. What was the rate of interest?

Answer

3%

Exercise 3.2E.60

Kenneth loaned his niece $1,200 to buy a computer. Two years later, she paid him back the $1,200, plus $96 interest. What was the rate of interest?

Exercise 3.2E.61

Lebron loaned his daughter $20,000 to help her buy a condominium. When she sold the condominium four years later, she paid him the $20,000, plus $3,000 interest. What was the rate of interest?

Answer

3.75%

Exercise 3.2E.62

Pablo borrowed $50,000 to start a business. Three years later, he repaid the $50,000, plus $9,375 interest. What was the rate of interest?

Exercise 3.2E.63

In 10 years, a bank account that paid 5.25% earned $18,375 interest. What was the principal of the account?

Answer

$35,000

Exercise 3.2E.64

In 25 years, a bond that paid 4.75% earned $2,375 interest. What was the principal of the bond?

Exercise 3.2E.65

Joshua's computer loan statement said he would pay $1,244.34 in interest for a 3-year loan at 12.4%. How much did Joshua borrow to buy the computer?

Answer

$3,345

Exercise 3.2E.66

Margaret's car loan statement said she would pay $7,683.20 in interest for a 5-year loan at 9.8%. How much did Margaret borrow to buy the car?

Solve Applications with Discount or Mark-up

In the following exercises, find the sale price.

Exercise 3.2E.67

Perla bought a cell phone that was on sale for $50 off. The original price of the cell phone was $189.

Answer

$139

Exercise 3.2E.68

Sophie saw a dress she liked on sale for $15 off. The original price of the dress was $96.

Exercise 3.2E.69

Rick wants to buy a tool set with original price $165. Next week the tool set will be on sale for $40 off.

Answer

$125

Exercise 3.2E.70

Angelo's store is having a sale on televisions. One television, with original price $859, is selling for $125 off.

In the following exercises, find

1. the amount of discount and
2. the sale price.

Exercise 3.2E.71

Janelle bought a beach chair on sale at 60% off. The original price was $44.95.

Answer

1. $26.97
2. $17.98

Exercise 3.2E.72

Errol bought a skateboard helmet on sale at 40% off. The original price was $49.95.

Exercise 3.2E.73

Kathy wants to buy a camera that lists for $389. The camera is on sale with a 33% discount.

Answer

1. $128.37
2. $260.63

Exercise 3.2E.74

Colleen bought a suit that was discounted 25% from an original price of $245.

Exercise 3.2E.75

Erys bought a treadmill on sale at 35% off. The original price was $949.95 (round to the nearest cent.)

Answer

1. $332.48
2. $617.47

Exercise 3.2E.76

Jay bought a guitar on sale at 45% off. The original price was $514.75 (round to the nearest cent.)

In the following exercises, find

1. the amount of discount and
2. the discount rate. (Round to the nearest tenth of a percent if needed.)

Exercise 3.2E.77

Larry and Donna bought a sofa at the sale price of $1,344. The original price of the sofa was $1,920.

Answer

1. $576
2. 30%

Exercise 3.2E.78

Hiroshi bought a lawnmower at the sale price of $240. The original price of the lawnmower is $300.

Exercise 3.2E.79

Patty bought a baby stroller on sale for $301.75. The original price of the stroller was $355.

Answer

1. $53.25
2. 15%

Exercise 3.2E.80

Bill found a book he wanted on sale for $20.80. The original price of the book was $32.

Exercise 3.2E.81

Nikki bought a patio set on sale for $480. The original price was $850. To the nearest tenth of a percent, what was the rate of discount?

Answer

1. $370
2. 43.5%

Exercise 3.2E.82

Stella bought a dinette set on sale for $725. The original price was $1,299. To the nearest tenth of a percent, what was the rate of discount?

In the following exercises, find ⓐ the amount of the mark-up and ⓑ the list price.

Exercise 3.2E.83

Daria bought a bracelet at original cost $16 to sell in her handicraft store. She marked the price up 45%.

Answer

1. $7.20
2. $23.20

Exercise 3.2E.84

Regina bought a handmade quilt at original cost $120 to sell in her quilt store. She marked the price up 55%.

Exercise 3.2E.85

Tom paid $0.60 a pound for tomatoes to sell at his produce store. He added a 33% mark-up.

Answer

1. $0.20
2. $0.80

Exercise 3.2E.86

Flora paid her supplier $0.74 a stem for roses to sell at her flower shop. She added an 85% mark-up.

Exercise 3.2E.87

Alan bought a used bicycle for $115. After re-conditioning it, he added 225% mark-up and then advertised it for sale.

Answer

1. $258.75
2. $373.75

Exercise 3.2E.88

Michael bought a classic car for $8,500. He restored it, then added 150% mark-up before advertising it for sale.

Everyday Math

Exercise 3.2E.89

Leaving a Tip At the campus coffee cart, a medium coffee costs $1.65. MaryAnne brings $2.00 with her when she buys a cup of coffee and leaves the change as a tip. What percent tip does she leave?

Answer

21.2%

Exercise 3.2E.90

Splitting a Bill Four friends went out to lunch and the bill came to $53.75. They decided to add enough tip to make a total of $64, so that they could easily split the bill evenly among themselves. What percent tip did they leave?

Writing Exercises

Exercise 3.2E.91

Without solving the problem "44 is 80% of what number" think about what the solution might be. Should it be a number that is greater than 44 or less than 44? Explain your reasoning.

Answer

The number should be greater than 44. Since 80% equals 0.8 in decimal form, 0.8 is less than one, and we must multiply the number by 0.8 to get 44, the number must be greater than 44.

Exercise 3.2E.92

Without solving the problem "What is 20% of 300?" think about what the solution might be. Should it be a number that is greater than 300 or less than 300? Explain your reasoning.

Exercise 3.2E.93

After returning from vacation, Alex said he should have packed 50% fewer shorts and 200% more shirts. Explain what Alex meant.

Answer

He meant that he should have packed half the shorts and twice the shirts.

Exercise 3.2E.94

Because of road construction in one city, commuters were advised to plan that their Monday morning commute would take 150% of their usual commuting time. Explain what this means.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
translate and solve basic percent equations.			
solve percent applications.			
find percent increase and percent decrease.			
solve simple interest applications.			
solve applications with discount or mark-up.			

ⓑ After reviewing this checklist, what will you do to become confident for all goals?

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3.3: Solve Mixture Applications

Learning Objectives

By the end of this section, you will be able to:

- Solve coin word problems
- Solve ticket and stamp word problems
- Solve mixture word problems
- Use the mixture model to solve investment problems using simple interest

Note

Before you get started, take this readiness quiz.

1. Multiply: $14(0.25)$.
   If you missed this problem, review Exercise 1.8.19.
2. Solve: $0.25x + 0.10(x + 4) = 2.5$.
   If you missed this problem, review Exercise 2.4.22.
3. The number of dimes is three more than the number of quarters. Let q represent the number of quarters. Write an expression for the number of dimes.
   If you missed this problem, review Exercise 1.3.43.

Solve Coin Word Problems

In **mixture problems**, we will have two or more items with different values to combine together. The mixture model is used by grocers and bartenders to make sure they set fair prices for the products they sell. Many other professionals, like chemists, investment bankers, and landscapers also use the mixture model.

Note

Doing the Manipulative Mathematics activity *Coin Lab* will help you develop a better understanding of mixture word problems.

We will start by looking at an application everyone is familiar with—money!

Imagine that we take a handful of coins from a pocket or purse and place them on a desk. How would we determine the value of that pile of coins? If we can form a step-by-step plan for finding the total value of the coins, it will help us as we begin solving coin word problems.

So what would we do? To get some order to the mess of coins, we could separate the coins into piles according to their value. Quarters would go with quarters, dimes with dimes, nickels with nickels, and so on. To get the total value of all the coins, we would add the total value of each pile.



Figure 3.3.1

How would we determine the value of each pile? Think about the dime pile—how much is it worth? If we count the number of dimes, we'll know how many we have—the *number* of dimes.

But this does not tell us the *value* of all the dimes. Say we counted 17 dimes, how much are they worth? Each dime is worth \$0.10—that is the *value* of one dime. To find the total value of the pile of 17 dimes, multiply 17 by \$0.10 to get \$1.70. This is the total

value of all 17 dimes. This method leads to the following model.

TOTAL VALUE OF COINS

For the same type of coin, the total value of a number of coins is found by using the model

$$number \cdot value = total\ value \tag{3.3.1}$$

where

number is the number of coins

value is the value of each coin

total value is the total value of all the coins

The number of dimes times the value of each dime equals the total value of the dimes.

$$\begin{aligned} \text{number.} \cdot \text{ value } &= \text{ total value} \\ 17 \cdot \$0.10 &= \$1.70 \end{aligned}$$

We could continue this process for each type of coin, and then we would know the total value of each type of coin. To get the total value of *all* the coins, add the total value of each type of coin.

Let's look at a specific case. Suppose there are 14 quarters, 17 dimes, 21 nickels, and 39 pennies.

Type	Number	• Value($)	= Total Value($)
Quarters	14	0.25	3.50
Dimes	17	0.10	1.70
Nickels	21	0.05	1.05
Pennies	39	0.01	0.39
			6.64

Table 3.3.1

The total value of all the coins is $6.64.

Notice how the chart helps organize all the information! Let's see how we use this method to solve a coin word problem.

Example 3.3.1

Adalberto has $2.25 in dimes and nickels in his pocket. He has nine more nickels than dimes. How many of each type of coin does he have?

Solution

Step 1. Read the problem. Make sure all the words and ideas are understood.

Determine the types of coins involved.
Think about the strategy we used to find the value of the handful of coins. The first thing we need is to notice what types of coins are involved. Adalberto has dimes and nickels. **Create a table** to organize the information. See chart below.

- Label the columns "type," "number," "value," "total value."
- List the types of coins.
- Write in the value of each type of coin.
- Write in the total value of all the coins.

We can work this problem all in cents or in dollars. Here we will do it in dollars and put in the dollar sign ($) in the table as a reminder.
The value of a dime is $0.10 and the value of a nickel is $0.05. The total value of all the coins is $2.25. The table below shows

this information.

Type	Number	•	Value($)	=	Total Value($)
Dimes			0.10		
Nickels			0.05		
					2.25

Step 2. Identify what we are looking for.

We are asked to find the number of dimes and nickels Adalberto has.

Step 3. Name what we are looking for. Choose a variable to represent that quantity.

Use variable expressions to represent the number of each type of coin and write them in the table.

Multiply the number times the value to get the total value of each type of coin.

Next we counted the number of each type of coin. In this problem we cannot count each type of coin—that is what you are looking for—but we have a clue. There are nine more nickels than dimes. The number of nickels is nine more than the number of dimes.

$$\begin{aligned} \text{Let } d &= \text{ number of dimes.} \\ d+9 &= \text{ number of nickels} \end{aligned}$$

Fill in the "number" column in the table to help get everything organized.

Type	Number	•	Value($)	=	Total Value($)
Dimes	d		0.10		
Nickels	$d+9$		0.05		
					2.25

Now we have all the information we need from the problem!

We multiply the number times the value to get the total value of each type of coin. While we do not know the actual number, we do have an expression to represent it.

And so now multiply $\text{number} \cdot \text{value} = \text{total value}$. See how this is done in the table below.

Type	Number	•	Value($)	=	Total Value($)
Dimes	d		0.10		$0.10d$
Nickels	$d+9$		0.05		$0.05(d+9)$
					2.25

Notice that we made the heading of the table show the model.

Step 4. Translate into an equation. It may be helpful to restate the problem in one sentence. Translate the English sentence into an algebraic equation.

Write the equation by adding the total values of all the types of coins.

	Value of dimes	+	value of nickels	=	total value of coins
Translate to an equation.	$0.10d$	+	$0.05(d+9)$	=	2.25

Step 5. Solve the equation using good algebra techniques.

Now solve this equation.	$0.10d + 0.05(d + 9) = 2.25$
Distribute.	$0.10d + 0.05d + 0.45 = 2.25$
Combine like terms.	$0.15d + 0.45 = 2.25$
Subtract 0.45 from each side.	$0.15d = 1.80$
Divide.	$d = 12$

So there are 12 dimes.	
The number of nickels is d+9d+9.	$d + 9$
	$12 + 9$
	21

Step 6. Check the answer in the problem and make sure it makes sense.

Does this check?

$$\begin{array}{lrcl} 12 \text{ dimes} & 12(0.10) & = & 1.20 \\ 21 \text{ nickels} & 21(0.05) & = & \underline{1.05} \\ & & & \$2.25 \checkmark \end{array} \qquad (3.3.2)$$

Step 7. Answer the question with a complete sentence.

Adalberto has twelve dimes and twenty-one nickels.

If this were a homework exercise, our work might look like the following.

Adalberto has $2.25 in dimes and nickels in his pocket. He has nine more nickels than dimes.

How many of each type does he have?

Type	Number • Value($)		= Total Value ($)
Dimes	d	0.10	0.10d
Nickels	d + 9	0.05	0.05(d + 9)
			2.25 ✓

12 dimes 12(0.10) = 1.20

21 nickels 21(0.05) = 1.05

$2.25 ✓

Adalberto has twelve dimes and twenty-one nickels.

$0.10d + 0.05d + 0.45 = 2.25$

$0.15d + 0.45 = 2.25$

$0.15d = 1.80$

$d = 12$ dimes

$d + 9$

$12 + 9$

21 nickels

Try It 3.3.1

Michaela has $2.05 in dimes and nickels in her change purse. She has seven more dimes than nickels. How many coins of each type does she have?

Answer

9 nickels, 16 dimes

Try It 3.3.2

Liliana has $2.10 in nickels and quarters in her backpack. She has 12 more nickels than quarters. How many coins of each type does she have?

Answer

17 nickels, 5 quarters

SOLVE COIN WORD PROBLEMS.

1. **Read** the problem. Make sure all the words and ideas are understood.
   - Determine the types of coins involved.
   - Create a table to organize the information.
   - Label the columns “type,” “number,” “value,” “total value.”
   - List the types of coins.
   - Write in the value of each type of coin.
   - Write in the total value of all the coins.

Type	Number • Value($) =	Total Value($)

2. **Identify** what we are looking for.
3. **Name** what we are looking for. Choose a variable to represent that quantity.
   - Use variable expressions to represent the number of each type of coin and write them in the table.
   - Multiply the number times the value to get the total value of each type of coin.
4. **Translate** into an equation.
   It may be helpful to restate the problem in one sentence with all the important information. Then, translate the sentence into an equation.
   Write the equation by adding the total values of all the types of coins.
5. **Solve** the equation using good algebra techniques.
6. **Check** the answer in the problem and make sure it makes sense.
7. **Answer** the question with a complete sentence.

Example 3.3.2

Maria has $2.43 in quarters and pennies in her wallet. She has twice as many pennies as quarters. How many coins of each type does she have?

Solution

Step 1. Read the problem.

Determine the types of coins involved.

We know that Maria has quarters and pennies.

Create a table to organize the information.

- **Step 2. Identify** what you are looking for.
  - We are looking for the number of quarters and pennies.

  Step 3. Name. Represent the number of quarters and pennies using variables.
  - Multiply the ‘number’ and the ‘value’ to get the ‘total value’ of each type of coin.

Type	Number • Value($) =	Total Value($)
Quarters	q	0.25	$0.25q$
Pennies	$2q$	0.01	$0.01(2q)$
		2.43

  Step 4. Translate. Write the equation by adding the ‘total value’ of all the types of coins.

Step5. Solve the equation.	$0.25q+0.01(2q)=2.43$
Multiply.	$0.25q+0.02q=2.43$
Combine like terms.	$0.27q=2.43$
Divide by 0.27	$q=9$ quarters
The number of pennies is 2q.	$2q$
	$2\cdot 9$
	18 pennies

Step 6. Check the answer in the problem.

Maria has 9 quarters and 18 pennies. Dies this make $2.43?

9 quarters	$9(0.25)$	$=$	2.25
18 pennies	$18(0.01)$	$=$	0.18
Total			$2.43✓

Step 7. Answer the question. Maria has nine quarters and eighteen pennies.

Try It 3.3.3

Sumanta has $4.20 in nickels and dimes in her piggy bank. She has twice as many nickels as dimes. How many coins of each type does she have?

Answer

42 nickels, 21 dimes

Try It 3.3.4

Alison has three times as many dimes as quarters in her purse. She has $9.35 altogether. How many coins of each type does she have?

Answer

51 dimes, 17 quarters

In the next example, we'll show only the completed table—remember the steps we take to fill in the table.

Example 3.3.3

Danny has $2.14 worth of pennies and nickels in his piggy bank. The number of nickels is two more than ten times the number of pennies. How many nickels and how many pennies does Danny have?

Solution

Step 1. Read the problem.	
Determine the types of coins involved.	pennies and nickels
Create a table.	
Write in the value of each type of coin.	Pennies are worth $0.01. Nickels are worth $0.05.
Step 2. Identify what we are looking for.	the number of pennies and nickels

Step 3. Name. Represent the number of each type of coin using variables.	
The number of nickels is defined in terms of the number of pennies, so start with pennies.	Let $p =$ number of pennies.
The number of nickels is two more than ten times the number of pennies.	And let $10p + 2 =$ number of nickels.
Multiply the number and the value to get the total value of each type of coin.	

Type	Number • Value ($) = Total Value ($)		
pennies	p	0.01	$0.01p$
nickels	$10p + 2$	0.05	$0.05(10p + 2)$
			$2.14

Step 4. Translate. Write the equation by adding the total value of all the types of coins.	$0.01p + 0.05(10p + 2) = 2.14$
Step 5. Solve the equation.	$0.01p + 0.50p + 0.10 = 2.14$ $0.51p + 0.10 = 2.14$ $0.51p = 2.04$ $p = 4$ pennies
How many nickels?	$10p + 2$ $10(4) + 2$ 42 nickels
Step 6. Check the answer in the problem and make sure it makes sense Danny has four pennies and 42 nickels. Is the total value $2.14? $4(0.01) + 42(0.05) \stackrel{?}{=} 2.14$ $2.14 = 2.14$✓	
Step 7. Answer the question.	Danny has four pennies and 42 nickels.

Try It 3.3.5

Jesse has $6.55 worth of quarters and nickels in his pocket. The number of nickels is five more than two times the number of quarters. How many nickels and how many quarters does Jesse have?

Answer

41 nickels, 18 quarters

Try It 3.3.6

Elane has $7.00 total in dimes and nickels in her coin jar. The number of dimes that Elane has is seven less than three times the number of nickels. How many of each coin does Elane have?

Answer

22 nickels, 59 dimes

Solve Ticket and Stamp Word Problems

Problems involving tickets or stamps are very much like coin problems. Each type of ticket and stamp has a value, just like each type of coin does. So to solve these problems, we will follow the same steps we used to solve coin problems.

Example 3.3.4

At a school concert, the total value of tickets sold was $1,506. Student tickets sold for $6 each and adult tickets sold for $9 each. The number of adult tickets sold was five less than three times the number of student tickets sold. How many student tickets and how many adult tickets were sold?

Solution

Step 1. Read the problem.

- Determine the types of tickets involved. There are student tickets and adult tickets.
- Create a table to organize the information.

Type	Number •	Value($) =	Total Value($)
Student		6	
Adult		9	
			1506

Step 2. Identify what we are looking for.

- We are looking for the number of student and adult tickets.

Step 3. Name. Represent the number of each type of ticket using variables.

We know the number of adult tickets sold was five less than three times the number of student tickets sold.

- Multiply the number times the value to get the total value of each type of ticket.

Type	Number •	Value($) =	Total Value($)
Student	s	6	$6s$
Adult	$3s-5$	9	$9(3s-5)$
			1506

Step 4. Translate. Write the equation by adding the total values of each type of ticket.

$$6s+9(3s-5)=1506$$

Step 5. Solve the equation.

$$\begin{aligned} 6s+27s-45 &= 1506 \\ 33s-45 &= 1506 \\ 33s &= 1551 \\ s &= 47 \text{ student tickets} \\ \text{Number of adult tickets} &= 3s-5 \\ &= 3(47)-5 \\ \text{So there were} \quad 136 &\quad \text{adult tickets} \end{aligned}$$

Step 6. Check the answer.

There were 47 student tickets at $6 each and 136 adult tickets at $9 each. Is the total value $1,506? We find the total value of each type of ticket by multiplying the number of tickets times its value then add to get the total value of all the tickets sold.

$$\begin{aligned} 47\cdot 6 &= 282 \\ 136\cdot 9 &= \underline{1224} \\ & \quad 1506\checkmark \end{aligned}$$

Step 7. Answer the question. They sold 47 student tickets and 136 adult tickets.

Try It 3.3.7

The first day of a water polo tournament the total value of tickets sold was $17,610. One-day passes sold for $20 and tournament passes sold for $30. The number of tournament passes sold was 37 more than the number of day passes sold. How many day passes and how many tournament passes were sold?

Answer

330 day passes, 367 tournament passes

Try It 3.3.8

At the movie theater, the total value of tickets sold was $2,612.50. Adult tickets sold for $10 each and senior/child tickets sold for $7.50 each. The number of senior/child tickets sold was 25 less than twice the number of adult tickets sold. How many senior/child tickets and how many adult tickets were sold?

Answer

112 adult tickets, 199 senior/child tickets

We have learned how to find the total number of tickets when the number of one type of ticket is based on the number of the other type. Next, we'll look at an example where we know the total number of tickets and have to figure out how the two types of tickets relate.

Suppose Bianca sold a total of 100 tickets. Each ticket was either an adult ticket or a child ticket. If she sold 20 child tickets, how many adult tickets did she sell?

- *Did you say '80'? How did you figure that out? Did you subtract 20 from 100?*

If she sold 45 child tickets, how many adult tickets did she sell?

- *Did you say '55'? How did you find it? By subtracting 45 from 100?*

What if she sold 75 child tickets? How many adult tickets did she sell?

- *The number of adult tickets must be 100−75. She sold 25 adult tickets.*

Now, suppose Bianca sold x child tickets. Then how many adult tickets did she sell? To find out, we would follow the same logic we used above. In each case, we subtracted the number of child tickets from 100 to get the number of adult tickets. We now do the same with x.

We have summarized this below.

Child tickets	Adult tickets
20	80
45	55
75	25
x	$100-x$

Table 3.3.2

We can apply these techniques to other examples

Example 3.3.5

Galen sold 810 tickets for his church's carnival for a total of $2,820. Children's tickets cost $3 each and adult tickets cost $5 each. How many children's tickets and how many adult tickets did he sell?

Solution

Step 1. Read the problem.

- Determine the types of tickets involved. There are children tickets and adult tickets.
- Create a table to organize the information.

Type	Number •	Value($) =	Total Value($)
Children		3	
Adult		5	
			2820

Step 2. Identify what we are looking for.

- We are looking for the number of children and adult tickets.

Step 3. Name. Represent the number of each type of ticket using variables.

- We know the total number of tickets sold was 810.
- This means the number of children's tickets plus the number of adult tickets must add up to 810.
- Let c be the number of children tickets.

Type	Number •	Value($) =	Total Value($)
Children	c	3	$3c$
Adult	$810-c$	5	$5(810-c)$
			2820

- Then $810-c$ is the number of adult tickets.
- Multiply the number times the value to get the total value of each type of ticket.

Step 4. Translate.

Write the equation by adding the total values of each type of ticket.

Step 5. Solve the equation.

$$\begin{aligned} 3c+5(810-c) &= 2,820 \\ 3c+4,050-5c &= 2,820 \\ -2c &= -1,230 \\ c &= 615 \text{ children tickets} \end{aligned}$$

How many adults?

$$\begin{aligned} &810-c \\ &810-615 \\ &195 \text{ adult tickets} \end{aligned}$$

Step 6. Check the answer. There were 615 children's tickets at $3 each and 195 adult tickets at $5 each. Is the total value $2,820?

$$\begin{aligned} 615\cdot 3 &= 1845 \\ 195\cdot 5 &= \underline{975} \\ & 2,820\checkmark \end{aligned}$$

Step 7. Answer the question. Galen sold 615 children's tickets and 195 adult tickets.

? Try It 3.3.9

During her shift at the museum ticket booth, Leah sold 115 tickets for a total of $1,163. Adult tickets cost $12 and student tickets cost $5. How many adult tickets and how many student tickets did Leah sell?

Answer

84 adult tickets, 31 student tickets

Try It 3.3.10

A whale-watching ship had 40 paying passengers on board. The total collected from tickets was $1,196. Full-fare passengers paid $32 each and reduced-fare passengers paid $26 each. How many full-fare passengers and how many reduced-fare passengers were on the ship?

Answer

26 full-fare, 14 reduced fare

Now, we'll do one where we fill in the table all at once.

Example 3.3.6

Monica paid $8.36 for stamps. The number of 41-cent stamps was four more than twice the number of two-cent stamps. How many 41-cent stamps and how many two-cent stamps did Monica buy?

Solution

The types of stamps are 41-cent stamps and two-cent stamps. Their names also give the value!

"The number of 41-cent stamps was four more than twice the number of two-cent stamps."

$$\text{Let } x = \text{ number of 2 -cent stamps.}$$
$$2x+4 = \text{ number of 41 - cent stamps}$$

Type	Number •	Value($) =	Total Value($)
41 cent stamps	$2x+4$	0.41	$0.41(2x+4)$
2 cent stamps	x	0.02	$0.02x$
			8.36

Write the equation from the total values.

$$0.41(2x+4)+0.02x = 8.36$$
$$0.82x+1.64+0.02x = 8.36$$
$$0.84x+1.64 = 8.36$$

Solve the equation.

$$0.84x = 6.72$$
$$x = 8$$

Monica bought eight two-cent stamps.

Find the number of 41-cent stamps she bought by evaluating $2x+4$ for $x=8$

$$2x+4$$
$$2(8)+4$$
$$20$$

Check.

$$\begin{aligned} 8(0.02)+20(0.41) &\stackrel{?}{=} 8.36 \\ 0.16+8.20 &\stackrel{?}{=} 8.36 \\ 8.36 &= 8.36\checkmark \end{aligned} \tag{3.3.3}$$

Monica bought eight two-cent stamps and 20 41-cent stamps

Try It 3.3.11

Eric paid $13.36 for stamps. The number of 41-cent stamps was eight more than twice the number of two-cent stamps. How many 41-cent stamps and how many two-cent stamps did Eric buy?

Answer

32 at $0.41, 12 at $0.02

Try It 3.3.12

Kailee paid $12.66 for stamps. The number of 41-cent stamps was four less than three times the number of 20-cent stamps. How many 41-cent stamps and how many 20-cent stamps did Kailee buy?

Answer

26 at $0.41, 10 at $0.20

Solve Mixture Word Problems

Now we'll solve some more general applications of the mixture model. Grocers and bartenders use the mixture model to set a fair price for a product made from mixing two or more ingredients. Financial planners use the mixture model when they invest money in a variety of accounts and want to find the overall interest rate. Landscape designers use the mixture model when they have an assortment of plants and a fixed budget, and event coordinators do the same when choosing appetizers and entrees for a banquet.

Our first mixture word problem will be making trail mix from raisins and nuts.

Example 3.3.7

Henning is mixing raisins and nuts to make 10 pounds of trail mix. Raisins cost $2 a pound and nuts cost $6 a pound. If Henning wants his cost for the trail mix to be $5.20 a pound, how many pounds of raisins and how many pounds of nuts should he use?

Solution

As before, we fill in a chart to organize our information.

The 10 pounds of trail mix will come from mixing raisins and nuts.

$$\text{Let } x = \text{ number of pounds of raisins.}$$
$$10 - x = \text{ number of pounds of nuts}$$

We enter the price per pound for each item.

We multiply the number times the value to get the total value.

Type	Number of pounds	• Price per pound ($)	= Total Value ($)
Raisins	x	2	$2x$
Nuts	$10 - x$	6	$6(10 - x)$
Trail mix	10	5.20	10(5.20)

Notice that the last line in the table gives the information for the total amount of the mixture.

We know the value of the raisins plus the value of the nuts will be the value of the trail mix.

Write the equation from the total values.	$2x + 6(10 - x) = 10(5.20)$
Solve the equation.	$2x + 60 - 6x = 52$
	$-4x = -8$

	$x = 2$ pounds of raisins
Find the number of pounds of nuts.	$10 - x$
	$10 - 2$
	8 pounds of nuts
Check. $2(\$2) + 8(\$6) \stackrel{?}{=} 10(\$5.20)$ $\$4 + \$48 \stackrel{?}{=} \$52$ $\$52 = \$52$✓	
	Henning mixed two pounds of raisins with eight pounds of nuts.

Try It 3.3.13

Orlando is mixing nuts and cereal squares to make a party mix. Nuts sell for $7 a pound and cereal squares sell for $4 a pound. Orlando wants to make 30 pounds of party mix at a cost of $6.50 a pound, how many pounds of nuts and how many pounds of cereal squares should he use?

Answer

5 pounds cereal squares, 25 pounds nuts

Try It 3.3.14

Becca wants to mix fruit juice and soda to make a punch. She can buy fruit juice for $3 a gallon and soda for $4 a gallon. If she wants to make 28 gallons of punch at a cost of $3.25 a gallon, how many gallons of fruit juice and how many gallons of soda should she buy?

Answer

21 gallons of fruit punch, 7 gallons of soda

We can also use the mixture model to solve investment problems using **simple interest**. We have used the simple interest formula, $I = Prt$, where t represented the number of years. When we just need to find the interest for one year, $t = 1$, so then $I = Pr$.

Example 3.3.8

Stacey has $20,000 to invest in two different bank accounts. One account pays interest at 3% per year and the other account pays interest at 5% per year. How much should she invest in each account if she wants to earn 4.5% interest per year on the total amount?

Solution

We will fill in a chart to organize our information. We will use the simple interest formula to find the interest earned in the different accounts.

The interest on the mixed investment will come from adding the interest from the account earning 3% and the interest from the account earning 5% to get the total interest on the $20,000.

$$\begin{aligned} \text{Let } x &= \text{amount invested at } 3\% \\ 20,000 - x &= \text{amount invested at } 5\% \end{aligned}$$

The amount invested is the *principal* for each account.

We enter the interest rate for each account.

We multiply the amount invested times the rate to get the interest.

Type	Amount Invested	• Rate	= Interest
3%	x	0.03	$0.03x$
5%	$20{,}000 - x$	0.05	$0.05(20{,}000 - x)$
4.5%	20,000	0.045	0.045(20,000)

Notice that the total amount invested, 20,000, is the sum of the amount invested at 3% and the amount invested at 5%. And the total interest, $0.045(20,000)$ is the sum of the interest earned in the 3% account and the interest earned in the 5% account.

As with the other mixture applications, the last column in the table gives us the equation to solve.

Write the equation from the interest earned. Solve the equation.	$0.03x + 0.05(20000 - x) = 0.045(20000)$ $0.03x + 1000 - 0.05x = 900$ $-0.02x = -100$ $x = 5000$ $\text{amount invested at } 3\%$
Find the amount invested at 5%.	$20{,}000 - x$ $20{,}000 - 5{,}000$ 15,000 = amount invested at 5%
Check. $0.03x + 0.05(15000 + x) \stackrel{?}{=} 0.045(20000)$ $150 + 750 \stackrel{?}{=} 900$ $900 = 900\checkmark$	
	Stacey should invest $5,000 in the account that earns 3% and $15,000 in the account that earns 5%.

Try It 3.3.15

Remy has $14,000 to invest in two mutual funds. One fund pays interest at 4% per year and the other fund pays interest at 7% per year. How much should she invest in each fund if she wants to earn 6.1% interest on the total amount?

Answer

$4,200 at 4%, $9,800 at 7%

Try It 3.3.16

Marco has $8,000 to save for his daughter's college education. He wants to divide it between one account that pays 3.2% interest per year and another account that pays 8% interest per year. How much should he invest in each account if he wants the interest on the total investment to be 6.5%?

Answer

$2,500 at 3.2%, $5,500 at 8%

Key Concepts

- **Total Value of Coins** For the same type of coin, the total value of a number of coins is found by using the model. number·value=total value where *number* is the number of coins and *value* is the value of each coin; *total value* is the total value of all the coins
- **Problem-Solving Strategy—Coin Word Problems**

1. **Read** the problem. Make all the words and ideas are understood. Determine the types of coins involved.
   - Create a table to organize the information.
   - Label the columns type, number, value, total value.
   - List the types of coins.
   - Write in the value of each type of coin.
   - Write in the total value of all the coins.
2. **Identify** what we are looking for.
3. **Name** what we are looking for. Choose a variable to represent that quantity.
   Use variable expressions to represent the number of each type of coin and write them in the table.
   Multiply the number times the value to get the total value of each type of coin.
4. **Translate** into an equation. It may be helpful to restate the problem in one sentence with all the important information.
   Then, translate the sentence into an equation.
   Write the equation by adding the total values of all the types of coins.
5. **Solve** the equation using good algebra techniques.
6. **Check** the answer in the problem and make sure it makes sense.
7. **Answer** the question with a complete sentence.

Glossary

mixture problems

Mixture problems combine two or more items with different values together.

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3.3E: Exercises

Practice Makes Perfect

Solve Coin Word Problems

In the following exercises, solve each coin word problem.

Exercise 3.3E.1

Jaime has $2.60 in dimes and nickels. The number of dimes is 14 more than the number of nickels. How many of each coin does he have?

Answer

8 nickels, 22 dimes

Exercise 3.3E.2

Lee has $1.75 in dimes and nickels. The number of nickels is 11 more than the number of dimes. How many of each coin does he have?

Exercise 3.3E.3

Ngo has a collection of dimes and quarters with a total value of $3.50. The number of dimes is seven more than the number of quarters. How many of each coin does he have?

Answer

15 dimes, 8 quarters

Exercise 3.3E.4

Connor has a collection of dimes and quarters with a total value of $6.30. The number of dimes is 14 more than the number of quarters. How many of each coin does he have?

Exercise 3.3E.5

A cash box of $1 and $5 bills is worth $45. The number of $1 bills is three more than the number of $5 bills. How many of each bill does it contain?

Answer

10 at $1, 7 at $5

Exercise 3.3E.6

Joe’s wallet contains $1 and $5 bills worth $47. The number of $1 bills is five more than the number of $5 bills. How many of each bill does he have?

Exercise 3.3E.7

Rachelle has $6.30 in nickels and quarters in her coin purse. The number of nickels is twice the number of quarters. How many coins of each type does she have?

Answer

18 quarters, 36 nickels

Exercise 3.3E.8

Deloise has $1.20 in pennies and nickels in a jar on her desk. The number of pennies is three times the number of nickels. How many coins of each type does she have?

Exercise 3.3E.9

Harrison has $9.30 in his coin collection, all in pennies and dimes. The number of dimes is three times the number of pennies. How many coins of each type does he have?

Answer

30 pennies, 90 dimes

Exercise 3.3E.10

Ivan has $8.75 in nickels and quarters in his desk drawer. The number of nickels is twice the number of quarters. How many coins of each type does he have?

Exercise 3.3E.11

In a cash drawer there is $125 in $5 and $10 bills. The number of $10 bills is twice the number of $5 bills. How many of each are in the drawer?

Answer

10 at $10, 5 at $5

Exercise 3.3E.12

John has $175 in $5 and $10 bills in his drawer. The number of $5 bills is three times the number of $10 bills. How many of each are in the drawer?

Exercise 3.3E.13

Carolyn has $2.55 in her purse in nickels and dimes. The number of nickels is nine less than three times the number of dimes. Find the number of each type of coin.

Answer

12 dimes and 27 nickels

Exercise 3.3E.14

Julio has $2.75 in his pocket in nickels and dimes. The number of dimes is 10 less than twice the number of nickels. Find the number of each type of coin.

Exercise 3.3E.15

Chi has $11.30 in dimes and quarters. The number of dimes is three more than three times the number of quarters. How many of each are there?

Answer

63 dimes, 20 quarters

Exercise 3.3E.16

Tyler has $9.70 in dimes and quarters. The number of quarters is eight more than four times the number of dimes. How many of each coin does he have?

Exercise 3.3E.17

Mukul has $3.75 in quarters, dimes and nickels in his pocket. He has five more dimes than quarters and nine more nickels than quarters. How many of each coin are in his pocket?

Answer

16 nickels, 12 dimes, 7 quarters

Exercise 3.3E.18

Vina has $4.70 in quarters, dimes and nickels in her purse. She has eight more dimes than quarters and six more nickels than quarters. How many of each coin are in her purse?

Solve Ticket and Stamp Word Problems

In the following exercises, solve each ticket or stamp word problem.

Exercise 3.3E.19

The school play sold $550 in tickets one night. The number of $8 adult tickets was 10 less than twice the number of $5 child tickets. How many of each ticket were sold?

Answer

30 child tickets, 50 adult tickets

Exercise 3.3E.20

If the number of $8 child tickets is seventeen less than three times the number of $12 adult tickets and the theater took in $584, how many of each ticket were sold?

Exercise 3.3E.21

The movie theater took in $1,220 one Monday night. The number of $7 child tickets was ten more than twice the number of $9 adult tickets. How many of each were sold?

Answer

110 child tickets, 50 adult tickets

Exercise 3.3E.22

The ball game sold $1,340 in tickets one Saturday. The number of $12 adult tickets was 15 more than twice the number of $5 child tickets. How many of each were sold?

Exercise 3.3E.23

The ice rink sold 95 tickets for the afternoon skating session, for a total of $828. General admission tickets cost $10 each and youth tickets cost $8 each. How many general admission tickets and how many youth tickets were sold?

Answer

34 general, 61 youth

Exercise 3.3E.24

For the 7:30 show time, 140 movie tickets were sold. Receipts from the $13 adult tickets and the $10 senior tickets totaled $1,664. How many adult tickets and how many senior tickets were sold?

Exercise 3.3E.25

The box office sold 360 tickets to a concert at the college. The total receipts were $4170. General admission tickets cost $15 and student tickets cost $10. How many of each kind of ticket was sold?

Answer

114 general, 246 student

Exercise 3.3E.26

Last Saturday, the museum box office sold 281 tickets for a total of $3954. Adult tickets cost $15 and student tickets cost $12. How many of each kind of ticket was sold?

Exercise 3.3E.27

Julie went to the post office and bought both $0.41 stamps and $0.26 postcards. She spent $51.40. The number of stamps was 20 more than twice the number of postcards. How many of each did she buy?

Answer

40 postcards, 100 stamps

Exercise 3.3E.28

Jason went to the post office and bought both $0.41 stamps and $0.26 postcards and spent $10.28. The number of stamps was four more than twice the number of postcards. How many of each did he buy?

Exercise 3.3E.29

Maria spent $12.50 at the post office. She bought three times as many $0.41 stamps as $0.02 stamps. How many of each did she buy?

Answer

30 at $0.41, 10 at $0.02

Exercise 3.3E.30

Hector spent $33.20 at the post office. He bought four times as many $0.41 stamps as $0.02 stamps. How many of each did he buy?

Exercise 3.3E.31

Hilda has $210 worth of $10 and $12 stock shares. The numbers of $10 shares is five more than twice the number of $12 shares. How many of each does she have?

Answer

15 $10 shares, 5 $12 shares

Exercise 3.3E.32

Mario invested $475 in $45 and $25 stock shares. The number of $25 shares was five less than three times the number of $45 shares. How many of each type of share did he buy?

Solve Mixture Word Problems

In the following exercises, solve each mixture word problem.

Exercise 3.3E.33

Lauren in making 15 liters of mimosas for a brunch banquet. Orange juice costs her $1.50 per liter and champagne costs her $12 per liter. How many liters of orange juice and how many liters of champagne should she use for the mimosas to cost Lauren $5 per liter?

Answer

5 liters champagne, 10 liters orange juice

Exercise 3.3E.34

Macario is making 12 pounds of nut mixture with macadamia nuts and almonds. Macadamia nuts cost $9 per pound and almonds cost $5.25 per pound. How many pounds of macadamia nuts and how many pounds of almonds should Macario use for the mixture to cost $6.50 per pound to make?

Exercise 3.3E.35

Kaapo is mixing Kona beans and Maui beans to make 25 pounds of coffee blend. Kona beans cost Kaapo $15 per pound and Maui beans cost $24 per pound. How many pounds of each coffee bean should Kaapo use for his blend to cost him $17.70 per pound?

Answer

7.5 lbs Maui beans, 17.5 Kona beans

Exercise 3.3E.36

Estelle is making 30 pounds of fruit salad from strawberries and blueberries. Strawberries cost $1.80 per pound and blueberries cost $4.50 per pound. If Estelle wants the fruit salad to cost her $2.52 per pound, how many pounds of each berry should she use?

Exercise 3.3E.37

Carmen wants to tile the floor of his house. He will need 1000 square feet of tile. He will do most of the floor with a tile that costs $1.50 per square foot, but also wants to use an accent tile that costs $9.00 per square foot. How many square feet of each tile should he plan to use if he wants the overall cost to be $3 per square foot?

Answer

800 at $1.50, 200 at $9.00

Exercise 3.3E.38

Riley is planning to plant a lawn in his yard. He will need nine pounds of grass seed. He wants to mix Bermuda seed that costs $4.80 per pound with Fescue seed that costs $3.50 per pound. How much of each seed should he buy so that the overall cost will be $4.02 per pound?

Exercise 3.3E.39

Vartan was paid $25,000 for a cell phone app that he wrote and wants to invest it to save for his son's education. He wants to put some of the money into a bond that pays 4% annual interest and the rest into stocks that pay 9% annual interest. If he wants to earn 7.4% annual interest on the total amount, how much money should he invest in each account?

Answer

$8000 at 4%, $17,000 at 9%

Exercise 3.3E.40

Vern sold his 1964 Ford Mustang for $55,000 and wants to invest the money to earn him 5.8% interest per year. He will put some of the money into Fund A that earns 3% per year and the rest in Fund B that earns 10% per year. How much should he invest into each fund if he wants to earn 5.8% interest per year on the total amount?

Exercise 3.3E.41

Stephanie inherited $40,000. She wants to put some of the money in a certificate of deposit that pays 2.1% interest per year and the rest in a mutual fund account that pays 6.5% per year. How much should she invest in each account if she wants to earn 5.4% interest per year on the total amount?

Answer

$10,000 in CD, $30,000 in mutual fund

Exercise 3.3E.42

Avery and Caden have saved $27,000 towards a down payment on a house. They want to keep some of the money in a bank account that pays 2.4% annual interest and the rest in a stock fund that pays 7.2% annual interest. How much should they put into each account so that they earn 6% interest per year?

Exercise 3.3E.43

Dominic pays 7% interest on his $15,000 college loan and 12% interest on his $11,000 car loan. What average interest rate does he pay on the total $26,000 he owes? (Round your answer to the nearest tenth of a percent.)

Answer

9.1%

Exercise 3.3E.44

Liam borrowed a total of $35,000 to pay for college. He pays his parents 3% interest on the $8,000 he borrowed from them and pays the bank 6.8% on the rest. What average interest rate does he pay on the total $35,000? (Round your answer to the nearest tenth of a percent.)

Everyday Math

Exercise 3.3E.45

As the treasurer of her daughter's Girl Scout troop, Laney collected money for some girls and adults to go to a 3-day camp. Each girl paid $75 and each adult paid $30. The total amount of money collected for camp was $765. If the number of girls is three times the number of adults, how many girls and how many adults paid for camp?

Answer

9 girls, 3 adults

Exercise 3.3E.46

Laurie was completing the treasurer's report for her son's Boy Scout troop at the end of the school year. She didn't remember how many boys had paid the $15 full-year registration fee and how many had paid the $10 partial-year fee. She knew that the number of boys who paid for a full-year was ten more than the number who paid for a partial-year. If $250 was collected for all the registrations, how many boys had paid the full-year fee and how many had paid the partial-year fee?

Writing Exercises

Exercise 3.3E.47

Suppose you have six quarters, nine dimes, and four pennies. Explain how you find the total value of all the coins.

Answer

Answers will vary.

Exercise 3.3E.48

Do you find it helpful to use a table when solving coin problems? Why or why not?

Exercise 3.3E.49

In the table used to solve coin problems, one column is labeled "number" and another column is labeled "value." What is the difference between the "number" and the "value?"

Answer

Answers will vary.

Exercise 3.3E.50

What similarities and differences did you see between solving the coin problems and the ticket and stamp problems?

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
solve coin word problems.			
solve ticket and stamp word problems.			
solve mixture word problems.			

ⓑ After reviewing this checklist, what will you do to become confident for all objectives?

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3.4: Triangles, Rectangles, and the Pythagorean Theorem

Learning Objectives

By the end of this section, you will be able to:

- Solve applications using properties of triangles
- Use the Pythagorean Theorem
- Solve applications using rectangle properties

Be prepared

Before you get started, take this readiness quiz.

1. Simplify: $12(6h)$.
   If you missed this problem, review Exercise 1.10.1.
2. The length of a rectangle is three less than the width. Let w represent the width. Write an expression for the length of the rectangle.
   If you missed this problem, review Exercise 1.3.43.
3. Solve: $A = \frac{1}{2}bh$ for b when A=260 and h=52.
   If you missed this problem, review Exercise 2.6.10.
4. Simplify: $\sqrt{144}$.
   If you missed this problem, review Exercise 1.9.10.

Solve Applications Using Properties of Triangles

In this section we will use some common geometry formulas. We will adapt our problem-solving strategy so that we can solve geometry applications. The geometry formula will name the variables and give us the equation to solve. In addition, since these applications will all involve shapes of some sort, most people find it helpful to draw a figure and label it with the given information. We will include this in the first step of the problem solving strategy for geometry applications.

SOLVE GEOMETRY APPLICATIONS

1. **Read** the problem and make sure all the words and ideas are understood. Draw the figure and label it with the given information.
2. **Identify** what we are looking for.
3. **Label** what we are looking for by choosing a variable to represent it.
4. **Translate** into an equation by writing the appropriate formula or model for the situation. Substitute in the given information.
5. **Solve** the equation using good algebra techniques.
6. **Check** the answer by substituting it back into the equation solved in step 5 and by making sure it makes sense in the context of the problem.
7. **Answer** the question with a complete sentence.

We will start geometry applications by looking at the properties of triangles. Let's review some basic facts about triangles. Triangles have three sides and three interior angles. Usually each side is labeled with a lowercase letter to match the uppercase letter of the opposite vertex.

The plural of the word *vertex* is *vertices*. All triangles have three **vertices**. Triangles are named by their vertices: The triangle in Figure 3.4.1 is called $\triangle ABC$.

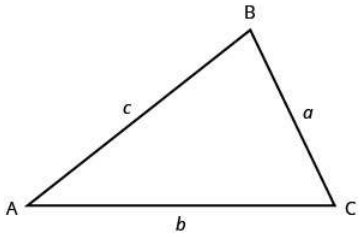


Figure 3.4.1: Triangle ABC has vertices A, B, and C. The lengths of the sides are a, b, and c.

The three angles of a triangle are related in a special way. The sum of their measures is 180°. Note that we read $m\angle A$ as "the measure of angle A." So in $\triangle ABC$ in Figure 3.4.1.

$$m\angle A + m\angle B + m\angle C = 180^\circ$$

Because the **perimeter** of a figure is the length of its boundary, the perimeter of $\triangle ABC$ is the sum of the lengths of its three sides.

$$P = a + b + c$$

To find the **area** of a triangle, we need to know its base and height. The height is a line that connects the base to the opposite vertex and makes a 90° angle with the base. We will draw $\triangle ABC$ again, and now show the height, h. See Figure 3.4.2.

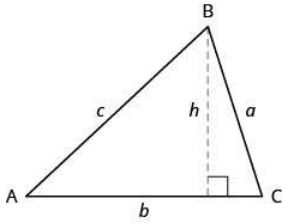
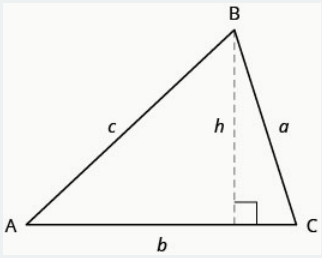


Figure 3.4.2: The formula for the area of $\triangle ABC$ is $A = \frac{1}{2}bh$ where b is the base and h is the height.

TRIANGLE PROPERTIES



For $\triangle ABC$

Angle measures:

$$m\angle A + m\angle B + m\angle C = 180^\circ \tag{3.4.1}$$

- The sum of the measures of the angles of a triangle is 180°.

Perimeter:

$$P = a + b + c \tag{3.4.2}$$

- The perimeter is the sum of the lengths of the sides of the triangle.

Area:

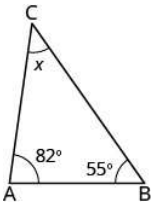
$A = \frac{1}{2}bh, b =$ base , $h =$ height

- The area of a triangle is one-half the base times the height.

Example 3.4.1

The measures of two angles of a triangle are 55 and 82 degrees. Find the measure of the third angle.

Solution

Step 1. Read the problem. Draw the figure and label it with the given information.	
Step 2. Identify what you are looking for.	the measure of the third angle in a triangle
Step 3. Name. Choose a variable to represent it.	Let $x =$ the measure of the angle.
Step 4. Translate.	
Write the appropriate formula and substitute.	$m\angle A + m\angle B + m\angle C = 180^\circ$
Step 5. Solve the equation.	$\begin{aligned} 55 + 82 + x &= 180 \\ 137 + x &= 180 \\ x &= 43 \end{aligned}$
Step 6. Check. $\begin{aligned} 55 + 82 + 43 &\stackrel{?}{=} 180 \\ 180 &= 180\checkmark \end{aligned}$	
Step 7. Answer the question.	The measure of the third angle is 43 degrees.

Try It 3.4.1

The measures of two angles of a triangle are 31 and 128 degrees. Find the measure of the third angle.

Answer

21 degrees

Try It 3.4.2

The measures of two angles of a triangle are 49 and 75 degrees. Find the measure of the third angle.

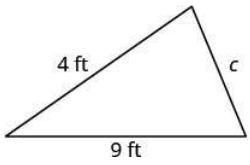
Answer

56 degrees

Example 3.4.2

The perimeter of a triangular garden is 24 feet. The lengths of two sides are four feet and nine feet. How long is the third side?

Solution

Step 1. Read the problem. Draw the figure and label it with the given information.	 P = 24 ft
Step 2. Identify what you are looking for.	length of the third side of a triangle
Step 3. Name. Choose a variable to represent it.	Let $c =$ the third side.
Step 4. Translate.	
Write the appropriate formula and substitute.	$P = a + b + c$
Substitute in the given information.	$24 \text{ ft} = 4 \text{ ft} + 9 \text{ ft} + c$
Step 5. Solve the equation.	$24 = 13 + c$ $11 = c$
Step 6. Check. $P = a+b+c$ $24 \stackrel{?}{=} 4+9+11$ $24 = 24\checkmark$	
Step 7. Answer the question.	The third side is 11 feet long.

Try It 3.4.3

The perimeter of a triangular garden is 48 feet. The lengths of two sides are 18 feet and 22 feet. How long is the third side?

Answer

8 feet

Try It 3.4.4

The lengths of two sides of a triangular window are seven feet and five feet. The perimeter is 18 feet. How long is the third side?

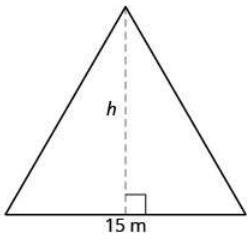
Answer

6 feet

Example 3.4.3

The area of a triangular church window is 90 square meters. The base of the window is 15 meters. What is the window's height?

Solution

Step 1. Read the problem. Draw the figure and label it with the given information.	 Area $= 90m^2$
Step 2. Identify what you are looking for.	height of a triangle
Step 3. Name. Choose a variable to represent it.	Let $h =$ the height.
Step 4. Translate.	
Write the appropriate formula.	$A = \frac{1}{2} \cdot b \cdot h$
Substitute in the given information.	$90\ \text{m}^2 = \frac{1}{2} \cdot 15\ \text{m} \cdot h$
Step 5. Solve the equation.	$90 = \frac{15}{2}h$ $12 = h$
Step 6. Check. $A = \frac{1}{2}bh$ $90 \stackrel{?}{=} \frac{1}{2} \cdot 15 \cdot 12$ $90 = 90\checkmark$	
Step 7. Answer the question.	The height of the triangle is 12 meters.

? Try It 3.4.5

The area of a triangular painting is 126 square inches. The base is 18 inches. What is the height?

Answer

14 inches

? Try It 3.4.6

A triangular tent door has area 15 square feet. The height is five feet. What is the base?

Answer

6 feet

The triangle properties we used so far apply to all triangles. Now we will look at one specific type of triangle—a right triangle. A **right triangle** has one 90° angle, which we usually mark with a small square in the corner.

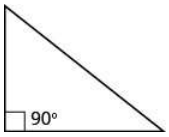


Figure 3.4.3

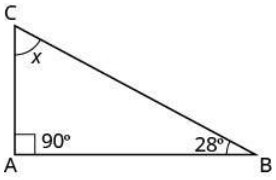
Definition: RIGHT TRIANGLE

A **right triangle** has one 90° angle, which is often marked with a square at the vertex.

Example 3.4.4

One angle of a right triangle measures 28°. What is the measure of the third angle?

Solution

Step 1. Read the problem. Draw the figure and label it with the given information.	
Step 2. Identify what you are looking for.	the measure of an angle
Step 3. Name. Choose a variable to represent it.	Let $x =$ the measure of an angle.
Step 4. Translate.	$m\angle A + m\angle B + m\angle C = 180$
Write the appropriate formula and substitute.	$x + 90 + 28 = 180$
Step 5. Solve the equation.	$x = 62$
Step 6. Check. $180 \stackrel{?}{=} 90 + 28 + 62$ $180 = 180\checkmark$	
Step 7. Answer the question.	The measure of the third angle is 62°.

Try It 3.4.7

One angle of a right triangle measures 56°. What is the measure of the other small angle?

Answer

34°

Try It 3.4.8

One angle of a right triangle measures 45°. What is the measure of the other small angle?

Answer

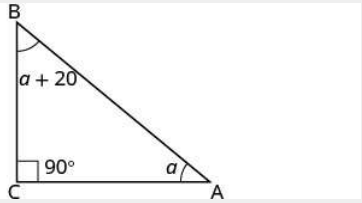
45°

In the examples we have seen so far, we could draw a figure and label it directly after reading the problem. In the next example, we will have to define one angle in terms of another. We will wait to draw the figure until we write expressions for all the angles we are looking for.

Example 3.4.5

The measure of one angle of a right triangle is 20 degrees more than the measure of the smallest angle. Find the measures of all three angles.

Solution

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	the measures of all three angles
Step 3. Name. Choose a variable to represent it.	Let $a = 1^{st}$ angle. $a + 20 = 2^{nd}$ angle $90 = 3^{rd}$ angle (the right angle)
Draw the figure and label it with the given information	
Step 4. Translate	$m\angle A + m\angle B + m\angle C = 180$
Write the appropriate formula. Substitute into the formula.	$a + (a + 20) + 90 = 180$
Step 5. Solve the equation.	$2a + 110 = 180$ $2a = 70$ $a = 35$ first angle $a + 20$ second angle $35 + 20 = 55$ And the third angle is 90.
Step 6. Check. $35 + 55 + 90 \stackrel{?}{=} 180$ $180 = 180$✓	
Step 7. Answer the question.	The three angles measure 35°, 55°, and 90°.

? Try It 3.4.9

The measure of one angle of a right triangle is 50° more than the measure of the smallest angle. Find the measures of all three angles.

Answer

20°,70°,90°

? Try It 3.4.10

The measure of one angle of a right triangle is 30° more than the measure of the smallest angle. Find the measures of all three angles.

Answer

30°,60°,90°

Use the Pythagorean Theorem

We have learned how the measures of the angles of a triangle relate to each other. Now, we will learn how the lengths of the sides relate to each other. An important property that describes the relationship among the lengths of the three sides of a right triangle is

called the **Pythagorean Theorem**. This theorem has been used around the world since ancient times. It is named after the Greek philosopher and mathematician, Pythagoras, who lived around 500 BC.

Before we state the Pythagorean Theorem, we need to introduce some terms for the sides of a triangle. Remember that a right triangle has a 90° angle, marked with a small square in the corner. The side of the triangle opposite the 90°90° angle is called the *hypotenuse* and each of the other sides are called *legs*.

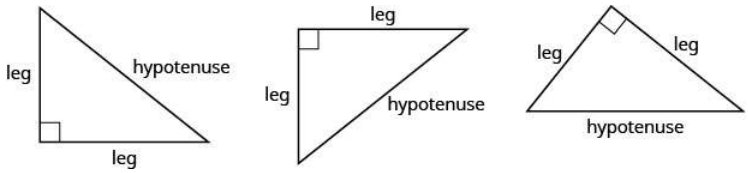


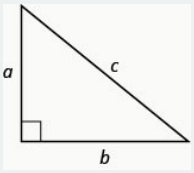
Figure 3.4.4

The Pythagorean Theorem tells how the lengths of the three sides of a right triangle relate to each other. It states that in any right triangle, the sum of the squares of the lengths of the two legs equals the square of the length of the hypotenuse. In symbols we say: in any right triangle, $a^2+b^2=c^2$, where a and b are the lengths of the legs and cc is the length of the hypotenuse.

Writing the formula in every exercise and saying it aloud as you write it, may help you remember the Pythagorean Theorem.

THE PYTHAGOREAN THEOREM

In any right triangle, where a and b are the lengths of the legs, c is the length of the hypotenuse.



Then

$$a^2+b^2=c^2 \tag{3.4.3}$$

To solve exercises that use the Pythagorean Theorem (Equation 3.4.3), we will need to find square roots. We have used the notation $\sqrt{m}$ and the definition:

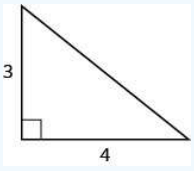
If $m=n^2$, then $\sqrt{m}=n$, for $n\geq 0$.

For example, we found that $\sqrt{25}$ is 5 because $25=5^2$.

Because the Pythagorean Theorem contains variables that are squared, to solve for the length of a side in a right triangle, we will have to use square roots.

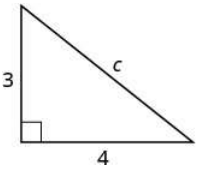
Example 3.4.6

Use the Pythagorean Theorem to find the length of the hypotenuse shown below.



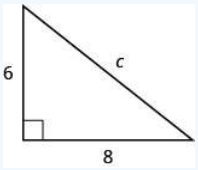
Solution

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	the length of the hypotenuse of the triangle

Step 3. Name. Choose a variable to represent it. Label side c on the figure.	Let c = the length of the hypotenuse. 
Step 4. Translate.	
Write the appropriate formula.	$a^2+b^2=c^2$
Substitute.	$3^2+4^2=c^2$
Step 5. Solve the equation.	$9+16=c^2$
Simplify.	$25=c^2$
Use the definition of square root.	$\sqrt{25}=c$
Simplify.	$5=c$
Step 6. Check. $3^2+4^2 \stackrel{?}{=} 5^2$ $9+16 \stackrel{?}{=} 25$ $25=25$ ✓	
Step 7. Answer the question.	The length of the hypotenuse is 5.

? Try It 3.4.11

Use the Pythagorean Theorem to find the length of the hypotenuse in the triangle shown below.

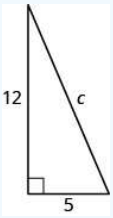


Answer

c=10

? Try It 3.4.12

Use the Pythagorean Theorem to find the length of the hypotenuse in the triangle shown below.

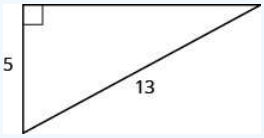


Answer

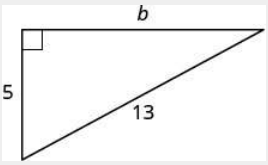
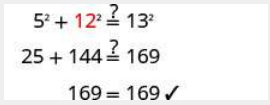
c=13

Example 3.4.7

Use the Pythagorean Theorem to find the length of the leg shown below.

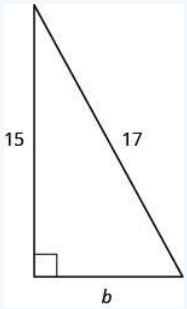


Solution

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	the length of the leg of the triangle
Step 3. Name. Choose a variable to represent it.	Let $b =$ the leg of the triangle.
Label side b.	
Step 4. Translate	
Write the appropriate formula.	$a^2 + b^2 = c^2$
Substitute.	$5^2 + b^2 = 13^2$
Step 5. Solve the equation.	$25 + b^2 = 169$
Isolate the variable term.	$b^2 = 144$
Use the definition of square root.	$b = \sqrt{144}$
Simplify.	$b = 12$
Step 6. Check.  $5^2 + 12^2 \stackrel{?}{=} 13^2$ $25 + 144 \stackrel{?}{=} 169$ $169 = 169$ ✓	
Step 7. Answer the question.	The length of the leg is 12.

Try It 3.4.13

Use the Pythagorean Theorem to find the length of the leg in the triangle shown below.

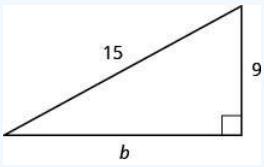


Answer

8

Try It 3.4.14

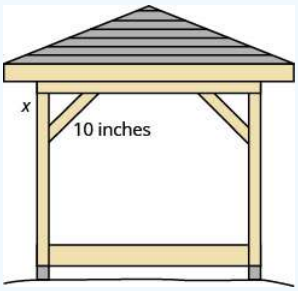
Use the Pythagorean Theorem to find the length of the leg in the triangle shown below.



Answer

12

Example 3.4.8



Kelvin is building a gazebo and wants to brace each corner by placing a 10″ piece of wood diagonally as shown above.

If he fastens the wood so that the ends of the brace are the same distance from the corner, what is the length of the legs of the right triangle formed? Approximate to the nearest tenth of an inch.

Solution

Step 1. Read the problem.

Step 2. Identify what we are looking for.	the distance from the corner that the bracket should be attached
Step 3. Name. Choose a variable to represent it.	Let x = distance from the corner.
Step 4. Translate Write the appropriate formula and substitute.	$a^2+b^2=c^2$ $x^2+x^2=10^2$
Step 5. Solve the equation.	
	$2x^2=100$
Isolate the variable.	$x^2=50$
Simplify. Approximate to the nearest tenth.	$x\approx 7.1$

Step 6. Check.
$a^2+b^2=c^2$
$(7.1)^2+(7.1)^2\approx 10^2$ Yes.

Step 7. Answer the question.	Kelven should fasten each piece of wood approximately 7.1'' from the corner.

? Try It 3.4.15

John puts the base of a 13-foot ladder five feet from the wall of his house as shown below. How far up the wall does the ladder reach?

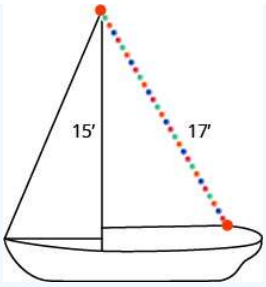


Answer

12 feet

? Try It 3.4.16

Randy wants to attach a 17 foot string of lights to the top of the 15 foot mast of his sailboat, as shown below. How far from the base of the mast should he attach the end of the light string?

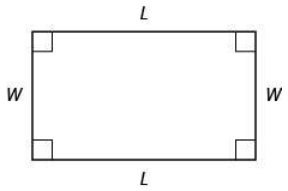


Answer

8 feet

Solve Applications Using Rectangle Properties

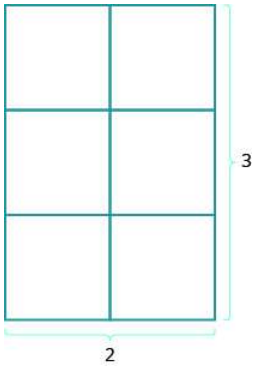
You may already be familiar with the properties of rectangles. Rectangles have four sides and four right (90°) angles. The opposite sides of a **rectangle** are the same length. We refer to one side of the rectangle as the length, L, and its adjacent side as the width, W.



The distance around this rectangle is $L+W+L+W$, or $2L+2W$. This is the **perimeter**, P, of the rectangle.

$$P=2L+2W \tag{3.4.4}$$

What about the **area** of a rectangle? Imagine a rectangular rug that is 2-feet long by 3-feet wide. Its area is 6 square feet. There are six squares in the figure.



$$\begin{aligned} A&=6\\ A&=2\cdot 3\\ A&=L\cdot W \end{aligned} \tag{3.4.5}$$

The area is the length times the width. The formula for the area of a rectangle is

$$A=LW. \tag{3.4.6}$$

PROPERTIES OF RECTANGLES

Rectangles have four sides and four right (90°) angles.

The lengths of opposite sides are equal.

The perimeter of a rectangle is the sum of twice the length and twice the width.

$$P = 2L + 2W \tag{3.4.7}$$

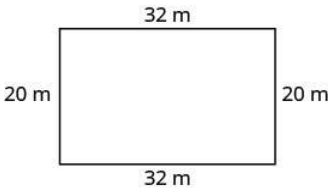
The area of a rectangle is the product of the length and the width.

$$A = L \cdot W \tag{3.4.8}$$

Example 3.4.9

The length of a rectangle is 32 meters and the width is 20 meters. What is the perimeter?

Solution

Step 1. Read the problem. Draw the figure and label it with the given information.	
Step 2. Identify what you are looking for.	the perimeter of a rectangle
Step 3. Name. Choose a variable to represent it.	Let $P =$ the perimeter.
Step 4. Translate.	
Write the appropriate formula.	$P = 2L + 2W$
Substitute.	$P = 2(32\text{ m}) + 2(20\text{ m})$
Step 5. Solve the equation.	$P = 64 + 40$ $P = 104$
Step 6. Check. $P \stackrel{?}{=} 104$ $20 + 32 + 20 + 32 \stackrel{?}{=} 104$ $104 = 104\checkmark$	
Step 7. Answer the question.	The perimeter of the rectangle is 104 meters.

Try It 3.4.17

The length of a rectangle is 120 yards and the width is 50 yards. What is the perimeter?

Answer

340 yards

Try It 3.4.18

The length of a rectangle is 62 feet and the width is 48 feet. What is the perimeter?

Answer

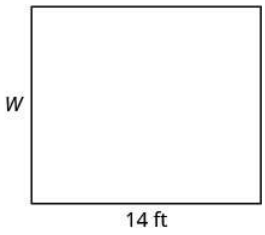
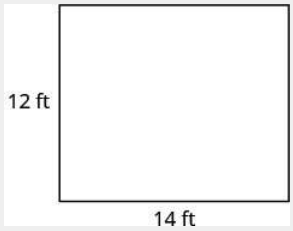
220 feet

https://math.libretexts.org/@go/page/61375

Example 3.4.10

The area of a rectangular room is 168 square feet. The length is 14 feet. What is the width?

Solution

Step 1. Read the problem. Draw the figure and label it with the given information.	
Step 2. Identify what you are looking for.	the width of a rectangular room
Step 3. Name. Choose a variable to represent it.	Let $W =$ the width.
Step 4. Translate.	
Write the appropriate formula.	$A = LW$
Substitute.	$168 = 14W$
Step 5. Solve the equation.	$\frac{168}{14} = \frac{14W}{14}$ $12 = W$
Step 6. Check.  $A = LW$ $168 \stackrel{?}{=} 14 \cdot 12$ $168 = 168$✓	
Step 7. Answer the question.	The width of the room is 12 feet.

Try It 3.4.19

The area of a rectangle is 598 square feet. The length is 23 feet. What is the width?

Answer

26 feet

Try It 3.4.20

The width of a rectangle is 21 meters. The area is 609 square meters. What is the length?

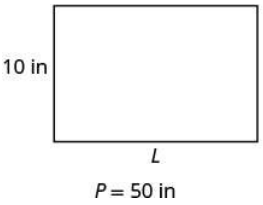
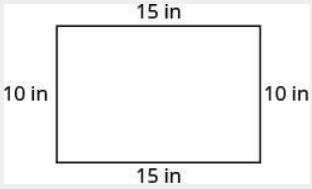
Answer

29 meters

Example 3.4.11

Find the length of a rectangle with perimeter 50 inches and width 10 inches.

Solution

Step 1. Read the problem. Draw the figure and label it with the given information.	
Step 2. Identify what you are looking for.	the length of the rectangle
Step 3. Name. Choose a variable to represent it.	Let $L =$ the length.
Step 4. Translate.	
Write the appropriate formula.	$P = 2L + 2W$
Substitute.	$50 = 2L + 2(10)$
Step 5. Solve the equation.	$50 - 20 = 2L + 20 - 20$ $30 = 2L$ $\frac{30}{2} = \frac{2L}{2}$ $15 = L$
Step 6. Check.  $P = 50$ $15 + 10 + 15 + 10 \stackrel{?}{=} 50$ $50 = 50\checkmark$	
Step 7. Answer the question.	The length is 15 inches.

Try It 3.4.21

Find the length of a rectangle with: perimeter 80 and width 25.

Answer

15

Try It 3.4.22

Find the length of a rectangle with: perimeter 30 and width 6.

Answer

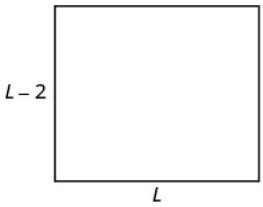
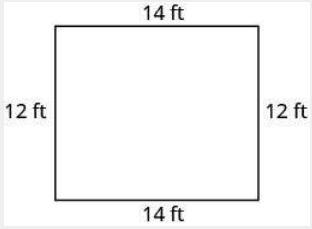
9

We have solved problems where either the length or width was given, along with the perimeter or area; now we will learn how to solve problems in which the width is defined in terms of the length. We will wait to draw the figure until we write an expression for the width so that we can label one side with that expression.

Example 3.4.12

The width of a rectangle is two feet less than the length. The perimeter is 52 feet. Find the length and width.

Solution

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	the length and width of a rectangle
Step 3. Name. Choose a variable to represent it. Since the width is defined in terms of the length, we let $L =$ length. The width is two feet less than the length, so we let $L-2$ width.	 $P = 52$ ft
Step 4. Translate.	
Write the appropriate formula. The formula for the perimeter of a rectangle relates all the information.	$P = 2L + 2W$
Substitute in the given information.	$52 = 2L + 2(L-2)$
Step 5. Solve the equation.	$52 = 2L + 2L - 4$
Combine like terms.	$52 = 4L - 4$
Add 4 to each side.	$56 = 4L$
Divide by 4.	$\frac{56}{4} = \frac{4L}{4}$ $14 = L$ The length is 14 feet.
Now we need to find the width.	The width is $L-2$. $L-2$ $14-2$ 12 The width is 12 feet.
Step 6. Check. Since $14+12+14+12=52$, this works! 	
Step 7. Answer the question.	The length is 14 feet and the width is 12 feet.

Try It 3.4.23

The width of a rectangle is seven meters less than the length. The perimeter is 58 meters. Find the length and width.

Answer

18 meters, 11 meters

Try It 3.4.24

The length of a rectangle is eight feet more than the width. The perimeter is 60 feet. Find the length and width.

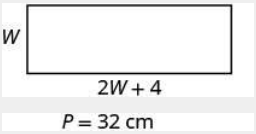
Answer

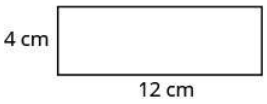
19 feet, 11 feet

Example 3.4.13

The length of a rectangle is four centimeters more than twice the width. The perimeter is 32 centimeters. Find the length and width.

Solution

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	the length and the width
Step 3. Name. Choose a variable to represent the width.	Let W = width
The length is four more than twice the width.	$2W + 4$ = length  $P = 32$ cm
Step 4. Translate	
Write the appropriate formula.	$P = 2L + 2W$
Substitute in the given information.	$32 = 2(2W + 4) + 2W$
Step 5. Solve the equation.	$32 = 4W + 8 + 2W$ $32 = 6W + 8$ $24 = 6W$ $4 = W$ (width) $2W + 4$ (length) $2(4) + 4$ 12 The length is 12 cm.

Step 6. Check.  $P = 2L + 2W$ $32 \stackrel{?}{=} 2 \cdot 12 + 2 \cdot 4$ $32 = 32\checkmark$	
Step 7. Answer the question.	The length is 12 cm and the width is 4 cm.

Try It 3.4.25

The length of a rectangle is eight more than twice the width. The perimeter is 64. Find the length and width.

Answer

24, 8

Try It 3.4.26

The width of a rectangle is six less than twice the length. The perimeter is 18. Find the length and width.

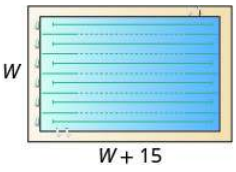
Answer

5, 4

Example 3.4.14

The perimeter of a rectangular swimming pool is 150 feet. The length is 15 feet more than the width. Find the length and width.

Solution

Step 1. Read the problem. Draw the figure and label it with the given information.	 $P = 150$ ft
Step 2. Identify what you are looking for.	the length and the width of the pool
Step 3. Name. Choose a variable to represent the width. The length is 15 feet more than the width.	Let W = width $W + 15$ = length
Step 4. Translate	
Write the appropriate formula.	$P = 2L + 2W$
Substitute.	$150 = 2(W + 15) + 2W$

Step 5. Solve the equation.	$150 = 2W + 30 + 2W$ $150 = 4W + 30$ $120 = 4W$ $30 = W$ (the width of the pool) $W + 15$ (the length of the pool) $30 + 15$ 45
Step 6. Check. $P = 2L + 2W$ $150 \stackrel{?}{=} 2(45) + 2(30)$ $150 = 150\checkmark$	
Step 7. Answer the question.	The length of the pool is 45 feet and the width is 30 feet.

? Try It 3.4.27

The perimeter of a rectangular swimming pool is 200 feet. The length is 40 feet more than the width. Find the length and width.

Answer

70 feet, 30 feet

? Try It 3.4.28

The length of a rectangular garden is 30 yards more than the width. The perimeter is 300 yards. Find the length and width.

Answer

90 yards, 60 yards

Key Concepts

- **Problem-Solving Strategy for Geometry Applications**
  1. **Read** the problem and make all the words and ideas are understood. Draw the figure and label it with the given information.
  2. **Identify** what we are looking for.
  3. **Name** what we are looking for by choosing a variable to represent it.
  4. **Translate** into an equation by writing the appropriate formula or model for the situation. Substitute in the given information.
  5. **Solve** the equation using good algebra techniques.
  6. **Check** the answer in the problem and make sure it makes sense.
  7. **Answer** the question with a complete sentence.
- **Triangle Properties For △ABC**
  Angle measures:
  - $m\angle A + m\angle B + m\angle C = 180$

  Perimeter:
  - $P = a + b + c$

  Area:
  - $A = \frac{1}{2}bh$, b=base,h=height

  A right triangle has one 90° angle.
- **The Pythagorean Theorem** In any right triangle, $a^2 + b^2 = c^2$ where c is the length of the hypotenuse and a and b are the lengths of the legs.

- **Properties of Rectangles**
  - Rectangles have four sides and four right (90°) angles.
  - The lengths of opposite sides are equal.
  - The perimeter of a rectangle is the sum of twice the length and twice the width: $P = 2L + 2W$.
  - The area of a rectangle is the length times the width: $A = LW$.

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3.4E: Exercises

Practice Makes Perfect

Solving Applications Using Triangle Properties

In the following exercises, solve using triangle properties.

Exercise 3.4E.1

The measures of two angles of a triangle are 26 and 98 degrees. Find the measure of the third angle.

Answer

56 degrees

Exercise 3.4E.2

The measures of two angles of a triangle are 61 and 84 degrees. Find the measure of the third angle.

Exercise 3.4E.3

The measures of two angles of a triangle are 105 and 31 degrees. Find the measure of the third angle.

Answer

44 degrees

Exercise 3.4E.4

The measures of two angles of a triangle are 47 and 72 degrees. Find the measure of the third angle.

Exercise 3.4E.5

The perimeter of a triangular pool is 36 yards. The lengths of two sides are 10 yards and 15 yards. How long is the third side?

Answer

11 feet

Exercise 3.4E.6

A triangular courtyard has perimeter 120 meters. The lengths of two sides are 30 meters and 50 meters. How long is the third side?

Exercise 3.4E.7

If a triangle has sides 6 feet and 9 feet and the perimeter is 23 feet, how long is the third side?

Answer

8 feet

Exercise 3.4E.8

If a triangle has sides 14 centimeters and 18 centimeters and the perimeter is 49 centimeters, how long is the third side?

Exercise 3.4E.9

A triangular flag has base one foot and height 1.5 foot. What is its area?

Answer

0.75 sq. ft.

Exercise 3.4E.10

A triangular window has base eight feet and height six feet. What is its area?

Exercise 3.4E.11

What is the base of a triangle with area 207 square inches and height 18 inches?

Answer

23 inches

Exercise 3.4E.12

What is the height of a triangle with area 893 square inches and base 38 inches?

Exercise 3.4E.13

One angle of a right triangle measures 33 degrees. What is the measure of the other small angle?

Answer

57

Exercise 3.4E.14

One angle of a right triangle measures 51 degrees. What is the measure of the other small angle?

Exercise 3.4E.15

One angle of a right triangle measures 22.5 degrees. What is the measure of the other small angle?

Answer

67.5

Exercise 3.4E.16

One angle of a right triangle measures 36.5 degrees. What is the measure of the other small angle?

Exercise 3.4E.17

The perimeter of a triangle is 39 feet. One side of the triangle is one foot longer than the second side. The third side is two feet longer than the second side. Find the length of each side.

Answer

13 ft., 12 ft., 14 ft.

Exercise 3.4E.18

The perimeter of a triangle is 35 feet. One side of the triangle is five feet longer than the second side. The third side is three feet longer than the second side. Find the length of each side.

Exercise 3.4E.19

One side of a triangle is twice the shortest side. The third side is five feet more than the shortest side. The perimeter is 17 feet. Find the lengths of all three sides.

Answer

3 ft., 6 ft., 8 ft.

Exercise 3.4E.20

One side of a triangle is three times the shortest side. The third side is three feet more than the shortest side. The perimeter is 13 feet. Find the lengths of all three sides.

Exercise 3.4E.21

The two smaller angles of a right triangle have equal measures. Find the measures of all three angles.

Answer

$45^\circ, 45^\circ, 90^\circ$

Exercise 3.4E.22

The measure of the smallest angle of a right triangle is 20° less than the measure of the next larger angle. Find the measures of all three angles.

Exercise 3.4E.23

The angles in a triangle are such that one angle is twice the smallest angle, while the third angle is three times as large as the smallest angle. Find the measures of all three angles.

Answer

$30^\circ, 60^\circ, 90^\circ$

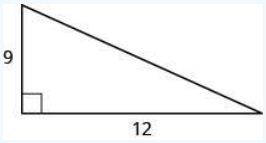
Exercise 3.4E.24

The angles in a triangle are such that one angle is 20° more than the smallest angle, while the third angle is three times as large as the smallest angle. Find the measures of all three angles.

Use the Pythagorean Theorem

In the following exercises, use the Pythagorean Theorem to find the length of the hypotenuse.

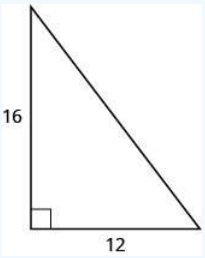
Exercise 3.4E. 25



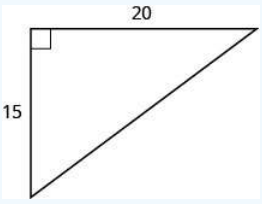
Answer

15

Exercise 3.4E. 26



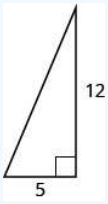
Exercise 3.4E. 27



Answer

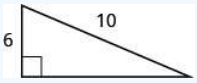
25

Exercise 3.4E. 28



In the following exercises, use the Pythagorean Theorem to find the length of the leg. Round to the nearest tenth, if necessary.

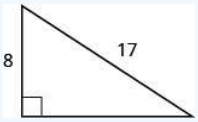
Exercise 3.4E. 29



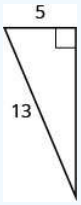
Answer

8

Exercise 3.4E.30



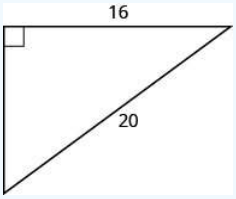
Exercise 3.4E.31



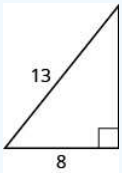
Answer

12

Exercise 3.4E.32



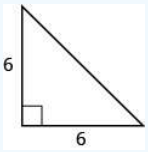
Exercise 3.4E.33



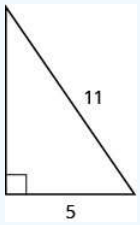
Answer

10.2

Exercise 3.4E.34



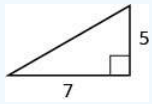
Exercise 3.4E.35



Answer

9.8

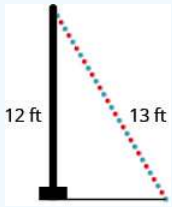
Exercise 3.4E.36



In the following exercises, solve using the Pythagorean Theorem. Approximate to the nearest tenth, if necessary.

Exercise 3.4E.37

A 13-foot string of lights will be attached to the top of a 12-foot pole for a holiday display, as shown below. How far from the base of the pole should the end of the string of lights be anchored?



Answer

5 feet

Exercise 3.4E.38

Pam wants to put a banner across her garage door, as shown below, to congratulate her son for his college graduation. The garage door is 12 feet high and 16 feet wide. How long should the banner be to fit the garage door?



Exercise 3.4E. 39

Chi is planning to put a path of paving stones through her flower garden, as shown below. The flower garden is a square with side 10 feet. What will the length of the path be?



Answer

14.1 feet

Exercise 3.4E. 40

Brian borrowed a 20 foot extension ladder to use when he paints his house. If he sets the base of the ladder 6 feet from the house, as shown below, how far up will the top of the ladder reach?



Solve Applications Using Rectangle Properties

In the following exercises, solve using rectangle properties.

Exercise 3.4E. 41

The length of a rectangle is 85 feet and the width is 45 feet. What is the perimeter?

Answer

260 feet

Exercise 3.4E. 42

The length of a rectangle is 26 inches and the width is 58 inches. What is the perimeter?

Exercise 3.4E. 43

A rectangular room is 15 feet wide by 14 feet long. What is its perimeter?

Answer

58 feet

Exercise 3.4E.44

A driveway is in the shape of a rectangle 20 feet wide by 35 feet long. What is its perimeter?

Exercise 3.4E.45

The area of a rectangle is 414 square meters. The length is 18 meters. What is the width?

Answer

23 meters

Exercise 3.4E.46

The area of a rectangle is 782 square centimeters. The width is 17 centimeters. What is the length?

Exercise 3.4E.47

The width of a rectangular window is 24 inches. The area is 624 square inches. What is the length?

Answer

26 inches

Exercise 3.4E.48

The length of a rectangular poster is 28 inches. The area is 1316 square inches. What is the width?

Exercise 3.4E.49

Find the length of a rectangle with perimeter 124 and width 38.

Answer

24

Exercise 3.4E.50

Find the width of a rectangle with perimeter 92 and length 19.

Exercise 3.4E.51

Find the width of a rectangle with perimeter 16.2 and length 3.2.

Answer

4.9

Exercise 3.4E.52

Find the length of a rectangle with perimeter 20.2 and width 7.8.

Exercise 3.4E.53

The length of a rectangle is nine inches more than the width. The perimeter is 46 inches. Find the length and the width.

Answer

16 in., 7 in.

Exercise 3.4E.54

The width of a rectangle is eight inches more than the length. The perimeter is 52 inches. Find the length and the width.

Exercise 3.4E.55

The perimeter of a rectangle is 58 meters. The width of the rectangle is five meters less than the length. Find the length and the width of the rectangle.

Answer

17 m, 12 m

Exercise 3.4E.56

The perimeter of a rectangle is 62 feet. The width is seven feet less than the length. Find the length and the width.

Exercise 3.4E.57

The width of the rectangle is 0.7 meters less than the length. The perimeter of a rectangle is 52.6 meters. Find the dimensions of the rectangle.

Answer

13.5 m length, 12.8 m width

Exercise 3.4E.58

The length of the rectangle is 1.1 meters less than the width. The perimeter of a rectangle is 49.4 meters. Find the dimensions of the rectangle.

Exercise 3.4E.59

The perimeter of a rectangle is 150 feet. The length of the rectangle is twice the width. Find the length and width of the rectangle.

Answer

50 ft., 25 ft.

Exercise 3.4E.60

The length of a rectangle is three times the width. The perimeter of the rectangle is 72 feet. Find the length and width of the rectangle.

Exercise 3.4E.61

The length of a rectangle is three meters less than twice the width. The perimeter of the rectangle is 36 meters. Find the dimensions of the rectangle.

Answer

7 m width, 11 m length

Exercise 3.4E.62

The length of a rectangle is five inches more than twice the width. The perimeter is 34 inches. Find the length and width.

Exercise 3.4E.63

The perimeter of a rectangular field is 560 yards. The length is 40 yards more than the width. Find the length and width of the field.

Answer

160 yd., 120 yd.

Exercise 3.4E.64

The perimeter of a rectangular atrium is 160 feet. The length is 16 feet more than the width. Find the length and width of the atrium.

Exercise 3.4E.65

A rectangular parking lot has perimeter 250 feet. The length is five feet more than twice the width. Find the length and width of the parking lot.

Answer

85 ft., 40 ft.

Exercise 3.4E.66

A rectangular rug has perimeter 240 inches. The length is 12 inches more than twice the width. Find the length and width of the rug.

Everyday Math

Exercise 3.4E.67

Christa wants to put a fence around her triangular flowerbed. The sides of the flowerbed are six feet, eight feet and 10 feet. How many feet of fencing will she need to enclose her flowerbed?

Answer

24 feet

Exercise 3.4E.68

Jose just removed the children's playset from his back yard to make room for a rectangular garden. He wants to put a fence around the garden to keep out the dog. He has a 50 foot roll of fence in his garage that he plans to use. To fit in the backyard, the width of the garden must be 10 feet. How long can he make the other length?

Writing Exercises

Exercise 3.4E.69

If you need to put tile on your kitchen floor, do you need to know the perimeter or the area of the kitchen? Explain your reasoning.

Answer

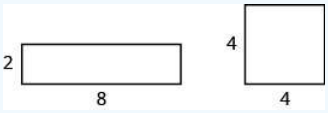
area; answers will vary

Exercise 3.4E.70

If you need to put a fence around your backyard, do you need to know the perimeter or the area of the backyard? Explain your reasoning.

Exercise 3.4E.71

Look at the two figures below.



1. Which figure looks like it has the larger area?
2. Which looks like it has the larger perimeter?
3. Now calculate the area and perimeter of each figure.
4. Which has the larger area?
5. Which has the larger perimeter?

Answer

1. Answers will vary.
2. Answers will vary.
3. Answers will vary.
4. The areas are the same.
5. The 2x8 rectangle has a larger perimeter than the 4x4 square.

Exercise 3.4E.72

Write a geometry word problem that relates to your life experience, then solve it and explain all your steps.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
solve applications using triangle properties.			
use the Pythagorean Theorem.			
solve applications using rectangle properties.			

ⓑ What does this checklist tell you about your mastery of this section? What steps will you take to improve?

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3.5: Solve Uniform Motion Applications

Learning Objectives

By the end of this section, you will be able to:

- Solve uniform motion applications

Note

Before you get started, take this readiness quiz.

1. Find the distance traveled by a car going 70 miles per hour for 3 hours.
   If you missed this problem, review Exercise 2.6.1.
2. Solve $x+1.2(x-10)=98$.
   If you missed this problem, review Exercise 2.4.7.
3. Convert 90 minutes to hours.
   If you missed this problem, review Exercise 1.11.1.

Solve Uniform Motion Applications

When planning a road trip, it often helps to know how long it will take to reach the destination or how far to travel each day. We would use the distance, rate, and time formula, D=rt, which we have already seen.

In this section, we will use this formula in situations that require a little more algebra to solve than the ones we saw earlier. Generally, we will be looking at comparing two scenarios, such as two vehicles traveling at different rates or in opposite directions. When the speed of each vehicle is constant, we call applications like this *uniform motion problems*.

Our problem-solving strategies will still apply here, but we will add to the first step. The first step will include drawing a diagram that shows what is happening in the example. Drawing the diagram helps us understand what is happening so that we will write an appropriate equation. Then we will make a table to organize the information, like we did for the money applications.

The steps are listed here for easy reference:

USE A PROBLEM-SOLVING STRATEGY IN DISTANCE, RATE, AND TIME APPLICATIONS.

1. **Read** the problem. Make sure all the words and ideas are understood.
   - Draw a diagram to illustrate what it happening.
   - Create a table to organize the information.
   - Label the columns rate, time, distance.
   - List the two scenarios.
   - Write in the information you know.

	Rate	•	Time	=	Distance

2. **Identify** what we are looking for.
3. **Name** what we are looking for. Choose a variable to represent that quantity.
   - Complete the chart.
   - Use variable expressions to represent that quantity in each row.
   - Multiply the rate times the time to get the distance.
4. **Translate** into an equation.
   - Restate the problem in one sentence with all the important information.
   - Then, translate the sentence into an equation.
5. **Solve** the equation using good algebra techniques.

6. **Check** the answer in the problem and make sure it makes sense.
7. **Answer** the question with a complete sentence.

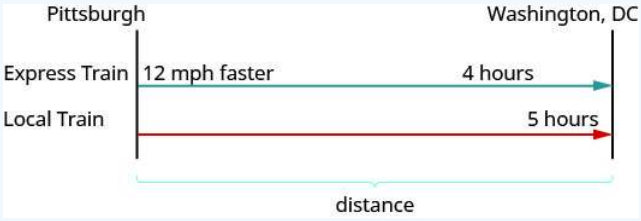
Example 3.5.1

An express train and a local train leave Pittsburgh to travel to Washington, D.C. The express train can make the trip in 4 hours and the local train takes 5 hours for the trip. The speed of the express train is 12 miles per hour faster than the speed of the local train. Find the speed of both trains.

Solution

Step 1. Read the problem. Make sure all the words and ideas are understood.

Draw a diagram to illustrate what it happening. Shown below is a sketch of what is happening in the example.



	Rate (mph)	•	Time (hrs)	=	Distance (miles)
Express			4		
Local			5		

Create a table to organize the information. Label the columns “Rate,” “Time,” and “Distance.” List the two scenarios. Write in the information you know.

Step 2. Identify what we are looking for.

We are asked to find the speed of both trains. Notice that the distance formula uses the word “rate,” but it is more common to use “speed” when we talk about vehicles in everyday English.

Step 3. Name what we are looking for. Choose a variable to represent that quantity.

Complete the chart Use variable expressions to represent that quantity in each row. We are looking for the speed of the trains. Let’s let r represent the speed of the local train. Since the speed of the express train is 12 mph faster, we represent that as r+12.

$$\begin{aligned} r &= \text{speed of the local train} \\ r+12 &= \text{speed of the express train} \end{aligned}$$

Fill in the speeds into the chart.

	Rate (mph)	•	Time (hrs)	=	Distance (miles)
Express	$r+12$		4		
Local	r		5		

Multiply the rate times the time to get the distance.

	Rate (mph)	•	Time (hrs)	=	Distance (miles)
Express	$r+12$		4		$4(r+12)$
Local	r		5		$5r$

Step 4. Translate into an equation.

Restate the problem in one sentence with all the important information. Then, translate the sentence into an equation.

	distance traveled by express train	=	distance traveled by local train
Translate to an equation.	$4(r+12)$	=	$5r$

- The equation to model this situation will come from the relation between the distances. Look at the diagram we drew above. How is the distance traveled by the express train related to the distance traveled by the local train?
- Since both trains leave from Pittsburgh and travel to Washington, D.C. they travel the same distance. So we write:

Step 5. Solve the equation using good algebra techniques.

Now solve this equation.	$4(r+12)=5r$ $4r+48=5r$ $48=r$ So the speed of the local train is 48 mph.
Find the speed of the express train.	$r+12$ $48+12$ 60 he speed of the express train is 60 mph.

Step 6. Check the answer in the problem and make sure it makes sense. \[\begin{array}{ll}{\text { express train }} & {60 \mathrm{mph}(4 \text { hours })=240 \mathrm{miles}} \\ {\text { local train }} & {48 \mathrm{mph}(5 \text { hours })=240 \mathrm{miles} \checkmark \end{array}\]

Step 7. Answer the question with a complete sentence.

The speed of the local train is 48 mph and the speed of the express train is 60 mph.

Try It 3.5.2

Wayne and Dennis like to ride the bike path from Riverside Park to the beach. Dennis's speed is seven miles per hour faster than Wayne's speed, so it takes Wayne 2 hours to ride to the beach while it takes Dennis 1.5 hours for the ride. Find the speed of both bikers.

Answer

Wayne 21 mph, Dennis 28 mph

Try It 3.5.3

Jeromy can drive from his house in Cleveland to his college in Chicago in 4.5 hours. It takes his mother 6 hours to make the same drive. Jeromy drives 20 miles per hour faster than his mother. Find Jeromy's speed and his mother's speed.

Answer

Jeromy 80 mph, mother 60 mph

In Exercise 3.5.4, the last example, we had two trains traveling the same distance. The diagram and the chart helped us write the equation we solved. Let's see how this works in another case.

Example 3.5.4

Christopher and his parents live 115 miles apart. They met at a restaurant between their homes to celebrate his mother's birthday. Christopher drove 1.5 hours while his parents drove 1 hour to get to the restaurant. Christopher's average speed was 10 miles per hour faster than his parents' average speed. What were the average speeds of Christopher and of his parents as they drove to the restaurant?

Solution

Step 1. Read the problem. Make sure all the words and ideas are understood.

Draw a diagram to illustrate what it happening. Below shows a sketch of what is happening in the example.



Create a table to organize the information.

Label the columns rate, time, distance.

List the two scenarios.

Write in the information you know.

	Rate (mph)	•	Time (hrs)	=	Distance (miles)
Christopher			1.5		
Parents			1		
					115

Step 2. Identify what we are looking for.

We are asked to find the average speeds of Christopher and his parents.

Step 3. Name what we are looking for. Choose a variable to represent that quantity.

Complete the chart.

Use variable expressions to represent that quantity in each row.

We are looking for their average speeds. Let's let r represent the average speed of the parents. Since the Christopher's speed is 10 mph faster, we represent that as r+10.

Fill in the speeds into the chart.

	Rate (mph)	•	Time (hrs)	=	Distance (miles)
Christopher	$r+10$		1.5		$1.5(r+10)$
Parents	r		1		r
					115

Multiply the rate times the time to get the distance.

Step 4. Translate into an equation.

Restate the problem in one sentence with all the important information. Then, translate the sentence into an equation. Again, we need to identify a relationship between the distances in order to write an equation. Look at the diagram we created above and notice the relationship between the distance Christopher traveled and the distance his parents traveled.

The distance Christopher travelled plus the distance his parents travel must add up to 115 miles. So we write:

	distance traveled by Christopher	+	distance traveled by his parents	= 115
Translate to an equation.	$1.5(r+10)$	+	r	$=115$

Step 5. Solve the equation using good algebra techniques.

	$1.5(r+10)+r=115$
	$1.5r+15+r=115$
Now solve this equation.	$2.5r+15=115$
	$2.5r=100$
	$r=40$
	so the parents' speed was 40 mph.
	$r+10$
Christopher's speed is r + 10	$40+10$
	50
	Christopher's speed was 50 mph.

Step 6. Check the answer in the problem and make sure it makes sense.

Christopher drove	50 mph (1.5 hours)	=	75 miles
His parents drove	40 mph (1 hour)	=	40 miles
			115 miles

Step 7. Answer the question with a complete sentence.

Christopher's speed was 50 mph.
His parents' speed was 40 mph.

? Try It 3.5.5

Carina is driving from her home in Anaheim to Berkeley on the same day her brother is driving from Berkeley to Anaheim, so they decide to meet for lunch along the way in Buttonwillow. The distance from Anaheim to Berkeley is 410 miles. It takes Carina 3 hours to get to Buttonwillow, while her brother drives 4 hours to get there. The average speed Carina's brother drove was 15 miles per hour faster than Carina's average speed. Find Carina's and her brother's average speeds.

Answer

Carina 50 mph, brother 65 mph

? Try It 3.5.6

Ashley goes to college in Minneapolis, 234 miles from her home in Sioux Falls. She wants her parents to bring her more winter clothes, so they decide to meet at a restaurant on the road between Minneapolis and Sioux Falls. Ashley and her parents both drove 2 hours to the restaurant. Ashley's average speed was seven miles per hour faster than her parents' average speed. Find Ashley's and her parents' average speed.

Answer

parents 55 mph, Ashley 62 mph

As you read the next example, think about the relationship of the distances traveled. Which of the previous two examples is more similar to this situation?

✓ Example 3.5.7

Two truck drivers leave a rest area on the interstate at the same time. One truck travels east and the other one travels west. The truck traveling west travels at 70 mph and the truck traveling east has an average speed of 60 mph. How long will they travel before they are 325 miles apart?

Solution

Step 1. Read the problem. Make sure all the words and ideas are understood.

Draw a diagram to illustrate what it happening.



Create a table to organize the information.

	Rate (mph)	• Time (hrs)	= Distance (miles)
West	70		
East	60		
			325

Step 2. Identify what we are looking for.

We are asked to find the amount of time the trucks will travel until they are 325 miles apart.

Step 3. Name what we are looking for. Choose a variable to represent that quantity.

We are looking for the time traveled. Both trucks will travel the same amount of time. Let's call the time t. Since their speeds are different, they will travel different distances. Complete the chart.

	Rate (mph)	• Time (hrs)	= Distance (miles)
West	70	t	$70t$
East	60	t	$60t$
			325

Step 4. Translate into an equation.

We need to find a relation between the distances in order to write an equation. Looking at the diagram, what is the relationship between the distance each of the trucks will travel? The distance traveled by the truck going west plus the distance traveled by the truck going east must add up to 325 miles. So we write:

	distance traveled by westbound truck	+	distance traveled by eastbound truck	= 32
Translate to an equation.	70t	+	60t	= 32

Step 5. Solve the equation using good algebra techniques.

$$\begin{aligned} \text{Now solve this equation.} \quad 70t+60t &= 325 \\ 130t &= 325 \\ t &= 2.5 \end{aligned} \tag{3.5.1}$$

Step 6. Check the answer in the problem and make sure it makes sense.

$$\begin{aligned} \text{Truck going West} \quad 70 \text{ mph } (2.5 \text{ hours}) &= 175 \text{ miles} \\ \text{Truck going East} \quad 60 \text{ mph } (2.5 \text{ hour}) &= \underline{150 \text{ miles}} \\ & \quad 325 \text{ miles} \end{aligned}$$

Step 7. Answer the question with a complete sentence. It will take the truck 2.5 hours to be 325 miles apart.

Try It 3.5.8

Pierre and Monique leave their home in Portland at the same time. Pierre drives north on the turnpike at a speed of 75 miles per hour while Monique drives south at a speed of 68 miles per hour. How long will it take them to be 429 miles apart?

Answer

3 hours

Try It 3.5.9

Thanh and Nhat leave their office in Sacramento at the same time. Thanh drives north on I-5 at a speed of 72 miles per hour. Nhat drives south on I-5 at a speed of 76 miles per hour. How long will it take them to be 330 miles apart?

Answer

2.2 hours

MATCHING UNITS IN PROBLEMS

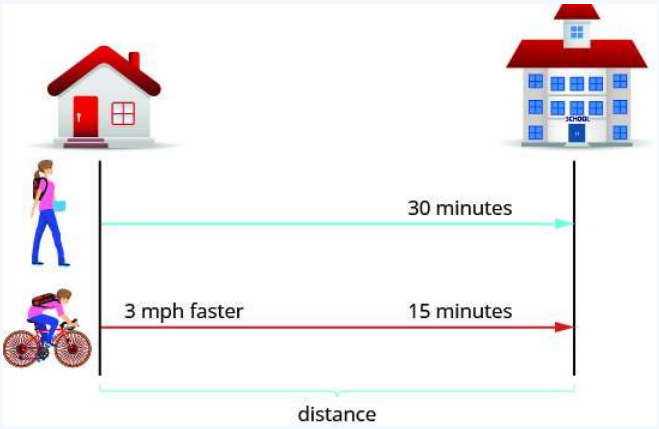
It is important to make sure the units match when we use the distance rate and time formula. For instance, if the rate is in miles per hour, then the time must be in hours.

Example 3.5.10

When Katie Mae walks to school, it takes her 30 minutes. If she rides her bike, it takes her 15 minutes. Her speed is three miles per hour faster when she rides her bike than when she walks. What are her walking speed and her speed riding her bike?

Solution

First, we draw a diagram that represents the situation to help us see what is happening.



We are asked to find her speed walking and riding her bike. Let's call her walking speed r. Since her biking speed is three miles per hour faster, we will call that speed r+3. We write the speeds in the chart.

The speed is in miles per hour, so we need to express the times in hours, too, in order for the units to be the same. Remember, one hour is 60 minutes. So:

$$\begin{aligned} 30 \text{ minutes is } \tfrac{30}{60} \text{ or } \tfrac{1}{2} \text{ hour} \\ 15 \text{ minutes is } \tfrac{15}{60} \text{ or } \tfrac{1}{4} \text{ hour} \end{aligned} \qquad (3.5.2)$$

Next, we multiply rate times time to fill in the distance column.

	Rate (mph)	· Time (hrs)	= Distance (miles)
Walk	r	$\frac{1}{2}$	$\frac{1}{2}r$
Bike	$r+3$	$\frac{1}{4}$	$\frac{1}{4}(r+3)$

The equation will come from the fact that the distance from Katie Mae's home to her school is the same whether she is walking or riding her bike.

So we say:

	distance walked = distance covered by bike
Translate into an equation.	$\frac{1}{2}r = \frac{1}{4}(r+3)$
Solve this equation.	$\frac{1}{2}r = \frac{1}{4}(r+3)$
Clear the fractions by multiplying by the LCD of all the fractions in the equation.	$8 \cdot \frac{1}{2}r = 8 \cdot \frac{1}{4}(r+3)$
Simplify.	$4r = 2(r+3)$ $4r = 2r + 6$ $2r = 6$ $r = 3$ mph (Katie Mae's walking speed) $r + 3$ biking speed $3 + 3$ 6 mph (Katie Mae's biking speed)
Let's check if this works. Walk 3 mph (0.5 hour) = 1.5 miles Bike 6 mph (0.25 hour) = 1.5 miles	
Yes, either way Katie Mae travels 1.5 miles to school.	Katie Mae's walking speed is 3 mph. Her speed riding her bike is 6 mph.

Try It 3.5.11

Suzy takes 50 minutes to hike uphill from the parking lot to the lookout tower. It takes her 30 minutes to hike back down to the parking lot. Her speed going downhill is 1.2 miles per hour faster than her speed going uphill. Find Suzy's uphill and downhill speeds.

Answer

uphill 1.8 mph, downhill three mph

Try It 3.5.12

Llewyn takes 45 minutes to drive his boat upstream from the dock to his favorite fishing spot. It takes him 30 minutes to drive the boat back downstream to the dock. The boat's speed going downstream is four miles per hour faster than its speed going upstream. Find the boat's upstream and downstream speeds.

Answer

upstream 8 mph, downstream 12 mph

In the distance, rate, and time formula, time represents the actual amount of **elapsed time** (in hours, minutes, etc.). If a problem gives us starting and ending times as clock times, we must find the elapsed time in order to use the formula.

Example 3.5.13

Hamilton loves to travel to Las Vegas, 255 miles from his home in Orange County. On his last trip, he left his house at 2:00 pm. The first part of his trip was on congested city freeways. At 4:00 pm, the traffic cleared and he was able to drive through the desert at a speed 1.75 times as fast as when he drove in the congested area. He arrived in Las Vegas at 6:30 pm. How fast was he driving during each part of his trip?

Solution

A diagram will help us model this trip.



Next, we create a table to organize the information.

We know the total distance is 255 miles. We are looking for the rate of speed for each part of the trip. The rate in the desert is 1.75 times the rate in the city. If we let r= the rate in the city, then the rate in the desert is 1.75r.

The times here are given as clock times. Hamilton started from home at 2:00 pm and entered the desert at 4:30 pm. So he spent two hours driving the congested freeways in the city. Then he drove faster from 4:00 pm until 6:30 pm in the desert. So he drove 2.5 hours in the desert.

Now, we multiply the rates by the times.

	Rate (mph)	• Time (hrs)	= Distance (miles)
City	r	2	$2r$
Desert	$1.75r$	2.5	$2.5(1.75r)$
			255

By looking at the diagram below, we can see that the sum of the distance driven in the city and the distance driven in desert is 255 miles.

	distance driven in the city + distance driven in desert = 255
Translate into an equation.	$2r + 2.5(1.75r) = 255$
Solve this equation.	$2r + 2.5(1.75r) = 255$ $2r + 4.375r = 255$ $6.375r = 255$ $r = 40$ mph city $1.75r$ desert speed $1.75(40)$ 70 mph
Check. City 40 mph (2 hours) = 80 miles Desert 70 mph (2.5 hours) = 175 miles 255 miles	
	Hamilton drove 40 mph in the city and 70 mph in the desert.

Try It 3.5.14

Cruz is training to compete in a triathlon. He left his house at 6:00 and ran until 7:30. Then he rode his bike until 9:45. He covered a total distance of 51 miles. His speed when biking was 1.6 times his speed when running. Find Cruz's biking and running speeds.

Answer

biking 16 mph, running 10 mph

Try It 3.5.15

Phuong left home on his bicycle at 10:00. He rode on the flat street until 11:15, then rode uphill until 11:45. He rode a total of 31 miles. His speed riding uphill was 0.6 times his speed on the flat street. Find his speed biking uphill and on the flat street.

Answer

uphill 12 mph, flat street 20 mph

Key Concepts

- **Distance, Rate, and Time**
  - $D = rt$ where D = distance, r = rate, t = time
- **Problem-Solving Strategy—Distance, Rate, and Time Applications**
  1. **Read** the problem. Make sure all the words and ideas are understood.
     Draw a diagram to illustrate what it happening.
     Create a table to organize the information: Label the columns rate, time, distance. List the two scenarios. Write in the information you know.
  2. **Identify** what we are looking for.
  3. **Name** what we are looking for. Choose a variable to represent that quantity.
     Complete the chart.
     Use variable expressions to represent that quantity in each row.
     Multiply the rate times the time to get the distance.
  4. **Translate** into an equation.
     Restate the problem in one sentence with all the important information.
     Then, translate the sentence into an equation.
  5. **Solve** the equation using good algebra techniques.
  6. **Check** the answer in the problem and make sure it makes sense.
  7. **Answer** the question with a complete sentence.

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3.5E: Exercises

Practice Makes Perfect

Solve Uniform Motion Applications

In the following exercises, solve.

Exercise 3.5E.1

Lilah is moving from Portland to Seattle. It takes her three hours to go by train. Mason leaves the train station in Portland and drives to the train station in Seattle with all Lilah's boxes in his car. It takes him 2.4 hours to get to Seattle, driving at 15 miles per hour faster than the speed of the train. Find Mason's speed and the speed of the train.

Answer

Mason 75 mph, train 60 mph

Exercise 3.5E.2

Kathy and Cheryl are walking in a fundraiser. Kathy completes the course in 4.8 hours and Cheryl completes the course in 8 hours. Kathy walks two miles per hour faster than Cheryl. Find Kathy's speed and Cheryl's speed.

Exercise 3.5E.3

Two busses go from Sacramento for San Diego. The express bus makes the trip in 6.8 hours and the local bus takes 10.2 hours for the trip. The speed of the express bus is 25 mph faster than the speed of the local bus. Find the speed of both busses.

Answer

express bus 75mph, local 50 mph

Exercise 3.5E.4

A commercial jet and a private airplane fly from Denver to Phoenix. It takes the commercial jet 1.1 hours for the flight, and it takes the private airplane 1.8 hours. The speed of the commercial jet is 210 miles per hour faster than the speed of the private airplane. Find the speed of both airplanes.

Exercise 3.5E.5

Saul drove his truck 3 hours from Dallas towards Kansas City and stopped at a truck stop to get dinner. At the truck stop he met Erwin, who had driven 4 hours from Kansas City towards Dallas. The distance between Dallas and Kansas City is 542 miles, and Erwin's speed was eight miles per hour slower than Saul's speed. Find the speed of the two truckers.

Answer

Saul 82 mph, Erwin 74 mph

Exercise 3.5E.6

Charlie and Violet met for lunch at a restaurant between Memphis and New Orleans. Charlie had left Memphis and drove 4.8 hours towards New Orleans. Violet had left New Orleans and drove 2 hours towards Memphis, at a speed 10 miles per hour faster than Charlie's speed. The distance between Memphis and New Orleans is 394 miles. Find the speed of the two drivers.

Exercise 3.5E.7

Sisters Helen and Anne live 332 miles apart. For Thanksgiving, they met at their other sister's house partway between their homes. Helen drove 3.2 hours and Anne drove 2.8 hours. Helen's average speed was four miles per hour faster than Anne's. Find Helen's average speed and Anne's average speed.

Answer

Helen 60 mph, Anne 56 mph

Exercise 3.5E.8

Ethan and Leo start riding their bikes at the opposite ends of a 65-mile bike path. After Ethan has ridden 1.5 hours and Leo has ridden 2 hours, they meet on the path. Ethan's speed is six miles per hour faster than Leo's speed. Find the speed of the two bikers.

Exercise 3.5E.9

Elvira and Aletheia live 3.1 miles apart on the same street. They are in a study group that meets at a coffee shop between their houses. It took Elvira half an hour and Aletheia two-thirds of an hour to walk to the coffee shop. Aletheia's speed is 0.6 miles per hour slower than Elvira's speed. Find both women's walking speeds.

Answer

Aletheia 2.4 mph, Elvira 3 mph

Exercise 3.5E.10

DaMarcus and Fabian live 23 miles apart and play soccer at a park between their homes. DaMarcus rode his bike for three-quarters of an hour and Fabian rode his bike for half an hour to get to the park. Fabian's speed was six miles per hour faster than DaMarcus' speed. Find the speed of both soccer players.

Exercise 3.5E.11

Cindy and Richard leave their dorm in Charleston at the same time. Cindy rides her bicycle north at a speed of 18 miles per hour. Richard rides his bicycle south at a speed of 14 miles per hour. How long will it take them to be 96 miles apart?

Answer

3 hours

Exercise 3.5E.12

Matt and Chris leave their uncle's house in Phoenix at the same time. Matt drives west on I-60 at a speed of 76 miles per hour. Chris drives east on I-60 at a speed of 82 miles per hour. How many hours will it take them to be 632 miles apart?

Exercise 3.5E.13

Two busses leave Billings at the same time. The Seattle bus heads west on I-90 at a speed of 73 miles per hour while the Chicago bus heads east at a speed of 79 miles an hour. How many hours will it take them to be 532 miles apart?

Answer

3.5 hours

Exercise 3.5E.14

Two boats leave the same dock in Cairo at the same time. One heads north on the Mississippi River while the other heads south. The northbound boat travels four miles per hour. The southbound boat goes eight miles per hour. How long will it take them to be 54 miles apart?

Exercise 3.5E.15

Lorena walks the path around the park in 30 minutes. If she jogs, it takes her 20 minutes. Her jogging speed is 1.5 miles per hour faster than her walking speed. Find Lorena's walking speed and jogging speed.

Answer

walking 3 mph, jogging 4.5 mph

Exercise 3.5E.16

Julian rides his bike uphill for 45 minutes, then turns around and rides back downhill. It takes him 15 minutes to get back to where he started. His uphill speed is 3.2 miles per hour slower than his downhill speed. Find Julian's uphill and downhill speed.

Exercise 3.5E.17

Cassius drives his boat upstream for 45 minutes. It takes him 30 minutes to return downstream. His speed going upstream is three miles per hour slower than his speed going downstream. Find his upstream and downstream speeds.

Answer

upstream 6 mph, downstream 9 mph

Exercise 3.5E.18

It takes Darline 20 minutes to drive to work in light traffic. To come home, when there is heavy traffic, it takes her 36 minutes. Her speed in light traffic is 24 miles per hour faster than her speed in heavy traffic. Find her speed in light traffic and in heavy traffic.

Exercise 3.5E.19

At 1:30 Marlon left his house to go to the beach, a distance of 7.6 miles. He rode his skateboard until 2:15, then walked the rest of the way. He arrived at the beach at 3:00. Marlon's speed on his skateboard is 2.5 times his walking speed. Find his speed when skateboarding and when walking.

Answer

skateboarding 8 mph, walking 3.2 mph

Exercise 3.5E.20

Aaron left at 9:15 to drive to his mountain cabin 108 miles away. He drove on the freeway until 10:45, and then he drove on the mountain road. He arrived at 11:05. His speed on the freeway was three times his speed on the mountain road. Find Aaron's speed on the freeway and on the mountain road.

Exercise 3.5E.21

Marisol left Los Angeles at 2:30 to drive to Santa Barbara, a distance of 95 miles. The traffic was heavy until 3:20. She drove the rest of the way in very light traffic and arrived at 4:20. Her speed in heavy traffic was 40 miles per hour slower than her speed in light traffic. Find her speed in heavy traffic and in light traffic.

Answer

heavy traffic 30 mph, light traffic 70 mph

Exercise 3.5E.22

Lizette is training for a marathon. At 7:00 she left her house and ran until 8:15, then she walked until 11:15. She covered a total distance of 19 miles. Her running speed was five miles per hour faster than her walking speed. Find her running and walking speeds.

Everyday Math

Exercise 3.5E.23

John left his house in Irvine at 8:35 am to drive to a meeting in Los Angeles, 45 miles away. He arrived at the meeting at 9:50. At 3:30 pm, he left the meeting and drove home. He arrived home at 5:18.

1. What was his average speed on the drive from Irvine to Los Angeles?
2. What was his average speed on the drive from Los Angeles to Irvine?
3. What was the total time he spent driving to and from this meeting?
4. John drove a total of 90 miles roundtrip. Find his average speed. (Round to the nearest tenth.)

Answer

1. 36 mph
2. 25 mph
3. 3.05 hours
4. 29.5 mph

Exercise 3.5E.24

Sarah wants to arrive at her friend's wedding at 3:00. The distance from Sarah's house to the wedding is 95 miles. Based on usual traffic patterns, Sarah predicts she can drive the first 15 miles at 60 miles per hour, the next 10 miles at 30 miles per hour, and the remainder of the drive at 70 miles per hour.

1. How long will it take Sarah to drive the first 15 miles?
2. How long will it take Sarah to drive the next 10 miles?
3. How long will it take Sarah to drive the rest of the trip?
4. What time should Sarah leave her house?

Writing Exercises

Exercise 3.5E.25

When solving a uniform motion problem, how does drawing a diagram of the situation help you?

Answer

Answers will vary.

Exercise 3.5E. 26

When solving a uniform motion problem, how does creating a table help you?

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
solve uniform motion applications.			

ⓑ What does this checklist tell you about your mastery of this section? What steps will you take to improve?

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3.6: Solve Applications with Linear Inequalities

Learning Objectives

By the end of this section, you will be able to:

- Solve applications with linear inequalities

Note

Before you get started, take this readiness quiz.

1. Write as an inequality: x is at least 30.
   If you missed this problem, review Exercise 2.7.34.
2. Solve $8-3y<41$.
   If you missed this problem, review Exercise 2.7.22.

Solve Applications with Linear Inequalities

Many real-life situations require us to solve inequalities. In fact, inequality applications are so common that we often do not even realize we are doing algebra. For example, how many gallons of gas can be put in the car for $20? Is the rent on an apartment affordable? Is there enough time before class to go get lunch, eat it, and return? How much money should each family member's holiday gift cost without going over budget?

The method we will use to solve applications with linear inequalities is very much like the one we used when we solved applications with equations. We will read the problem and make sure all the words are understood. Next, we will identify what we are looking for and assign a variable to represent it. We will restate the problem in one sentence to make it easy to translate into an inequality. Then, we will solve the inequality.

Example 3.6.1

Emma got a new job and will have to move. Her monthly income will be $5,265. To qualify to rent an apartment, Emma's monthly income must be at least three times as much as the rent. What is the highest rent Emma will qualify for?

Solution

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	the highest rent Emma will qualify for
Step 3. Name what we are looking for.	
Choose a variable to represent that quantity.	Let r = rent
Step 4. Translate into an inequality.	
First write a sentence that gives the information to find it.	Emma's monthly income must be at least three times the rent.
Step 5. Solve the inequality.	$5265 \geq 3r$
Remember, $a>x$ has the same meaning as $x<a$	$1755 \geq r$
	$r \leq 1755$
Step 6. Check the answer in the problem and make sure it makes sense.	
A maximum rent of $1,755 seems reasonable for an income of $5,265.	
Step 7. Answer the answer in the problem complete sentence.	the question with a The maximum rent is $1,755.

Try It 3.6.2

Alan is loading a pallet with boxes that each weighs 45 pounds. The pallet can safely support no more than 900 pounds. How many boxes can he safely load onto the pallet?

Answer

There can be no more than 20 boxes.

Try It 3.6.3

The elevator in Yehire's apartment building has a sign that says the maximum weight is 2,100 pounds. If the average weight of one person is 150 pounds, how many people can safely ride the elevator?

Answer

A maximum of 14 people can safely ride in the elevator.

Sometimes an application requires the solution to be a whole number, but the algebraic solution to the inequality is not a whole number. In that case, we must round the algebraic solution to a whole number. The context of the application will determine whether we round up or down. To check applications like this, we will round our answer to a number that is easy to compute with and make sure that number makes the inequality true.

Example 3.6.4

Dawn won a mini-grant of $4,000 to buy tablet computers for her classroom. The tablets she would like to buy cost $254.12 each, including tax and delivery. What is the maximum number of tablets Dawn can buy?

Solution

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	the maximum number of tablets Dawn can buy
Step 3. Name what we are looking for. Choose a variable to represent that quantity.	Let n = the number of tablets.
Step 4. Translate. write a sentence that gives the information to find it.	$254.12 times the number of tablets is no more than $4000.
Translate into an inequality.	$254.12n \leq 4000$
Step 5. Solve the inequality.	$n \leq 15.74$
But n must be a whole number of tablets, so round to 15.	$n \leq 15$
Step 6. Check the answer in the problem and make sure it makes sense. Rounding down the price to $250, 15 tablets would cost $3750, while 16 tablets would be $4000. So a maximum of 15 tablets at $254.12 seems reasonable.	
Step 7. Answer the answer in the problem complete sentence.	the question with a Dawn can buy a maximum of 15 tablets.

Try It 3.6.5

Angie has $20 to spend on juice boxes for her son's preschool picnic. Each pack of juice boxes costs $2.63. What is the maximum number of packs she can buy?

Answer

seven packs

Try It 3.6.6

Daniel wants to surprise his girlfriend with a birthday party at her favorite restaurant. It will cost $42.75 per person for dinner, including tip and tax. His budget for the party is $500. What is the maximum number of people Daniel can have at the party?

Answer

11 people

Example 3.6.7

Pete works at a computer store. His weekly pay will be either a fixed amount, $925, or $500 plus 12% of his total sales. How much should his total sales be for his variable pay option to exceed the fixed amount of $925?

Solution

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	the total sales needed for his variable pay option to exceed the fixed amount of $925
Step 3. Name what we are looking for. Choose a variable to represent that quantity.	Let s = the total sales.
Step 4. Translate. write a sentence that gives the information to find it. Translate into an inequality. Remember to convert the percent to a decimal.	$500 plus 12% of total sales is more than $925. $500 + 0.12s > 925$
Step 5. Solve the inequality.	$0.12s > 425$ $s > 3541.\overline{66}$
Step 6. Check the answer in the problem and make sure it makes sense. Rounding down the price to $250, 15 tablets would cost $3750, while If we round the total sales up to $4000, we see that $500+0.12(4000) = 980$, which is more than $925.	
Step 7. Answer the the question with a complete sentence.	The total sales must be more than $3541.67

Try It 3.6.8

Tiffany just graduated from college and her new job will pay her $20000 per year plus 2% of all sales. She wants to earn at least $100000 per year. For what total sales will she be able to achieve her goal?

Answer

at least $4000000

Try It 3.6.9

Christian has been offered a new job that pays $24000 a year plus 3% of sales. For what total sales would this new job pay more than his current job which pays $60000?

Answer

at least $1200000

Example 3.6.10

Sergio and Lizeth have a very tight vacation budget. They plan to rent a car from a company that charges $75 a week plus $0.25 a mile. How many miles can they travel and still keep within their $200 budget?

Solution

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	the number of miles Sergio and Lizeth can travel
Step 3. Name what we are looking for. Choose a variable to represent that quantity.	Let m = the number of miles.
Step 4. Translate. write a sentence that gives the information to find it. Translate into an inequality.	$75 plus 0.25 times the number of miles is less than or equal to $200. $75 + 25m \leq 200$
Step 5. Solve the inequality.	$0.25m \leq 125$ $m \leq 500$ miles
Step 6. Check the answer in the problem and make sure it makes sense. Yes, $75 + 0.25(500) = 200$.	
Step 7. Answer the the question with a complete sentence.	Sergio and Lizeth can travel 500 miles and still stay on budget.

Try It 3.6.11

Taleisha's phone plan costs her $28.80 a month plus $0.20 per text message. How many text messages can she use and keep her monthly phone bill no more than $50?

Answer

no more than 106 text messages

Try It 3.6.12

Rameen's heating bill is $5.42 per month plus $1.08 per therm. How many therms can Rameen use if he wants his heating bill to be a maximum of $87.50?

Answer

no more than 76 therms

A common goal of most businesses is to make a profit. *Profit* is the money that remains when the expenses have been subtracted from the money earned. In the next example, we will find the number of jobs a small businessman needs to do every month in order to make a certain amount of **profit**.

Example 3.6.13

Elliot has a landscape maintenance business. His monthly expenses are $1,100. If he charges $60 per job, how many jobs must he do to earn a profit of at least $4,000 a month?

Solution

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	the number of jobs Elliot needs
Step 3. Name what we are looking for.	
Choose a variable to represent it	Let j = the number of jobs.
Step 4. Translate. write a sentence that	$60 times the number of jobs minus $1,100 is at least $4,000.
gives the information to find it.	less than or equal to $200.
Translate into an inequality.	$60j - 1100 \geq 4000$
Step 5. Solve the inequality.	$60j \geq 5100$
	$j \geq 85$ jobs
Step 6. Check the answer in the problem and make sure it makes sense.	If Elliot did 90 jobs, his profit would be $60(90) - 1{,}100$, or $4,300. This is more than $4,000.
Step 7. Answer the the question with a complete sentence.	Elliot must work at least 85 jobs.

Try It 3.6.14

Caleb has a pet sitting business. He charges $32 per hour. His monthly expenses are $2272. How many hours must he work in order to earn a profit of at least $800 per month?

Answer

at least 96 hours

Try It 3.6.15

Felicity has a calligraphy business. She charges $2.50 per wedding invitation. Her monthly expenses are $650. How many invitations must she write to earn a profit of at least $2800 per month?

Answer

at least 1380 invitations

Sometimes life gets complicated! There are many situations in which several quantities contribute to the total expense. We must make sure to account for all the individual expenses when we solve problems like this.

Example 3.6.16

Brenda's best friend is having a destination wedding and the event will last 3 days. Brenda has $500 in savings and can earn $15 an hour babysitting. She expects to pay $350 airfare, $375 for food and entertainment and $60 a night for her share of a hotel room. How many hours must she babysit to have enough money to pay for the trip?

Solution

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	the number of hours Brenda must babysit
Step 3. Name what we are looking for. Choose a variable to represent that quantity.	Let h = the number of hours.
Step 4. Translate. write a sentence that gives the information to find it.	The expenses must be less than or equal to the income. The cost of airfare plus the cost of food and entertainment and the hotel bill must be less than or equal to the savings plus the amount earned babysitting.
Translate into an inequality.	$\$350 + \$375 + \$60(3) \leq \$500 + \$15h$
Step 5. Solve the inequality.	$905 \leq 500 + 15h$ $405 \leq 15h$ $27 \leq h$ $h \geq 27$
Step 6. Check the answer in the problem and make sure it makes sense.	
We substitute 27 into the inequality.	
$905 \leq 500 + 15h$	
$905 \leq 500 + 15(27)$	
$905 \leq 905$	
Step 7. Answer the the question with a complete sentence.	Brenda must babysit at least 27 hours.

? Try It 3.6.17

Malik is planning a 6-day summer vacation trip. He has $840 in savings, and he earns $45 per hour for tutoring. The trip will cost him $525 for airfare, $780 for food and sightseeing, and $95 per night for the hotel. How many hours must he tutor to have enough money to pay for the trip?

Answer

at least 23 hours

? Try It 3.6.18

Josue wants to go on a 10-day road trip next spring. It will cost him $180 for gas, $450 for food, and $49 per night for a motel. He has $520 in savings and can earn $30 per driveway shoveling snow. How many driveways must he shovel to have enough money to pay for the trip?

Answer

at least 20 driveways

Key Concepts

- **Solving inequalities**
  1. **Read** the problem.
  2. **Identify** what we are looking for.
  3. **Name** what we are looking for. Choose a variable to represent that quantity.
  4. **Translate.** Write a sentence that gives the information to find it. Translate into an inequality.

5. **Solve** the inequality.
6. **Check** the answer in the problem and make sure it makes sense.
7. **Answer** the question with a complete sentence.

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3.6E: Exercises

Practice Makes Perfect

Solve Applications with Linear Inequalities

In the following exercises, solve.

Exercise 3.6E.1

Mona is planning her son's birthday party and has a budget of $285. The Fun Zone charges $19 per child. How many children can she have at the party and stay within her budget?

Answer

15 children

Exercise 3.6E.2

Carlos is looking at apartments with three of his friends. They want the monthly rent to be no more than $2360. If the roommates split the rent evenly among the four of them, what is the maximum rent each will pay?

Exercise 3.6E.3

A water taxi has a maximum load of 1,800 pounds. If the average weight of one person is 150 pounds, how many people can safely ride in the water taxi?

Answer

12 people

Exercise 3.6E.4

Marcela is registering for her college classes, which cost $105 per unit. How many units can she take to have a maximum cost of $1,365?

Exercise 3.6E.5

Arleen got a $20 gift card for the coffee shop. Her favorite iced drink costs $3.79. What is the maximum number of drinks she can buy with the gift card?

Answer

five drinks

Exercise 3.6E.6

Teegan likes to play golf. He has budgeted $60 next month for the driving range. It costs him $10.55 for a bucket of balls each time he goes. What is the maximum number of times he can go to the driving range next month?

Exercise 3.6E.7

Joni sells kitchen aprons online for $32.50 each. How many aprons must she sell next month if she wants to earn at least $1,000?

Answer

31 aprons

Exercise 3.6E.8

Ryan charges his neighbors $17.50 to wash their car. How many cars must he wash next summer if his goal is to earn at least $1,500?

Exercise 3.6E.9

Keshad gets paid $2,400 per month plus 6% of his sales. His brother earns $3,300 per month. For what amount of total sales will Keshad's monthly pay be higher than his brother's monthly pay?

Answer

$15,000

Exercise 3.6E.10

Kimuyen needs to earn $4,150 per month in order to pay all her expenses. Her job pays her $3,475 per month plus 4% of her total sales. What is the minimum Kimuyen's total sales must be in order for her to pay all her expenses?

Exercise 3.6E.11

Andre has been offered an entry-level job. The company offered him $48,000 per year plus 3.5% of his total sales. Andre knows that the average pay for this job is $62,000. What would Andre's total sales need to be for his pay to be at least as high as the average pay for this job?

Answer

$400,000

Exercise 3.6E.12

Nataly is considering two job offers. The first job would pay her $83,000 per year. The second would pay her $66,500 plus 15% of her total sales. What would her total sales need to be for her salary on the second offer be higher than the first?

Exercise 3.6E.13

Jake's water bill is $24.80 per month plus $2.20 per ccf (hundred cubic feet) of water. What is the maximum number of ccf Jake can use if he wants his bill to be no more than $60?

Answer

16 ccf

Exercise 3.6E.14

Kiyoshi's phone plan costs $17.50 per month plus $0.15 per text message. What is the maximum number of text messages Kiyoshi can use so the phone bill is no more than $56.50?

Exercise 3.6E.15

Marlon's TV plan costs $49.99 per month plus $5.49 per first-run movie. How many first-run movies can he watch if he wants to keep his monthly bill to be a maximum of $100?

Answer

nine movies

Exercise 3.6E.16

Kellen wants to rent a banquet room in a restaurant for her cousin's baby shower. The restaurant charges $350 for the banquet room plus $32.50 per person for lunch. How many people can Kellen have at the shower if she wants the maximum cost to be $1,500?

Exercise 3.6E.17

Moshde runs a hairstyling business from her house. She charges $45 for a haircut and style. Her monthly expenses are $960. She wants to be able to put at least $1,200 per month into her savings account order to open her own salon. How many "cut & styles" must she do to save at least $1,200 per month?

Answer

48 cut & styles

Exercise 3.6E.18

Noe installs and configures software on home computers. He charges $125 per job. His monthly expenses are $1,600. How many jobs must he work in order to make a profit of at least $2,400?

Exercise 3.6E.19

Katherine is a personal chef. She charges $115 per four-person meal. Her monthly expenses are $3,150. How many four-person meals must she sell in order to make a profit of at least $1,900?

Answer

44 meals

Exercise 3.6E.20

Melissa makes necklaces and sells them online. She charges $88 per necklace. Her monthly expenses are $3745. How many necklaces must she sell if she wants to make a profit of at least $1,650?

Exercise 3.6E.21

Five student government officers want to go to the state convention. It will cost them $110 for registration, $375 for transportation and food, and $42 per person for the hotel. There is $450 budgeted for the convention in the student government savings account. They can earn the rest of the money they need by having a car wash. If they charge $5 per car, how many cars must they wash in order to have enough money to pay for the trip?

Answer

49 cars

Exercise 3.6E.22

Cesar is planning a 4-day trip to visit his friend at a college in another state. It will cost him $198 for airfare, $56 for local transportation, and $45 per day for food. He has $189 in savings and can earn $35 for each lawn he mows. How many lawns must he mow to have enough money to pay for the trip?

Exercise 3.6E.23

Alonzo works as a car detailer. He charges $175 per car. He is planning to move out of his parents' house and rent his first apartment. He will need to pay $120 for application fees, $950 for security deposit, and first and last months' rent at $1,140 per month. He has $1,810 in savings. How many cars must he detail to have enough money to rent the apartment?

Answer

9 cars

Exercise 3.6E.24

Eun-Kyung works as a tutor and earns $60 per hour. She has $792 in savings. She is planning an anniversary party for her parents. She would like to invite 40 guests. The party will cost her $1,520 for food and drinks and $150 for the photographer. She will also have a favor for each of the guests, and each favor will cost $7.50. How many hours must she tutor to have enough money for the party?

Everyday Math

Exercise 3.6E.25

Maximum Load on a Stage In 2014, a high school stage collapsed in Fullerton, California, when 250 students got on stage for the finale of a musical production. Two dozen students were injured. The stage could support a maximum of 12,750 pounds. If the average weight of a student is assumed to be 140 pounds, what is the maximum number of students who could safely be on the stage?

Answer

91 students

Exercise 3.6E.26

Maximum Weight on a Boat In 2004, a water taxi sank in Baltimore harbor and five people drowned. The water taxi had a maximum capacity of 3,500 pounds (25 people with average weight 140 pounds). The average weight of the 25 people on the water taxi when it sank was 168 pounds per person. What should the maximum number of people of this weight have been?

Exercise 3.6E.27

Wedding Budget Adele and Walter found the perfect venue for their wedding reception. The cost is $9,850 for up to 100 guests, plus $38 for each additional guest. How many guests can attend if Adele and Walter want the total cost to be no more than $12,500?

Answer

169 guests

Exercise 3.6E.28

Shower Budget Penny is planning a baby shower for her daughter-in-law. The restaurant charges $950 for up to 25 guests, plus $31.95 for each additional guest. How many guests can attend if Penny wants the total cost to be no more than $1,500?

Writing Exercises

Exercise $3.6E.29$

Find your last month's phone bill and the hourly salary you are paid at your job. (If you do not have a job, use the hourly salary you would realistically be paid if you had a job.) Calculate the number of hours of work it would take you to earn at least enough money to pay your phone bill by writing an appropriate inequality and then solving it.

Answer

Answers will vary.

Exercise $3.6E.30$

Find out how many units you have left, after this term, to achieve your college goal and estimate the number of units you can take each term in college. Calculate the number of terms it will take you to achieve your college goal by writing an appropriate inequality and then solving it.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
solve applications with linear inequalities.			

ⓑ What does this checklist tell you about your mastery of this section? What steps will you take to improve?

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3.7: Chapter 3 Review Exercises

Chapter 3 Review Exercises

3.1 Using a Problem Solving Strategy

Approach Word Problems with a Positive Attitude

In the following exercises, reflect on your approach to word problems.

Exercise 3.7.1

How has your attitude towards solving word problems changed as a result of working through this chapter? Explain.

Answer

answers will vary

Exercise 3.7.2

Did the problem-solving strategy help you solve word problems in this chapter? Explain.

Use a Problem-Solving Strategy for Word Problems

In the following exercises, solve using the problem-solving strategy for word problems. Remember to write a complete sentence to answer each question.

Exercise 3.7.3

Three-fourths of the people at a concert are children. If there are 87 children, what is the total number of people at the concert?

Answer

116

Exercise 3.7.4

There are nine saxophone players in the band. The number of saxophone players is one less than twice the number of tuba players. Find the number of tuba players.

Solve Number Problems

In the following exercises, solve each number word problem.

Exercise 3.7.5

The sum of a number and three is forty-one. Find the number.

Answer

38

Exercise 3.7.6

Twice the difference of a number and ten is fifty-four. Find the number.

Exercise 3.7.7

One number is nine less than another. Their sum is negative twenty-seven. Find the numbers.

Answer

$-18,-9$

Exercise 3.7.8

One number is eleven more than another. If their sum is increased by seventeen, the result is 90. Find the numbers.

Exercise 3.7.9

One number is two more than four times another. Their sum is -13. Find the numbers.

Answer

$-3,-10$

Exercise 3.7.10

The sum of two consecutive integers is -135. Find the numbers.

Exercise 3.7.11

Find three consecutive integers whose sum is -141.

Answer

$-48,-47,-46$

Exercise 3.7.12

Find three consecutive even integers whose sum is 234.

Exercise 3.7.13

Find three consecutive odd integers whose sum is 51.

Answer

15, 17, 19

Exercise 3.7.14

Koji has $5,502 in his savings account. This is $30 less than six times the amount in his checking account. How much money does Koji have in his checking account?

3.2 Solve Percent Applications

Translate and Solve Basic Percent Equations

In the following exercises, translate and solve.

Exercise 3.7.15

What number is 67% of 250?

Answer

167.5

Exercise 3.7.16

300% of 82 is what number?

Exercise 3.7.17

12.5% of what number is 20?

Answer

160

Exercise 3.7.18

72 is 30% of what number?

Exercise 3.7.19

What percent of 125 is 150?

Answer

120%

Exercise 3.7.20

127.5 is what percent of 850?

Solve Percent Applications

In the following exercises, solve.

Exercise 3.7.21

The bill for Dino's lunch was $19.45. He wanted to leave 20% of the total bill as a tip. How much should the tip be?

Answer

$3.89

Exercise 3.7.22

Reza was very sick and lost 15% of his original weight. He lost 27 pounds. What was his original weight?

Exercise 3.7.23

Dolores bought a crib on sale for $350. The sale price was 40% of the original price. What was the original price of the crib?

Answer

$875

Exercise 3.7.24

Jaden earns $2,680 per month. He pays $938 a month for rent. What percent of his monthly pay goes to rent?

Find Percent Increase and Percent Decrease

In the following exercises, solve.

Exercise 3.7.25

Angel's got a raise in his annual salary from $55,400 to $56,785. Find the percent increase.

Answer

2.5%

Exercise 3.7.26

Rowena's monthly gasoline bill dropped from $83.75 last month to $56.95 this month. Find the percent decrease.

Solve Simple Interest Applications

In the following exercises, solve.

Exercise 3.7.27

Winston deposited $3,294 in a bank account with interest rate 2.6%. How much interest was earned in 5 years?

Answer

$428.22

Exercise 3.7.28

Moira borrowed $4,500 from her grandfather to pay for her first year of college. Three years later, she repaid the $4,500 plus $243 interest. What was the rate of interest?

Exercise 3.7.29

Jaime's refrigerator loan statement said he would pay $1,026 in interest for a 4-year loan at 13.5%. How much did Jaime borrow to buy the refrigerator?

Answer

$1,900

Exercise 3.7.30

In 12 years, a bond that paid 6.35% interest earned $7,620 interest. What was the principal of the bond?

Solve Applications with Discount or Mark-up

In the following exercises, find the sale price.

Exercise 3.7.31

The original price of a handbag was $84. Carole bought it on sale for $21 off.

Answer

$63

Exercise 3.7.32

Marian wants to buy a coffee table that costs $495. Next week the coffee table will be on sale for $149 off.

In the following exercises, find

1. the amount of discount and
2. the sale price.

Exercise 3.7.33

Emmett bought a pair of shoes on sale at 40% off from an original price of $138.

Answer

1. $55.20
2. $82.80

Exercise 3.7.34

Anastasia bought a dress on sale at 75% off from an original price of $280.

In the following exercises, find ⓐ the amount of discount and ⓑ the discount rate. (Round to the nearest tenth of a percent, if needed.)

Exercise 3.7.35

Zack bought a printer for his office that was on sale for $380. The original price of the printer was $450.

Answer

1. $70
2. 15.6%

Exercise 3.7.36

Lacey bought a pair of boots on sale for $95. The original price of the boots was $200.

In the following exercises, find

1. the amount of the mark-up and
2. the list price.

Exercise 3.7.37

Nga and Lauren bought a chest at a flea market for $50. They re-finished it and then added a 350% mark-up.

Answer

1. $175
2. $225

Exercise 3.7.38

Carly bought bottled water for $0.24 per bottle at the discount store. She added a 75% mark-up before selling them at the football game.

3.3 Solve Mixture Applications

Solve Coin Word Problems

In the following exercises, solve each coin word problem.

Exercise 3.7.39

Francie has $4.35 in dimes and quarters. The number of dimes is five more than the number of quarters. How many of each coin does she have?

Answer

16 dimes, 11 quarters

Exercise 3.7.40

Scott has $0.39 in pennies and nickels. The number of pennies is eight times the number of nickels. How many of each coin does he have?

Exercise 3.7.41

Paulette has $140 in $5 and $10 bills. The number of $10 bills is one less than twice the number of $5 bills. How many of each does she have?

Answer

six $5 bills, 11 $10 bills

Exercise 3.7.42

Lenny has $3.69 in pennies, dimes, and quarters. The number of pennies is three more than the number of dimes. The number of quarters is twice the number of dimes. How many of each coin does he have?

Solve Ticket and Stamp Word Problems

In the following exercises, solve each ticket or stamp word problem.

Exercise 3.7.43

A church luncheon made $842. Adult tickets cost $10 each and children's tickets cost $6 each. The number of children was 12 more than twice the number of adults. How many of each ticket were sold?

Answer

35 adults, 82 children

Exercise 3.7.44

Tickets for a basketball game cost $2 for students and $5 for adults. The number of students was three less than 10 times the number of adults. The total amount of money from ticket sales was $619. How many of each ticket were sold?

Exercise 3.7.45

125 tickets were sold for the jazz band concert for a total of $1,022. Student tickets cost $6 each and general admission tickets cost $10 each. How many of each kind of ticket were sold?

Answer

57 students, 68 adults

Exercise 3.7.46

One afternoon the water park sold 525 tickets for a total of $13,545. Child tickets cost $19 each and adult tickets cost $40 each. How many of each kind of ticket were sold?

Exercise 3.7.47

Ana spent $4.06 buying stamps. The number of $0.41 stamps she bought was five more than the number of $0.26 stamps. How many of each did she buy?

Answer

three $0.26 stamps, eight $0.41 stamps

Exercise 3.7.48

Yumi spent $34.15 buying stamps. The number of $0.56 stamps she bought was 10 less than four times the number of $0.41 stamps. How many of each did she buy?

Solve Mixture Word Problems

In the following exercises, solve each mixture word problem.

Exercise 3.7.49

Marquese is making 10 pounds of trail mix from raisins and nuts. Raisins cost $3.45 per pound and nuts cost $7.95 per pound. How many pounds of raisins and how many pounds of nuts should Marquese use for the trail mix to cost him $6.96 per pound?

Answer

2.2 lb. of raisins, 7.8 lb. of nuts

Exercise 3.7.50

Amber wants to put tiles on the backsplash of her kitchen counters. She will need 36 square feet of tile. She will use basic tiles that cost $8 per square foot and decorator tiles that cost $20 per square foot. How many square feet of each tile should she use so that the overall cost of the backsplash will be $10 per square foot?

Exercise 3.7.51

Shawn has $15,000 to invest. She will put some of it into a fund that pays 4.5% annual interest and the rest in a certificate of deposit that pays 1.8% annual interest. How much should she invest in each account if she wants to earn 4.05% annual interest on the total amount?

Answer

$12,500 at 4.5%, $2,500 at 1.8%

Exercise 3.7.52

Enrique borrowed $23,500 to buy a car. He pays his uncle 2% interest on the $4,500 he borrowed from him, and he pays the bank 11.5% interest on the rest. What average interest rate does he pay on the total $23,500? (Round your answer to the nearest tenth of a percent.)

3.4 Solve Geometry Applications: Triangles, Rectangles and the Pythagorean Theorem

Solve Applications Using Triangle Properties

In the following exercises, solve using triangle properties.

Exercise 3.7.53

The measures of two angles of a triangle are 22 and 85 degrees. Find the measure of the third angle.

Answer

73°

Exercise 3.7.54

The playground at a shopping mall is a triangle with perimeter 48 feet. The lengths of two sides are 19 feet and 14 feet. How long is the third side?

Exercise 3.7.55

A triangular road sign has base 30 inches and height 40 inches. What is its area?

Answer

600 square inches

Exercise 3.7.56

What is the height of a triangle with area 67.5 square meters and base 9 meters?

Exercise 3.7.57

One angle of a triangle is 30° more than the smallest angle. The largest angle is the sum of the other angles. Find the measures of all three angles.

Answer

30°,60°,90°

Exercise 3.7.58

One angle of a right triangle measures 58°. What is the measure of the other angles of the triangle?

Exercise 3.7.59

The measure of the smallest angle in a right triangle is 45° less than the measure of the next larger angle. Find the measures of all three angles.

Answer

22.5°,67.5°,90°

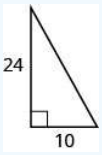
Exercise 3.7.60

The perimeter of a triangle is 97 feet. One side of the triangle is eleven feet more than the smallest side. The third side is six feet more than twice the smallest side. Find the lengths of all sides.

Use the Pythagorean Theorem

In the following exercises, use the Pythagorean Theorem to find the length of the hypotenuse.

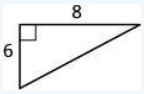
Exercise 3.7.61



Answer

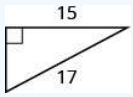
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Exercise 3.7.62



In the following exercises, use the Pythagorean Theorem to find the length of the missing side. Round to the nearest tenth, if necessary.

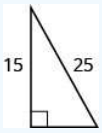
Exercise 3.7.63



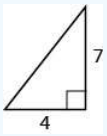
Answer

8

Exercise 3.7.64



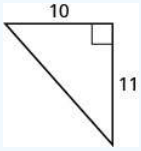
Exercise 3.7.65



Answer

8.1

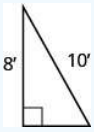
Exercise 3.7.66



In the following exercises, solve. Approximate to the nearest tenth, if necessary.

Exercise 3.7.67

Sergio needs to attach a wire to hold the antenna to the roof of his house, as shown in the figure. The antenna is 8 feet tall and Sergio has 10 feet of wire. How far from the base of the antenna can he attach the wire?

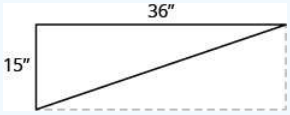


Answer

$6'$

Exercise 3.7.68

Seong is building shelving in his garage. The shelves are 36 inches wide and 15 inches tall. He wants to put a diagonal brace across the back to stabilize the shelves, as shown. How long should the brace be?



Solve Applications Using Rectangle Properties

In the following exercises, solve using rectangle properties.

Exercise 3.7.69

The length of a rectangle is 36 feet and the width is 19 feet. Find the

1. perimeter
2. area.

Answer

1. 110 ft.
2. 684 sq. ft.

Exercise 3.7.70

A sidewalk in front of Kathy's house is in the shape of a rectangle four feet wide by 45 feet long. Find the

1. perimeter
2. area.

Exercise 3.7.71

The area of a rectangle is 2356 square meters. The length is 38 meters. What is the width?

Answer

62 m

Exercise 3.7.72

The width of a rectangle is 45 centimeters. The area is 2,700 square centimeters. What is the length?

Exercise 3.7.73

The length of a rectangle is 12 cm more than the width. The perimeter is 74 cm. Find the length and the width.

Answer

24.5 cm, 12.5 cm

Exercise 3.7.74

The width of a rectangle is three more than twice the length. The perimeter is 96 inches. Find the length and the width.

3.5 Solve Uniform Motion Applications

Solve Uniform Motion Applications

In the following exercises, solve.

Exercise 3.7.75

When Gabe drives from Sacramento to Redding it takes him 2.2 hours. It takes Elsa 2 hours to drive the same distance. Elsa's speed is seven miles per hour faster than Gabe's speed. Find Gabe's speed and Elsa's speed.

Answer

Gabe 70 mph, Elsa 77 mph

Exercise 3.7.76

Louellen and Tracy met at a restaurant on the road between Chicago and Nashville. Louellen had left Chicago and drove 3.2 hours towards Nashville. Tracy had left Nashville and drove 4 hours towards Chicago, at a speed one mile per hour faster than Louellen's speed. The distance between Chicago and Nashville is 472 miles. Find Louellen's speed and Tracy's speed.

Exercise 3.7.77

Two busses leave Amarillo at the same time. The Albuquerque bus heads west on the I-40 at a speed of 72 miles per hour, and the Oklahoma City bus heads east on the I-40 at a speed of 78 miles per hour. How many hours will it take them to be 375 miles apart?

Answer

2.5 hours

Exercise 3.7.78

Kyle rowed his boat upstream for 50 minutes. It took him 30 minutes to row back downstream. His speed going upstream is two miles per hour slower than his speed going downstream. Find Kyle's upstream and downstream speeds.

Exercise 3.7.79

At 6:30, Devon left her house and rode her bike on the flat road until 7:30. Then she started riding uphill and rode until 8:00. She rode a total of 15 miles. Her speed on the flat road was three miles per hour faster than her speed going uphill. Find Devon's speed on the flat road and riding uphill.

Answer

flat road 11 mph, uphill 8 mph

Exercise 3.7.80

Anthony drove from New York City to Baltimore, a distance of 192 miles. He left at 3:45 and had heavy traffic until 5:30. Traffic was light for the rest of the drive, and he arrived at 7:30. His speed in light traffic was four miles per hour more than twice his speed in heavy traffic. Find Anthony's driving speed in heavy traffic and light traffic.

3.6 Solve Applications with Linear Inequalities

Solve Applications with Linear Inequalities

In the following exercises, solve.

Exercise 3.7.81

Julianne has a weekly food budget of $231 for her family. If she plans to budget the same amount for each of the seven days of the week, what is the maximum amount she can spend on food each day?

Answer

$33 per day

Exercise 3.7.82

Rogelio paints watercolors. He got a $100 gift card to the art supply store and wants to use it to buy $12'' \times 16''$ canvases. Each canvas costs $10.99. What is the maximum number of canvases he can buy with his gift card?

Exercise 3.7.83

Briana has been offered a sales job in another city. The offer was for $42,500 plus 8% of her total sales. In order to make it worth the move, Briana needs to have an annual salary of at least $66,500. What would her total sales need to be for her to move?

Answer

at least $300,000

Exercise 3.7.84

Renee's car costs her $195 per month plus $0.09 per mile. How many miles can Renee drive so that her monthly car expenses are no more than $250?

Exercise 3.7.85

Costa is an accountant. During tax season, he charges $125 to do a simple tax return. His expenses for buying software, renting an office, and advertising are $6,000. How many tax returns must he do if he wants to make a profit of at least $8,000?

Answer

at least 112 jobs

Exercise 3.7.86

Jenna is planning a 5-day resort vacation with three of her friends. It will cost her $279 for airfare, $300 for food and entertainment, and $65 per day for her share of the hotel. She has $550 saved towards her vacation and can earn $25 per hour as an assistant in her uncle's photography studio. How many hours must she work in order to have enough money for her vacation?

Practice Test

Exercise 3.7.87

Four-fifths of the people on a hike are children. If there are 12 children, what is the total number of people on the hike?

Answer

15

Exercise 3.7.88

One number is three more than twice another. Their sum is −63. Find the numbers.

Exercise 3.7.89

The sum of two consecutive odd integers is −96. Find the numbers.

Answer

−49,−47

Exercise 3.7.90

Marla's breakfast was 525 calories. This was 35% of her total calories for the day. How many calories did she have that day?

Exercise 3.7.91

Humberto's hourly pay increased from $16.25 to $17.55. Find the percent increase.

Answer

8%

Exercise 3.7.92

Melinda deposited $5,985 in a bank account with an interest rate of 1.9%. How much interest was earned in 2 years?

Exercise 3.7.93

Dotty bought a freezer on sale for $486.50. The original price of the freezer was $695. Find

1. the amount of discount and
2. the discount rate.

Answer

1. $208.50
2. 30%

Exercise 3.7.94

Bonita has $2.95 in dimes and quarters in her pocket. If she has five more dimes than quarters, how many of each coin does she have?

Exercise 3.7.95

At a concert, $1,600 in tickets were sold. Adult tickets were $9 each and children's tickets were $4 each. If the number of adult tickets was 30 less than twice the number of children's tickets, how many of each kind were sold?

Answer

140 adult, 85 children

Exercise 3.7.96

Kim is making eight gallons of punch from fruit juice and soda. The fruit juice costs $6.04 per gallon and the soda costs $4.28 per gallon. How much fruit juice and how much soda should she use so that the punch costs $5.71 per gallon?

Exercise 3.7.97

The measure of one angle of a triangle is twice the measure of the smallest angle. The measure of the third angle is 14 more than the measure of the smallest angle. Find the measures of all three angles.

Answer

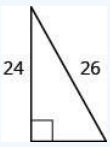
41.5°,55.5°,83°

Exercise 3.7.98

What is the height of a triangle with area 277.2 square inches and base 44 inches?

In the following exercises, use the Pythagorean Theorem to find the length of the missing side. Round to the nearest tenth, if necessary.

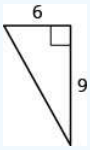
Exercise 3.7.99



Answer

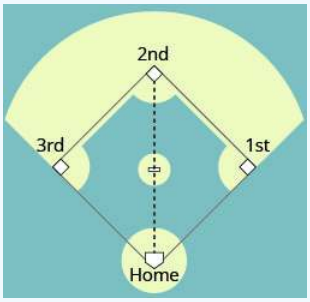
10

Exercise 3.7.100



Exercise 3.7.101

A baseball diamond is really a square with sides of 90 feet. How far is it from home plate to second base, as shown?



Answer

127.3 ft.

Exercise 3.7.102

The length of a rectangle is two feet more than five times the width. The perimeter is 40 feet. Find the dimensions of the rectangle.

Exercise 3.7.103

Two planes leave Dallas at the same time. One heads east at a speed of 428 miles per hour. The other plane heads west at a speed of 382 miles per hour. How many hours will it take them to be 2,025 miles apart?

Answer

2.5 hours

Exercise 3.7.104

Leon drove from his house in Cincinnati to his sister's house in Cleveland, a distance of 252 miles. It took him 412412 hours. For the first half hour he had heavy traffic, and the rest of the time his speed was five miles per hour less than twice his speed in heavy traffic. What was his speed in heavy traffic?

Exercise 3.7.105

Chloe has a budget of $800 for costumes for the 18 members of her musical theater group. What is the maximum she can spend for each costume?

Answer

at most $44.44 per costume

Exercise 3.7.106

Frank found a rental car deal online for $49 per week plus $0.24 per mile. How many miles could he drive if he wants the total cost for one week to be no more than $150?

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CHAPTER OVERVIEW

4: Graphing Lines

4.1: Rectangular Coordinate System
4.2: Graph by Plotting Points
4.3: Graph Using Intercepts
4.4: Graph Using the y-Intercept and Slope
4.5: Finding Linear Equations
4.6: Parallel and Perpendicular Lines
4.7: Introduction to Functions
4.8: Linear Inequalities (Two Variables)
4.E: Review Exercises and Sample Exam

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4.1: Rectangular Coordinate System

Learning Objectives

- Plot points using the rectangular coordinate system.
- Calculate the distance between any two points in the rectangular coordinate plane.
- Determine the midpoint between any two points.

Rectangular Coordinate System

The rectangular coordinate system consists of two real number lines that intersect at a right angle. The horizontal number line is called the x-axis, and the vertical number line is called the y-axis. These two number lines define a flat surface called a plane, and each point on this plane is associated with an ordered pair of real numbers (x, y). The first number is called the **x-coordinate**, and the second number is called the **y-coordinate**. The intersection of the two axes is known as the origin, which corresponds to the point $(0, 0)$.

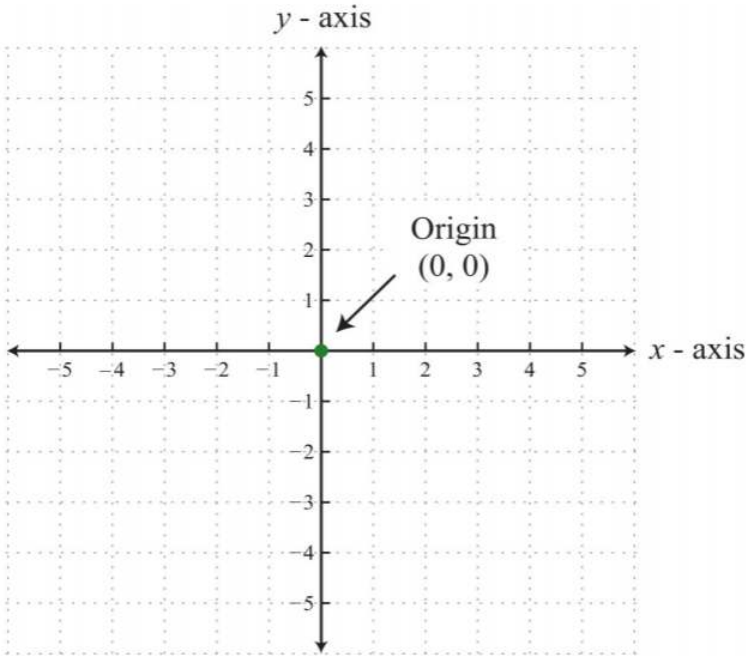


Figure 4.1.1

An ordered pair (x, y) represents the position of a point relative to the origin. The x-coordinate represents a position to the right of the origin if it is positive and to the left of the origin if it is negative. The y-coordinate represents a position above the origin if it is positive and below the origin if it is negative. Using this system, every position (point) in the plane is uniquely identified. For example, the pair $(2, 3)$ denotes the position relative to the origin as shown:

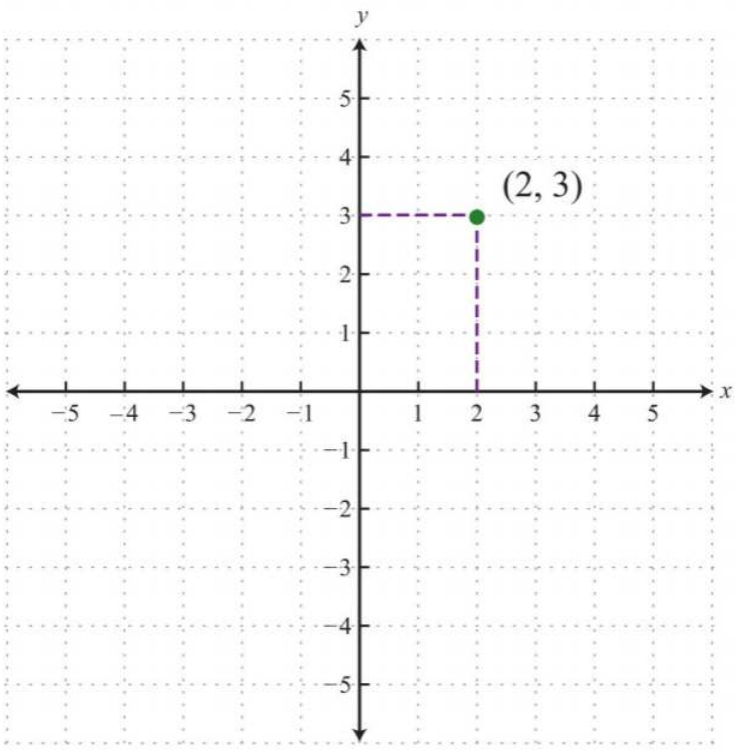


Figure 4.1.2

This system is often called the Cartesian coordinate system, named after the French mathematician René Descartes (1596– 1650).

The x- and y-axes break the plane into four regions called quadrants, named using roman numerals I, II, III, and IV, as pictured. In quadrant I, both coordinates are positive. In quadrant II, the x-coordinate is negative and the y-coordinate is positive. In quadrant III, both coordinates are negative. In quadrant IV, the x-coordinate is positive and the y-coordinate is negative.

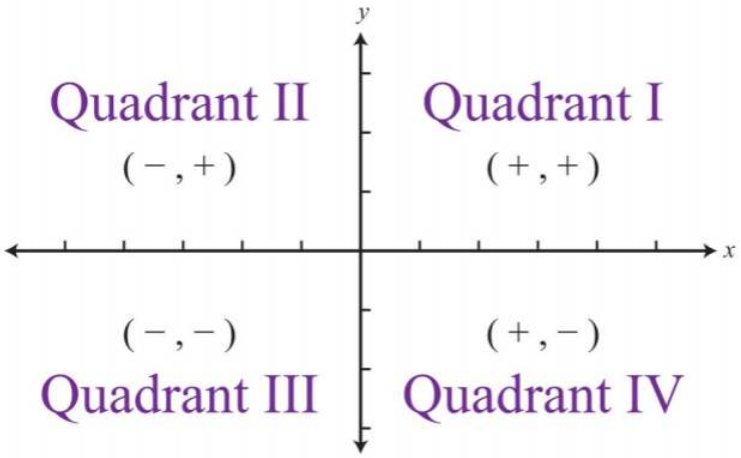


Figure 4.1.3

Example 4.1.1

Plot the ordered pair $(-3, 5)$ and determine the quadrant in which it lies.

Solution:

The coordinates $x = -3$ and $y = 5$ indicate a point 3 units to the left of and 5 units above the origin.

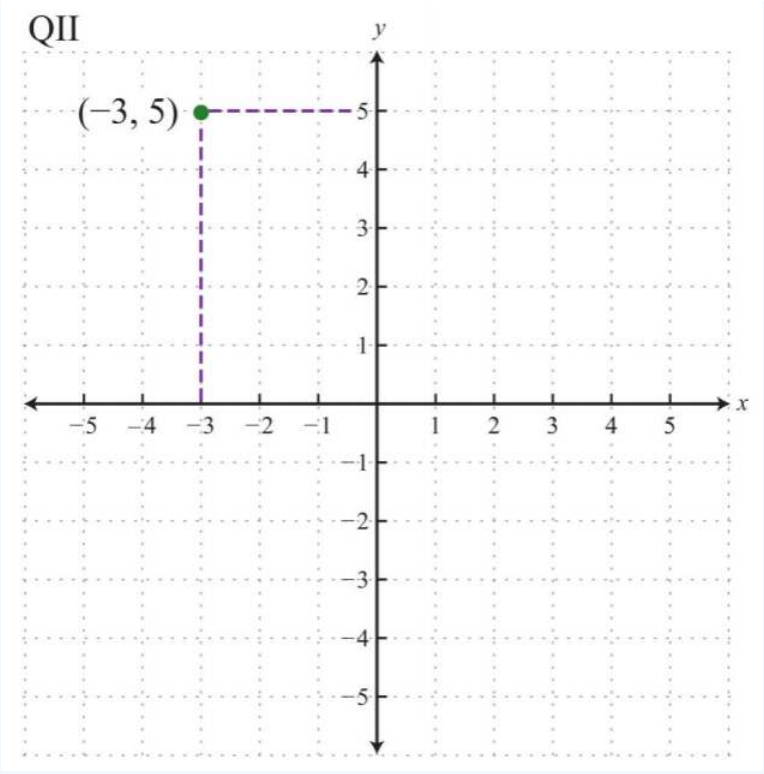


Figure 4.1.4

Answer:

The point is plotted in quadrant II (QII) because the x-coordinate is negative and the y-coordinate is positive.

Ordered pairs with 0 as one of the coordinates do not lie in a quadrant; these points are on one axis or the other (or the point is the origin if both coordinates are 0). Also, the scale indicated on the x-axis may be different from the scale indicated on the y-axis. Choose a scale that is convenient for the given situation.

Example 4.1.2

Plot this set of ordered pairs:

$$\{(4,0),(-6,0),(0,3),(-2,6),(-4,-6)\}$$

Solution:

Each tick mark on the x-axis represents 2 units and each tick mark on the y-axis represents 3 units.

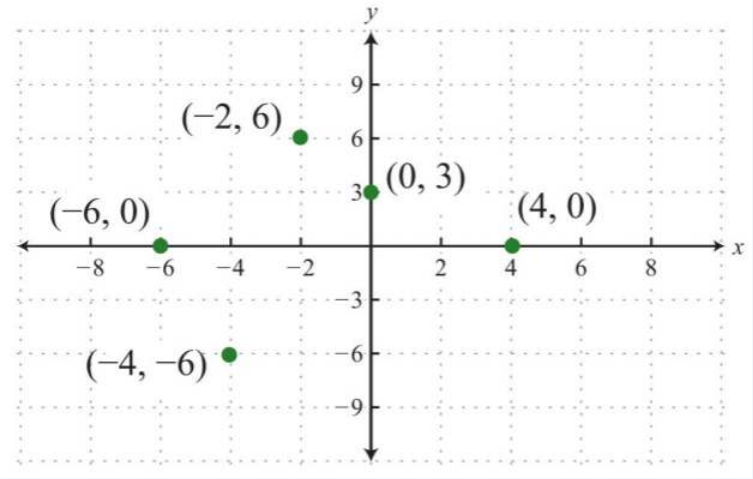


Figure 4.1.5

Example 4.1.3

Plot this set of ordered pairs:

$$\{(-6,-5),(-3,-3),(0,-1),(3,1),(6,3)\}$$

Solution:

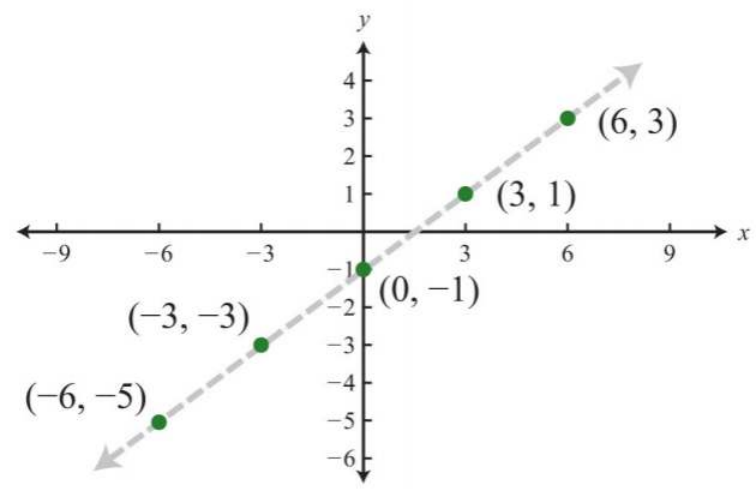


Figure 4.1.6

In this example, the points appear to be collinear, or to lie on the same line. The entire chapter focuses on finding and expressing points with this property.

Exercise 4.1.1

Plot the set of points $\{(5,3),(-3,2),(-2,-4),(4,-3)\}$and indicate in which quadrant they lie.

Answer

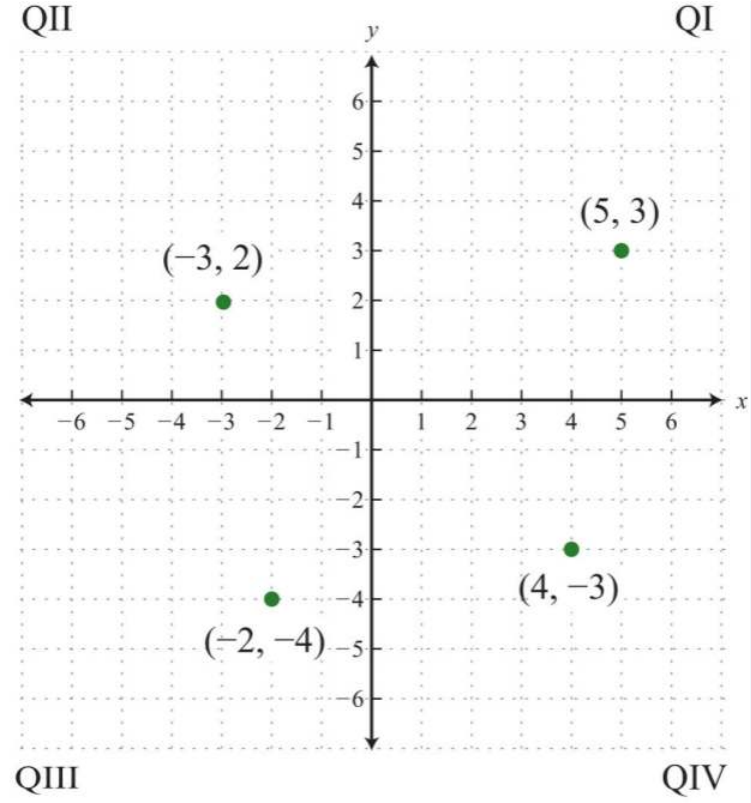


Figure 4.1.7

Graphs are used in everyday life to display data visually. A line graph consists of a set of related data values graphed on a coordinate plane and connected by line segments. Typically, the independent quantity, such as time, is displayed on the x-axis and the dependent quantity, such as distance traveled, on the y-axis.

Example 4.1.4

The following line graph shows the number of mathematics and statistics bachelor's degrees awarded in the United States each year since 1970.



Figure 4.1.8*: Source: Digest of Education Statistics.*

a. How many mathematics and statistics bachelor's degrees were awarded in 1975?

b. In which years were the number of mathematics and statistics degrees awarded at the low of $11,000$?

Solution:

a. The scale on the x-axis represents time since 1970, so to determine the number of degrees awarded in 1975, read the y-value of the graph at $x = 5$.

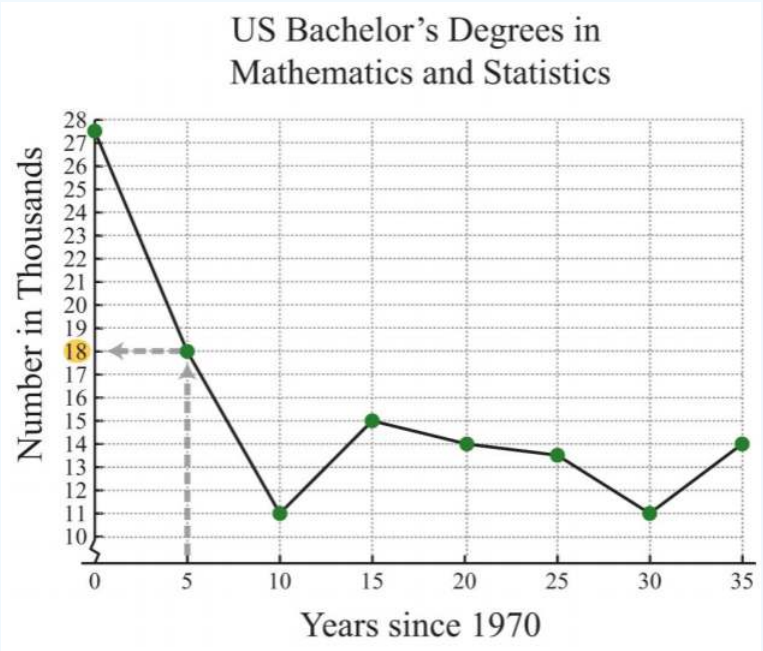


Figure 4.1.9*: Source: Digest of Education Statistics.*

The y-value corresponding to $x = 5$ is 18. The graph indicates that this is in thousands; there were $18,000$ mathematics and statistics degrees awarded in 1975.

b. To find the year a particular number of degrees was awarded, first look at the y-axis. In this case, $11,000$ degrees is represented by 11 on the y-axis; look to the right to see in which years this occurred.

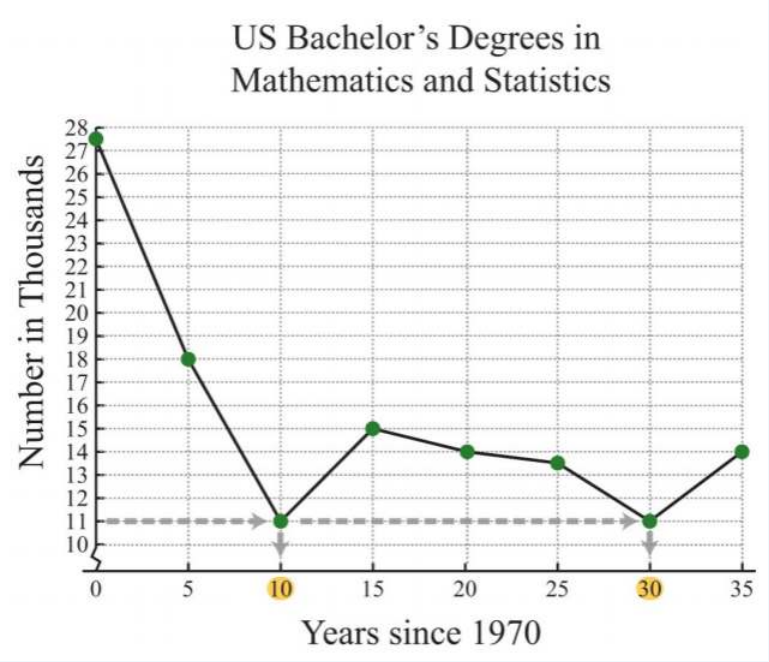


Figure 4.1.10*: Source: Digest of Education Statistics.*

The y-value of 11 occurs at two data points, one where $x = 10$ and the other where $x = 30$. These values correspond to the years 1980 and 2000, respectively.

Answers:

a. In the year 1975, $18,000$ mathematics and statistics degrees were awarded.
b. In the years 1980 and 2000, the lows of $11,000$ mathematics and statistics degrees were awarded.

Distance Formula

Frequently you need to calculate the distance between two points in a plane. To do this, form a right triangle using the two points as vertices of the triangle and then apply the Pythagorean theorem. Recall that the **Pythagorean theorem** states that if given any right triangle with legs measuring a and b units, then the square of the measure of the hypotenuse c is equal to the sum of the squares of the legs: $a^2 + b^2 = c^2$. In other words, the hypotenuse of any right triangle is equal to the square root of the sum of the squares of its legs.

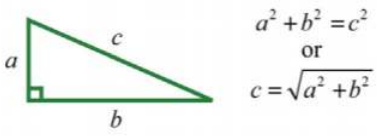


Figure 4.1.11

Example 4.1.5

Find the distance between $(-1, 2)$ and $(3, 5)$.

Solution:

Form a right triangle by drawing horizontal and vertical lines through the two points. This creates a right triangle as shown below:

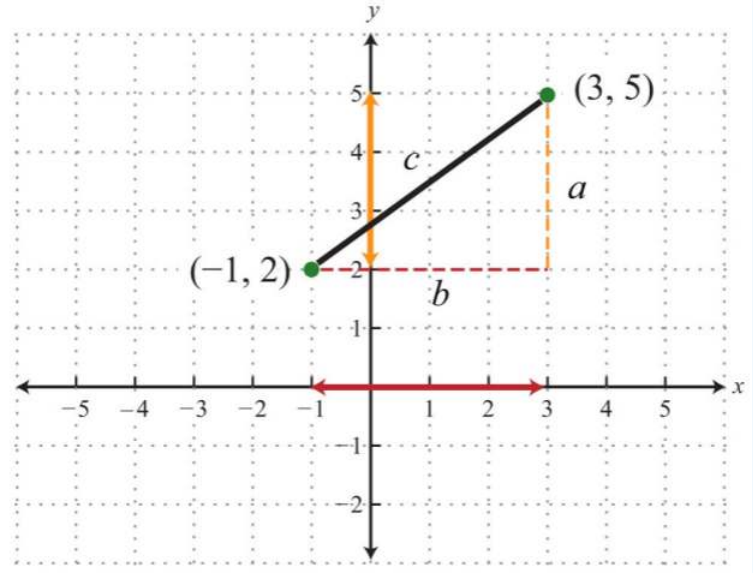


Figure 4.1.12

The length of leg b is calculated by finding the distance between the x-values of the given points, and the length of leg a is calculated by finding the distance between the given y-values.

$$a = 5 - 2 = 3 \text{units}$$
$$b = 3 - (-1) = 3 + 1 = 4 \text{units}$$

Next, use the Pythagorean theorem to find the length of the hypotenuse.

$$\begin{aligned} c &= \sqrt{3^2 + 4^2} \\ &= \sqrt{9 + 16} \\ &= \sqrt{25} \\ &= 5 \text{ units} \end{aligned}$$

Answer:

The distance between the two points is 5 units.

Generalize this process to produce a formula that can be used to algebraically calculate the distance between any two given points.

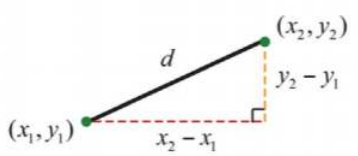


Figure 4.1.13

Given two points, (x_1, y_1) and (x_2, y_2), then the distance, d, between them is given by the distance formula:

$$\sqrt{(x_2 + x_1)^2 + (y_2 + y_1)^2} \tag{4.1.1}$$

Example 4.1.6

Calculate the distance between $(-3, -1)$ and $(-2, 4)$.

Solution:

Use the distance formula.

x_1, y_1	x_2, y_2
$(-3, -1)$	$(-2, 4)$

It is a good practice to include the formula in its general form as a part of the written solution before substituting values for the variables. This improves readability and reduces the chance for errors.

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-2-(-3))^2+(4-(-1))^2} \\ &= \sqrt{(-2+3)^2 + (4+1)^2} \\ &= \sqrt{(1)^2 + (5)^2} \\ &= \sqrt{1+25} \\ &= \sqrt{26} \end{aligned}$$

Answer:

$\sqrt{26}$ units

Exercise 4.1.2

Calculate the distance between $(-7, 5)$ and $(-1, 13)$.

Answer

10 units

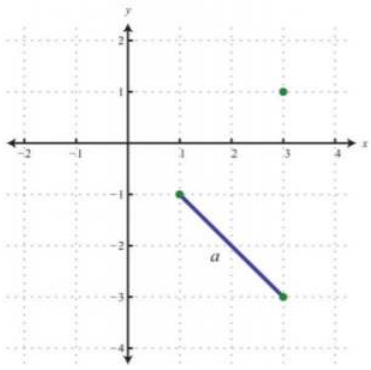
Example 4.1.7

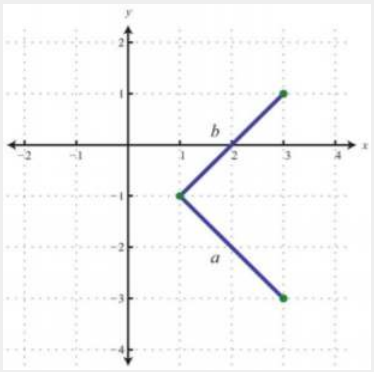
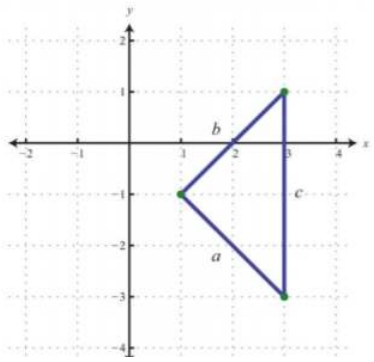
Do the three points $(1, -1)$, $(3, -3)$, and $(3, 1)$ form a right triangle?

Solution:

The Pythagorean theorem states that having side lengths that satisfy the property $a^2 + b^2 = c^2$ is a necessary and sufficient condition of right triangles. In other words, if you can show that the sum of the squares of the leg lengths of the triangle is equal to the square of the length of the hypotenuse, then the figure must be a right triangle. First, calculate the length of each side using the distance formula.

Table 4.1.1

Geometry	Calculation
 Figure 4.1.14	Points: $(1, -1)$ and $(3, -3)$ $a = \sqrt{(3-1)^2 + (-3-(-1))^2}$ $= \sqrt{(a)^2 + (-3+1)^2}$ $= \sqrt{4+(-2)^2}$ $= \sqrt{4+4}$ $= \sqrt{8}$

Geometry	Calculation
 Figure 4.1.15	Points: $(1,-1)$ and $(3,1)$ $b=\sqrt{(3-1)^2+(1-(-1))^2}$ $=\sqrt{2^2+(1+1)^2}$ $=\sqrt{4+(2)^2}$ $=\sqrt{4+4}$ $=\sqrt{8}$
 Figure 4.1.16	Points: $(3,-3)$ and $(3,1)$ $c=\sqrt{(3-3)^2+(1-(-3))^2}$ $=\sqrt{(0)^2+(1+3)^2}$ $=\sqrt{0+(4)^2}$ $=\sqrt{16}$ $=4$

Now we check to see if $a^2+b^2=c^2$.

$$\begin{aligned} a^2+b^2&=c^2\\ (\sqrt{8})^2+(\sqrt{8})^2&=(4)^2\\ \sqrt{64}+\sqrt{64}&=16\\ 8+8&=16\\ 16&=16 \quad \checkmark \end{aligned}$$

Answer:

Yes, the three points form a right triangle. In fact, since two of the legs are equal in length, the points form an isosceles right triangle.

Midpoint Formula

The point that bisects the line segment formed by two points, (x_1,y_1) and (x_2,y_2), is called the midpoint and is given by the following formula:

$$\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right) \tag{4.1.2}$$

The midpoint is an ordered pair formed by finding the average of the x-values and the average of the y-values of the given points.

Example 4.1.8

Calculate the midpoint between $(-1,-2)$ and $(7,4)$.

Solution:

First, calculate the average of the x- and y-values of the given points.

$$\begin{array}{cc} (x_1, y_1) & (x_2, y_2) \\ (-1, -2) & (7, 4) \end{array}$$

$$\frac{x_1 + x_2}{2} = \frac{-1 + 7}{2} = \frac{6}{2} = 3$$
$$\frac{y_1 + y_2}{2} = \frac{-2 + 4}{2} = \frac{2}{2} = 1$$

Next, form the midpoint as an ordered pair using the averaged coordinates.

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
$$(3, 1)$$

Answer:

$$(3, 1)$$

To verify that this is indeed the midpoint, calculate the distance between the two given points and verify that the result is equal to the sum of the two equal distances from the endpoints to this midpoint. This verification is left to the reader as an exercise.

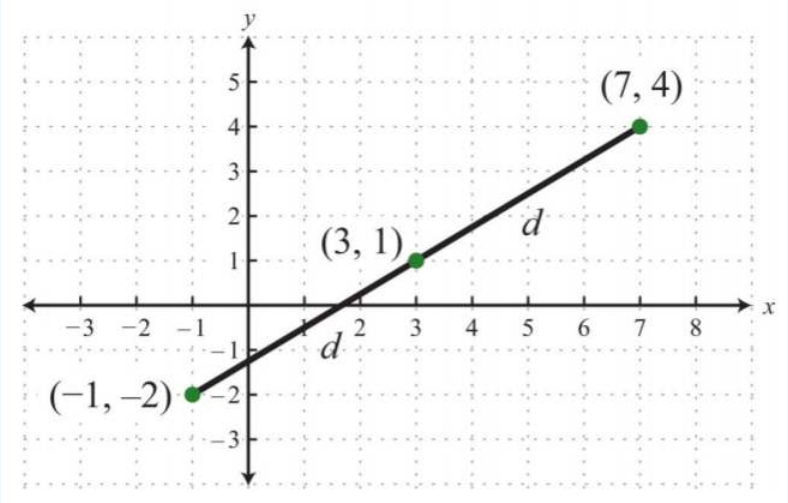


Figure 4.1.17

? Exercise 4.1.3

Find the midpoint between $(-6, 5)$ and $(6, -11)$.

Answer

$(0, -3)$

Key Takeaways

- Use the rectangular coordinate system to uniquely identify points in a plane using ordered pairs (x, y). Ordered pairs indicate position relative to the origin. The x-coordinate indicates position to the left and right of the origin. The y-coordinate indicates position above or below the origin.
- The scales on the x-axis and y-axis may be different. Choose a scale for each axis that is appropriate for the given problem.
- Graphs are used to visualize real-world data. Typically, independent data is associated with the x-axis and dependent data is associated with the y-axis.
- The Pythagorean theorem gives us a necessary and sufficient condition of right triangles. Given a right triangle, then the measures of the sides satisfy $a^2 + b^2 = c^2$. Conversely, if the sides satisfy $a^2 + b^2 = c^2$, then the triangle must be a right triangle.
- The distance formula, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, is derived from the Pythagorean theorem and gives us the distance between any two points, (x_1, y_1) and (x_2, y_2), in a rectangular coordinate plane.

- The midpoint formula, $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$, is derived by taking the average of each coordinate and forming an ordered pair.

Exercise 4.1.4 Ordered Pairs

Give the coordinates of points A, B, C, D, and E.

1.

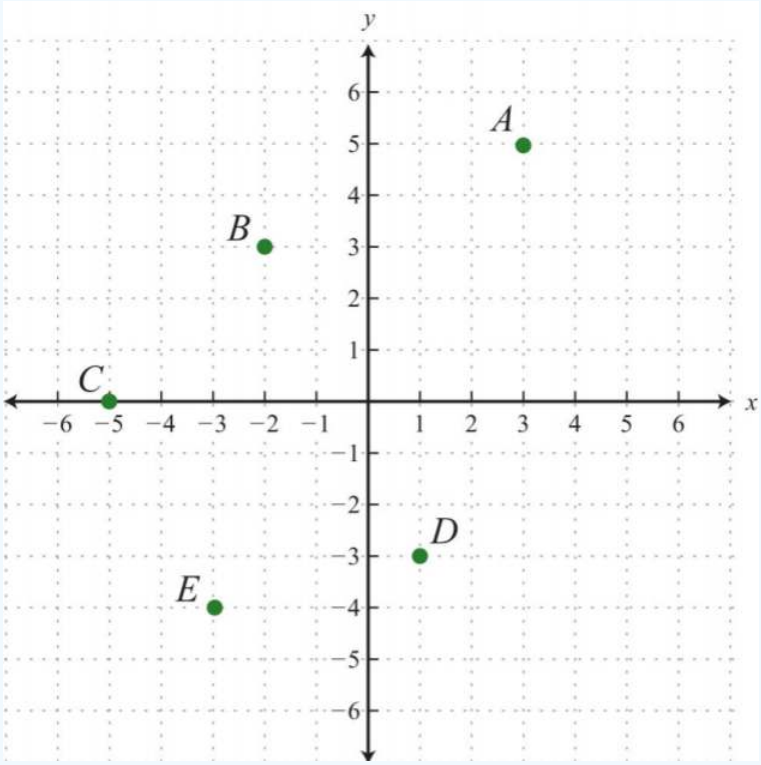


Figure 4.1.18

2.

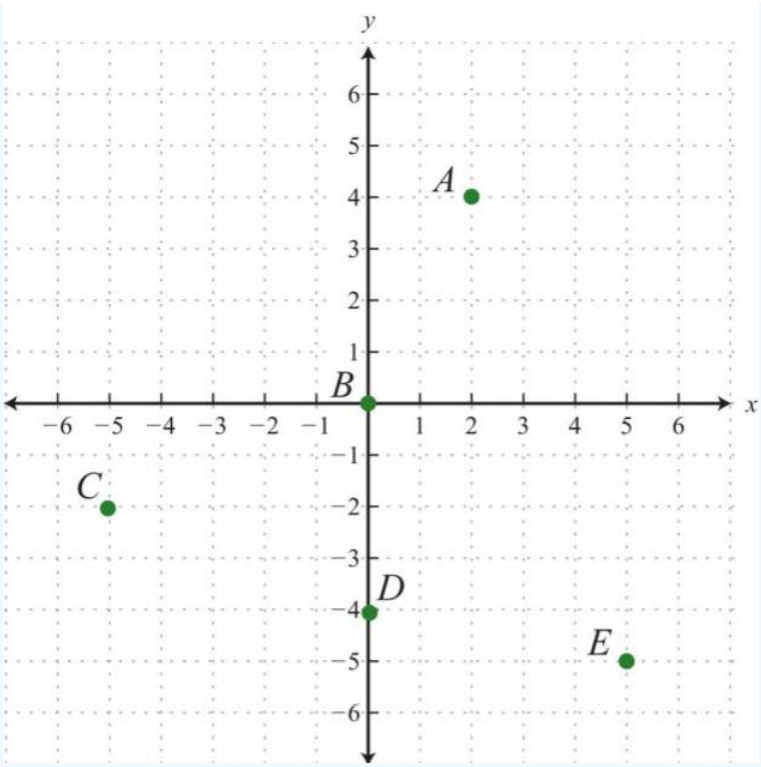


Figure 4.1.19

3.

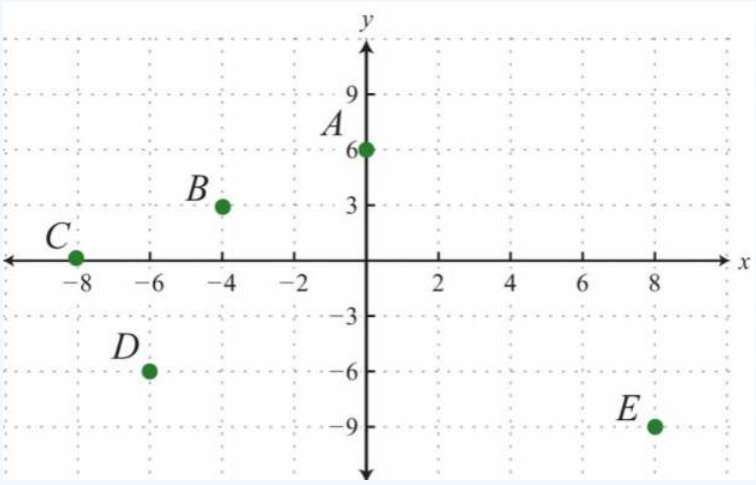


Figure 4.1.20

4.

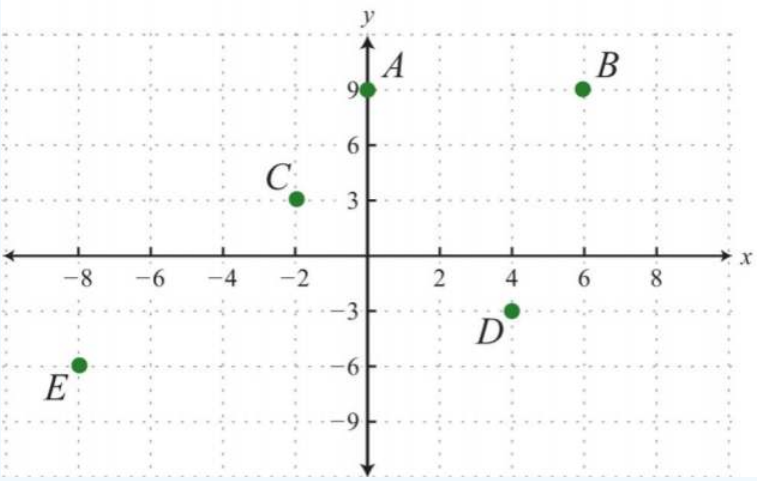


Figure 4.1.21

5.

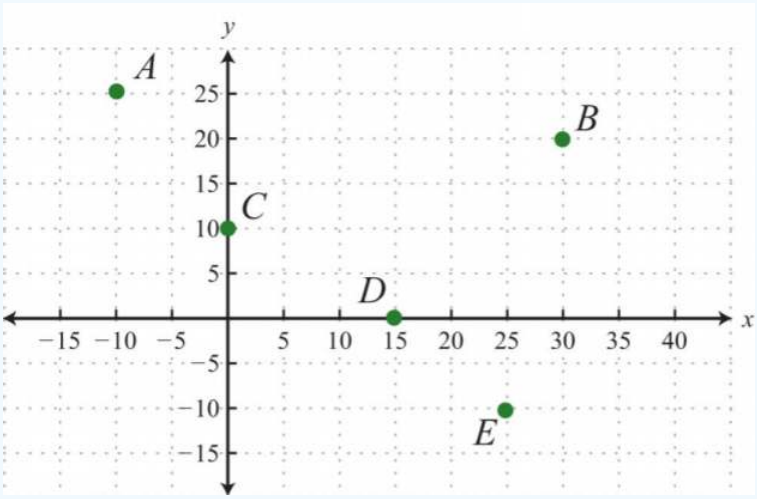


Figure 4.1.22

6.

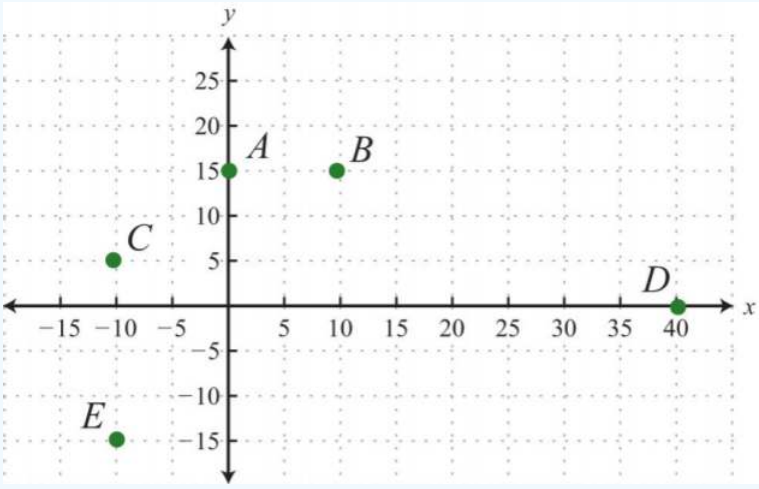


Figure 4.1.23

Answer

1. $A:(3,5); B:(-2,3); C:(-5,0); D:(1,-3); E:(-3,-4)$

3. $A:(0,6); B:(-4,3); C:(-8,0); D:(-6,-6); E:(8,-9)$
5. $A:(-10,25); B:(30,20); C:(0,10); D:(15,0); E:(25,-10)$

Exercise 4.1.5 Ordered Pairs

Graph the given set of ordered pairs.

1. $\{(-4,5),(-1,1),(-3,-2),(5,-1)\}$
2. $\{(-15,-10),(-5,10),(15,10),(5,-10)\}$
3. $\{(-2,5),(10,0),(2,-5),(6,-10)\}$
4. $\{(-8,3),(-4,6),(0,-6),(6,9)\}$
5. $\{(-10,5),(20,-10),(30,15),(50,0)\}$
6. $\{(-\frac{5}{3},-\frac{1}{2}),(-\frac{1}{3},\frac{1}{2}),(\frac{2}{3},-1),(\frac{5}{3},1)\}$
7. $\{(-\frac{3}{5},-\frac{4}{3}),(\frac{2}{5},\frac{4}{3}),(1,-\frac{2}{3}),(0,1)\}$
8. $\{(-3.5,0),(-1.5,2),(0,1.5),(2.5,-1.5)\}$
9. $\{(-0.8,0.2),(-0.2,-0.4),(0,-1),(0.6,-0.4)\}$
10. $\{(-1.2,-1.2),(-0.3,-0.3),(0,0),(0.6,0.6),(1.2,1.2)\}$

Answer

1.

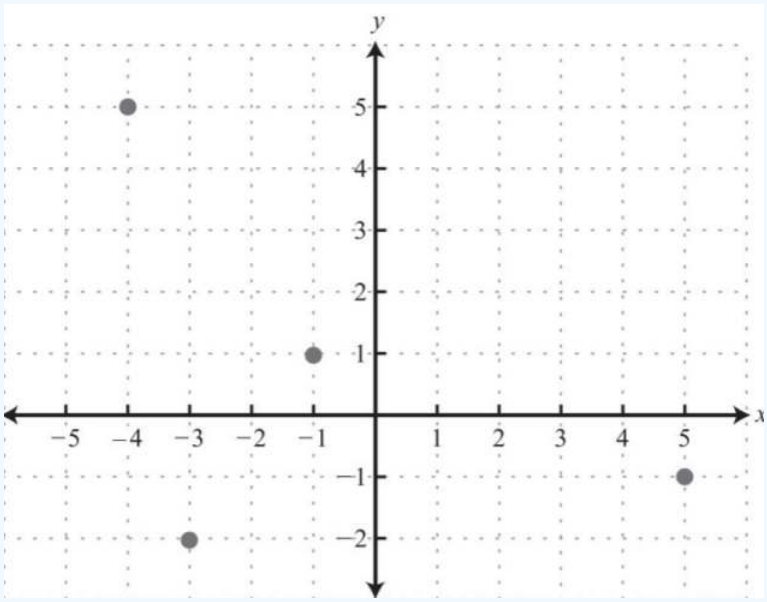


Figure 4.1.24

3.

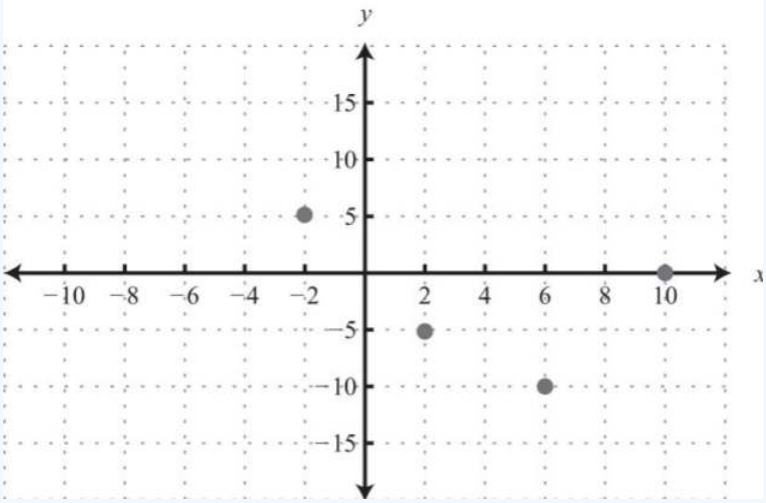


Figure 4.1.25

5.

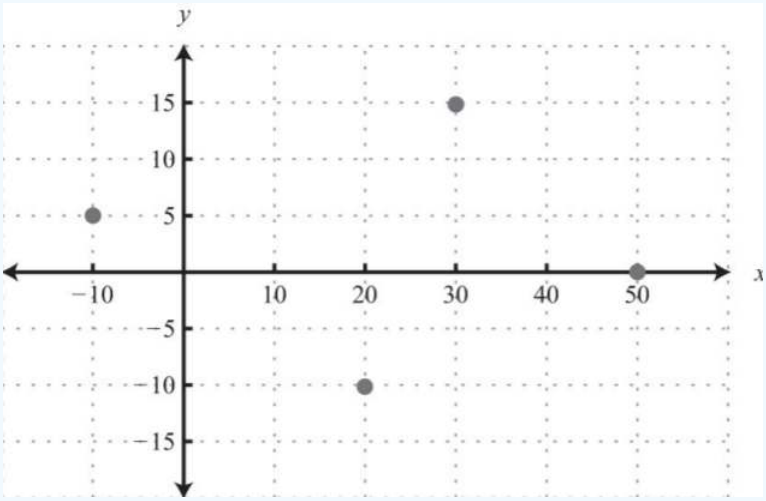


Figure 4.1.26

7.

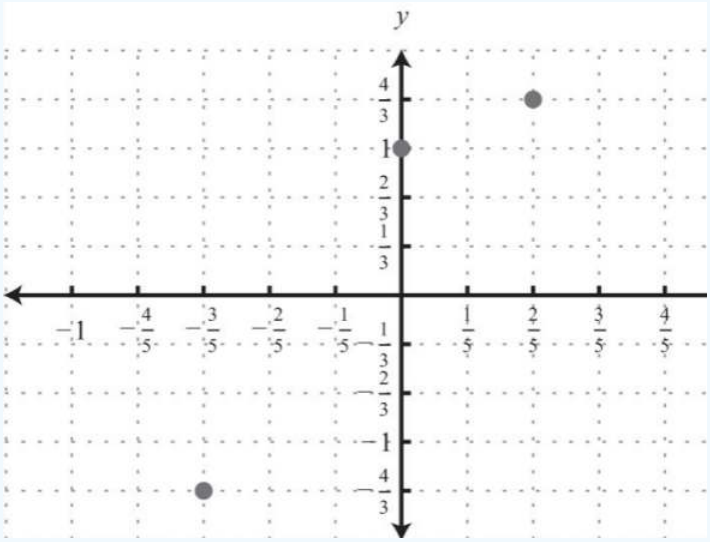


Figure 4.1.27

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9.

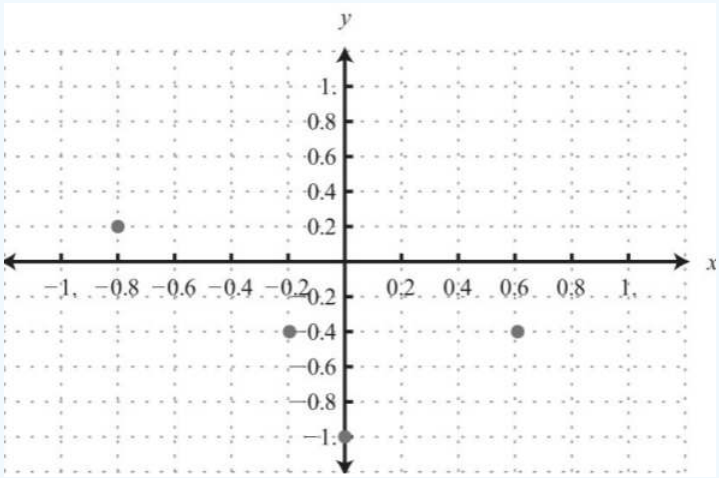


Figure 4.1.28

Exercise 4.1.6 Ordered Pairs

State the quadrant in which the given point lies.

1. $(-3, 2)$
2. $(5, 7)$
3. $(-12, -15)$
4. $(7, -8)$
5. $(-3.8, 4.6)$
6. $(17.3, 1.9)$
7. $(-\frac{1}{8}, -\frac{5}{8})$
8. $(\frac{3}{4}, -\frac{1}{4})$
9. $x > 0$ and $y < 0$
10. $x < 0$ and $y < 0$
11. $x < 0$ and $y > 0$
12. $x > 0$ and $y > 0$

Answer

1. QII
3. QIII
5. QII
7. QIII
9. QIV
11. QII

Exercise 4.1.7 Ordered Pairs

The average price of a gallon of regular unleaded gasoline in US cities is given in the following line graph. Use the graph to answer the following questions.

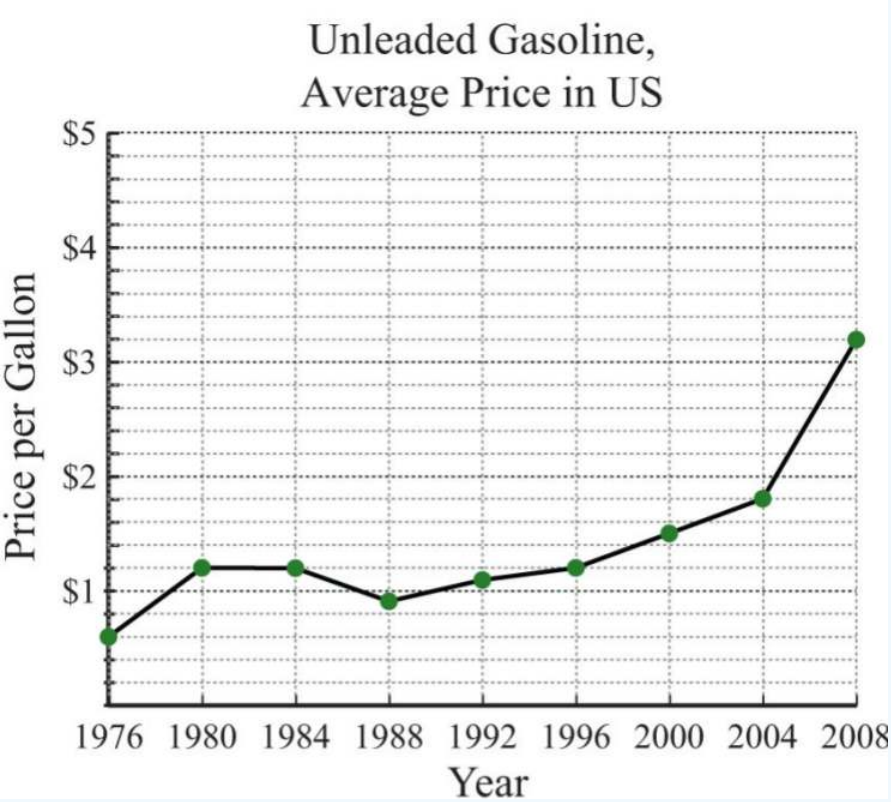


Figure 4.1.29*: Source: Bureau of Labor Statistics*

1. What was the average price of a gallon of unleaded gasoline in 2004?
2. What was the average price of a gallon of unleaded gasoline in 1976?
3. In which years were the average price of a gallon of unleaded gasoline $1.20?
4. What is the price increase of a gallon of gasoline from 1980 to 2008?
5. What was the percentage increase in the price of a gallon of unleaded gasoline from 1976 to 1980?
6. What was the percentage increase in the price of a gallon of unleaded gasoline from 2000 to 2008?

Answer

1. $1.80

3. 1980 to 1984, 1996

3. 100%

? Exercise 4.1.8 Ordered Pairs

The average price of all-purpose white flour in US cities from 1980 to 2008 is given in the following line graph. Use the graph to answer the questions that follow.

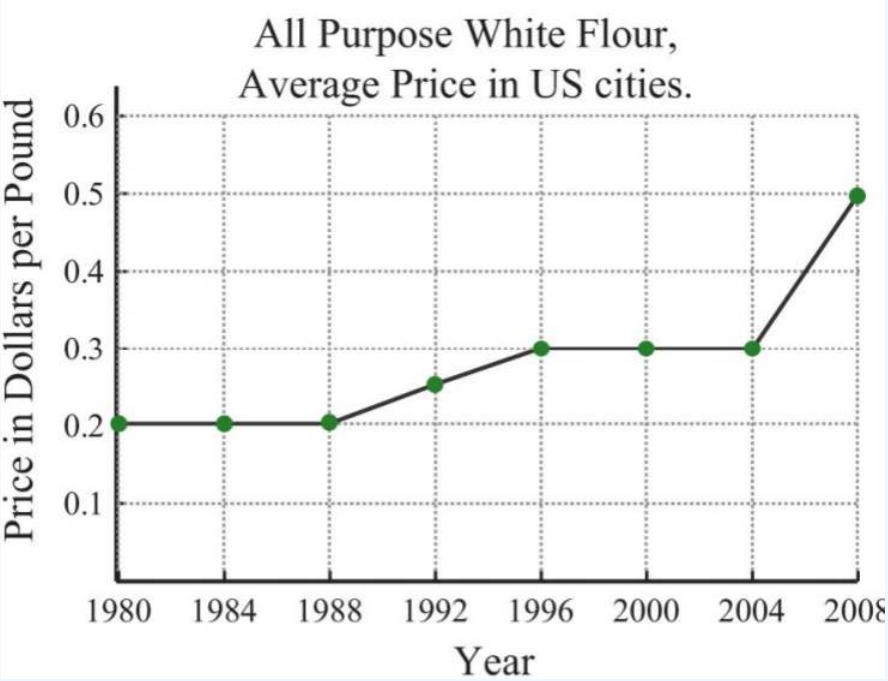


Figure 4.1.30*: Source: Bureau of Labor Statistics*

1. What was the average price per pound of all-purpose white flour in 2000?
2. What was the average price per pound of all-purpose white flour in 2008?
3. In which year did the price of flour average $0.25 per pound?
4. In which years did the price of flour average $0.20 per pound?
5. What was the percentage increase in flour from the year 2000 to 2008?
6. What was the percentage increase in flour from the year 1992 to 2000?

