MATH 081: INTRODUCTION TO ALGEBRA



Patrick Kwoni
Highline College

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Patrick Kwoni

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Glossary

Detailed Licensing

Licensing

*A detailed breakdown of this resource's licensing can be found in **Back Matter/Detailed Licensing**.*

CHAPTER OVERVIEW

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1.1: It's All in the Mindset

Questions to consider

- What is a growth mindset, and how does it affect my learning?
- What are performance goals versus learning goals?

In the previous sections of this chapter you have focused on a number of concepts and models about learning. One of the things they all have in common is that they utilize different approaches to education by presenting new ways to think about learning. In each of these, the common element has been a better understanding of yourself as a learner and how to apply what you know about yourself to your own learning experience. If you were to distill all that you have learned in this chapter so far down to a single factor, it would be about using your mindset to your best advantage. In this next section, you will examine how all of this works in a broader sense by learning about the significance of certain mindsets and how they can hinder or promote your own learning efforts.



Figure 1.1.7: Many fields of study and work create intersections of growth and fixed mindset. People may feel great ability to grow and learn in some areas, like art and communication, but feel more limited in others, such as planning and financials. Recognizing these intersections will help you approach new topics and tasks. (Credit: mentatdgt / Pexels)

Performance vs. Learning Goals

As you have discovered in this chapter, much of our ability to learn is governed by our motivations and goals. What has not yet been covered in detail has been how sometimes hidden goals or mindsets can impact the learning process. In truth, we all have goals that we might not be fully aware of, or if we are aware of them, we might not understand how they help or restrict our ability to learn. An illustration of this can be seen in a comparison of a student that has *performance*-based goals with a student that has *learning*-based goals.

If you are a student with strict performance goals, your primary psychological concern might be to appear intelligent to others. At first, this might not seem to be a bad thing for college, but it can truly limit your ability to move forward in your own learning. Instead, you would tend to play it safe without even realizing it. For example, a student who is strictly performance-goal-oriented will often only says things in a classroom discussion when they think it will make them look knowledgeable to the instructor or their classmates. For example, a performance-oriented student might ask a question that she knows is beyond the topic being covered (e.g., asking about the economics of Japanese whaling while discussing the book *Moby Dick* in an American literature course). Rarely will they ask a question in class because they actually do not understand a concept. Instead they will ask questions that make them look intelligent to others or in an effort to “stump the teacher.” When they do finally ask an honest question, it may be because they are more afraid that their lack of understanding will result in a poor performance on an exam rather than simply wanting to learn.

If you are a student who is driven by learning goals, your interactions in classroom discussions are usually quite different. You see the opportunity to share ideas and ask questions as a way to gain knowledge quickly. In a classroom discussion you can ask for

clarification immediately if you don't quite understand what is being discussed. If you are a person guided by learning goals, you are less worried about what others think since you are there to learn and you see that as the most important goal.

Another example where the difference between the two mindsets is clear can be found in assignments and other coursework. If you are a student who is more concerned about performance, you may avoid work that is challenging. You will take the "easy A" route by relying on what you already know. You will not step out of your comfort zone because your psychological goals are based on approval of your performance instead of being motivated by learning.

This is very different from a student with a learning-based psychology. If you are a student who is motivated by learning goals, you may actively seek challenging assignments, and you will put a great deal of effort into using the assignment to expand on what you already know. While getting a good grade is important to you, what is even more important is the learning itself.

If you find that you sometimes lean toward performance-based goals, do not feel discouraged. Many of the best students tend to initially focus on performance until they begin to see the ways it can restrict their learning. The key to switching to learning-based goals is often simply a matter of first recognizing the difference and seeing how making a change can positively impact your own learning.

What follows in this section is a more in-depth look at the difference between performance- and learning-based goals. This is followed by an exercise that will give you the opportunity to identify, analyze, and determine a positive course of action in a situation where you believe you could improve in this area.

WHAT STUDENTS SAY

1. In the past, did you feel like you had control over your own learning?
    a. No. Someone has always dictated how and what I learned.
    b. Yes. I always look for ways to take control of what and how I learned.
    c. I am uncertain. I never thought about it before.
2. Have you ever heard of learning styles or do you know your own learning style?
    a. No. I have never heard of learning styles.
    b. Yes. I have heard of learning styles and know my own.
    c. Yes. I have heard of learning styles, but I don't think they're accurate or relate to me.
3. Which factors other than intelligence do you think have the greatest influence on learning?
    a. Motivation
    b. Perseverance
    c. Understanding how I learn
    d. Good teachers and support

You can also take the anonymous What Students Say surveys to add your voice to this textbook. Your responses will be included in updates.

Students offered their views on these questions, and the results are displayed in the graphs below.

In the past, did you feel like you had control over your own learning?

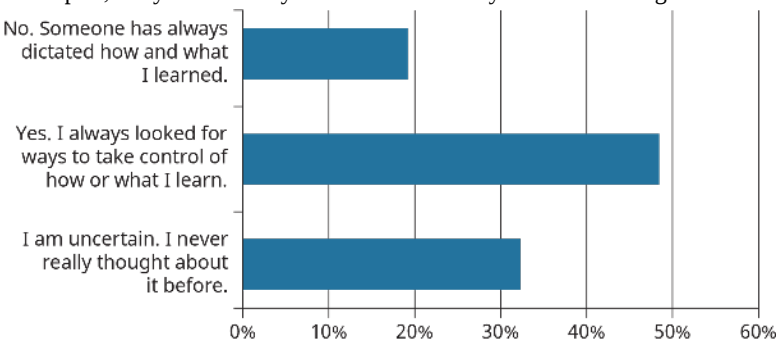


Figure 1.1.8

Have you ever heard of learning styles or do you know your own learning style?

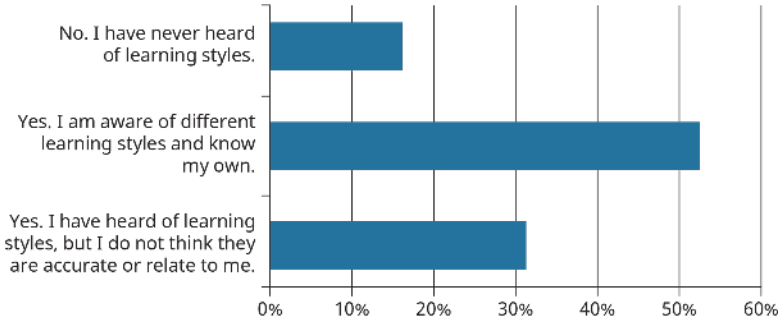


Figure 1.1.9

Which factors other than intelligence do you think have the greatest influence on learning?

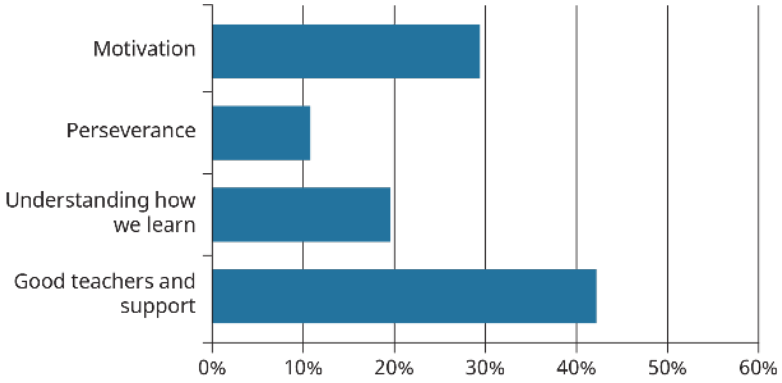


Figure 1.1.10

Fixed vs. Growth Mindset

The research-based model of these two mindsets and their influence on learning was presented in 1988 by Carol Dweck.[7] In Dr. Dweck's work, she determined that a student's perception about their own learning accompanied by a broader goal of learning had a significant influence on their ability to overcome challenges and grow in knowledge and ability. This has become known as the Fixed vs. Growth Mindset model. In this model, the *performance*-goal-oriented student is represented by the *fixed* mindset, while the *learning*-goal-oriented student is represented by the *growth* mindset.

In the following graphic, based on Dr. Dweck's research, you can see how many of the components associated with learning are impacted by these two mindsets.

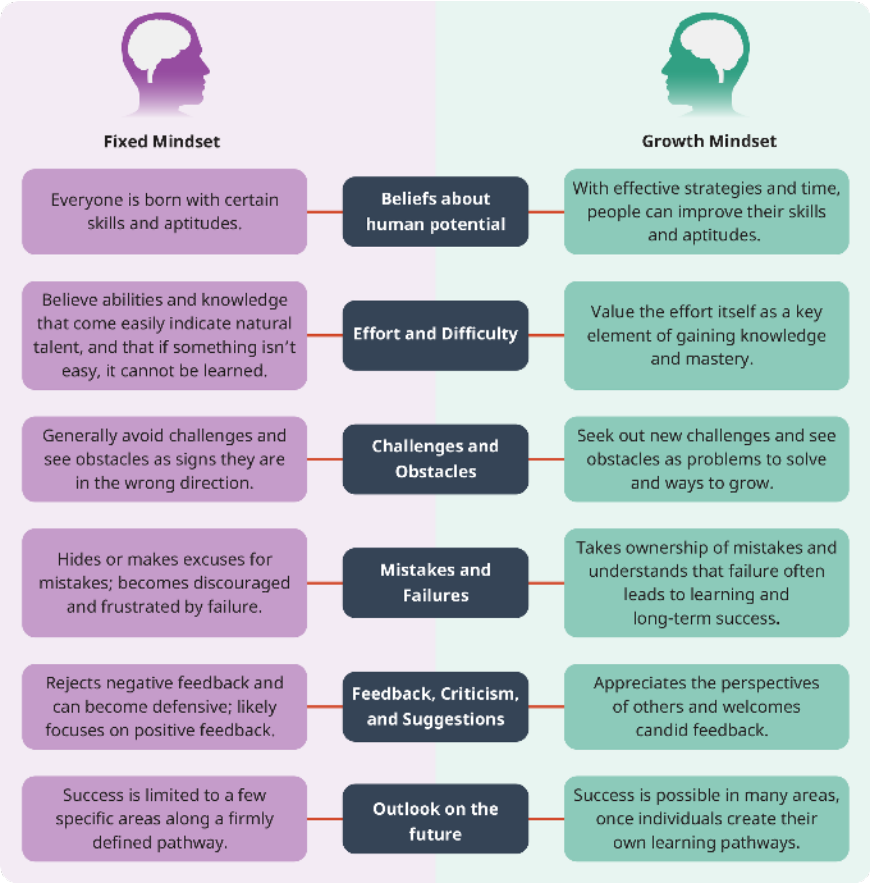


Figure 1.1.11: The differences between fixed and growth mindset are clear when aligned to key elements of learning and personality. (Credit: Based on work by Dr. Carol Dweck)

The Growth Mindset and Lessons About Failing

Something you may have noticed is that a growth mindset would tend to give a learner grit and persistence. If you had learning as your major goal, you would normally keep trying to attain that goal even if it took you multiple attempts. Not only that, but if you learned a little bit more with each try you would see each attempt as a success, even if you had not achieved complete mastery of whatever it was you were working to learn.

With that in mind, it should come as no surprise that Dr. Dweck found that those people who believed their abilities could change through learning (growth vs. a fixed mindset) readily accepted learning challenges and persisted despite early failures.

Improving Your Ability to Learn

As strange as it may seem, research into fixed vs. growth mindsets has shown that if you believe you can learn something new, you greatly improve your ability to learn. At first, this may seem like the sort of feel-good advice we often encounter in social media posts or quotes that are intended to inspire or motivate us (e.g., *believe in yourself!*), but in looking at the differences outlined between a fixed and a growth mindset, you can see how each part of the growth mindset path would increase your probability of success when it came to learning.

ACTIVITY

Very few people have a strict fixed or growth mindset all of the time. Often we tend to lean one way or another in certain situations. For example, a person trying to improve their ability in a sport they enjoy may exhibit all of the growth mindset traits and characteristics, but they find themselves blocked in a fixed mindset when they try to learn something in another area like computer programming or arithmetic.

In this exercise, do a little self-analysis and think of some areas where you may find yourself hindered by a fixed mindset. Using the outline presented below, in the far right column, write down how you can change your own behavior for each of the parts of the learning process. What will you do to move from a fixed to a growth mindset? For example, say you were trying to

learn to play a musical instrument. In the *Challenges* row, you might pursue a growth path by trying to play increasingly more difficult songs rather than sticking to the easy ones you have already mastered. In the *Criticism* row, you might take someone's comment about a weakness in timing as a motivation for you to practice with a metronome. For *Success of others* you could take inspiration from a famous musician that is considered a master and study their techniques.

Whatever it is that you decide you want to use for your analysis, apply each of the Growth characteristics to determine a course of action to improve.

Table 2.4

Parts of the learning process	Growth characteristic	What will you do to adopt a growth mindset?
Challenges	Embraces challenges	
Obstacles	Persists despite setbacks	
Effort	Sees effort as a path to success	
Criticism	Learns from criticism	
Success of Others	Finds learning and inspiration in the success of others	

Footnotes

- Dweck, C.S. & Leggett, E.L. (1988). A Social-Cognitive Approach to Motivation and Personality

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1.2: Failure Is Not an Option

Failure Is Not an Option

Nathan Wallace

In the movie Apollo 13, Ed Harris portrays NASA flight director Gene Kranz as he successfully guides the crew of a damaged spacecraft to safety. In a famous scene during which Kranz and his staff are attempting to overcome some extremely daunting challenges, Harris shouts, “Failure is not an option!” This singular statement perfectly articulated the determination of Kranz to bring the Apollo astronauts back to Earth.

This “failure is not an option” credo was perfect for the life and death situation that NASA was facing. Failure meant that the astronauts on Apollo 13 would never come home, and that outcome was unacceptable. Attending college, on the other hand, shouldn’t be a life or death experience, though it sometimes might feel like one. Failure, though never the intended outcome, can and sometimes does happen. Sometimes failure manifests itself in election results for a student government post, in a test score, or even in a final grade.

Throughout my life I have had many failures. In high school I drove my parents and teachers crazy because of my lack of academic achievement. I even managed to get an F- in Spanish on my report card. When I told my mom that it was a typo she responded, “So you didn’t get an F?” “No,” I said, “I definitely earned the F, but there’s no such thing as an F-.” To this day I’m not so sure that my reply was accurate. I might have earned that minus after all.

My failures in high school led to only one acceptance from of all the colleges I applied to attend. Furthermore, I was not accepted to the school’s main campus, but to their branch campus. During my first semester there my effort wasn’t much better than in high school, but since my parents were now paying for my education I did enough work to avoid academic probation. It wasn’t until my second semester that I found my niche as a Religious Studies major and started getting good grades, moved to the main campus, and eventually graduated with honors.

Since graduating from college, my career path has taken me into higher education as a Student Affairs administrator. This career has exposed me to many great theories regarding student success, and many of them gave me insight into my own college experience. But it was Stanford psychologist Carol Dweck who appeared to be thinking of me when she wrote the following about fixed mindsets in the introduction to her book titled *Mindset: The New Psychology of Success*:

Believing that your qualities are carved in stone—the fixed mindset—creates an urgency to prove yourself over and over. If you have only a certain amount of intelligence, a certain personality, and a certain moral character—well, then you’d better prove that you have a healthy dose of them. It simply wouldn’t do to look or feel deficient in these most basic characteristics. (Dweck, 2006)

This statement was a revelation to me. I finally understood my problem throughout high school and even in college. I earned good grades because I liked Religious Studies but never really challenged myself inside or outside of the classroom. My problem was that I had a fixed mindset about academic success. I believed that a person is either smart or they’re not, and nothing could be done to significantly change that. I also believed that I was one of the fortunate ones to be “gifted” with an abundance of intelligence.

One might think that having confidence in your intelligence is a whole lot better than thinking that you’re stupid, but the result was the same. My fixed mindset was holding me back because it led to a paralyzing fear of failure. Since as far back as I could remember, my family, friends, and teachers were always telling me how smart I was, and I believed them. But that belief was a double-edged sword. High school and college offered many occasions when self-confidence in my inherent intelligence could be threatened. If I fail on this test or in this course it means that I’m not the smart person I thought I was. If I fail, my family and friends will find out that they were wrong about me.

However, there was a way to avoid all of the risks of academic rigor. I could just not try. If I don’t try I’ll get bad marks on my report card, but those won’t be true indicators of my intelligence. By not putting forth any effort, my intelligence would never be disproven. I would always be able to say to myself and others that, “I could do the work and be a straight A student, but I’m just not interested.” Looking back on this time in my life, it is clear to me that this wasn’t a conscious decision to save face. It was fear, not logic, which was guiding my behavior.

After reading *Mindset* I have made a conscious effort to identify and thwart any remaining fixed mindset thoughts that I continue to hold. Dweck’s book acts as a manual for rooting out fixed mindset thoughts, because she explains that the idea of fixed mindsets is only half of her mindset theory. There is another kind of mindset, and she calls it growth mindset. Dweck writes that, “This growth

mindset is based on the belief that your basic qualities are things you can cultivate through your efforts" (Dweck, 2006). Dweck goes on to explain that we can choose to have a growth mindset about any type of ability, whether it's math, art, athletics, or any other skill that one wishes to cultivate.

I put this theory to the test not long after reading the book. A few years ago I attended a meeting only to find out that it wasn't any ordinary meeting. During this meeting we would be brainstorming solutions to a specific problem. This was going to be a true brainstorming session, led by a facilitator trained in the science of soliciting uninhibited ideas from an audience. As soon as I heard the word *brainstorming* I froze. I have always hated brainstorming. I'm the type of person that likes to think things through two or three times before expressing an opinion. My fear of failing at this task in front of my coworkers paralyzed my mind. I couldn't think.

That's when it hit me. This was fixed mindset thinking. My belief in my brainstorming inadequacies was preventing me from even trying. So I flipped this thinking on its head and decided the best way to improve my brainstorming abilities was to clear my mind and start firing out ideas. I gave it a shot, and though the ideas didn't come out at the prolific rate of some of my colleagues, I had never before had such a positive outcome and experience while brainstorming. Through this experience I found that I really could choose to have a growth mindset, and that this choice produces a greater chance of success. With a greater chance of success comes a smaller chance of failure.