Answer

1. $0.30
3. 1992
5. 67%

? Exercise 4.1.9 Ordered Pairs

Given the following data, create a line graph.

1. The percentage of total high school graduates who enrolled in college.

Table 4.1.2*: Source: Digest of Education Statistics*

Year	Percentage
1969	36%
1979	40%
1989	47%
1999	42%

2. The average daily temperature given in degrees Fahrenheit in May.

Table 4.1.3

Exam	Temperature

Exam	Temperature
8:00 am	60
12:00 pm	72
4:00 pm	75
8:00 pm	67
12:00 am	60
4:00 am	55

Answer

1.

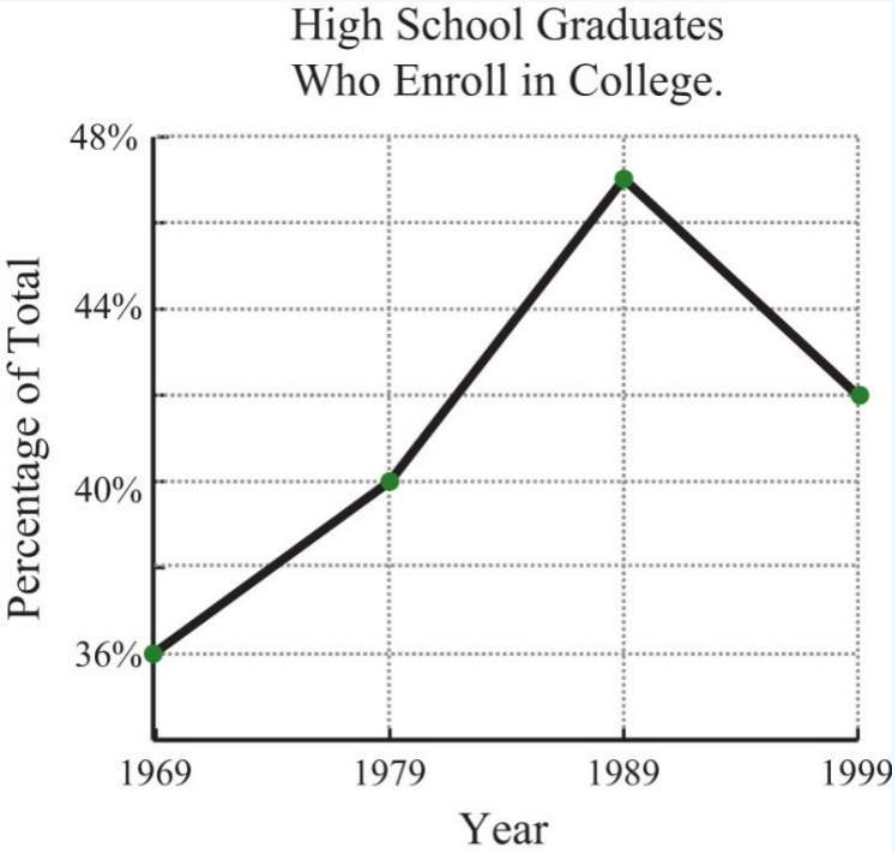


Figure 4.1.31

Exercise 4.1.10 Ordered Pairs

Calculate the area of the shape formed by connecting the following set of vertices.

1. $\{(0,0),(0,3),(5,0),(5,3)\}$
2. $\{(-1,-1),(-1,1),(1,-1),(1,1)\}$
3. $\{(-2,-1),(-2,3),(5,3),(5,-1)\}$
4. $\{(-5,-4),(-5,5),(3,5),(3,-4)\}$
5. $\{(0,0),(4,0),(2,2)\}$
6. $\{(-2,-2),(2,-2),(0,2)\}$
7. $\{(0,0),(0,6),(3,4)\}$
8. $\{(-2,0),(5,0),(3,-3)\}$

Answer

1. 15 square units
3. 28 square units
5. 4 square units
7. 9 square units

Exercise 4.1.11 Distance Formula

Calculate the distance between the given two points.

1. $(-5, 3)$ and $(-1, 6)$
2. $(6, -2)$ and $(-2, 4)$
3. $(0, 0)$ and $(5, 12)$
4. $(-6, -8)$ and $(0, 0)$
5. $(-7, 8)$ and $(5, -1)$
6. $(-1, -2)$ and $(9, 22)$
7. $(-1, 2)$ and $(-\frac{7}{2}, -4)$
8. $(-\frac{1}{2}, \frac{1}{3})$ and $(\frac{5}{2}, -\frac{11}{3})$
9. $(-\frac{1}{3}, \frac{2}{3})$ and $(1, -\frac{1}{3})$
10. $(\frac{1}{2}, -\frac{3}{4})$ and $(\frac{3}{2}, \frac{1}{4})$
11. $(1, 2)$ and $(4, 3)$
12. $(2, -4)$ and $(-3, -2)$
13. $(-1, 5)$ and $(1, -3)$
14. $(1, -7)$ and $(5, -1)$
15. $(-7, -3)$ and $(-1, 6)$
16. $(0, 1)$ and $(1, 0)$
17. $(-0.2, -0.2)$ and $(1.8, 1.8)$
18. $(1.2, -3.3)$ and $(2.2, -1.7)$

Answer

1. 5 units
3. 13 units
5. 15 units
7. $\frac{13}{2}$ units
9. $\frac{5}{3}$ units
11. $\sqrt{10}$ units
13. $2\sqrt{17}$ units
15. $3\sqrt{13}$ units
17. 2.8 units

Exercise 4.1.12 Distance Formula

For each problem, show that the three points form a right triangle.

1. $(-3, -2), (0, -2)$, and $(0, 4)$
2. $(7, 12), (7, -13)$,and $(-5, -4)$
3. $(-1.4, 0.2), (1, 2)$,and $(1, -3)$
4. $(2, -1), (-1, 2)$, and $(6, 3)$

5. $(-5, 2)$, $(-1, -2)$, and $(-2, 5)$
6. $(1, -2)$, $(2, 3)$, and $(-3, 4)$

Answer

1. Proof
3. Proof
5. Proof

Exercise 4.1.13 Distance Formula

Isosceles triangles have two legs of equal length. For each problem, show that the following points form an isosceles triangle.

1. $(1, 6)$, $(-1, 1)$, and $(3, 1)$
2. $(-6, -2)$, $(-3, -5)$, and $(-9, -5)$
3. $(-3, 0)$, $(0, 3)$, and $(3, 0)$
4. $(0, -1)$, $(0, 1)$, and $(1, 0)$

Answer

1. Proof
3. Proof

Exercise 4.1.14 Distance Formula

Calculate the area and the perimeter of the triangles formed by the following set of vertices.

1. $\{(-4, -5), (-4, 3), (2, 3)\}$
2. $\{(-1, 1), (3, 1), (3, -2)\}$
3. $\{(-3, 1), (-3, 5), (1, 5)\}$
4. $\{(-3, -1), (-3, 7), (1, -1)\}$

Answer

1. Perimeter: 24 units; area: 24 square units
3. Perimeter: $8 + 4\sqrt{2}$ units; area: 8 square units

Exercise 4.1.15 Midpoint Formula

Find the midpoint between the given two points.

1. $(-1, 6)$ and $(-7, -2)$
2. $(8, 0)$ and $(4, -3)$
3. $(-10, 0)$ and $(10, 0)$
4. $(-3, -6)$ and $(-3, 6)$
5. $(-10, 5)$ and $(14, -5)$
6. $(0, 1)$ and $(2, 2)$
7. $(5, -3)$ and $(4, -5)$
8. $(0, 0)$ and $(1, 1)$
9. $(-1, -1)$ and $(4, 4)$
10. $(3, -5)$ and $(3, 5)$
11. $(-\frac{1}{2}, -\frac{1}{3})$ and $(\frac{3}{2}, \frac{7}{3})$
12. $(\frac{3}{4}, -\frac{2}{3})$ and $(\frac{1}{8}, -\frac{1}{2})$
13. $(\frac{5}{3}, \frac{1}{4})$ and $(-\frac{1}{6}, -\frac{3}{2})$

14. $(-\frac{1}{5}, -\frac{5}{2})$ and $(\frac{7}{10}, -\frac{1}{4})$
15. Given the right triangle formed by the vertices $(0, 0)$, $(6, 0)$, and $(6, 8)$, show that the midpoints of the sides form a right triangle.
16. Given the isosceles triangle formed by the vertices $(-10, -12)$, $(0, 12)$, and $(10, -12)$, show that the midpoints of the sides also form an isosceles triangle.
17. Calculate the area of the triangle formed by the vertices $(-4, -3)$, $(-1, 1)$, and $(2, -3)$. (Hint: The vertices form an isosceles triangle.)
18. Calculate the area of the triangle formed by the vertices $(-2, 1)$, $(4, 1)$, and $(1, -5)$.

Answer

1. $(-4, 2)$
3. $(0, 0)$
5. $(2, 0)$
7. $(\frac{9}{2}, -4)$
9. $(\frac{3}{2}, \frac{3}{2})$
11. $(\frac{1}{2}, 1)$
13. $(\frac{3}{4}, -\frac{5}{8})$
15. 12 square units

? Exercise 4.1.16 Discussion Board Topics

1. Research and discuss the life and contributions to mathematics of René Descartes.
2. Research and discuss the history of the right triangle and the Pythagorean theorem.
3. What is a Pythagorean triple? Provide some examples.
4. Explain why you cannot use a ruler to calculate distance on a graph.
5. How do you bisect a line segment with only a compass and a straightedge?

Answer

1. Answers may vary
3. Answers may vary
5. Answers may vary

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4.2: Graph by Plotting Points

Learning Objectives

- Verify solutions to linear equations with two variables.
- Graph lines by plotting points.
- Identify and graph horizontal and vertical lines.

Solutions to Equations with Two Variables

A linear equation with two variables has standard form $ax+by=c$, where a, b, and c are real numbers and a and b are not both 0. Solutions to equations of this form are ordered pairs (x, y), where the coordinates, when substituted into the equation, produce a true statement.

Example 4.2.1

Determine whether $(1,-2)$ and $(-4,1)$ are solutions to $6x-3y=12$.

Solution:

Substitute the x- and y-values into the equation to determine whether the ordered pair produces a true statement.

$\underline{Check\ (1,-2)}$	$\underline{Check\ (-4,1)}$
$6x-3y=12$	$6x-3y=12$
$6(1)-3(-2)=12$	$6(-4)-3(1)=12$
$6+6=12$	$-24-3=12$
$12=12$ ✓	$-27=12$ x

Answer:

$(1,-2)$ is a solution, and $(-4,1)$ is not.

It is often the case that a linear equation is given in a form where one of the variables, usually y, is isolated. If this is the case, then we can check that an ordered pair is a solution by substituting in a value for one of the coordinates and simplifying to see if we obtain the other.

Example 4.2.2

Are $(\frac{1}{2},-3)$ and $(-5,14)$ solutions to $y=2x-4$?

Solution:

Substitute the x-values and simplify to see if the corresponding y-values are obtained.

$\underline{x=\frac{1}{2}}$	$\underline{x=-5}$
$y=2x-4$	$y=2x-4$
$=2\left(\frac{1}{2}\right)-4$	$=2(-5)-4$
$=1-4$	$=-10-4$
$=-3$ ✓	$=-14$ x

Answer:

$(\frac{1}{2},-3)$ is a solution, and $(-5,14)$ is not.

Exercise 4.2.1

Is $(6, -1)$ a solution to $y = -\frac{2}{3}x + 3$?

Answer

Yes

When given linear equations with two variables, we can solve for one of the variables, usually y, and obtain an equivalent equation as follows:

Written in this form, we can see that y depends on x. Here x is the independent variable and y is the dependent variable.

$$\left.\begin{aligned} 6x - 3y &= 12 \\ y &= 2x - 4 \end{aligned}\right\} \textit{Equivalent equations}$$

The linear equation $y = 2x - 4$ can be used to find ordered pair solutions. If we substitute any real number for x, then we can simplify to find the corresponding y-value. For example, if $x = 3$, then $y = 2(3) - 4 = 6 - 4 = 2$, and we can form an ordered pair solution, $(3, 2)$. Since there are infinitely many real numbers to choose for x, the linear equation has infinitely many ordered pair solutions (x, y).

Example 4.2.3

Find ordered pair solutions to the equation $5x - y = 14$ with the given x-values $\{-2, -1, 0, 4, 6\}$.

Solution:

First, solve for y.

Next, substitute the x-values in the equation $y = 5x - 14$ to find the corresponding y-values.

$x - value$ $y - value$ $Solution$ $x = -2$ $(-2, -24)$ $x = -1$ $(-1, -19)$ $x = 0$ $(0, -14)$ $x = 4$ $(4, 6)$ $x = 6$ $(6, 16)$ Table 4.2.1

Answer:

$$\{(-2, -24), (-1, -19), (0, -14), (4, 6), (6, 16)\}$$

In the previous example, certain x-values are given, but that is not always going to be the case. When treating x as the independent variable, we can choose any values for x and then substitute them into the equation to find the corresponding y-values. This method produces as many ordered pair solutions as we wish.

Example 4.2.4

Find five ordered pair solutions to $6x + 2y = 10$.

Solution:

First, solve for y.

$$\begin{aligned} 6x + 2y &= 10 \\ 6x + 2y - 6x &= 10 - 6x \\ 2y &= -6x + 10 \\ \frac{2y}{2} &= \frac{-6x + 10}{2} \\ y &= \frac{-6x}{2} + \frac{10}{2} \\ y &= -3x + 5 \end{aligned}$$

Next, choose any set of x-values. Usually we choose some negative values and some positive values. In this case, we will find the corresponding y-values when x is $\{-2, -1, 0, 1, 2\}$. Make the substitutions required to fill in the following table (often referred to as a t-chart):

x	y		Solutions
-2	11	$y=-3(-2)+5=6+5=11$	$(-2, 11)$
-1	8	$y=-3(-1)+5=3+5=8$	$(-1, 8)$
0	5	$y=-3(0)+5=0+5=5$	$(0, 5)$
1	2	$y=-3(1)+5=-3+5=2$	$(1, 2)$
2	-1	$y=-3(2)+5=-6+5=-1$	$(2, -1)$

Figure 4.2.1

Answer:

$\{(-2, 11), (-1, 8), (0, 5), (1, 2), (2, -1)\}$ Since there are infinitely many ordered pair solutions, answers may vary depending on the choice of values for the independent variable.

? Exercise 4.2.2

Find five ordered pair solutions to $10x - 2y = 2$.

Answer

$\{(-2, -11), (-1, -6), (0, -1), (1, 4), (2, 9)\}$ (answers may vary)

Graph by Plotting Points

Since the solutions to linear equations are ordered pairs, they can be graphed using the rectangular coordinate system. The set of all solutions to a linear equation can be represented on a rectangular coordinate plane using a straight line connecting at least two points; this line is called its graph. To illustrate this, plot five ordered pair solutions, $\{(-2, 11), (-1, 8), (0, 5), (1, 2), (2, -1)\}$ to the linear equation $6x + 2y = 10$.

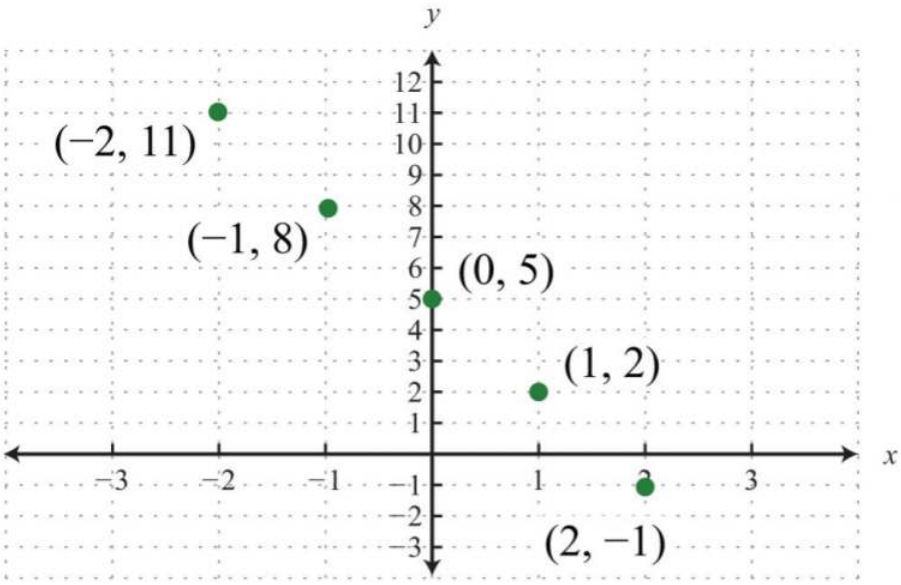


Figure 4.2.2

Notice that the points are collinear; this will be the case for any linear equation. Draw a line through the points with a straightedge, and add arrows on either end to indicate that the graph extends indefinitely.

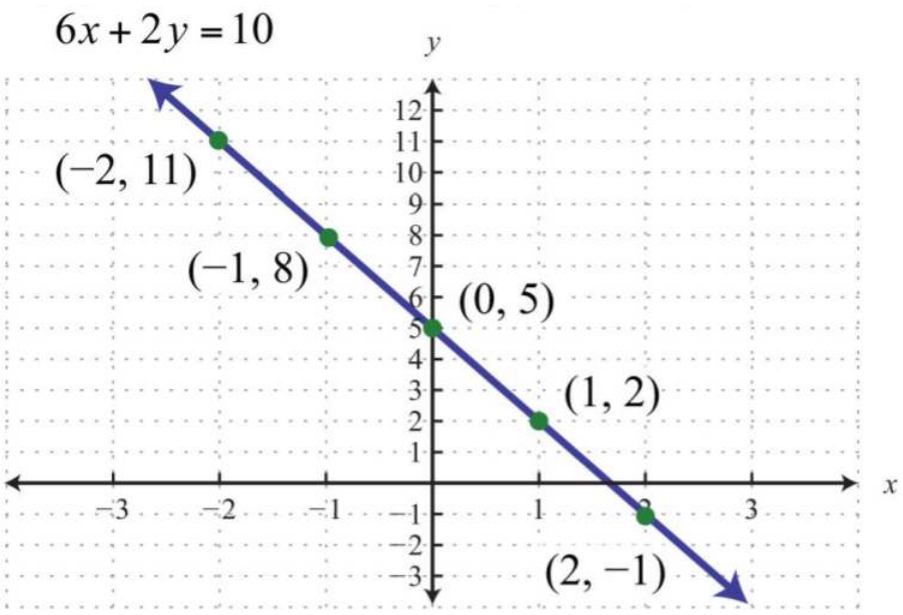


Figure 4.2.3

The resulting line represents all solutions to $6x + 2y = 10$, of which there are infinitely many. The steps for graphing lines by plotting points are outlined in the following example.

Example 4.2.5

Find five ordered pair solutions and graph:

$$10x - 5y = 10.$$

Solution:

Step 1: Solve for y.

Step 2: Choose at least two x-values and find the corresponding y-values. In this section, we will choose five real numbers to use as x-values. It is a good practice to choose 0 and some negative numbers, as well as some positive numbers.

Figure 4.2.4

Five ordered pair solutions are $\{(-2,-6),(-1,-4),(0,-2),(1,0),(2,2)\}$

Step 3: Choose an appropriate scale, plot the points, and draw a line through them using a straightedge. In this case, choose a scale where each tick mark on the y-axis represents 2 units because all the y-values are multiples of 2.

Answer:

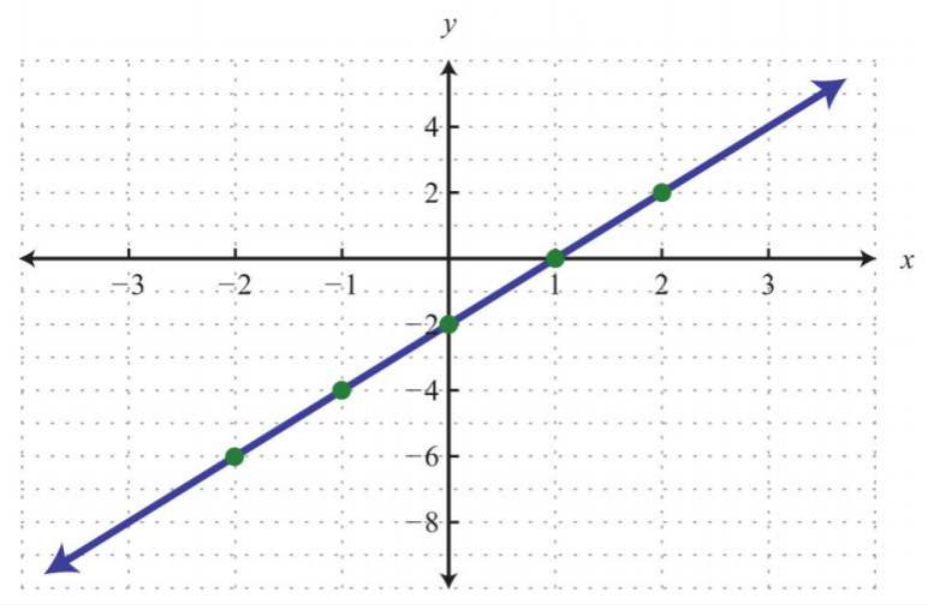


Figure 4.2.5

It will not always be the case that y can be solved in terms of x with integer coefficients. In fact, the coefficients often turn out to be fractions.

Example 4.2.6

Find five ordered pair solutions and graph:

$$-5x+2y=10.$$

Solution:

$$\begin{aligned} -5x+2y &= 10 \\ -5x+2y+5x &= 10+5x \\ 2y &= 5x+10 \\ frac2y2 &= \frac{5x+10}{2} \\ y &= \frac{5x}{2}+\frac{10}{2} \\ y &= \frac{5}{2}x+5 \end{aligned}$$

Remember that you can choose any real number for the independent variable x, so choose wisely here. Since the denominator of the coefficient of the variable x is 2, you can avoid fractions by choosing multiples of 2 for the x-values. In this case, choose the set of x-values $\{-6,-4,-2,0,2\}$and find the corresponding y-values.

x	y
-6	-10
-4	-5
-2	0
0	5
2	10

$$y=\frac{5}{2}(-6)+5=-15+5=-10$$

$$y=\frac{5}{2}(-4)+5=-10+5=-5$$

$$y=\frac{5}{2}(-2)+5=-5+5=0$$

$$y=\frac{5}{2}(0)+5=0+5=5$$

$$y=\frac{5}{2}(2)+5=5+5=10$$

Figure 4.2.6

Five solutions are $\{(-6,-10),(-4,-5),(-2,0),(0,5),(2,10)\}$ Here we choose to scale the x-axis with multiples of 2 and the y-axis with multiples of 5.

Answer:

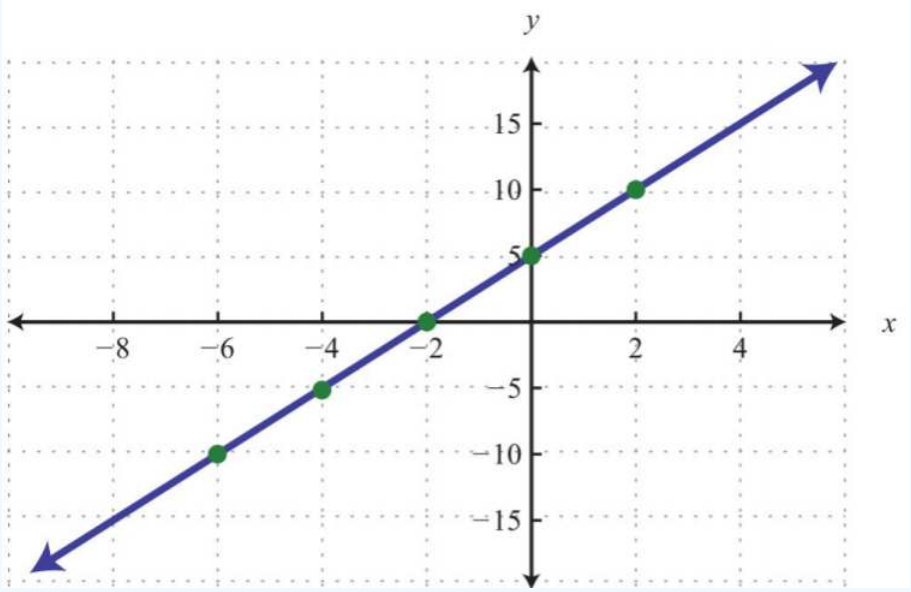


Figure 4.2.7

Exercise 4.2.3

Find five ordered pair solutions and graph: $x+2y=6$.

Answer

$\{(-2,4),(0,3),(2,2),(4,1),(6,0)\}$

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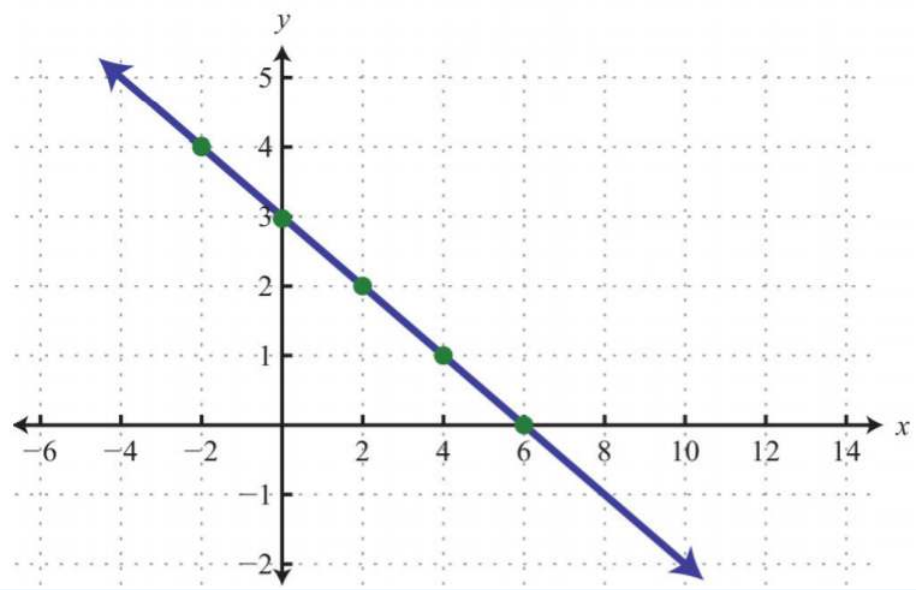


Figure 4.2.8

Horizontal and Vertical Lines

We need to recognize by inspection linear equations that represent a vertical or horizontal line.

Example 4.2.7

Graph by plotting five points: $y = -2$.

Solution:

Since the given equation does not have a variable x, we can rewrite it with a 0 coefficient for x.

$$y = 0x - 2$$

Choose any five values for x and see that the corresponding y-value is always -2.

x	y	
−2	−2	$y = 0(-2) - 2 = 0 - 2 = -2$
−1	−2	$y = 0(-1) - 2 = 0 - 2 = -2$
0	−2	$y = 0(0) - 2 = 0 - 2 = -2$
1	−2	$y = 0(1) - 2 = 0 - 2 = -2$
2	−2	$y = 0(2) - 2 = 0 - 2 = -2$

Figure 4.2.9

We now have five ordered pair solutions to plot $\{(-2, -2), (-1, -2), (0, -2), (1, -2), (2, -2)\}$

Answer:

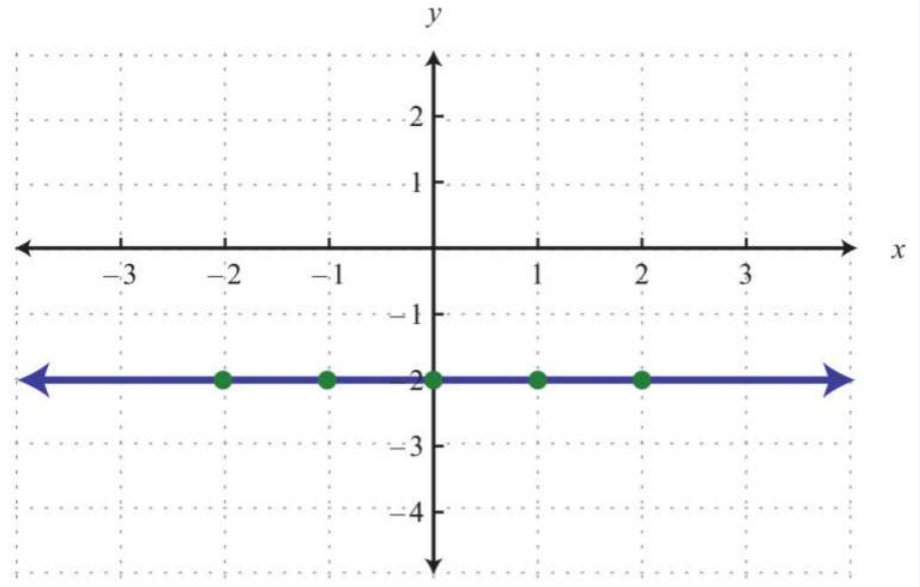


Figure 4.2.10

When the coefficient for the variable x is 0, the graph is a horizontal line. In general, the equation for a horizontal line can be written in the form $y = k$, where k represents any real number.

Example 4.2.8

Graph by plotting five points: $x = 3$.

Solution:

Since the given equation does not have a variable y, rewrite it with a 0 coefficient for y.

$$x = 0y + 3$$

Choose any five values for y and see that the corresponding x-value is always 3.

x	y	
3	−2	$x = 0(-2) + 3 = 0 + 3 = 3$
3	−1	$x = 0(-1) + 3 = 0 + 3 = 3$
3	0	$x = 0(0) + 3 = 0 + 3 = 3$
3	1	$x = 0(1) + 3 = 0 + 3 = 3$
3	2	$x = 0(2) + 3 = 0 + 3 = 3$

Figure 4.2.11

We now have five ordered pair solutions to plot: $\{(3, -2), (3, -1), (3, 0), (3, 1), (3, 2)\}$

Answer:

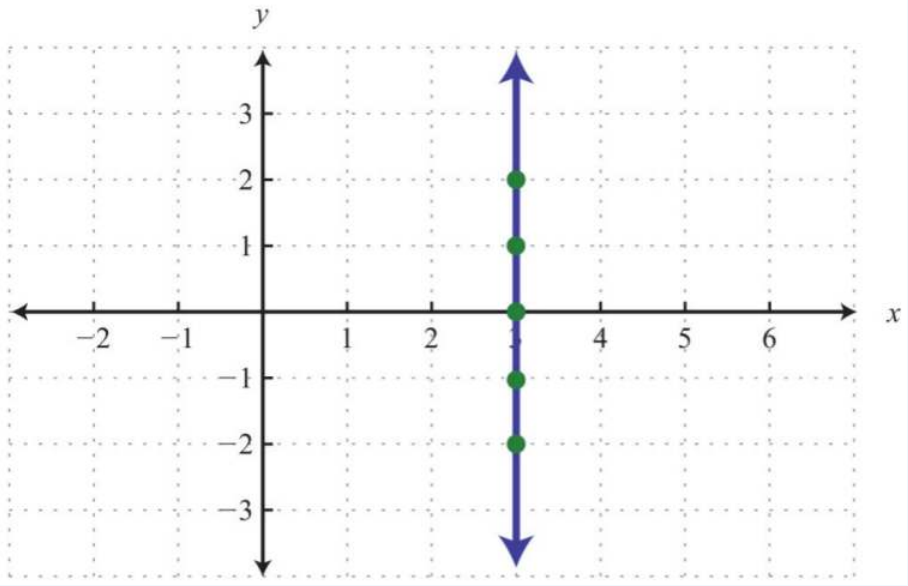


Figure 4.2.12

When the coefficient for the variable y is 0, the graph is a vertical line. In general, the equation for a vertical line can be written as $x = k$, where k represents any real number.

To summarize, if k is a real number.

$$y = k \quad \textit{Horizontal line} \tag{4.2.1}$$

$$x = k \quad \textit{Vertical line} \tag{4.2.2}$$

? Exercise 4.2.4

Graph $y = 5$ and $x = -2$ on the same set of axes and determine where they intersect.

Answer

$(-2, 5)$

Key Takeaways

- Solutions to linear equations with two variables $ax + by = c$ are ordered pairs (x, y), where the coordinates, when substituted into the equation, result in a true statement.
- Linear equations with two variables have infinitely many ordered pair solutions. When the solutions are graphed, they are collinear.
- To find ordered pair solutions, choose values for the independent variable, usually x, and substitute them in the equation to find the corresponding y-values.
- To graph linear equations, determine at least two ordered pair solutions and draw a line through them with a straightedge.
- Horizontal lines are described by $y = k$, where k is any real number.
- Vertical lines are described by $x = k$, where k is any real number.

? Exercise 4.2.5 Solutions to Linear Systems

Determine whether the given point is a solution.

1. $5x - 2y = 4; (-1, 1)$
2. $3x - 4y = 10; (2, -1)$
3. $-3x + y = -6; (4, 6)$
4. $-8x - y = 24; (-2, -3)$
5. $-x + y = -7; (5, -2)$

6. $9x-3y=6;(0,-2)$
7. $\frac{1}{2}x+\frac{1}{3}y=-\frac{1}{6};(1,-2)$
8. $\frac{3}{4}x-\frac{1}{2}y=-1;(2,1)$
9. $4x-3y=1;(\frac{1}{2},\frac{1}{3})$
10. $-10x+2y=-\frac{9}{5};(\frac{1}{5},\frac{1}{10})$
11. $y=\frac{1}{3}x+3;(6,3)$
12. $y=-4x+1;(-2,9)$
13. $y=\frac{2}{3}x-3;(0,-3)$
14. $y=-\frac{5}{8}x+1;(8,-5)$
15. $y=-\frac{1}{2}x+\frac{3}{4};(-\frac{1}{2},1)$
16. $y=-\frac{1}{3}x-\frac{1}{2};(\frac{1}{2},-\frac{2}{3})$
17. $y=2;(-3,2)$
18. $y=4;(4,-4)$
19. $x=3;(3,-3)$
20. $x=0;(1,0)$

Answer

1. No
3. Yes
5. Yes
7. Yes
9. Yes
11. No
13. Yes
15. Yes
17. Yes
19. Yes

? Exercise 4.2.6 Solutions to Linear Systems

Find the ordered pair solutions given the set of x-values.

1. $y=-2x+4;\{-2,0,2\}$
2. $y=12x-3;\{-4,0,4\}$
3. $y=-\frac{3}{4}x+\frac{1}{2};\{-2,0,2\}$
4. $y=-3x+1;\{-\frac{1}{2},0,\frac{1}{2}\}$
5. $y=-4;\{-3,0,3\}$
6. $y=\frac{1}{2}x+\frac{3}{4};\{-\frac{1}{4},0,\frac{1}{4}\}$
7. $2x-3y=1;\{0,1,2\}$
8. $3x-5y=-15;\{-5,0,5\}$
9. $-x+y=3;\{-5,-1,0\}$
10. $\frac{1}{2}x-\frac{1}{3}y=-4;\{-4,-2,0\}$
11. $\frac{3}{5}x+\frac{1}{10}y=2;\{-15,-10,-5\}$
12. $x-y=0;\{10,20,30\}$

Answer

1. $\{(-2,8),(0,4),(2,0)\}$
3. $\{(-2,2),(0,1/2),(2,-1)\}$

5. $\{(-3,-4),(0,-4),(3,-4)\}$
7. $\{(0,-\frac{1}{3}),(1,\frac{1}{3}),(2,1)\}$
9. $\{(-5,-2),(-1,2),(0,3)\}$
11. $\{(-15,110),(-10,80),(-5,50)\}$

Exercise 4.2.7 Solutions to Linear Systems

Find the ordered pair solutions, given the set of y-values.

1. $y=\frac{1}{2}x-1;\{-5,0,5\}$
2. $y=-\frac{3}{4}x+2;\{0,2,4\}$
3. $3x-2y=6;\{-3,-1,0\}$
4. $-x+3y=4;\{-4,-2,0\}$
5. $\frac{1}{3}x-\frac{1}{2}y=-4;\{-1,0,1\}$
6. $\frac{3}{5}x+\frac{1}{10}y=2;\{-20,-10,-5\}$

Answer

1. $\{(-8,-5),(2,0),(12,5)\}$
3. $\{(0,-3),(\frac{4}{3},-1),(2,0)\}$
5. $\{(-\frac{27}{2},-1),(-12,0),(-\frac{21}{2},1)\}$

Exercise 4.2.8 Graphing Lines

Given the set of x-values $\{-2,-1,0,1,2\}$, find the corresponding y-values and graph them.

1. $y=x+1$
2. $y=-x+1$
3. $y=2x-1$
4. $y=-3x+2$
5. $y=5x-10$
6. $5x+y=15$
7. $3x-y=9$
8. $6x-3y=9$
9. $y=-5$
10. $y=3$

Answer

1.

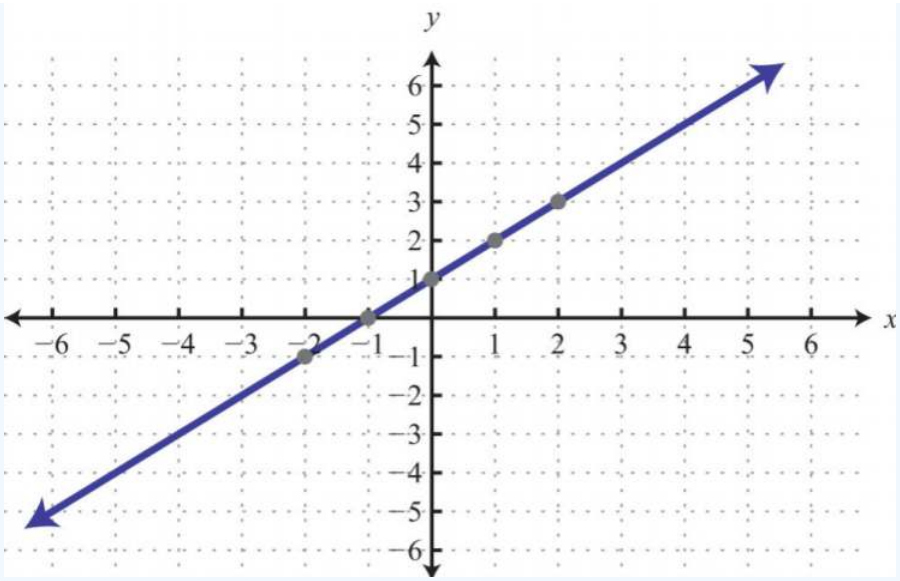


Figure 4.2.13

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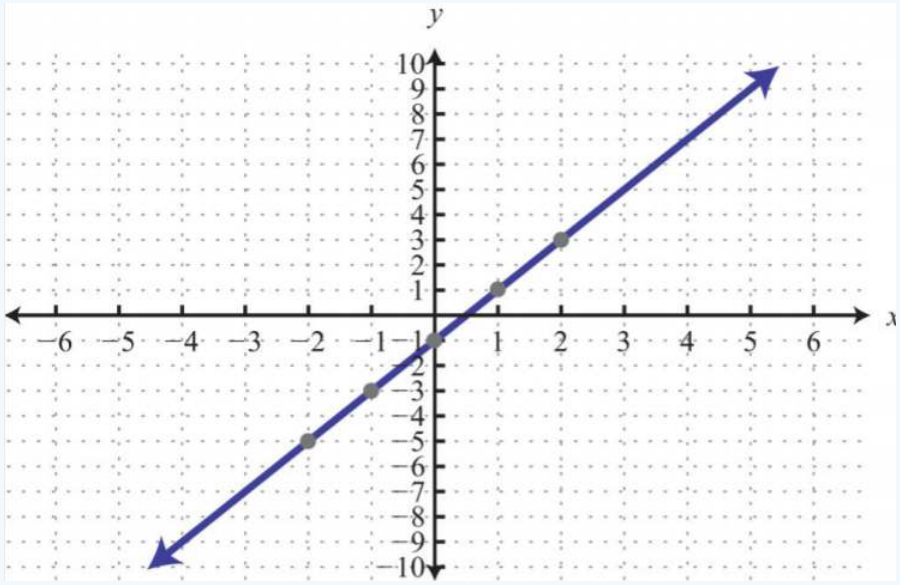


Figure 4.2.14

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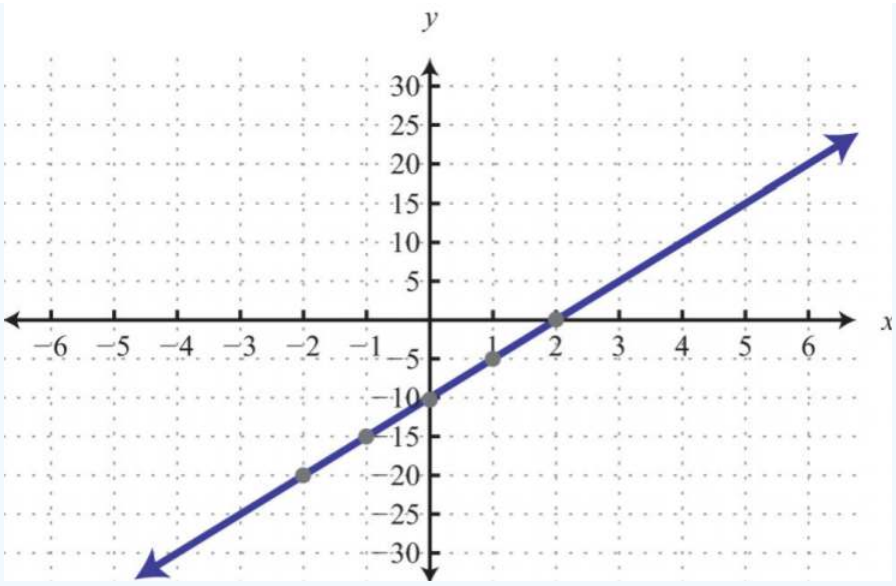


Figure 4.2.15

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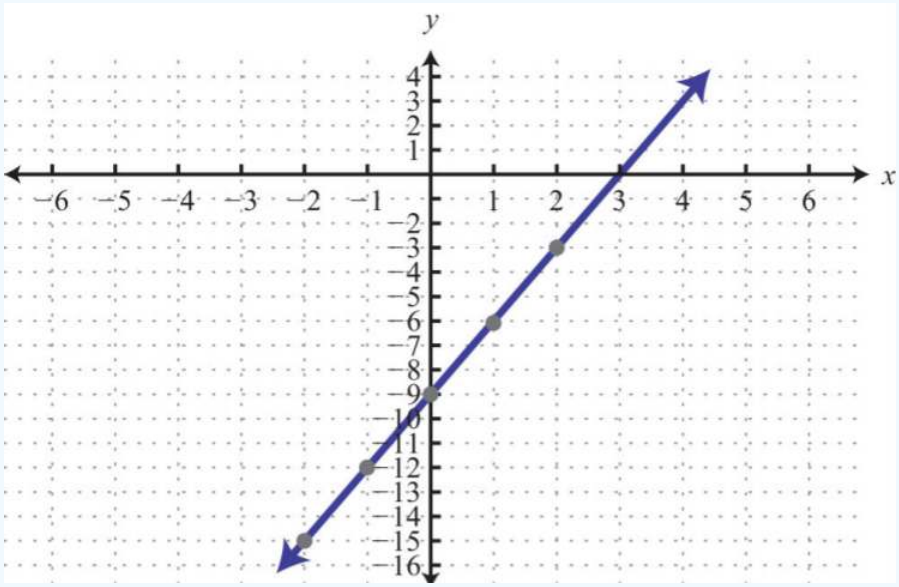


Figure 4.2.16

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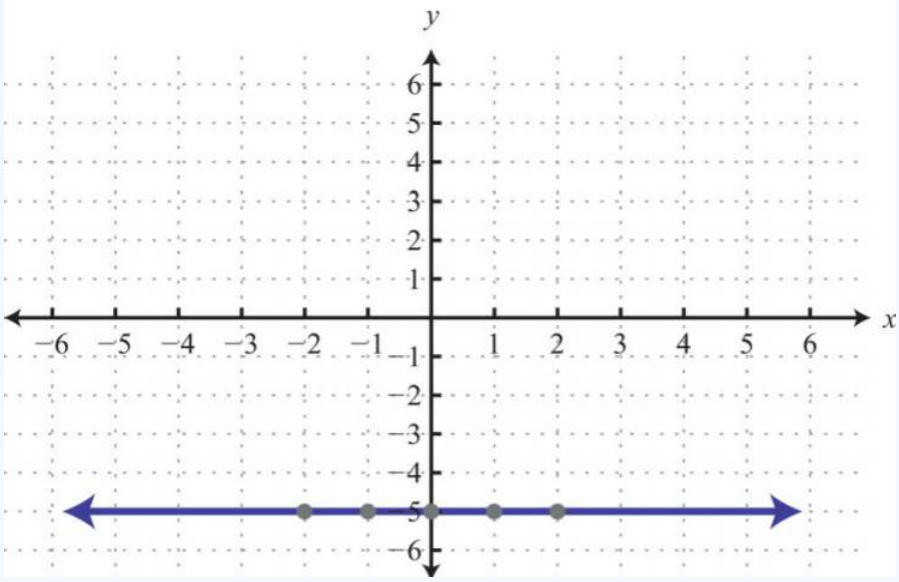


Figure 4.2.17

Exercise 4.2.9 Graphing Lines

Find at least five ordered pair solutions and graph.

1. $y = 2x - 1$
2. $y = -5x + 3$
3. $y = -4x + 2$
4. $y = 10x - 20$
5. $y = -\frac{1}{2}x + 2$
6. $y = \frac{1}{3}x - 1$
7. $y = \frac{2}{3}x - 6$
8. $y = -\frac{2}{3}x + 2$
9. $y = x$
10. $y = -x$
11. $-2x + 5y = -15$
12. $x + 5y = 5$
13. $6x - y = 2$
14. $4x + y = 12$
15. $-x + 5y = 0$
16. $x + 2y = 0$
17. $\frac{1}{10}x - y = 3$
18. $\frac{3}{2}x + 5y = 30$

Answer

1.

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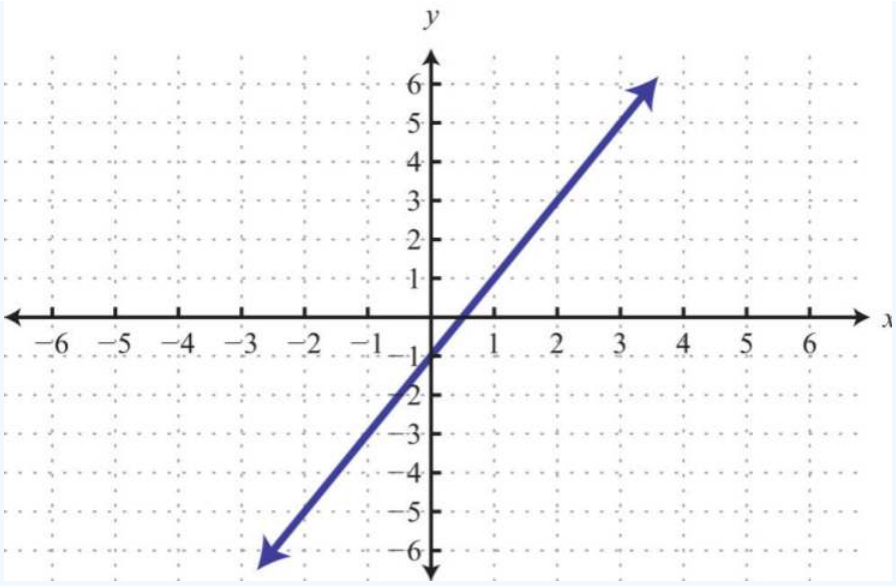


Figure 4.2.18

3.

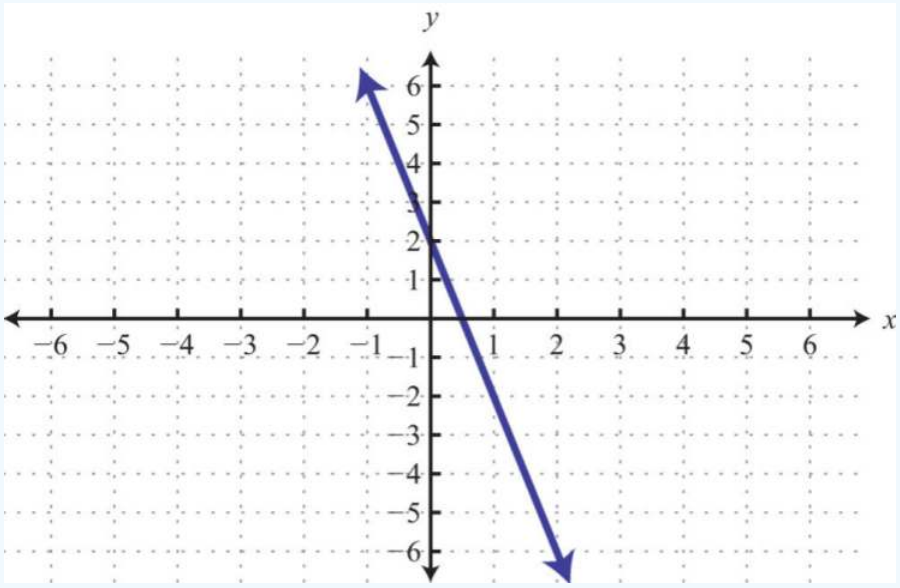


Figure 4.2.19

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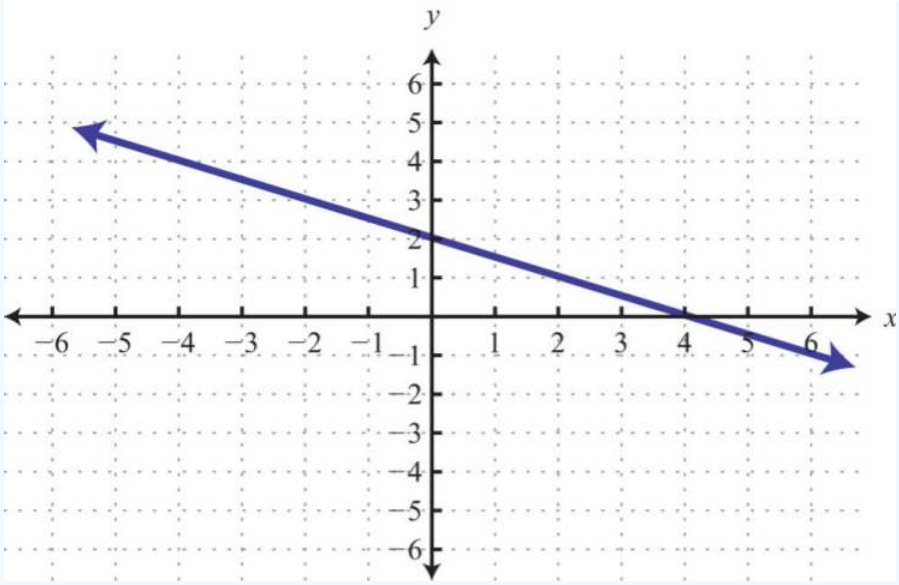


Figure 4.2.20

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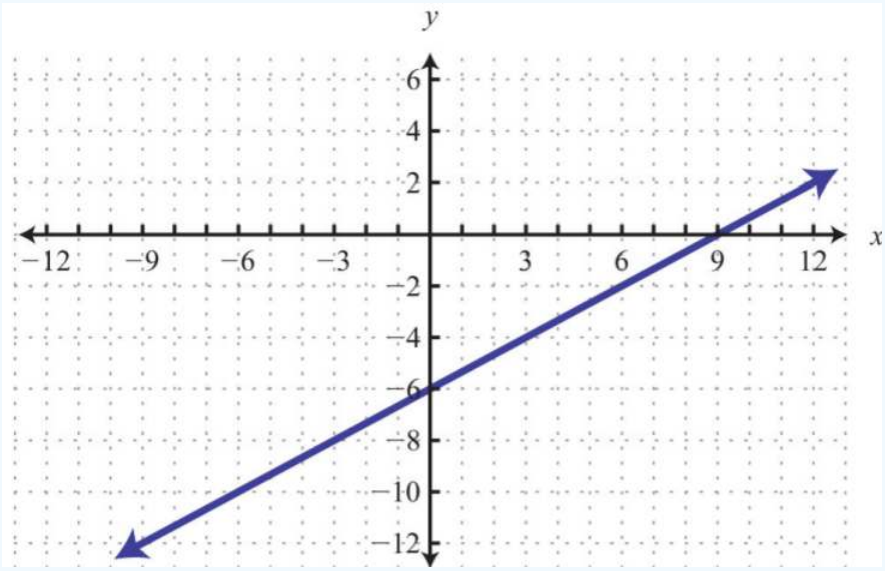


Figure 4.2.21

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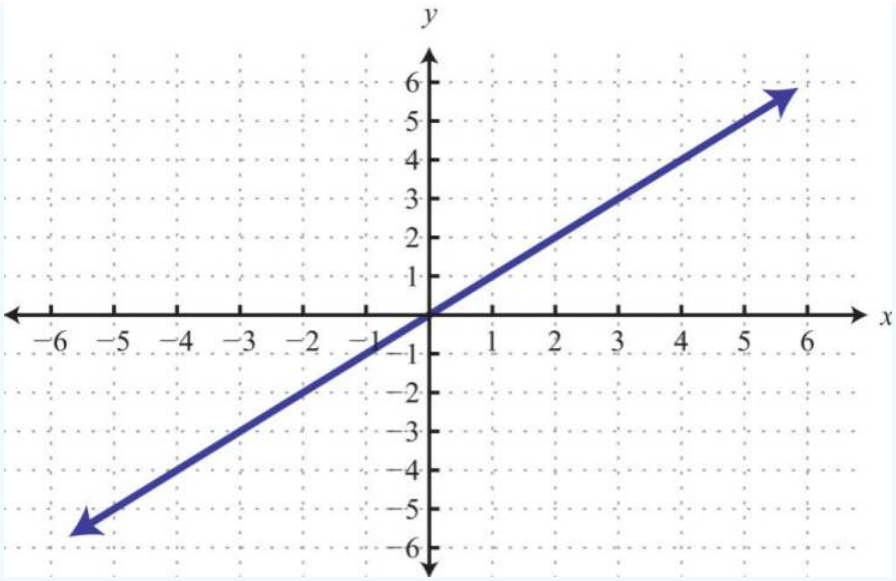


Figure 4.2.22

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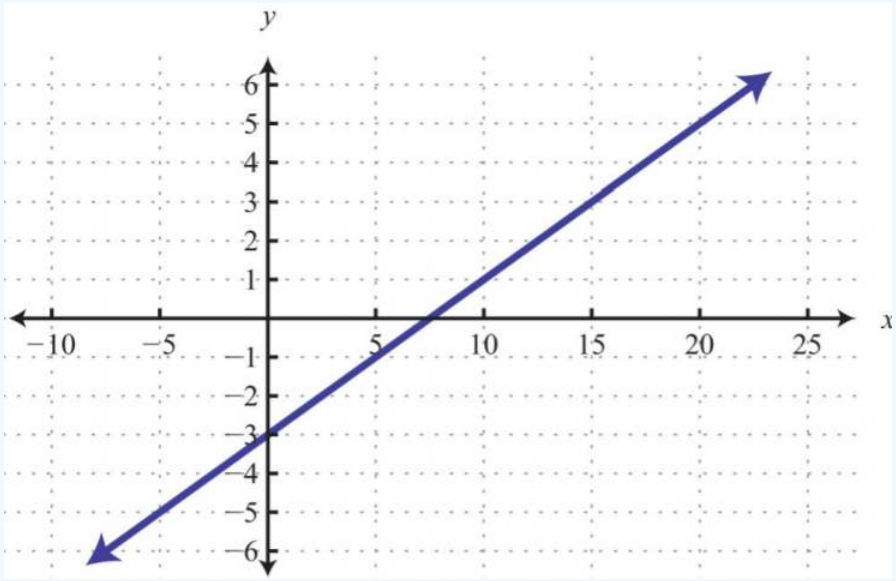


Figure 4.2.23

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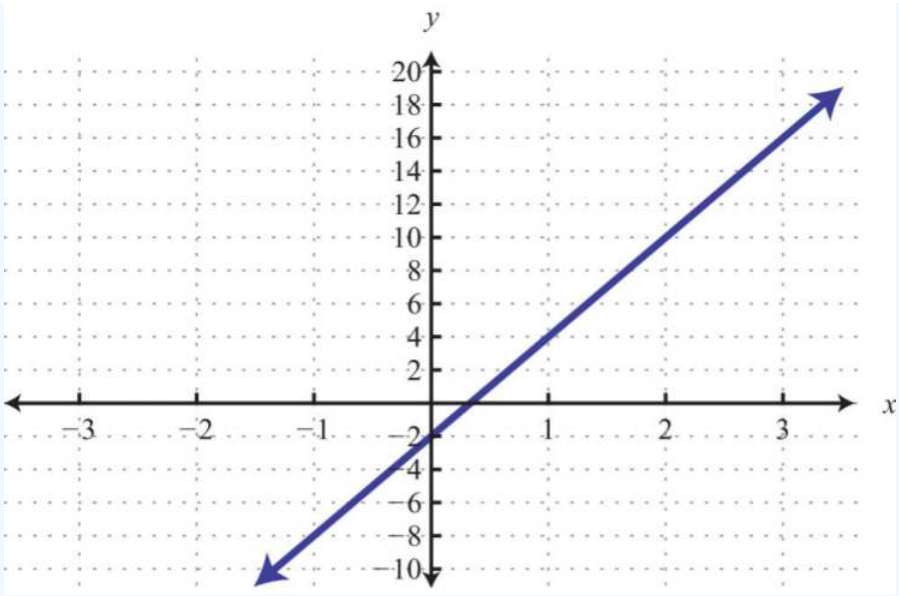


Figure 4.2.24

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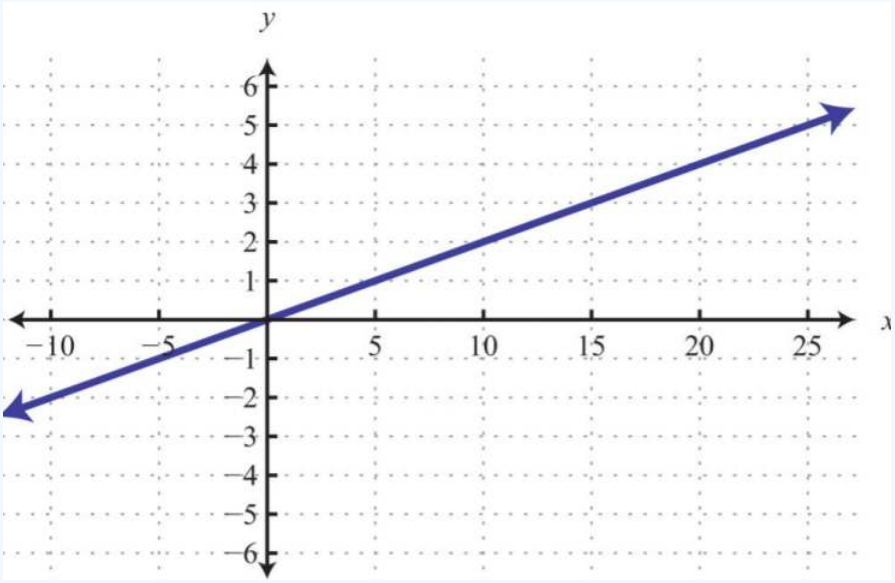


Figure 4.2.25

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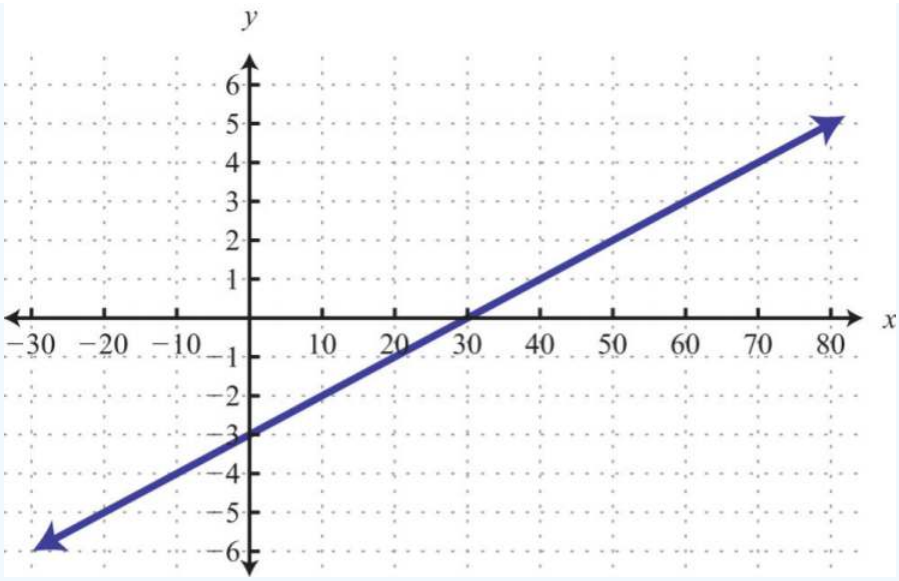


Figure 4.2.26

Exercise 4.2.10 Horizontal and Vertical Lines

Find at least five ordered pair solutions and graph them.

1. $y = 4$
2. $y = -10$
3. $x = 4$
4. $x = -1$
5. $y = 0$
6. $x = 0$
7. $y = \frac{3}{4}$
8. $x = -\frac{5}{4}$
9. Graph the lines $y = -4$ and $x = 2$ on the same set of axes. Where do they intersect?
10. Graph the lines $y = 5$ and $x = -5$ on the same set of axes. Where do they intersect?
11. What is the equation that describes the x-axis?
12. What is the equation that describes the y-axis?

Answer

1.

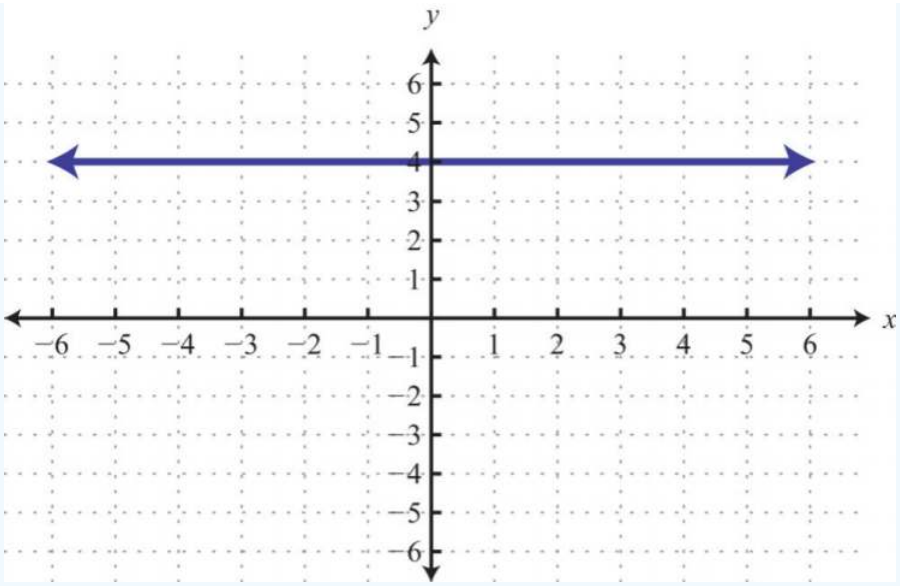


Figure 4.2.27

3.

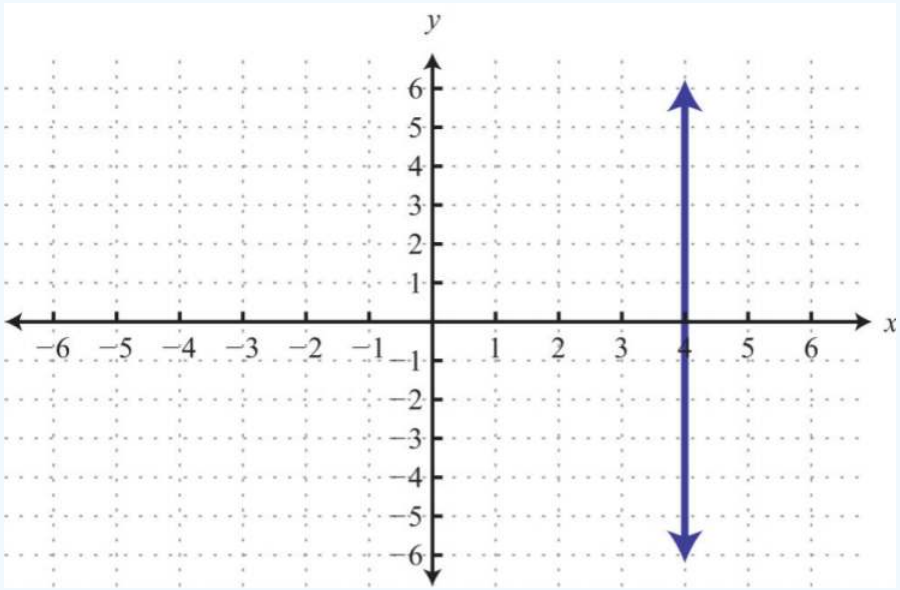


Figure 4.2.28

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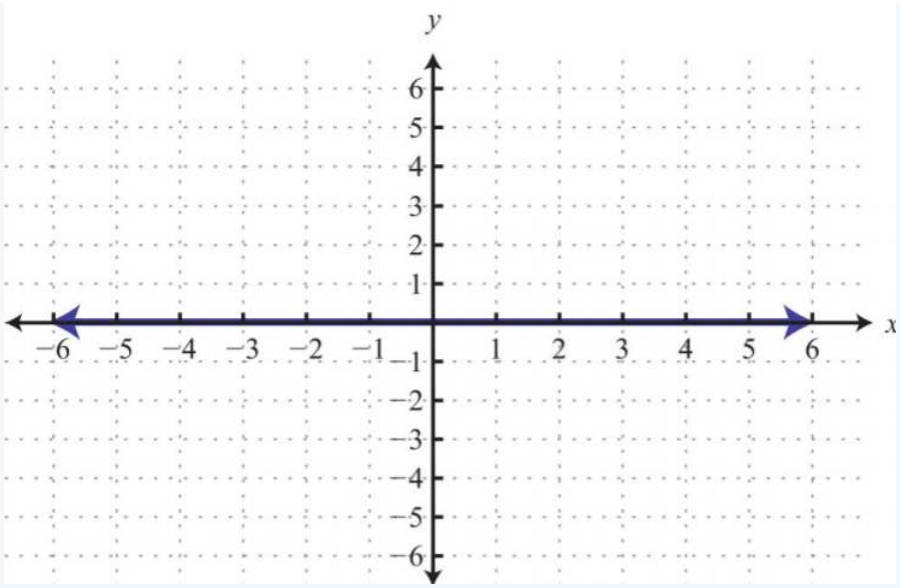


Figure 4.2.29

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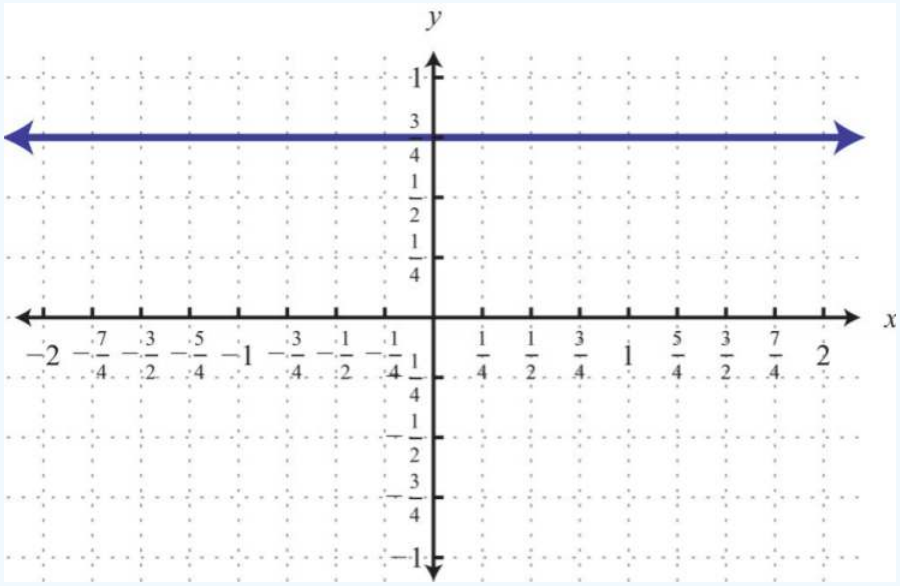


Figure 4.2.30

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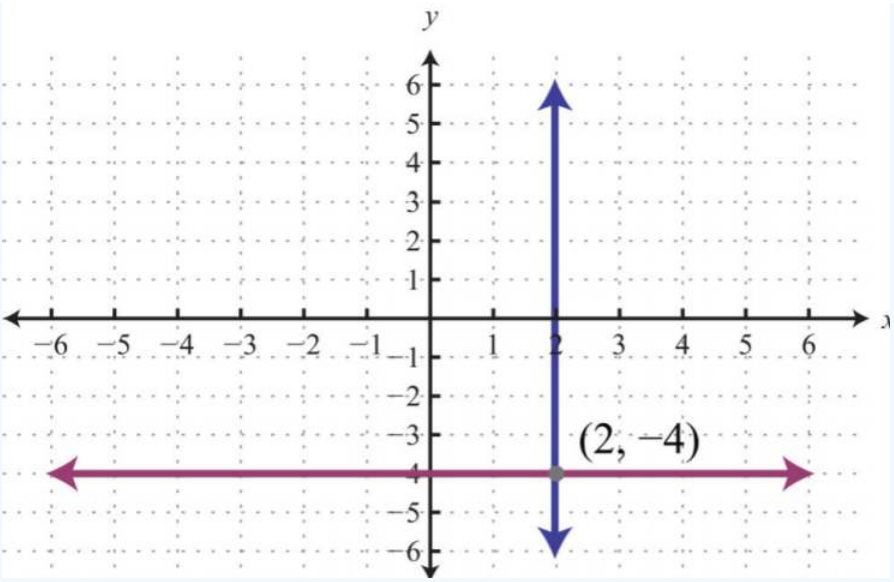


Figure 4.2.31

11. $y = 0$

Exercise 4.2.11 Mixed Practice

Graph by plotting points.

1. $y = -\frac{3}{5}x + 6$
2. $y = \frac{3}{5}x - 3$
3. $y = -3$
4. $x = -5$
5. $3x - 2y = 6$
6. $-2x + 3y = -12$

Answer

1.

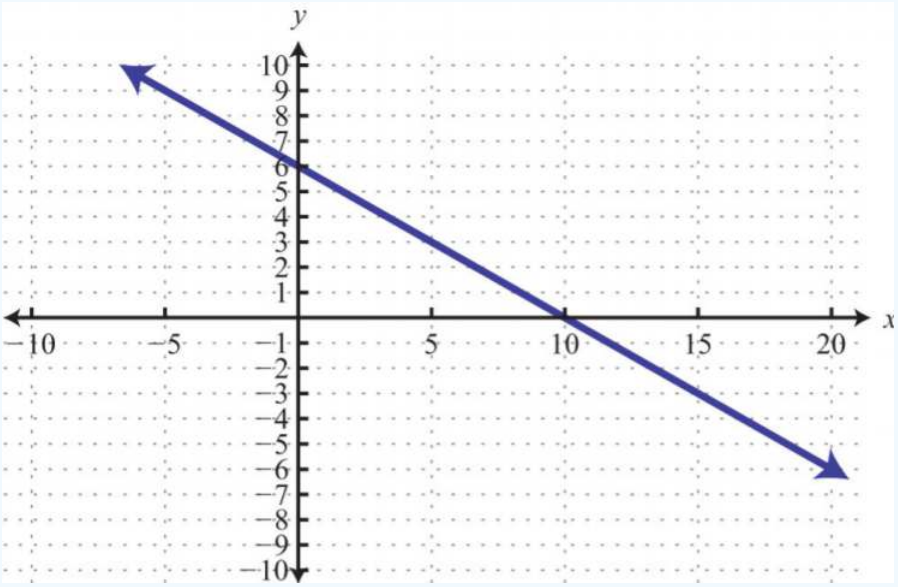


Figure 4.2.32

3.

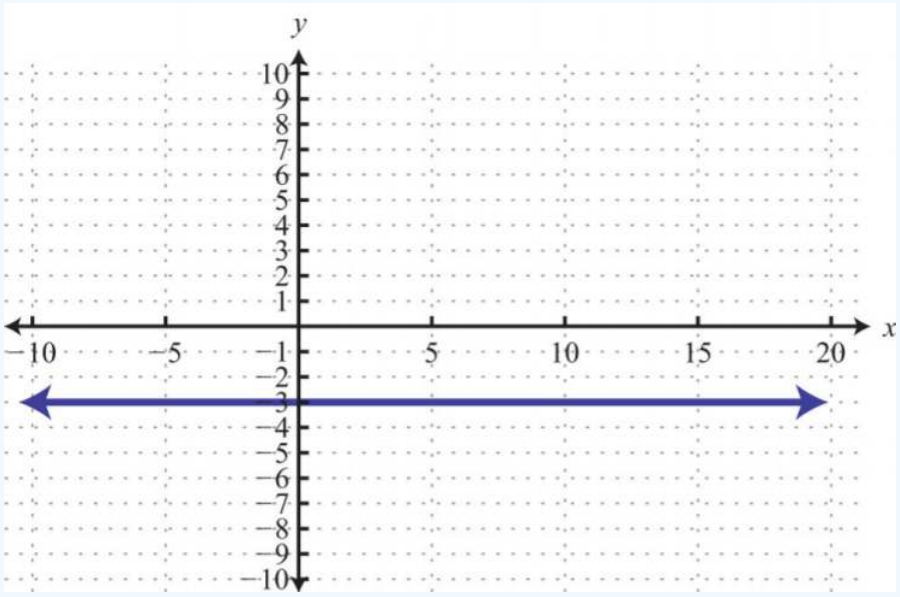


Figure 4.2.33

5.

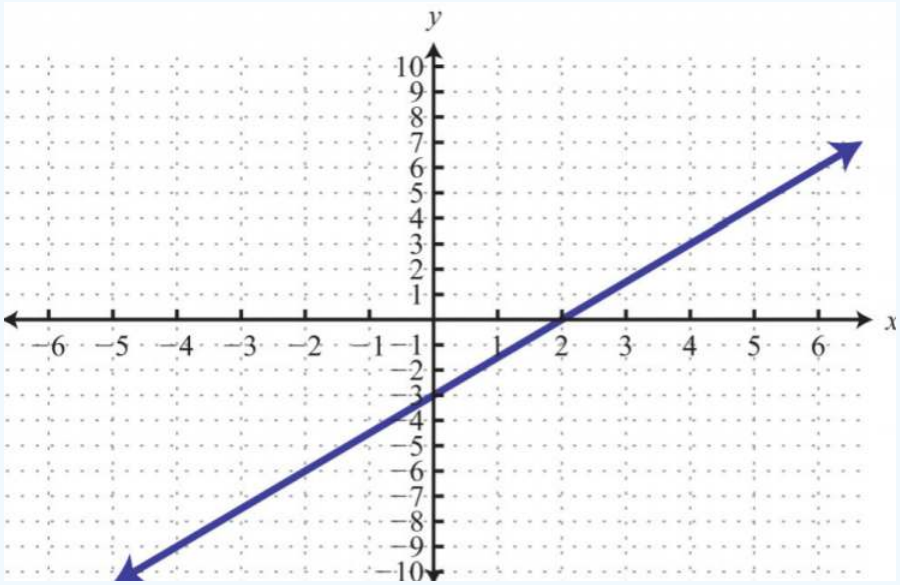


Figure 4.2.34

Exercise 4.2.12 Discussion Board Topics

1. Discuss the significance of the relationship between algebra and geometry in describing lines.
2. Give real-world examples relating two unknowns

Answer

1. Answers may vary

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4.3: Graph Using Intercepts

Learning Objectives

- Identify and find x- and y-intercepts of a graph.
- Graph a line using x- and y-intercepts

Definition of x- and y-intercepts

The ***x*-intercept** is the point where the graph of a line intersects the x-axis.

The ***y*-intercept** is the point where the graph of a line intersects the y-axis.

These points have the form $(x, 0)$ and $(0, y)$, respectively.

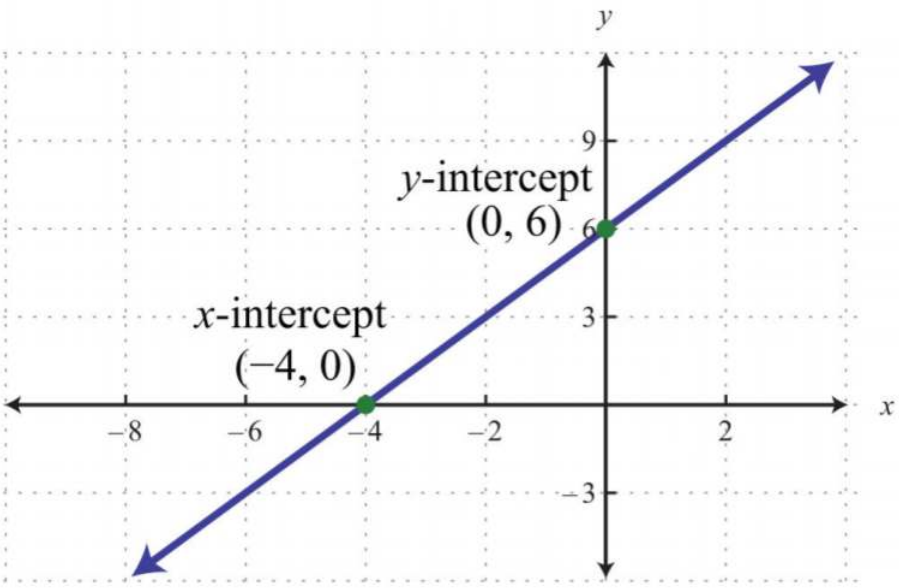


***Figure* 4.3.1**

To find the x- and y-intercepts algebraically, use the fact that all x-intercepts have a y-value of zero and all y-intercepts have an x-value of zero. To find the y-intercept, set $x = 0$ and determine the corresponding y-value. Similarly, to find the x-intercept, set $y = 0$ and determine the corresponding x-value.

Example 4.3.1

Find the x- and y-intercepts:

$$-3x + 2y = 12.$$

Solution:

To find the x-intercept, set $y = 0$.

Therefore, the x-intercept is $(-4, 0)$. To find the y-intercept, set $x = 0$.

$$\begin{aligned} -3x + 2y &= 12 \qquad \textit{To find the } y-\textit{intercept,} \\ \downarrow & \qquad\qquad\qquad\quad \textit{set } x = 0. \\ -3(0) + 2y &= 12 \\ 2y &= 12 \\ y &= 6 \end{aligned}$$

Hence the y-intercept is $(0, 6)$. Note that this linear equation is graphed above.

Answer:

x-intercept: $(-4, 0)$; y-intercept: $(0, 6)$

Example 4.3.2

Find the x- and y-intercepts:

$$y = -3x + 9$$

Solution:

Begin by finding the x-intercept.

$$\begin{aligned} y &= -3x + 9 && \textit{Set } y = 0. \\ &\downarrow \\ 0 &= -3x + 9 && \textit{Solve for } x. \\ 3x &= 9 \\ x &= 3 \end{aligned}$$

The x-intercept is $(3, 0)$. Next, determine the y-intercept.

$$\begin{aligned} y &= -3x + 9 && \textit{Set } x = 0. \\ &\downarrow \\ y &= -3(0) + 9 && \textit{Solve for } y. \\ y &= 9 \end{aligned}$$

The y-intercept is $(0, 9)$.

Answer:

x-intercept: $(3, 0)$; y-intercept: $(0, 9)$

Keep in mind that the intercepts are ordered pairs and not numbers. In other words, the x-intercept is not $x = 2$ but rather $(2, 0)$. In addition, not all graphs necessarily have both intercepts: for example,

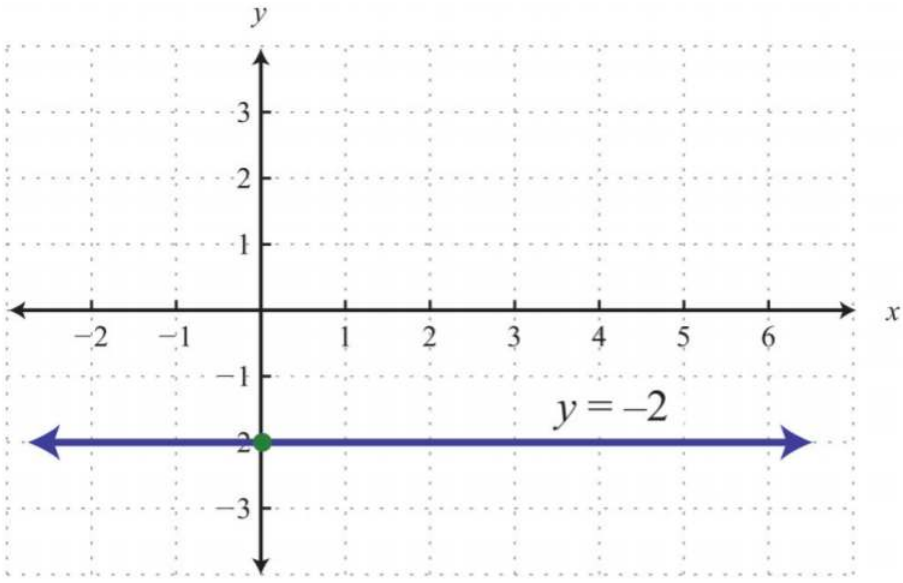


***Figure* 4.3.2**

The horizontal line graphed above has a y-intercept of $(0, -2)$ and no x-intercept.

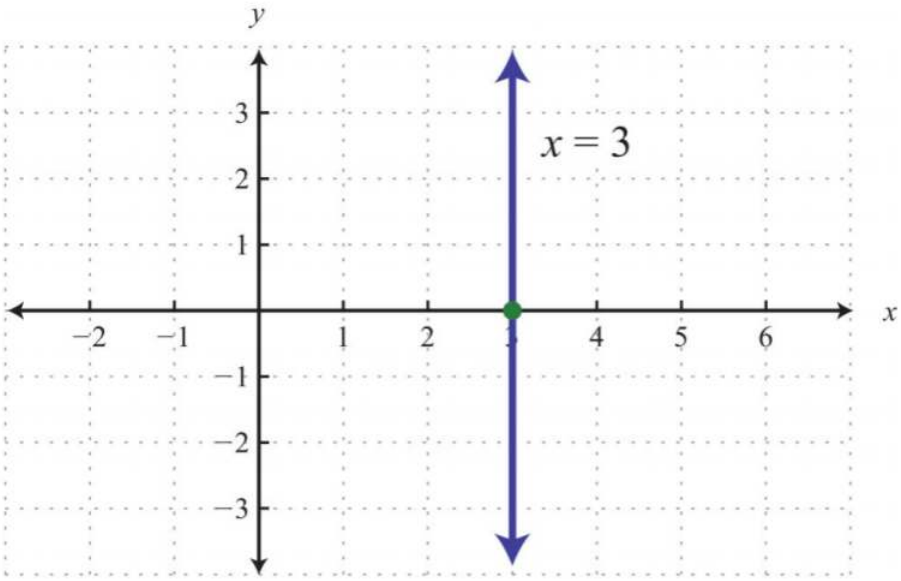


***Figure* 4.3.3**

The vertical line graphed above has an x-intercept $(3, 0)$ and no y-intercept.

? Exercise 4.3.1

Find the x- and y-intercepts:

$$4x - y = 2\,.$$

Answer

x-intercept: $(\frac{1}{2}, 0)$; y-intercept: $(0, -2)$

Graphing Lines Using Intercepts

Since two points determine a line, we can use the x- and y-intercepts to graph linear equations. We have just outlined an easy method for finding intercepts; now we outline the steps for graphing lines using the intercepts.

✔ Example 4.3.3

Graph using intercepts:

$$2x - 3y = 12.$$

Solution:

Step 1: Find the x- and y-intercepts.

$To\ find\ the\ x-intercept,\ set\ y = 0.$	$To\ find\ the\ y-intercept,\ set\ x = 0.$
$2x - 3y = 12$	$2x - 3y = 12$
$2x - 3(0) = 12$	$2(0) - 3y = 12$
$2x = 12$	$-3y = 12$
$x = 6$	$y = -4$
$x-intercept:\ (6, 0)$	$y-intercept:\ (0, -4)$

Step 2: Plot the intercepts and draw the line through them. Use a straightedge to create a nice straight line. Add an arrow on either end to indicate that the line continues indefinitely in either direction.

Answer:

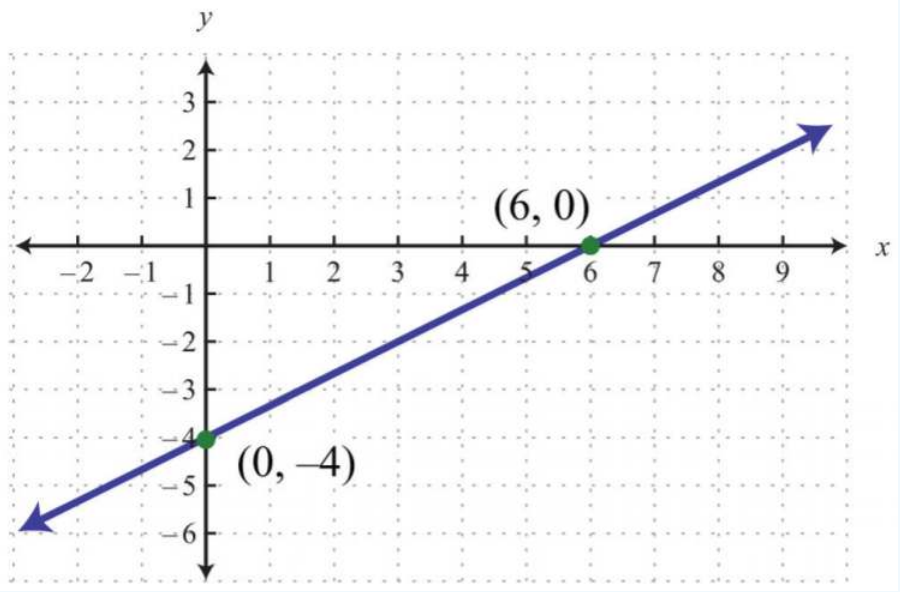


***Figure* 4.3.4**

Example 4.3.4

Graph using intercepts:

$$y = -\tfrac{1}{5}x + 3\,.$$

Solution:

Begin by determining the x- and \(y\-intercepts.

$$\begin{array}{c|c} \underline{x-intercept} & \underline{y-intercept} \\ y = -\tfrac{1}{5}x + 3 & y = -\tfrac{1}{5}x + 3 \\ 0 = -\tfrac{1}{5}x + 3 & y = -\tfrac{1}{5}(0) + 3 \\ \tfrac{1}{5}x = 3 & y = 3 \\ 5 \cdot \tfrac{1}{5}x = 5 \cdot 3 & y-intercept:\ (0,3) \\ x = 15 & \\ x-intercept:\ (15,0) & \end{array}$$

Next, graph the two points and draw a line through them with a straighedge.

Answer:

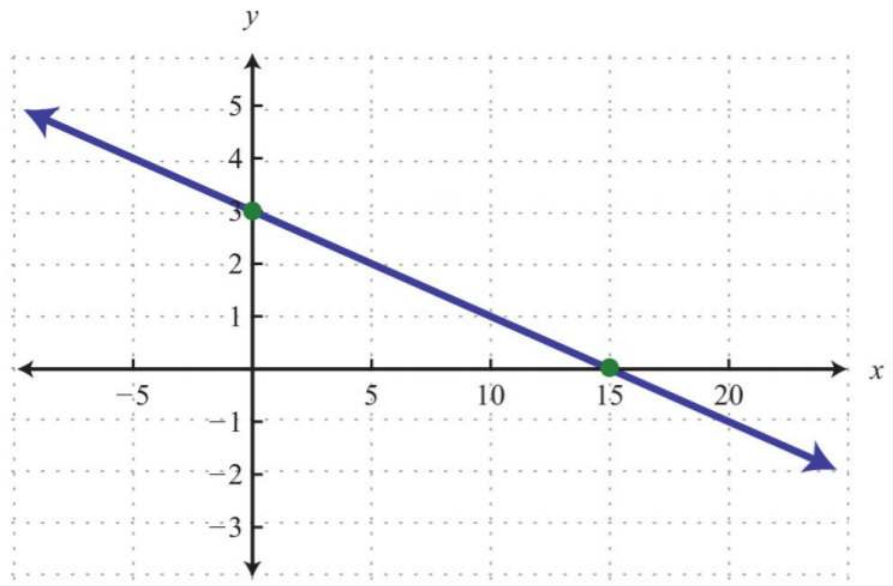


***Figure* 4.3.5**

Example 4.3.5

Graph using intercepts:

$$y = -2x.$$

Solution:

$x-intercept$	$y-intercept$
$y = -2x$	$y = -2x$
$0 = -2x$	$y = -2(0)$
$\frac{0}{-2} = \frac{-2x}{-2}$	$y = 0$
$0 = x$	$y-intercept: \ (0,0)$
$x-intercept: \ (0,0)$	

Here the x- and y-intercepts are actually the same point, the origin. We will need at least one more point so that we can graph the line. Choose any value for x and determine the corresponding value for y.

x	y	
-1	2	$y = -2(-1) = 2 = 2$
1	-2	$y = -2(1) = -2$

***Figure* 4.3.6**

Use the ordered pair solutions $(0,0)$, $(-1,2)$, and $(1,-2)$ to graph the line.

Answer:

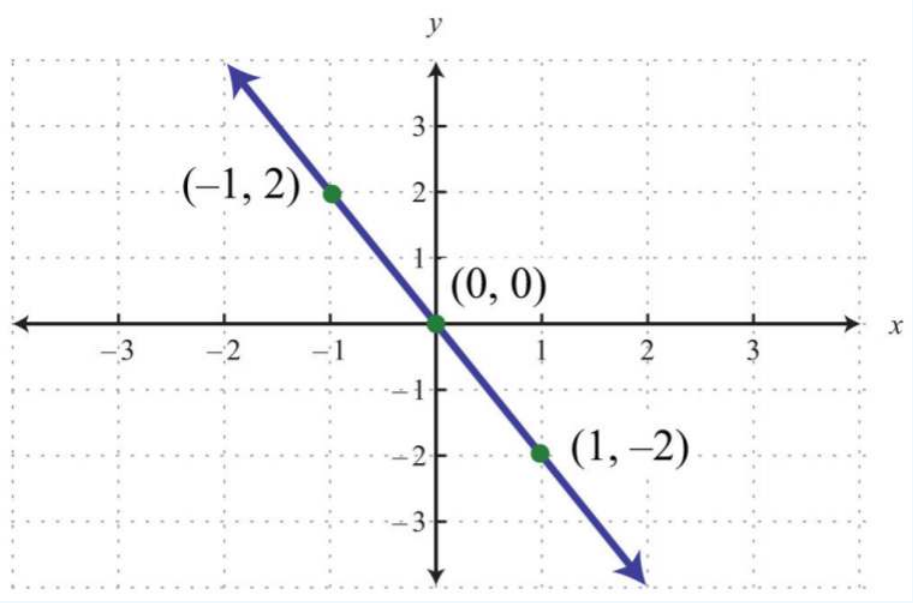


***Figure* 4.3.7**

To summarize, any linear equation can be graphed by finding two points and connecting them with a line drawn with a straightedge. Two important and useful points are the x- and y-intercepts; find these points by substituting $y = 0$ and $x = 0$, respectively. This method for finding intercepts will be used throughout our study of algebra.

? Exercise 4.3.2

Graph using intercepts:

$$3x - 5y = 15.$$

Answer

x-intercept: $(5, 0)$; y-intercept: $(0, -3)$

Finding Intercepts Given the Graph

The x- and y-intercepts are important points on any graph. This chapter will focus on the graphs of linear equations. However, at this point, we can use these ideas to determine intercepts of nonlinear graphs. Remember that intercepts are ordered pairs that indicate where the graph intersects the axes.

✓ Example 4.3.6

Find the x- and y-intercepts given the following graph:

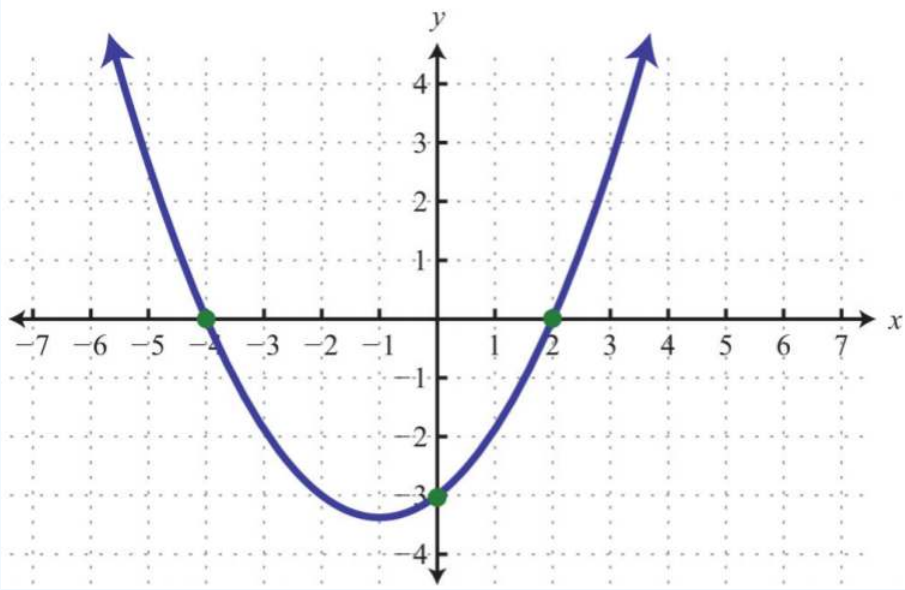


***Figure* 4.3.8**

Solution:

We see that the graph intersects the x-axis in two places. This graph has two x-intercepts, namely, $(-4, 0)$ and $(2, 0)$. Furthermore, the graph intersects the y-axis in one place. The only y-intercept is $(0, -3)$.

Answer:

x-intercepts: $(-4, 0), (2, 0)$; y-intercept: $(0, -3)$

In our study of algebra, we will see that some graphs have many intercepts. Also, we will see that some graphs do not have any.

✔ Example 4.3.7

Given the following graph, find the x- and y-intercepts:

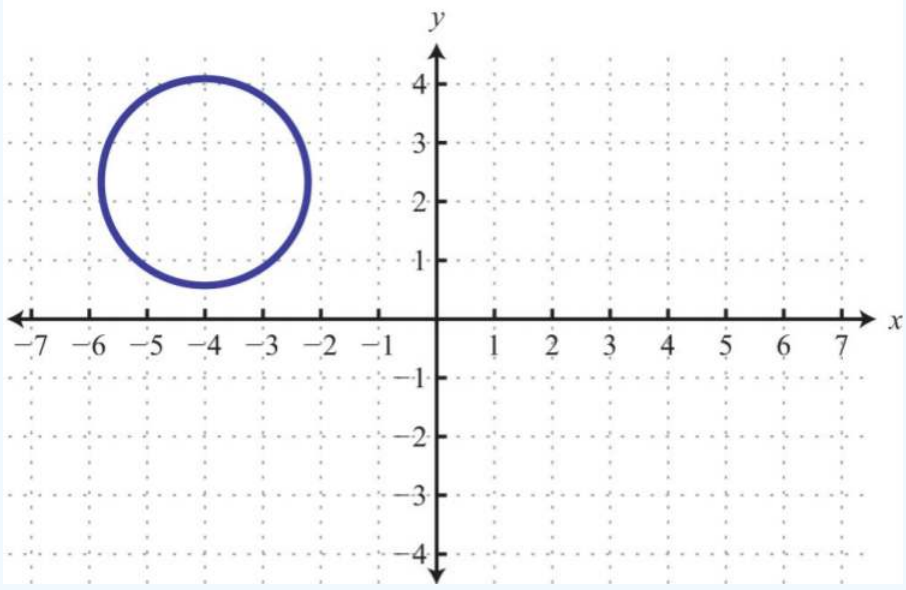


***Figure* 4.3.9**

Solution:

This is a graph of a circle; we can see that it does not intersect either axis. Therefore, this graph does not have any intercepts.

Answer:

None

Key Takeaways

- Since two points determine any line, we can graph lines using the x- and y-intercepts.
- To find the x-intercept, set $y = 0$ and solve for x.
- To find the y-intercept, set $x = 0$ and solve for y.
- This method of finding x- and y-intercepts will be used throughout our study of algebra because it works for any equation.
- To graph a line, find the intercepts, if they exist, and draw a straight line through them. Use a straightedge to create the line and include arrows on either end to indicate that the line extends infinitely in either direction.
- Horizontal and vertical lines do not always have both x- and y-intercepts.

? Exercise 4.3.3 Intercepts

Given the graph, find the x- and y-intercepts.

1.

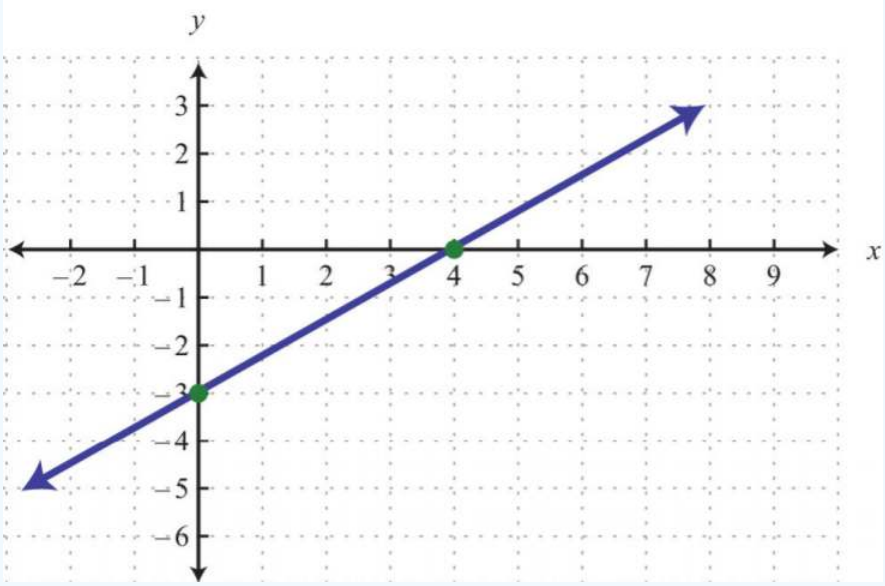


***Figure* 4.3.10**

2.

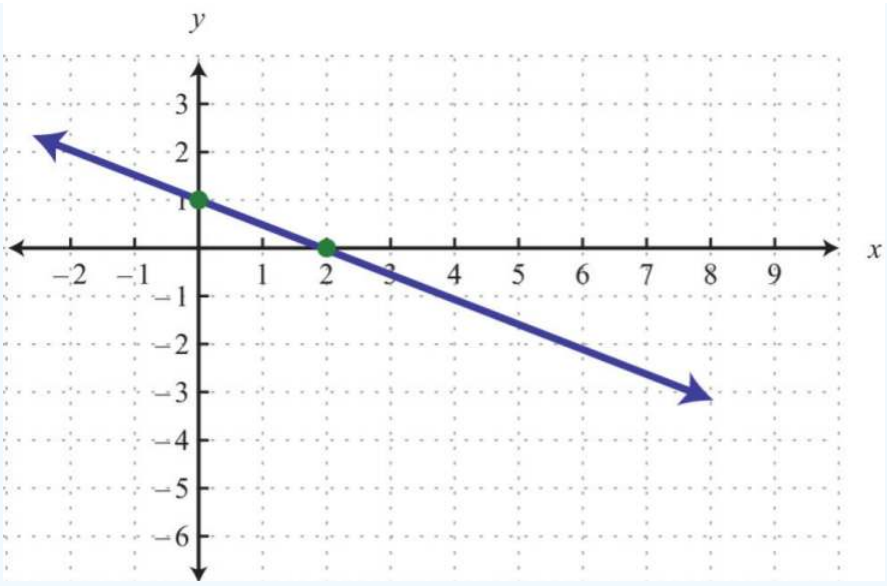


Figure **4.3.11**

3.

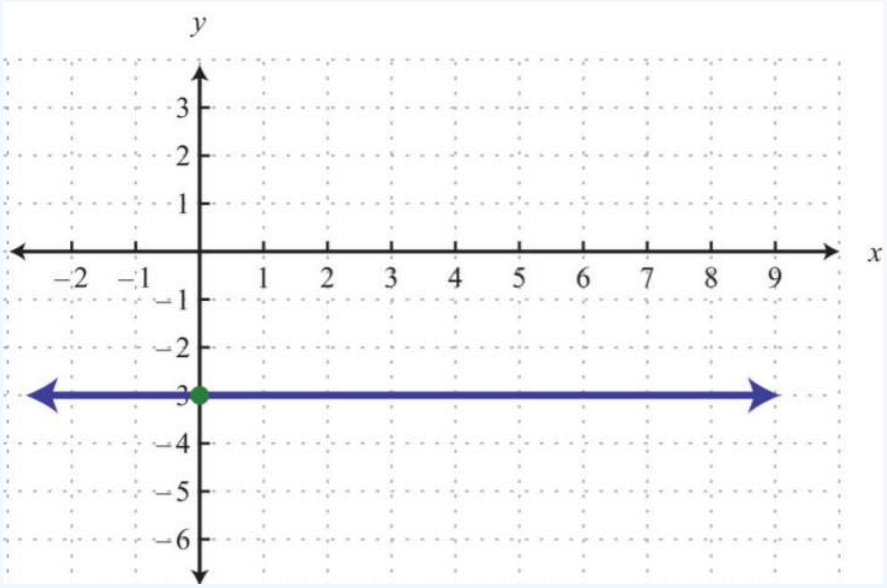


Figure **4.3.12**

4.

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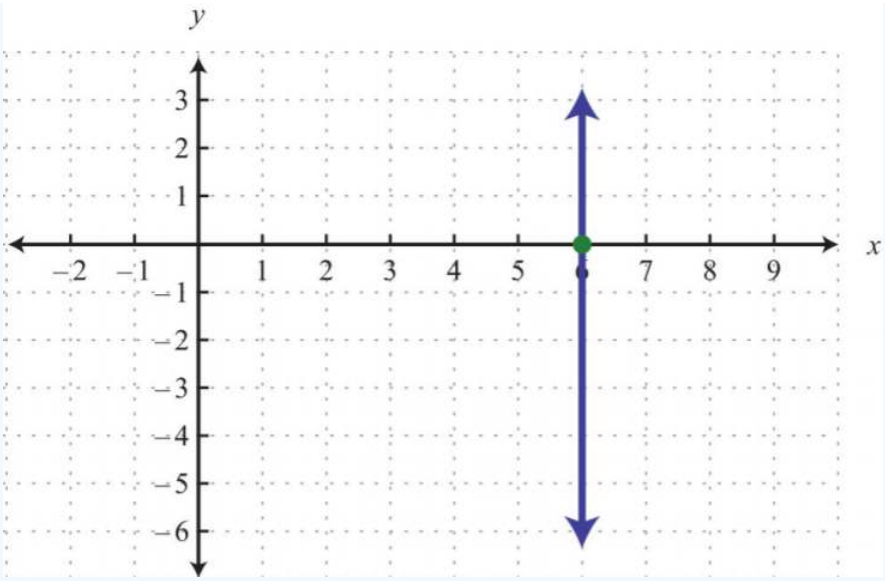


Figure **4.3.13**

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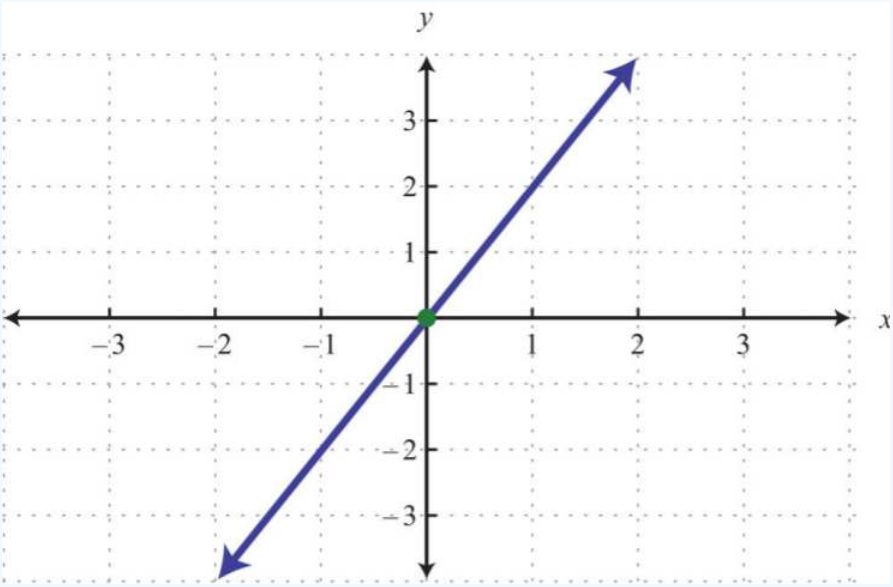


Figure **4.3.14**

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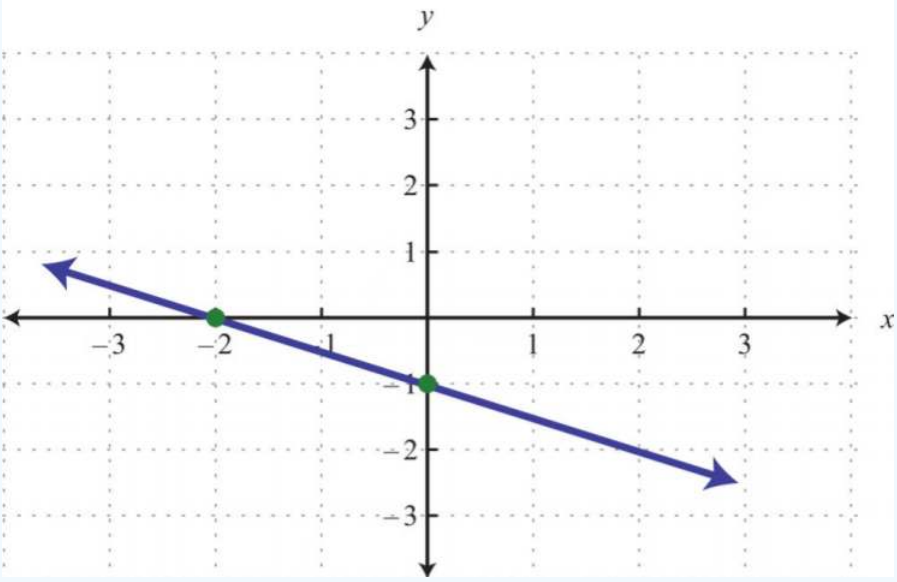


***Figure* 4.3.15**

Answer

1. y-intercept: $(0, -3)$; x-intercept: $(4, 0)$
3. y-intercept: $(0, -3)$; x-intercept: none
5. y-intercept: $(0, 0)$; x-intercept: $(0, 0)$

? Exercise 4.3.4 Intercepts

Find the x- and y-intercepts.

1. $5x - 4y = 20$
2. $-2x + 7y = -28$
3. $x - y = 3$
4. $-x + y = 0$
5. $3x - 4y = 1$
6. $-2x + 5y = 3$
7. $\frac{1}{4}x - \frac{1}{3}y = 1$
8. $-\frac{2}{5}x + \frac{3}{4}y = 2$
9. $y = 6$
10. $y = -3$
11. $x = 2$
12. $x = -1$
13. $y = mx + b$
14. $ax + by = c$

Answer

1. x-intercept: $(4, 0)$; y-intercept: $(0, -5)$
3. x-intercept: $(3, 0)$; y-intercept: $(0, -3)$
5. x-intercept: $(\frac{1}{3}, 0)$; y-intercept: $(0, -\frac{1}{4})$
7. x-intercept: $(4, 0)$; y-intercept: $(0, -3)$
9. x-intercept: none; y-intercept: $(0, 6)$

11. x-intercept: $(2, 0)$; y-intercept: none

13. x-intercept: $(-\frac{b}{m}, 0)$; y-intercept: $(0, b)$

Exercise 4.3.5 Graph Using Intercepts

Find the intercepts and graph them.

1. $3x + 4y = 12$
2. $-2x + 3y = 6$
3. $5x - 2y = 10$
4. $-4x - 8y = 16$
5. $-\frac{1}{2}x + \frac{1}{3}y = 1$
6. $\frac{3}{4}x - \frac{1}{2}y = -3$
7. $2x - \frac{5}{2}y = 10$
8. $2x - \frac{7}{3}y = -14$
9. $4x - y = -8$
10. $6x - y = 6$
11. $-x + 2y = 1$
12. $3x + 4y = 6$
13. $2x + y = -1$
14. $-2x + 6y = 3$
15. $15x + 4y = -60$
16. $-25x + 3y = 75$
17. $4x + 2y = 0$
18. $3x - y = 0$
19. $-12x + 6y = -4$
20. $3x + 12y = -4$
21. $y = 2x + 4$
22. $y = -x + 3$
23. $y = \frac{1}{2}x + 1$
24. $y = \frac{2}{3}x - 3$
25. $y = -\frac{2}{5}x + 1$
26. $y = -\frac{5}{8}x - \frac{5}{4}$
27. $y = -\frac{7}{8}x - \frac{7}{2}$
28. $y = -x + \frac{3}{2}$
29. $y = 3$
30. $y = \frac{3}{2}$
31. $x = 5$
32. $x = -2$
33. $y = 5x$
34. $y = -x$

Answer

1.

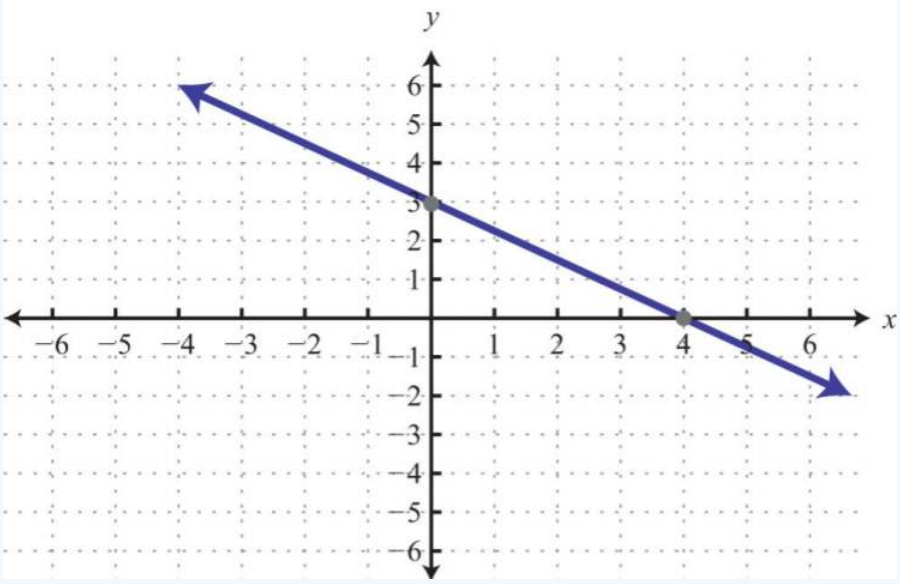


Figure 4.3.16

3.

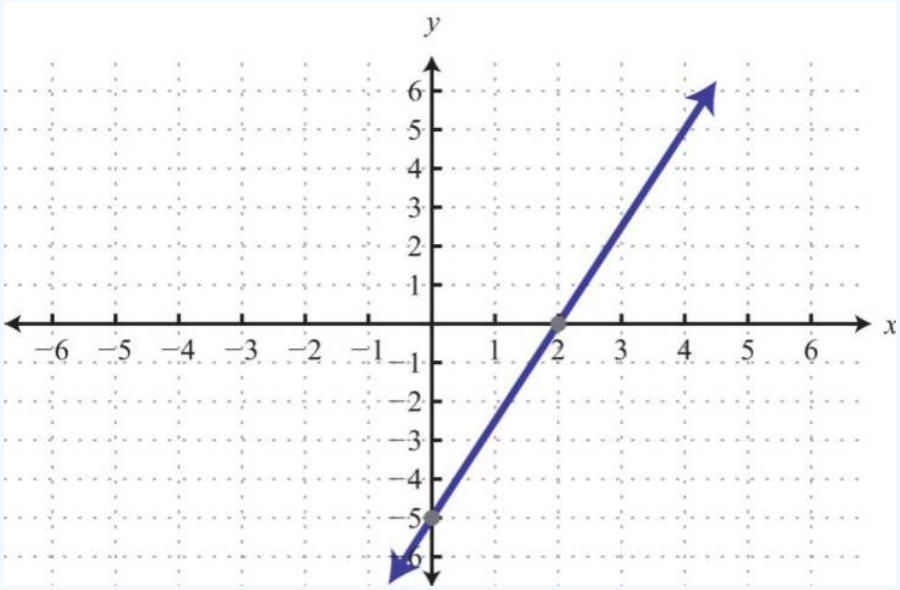


Figure 4.3.17

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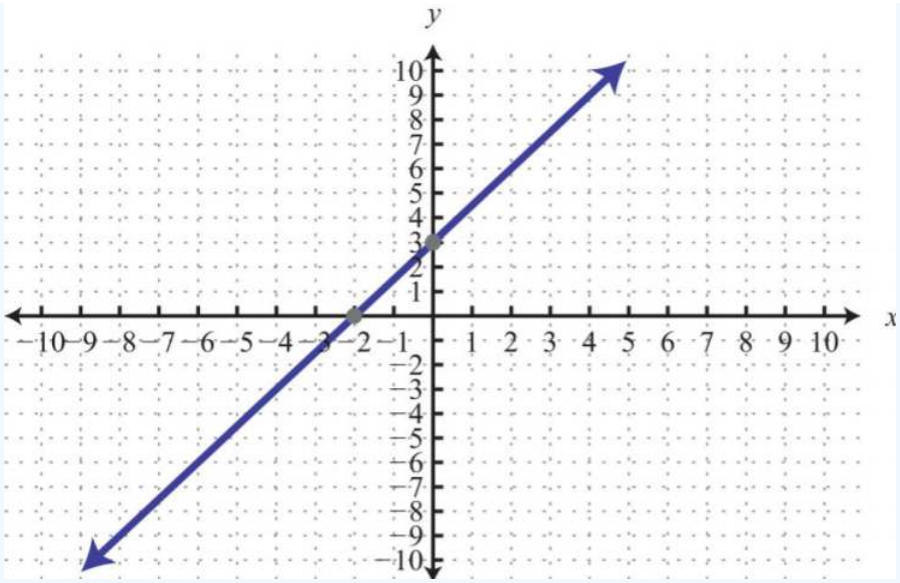


Figure 4.3.18

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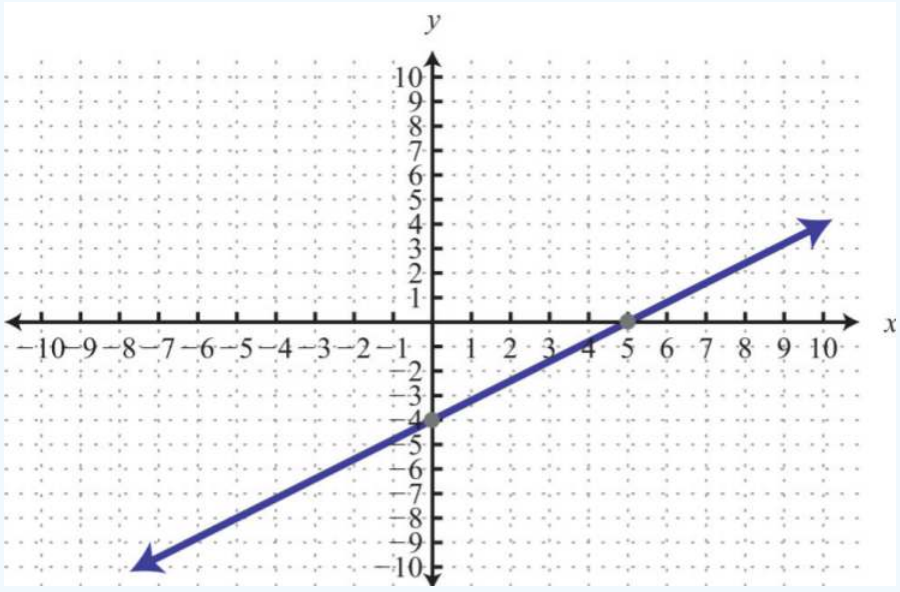


Figure 4.3.19

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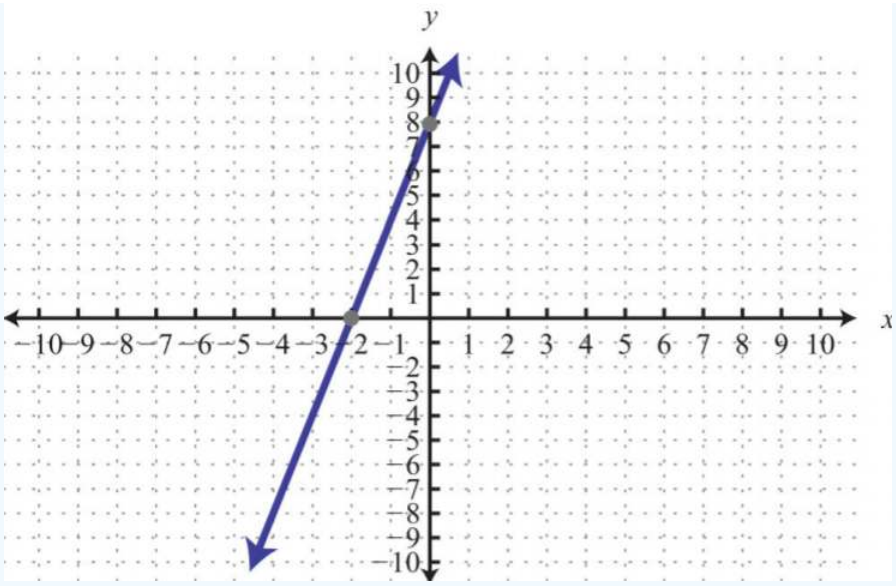


Figure 4.3.20

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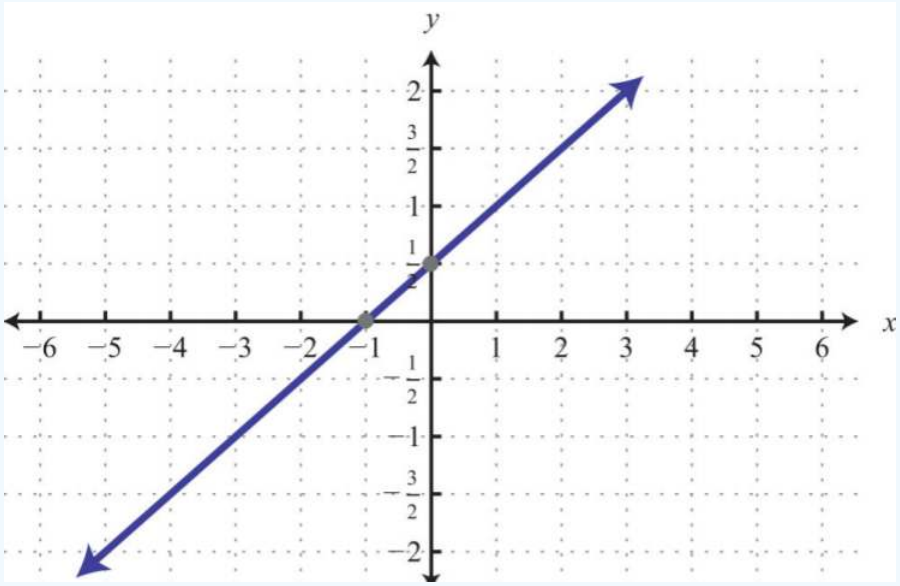


Figure 4.3.21

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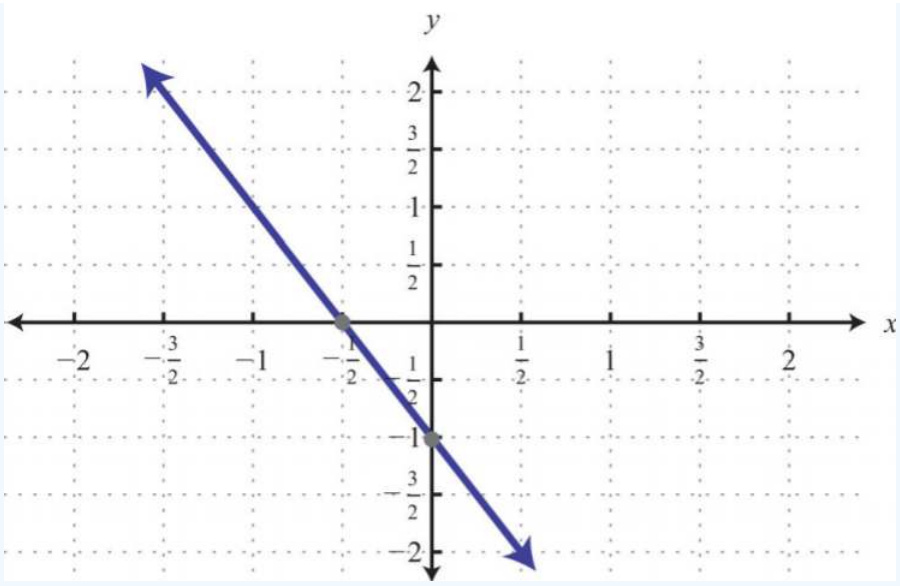


Figure 4.3.22

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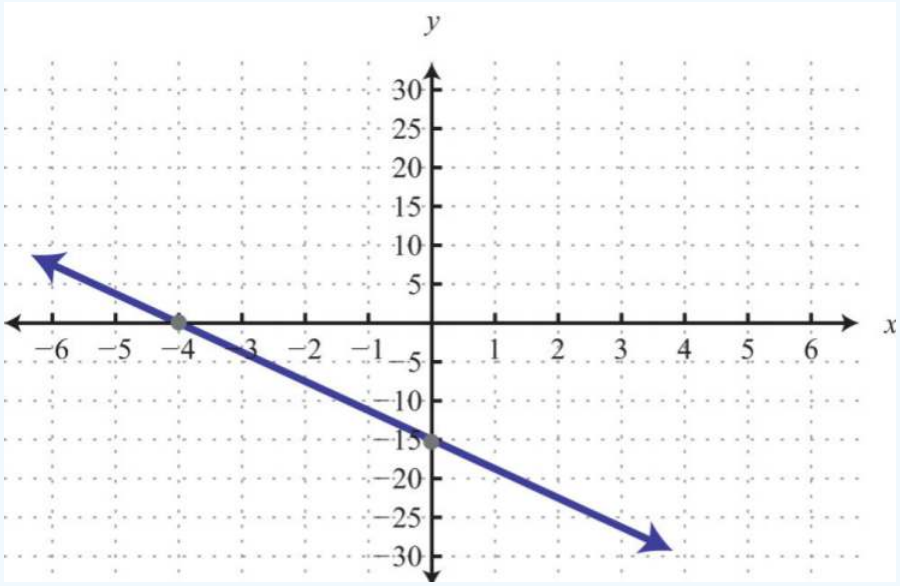


Figure 4.3.23

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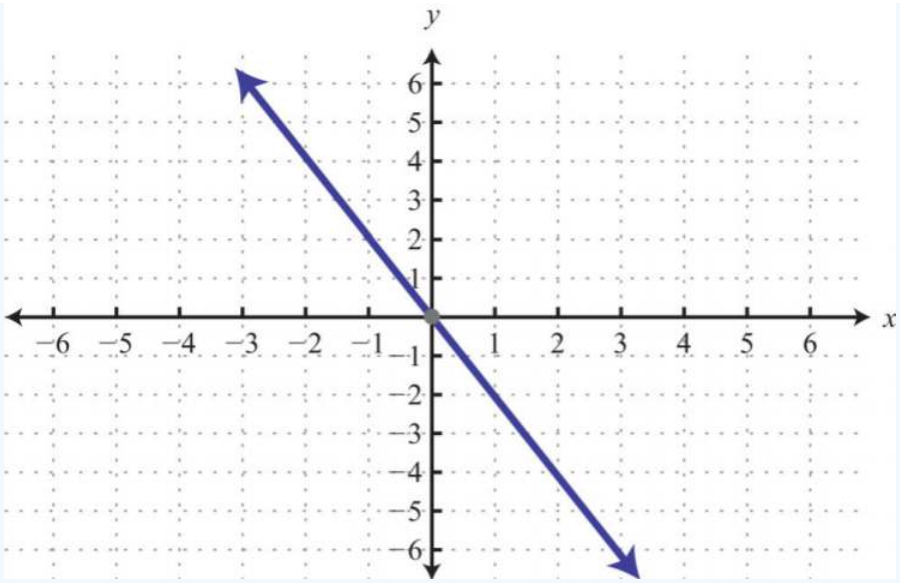


Figure 4.3.24

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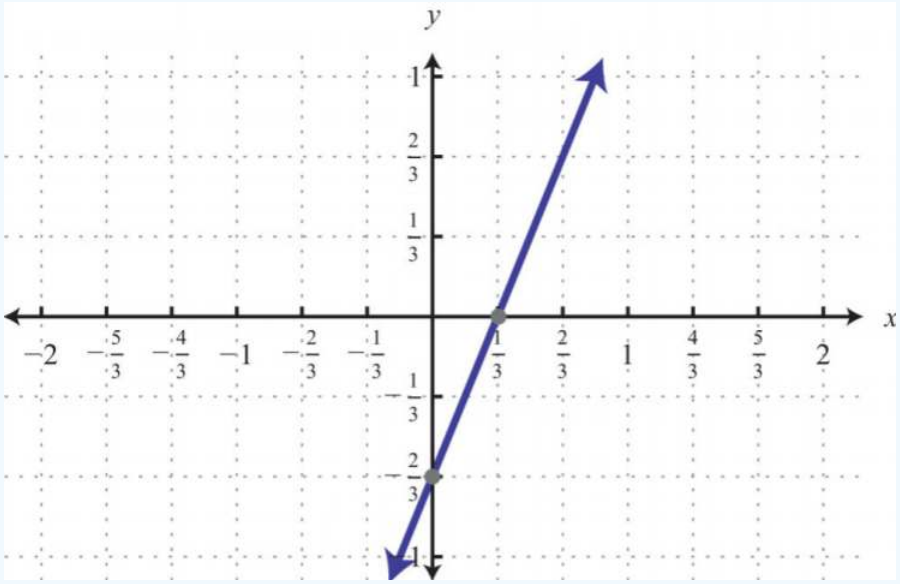


Figure 4.3.25

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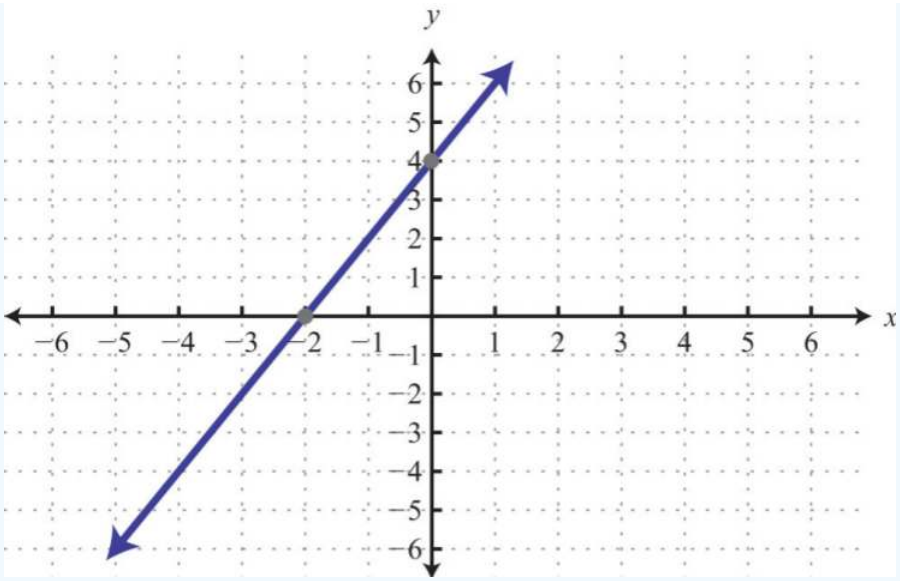


Figure 4.3.26

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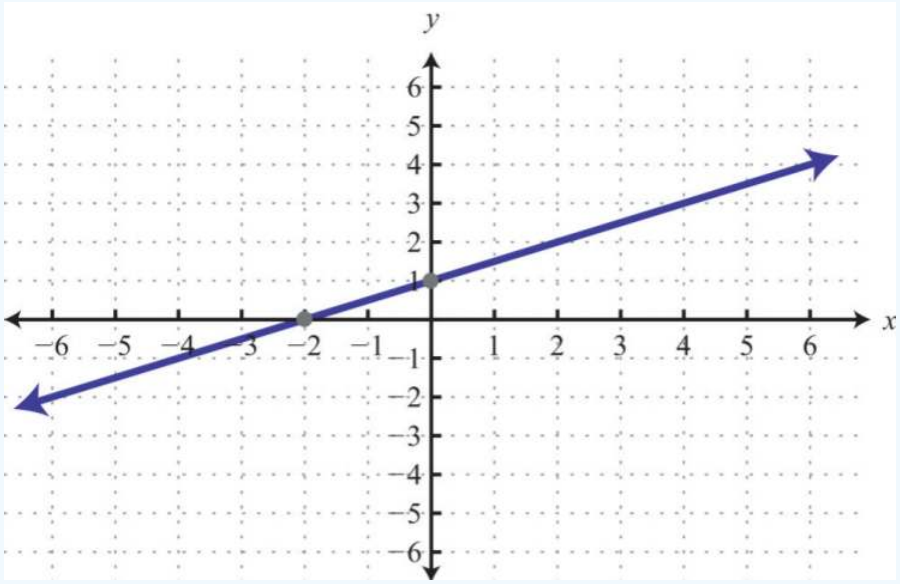


Figure 4.3.27

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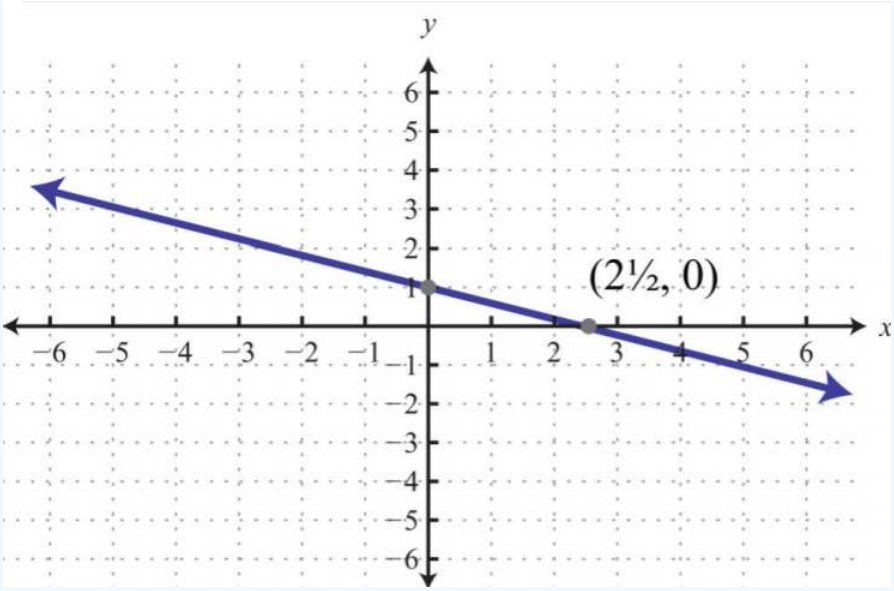


Figure 4.3.28

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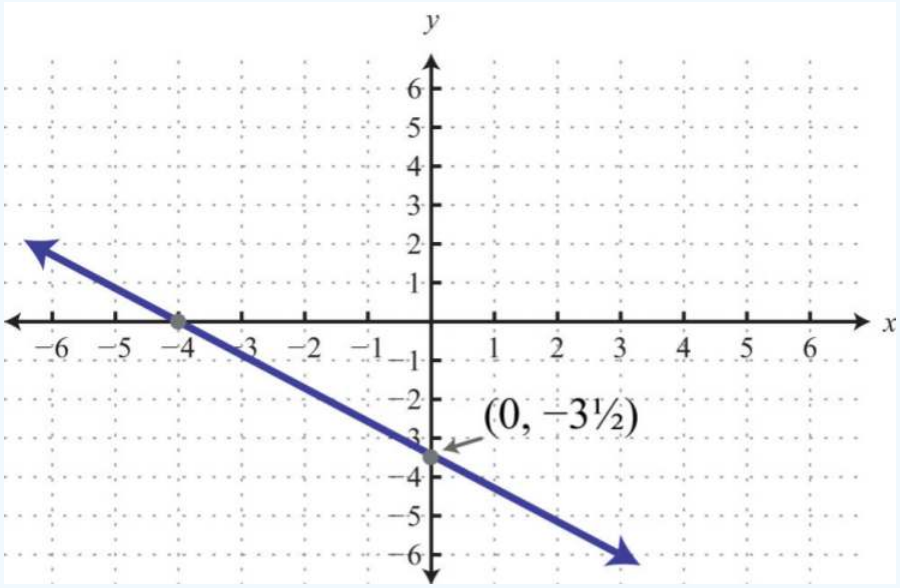


Figure 4.3.29

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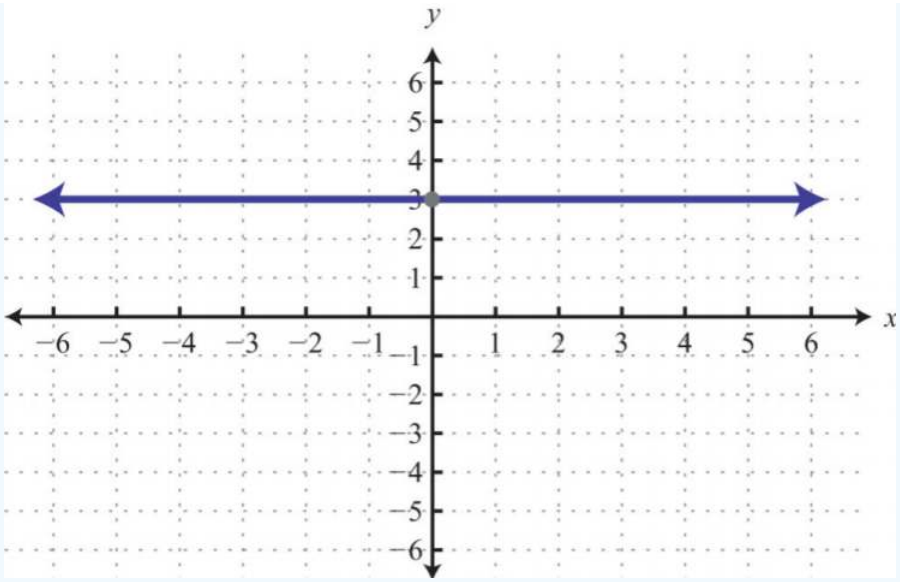


Figure 4.3.30

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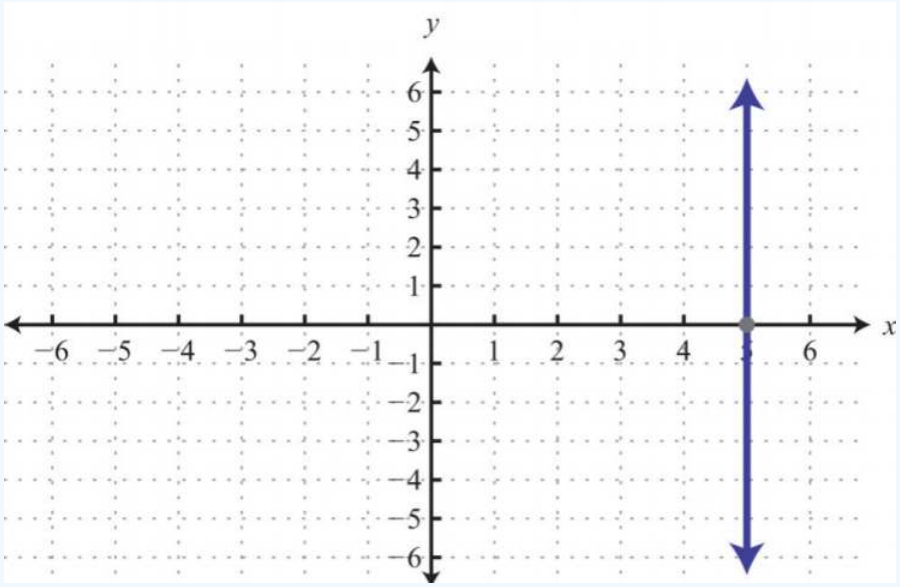


Figure 4.3.31

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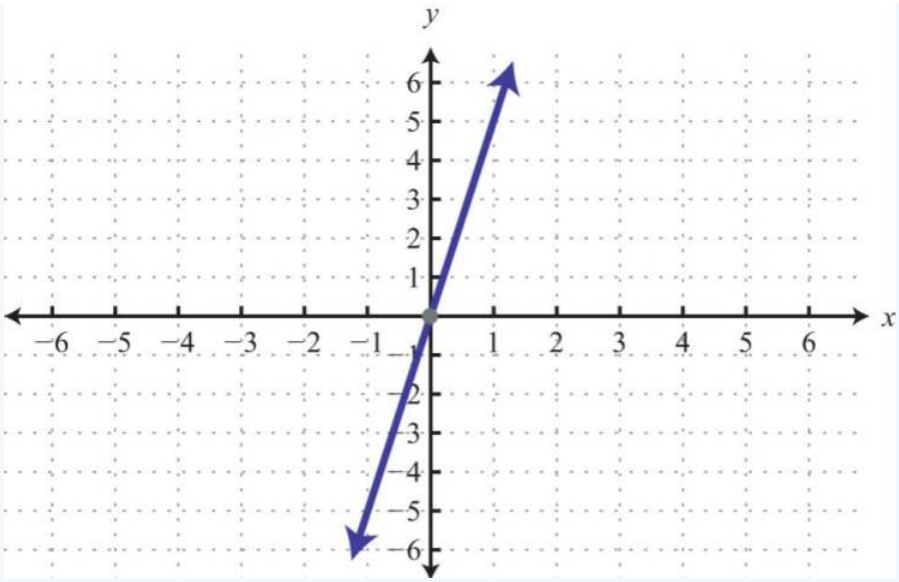


Figure 4.3.32

? Exercise 4.3.6 Intercepts of Nonlinear Graphs

Given the graph find the x- and y-intercepts.

1.

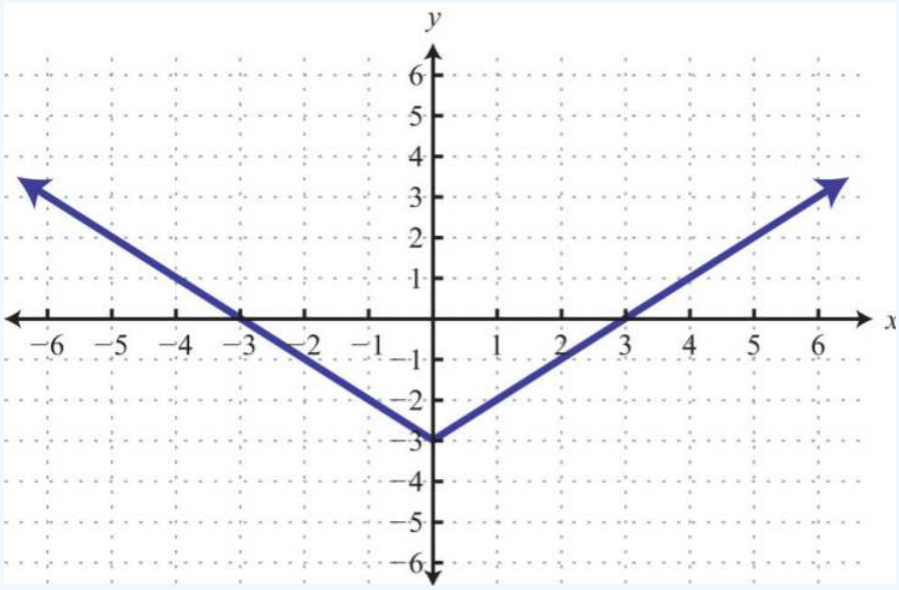


***Figure* 4.3.33**

2.

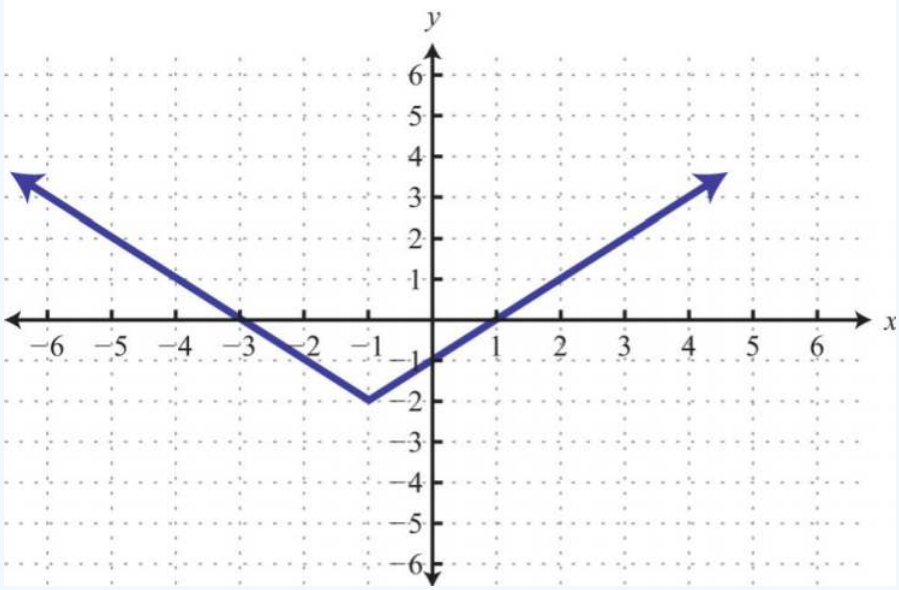


Figure **4.3.34**

3.

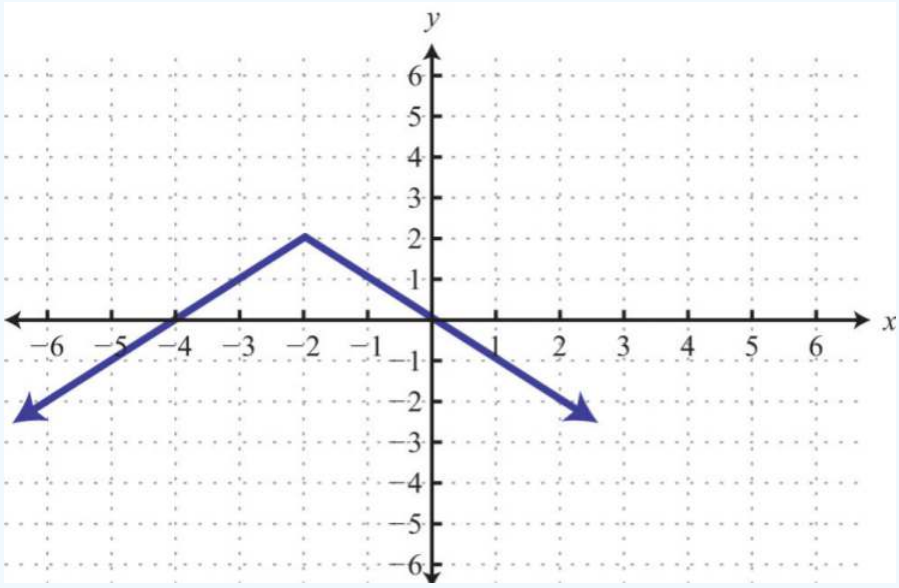


Figure **4.3.35**

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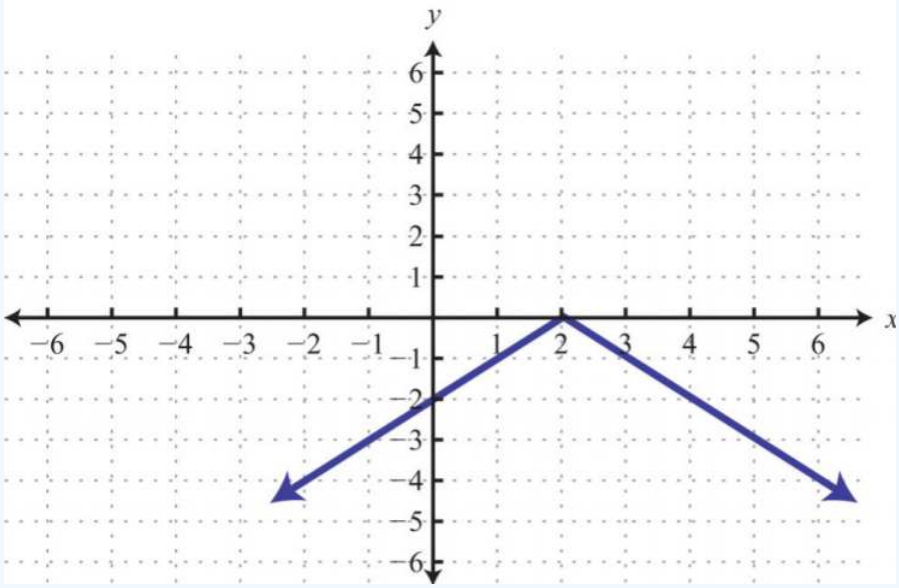


Figure **4.3.36**

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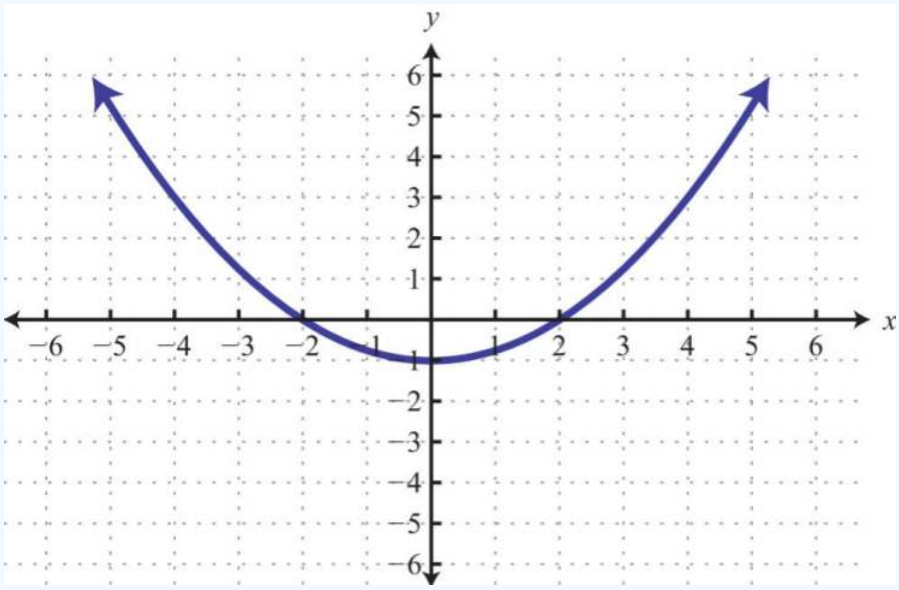


Figure **4.3.37**

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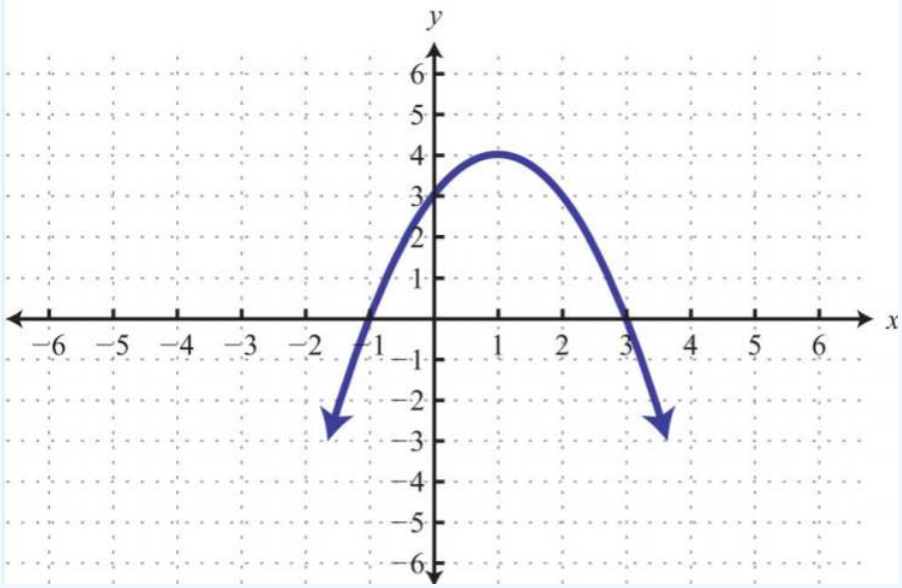


Figure **4.3.38**

7.

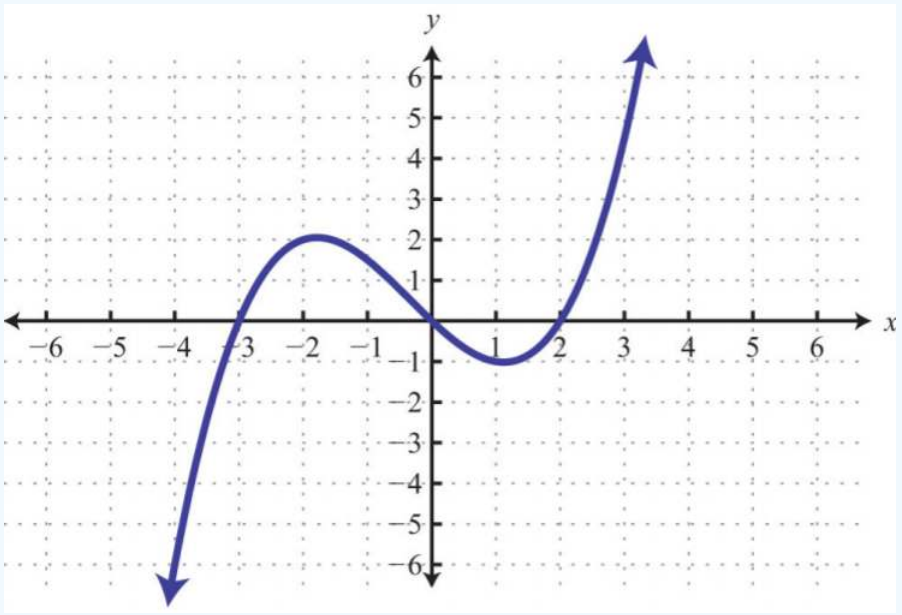


Figure **4.3.39**

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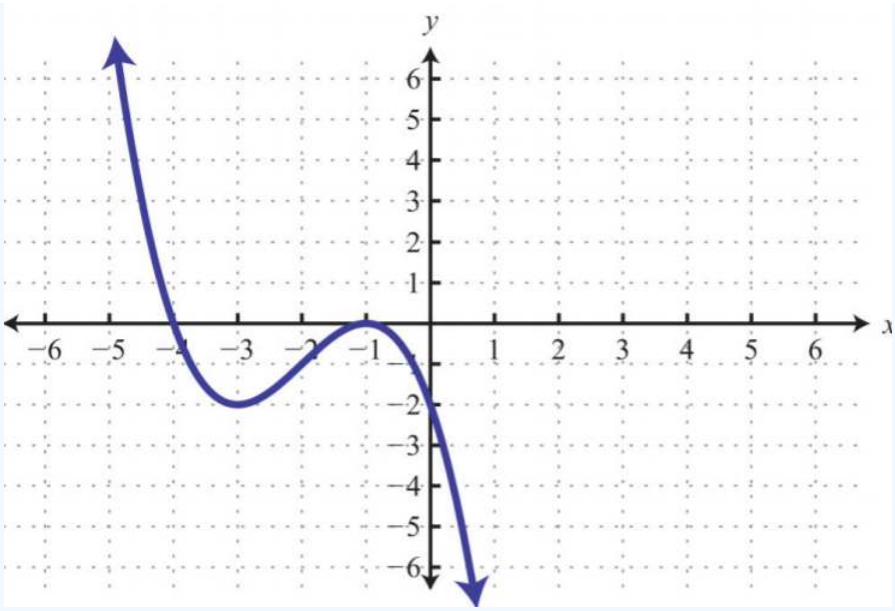


Figure **4.3.40**

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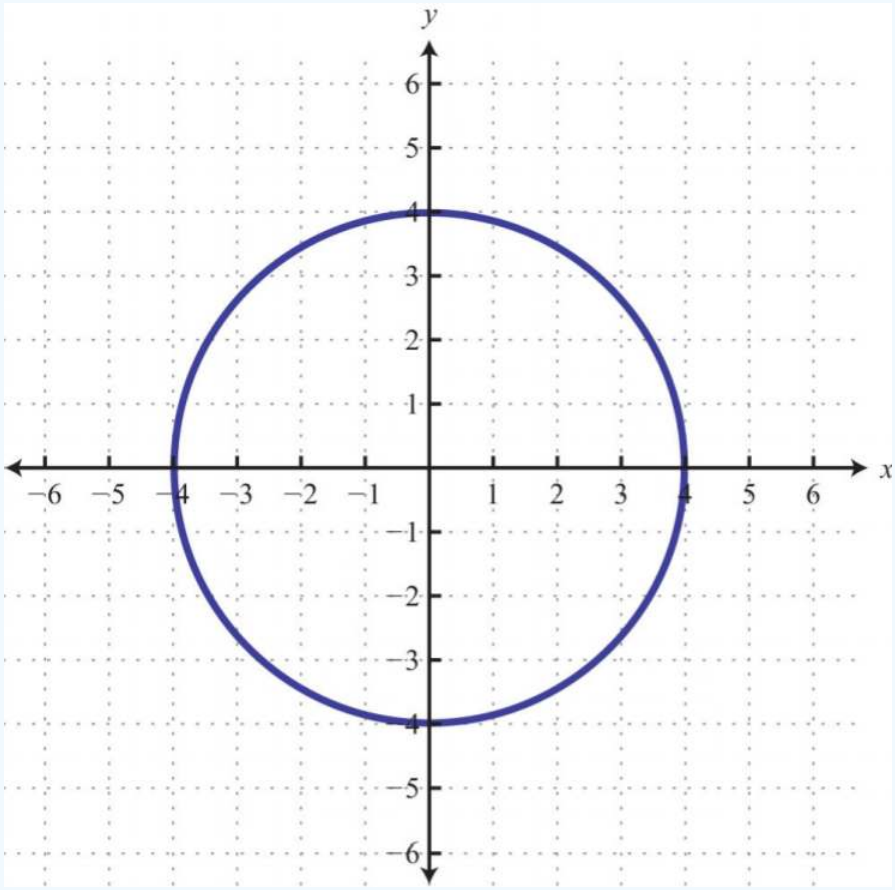


Figure **4.3.41**

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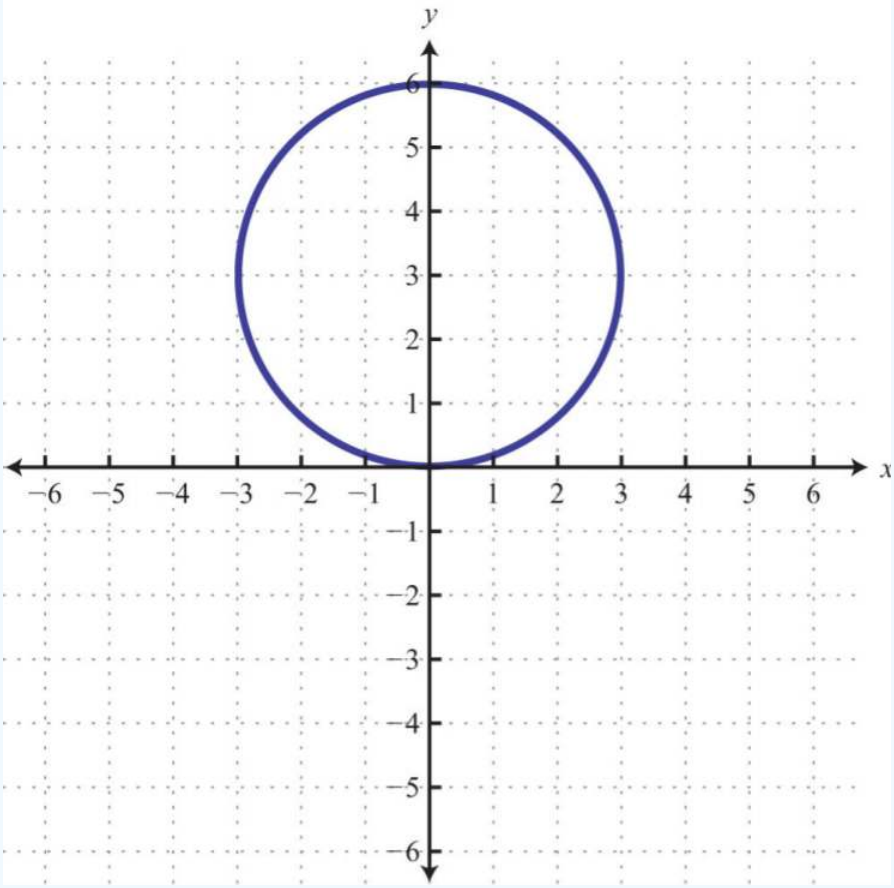


***Figure* 4.3.42**

Answer

1. x-intercepts: $(-3, 0), (3, 0)$; y-intercept: $(0, -3)$
3. x-intercepts: $(-4, 0), (0, 0)$; y-intercept: $(0, 0)$
5. x-intercepts: $(-2, 0), (2, 0)$; y-intercept: $(0, -1)$
7. x-intercepts: $(-3, 0), (0, 0), (2, 0)$; y-intercept: $(0, 0)$
9. x-intercepts: $(-4, 0), (4, 0)$; y-intercepts: $(0, -4), (0, 4)$

? Exercise 4.3.7 Discussion Board Topics

1. What are the x-intercepts of the line $y = 0$?
2. What are the y-intercepts of the line $x = 0$?
3. Do all lines have intercepts?
4. How many intercepts can a circle have? Draw circles showing all possible numbers of intercepts.
5. Research and post the definitions of line segment, ray, and line. Why are the arrows important?

Answer

1. Answers may vary
3. Answers may vary
5. Answers may vary

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4.4: Graph Using the y-Intercept and Slope

Learning Objectives

- Identify and find the slope of a line.
- Graph a line using the slope and y-intercept.

Slope

The steepness of any incline can be measured as the ratio of the vertical change to the horizontal change. For example, a 5% incline can be written as $\frac{5}{100}$, which means that for every 100 feet forward, the height increases 5 feet.

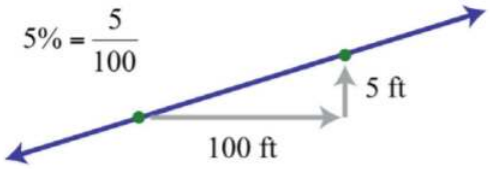


***Figure* 4.4.1**

In mathematics, we call the incline of a line the slope and use the letter m to denote it. The vertical change is called the rise and the horizontal change is called the run.

$$\textit{Slope} \quad m = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{rise}{run} \tag{4.4.1}$$

The rise and the run can be positive or negative. A positive rise corresponds to a vertical change up and a negative rise corresponds to a vertical change down. A positive run denotes a horizontal change to the right and a negative run corresponds to a horizontal change to the left. Given the graph, we can calculate the slope by determining the vertical and horizontal changes between any two points.

Example 4.4.1

Find the slope of the given line:

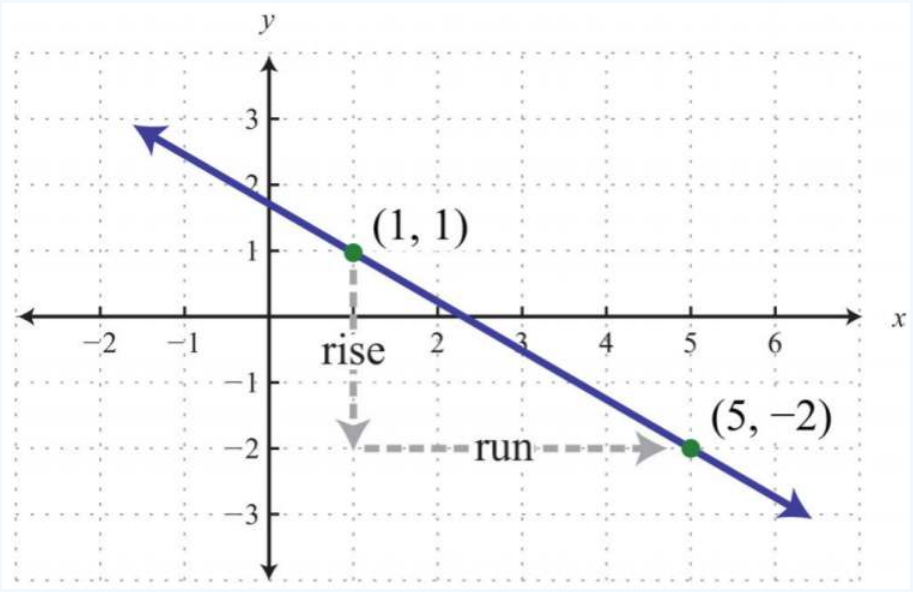


***Figure* 4.4.2**

Solution:

From the given points on the graph, count 3 units down and 4 units right.

$$m = \frac{rise}{run} = \frac{-3 \text{ units}}{4 \text{ units}} = -\frac{3}{4}$$

Answer:

$$m = -\frac{3}{4}$$

Here we have a negative slope, which means that for every 4 units of movement to the right, the vertical change is 3 units downward. There are four geometric cases for the value of the slope.

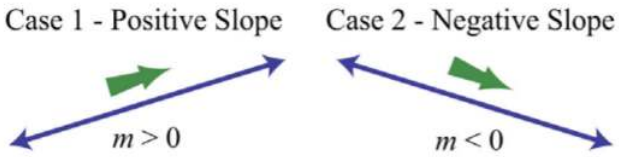


***Figure* 4.4.3**

Reading the graph from left to right, we see that lines with an upward incline have positive slopes and lines with a downward incline have negative slopes.

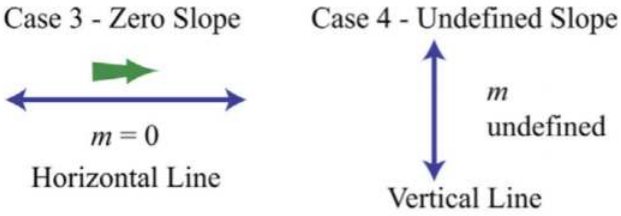


***Figure* 4.4.4**

If the line is horizontal, then the rise is 0:

$$m = \frac{rise}{run} = \frac{0}{run} = 0$$

The slope of a horizontal line is 0. If the line is vertical, then the run is 0:

$$m = \frac{rise}{run} = \frac{rise}{0} \quad Undefined$$

The slope of a vertical line is undefined.

? Exercise 4.4.1

Find the slope of the given line:

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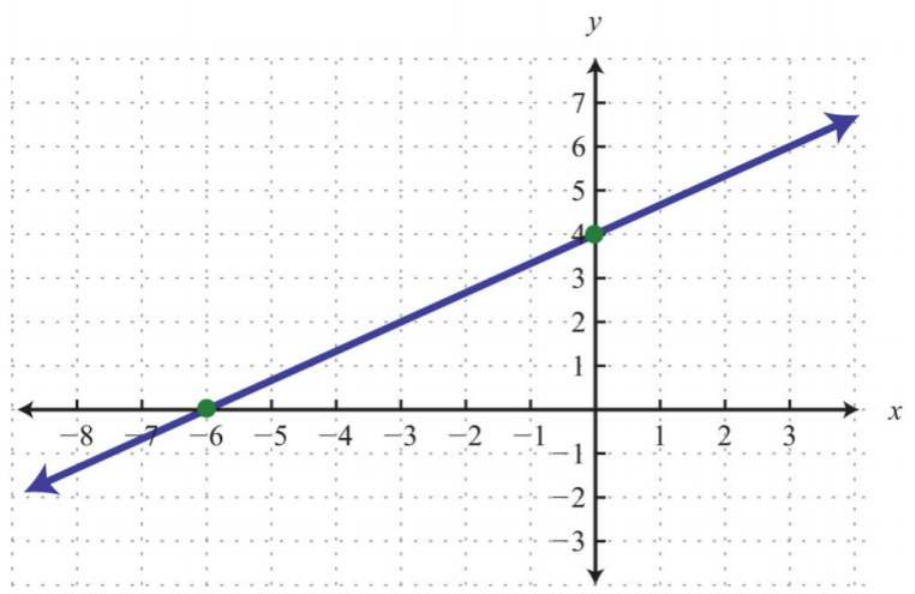


***Figure* 4.4.5**

Answer

$m = \frac{2}{3}$

Calculating the slope can be difficult if the graph does not have points with integer coordinates. Therefore, we next develop a formula that allows us to calculate the slope algebraically. Given any two points (x_1, y_1) and (x_2, y_2), we can obtain the rise and run by subtracting the corresponding coordinates.

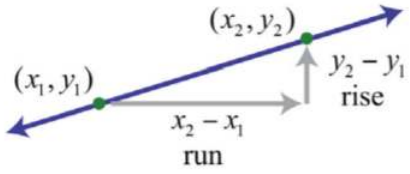


***Figure* 4.4.6**

This leads us to the slope formula. Given any two points (x_1, y_1) and (x_2, y_2), the slope is given by

$$m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$$

✔ Example 4.4.2

Find the slope of the line passing through $(-3, -5)$ and $(2, 1)$.

Solution:

Given $(-3, -5)$ and $(2, 1)$, calculate the difference of the y-values divided by the difference of the x-values. Since subtraction is not commutative, take care to be consistent when subtracting the coordinates.

$$\begin{array}{cc} (x_1, y_1) & (x_2, y_2) \\ (-3, -5) & (2, 1) \end{array}$$

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{1-(-5)}{2-(-3)} \\ &= \frac{1+5}{2+3} \\ &= \frac{6}{5} \end{aligned}$$

Answer:

$$m = \tfrac{6}{5}$$

We can graph the line described in the previous example and verify that the slope is $\frac{6}{5}$.

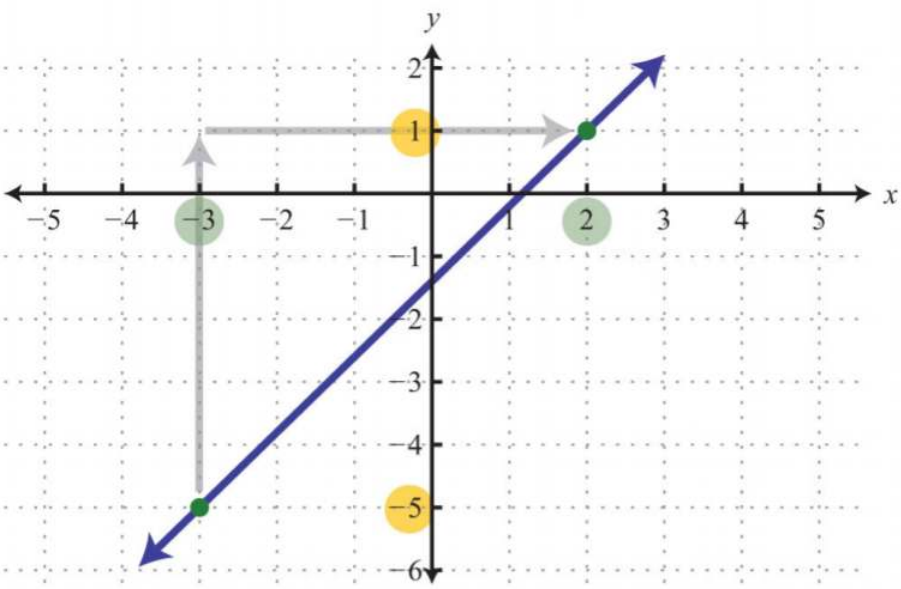


***Figure* 4.4.7**

Certainly the graph is optional; the beauty of the slope formula is that we can obtain the slope, given two points, using only algebra.

Example 4.4.3

Find the slope of the line passing through $(-4, 3)$ and $(-1, -7)$.

Solution:

$$\begin{array}{cc} (x_1, y_1) & (x_2, y_2) \\ (-4, 3) & (-1, -7) \end{array}$$

$$m = \tfrac{y_2 - y_1}{x_2 - x_1} = \tfrac{-7-(3)}{-1-(-4)} = \tfrac{-7-3}{-1+4} = \tfrac{-10}{3}$$

Answer:

$$m = -\tfrac{10}{3}$$

When using the slope formula, take care to be consistent since order does matter. You must subtract the coordinates of the first point from the coordinates of the second point for both the numerator and the denominator in the ***same order***.

Example 4.4.4

Find the slope of the line passing through $(7, -2)$ and $(-5, -2)$.

Solution:

$$\begin{array}{cc} (x_1, y_1) & (x_2, y_2) \\ (7, -2) & (-5, -2) \end{array}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-(-2)}{-5-(7)} = \frac{-2+2}{-5-7} = \frac{0}{-12} = 0$$

Answer:

$m = 0$. As an exercise, plot the given two points and verify that they lie on a horizontal line.

Example 4.4.5

Find the slope of the line passing through $(-4, -3)$ and $(-4, 5)$.

Solution:

$$\begin{array}{cc} (x_1, y_1) & (x_2, y_2) \\ (-4, -3) & (-4, 5) \end{array}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-(-3)}{-4-(-4)} = \frac{5+3}{-4+4} = \frac{8}{0} \quad Undefined$$

Answer:

The slope m is undefined. As an exercise, plot the given two points and verify that they lie on a vertical line.

Exercise 4.4.2

Calculate the slope of the line passing through $(-2, 3)$ and $(5, -5)$.

Answer

$m = -\frac{8}{7}$

When considering the slope as a rate of change it is important to include the correct units.

Example 4.4.6

A Corvette Coupe was purchased new in 1970 for about $\$5,200$ and depreciated in value over time until it was sold in 1985 for $\$1,300$. At this point, the car was beginning to be considered a classic and started to increase in value. In the year 2000, when the car was 30 years old, it sold at auction for $\$10,450$. The following line graph depicts the value of the car over time.

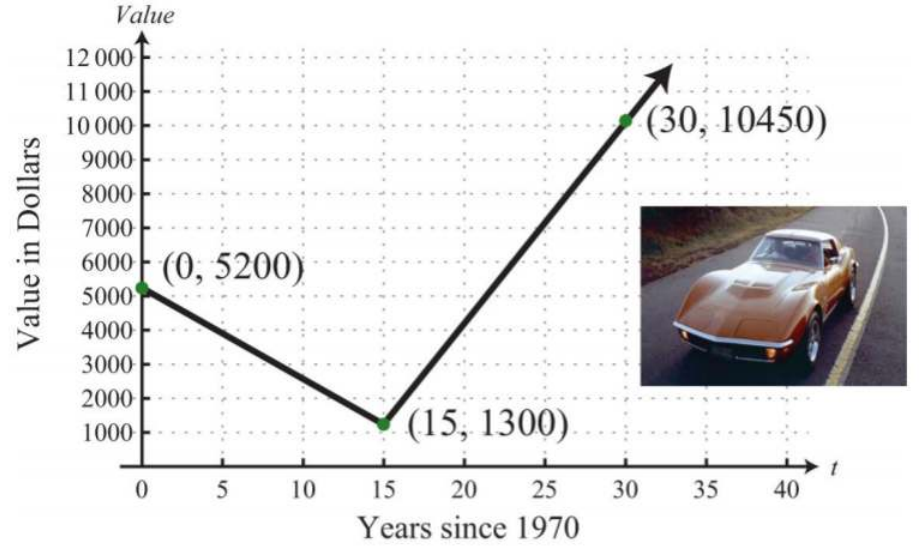


***Figure* 4.4.8**

a. Determine the rate at which the car depreciated in value from 1970 to 1985.
b. Determine the rate at which the car appreciated in value from 1985 to 2000.

Solution:

Notice that the value depends on the age of the car and that the slope measures the rate in dollars per year.

a. The slope of the line segment depicting the value for the first 15 years is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\$1{,}300 - \$5{,}200}{15 \text{ years} - 0 \text{ years}} = \frac{-\$3{,}900}{15 \text{ years}} = -\$260 \text{ per year}$$

b. The slope of the line segment depicting the value for the next 15 years is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\$10{,}450 - \$1{,}300}{30 \text{ years} - 15 \text{ years}} = \frac{\$9{,}150}{15 \text{ years}} = \$610 \text{ per year}$$

Answer:

a. The value of the car depreciated $\$260$ per year from 1970 to 1985.
b. The value of the car appreciated $\$610$ per year from 1985 to 2000.

Slope-Intercept Form of a Line

To this point, we have learned how to graph lines by plotting points and by using the x- and y-intercepts. In addition, we have seen that we need only two points to graph a line. In this section, we outline a process to easily determine two points using the y-intercept and the slope. The equation of any nonvertical line can be written in slope-intercept form $y = mx + b$. In this form, we can identify the slope, m, and the y-intercept, $(0, b)$.

Example 4.4.7

Determine the slope and y-intercept:

$$y = -\frac{4}{5}x + 7.$$

Solution:

In this form, the coefficient of x is the slope, and the constant is the y-value of the y-intercept. Therefore, by inspection, we have

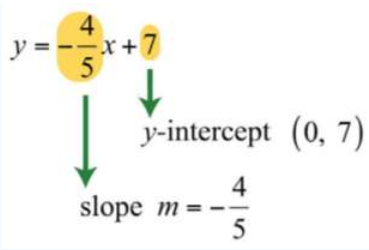


***Figure* 4.4.9**

Answer:

The y-intercept is $(0, 7)$, and the slope is $m = -\frac{4}{5}$.

It is not always the case that the linear equation is given in slope-intercept form. When it is given in standard form, you have to first solve for y to obtain slope-intercept form.

Example 4.4.8

Express $3x + 5y = 30$ in slope-intercept form and then identify the slope and y-intercept.

Solution:

Begin by solving for y. To do this, apply the properties of equality to first isolate $5y$ and then divide both sides by 5.

$$\begin{aligned} 3x + 5y &= 30 \\ 3x + 5y - 3x &= 30 - 3x \\ 5y &= -3x + 30 \\ \frac{5y}{5} &= \frac{-3x + 30}{5} \\ y &= \frac{-3x}{5} + \frac{30}{5} \\ y &= -\frac{3}{5}x + 6 \end{aligned}$$

Answer:

Slope-intercept form: $y = -\frac{3}{5}x + 6$; y-intercept: $(0, 6)$; slope: $m = -\frac{3}{5}$

Once the equation is in slope-intercept form, we immediately have one point to plot, the y-intercept. From the intercept, you can mark off the slope to plot another point on the line. From the previous example we have

$$y = -\frac{3}{5}x + 6$$

y-intercept: $(0, 6)$

slope: $m = -\frac{3}{5} = \frac{-3}{5} = \frac{rise}{run}$

Starting from the point $(0, 6)$, use the slope to mark another point 3 units down and 5 units to the right.

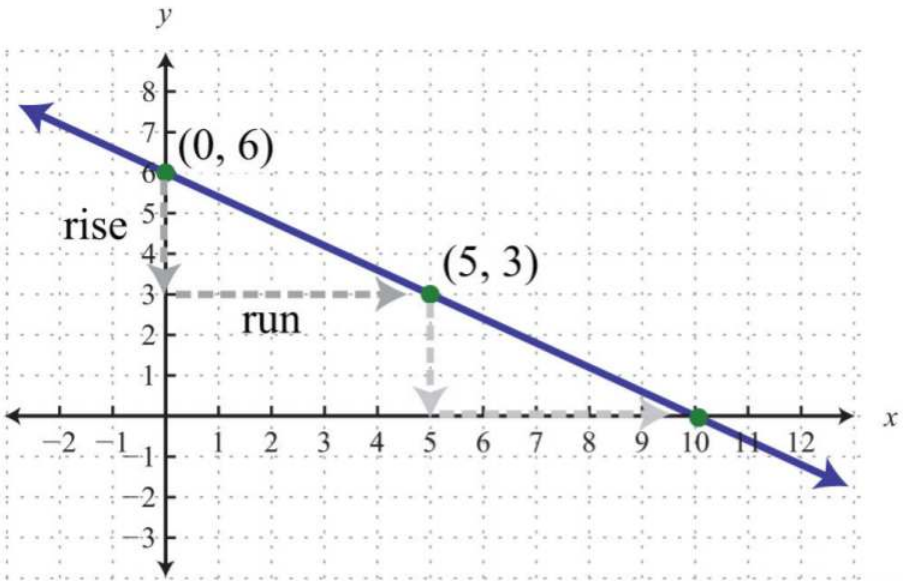


***Figure* 4.4.10**

It is not necessary to check that the second point, (5, 3), solves the original linear equation. However, we do it here for the sake of completeness.

$$\begin{aligned} 3x+5y &= 30 \\ 3(5)+5(3) &= 30 \\ 15+15 &= 30 \\ 30 &= 30 \quad \checkmark \end{aligned}$$

Marking off the slope in this fashion produces as many ordered pair solutions as we desire. Notice that if we mark off the slope again, from the point $(5, 3)$, then we obtain the x-intercept, $(10, 0)$.

Example 4.4.9

Graph:

$$-x+2y=4\,.$$

Solution:

In this example, we outline the general steps for graphing a line using slope-intercept form.

Step 1: Solve for y to obtain slope-intercept form.

$$\begin{aligned} -x+2y &= 4 \\ -x+2y+x &= 4+x \\ 2y &= x+4 \\ \frac{2y}{2} &= \frac{x+4}{2} \\ y &= \frac{1x}{2}+\frac{4}{2} \\ y &= \frac{1}{2}x+2 \end{aligned}$$

Step 2: Identify the y-intercept and slope.

$$y\text{-intercept: } (0,2)$$

$$\text{slope: } m=\tfrac{1}{2}=\tfrac{rise}{run}$$

Step 3: Plot the y-intercept and use the slope to find another ordered pair solution. Starting from the y-intercept, mark off the slope and identify a second point. In this case, mark a point after a rise of 1 unit and a run of 2 units.

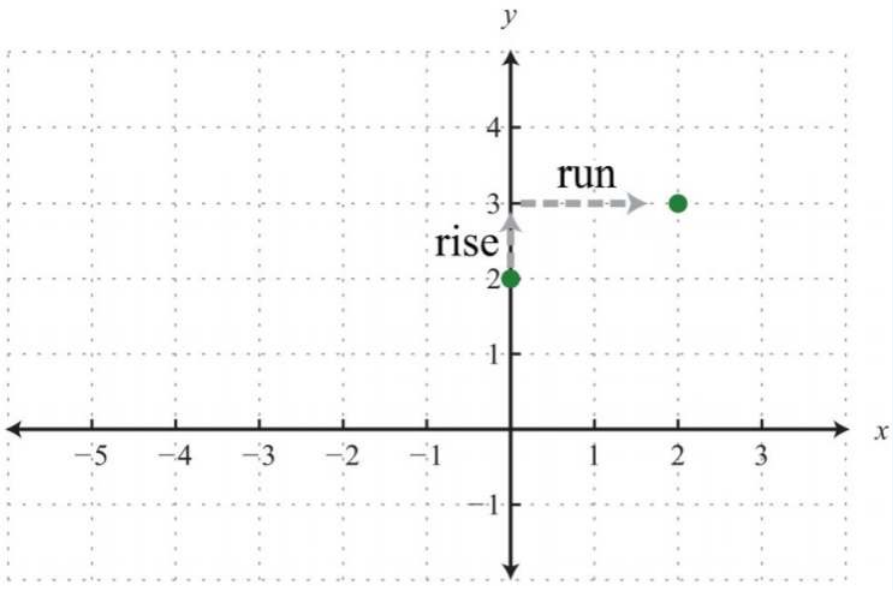


Figure **4.4.11**

Step 4: Draw the line through the two points with a straightedge.

Answer:

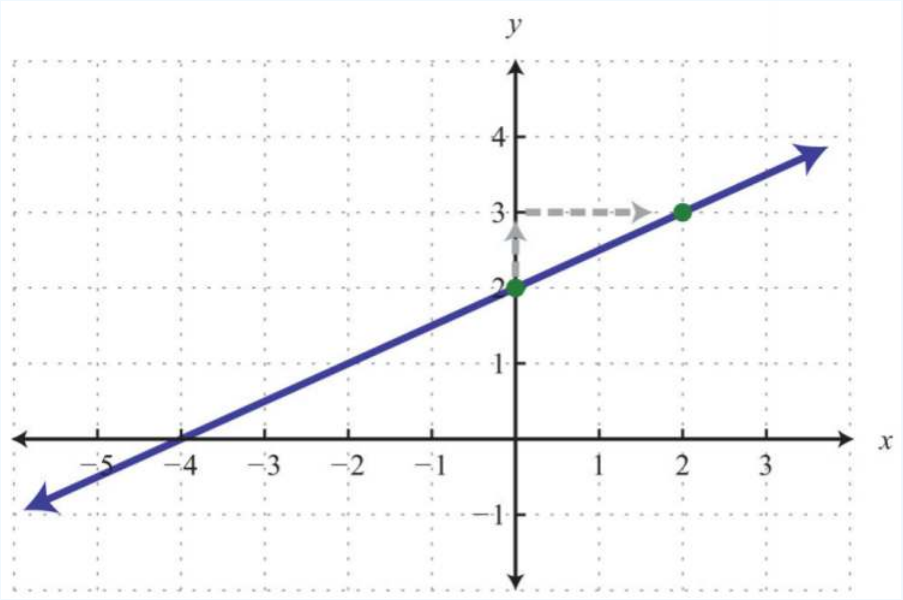


Figure **4.4.12**

In this example, we notice that we could get the x-intercept by marking off the slope in a different but equivalent manner. Consider the slope as the ratio of two negative numbers as follows:

$$m = \frac{1}{2} = \frac{-1}{-2} = \frac{rise}{run}$$

We could obtain another point on the line by marking off the equivalent slope down 1 unit and left 2 units. We do this twice to obtain the x-intercept, $(-4, 0)$.

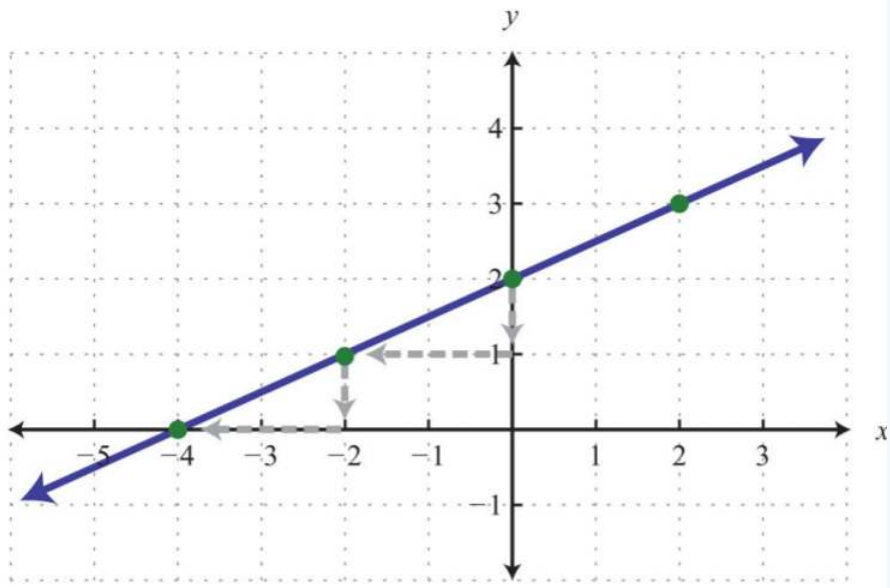


***Figure* 4.4.13**

Marking off the slope multiple times is not necessarily always going to give us the x-intercept, but when it does, we obtain a valuable point with little effort. In fact, it is a good practice to mark off the slope multiple times; doing so allows you to obtain more points on the line and produce a more accurate graph.

Example 4.4.10

Graph and find the x-intercept:

$$y = \tfrac{3}{4}x - 2\,.$$

Solution:

The equation is given in slope-intercept form. Therefore, by inspection, we have the y-intercept and slope.

$$y\text{-intercept: } (0, -2)$$

$$\text{slope: } m = \tfrac{3}{4} = \tfrac{rise}{run}$$

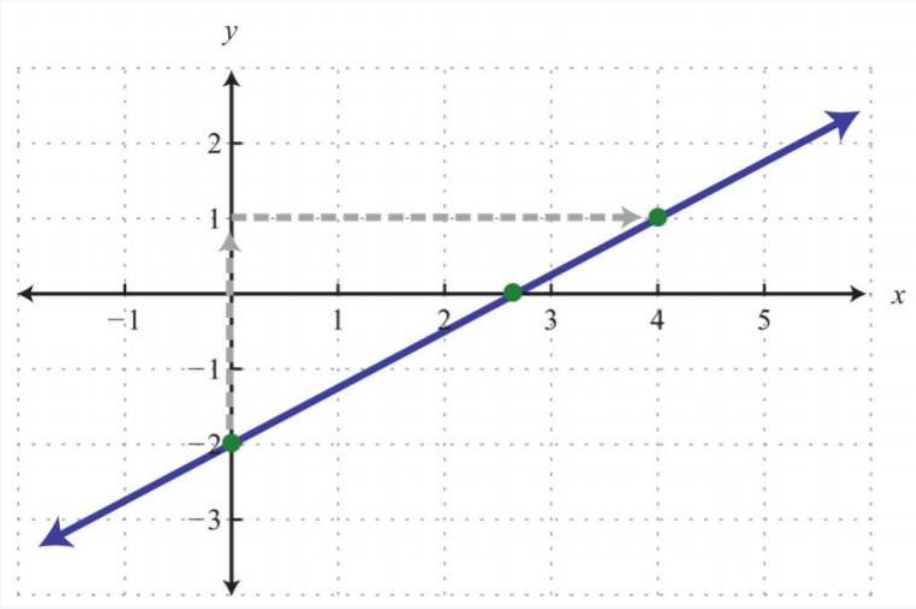


Figure **4.4.14**

We can see that the x-value of the x-intercept is a mixed number between 2 and 3. To algebraically find x-intercepts, recall that we must set $y = 0$ and solve for x.

Answer:

The x-intercept is $(2\frac{2}{3}, 0)$.

Example 4.4.11

Graph:

$$x - y = 0\,.$$

Solution:

Begin by solving for y.

$$\begin{aligned} x - y &= 0 \\ x - y\color{cyan}{-x} &= 0\color{cyan}{-x} \\ -y &= -x \\ \color{cyan}{-1}\cdot(-y) &= \color{cyan}{-1}\cdot(-x) \\ y &= x \end{aligned}$$

The equation $y = x$ can be written $y = 1x + 0$, and we have

$$y\text{-intercept: } (0, 0)$$

$$\text{slope: } m = 1 = \tfrac{1}{1} = \tfrac{rise}{run}$$

Answer:

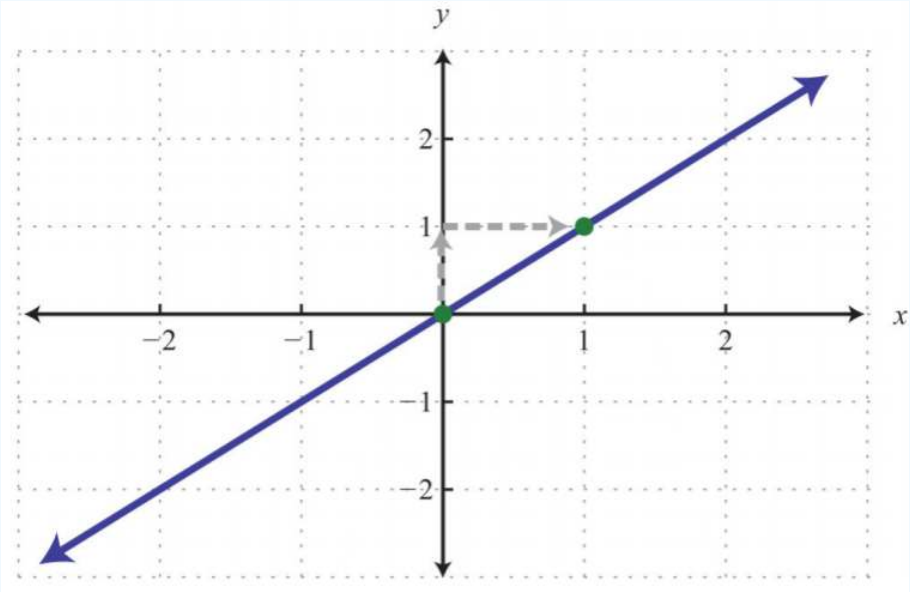


Figure **4.4.15**

Exercise 4.4.3

Graph $-2x + 5y = 20$ and label the x-intercept.

Answer

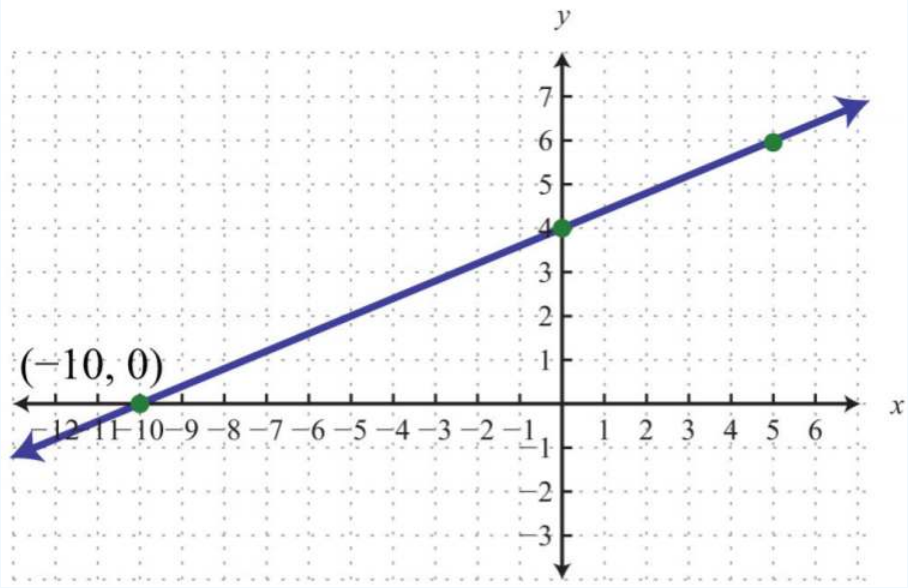


Figure 4.4.16

Key Takeaways

- Slope measures the steepness of a line as *rise over run*. A positive rise denotes a vertical change up, and a negative rise denotes a vertical change down. A positive run denotes a horizontal change right, and a negative run denotes a horizontal change left.
- Horizontal lines have a slope of zero, and vertical lines have undefined slopes.
- Given any two points on a line, we can algebraically calculate the slope using the slope formula, $m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$.
- Any nonvertical line can be written in slope-intercept form, $y = mx + b$, from which we can determine, by inspection, the slope m and y-intercept $(0, b)$.
- If we know the y-intercept and slope of a line, then we can easily graph it. First, plot the y-intercept, and from this point use the slope as rise over run to mark another point on the line. Finally, draw a line through these two points with a straightedge and add an arrow on either end to indicate that it extends indefinitely.
- We can obtain as many points on the line as we wish by marking off the slope multiple times.

? Exercise 4.4.4 Slope

Determine the slope and the y-intercept of the given graph.

1.

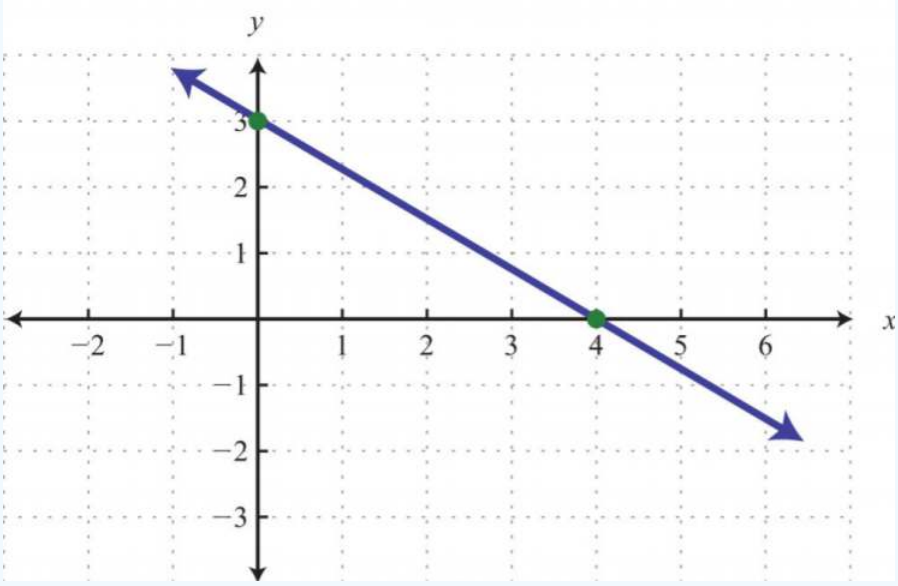


Figure **4.4.17**

2.

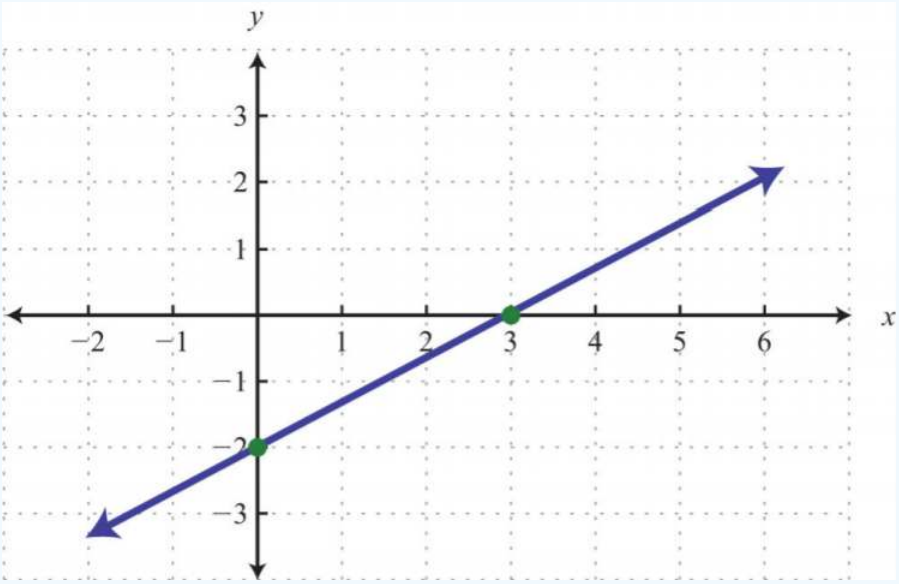


Figure **4.4.18**

3.

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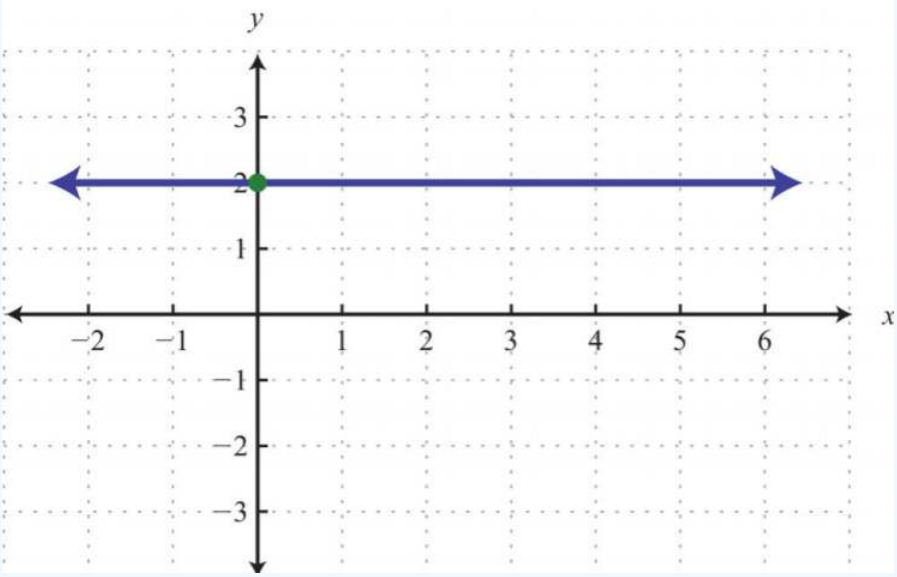


Figure **4.4.19**

4.

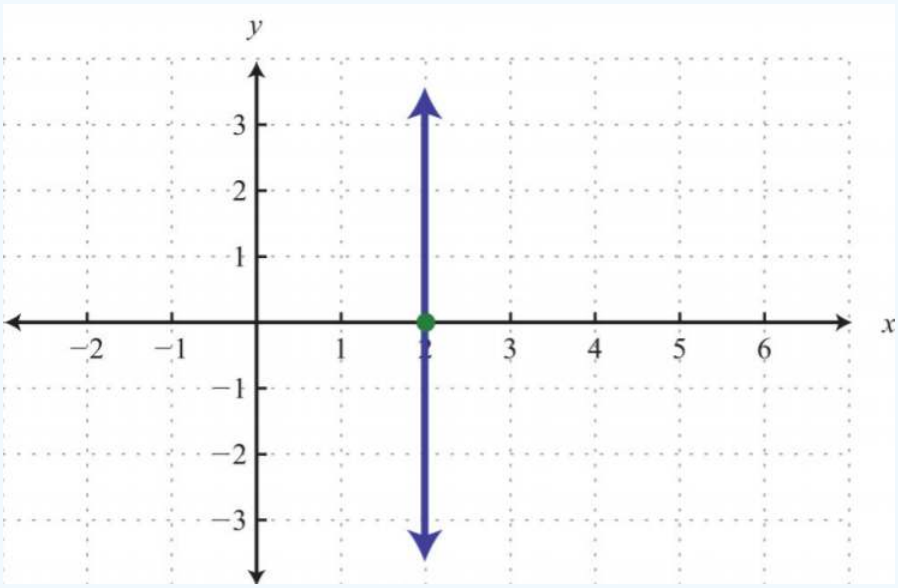


Figure **4.4.20**

5.

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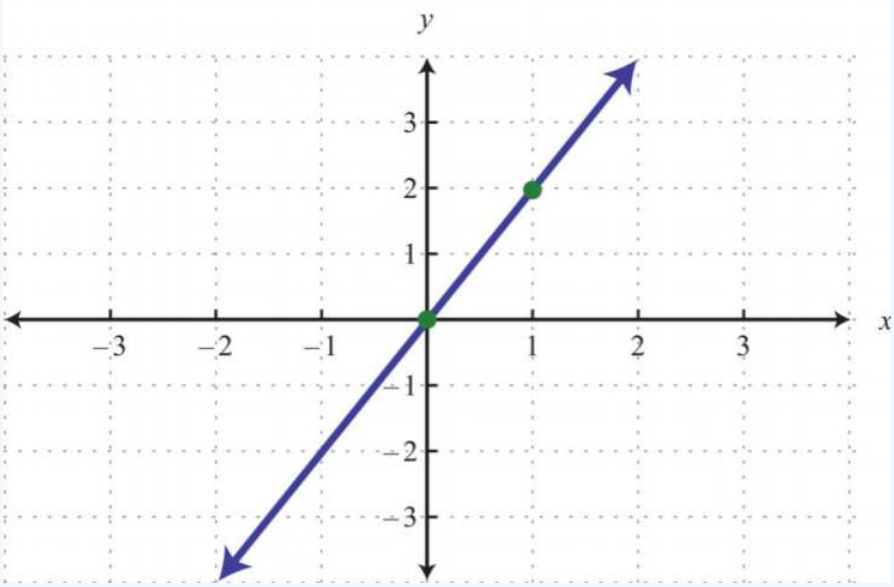


Figure **4.4.21**

6.

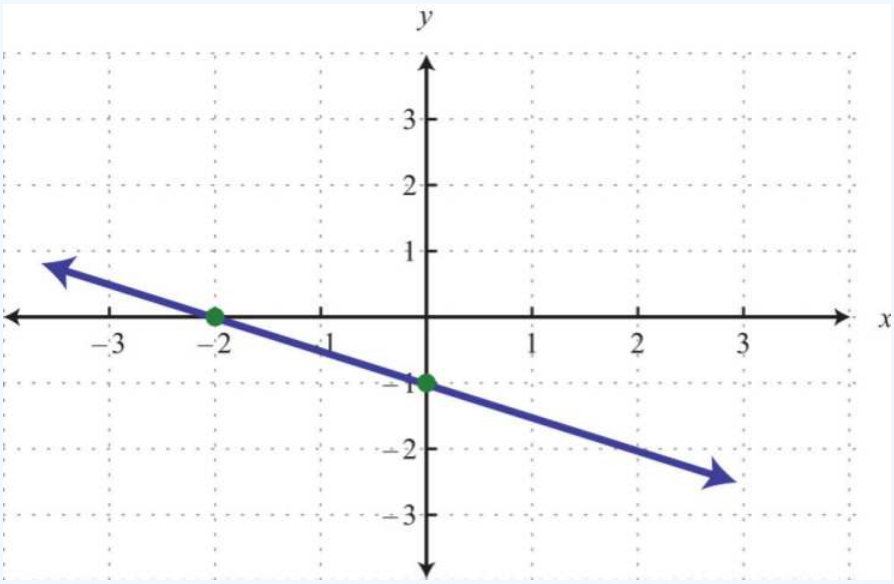


Figure **4.4.22**

Answer

1. y-intercept: $(0, 3)$; slope: $m = -\frac{3}{4}$
3. y-intercept: $(0, 2)$; slope: $m = 0$
5. y-intercept: $(0, 0)$; slope: $m = 2$

? Exercise 4.4.5 Slope

Determine the slope, given two points.

1. $(3, 2)$ and $(5, 1)$
2. $(7, 8)$ and $(-3, 5)$
3. $(2, -3)$ and $(-3, 2)$

4. $(-3, 5)$ and $(7, -5)$
5. $(-1, -6)$ and $(3, 2)$
6. $(5, 3)$ and $(4, 12)$
7. $(-9, 3)$ and $(-6, -5)$
8. $(-22, 4)$ and $(-8, -12)$
9. $(\frac{1}{2}, -\frac{1}{3})$ and $(-\frac{1}{2}, \frac{2}{3})$
10. $(-\frac{3}{4}, \frac{3}{2})$ and $(\frac{1}{4}, -\frac{1}{2})$
11. $(-\frac{1}{3}, \frac{5}{8})$ and $(\frac{1}{2}, -\frac{3}{4})$
12. $(-\frac{3}{5}, -\frac{3}{2})$ and $(\frac{1}{10}, \frac{4}{5})$
13. $(3, -5)$ and $(5, -5)$
14. $(-3, 1)$ and $(-14, 1)$
15. $(-2, 3)$ and $(-2, -4)$
16. $(-4, -4)$ and $(5, 5)$
17. A roof drops 4 feet for every 12 feet forward. Determine the slope of the roof.
18. A road drops 300 feet for every $5,280$ feet forward. Determine the slope of the road.
19. The following graph gives the US population of persons 65 years old and over. At what rate did this population increase from 2000 to 2008?