Nevertheless, when it comes to academic success and success in all phases of life, failure is always an option. Though it can be painful, failure can lead to great learning and progress when a specific failure is analyzed through the lens of a growth mindset. By focusing more on effort than on outcomes anyone can learn and grow, regardless of their skill level. Therefore, to make the most of their time in college, students must seek out challenges that will stretch their abilities. These challenges can take many forms and they can occur in a variety of settings, both inside and outside of the classroom. When seeking out challenges there is always the possibility of agonizing defeat, but out of that defeat can be the seeds of great success in the future.

Reference

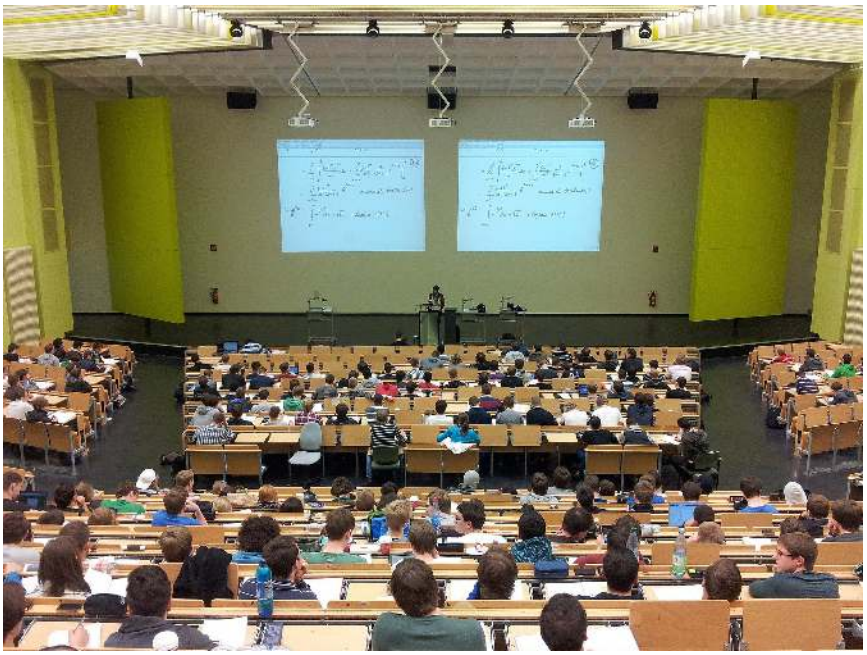
Dweck, C. (2006). *Mindset: The new psychology of success.* New York: Ballantine Books.

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1.3: Active Learning from Lectures

The learning of information and skills presented in class lectures or discussions should be viewed as a process: preparation to take in the new information, the act of taking in the new information, and then reviewing the information so that it is later accessible (recalled from memory) to use for a project, paper, or test.



The activities suggested for “Before Class” in Table 5-2 help students develop a mental framework into which this new information can be “filed” or organized which aids information recall (Bower, Clark, Lesgold, & Winzenz, 1969). It also makes note taking easier because you are familiar with the material and feel less panicked about getting everything noted in full detail.

Attending class and taking the notes is important primarily because the professor may be presenting information that is not presented in the text. It should also be seen as a structured opportunity to engage with the material that will further aid recall when needed. Oftentimes, students can also get clues as to what a professor perceives as important material from attending class that will aid in test preparation.

Table 5-2: Effective Techniques for Learning from Class

Before Class	During Class	After Class
Complete assigned readings to prepare for class. Take notes or mark your text for easy reference.	Take notes. Active engagement with the material (like writing down what you are hearing) helps improve memory and concentration and material to later review. Have a positive mindset about taking notes you are writing down answers to questions on the test.”	Review your notes and handouts within 24 hours of taking them. Fill in details that you remember but did not have time to write down. Write questions for the notes that you can use for self-testing later. Write a summary of the day’s notes.
Prepare questions that you may have about the reading.	Put a date on your note paper. It helps to also include the text chapter or general topic that might be related to these notes as well.	Compare your notes with a reliable classmate who may help give you ideas on what you missed that might be important or how to better organize the information. Discuss the notes to add another-layer of processing.

Before Class	During Class	After Class
Review notes from the previous class period to get a sense of where you may have left off.	Use active listening techniques. Refer to the informational box called “Ten Tips for Active Listening” for more information.	Follow up on questions that you have written down with your professor, the teaching assistant, or another reliable classmate.
Be sure you have your materials ready for class: text, three-ring notebook with paper, writing utensils, highlighters, charged laptop, etc.	Have your textbook accessible in case your faculty member refers to specific information. Write down that reference in your notes as it is probably important information.	Integrate your class notes with your text notes or markings. If you highlighted in your book, add some of that information to your notes for the “whole” picture.
Get a good night’s sleep. It is easier to concentrate on the lecture when you are alert.	Write down the main points a speaker is making along with any supple-mental details like examples or experiments. Do not expect to be able to write down every word the speaker says.	Develop tools that swill serve to help you review the material now, as well as later. Create visual diagrams* of concepts, develop ram cards* of concepts, reorganize your notes, etc. *Refer to informational boxes on “Visualize Your Information” and “Cram Cards for Long-Term Review.”
Make it a habit to attend every class. Put your classes on your schedule and treat them as a limited opportunity to engage with your professor and this material.	Use abbreviations for commonly used words to increase your speed in note taking. Create some abbreviations of your own.	Self-test your comprehension of the information on an ongoing basis. Do not just “look over” or read what you have written. Use the questions you have created. Cover up the “answer” and see if you can explain what is in the notes. Use the cram cards or visual diagrams to test yourself.
If you cannot attend class due to illness or an-other extraordinary reason, contact a classmate to arrange for notes. It is always a good idea to send an e-mail or call your professor as well.	Develop a system of organization for your notes that works for you. Common methods include the Cornell, Outlining, Mapping, Text-Class, and Sentence Methods. Refer to informational box called “Note Taking Methods: What Is Right for You?”	
	Leave space between main points so that you can add in new information that you may have missed or from the text.	
	If the professor writes it down or puts it on a PowerPoint slide, it is probably important, so write it down.	
	If the professor repeats it, it is important, so write it down.	
	Listen for cues like “very important,” “in summary,” or sequence words like “first,” “second,” etc. This information provides cues for what is important and possible note organization.	
	Ask questions you may have. If you are not comfortable talking in class, write them down and talk with your professor later.	

Adapted from Dembo Sell (2008) and Downing (2008).

Ten Tips for Active Listening

Effective note taking in class requires the use of active listening techniques for optimal results.

1. Accept responsibility as a listener.
2. Adopt a positive attitude toward listening. Listening is a choice.
3. Sit somewhere in the classroom that will minimize distractions and allow you to focus on the professor.
4. Maintain eye contact with your professor.
5. Focus on the content being presented rather than the delivery.
6. Ask questions in class.
7. Ask mental questions and search for the answer in the lecture or discussion.
8. Avoid emotional involvement that may impair your ability to concentrate or filter important information.
9. Clue in on non-verbal communication, such as exaggerated movements, excitement, etc., as it may serve as a cue to important information.
10. Monitor your concentration throughout the class period and continually refocus.

Adapted from Kline (2002) and Treuer (2006).



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1.4: Active Learning from Reading

As with learning from lectures and discussions, the learning of information and skills presented in readings, texts, and written text from a Web site should be viewed as a process: preparation to take in the new information, the act of taking in the new information, and then reviewing the information so that it is later accessible (recalled from memory) to use for a project, paper, or test.



Students who simply start and finish a reading with no further actions taken are really wasting their time. Reading a text needs to be approached properly in order to ensure comprehension and retention of the information. Planning for an active reading session by engaging in the "before reading" activities facilitates an active mindset to the reading session and discourages "auto-pilot reading" that results in scanning of the words but no real careful thought about what is being said.

Preparing for class by reading assigned texts related to the lecture material is extremely important to optimize the lecture experience. If you have a professor who tends to lecture at a fast pace, you have the foundational information and only need to focus on writing down unfamiliar information. If your professor expects class participation in discussions or utilizes active classroom techniques, such as case studies, small group activities and discussions, simulations, or demonstrations, you will understand what is happening and be able to contribute to the learning experience. Marking or highlighting your text for main ideas while reading not only keeps your mind focused on the reading but also serves to prepare you for easier exam review.

Table 5-3. Effective Techniques for Reading

Before Reading	During Reading	After Reading
Preview the reading headings to get a "big picture" of the outline of the reading. Look at the pictures and figures. Check out the bold and italicized words.	As you are reading, seek out the answers to the questions that the reading or you generated rather than passively scanning the words.	Take reading notes from the highlighted material from the text. Utilize visual organizers and summaries to capture information. Revisit the informational box on "Note-Taking Methods: What Is Right for You?" and visit "Visualize Your information and "Cram Cards for Long-term Review."
Preview any questions that the chapter may offer so that you can actively seek answers to those questions.	Monitor your concentration and comprehension. After each paragraph, ask yourself, "What was the main point of that paragraph?" After each section, summarize what you have read.	Answer the questions that a the reading may have provided or that you developed. Reread sections in which you remember little information.
Turn text headings into questions so that you can actively seek answers to those questions. Jot those questions down in the margin of the text or in your notes if you plan to take reading notes.	Mark your text. Highlight the main idea of a paragraph or write down the main idea of a paragraph in your notes. If you cannot mark your text, take reading notes on the main ideas. Refer to the informational box called "In Search of Main Ideas" for more information	Integrate your reading notes and lecture/discussion notes into one location for easier review for an exam.
Plan for a high energy time of the day to read. Consider breaking up a long reading assignment into multiple, shorter reading sessions.	Look up the definitions to words that you do not understand to help build your vocabulary and facilitate understanding of the topic.	Discuss readings with classmates on a regular basis. Take turns explaining to each other sections of the reading. Ask and answer reading questions. Jot down questions that are unclear to the study group to ask the professor in class or during office hours.
Be sure you have your materials ready for reading: text, notebook paper, writing utensils, high-lighters, charged laptop, etc.		

Adapted from Dembo and Seli (2008) and Downing (2008).

In Search of Main Ideas

Some students do not actively take notes or mark their readings and texts because they determine it is a lot of extra work to write down so much information. However, the problem is that students are usually writing down too much information and not really cluing in to the main ideas. Here are some tips for identifying the main ideas when reading:

1. The Table of Contents and chapter outlines provide a broad view of the main points that will be covered in a reading. Flesh out the outlines that are already provided for you.
2. Oftentimes in a textbook, the main idea is the first or last sentence of a paragraph. If it is not the first or the last sentence, then look back at the entire paragraph to see what the overall issue seems to be. Look for the overall pat- terns of your textbooks.
3. Titles, headings, and subheadings announce the major subject. Make these headings into questions, and the answers to the questions will likely be the main ideas.
4. Bold and italic words point to a main idea or key concept that you need to understand.
5. Repetition of key words or phrases throughout the text point to a main idea.
6. Chapter questions at the end of the chapter are about the main ideas of the text. Answer those questions and you will identify your main ideas.
7. Summaries presented at the end of the chapter also tend to restate the main ideas briefly. Flesh these ideas out with some supporting ideas, and you would have a good view of the entire chapter.
8. Stop and look at the visuals—pictures, diagrams, tables, etc. Oftentimes, the message depicted in the graph or picture is typically a main idea.
9. Detailed statistics, several examples in a row, and other details often signal that a main idea is being clarified, proven, supported, etc. Track back or ahead to find the main idea they are trying to illustrate.
10. Text that includes bullet points, numbering, or sequences is often a main idea.
11. Look for organizational patterns in the reading that might highlight the main ideas. For instance, are two issues being compared or contrasted? What was the effect of a certain event? Are problems and various solutions being presented? Is there a timeline of events that is important?
12. Be intentional about searching for the main ideas. Ask yourself at the end of each section or paragraph, "What is the point?" or "What is it that the author wants me to know?"
13. If you are reading a narrative, ask yourself questions like, "Who are the main characters?" "Why is this character important to this story?" "Why did the author chose to tell this part of the story?"

Finally, taking the time to think critically about the reading you have done further solidifies that information in your memory and can serve to prepare you for exam review. Summarizing material using visual organizers (refer to informational box "Visualize Your Information" and Figure 5-1) moves your thinking from low levels of thinking to high levels of thinking on Bloom's Taxonomy. Integrating text and lecture notes and discussing readings with your classmates and professor promotes memory storage of that information and thinking at higher levels as well.

Visualize Your Information

Summarizing main ideas of the readings or lectures can be difficult without the right tools. Visual organizers can help students condense important information into visual diagrams that can be easily reviewed for exams or help organize a writing assignment. Muskingum College's Center for Advancement and Learning (n.d.) hosts a useful Web site that summarizes and provides examples of various visual organizers (a.k.a. Information Organization Strategies) at

muskingum.edu/~cal/database/g...anization.html (Web site wants username and password now.)

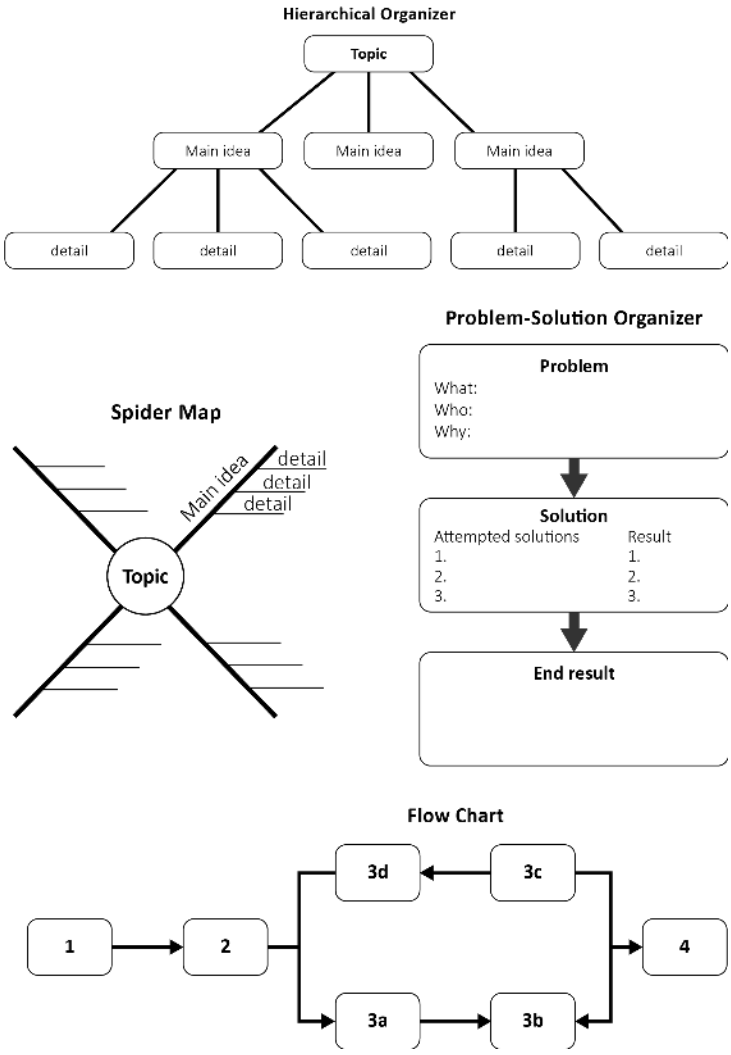
Consider how you might be able to apply several pages of text to a simple flowchart, hierarchical organizer, comparison-contrast organizer, spider map, etc. (See Figure 5-1.)

An additional benefit of using visual organizers is that you are thinking about the material at a deeper level as you identify the proper visual tool and apply the information. While at first it may take more time to determine the proper tool, it will get easier with practice and become second nature.

Activity 5-2

Utilizing what you have learned from the informational boxes "In Search of Main Ideas" and "Visualize Your Information," select a text chapter or a week of class notes and create some visual organizers that summarize the main ideas of that material. Ask a classmate to share in the same exercise and compare your work in a critical manner. What have you learned from this exercise and each other?

Figure 5-1. Examples of visual organizers



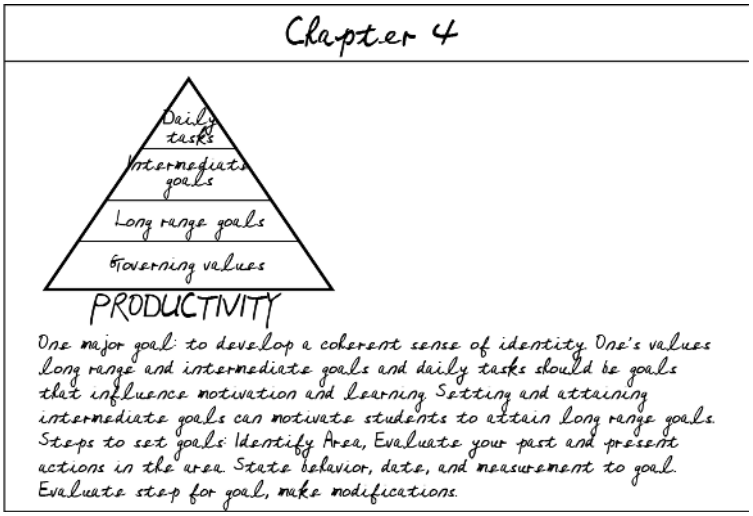
Source: Muskingum College's Center for Advancement and Learning (n.d)

Cram Cards for Long-term Review

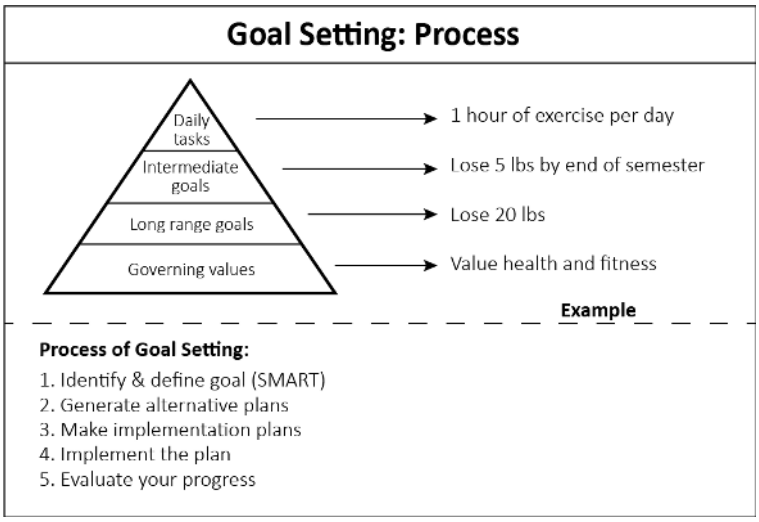
The use of "cram" in cram cards is deceiving. Cram cards are a study tool that promote the active learning of reading or lecture notes and allows for the easy and portable review of material over a longer period of time, therefore promoting later recall. It is a technique that will take some practice to hone your skills in selecting main ideas, but students who use cram cards indicate it is well worth their time.