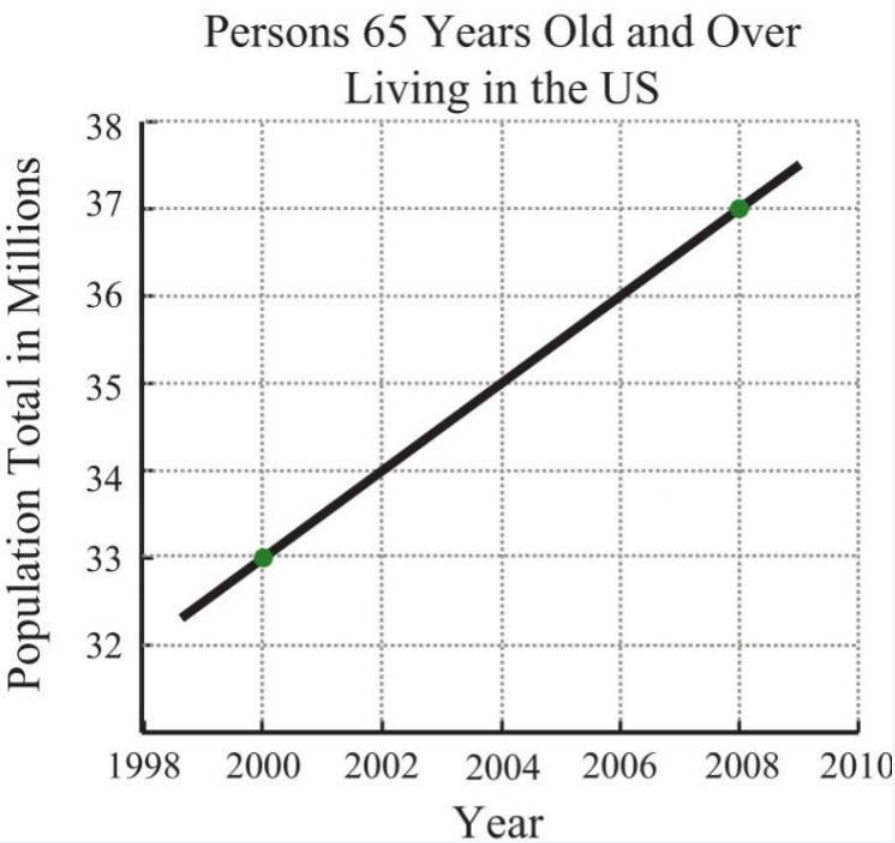


***Figure* 4.4.23** *: Source: US Census Bureau.*

20. The following graph gives total consumer credit outstanding in the United States. At what rate did consumer credit increase from 2002 to 2008?

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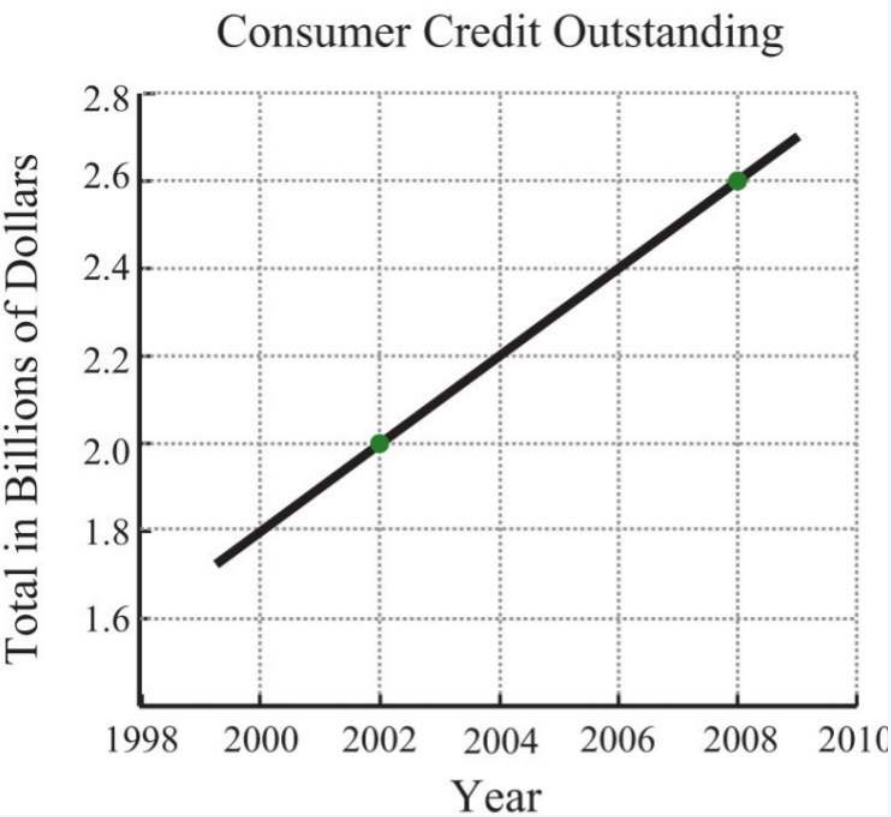


***Figure* 4.4.24**: *Source: US Census Bureau.*

21. A commercial van was purchased new for $\$20,000$ and is expected to be worth $\$4,000$ in 8 years. Determine the rate at which the van depreciates in value.

22. A commercial-grade copy machine was purchased new for $\$4,800$ and will be considered worthless in 6 years. Determine the rate at which the copy machine depreciates in value.

23. Find y if the slope of the line passing through $(-2,3)$ and $(4,y)$ is 12.

24. Find y if the slope of the line passing through $(5,y)$ and $(6,-1)$ is 10.

25. Find y if the slope of the line passing through $(5,y)$ and $(-4,2)$ is 0.

26. Find x if the slope of the line passing through $(-3,2)$ and $(x,5)$ is undefined.

Answer

1. $-\frac{1}{2}$
3. -1
5. 2
7. $-\frac{8}{3}$
9. -1
11. $-\frac{33}{20}$
13. 0
15. Undefined
17. $-\frac{1}{3}$
19. $\frac{1}{2}$ million per year

21. $\$2,000$ per year

23. 75

25. 2

Exercise 4.4.6 Slope-Intercept Form

Express the given linear equation in slope-intercept form and identify the slope and y-intercept.

1. $6x-5y=30$
2. $-2x+7y=28$
3. $9x-y=17$
4. $x-3y=18$
5. $2x-3y=0$
6. $-6x+3y=0$
7. $\frac{2}{3}x-\frac{5}{4}y=10$
8. $-\frac{4}{3}x+\frac{1}{5}y=-5$

Answer

1. $y=\frac{6}{5}x-6$; slope: $\frac{6}{5}$; y-intercept: $(0,-6)$

3. $y=9x-17$; slope: 9; y-intercept: $(0,-17)$

5. $y=\frac{2}{3}x$; slope: $\frac{2}{3}$; y-intercept: $(0,0)$

7. $y=\frac{8}{15}x-8$; slope: $\frac{8}{15}$; y-intercept: $(0,-8)$

Exercise 4.4.7 Slope-Intercept Form

Graph the line given the slope and the y-intercept.

1. $m=\frac{1}{3}$ and $(0,-2)$
2. $m=-\frac{2}{3}$ and $(0,4)$
3. $m=3$ and $(0,1)$
4. $m=-2$ and $(0,-1)$
5. $m=0$ and $(0,5)$
6. m undefined and $(0,0)$
7. $m=1$ and $(0,0)$
8. $m=-1$ and $(0,0)$
9. $m=-\frac{15}{3}$ and $(0,20)$
10. $m=-10$ and $(0,-5)$

Answer

1.

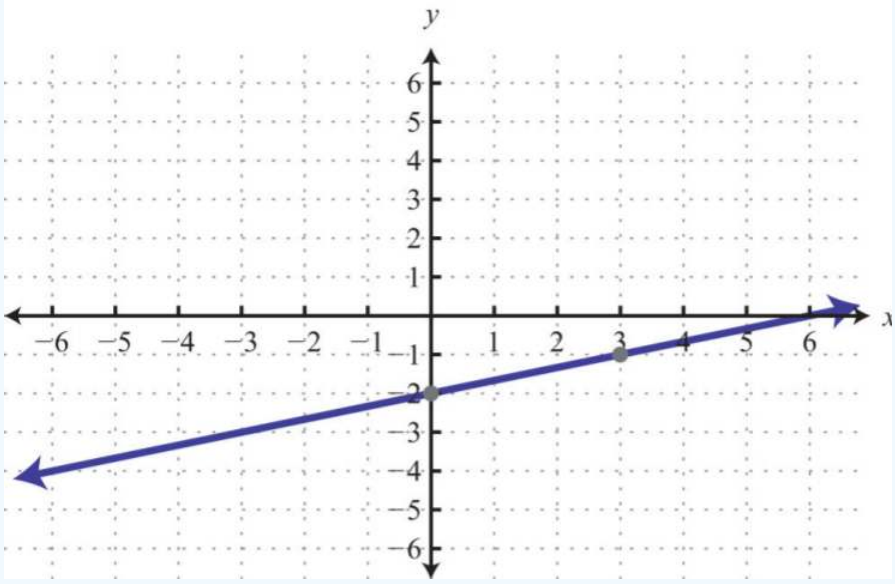


Figure 4.4.25

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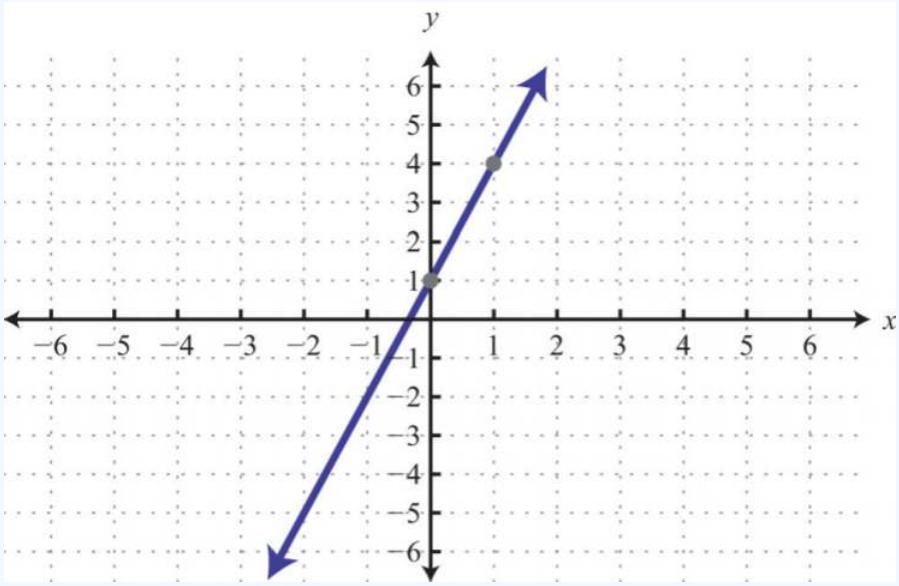


Figure 4.4.26

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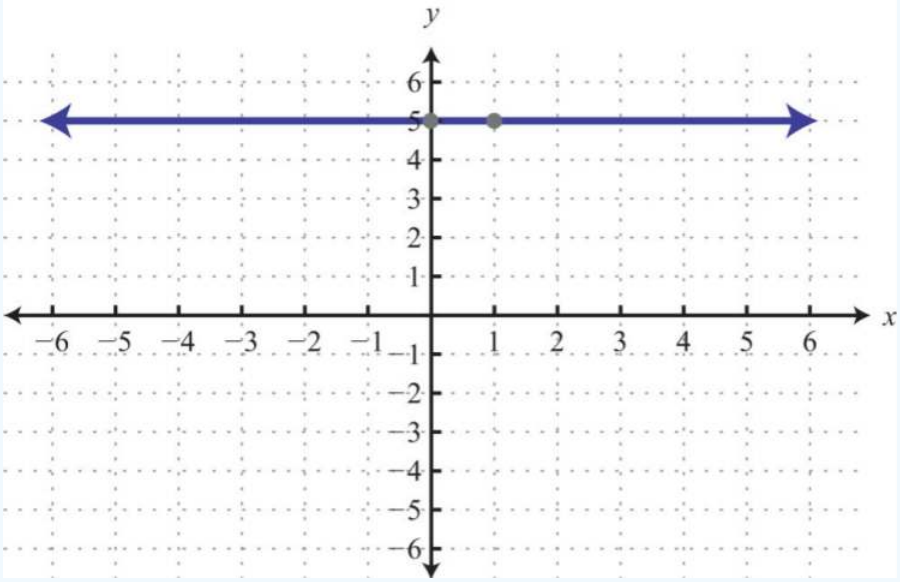


Figure 4.4.27

7.

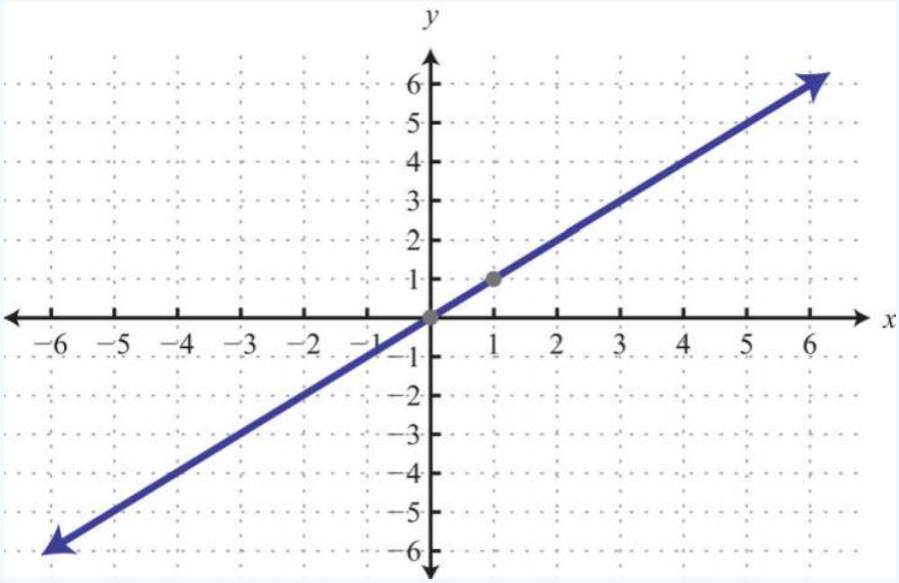


Figure 4.4.28

9.

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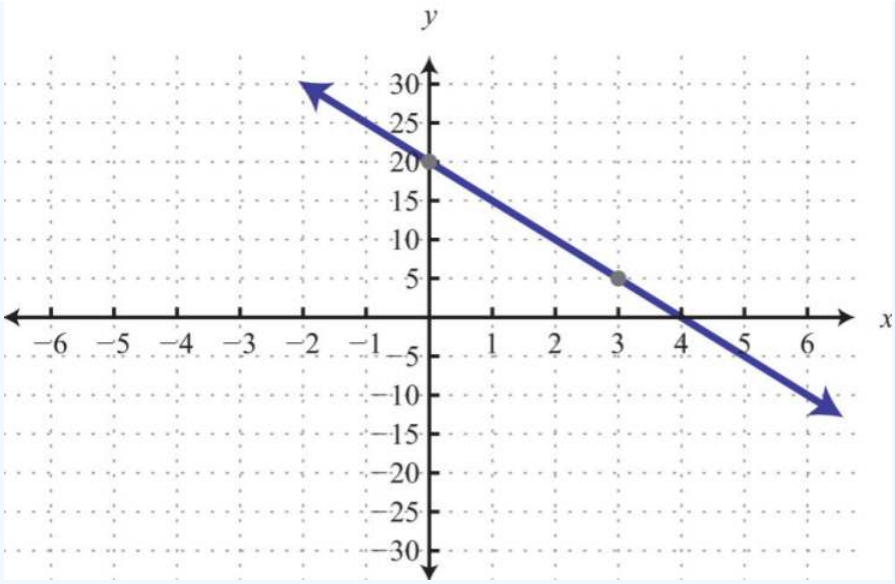


Figure 4.4.29

Exercise 4.4.8 Slope-Intercept Form

Graph using the slope and y-intercept.

1. $y = \frac{2}{3}x - 2$
2. $y = -\frac{1}{3}x + 1$
3. $y = -3x + 6$
4. $y = 3x + 1$
5. $y = \frac{3}{5}x$
6. $y = -\frac{3}{7}x$
7. $y = -8$
8. $y = 7$
9. $y = -x + 2$
10. $y = x + 1$
11. $y = \frac{1}{2}x + \frac{3}{2}$
12. $y = -\frac{3}{4}x + \frac{5}{2}$
13. $4x + y = 7$
14. $3x - y = 5$
15. $5x - 2y = 10$
16. $-2x + 3y = 18$
17. $x - y = 0$
18. $x + y = 0$
19. $\frac{1}{2}x - \frac{1}{3}y = 1$
20. $-\frac{2}{3}x + \frac{1}{2}y = 2$
21. $3x + 2y = 1$
22. $5x + 3y = 1$
23. On the same set of axes, graph the three lines, where $y = \frac{3}{2}x + b$ and $b = \{-2, 0, 2\}$.
24. On the same set of axes, graph the three lines, where $y = mx + 1$ and $m = \{-\frac{1}{2}, 0, \frac{1}{2}\}$.

Answer

1.

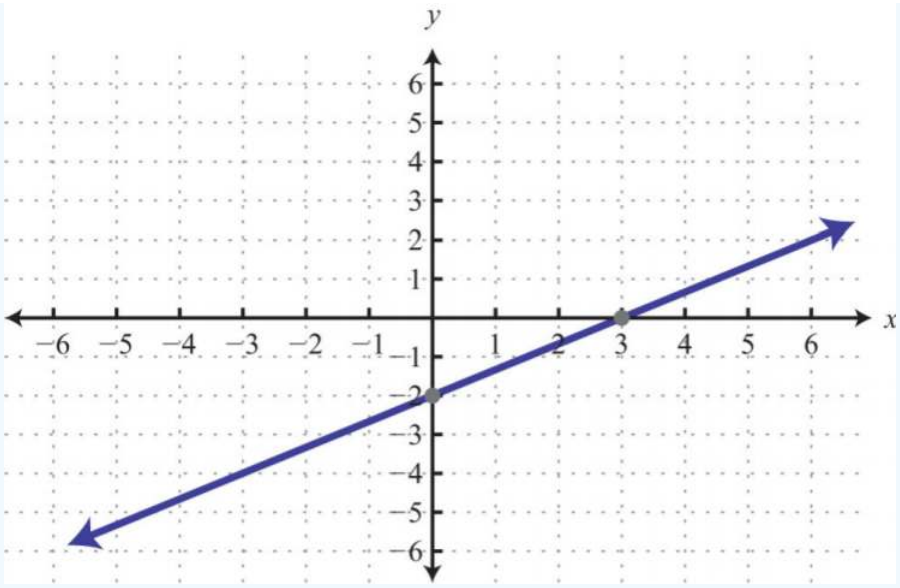


Figure 4.4.30

3.

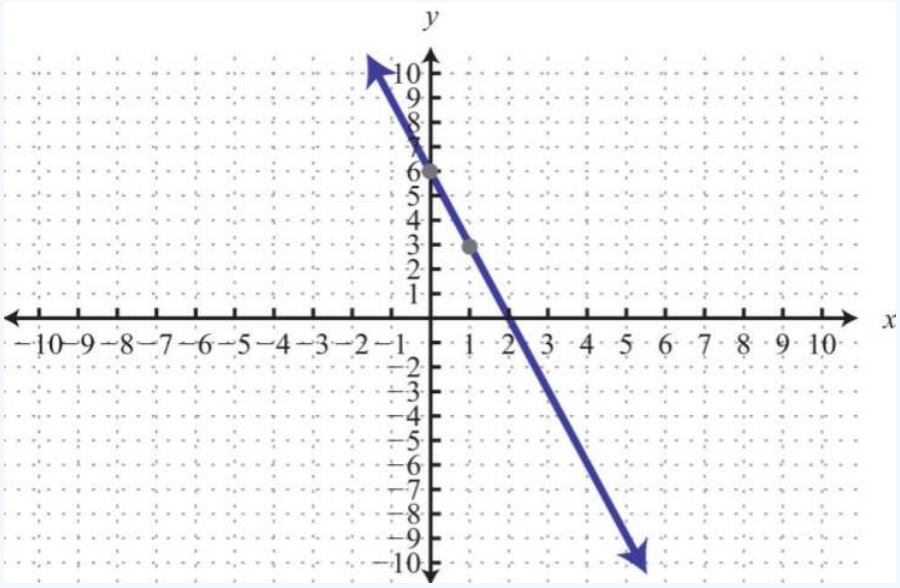


Figure 4.4.31

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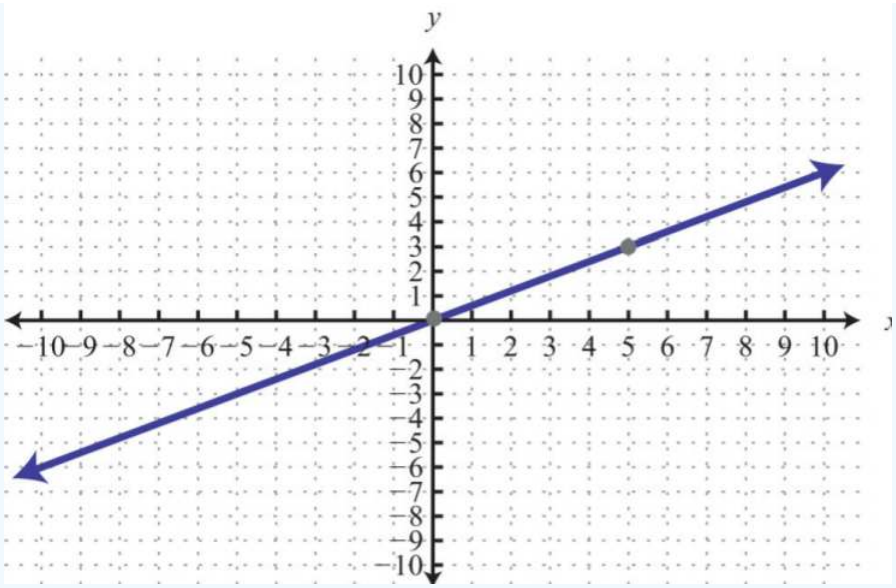


Figure 4.4.32

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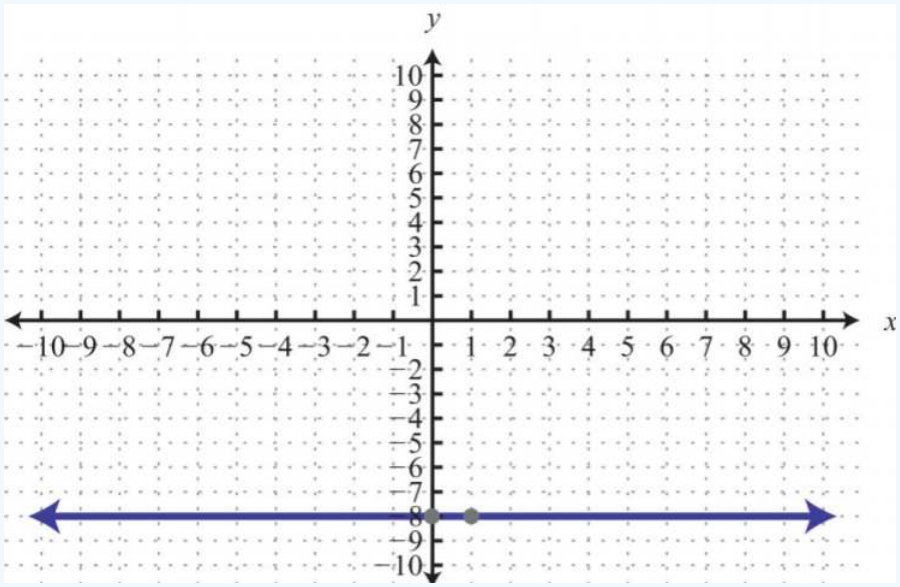


Figure 4.4.33

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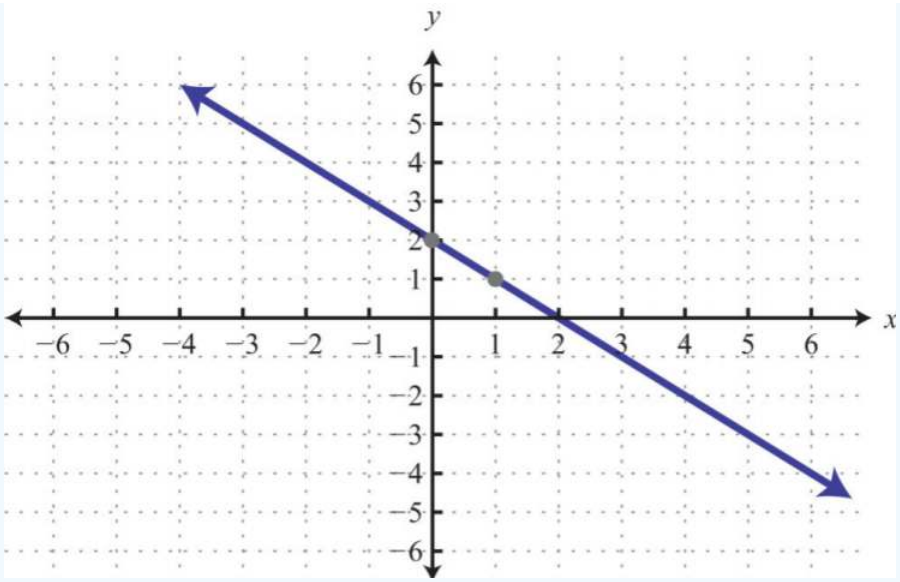


Figure 4.4.34

11.

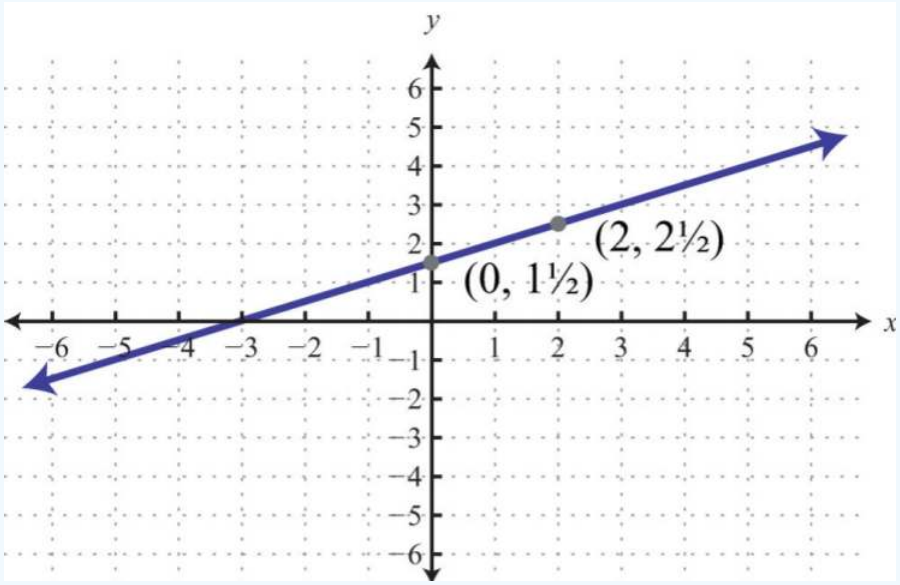


Figure 4.4.35

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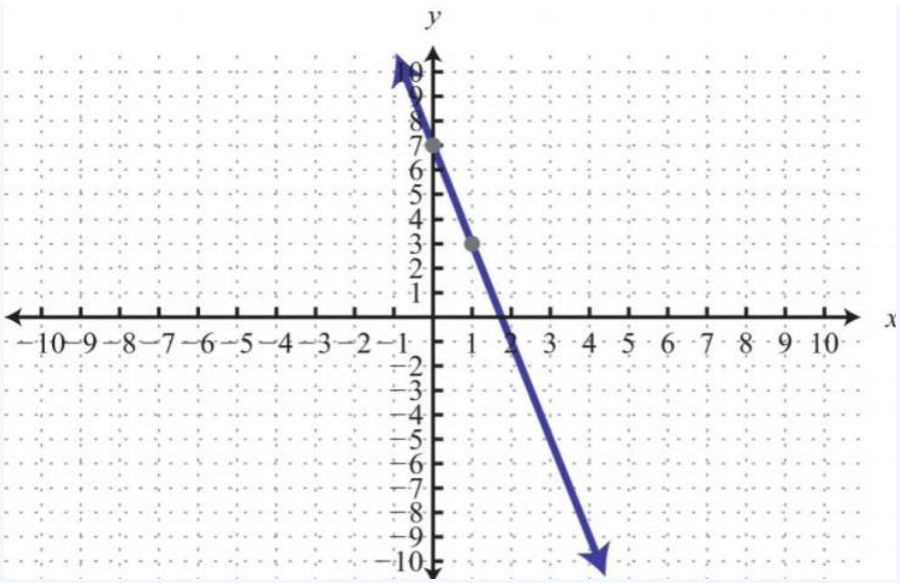


Figure 4.4.36

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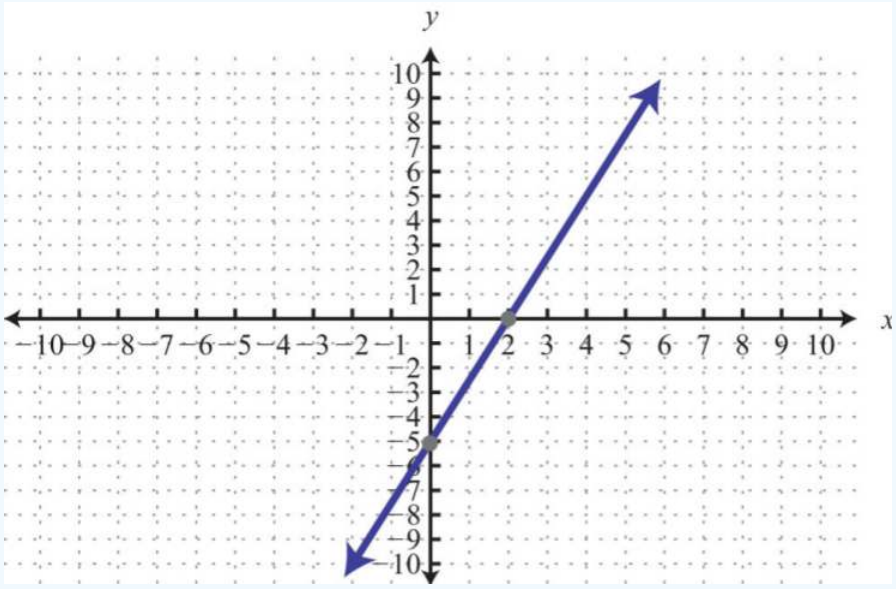


Figure 4.4.37

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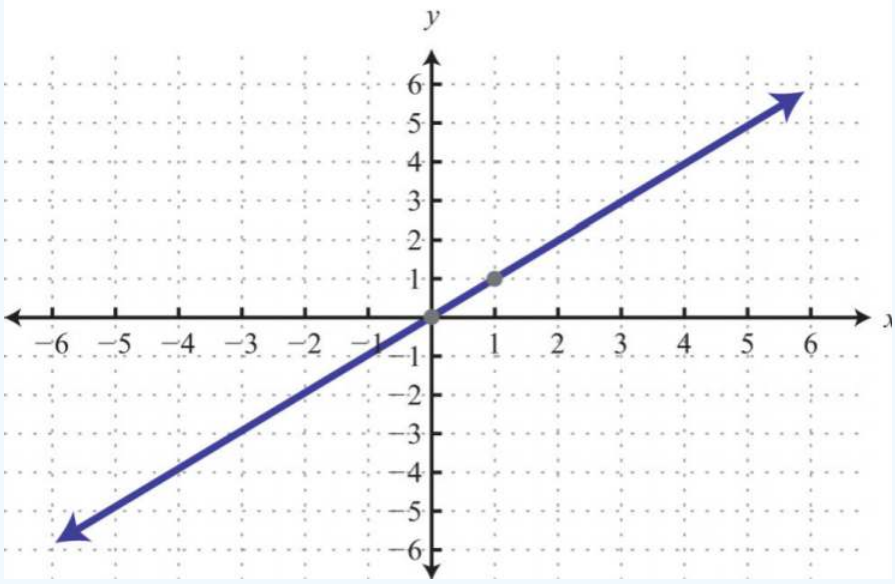


Figure 4.4.38

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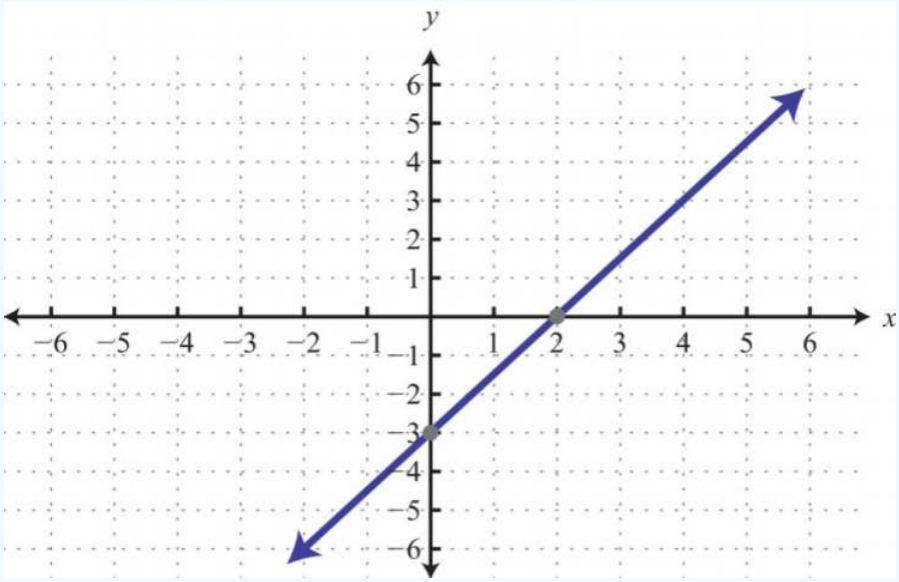


Figure 4.4.39

21.

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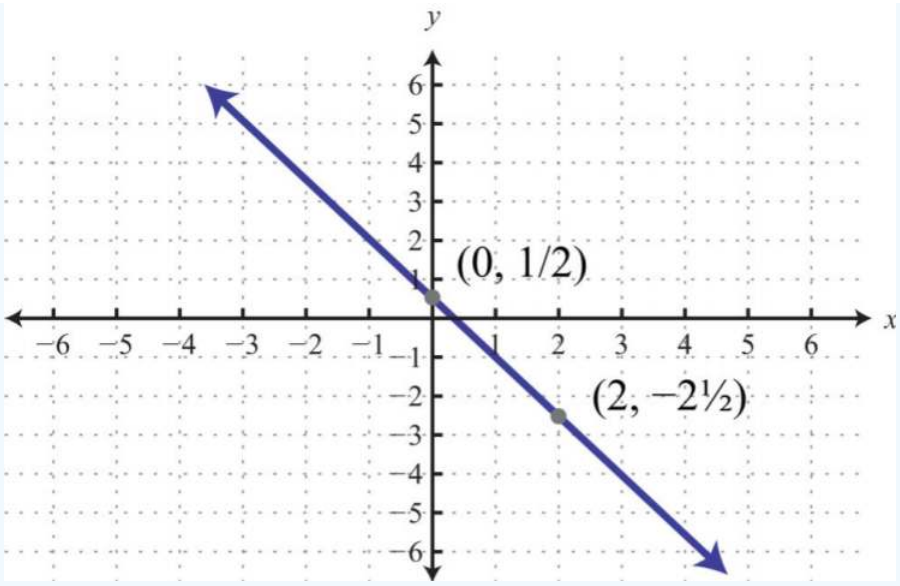


Figure 4.4.40

23.

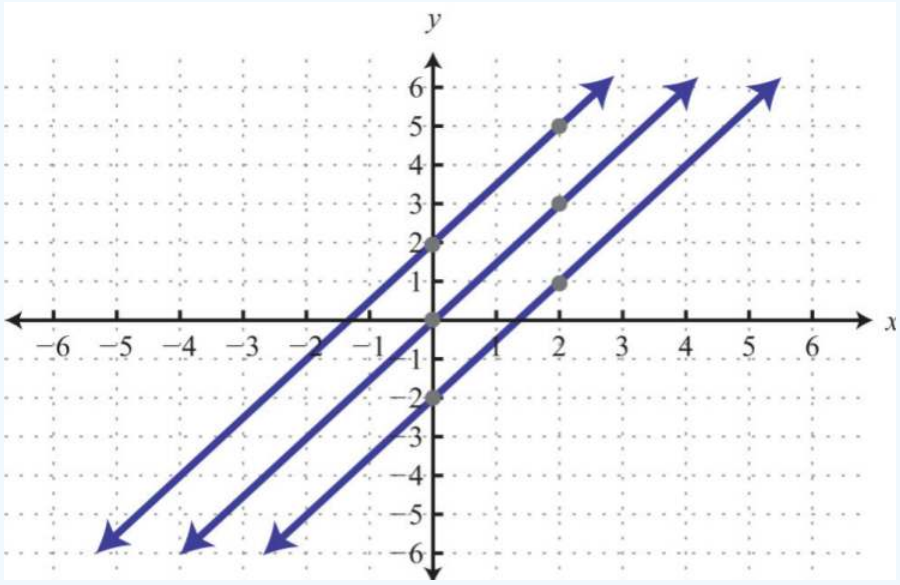


Figure 4.4.41

? Exercise 4.4.9 Discussion Board Topics

1. Name three methods for graphing lines. Discuss the pros and cons of each method.
2. Choose a linear equation and graph it three different ways. Scan the work and share it on the discussion board.
3. Why do we use the letter m for slope?
4. How are equivalent fractions useful when working with slopes?
5. Can we graph a line knowing only its slope?
6. Research and discuss the alternative notation for slope: $m = \frac{\Delta y}{\Delta x}$.
7. What strategies for graphing lines should be brought to an exam? Explain

Answer

1. Answers may vary

3. Answers may vary

5. Answers may vary

7. Answers may vary

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4.5: Finding Linear Equations

Learning Objectives

- Given a graph, identify the slope and y-intercept.
- Find the equation of the line using the slope and y-intercept.
- Find the equation of the line using point-slope form.

Finding Equations Using Slope-Intercept Form

Given the algebraic equation of a line, we are able to graph it in a number of ways. In this section, we will be given a geometric description of a line and be asked to find the algebraic equation. Finding the equation of a line can be accomplished in a number of ways, the first of which makes use of slope-intercept form, $y = mx + b$. If we know the slope, m, and the y-intercept, $(0, b)$, we can construct the equation.

Example 4.5.1

Find the equation of a line with slope $m = -\frac{5}{8}$ and y-intercept $(0, 1)$.

Solution:

The given y-intercept implies that $b = 1$. Substitute the slope m and the y-value of the y-intercept b into the equation $y = mx + b$.

$$\begin{aligned} y &= m\ x + b \\ &\downarrow \quad \downarrow \\ y &= -\frac{5}{8}x + 1 \end{aligned}$$

Answer:

$$y = -\frac{5}{8}x + 1$$

Finding a linear equation is very straightforward if the slope and y-intercept are given. This is certainly not always the case; however, the example demonstrates that the algebraic equation of a line depends on these two pieces of information. If the graph is given, then we can often read it to determine the y-intercept and slope.

Example 4.5.2

Find the equation of the line given the graph:

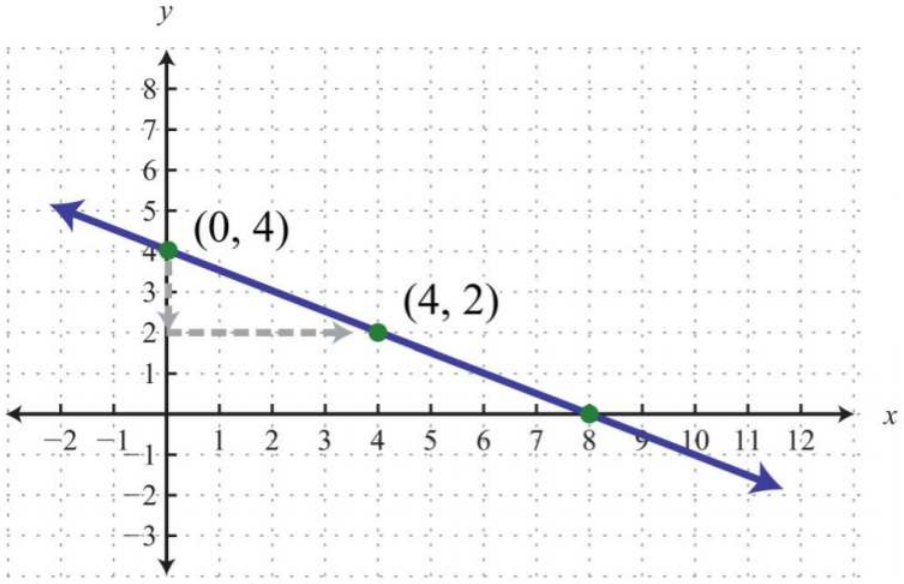


***Figure* 4.5.1**

Solution:

By reading the graph, we can see that the y-intercept is $(0, 4)$, and thus

$$b = 4$$

Furthermore, from the points $(0, 4)$ to $(4, 2)$, we can see that the rise is -2 units and the run is 4 units.

$$m = \frac{rise}{run} = \frac{-2}{4} = -\frac{1}{2}$$

Now substitute m and b into slope-intercept form:

$$y = mx+b$$
$$y = -\frac{1}{2}x+4$$

Answer:

$$y = -\tfrac{1}{2}x + 4$$

Often the y-intercept and slope will not be given or are not easily discernible from the graph. For this reason, we will develop some algebraic techniques that allow us to calculate these quantities.

✔ Example 4.5.3

Find the equation of the line with slope $m = -\frac{2}{3}$ passing through $(-6, 3)$.

Solution:

Begin by substituting the given slope into slope-intercept form.

$$y = mx + b$$
$$y = -\frac{2}{3}x + b$$

For the ordered pair $(-6, 3)$ to be a solution, it must solve the equation. Therefore, we can use it to find b. Substitute the appropriate x- and y-values as follows:

$$y = -\frac{2}{3}x + b$$
$$\downarrow \quad \downarrow$$
$$(3) = -\frac{2}{3}(-6) + b$$

After substituting the appropriate values, solve for the only remaining variable, b.

$$\begin{aligned} 3 &= -\frac{2}{3}(-6) + b \\ 3 &= -2(-2) + b \\ 3 &= 4 + b \\ -1 &= b \end{aligned}$$

Once we have b, we can then complete the equation:

$$y = mx + b$$
$$y = -\frac{2}{3}x - 1$$

As a check, verify that $(-6, 3)$ solves this linear equation as follows:

Answer:

$y = -\frac{2}{3}x - 1$

Example 4.5.4

Find the equation of the line given the graph:

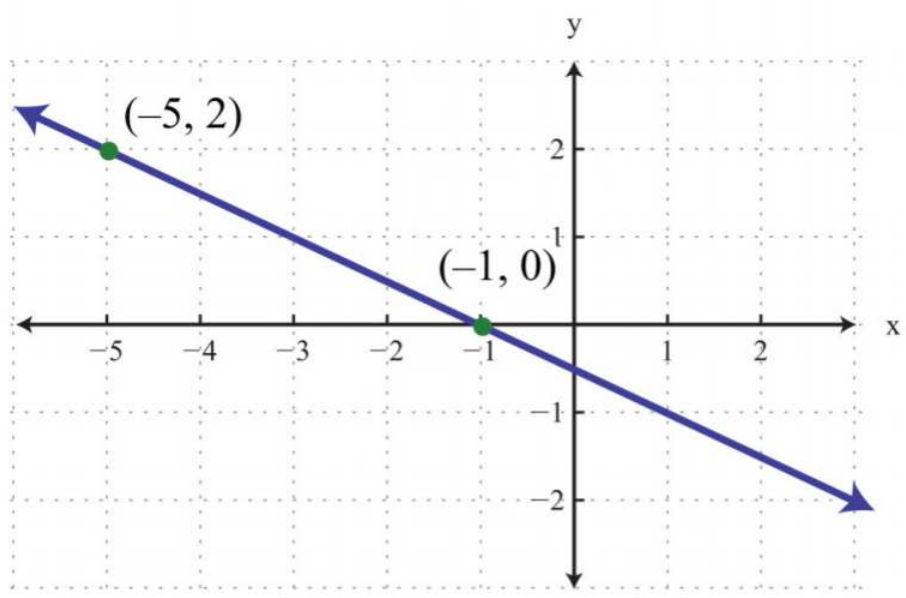


***Figure* 4.5.2**

Solution:

Use the graph to determine the slope. From the points $(-5, 2)$ to $(-1, 0)$, we can see that the rise between the points is -2 units and the run is 4 units. Therefore, we calculate the slope as follows:

$$m = \frac{rise}{run} = \frac{-2}{4} = -\frac{1}{2}$$

Substitute the slope into slope-intercept form.

$$y = mx + b$$
$$y = -\frac{1}{2}x + b$$

Now substitute the coordinates of one of the given points to find b. It does not matter which one you choose. Here choose $(-1, 0)$:

$$y = -\frac{1}{2}x + b$$
$$0 = -\frac{1}{2}(-1) + b$$
$$0 = \frac{1}{2} + b$$
$$-\frac{1}{2} = b$$

Next, put it all together.

$$y = mx + b$$
$$y = -\frac{1}{2}x - \frac{1}{2}$$

Answer:

$$y = -\tfrac{1}{2}x - \tfrac{1}{2}$$

As an exercise, substitute the coordinates of the point $(-5, 2)$ to see that b will turn out to be the same value. In fact, you can substitute any ordered pair solution of the line to find b. We next outline an algebraic technique for finding the equation of a nonvertical line passing through two given points.

Example 4.5.5

Find the equation of the line passing through $(-4, -2)$ and $(1, 3)$.

Solution:

When finding a linear equation using slope-intercept form $y = mx + b$, the goal is to find m and then b.

Step 1: Find the slope m. In this case, given two points, use the slope formula.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{3 - (-2)}{1 - (-4)} \\ &= \frac{3 + 2}{1 + 4} \\ &= \frac{5}{5} \\ &= 1 \end{aligned}$$

Substitute $m = 1$ into slope-intercept form.

$$y = mx + b$$
$$y = 1x + b$$

Step 2: Find b. To do this, substitute the coordinates of any given ordered pair solution. Use $(1, 3)$:

$$y = 1x + b$$
$$3 = 1(1) + b$$
$$3 = 1 + b$$
$$2 = b$$

Step 3: Finish building the equation by substituting in the value for b. In this case, we use $b = 2$.

$$y = 1x + b$$
$$y = 1x + 2$$

Answer:

$$y = x + 2$$

These three steps outline the process for finding the equation of any nonvertical line in slope-intercept form. This is a completely algebraic method, but always keep in mind the geometry behind the technique.

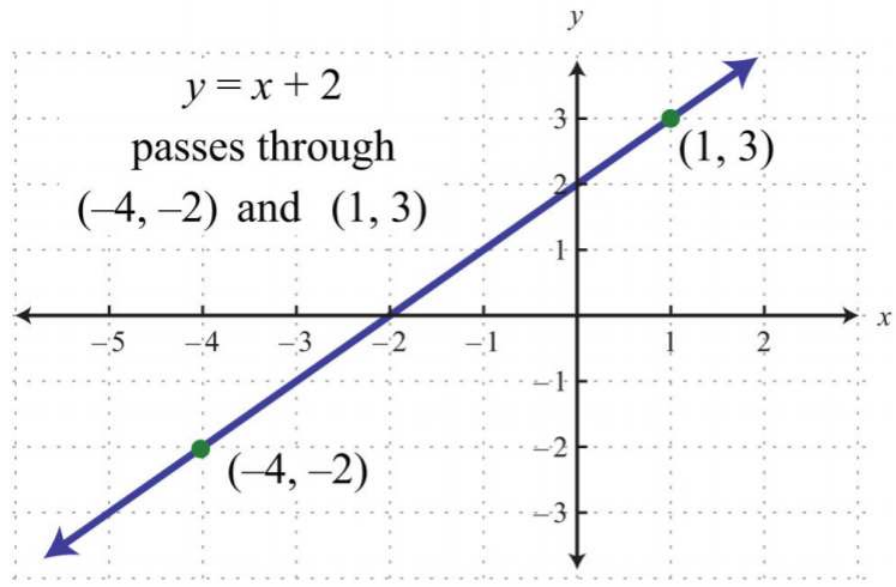


***Figure* 4.5.3**

Note that the line has a y-intercept at $(0, 2)$, with slope $m = 1$.

Example 4.5.6

Find the equation of the line passing through $(-1, 3)$ and $(5, 1)$.

Solution:

First, find m, the slope. Given two points, use the slope formula as follows:

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{1 - (3)}{5 - (-1)} \\ &= \frac{1 - 3}{5 + 1} \\ &= \frac{-2}{6} \\ &= -\frac{1}{3} \end{aligned}$$

Substitute $m = -\frac{1}{3}$ into slope-intercept form.

$$y = mx + b$$
$$y = -\frac{1}{3}x + b$$

Next, find b. Substitute the coordinates of the point $(-1, 3)$.

$$y = -\frac{1}{3}x + b$$
$$3 = -\frac{1}{3}(-1) + b$$
$$3 = \frac{1}{3} + b$$
$$3 - \frac{1}{3} = b$$
$$\frac{3 \cdot 3}{1 \cdot 3} - \frac{1}{3} = b$$
$$\frac{8}{3} = b$$

Finally, substitute $b = \frac{8}{3}$ into the equation.

$$y = -\frac{1}{3}x + b$$
$$y = -\frac{1}{3}x + \frac{8}{3}$$

Answer:

$$y = -\tfrac{1}{3}x + \tfrac{8}{3}$$

? Exercise 4.5.1

Find the equation of the line passing through $(-3, 4)$ and $(6, -2)$.

Answer

$y = -\frac{2}{3}x + 2$

Finding Equations Using a Point and the Slope

Given any point on a line and its slope, we can find the equation of that line. Begin by applying the slope formula with a given point (x_1, y_1) and a variable point (x, y).

$$m = \frac{y - y_1}{x - x_1}$$
$$\frac{m}{1} = \frac{y - y_1}{x - x_1} \qquad \textit{Cross multiply.}$$
$$m(x - x_1) = y - y_1 \qquad \textit{Apply the symmetric property.}$$
$$y - y_1 = m(x - x_1)$$

The equation $y - y_1 = m(x - x_1)$ is called the point-slope form of a line. Any nonvertical linear equation can be written in this form. It is useful for finding the equation of a line given the slope and any ordered pair solution.

✓ Example 4.5.7

Find the equation of the line with slope $m = \frac{1}{2}$ passing through $(4, -1)$.

Solution:

Use point-slope form, where $m = \frac{1}{2}$ and $(x_1, y_1) = (4, -1)$.

At this point, we must choose to present the equation of our line in either standard form or slope-intercept form.

Standard form	*Slope − intercept form*
$y+1=\frac{1}{2}x-2$	$y+1=\frac{1}{2}x=2$
$y+1-1=\frac{1}{2}x-2-1$	$y+1-1=\frac{1}{2}x-2-1$
$y-\frac{1}{2}x=\frac{1}{2}x-3-\frac{1}{2}x$	$y=\frac{1}{2}x-3$
$-\frac{1}{2}x+y=-3$	

In this textbook, we will present our lines in slope-intercept form. This facilitates future graphing.

Answer:

$$y=\tfrac{1}{2}x-3$$

Example 4.5.8

Find the equation of the line passing through $(-5,3)$ with slope $m=-\frac{2}{5}$.

Solution:

Substitute $(-5,3)$ and $m=-\frac{2}{5}$ into point-slope form.

$$\begin{aligned} y-y_1 &= m(x-x_1) \\ y-(3) &= -\frac{2}{5}(x-(-5)) && \textit{Solve for } y. \\ y-3 &= -\frac{2}{5}(x+5) && \textit{Distribute } -\frac{2}{5}. \\ y-3 &= -\frac{2}{5}x-2 \\ y-3+3 &= -\frac{2}{5}x-2+3 \\ y &= -\frac{2}{5}x+1 \end{aligned}$$

Answer:

$$y=-\tfrac{2}{5}x+1$$

It is always important to understand what is occurring geometrically. Compare the answer for the last example to the corresponding graph below.

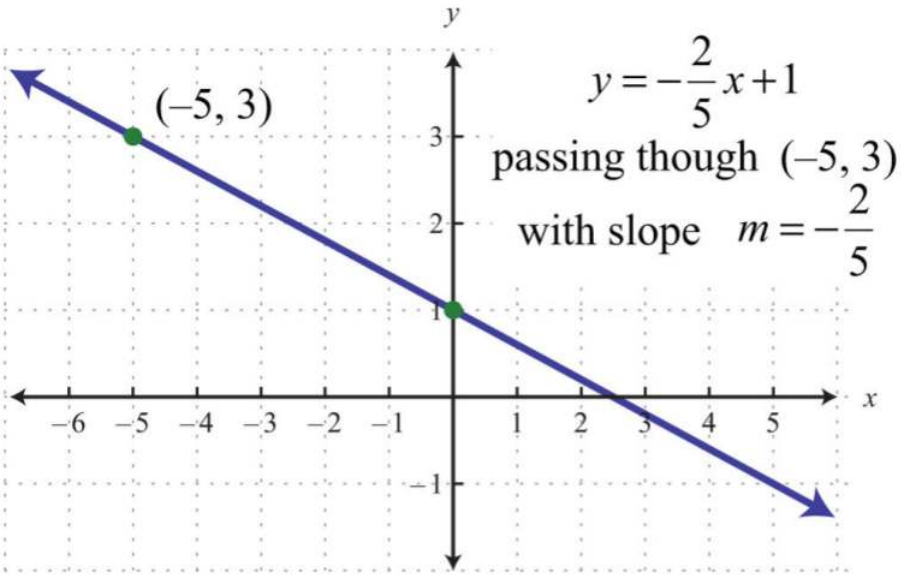


Figure **4.5.4**

The geometric understanding is important because you will often be given graphs from which you will need to determine a point on the line and the slope.

Example 4.5.9

Find an equation of the given graph:

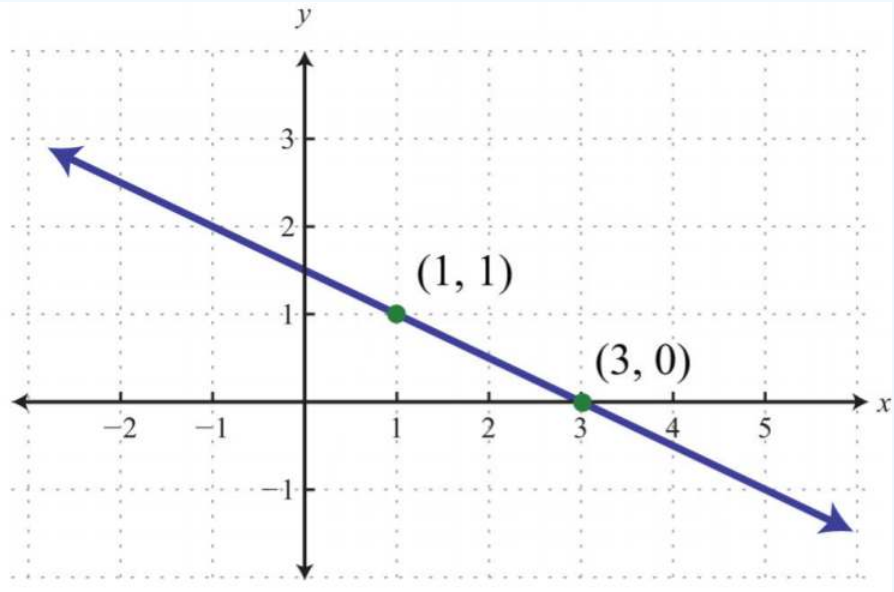


Figure **4.5.5**

Solution:

Between the points $(1,1)$ to $(3,0)$, we can see that the rise is -1 unit and the run is 2 units. The slope of the line is $m = \frac{rise}{run} = \frac{-1}{2} = -\frac{1}{2}$. Use this and the point $(3,0)$ to find the equation as follows:

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 0 &= -\frac{1}{2}(x-3) \\ y &= -\frac{1}{2}x + \frac{3}{2} \end{aligned}$$

Answer:

$$y = -\tfrac{1}{2}x + \tfrac{3}{2}$$

Example 4.5.10

Find the equation of the line passing through $(-1,1)$ and $(7,-1)$.

Solution:

Begin by calculating the slope using the slope formula.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-1-1}{7-(-1)} \\ &= \frac{-2}{7+1} \\ &= \frac{-2}{8} \\ &= -\frac{1}{4} \end{aligned}$$

Next, substitute into point-slope form using one of the given points; it does not matter which point is used. Use $m = -\frac{1}{4}$ and the point $(-1, 1)$.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y-1 &= -\frac{1}{4}(x-(-1)) \\ y - 1 &= -\frac{1}{4}(x+1) \\ y - 1 &= -\frac{1}{4}x - \frac{1}{4} \\ y &= -\frac{1}{4}x - \frac{1}{4} + 1 \\ y &= -\frac{1}{4}x + \frac{3}{4} \end{aligned}$$

Answer:

$$y = -\tfrac{1}{4}x + \tfrac{3}{4}$$

? Exercise 4.5.2

Find the equation of the line passing through $(4, -5)$ and $(-4, 1)$.

Answer

$y = -\frac{3}{4}x - 2$

Key Takeaways

- Given the graph of a line, you can determine the equation in two ways, using slope-intercept form, $y = mx + b$, or point-slope form, $y - y_1 = m(x - x_1)$.
- The slope and one point on the line is all that is needed to write the equation of a line.
- All nonvertical lines are completely determined by their y-intercept and slope.
- If the slope and y-intercept can be determined, then it is best to use slope-intercept form to write the equation.
- If the slope and a point on the line can be determined, then it is best to use point-slope form to write the equation.

? Exercise 4.5.3 Slope-Intercept Form

Determine the slope and y-intercept.

1. $5x - 3y = 18$
2. $-6x + 2y = 12$
3. $x - y = 5$
4. $-x + y = 0$
5. $4x - 5y = 15$
6. $-7x + 2y = 3$
7. $y = 3$
8. $y = -\frac{3}{4}$

9. $\frac{1}{5}x - \frac{1}{3}y = -1$
10. $\frac{5}{16}x + \frac{3}{8}y = 9$
11. $-\frac{2}{3}x + \frac{5}{2}y = \frac{5}{4}$
12. $\frac{1}{2}x - \frac{3}{4}y = -\frac{1}{2}$

Answer

1. $m = \frac{5}{3}$; $(0, -6)$
3. $m = 1$; $(0, -5)$
5. $m = \frac{4}{5}$; $(0, -3)$
7. $m = 0$; $(0, 3)$
9. $m = \frac{3}{5}$; $(0, 3)$
11. $m = \frac{4}{15}$; $(0, \frac{1}{2})$

? Exercise 4.5.4 Finding Equations in Slope-Intercept Form

Given the slope and y-intercept, determine the equation of the line.

1. $m = \frac{1}{2}$; $(0, 5)$
2. $m = 4$; $(0, -1)$
3. $m = -\frac{2}{3}$; $(0, -4)$
4. $m = -3$; $(0, 9)$
5. $m = 0$; $(0, -1)$
6. $m = 5$; $(0, 0)$

Answer

1. $y = \frac{1}{2}x + 5$
3. $y = -\frac{2}{3}x - 4$
5. $y = -1$

? Exercise 4.5.5 Finding Equations in Slope-Intercept Form

Given the graph, find the equation in slope-intercept form.

1.

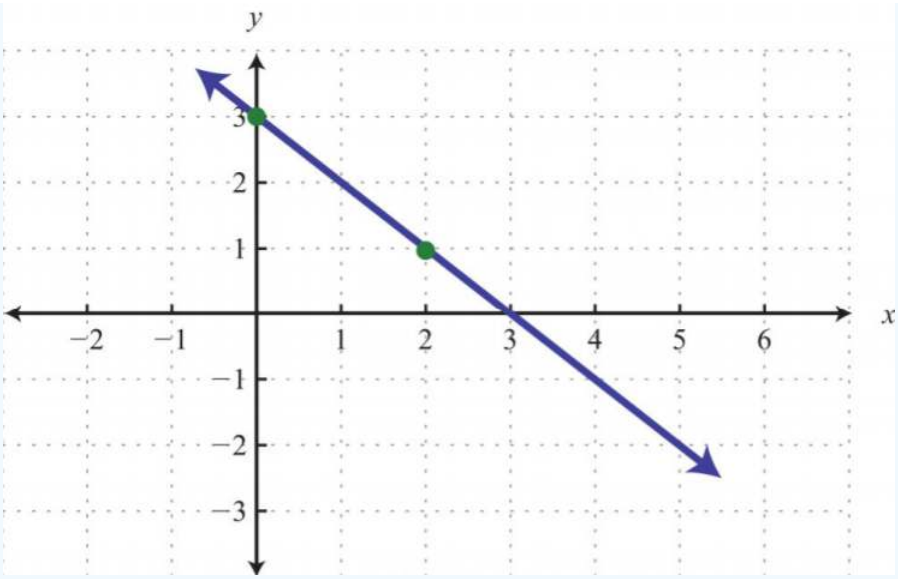


Figure **4.5.6**

2.

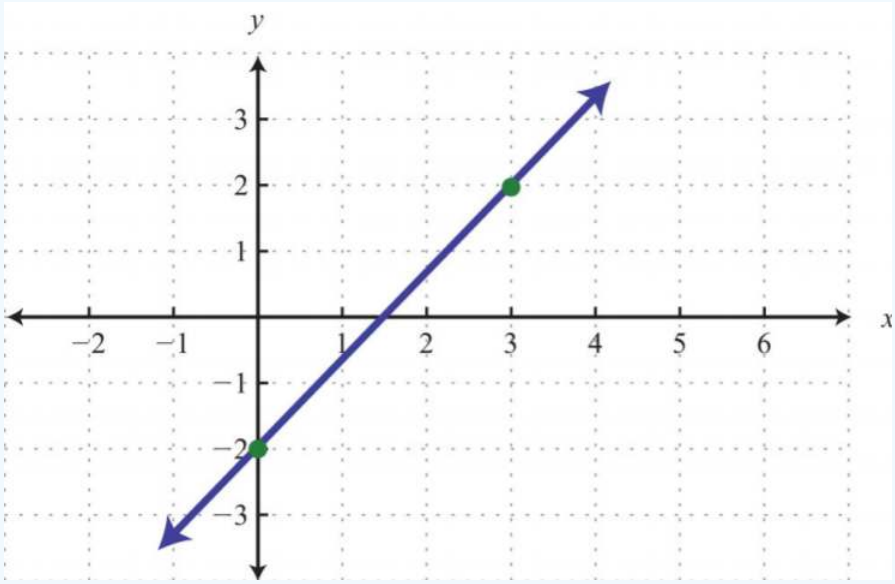


Figure **4.5.7**

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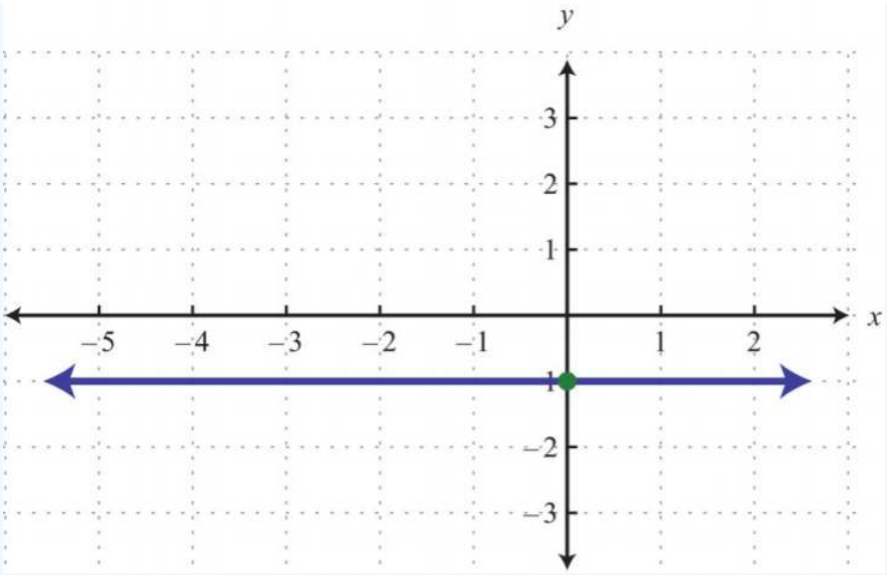


Figure **4.5.8**

4.

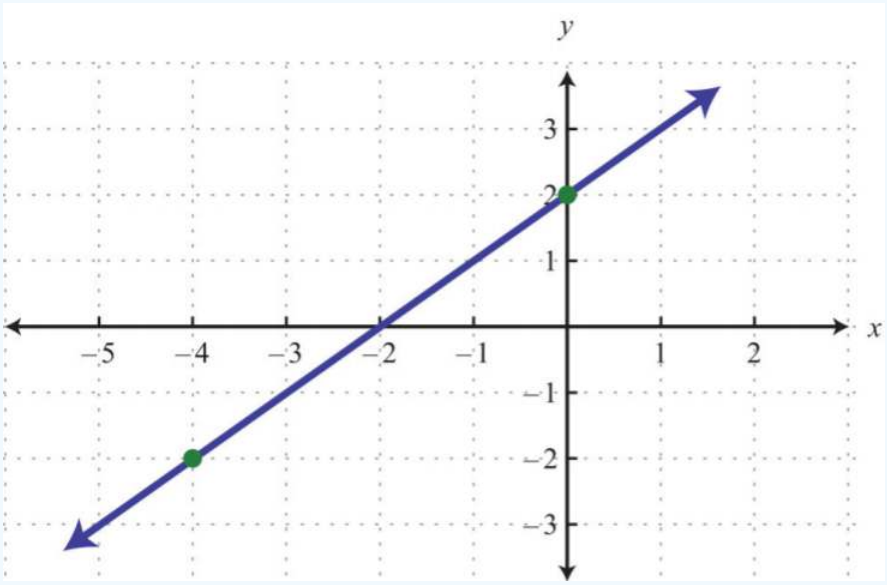


Figure **4.5.9**

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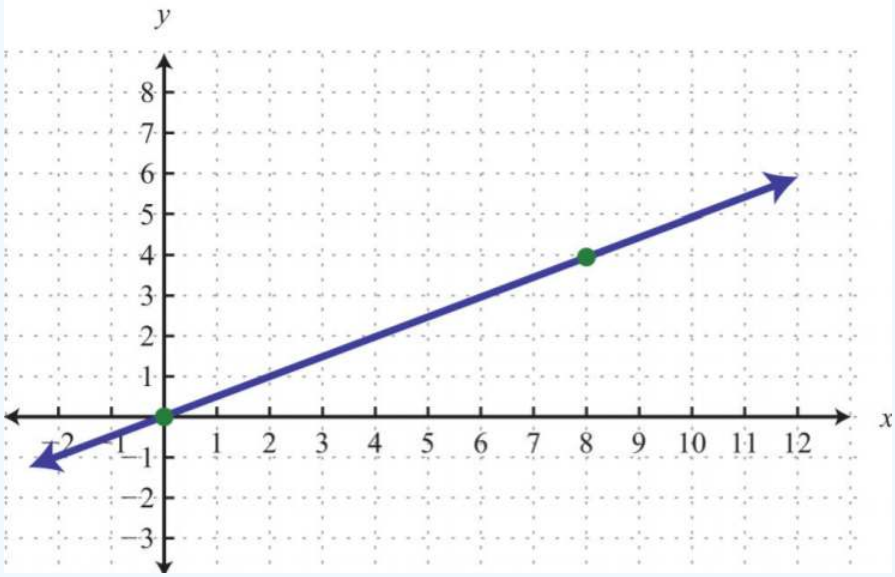


Figure **4.5.10**

6.

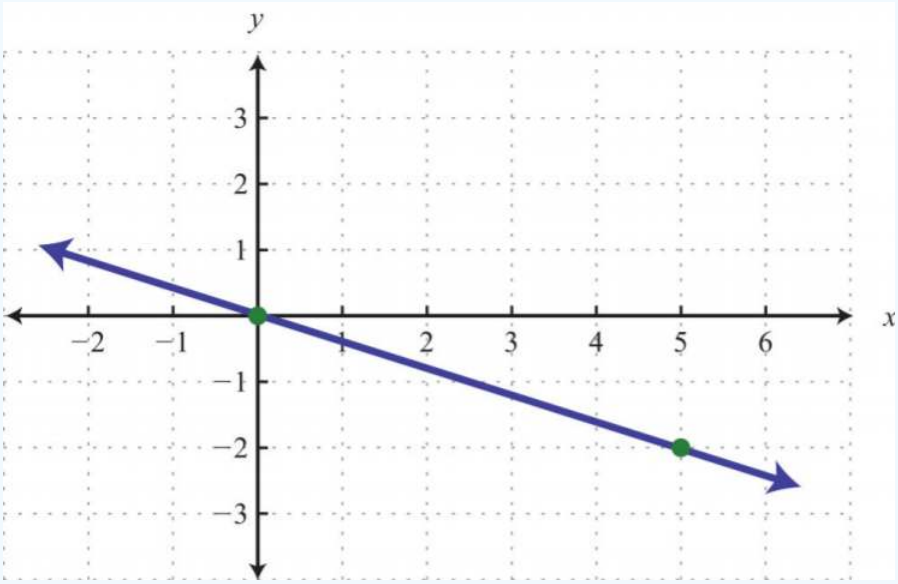


Figure **4.5.11**

Answer

1. $y = -x + 3$
3. $y = -1$
5. $y = \frac{1}{2}x$

? Exercise 4.5.6 Finding Equations in Slope-Intercept Form

Find the equation, given the slope and a point.

1. $m = \frac{2}{3}$; $(-9, 2)$
2. $m = -\frac{1}{5}$; $(5, -5)$
3. $m = 0$; $(-4, 3)$

4. $m = 3$; $(-2, 1)$
5. $m = -5$; $(-2, 8)$
6. $m = -4$; $(\frac{1}{2}, -\frac{3}{2})$
7. $m = -\frac{1}{2}$; $(3, 2)$
8. $m = \frac{3}{4}$; $(\frac{1}{3}, \frac{5}{4})$
9. $m = 0$; $(3, 0)$
10. m undefined; $(3, 0)$

Answer

1. $y = \frac{2}{3}x + 8$
3. $y = 3$
5. $y = -5x - 2$
7. $y = -\frac{1}{2}x + \frac{7}{2}$
9. $y = 0$

Exercise 4.5.7 Finding Equations in Slope-Intercept Form

Given two points, find the equation of the line.

1. $(-6, 6), (2, 2)$
2. $(-10, -3), (5, 0)$
3. $(0, \frac{1}{2}), (\frac{1}{2}, -1)$
4. $(\frac{1}{3}, \frac{1}{3}), (\frac{2}{3}, 1)$
5. $(3, -4), (-6, -7)$
6. $(-5, 2), (3, 2)$
7. $(-6, 4), (-6, -3)$
8. $(-4, -4), (-1, -1)$
9. $(3, -3), (-5, 5)$
10. $(0, 8), (-4, 0)$

Answer

1. $y = -\frac{1}{2}x + 3$
3. $y = -3x + \frac{1}{2}$
5. $y = \frac{1}{3}x - 5$
7. $x = -6$
9. $y = -x$

Exercise 4.5.8 Equations Using Point-Slope Form

Find the equation, given the slope and a point.

1. $m = \frac{1}{2}$; $(4, 3)$
2. $m = -\frac{1}{3}$; $(9, -2)$
3. $m = 6$; $(1, -5)$
4. $m = -10$; $(1, -20)$
5. $m = -3$; $(2, 3)$
6. $m = \frac{2}{3}$; $(-3, -5)$
7. $m = -\frac{3}{4}$; $(-8, 3)$

8. $m = 5; (\frac{1}{5}, -3)$
9. $m = -3; (-\frac{1}{9}, 2)$
10. $m = 0; (4, -6)$
11. $m = 0; (-5, 10)$
12. $m = \frac{5}{8}; (4, 3)$
13. $m = -\frac{3}{5}; (-2, -1)$
14. $m = \frac{1}{4}; (12, -2)$
15. $m = 1; (0, 0)$
16. $m = -\frac{3}{4}; (0, 0)$

Answer

1. $y = \frac{1}{2}x + 1$
3. $y = 6x - 11$
5. $y = -3x + 9$
7. $y = -\frac{3}{4}x - 3$
9. $y = -3x + \frac{5}{3}$
11. $y = 10$
13. $y = -\frac{3}{5}x - \frac{1}{15}$
15. $y = x$

Exercise 4.5.9 Equations Using Point-Slope Form

Given the graph, use the point-slope formula to find the equation.

1.

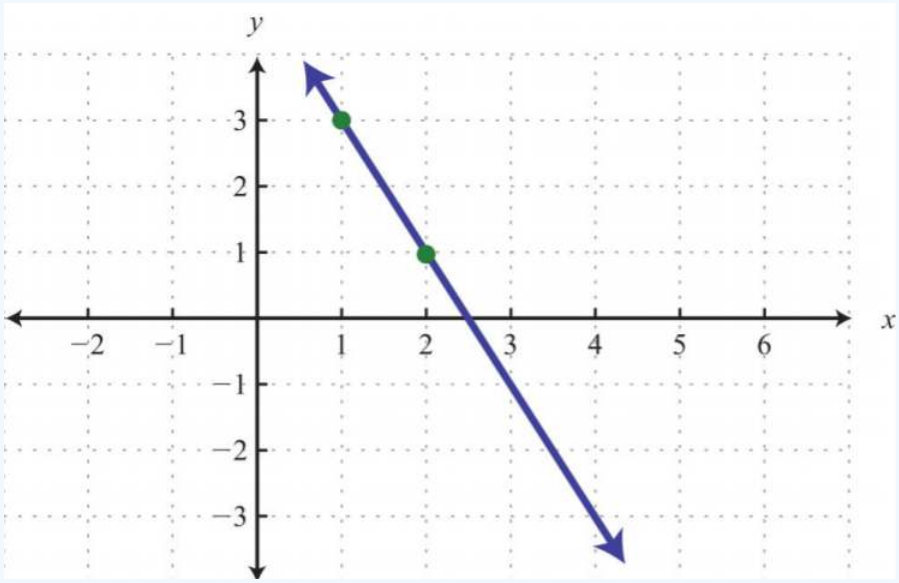


***Figure* 4.5.12**

2.

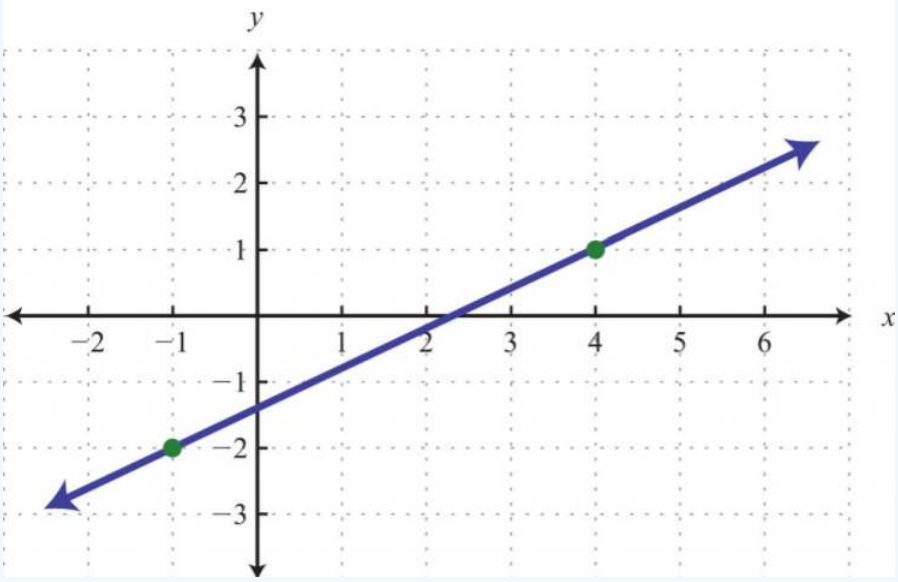


Figure **4.5.13**

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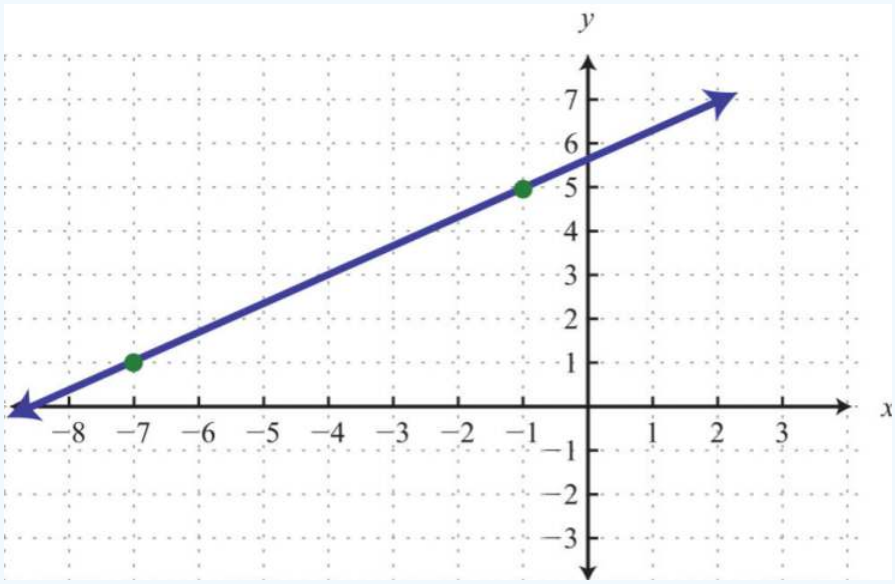


Figure **4.5.14**

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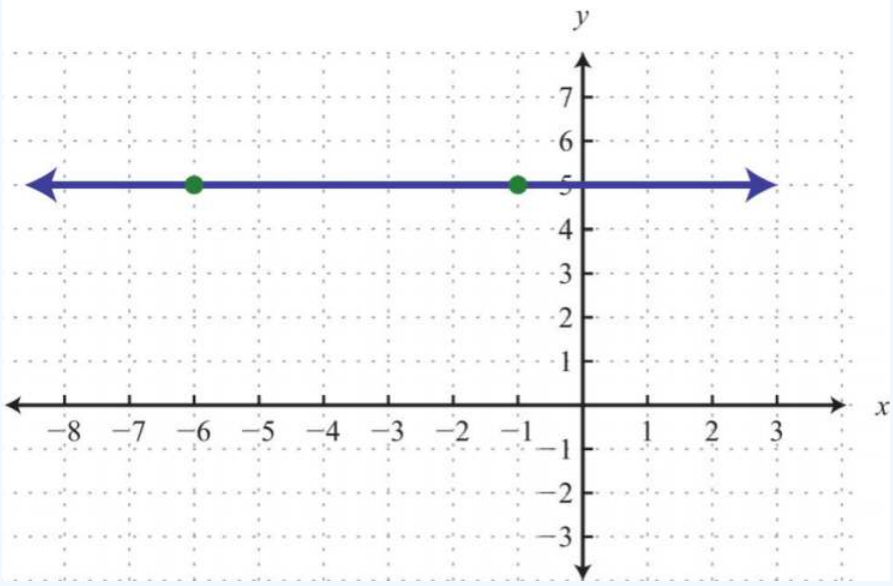


Figure **4.5.15**

5.

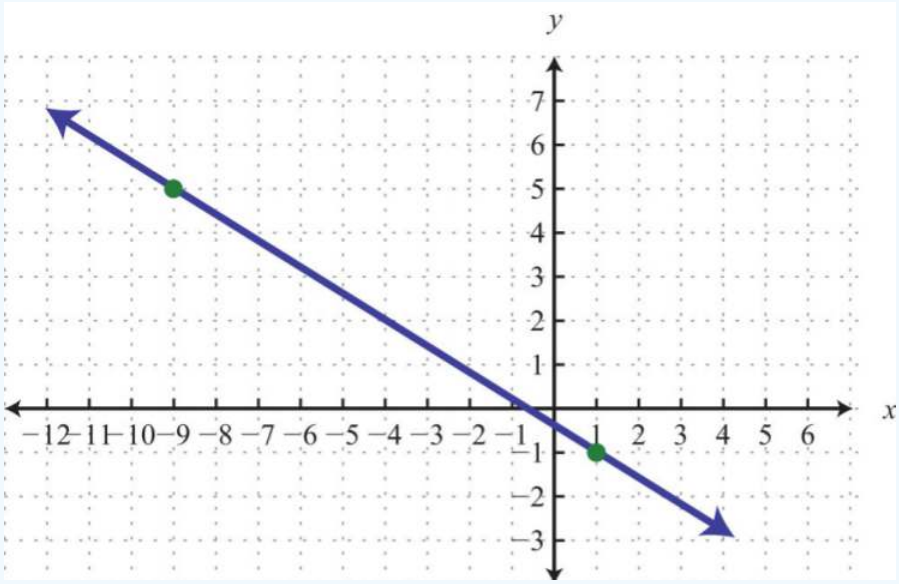


Figure **4.5.16**

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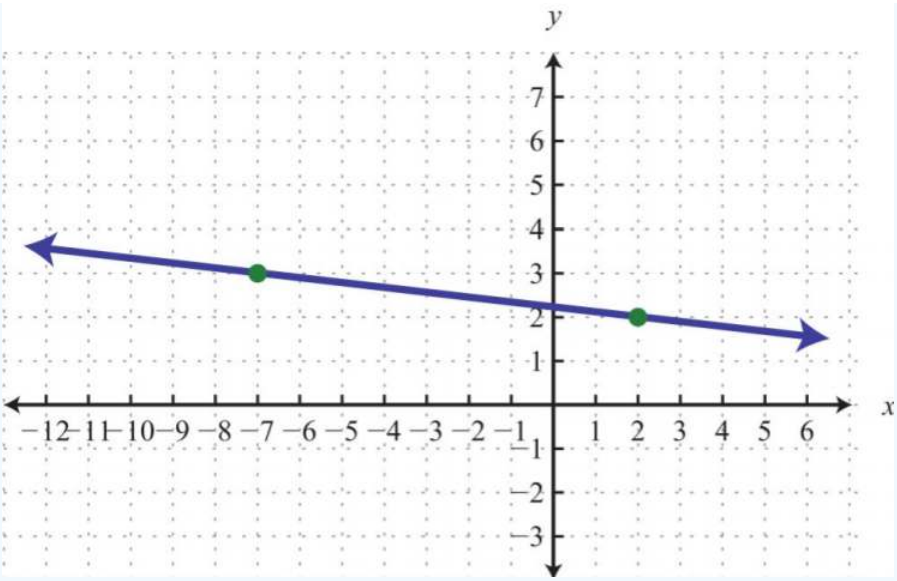


***Figure* 4.5.17**

Answer

1. $y = -2x + 5$
3. $y = \frac{2}{3}x + \frac{17}{3}$
5. $y = -\frac{3}{5}x - \frac{2}{5}$

? Exercise 4.5.10 Equations Using Point-Slope Form

Use the point-slope formula to find the equation of the line passing through the two points.

1. $(-4, 0), (0, 5)$
2. $(-1, 2), (0, 3)$
3. $(-3, -2), (3, 2)$
4. $(3, -1), (2, -3)$
5. $(-2, 4), (2, -4)$
6. $(-5, -2), (5, 2)$
7. $(-3, -1), (3, 3)$
8. $(1, 5), (0, 5)$
9. $(1, 2), (2, 4)$
10. $(6, 3), (2, -3)$
11. $(10, -3), (5, -4)$
12. $(-3, 3), (-1, 12)$
13. $(\frac{4}{5}, -\frac{1}{3}), (-\frac{1}{5}, \frac{2}{3})$
14. $(\frac{5}{3}, \frac{1}{3}), (-\frac{10}{3}, -\frac{5}{3})$
15. $(3, -\frac{1}{4}), (4, -\frac{1}{2})$
16. $(0, 0), (-5, 1)$
17. $(2, -4), (0, 0)$
18. $(3, 5), (3, -2)$
19. $(-4, 7), (-1, 7)$
20. $(-8, 0), (6, 0)$

Answer

1. $y = \frac{5}{4}x + 5$

3. $y = \frac{2}{3}x$
5. $y = -2x$
7. $y = \frac{2}{3}x + 1$
9. $y = 2x$
11. $y = \frac{1}{5}x - 5$
13. $y = -x + \frac{7}{15}$
15. $y = -\frac{1}{4}x + \frac{1}{2}$
17. $y = -2x$
19. $y = 7$

Exercise 4.5.11 Applications

1. Joe has been keeping track of his cellular phone bills for the last two months. The bill for the first month was $\$38.00$ for 100 minutes of usage. The bill for the second month was $\$45.50$ for 150 minutes of usage. Find a linear equation that gives the total monthly bill based on the minutes of usage.
2. A company in its first year of business produced 150 training manuals for a total cost of $\$2,350$. The following year, the company produced 50 more manuals at a cost of $\$1,450$. Use this information to find a linear equation that gives the total cost of producing training manuals from the number of manuals produced.
3. A corn farmer in California was able to produce 154 bushels of corn per acre 2 years after starting his operation. Currently, after 7 years of operation, he has increased his yield to 164 bushels per acre. Use this information to write a linear equation that gives the total yield per acre based on the number of years of operation, and use it to predict the yield for next year.
4. A Webmaster has noticed that the number of registered users has been steadily increasing since beginning an advertising campaign. Before starting to advertise, he had $1,200$ registered users, and after 3 months of advertising he now has $1,590$ registered users. Use this data to write a linear equation that gives the total number of registered users, given the number of months after starting to advertise. Use the equation to predict the number of users 7 months into the advertising campaign.
5. A car purchased new cost $\$22,000$ and was sold 10 years later for $\$7,000$. Write a linear equation that gives the value of the car in terms of its age in years.
6. An antique clock was purchased in 1985 for $\$1,500$ and sold at auction in 1997 for $\$5,700$. Determine a linear equation that models the value of the clock in terms of years since 1985.

Answer

1. cost$= 0.15x + 23$
3. yield$= 2x + 150$; 166 bushels
5. value$= -1,500x + 22,000$

Exercise 4.5.12 Discussion Board Topics

1. Discuss the merits and drawbacks of point-slope form and y-intercept form.
2. Research and discuss linear depreciation. In a linear depreciation model, what do the slope and y-intercept represent?

Answer

1. Answers may vary

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4.6: Parallel and Perpendicular Lines

Learning Objectives

- Determine the slopes of parallel and perpendicular lines.
- Find equations of parallel and perpendicular lines

Definition of Parallel and Perpendicular

Parallel lines are lines in the same plane that never intersect. Two nonvertical lines in the same plane, with slopes m_1 and m_2, are parallel if their slopes are the same, $m_1 = m_2$. Consider the following two lines:

Consider their corresponding graphs:

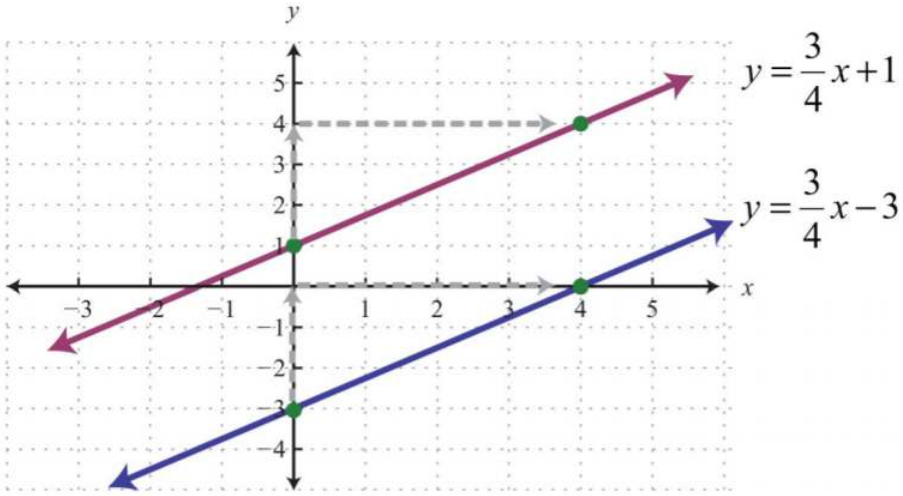


***Figure* 4.6.1**

Both lines have a slope $m = \frac{3}{4}$ and thus are parallel.

Perpendicular lines are lines in the same plane that intersect at right angles (90 degrees). Two nonvertical lines in the same plane, with slopes m_1 and m_2, are perpendicular if the product of their slopes is $-1 : m1 \cdot m2 = -1$. We can solve for m_1 and obtain $m_1 = \frac{-1}{m_2}$. In this form, we see that perpendicular lines have slopes that are **negative reciprocals**, or opposite reciprocals. For example, if given a slope

$$m = -\frac{5}{8}$$

then the slope of a perpendicular line is the opposite reciprocal:

$$m_\perp = \frac{8}{5}$$

The mathematical notation $m_\perp$ reads "m perpendicular." We can verify that two slopes produce perpendicular lines if their product is -1.

$$m \cdot m_\perp = -\frac{5}{8} \cdot \frac{8}{5} = -\frac{40}{40} = -1 \quad \checkmark$$

Geometrically, we note that if a line has a positive slope, then any perpendicular line will have a negative slope. Furthermore, the rise and run between two perpendicular lines are interchanged.

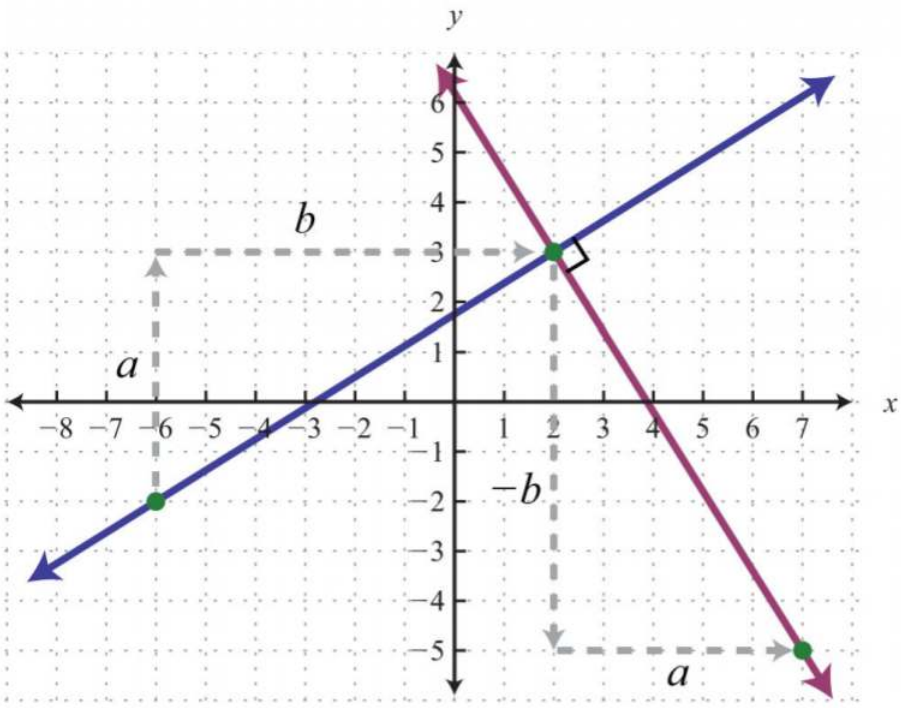


***Figure* 4.6.2**

Perpendicular lines have slopes that are opposite reciprocals, so remember to find the reciprocal and change the sign. In other words,

$$\text{If } m = \frac{a}{b}\text{, then } m_{\perp} = -\frac{b}{a}$$

Determining the slope of a perpendicular line can be performed mentally. Some examples follow

Table 4.6.1

Given slope	Slope of perpendicular line
$m = \frac{1}{2}$	$m_{\perp} = -2$
$m = -\frac{3}{4}$	$m_{\perp} = \frac{4}{3}$
$m = 3$	$m_{\perp} = -\frac{1}{3}$
$m = -4$	$m_{\perp} = \frac{1}{4}$

Example 4.6.1

Determine the slope of a line parallel to $y = -5x + 3$.

Solution:

Since the given line is in slope-intercept form, we can see that its slope is $m = -5$. Thus the slope of any line parallel to the given line must be the same, $m_{\parallel} = -5$. The mathematical notation $m_{\parallel}$ reads "m parallel."

Answer:

$$m_{\parallel} = -5$$

Example 4.6.2

Determine the slope of a line perpendicular to $3x - 7y = 21$.

Solution:

First, solve for y and express the line in slope-intercept form.

In this form, we can see that the slope of the given line is $m = \frac{3}{7}$, and thus $m_{\perp} = -\frac{7}{3}$.

Answer:

$m_{\perp} = -\frac{7}{3}$

Exercise 4.6.1

Find the slope of the line perpendicular to $15x + 5y = 20$.

Answer

$m_{\perp} = \frac{1}{3}$

Finding Equations of Parallel and Perpendicular Lines

We have seen that the graph of a line is completely determined by two points or one point and its slope. Often you will be asked to find the equation of a line given some geometric relationship—for instance, whether the line is parallel or perpendicular to another line.

Example 4.6.3

Find the equation of the line passing through $(6, -1)$ and parallel to $y = \frac{1}{2}x + 2$

Solution

Here the given line has slope $m = \frac{1}{2}$, and the slope of a line parallel is $m_{\parallel} = \frac{1}{2}$. Since you are given a point and the slope, use the point-slope form of a line to determine the equation.

Point	*Slope*
$(6, -1)$	$m_{\parallel} = \frac{1}{2}$

Answer:

$y = \frac{1}{2}x - 4$

It is important to have a geometric understanding of this question. We were asked to find the equation of a line parallel to another line passing through a certain point.

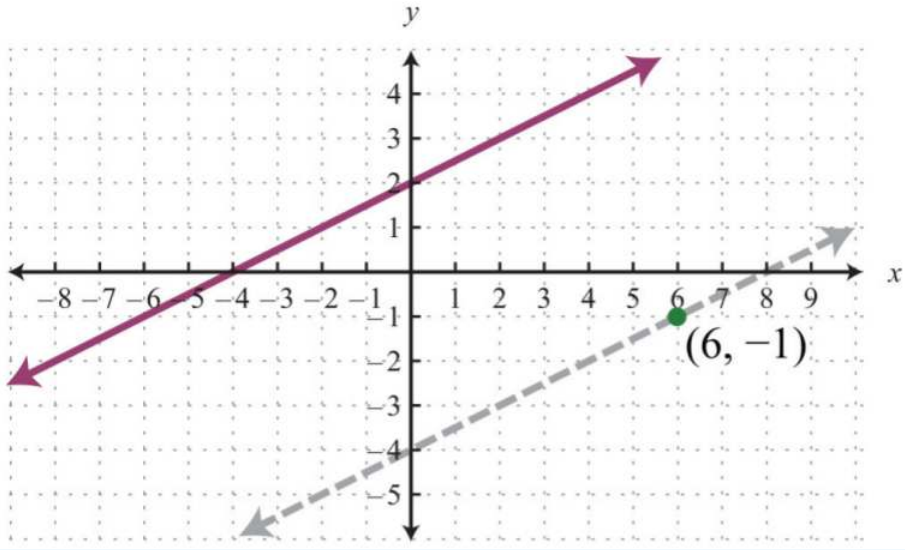


***Figure* 4.6.3**

Through the point $(6,-1)$ we found a parallel line, $y=\frac{1}{2}x-4$, shown dashed. Notice that the slope is the same as the given line, but the y-intercept is different. If we keep in mind the geometric interpretation, then it will be easier to remember the process needed to solve the problem.

Example 4.6.4

Find the equation of the line passing through $(-1,-5)$ and perpendicular to $y=-\frac{1}{4}x+2$.

Solution:

The given line has slope $m=-\frac{1}{4}$, and thus $m_{\perp}=+\frac{4}{1}=4$. Substitute this slope and the given point into point-slope form.

Point	*Slope*
$(-1,-5)$	$m_{\perp}=4$

Answer:

$y=4x-1$

Geometrically, we see that the line $y=4x-1$, shown dashed below, passes through $(-1,-5)$ and is perpendicular to the given line.

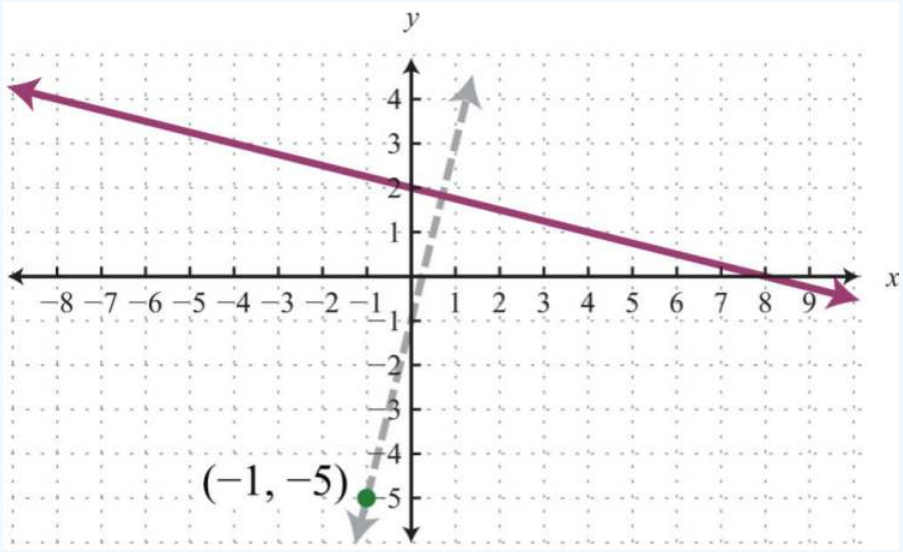


Figure **4.6.4**

It is not always the case that the given line is in slope-intercept form. Often you have to perform additional steps to determine the slope. The general steps for finding the equation of a line are outlined in the following example.

Example 4.6.5

Find the equation of the line passing through $(8,-2)$ and perpendicular to $6x+3y=1$.

Solution:

Step 1: Find the slope m. First, find the slope of the given line. To do this, solve for y to change standard form to slope-intercept form, $y=mx+b$.

$$\begin{aligned} 6x+3y &= 1 \\ 6x+3y-6x &= 1-6x \\ 3y &= -6x+1 \\ \frac{3y}{3} &= \frac{-6x+1}{3} \\ y &= \frac{-6x}{3}+\frac{1}{3} \\ y &= -2x+\frac{1}{3} \end{aligned}$$

In this form, you can see that the slope is $m=-2=-\frac{2}{1}$, and thus $m_{\perp}=\frac{-1}{-2}=+\frac{1}{2}$.

Step 2: Substitute the slope you found and the given point into the point-slope form of an equation for a line. In this case, the slope is $m_{\perp}=\frac{1}{2}$ and the given point is $(8,-2)$.

$$\begin{aligned} y-y_1 &= m(x-x_1) \\ y-(-2) &= \frac{1}{2}(x-8) \end{aligned}$$

Step 3: Solve for y.

Answer:

$y=\frac{1}{2}x-6$

Example 4.6.6

Find the equation of the line passing through $(\frac{7}{2},1)$ and parallel to $2x+14y=7$.

Solution:

Find the slope m by solving for y.

$$\begin{aligned} 2x+14y &= 7 \\ 2x+14y-2x &= 7-2x \\ 14y &= -2x+7 \\ \frac{14y}{14} &= \frac{-2x+7}{14} \\ y &= \frac{-2x}{14}+\frac{7}{14} \\ y &= -\frac{1}{7}x+\frac{1}{2} \end{aligned}$$

The given line has the slope $m=-\frac{1}{7}$, and so $m_{\parallel}=-\frac{1}{7}$. We use this and the point $(\frac{7}{2},1)$ in point-slope form.

$$\begin{aligned} y-y_1 &= m(x-x_1) \\ y-1 &= -\frac{1}{7}\left(x-\frac{7}{2}\right) \\ y-1 &= -\frac{1}{7}x+\frac{1}{2} \\ y-1+1 &= -\frac{1}{7}x+\frac{1}{2}+1 \\ y &= -\frac{1}{7}x+\frac{1}{2}+\frac{2}{2} \\ y &= -\frac{1}{7}x+\frac{3}{2} \end{aligned}$$

Answer:

$$y = -\tfrac{1}{7}x+\tfrac{3}{2}$$

Exercise 4.6.2

Find the equation of the line perpendicular to $x-3y=9$ and passing through $(-\frac{1}{2},2)$.

Answer

$y=-3x+\frac{1}{2}$

When finding an equation of a line perpendicular to a horizontal or vertical line, it is best to consider the geometric interpretation.

Example 4.6.7

Find the equation of the line passing through $(-3,-2)$ and perpendicular to $y=4$.

Solution:

We recognize that $y=4$ is a horizontal line and we want to find a perpendicular line passing through $(-3,-2)$.

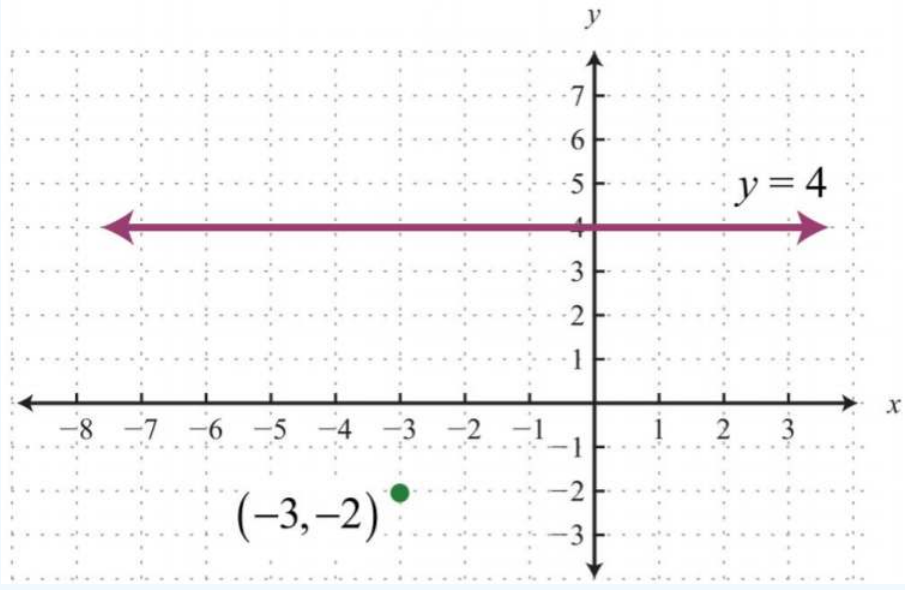


***Figure* 4.6.5**

If we draw the line perpendicular to the given horizontal line, the result is a vertical line.

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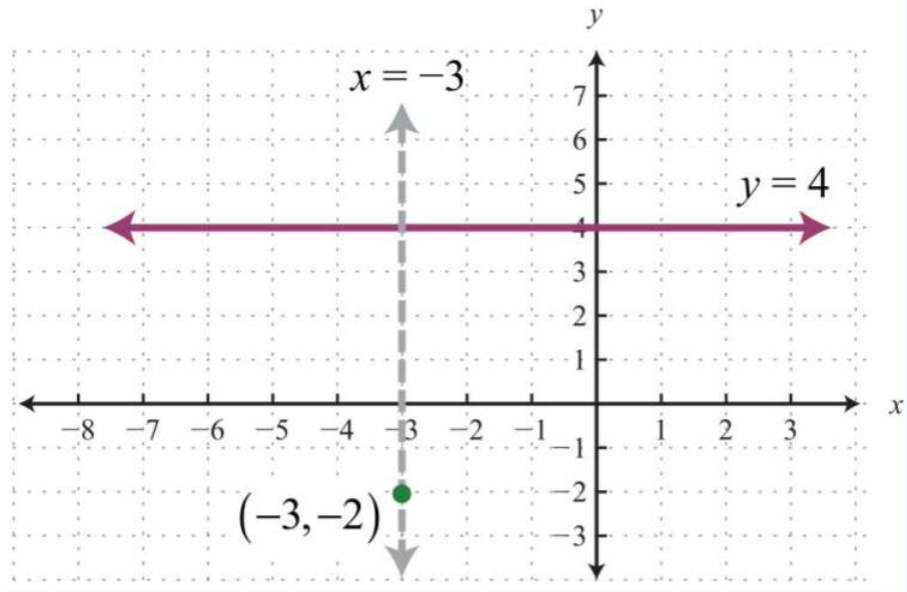


***Figure* 4.6.6**

Equations of vertical lines look like $x = k$. Since it must pass through $(-3, -2)$, we conclude that $x = -3$ is the equation. All ordered pair solutions of a vertical line must share the same x-coordinate.

Answer:

$$x = -3$$

We can rewrite the equation of any horizontal line, $y = k$, in slope-intercept form as follows:

$$y = 0x + k$$

Written in this form, we see that the slope is $m = 0 = \frac{0}{1}$. If we try to find the slope of a perpendicular line by finding the opposite reciprocal, we run into a problem: $m_{\perp} = -\frac{1}{0}$, which is undefined. This is why we took care to restrict the definition to two nonvertical lines. Remember that horizontal lines are perpendicular to vertical lines.

Key Takeaways

- Parallel lines have the same slope.
- Perpendicular lines have slopes that are opposite reciprocals. In other words, if $m = \frac{a}{b}$, then $m_{\perp} = -\frac{b}{a}$.
- To find an equation of a line, first use the given information to determine the slope. Then use the slope and a point on the line to find the equation using point-slope form.
- Horizontal and vertical lines are perpendicular to each other.

? Exercise 4.6.3 Parallel and Perpendicular Lines

Determine the slope of parallel lines and perpendicular lines.

1. $y = -\frac{3}{4}x + 8$
2. $y = \frac{1}{2}x - 3$
3. $y = 4x + 4$
4. $y = -3x + 7$
5. $y = -\frac{5}{8}x - 12$
6. $y = \frac{7}{3}x + \frac{3}{2}$
7. $y = 9x - 25$
8. $y = -10x + 15$
9. $y = 5$
10. $x = -12$

11. $x-y=0$
12. $x+y=0$
13. $4x+3y=0$
14. $3x-5y=10$
15. $-2x+7y=14$
16. $-x-y=\frac{1}{5}$
17. $\frac{1}{2}x-\frac{1}{3}y=-1$
18. $-\frac{2}{3}x+\frac{4}{5}y=8$
19. $2x-\frac{1}{5}y=\frac{1}{10}$
20. $-\frac{4}{5}x-2y=7$

Answer

1. $m_{\parallel}=-\frac{3}{4}$ and $m_{\perp}=\frac{4}{3}$
3. $m_{\parallel}=4$ and $m_{\perp}=-\frac{1}{4}$
5. $m_{\parallel}=-\frac{5}{8}$ and $m_{\perp}=\frac{8}{5}$
7. $m_{\parallel}=9$ and $m_{\perp}=-\frac{1}{9}$
9. $m_{\parallel}=0$ and $m_{\perp}$ undefined
11. $m_{\parallel}=1$ and $m_{\perp}=-1$
13. $m_{\parallel}=-\frac{4}{3}$ and $m_{\perp}=\frac{3}{4}$
15. $m_{\parallel}=\frac{2}{7}$ and $m_{\perp}=-\frac{7}{2}$
17. $m_{\parallel}=\frac{3}{2}$ and $m_{\perp}=-\frac{2}{3}$
19. $m_{\parallel}=10$ and $m_{\perp}=-\frac{1}{10}$

? Exercise 4.6.4 Parallel and Perpendicular Lines

Determine if the lines are parallel, perpendicular, or neither.

1. $\begin{cases} y=\frac{2}{3}x+3 \\ y=\frac{2}{3}x-3 \end{cases}$
2. $\begin{cases} y=\frac{3}{4}x-1 \\ y=\frac{4}{3}x+3 \end{cases}$
3. $\begin{cases} y=-2x+1 \\ y=\frac{1}{2}x+8 \end{cases}$
4. $\begin{cases} y=3x-\frac{1}{2} \\ y=3x+2 \end{cases}$
5. $\begin{cases} y=5 \\ x=-2 \end{cases}$
6. $\begin{cases} y=7 \\ y=-\frac{1}{7} \end{cases}$
7. $\begin{cases} 3x-5y=15 \\ 5x+3y=9 \end{cases}$

8. $\begin{cases} x-y=7 \\ 3x+3y=2 \end{cases}$
9. $\begin{cases} 2x-6y=4 \\ -x+3y=-2 \end{cases}$
10. $\begin{cases} -4x+2y=3 \\ 6x-3y=-3 \end{cases}$
11. $\begin{cases} x+3y=9 \\ 2x+3y=6 \end{cases}$
12. $\begin{cases} y-10=0 \\ x-10=0 \end{cases}$
13. $\begin{cases} y+2=0 \\ 2y-10=0 \end{cases}$
14. $\begin{cases} 3x+2y=6 \\ 2x+3y=6 \end{cases}$
15. $\begin{cases} -5x+4y=20 \\ 10x-8y=16 \end{cases}$
16. $\begin{cases} \frac{1}{2}x-\frac{1}{3}y=1 \\ \frac{1}{6}x+\frac{1}{4}y=-2 \end{cases}$

Answer

1. Parallel
3. Perpendicular
5. Perpendicular
7. Perpendicular
9. Parallel
11. Neither
13. Parallel
15. Parallel

Exercise 4.6.5 Equations in Point-Slope Form

Find the equation of the line

1. Parallel to $y=\frac{1}{2}x+2$ and passing through $(6,-1)$.
2. Parallel to $y=-\frac{3}{4}x-3$ and passing through $(-8,2)$.
3. Perpendicular to $y=3x-1$ and passing through $(-3,2)$.
4. Perpendicular to $y=-\frac{1}{3}x+2$ and passing through $(4,-3)$.
5. Perpendicular to $y=-2$ and passing through $(-1,5)$.
6. Perpendicular to $x=\frac{1}{5}$ and passing through $(5,-3)$.
7. Parallel to $y=3$ and passing through $(2,4)$.
8. Parallel to $x=2$ and passing through (7, −3)\).
9. Perpendicular to $y=x$ and passing through $(7,-13)$.
10. Perpendicular to $y=2x+9$ and passing through $(3,-1)$.
11. Parallel to $y=\frac{1}{4}x-5$ and passing through $(-2,1)$.
12. Parallel to $y=-\frac{3}{4}x+1$ and passing through $(4,\frac{1}{4})$.
13. Parallel to $2x-3y=6$ and passing through $(6,-2)$.
14. Parallel to $-x+y=4$ and passing through $(9,7)$.
15. Perpendicular to $5x-3y=18$ and passing through $(-9,10)$.

16. Perpendicular to $x-y=11$ and passing through $(6,-8)$.
17. Parallel to $\frac{1}{5}x-\frac{1}{3}y=2$ and passing through $(-15,6)$.
18. Parallel to $-10x-\frac{5}{7}y=12$ and passing through $(-1,\frac{1}{2})$.
19. Perpendicular to $\frac{1}{2}x-\frac{1}{3}y=1$ and passing through $(-10,3)$.
20. Perpendicular to $-5x+y=-1$ and passing through $(-4,0)$.
21. Parallel to $x+4y=8$ and passing through $(-1,-2)$.
22. Parallel to $7x-5y=35$ and passing through $(2,-3)$.
23. Perpendicular to $6x+3y=1$ and passing through $(8,-2)$.
24. Perpendicular to $-4x-5y=1$ and passing through $(-1,-1)$.
25. Parallel to $-5x-2y=4$ and passing through $(\frac{1}{5},-\frac{1}{4})$.
26. Parallel to $6x-\frac{3}{2}y=9$ and passing through $(\frac{1}{3},\frac{2}{3})$.
27. Perpendicular to $y-3=0$ and passing through $(-6,12)$.
28. Perpendicular to $x+7=0$ and passing through $(5,-10)$.

Answer

1. $y=\frac{1}{2}x-4$
3. $y=-\frac{1}{3}x+1$
5. $x=-1$
7. $y=4$
9. $y=-x-6$
11. $y=\frac{1}{4}x+\frac{3}{2}$
13. $y=\frac{2}{3}x-6$
15. $y=-\frac{3}{5}x+\frac{23}{5}$
17. $y=\frac{3}{5}x+15$
19. $y=-\frac{2}{3}x-\frac{11}{3}$
21. $y=-\frac{1}{4}x-\frac{9}{4}$
23. $y=\frac{1}{2}x-6$
25. $y=-\frac{5}{2}x+\frac{1}{4}$
27. $x=-6$

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4.7: Introduction to Functions

Learning Objectives

- Identify a function.
- State the domain and range of a function.
- Use function notation.

Relations, Functions, Domain, and Range

Relationships between sets occur often in everyday life. For example, for each month in Cape Canaveral, we can associate an average amount of rainfall. In this case, the amount of precipitation depends on the month of the year, and the data can be written in tabular form or as a set of ordered pairs.

Table 4.7.1

Month	Precipitation	Ordered pairs
January	2.4 in	(January, 2.4)
February	3.3 in	(February, 3.3)
March	3.1 in	(March, 3.1)
April	2.0 in	(April, 2.0)
May	3.8 in	(May, 3.8)
June	6.8 in	(June, 6.8)
July	8.1 in	(July, 8.1)
August	7.6 in	(August, 7.6)
September	7.3 in	(September, 7.3)
October	4.1 in	(October, 4.1)
November	3.3 in	(November, 3.3)
December	2.4 in	(December, 2.4)

We define a relation as any set of ordered pairs. Usually we write the independent component of the relation in the first column and the dependent component in the second column. In the opening example, notice that it makes sense to relate the average amount of precipitation as dependent on the month of year. The set of all elements in the first column of a relation is called the domain. The set of all elements that compose the second column is called the range. In this example, the domain consists of the set of all months of the year, and the range consists of the values that represent the average rainfall for each month.

In the context of algebra, the relations of interest are sets of ordered pairs (x, y) in the rectangular coordinate plane. In this case, the x-values define the domain and the y-values define the range. Of special interest are relations where every x-value corresponds to exactly one y-value; these relations are called functions.

Example 4.7.1

Determine the domain and range of the following relation and state whether or not it is a function:

$$\{(-1, 4), (0, 7), (2, 3), (3, 3), (4, -2)\}$$

Solution:

Here we separate the domain and range and depict the correspondence between the values with arrows.

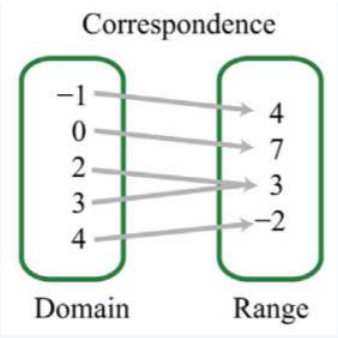


***Figure* 4.7.1**

Answer:

The domain is $\{-1, 0, 2, 3, 4\}$, and the range is $\{-2, 3, 4, 7\}$. The relation is a function because each x-value corresponds to exactly one y-value.

Example 4.7.2

Determine the domain and range of the following relation and state whether or not it is a function:

$$\{(-4, -3), (-2, 6), (0, 3), (3, 5), (3, 7)\}$$

Solution:

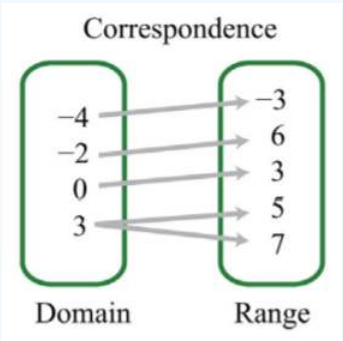


***Figure* 4.7.2**

Answer:

The domain is $\{-4, -2, 0, 3\}$, and the range is $\{-3, 3, 5, 6, 7\}$. This relation is not a function because the x-value 3 has two corresponding y-values.

In the previous example, the relation is not a function because it contains ordered pairs with the same x-value, $(3, 5)$ and $(3, 7)$. We can recognize functions as relations where no x-values are repeated.

In algebra, equations such as $y = \frac{3}{4}x - 2$ define relations. This linear equation can be graphed as follows:

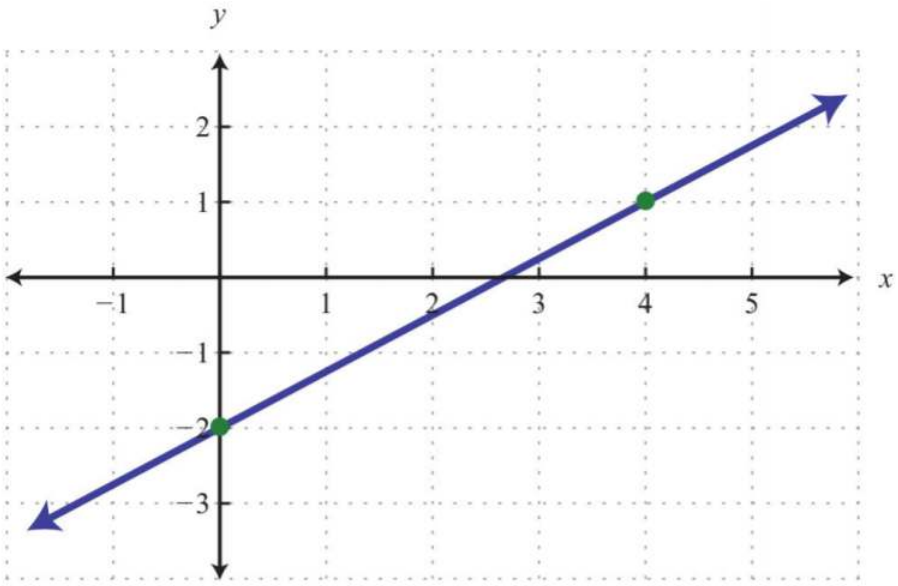


***Figure* 4.7.3**

The graph is a relation since it represents the infinite set of ordered pair solutions to $y = \frac{3}{4}x - 2$. The domain is the set of all x-values, and in this case consists of all real numbers. The range is the set of all possible y-values, and in this case also consists of all real numbers. Furthermore, the graph is a function because for each x-value there is only one corresponding y-value. In fact, any nonvertical or nonhorizontal line is a function with domain and range consisting of all real numbers.

Any graph is a set of ordered pairs and thus defines a relation. Consider the following graph of a circle:

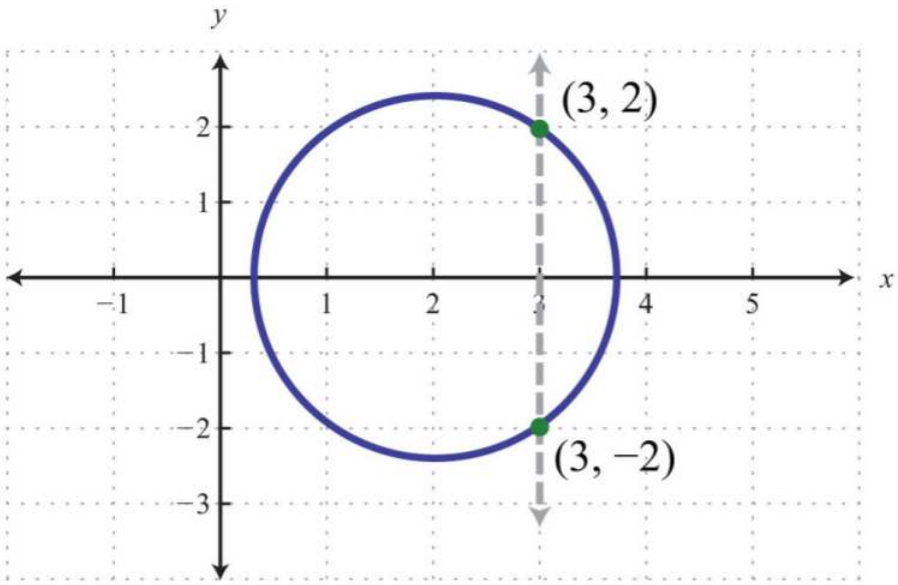


***Figure* 4.7.4**

Here the graph represents a relation where many x-values in the domain correspond to two y-values. If we draw a vertical line, as illustrated, we can see that $(3, 2)$ and $(3, -2)$ are two ordered pairs with the same x-value. Therefore, the x-value 3 corresponds to two y-values; hence the graph does not represent a function. The illustration suggests that if any vertical line intersects a graph more than once, then the graph does not represent a function. This is called the vertical line test.

Example 4.7.3

Given the following graph, determine the domain and range and state whether or not it is a function.

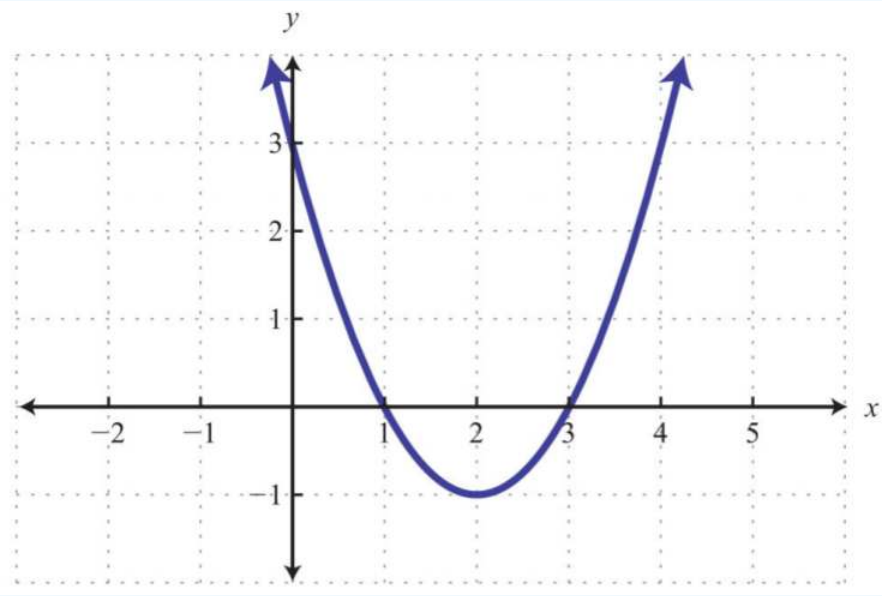


***Figure* 4.7.5**

Solution:

The given shape is called a parabola and extends indefinitely to the left and right as indicated by the arrows. This suggests that if we choose any x-value, then we will be able to find a corresponding point on the graph; therefore, the domain consists of all real numbers. Furthermore, the graph shows that -1 is the minimum y-value, and any y-value greater than that is represented in the relation. Hence the range consists of all y-values greater than or equal to -1, or in interval notation, $[-1, \infty)$.

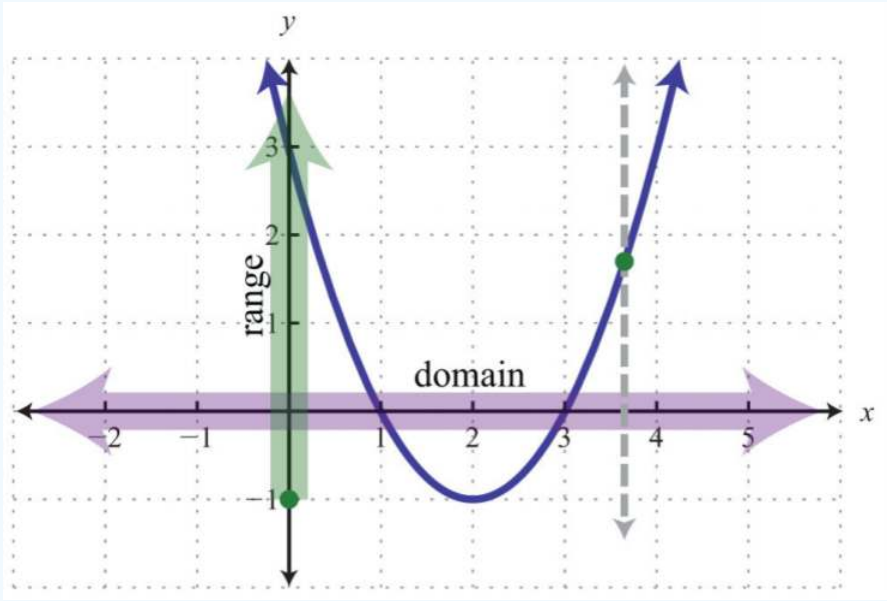


***Figure* 4.7.6**

Lastly, any vertical line will intersect the graph only once; therefore, it is a function.

Answer:

The domain is all real numbers $R = (-\infty, \infty)$, and the range is $[-1, \infty)$. The graph represents a function because it passes the vertical line test.

? Exercise 4.7.1

Given the graph, determine the domain and range and state whether or not it is a function:

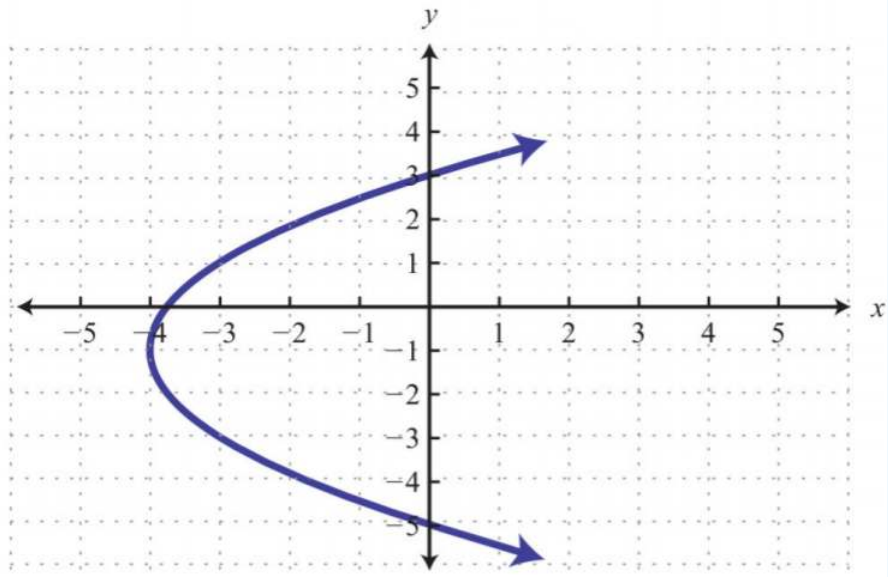


***Figure* 4.7.7**

Answer

Domain: $[-4, \infty)$; range: $(-\infty, \infty)$; function: no

Function Notation and Linear Functions

With the definition of a function comes special notation. If we consider each x-value to be the input that produces exactly one output, then we can use the notation

$$f(x) = y \tag{4.7.1}$$

The notation $f(x)$ reads "f of x" and should not be confused with multiplication. Most of our study of algebra involves functions, so the notation becomes very useful when performing common tasks. Functions can be named with different letters; some common names for functions are $g(x), h(x), C(x)$, and $R(x)$. First, consider nonvertical lines that we know can be expressed using slope-intercept form, $y = mx + b$. For any real numbers m and b, the equation defines a function, and we can replace y with the new notation $f(x)$ as follows:

$$y = mx + b \tag{4.7.2}$$

$$f(x) = mx + b \tag{4.7.3}$$

Therefore, a linear function is any function that can be written in the form $f(x) = mx + b$. In particular, we can write the following:

The notation also shows values to evaluate in the equation. If the value for x is given as 8, then we know that we can find the corresponding y-value by substituting 8 in for x and simplifying. Using function notation, this is denoted $f(8)$ and can be interpreted as follows:

Finally, simplify:

We have $f(8) = 4$. This notation tells us that when $x = 8$ (***the input***), the function results in 4 (***the output***).

Given the linear function $f(x) = -5x + 7$, find $f(-2)$.

Solution:

In this case, $f(-2)$ indicates that we should evaluate when $x = -2$.

$$\begin{aligned} f(x) &= -5x + 7 \\ f(-2) &= -5(-2) + 7 \qquad \textit{Replace } x \textit{ with } -2. \\ &= 10 + 7 \\ &= 17 \end{aligned}$$

Answer:

$$f(-2) = 17$$

Example 4.7.5

Given the linear function $f(x) = -5x + 7$, find x when $f(x) = 10$.

Solution:

In this case, $f(x) = 10$ indicates that the function should be set equal to 10.

$$\begin{aligned} f(x) &= -5x + 7 \\ 10 &= -5x + 7 \qquad \textit{Replace } f(x) \textit{ with } 10. \\ 10 - 7 &= -5x + 7 - 7 \qquad \textit{Solve for } x. \\ 3 &= -5x \\ \frac{3}{-5} &= \frac{-5x}{-5} - \frac{3}{5} \qquad = x \end{aligned}$$

Answer:

Here $x = -\frac{3}{5}$, and we can write $f(-\frac{3}{5}) = 10$.

Example 4.7.6

Given the graph of a linear function $g(x)$,

a. Find $g(2)$.
b. Find x when $g(x) = 3$.

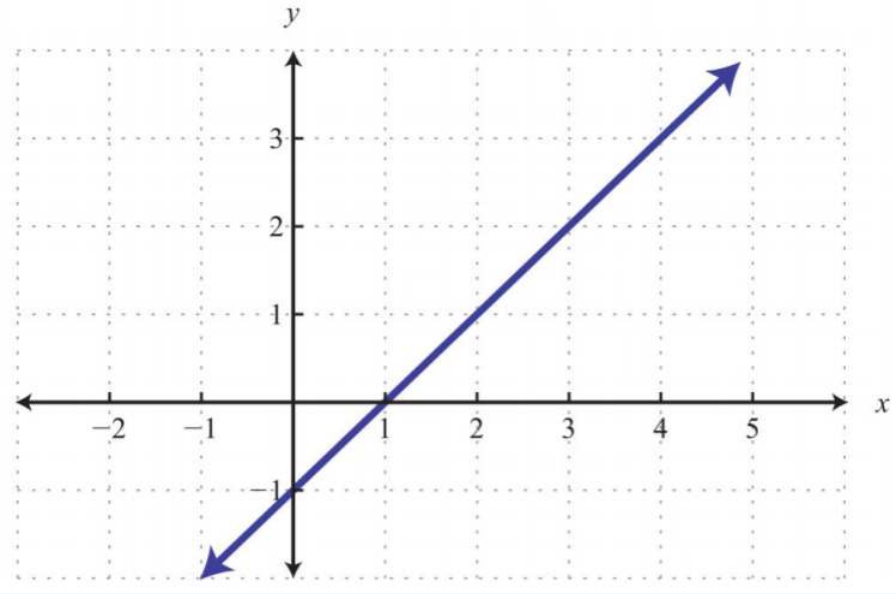


Figure **4.7.8**

Solution:

a. The notation $g(2)$ implies that $x = 2$. Use the graph to determine the corresponding y-value.

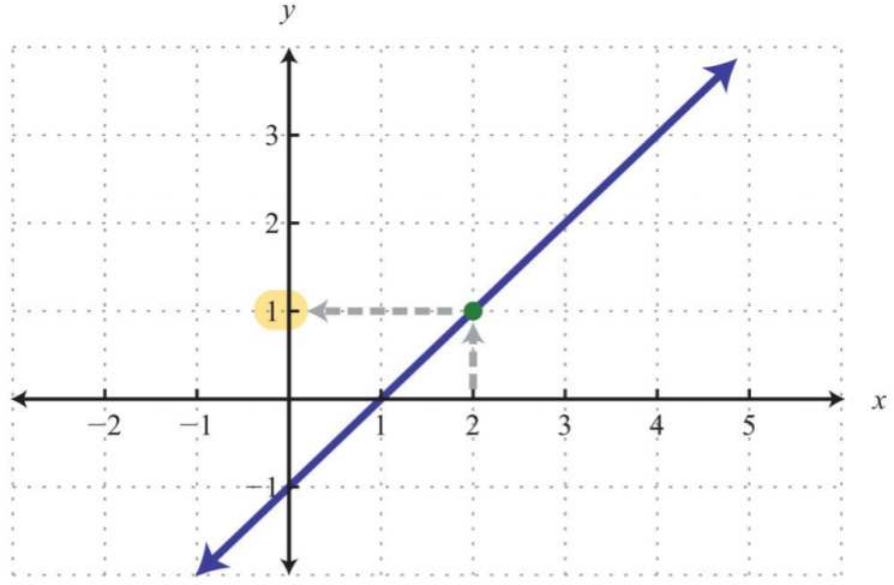


Figure **4.7.9**

b. The notation $g(x) = 3$ implies that the y-value is given as 3. Use the graph to determine the corresponding x-value.

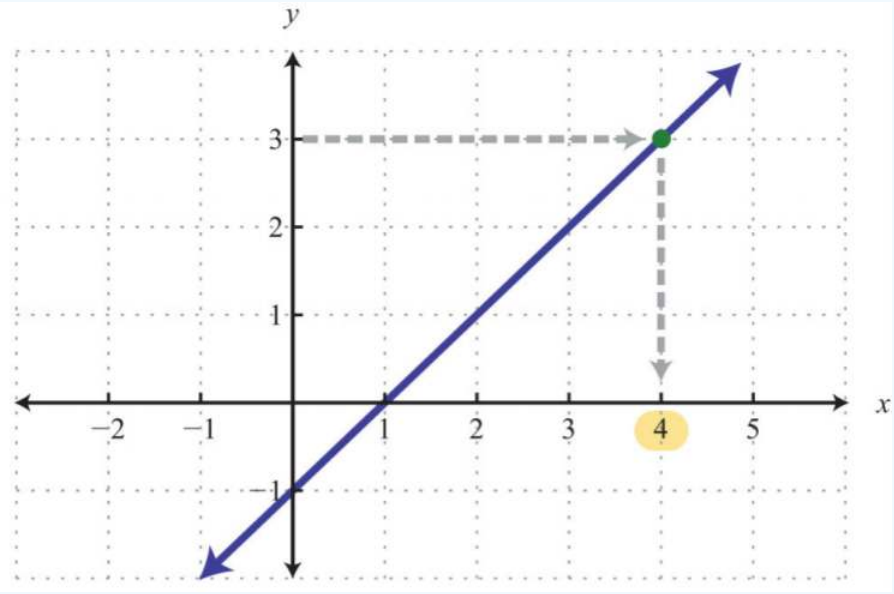


Figure **4.7.10**

Answer:

a. $g(2) = 1$
b. $x = 4$

Example 4.7.7

Graph the linear function $f(x) = -\frac{5}{3}x + 6$ and state the domain and range.

Solution:

From the function, we see that $b = 6$ and thus the y-intercept is $(0, 6)$. Also, we can see that the slope is $m = \frac{-5}{3} = -\frac{5}{3} = \frac{rise}{run}$. Starting from the y-intercept, mark a second point down 5 units and right 3 units.

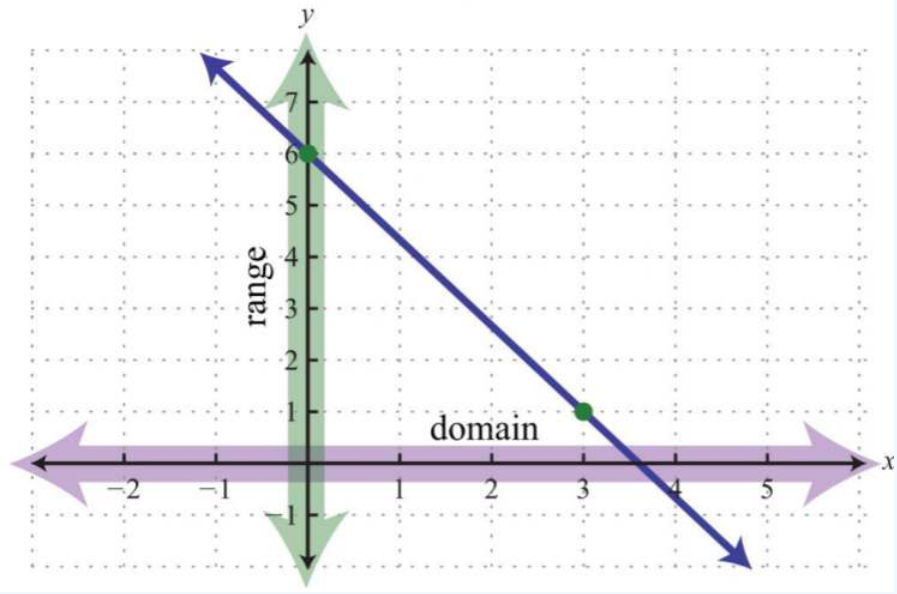


***Figure* 4.7.11**

Given any coordinate on the x-axis, we can find a corresponding point on the graph; the domain consists of all real numbers. Also, for any coordinate on the y-axis, we can find a point on the graph; the range consists of all real numbers.

Answer:

Both the domain and range consist of all real numbers R.

Exercise 4.7.2

Given the linear function $g(x) = -x + 5$,

a. Find $g(-\frac{1}{2})$.
b. Find x when $g(x) = 18$.

Answer

a. $g(-\frac{1}{2}) = \frac{1}{12}$

b. $x = -13$

Key Takeaways

- A relation is any set of ordered pairs. However, in the context of this course, we will be working with sets of ordered pairs (x, y) in the rectangular coordinate system. The set of x-values defines the domain and the set of y-values defines the range.
- Special relations where every x-value (*input*) corresponds to exactly one y-value (*output*) are called functions.
- We can easily determine whether an equation represents a function by performing the vertical line test on its graph. If any vertical line intersects the graph more than once, then the graph does not represent a function. In this case, there will be more than one point with the same x-value.

- Any nonvertical or nonhorizontal line is a function and can be written using function notation $f(x) = mx + b$. Both the domain and range consist of all real numbers.
  - If asked to find $f(a)$, we substitute a in for the variable and then simplify.
  - If asked to find x when $f(x) = a$, we set the function equal to a and then solve for x.

? Exercise 4.7.3 Functions

For each problem below, does the correspondence represent a function?

1. Algebra students to their scores on the first exam.
2. Family members to their ages.
3. Lab computers to their users.
4. Students to the schools they have attended.
5. People to their citizenships.
6. Local businesses to their number of employees.

Answer

1. Yes

3. No

5. No

? Exercise 4.7.4 Functions

Determine the domain and range and state whether the relation is a function or not.

1. $\{(3, 2), (5, 3), (7, 4)\}$
2. $\{(-5, -3), (0, 0), (5, 0)\}$
3. $\{(-10, 2), (-8, 1), (-8, 0)\}$
4. $\{(9, 12), (6, 6), (6, 3)\}$

5.

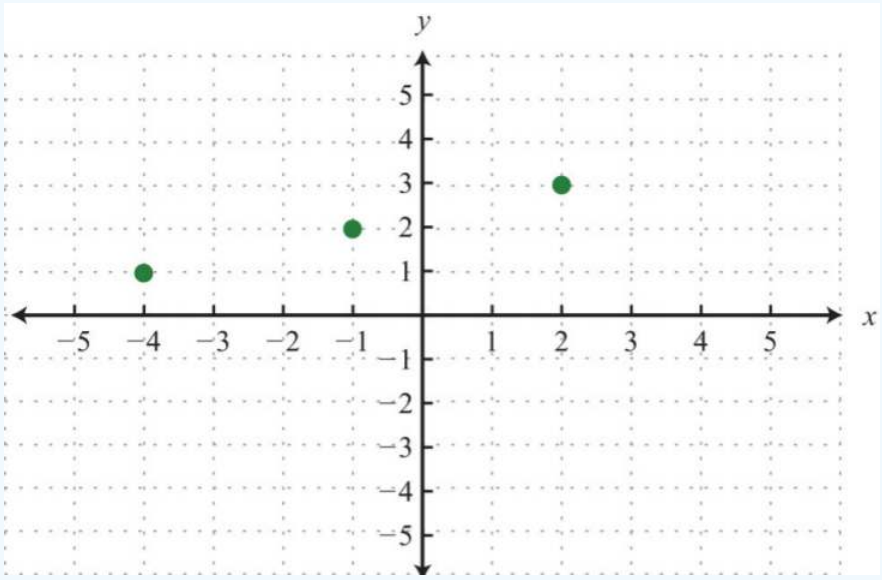


***Figure* 4.7.12**

6.

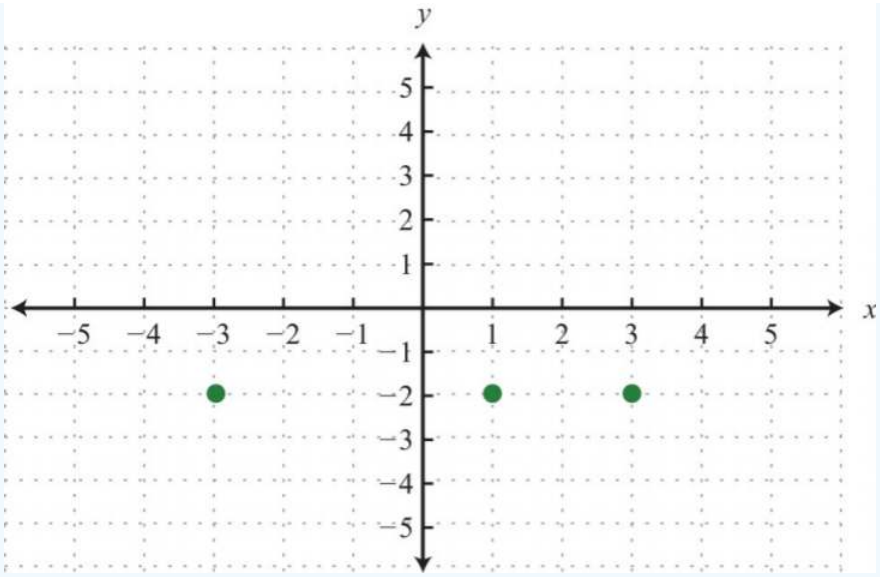


Figure **4.7.13**

7.

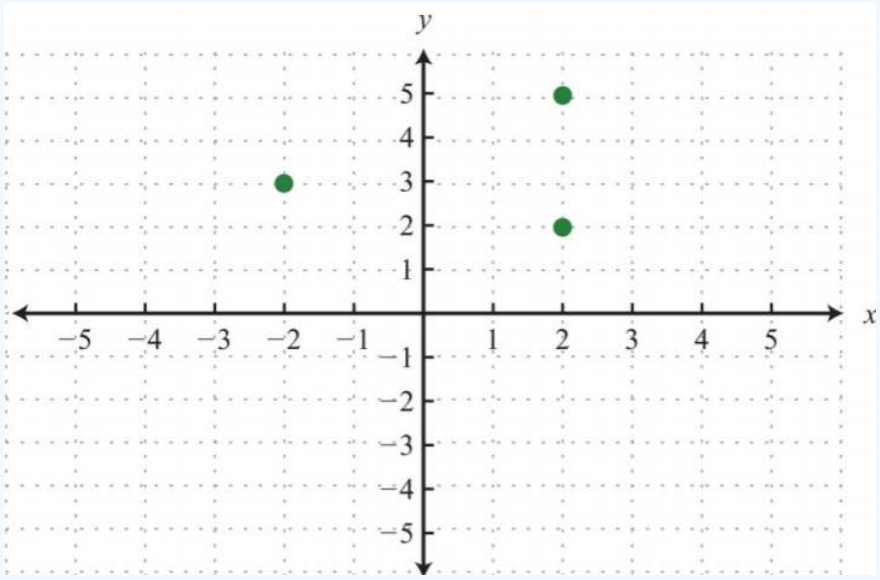


Figure **4.7.14**

8.

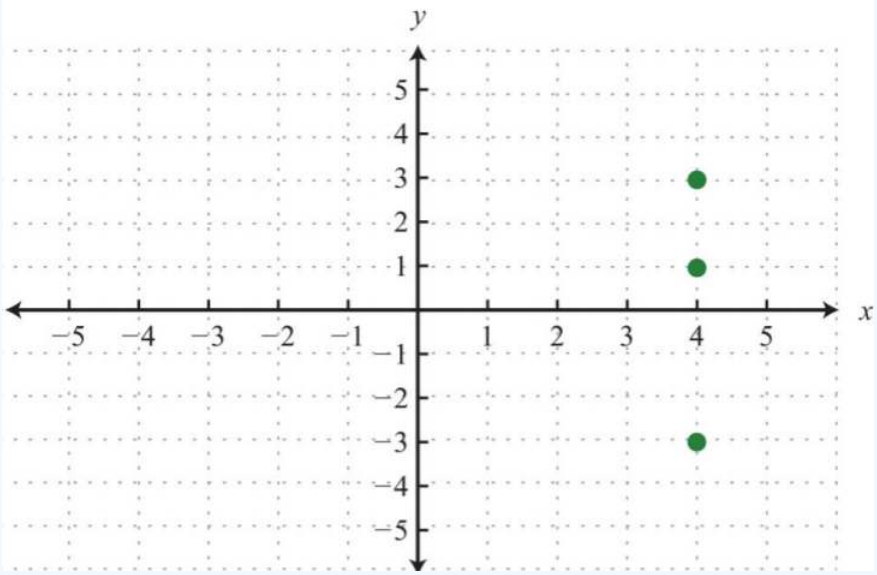


Figure **4.7.15**

9.

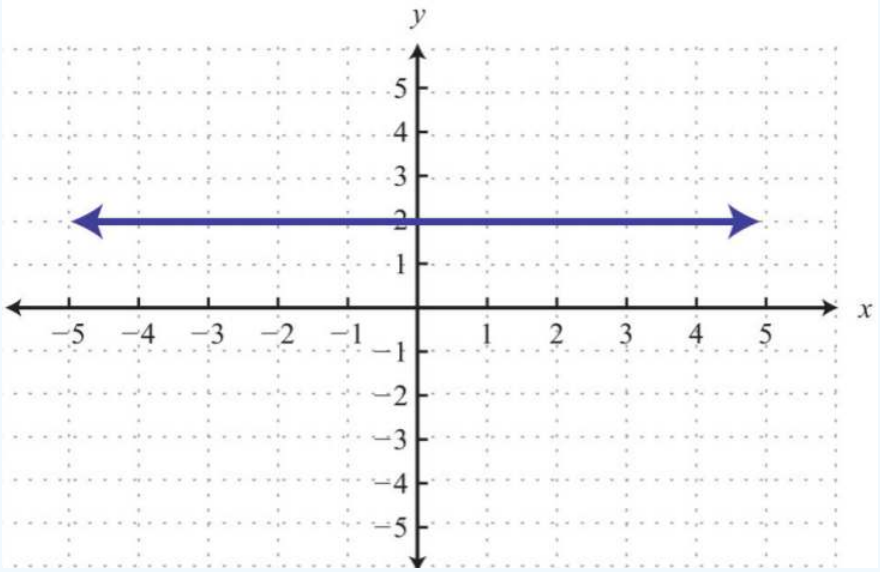


Figure **4.7.16**

10.

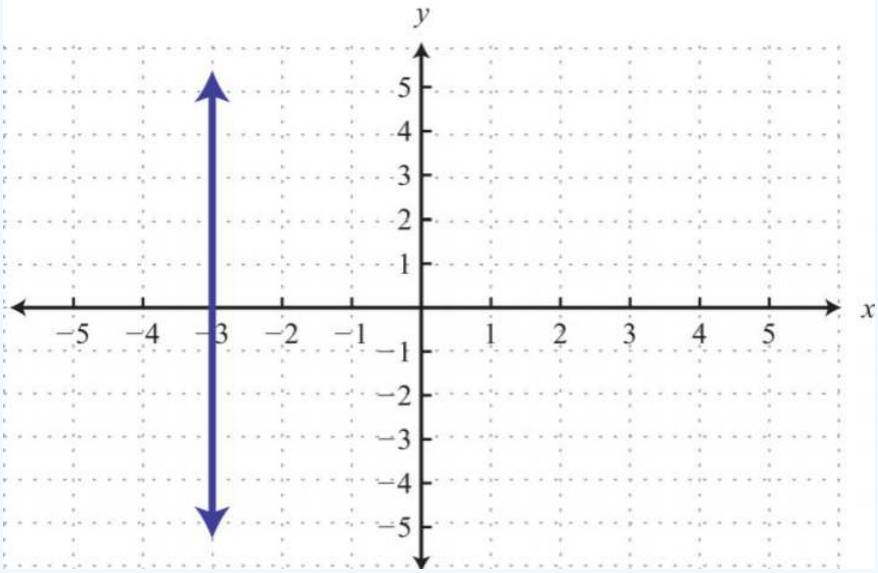


Figure **4.7.17**

11.

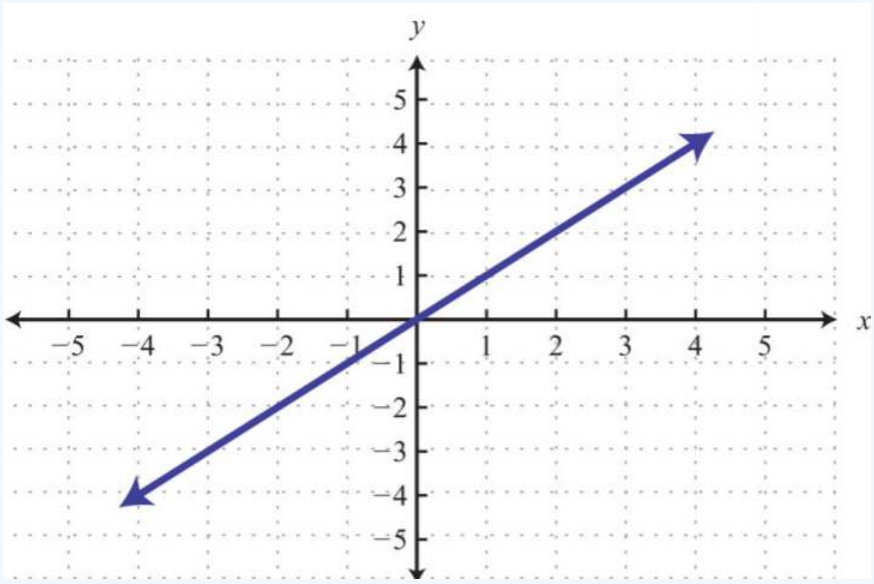


Figure **4.7.18**

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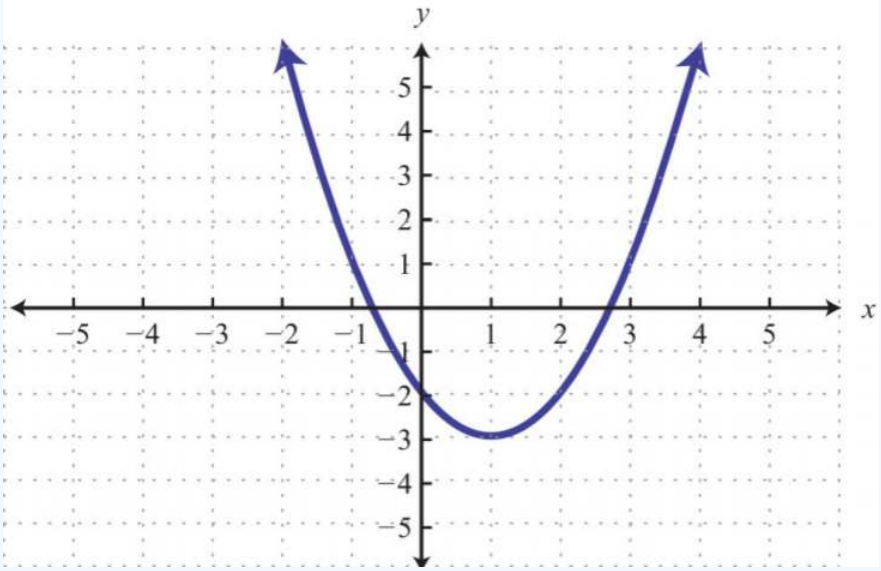


Figure **4.7.19**

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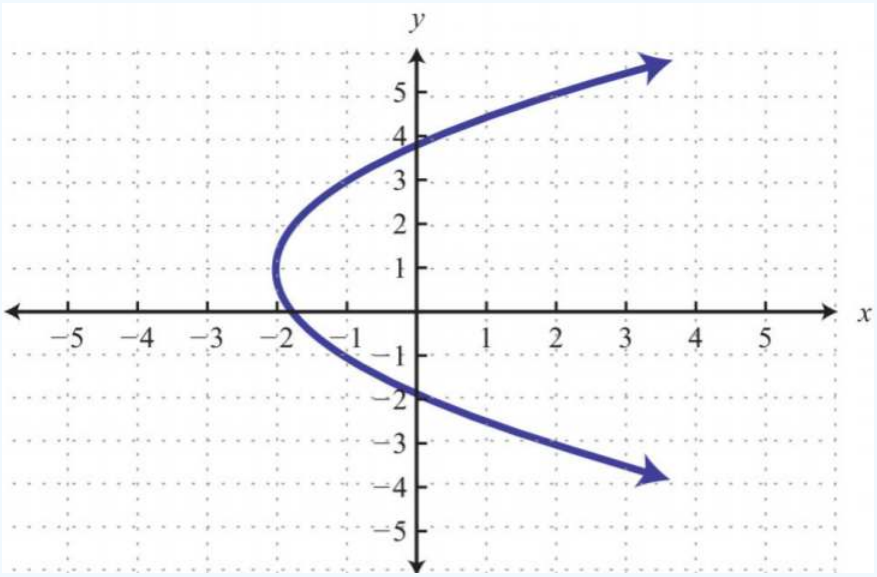


Figure **4.7.20**

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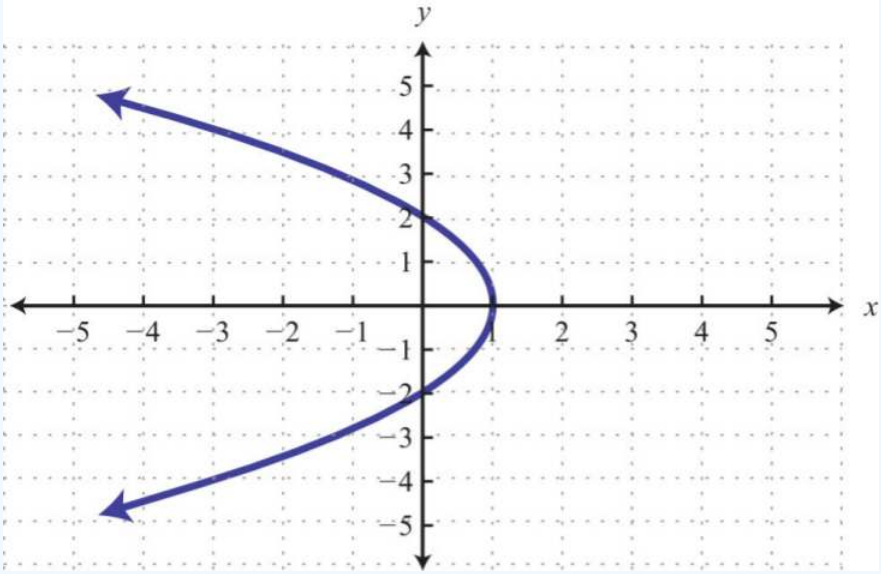


Figure **4.7.21**

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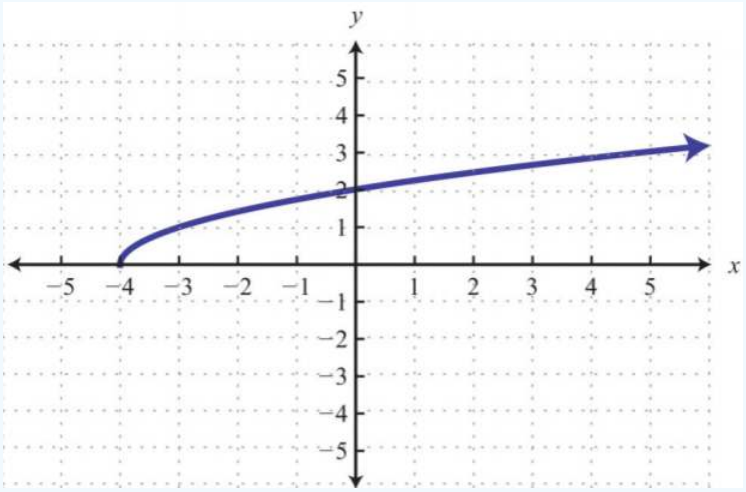


Figure **4.7.22**

16.

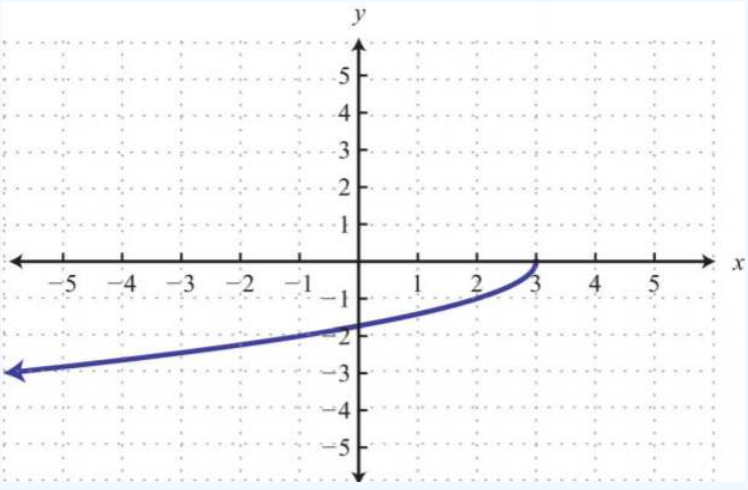


Figure **4.7.23**

17.

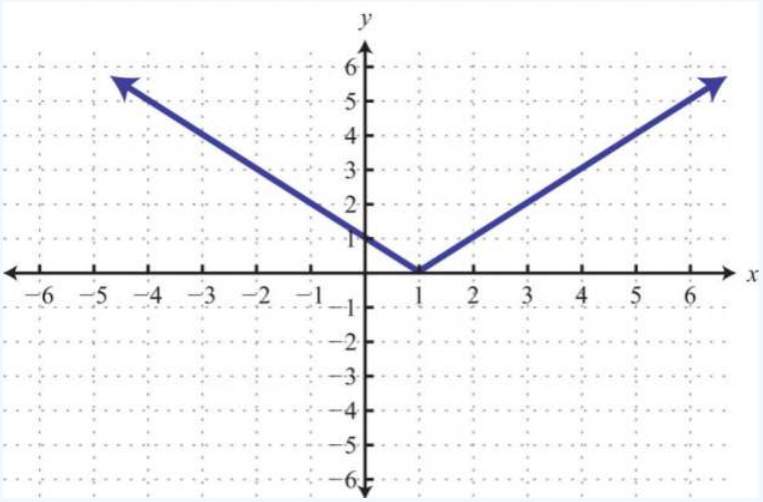


Figure **4.7.24**

18.

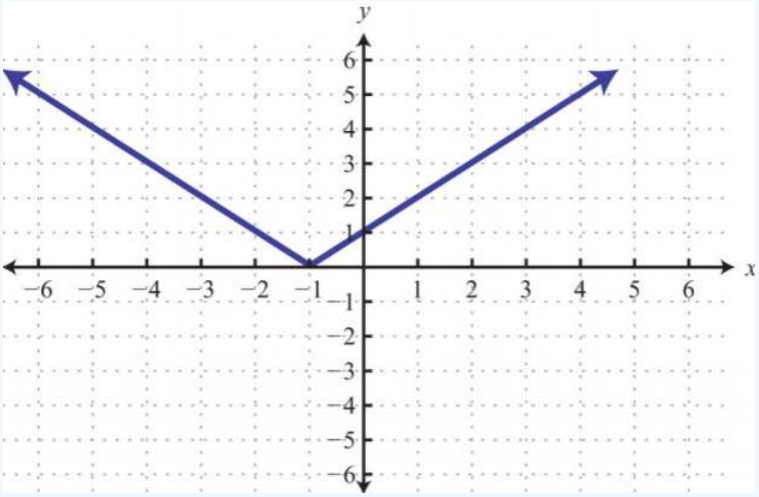


Figure **4.7.25**

19.

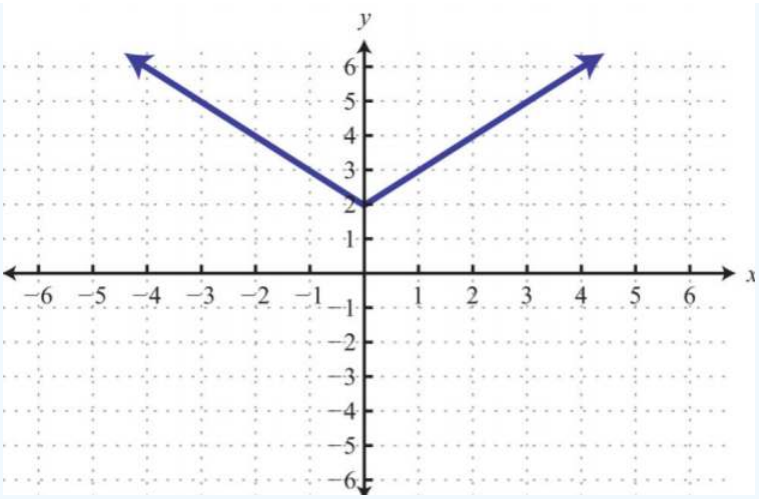


Figure **4.7.26**

20.

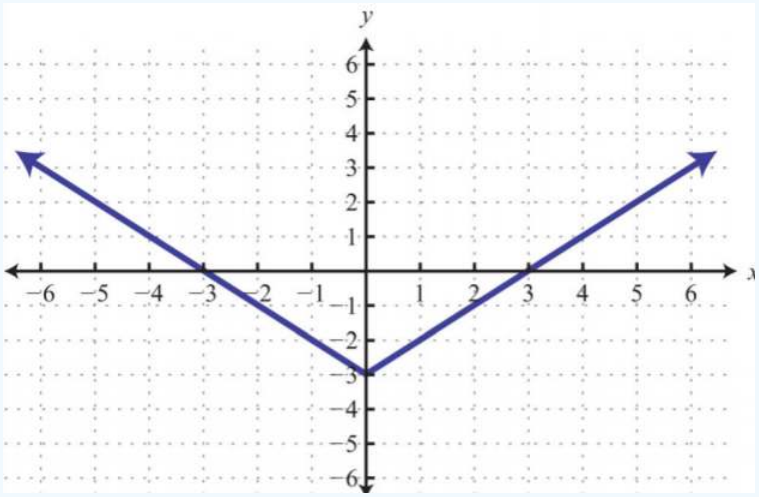


Figure **4.7.27**

Answer

1. Domain: $\{3, 5, 7\}$ range: $\{2, 3, 4\}$ function: yes
3. Domain: $\{-10, -8\}$; range: $\{0, 1, 2\}$ function: no
5. Domain: $\{-4, -1, 2\}$; range: $\{1, 2, 3\}$ function: yes
7. Domain: $\{-2, 2\}$; range: $\{2, 3, 5\}$ function: no
9. Domain: $(-\infty, \infty)$; range: $\{2\}$; function: yes
11. Domain: $(-\infty, \infty)$; range: $(-\infty, \infty)$; function: yes
13. Domain: $[-2, \infty)$; range: $(-\infty, \infty)$; function: no
15. Domain: $[-4, \infty)$; range: $[0, \infty)$; function: yes
17. Domain: $(-\infty, \infty)$; range: $[0, \infty)$; function: yes
19. Domain: $(-\infty, \infty)$; range: $[2, \infty)$; function: yes

Exercise 4.7.5 Function Notation

Given the following functions, find the function values.

1. $f(x) = 3x$, find $f(-2)$.
2. $f(x) = -5x + 1$, find $f(-1)$.
3. $f(x) = \frac{3}{5}x - 4$, find $f(15)$.
4. $f(x) = \frac{2}{5}x - \frac{1}{5}$, find $f(3)$.
5. $f(x) = \frac{5}{2}x - \frac{1}{3}$, find $f(-\frac{1}{3})$.
6. $f(x) = -6$, find $f(7)$.
7. $g(x) = 5$, find $g(-4)$.
8. $g(x) = -5x$, find $g(-3)$.
9. $g(x) = -\frac{1}{8}x + \frac{5}{8}$, find $g(\frac{5}{8})$.
10. $g(x) = \frac{5}{3}x - 5$, find $g(3)$.
11. $f(x) = 5x - 9$, find x when $f(x) = 1$.
12. $f(x) = -7x + 2$, find x when $f(x) = 0$.
13. $f(x) = -\frac{7}{5}x - 2$, find x when $f(x) = -9$.
14. $f(x) = -x - 4$, find x when $f(x) = 12$.
15. $g(x) = x$, find x when $g(x) = 12$.
16. $g(x) = -x + 1$, find x when $g(x) = \frac{2}{3}$.
17. $g(x) = -5x + \frac{1}{3}$, find x when $g(x) = -\frac{1}{2}$.
18. $g(x) = -\frac{5}{8}x + 3$, find x when $g(x) = 3$.

Answer

1. $f(-2) = -6$
3. $f(15) = 5$
5. $f(-\frac{1}{3}) = -\frac{7}{6}$
7. $g(-4) = 5$
9. $g(\frac{5}{8}) = \frac{35}{64}$
11. $x = 2$
13. $x = 5$
15. $x = 12$
17. $x = \frac{1}{6}$

Exercise 4.7.6 Function Notation

Given $f(x) = \frac{2}{3}x - 1$ and $g(x) = -3x + 2$ calculate the following.

1. $f(6)$
2. $f(-\frac{1}{2})$
3. $f(0)$
4. $f(1)$
5. $g(\frac{2}{3})$
6. $g(0)$
7. $g(-1)$
8. $g(-\frac{1}{2})$
9. Find x when $f(x) = 0$.
10. Find x when $f(x) = -3$.
11. Find x when $g(x) = -1$.

12. Find x when $g(x) = 0$.

Answer

1. $f(6) = 3$
3. $f(0) = -1$
5. $g(\frac{2}{3}) = 0$
7. $g(-1) = 5$
9. $x = \frac{3}{2}$
11. $x = 1$

Exercise 4.7.7 Function Notation

Given the graph, find the function values.

1. Given the graph of $f(x)$, find $f(-4)$, $f(-1)$, $f(0)$,and $f(2)$.

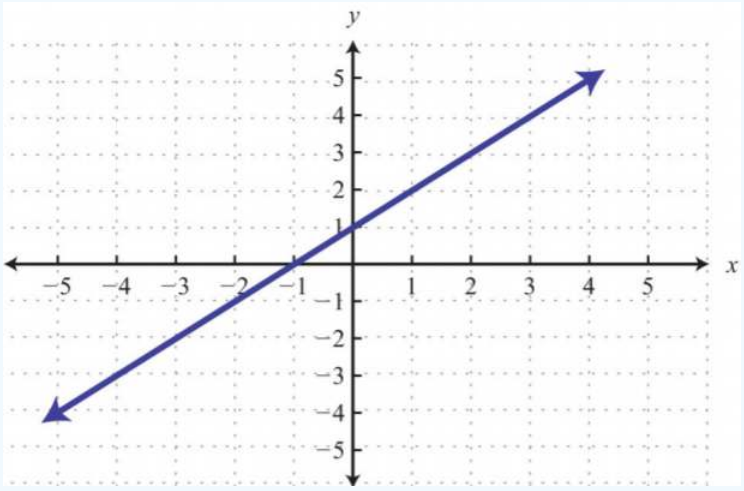


***Figure* 4.7.28**

2. Given the graph of $g(x)$, find $g(-3)$, $g(-1)$, $g(0)$, and $g(1)$.

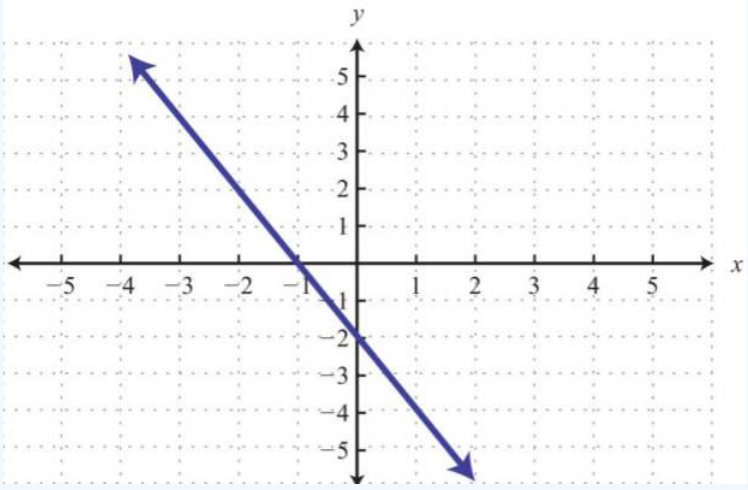


***Figure* 4.7.29**

3. Given the graph of $f(x)$, find $f(-4)$, $f(-1)$, $f(0)$,and $f(2)$.

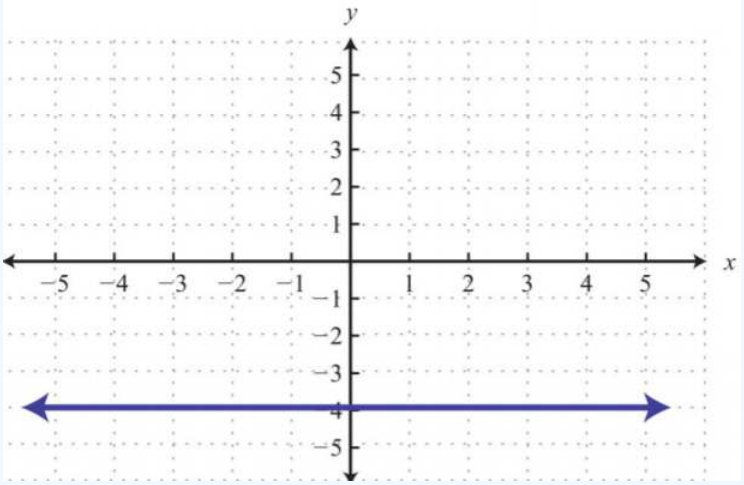


***Figure* 4.7.30**

4. Given the graph of $g(x)$, find $g(-4)$, $g(-1)$, $g(0)$, and $g(2)$.

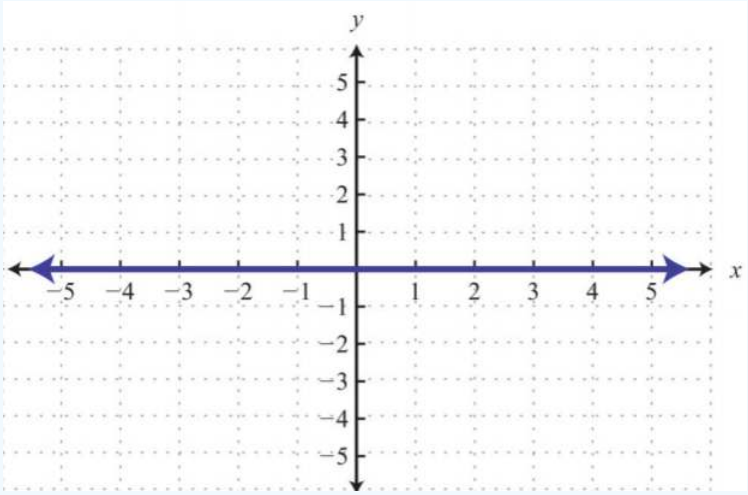


***Figure* 4.7.31**

5. Given the graph of $f(x)$, find $f(-1)$, $f(0)$, $f(1)$, and $f(3)$.

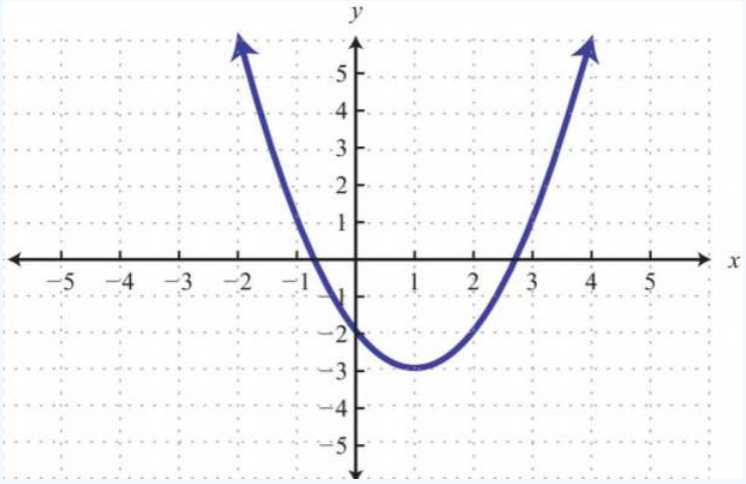


***Figure* 4.7.32**

6. Given the graph of $g(x)$, find $g(-2)$, $g(0)$, $g(2)$, and $g(6)$.

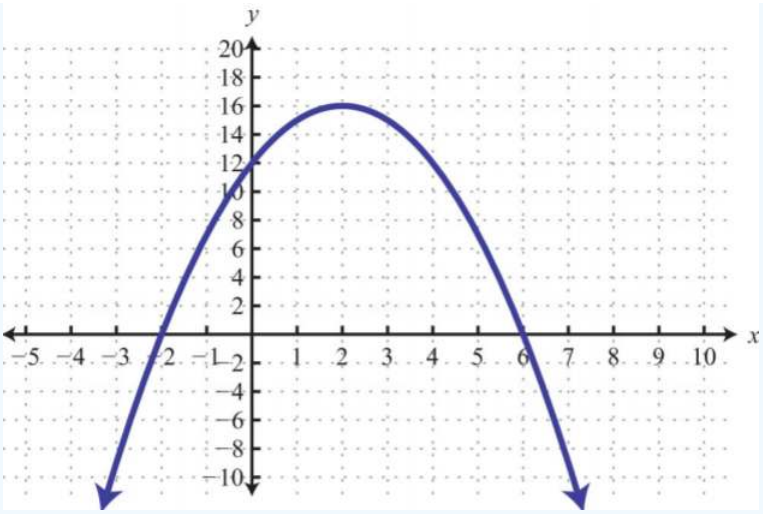


***Figure* 4.7.33**

7. Given the graph of $g(x)$, find $g(-4)$, $g(-3)$, $g(0)$, and $g(4)$.

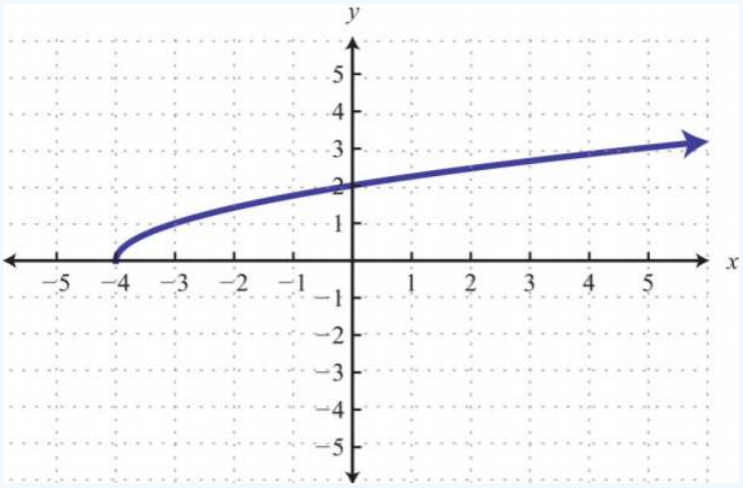


***Figure* 4.7.34**

8. Given the graph of $f(x)$, find $f(-4)$, $f(0)$, $f(1)$, and $f(3)$.

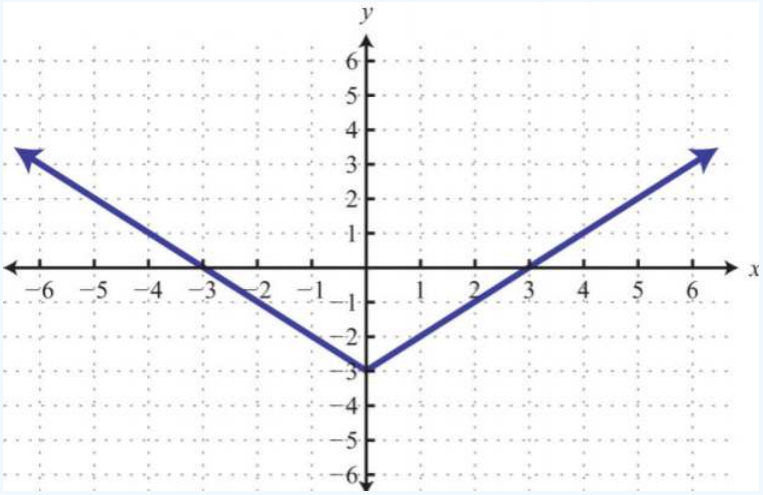


***Figure* 4.7.35**

Answer

1. $f(-4)=-3, f(-1)=0, f(0)=1,$ and $f(2)=3$
3. $f(-4)=-4, f(-1)=-4, f(0)=-4,$ and $f(2)=-4$
5. $f(-1)=1, f(0)=-2, f(1)=-3,$ and $f(3)=1$
7. $g(-4)=0, g(-3)=1, g(0)=2,$ and $g(4)=3$

Exercise 4.7.8 Function Notation

Given the graph, find the x-values.

1. Given the graph of $f(x)$, find x when $f(x)=3, f(x)=1$, and $f(x)=-3$.

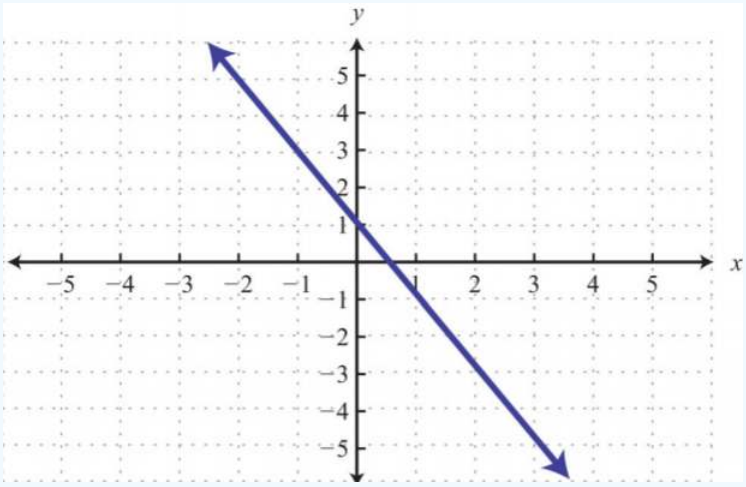


***Figure* 4.7.36**

2. Given the graph of $g(x)$, find x when $g(x)=-1, g(x)=0,$ and $g(x)=1$.

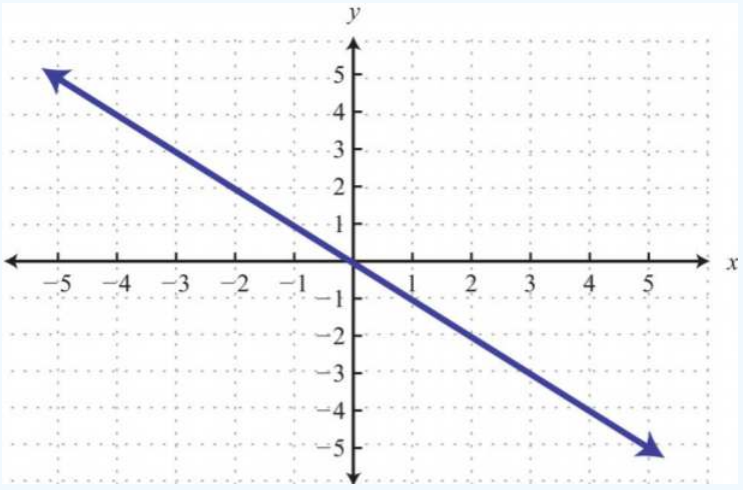


***Figure* 4.7.37**

3. Given the graph of $f(x)$, find x when $f(x)=3$.

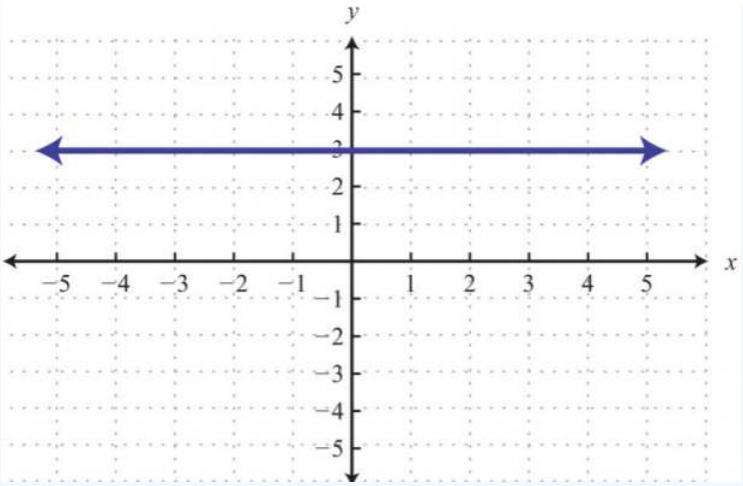


***Figure* 4.7.38**

4. Given the graph of $g(x)$, find x when $g(x) = -2$, $g(x) = 0$, and $g(x) = 4$.

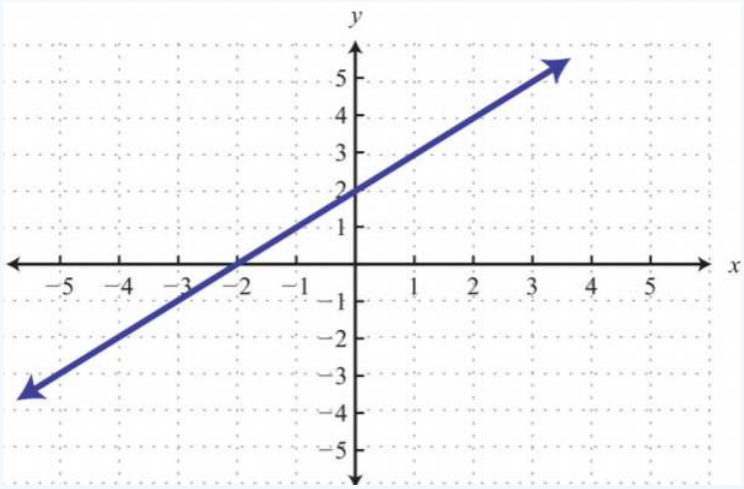


***Figure* 4.7.39**

5. Given the graph of $f(x)$, find x when $f(x) = -16$, $f(x) = -12$, and $f(x) = 0$.

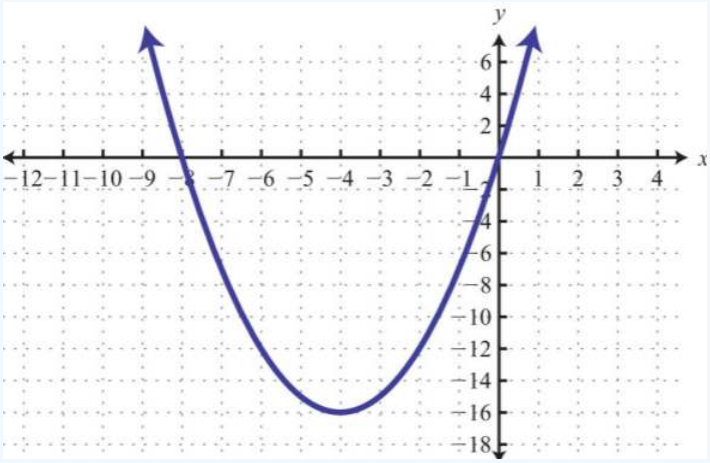


***Figure* 4.7.40**

6. Given the graph of $g(x)$, find x when $g(x) = -3$, $g(x) = 0$, and $g(x) = 1$.

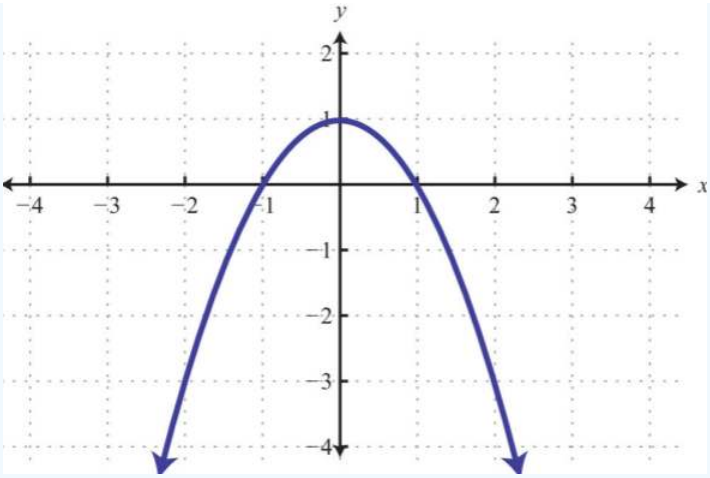


***Figure* 4.7.41**

7. Given the graph of $f(x)$, find x when $f(x) = -4$, $f(x) = 0$, and $f(x) = -2$.

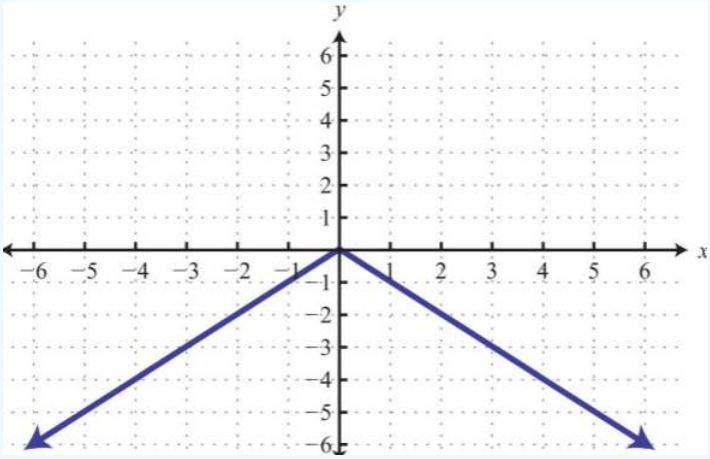


***Figure* 4.7.42**

8. Given the graph of $g(x)$, find x when $g(x) = 5$, $g(x) = 3$, and $g(x) = 2$.

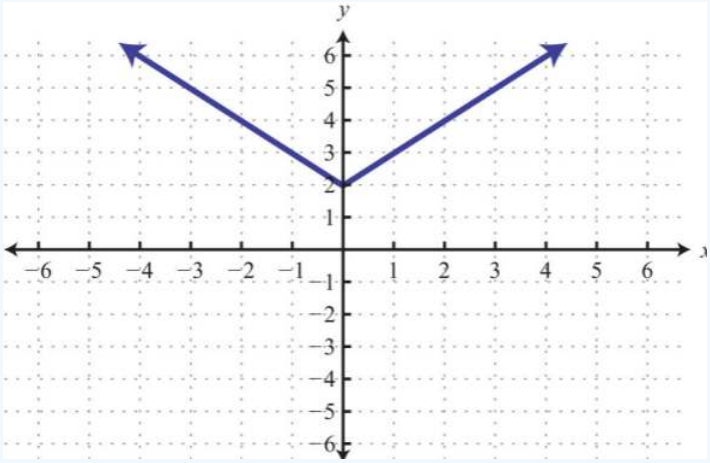


***Figure* 4.7.43**

9. The cost in dollars of producing pens with a company logo is given by the function $C(x) = 1.65x + 120$, where x is the number of pens produced. Use the function to calculate the cost of producing 200 pens.

10. The revenue in dollars from selling sweat shirts is given by the function $R(x) = 29.95x$, where x is the number of sweat shirts sold. Use the function to determine the revenue if 20 sweat shirts are sold.

11. The value of a new car in dollars is given by the function $V(t) = -2,500t + 18,000$, where t represents the age of the car in years. Use the function to determine the value of the car when it is 5 years old. What was the value of the car when new?

12. The monthly income in dollars of a commissioned car salesman is given by the function $I(n) = 550n + 1,250$, where n represents the number of cars sold in the month. Use the function to determine the salesman's monthly income if he sells 3 cars this month. What is his income if he does not sell any cars in a month?

13. The perimeter of an isosceles triangle with a base measuring 10 centimeters is given by the function $P(x) = 2x + 10$, where x represents the length of each of the equal sides. Find the length of each side if the perimeter is 40 centimeters.

14. The perimeter of a square depends on the length of each side s and is modeled by the function $P(s) = 4s$. If the perimeter of a square measures 140 meters, then use the function to calculate the length of each side.

15. A certain cellular phone plan charges \$18 per month and \$0.10 per minute of usage. The cost of the plan is modeled by the function $C(x) = 0.10x + 18$, where x represents the number of minutes of usage per month. Determine the minutes of usage if the cost for the month was \$36.

16. The monthly revenue generated by selling subscriptions to a tutoring website is given by the function $R(x) = 29x$, where x represents the number of subscription sales per month. How many subscriptions were sold if the revenues for the month totaled $\$1,508$?

Answer

1. $f(-1) = 3, f(0) = 1,$ and $f(2) = -3$
3. $f(1) = 3$ (answers may vary)
5. $f(-4) = -16$; $f(-6) = -12$ and $f(-2) = -12$; $f(-8) = 0$ and $f(0) = 0$
7. $f(-4) = -4$ and $f(4) = -4$; $f(0) = 0$; $f(-2) = -2$ and $f(2) = -2$
9. \$450
11. New: $\$18,000$; 5 years old: $\$5,500$
13. 15 centimeters
15. 180 minutes

Exercise 4.7.9 Function Notation

Graph the linear function and state the domain and range.

1. $f(x) = -\frac{5}{2}x + 10$
2. $f(x) = \frac{3}{5}x - 10$
3. $g(x) = 6x + 2$
4. $g(x) = -4x + 6$
5. $h(t) = \frac{1}{2}t - 3$
6. $h(t) = -\frac{3}{4}t + 3$
7. $C(x) = 100 + 50x$
8. $C(x) = 50 + 100x$

Answer

1. Domain and range: R

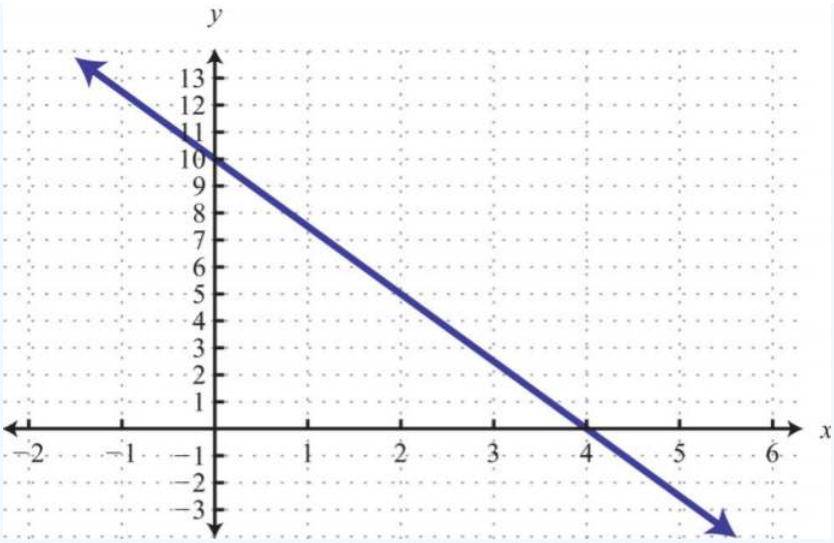


Figure 4.7.44

3. Domain and range: R

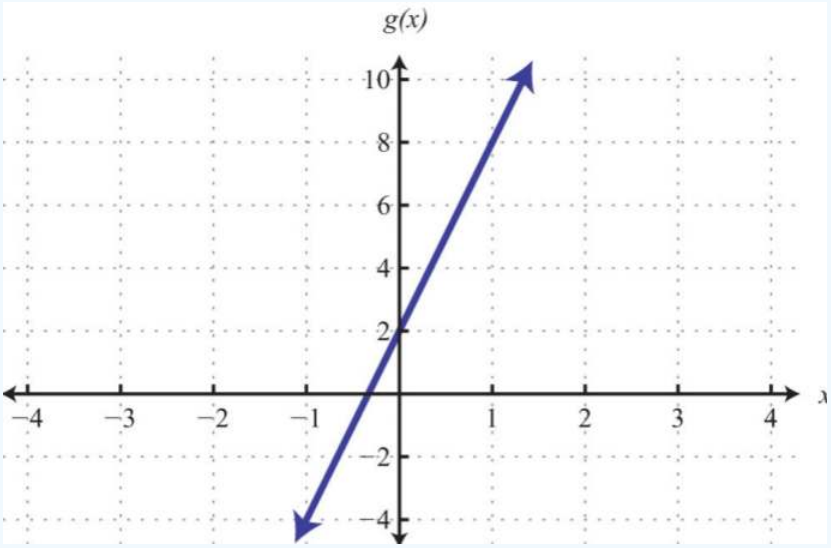


Figure 4.7.45

5. Domain and range: R

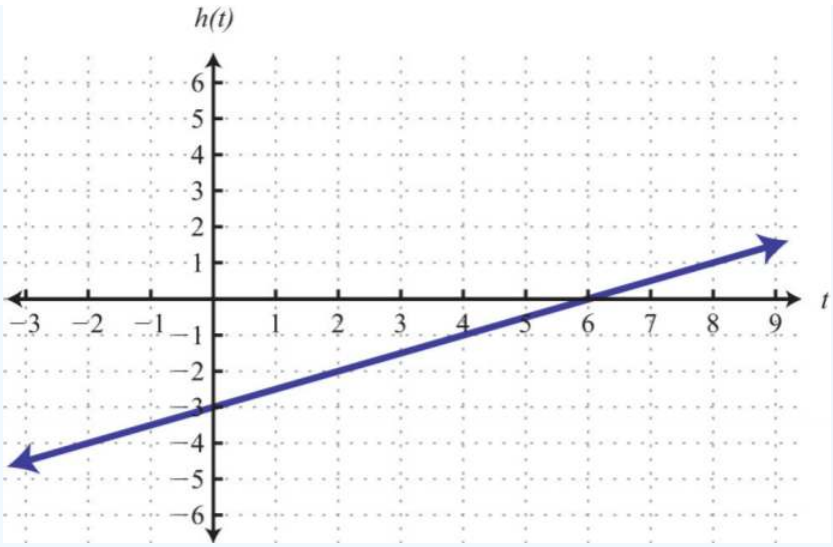


Figure 4.7.46

7. Domain and range: R

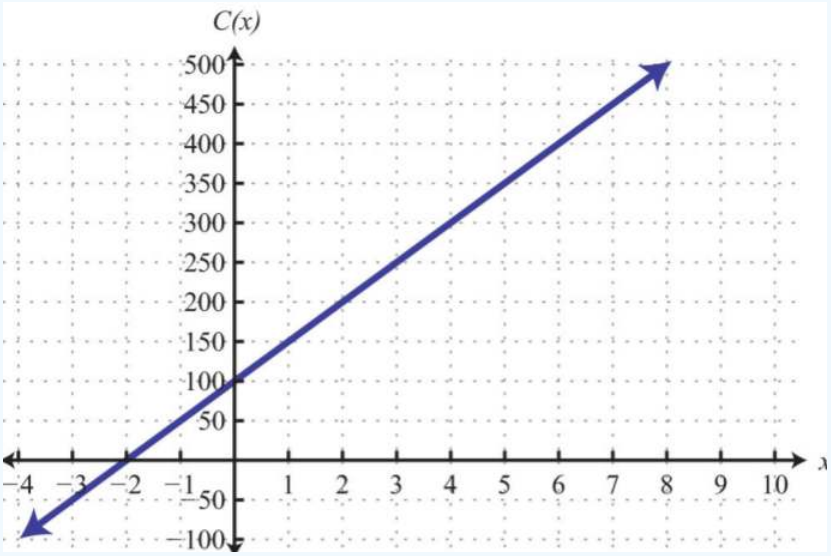


Figure 4.7.47

? Exercise 4.7.10 Discussion Board Topics

1. Is a vertical line a function? What are the domain and range of a vertical line?
2. Is a horizontal line a function? What are the domain and range of a horizontal line?
3. Come up with your own correspondence between real-world sets. Explain why it does or does not represent a function.
4. Can a function have more than one y-intercept? Explain.

Answer

1. Answers may vary
3. Answers may vary

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4.8: Linear Inequalities (Two Variables)

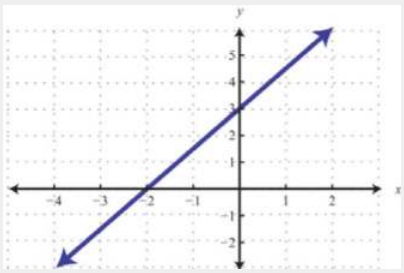
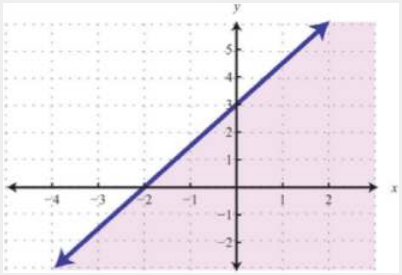
Learning Objectives

- Identify and check solutions to linear inequalities with two variables.
- Graph solution sets of linear inequalities with two variables.

Solutions to Linear Inequalities

We know that a linear equation with two variables has infinitely many ordered pair solutions that form a line when graphed. A linear inequality with two variables, on the other hand, has a solution set consisting of a region that defines half of the plane.

Table 4.8.1

Linear Equation	Linear Inequality
$y = \frac{3}{2}x + 3$	$y \leq \frac{3}{2}x + 3$
	
***Figure* 4.8.1**	***Figure* 4.8.2**

For the inequality, the line defines one boundary of the region that is shaded. This indicates that any ordered pair that is in the shaded region, including the boundary line, will satisfy the inequality. To see that this is the case, choose a few test points and substitute them into the inequality.

Table 4.8.2

Test point	$y \leq \frac{3}{2}x + 3$
$(0, 0)$	$0 \leq \frac{3}{2}(0) + 3$ $0 \leq 3$ ✓
$(2, 1)$	$1 \leq \frac{3}{2}(2) + 3$ $1 \leq 3 + 3$ $1 \leq 6$ ✓
$(-2, -1)$	$-1 \leq \frac{3}{2}(-2) + 3$ $-1 \leq -3 + 3$ $-1 \leq 0$ ✓

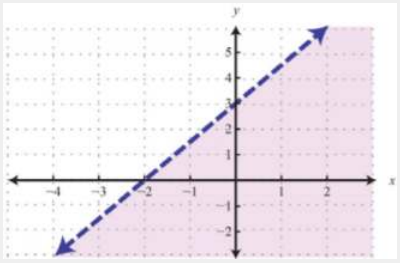
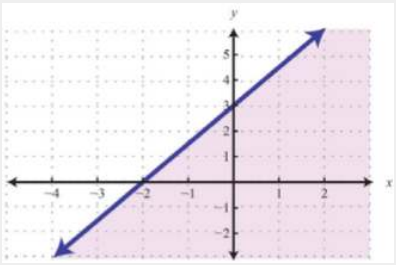
Also, we can see that ordered pairs outside the shaded region do not solve the linear inequality.

Test point	$y \leq \frac{3}{2}x + 3$
$(-2, 3)$	$3 \leq \frac{3}{2}(-2) + 3$ $3 \leq -3 + 3$ $3 \leq 0$ x

The graph of the solution set to a linear inequality is always a region. However, the boundary may not always be included in that set. In the previous example, the line was part of the solution set because of the "or equal to" part of the inclusive inequality $\leq$. If

we have a strict inequality $<$, we would then use a dashed line to indicate that those points are not included in the solution set.

Table 4.8.3

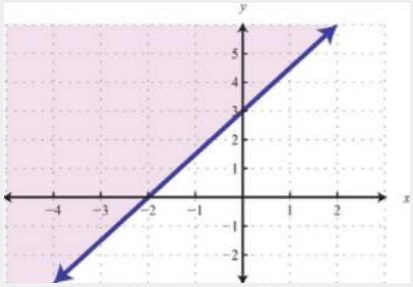
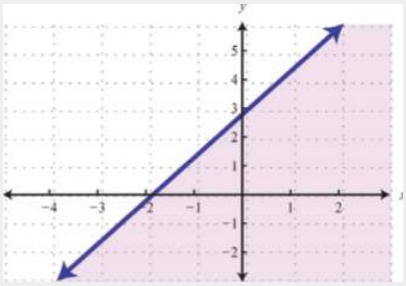
Non-Inclusive Boundary	Inclusive Boundary
$y < \frac{3}{2}x + 3$	$y \leq \frac{3}{2}x + 3$
	
***Figure* 4.8.3**	***Figure* 4.8.4**

Consider the point $(0, 3)$ on the boundary; this ordered pair satisfies the linear equation. It is the "or equal to" part of the inclusive inequality that makes it part of the solution set.

$y < \frac{3}{2}x + 3$	$y \leq \frac{3}{2}x + 3$
$3 < \frac{3}{2}(0) + 3$	$3 \leq \frac{3}{2}(0) + 3$
$3 < 0 + 3$	$3 \leq 0 + 3$
$3 < 3$ x	$3 \leq 3$ ✓

So far, we have seen examples of inequalities that were "less than." Now consider the following graphs with the same boundary:

Table 4.8.4

Greater Than (Above)	Less Than (Below)
$y \geq \frac{3}{2}x + 3$	$y \leq \frac{3}{2}x + 3$
	
***Figure* 4.8.5**	***Figure* 4.8.6**

Given the graphs above, what might we expect if we use the origin $(0, 0)$ as a test point?

$y \geq \frac{3}{2}x + 3$	$y \leq \frac{3}{2}x + 3$
$0 \geq \frac{3}{2}(0) + 3$	$0 \leq \frac{3}{2}(0) + 3$
$0 \geq 0 + 3$	$0 \leq 0 + 3$
$0 \geq 3$ x	$0 \leq 3$ ✓

? Exercise 4.8.1

Which of the ordered pairs $(-2, -1), (0, 0), (-2, 8), (2, 1)$,and $(4, 2)$ solve the inequality $y > -\frac{1}{2}x + 2$?

Answer

$(-2, 8)$ and $(4, 2)$

Graphing Solutions to Linear Inequalities

Solutions to linear inequalities are a shaded half-plane, bounded by a solid line or a dashed line. This boundary is either included in the solution or not, depending on the given inequality. If we are given a strict inequality, we use a dashed line to indicate that the boundary is not included. If we are given an inclusive inequality, we use a solid line to indicate that it is included. The steps for graphing the solution set for an inequality with two variables are outlined in the following example.

Example 4.8.1

Graph the solution set:

$$y > -3x + 1\,.$$

Solution:

Step 1: Graph the boundary line. In this case, graph a dashed line $y = -3x + 1$ because of the strict inequality. By inspection, we see that the slope is $m = -3 = -\frac{3}{1} = \frac{rise}{run}$ and the y-intercept is $(0, 1)$.

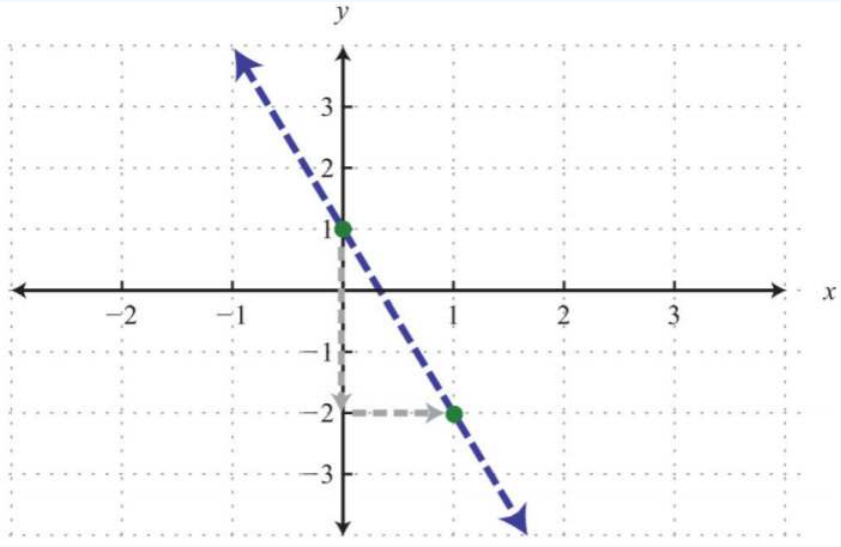


***Figure* 4.8.7**

Step 2: Test a point ***not*** on the boundary. A common test point is the origin $(0, 0)$. The test point helps us determine which half of the plane to shade.

Test point	$y > -3x + 1$
$(0, 0)$	$0 > -3(0) + 1$
	$0 > 1$ x

Step 3: Shade the region containing the solutions. Since the test point $(0, 0)$ was not a solution, it does not lie in the region containing all the ordered pair solutions. Therefore, shade the half of the plane that does not contain this test point. In this case, shade above the boundary line.

Answer:

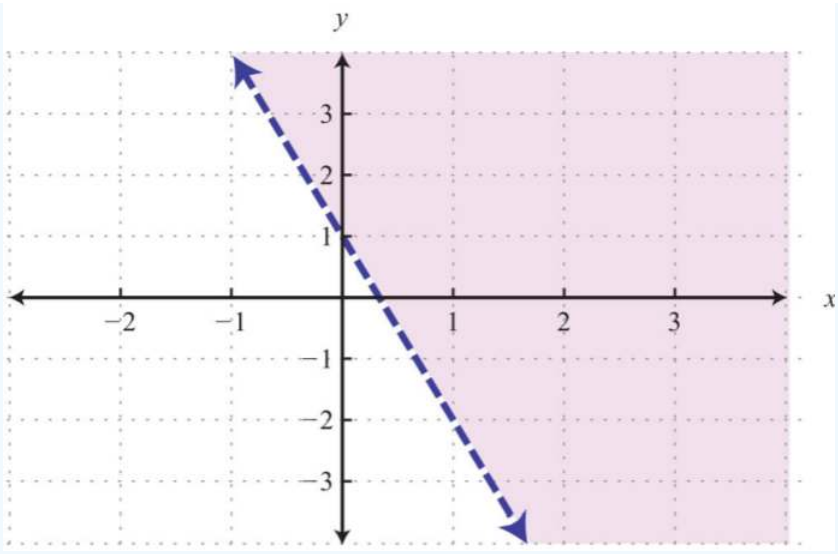


***Figure* 4.8.8**

Consider the problem of shading above or below the boundary line when the inequality is in slope-intercept form. If $y > mx + b$, then shade above the line. If $y < mx + b$, then shade below the line. Use this with caution; sometimes the boundary is given in standard form, in which case these rules do not apply.

Example 4.8.2

Graph the solution set:

$$2x - 5y \geq -10.$$

Solution:

Here the boundary is defined by the line $2x - 5y = -10$. Since the inequality is inclusive, we graph the boundary using a solid line. In this case, graph the boundary line using intercepts.

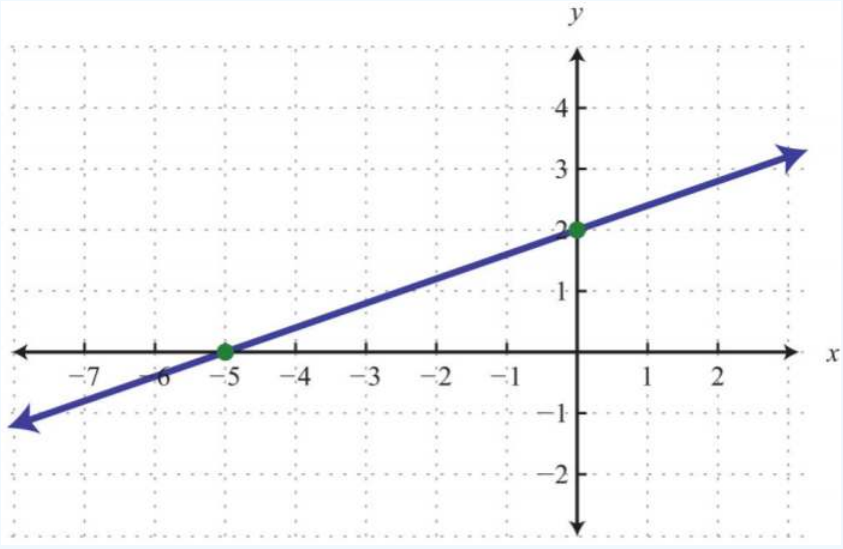


***Figure* 4.8.9**

Next, test a point; this helps decide which region to shade.

Test point	$2x-5y \geq -10$
$(0,0)$	$2(0)-5(0) \geq -10$
	$0 \geq -10$ ✓

Since the test point is in the solution set, shade the half of the plane that contains it.

Answer:

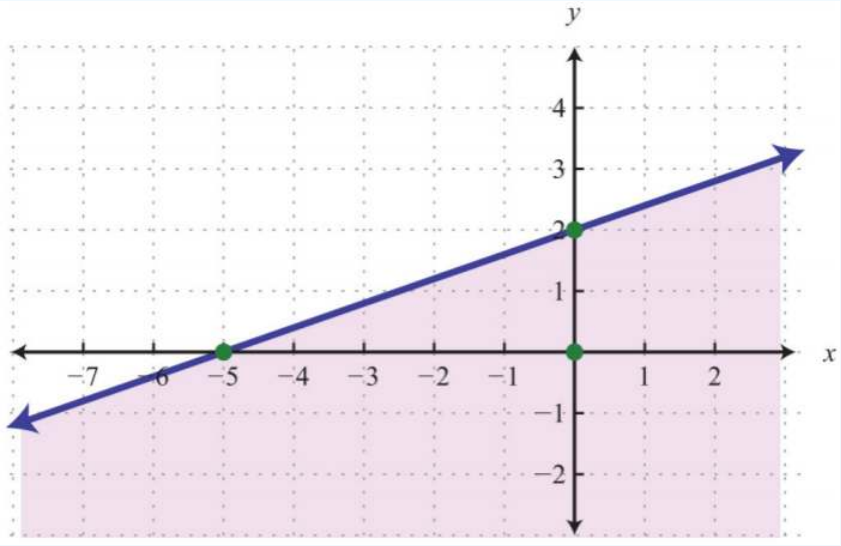


***Figure* 4.8.10**

In this example, notice that the solution set consists of all the ordered pairs below the boundary line. This may be counterintuitive because of the original $\geq$ in the inequality. This illustrates that it is a best practice to actually test a point. Solve for y and you see that the shading is correct.

In slope-intercept form, you can see that the region below the boundary line should be shaded. An alternate approach is to first express the boundary in slope-intercept form, graph it, and then shade the appropriate region.

Graph the solution set:

$$y < 2.$$

Solution:

First, graph the boundary line $y = 2$ with a dashed line because of the strict inequality.

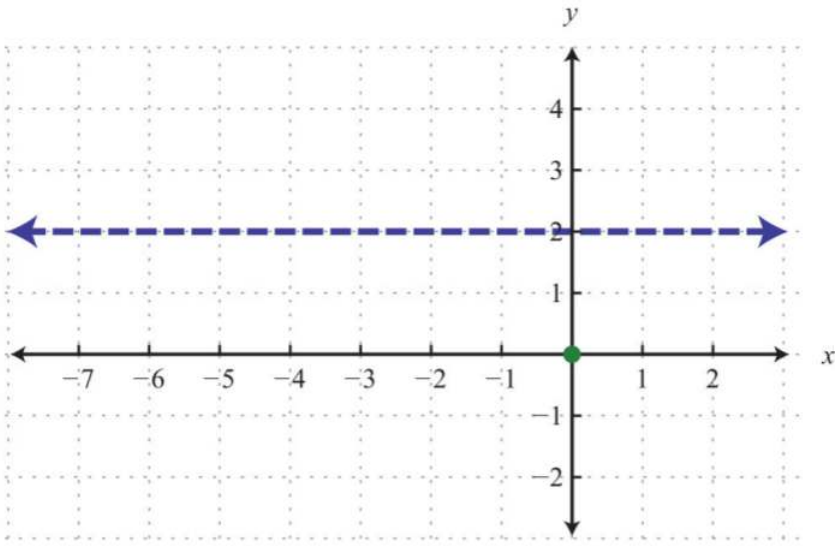


***Figure* 4.8.11**

Now, test a point.

Test point	$y < 2$
$(0, 0)$	$0 < 2$ ✓

In this case, shade the region that contains the test point.

Answer:

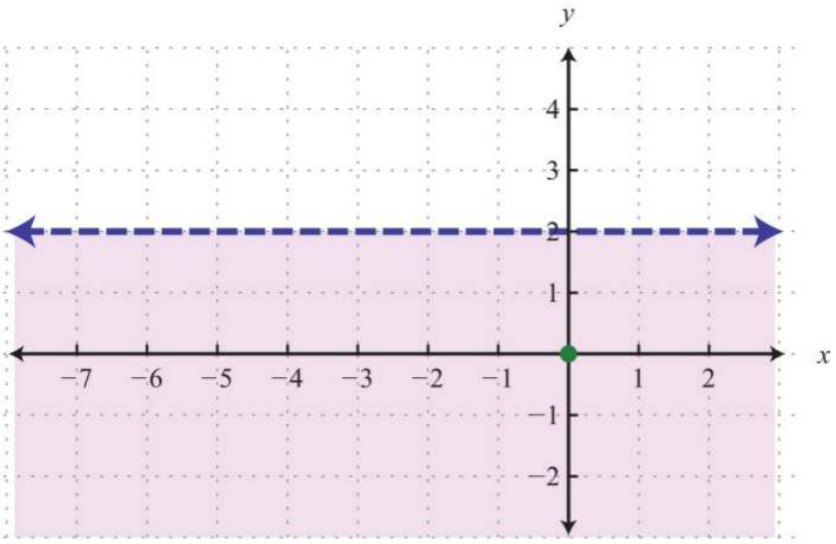


***Figure* 4.8.12**

? Exercise 4.8.2

Graph the solution set:

$$5x - y \leq 10.$$

Answer

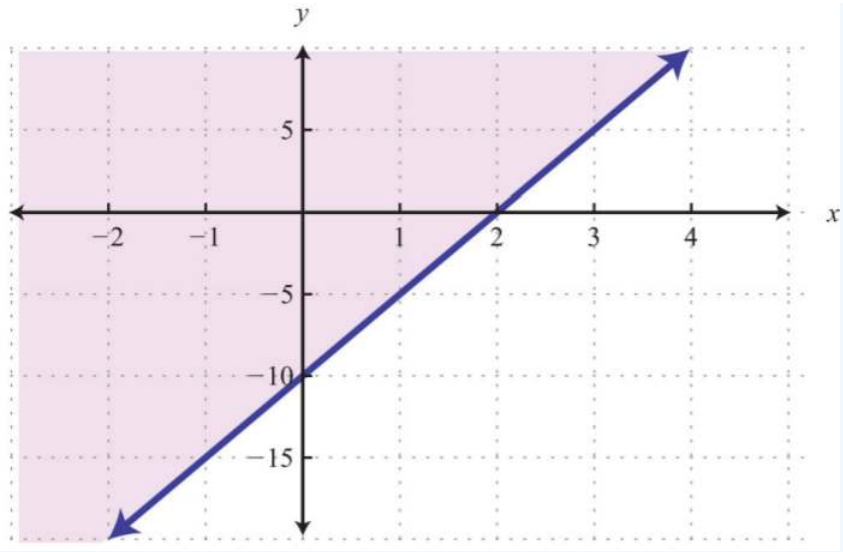


Figure 4.8.13

Key Takeaways

- Linear inequalities with two variables have infinitely many ordered pair solutions, which can be graphed by shading in the appropriate half of a rectangular coordinate plane.
- To graph the solution set of a linear inequality with two variables, first graph the boundary with a dashed or solid line depending on the inequality. If given a strict inequality, use a dashed line for the boundary. If given an inclusive inequality, use a solid line. Next, choose a test point not on the boundary. If the test point solves the inequality, then shade the region that contains it; otherwise, shade the opposite side.
- When graphing the solution sets of linear inequalities, it is a good practice to test values in and out of the solution set as a check.

? Exercise 4.8.3 Solutions to Linear Inequalities (Two Variables)

Is the ordered pair a solution to the given inequality?

1. $y < 5x + 1\,;\,(0, 0)$
2. $y > -\frac{1}{2}x - 4\,;\,(0, -2)$
3. $y \leq \frac{2}{3}x + 1\,;\,(6, 5)$
4. $y \geq -\frac{1}{3}x - 5\,;\,(-3, -8)$
5. $y < \frac{1}{5}x - \frac{1}{3}\,;\,(-\frac{1}{2}, -1)$
6. $4x - 3y \leq 2;\,(-2, -1)$
7. $-x + 4y > 7;\,(0, 0)$
8. $7x - 3y < 21;\,(5, -3)$
9. $y > -5;\,(-3, -1)$
10. $x \leq 0;\,(0, 7)$

Answer

1. Yes
3. Yes
5. Yes
7. No
9. Yes

Exercise 4.8.4 Graphing Solutions to Linear Inequalities

Graph the solution set.

1. $y < -3x + 3$
2. $y < -\frac{2}{3}x + 4$
3. $y \geq -\frac{1}{2}x$
4. $y \geq \frac{4}{5}x - 8$
5. $y \leq 8x - 7$
6. $y > -5x + 3$
7. $y > -x + 4$
8. $y > x - 2$
9. $y \geq -1$
10. $y < -3$
11. $x < 2$
12. $x \geq 2$
13. $y \leq \frac{3}{4}x - \frac{1}{2}$
14. $y > -\frac{3}{2}x + \frac{5}{2}$
15. $-2x + 3y > 6$
16. $7x - 2y > 14$
17. $5x - y < 10$
18. $x - y < 0$
19. $3x - 2y \geq 0$
20. $x - 5y \leq 0$
21. $-x + 2y \leq -4$
22. $-x + 2y \leq 3$
23. $2x - 3y \geq -1$
24. $5x - 4y < -3$
25. $\frac{1}{2}x - \frac{1}{3}y < 1$
26. $\frac{1}{2}x - \frac{1}{10}y \geq \frac{1}{2}$
27. $x \geq -2y$
28. $x < 2y + 3$
29. $3x - y + 2 > 0$
30. $3 - y - 2x < 0$
31. $-4x \leq 12 - 3y$
32. $5x \leq -4y - 12$
33. Write an inequality that describes all points in the upper half-plane above the x-axis.
34. Write an inequality that describes all points in the lower half-plane below the x-axis.
35. Write an inequality that describes all points in the half-plane left of the y-axis.
36. Write an inequality that describes all points in the half-plane right of the y-axis.
37. Write an inequality that describes all ordered pairs whose y-coordinates are at least 2.
38. Write an inequality that describes all ordered pairs whose x-coordinate is at most 5.

Answer

1.

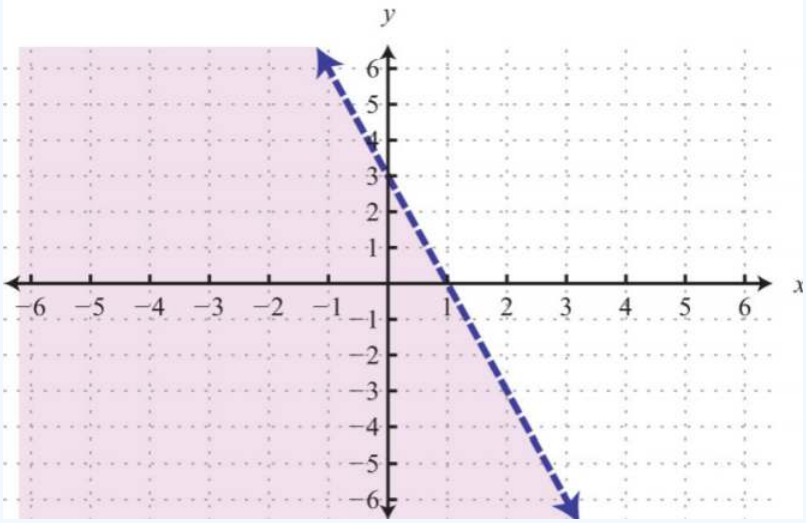


Figure 4.8.14

3.

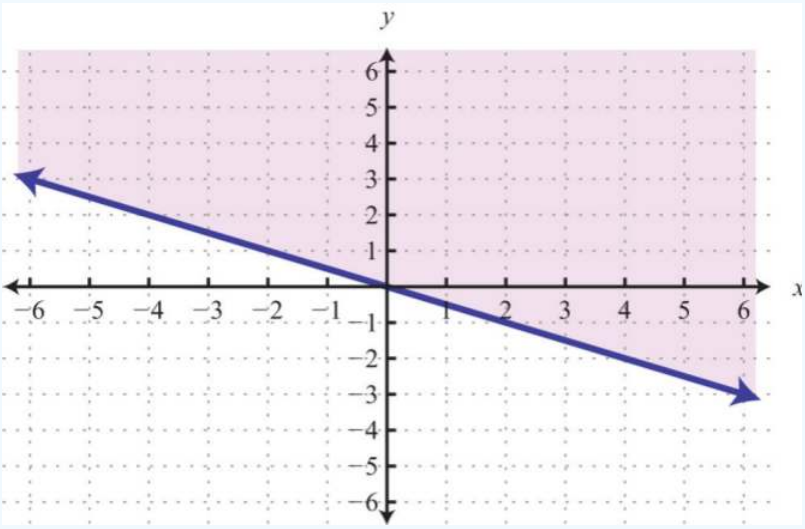


Figure 4.8.15

5.

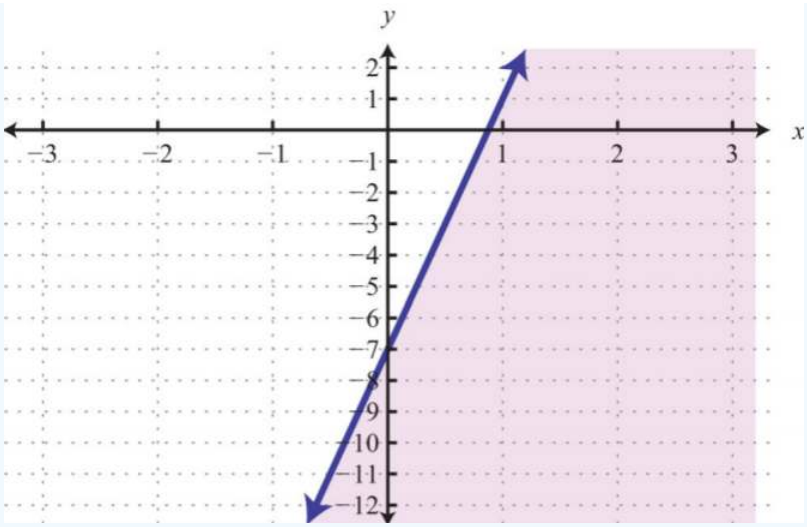


Figure 4.8.16

7.

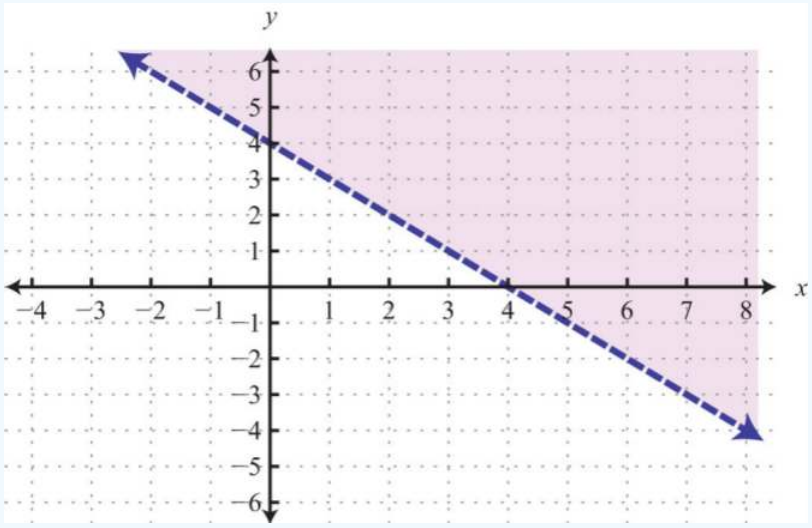


Figure 4.8.17

9.

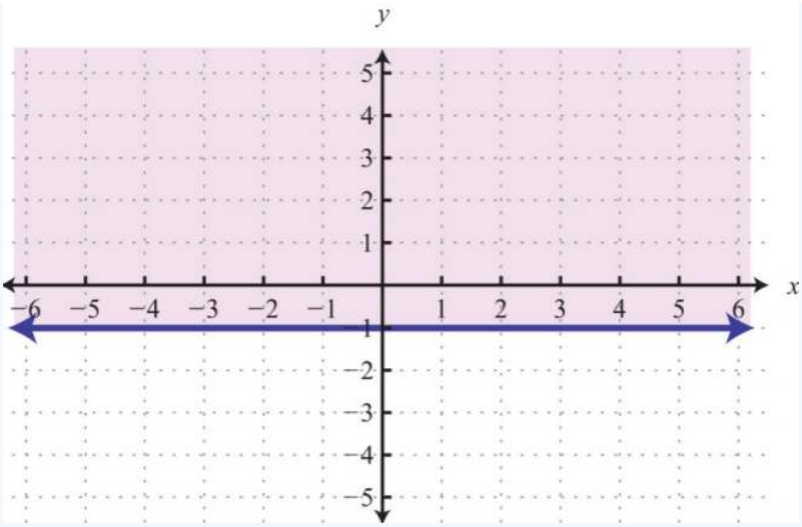


Figure 4.8.18

11.

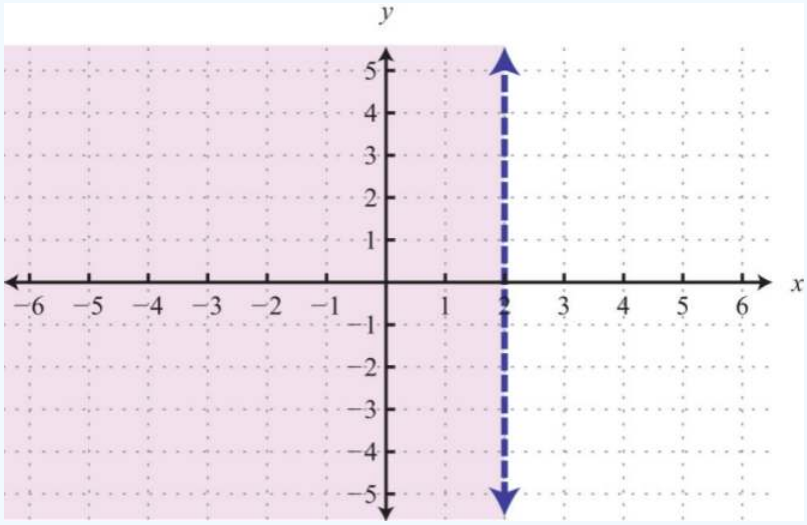


Figure 4.8.19

13.

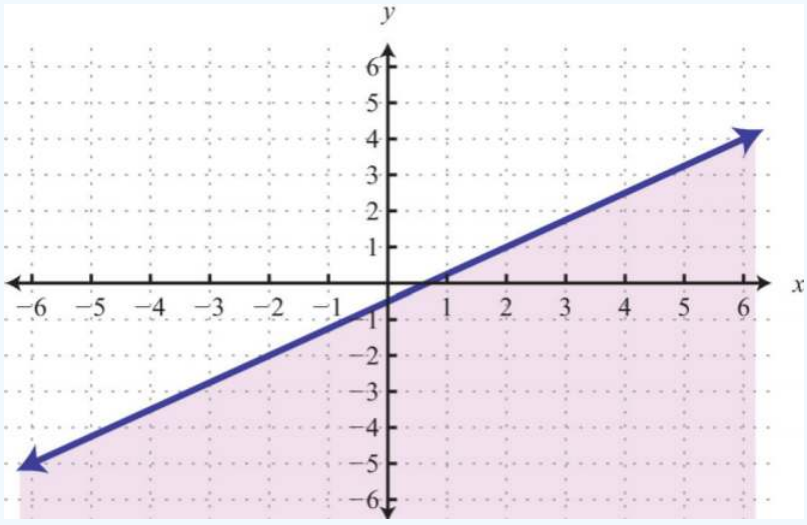


Figure 4.8.20

15.

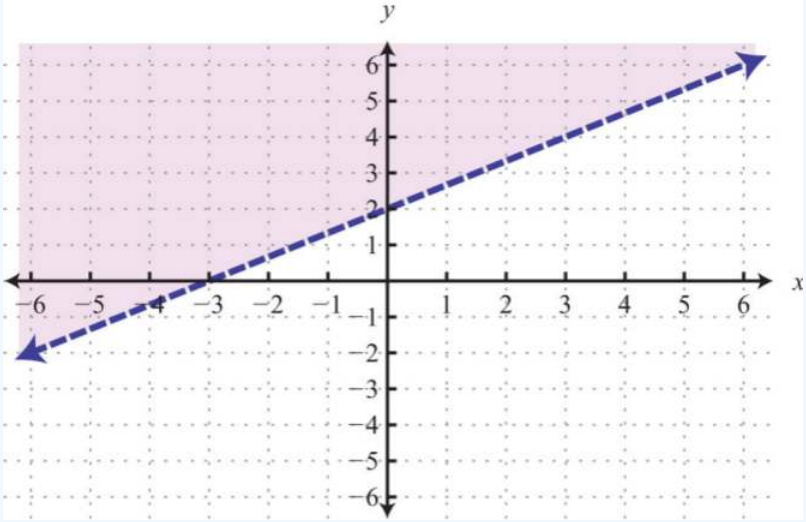


Figure 4.8.21

17.

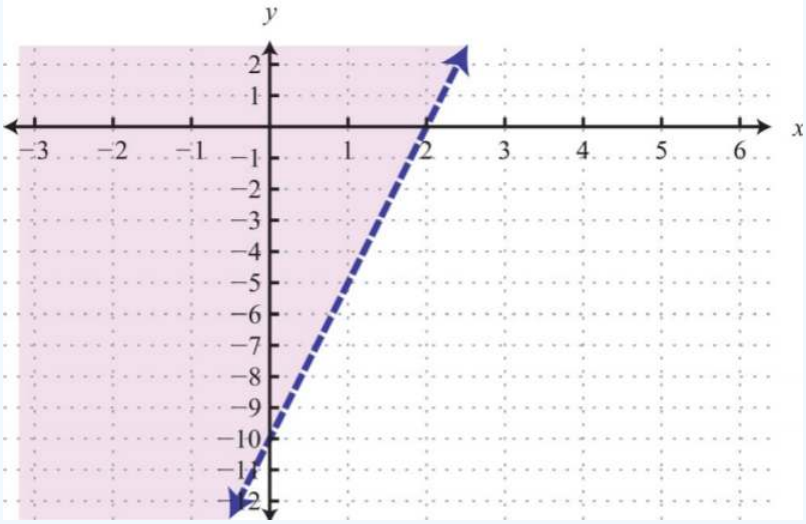


Figure 4.8.22

19.

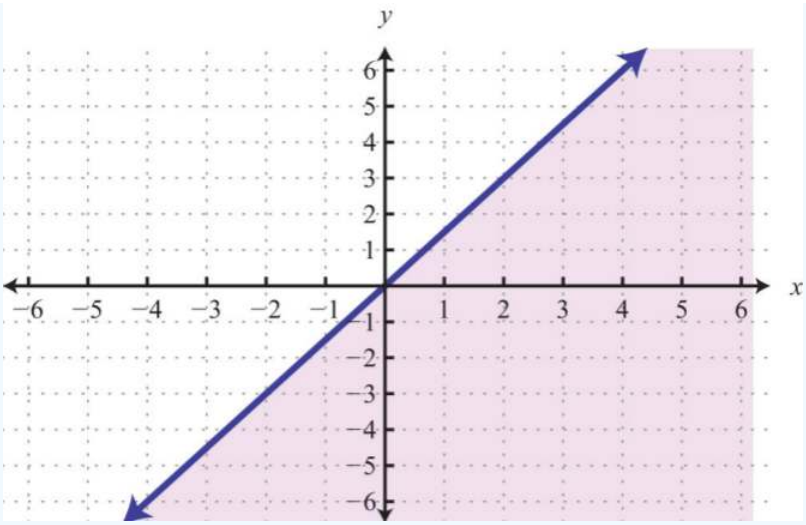


Figure 4.8.23

21.

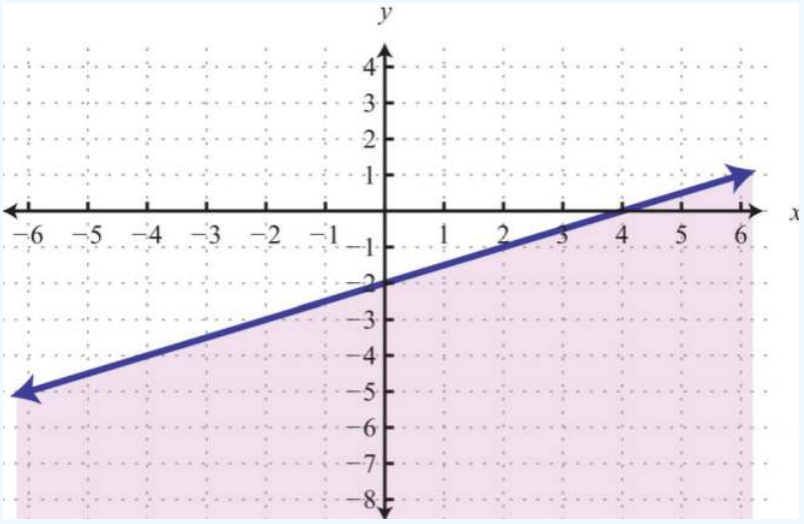


Figure 4.8.24

23.

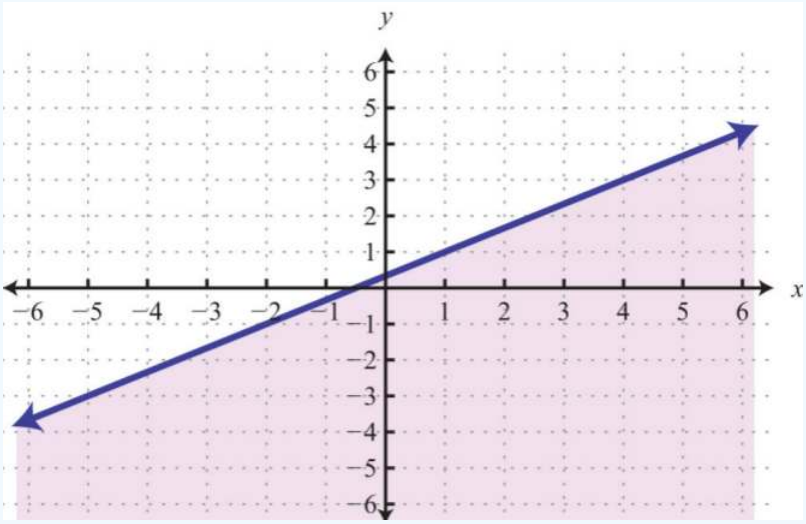


Figure 4.8.25

25.

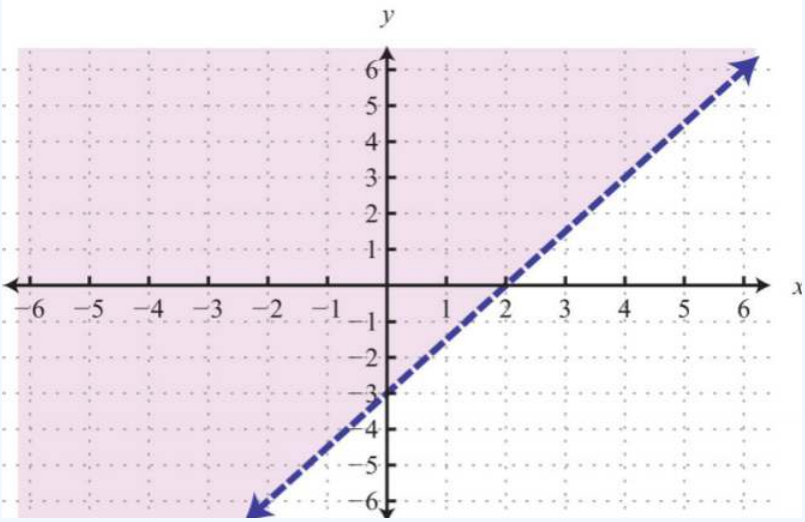


Figure 4.8.26

27.

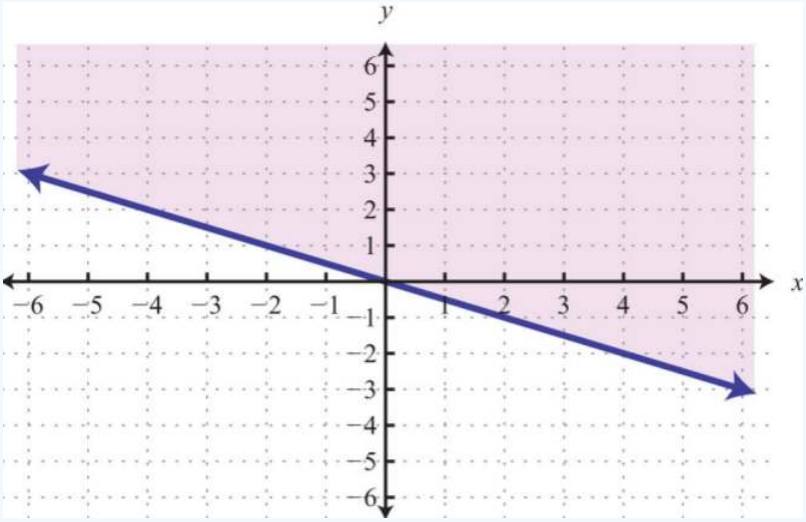


Figure 4.8.27

29.

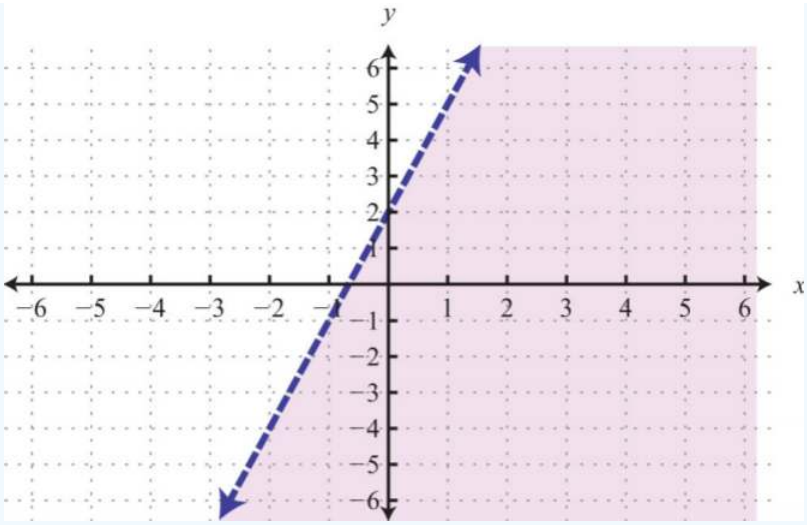


Figure 4.8.28

31.

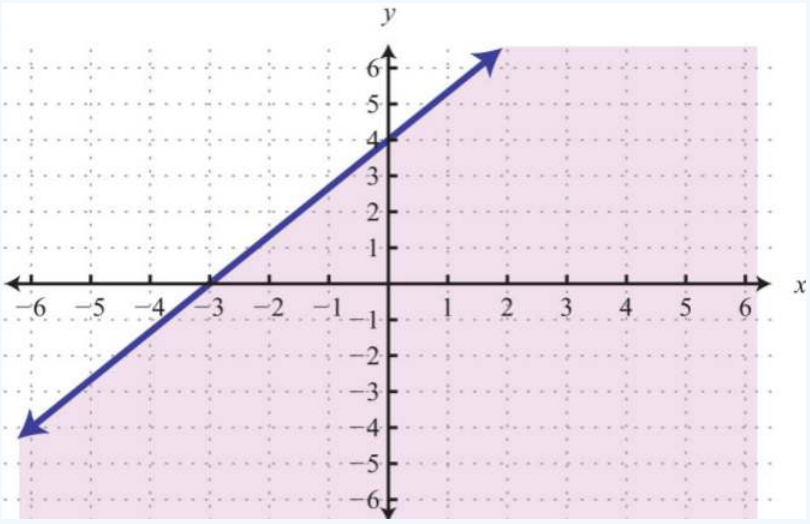


Figure 4.8.29

33. $y > 0$

35. $x < 0$

37. $y \geq 2$

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4.E: Review Exercises and Sample Exam

Review Exercises

Exercise $4.E.1$ Rectangular Coordinate System

Graph the given set of ordered pairs.

1. $\{(-3,4),(-4,0),(0,3),(2,4)\}$
2. $\{(-5,5),(-3,-1),(0,0),(3,2)\}$
3. Graph the points $(-3,5),(-3,-3),$ and $(3,-3)$ on a rectangular coordinate plane. Connect the points and calculate the area of the shape.
4. Graph the points $(-4,1),(0,1),(0,-2),$and $(-4,-2)$ on a rectangular coordinate plane. Connect the points and calculate the area of the shape.
5. Graph the points $(1,0),(4,0),(1,-5),$and $(4,-5)$ on a rectangular coordinate plane. Connect the points and calculate the perimeter of the shape.
6. Graph the points $(-5,2),(-5,-3),(1,2),$and $(1,-3)$ on a rectangular coordinate plane. Connect the points and calculate the perimeter of the shape.

Answer

1.

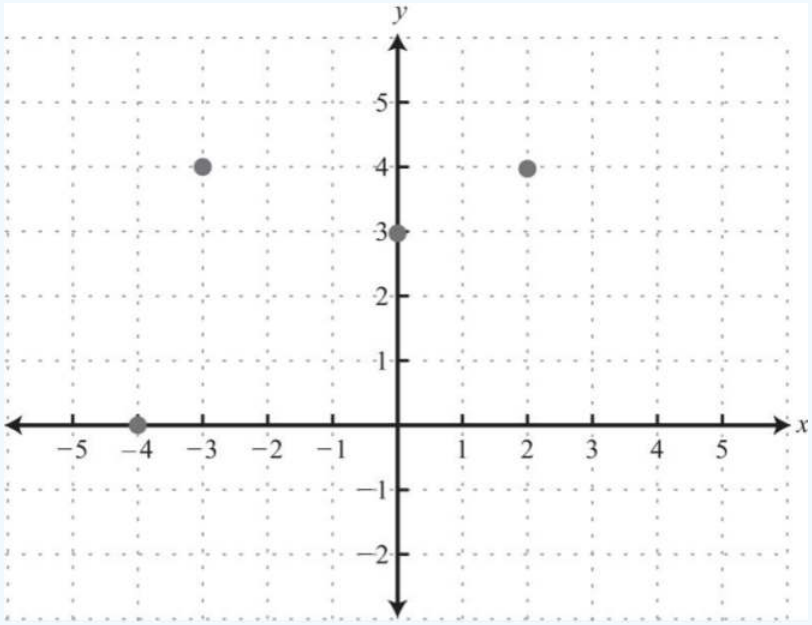


Figure 3.E.1

3. Area: 24 square units

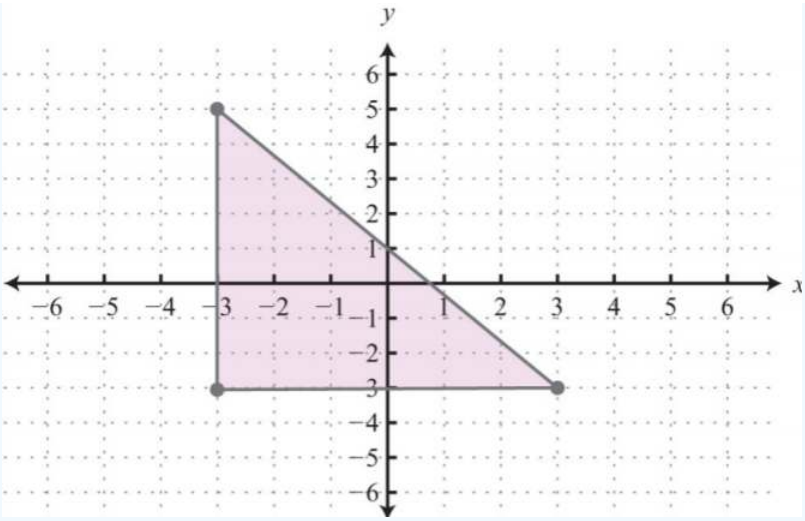


Figure 3.E.2

5. Perimeter: 16 units

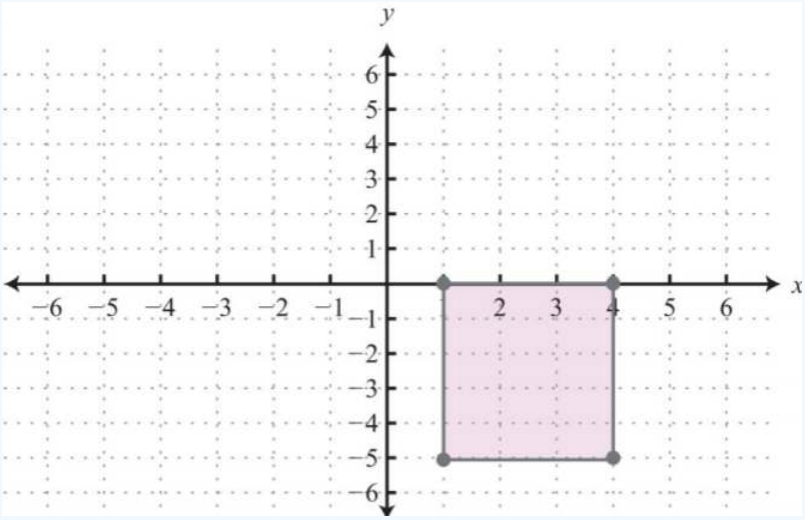


Figure 3.E.3

? Exercise $4.E.2$ Rectangular Coordinate System

Calculate the distance between the given two points.

1. $(-1,-2)$ and $(5,6)$
2. $(2,-5)$ and $(-2,-2)$
3. $(-9,-3)$ and $(-8,4)$
4. $(-1,3)$ and $(1,-3)$

Answer

1. 10 units
3. $5\sqrt{2}$ units

Exercise $4.E.3$ Rectangular Coordinate System

Calculate the midpoint between the given points.

1. $(-1, 3)$ and $(5, -7)$
2. $(6, -3)$ and $(-8, -11)$
3. $(7, -2)$ and $(-6, -1)$
4. $(-6, 0)$ and $(0, 0)$
5. Show algebraically that the points $(-1, -1), (1, -3)$, and $(2, 0)$ form an isosceles triangle.
6. Show algebraically that the points $(2, -1), (6, 1)$, and $(5, 3)$ form a right triangle.

Answer

1. $(2, -2)$
3. $(\frac{1}{2}, -\frac{3}{2})$
5. Answers may vary

Exercise $4.E.4$ Graph by Plotting Points

Determine whether the given point is a solution.

1. $-5x + 2y = 7$; $(1, -1)$
2. $6x - 5y = 4$; $(-1, -2)$
3. $y = \frac{3}{4}x + 1$; $(-\frac{2}{3}, \frac{1}{2})$
4. $y = -\frac{3}{5}x - 2$; $(10, -8)$

Answer

1. No
3. Yes

Exercise $4.E.5$ Graph by Plotting Points

Find at least five ordered pair solutions and graph.

1. $y = -x + 2$
2. $y = 2x - 3$
3. $y = \frac{1}{2}x - 2$
4. $y = -\frac{2}{3}x$
5. $y = 3$
6. $x = -3$
7. $x - 5y = 15$
8. $2x - 3y = 12$

Answer

1.

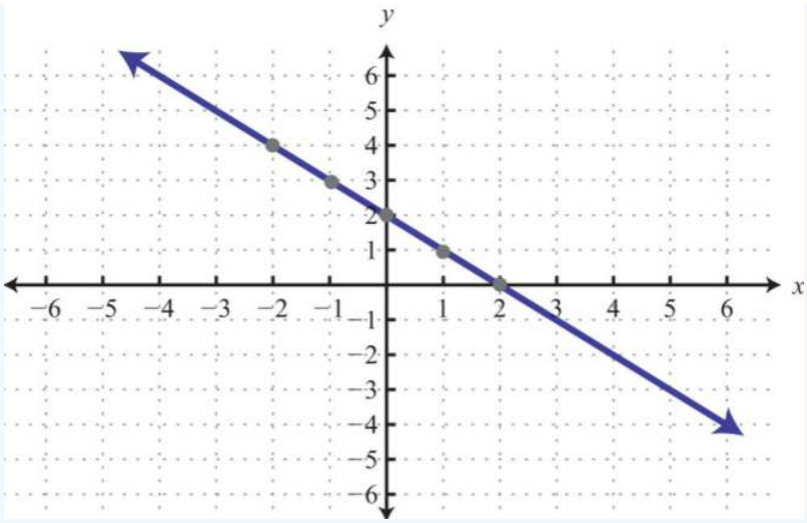


Figure 3.E.4

3.

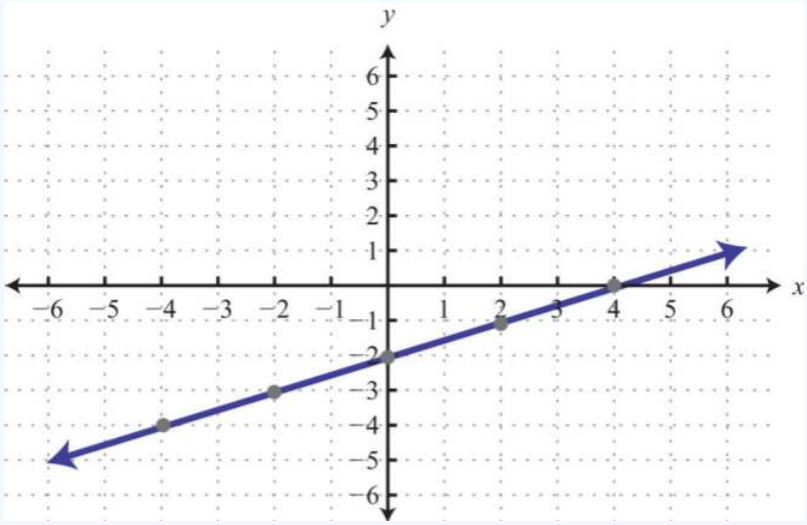


Figure 3.E.5

5.

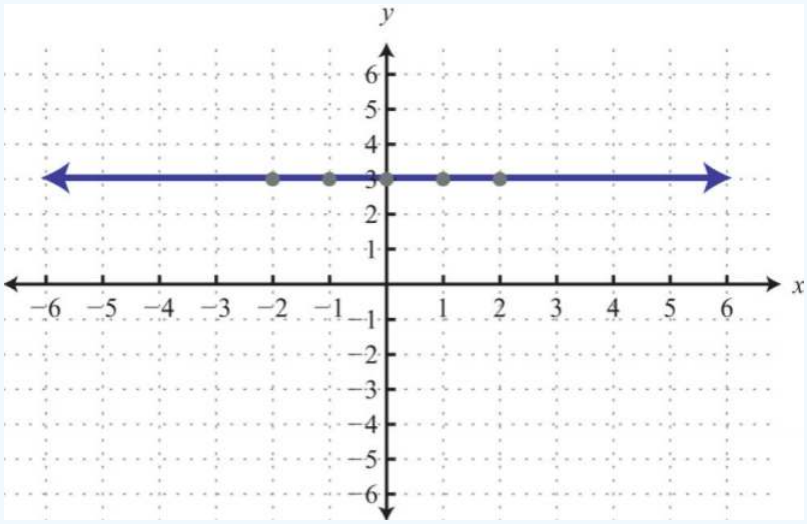


Figure 3.E.6

7.

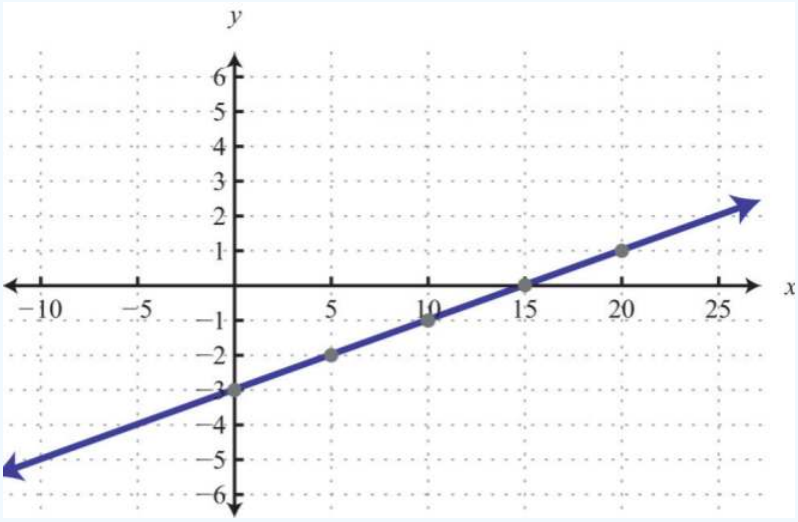


Figure 3.E.7

Exercise $4.E.6$ Graph Using Intercepts

Given the graph, find the x- and y- intercepts.

1.

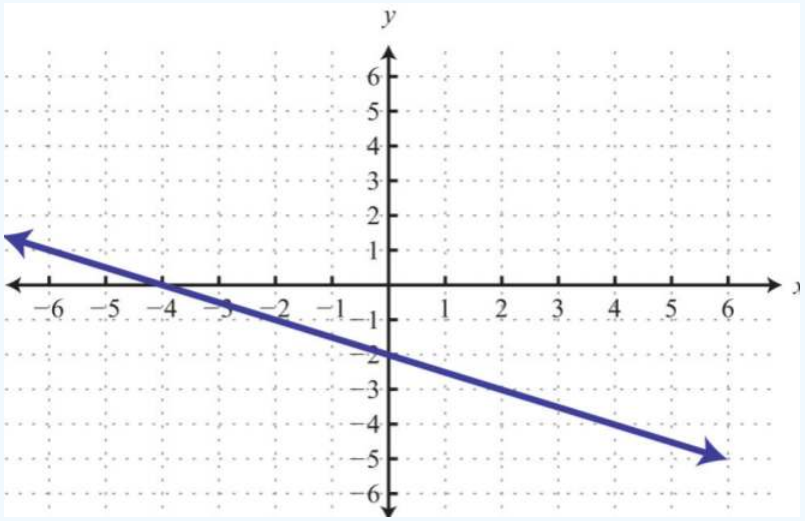


Figure 3.E.8

2.

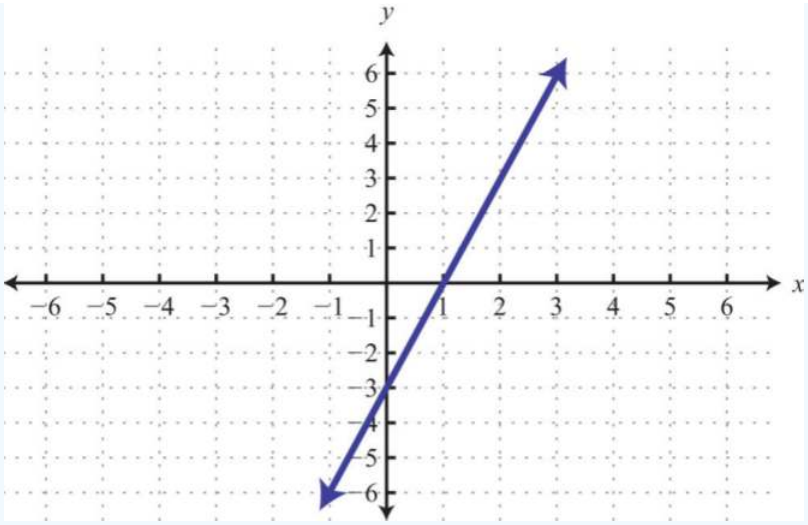


Figure 3.E.9

3.

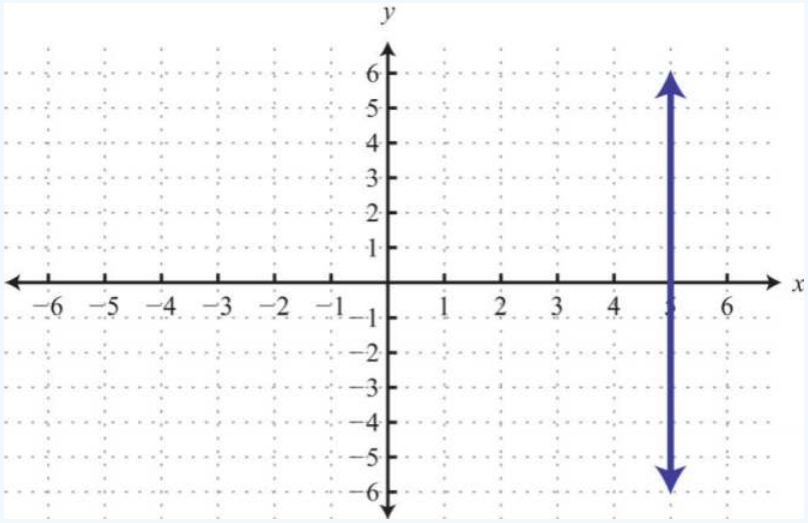


Figure 3.E.10

4.

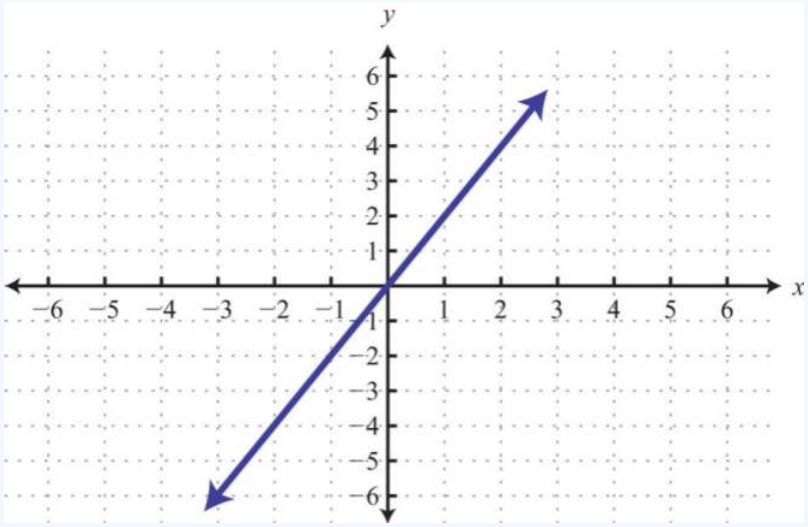


Figure 3.E.11

Answer

1. y-intercept: $(0, -2)$; x-intercept: $(-4, 0)$
3. y-intercept: none; x-intercept: $(5, 0)$

Exercise $4.E.7$ Graph Using Intercepts

Find the intercepts and graph them.

1. $3x - 4y = 12$
2. $2x - y = -4$
3. $\frac{1}{2}x - \frac{1}{3}y = 1$
4. $-\frac{1}{2}x + \frac{2}{3}y = 2$
5. $y = -\frac{5}{3}x + 5$
6. $y = -3x + 4$

Answer

1.

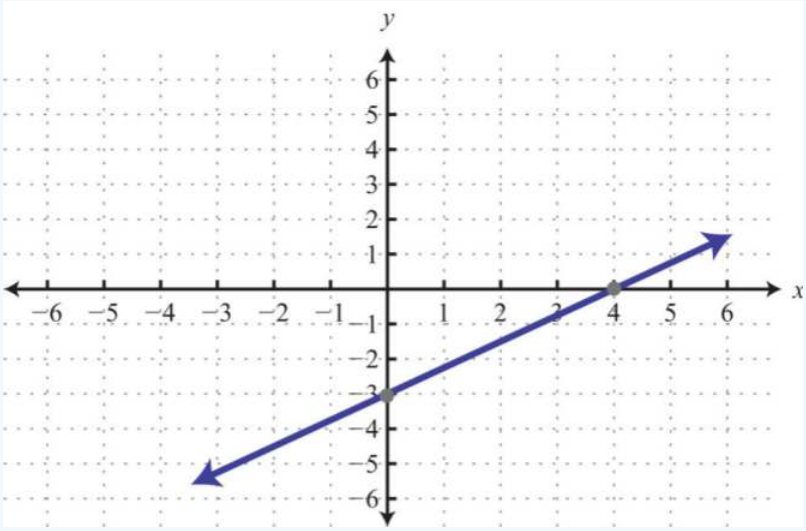


Figure 3.E.12

3.

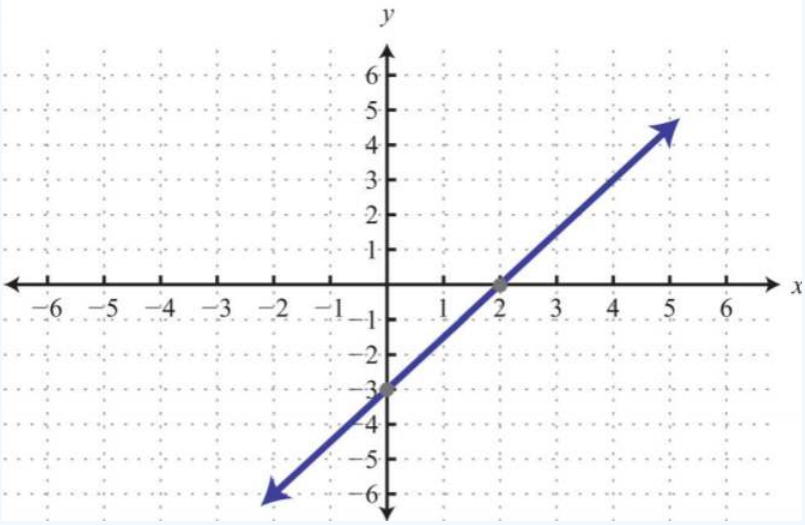


Figure 3.E.13

5.

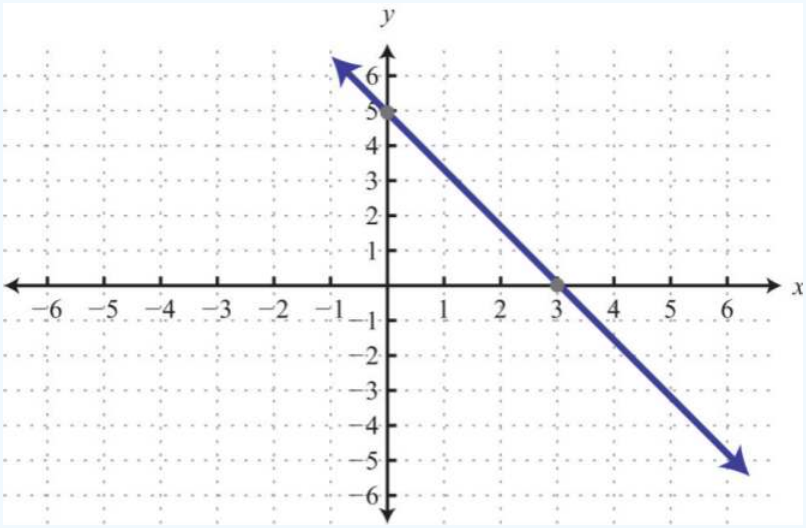


Figure 3.E.14

? Exercise $4.E.8$ Graph Using the y-Intercept and Slope

Given the graph, determine the slope and y-intercept.

1.

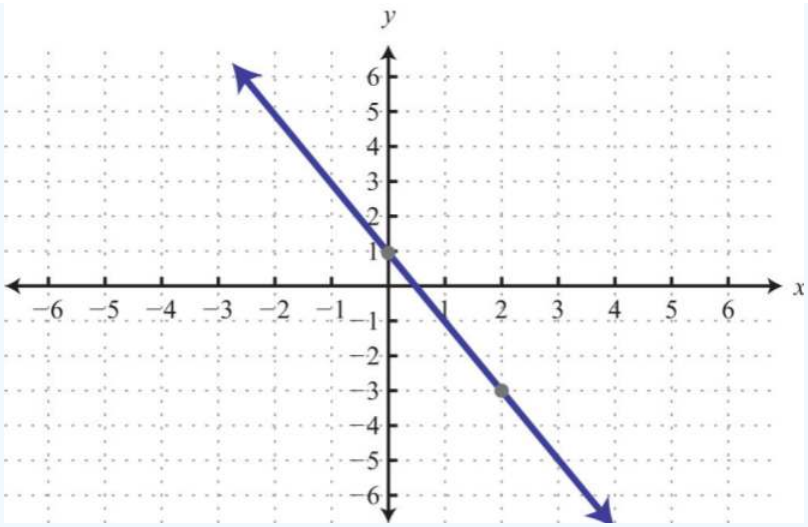


Figure 3.E.15

2.

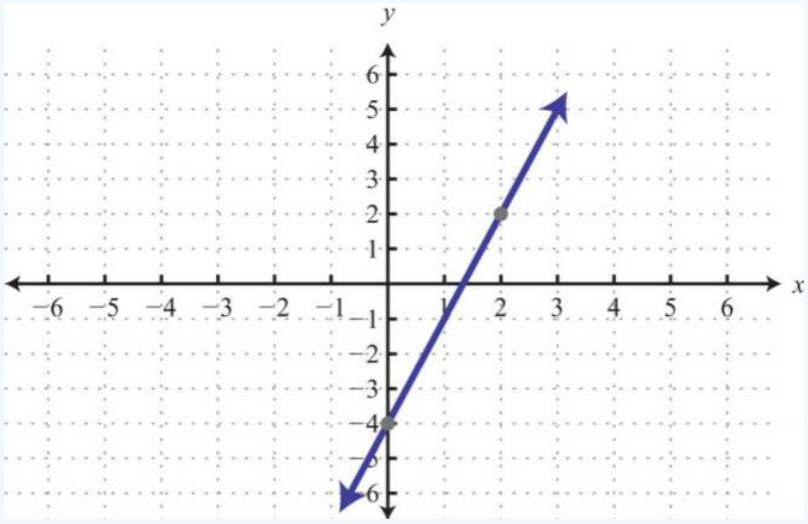


Figure 3.E.16

Answer

1. y-intercept: $(0, 1)$; slope: -2

Exercise $4.E.9$ Graph Using the y-Intercept and Slope

Determine the slope, given two points.

1. $(-3, 8)$ and $(5, -6)$
2. $(0, -5)$ and $(-6, 3)$
3. $(\frac{1}{2}, -\frac{2}{3})$ and $(\frac{1}{4}, -\frac{1}{3})$
4. $(5, -\frac{3}{4})$ and $(2, -\frac{3}{4})$

Answer

1. $-\frac{7}{4}$
3. $-\frac{4}{3}$

Exercise $4.E.10$ Graph Using the y-Intercept and Slope

Express in slope-intercept form and identify the slope and y-intercept.

1. $12x - 4y = 8$
2. $3x - 6y = 24$
3. $-\frac{1}{3}x + \frac{3}{4}y = 1$
4. $-5x + 3y = 0$

Answer

1. $y = 3x - 2$; slope: 3; y-intercept $(0, -2)$
3. $y = \frac{4}{9}x + \frac{4}{3}$; slope: $\frac{4}{9}$; y-intercept $(0, \frac{4}{3})$

Exercise $4.E.11$ Graph Using the y-Intercept and Slope

1. $y = -x + 3$
2. $y = 4x - 1$
3. $y = -2x$
4. $y = -\frac{5}{2}x + 3$
5. $2x - 3y = 9$
6. $2x + \frac{3}{2}y = 3$
7. $y = 0$
8. $x - 4y = 0$

Answer

1.

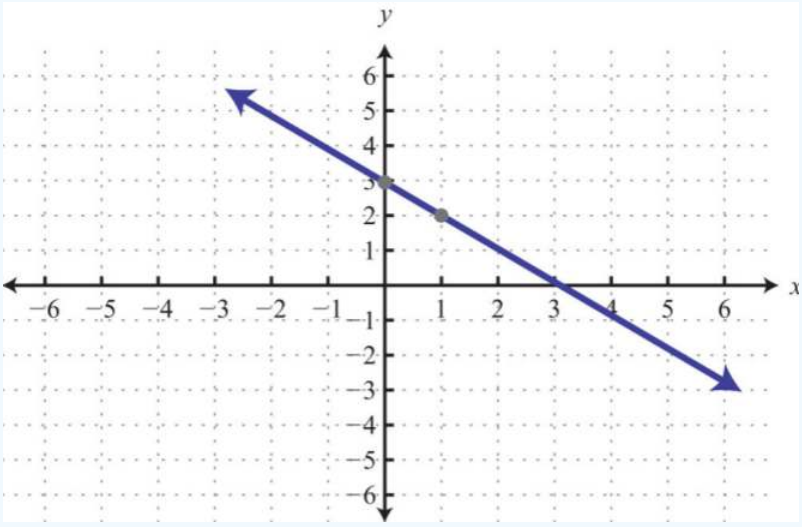


Figure 3.E.17

3.

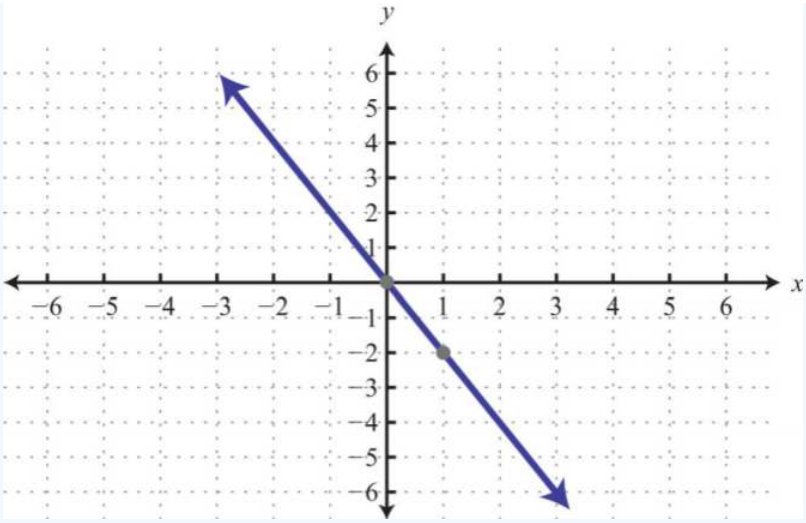


Figure 3.E.18

5.

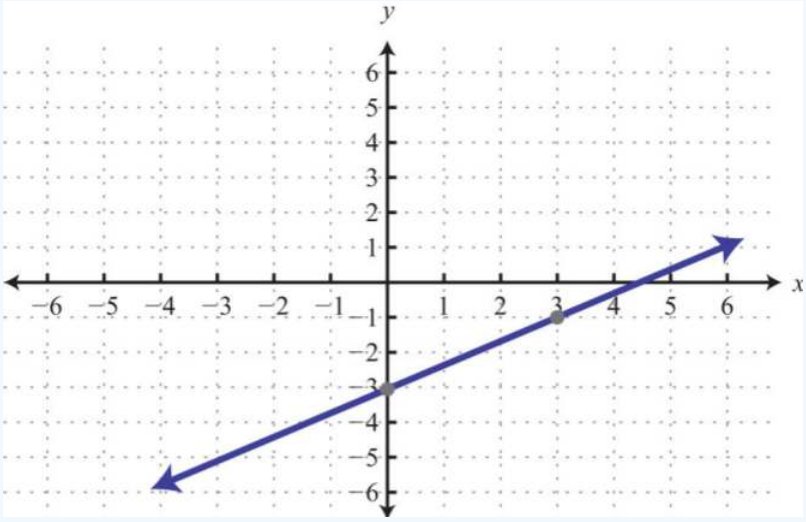


Figure 3.E.19

7.

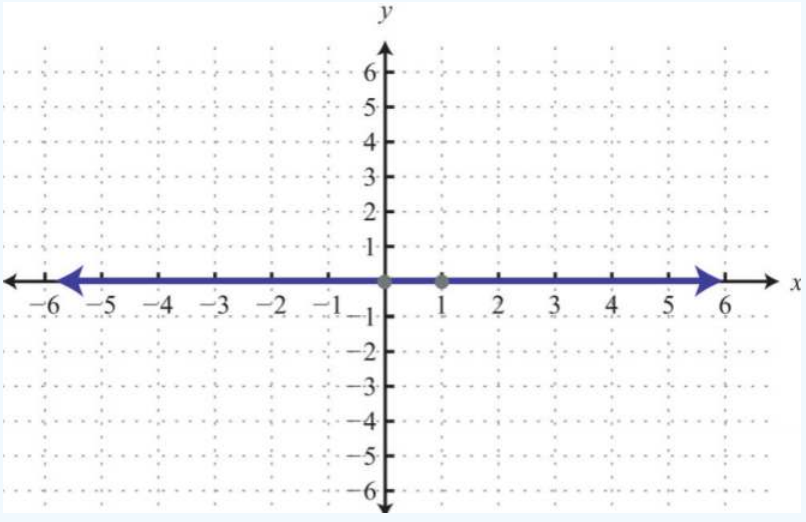


Figure 3.E.20

Exercise $4.E.12$ Finding Linear Equations

Given the graph, determine the equation of the line.

1.

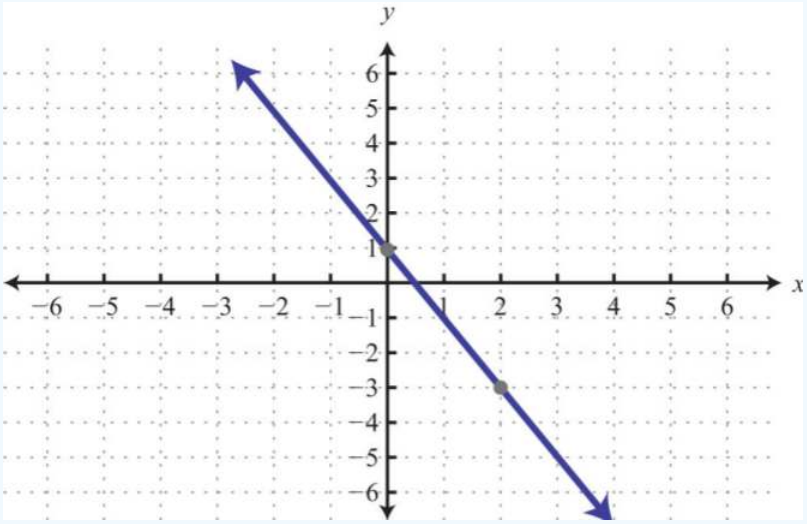


Figure 3.E.21

2.

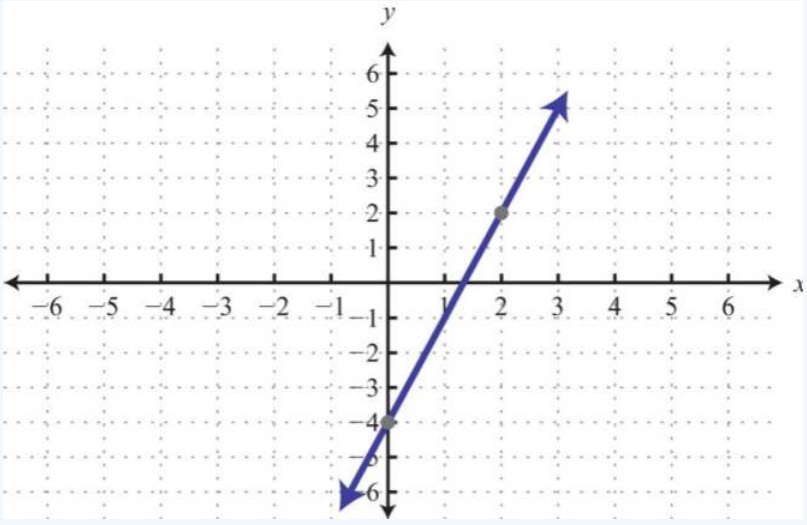


Figure 3.E.22

3.

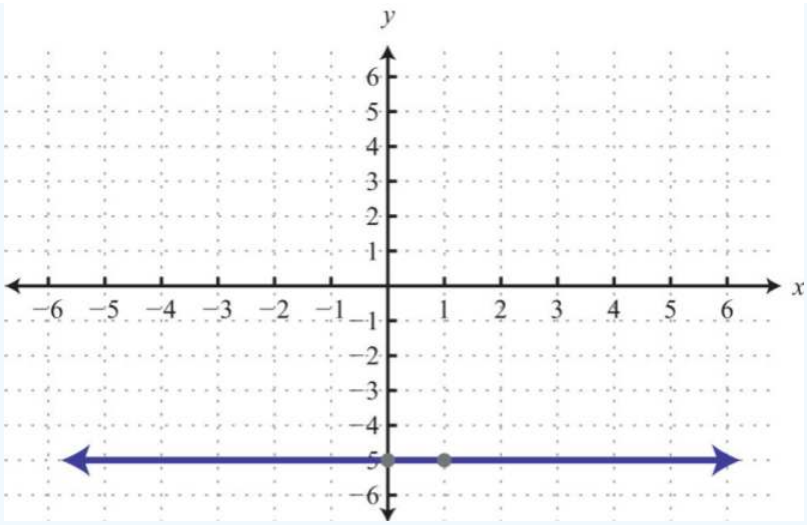


Figure 3.E.23

4.

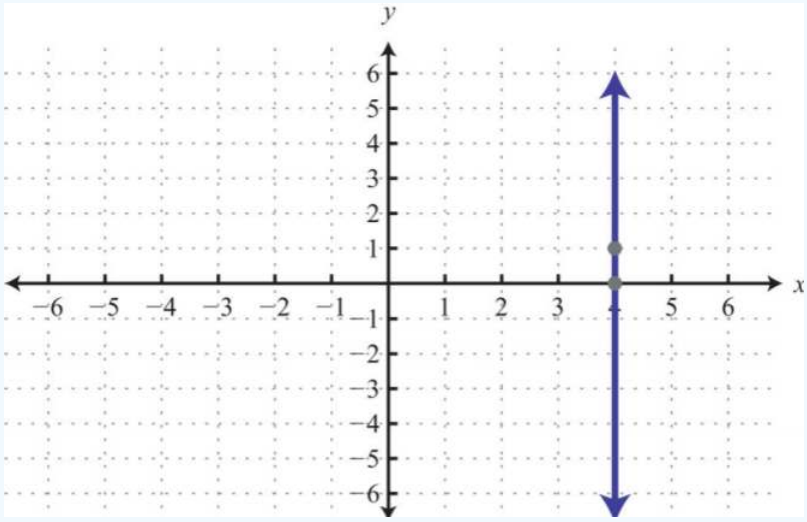


Figure 3.E.24

Answer

1. $y = -2x + 1$
3. $y = -5$

Exercise $4.E.13$ Finding Linear Equations

Find the equation of a line, given the slope and a point on the line.

1. $m = \frac{1}{2}; (-4, 8)$
2. $m = -\frac{1}{5}; (-5, -9)$
3. $m = \frac{2}{3}; (1, -2)$
4. $m = -\frac{3}{4}; (2, -3)$

Answer

1. $y = \frac{1}{2}x + 10$

3. $y = \frac{2}{3}x - \frac{8}{3}$

Exercise $4.E.14$ Finding Linear Equations

Find the equation of the line given two points on the line.

1. $(-5, -5)$ and $(10, 7)$
2. $(-6, 12)$ and $(3, -3)$
3. $(2, -1)$ and $(-2, 2)$
4. $(\frac{5}{2}, -2)$ and $(-5, \frac{5}{2})$
5. $(7, -6)$ and $(3, -6)$
6. $(10, 1)$ and $(10, -3)$

Answer

1. $y = \frac{4}{5}x - 1$
3. $y = -\frac{3}{4}x + \frac{1}{2}$
5. $y = -6$

Exercise $4.E.15$ Parallel and Perpendicular Lines

Determine if the lines are parallel, perpendicular, or neither.

1. $\begin{cases} -3x + 7y = 14 \\ 6x - 14y = 42 \end{cases}$
2. $\begin{cases} 2x + 3y = 18 \\ 2x - 3y = 36 \end{cases}$
3. $\begin{cases} x + 4y = 2 \\ 8x - 2y = -1 \end{cases}$
4. $\begin{cases} y = 2 \\ x = 2 \end{cases}$

Answer

1. Parallel
3. Perpendicular

Exercise $4.E.16$ Parallel and Perpendicular Lines

Find the equation of the line in slope-intercept form.

1. Parallel to $5x - y = 15$ and passing through $(-10, -1)$.
2. Parallel to $x - 3y = 1$ and passing through $(2, -2)$.
3. Perpendicular to $8x - 6y = 4$ and passing through $(8, -1)$.
4. Perpendicular to $7x + y = 14$ and passing through $(5, 1)$.
5. Parallel to $y = 1$ and passing through $(4, -1)$.
6. Perpendicular to $y = 1$ and passing through $(4, -1)$.

Answer

1. $y = 5x + 49$
3. $y = -\frac{3}{4}x + 5$
5. $y = -1$

Exercise $4.E.17$ Introduction to Functions

Determine the domain and range and state whether it is a function or not.

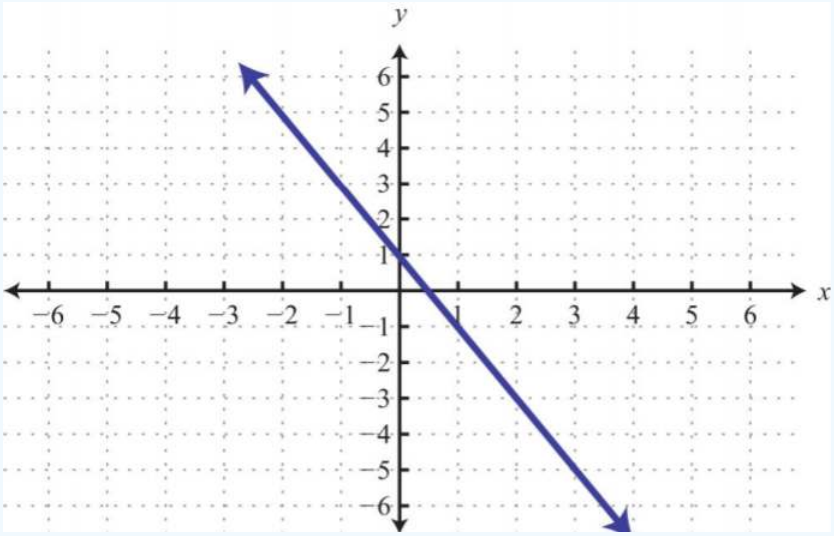
1. $\{(-10,-1),(-5,2),(5,2)\}$
2. $\{(-12,4),(-1,-3),(-1,-2)\}$
3. 

Figure 3.E.25

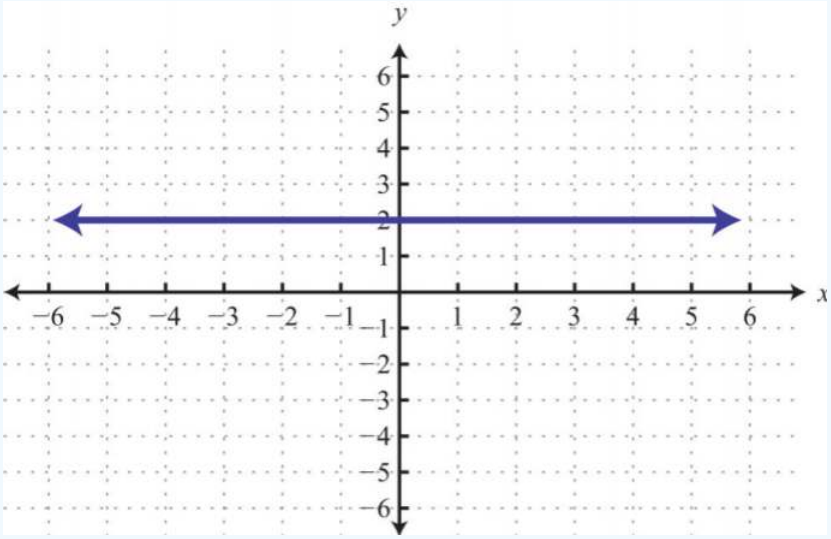
4. 

Figure 3.E.26

5.

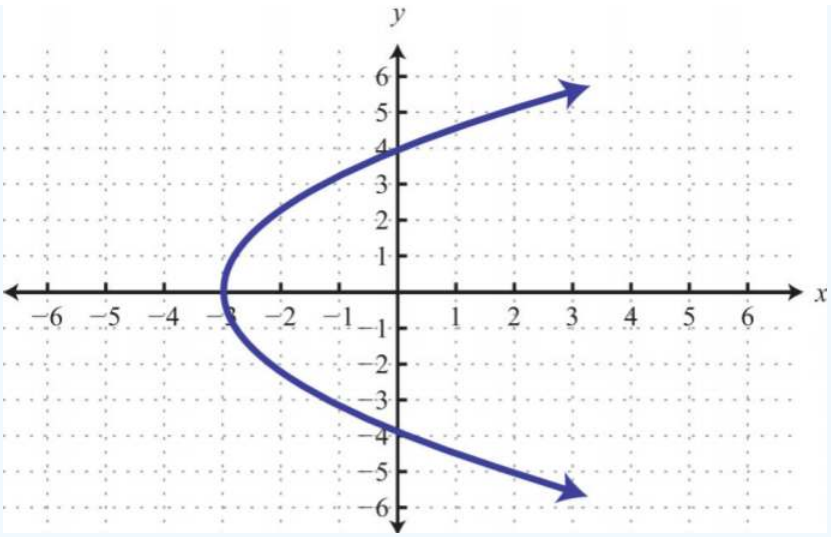


Figure 3.E.27

6.

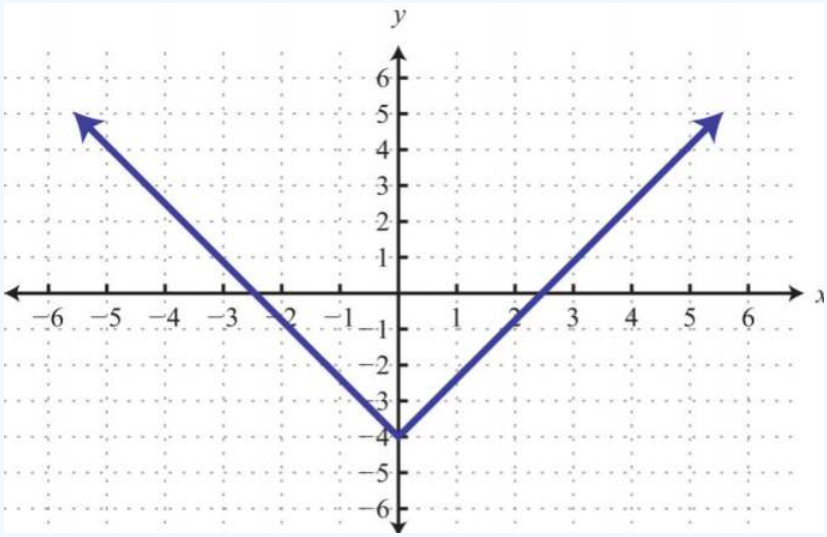


Figure 3.E.28

Answer

1. Domain: $\{-10, -5, 5\}$; range: $\{-1, 2\}$; function: yes
3. Domain: R; range: R; function: yes
5. Domain: $[-3, \infty)$; range: R; function: no

? Exercise $4.E.18$ Introduction to Functions

Given the following,

1. $f(x) = 9x - 4$, find $f(-1)$.
2. $f(x) = -5x + 1$, find $f(-3)$.
3. $g(x) = \frac{1}{2}x - \frac{1}{3}$, find $g(-\frac{1}{3})$.
4. $g(x) = -\frac{3}{4}x + \frac{1}{3}$, find $g(\frac{2}{3})$.
5. $f(x) = 9x - 4$, find x when $f(x) = 0$.
6. $f(x) = -5x + 1$, find x when $f(x) = 2$.

7. $g(x)=\frac{1}{2}x-\frac{1}{3}$, find x when $g(x)=1$.
8. $g(x)=-\frac{3}{4}x+\frac{1}{3}$, find x when $g(x)=-1$.

Answer

1. $f(-1)=-13$
3. $g(-\frac{1}{3})=-\frac{1}{2}$
5. $x=\frac{4}{9}$
7. $x=\frac{8}{3}$

Exercise $4.E.19$ Introduction to Functions

Given the graph of a function $f(x)$, determine the following.

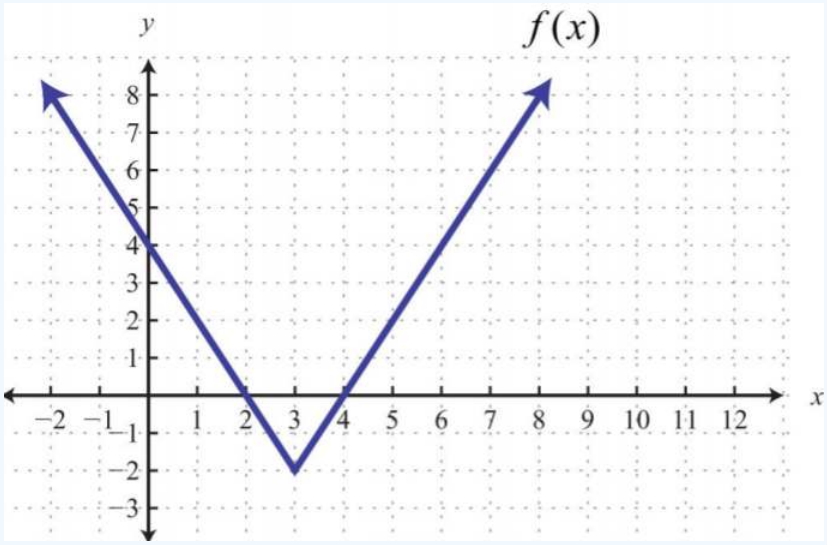


Figure 3.E.29

1. $f(3)$
2. x when $f(x)=4$

Answer

1. $f(3)=-2$

Exercise $4.E.20$ Linear Inequalities (Two Variables)

Is the ordered pair a solution to the given inequality?

1. $6x-2y\leq 1$; $(-3,-7)$
2. $-3x+y>2$; $(0,2)$
3. $6x-10y<-1$; $(5,-3)$
4. $x-\frac{1}{3}y>0$; $(1,4)$
5. $y>0$; $(-3,-1)$
6. $x\leq -5$; $(-6,4)$

Answer

1. Yes
3. No

5. Yes

Exercise $4.E.21$ Linear Inequalities (Two Variables)

Graph the solution set.

1. $y \geq -2x + 1$
2. $y < 3x - 4$
3. $-x + y \leq 3$
4. $\frac{5}{2}x + \frac{1}{2}y \leq 2$
5. $3x - 5y > 0$
6. $y > 0$

Answer

1.

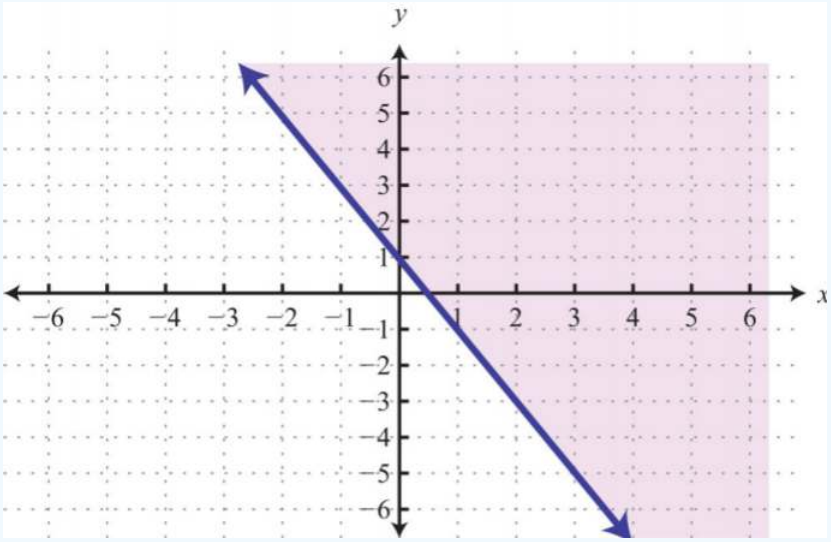


Figure 3.E.30

3.

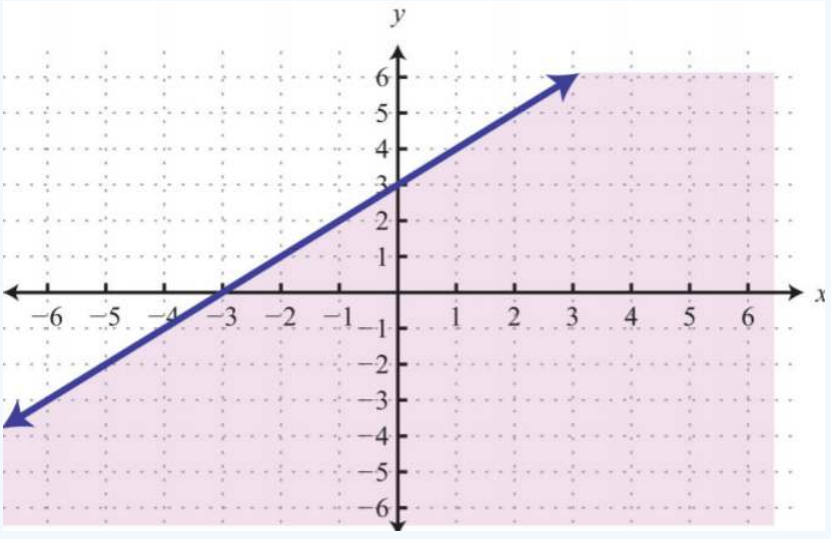


Figure 3.E.31

5.

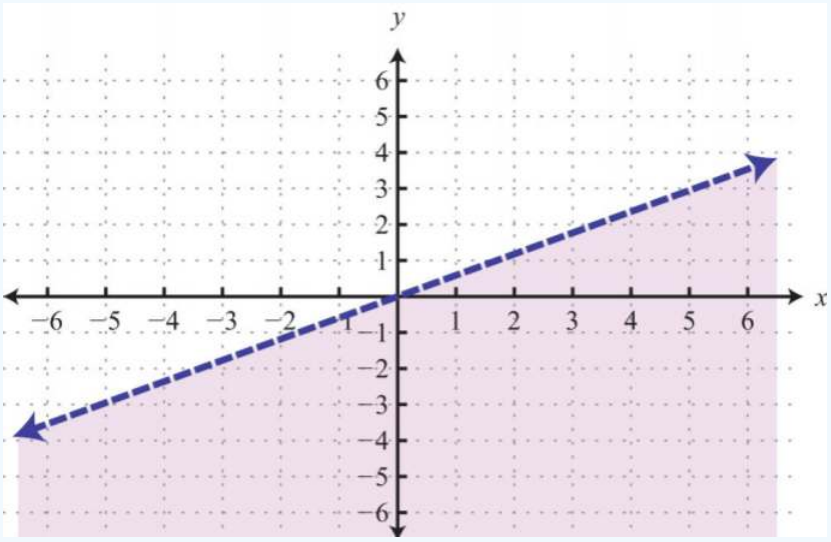


Figure 3.E.32

Sample Exam

? Exercise $4.E.22$

1. Graph the points $(-4,-2)$, $(-4,1)$, and $(0,-2)$ on a rectangular coordinate plane. Connect the points and calculate the area of the shape.
2. Is $(-2,4)$ a solution to $3x-4y=-10$? Justify your answer.

Answer

1. Area: 6 square units

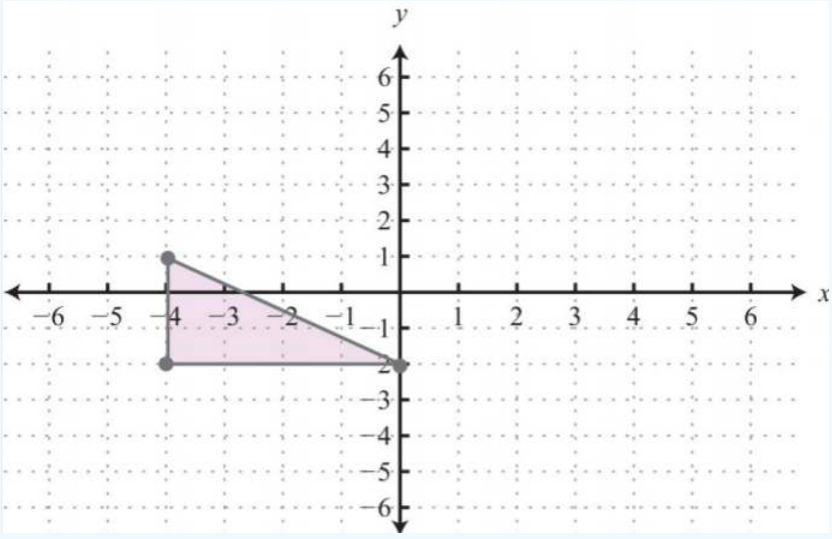


Figure 3.E.33

Exercise $4.E.23$

Given the set of x-values $\{-2, -1, 0, 1, 2\}$, find the corresponding y-values and graph the following.

1. $y = x - 1$
2. $y = -x + 1$
3. On the same set of axes, graph $y = 4$ and $x = -3$. Give the point where they intersect.

Answer

1.

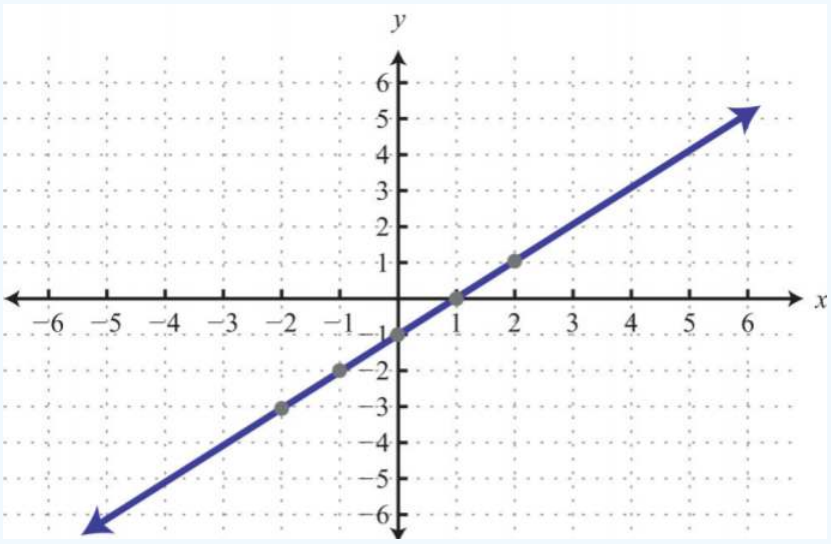


Figure 3.E.34

3. Intersection: $(-3, 4)$

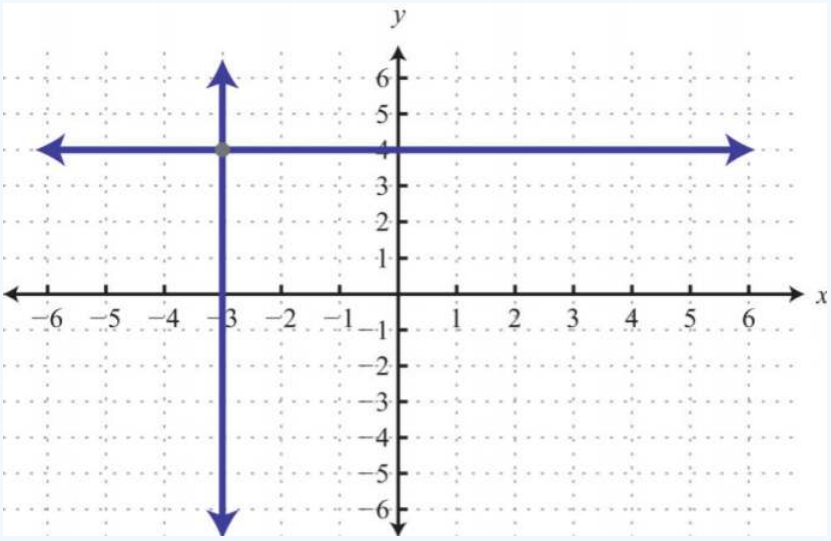


Figure 3.E.35

Exercise $4.E.24$

Find the x- and y-intercepts and use those points to graph the following.

1. $2x - y = 8$
2. $12x + 5y = 15$

3. Calculate the slope of the line passing through $(-4,-5)$ and $(-3,1)$.

Answer

2.

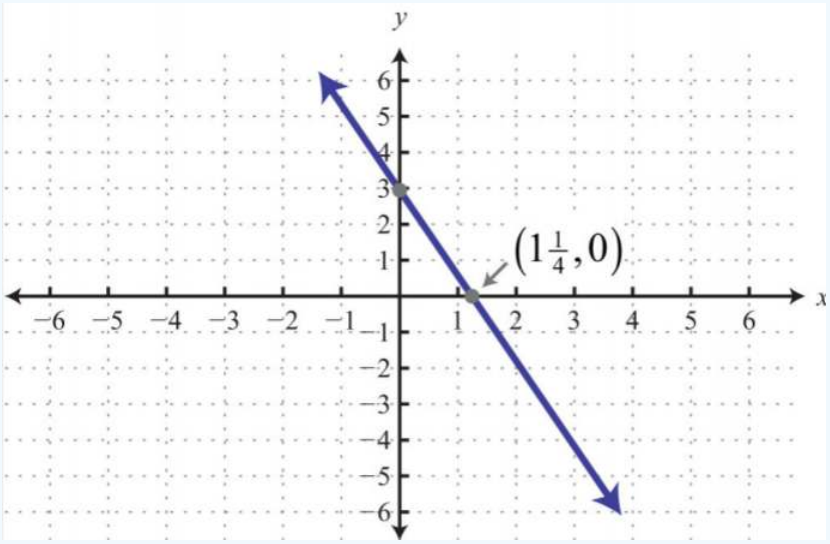


Figure 3.E.36

Exercise $4.E.25$

Determine the slope and y-intercept. Use them to graph the following.

1. $y=-\frac{3}{2}x+6$
2. $5x-2y=6$
3. Given $m=-3$, determine $m_{\perp}$.
4. Are the given lines parallel, perpendicular, or neither? $\begin{cases} -2x+3y=-12 \\ 4x-6y=30 \end{cases}$
5. Determine the slope of the given lines.
   a. $y=-2$
   b. $x=\frac{1}{3}$
   c. Are these lines parallel, perpendicular, or neither?
6. Determine the equation of the line with slope $m=-\frac{3}{4}$ passing through $(8,1)$.
7. Find the equation to the line passing through $(-2,3)$ and $(4,1)$.
8. Find the equation of the line parallel to $5x-y=6$ passing through $(-1,-2)$.
9. Find the equation of the line perpendicular to $-x+2y=4$ passing through $(\frac{1}{2},5)$.

Answer

1. Slope: $-\frac{3}{2}$; y-intercept: $(0,6)$

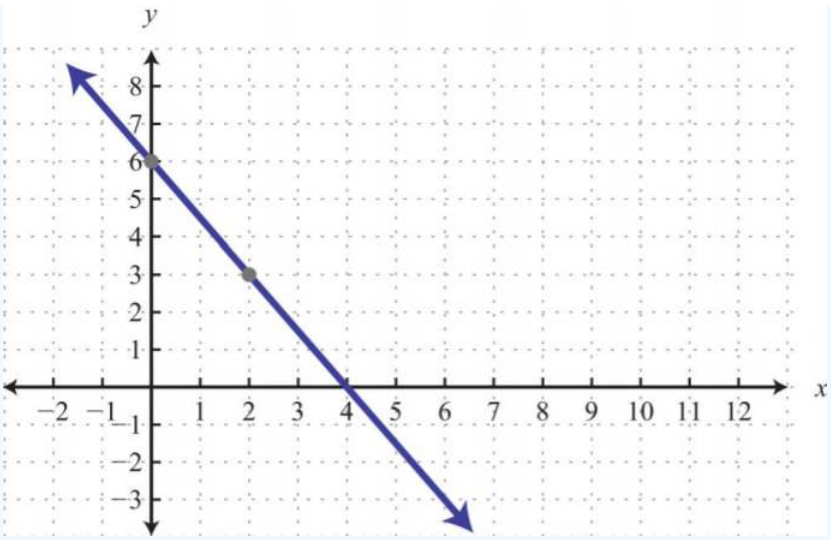


Figure 3.E.37

3. $m_{\perp} = \frac{1}{3}$

5. a. 0; b. Undefined; c. Perpendicular

6. $y = -\frac{3}{4}x + 7$

8. $y = 5x + 3$

? Exercise $4.E.26$

Given a linear function $f(x) = -\frac{4}{5}x + 2$, determine the following.

1. $f(10)$
2. x when $f(x) = 0$
3. Graph the solution set: $3x - 4y > 4$.
4. Graph the solution set: $y - 2x \geq 0$.
5. A rental car company charges $\$32.00$ plus $\$0.52$ per mile driven. Write an equation that gives the cost of renting the car in terms of the number of miles driven. Use the formula to determine the cost of renting the car and driving it 46 miles.
6. A car was purchased new for $\$12,000$ and was sold 5 years later for $\$7,000$. Write a linear equation that gives the value of the car in terms of its age in years.
7. The area of a rectangle is 72 square meters. If the width measures 4 meters, then determine the length of the rectangle.

Answer

1. $f(10) = -6$
3.

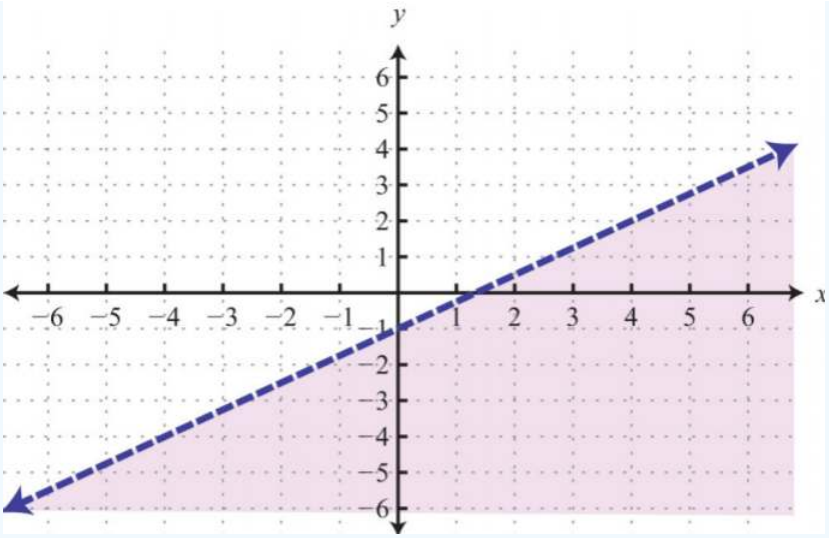


Figure 3.E.38

5. cost$= 0.52x + 32$; $\$55.92$

7. 18 meters

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CHAPTER OVERVIEW

5: Systems of Linear Equations

- 5.1: Solve Systems of Equations by Graphing
  - 5.1E: Exercises
- 5.2: Solve Systems of Equations by Substitution
  - 5.2E: Exercises
- 5.3: Solve Systems of Equations by Elimination
  - 5.3E: Exercises
- 5.4: Solve Applications with Systems of Equations
  - 5.4E: Exercises
- 5.5: Solve Mixture Applications with Systems of Equations
  - 5.5E: Exercises
- 5.6: Graphing Systems of Linear Inequalities
  - 5.6E: Exercises
- 5.7: Chapter 5 Review Exercises

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5.1: Solve Systems of Equations by Graphing

Learning Objectives

By the end of this section, you will be able to:

- Determine whether an ordered pair is a solution of a system of equations
- Solve a system of linear equations by graphing
- Determine the number of solutions of linear system
- Solve applications of systems of equations by graphing

Note

Before you get started, take this readiness quiz.

1. For the equation $y = \frac{2}{3}x - 4$
   ⓐ is (6,0) a solution? ⓑ is (−3,−2) a solution?
   If you missed this problem, review Example 2.1.1.
2. Find the slope and y-intercept of the line 3x−y=12.
   If you missed this problem, review Example 4.5.7.
3. Find the x- and y-intercepts of the line 2x−3y=12.
   If you missed this problem, review Example 4.3.7.

Determine Whether an Ordered Pair is a Solution of a System of Equations

In the section on Solving Linear Equations and Inequalities we learned how to solve linear equations with one variable. Remember that the solution of an equation is a value of the variable that makes a true statement when substituted into the equation. Now we will work with **systems of linear equations**, two or more linear equations grouped together.

Definition: SYSTEM OF LINEAR EQUATIONS

When two or more linear equations are grouped together, they form a system of linear equations.

We will focus our work here on systems of two linear equations in two unknowns. Later, you may solve larger systems of equations.

An example of a system of two linear equations is shown below. We use a brace to show the two equations are grouped together to form a system of equations.

$$\begin{cases} 2x + y = 7 \\ x - 2y = 6 \end{cases} \tag{5.1.1}$$

A linear equation in two variables, like $2x + y = 7$, has an infinite number of solutions. Its graph is a line. Remember, every point on the line is a solution to the equation and every solution to the equation is a point on the line.

To solve a system of two linear equations, we want to find the values of the variables that are solutions to both equations. In other words, we are looking for the ordered pairs (x, y) that make both equations true. These are called the **solutions to a system of equations**.

Definition: SolutionS OF A SYSTEM OF EQUATIONS

Solutions of a system of equations are the values of the variables that make all the equations true. A solution of a system of two linear equations is represented by an ordered pair (x, y).

To determine if an ordered pair is a solution to a system of two equations, we substitute the values of the variables into each equation. If the ordered pair makes both equations true, it is a solution to the system.

Let's consider the system below:

$$\begin{cases} 3x-y=7 \\ x-2y=4 \end{cases} \tag{5.1.2}$$

Is the ordered pair (2,−1) a solution?

We substitute $x=2$ and $y=-1$ into both equations.

$$\begin{aligned} 3x-y &= 7 & \qquad x-2y &= 4 \\ 3(2)-(-1) &\stackrel{?}{=} 7 & \qquad 2-2(-1) &\stackrel{?}{=} 4 \\ 7 &= 7 \text{ true} & \qquad 4 &= 4 \text{ true} \end{aligned}$$

The ordered pair (2, −1) made both equations true. Therefore (2, −1) is a solution to this system.

Let's try another ordered pair. Is the ordered pair (3, 2) a solution?

We substitute $x=3$ and $y=2$ into both equations.

$$\begin{aligned} 3x-y &= 7 & \qquad x-2y &= 4 \\ 3(3)-2 &\stackrel{?}{=} 7 & \qquad 2-2(2) &\stackrel{?}{=} 4 \\ 7 &= 7 \text{ true} & \qquad -2 &= 4 \text{ false} \end{aligned}$$

The ordered pair (3, 2) made one equation true, but it made the other equation false. Since it is not a solution to **both** equations, it is not a solution to this system.

Example 5.1.1

Determine whether the ordered pair is a solution to the system: $\begin{cases} x-y=-1 \\ 2x-y=-5 \end{cases}$

1. (−2,−1)
2. (−4,−3)

Solution

1.

$$\begin{cases} x-y=-1 \\ 2x-y=-5 \end{cases}$$

We substitute $x=-2$ and $y=-1$ into both equations

$$\begin{aligned} x-y &= 1 & \qquad 2x-y &= -5 \\ -2-(-1) &= -1 & \qquad 2(-2)-(-1) &\stackrel{?}{=} -5 \\ -1 &= -1 \checkmark & \qquad -3 &= -5 \times \end{aligned}$$

(−2, −1) does not make both equations true. (−2, −1) is not a solution.

2.

We substitute $x=-4$ and $y=-3$ into both equations.

$$\begin{aligned} x-y &= -1 & \qquad 2x-y &= -5 \\ -4-(-3) &\stackrel{?}{=} -1 & \qquad 2(-4)-(-3) &\stackrel{?}{=} -5 \\ -1 &= -1 \checkmark & \qquad -5 &= -5 \checkmark \end{aligned}$$

(−4, −3) makes both equations true. (−4, −3) is a solution.

Try It 5.1.2

Determine whether the ordered pair is a solution to the system: $\begin{cases} 3x+y=0 \\ x+2y=-5 \end{cases}$

1. (1,−3)

2. (0,0)

Answer

1. yes
2. no

? Try It 5.1.3

Determine whether the ordered pair is a solution to the system: $\begin{cases} x-3y=-8 \\ -3x-y=4 \end{cases}$

1. (2,−2)
2. (−2,2)

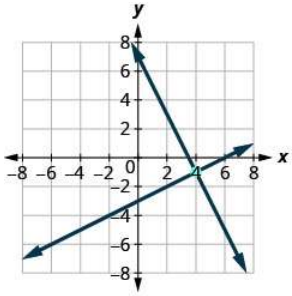
Answer

1. no
2. yes

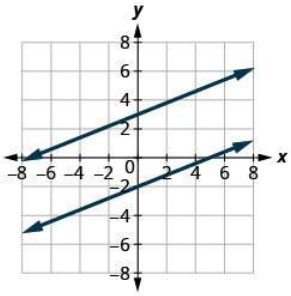
Solve a System of Linear Equations by Graphing

In this chapter we will use three methods to solve a system of linear equations. The first method we'll use is graphing. The graph of a linear equation is a line. Each point on the line is a solution to the equation. For a system of two equations, we will graph two lines. Then we can see all the points that are solutions to each equation. And, by finding what the lines have in common, we'll find the solution to the system.

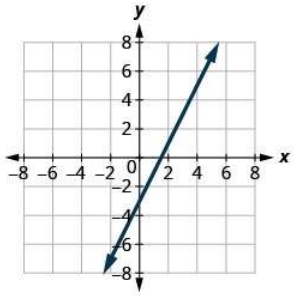
Most linear equations in one variable have one solution, but we saw that some equations, called **contradictions**, have no solutions and for other equations, called **identities**, all numbers are solutions. Similarly, when we solve a system of two linear equations represented by a graph of two lines in the same plane, there are three possible cases, as shown in Figure 5.1.1:



The lines intersect.
Intersecting lines have one point in common.
There is one solution to this system.



The lines are parallel.
Parallel lines have no points in common.
There is no solution to this system.



Both equations give the same line.
Because we have just one line, there are infinitely many solutions.

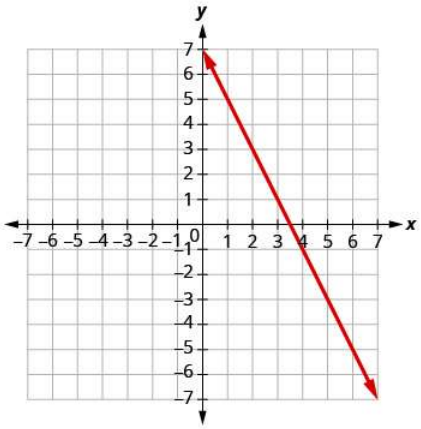
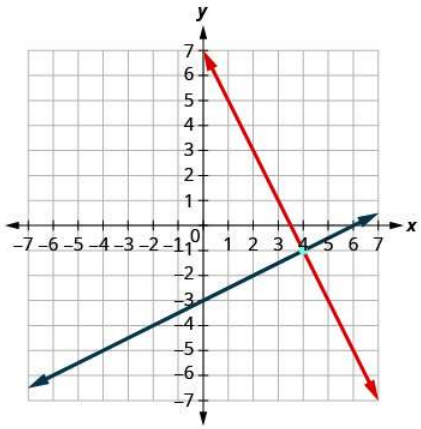
Figure 5.1.1

For the first example of solving a system of linear equations in this section and in the next two sections, we will solve the same system of two linear equations. But we'll use a different method in each section. After seeing the third method, you'll decide which method was the most convenient way to solve this system.

✔ Example 5.1.4: How to Solve a System of Linear Equations by Graphing

Solve the system by graphing: $\begin{cases} 2x+y=7 \\ x-2y=6 \end{cases}$

Solution

Step 1. Graph the first equation.	To graph the first line, write the equation in slope–intercept form. $2x + y = 7$ $y = -2x + 7$ $m = -2$ $b = 7$	$2x + y = 7$ $x - 2y = 6$ 
Step 2. Graph the second equation on the same rectangular coordinate system.	To graph the second line, use intercepts. $x - 2y = 6$ (0, –3) (6, 0)	
Step 3. Determine whether the lines intersect, are parallel, or are the same line.	Look at the graph of the lines.	The lines intersect.
Step 4. Identify the solution to the system. If the lines intersect, identify the point of intersection. Check to make sure it is a solution to both equations. This is the solution to the system. If the lines are parallel, the system has no solution If the lines are the same, the system has an infinite number of solutions.	Since the lines intersect, find the point of intersection. Check the point in both equations.	The lines intersect at (4, –1). $2x + y = 7$ $2(4) + (-1) \stackrel{?}{=} 7$ $8 - 1 \stackrel{?}{=} 7$ $7 = 7$ ✓ $x - 2y = 6$ $4 - 2(-1) \stackrel{?}{=} 6$ $6 = 6$ ✓ The solution is (4, –1).

? Try It 5.1.5

Solve each system by graphing: $\begin{cases} x - 3y = -3 \\ x + y = 5 \end{cases}$

Answer

(3,2)

Try It 5.1.6

Solve each system by graphing: $\begin{cases} -x+y=1 \\ 3x+2y=12 \end{cases}$

Answer

(2,3)

The steps to use to solve a system of linear equations by graphing are shown below.

TO SOLVE A SYSTEM OF LINEAR EQUATIONS BY GRAPHING.

1. Graph the first equation.
2. Graph the second equation on the same rectangular coordinate system.
3. Determine whether the lines intersect, are parallel, or are the same line.
4. Identify the solution to the system.
   - If the lines intersect, identify the point of intersection. Check to make sure it is a solution to both equations. This is the solution to the system.
   - If the lines are parallel, the system has no solution.
   - If the lines are the same, the system has an infinite number of solutions.

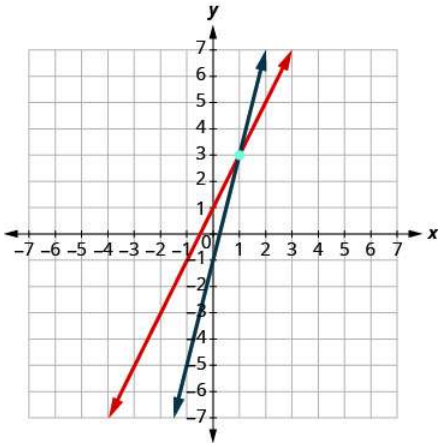
Example 5.1.7

Solve the system by graphing: $\begin{cases} y=2x+1 \\ y=4x-1 \end{cases}$

Solution

Both of the equations in this system are in slope-intercept form, so we will use their slopes and y-intercepts to graph them.
$\begin{cases} y=2x+1 \\ y=4x-1 \end{cases}$

Find the slope and y-intercept of the first equation.	$y=2x+1$ $m=2$ $b=1$
Find the slope and y-intercept of the first equation.	$y=4x-1$ $m=4$ $b=-1$
Graph the two lines.	
Determine the point of intersection.	The lines intersect at (1, 3).

	
Check the solution in both equations.	$y = 2x + 1$ $\quad$ $y = 4x - 1$ $3 \stackrel{?}{=} 2 \cdot 1 + 1$ $\quad$ $3 \stackrel{?}{=} 4 \cdot 1 - 1$ $3 = 3\checkmark$ $\quad$ $3 = 3\checkmark$
	The solution is (1, 3).

Try It 5.1.8

Solve the system by graphing: $\begin{cases} y = 2x + 2 \\ y = -x - 4 \end{cases}$

Answer

(−2,−2)

Try It 5.1.9

Solve the system by graphing: $\begin{cases} y = 3x + 3 \\ y = -x + 7 \end{cases}$

Answer

(1,6)

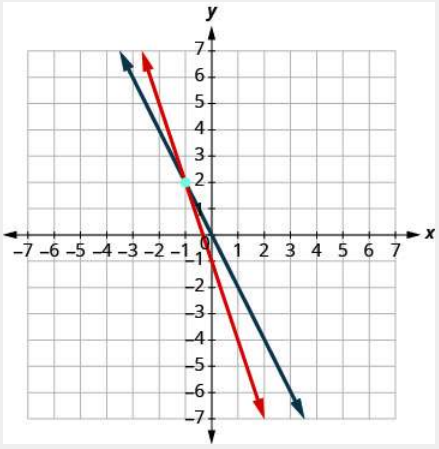
Both equations in Example 5.1.7 were given in slope–intercept form. This made it easy for us to quickly graph the lines. In the next example, we'll first re-write the equations into slope–intercept form.

Example 5.1.10

Solve the system by graphing: $\begin{cases} 3x + y = -1 \\ 2x + y = 0 \end{cases}$

Solution

We'll solve both of these equations for y so that we can easily graph them using their slopes and y-intercepts. $\begin{cases} 3x + y = -1 \\ 2x + y = 0 \end{cases}$

Solve the first equation for y.	
Find the slope and y-intercept.	$3x+y=-1$ $y=-3x-1$ $m=-3$ $b=-1$
Solve the second equation for y.	$2x+y=0$ $y=-2x$
Find the slope and y-intercept.	$b=0$
Graph the lines.	
Determine the point of intersection.	The lines intersect at $(-1, 2)$.
Check the solution in both equations.	$3x+y=-1$ $\quad 3(-1)+2 \stackrel{?}{=} -1 \quad -1=-1\checkmark$ $2x+y=0$ $\quad 2(-1)+2 \stackrel{?}{=} 0 \quad 0=0\checkmark$
	The solution is $(-1, 2)$.

? Try It 5.1.11

Solve each system by graphing: $\begin{cases} -x+y=1 \\ 2x+y=10 \end{cases}$

Answer

(3,4)

? Try It 5.1.12

Solve each system by graphing: $\begin{cases} 2x+y=6 \\ x+y=1 \end{cases}$

Answer

(5,−4)

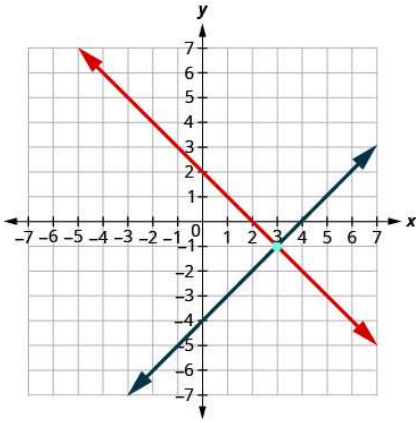
Usually when equations are given in standard form, the most convenient way to graph them is by using the intercepts. We'll do this in Example 5.1.13.

Example 5.1.13

Solve the system by graphing: $\begin{cases} x+y=2 \\ x-y=4 \end{cases}$

Solution

We will find the *x*- and *y*-intercepts of both equations and use them to graph the lines.

	$x+y=2$	
To find the intercepts, let $x = 0$ and solve for y, then let $y = 0$ and solve for x.	$x+y=2 \quad x+y=2$ $0+y=2 \quad x+0=2$ $y=2 \quad x=2$	x | y 0 | 2 2 | 0
	$x-y=4$	
To find the intercepts, let $x = 0$ then let $y = 0$.	$x-y=4 \quad x-y=4$ $0-y=4 \quad x-0=4$ $-y=4 \quad x=4$ $y=-4$	x | y 0 | –4 4 | 0
Graph the line.		
Determine the point of intersection.	The lines intersect at (3, −1).	
Check the solution in both equations.	$x+y = 2 \quad x-y = 4$ $3+(-1) \stackrel{?}{=} 2 \quad 3-(-1) \stackrel{?}{=} 4$ $2 = 2\checkmark \quad 4 = 4\checkmark$ The solution is (3, −1).	

(5.1.3)

Try It 5.1.14

Solve each system by graphing: $\begin{cases} x+y=6 \\ x-y=2 \end{cases}$

Answer

(4,2)

Try It 5.1.15

Solve each system by graphing: $\begin{cases} x+y=2 \\ x-y=8 \end{cases}$

Answer

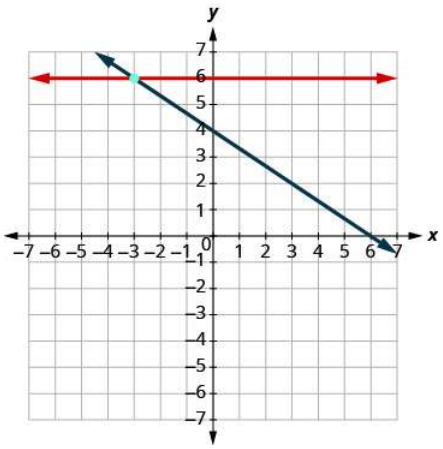
(5,−3)

Do you remember how to graph a linear equation with just one variable? It will be either a vertical or a horizontal line.

Example 5.1.16

Solve the system by graphing: $\begin{cases} y=6 \\ 2x+3y=12 \end{cases}$

Solution

	$y=6$ $2x+3y=12$
We know the first equation represents a horizontal line whose y-intercept is 6.	$y=6$
The second equation is most conveniently graphed using intercepts.	$2x+3y=12$
To find the intercepts, let $x=0$ and then $y=0$.	x | y 0 | 4 6 | 0
Graph the lines.	
Determine the point of intersection.	The lines intersect at (−3, 6).
Check the solution to both equations.	$y = 6$, $6 \stackrel{?}{=} 6$, $6 = 6\checkmark$ $2x+3y = 12$, $2(-3)+3(6) \stackrel{?}{=} 12$, $-6+18 \stackrel{?}{=} 12$, $12 = 12\checkmark$
	The solution is (−3, 6).

Try It 5.1.17

Solve each system by graphing: $\begin{cases} y = -1 \\ x + 3y = 6 \end{cases}$

Answer

(9,−1)

Try It 5.1.18

Solve each system by graphing: $\begin{cases} x = 4 \\ 3x - 2y = 24 \end{cases}$

Answer

(4,−6)

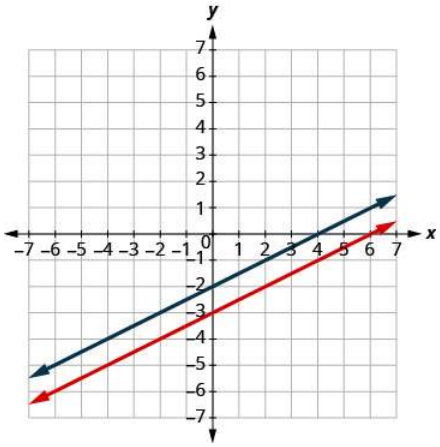
In all the systems of linear equations so far, the lines intersected and the solution was one point. In the next two examples, we'll look at a system of equations that has no solution and at a system of equations that has an infinite number of solutions.

Example 5.1.19

Solve the system by graphing: $\begin{cases} y = \frac{1}{2}x - 3 \\ x - 2y = 4 \end{cases}$

Solution

	$y = \frac{1}{2}x - 3$ $x - 2y = 4$
To graph the first equation, we will use its slope and y-intercept.	$y = \frac{1}{2}x - 3$
	$m = \frac{1}{2}$
	$b = -3$
To graph the second equation, we will use the intercepts.	$x - 2y = 4$
	x | y 0 | −2 4 | 0

Graph the lines.	
Determine the point of intersection.	The lines are parallel.
	Since no point is on both lines, there is no ordered pair that makes both equations true. There is no solution to this system.

Try It 5.1.20

Solve each system by graphing: $\begin{cases} y = -\frac{1}{4}x + 2 \\ x + 4y = -8 \end{cases}$

Answer

no solution

Try It 5.1.21

Solve each system by graphing: $\begin{cases} y = 3x - 1 \\ 6x - 2y = 6 \end{cases}$

Answer

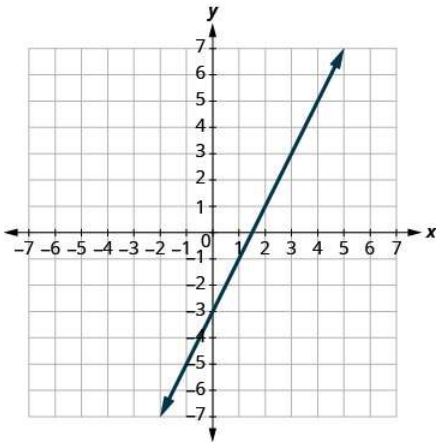
no solution

Example 5.1.22

Solve the system by graphing: $\begin{cases} y = 2x - 3 \\ -6x + 3y = -9 \end{cases}$

Solution

	$y = 2x - 3$ $-6x + 3y = -9$
Find the slope and y-intercept of the first equation.	$y = 2x - 3$ $m = 2$ $b = -3$
Find the intercepts of the second equation.	$-6x + 3y = -9$

	$\begin{array}{\|c\|c\|}\hline x & y \\ \hline 0 & -3 \\ \hline \frac{3}{2} & 0 \\ \hline \end{array}$
Graph the lines.	
Determine the point of intersection.	The lines are the same!
	Since every point on the line makes both equations true, there are infinitely many ordered pairs that make both equations true.
	There are infinitely many solutions to this system.

Try It 5.1.23

Solve each system by graphing: $\begin{cases} y = -3x - 6 \\ 6x + 2y = -12 \end{cases}$

Answer

infinitely many solutions

Try It 5.1.24

Solve each system by graphing: $\begin{cases} y = \frac{1}{2}x - 4 \\ 2x - 4y = 16 \end{cases}$

Answer

infinitely many solutions

If you write the second equation in Example 5.1.22 in slope-intercept form, you may recognize that the equations have the same slope and same *y*-intercept.

When we graphed the second line in the last example, we drew it right over the first line. We say the two lines are coincident. Coincident lines have the same slope and same *y*-intercept.

COINCIDENT LINES

Coincident lines have the same slope and same *y*-intercept.

Determine the Number of Solutions of a Linear System

There will be times when we will want to know how many solutions there will be to a system of linear equations, but we might not actually have to find the solution. It will be helpful to determine this without graphing.

We have seen that two lines in the same plane must either intersect or are parallel. The systems of equations in Example 5.1.4 through Example 5.1.18 all had two intersecting lines. Each system had one solution.

A system with parallel lines, like Example 5.1.19, has no solution. What happened in Example 5.1.22? The equations have **coincident lines**, and so the system had infinitely many solutions.

We'll organize these results in Figure 5.1.2 below:

Graph	Number of solutions
2 intersecting lines	1
Parallel lines	None
Same line	Infinitely many

Figure 5.1.2

Parallel lines have the same slope but different *y*-intercepts. So, if we write both equations in a system of linear equations in slope–intercept form, we can see how many solutions there will be without graphing! Look at the system we solved in Example 5.1.19.

$$\begin{cases} y = \frac{1}{2}x - 3 \\ x - 2y = 4 \end{cases}$$

The first line is in slope–intercept form. If we solve the second equation for y, we get

$$y = \frac{1}{2}x - 3 \qquad x - 2y = 4$$
$$x - 2y = -x + 4$$
$$y = \frac{1}{2}x - 2$$
$$m = \frac{1}{2}, b = -3 \qquad m = \frac{1}{2}, b = -2$$

The two lines have the same slope but different *y*-intercepts. They are parallel lines.

Figure 5.1.3 shows how to determine the number of solutions of a linear system by looking at the slopes and intercepts.

Number of Solutions of a Linear System of Equations			
Slopes	**Intercepts**	**Type of Lines**	**Number of Solutions**
Different		Intersecting	1 point
Same	Different	Parallel	No solution
Same	Same	Coincident	Infinitely many solutions

Figure 5.1.3

Let's take one more look at our equations in Example 5.1.19 that gave us parallel lines.

$$\begin{cases} y = \frac{1}{2}x - 3 \\ x - 2y = 4 \end{cases} \tag{5.1.4}$$

When both lines were in slope-intercept form we had:

$$y = \frac{1}{2}x - 3 \quad y = \frac{1}{2}x - 2 \tag{5.1.5}$$

Do you recognize that it is impossible to have a single ordered pair (x,y) that is a solution to both of those equations?

We call a system of equations like this **an inconsistent system**. It has no solution.

A system of equations that has at least one solution is called a **consistent system**.

CONSISTENT AND INCONSISTENT SYSTEMS

A **consistent system** of equations is a system of equations with at least one solution.

An **inconsistent system** of equations is a system of equations with no solution.

We also categorize the equations in a system of equations by calling the equations *independent* or *dependent*. If two equations are **independent equations**, they each have their own set of solutions. Intersecting lines and parallel lines are independent.

If two equations are dependent, all the solutions of one equation are also solutions of the other equation. When we graph two **dependent equations**, we get coincident lines.

INDEPENDENT AND DEPENDENT EQUATIONS

Two equations are **independent** if they have different solutions.

Two equations are **dependent** if all the solutions of one equation are also solutions of the other equation.

Let's sum this up by looking at the graphs of the three types of systems. See Figure 5.1.4 and Figure 5.1.5.

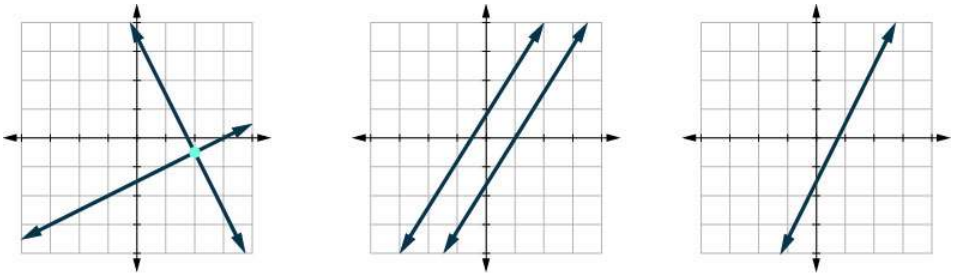


Figure 5.1.4

Lines	Intersecting	Parallel	Coincident
Number of solutions	1 point	No solution	Infinitely many
Consistent/inconsistent	Consistent	Inconsistent	Consistent
Dependent/independent	Independent	Independent	Dependent

Figure 5.1.5

Example 5.1.25

Without graphing, determine the number of solutions and then classify the system of equations: $\begin{cases} y = 3x - 1 \\ 6x - 2y = 12 \end{cases}$

Solution

We will compare the slopes and intercepts of the two lines.

$$\begin{cases} y = 3x - 1 \\ 6x - 2y = 12 \end{cases}$$

The first equation is already in slope-intercept form.

$$y = 3x - 1$$

Write the second equation inslope–intercept form.

$$\begin{aligned} 6x - 2y &= 12 \\ -2y &= -6x - 12 \\ \tfrac{-2y}{-2} &= \tfrac{-6x+12}{-2} \\ y &= 3x - 6 \end{aligned}$$

Find the slope and intercept of each line.

$$\begin{aligned} y &= 3x - 1 & y &= 3x - 6 \\ m &= 3 & m &= 3 \\ b &= -1 & b &= -6 \end{aligned}$$

Since the slopes are the same and y-intercepts are different, the lines are parallel.

A system of equations whose graphs are parallel lines has no solution and is inconsistent and independent.

Try It 5.1.26

Without graphing, determine the number of solutions and then classify the system of equations.

$$\begin{cases} y = -2x - 4 \\ 4x + 2y = 9 \end{cases}$$

Answer

no solution, inconsistent, independent

Try It 5.1.27

Without graphing, determine the number of solutions and then classify the system of equations.

$$\begin{cases} y = \frac{1}{3}x - 5 \\ x - 3y = 6 \end{cases}$$

Answer

no solution, inconsistent, independent

Example 5.1.28

Without graphing, determine the number of solutions and then classify the system of equations: $\begin{cases} 2x + y = -3 \\ x - 5y = 5 \end{cases}$

Solution

We will compare the slopes and intercepts of the two lines.

$$\begin{cases} 2x + y = -3 \\ x - 5y = 5 \end{cases}$$

Write the second equation in slope–intercept form.

$$\begin{aligned} 2x + y &= -3 \\ y &= -2x - 3 \end{aligned} \qquad \begin{aligned} x - 5y &= 5 \\ -5y &= -x + 5 \\ \tfrac{-5y}{-5} &= \tfrac{-x+5}{-5} \\ y &= \tfrac{1}{5}x - 1 \end{aligned}$$

Find the slope and intercept of each line.

$$\begin{aligned} y &= -2x - 3 \\ m &= -2 \\ b &= -3 \end{aligned} \qquad \begin{aligned} y &= \tfrac{1}{5}x - 1 \\ m &= \tfrac{1}{5} \\ b &= -1 \end{aligned}$$

Since the slopes are different, the lines intersect.

A system of equations whose graphs are intersect has 1 solution and is consistent and independent.

Try It 5.1.29

Without graphing, determine the number of solutions and then classify the system of equations.

$$\begin{cases} 3x + 2y = 2 \\ 2x + y = 1 \end{cases}$$

Answer

one solution, consistent, independent

Try It 5.1.30

Without graphing, determine the number of solutions and then classify the system of equations.

$$\begin{cases} x+4y=12 \\ -x+y=3 \end{cases}$$

Answer

one solution, consistent, independent

Example 5.1.31

Without graphing, determine the number of solutions and then classify the system of equations. $\begin{cases} 3x-2y=4 \\ y=\frac{3}{2}x-2 \end{cases}$

Solution

We will compare the slopes and intercepts of the two lines. $\begin{cases} 3x-2y &= 4 \\ y &= \frac{3}{2}x-2 \end{cases}$

Write the second equation in slope–intercept form.

$$\begin{aligned} 3x-2y &= 4 \\ -2y &= -3x+4 \\ \frac{-2y}{-2} &= \frac{-3x+4}{-2} \\ y &= \frac{3}{2}x-2 \end{aligned}$$

Find the slope and intercept of each line. $y = \frac{3}{2}x-2$

Since the equations are the same, they have the same slope and same y-intercept and so the lines are coincident.

A system of equations whose graphs are coincident lines has infinitely many solutions and is consistent and dependent.

Try It 5.1.32

Without graphing, determine the number of solutions and then classify the system of equations.

$$\begin{cases} 4x-5y=20 \\ y=\frac{4}{5}x-4 \end{cases}$$

Answer

infinitely many solutions, consistent, dependent

Try It 5.1.33

Without graphing, determine the number of solutions and then classify the system of equations.

$$\begin{cases} -2x-4y=8 \\ y=-\frac{1}{2}x-2 \end{cases}$$

Answer

infinitely many solutions, consistent, dependent

Solve Applications of Systems of Equations by Graphing

We will use the same problem solving strategy we used in Math Models to set up and solve applications of systems of linear equations. We'll modify the strategy slightly here to make it appropriate for systems of equations.

USE A PROBLEM SOLVING STRATEGY FOR SYSTEMS OF LINEAR EQUATIONS.

1. **Read** the problem. Make sure all the words and ideas are understood.
2. **Identify** what we are looking for.
3. **Name** what we are looking for. Choose variables to represent those quantities.
4. **Translate** into a system of equations.
5. **Solve** the system of equations using good algebra techniques.
6. **Check** the answer in the problem and make sure it makes sense.
7. **Answer** the question with a complete sentence.

Step 5 is where we will use the method introduced in this section. We will graph the equations and find the solution.

Example 5.1.34

Sondra is making 10 quarts of punch from fruit juice and club soda. The number of quarts of fruit juice is 4 times the number of quarts of club soda. How many quarts of fruit juice and how many quarts of club soda does Sondra need?

Solution

Step 1. Read the problem.

Step 2. Identify what we are looking for.

We are looking for the number of quarts of fruit juice and the number of quarts of club soda that Sondra will need.

Step 3. Name what we are looking for. Choose variables to represent those quantities.

Let f= number of quarts of fruit juice.
c= number of quarts of club soda

Step 4. Translate into a system of equations.

The number of quarts of fruit juice and the number of quarts of club soda is 10

$$f + c = 10$$

The number of quarts of fruit juice is four times the number of quarts of club soda

$$f = 4c$$

We now have the system. $\begin{cases} f + c = 10 \\ f = 4c \end{cases}$

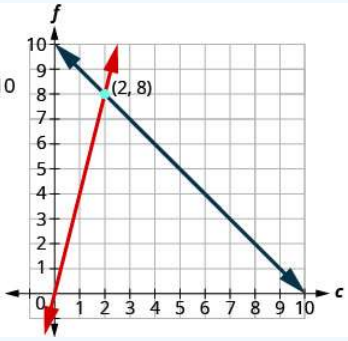
Step 5. Solve the system of equations using good algebra techniques.

$f = 4c$ $\quad$ $f + c = 10$

$m = 4$ $\quad$ $f = -c + 10$

$b = 0$ $\quad$ $m = -1$

$b = 10$



The point of intersection (2, 8) is the solution. This means Sondra needs 2 quarts of club soda and 8 quarts of fruit juice.

Step 6. Check the answer in the problem and make sure it makes sense.

Does this make sense in the problem?

Yes, the number of quarts of fruit juice, 8 is 4 times the number of quarts of club soda, 2.

Yes, 10 quarts of punch is 8 quarts of fruit juice plus 2 quarts of club soda.

Step 7. Answer the question with a complete sentence.

Sondra needs 8 quarts of fruit juice and 2 quarts of soda.

Try It 5.1.35

Manny is making 12 quarts of orange juice from concentrate and water. The number of quarts of water is 3 times the number of quarts of concentrate. How many quarts of concentrate and how many quarts of water does Manny need?

Answer

Manny needs 3 quarts juice concentrate and 9 quarts water.

Try It 5.1.36

Alisha is making an 18 ounce coffee beverage that is made from brewed coffee and milk. The number of ounces of brewed coffee is 5 times greater than the number of ounces of milk. How many ounces of coffee and how many ounces of milk does Alisha need?

Answer

Alisha needs 15 ounces of coffee and 3 ounces of milk.

Note

Access these online resources for additional instruction and practice with solving systems of equations by graphing.

- Instructional Video Solving Linear Systems by Graphing
- Instructional Video Solve by Graphing

Key Concepts

- **To solve a system of linear equations by graphing**
  1. Graph the first equation.
  2. Graph the second equation on the same rectangular coordinate system.
  3. Determine whether the lines intersect, are parallel, or are the same line.
  4. Identify the solution to the system.
     If the lines intersect, identify the point of intersection. Check to make sure it is a solution to both equations. This is the solution to the system.
     If the lines are parallel, the system has no solution.
     If the lines are the same, the system has an infinite number of solutions.
  5. Check the solution in both equations.
- Determine the number of solutions from the graph of a linear system

Graph	Number of solutions
2 intersecting lines	1
Parallel lines	None
Same line	Infinitely many

- Determine the number of solutions of a linear system by looking at the slopes and intercepts

Number of Solutions of a Linear System of Equations			
Slopes	**Intercepts**	**Type of Lines**	**Number of Solutions**
Different		Intersecting	1 point
Same	Different	Parallel	No solution
Same	Same	Coincident	Infinitely many solutions

- Determine the number of solutions and how to classify a system of equations

Lines	**Intersecting**	**Parallel**	**Coincident**
Number of solutions	1 point	No solution	Infinitely many
Consistent/inconsistent	Consistent	Inconsistent	Consistent
Dependent/independent	Independent	Independent	Dependent

- **Problem Solving Strategy for Systems of Linear Equations**
  1. **Read** the problem. Make sure all the words and ideas are understood.
  2. **Identify** what we are looking for.
  3. **Name** what we are looking for. Choose variables to represent those quantities.
  4. **Translate** into a system of equations.
  5. **Solve** the system of equations using good algebra techniques.
  6. **Check** the answer in the problem and make sure it makes sense.
  7. **Answer** the question with a complete sentence.

Glossary

coincident lines

Coincident lines are lines that have the same slope and same y-intercept.

consistent system

A consistent system of equations is a system of equations with at least one solution.

dependent equations

Two equations are dependent if all the solutions of one equation are also solutions of the other equation.

inconsistent system

An inconsistent system of equations is a system of equations with no solution.

independent equations

Two equations are independent if they have different solutions.

solutions of a system of equations

Solutions of a system of equations are the values of the variables that make all the equations true. A solution of a system of two linear equations is represented by an ordered pair (x, y).

system of linear equations

When two or more linear equations are grouped together, they form a system of linear equations.

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5.1E: Exercises

Practice Makes Perfect

Determine Whether an Ordered Pair is a Solution of a System of Equations. In the following exercises, determine if the following points are solutions to the given system of equations.

Exercise 5.1E.1

$$\begin{cases} 2x-6y=0 \\ 3x-4y=5 \end{cases}$$

1. (3,1)
2. (−3,4)

Answer

1. yes
2. no

Exercise 5.1E.2

$$\begin{cases} 7x-4y=-1 \\ -3x-2y=1 \end{cases}$$

1. $(-\frac{3}{13}, -\frac{2}{13})$
2. (1,-2)

Answer

1. yes
2. no

Exercise 5.1E.3

$$\begin{cases} 2x+y=5 \\ x+y=1 \end{cases}$$

1. (4,−3)
2. (2,0)

Answer

1. yes
2. no

Exercise 5.1E.4

$$\begin{cases} -3x+y=8 \\ -x+2y=-9 \end{cases}$$

1. (−5,−7)
2. (−5,7)

Exercise 5.1E.5

$$\begin{cases} x+y=2 \\ y=\frac{3}{4}x \end{cases}$$

1. $\left(\frac{8}{7},\frac{6}{7}\right)$
2. $\left(1,\frac{3}{4}\right)$

Answer

1. yes
2. no

Exercise 5.1E.6

$$\begin{cases} x+y=1 \\ y=\frac{2}{5}x \end{cases}$$

1. $\left(\frac{5}{7},\frac{2}{7}\right)$
2. (5,2)

Exercise 5.1E.7

$$\begin{cases} x+5y=10 \\ y=\frac{3}{5}x+1 \end{cases}$$

1. (−10,4)
2. $\left(\frac{5}{4},\frac{7}{4}\right)$

Answer

1. no
2. yes

Exercise 5.1E.8

$$\begin{cases} x+3y=9 \\ y=\frac{2}{3}x-2 \end{cases}$$

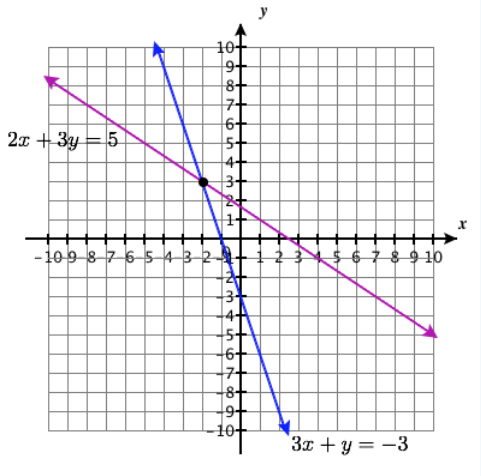
1. (−6,5)
2. $\left(5,\frac{4}{3}\right)$

Solve a System of Linear Equations by Graphing In the following exercises, solve the following systems of equations by graphing.

Exercise 5.1E.9

$$\begin{cases} 3x+y=-3 \\ 2x+3y=5 \end{cases}$$

Answer



Solution Point: $(-2, 3)$

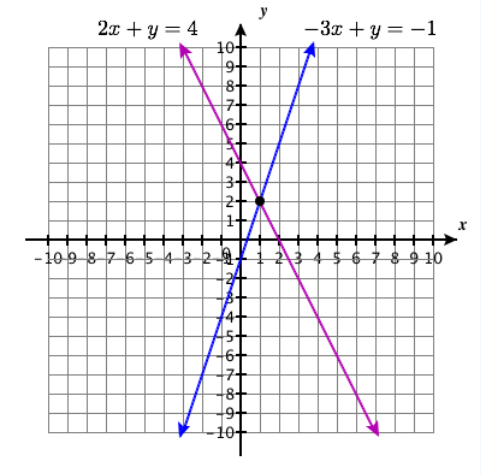
Exercise 5.1E. 10

$$\begin{cases} -x + y = 2 \\ 2x + y = -4 \end{cases}$$

Exercise 5.1E. 11

$$\begin{cases} -3x + y = -1 \\ 2x + y = 4 \end{cases}$$

Answer



Solution Point: $(1, 2)$

Exercise 5.1E. 12

$$\begin{cases} -2x + 3y = -3 \\ x + y = 4 \end{cases}$$

Exercise 5.1E.13

$$\begin{cases} y = x + 2 \\ y = -2x + 2 \end{cases}$$

Answer

Solution Point: $(0, 2)$

Exercise 5.1E.14

$$\begin{cases} y = x - 2 \\ y = -3x + 2 \end{cases}$$

Exercise 5.1E.15

$$\begin{cases} y = \frac{3}{2}x + 1 \\ y = -\frac{1}{2}x + 5 \end{cases}$$

Answer

Solution Point: $(2, 4)$

Exercise 5.1E.16

$$\begin{cases} y = \frac{2}{3}x - 2 \\ y = -\frac{1}{3}x - 5 \end{cases}$$

Exercise 5.1E.17

$$\begin{cases} -x + y = -3 \\ 4x + 4y = 4 \end{cases}$$

Answer

Solution Point: $(2, -1)$

Exercise 5.1E.18

$$\begin{cases} x - y = 3 \\ 2x - y = 4 \end{cases}$$

Exercise 5.1E.19

$$\begin{cases} -3x + y = -1 \\ 2x + y = 4 \end{cases}$$

Answer

Solution Point: $(1, 2)$

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Exercise 5.1E.20

$$\begin{cases} -3x + y = -2 \\ 4x - 2y = 6 \end{cases}$$

Exercise 5.1E.21

$$\begin{cases} x + y = 5 \\ 2x - y = 4 \end{cases}$$

Answer

Solution Point: $(3, 2)$

Exercise 5.1E.22

$$\begin{cases} x - y = 2 \\ 2x - y = 6 \end{cases}$$

Exercise 5.1E.23

$$\begin{cases} x + y = 2 \\ x - y = 0 \end{cases}$$

Answer

Solution Point: $(1, 1)$

Exercise 5.1E.24

$$\begin{cases} x + y = 6 \\ x - y = -8 \end{cases}$$

Exercise 5.1E.25

$$\begin{cases} x + y = -5 \\ x - y = 3 \end{cases}$$

Answer

Solution Point: $(-1, -4)$

Exercise 5.1E.26

$$\begin{cases} x + y = 4 \\ x - y = 0 \end{cases}$$

Exercise 5.1E.27

$$\begin{cases} x + y = -4 \\ -x + 2y = -2 \end{cases}$$

Answer

Solution Point: $(3, 3)$

Exercise $5.1E.28$

$$\begin{cases} -x+3y=3 \\ x+3y=3 \end{cases}$$

Exercise $5.1E.29$

$$\begin{cases} -2x+3y=3 \\ x+3y=12 \end{cases}$$

Answer

Solution Point: $(-5, 6)$

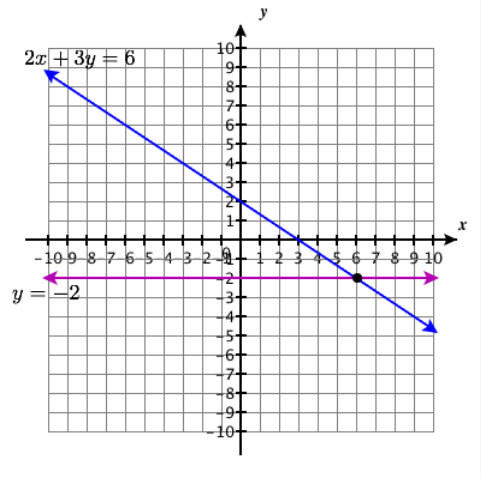
Exercise $5.1E.30$

$$\begin{cases} 2x-y=4 \\ 2x+3y=12 \end{cases}$$

Exercise $5.1E.31$

$$\begin{cases} 2x+3y=6 \\ y=-2 \end{cases}$$

Answer



Solution Point: $(6, -2)$

Exercise $5.1E.32$

$$\begin{cases} -2x+y=2 \\ y=4 \end{cases}$$

Exercise $5.1E.33$

$$\begin{cases} x-3y=-3 \\ y=2 \end{cases}$$

Answer

Solution Point: $(3, 2)$

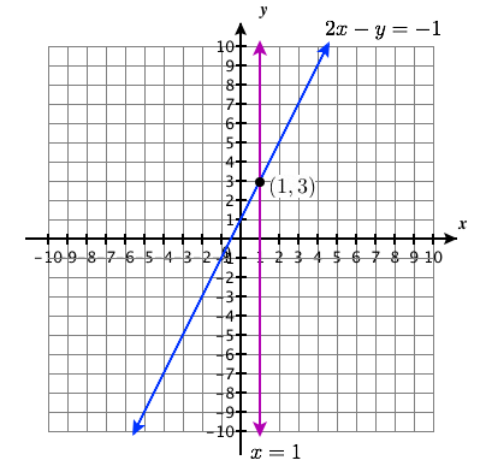
Exercise 5.1E.34

$$\begin{cases} 2x - 2y = 8 \\ y = -3 \end{cases}$$

Exercise 5.1E.35

$$\begin{cases} 2x - y = -1 \\ x = 1 \end{cases}$$

Answer



Solution Point: $(1, 3)$

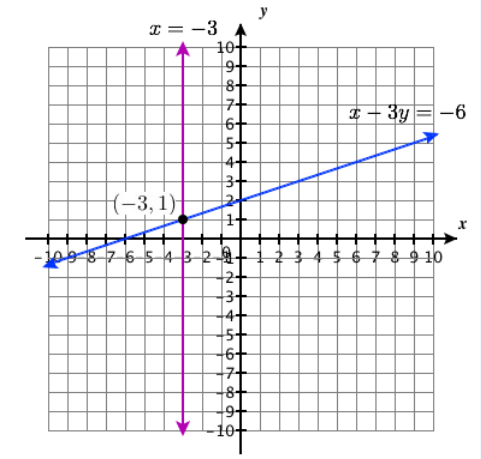
Exercise 5.1E.36

$$\begin{cases} x + 2y = 2 \\ x = -2 \end{cases}$$

Exercise 5.1E.37

$$\begin{cases} x - 3y = -6 \\ x = -3 \end{cases}$$

Answer



Solution Point: $(-3, 1)$

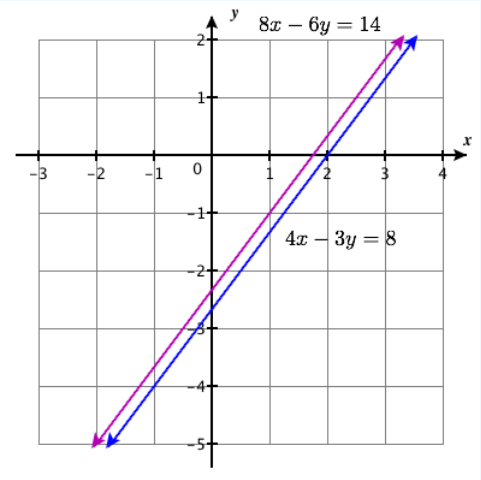
Exercise 5.1E. 38

$$\begin{cases} x + y = 4 \\ x = 1 \end{cases}$$

Exercise 5.1E. 39

$$\begin{cases} 4x - 3y = 8 \\ 8x - 6y = 14 \end{cases}$$

Answer



no solution

Exercise 5.1E. 40

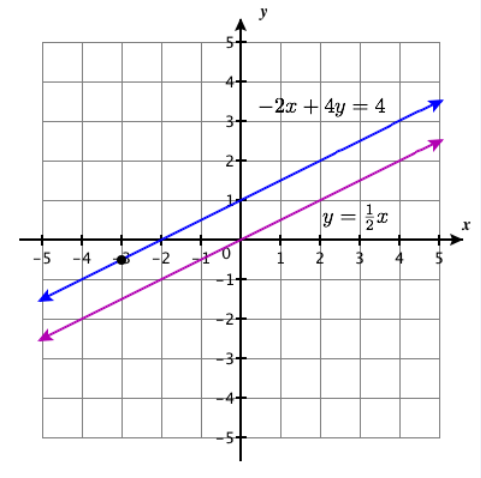
$$\begin{cases} x + 3y = 4 \\ -2x - 6y = 3 \end{cases}$$

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Exercise 5.1E. 41

$$\begin{cases} -2x+4y=4 \\ y=\frac{1}{2}x \end{cases}$$

Answer



no solution

Exercise 5.1E. 42

$$\begin{cases} 3x+5y=10 \\ y=-\frac{3}{5}x+1 \end{cases}$$

Exercise 5.1E. 43

$$\begin{cases} x=-3y+4 \\ 2x+6y=8 \end{cases}$$

Answer

no solution

Exercise 5.1E. 44

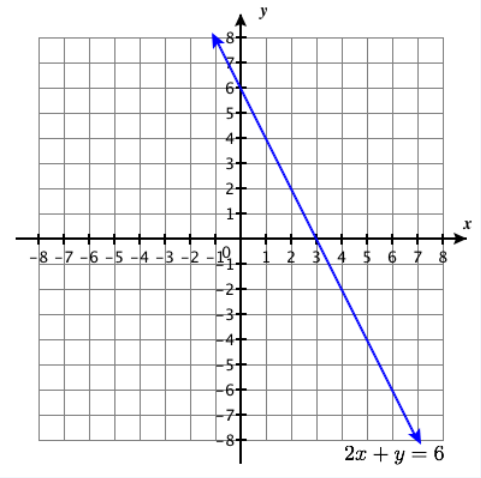
$$\begin{cases} 4x=3y+7 \\ 8x-6y=14 \end{cases}$$

Exercise 5.1E. 45

$$\begin{cases} 2x+y=6 \\ -8x-4y=-24 \end{cases}$$

Answer

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infinitely many solutions with solution set: $\{(x,y)|2x+y=6\}$

Exercise 5.1E.46

$$\begin{cases} 5x+2y=7 \\ -10x-4y=-14 \end{cases}$$

Exercise 5.1E.47

$$\begin{cases} x+3y=-6 \\ 4y=-\frac{4}{3}x-8 \end{cases}$$

Answer

infinitely many solutions with solution set: $\{(x,y)|x+3y=-6\}$

Exercise 5.1E.48

$$\begin{cases} -x+2y=-6 \\ y=-\frac{1}{2}x-1 \end{cases}$$

Exercise 5.1E.49

$$\begin{cases} -3x+2y=-2 \\ y=-x+4 \end{cases}$$

Answer

Solution Point: $(2,2)$

Exercise 5.1E.50

$$\begin{cases} -x+2y=-2 \\ y=-x-1 \end{cases}$$

Determine the Number of Solutions of a Linear System Without graphing the following systems of equations, determine the number of solutions and then classify the system of equations.

Exercise 5.1E.51

$$\begin{cases} y = \frac{2}{3}x + 1 \\ -2x + 3y = 5 \end{cases}$$

Answer

0 solutions

Exercise 5.1E.52

$$\begin{cases} y = \frac{1}{3}x + 2 \\ x - 3y = 9 \end{cases}$$

Exercise 5.1E.53

$$\begin{cases} y = -2x + 1 \\ 4x + 2y = 8 \end{cases}$$

Answer

0 solutions

Exercise 5.1E.54

$$\begin{cases} y = 3x + 4 \\ 9x - 3y = 18 \end{cases}$$

Exercise 5.1E.55

$$\begin{cases} y = \frac{2}{3}x + 1 \\ 2x - 3y = 7 \end{cases}$$

Answer

0 solutions

Exercise 5.1E.56

$$\begin{cases} 3x + 4y = 12 \\ y = -3x - 1 \end{cases}$$

Exercise 5.1E.57

$$\begin{cases} 4x + 2y = 10 \\ 4x - 2y = -6 \end{cases}$$

Answer

consistent, 1 solution

Exercise 5.1E.58

$$\begin{cases} 5x+3y=4 \\ 2x-3y=5 \end{cases}$$

Exercise 5.1E.59

$$\begin{cases} y=-\frac{1}{2}x+5 \\ x+2y=10 \end{cases}$$

Answer

infinitely many solutions

Exercise 5.1E.60

$$\begin{cases} y=x+1 \\ -x+y=1 \end{cases}$$

Exercise 5.1E.61

$$\begin{cases} y=2x+3 \\ 2x-y=-3 \end{cases}$$

Answer

infinitely many solutions

Exercise 5.1E.62

$$\begin{cases} 5x-2y=10 \\ y=\frac{5}{2}x-5 \end{cases}$$

Solve Applications of Systems of Equations by Graphing In the following exercises, solve.

Exercise 5.1E.63

Molly is making strawberry infused water. For each ounce of strawberry juice, she uses three times as many ounces of water. How many ounces of strawberry juice and how many ounces of water does she need to make 64 ounces of strawberry infused water?

Answer

Molly needs 16 ounces of strawberry juice and 48 ounces of water.

Exercise 5.1E.64

Jamal is making a snack mix that contains only pretzels and nuts. For every ounce of nuts, he will use 2 ounces of pretzels. How many ounces of pretzels and how many ounces of nuts does he need to make 45 ounces of snack mix?

Exercise 5.1E.65

Enrique is making a party mix that contains raisins and nuts. For each ounce of nuts, he uses twice the amount of raisins. How many ounces of nuts and how many ounces of raisins does he need to make 24 ounces of party mix?

Answer

Enrique needs 8 ounces of nuts and 16 ounces of water.

Exercise 5.1E.66

Owen is making lemonade from concentrate. The number of quarts of water he needs is 4 times the number of quarts of concentrate. How many quarts of water and how many quarts of concentrate does Owen need to make 100 quarts of lemonade?

Everyday Math

Exercise 5.1E.67

Leo is planning his spring flower garden. He wants to plant tulip and daffodil bulbs. He will plant 6 times as many daffodil bulbs as tulip bulbs. If he wants to plant 350 bulbs, how many tulip bulbs and how many daffodil bulbs should he plant?

Answer

Leo should plant 50 tulips and 300 daffodils.

Exercise 5.1E.68

A marketing company surveys 1,200 people. They surveyed twice as many females as males. How many males and females did they survey?

Writing Exercises

Exercise 5.1E.69

In a system of linear equations, the two equations have the same slope. Describe the possible solutions to the system.

Answer

Given that it is only known that the slopes of both linear equations are the same, there are either no solutions (the graphs of the equations are parallel) or infinitely many.

Exercise 5.1E.70

In a system of linear equations, the two equations have the same intercepts. Describe the possible solutions to the system.

Self Check

After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
determine whether an ordered pair is a solution of a system of equations.			
solve a system of linear equations by graphing.			
determine the number of solutions of a linear system.			
solve applications of systems of equations by graphing.			

If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them. What did you do to become confident of your ability to do these things? Be specific.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math every topic builds upon previous work. It is important to make sure you have a strong foundation before you move on. Who can you ask for help? Your fellow classmates and instructor are good resources. Is there a place on campus where math tutors are available? Can your study skills be improved?

…no - I don't get it! This is a warning sign and you must not ignore it. You should get help right away or you will quickly be overwhelmed. See your instructor as soon as you can to discuss your situation. Together you can come up with a plan to get you the help you need.

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5.2: Solve Systems of Equations by Substitution

Learning Objectives

By the end of this section, you will be able to:

- Solve a system of equations by substitution
- Solve applications of systems of equations by substitution

Note

Before you get started, take this readiness quiz.

1. Simplify −5(3−x).
   If you missed this problem, review Example 1.10.43.
2. Simplify 4−2(n+5).
   If you missed this problem, review Example 1.10.49.
3. Solve for y. 8y−8=32−2y
   If you missed this problem, review Example 2.3.22.
4. Solve for x. 3x−9y=−3
   If you missed this problem, review Example 2.6.22.

Solving systems of linear equations by graphing is a good way to visualize the types of solutions that may result. However, there are many cases where solving a system by graphing is inconvenient or imprecise. If the graphs extend beyond the small grid with x and y both between −10 and 10, graphing the lines may be cumbersome. And if the solutions to the system are not integers, it can be hard to read their values precisely from a graph.

In this section, we will solve systems of linear equations by the substitution method.

Solve a System of Equations by Substitution

We will use the same system we used first for graphing.

$$\begin{cases} 2x+y=7 \\ x-2y=6 \end{cases}$$

We will first solve one of the equations for either x or y. We can choose either equation and solve for either variable—but we'll try to make a choice that will keep the work easy.

Then we substitute that expression into the other equation. The result is an equation with just one variable—and we know how to solve those!

After we find the value of one variable, we will substitute that value into one of the original equations and solve for the other variable. Finally, we check our solution and make sure it makes both equations true.

We'll fill in all these steps now in Example 5.2.1.

Example 5.2.1: How to Solve a System of Equations by Substitution

Solve the system by substitution. $\begin{cases} 2x+y=7 \\ x-2y=6 \end{cases}$

Solution

Step 1. Solve one of the equations for either variable.	We'll solve the first equation for y.	$2x+y=7$ $y=7-2x$
Step 2. Substitute the expression from Step 1 into the other equation.	We replace y in the second equation with the expression $7-2x$.	$x-2y=6$ $x-2(7-2x)=6$
Step 3. Solve the resulting equation.	Now we have an equation with just 1 variable. We know how to solve this!	$x-2(7-2x)=6$ $x-14+4x=6$ $5x=20$ $x=4$
Step 4. Substitute the solution in Step 3 into one of the original equations to find the other variable.	We'll use the first equation and replace x with 4.	$2x+y=7$ $2(4)+y=7$ $8+y=7$ $y=-1$
Step 5. Write the solution as an ordered pair.	The ordered pair is (x, y).	$(4,-1)$
Step 6. Check that the ordered pair is a solution to **both** original equations.	Substitute $(4,-1)$ into both equations and make sure they are both true.	$2x+y=7$ $2(4)+(-1)\stackrel{?}{=}7$ $7=7$ ✓ $x-2y=6$ $4-2(-1)\stackrel{?}{=}6$ $6=6$ ✓ Both equations are true. $(4,-1)$ is the solution to the system.

Try It 5.2.2

Solve the system by substitution. $\begin{cases} -2x+y=-11 \\ x+3y=9 \end{cases}$

Answer

(6,1)

Try It 5.2.3

Solve the system by substitution. $\begin{cases} x+3y=10 \\ 4x+y=18 \end{cases}$

Answer

(4,2)

SOLVE A SYSTEM OF EQUATIONS BY SUBSTITUTION.

1. Solve one of the equations for either variable.
2. Substitute the expression from Step 1 into the other equation.
3. Solve the resulting equation.
4. Substitute the solution in Step 3 into one of the original equations to find the other variable.

5. Write the solution as an ordered pair.
6. Check that the ordered pair is a solution to **both** original equations.

If one of the equations in the system is given in slope–intercept form, Step 1 is already done! We'll see this in Example 5.2.4.

Example 5.2.4

Solve the system by substitution. $\begin{cases} x+y=-1 \\ y=x+5 \end{cases}$

Solution

The second equation is already solved for y. We will substitute the expression in place of y in the first equation.

	$x+y=-1$ $y=x+5$
The second equation is already solved for y. We will substitute into the first equation.	
Replace the y with $x+5$.	$y=x+5$ $x+y=-1$
Solve the resulting equation for x.	$x+x+5=-1$
	$2x+5=-1$
	$2x=-6$
Substitute $x=-3$ into $y=x+5$ to find y.	$x=-3$ $y=x+5$
	$y=-3+5$
The ordered pair is $(-3, 2)$.	$y=2$
Check the ordered pair in both equations: $x+y=-1 \quad y=x+5$ $-3+2 \stackrel{?}{=} -1 \quad 2 \stackrel{?}{=} -3+5$ $-1=-1\checkmark \quad 2=2\checkmark$	
	The solution is $(-3, 2)$.

Try It 5.2.5

Solve the system by substitution. $\begin{cases} x+y=6 \\ y=3x-2 \end{cases}$

Answer

(2,4)

Try It 5.2.6

Solve the system by substitution. $\begin{cases} 2x-y=1 \\ y=-3x-6 \end{cases}$

Answer

(−1,−3)

If the equations are given in standard form, we'll need to start by solving for one of the variables. In this next example, we'll solve the first equation for y.

Example 5.2.7

Solve the system by substitution. $\begin{cases} 3x+y=5 \\ 2x+4y=-10 \end{cases}$

Solution

We need to solve one equation for one variable. Then we will substitute that expression into the other equation.

Solve for y. Substitute into the other equation.	$3x+y=5$ $y=-3x+5$ $2x+4y=-10$
Replace the y with $-3x+5$.	$2x+4(-3x+5)=-10$
Solve the resulting equation for x.	$2x-12x+20=-10$
	$-10x+20=-10$ $-10x=-30$
Substitute $x=3$ into $3x+y=5$ to find y.	$x=3$ $3x+y=5$
	$3(3)+y=5$ $9+y=5$
The ordered pair is $(3,-4)$.	$y=-4$
Check the ordered pair in both equations: $3x+y=5$ $\quad$ $2x+4y=-10$ $3\cdot 3+(-4)\stackrel{?}{=}5$ $\quad$ $2\cdot 3+4(-4)\stackrel{?}{=}-10$ $9-4\stackrel{?}{=}5$ $\quad$ $6-16\stackrel{?}{=}-10$ $5=5\checkmark$ $\quad$ $-10=-10\checkmark$	
	The solution is $(3,-4)$.

Try It 5.2.8

Solve the system by substitution. $\begin{cases} 4x+y=2 \\ 3x+2y=-1 \end{cases}$

Answer

$(1,-2)$

Try It 5.2.9

Solve the system by substitution. $\begin{cases} -x+y=4 \\ 4x-y=2 \end{cases}$

Answer

$(2,6)$

In Example 5.2.7 it was easiest to solve for y in the first equation because it had a coefficient of 1. In Example 5.2.10 it will be easier to solve for x.

Example 5.2.10

Solve the system by substitution. $\begin{cases} x-2y=-2 \\ 3x+2y=34 \end{cases}$

Solution

We will solve the first equation for x and then substitute the expression into the second equation.

	$x-2y=-2$
Solve for x. Substitute into the other equation.	$x=2y-2$ $3x+2y=34$
Replace the x with $2y-2$.	$3(2y-2)+2y=34$
Solve the resulting equation for y.	$6y-6+2y=34$
Substitute $y=5$ into $x-2y=-2$ to find x.	$8y-6=34$ $8y=40$ $y=5$ $x-2y=-2$ $x-2\cdot 5=-2$ $x-10=-2$ $x=8$
The ordered pair is (8, 5).	
Check the ordered pair in both equations: $x-2y=-2$ and $3x+2y=34$ $8-2\cdot 5\stackrel{?}{=}-2$ and $3\cdot 8+2\cdot 5\stackrel{?}{=}34$ $8-10\stackrel{?}{=}-2$ and $24+10\stackrel{?}{=}34$ $-2=-2\checkmark$ and $34=34\checkmark$	
	The solution is (8, 5).

Try It 5.2.11

Solve the system by substitution. $\begin{cases} x-5y=13 \\ 4x-3y=1 \end{cases}$

Answer

(−2,−3)

Try It 5.2.12

Solve the system by substitution. $\begin{cases} x-6y=-6 \\ 2x-4y=4 \end{cases}$

Answer

(6,2)

When both equations are already solved for the same variable, it is easy to substitute!

Example 5.2.13

Solve the system by substitution. $\begin{cases} y = -2x + 5 \\ y = \frac{1}{2}x \end{cases}$

Solution

Since both equations are solved for *y*, we can substitute one into the other.

Substitute $\frac{1}{2}x$ for *y* in the first equation.	$y = \frac{1}{2}x$ $y = -2x + 5$
Replace the *y* with $\frac{1}{2}x$	$\frac{1}{2}x = -2x + 5$
Solve the resulting equation. Start by clearing the fraction.	$2\left(\frac{1}{2}x\right) = 2(-2x + 5)$
Solve for *x*.	$x = -4x + 10$
	$5x = 10$
Substitute $x = 2$ into $y = \frac{1}{2}x$ to find *y*.	$x = 2$ $y = \frac{1}{2}x$ $y = \frac{1}{2} \cdot 2$ $y = 1$
The ordered pair is (2,1).	
Check the ordered pair in both equations: $y = \frac{1}{2}x \quad y = -2x + 5$ $1 \stackrel{?}{=} \frac{1}{2} \cdot 2 \quad 1 \stackrel{?}{=} -2 \cdot 2 + 5$ $1 = 1\checkmark \quad 1 = -4 + 5$ $1 = 1\checkmark$	
	The solution is (2,1).

Try It 5.2.14

Solve the system by substitution. $\begin{cases} y = 3x - 16 \\ y = \frac{1}{3}x \end{cases}$

Answer

(6,2)

Try It 5.2.15

Solve the system by substitution. $\begin{cases} y = -x + 10 \\ y = \frac{1}{4}x \end{cases}$

Answer

(8,2)

Be very careful with the signs in the next example.

Example 5.2.16

Solve the system by substitution. $\begin{cases} 4x+2y=4 \\ 6x-y=8 \end{cases}$

Solution

We need to solve one equation for one variable. We will solve the first equation for y.

	$4x+2y=4$
Solve the first equation for y.	$2y=-4x+4$
Substitute $-2x+2$ for y in the second equation.	$y=-2x+2$ $6x-y=8$
Replace the y with $-2x+2$.	$6x-(-2x+2)=8$
Solve the equation for x.	$6x+2x-2=8$
	$8x-2=8$ $8x=10$
Substitute $x=\frac{5}{4}$ into $4x+2y=4$ to find y.	$x=\frac{5}{4}$ $4x+2y=4$ $4\left(\frac{5}{4}\right)+2y=4$ $5+2y=4$ $2y=-1$ $y=-\frac{1}{2}$
The ordered pair is $(\frac{5}{4},-\frac{1}{2})$.	
Check the ordered pair in both equations.	$\begin{aligned} 4x+2y &= 4 \\ 4(\tfrac{5}{4})+2(-\tfrac{1}{2}) &\stackrel{?}{=} 4 \\ 5-1 &\stackrel{?}{=} 4 \\ 4 &= 4\checkmark \end{aligned}$ $\begin{aligned} 6x-y &= 8 \\ 6(\tfrac{5}{4})-(-\tfrac{1}{2}) &\stackrel{?}{=} 8 \\ \tfrac{15}{4}-(-\tfrac{1}{2}) &\stackrel{?}{=} 8 \\ \tfrac{16}{2} &\stackrel{?}{=} 8 \\ 8 &= 8\checkmark \end{aligned}$
	The solution is $\left(\frac{5}{4},-\frac{1}{2}\right)$.

Try It 5.2.17

Solve the system by substitution. $\begin{cases} x-4y=-4 \\ -3x+4y=0 \end{cases}$

Answer

$(2,\frac{3}{2})$

Try It 5.2.18

Solve the system by substitution. $\begin{cases} 4x-y=0 \\ 2x-3y=5 \end{cases}$

Answer

$(-\frac{1}{2}, -2)$

In Example, it will take a little more work to solve one equation for x or y.

Example 5.2.19

Solve the system by substitution. $\begin{cases} 4x-3y=6 \\ 15y-20x=-30 \end{cases}$

Solution

We need to solve one equation for one variable. We will solve the first equation for x.

	$4x-3y=6$
Solve the first equation for x.	$4x=3y+6$
Substitute $\frac{3}{4}y+\frac{3}{2}$ for x in the second equation.	$x=\frac{3}{4}y+\frac{3}{2}$ $15y-20x=-30$
Replace the x with $\frac{3}{4}y+\frac{3}{2}$	$15y-20\left(\frac{3}{4}y+\frac{3}{2}\right)=-30$
Solve for y.	$15y-15y-30=-30$
	$0-30=-30$
	$0=0$

Since 0 = 0 is a true statement, the system is consistent. The equations are dependent. The graphs of these two equations would give the same line. The system has infinitely many solutions.

Try It 5.2.20

Solve the system by substitution. $\begin{cases} 2x-3y=12 \\ -12y+8x=48 \end{cases}$

Answer

infinitely many solutions

Try It 5.2.21

Solve the system by substitution. $\begin{cases} 5x+2y=12 \\ -4y-10x=-24 \end{cases}$

Answer

infinitely many solutions

Look back at the equations in Example 5.2.22 Is there any way to recognize that they are the same line?

Let's see what happens in the next example.

Example 5.2.22

Solve the system by substitution. $\begin{cases} 5x-2y=-10 \\ y=\frac{5}{2}x \end{cases}$

Solution

The second equation is already solved for y, so we can substitute for y in the first equation.

Substitute x for y in the first equation.	$y=\left(\frac{5}{2}x\right)$ $5x-2y=-10$
Replace the y with $\frac{5}{2}x$.	$5x-2\left(\frac{5}{2}x\right)=-10$
Solve for x.	$5x-5x=-10$
	$0\neq-10$

Since $0 = -10$ is a false statement the equations are inconsistent. The graphs of the two equation would be parallel lines. The system has no solutions.

Try It 5.2.23

Solve the system by substitution. $\begin{cases} 3x+2y=9 \\ y=-\frac{3}{2}x+1 \end{cases}$

Answer

no solution

Try It 5.2.24

Solve the system by substitution. $\begin{cases} 5x-3y=2 \\ y=\frac{5}{3}x-4 \end{cases}$

Answer

no solution

Solve Applications of Systems of Equations by Substitution

We'll copy here the problem solving strategy we used in the Solving Systems of Equations by Graphing section for solving systems of equations. Now that we know how to solve systems by substitution, that's what we'll do in Step 5.

HOW TO USE A PROBLEM SOLVING STRATEGY FOR SYSTEMS OF LINEAR EQUATIONS.

1. **Read** the problem. Make sure all the words and ideas are understood.
2. **Identify** what we are looking for.
3. **Name** what we are looking for. Choose variables to represent those quantities.
4. **Translate** into a system of equations.
5. **Solve** the system of equations using good algebra techniques.
6. **Check** the answer in the problem and make sure it makes sense.
7. **Answer** the question with a complete sentence.

Some people find setting up word problems with two variables easier than setting them up with just one variable. Choosing the variable names is easier when all you need to do is write down two letters. Think about this in the next example—how would you have done it with just one variable?

Example 5.2.25

The sum of two numbers is zero. One number is nine less than the other. Find the numbers.

Solution

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	We are looking for two numbers.
Step 3. Name what we are looking for.	Let n= the first number Let m= the second number
Step 4. Translate into a system of equations.	The sum of two numbers is zero.
	$n+m=0$
	One number is nine less than the other.
	$n=m-9$
The system is:	$\begin{aligned} n+m&=0\\ n&=m-9\end{aligned}$
Step 5. Solve the system of equations. We will use substitution since the second equation is solved for n.	
Substitute $m-9$ for n in the first equation.	$n=m-9$ $n+m=0$
Solve for m.	$m-9+m=0$
	$2m-9=0$
	$2m=9$
Substitute $m=\frac{9}{2}$ into the second equation and then solve for n.	$m=\frac{9}{2}$ $n=m-9$
	$m=\frac{9}{2}-9$
	$m=\frac{9}{2}-\frac{18}{2}$
	$n=-\frac{9}{2}$
Step 6. Check the answer in the problem.	Do these numbers make sense in the problem? We will leave this to you!
Step 7. Answer the question.	The numbers are $\frac{9}{2}$ and $-\frac{9}{2}$.

Try It 5.2.26

The sum of two numbers is 10. One number is 4 less than the other. Find the numbers.

Answer

The numbers are 3 and 7.

Try It 5.2.27

The sum of two number is −6. One number is 10 less than the other. Find the numbers.

Answer

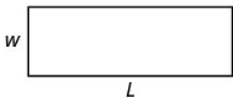
The numbers are 2 and −8.

In the Example 5.2.28, we'll use the formula for the perimeter of a rectangle, $P = 2L + 2W$.

Example 5.2.28

The perimeter of a rectangle is 88. The length is five more than twice the width. Find the length and width of the rectangle.

Solution

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	We are looking for the length and width.
Step 3. Name what we are looking for.	Let L= the length W= the width
Step 4. Translate into a system of equations.	The perimeter of a rectangle is 88.
	$2L + 2W = P$ $2L + 2W = 88$
	The length is five more than twice the width.
	$L = 2W + 5$
The system is:	$2L + 2W = 88$ $L = 2W + 5$
Step 5. Solve the system of equations. We will use substitution since the second equation is solved for L. Substitute $2W + 5$ for L in the first equation.	$L = 2W + 5$ $2L + 2W = 88$
Solve for W.	$2(2W + 5) + 2W = 88$
	$4W + 10 + 2W = 88$
	$6W + 10 = 88$
	$6W = 78$
Substitute $W = 13$ into the second equation and then solve for L.	$W = 13$ $L = 2W + 5$
	$L = 2 \cdot 13 + 5$
	$L = 31$
Step 6. Check the answer in the problem.	Does a rectangle with length 31 and width 13 have perimeter 88? Yes.
Step 7. Answer the equation.	The length is 31 and the width is 13.

Try It 5.2.29

The perimeter of a rectangle is 40. The length is 4 more than the width. Find the length and width of the rectangle.

Answer

The length is 12 and the width is 8.

Try It 5.2.30

The perimeter of a rectangle is 58. The length is 5 more than three times the width. Find the length and width of the rectangle.

Answer

The length is 23 and the width is 6.

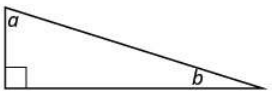
For Example 5.2.31 we need to remember that the sum of the measures of the angles of a triangle is 180 degrees and that a right triangle has one 90 degree angle.

Example 5.2.31

The measure of one of the small angles of a right triangle is ten more than three times the measure of the other small angle. Find the measures of both angles.

Solution

We will draw and label a figure.

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	We are looking for the measures of the angles.
Step 3. Name what we are looking for.	Let a= the measure of the 1st angle b= the measure of the 2nd angle
Step 4. Translate into a system of equations.	The measure of one of the small angles of a right triangle is ten more than three times the measure of the other small angle.
	$a = 3b + 10$
	The sum of the measures of the angles of a triangle is 180.
	$a + b + 90 = 180$
The system is:	$a = 3b + 10$ $a + b + 90 = 180$
Step 5. Solve the system of equations. We will use substitution since the first equation is solved for a.	$a = 3b + 10$ $a + b + 90 = 180$
Substitute $3b + 10$ for a in the second equation.	$(3b + 10) + b + 90 = 180$
Solve for b.	$4b + 100 = 180$

	$4b = 80$
	$b = 20$ $a = 3b + 10$
Substitute $b = 20$ into the first equation and then solve for a.	$a = 3 \cdot 20 + 10$ $a = 70$
Step 6. Check the answer in the problem.	We will leave this to you!
Step 7. Answer the question.	The measures of the small angles are 20 and 70.

Try It 5.2.32

The measure of one of the small angles of a right triangle is 2 more than 3 times the measure of the other small angle. Find the measure of both angles.

Answer

The measure of the angles are 22 degrees and 68 degrees.

Try It 5.2.33

The measure of one of the small angles of a right triangle is 18 less than twice the measure of the other small angle. Find the measure of both angles.

Answer

The measure of the angles are 36 degrees and 54 degrees.

Example 5.2.34

Heather has been offered two options for her salary as a trainer at the gym. Option A would pay her $25,000 plus $15 for each training session. Option B would pay her $10,000 + $40 for each training session. How many training sessions would make the salary options equal?

Solution

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	We are looking for the number of training sessions that would make the pay equal.
Step 3. Name what we are looking for.	Let s= Heather's salary. n= the number of training sessions
Step 4. Translate into a system of equations.	Option A would pay her $25,000 plus $15 for each training session.
	$s = 25{,}000 + 15n$
	Option B would pay her $10,000 + $40 for each training session
	$s = 10{,}000 + 40n$
The system is:	$s = 25{,}000 + 15n$ $s = 10{,}000 + 40n$

Step 5. Solve the system of equations. We will use substitution.	$s = 25{,}000 + 15n$ $s = 10{,}000 + 40n$
Substitute $25{,}000 + 15n$ for s in the second equation.	$25{,}000 + 15n = 10{,}000 + 40n$
Solve for n.	$25{,}000 = 10{,}000 + 25n$
	$15{,}000 = 25n$
	$600 = n$
Step 6. Check the answer.	Are 600 training sessions a year reasonable? Are the two options equal when $n = 600$?
Step 7. Answer the question.	The salary options would be equal for 600 training sessions.

Try It 5.2.35

Geraldine has been offered positions by two insurance companies. The first company pays a salary of $12,000 plus a commission of $100 for each policy sold. The second pays a salary of $20,000 plus a commission of $50 for each policy sold. How many policies would need to be sold to make the total pay the same?

Answer

There would need to be 160 policies sold to make the total pay the same.

Try It 5.2.36

Kenneth currently sells suits for company A at a salary of $22,000 plus a $10 commission for each suit sold. Company B offers him a position with a salary of $28,000 plus a $4 commission for each suit sold. How many suits would Kenneth need to sell for the options to be equal?

Answer

Kenneth would need to sell 1,000 suits.

Note

Access these online resources for additional instruction and practice with solving systems of equations by substitution.

- Instructional Video-Solve Linear Systems by Substitution
- Instructional Video-Solve by Substitution

Key Concepts

- **Solve a system of equations by substitution**
  1. Solve one of the equations for either variable.
  2. Substitute the expression from Step 1 into the other equation.
  3. Solve the resulting equation.
  4. Substitute the solution in Step 3 into one of the original equations to find the other variable.
  5. Write the solution as an ordered pair.
  6. Check that the ordered pair is a solution to both original equations.

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5.2E: Exercises

Practice Makes Perfect

Solve a System of Equations by Substitution

In the following exercises, solve the systems of equations by substitution.

Exercise 5.2E.1

$$\begin{cases} 2x+y=-4 \\ 3x-2y=-6 \end{cases}$$

Answer

$(-2,0)$

Exercise 5.2E.2

$$\begin{cases} 2x+y=-2 \\ 3x-y=7 \end{cases}$$

Exercise 5.2E.3

$$\begin{cases} x-2y=-5 \\ 2x-3y=-4 \end{cases}$$

Answer

$(7,6)$

Exercise 5.2E.4

$$\begin{cases} x-3y=-9 \\ 2x+5y=4 \end{cases}$$

Exercise 5.2E.5

$$\begin{cases} 5x-2y=-6 \\ y=3x+3 \end{cases}$$

Answer

$(0,3)$

Exercise 5.2E.6

$$\begin{cases} -2x+2y=6 \\ y=-3x+1 \end{cases}$$

Exercise 5.2E.7

$$\begin{cases} 2x+3y=3 \\ y=-x+3 \end{cases}$$

Answer

$(6, -3)$

Exercise 5.2E.8

$$\begin{cases} 2x + 5y = -14 \\ y = -2x + 2 \end{cases}$$

Exercise 5.2E.9

$$\begin{cases} 2x + 5y = 1 \\ y = \frac{1}{3}x - 2 \end{cases}$$

Answer

$(3, -1)$

Exercise 5.2E.10

$$\begin{cases} 3x + 4y = 1 \\ y = -\frac{2}{5}x + 2 \end{cases}$$

Exercise 5.2E.11

$$\begin{cases} 3x - 2y = 6 \\ y = \frac{2}{3}x + 2 \end{cases}$$

Answer

$(6, 6)$

Exercise 5.2E.12

$$\begin{cases} -3x - 5y = 3 \\ y = \frac{1}{2}x - 5 \end{cases}$$

Exercise 5.2E.13

$$\begin{cases} 2x + y = 10 \\ -x + y = -5 \end{cases}$$

Answer

$(5, 0)$

Exercise 5.2E.14

$$\begin{cases} -2x + y = 10 \\ -x + 2y = 16 \end{cases}$$

Exercise 5.2E.15

$$\begin{cases} 3x+y=1 \\ -4x+y=15 \end{cases}$$

Answer

$(-2,7)$

Exercise 5.2E.16

$$\begin{cases} x+y=0 \\ 2x+3y=-4 \end{cases}$$

Exercise 5.2E.17

$$\begin{cases} x+3y=1 \\ 3x+5y=-5 \end{cases}$$

Answer

$(-5,2)$

Exercise 5.2E.18

$$\begin{cases} x+2y=-1 \\ 2x+3y=1 \end{cases}$$

Exercise 5.2E.19

$$\begin{cases} 2x+y=5 \\ x-2y=-15 \end{cases}$$

Answer

$(-1,7)$

Exercise 5.2E.20

$$\begin{cases} 4x+y=10 \\ x-2y=-20 \end{cases}$$

Exercise 5.2E.21

$$\begin{cases} y=-2x-1 \\ y=-\frac{1}{3}x+4 \end{cases}$$

Answer

$(-3,5)$

Exercise 5.2E.22

$$\begin{cases} y=x-6 \\ y=-\frac{3}{2}x+4 \end{cases}$$

Exercise 5.2E.23

$$\begin{cases} y = 2x - 8 \\ y = \frac{3}{5}x + 6 \end{cases}$$

Answer

$(10, 12)$

Exercise 5.2E.24

$$\begin{cases} y = -x - 1 \\ y = x + 7 \end{cases}$$

Exercise 5.2E.25

$$\begin{cases} 4x + 2y = 8 \\ 8x - y = 1 \end{cases}$$

Answer

$\left(\frac{1}{2}, 3\right)$

Exercise 5.2E.26

$$\begin{cases} -x - 12y = -1 \\ 2x - 8y = -6 \end{cases}$$

Exercise 5.2E.27

$$\begin{cases} 15x + 2y = 6 \\ -5x + 2y = -4 \end{cases}$$

Answer

$\left(\frac{1}{2}, -\frac{3}{4}\right)$

Exercise 5.2E.28

$$\begin{cases} 2x - 15y = 7 \\ 12x + 2y = -4 \end{cases}$$

Exercise 5.2E.29

$$\begin{cases} y = 3x \\ 6x - 2y = 0 \end{cases}$$

Answer

Infinitely many solutions. The two equations represent the same line.
The solution set is: $\{(x, y) \mid y = 3x\}$
All points that are solutions of the equation $y = 3x$ are solutions of this system.

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Exercise 5.2E.30

$$\begin{cases} x = 2y \\ 4x - 8y = 0 \end{cases}$$

Exercise 5.2E.31

$$\begin{cases} 2x + 16y = 8 \\ -x - 8y = -4 \end{cases}$$

Answer

Infinitely many solutions. The two equations represent the same line.
The solution set is: $\{(x, y) \mid 2x + 16y = 8\}$
All points that are solutions of the equation $2x + 16y = 8$ are solutions of this system.

Exercise 5.2E.32

$$\begin{cases} 15x + 4y = 6 \\ -30x - 8y = -12 \end{cases}$$

Exercise 5.2E.33

$$\begin{cases} y = -4x \\ 4x + y = 1 \end{cases}$$

Answer

No solution

Exercise 5.2E.34

$$\begin{cases} y = -\frac{1}{4}x \\ x + 4y = 8 \end{cases}$$

Exercise 5.2E.35

$$\begin{cases} y = \frac{7}{8}x + 4 \\ -7x + 8y = 6 \end{cases}$$

Answer

No solution

Exercise 5.2E.36

$$\begin{cases} y = -\frac{2}{3}x + 5 \\ 2x + 3y = 11 \end{cases}$$

Solve Applications of Systems of Equations by Substitution

In the following exercises, translate to a system of equations and solve.

https://math.libretexts.org/@go/page/61406

Exercise 5.2E.37

The sum of two numbers is 15. One number is 3 less than the other. Find the numbers.

Answer

The numbers are 6 and 9.

Exercise 5.2E.38

The sum of two numbers is 30. One number is 4 less than the other. Find the numbers.

Exercise 5.2E.39

The sum of two numbers is −26. One number is 12 less than the other. Find the numbers.

Answer

The numbers are −7 and −19.

Exercise 5.2E.40

The perimeter of a rectangle is 50. The length is 5 more than the width. Find the length and width.

Exercise 5.2E.41

The perimeter of a rectangle is 60. The length is 10 more than the width. Find the length and width.

Answer

The length is 20 and the width is 10.

Exercise 5.2E.42

The perimeter of a rectangle is 58. The length is 5 more than three times the width. Find the length and width.

Exercise 5.2E.43

The perimeter of a rectangle is 84. The length is 10 more than three times the width. Find the length and width.

Answer

The length is 34 and the width is 8.

Exercise 5.2E.44

The measure of one of the small angles of a right triangle is 14 more than 3 times the measure of the other small angle. Find the measure of both angles.

Exercise 5.2E.45

The measure of one of the small angles of a right triangle is 26 more than 3 times the measure of the other small angle. Find the measure of both angles.

Answer

The measures are 16° and 74°

Exercise 5.2E.46

The measure of one of the small angles of a right triangle is 15 less than twice the measure of the other small angle. Find the measure of both angles.

Exercise 5.2E.47

The measure of one of the small angles of a right triangle is 45 less than twice the measure of the other small angle. Find the measure of both angles.

Answer

The measures are 45° and 45°.

Exercise 5.2E.48

Maxim has been offered positions by two car dealers. The first company pays a salary of $10,000 plus a commission of $1,000 for each car sold. The second pays a salary of $20,000 plus a commission of $500 for each car sold. How many cars would need to be sold to make the total pay the same?

Exercise 5.2E.49

Jackie has been offered positions by two cable companies. The first company pays a salary of $ 14,000 plus a commission of $100 for each cable package sold. The second pays a salary of $20,000 plus a commission of $25 for each cable package sold. How many cable packages would need to be sold to make the total pay the same?

Answer

80 cable packages would need to be sold.

Exercise 5.2E.50

Amara currently sells televisions for company A at a salary of $17,000 plus a $100 commission for each television she sells. Company B offers her a position with a salary of $29,000 plus a $20 commission for each television she sells. How televisions would Amara need to sell for the options to be equal?

Exercise 5.2E.51

Mitchell currently sells stoves for company A at a salary of $12,000 plus a $150 commission for each stove he sells. Company B offers him a position with a salary of $24,000 plus a $50 commission for each stove he sells. How many stoves would Mitchell need to sell for the options to be equal?

Answer

Mitchell would need to sell 120 stoves.

Everyday Math

Exercise 5.2E.52

When Gloria spent 15 minutes on the elliptical trainer and then did circuit training for 30 minutes, her fitness app says she burned 435 calories. When she spent 30 minutes on the elliptical trainer and 40 minutes circuit training she burned 690

calories. Solve the system $\begin{cases} 15e+30c=435 \\ 30e+40c=690 \end{cases}$ for e, the number of calories she burns for each minute on the elliptical trainer, and cc, the number of calories she burns for each minute of circuit training.

Exercise 5.2E.53

Stephanie left Riverside, California, driving her motorhome north on Interstate 15 towards Salt Lake City at a speed of 56 miles per hour. Half an hour later, Tina left Riverside in her car on the same route as Stephanie, driving 70 miles per hour. Solve the system $\begin{cases} 56s=70t \\ s=t+\frac{1}{2} \end{cases}$

1. for t to find out how long it will take Tina to catch up to Stephanie.
2. what is the value of ss, the number of hours Stephanie will have driven before Tina catches up to her?

Answer

1. $t=2$ hours
2. $s=2\frac{1}{2}$ hours

Writing Exercises

Exercise 5.2E.54

Solve the system of equations
$\begin{cases} x+y=10 \\ x-y=6 \end{cases}$

1. by graphing.
2. by substitution.
3. Which method do you prefer? Why?

Exercise 5.2E.55

Solve the system of equations
$\begin{cases} 3x+y=12 \\ x=y-8 \end{cases}$ by substitution and explain all your steps in words.

Answer

Answers will vary.

Self Check

a. After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
solve a system of equations by substitution.			
solve applications of systems of equations by substitution.			

b. After reviewing this checklist, what will you do to become confident for all objectives?

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5.3: Solve Systems of Equations by Elimination

Learning Objectives

By the end of this section, you will be able to:

- Solve a system of equations by elimination
- Solve applications of systems of equations by elimination
- Choose the most convenient method to solve a system of linear equations

Note

Before you get started, take this readiness quiz.

1. Simplify −5(6−3a).
   If you missed this problem, review Example 1.10.43.
2. Solve the equation $\frac{1}{3}x+\frac{5}{8}=\frac{31}{24}$.
   If you missed this problem, review Example 2.5.1.

We have solved systems of linear equations by graphing and by substitution. Graphing works well when the variable coefficients are small and the solution has integer values. Substitution works well when we can easily solve one equation for one of the variables and not have too many fractions in the resulting expression.

The third method of solving systems of linear equations is called the Elimination Method. When we solved a system by substitution, we started with two equations and two variables and reduced it to one equation with one variable. This is what we'll do with the elimination method, too, but we'll have a different way to get there.

Solve a System of Equations by Elimination

The **Elimination Method** is based on the Addition Property of Equality. The Addition Property of Equality says that when you add the same quantity to both sides of an equation, you still have equality. We will extend the Addition Property of Equality to say that when you add equal quantities to both sides of an equation, the results are equal.

For any expressions *a*, *b*, *c*, and *d*,

$$\begin{array}{ll}\text{if} & a=b\\ \text{and} & c=d\\ \text{then} & a+c=b+d\end{array} \tag{5.3.1}$$

To solve a system of equations by elimination, we start with both equations in standard form. Then we decide which variable will be easiest to eliminate. How do we decide? We want to have the coefficients of one variable be opposites, so that we can add the equations together and eliminate that variable.

Notice how that works when we add these two equations together:

$$\begin{array}{l}3x+y=5\\ \underline{2x-y=0}\\ 5x\quad\quad=5\end{array} \tag{5.3.2}$$

The *y*'s add to zero and we have one equation with one variable.

Let's try another one:

$$\begin{cases}x+4y=2\\ 2x+5y=-2\end{cases} \tag{5.3.3}$$

This time we don't see a variable that can be immediately eliminated if we add the equations.

But if we multiply the first equation by −2, we will make the coefficients of *x* opposites. We must multiply every term on both sides of the equation by −2.

$$\begin{cases} -2(x+4y) = -2(2) \\ 2x+5y = -2 \end{cases}$$

$$\begin{cases} -2x-8y = -4 \\ 2x+5y = -2 \end{cases}$$

Now we see that the coefficients of the x terms are opposites, so x will be eliminated when we add these two equations.

Add the equations yourself—the result should be $-3y = -6$. And that looks easy to solve, doesn't it? Here is what it would look like.

$$\begin{array}{r} -2x-8y=-4 \\ 2x+5y=-2 \\ \hline -3y=-6 \end{array}$$

We'll do one more:

$$\begin{cases} 4x-3y=10 \\ 3x+5y=-7 \end{cases} \tag{5.3.4}$$

It doesn't appear that we can get the coefficients of one variable to be opposites by multiplying one of the equations by a constant, unless we use fractions. So instead, we'll have to multiply both equations by a constant.

We can make the coefficients of x be opposites if we multiply the first equation by 3 and the second by -4, so we get $12x$ and $-12x$.

$$3(4x-3y) = 3(10)$$
$$-4(3x+5y) = -4(-7)$$

This gives us these two new equations:

$$\begin{cases} 12x-9y=30 \\ -12x-20y=28 \end{cases}$$

When we add these equations,

$$\begin{array}{r} 12x-9y=30 \\ -12x-20y=28 \\ \hline -29y=58 \end{array} \tag{5.3.5}$$

the x's are eliminated and we just have $-29y = 58$.

Once we get an equation with just one variable, we solve it. Then we substitute that value into one of the original equations to solve for the remaining variable. And, as always, we check our answer to make sure it is a solution to both of the original equations.

Now we'll see how to use elimination to solve the same system of equations we solved by graphing and by substitution.

Example 5.3.1: How to Solve a System of Equations by Elimination

Solve the system by elimination. $\begin{cases} 2x+y=7 \\ x-2y=6 \end{cases}$

Solution

Step 1. Write both equations in standard form. If any coefficients are fractions, clear them.	Both equations are in standard form, $Ax + By = C$. There are no fractions.	$\begin{cases} 2x + y = 7 \\ x - 2y = 6 \end{cases}$
Step 2. Make the coefficients of one variable opposites. Decide which variable you will eliminate. Multiply one or both equations so that the coefficients of that variable are opposites.	We can eliminate the y's by multiplying the first equation by 2. Multiply both sides of $2x + y = 7$ by 2.	$\begin{cases} 2x + y = 7 \\ x - 2y = 6 \end{cases}$ $\begin{cases} 2(2x + y) = 2(7) \\ x - 2y = 6 \end{cases}$
Step 3. Add the equations resulting from Step 2 to eliminate one variable.	We add the x's, y's, and constants.	$\begin{array}{r} 4x + 2y = 14 \\ x - 2y = 6 \\ \hline 5x \quad\quad = 20 \end{array}$
Step 4. Solve for the remaining variable.	Solve for x.	$x = 4$
Step 5. Substitute the solution from Step 4 into one of the original equations. Then solve for the other variable.	Substitute $x = 4$ into the second equation, $x - 2y = 6$. Then solve for y.	$x - 2y = 6$ $4 - 2y = 6$ $-2y = 2$ $y = -1$
Step 6. Write the solution as an ordered pair.	Write it as (x, y).	$(4, -1)$
Step 7. Check that the ordered pair is a solution to **both** original equations.	Substitute $(4, -1)$ into $2x + y = 7$ and $x - 2y = 6$ Do they make both equations true? Yes!	$2x + y = 7$: $2(4) + (-1) \stackrel{?}{=} 7$: $7 = 7$ ✓ $x - 2y = 6$: $4 - 2(-1) \stackrel{?}{=} 6$: $6 = 6$ ✓ The solution is $(4, -1)$.

Try It 5.3.2

Solve the system by elimination. $\begin{cases} 3x + y = 5 \\ 2x - 3y = 7 \end{cases}$

Answer

$(2, -1)$

Try It 5.3.3

Solve the system by elimination. $\begin{cases} 4x + y = -5 \\ -2x - 2y = -2 \end{cases}$

Answer

$(-2, 3)$

The steps are listed below for easy reference.

HOW TO SOLVE A SYSTEM OF EQUATIONS BY ELIMINATION.

1. Write both equations in standard form. If any coefficients are fractions, clear them.
2. Make the coefficients of one variable opposites.
   - Decide which variable you will eliminate.
   - Multiply one or both equations so that the coefficients of that variable are opposites.
3. Add the equations resulting from Step 2 to eliminate one variable.
4. Solve for the remaining variable.
5. Substitute the solution from Step 4 into one of the original equations. Then solve for the other variable.
6. Write the solution as an ordered pair.
7. Check that the ordered pair is a solution to **both** original equations.

First we'll do an example where we can eliminate one variable right away.

Example 5.3.4

Solve the system by elimination. $\begin{cases} x+y=10 \\ x-y=12 \end{cases}$

Solution

	$\begin{array}{r} x+y=10 \\ x-y=12 \end{array}$
Both equations are in standard form.	
The coefficients of y are already opposites.	
Add the two equations to eliminate y. The resulting equation has only 1 variable, x.	$\begin{array}{r} x+y=10 \\ x-y=12 \\ \hline 2x \quad =22 \end{array}$
Solve for x, the remaining variable. Substitute $x = 11$ into one of the original equations.	$x=11$ $x+y=10$
	$11+y=10$
Solve for the other variable, y.	$y=-1$
Write the solution as an ordered pair.	The ordered pair is (11, −1).
Check that the ordered pair is a solution to **both** original equations. $\begin{array}{rcl} x+y & = & 10 \\ 11+(-1) & \stackrel{?}{=} & 10 \\ 10 & = & 10\checkmark \end{array}$ $\quad$ $\begin{array}{rcl} x-y & = & 12 \\ 11-(-1) & \stackrel{?}{=} & 12 \\ 12 & = & 12\checkmark \end{array}$	
	The solution is (11, −1).

Try It 5.3.5

Solve the system by elimination. $\begin{cases} 2x+y=5 \\ x-y=4 \end{cases}$

Answer

(3,−1)

Try It 5.3.6

Solve the system by elimination. $\begin{cases} x+y=3 \\ -2x-y=-1 \end{cases}$

Answer

$(-2,5)$

In Example 5.3.7, we will be able to make the coefficients of one variable opposites by multiplying one equation by a constant.

Example 5.3.7

Solve the system by elimination. $\begin{cases} 3x-2y=-2 \\ 5x-6y=10 \end{cases}$

Solution

	$\begin{aligned} 3x-2y&=-2\\ 5x-6y&=10 \end{aligned}$
Both equations are in standard form.	
None of the coefficients are opposites.	
We can make the coefficients of y opposites by multiplying the first equation by -3.	$\begin{aligned} -3(3x-2y)&=-3(-2)\\ 5x-6y&=10 \end{aligned}$
Simplify.	$\begin{aligned} -9x+6y&=6\\ 5x-6y&=10 \end{aligned}$
Add the two equations to eliminate y.	$\begin{array}{rl} -9x+6y&=6\\ 5x-6y&=10\\ \hline -4x&=16 \end{array}$
Solve for the remaining variable, x. Substitute $x=-4$ into one of the original equations.	$x=-4$ $3x-2y=-2$
	$3(-4)-2y=-2$
Solve for y.	$\begin{aligned} -12-2y&=-2\\ -2y&=10\\ y&=-5 \end{aligned}$
Write the solution as an ordered pair.	The ordered pair is $(-4,-5)$.
Check that the ordered pair is a solution to both original equations.	$\begin{array}{rcl\|rcl} 3x-2y&=&-2 & 5x-6y&=&10\\ 3(-4)-2(-5)&\stackrel{?}{=}&-2 & 5(-4)-6(-5)&\stackrel{?}{=}&10\\ -12+10&\stackrel{?}{=}&-2 & -20+30&\stackrel{?}{=}&10\\ -2&=&-2\checkmark & 10&=&10\checkmark \end{array}$	
	The solution is $(-4,-5)$.	

Try It 5.3.8

Solve the system by elimination. $\begin{cases} 4x-3y=1 \\ 5x-9y=-4 \end{cases}$

Answer

(1,1)

Try It 5.3.9

Solve the system by elimination. $\begin{cases} 3x+2y=2 \\ 6x+5y=8 \end{cases}$

Answer

(−2,4)

Now we'll do an example where we need to multiply both equations by constants in order to make the coefficients of one variable opposites.

Example 5.3.10

Solve the system by elimination. $\begin{cases} 4x-3y=9 \\ 7x+2y=-6 \end{cases}$

Solution

In this example, we cannot multiply just one equation by any constant to get opposite coefficients. So we will strategically multiply both equations by a constant to get the opposites.

	$\begin{aligned} 4x-3y&=9 \\ 7x+2y&=-6 \end{aligned}$
Both equations are in standard form. To get opposite coefficients of y, we will multiply the first equation by 2 and the second equation by 3.	$\begin{aligned} 2(4x-3y)&=2(9) \\ 3(7x+2y)&=3(-6) \end{aligned}$
Simplify.	$\begin{aligned} 8x-6y&=18 \\ 21x+6y&=-18 \end{aligned}$
Add the two equations to eliminate y.	$\begin{aligned} 8x-6y&=18 \\ 21x+6y&=-18 \\ \hline 39x&=0 \end{aligned}$
Solve for x. Substitute $x = 0$ into one of the original equations.	$x=0$ $7x+2y=-6$
	$7\cdot 0+2y=-6$
Solve for y.	$2y=-6$
	$y=-3$
Write the solution as an ordered pair.	The ordered pair is (0, −3).
Check that the ordered pair is a solution to **both** original equations. $\begin{aligned} 4x-3y &= 9 \\ 4(0)-3(-3) &\stackrel{?}{=} 9 \\ 9 &= 9\checkmark \end{aligned}$ $\quad$ $\begin{aligned} 7x+2y &= -6 \\ 7(0)+2(-3) &\stackrel{?}{=} -6 \\ -6 &= -6\checkmark \end{aligned}$	
	The solution is (0, −3).

Try It 5.3.11

Solve the system by elimination. $\begin{cases} 3x-4y=-9 \\ 5x+3y=14 \end{cases}$

Answer

(1,3)

Try It 5.3.12

Solve the system by elimination. $\begin{cases} 7x+8y=4 \\ 3x-5y=27 \end{cases}$

Answer

(4,−3)

When the system of equations contains fractions, we will first clear the fractions by multiplying each equation by its LCD.

Example 5.3.13

Solve the system by elimination. $\begin{cases} x+\frac{1}{2}y=6 \\ \frac{3}{2}x+\frac{2}{3}y=\frac{17}{2} \end{cases}$

Solution

In this example, both equations have fractions. Our first step will be to multiply each equation by its LCD to clear the fractions.

	$x+\frac{1}{2}y=6$ $\frac{3}{2}x+\frac{2}{3}y=\frac{17}{2}$
To clear the fractions, multiply each equation by its LCD.	$2\left(x+\frac{1}{2}y\right)=2(6)$ $6\left(\frac{3}{2}x+\frac{2}{3}y\right)=6\left(\frac{17}{2}\right)$
Simplify.	$2x+y=12$ $9x+4y=51$
Now we are ready to eliminate one of the variables. Notice that both equations are in standard form.	
We can eliminate y multiplying the top equation by −4.	$-4(2x+y)=-4(12)$ $9x+4y=51$
Simplify and add.	$-8x-4y=-48$ $9x+4y=51$ $x=3$
Substitute $x = 3$ into one of the original equations.	$x+\frac{1}{2}y=6$
Solve for y.	$3+\frac{1}{2}y=6$
	$\frac{1}{2}y=3$
	$y=6$
Write the solution as an ordered pair.	The ordered pair is (3, 6).

Check that the ordered pair is a solution to **both** original equations.

$$\begin{aligned} x+\tfrac{1}{2}y &= 6 \\ 3+\tfrac{1}{2}(6) &\stackrel{?}{=} 6 \\ 3+3 &\stackrel{?}{=} 6 \\ 6 &= 6\checkmark \end{aligned} \qquad \begin{aligned} \tfrac{3}{2}x+\tfrac{2}{3}y &= \tfrac{17}{2} \\ \tfrac{3}{2}(3)+\tfrac{2}{3}(6) &\stackrel{?}{=} \tfrac{17}{2} \\ \tfrac{9}{2}+4 &\stackrel{?}{=} \tfrac{17}{2} \\ \tfrac{9}{2}+\tfrac{8}{2} &\stackrel{?}{=} \tfrac{17}{2} \\ \tfrac{17}{2} &= \tfrac{17}{2}\checkmark \end{aligned}$$

The solution is (3, 6).