Tips for Writing Cram Cards:

- Use 4 x 6 index cards (3 x 5 size tends to be too small)—some enthusiasts will even use the colored index cards for different sources of information (i.e., notes versus text reading) or topics (e.g., blue for English and green for Biology).
- Make one cram card per major concept or main idea.
- Do not simply take notes on the card; capture a summary of the main ideas maintaining the proper relationships of the concepts.
- Write the information in your own words.
- Include book examples or provide real-life examples for difficult concepts.
- Include key words.
- Be sure to write legibly.



Example of a Bad Cram Card



Example of a Good Cram Card

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1.5: Taking Notes

Questions to consider

- How can you prepare to take notes to maximize the effectiveness of the experience?
- What are some specific strategies you can employ for better notetaking?
- Why is annotating your notes after the notetaking session a critical step to follow?

Beyond providing a record of the information you are reading or hearing, notes help you organize the ideas and help you make meaning out of something about which you may not be familiar, so notetaking and reading are two compatible skill sets. Taking notes also helps you stay focused on the question at hand. Nanami often takes notes during presentations or class lectures so she can follow the speaker's main points and condense the material into a more readily usable format. Strong notes build on your prior knowledge of a subject, help you discuss trends or patterns present in the information, and direct you toward areas needing further research or reading.



Figure 1.5.9 Strong notes build on your prior knowledge of a subject, help you discuss trends or patterns present in the information, and direct you toward areas needing further research or reading.

It is not a good habit to transcribe every single word a speaker utters—even if you have an amazing ability to do that. Most of us don't have that court-reporter-esque skill level anyway, and if we try, we would end up missing valuable information. Learn to listen for main ideas and distinguish between these main ideas and details that typically support the ideas. Include examples that explain the main ideas, but do so using understandable abbreviations.

Think of all notes as potential study guides. In fact, if you only take notes without actively working on them after the initial notetaking session, the likelihood of the notes helping you is slim. Research on this topic concludes that without active engagement after taking notes, most students forget 60–75 percent of material over which they took the notes—within two days! That sort of defeats the purpose, don't you think? This information about memory loss was first brought to light by 19th-century German psychologist Hermann Ebbinghaus. Fortunately, you do have the power to thwart what is sometimes called the Ebbinghaus Forgetting Curve by reinforcing what you learned through review at intervals shortly after you take in the material and frequently thereafter.

If you are a musician, you'll understand this phenomenon well. When you first attempt a difficult piece of music, you may not remember the chords and notes well at all, but after frequent practice and review, you generate a certain muscle memory and cognitive recall that allows you to play the music more easily.

Notetaking may not be the most glamorous aspect of your higher-education journey, but it is a study practice you will carry throughout college and into your professional life. Setting yourself up for successful notetaking is almost as important as the actual taking of notes, and what you do after your notetaking session is equally significant. Well-written notes help you organize your thoughts, enhance your memory, and participate in class discussion, and they prepare you to respond successfully on exams. With all that riding on your notes, it would behoove you to learn how to take notes properly and continue to improve your notetaking skills.

Analysis Question

Do you currently have a preferred way to take notes? When did you start using it? Has it been effective? What other strategy might work for you?

Preparing to Take Notes

Preparing to take notes means more than just getting out your laptop or making sure you bring pen and paper to class. You'll do a much better job with your notes if you understand why we take notes, have a strong grasp on your preferred notetaking system, determine your specific priorities depending on your situation, and engage in some version of efficient shorthand.

Like handwriting and fingerprints, we all have unique and fiercely independent notetaking habits. These understandably and reasonably vary from one situation to the next, but you can only improve your skills by learning more about ways to take effective notes and trying different methods to find a good fit.

The very best notes are the ones you take in an organized manner that encourages frequent review and use as you progress through a topic or course of study. For this reason, you need to develop a way to organize all your notes for each class so they remain together and organized. As old-fashioned as it sounds, a clunky three-ring binder is an excellent organizational container for class notes. You can easily add to previous notes, insert handouts you may receive in class, and maintain a running collection of materials for each separate course. If the idea of carrying around a heavy binder has you rolling your eyes, then transfer that same structure into your computer files. If you don't organize your many documents into some semblance of order on your computer, you will waste significant time searching for improperly named or saved files.

You may be interested in relatively new research on what is the more effective notetaking strategy: handwriting versus typing directly into a computer. While individuals have strong personal opinions on this subject, most researchers agree that the format of student notes is less important than what students do with the notes they take afterwards. Both handwriting notes and using a computer for notetaking have pros and cons.

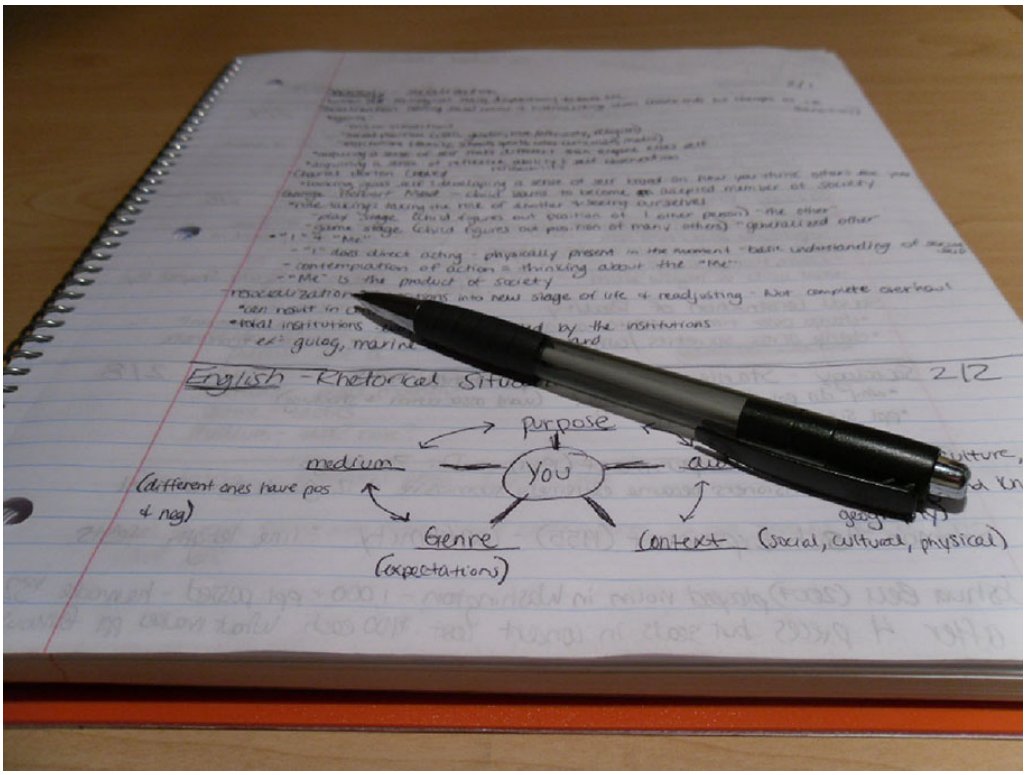


Figure 1.5.10 The best notes are the ones you take in an organized manner. Frequent review and further annotation are important to build a deep and useful understanding of the material. (Credit: English106 / Flickr / Attribution 2.0 Generic (CC-BY 2.0))

Managing Notetaking Systems (Computer, Paper/Pen, Note Cards, Textbook)

Whichever of the many notetaking systems you choose (and new ones seem to come out almost daily), the very best one is the one that you will use consistently. The skill and art of notetaking is not automatic for anyone; it takes a great deal of practice, patience, and continuous attention to detail. Add to that the fact that you may need to master multiple notetaking techniques for different classes, and you have some work to do. Unless you are specifically directed by your instructor, you are free to combine the best parts of different systems if you are most comfortable with that hybrid system.

Just to keep yourself organized, all your notes should start off with an identifier, including at the very least the date, the course name, the topic of the lecture/presentation, and any other information you think will help you when you return to use the notes for further study, test preparation, or assignment completion. Additional, optional information may be the number of notetaking sessions about this topic or reminders to cross-reference class handouts, textbook pages, or other course materials. It's also always a good idea to leave some blank space in your notes so you can insert additions and questions you may have as you review the material later.

Notetaking Strategies

You may have a standard way you take all your notes for all your classes. When you were in high school, this one-size-fits-all approach may have worked. Now that you're in college, reading and studying more advanced topics, your general method may still work some of the time, but you should have some different strategies in place if you find that your method isn't working as well with college content. You probably will need to adopt different notetaking strategies for different subjects. The strategies in this section represent various ways to take notes in such a way that you are able to study after the initial notetaking session.

Cornell Method

One of the most recognizable notetaking systems is called the *Cornell Method*, a relatively simple way to take effective notes devised by Cornell University education professor Dr. Walter Pauk in the 1940s. In this system, you take a standard piece of note paper and divide it into three sections by drawing a horizontal line across your paper about one to two inches from the bottom of the page (the summary area) and then drawing a vertical line to separate the rest of the page above this bottom area, making the left side about two inches (the recall column) and leaving the biggest area to the right of your vertical line (the notes column). You may want to make one page and then copy as many pages as you think you'll need for any particular class, but one advantage of this

system is that you can generate the sections quickly. Because you have divided up your page, you may end up using more paper than you would if you were writing on the entire page, but the point is not to keep your notes to as few pages as possible. The Cornell Method provides you with a well-organized set of notes that will help you study and review your notes as you move through the course. If you are taking notes on your computer, you can still use the Cornell Method in Word or Excel on your own or by using a template someone else created.

Topic/Objective:	Name:
	Class/Period:
	Date:
Essential Question:	

Questions:	Notes:

Summary:

Figure 1.5.11: The Cornell Method provides a straightforward, organized, and flexible approach

Now that you have the notetaking format generated, the beauty of the Cornell Method is its organized simplicity. Just write on one side of the page (the right-hand notes column)—this will help later when you are reviewing and revising your notes. During your notetaking session, use the notes column to record information over the main points and concepts of the lecture; try to put the ideas into your own words, which will help you not transcribe the speaker's words verbatim. Skip lines between each idea in this column. Practice the shortcut abbreviations covered in the next section and avoid writing in complete sentences. Don't make your notes too cryptic, but you can use bullet points or phrases equally well to convey meaning—we do it all the time in conversation. If you know you will need to expand the notes you are taking in class but don't have time, you can put reminders directly in the notes by adding and underlining the word *expand* by the ideas you need to develop more fully.

As soon as possible after your notetaking session, preferably within eight hours but no more than twenty-four hours, read over your notes column and fill in any details you missed in class, including the places where you indicated you wanted to expand your notes. Then in the recall column, write any key ideas from the corresponding notes column—you can't stuff this smaller recall column as if you're explaining or defining key ideas. Just add the one- or two-word main ideas; these words in the recall column serve as cues to help you remember the detailed information you recorded in the notes column.

Once you are satisfied with your notes and recall columns, summarize this page of notes in two or three sentences using the summary area at the bottom of the sheet. This is an excellent time to get with another classmate or a group of students who all heard the same lecture to make sure you all understood the key points. Now, before you move onto something else, cover the large notes column, and quiz yourself over the key ideas you recorded in the recall column. Repeat this step often as you go along, not just immediately before an exam, and you will help your memory make the connections between your notes, your textbook reading, your in-class work, and assignments that you need to succeed on any quizzes and exams.

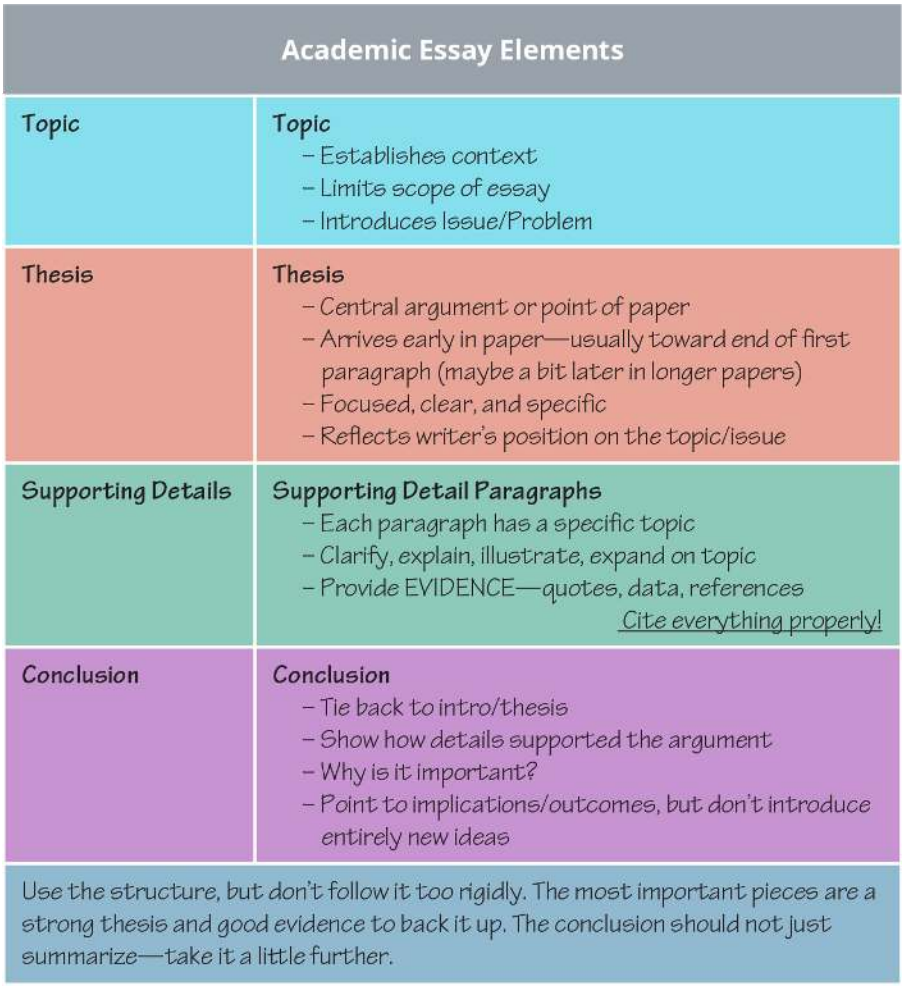


Figure 1.5.12 This sample set of notes in the Cornell Method is designed to make sense of a large amount of information. The process of organizing the notes can help you retain the information more effectively than less consistent methods.

The main advantage of the Cornell Method is that you are setting yourself up to have organized, workable notes. The neat format helps you move into study-mode without needing to re-copy less organized notes or making sense of a large mass of information you aren't sure how to process because you can't remember key ideas or what you meant. If you write notes in your classes without any sort of system and later come across something like "Napoleon—short" in the middle of a glob of notes, what can you do at this point? Is that important? Did it connect with something relevant from the lecture? How would you possibly know? You are your best advocate for setting yourself up for success in college.

Outlining

Other note organizing systems may help you in different disciplines. You can take notes in a formal outline if you prefer, using Roman numerals for each new topic, moving down a line to capital letters indented a few spaces to the right for concepts related to the previous topic, then adding details to support the concepts indented a few more spaces over and denoted by an Arabic numeral. You can continue to add to a formal outline by following these rules.

You don't absolutely have to use the formal numerals and letter, but you have to then be careful to indent so you can tell when you move from a higher level topic to the related concepts and then to the supporting information. The main benefit of an outline is how

organized it is. You have to be on your toes when you are taking notes in class to ensure you keep up the organizational format of the outline, which can be tricky if the lecture or presentation is moving quickly or covering many diverse topics.

The following formal outline example shows the basic pattern:

I. Dogs *(main topic–usually general)*
   A. German Shepherd *(concept related to main topic)*
      1. Protection *(supporting info about the concept)*
      2. Assertive
      3. Loyal
   B. Weimaraner *(concept related to main topic)*
      1. Family-friendly *(supporting info about the concept)*
      2. Active
      3. Healthy
II. Siamese

You would just continue on with this sort of numbering and indenting format to show the connections between main ideas, concepts, and supporting details. Whatever details you do not capture in your notetaking session, you can add after the lecture as you review your outline.

Chart or table

Similar to creating an outline, you can develop a chart to compare and contrast main ideas in a notetaking session. Divide your paper into four or five columns with headings that include either the main topics covered in the lecture or categories such as How?, What?, When used?, Advantages/Pros, Disadvantages/Cons, or other divisions of the information. You write your notes into the appropriate columns as that information comes to light in the presentation.

Example of a Chart to Organize Ideas and Categories

	Structure	Types	Functions in Body	Additional Notes
Carbohydrates				
Lipids				
Proteins				
Nucleic Acid				

This format helps you pull out the salient ideas and establishes an organized set of notes to study later. (If you haven't noticed that this *reviewing later* idea is a constant across all notetaking systems, you should…take note of that.) Notes by themselves that you never reference again are little more than scribblings. That would be a bit like compiling an extensive grocery list so you stay on budget when you shop, work all week on it, and then just throw it away before you get to the store. You may be able to recall a few items, but likely won't be as efficient as you could be if you had the notes to reference. Just as you cannot read all the many books, articles, and documents you need to peruse for your college classes, you cannot remember the most important ideas of all the notes you will take as part of your courses, so you must review.