Try It 5.3.14

Solve the system by elimination. $\begin{cases} \frac{1}{3}x-\frac{1}{2}y=1 \\ \frac{3}{4}x-y=\frac{5}{2} \end{cases}$

Answer

(6,2)

Try It 5.3.15

Solve the system by elimination. $\begin{cases} x+\frac{3}{5}y=-\frac{1}{5} \\ -\frac{1}{2}x-\frac{2}{3}y=\frac{5}{6} \end{cases}$

Answer

(1,−2)

In the Solving Systems of Equations by Graphing we saw that not all systems of linear equations have a single ordered pair as a solution. When the two equations were really the same line, there were infinitely many solutions. We called that a consistent system. When the two equations described parallel lines, there was no solution. We called that an inconsistent system.

Example 5.3.16

Solve the system by elimination. $\begin{cases} 3x+4y=12 \\ y=3-\frac{3}{4}x \end{cases}$

Solution

$$\begin{cases} 3x+4y=12 \\ y=3-\frac{3}{4}x \end{cases}$$

Write the second equation in standard form.

$$\begin{cases} 3x+4y=12 \\ \frac{3}{4}x+y=3 \end{cases}$$

Clear the fractions by multiplying thesecond equation by 4.

$$\begin{cases} 3x+4y=12 \\ 4\left(\frac{3}{4}x+y\right)=4(3) \end{cases}$$

Simplify.

$$\begin{cases} 3x+4y=12 \\ 3x+4y=12 \end{cases}$$

To eliminate a variable, we multiply thesecond equation by -1.

$$\begin{cases} 3x+4y=12 \\ \underline{-3x-4y=-12} \end{cases}$$

Simplify and add.

$$0=0$$

This is a true statement. The equations are consistent but dependent. Their graphs would be the same line. The system has infinitely many solutions.

After we cleared the fractions in the second equation, did you notice that the two equations were the same? That means we have coincident lines.

Try It 5.3.17

Solve the system by elimination. $\begin{cases} 5x-3y=15 \\ y=-5+\frac{5}{3}x \end{cases}$

Answer

infinitely many solutions

Try It 5.3.18

Solve the system by elimination. $\begin{cases} x+2y=6 \\ y=-\frac{1}{2}x+3 \end{cases}$

Answer

infinitely many solutions

Example 5.3.19

Solve the system by elimination. $\begin{cases} -6x+15y=10 \\ 2x-5y=-5 \end{cases}$

Solution

The equations are in standard form.	$\begin{cases} -6x+15y=10 \\ 2x-5y=-5 \end{cases}$
Multiply the second equation by 3 to eliminate a variable.	$\begin{cases} -6x+15y=10 \\ 3(2x-5y)=3(-5) \end{cases}$
Simplify and add.	$\begin{cases} -6x+15y=10 \\ 6x-15y=-15 \end{cases}$ $0 \neq 5$

This statement is false. The equations are inconsistent and so their graphs would be parallel lines.

The system does not have a solution.

? Try It 5.3.20

Solve the system by elimination. $\begin{cases} -3x+2y=8 \\ 9x-6y=13 \end{cases}$

Answer

no solution

? Try It 5.3.21

Solve the system by elimination. $\begin{cases} 7x-3y=-2 \\ -14x+6y=8 \end{cases}$

Answer

no solution

Solve Applications of Systems of Equations by Elimination

Some applications problems translate directly into equations in standard form, so we will use the elimination method to solve them. As before, we use our Problem Solving Strategy to help us stay focused and organized.

✓ Example 5.3.22

The sum of two numbers is 39. Their difference is 9. Find the numbers.

Solution

Step 1. Read the problem	
Step 2. Identify what we are looking for.	We are looking for two numbers.
Step 3. Name what we are looking for.	Let n = the first number. m = the second number
Step 4. Translate into a system of equations.	
	The sum of two numbers is 39. $n+m=39$ Their difference is 9. $n-m=9$
The system is:	$\begin{cases} n+m=39 \\ n-m=9 \end{cases}$
Step 5. Solve the system of equations. To solve the system of equations, use elimination. The equations are in standard form and the coefficients of m are opposites. Add.	$\begin{cases} n+m=39 \\ \underline{n-m=9} \\ 2n \quad\quad =48 \end{cases}$
Solve for n.	$n=24$
Substitute n=24 into one of the original equations and solve form.	$n+m=39$ $24+m=39$ $m=15$
Step 6. Check the answer.	Since 24+15=39 and 24−15=9, the answers check.
Step 7. Answer the question.	The numbers are 24 and 15.

? Try It 5.3.23

The sum of two numbers is 42. Their difference is 8. Find the numbers.

Answer

The numbers are 25 and 17.

? Try It 5.3.24

The sum of two numbers is −15. Their difference is −35. Find the numbers.

Answer

The numbers are −25 and 10.

✓ Example 5.3.25

Joe stops at a burger restaurant every day on his way to work. Monday he had one order of medium fries and two small sodas, which had a total of 620 calories. Tuesday he had two orders of medium fries and one small soda, for a total of 820 calories. How many calories are there in one order of medium fries? How many calories in one small soda?

Solution

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	We are looking for the number of calories in one order of medium fries and in one small soda.
Step 3. Name what we are looking for.	Let f = the number of calories in 1 order of medium fries. s = the number of calories in 1 small soda.
Step 4. Translate into a system of equations:	one medium fries and two small sodas had a total of 620 calories
	$f+2s=620$
	two medium fries and one small soda had a total of 820 calories.
	$2f+s=820$
Our system is:	$\begin{cases} f+2s=620 \\ 2f+s=820 \end{cases}$
Step 5. Solve the system of equations. To solve the system of equations, use elimination. The equations are in standard form. To get opposite coefficients of f, multiply the top equation by -2.	$\begin{cases} -2(f+2s)=-2(620) \\ 2f+s=820 \end{cases}$
Simplify and add.	$\begin{array}{r} -2f-4s=-1240 \\ 2f+s=820 \\ \hline -3s=-420 \end{array}$
Solve for s.	$s=140$
Substitute $s=140$ into one of the original equations and then solve for f.	$f+2s=620$
	$f+2\cdot 140=620$
	$f+280=620$
	$f=340$
Step 6. Check the answer.	Verify that these numbers make sense in the problem and that they are solutions to both equations. We leave this to you!
Step 7. Answer the question.	The small soda has 140 calories and the fries have 340 calories.

? Try It 5.3.26

Malik stops at the grocery store to buy a bag of diapers and 2 cans of formula. He spends a total of $37. The next week he stops and buys 2 bags of diapers and 5 cans of formula for a total of $87. How much does a bag of diapers cost? How much is one can of formula?

Answer

The bag of diapers costs $11 and the can of formula costs $13.

? Try It 5.3.27

To get her daily intake of fruit for the day, Sasha eats a banana and 8 strawberries on Wednesday for a calorie count of 145. On the following Wednesday, she eats two bananas and 5 strawberries for a total of 235 calories for the fruit. How many calories are there in a banana? How many calories are in a strawberry?

Answer

There are 105 calories in a banana and 5 calories in a strawberry.

Choose the Most Convenient Method to Solve a System of Linear Equations

When you will have to solve a system of linear equations in a later math class, you will usually not be told which method to use. You will need to make that decision yourself. So you'll want to choose the method that is easiest to do and minimizes your chance of making mistakes.

Graphing	Substitution	Elimination
Use when you need a picture of the situation.	Use when one equation is already solved for one variable.	Use when the equations are in standard form.

✓ Example 5.3.28

For each system of linear equations decide whether it would be more convenient to solve it by substitution or elimination. Explain your answer.

1. $\begin{cases} 3x+8y=40 \\ 7x-4y=-32 \end{cases}$
2. $\begin{cases} 5x+6y=12 \\ y=\frac{2}{3}x-1 \end{cases}$

Solution

1. $\begin{cases} 3x+8y=40 \\ 7x-4y=-32 \end{cases}$

Since both equations are in standard form, using elimination will be most convenient.

2. $\begin{cases} 5x+6y=12 \\ y=\frac{2}{3}x-1 \end{cases}$

Since one equation is already solved for y, using substitution will be most convenient.

? Try It 5.3.29

For each system of linear equations, decide whether it would be more convenient to solve it by substitution or elimination. Explain your answer.

1. $\begin{cases} 4x-5y=-32 \\ 3x+2y=-1 \end{cases}$
2. $\begin{cases} x=2y-1 \\ 3x-5y=-7 \end{cases}$

Answer

1. Since both equations are in standard form, using elimination will be most convenient.
2. Since one equation is already solved for xx, using substitution will be most convenient.

Try It 5.3.30

For each system of linear equations, decide whether it would be more convenient to solve it by substitution or elimination. Explain your answer.

1. $\begin{cases} y = 2x - 1 \\ 3x - 4y = -6 \end{cases}$
2. $\begin{cases} 6x - 2y = 12 \\ 3x + 7y = -13 \end{cases}$

Answer

1. Since one equation is already solved for y, using substitution will be most convenient;
2. Since both equations are in standard form, using elimination will be most convenient.

Note

Access these online resources for additional instruction and practice with solving systems of linear equations by elimination.

- Instructional Video-Solving Systems of Equations by Elimination
- Instructional Video-Solving by Elimination
- Instructional Video-Solving Systems by Elimination

Key Concepts

- **To Solve a System of Equations by Elimination**
  1. Write both equations in standard form. If any coefficients are fractions, clear them.
  2. Make the coefficients of one variable opposites.
     - Decide which variable you will eliminate.
     - Multiply one or both equations so that the coefficients of that variable are opposites.
  3. Add the equations resulting from Step 2 to eliminate one variable.
  4. Solve for the remaining variable.
  5. Substitute the solution from Step 4 into one of the original equations. Then solve for the other variable.
  6. Write the solution as an ordered pair.
  7. Check that the ordered pair is a solution to **both** original equations.

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5.3E: Exercises

Practice Makes Perfect

Solve a System of Equations by Elimination

In the following exercises, solve the systems of equations by elimination.

Exercise 5.3E.1

$$\begin{cases} 5x+2y=2 \\ -3x-y=0 \end{cases}$$

Exercise 5.3E.2

$$\begin{cases} -3x+y=-9 \\ x-2y=-12 \end{cases}$$

Answer

$(6,9)$

Exercise 5.3E.3

$$\begin{cases} 6x-5y=-1 \\ 2x+y=13 \end{cases}$$

Exercise 5.3E.4

$$\begin{cases} 3x-y=-7 \\ 4x+2y=-6 \end{cases}$$

Answer

$(-2,1)$

Exercise 5.3E.5

$$\begin{cases} x+y=-1 \\ x-y=-5 \end{cases}$$

Exercise 5.3E.6

$$\begin{cases} x+y=-8 \\ x-y=-6 \end{cases}$$

Answer

$(-7,-1)$

Exercise 5.3E.7

$$\begin{cases} 3x-2y=1 \\ -x+2y=9 \end{cases}$$

Exercise 5.3E.8

$$\begin{cases} -7x+6y=-10 \\ x-6y=22 \end{cases}$$

Answer

$(-2,-4)$

Exercise 5.3E.9

$$\begin{cases} 3x+2y=-3 \\ -x-2y=-19 \end{cases}$$

Exercise 5.3E.10

$$\begin{cases} 5x+2y=1 \\ -5x-4y=-7 \end{cases}$$

Answer

$(-1,3)$

Exercise 5.3E.11

$$\begin{cases} 6x+4y=-4 \\ -6x-5y=8 \end{cases}$$

Exercise 5.3E.12

$$\begin{cases} 3x-4y=-11 \\ x-2y=-5 \end{cases}$$

Answer

$(-1,2)$

Exercise 5.3E.13

$$\begin{cases} 5x-7y=29 \\ x+3y=-3 \end{cases}$$

Exercise 5.3E.14

$$\begin{cases} 6x-5y=-75 \\ -x-2y=-13 \end{cases}$$

Answer

$(-5,9)$

Exercise 5.3E.15

$$\begin{cases} -x+4y=8 \\ 3x+5y=10 \end{cases}$$

Exercise 5.3E.16

$$\begin{cases} 2x-5y=7 \\ 3x-y=17 \end{cases}$$

Answer

$(6,1)$

Exercise 5.3E.17

$$\begin{cases} 5x-3y=-1 \\ 2x-y=2 \end{cases}$$

Exercise 5.3E.18

$$\begin{cases} 7x+y=-4 \\ 13x+3y=4 \end{cases}$$

Answer

$(-2,10)$

Exercise 5.3E.19

$$\begin{cases} -3x+5y=-13 \\ 2x+y=-26 \end{cases}$$

Exercise 5.3E.20

$$\begin{cases} 3x-5y=-9 \\ 5x+2y=16 \end{cases}$$

Answer

$(2,3)$

Exercise 5.3E.21

$$\begin{cases} 4x-3y=3 \\ 2x+5y=-31 \end{cases}$$

Exercise 5.3E.22

$$\begin{cases} 4x+7y=14 \\ -2x+3y=32 \end{cases}$$

Answer

$(-7,6)$

Exercise 5.3E.23

$$\begin{cases} 5x+2y=21 \\ 7x-4y=9 \end{cases}$$

Exercise 5.3E.24

$$\begin{cases} 3x+8y=-3 \\ 2x+5y=-3 \end{cases}$$

Answer

$(-9,3)$

Exercise 5.3E.25

$$\begin{cases} 11x+9y=-5 \\ 7x+5y=-1 \end{cases}$$

Exercise 5.3E.26

$$\begin{cases} 3x+8y=67 \\ 5x+3y=60 \end{cases}$$

Answer

$(9,5)$

Exercise 5.3E.27

$$\begin{cases} 2x+9y=-4 \\ 3x+13y=-7 \end{cases}$$

Exercise 5.3E.28

$$\begin{cases} \frac{1}{3}x-y=-3 \\ x+\frac{5}{2}y=2 \end{cases}$$

Answer

$(-3,2)$

Exercise 5.3E.29

$$\begin{cases} x+\frac{1}{2}y=\frac{3}{2} \\ \frac{1}{5}x-\frac{1}{5}y=3 \end{cases}$$

Exercise 5.3E.30

$$\begin{cases} x+\frac{1}{3}y=-1 \\ \frac{1}{2}x-\frac{1}{3}y=-2 \end{cases}$$

Answer

$(-2,3)$

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Exercise 5.3E.31

$$\begin{cases} \frac{1}{3}x - y = -3 \\ \frac{2}{3}x + \frac{5}{2}y = 3 \end{cases}$$

Exercise 5.3E.32

$$\begin{cases} 2x + y = 3 \\ 6x + 3y = 9 \end{cases}$$

Answer

infinitely many solutions

Exercise 5.3E.33

$$\begin{cases} x - 4y = -1 \\ -3x + 12y = 3 \end{cases}$$

Exercise 5.3E.34

$$\begin{cases} -3x - y = 8 \\ 6x + 2y = -16 \end{cases}$$

Answer

infinitely many solutions

Exercise 5.3E.35

$$\begin{cases} 4x + 3y = 2 \\ 20x + 15y = 10 \end{cases}$$

Exercise 5.3E.36

$$\begin{cases} 3x + 2y = 6 \\ -6x - 4y = -12 \end{cases}$$

Answer

infinitely many solutions

Exercise 5.3E.37

$$\begin{cases} 5x - 8y = 12 \\ 10x - 16y = 20 \end{cases}$$

Exercise 5.3E.38

$$\begin{cases} -11x + 12y = 60 \\ -22x + 24y = 90 \end{cases}$$

Answer

inconsistent, no solution

Exercise 5.3E.39

$$\begin{cases} 7x-9y=16 \\ -21x+27y=-24 \end{cases}$$

Exercise 5.3E.40

$$\begin{cases} 5x-3y=15 \\ y=\frac{5}{3}x-2 \end{cases}$$

Answer

inconsistent, no solution

Exercise 5.3E.41

$$\begin{cases} 2x+4y=7 \\ y=-\frac{1}{2}x-4 \end{cases}$$

Solve Applications of Systems of Equations by Elimination

In the following exercises, translate to a system of equations and solve.

Exercise 5.3E.42

The sum of two numbers is 65. Their difference is 25. Find the numbers.

Answer

The numbers are 20 and 45.

Exercise 5.3E.43

The sum of two numbers is 37. Their difference is 9. Find the numbers.

Exercise 5.3E.44

The sum of two numbers is −27. Their difference is −59. Find the numbers.

Answer

The numbers are 16 and −43.

Exercise 5.3E.45

The sum of two numbers is −45. Their difference is −89. Find the numbers.

Exercise 5.3E.46

Andrea is buying some new shirts and sweaters. She is able to buy 3 shirts and 2 sweaters for $114 or she is able to buy 2 shirts and 4 sweaters for $164. How much does a shirt cost? How much does a sweater cost?

Answer

A shirt costs $16 and a sweater costs $33.

Exercise 5.3E.47

Peter is buying office supplies. He is able to buy 3 packages of paper and 4 staplers for $40 or he is able to buy 5 packages of paper and 6 staplers for $62. How much does a package of paper cost? How much does a stapler cost?

Exercise 5.3E.48

The total amount of sodium in 2 hot dogs and 3 cups of cottage cheese is 4720 mg. The total amount of sodium in 5 hot dogs and 2 cups of cottage cheese is 6300 mg. How much sodium is in a hot dog? How much sodium is in a cup of cottage cheese?

Answer

There are 860 mg in a hot dog. There are 1,000 mg in a cup of cottage cheese.

Exercise 5.3E.49

The total number of calories in 2 hot dogs and 3 cups of cottage cheese is 960 calories. The total number of calories in 5 hot dogs and 2 cups of cottage cheese is 1190 calories. How many calories are in a hot dog? How many calories are in a cup of cottage cheese?

Choose the Most Convenient Method to Solve a System of Linear Equations

In the following exercises, decide whether it would be more convenient to solve the system of equations by substitution or elimination.

Exercise 5.3E.50

1. $\begin{cases} 8x-15y=-32 \\ 6x+3y=-5 \end{cases}$
2. $\begin{cases} x=4y-3 \\ 4x-2y=-6 \end{cases}$

Answer

1. elimination
2. substitution

Exercise 5.3E.51

1. $\begin{cases} y=7x-5 \\ 3x-2y=16 \end{cases}$
2. $\begin{cases} 12x-5y=-42 \\ 3x+7y=-15 \end{cases}$

Exercise 5.3E.52

1. $\begin{cases} y=4x+9 \\ 5x-2y=-21 \end{cases}$
2. $\begin{cases} 9x-4y=24 \\ 3x+5y=-14 \end{cases}$

Answer

1. substitution
2. elimination

Exercise 5.3E.53

1. $\begin{cases} 14x-15y=-30 \\ 7x+2y=10 \end{cases}$
2. $\begin{cases} x=9y-11 \\ 2x-7y=-27 \end{cases}$

Everyday Math

Exercise 5.3E.54

In one hour Norris can row 3 miles upstream against the current. In the same amount of time he can row 5 miles downstream, with the current. Solve the system. $\begin{cases} r-c=3 \\ r+c=5 \end{cases}$

1. for r, his rowing speed in still water.
2. Then solve for c, the speed of the river current.

Answer

1. r=4
2. c=1

Exercise 5.3E.55

Josie wants to make 10 pounds of trail mix using nuts and raisins, and she wants the total cost of the trail mix to be $54. Nuts cost $6 per pound and raisins cost $3 per pound. Solve the system $\begin{cases} n+r=10 \\ 6n+3r=54 \end{cases}$ to find n, the number of pounds of nuts, and rr, the number of pounds of raisins she should use.

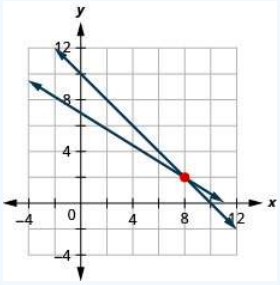
Writing Exercises

Exercise 5.3E.56

Solve the system
$\begin{cases} x+y=10 \\ 5x+8y=56 \end{cases}$

1. by substitution
2. by graphing
3. Which method do you prefer? Why?

Answer

1. (8, 2)
2. 

3. Answers will vary.

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Exercise $5.3E.57$

Solve the system $\begin{cases} x+y=-12 \\ y=4-\frac{1}{2}x \end{cases}$

1. by substitution
2. by graphing
3. Which method do you prefer? Why?

Self Check

a. After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
solve a system of equations by elimination.			
solve applications of systems of equations by elimination.			
choose the most convenient method to solve a system of linear equations.			

b. What does this checklist tell you about your mastery of this section? What steps will you take to improve?

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5.4: Solve Applications with Systems of Equations

Learning Objectives

By the end of this section, you will be able to:

- Translate to a system of equations
- Solve direct translation applications
- Solve geometry applications
- Solve uniform motion applications

Note

Before you get started, take this readiness quiz.

1. The sum of twice a number and nine is 31. Find the number.
   If you missed this problem, review Example 3.1.10.
2. Twins Jon and Ron together earned $96,000 last year. Ron earned $8,000 more than three times what Jon earned. How much did each of the twins earn?
   If you missed this problem, review Example 3.1.31.
3. Alessio rides his bike $3\frac{1}{2}$ hours at a rate of 10 miles per hour. How far did he ride?
   If you missed this problem, review Example 2.6.1.

Previously in this chapter we solved several applications with systems of linear equations. In this section, we'll look at some specific types of applications that relate two quantities. We'll translate the words into linear equations, decide which is the most convenient method to use, and then solve them.

We will use our Problem Solving Strategy for Systems of Linear Equations.

USE A PROBLEM SOLVING STRATEGY FOR SYSTEMS OF LINEAR EQUATIONS.

1. **Read** the problem. Make sure all the words and ideas are understood.
2. **Identify** what we are looking for.
3. **Name** what we are looking for. Choose variables to represent those quantities.
4. **Translate** into a system of equations.
5. **Solve** the system of equations using good algebra techniques.
6. **Check** the answer in the problem and make sure it makes sense.
7. **Answer** the question with a complete sentence.

Translate to a System of Equations

Many of the problems we solved in earlier applications related two quantities. Here are two of the examples from the chapter on Math Models.

- The sum of two numbers is negative fourteen. One number is four less than the other. Find the numbers.
- A married couple together earns $110,000 a year. The wife earns $16,000 less than twice what her husband earns. What does the husband earn?

In that chapter we translated each situation into one equation using only one variable. Sometimes it was a bit of a challenge figuring out how to name the two quantities, wasn't it?

Let's see how we can translate these two problems into a system of equations with two variables. We'll focus on Steps 1 through 4 of our Problem Solving Strategy.

Example 5.4.1: How to Translate to a System of Equations

Translate to a system of equations:

The sum of two numbers is negative fourteen. One number is four less than the other. Find the numbers.

Solution

Step 1. Read the problem. Make sure you understand all the words and ideas.	This is a number problem.	The sum of two numbers is negative fourteen. One number is four less than the other. Find the numbers.
Step 2. Identify what you are looking for.	"Find the numbers."	We are looking for 2 numbers.
Step 3. Name what you are looking for. Choose variables to represent those quantities.	We will use two variables, m and n.	Let m = one number n = second number
Step 4. Translate into a system of equations.	We will write one equation for each sentence.	The sum of the numbers ($m+n$) is ($=$) -14 (-14) One number (m) is ($=$) four less than the other ($n-4$) The system is: $\begin{cases} m+n=-14 \\ m=n-4 \end{cases}$

Try It 5.4.2

Translate to a system of equations:

The sum of two numbers is negative twenty-three. One number is 7 less than the other. Find the numbers.

Answer

$$\begin{cases} m+n=-23 \\ m=n-7 \end{cases}$$

Try It 5.4.3

Translate to a system of equations:

The sum of two numbers is negative eighteen. One number is 40 more than the other. Find the numbers.

Answer

$$\begin{cases} m+n=-18 \\ m=n+40 \end{cases}$$

We'll do another example where we stop after we write the system of equations.

Example 5.4.4

Translate to a system of equations:

A married couple together earns $110,000 a year. The wife earns $16,000 less than twice what her husband earns. What does the husband earn?

Solution

We are looking for the amount that the husband and wife each earn.	Let $h =$ the amount the husband earns.
	$w =$ the amount the wife earns
Translate.	A married couple together earns $110,000.
	$w + h = 110000$
	The wife earns $16,000 less than twice what husband earns.
	$w = 2h - 16,000$
The system of equations is:	$\begin{cases} w + h = 110,000 \\ w = 2h - 16,000 \end{cases}$

Try It 5.4.5

Translate to a system of equations:

A couple has a total household income of $84,000. The husband earns $18,000 less than twice what the wife earns. How much does the wife earn?

Answer

$$\begin{cases} w + h = 84,000 \\ h = 2w - 18,000 \end{cases}$$

Try It 5.4.6

Translate to a system of equations:

A senior employee makes $5 less than twice what a new employee makes per hour. Together they make $43 per hour. How much does each employee make per hour?

Answer

$$\begin{cases} s = 2n - 5 \\ s + n = 43 \end{cases}$$

Solve Direct Translation Applications

We set up, but did not solve, the systems of equations in Example 5.4.1 and Example 5.4.4 Now we'll translate a situation to a system of equations and then solve it.

Example 5.4.7

Translate to a system of equations and then solve:

Devon is 26 years older than his son Cooper. The sum of their ages is 50. Find their ages.

Solution

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	We are looking for the ages of Devon and Cooper.
Step 3. Name what we are looking for.	Let d= Devon's age. c= Cooper's age
Step 4. Translate into a system of equations.	Devon is 26 years older than Cooper.
	$d = c + 26$
	The sum of their ages is 50.

	$d + c = 50$
The system is:	$d = c + 26$ $d + c = 50$
Step 5. Solve the system of equations. Solve by substitution.	$d = c + 26$ $d + c = 50$ $d + c = 50$
Substitute $c + 26$ into the second equation.	$c + 26 + c = 50$
Solve for c.	$2c + 26 = 50$
	$2c = 24$
	$c = 12$ $d = c + 26$
Substitute $c = 12$ into the first equation and then solve for d.	$d = 12 + 26$
	$d = 38$
Step 6. Check the answer in the problem.	Is Devon's age 26 more than Cooper's? Yes, 38 is 26 more than 12. Is the sum of their ages 50? Yes, 38 plus 12 is 50.
Step 7. Answer the question.	Devon is 38 and Cooper is 12 years old.

Try It 5.4.8

Translate to a system of equations and then solve:

Ali is 12 years older than his youngest sister, Jameela. The sum of their ages is 40. Find their ages.

Answer

Ali is 26 and Jameela is 14.

Try It 5.4.9

Translate to a system of equations and then solve:

Jake's dad is 6 more than 3 times Jake's age. The sum of their ages is 42. Find their ages.

Answer

Jake is 9 and his dad is 33.

Example 5.4.10

Translate to a system of equations and then solve:

When Jenna spent 10 minutes on the elliptical trainer and then did circuit training for 20 minutes, her fitness app says she burned 278 calories. When she spent 20 minutes on the elliptical trainer and 30 minutes circuit training she burned 473 calories. How many calories does she burn for each minute on the elliptical trainer? How many calories does she burn for each minute of circuit training?

Solution

Step 1. Read the problem.	

Step 2. Identify what we are looking for.	We are looking for the number of calories burned each minute on the elliptical trainer and each minute of circuit training.
Step 3. Name what we are looking for.	Let e= number of calories burned per minute on the elliptical trainer. c= number of calories burned per minute while circuit training
Step 4. Translate into a system of equations.	10 minutes on the elliptical and circuit training for 20 minutes, burned 278 calories
	$10e + 20c = 278$
	20 minutes on the elliptical and 30 minutes of circuit training burned 473 calories
	$20e + 30c = 473$
The system is:	$\begin{aligned} 10e + 20c &= 278 \\ 20e + 30c &= 473 \end{aligned}$
Step 5. Solve the system of equations.	
Multiply the first equation by −2 to get opposite coefficients of e.	$\begin{aligned} -2(10e + 20c) &= -2(278) \\ 20e + 30c &= 473 \end{aligned}$
Simplify and add the equations. Solve for c.	$\begin{aligned} -20e - 40c &= -556 \\ 20e + 30c &= 473 \\ \hline -10c &= -83 \\ c &= 8.3 \end{aligned}$
Substitute $c = 8.3$ into one of the original equations to solve for e.	$10e + 20c = 278$
	$10e + 20(8.3) = 278$
	$10e + 166 = 278$
	$10e = 112$
	$e = 11.2$
Step 6. Check the answer in the problem.	Check the math on your own.
$\begin{aligned} 10(11.2) + 20(8.3) &\stackrel{?}{=} 278 \\ 20(11.2) + 30(8.3) &\stackrel{?}{=} 473 \end{aligned}$	
Step 7. Answer the question.	Jenna burns 8.3 calories per minute circuit training and 11.2 calories per minute while on the elliptical trainer.

Try It 5.4.11

Translate to a system of equations and then solve:

Mark went to the gym and did 40 minutes of Bikram hot yoga and 10 minutes of jumping jacks. He burned 510 calories. The next time he went to the gym, he did 30 minutes of Bikram hot yoga and 20 minutes of jumping jacks burning 470 calories. How many calories were burned for each minute of yoga? How many calories were burned for each minute of jumping jacks?

Answer

Mark burned 11 calories for each minute of yoga and 7 calories for each minute of jumping jacks.

Try It 5.4.12

Translate to a system of equations and then solve:

Erin spent 30 minutes on the rowing machine and 20 minutes lifting weights at the gym and burned 430 calories. During her next visit to the gym she spent 50 minutes on the rowing machine and 10 minutes lifting weights and burned 600 calories. How many calories did she burn for each minutes on the rowing machine? How many calories did she burn for each minute of weight lifting?

Answer

Erin burned 11 calories for each minute on the rowing machine and 5 calories for each minute of weight lifting.

Solve Geometry Applications

When we learned about Math Models, we solved geometry applications using properties of triangles and rectangles. Now we'll add to our list some properties of angles.

The measures of two **complementary angles** add to 90 degrees. The measures of two **supplementary angles** add to 180 degrees.

COMPLEMENTARY AND SUPPLEMENTARY ANGLES

Two angles are **complementary** if the sum of the measures of their angles is 90 degrees.

Two angles are **supplementary** if the sum of the measures of their angles is 180 degrees.

If two angles are complementary, we say that *one angle is the complement of the other.*

If two angles are supplementary, we say that *one angle is the supplement of the other.*

Example 5.4.13

Translate to a system of equations and then solve:

The difference of two complementary angles is 26 degrees. Find the measures of the angles.

Solution

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	We are looking for the measure of each angle.
Step 3. Name what we are looking for.	Let x = the measure of the first angle. y = the measure of the second angle
Step 4. Translate into a system of equations.	The angles are complementary. $x+y=90$ The difference of the two angles is 26 degrees. $x-y=26$
The system is	$\begin{cases} x+y=90 \\ x-y=26 \end{cases}$
Step 5. Solve the system of equations by elimination.	$\begin{cases} x+y=90 \\ x-y=26 \end{cases}$ $2x=116$ $x=58$
Substitute $x=58$ into the first equation.	$x+y=90$ $58+y=90$ $y=32$

Step 6. Check the answer in the problem. $58+32=90$✓ $58-32=36$✓	
Step 7. Answer the question.	The angle measures are 58 degrees and 32 degrees.

Try It 5.4.14

Translate to a system of equations and then solve:

The difference of two complementary angles is 20 degrees. Find the measures of the angles.

Answer

The angle measures are 55 degrees and 35 degrees.

Try It 5.4.15

Translate to a system of equations and then solve:

The difference of two complementary angles is 80 degrees. Find the measures of the angles.

Answer

The angle measures are 5 degrees and 85 degrees.

Example 5.4.16

Translate to a system of equations and then solve:

Two angles are supplementary. The measure of the larger angle is twelve degrees less than five times the measure of the smaller angle. Find the measures of both angles.

Solution

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	We are looking for the measure of each angle.
Step 3. Name what we are looking for.	Let x= the measure of the first angle. y= the measure of the second angle
Step 4. Translate into a system of equations.	The angles are supplementary.
	$x+y=180$
	The larger angle is twelve less than five times the smaller angle
	$y=5x-12$
The system is: **Step 5. Solve** the system of equations substitution.	$x+y=180$ $y=5x-12$ $x+y=180$
Substitute $5x-12$ for y in the first equation.	$x+5x-12=180$
Solve for x.	$6x-12=180$
	$6x=192$

	$x = 32$ $y = 5x - 12$
Substitute 32 for in the second equation, then solve for y.	$y = 5 \cdot 32 - 12$
	$y = 160 - 12$
	$y = 148$
Step 6. Check the answer in the problem. $32 + 158 = 180$✓ $5 \cdot 32 - 12 = 148$✓	
Step 7. Answer the question.	The angle measures are 148 and 32.

Try It 5.4.17

Translate to a system of equations and then solve:

Two angles are supplementary. The measure of the larger angle is 12 degrees more than three times the smaller angle. Find the measures of the angles.

Answer

The angle measures are 42 degrees and 138 degrees.

Try It 5.4.18

Translate to a system of equations and then solve:

Two angles are supplementary. The measure of the larger angle is 18 less than twice the measure of the smaller angle. Find the measures of the angles.

Answer

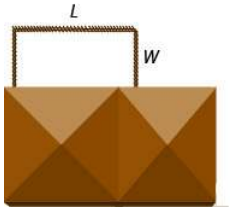
The angle measures are 66 degrees and 114 degrees.

Example 5.4.19

Translate to a system of equations and then solve:

Randall has 125 feet of fencing to enclose the rectangular part of his backyard adjacent to his house. He will only need to fence around three sides, because the fourth side will be the wall of the house. He wants the length of the fenced yard (parallel to the house wall) to be 5 feet more than four times as long as the width. Find the length and the width.

Solution

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	We are looking for the length and width.
	

Step 3. Name what we are looking for.	Let L= the length of the fenced yard. W= the width of the fenced yard
Step 4. Translate into a system of equations.	One length and two widths equal 125.
	$L+2W=125$
	The length will be 5 feet more than four times the width.
	$L=4W+5$
The system is: **Step 5. Solve** the system of equations by substitution.	$L+2W=125$ $L=4W+5$ $L+2W=125$
Substitute $L = 4W + 5$ into the first equation, then solve for W.	$4W+5+2W=125$
	$6W+5=125$
	$6W=120$
Substitute 20 for W in the second equation, then solve for L.	$W=20$ $L=4W+5$
	$L=4\cdot 20+5$
	$L=80+5$
	$L=85$
Step 6. Check the answer in the problem. $20+85+20 = 125\checkmark$ $85 = 4\cdot 20+5\checkmark$	
Step 7. Answer the equation.	The length is 85 feet and the width is 20 feet.

Try It 5.4.20

Translate to a system of equations and then solve:

Mario wants to put a rectangular fence around the pool in his backyard. Since one side is adjacent to the house, he will only need to fence three sides. There are two long sides and the one shorter side is parallel to the house. He needs 155 feet of fencing to enclose the pool. The length of the long side is 10 feet less than twice the width. Find the length and width of the pool area to be enclosed.

Answer

The length is 60 feet and the width is 35 feet.

Try It 5.4.21

Translate to a system of equations and then solve:

Alexis wants to build a rectangular dog run in her yard adjacent to her neighbor's fence. She will use 136 feet of fencing to completely enclose the rectangular dog run. The length of the dog run along the neighbor's fence will be 16 feet less than twice the width. Find the length and width of the dog run.

Answer

The length is 60 feet and the width is 38 feet.

Solve Uniform Motion Applications

We used a table to organize the information in uniform motion problems when we introduced them earlier. We'll continue using the table here. The basic equation was $D = rt$ where D is the distance traveled, r is the rate, and t is the time.

Our first example of a uniform motion application will be for a situation similar to some we have already seen, but now we can use two variables and two equations.

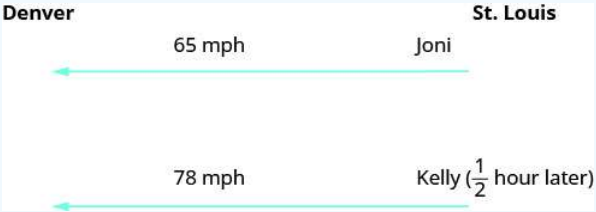
Example 5.4.22

Translate to a system of equations and then solve:

Joni left St. Louis on the interstate, driving west towards Denver at a speed of 65 miles per hour. Half an hour later, Kelly left St. Louis on the same route as Joni, driving 78 miles per hour. How long will it take Kelly to catch up to Joni?

Solution

A diagram is useful in helping us visualize the situation.



Identify and name what we are looking for. A chart will help us organize the data. We know the rates of both Joni and Kelly, and so we enter them in the chart.	
We are looking for the length of time Kelly, k, and Joni, j, will each drive. Since D=r·t we can fill in the Distance column.	

Type	Rate	• Time	= Distance
Joni	65	j	$65j$
Kelly	78	k	$78k$

Translate into a system of equations.

To make the system of equations, we must recognize that Kelly and Joni will drive the same distance. So, 65j=78k.

Also, since Kelly left later, her time will be $\frac{1}{2}$ hour less than Joni's time.

So, $k = j - \frac{1}{2}$.

Now we have the system.	$k = j - \frac{1}{2}$ $65j = 78k$
Solve the system of equations by substitution.	$65j = 78k$
Substitute k=j−12 into the second equation, then solve for j.	$65j = 78\left(j - \frac{1}{2}\right)$
	$65j = 78j - 39$
	$-13j = -39$
	$j = 3$
To find Kelly's time, substitute j = 3 into the first equation, then solve for k.	$k = j - \frac{1}{2}$
	$k = 3 - \frac{1}{2}$

	$k=\frac{5}{2}$ or $k=2\frac{1}{2}$
Check the answer in the problem. Joni 3 hours (65 mph) = 195 miles. Kelly $2\frac{1}{2}$ hours (78 mph) = 195 miles. Yes, they will have traveled the same distance when they meet.	
Answer the question.	Kelly will catch up to Joni in $2\frac{1}{2}$ hours. By then, Joni will have traveled 3 hours.

? Try It 5.4.23

Translate to a system of equations and then solve: Mitchell left Detroit on the interstate driving south towards Orlando at a speed of 60 miles per hour. Clark left Detroit 1 hour later traveling at a speed of 75 miles per hour, following the same route as Mitchell. How long will it take Clark to catch Mitchell?

Answer

It will take Clark 4 hours to catch Mitchell.

? Try It 5.4.24

Translate to a system of equations and then solve: Charlie left his mother's house traveling at an average speed of 36 miles per hour. His sister Sally left 15 minutes (1/4 hour) later traveling the same route at an average speed of 42 miles per hour. How long before Sally catches up to Charlie?

Answer

It will take Sally $1\frac{1}{2}$ hours to catch up to Charlie.

Many real-world applications of uniform motion arise because of the effects of currents—of water or air—on the actual speed of a vehicle. Cross-country airplane flights in the United States generally take longer going west than going east because of the prevailing wind currents.

Let's take a look at a boat traveling on a river. Depending on which way the boat is going, the current of the water is either slowing it down or speeding it up.

Figure 5.4.1 and Figure 5.4.2 show how a river current affects the speed at which a boat is actually traveling. We'll call the speed of the boat in still water b and the speed of the river current c.

In Figure 5.4.1 the boat is going downstream, in the same direction as the river current. The current helps push the boat, so the boat's actual speed is faster than its speed in still water. The actual speed at which the boat is moving is $b + c$.

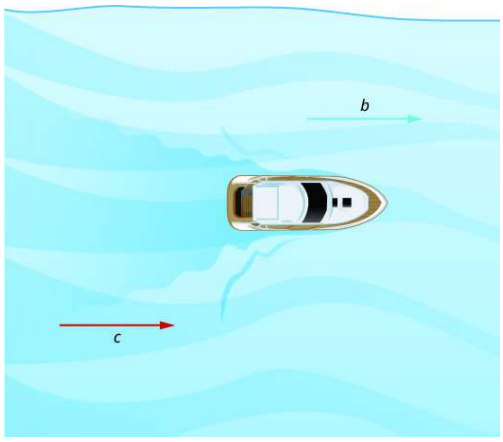


Figure 5.4.1

In Figure 5.4.2 the boat is going upstream, opposite to the river current. The current is going against the boat, so the boat's actual speed is slower than its speed in still water. The actual speed of the boat is b−c.

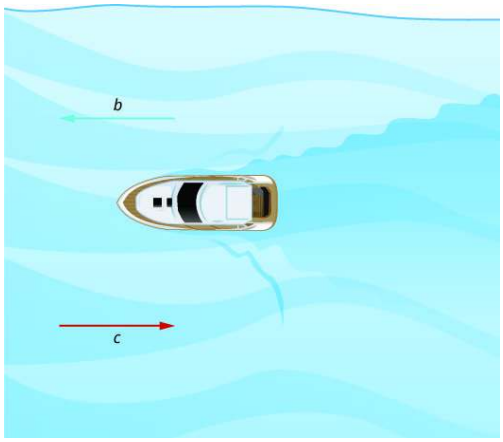


Figure 5.4.2

We'll put some numbers to this situation in Example 5.4.25

Example 5.4.25

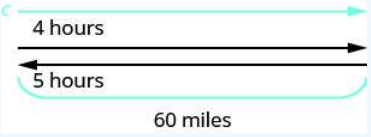
Translate to a system of equations and then solve:

A river cruise ship sailed 60 miles downstream for 4 hours and then took 5 hours sailing upstream to return to the dock. Find the speed of the ship in still water and the speed of the river current.

Solution

Read the problem.

This is a uniform motion problem and a picture will help us visualize the situation.



Identify what we are looking for.	We are looking for the speed of the ship in still water and the speed of the current.
Name what we are looking for.	Let s=s= the rate of the ship in still water. c=c= the rate of the current

A chart will help us organize the information. The ship goes downstream and then upstream. Going downstream, the current helps the ship; therefore, the ship's actual rate is $s + c$. Going upstream, the current slows the ship; therefore, the actual rate is $s - c$.	
Downstream it takes 4 hours. Upstream it takes 5 hours. Each way the distance is 60 miles.	
Translate into a system of equations. Since rate times time is distance, we can write the system of equations.	$\begin{cases} 4(s+c)=60 \\ 5(s-c)=60 \end{cases}$
Solve the system of equations. Distribute to put both equations in standard form, then solve by elimination.	$\begin{cases} 4s+4c=60 \\ 5s-5c=60 \end{cases}$
Multiply the top equation by 5 and the bottom equation by 4. Add the equations, then solve for s.	$\begin{array}{r} 20s+20c=300 \\ 20s-20c=240 \\ \hline 40s \quad\quad =540 \end{array}$
Substitute $s = 13.5$ into one of the original equations.	$s = 13.5$ $4(s+c)=60$
	$4(13.5+c)=60$
	$54+4c=60$
	$4c=6$
	$c=1.5$
Check the answer in the problem. The downstream rate would be $13.5 + 1.5 = 15$ mph. In 4 hours the ship would travel $15 \cdot 4 = 60$ miles. The upstream rate would be $13.5 - 1.5 = 12$ mph. In 5 hours the ship would travel $12 \cdot 5 = 60$ miles.	
Answer the question.	The rate of the ship is 13.5 mph and the rate of the current is 1.5 mph.

	Rate	• Time =	Distance
downstream	$s + c$	4	60
upstream	$s - c$	5	60

? Try It 5.4.26

Translate to a system of equations and then solve: A Mississippi river boat cruise sailed 120 miles upstream for 12 hours and then took 10 hours to return to the dock. Find the speed of the river boat in still water and the speed of the river current.

Answer

The rate of the boat is 11 mph and the rate of the current is 1 mph.

Try It 5.4.27

Translate to a system of equations and then solve: Jason paddled his canoe 24 miles upstream for 4 hours. It took him 3 hours to paddle back. Find the speed of the canoe in still water and the speed of the river current.

Answer

The speed of the canoe is 7 mph and the speed of the current is 1 mph.

Wind currents affect airplane speeds in the same way as water currents affect boat speeds. We'll see this in Example 5.4.28 A wind current in the same direction as the plane is flying is called a *tailwind*. A wind current blowing against the direction of the plane is called a *headwind*.

Example 5.4.28

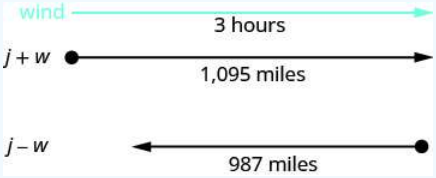
Translate to a system of equations and then solve:

A private jet can fly 1095 miles in three hours with a tailwind but only 987 miles in three hours into a headwind. Find the speed of the jet in still air and the speed of the wind.

Solution

Read the problem.

This is a uniform motion problem and a picture will help us visualize.



Identify what we are looking for.	We are looking for the speed of the jet in still air and the speed of the wind.
Name what we are looking for.	Let j= the speed of the jet in still air. w= the speed of the wind
A chart will help us organize the information. The jet makes two trips-one in a tailwind and one in a headwind. In a tailwind, the wind helps the jet and so the rate is $j + w$. In a headwind, the wind slows the jet and so the rate is $j - w$.	

	Rate • Time = Distance		
tailwind	$j + w$	3	1095
headwind	$j - w$	3	987

Each trip takes 3 hours. In a tailwind the jet flies 1095 miles. In a headwind the jet flies 987 miles.	
Translate into a system of equations. Since rate times time is distance, we get the system of equations.	$\begin{aligned} 3(j+w) &= 1095 \\ 3(j-w) &= 987 \end{aligned}$
Solve the system of equations. Distribute, then solve by elimination.	$\begin{aligned} 3j + 3w &= 1095 \\ 3j - 3w &= 987 \\ \hline 6j &= 2082 \end{aligned}$

Add, and solve for j. Substitute $j = 347$ into one of the original equations, then solve for w.	$j = 347$ $3(j + w) = 1095$
	$3(347 + w) = 1095$
	$1041 + 3w = 1095$
	$3w = 54$
	$w = 18$
Check the answer in the problem. With the tailwind, the actual rate of the jet would be $347 + 18 = 365$ mph. In 3 hours the jet would travel $365 \cdot 3 = 1095$ miles. Going into the headwind, the jet's actual rate would be $347 - 18 = 329$ mph. In 3 hours the jet would travel $329 \cdot 3 = 987$ miles.	
Answer the question.	The rate of the jet is 347 mph and the rate of the wind is 18 mph.

Try It 5.4.29

Translate to a system of equations and then solve: A small jet can fly 1,325 miles in 5 hours with a tailwind but only 1025 miles in 5 hours into a headwind. Find the speed of the jet in still air and the speed of the wind.

Answer

The speed of the jet is 235 mph and the speed of the wind is 30 mph.

Try It 5.4.30

Translate to a system of equations and then solve: A commercial jet can fly 1728 miles in 4 hours with a tailwind but only 1536 miles in 4 hours into a headwind. Find the speed of the jet in still air and the speed of the wind.

Answer

The speed of the jet is 408 mph and the speed of the wind is 24 mph.

Glossary

complementary angles

Two angles are complementary if the sum of the measures of their angles is 90 degrees.

supplementary angles

Two angles are supplementary if the sum of the measures of their angles is 180 degrees.

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5.4E: Exercises

Practice Makes Perfect

Translate to a System of Equations

In the following exercises, translate to a system of equations and solve the system.

Exercise 5.4E.1

The sum of two numbers is fifteen. One number is three less than the other. Find the numbers.

Answer

The numbers are 6 and 9.

Exercise 5.4E.2

The sum of two numbers is twenty-five. One number is five less than the other. Find the numbers.

Exercise 5.4E.3

The sum of two numbers is negative thirty. One number is five times the other. Find the numbers.

Answer

The numbers are -5 and -25.

Exercise 5.4E.4

The sum of two numbers is negative sixteen. One number is seven times the other. Find the numbers.

Exercise 5.4E.5

Twice a number plus three times a second number is twenty-two. Three times the first number plus four times the second is thirty-one. Find the numbers.

Answer

The numbers are 5 and 4.

Exercise 5.4E.6

Six times a number plus twice a second number is four. Twice the first number plus four times the second number is eighteen. Find the numbers.

Exercise 5.4E.7

Three times a number plus three times a second number is fifteen. Four times the first plus twice the second number is fourteen. Find the numbers.

Answer

The numbers are 2 and 3.

Exercise 5.4E.8

Twice a number plus three times a second number is negative one. The first number plus four times the second number is two. Find the numbers.

Exercise 5.4E.9

A married couple together earn $75,000. The husband earns $15,000 more than five times what his wife earns. What does the wife earn?

Answer

$10,000

Exercise 5.4E.10

During two years in college, a student earned $9,500. The second year she earned $500 more than twice the amount she earned the first year. How much did she earn the first year?

Exercise 5.4E.11

Daniela invested a total of $50,000, some in a certificate of deposit (CD) and the remainder in bonds. The amount invested in bonds was $5000 more than twice the amount she put into the CD. How much did she invest in each account?

Answer

She put $15,000 into a CD and $35,000 in bonds.

Exercise 5.4E.12

Jorge invested $28,000 into two accounts. The amount he put in his money market account was $2,000 less than twice what he put into a CD. How much did he invest in each account?

Exercise 5.4E.13

In her last two years in college, Marlene received $42,000 in loans. The first year she received a loan that was $6,000 less than three times the amount of the second year's loan. What was the amount of her loan for each year?

Answer

The amount of the first year's loan was $30,000 and the amount of the second year's loan was $12,000.

Exercise 5.4E.14

Jen and David owe $22,000 in loans for their two cars. The amount of the loan for Jen's car is $2000 less than twice the amount of the loan for David's car. How much is each car loan?

Solve Direct Translation Applications

In the following exercises, translate to a system of equations and solve.

Exercise 5.4E.15

Alyssa is twelve years older than her sister, Bethany. The sum of their ages is forty-four. Find their ages.

Answer

Bethany is 16 years old and Alyssa is 28 years old.

Exercise 5.4E.16

Robert is 15 years older than his sister, Helen. The sum of their ages is sixty-three. Find their ages.

Exercise 5.4E.17

The age of Noelle's dad is six less than three times Noelle's age. The sum of their ages is seventy-four. Find their ages.

Answer

Noelle is 20 years old and her dad is 54 years old.

Exercise 5.4E.18

The age of Mark's dad is 4 less than twice Marks's age. The sum of their ages is ninety-five. Find their ages.

Exercise 5.4E.19

Two containers of gasoline hold a total of fifty gallons. The big container can hold ten gallons less than twice the small container. How many gallons does each container hold?

Answer

The small container holds 20 gallons and the large container holds 30 gallons.

Exercise 5.4E.20

June needs 48 gallons of punch for a party and has two different coolers to carry it in. The bigger cooler is five times as large as the smaller cooler. How many gallons can each cooler hold?

Exercise 5.4E.21

Shelly spent 10 minutes jogging and 20 minutes cycling and burned 300 calories. The next day, Shelly swapped times, doing 20 minutes of jogging and 10 minutes of cycling and burned the same number of calories. How many calories were burned for each minute of jogging and how many for each minute of cycling?

Answer

There were 10 calories burned jogging and 10 calories burned cycling.

Exercise 5.4E.22

Drew burned 1800 calories Friday playing one hour of basketball and canoeing for two hours. Saturday he spent two hours playing basketball and three hours canoeing and burned 3200 calories. How many calories did he burn per hour when playing basketball?

Exercise 5.4E.23

Troy and Lisa were shopping for school supplies. Each purchased different quantities of the same notebook and thumb drive. Troy bought four notebooks and five thumb drives for $116. Lisa bought two notebooks and three thumb dives for $68. Find the cost of each notebook and each thumb drive.

Answer

Notebooks are $4 and thumb drives are $20.

Exercise 5.4E.24

Nancy bought seven pounds of oranges and three pounds of bananas for $17. Her husband later bought three pounds of oranges and six pounds of bananas for $12. What was the cost per pound of the oranges and the bananas?

Solve Geometry Applications In the following exercises, translate to a system of equations and solve.

Exercise 5.4E.25

The difference of two complementary angles is 30 degrees. Find the measures of the angles.

Answer

The measures are 60 degrees and 30 degrees.

Exercise 5.4E.26

The difference of two complementary angles is 68 degrees. Find the measures of the angles.

Exercise 5.4E.27

The difference of two supplementary angles is 70 degrees. Find the measures of the angles.

Answer

The measures are 125 degrees and 55 degrees.

Exercise 5.4E.28

The difference of two supplementary angles is 24 degrees. Find the measure of the angles.

Exercise 5.4E.29

The difference of two supplementary angles is 8 degrees. Find the measures of the angles.

Answer

94 degrees and 86 degrees

Exercise 5.4E.30

The difference of two supplementary angles is 88 degrees. Find the measures of the angles.

Exercise 5.4E.31

The difference of two complementary angles is 55 degrees. Find the measures of the angles.

Answer

72.5 degrees and 17.5 degrees

Exercise 5.4E.32

The difference of two complementary angles is 17 degrees. Find the measures of the angles.

Exercise 5.4E.33

Two angles are supplementary. The measure of the larger angle is four more than three times the measure of the smaller angle. Find the measures of both angles.

Answer

The measures are 44 degrees and 136 degrees.

Exercise 5.4E.34

Two angles are supplementary. The measure of the larger angle is five less than four times the measure of the smaller angle. Find the measures of both angles.

Exercise 5.4E.35

Two angles are complementary. The measure of the larger angle is twelve less than twice the measure of the smaller angle. Find the measures of both angles.

Answer

The measures are 34 degrees and 56 degrees.

Exercise 5.4E.36

Two angles are complementary. The measure of the larger angle is ten more than four times the measure of the smaller angle. Find the measures of both angles.

Exercise 5.4E.37

Wayne is hanging a string of lights 45 feet long around the three sides of his rectangular patio, which is adjacent to his house. The length of his patio, the side along the house, is five feet longer than twice its width. Find the length and width of the patio.

Answer

The width is 10 feet and the length is 25 feet.

Exercise 5.4E.38

Darrin is hanging 200 feet of Christmas garland on the three sides of fencing that enclose his rectangular front yard. The length, the side along the house, is five feet less than three times the width. Find the length and width of the fencing.

Exercise 5.4E.39

A frame around a rectangular family portrait has a perimeter of 60 inches. The length is fifteen less than twice the width. Find the length and width of the frame.

Answer

The width is 15 feet and the length is 15 feet.

Exercise 5.4E.40

The perimeter of a rectangular toddler play area is 100 feet. The length is ten more than three times the width. Find the length and width of the play area.

Solve Uniform Motion Applications In the following exercises, translate to a system of equations and solve.

Exercise 5.4E.41

Sarah left Minneapolis heading east on the interstate at a speed of 60 mph. Her sister followed her on the same route, leaving two hours later and driving at a rate of 70 mph. How long will it take for Sarah's sister to catch up to Sarah?

Answer

It took Sarah's sister 12 hours.

Exercise 5.4E.42

College roommates John and David were driving home to the same town for the holidays. John drove 55 mph, and David, who left an hour later, drove 60 mph. How long will it take for David to catch up to John?

Exercise 5.4E.43

At the end of spring break, Lucy left the beach and drove back towards home, driving at a rate of 40 mph. Lucy's friend left the beach for home 30 minutes (half an hour) later, and drove 50 mph. How long did it take Lucy's friend to catch up to Lucy?

Answer

It took Lucy's friend 2 hours.

Exercise 5.4E.44

Felecia left her home to visit her daughter driving 45 mph. Her husband waited for the dog sitter to arrive and left home twenty minutes (1/3 hour) later. He drove 55 mph to catch up to Felecia. How long before he reaches her?

Exercise 5.4E.45

The Jones family took a 12 mile canoe ride down the Indian River in two hours. After lunch, the return trip back up the river took three hours. Find the rate of the canoe in still water and the rate of the current.

Answer

The canoe rate is 5 mph and the current rate is 1 mph.

Exercise 5.4E.46

A motor boat travels 60 miles down a river in three hours but takes five hours to return upstream. Find the rate of the boat in still water and the rate of the current.

Exercise 5.4E.47

A motor boat traveled 18 miles down a river in two hours but going back upstream, it took 4.5 hours due to the current. Find the rate of the motor boat in still water and the rate of the current.

Answer

The boat rate is 6.5 mph and the current rate is 2.5 mph.

Exercise 5.4E.48

A river cruise boat sailed 80 miles down the Mississippi River for four hours. It took five hours to return. Find the rate of the cruise boat in still water and the rate of the current.

Exercise 5.4E.49

A small jet can fly 1,072 miles in 4 hours with a tailwind but only 848 miles in 4 hours into a headwind. Find the speed of the jet in still air and the speed of the wind.

Answer

The jet rate is 240 mph and the wind speed is 28 mph.

Exercise 5.4E.50

A small jet can fly 1,435 miles in 5 hours with a tailwind but only 1215 miles in 5 hours into a headwind. Find the speed of the jet in still air and the speed of the wind.

Exercise 5.4E.51

A commercial jet can fly 868 miles in 2 hours with a tailwind but only 792 miles in 2 hours into a headwind. Find the speed of the jet in still air and the speed of the wind.

Answer

The jet rate is 415 mph and the wind speed is 19 mph.

Exercise 5.4E.52

A commercial jet can fly 1,320 miles in 3 hours with a tailwind but only 1,170 miles in 3 hours into a headwind. Find the speed of the jet in still air and the speed of the wind.

Everyday Math

Exercise 5.4E.53

At a school concert, 425 tickets were sold. Student tickets cost $5 each and adult tickets cost $8 each. The total receipts for the concert were $2,851. Solve the system

$$\begin{cases} s+a=425 \\ 5s+8a=2,851 \end{cases}$$

to find s, the number of student tickets and aa, the number of adult tickets.

Answer

s=183,a=242

Exercise 5.4E.54

The first graders at one school went on a field trip to the zoo. The total number of children and adults who went on the field trip was 115. The number of adults was $\frac{1}{4}$ the number of children. Solve the system

$$\begin{cases} c+a=115 \\ a=\frac{1}{4}c \end{cases}$$

to find c, the number of children and aa, the number of adults.

Writing Exercises

Exercise 5.4E.55

Write an application problem similar to Example using the ages of two of your friends or family members. Then translate to a system of equations and solve it.

Answer

Answers will vary.

Exercise 5.4E.56

Write a uniform motion problem similar to Example that relates to where you live with your friends or family members. Then translate to a system of equations and solve it.

Self Check

a. After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
translate to a system of equations.			
solve direct translation applications.			
solve geometry applications.			
solve uniform motion applications.			

b. On a scale of 1-10, how would you rate your mastery of this section in light of your responses on the checklist? How can you improve this?

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5.5: Solve Mixture Applications with Systems of Equations

Learning Objectives

By the end of this section, you will be able to:

- Solve mixture applications
- Solve interest applications

Note

Before you get started, take this readiness quiz.

1. Multiply 4.025(1,562).
   If you missed this problem, review Example 1.8.22.
2. Write 8.2% as a decimal.
   If you missed this problem, review Example 1.8.46.
3. Earl's dinner bill came to $32.50 and he wanted to leave an 18% tip. How much should the tip be?
   If you missed this problem, review Example 3.2.10.

Solve Mixture Applications

When we solved mixture applications with coins and tickets earlier, we started by creating a table so we could organize the information. For a coin example with nickels and dimes, the table looked like this:

Type	Number	• Value($)	= Total Value($)
nickels		0.05	
dimes		0.10	

Using one variable meant that we had to relate the number of nickels and the number of dimes. We had to decide if we were going to let n be the number of nickels and then write the number of dimes in terms of n, or if we would let d be the number of dimes and write the number of nickels in terms of d.

Now that we know how to solve systems of equations with two variables, we'll just let n be the number of nickels and d be the number of dimes. We'll write one equation based on the total value column, like we did before, and the other equation will come from the number column.

For the first example, we'll do a ticket problem where the ticket prices are in whole dollars, so we won't need to use decimals just yet.

Example 5.5.1

Translate to a system of equations and solve:

The box office at a movie theater sold 147 tickets for the evening show, and receipts totaled $1,302. How many $11 adult and how many $8 child tickets were sold?

Solution

Step 1. Read the problem.	We will create a table to organize the information.
Step 2. Identify what we are looking for.	We are looking for the number of adult tickets and the number of child tickets sold.
Step 3. Name what we are looking for.	Let a= the number of adult tickets. c= the number of child tickets
A table will help us organize the data. We have two types of tickets: adult and child.	Write a and c for the number of tickets.

Write the total number of tickets sold at the bottom of the Number column.	Altogether 147 were sold.
Write the value of each type of ticket in the Value column.	The value of each adult ticket is $11. The value of each child tickets is $8.
The number times the value gives the total value, so the total value of adult tickets is $a \cdot 11 = 11a$, and the total value of child tickets is $c \cdot 8 = 8c$.	<table><tr><th>Type</th><th>Number</th><th>•</th><th>Value ($)</th><th>=</th><th>Total Value ($)</th></tr><tr><td>adult</td><td>a</td><td></td><td>11</td><td></td><td>$11a$</td></tr><tr><td>child</td><td>c</td><td></td><td>8</td><td></td><td>$8c$</td></tr><tr><td></td><td>147</td><td></td><td></td><td></td><td>1302</td></tr></table>
Altogether the total value of the tickets was $1,302.	Fill in the Total Value column.
Step 4. Translate into a system of equations.	
The Number column and the Total Value column give us the system of equations. We will use the elimination method to solve this system.	$a + c = 147$ $11a + 8c = 1302$
Multiply the first equation by −8.	$-8(a + c) = -8(147)$ $11a + 8c = 1302$
Simplify and add, then solve for a.	$-8a - 8c = -1176$ $11a + 8c = 1302$ $3a = 126$
	$a = 42$
Substitute $a = 42$ into the first equation, then solve for c.	$42 + c = 147$
	$c = 105$
Step 5. Check the answer in the problem. 42 adult tickets at $11 per ticket makes $462 105 child tickets at $8 per ticket makes $840. The total receipts are $1,302.✓	
Step 6. Answer the question.	The movie theater sold 42 adult tickets and 105 child tickets.

Try It 5.5.2

Translate to a system of equations and solve:

The ticket office at the zoo sold 553 tickets one day. The receipts totaled $3,936. How many $9 adult tickets and how many $6 child tickets were sold?

Answer

There were 206 adult tickets sold and 347 children tickets sold.

Try It 5.5.3

Translate to a system of equations and solve:

A science center sold 1,363 tickets on a busy weekend. The receipts totaled $12,146. How many $12 adult tickets and how many $7 child tickets were sold?

Answer

There were 521 adult tickets sold and 842 children tickets sold.

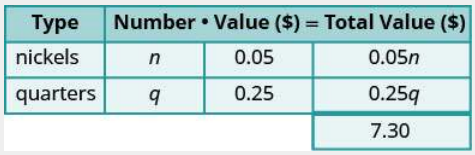
In Example 5.5.4 we'll solve a coin problem. Now that we know how to work with systems of two variables, naming the variables in the 'number' column will be easy.

Example 5.5.4

Translate to a system of equations and solve:

Priam has a collection of nickels and quarters, with a total value of $7.30. The number of nickels is six less than three times the number of quarters. How many nickels and how many quarters does he have?

Solution

Step 1. Read the problem.	We will create a table to organize the information.
Step 2. Identify what we are looking for.	We are looking for the number of nickels and the number of quarters.
Step 3. Name what we are looking for.	Let n= the number of nickels. q= the number of quarters
A table will help us organize the data. We have two types of coins, nickels and quarters.	Write n and q for the number of each type of coin.
Fill in the Value column with the value of each type of coin.	The value of each nickel is $0.05. The value of each quarter is $0.25.
The number times the value gives the total value, so, the total value of the nickels is n (0.05) = 0.05n and the total value of quarters is q(0.25) = 0.25q. Altogether the total value of the coins is $7.30.	
Step 4. Translate into a system of equations.	
The Total value column gives one equation.	$0.05n + 0.25q = 7.30$
We also know the number of nickels is six less than three times the number of quarters. Translate to get the second equation.	$n = 3q - 6$
Now we have the system to solve.	$0.05n + 0.25q = 7.30$ $n = 3q - 6$
Step 5. Solve the system of equations We will use the substitution method. Substitute $n = 3q - 6$ into the first equation. Simplify and solve for q.	$0.05n + 0.25q = 7.30$
	$0.05(3q - 6) + 0.25q = 7.3$

	$0.15q - 0.3 + 0.25q = 7.3$
	$0.4q - 0.3 = 7.3$
	$0.4q = 7.6$
	$q = 19$
To find the number of nickels, substitute $q = 19$ into the second equation.	$n = 3q - 6$
	$n = 3 \cdot 19 - 6$
	$n = 51$
Step 6. Check the answer in the problem. $19 \text{ quarters at } \$0.25 = \$4.75$ $51 \text{ ickels at } \$0.05 = \$2.55$ $\text{Total } = \$7.30\checkmark$ $3 \cdot 19 - 16 = 51\checkmark$	
Step 7. Answer the question.	Priam has 19 quarters and 51 nickels.

Try It 5.5.5

Translate to a system of equations and solve:

Matilda has a handful of quarters and dimes, with a total value of $8.55. The number of quarters is 3 more than twice the number of dimes. How many dimes and how many quarters does she have?

Answer

Matilda has 13 dimes and 29 quarters.

Try It 5.5.6

Translate to a system of equations and solve:

Juan has a pocketful of nickels and dimes. The total value of the coins is $8.10. The number of dimes is 9 less than twice the number of nickels. How many nickels and how many dimes does Juan have?

Answer

Juan has 36 nickels and 63 dimes.

Some mixture applications involve combining foods or drinks. Example situations might include combining raisins and nuts to make a trail mix or using two types of coffee beans to make a blend.

Example 5.5.7

Translate to a system of equations and solve:

Carson wants to make 20 pounds of trail mix using nuts and chocolate chips. His budget requires that the trail mix costs him $7.60 per pound. Nuts cost $9.00 per pound and chocolate chips cost $2.00 per pound. How many pounds of nuts and how many pounds of chocolate chips should he use?

Solution

Step 1. Read the problem.	We will create a table to organize the information.

Step 2. Identify what we are looking for.	We are looking for the number of pounds of nuts and the number of pounds of chocolate chips.
Step 3. Name what we are looking for.	Let n= the number of pound of nuts. c= the number of pounds of chips
Carson will mix nuts and chocolate chips to get trail mix. Write in n and c for the number of pounds of nuts and chocolate chips. There will be 20 pounds of trail mix. Put the price per pound of each item in the Value column. Fill in the last column using	

Type	Number of pounds	• Value ($)	= Total Value ($)
nuts	n	9.00	$9n$
chocolate chips	c	2.00	$2c$
trail mix	20	7.60	$7.60(20) = 152$

Number · Value = Total Value	
Step 4. Translate into a system of equations. We get the equations from the Number and Total Value columns.	$\begin{aligned} n + c &= 20 \\ 9n + 2c &= 152 \end{aligned}$
Step 5. Solve the system of equations We will use elimination to solve the system.	
Multiply the first equation by −2 to eliminate c.	$\begin{aligned} -2(n + c) &= -2(20) \\ 9n + 2c &= 152 \end{aligned}$
Simplify and add. Solve for n.	$\begin{aligned} -2n - 2c &= -40 \\ 9n + 2c &= 152 \\ \hline 7n \quad &= 112 \end{aligned}$
	$n = 16$
To find the number of pounds of chocolate chips, substitute n = 16 into the first equation, then solve for c.	$\begin{aligned} n + c &= 20 \\ 16 + c &= 20 \end{aligned}$
	c=4
Step 6. Check the answer in the problem. $16 + 4 = 20$✓ $9 \cdot 16 + 2 \cdot 4 = 152$✓	
Step 7. Answer the question.	Carson should mix 16 pounds of nuts with 4 pounds of chocolate chips to create the trail mix.

? Try It 5.5.8

Translate to a system of equations and solve:

Greta wants to make 5 pounds of a nut mix using peanuts and cashews. Her budget requires the mixture to cost her $6 per pound. Peanuts are $4 per pound and cashews are $9 per pound. How many pounds of peanuts and how many pounds of cashews should she use?

Answer

Greta should use 3 pounds of peanuts and 2 pounds of cashews.

Try It 5.5.9

Translate to a system of equations and solve:

Sammy has most of the ingredients he needs to make a large batch of chili. The only items he lacks are beans and ground beef. He needs a total of 20 pounds combined of beans and ground beef and has a budget of $3 per pound. The price of beans is $1 per pound and the price of ground beef is $5 per pound. How many pounds of beans and how many pounds of ground beef should he purchase?

Answer

Sammy should purchase 10 pounds of beans and 10 pounds of ground beef.

Another application of mixture problems relates to concentrated cleaning supplies, other chemicals, and mixed drinks. The concentration is given as a percent. For example, a 20% concentrated household cleanser means that 20% of the total amount is cleanser, and the rest is water. To make 35 ounces of a 20% concentration, you mix 7 ounces (20% of 35) of the cleanser with 28 ounces of water.

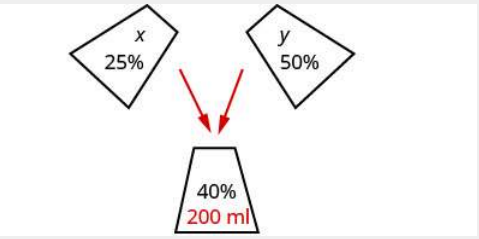
For these kinds of mixture problems, we'll use percent instead of value for one of the columns in our table.

Example 5.5.10

Translate to a system of equations and solve:

Sasheena is a lab assistant at her community college. She needs to make 200 milliliters of a 40% solution of sulfuric acid for a lab experiment. The lab has only 25% and 50% solutions in the storeroom. How much should she mix of the 25% and the 50% solutions to make the 40% solution?

Solution

Step 1. Read the problem.	A figure may help us visualize the situation, then we will create a table to organize the information.
Sasheena must mix some of the 25% solution and some of the 50% solution together to get 200 ml of the 40% solution.	
Step 2. Identify what we are looking for.	We are looking for how much of each solution she needs.
Step 3. Name what we are looking for.	Let x= number of ml of 25% solution. y= number of ml of 50% solution
A table will help us organize the data. She will mix x ml of 25% with y ml of 50% to get 200 ml of 40% solution. We write the percents as decimals in the chart. We multiply the number of units times the concentration to get the total amount of sulfuric acid in each solution.	

Type	Number of units	• Concentration %	= Amount
25%	x	0.25	$0.25x$
50%	y	0.50	$0.50y$
40%	200	0.40	0.40(200)

Step 4. Translate into a system of equations. We get the equations from the Number column and the Amount column.	
Now we have the system.	$\begin{cases} x+y=200 \\ 0.25x+0.50y=0.40(200) \end{cases}$
Step 5. Solve the system of equations. We will solve the system by elimination. Multiply the first equation by −0.5 to eliminate y.	$\begin{cases} -0.5(x+y)=-0.5(200) \\ 0.25x+0.50y=80 \end{cases}$
Simplify and add to solve for x.	$\begin{cases} -0.5x-0.5y=-100 \\ 0.25x+0.5y=80 \end{cases}$ $-0.25x = -20$ $x = 80$
To solve for y, substitute $x = 80$ into the first equation.	$x+y=200$
	$80+y=200$
	$y=120$
Step 6. Check the answer in the problem. $80+120 = 120$✓ $0.25(80)+0.50(120) = 80$✓ Yes!	
Step 7. Answer the question.	Sasheena should mix 80 ml of the 25% solution with 120 ml of the 50% solution to get the 200 ml of the 40% solution.

Try It 5.5.11

Translate to a system of equations and solve:

LeBron needs 150 milliliters of a 30% solution of sulfuric acid for a lab experiment but only has access to a 25% and a 50% solution. How much of the 25% and how much of the 50% solution should he mix to make the 30% solution?

Answer

LeBron needs 120 ml of the 25% solution and 30 ml of the 50% solution.

Try It 5.5.12

Translate to a system of equations and solve:

Anatole needs to make 250 milliliters of a 25% solution of hydrochloric acid for a lab experiment. The lab only has a 10% solution and a 40% solution in the storeroom. How much of the 10% and how much of the 40% solutions should he mix to make the 25% solution?

Answer

Anatole should mix 125 ml of the 10% solution and 125 ml of the 40% solution.

Solve Interest Applications

The formula to model interest applications is $I = Prt$. Interest, I, is the product of the principal, P, the rate, r, and the time, t. In our work here, we will calculate the interest earned in one year, so t will be 1.

We modify the column titles in the mixture table to show the formula for interest, as you'll see in Example 5.5.13.

Example 5.5.13

Translate to a system of equations and solve:

Adnan has $40,000 to invest and hopes to earn 7.1% interest per year. He will put some of the money into a stock fund that earns 8% per year and the rest into bonds that earns 3% per year. How much money should he put into each fund?

Solution

Step 1. Read the problem.	A chart will help us organize the information.
Step 2. Identify what we are looking for.	We are looking for the amount to invest in each fund.
Step 3. Name what we are looking for.	Let s= the amount invested in stocks. b= the amount invested in bonds.
Write the interest rate as a decimal for each fund. Multiply: Principal · Rate · Time to get the Interest.	<table><tr><th>Account</th><th>Principal</th><th>Rate</th><th>Time</th><th>Interest</th></tr><tr><td>stock fund</td><td>s</td><td>0.08</td><td>1</td><td>0.08s</td></tr><tr><td>bonds</td><td>b</td><td>0.03</td><td>1</td><td>0.03b</td></tr><tr><td>Total</td><td>40,000</td><td>0.071</td><td></td><td>0.071(40,000)</td></tr></table>
Step 4. Translate into a system of equations. We get our system of equations from the Principal column and the Interest column.	$\begin{cases} s + b = 40{,}000 \\ 0.08s + 0.03b = 0.071(40{,}000) \end{cases}$
Step 5. Solve the system of equations Solve by elimination. Multiply the top equation by −0.03.	$\begin{cases} -0.03(s + b) = -0.03(40{,}000) \\ 0.08s + 0.03b = 2{,}840 \end{cases}$
Simplify and add to solve for s.	$\begin{array}{r} -0.03s - 0.03b = -1{,}200 \\ 0.08s + 0.03b = 2{,}840 \\ \hline 0.05s = 1{,}640 \end{array}$
	$s = 32{,}800$
To find b, substitute s = 32,800 into the first equation.	$s + b = 40{,}000$ $32{,}800 + b = 40{,}000$
	$b = 7{,}200$
Step 6. Check the answer in the problem.	We leave the check to you.
Step 7. Answer the question.	Adnan should invest $32,800 in stock and $7,200 in bonds.

Did you notice that the Principal column represents the total amount of money invested while the Interest column represents only the interest earned? Likewise, the first equation in our system, $s + b = 40{,}000$, represents the total amount of money invested and the second equation, $0.08s + 0.03b = 0.071(40{,}000)$, represents the interest earned.

Try It 5.5.14

Translate to a system of equations and solve:

Leon had $50,000 to invest and hopes to earn 6.2 % interest per year. He will put some of the money into a stock fund that earns 7% per year and the rest in to a savings account that earns 2% per year. How much money should he put into each fund?

Answer

Leon should put $42,000 in the stock fund and $8000 in the savings account.

Try It 5.5.15

Translate to a system of equations and solve:

Julius invested $7,000 into two stock investments. One stock paid 11% interest and the other stock paid 13% interest. He earned 12.5% interest on the total investment. How much money did he put in each stock?

Answer

Julius invested $1,750 at 11% and $5,250 at 13%.

Example 5.5.16

Translate to a system of equations and solve:

Rosie owes $21,540 on her two student loans. The interest rate on her bank loan is 10.5% and the interest rate on the federal loan is 5.9%. The total amount of interest she paid last year was $1,669.68. What was the principal for each loan?

Solution

Step 1. Read the problem.	A chart will help us organize the information.
Step 2. Identify what we are looking for.	We are looking for the principal of each loan.
Step 3. Name what we are looking for.	Let b= the principal for the bank loan. f= the principal on the federal loan
The total loans are $21,540.	
Record the interest rates as decimals in the chart.	

Account	Principal •	Rate •	Time =	Interest
bank	b	0.105	1	$0.105b$
federal	f	0.059	1	$0.059f$
Total	21,540			1669.68

Multiply using the formula $I = Prt$ to get the Interest.	
Step 4. Translate into a system of equations. The system of equations comes from the Principal column and the Interest column.	$$\begin{aligned} b+f &= 21{,}540 \\ 0.105b+0.059f &= 1669.68 \end{aligned}$$
Step 5. Solve the system of equations We will use substitution to solve. Solve the first equation for b.	$$\begin{aligned} b+f &= 21{,}540 \\ b &= -f+21{,}540 \end{aligned}$$
Substitute $b = -f + 21{,}540$ into the second equation.	$$\begin{aligned} 0.105b+0.059f &= 1669.68 \\ 0.105(-f+21{,}540)+0.059f &= 1669.68 \end{aligned}$$

Simplify and solve for f.	$-0.105f + 2261.70 + 0.059f = 1669.68$
	$-0.046f + 2261.70 = 1669.68$
	$-0.046f = -592.02$
	$f = 12{,}870$
To find b, substitute $f = 12{,}870$ into the first equation.	$b + f = 21{,}540$ $12{,}870 + f = 21{,}540$
	$f = 8{,}670$
Step 6. Check the answer in the problem.	We leave the check to you.
Step 7. Answer the question.	The principal of the bank loan is \$12,870 and the principal for the federal loan is \$8,670.

Try It 5.5.17

Translate to a system of equations and solve:

Laura owes \$18,000 on her student loans. The interest rate on the bank loan is 2.5% and the interest rate on the federal loan is 6.9 %. The total amount of interest she paid last year was \$1,066. What was the principal for each loan?

Answer

The principal amount for the bank loan was \$4,000. The principal amount for the federal loan was \$14,000.

Try It 5.5.18

Translate to a system of equations and solve:

Jill's Sandwich Shoppe owes \$65,200 on two business loans, one at 4.5% interest and the other at 7.2% interest. The total amount of interest owed last year was \$3,582. What was the principal for each loan?

Answer

The principal amount for was \$41,200 at 4.5%. The principal amount was, \$24,000 at 7.2%.

Note

Access these online resources for additional instruction and practice with solving application problems with systems of linear equations.

- Cost and Mixture Word Problems
- Mixture Problems

Key Concepts

- **Table for coin and mixture applications**

Type	Number	• Value(\$)	= Total Value(\$)
Total			

- **Table for concentration applications**

Type	Number of units	•	Concentration %	=	Amount
Total					

- **Table for interest applications**

Account	Principal	•	Rate	•	Time	=	Interest
					1		
					1		
Total							

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5.5E: Exercises

Practice Makes Perfect

Solve Mixture Applications

In the following exercises, translate to a system of equations and solve.

Exercise 5.5E.1

Tickets to a Broadway show cost $35 for adults and $15 for children. The total receipts for 1650 tickets at one performance were $47,150. How many adult and how many child tickets were sold?

Answer

There 1120 adult tickets and 530 child tickets sold.

Exercise 5.5E.2

Tickets for a show are $70 for adults and $50 for children. One evening performance had a total of 300 tickets sold and the receipts totaled $17,200. How many adult and how many child tickets were sold?

Exercise 5.5E.3

Tickets for a train cost $10 for children and $22 for adults. Josie paid $1,200 for a total of 72 tickets. How many children's tickets and how many adult tickets did Josie buy?

Answer

Josie bought 40 adult tickets and 32 children tickets.

Exercise 5.5E.4

Tickets for a baseball game are $69 for Main Level seats and $39 for Terrace Level seats. A group of sixteen friends went to the game and spent a total of $804 for the tickets. How many of Main Level and how many Terrace Level tickets did they buy?

Exercise 5.5E.5

Tickets for a dance recital cost $15 for adults and $7 for children. The dance company sold 253 tickets and the total receipts were $2,771. How many adult tickets and how many child tickets were sold?

Answer

There were 125 adult tickets and 128 children tickets sold.

Exercise 5.5E.6

Tickets for the community fair cost $12 for adults and $5 dollars for children. On the first day of the fair, 312 tickets were sold for a total of $2,204. How many adult tickets and how many child tickets were sold?

Exercise 5.5E.7

Brandon has a cup of quarters and dimes with a total value of $3.80. The number of quarters is four less than twice the number of dimes. How many quarters and how many dimes does Brandon have?

Answer

Brandon has 12 quarters and 8 dimes.

Exercise 5.5E.8

Sherri saves nickels and dimes in a coin purse for her daughter. The total value of the coins in the purse is $0.95. The number of nickels is two less than five times the number of dimes. How many nickels and how many dimes are in the coin purse?

Exercise 5.5E.9

Peter has been saving his loose change for several days. When he counted his quarters and dimes, he found they had a total value $13.10. The number of quarters was fifteen more than three times the number of dimes. How many quarters and how many dimes did Peter have?

Answer

Peter had 11 dimes and 48 quarters.

Exercise 5.5E.10

Lucinda had a pocketful of dimes and quarters with a value of $ $6.20. The number of dimes is eighteen more than three times the number of quarters. How many dimes and how many quarters does Lucinda have?

Exercise 5.5E.11

A cashier has 30 bills, all of which are $10 or $20 bills. The total value of the money is $460. How many of each type of bill does the cashier have?

Answer

The cashier has fourteen $10 bills and sixteen $20 bills.

Exercise 5.5E.12

A cashier has 54 bills, all of which are $10 or $20 bills. The total value of the money is $910. How many of each type of bill does the cashier have?

Exercise 5.5E.13

Marissa wants to blend candy selling for $1.80 per pound with candy costing $1.20 per pound to get a mixture that costs her $1.40 per pound to make. She wants to make 90 pounds of the candy blend. How many pounds of each type of candy should she use?

Answer

Marissa should use 60 pounds of the $1.20/lb candy and 30 pounds of the $1.80/lb candy.

Exercise 5.5E.14

How many pounds of nuts selling for $6 per pound and raisins selling for $3 per pound should Kurt combine to obtain 120 pounds of trail mix that cost him $5 per pound?

Exercise 5.5E.15

Hannah has to make twenty-five gallons of punch for a potluck. The punch is made of soda and fruit drink. The cost of the soda is $1.79 per gallon and the cost of the fruit drink is $2.49 per gallon. Hannah's budget requires that the punch cost $2.21 per gallon. How many gallons of soda and how many gallons of fruit drink does she need?

Answer

Hannah needs 10 gallons of soda and 15 gallons of fruit drink.

Exercise 5.5E.16

Joseph would like to make 12 pounds of a coffee blend at a cost of $6.25 per pound. He blends Ground Chicory at $4.40 a pound with Jamaican Blue Mountain at $8.84 per pound. How much of each type of coffee should he use?

Exercise 5.5E.17

Julia and her husband own a coffee shop. They experimented with mixing a City Roast Columbian coffee that cost $7.80 per pound with French Roast Columbian coffee that cost $8.10 per pound to make a 20 pound blend. Their blend should cost them $7.92 per pound. How much of each type of coffee should they buy?

Answer

Julia and her husband should buy 12 pounds of City Roast Columbian coffee and 8 pounds of French Roast Columbian coffee.

Exercise 5.5E.18

Melody wants to sell bags of mixed candy at her lemonade stand. She will mix chocolate pieces that cost $4.89 per bag with peanut butter pieces that cost $3.79 per bag to get a total of twenty-five bags of mixed candy. Melody wants the bags of mixed candy to cost her $4.23 a bag to make. How many bags of chocolate pieces and how many bags of peanut butter pieces should she use?

Exercise 5.5E.19

Jotham needs 70 liters of a 50% alcohol solution. He has a 30% and an 80% solution available. How many liters of the 30% and how many liters of the 80% solutions should he mix to make the 50% solution?

Answer

Jotham should mix 42 liters of the 30% solution and 28 liters of the 80% solution.

Exercise 5.5E.20

Joy is preparing 15 liters of a 25% saline solution. She only has 40% and 10% solution in her lab. How many liters of the 40% and how many liters of the 10% should she mix to make the 25% solution?

Exercise 5.5E.21

A scientist needs 65 liters of a 15% alcohol solution. She has available a 25% and a 12% solution. How many liters of the 25% and how many liters of the 12% solutions should she mix to make the 15% solution?

Answer

The scientist should mix 15 liters of the 25% solution and 50 liters of the 12% solution.

Exercise 5.5E.22

A scientist needs 120 liters of a 20% acid solution for an experiment. The lab has available a 25% and a 10% solution. How many liters of the 25% and how many liters of the 10% solutions should the scientist mix to make the 20% solution?

Exercise 5.5E.23

A 40% antifreeze solution is to be mixed with a 70% antifreeze solution to get 240 liters of a 50% solution. How many liters of the 40% and how many liters of the 70% solutions will be used?

Answer

160 liters of the 40% solution and 80 liters of the 70% solution will be used.

Exercise 5.5E.24

A 90% antifreeze solution is to be mixed with a 75% antifreeze solution to get 360 liters of a 85% solution. How many liters of the 90% and how many liters of the 75% solutions will be used?

Solve Interest Applications

In the following exercises, translate to a system of equations and solve.

Exercise 5.5E.25

Hattie had $3,000 to invest and wants to earn 10.6% interest per year. She will put some of the money into an account that earns 12% per year and the rest into an account that earns 10% per year. How much money should she put into each account?

Answer

Hattie should invest $900 at 12% and $2,100 at 10%.

Exercise 5.5E.26

Carol invested $2,560 into two accounts. One account paid 8% interest and the other paid 6% interest. She earned 7.25% interest on the total investment. How much money did she put in each account?

Exercise 5.5E.27

Sam invested $48,000, some at 6% interest and the rest at 10%. How much did he invest at each rate if he received $4,000 in interest in one year?

Answer

Sam invested $28,000 at 10% and $20,000 at 6%.

Exercise 5.5E.28

Arnold invested $64,000, some at 5.5% interest and the rest at 9%. How much did he invest at each rate if he received $4,500 in interest in one year?

Exercise 5.5E.29

After four years in college, Josie owes $65,800 in student loans. The interest rate on the federal loans is 4.5% and the rate on the private bank loans is 2%. The total interest she owed for one year was $2,878.50. What is the amount of each loan?

Answer

The federal loan is $62,500 and the bank loan is $3,300.

Exercise 5.5E.30

Mark wants to invest $10,000 to pay for his daughter's wedding next year. He will invest some of the money in a short term CD that pays 12% interest and the rest in a money market savings account that pays 5% interest. How much should he invest at each rate if he wants to earn $1,095 in interest in one year?

Exercise 5.5E.31

A trust fund worth $25,000 is invested in two different portfolios. This year, one portfolio is expected to earn 5.25% interest and the other is expected to earn 4%. Plans are for the total interest on the fund to be $1150 in one year. How much money should be invested at each rate?

Answer

$12,000 should be invested at 5.25% and $13,000 should be invested at 4%.

Exercise 5.5E.32

A business has two loans totaling $85,000. One loan has a rate of 6% and the other has a rate of 4.5%. This year, the business expects to pay $4650 in interest on the two loans. How much is each loan?

Everyday Math

In the following exercises, translate to a system of equations and solve.

Exercise 5.5E.33

Laurie was completing the treasurer's report for her son's Boy Scout troop at the end of the school year. She didn't remember how many boys had paid the $15 full-year registration fee and how many had paid the $10 partial-year fee. She knew that the number of boys who paid for a full-year was ten more than the number who paid for a partial-year. If $250 was collected for all the registrations, how many boys had paid the full-year fee and how many had paid the partial-year fee?

Answer

14 boys paid the full-year fee. 4 boys paid the partial-year fee

Exercise 5.5E.34

As the treasurer of her daughter's Girl Scout troop, Laney collected money for some girls and adults to go to a three-day camp. Each girl paid $75 and each adult paid $30. The total amount of money collected for camp was $765. If the number of girls is three times the number of adults, how many girls and how many adults paid for camp?

Writing Exercises

Exercise 5.5E.35

Take a handful of two types of coins, and write a problem similar to Example relating the total number of coins and their total value. Set up a system of equations to describe your situation and then solve it.

Answer

Answers will vary.

Exercise $5.5E.36$

In Example we solved the system of equations $\begin{cases} b+f=21,540 \\ 0.105b+0.059f=1669.68 \end{cases}$ by substitution. Would you have used substitution or elimination to solve this system? Why?

Self Check

a. After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
solve mixture applications.			
solve interest applications.			

b. After looking at the checklist, do you think you are well-prepared for the next section? Why or why not?

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5.6: Graphing Systems of Linear Inequalities

Learning Objectives

By the end of this section, you will be able to:

- Determine whether an ordered pair is a solution of a system of linear inequalities
- Solve a system of linear inequalities by graphing
- Solve applications of systems of inequalities

Note

Before you get started, take this readiness quiz.

1. Graph x>2 on a number line.
   If you missed this problem, review Example 2.7.1.
2. Solve the inequality 2a<5a+12.
   If you missed this problem, review Example 2.7.22.
3. Determine whether the ordered pair $(3, \frac{1}{2})$ is a solution to the system $\begin{cases} x+2y=4 \\ y=6x \end{cases}$.
   If you missed this problem, review Example 5.1.1.

Determine Whether an Ordered Pair is a Solution of a System of Linear Inequalities

The definition of a system of linear inequalities is very similar to the definition of a system of linear equations.

SYSTEM OF LINEAR INEQUALITIES

Two or more linear inequalities grouped together form a **system of linear inequalities**.

A system of linear inequalities looks like a system of linear equations, but it has inequalities instead of equations. A system of two linear inequalities is shown below.

$$\begin{cases} x+4y \geq 10 \\ 3x-2y < 12 \end{cases} \tag{5.6.1}$$

To solve a system of linear inequalities, we will find values of the variables that are solutions to both inequalities. We solve the system by using the graphs of each inequality and show the solution as a graph. We will find the region on the plane that contains all ordered pairs (x,y)(x,y) that make both inequalities true.

SolutionS OF A SYSTEM OF LINEAR INEQUALITIES

Solutions of a system of linear inequalities are the values of the variables that make all the inequalities true.

The solution of a system of linear inequalities is shown as a shaded region in the *x*-*y* coordinate system that includes all the points whose ordered pairs make the inequalities true.

To determine if an ordered pair is a solution to a system of two inequalities, we substitute the values of the variables into each inequality. If the ordered pair makes both inequalities true, it is a solution to the system.

Example 5.6.1

Determine whether the ordered pair is a solution to the system. $\begin{cases} x+4y \geq 10 \\ 3x-2y < 12 \end{cases}$

1. (−2, 4)
2. (3,1)

Solution

1. Is the ordered pair (−2, 4) a solution?
We substitute $x = -2$ and $y = 4$ into both inequalities.

$$x + 4y \geq 10 \qquad 3x - 2y < 12$$
$$-2 + 4(4) \stackrel{?}{\geq} 10 \qquad 3(-2) - 2(4) \stackrel{?}{<} 12$$
$$14 \geq 10 \text{ true} \qquad -14 < 12 \text{ true}$$

The ordered pair (−2, 4) made both inequalities true. Therefore (−2, 4) is a solution to this system.

2. Is the ordered pair (3,1) a solution?
We substitute $x = 3$ and $y = 1$ into both inequalities.

$$x + 4y \geq 10 \qquad 3x - 2y < 12$$
$$3 + 4(1) \stackrel{?}{\geq} 10 \qquad 3(3) - 2(1) \stackrel{?}{<} 12$$
$$7 \geq 10 \text{ false} \qquad 7 < 12 \text{ true}$$

The ordered pair (3,1) made one inequality true, but the other one false. Therefore (3,1) is not a solution to this system.

Try It 5.6.2

Determine whether the ordered pair is a solution to the system. $\begin{cases} x - 5y > 10 \\ 2x + 3y > -2 \end{cases}$

1. (3,−1)
2. (6,−3)

Answer

1. no
2. yes

Try It 5.6.3

Determine whether the ordered pair is a solution to the system. $\begin{cases} y > 4x - 2 \\ 4x - y < 20 \end{cases}$

1. (2,1)
2. (4,−1)

Answer

1. no
2. no

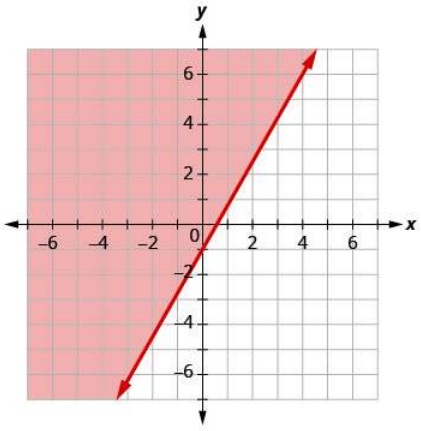
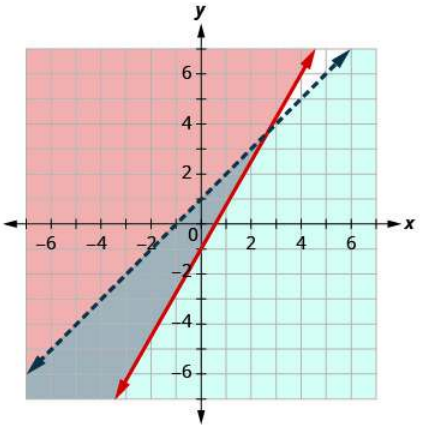
Solve a System of Linear Inequalities by Graphing

The solution to a single linear inequality is the region on one side of the boundary line that contains all the points that make the inequality true. The solution to a system of two linear inequalities is a region that contains the solutions to both inequalities. To find this region, we will graph each inequality separately and then locate the region where they are both true. The solution is always shown as a graph.

Example 5.6.4: How to Solve a System of Linear inequalities

Solve the system by graphing. $\begin{cases} y \geq 2x - 1 \\ y < x + 1 \end{cases}$

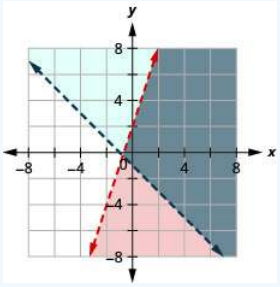
Solution

Step 1. Graph the first inequality. Graph the boundary line. Shade in the side of the boundary line where the inequality is true.	We will graph $y \geq 2x - 1$. We graph the line $y = 2x - 1$. It is a solid line because the inequality sign is $\geq$. We choose (0,0) as a test point. It is a solution to $y \geq 2x - 1$, so we shade in the left side of the boundary line.	$\begin{cases} y \geq 2x - 1 \\ y < x + 1 \end{cases}$ 
Step 2. On the same grid, graph the second inequality. Graph the boundary line. Shade in the side of that boundary line where the inequality is true.	We will graph $y < x + 1$ on the same grid. We graph the line $y = x + 1$. It is a dashed line because the inequality sign is $<$. Again, we use (0,0) as a test point. It is a solution so we shade in that side of the line $y = x + 1$.	
Step 3. The solution is the region where the shading overlaps.	The point where the boundary lines intersect is not a solution because it is not a solution to $y < x + 1$.	The solution is all points in the darker shaded region.
Step 4. Check by choosing a test point.	We'll use (–1, –1) as a test point.	Is (–1, –1) a solution to $y \geq 2x - 1$? $-1 \stackrel{?}{\geq} 2(-1) - 1$ $-1 \geq -3$ true Is (–1, –1) a solution to $y < x + 1$? $-1 \stackrel{?}{<} -1 + 1$ $-1 < 0$ true The region containing (–1, –1) is the solution to this system.

? Try It 5.6.5

Solve the system by graphing. $\begin{cases} y < 3x + 2 \\ y > -x - 1 \end{cases}$

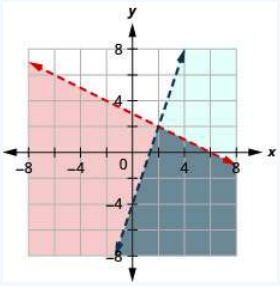
Answer



Try It 5.6.6

Solve the system by graphing. $\begin{cases} y < -\frac{1}{2}x + 3 \\ y < 3x - 4 \end{cases}$

Answer



SOLVE A SYSTEM OF LINEAR INEQUALITIES BY GRAPHING.

1. Graph the first inequality.
   - Graph the boundary line.
   - Shade in the side of the boundary line where the inequality is true.
2. On the same grid, graph the second inequality.
   - Graph the boundary line.
   - Shade in the side of that boundary line where the inequality is true.
3. The solution is the region where the shading overlaps.
4. Check by choosing a test point.

Example 5.6.7

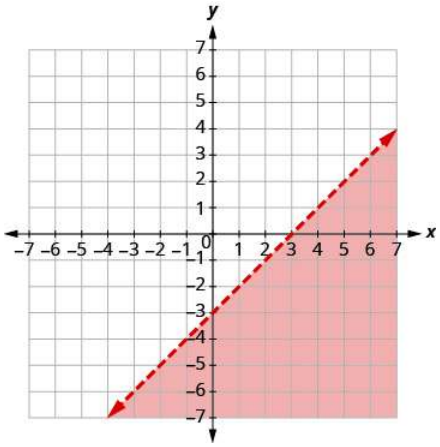
Solve the system by graphing. $\begin{cases} x - y > 3 \\ y < -\frac{1}{5}x + 4 \end{cases}$

Solution

Graph $x - y > 3$, by graphing $x - y = 3$ and testing a point.

The intercepts are $x = 3$ and $y = -3$ and the boundary line will be dashed.

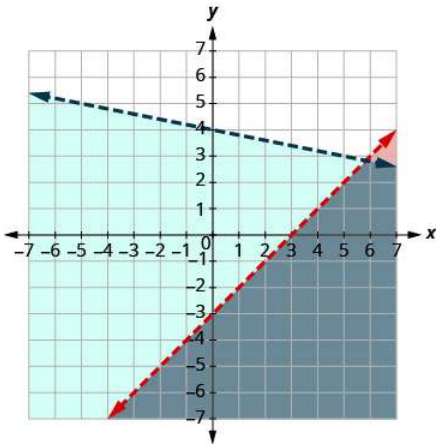
Test (0, 0). It makes the inequality false. So, shade the side that does not contain (0, 0) red.



Graph $y<-15x+4$ by graphing $y=-15x+4$ using the slope $m=-15$ and y−intercept $b = 4$. The boundary line will be dashed.

Test (0, 0). It makes the inequality true, so shade the side that contains (0, 0) blue.

Choose a test point in the solution and verify that it is a solution to both inequalities.

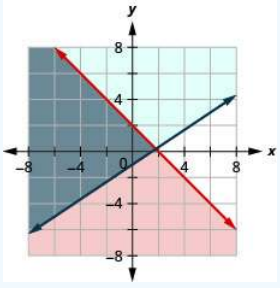


The point of intersection of the two lines is not included as both boundary lines were dashed. The solution is the area shaded twice which is the darker-shaded region.

Try It 5.6.8

Solve the system by graphing. $\begin{cases} x + y \leq 2 \\ y \geq \frac{2}{3}x - 1 \end{cases}$

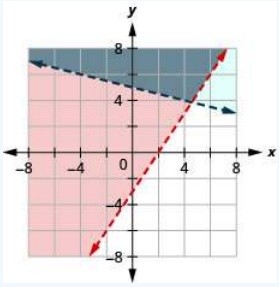
Answer



Try It 5.6.9

Solve the system by graphing. $\begin{cases} 3x - 2y \leq 6 \\ y > -\frac{1}{4}x + 5 \end{cases}$

Answer



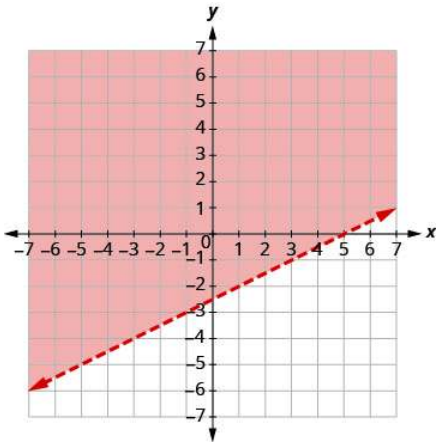
Example 5.6.10

Solve the system by graphing. $\begin{cases} x - 2y < 5 \\ y > -4 \end{cases}$

Solution

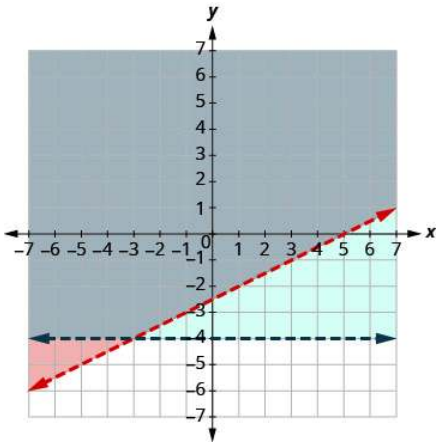
Graph x−2y<5, by graphing x−2y=5 and testing a point.
The intercepts are $x = 5$ and $y = -2.5$ and the boundary line will be dashed.

Test (0, 0). It makes the inequality true. So, shade the side that contains (0, 0) red.



Graph $y > -4$, by graphing $y = -4$ and recognizing that it is a horizontal line through $y = -4$. The boundary line will be dashed.

Test (0, 0). It makes the inequality true. So, shade (blue) the side that contains (0, 0) blue.



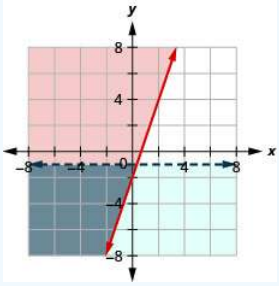
The point (0, 0) is in the solution and we have already found it to be a solution of each inequality. The point of intersection of the two lines is not included as both boundary lines were dashed.

The solution is the area shaded twice which is the darker-shaded region.

Try It 5.6.11

Solve the system by graphing. $\begin{cases} y \geq 3x - 2 \\ y < -1 \end{cases}$

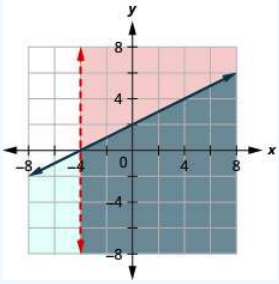
Answer



Try It 5.6.12

Solve the system by graphing. $\begin{cases} x > -4 \\ x - 2y \leq -4 \end{cases}$

Answer



Systems of linear inequalities where the boundary lines are parallel might have no solution. We'll see this in Example 5.6.13

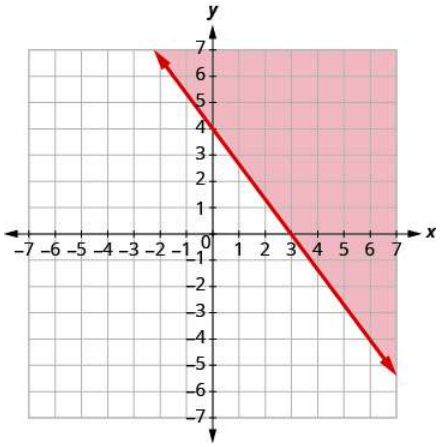
Example 5.6.13

Solve the system by graphing. $\begin{cases} 4x + 3y \geq 12 \\ y < -\frac{4}{3}x + 1 \end{cases}$

Solution

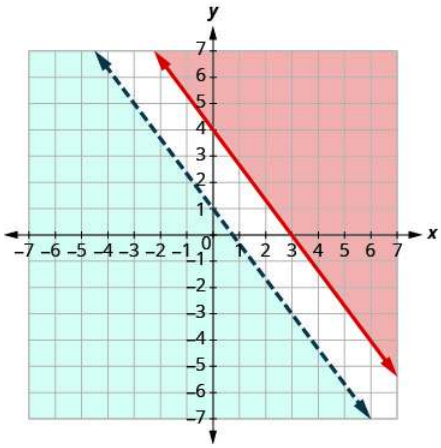
Graph $4x + 3y \geq 12$, by graphing 4x+3y=12 and testing a point.
The intercepts are $x = 3$ and $y = 4$ and the boundary line will be solid.

Test (0, 0). It makes the inequality false. So,
shade the side that does not contain (0, 0) red.



Graph $y < -\frac{4}{3}x + 1$ by graphing $y = -\frac{4}{3}x + 1$ using the slope $m = \frac{4}{3}$ and the y-intercept $b = 1$. The boundary line will be dashed.

Test (0, 0). It makes the inequality true. So,
shade the side that contains (0, 0) blue.



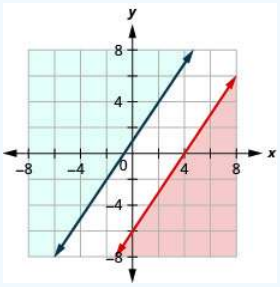
There is no point in both shaded regions, so the system has no solution. This system has no solution.

Try It 5.6.14

Solve the system by graphing. $\begin{cases} 3x - 2y \leq 12 \\ y \geq \frac{3}{2}x + 1 \end{cases}$

Answer

no solution

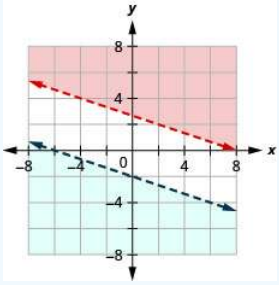


Try It 5.6.15

Solve the system by graphing. $\begin{cases} x+3y>8 \\ y<-\frac{1}{3}x-2 \end{cases}$

Answer

no solution

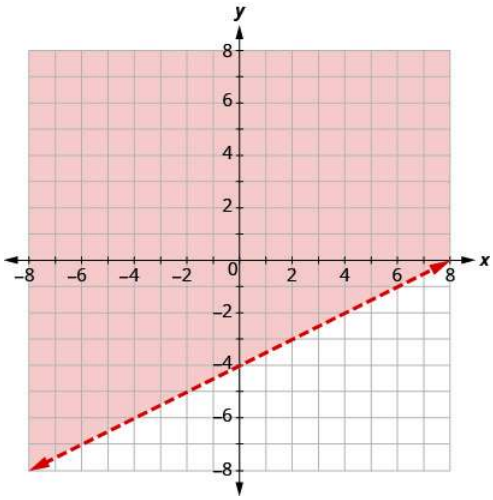


Example 5.6.16

Solve the system by graphing. $\begin{cases} y>\frac{1}{2}x-4 \\ x-2y<-4 \end{cases}$

Solution

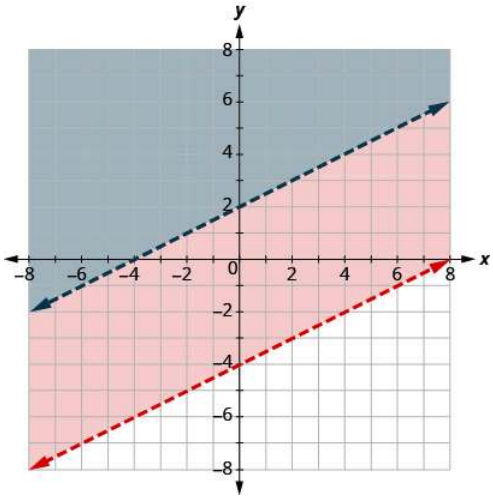
Graph $y>\frac{1}{2}x-4$ by graphing $y=\frac{1}{2}x-4$ using the slope $m=\frac{1}{2}$ and the intercept $b=-4$. The boundary line will be dashed. Test (0, 0). It makes the inequality true. So, shade the side that contains (0, 0) red.



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Graph x−2y<−4x−2y<−4 by graphing x−2y=−4x−2y=−4 and testing a point.
The intercepts are $x = -4$ and $y = 2$ and the boundary line will be dashed.

Choose a test point in the solution and verify that it is a solution to both inequalities.



No point on the boundary lines is included in the solution as both lines are dashed.

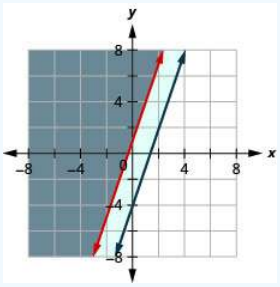
The solution is the region that is shaded twice, which is also the solution to x−2y<−4.

Try It 5.6.17

Solve the system by graphing. $\begin{cases} y \geq 3x+1 \\ -3x+y \geq -4 \end{cases}$

Answer

$y \geq 3x+1$

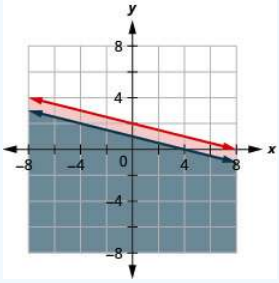


Try It 5.6.18

Solve the system by graphing. $\begin{cases} y \leq -\frac{1}{4}x+2 \\ x+4y \leq 4 \end{cases}$

Answer

$x+4y \leq 4$



Solve Applications of Systems of Inequalities

The first thing we'll need to do to solve applications of systems of inequalities is to translate each condition into an inequality. Then we graph the system as we did above to see the region that contains the solutions. Many situations will be realistic only if both variables are positive, so their graphs will only show Quadrant I.

Example 5.6.19

Christy sells her photographs at a booth at a street fair. At the start of the day, she wants to have at least 25 photos to display at her booth. Each small photo she displays costs her $4 and each large photo costs her $10. She doesn't want to spend more than $200 on photos to display.

1. Write a system of inequalities to model this situation.
2. Graph the system.
3. Could she display 15 small and 5 large photos?
4. Could she display 3 large and 22 small photos?

Solution

1.Let x= the number of small photos.

y= the number of large photos

To find the system of inequalities, translate the information.

$$\text{She wants to have at least 25 photos.}$$

$$\text{The number of small plus the number of large should be at least 25.}$$

$$x+y \geq 25$$

$$\$4 \text{ for each small and } \$10 \text{ for each large must be no more than } \$200$$

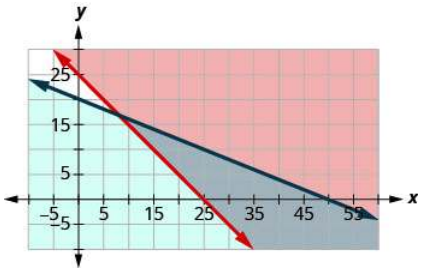
$$4x+10y \leq 200$$

We have our system of inequalities. $\begin{cases} x+y \geq 25 \\ 4x+10y \leq 200 \end{cases}$

2.

To graph $x+y \geq 25$, graph $x + y = 25$ as a solid line.
Choose (0, 0) as a test point. Since it does not make the inequality
true, shade the side that does not include the point (0, 0) red.

To graph $4x+10y \leq 200$, graph $4x + 10y = 200$ as a solid line.
Choose (0, 0) as a test point. Since it does not make the inequality
true, shade the side that includes the point (0, 0) blue.



The solution of the system is the region of the graph that is double shaded and so is shaded darker.

3. To determine if 10 small and 20 large photos would work, we see if the point (10, 20) is in the solution region. It is not. Christy would not display 10 small and 20 large photos.

4. To determine if 20 small and 10 large photos would work, we see if the point (20, 10) is in the solution region. It is. Christy could choose to display 20 small and 10 large photos.

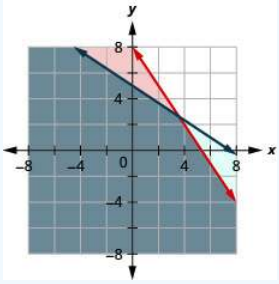
Notice that we could also test the possible solutions by substituting the values into each inequality.

Try It 5.6.20

A trailer can carry a maximum weight of 160 pounds and a maximum volume of 15 cubic feet. A microwave oven weighs 30 pounds and has 2 cubic feet of volume, while a printer weighs 20 pounds and has 3 cubic feet of space.

1. Write a system of inequalities to model this situation.
2. Graph the system.
3. Could 4 microwaves and 2 printers be carried on this trailer?
4. Could 7 microwaves and 3 printers be carried on this trailer?

Answer

1. $\begin{cases} 30m+20p \le 160 \\ 2m+3p \le 15 \end{cases}$
2. 

3. yes
4. no

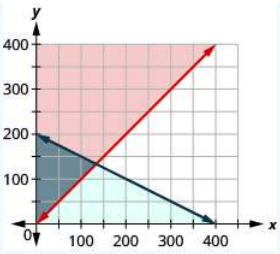
Try It 5.6.21

Mary needs to purchase supplies of answer sheets and pencils for a standardized test to be given to the juniors at her high school. The number of the answer sheets needed is at least 5 more than the number of pencils. The pencils cost $2 and the answer sheets cost $1. Mary's budget for these supplies allows for a maximum cost of $400.

1. Write a system of inequalities to model this situation.
2. Graph the system.
3. Could Mary purchase 100 pencils and 100 answer sheets?
4. Could Mary purchase 150 pencils and 150 answer sheets?

Answer

1. $\begin{cases} a \ge p+5 \\ a+2p \le 400 \end{cases}$
2.



3. no

4. no

Example 5.6.22

Omar needs to eat at least 800 calories before going to his team practice. All he wants is hamburgers and cookies, and he doesn't want to spend more than $5. At the hamburger restaurant near his college, each hamburger has 240 calories and costs $1.40. Each cookie has 160 calories and costs $0.50.

1. Write a system of inequalities to model this situation.
2. Graph the system.
3. Could he eat 1 cookie and 3 hamburgers?
4. Could he eat 4 cookies and 2 hamburgers?

Solution

1.

Let h= the number of hamburgers.
c= the number of cookies
To find the system of inequalities, translate the information.
The calories from hamburgers at 240 calories each, plus the calories from cookies at 160 calories each must be more that 800.

$$240h + 160c \geq 800 \tag{5.6.2}$$

The amount spent on hamburgers at $1.40 each, plus the amount spent on cookies at $0.50 each must be no more than $5.00.

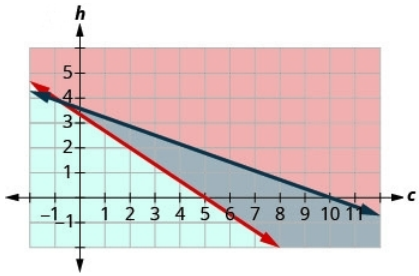
$$1.40h + 0.50c \leq 5 \tag{5.6.3}$$

We have our system of inequalities. $\begin{cases} 240h + 160c \geq 800 \\ 1.40h + 0.50c \leq 5 \end{cases}$

2.

To graph $240h + 160c \geq 800$ graph 240h+160c=800 as a solid line.
Choose (0, 0) as a test point. it does not make the inequality true.
So, shade (red) the side that does not include the point (0, 0).

To graph $1.40h + 0.50c \leq 5$, graph 1.40h+0.50c=5 as a solid line.
Choose (0,0) as a test point. It makes the inequality true. So, shade
(blue) the side that includes the point.



The solution of the system is the region of the graph that is double shaded and so is shaded darker.

3. To determine if 1 cookie and 3 hamburgers would meet Omar's criteria, we see if the point (1, 3) is in the solution region. It is not.
4. To determine if 4 cookies and 2 hamburgers would meet Omar's criteria, we see if the point (4, 2) is in the solution region. It is. He might choose to eat 4 cookies and 2 hamburgers.

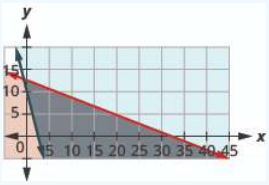
We could also test the possible solutions by substituting the values into each inequality.

Try It 5.6.23

Tension needs to eat at least an extra 1,000 calories a day to prepare for running a marathon. He has only $25 to spend on the extra food he needs and will spend it on $0.75 donuts which have 360 calories each and $2 energy drinks which have 110 calories.

1. Write a system of inequalities that models this situation.
2. Graph the system.
3. Can he buy 8 donuts and 4 energy drinks?
4. Can he buy 1 donut and 3 energy drinks?

Answer

1. $\begin{cases} 0.75d + 2e \leq 25 \\ 360d + 110e \geq 1000 \end{cases}$
2. 

3. yes
4. no

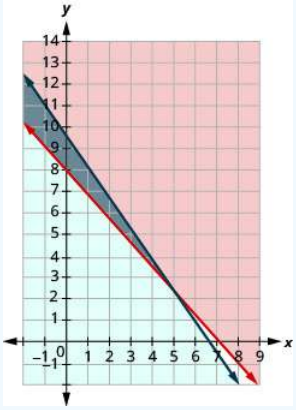
Try It 5.6.24

Philip's doctor tells him he should add at least 1000 more calories per day to his usual diet. Philip wants to buy protein bars that cost $1.80 each and have 140 calories and juice that costs $1.25 per bottle and have 125 calories. He doesn't want to spend more than $12.

1. Write a system of inequalities that models this situation.
2. Graph the system.
3. Can he buy 3 protein bars and 5 bottles of juice?
4. Can he buy 5 protein bars and 3 bottles of juice?

Answer

1. $\begin{cases} 140p + 125j \geq 1000 \\ 1.80p + 1.25j \leq 12 \end{cases}$
2.



3. yes

4. no

Note

Access these online resources for additional instruction and practice with graphing systems of linear inequalities.

- Graphical System of Inequalities
- Systems of Inequalities
- Solving Systems of Linear Inequalities by Graphing

Key Concepts

- **To Solve a System of Linear Inequalities by Graphing**
  1. Graph the first inequality.
     - Graph the boundary line.
     - Shade in the side of the boundary line where the inequality is true.
  2. On the same grid, graph the second inequality.
     - Graph the boundary line.
     - Shade in the side of that boundary line where the inequality is true.
  3. The solution is the region where the shading overlaps.
  4. Check by choosing a test point.

Glossary

system of linear inequalities

Two or more linear inequalities grouped together form a system of linear inequalities.

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5.6E: Exercises

Practice Makes Perfect

Determine Whether an Ordered Pair is a Solution of a System of Linear Inequalities

In the following exercises, determine whether each ordered pair is a solution to the system.

Exercise 5.6E.1

$$\begin{cases} 3x+y>5 \\ 2x-y\leq 10 \end{cases}$$

1. (3,−3)
2. (7,1)

Answer

1. true
2. false

Exercise 5.6E.2

$$\begin{cases} 4x-y<10 \\ -2x+2y>-8 \end{cases}$$

1. (5,−2)
2. (−1,3)

Exercise 5.6E.3

$$\begin{cases} y>\frac{2}{3}x-5 \\ x+\frac{1}{2}y\leq 4 \end{cases}$$

1. (6,−4)
2. (3,0)

Answer

1. false
2. true

Exercise 5.6E.4

$$\begin{cases} y<\frac{3}{2}x+3 \\ \frac{3}{4}x-2y<5 \end{cases}$$

1. (−4,−1)
2. (8,3)

Exercise 5.6E.5

$$\begin{cases} 7x+2y>14 \\ 5x-y\leq 8 \end{cases}$$

1. (2,3)
2. (7,−1)

Answer

1. true

false

Exercise 5.6E.6

$$\begin{cases} 6x-5y<20 \\ -2x+7y>-8 \end{cases}$$

1. (1,−3)
2. (−4,4)

Exercise 5.6E.7

$$\begin{cases} 2x+3y\geq 2 \\ 4x-6y<-1 \end{cases}$$

1. $\left(\frac{3}{2},\frac{4}{3}\right)$
2. $\left(\frac{1}{4},\frac{7}{6}\right)$

Answer

1. true
2. true

Exercise 5.6E.8

$$\begin{cases} 5x-3y<-2 \\ 10x+6y>4 \end{cases}$$

1. $\left(\frac{1}{5},\frac{2}{3}\right)$
2. $\left(-\frac{3}{10},\frac{7}{6}\right)$

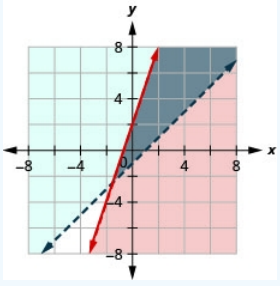
Solve a System of Linear Inequalities by Graphing

In the following exercises, solve each system by graphing.

Exercise 5.6E.9

$$\begin{cases} y\leq 3x+2 \\ y>x-1 \end{cases}$$

Answer



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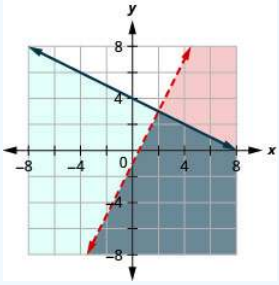
Exercise 5.6E.10

$$\begin{cases} y < -2x + 2 \\ y \geq -x - 1 \end{cases}$$

Exercise 5.6E.11

$$\begin{cases} y < 2x - 1 \\ y \leq -\frac{1}{2}x + 4 \end{cases}$$

Answer



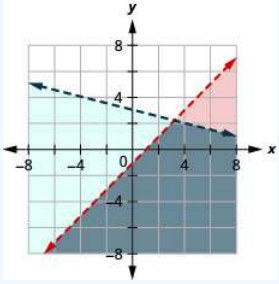
Exercise 5.6E.12

$$\begin{cases} y \geq -\frac{2}{3}x + 2 \\ y > 2x - 3 \end{cases}$$

Exercise 5.6E.13

$$\begin{cases} x - y > 1 \\ y < -\frac{1}{4}x + 3 \end{cases}$$

Answer



Exercise 5.6E.14

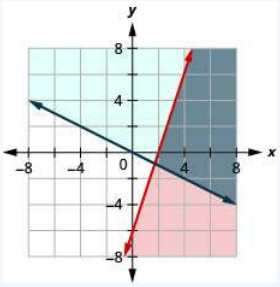
$$\begin{cases} x + 2y < 4 \\ y < x - 2 \end{cases}$$

Exercise 5.6E.15

$$\begin{cases} 3x - y \leq 6 \\ y \geq -\frac{1}{2}x \end{cases}$$

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Answer



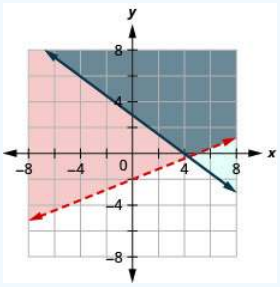
Exercise 5.6E.16

$$\begin{cases} 2x+4y \geq 8 \\ y \leq \frac{3}{4}x \end{cases}$$

Exercise 5.6E.17

$$\begin{cases} 2x-5y < 10 \\ 3x+4y \geq 12 \end{cases}$$

Answer



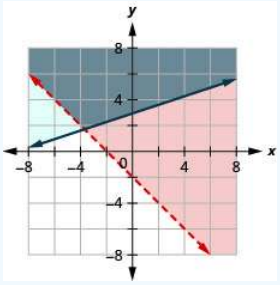
Exercise 5.6E.18

$$\begin{cases} 3x-2y \leq 6 \\ -4x-2y > 8 \end{cases}$$

Exercise 5.6E.19

$$\begin{cases} 2x+2y > -4 \\ -x+3y \geq 9 \end{cases}$$

Answer



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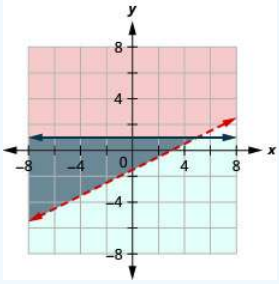
Exercise 5.6E. 20

$$\begin{cases} 2x+y>-6 \\ -x+2y \geq -4 \end{cases}$$

Exercise 5.6E. 21

$$\begin{cases} x-2y<3 \\ y \leq 1 \end{cases}$$

Answer



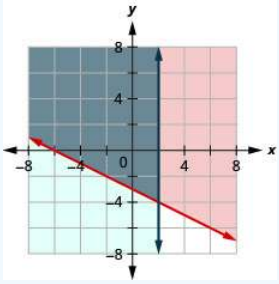
Exercise 5.6E. 22

$$\begin{cases} x-3y>4 \\ y \leq -1 \end{cases}$$

Exercise 5.6E. 23

$$\begin{cases} y \geq -\frac{1}{2}x-3 \\ x \leq 2 \end{cases}$$

Answer



Exercise 5.6E. 24

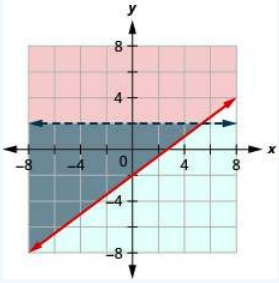
$$\begin{cases} y \leq -\frac{2}{3}x+5 \\ x \geq 3 \end{cases}$$

Exercise 5.6E. 25

$$\begin{cases} y \geq \frac{3}{4}x-2 \\ y<2 \end{cases}$$

Answer

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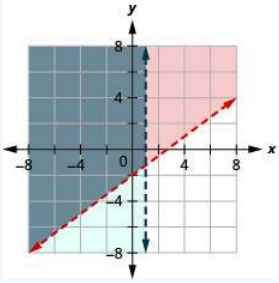
Exercise 5.6E.26

$$\begin{cases} y \leq -\frac{1}{2}x + 3 \\ y < 1 \end{cases}$$

Exercise 5.6E.27

$$\begin{cases} 3x - 4y < 8 \\ x < 1 \end{cases}$$

Answer



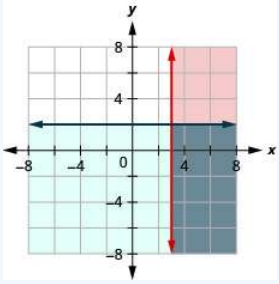
Exercise 5.6E.28

$$\begin{cases} -3x + 5y > 10 \\ x > -1 \end{cases}$$

Exercise 5.6E.29

$$\begin{cases} x \geq 3 \\ y \leq 2 \end{cases}$$

Answer



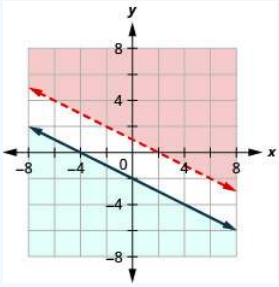
Exercise $5.6E.30$

$$\begin{cases} x \leq -1 \\ y \geq 3 \end{cases}$$

Exercise $5.6E.31$

$$\begin{cases} 2x + 4y > 4 \\ y \leq -\frac{1}{2}x - 2 \end{cases}$$

Answer



Exercise $5.6E.32$

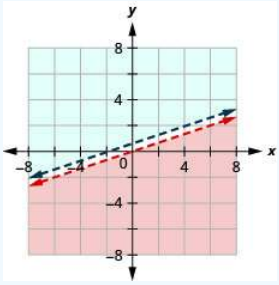
$$\begin{cases} x - 3y \geq 6 \\ y > \frac{1}{3}x + 1 \end{cases}$$

Exercise $5.6E.33$

$$\begin{cases} -2x + 6y < 0 \\ 6y > 2x + 4 \end{cases}$$

Answer

No solution



Exercise $5.6E.34$

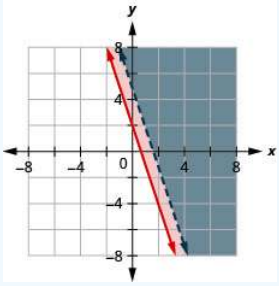
$$\begin{cases} -3x + 6y > 12 \\ 4y \leq 2x - 4 \end{cases}$$

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Exercise 5.6E.35

$$\begin{cases} y \geq -3x + 2 \\ 3x + y > 5 \end{cases}$$

Answer



Exercise 5.6E.36

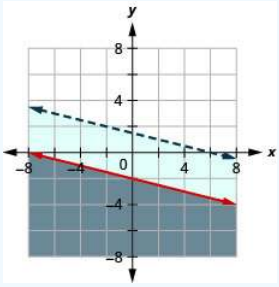
$$\begin{cases} y \geq \frac{1}{2}x - 1 \\ -2x + 4y \geq 4 \end{cases}$$

Exercise 5.6E.37

$$\begin{cases} y \leq -\frac{1}{4}x - 2 \\ x + 4y < 6 \end{cases}$$

Answer

x+4y<6



Exercise 5.6E.38

$$\begin{cases} y \geq 3x - 1 \\ -3x + y > -4 \end{cases}$$

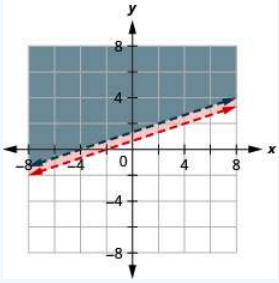
Exercise 5.6E.39

$$\begin{cases} 3y > x + 2 \\ -2x + 6y > 8 \end{cases}$$

Answer

$-2x + 6y > 8$

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Exercise 5.6E. 40

$$\begin{cases} y < \frac{3}{4}x - 2 \\ -3x + 4y < 7 \end{cases}$$

Solve Applications of Systems of Inequalities

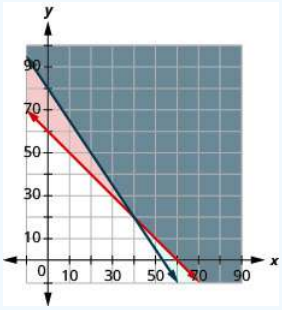
In the following exercises, translate to a system of inequalities and solve.

Exercise 5.6E. 41

Caitlyn sells her drawings at the county fair. She wants to sell at least 60 drawings and has portraits and landscapes. She sells the portraits for $15 and the landscapes for $10. She needs to sell at least $800 worth of drawings in order to earn a profit.

1. Write a system of inequalities to model this situation.
2. Graph the system.
3. Will she make a profit if she sells 20 portraits and 35 landscapes?
4. Will she make a profit if she sells 50 portraits and 20 landscapes?

Answer

1. $\begin{cases} p + l \geq 60 \\ 15p + 10l \geq 800 \end{cases}$
2. 

3. No
4. Yes

Exercise 5.6E. 42

Jake does not want to spend more than $50 on bags of fertilizer and peat moss for his garden. Fertilizer costs $2 a bag and peat moss costs $5 a bag. Jake's van can hold at most 20 bags.

1. Write a system of inequalities to model this situation.
2. Graph the system.
3. Can he buy 15 bags of fertilizer and 4 bags of peat moss?

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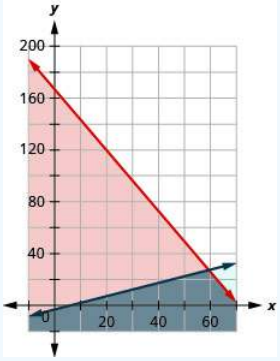
4. Can he buy 10 bags of fertilizer and 10 bags of peat moss?

Exercise $5.6E.43$

Reiko needs to mail her Christmas cards and packages and wants to keep her mailing costs to no more than $500. The number of cards is at least 4 more than twice the number of packages. The cost of mailing a card (with pictures enclosed) is $3 and for a package the cost is $7.

1. Write a system of inequalities to model this situation.
2. Graph the system.
3. Can she mail 60 cards and 26 packages?
4. Can she mail 90 cards and 40 packages?

Answer

1. $\begin{cases} 7p+3c \leq 500 \\ p \geq 2c+4 \end{cases}$
2. 

3. Yes
4. No

Exercise $5.6E.44$

Juan is studying for his final exams in Chemistry and Algebra. He knows he only has 24 hours to study, and it will take him at least three times as long to study for Algebra than Chemistry.

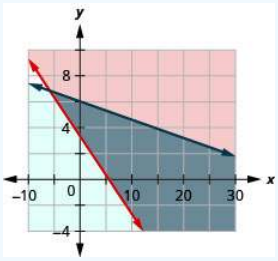
1. Write a system of inequalities to model this situation.
2. Graph the system.
3. Can he spend 4 hours on Chemistry and 20 hours on Algebra?
4. Can he spend 6 hours on Chemistry and 18 hours on Algebra?

Exercise $5.6E.45$

Jocelyn is pregnant and needs to eat at least 500 more calories a day than usual. When buying groceries one day with a budget of $15 for the extra food, she buys bananas that have 90 calories each and chocolate granola bars that have 150 calories each. The bananas cost $0.35 each and the granola bars cost $2.50 each.

1. Write a system of inequalities to model this situation.
2. Graph the system.
3. Could she buy 5 bananas and 6 granola bars?
4. Could she buy 3 bananas and 4 granola bars?

Answer

1. $\begin{cases} 90b+150g \geq 500 \\ 0.35b+2.50g \leq 15 \end{cases}$
2. 

3. No
4. Yes

? Exercise $5.6E.46$

Mark is attempting to build muscle mass and so he needs to eat at least an additional 80 grams of protein a day. A bottle of protein water costs $3.20 and a protein bar costs $1.75. The protein water supplies 27 grams of protein and the bar supplies 16 gram. If he has $ 10 dollars to spend

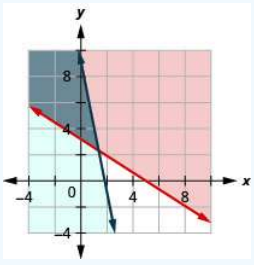
1. Write a system of inequalities to model this situation.
2. Graph the system.
3. Could he buy 3 bottles of protein water and 1 protein bar?
4. Could he buy no bottles of protein water and 5 protein bars?

? Exercise $5.6E.47$

Jocelyn desires to increase both her protein consumption and caloric intake. She desires to have at least 35 more grams of protein each day and no more than an additional 200 calories daily. An ounce of cheddar cheese has 7 grams of protein and 110 calories. An ounce of parmesan cheese has 11 grams of protein and 22 calories.

1. Write a system of inequalities to model this situation.
2. Graph the system.
3. Could she eat 1 ounce of cheddar cheese and 3 ounces of parmesan cheese?
4. Could she eat 2 ounces of cheddar cheese and 1 ounce of parmesan cheese?

Answer

1. $\begin{cases} 7c+11p \geq 35 \\ 110c+22p \leq 200 \end{cases}$
2. 

3. Yes
4. No

Exercise 5.6E. 48

Mark is increasing his exercise routine by running and walking at least 4 miles each day. His goal is to burn a minimum of 1,500 calories from this exercise. Walking burns 270 calories/mile and running burns 650 calories.

1. Write a system of inequalities to model this situation.
2. Graph the system.
3. Could he meet his goal by walking 3 miles and running 1 mile?
4. Could he meet his goal by walking 2 miles and running 2 mile?

Everyday Math

Exercise 5.6E. 49

Tickets for an American Baseball League game for 3 adults and 3 children cost less than $75, while tickets for 2 adults and 4 children cost less than $62.

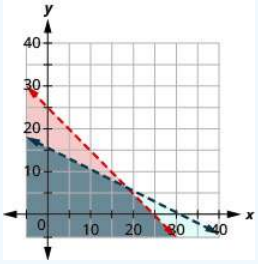
1. Write a system of inequalities to model this problem.
2. Graph the system.
3. Could the tickets cost $20 for adults and $8 for children?
4. Could the tickets cost $15 for adults and $5 for children?

Answer

1. \(\left\{

$$\begin{aligned} 3a+3c &< 75 \\ 2a+4c &< 62 \end{aligned} \tag{5.6E.1}$$

\right.\

2. 

3. No
4. Yes

Exercise 5.6E. 50

Grandpa and Grandma are treating their family to the movies. Matinee tickets cost $4 per child and $4 per adult. Evening tickets cost $6 per child and $8 per adult. They plan on spending no more than $80 on the matinee tickets and no more than $100 on the evening tickets.

1. Write a system of inequalities to model this situation.
2. Graph the system.
3. Could they take 9 children and 4 adults to both shows?
4. Could they take 8 children and 5 adults to both shows?

Writing Exercises

Exercise $5.6E.51$

Graph the inequality $x - y \geq 3$. How do you know which side of the line $x - y = 3$ should be shaded?

Answer

Answers will vary.

Exercise $5.6E.52$

Graph the system $\begin{cases} x + 2y \leq 6 \\ y \geq -\frac{1}{2}x - 4 \end{cases}$. What does the solution mean?

Self Check

a. After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
determine whether an ordered pair is a solution of a system of linear inequalities.			
solve a system of linear inequalities by graphing.			
solve applications of systems of inequalities.			

b. After reviewing this checklist, what will you do to become confident for all objectives?

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5.7: Chapter 5 Review Exercises

Chapter 5 Review Exercises

Solve Systems of Equations by Graphing

Determine Whether an Ordered Pair is a Solution of a System of Equations.

In the following exercises, determine if the following points are solutions to the given system of equations.

Exercise 5.7.1

$$\begin{cases} x+3y=-9 \\ 2x-4y=12 \end{cases}$$

1. $(-3,-2)$
2. $(0,-3)$

Answer

1. no
2. yes

Exercise 5.7.2

$$\begin{cases} x+y=8 \\ y=x-4 \end{cases}$$

1. $(6,2)$
2. $(9,-1)$

Solve a System of Linear Equations by Graphing

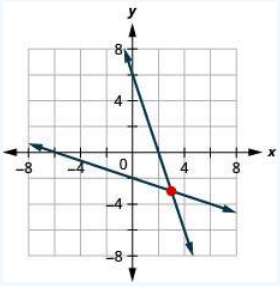
In the following exercises, solve the following systems of equations by graphing.

Exercise 5.7.3

$$\begin{cases} 3x+y=6 \\ x+3y=-6 \end{cases}$$

Answer

$(3,-3)$



Exercise 5.7.4

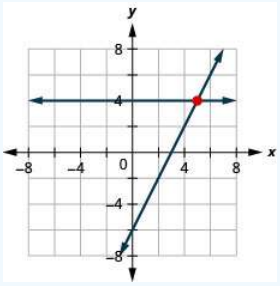
$$\begin{cases} y=x-2 \\ y=-2x-2 \end{cases}$$

Exercise 5.7.5

$$\begin{cases} 2x - y = 6 \\ y = 4 \end{cases}$$

Answer

(5,4)



Exercise 5.7.6

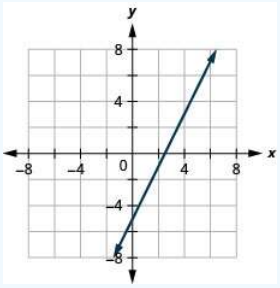
$$\begin{cases} x + 4y = -1 \\ x = 3 \end{cases}$$

Exercise 5.7.7

$$\begin{cases} 2x - y = 5 \\ 4x - 2y = 10 \end{cases}$$

Answer

coincident lines



Exercise 5.7.8

$$\begin{cases} -x + 2y = 4 \\ y = \frac{1}{2}x - 3 \end{cases}$$

Determine the Number of Solutions of a Linear System

In the following exercises, without graphing determine the number of solutions and then classify the system of equations.

Exercise 5.7.9

$$\begin{cases} y = \frac{2}{5}x + 2 \\ -2x + 5y = 10 \end{cases}$$

Answer

infinitely many solutions, consistent system, dependent equations

Exercise 5.7.10

$$\begin{cases} 3x+2y=6 \\ y=-3x+4 \end{cases}$$

Exercise 5.7.11

$$\begin{cases} 5x-4y=0 \\ y=\frac{5}{4}x-5 \end{cases}$$

Answer

no solutions, inconsistent system, independent equations

Exercise 5.7.12

$$\begin{cases} y=-\frac{3}{4}x+1 \\ 6x+8y=8 \end{cases}$$

Solve Applications of Systems of Equations by Graphing

Exercise 5.7.13

LaVelle is making a pitcher of caffe mocha. For each ounce of chocolate syrup, she uses five ounces of coffee. How many ounces of chocolate syrup and how many ounces of coffee does she need to make 48 ounces of caffe mocha?

Answer

LaVelle needs 8 ounces of chocolate syrup and 40 ounces of coffee.

Exercise 5.7.14

Eli is making a party mix that contains pretzels and chex. For each cup of pretzels, he uses three cups of chex. How many cups of pretzels and how many cups of chex does he need to make 12 cups of party mix?

Solve Systems of Equations by Substitution

Solve a System of Equations by Substitution

In the following exercises, solve the systems of equations by substitution.

Exercise 5.7.15

$$\begin{cases} 3x-y=-5 \\ y=2x+4 \end{cases}$$

Answer

$(-1,2)$

Exercise 5.7.16

$$\begin{cases} 3x-2y=2 \\ y=\frac{1}{2}x+3 \end{cases}$$

Exercise 5.7.17

$$\begin{cases} x-y=0 \\ 2x+5y=-14 \end{cases}$$

Answer

$(-2,-2)$

Exercise 5.7.18

$$\begin{cases} y=-2x+7 \\ y=\frac{2}{3}x-1 \end{cases}$$

Exercise 5.7.19

$$\begin{cases} y=-5x \\ 5x+y=6 \end{cases}$$

Answer

no solution

Exercise 5.7.20

$$\begin{cases} y=-\frac{1}{3}x+2 \\ x+3y=6 \end{cases}$$

Solve Applications of Systems of Equations by Substitution

In the following exercises, translate to a system of equations and solve.

Exercise 5.7.21

The sum of two number is 55. One number is 11 less than the other. Find the numbers.

Answer

The numbers are 22 and 33.

Exercise 5.7.22

The perimeter of a rectangle is 128. The length is 16 more than the width. Find the length and width.

Exercise 5.7.23

The measure of one of the small angles of a right triangle is 2 less than 3 times the measure of the other small angle. Find the measure of both angles.

Answer

The measures are 23 degrees and 67 degrees.

Exercise 5.7.24

Gabriela works for an insurance company that pays her a salary of $32,000 plus a commission of $100 for each policy she sells. She is considering changing jobs to a company that would pay a salary of $40,000 plus a commission of $80 for each policy sold. How many policies would Gabriela need to sell to make the total pay the same?

Solve Systems of Equations by Elimination

Solve a System of Equations by Elimination In the following exercises, solve the systems of equations by elimination.

Exercise 5.7.25

$$\begin{cases} x+y=12 \\ x-y=-10 \end{cases}$$

Answer

(1,11)

Exercise 5.7.26

$$\begin{cases} 4x+2y=2 \\ -4x-3y=-9 \end{cases}$$

Exercise 5.7.27

$$\begin{cases} 3x-8y=20 \\ x+3y=1 \end{cases}$$

Answer

(4,−1)

Exercise 5.7.28

$$\begin{cases} 3x-2y=6 \\ 4x+3y=8 \end{cases}$$

Exercise 5.7.29

$$\begin{cases} 9x+4y=2 \\ 5x+3y=5 \end{cases}$$

Answer

(−2,5)

Exercise 5.7.30

$$\begin{cases} -x+3y=8 \\ 2x-6y=-20 \end{cases}$$

Solve Applications of Systems of Equations by Elimination

In the following exercises, translate to a system of equations and solve.

Exercise 5.7.31

The sum of two numbers is −90. Their difference is 16. Find the numbers.

Answer

The numbers are −37 and −53.

Exercise 5.7.32

Omar stops at a donut shop every day on his way to work. Last week he had 8 donuts and 5 cappuccinos, which gave him a total of 3,000 calories. This week he had 6 donuts and 3 cappuccinos, which was a total of 2,160 calories. How many calories are in one donut? How many calories are in one cappuccino?

Choose the Most Convenient Method to Solve a System of Linear Equations

In the following exercises, decide whether it would be more convenient to solve the system of equations by substitution or elimination.

Exercise 5.7.33

$$\begin{cases} 6x-5y=27 \\ 3x+10y=-24 \end{cases}$$

Answer

elimination

Exercise 5.7.34

$$\begin{cases} y=3x-9 \\ 4x-5y=23 \end{cases}$$

Solve Applications with Systems of Equations

Translate to a System of Equations

In the following exercises, translate to a system of equations. Do not solve the system.

Exercise 5.7.35

The sum of two numbers is −32. One number is two less than twice the other. Find the numbers.

Answer

$$\begin{cases} x+y=-32 \\ x=2y-2 \end{cases}$$

Exercise 5.7.36

Four times a number plus three times a second number is −9. Twice the first number plus the second number is three. Find the numbers.

Exercise 5.7.37

Last month Jim and Debbie earned $7,200. Debbie earned $1,600 more than Jim earned. How much did they each earn?

Answer

$$\begin{cases} j+d=7200 \\ d=j+1600 \end{cases}$$

Exercise 5.7.38

Henri has $24,000 invested in stocks and bonds. The amount in stocks is $6,000 more than three times the amount in bonds. How much is each investment?

Solve Direct Translation Applications

In the following exercises, translate to a system of equations and solve.

Exercise 5.7.39

Pam is 3 years older than her sister, Jan. The sum of their ages is 99. Find their ages.

Answer

Pam is 51 and Jan is 48.

Exercise 5.7.40

Mollie wants to plant 200 bulbs in her garden. She wantsall irises and tulips. She wants to plant three times as many tulips as irises. How many irises and how many tulips should she plant?

Solve Geometry Applications

In the following exercises, translate to a system of equations and solve.

Exercise 5.7.41

The difference of two supplementary angles is 58 degrees. Find the measures of the angles.

Answer

The measures are 119 degrees and 61 degrees.

Exercise 5.7.42

Two angles are complementary. The measure of the larger angle is five more than four times the measure of the smaller angle. Find the measures of both angles.

Exercise 5.7.43

Becca is hanging a 28 foot floral garland on the two sides and top of a pergola to prepare for a wedding. The height is four feet less than the width. Find the height and width of the pergola.

Answer

The pergola is 8 feet high and 12 feet wide.

Exercise 5.7.44

The perimeter of a city rectangular park is 1428 feet. The length is 78 feet more than twice the width. Find the length and width of the park.

Solve Uniform Motion Applications

In the following exercises, translate to a system of equations and solve.

Exercise 5.7.45

Sheila and Lenore were driving to their grandmother's house. Lenore left one hour after Sheila. Sheila drove at a rate of 45 mph, and Lenore drove at a rate of 60 mph. How long will it take for Lenore to catch up to Sheila?

Answer

It will take Lenore 3 hours.

Exercise 5.7.46

Bob left home, riding his bike at a rate of 10 miles per hour to go to the lake. Cheryl, his wife, left 45 minutes ($\frac{3}{4}$ hour) later, driving her car at a rate of 25 miles per hour. How long will it take Cheryl to catch up to Bob?

Exercise 5.7.47

Marcus can drive his boat 36 miles down the river in three hours but takes four hours to return upstream. Find the rate of the boat in still water and the rate of the current.

Answer

The rate of the boat is 10.5 mph. The rate of the current is 1.5 mph.

Exercise 5.7.48

A passenger jet can fly 804 miles in 2 hours with a tailwind but only 776 miles in 2 hours into a headwind. Find the speed of the jet in still air and the speed of the wind.

Solve Mixture Applications with Systems of Equations

Solve Mixture Applications

In the following exercises, translate to a system of equations and solve.

Exercise 5.7.49

Lynn paid a total of $2,780 for 261 tickets to the theater. Student tickets cost $10 and adult tickets cost $15. How many student tickets and how many adult tickets did Lynn buy?

Answer

Lynn bought 227 student tickets and 34 adult tickets.

Exercise 5.7.50

Priam has dimes and pennies in a cup holder in his car. The total value of the coins is $4.21. The number of dimes is three less than four times the number of pennies. How many dimes and how many pennies are in the cup?

Exercise 5.7.51

Yumi wants to make 12 cups of party mix using candies and nuts. Her budget requires the party mix to cost her $1.29 per cup. The candies are $2.49 per cup and the nuts are $0.69 per cup. How many cups of candies and how many cups of nuts should she use?

Answer

Yumi should use 4 cups of candies and 8 cups of nuts.

Exercise 5.7.52

A scientist needs 70 liters of a 40% solution of alcohol. He has a 30% and a 60% solution available. How many liters of the 30% and how many liters of the 60% solutions should he mix to make the 40% solution?

Solve Interest Applications

In the following exercises, translate to a system of equations and solve.

Exercise 5.7.53

Jack has $12,000 to invest and wants to earn 7.5% interest per year. He will put some of the money into a savings account that earns 4% per year and the rest into CD account that earns 9% per year. How much money should he put into each account?

Answer

Jack should put $3600 into savings and $8400 into the CD.

Exercise 5.7.54

When she graduates college, Linda will owe $43,000 in student loans. The interest rate on the federal loans is 4.5% and the rate on the private bank loans is 2%. The total interest she owes for one year was $1585. What is the amount of each loan?

Graphing Systems of Linear Inequalities

Determine Whether an Ordered Pair is a Solution of a System of Linear Inequalities

In the following exercises, determine whether each ordered pair is a solution to the system.

Exercise 5.7.55

$$\begin{cases} 4x+y>6 \\ 3x-y\leq 12 \end{cases}$$

1. (2,−1)
2. (3,−2)

Answer

1. yes
2. no

Exercise 5.7.56

$$\begin{cases} y>\frac{1}{3}x+2 \\ x-\frac{1}{4}y\leq 10 \end{cases}$$

1. (6,5)
2. (15,8)

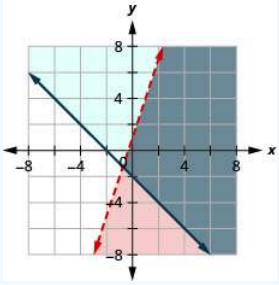
Solve a System of Linear Inequalities by Graphing

In the following exercises, solve each system by graphing.

Exercise 5.7.57

$$\begin{cases} y < 3x + 1 \\ y \geq -x - 2 \end{cases}$$

Answer



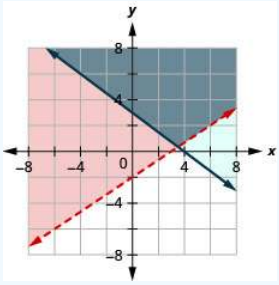
Exercise 5.7.58

$$\begin{cases} x - y > -1 \\ y < \frac{1}{3}x - 2 \end{cases}$$

Exercise 5.7.59

$$\begin{cases} 2x - 3y < 6 \\ 3x + 4y \geq 12 \end{cases}$$

Answer



Exercise 5.7.60

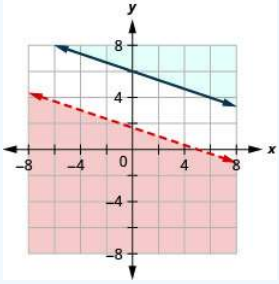
$$\begin{cases} y \leq -\frac{3}{4}x + 1 \\ x \geq -5 \end{cases}$$

Exercise 5.7.61

$$\begin{cases} x + 3y < 5 \\ y \geq -\frac{1}{3}x + 6 \end{cases}$$

Answer

No solution



Exercise 5.7.62

$$\begin{cases} y \geq 2x - 5 \\ -6x + 3y > -4 \end{cases}$$

Solve Applications of Systems of Inequalities

In the following exercises, translate to a system of inequalities and solve.

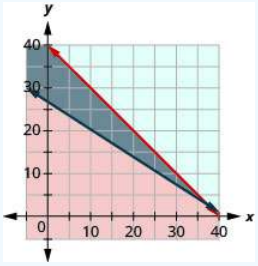
Exercise 5.7.63

Roxana makes bracelets and necklaces and sells them at the farmers' market. She sells the bracelets for $12 each and the necklaces for $18 each. At the market next weekend she will have room to display no more than 40 pieces, and she needs to sell at least $500 worth in order to earn a profit.

1. Write a system of inequalities to model this situation.
2. Graph the system.
3. Should she display 26 bracelets and 14 necklaces?
4. Should she display 39 bracelets and 1 necklace?

Answer

1. $\begin{cases} b + n \leq 40 \\ 12b + 18n \geq 500 \end{cases}$
2.



3. yes
4. no

Exercise 5.7.64

Annie has a budget of $600 to purchase paperback books and hardcover books for her classroom. She wants the number of hardcover to be at least 5 more than three times the number of paperback books. Paperback books cost $4 each and hardcover books cost $15 each.

1. Write a system of inequalities to model this situation.
2. Graph the system.

3. Can she buy 8 paperback books and 40 hardcover books?
4. Can she buy 10 paperback books and 37 hardcover books?

Practice Test

Exercise 5.7.1

$$\begin{cases} x-4y=-8 \\ 2x+5y=10 \end{cases}$$

1. (0,2)
2. (4,3)

Answer

1. yes
2. no

In the following exercises, solve the following systems by graphing.

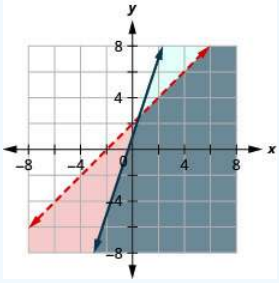
Exercise 5.7.2

$$\begin{cases} x-y=5 \\ x+2y=-4 \end{cases}$$

Exercise 5.7.3

$$\begin{cases} x-y>-2 \\ y\leq 3x+1 \end{cases}$$

Answer



In the following exercises, solve each system of equations. Use either substitution or elimination.

Exercise 5.7.4

$$\begin{cases} 3x-2y=3 \\ y=2x-1 \end{cases}$$

Exercise 5.7.5

$$\begin{cases} x+y=-3 \\ x-y=11 \end{cases}$$

Answer

(4,−7)

Exercise 5.7.6

$$\begin{cases} 4x-3y=7 \\ 5x-2y=0 \end{cases}$$

Exercise 5.7.7

$$\begin{cases} y=-\frac{4}{5}x+1 \\ 8x+10y=10 \end{cases}$$

Answer

infinitely many solutions

Exercise 5.7.8

$$\begin{cases} 2x+3y=12 \\ -4x+6y=-16 \end{cases}$$

In the following exercises, translate to a system of equations and solve.

Exercise 5.7.9

The sum of two numbers is −24. One number is 104 less than the other. Find the numbers.

Answer

The numbers are 40 and 64

Exercise 5.7.10

Ramon wants to plant cucumbers and tomatoes in his garden. He has room for 16 plants, and he wants to plant three times as many cucumbers as tomatoes. How many cucumbers and how many tomatoes should he plant?

Exercise 5.7.11

Two angles are complementary. The measure of the larger angle is six more than twice the measure of the smaller angle. Find the measures of both angles.

Answer

The measures of the angles are 28 degrees and 62 degrees.

Exercise 5.7.12

On Monday, Lance ran for 30 minutes and swam for 20 minutes. His fitness app told him he had burned 610 calories. On Wednesday, the fitness app told him he burned 695 calories when he ran for 25 minutes and swam for 40 minutes. How many calories did he burn for one minute of running? How many calories did he burn for one minute of swimming?

Exercise 5.7.13

Kathy left home to walk to the mall, walking quickly at a rate of 4 miles per hour. Her sister Abby left home 15 minutes later and rode her bike to the mall at a rate of 10 miles per hour. How long will it take Abby to catch up to Kathy?

Answer

It will take Kathy $\frac{1}{6}$ of an hour (or 10 minutes)

Exercise 5.7.14

It takes $5\frac{1}{2}$ hours for a jet to fly 2,475 miles with a headwind from San Jose, California to Lihue, Hawaii. The return flight from Lihue to San Jose with a tailwind, takes 5 hours. Find the speed of the jet in still air and the speed of the wind.

Exercise 5.7.15

Liz paid $160 for 28 tickets to take the Brownie troop to the science museum. Children's tickets cost $5 and adult tickets cost $9. How many children's tickets and how many adult tickets did Liz buy?

Answer

Liz bought 23 children's tickets and 5 adult tickets.

Exercise 5.7.16

A pharmacist needs 20 liters of a 2% saline solution. He has a 1% and a 5% solution available. How many liters of the 1% and how many liters of the 5% solutions should she mix to make the 2% solution?

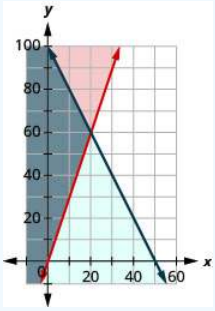
Exercise 5.7.1

Translate to a system of inequalities and solve.

Andi wants to spend no more than $50 on Halloween treats. She wants to buy candy bars that cost $1 each and lollipops that cost $0.50 each, and she wants the number of lollipops to be at least three times the number of candy bars.

1. Write a system of inequalities to model this situation.
2. Graph the system.
3. Can she buy 20 candy bars and 70 lollipops?
4. Can she buy 15 candy bars and 65 lollipops?

Answer

1. $\begin{cases} C+0.5L \leq 50 \\ L \geq 3C \end{cases}$
2. 

3. No
4. Yes

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CHAPTER OVERVIEW

6: Polynomial Functions

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6.1: Functions

We begin with the definition of a *relation*.

Relation

A relation is a collection of ordered pairs.

The collection of ordered pairs

$$R = \{(0,3),(0,4),(1,5),(2,6)\}$$

is an example of a relation.

If we collect the first element of each ordered pair in a set, we have what is called the *domain* of the relation.

Domain

The domain of a relation is the set of all first elements of the ordered pairs.

For example, in the relation

$$R = \{(0,3),(0,4),(1,5),(2,6)\}$$

if we collect the first element of each ordered pair in R, we get the domain:

$$\text{Domain of R} = \{0,1,2\}$$

Although the number zero appears twice as a first element in the ordered pairs of R, note that we list it only once when listing the elements in the domain of R.

In similar fashion, if we collect the second elements of each ordered pair in a set, we have what is called the *range* of a relation.

Range

The range of a relation is the set of all second elements of the ordered pairs.

For example, in the relation

$$R = \{(0,3),(0,4),(1,5),(2,6)\}$$

if we collect the second element of each ordered pair in R, we get the range:

$$\text{Range of R} = \{3,4,5,6\}$$

Example 6.1.1

State the domain and range of the relation

$$T = \{(5,3),(6,3),(7,4)\}$$

Solution

Collect the first element of each ordered pair in T to list the domain:

$$\text{Domain of } T = \{5,6,7\}$$

Collect the second element of each ordered pair in T to list the range:

$$\text{Range of } T = \{3,4\}$$

Note that even though the number three appears in the second position twice, we list it only once in describing the range.

Exercise 6.1.1

State the domain and range of the relation

$$S = \{(-1, 7), (2, 5), (2, 3)\}$$

Answer

Domain of $S = \{-1, 2\}$, Range of $S = \{3, 5, 7\}$

Example 6.1.2

State the domain and range of the relation shown in Figure 6.1.1.

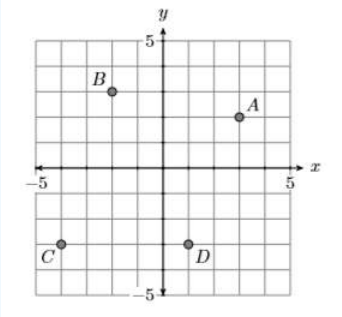


Figure 6.1.1: We can present a relation as a collection of ordered pairs in a graph.

Solution

Point A has coordinates $(3, 2)$, point B has coordinates $(-2, 3)$, point C has coordinates $(-4, -3)$, and point D has coordinates $(1, -3)$. We can collect these points in a set.

$$S = \{(3, 2), (-2, 3), (-4, -3), (1, -3)\}$$

If we collect each element in the first position of each ordered pair, we have the domain.

$$\text{Domain of } S = \{-4, -2, 1, 3\}$$

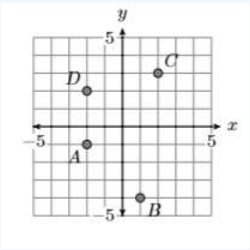
Note that it is traditional to list the domain elements in order (smallest to largest). Next, if we collect each element in the second position of each ordered pair, we have the range.

$$\text{Range of } S = \{-3, 2, 3\}$$

Again, it is traditional to list the elements in order. Note again that we did not repeat the number -3 in listing the range, even though it is used twice as a second element of an ordered pair in the set S.

Exercise 6.1.2

State the domain and range of the relation shown below.



Answer

Domain of $S = \{-2, 1, 2\}$, Range of $S = \{-4, -1, 2, 3\}$

Mapping Diagrams

A mapping diagram is a useful construction that helps one to analyze a relation. Consider the earlier relation $R = \{(0, 3), (0, 4), (1, 5), (2, 6)\}$, which had domain $\mathcal{D} = \{0, 1, 2\}$ and range $\mathcal{R} = \{3, 4, 5, 6\}$. To construct a mapping diagram for R, list the elements in the domain of R on the left, list the elements of the range of R on the right, then use arrows to indicate the ordered pairs (see Figure 6.1.2).

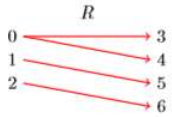


Figure 6.1.2: Using a mapping diagram to describe the relation R.

Note how the ordred pair $(0, 3)$ is indicated by drawing an arrow connecting 0 on the left to 3 on the right. We say that the relation "maps 0 to 3" and write $R : 0 \to 3$. In similar fashion:

- The ordered pair $(0, 4)$ is indicated by drawing an arrow connecting 0 on the left to 4 on the right; i.e., R "maps 0 to 4" or $R : 0 \to 4$.
- The ordered pair $(1, 5)$ is indicated by drawing an arrow connecting 1 on the left to 5 on the right; i.e., R "maps 1 to 5" or $R : 1 \to 5$.
- The ordered pair $(2, 6)$ is indicated by drawing an arrow connecting 2 on the left to 6 on the right; i.e., R "maps 2 to 6" or $R : 2 \to 6$.

Example 6.1.3

Create a mapping diagram for the relation in Example 6.1.1.

Solution

The relation of Example 6.1.1 is $T = \{(5, 3), (6, 3), (7, 4)\}$. List the domain $\mathcal{D} = \{5, 6, 7\}$ on the left, the range $\mathcal{R} = \{3, 4\}$ on the right, then use arrows to indicate the ordered pairs (see Figure 6.1.3).

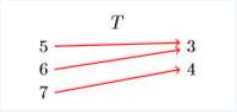


Figure 6.1.3: A mapping diagram for the relation T.

Function Definition

A *function* is a very special type of relation.

Function

A relation is a function if and only if each object in the domain is paired with exactly one object in the range.

As a first example, consider the relation $R = \{(0, 3), (0, 4), (1, 5), (2, 6)\}$ whose mapping diagram is pictured in Figure 6.1.2. Note that 0 in the domain is paired with two objects, 3 and 4, in the range. Hence, relation R is is not a function.

As a second example, consider the relation $T = \{(5, 3), (6, 3), (7, 4)\}$, whose mapping diagram is pictured in Figure 6.1.3. In this example, each domain object is paired with exactly one range object: 5 only gets sent to 3, 6 only gets sent to 3, and 7 only gets sent to 4. Hence, the relation T is a function. The fact that the range object 3 is used twice does not matter. It's the fact that each domain object gets sent to exactly one range object that matters.

Example 6.1.4

Consider the relation pictured in Figure 6.1.4. Is it a function?

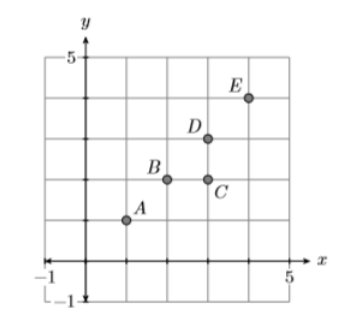


Figure 6.1.4: Is this relation a function?

Solution

The graph in Figure 6.1.4 consists of the points $A(1,1)$, $B(2,2)$, $C(3,2)$, $D(3,3)$, and $E(4,4)$. The domain is $\mathcal{D}=\{1,2,3,4\}$ and the range is $\mathcal{R}=\{1,2,3,4\}$. A mapping diagram (see Figure 6.1.5) will help us decide if the relation represented by the graph is a function. Put the domain on the left, the range on the right, then use arrows to indicate the ordered pairs. Let's call the relation f.

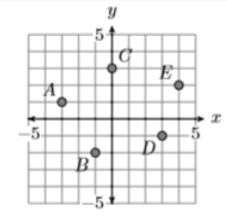
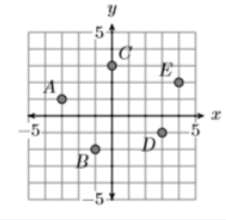


Figure 6.1.5: A mapping diagram for the relation f depicted in Figure 6.1.4.

In Figure 6.1.5, note how the domain object 3 is "sent to" or paired with two range objects, 2 and 3. Hence, the relation f is not a function.

Exercise 6.1.4

Consider the relation pictured below. Is it a function?



Answer

Yes, the relation is a function.

Example 6.1.5

The following relation pairs automobiles with their gas mileage. Determine if the relation is a function.

$$T=\{(\text{Bentley Mulsanne},18),(\text{Kia Soul},30),(\text{Lamborghini Gallardo},20),(\text{Smart Fortwo},41),(\text{Jaguar XF},23)\}$$

Solution

In Figure 6.1.6, we create a mapping diagram indicating the relation between cars and their gas mileage. Note that each domain object on the left is paired with exactly one range object on the right. Hence, this relation is a function.

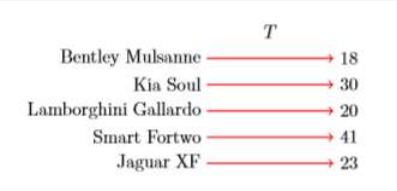


Figure 6.1.6: A mapping diagram for the relation T.

Exercise 6.1.5

The following relation pairs people with their age. Determine if the relation is a function.

$$S = \{(\text{Mary},23),(\text{Joe},18),\ (\text{Alfonzo},20),(\text{Zoe},18),\ (\text{Maria},22),(\text{Chris},23)\ \}$$

Answer

Yes, the relation is a function.

Example 6.1.6

The following relation pairs a particular bird with the state that has adopted that bird as its state bird. Determine if the relation is a function.

$$R = \{\ (\text{Yellowhammer, Alabama}),\ (\text{Robin, Connecticut}),\ (\text{Nene, Hawaii}),\ (\text{Robin, Michigan})\ \}$$

Solution

In Figure 6.1.7, we create a mapping diagram indicating the relation between birds and their state adoptions. Note that the domain object "Robin" is paired with two range objects, "Connecticut" and "Michigan," hence this relation is not a function.

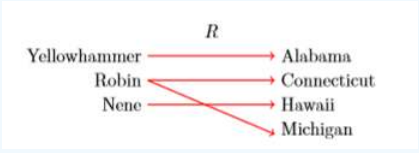


Figure 6.1.7: A mapping diagram for the relation R.

Exercise 6.1.6

The following relation pairs people with the types of cars they own. Determine if the relation is a function.

$$S = \{(\text{Bernard},\text{station wagon}),\ (\text{Tina},\text{truck}),\ (\text{Gilberto},\text{sedan}),\ (\text{Kate},\text{sport utility}),\ (\text{Bernard},\text{sedan}),\ (\text{Kate},\text{minivan})\}$$

Answer

No, the relation is not a function.

Mapping Diagram Notation

The goal of this section is to introduce function notation. Let's begin with the mapping diagram in Figure 6.1.8.

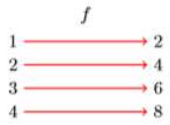


Figure 6.1.8: Mapping diagram.

The mapping diagram in Figure 6.1.8 reveals the following facts:

- f maps 1 to 2 or $f : 1 \to 2$
- f maps 2 to 4 or $f : 2 \to 4$
- f maps 3 to 6 or $f : 3 \to 6$
- f maps 4 to 8 or $f : 4 \to 8$

Note how the notation $f : 4 \to 8$ correlates nicely with the mapping diagram in Figure 6.1.8. The notation $f : 4 \to 8$ is read "f maps 4 to 8" or " f sends 4 to 8."

A closer look at the mapping diagram in Figure 6.1.8 reveals an interesting pattern. The "rule" seems to be that the relation f doubles each entry in its domain: twice 1 is 2, twice 2 is 4, twice 3 is 6, etc. It's possible to give a general description of this "rule" by writing:

$$f : x \to 2x \quad (5.1) \tag{6.1.1}$$

That is, f sends x to twice x, or equivalently, $2x$. For example, we might ask "where does f send 15?" To answer this question, we would replace x with 15 in the rule 6.1.1 to get

$$f : 15 \to 2(15)$$

or equivalently,

$$f : 15 \to 30$$

We could also ask "where does f send -7?" To answer this question, we would replace x with -7 in the rule 6.1.1 to get

$$f : -7 \to 2(-7)$$

or equivalently,

$$f : -7 \to -14$$

Example 6.1.7

Given the rule $f : x \to 2x + 3$, answer the question "where does f send 8?"

Solution

To find where "f sends 8," substitute 8 for x in the rule $f : x \to 2x + 3$ to get

$$f : 8 \to 2(8) + 3$$

or equivalently,

$$f : 8 \to 19$$

Exercise 6.1.7

Given the rule $f : x \to 3x - 5$, answer the question "where does f send -2?"

Answer

$f : -2 \to -11$

Example 6.1.8

Given the rule $f : x \to x/(x+3)$, answer the question "where does f send -1?"

Solution

To find where "f sends -1," substitute -1 for x in the rule $f : x \to x/(x+3)$ to get

$$f : -1 \to \frac{-1}{-1+3}$$

or equivalently,

$$f : -1 \to -\frac{1}{2}$$

Exercise 6.1.8

Given the rule \(f : x → 2x^2 +5x\), answer the question "where does f send 3?"

Answer

$f : 3 \to 33$

In Examples $6.1.7$ and $6.1.8$, note that each time you substitute a value for x in the given rule, you get a unique answer. This means that each object in the domain of f is sent to a unique object in the range of f, making the rules in Examples $6.1.7$ and $6.1.8$ functions. This leads us to an itemized description of a function.

Rule of Three

A function consists of three parts:

- a set of objects which mathematicians call the *domain*
- a second set of objects which mathematicians call the *range*
- a *rule* that describes how to assign each object in the domain to exactly one object in the range.

Function Notation

Although the mapping diagram notation $f : x \to 3-4x$ is quite easy to understand, the standard function notation used is $f(x) = 3-4x$. With mapping diagram notation, if we want to answer the question "where does f send 12?", we write:

$$\begin{aligned} f &: x \to 3-4x \\ f &: 12 \to 3-4(12) \\ f &: 12 \to 3-48 \\ f &: 12 \to -45 \end{aligned}$$

Hence, $f : 12 \to -45$; i.e., f sends 12 to -45. Function notation uses exactly the same concept; i.e, substitute 12 for x.

Tips for Using Function Notation

1. Replace all occurrences of variables in the notation with open parentheses. Leave room between the parentheses to substitute the given value of the variable.
2. Substitute the given values of variables in the open parentheses prepared in the first step.
3. Evaluate the resulting expression according to the Rules Guiding Order of Operations.

Given $f(x) = 3-4x$, to evaluate $f(12)$, first restate the function notation, then replace each occurrence of the variable with open parentheses.

$$\begin{aligned} f(x) &= 3-4x \quad \text{Original function notation.} \\ f(\ ) &= 3-4(\ ) \quad \text{Replace each occurrence of } x \text{ with open parentheses.} \end{aligned}$$

Now substitute 12 for x in the open parentheses prepared in the last step.

$$\begin{aligned} f(12) &= 3-4(12) \quad \text{Substitute 12 for } x \text{ in the open parentheses positions.} \\ f(12) &= 3-48 \quad \text{Multiply.} \\ f(12) &= -45 \quad \text{Subtract.} \end{aligned}$$

Hence, $f(12) = -45$; i.e., f sends 12 to -45.

Example 6.1.9

Given $f(x) = 4-5x-x^2$, evaluate $f(-3)$.

Solution

Start by replacing each occurrence of the variable x with open parentheses.

$$\begin{aligned} f(x) &= 4-5x-x^2 \quad \text{Original function notation.} \\ f(\,) &= 4-5(\quad)-(\quad)^2 \quad \text{Replace each occurrence of } x \text{ with open parentheses.} \end{aligned}$$

Now substitute -3 for x in the open parentheses prepared in the last step.

$$\begin{aligned} f(-3) &= 4-5(-3)-(-3)^2 \quad \text{Substitute } -3 \text{ for } x \text{ in the open parentheses positions.} \\ f(-3) &= 4-5(-3)-9 \quad \text{Evaluate exponent: } (-3)^2 = 9 \\ f(-3) &= 4+15-9 \quad \text{Multiply: } -5(-3) = 15 \\ f(-3) &= 10 \quad \text{Simplify.} \end{aligned}$$

Thus, $f(-3) = 10$. Check this on your calculator (see Figure 6.1.9).

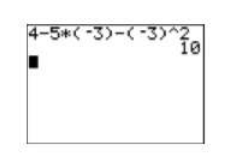


Figure 6.1.9: Calculator check.

The next example demonstrates one of the advantages of function notation. For example, it is easy to refer to the function in which you want to substitute the given x-value.

Example 6.1.10

Given $f(x) = 5-x$ and $g(x) = x^2-9$, find $f(-1)$ and $g(-2)$.

Solution

We're given two function definitions, f and g, but we're first asked to find $f(-1)$. This means that we must replace each occurrence of x with -1 in the function $f(x) = 5-x$.

$$\begin{aligned} f(x) &= 5-x \quad \text{Original function notation.} \\ f(\,) &= 5-(\,) \quad \text{Replace each occurrence of } x \text{ with open parentheses.} \end{aligned}$$

Now substitute -1 for x in the open parentheses prepared in the last step.

$$\begin{aligned} f(-1) &= 5-(-1) \quad \text{Substitute } -1 \text{ for } x \text{ in the open} \\ &= 5+1 \quad \text{Add the opposite.} \\ &= 6 \quad \text{Simplify.} \end{aligned}$$

Thus, $f(-1) = 6$. We're next asked to find $g(-2)$. This means that we must replace each occurrence of x with -2 in the function $g(x) = x^2-9$.

$$\begin{aligned} g(x) &= x^2-9 \quad \text{Original function notation.} \\ g(\,) &= (\,)^2-9 \quad \text{Replace each occurrence of } x \text{ with open parentheses.} \end{aligned}$$

Now substitute -2 for x in the open parentheses prepared in the last step.

$$\begin{aligned} g(-2) &= (-2)^2 - 9 \quad \text{Substitute } -2 \text{ for } x \text{ in the open parentheses position.} \\ &= 4 - 9 \quad \text{Exponent first: } (-2)^2 = 4 \\ &= -5 \quad \text{Simplify.} \end{aligned}$$

Thus, $g(-2) = -5$.

Exercise 6.1.10

Given f\((x)=3x^2 −20\) and \(g(x)=4x +6/x\), find $f(-3)$ and $g(2)$.

Answer

$f(-3) = 7$ and $g(2) = 10$

Interchanging y and $f(x)$

In most cases, y and $f(x)$ are completely interchangeable. For example, compare and contrast the following two examples.

Question: Given $y = 3x + 7$, find y when x equals 5.

Solution: Replace x with 5.

$$\begin{aligned} y &= 3x + 7 \\ y &= 3(5) + 7 \\ y &= 15 + 7 \\ y &= 22 \end{aligned}$$

Question: Given $f(x) = 3x + 7$, evaluate $f(5)$

Solution: Replace x with 5.

$$\begin{aligned} f(x) &= 3x + 7 \\ f(5) &= 3(5) + 7 \\ f(5) &= 15 + 7 \\ f(5) &= 22 \end{aligned}$$

In each case, the answer is 22. However, in the first case, the answer $y = 22$ disguises the fact that an x-value of 5 was used to arrive at the result. On the other hand, when we use function notation, the final answer $f(5) = 22$ indicates that we used an x-value of 5 to determine that the y-value is 22. This is another advantage of function notation.

Let's look at one final application demonstrating that y and $f(x)$ are interchangeable.

Example 6.1.11

Sketch the graph of $f(x) = 2x - 3$.

Solution

Because y and $f(x)$ are interchangeable, the instruction is identical to "sketch the graph of $y = 2x - 3$." The graph is a line, with slope 2 and y -intercept at $(0, -3)$. Plot the y-intercept at $(0, -3)$, then move up 2 and right 1 to create a line with slope 2 (see Figure $6.1.10$). Note how we've labeled the graph with its equation using function notation.

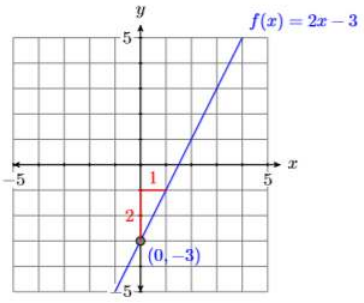
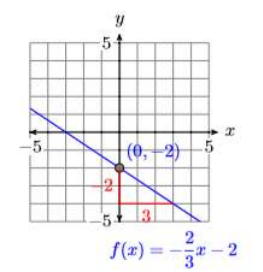


Figure 6.1.10: The graph of $f(x) = 2x - 3$ is a line.

Exercise 6.1.11

Sketch the graph of $f(x) = -\frac{2}{3}x - 2$

Answer



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6.2: Polynomials

We begin with the definition of a term.

Definition: Term

A *term* is either a single number (called a *constant* term) or the product of a number and one or more variables.

For example, each of the following is a term.

$$-5 \quad -3x^2 \quad 12y^2z^3 \quad 13a^2bc^3$$

Note how the first term is a single number, while the remaining terms are products of a number and one or more variables. For example, $-3x^2$ is the product of -3, x, and x.

Definition: Coefficient

When a term is a product of a number and one or more variables, the number is called the *coefficient* of the term. In the case of a term that is a single number, the number itself is called the coefficient.

Thus, for example, the coefficients of the terms

$$-5 \quad -3x^2 \quad 12y^2z^3 \quad 13a^2bc^3$$

are -5, -3, 12, and 13, respectively.

Definition: Degree

The *degree* of a term is the sum of the exponents on each variable of the term. A constant term (single number with no variables) has degree zero.

Thus, for example, the degrees of the terms

$$-5 \quad -3x^2 \quad 12y^2z^3 \quad 13a^2bc^3$$

are 0, 2, 5, and 6, respectively. In the last example, note that $13a^2bc^3$ is equivalent to $13a^2b^1c^3$, so adding exponents, we get:

$$\begin{aligned} \text{Degree of } 13a^2bc^3 &= \text{ Degree of } 13a^2b^1c^3 \\ &= 2+1+3 \\ &= 6 \end{aligned}$$

Definition: Monomial

The words *monomial* and *term* are equivalent.

Thus,

$$-5 \quad -3x^2 \quad 12y^2z^3 \quad 13a^2bc^3$$

are monomials.

Definition: Binomial

A *binomial* is a mathematical expression containing exactly two terms, separated by plus or minus signs.

For example, each of the mathematical expressions

$$2x+3y \quad -3a^2-3b^2 \quad xy+7 \quad -3x^2y+5xy^2$$

is a binomial. Each expression has exactly two terms.

Definition: Trinomial

A *trinomial* is a mathematical expression containing exactly three terms, separated by plus or minus signs.

For example, each of the mathematical expressions

$$2x^2+3x+7 \quad a^2+2ab+b^2 \quad x^4-2x^2y^2+3y^4$$

is a trinomial. Each expression has exactly three terms.

A bicycle has two wheels, a binomial has two terms. A tricycle has three wheels, a trinomial has three terms. But once we get past three terms, the assignment of special names ceases and we use the generic word *polynomial*, which means "many terms."

Definition: Polynomial

A *polynomial* is a many-termed mathematical expression, with terms separated by plus or minus signs. The coefficients of a polynomial are the coefficients of its terms.

Each of the previous expressions,

$$12y^2z^3 \quad -3a^2-3b^2 \quad x^4-2x^2y^2+3y^4$$

though assigned the particular names monomial, binomial, and trinomial, respectively, are also "many-termed" expressions and can also be called polynomials. However, because the word polynomial means "many terms," we can also use the word polynomial to describe mathematical expressions with more than three terms, such as:

$$x^4-4x^3y+6x^2y^2-4xy^3+y^4$$

The coefficients of $x^4-4x^3y+6x^2y^2-4xy^3+y^4$ are 1, -4, 6, -4, and 1.

Ascending and Descending Powers

When asked to simplify a polynomial expression, we should combine any like terms we find, and when possible, arrange the answer in *ascending* or *descending* powers.

Example 6.2.1

Simplify the following polynomial expression, arranging your answer in the descending powers of x. Once you've completed that task, make a second arrangement, ordering your terms in ascending powers of x.

$$2x^3+7x-3x^2+11x+8x^2+11+15x$$

Solution

In order to arrange our answer in descending powers of x, we want to place the term with the highest power of x first and the term with the lowest power of x last. We use the commutative and associative properties to change the order and regroup, then we combine like terms.

$$\begin{aligned} 2x^3+7x & -3x^2+11x+8x^2+11+15x \\ &= 2x^3+\left(-3x^2+8x^2\right)+(7x+11x+15x)+11 \\ &= 2x^3+5x^2+33x+11 \end{aligned}$$

Note how the powers of x start at 3, then go down in order.

To arrange our final answer in ascending powers of x, we put the lowest power of x first, then the highest power of x last, regrouping and combining like terms.

$$\begin{aligned} 2x^3+7x & -3x^2+11x+8x^2+11+15x \\ &= 11+(7x+11x+15x)+\left(-3x^2+8x^2\right)+2x^3 \\ &= 11+33x+5x^2+2x^3 \end{aligned}$$

Note how we start with the constant term, then the powers of x increase in order.

Exercise 6.2.1

Simplify the following polynomial, and arrange your answer in ascending powers of

$$3x^2 - 5x^3 + 8x + 9x^2 - 7x + 2x^3$$

Answer

$x + 12x^2 - 3x^3$

When we have a polynomial in a single variable, such as the polynomial in Example 6.2.1, arranging the terms in ascending or descending order is fairly straightforward. However, a polynomial in two or more variables is a bit more difficult, and sometimes impossible, to arrange in a decent order.

Example 6.2.2

Simplify the following polynomial expression, then arrange your answer in descending powers of x.

$$x^3 + 2xy^2 - 6x^2y + y^3 - 3xy^2 + 4x^2y$$

Solution

We'll again use the commutative and associative properties to change the order and regroup, putting the terms with the highest powers of x first, then follow with terms containing lower powers of x in order.

$$\begin{aligned} x^3 + 2xy^2 \ & - 6x^2y + y^3 - 3xy^2 + 4x^2y \\ &= x^3 + \left(-6x^2y + 4x^2y\right) + \left(2xy^2 - 3xy^2\right) + y^3 \\ &= x^3 - 2x^2y - xy^2 + y^3 \end{aligned}$$

Note that this is a very natural order, the powers of x decrease while simultaneously the powers of y increase.

Exercise 6.2.2

Simplify the following polynomial, and arrange your answer in descending powers of x:

$$-4x^2y^2 + 3xy^3 + 6x^3y - xy^3 + 2x^2y^2$$

Answer

$6x^3y - 2x^2y^2 + 2xy^3$

Not all examples will have nice ordering presented in Example 6.2.2, with the powers of one variable descending while the powers of the other variable simultaneously ascends. Sometimes we have to make some very subjective choices on the ordering of terms.

Example 6.2.3

Simplify the following polynomial expression, then arrange your answer in some sort of reasonable order.

$$a^3b^3 + 2a^2b - 3a^2b^3 + 4a^3b^3 + 5a^4 + 3a^2b + b^5$$

Solution

Let's try to arrange the terms so that the powers of a descend. Again, we use the commutative and associative properties to change the order and regroup.

$$\begin{aligned} a^3b^3 + 2a^2b \ & - 3a^2b^3 + 4a^3b^3 + 5a^4 + 3a^2b + b^5 \\ &= 5a^4 + \left(a^3b^3 + 4a^3b^3\right) + \left(2a^2b + 3a^2b\right) - 3a^2b^3 + b^5 \\ &= 5a^4 + 5a^3b^3 + 5a^2b - 3a^2b^3 + b^5 \end{aligned}$$

Note that in our final arrangement, the powers of a descend, but the powers of b bounce up and down, but at least we have the powers of a descending. That should help us spot if we've missed a term while simplifying the given problem.

Exercise 6.2.3

Simplify the following polynomial, and arrange your answer in ascending powers of b:

$$5a^3b^2+4ab^3-2a^2b+3a^3b^2-ab^3$$

Answer

$-2a^2b+8a^3b^2+3ab^3$

The Degree of a Polynomial

To find the degree of a polynomial, locate the term of the polynomial having the highest degree.

The degree of a polynomial

The *degree* of a polynomial is the degree of the term having the highest degree.

Finding the degree of a polynomial of a single variable is pretty easy.

Example 6.2.4

What is the degree of the polynomial $x^3-4x^2+5-6x+2x^7$?

Solution

First, let's arrange the polynomial in descending powers of x.

$$2x^7+x^3-4x^2-6x+5$$

Arranging the polynomial in descending powers of x makes it easier to see that the term of the polynomial with the highest degree is $2x^7$. Therefore, the degree of the polynomial is 7.

Exercise 6.2.4

What is the degree of the polynomial $2x^3+8x^2+3x^4+2x+10$?

Answer

4

Finding the degree of a polynomial of more than one variable is a little bit trickier.

Example 6.2.5

What is the degree of the polynomial $x^4-2x^3y^7+y^5$?

Solution

Note that the polynomial is already arranged in descending powers of x, an arrangement that is probably as good as we are going to get. In the following table, we list the degree of each term. Remember, the degree of any term is found by summing the exponents on its variables.

Term	Degree
x^4	4
$-2x^3y^7$	10
y^5	5

Hence, the term with the highest degree is $-2x^3y^7$, making 10 the degree of the polynomial.

Exercise 6.2.5

What is the degree of the polynomial $x^2y^4 - 6x^2y^2 + 5x^2y^5 - 2xy$?

Answer

7

Polynomial Functions

First we define what we mean by a polynomial function.

Polynomial function

A polynomial function is a function defined by a rule that assigns to each domain object a range object defined by a polynomial expression.

Advanced courses, such as multivariate calculus, frequently use polynomial functions of more than one variable such as $f(x, y) = x^2 + y^2$. However, in this course, our focus will be on polynomial functions of a single variable, such as $p(x) = 3 - 4x - 9x^2$ and $q(x) = x^3 - 9x^2 + 11$.

Example 6.2.6

Given the polynomial function $p(x) = x^3 - 8x - 11$, evaluate $p(-3)$.

Solution

To evaluate $p(-3)$, first restate the function definition, then replace each occurrence of the variable x with open parentheses.

$$\begin{aligned} p(x) &= x^3 - 8x - 11 && \text{Original function definition.} \\ p(\ ) &= (\ )^3 - 8(\ ) - 11 && \text{Replace each occurrence of } x \text{ with open parentheses.} \end{aligned}$$

Next, substitute -3 for x in the open parentheses prepared in the last step.

$$\begin{aligned} p(-3) &= (-3)^3 - 8(-3) - 11 && \text{Substitute } -3 \text{ for } x \text{ in the open parentheses positions.} \\ p(-3) &= -27 - 8(-3) - 11 && \text{Exponent first: } (-3)^3 = -27 \\ p(-3) &= -27 + 24 - 11 && \text{Multiply: } -8(-3) = 24 \\ p(-3) &= -14 && \text{Add.} \end{aligned}$$

Hence, $p(-3) = -14$. You can easily check this result on your calculator (see Figure 6.2.1).

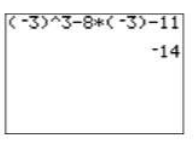


Figure 6.2.1: Calculator check.

Exercise 6.2.6

Given the polynomial function $p(x) = -3x^2 + 7x + 4$, evaluate $p(2)$.

Answer

6

The Graph of a Polynomial Function

One of the most important polynomial functions in all of mathematics and science is the polynomial having degree two.

Quadratic polynomial

The second degree polynomial having the form

$$p(x) = ax^2 + bx + c$$

is called a *quadratic polynomial*. The graph of this polynomial is called a *parabola*.

The parabola is approximately U-shaped. Some open upwards, some open downwards, depending on the sign of the leading term.

In Figure 6.2.2, the leading term of the parabola $p(x) = 2x^2 - 8x + 6$ has positive two as its coefficient, so it opens upward.

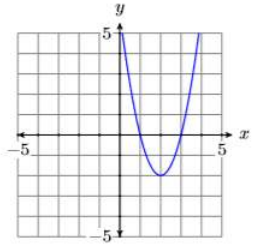


Figure 6.2.2: The graph of $p(x) = 2x^2 - 8x + 6$ opens up.

In Figure 6.2.3, the leading term of the parabola $p(x) = -2x^2 - 8x - 6$ has negative two as its coefficient, so it opens downward.

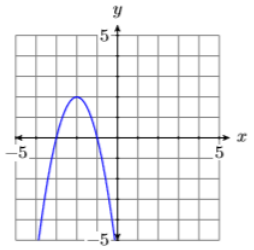


Figure 6.2.3: The graph of $p(x) = -2x^2 - 8x - 6$ opens down.

Note

The sign of the leading term of $p(x) = ax^2 + bx + c$ determines whether the parabola opens up or down.

- If $a > 0$, the parabola opens upward.
- If $a < 0$, the parabola opens downward.

The turning point of a parabola has a special name.

The vertex of a parabola

The graph of the second degree polynomial $p(x) = ax^2 + bx + c$ has a single turning point, called the *vertex* of the parabola.

Example 6.2.7

Use your graphing calculator to sketch the graph of the quadratic polynomial $p(x) = -3x^2 + 12x + 25$.

Solution

The degree of the polynomial $p(x) = -3x^2 + 12x + 25$ is two, so it is a quadratic polynomial and its graph is a parabola. Moreover, its leading term has negative three as its coefficient, so we know that the parabola opens downward. Enter $y = -3x^2 + 12x + 25$ as $Y1 = -3 * X \wedge 2 + 12 * X + 25$ in the **Y= menu** (see the first image in Figure 6.2.4), then select **6:ZStandard** from the **ZOOM** menu to produce the third image in Figure 6.2.4.

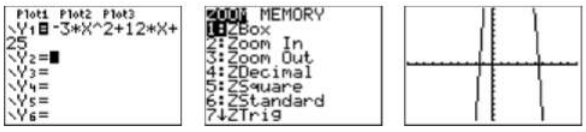


Figure 6.2.4: Sketching the graph of $p(x) = -3x^2 + 12x + 25$.

Note that the graph in Figure 6.2.4 appears to have the U-shape of a parabola that opens downwards. Its vertex (turning point) is not visible, but one would surmise that it lies off the top of the screen. We need to adjust the **WINDOW** parameters so that the vertex of the parabola is visible in the viewing screen. After some experimentation, we settle on the parameters shown in the first image in Figure 6.2.5, then push the **GRAPH** button to produce the second image in Figure 6.2.5.

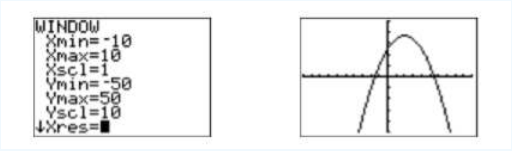
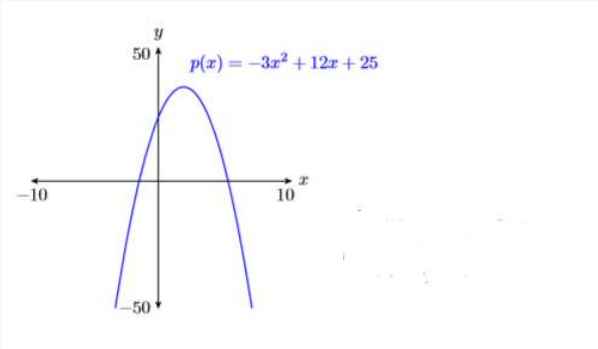


Figure 6.2.5: Adjust the **WINDOW** parameters so that the vertex is visible in the viewing screen.

In reporting your result on your homework, follow the *Calculator Submission Guidelines* from Chapter 3, Section2.

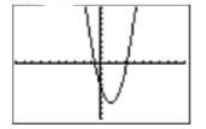


1. Draw axes with a ruler.
2. Label the horizontal axis x and the vertical axis y.
3. Indicate the **WINDOW** parameters Xmin, Xmax, Ymin, and Ymax\) at the end of each axis.
4. Freehand the curve and label it with its equation.

Exercise 6.2.7

Use your graphing calculator to sketch the graph of the quadratic polynomial $p(x) = 2x^2 - 5x - 4$.

Answer



When the degree of the polynomial is larger than two, the number of turning points of the graph might increase. This makes for some very interesting curves. In more advanced courses, such as intermediate and college algebra, you will be introduced to a variety of techniques that will help you determine appropriate viewing windows for the graphs of these higher degree polynomials. However, in this introductory section, we will assist you by suggesting a good viewing window for each polynomial, one that will allow you to see all of the turning points of the graph of the polynomial.

Example 6.2.8

Use your graphing calculator to sketch the graph of the polynomial function $p(x) = x^4 - 37x^2 + 24x + 180$. Set your window parameters as follows: $\mathbf{X}\min = -10, \mathbf{X}\max = 10, \mathbf{X}\text{scl} = 1, \mathbf{Y}\min = -1000, \mathbf{Y}\max = 1000,$ and $\mathbf{Y}\text{scl} = 100.$

Solution

Enter the polynomial function in $\mathbf{Y1}$ of the **Y= menu**, then enter the suggested window parameters in the **WINDOW** menu (see Figure 6.2.6).

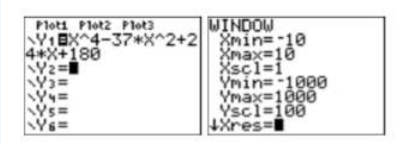


Figure 6.2.6: Enter the polynomial and adjust the WINDOW parameters.

Push the **GRAPH** button on the top row of your calculator to produce the graph of the polynomial function shown in Figure 6.2.7.

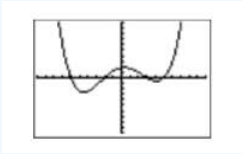


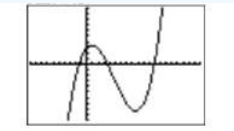
Figure 6.2.7: The graph of $p(x) = x^4 - 37x^2 + 24x + 180$.

Sweet-looking curve!

Exercise 6.2.8

Use your graphing calculator to sketch the graph of the quadratic polynomial p(x)=x3 −14x2 + 20x+ 60. Set your window parameters as follows: $\mathbf{X}\min = -10, \mathbf{X}\max = 20, \mathbf{X}\text{scl} = 1, \mathbf{Y}\min = -200, \mathbf{Y}\max = 200,$ and $\mathbf{Y}\text{scl} = 20.$

Answer



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6.3: Applications of Polynomials

In this section we investigate real-world applications of polynomial functions.

Example 6.3.1

The average price of a gallon of gas at the beginning of each month for the period starting in November 2010 and ending in May 2011 are given in the margin. The data is plotted in Figure 6.3.1 and fitted with the following third degree polynomial, where t is the number of months that have passed since October of 2010.

$$p(t) = -0.0080556t^3 + 0.11881t^2 - 0.30671t + 3.36 \tag{6.3.1}$$

Use the graph and then the polynomial to estimate the price of a gallon of gas in California in February 2011.



Figure 6.3.1: Fitting gas price versus month with a cubic polynomial.

Month	Price
Nov.	3.14
Dec.	3.21
Jan	3.31
Mar.	3.87
Apr.	4.06
May	4.26

Solution

Locate February ($t = 4$) on the horizontal axis. From there, draw a vertical arrow up to the graph, and from that point of intersection, a second horizontal arrow over to the vertical axis (see Figure 6.3.2). It would appear that the price per gallon in February was approximately $\$3.51$.

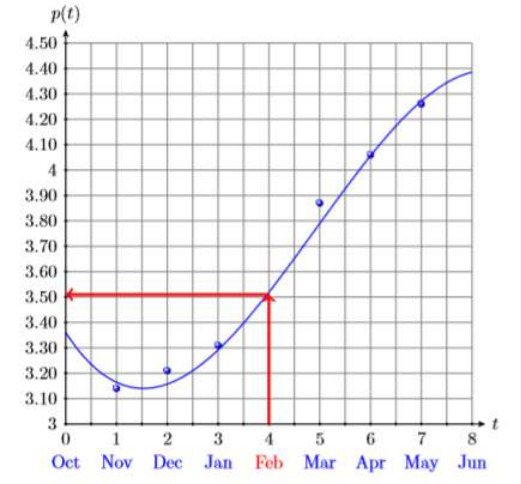


Figure 6.3.2: Approximating price of gas during February.

Next, we'll use the fitted third degree polynomial to approximate the price per gallon for the month of February, 2011. Start with the function defined by Equation 6.3.1 and substitute 4 for t.

$$\begin{aligned} p(t) &= -0.0080556t^3 + 0.11881t^2 - 0.30671t + 3.36 \\ p(4) &= -0.0080556(4)^3 + 0.11881(4)^2 - 0.30671(4) + 3.36 \end{aligned}$$

Use the calculator to evaluate $p(4)$ (see Figure 6.3.3). Rounding to the nearest

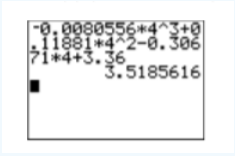


Figure 6.3.3: Evaluating $p(4)$.

penny, the price in February was $\$3.52$ per gallon.

Example 6.3.2

If a projectile is fired into the air, its height above ground at any time is given by the formula

$$y = y_0 + v_0 t - \frac{1}{2} g t^2 \tag{6.3.2}$$

where

y = height above ground at time t

y_0 = initial height above ground at time $t = 0$

v_0 = initial velocity at time $t = 0$

g = acceleration due to gravity

t = time passed since projectile's firing

If a projectile is launched with an initial velocity of 100 meters per second (100m/s) from a rooftop 8 meters (8m) above ground level, at what time will the projectile first reach a height of 400 meters (400m)? Note: Near the earth's surface, the acceleration due to gravity is approximately 9.8 meters per second per second (9.8(m/s)/s or 9.8m/s^2).

Solution

We're given the initial height is $y_0 = 8\,\text{m}$, the initial velocity is $v_0 = 100\,\text{m/s}$, and the acceleration due to gravity is $g = 9.8\,\text{m/s}^2$. Substitute these values in Equation 6.3.2, then simplify to produce the following result:

$$y = y_0 + v_0 t - \frac{1}{2} g t^2$$
$$y = 8 + 100t - \frac{1}{2}(9.8)t^2$$
$$y = 8 + 100t - 4.9t^2$$

Enter $y = 8 + 100t - 4.9t^2$ as $\mathbf{Y1 = 8 + 100^{*}X - 4.9^{*}X \wedge 2}$ in the **Y= menu** (see the first image in Figure 6.3.4). After some experimentation, we settled on the **WINDOW** parameters shown in the second image in Figure 6.3.4. Push the GRAPH button to produce the graph of $y = 8 + 100t - 4.9t^2$ shown in the third image Figure 6.3.4.

In this example, the horizontal axis is actually the t-axis. So when we set Xmin and Xmax, we're actually setting bounds on the t-axis.

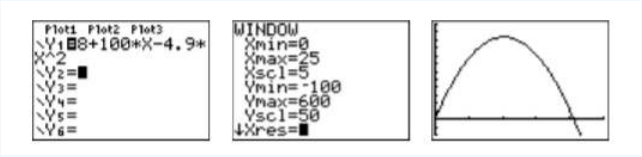


Figure 6.3.4: Sketching the graph of $y = 8 + 100t - 4.9t^2$.

To find when the projectile reaches a height of 400 meters (400m), substitute 400 for y to obtain:

$$400 = 8 + 100t - 4.9t^2 \tag{6.3.3}$$

Enter the left-hand side of Equation 6.3.3 into $\mathbf{Y2}$ in the **Y= menu**, as shown in the first image in Figure 6.3.5. Push the **GRAPH** button to produce the result shown in the second image in Figure 6.3.5. Note that there are two points of intersection, which makes sense as the projectile hits 400 meters on the way up and 400 meters on the way down.

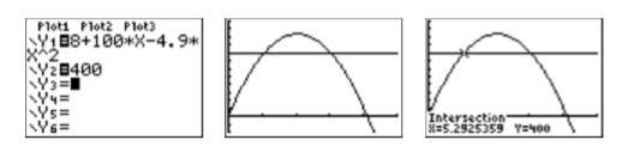


Figure 6.3.5: Determining when the object first reaches 400 meters.

To find the first point of intersection, select **5:intersect** from the **CALC** menu. Press **ENTER** in response to "First curve," then press ENTER again in response to "Second curve." For your guess, use the arrow keys to move the cursor closer to the first point of intersection than the second. At this point, press **ENTER** in response to "Guess." The result is shown in the third image in Figure 6.3.5. The projectile first reaches a height of 400 meters at approximately 5.2925359seconds after launch.

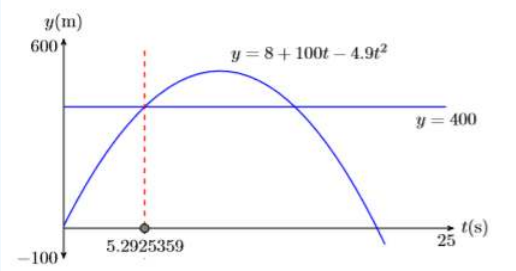


Figure 6.3.6: Reporting your graphical solution on your homework.

The parabola shown in Figure 6.3.6 is not the actual flight path of the projectile. The graph only predicts the height of the projectile as a function of time.

Reporting the solution on your homework: Duplicate the image in your calculator's viewing window on your homework page. Use a ruler to draw all lines, but freehand any curves.

- Label the horizontal and vertical axes with t and y, respectively (see Figure 6.3.6). Include the units (seconds (s) and meters (m)).
- Place your **WINDOW** parameters at the end of each axis (see Figure 6.3.6). Include the units (seconds (s) and meters (m)).

- Label each graph with its equation (see Figure 6.3.6).
- Draw a dashed vertical line through the first point of intersection. Shade and label the point (with its t-value) where the dashed vertical line crosses the t-axis. This is the first solution of the equation $400 = 8 + 100t - 4.9t^2$ (see Figure 6.3.6).

Rounding to the nearest tenth of a second, it takes the projectile approximately $t \approx 5.3$ seconds to first reach a height of 400 meters.

The phrase "shade and label the point" means fill in the point on the t-axis, then write the t-value of the point just below the shaded point.

Exercise 6.3.2

If a projectile is launched with an initial velocity of 60 meters per second from a rooftop 12 meters above ground level, at what time will the projectile first reach a height of 150 meters?

Answer

≈ 3.0693987seconds

Zeros and x-intercepts of a Function

Recall that $f(x)$ and y are interchangeable. Therefore, if we are asked to find where a function is equal to zero, then we need to find the points on the graph of the function that have a y-value equal to zero (see Figure 6.3.7).

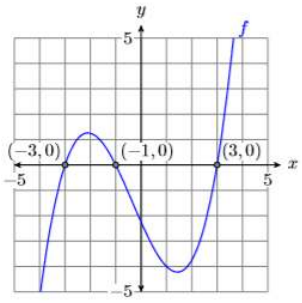


Figure 6.3.7: Locating the zeros of a function.

Zeros and x-intercepts

The points where the graph off crosses the x-axis are called the x-intercepts of the graph of f. The x-value of each x-intercept is called a zero of the function f.

The graph of f crosses the x-axis in Figure 6.3.7 at $(-3, 0)$, $(-1, 0)$, and $(3, 0)$. Therefore:

- The x-intercepts of f are: $(-3, 0)$, $(-1, 0)$, and $(3, 0)$
- The zeros of f are: -3, -1, and 3

Key idea

A function is zero where its graph crosses the x-axis.

Example 6.3.3

Find the zero(s) of the function $f(x) = 1.5x + 5.25$.

Solution

Remember, $f(x) = 1.5x + 5.25$ and $y = 1.5x + 5.25$ are equivalent. We're looking for the value of x that makes $y = 0$ or $f(x) = 0$. So, we'll start with $f(x) = 0$, then replace $f(x)$ with $1.5x + 5.25$.

$$\begin{aligned} f(x) &= 0 && \text{We want the value of } x \text{ that makes the function equal to zero.} \\ 1.5x + 5.25 &= 0 && \text{Replace } f(x) \text{ with } 1.5x + 5.25 \end{aligned}$$

Now we solve for x.

$$\begin{aligned} 1.5x &= -5.25 && \text{Subtract } 5.25 \text{ from both sides.} \\ x &= \frac{-5.25}{1.5} && \text{Divide both sides by } 1.5 \\ x &= -3.5 && \text{Divide: } -5.25/1.5 = -3.5 \end{aligned}$$

Check: Substitute -3.5 for x in the function $f(x) = 1.5x + 5.25$.

$$\begin{aligned} f(x) &= 1.5x + 5.25 && \text{The original function.} \\ f(-3.5) &= 1.5(-3.5) + 5.25 && \text{Substitute } -3.5 \text{ for } x \\ f(-3.5) &= -5.25 + 5.25 && \text{Multiply: } 1.5(-3.5) = -5.25 \\ f(-3.5) &= 0 && \text{Add.} \end{aligned}$$

Note that -3.5, when substituted for x, makes the function $f(x) = 1.5x + 5.25$ equal to zero. This is why -3.5 is called a zero of the function.

Graphing calculator solution: We should be able to find the zero by sketching the graph of f and noting where it crosses the x-axis. Start by loading the function $f(x) = 1.5x + 5.25$ into $\mathbf{Y1}$ in the **Y= menu** (see the first image in Figure 6.3.8).

Select **6:ZStandard** from the **ZOOM** menu to produce the graph of f (see the second image in Figure 6.3.8). Press **2ND CALC** to open the the **CALCULATE** menu (see the third image in Figure 6.3.8). To find the zero of the function f:

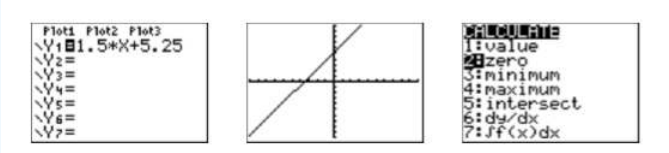


Figure 6.3.8: Finding the zero of $f(x) = 1.5x + 5.25$.

1. Select **2:zero** from the **CALCULATE** menu. The calculator responds by asking for a "Left Bound?" (see the first image in Figure 6.3.9). Use the left arrow button to move the cursor so that it lies to the left of the x-intercept of f and press **ENTER**.
2. The calculator responds by asking for a "Right Bound?" (see the second image in Figure 6.3.9). Use the right arrow button to move the cursor so that it lies to the right of the x-intercept of f and press **ENTER**.
3. The calculator responds by asking for a "Guess?" (see the third image in Figure 6.3.9). As long as your cursor lies between the left- and right-bound marks at the top of the screen (see the third image in Figure 6.3.9), you have a valid guess. Since the cursor already lies between the left- and right-bounds, simply press **ENTER** to use the current position of the cursor as your guess.