Concept Mapping and Visual Notetaking

One final notetaking method that appeals to learners who prefer a visual representation of notes is called *mapping* or sometimes *mind mapping* or *concept mapping*, although each of these names can have slightly different uses. Variations of this method abound, so you may want to look for more versions online, but the basic principles are that you are making connections between main ideas through a graphic depiction; some can get rather elaborate with colors and shapes, but a simple version may be more useful at least to begin. Main ideas can be circled or placed in a box with supporting concepts radiating off these ideas shown with a connecting line and possibly details of the support further radiating off the concepts. You can present your main ideas vertically or horizontally, but turning your paper long-ways, or in landscape mode, may prove helpful as you add more main ideas.

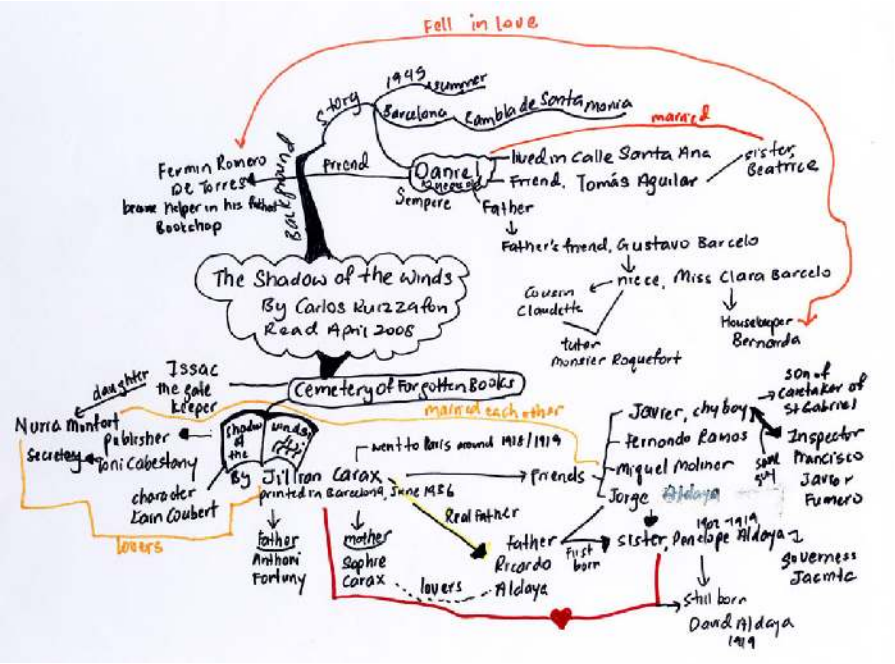


Figure 1.5.13 Concept mapping, sometimes referred to as mind mapping, can be an effective and very personalized approach to capturing information. (Credit: ArtistIvanChew / Flickr / Attribution 2.0 Generic (CC-BY 2.0))

You may be interested in trying visual notetaking or adding pictures to your notes for clarity. Sometimes when you can't come up with the exact wording to explain something or you're trying to add information for complex ideas in your notes, sketching a rough image of the idea can help you remember. According to educator Sherrill Knezel in an article entitled "The Power of Visual Notetaking," this strategy is effective because "When students use images and text in notetaking, it gives them two different ways to pull up the information, doubling their chances of recall." Don't shy away from this creative approach to notetaking just because you believe you aren't an artist; the images don't need to be perfect. You may want to watch Rachel Smith's TEDx Talk called "Drawing in Class" to learn more about visual notetaking.

You can play with different types of notetaking suggestions and find the method(s) you like best, but once you find what works for you, stick with it. You will become more efficient with the method the more you use it, and your notetaking, review, and test prep will become, if not easier, certainly more organized, which can delete decrease your anxiety.

Practicing Decipherable Shorthand

Most college students don't take a class in shorthand, once the domain of secretaries and executive assistants, but maybe they should. That almost-lost art in the age of computers could come in very handy during intense notetaking sessions. Elaborate shorthand systems do exist, but you would be better served in your college notetaking adventures to hone a more familiar, personalized form of shorthand to help you write more in a shorter amount of time. Seemingly insignificant shortcuts can add up to ease the stress notetaking can induce—especially if you ever encounter an "I'm not going to repeat this" kind of presenter! Become familiar with these useful abbreviations:

Shortcut symbol	Meaning
w/, w/o, w/in	with, without, within
&	and
#	number
b/c	because
X, √	incorrect, correct
Diff	different, difference
etc.	and so on

ASAP	as soon as possible
US, UK	United States, United Kingdom
info	information
Measurements: ft, in, k, m	foot, inch, thousand, million
¶	paragraph or new paragraph
Math symbols: =, +, >, <, ÷	equal, plus, greater, less, divided by
WWI, WWII	World Wars I and II
impt	important
?, !, **	denote something is very significant; don't over use

Do you have any other shortcuts or symbols that you use in your notes? Ask your parents if they remember any that you may be able to learn.

Annotating Notes After Initial Notetaking Session

Annotating notes after the initial notetaking session may be one of the most valuable study skills you can master. Whether you are highlighting, underlining, or adding additional notes, you are reinforcing the material in your mind and memory.

Admit it—who can resist highlighting markers? Gone are the days when yellow was the star of the show, and you had to be very careful not to press too firmly for fear of obliterating the words you were attempting to emphasize. Students now have a veritable rainbow of highlighting options and can color-code notes and text passages to their hearts' content. Technological advances may be important, but highlighter color choice is monumental! Maybe.

The only reason to highlight anything is to draw attention to it, so you can easily pick out that ever-so-important information later for further study or reflection. One problem many students have is not knowing when to stop. If what you need to recall from the passage is a particularly apt and succinct definition of the term important to your discipline, highlighting the entire paragraph is less effective than highlighting just the actual term. And if you don't rein in this tendency to color long passages (possibly in multiple colors) you can end up with a whole page of highlighted text. Ironically, that is no different from a page that is not highlighted at all, so you have wasted your time. Your mantra for highlighting text should be *less is more*. Always read your text selection first before you start highlighting anything. You need to know what the overall message is before you start placing emphasis in the text with highlighting.

Another way to annotate notes after initial notetaking is underlying significant words or passages. Albeit not quite as much fun as its colorful cousin highlighting, underlining provides precision to your emphasis.

Some people think of annotations as only using a colored highlighter to mark certain words or phrases for emphasis. Actually, annotations can refer to anything you do with a text to enhance it for your particular use (either a printed text, handwritten notes, or other sort of document you are using to learn concepts). The annotations may include highlighting passages or vocabulary, defining those unfamiliar terms once you look them up, writing questions in the margin of a book, underlining or circling key terms, or otherwise marking a text for future reference. You can also annotate some electronic texts.

Realistically, you may end up doing all of these types of annotations at different times. We know that repetition in studying and reviewing is critical to learning, so you may come back to the same passage and annotate it separately. These various markings can be invaluable to you as a study guide and as a way to see the evolution of your learning about a topic. If you regularly begin a reading session writing down any questions you may have about the topic of that chapter or section and also write out answers to those questions at the end of the reading selection, you will have a good start to what that chapter covered when you eventually need to study for an exam. At that point, you likely will not have time to reread the entire selection especially if it is a long reading selection, but with strong annotations in conjunction with your class notes, you won't need to do that. With experience in reading discipline-specific texts and writing essays or taking exams in that field, you will know better what sort of questions to ask in your annotations