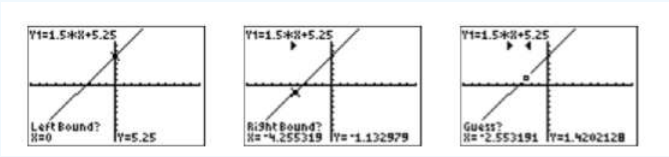


Figure 6.3.9: Using **2:zero** from the **CALCULATE** menu.

The calculator responds by approximating the zero of the function as shown in Figure 6.3.10.

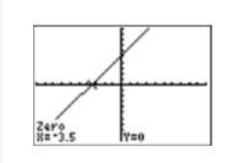


Figure 6.3.10: -3.5 is a zero off.

Note that the approximation found using the calculator agrees nicely with the zero found using the algebraic technique.

Exercise 6.3.3

Find the zero(s) of the function $f(x) = 2.6x - 9.62$.

Answer

3.7

Example 6.3.4

How long will it take the projectile in Example 6.3.2 to return to ground level?

Solution

In Example 6.3.2, the height of the projectile above the ground as a function of time is given by the equation

$$y = 8 + 100t - 4.9t^2$$

When the projectile returns to the ground, its height above ground will be zero meters. To find the time that this happens, substitute $y = 0$ in the last equation and solve for t.

$$0 = 8 + 100t - 4.9t^2$$

Enter the equation $y = 8 + 100t - 4.9t^2$ into $\mathbf{Y1}$ in the **Y= menu** of your calculator (see the first image in Figure 6.3.11), then set the **WINDOW** parameters shown in the second image in Figure 6.3.11. Push the **GRAPH** button to produce the graph of the function shown in the third image in Figure 6.3.11.

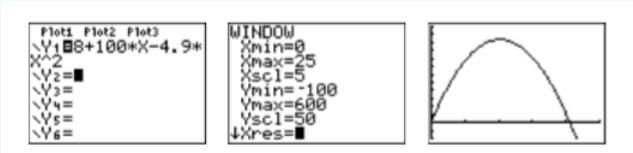


Figure 6.3.11: Sketching the graph of $y = 8 + 100t - 4.9t^2$.

In this example, the horizontal axis is actually the t-axis. So when we set Xmin and Xmax, we're actually setting bounds on the t-axis.

To find the time when the projectile returns to ground level, we need to find where the graph of $y = 8 + 100t - 4.9t^2$ crosses the horizontal axis (in this case the t-axis) . Select **2:zero** from the **CALC** menu. Use the arrow keys to move the cursor slightly to the left of the t-intercept, then press **ENTER** in response to "Left bound." Move your cursor slightly to the right of the t-intercept, then press **ENTER** in response to "Right bound." Leave your cursor where it is and press **ENTER** in response to "Guess." The result is shown in Figure 6.3.11.

- Label the horizontal and vertical axes with t and y, respectively (see Figure 6.3.12). Include the units (seconds (s) and meters (m)).
- Place your **WINDOW** parameters at the end of each axis (see Figure 6.3.12).
- Label the graph with its equation (see Figure 6.3.12).
- Draw a dashed vertical line through the t-intercept. Shade and label the t-value of the point where the dashed vertical line crosses the t-axis. This is the solution of the equation $0 = 8 + 100t - 4.9t^2$ (see Figure 6.3.12).

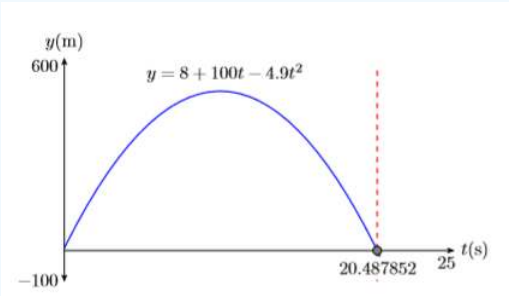


Figure 6.3.12: Reporting your graphical solution on your homework.

Rounding to the nearest tenth of a second, it takes the projectile approximately $t \approx 20.5$ seconds to hit the ground.

Exercise 6.3.4

If a projectile is launched with an initial velocity of 60 meters per second from a rooftop 12 meters above ground level, at what time will the projectile return to ground level?

Answer

≈ 12.441734seconds

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6.4: Adding and Subtracting Polynomials

In this section we concentrate on adding and subtracting polynomial expressions, based on earlier work combining like terms in Ascending and Descending Powers. Let's begin with an addition example.

Example 6.4.1

Simplify:

$$\left(a^2+3ab-b^2\right)+\left(4a^2+11ab-9b^2\right)$$

Solution

Use the commutative and associative properties to change the order and regroup. Then combine like terms.

$$\begin{aligned}\left(a^2+3ab-b^2\right) &+\left(4a^2+11ab-9b^2\right)\\ &=\left(a^2+4a^2\right)+(3ab+11ab)+\left(-b^2-9b^2\right)\\ &=5a^2+14ab-10b^2\end{aligned}$$

Exercise 6.4.1

Simplify: $\left(3s^2-2st+4t^2\right)+\left(s^2+7st-5t^2\right)$

Answer

$4s^2+5st-t^2$

Let's combine some polynomial functions.

Example 6.4.2

Given $f(x)=3x^2-4x-8$ and $g(x)=x^2-11x+15$, simplify $f(x)+g(x)$.

Solution

First, replace $f(x)$ and $g(x)$ with their definitions. Be sure to surround each polynomial with parentheses, because we are asked to add all of $f(x)$ to all of $g(x)$.

$$f(x)+g(x)=(3x^2-4x-8)+(x^2-11x+15)$$

Now use the commutative and associative properties to change the order and regroup. Combine like terms.

$$\begin{aligned}&=\left(3x^2+x^2\right)+(-4x-11x)+(-8+15)\\ &=4x^2-15x+7\end{aligned}$$

Hence, $f(x)+g(x)=4x^2-15x+7$.

Exercise 6.4.2

Given $f(x)=2x^2+9x-5$ and $g(x)=-x^2-4x+3$, simplify $f(x)+g(x)$.

Answer

x^2+5x-2

If you are comfortable skipping a step or two, it is not necessary to write down all of the steps shown in Examples 6.4.1 and 6.4.2. Let's try combining like terms mentally in the next example.

Example 6.4.3

Simplify:

$$\left(x^3-2x^2y+3xy^2+y^3\right)+\left(2x^3-4x^2y-8xy^2+5y^3\right)$$

Solution

If we use the associative and commutative property to reorder and regroup, then combine like terms, we get the following result.

$$\begin{aligned}\left(x^3-2x^2y\right.&\left.+3xy^2+y^3\right)+\left(2x^3-4x^2y-8xy^2+5y^3\right)\\&=\left(x^3+2x^3\right)+\left(-2x^2y-4x^2y\right)+\left(3xy^2-8xy^2\right)+\left(y^3+5y^3\right)\\&=3x^3-6x^2y-5xy^2+6y^3\end{aligned}$$

However, if we can combine like terms mentally, eliminating the middle step, it is much more efficient to write:

$$\begin{aligned}&\left(x^3-2x^2y+3xy^2+y^3\right)+\left(2x^3-4x^2y-8xy^2+5y^3\right)\\&\quad=3x^3-6x^2y-5xy^2+6y^3\end{aligned}$$

Exercise 6.4.3

Simplify: $\left(-5a^2b+4ab-3ab^2\right)+\left(2a^2b+7ab-ab^2\right)$

Answer

$-3a^2b+11ab-4ab^2d$

Negating a Polynomial

Before attempting subtraction of polynomials, let's first address how to negate or "take the opposite" of a polynomial. First recall that negating is equivalent to multiplying by -1.

Negating

If a is any number, then

$$-a=(-1)a.$$

That is, negating is equivalent to multiplying by -1.

We can use this property to simplify $-(a+b)$. First, negating is identical to multiplying by -1. Then we can distribute the -1.

$$\begin{aligned}-(a+b)&=(-1)(a+b)\quad\text{Negating is equivalent to multiplying by }-1\\&=(-1)a+(-1)b\quad\text{Distribute the }-1.\\&=-a+(-b)\quad\text{Simplify: }(-1)a=-a\text{ and }(-1)b=-b\\&=-a-b\quad\text{Subtraction means add the opposite.}\end{aligned}$$

Thus, $-(a+b)=-a-b$. However, it is probably simpler to note that the minus sign in front of the parentheses simply changed the sign of each term inside the parentheses.

Negating a sum

When negating a sum of terms, the effect of the minus sign is to change each term in the parentheses to the opposite sign.

$$-(a+b)=-a-b$$

Let's look at this principle in the next example.

Example 6.4.4

Simplify: $-\left(-3x^2+4x-8\right)$

Solution

First, negating is equivalent to multiplying by -1. Then distribute the -1.

$$\begin{aligned}&-\left(-3x^2+4x-8\right)\\&=(-1)\left(-3x^2+4x-8\right) \quad \text{Negating is equivalent to multiplying by } -1\\&=(-1)\left(-3x^2\right)+(-1)(4x)-(-1)(8) \quad \text{Distribute the } -1\\&=3x^2+(-4x)-(-8) \quad \text{Simplify: } (-1)(-3x^2)=3x^2, (-1)(4x)=-4x, \text{and} (-1)(8)=-8\\&=3x^2-4x+8 \quad \text{Subtraction means add the opposite.}\end{aligned}$$

Alternate solution:

As we saw above, a negative sign in front of a parentheses simply changes the sign of each term inside the parentheses. So it is much more efficient to write

$$-(-3x^2+4x-8)=3x^2-4x+8$$

simply changing the sign of each term inside the parentheses.

Exercise 6.4.4

Simplify: $-\left(2x^2-3x+9\right)$

Answer

$-2x^2+3x-9$

Subtracting Polynomials

Now that we know how to negate a polynomial (change the sign of each term of the polynomial), we're ready to subtract polynomials.

Example 6.4.5

Simplify: $\left(y^3-3y^2z+4yz^2+z^3\right)-\left(2y^3-8y^2z+2yz^2-8z^3\right)$

Solution

First, distribute the minus sign, changing the sign of each term of the second polynomial.

$$\left(y^3-3y^2z+4yz^2+z^3\right)-\left(2y^3-8y^2z+2yz^2-8z^3\right)=y^3-3y^2z+4yz^2+z^3-2y^3+8y^2z-2yz^2+8z^3$$

Regroup, combining like terms. You may perform this next step mentally if you wish.

$$\begin{aligned}&=\left(y^3-2y^3\right)+\left(-3y^2z+8y^2z\right)+\left(4yz^2-2yz^2\right)+\left(z^3+8z^3\right)\\&=-y^3+5y^2z+2yz^2+9z^3\end{aligned}$$

Exercise 6.4.5

Simplify: $\left(4a^2b+2ab-7ab^2\right)-\left(2a^2b-ab-5ab^2\right)$

Answer

$2a^2b+3ab-2ab^2$

Let's subtract two polynomial functions.

Example 6.4.6

Given $p(x) = -5x^3 + 6x - 9$ and $q(x) = 6x^2 - 7x - 11$, simplify $p(x) - q(x)$.

Solution

First, replace $p(x)$ and $q(x)$ with their definitions. Because we are asked to subtract all of $q(x)$ from all of $p(x)$, it is critical to surround each polynomial with parentheses.

$$p(x) - q(x) = \left(-5x^3 + 6x - 9\right) - \left(6x^2 - 7x - 11\right)$$

Distribute the minus sign, changing the sign of each term in the second polynomial, then regroup and combine like terms.

$$\begin{aligned} &= -5x^3 + 6x - 9 - 6x^2 + 7x + 11 \\ &= -5x^3 - 6x^2 + (6x + 7x) + (-9 + 11) \\ &= -5x^3 - 6x^2 + 13x + 2 \end{aligned}$$

However, after distributing the minus sign, if we can combine like terms mentally, eliminating the middle step, it is much more efficient to write:

$$\begin{aligned} p(x) - q(x) &= \left(-5x^3 + 6x - 9\right) - \left(6x^2 - 7x - 11\right) \\ &= -5x^3 + 6x - 9 - 6x^2 + 7x + 11 \\ &= -5x^3 - 6x^2 + 13x + 2 \end{aligned}$$

Exercise 6.4.6

Given $f(x) = 3x^2 + 9x - 4$ and $g(x) = -5x2 + 4x - 6$, simplify $f(x) - g(x)$.

Answer

$8x^2 + 5x + 2$

Some Applications

Recall that the area of a rectangle having length L and width W is found using the formula $A = LW$. The area of a square having side s is found using the formula $A = s^2$ (see Figure 6.4.1).

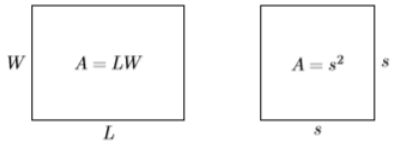


Figure 6.4.1: Area formulae for the rectangle and square.

Example 6.4.7

Find the area of the square in Figure 6.4.2 by summing the area of its parts.

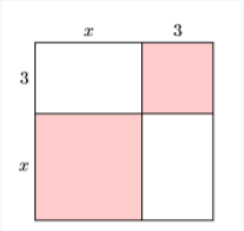


Figure 6.4.2: Find the sum of the parts.

Solution

Let's separate each of the four pieces and label each with its area (see Figure 6.4.3).

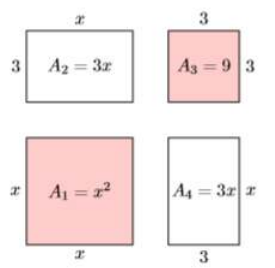


Figure 6.4.3: Finding the area of each of the four parts.

The two shaded squares in Figure 6.4.3 have areas $A_1 = x^2$ and $A_3 = 9$, respectively. The two unshaded rectangles in Figure 6.4.3 have areas $A_2 = 3x$ and $A_4 = 3x$. Summing these four areas gives us the area of the entire figure.

$$\begin{aligned} A &= A_1 + A_2 + A_3 + A_4 \\ &= x^2 + 3x + 9 + 3x \\ &= x^2 + 6x + 9 \end{aligned}$$

Exercise 6.4.7

Find the area of the square shown below by summing the area of its parts.

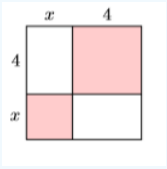


Figure 6.4.4

Answer

$x^2 + 8x + 16$

Example 6.4.8

Ginger runs a business selling wicker baskets. Her business costs for producing and selling x wicker baskets are given by the polynomial function $C(x) = 100 + 3x - 0.02x^2$. The revenue she earns from selling x wicker baskets is given by the polynomial function $R(x) = 2.75x$. Find a formula for $P(x)$, the profit made from selling x wicker baskets. Use your formula to determine Ginger's profit if she sells 123 wicker baskets.

Solution

The profit made from selling x wicker baskets is found by subtracting the costs incurred from the revenue received. In symbols:

$$P(x) = R(x) - C(x)$$

Next, replace $R(x)$ and $C(x)$ with their definitions. Because we are supposed to subtract all of the cost from the revenue, be sure to surround the cost polynomial with parentheses.

$$P(x) = 2.75x - (100 + 3x - 0.02x^2)$$

Distribute the minus sign and combine like terms.

$$= 2.75x - 100 - 3x + 0.02x^2$$
$$= 0.02x^2 - 0.25x - 100$$

Thus, the profit function is $P(x) = 0.02x^2 - 0.25x - 100$.

Next, to determine the profit if 123 wicker baskets are sold, substitute 123 for x in the profit function $P(x)$.

$$\begin{aligned} P(x) &= 0.02x^2 - 0.25x - 100 \\ P(123) &= 0.02(123)^2 - 0.25(123) - 100 \end{aligned}$$

You can now use your graphing calculator to determine the profit (see Figure $6.4.5$). Hence, the profit made from selling 123 wicker baskets is $\$171.83$

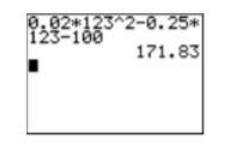


Figure $6.4.5$: Determining the profit from selling 123 wicker baskets.

Exercise $6.4.8$

The costs for producing and selling x widgets are given by the polynomial function $C(x) = 50 + 5x - 0.5x^2$, and the revenue for selling x widgets is given by the polynomial function $R(x) = 3.5x$. Determine the profit if 75 widgets are sold.

Answer

$\$2,650$

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6.5: Laws of Exponents

In Chapter 1, section 1, we first introduced the definition of an exponent. For convenience, we repeat that definition.

In the exponential expression a^n, the number a is called the base, while the number n is called the exponent.

Exponents

Let a be any real number and let n be any whole number. If $n \neq 0$, then:

$$a^n = \underbrace{a \cdot a \cdot a \cdot \cdots \cdot a}_{n \text{ times}}$$

That is, to calculate a^n, write a as a factor n times. In the case where $a \neq 0$, but $n = 0$, then we define:

$$a^0 = 1$$

For example, raising a number to the fifth power requires that we repeat the number as a factor five times (see Figure 6.5.1).

$$\begin{aligned}(-2)^5 &= (-2)(-2)(-2)(-2)(-2) \\ &= -32\end{aligned}$$

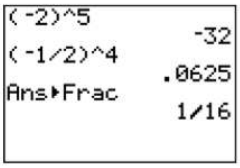


Figure 6.5.1: Evaluating $(-2)^5$ and $(-1/2)^4$. Recall: Use the **MATH** key to locate the ▶**Frac** command.

Raising a number to the fourth power requires that we repeat that number as a factor four times (see Figure 6.5.1).

$$\begin{aligned}\left(-\frac{1}{2}\right)^4 &= \left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right) \\ &= \frac{1}{16}\end{aligned}$$

As a final example, note that $10^0 = 1$, but 0^0 is undefined (see Figure 6.5.2).



Figure 6.5.2: Evaluating 10^0 and 0^0 on the graphing calculator.

Note

For those who may be wondering why $a^0 = 1$, provided $a \neq 0$, here is a nice argument. First, note that $a^1 = a$, so:

$$a \cdot a^0 = a^1 \cdot a^0$$

On the right, repeat the base and add the exponents.

$$a \cdot a^0 = a^1$$

Or equivalently:

$$a \cdot a^0 = a$$

Now, divide both sides by a, which is permissible if $a \neq 0$.

$$\frac{a \cdot a^0}{a} = \frac{a}{a}$$

Simplify both sides:

$$a^0 = 1$$

Multiplying With Like Bases

In the expression a^n, the number a is called the base and the number n is called the *exponent*. Frequently, we'll be required to multiply two exponential expressions with like bases, such as $x^3 \cdot x^4$. Recall that the exponent tells us how many times to write each base as a factor, so we can write:

$$\begin{aligned} x^3 \cdot x^4 &= (x \cdot x \cdot x) \cdot (x \cdot x \cdot x \cdot x) \\ &= x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \\ &= x^7 \end{aligned}$$

Note that we are simply counting the number of times that x occurs as a factor. First, we have three $x's$, then four $x's$, for a total of seven $x's$. However a little thought tells us that it is much quicker to simply add the exponents to reveal the total number of times x occurs as a factor.

$$\begin{aligned} x^3 \cdot x^4 &= x^{3+4} \\ &= x^7 \end{aligned}$$

The preceding discussion is an example of the following general law of exponents.

Multiplying With Like Bases

To multiply two exponential expressions with like bases, repeat the base and add the exponents.

$$a^m \cdot a^n = a^{m+n}$$

Example 6.5.1

Simplify each of the following expressions:

a. $y^4 \cdot y^8$
b. $2^3 \cdot 2^5$
c. $(x+y)^2(x+y)^7$

Solution

In each example we have like bases. Thus, the approach will be the same for each example: repeat the base and add the exponents.

a. $\begin{aligned} y^4 \cdot y^8 &= y^{4+8} \\ &= y^{12} \end{aligned}$

b. $\begin{aligned} 2^3 \cdot 2^5 &= 2^{3+5} \\ &= 2^8 \end{aligned}$

c. $\begin{aligned} (x+y)^2(x+y)^7 &= (x+y)^{2+7} \\ &= (x+y)^9 \end{aligned}$

With a little practice, each of the examples can be simplified mentally. Repeat the base and add the exponents in your head: $y^4 \cdot y^8 = y^{12}, 2^3 \cdot 2^5 = 2^8$ and $(x+y)^2(x+y)^7 = (x+y)^9$.

Exercise 6.5.1

$3^4 \cdot 3^2$

Answer

3^6

Example 6.5.2

Simplify: $\left(a^6b^4\right)\left(a^3b^2\right)$

Solution

We'll use the commutative and associative properties to change the order of operation, then repeat the common bases and add the exponents.

$$\begin{aligned}\left(a^6b^4\right)\left(a^3b^2\right) &= a^6b^4a^3b^2 \quad \text{The associative property allows us to regroup in the order we prefer.}\\ &= a^6a^3b^4b^2 \quad \text{The commutative property allows us to change the order of multiplication.}\\ &= a^9b^6 \quad \text{Repeat the common bases and add the exponents.}\end{aligned}$$

With practice, we realize that if all of the operators are multiplication, then we can multiply in the order we prefer, repeating the common bases and adding the exponents mentally: $\left(a^6b^4\right)\left(a^3b^2\right) = a^9b^6$.

Exercise 6.5.2

$\left(x^2y^6\right)\left(x^4y^3\right)$

Answer

x^6y^9

Example 6.5.3

Simplify: $x^{n+3}\cdot x^{3-2n}$

Solution

Again, we repeat the base and add the exponents.

$$\begin{aligned}x^{n+3}\cdot x^{3-2n} &= x^{(n+3)+(3-2n)} \quad \text{Repeat the base, add the exponents.}\\ &= x^{6-n} \quad \text{Simplify. Combine like terms.}\end{aligned}$$

Exercise 6.5.3

$x^{5-n}\cdot x^{4n+2}$

Answer

x^{3n+7}

Dividing With Like Bases

Like multiplication, we will also be frequently asked to divide exponential expressions with like bases, such as x^7/x^4. Again, the key is to remember that the exponent tells us how many times to write the base as a factor, so we can write:

$$\begin{aligned}\frac{x^7}{x^4} &= \frac{x\cdot x\cdot x\cdot x\cdot x\cdot x\cdot x}{x\cdot x\cdot x\cdot x}\\ &= \frac{\cancel{x}\cdot\ \cancel{x}\cdot\ \cancel{x}\cdot\ \cancel{x}\cdot x\cdot x\cdot x}{\cancel{x}\cdot\ \cancel{x}\cdot\ \cancel{x}\cdot\ \cancel{x}}\\ &= x^3\end{aligned}$$

Note how we cancel four $x's$ in the numerator for four $x's$ in the denominator. However, in a sense we are "subtracting four $x's$" from the numerator, so a faster way to proceed is to repeat the base and subtract the exponents, as follows:

$$\begin{aligned}\frac{x^7}{x^4} &= x^{7-4}\\ &= x^3\end{aligned}$$

The preceding discussion is an example of the second general law of exponents.

How to Divide with Like Bases

To divide two exponential expressions with like bases, repeat the base and subtract the exponents. Given $a \neq 0$,

$$\frac{a^m}{a^n} = a^{m-n}$$

Note that the subtraction of the exponents follows the rule "top minus bottom."

Note

Here is another nice argument why $a^0 = 1$, provided $a \neq 0$. Start with:

$$\frac{a^1}{a^1} = 1$$

Repeat the base and subtract the exponents.

$$a^{1-1} = 1$$

Simplify.

$$a^0 = 1$$

Example 6.5.4

Simplify each of the following expressions:

a. $\frac{x^{12}}{x^3}$

b. $\frac{5^7}{5^7}$

c. $\frac{(2x+1)^8}{(2x+1)^3}$

Solution

In each example we have like bases. Thus, the approach will be the same for each example: repeat the base and subtract the exponents.

a. $\frac{x^{12}}{x^3} = x^{12-3}$
$= x^9$

b. $\frac{5^7}{5^7} = 5^{7-7}$
$= 5^0$
$= 1$

c. $\frac{(2x+1)^8}{(2x+1)^3} = (2x+1)^{8-3}$
$= (2x+1)^5$

With a little practice, each of the examples can be simplified mentally. Repeat the base and subtract the exponents in your head: $x^{12}/x^3 = x^9, 5^7/5^4 = 5^3$ and $(2x+1)^8/(2x+1)^3 = (2x+1)^5$.

Exercise 6.5.4

$\frac{4^5}{4^3}$

Answer

4^2

Example 6.5.5

Simplify: $\dfrac{12x^5y^7}{4x^3y^2}$

Solution

We first express the fraction as a product of three fractions, the latter two with a common base. In the first line of the following solution, note that if you multiply numerators and denominators of the three separate fractions, the product equals the original fraction on the left.

$$\begin{aligned}\frac{12x^5y^7}{4x^3y^2} &= \frac{12}{4}\cdot\frac{x^5}{x^3}\cdot\frac{y^7}{y^2} && \text{Break into a product of three fractions.}\\ &= 3x^{5-3}y^{7-2} && \text{Simplify: } 12/4=3.\ \text{Then repeat the common}\\ &= 3x^2y^5 && \text{Simplify.}\end{aligned}$$

Exercise 6.5.5

Simplify: $\dfrac{15a^6b^9}{3ab^5}$

Answer

$5a^5b^4$

Example 6.5.6

Simplify: $\dfrac{x^{5n-4}}{x^{3-2n}}$

Solution

Again, we repeat the base and subtract the exponents.

$$\begin{aligned}\frac{x^{5n-4}}{x^{3-2n}} &= x^{(5n-4)-(3-2n)} && \text{Repeat the base, subtract exponents.}\\ &= x^{5n-4-3+2n} && \text{Distribute the minus sign.}\\ &= x^{7n-7} && \text{Simplify. Combine like terms.}\end{aligned}$$

Exercise 6.5.6

Simplify: $\dfrac{x^{3n-6}}{x^{n+2}}$

Answer

x^{2n-8}

Raising a Power to a Power

Suppose we have an exponential expression raised to a second power, such as $(x^2)3$. The second exponent tells us to write x^2 as a factor three times:

$$\begin{aligned}\left(x^2\right)^3 &= x^2\cdot x^2\cdot x^2 && \text{Write } x^2 \text{ as a factor three times.}\\ &= x^6 && \text{Repeat the base, add the exponents.}\end{aligned}$$

Note how we added $2+2+2$ to get 6. However, a much faster way to add "three twos" is to multiply: $3\cdot 2=6$. Thus, when raising a "power to a second power," repeat the base and multiply the exponents, as follows:

$$\begin{aligned}\left(x^2\right)^3 &= x^{2\cdot 3}\\ &= x^6\end{aligned}$$

The preceding discussion gives rise to the following third law of exponents.

Raising a Power to a Power

When raising a power to a power, repeat the base and multiply the exponents. In symbols:

$$(a^m)^n = a^{mn}$$

Note that juxtaposing two variables, as in mn, means "m times n."

Example 6.5.7

Simplify each of the following expressions:

a. $\left(z^3\right)^5$
b. $\left(7^3\right)^0$
c. $\left[(x-y)^3\right]^6$

Solution

In each example we are raising a power to a power. Hence, in each case, we repeat the base and multiply the exponents.

a. $\begin{aligned}\left(z^3\right)^5 &= z^{3 \cdot 5} \\ &= z^{15}\end{aligned}$

b. $\begin{aligned}\left(7^3\right)^0 &= 7^{3 \cdot 0} \\ &= 7^0 \\ &= 1\end{aligned}$

c. $\begin{aligned}\left[(x-y)^3\right]^6 &= (x-y)^{3 \cdot 6} \\ &= (x-y)^{18}\end{aligned}$

With a little practice, each of the examples can be simplified mentally. Repeat the base and multiply the exponents in your head: $\left(z^3\right)^5 = z^{15}$, $\left(7^3\right)^4 = 7^{12}$ and $\left[(x-y)^3\right]^6 = (x-y)^{18}$.

Exercise 6.5.7

Simplify: $\left(2^3\right)^4$

Answer

2^{12}

Example 6.5.8

Simplify: $\left(x^{2n-3}\right)^4$

Solution

Again, we repeat the base and multiply the exponents.

$$\begin{aligned}\left(x^{2n-3}\right)^4 &= x^{4(2n-3)} && \text{Repeat the base, multiply exponents.} \\ &= x^{8n-12} && \text{Distribute the 4.}\end{aligned}$$

Exercise 6.5.8

Simplify: $\left(a^{2-n}\right)^3$

Answer

a^{6-3n}

Raising a Product to a Power

We frequently have need to raise a product to a power, such as $(xy)^3$. Again, remember the exponent is telling us to write xy as a factor three times, so:

$$\begin{aligned}(xy)^3 &= (xy)(xy)(xy) \quad \text{Write } xy \text{ as a factor three times.}\\ &= xyxyxy \quad \text{The associative property allows us to group as we please.}\\ &= xxxyyy \quad \text{The commutative property allows us to change the order as we please.}\\ &= x^3y^3 \quad \text{Invoke the exponent definition: } xxx = x^3 \text{ and } yyy = y^3\end{aligned}$$

However, it is much simpler to note that when you raise a product to a power, you raise each factor to that power. In symbols:

$$(xy)^3 = x^3y^3$$

The preceding discussion leads us to a fourth law of exponents.

Raising a Product to a Power

To raise a product to a power, raise each factor to that power. In symbols:

$$(ab)^n = a^nb^n$$

Example 6.5.9

Simplify each of the following expressions:

a. $(yz)^5$
b. $(-2x)^3$
c. $(-3y)^2$

Solution

In each example we are raising a product to a power. Hence, in each case, we raise each factor to that power.

a. $(yz)^5 = y^5z^5$
b. $\begin{aligned}(-2x)^3 &= (-2)^3x^3\\ &= -8x^3\end{aligned}$
c. $\begin{aligned}(-3y)^2 &= (-3)^2y^2\\ &= 9y^2\end{aligned}$

With a little practice, each of the examples can be simplified mentally. Raise each factor to the indicated power in your head: $(yz)^5 = y^5z^5$, $(-2x)^3 = -8x^3$ and $(-3y)^2 = 9y^2$

Exercise 6.5.9

Simplify: $(-2b)^4$

Answer

$16b^4$

When raising a product of three factors to a power, it is easy to show that we should raise each factor to the indicated power. For example, $(abc)^3 = a^3b^3c^3$. In general, this is true regardless of the number of factors. When raising a product to a power, raise each of the factors to the indicated power.

Example 6.5.10

Simplify: $\left(-2a^3b^2\right)^3$

Solution

Raise each factor to the third power, then simplify.

$$\begin{aligned}\left(-2a^3b^2\right)^3 &= (-2)^3\left(a^3\right)^3\left(b^2\right)^3 \quad \text{Raise each factor to the third power.}\\ &= -8a^9b^6\end{aligned}$$

Simplify: $(-2)^3 = 8$. In the remaining factors, raising a power to a power requires that we multiply the exponents.

Exercise 6.5.10

Simplify: $\left(-3xy^4\right)^5$

Answer

$-243x^5y^{20}$

Example 6.5.11

Simplify: $\left(-2x^2y\right)^2\left(-3x^3y\right)$

Solution

In the first grouped product, raise each factor to the second power.

$$\left(-2x^2y\right)^2\left(-3x^3y\right) = \left((-2)^2\left(x^2\right)^2y^2\right)\left(-3x^3y\right) \quad \text{Raise each factor in the first grouped product to the second power.}$$
$$= \left(4x^4y^2\right)\left(-3x^3y\right) \quad \text{Simplify: } (-2)^2 = 4 \text{ and } \left(x^2\right)^2 = x^4$$

The associative and commutative property allows us to multiply all six factors in the order that we please. Hence, we'll multiply 4 and -3, then x^4 and x^3, and \(y^2 and y, in that order. In this case, we repeat the base and add the exponents.

$$= -12x^7y^3 \quad \text{Simplify: } (4)(-3) = -12. \text{ Also, } x^4x^3 = x^7 \text{ and } y^2y = y^3$$

Exercise 6.5.11

Simplify: $\left(-a^3b^2\right)^3\left(-2a^2b^4\right)^2$

Answer

$-4a^{13}b^{14}$

Raising a Quotient to a Power

Raising a quotient to a power is similar to raising a product to a power. For example, raising $(x/y)^3$ requires that we write x/y as a factor three times.

$$\left(\frac{x}{y}\right)^3 = \frac{x}{y}\cdot\frac{x}{y}\cdot\frac{x}{y}$$
$$= \frac{x\cdot x\cdot x}{y\cdot y\cdot y}$$
$$= \frac{x^3}{y^3}$$

However, it is much simpler to realize that when you raise a quotient to a power, you raise both numerator and denominator to that power. In symbols:

$$\left(\frac{x}{y}\right)^3 = \frac{x^3}{y^3}$$

This leads to the fifth and final law of exponents.

Raising a Quotient to a Power

To raise a quotient to a power, raise both numerator and denominator to that power. Given $b \neq 0$,

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

Example 6.5.12

Simplify each of the following expressions:

a. $\left(\frac{2}{3}\right)^2$

b. $\left(\frac{x}{3}\right)^3$

c. $\left(-\frac{2}{y}\right)^4$

Solution

In each example we are raising a quotient to a power. Hence, in each case, we raise both numerator and denominator to that power.

a. $\left(\frac{2}{3}\right)^2 = \frac{2^2}{3^2}$
$= \frac{4}{9}$

b. $\left(\frac{x}{3}\right)^3 = \frac{x^3}{3^3}$
$= \frac{x^3}{27}$

c. $\left(-\frac{2}{y}\right)^4 = \frac{2^4}{y^4}$
$= \frac{16}{y^4}$

Note that in example (c), raising a negative base to an even power produces a positive result. With a little practice, each of the examples can be simplified mentally. Raise numerator and denominator to the indicated power in your head: $(2/3)^2 = 4/9$, $(x/3)^3 = x^3/27$, and $(-2/y)^4 = 16/y^4$

Exercise 6.5.12

Simplify: $\left(\frac{5}{4}\right)^3$

Answer

$\frac{125}{64}$

Example 6.5.13

Simplify: $\left(\frac{2x^5}{y^3}\right)^2$

Solution

Raise both numerator and denominator to the second power, then simplify:

$$\left(\frac{2x^5}{y^3}\right)^2 = \frac{\left(2x^5\right)^2}{\left(y^3\right)^2} \quad \text{Raise numerator and denominator to the second power.}$$

In the numerator, we need to raise each factor of the product to the second power. Then we need to remind ourselves that when we raise a power to a power, we multiply the exponents.

$$= \frac{2^2\left(x^5\right)^2}{\left(y^3\right)^2} \quad \text{Raise each factor in the numerator and denominator to the second power.}$$

$$= \frac{4x^{10}}{y^6} \quad \text{Simplify: } 2^2 = 4,\ \left(x^5\right)^2 = x^{10}, \text{ and } \left(y^3\right)^2 = y^6$$

Exercise 6.5.13

Simplify: $\left(\frac{a^4}{3b^2}\right)^3$

Answer

$$\frac{a^{12}}{27b^6}$$

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6.6: Multiplying Polynomials

In this section we will find the products of polynomial expressions and functions. We start with the product of two monomials, then graduate to the product of a monomial and polynomial, and complete the study by finding the product of any two polynomials.

The Product of Monomials

As long as all of the operations are multiplication, we can use the commutative and associative properties to change the order and regroup.

Example 6.6.1

Multiply: $-5(7y)$. Simplify: $-5(7y)$.

Solution

Use the commutative and associative properties to change the order and regroup.

$$\begin{aligned} -5(7y) &= [(-5)(7)]y && \text{Reorder. Regroup.} \\ &= -35y && \text{Multiply: } (-5)(7) = -35 \end{aligned}$$

Exercise 6.6.1

Multiply: $3(4x)$

Answer

$12x$

Example 6.6.2

Multiply: $(-3x^2)(4x^3)$. Simplify: $(-3x^2)(4x^3)$.

Solution

Use the commutative and associative properties to change the order and regroup.

$$\begin{aligned} \left(-3x^2\right)\left(4x^3\right) &= [(-3)(4)]\left(x^2x^3\right) && \text{Reorder. Regroup.} \\ &= -12x^5 && \text{Multiply: } (-3)(4) = 12, x^2x^3 = x^5 \end{aligned}$$

Exercise 6.6.2

Multiply: $(7y^5)(-2y^2)$

Answer

$-14y^7$

Example 6.6.3

Multiply: $(-2a^2b^3)(-5a^3b)$. Simplify: $(-2a^2b^3)(-5a^3b)$.

Solution

Use the commutative and associative properties to change the order and regroup.

$$\begin{aligned} \left(-2a^2b^3\right)\left(-5a^3b\right) &= [(-2)(-5)]\left(a^2a^3\right)\left(b^3b\right) && \text{Reorder. Regroup.} \\ &= 10a^5b^4 && \text{Multiply: } (-2)(-5) = 10, a^2a^3 = a^5, \text{ and } b^3b = b^4 \end{aligned}$$

Exercise 6.6.3

Multiply: $(-6st^2)(3s^3t^4)$

Answer

$-18s^4t^6$

When multiplying monomials, it is much more efficient to make the required calculations mentally. In the case of Example 6.6.1, multiply -5 and 7 mentally to get

$$-5(7y) = -35y$$

In the case of Example 6.6.2, multiply -3 and 4 to get -12, then repeat the base x and add exponents to get

$$(-3x^2)(4x^3) = -12x^5$$

Finally, in the case of Example 6.6.3, multiply -2 and -5 to get 10, then repeat the bases and add their exponents.

$$(-2a^2b^3)(-5a^3b) = 10a^5b^4$$

Multiplying a Monomial and a Polynomial

Now we turn our attention to the product of a monomial and a polynomial.

Example 6.6.4

Multiply: $5\left(3x^2-4x-8\right)$

Solution

We need to first distribute the 5 times each term of the polynomial. Then we multiply the resulting monomials mentally.

$$\begin{aligned} 5\left(3x^2-4x-8\right) &= 5\left(3x^2\right)-5(4x)-5(8) \\ &= 15x^2-20x-40 \end{aligned}$$

Exercise 6.6.4

Multiply: $4\left(-2y^2+3y+5\right)$

Answer

$-8y^2+12y+20$

Example 6.6.5

Multiply: $2y(-3y+5)$

Solution

We need to first distribute the $2y$ times each term of the polynomial. Then we multiply the resulting monomials mentally.

$$\begin{aligned} 2y(-3y+5) &= 2y(-3y)+2y(5) \\ &= -6y^2+10y \end{aligned}$$

Exercise 6.6.5

Multiply: $-3x(4-2x)$

Answer

$-12x+6x^2$

Example 6.6.6

Multiply: $-3ab\left(a^2+2ab-b^2\right)$

Solution

We need to first distribute the $-3ab$ times each term of the polynomial. Then we multiply the resulting monomials mentally.

$$\begin{aligned}-3ab\left(a^2+2ab-b^2\right) &= -3ab\left(a^2\right)+(-3ab)(2ab)-(-3ab)\left(b^2\right)\\ &= -3a^3b+\left(-6a^2b^2\right)-\left(-3ab^3\right)\\ &= -3a^3b-6a^2b^2+3ab^3\end{aligned}$$

Alternate solution:

It is far more efficient to perform most of the steps of the product $-3ab(a^2+2ab-b^2)$ mentally. We know we must multiply $-3ab$ times each of the terms of the polynomial $a^2+2ab-b^2$. Here are our mental calculations:

i. $-3ab$ times a^2 equals $-3a3b$.
ii. $-3ab$ times $2ab$ equals $-6a^2b^2$.
iii. $-3ab$ times $-b^2$ equals $3ab^3$.

Thinking in this manner allows us to write down the answer without any of the steps shown in our first solution. That is, we immediately write:

$$-3ab\left(a^2+2ab-b^2\right)=-3a^3b-6a^2b^2+3ab^3$$

Exercise 6.6.6

Multiply: $2xy\left(x^2-4xy^2+7x\right)$

Answer

$2x^3y-8x^2y^3+14x^2y$

Example 6.6.7

Multiply: $-2z^2\left(z^3+4z^2-11\right)$

Solution

We need to first distribute the $-2z^2$ times each term of the polynomial. Then we multiply the resulting monomials mentally.

$$\begin{aligned}-2z^2\left(z^3+4z^2-11\right) &= -2z^2\left(z^3\right)+\left(-2z^2\right)\left(4z^2\right)-\left(-2z^2\right)(11)\\ &= -2z^5+\left(-8z^4\right)-\left(-22z^2\right)\\ &= -2z^5-8z^4+22z^2\end{aligned}$$

Alternate solution:

It is far more efficient to perform most of the steps of the product $-2z^2(z^3+4z^2-11)$ mentally. We know we must multiply $-2z^2$ times each of the terms of the polynomial z^3+4z^2-11 . Here are our mental calculations:

i. $-2z^2$ times z^3 equals $-2z^5$.
ii. $-2z^2$ times $4z^2$ equals $-8z^4$.
iii. $-2z^2$ times -11 equals $22z^2$.

Thinking in this manner allows us to write down the answer without any of the steps shown in our first solution. That is, we immediately write:

$$-2z^2\left(z^3+4z^2-11\right)=-2z^5-8z^4+22z^2$$

Exercise 6.6.7

Multiply: $-5y^3\left(y^2-2y+1\right)$

Answer

$-5y^5+10y^4-5y^3$

Multiplying Polynomials

Now that we've learned how to take the product of two monomials or the product of a monomial and a polynomial, we can apply what we've learned to multiply two arbitrary polynomials.

Before we begin with the examples, let's revisit the distributive property. We know that

$$2\cdot(3+4)=2\cdot3+2\cdot4$$

Both sides are equal to 14. We're used to having the monomial factor on the left, but it can also appear on the right.

$$(3+4)\cdot2=3\cdot2+4\cdot2$$

Again, both sides equal 14. So, whether the monomial appears on the left or right makes no difference; that is, $a(b+c)=ab+ac$ and $(b+c)a=ba+ca$ give the same result. In each case, you multiply a times both terms of $b+c$.

Example 6.6.8

Multiply: $(2x+5)(3x+2)$

Solution

Note that $(2x+5)(3x+2)$ has the form $(b+c)a$, where a is the binomial $3x+2$. Because $(b+c)a=ba+ca$, we will multiply $3x+2$ times both terms of $2x+5$ in the following manner.

$$(2x+5)(3x+2)=2x(3x+2)+5(3x+2)$$

Now we distribute monomials times polynomials, then combine like terms.

$$\begin{aligned}&=6x^2+4x+15x+10\\&=6x^2+19x+10\end{aligned}$$

Thus, $(2x+5)(3x+2)=6x^2+19x+10$

Exercise 6.6.8

Multiply: $(3x+4)(5x+1)$

Answer

$15x^2+23x+4$

Example 6.6.9

Multiply: $(x+5)\left(x^2+2x+7\right)$

Solution

Note that $(x+5)(x^2+2x+7)$ has the form $(b+c)a$, where a is the trinomial x^2+2x+7. Because $(b+c)a=ba+ca$, we will multiply x^2+2x+7 times both terms of $x+5$ in the following manner.

$$(x+5)\left(x^2+2x+7\right)=x\left(x^2+2x+7\right)+5\left(x^2+2x+7\right)$$

Now we distribute monomials times polynomials, then combine like terms.

$$= x^3 + 2x^2 + 7x + 5x^2 + 10x + 35$$
$$= x^3 + 7x^2 + 17x + 35$$

Thus, $(x+5)\left(x^2+2x+7\right) = x^3 + 7x^2 + 17x + 35$

Exercise 6.6.9

Multiply: $(z+4)\left(z^2+3z+9\right)$

Answer

$z^3 + 7z^2 + 21z + 36$

Speeding Things Up a Bit

Let's re-examine Example 6.6.9 with the hope of unearthing a pattern that will allow multiplication of polynomials to proceed more quickly with less work. Note the first step of Example 6.6.9.

$$(x+5)\left(x^2+2x+7\right) = x\left(x^2+2x+7\right) + 5\left(x^2+2x+7\right)$$

Note that the first product on the right is the result of taking the product of the first term of $x+5$ and x^2+2x+7. Similarly, the second product on the right is the result of taking the product of the second term of $x+5$ and x^2+2x+7. Next, let's examine the result of the second step.

$$(x+5)\left(x^2+2x+7\right) = x^3 + 2x^2 + 7x + 5x^2 + 10x + 35$$

The first three terms on the right are the result of multiplying x times x^2+2x+7.

$$(x+5)(x^2+2x+7) = x^3 + 2x^2 + 7x + \cdots$$

The second set of three terms on the right are the result of multiplying 5 times x^2+2x+7.

$$(x+5)(x^2+2x+7) = \cdots + 5x^2 + 10x + 35$$

These notes suggest an efficient shortcut. To multiply $x+5$ times x^2+2x+7,

- Multiply x times each term of x^2+2x+7.
- Multiply 5 times each term of x^2+2x+7.
- Combine like terms.

This process would have the following appearance.

$$\begin{aligned}(x+5)\left(x^2+2x+7\right) &= x^3 + 2x^2 + 7x + 5x^2 + 10x + 35\\ &= x^3 + 7x^2 + 17x + 35\end{aligned}$$

Example 6.6.10

Use the shortcut technique described above to simplify: $(2y-6)\left(3y^2+4y+11\right)$

Solution

Multiply $2y$ times each term of $3y^2+4y+11$, then multiply -6 times each term of $3y^2+4y+11$. Finally, combine like terms.

$$\begin{aligned}(2y-6)\left(3y^2+4y+11\right) &= 6y^3 + 8y^2 + 22y - 18y^2 - 24y - 66\\ &= 6y^3 - 10y^2 - 2y - 66\end{aligned}$$

Exercise 6.6.10

Multiply: $(3x+2)\left(4x^2-x+10\right)$

Answer

$12x^3+5x^2+28x+20$

Example 6.6.11

Use the shortcut technique to simplify $(a+b)^2$

Solution

To simplify $(a+b)^2$, we first must write $a+b$ as a factor two times.

$$(a+b)^2=(a+b)(a+b)$$

Next, multiply a times both terms of $a+b$, multiply b times both terms of $a+b$, then combine like terms.

$$\begin{aligned}&=a^2+ab+ba+b^2\\&=a^2+2ab+b^2\end{aligned}$$

Note that $ab=ba$ because multiplication is commutative, so $ab+ba=2ab$.

Exercise 6.6.11

Multiply: $(3y-2)^2$

Answer

$9y^2-12y+4$

Example 6.6.12

Use the shortcut technique to simplify: $\left(x^2+x+1\right)\left(x^2-x-1\right)$

Solution

This time the first factor contains three terms, x^2, x, and 1, so we first multiply x^2 times each term of x^2-x-1 , then x times each term of x^2-x-1 , and 1 times each term of x^2-x-1 . Then we combine like terms.

$$\begin{aligned}\left(x^2+x+1\right)\left(x^2-x-1\right)\quad&=x^4-x^3-x^2+x^3-x^2-x+x^2-x-1\\&=x^4-x^2-2x-1\end{aligned}$$

Exercise 6.6.12

Multiply: $\left(a^2-2a+3\right)\left(a^2+2a-3\right)$

Answer

$a^4-4a^2+12a-9$

Some Applications

Recall that the area of a circle of radius r is found using the formula $A=\pi r^2$. The circumference (distance around) of a circle of radius r is found using the formula $C=2\pi r$ (see Figure 6.6.1).

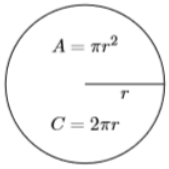


Figure 6.6.1: Formulae for the area and circumference of a circle of radius r.

Example 6.6.13

In Figure 6.6.2 are pictured two concentric circles (same center). The inner circle has radius x and the outer circle has radius $x+1$. Find the area of the shaded region (called an annulus) as a function of x.

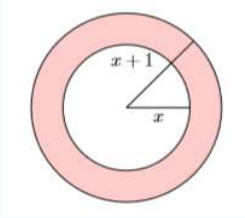


Figure 6.6.2: Find the area of the shaded region.

Solution

We can find the area of the shaded region by subtracting the area of the inner circle from the area of the outer circle.

$$A = \text{ Area of outer circle - Area of inner circle}$$

We use the formula $A = \pi r^2$ to compute the area of each circle. Because the outer circle has radius $x+1$, it has area $\pi(x+1)^2$. Because the inner circle has radius x, it has area πx^2.

$$= \pi(x+1)^2 - \pi x^2$$

Next, we'll expand $(x+1)^2$, then combine like terms.

$$\begin{aligned} &= \pi(x+1)(x+1) - \pi x^2 \\ &= \pi\left(x^2 + x + x + 1\right) - \pi x^2 \\ &= \pi\left(x^2 + 2x + 1\right) - \pi x^2 \end{aligned}$$

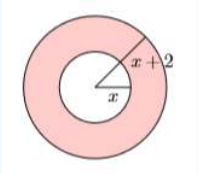
Finally, distribute π times each term of $x^2 + 2x + 1$, then combine like terms.

$$\begin{aligned} &= \pi x^2 + 2\pi x + \pi - \pi x^2 \\ &= 2\pi x + \pi \end{aligned}$$

Hence, the area of the shaded region is $A = 2\pi x + \pi$.

Exercise 6.6.13

Two concentric circles are shown below. The inner circle has radius x and the outer circle has radius $x+2$. Find the area of the shaded region as a function of x.



Answer

$4\pi x + 4\pi$

Example 6.6.14

The demand for widgets is a function of the unit price, where the demand is the number of widgets the public will buy and the unit price is the amount charged for a single widget. Suppose that the demand is given by the function $x = 270 - 0.75p$, where x is the demand and p is the unit price. Note how the demand decreases as the unit price goes up (makes sense). Use the graphing calculator to determine the unit price a retailer should charge for widgets so that his revenue from sales equals $\$20,000$.

Solution

To determine the revenue (R), you multiply the number of widgets sold (x) by the unit price (p).

$$R = xp \tag{6.6.1}$$

However, we know that the number of units sold is the demand, given by the formula

$$x = 270 - 0.75p \tag{6.6.2}$$

Substitute equation 6.6.2 into equation 6.6.1 to obtain the revenue as a function of the unit price.

$$R = (270 - 0.75p)p$$

Expand.

$$R = 270p - 0.75p^2 \tag{6.6.3}$$

We're asked to determine the unit price that brings in a revenue of $\$20,000$. Substitute $\$20,000$ for R in equation 6.6.3.

$$20000 = 270p - 0.75p^2 \tag{6.6.4}$$

Enter each side of equation 6.6.4 into the **Y= menu** of your calculator (see the first image in Figure 6.6.3). After some experimentation, we settled on the **WINDOW** parameters shown in the second image of Figure 6.6.3. After making these settings, push the **GRAPH** button to produce the graph shown in the third image in Figure 6.6.3.

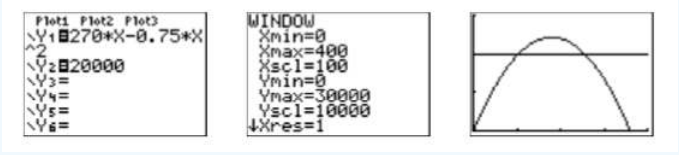


Figure 6.6.3: Solving $20000 = 270p - 0.75p^2$.

Note that the third image in Figure 6.6.3 shows that there are two solutions, that is, two ways we can set the unit price to obtain a revenue of $\$20,000$. To find the first solution, select 5:intersect from the **CALC** menu, press **ENTER** in response to "First curve," press **ENTER** in response to "Second curve," then move your cursor closer to the point of intersection on the left and press **ENTER** in response to "Guess." The result is shown in the first image in Figure 6.6.4.

Repeat the process to find the second point of intersection. The result is shown in the second image in Figure 6.6.4.

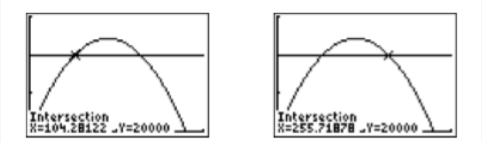


Figure 6.6.4: Finding the points of intersection.

Note

In this example, the horizontal axis is actually the p-axis. So when we set Xmin and Xmax, we're actually setting bounds on the p-axis.

Reporting the solution on your homework:

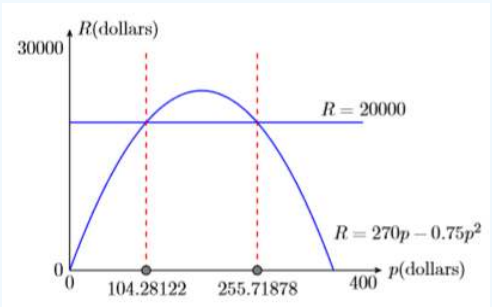


Figure 6.6.5: Reporting your graphical solution on your homework.

Duplicate the image in your calculator's viewing window on your homework page. Use a ruler to draw all lines, but freehand any curves.

- Label the horizontal and vertical axes with p and R, respectively (see Figure 6.6.5). Include the units (dollars and dollars).
- Place your **WINDOW** parameters at the end of each axis (see Figure 6.6.5).
- Label each graph with its equation (see Figure 6.6.5).
- Drop a dashed vertical line through each point of intersection. Shade and label the p-values of the points where the dashed vertical lines cross the p-axis. These are the solutions of the equation $20000 = 270p - 0.75p^2$ (see Figure 6.6.5).

Rounding to the nearest penny, setting the unit price at either $\$104.28$ or $\$255.72$ will bring in a revenue of $\$20,000$.

Exercise 6.6.14

Suppose that the demand for gadgets is given by the function $x = 320 - 1.5p$, where x is the demand and p is the unit price. Use the graphing calculator to determine the unit price a retailer should charge for gadgets so that her revenue from sales equals $\$12,000$.

Answer

$\$48.55$ or $\$164.79$

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6.7: Special Products

This section is dedicated to explaining a number of important shortcuts for multiplying binomials. These are extremely important patterns that will produce the same products computed in previous sections. It is essential that readers practice until they become proficient using each of the patterns presented in this section.

The FOIL Method

Consider the product of two binomials $(x+3)(x+6)$. We already know how to find the product of these two binomials; we multiply x times both terms of $x+6$, then we multiply 3 times both terms of $x+6$.

$$(x+3)(x+6)=x^2+6x+3x+18$$

Normally we combine like terms, but we halt the process at this point so as to introduce the pattern called the FOIL method. The letters in the word FOIL stand for "First," "Outer," "Inner," and "Last."

Let's see how we can connect these terms to the product $(x+3)(x+6)$.

- The arrows indicate the terms in the "First" positions in each binomial. If you multiply the terms in the "First" position, you get x^2.

$$\underbrace{(x+3)(x}_{F}+6)$$

- The arrows indicate the terms in the "Outer" positions in each binomial. If you multiply the terms in the "Outer" positions, you get $6x$.

$$\underbrace{(x+3)(x+6}_{O})$$

- The arrows indicate the terms in the "Inner" positions in each binomial. If you multiply the terms in the "Inner" positions, you get $3x$.

$$(x+3\underbrace{)(x}_{I}+6)$$

- The arrows indicate the terms in the "Last" positions in each binomial. If you multiply the terms in the "Last" positions, you get 18.

$$(x+3\underbrace{)(x+6}_{L})$$

The following diagram shows the connection between "First," "Outer," "Inner," "Last," and the answer.

$$(x+3)(x+6)=\begin{array}{ccccccccc} F & & O & & I & & L \\ x^2 & + & 6x & + & 3x & + & 18 \end{array}$$

Example 6.7.1

Use the FOIL method to simplify: $(x+5)(x+7)$

Solution

Multiply the "First" positions: x^2. Multiply the "Outer" positions: $7x$. Multiply the "Inner" positions: $5x$. Multiply the "Last" positions: 35.

$$(x+5)(x+7)=\begin{array}{ccccccccc} F & & O & & I & & L \\ x^2 & + & 7x & + & 5x & + & 35 \end{array}$$

Combining like terms, $(x+5)(x+7)=x^2+12x+35$

Exercise 6.7.1

Simplify: $(x+2)(x+11)$

Answer

$x^2+13x+22$

Example 6.7.2

Use the FOIL method to simplify: $(2x-7)(x-4)$

Solution

Multiply the "First" positions: $2x^2$. Multiply the "Outer" positions: $-8x$. Multiply the "Inner" positions: $-7x$. Multiply the "Last" positions: 28.

$$(2x-7)(x-4) = \underset{2x^2}{F} \quad \underset{-8x}{O} \quad \underset{-7x}{I} \quad \underset{28}{L}$$

Combining like terms, $(2x-7)(x-4) = 2x^2-15x+28$

Exercise 6.7.2

Simplify: $(x-1)(4x+5)$

Answer

$4x^2+x-5$

At first glance, the FOIL method doesn't look like much of a shortcut. After all, if we simply use the distributive property on the product of Example 6.7.2, we get the same quick result.

$$\begin{aligned}(2x-7)(x-4) &= 2x(x-4)-7(x-4)\\ &= 2x^2-8x-7x+28\\ &= 2x^2-15x+28\end{aligned}$$

The FOIL method becomes a true shortcut when we add the "Outer" and "Inner" results in our head.

FOIL Shortcut

To multiply two binomials, follow these steps:

1. Multiply the terms in the "First" positions.
2. Multiply the terms in the "Outer" and "Inner" positions and combine the results mentally (if they are like terms).
3. Multiply the terms in the "Last" positions.

Example 6.7.3

Use the FOIL shortcut to simplify: $(3x+8)(2x-1)$

Solution

Each of the following steps is performed mentally.

1. Multiply the terms in the "First" positions: $6x^2$
2. Multiply the terms in the "Outer" and "Inner" positions and add the results mentally: $-3x+16x = 13x$
3. Multiply the terms in the "Last" positions: -8

Write the answer with no intermediate steps: $(3x+8)(2x-1) = 6x^2+13x-8$

Exercise 6.7.3

Simplify: $(2z-3)(5z-1)$

Answer

$10z^2-17z+3$

Example 6.7.4

Use the FOIL shortcut to simplify: $(4y-3)(5y+2)$

Solution

Each of the following steps is performed mentally.

1. Multiply the terms in the "First" positions: $20y^2$
2. Multiply the terms in the "Outer" and "Inner" positions and add the results mentally: $8y-15y=-7y$
3. Multiply the terms in the "Last" positions: -6

Write the answer with no intermediate steps: $(4y-3)(5y+2)=20y^2-7y-6$

Exercise 6.7.4

Simplify: $(7x+2)(2x-3)$

Answer

$14x^2-17x-6$

The Difference of Squares

We can use the FOIL shortcut to multiply $(a+b)(a-b)$.

1. Multiply the terms in the "First" positions: a^2
2. Multiply the terms in the "Outer" and "Inner" positions and add the results mentally: $ab-ab=0$
3. Multiply the terms in the "Last" positions: $-b^2$

Thus, $(a+b)(a-b)=a^2-b^2$. Note how the right-hand side a^2-b^2 is the difference of two squares. This leads to the following shortcut.

The difference of squares

If you have identical terms in the "First" positions and identical terms in the "Last" positions, but one set is separated with a plus sign while the other is separated by a minus sign, then proceed as follows:

1. Square the "First" term.
2. Square the "Last" term.
3. Place a minus sign between the results

That is,

$$(a+b)(a-b)=a^2-b^2$$

Note

If you don't have identical terms in the "First" and "Last" positions, with one set separated with a plus sign and the other with a minus sign, then you do not have the difference of squares pattern and you must find some other way to multiply. For example, $(x+3)(x-3)$ is an example of the difference of squares pattern, but $(2y+3)(2y-5)$ is not.

Example 6.7.5

Use the difference of squares shortcut to simplify: $(x+3)(x-3)$

Solution

Note how the terms in the "First" position are identical, as are the terms in the "Last" position, with one set separated by a plus sign and the other with a minus sign. Hence, this is the difference of squares pattern and we proceed as follows:

1. Square the term in the "First" position: x^2
2. Square the term in the "Last" position: $(-3)^2 = 9$
3. Separate the squares with a minus sign.

That is:

$$\begin{aligned}(x+3)(x-3) &= x^2-(3)^2 \\ &= x^2-9\end{aligned}$$

Note

You should practice this pattern until you can go straight from the problem statement to the answer without writing down any intermediate work, as in $(x+3)(x-3) = x^2-9$.

Exercise 6.7.5

Simplify: $(x+5)(x-5)$

Answer

x^2-25

Example 6.7.6

Use the difference of squares shortcut to simplify: $(8y+7z)(8y-7z)$

Solution

Note how the terms in the "First" position are identical, as are the terms in the "Last" position, with one set separated by a plus sign and the other with a minus sign. Hence, this is the difference of squares pattern and we proceed as follows:

1. Square the term in the "First" position: $(8y)^2 = 64y^2$
2. Square the term in the "Last" position: $(7z)^2 = 49z^2$
3. Separate the squares with a minus sign.

That is:

$$\begin{aligned}(8y+7z)(8y-7z) &= (8y)^2-(7z)^2 \\ &= 64y^2-49z^2\end{aligned}$$

Note

You should practice this pattern until you can go straight from the problem statement to the answer without writing down any intermediate work, as in $(8y+7z)(8y-7z) = 64y^2-49z^2$

Exercise 6.7.6

Simplify: $(3a-6b)(3a+6b)$

Answer

$9a^2-36b^2$

Example 6.7.7

Use the difference of squares shortcut to simplify: $\left(x^3-5y^2\right)\left(x^3+5y^2\right)$

Solution

Note how the terms in the "First" position are identical, as are the terms in the "Last" position, with one set separated by a plus sign and the other with a minus sign. Hence, this is the difference of squares pattern and we proceed as follows:

1. Square the term in the "First" position: $(x^3)^2=x^6$
2. Square the term in the "Last" position: $(5y^2)^2=25y^4$
3. Separate the squares with a minus sign.

That is:

$$\begin{aligned}\left(x^3-5y^2\right)\left(x^3+5y^2\right) &= \left(x^3\right)^2-\left(5y^2\right)^2\\ &= x^6-25y^4\end{aligned}$$

Note

You should practice this pattern until you can go straight from the problem statement to the answer without writing down any intermediate work, as in $\left(x^3-5y^2\right)\left(x^3+5y^2\right)=x^6-25y^4$

Exercise 6.7.7

Simplify: $\left(2y^4+z^3\right)\left(2y^4-z^3\right)$

Answer

$4y^8-z^6$

Squaring a Binomial

Before demonstrating the correct procedure for squaring a binomial, we will first share one of the most common mistakes made in algebra.

Warning! This is incorrect!

One of the most common mistakes made in algebra is the assumption that:

$$(a+b)^2=a^2+b^2$$

The fact that this is incorrect is easily checked. Substitute 3 for a and 4 for b.

$$\begin{aligned}(3+4)^2 &= 3^2+4^2\\ 7^2 &= 3^2+4^2\\ 49 &= 9+16\end{aligned}$$

Clearly this is incorrect!

So what is the correct answer? First, $(a+b)^2=(a+b)(a+b)$. We can now use the FOIL shortcut.

1. Multiply the terms in the "First" positions: a^2
2. Multiply the terms in the "Outer" and "Inner" positions and add the results mentally: $ab+ab=2ab$
3. Multiply the terms in the "Last" positions: b^2

Hence, the correct answer is $(a+b)^2=a^2+2ab+b^2$. This leads us to the following shortcut for squaring a binomial.

Squaring a binomial

To square a binomial, such as $(a+b)^2$, perform the following steps:

1. Square the “First” term: a^2
2. Multiply the “First” and “Last” terms and double the result: $2ab$
3. Square the “Last” term: b^2

That is:

$$(a+b)^2 = a^2 + 2ab + b^2$$

Example 6.7.8

Use the squaring a binomial shortcut to expand: $(x+5)^2$

Solution

Follow these steps:

1. Square the first term: x^2
2. Multiply the “First” and “Last” terms and double the result: $2(x)(5) = 10x$
3. Square the “Last” term: $5^2 = 25$

Thus:

$$\begin{aligned}(x+5)^2 &= x^2 + 2(x)(5) + (5)^2 \\ &= x^2 + 10x + 25\end{aligned}$$

Note

You should practice this pattern until you can go straight from the problem statement to the answer without writing down any intermediate work, as in $(x+5)^2 = x^2 + 10x + 25$

Exercise 6.7.8

Simplify: $(x+3)^2$

Answer

$x^2 + 6x + 9$

Comment

Students often refuse to learn the “squaring a binomial” shortcut, noting that they can just as easily use the FOIL technique or a simple application of the distributive property to arrive at the same result. Unfortunately, failure to learn the “squaring a binomial” shortcut will severely handicap students, as this pattern is an important component of many procedures in future mathematics courses.

Example 6.7.9

Use the squaring a binomial shortcut to expand: $(3x+7y)^2$

Solution

Follow these steps:

1. Square the first term: $(3x)^2 = 9x^2$
2. Multiply the “First” and “Last” terms and double the result: $2(3x)(7y) = 42xy$
3. Square the “Last” term: $(7y)^2 = 49y^2$

Thus:

$$\begin{aligned}(3x+7y)^2 &= (3x)^2+2(3x)(7y)+(7y)^2\\ &= 9x^2+42xy+49y^2\end{aligned}$$

Note

You should practice this pattern until you can go straight from the problem statement to the answer without writing down any intermediate work, as in $(3x+7y)^2 = 9x^2+42xy+49y^2$

Exercise 6.7.9

Simplify: $(2y+3z)^2$

Answer

$4y^2+12yz+9z^2$

In the next example, when squaring a binomial with a minus sign, we take care of the minus sign by "adding the opposite."

Example 6.7.10

Use the squaring a binomial shortcut to expand: $\left(4a^2-5b^3\right)^2$

Solution

Add the opposite: $(4a^2-5b^3)^2 = (4a^2+(-5b^3))^2$. Now follow these steps:

1. Square the first term: $(4a^2)^2 = 16a^4$
2. Multiply the "First" and "Last" terms and double the result: $2(4a^2)(-5b^3) = -40a^2b^3$
3. Square the "Last" term: $(-5b^3)^2 = 25b^6$

Thus:

$$\begin{aligned}\left(4a^2-5b^3\right)^2 &= \left(4a^2+\left(-5b^3\right)\right)^2\\ &= \left(4a^2\right)^2+2\left(4a^2\right)\left(-5b^3\right)+\left(-5b^3\right)^2\\ &= 16a^4-40a^2b^3+25b^6\end{aligned}$$

Note

You should practice this pattern until you can go straight from the problem statement to the answer without writing down any intermediate work, as in

$$\left(4a^2-5b^3\right)^2 = 16a^4-40a^2b^3+25b^6$$

Exercise 6.7.10

Simplify: $\left(3x^4-5z^2\right)^2$

Answer

$9x^8-30x^4z^2+25z^4$

Example 6.7.10 shows us that if we are squaring a difference, the middle term will be minus. That is, the only difference between $(a+b)^2$ and $(a-b)^2$ is the sign of the middle term.

Squaring a binomial

The shortcuts for squaring a binomial are:

$$(a+b)^2 = a^2 + 2ab + b^2$$
$$(a-b)^2 = a^2 - 2ab + b^2$$

Example 6.7.11

Use the squaring a binomial shortcut to expand: $\left(x^3 - y^3\right)^2$

Solution

Use the pattern $(a-b)^2 = a^2 - 2ab + b^2$. Square the "First" term, multiply the "First" and "Last" terms and double the result, then square the "Last" term. Because of the minus sign, the middle term will be minus, but all other terms are plus.

$$\begin{aligned}\left(x^3 - y^3\right)^2 &= \left(x^3\right)^2 - 2\left(x^3\right)\left(y^3\right) + \left(y^3\right)^2 \\ &= x^6 - 2x^3y^3 + y^6\end{aligned}$$

Exercise 6.7.11

Simplify: $\left(a^2 - 3b^5\right)^2$

Answer

$a^4 - 6a^2b^5 + 9b^{10}$

An Application

In Example 6.7.7, we found the area of the outer square by summing the areas of its parts (see Figure 6.7.1). Recall that the answer was $A = x^2 + 6x + 9$.

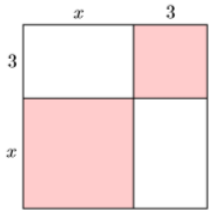


Figure 6.7.1: Find the area of the square.

Now that we have the squaring a binomial shortcut, we can simplify the process of finding the area of the outer square by squaring its side. That is:

$$A = (x+3)^2$$

Now we can use the squaring the binomial technique to expand.

$$\begin{aligned}&= x^2 + 2(x)(3) + (3)^2 \\ &= x^2 + 6x + 9\end{aligned}$$

Note that this is the same as the answer found by summing the four parts of the square in Example 6.7.7.

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6.E: Polynomial Functions (Exercises)

5.1: Functions

In Exercises 1-6, state the domain and range of the given relation.

1) $R = \{(7,4),(2,4),(4,2),(8,5)\}$

Answer

Domain $= \{2,4,7,8\}$and Range $= \{2,4,5\}$

2) $S = \{(6,4),(3,3),(2,5),(8,7)\}$

3) $T = \{(7,2),(3,1),(9,4),(8,1)\}$

Answer

Domain $= \{3,7,8,9\}$and Range $= \{1,2,4\}$

4) $R = \{(0,1),(8,2),(6,8),(9,3)\}$

5) $T = \{(4,7),(4,8),(5,0),(0,7)\}$

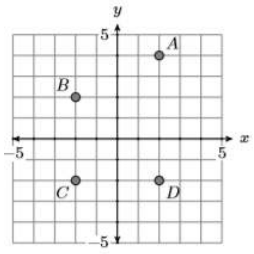
Answer

Domain $= \{0,4,5\}$and Range $= \{0,7,8\}$

6) $T = \{(9,0),(3,6),(8,0),(3,8)\}$

In Exercises 7-10, state the domain and range of the given relation.

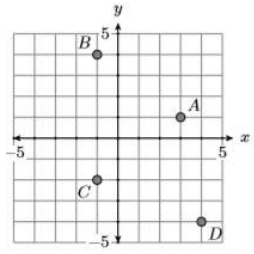
7)



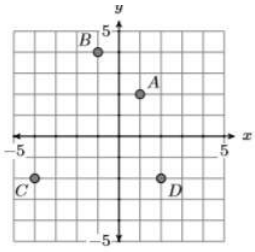
Answer

Domain $= \{-2,2\}$ and Range $= \{-2,2,4\}$

8)



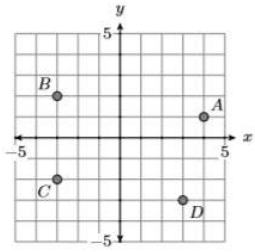
9)



Answer

Domain $= \{-4, -1, 1, 2\}$ and Range $= \{-2, 2, 4\}$

10)



In Exercises 11-18, determine whether the given relation is a function.

11) $R = \{(-6, -4), (-4, -4), (1, -4)\}$

Answer

Function

12) $T = \{(-8, -3), (-4, -3), (2, -3)\}$

13) $T = \{(-1, -7), (2, -5), (4, -2)\}$

Answer

Function

14) $S = \{(-6, -6), (-4, 0), (9, 1)\}$

15) $T = \{(-9, 1), (1, 6), (1, 8)\}$

Answer

Not a function

16) $S = \{(-7, 0), (1, 1), (1, 2)\}$

17) $R = \{(-7, -8), (-7, -6), (-5, 0)\}$

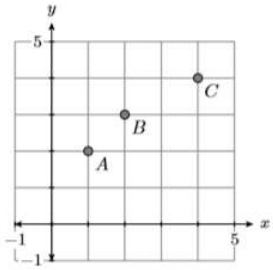
Answer

Not a function

18) $T = \{(-8, -9), (-8, -4), (-5, 9)\}$

In Exercises 19-22, determine whether the given relation is a function.

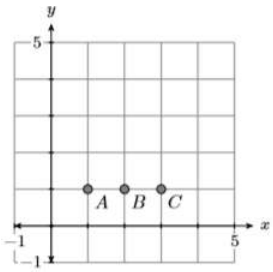
19)



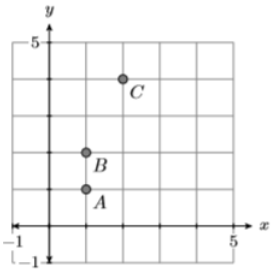
Answer

Function

20)



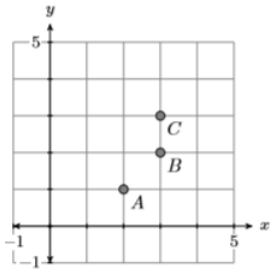
21)



Answer

Not a function

22)



23) Given $f(x) = |6x - 9|$, evaluate $f(8)$.

Answer

39

24) Given $f(x) = |8x - 3|$, evaluate $f(5)$.

25) Given $f(x) = -2x^2 + 8$, evaluate $f(3)$.

Answer

-10

26) Given $f(x) = 3x^2 + x + 6$, evaluate $f(-3)$.

27) Given $f(x) = -3x^2 + 4x + 1$, evaluate $f(2)$.

Answer

-3

28) Given $f(x) = -3x^2 + 4x - 2$, evaluate $f(2)$.

29) Given $f(x) = |5x + 9|$, evaluate $f(-8)$.

Answer

31

30) Given $f(x) = |9x - 6|$, evaluate $f(4)$.

31) Given $f(x) = \sqrt{x - 6}$, evaluate $f(42)$.

Answer

6

32) Given $f(x) = \sqrt{x + 8}$, evaluate $f(41)$.

33) Given $f(x) = \sqrt{x - 7}$, evaluate $f(88)$.

Answer

9

34) Given $f(x) = \sqrt{x + 9}$, evaluate $f(16)$.

35) Given $f(x) = -4x + 6$, evaluate $f(8)$.

Answer

-26

36) Given $f(x) = -9x + 2$, evaluate $f(-6)$.

37) Given $f(x) = -6x + 7$, evaluate $f(8)$.

Answer

-41

38) Given $f(x) = -6x - 2$, evaluate $f(5)$.

39) Given $f(x) = -2x^2 + 3x + 2$ and $g(x) = 3x^2 + 5x - 5$, evaluate $f(3)$ and $g(3)$.

Answer

$f(3) = -7$ and $g(3) = 37$

40) Given $f(x) = 3x^2 - 3x - 5$ and $g(x) = 2x^2 - 5x - 8$, evaluate $f(-2)$ and $g(-2)$.

41) Given $f(x) = 6x - 2$ and $g(x) = -8x + 9$, evaluate $f(-7)$ and $g(-7)$.

Answer

$f(-7) = -44$ and $g(-7) = 65$

42) Given $f(x) = 5x - 3$ and $g(x) = 9x - 9$, evaluate $f(-2)$ and $g(-2)$.

43) Given $f(x) = 4x - 3$ and $g(x) = -3x + 8$, evaluate $f(-3)$ and $g(-3)$.

Answer

$f(-3) = -15$ and $g(-3) = 17$

44) Given $f(x) = 8x + 7$ and $g(x) = 2x - 7$, evaluate $f(-9)$ and $g(-9)$.

45) Given $f(x) = -2x^2 + 5x - 9$ and $g(x) = -2x^2 + 3x - 4$, evaluate $f(-2)$ and $g(-2)$.

Answer

$f(-2) = -27$ and $g(-2) = -18$

46) Given $f(x) = -3x^2 + 5x - 2$ and $g(x) = 3x^2 - 4x + 2$, evaluate $f(-1)$ and $g(-1)$.

5.2: Polynomials

In Exercises 1-6, state the coefficient and the degree of each of the following terms.

1) $3v^5u^6$

Answer

Coefficient $= 3$, Degree $= 11$

2) $-3b^5z^8$

3) $-5v^6$

Answer

Coefficient $= -5$, Degree $= 6$

4) $-5c^3$

5) $2u^7x^4d^5$

Answer

Coefficient $= 2$, Degree $= 16$

6) $9w^4c^5u^7$

In Exercises 7-16, state whether each of the following expressions is a monomial, binomial, or trinomial.

7) $-7b^9c^3$

Answer

Monomial

8) $7b^6c^2$

9) $4u + 7v$

Answer

Binomial

10) $-3b + 5c$

11) $3b^4 - 9bc + 9c^2$

Answer

Trinomial

12) $8u^4 + 5uv + 3v^4$

13) $5s^2 + 9t^7$

Answer

Binomial

14) $-8x^6 - 6y^7$

15) $2u^3 - 5uv - 4v^4$

Answer

Trinomial

16) $6y^3 - 4yz + 7z^3$

In Exercises 17-20, sort each of the given polynomials in descending powers of x.

17) $-2x^7 - 9x^{13} - 6x^{12} - 7x^{17}$

Answer

$-7x^{17} - 9x^{13} - 6x^{12} - 2x^7$

18) $2x^4 - 8x^{19} + 3x^{10} - 4x^2$

19) $8x^6 + 2x^{15} - 3x^{11} - 2x^2$

Answer

$2x^{15} - 3x^{11} + 8x^6 - 2x^2$

20) $2x^6 - 6x^7 - 7x^{15} - 9x^{18}$

In Exercises 21-24, sort each of the given polynomials in ascending powers of x.

21) $7x^{17} + 3x^4 - 2x^{12} + 8x^{14}$

Answer

$3x^4 - 2x^{12} + 8x^{14} + 7x^{17}$

22) $6x^{18} - 6x^4 - 2x^{19} - 7x^{14}$

23) $2x^{13} + 3x^{18} + 8x^7 + 5x^4$

Answer

$5x^4 + 8x^7 + 2x^{13} + 3x^{18}$

24) $-6x^{18} - 8x^{11} - 9x^{15} + 5x^{12}$

In Exercises 25-32, simplify the given polynomial, combining like terms, then arranging your answer in descending powers of x.

25) $-5x + 3 - 6x^3 + 5x^2 - 9x + 3 - 3x^2 + 6x^3$

Answer

$2x^2 - 14x + 6$

26) $-2x^3+8x-x^2+5+7+6x^2+4x^3-9x$

27) $4x^3+6x^2-8x+1+8x^3-7x^2+5x-8$

Answer

$12x^3-x^2-3x-7$

28) $-8x^3-2x^2-7x-3+7x^3-9x^2-8x+9$

29) $x^2+9x-3+7x^2-3x-8$

Answer

$8x^2+6x-11$

30) $-4x^2-6x+3-3x^2+3x-6$

31) $8x+7+2x^2-8x-3x^3-x^2$

Answer

$-3x^3+x^2+7$

32) $-x^2+8-7x+8x-5x^2+4x^3$

In Exercises 33-44, simplify the given polynomial, combining like terms, then arranging your answer in a reasonable order, perhaps in descending powers of either variable. ***Note: Answers may vary, depending on which variable you choose to dictate the order.***

33) $-8x^2-4xz-2z^2-3x^2-8xz+2z^2$

Answer

$-11x^2-12xz$

34) $-5x^2+9xz-4z^2-6x^2-7xz+7z^2$

35) $-6u^3+4uv^2-2v^3-u^3+6u^2v-5uv^2$

Answer

$-7u^3+6u^2v-uv^2-2v^3$

36) $7a^3+6a^2b-5ab^2+4a^3+6a^2b+6b^3$

37) $-4b^2c-3bc^2-5c^3+9b^3-3b^2c+5bc^2$

Answer

$9b^3-7b^2c+2bc^2-5c^3$

38) $4b^3-6b^2c+9bc^2-9b^3-8bc^2+3c^3$

39) $-8y^2+6yz-7z^2-2y^2-3yz-9z^2$

Answer

$-10y^2+3yz-16z^2$

40) $8x^2+xy+3y^2-x^2+7xy+y^2$

41) $7b^2c+8bc^2-6c^3-4b^3+9bc^2-6c^3$

Answer

$-4b^3+7b^2c+17bc^2-12c^3$

42) $7x^3 - 9x^2y + 3y^3 + 7x^3 + 3xy^2 - 7y^3$

43) $9a^2 + ac - 9c^2 - 5a^2 - 2ac + 2c^2$

Answer

$4a^2 - ac - 7c^2$

44) $7u^2 + 3uv - 6v^2 - 6u^2 + 7uv + 6v^2$

In Exercises 45-50, state the degree of the given polynomial.

45) $3x^{15} + 4 + 8x^3 - 8x^{19}$

Answer

19

46) $-4x^6 - 7x^{16} - 5 + 3x^{18}$

47) $7x^{10} - 3x^{18} + 9x^4 - 6$

Answer

18

48) $3x^{16} - 8x^5 + x^8 + 7$

49) $-2 - x^7 - 5x^5 + x^{10}$

Answer

10

50) $x^{11} + 7x^{16} + 8 - 7x^{10}$

51) Given $f(x) = 5x^3 + 4x^2 - 6$, evaluate $f(-1)$.

Answer

-7

52) Given $f(x) = -3x^3 + 3x^2 - 9$, evaluate $f(-1)$.

53) Given $f(x) = 5x^4 - 4x - 6$, evaluate $f(-2)$.

Answer

82

54) Given $f(x) = -2x^4 - 4x - 9$, evaluate $f(2)$.

55) Given $f(x) = 3x^4 + 5x^3 - 9$, evaluate $f(-2)$.

Answer

-1

56) Given $f(x) = -3x^4 + 2x^3 - 6$, evaluate $f(-1)$.

57) Given $f(x) = 3x^4 - 5x^2 + 8$, evaluate $f(-1)$.

Answer

6

58) Given $f(x) = -4x^4 - 5x^2 - 3$, evaluate $f(3)$.

59) Given $f(x) = -2x^3 + 4x - 9$, evaluate $f(2)$.

Answer

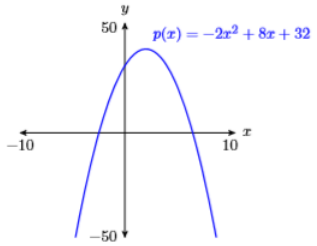
-17

60) Given $f(x) = 4x^3 + 3x + 7$, evaluate $f(-2)$.

In Exercises 61-64, use your graphing calculator to sketch the the given quadratic polynomial. In each case the graph is a parabola, so adjust the WINDOW parameters until the vertex is visible in the viewing window, then follow the Calculator Submission Guidelines when reporting your solution on your homework.

61) $p(x) = -2x^2 + 8x + 32$

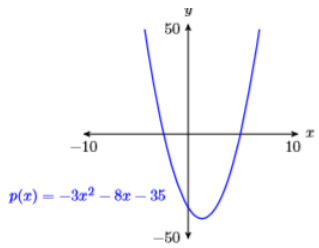
Answer



62) $p(x) = 2x^2 + 6x - 18$

63) $p(x) = 3x^2 - 8x - 35$

Answer

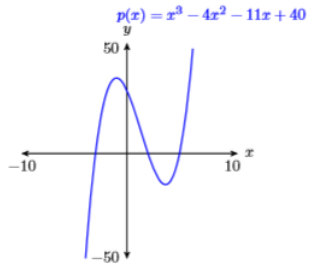


64) $p(x) = -4x^2 - 9x + 50$

In Exercises 65-68, use your graphing calculator to sketch the polynomial using the given WINDOW parameters. Follow the Calculator Submission Guidelines when reporting your solution on your homework.

65) $p(x) = x^3 - 4x^2 - 11x + 30$
$\mathbf{X}\min = -10 \quad \mathbf{X}\max = 10$
$\mathbf{Y}\min = -50 \quad \mathbf{Y}\max = 50$

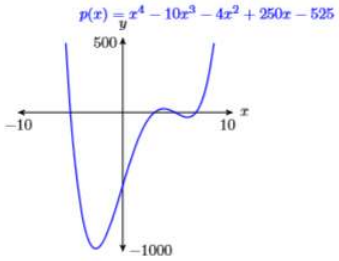
Answer



66) $p(x) = -x^3 + 4x^2 + 27x - 90$
$\mathbf{X}\min = -10 \quad \mathbf{X}\max = 10$
$\mathbf{Y}\min = -150 \quad \mathbf{Y}\max = 50$

67) $p(x) = x^4 - 10x^3 - 4x^2 + 250x - 525$
$\mathbf{X}\min = -10 \quad \mathbf{X}\max = 10$
$\mathbf{Y}\min = -1000 \quad \mathbf{Y}\max = 500$

Answer



68) $p(x) = -x^4 + 2x^3 + 35x^2 - 36x - 180$
$\mathbf{X}\min = -10 \quad \mathbf{X}\max = 10$
$\mathbf{Y}\min = -50 \quad \mathbf{Y}\max = 50$

5.3: Applications of Polynomials

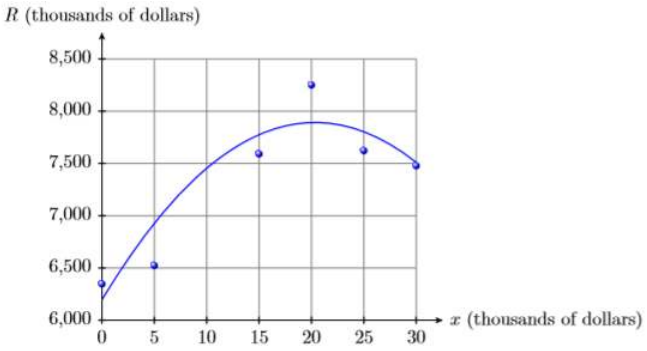
1) A firm collects data on the amount it spends on advertising and the resulting revenue collected by the firm. Both pieces of data are in thousands of dollars.

x (advertising costs)	0	5	15	20	25	30
R (revenue)	6347	6524	7591	8251	7623	7478

The data is plotted then fitted with the following second degree polynomial, where x is the amount invested in thousands of dollars and $R(x)$ is the amount of revenue earned by the firm (also in thousands of dollars).

$$R(x) = -4.1x^2 + 166.8x + 6196$$

Use the graph and then the polynomial to estimate the firm's revenue when the firm invested $\$10,000$ in advertising.



Answer

Approximately $\$7,454,000$

2) The table below lists the estimated number of aids cases in the United States for the years 1999-2003.

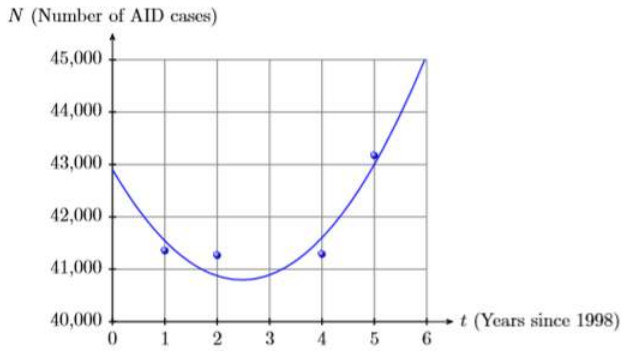
Year	1999	2000	2002	2003

AIDS Cases	41,356	41,267	41,289	43,171

The data is plotted then fitted with the following second degree polynomial, where t is the number of years that have passed since 1998 and $N(t)$ is the number of aids case reported t years after 1998.

$$N(t) = 345.14t^2 - 1705.7t + 42904$$

Use the graph and then the polynomial to estimate the number of AIDS cases in the year 2001.



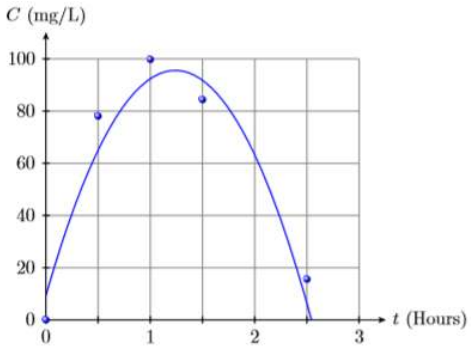
3) The following table records the concentration (in milligrams per liter) of medication in a patient's blood after indicated times have passed.

Time (Hours)	0	0.5	1	0.5	2.5
Concentration(mg/L)	0	78.1	99.8	84.4	15.6

The data is plotted then fitted with the following second degree polynomial, where t is the number of hours that have passed since taking the medication and $C(t)$ is the concentration (in milligrams per liter) of the medication in the patient's blood after t hours have passed.

$$C(t) = -56.214t^2 + 139.31t + 9.35$$

Use the graph and then the polynomial to estimate the the concentration of medication in the patient's blood 2 hours after taking the medication.



Answer

Approximately $63\text{mg}/\text{L}$

4) The following table records the population (in millions of people) of the United States for the given year.

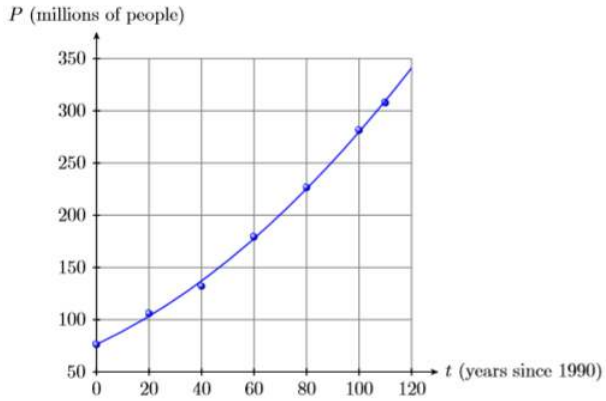
Year	1900	1920	1940	1960	1980	2000	2010

Population (millions)	76.2	106.0	132.2	179.3	226.5	281.4	307.7

The data is plotted then fitted with the following second degree polynomial, where t is the number of years that have passed since 1990 and $P(t)$ is the population (in millions) t years after 1990.

$P(t) = 0.008597t^2 + 1,1738t + 76.41$

Use the graph and then the polynomial to estimate the the population of the United States in the year 1970.



5) If a projectile is launched with an initial velocity of 457 meters per second (457m/s) from a rooftop 75 meters (75m) above ground level, at what time will the projectile first reach a height of 6592 meters (6592m)? Round your answer to the nearest second.

Note: The acceleration due to gravity near the earth's surface is 9.8 *meters per second per second (*9.8m/s^2*).*

Answer

17.6 seconds

6) If a projectile is launched with an initial velocity of 236 meters per second (236m/s) from a rooftop 15 meters (15m) above ground level, at what time will the projectile first reach a height of 1838 meters (1838m)? Round your answer to the nearest second.

Note: The acceleration due to gravity near the earth's surface is 9.8 *meters per second per second (*9.8m/s^2*).*

7) If a projectile is launched with an initial velocity of 229 meters per second (229m/s) from a rooftop 58 meters (58m) above ground level, at what time will the projectile first reach a height of 1374 meters (1374m)? Round your answer to the nearest second.

Note: The acceleration due to gravity near the earth's surface is 9.8 meters per second per second (9.8m/s^2*).*

Answer

6.7 seconds

8) If a projectile is launched with an initial velocity of 234 meters per second (234m/s) from a rooftop 16 meters (16m) above ground level, at what time will the projectile first reach a height of 1882 meters (1882m)? Round your answer to the nearest second.

Note: The acceleration due to gravity near the earth's surface is 9.8 *meters per second per second (*9.8m/s^2*).*

In Exercises 9-12, first use an algebraic technique to find the zero of the given function, then use the 2:zero utility on your graphing calculator to locate the zero of the function. Use the Calculator Submission Guidelines when reporting the zero found using your graphing calculator.

9) $f(x) = 3.25x - 4.875$

Answer

Zero: 1.5

10) $f(x) = 3.125 - 2.5x$

11) $f(x) = 3.9 - 1.5x$

Answer

Zero: 2.6

12) $f(x) = 0.75x + 2.4$

13) If a projectile is launched with an initial velocity of 203 meters per second (203m/s) from a rooftop 52 meters (52m) above ground level, at what time will the projectile return to ground level? Round your answer to the nearest tenth of a second.

Note: The acceleration due to gravity near the earth's surface is 9.8 *meters per second per second* (9.8m/s^2).

Answer

41.7seconds

14) If a projectile is launched with an initial velocity of 484meters per second (484m/s) from a rooftop 17 meters (17m) above ground level, at what time will the projectile return to ground level? Round your answer to the nearest tenth of a second.

Note: The acceleration due to gravity near the earth's surface is 9.8 *meters per second per second* (9.8m/s^2).

15) If a projectile is launched with an initial velocity of 276 meters per second (276m/s) from a rooftop 52 meters (52m) above ground level, at what time will the projectile return to ground level? Round your answer to the nearest tenth of a second.

Note: The acceleration due to gravity near the earth's surface is 9.8 *meters per second per second* (9.8m/s^2).

Answer

56.5seconds

16) If a projectile is launched with an initial velocity of 204 meters per second (204m/s) from a rooftop 92 meters (92m) above ground level, at what time will the projectile return to ground level? Round your answer to the nearest tenth of a second.

Note: The acceleration due to gravity near the earth's surface is 9.8 *meters per second per second* (9.8m/s^2).

5.4: Adding and Subtracting Polynomials

In Exercises 1-8, simplify the given expression. Arrange your answer in some sort of reasonable order.

1) $\left(-8r^2t + 7rt^2 + 3t^3\right) + \left(9r^3 + 2rt^2 + 4t^3\right)$

Answer

$9r^3 - 8r^2t + 9rt^2 + 7t^3$

2) $\left(-a^3 - 8ac^2 - 7c^3\right) + \left(-7a^3 - 8a^2c + 8ac^2\right)$

3) $\left(7x^2 - 6x - 9\right) + \left(8x^2 + 10x + 9\right)$

Answer

$15x^2 + 4x$

4) $\left(-7x^2 + 5x - 6\right) + \left(-10x^2 - 1\right)$

5) $\left(-2r^2 + 7rs + 4s^2\right) + \left(-9r^2 + 7rs - 2s^2\right)$

Answer

$-11r^2 + 14rs + 2s^2$

6) $\left(-2r^2+3rt-4t^2\right)+\left(7r^2+4rt-7t^2\right)$

7) $\left(-8y^3-3y^2z-6z^3\right)+\left(-3y^3+7y^2z-9yz^2\right)$

Answer

$-11y^3+4y^2z-9yz^2-6z^3$

8) $\left(7y^2z+8yz^2+2z^3\right)+\left(8y^3-8y^2z+9yz^2\right)$

In Exercises 9-14, simplify the given expression by distributing the minus sign.

9) $-\left(5x^2-4\right)$

Answer

$-5x^2+4$

10) $-\left(-8x^2-5\right)$

11) $-\left(9r^3-4r^2t-3rt^2+4t^3\right)$

Answer

$-9r^3+4r^2t+3rt^2-4t^3$

12) $-\left(7u^3-8u^2v+6uv^2+5v^3\right)$

13) $-\left(-5x^2+9xy+6y^2\right)$

Answer

$5x^2-9xy-6y^2$

14) $-\left(-4u^2-6uv+5v^2\right)$

In Exercises 15-22, simplify the given expression. Arrange your answer in some sort of reasonable order.

15) $\left(-u^3-4u^2w+7w^3\right)-\left(u^2w+uw^2+3w^3\right)$

Answer

$-u^3-5u^2w-uw^2+4w^3$

16) $\left(-b^2c+8bc^2+8c^3\right)-\left(6b^3+b^2c-4bc^2\right)$

17) $\left(2y^3-2y^2z+3z^3\right)-\left(-8y^3+5yz^2-3z^3\right)$

Answer

$10y^3-2y^2z-5yz^2+6z^3$

18) $\left(4a^3+6ac^2+5c^3\right)-\left(2a^3+8a^2c-7ac^2\right)$

19) $\left(-7r^2-9rs-2s^2\right)-\left(-8r^2-7rs+9s^2\right)$

Answer

$r^2-2rs-11s^2$

20) $\left(-4a^2+5ab-2b^2\right)-\left(-8a^2+7ab+2b^2\right)$

21) $\left(10x^2+2x-6\right)-\left(-8x^2+14x+17\right)$

Answer

$18x^2-12x-23$

22) $\left(-5x^2+19x-5\right)-\left(-15x^2+19x+8\right)$

In Exercises 23-28, for the given polynomial functions $f(x)$ and $g(x)$, simplify $f(x)+g(x)$. Arrange your answer in descending powers of x.

23) $f(x)=-2x^2+9x+7$
$g(x)=8x^3-7x^2+5$

Answer

$8x^3-9x^2+9x+12$

24) $f(x)=-8x^3+6x-9$
$g(x)=x^3-x^2+3x$

25) $f(x)=5x^3-5x^2+8x$
$g(x)=7x^2-2x-9$

Answer

$5x^3+2x^2+6x-9$

26) $f(x)=-x^2+8x+1$
$g(x)=-7x^3+8x-9$

27) $f(x)=-3x^2-8x-9$
$g(x)=5x^2-4x+4$

Answer

$2x^2-12x-5$

28) $f(x)=-3x^2+x-8$
$g(x)=7x^2-9$

In Exercises 29-34, for the given polynomial functions $f(x)$ and $g(x)$, simplify $f(x)-g(x)$. Arrange your answer in descending powers of x.

29) $f(x)=-6x^3-7x+7$
$g(x)=-3x^3-3x^2-8x$

Answer

$-3x^3+3x^2+x+7$

30) $f(x)=5x^3-5x+4$
$g(x)=-8x^3-2x^2-3x$

31) $f(x)=12x^2-5x+4$
$g(x)=8x^2-16x-7$

Answer

$4x^2+11x+11$

32) $f(x)=-7x^2+12x+17$
$g(x)=-10x^2-17$

33) $f(x)=-3x^3-4x+2$
$g(x)=-4x^3-7x^2+6$

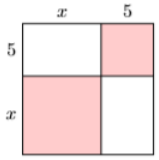
Answer

x^3+7x^2-4x-4

34) $f(x)=-9x^2+9x+3$
$g(x)=7x^3+7x^2+5$

In Exercises 35-36, find the area of the given square by summing the areas of its four parts.

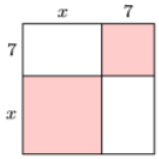
35)



Answer

$x^2+10x+25$

36)



37) Rachel runs a small business selling wicker baskets. Her business costs for producing and selling x wicker baskets are given by the polynomial function $C(x)=232+7x-0.0085x^2$. The revenue she earns from selling x wicker baskets is given by the polynomial function $R(x)=33.45x$. Find a formula for $P(x)$, the profit made from selling x wicker baskets. Use your formula to determine Rachel's profit if she sells 233 wicker baskets. Round your answer to the nearest cent.

Answer

$\$6,392.31$

38) Eloise runs a small business selling baby cribs. Her business costs for producing and selling x baby cribs are given by the polynomial function $C(x)=122+8x-0.0055x^2$. The revenue she earns from selling x baby cribs is given by the polynomial function $R(x)=33.45x$. Find a formula for $P(x)$, the profit made from selling x baby cribs. Use your formula to determine Eloise's profit if she sells 182 baby cribs. Round your answer to the nearest cent.

5.5: Laws of Exponents

In Exercises 1-8, simplify each of the given exponential expressions.

1) $(-4)^3$

Answer

-64

2) $(-9)^2$

3) $\left(-\frac{5}{7}\right)^0$

Answer

1

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4) $\left(-\frac{2}{5}\right)^0$

5) $\left(-\frac{4}{3}\right)^2$

Answer

$\frac{16}{9}$

6) $\left(-\frac{2}{3}\right)^2$

7) $(-19)^0$

Answer

1

8) $(-17)^0$

In Exercises 9-18, simplify each of the given exponential expressions.

9) $(7v-6w)^{18} \cdot (7v-6w)^{17}$

Answer

$(7v-6w)^{35}$

10) $(8a+7c)^3 \cdot (8a+7c)^{19}$

11) $3^4 \cdot 3^0$

Answer

3^4

12) $5^7 \cdot 5^0$

13) $4^n \cdot 4^{8n+3}$

Answer

4^{9n+3}

14) $4^{6m+5} \cdot 4^{m-5}$

15) $x^8 \cdot x^3$

Answer

x^{11}

16) $a^9 \cdot a^{15}$

17) $2^5 \cdot 2^3$

Answer

2^8

18) $2^{10} \cdot 2^3$

In Exercises 19-28, simplify each of the given exponential expressions.

19) $\frac{4^{16}}{4^{16}}$

Answer

1

20) $\frac{3^{12}}{3^{12}}$

21) $\frac{w^{11}}{w^{7}}$

Answer

w^{4}

22) $\frac{c^{10}}{c^{8}}$

23) $\frac{(9a-8c)^{15}}{(9a-8c)^{8}}$

Answer

$(9a-8c)^{7}$

24) $\frac{(4b+7c)^{15}}{(4b+7c)^{5}}$

25) $\frac{2^{9n+5}}{2^{3n-4}}$

Answer

2^{6n+9}

26) $\frac{2^{4k-9}}{2^{3k-8}}$

27) $\frac{4^{17}}{4^{9}}$

Answer

4^{8}

28) $\frac{2^{17}}{2^{6}}$

In Exercises 29-38, simplify each of the given exponential expressions.

29) $\left(4^{8m-6}\right)^{7}$

Answer

4^{56m-42}

30) $\left(2^{2m-9}\right)^{3}$

31) $\left[(9x+5y)^{3}\right]^{7}$

Answer

$(9x+5y)^{21}$

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32) $\left[(4u-v)^8\right]^9$

33) $\left(4^3\right)^2$

Answer

4^6

34) $\left(3^4\right)^2$

35) $\left(c^4\right)^7$

Answer

c^{28}

36) $\left(w^9\right)^5$

37) $\left(6^2\right)^0$

Answer

1

38) $\left(8^9\right)^0$

In Exercises 39-48, simplify each of the given exponential expressions.

39) $(uw)^5$

Answer

u^5w^5

40) $(ac)^4$

41) $(-2y)^3$

Answer

$-8y^3$

42) $(-2b)^3$

43) $\left(3w^9\right)^4$

Answer

$81w^{36}$

44) $\left(-3u^9\right)^4$

45) $\left(-3x^8y^2\right)^4$

Answer

$81x^{32}y^8$

46) $\left(2x^8z^6\right)^4$

47) $\left(7s^{6n}\right)^3$

Answer

$343s^{18n}$

48) $\left(9b^{6n}\right)^3$

In Exercises 49-56, simplify each of the given exponential expressions.

49) $\left(\frac{v}{2}\right)^3$

Answer

$\frac{v^3}{8}$

50) $\left(\frac{t}{9}\right)^2$

51) $\left(-\frac{2}{u}\right)^2$

Answer

$\frac{4}{u^2}$

52) $\left(-\frac{3}{w}\right)^3$

53) $\left(-\frac{r^8}{5}\right)^4$

Answer

$\frac{r^{32}}{625}$

54) $\left(-\frac{x^{11}}{5}\right)^5$

55) $\left(\frac{5}{c^9}\right)^4$

Answer

$\frac{625}{c^{36}}$

56) $\left(\frac{5}{u^{12}}\right)^2$

57) Complete each of the laws of exponents presented in the first column, then use the results to simplify the expressions in the second column.

$a^m a^n = ?$	$a^3 a^5 = ?$
$\frac{a^m}{a^n} = ?$	$\frac{a^6}{a^2} = ?$
$(a^m)^n = ?$	$\left(a^5\right)^7 = ?$
$(ab)^m = ?$	$(ab)^9 = ?$
$\left(\frac{a}{b}\right)^m = ?$	$\left(\frac{a}{b}\right)^3 = ?$

Answer

The general answers are: a^{m+n}, a^{m-n}, a^{mn}, $a^m b^m$, $\frac{a^m}{b^m}$.

The specific answers are: a^8, a^4, a^{35}, $a^9 b^9$, $\frac{a^3}{b^3}$.

5.6: Multiplying Polynomials

In Exercises 1-10, simplify the given expression.

1) $-3(7r)$

Answer

$-21r$

2) $7(3a)$

3) $\left(-9b^3\right)\left(-8b^6\right)$

Answer

$72b^9$

4) $\left(8s^3\right)\left(-7s^4\right)$

5) $\left(-7r^2t^4\right)\left(7r^5t^2\right)$

Answer

$-49r^7t^6$

6) $\left(-10s^2t^8\right)\left(-7s^4t^3\right)$

7) $\left(-5b^2c^9\right)\left(-8b^4c^4\right)$

Answer

$40b^6c^{13}$

8) $\left(-9s^2t^8\right)\left(7s^5t^4\right)$

9) $\left(-8v^3\right)\left(4v^4\right)$

Answer

$-32v^7$

10) $\left(-9y^3\right)\left(3y^5\right)$

In Exercises 11-22, use the distributive property to expand the given expression.

11) $9\left(-2b^2+2b+9\right)$

Answer

$-18b^2+18b+81$

12) $9\left(-4b^2+7b-8\right)$

13) $-4\left(10t^2-7t-6\right)$

Answer

$-40t^2+28t+24$

14) $-5\left(-7u^2-7u+2\right)$

15) $-8u^2\left(-7u^3-8u^2-2u+10\right)$

Answer

$56u^5+64u^4+16u^3-80u^2$

16) $-3s^2\left(-7s^3-9s^2+6s+3\right)$

17) $10s^2\left(-10s^3+2s^2+2s+8\right)$

Answer

$-100s^5+20s^4+20s^3+80s^2$

18) $8u^2\left(9u^3-5u^2-2u+5\right)$

19) $2st\left(-4s^2+8st-10t^2\right)$

Answer

$-8s^3t+16s^2t^2-20st^3$

20) $7uv\left(-9u^2-3uv+4v^2\right)$

21) $-2uw\left(10u^2-7uw-2w^2\right)$

Answer

$-20u^3w+14u^2w^2+4uw^3$

22) $-6vw\left(-5v^2+9vw+5w^2\right)$

In Exercises 23-30, use the technique demonstrated in Example 5.6.8 and Example 5.6.9 to expand each of the following expressions using the distributive property.

23) $(-9x-4)(-3x+2)$

Answer

$27x^2-6x-8$

24) $(4x-10)(-2x-6)$

25) $(3x+8)(3x-2)$

Answer

$9x^2+18x-16$

26) $(-6x+8)(-x+1)$

27) $-12x^3+14x^2+6x-5$

Answer

$-\frac{930}{289}$

28) $(4x-6)\left(-7x^2-10x+10\right)$

29) $(x-6)\left(-2x^2-4x-4\right)$

Answer

$-2x^3+8x^2+20x+24$

30) $(5x-10)\left(-3x^2+7x-8\right)$

In Exercises 31-50, use the shortcut technique demonstrated in Example 5.6.10, Example 5.6.11, and Example 5.6.12 to expand each of the following expressions using the distributive property.

31) $(8u-9w)(8u-9w)$

Answer

$64u^2-144uw+81w^2$

32) $(3b+4c)(-8b+10c)$

33) $(9r-7t)(3r-9t)$

Answer

$27r^2-102rt+63t^2$

34) $(-6x-3y)(-6x+9y)$

35) $(4r-10s)\left(-10r^2+10rs-7s^2\right)$

Answer

$-40r^3+140r^2s-128rs^2+70s^3$

36) $(5s-9t)\left(-3s^2+4st-9t^2\right)$

37) $(9x-2z)\left(4x^2-4xz-10z^2\right)$

Answer

$36x^3-44x^2z-82xz^2+20z^3$

38) $(r-4t)\left(7r^2+4rt-2t^2\right)$

39) $(9r+3t)^2$

Answer

$81r^2+54rt+9t^2$

40) $(4x+8z)^2$

41) $(4y+5z)(4y-5z)$

Answer

$16y^2-25z^2$

42) $(7v+2w)(7v-2w)$

43) $(7u+8v)(7u-8v)$

Answer

$49u^2-64v^2$

44) $(6b+8c)(6b-8c)$

45) $(7b+8c)^2$

Answer

$49b^2+112bc+64c^2$

46) $(2b+9c)^2$

47) $\left(2t^2+9t+4\right)\left(2t^2+9t+4\right)$

Answer

$4t^4+36t^3+97t^2+72t+16$

48) $\left(3a^2-9a+4\right)\left(3a^2-9a+2\right)$

49) $\left(4w^2+3w+5\right)\left(3w^2-6w+8\right)$

Answer

$12w^4-15w^3+29w^2-6w+40$

50) $\left(4s^2+3s+8\right)\left(2s^2+4s-9\right)$

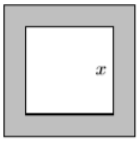
51) The demand for widgets is given by the function $x=320-0.95p$, where x is the demand and p is the unit price. What unit price should a retailer charge for widgets in order that his revenue from sales equals $\$7,804$? Round your answers to the nearest cent.

Answer

$\$26.47, \$310.37$

52) The demand for widgets is given by the function $x=289-0.91p$, where x is the demand and p is the unit price. What unit price should a retailer charge for widgets in order that his revenue from sales equals $\$7,257$? Round your answers to the nearest cent.

53) In the image that follows, the edge of the outer square is 6 inches longer than 3 times the edge of the inner square.

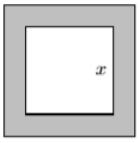


a. Express the area of the shaded region as a polynomial in terms of x, the edge of the inner square. Your final answer must be presented as a second degree polynomial in the form $A(x)=ax^2+bx+c$.
b. Given that the edge of the inner square is 5 inches, use the polynomial in part (a) to determine the area of the shaded region.

Answer

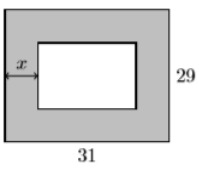
$A(x)=8x^2+36x+36$, $A(5)=416$ square inches

54) In the image that follows, the edge of the outer square is 3 inches longer than 2 times the edge of the inner square.



a. Express the area of the shaded region as a polynomial in terms of x, the edge of the inner square. Your final answer must be presented as a second degree polynomial in the form $A(x)=ax^2+bx+c$.
b. Given that the edge of the inner square is 4 inches, use the polynomial in part (a) to determine the area of the shaded region.

55) A rectangular garden is surrounded by a uniform border of lawn measuring x units wide. The entire rectangular plot measures 31 by 29 feet.

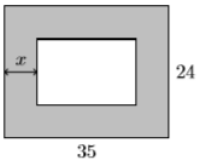


a. Find the area of the interior rectangular garden as a polynomial in terms of x. Your final answer must be presented as a second degree polynomial in the form $A(x) = ax^2 + bx + c$.
b. Given that the width of the border is 9.3 feet, use the polynomial in part (a) to determine the area of the interior rectangular garden.

Answer

$899 - 120x + 4x^2$, 128.96square feet

56) A rectangular garden is surrounded by a uniform border of lawn measuring x units wide. The entire rectangular plot measures 35 by 24 feet.



a. Find the area of the interior rectangular garden as a polynomial in terms of x. Your final answer must be presented as a second degree polynomial in the form $A(x) = ax^2 + bx + c$.
b. Given that the width of the border is 1.5 feet, use the polynomial in part (a) to determine the area of the interior rectangular garden.

5.7: Special Products

In Exercises 1-12, use the FOIL shortcut as in Example 5.7.3 and Example 5.7.4 to multiply the given binomials.

1) $(5x + 2)(3x + 4)$

Answer

$15x^2 + 26x + 8$

2) $(5x + 2)(4x + 3)$

3) $(6x - 3)(5x + 4)$

Answer

$30x^2 + 9x - 12$

4) $(6x - 2)(4x + 5)$

5) $(5x - 6)(3x - 4)$

Answer

$15x^2 - 38x + 24$

6) $(6x - 4)(3x - 2)$

7) $(6x - 2)(3x - 5)$

Answer

$18x^2 - 36x + 10$

8) $(2x - 3)(6x - 4)$

9) $(6x + 4)(3x + 5)$

Answer

$18x^2 + 42x + 20$

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10) $(3x+2)(4x+6)$

11) $(4x-5)(6x+3)$

Answer

$24x^2-18x-15$

12) $(3x-5)(2x+6)$

In Exercises 13-20, use the difference of squares shortcut as in Example 5.7.5 to multiply the given binomials.

13) $(10x-12)(10x+12)$

Answer

$100x^2-144$

14) $(10x-11)(10x+11)$

15) $(6x+9)(6x-9)$

Answer

$36x^2-81$

16) $(9x+2)(9x-2)$

17) $(3x+10)(3x-10)$

Answer

$9x^2-100$

18) $(12x+12)(12x-12)$

19) $(10x-9)(10x+9)$

Answer

$100x^2-81$

20) $(4x-6)(4x+6)$

In Exercises 21-28, use the squaring a binomial shortcut as in Example 5.7.8 to expand the given expression.

21) $(2x+3)^2$

Answer

$4x^2+12x+9$

22) $(8x+9)^2$

23) $(9x-8)^2$

Answer

$81x^2-144x+64$

24) $(4x-5)^2$

25) $(7x+2)^2$

Answer

$49x^2+28x+4$

26) $(4x+2)^2$

27) $(6x-5)^2$

Answer

$36x^2-60x+25$

28) $(4x-3)^2$

In Exercises 29-76, use the appropriate shortcut to multiply the given binomials.

29) $(11x-2)(11x+2)$

Answer

$121x^2-4$

30) $(6x-7)(6x+7)$

31) $(7r-5t)^2$

Answer

$49r^2-70rt+25t^2$

32) $(11u-9w)^2$

33) $(5b+6c)(3b-2c)$

Answer

$15b^2+8bc-12c^2$

34) $(3r+2t)(5r-3t)$

35) $(3u+5v)(3v-5v)$

Answer

$9u^2-25v^2$

36) $(11a+4c)(11a-4c)$

37) $\left(9b^3+10c^5\right)\left(9b^3-10c^5\right)$

Answer

$81b^6-100c^{10}$

38) $\left(9r^5+7t^2\right)\left(9r^5-7t^2\right)$

39) $(9s-4t)(9s+4t)$

Answer

$81s^2-16t^2$

40) $(12x-7y)(12x+7y)$

41) $(7x-9y)(7x+9y)$

Answer

$49x^2-81y^2$

42) $(10r-11t)(10r+11t)$

43) $(6a-6b)(2a+3b)$

Answer

$12a^2+6ab-18b^2$

44) $(6r-5t)(2r+3t)$

45) $(10x-10)(10x+10)$

Answer

$100x^2-100$

46) $(12x-8)(12x+8)$

47) $(4a+2b)(6a-3b)$

Answer

$24a^2-6b^2$

48) $(3b+6c)(2b-4c)$

49) $(5b-4c)(3b+2c)$

Answer

$15b^2-2bc-8c^2$

50) $(3b-2c)(4b+5c)$

51) $(4b-6c)(6b-2c)$

Answer

$24b^2-44bc+12c^2$

52) $(4y-4z)(5y-3z)$

53) $\left(11r^5+9t^2\right)^2$

Answer

$121r^{10}+198r^5t^2+81t^4$

54) $\left(11x^3+10z^5\right)^2$

55) $(4u-4v)(2u-6v)$

Answer

$8u^2-32uv+24v^2$

56) $(4u-5w)(5u-6w)$

57) $\left(8r^4+7t^5\right)^2$

Answer

$64r^8+112r^4t^5+49t^{10}$

58) $\left(2x^5+5y^2\right)^2$

59) $(4r+3t)(4r-3t)$

Answer

$16r^2 - 9t^2$

60) $(3r + 4s)(3r - 4s)$

61) $(5r + 6t)^2$

Answer

$25r^2 + 60rt + 36t^2$

62) $(12v + 5w)^2$

63) $(3x - 4)(2x + 5)$

Answer

$6x^2 + 7x - 20$

64) $(5x - 6)(4x + 2)$

65) $(6b + 4c)(2b + 3c)$

Answer

$12b^2 + 26bc + 12c^2$

66) $(3v + 6w)(2v + 4w)$

67) $\left(11u^2 + 8w^3\right)\left(11u^2 - 8w^3\right)$

Answer

$121u^4 - 64w^6$

68) $\left(3u^3 + 11w^4\right)\left(3u^3 - 11w^4\right)$

69) $(4y + 3z)^2$

Answer

$16y^2 + 24yz + 9z^2$

70) $(11b + 3c)^2$

71) $(7u - 2v)^2$

Answer

$49u^2 - 28uv + 4v^2$

72) $(4b - 5c)^2$

73) $(3v + 2w)(5v + 6w)$

Answer

$15v^2 + 28vw + 12w^2$

74) $(5y + 3z)(4y + 2z)$

75) $(5x - 3)(6x + 2)$

Answer

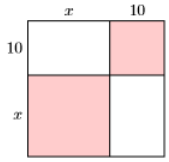
$30x^2 - 8x - 6$

76) $(6x - 5)(3x + 2)$

For each of the following figure, compute the area of the square using two methods.

a. **Find the area by summing the areas of its parts (see Example 5.5.7).**
b. **Find the area by squaring the side of the square using the squaring a binomial shortcut.**

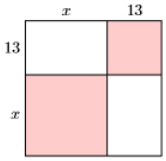
77)



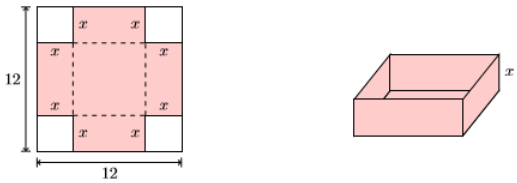
Answer

$A = x^2 + 20x + 100$

78)



79) A square piece of cardboard measures 12 inches on each side. Four squares, each having a side of x inches, are cut and removed from each of the four corners of the square piece of cardboard. The sides are then folded up along the dashed lines to form a box with no top.



a. Find the volume of the box as a function of x, the measure of the side of each square cut from the four corner of the original piece of cardboard. Multiply to place your answer in standard polynomial form, simplifying your answer as much as possible.
b. Use the resulting polynomial to determine the volume of the box if squares of length 1.25 inches are cut from each corner of the original piece of cardboard. Round your answer to the nearest cubic inch.

Answer

a. $V(x) = 144x - 48x^2 + 4x^3$
b. $V(1.25) \approx 113$ cubic inches

80) Consider again the box formed in Exercise 79.

a. Find the surface area of the box as a function of x, the measure of the side of each square cut from the four corner of the original piece of cardboard. Multiply to place your answer in standard polynomial form, simplifying your answer as much as possible.
b. Use the resulting polynomial to determine the surface area of the box if squares of length 1.25 inches are cut from each corner of the original piece of cardboard. Round your answer to the nearest square inch.

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CHAPTER OVERVIEW

7: Factoring and Solving by Factoring

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7.1: Introduction to Factoring

Learning Objectives

- Determine the greatest common factor (GCF) of natural numbers.
- Determine the GCF of monomials.
- Factor out the GCF of a polynomial.
- Factor a four-term polynomial by grouping.

GCF of Natural Numbers

The process of writing a number or expression as a product is called factoring. If we write $60 = 5 \cdot 12$, we say that the product $5 \cdot 12$ is a factorization of 60 and that 5 and 12 are factors. Typically, there are many ways to factor a number. For example,

$$\begin{aligned} 60 &= 6 \cdot 10 \\ 60 &= 2 \cdot 30 \qquad \textit{Factorizations of } 60 \\ 60 &= 4 \cdot 3 \cdot 5 \end{aligned}$$

Recall that a prime number is defined as a natural number with exactly two natural number factors, 1 and itself. The first ten prime numbers follow:

$$2,\ 3,\ 5,\ 7,\ 11,\ 13,\ 17,\ 19,\ 23,\ 29,\ \ldots$$

Any natural number greater than 1 can be uniquely written as a product of prime numbers. This product is called the prime factorization. The prime factorization of 60 can be determined by continuing to factor until only a product of prime numbers remains.

$$\begin{aligned} 60 &= 2 \cdot 30 \\ &= 2 \cdot 2 \cdot 15 \\ &= 2 \cdot 2 \cdot 3 \cdot 5 \end{aligned}$$

Since the prime factorization is unique, it does not matter how we choose to initially factor the number; the end result will be the same. The prime factorization of 60 follows:

$$60 = 2^2 \cdot 3 \cdot 5 \qquad \textit{The prime factorization of } 60$$

Recall that the greatest common factor (GCF) of any two natural numbers is the product of all the common prime factors.

Example 7.1.1

Find the GCF of 60 and 140.

Solution:

First, determine the prime factorizations of both integers.

$$\begin{array}{r|l} 140 = 10 \cdot 14 & 60 = 6 \cdot 10 \\ = 2 \cdot 5 \cdot 2 \cdot 7 & = 2 \cdot 3 \cdot 2 \cdot 5 \\ = 2^2 \cdot 5 \cdot 7 & = 2^2 \cdot 3 \cdot 5 \end{array}$$

The product of the common prime factors is $2^2 \cdot 5$; hence the $\text{GCF}(60, 140) = 2^2 \cdot 5 = 20$. To see that it is the greatest common factor, we can write the following:

$$\begin{aligned} 140 &= 20 \cdot 7 \\ 60 &= 20 \cdot 3 \end{aligned}$$

Answer:

The greatest common factor of 60 and 140 is 20.

Example 7.1.2

Find the GCF of 504 and 1,080.

Solution:

First, determine the prime factorizations of both integers.

$$\begin{aligned} 504 &= 9 \cdot 56 \\ &= 3 \cdot 3 \cdot 7 \cdot 8 \\ &= 3 \cdot 3 \cdot 7 \cdot 2 \cdot 2 \cdot 2 \\ &= 2^3 \cdot 3^2 \cdot 7 \end{aligned} \qquad \begin{aligned} 1080 &= 10 \cdot 108 \\ &= 2 \cdot 5 \cdot 9 \cdot 12 \\ &= 2 \cdot 5 \cdot 3 \cdot 3 \cdot 2 \cdot 2 \cdot 3 \\ &= 2^3 \cdot 3^3 \cdot 5 \end{aligned}$$

The product of the common prime factors is $2^3 \cdot 3^2$.

The GCF$(504, 1080) = 2^3 \cdot 3^2 = 72$. Note that we multiplied the common prime factors with the smallest exponent.

$$\begin{aligned} 5040 &= 72 \cdot 7 \\ 1080 &= 72 \cdot 15 \end{aligned}$$

The numbers 7 and 15 share no common natural number factor other than 1; we say that they are relatively prime.

Answer:

The greatest common factor of 504 and $1,080$ is 72.

GCF of Monomials

We next consider factorizations of monomials. For example, $6x$ and x^4 are factors of $6x^5$ because $6x^5 = 6x \cdot x^4$. Typically, there are many ways to factor a monomial. Some factorizations of $6x^5$ follow:

$$\begin{aligned} 6x^5 &= 2x^3 \cdot 3x^2 \\ 6x^5 &= 6x^4 \cdot x \qquad \textit{Factorizations of } 6x^5 \\ 6x^5 &= 2x \cdot 3x \cdot x^3 \end{aligned}$$

Given two or more monomials, it will be useful to find the greatest common monomial factor of each. For example, consider $6x^5y^3z$ and $8x^2y^3z^2$. The variable part of these two monomials look very much like the prime factorization of natural numbers and, in fact, can be treated the same way.

Steps for finding the GCF of monomials are outlined in the following example.

Example 7.1.3

Find the GCF of $6x^5y^3z$ and $8x^2y^3z^2$.

Solution:

Step 1: Find the GCF of the coefficients.

$$6x^5y^3z \qquad \text{and} \qquad 8x^2y^3z^2$$

In this case, the GCF$(6, 8) = 2$.

Step 2: Determine the common variable factors with smallest exponents.

$$6x^5y^3z \qquad \text{and} \qquad 8x^2y^3z^2$$

In this case, the common variables with the smallest exponents are $x^2, y^3, and z^1$.

Step 3: The GCF of the monomials is the product of the common variable factors and the GCF of the coefficients. Therefore,

$$\text{GCF}\left(6x^5y^3z, 8x^2y^3z^2\right) = 2 \cdot x^2 \cdot y^3 \cdot z$$

Answer:

$$2x^2y^3z$$

It is worth pointing out that the GCF in the previous example divides both expressions evenly:

$$\frac{6x^5y^3z}{2x^2y^3z}=3x^3 \qquad \text{and} \qquad \frac{8x^2y^3z^2}{2x^2y^3z}=4z$$

Furthermore, we can write the following:

$$6x^5y^3z=2x^2y^3z\cdot 3x^3 \qquad \text{and} \qquad 8x^2y^3z^2=2x^2y^3z\cdot 4z$$

The factors $3x^3$ and $4z$ share no common monomial factors other than 1; they are relatively prime.

Example 7.1.4

Determine the GCF of the following expressions:

$$30x^6y \text{ and } 18x^4y^2z.$$

Solution:

The prime factorizations of the coefficients are

$$30=2\cdot 3\cdot 5$$
$$18=2\cdot 3\cdot 3$$

Thus the GCF$(30,18)=2\cdot 3=6$. Next, consider the variable part:

$$30x^6y \qquad \text{and} \qquad 18x^4y^2z$$

The variable factors in common are x^4 and y. The factor z is not in common and we have

$$\text{GCF}=6\cdot x^4\cdot y$$

Answer:

$$6x^4y$$

Example 7.1.5

Determine the GCF of the following three expressions:

$$12a^5b^2(a+b)^5\ ,\ 60a^4b^3c(a+b)^3 \text{ , and } 24a^2b^7c^3(a+b)^2\,.$$

Solution:

First, determine the GCF of the coefficients.

$$12=2^2\cdot 3$$
$$60=2^2\cdot 3\cdot 5$$
$$24=2^3\cdot 3$$

The GCF $(12,60,24)=2^2\cdot 3=12$. Next, determine the common factors of the variable part:

$$12a^5b^2(a+b)^5 \qquad \text{and} \qquad 60a^4b^3c(a+b)^3 \qquad \text{and} \qquad 24a^2b^7c^3(a+b)^2$$

The variable factors in common are a^2, b^2, and $(a+b)^2$. Therefore,

$$\text{GCF}=12\cdot a^2\cdot b^2\cdot (a+b)^2$$

Answer:

$12a^2b^2(a+b)^2$. Note that the variable c is not common to all three expressions and thus is not included in the GCF.

Exercise 7.1.1

Determine the GCF of the following:

$$60x^4y^3(x+2y)^7, 45x^2y^5(x+2y)^4, \text{ and } 30x^7y^7(x+2y)^3.$$

Answer

$15x^2y^3(x+2y)^3$

Factoring out the GCF

We have seen that application of the distributive property is the key to multiplying polynomials. The process of factoring a polynomial involves using the distributive property in reverse to write each polynomial as a product of polynomial factors.

$$a(b+c)=ab+ac \quad Multiplying$$
$$ab+ac=a(b+c) \quad Factoring$$

To demonstrate this idea, we multiply and factor side by side. Factoring utilizes the GCF of the terms.

Table 7.1.1

Multiplying	Factoring
$3(5x+1)=3\cdot 5x+3\cdot 1$ $=15x+3$	$15x+3=3\cdot 5x+3\cdot 1$ $=3(5x+1)$
$2x^2(3x^3+4)=2x^2\cdot 3x^3+2x^2\cdot 4$ $=6x^5+8x^2$	$6x^5+8x^2=2x^2\cdot 3x^3+2x^2\cdot 4$ $=2x^2(3x^3+4)$

In the previous example, we see that the distributive property allows us to write the polynomial $6x^5+8x^2$ as a product of the two factors $2x^2$ and $(3x^3+4)$. Note that in this case, $2x^2$ is the GCF of the terms of the polynomial:

$$\text{GCF } 6x^5, 8x^2)=2x^2$$

Factoring out the GCF involves rewriting a polynomial as a product where a factor is the GCF of all of its terms:

$$15x+3=3(5x+1) \quad Factoring\ out\ the\ GCF$$
$$6x^5+8x^2=2x^2(3x^3+4)$$

The steps for factoring out the GCF of a polynomial are outlined in the following example.

Example 7.1.6

Factor out the GCF:

$$7x^4+21x^3-14x^2\,.$$

Solution:

Step 1: Identify the GCF of all the terms. In this case, the GCF$(7,21,14)=7$, and the common variable factor with the smallest exponent is x^2. The GCF of the polynomial is $7x^2$.

$$7x^4+21x^3-14x^2=7x^2(\quad ?\quad)$$

Step 2: Determine the terms of the missing factor by dividing each term of the original expression by the GCF. (**This step is usually performed mentally**.)

$$\frac{7x^4}{7x^2}=x^2 \qquad \frac{21x^3}{7x^2}=3x \qquad \frac{-14x^2}{7x^2}=-2$$

Step 3: Apply the distributive property (in reverse) using the terms found in the previous step.

$$7x^4+21x^3-14x^2=7x^2(x^2+3x-2)$$

Step 4: As a check, multiply using the distributive property to verify that the product equals the original expression. (**This step is optional and can be performed mentally**.)

$$7x^2(x^2+3x-2)=7x^2\cdot x^2+7x^2\cdot 3x-7x^2\cdot 2$$
$$=7x^4+21x^3-14x^2 \quad \checkmark$$

Answer:

$$7x^2(x^2+3x-2)$$

Example 7.1.7

Factor out the GCF:

$$48a-16b+4c$$

Solution:

There are no variable factors in common and the GCF$(48,16,4)=4$.

$$\begin{aligned} 48a-16b+4c &= 4(\quad ? \quad) \\ &= 4(12a-4b+c) \end{aligned}$$

Answer:

$$4(12a-4b+c)$$

Example 7.1.8

Factor out the GCF:

$$25x^3+15x^2+5x$$

Solution:

The GCF$(25,15,5)=5$, and the common variable factor with smallest exponents is x^1. The GCF of all the terms is $5x$.

$$\begin{aligned} 25x^3+15x^2+5x &= 5x(\quad ? \quad) \\ &= 5x(5x^2+3x+1) \end{aligned}$$

Answer:

$$5x(5x^2+3x+1)$$

If the GCF is the same as one of the terms, then, after the GCF is factored out, a constant term 1 will remain. In the previous example, we can see that $\frac{5x}{5x}=1$. The importance of remembering the constant term becomes clear when performing the check using the distributive property:

$$\begin{aligned} 5x(5x^2+3x+1) &= 5x\cdot 5x^2+5x\cdot 3x+5x\cdot 1 \\ &= 25x^3+15x^2+5x \quad \checkmark \end{aligned}$$

The constant term 1 allows us to obtain the same original expression after we distribute.

Example 7.1.9

Factor out the GCF:

$$15x^6y^4+10x^5y^3z^2-20xy^6z^3.$$

Solution:

The GCF$(10,15,20)=5$, and the common variables with smallest exponent are x^1 and y^3. Therefore, the GCF of the terms is $5xy^3$. The first term does not have a variable factor of z and thus cannot be a part of the greatest common factor. If we divide each term by $5xy^3$, we obtain

$$\frac{15x^6y^4}{5xy^3}=3x^5y \qquad \frac{10x^5y^3z^2}{5xy^3}=2x^4z^2 \qquad \frac{-20xy^6z^3}{5xy^3}=-4y^3z^3$$

and can write

$$\begin{aligned} 15x^6y^4+10x^5y^3z^2-20xy^6z^3 &= 5xy^3(\quad ? \quad) \\ &= 5xy^3(3x^5y+2x^4z^2-4y^3z^3) \end{aligned}$$

Answer:

$$5xy^3(3x^5y+2x^4z^2-4y^3z^3)$$

Example 7.1.10

Factor out the GCF:

$$24a^6b^2c^5+8a^7b^5c$$

Solution:

The GCF$(24,8)=8$, and the variable factors with smallest exponents are a^6, b^2, and c. Therefore, the GCF of all the terms is $8a^6b^2c$.

$$\begin{aligned}24a^6b^2c^5+8a^7b^5c &= 8a^6b^2c(\quad ?\quad)\\ &= 8a^6b^2c(3c^4+ab^3)\end{aligned}$$

Answer:

$$8a^6b^2c(3c^4+ab^3)$$

Of course, not all polynomials with integer coefficients can be factored as a product of polynomials with integer coefficients other than 1 and itself. If this is the case, then we say that it is a prime polynomial.

Example 7.1.11

Factor:

$$3x-5$$

Solution:

Prime: there are no polynomial factors other than 1 and itself.

Answer:

Prime

Exercise 7.1.2

Factor out the GCF:

$$16x^4y^3-8x^2y^5-4x^2y\,.$$

Answer

$4x^2y(4x^2y^2-2y^4-1)$

Factor by Grouping

In this section, we outline a technique for factoring polynomials with four terms. First, review some preliminary examples where the terms have a common binomial factor.

Example 7.1.12

Factor:

$$5x(x-3)+2(x-3)\,.$$

Solution:

This expression is a binomial with terms $5x(x-3)$ and $2(x-3)$. In this case, $(x-3)$ is a common factor. Begin by factoring this common factor out:

$$5x(x-3)+2(x-3)=(x-3)(\quad ?\quad)$$

To determine the terms of the remaining factor, divide each term by $(x-3)$:

$$\frac{5x(x-3)}{(x-3)}=5x \qquad \frac{2(x-3)}{(x-3)}=2$$

This step is typically performed mentally. We have

$$\begin{aligned} 5x(x-3)+2(x-3) &= (x-3)(\quad ?\quad) \\ &= (x-3)(5x+2) \end{aligned}$$

Answer:

$$(x-3)(5x+2)$$

Recall that 1 is always a common factor. If the GCF is the same as a term, then the factor 1 remains after we factor out that GCF.

Example 7.1.13

Factor:

$$3x(4x+1)-(4x+1)\,.$$

Solution:

Rewrite the second term $-(4x+1)$ as $-1(4x+1)$ and then factor out the common binomial factor $(4x+1)$.

$$\begin{aligned} 3x(4x+1)-(4x+1) &= 3x(4x+1)-1(4x+1) \\ &= (4x+1)(\quad ?\quad) \\ &= (4x+1)(3x-1) \end{aligned}$$

Answer:

$$(4x+1)(3x-1)$$

Remember that the goal for this section is to develop a technique that enables us to factor polynomials with four terms into a product of binomials. The intermediate step of this process looks like the previous two examples. For example, we wish to factor

$$5x^2-15x+2x-6$$

Begin by grouping the first two terms and the last two terms. Then factor out the GCF of each grouping:

$$\begin{aligned} &\underbrace{5x^2-15x}_{group} \quad + \quad \underbrace{2x-6}_{group} \\ &= 5x(x-3)+2(x-3) \end{aligned}$$

In this form, it is a binomial with a common binomial factor, (x−3).

$$\begin{aligned} &= (x-3)(\quad ?\quad) \\ &= (x-3)(5x+2) \end{aligned}$$

The steps that follow outline a technique for factoring four-term polynomials called factor by grouping.

Example 7.1.14

Factor:

$$2x^3+4x^2+3x+6\ .$$

Solution:

Group terms in such a way as to obtain a binomial with common factors.

Step 1: Group the first two and last two terms and then factor out the GCF of each.

$$\begin{aligned} &\underbrace{2x^3+4x^2}_{group} \quad + \quad \underbrace{3x+6}_{group} \end{aligned}$$

The GCF of the first two terms is $2x^2$, and the GCF of the second two terms is 3.

$$2x^3+4x^2+3x+6 = \underbrace{2x^3+4x^2}_{group} + \underbrace{3x+6}_{group}$$
$$= 2x^2(\quad ?\quad)+3(\quad ?\quad)$$
$$= 2x^2(x+2)+3(x+2)$$

Step 2: At this point, the polynomial is a binomial. Factor out any factors common to both terms. Here $(x+2)$ is a common factor.

$$= 2x^2(x+2)+3(x+2)$$
$$= (x+2)(\quad ?\quad)$$
$$= (x+2)(2x^2+3)$$

Step 3: Optional check: multiply to verify that we obtain the original expression.

$$(x+2)(2x^2+3) = 2x^3+3x+4x^2+6$$
$$= 2x^3+4x^2+3x+6 \quad \checkmark$$

Answer:

$$(x+2)(2x^2+3)$$

Example 7.1.15

Factor:

$$2a^3-3a^2+2a-3\ .$$

Solution:

The GCF of the first two terms is a^2 and the GCF of the second two terms is 1.

$$2a^3-3a^2+2a-3 = \underbrace{2a^3-3a^2}_{group} + \underbrace{2a-3}_{group}$$
$$= a^2(\quad ?\quad)+1(\quad ?\quad)$$
$$= a^2(2a-3)+1(2a-3)$$
$$= (2a-3)(\quad ?\quad)$$
$$= (2a-3)(a^2+1)$$

Answer:

$2a-3)(a^2+1)$. The check is left to the reader.

Example 7.1.16

Factor:

$$6x^4-24x^3-5x+20\ .$$

Solution:

The GCF for the first group is $6x^3$. We have to choose 5 or -5 to factor out of the second group.

$$\underbrace{6x^4-24x^3}_{group} - \underbrace{5x+20}_{group}$$
$$= 6x^3(x-4)+5(-x+4) \qquad x$$
$$= 6x^3(x-4)-5(x-4) \qquad \checkmark$$

Factoring out a $+5$ does not result in a common binomial factor. If we choose to factor out -5, then we obtain a common binomial factor and can proceed. Note that when factoring out a negative number, we change the signs of the factored terms.

$$6x^4-24x^3-5x+20=\underbrace{6x^4-24x^3}_{group}-\underbrace{5x+20}_{group}$$
$$=6x^3(\quad ?\quad)-5(\quad ?\quad)$$
$$=6x^3(x-4)-5(x-4)$$
$$=(x-4)(\quad ?\quad)$$
$$=(x-4)(6x^3-5)$$

Answer:

$(x-4)(6x^3-5)$. The check is left to the reader.

Note

The sign of the leading coefficient in the second grouping usually indicates whether or not to factor out a negative factor. If that coefficient is positive, factor out a positive factor. If it is negative, factor out a negative factor.

When all the terms of a polynomial have a GCF other than 1, it is a best practice to factor that out before factoring by grouping.

Example 7.1.17

Factor:

$$3y^4+9y^2-6y^3-18y\ .$$

Solution:

Here we notice that the greatest common factor of all the terms is $3y$. Begin by factoring out the GCF and then factor the result by grouping.

$$\begin{aligned}&3y^4+9y^2-6y^3-18y\\&=3y[y^3+3y-2y^2-6] && \textit{Factor out the GCF.}\\&=3y[y(y^2+3)-2(y^2+3)] && \textit{Factor by grouping.}\\&=3y[(y^2+3)(y-2)]\\&=3y(y^2+3)(y-2)\end{aligned}$$

Answer:

$$3y(y^2+3)(y-2)$$

Sometimes we must first rearrange the terms in order to obtain a common factor.

Example 7.1.18

Factor:

$$ab-2a^2b+a^3-2b^3\ .$$

Solution:

Simply factoring the GCF out of the first group and last group does not yield a common binomial factor.

$$\underbrace{ab-2a^2b}_{group}+\underbrace{a^3-2b^3}_{group}$$
$$=ab(1-2a)+1(a^3-2b^3)$$

We must rearrange the terms, searching for a grouping that produces a common factor. In this example, we have a workable grouping if we switch the terms a^3 and ab.

$$ab-2a^2b+a^3-2b^3 = \underbrace{a^3-2a^2b}_{group} + \underbrace{ab-2b^3}_{group}$$
$$= a^2(a-2b)+b(a-2b)$$
$$= (a-2b)(a^2+b)$$

Answer:

$$(a-2b)(a^2+b)$$

Not all factorable four-term polynomials can be factored with this technique. For example,

$$3x^3+5x^2-x+2$$

This four-term polynomial cannot be grouped in any way as to produce a common binomial factor. Despite this, the polynomial is not prime and can be written as a product of polynomials. It can be factored as follows:

$$3x^3+5x^2-x+2 = (x+2)(3x^2-x+1)$$

Factoring such polynomials is something that we will learn to do as we move further along in our study of algebra. For now, we will limit our attempt to factor four-term polynomials to using the factor by grouping technique.

? Exercise 7.1.3

Factor:

$$x^3-x^2y-xy+y^2$$

Answer

$(x-y)(x^2-y)$

Key Takeaways

- To find the greatest common factor (GCF) of any collection of natural numbers, first find the prime factorization of each. The GCF is the product of all the common prime factors.
- The GCF of two or more monomials is the product of the GCF of the coefficients and the common variable factors with the smallest power.
- If the terms of a polynomial have a greatest common factor, then factor out that GCF using the distributive property. Divide each term of the polynomial by the GCF to determine the terms of the remaining factor.
- Some four-term polynomials can be factored by grouping the first two terms and the last two terms. Factor out the GCF of each group and then factor out the common binomial factor.
- When factoring by grouping, you sometimes have to rearrange the terms to find a common binomial factor. After factoring out the GCF, the remaining binomial factors must be the same for the technique to work.
- Not all polynomials can be factored as the product of polynomials with integer coefficients. In this case, we call it a prime polynomial.

? Exercise 7.1.4 GCF of Natural Numbers

Give the prime factorization of each number and determine the GCF.

1. $18, 24$
2. $45, 75$
3. $72, 60$
4. $168, 175$
5. $144, 245$
6. $15, 50, 60$
7. $14, 63, 70$
8. $12, 48, 125$
9. $60, 72, 900$

10. $252, 336, 360$

Answer

1. $18 = 2 \cdot 32, 24 = 23 \cdot 3, \text{GCF} = 6$
3. $72 = 23 \cdot 32, 60 = 22 \cdot 3 \cdot 5, \text{GCF} = 12$
5. $144 = 24 \cdot 32, 245 = 5 \cdot 72, \text{GCF} = 1$
7. $14 = 2 \cdot 7, 63 = 32 \cdot 7, 70 = 2 \cdot 5 \cdot 7, \text{ GCF} = 7$
9. $60 = 22 \cdot 3 \cdot 5, 72 = 23 \cdot 32, 900 = 22 \cdot 32 \cdot 52, \text{GCF} = 12$

Exercise 7.1.5 GCF of Variable Expressions

Determine the GCF of all the terms.

1. $15x, 30$
2. $14x, 21$
3. $45x^4, 8x^3$
4. $36x^5, 35y^2$
5. $6x, 27x, 36x$
6. $12x^3, 4x^2, 6x$
7. $12x^2y, 60xy^3$
8. $7ab^2, 2a^2b, 3a^3b^3$
9. $6a^2b^2, 18a^3b^2, 9ab^2$
10. $15x(x+2), 9(x+2)$
11. $20x(2x-1), 16(2x-1)$
12. $20x^3(x+y)^5, 10x^5(x+y)^2$

Answer

1. 15
3. x^3
5. $3x$
7. $12xy$
9. $3ab^2$
11. $4(2x-1)$

Exercise 7.1.6 Factoring out the GCF

Given the GCF, determine the missing factor.

1. $25x^2 + 10x = 5x(\quad ? \quad)$
2. $12y^5 + 7y^2 = y^2(\quad ? \quad)$
3. $22x^4 - 121x^2 + 11x = 11x(\quad ? \quad)$
4. $30y^3 - 45y^2 - 3y = 3y(\quad ? \quad)$
5. $36a^5b^7 - 60a^6b^5 = 12a^5b^5(\quad ? \quad)$
6. $24x^2y + 48xy^2 - 12xy = 12xy(\quad ? \quad)$

Answer

1. $(5x+2)$
3. $(2x^3 - 11x + 1)$
5. $(3b^2 - 5a)$

Exercise 7.1.7 Factoring out the GCF

Factor out the GCF.

1. $4x-8$
2. $27x-9$
3. $3x-18$
4. $5x-10$
5. $25x-16$
6. $72x-35$
7. $15x^2+30x$
8. $14a^2-7a$
9. $30a^5-10a^2$
10. $8x^4-16x^2$
11. $5x^6+x^3$
12. $3x^7-9x^5$
13. $18a^2+30a-6$
14. $24a^2-36a-12$
15. $27x^3-6x^2+3x$
16. $8x^3-12x^2+2x$
17. $9x^4+18x^3-3x^2$
18. $12y^4-16y^3+20y^2$
19. $7x^5-21x^3-14x^2+28x$
20. $36y^{10}+12y^8-18y^4-6y^3$
21. $12x^5y^2-8x^3y$
22. $125a^8b^4c^3-25a^2b^3c^3$
23. $6x^4y^3-4x^3y^2+8x^2y$
24. $15x^4y^2-30x^3y^3+15x^2y^4$
25. $81x^7y^6z^2-18x^2y^8z^4+9x^2y^5z^2$
26. $4x^5y^4z^9+26x^5y^3z^4-14x^6y^8z^5$
27. $2x(x-3)+5(x-3)$
28. $3x(2x+1)-4(2x+1)$
29. $5x(5x+2)-(5x+2)$
30. $2x(3x+4)+(3x+4)$
31. $x^2(4x-7)-5(4x-7)$
32. $(x+6)-3x^2(x+6)$
33. $(a+b)^2-3a(a+b)^2$
34. $(ab+2)^3+3ab(ab+2)^3$
35. $7x(x+7)^5+14x^2(x+7)^5$
36. $36x^5(3x+2)^4-12x^3(3x+2)^4$

Answer

1. $4(x-2)$
3. $3(x-6)$
5. Prime
7. $15x(x+2)$
9. $10a^2(3a^3-1)$
11. $x^3(5x^3+1)$
13. $6(3a^2+5a-1)$

15. $3x(9x^2-2x+1)$
17. $3x^2(3x^2+6x-1)$
19. $7x(x^4-3x^2-2x+4)$
21. $4x^3y(3x^2y-2)$
23. $2x^2y(3x^2y^2-2xy+4)$
25. $9x^2y^5z^2(9x^5y-2y^3z^2+1)$
27. $(x-3)(2x+5)$
29. $(5x+2)(5x-1)$
31. $(4x-7)(x^2-5)$
33. $(a+b)^2(1-3a)$
35. $7x(x+7)^5(1+2x)$

? Exercise 7.1.8 Factoring out the GCF

Are the following factored correctly? Check by multiplying.

1. $4x^2-16x=4x(x-4)$
2. $3a^3-3a=3a(a^2)$
3. $3x^3-5x^6=x^3(3-x^2)$
4. $5x^3-10x^4+15x^5=5x^3(1-2x+3x^2)$
5. $x^3-x^2+x=x(x^2-x)$
6. $12x^4y^3-16x^5y^2+8x^6y^7=4x^4y^2(3y-4x+2x^2y^5)$

Answer

1. Yes
3. No
5. No

? Exercise 7.1.9 Factoring out the GCF

Use polynomial long division to show that the given factor divides the polynomial evenly.

1. Show that $(x-1)$ is a factor of $(2x^3-5x^2+4x-1)$.
2. Show that $(x+3)$ is a factor of $(3x^3+7x^2-4x+6)$.
3. Show that $(3x-2)$ is a factor of $(3x^3+4x^2-7x+2)$.
4. Show that $(2x+1)$ is a factor of $(2x^3-5x^2+x+2)$.
5. The height in feet of an object tossed into the air is given by the function $h(t)=-16t^2+32t$, where t is the time in seconds after it is tossed. Write the function in factored form.
6. The height in feet of an object dropped from a 16-foot ladder is given by the function $h(t)=-16t^2+16$, where t is the time in seconds after it is tossed. Write the function in factored form.

7. The surface area of a cylinder is given by the formula $SA = 2\pi r^2 + 2\pi rh$, where r represents the radius of the base and h is the height of the cylinder. Express this formula in factored form.

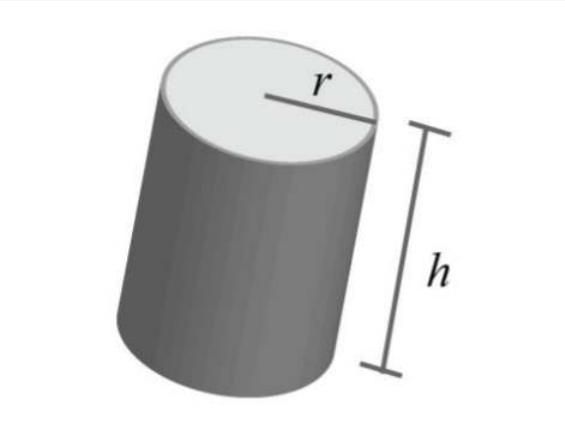


***Figure* 7.1.1**

8. The surface area of a cone is given by the formula $SA = \pi r^2 + \pi rs$, where r represents the radius of the base and s represents the slant height. Express this formula in factored form.

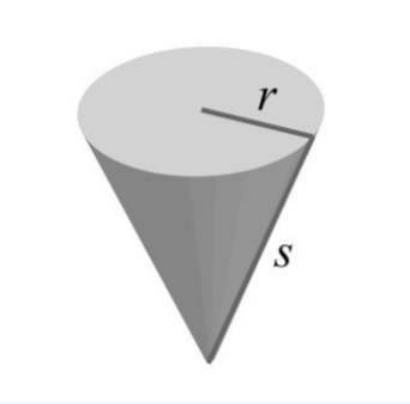


***Figure* 7.1.2**

Answer

5. $h(t) = -16t(t-2)$

7. $SA = 2\pi r(r+h)$

? Exercise 7.1.10 Factor by Grouping

Factor by grouping.

1. $x^2 - 10x + 2x - 20$
2. $x^2 - 6x - 3x + 18$
3. $x^3 + 2x^2 + 2x + 4$
4. $x^3 - 3x^2 + 5x - 15$
5. $x^3 + 7x^2 - 2x - 14$
6. $2x^3 + 2x^2 - x - 1$
7. $x^3 - 5x^2 + 4x - 20$
8. $6x^3 - 3x^2 + 2x - 1$
9. $9x^3 - 6x^2 - 3x + 2$
10. $2x^4 - x^3 - 6x + 3$
11. $x^5 + x^3 + 2x^2 + 2$
12. $6x^5 - 4x^3 - 9x^2 + 6$

13. $3a^3b+3ab^2+2a^2+2b$
14. $2a^3+2ab^3-3a^2b-3b^4$
15. $2a^3-a^2b^2-2ab+b^3$
16. $a^4-3a^3b^2+ab^2-3b^4$
17. $3a^2b^4-6b^3-a^2b+2$
18. $3x^3+2y^3+x^3y^3+6$
19. $-3x^3-5y^3+x^3y^3+15$
20. $2x^3y^3+2-y^3-4x^3$
21. $3x^2-y^3+xy^2-3xy$
22. $2x^2+y^3-2xy-xy^2$

Answer

1. $(x-10)(x+2)$
3. $(x+2)(x^2+2)$
5. $(x+7)(x^2-2)$
7. $(x-5)(x^2+4)$
9. $(3x-2)(3x^2-1)$
11. $(x^2+1)(x^3+2)$
13. $(a^2+b)(3ab+2)$
15. $(a^2-b)(2a-b^2)$
17. $(3b^3-1)(a^2b-2)$
19. $(x^3-5)(y^3-3)$
21. $(x-y)(3x+y^2)$

? Exercise 7.1.11 Factor by Grouping

Factor out the GCF first and then factor by grouping.

1. $5x^2-35x-15x+105$
2. $12x^2-30x-12x+30$
3. $2x^3+6x^2-10x-30$
4. $6x^3-3x^2-42x+21$
5. $4x^4+4x^3-12x^2-12x$
6. $-9x^4+6x^3-45x^2+30x$
7. $-12x^5+4x^4+6x^3-2x^2$
8. $24x^5-36x^4+8x^3-12x^2$
9. $24a^3b^2-60a^3b+40ab^2-100ab$
10. $a^4b^2-2a^3b^3+a^2b^3-2ab^4$

Answer

1. $5(x-7)(x-3)$
3. $2(x+3)(x^2-5)$
5. $4x(x+1)(x^2-3)$
7. $-2x^2(3x-1)(2x^2-1)$
9. $4ab(3a^2+5)(2b-5)$

Exercise 7.1.12 Discussion Board Topics

1. Research the Euclidean algorithm for finding the GCF of two natural numbers. Give an example that illustrates the steps.
2. Research and discuss the contributions of Euclid of Alexandria.
3. Explain what factoring is and give an example.
4. Is $5x(x+2)-3(x+2)$ fully factored? Explain.
5. Make up a factoring problem of your own and provide the answer. Post the problem and the solution on the discussion board.

Answer

1. Answers may vary

3. Answers may vary

5. Answers may vary

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7.2: Factoring Trinomials of the Form x²+bx+c

Learning Objectives

- Factor trinomials of the form x^2+bx+c .
- Factor trinomials using the AC method.

Factoring Trinomials of the Form x^2+bx+c

Some trinomials of the form x^2+bx+c can be factored as a product of binomials. For example,

$$x^2+7x+10=(x+2)(x+5)$$

We can verify this factorization by multiplying:

$$\begin{aligned}(x+2)(x+5) &= x^2+5x+2x+10\\ &= x^2+7x+10 \quad \checkmark\end{aligned}$$

Factoring trinomials requires that we work the distributive process in reverse. Notice that the product of the first terms of each binomial is equal to the first term of the trinomial.

$$x\cdot x=x^2$$

The middle term of the trinomial, $7x$, is the sum of the products of the outer and inner terms of the binomials:

$$5x+2x=7x$$

And the product of the last terms of each binomial is equal to the last term of the trinomial.

$$2\cdot 5=10$$

This can be visually interpreted as follows:

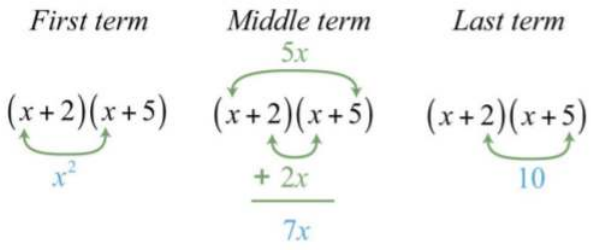


Figure 7.2.1

If a trinomial of this type factors, then these relationships will be true:

$$\begin{aligned}x^2+bx+c &= (x+m)(x+n)\\ &= x^2+nx+mx+mn\\ &= x^2+(n+m)x+mn\end{aligned}$$

This gives us

$$b=n+m \quad \text{and} \quad c=mn \tag{7.2.1}$$

In short, if the leading coefficient of a factorable trinomial is one, then the factors of the last term must add up to the coefficient of the middle term. This observation is the key to factoring trinomials using the technique known astrial and error (or guess and check). The steps are outlined in the following example.

Example 7.2.1

Factor:

$$x^2+7x+12$$

Solution:

Note that the polynomial to be factored has three terms; it is a trinomial with a leading coefficient of 1. Use trial and error to factor as follows:

Step 1: Write two sets of blank parentheses. If a trinomial of this form factors, then it will factor into two linear binomial factors.

$$x^2+7x+12=(\quad)(\quad)$$

Step 2: Write the factors of the first term in the first space of each set of parentheses. In this case, factor $x^2=x\cdot x$.

$$x^2+7x+12=(x\quad)(x\quad)$$

Step 3: Determine the factors of the last term whose sum equals the coefficient of the middle term. To do this, list all of the factorizations of 12 and search for factors whose sum equals the coefficient of the middle term, 7.

$$\begin{aligned} 12&=1\cdot 12 &&\rightarrow 1+12=13\\ &=2\cdot 6 &&\rightarrow 2+6=8\\ &=3\cdot 4 &&\rightarrow 3+4=7 \end{aligned}$$

Choose $12=3\cdot 4$ because $3+4=7$.

Step 4: Write in the last term of each binomial using the factors determined in the previous step.

$$x^2+7x+12=(x+3)(x+4)$$

Step 5: Check by multiplying the two binomials.

$$\begin{aligned} (x+3)(x+4)&=x^2+4x+3x+12\\ &=x^2+7x+12\quad\checkmark \end{aligned}$$

Answer:

$$(x+3)(x+4)$$

Since multiplication is commutative, the order of the factors does not matter.

$$\begin{aligned} x^2+7x+12&=(x+3)(x+4)\\ &=(x+4)(x+3) \end{aligned}$$

If the last term of the trinomial is positive, then either both of the constant factors must be negative or both must be positive. Therefore, when looking at the list of factorizations of the last term, we are searching for sums that are equal to the coefficient of the middle term.

Example 7.2.2

Factor:

$$x^2-9x+20.$$

Solution:

First, factor $x^2=x\cdot x$.

$$x^2-9x+20=(x\quad ?)(x\quad ?)$$

Next, determine which factors of 20 add up to -9:

$$\begin{aligned} 20&=1\cdot 20\\ &-2\cdot 10\\ &=4\cdot 5 \end{aligned}$$

In this case, choose -4 and -5 because $(-4)(-5)=+20$ and $-4+(-5)=-9$.

$$x^2-9x+20=(x-4)(x-5)$$

Check.

$$\begin{aligned}(x-4)(x-5) &= x^2-5x-4x+20\\ &= x^2-9x+20 \quad \checkmark\end{aligned}$$

Answer:

$$(x-4)(x-5)$$

If the last term of the trinomial is negative, then one of its factors must be negative. In this case, search the list of factorizations of the last term for differences that equal the coefficient of the middle term.

Example 7.2.3

Factor:

$$x^2-4x-12\,.$$

Solution:

Begin by factoring the first term $x^2 = x \cdot x$.

$$x^2-4x-12 = (x \quad ?)(x \quad ?)$$

The factors of 12 are listed below. In this example, we are looking for factors whose difference is -4.

Here choose the factors 2 and -6 because the coefficient of the middle term, -4, is obtained if we add $2+(-6)$.

$x^2-4x-12 = (x+2)(x-6)$

Multiply to check.

Answer:

$(x+2)(x-6)$

Often our first guess will not produce a correct factorization. This process may require repeated trials. For this reason, the check is very important and is not optional.

Example 7.2.4

Factor:

$$x^2+5x-6\,.$$

Solution:

The first term of this trinomial x^2 factors as $x \cdot x$.

\(x^{2}+5x-6=(x\quad ?)(x\quad ?)

Consider the factors of 6:

$$\begin{aligned}6 &= 1 \cdot 6\\ &= 2 \cdot 3\end{aligned}$$

Suppose we choose the factors 2 and 3 because $2+3=5$, the coefficient of the middle term. Then we have the following incorrect factorization:

$$x^2+5x-6 \stackrel{?}{=} (x+2)(x+3)$$

When we multiply to check, we find the error.

$$\begin{aligned}(x+2)(x+3) &= x^2+3x+2x+6\\ &= x^2+5x+6 \quad x\end{aligned}$$

In this case, the middle term is correct but the last term is not. Since the last term in the original expression is negative, we need to choose factors that are opposite in sign. Therefore, we must try again. This time we choose the factors -1 and 6 because $-1+6=5$.

$$x^2+5x-6=(x-1)(x+6)$$

Now the check shows that this factorization is correct.

Answer:

$(x-1)(x+6)$

If we choose the factors wisely, then we can reduce much of the guesswork in this process. However, if a guess is not correct, do not get discouraged; just try a different set of factors.

Example 7.2.5

Factor:

$$x^2+3x+20\,.$$

Solution:

$$\begin{aligned} x^2+3x+20 &= (x\quad ?)(x\quad ?) \\ 20 &= 1\cdot 20 \\ &= 2\cdot 10 \\ &= 4\cdot 5 \end{aligned}$$

Here there are no factors of 20 whose sum is 3. Therefore, the original trinomial cannot be factored as a product of two binomials. This trinomial is prime.

Answer:

Prime

Exercise 7.2.1

Factor:

$$x^2-13x-30\,.$$

Answer

$(x+2)(x-15)$

The techniques described can also be used to factor trinomials with more than one variable.

Example 7.2.6

Factor:

$$x^2-14xy-72y^2$$

Solution:

The first term x^2 factors as $x\cdot x$.

$$x^2+xy-72y^2=(x\quad ?)(x\quad ?)$$

Next, look for factors of the coefficient of the last term, 72, whose sum is -14.

$$\begin{aligned} 72 &= 1\cdot 72 \\ &= 2\cdot 36 \\ &= 3\cdot 24 \\ &= 4\cdot 18 \qquad \to 4+(-18)=-14 \\ &= 6\cdot 12 \\ &= 8\cdot 9 \end{aligned}$$

Therefore, the coefficient of the last term can be factored $-72 = 4(-18)$, where $4 + (-18) = -14$. Because the last term has a variable factor of y^2, factor $72y^2$ as $4y(-18y)$ and try the following factorization:

$$x^2 - 14xy - 72y^2 = (x + 4y)(x - 18y)$$

Multiply to check.

$$\begin{aligned}(x + 4y)(x - 18y) &= x^2 - 18xy + 4xy - 72y^2 \\ &= x^2 - 14xy - 72y^2 \quad \checkmark\end{aligned}$$

Visually, we have the following:

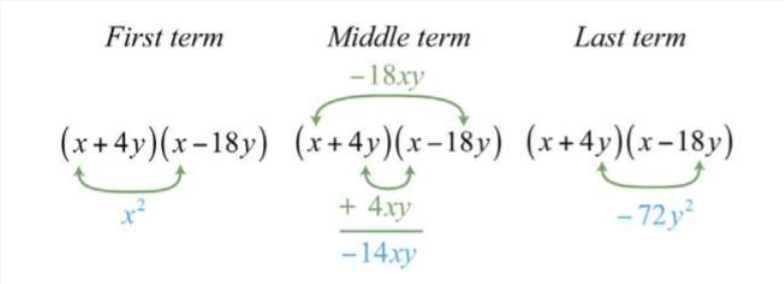


Figure 7.2.2

Answer:

$$(x + 4y)(x - 18y)$$

Exercise 7.2.2

Factor:

$$x^2y^2 + 9xy - 10\,.$$

Answer

$(xy - 1)(xy + 10)$

Factoring Using the AC Method

An alternate technique for factoring trinomials, called the AC method, makes use of the grouping method for factoring four-term polynomials. If a trinomial in the form $ax^2 + bx + c$ can be factored, then the middle term, bx, can be replaced with two terms with coefficients whose sum is b and product ac. This substitution results in an equivalent expression with four terms that can be factored by grouping. The steps are outlined in the following example.

Example 7.2.7

Factor using the AC method:

$$x^2 - x - 30\,.$$

Solution:

In this example $a = 1, b = -1$, and $c = -30$.

Step 1: Determine the product ac.

Step 2: Find factors of ac whose sum equals the coefficient of the middle term, b.

$$\begin{aligned}-30 &= 1(-30) \\ &= 2(-15) \\ &= 3(-10) \\ &= 5(-5) \qquad \rightarrow 5 + (-6) = -1\end{aligned}$$

We can see that the sum of the factors 5 and -6 is equal to the coefficient of the middle term, -1.

Step 3: Use the factors as coefficients for the terms that replace the middle term. Here $-x = -6x + 5x$. Write

$$x^2 - x - 30 = x^2 - 6x + 5x - 30$$

Step 4: Factor the equivalent expression by grouping.

$$\begin{aligned} x^2 - x - 30 = & \underbrace{x^2 - 6x}_{group} + \underbrace{5x - 20}_{group} \\ & = x(x-6) + 5(x-6) \\ & = (x-6)(x+5) \end{aligned}$$

Answer:

$$(x-6)(x+5)$$

Notice that the AC method is consistent with the trial and error method. Both methods require that $b = m + n$, where $c = mn$. In the example above, $-30 = (-6)(5)$ and $-1 = (-6) + 5$. The only difference between the methods, when the leading coefficient is 1, is in the process used to obtain the final factorization.

Example 7.2.8

Factor:

$$y^2 - 14x + 48\,.$$

Solution:

Here $ac = 48$ and we search for factors whose sum is -14.

$$\begin{aligned} 48 &= 1(48) \\ &= 2(24) \\ &= 3(16) \\ &= 4(12) \\ &= 6(8) \qquad \rightarrow -6 + (-8) = -14 \end{aligned}$$

Therefore, $-14x = -6x - 8x$. Substitute the new terms and factor by grouping.

$$\begin{aligned} y^2 - 14x + 48 &= x^2 - 6x - 8x + 48 \\ &= x(x-6) - 8(x-6) \\ &= (x-6)(x-8) \end{aligned}$$

Answer:

$(x-6)(x-8)$. The check is left to the reader.

At this point, it is recommended that the reader stop and factor as many trinomials of the form $x^2 + bx + c$ as time allows before moving on to the next section. Factoring trinomials is one of the more important skills that we learn in this course and should be mastered.

Key Takeaways

- Factor a trinomial by systematically guessing what factors give two binomials whose product is the original trinomial.
- If a trinomial of the form $x^2 + bx + c$ factors into the product of two binomials, then the coefficient of the middle term is the sum of factors of the last term.
- Not all trinomials can be factored as the product of binomials with integer coefficients. In this case, we call it a prime trinomial.
- Factoring is one of the more important skills required in algebra. For this reason, you should practice working as many problems as it takes to become proficient.

Exercise 7.2.3 Factoring Trinomials with Leading Coefficient 1

Are the following factored correctly? Check by multiplying.

1. $x^2+5x-6=(x+2)(x+3)$
2. $x^2+6x+16=(x+8)(x-2)$
3. $y^2+2y-8=(y+4)(y-2)$
4. $y^2-10y+21=(y-3)(y-7)$
5. $a^2-10a+25=(a-5)^2$
6. $a^2+6a+9=(a-3)^2$
7. $x^2+10x-25=(x+5)(x-5)$
8. $x^2+5x+14=(x-2)(x+7)$
9. $y^2+50y-600=(y+60)(y-10)$
10. $y^2-3y+2=(y-2)(y-1)$

Answer

1. No
3. Yes
5. Yes
7. No
9. Yes

Exercise 7.2.4 Factoring Trinomials with Leading Coefficient 1

Factor.

1. x^2+6x+8
2. x^2+4x+3
3. x^2+3x+2
4. x^2+x-2
5. $x^2+3x-10$
6. $x^2-2x-35$
7. $x^2-13x+12$
8. $x^2-15x+36$
9. $x^2-12x+36$
10. $x^2+18x+81$
11. x^2-2x+1
12. $x^2-18x+81$
13. x^2+5x+5
14. x^2-4x+6
15. $x^2-20x+91$
16. $x^2+20x+91$
17. $x^2-2x-48$
18. $x^2+16x+48$
19. $x^2+22x+48$
20. $x^2+22x-48$
21. $y^2+7y+12$
22. $y^2+8y-20$
23. $y^2-16y+60$
24. $y^2-31y-32$
25. $a^2-11a-42$
26. $a^2-14a-51$

27. $a^2+26a+25$
28. $a^2-22a+120$
29. a^2+4a-1
30. a^2-6a+2
31. $y^2-14x+40$
32. $y^2-4y-96$
33. $x^2-2xy+y^2$
34. $x^2+2xy+y^2$
35. $x^2-16xy+15y^2$
36. $x^2-4xy-32y^2$
37. $x^2+2xy-15y^2$
38. $x^2-12xy+32y^2$
39. $x^2y^2-6xy+9$
40. $x^2y^2+25xy-26$
41. $a^2+4ab+4b^2$
42. $a^2-19ab-20b^2$
43. $a^2-ab-12b^2$
44. $a^2-10ab-56b^2$
45. $a^2b^2-2ab-15$
46. $a^2b^2-10ab+24$
47. The area of a square is given by the function $A(x)=x^2-14x+49$, where x is measured in meters. Rewrite this function in factored form.
48. The area of a square is given by the function $A(x)=x^2+16x+64$, where x is measured in meters. Rewrite this function in factored form.

Answer

1. $(x+2)(x+4)$
3. $(x+1)(x+2)$
5. $(x-2)(x+5)$
7. $(x-1)(x-12)$
9. $(x-6)^2$
11. $(x-1)^2$
13. Prime
15. $(x-7)(x-13)$
17. $(x+6)(x-8)$
19. Prime
21. $(y+3)(y+4)$
23. $(y-6)(y-10)$
25. $(a+3)(a-14)$
27. $(a+1)(a+25)$
29. Prime
31. $(y-10)(y-4)$
33. $(x-y)^2$
35. $(x-15y)(x-y)$
37. $(x+5y)(x-3y)$

39. $(xy-3)^2$
41. $(a+2b)^2$
43. $(a+3b)(a-4b)$
45. $(ab+3)(ab-5)$
45. $A(x)=(x-7)^2$

Exercise 7.2.5 Factor Using the AC Method

Factor using the AC method.

1. $x^2+5x-14$
2. $x^2+2x-48$
3. x^2-9x+8
4. $x^2-14x+24$
5. x^2-x-72
6. x^2-x-90
7. $y^2-8y+16$
8. $y^2+16y+64$
9. $x^2+4x+12$
10. x^2+5x-8
11. $x^2+3xy-18y2$
12. $x^2-15xy+50y^2$

Answer

1. $(x+7)(x-2)$
3. $(x-8)(x-1)$
5. $(x-9)(x+8)$
7. $(y-4)^2$
9. Prime
11. $(x+6y)(x-3y)$

Exercise 7.2.6 Discussion Board Topics

1. Create your own trinomial of the form x^2+bx+c that factors. Share it along with the solution on the discussion board.
2. Write out your own list of steps for factoring a trinomial of the form x^2+bx+c and share your steps on the discussion board.
3. Create a trinomial that does not factor and share it along with an explanation of why it does not factor.

Answer

1. Answers may vary
3. Answers may vary

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7.3: Factoring Trinomials of the Form ax²+bx+c

Learning Objectives

- Factor trinomials of the form ax^2+bx+c .
- Factor trinomials with a common factor.

Factoring Trinomials of the Form ax^2+bx+c

Factoring trinomials of the form ax^2+bx+c can be challenging because the middle term is affected by the factors of both a and c. To illustrate this, consider the following factored trinomial:

$$10x^2+17x+3=(2x+3)(5x+1)$$

We can multiply to verify that this is the correct factorization.

$$\begin{aligned}(2x+3)(5x+1) &= 10x^2+2x+15x+3\\ &= 10x^2+17x+3 \quad \checkmark\end{aligned}$$

As we have seen before, the product of the first terms of each binomial is equal to the first term of the trinomial. The middle term of the trinomial is the sum of the products of the outer and inner terms of the binomials. The product of the last terms of each binomial is equal to the last term of the trinomial. Visually, we have the following:

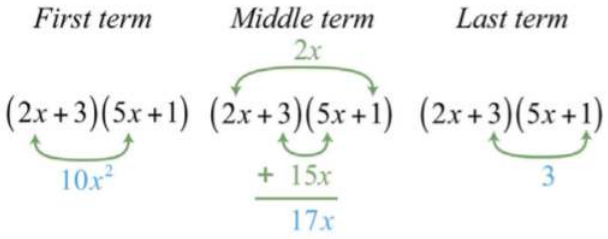


Figure 7.3.1

In general,

$$\begin{aligned}ax^2+bx+c &= (px+m)(qx+n)\\ &= pqx^2+pnx+qmx+mn\\ &= pqx^2+(pn+qm)x+mn\end{aligned}$$

This gives us,

$$a=pq \quad \text{and} \quad b=pn+qm, \quad \text{where} \quad c=mn \tag{7.3.1}$$

In short, when the leading coefficient of a trinomial is something other than 1, there will be more to consider when determining the factors using the trial and error method. The key lies in the understanding of how the middle term is obtained. Multiply $(2x+5)(3x+7)$ and carefully follow the formation of the middle term.

$$\begin{aligned}(2x+5)(3x+7) &= 2x\cdot 3x \quad \underbrace{+2x\cdot 7+5\cdot 3x}_{middle\ term} \quad +5\cdot 7\\ &= 6x^2+14x+15x+35\\ &= 6x^2+29x+35\end{aligned}$$

If we think of the FOIL method for multiplying binomials, then the middle term results from the sum of the inner product and the outer product. In this case, $14x+15x=29x$, as illustrated below:

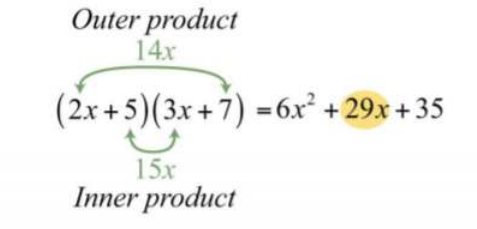


Figure 7.3.2

For this reason, we need to look for products of the factors of the first and last terms whose sum is equal to the coefficient of the middle term. For example, to factor $6x^2+29x+35$, look at the factors of 6 and 35.

$$\begin{aligned} 6 &= 1\cdot 6 & 35 &= 1\cdot 35 \\ &= 2\cdot 3 & &= 5\cdot 7 \end{aligned}$$

The combination that produces the coefficient of the middle term is $2\cdot 7+3\cdot 5=14+15=29$. Make sure that the outer terms have coefficients 2 and 7, and that the inner terms have coefficients 5 and 3. Use this information to factor the trinomial:

$$\begin{aligned} 6x^2+29x+35 &= (2x\quad ?)(3x\quad ?) \\ &= (2x+5)(3x+7) \end{aligned}$$

Example 7.3.1

Factor:

$$3x^2+7x+2\,.$$

Solution:

Since the leading coefficient and the last term are both prime, there is only one way to factor each.

$$3=1\cdot 3 \quad \text{and} \quad 2=1\cdot 2$$

Begin by writing the factors of the first term, $3x^2$, as follows:

$$3x^2+7x+2=(x\quad ?)(3x\quad ?)$$

The middle and last term are both positive; therefore, the factors of 2 are chosen as positive numbers. In this case, the only choice is in which grouping to place these factors.

$$(x+1)(3x+2) \quad \text{or} \quad (x+2)(3x+1)$$

Determine which grouping is correct by multiplying each expression.

$$\begin{aligned} (x+1)(3x+2) &= 3x^2+2x+3x+2 \\ &= 3x^2+5x+2 \quad x \\ (x+2)(3x+1) &= 3x^2+x+6x+2 \\ &= 3x^2+7x+2 \quad \checkmark \end{aligned}$$

Notice that these products differ only in their middle terms. Also, notice that the middle term is the sum of the inner and outer product, as illustrated below:

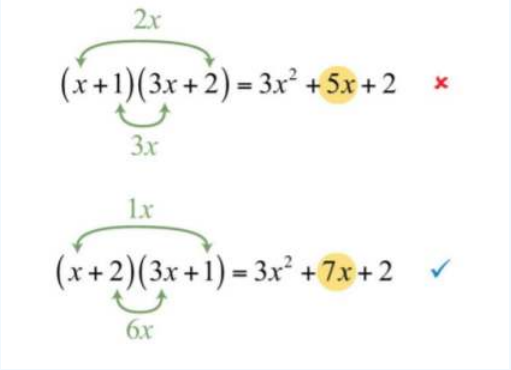


Figure 7.3.3

Answer:

$$(x+2)(3x+1)$$

Example 7.3.2

Factor:

$$12x^2+38x+20.$$

Solution:

First, consider the factors of the first and last terms.

$$\begin{aligned} 12 &= 1\cdot 12 \\ &= 2\cdot 6 \\ &= 3\cdot 4 \end{aligned} \qquad \begin{aligned} 20 &= 1\cdot 20 \\ &= 2\cdot 10 \\ &= 4\cdot 5 \end{aligned}$$

We search for products of factors whose sum equals the coefficient of the middle term, 38. For brevity, the thought process is illustrated starting with the factors 2 and 6. Factoring begins at this point with the first term.

$$12x^2+38x+20 = (2x \quad ?)(6x \quad ?)$$

We search for factors of 20 that along with the factors of 12 produce a middle term of 38x

$Factors\ of\ 20$	$Possible\ factorization$	
$1\cdot 20$	$(2x+1)(6x+20)$	$middle\ term \Rightarrow 46x$
$1\cdot 20$	$(2x+20)(6x+1)$	$middle\ term \Rightarrow 122x$
$2\cdot 10$	$(2x+2)(6x+10)$	$middle\ term \Rightarrow 32x$
$2\cdot 10$	$(2x+10)(6x+2)$	$middle\ term \Rightarrow 64x$
$4\cdot 5$	$(2x+4)(6x+5)$	$middle\ term \Rightarrow 34x$
$4\cdot 5$	$(2x+5)(6x+4)$	$middle\ term \Rightarrow 38x$ ✓

Here the last combination produces a middle term of $38x$.

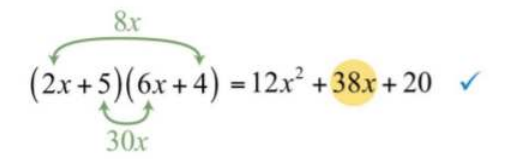


Figure 7.3.4

Answer:

$$(2x+5)(6x+4)$$

Example 7.3.3

Factor:

$$10x^2 - 23x + 6\,.$$

Solution

First, consider the factors of the first and last terms.

$$\begin{aligned} 10 &= 1 \cdot 10 & 6 &= 1 \cdot 6 \\ &= 2 \cdot 5 & &= 2 \cdot 3 \end{aligned}$$

We are searching for products of factors whose sum equals the coefficient of the middle term, -23. Factoring begins at this point with two sets of blank parentheses:

$$10x^2 - 23x + 6 = (\quad)(\quad)$$

Since the last term is positive and the middle term is negative, we know that both factors of the last term must be negative. Here we list all possible combinations with the factors of $10x^2 = 2x \cdot 5x$.

$$10x^2 - 23x + 6 = (2x \quad ?)(5x \quad ?)$$

$$\begin{aligned} &(2x-1)(5x-6) && middle\ term \Rightarrow -17x \\ &(2x-6)(5x-1) && middle\ term \Rightarrow -32x \\ &(2x-2)(5x-3) && middle\ term \Rightarrow -16x \\ &(2x-3)(5x-2) && middle\ term \Rightarrow -19x \end{aligned}$$

There is no combination that produces a middle term of $-23x$. We then move on to the factors of $10x^2 = 10x \cdot x$ and list all possible combinations:

$$10x^2 - 23x + 6 = (10x \quad ?)(x \quad ?)$$

$$\begin{aligned} &(10x-1)(x-6) && middle\ term \Rightarrow -61x \\ &(10x-6)(x-1) && middle\ term \Rightarrow -162x \\ &(10x-2)(x-3) && middle\ term \Rightarrow -32x \\ &(10x-3)(x-2) && middle\ term \Rightarrow -23x \quad \checkmark \end{aligned}$$

And we can write

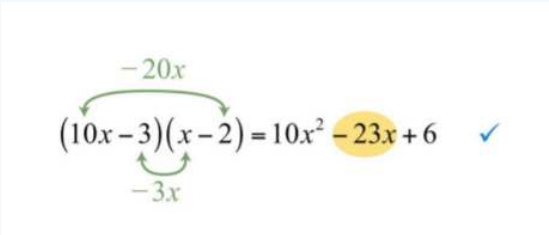


Figure 7.3.5

Answer:

$(10x-3)(x-2)$. The complete check is left to the reader.

We can reduce much of the guesswork involved in factoring trinomials if we consider all of the factors of the first and last terms and their products.

Example 7.3.4

Factor:

$$5x^2 + 38x - 16\,.$$

Solution:

We begin with the factors of 5 and 16.

$$\begin{aligned} 16 &= 1 \cdot 16 \\ 5 = 1 \cdot 5 \qquad &= 2 \cdot 8 \\ &= 4 \cdot 4 \end{aligned}$$

Since the leading coefficient is prime, we can begin with the following:

$$5x^2 + 38x - 16 = (x \quad ?)(5x \quad ?)$$

We look for products of the factors of 5 and 16 that could possibly add to 38.

Factors of 16	*Possible*	*products*	
$1 \cdot 16$	$1 \cdot 1$ *and* $5 \cdot 16$	*products* $\Rightarrow$ 1 *and* 80	
$1 \cdot 16$	$1 \cdot 16$ *and* $5 \cdot 1$	*products* $\Rightarrow$ 16 *and* 5	
$2 \cdot 8$	$1 \cdot 2$ *and* $5 \cdot 8$	*products* $\Rightarrow$ 2 *and* 40	✓
$2 \cdot 8$	$1 \cdot 8$ *and* $5 \cdot 2$	*products* $\Rightarrow$ 8 *and* 10	
$4 \cdot 4$	$1 \cdot 4$ *and* $5 \cdot 4$	*products* $\Rightarrow$ 4 *and* 20	

Since the last term is negative, we must look for factors with opposite signs. Here we can see that the products 2 and 40 add up to 38 if they have opposite signs:

$$1 \cdot (-2) + 5 \cdot 8 = -2 + 40 = 38$$

Therefore, use -2 and 8 as the factors of 16, making sure that the inner and outer products are $-2x$ and $40x$:

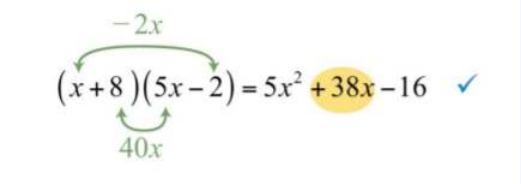


Figure 7.3.6

Answer:

$(x+8)(5x-2)$. The complete check is left to the reader.

After lots of practice, the process described in the previous example can be performed mentally.

? Exercise 7.3.1

Factor:

$$12x^2 - 31x - 30$$

Answer

$(3x-10)(4x+3)$

When given trinomials with multiple variables, the process is similar.

✓ Example 7.3.5

Factor:

$$9x^2 + 30xy + 25y^2.$$

Solution:

Search for factors of the first and last terms such that the sum of the inner and outer products equals the middle term.

$$\begin{aligned} 9x^2 &= 1x \cdot 9x \qquad 25y^2 = 1y \cdot 25y \\ &= 3x \cdot 3x \qquad\quad = 5y \cdot 5y \end{aligned}$$

Add the following products to obtain the middle term: $3x \cdot 5y + 3x \cdot 5y = 30xy$.

$$\begin{aligned} 9x^2 + 30xy + 25y^2 &= (3x \quad)(3x \quad) \\ &= (3x+5y)(3x+5y) \\ &= (3x+5y)^2 \end{aligned}$$

In this example, we have a perfect square trinomial. Check.

$$\begin{aligned} (3x+5y)^2 &= 9x^2 + 2 \cdot 3x \cdot 5y + 25y^2 \\ &= 9x^2 + 30xy + 25y^2 \quad \checkmark \end{aligned}$$

Answer:

$$(3x+5y)^2$$

? Exercise 7.3.2

Factor:

$$16x^2 - 24xy + 9y^2.$$

Answer

$(4x-3y)^2$

Factoring Trinomials with Common Factors

It is a good practice to first factor out the GCF, if there is one. Doing this produces a trinomial factor with smaller coefficients. As we have seen, trinomials with smaller coefficients require much less effort to factor. This commonly overlooked step is worth identifying early.

✓ Example 7.3.6

Factor:

$$12x^2 - 27x + 6.$$

Solution:

Begin by factoring out the GCF.

$$12x^2 - 27x + 6 = 3(4x^2 - 9x + 2)$$

After factoring out 3, the coefficients of the resulting trinomial are smaller and have fewer factors.

$$\begin{aligned} 4 &= 1 \cdot 4 \quad 2 = 1 \cdot 2 \\ &= 2 \cdot 2 \end{aligned}$$

After some thought, we can see that the combination that gives the coefficient of the middle term is $4(-2) + 1(-1) = -8 - 1 = -9$.

$$\begin{aligned} 3(4x^2 - 9x + 2) &= 3(4x \quad ?)(x \quad ?) \\ &= 3(4x-1)(x-2) \end{aligned}$$

Check.

$$\begin{aligned} 3(4x-1)(x-2) &= 3(4x^2 - 8x - x + 2) \\ &= 3(4x^2 - 9x + 2) \\ &= 12x^2 - 27x + 6 \quad \checkmark \end{aligned}$$

The factor 3 is part of the factored form of the original expression; be sure to include it in the answer.

Answer:

$$3(4x-1)(x-2)$$

It is a good practice to consistently work with trinomials where the leading coefficient is positive.

Example 7.3.7

Factor:

$$-x^2+2x+15\,.$$

Solution

In this example, the leading coefficient is -1. Before beginning the factoring process, factor out the -1:

$$-x^2+2x+15=-1(x^2-2x-15)$$

At this point, factor the remaining trinomial as usual, remembering to write the -1 as a factor in your final answer. Because $3+(-5)=-2$, use 3 and 5 as the factors of 15.

$$\begin{aligned}-x^2+2x=15 \;&=-1(x^2-2x-15)\\&=-1(x\quad)(x\quad)\\&=-(x+3)(x-5)\end{aligned}$$

Answer:

$-1(x+3)(x-5)$. The check is left to the reader.

Example 7.3.8

Factor:

$$-60a^2-5a+30$$

Solution

The GCF of all the terms is 5. However, in this case factor out -5 because this produces a trinomial factor where the leading coefficient is positive.

$$-60a^2-5a+30=-5(12a^2+a-6)$$

Focus on the factors of 12 and 6 that combine to give the middle coefficient, 1.

$$\begin{aligned}12&=1\cdot 12 \quad 6=1\cdot 6\\&=2\cdot 6 \qquad =2\cdot 3\\&=3\cdot 4\end{aligned}$$

After much thought, we find that $3\cdot 3-4\cdot 2=9-8=1$. Factor the remaining trinomial.

$$\begin{aligned}-60a^2-5a+30 \;&=-5(12a^2+a-6)\\&=-5(4a\quad)(3a\quad)\\&=-5(4a+3)(3a-2)\end{aligned}$$

Answer:

$-5(4a+3)(3a-2)$. The check is left to the reader.

Exercise 7.3.3

Factor:

$$24+2x-x^2\,.$$

Answer

$-1(x-6)(x+4)$

Factoring Using the AC Method

In this section, we factor trinomials of the form ax^2+bx+c using the AC method described previously.

Example 7.3.9

Factor using the AC method:

$$18x^2-21x+5\,.$$

Solution:

Here $a=18, b=-21$, and $c=5$.

$$\begin{aligned} ac &= 18(5) \\ &= 90 \end{aligned}$$

Factor 90 and search for factors whose sum is -21.

$$\begin{aligned} 90 &= 1(90) \\ &= 2(45) \\ &= 3(30) \\ &= 5(18) \\ &= 6(15) \quad \checkmark \\ &= 9(10) \end{aligned}$$

In this case, the sum of the factors -6 and -15 equals the middle coefficient, -21. Therefore, $-21x=-6x-15x$, and we can write

$$18x^2-21x+5 = 18x^2-6x-15x+5$$

Factor the equivalent expression by grouping.

$$\begin{aligned} 18x^2-21x+5 &= 18x^2-6x-15x+5 \\ &= 6x(3x-1)-5(3x-1) \\ &= (3x-1)(6x-5) \end{aligned}$$

Answer:

$$(3x-1)(6x-5)$$

Example 7.3.10

Factor using the AC method: $9x^2-61x-14$.

Solution:

Here $a=9, b=-61$, and $c=-14$.

We factor -126 as follows:

$$\begin{aligned} -126 &= 1(-126) \\ &= 2(-63) \quad \checkmark \\ &= 3(-42) \\ &= 6(-21) \\ &= 7(-18) \\ &= 9(-14) \end{aligned}$$

The sum of factors 2 and -63 equals the middle coefficient, -61. Replace $-61x$ with $2x-63x$:

$$\begin{aligned} 9x^2-61x-14 &= 9x^2+2x-63x-14 \quad \textit{Rearrange the terms.} \\ &= 9x^2-63x+2x-14 \quad \textit{Factor by grouping.} \\ &= 9x(x-7)+2(x-7) \\ &= (x-7)(9x+2) \end{aligned}$$

Answer:

$(x-7)(9x+2)$. The check is left to the reader.

Key Takeaways

- If a trinomial of the form ax^2+bx+c factors into the product of two binomials, then the coefficient of the middle term will be the sum of certain products of factors of the first and last terms.
- If the trinomial has a greatest common factor, then it is a best practice to first factor out the GCF before attempting to factor it into a product of binomials.
- If the leading coefficient of a trinomial is negative, then it is a best practice to factor that negative factor out before attempting to factor the trinomial.
- Factoring trinomials of the form ax^2+bx+c takes lots of practice and patience. It is extremely important to take the time to become proficient by working lots of exercises.

? Exercise 7.3.4 Factoring Trinomials

Factor.

1. $3x^2-14x-5$
2. $5x^2+7x+2$
3. $2x^2+5x-3$
4. $2x^2+13x-7$
5. $2x^2+9x-5$
6. $7x^2+20x-3$
7. $7x^2-46x-21$
8. $3x^2+x-2$
9. $5x^2+34x-7$
10. $5x^2-28x-12$
11. $9x^2-12x+4$
12. $4x^2-20x+25$
13. $49x^2+14x+1$
14. $25x^2-10x+1$
15. $2x^2+7x+16$
16. $6x^2-19x-10$
17. $27x^2+66x-16$
18. $12x^2-88x-15$
19. $12y^2-8y+1$
20. $16y^2-66y-27$
21. $9x^2-12xy+4y^2$
22. $25x^2+40x+16$
23. $15x^2-26xy+8y^2$
24. $12a^2-4ab-5b^2$
25. $4x^2y^2+16xy-9$
26. $20x^2y^2+4xy-7$
27. The area of a rectangle is given by the function $A(x)=3x^2-10x+3$, where x is measured in meters. Rewrite this function in factored form.
28. The area of a rectangle is given by the function $A(x)=10x^2-59x-6$, where x is measured in meters. Rewrite this function in factored form.

Answer

1. $(x-5)(3x+1)$
3. $(x+3)(2x-1)$
5. $(x+5)(2x-1)$

7. $(x-7)(7x+3)$
9. $(x+7)(5x-1)$
11. $(3x-2)^2$
13. $(7x+1)^2$
15. Prime
17. $(3x+8)(9x-2)$
19. $(6y-1)(2y-1)$
21. $(3x-2y)^2$
23. $(3x-4y)(5x-2y)$
25. $(2xy-1)(2xy+9)$
27. $A(x)=(3x-1)(x-3)$

Exercise 7.3.5 Factoring Trinomials with Common Factors

Factor.

1. $6x^2-20x-16$
2. $45x^2+27x-18$
3. $20x^2-20x+5$
4. $3x^2+39x-90$
5. $16x^2+26x-10$
6. $54x^2-15x+6$
7. $45x^2-45x-20$
8. $90x^2+300x+250$
9. $40x^2-36xy+8y^2$
10. $24a^2b^2+18ab-81$
11. $6x^2y^2+46xy+28$
12. $2x^5+44x^4+144x^3$
13. $5x^3-65x^2+60x$
14. $15a^4b^2-25a^3b-10a^2$
15. $6a^4b+2a^3b^2-4a^2b^3$
16. $20a^3b^2-60a^2b^3+45ab^4$

Answer

1. $2(x-4)(3x+2)$
3. $5(2x-1)^2$
5. $2(8x^2+13x-5)$
7. $5(3x-4)(3x+1)$
9. $4(5x-2y)(2x-y)$
11. $2(xy+7)(3xy+2)$
13. $5x(x-12)(x-1)$
15. $2a^2b(3a-2b)(a+b)$

Exercise 7.3.6 Factoring Trinomials with Common Factors

Factor out -1 and then factor further.

1. $-x^2-4x+21$
2. $-x^2+x+12$
3. $-x^2+15x-56$
4. $-x^2+x+72$
5. $-y^2+10y-25$
6. $-y^2-16y-64$
7. $36-9a-a^2$
8. $72-6a-a^2$
9. $32+4x-x^2$
10. $200+10x-x^2$

Answer

1. $-1(x-3)(x+7)$
3. $-1(x-7)(x-8)$
5. $-1(y-5)^2$
7. $-1(a-3)(a+12)$
9. $-1(x-8)(x+4)$

Exercise 7.3.7 Factoring Trinomials with Common Factors

Factor out a negative common factor first and then factor further if possible.

1. $-8x^2+6x+9$
2. $-4x^2+28x-49$
3. $-18x^2-6x+4$
4. $2+4x-30x^2$
5. $15+39x-18x^2$
6. $90+45x-10x^2$
7. $-2x^2+26x+28$
8. $-18x^3-51x^2+9x$
9. $-3x^2y^2+18xy^2-24y^2$
10. $-16a^4+16a^3b-4a^2b^2$
11. The height in feet of a projectile launched from a tower is given by the function $h(t)=-16t^2+64t+80$, where t represents the number of seconds after launch. Rewrite the given function in factored form.
12. The height in feet of a projectile launched from a tower is given by the function $h(t)=-16t^2+64t+192$, where t represents the number of seconds after launch. Rewrite the given function in factored form.

Answer

1. $-(2x-3)(4x+3)$
3. $-2(3x-1)(3x+2)$
5. $-3(2x-5)(3x+1)$
7. $-2(x-14)(x+1)$
9. $-3y^2(x-4)(x-2)$
11. $h(t)=-16(t+1)(t-5)$

Exercise 7.3.8 Factoring Using the AC Method

Factor using the AC method.

1. $2x^2+5x-7$
2. $3x^2+7x-10$
3. $4x^2-25x+6$
4. $16x^2-38x-5$
5. $6x^2+23x-18$
6. $8x^2+10x-25$
7. $4x^2+28x+40$
8. $-6x^2-3x+30$
9. $12x^2-56xy+60y^2$
10. $20x^2+80xy+35y^2$

Answer

1. $(x-1)(2x+7)$
3. $(x-6)(4x-1)$
5. $(2x+9)(3x-2)$
7. $4(x+2)(x+5)$
9. $4(x-3y)(3x-5y)$

Exercise 7.3.9 Discussion Board Topics

1. Create your own trinomial of the form ax^2+bx+c that factors. Share it, along with the solution, on the discussion board.
2. Write out your own list of steps for factoring a trinomial of the form ax^2+bx+c and share it on the discussion board.
3. Create a trinomial of the form ax^2+bx+c that does not factor and share it along with the reason why it does not factor.

Answer

1. Answers may vary
3. Answers may vary

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7.4: Factoring Special Binomials

Learning Objectives

- Factor binomials that are differences of squares.
- Factor binomials that are sums and differences of cubes.

Difference of Squares

A binomial is a polynomial with two terms. We begin with our first special binomial called difference of squares:

$$a^2 - b^2 = (a+b)(a-b) \tag{7.4.1}$$

To verify the above formula, multiply:

$$\begin{aligned}(a+b)(a-b) &= a^2 - ab + ba - b^2 \\ &= a^2 - ab + ab - b^2 \\ &= a^2 - b^2\end{aligned}$$

We use this formula to factor certain special binomials.

Example 7.4.1

Factor:

$$x^2 - 16.$$

Solution:

Step 1: Identify the binomial as difference of squares and determine the square factors of each term.

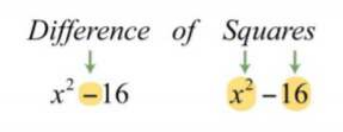


Figure 7.4.1

Here we can write

$$x^2 - 16 = (x)^2 - (4)^2$$

The terms are squares of x and 4. Hence $a = x$ and $b = 4$.

Step 2: Substitute into the difference of squares formula.

$$\begin{aligned}a^2 - b^2 &= (a+b)(a-b) \\ &\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\ x^2 - 16 &= (x+4)(x-4)\end{aligned}$$

Step 3: Multiply to check. This step is optional.

Answer:

$(x+4)(x-4)$

It is worth taking some extra time at this point to review all of the squares of integers from 1 to 12.

$$\begin{aligned} 1^2 &= 1 & 7^2 &= 49 \\ 2^2 &= 4 & 8^2 &= 64 \\ 3^2 &= 9 & 9^2 &= 81 \\ 4^2 &= 16 & 10^2 &= 100 \\ 5^2 &= 25 & 11^2 &= 121 \\ 6^2 &= 36 & 12^2 &= 144 \end{aligned}$$

Recognizing these perfect square integers helps speed the factoring process.

Example 7.4.2

Factor:

$$9x^2-121.$$

Solution:

The subtraction indicates that this is a difference. Furthermore, we recognize that the terms are squares.

$$9x^2-121=(3x)^2-(11)^2$$

In this case, $a=3x$ and $b=11$. Substitute into the formula for difference of squares.

$$\begin{aligned} a^2-b^2 &= (a+b)(a-b) \\ 9x^2-121 &= (3x+11)(3x-11) \end{aligned}$$

Answer:

$$(3x+11)(3x-11)$$

It may be the case that the terms of the binomial have a common factor. If so, it will be difficult to identify the perfect squares until we first factor out the GCF.

Example 7.4.3

Factor:

$$12y^2-75.$$

Solution:

The terms are not perfect squares. However, notice that they do have a common factor. First, factor out the GCF, 3.

$$12y^2-75=3(4y^2-25)$$

The resulting binomial factor is a difference of squares with $a=2y$ and $b=5$.

$$\begin{aligned} 12y^2-75 &= 3(4y^2-25) \\ &= 3(2y+5)(2y-5) \end{aligned}$$

Answer:

$$3(2y+5)(2y-5)$$

Example 7.4.4

Factor:

$$49x^2-100y^2.$$

Solution:

Here we have a binomial with two variables and recognize that it is a difference of squares.

$$49x^2-100y^2=(7x)^2-(10y)^2$$

Therefore, $a=7x$ and $b=10y$. Substitute into the formula for difference of squares.

$$\begin{aligned} a^2-b^2 &= (a+b)(a-b) \\ 49x^2-100y^2 &= (7x+10y)(7x-10y) \end{aligned}$$

Answer:

$$(7x+10y)(7x-10y)$$

Exercise 7.4.1

Factor:

$$36x^2-1.$$

Answer

$(6x+1)(6x-1)$

Given any real number b, a polynomial of the form x^2+b^2 is prime.

Furthermore, the sum of squares a^2+b^2 does not have a general factored equivalent. Care should be taken not to confuse this with a perfect square trinomial:

$$\begin{aligned}(a+b)^2 &= (a+b)(a+b)\\ &= a^2+ab+ba+b^2\\ &= a^2+2ab+b^2\end{aligned}$$

Therefore,

$$(a+b)^2 \neq a^2+b^2 \tag{7.4.2}$$

When the degree of the special binomial is greater than two, we may need to apply the difference of squares formula multiple times. A polynomial is completely factored when none of the factors can be factored any further.

Example 7.4.5

Factor completely:

$$x^4-16.$$

Solution:

First, identify what is being squared:

$$x^4-16=(\quad)^2-(\quad)^2$$

To do this, recall the power rule for exponents, $(x^m)^n=x^{mn}$. When exponents are raised to a power, multiply them. With this in mind, determine that $(x^2)^2=x^4$ and write

$$x^4-16=(x^2)^2-(4)^2$$

Therefore, $a=x^2$ and $b=4$. Substitute into the formula for difference of squares.

$$x^4-16=(x^2+4)(x^2-4)$$

At this point, notice that the factor (x^2-4) is itself a difference of two squares and thus can be further factored using $a=x$ and $b=2$. The factor (x^2+4) is a sum of squares, which cannot be factored using real numbers.

$$\begin{aligned}x^4-16 &= (x^2+4)(x^2-4)\\ &= (x^2+4)(x+2)(x-2)\end{aligned}$$

Answer:

$$(x^2+4)(x+2)(x-2)$$

Exercise 7.4.2

Factor completely:

$$81x^4-1.$$

Answer

$(9x^2+1)(3x+1)(3x-1)$

Sum and Difference of Cubes

Two other special binomials of interest are the sum and difference of cubes:

$$a^3+b^3=(a+b)(a^2-ab+b^2) \tag{7.4.3}$$

$$a^3-b^3=(a-b)(a^2+ab+b^2) \tag{7.4.4}$$

We can verify these formulas by multiplying:

$$\begin{aligned}(a+b)(a^2-ab+b^2) &= a^3-a^2b+ab^2-ab^2+b^3\\ &= a^3+b^3 \quad \checkmark\\ (a-b)(a^2+ab+b^2) &= a^3+a^2b+ab^2=a^2b-ab^2-b^3\\ &= a^3-b^3 \quad \checkmark\end{aligned}$$

The process for factoring the sum and difference of cubes is very similar to that for the difference of squares. We first identify a and b and then substitute into the appropriate formula. The separate formulas for sum and difference of cubes allow us to always choose a and b to be positive.

Example 7.4.6

Factor:

$$x^3+8.$$

Solution:

The plus sign and the fact that the terms are cubes indicate to us that this is a sum of cubes.

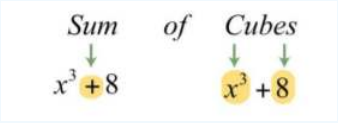


Figure 7.4.2

Next, identify what is being cubed.

$$x^3+8=(x)^3+(2)^3$$

In this case, $a=x$ and $b=2$. Substitute into the sum of cubes formula.

$$\begin{aligned}a^3+b^3 &= (a+b)(a^2-a\ b+\ b^2)\\ &\quad\ \downarrow\ \ \downarrow\ \downarrow\ \ \downarrow\ \downarrow\ \ \downarrow\\ x^3+8 &= (x+2)(x^2-x\cdot 2+2^2)\\ &= (x+2)(x^2-2x+4)\end{aligned}$$

The resulting trinomial is prime and the factoring is complete. We can check this factorization by multiplying.

$$\begin{aligned}(x+2)(x^2-2x+4) &= x^3-2x^2+4x+2x^2-4x+8\\ &= x^3-\ 2x^2+\ 4x+\ 2x^2-\ 4x+8\\ &= x^3+8 \quad \checkmark\end{aligned}$$

Answer:

$$(x-2)(x^2-2x+4)$$

It is helpful to review the perfect cubes of integers from 1 to 12. This will aid you in identifying sums and differences of cubes.

$$
\begin{array}{ll}
1^3 = 1 & y^3 = 343 \\
2^3 = 8 & 8^3 = 512 \\
3^3 = 27 & 9^3 = 729 \\
4^3 = 64 & 10^3 = 1000 \\
5^3 = 125 & 11^3 = 1331 \\
6^3 = 216 & 12^3 = 1728
\end{array}
$$

Example 7.4.7

Factor:

$$y^3 - 125.$$

Solution:

In this case, we have a difference of cubes.

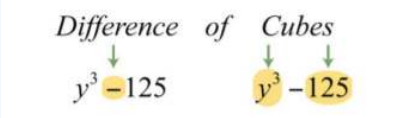


Figure 7.4.3

We can write

$$y^3 - 125 = (y)^3 - (5)^3$$

Substitute $a = y$ and $b = 5$ into the formula for difference of cubes.

$$
\begin{aligned}
a^3 - b^3 &= (a-b)(a^2 + a\ b + \ b^2) \\
&\quad\downarrow\ \downarrow\ \downarrow\ \ \downarrow\ \downarrow\ \ \downarrow \\
y^3 - 1258 &= (y-5)(y^2 + y \cdot 5 + 5^2) \\
&= (y-5)(y^2 + 5y + 25)
\end{aligned}
$$

Answer:

$$(y-5)(y^2 + 5y + 25)$$

Always look for common factors when factoring. If the terms of the binomial have a GCF other than 1, then factor that out first.

Example 7.4.8

Factor:

$$54x^4 + 128x.$$

Solution:

Begin by factoring out the GCF $2x$.

$$54x^4 + 128x = 2x(27x^3 + 64)$$

The resulting binomial factor is a sum of cubes, where $a = 3x$ and $b = 4$.

$$
\begin{aligned}
54x^4 + 128x &= 2x(27x^3 + 64) \\
&= 2x(3x + 4)((3x)^2 - (3x)(4) + 4^2) \\
&= 2x(3x + 4)(9x^2 - 12x + 16)
\end{aligned}
$$

Answer:

$$2x(3x + 4)(9x^2 - 12x + 16)$$

Example 7.4.9

Factor:

$$x^3y^3-1.$$

Solution

This binomial is a difference of cubes with two variables. Identify what is being cubed.

$$x^3y^3-1=(xy)^3-(1)^3$$

Here $a=xy$ and $b=1$. Substitute into the appropriate formula and simplify.

$$\begin{aligned} a^3-b^3 &= (a-b)(a^2+ab+b^2) \\ x^3y^3-1 &= (xy-1)((xy)^2+xy\cdot 1+1^2) \\ &= (xy-1)(x^2y^2+xy+1) \end{aligned}$$

Answer:

$$(xy-1)(x^2y^2+xy+1)$$

Exercise 7.4.3

Factor:

$$8x^3+343.$$

Answer

$(2x+7)(4x^2-14x+49)$

When factoring, always look for resulting factors to factor further.

Example 7.4.10

Factor completely:

$$x^6-64.$$

Solution:

When confronted with a binomial that is both a difference of squares and cubes, as this is, make it a rule to factor using difference of squares first.

$$x^6-64=(x^3)^2-(8)^2$$

Therefore, $a=x^3$ and $b=8$. Substitute into the difference of squares formula.

$$x^6-64=(x^3+8)(x^3-8)$$

The resulting two binomial factors are a sum and a difference of cubes. Each can be factored further.

$$\begin{aligned} x^3+8 &= (x+2)(x^2-2x+2^2) \\ x^3-8 &= (x-2)(x^2+2x+2^2) \end{aligned}$$

Therefore, we have

$$\begin{aligned} x^6-64 &= \overbrace{(x^3+8)}^{} \cdot \overbrace{(x^3-8)}^{} \\ &= \overbrace{(x+2)(x^2-2x+4)}^{} \cdot \overbrace{(x-2)(x^2+2x+4)}^{} \end{aligned}$$

The trinomial factors are prime and the expression is completely factored.

Answer:

$$(x+2)(x^2-2x+4)(x-2)(x^2+2x+4)$$

As an exercise, factor the previous example as a difference of cubes first and then compare the results. Why do you think we make it a rule to factor using difference of squares first?

Exercise 7.4.4

Factor:

$$x^6 - y^6 .$$

Answer

$(x+y)(x^2-xy+y2)(x-y)(x^2+xy+y2)$

Key Takeaways

- When factoring special binomials, the first step is to identify it as a sum or difference. Once we identify the binomial, we then determine the values of a and b and substitute into the appropriate formula.
- The formulas for all of the special binomials should be memorized. In addition, to help facilitate the identification of special binomials, memorize the squares and cubes of integers up to at least 12.
- If a binomial is both a difference of squares and cubes, then first factor it as a difference of squares.

Exercise 7.4.5 Difference of Squares

Factor completely.

1. $x^2 - 9$
2. $x^2 - 100$
3. $y^2 - 36$
4. $y^2 - 144$
5. $x^2 + 4$
6. $x^2 - 5$
7. $m^2 - 81$
8. $m^2 - 64$
9. $16x^2 - 9$
10. $25x^2 - 4$
11. $144x^2 - 1$
12. $9x^2 - 121$
13. $4y^2 - 81$
14. $100y^2 - 49$
15. $9 - 4x^2$
16. $100 - x^2$
17. $1 - y^2$
18. $25 - 9y^2$
19. $-3x^2 + 75$
20. $-16x^2 + 25$
21. $2x^2 - 72$
22. $20x^3 - 45x$
23. $-48x + 27x^3$
24. $36x^2 - 100$
25. $x^2 - y^2$
26. $25x^2 - 9y^2$
27. $a^2 - 4b^2$
28. $a^2b^2 - 36$
29. $4x^2y^2 - 1$
30. $x^2y^2 - 25$
31. $2a^3 - 8ab^2$

32. $3a^3b^4-75ab^2$
33. $-100xy^3+4x^3y$
34. $-18x^3y^3+32xy$
35. $(x+1)^2-y^2$
36. $x^2-(y-2)^2$
37. $(x-3)^2-(y+3)^2$
38. $(x^2+2)^2-(x-1)^2$
39. $(x^2-1)^2-(2x+3)^2$
40. x^4-1
41. x^4-y^4
42. $16x^4-81$
43. a^4b^4-16
44. a^4-16b^4
45. x^8-1
46. $25x^8-1$
47. a^8-b^2
48. a^4-9
49. x^8-y^8
50. $81x^8-1$
51. The height of a projectile dropped from a 64-foot tower is given by the function $h(t)=-16t^2+64$, where t represents the time in seconds after it is dropped. Rewrite this function in factored form. (Hint: Factor out -16 first.)
52. The height of a projectile dropped from a 36-foot tower is given by the function $h(t)=-16t^2+36$, where t represents the time in seconds after it is dropped. Rewrite this function in factored form.

Answer

1. $(x+3)(x-3)$
3. $(y+6)(y-6)$
5. Prime
7. $(m+9)(m-9)$
9. $(4x+3)(4x-3)$
11. $(12x+1)(12x-1)$
13. $(2y+9)(2y-9)$
15. $(3+2x)(3-2x)$
17. $(1+y)(1-y)$
19. $-3(x+5)(x-5)$
21. $2(x+6)(x-6)$
23. $3x(3x+4)(3x-4)$
25. $(x+y)(x-y)$
27. $(a+2b)(a-2b)$
29. $(2xy+1)(2xy-1)$
31. $2a(a+2b)(a-2b)$
33. $4xy(x+5y)(x-5y)$
35. $(x+1+y)(x+1-y)$
37. $(x+y)(x-y-6)$
39. $(x^2+2x+2)(x^2-2x-4)$

41. $(x^2+y^2)(x+y)(x-y)$
43. $(a^2b^2+4)(ab+2)(ab-2)$
45. $(x^4+1)(x^2+1)(x+1)(x-1)$
47. $(a^4+b)(a^4-b)$
49. $(x^4+y^4)(x^2+y^2)(x+y)(x-y)$
51. $h(t)=-16(t+2)(t-2)$

? Exercise 7.4.6 Sum and Difference of Cubes

Factor completely.

1. x^3-1
2. x^3+1
3. y^3-27
4. y^3-8
5. $8y^3+1$
6. $27y^3-1$
7. $64a^3-125$
8. $8a^3+27$
9. a^3+216
10. a^3-125
11. x^3-1000
12. $343m^3-1$
13. $512n^3+1$
14. $8x^3+343$
15. $40y^3-135$
16. $27y^3+729$
17. $27y^3-64$
18. x^3+3
19. $5x^3+1$
20. $1-y^3$
21. $27-1,000y^3$
22. $343+125a^3$
23. x^3-y^3
24. x^3+y^3
25. x^3y^3+125
26. $8x^3y^3-27$
27. $27a^3-8b^3$
28. $16x^3-250y^3$
29. $128x^3+2y^3$
30. $54x^3-2y^3$
31. $3a^4b-24ab^4$
32. $a^3b^3c^3-1$
33. $(x+1)^3-8$
34. $8x^3-(x-5)^3$
35. $(x-2)^3+(x+2)^3$
36. $(a+3)^3+(b-3)^3$
37. x^6-1
38. x^6+1
39. $64a^6-1$
40. a^6-81b^6

41. $x^6 - y^6$
42. $x^6 + y^6$

Answer

1. $(x-1)(x^2+x+1)$
3. $(y-3)(y^2+3y+9)$
5. $(2y+1)(4y^2-2y+1)$
7. $(4a-5)(16a^2+20a+25)$
9. $(a+6)(a^2-6a+36)$
11. $(x-10)(x^2+10x+100)$
13. $(8n+1)(64n^2-8n+1)$
15. $5(2y-3)(4y^2+6y+9)$
17. $(3y-4)(9y^2+12y+16)$
19. Prime
21. $(3-10y)(9+30y+100y^2)$
23. $(x-y)(x^2+xy+y^2)$
25. $(xy+5)(x^2y^2-5xy+25)$
27. $(3a-2b)(9a^2+6ab+4b^2)$
29. $2(4x+y)(16x^2-4xy+y^2)$
31. $3ab(a-2b)(a^2+2ab+4b^2)$
33. $(x-1)(x^2+4x+7)$
35. $2x(x^2+12)$
37. $(x+1)(x^2-x+1)(x-1)(x^2+x+1)$
39. $(2a+1)(4a^2-2a+1)(2a-1)(4a^2+2a+1)$
41. $(x+y)(x^2-xy+y^2)(x-y)(x^2+xy+y^2)$

? Exercise 7.4.7 Discussion Board Topics

1. If a binomial falls into both categories, difference of squares and difference of cubes, which would be best to factor it as, and why? Create an example that illustrates this situation and factor it using both formulas.
2. What can be said about the degrees of the factors of a polynomial? Give an example.
3. Make up your own difference of squares factoring exercise and provide the answer. Explain how you solved it.
4. Make up your own sum or difference of cubes factoring exercise and provide the answer. Explain how you solved it.

Answer

1. Answers may vary
3. Answers may vary

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7.5: General Guidelines for Factoring Polynomials

Learning Objectives

- Develop a general strategy for factoring polynomials.

General Factoring Strategy

We have learned various techniques for factoring polynomials with up to four terms. The challenge is to identify the type of polynomial and then decide which method to apply. The following outlines a **general guideline for factoring** polynomials:

1. Check for common factors. If the terms have common factors, then factor out the greatest common factor (GCF) and look at the resulting polynomial factors to factor further.
2. Determine the number of terms in the polynomial.
   a. Factor four-term polynomials by grouping.
   b. Factor trinomials (three terms) using "trial and error" or the AC method.
   c. Factor binomials (two terms) using the following special products:

Table 7.5.1

Difference of Squares:	$a^2-b^2+(a+b)(a-b)$
Sum of Squares:	a^2+b^2 no general formula
Difference of Cubes:	$a^3-b^3=(a-b)(a^2+ab+b^2)$
Sum of Cubes:	$a^3+b^3=(a+b)(a^2-ab+b^2)$

3. Look for factors that can be factored further.
4. Check by multiplying.

Note

- If a binomial is both a difference of squares and a difference of cubes, then first factor it as difference of squares and then as a sum and difference of cubes to obtain a more complete factorization.
- Not all polynomials with integer coefficients factor. When this is the case, we say that the polynomial is prime.

If an expression has a GCF, then factor this out first. Doing so is often overlooked and typically results in factors that are easier to work with. Also, look for the resulting factors to factor further; many factoring problems require more than one step. A polynomial is completely factored when none of the factors can be factored further.

Example 7.5.1

Factor:

$$6x^4-3x^3-24x^2+12x\ .$$

Solution:

This four-term polynomial has a GCF of $3x$. Factor this out first.

$$6x^4-3x^3-24x^2+12x=3x(2x^3-x^2-8x+4)$$

Now factor the resulting four-term polynomial by grouping.

$$\begin{aligned}6x^4-3x^3-24x^2+12x &= 3x\quad \underbrace{(2x^3-x^2}_{group}\quad \underbrace{-8x+4)}_{group}\\ &=3x(x^2(2x-1)-4(2x-1))\\ &=3x((2x-1)(x^2-4))\\ &=3x(2x-1)(x^2-4)\end{aligned}$$

The factor (x^2-4) is a difference of squares and can be factored further.

$$\begin{aligned} &= 3x(2x-1)(x^2-4) \\ &= 3x(2x-1)(x+2)(x-2) \end{aligned}$$

Answer:

$$3x(2x-1)(x+2)(x-2)$$

Example 7.5.2

Factor:

$$18x^3y-60x^2y+50xy.$$

Solution:

This trinomial has a GCF of $2xy$. Factor this out first.

$$18x^3y-60x^2y+50xy = 2xy(9x^2-30x+25)$$

The trinomial factor can be factored further using the trial and error method. Use the factors $9=3\cdot 3$ and $25=(-5)\cdot(-5)$. These combine to generate the correct coefficient for the middle term: $3(-5)+3(-5)=-15-15=-30$.

$$\begin{aligned} 18x^3y-60x^2y+50xy &= 2xy(9x^2-30x+25) \\ &= 2xy(3x \quad ?)(3x \quad ?) \\ &= 2xy(3x-5)(3x-5) \\ &= 2xy(3x-5)^2 \end{aligned}$$

Check.

$$\begin{aligned} 2xy(3x-5)^2 &= 2xy(9x^2-2\cdot 3x\cdot 5+5^2) \\ &= 2xy(9x^2-30x+25) \\ &= 18x^3y-60x^2y+50xy \quad \checkmark \end{aligned}$$

Answer:

$$2xy(3x-5)^2$$

Example 7.5.3

Factor:

$$5a^3b^4+10a^2b^3-75ab^2\,.$$

Solution:

This trinomial has a GCF of 5ab2. Factor this out first.

$$5a^3b^4+10a^2b^3-75ab^2 = 5ab^2(a^2b^2+2ab-15)$$

The resulting trinomial factor can be factored as follows:

$$\begin{aligned} 5a^3b^4+10a^2b^3-75ab^2 &= 5ab^2(a^2b^2+2ab-15) \\ &= 5ab^2(ab \quad ?)(ab \quad ?) \\ &= 5ab^2(ab+5)(ab-3) \end{aligned}$$

Answer:

$$5ab^2(ab+5)(ab-3)$$

Exercise 7.5.1

Factor:

$$3x^3y-12x^2y^2+12xy^3.$$

Answer

$3xy(x-2y)^2$

Example 7.5.4

Factor:

$$16y^4-1.$$

Solution:

This binomial does not have a GCF. Therefore, begin factoring by identifying it as a difference of squares.

$$16y^4-1=(4y^2)^2-(1)^2$$

Here $a=4y^2$ and $b=1$. Substitute into the formula for difference of squares.

The factor $(4y^2+1)$ is a sum of squares and is prime. However, $(4y^2-1)$ is a difference of squares and can be factored further.

$$\begin{aligned}16y^4-1 &=(4y^2+1)(4y^2-1)\\ &=(4y^2+1)(2y+1)(2y-1)\end{aligned}$$

Answer:

$$(4y^2+1)(2y+1)(2y-1)$$

Example 7.5.5

Factor:

$$x^6-64y^6.$$

Solution:

This binomial is a difference of squares and a difference of cubes. When this is the case, first factor it as a difference of squares.

$$x^6-64y^6=(x^3)^2-(8y^3)^2$$

We can write

$$x^6-64y^6=(x^3+8y^3)(x^3-8y^3)$$

Each factor can be further factored either as a sum or difference of cubes, respectively.

$$\begin{aligned}x^3+8y^3 &=(x)^3+(2y)^3=(x+2y)(x^2-2xy+4y^2)\\ x^3-8y^3 &=(x)^3-(2y)^3=(x-2y)(x^2+2xy+4y^2)\end{aligned}$$

Therefore,

$$\begin{aligned}x^6-64y^6 &=(x^3+8y^3)(x^3-8y^3)\\ &=(x+2y)(x^2-2xy+4y^2)(x-2y)(x^2+2xy+4y^2)\end{aligned}$$

Answer:

$$(x+2y)(x^2-2xy+4y^2)(x-2y)(x^2+2xy+4y^2)$$

Example 7.5.6

First, identify this expression as a difference of squares.

$$x^2-(2x-1)^2=(x)^2-(2x-1)^2$$

Solution:

Here use $a=x$ and $b=2x-1$ in the formula for a difference of squares.

$$\begin{aligned} a^2-b^2 &= (a+b)(a-b) \\ x^2-(2x-1)^2 &= [(x+(2x-1))][(x-(2x-1))] \\ &= (x+2x-1)(x-2x+1) \\ &= (3x-1)(-x+1) \end{aligned}$$

Answer:

$$(3x-1)(-x+1)$$

Exercise 7.5.2

Factor:

$$x^4+2x^3+27x+54\ .$$

Answer

$(x+2)(x+3)(x^2-3x+9)$

Key Takeaways

- Use the polynomial type to determine the method used to factor it.
- It is a best practice to look for and factor out the greatest common factor (GCF) first. This will facilitate further factoring and simplify the process. Be sure to include the GCF as a factor in the final answer.
- Look for resulting factors to factor further. It is often the case that factoring requires more than one step.
- If a binomial can be considered as both a difference of squares and a difference of cubes, then first factor it as a difference of squares. This results in a more complete factorization.

Exercise 7.5.3 Mixed Factoring

Factor completely.

1. $2x^5y^2-12x^4y^3$
2. $18x^5y^3-6x^4y^5$
3. $5x^2+20x-25$
4. $4x^2+10x-6$
5. $24x^3-30x^2-9x$
6. $30x^3-65x^2+10x$
7. $6x^3+27x^2-9x$
8. $21x^3+49x^2-28x$
9. $5x^3-30x^2-15x+90$
10. $6x^4+24x^3-2x^2-8x$
11. $x^4-6x^3+8x-48$
12. $x^4-5x^3+27x-135$
13. $4x^3-4x^2-9x+9$
14. $50x^3+25x^2-32x-16$
15. $2x^3+250$
16. $3x^5-81x^2$
17. $2x^5-162x^3$

18. $4x^4 - 36$
19. $x^4 + 16$
20. $x^3 + 9$
21. $72 - 2x^2$
22. $5x^4 - 25x^2$
23. $7x^3 - 14x$
24. $36x^2 - 12x + 1$
25. $25x^2 + 10x + 1$
26. $250x^3 + 200x^4 + 40x^5$
27. $-7x^2 + 19x + 6$
28. $-8x^4 + 40x^3 - 50x^2$
29. $a^4 - 16$
30. $16a^4 - 81b^4$
31. $y^5 + y^4 - y - 1$
32. $4y^5 + 2y^4 - 4y^2 - 2y$
33. $3x^8 - 192x^2$
34. $4x^7 + 4x$
35. $4x^2 - 19xy + 12y^2$
36. $16x^2 - 66xy - 27y^2$
37. $5x^5 - 3x^4 - 5x^3 + 3x^2$
38. $4a^2b^2 - 4a^2 - 9b^2 + 9$
39. $15a^2 - 4ab - 4b^2$
40. $6a^2 - 25ab + 4b^2$
41. $6x^2 + 5xy + 6y^2$
42. $9x^2 + 5xy - 12y^2$
43. $(3x - 1)^2 - 64$
44. $(x - 5)^2 - (x - 2)^2$
45. $(x + 1)^3 + 8$
46. $(x - 4)^3 - 27$
47. $(2x - 1)^2 - (2x - 1) - 12$
48. $(x - 4)^2 + 5(x - 4) + 6$
49. $a^3b - 10a^2b^2 + 25ab^3$
50. $2a^3b^2 - 12a^2b + 18a$
51. $15a^2b^2 - 57ab - 12$
52. $-60x^3 + 4x^2 + 24x$
53. $-24x^5 + 78x^3 - 54x$
54. $9y^6 - 13y^4 + 4y^2$
55. $36 - 15a - 6a^2$
56. $60ab^2 + 5a^2b^2 - 5a^3b^2$
57. $x^4 - 1$
58. $16x^4 - 64$
59. $x^8 - 1$
60. $81x^8 - 1$
61. $x^{16} - 1$
62. $x^{12} - 1$
63. $54x^6 - 216x^4 - 2x^3 + 8x$
64. $4a^4 - 4a^2b^2 - a^2 + b^2$
65. $32y^3 + 32y^2 - 18y - 18$
66. $3a^3 + a^2b - 12ab - 4b^2$
67. $18m^2 - 21mn - 9n^2$
68. $5m^2n^2 + 10mn - 15$

69. The volume of a certain rectangular solid is given by the function $V(x) = x^3 - 2x^2 - 3x$. Write the function in its factored form.
70. The volume of a certain right circular cylinder is given by the function $V(x) = 4\pi x^3 - 4\pi x^2 + \pi x$. Write the function in its factored form.

Answer

1. $2x^4y^2(x-6y)$
3. $5(x-1)(x+5)$
5. $3x(2x-3)(4x+1)$
7. $3x(2x^2+9x-3)$
9. $5(x-6)(x^2-3)$
11. $(x-6)(x+2)(x^2-2x+4)$
13. $(x-1)(2x-3)(2x+3)$
15. $2(x+5)(x^2-5x+25)$
17. $2x^3(x+9)(x-9)$
19. Prime
21. $2(6+x)(6-x)$
23. $7x(x^2-2)$
25. $(5x+1)^2$
27. $-(x-3)(7x+2)$
29. $(a^2+4)(a+2)(a-2)$
31. $(y^2+1)(y-1)(y+1)^2$
33. $3x^2(x+2)(x^2-2x+4)(x-2)(x^2+2x+4)$
35. $(x-4y)(4x-3y)$
37. $x^2(5x-3)(x+1)(x-1)$
39. $(3a-2b)(5a+2b)$
41. Prime
43. $3(x-3)(3x+7)$
45. $(x+3)(x^2+3)$
47. $2(x+1)(2x-5)$
49. $ab(a-5b)^2$
51. $3(ab-4)(5ab+1)$
53. $-6x(x+1)(x-1)(2x+3)(2x-3)$
55. $-3(a+4)(2a-3)$
57. $(x^2+1)(x+1)(x-1)$
59. $(x^4+1)(x^2+1)(x+1)(x-1)$
61. $(x^8+1)(x^4+1)(x^2+1)(x+1)(x-1)$
63. $2x(x+2)(x-2)(3x-1)(9x^2+3x+1)$
65. $2(y+1)(4y-3)(4y+3)$
67. $3(2m-3n)(3m+n)$

69. $V(x) = x(x+1)(x-3)$

Exercise 7.5.4 Discussion Board

1. First, factor the trinomial $24x^2 - 28x - 40$. Then factor out the GCF. Discuss the significance of factoring out the GCF first. Do you obtain the same result?
2. Discuss a plan for factoring polynomial expressions on an exam. What should you be looking for and what should you be expecting?

Answer

1. Answer may vary

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7.6: Solving Equations by Factoring

Learning Objectives

- Verify solutions to quadratic equations.
- Solve quadratic equations by factoring.
- Determine a quadratic equation with given solutions.
- Solve polynomial equations by factoring.

Solving Quadratic Equations by Factoring

Learning how to solve equations is one of our main goals in algebra. Up to this point, we have solved linear equations, which are of degree 1. In this section, we will learn a technique that can be used to solve certain equations of degree 2. A quadratic equation is any equation that can be written in the standard form

$$ax^2+bx+c=0 \tag{7.6.1}$$

,

where a, b, and c are real numbers and $a \neq 0$. The following are some examples of quadratic equations, all of which will be solved in this section:

$$x^2+x-6=0$$
$$4x^2-9=0$$
$$2x^2+10x+20=-3x+5$$

A solution of a quadratic equation in standard form is called a root. Quadratic equations can have two real solutions, one real solution, or no real solution. The quadratic equation $x^2+x-6=0$ has two solutions, namely, $x=-3$ and $x=2$.

Example 7.6.1

Verify that $x=-3$ and $x=2$ are solutions to $x^2+x-6=0$.

Solution:

To verify solutions, substitute the values for x and then simplify to see if a true statement results.

$Check: x=-3$	$Check: x=2$
$x^2+x-6=0$	$x^2+x-6=0$
$(-3)^2+(-3)-6=0$	$(2)^2+(2)-6=0$
$9-3-6=0$	$4+2-6=0$
$0=0$ ✓	$0=0$ ✓

Answer:

Both values produce true statements. Therefore, they are both solutions to the equation.

Our goal is to develop algebraic techniques for finding solutions to quadratic equations. The first technique requires the zero-product property:

$$\text{If } a \cdot b=0 \text{ , then } a=0 \text{ or } b=0$$

In other words, if any product is equal to zero, then one or both of the variable factors must be equal to zero.

Example 7.6.2

Solve:

$$(x-8)(x+7)=0 \text{ .}$$

Solution:

This equation consists of a product of two quantities equal to zero; therefore, the zero-product property applies. One or both of the quantities must be zero.

$$\begin{array}{lll} (x-8)=0 & \text{or} & (x+7)=0 \\ x-8+8=0+8 & & x+7-7=0-7 \\ x=8 & & x=-7 \end{array}$$

To verify that these are solutions, substitute them for the variable x.

$$\begin{array}{r|r} Check: x=8 & Check: x=-7 \\ (x-8)(x+7)=0 & (x-8)(x+7)=0 \\ (8-8)(8+7)=0 & (-7+8)(-7+7)=0 \\ (0)(15)=0 & (1)(0)=0 \\ 0=0 \quad \checkmark & 0=0 \quad \checkmark \end{array}$$

Notice that each solution produces a factor that is equal to zero.

Answer:

The solutions are 8 and -7.

The quadratic equation may not be given in its factored form.

Example 7.6.3

Solve:

$$x^2+3x-10=0\ .$$

Solution:

The goal is to produce a product that is equal to zero. We can do that by factoring the trinomial on the left side of the equation.

$$\begin{array}{ll} x^2+3x-10=0 & \\ (x\quad ?)(x\quad ?)=0 & -10=5(-2) \\ & and\ 3=5+(-2) \\ (x+5)(x-2)=0 & \end{array}$$

Next, apply the zero-product property and set each factor equal to zero.

$$x+5=0 \quad \text{or} \quad x-2=0$$

This leaves us with two linear equations, each of which can be solved for x.

$$\begin{array}{ll} x+5-5=0-5 & x-2+2=0+2 \\ x=-5 & x=2 \end{array}$$

Check the solutions by substituting into the original equation to verify that we obtain true statements.

$$\begin{array}{r|r} Check: x=-5 & Check: x=2 \\ x^2+3x-10=0 & x^2+3x-10=0 \\ (-5)^2+3(-5)-10=0 & (2)^2+3(2)-10=0 \\ 25-15-10=0 & 4+6-10=0 \\ 0=0 \quad \checkmark & 0=0 \quad \checkmark \end{array}$$

Answer:

The solutions are -5 and 2.

Using the zero-product property after factoring a quadratic equation in standard form is the key to this technique. However, the quadratic equation may not be given in standard form, and so there may be some preliminary steps before factoring. The steps required to solve by factoring are outlined in the following example.

Example 7.6.4

Solve:

$$2x^2+10x+20=-3x+5\ .$$

Solution:

Step 1: Express the quadratic equation in standard form. For the zero-product property to apply, the quadratic expression must be equal to zero. Use the addition and subtraction properties of equality to combine opposite-side like terms and obtain zero on one side of the equation. In this example, add $3x$ to and subtract 5 from both sides.

$$\begin{aligned} 2x^2+10x+20+3x &= -3x+5+3x \\ 2x^2+13x+20 &= 5 \\ 2x^2+13x+20-5 &= 5-5 \\ 2x^2+13x+15 &= 0 \end{aligned}$$

Step 2: Factor the quadratic expression.

$$(2x+3)(x+5)=0$$

Step 3: Apply the zero-product property and set each variable factor equal to zero.

$$2x+3=0 \quad \text{or} \quad x+5=0$$

Step 4: Solve the resulting linear equations.

$$\begin{array}{ll} 2x+3=0 & \text{or} \quad x+5=0 \\ 2x=-3 & x=-5 \\ \frac{2x}{2}=\frac{-3}{2} & \\ x=-\frac{3}{2} & \end{array}$$

Answer:

The solutions are -5 and $-\frac{3}{2}$. The check is optional.

Example 7.6.5

Solve:

$$9x^2+1=6x\ .$$

Solution:

Write this in standard form by subtracting \(6x\ from both sides.

$$\begin{aligned} 9x^2+1-6x &= 6x-6x \\ 9x^2-6x+1 &= 0 \end{aligned}$$

Once the equation is in standard form, equal to zero, factor.

$$(3x-1)(3x-1)=0$$

This is a perfect square trinomial. Hence setting each factor equal to zero results in a repeated solution.

$$\begin{array}{ll} 3x-1=0 & \text{or} \quad 3x-1=0 \\ 3x=1 & 3x=1 \\ x=\frac{1}{3} & x=\frac{1}{3} \end{array}$$

A repeated solution is called a double root and does not have to be written twice.

Answer:

The solution is $\frac{1}{3}$.

Exercise 7.6.1

Solve:

$$x^2 - 3x = 28\,.$$

Answer

$x = -4$ or $x = 7$

Not all quadratic equations in standard form are trinomials. We often encounter binomials.

Example 7.6.6

Solve:

$$x^2 - 9 = 0\,.$$

Solution:

This quadratic equation is given in standard form, where the binomial on the left side is a difference of squares. Factor as follows:

$$\begin{aligned} x^2 - 9 &= 0 \\ (x+3)(x-3) &= 0 \end{aligned}$$

Next, set each factor equal to zero and solve.

$$\begin{aligned} x+3 = 0 \quad &\text{or} \quad x-3 = 0 \\ x = -3 \quad &\qquad\quad x = 3 \end{aligned}$$

Answer:

The solutions are 3 and -3, which can also be written as ± 3.

Example 7.6.7

Solve:

$$5x^2 = 15x$$

Solution:

By inspection, we see that $x = 0$ is a solution to this quadratic equation. Since dividing by zero is undefined, we want to avoid dividing both sides of this equation by x. In general, we wish to avoid dividing both sides of any equation by a variable or an expression containing a variable. We will discuss this in more detail later. The first step is to rewrite this equation in standard form with zero on one side.

$$\begin{aligned} 5x^2 &= 15x \\ 5x^2 - 15x &= 15x - 15x \quad \textit{Subtract } 15x \textit{ from both sides.} \\ 5x^2 - 15x &= 0 \end{aligned}$$

Next, factor the expression. Notice that the binomial on the left has a GCF of $5x$.

$$\begin{aligned} 5x^2 - 15x &= 0 \\ 5x(x-3) &= 0 \end{aligned}$$

Set each factor to equal to zero.

$$\begin{aligned} 5x = 0 \quad &\text{or} \quad x-3 = 0 \\ \tfrac{5x}{5} = \tfrac{0}{5} \quad &\qquad\quad x = 3 \\ x = 0 \quad & \end{aligned}$$

Answer:

The solutions are 0 and 3.

Example 7.6.8

Solve:

$$(2x+1)(x+5)=11\,.$$

Solution:

This quadratic equation appears to be factored; hence it might be tempting to set each factor equal to 11. However, this would lead to incorrect results. We must rewrite the equation in standard form, equal to zero, so that we can apply the zero-product property.

$$\begin{aligned}(2x+1)(x+5) &= 11\\ 2x^2+10x+x+5 &= 11\\ 2x^2+11x+5-11 &= 11-11\\ 2x^2+11x-6 &= 0\end{aligned}$$

Once it is in standard form, we can factor and then set each factor equal to zero.

$$(2x-1)(x+6)=0$$

$$\begin{aligned}2x-1=0 \quad &\text{or} \quad x+6=0\\ 2x=1 \quad &\qquad\quad x=-6\\ x=\tfrac{1}{2}\end{aligned}$$

Answer:

The solutions are $\frac{1}{2}$ and -6.

Example 7.6.9

Solve:

$$15x^2-25x+10=0\,.$$

Solution:

We begin by factoring out the GCF of 5. Then factor the resulting trinomial.

$$\begin{aligned}15x^2-25x+10 &= 0\\ 5(3x^2-5x+2) &= 0\\ 5(3x-2)(x-1) &= 0\end{aligned}$$

Next, we set each ***variable factor*** equal to zero and solve for x.

$$\begin{aligned}3x-2=0 \quad &\text{or} \quad x-1=0\\ 3x=2 \quad &\qquad\quad x=1\\ x=\tfrac{2}{3}\end{aligned}$$

Notice that the factor 5 is not a variable factor and thus did not contribute to the solution set.

Answer:

The solutions are $\frac{2}{3}$ and 1.

Example 7.6.10

Factor:

$$52x^2+76x-13=0\,.$$

Solution:

Clear the fractions by multiplying both sides of the equation by the LCD, which is equal to 6.

$$\begin{aligned} \frac{5}{2}x^2+\frac{7}{6}x-\frac{1}{3} &= 0 \\ 6\cdot\left(\frac{5}{2}x^2+\frac{7}{6}x-\frac{1}{3}\right) &= 6\cdot(0) \\ 15x^2+7x-2 &= 0 \end{aligned}$$

At this point, we have an equivalent equation with integer coefficients and can factor as usual. Begin with the factors of 15 and 2.

$$\begin{aligned} 15 &= 1\cdot 15 \quad 2 = 1\cdot 2 \\ &= 3\cdot 5 \end{aligned}$$

The coefficient of the middle term is $7 = 3(-1)+5(2)$. Factor as follows:

$$\begin{aligned} 15x^2+7x-2 &= 0 \\ (3x+2)(5x-1) &= 0 \end{aligned}$$

Set each factor equal to zero and solve.

$$\begin{aligned} 3x+2=0 \quad &\text{or} \quad 5x-1=0 \\ 3x=-2 \quad &\qquad 5x=1 \\ x=-\tfrac{2}{3} \quad &\qquad x=\tfrac{1}{5} \end{aligned}$$

Answer:

The solutions are $-\frac{2}{3}$ and $\frac{1}{5}$.

? Exercise 7.6.2

Solve:

$$4x^2-9=0\,.$$

Answer

$-\frac{3}{2}$ and $\frac{3}{2}$

Finding Equations with Given Solutions

The zero-product property states,

$$\text{If } a\cdot b=0\text{, then } a=0 \text{ or } b=0$$

And, in fact, the converse is true as well:

$$\text{If } a=0 \text{ or } b=0\text{, then } ab=0$$

When this is the case, we can write the following:

$$a\cdot b=0\text{, if and only if } a=0 \text{ or } b=0$$

We use this property to find equations, given the solutions. To do this, the steps for solving by factoring are performed in reverse.

✓ Example 7.6.11

Find a quadratic equation with solutions -7 and 2.

Solution:

Given the solutions, we can determine two linear factors.

$$\begin{aligned} x=-7 \quad &\text{or} \quad x=2 \\ x+7=0 \quad &\qquad x-2=0 \end{aligned}$$

The product of these linear factors is equal to zero when $x = -7$ or $x = 2$:

$$(x+7)(x-2) = 0$$

Multiply the binomials and present the equation in standard form.

$$\begin{aligned} x^2 - 2x + 7x - 14 &= 0 \\ x^2 + 5x - 14 &= 0 \end{aligned}$$

Answer:

$$x^2 + 5x - 14 = 0\ .$$

We may check our equation by substituting the given answers to see if we obtain a true statement. Also, the equation found aboveis not unique and so the check becomes essential when our equation looks different from someone else's. This is left as an exercise.

Example 7.6.12

Find a quadratic equation with integer coefficients, given solutions $\frac{1}{2}$ and $-\frac{3}{4}$.

Solution:

To avoid fractional coefficients, we first clear the fractions by multiplying both sides by the denominator.

$$\begin{array}{ccc} x = \frac{1}{2} & \text{or} & x = -\frac{3}{4} \\ 2 \cdot x = 2 \cdot \frac{1}{2} & & 4 \cdot x = -\frac{3}{4} \cdot 4 \\ 2x = 1 & & 4x = -3 \\ 2x - 1 = 0 & & 4x + 3 = 0 \end{array}$$

Apply the zero-product property and multiply.

$$\begin{aligned} (2x-1)(4x+3) &= 0 \\ 8x^2 + 6x - 4x - 3 &= 0 \\ 8x^2 + 2x - 3 &= 0 \end{aligned}$$

Answer:

$$8x^2 + 2x - 3 = 0$$

Exercise 7.6.3

Find a quadratic equation with integer coefficients, given solutions -1 and $\frac{2}{3}$.

Answer

$3x^2 + x - 2 = 0$

Solving Polynomial Equations by Factoring

The zero-product property is true for any number of factors that make up an equation. If an expression is equal to zero and can be factored into linear factors, then we will be able to set each factor equal to zero and solve for each equation.

Example 7.6.13

Solve:

$$3x(x-5)(3x-2) = 0\,.$$

Solution:

Set each variable factor equal to zero and solve.

$$3x=0 \quad \text{or} \quad x-5=0 \quad \text{or} \quad 3x-2=0$$
$$\frac{3x}{3}=\frac{0}{3} \qquad x=5 \qquad \frac{3x}{3}=\frac{2}{3}$$
$$x=0 \qquad\qquad x=\frac{2}{3}$$

Answer:

The solutions are $0, 5$, and $\frac{2}{3}$.

Of course, we cannot expect the equation to be given in factored form.

Example 7.6.14

Solve:

$$x^3+2x^2-9x-18=0\ .$$

Solution

Begin by factoring the left side completely.

$$\begin{aligned} x^3+2x^2-9x-18&=0 && \textit{Factor by grouping.}\\ x^2(x+2)-9(x+2)&=0 &&\\ (x+2)(x^2-9)&=0 && \textit{Factor as a difference of squares.}\\ (x+2)(x+3)(x-3)&=0 &&\end{aligned}$$

Set each factor equal to zero and solve.

$$x+2=0 \quad \text{or} \quad x+3=0 \quad \text{or} \quad x-3=0$$
$$x=-2 \qquad x=-3 \qquad x=3$$

Answer:

The solutions are $-2, -3$, and 3.

Notice that the degree of the polynomial is 3 and we obtained three solutions. In general, for any polynomial equation with one variable of degree n, the fundamental theorem of algebra guarantees n real solutions or fewer. We have seen that many polynomials do not factor. This does not imply that equations involving these unfactorable polynomials do not have real solutions. In fact, many polynomial equations that do not factor do have real solutions. We will learn how to solve these types of equations as we continue in our study of algebra.

Exercise 7.6.4

Solve:

$$-10x^3-18x^2+4x=0\ .$$

Answer

$-2, 0, \frac{1}{5}$

Key Takeaways

- A polynomial can have at most a number of solutions equal to its degree. Therefore, quadratic equations can have up to two real solutions.
- To solve a quadratic equation, first write it in standard form. Once the quadratic expression is equal to zero, factor it and then set each variable factor equal to zero. The solutions to the resulting linear equations are the solutions to the quadratic equation.
- Not all quadratic equations can be solved by factoring. We will learn how to solve quadratic equations that do not factor later in the course.
- To find a quadratic equation with given solutions, perform the process of solving by factoring in reverse.

- If any polynomial is factored into linear factors and is set to zero, then we can determine the solutions by setting each variable factor equal to zero and solving each individually.

Exercise 7.6.5 Solutions to Quadratic Equations

Determine whether the given set of values are solutions to the quadratic equation.

1. $-3, 5; x^2 - 2x - 15 = 0$
2. $7, -1; x^2 - 6x - 7 = 0$
3. $-\frac{1}{2}, \frac{1}{2}; x^2 - 14 = 0$
4. $-\frac{3}{4}, \frac{3}{4}; x^2 - 916 = 0$
5. $-3, 2; x^2 - x - 6 = 0$
6. $-5, 1; x^2 - 4x - 5 = 0$

Answer

1. Yes
3. Yes
5. No

Exercise 7.6.6 Solutions to Quadratic Equations

Solve.

1. $(x-3)(x+2) = 0$
2. $(x+5)(x+1) = 0$
3. $(2x-1)(x-4) = 0$
4. $(3x+1)(3x-1) = 0$
5. $(x-2)^2 = 0$
6. $(5x+3)^2 = 0$
7. $7x(x-5) = 0$
8. $-2x(2x-3) = 0$
9. $(x-12)(x+34) = 0$
10. $(x+58)(x-38) = 0$
11. $(14x+12)(16x-23) = 0$
12. $(15x-3)^2 = 0$
13. $-5(x+1)(x-2) = 0$
14. $12(x-7)(x-6) = 0$
15. $(x+5)(x-1) = 0$
16. $(x+5)(x+1) = 0$
17. $-2(3x-2)(2x+5) = 0$
18. $5(7x-8)^2 = 0$

Answer

1. $-2, 3$
3. $\frac{1}{2}, 4$
5. 2
7. $0, 5$
9. $-\frac{3}{4}, \frac{1}{2}$
11. $-2, 4$
13. $-1, 2$

15. $-5, 1$

17. $-\frac{5}{2}, \frac{2}{3}$

Exercise 7.6.7 Solve by Factoring

Solve.

1. $x^2 - x - 6 = 0$
2. $x^2 + 3x - 10 = 0$
3. $y^2 - 10y + 24 = 0$
4. $y^2 + 6y - 27 = 0$
5. $x^2 - 14x + 40 = 0$
6. $x^2 + 14x + 49 = 0$
7. $x^2 - 10x + 25 = 0$
8. $3x^2 + 2x - 1 = 0$
9. $5x^2 - 9x - 2 = 0$
10. $7y^2 + 20y - 3 = 0$
11. $9x^2 - 42x + 49 = 0$
12. $25x^2 + 30x + 9 = 0$
13. $2y^2 + y - 3 = 0$
14. $7x^2 - 11x - 6 = 0$
15. $2x^2 = -15x + 8$
16. $8x - 5 = 3x^2$
17. $x^2 - 36 = 0$
18. $x^2 - 100 = 0$
19. $4x^2 - 81 = 0$
20. $49x^2 - 4 = 0$
21. $x^2 = 4$
22. $9y^2 = 1$
23. $16y^2 = 25$
24. $36x^2 = 25$
25. $4x^2 - 36 = 0$
26. $2x^2 - 18 = 0$
27. $10x^2 + 20x = 0$
28. $-3x^2 + 6x = 0$
29. $25x^2 = 50x$
30. $x^2 = 0$
31. $(x + 1)^2 - 25 = 0$
32. $(x - 2)^2 - 36 = 0$
33. $5x(x - 4) = -4 + x$
34. $(x - 1)(x - 10) = 22$
35. $(x - 3)(x - 5) = 24$
36. $-2x(x - 9) = x + 21$
37. $(x + 1)(6x + 1) = 2x$
38. $(x - 2)(x + 12) = 15x$
39. $(x + 1)(x + 2) = 2(x + 16)$
40. $(x - 9)(2x + 3) = 2(x - 9)$

Answer

1. $-2, 3$

3. $4, 6$

5. $4, 10$

7. 5

9. $-\frac{1}{5}, 2$

11. $\frac{7}{3}$

13. $-\frac{3}{2}, 1$

15. $-8, \frac{1}{2}$

17. $-6, 6$

19. $-\frac{9}{2}, \frac{9}{2}$

21. $-2, 2$

23. $-\frac{5}{4}, \frac{5}{4}$

25. $-3, 3$

27. $-2, 0$

29. $0, 2$

31. $-6, 4$

33. $\frac{1}{5}, 4$

35. $-1, 9$

37. $-\frac{1}{2}, -\frac{1}{3}$

39. $-6, 5$

? Exercise 7.6.8 Solve by Factoring

Clear the fractions by first multiplying both sides by the LCD and then solve.

1. $115x^2 + 13x + 25 = 0$
2. $114x^2 - 12x + 37 = 0$
3. $32x^2 - 23 = 0$
4. $52x^2 - 110 = 0$
5. $314x^2 - 212 = 0$
6. $13x^2 - 15x = 0$
7. $132x^2 - 12x + 2 = 0$
8. $13x^2 + 56x - 12 = 0$
9. The sides of a square measure $x + 3$ units. If the area is 25 square units, then find x.
10. The height of a triangle is 2 units more than its base. If the area is 40 square units, then find the length of the base.
11. The sides of a right triangle have measures that are consecutive integers. Find the length of the hypotenuse. (Hint: The hypotenuse is the longest side. Apply the Pythagorean theorem.)
12. The profit in dollars generated by producing and selling x custom lamps is given by the function $P(x) = -10x^2 + 800x - 12000$. How many lamps must be sold and produced to break even? (Hint: We break even when the profit is zero.)

Answer

1. $-3, -2$

3. $-\frac{2}{3}, \frac{2}{3}$

5. ± 7

7. 8

9. 2 units

11. 5 units

Exercise 7.6.9 Solve by Factoring

Assuming dry road conditions and average reaction times, the safe stopping distance, d in feet of an average car is given using the formula $d = 120v^2 + v$, where v represents the speed of the car in miles per hour. For each problem below, given the stopping distance, determine the safe speed.

1. 15 feet
2. 40 feet
3. 75 feet
4. 120 feet

Answer

1. 10 miles per hour

3. 30 miles per hour

Exercise 7.6.10 Finding Equations with Given Solutions

Find a quadratic equation with integer coefficients, given the following solutions.

1. $-3, 1$
2. $-5, 3$
3. $-10, -3$
4. $-7, -4$
5. $-1, 0$
6. $0, \frac{3}{5}$
7. $-2, 2$
8. $-\frac{1}{2}, \frac{1}{2}$
9. $-4, \frac{1}{3}$
10. $\frac{2}{3}, \frac{2}{5}$
11. $-\frac{1}{5}, -\frac{2}{3}$
12. $-\frac{3}{2}, \frac{3}{4}$
13. 3, double root
14. -5, double root

Answer

1. $x^2 + 2x - 3 = 0$

3. $x^2 + 13x + 30 = 0$

5. $x^2 + x = 0$

7. $x^2 - 4 = 0$

9. $3x^2 + 11x - 4 = 0$

11. $15x^2 + 13x + 2 = 0$

13. $x^2 - 6x + 9 = 0$

Exercise 7.6.11 Solving Polynomial Equations

Solve.

1. $7x(x+5)(x-9)=0$
2. $(x-1)(x-2)(x-3)=0$
3. $-2x(x-10)(x-1)=0$
4. $8x(x-4)^2=0$
5. $4(x+3)(x-2)(x+1)=0$
6. $-2(3x+1)(3x-1)(x-1)(x+1)=0$
7. $x^3-x^2-2x=0$
8. $2x^3+5x^2-3x=0$
9. $5x^3-15x^2+10x=0$
10. $-2x^3+2x^2+12x=0$
11. $3x^3-27x=0$
12. $-2x^3+8x=0$
13. $x^3+x^2-x-1=0$
14. $x^3+2x^2-16x-32=0$
15. $8x^3-4x^2-18x+9=0$
16. $12x^3=27x$

Answer

1. $-5, 0, 9$
3. $0, 1, 10$
5. $-3, -1, 2$
7. $-1, 0, 2$
9. $0, 1, 2$
11. $-3, 0, 3$
13. $-1, 1$
15. $-\frac{3}{2}, \frac{1}{2}, \frac{3}{2}$

Exercise 7.6.12 Discussion Board Topics

1. Explain why $2(x+5)(x-5)=0$ has two solutions and $2x(x+5)(x-5)=0$ has three solutions.
2. Make up your own quadratic equation and post it and the solutions on the discussion board.
3. Explain, in your own words, how to solve a quadratic equation in standard form.

Answer

1. Answers may vary
3. Answers may vary

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7.7: Applications Involving Quadratic Equations

Learning Objectives

- Set up and solve applications involving relationships between real numbers.
- Set up and solve applications involving geometric relationships involving area and the Pythagorean theorem.
- Set up and solve applications involving the height of projectiles.

Number Problems

The algebraic setups of the word problems that we have previously encountered led to linear equations. When we translate the applications to algebraic setups in this section, the setups lead to quadratic equations. Just as before, we want to avoid relying on the "guess and check" method for solving applications. Using algebra to solve problems simplifies the process and is more reliable.

Example 7.7.1

One integer is 4 less than twice another integer, and their product is 96. Set up an algebraic equation and solve it to find the two integers.

Solution:

First, identify the variables. Avoid two variables by using the relationship between the two unknowns.

Let n represent one of the integers.

$$\overbrace{2n}^{\text{twice}} \quad \overbrace{-\ 4}^{\text{4 less than}}$$

Let $2n-4$ represent the other integer.

Figure 7.7.1

The key phrase, "their product is 96," indicates that we should multiply and set the product equal to 96.

$$n \cdot (2n-4) = 96$$

Once we have the problem translated to a mathematical equation, we then solve. In this case, we can solve by factoring. The first step is to write the equation in standard form:

$$\begin{aligned} n \cdot (2n-4) &= 96 && \textit{Distribute } n. \\ 2n^2 - 4n &= 96 && \textit{Subtract } 96 \textit{ from both sides.} \\ 2n^2 - 4n - 96 &= 0 \end{aligned}$$

Next, factor completely and set each variable factor equal to zero.

$$\begin{aligned} 2n^2 - 4n - 96 &= 0 && \textit{Factor out the GCF, } 2. \\ 2(n^2 - 2n - 48) &= 0 && \textit{Factor the resulting trinomial.} \\ 2(n+6)(n-8) &= 0 && \textit{Set each variable factor equal to zero.} \end{aligned}$$

$$\begin{aligned} n+6 &= 0 \quad \text{or} \quad n-8 = 0 \\ n &= -6 \qquad\quad n = 8 \end{aligned}$$

The problem calls for two integers whose product is $+96$. The product of two positive numbers is positive and the product of two negative numbers is positive. Hence we can have two sets of solutions. Use $2n-4$ to determine the other integers.

$$\begin{aligned} n &= -6 & n &= 8 \\ 2n-4 &= 2(-6)-4 & 2n-4 &= 2(8)-4 \\ &= -12-4 & &= 16-4 \\ &= -16 & &= 12 \end{aligned}$$

Answer:

Two sets of integers solve this problem: $\{8, 12\}$ and $\{-6, -16\}$. Notice that $(8)(12) = 96$ and $(-6)(-16) = 96$; our solutions check out.

With quadratic equations, we often obtain two solutions for the identified unknown. Although it may be the case that both are solutions to the equation, they may not be solutions to the problem. If a solution does not solve the original application, then we disregard it.

Recall that consecutive odd and even integers both are separated by two units.

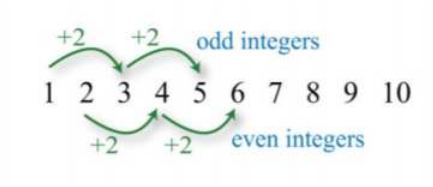


Figure 7.7.2

Example 7.7.2

The product of two consecutive positive odd integers is 99. Find the integers.

Solution:

Let n represent the first positive odd integer.

Let $n+2$ represent the next positive odd integer.

The key phase, "product…is 99," indicates that we should multiply and set the product equal to 99.

$$n \cdot (n+2) = 99$$

Rewrite the quadratic equation in standard form and solve by factoring.

$$\begin{aligned} n^2+2n &= 99 \\ n^2+2n-99 &= 0 \\ (n-9)(n+11) &= 0 \end{aligned}$$

$$\begin{aligned} n-9=0 \quad &\text{or} \quad n+11=0 \\ n=9 \quad &\quad\;\; n=-11 \end{aligned}$$

Because the problem asks for positive integers, $n = 9$ is the only solution. Back substitute to determine the next odd integer.

$$\begin{aligned} n+2 &= 9+2 \\ &= 11 \end{aligned}$$

Answer:

The consecutive positive odd integers are 9 and 11.

Example 7.7.3

Given two consecutive positive odd integers, the product of the larger and twice the smaller is equal to 70. Find the integers.

Solution:

Let n represent the smaller positive odd integer.

Let $n+2$ represent the next positive odd integer.

The key phrase “twice the smaller” can be translated to $2n$. The phrase “product…is 70” indicates that we should multiply this by the larger odd integer and set the product equal to 70.

$$(n+2)\cdot 2n = 70$$

Solve by factoring.

$$\begin{aligned}
(n+2)\cdot 2n &= 70 && \textit{Distribute.}\\
2n^2+4n &= 70 && \textit{Subtract 70 from both sides.}\\
2n^2+4n-70 &= 0 && \textit{Factor out the GCF, 2.}\\
2(n^2+2n-35) &= 0 && \textit{Factor the resulting trinomial.}\\
2(n-5)(n+7) &= 0 && \textit{Set each variable factor equal to zero.}
\end{aligned}$$

$$\begin{aligned}
n-5=0 \quad &\text{or} \quad n+7=0\\
n=5 \quad &\quad\;\; n=-7
\end{aligned}$$

Because the problem asks for positive integers, $n=5$ is the only solution.

Back substitute into $n+2$ to determine the next odd integer.

$$\begin{aligned}
n+2 &= 5+2\\
&= 7
\end{aligned}$$

Answer:

The positive odd integers are 5 and 7.

? Exercise 7.7.1

The product of two consecutive positive even integers is 168. Find the integers.

Answer

The positive even integers are 12 and 14.

Geometry Problems

When working with geometry problems, it is helpful to draw a picture. Below are some area formulas that you are expected to know. (Recall that $\pi \approx 3.14$.)

Table 7.7.1

Area of a rectangle:	$A = l\cdot w$
Area of a square:	$A = s^2$
Area of a triangle:	$A = \frac{1}{2}bh$
Area of a circle:	$A = \pi r^2$

✓ Example 7.7.4

The floor of a rectangular room has a length that is 4 feet more than twice its width. If the total area of the floor is 240 square feet, then find the dimensions of the floor.

Solution:

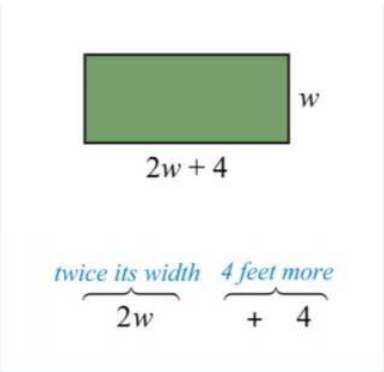


Figure 7.7.3

Use the formula $A = l \cdot w$ and the fact that the area is 240 square feet to set up an algebraic equation.

$$\begin{aligned} A &= l \cdot w \\ 240 &= (2w + 4) \cdot w \end{aligned}$$

Solve by factoring.

$$\begin{aligned} w - 10 &= 0 \quad \text{or} \quad w + 12 = 0 \\ w &= 10 \qquad\quad\ \ w = -12 \end{aligned}$$

At this point we have two possibilities for the width of the rectangle. However, since a negative width is not defined, choose the positive solution, $w = 10$. Back substitute to find the length.

$$\begin{aligned} 2w + 4 &= 2(10) + 4 \\ &= 20 + 4 \\ &= 24 \end{aligned}$$

Answer:

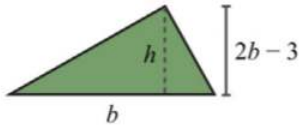
The width is 10 feet and the length is 24 feet.

It is important to include the correct units in the final presentation of the answer. In the previous example, it would not make much sense to say the width is 10. Make sure to indicate that the width is 10 feet.

Example 7.7.5

The height of a triangle is 3 inches less than twice the length of its base. If the total area of the triangle is 7 square inches, then find the lengths of the base and height.

Solution:



Let b represent the length of the base of the triangle.

twice the base: $2b$ — 3 inches less than: $-\ 3$

Let $2b - 3$ represent the height of the triangle.

Figure 7.7.4

Use the formula $A = \frac{1}{2}bh$ and the fact that the area is 7 square inches to set up an algebraic equation.

$$A = \frac{1}{2} b \cdot h$$
$$7 = \frac{1}{2} b(2b - 3)$$

To avoid fractional coefficients, multiply both sides by 2 and then rewrite the quadratic equation in standard form.

Factor and then set each factor equal to zero.

$$\begin{array}{ll} 2b - 7 = 0 & \text{or} \quad b + 2 = 0 \\ 2b = 7 & b = -2 \\ b = \frac{7}{2} & \end{array}$$

In this case, disregard the negative answer; the length of the base is $\frac{7}{2}$ inches long. Use $2b - 3$ to determine the height of the triangle.

Answer:

The base measures $\frac{7}{2} = 3\frac{1}{2}$ inches and the height is 4 inches.

Exercise 7.7.1

The base of a triangle is 5 units less than twice the height. If the area is 75 square units, then what is the length of the base and height?

Answer

The height is 10 units and the base is 15 units.

Recall that a right triangle is a triangle where one of the angles measures 90°. The side opposite of the right angle is the longest side of the triangle and is called the hypotenuse. The Pythagorean theorem gives us a relationship between the legs and hypotenuse of any right triangle, where a and b are the lengths of the legs and c is the length of the hypotenuse:

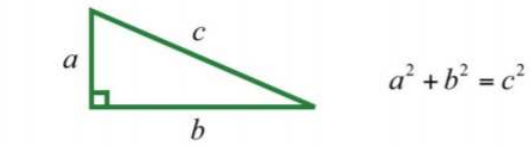


Figure 7.7.5

Given certain relationships, we use this theorem when determining the lengths of sides of right triangles.

Example 7.7.6

The hypotenuse of a right triangle is 10 inches. If the short leg is 2 inches less than the long leg, then find the lengths of the legs.

Solution:

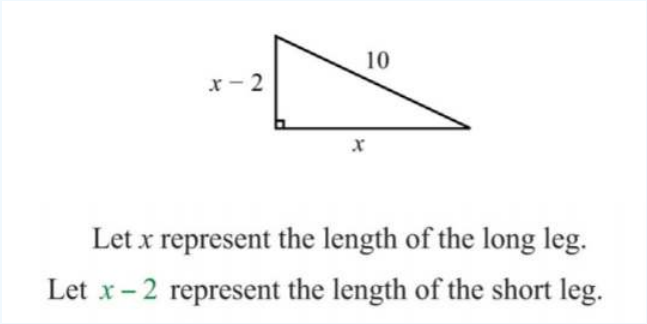


Figure 7.7.6

Given that the hypotenuse measures 10 inches, substitute its value into the Pythagorean theorem and obtain a quadratic equation in terms of x.

$$\begin{aligned} a^2+b^2 &= c^2 \\ (x-2)^2+x^2 &= 10^2 \end{aligned}$$

Multiply and rewrite the equation in standard form.

$$\begin{aligned} (x-2)^2+x^2 &= 10^2 \\ x^2-4x+4+x^2 &= 100 \\ 2x^2-4x-96 &= 0 \end{aligned}$$

Once it is in standard form, factor and set each variable factor equal to zero.

$$\begin{aligned} 2x^2-4x-96 &= 0 \\ 2(x^2-2x-48) &= 0 \\ 2(x+6)(x-8) &= 0 \end{aligned}$$

$$\begin{aligned} x+6=0 \quad &\text{or} \quad x-8=0 \\ x=-6 \quad &\qquad x=8 \end{aligned}$$

Because lengths cannot be negative, disregard the negative answer. In this case, the long leg measures 8 inches. Use $x-2$ to determine the length of the short leg.

$$\begin{aligned} x-2 &= 8-2 \\ &= 6 \end{aligned}$$

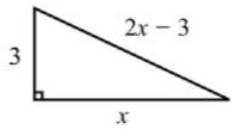
Answer:

The short leg measures 6 inches and the long leg measures 8 inches.

Example 7.7.7

One leg of a right triangle measures 3 centimeters. The hypotenuse of the right triangle measures 3 centimeters less than twice the length of the unknown leg. Find the measure of all the sides of the triangle.

Solution:



Let x represent the length of the unknown leg.

Let $2x-3$ represent the length of the unknown leg.

Figure 7.7.7

To set up an algebraic equation, we use the Pythagorean theorem.

$$\begin{aligned} a^2+b^2 &= c^2 \\ 3^2+x^2 &= (2x-3)^2 \end{aligned}$$

Solve by factoring.

$$\begin{aligned} 3^2+x^2 &= (2x-3)^2 \\ 9+x^2 &= 4x^2-12x+9 \\ 0 &= 3x^2-12x \\ 0 &= 3x(x-4) \end{aligned}$$

$$3x = 0 \quad \text{or} \quad x - 4 = 0$$
$$x = 0 \qquad\qquad x = 4$$

Disregard 0. The length of the unknown leg is 4 centimeters. Use $2x - 3$ to determine the length of the hypotenuse.

Answer:

The sides of the triangle measure 3 centimeters, 4 centimeters, and 5 centimeters.

? Exercise 7.7.2

The hypotenuse of a right triangle measures 13 units. If one leg is 2 units more than twice that of the other, then find the length of each leg.

Answer

The two legs measure 5 units and 12 units.

Projectile Problems

The height of an object launched upward, ignoring the effects of air resistance, can be modeled with the following formula:

$$\text{height} = -\frac{1}{2}gt^2 + v_0 t + s_0 \tag{7.7.1}$$

Using function notation, which is more appropriate, we have

$$h(t) = -\frac{1}{2}gt^2 + v_0 t + s_0 \tag{7.7.2}$$

With this formula, the height can be calculated at any given time t after the object is launched. The coefficients represent the following:

Table 7.7.2

$-\frac{1}{2}g$	The letter g represents the acceleration due to gravity.
v_0	"v-naught" represents the initial velocity of the object.
s_0	"s-naught" represents the initial height from which the object is launched.

We consider only problems where the acceleration due to gravity can be expressed as $g = 32$ ft/sec^2. Therefore, in this section time will be measured in seconds and the height in feet. Certainly though, the formula is valid using units other than these.

✓ Example 7.7.8

The height of a projectile launched upward at a speed of 32 feet/second from a height of 128 feet is given by the function $h(t) = -16t^2 + 32t + 128$. How long does it take to hit the ground?

Solution:

An inefficient method for finding the time to hit the ground is to simply start guessing at times and evaluating. To do this, construct a chart.

t	h		*Points*
0	128	$h(0) = -16(0)^2 + 32(0) + 128 = 128$	(0, 128)
1	144	$h(1) = -16(1)^2 + 32(1) + 128 = 144$	(1, 144)
2	128	$h(2) = -16(2)^2 + 32(2) + 128 = 128$	(2, 128)
3	80	$h(3) = -16(3)^2 + 32(3) + 128 = 80$	(3, 80)
4	0	$h(4) = -16(4)^2 + 32(4) + 128 = 0$	(4, 0)

Figure 7.7.8

Use the table to sketch the height of the projectile over time.

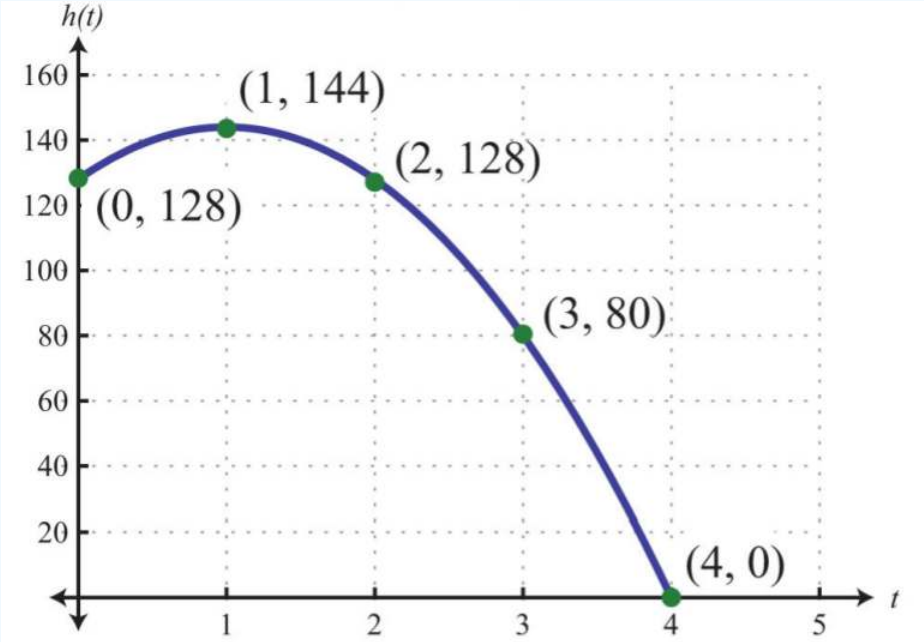


Figure 7.7.9

We see that at 4 seconds, the projectile hits the ground. Note that when this occurs, the height is equal to 0. Now we need to solve this problem algebraically. To find the solution algebraically, use the fact that the height is 0 when the object hits the ground. We need to find the time, t, when $h(t) = 0$.

$$h(t) = -16t^2 + 32t + 128$$
$$\downarrow$$
$$0 = -16t^2 + 32t + 128$$

Solve the equation by factoring

$$0 = -16t^2 + 32t + 128$$
$$0 = -16(t^2 - 2t - 8)$$
$$0 = -16(t-4)(t+2)$$

Now set each variable factor to zero.

$$t - 4 = 0 \quad \text{or} \quad t + 2 = 0$$
$$t = 4 \qquad\qquad t = -2$$

As expected, the projectile hits the ground at $t = 4$ seconds. Disregard -2 as a solution because negative time is not defined.

Answer:

The projectile hits the ground 4 seconds after it is launched.

Example 7.7.9

The height of a certain book dropped from the top of a 144 foot building is given by $h(t) = -16t^2 + 144$. How long does it take to hit the ground?

Solution:

Find the time t when the height $h(t) = 0$.

$$\begin{aligned} 0 &= -16t^2 + 144 \\ 0 &= -16(t^2 - 9) \\ 0 &= -16(t+3)(t-3) \end{aligned}$$

$$\begin{aligned} t+3 &= 0 \quad \text{or} \quad t-3 = 0 \\ t &= -3 \qquad\quad t = 3 \end{aligned}$$

Answer:

The book takes 3 seconds to hit the ground when dropped from the top of a 144-foot building.

Exercise 7.7.3

The height of a projectile, shot straight up into the air from the ground, is given by $h(t) = -16t^2 + 80t$. How long does it take to come back down to the ground?

Answer

It will take 5 seconds to come back down to the ground.

Key Takeaways

- It is best to translate a word problem to a mathematical setup and then solve using algebra. Avoid using the "guess and check" method of solving applications in this section.
- When solving applications, check that your solutions make sense in the context of the question. For example, if you wish to find the length of the base of a triangle, then you would disregard any negative solutions.
- It is important to identify each variable and state in a sentence what each variable represents. It is often helpful to draw a picture.

Exercise 7.7.4 Number Problems

Set up an algebraic equation and then solve.

1. One integer is five times another. If the product of the two integers is 80, then find the integers.
2. One integer is four times another. If the product of the two integers is 36, then find the integers.
3. An integer is one more than four times another. If the product of the two integers is 39, then find the integers.
4. An integer is 3 more than another. If the product of the two integers is 130, then find the integers.
5. An integer is 2 less than twice another. If the product of the two integers is 220, then find the integers.
6. An integer is 3 more than twice another. If the product of the two integers is 90, then find the integers.
7. One integer is 2 units more than another. If the product of the two integers is equal to five times the larger, then find the two integers.
8. A positive integer is 1 less than twice another. If the product of the two integers is equal to fifteen times the smaller, then find the two integers.
9. A positive integer is 3 more than twice a smaller positive integer. If the product of the two integers is equal to six times the larger, then find the integers.
10. One positive integer is 3 more than another. If the product of the two integers is equal to twelve times the smaller, then find the integers.
11. An integer is 3 more than another. If the product of the two integers is equal to 2 more than four times their sum, then find the integers.

12. An integer is 5 more than another. If the product of the two integers is equal to 2 more than twice their sum, then find the integers.
13. The product of two consecutive positive even integers is 120. Find the integers.
14. The product of two consecutive positive odd integers is 99. Find the integers.
15. The product of two consecutive positive integers is 110. Find the integers.
16. The product of two consecutive positive integers is 42. Find the integers.
17. The product of two consecutive positive odd integers is equal to 1 less than seven times the sum of the integers. Find the integers.
18. The product of two consecutive positive even integers is equal to 22 more than eleven times the sum of the integers. Find the integers.
19. The sum of the squares of two consecutive positive odd integers is 74. Find the integers.
20. The sum of the squares of two consecutive positive even integers is 100. Find the integers.
21. The sum of the squares of two consecutive positive integers is 265. Find the integers.
22. The sum of the squares of two consecutive positive integers is 181. Find the integers.
23. For two consecutive positive odd integers, the product of twice the smaller and the larger is 126. Find the integers.
24. For two consecutive positive even integers, the product of the smaller and twice the larger is 160. Find the integers.

Answer

1. $\{4, 20\}$ or $\{-4, -20\}$

3. $3, 13$

5. $\{11, 20\}$ or $\{-22, -10\}$

7. $\{5, 7\}$ or $\{-2, 0\}$

9. $6, 15$

11. $\{7, 10\}$ or $\{-2, 1\}$

13. $10, 12$

15. $10, 11$

17. $13, 15$

19. $5, 7$

21. $11, 12$

23. $7, 9$

? Exercise 7.7.5 Geometry Problems

Set up an algebraic equation and then solve.

1. The width of a rectangle is 7 feet less than its length. If the area of the rectangle is 170 square feet, then find the length and width.
2. The length of a rectangle is 2 feet more than its width. If the area of the rectangle is 48 square feet, then find the length and width.
3. The width of a rectangle is 3 units less than the length. If the area is 70 square units, then find the dimensions of the rectangle.
4. The width of a rectangle measures one half of the length. If the area is 72 square feet, then find the dimensions of the rectangle.
5. The length of a rectangle is twice that of its width. If the area of the rectangle is 72 square inches, then find the length and width.
6. The length of a rectangle is three times that of its width. If the area of the rectangle is 75 square centimeters, then find the length and width.
7. The length of a rectangle is 2 inches more than its width. The area of the rectangle is equal to 12 inches more than three times the perimeter. Find the length and width of the rectangle.

8. The length of a rectangle is 3 meters more than twice the width. The area of the rectangle is equal to 10 meters less than three times the perimeter. Find the length and width of the rectangle.
9. A uniform border is to be placed around an 8-inch-by-10-inch picture. If the total area including the border must be 224 square inches, then how wide should the border be?

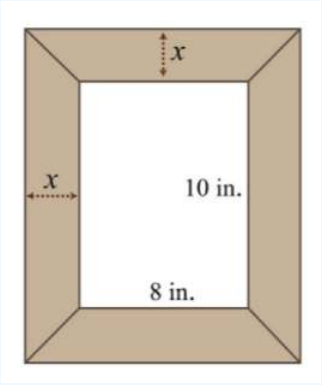


Figure 7.7.10

10. A 2-foot brick border is constructed around a square cement slab. If the total area, including the border, is 121 square feet, then what are the dimensions of the slab?

11. The area of a picture frame including a 2-inch wide border is 99 square inches. If the width of the inner area is 2 inches more than its length, then find the dimensions of the inner area.

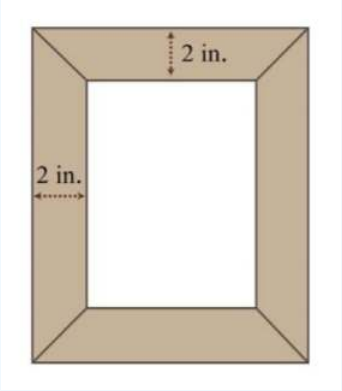


Figure 7.7.11

12. A box can be made by cutting out the corners and folding up the edges of a square sheet of cardboard. A template for a cardboard box with a height of 2 inches is given. What is the length of each side of the cardboard sheet if the volume of the box is to be 50 cubic inches?

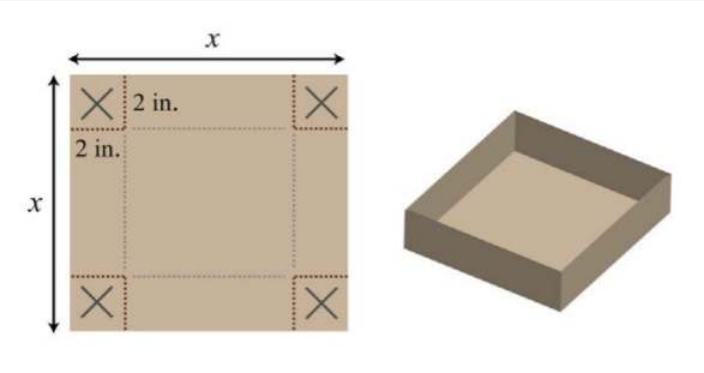


Figure 7.7.12

13. The height of a triangle is 3 inches more than the length of its base. If the area of the triangle is 44 square inches, then find the length of its base and height.

14. The height of a triangle is 4 units less than the length of the base. If the area of the triangle is 48 square units, then find the length of its base and height.

15. The base of a triangle is twice that of its height. If the area is 36 square centimeters, then find the length of its base and height.

16. The height of a triangle is three times the length of its base. If the area is $73\frac{1}{2}$ square feet, then find the length of the base and height.

17. The height of a triangle is 1 unit more than the length of its base. If the area is 5 units more than four times the height, then find the length of the base and height of the triangle.

18. The base of a triangle is 4 times that of its height. If the area is 3 units more than five times the height, then find the length of the base and height of the triangle.

19. The diagonal of a rectangle measures 5 inches. If the length is 1 inch more than its width, then find the dimensions of the rectangle.

20. The diagonal of a rectangle measures 10 inches. If the width is 2 inches less than the length, then find the area of the rectangle.

21. If the sides of a right triangle are consecutive even integers, then what are their measures?

22. The hypotenuse of a right triangle is 13 units. If the length of one leg is 2 more than twice the other, then what are their lengths?

23. The shortest leg of a right triangle measures 9 centimeters and the hypotenuse measures 3 centimeters more than the longer leg. Find the length of the hypotenuse.

24. The long leg of a right triangle measures 24 centimeters and the hypotenuse measures 4 centimeters more three times the short leg. Find the length of the hypotenuse.

Answer

1. Length: 17 feet; width: 10 feet

3. Length: 10 units; width: 7 units

5. Length: 12 inches; width: 6 inches

7. Length: 14 inches; width: 12 inches

9. 3 inches

11. 5 inches by 7 inches

13. Base: 8 inches; height: 11 inches

15. Base: 12 centimeters; height: 6 centimeters

17. Base: 9 units; height: 10 units

19. 3 inches by 4 inches

21. 6 units, 8 units, and 10 units

23. 15 centimeters

? Exercise 7.7.6 Projectile Problems

Set up an algebraic equation and then solve.

1. The height of a projectile launched upward at a speed of 32 feet/second from a height of 48 feet is given by the function \(h(t)=−16t^{2}+32t+48. How long will it take the projectile to hit the ground?
2. The height of a projectile launched upward at a speed of 16 feet/second from a height of 192 feet is given by the function $h(t) = −16t^2 + 16t + 192$. How long will it take to hit the ground?

3. An object launched upward at a speed of 64 feet/second from a height of 80 feet. How long will it take the projectile to hit the ground?
4. An object launched upward at a speed of 128 feet/second from a height of 144 feet. How long will it take the projectile to hit the ground?
5. The height of an object dropped from the top of a 64-foot building is given by $h(t) = -16t^2 + 64$. How long will it take the object to hit the ground?
6. The height of an object dropped from an airplane at $1,600$ feet is given by $h(t) = -16t^2 + 1,600$. How long will it take the object to hit the ground?
7. An object is dropped from a ladder at a height of 16 feet. How long will it take to hit the ground?
8. An object is dropped from a 144-foot building. How long will it take to hit the ground?
9. The height of a projectile, shot straight up into the air from the ground at 128 feet/second, is given by $h(t) = -16t^2 + 128t$. How long does it take to come back down to the ground?
10. A baseball, tossed up into the air from the ground at 32 feet/second, is given by $h(t) = -16t^2 + 32t$. How long does it take to come back down to the ground?
11. How long will it take a baseball thrown into the air at 48 feet/second to come back down to the ground?
12. A football is kicked up into the air at 80 feet/second. Calculate how long will it hang in the air.

Answer

1. 3 seconds

3. 5 seconds

5. 2 seconds

7. 1 second

9. 8 seconds

11. 3 seconds

? Exercise 7.7.7 Discussion Board Topics

1. Research and discuss the life of Pythagoras.
2. If the sides of a square are doubled, then by what factor is the area increased? Why?
3. Design your own geometry problem involving the area of a rectangle or triangle. Post the question and a complete solution on the discussion board.
4. Write down your strategy for setting up and solving word problems. Share your strategy on the discussion board.

Answer

1. Answers may vary

3. Answers may vary

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7.E: Review Exercises and Sample Exam

Review Exercises

Exercise 7.E.1 Introduction to Factoring

Determine the missing factor.

1. $12x^3-24x^2+4x=4x($? $)$
2. $10y^4-35y^3-5y^2=5y^2($? $)$
3. $-18a^5+9a^4-27a^3=-9a^3($? $)$
4. $-21x^2y+7xy^2-49xy=-7xy($? $)$

Answer

1. $(3x^2-6x+1)$
3. $(2a^2-a+3)$

Exercise 7.E.2 Introduction to Factoring

Factor out the GCF.

1. $22x^2+11x$
2. $15y^4-5y^3$
3. $18a^3-12a^2+30a$
4. $12a^5+20a^3-4a$
5. $9x^3y^2-18x^2y^2+27xy^2$
6. $16a^5b^5c-8a^3b^6+24a^3b^2c$

Answer

1. $11x(2x+1)$
3. $6a(3a^2-2a+5)$
5. $9xy2(x^2-2x+3)$

Exercise 7.E.3 Introduction to Factoring

Factor by grouping.

1. $x^2+2x-5x-10$
2. $2x^2-2x-3x+3$
3. $x^3+5x^2-3x-15$
4. x^3-6x^2+x-6
5. $x^3-x^2y-2x+2y$
6. $a^2b^2-2a^3+6ab-3b^3$

Answer

1. $(x+2)(x-5)$
3. $(x+5)(x^2-3)$
5. $(x-y)(x^2-2)$

Exercise 7.E.4 Factoring Trinomials of the Form $x^2 + bx + c$

Are the following factored correctly? Check by multiplying.

1. $x^2+5x+6=(x+6)(x-1)$
2. $x^2+3x-10=(x+5)(x-2)$
3. $x^2+6x+9=(x+3)^2$
4. $x^2-6x-9=(x-3)(x+3)$

Answer

1. No

3. Yes

Exercise 7.E.5 Factoring Trinomials of the Form $x^2 + bx + c$

Factor.

1. $x^2-13x-14$
2. $x^2+13x+12$
3. $y^2+10y+25$
4. $y^2-20y+100$
5. $a^2-8a-48$
6. $b^2-18b+45$
7. $x^2+2x+24$
8. $x^2-10x-16$
9. $a^2+ab-2b^2$
10. $a^2b^2+5ab-50$

Answer

1. $(x-14)(x+1)$

3. $(y+5)^2$

5. $(a-12)(a+4)$

7. Prime

9. $(a-b)(a+2b)$

Exercise 7.E.6 Factoring Trinomials of the Form $ax^2 + bx + c$

Factor.

1. $5x^2-27x-18$
2. $3x^2-14x+8$
3. $4x^2-28x+49$
4. $9x^2+48x+64$
5. $6x^2-29x-9$
6. $8x^2+6x+9$
7. $60x^2-65x+15$
8. $16x^2-40x+16$
9. $6x^3-10x^2y+4xy^2$
10. $10x^3y-82x^2y^2+16xy^3$
11. $-y^2+9y+36$
12. $-a^2-7a+98$
13. $16+142x-18x^2$

14. $45-132x-60x^2$

Answer

1. $(5x+3)(x-6)$
3. $(2x-7)^2$
5. Prime
7. $5(3x-1)(4x-3)$
9. $2x(3x-2y)(x-y)$
11. $-1(y-12)(y+3)$
13. $-2(9x+1)(x-8)$

Exercise $7.E.7$ Factoring Special Binomials

Factor completely.

1. x^2-81
2. $25x^2-36$
3. $4x^2-49$
4. $81x^2-1$
5. x^2-64y^2
6. $100x^2y^2-1$
7. $16x^4-y^4$
8. x^4-81y^4
9. $8x^3-125$
10. $27+y^3$
11. $54x^4y-2xy^4$
12. $3x^4y^2+24xy^5$
13. $64x^6-y^6$
14. x^6+1

Answer

1. $(x+9)(x-9)$
3. $(2x+7)(2x-7)$
5. $(x+8y)(x-8y)$
7. $(4x^2+y^2)(2x+y)(2x-y)$
9. $(2x-5)(4x^2+10x+25)$
11. $2xy(3x-y)(9x^2+3xy+y^2)$
13. $(2x+y)(4x^2-2xy+y^2)(2x-y)(4x^2+2xy+y^2)$

Exercise $7.E.8$ General Guidelines for Factoring Polynomials

Factor completely.

1. $8x^3-4x^2+20x$
2. $50a^4b^4c+5a^3b^5c^2$
3. x^3-12x^2-x+12
4. $a^3-2a^2-3ab+6b$
5. $-y^2-15y+16$
6. $x^2-18x+72$

7. $144x^2-25$
8. $3x^4-48$
9. $20x^2-41x-9$
10. $24x^2+14x-20$
11. $a^4b-343ab^4$
12. $32x^7y^2+4xy^8$

Answer

1. $4x(2x^2-x+5)$
3. $(x-12)(x+1)(x-1)$
5. $-1(y+16)(y-1)$
7. $(12x+5)(12x-5)$
9. $(4x-9)(5x+1)$
11. $ab(a-7b)(a^2+7ab+49b^2)$

Exercise 7.E.9 Solving Equations by Factoring

Solve.

1. $(x-9)(x+10)=0$
2. $-3x(x+8)=0$
3. $6(x+1)(x-1)=0$
4. $(x-12)(x+4)(2x-1)=0$
5. $x^2+5x-50=0$
6. $3x^2-13x+4=0$
7. $3x^2-12=0$
8. $16x^2-9=0$
9. $(x-2)(x+6)=20$
10. $2(x-2)(x+3)=7x-9$
11. $52x^2-203x=0$
12. $23x^2-512x+124=0$

Answer

1. $9,-10$
3. $-1,1$
5. $-10,5$
7. ± 2
9. $-8,4$
11. $0,\frac{8}{3}$

Exercise 7.E.10 Solving Equations by Factoring

Find a quadratic equation with integer coefficients, given the following solutions.

1. $-7,6$
2. $0,-10$
3. $-\frac{1}{9},\frac{1}{2}$
4. $\pm\frac{3}{2}$

Answer

1. $x^2+x-42=0$
3. $18x^2-7x-1=0$

Exercise 7.E.11 Applications Involving Quadratic Equations

Set up an algebraic equation and then solve the following.

1. An integer is 4 less than twice another. If the product of the two integers is 96, then find the integers.
2. The sum of the squares of two consecutive positive even integers is 52. Find the integers.
3. A 20-foot ladder leaning against a wall reaches a height that is 4 feet more than the distance from the wall to the base of the ladder. How high does the ladder reach?
4. The height of an object dropped from the top of a 196-foot building is given by $h(t)=-16t^2+196$, where t represents the number of seconds after the object has been released. How long will it take the object to hit the ground?
5. The length of a rectangle is 1 centimeter less than three times the width. If the area is 70 square centimeters, then find the dimensions of the rectangle.
6. The base of a triangle is 4 centimeters more than twice the height. If the area of the triangle is 80 square centimeters, then find the measure of the base.

Answer

1. $\{8,12\}$ or $\{-6,-16\}$
3. 16 feet
5. Length: 14 centimeters; width: 5 centimeters

Sample Exam

Exercise 7.E.12

1. Determine the GCF of the terms $25a^2b^2c$, $50ab^4$, and $35a^3b^3c^2$.
2. Determine the missing factor: $24x^2y^3-16x^3y^2+8x^2y=8x^2y($? $)$.

Answer

1. $5ab^2$

Exercise 7.E.13

Factor.

1. $12x^5-15x^4+3x^2$
2. x^3-4x^2-2x+8
3. $x^2-7x+12$
4. $9x^2-12x+4$
5. x^2-81
6. x^3+27y^3

Answer

1. $3x^2(4x^3-5x^2+1)$
3. $(x-4)(x-3)$
5. $(x+9)(x-9)$

Exercise 7.E.14

Factor completely.

1. x^3+2x^2-4x-8
2. x^4-1
3. $-6x^3+20x^2-6x$
4. x^6-1

Answer

1. $(x+2)^2(x-2)$
3. $-2x(3x-1)(x-3)$

Exercise 7.E.15

Solve.

1. $(2x+1)(x-7)=0$
2. $3x(4x-3)(x+1)=0$
3. $x^2-64=0$
4. $x^2+4x-12=0$
5. $23x^2+89x-16=0$
6. $(x-5)(x-3)=-1$
7. $3x(x+3)=14x+2$
8. $(3x+1)(3x+2)=9x+3$

Answer

1. $-\frac{1}{2}, 7$
3. ± 8
5. $-\frac{3}{2}, \frac{1}{6}$
7. $-\frac{1}{3}, 2$

Exercise 7.E.16

For each problem, set up an algebraic equation and then solve.

1. An integer is 4 less than twice another. If the product of the two integers is 70, then find the integers.
2. The sum of the squares of two consecutive positive odd integers is 130. Find the integers.
3. The length of a rectangle is 4 feet more than twice its width. If the area is 160 square feet, then find the dimensions of the rectangle.
4. The height of a triangle is 6 centimeters less than four times the length of its base. If the area measures 27 square centimeters, then what is the height of the triangle?
5. The height of a projectile launched upward at a speed of 64 feet/second from a height of 36 feet is given by the function $h(t)=-16t^2+64t+36$. How long will it take the projectile to hit the ground?

Answer

1. $\{7, 10\}$ or $\{-14, -5\}$
3. Width: 8 feet; length: 20 feet
5. $4\frac{1}{2}$ sec

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CHAPTER OVERVIEW

8: Rational Expressions and Equations

8.1: Simplifying Rational Expressions
8.2: Multiplying and Dividing Rational Expressions
8.3: Adding and Subtracting Rational Expressions
8.4: Solving Rational Equations

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8.1: Simplifying Rational Expressions

Learning Objectives

- Determine the restrictions to the domain of a rational expression.
- Simplify rational expressions.
- Simplify expressions with opposite binomial factors.
- Simplify and evaluate rational functions.

Rational Expressions, Evaluation, and Restrictions

A rational number, or fraction $\frac{a}{b}$, is a real number defined as a quotient of two integers ***a*** and ***b***, where $b \neq 0$. Similarly, we define a rational expression, or algebraic fraction $\frac{P}{Q}$, as the quotient of two polynomials ***P*** and ***Q***, where $Q \neq 0$. Some examples of rational expressions follow:

The example $\frac{x+3}{x-5}$ consists of linear expressions in both the numerator and denominator. Because the denominator contains a variable, this expression is not defined for all values of ***x***.

Evaluate $\frac{x+3}{x-5}$ for the set of ***x***-values $\{-3, 4, 5\}$.

Solution:

Substitute the values in for ***x***.

$x=-3$	$x=4$	$x=5$
$\frac{x+3}{x-5}=\frac{(-3)+3}{(-3)-5}$	$\frac{x+3}{x-5}=\frac{(4)+3}{(4)-5}$	$\frac{x+3}{x-5}=\frac{(5)+3}{(5)-5}$
$=\frac{0}{-8}$	$=\frac{7}{-1}$	$=\frac{8}{0}$ *Undefined*
$=0$	$=-7$	

Answer:

When $x=-3$, the value of the rational expression is 0; when $x=4$, the value of the rational expression is -7; and when $x=5$, the value of the rational expression is undefined.

This example illustrates that variables are restricted to values that do not make the denominator equal to 0. The domain of a rational expression is the set of real numbers for which it is defined, and restrictions are the real numbers for which the expression is not defined. We often express the domain of a rational expression in terms of its restrictions.

Example 8.1.2

Find the domain of the following:

$$\frac{x+7}{2x^2+x-6}$$

Solution:

In this example, the numerator $x+7$ is a linear expression and the denominator $2x^2+x-6$ is a quadratic expression. If we factor the denominator, then we will obtain an equivalent expression.

$$\frac{x+7}{2x^2+x-6}=\frac{x+7}{(2x-3)(x+2)}$$

Because rational expressions are undefined when the denominator is 0, we wish to find the values for ***x*** that make it 0. To do this, apply the zero product property. Set each factor in the denominator equal to 0 and solve.

$$(2x-3)(x+2)=0$$

$$\begin{aligned} 2x-3=0 \quad &\text{or} \quad x+2=0 \\ 2x=3 \quad &\quad\quad x=-2 \\ x=\tfrac{3}{2} \quad &\end{aligned}$$

We conclude that the original expression is defined for any real number except $\frac{3}{2}$ and -2. These two values are the restrictions to the domain. It is important to note that -7 is ***not*** a restriction to the domain because the expression is defined as 0 when the numerator is 0.

$$\begin{aligned} \frac{x+7}{2x^2+x-6} &= \frac{(-7)+7}{2(-7)^2+(-7)-6} \\ &= \frac{0}{98-7-6} \\ &= \frac{0}{85} \\ &= 0 \end{aligned}$$

Answer:

The domain consists of any real number ***x***, where $x \neq \frac{3}{2}$ and $x \neq -2$.

We can express the domain of the previous example using notation as follows:

Set − builder notation	*Interval notation*
$\left\{x \middle\| x \neq -2, \frac{3}{2}\right\}$	$(-\infty,-2)\cup\left(-2,\frac{3}{2}\right)\cup\left(\frac{3}{2},\infty\right)$

The restrictions to the domain of a rational expression are determined by the denominator. Ignore the numerator when finding those restrictions.

Example 8.1.3

Determine the domain:

$$\frac{x^4+x^3-2x^2-x}{x^2-1}$$

Solution:

To find the restrictions to the domain, set the denominator equal to 0 and solve:

$$\begin{aligned} x^2-1 &= 0 \\ (x+1)(x-1) &= 0 \\ x+1 = 0 \quad &\text{or} \quad x-1=0 \\ x=-1 \quad &\quad\quad x=1 \end{aligned}$$

These two values cause the denominator to be 0. Hence they are restricted from the domain.

Answer:

The domain consists of any real number ***x***, where $x \neq \pm 1$.

Example 8.1.4

Determine the domain:

$$\frac{x^2-25}{4}$$

Solution:

There is no variable in the denominator and thus no restriction to the domain.

Answer:

The domain consists of all real numbers, **R**.

Simplifying Rational Expressions

When simplifying fractions, look for common factors that cancel. For example,

$$\frac{12}{60}=\frac{1\cdot \cancel{12}}{5\cdot \cancel{12}}=\frac{1}{5}$$

We say that the fraction $\frac{12}{60}$ is equivalent to $\frac{1}{5}$. Fractions are in simplest form if the numerator and denominator share no common factor other than 1. Similarly, when working with rational expressions, look for factors to cancel. For example,

$$\frac{x+4}{(x-3)(x+4)}=\frac{1\cdot \cancel{(x+4)}}{(x-3)\ \cancel{(x+4)}}=\frac{1}{x-3}$$

The resulting rational expression is equivalent if it shares the same domain. Therefore, we must make note of the restrictions and write

$$\frac{x+4}{(x-3)(x+4)}=\frac{1}{x-3},\ \text{where } x\neq 3 \text{ and } x\neq -4$$

In words, $\frac{x+4}{(x-3)(x+4)}$ is equivalent to $\frac{1}{x-3}$, if $x\neq 3$ and $x\neq -4$. We can verify this by choosing a few values with which to evaluate both expressions to see if the results are the same. Here we choose $x=7$ and evaluate as follows:

$$\begin{aligned}\frac{x+4}{(x-3)(x+4)}&=\frac{1}{x-3}\\ \frac{(7)+4}{(7-3)(7+4)}&=\frac{1}{(7)-3}\\ \frac{11}{(4)(11)}&=\frac{1}{4}\\ \frac{1}{4}&=\frac{1}{4}\ \checkmark\end{aligned}$$

It is important to state the restrictions before simplifying rational expressions because the simplified expression may be defined for restrictions of the original. In this case, the expressions are not equivalent. Here -4 is defined for the simplified equivalent but not for the original, as illustrated below:

$$\begin{aligned}\frac{x+4}{(x-3)(x+4)}&=\frac{1}{x-3}\\ \frac{(-4)+4}{(-4-3)(-4+4)}&=\frac{1}{(-4)-3}\\ \frac{0}{(-7)(0)}&=\frac{1}{-7}\\ \frac{0}{0}&=-\frac{1}{7}\quad x\end{aligned}$$

Example 8.1.5

Simplify and state the restriction:

$$\frac{25x^2}{15x^3}$$

Solution:

In this example, the expression is undefined when ***x*** is 0.

$$\frac{25x^2}{15x^3}=\frac{25(0)^2}{15(0)^3}=\frac{0}{0}\ \textit{Undefined}$$

Therefore, the domain consists of all real numbers ***x***, where $x\neq 0$. With this understanding, we can cancel common factors.

$$\begin{aligned}\frac{25x^2}{15x^3}&=\frac{5\cdot \cancel{5x^2}}{3x\cdot \cancel{5x^2}}\\ &=\frac{5}{3x}\end{aligned}$$

Answer:

$\frac{5}{3x}$, where $x \neq 0$

Example 8.1.6

State the restrictions and simplify:

$$\frac{3x(x-5)}{(2x-1)(x-5)}$$

Solution:

To determine the restrictions, set the denominator equal to 0 and solve.

$$(2x+1)(x-5)=0$$

$$\begin{aligned} 2x+1 &= 0 \quad \text{or} \quad x-5=0 \\ 2x &= -1 \qquad\quad x=5 \\ x &= -\tfrac{1}{2} \end{aligned}$$

The domain consists of all real numbers except for $-\frac{1}{2}$ and 5. Next, we find an equivalent expression by canceling common factors.

$$\begin{aligned} \frac{3x(x-5)}{(2x+1)(x-5)} &= \frac{3x\ \cancel{(x-5)}}{(2x+1)\ \cancel{(x-5)}} \\ &= \frac{3x}{2x+1} \end{aligned}$$

Answer:

$\frac{3x}{2x+1}$, where $x \neq -\frac{1}{2}$ and $x \neq 5$

Typically, rational expressions are not given in factored form. If this is the case, factor first and then cancel. The steps are outlined in the following example.

Example 8.1.7

State the restrictions and simplify:

$$\frac{3x+6}{x^2+x-2}$$

Solution:

Step 1: Completely factor the numerator and denominator.

$$\frac{3x+6}{x^2+x-2} = \frac{3(x+2)}{(x-1)(x+2)}$$

Step 2: Determine the restrictions to the domain. To do this, set the denominator equal to 0 and solve.

$$(x-1)(x+2)=0$$

$$\begin{aligned} x-1 &= 0 \quad \text{or} \quad x+2=0 \\ x &= 1 \qquad\quad x=-2 \end{aligned}$$

The domain consists of all real numbers except -2 and 1.

Step 3: Cancel common factors, if any.

$$\begin{aligned} \frac{3x+6}{x^2+x-2} &= \frac{3\ \cancel{(x+2)}}{(x-1)\ \cancel{(x+2)}} \\ &= \frac{3}{x-1} \end{aligned}$$

Answer:

$\frac{3}{x-1}$, where $x \neq 1$ and $x \neq -2$

Example 8.1.8

State the restrictions and simplify:

Solution:

First, factor the numerator and denominator.

Any value of x that results in a value of 0 in the denominator is a restriction. By inspection, we determine that the domain consists of all real numbers except 4 and 3. Next, cancel common factors.

$$= \frac{\cancel{(x-3)}(x+10)}{(x-4)\ \cancel{(x-3)}}$$
$$= \frac{x+10}{x-4}$$

Answer:

$\frac{x+10}{x-4}$, where $x \neq 3$ and $x \neq 4$

It is important to remember that we can only cancel factors of a product. A common mistake is to cancel terms. For example,

State the restrictions and simplify:

$$\frac{x^2-16}{5x^2-20x}$$

Answer

$\frac{x+4}{5x}$, where $x \neq 0$ and $x \neq 4$

In some examples, we will make a broad assumption that the denominator is nonzero. When we make that assumption, we do not need to determine the restrictions.

Example 8.1.9

Simplify:

(Assume all denominators are nonzero.)

Solution:

Factor the numerator by grouping. Factor the denominator using the formula for a difference of squares.

Next, cancel common factors.

$$= \frac{\cancel{(x+y)}(y-3)}{\cancel{(x+y)}(x-y)}$$
$$= \frac{y-3}{x-y}$$

Answer:

$$\frac{y-3}{x-y}$$

Opposite Binomial Factors

Recall that the opposite of the real number a is $-a$. Similarly, we can define the opposite of a polynomial P to be $-P$. We first consider the opposite of the binomial $a-b$:

$$-(a-b)=-a+b=b-a \tag{8.1.1}$$

This leads us to the opposite binomial property:

$$-(a-b)=(b-a) \tag{8.1.2}$$

This is equivalent to factoring out a -1.

$$(b-a)=-1(a-b) \tag{8.1.3}$$

If $a \neq b$, then we can divide both sides by $(a-b)$ and obtain the following:

$$\frac{b-a}{a-b}=-1$$

Example 8.1.10

State the restrictions and simplify:

$$\frac{3-x}{x-3}$$

Solution:

By inspection, we can see that the denominator is 0 if $x=3$. Therefore, 3 is the restriction to the domain. Apply the opposite binomial property to the numerator and then cancel.

Answer:

$\frac{3-x}{x-3}$, where $x \neq 3$

Since addition is commutative, we have

$$(a+b)=(b+a) \tag{8.1.4}$$

or

$$\frac{b+a}{a+b}=1$$

Take care not to confuse this with the opposite binomial property. Also, it is important to recall that

$$\frac{-a}{b}=-\frac{a}{b}=\frac{a}{-b}$$

In other words, show a negative fraction by placing the negative sign in the numerator, in front of the fraction bar, or in the denominator. Generally, negative denominators are avoided.

Example 8.1.11

Simplify and state the restrictions:

$$\frac{4-x^2}{x^2+3x-10}$$

Solution:

Begin by factoring the numerator and denominator.

$$\frac{4-x^2}{x^2+3x-10}=\frac{(2+x)(2-x)}{(x-2)(x+5)} \qquad \textit{The restrictions are } x \neq 2 \textit{ and } x \neq -5.$$

$$=\frac{(2+x)\cdot(-1)\,(x-2)}{(x-2)(x+5)} \qquad \textit{Apply the opposite binomial property, then cancel.}$$

$$=\frac{(2+x)\cdot(-1)}{(x+5)}$$

$$=-\frac{2+x}{x+5} \quad \text{or} \quad =-\frac{x+2}{x+5}$$

Answer:

$-\frac{x+2}{x+5}$, where $x \neq 2$ and $x \neq -5$

Exercise 8.1.2

Simplify and state the restrictions:

Answer

$\frac{-2x+3}{x+5}$, **where** $x \neq \pm 5$

Rational Functions

Rational functions have the form

$$r(x) = \frac{p(x)}{q(x)} \tag{8.1.5}$$

,

where *p(x)* and *q(x)* are polynomials and *q(x)≠0*. The domain of a rational function consists of all real numbers ***x*** such that the denominator *q(x)≠0*.

Example 8.1.12

a. Simplify: $r(x) = \frac{2x^2+5x-3}{6x^2+18x}$.
b. State the domain.
c. Calculate $r(-2)$.

Solution:

a. To simplify the rational function, first factor and then cancel.

$$\begin{aligned} r(x) &= \frac{2x^2+5x-3}{6x^2+18x} \\ &= \frac{(2x-1)\,(x+3)}{6x\,(x+3)} \\ &= \frac{2x-1}{6x} \end{aligned}$$

b. To determine the restrictions, set the denominator of the original function equal to 0 and solve.

$$\begin{aligned} 6x^2+18x &= 0 \\ 6x(x+3) &= 0 \\ 6x = 0 \quad &\text{or} \quad x+3 = 0 \\ x = 0 \quad &\quad\;\; x = -3 \end{aligned}$$

The domain consists of all real numbers ***x***, where $x \neq 0$ and $x \neq -3$.

c. Since -2 is not a restriction, substitute it for the variable ***x*** using the simplified form.

$$r(x) = \frac{2x-1}{6x}$$
$$r(-2) = \frac{2(-2)-1}{6(-2)}$$
$$= \frac{-4-1}{-12}$$
$$= \frac{-5}{-12}$$
$$= \frac{5}{12}$$

Answer:

a. $\frac{2x-1}{6x}$ b. The domain is all real numbers except 0 and -3. c. $r(-2) = \frac{5}{12}$

If a cost function $C(x)$ represents the cost of producing ***x*** units, then the average cost $c(x)$ is the cost divided by the number of units produced.

$$\textit{Average cost}: c(x) = \frac{C(x)}{x} \tag{8.1.6}$$

Example 8.1.13

The cost in dollars of producing t-shirts with a company logo is given by $C(x) = 7x + 200$, where ***x*** represents the number of shirts produced. Determine the average cost of producing

a. 40 t-shirts
b. 250 t-shirts
c. 1,000 t-shirts

Solution:

Set up a function representing the average cost.

$$c(x) = \frac{C(x)}{x} = \frac{7x+200}{x}$$

Next, calculate c(40), c(250), and c(1000).

$$c(40) = \frac{7(40)+200}{40} = \frac{280+200}{40} = \frac{480}{40} = 12.00$$
$$c(250) = \frac{7(250)+200}{250} = \frac{1750+200}{250} = \frac{1950}{250} = 7.80$$
$$c(1000) = \frac{7(1000)+200}{1000} = \frac{7000+200}{1000} = \frac{7200}{1000} = 7.20$$

Answer:

a. If 40 t-shirts are produced, then the average cost per t-shirt is $12.00.
b. If 250 t-shirts are produced, then the average cost per t-shirt is $7.80.
c. If 1,000 t-shirts are produced, then the average cost per t-shirt is $7.20.

Key Takeaways

- Rational expressions usually are not defined for all real numbers. The real numbers that give a value of 0 in the denominator are not part of the domain. These values are called restrictions.
- Simplifying rational expressions is similar to simplifying fractions. First, factor the numerator and denominator and then cancel the common factors. Rational expressions are simplified if there are no common factors other than 1 in the numerator and the denominator.
- Simplified rational expressions are equivalent for values in the domain of the original expression. Be sure to state the restrictions if the denominators are not assumed to be nonzero.

- Use the opposite binomial property to cancel binomial factors that involve subtraction. Use $-(a-b)=b-a$ to replace factors that will then cancel. Do not confuse this with factors that involve addition, such as $(a+b)=(b+a)$.

Exercise 8.1.3 Rational Expressions

Evaluate for the given set of *x*-values.

1. $5x;\ -1, 0, 1$
2. $\frac{4x^3}{x^2};\ -1, 0, 1$
3. $\frac{1}{x+9};\ -10, -9, 0$
4. $\frac{x+6}{x-5};\ -6, 0, 5$
5. $\frac{3x(x-2)}{2x-1};\ 0, \frac{1}{2}, 2$
6. $\frac{9x^2-1}{x-7};\ 0, \frac{1}{3}, 7$
7. $5x^2-9;\ -3, 0, 3$
8. $\frac{x^2-2}{5x^2-3x-10};\ -5, -4, 5$
9. Fill in the following chart:

x	$\frac{1}{x}$
-3	
-2	
0	
2	
3	

***Figure* 8.1.1**

10. Fill in the following chart:

x	$\frac{1}{x-2}$
-1	
0	
1	
2	
3	

***Figure* 8.1.2**

11. Fill in the following chart:

x	$\frac{1}{x+2}$
-4	
-3	
-2	
-1	
0	

***Figure* 8.1.3**

12. Fill in the following chart:

x	$\frac{1}{x}-2$
-2	
-1	
0	
1	
2	

***Figure* 8.1.4**

Answer

1. -5, undefined, 5
3. -1, undefined, $\frac{1}{9}$
5. 0, undefined, 0
7. Undefined, $-\frac{5}{9}$, undefined
9.

x	$\frac{1}{x}$
-3	$-1/3$
-2	$-1/2$
0	undefined
2	$1/2$
3	$1/3$

Figure 8.1.5

11.

x	$\frac{1}{x+2}$
-4	$-1/2$
-3	-1
-2	undefined
-1	1
0	$1/2$

Figure 8.1.6

? Exercise 8.1.4 Rational Expressions

An object's weight depends on its height above the surface of earth. If an object weighs 120 pounds on the surface of earth, then its weight in pounds, W, x miles above the surface is approximated by the formula

$$W = \frac{120 \cdot 4000^2}{(4000+x)^2}$$

For each problem below, approximate the weight of a 120-pound object at the given height above the surface of earth. (1 mile = 5,280 feet)

1. 100 miles
2. 1,000 miles
3. 44,350 feet
4. 90,000 feet

Answer

1. 114 pounds

3. 119.5 pounds

Exercise 8.1.5 Rational Expressions

The price to earnings ratio (P/E) is a metric used to compare the valuations of similar publicly traded companies. The P/E ratio is calculated using the stock price and the earnings per share (EPS) over the previous 12-month period as follows:

$$\text{P/E}=\text{price per share earnings per share}$$

If each share of a company stock is priced at $22.40, then calculate the P/E ratio given the following values for the earnings per share.

1. $1.40
2. $1.21
3. What happens to the P/E ratio when earnings decrease?
4. What happens to the P/E ratio when earnings increase?

Answer

1. 16

3. The P/E ratio increases.

Exercise 8.1.6 Rational Expressions

State the restrictions to the domain.

1. $\frac{1}{3x}$
2. $\frac{3x^2}{7x^5}$
3. $\frac{3x(x+1)}{x+4}$
4. $\frac{2x^2(x-3)}{x-1}$
5. $\frac{1}{5x-1}$
6. $\frac{x-2}{3x-2}$
7. $\frac{x-9}{5x(x-2)}$
8. $\frac{1}{(x-3)(x+6)}$
9. $\frac{x}{1-x^2}$
10. $\frac{x^2-9}{x^2-36}$
11. $\frac{1}{2x(x+3)(2x-1)}$
12. $\frac{x-3}{(3x-1)(2x+3)}$
13. $\frac{4x(2x+1)}{12x^2+x-1}$
14. $\frac{x-5}{3x^2-15x}$

Answer

1. $x \neq 0$

3. $x \neq -4$

5. $x \neq \frac{1}{5}$

7. $x \neq 0$ and $x \neq 2$

9. $x \neq \pm 1$

11. $x \neq 0$, $x \neq -3$, and $x \neq \frac{1}{2}$

13. $x \neq -\frac{1}{3}$ and $x \neq \frac{1}{4}$

Exercise 8.1.7 Simplifying Rational Expressions

State the restrictions and then simplify.

1. $\frac{5x^2}{20x^3}$
2. $\frac{12x^6}{60x}$
3. $\frac{3x^2(x-2)}{9x(x-2)}$
4. $\frac{20(x-3)(x-5)}{6(x-3)(x+1)}$
5. $\frac{6x^2(x-8)}{36x(x+9)(x-8)}$
6. $\frac{16x^2-1}{(4x+1)^2}$
7. $\frac{9x^2-6x+1}{(3x-1)^2}$
8. $\frac{x-7}{x^2-49}$
9. $\frac{x^2-64}{x^2+8x}$
10. $\frac{x+10}{x^2-100}$
11. $\frac{2x^3-12x^2}{5x^2-30x}$
12. $\frac{30x^5+60x^4}{2x^3-8x}$
13. $\frac{2x-12}{x^2+x-6}$
14. $\frac{x^2-x-6}{3x^2-8x-3}$
15. $\frac{6x^2-25x+25}{3x^2+16x-35}$
16. $\frac{3x^2+4x-1}{5x^2-9}$
17. $\frac{x^2-10x+21}{x^2-4x-21}$
18. $\frac{x^3-1}{x^2-1}$
19. $\frac{x^3+8}{x^2-4}$
20. $\frac{x^4-1}{6x^2-4}$

Answer

1. $\frac{1}{4x}; x \neq 0$
3. $\frac{x}{3}; x \neq 0, 2$
5. $\frac{x}{6(x+9)}; x \neq 0, -9, 8$
7. $1; x \neq \frac{1}{3}$
9. $\frac{x-8}{x}; x \neq 0, -8$
11. $\frac{2x}{5}; x \neq 0, 6$
13. $\frac{2x-12}{x^2+x-6}$ $x \neq -2, \frac{3}{2}$
15. $\frac{2x-5}{x+7}; x \neq -7, \frac{5}{3}$
17. $\frac{x-3}{x+3}; x \neq -3, 7$
19. $\frac{x^2-2x+4}{x-2}; x \neq \pm 2$

Exercise 8.1.8 Simplifying Rational Expressions with Opposite Binomial Factors

State the restrictions and then simplify.

1. $\frac{x-9}{9-x}$
2. $\frac{3x-2}{2-3x}$
3. $\frac{x+6}{6+x}$
4. $\frac{3x+1}{1+3x}$
5. $\frac{(2x-5)(x-7)}{(7-x)(2x-1)}$
6. $\frac{(3x+2)(x+5)}{(x-5)(2+3x)}$
7. $\frac{x^2-4}{(2-x)^2}$
8. $\frac{16-9x^2}{(3x+4)^2}$
9. $\frac{4x^2(10-x)}{3x^3-300x}$
10. $-\frac{2x+1}{4x^3-49x}$
11. $\frac{2x^2-7x-4}{1-4x^2}$
12. $\frac{9x^2-4}{4x-6x^2}$
13. $\frac{x^2-5x-14}{7-15x+2x^2}$
14. $\frac{2x^3+x^2-2x-1}{1+x-2x^2}$
15. $\frac{x^3+2x-3x^2-6}{2+x^2}$
16. $\frac{27+x^3}{x^2+6x+9}$
17. $\frac{64-x^3}{x^2-8x+16}$
18. $\frac{x^2+4}{4-x^2}$

Answer

1. $-1; x \neq 9$
3. $1; x \neq -6$
5. $\frac{-2x-5}{2x-1}; x \neq \frac{1}{2}, 7$
7. $\frac{x+2}{x-2}; x \neq 2$
9. $-\frac{4x}{3(x+10)}; x \neq \pm 10, 0$
11. $\frac{x-4}{1-2x}; x \neq \pm\frac{1}{2}$
13. $\frac{x+2}{2x-1}; x \neq \frac{1}{2}, 7$
15. $x-3$; none
17. $\frac{-16+4x+x^2}{x-4}; x \neq 4$

Exercise 8.1.9 Simplifying Rational Expressions with Opposite Binomial Factors

Simplify. (Assume all denominators are nonzero.)

1. $-\frac{15x^3y^2}{5xy^2(x+y)}$
2. $\frac{14x^7y^2(x-2y)^4}{7x^8y(x-2y)^2}$
3. $\frac{y+x}{x^2-y^2}$

4. $\frac{y-x}{x^2-y^2}$
5. $\frac{x^2-y^2}{(x-y)^2}$
6. $\frac{a^2-ab-6b^2}{a^2-6ab+9b^2}$
7. $\frac{2a^2-11a+12}{-32+2a^2}$
8. $\frac{a^2b-3a^2}{3a^2-3ab}$
9. $\frac{xy^2-x+y^3-y}{x-xy^2}$
10. $\frac{x^3-xy^2-x^2y+y}{3x^2-2xy+y^2}$
11. $\frac{x^3-27}{x^2+3x+9}$
12. $\frac{x^2-x+1}{x^3+1}$

Answer

1. $-\frac{3x^2}{x+y}$
3. $\frac{1}{x-y}$
5. $\frac{x+y}{x-y}$
7. $\frac{2a-3}{2(a+4)}$
9. $-\frac{x+y}{x}$
11. $x-3$

Exercise 8.1.10 Rational Functions

Calculate the following.

1. $f(x)=\frac{5x}{x-3}; f(0), f(2), f(4)$
2. $f(x)=\frac{x+7}{x^2+1}; f(-1), f(0), f(1)$
3. $g(x)=\frac{x^3}{(x-2)^2}; g(0), g(2), g(-2)$
4. $g(x)=\frac{x^2-9}{9-x^2}; g(-2), g(0), g(2)$
5. $g(x)=\frac{x^3}{x^2+1}; g(-1), g(0), g(1)$
6. $g(x)=\frac{5x+1}{x^2-25}; g(-\frac{1}{5}), g(-1), g(-5)$

Answer

1. f(0)=0, f(2)=−10, f(4)=20
3. g(0)=0, g(2) undefined, g(−2)=$-\frac{1}{2}$
5. g(−1)=$-\frac{1}{2}$, g(0)=0, g(1)=$\frac{1}{2}$

Exercise 8.1.11 Rational Functions

State the restrictions to the domain and then simplify.

1. $f(x)=-\frac{3x^2-6x}{x^2+4x+4}$
2. $f(x)=\frac{x^2+6x+9}{2x^2+5x-3}$
3. $g(x)=\frac{9-x}{x^2-81}$
4. $g(x)=\frac{x^3-27}{3-x}$

5. $g(x)=\frac{3x-15}{10-2x}$
6. $g(x)=\frac{25-5x}{4x-20}$
7. The cost in dollars of producing coffee mugs with a company logo is given by $C(x)=x+40$, where x represents the number of mugs produced. Calculate the average cost of producing 100 mugs and the average cost of producing 500 mugs.
8. The cost in dollars of renting a moving truck for the day is given by $C(x)=0.45x+90$, where x represents the number of miles driven. Calculate the average cost per mile if the truck is driven 250 miles in one day.
9. The cost in dollars of producing sweat shirts with a custom design on the back is given by $C(x)=1200+(12-0.05x)x$, where x represents the number of sweat shirts produced. Calculate the average cost of producing 150 custom sweat shirts.
10. The cost in dollars of producing a custom injected molded part is given by $C(x)=500+(3-0.001x)x$, where x represents the number of parts produced. Calculate the average cost of producing 1,000 custom parts.

Answer

1. $f(x)=-\frac{3x}{x+2}; x\neq -2$
3. $g(x)=-\frac{1}{x+9}; x\neq \pm 9$
5. $g(x)=-\frac{3}{2}; x\neq 5$
7. The average cost of producing 100 mugs is $1.40 per mug. The average cost of producing 500 mugs is $1.08 per mug.
9. $12.50

? Exercise 8.1.12 Discussion Board

1. Explain why $\frac{b-a}{a-b}=-1$ and illustrate this fact by substituting some numbers for the variables.
2. Explain why $\frac{b+a}{a+b}=1$ and illustrate this fact by substituting some numbers for the variables.
3. Explain why we cannot cancel x in the expression $\frac{x}{x+1}$.

Answer

1. Answers may vary
3. Answers may vary

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8.2: Multiplying and Dividing Rational Expressions

Learning Objectives

- Multiply rational expressions.
- Divide rational expressions.
- Multiply and divide rational functions.

Multiplying Rational Expressions

When multiplying fractions, we can multiply the numerators and denominators together and then reduce, as illustrated:

$$\frac{3}{5}\cdot\frac{5}{9}=\frac{3\cdot 5}{5\cdot 9}=\frac{\overset{1}{\cancel{3}}\cdot\overset{1}{\cancel{5}}}{\underset{1}{\cancel{5}}\cdot\underset{3}{\cancel{9}}}=\frac{1}{3}$$

Multiplying rational expressions is performed in a similar manner. For example,

$$\frac{y}{x}\cdot\frac{x}{y^2}=\frac{y\cdot x}{x\cdot y^2}=\frac{\overset{1}{\cancel{y}}\cdot\overset{1}{\cancel{x}}}{\underset{1}{\cancel{x}}\cdot\underset{3}{\cancel{y^2}}}=\frac{1}{y}$$

In general, given polynomials P, Q, R, and S, where $Q\neq 0$ and $S\neq 0$, we have

$$\frac{P}{Q}\cdot\frac{R}{S}=\frac{PR}{QS} \tag{8.2.1}$$

In this section, assume that all variable expressions in the denominator are nonzero unless otherwise stated.

Example 8.2.1

Multiply:

$$\frac{12x^2}{5y^3}\cdot\frac{20y^4}{6x^3}$$

Solution:

Multiply numerators and denominators and then cancel common factors.

$$\begin{aligned}\frac{12x^2}{5y^3}\cdot\frac{20y^4}{6x^3}&=\frac{240x^2y^4}{30x^3y^3} && \textit{Multiply.}\\ &=\frac{\overset{8}{\cancel{240}}\ \overset{1}{\cancel{x^2}}\ \overset{y}{\cancel{y^4}}}{\underset{1}{\cancel{30}}\ \underset{x}{\cancel{x^3}}\ \underset{1}{\cancel{y^3}}} && \textit{Cancel.}\\ &=\frac{8y}{x}\end{aligned}$$

Answer:

$$\frac{8y}{x}$$

Example 8.2.2

Multiply:

$$\frac{x-3}{x+5}\cdot\frac{x+5}{x+7}$$

Solution:

Leave the product in factored form and cancel the common factors.

$$\frac{x-3}{x+5}\cdot\frac{x+5}{x+7}=\frac{(x-3)\cdot\cancel{(x+5)}}{\cancel{(x+5)}\cdot(x+7)}$$
$$=\frac{x-3}{x+7}$$

Answer:

$$\frac{x-3}{x+7}$$

Example 8.2.3

Multiply:

$$\frac{15x^2y^3}{(2x-1)}\cdot\frac{x(2x-1)}{3x^2y(x+3)}$$

Solution:

Leave the polynomials in the numerator and denominator factored so that we can cancel the factors. In other words, do not apply the distributive property.

$$\frac{15x^2y^3}{(2x-1)}\cdot\frac{x(2x-1)}{3x^2y(x+3)}=\frac{15x^3y^3(2x-1)}{3x^2y(2x-1)(x+3)}\qquad \textit{Multiply.}$$
$$=\frac{\overset{5}{\cancel{15}}\ \overset{x}{\cancel{x^3}}\ \overset{y^2}{\cancel{y^3}}\ \overset{1}{\cancel{(2x-1)}}}{\cancel{3}\ \cancel{x^2}\ \cancel{y}\ \cancel{(2x-1)}\ (x+3)}\qquad \textit{Cancel.}$$
$$=\frac{5xy^2}{x+3}$$

Answer:

$$\frac{5xy^2}{x+3}$$

Typically, rational expressions will not be given in factored form. In this case, first factor all numerators and denominators completely. Next, multiply and cancel any common factors, if there are any.

Example 8.2.4

Multiply:

$$\frac{x+5}{x-5}\cdot\frac{x-5}{x^2-25}$$

Solution

Factor the denominator x^2-25 as a difference of squares. Then multiply and cancel.

$$\frac{x+5}{x-5}\cdot\frac{x-5}{x^2-25}=\frac{x+5}{x-5}\cdot\frac{x-5}{(x+5)(x-5)}\qquad \textit{Factor.}$$
$$=\frac{\overset{1}{\cancel{(x+5)}}\ \overset{1}{\cancel{(x-5)}}}{\cancel{(x-5)}\ \cancel{(x+5)}\ (x-5)}\qquad \textit{Cancel.}$$
$$=\frac{1}{x-5}$$

Keep in mind that 1 is always a factor; so when the entire numerator cancels out, make sure to write the factor 1.

Answer:

$$\frac{1}{x-5}$$

Example 8.2.5

Multiply:

Solution:

It is a best practice to leave the final answer in factored form.

Answer:

$$\frac{(x+2)(x-4)}{(x-2)(x+7)}$$

Example 8.2.6

Multiply:

Solution:

The trinomial $-2x^2+x+3$ in the numerator has a negative leading coefficient. Recall that it is a best practice to first factor out a -1 and then factor the resulting trinomial.

Answer:

$$-\frac{3(2x-3)}{x(x+4)}$$

Example 8.2.7

Multiply:

$$\frac{7-x}{x^2+3x}\cdot\frac{x^2+10x+21}{x^2-49}$$

Solution:

We replace $7-x$ with $-1(x-7)$ so that we can cancel this factor.

$$\begin{aligned}\frac{7-x}{x^2+3x}\cdot\frac{x^2+10x+21}{x^2-49} &= \frac{-1(x-7)}{x(x+3)}\cdot\frac{(x+3)(x+7)}{(x+7)(x-7)}\\ &= \frac{-1\ \cancel{(x-7)}\ \cancel{(x+3)}\ \cancel{(x+7)}}{x\ \cancel{(x+3)}\ \cancel{(x+7)}\ \cancel{(x-7)}}\\ &= \frac{-1}{x}\\ &= -\frac{1}{x}\end{aligned}$$

Answer:

$$-\frac{1}{x}$$

Exercise 8.2.1

Multiply:

$$\frac{x^2-64}{8-x}\cdot\frac{x+x^2}{x^2+9x+8}$$

Answer

$-x$

Dividing Rational Expressions

To divide two fractions, we multiply by the reciprocal of the divisor, as illustrated:

$$\frac{5}{8} \div \frac{1}{2} = \frac{5}{8} \cdot \frac{2}{1} = \frac{5 \cdot \overset{1}{\cancel{2}}}{\underset{4}{\cancel{8}} \cdot 1} = \frac{5}{4}$$

Dividing rational expressions is performed in a similar manner. For example,

$$\frac{x}{y^2} \div \frac{1}{y} = \frac{x}{y^2} \cdot \frac{y}{1} = \frac{x \cdot \overset{1}{\cancel{y}}}{\underset{\cancel{y}}{y^2} \cdot 1} = \frac{x}{y}$$

In general, given polynomials ***P***, ***Q***, ***R***, and ***S***, where $Q \neq 0$, $R \neq 0$, and $S \neq 0$, we have

$$\frac{P}{Q} \div \frac{R}{S} = \frac{P}{Q} \cdot \frac{S}{R} = \frac{PS}{QR} \tag{8.2.2}$$

Example 8.2.8

Divide:

$$\frac{8x^5y}{25z^6} \div \frac{20xy^4}{15z^3}$$

Solution:

First, multiply by the reciprocal of the divisor and then cancel.

$$\begin{aligned} \frac{8x^5y}{25z^6} \div \frac{20xy^4}{15z^3} &= \frac{8x^5y}{25z^6} \cdot \frac{15z^3}{20xy^4} && \textit{Multiply by the reciprocal of the divisor.} \\ &= \frac{120x^5yz^3}{500xy^4z^6} \\ &= \frac{\overset{6}{\cancel{120}}\ \overset{x^4}{\cancel{x^5}}\ \cancel{y}\ \cancel{z^3}}{\underset{25}{\cancel{500}}\ \cancel{x}\ \underset{y^3}{\cancel{y^4}}\ \underset{z^3}{\cancel{z^6}}} && \textit{Cancel.} \\ &= \frac{6x^4}{25y^3z^3} \end{aligned}$$

Answer:

$$\frac{6x^4}{25y^3z^3}$$

Example 8.2.9

Divide:

$$\frac{x+2}{x^2-4} \div \frac{x+3}{x-2}$$

Solution:

After multiplying by the reciprocal of the divisor, factor and cancel.

$$\begin{aligned}\frac{x+2}{x^2-4} \div \frac{x+3}{x-2} &= \frac{x+2}{x^2-4} \cdot \frac{x-2}{x+3} && \textit{Multiply by the reciprocal of the divisor.}\\ &= \frac{(x+2)}{(x+2)(x-2)} \cdot \frac{(x-2)}{(x+3)} && \textit{Factor.}\\ &= \frac{\cancel{(x+2)}\ \cancel{(x-2)}}{\cancel{(x+2)}\ \cancel{(x-2)}(x+3)} && \textit{Cancel.}\\ &= \frac{1}{x+3}\end{aligned}$$

Answer:

$$\frac{1}{x+3}$$

Example 8.2.10

Divide:

Solution:

Begin by multiplying by the reciprocal of the divisor. After doing so, factor and cancel.

Answer:

$$\frac{(x-8)(x-5)}{(x+7)^2}$$

Example 8.2.11

Divide:

Solution:

Just as we do with fractions, think of the divisor $(2x-3)$ as an algebraic fraction over 1.

Answer:

$$-\frac{2x+3}{x+2}$$

Exercise 8.2.2

Divide:

Answer

$-4x^3-8x^2$

Multiplying and Dividing Rational Functions

The product and quotient of two rational functions can be simplified using the techniques described in this section. The restrictions to the domain of a product consist of the restrictions of each function.

Example 8.2.12

Calculate $(f \cdot g)(x)$ and determine the restrictions to the domain.

Solution:

In this case, the domain of $f(x)$ consists of all real numbers except 0, and the domain of $g(x)$ consists of all real numbers except $\frac{1}{4}$.

Therefore, the domain of the product consists of all real numbers except 0 and $\frac{1}{4}$. Multiply the functions and then simplify the result.

Answer:

$$(f \cdot g)(x) = -\frac{4x+1}{5x}\text{, where } x \neq 0, \frac{1}{4}$$

The restrictions to the domain of a quotient will consist of the restrictions of each function as well as the restrictions on the reciprocal of the divisor.

Example 8.2.13

Calculate $(f/g)(x)$ and determine the restrictions.

Solution:

In this case, the domain of $f(x)$ consists of all real numbers except 3 and 8, and the domain of $g(x)$ consists all real numbers except 3. In addition, the reciprocal of $g(x)$ has a restriction of -8. Therefore, the domain of this quotient consists of all real numbers except 3, 8, and -8.

Answer:

$(f/g)(x) = 1$, where $x \neq 3, 8, -8$

Key Takeaways

- After multiplying rational expressions, factor both the numerator and denominator and then cancel common factors. Make note of the restrictions to the domain. The values that give a value of 0 in the denominator are the restrictions.
- To divide rational expressions, multiply by the reciprocal of the divisor.
- The restrictions to the domain of a product consist of the restrictions to the domain of each factor.
- The restrictions to the domain of a quotient consist of the restrictions to the domain of each rational expression as well as the restrictions on the reciprocal of the divisor.

Exercise 8.2.2 Multiplying Rational Expressions

Multiply. (Assume all denominators are nonzero.)

1. $\frac{2x}{3} \cdot \frac{9}{4x^2}$
2. $-\frac{5x}{3y} \cdot \frac{y^2}{25x}$
3. $\frac{5x^2}{2y} \cdot \frac{4y^2}{15x^3}$
4. $\frac{16a^4}{7b^2} \cdot \frac{49b^3}{2a^3}$
5. $\frac{x-6}{12x^3} \cdot \frac{24x^2}{x-6}$
6. $\frac{x+10}{2x-1} \cdot \frac{x-2}{x+10}$
7. $\frac{(y-1)^2}{y+1} \cdot \frac{1}{y-1}$
8. $\frac{y^2-9}{y+3} \cdot \frac{2y-3}{y-3}$
9. $\frac{2a-5}{a-5} \cdot \frac{2a+5}{4a^2-25}$
10. $\frac{2a^2-9a+4}{a^2-16} \cdot \left(a^2+4a\right)$

11. $\frac{2x^2+3x-2}{(2x-1)^2} \cdot \frac{2x}{x+2}$
12. $\frac{9x^2+19x+2}{4-x^2} \cdot \frac{x^2-4x+4}{9x^2-8x-1}$
13. $\frac{x^2+8x+16}{16-x^2} \cdot \frac{x^2-3x-4}{x^2+5x+4}$
14. $\frac{x^2-x-2}{x^2+8x+7} \cdot \frac{x^2+2x-15}{x^2-5x+6}$
15. $\frac{x+1}{x-3} \cdot \frac{3-x}{x+5}$
16. $\frac{2x-1}{x-1} \cdot \frac{x+6}{1-2x}$
17. $\frac{9+x}{3x+1} \cdot \frac{3}{x+9}$
18. $\frac{1}{2+5x} \cdot \frac{5x+2}{5x}$
19. $\frac{100-y^2}{y-10} \cdot \frac{25y^2}{y+10}$
20. $\frac{3y^3}{6y-5} \cdot \frac{36y^2-25}{5+6y}$
21. $\frac{3a^2+14a-5}{a^2+1} \cdot \frac{3a+1}{1-9a^2}$
22. $\frac{4a^2-16a}{4a-1} \cdot \frac{1-16a^2}{4a^2-15a-4}$
23. $\frac{x+9}{-x^2+14x-45} \cdot \left(x^2-81\right)$
24. $\frac{1}{2+5x} \cdot \left(25x^2+20x+4\right)$
25. $\frac{x^2+x-6}{3x^2+15x+18} \cdot \frac{2x^2-8}{x^2-4x+4}$
26. $\frac{5x^2-4x-1}{5x^2-6x+1} \cdot \frac{25x^2-10x+1}{3-75x^2}$

Answer

1. $\frac{3}{2x}$
3. $\frac{2y}{3x}$
5. $\frac{2}{x}$
7. $\frac{y-1}{y+1}$
9. $\frac{1}{a-5}$
11. $\frac{2x}{2x-1}$
13. -1
15. $-\frac{x+1}{x+5}$
17. $\frac{3}{3x+1}$
19. $-25y^2$

21. $-\dfrac{a+5}{a^2+1}$

23. $-\dfrac{(x+9)^2}{x-5}$

25. $\dfrac{2}{3}$

? Exercise 8.2.3 Dividing Rational Expressions

Divide. (Assume all denominators are nonzero.)

1. $\dfrac{5x}{8} \div \dfrac{15x^2}{4}$
2. $\dfrac{3}{8y} \div \dfrac{15}{2y^2}$
3. $\dfrac{\dfrac{5x^9}{3y^3}}{\dfrac{25x^{10}}{9y^5}}$
4. $\dfrac{\dfrac{12x^4y^2}{21z^5}}{\dfrac{6x^3y^2}{7z^3}}$
5. $\dfrac{(x-4)^2}{30x^4} \div \dfrac{x-4}{15x}$
6. $\dfrac{5y^4}{10(3y-5)^2} \div \dfrac{10y^5}{2(3y-5)^3}$
7. $\dfrac{x^2-9}{5x} \div (x-3)$
8. $\dfrac{y^2-64}{8y} \div (8+y)$
9. $\dfrac{(a-8)^2}{2a^2+10a} \div \dfrac{a-8}{a}$
10. $\dfrac{2}{4a^2b^3(a-2b)} \div 12ab(a-2b)^5$
11. $\dfrac{x^2+7x+10}{x^2+4x+4} \div \dfrac{1}{x^2-4}$
12. $\dfrac{2x^2-x-1}{2x^2-3x+1} \div \dfrac{1}{4x^2-1}$
13. $\dfrac{y+1}{y^2-3y} \div \dfrac{y^2-1}{y^2-6y+9}$
14. $\dfrac{9-a^2}{a^2-8a+15} \div \dfrac{2a^2-10a}{a^2-10a+25}$
15. $\dfrac{a^2-3a-18}{2a^2-11a-6} \div \dfrac{a^2+a-6}{2a^2-a-1}$
16. $\dfrac{y^2-7y+10}{y^2+5y-14} \div \dfrac{2y^2-9y-5}{y^2+14y+49}$
17. $\dfrac{6y^2+y-1}{4y^2+4y+1} \div \dfrac{3y^2+2y-1}{2y^2-7y-4}$
18. $\dfrac{x^2-7x-18}{x^2+8x+12} \div \dfrac{x^2-81}{x^2+12x+36}$
19. $\dfrac{4a^2-b^2}{b+2a} \div (b-2a)^2$

20. $\frac{x^2-y^2}{y+x} \div (y-x)^2$
21. $\frac{5y^2(y-3)}{4x^3} \div \frac{25y(3-y)}{2x^2}$
22. $\frac{15x^3}{3(y+7)} \div \frac{25x^6}{9(7+y)^2}$
23. $\frac{3x+4}{x-8} \div \frac{7x}{8-x}$
24. $\frac{3x-2}{2x+1} \div \frac{2-3x}{3x}$
25. $\frac{(7x-1)^2}{4x+1} \div \frac{28x^2-11x+1}{1-4x}$
26. $\frac{4x}{(x+2)^2} \div \frac{2-x}{x^2-4}$
27. $\frac{a^2-b^2}{a} \div (b-a)^2$
28. $\frac{(a-2b)^2}{2b} \div (2b^2+ab-a^2)$
29. $\frac{x^2-6x+9}{x^2+7x+12} \div \frac{9-x^2}{x^2+8x+16}$
30. $\frac{2x^2-9x-5}{25-x^2} \div \frac{1-4x+4x^2}{-2x^2-9x+5}$
31. $\frac{3x^2-16x+5}{100-4x^2} \div \frac{9x^2-6x+1}{3x^2+14x-5}$
32. $\frac{10x^2-25x-15}{x^2-6x+9} \div \frac{9-x^2}{x^2+6x+9}$

Answer

1. $\frac{1}{6x}$
3. $\frac{3y^2}{5x}$
5. $\frac{x-4}{2x3}$
7. $\frac{x+3}{5x}$
9. $\frac{a-8}{2(a+5)}$
11. $(x+5)(x-2)$
13. $\frac{y-3}{y(y-1)}$
15. $\frac{a-1}{a-2}$
17. $\frac{y-4}{y+1}$
19. $\frac{1}{2a-b}$
21. $-\frac{y}{10x}$
23. $-\frac{3x+4}{7x}$

25. $-\dfrac{7x-1}{4x+1}$
27. $\dfrac{a+b}{a(a-b)}$
29. $-\dfrac{(x-3)(x+4)}{(x+3)^2}$
31. $-\dfrac{1}{4}$

Exercise 8.2.4 Dividing Rational Expressions

Recall that multiplication and division are to be performed in the order they appear from left to right. Simplify the following.

1. $\dfrac{1}{x^2}\cdot\dfrac{x-1}{x+3}\div\dfrac{x-1}{x^3}$
2. $\dfrac{x-7}{x+9}\cdot\dfrac{1}{x^3}\div\dfrac{x-7}{x}$
3. $\dfrac{x+1}{x-2}\div\dfrac{x}{x-5}\cdot\dfrac{x^2}{x+1}$
4. $\dfrac{x+4}{2x+5}\div\dfrac{x-3}{2x+5}\cdot\dfrac{x+4}{x-3}$
5. $\dfrac{2x-1}{x+1}\div\dfrac{x-4}{x^2+1}\cdot\dfrac{x-4}{2x-1}$
6. $\dfrac{4x^2-1}{3x+2}\div\dfrac{2x-1}{x+5}\cdot\dfrac{3x+2}{2x+1}$

Answer

1. $\dfrac{x}{x+3}$
3. $\dfrac{x(x-5)}{x-2}$
5. $\dfrac{x^2+1}{x+1}$

Exercise 8.2.5 Multiplying and Dividing Rational Functions

Calculate $(f\cdot g)(x)$ and determine the restrictions to the domain.

1. $f(x)=\dfrac{1}{x}$ and $g(x)=\dfrac{1}{x-1}$
2. $f(x)=\dfrac{x+1}{x-1}$ and $g(x)=x^2-1$
3. $f(x)=\dfrac{3x+2}{x+2}$ and $g(x)=\dfrac{x^2-4}{(3x+2)^2}$
4. $f(x)=\dfrac{(1-3x)}{2x-6}$ and $g(x)=\dfrac{(x-6)^2}{9x^2-1}$
5. $f(x)=\dfrac{25x^2-1}{x^2+6x+9}$ and $g(x)=\dfrac{x^2-9}{5x+1}$
6. $f(x)=\dfrac{x^2-49}{2x^2+13x-7}$ and $g(x)=\dfrac{4x^2-4x+1}{7-x}$

Answer

1. $(f\cdot g)(x)=\dfrac{1}{x(x-1)};x\neq 0,1$

3. $(f \cdot g)(x) = \dfrac{x-2}{3x+2}; x \neq -2, -\dfrac{2}{3}$

5. $(f \cdot g)(x) = \dfrac{(x-3)}{(5x-1)x+3}; x \neq -3, -\dfrac{1}{5}$

Exercise 8.2.6 Multiplying and Dividing Rational Functions

Calculate $(f/g)(x)$ and state the restrictions.

1. $f(x) = \dfrac{1}{x}$ and $g(x) = \dfrac{x-2}{x-1}$
2. $f(x) = \dfrac{(5x+3)^2}{x^2}$ and $g(x) = \dfrac{5x+3}{6-x}$
3. $f(x) = \dfrac{5-x}{(x-8)^2}$ and $g(x) = \dfrac{x^2-2}{5x-8}$
4. $f(x) = \dfrac{x^2-2x-1}{5x^2-3x-10}$ and $g(x) = \dfrac{2x^2-5x-3}{x^2-7x+12}$
5. $f(x) = \dfrac{3x^2+11x-4}{9x^2-6x+1}$ and $g(x) = \dfrac{x^2-2x+1}{3x^2-4x+1}$
6. $f(x) = \dfrac{36-x^2}{x^2+12x+36}$ and $g(x) = \dfrac{x^2-12x+3}{6x^2+4x-12}$

Answer

1. $(f/g)(x) = \dfrac{x-1}{x(x-2)}; x \neq 0, 1, 2$

3. $(f/g)(x) = -\dfrac{1}{(x-8)(x+5)}; x \neq \pm 5, 8$

5. $(f/g)(x) = \dfrac{(x+4)}{(x-1)}; x \neq \dfrac{1}{3}, 1$

Exercise 8.2.7 Discussion Board Topics

1. In the history of fractions, who is credited for the first use of the fraction bar?
2. How did the ancient Egyptians use fractions?
3. Explain why $x = 7$ is a restriction to $\dfrac{1}{x} \div \dfrac{x-7}{x-2}$.

Answer

1. Answer may vary

3. Answer may vary

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8.3: Adding and Subtracting Rational Expressions

Learning Objectives

- Add and subtract rational expressions with common denominators.
- Add and subtract rational expressions with unlike denominators.
- Add and subtract rational functions.

Adding and Subtracting with Common Denominators

Adding and subtracting rational expressions is similar to adding and subtracting fractions. Recall that if the denominators are the same, we can add or subtract the numerators and write the result over the common denominator.

$$\begin{aligned}\frac{3}{13}+\frac{7}{13} &= \frac{3+7}{13}\\ &= \frac{10}{13}\end{aligned}$$

When working with rational expressions, the common denominator will be a polynomial. In general, given polynomials ***P***, ***Q***, and ***R***, where $Q \neq 0$, we have the following:

$$\frac{P}{Q} \pm \frac{R}{Q} = \frac{P \pm R}{Q} \tag{8.3.1}$$

In this section, assume that all variable factors in the denominator are nonzero.

Example 8.3.1

Add:

$$\tfrac{3}{y}+\tfrac{7}{y}$$

Solution:

Add the numerators 3 and 7, and write the result over the common denominator, ***y***.

$$\begin{aligned}\frac{3}{y}+\frac{7}{y} &= \frac{3+7}{y}\\ &= \frac{10}{y}\end{aligned}$$

Answer:

$$\tfrac{10}{y}$$

Example 8.3.2

Subtract:

$$\tfrac{x-5}{2x-1}-\tfrac{1}{2x-1}$$

Solution:

Subtract the numerators $x-5$ and 1, and write the result over the common denominator, $2x-1$.

$$\begin{aligned}\frac{x-5}{2x-1}-\frac{1}{2x-1} &= \frac{x-5-1}{2x-1} \qquad \textit{Simplify the numerator.}\\ &= \frac{x-6}{2x-1}\end{aligned}$$

Answer:

$$\tfrac{x-6}{2x-1}$$

Example 8.3.3

Subtract:

$$\frac{2x+7}{(x+5)(x-3)} - \frac{x+10}{(x+5)(x-3)}$$

Solution:

We use parentheses to remind us to subtract the entire numerator of the second rational expression.

$$\begin{aligned} \frac{2x+7}{(x+5)(x-3)} - \frac{x+10}{(x+5)(x-3)} &= \frac{(2x+7)-(x+10)}{(x+5)(x-3)} && \textit{Simplify the numerator.} \\ &= \frac{2x+7-x-10}{(x+5)(x-3)} && \textit{Leave the denominator factored.} \\ &= \frac{\overset{1}{\cancel{x-3}}}{(x+5)\ \cancel{(x-3)}} && \textit{Cancel common factors.} \\ &= \frac{1}{x+5} \end{aligned}$$

Answer:

$$\frac{1}{x+5}$$

Example 8.3.4

Simplify:

$$\frac{2x^2+10x+3}{x^2-36} - \frac{x^2+6x+5}{x^2-36} + \frac{x-4}{x^2-36}$$

Solution:

Subtract and add the numerators. Make use of parentheses and write the result over the common denominator, x^2-36.

Answer:

$$\frac{x-1}{x-6}$$

Exercise 8.3.1

Subtract:

Answer

$$\frac{1}{x-4}$$

Adding and Subtracting with Unlike Denominators

To add rational expressions with unlike denominators, first find equivalent expressions with common denominators. Do this just as you have with fractions. If the denominators of fractions are relatively prime, then the least common denominator (LCD) is their product. For example,

$$\frac{1}{3} + \frac{1}{5} \Rightarrow \text{LCD} = 3 \cdot 5 = 15$$

Multiply each fraction by the appropriate form of 1 to obtain equivalent fractions with a common denominator.

$$\begin{aligned}\frac{1}{3}+\frac{1}{5} &= \frac{1\cdot 5}{3\cdot 5}+\frac{1\cdot 3}{5\cdot 3} \\ &= \frac{5}{15}+\frac{3}{15} && \textit{Equivalent fractions with a common denominator} \\ &= \frac{5+3}{15} \\ &= \frac{8}{15}\end{aligned}$$

The process of adding and subtracting rational expressions is similar. In general, given polynomials ***P***, ***Q***, ***R***, and ***S***, where $Q \neq 0$ and $S \neq 0$, we have the following:

$$\frac{P}{Q} \pm \frac{R}{S} = \frac{PS \pm QR}{QS} \tag{8.3.2}$$

In this section, assume that all variable factors in the denominator are nonzero.

Example 8.3.5

Add:

$$\tfrac{1}{x}+\tfrac{1}{y}$$

Solution:

In this example, the $LCD = xy$. To obtain equivalent terms with this common denominator, multiply the first term by $\frac{y}{y}$ and the second term by $\frac{x}{x}$.

$$\begin{aligned}\frac{1}{x}+\frac{1}{y} &= \frac{1}{x}\cdot\frac{y}{y}+\frac{1}{y}\cdot\frac{x}{x} \\ &= \frac{y}{xy}+\frac{x}{xy} && \textit{Equivalent terms with a common denominator.} \\ &= \frac{y+x}{xy} && \textit{Add the numerators and place the result over the common denominator, } xy.\end{aligned}$$

Answer:

$$\tfrac{y+x}{xy}$$

Example 8.3.6

Subtract:

$$\tfrac{1}{y}-\tfrac{1}{y-3}$$

Solution:

Since the $LCD = y(y-3)$, multiply the first term by 1 in the form of $\frac{(y-3)}{(y-3)}$ and the second term by $\frac{y}{y}$.

$$\begin{aligned}\frac{1}{y}-\frac{1}{y-3} &= \frac{1}{y}\cdot\frac{(y-3)}{(y-3)}-\frac{1}{y-3}\cdot\frac{y}{y} \\ &= \frac{(y-3)}{y(y-3)}-\frac{y}{y(y-3)} \\ &= \frac{y-3-y}{y(y-3)} \\ &= \frac{-3}{y(y-3)} \quad \text{or} \quad = -\frac{3}{y(y-3)}\end{aligned}$$

Answer:

$$\tfrac{-3}{y(y-3)}$$

It is not always the case that the LCD is the product of the given denominators. Typically, the denominators are not relatively prime; thus determining the LCD requires some thought. Begin by factoring all denominators. The LCD is the product of all factors with the highest power. For example, given

$$\frac{1}{x^3(x+2)(x-3)} \quad \text{and} \quad \frac{1}{x(x+2)^2}$$

there are three base factors in the denominator: x, $(x+2)$, and $(x-3)$. The highest powers of these factors are x^3, $(x+2)^2$, and $(x-3)^1$. Therefore,

$$\text{LCD} = x^3(x+2)^2(x-3)$$

The general steps for adding or subtracting rational expressions are illustrated in the following example.

Example 8.3.7

Subtract:

Solution:

Step 1: Factor all denominators to determine the LCD.

The LCD is $(x+1)(x+3)(x-5)$.

Step 2: Multiply by the appropriate factors to obtain equivalent terms with a common denominator. To do this, multiply the first term by $\frac{(x-5)}{(x-5)}$ and the second term by $\frac{(x+3)}{(x+3)}$.

$$= \frac{x}{(x+1)(x+3)} \cdot \frac{(x-5)}{(x-5)} - \frac{3}{(x+1)(x-5)} \cdot \frac{(x+3)}{(x+3)}$$
$$= \frac{x(x-5)}{(x+1)(x+3)(x-5)} - \frac{3(x+3)}{(x+1)(x+3)(x-5)}$$

Step 3: Add or subtract the numerators and place the result over the common denominator.

$$= \frac{x(x-5)-3(x+3)}{(x+1)(x+3)(x-5)}$$

Step 4: Simplify the resulting algebraic fraction.

Answer:

$$\frac{(x-9)}{(x+3)(x-5)}$$

Example 8.3.8

Subtract:

Solution:

It is best not to factor the numerator, $x^2-9x+18$, because we will most likely need to simplify after we subtract.

Answer:

$$\frac{18}{(x-4)(x-9)}$$

Example 8.3.9

Subtract:

$$\frac{1}{x^2-4} - \frac{1}{2-x}$$

Solution:

First, factor the denominators and determine the LCD. Notice how the opposite binomial property is applied to obtain a more workable denominator.

$$\frac{1}{x^2-4}-\frac{1}{2-x}=\frac{1}{(x+2)(x-2)}-\frac{1}{-1(x-2)}$$
$$=\frac{1}{(x+2)(x-2)}+\frac{1}{(x-2)}$$

The LCD is $(x+2)(x-2)$. Multiply the second term by 1 in the form of $\frac{(x+2)}{(x+2)}$.

$$=\frac{1}{(x+2)(x-2)}+\frac{1}{(x-2)}\cdot\frac{(x+2)}{(x+2)}$$
$$=\frac{1}{(x+2)(x-2)}+\frac{x+2}{(x-2)(x+2)}$$

Now that we have equivalent terms with a common denominator, add the numerators and write the result over the common denominator.

$$=\frac{1}{(x+2)(x-2)}+\frac{x+2}{(x-2)(x+2)}$$
$$=\frac{1+x+2}{(x+2)(x-2)}$$
$$=\frac{x+3}{(x+2)(x-2)}$$

Answer:

$$\frac{x+3}{(x+2)(x-2)}$$

Example 8.3.10

Simplify:

$$\frac{y-1}{y+1}-\frac{y+1}{y-1}+\frac{y^2-5}{y^2-1}$$

Solution:

Begin by factoring the denominator.

$$\frac{y-1}{y+1}-\frac{y+1}{y-1}+\frac{y^2-5}{y^2-1}$$
$$=\frac{y-1}{y+1}-\frac{y+1}{y-1}+\frac{y^2-5}{(y+1)(y-1)}$$

We can see that the LCD is $(y+1)(y-1)$. Find equivalent fractions with this denominator.

Next, subtract and add the numerators and place the result over the common denominator.

Finish by simplifying the resulting rational expression.

Answer:

$\frac{y-5}{y-1}$

Exercise 8.3.2

Simplify:

$$-\frac{2x^2-1+x}{1+x}-\frac{5}{1-x}$$

Answer

$$\frac{x+3}{x-1}$$

Rational expressions are sometimes expressed using negative exponents. In this case, apply the rules for negative exponents before simplifying the expression.

Example 8.3.11

Simplify:

$$y^{-2}+(y-1)^{-1}$$

Solution:

Recall that $x^{-n}=\frac{1}{x^n}$. We begin by rewriting the negative exponents as rational expressions.

$$\begin{aligned} & y^{-2}+(y-1)^{-1} \\ =&\frac{1}{y^2}+\frac{1}{(y-1)^1} && \textit{Replace negative exponents.} \\ =&\frac{1}{y^2}\cdot\frac{(y-1)}{(y-1)}+\frac{1}{(y-1)}\cdot\frac{y^2}{y^2} && \textit{Multiply by factors to obtain equivalent} \\ & && \textit{expressions with a common denominator.} \\ =&\frac{(y-1)}{y^2(y-1)}+\frac{y^2}{y^2(y-1)} \\ =&\frac{(y-1)+y^2}{y^2(y-1)} && \textit{Add and simplify.} \\ =&\frac{y^2+y-1}{y^2(y-1)} && \textit{The trinomial does not factor.} \end{aligned}$$

Answer:

$\frac{y^2+y-1}{y^2(y-1)}$

Adding and Subtracting Rational Functions

We can simplify sums or differences of rational functions using the techniques learned in this section. The restrictions of the result consist of the restrictions to the domains of each function.

Example 8.3.12

Calculate $(f+g)(x)$, given $f(x)=\frac{1}{x+3}$ and $g(x)=\frac{1}{x-2}$, and state the restrictions.

Solution:

$$\begin{aligned} (f+g)(x) &= f(x)+g(x) \\ &= \frac{1}{x+3}+\frac{1}{x-2} \\ &= \frac{1}{x+3}\cdot\frac{(x-2)}{(x-2)}+\frac{1}{x-2}\cdot\frac{(x+3)}{(x+3)} \\ &= \frac{x-2}{(x+3)(x-2)}+\frac{x+3}{(x-2)(x+3)} \\ &= \frac{x-2+x+3}{(x+3)(x-2)} \\ &= \frac{2x+1}{(x+3)(x-2)} \end{aligned}$$

Here the domain of f consists of all real numbers except -3, and the domain of g consists of all real numbers except 2. Therefore, the domain of $f+g$ consists of all real numbers except -3 and 2.

Answer:

$\frac{2x+1}{(x+3)(x-2)}$, where $x\neq -3, 2$

Example 8.3.13

Calculate $(f-g)(x)$, given $f(x)=\frac{x(x-1)}{x^2-25}$ and $g(x)=\frac{x-3}{x-5}$, and state the restrictions to the domain.

Solution:

$$\begin{aligned}(f-g)(x) &= f(x)-g(x)\\ &= \frac{x(x-1)}{x^2-25}-\frac{x-3}{x-5}\\ &= \frac{x(x-1)}{(x+5)(x-5)}-\frac{(x-3)}{(x-5)}\cdot\frac{(x+5)}{(x+5)}\\ &= \frac{x(x-1)-(x-3)(x+5)}{(x+5)(x-5)}\\ &= \frac{x^2-x-(x^2+5x-3x-15)}{(x+5)(x-5)}\\ &= \frac{x^2-x-(x^2+2x-15)}{(x+5)(x-5)}\\ &= \frac{x^2-x-x^2-2x+15}{(x+5)(x-5)}\\ &= \frac{-3x+15}{(x+5)(x-5)}\\ &= \frac{-3\ \cancel{(x-5)}}{(x+5)\ \cancel{(x-5)}}\\ &= \frac{-3}{x+5}\end{aligned}$$

The domain of f consists of all real numbers except 5 and -5, and the domain of g consists of all real numbers except 5. Therefore, the domain of $f-g$ consists of all real numbers except -5 and 5.

Answer:

$\frac{-3}{x+5}$, where $x\neq\pm5$

Key Takeaways

- When adding or subtracting rational expressions with a common denominator, add or subtract the expressions in the numerator and write the result over the common denominator.
- To find equivalent rational expressions with a common denominator, first factor all denominators and determine the least common multiple. Then multiply numerator and denominator of each term by the appropriate factor to obtain a common denominator. Finally, add or subtract the expressions in the numerator and write the result over the common denominator.
- The restrictions to the domain of a sum or difference of rational functions consist of the restrictions to the domains of each function.

Exercise 8.3.3 Adding and Subtracting with Common Denominators

Simplify. (Assume all denominators are nonzero.)

1. $\frac{3}{x}+\frac{7}{x}$
2. $\frac{9}{x}-\frac{10}{x}$
3. $\frac{x}{y}-\frac{3}{y}$
4. $\frac{4}{x-3}+\frac{6}{x-3}$
5. $\frac{7}{2x-1}-\frac{x}{2x-1}$
6. $\frac{8}{3x-8}-\frac{3x}{3x-8}$
7. $\frac{2}{x-9}+\frac{x-11}{x-9}$

8. $\frac{y+2}{2y+3}-\frac{y+3}{2y+3}$
9. $\frac{2x-3}{4x-1}-\frac{x-4}{4x-1}$
10. $\frac{2x}{x-1}-\frac{3x+4}{x-1}+\frac{x-2}{x-1}$
11. $\frac{1}{3y}-\frac{2y-9}{3y}-\frac{13-5y}{3y}$
12. $\frac{-3y+2}{5y-10}+\frac{y+7}{5y-10}-\frac{3y+4}{5y-10}$
13. $\frac{x}{(x+1)(x-3)}-\frac{3}{(x+1)(x-3)}$
14. $\frac{3x+5}{(2x-1)(x-6)}-\frac{x+6}{(2x-1)(x-6)}$
15. $\frac{x}{x^2-36}+\frac{6}{x^2-36}$
16. $\frac{x}{x^2-81}-\frac{9}{x^2-81}$
17. $\frac{x^2+2}{x^2+3x-28}+\frac{x-2}{2x^2+3x-28}$
18. $\frac{x^2}{x^2-x-3}-\frac{3-x^2}{x^2-x-3}$

Answer

1. $\frac{10}{x}$
3. $\frac{x-3}{y}$
5. $\frac{7-x}{2x-1}$
7. 1
9. $\frac{x+1}{4x-1}$
11. $\frac{y-1}{y}$
13. $\frac{1}{x+1}$
15. $\frac{1}{x-6}$
17. $\frac{x+5}{x+7}$

? Exercise 8.3.4 Adding and Subtracting with Unlike Denominators

Simplify. (Assume all denominators are nonzero.)

1. $\frac{1}{2}+\frac{1}{3x}$
2. $\frac{1}{5x^2}-\frac{1}{x}$
3. $\frac{1}{12y^2}+\frac{3}{10y^3}$
4. $\frac{1}{x}-\frac{1}{2y}$
5. $\frac{1}{y}-2$
6. $\frac{3}{y+2}-4$
7. $\frac{2}{x+4}+2$
8. $\frac{2}{y}-\frac{1}{y^2}$
9. $\frac{3}{x+1}+\frac{1}{x}$
10. $\frac{1}{x-1}-\frac{2}{x}$
11. $\frac{1}{x-3}+\frac{1}{x+5}$
12. $\frac{1}{x+2}-\frac{1}{x-3}$
13. $\frac{x}{x+1}-\frac{2}{x-2}$
14. $\frac{2x-3}{x+5}-\frac{x}{x-3}$
15. $\frac{y+1}{y-1}+\frac{y-1}{y+1}$

16. $\frac{3y-1}{3y}-\frac{y+4}{y-2}$
17. $\frac{2x-5}{2x+5}-\frac{2x+5}{2x-5}$
18. $\frac{2}{2x-1}-\frac{2x+1}{1-2x}$
19. $\frac{3x+4}{x-8}-\frac{2}{8-x}$
20. $\frac{1}{y-1}+\frac{1}{1-y}$
21. $\frac{2x^2}{x^2-9}+\frac{x+15}{9-x^2}$
22. $\frac{x}{x+3}+\frac{1}{x-3}-\frac{1}{5}-\frac{x}{(x+3)(x-3)}$
23. $\frac{2x}{3x-1}-\frac{1}{3x+1}+\frac{2(x-1)}{(3x-1)(3x+1)}$
24. $\frac{4x}{2x+1}-\frac{x}{x-5}+\frac{16x-3}{(2x+1)(x-5)}$
25. $\frac{x}{3x}+\frac{2}{x-2}+\frac{4}{3x(x-2)}$
26. $-\frac{2x}{x+6}-\frac{3x}{6-x}-\frac{18(x-2)}{(x+6)(x-6)}$
27. $\frac{x}{x+5}-\frac{1}{x-7}-\frac{25-7x}{(x+5)(x-7)}$
28. $\frac{x}{x^2}-\frac{2}{x-3}+\frac{2}{x-3}$
29. $\frac{1}{x+5}-\frac{x^2}{x^2-25}$
30. $\frac{5x-2}{x^2-4}-\frac{2}{x-2}$
31. $\frac{1}{x+1}-\frac{6x-3}{x^2-7x-8}$
32. $\frac{3x}{9x^2-16}-\frac{1}{3x+4}$
33. $\frac{2x}{x^2-1}+\frac{1}{x^2+x}$
34. $\frac{x(4x-1)}{2x^2}+\frac{7}{x-4}-\frac{x}{4+x}$
35. $\frac{3x^2}{3x^2+5x-2}-\frac{2x}{3x-1}$
36. $\frac{2x}{x-4}-\frac{11x+4}{x^2-2x-8}$
37. $\frac{x}{2x+1}+\frac{6x-24}{2x^2-7x-4}$
38. $\frac{1}{x^2-x-6}+\frac{1}{x^2-3x-10}$
39. $\frac{x}{x^2+4x+3}-\frac{3}{x^2-4x-5}$
40. $\frac{y+1}{2y^2+5y-3}-\frac{y}{4y^2-1}$
41. $\frac{y-1}{y^2-25}-\frac{2}{y^2-10y+25}$
42. $\frac{3x^2+2}{4x^2-2x-8}-\frac{1}{2x-4}$
43. $\frac{4x^2+2}{8x^2-6x-7}-\frac{2}{8x-7}$
44. $\frac{a}{4-a+a^2}-\frac{9a+18}{a^2-13a+36}$
45. $\frac{3a-12}{a^2-8a+16}-\frac{a+2}{4-a}$
46. $\frac{a^2-14}{2a^2-7a-4}-\frac{5}{1+2a}$
47. $\frac{1}{x+3}-\frac{x}{x^2-6x+9}+\frac{3}{x^2-9}$
48. $\frac{3x}{x+7}-\frac{2x}{x-2}+\frac{23x-10}{x^2+5x-14}$
49. $\frac{x+3}{x-1}+\frac{x-1}{x+2}-\frac{x(x+11)}{x^2+x-2}$
50. $-\frac{2x}{3x+1}-\frac{4}{x-2}+\frac{4(x+5)}{3x^2-5x-2}$
51. $\frac{x-1}{4x-1}-\frac{x+3}{2x+3}-\frac{3(x+5)}{8x^2+10x-3}$
52. $\frac{3x}{2x-3}-\frac{2}{2x+3}-\frac{6x^2-5x-9}{4x^2-9}$
53. $\frac{1}{y+1}+\frac{1}{y}+\frac{2}{y^2-1}$
54. $\frac{1}{y}-\frac{1}{y+1}+\frac{1}{y-1}$
55. $5^{-2}+2^{-1}$
56. $6^{-1}+4^{-2}$
57. $x^{-1}+y^{-1}$

58. $x^{-2}-y^{-1}$
59. $(2x-1)^{-1}-x^{-2}$
60. $(x-4)^{-1}-(x+1)^{-1}$
61. $3x^2(x-1)^{-1}-2x$
62. $2(y-1)^{-2}-(y-1)^{-1}$

Answer

1. $\frac{3x+2}{6x}$
3. $\frac{5y+18}{60y^3}$
5. $\frac{1-2y}{y}$
7. $\frac{2(x+5)}{x+4}$
9. $\frac{4x+1}{x(x+1)}$
11. $\frac{2(x+1)}{(x-3)(x+5)}$
13. $\frac{x^2-4x-2}{(x-2)(x+1)}$
15. $\frac{2(y2+1)}{(y+1)(y-1)}$
17. $-\frac{40x}{(2x+5)(2x-5)}$
19. $\frac{3(x+2)}{x-8}$
21. $\frac{2x+5}{x+3}$
23. $\frac{2x+1}{3x+1}$
25. $\frac{x^2+4x+4}{3x(x-2)}$
27. $\frac{x-6}{x-7}$
29. $\frac{x-5-x^2}{(x+5)(x-5)}$
31. $-\frac{5}{x-8}$
33. $\frac{2x-1}{x(x-1)}$
35. $\frac{x(x-4)}{(x+2)(3x-1)}$
37. $\frac{x+6}{2x+1}$
39. $\frac{x-9}{(x-5)(x+3)}$
41. $\frac{y^2-8y-5}{(y+5)(y-5)^2}$
43. $\frac{4x}{x+1}$
45. $\frac{a+5}{a-4}$
47. $-\frac{6x}{(x+3)(x-3)^2}$
49. $\frac{x-7}{x+2}$
51. $-\frac{x-5}{4x-1}$

53. $\frac{2y-1}{y(y-1)}$

55. $\frac{27}{50}$

57. $\frac{x+y}{xy}$

59. $\frac{(x-1)^2}{x^2(2x-1)}$

61. $\frac{x(x+2)}{x-1}$

Exercise 8.3.5 Adding and Subtracting Rational Functions

Calculate $(f+g)(x)$ and $(f-g)(x)$ and state the restrictions to the domain.

1. $f(x)=\frac{1}{3x}$ and $g(x)=\frac{1}{x-2}$
2. $f(x)=\frac{1}{x-1}$ and $g(x)=\frac{1}{x+5}$
3. $f(x)=\frac{x}{x-4}$ and $g(x)=\frac{1}{4-x}$
4. $f(x)=\frac{x}{x-5}$ and $g(x)=\frac{1}{2x-3}$
5. $f(x)=\frac{x-1}{x^2-4}$ and $g(x)=\frac{4}{x^2-6x-16}$
6. $f(x)=\frac{5}{x+2}$ and $g(x)=\frac{3}{x+4}$

Answer

1. $(f+g)(x)=\frac{2(2x-1)}{3x(x-2)}; (f-g)(x)=-\frac{2(x+1)}{3x(x-2)}; x\neq 0,2$
3. $(f+g)(x)=\frac{x-1}{x-4}; (f-g)(x)=\frac{x+1}{x-4}; x\neq 4$
5. $(f+g)(x)=\frac{x(x-5)}{(x+2)(x-2)(x-8)}; (f-g)(x)=\frac{x^2-13x+16}{(x+2)(x-2)(x-8)}; x\neq -2,2,8$

Exercise 8.3.6 Adding and Subtracting Rational Functions

Calculate $(f+f)(x)$ and state the restrictions to the domain.

1. $f(x)=\frac{1}{x}$
2. $f(x)=\frac{1}{2x}$
3. $f(x)=\frac{x}{2x-1}$
4. $f(x)=\frac{1}{x+2}$

Answer

1. $(f+f)(x)=\frac{2}{x}; x\neq 0$
3. $(f+f)(x)=\frac{2x}{2x-1}; x\neq \frac{1}{2}$

Exercise 8.3.7 Discussion Board

1. Explain to a classmate why this is incorrect: $\frac{1}{x^2}+\frac{2}{x^2}=\frac{3}{2x^2}$.
2. Explain to a classmate how to find the common denominator when adding algebraic expressions. Give an example.

Answer

1. Answer may vary

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8.4: Solving Rational Equations

Learning Objectives

- Solve rational equations.
- Solve literal equations, or formulas, involving rational expressions.

Solving Rational Equations

A rational equation is an equation containing at least one rational expression. Rational expressions typically contain a variable in the denominator. For this reason, we will take care to ensure that the denominator is not 0 by making note of restrictions and checking our solutions.

Solve rational equations by clearing the fractions by multiplying both sides of the equation by the least common denominator (LCD).

Example 8.4.1

Solve:

$$\frac{5}{x}-\frac{1}{3}=\frac{1}{x}$$

Solution:

We first make a note that $x \neq 0$ and then multiply both sides by the LCD, $3x$:

Check your answer by substituting 12 for x to see if you obtain a true statement.

$$\begin{aligned}\frac{5}{x}-\frac{1}{3}&=\frac{1}{x}\\ \frac{5}{12}-\frac{1}{3}&=\frac{1}{12}\\ \frac{5}{12}-\frac{4}{12}&=\frac{1}{12}\\ \frac{1}{12}&=\frac{1}{12}\end{aligned}$$ ✓

Answer:

The solution is 12.

After multiplying both sides of the previous example by the LCD, we were left with a linear equation to solve. This is not always the case; sometimes we will be left with a quadratic equation.

Example 8.4.2

Solve:

$$2-\frac{1}{x(x+1)}=\frac{3}{x+1}$$

Solution:

In this example, there are two restrictions, $x \neq 0$ and $x \neq -1$. Begin by multiplying both sides by the LCD, $x(x+1)$.

After distributing and dividing out the common factors, a quadratic equation remains. To solve it, rewrite it in standard form, factor, and then set each factor equal to 0.

$$\begin{aligned}2x+1&=0 \quad \text{or} \quad x-1=0\\ 2x&=-1 \qquad\quad x=1\\ x&=-\tfrac{1}{2}\end{aligned}$$

Check to see if these values solve the original equation.

$$2-\frac{1}{x(x+1)}=\frac{3}{x+1}$$

$Check\ x=-\frac{1}{2}$	$Check\ x=1$
$2-\frac{1}{(-\frac{1}{2})((-\frac{1}{2})+1)}=\frac{3}{(-\frac{1}{2})+1}$	$2-\frac{1}{1(1+1)}=\frac{3}{1+1}$
$2-\frac{1}{(-\frac{1}{4})}=\frac{3}{\frac{1}{2}}$	$\frac{4}{2}-\frac{1}{2}=\frac{3}{2}$
$2+1\cdot\frac{4}{1}=3\cdot\frac{2}{1}$	$\frac{3}{2}=\frac{3}{2}$ ✓
$2+4=6$	
$6=6$ ✓	

Answer:

The solutions are $-\frac{1}{2}$ and 1.

Up to this point, all of the possible solutions have solved the original equation. However, this may not always be the case. Multiplying both sides of an equation by variable factors may lead to extraneous solutions, which are solutions that do not solve the original equation. A complete list of steps for solving a rational equation is outlined in the following example.

Example 8.4.3

Solve:

$$\frac{x}{x+2}+\frac{2}{x^2+5x+6}=\frac{5}{x+3}$$

Solution:

Step 1: Factor all denominators and determine the LCD.

$$\frac{x}{x+2}+\frac{2}{x^2+5x+6}=\frac{5}{x+3}$$
$$\frac{x}{(x+2)}+\frac{2}{(x+2)(x+3)}=\frac{5}{(x+3)}$$

The LCD is $(x+2)(x+3)$.

Step 2: Identify the restrictions. In this case, they are $x\neq -2$ and $x\neq -3$.

Step 3: Multiply both sides of the equation by the LCD. Distribute carefully and then simplify.

$$(x+2)(x+3)\cdot\left(\frac{x}{(x+2)}+\frac{2}{(x+2)(x+3)}\right)=(x+2)(x+3)\cdot\frac{5}{(x+3)}$$
$$\frac{x\cdot\cancel{(x+2)}\cancel{(x+3)}}{\cancel{(x+2)}}+\frac{2\cdot\cancel{(x+2)}\cancel{(x+3)}}{\cancel{(x+2)}\cancel{(x+3)}}=\frac{5\cdot(x+2)\cancel{(x+3)}}{\cancel{(x+3)}}$$
$$x(x+3)+2=5(x+2)$$
$$x^2+3x+2=5x+10$$

Step 4: Solve the resulting equation. Here the result is a quadratic equation. Rewrite it in standard form, factor, and then set each factor equal to 0.

$x+2=0$ or $x-4=0$

$x=-2$ $\quad$ $x=4$

Step 5: Check for extraneous solutions. Always substitute into the original equation, or the factored equivalent. In this case, choose the factored equivalent to check:

$$\frac{x}{(x+2)}+\frac{2}{(x+2)(x+3)}=\frac{5}{(x+3)}$$

$$\text{Check } x=-2 \qquad \text{Check } x=4$$

$$\frac{-2}{(-2+2)}+\frac{2}{(-2+2)(-2+3)}=\frac{5}{(-2+3)} \qquad \frac{4}{(4+2)}+\frac{2}{(4+2)(4+3)}=\frac{5}{(4+3)}$$

$$-\frac{2}{0}+\frac{2}{0(1)}=\frac{5}{1} \quad x \qquad \frac{4}{6}+\frac{2}{6\cdot 7}=\frac{5}{7}$$

$$\text{Undefined terms!} \qquad \frac{2}{3}+\frac{1}{21}=\frac{5}{7}$$

$$\frac{14}{21}+\frac{1}{21}=\frac{5}{7}$$

$$\frac{15}{21}=\frac{5}{7}$$

$$\frac{5}{7}=\frac{5}{7} \quad \checkmark$$

Here -2 is an extraneous solution and is not included in the solution set. It is important to note that -2 is a restriction.

Answer:

The solution is 4.

If this process produces a solution that happens to be a restriction, then disregard it as an extraneous solution.

? Exercise 8.4.1

Solve:

Answer

-3

Sometimes all potential solutions are extraneous, in which case we say that there is no solution to the original equation. In the next two examples, we demonstrate two ways in which a rational equation can have no solutions.

✓ Example 8.4.4

Solve:

$$\frac{3x}{x^2-4}-\frac{2}{x+2}=\frac{1}{x+2}$$

Solution:

To identify the LCD, first factor the denominators.

$$\frac{3x}{x^2-4}-\frac{2}{x+2}=\frac{1}{x+2}$$

$$\frac{3x}{(x+2)(x-2)}-\frac{2}{(x+2)}=\frac{1}{(x+2)}$$

Multiply both sides by the least common denominator (LCD), $(x+2)(x-2)$, distributing carefully.

The equation is a contradiction and thus has no solution.

Answer:

No solution, $\emptyset$

✓ Example 8.4.5

Solve:

$$\frac{x}{x-4}-\frac{4}{x+5}=\frac{36}{x^2+x-20}$$

Solution:

First, factor the denominators.

$$\frac{x}{(x-4)} - \frac{4}{(x+5)} = \frac{36}{(x-4)(x+5)}$$

Take note that the restrictions are $x \neq 4$ and $x \neq -5$. To clear the fractions, multiply by the LCD, $(x-4)(x+5)$.

$x-4=0$ or $x+5=0$
$x=4$ $x=-5$

Both of these values are restrictions of the original equation; hence both are extraneous.

Answer:

No solution, $\emptyset$

? Exercise 8.4.2

Solve:

Answer

$\emptyset$

It is important to point out that this technique for clearing algebraic fractions only works for equations. Do not try to clear algebraic fractions when simplifying expressions. As a reminder, we have

Expression	*Equation*
$\frac{1}{x} + \frac{x}{2x+1}$	$\frac{1}{x} + \frac{x}{2x+1} = 0$

Expressions are to be simplified and equations are to be solved. If we multiply the expression by the LCD, $x(2x+1)$, we obtain another expression that is not equivalent.

Incorrect	*Correct*
$\frac{1}{x} + \frac{x}{2x+1}$	$\frac{1}{x} + \frac{x}{2x+1} = 0$
$\neq x(2x+1)\cdot\left(\frac{1}{x} + \frac{x}{2x+1}\right)$	$x(2x+1)\cdot\left(\frac{1}{x} + \frac{x}{2x+1}\right) = x(2x+1)\cdot 0$
$= 2x+1+x^2$ x	$2x+1+x^2=0$
	$x^2+2x+1=0$ ✓

Literal Equations

Literal equations, or formulas, are often rational equations. Hence the techniques described in this section can be used to solve for particular variables. Assume that all variable expressions in the denominator are nonzero.

✓ Example 8.4.6

Solve for x:

$$x = \frac{x-5}{y}$$

Solution:

The goal is to isolate ***x***. Assuming that ***y*** is nonzero, multiply both sides by ***y*** and then add 5 to both sides.

Answer:

$x = yz+5$

✓ Example 8.4.7

Solve for c:

$$\frac{1}{c} = \frac{1}{a} + \frac{1}{b}$$

Solution:

In this example, the goal is to isolate c. We begin by multiplying both sides by the LCD, $a \cdot b \cdot c$, distributing carefully.

$$\frac{1}{c} = \frac{1}{a} + \frac{1}{b}$$
$$abc \cdot \frac{1}{c} = abc \cdot \frac{1}{a} + abc \cdot \frac{1}{b}$$
$$ab = bc + ac$$

On the right side of the equation, factor out c.

$$ab = c(b + a)$$

Next, divide both sides of the equation by the quantity $(b + a)$.

$$\frac{ab}{(b+a)} = \frac{c(b+a)}{(b+a)}$$
$$\frac{ab}{b+a} = c$$

Answer:

$$c = \frac{ab}{b+a}$$

Exercise 8.4.3

Solve for y:

$$x = \frac{y+1}{y-1}$$

Answer

$y = \frac{1+x}{x-1}$

Key Takeaways

- Begin solving rational equations by multiplying both sides by the LCD. The resulting equivalent equation can be solved using the techniques learned up to this point.
- Multiplying both sides of a rational equation by a variable expression introduces the possibility of extraneous solutions. Therefore, we must check the solutions against the set of restrictions. If a solution is a restriction, then it is not part of the domain and is extraneous.
- When multiplying both sides of an equation by an expression, distribute carefully and multiply each term by that expression.
- If all of the resulting solutions are extraneous, then the original equation has no solutions.

Exercise 8.4.4 Rational Equations

Solve.

1. $\frac{1}{2} + \frac{1}{x} = \frac{1}{8}$
2. $\frac{1}{3} - \frac{1}{x} = \frac{2}{9}$
3. $\frac{1}{3x} - \frac{2}{3} = \frac{1}{x}$
4. $\frac{2}{5x} - \frac{1}{x} = \frac{3}{10}$
5. $\frac{1}{2x+1} = 5$
6. $\frac{3}{3x-1} + 4 = 5$
7. $\frac{2x-3}{x+5} = \frac{2}{x+5}$
8. $\frac{5x}{2x-1} = \frac{x-1}{2x-1}$
9. $\frac{5}{x-7} = \frac{6}{x-9}$
10. $\frac{5}{x+5} = \frac{3}{x+1}$
11. $\frac{x}{6} - \frac{6}{x} = 0$
12. $\frac{5x+x}{5} = -2$

13. $\frac{x}{x+12}=\frac{2}{x}$
14. $\frac{2x}{x+5}=\frac{1}{6-x}$
15. $\frac{1}{x}+\frac{x}{2x+1}=0$
16. $\frac{9x}{3x-1}-\frac{4}{x}=0$
17. $1-\frac{2}{x}=\frac{48}{x^2}$
18. $2-\frac{9}{x}=\frac{5}{x^2}$
19. $1+\frac{12}{x}=\frac{12}{x-2}$
20. $1-\frac{3x-5}{x(3x-4)}=-\frac{1}{x}$
21. $\frac{x}{2}=\frac{14}{x+3}$
22. $\frac{3x}{2}=\frac{x+1}{3-x}$
23. $6=\frac{-3x+3}{x-1}$
24. $\frac{1}{2x-2}=2+\frac{6(4-x)}{x-2}$
25. $2+\frac{2x}{x-3}=\frac{3(x-1)}{x-3}$
26. $\frac{x}{x-1}+\frac{1}{6x-1}=\frac{x(x-1)}{(6x-1)}$
27. $\frac{12}{x^2-81}=\frac{1}{x+9}-\frac{2}{x-9}$
28. $\frac{14}{x^2-49}=\frac{2}{x-7}-\frac{3}{x+7}$
29. $\frac{6x}{x+3}+\frac{4}{x-3}=\frac{3x}{x^2-9}$
30. $\frac{3x}{x+2}-\frac{17}{x-2}=-\frac{48}{x^2-4}$
31. $x^{-1}+3=0$
32. $4^{-y}-1=0$
33. $y^{-2}-4=0$
34. $9x^{-2}-1=0$
35. $3(x-1)^{-1}+5=0$
36. $5-2(3x+1)^{-1}=0$
37. $3+2x^{-3}=2x^{-3}$
38. $\frac{1}{x}=\frac{1}{x+1}$
39. $\frac{x}{x+1}=\frac{x+1}{x}$
40. $\frac{3x-1}{3x}=\frac{x}{x+3}$
41. $\frac{4x-7}{x-5}=\frac{3x-2}{x-5}$
42. $\frac{x}{x^2-9}=\frac{1}{x-3}$
43. $\frac{3x+4}{x-8}-\frac{2}{8-x}=1$
44. $\frac{1}{x}=\frac{6}{x(x+3)}$
45. $\frac{3}{x}=\frac{1}{x+1}+\frac{13}{x(x+1)}$
46. $\frac{x}{x-1}-\frac{3}{4x-1}=\frac{9x}{(4x-1)(x-1)}$
47. $\frac{1}{x-4}+\frac{x}{x-2}=2x^2-6x+8$
48. $\frac{x}{x-5}+\frac{x-1}{x^2-11x+30}=\frac{5}{x-6}$
49. $\frac{x}{x+1}-\frac{6}{5x^2+4x-1}=-\frac{5}{5x-1}$
50. $\frac{-8x^2-4}{x-12}+\frac{2(x+2)}{x^2+4x-60}=\frac{1}{x+2}$
51. $\frac{x}{x+2}-\frac{20}{x^2-6-x}=\frac{-4}{x-3}$
52. $\frac{x+7}{x-1}+\frac{x-1}{x+1}=\frac{4}{x^2-1}$
53. $\frac{x-1}{x-3}+\frac{x-3}{x-1}=-\frac{x+5}{x-3}$
54. $\frac{x-2}{x-5}-\frac{x-5}{x-2}=\frac{8-x}{x-5}$
55. $\frac{x+7}{x-2}-\frac{81}{x^2+5x-14}=\frac{9}{x+7}$
56. $\frac{x}{x-6}+1=\frac{5x+30}{36-x^2}$
57. $\frac{2x}{x+1}-\frac{4}{4x-3}=-\frac{7}{4x^2+x-3}$

58. $\frac{x-5}{x-10}+\frac{5}{x-5}=-\frac{5x}{x^2-15x+50}$
59. $\frac{5}{x^2+5x+4}+\frac{x+1}{x^2+3x-4}=\frac{5}{x^2-1}$
60. $\frac{1}{x^2-2x-63}+\frac{x-9x^2+10}{x+21}=\frac{1}{x^2-6x-27}$
61. $\frac{4}{x^2-4}+\frac{2(x-2)}{x^2-4x-12}=\frac{x+2}{x^2-8x+12}$
62. $\frac{x+2}{x^2-5x+4}+\frac{x+2}{x^2+x-2}=\frac{x-1}{x^2-2x-8}$
63. $\frac{6x}{x-1}-\frac{11x+1}{2x^2-x-1}=\frac{6x}{2x+1}$
64. $\frac{8x}{2x-3}+\frac{4x^2}{x^2-7x+6}=\frac{1}{x-2}$

Answer

1. $-\frac{8}{3}$
3. -1
5. $-\frac{2}{5}$
7. $\frac{5}{2}$
9. -3
11. $-6, 6$
13. $-4, 6$
15. -1
17. $-6, 8$
19. $-4, 6$
21. $-7, 4$
23. $\emptyset$
25. $\emptyset$
27. -39
29. $\frac{4}{3}, \frac{3}{2}$
31. $-\frac{1}{3}$
33. $-\frac{1}{2}, \frac{1}{2}$
35. $\frac{2}{5}$
37. $\emptyset$
39. $-\frac{1}{2}$
41. $\emptyset$
43. -7
45. 5
47. -1
49. $\emptyset$
51. -4
53. $\frac{5}{3}$
55. $\emptyset$
57. $\frac{1}{2}$

59. $-6, 4$

61. 10

63. $\frac{1}{3}$

Exercise 8.4.5 Literal Equations

Solve for the indicated variable.

1. Solve for r: $t = \frac{D}{r}$
2. Solve for b: $h = \frac{2A}{b}$
3. Solve for P: $t = \frac{I}{Pr}$
4. Solve for π: $r = \frac{C}{2\pi}$
5. Solve for c: $\frac{1}{a} = \frac{1}{b} + \frac{1}{c}$
6. Solve for y: $m = \frac{y-y_1}{x-x_1}$
7. Solve for w: $P = 2(l+w)$
8. Solve for t: $A = P(1+rt)$
9. Solve for m: $s = \frac{1}{n+m}$
10. Solve for S: $h = S2\pi r - r$
11. Solve for x: $y = \frac{x}{x+2}$
12. Solve for x: $y = 2x + 15x$
13. Solve for R: $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$
14. Solve for S_1: $\frac{1}{f} = \frac{1}{S_1} + \frac{1}{S_2}$

Answer

1. $r = \frac{D}{t}$

3. $P = \frac{I}{tr}$

5. $c = \frac{ab}{b-a}$

7. $w = \frac{P}{2} - l$

9. $m = \frac{1}{s} - n$

11. $x = -\frac{2y}{y-1}$

13. $R = \frac{R_1 R_2}{R_1+R_2}$

Exercise 8.4.6 Discussion Board

1. Explain why multiplying both sides of an equation by the LCD sometimes produces extraneous solutions.
2. Explain the connection between the technique of cross multiplication and multiplying both sides of a rational equation by the LCD.
3. Explain how we can tell the difference between a rational expression and a rational equation. How do we treat them differently?

Answer

1. Answers may vary

3. Answers may vary

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CHAPTER OVERVIEW

9: Roots and Radicals

9.1: Simplify and Use Square Roots

9.1E: Exercises

9.2: Simplify Square Roots

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9.3: Add and Subtract Square Roots

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9.1: Simplify and Use Square Roots

Learning Objectives

By the end of this section, you will be able to:

- Simplify expressions with square roots
- Estimate square roots
- Approximate square roots
- Simplify variable expressions with square roots

BE PREPARED

Before you get started, take this readiness quiz.

1. Simplify: ⓐ 9^2 ⓑ $(-9)^2$ ⓒ -9^2.
   If you missed this problem, review [link].
2. Round 3.846 to the nearest hundredth.
   If you missed this problem, review [link].
3. For each number, identify whether it is a real number or not a real number:
   ⓐ $-\sqrt{100}$ ⓑ $\sqrt{-100}$.
   If you missed this problem, review [link].

Simplify Expressions with Square Roots

Remember that when a number n is multiplied by itself, we write n^2 and read it "n squared." For example, 15^2 reads as "15 squared," and 225 is called the square of 15, since $15^2 = 225$.

Definition: SQUARE OF A NUMBER

If $n^2 = m$, then m is the square of n.

Sometimes we will need to look at the relationship between numbers and their squares in reverse. Because 225 is the square of 15, we can also say that 15 is a square root of 225. A number whose square is m is called a *square root* of m.

Definition: SQUARE ROOT OF A NUMBER

If $n^2 = m$, then n is a square root of m.

Notice $(-15)^2 = 225$ also, so −15 is also a square root of 225. Therefore, both 15 and −15 are square roots of 225.

So, every positive number has two square roots—one positive and one negative. What if we only wanted the positive square root of a positive number? The *radical sign*, $\sqrt{m}$, denotes the positive square root. The positive square root is also called the principal square root.

We also use the radical sign for the square root of zero. Because $0^2 = 0$, $\sqrt{0} = 0$. Notice that zero has only one square root.

Definition: SQUARE ROOT NOTATION



$\sqrt{m}$ is read as "the square root of m."

If $m = n^2$, then $\sqrt{m} = n$, for $n \geq 0$.

The square root of m, $\sqrt{m}$, is the positive number whose square is m.

Since 15 is the positive square root of 225, we write $\sqrt{225}=15$. Fill in Figure to make a table of square roots you can refer to as you work this chapter.

$\sqrt{1}$	$\sqrt{4}$	$\sqrt{9}$	$\sqrt{16}$	$\sqrt{25}$	$\sqrt{36}$	$\sqrt{49}$	$\sqrt{64}$	$\sqrt{81}$	$\sqrt{100}$	$\sqrt{121}$	$\sqrt{144}$	$\sqrt{169}$	$\sqrt{196}$	$\sqrt{225}$
														15

We know that every positive number has two square roots and the radical sign indicates the positive one. We write $\sqrt{225}=15$. If we want to find the negative square root of a number, we place a negative in front of the radical sign. For example, $-\sqrt{225}=-15$.

Example 9.1.1

Simplify:

1. $\sqrt{36}$
2. $\sqrt{196}$
3. $-\sqrt{81}$
4. $-\sqrt{289}$.

Answer

1.

$$\begin{array}{ll} & \sqrt{36} \\ \text{Since} 6^2=36 & 6 \end{array} \tag{9.1.1}$$

2.

$$\begin{array}{ll} & \sqrt{196} \\ \text{Since} 14^2=196 & 14 \end{array} \tag{9.1.2}$$

3.

$$\begin{array}{ll} & -\sqrt{81} \\ \text{The negative is in front of the radical sign} & -9 \end{array} \tag{9.1.3}$$

4.

$$\begin{array}{ll} & -\sqrt{289} \\ \text{The negative is in front of the radical sign} & -17 \end{array} \tag{9.1.4}$$

Example 9.1.2

Simplify:

1. $-\sqrt{49}$
2. $\sqrt{225}$.

Answer

1. -7
2. 15

Example 9.1.3

implify:

1. $\sqrt{64}$
2. $-\sqrt{121}$.

Answer

1. 8
2. −11

Example 9.1.4

Simplify:

1. $\sqrt{-169}$
2. $-\sqrt{64}$

Answer

1.

$$\begin{array}{ll} & \sqrt{-169} \\ \text{There is no real number whose square is} s-169 & \sqrt{-169}\text{is not a real number.} \end{array} \tag{9.1.5}$$

2.

$$\begin{array}{ll} & -\sqrt{64} \\ \text{The negative is in front of the radical sign} & -8 \end{array} \tag{9.1.6}$$

Example 9.1.5

Simplify:

1. $\sqrt{-196}$
2. $-\sqrt{81}$.

Answer

1. not a real number
2. −9

Example 9.1.6

Simplify:

1. $-\sqrt{49}$
2. $\sqrt{-121}$.

Answer

1. −7
2. not a real number

When using the order of operations to simplify an expression that has square roots, we treat the radical as a grouping symbol.

Example 9.1.7

Simplify:

1. $\sqrt{25}+\sqrt{144}$
2. $\sqrt{25+144}$.

Answer

1.

$$\begin{array}{ll} & \sqrt{25}+\sqrt{144} \\ \text{Use the order of operations} & 5+12 \\ \text{Simplify.} & 17 \end{array} \tag{9.1.7}$$

2.

$$\begin{array}{ll} & \sqrt{25+144} \\ \text{Simplify under the radical sign.} & \sqrt{169} \\ \text{Simplify.} & 13 \end{array} \tag{9.1.8}$$

Notice the different answers in parts 1 and 2!

Example 9.1.8

Simplify:

1. $\sqrt{9}+\sqrt{16}$
2. $\sqrt{9+16}$.

Answer

1. 7
2. 5

Example 9.1.9

Simplify:

1. $\sqrt{64+225}$
2. $\sqrt{64}+\sqrt{225}$.

Answer

1. 17
2. 23

Estimate Square Roots

So far we have only considered square roots of perfect square numbers. The square roots of other numbers are not whole numbers. Look at Table below.

Number	Square Root
4	$\sqrt{4}=2$
5	$\sqrt{5}$

Number	Square Root
6	$\sqrt{6}$
7	$\sqrt{7}$
8	$\sqrt{8}$
9	$\sqrt{9} = 3$

The square roots of numbers between 4 and 9 must be between the two consecutive whole numbers 2 and 3, and they are not whole numbers. Based on the pattern in the table above, we could say that $\sqrt{5}$ must be between 2 and 3. Using inequality symbols, we write:

$2 < \sqrt{5} < 3$

Example 9.1.10

Estimate $\sqrt{60}$ between two consecutive whole numbers.

Answer

Think of the perfect square numbers closest to 60. Make a small table of these perfect squares and their squares roots.

Number	Square root
36	6
49	7
64	8
81	9

60 $\sqrt{60}$

Locate 60 between two consecutive perfect squares.	$49 < 60 < 64$
$\sqrt{60}$ is between their square roots.	$7 < \sqrt{60} < 8$

Example 9.1.11

Estimate the square root $\sqrt{38}$ between two consecutive whole numbers.

Answer

$6 < \sqrt{38} < 7$

Example 9.1.12

Estimate the square root $\sqrt{84}$ between two consecutive whole numbers.

Answer

$9 < \sqrt{84} < 10$

Approximate Square Roots

There are mathematical methods to approximate square roots, but nowadays most people use a calculator to find them. Find the $\sqrt{x}$ key on your calculator. You will use this key to approximate square roots.

When you use your calculator to find the square root of a number that is not a perfect square, the answer that you see is not the exact square root. It is an approximation, accurate to the number of digits shown on your calculator's display. The symbol for an approximation is $\approx$ and it is read 'approximately.'

Suppose your calculator has a 10-digit display. You would see that

$\sqrt{5} \approx 2.236067978$

If we wanted to round $\sqrt{5}$ to two decimal places, we would say

$\sqrt{5} \approx 2.24$

How do we know these values are approximations and not the exact values? Look at what happens when we square them:

$$\begin{aligned} (2.236067978)^2 &= 5.000000002 \\ (2.24)^2 &= 5.0176 \end{aligned} \tag{9.1.9}$$

Their squares are close to 5, but are not exactly equal to 5.

Using the square root key on a calculator and then rounding to two decimal places, we can find:

$$\begin{aligned} \sqrt{4} &= 2 \\ \sqrt{5} &\approx 2.24 \\ \sqrt{6} &\approx 2.45 \\ \sqrt{7} &\approx 2.65 \\ \sqrt{8} &\approx 2.83 \\ \sqrt{9} &= 3 \end{aligned} \tag{9.1.10}$$

Example 9.1.13

Round $\sqrt{17}$ to two decimal places.

Answer

$$\begin{array}{ll} & \sqrt{17} \\ \text{Use the calculator square root key.} & 4.123105626\ldots \\ \text{Round to two decimal places.} & 4.12 \\ & \sqrt{17} \approx 4.12 \end{array} \tag{9.1.11}$$

Example 9.1.14

Round $\sqrt{11}$ to two decimal places.

Answer

≈ 3.32

Example 9.1.15

Round $\sqrt{13}$ to two decimal places.

Answer

≈ 3.61

Simplify Variable Expressions with Square Roots

What if we have to find a square root of an expression with a variable? Consider $\sqrt{9x^2}$. Can you think of an expression whose square is $9x^2$?

$$\begin{aligned} (?)^2 &= 9x^2 \\ (3x)^2 &= 9x^2 \quad \text{so} \sqrt{9x^2} = 3x \end{aligned} \tag{9.1.12}$$

When we use the radical sign to take the square root of a variable expression, we should specify that x≥0x≥0 to make sure we get the *principal square root*.

However, in this chapter we will assume that each variable in a square-root expression represents a non-negative number and so we will not write $x \geq 0$ next to every radical.

What about square roots of higher powers of variables? Think about the Power Property of Exponents we used in Chapter 6.

$(a^m)^n = a^{m \cdot n}$

If we square a^m, the exponent will become 2m.

$(a^m)^2 = a^{2m}$

How does this help us take square roots? Let's look at a few:

$$\begin{aligned} \sqrt{25u^8} &= 5u^4 \quad \text{Because} (5u^4)^2 = 25u^8 \\ \sqrt{16r^{20}} &= 4r^{10} \quad \text{Because} (4r^{10})^2 = 16r^{20} \\ \sqrt{196q^{36}} &= 14q^{18} \quad \text{Because} (14r^{18})^2 = 196q^{36} \end{aligned} \tag{9.1.13}$$

? Example 9.1.16

Simplify:

1. $\sqrt{x^6}$
2. $\sqrt{y^{16}}$

Answer

1.

$$\begin{aligned} & \sqrt{x^6} \\ \text{Since} (x^3)^2 = x^6 \quad & x^3 \end{aligned} \tag{9.1.14}$$

2.

$$\begin{aligned} & \sqrt{y^{16}} \\ \text{Since} (y^8)^2 = y^{16} \quad & y^8 \end{aligned} \tag{9.1.15}$$

? Example 9.1.17

Simplify:

1. $\sqrt{y^8}$
2. $\sqrt{z^{12}}$.

Answer

1. y^4
2. z^6

? Example 9.1.18

Simplify:

1. $\sqrt{m^4}$
2. $\sqrt{b^{10}}$.

Answer

1. m^2
2. b^5

Example 9.1.19

Simplify: $\sqrt{16n^2}$

Answer

$$\begin{array}{cc} & \sqrt{16n^2} \\ \text{Since} (4n)^2 = 16n^2 & 4n \end{array} \tag{9.1.16}$$

Example 9.1.20

Simplify: $\sqrt{64x^2}$.

Answer

$8x$

Example 9.1.21

Simplify: $\sqrt{169y^2}$.

Answer

$13y$

Example 9.1.22

Simplify: $-\sqrt{81c^2}$.

Answer

$$\begin{array}{cc} & -\sqrt{81c^2} \\ \text{Since} (9c)^2 = 81c^2 & -9c \end{array} \tag{9.1.17}$$

Example 9.1.23

Simplify: $-\sqrt{121y^2}$.

Answer

$-11y$

Example 9.1.24

Simplify: $-\sqrt{100p^2}$.

Answer

$-10p$

Example 9.1.25

Simplify: $\sqrt{36x^2y^2}$.

Answer

$$\text{Since}(6xy)^2 = \sqrt{36x^2y^2} \quad \begin{matrix}\sqrt{36x^2y^2} \\ 6xy\end{matrix} \tag{9.1.18}$$

Example 9.1.26

Simplify: $\sqrt{100a^2b^2}$.

Answer

10ab

Example 9.1.27

Simplify: $\sqrt{225m^2n^2}$.

Answer

15mn

Example 9.1.28

Simplify: $\sqrt{64p^{64}}$.

Answer

$$\text{Since}(8p^8)^2 = \sqrt{64p^{64}} \quad \begin{matrix}\sqrt{64p^{64}} \\ 8p^8\end{matrix} \tag{9.1.19}$$

Example 9.1.29

Simplify: $\sqrt{49x^{30}}$.

Answer

$7x^{15}$

Example 9.1.30

Simplify: $\sqrt{81w^{36}}$

Answer

$9w^{18}$

Example 9.1.31

Simplify: $\sqrt{121a^6b^8}$

Answer

$$\text{Since}(11a^3b^4)^2 = \sqrt{121a^6b^8} \quad \begin{matrix}\sqrt{121a^6b^8} \\ 11a^3b^4\end{matrix} \tag{9.1.20}$$

? Example 9.1.32

Simplify: $\sqrt{169x^{10}y^{14}}$

Answer

$13x^5y^7$

? Example 9.1.33

Simplify: $\sqrt{144p^{12}q^{20}}$

Answer

$\sqrt{12p^6q^{10}}$

Access this online resource for additional instruction and practice with square roots.

- Square Roots

Key Concepts

- Note that the square root of a negative number is not a real number.
- Every positive number has two square roots, one positive and one negative. The positive square root of a positive number is the principal square root.
- We can estimate square roots using nearby perfect squares.
- We can approximate square roots using a calculator.
- When we use the radical sign to take the square root of a variable expression, we should specify that $x \geq 0$ to make sure we get the principal square root.

Glossary

square of a number

- If $n^2 = m$, then m is the square of n

square root of a number

- If $n^2 = m$, then n is a square root of m

square root notation

- If $m = n^2$, then $\sqrt{m} = n$. We read $\sqrt{m}$ as 'the square root of m.'

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9.1E: Exercises

Practice Makes Perfect

Simplify Expressions with Square Roots

In the following exercises, simplify.

Example 9.1E.34

$\sqrt{36}$

Answer

6

Example 9.1E.35

$\sqrt{4}$

Example 9.1E.36

$\sqrt{64}$

Answer

8

Example 9.1E.37

$\sqrt{169}$

Example 9.1E.38

$\sqrt{9}$

Answer

3

Example 9.1E.39

$\sqrt{16}$

Example 9.1E.40

$\sqrt{100}$

Answer

10

Example 9.1E.41

$\sqrt{144}$

Example 9.1E.42

$-\sqrt{4}$

Answer

-2

Example 9.1E.43

$-\sqrt{100}$

Example 9.1E.44

$-\sqrt{1}$

Answer

-1

Example 9.1E.45

$-\sqrt{121}$

Example 9.1E.46

$\sqrt{-121}$

Answer

not a real number

Example 9.1E.47

$\sqrt{-36}$

Example 9.1E.48

$\sqrt{-9}$

Answer

not a real number

Example 9.1E.49

$\sqrt{-49}$

Example 9.1E.50

$\sqrt{9+16}$

Answer

5

Example 9.1E.51

$\sqrt{25+144}$

Example 9.1E.52

$\sqrt{9}+\sqrt{16}$

Answer

7

Example 9.1E.53

$\sqrt{25}+\sqrt{144}$

Estimate Square Roots

In the following exercises, estimate each square root between two consecutive whole numbers.

Example 9.1E.54

$\sqrt{70}$

Answer

$8<\sqrt{70}<9$

Example 9.1E.55

$\sqrt{55}$

Example 9.1E.56

$\sqrt{200}$

Answer

$14<\sqrt{200}<15$

Example 9.1E.57

$\sqrt{172}$

Approximate Square Roots

In the following exercises, approximate each square root and round to two decimal places.

Example 9.1E.58

$\sqrt{19}$

Answer

4.36

Example 9.1E.59

$\sqrt{21}$

Example 9.1E.60

$\sqrt{53}$

Answer

7.28

Example 9.1E.61

$\sqrt{47}$

Simplify Variable Expressions with Square Roots

In the following exercises, simplify.

Example 9.1E.62

$\sqrt{y^2}$

Answer

y

Example 9.1E.63

$\sqrt{b^2}$

Example 9.1E.64

$\sqrt{a^{14}}$

Answer

a^7

Example 9.1E.65

$\sqrt{w^{24}}$

Example 9.1E.66

$\sqrt{49x^2}$

Answer

$7x$

Example 9.1E.67

$\sqrt{100y^2}$

Example 9.1E.68

$\sqrt{121m^{20}}$

Answer

$11m^{10}$

Example 9.1E.69

$25h^{44}$

Example 9.1E.70

$\sqrt{81x^{36}}$

Answer

$9x^{18}$

Example 9.1E.71

$\sqrt{144z^{84}}$

Example 9.1E.72

$-\sqrt{81x^{18}}$

Answer

$-9x^{9}$

Example 9.1E.73

$-\sqrt{100m^{32}}$

Example 9.1E.74

$-\sqrt{64a^2}$

Answer

$-8a$

Example 9.1E.75

$-\sqrt{25x^2}$

Example 9.1E.76

$\sqrt{144x^2y^2}$

Answer

$12xy$

https://math.libretexts.org/@go/page/61441

Example 9.1E.77

$\sqrt{196a^2b^2}$

Example 9.1E.78

$\sqrt{169w^8y^{10}}$

Answer

$13w^4y^5$

Example 9.1E.79

$\sqrt{81p^{24}q^6}$

Example 9.1E.80

$\sqrt{9c^8d^{12}}$

Answer

$3c^4d^6$

Examlpe 9.1E.81

$\sqrt{36r^6s^{20}}$

Everyday Math

Example 9.1E.82

Decorating Denise wants to have a square accent of designer tiles in her new shower. She can afford to buy 625 square centimeters of the designer tiles. How long can a side of the accent be?

Answer

25 centimeters

Example 9.1E.83

Decorating Morris wants to have a square mosaic inlaid in his new patio. His budget allows for 2025 square inch tiles. How long can a side of the mosaic be?

Writing Exercises

Example 9.1E.84

Why is there no real number equal to $\sqrt{-64}$?

Answer

Answers will vary.

? Example $9.1E.85$

What is the difference between 9^2 and $\sqrt{9}$?

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
simplify expressions with square roots.			
estimate square roots.			
approximate square roots.			
simplify variable expressions with square roots.			

ⓑ On a scale of 1–10, how would you rate your mastery of this section in light of your responses on the checklist? How can you improve this?

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9.2: Simplify Square Roots

Learning Objectives

By the end of this section, you will be able to:

- Use the Product Property to simplify square roots
- Use the Quotient Property to simplify square roots

BE PREPARED

Before you get started take this readiness quiz.

1. Simplify: $\frac{80}{176}$.
   If you missed this problem, review [link].
2. Simplify: $\frac{n^9}{n^3}$.
   If you missed this problem, review [link].
3. Simplify: $\frac{q^4}{q^{12}}$.
   If you missed this problem, review [link].

In the last section, we estimated the square root of a number between two consecutive whole numbers. We can say that $\sqrt{50}$ is between 7 and 8. This is fairly easy to do when the numbers are small enough that we can use [link].

But what if we want to estimate $\sqrt{500}$? If we simplify the square root first, we'll be able to estimate it easily. There are other reasons, too, to simplify square roots as you'll see later in this chapter.

A square root is considered *simplified* if its radicand contains no perfect square factors.

Definition: SIMPLIFIED SQUARE ROOT

$\sqrt{a}$ is considered simplified if a has no perfect square factors.

So $\sqrt{31}$ is simplified. But $\sqrt{32}$ is not simplified, because 16 is a perfect square factor of 32.

Use the Product Property to Simplify Square Roots

The properties we will use to simplify expressions with square roots are similar to the properties of exponents. We know that $(ab)^m = a^m b^m$. The corresponding property of square roots says that $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$.

Definition: PRODUCT PROPERTY OF SQUARE ROOTS

If a, b are non-negative real numbers, then $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$.

We use the Product Property of Square Roots to remove all perfect square factors from a radical. We will show how to do this in Example.

How To Use the Product Property to Simplify a Square Root

Example 9.2.1

Simplify: $\sqrt{50}$.

Answer

Step 1. Find the largest perfect square factor of the radicand.	25 is the largest perfect square factor of 50.	$\sqrt{50}$
Rewrite the radicand as a product using the perfect square factor.	$50 = 25 \cdot 2$	
	Always write the perfect square factor first.	$\sqrt{25 \cdot 2}$
Step 2. Use the product rule to rewrite the radical as the product of two radicals.		$\sqrt{25} \cdot \sqrt{2}$
Step 3. Simplify the square root of the perfect square.		$5\sqrt{2}$

Example 9.2.2

Simplify: $\sqrt{48}$.

Answer

$4\sqrt{3}$

Example 9.2.3

Simplify: $\sqrt{45}$.

Answer

$3\sqrt{5}$

Notice in the previous example that the simplified form of $\sqrt{50}$ is $5\sqrt{2}$, which is the product of an integer and a square root. We always write the integer in front of the square root.

Definition: SIMPLIFY A SQUARE ROOT USING THE PRODUCT PROPERTY.

1. Find the largest perfect square factor of the radicand. Rewrite the radicand as a product using the perfect-square factor.
2. Use the product rule to rewrite the radical as the product of two radicals.
3. Simplify the square root of the perfect square.

Example 9.2.4

Simplify: $\sqrt{500}$.

Answer

	$\sqrt{500}$	
Rewrite the radicand as a product using the largest perfect square factor	$\sqrt{100 \cdot 5}$	(9.2.1)
Rewrite the radical as the product of two radicals	$\sqrt{100} \cdot \sqrt{5}$	
Simplify	$10\sqrt{5}$	

Example 9.2.5

Simplify: $\sqrt{288}$.

Answer

$12\sqrt{2}$

Example 9.2.6

Simplify: $\sqrt{432}$.

Answer

$12\sqrt{3}$

We could use the simplified form $10\sqrt{5}$ to estimate $\sqrt{500}$. We know $\sqrt{5}$ is between 2 and 3, and $\sqrt{500}$ is $10\sqrt{5}$. So $\sqrt{500}$ is between 20 and 30.

The next example is much like the previous examples, but with variables.

Example 9.2.7

Simplify: $\sqrt{x^3}$.

Answer

	$\sqrt{x^3}$	
Rewrite the radicand as a product using the largest perfect square factor	$\sqrt{x^2 \cdot x}$	(9.2.2)
Rewrite the radical as the product of two radicals	$\sqrt{x^2} \cdot \sqrt{x}$	
Simplify	$x\sqrt{x}$	

Example 9.2.8

Simplify: $\sqrt{b^5}$.

Answer

$b^2\sqrt{b}$

Example 9.2.9

Simplify: $\sqrt{p^9}$.

Answer

$p^4\sqrt{p}$

We follow the same procedure when there is a coefficient in the radical, too.

Example 9.2.10

Simplify: $\sqrt{25y^5}$.

Answer

	$\sqrt{25y^5}$	
Rewrite the radicand as a product using the largest perfect square factor.	$\sqrt{25y^4 \cdot y}$	(9.2.3)
Rewrite the radical as the product of two radicals.	$\sqrt{25y^4} \cdot \sqrt{y}$	
Simplify.	$5y^2\sqrt{y}$	

Example 9.2.11

Simplify: $\sqrt{16x^7}$.

Answer

$4x^3\sqrt{x}$

Example 9.2.12

Simplify: $\sqrt{49v^9}$.

Answer

$7v^4\sqrt{v}$

In the next example both the constant and the variable have perfect square factors.

Example 9.2.13

Simplify: $\sqrt{72n^7}$.

Answer

	$\sqrt{72n^7}$	
Rewrite the radicand as a product using the largest perfect square factor.	$\sqrt{36n^6 \cdot 2n}$	(9.2.4)
Rewrite the radical as the product of two radicals.	$\sqrt{36n^6} \cdot \sqrt{2n}$	
Simplify.	$6n^3\sqrt{2n}$	

Example 9.2.14

Simplify: $\sqrt{32y^5}$.

Answer

$4y^2\sqrt{2y}$

Example 9.2.15

Simplify: $\sqrt{75a^9}$.

Answer

$5a^4\sqrt{3a}$

Example 9.2.16

Simplify: $\sqrt{63u^3v^5}$.

Answer

	$\sqrt{63u^3v^5}$
Rewrite the radicand as a product using the largest perfect square factor.	$\sqrt{9u^2v^4 \cdot 7uv}$
Rewrite the radical as the product of two radicals.	$\sqrt{9u^2v^4} \cdot \sqrt{7uv}$
Simplify.	$3uv^2\sqrt{7uv}$

(9.2.5)

Example 9.2.17

Simplify: $\sqrt{98a^7b^5}$.

Answer

$7a^3b^2\sqrt{2ab}$

Example 9.2.18

Simplify: $\sqrt{180m^9n^{11}}$.

Answer

$6m^4n^5\sqrt{5mn}$

We have seen how to use the Order of Operations to simplify some expressions with radicals. To simplify $\sqrt{25}+\sqrt{144}$ we must simplify each square root separately first, then add to get the sum of 17.

The expression $\sqrt{17}+\sqrt{7}$ cannot be simplified—to begin we'd need to simplify each square root, but neither 17 nor 7 contains a perfect square factor.

In the next example, we have the sum of an integer and a square root. We simplify the square root but cannot add the resulting expression to the integer.

Example 9.2.19

Simplify: $3+\sqrt{32}$.

Answer

	$3+\sqrt{32}$
Rewrite the radicand as a product using the largest perfect square factor.	$3+\sqrt{16 \cdot 2}$
Rewrite the radical as the product of two radicals.	$3+\sqrt{16} \cdot \sqrt{2}$
Simplify.	$3+4\sqrt{2}$

(9.2.6)

The terms are not like and so we cannot add them. Trying to add an integer and a radical is like trying to add an integer and a variable—they are not like terms!

Example 9.2.20

Simplify: $5+\sqrt{75}$.

Answer

$5+5\sqrt{3}$

Example 9.2.21

Simplify: $2+\sqrt{98}$.

Answer

$2+7\sqrt{2}$

The next example includes a fraction with a radical in the numerator. Remember that in order to simplify a fraction you need a common factor in the numerator and denominator.

Example 9.2.22

Simplify: $\frac{4-\sqrt{48}}{2}$.

Answer

	$\frac{4-\sqrt{48}}{2}$	(9.2.7)
Rewrite the radicand as a product using thelargest perfect square factor.	$\frac{4-\sqrt{16\cdot 3}}{2}$	
Rewrite the radical as the product of two radicals.	$\frac{4-\sqrt{16}\cdot\sqrt{3}}{2}$	
Simplify.	$\frac{4-4\sqrt{3}}{2}$	
Factor the common factor from thenumerator.	$\frac{4(1-\sqrt{3})}{2}$	
Remove the common factor, 2, from thenumerator and denominator.	$2(1-\sqrt{3})$	

Example 9.2.23

Simplify: $\frac{10-\sqrt{75}}{5}$.

Answer

$2-\sqrt{3}$

Example 9.2.24

Simplify: $\frac{6-\sqrt{45}}{3}$.

Answer

$2-\sqrt{5}$

Use the Quotient Property to Simplify Square Roots

Whenever you have to simplify a square root, the first step you should take is to determine whether the radicand is a perfect square. A *perfect square fraction* is a fraction in which both the numerator and the denominator are perfect squares.

Example 9.2.25

Simplify: $\sqrt{\frac{9}{64}}$.

Answer

$$\text{Since} (\tfrac{3}{8})^2 \quad \begin{matrix} \sqrt{\frac{9}{64}} \\ \frac{3}{8} \end{matrix} \tag{9.2.8}$$

Example 9.2.26

Simplify: $\sqrt{\frac{25}{16}}$.

Answer

$\frac{5}{4}$

Example 9.2.27

Simplify: $\sqrt{\frac{49}{81}}$.

Answer

$\frac{7}{9}$

If the numerator and denominator have any common factors, remove them. You may find a perfect square fraction!

Example 9.2.28

Simplify: $\sqrt{\frac{45}{80}}$.

Answer

$$\begin{matrix} \text{Simplify inside the radical first. Rewrite showing the common factors of the numerator and denominator.} \\ \text{Simplify the fraction by removing common factors.} \\ \text{Simplify.} (\tfrac{3}{4})^2 = \tfrac{9}{16} \end{matrix} \quad \begin{matrix} \sqrt{\frac{45}{80}} \\ \sqrt{\frac{5 \cdot 9}{5 \cdot 16}} \\ \sqrt{\frac{9}{16}} \\ \frac{3}{4} \end{matrix} \tag{9.2.9}$$

Example 9.2.29

Simplify: $\sqrt{\frac{75}{48}}$.

Answer

$\frac{5}{4}$

Example 9.2.30

Simplify: $\sqrt{\frac{98}{162}}$.

Answer

$\frac{7}{9}$

In the last example, our first step was to simplify the fraction under the radical by removing common factors. In the next example we will use the Quotient Property to simplify under the radical. We divide the like bases by subtracting their exponents, $\frac{a^m}{a^n} = a^{m-n}$, $a \neq 0$.

Example 9.2.31

Simplify: $\sqrt{\frac{m^6}{m^4}}$.

Answer

	$\sqrt{\frac{m^6}{m^4}}$
Simplify the fraction inside the radical first	$\sqrt{m^2}$
Divide the like bases by subtracting the exponents.	
Simplify.	m

(9.2.10)

Example 9.2.32

Simplify: $\sqrt{\frac{a^8}{a^6}}$.

Answer

a

Example 9.2.33

Simplify: $\sqrt{\frac{x^{14}}{x^{10}}}$.

Answer

x^2

Example 9.2.34

Simplify: $\sqrt{\frac{48p^7}{3p^3}}$.

Answer

	$\sqrt{\frac{48p^7}{3p^3}}$
Simplify the fraction inside the radical first.	$\sqrt{16p^4}$
Simplify.	$4p^2$

(9.2.11)

Example 9.2.35

Simplify: $\sqrt{\frac{75x^5}{3x}}$.

Answer

$5x^2$

Example 9.2.36

Simplify: $\sqrt{\frac{72z^{12}}{2z^{10}}}$.

Answer

6z

Remember the Quotient to a Power Property? It said we could raise a fraction to a power by raising the numerator and denominator to the power separately.

$(\frac{a}{b})^m = \frac{a^m}{b^m}, b \neq 0$

We can use a similar property to simplify a square root of a fraction. After removing all common factors from the numerator and denominator, if the fraction is not a perfect square we simplify the numerator and denominator separately.

Definition: QUOTIENT PROPERTY OF SQUARE ROOTS

If a, b are non-negative real numbers and $b \neq 0$, then

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

Example 9.2.37

Simplify: $\sqrt{\frac{21}{64}}$.

Answer

We cannot simplify the fraction inside the radical. Rewrite using the quotient property.	$\frac{\sqrt{21}}{\sqrt{64}}$	(9.2.12)
Simplify the square root of 64. The numerator cannot be simplified.	$\frac{\sqrt{21}}{8}$	

Example 9.2.38

Simplify: $\sqrt{\frac{19}{49}}$.

Answer

$\frac{\sqrt{19}}{7}$

Example 9.2.39

Simplify: $\sqrt{\frac{28}{81}}$

Answer

$\frac{2\sqrt{7}}{9}$

How to Use the Quotient Property to Simplify a Square Root

Example 9.2.40

Simplify: $\sqrt{\frac{27m^3}{196}}$.

Answer

Step 1. Simplify the fraction in the radicand, if possible.	$\frac{27m^3}{196}$ cannot be simplified.	$\sqrt{\frac{27m^3}{196}}$
Step 2. Use the Quotient Property to rewrite the radical as the quotient of two radicals.	We rewrite $\sqrt{\frac{27m^3}{196}}$ as the quotient of $\sqrt{27m^3}$ and $\sqrt{196}$.	$\frac{\sqrt{27m^3}}{\sqrt{196}}$
Step 3. Simplify the radicals in the numerator and the denominator.	$9m^2$ and 196 are perfect squares.	$\frac{\sqrt{9m^2} \cdot \sqrt{3m}}{\sqrt{196}}$ $\frac{3m\sqrt{3m}}{14}$

Example 9.2.41

Simplify: $\sqrt{\frac{24p^3}{49}}$

Answer

$\frac{2p\sqrt{6p}}{7}$

Example 9.2.42

Simplify: $\sqrt{\frac{48x^5}{100}}$

Answer

$\frac{2x^2\sqrt{3x}}{5}$

Definition: SIMPLIFY A SQUARE ROOT USING THE QUOTIENT PROPERTY.

1. Simplify the fraction in the radicand, if possible.
2. Use the Quotient Property to rewrite the radical as the quotient of two radicals.
3. Simplify the radicals in the numerator and the denominator.

Example 9.2.43

Simplify: $\sqrt{\frac{45x^5}{y^4}}$.

Answer

We cannot simplify the fraction inside the radical. Rewrite using the quotient property. $\frac{\sqrt{45x^5}}{\sqrt{y^4}}$

Simplify the radicals in the numerator and the denominator. $\frac{\sqrt{9x^4}\sqrt{5x}}{y^2}$

Simplify. $\frac{3x^2\sqrt{5x}}{y^2}$ (9.2.13)

Example 9.2.44

Simplify: $\sqrt{\frac{80m^3}{n^6}}$

Answer

$\frac{4m\sqrt{5m}}{n^3}$

Example 9.2.45

Simplify: $\sqrt{\frac{54u^7}{v^8}}$.

Answer

$\frac{3u^3\sqrt{6u}}{v^4}$

Be sure to simplify the fraction in the radicand first, if possible.

Example 9.2.46

Simplify: $\sqrt{\frac{81d^9}{25d^4}}$.

Answer

$$\begin{aligned}
&\sqrt{\frac{81d^9}{25d^4}} \\
&\text{Simplify the fraction in the radicand.} \quad \sqrt{\frac{81d^5}{25}} \\
&\text{Rewrite using the quotient property.} \quad \frac{\sqrt{81d^5}}{\sqrt{25}} \\
&\text{Simplify the radicals in the numerator and the denominator.} \quad \frac{\sqrt{81d^4}\sqrt{d}}{5} \\
&\text{Simplify.} \quad \frac{9d^2\sqrt{d}}{5}
\end{aligned} \tag{9.2.14}$$

? Example 9.2.47

Simplify: $\sqrt{\frac{64x^7}{9x^3}}$.

Answer

$\frac{8x^2}{3}$

? Example 9.2.48

Simplify: $\sqrt{\frac{16a^9}{100a^5}}$.

Answer

$\frac{2a^2}{5}$

? Example 9.2.49

Simplify: $\sqrt{\frac{18p^5q^7}{32pq^2}}$.

Answer

$$\begin{aligned}
&\sqrt{\frac{18p^5q^7}{32pq^2}} \\
&\text{Simplify the fraction in the radicand.} \quad \sqrt{\frac{9p^4q^5}{16}} \\
&\text{Rewrite using the quotient property.} \quad \frac{\sqrt{9p^4q^5}}{\sqrt{16}} \\
&\text{Simplify the radicals in the numerator and the denominator.} \quad \frac{\sqrt{9p^4q^4}\sqrt{q}}{4} \\
&\text{Simplify.} \quad \frac{3p^2q^2\sqrt{q}}{4}
\end{aligned} \tag{9.2.15}$$

? ExAMPLe 9.2.50

Simplify: $\sqrt{\frac{50x^5y^3}{72x^4y}}$.

Answer

$\frac{5y\sqrt{x}}{6}$

? Example 9.2.51

Simplify: $\sqrt{\frac{48m^7n^2}{125m^5n^9}}$.

Answer

$\frac{4m\sqrt{3}}{5n^3\sqrt{5n}}$

Key Concepts

- **Simplified Square Root** $\sqrt{a}$ is considered simplified if a has no perfect-square factors.
- **Product Property of Square Roots** If a, b are non-negative real numbers, then

  $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$
- **Simplify a Square Root Using the Product Property** To simplify a square root using the Product Property:
  1. Find the largest perfect square factor of the radicand. Rewrite the radicand as a product using the perfect square factor.
  2. Use the product rule to rewrite the radical as the product of two radicals.
  3. Simplify the square root of the perfect square.
- **Quotient Property of Square Roots** If a, b are non-negative real numbers and $b \neq 0$, then

  $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$
- **Simplify a Square Root Using the Quotient Property** To simplify a square root using the Quotient Property:
  1. Simplify the fraction in the radicand, if possible.
  2. Use the Quotient Rule to rewrite the radical as the quotient of two radicals.
  3. Simplify the radicals in the numerator and the denominator.

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9.2E: Exercises

Practice Makes Perfect

Use the Product Property to Simplify Square Roots

In the following exercises, simplify.

Example 9.2E.52

$\sqrt{27}$

Answer

$3\sqrt{3}$

Example 9.2E.53

$\sqrt{80}$

Example 9.2E.54

$\sqrt{125}$

Answer

$5\sqrt{5}$

Example 9.2E.55

$\sqrt{96}$

Example 9.2E.56

$\sqrt{200}$

Answer

$10\sqrt{2}$

Example 9.2E.57

$\sqrt{147}$

Example 9.2E.58

$\sqrt{450}$

Answer

$15\sqrt{2}$

Example 9.2E.59

$\sqrt{252}$

Example 9.2E.60

$\sqrt{800}$

Answer

$20\sqrt{2}$

Example 9.2E.61

$\sqrt{288}$

Example 9.2E.62

$\sqrt{675}$

Answer

$15\sqrt{3}$

Example 9.2E.63

$\sqrt{1250}$

Example 9.2E.64

$\sqrt{x^7}$

Answer

$x^3\sqrt{x}$

Example 9.2E.65

$\sqrt{y^{11}}$

Example 9.2E.66

$\sqrt{p^3}$

Answer

$p\sqrt{p}$

Example 9.2E.67

$\sqrt{q^5}$

Example 9.2E.68

$\sqrt{m^{13}}$

Answer

$m^6\sqrt{m}$

Example 9.2E.69

$\sqrt{n^{21}}$

Example 9.2E.70

$\sqrt{r^{25}}$

Answer

$r^{12}\sqrt{r}$

Example 9.2E.71

$\sqrt{s^{33}}$

Example 9.2E.72

$\sqrt{49n^{17}}$

Answer

$7n^{8}\sqrt{n}$

Example 9.2E.73

$\sqrt{25m^{9}}$

Example 9.2E.74

$\sqrt{81r^{15}}$

Answer

$9r^{7}\sqrt{r}$

Example 9.2E.75

$\sqrt{100s^{19}}$

Example 9.2E.76

$\sqrt{98m^{5}}$

Answer

$7m^{2}\sqrt{2m}$

Example 9.2E.77

$\sqrt{32n^{11}}$

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Example 9.2E.78

$\sqrt{125r^{13}}$

Answer

$5r^6\sqrt{5r}$

Example 9.2E.79

$\sqrt{80s^{15}}$

Example 9.2E.80

$\sqrt{200p^{13}}$

Answer

$10p^6\sqrt{2p}$

Example 9.2E.81

$\sqrt{128q^3}$

Example 9.2E.82

$\sqrt{242m^{23}}$

Answer

$11m^{11}\sqrt{2m}$

Example 9.2E.83

$\sqrt{175n^{13}}$

Exercise 9.2E.84

$\sqrt{147m^7n^{11}}$

Answer

$7m^3n^5\sqrt{3mn}$

Example 9.2E.85

$\sqrt{48m^7n^5}$

Example 9.2E.86

$\sqrt{75r^{13}s^9}$

Answer

$5r^6s^4\sqrt{3rs}$

Example 9.2E.87

$\sqrt{96r^3s^3}$

Example 9.2E.88

$\sqrt{300p^9q^{11}}$

Answer

$10p^4q^5\sqrt{3pq}$

Example 9.2E.89

$\sqrt{192q^3r^7}$

Example 9.2E.90

$\sqrt{242m^{13}n^{21}}$

Answer

$11m^6n^{10}\sqrt{2mn}$

Example 9.2E.91

$\sqrt{150m^9n^3}$

Example 9.2E.92

$5+\sqrt{12}$

Answer

$5+2\sqrt{3}$

Example 9.2E.93

$8+\sqrt{96}$

Example 9.2E.94

$1+\sqrt{45}$

Answer

$1+3\sqrt{5}$

Example 9.2E.95

$3+\sqrt{125}$

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Example 9.2E.96

$\frac{10-\sqrt{24}}{2}$

Answer

$5-\sqrt{6}$

Example 9.2E.97

$\frac{8-\sqrt{80}}{4}$

Example 9.2E.98

$\frac{3+\sqrt{90}}{3}$

Answer

$1+\sqrt{10}$

Example 9.2E.99

$\frac{15+\sqrt{75}}{5}$

Use the Quotient Property to Simplify Square Roots

In the following exercises, simplify.

Example 9.2E.100

$\sqrt{\frac{49}{64}}$

Answer

$\frac{7}{8}$

Example 9.2E.101

$\sqrt{\frac{100}{36}}$

Example 9.2E.102

$\sqrt{\frac{121}{16}}$

Answer

$\frac{11}{4}$

Example 9.2E.103

$\sqrt{\frac{144}{169}}$

Example 9.2E.104

$\sqrt{\frac{72}{98}}$

Answer

$\frac{6}{7}$

Example 9.2E.105

$\sqrt{\frac{75}{12}}$

Example 9.2E.106

$\sqrt{\frac{45}{125}}$

Answer

$\frac{3}{5}$

Example 9.2E.107

$\sqrt{\frac{300}{243}}$

Example 9.2E.108

$\sqrt{\frac{x^{10}}{x^6}}$

Answer

x^2

Example 9.2E.109

$\sqrt{\frac{p^{20}}{p^{10}}}$

Example 9.2E.110

$\sqrt{\frac{y^4}{y^8}}$

Answer

$\frac{1}{y^2}$

Example 9.2E.111

$\sqrt{\frac{q^8}{q^{14}}}$

Example 9.2E.112

$\sqrt{\frac{200x^7}{2x^3}}$

Answer

$10x^2$

Example 9.2E.113

$\sqrt{\frac{98y^{11}}{2y^5}}$

Example 9.2E.114

$\sqrt{\frac{96p^9}{6p}}$

Answer

$4p^4$

Example 9.2E.115

$\sqrt{\frac{108q^{10}}{3q^2}}$

Example 9.2E.116

$\sqrt{\frac{36}{35}}$

Answer

$\frac{6}{\sqrt{35}}$

Example 9.2E.117

$\sqrt{\frac{144}{65}}$

Example 9.2E.118

$\sqrt{\frac{20}{81}}$

Answer

$\frac{2\sqrt{5}}{9}$

Example 9.2E.119

$\sqrt{\frac{211}{96}}$

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Example 9.2E.120

$\sqrt{\frac{96x^7}{121}}$

Answer

$\frac{4x^3\sqrt{6x}}{11}$

Example 9.2E.121

$\sqrt{\frac{108y^4}{49}}$

Example 9.2E.122

$\sqrt{\frac{300m^5}{64}}$

Answer

$\frac{5m^2\sqrt{3m}}{4}$

Example 9.2E.123

$\sqrt{\frac{125n^7}{169}}$

Example 9.2E.124

$\sqrt{\frac{98r^5}{100}}$

Answer

$\frac{7r^2\sqrt{2r}}{10}$

Example 9.2E.125

$\sqrt{\frac{180s^{10}}{144}}$

Example 9.2E.126

$\sqrt{\frac{28q^6}{225}}$

Answer

$\frac{2q^3\sqrt{7}}{15}$

Example 9.2E.127

$\sqrt{\frac{150r^3}{256}}$

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Example 9.2E.128

$\sqrt{\frac{75r^9}{s^8}}$

Answer

$\frac{5r^4\sqrt{3r}}{s^4}$

Example 9.2E.129

$\sqrt{\frac{72x^5}{y^6}}$

Example 9.2E.130

$\sqrt{\frac{28p^7}{q^2}}$

Answer

$\frac{4p^3\sqrt{7p}}{q}$

Example 9.2E.131

$\sqrt{\frac{45r^3}{s^{10}}}$

Example 9.2E.132

$\sqrt{\frac{100x^5}{36x^3}}$

Answer

$\frac{5x}{3}$

Example 9.2E.133

$\sqrt{\frac{49r^{12}}{16r^6}}$

Example 9.2E.134

$\sqrt{\frac{121p^5}{81p^2}}$

Answer

$\frac{11p\sqrt{p}}{9}$

Example 9.2E.135

$\sqrt{\frac{25r^8}{64r}}$

Example 9.2E.136

$\sqrt{\frac{32x^5y^3}{18x^3y}}$.

Answer

$\frac{4xy}{3}$

Example 9.2E.137

$\sqrt{\frac{75r^6s^8}{48rs^4}}$

Example 9.2E.138

$\sqrt{\frac{27p^2q^{10}}{8p^5q^3}}$

Answer

$\frac{1}{2pq\sqrt{p}}$

Example 9.2E.139

$\sqrt{\frac{50r^5s^2}{128r^2s^5}}$

Everyday Math

Example 9.2E.140

1. Elliott decides to construct a square garden that will take up 288 square feet of his yard. Simplify $\sqrt{288}$ to determine the length and the width of his garden. Round to the nearest tenth of a foot.
2. Suppose Elliott decides to reduce the size of his square garden so that he can create a 5-foot-wide walking path on the north and east sides of the garden. Simplify $\sqrt{288}-5$ to determine the length and width of the new garden. Round to the nearest tenth of a foot.

Answer

1. 17.0 feet
2. 15.0 feet

Example 9.2E.141

1. Melissa accidentally drops a pair of sunglasses from the top of a roller coaster, 64 feet above the ground. Simplify $\sqrt{\frac{64}{16}}$ to determine the number of seconds it takes for the sunglasses to reach the ground.
2. Suppose the sunglasses in the previous example were dropped from a height of 144 feet. Simplify $\sqrt{\frac{144}{16}}$ to determine the number of seconds it takes for the sunglasses to reach the ground.

Writing Exercises

? Example 9.2E.142

Explain why $\sqrt{x^4} = x^2$. Then explain why $\sqrt{x^{16}} = x^8$.

Answer

Answers will vary.

? Example 9.2E.143

Explain why $7 + \sqrt{9}$ is not equal to $\sqrt{7+9}$.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
use the Product Property to simplify square roots.			
use the Quotient Property to simplify square roots.			

ⓑ After reviewing this checklist, what will you do to become confident for all objectives?

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9.3: Add and Subtract Square Roots

Learning Objectives

By the end of this section, you will be able to:

- Add and subtract like square roots
- Add and subtract square roots that need simplification

BE PREPARED

Before you get started, take this readiness quiz.

1. Add: ⓐ $3x+9x$ ⓑ $5m+5n$.
   If you missed this problem, review [link].
2. Simplify: $\sqrt{50x^3}$.
   If you missed this problem, review [link].

We know that we must follow the order of operations to simplify expressions with square roots. The radical is a grouping symbol, so we work inside the radical first. We simplify $\sqrt{2+7}$ in this way:

$$\begin{array}{ll} & \sqrt{2+7} \\ \text{Add inside the radical.} & \sqrt{9} \\ \text{Simplify.} & 3 \end{array} \tag{9.3.1}$$

So if we have to add $\sqrt{2}+\sqrt{7}$, we must not combine them into one radical.

$\sqrt{2}+\sqrt{7} \neq \sqrt{2+7}$

Trying to add square roots with different radicands is like trying to add unlike terms.

$$\begin{array}{llll} \text{But, just like we can} & x+x & \text{we can add} & \sqrt{3}+\sqrt{3} \\ & x+x=2x & & \sqrt{3}+\sqrt{3}=2\sqrt{3} \end{array} \tag{9.3.2}$$

Adding square roots with the same radicand is just like adding like terms. We call square roots with the same radicand like square roots to remind us they work the same as like terms.

Definition: LIKE SQUARE ROOTS

Square roots with the same radicand are called **like square roots**.

We add and subtract like square roots in the same way we add and subtract like terms. We know that 3x+8x is 11x. Similarly we add $3\sqrt{x}+8\sqrt{x}$ and the result is $11\sqrt{x}$.

Add and Subtract Like Square Roots

Think about adding like terms with variables as you do the next few examples. When you have like radicands, you just add or subtract the coefficients. When the radicands are not like, you cannot combine the terms.

Example 9.3.1

Simplify: $2\sqrt{2}-7\sqrt{2}$.

Answer

$$\begin{array}{ll} & 2\sqrt{2}-7\sqrt{2} \\ \text{Since the radicals are like, we subtract the coefficients.} & -5\sqrt{2} \end{array} \tag{9.3.3}$$

Example 9.3.2

Simplify: $8\sqrt{2}-9\sqrt{2}$.

Answer

$-\sqrt{2}$

Example 9.3.3

Simplify: $5\sqrt{3}-9\sqrt{3}$.

Answer

$-4\sqrt{3}$

Example 9.3.4

Simplify: $3\sqrt{y}+4\sqrt{y}$.

Answer

$$\begin{array}{ll} & 3\sqrt{y}+4\sqrt{y} \\ \text{Since the radicals are like, we add the coefficients.} & 7\sqrt{y} \end{array} \tag{9.3.4}$$

Example 9.3.5

Simplify: $2\sqrt{x}+7\sqrt{x}$.

Answer

$9\sqrt{x}$

Example 9.3.6

Simplify: $5\sqrt{u}+3\sqrt{u}$.

Answer

$8\sqrt{u}$

Example 9.3.7

Simplify: $4\sqrt{x}-2\sqrt{y}$

Answer

$$\begin{array}{ll} & 4\sqrt{x}-2\sqrt{y} \\ \text{Since the radicals are not like, we cannot subtract them. We leave the expression as is.} & 4\sqrt{x}-2\sqrt{y} \end{array} \tag{9.3.5}$$

Example 9.3.8

Simplify: $7\sqrt{p}-6\sqrt{q}$.

Answer

$7\sqrt{p}-6\sqrt{q}$

Example 9.3.9

Simplify: $6\sqrt{a}-3\sqrt{b}$.

Answer

$6\sqrt{a}-3\sqrt{b}$

Example 9.3.10

Simplify: $5\sqrt{13}+4\sqrt{13}+2\sqrt{13}$.

Answer

$$\begin{aligned} & & 5\sqrt{13}+4\sqrt{13}+2\sqrt{13} & \\ \text{Since the radicals are like, we add the coefficients.} & & 11\sqrt{13} & (9.3.6) \end{aligned}$$

Example 9.3.11

Simplify: $4\sqrt{11}+2\sqrt{11}+3\sqrt{11}$.

Answer

$9\sqrt{11}$

Example 9.3.12

Simplify: $6\sqrt{10}+2\sqrt{10}+3\sqrt{10}$.

Answer

$11\sqrt{10}$

Example 9.3.13

Simplify: $2\sqrt{6}-6\sqrt{6}+3\sqrt{3}$.

Answer

$$\begin{aligned} & & 2\sqrt{6}-6\sqrt{6}+3\sqrt{3} & \\ \text{Since the first two radicals are like, we subtract their coefficients.} & & -4\sqrt{6}+3\sqrt{3} & (9.3.7) \end{aligned}$$

Example 9.3.14

Simplify: $5\sqrt{5}-4\sqrt{5}+2\sqrt{6}$.

Answer

$\sqrt{5}+2\sqrt{6}$

Example 9.3.15

Simplify: $3\sqrt{7}-8\sqrt{7}+2\sqrt{5}$.

Answer

$-5\sqrt{7}+2\sqrt{5}$

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Example 9.3.16

Simplify: $2\sqrt{5n}-6\sqrt{5n}+4\sqrt{5n}$.

Answer

$$\begin{array}{ll} & 2\sqrt{5n}-6\sqrt{5n}+4\sqrt{5n} \\ \text{Since the radicals are like, we combine them.} & -0\sqrt{5n} \\ \text{Simplify.} & 0 \end{array} \tag{9.3.8}$$

Example 9.3.17

Simplify: $\sqrt{7x}-7\sqrt{7x}+4\sqrt{7x}$.

Answer

$-2\sqrt{7x}$

Example 9.3.18

Simplify: $4\sqrt{3y}-7\sqrt{3y}+2\sqrt{3y}$.

Answer

$-3\sqrt{y}$

When radicals contain more than one variable, as long as all the variables and their exponents are identical, the radicals are like.

Example 9.3.19

Simplify: $\sqrt{3xy}+5\sqrt{3xy}-4\sqrt{3xy}$.

Answer

$$\begin{array}{ll} & \sqrt{3xy}+5\sqrt{3xy}-4\sqrt{3xy} \\ \text{Since the radicals are like, we combine them.} & 2\sqrt{3xy} \end{array} \tag{9.3.9}$$

Example 9.3.20

Simplify: $\sqrt{5xy}+4\sqrt{5xy}-7\sqrt{5xy}$.

Answer

$-2\sqrt{5xy}$

Example 9.3.21

Simplify: $3\sqrt{7mn}+\sqrt{7mn}-4\sqrt{7mn}$.

Answer

0

Add and Subtract Square Roots that Need Simplification

Remember that we always simplify square roots by removing the largest perfect-square factor. Sometimes when we have to add or subtract square roots that do not appear to have **like radicals**, we find like radicals after simplifying the square roots.

Example 9.3.22

Simplify: $\sqrt{20}+3\sqrt{5}$.

Answer

$$\begin{array}{ll} & \sqrt{20}+3\sqrt{5} \\ \text{Simplify the radicals, when possible.} & \sqrt{4}\cdot\sqrt{5}+3\sqrt{5} \\ & 2\sqrt{5}+3\sqrt{5} \\ \text{Combine the like radicals.} & 5\sqrt{5} \end{array} \tag{9.3.10}$$

Example 9.3.23

Simplify: $\sqrt{18}+6\sqrt{2}$.

Answer

$9\sqrt{2}$

Example 9.3.24

Simplify: $\sqrt{27}+4\sqrt{3}$.

Answer

$7\sqrt{3}$

Example 9.3.25

Simplify: $\sqrt{48}-\sqrt{75}$

Answer

$$\begin{array}{ll} & \sqrt{48}-\sqrt{75} \\ \text{Simplify the radicals.} & \sqrt{16}\cdot\sqrt{3}-\sqrt{25}\cdot\sqrt{3} \\ & 4\sqrt{3}-5\sqrt{3} \\ \text{Combine the like radicals.} & -\sqrt{3} \end{array} \tag{9.3.11}$$

Example 9.3.26

Simplify: $\sqrt{32}-\sqrt{18}$.

Answer

$\sqrt{2}$

Example 9.3.27

Simplify: $\sqrt{20}-\sqrt{45}$.

Answer

$-\sqrt{5}$

Just like we use the Associative Property of Multiplication to simplify 5(3x) and get 15x, we can simplify $5(3\sqrt{x})$ and get $15\sqrt{x}$. We will use the Associative Property to do this in the next example.

Example 9.3.28

Simplify: $5\sqrt{18}-2\sqrt{8}$.

Answer

$$\begin{aligned} & & 5\sqrt{18}-2\sqrt{8} \\ & \text{Simplify the radicals.} & 5\cdot\sqrt{9}\cdot\sqrt{2}-2\cdot\sqrt{4}\cdot\sqrt{2} \\ & & 5\cdot 3\cdot\sqrt{2}-2\cdot 2\cdot\sqrt{2} \\ & & 15\sqrt{2}-4\sqrt{2} \\ & \text{Combine the like radicals.} & 11\sqrt{2} \end{aligned} \tag{9.3.12}$$

Example 9.3.29

Simplify: $4\sqrt{27}-3\sqrt{12}$.

Answer

$6\sqrt{3}$

Example 9.3.30

Simplify: $3\sqrt{20}-7\sqrt{45}$.

Answer

$-15\sqrt{5}$

Example 9.3.31

Simplify: $\frac{3}{4}\sqrt{192}-\frac{5}{6}\sqrt{108}$.

Answer

$$\begin{aligned} & & \tfrac{3}{4}\sqrt{192}-\tfrac{5}{6}\sqrt{108} \\ & \text{Simplify the radicals.} & \tfrac{3}{4}\sqrt{64}\cdot\sqrt{3}-\tfrac{5}{6}\sqrt{36}\cdot\sqrt{3} \\ & & \tfrac{3}{4}\cdot 8\cdot\sqrt{3}-\tfrac{5}{6}\cdot 6\cdot\sqrt{3} \\ & & 6\sqrt{3}-5\sqrt{3} \\ & \text{Combine the like radicals.} & \sqrt{3} \end{aligned} \tag{9.3.13}$$

Example 9.3.32

Simplify: $\frac{2}{3}\sqrt{108}-\frac{5}{7}\sqrt{147}$.

Answer

$-\sqrt{3}$

Example 9.3.33

Simplify: $\frac{3}{5}\sqrt{200}-\frac{3}{4}\sqrt{128}$.

Answer

0

Example 9.3.34

Simplify: $\frac{2}{3}\sqrt{48}-\frac{3}{4}\sqrt{12}$.

Answer

$$\begin{aligned}
&\text{Simplify the radicals.} && \tfrac{2}{3}\sqrt{48}-\tfrac{3}{4}\sqrt{12}\\
& && \tfrac{2}{3}\sqrt{16}\cdot\sqrt{3}-\tfrac{3}{4}\sqrt{4}\cdot\sqrt{3}\\
& && \tfrac{2}{3}\cdot 4\cdot\sqrt{3}-\tfrac{3}{4}\cdot 2\cdot\sqrt{3}\\
& && \tfrac{8}{3}\sqrt{3}-\tfrac{3}{2}\sqrt{3}\\
&\text{Find a common denominator to subtract the coefficients of the like radicals.} && \tfrac{16}{6}\sqrt{3}-\tfrac{9}{6}\sqrt{3}\\
&\text{Simplify.} && \tfrac{7}{6}\sqrt{3}
\end{aligned} \tag{9.3.14}$$

Example 9.3.35

Simplify: $\frac{2}{5}\sqrt{32}-\frac{1}{3}\sqrt{8}$

Answer

$\frac{14}{15}\sqrt{2}$

Example 9.3.36

Simplify: $\frac{1}{3}\sqrt{80}-\frac{1}{4}\sqrt{125}$

Answer

$\frac{1}{12}[\sqrt{5}$

In the next example, we will remove constant and variable factors from the square roots.

Example 9.3.37

Simplify: $\sqrt{18n^5}-\sqrt{32n^5}$

Answer

$$\begin{aligned}
& && \sqrt{18n^5}-\sqrt{32n^5}\\
&\text{Simplify the radicals.} && \sqrt{9n^4}\cdot\sqrt{2n}-\sqrt{16n^4}\cdot\sqrt{2n}\\
& && 3n^2\sqrt{2n}-4n^2\sqrt{2n}\\
&\text{Combine the like radicals.} && -n^2\sqrt{2n}
\end{aligned} \tag{9.3.15}$$

Example 9.3.38

Simplify: $\sqrt{32m^7}-\sqrt{50m^7}$.

Answer

$-m^3\sqrt{2m}$

Example 9.3.39

Simplify: $\sqrt{27p^3}-\sqrt{48p^3}$

Answer

$-p^3\sqrt{p}$

Example 9.3.40

Simplify: $9\sqrt{50m^2}-6\sqrt{48m^2}$.

Answer

\[\begin{array}{ll} {}&{9\sqrt{50m^{2}}−6\sqrt{48m^{2}}}\\ {\text{Simplify the radicals.}}&{9\sqrt{25m^{2}}·\sqrt{2}−6·\sqrt{16m^{2}}·\sqrt{3}}\\ {}&{9·5m·\sqrt{2}−6·4m·\sqrt{3}}\\ {}&{45m\sqrt{2}−24m\sqrt{3}}\\ \end{array}\]

Example 9.3.41

Simplify: $5\sqrt{32x^2}-3\sqrt{48x^2}$.

Answer

$20x\sqrt{2}-12x\sqrt{3}$

Example 9.3.42

Simplify: $7\sqrt{48y^2}-4\sqrt{72y^2}$.

Answer

$28y\sqrt{3}-24y\sqrt{2}$

Example 9.3.43

Simplify: $2\sqrt{8x^2}-5x\sqrt{32}+5\sqrt{18x^2}$.

Answer

$$\begin{array}{ll} & 2\sqrt{8x^2}-5x\sqrt{32}+5\sqrt{18x^2} \\ \text{Simplify the radicals.} & 2\sqrt{4x^2}\cdot\sqrt{2}-5x\sqrt{16}\cdot\sqrt{2}+5\sqrt{9x^2}\cdot\sqrt{2} \\ & 2\cdot 2x\cdot\sqrt{2}-5x\cdot 4\cdot\sqrt{2}+5\cdot 3x\cdot\sqrt{2} \\ & 4x\sqrt{2}-20x\sqrt{2}+15x\sqrt{2} \\ \text{Combine the like radicals.} & -x\sqrt{2} \end{array} \tag{9.3.16}$$

Example 9.3.44

Simplify: $3\sqrt{12x^2}-2x\sqrt{48}+4\sqrt{27x^2}$

Answer

$10x\sqrt{3}$

Example 9.3.45

Simplify: $3\sqrt{18x^2}-6x\sqrt{32}+2\sqrt{50x^2}$.

Answer

$-5x\sqrt{2}$

Access this online resource for additional instruction and practice with the adding and subtracting square roots.

- Adding/Subtracting Square Roots

Glossary

like square roots

Square roots with the same radicand are called like square roots.

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9.3E: Exercises

Practice Makes Perfect

Add and Subtract Like Square Roots

In the following exercises, simplify.

Example 9.3E.46

$8\sqrt{2}-5\sqrt{2}$

Answer

$3\sqrt{2}$

Example 9.3E.47

$7\sqrt{2}-3\sqrt{2}$

Example 9.3E.48

$3\sqrt{5}+6\sqrt{5}$

Answer

$9\sqrt{5}$

Example 9.3E.49

$4\sqrt{5}+8\sqrt{5}$

Example 9.3E.50

$9\sqrt{7}-10\sqrt{7}$

Answer

$-\sqrt{7}$

Example 9.3E.51

$11\sqrt{7}-12\sqrt{7}$

Example 9.3E.52

$7\sqrt{y}+2\sqrt{y}$

Answer

$9\sqrt{y}$

Example 9.3E.53

$9\sqrt{n}+3\sqrt{n}$

Example 9.3E.54

$\sqrt{a}-4\sqrt{a}$

Answer

$-3\sqrt{a}$

Example 9.3E.55

$\sqrt{b}-6\sqrt{b}$

Example 9.3E.56

$5\sqrt{c}+2\sqrt{c}$

Answer

$7\sqrt{c}$

Example 9.3E.57

$7\sqrt{d}+2\sqrt{d}$

Example 9.3E.58

$8\sqrt{a}-2\sqrt{b}$

Answer

$8\sqrt{a}-2\sqrt{b}$

Example 9.3E.59

$5\sqrt{c}-3\sqrt{d}$

Example 9.3E.60

$5\sqrt{m}+\sqrt{n}$

Answer

$5\sqrt{m}+\sqrt{n}$

Example 9.3E.61

$\sqrt{n}+3\sqrt{p}$

Example 9.3E.62

$8\sqrt{7}+2\sqrt{7}+3\sqrt{7}$

Answer

$13\sqrt{7}$

Example 9.3E.63

$6\sqrt{5}+3\sqrt{5}+\sqrt{5}$

Example 9.3E.64

$3\sqrt{11}+2\sqrt{11}-8\sqrt{11}$

Answer

$-3\sqrt{11}$

Example 9.3E.65

$2\sqrt{15}+5\sqrt{15}-9\sqrt{15}$

Example 9.3E.66

$3\sqrt{3}-8\sqrt{3}+7\sqrt{5}$

Answer

$-5\sqrt{3}+7\sqrt{5}$

Example 9.3E.67

$5\sqrt{7}-8\sqrt{7}+6\sqrt{3}$

Example 9.3E.68

$6\sqrt{2}+2\sqrt{2}-3\sqrt{5}$

Answer

$8\sqrt{2}-3\sqrt{5}$

Example 9.3E.69

$7\sqrt{5}+\sqrt{5}-8\sqrt{10}$

Example 9.3E.70

$3\sqrt{2a}-4\sqrt{2a}+5\sqrt{2a}$

Answer

$4\sqrt{2a}$

Example 9.3E.71

$\sqrt{11b}-5\sqrt{11b}+3\sqrt{11b}$

Example 9.3E.72

$8\sqrt{3c}+2\sqrt{3c}-9\sqrt{3c}$

Answer

$\sqrt{3c}$

Example 9.3E.73

$3\sqrt{5d}+8\sqrt{5d}-11\sqrt{5d}$

Example 9.3E.74

$5\sqrt{3ab}+\sqrt{3ab}-2\sqrt{3ab}$

Answer

\(4\sqrt{3ab}\

Example 9.3E.75

$8\sqrt{11cd}+5\sqrt{11cd}-9\sqrt{11cd}$

Example 9.3E.76

$2\sqrt{pq}-5\sqrt{pq}+4\sqrt{pq}$

Answer

$\sqrt{pq}$

Example 9.3E.77

$11\sqrt{2rs}-9\sqrt{2rs}+3\sqrt{2rs}$

Add and Subtract Square Roots that Need Simplification

In the following exercises, simplify.

Example 9.3E.78

$\sqrt{50}+4\sqrt{2}$

Answer

$9\sqrt{2}$

Example 9.3E.79

$\sqrt{48}+2\sqrt{3}$

Example 9.3E.80

$\sqrt{80}-3\sqrt{5}$

Answer

$\sqrt{5}$

Example 9.3E.81

$\sqrt{28}-4\sqrt{7}$

Example 9.3E.82

$\sqrt{27}-\sqrt{75}$

Answer

$-2\sqrt{3}$

Example 9.3E.83

$\sqrt{72}-\sqrt{98}$

Example 9.3E.84

$\sqrt{48}+\sqrt{27}$

Answer

$7\sqrt{3}$

Example 9.3E.85

$\sqrt{45}+\sqrt{80}$

Example 9.3E.86

$2\sqrt{50}-3\sqrt{72}$

Answer

$-8\sqrt{2}$

Example 9.3E.87

$3\sqrt{98}-\sqrt{128}$

Example 9.3E.88

$2\sqrt{12}+3\sqrt{48}$

Answer

$16\sqrt{3}$

Example 9.3E.89

$4\sqrt{75}+2\sqrt{108}$

Example 9.3E.90

$\frac{2}{3}\sqrt{72}+\frac{1}{5}\sqrt{50}$

Answer

$5\sqrt{2}$

Example 9.3E.91

$\frac{2}{5}\sqrt{75}+\frac{3}{4}\sqrt{48}$

Example 9.3E.92

$\frac{1}{2}\sqrt{20}-\frac{2}{3}\sqrt{45}$

Answer

$-\sqrt{5}$

Example 9.3E.93

$\frac{2}{3}\sqrt{54}-\frac{3}{4}\sqrt{96}$

Example 9.3E.94

$\frac{1}{6}\sqrt{27}-\frac{3}{8}\sqrt{48}$

Answer

$-\sqrt{3}$

Example 9.3E.95

$\frac{1}{8}\sqrt{32}-\frac{1}{10}\sqrt{50}$

Example 9.3E.96

$\frac{1}{4}\sqrt{98}-\frac{1}{3}\sqrt{128}$

Answer

$-\frac{3}{4}\sqrt{2}$

Example 9.3E.97

$\frac{1}{3}\sqrt{24}+\frac{1}{4}\sqrt{54}$

Example 9.3E.98

$\sqrt{72a^5}-\sqrt{50a^5}$

Answer

$a^2\sqrt{2a}$

https://math.libretexts.org/@go/page/61445

Example 9.3E.99

$\sqrt{48b^5} - \sqrt{75b^5}$

Example 9.3E.100

$\sqrt{80c^7} - \sqrt{20c^7}$

Answer

$2c^3\sqrt{5c}$

Example 9.3E.101

$\sqrt{96d^9} - \sqrt{24d^9}$

Example 9.3E.102

$9\sqrt{80p^4} - 6\sqrt{98p^4}$

Answer

$36p^2\sqrt{5} - 42p^2\sqrt{2}$

Example 9.3E.103

$8\sqrt{72q^6} - 3\sqrt{75q^6}$

Example 9.3E.104

$2\sqrt{50r^8} + 4\sqrt{54r^8}$

Answer

$10r^4\sqrt{2} + 12r^4\sqrt{6}$

Example 9.3E.105

$5\sqrt{27s^6} + 2\sqrt{20s^6}$

Example 9.3E.106

$3\sqrt{20x^2} - 4\sqrt{45x^2} + 5x\sqrt{80}$

Answer

$14x\sqrt{5}$

Example 9.3E.107

$2\sqrt{28x^2} - 6\sqrt{3x^2} + 6x\sqrt{7}$

Example 9.3E.108

$3\sqrt{128y^2}+4y\sqrt{162}-8\sqrt{98y^2}$

Answer

$-12y\sqrt{2}$

Example 9.3E.109

$3\sqrt{75y^2}+8y\sqrt{48}-\sqrt{300y^2}$

Mixed Practice

Example 9.3E.110

$2\sqrt{8}+6\sqrt{8}-5\sqrt{8}$

Answer

$3\sqrt{8}$

Example 9.3E.111

$\frac{2}{3}\sqrt{27}+\frac{3}{4}\sqrt{48}$

Example 9.3E.112

$\sqrt{175k^4}-\sqrt{63k^4}$

Answer

$2k^2\sqrt{7}$

Example 9.3E.113

$\frac{5}{6}\sqrt{162}+\frac{3}{16}\sqrt{128}$

Example 9.3E.114

$2\sqrt{363}-2\sqrt{300}$

Answer

$2\sqrt{3}$

Example 9.3E.115

$\sqrt{150}+4\sqrt{6}$

Example 9.3E.116

$9\sqrt{2}-8\sqrt{2}$

Answer

$\sqrt{2}$

Example 9.3E.117

$5\sqrt{x}-8\sqrt{y}$

Example 9.3E.118

$8\sqrt{13}-4\sqrt{13}-3\sqrt{13}$

Answer

$\sqrt{13}$

Example 9.3E.119

$5\sqrt{12c^4}-3\sqrt{27c^6}$

Example 9.3E.120

$\sqrt{80a^5}-\sqrt{45a^5}$

Answer

$a^2\sqrt{5a}$

Example 9.3E.121

$\frac{3}{5}\sqrt{75}-\frac{1}{4}\sqrt{48}$

Example 9.3E.122

$21\sqrt{19}-2\sqrt{19}$

Answer

$19\sqrt{19}$

Example 9.3E.123

$\sqrt{500}+\sqrt{405}$

Example 9.3E.124

$\frac{5}{6}\sqrt{27}+\frac{5}{8}\sqrt{48}$

Answer

$5\sqrt{3}$

Example 9.3E.125

$11\sqrt{11}-10\sqrt{11}$

Example 9.3E.126

$\sqrt{75}-\sqrt{108}$

Answer

$-\sqrt{3}$

Example 9.3E.127

$2\sqrt{98}-4\sqrt{72}$

Example 9.3E.128

$4\sqrt{24x^2}-\sqrt{54x^2}+3x\sqrt{6}$

Answer

$8x\sqrt{6}$

Example 9.3E.129

$8\sqrt{80y^6}-6\sqrt{48y^6}$

Everyday Math

Example 9.3E.130

A decorator decides to use square tiles as an accent strip in the design of a new shower, but she wants to rotate the tiles to look like diamonds. She will use 9 large tiles that measure 8 inches on a side and 8 small tiles that measure 2 inches on a side. Determine the width of the accent strip by simplifying the expression $9(8\sqrt{2})+8(2\sqrt{2})$. (Round to the nearest tenth of an inch.)

Answer

124.5 inches

Example 9.3E.131

Suzy wants to use square tiles on the border of a spa she is installing in her backyard. She will use large tiles that have area of 12 square inches, medium tiles that have area of 8 square inches, and small tiles that have area of 4 square inches. Once section of the border will require 4 large tiles, 8 medium tiles, and 10 small tiles to cover the width of the wall. Simplify the expression $4\sqrt{12}+8\sqrt{8}+10\sqrt{4}$ to determine the width of the wall.

Writing Exercises

Example 9.3E.132

Explain the difference between like radicals and unlike radicals. Make sure your answer makes sense for radicals containing both numbers and variables.

Answer

Answers will vary.

? Example $9.3E.133$

Explain the process for determining whether two radicals are like or unlike. Make sure your answer makes sense for radicals containing both numbers and variables.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
add and subtract like square roots.			
add and subtract square roots that need simplification.			

ⓑ What does this checklist tell you about your mastery of this section? What steps will you take to improve?

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9.4: Multiply Square Roots

Learning Objectives

By the end of this section, you will be able to:

- Multiply square roots
- Use polynomial multiplication to multiply square roots

Note

Before you get started, take this readiness quiz.

1. Simplify: (3u)(8v).
   If you missed this problem, review Example 6.2.31.
2. Simplify: 6(12−7n).
   If you missed this problem, review Example 6.3.1.
3. Simplify: (2+a)(4−a).
   If you missed this problem, review Example 6.3.34.

Multiply Square Roots

We have used the Product Property of Square Roots to simplify square roots by removing the perfect square factors. The Product Property of Square Roots says

$$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$$

We can use the Product Property of Square Roots 'in reverse' to multiply square roots.

$$\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$$

Remember, we assume all variables are greater than or equal to zero.

We will rewrite the Product Property of Square Roots so we see both ways together.

Definition: PRODUCT PROPERTY OF SQUARE ROOTS

If a, b are nonnegative real numbers, then

$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$ and $\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$.

So we can multiply $\sqrt{3} \cdot \sqrt{5}$ in this way:

$$\sqrt{3} \cdot \sqrt{5}$$
$$\sqrt{3 \cdot 5}$$
$$\sqrt{15}$$

Sometimes the product gives us a perfect square:

$$\sqrt{2} \cdot \sqrt{8}$$
$$\sqrt{2 \cdot 8}$$
$$\sqrt{16}$$
$$4$$

Even when the product is not a perfect square, we must look for perfect-square factors and simplify the radical whenever possible.

Multiplying radicals with coefficients is much like multiplying variables with coefficients. To multiply 4x·3y we multiply the coefficients together and then the variables. The result is 12xy. Keep this in mind as you do these examples.

Example 9.4.1

Simplify:

1. $\sqrt{2} \cdot \sqrt{6}$
2. $(4\sqrt{3})(2\sqrt{12})$.

Answer

1.	$\sqrt{2} \cdot \sqrt{6}$
Multiply using the Product Property.	$\sqrt{12}$
Simplify the radical.	$\sqrt{4} \cdot \sqrt{3}$
Simplify.	$2\sqrt{3}$
2.	$(4\sqrt{3})(2\sqrt{12})$
Multiply using the Product Property.	$8\sqrt{36}$
Simplify the radical.	$8 \cdot 6$
Simplify.	48

Example 9.4.2

Simplify:

1. $\sqrt{3} \cdot \sqrt{6}$
2. $(2\sqrt{6})(3\sqrt{12})$.

Answer

1. $3\sqrt{2}$
2. $36\sqrt{2}$

Example 9.4.3

Simplify:

1. $\sqrt{5} \cdot \sqrt{10}$
2. $(6\sqrt{3})(5\sqrt{6})$

Answer

1. $5\sqrt{2}$
2. $90\sqrt{2}$

Example 9.4.4

Simplify: $(6\sqrt{2})(3\sqrt{10})$

Answer

	$(6\sqrt{2})(3\sqrt{10})$
Multiply using the Product Property.	$18\sqrt{20}$
Simplify the radical.	$18\sqrt{4} \cdot \sqrt{5}$
Simplify.	$18 \cdot 2 \cdot \sqrt{5}$

	$36\sqrt{5}$

Example 9.4.5

Simplify: $(3\sqrt{2})(2\sqrt{30})$

Answer

$12\sqrt{15}$

Example 9.4.6

Simplify: $(3\sqrt{3})(3\sqrt{6})$.

Answer

$27\sqrt{2}$

When we have to multiply square roots, we first find the product and then remove any perfect square factors.

Example 9.4.7

Simplify:

1. $(\sqrt{8x^3})(\sqrt{3x})$
2. $(\sqrt{20y^2})(\sqrt{5y^3})$

Answer

1.	$(\sqrt{8x^3})(\sqrt{3x})$
Multiply using the Product Property.	$\sqrt{24x^4}$
Simplify the radical.	$\sqrt{4x^4}\cdot\sqrt{6}$
Simplify.	$2x^2\sqrt{6}$
2.	$(\sqrt{20y^2})(\sqrt{5y^3})$
Multiply using the Product Property.	$\sqrt{100y^5}$
Simplify the radical.	$10y^2\sqrt{y}$

Example 9.4.8

Simplify:

1. $(\sqrt{6x^3})(\sqrt{3x})$
2. $(\sqrt{2y^3})(\sqrt{50y^2})$.

Answer

1. $3x^2\sqrt{2}$
2. $10y^2\sqrt{y}$

Example 9.4.9

Simplify:

1. $(\sqrt{6x^5})(\sqrt{2x})$
2. $(\sqrt{12y^2})(\sqrt{3y^5})$

Answer

1. $2x^3\sqrt{3}$
2. $6y^2\sqrt{y}$

Example 9.4.10

Simplify: $(10\sqrt{6p^3})(3\sqrt{18p})$

Answer

	$(10\sqrt{6p^3})(3\sqrt{18p})$
Multiply.	$30\sqrt{108p^4}$
Simplify the radical.	$30\sqrt{36p^4}\cdot\sqrt{3}$
	$30\cdot 6p^2\cdot\sqrt{3}$
	$180p^2\sqrt{3}$

Example 9.4.11

Simplify: $(\sqrt{62x^2})(8\sqrt{45x^4})$

Answer

$144x^3\sqrt{10}$

Example 9.4.12

Simplify: $(2\sqrt{6y^4})(12\sqrt{30y})$.

Answer

$144y^2\sqrt{5y}$

Example 9.4.13

Simplify:

1. $(\sqrt{2})^2$
2. $(-\sqrt{11})^2$.

Answer

1.	$(\sqrt{2})^2$
Rewrite as a product.	$(\sqrt{2})(\sqrt{2})$
Multiply.	$\sqrt{4}$
Simplify.	2

2.	$(-\sqrt{11})^2$
Rewrite as a product.	$(-\sqrt{11})(-\sqrt{11})$
Multiply.	$\sqrt{121}$
Simplify.	11

? Example 9.4.14

Simplify:

1. $(\sqrt{12})^2$
2. $(-\sqrt{15})^2$.

Answer

1. 12
2. 15

? Example 9.4.15

Simplify:

1. $(\sqrt{16})^2$
2. $(-\sqrt{20})^2$.

Answer

1. 16
2. 20

The results of the previous example lead us to this property.

Definition: SQUARING A SQUARE ROOT

If a is a nonnegative real number, then

$(\sqrt{a})^2 = a$

By realizing that squaring and taking a square root are 'opposite' operations, we can simplify $(\sqrt{2})^2$ and get 2 right away. When we multiply the two **like square roots** in part (a) of the next example, it is the same as squaring.

? Example 9.4.16

Simplify:

1. $(2\sqrt{3})(8\sqrt{3})$
2. $(3\sqrt{6})^2$.

Answer

1.	$(2\sqrt{3})(8\sqrt{3})$
Multiply. Remember, $(\sqrt{3^2})$	16·3
Simplify.	48
2.	$(3\sqrt{6})^2$
Multiply.	9·6

Simplify.	54

Example 9.4.17

Simplify:

1. $(6\sqrt{11})(5\sqrt{11})$
2. $(5\sqrt{8})^2$.

Answer

1. 330
2. 200

Example 9.4.18

Simplify:

1. $(3\sqrt{7})(10\sqrt{7})$
2. $(-4\sqrt{6})^2$.

Answer

1. 210
2. 96

Use Polynomial Multiplication to Multiply Square Roots

In the next few examples, we will use the Distributive Property to multiply expressions with square roots.

We will first distribute and then simplify the square roots when possible.

Example 9.4.19

Simplify:

1. $3(5-\sqrt{2})$
2. $\sqrt{2}(4-\sqrt{10})$.

Answer

1.	$3(5-\sqrt{2})$
Distribute.	$15-3\sqrt{2})$
2.	$\sqrt{2}(4-\sqrt{10})$
Distribute.	$4\sqrt{2}-\sqrt{20}$
Simplify.	$4\sqrt{2}-2\sqrt{5}$

Example 9.4.20

Simplify:

1. $2(3-\sqrt{5})$
2. $\sqrt{3}(2-\sqrt{18})$.

Answer

1. $6-2\sqrt{5}$
2. $2\sqrt{3}-3\sqrt{6}$

Example 9.4.21

Simplify:

1. $6(2+\sqrt{6})$
2. $\sqrt{7}(1+\sqrt{14})$.

Answer

1. $12+\sqrt{6}$
2. $\sqrt{7}+7\sqrt{2}$

Example 9.4.22

Simplify:

1. $\sqrt{5}(7+2\sqrt{5})$
2. $\sqrt{6}(\sqrt{2}+\sqrt{18})$.

Answer

1.	$\sqrt{5}(7+2\sqrt{5})$
Distribute.	$7\sqrt{5}+2\cdot 5$
Simplify.	$7\sqrt{5}+10$
	$10+7\sqrt{5}$
2.	$\sqrt{6}(\sqrt{2}+\sqrt{18})$
Multiply.	$\sqrt{12}+\sqrt{108}$
Simplify.	$(\sqrt{4}\cdot\sqrt{3}+\sqrt{36}\cdot\sqrt{3})$
	$2\sqrt{3}+6\sqrt{3}$
Combine like radicals.	$8\sqrt{3}$

Example 9.4.23

Simplify:

1. $\sqrt{6}(1+3\sqrt{6})$
2. $\sqrt{12}(\sqrt{3}+\sqrt{24})$

Answer

1. $18+\sqrt{6}$
2. $6+12\sqrt{2}$

Example 9.4.24

Simplify:

1. $\sqrt{8}(2-5\sqrt{8})$
2. $\sqrt{14}(\sqrt{2}+\sqrt{42})$

Answer

1. $-40+4\sqrt{2}$
2. $2\sqrt{7}+14\sqrt{3}$

When we worked with polynomials, we multiplied binomials by binomials. Remember, this gave us four products before we combined any like terms. To be sure to get all four products, we organized our work—usually by the FOIL method.

Example 9.4.25

Simplify: $(2+\sqrt{3})(4-\sqrt{3})$

Answer

	$(2+\sqrt{3})(4-\sqrt{3})$
Multiply.	$8-2\sqrt{3}+4\sqrt{3}-3$
Combine like terms.	$5+2\sqrt{3}$

Example 9.4.26

Simplify: $(1+\sqrt{6})(3-\sqrt{6})$.

Answer

$-3+2\sqrt{6}$

Example 9.4.27

Simplify: $(4-\sqrt{10})(2+\sqrt{10})$.

Answer

$-2+2\sqrt{10}$

Example 9.4.28

Simplify: $(3-2\sqrt{7})(4-2\sqrt{7})$.

Answer

	$(3-2\sqrt{7})(4-2\sqrt{7})$
Multiply.	$12-6\sqrt{7}-8\sqrt{7}+4\cdot 7$
Simplify.	$12-6\sqrt{7}-8\sqrt{7}+28$
Combine like terms.	$40-14\sqrt{7}$

Example 9.4.29

Simplify: $(6-3\sqrt{7})(3+4\sqrt{7})$.

Answer

$-66+15\sqrt{7}$

Example 9.4.30

Simplify: $(2-3\sqrt{11})(4-\sqrt{11})$

Answer

$41+14\sqrt{11}$

Example 9.4.31

Simplify: $(3\sqrt{2}-\sqrt{5})(\sqrt{2}+4\sqrt{5})$.

Answer

	$3\sqrt{2}-\sqrt{5})(\sqrt{2}+4\sqrt{5})$
Multiply.	$3\cdot 2+12\sqrt{10}-\sqrt{10}-4\cdot 5$
Simplify.	$6+12\sqrt{10}-\sqrt{10}-20$
Combine like terms.	$-14+11\sqrt{10}$

Example 9.4.32

Simplify: $(5\sqrt{3}-\sqrt{7})(\sqrt{3}+2\sqrt{7})$

Answer

$1+9\sqrt{21}$

Example 9.4.33

Simplify: $(\sqrt{6}-3\sqrt{8})(2\sqrt{6}+\sqrt{8})$

Answer

$-12-20\sqrt{3}$

Example 9.4.34

Simplify: $(4-2\sqrt{x})(1+3\sqrt{x})$.

Answer

	$(4-2\sqrt{x})(1+3\sqrt{x})$.
Multiply.	$4+12\sqrt{x}-2\sqrt{x}-6x$
Combine like terms.	$4+10\sqrt{x}-6x$

Example 9.4.35

Simplify: $(6-5\sqrt{m})(2+3\sqrt{m})$.

Answer

$12+8\sqrt{m}-15m$

Example 9.4.36

Simplify: $(10+3\sqrt{n})(1-5\sqrt{n})$

Answer

$10-47\sqrt{n}-15n$

Note that some special products made our work easier when we multiplied binomials earlier. This is true when we multiply square roots, too. The special product formulas we used are shown below.

Definition: SPECIAL PRODUCT FORMULAS

Binomial Squares	Product of Conjugates
$(a+b)^2=a^2+2ab+b^2$	$(a-b)(a+b)=a^2-b^2$
$(a-b)^2=a^2-2ab+b^2$	

We will use the special product formulas in the next few examples. We will start with the Binomial Squares formula.

Example 9.4.37

Simplify:

1. $(2+\sqrt{3})^2$
2. $(4-2\sqrt{5})^2$.

Answer

Be sure to include the 2ab term when squaring a binomial.

1.

	$\overset{(a+b)^2}{(2+\sqrt{3})^2}$
Multiply using the binomial square pattern.	$\overset{a^2+2ab+b^2}{2^2+2\cdot 2\cdot\sqrt{3}+(\sqrt{3})^2}$
Simplify.	$4+4\sqrt{3}+3$
Combine like terms.	$7+4\sqrt{3}$

2.

	$\overset{(a-b)^2}{(4-2\sqrt{5})^2}$
Multiply using the binomial square pattern.	$\overset{a^2-2ab+b^2}{4^2-2\cdot 4\cdot 2\sqrt{5}+(2\sqrt{5})^2}$
Simplify.	$16-16\sqrt{5}+4\cdot 5$ $16-16\sqrt{5}+20$
Combine like terms.	$36-16\sqrt{5}$

Example 9.4.38

Simplify:

1. $(10+\sqrt{2})^2$
2. $(1+3\sqrt{6})^2$.

Answer

1. $102+20\sqrt{2}$
2. $55+6\sqrt{6}$

Exercise 9.4.39

Simplify:

1. $(6-\sqrt{5})^2$
2. $(9-2\sqrt{10})^2$.

Answer

1. $41-12\sqrt{5}$
2. $121-36\sqrt{10}$

Example 9.4.40

Simplify: $(1+3\sqrt{x})^2$.

Answer

	$\overset{(a+b)^2}{(1+3\sqrt{x})^2}$
Multiply using the binomial square pattern.	$\overset{a^2+2ab+b^2}{1^2+2\cdot 1\cdot 3\sqrt{x}+(3\sqrt{x})^2}$
Simplify.	$1+6\sqrt{x}+9x$

Example 9.4.41

Simplify: $(2+5\sqrt{m})^2$.

Answer

$4+20\sqrt{m}+25m$

Example 9.4.42

Simplify: $(3-4\sqrt{n})^2$.

Answer

$9-24\sqrt{n}+16n$

In the next two examples, we will find the product of conjugates.

Example 9.4.43

Simplify: $(4-\sqrt{2})(4+\sqrt{2})$.

Answer

	$\overset{(a-b)(a+b)}{(4-\sqrt{2})(4+\sqrt{2})}$
Multiply using the binomial square pattern.	$\overset{a^2-b^2}{4^2-(\sqrt{2})^2}$

Simplify.	$16-2$ 14

Example 9.4.44

Simplify: $(2-\sqrt{3})(2+\sqrt{3})$

Answer

1

ExAMPLe 9.4.45

Simplify: $(1+\sqrt{5})(1-\sqrt{5})$

Answer

-4

Example 9.4.46

Simplify: $(5-2\sqrt{3})(5+2\sqrt{3})$

Answer

	$(a-b)(a+b)$ $(5-2\sqrt{3})(5+2\sqrt{3})$
Multiply using the binomial square pattern.	a^2-b^2 $5^2-(2\sqrt{3})^2$
Simplify.	$25-4\cdot 3$ 13

Example 9.4.47

Simplify: $(3-2\sqrt{5})(3+2\sqrt{5})$.

Answer

-11

Example 9.4.48

Simplify: $(4+5\sqrt{7})(4-5\sqrt{7})$.

Answer

-159

Access these online resources for additional instruction and practice with multiplying square roots.

- Product Property
- Multiply Binomials with Square Roots

Key Concepts

- **Product Property of Square Roots** If a, b are nonnegative real numbers, then

  $\sqrt{ab}=\sqrt{a\cdot b}$ and $\sqrt{a}\cdot\sqrt{b}=\sqrt{ab}$

- **Special formulas** for multiplying binomials and conjugates:

$$(a+b)^2 = a^2 + 2ab + b^2 \qquad (a-b)(a+b) = a^2 - b^2$$
$$(a-b)^2 = a^2 - 2ab + b^2$$

- The FOIL method can be used to multiply binomials containing radicals.

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9.4E: Exercises

Practice Makes Perfect

Multiply Square Roots

In the following exercises, simplify.

Example 9.4E.48

1. $\sqrt{2} \cdot \sqrt{8}$
2. $(3\sqrt{3})(2\sqrt{18})$

Answer

1. 44
2. $18\sqrt{6}$

Example 9.4E.49

1. $\sqrt{6} \cdot \sqrt{6}$
2. $(3\sqrt{2})(2\sqrt{32})$

Example 9.4E.50

1. $\sqrt{7} \cdot \sqrt{14}$
2. $(4\sqrt{8})(5\sqrt{8})$

Answer

1. $7\sqrt{2}$
2. 160

Example 9.4E.51

1. $\sqrt{6} \cdot \sqrt{12}$
2. $(2\sqrt{5})(2\sqrt{10})$

Example 9.4E.52

$(5\sqrt{2})(3\sqrt{6})$

Answer

$30\sqrt{3}$

Example 9.4E.53

$(2\sqrt{3})(4\sqrt{6})$

Example 9.4E.54

$(-2\sqrt{3})(3\sqrt{18})$

Answer

$-18\sqrt{6}$

Example 9.4E.55

$(-4\sqrt{5})(5\sqrt{10})$

Example 9.4E.56

$(5\sqrt{6})(-\sqrt{12})$

Answer

$-30\sqrt{2}$

Example 9.4E.57

$(6\sqrt{2})(-\sqrt{10})$

Example 9.4E.58

$(-2\sqrt{7})(-2\sqrt{14})$

Answer

$28\sqrt{2}$

Example 9.4E.59

$(-2\sqrt{11})(-4\sqrt{22})$

Example 9.4E.60

1. $(\sqrt{15y})(\sqrt{5y^3})$
2. $(\sqrt{2n^2})(\sqrt{18n^3})$

Answer

1. $5y^2\sqrt{3}$
2. $6n^2\sqrt{n}$

Example 9.4E.61

1. $(\sqrt{14x^3})(\sqrt{7x^3})$
2. $(\sqrt{3q^2})(\sqrt{48q^3})$

Example 9.4E.62

1. $(\sqrt{16y^2})(\sqrt{8y^4})$
2. $(\sqrt{11s^6})(\sqrt{11s})$

Answer

1. $8y^3\sqrt{2}$
2. $11s^3\sqrt{s}$

Example 9.4E.63

ⓐ $(\sqrt{8x^3})(\sqrt{3x})$
ⓑ $(\sqrt{7r})(\sqrt{7r^8})$

Example 9.4E.64

$(2\sqrt{5b^3})(4\sqrt{15b})$

Answer

$40b^2\sqrt{3}$

Example 9.4E.65

$(\sqrt{38c^5})(\sqrt{26c^3})$

Example 9.4E.66

$(6\sqrt{3d^3})(4\sqrt{12d^5})$

Answer

$144d^4$

Example 9.4E.67

$(2\sqrt{5b^3})(4\sqrt{15b})$

Example 9.4E.68

$(2\sqrt{5d^6})(3\sqrt{20d^2})$

Answer

$60d^4$

Example 9.4E.69

$(-2\sqrt{7z^3})(3\sqrt{14z^8})$

Example 9.4E.70

$(4\sqrt{2k^5})(-3\sqrt{32k^6})$

Answer

$-96k^5\sqrt{k}$

Example 9.4E.71

1. $(\sqrt{7})^2$
2. $(-\sqrt{15})^2$

Example 9.4E.72

1. $(\sqrt{11})^2$
2. $(-\sqrt{21})^2$

Answer

1. 11
2. 21

Example 9.4E.73

1. $(\sqrt{19})^2$
2. $(-\sqrt{5})^2$

Exercise 9.4E.74

1. $(\sqrt{23})^2$
2. $(-\sqrt{3})^2$

Answer

1. 23
2. 3

Example 9.4E.75

1. $(4\sqrt{11})(-3\sqrt{11})$
2. $(5\sqrt{3})^2$

Example 9.4E.76

1. $(2\sqrt{13})(-9\sqrt{13})$
2. $(6\sqrt{5})^2$

Answer

1. -234
2. 180

Example 9.4E.77

1. $(-3\sqrt{12})(-2\sqrt{6})$
2. $(-4\sqrt{10})^2$

Example 9.4E.78

1. $(-7\sqrt{5})(-3\sqrt{10})$
2. $(-2\sqrt{14})^2$

Answer

1. $105\sqrt{2}$
2. 56

Use Polynomial Multiplication to Multiply Square Roots

In the following exercises, simplify.

Example 9.4E.79

1. $3(4-\sqrt{3})$
2. $\sqrt{2}(4-\sqrt{6})$

Example 9.4E.80

1. $4(6-\sqrt{11})$
2. $\sqrt{2}(5-\sqrt{12})$

Answer

1. $24-4\sqrt{11}$
2. $5\sqrt{2}-2\sqrt{6}$

Example 9.4E.81

1. $5(3-\sqrt{7})$
2. $\sqrt{3}(4-\sqrt{15})$

Example 9.4E.82

1. $7(-2-\sqrt{11})$
2. $\sqrt{7}(6-\sqrt{14})$

Answer

1. $-14-7\sqrt{11}$
2. $6\sqrt{7}-7\sqrt{2}$

Example 9.4E.83

1. $\sqrt{7}(5+2\sqrt{7})$
2. $\sqrt{5}(\sqrt{10}+\sqrt{18})$

Example 9.4E.84

1. $\sqrt{11}(8+4\sqrt{11})$
2. $\sqrt{3}(\sqrt{12}+\sqrt{27})$

Answer

1. $44+8\sqrt{11}$
2. 15

Example 9.4E.85

1. $\sqrt{11}(-3+4\sqrt{1})$
2. $\sqrt{3}(\sqrt{15}-\sqrt{18})$

Example 9.4E.86

1. $\sqrt{2}(-5+9\sqrt{2})$
2. $\sqrt{7}(\sqrt{3}-\sqrt{21})$

Answer

1. $18-5\sqrt{2}$

2. $\sqrt{21}-7\sqrt{3}$

Example 9.4E.87

$(8+\sqrt{3})(2-\sqrt{3})$

Example 9.4E.88

$(7+\sqrt{3})(9-\sqrt{3})$

Answer

$60+2\sqrt{3}$

Example 9.4E.89

$(8-\sqrt{2})(3+\sqrt{2})$

Example 9.4E.90

$(9-\sqrt{2})(6+\sqrt{2})$

Answer

$52+3\sqrt{2}$

Example 9.4E.91

$(3-\sqrt{7})(5-\sqrt{7})$

Example 9.4E.92

$(5-\sqrt{7})(4-\sqrt{7})$

Answer

$27-9\sqrt{7}$

Example 9.4E.93

$(1+3\sqrt{10})(5-2\sqrt{10})$

Exercise 9.4E.94

$(7-2\sqrt{5})(4+9\sqrt{5})$

Answer

$-62+55\sqrt{5}$

Example 9.4E.95

$(\sqrt{3}+\sqrt{10})(\sqrt{3}+2\sqrt{10})$

Example 9.4E.96

$(\sqrt{11}+\sqrt{5})(\sqrt{11}+6\sqrt{5})$

Answer

$41+7\sqrt{55}$

Example 9.4E.97

$(2\sqrt{7}-5\sqrt{11})(4\sqrt{7}+9\sqrt{11})$

Example 9.4E.98

$(4\sqrt{6}+7\sqrt{13})(8\sqrt{6}-3\sqrt{13})$

Answer

$-81+44\sqrt{78}$

Example 9.4E.99

$(5-\sqrt{u})(3+\sqrt{u})$

Example 9.4E.100

$(9-\sqrt{w})(2+\sqrt{w})$

Answer

$18+7\sqrt{w}$

Example 9.4E.101

$(7+2\sqrt{m})(4+9\sqrt{m})$

Example 9.4E.102

$(6+5\sqrt{n})(11+3\sqrt{n})$

Answer

$66+73\sqrt{n}+15n$

Example 9.4E.103

1. $(3+\sqrt{5})^2$
2. $(2-5\sqrt{3})^2$

Example 9.4E.104

1. $(4+\sqrt{11})^2$
2. $(3-2\sqrt{5})^2$

Answer

1. $27+8\sqrt{11}$

2. $29-12\sqrt{5}$

Example 9.4E.105

1. $(9-\sqrt{6})^2$
2. $(10+3\sqrt{7})^2$

Example 9.4E.106

1. $(5-\sqrt{10})^2$
2. $(8+3\sqrt{2})^2$

Answer

1. $35-10\sqrt{10}$
2. $82+48\sqrt{2}$

Example 9.4E.107

$(3-\sqrt{5})(3+\sqrt{5})$

Example 9.4E.108

$(10-\sqrt{3})(10+\sqrt{3})$

Answer

97

Example 9.4E.109

$(4+\sqrt{2})(4-\sqrt{2})$

Example 9.4E.110

$(7+\sqrt{10})(7-\sqrt{10})$

Answer

39

Example 9.4E.111

$(4+9\sqrt{3})(4-9\sqrt{3})$

Example 9.4E.112

$(1+8\sqrt{2})(1-8\sqrt{2})$

Answer

-127

Example 9.4E.113

$(12-5\sqrt{5})(12+5\sqrt{5})$

Example 9.4E.114

$(9-4\sqrt{3})(9+4\sqrt{3})$

Answer

33

Mixed Practice

In the following exercises, simplify.

Example 9.4E.115

$\sqrt{3}\cdot\sqrt{21}$

Example 9.4E.116

$(4\sqrt{6})(-\sqrt{18})$

Answer

$-24\sqrt{3}$

Example 9.4E.117

$(-5+\sqrt{7})(6+\sqrt{21})$

Example 9.4E.118

$(-5\sqrt{7})(6\sqrt{21})$

Answer

$-210\sqrt{3}$

Example 9.4E.119

$(-4\sqrt{2})(2\sqrt{18})$

Example 9.4E.120

$(\sqrt{35y^3})(\sqrt{7y^3})$

Answer

$7y^3\sqrt{5}$

Example 9.4E.121

$(4\sqrt{12x^5})(2\sqrt{6x^3})$

Example 9.4E.122

$(\sqrt{29})^2$

Answer

29

Example 9.4E.123

$(-4\sqrt{17})(-3\sqrt{17})$

Example 9.4E.124

$(-4+\sqrt{17})(-3+\sqrt{17})$

Answer

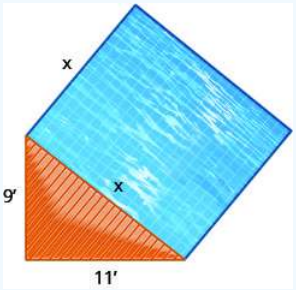
$29-7\sqrt{17}$

Everyday Math

Example 9.4E.125

A landscaper wants to put a square reflecting pool next to a triangular deck, as shown below. The triangular deck is a right triangle, with legs of length 9 feet and 11 feet, and the pool will be adjacent to the hypotenuse.

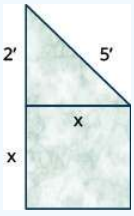
1. Use the Pythagorean Theorem to find the length of a side of the pool. Round your answer to the nearest tenth of a foot.
2. Find the exact area of the pool.



Example 9.4E.126

An artist wants to make a small monument in the shape of a square base topped by a right triangle, as shown below. The square base will be adjacent to one leg of the triangle. The other leg of the triangle will measure 2 feet and the hypotenuse will be 5 feet.

1. Use the Pythagorean Theorem to find the length of a side of the square base. Round your answer to the nearest tenth of a foot.



2. Find the exact area of the face of the square base.

Answer

1. 4.6feet
2. 21 sq. feet

Example 9.4E.127

A square garden will be made with a stone border on one edge. If only $3+\sqrt{10}$ feet of stone are available, simplify $(3+\sqrt{10})^2$ to determine the area of the largest such garden.

Example 9.4E.128

A garden will be made so as to contain two square sections, one section with side length $\sqrt{5}+\sqrt{6}$ yards and one section with side length $\sqrt{2}+\sqrt{3}$ yards. Simplify $(\sqrt{5}+\sqrt{6})(\sqrt{2}+\sqrt{3})$ to determine the total area of the garden.

Example 9.4E.129

Suppose a third section will be added to the garden in the previous exercise. The third section is to have a width of $\sqrt{432}$ feet. Write an expression that gives the total area of the garden.

Writing Exercises

Example 9.4E.130

1. Explain why $(-\sqrt{n})^2$ is always positive, for $n \geq 0$.
2. Explain why $-(\sqrt{n})^2$ is always negative, for $n \geq 0$.

Answer

1. when squaring a negative, it becomes a positive
2. since the negative is not included in the parenthesis, it is not squared, and remains negative

Example 9.4E.131

Use the binomial square pattern to simplify $(3+\sqrt{2})^2$. Explain all your steps.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
multiply square roots.			
use polynomial multiplication to multiply square roots.			

ⓑ On a scale of 1–10, how would you rate your mastery of this section in light of your responses on the checklist? How can you improve this?

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9.5: Divide Square Roots

Learning Objectives

By the end of this section, you will be able to:

- Divide square roots
- Rationalize a one-term denominator
- Rationalize a two-term denominator

Note

Before you get started, take this readiness quiz.

1. Find a fraction equivalent to $\frac{5}{8}$ with denominator 48.
   If you missed this problem, review Exercise 1.6.1.
2. Simplify: $(\sqrt{5})^2$.
   If you missed this problem, review Example 9.4.13.
3. Multiply: (7+3x)(7−3x).
   If you missed this problem, review Exercise 6.4.22.

Divide Square Roots

We know that we simplify fractions by removing factors common to the numerator and the denominator. When we have a fraction with a square root in the numerator, we first simplify the square root. Then we can look for common factors.

Common Factors	No common factors
$\frac{3\sqrt{2}}{3 \cdot 5}$	$\frac{2\sqrt{3}}{3 \cdot 5}$

Example 9.5.1

Simplify: $\frac{\sqrt{54}}{6}$

Answer

	$\frac{\sqrt{54}}{6}$
Simplify the radical.	$\frac{\sqrt{9} \cdot \sqrt{6}}{6}$
Simplify.	$\frac{3\sqrt{6}}{6}$
Remove the common factors.	$\frac{3\sqrt{6}}{3 \cdot 2}$
Simplify.	$\frac{\sqrt{6}}{2}$

Example 9.5.2

Simplify: $\frac{\sqrt{32}}{8}$.

Answer

$\frac{\sqrt{2}}{2}$

Example 9.5.3

Simplify: $\frac{\sqrt{75}}{15}$.

Answer

$\frac{\sqrt{3}}{3}$

Example 9.5.4

Simplify: $\frac{6-\sqrt{24}}{12}$.

Answer

	$\frac{6-\sqrt{24}}{12}$
Simplify the radical.	$\frac{6-\sqrt{4}\cdot\sqrt{6}}{12}$
Simplify.	$\frac{6-2\sqrt{6}}{12}$
Factor the common factor from the numerator.	$\frac{2(3-\sqrt{6})}{12}$
Remove the common factors.	$\frac{2(3-\sqrt{6})}{2\cdot 6}$
Simplify.	$\frac{3-\sqrt{6}}{6}$

Example 9.5.5

Simplify: $\frac{8-\sqrt{40}}{10}$.

Answer

$\frac{4-\sqrt{10}}{5}$

Example 9.5.6

Simplify: $\frac{10-\sqrt{75}}{20}$.

Answer

$\frac{5-\sqrt{3}}{4}$

We have used the Quotient Property of Square Roots to simplify square roots of fractions. The Quotient Property of Square Roots says

$$\sqrt{\frac{a}{b}}=\frac{\sqrt{a}}{\sqrt{b}}\text{, } b\neq 0.$$

Sometimes we will need to use the Quotient Property of Square Roots 'in reverse' to simplify a fraction with square roots.

$$\frac{\sqrt{a}}{\sqrt{b}}=\sqrt{\frac{a}{b}}\text{, } b\neq 0.$$

We will rewrite the Quotient Property of Square Roots so we see both ways together. Remember: we assume all variables are greater than or equal to zero so that their square roots are real numbers.

Definition: QUOTIENT PROPERTY OF SQUARE ROOTS

If a, b are non-negative real numbers and $b \neq 0$, then

$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ and $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$

We will use the Quotient Property of Square Roots 'in reverse' when the fraction we start with is the quotient of two square roots, and neither radicand is a perfect square. When we write the fraction in a single square root, we may find common factors in the numerator and denominator.

Example 9.5.7

Simplify: $\frac{\sqrt{27}}{\sqrt{75}}$

Answer

	$\frac{\sqrt{27}}{\sqrt{75}}$
Neither radicand is a perfect square, so rewrite using the quotient property of square root.	$\sqrt{\frac{27}{75}}$
Remove common factors in the numerator and denominator.	$\sqrt{\frac{9}{25}}$
Simplify.	$\frac{3}{5}$

Example 9.5.8

Simplify: $\frac{\sqrt{48}}{\sqrt{108}}$

Answer

$\frac{2}{3}$

Example 9.5.9

Simplify: $\frac{\sqrt{96}}{\sqrt{54}}$

Answer

$\frac{4}{3}$

We will use the Quotient Property for Exponents, $\frac{a^m}{a^n} = a^{m-n}$, when we have variables with exponents in the radicands.

Example 9.5.10

Simplify: $\frac{\sqrt{6y^5}}{\sqrt{2y}}$

Answer

	$\frac{\sqrt{6y^5}}{\sqrt{2y}}$
Neither radicand is a perfect square, so rewrite using the quotient property of square root.	$\sqrt{\frac{6y^5}{2y}}$

Remove common factors in the numerator and denominator.	$\sqrt{3y^4}$
Simplify.	$y^2\sqrt{3}$

Example 9.5.11

Simplify: $\frac{\sqrt{12r^3}}{\sqrt{6r}}$.

Answer

$r\sqrt{2}$

Example 9.5.12

Simplify: $\frac{\sqrt{14p^9}}{\sqrt{2p^5}}$

Answer

$p^2\sqrt{7}$

Example 9.5.13

Simplify: $\frac{\sqrt{72x^3}}{\sqrt{162x}}$

Answer

	$\frac{\sqrt{72x^3}}{\sqrt{162x}}$
Rewrite using the quotient property of square roots.	$\sqrt{\frac{72x^3}{162x}}$
Remove common factors.	$\sqrt{\frac{18\cdot 4\cdot x^2\cdot x}{18\cdot 9\cdot x}}$
Simplify.	$\sqrt{\frac{4x^2}{9}}$
Simplify the radical.	$\frac{2x}{3}$

ExAMPLe 9.5.14

Simplify: $\frac{\sqrt{50s^3}}{\sqrt{128s}}$.

Answer

$\frac{5s}{8}$

Example 9.5.15

Simplify: $\frac{\sqrt{75q^5}}{\sqrt{108q}}$.

Answer

$\frac{5q^2}{6}$

Example 9.5.16

Simplify: $\frac{\sqrt{147ab^8}}{\sqrt{3a^3b^4}}$.

Answer

	$\frac{\sqrt{147ab^8}}{\sqrt{3a^3b^4}}$
Rewrite using the quotient property of square roots.	$\sqrt{\frac{147ab^8}{3a^3b^4}}$
Remove common factors.	\(\sqrt{\frac{49b^4}{a^2}}\
Simplify the radical.	$\frac{7b^2}{a}$

Example 9.5.17

Simplify: $\frac{\sqrt{162x^{10}y^2}}{\sqrt{2x^6y^6}}$.

Answer

$\frac{9x^2}{y^2}$

Example 9.5.18

Simplify: $\frac{\sqrt{300m^3n^7}}{\sqrt{3m^5n}}$.

Answer

$\frac{10n^3}{m}$

Rationalize a One Term Denominator

Before the calculator became a tool of everyday life, tables of square roots were used to find approximate values of square roots. Figure shows a portion of a table of squares and square roots. Square roots are approximated to five decimal places in this table.

n	n^2	$\sqrt{n}$
200	40,000	14.14214
201	40,401	14.17745
202	40,804	14.21267
203	41,209	14.24781
204	41,616	14.28286
205	42,025	14.31782
206	42,436	14.35270
207	42,849	14.38749
208	43,264	14.42221
209	43,681	14.45683
210	44,100	14.49138

A table of square roots was used to find approximate values of square roots before there were calculators.

If someone needed to approximate a fraction with a square root in the denominator, it meant doing long division with a five decimal-place divisor. This was a very cumbersome process.

For this reason, a process called rationalizing the denominator was developed. A fraction with a radical in the denominator is converted to an equivalent fraction whose denominator is an integer. This process is still used today and is useful in other areas of mathematics, too.

Definition: RATIONALIZING THE DENOMINATOR

The process of converting a fraction with a radical in the denominator to an equivalent fraction whose denominator is an integer is called **rationalizing the denominator**.

Square roots of numbers that are not perfect squares are irrational numbers. When we **rationalize the denominator**, we write an equivalent fraction with a rational number in the denominator.

Let's look at a numerical example.

Suppose we need an approximate value for the fraction.	$\frac{1}{\sqrt{2}}$
A five decimal place approximation to $\sqrt{2}$ is 1.41421	$\frac{1}{1.41421}$
Without a calculator, would you want to do this division?	$1.41421\overline{)1.0}$

But we can find a fraction equivalent to $\frac{1}{\sqrt{2}}$ by multiplying the numerator and denominator by $\sqrt{2}$.

$$\frac{1}{\sqrt{2}}$$
$$\frac{1 \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}}$$
$$\frac{\sqrt{2}}{2}$$

Now if we need an approximate value, we divide $2\overline{)1.41421}$. This is much easier.

Even though we have calculators available nearly everywhere, a fraction with a radical in the denominator still must be rationalized. It is not considered simplified if the denominator contains a square root.

Similarly, a square root is not considered simplified if the radicand contains a fraction.

Definition: SIMPLIFIED SQUARE ROOTS

A square root is considered simplified if there are

- no perfect-square factors in the radicand
- no fractions in the radicand
- no square roots in the denominator of a fraction

To rationalize a denominator, we use the property that $(\sqrt{a})^2 = a$. If we square an irrational square root, we get a rational number.

We will use this property to rationalize the denominator in the next example.

Example 9.5.19

Simplify: $\frac{4}{\sqrt{3}}$.

Answer

To remove the square root from the denominator, we multiply it by itself. To keep the fractions equivalent, we multiply both the numerator and denominator by the same factor.

	$\frac{4}{\sqrt{3}}$
Multiply both the numerator and denominator by $\sqrt{3}$	$\frac{4 \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}}$
Simplify.	$\frac{4\sqrt{3}}{3}$

Example 9.5.20

Simplify: $\frac{5}{\sqrt{3}}$.

Answer

$\frac{5\sqrt{3}}{3}$

Example 9.5.21

Simplify: $\frac{6}{\sqrt{5}}$.

Answer

$\frac{6\sqrt{5}}{5}$

Example 9.5.22

Simplify: $-\frac{8}{3\sqrt{6}}$

Answer

To remove the square root from the denominator, we multiply it by itself. To keep the fractions equivalent, we multiply both the numerator and denominator by $\sqrt{6}$.

	$-\frac{8}{3\sqrt{6}}$
Multiply both the numerator and the denominator by $\sqrt{6}$.	$-\frac{8 \cdot \sqrt{6}}{3\sqrt{6} \cdot \sqrt{6}}$
Simplify.	$-\frac{8\sqrt{6}}{3 \cdot 6}$
Remove common factors.	$-\frac{4 \cdot \cancel{2}\sqrt{6}}{3 \cdot \cancel{2} \cdot 3}$
Simplify.	$-\frac{4\sqrt{6}}{9}$

Example 9.5.23

Simplify: $\frac{5}{2\sqrt{5}}$.

Answer

$\frac{\sqrt{5}}{2}$

Example 9.5.24

Simplify: $-\frac{9}{4\sqrt{3}}$.

Answer

$-\frac{3\sqrt{3}}{4}$

Always simplify the radical in the denominator first, before you rationalize it. This way the numbers stay smaller and easier to work with.

Example 9.5.25

Simplify: $\sqrt{\frac{5}{12}}$.

Answer

	$\sqrt{\frac{5}{12}}$
The fraction is not a perfect square, so rewrite using the Quotient Property.	$\frac{\sqrt{5}}{\sqrt{12}}$
Simplify the denominator.	$\frac{\sqrt{5}}{2\sqrt{3}}$
Rationalize the denominator.	$\frac{\sqrt{5}\cdot\sqrt{3}}{2\sqrt{3}\cdot\sqrt{3}}$
Simplify.	$\frac{\sqrt{15}}{2\cdot 3}$
Simplify.	$\frac{\sqrt{15}}{6}$

Example 9.5.26

Simplify: $\sqrt{\frac{7}{18}}$.

Answer

$\frac{\sqrt{14}}{6}$

Example 9.5.27

Simplify: $\sqrt{\frac{3}{32}}$.

Answer

$\frac{\sqrt{6}}{8}$

Example 9.5.28

Simplify: $\sqrt{\frac{11}{28}}$

Answer

	$\sqrt{\frac{11}{28}}$
Rewrite using the Quotient Property.	$\frac{\sqrt{11}}{\sqrt{28}}$
Simplify the denominator.	$\frac{\sqrt{11}}{2\sqrt{7}}$

Rationalize the denominator.	$\frac{\sqrt{11}\cdot\sqrt{7}}{2\sqrt{7}\cdot\sqrt{7}}$
Simplify.	$\frac{\sqrt{77}}{2\cdot 7}$
Simplify.	$\frac{\sqrt{77}}{14}$

Example 9.5.29

Simplify: $\sqrt{\frac{3}{27}}$.

Answer

$\frac{1}{3}$

Example 9.5.30

Simplify: $\sqrt{\frac{10}{50}}$

Answer

$\frac{\sqrt{5}}{5}$

Rationalize a Two-Term Denominator

When the denominator of a fraction is a sum or difference with square roots, we use the Product of Conjugates pattern to rationalize the denominator.

$$\begin{array}{ll} (a-b)(a+b) & (2-\sqrt{5})(2+\sqrt{5}) \\ a^2-b^2 & 2^2-(\sqrt{5})^2 \\ & 4-5 \\ & -1 \end{array}$$

When we multiply a binomial that includes a square root by its conjugate, the product has no square roots.

Example 9.5.31

Simplify: $\frac{4}{4+\sqrt{2}}$.

Answer

	$\frac{4}{4+\sqrt{2}}$
Multiply the numerator and denominator by the conjugate of the denominator.	$\frac{4(4-\sqrt{2})}{(4+\sqrt{2})(4-\sqrt{2})}$
Multiply the conjugates in the denominator.	$\frac{4(4-\sqrt{2})}{4^2-(\sqrt{2})^2}$
Simplify the denominator.	$\frac{4(4-\sqrt{2})}{16-2}$
Simplify the denominator.	$\frac{4(4-\sqrt{2})}{14}$

Remove common factors from the numerator and denominator.	$\frac{2(4-\sqrt{2})}{7}$
We leave the numerator in factored form to make it easier to look for common factors after we have simplified the denominator.	

Example 9.5.32

Simplify: $\frac{2}{2+\sqrt{3}}$.

Answer

$\frac{2(2-\sqrt{3})}{1}$

Example 9.5.33

Simplify: $\frac{5}{5+\sqrt{3}}$.

Answer

$\frac{5(5-\sqrt{3})}{22}$

Example 9.5.34

Simplify: $\frac{5}{2-\sqrt{3}}$.

Answer

	$\frac{5}{2-\sqrt{3}}$
Multiply the numerator and denominator by the conjugate of the denominator.	$\frac{5(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})}$
Multiply the conjugates in the denominator.	$\frac{5(2+\sqrt{3})}{2^2-(\sqrt{3})^2}$
Simplify the denominator.	$\frac{5(2+\sqrt{3})}{4-3}$
Simplify the denominator.	$\frac{5(2+\sqrt{3})}{1}$
Simplify.	$5(2+\sqrt{3})$

Example 9.5.35

Simplify: $\frac{3}{1-\sqrt{5}}$.

Answer

$-\frac{3(1+\sqrt{5})}{4}$

Example 9.5.36

Simplify: $\frac{2}{4-\sqrt{6}}$.

Answer

$\frac{4+\sqrt{6}}{5}$

Example 9.5.37

Simplify: $\frac{\sqrt{3}}{\sqrt{u}-\sqrt{6}}$.

Answer

	$\frac{\sqrt{3}}{\sqrt{u}-\sqrt{6}}$
Multiply the numerator and denominator by the conjugate of the denominator.	$\frac{\sqrt{3}(\sqrt{u}+\sqrt{6})}{(\sqrt{u}-\sqrt{6})(\sqrt{u}+\sqrt{6})}$
Multiply the conjugates in the denominator.	$\frac{\sqrt{3}(\sqrt{u}+\sqrt{6})}{(\sqrt{u})^2-(\sqrt{6})^2}$
Simplify the denominator.	$\frac{\sqrt{3}(\sqrt{u}+\sqrt{6})}{u-6}$

Example 9.5.38

Simplify: $\frac{\sqrt{5}}{\sqrt{x}+\sqrt{2}}$.

Answer

$\frac{\sqrt{5}(\sqrt{x}-\sqrt{2})}{x-2}$

Example 9.5.39

Simplify: $\frac{\sqrt{10}}{\sqrt{y}-\sqrt{3}}$.

Answer

$\frac{\sqrt{10}(\sqrt{y}+\sqrt{3})}{y-3}$

Example 9.5.40

Simplify: $\frac{\sqrt{x}+\sqrt{7}}{\sqrt{x}-\sqrt{7}}$.

Answer

	$\frac{\sqrt{x}+\sqrt{7}}{\sqrt{x}-\sqrt{7}}$
Multiply the numerator and denominator by the conjugate of the denominator.	$\frac{(\sqrt{x}+\sqrt{7})(\sqrt{x}+\sqrt{7})}{(\sqrt{x}-\sqrt{7})(\sqrt{x}+\sqrt{7})}$

Multiply the conjugates in the denominator.	$\frac{(\sqrt{x}+\sqrt{7})(\sqrt{x}+\sqrt{7})}{(\sqrt{x})^2-(\sqrt{7})^2}$
Simplify the denominator.	$\frac{(\sqrt{x}+\sqrt{7})^2}{x-7}$
We do not square the numerator. In factored form, we can see there are no common factors to remove from the numerator and denominator.	

Example 9.5.41

Simplify: $\frac{\sqrt{p}+\sqrt{2}}{\sqrt{p}-\sqrt{2}}$.

Answer

$\frac{(\sqrt{p}+\sqrt{2})^2}{p-2}$

Example 9.5.42

Simplify: $\frac{\sqrt{q}-\sqrt{10}}{\sqrt{q}+\sqrt{10}}$.

Answer

$\frac{(\sqrt{q}-\sqrt{10})^2}{q-10}$

Access this online resource for additional instruction and practice with dividing and rationalizing.

- Dividing and Rationalizing

Key Concepts

- **Quotient Property of Square Roots**
  - If a, b are non-negative real numbers and $b \neq 0$, then

    $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ and $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$
- **Simplified Square Roots**
  A square root is considered simplified if there are
  - no perfect square factors in the radicand
  - no fractions in the radicand
  - no square roots in the denominator of a fraction

Glossary

rationalizing the denominator

The process of converting a fraction with a radical in the denominator to an equivalent fraction whose denominator is an integer is called rationalizing the denominator.

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9.5E: Exercises

Practice Makes Perfect

Divide Square Roots

In the following exercises, simplify.

Example 9.5E.43

$\frac{\sqrt{27}}{6}$

Answer

$\frac{\sqrt{3}}{2}$

Example 9.5E.44

$\frac{\sqrt{50}}{10}$

Example 9.5E.45

$\frac{\sqrt{72}}{9}$

Answer

$\frac{2\sqrt{2}}{3}$

Example 9.5E.46

$\frac{\sqrt{243}}{6}$

Example 9.5E.47

$\frac{2-\sqrt{32}}{8}$

Answer

$\frac{1-2\sqrt{2}}{4}$

Example 9.5E.48

$\frac{3+\sqrt{27}}{9}$

Example 9.5E.49

$\frac{6+\sqrt{45}}{6}$

Answer

$\frac{2+\sqrt{5}}{2}$

Example 9.5E.50

$\frac{10-\sqrt{200}}{20}$

Example 9.5E.51

$\frac{\sqrt{80}}{\sqrt{125}}$

Answer

$\frac{4}{5}$

Example 9.5E.52

$\frac{\sqrt{72}}{\sqrt{200}}$

Example 9.5E.53

$\frac{\sqrt{128}}{\sqrt{72}}$

Answer

$\frac{4}{3}$

Example 9.5E.54

$\frac{\sqrt{48}}{\sqrt{75}}$

Example 9.5E.55

1. $\frac{\sqrt{8x^6}}{2x^2}$
2. $\frac{\sqrt{200m^5}}{98m}$

Answer

1. $2x^2$
2. $\frac{10m^2}{7}$

Example 9.5E.56

1. $\frac{\sqrt{10y^3}}{5y}$
2. $\frac{\sqrt{108n^7}}{243n^3}$

Example 9.5E.57

$\frac{\sqrt{75r^3}}{108r}$

Answer

$\frac{5r}{6}$

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Example 9.5E.58

$\frac{\sqrt{196q^5}}{484q}$

Example 9.5E.59

$\frac{\sqrt{108p^5q^2}}{\sqrt{34p^3q^6}}$

Answer

$\frac{3p\sqrt{102}}{17q^2}$

Example 9.5E.60

$\frac{\sqrt{98rs^{10}}}{\sqrt{2r^3s^4}}$

Example 9.5E.61

$\frac{\sqrt{320mn^5}}{\sqrt{45m^7n^3}}$

Answer

$\frac{8n}{3m^3}$

Example 9.5E.62

$\frac{\sqrt{810c^3d^7}}{\sqrt{1000c^5d}}$

Example 9.5E.63

$\frac{\sqrt{98}}{14}$

Answer

$\frac{\sqrt{2}}{2}$

Example 9.5E.64

$\frac{\sqrt{72}}{18}$

Example 9.5E.65

$\frac{5+\sqrt{125}}{15}$

Answer

$\frac{1+\sqrt{3}}{3}$

Example 9.5E.66

$\frac{6-\sqrt{45}}{12}$

Example 9.5E.67

$\frac{\sqrt{96}}{\sqrt{150}}$

Answer

$\frac{4}{5}$

Example 9.5E.68

$\frac{\sqrt{28}}{\sqrt{63}}$

Example 9.5E.69

$\frac{\sqrt{26y^7}}{2y}$

Answer

$y^3\sqrt{13}$

Example 9.5E.70

$\frac{\sqrt{15x^3}}{\sqrt{3x}}$

Rationalize a One-Term Denominator

In the following exercises, simplify and rationalize the denominator.

Example 9.5E.71

$\frac{10}{\sqrt{6}}$

Answer

$\frac{5\sqrt{6}}{3}$

Example 9.5E.72

$\frac{8}{\sqrt{3}}$

Example 9.5E.73

$\frac{6}{\sqrt{7}}$

Answer

$\frac{6\sqrt{7}}{7}$

Example 9.5E.74

$\frac{4}{\sqrt{5}}$

Example 9.5E.75

$\frac{3}{\sqrt{13}}$

Answer

$\frac{3\sqrt{13}}{13}$

Example 9.5E.76

$\frac{10}{\sqrt{11}}$

Example 9.5E.77

$\frac{10}{3\sqrt{10}}$

Answer

$\frac{\sqrt{10}}{3}$

Example 9.5E.78

$\frac{2}{5\sqrt{2}}$

Example 9.5E.79

$\frac{4}{9\sqrt{5}}$

Answer

$\frac{4\sqrt{5}}{45}$

Example 9.5E.80

$\frac{9}{2\sqrt{7}}$

Example 9.5E.81

$-\frac{9}{2\sqrt{3}}$

Answer

$-\frac{3\sqrt{3}}{2}$

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Example 9.5E.82

$-\frac{8}{3\sqrt{6}}$

Example 9.5E.83

$\sqrt{\frac{3}{20}}$

Answer

$\frac{\sqrt{15}}{10}$

Example 9.5E.84

$\sqrt{\frac{4}{27}}$

Example 9.5E.85

$\sqrt{\frac{7}{40}}$

Answer

$\frac{\sqrt{70}}{20}$

Example 9.5E.86

$\sqrt{\frac{8}{45}}$

Example 9.5E.87

$\sqrt{\frac{19}{175}}$

Answer

$\frac{\sqrt{133}}{35}$

Example 9.5E.88

$\sqrt{\frac{17}{192}}$

Rationalize a Two-Term Denominator

In the following exercises, simplify by rationalizing the denominator.

Example 9.5E.89

1. $\frac{3}{3+\sqrt{11}}$
2. $\frac{8}{1-\sqrt{5}}$

Answer

1. $\frac{3(3-\sqrt{11})}{-2}$

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2. $-2(1+\sqrt{5})$

Example 9.5E.90

1. $\frac{4}{4+\sqrt{7}}$
2. $\frac{7}{2-\sqrt{6}}$

Example 9.5E.91

1. $\frac{5}{5+\sqrt{6}}$
2. $\frac{6}{3-\sqrt{7}}$

Answer

1. $\frac{5(5-\sqrt{6})}{19}$
2. $3(3+\sqrt{7})$

Example 9.5E.92

1. $\frac{6}{6+\sqrt{5}}$
2. $\frac{5}{4-\sqrt{11}}$

Example 9.5E.93

$\frac{\sqrt{3}}{\sqrt{m}-\sqrt{5}}$

Answer

$\frac{\sqrt{3}(\sqrt{m}+\sqrt{5})}{m-5}$

Example 9.5E.94

$\frac{\sqrt{5}}{\sqrt{n}-\sqrt{7}}$

Example 9.5E.95

$\frac{\sqrt{2}}{\sqrt{x}-\sqrt{6}}$

Answer

$\frac{\sqrt{2}(\sqrt{x}+\sqrt{3})}{x-6}$

Example 9.5E.96

$\frac{\sqrt{7}}{\sqrt{y}+\sqrt{3}}$

Example $9.5E.97$

$\frac{\sqrt{r}+\sqrt{5}}{\sqrt{r}-\sqrt{5}}$

Answer

$\frac{(\sqrt{r}+\sqrt{5})^2}{r-5}$

Example $9.5E.98$

$\frac{\sqrt{s}-\sqrt{6}}{\sqrt{s}+\sqrt{6}}$

Example $9.5E.99$

$\frac{\sqrt{150x^2y^6}}{\sqrt{6x^4y^2}}$

Answer

$\frac{5y^2}{x}$

Example $9.5E.100$

$\frac{\sqrt{80p^3q}}{\sqrt{5pq^5}}$

Example $9.5E.101$

$\frac{15}{\sqrt{5}}$

Answer

$3\sqrt{5}$

Example $9.5E.102$

$\frac{3}{5\sqrt{8}}$

Example $9.5E.103$

$\sqrt{\frac{8}{54}}$

Answer

$\frac{2\sqrt{3}}{9}$

Example $9.5E.104$

$\sqrt{\frac{12}{20}}$

Example 9.5E.105

$\frac{3}{5+\sqrt{5}}$

Answer

$\frac{3(5-\sqrt{5})}{20}$

Example 9.5E.106

$\frac{20}{4-\sqrt{3}}$

Example 9.5E.107

$\frac{\sqrt{2}}{\sqrt{x}-\sqrt{3}}$

Answer

$\frac{\sqrt{2}(\sqrt{x}+\sqrt{3})}{x-3}$

Example 9.5E.108

$\frac{\sqrt{5}}{\sqrt{y}-\sqrt{7}}$

Example 9.5E.109

$\frac{\sqrt{x}+\sqrt{8}}{\sqrt{x}-\sqrt{8}}$

Answer

$\frac{(\sqrt{x}+2\sqrt{2})^2}{x-8}$

Example 9.5E.110

$\frac{\sqrt{m}-\sqrt{3}}{\sqrt{m}+\sqrt{3}}$

Everyday Math

Example 9.5E.111

A supply kit is dropped from an airplane flying at an altitude of 250 feet. Simplify $\sqrt{\frac{250}{16}}$ to determine how many seconds it takes for the supply kit to reach the ground.

Answer

$\frac{5\sqrt{10}}{4}$ seconds

Example 9.5E.112

A flare is dropped into the ocean from an airplane flying at an altitude of 1,200 feet. Simplify $\sqrt{\frac{1200}{16}}$ to determine how many seconds it takes for the flare to reach the ocean.

Writing Exercises

Example 9.5E.113

1. Simplify $\sqrt{\frac{27}{3}}$ and explain all your steps.
2. Simplify $\sqrt{\frac{27}{5}}$ and explain all your steps.
3. Why are the two methods of simplifying square roots different?

Answer

Answers will vary.

Example 9.5E.114

1. Approximate $\frac{1}{\sqrt{2}}$ by dividing $\frac{1}{1.414}$ using long division without a calculator.
2. Rationalizing the denominator of $\frac{1}{\sqrt{2}}$ gives $\frac{\sqrt{2}}{2}$. Approximate $\frac{\sqrt{2}}{2}$ by dividing $\frac{1.414}{2}$ using long division without a calculator.
3. Do you agree that rationalizing the denominator makes calculations easier? Why or why not?

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
divide square roots.			
rationalize a one-term denominator.			
rationalize a two-term denominator.			

ⓑ After looking at the checklist, do you think you are well-prepared for the next section? Why or why not?

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CHAPTER OVERVIEW

10: Solving Quadratic Equations and Graphing Parabolas

10.1: Extracting Square Roots

10.2: Quadratic Formula

10.3: Graphing Parabolas

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10.1: Extracting Square Roots

Learning Objectives

- Solve quadratic equations by extracting square roots.

Extracting Square Roots

Recall that a quadratic equation is in **standard form** if it is equal to 0:

$$ax^2+bx+c=0 \tag{10.1.1}$$

where ***a***, ***b***, and ***c*** are real numbers and $a \neq 0$. A solution to such an equation is called a **root**. Quadratic equations can have two real solutions, one real solution, or no real solution. If the quadratic expression on the left factors, then we can solve it by factoring. A review of the steps used to solve by factoring follow:

Step 1: Express the quadratic equation in standard form.

Step 2: Factor the quadratic expression.

Step 3: Apply the zero-product property and set each variable factor equal to 0.

Step 4: Solve the resulting linear equations.

For example, we can solve $x^2-4=0$ by factoring as follows:

The two solutions are −2 and 2. The goal in this section is to develop an alternative method that can be used to easily solve equations where ***b*** = 0, giving the form

$$ax^2+c=0 \tag{10.1.2}$$

The equation $x^2-4=0$ is in this form and can be solved by first isolating x^2.

If we take the square root of both sides of this equation, we obtain the following:

$$\begin{aligned}\sqrt{x^2} &= \sqrt{4}\\ |x| &= 2\end{aligned}$$

Here we see that $x=-2$ and $x=2$ are solutions to the resulting equation. In general, this describes the **square root property**; for any real number ***k***,

$$\text{If } x^2=k, \text{ then } x=\pm\sqrt{k} \tag{10.1.3}$$

The notation "±" is read "plus or minus" and is used as compact notation that indicates two solutions. Hence the statement $x=\pm\sqrt{k}$ indicates that $x=-\sqrt{k}$ or $x=+\sqrt{k}$. Applying the square root property as a means of solving a quadratic equation is called **extracting the roots**.

Example 10.1.1

Solve:

$$x^2-25=0$$

Solution:

Begin by isolating the square.

Next, apply the square root property.

$$\begin{aligned}x^2 &= 25\\ x &= \pm\sqrt{25}\\ x &= \pm 5\end{aligned}$$

Answer:

The solutions are −5 and 5. The check is left to the reader.

Certainly, the previous example could have been solved just as easily by factoring. However, it demonstrates a technique that can be used to solve equations in this form that do not factor.

Example 10.1.2

Solve:

$$x^2 - 5 = 0$$

Solution:

Notice that the quadratic expression on the left does not factor. We can extract the roots if we first isolate the leading term, x^2.

Apply the square root property.

$x = \pm\sqrt{5}$

For completeness, check that these two real solutions solve the original quadratic equation. Generally, the check is optional.

Check $x = -\sqrt{5}$	Check $x = \sqrt{5}$
$x^2 - 5 = 0$	$x^2 - 5 = 0$
$(-\sqrt{5})^2 - 5 = 0$	$(\sqrt{5})^2 - 5 = 0$
$5 - 5 = 0$	$5 - 5 = 0$
$0 = 0$✓	$0 = 0$✓

Answer:

The solutions are $-\sqrt{5}$ and $\sqrt{5}$.

Example 10.1.3

Solve

Solution

Begin by isolating x^2.

Apply the square root property and then simplify.

$$x = \pm\sqrt{\frac{45}{4}}$$
$$x = \pm\frac{\sqrt{9\cdot 5}}{\sqrt{4}}$$
$$x = \pm\frac{3\sqrt{5}}{2}$$

Answer:

The solutions are $-\frac{3\sqrt{5}}{2}$ and $\frac{3\sqrt{5}}{2}$.

Sometimes quadratic equations have no real solution.

Example 10.1.4

Solve:

$$x^2 + 9 = 0$$

Solution

Begin by isolating x^2.

After applying the square root property, we are left with the square root of a negative number. Therefore, there is no real solution to this equation.

Answer:

No real solution

Reverse this process to find equations with given solutions of the form **±*k***.

Find an equation with solutions $-2\sqrt{3}$ and $2\sqrt{3}$.

Solution:

Begin by squaring both sides of the following equation:

$$x = \pm 2\sqrt{3}$$
$$x^2 = (\pm 2\sqrt{3})^2$$
$$x^2 = 4 \cdot 3$$
$$x^2 = 12$$

Lastly, subtract 12 from both sides and present the equation in standard form.

Answer:

$x^2 - 12 = 0$

? Exercise 10.1.1

Solve:

$$9x^2 - 8 = 0\,.$$

Answer

$x = -\frac{2\sqrt{2}}{3}$ or $x = \frac{2\sqrt{2}}{3}$

Consider solving the following equation:

$$(x+2)^2 = 25$$

To solve this equation by factoring, first square x+2 and then put it in standard form, equal to zero, by subtracting 25 from both sides.

Factor and then apply the zero-product property.

$x^2 + 4x - 21 = 0$
$(x+7)(x-3) = 0$

$$x+7=0 \quad \text{or} \quad x-3=0$$
$$x=-7 \qquad\qquad x=3$$

The two solutions are -7 and 3.

When an equation is in this form, we can obtain the solutions in fewer steps by extracting the roots.

✔ Example 10.1.6

Solve:

$$(x=2)^2 = 25\,.$$

Solution

Solve by extracting the roots.

$$\begin{aligned} (x+2)^2 &= 25 && \textit{Apply the square root property.} \\ x+2 &= \pm\sqrt{25} && \textit{Simplify.} \\ x+2 &= \pm 5 \\ x &= -2 \pm 5 \end{aligned}$$

At this point, separate the "plus or minus" into two equations and simplify each individually.

$$\begin{aligned} x &= -2-5 \quad \text{or } x = -2+5 \\ x &= -7 \qquad\quad x = 3 \end{aligned}$$

Answer:

The solutions are -7 and 3.

In addition to fewer steps, this method allows us to solve equations that do not factor.

Example 10.1.7

Solve:

$$(3x+3)^2 - 27 = 0$$

Solution

Begin by isolating the square.

Next, extract the roots and simplify.

$$\begin{aligned} (3x+3)^2 &= 27 \\ 3x+3 &= \pm\sqrt{27} \\ 3x+3 &= \pm\sqrt{9\cdot 3} \\ 3x+3 &= \pm 3\sqrt{3} \end{aligned}$$

Solve for ***x***.

Answer:

The solutions are $-1-\sqrt{3}$ and $-1+\sqrt{3}$

Solve:

$$9(2x-1)^2 - 8 = 0$$

Solution:

Begin by isolating the square factor.

Apply the square root property and solve.

Answer:

The solutions are $\frac{3-2\sqrt{2}}{6}$ and $\frac{3+2\sqrt{2}}{6}$

Exercise 10.1.2

Solve:

$$3(x-5)^2 - 2 = 0$$

Answer

$15 \pm 6\sqrt{3}$

Example 10.1.9

The length of a rectangle is twice its width. If the diagonal measures 2 feet, then find the dimensions of the rectangle.

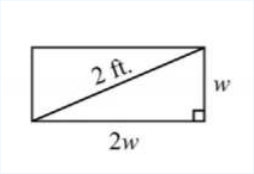


Figure 10.1.1

Solution

Let w represent the width.

Let $2w$ represent the length.

The diagonal of any rectangle forms two right triangles. Thus the Pythagorean theorem applies. The sum of the squares of the legs of a right triangle is equal to the square of the hypotenuse:

$$(\text{leg})^2+(\text{leg})^2=\text{ hypotenuse }^2$$
$$(2w)^2+(w)^2=(2)^2$$

Solve.

$$\begin{aligned}(2w)^2+(w)^2&=(2)^2\\ 4w^2+w^2&=4\\ 5w^2&=4 \qquad \textit{Isolate } w^2.\\ \frac{5w^2}{5}&=\frac{4}{5}\\ w^2&=\frac{4}{5} \qquad \textit{Extract the roots.}\\ w&=\pm\sqrt{\frac{4}{5}}\\ w&=\pm\frac{2}{\sqrt{5}}\end{aligned}$$

Here we obtain two solutions, $w=-\frac{2}{\sqrt{5}}$ and $w=\frac{2}{\sqrt{5}}$. Since the problem asked for a length of a rectangle, we disregard the negative answer. Furthermore, we will rationalize the denominator and present our solutions without any radicals in the denominator.

$$\begin{aligned}w&=\frac{2}{\sqrt{5}} \qquad \textit{Rationalize the denominator.}\\ &=\frac{2}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}}\\ &=\frac{2\sqrt{5}}{\sqrt{25}}\\ &=\frac{2\sqrt{5}}{5}\end{aligned}$$

Back substitute to find the length.

$$\begin{aligned}l&=2w\\ &=2\left(\frac{2\sqrt{5}}{5}\right)\\ &=\frac{4\sqrt{5}}{5}\end{aligned}$$

Answer:

The length of the rectangle is $\frac{4\sqrt{5}}{5}$ feet and the width is $\frac{2\sqrt{5}}{5}$ feet.

Key Takeaways

- Solve equations of the form $ax^2+c=0$ by extracting the roots.
- Extracting roots involves isolating the square and then applying the square root property. After applying the square root property, you have two linear equations that each can be solved. Be sure to simplify all radical expressions and rationalize the denominator if necessary.

Exercise 10.1.3 extracting square roots

Solve by factoring and then solve by extracting roots. Check answers.

1. $x^2-36=0$
2. $x^2-81=0$
3. $4y^2-9=0$
4. $9y^2-25=0$
5. $(x-2)^2-1=0$
6. $(x+1)^2-4=0$
7. $4(y-2)^2-9=0$
8. $9(y+1)^2-4=0$
9. $-3(t-1)^2+12=0$
10. $-2(t+1)^2+8=0$
11. $(x-5)^2-25=0$
12. $(x+2)^2-4=0$

Answer

1. $-6,6$
3. $-\frac{3}{2},\frac{3}{2}$
5. $1,3$
7. $\frac{1}{2},\frac{7}{2}$
9. $-1,3$
11. $0,10$

Exercise 10.1.4 extracting square roots

Solve by extracting the roots.

1. $x^2=16$
2. $x^2=1$
3. $y^2=9$
4. $y^2=64$
5. $x^2=14$
6. $x^2=19$
7. $y^2=0.25$
8. $y^2=0.04$
9. $x^2=12$
10. $x^2=18$
11. $16x^2=9$
12. $4x^2=25$
13. $2t^2=1$
14. $3t^2=2$

15. $x^2 - 100 = 0$
16. $x^2 - 121 = 0$
17. $y^2 + 4 = 0$
18. $y^2 + 1 = 0$
19. $x^2 - 49 = 0$
20. $x^2 - 925 = 0$
21. $y^2 - 0.09 = 0$
22. $y^2 - 0.81 = 0$
23. $x^2 - 7 = 0$
24. $x^2 - 2 = 0$
25. $x^2 - 8 = 0$
26. $t^2 - 18 = 0$
27. $x^2 + 8 = 0$
28. $x^2 + 125 = 0$
29. $16x^2 - 27 = 0$
30. $9x^2 - 8 = 0$
31. $2y^2 - 3 = 0$
32. $5y^2 - 2 = 0$
33. $3x^2 - 1 = 0$
34. $6x^2 - 3 = 0$
35. $(x + 7)^2 - 4 = 0$
36. $(x + 9)^2 - 36 = 0$
37. $(2y - 3)^2 - 81 = 0$
38. $(2y + 1)^2 - 25 = 0$
39. $(x - 5)^2 - 20 = 0$
40. $(x + 1)^2 - 28 = 0$
41. $(3t + 2)^2 - 6 = 0$
42. $(3t - 5)^2 - 10 = 0$
43. $4(y + 2)^2 - 3 = 0$
44. $9(y - 7)^2 - 5 = 0$
45. $4(3x + 1)^2 - 27 = 0$
46. $9(2x - 3)^2 - 8 = 0$
47. $2(3x - 1)^2 + 3 = 0$
48. $5(2x - 1)^2 - 3 = 0$
49. $3(y - 23)^2 - 32 = 0$
50. $2(3y - 13)^2 - 52 = 0$

Answer

1. ± 4
3. ± 3
5. $\pm \frac{1}{2}$
7. ± 0.5
9. $\pm 2\sqrt{3}$
11. $\pm \frac{3}{4}$
13. $\pm \sqrt{\frac{1}{2}}$
15. ± 10
17. No real solution
19. $\pm \frac{2}{3}$

21. ± 0.3
23. $\pm\sqrt{7}$
25. $\pm 2\sqrt{2}$
27. No real solution
29. $\pm\frac{3\sqrt{3}}{4}$
31. $\pm 6\sqrt{2}$
33. $\pm 3\sqrt{3}$
35. $-9, -5$
37. $-3, 6$
39. $5 \pm 2\sqrt{5}$
41. $\pm\frac{\sqrt{6}-2}{3}$
43. $\pm\frac{\sqrt{3}}{2} - 2$
45. $\pm\frac{3\sqrt{3}-2}{6}$
47. No real solution
49. $\pm\frac{4\sqrt{6}}{3} + 23$

Exercise 10.1.5 extracting square roots

Find a quadratic equation in standard form with the following solutions.

1. ± 7
2. ± 13
3. $\pm\sqrt{7}$
4. $\pm\sqrt{3}$
5. $\pm 3\sqrt{5}$
6. $\pm 5\sqrt{2}$
7. $1 \pm \sqrt{2}$
8. $2 \pm \sqrt{3}$

Answer

1. $x^2 - 49 = 0$
3. $x^2 - 7 = 0$
5. $x^2 - 45 = 0$
7. $x^2 - 2x - 1 = 0$

Exercise 10.1.6 extracting square roots

Solve and round off the solutions to the nearest hundredth.

1. $9x(x+2) = 18x + 1$
2. $x^2 = 10(x^2 - 2) - 5$
3. $(x+3)(x-7) = 11 - 4x$
4. $(x-4)(x-3) = 66 - 7x$
5. $(x-2)^2 = 67 - 4x$
6. $(x+3)^2 = 6x + 59$

7. $(2x+1)(x+3)-(x+7)=(x+3)^2$
8. $(3x-1)(x+4)=2x(x+6)-(x-3)$

Answer

1. ±0.33
3. ±5.66
5. ±7.94
7. ±3.61

Exercise 10.1.7 extracting square roots

Set up an algebraic equation and use it to solve the following.

1. If 9 is subtracted from 4 times the square of a number, then the result is 3. Find the number.
2. If 20 is subtracted from the square of a number, then the result is 4. Find the number.
3. If 1 is added to 3 times the square of a number, then the result is 2. Find the number.
4. If 3 is added to 2 times the square of a number, then the result is 12. Find the number.
5. If a square has an area of 8 square centimeters, then find the length of each side.
6. If a circle has an area of 32π square centimeters, then find the length of the radius.
7. The volume of a right circular cone is 36π cubic centimeters when the height is 6 centimeters. Find the radius of the cone. (The volume of a right circular cone is given by $V=13\pi r^2 h$.)
8. The surface area of a sphere is 75π square centimeters. Find the radius of the sphere. (The surface area of a sphere is given by SA=$4\pi r^2$.)
9. The length of a rectangle is 6 times its width. If the area is 96 square inches, then find the dimensions of the rectangle.
10. The base of a triangle is twice its height. If the area is 16 square centimeters, then find the length of its base.
11. A square has an area of 36 square units. By what equal amount will the sides have to be increased to create a square with double the given area?
12. A circle has an area of 25π square units. By what amount will the radius have to be increased to create a circle with double the given area?
13. If the sides of a square measure 1 unit, then find the length of the diagonal.
14. If the sides of a square measure 2 units, then find the length of the diagonal.
15. The diagonal of a square measures 5 inches. Find the length of each side.
16. The diagonal of a square measures 3 inches. Find the length of each side.
17. The length of a rectangle is twice its width. If the diagonal measures 10 feet, then find the dimensions of the rectangle.
18. The length of a rectangle is twice its width. If the diagonal measures 8 feet, then find the dimensions of the rectangle.
19. The length of a rectangle is 3 times its width. If the diagonal measures 5 meters, then find the dimensions of the rectangle.
20. The length of a rectangle is 3 times its width. If the diagonal measures 2 feet, then find the dimensions of the rectangle.
21. The height in feet of an object dropped from a 9-foot ladder is given by h(t)= $-16t^2+9$, where t represents the time in seconds after the object has been dropped. How long does it take the object to hit the ground? (Hint: The height is 0 when the object hits the ground.)
22. The height in feet of an object dropped from a 20-foot platform is given by h(t)= $-16t^2+20$, where t represents the time in seconds after the object has been dropped. How long does it take the object to hit the ground?
23. The height in feet of an object dropped from the top of a 144-foot building is given by h(t)= $-16t^2+144$, where t is measured in seconds.
    a. How long will it take to reach half of the distance to the ground, 72 feet?
    b. How long will it take to travel the rest of the distance to the ground? Round off to the nearest hundredth of a second.
24. The height in feet of an object dropped from an airplane at 1,600 feet is given by h(t)= $-16t^2+1,600$, where t is in seconds.
    a. How long will it take to reach half of the distance to the ground?
    b. How long will it take to travel the rest of the distance to the ground? Round off to the nearest hundredth of a second.

25. Create an equation of your own that can be solved by extracting the root. Share it, along with the solution, on the discussion board.
26. Explain why the technique of extracting roots greatly expands our ability to solve quadratic equations.
27. Explain in your own words how to solve by extracting the roots. 106. Derive a formula for the diagonal of a square in terms of its sides.

Answer

1. $-\sqrt{3}$ or $\sqrt{3}$
3. $-3\sqrt{3}$ or $3\sqrt{3}$
5. $2\sqrt{2}$ centimeters
7. $3\sqrt{2}$ centimeters
9. Length: 24 inches; width: 4 inches
11. $-6+6\sqrt{2}\approx 2.49$ units
13. $\sqrt{2}$ units
15. $52\sqrt{2}$ inches
17. Length: $4\sqrt{5}$ feet; width: $2\sqrt{5}$ feet
19. Length: $310--\sqrt{2}$ meters; width: $10--\sqrt{2}$ meters
21. $\frac{3}{4}$ second
23. a. 2.12 seconds b. 0.88 second
25. Answers may vary

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10.2: Quadratic Formula

Learning Objectives

- Solve quadratic equations with real solutions using the quadratic formula.

The Quadratic Formula

In this section, we will develop a formula that gives the solutions to any quadratic equation in standard form. To do this, we begin with a general quadratic equation in standard form and solve for ***x*** by completing the square. Here ***a***, ***b***, and ***c*** are real numbers and $a \neq 0$:

$$\begin{aligned} ax^2+bx+c &= 0 \\ \frac{ax^2+bx+c}{a} &= \frac{0}{a} && \textit{Divide both sides by } a. \\ x^2+\frac{b}{a}x+\frac{c}{a} &= 0 && \textit{Subtract } \frac{c}{a} \textit{ from both sides.} \\ x^2+\frac{b}{a}x &= -\frac{c}{a} \end{aligned}$$

Determine the constant that completes the square: take the coefficient of ***x***, divide it by 2, and then square it.

$$\left(\frac{b/a}{2}\right)^2 = \left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} \tag{10.2.1}$$

Add this to both sides of the equation and factor.

Solve by extracting roots.

This derivation gives us a formula that solves any quadratic equation in standard form. Given $ax^2+bx+c=0$, where ***a***, ***b***, and ***c*** are real numbers and $a \neq 0$, then the solutions can be calculated using the quadratic formula:

Consider the quadratic equation $2x^2-7x+3=0$. It can be solved by factoring as follows:

$$\begin{aligned} 2x-1=0 \quad & \text{or } x-3=0 \\ 2x=1 \quad & \qquad x=3 \\ x=\tfrac{1}{2} \quad & \end{aligned}$$

The solutions are 1/2 and 3. The following example shows that we can obtain the same results using the quadratic formula.

Example 10.2.1

Solve using the quadratic formula:

$$2x^2-7x+3=0 \text{ .}$$

Solution

Begin by identifying ***a***, ***b***, and ***c*** as the coefficients of each term.

Substitute these values into the quadratic formula and then simplify.

Separate the “plus or minus” into two equations and simplify each individually.

$$\begin{aligned} x=\tfrac{7-5}{4} \quad & \text{or} \quad x=\tfrac{7+5}{4} \\ x=\tfrac{2}{4} \quad & \qquad x=\tfrac{12}{4} \\ x=\tfrac{1}{2} \quad & \qquad x=3 \end{aligned}$$

Answer:

The solutions are 1/2 and 3.

Of course, if the quadratic factors, then it is a best practice to solve it by factoring. However, not all quadratic polynomials factor; nevertheless, the quadratic formula provides us with a means to solve such equations.

Example 10.2.2

Solve using the quadratic formula:

$$5x^2+2x-1=0\ .$$

Solution:

Begin by identifying ***a***, ***b***, and ***c***.

$$a=5 \qquad b=2 \qquad c=-1$$

Substitute these values into the quadratic formula.

Answer:

The solutions are $\frac{-1\pm\sqrt{6}}{5}$

Often terms are missing. When this is the case, use 0 as the coefficient.

Solve using the quadratic formula:

$$x^2-18=0\ .$$

Solution:

Think of this equation with the following coefficients:

$$1x^2+0x-18=0$$

Here

$$a=1 \qquad b=0 \qquad c=-18$$

Substitute these values into the quadratic formula.

Answer:

The solutions are $\pm 3\sqrt{2}$

Since the coefficient of ***x*** was 0, we could have solved the equation by extracting the roots. As an exercise, solve the previous example using this method and verify that the results are the same.

Solve using the quadratic formula:

$$9x^2-12x+4=0\ .$$

Solution:

In this case,

Substitute these values into the quadratic formula and then simplify.

In this example, notice that the radicand of the square root is 0. This results in only one solution to this quadratic equation. Normally, we expect two solutions. When we find only one solution, the solution is called a double root. If we solve this equation by factoring, then the solution appears twice.

$$\begin{aligned} 3x-2&=0 \quad \text{or} \quad 3x-2=0\\ 3x&=2 \qquad\qquad 3x=2\\ x&=\tfrac{2}{3} \qquad\qquad x=\tfrac{2}{3} \end{aligned}$$

Answer: $\frac{2}{3}$, double root

Example 10.2.5

Solve using the quadratic formula:

$$x^2+x+1=0\ .$$

Solution:

In this case,

$$a=1 \qquad b=1 \qquad c=1$$

Substitute these values into the quadratic formula.

The solution involves the square root of a negative number; hence the solutions are not real. This quadratic equation has two non real solutions and will be discussed in further detail as we continue in our study of algebra. For now, simply state that the equation does not have real solutions.

Answer:

No real solutions

Solve $x^2-2x-2=0$.

Answer

$1\pm\sqrt{3}$

It is important to place the quadratic equation in standard form before using the quadratic formula.

Example 10.2.6

Solve using the quadratic formula:

$$(2x+1)(2x-1)=24x+8$$

Solution:

Begin by using the distributive property to expand the left side and combining like terms to obtain an equation in standard form, equal to 0.

Once the equation is in standard form, identify ***a***, ***b***, and ***c***. Here

Substitute these values into the quadratic formula and then simplify.

Answer:

The solutions are $\frac{6\pm3\sqrt{5}}{2}$

Solve $3x(x-2)=1$

Answer

$\frac{3\pm2\sqrt{3}}{3}$

Key Takeaways

- Use the quadratic formula to solve any quadratic equation in standard form.
- To solve any quadratic equation, first rewrite in standard form, $ax^2+bx+c=0$, substitute the appropriate coefficients into the quadratic formula, $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$, and then simplify.

Exercise 10.2.3 quadratic formula

Identify the coefficients a, b, and c used in the quadratic formula. Do not solve.

1. $x^2 - x + 5 = 0$
2. $x^2 - 3x - 1 = 0$
3. $3x^2 - 10 = 0$
4. $-y^2 + 5 = 0$
5. $5t^2 - 7t = 0$
6. $-y^2 + y = 0$
7. $-x^2 + x = -6$
8. $-2x^2 - x = -15$
9. $(3x + 1)(2x + 5) = 19x + 4$
10. $(4x + 1)(2x + 1) = 16x + 4$

Answer

1. a=1, b=−1, and c=5
3. a=3, b=0, and c=−10
5. a=5, b=−7, and c=0
7. a=−1, b=1, and c=6
9. a=6, b=−2, and c=1

Exercise 10.2.4 quadratic formula

Solve by factoring and then solve using the quadratic formula. Check answers.

1. $x^2 - 10x + 24 = 0$
2. $x^2 - 3x - 18 = 0$
3. $t^2 + 6t + 5 = 0$
4. $t^2 + 9t + 14 = 0$
5. $2x^2 - 7x - 4 = 0$
6. $3x^2 - x - 2 = 0$
7. $-2x^2 - x + 3 = 0$
8. $-6x^2 + x + 1 = 0$
9. $y^2 - 2y + 1 = 0$
10. $y^2 - 1 = 0$

Answer

1. 4, 6
3. −5, −1
5. −1/2, 4
7. −3/2, 1
9. 1, double root

Exercise 10.2.5 quadratic formula

Use the quadratic formula to solve the following.

1. $x^2 - 6x + 4 = 0$
2. $x^2 - 4x + 1 = 0$
3. $x^2 + 2x - 5 = 0$

4. $x^2+4x-6=0$
5. $t^2-4t-1=0$
6. $t^2-8t-2=0$
7. $-y^2+y+1=0$
8. $-y^2-3y+2=0$
9. $-x^2+16x-62=0$
10. $-x^2+14x-46=0$
11. $2t^2-4t-3=0$
12. $4t^2-8t-1=0$
13. $-4y^2+12y-9=0$
14. $-25x^2+10x-1=0$
15. $3x^2+6x+2=0$
16. $5x^2+10x+2=0$
17. $9t^2+6t-11=0$
18. $8t^2+8t+1=0$
19. $x^2-2=0$
20. $x^2-18=0$
21. $9x^2-3=0$
22. $2x^2-5=0$
23. $y^2+9=0$
24. $y^2+1=0$
25. $2x^2=0$
26. $x^2=0$
27. $-2y^2+5y=0$
28. \(-3y^{2] + 7y = 0\)
29. $t^2-t=0$
30. \(t^{2] + 2 t = 0\)
31. $x^2-0.6x-0.27=0$
32. $x^2-1.6x-0.8=0$
33. $y^2-1.4y-0.15=0$
34. $y^2-3.6y+2.03=0$
35. $12t^2+5t+32=0$
36. $-t^2+3t-34=0$
37. $3y^2+12y-13=0$
38. $-2y^2+13y+12=0$
39. $2x^2-10x+3=4$
40. $3x^2+6x+1=8$
41. $-2y^2=3(y-1)$
42. $3y^2=5(2y-1)$
43. $(t+1)^2=2t+7$
44. $(2t-1)^2=73-4t$
45. $(x+5)(x-1)=2x+1$
46. $(x+7)(x-2)=3(x+1)$
47. $x(x+5)=3(x-1)$
48. $x(x+4)=-7$
49. $(5x+3)(5x-3)-10(x-1)=0$
50. $(3x+4)(3x-1)-33x=-20$
51. $27y(y+1)+2(3y-2)=0$
52. $8(4y^2+3)-3(28y-1)=0$
53. $(x+2)^2-2(x+7)=4(x+1)$
54. $(x+3)^2-10(x+5)=-2(x+1)$

Answer

1. $3 \pm \sqrt{5}$
3. $-1 \pm \sqrt{6}$
5. $2 \pm \sqrt{5}$
7. $\frac{1 \pm \sqrt{5}}{2}$
9. $8 \pm \sqrt{2}$
11. $\frac{2 \pm \sqrt{10}}{2}$
13. $\frac{3}{2}$, double root
15. $\frac{-3 \pm \sqrt{3}}{3}$
17. $\frac{-1 \pm 2\sqrt{3}}{3}$
19. $\pm \sqrt{2}$
21. $\pm \sqrt{\frac{1}{3}}$
23. No real solutions
25. 0, double root
27. $0, \frac{5}{2}$
29. $0, 1$
31. $-0.3, 0.9$
33. $-0.1, 1.5$
35. No real solutions
37. $\frac{-6 \pm 5\sqrt{3}}{3}$
39. $\frac{5 \pm 3\sqrt{3}}{2}$
41. $\frac{-3 \pm \sqrt{33}}{4}$
43. $\pm \sqrt{6}$
45. $-1 \pm \sqrt{7}$
47. No real solutions
49. $\frac{1}{5}$, double root
51. $-\frac{4}{3}, \frac{1}{9}$
53. $1 \pm 2\sqrt{10}$

Exercise 10.2.6 discussion board

1. When talking about a quadratic equation in standard form, $ax^2 + bx + c = 0$, why is it necessary to state that a≠0? What happens if a is equal to 0?
2. Research and discuss the history of the quadratic formula and solutions to quadratic equations.

Answer

1. Answers may vary

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10.3: Graphing Parabolas

Learning Objectives

- Graph a parabola.
- Find the intercepts and vertex of a parabola.
- Find the vertex of a parabola by completing the square.

The Graph of a Quadratic Equation

We know that any linear equation with two variables can be written in the form $y = mx + b$ and that its graph is a line. In this section, we will see that any quadratic equation of the form $y = ax^2 + bx + c$ has a curved graph called a parabola.

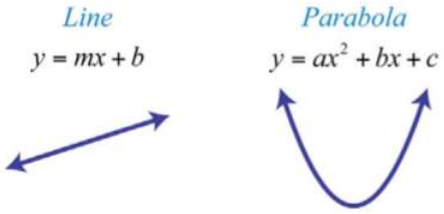


Figure 10.3.1

Two points determine any line. However, since a parabola is curved, we should find more than two points. In this text, we will determine at least five points as a means to produce an acceptable sketch. To begin, we graph our first parabola by plotting points. Given a quadratic equation of the form $y = ax^2 + bx + c$, ***x*** is the independent variable and ***y*** is the dependent variable. Choose some values for ***x*** and then determine the corresponding ***y***-values. Then plot the points and sketch the graph.

Example 10.3.1

Graph by plotting points:

$$y = x^2 - 2x - 3$$

Solution:

In this example, choose the ***x***-values $\{-2, -1, 0, 1, 2, 3, 4\}$ and calculate the corresponding ***y***-values.

x	y		*Points*
−2	5	$y = (-2)^2 - 2(-2) - 3 = 4 + 4 - 3 = 5$	(−2, 5)
−1	0	$y = (-1)^2 - 2(-1) - 3 = 1 + 2 - 3 = 0$	(−1, 0)
0	−3	$y = (0)^2 - 2(0) - 3 = 0 - 0 - 3 = -3$	(0, −3)
1	−4	$y = (1)^2 - 2(1) - 3 = 1 - 2 - 3 = -4$	(1, −4)
2	−3	$y = (2)^2 - 2(2) - 3 = 4 - 4 - 3 = -3$	(2, −3)
3	0	$y = (3)^2 - 2(3) - 3 = 9 - 6 - 3 = 0$	(3, 0)
4	5	$y = (4)^2 - 2(4) - 3 = 16 - 8 - 3 = 5$	(4, 5)

Figure 10.3.2

Plot these points and determine the shape of the graph.

Answer:

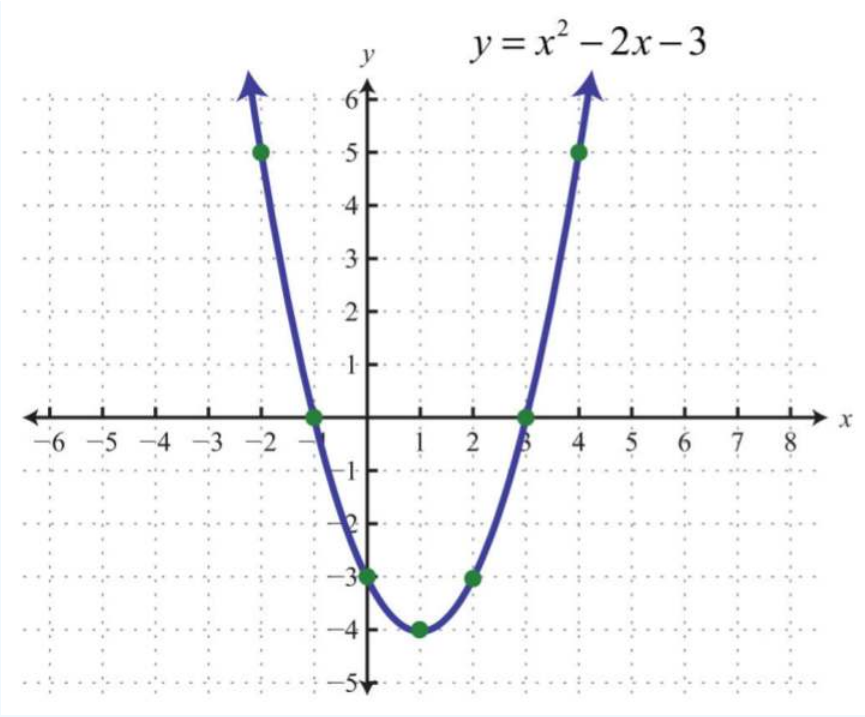


Figure 10.3.3

When graphing, we want to include certain special points in the graph. The ***y***-intercept is the point where the graph intersects the ***y***-axis. The ***x***-intercepts are the points where the graph intersects the ***x***-axis. The vertex is the point that defines the minimum or maximum of the graph. Lastly, the line of symmetry(also called the axis of symmetry) is the vertical line through the vertex, about which the parabola is symmetric.

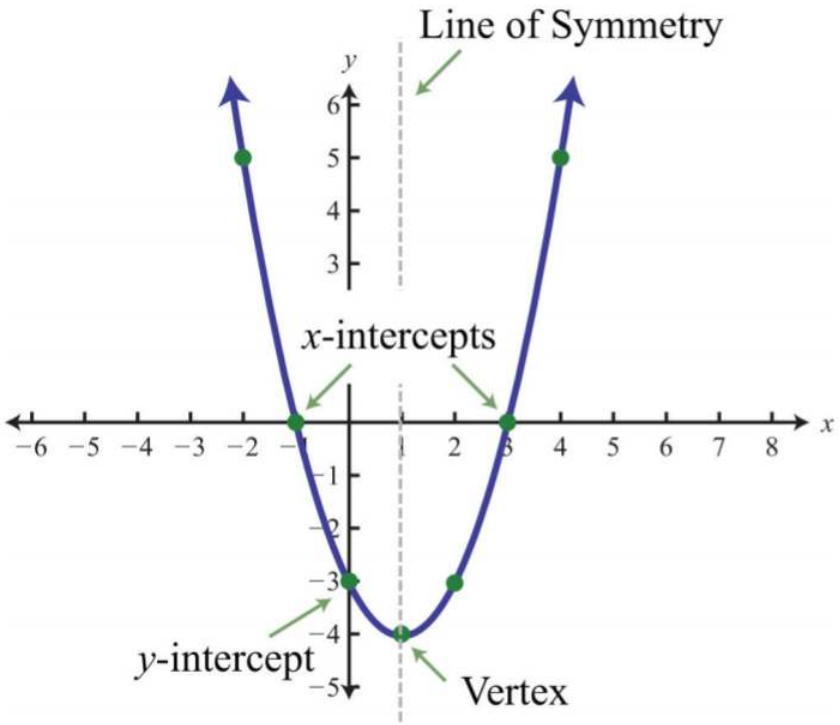


Figure 10.3.4

For any parabola, we will find the vertex and ***y***-intercept. In addition, if the ***x***-intercepts exist, then we will want to determine those as well. Guessing at the ***x***-values of these special points is not practical; therefore, we will develop techniques that will facilitate finding them. Many of these techniques will be used extensively as we progress in our study of algebra. Given a quadratic equation of the form $y = ax^2 + bx + c$, find the ***y***-intercept by setting x=0 and solving. In general, $y = a(0)^2 + b(0) + c = c$, and we have

\(\color{Cerulean}{y-intercept}

\[(0,c]\)

Next, recall that the ***x***-intercepts, if they exist, can be found by setting y=0. Doing this, we have $0 = a^2 + bx + c$, which has general solutions given by the quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. Therefore, the x-intercepts have this general form:

$$x - intercepts$$

Using the fact that a parabola is symmetric, we can determine the vertical line of symmetry using the ***x***-intercepts. To do this, we find the ***x***-value midway between the ***x***-intercepts by taking an average as follows:

Therefore, the line of symmetry is the vertical line:

$$Line\ of\ symmetry$$

$$x = -\frac{b}{2a} \tag{10.3.1}$$

We can use the line of symmetry to find the ***x***-value of the vertex. The steps for graphing a parabola are outlined in the following example.

Example 10.3.2

Graph:

Solution

Step 1: Determine the ***y***-intercept. To do this, set ***x*** = 0 and solve for ***y***.

The ***y***-intercept is (0, 3).

Step 2: Determine the ***x***-intercepts. To do this, set ***y*** = 0 and solve for ***x***.

$$\begin{aligned} x+3 &= 0 \quad \text{or} \quad x-1=0 \\ x &= -3 \qquad\quad x=1 \end{aligned}$$

Here when ***y*** = 0, we obtain two solutions. There are two ***x***-intercepts, (−3, 0) and (1, 0).

Step 3: Determine the vertex. One way to do this is to use the equation for the line of symmetry, $x = \frac{-b}{2a}$, to find the ***x***-value of the vertex. In this example, ***a*** = −1 and ***b*** = −2:

Substitute −1 into the original equation to find the corresponding ***y***-value.

The vertex is (−1, 4).

Step 4: Determine extra points so that we have at least five points to plot. In this example, one other point will suffice. Choose ***x*** = −2 and find the corresponding ***y***-value.

x	y		*Point*
−2	3	$y = -(-2)^2 - 2(-2) + 3 = -4 + 4 + 3 = 3$	(−2, 3)

Figure 10.3.5

Our fifth point is (-2,3).

Step 5: Plot the points and sketch the graph. To recap, the points that we have found are

Table 10.3.1

y-intercept:	(0,3)

x-intercept:	(-3,0) and (1,0)
Vertex:	(-1,4)
Extra point:	(-2,3)

Answer:

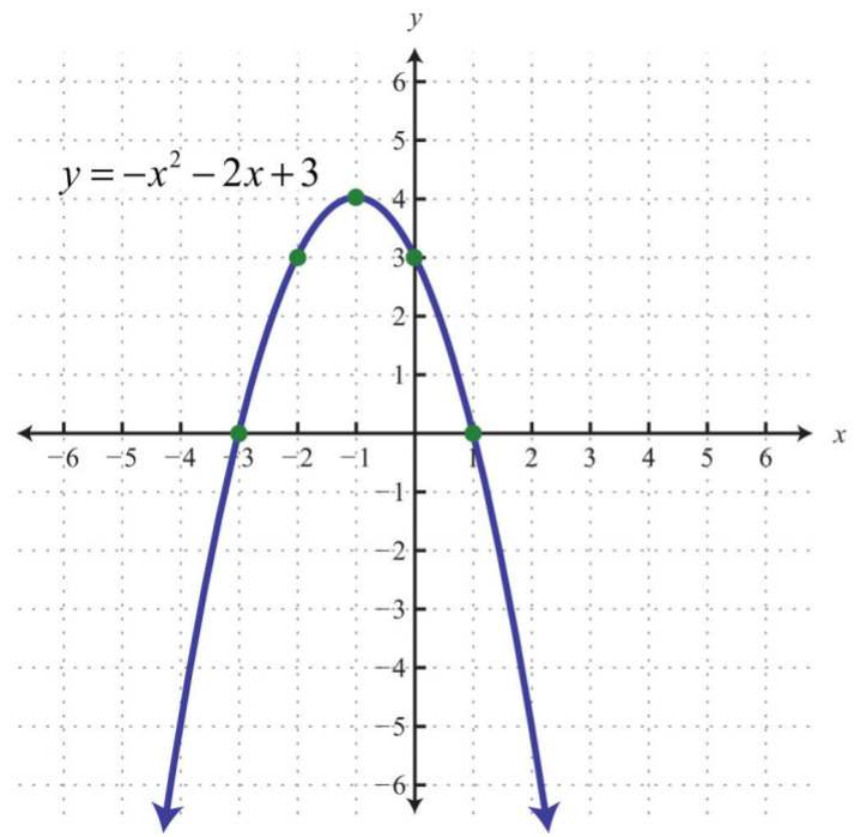


Figure 10.3.6

The parabola opens downward. In general, use the leading coefficient to determine whether the parabola opens upward or downward. If the leading coefficient is negative, as in the previous example, then the parabola opens downward. If the leading coefficient is positive, then the parabola opens upward.

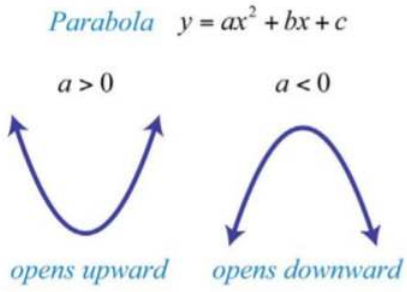


Figure 10.3.7

All quadratic equations of the form $y = ax^2 + bx + c$ have parabolic graphs with ***y***-intercept (0, ***c***). However, not all parabolas have ***x*** intercepts.

Example 10.3.3

Graph:

$$y = 2x^2 + 4x + 5$$

Solution:

Because the leading coefficient 2 is positive, note that the parabola opens upward. Here ***c*** = 5 and the ***y***-intercept is (0, 5). To find the ***x***-intercepts, set ***y*** = 0.

$$y = 2x^2 + 4x + 5$$
$$0 = 2x^2 + 4x + 5$$

In this case, ***a*** = 2, ***b*** = 4, and ***c*** = 5. Use the discriminant to determine the number and type of solutions.

Since the discriminant is negative, we conclude that there are no real solutions. Because there are no real solutions, there are no ***x***-intercepts. Next, we determine the ***x***-value of the vertex.

Given that the ***x***-value of the vertex is −1, substitute into the original equation to find the corresponding ***y***-value.

$$\begin{aligned} y &= 2x^2 + 4x + 5 \\ &= 2(-1)^2 + 4(-1) + 5 \\ &= 2 - 4 + 5 \\ &= 3 \end{aligned}$$

The vertex is (−1, 3). So far, we have only two points. To determine three more, choose some ***x***-values on either side of the line of symmetry, ***x*** = −1. Here we choose ***x***-values −3, −2, and 1.

x	y		*Points*
−3	11	$y = 2(-3)^2 + 4(-3) + 5 = 18 - 12 + 5 = 11$	(−3, 11)
−2	5	$y = 2(-2)^2 + 4(-2) + 5 = 8 - 8 + 5 = 5$	(−2, 5)
1	11	$y = 2(1)^2 + 4(1) + 5 = 2 + 4 + 5 = 11$	(1, 11)

Figure 10.3.8

To summarize, we have

Table 10.3.2

y-intercept:	(0,5)
x-intercepts:	None
Vertex:	(-1,3)
Extra points	(-3,11), (-2,5), (1,11)

Plot the points and sketch the graph.

Answer:

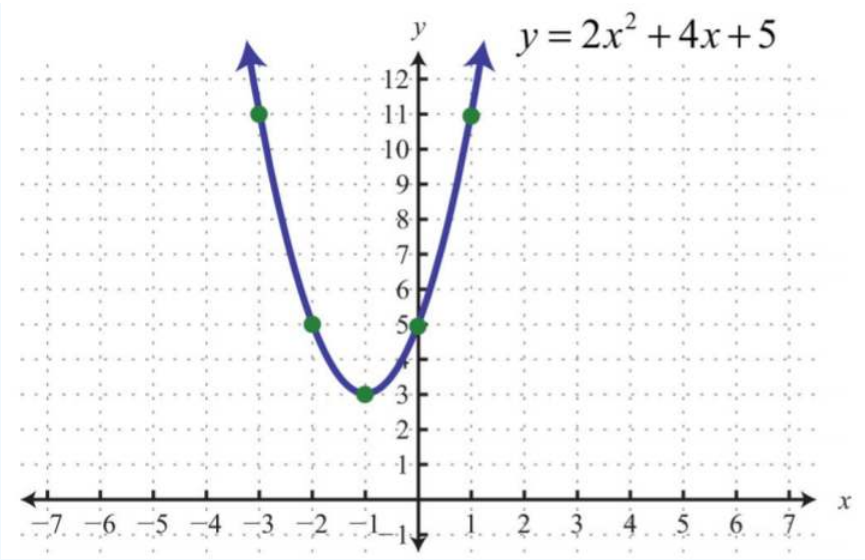


Figure 10.3.9

Example 10.3.4

Graph:

Solution

Note that ***a*** = −2: the parabola opens downward. Since ***c*** = −18, the ***y***-intercept is (0, −18). To find the ***x***-intercepts, set ***y*** = 0.

Solve by factoring.

$$x - 3 = 0 \quad \text{or} \quad x - 3 = 0$$
$$x = 3 \qquad\qquad x = 3$$

Here ***x*** = 3 is a double root, so there is only one ***x***-intercept, (3, 0). From the original equation, ***a*** = −2, ***b*** = 12, and ***c*** = −18. The ***x***-value of the vertex can be calculated as follows:

$$\begin{aligned} x &= \frac{-b}{2a} \\ &= \frac{-(12)}{2(-2)} \\ &= \frac{-12}{-4} \\ &= 3 \end{aligned}$$

Given that the ***x***-value of the vertex is 3, substitute into the original equation to find the corresponding ***y***-value.

Therefore, the vertex is (3, 0), which happens to be the same point as the ***x***-intercept. So far, we have only two points. To determine three more, choose some ***x***-values on either side of the line of symmetry, ***x*** = 3 in this case.

Choose ***x***-values 1, 5, and 6.

Figure 10.3.10

To summarize, we have

Table 10.3.3

y-intercept:	(0, -18)
x-intercepts:	(3, 0)
Vertex:	(3, 0)
Extra points:	(1, -8), (5, -8), (6, -18)

Plot the points and sketch the graph.

Answer:

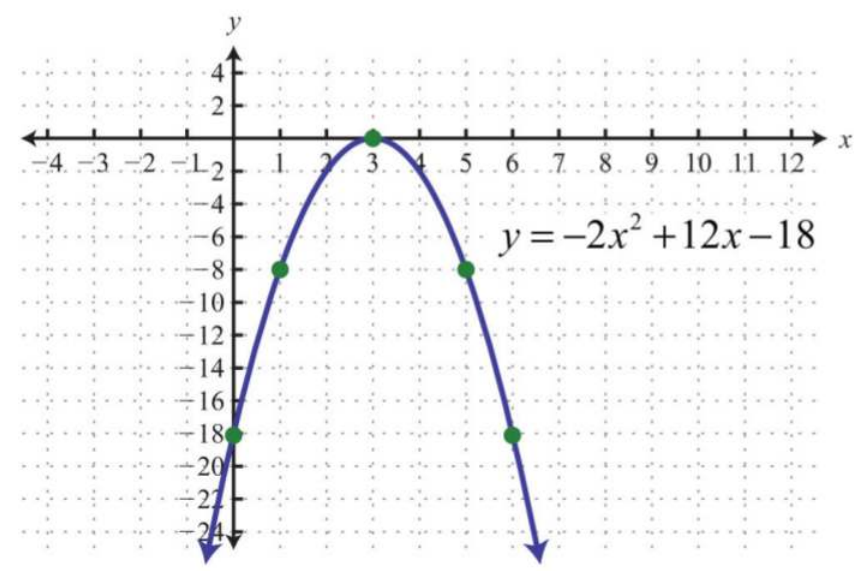


Figure 10.3.11

Example 10.3.5

Graph:

Solution:

Since $a = 1$, the parabola opens upward. Furthermore, $c = -1$, so the y-intercept is $(0, -1)$. To find the x-intercepts, set $y = 0$.

In this case, solve using the quadratic formula with $a = 1$, $b = -2$, and $c = -1$.

Here we obtain two real solutions for x, and thus there are two x-intercepts:

$(1-\sqrt{2},0)$ and $(1+\sqrt{2},0)$

Approximate values using a calculator:

$$(-0.41,0) \quad \text{and} \quad (2.41,0)$$

Use the approximate answers to place the ordered pair on the graph.

However, we will present the exact x-intercepts on the graph. Next, find the vertex.

$$\begin{aligned} x &= \frac{-b}{2a} \\ &= \frac{-(-2)}{2(1)} \\ &= \frac{2}{2} \\ &= 1 \end{aligned}$$

Given that the ***x***-value of the vertex is 1, substitute into the original equation to find the corresponding ***y***-value.

The vertex is (1, -2). We need one more point.

Figure 10.3.12

To summarize, we have

Table 10.3.4

y-intercept:	(0, -1)
x-intercepts:	$(1-\sqrt{2}, 0)$ and $(1+\sqrt{2}, 0)$
Vertex:	$(1, -2)$
Extra point:	$(2, -1)$

Plot the points and sketch the graph.

Answer:

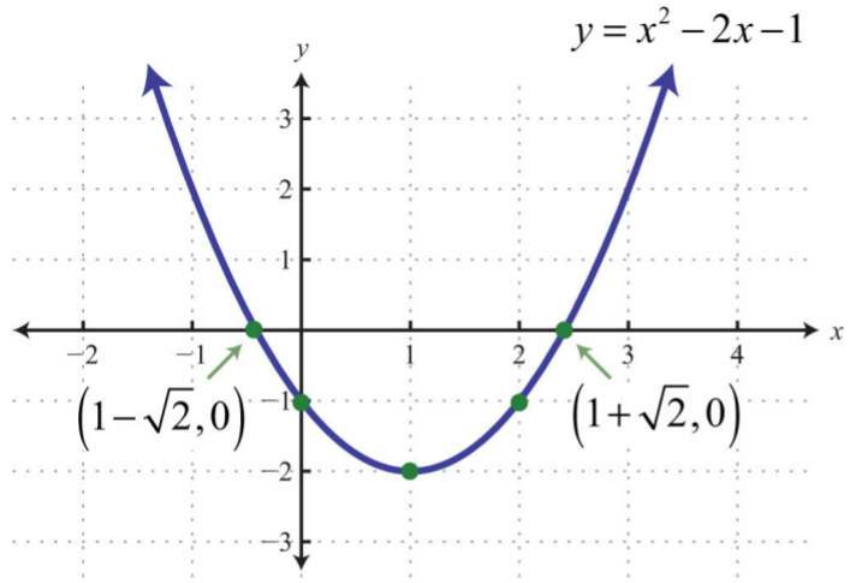


Figure 10.3.13

Exercise 10.3.1

Graph:

$$y = 9x^2 - 5$$

Answer

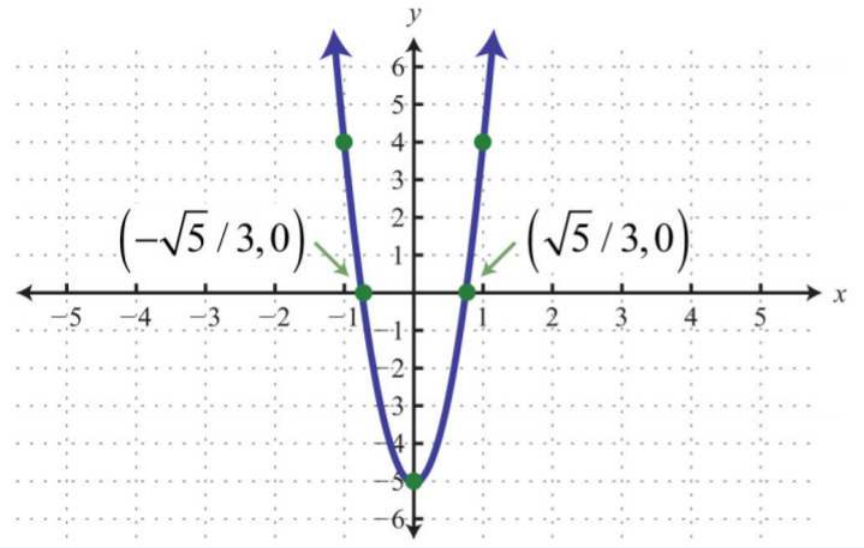


Figure 10.3.14

Finding the Maximum and Minimum

It is often useful to find the maximum and/or minimum values of functions that model real-life applications. To find these important values given a quadratic function, we use the vertex. If the leading coefficient ***a*** is positive, then the parabola opens upward and there will be a minimum ***y***-value. If the leading coefficient ***a*** is negative, then the parabola opens downward and there will be a maximum ***y***-value.

Example 10.3.6

Determine the maximum or minimum:

$$y = -4x^2 + 24x - 35$$

Solution:

Since ***a*** = −4, we know that the parabola opens downward and there will be a maximum ***y***-value. To find it, we first find the ***x***-value of the vertex.

$$\begin{aligned} x &= -\frac{b}{2a} && x-value\ of\ the\ vertex. \\ &= -\frac{24}{2(-4)} && Substitute\ a=-4\ and\ b=24. \\ &= -\frac{24}{-8} && Simplify. \\ &= 3 \end{aligned}$$

The ***x***-value of the vertex is 3. Substitute this value into the original equation to find the corresponding ***y***-value.

The vertex is (3, 1). Therefore, the maximum ***y***-value is 1, which occurs when ***x*** = 3, as illustrated below:

Figure 10.3.15

Note

The graph is not required to answer this question.

Answer:

The maximum is 1.

Example 10.3.7

Determine the maximum or minimum:

$$y = 4x^2 - 32x + 62$$

Solution:

Since ***a*** = +4, the parabola opens upward and there is a minimum ***y***-value. Begin by finding the ***x***-value of the vertex.

$$\begin{aligned} x &= -\frac{b}{2a} \\ &= -\frac{-32}{2(4)} && \textit{Substitute } a = 4 \textit{ and } b = -32. \\ &= -\frac{-32}{8} && \textit{Simplify.} \\ &= 4 \end{aligned}$$

Substitute ***x*** = 4 into the original equation to find the corresponding ***y***-value.

The vertex is (4, −2). Therefore, the minimum ***y***-value of −2 occurs when ***x*** = 4, as illustrated below:

Figure 10.3.16

Answer:

The minimum is -2.

Exercise 10.3.2

Determine the maximum or minimum:

$$y = (x-3)^2 - 9$$

Answer

The minimum is −9.

A parabola, opening upward or downward (as opposed to sideways), defines a function and extends indefinitely to the right and left as indicated by the arrows. Therefore, the domain (the set of ***x***-values) consists of all real numbers. However, the range (the set of ***y***-values) is bounded by the ***y***-value of the vertex.

Example 10.3.8

Determine the domain and range:

$$y = x^2 - 4x + 3$$

Solution:

First, note that since a=1 is positive, the parabola opens upward. Hence there will be a minimum ***y***-value. To find that value, find the ***x***-value of the vertex:

$$x = -\frac{b}{2a} = -\frac{-4}{2(1)} = 2$$

Then substitute into the equation to find the corresponding ***y***-value.

The vertex is (2, −1). The range consists of the set of ***y***-values greater than or equal to the minimum ***y***-value −1.

Figure 10.3.17

Answer:

Domain: **R** = $(-\infty, \infty)$; Range: $[-1, \infty)$

Example 10.3.9

The height in feet of a projectile is given by the function $h(t) = -16t^2 + 72t$, where t represents the time in seconds after launch. What is the maximum height reached by the projectile?

Solution:

Here a=−16, and the parabola opens downward. Therefore, the ***y***-value of the vertex determines the maximum height. Begin by finding the ***x***-value of the vertex:

$$x = -\frac{b}{2a} = -\frac{72}{2(-16)} = \frac{72}{32} = \frac{9}{4}$$

The maximum height will occur in 9/4 = 2¼ seconds. Substitute this time into the function to determine the height attained.

$$\begin{aligned} h\left(\frac{9}{4}\right) &= -16\left(\frac{9}{4}\right)^2 + 72\left(\frac{9}{4}\right) \\ &= -16\left(\frac{81}{16}\right) + 72\left(\frac{9}{4}\right) \\ &= -81 + 162 \\ &= 81 \end{aligned}$$

Answer:

The maximum height of the projectile is 81 feet.

Finding the Vertex by Completing the Square

In this section, we demonstrate an alternate approach for finding the vertex. Any quadratic equation $y = ax^2 + bx + c$ can be rewritten in the form

$$y = a(x-h)^2 + k \tag{10.3.2}$$

In this form, the vertex is *(h, k)*.

Example 10.3.10

Determine the vertex:

$$y = -4(x-3)^2 + 1$$

Solution:

When the equation is in this form, we can read the vertex directly from the equation.

$$\begin{aligned} y = &\ a\,(\,x-h)^2 + k \\ &\quad\ \downarrow \quad \downarrow \\ y = &-4(x-3)^2 + 1 \end{aligned}$$

Here ***h***=3 and ***k***=1.

Answer:

The vertex is (3, 1).

Example 10.3.11

Determine the vertex:

$$y = 2(x+3)^2 - 2$$

Solution:

Rewrite the equation as follows before determining ***h*** and ***k***.

$$y = a(x-h)^2 + k$$
$$\downarrow \quad \downarrow$$
$$y = 2(x-(-3))^2 + (-2)$$

Here ***h***=-3 and ***k***=-2.

Answer:

The vertex is (-3,-2).

Often the equation is not given in this form. To obtain this form, complete the square.

Example 10.3.12

Rewrite in $y = a(x-h)^2 + k$ form and determine the vertex: $y = x^2 + 4x + 9$.

Solution:

Begin by making room for the constant term that completes the square.

$$\begin{aligned} y &= x^2 + 4x + 9 \\ &= x^2 + 4x + \_\_ + 9 - \_\_ \end{aligned}$$

The idea is to add and subtract the value that completes the square, $\frac{b^2}{2}$, and then factor. In this case, add and subtract $\frac{4^2}{2} = 2^2 = 4$.

$$\begin{aligned} y &= x^2 + 4x + 9 && \textit{Add and subtract 4.} \\ &= \underbrace{x^2 + 4x + 4}_{\text{factor}} + 9 - 4 && \textit{Factor.} \\ &= (x+2)(x+2) + 5 \\ &= (x+2)^2 + 5 \end{aligned}$$

Adding and subtracting the same value within an expression does not change it. Doing so is equivalent to adding 0. Once the equation is in this form, we can easily determine the vertex.

$$y = a(x-h)^2 + k$$
$$\downarrow \quad \downarrow$$
$$y = (x-(-2))^2 + 5$$

Here ***h***=-2 and ***k***=5.

Answer:

The vertex is (-2, 5).

If there is a leading coefficient other than 1, then we must first factor out the leading coefficient from the first two terms of the trinomial.

Example 10.3.13

Rewrite in $y = a(x-h)^2 + k$ form and determine the vertex: $y = 2x^2 - 4x + 8$.

Solution:

Since ***a*** = 2, factor this out of the first two terms in order to complete the square. Leave room inside the parentheses to add a constant term.

Now use −2 to determine the value that completes the square. In this case, $\frac{(-2)^2}{2} = \backslash(-1)^2 = 1$. Add and subtract 1 and factor as follows:

In this form, we can easily determine the vertex.

$$y = a(x-h)^2 + k$$
$$\downarrow \quad \downarrow$$
$$y = 2(x-1)^2 + 6$$

Here ***h***=1 and ***k***=6.

Answer:

The vertex is (1, 6).

? Exercise 10.3.3

Rewrite in $y = a(x-h)^2 + k$ form and determine the vertex:

$$y = -2x^2 - 12x + 3\ .$$

Answer

$y = -2(x+3)^2 + 21$; vertex: $(-3, 21)$

Key Takeaways

- The graph of any quadratic equation $y = ax^2 + bx + c$, where *a*, *b*, and *c* are real numbers and *a≠0*, is called a parabola.
- When graphing parabolas, find the vertex and *y*-intercept. If the *x*-intercepts exist, find those as well. Also, be sure to find ordered pair solutions on either side of the line of symmetry, $x = \frac{-b^2}{a}$.
- Use the leading coefficient, *a*, to determine if a parabola opens upward or downward. If *a* is positive, then it opens upward. If *a* is negative, then it opens downward.
- The vertex of any parabola has an x-value equal to $x = \frac{-b^2}{a}$. After finding the *x*-value of the vertex, substitute it into the original equation to find the corresponding *y*-value. This *y*-value is a maximum if the parabola opens downward, and it is a minimum if the parabola opens upward.
- The domain of a parabola opening upward or downward consists of all real numbers. The range is bounded by the *y*-value of the vertex.
- An alternate approach to finding the vertex is to rewrite the quadratic equation in the form $y = a(x-h)^2 + k$. When in this form, the vertex is *(h, k)* and can be read directly from the equation. To obtain this form, take $y = ax^2 + bx + c$ and complete the square.

? Exercise 10.3.4 the graph of quadratic equations

Does the parabola open upward or downward? Explain.

1. $y = x^2 - 9x + 20$
2. $y = x^2 - 12x + 32$
3. $y = -2x^2 + 5x + 12$
4. $y = -6x^2 + 13x - 6$
5. $y = 64 - x^2$
6. $y = -3x + 9x^2$

Answer

1. Upward
3. Downward
5. Downward

Exercise 10.3.5 the graph of quadratic equations

Determine the *x*- and *y*-intercepts.

1. $y = x^2 + 4x - 12$
2. $y = x^2 - 13x + 12$
3. $y = 2x^2 + 5x - 3$
4. $y = 3x^2 - 4x - 4$
5. $y = -5x^2 - 3x + 2$
6. $y = -6x^2 + 11x - 4$
7. $y = 4x^2 - 25$
8. $y = 9x^2 - 49$
9. $y = x^2 - x + 1$
10. $y = 5x^2 + 15x$

Answer

1. *x*-intercepts: $(-6, 0), (2, 0)$; *y*-intercept: $(0, -12)$
3. *x*-intercepts: $(-3, 0), (\frac{1}{2}, 0)$; *y*-intercept: $(0, -3)$
5. *x*-intercepts: $(-1, 0), (\frac{2}{5}, 0)$; *y*-intercept: $(0, 2)$
7. *x*-intercepts: $(-\frac{5}{2}, 0), (\frac{5}{2}, 0)$; *y*-intercept: $(0, -25)$
9. *x*-intercepts: none; *y*-intercept: $(0, 1)$

Exercise 10.3.6 the graph of quadratic equations

Find the vertex and the line of symmetry.

1. $y = -x^2 + 10x - 34$
2. $y = -x^2 - 6x + 1$
3. $y = -4x^2 + 12x - 7$
4. $y = -9x^2 + 6x + 2$
5. $y = 4x^2 - 1$
6. $y = x^2 - 16$

Answer

1. Vertex: $(5, -9)$; line of symmetry: $x = 5$
3. Vertex: $(\frac{3}{2}, 2)$; line of symmetry: $x = \frac{3}{2}$
5. Vertex: $(0, -1)$; line of symmetry: $x = 0$

Exercise 10.3.7 the graph of quadratic equations

Graph. Find the vertex and the *y*-intercept. In addition, find the *x*-intercepts if they exist.

1. $y = x^2 - 2x - 8$
2. $y = x^2 - 4x - 5$
3. $y = -x^2 + 4x + 12$
4. $y = -x^2 - 2x + 15$
5. $y = x^2 - 10x$
6. $y = x^2 + 8x$
7. $y = x^2 - 9$
8. $y = x^2 - 25$
9. $y = 1 - x^2$

10. $y = 4 - x^2$
11. $y = x^2 - 2x + 1$
12. $y = x^2 + 4x + 4$
13. $y = -4x^2 + 12x - 9$
14. $y = -4x^2 - 4x + 3$
15. $y = x^2 - 2$
16. $y = x^2 - 3$
17. $y = -4x^2 + 4x - 3$
18. $y = 4x^2 + 4x + 3$
19. $y = x^2 - 2x - 2$
20. $y = x^2 - 6x + 6$
21. $y = -2x^2 + 6x - 3$
22. $y = -4x^2 + 4x + 1$
23. $y = x^2 + 3x + 4$
24. $y = -x^2 + 3x - 4$
25. $y = -2x^2 + 3$
26. $y = -2x^2 - 1$
27. $y = 2x^2 + 4x - 3$
28. $y = 3x^2 + 2x - 2$

Answer

1.

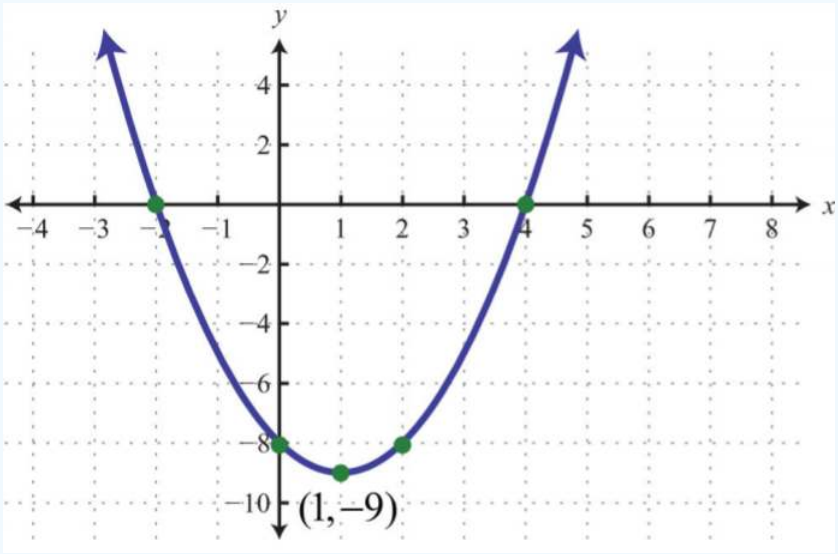


Figure 10.3.18

3.

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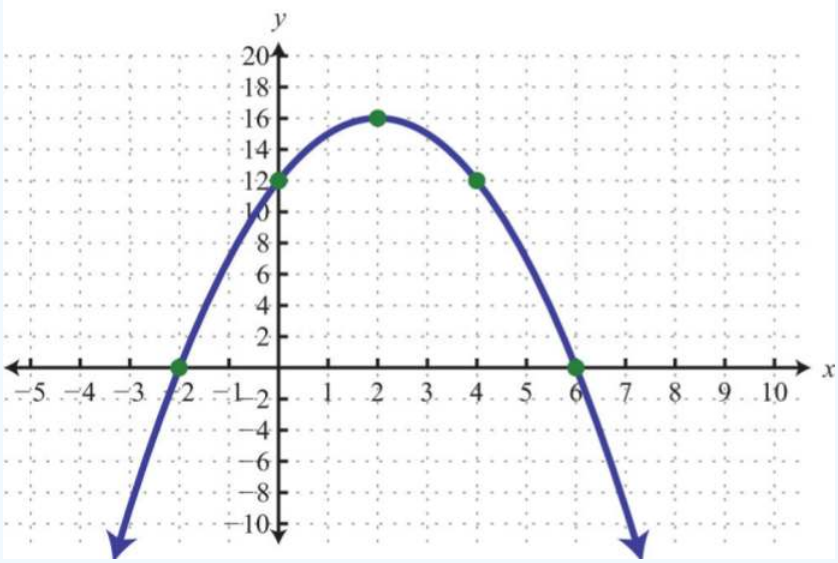


Figure 10.3.19

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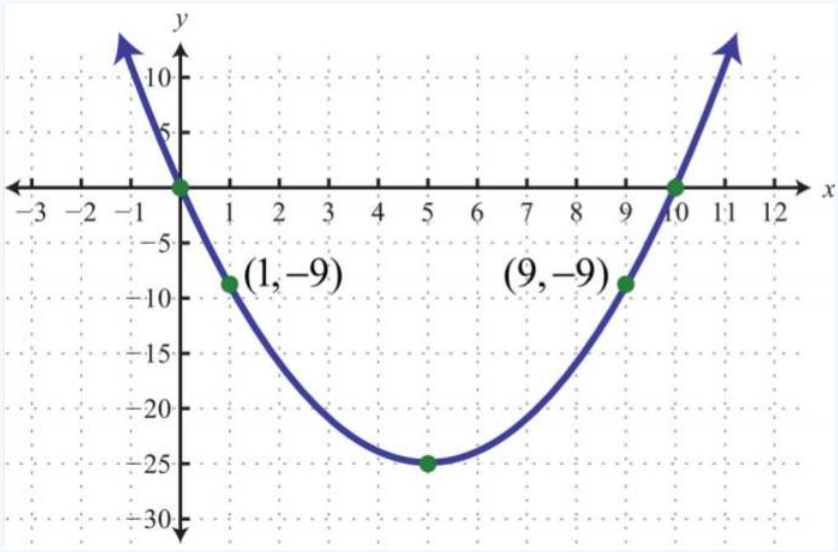


Figure 10.3.20

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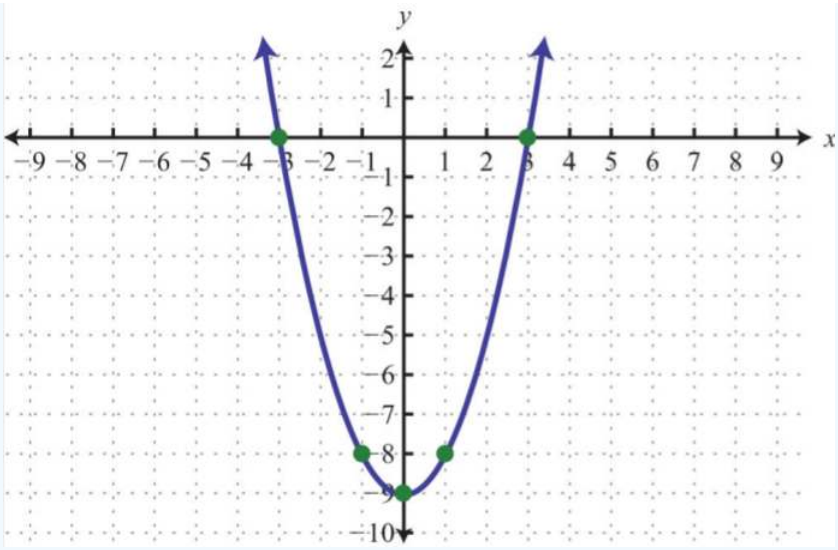


Figure 10.3.21

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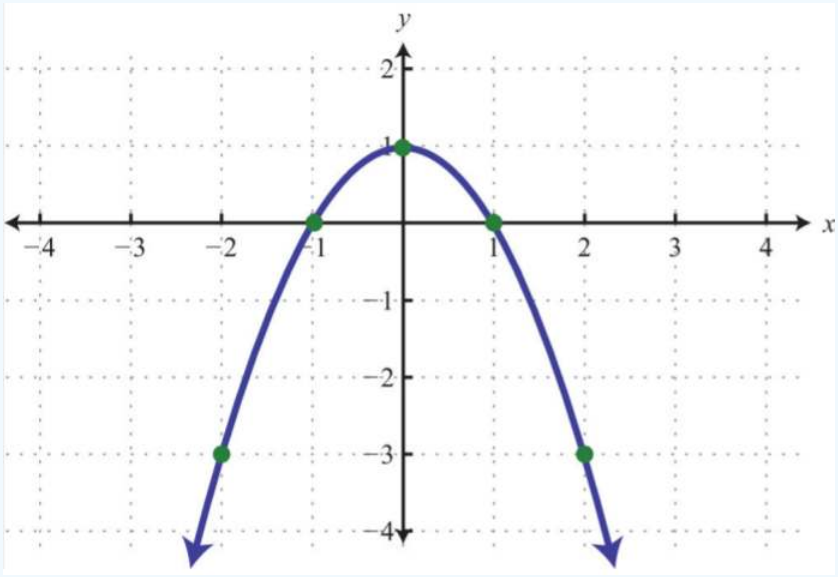


Figure 10.3.22

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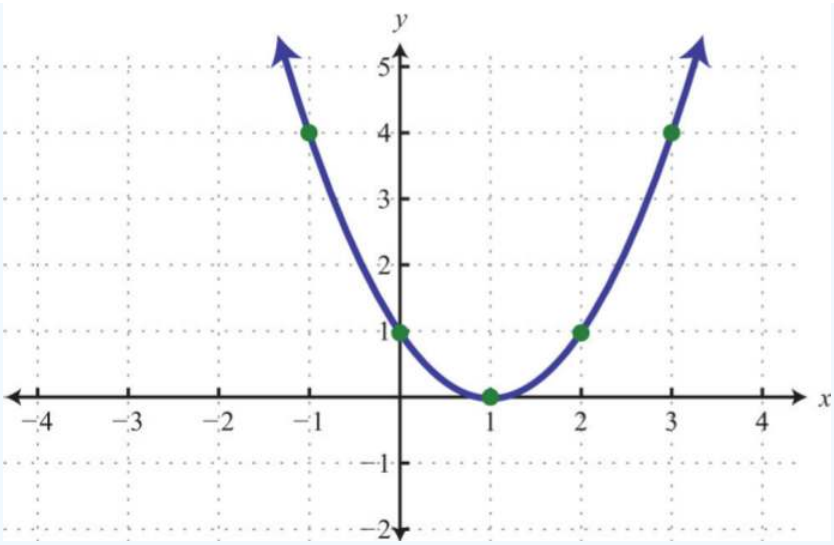


Figure 10.3.23

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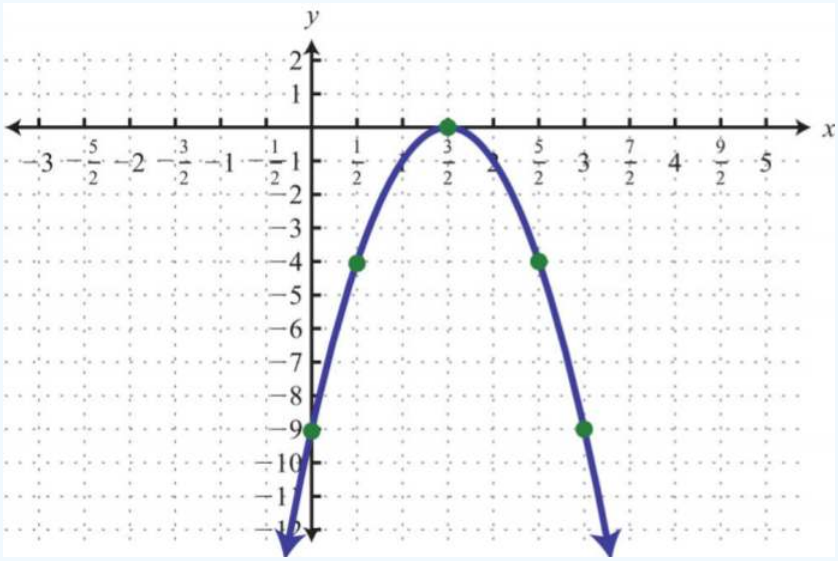


Figure 10.3.24

15.

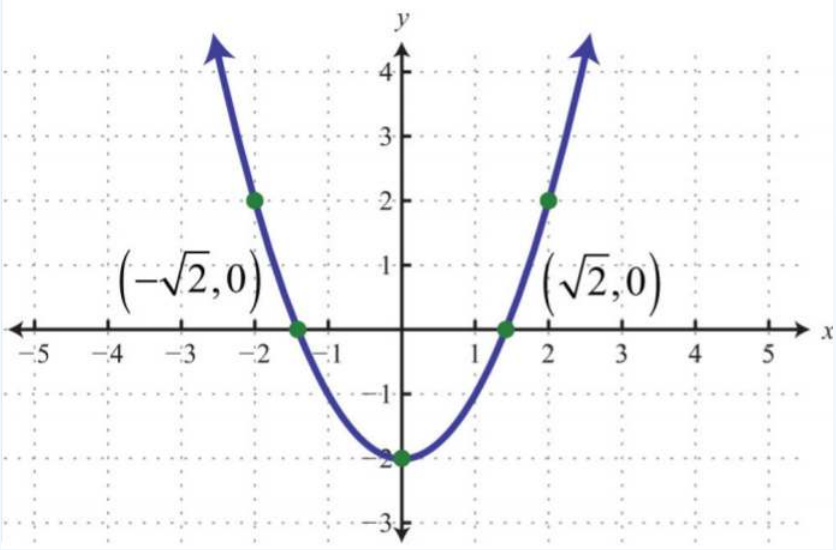


Figure 10.3.25

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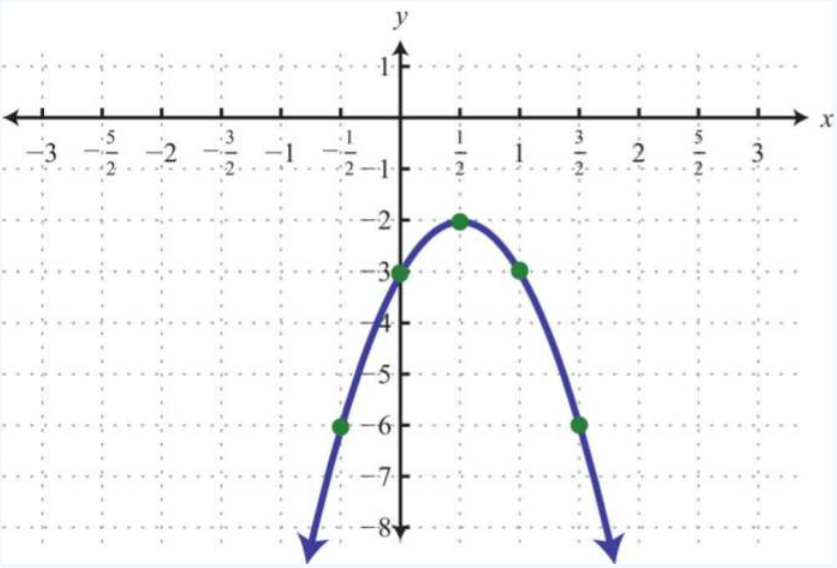


Figure 10.3.26

19.

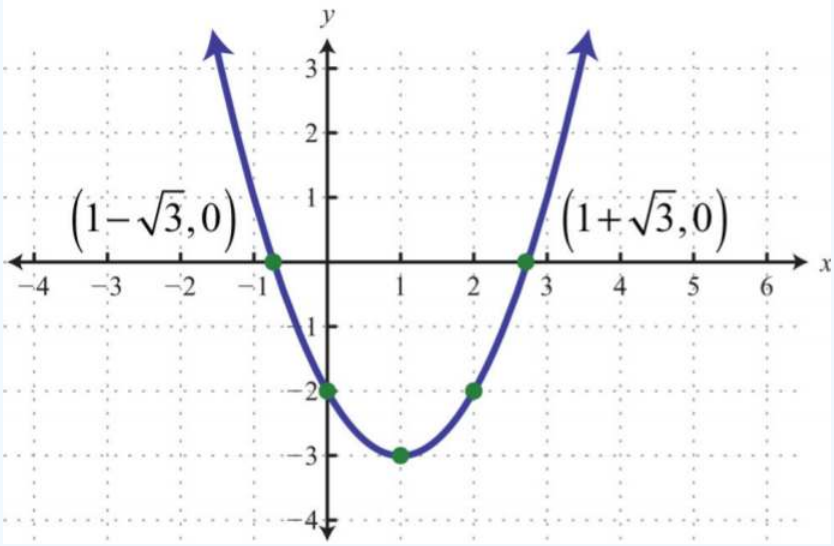


Figure 10.3.27

21.

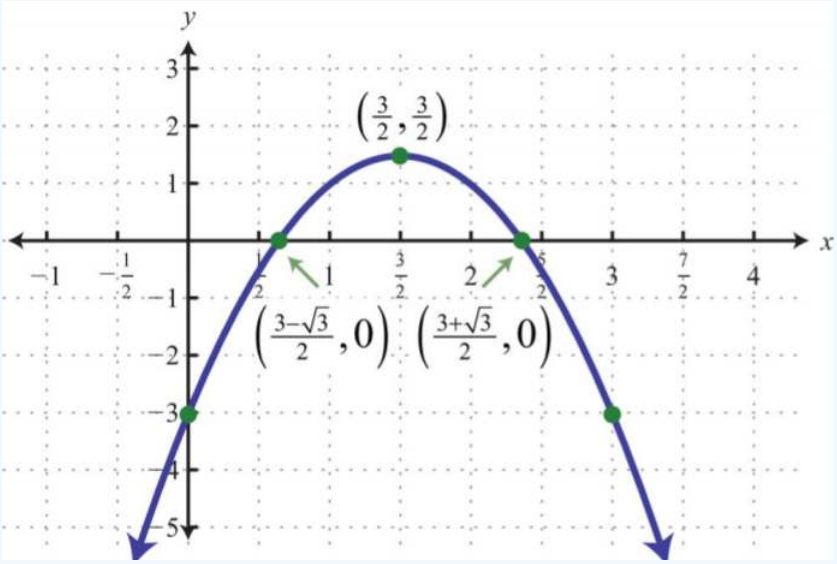


Figure 10.3.28

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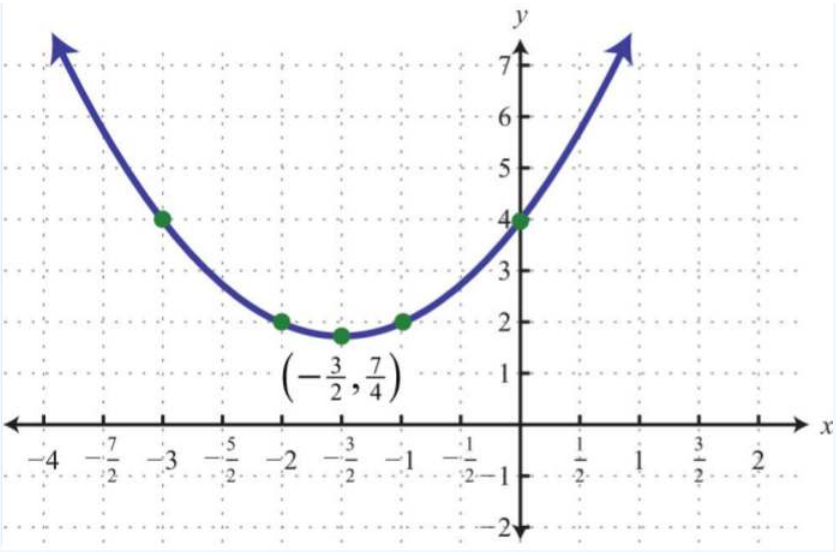


Figure 10.3.29

25.

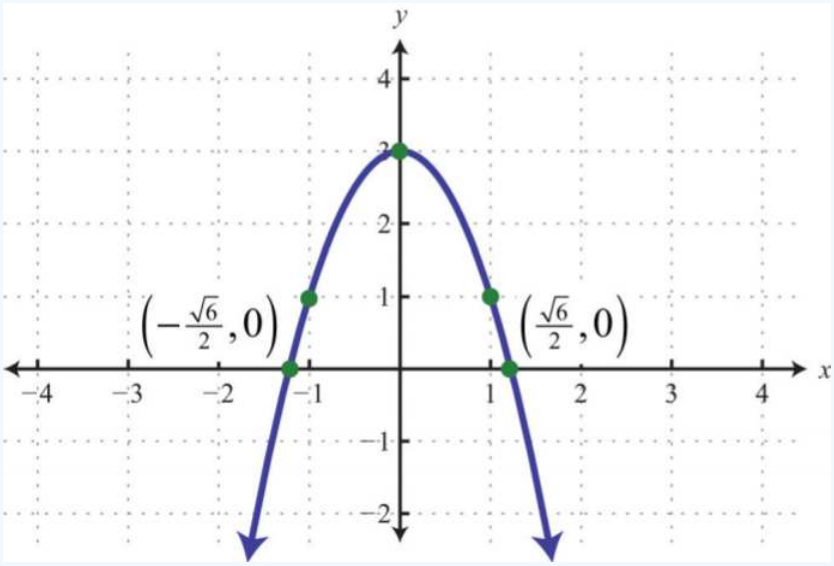


Figure 10.3.30

27.

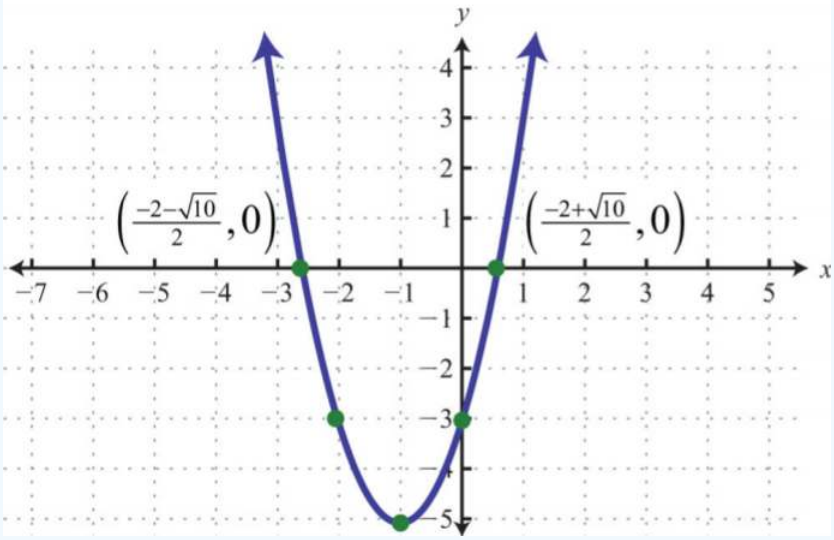


Figure 10.3.31

Exercise 10.3.8 maximum or minimum

Determine the maximum or minimum *y*-value.

1. $y=-x^2-6x+1$
2. $y=-x^2-4x+8$
3. $y=25x^2-10x+5$
4. $y=16x^2-24x+7$
5. \(y=−x^{2}
6. $y=1-9x^2$
7. $y=20x-10x^2$
8. $y=12x+4x^2$
9. $y=3x^2-4x-2$
10. $y=6x^2-8x+5$

Answer

1. Maximum: $y=10$
3. Minimum: $y=4$
5. Maximum: $y=0$
7. Maximum: $y=10$
9. Minimum: $y=-\frac{10}{3}$

Exercise 10.3.9 maximum or minimum

Given the following quadratic functions, determine the domain and range.

1. $f(x)=3x^2+30x+50$
2. $f(x)=5x^2-10x+1$
3. $g(x)=-2x^2+4x+1$
4. $g(x)=-7x^2-14x-9$
5. The height in feet reached by a baseball tossed upward at a speed of 48 feet/second from the ground is given by the function $h(t)=-16t^2+48t$, where *t* represents time in seconds. What is the baseball's maximum height and how long will it take to attain that height?

6. The height of a projectile launched straight up from a mound is given by the function $h(t) = -16t^2 + 96t + 4$, where t represents seconds after launch. What is the maximum height?
7. The profit in dollars generated by producing and selling x custom lamps is given by the function $P(x) = -10x^2 + 800x - 12,000$. What is the maximum profit?
8. The revenue in dollars generated from selling a particular item is modeled by the formula $R(x) = 100x - 0.0025x^2$, where x represents the number of units sold. What number of units must be sold to maximize revenue?
9. The average number of hits to a radio station website is modeled by the formula $f(x) = 450t^2 - 3,600t + 8,000$, where t represents the number of hours since 8:00 a.m. At what hour of the day is the number of hits to the website at a minimum?
10. The value in dollars of a new car is modeled by the formula $V(t) = 125t^2 - 3,000t + 22,000$, where t represents the number of years since it was purchased. Determine the minimum value of the car.
11. The daily production costs in dollars of a textile manufacturing company producing custom uniforms is modeled by the formula $C(x) = 0.02x^2 - 20x + 10,000$, where x represents the number of uniforms produced.
    a. How many uniforms should be produced to minimize the daily production costs?
    b. What is the minimum daily production cost?
12. The area of a certain rectangular pen is given by the formula $A = 14w - w^2$, where w represents the width in feet. Determine the width that produces the maximum area.

Answer

1. Domain: R; range: $[-25, \infty)$
3. Domain: R; range: $(-\infty, 3]$
5. The maximum height of 36 feet occurs after 1.5 seconds.
7. $4,000
9. 12:00 p.m.
11. a. 500 uniforms; b. $5,000

? Exercise 10.3.10 vertex by completing the square

Determine the vertex.

1. $y = -(x-5)^2 + 3$
2. $y = -2(x-1)^2 + 7$
3. $y = 5(x+1)^2 + 6$
4. $y = 3(x+4)^2 + 10$
5. $y = -5(x+8)^2 - 1$
6. $y = (x+2)^2 - 5$

Answer

1. $(5, 3)$
3. $(-1, 6)$
5. $(-8, -1)$

? Exercise 10.3.11 vertex by completing the square

Rewrite in $y = a(x-h)^2 + k$ form and determine the vertex.

1. $y = x^2 - 14x + 24$
2. $y = x^2 - 12x + 40$
3. $y = x^2 + 4x - 12$
4. $y = x^2 + 6x - 1$
5. $y = 2x^2 - 12x - 3$

6. $y = 3x^2 - 6x + 5$
7. $y = -x^2 + 16x + 17$
8. $y = -x^2 + 10x$

Answer

1. $y = (x-7)^2 - 25$; vertex: $(7, -25)$
3. $y = (x+2)^2 - 16$; vertex: $(-2, -16)$
5. $y = 2(x-3)^2 - 21$; vertex: $(3, -21)$
7. $y = -(x-8)^2 + 81$; vertex: $(8, 81)$

Exercise 10.3.12 vertex by completing the square

Graph.

1. $y = x^2 - 1$
2. $y = x^2 + 1$
3. $y = (x-1)^2$
4. $y = (x+1)^2$
5. $y = (x-4)^2 - 9$
6. $y = (x-1)^2 - 4$
7. $y = -2(x+1)^2 + 8$
8. $y = -3(x+2)^2 + 12$
9. $y = -5(x-1)^2$
10. $y = -(x+2)^2$
11. $y = -4(x-1)^2 - 2$
12. $y = 9(x+1)^2 + 2$
13. $y = (x+5)^2 - 15$
14. $y = 2(x-5)^2 - 3$
15. $y = -2(x-4)^2 + 22$
16. $y = 2(x+3)^2 - 13$

Answer

1.

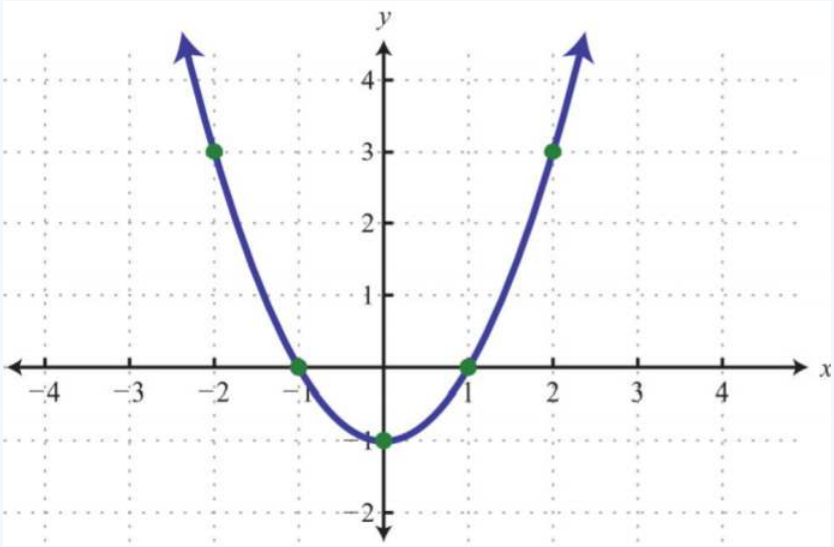


Figure 10.3.32

3.

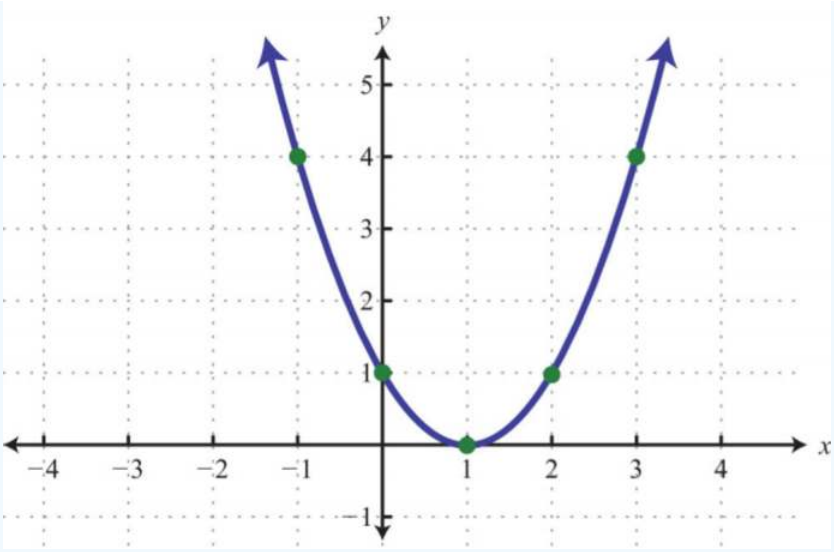


Figure 10.3.33

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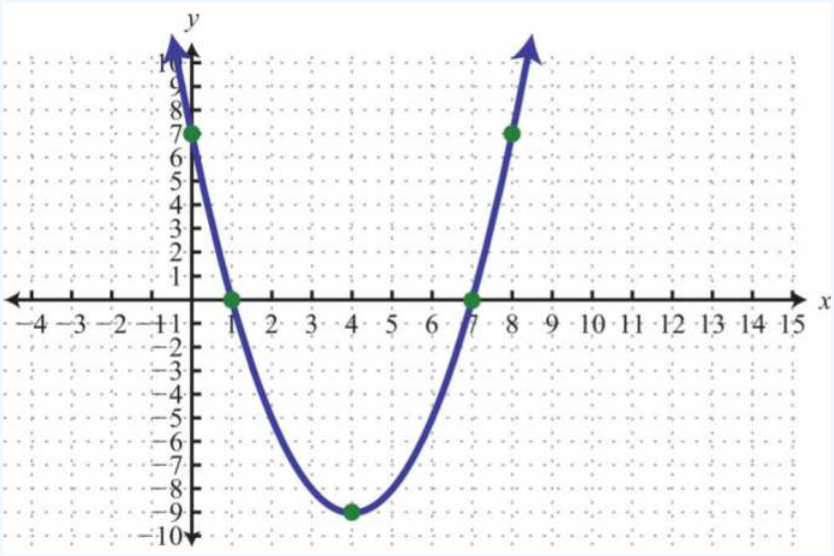


Figure 10.3.34

7.

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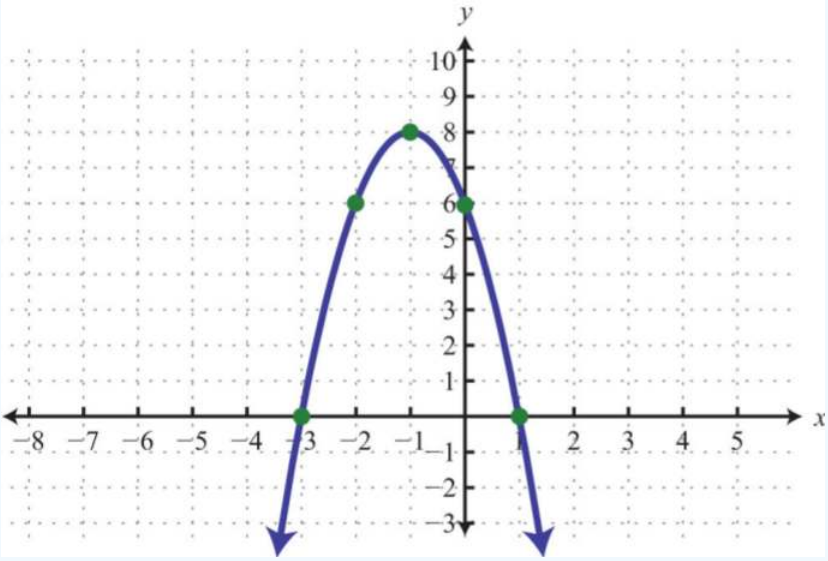


Figure 10.3.35

9.

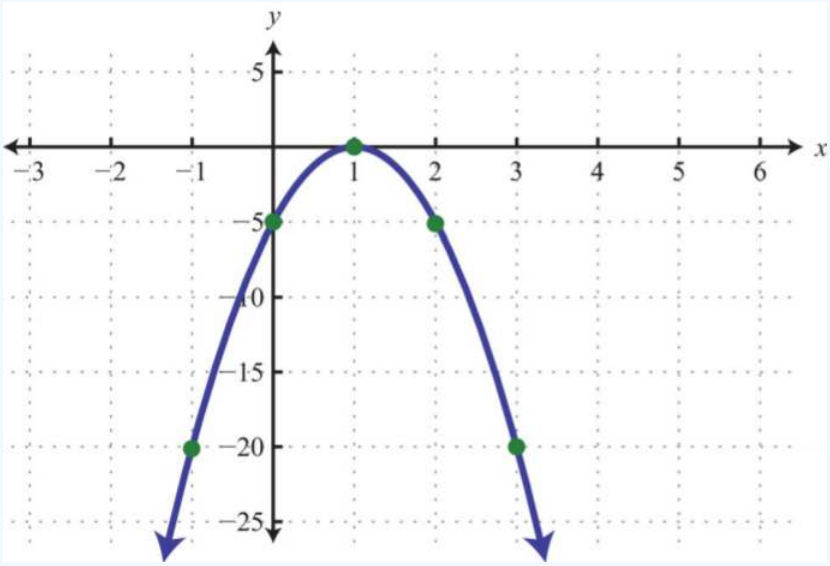


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11.

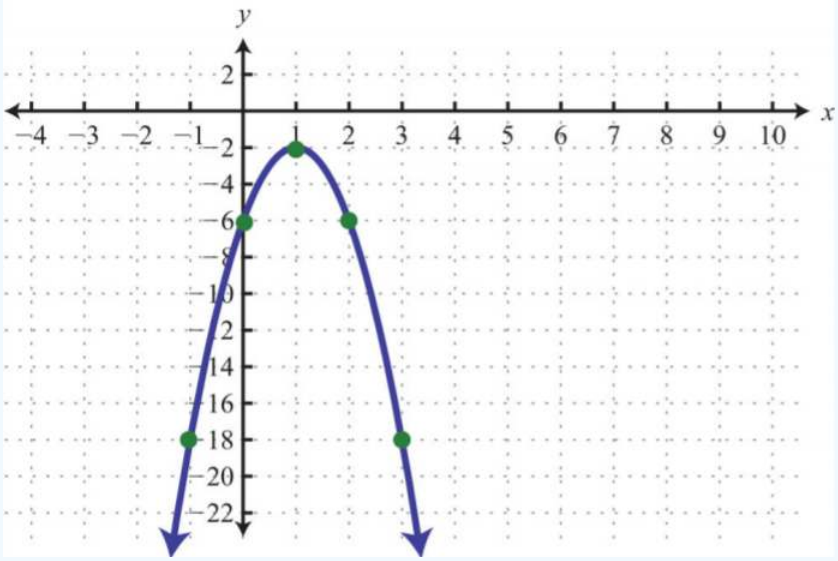


Figure 10.3.37

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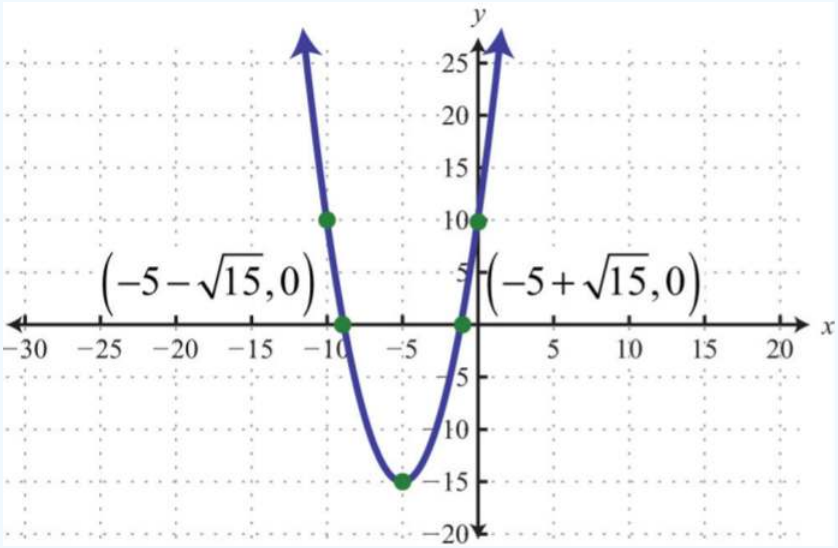


Figure 10.3.38

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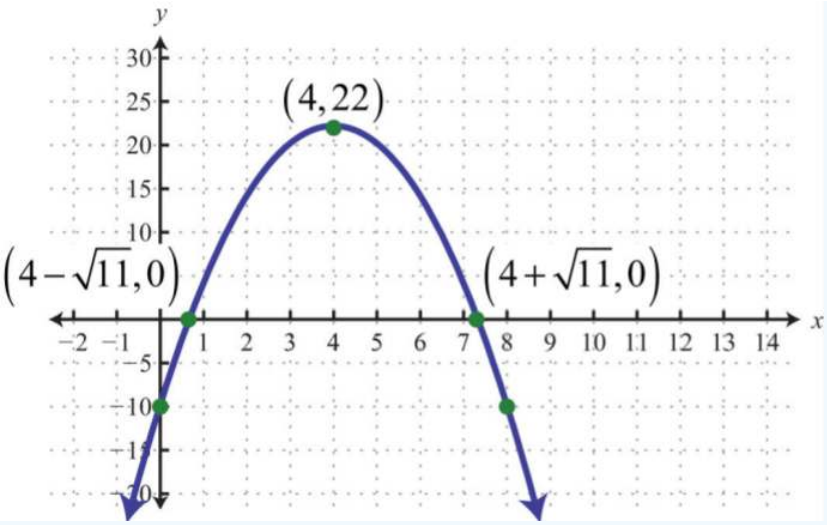


Figure 10.3.39

Exercise 10.3.13 discussion board

1. Write down your plan for graphing a parabola on an exam. What will you be looking for and how will you present your answer? Share your plan on the discussion board.
2. Why is any parabola that opens upward or downward a function? Explain to a classmate how to determine the domain and range.

Answer

1. Answers may vary

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