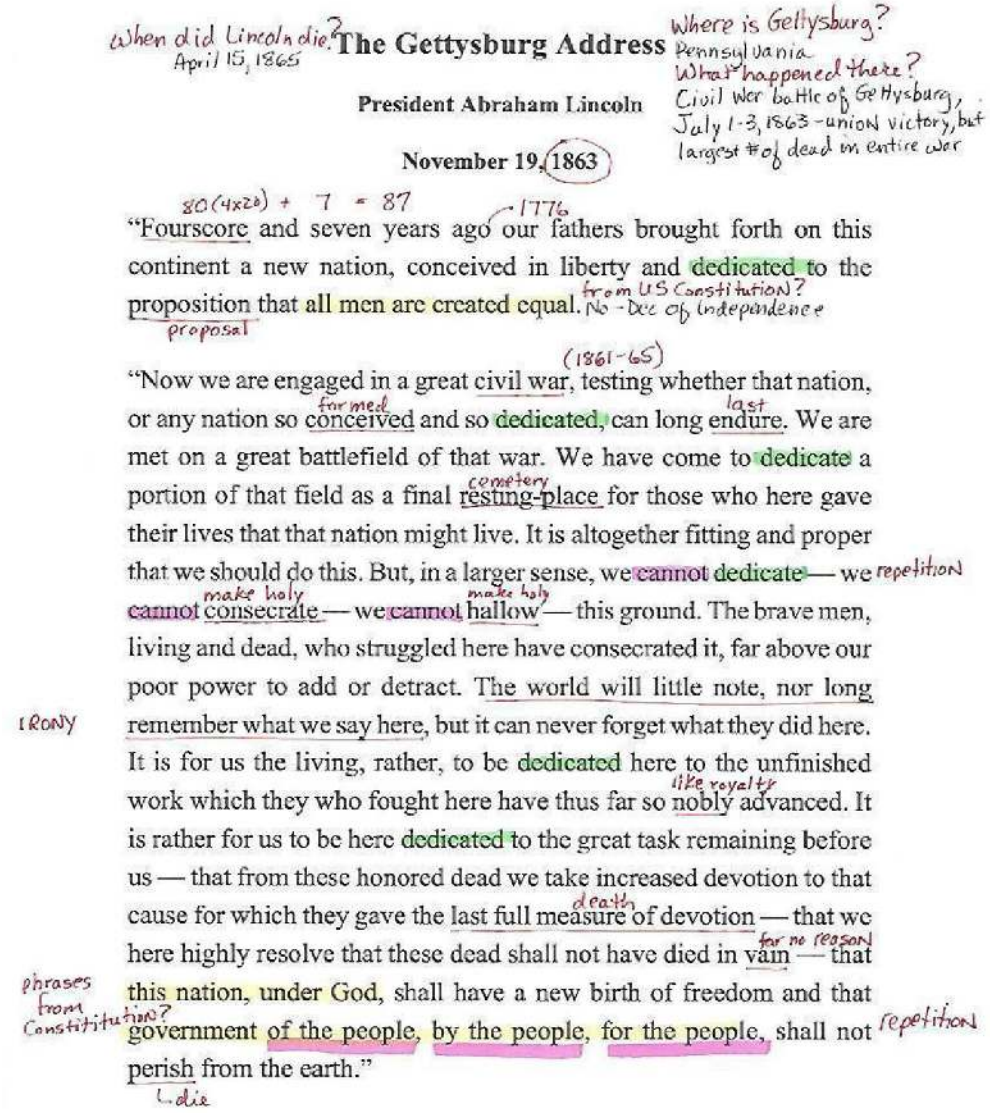


Figure 1.5.14 Annotations may include highlighting important topics, defining unfamiliar terms, writing questions in, underlining or circling key terms, or otherwise marking a text for future reference. Whichever approach you choose, try not to overdo it; neat, organized, and efficient notes are more effective than crowded or overdone notes.

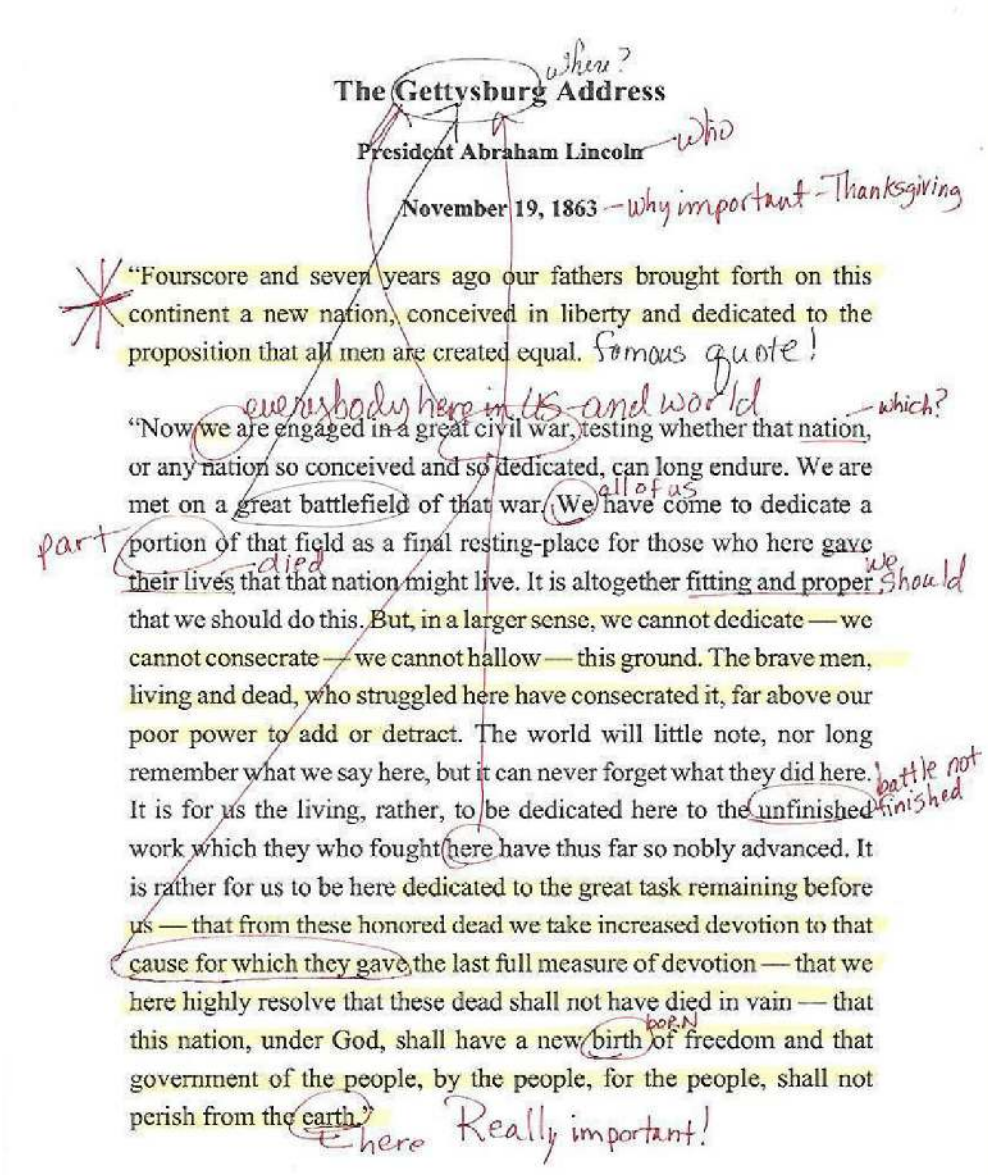


Figure 1.5.15 While these notes may be meaningful to the person who took them, they are neither organized nor consistent. For example, note that some of the more commonly used terms, like "we" and "unfinished," are defined, but less common ones -- "consecrate" and "hallow" -- are not.

What you have to keep in the front of your mind while you are annotating, especially if you are going to conduct multiple annotation sessions, is to not overdo whatever method you use. Be judicious about what you annotate and how you do it on the page, which means you must be neat about it. Otherwise, you end up with a mess of either color or symbols combined with some cryptic notes that probably took you quite a long time to create, but won't be worth as much to you as a study aid as they could be. This is simply a waste of time and effort.

You cannot eat up every smidgen of white space on the page writing out questions or summaries and still have a way to read the original text. If you are lucky enough to have a blank page next to the beginning of the chapter or section you are annotating, use this, but keep in mind that when you start writing notes, you aren't exactly sure how much space you'll need. Use a decipherable shorthand and write only what you need to convey the meaning in very small print. If you are annotating your own notes, you can make a habit of using only one side of the paper in class, so that if you need to add more notes later, you could use the other side. You can also add a blank page to your notes before beginning the next class date in your notebook so you'll end up with extra paper for annotations when you study.

Professional resources may come with annotations that can be helpful to you as you work through the various documentation requirements you'll encounter in college as well. Purdue University's Online Writing Lab (OWL) provides an annotated sample for

how to format a college paper according to guidelines in the Modern Language Association (MLA) manual that you can see, along with other annotations.

Adding Needed Additional Explanations to Notes

Marlon was totally organized and ready to take notes in a designated course notebook at the beginning of every philosophy class session. He always dated his page and indicated what the topic of discussion was. He had various colored highlighters ready to denote the different note purposes he had defined: vocabulary in pink, confusing concepts in green, and note sections that would need additional explanations later in yellow. He also used his own shorthand and an impressive array of symbols to indicate questions (red question mark), highly probable test material (he used a tiny bomb exploding here), additional reading suggestions, and specific topics he would ask his instructor before the next class. Doing everything so precisely, Marlon's methods seemed like a perfect example of how to take notes for success. Inevitably though, by the end of the hour-and-a-half class session, Marlon was frantically switching between writing tools, near to tears, and scouring his notes as waves of yellow teased him with uncertainty. What went wrong?

As with many of us who try diligently to do everything we know how to do for success or what we think we know because we read books and articles on success in between our course work, Marlon is suffering from trying to do too much simultaneously. It's an honest mistake we can make when we are trying to save a little time or think we can multitask and kill two birds with one stone.

Unfortunately, this particular error in judgement can add to your stress level exponentially if you don't step back and see it for what it is. Marlon attempted to take notes in class as well as annotate his notes to get them ready for his test preparation. It was too much to do at one time, but even if he could have done all those things during class, he's missing one critical point about notetaking.

As much as we may want to hurry and get it over with, notetaking in class is just the beginning. Your instructor likely gave you a pre-class assignment to read or complete before coming to that session. The intention of that preparatory lesson is for you to come in with some level of familiarity for the topic under consideration and questions of your own. Once you're in class, you may also need to participate in a group discussion, work with your classmates, or perform some other sort of lesson-directed activity that would necessarily take you away from taking notes. Does that mean you should ignore taking notes for that day? Most likely not. You may just need to indicate in your notes that you worked on a project or whatever other in-class event you experienced that date.

Very rarely in a college classroom will you engage in an activity that is not directly related to what you are studying in that course. Even if you enjoyed every minute of the class session and it was an unusual format for that course, you still need to take some notes. Maybe your first note could be to ask yourself why you think the instructor used that unique teaching strategy for the class that day. Was it effective? Was it worth using the whole class time? How will that experience enhance what you are learning in that course?

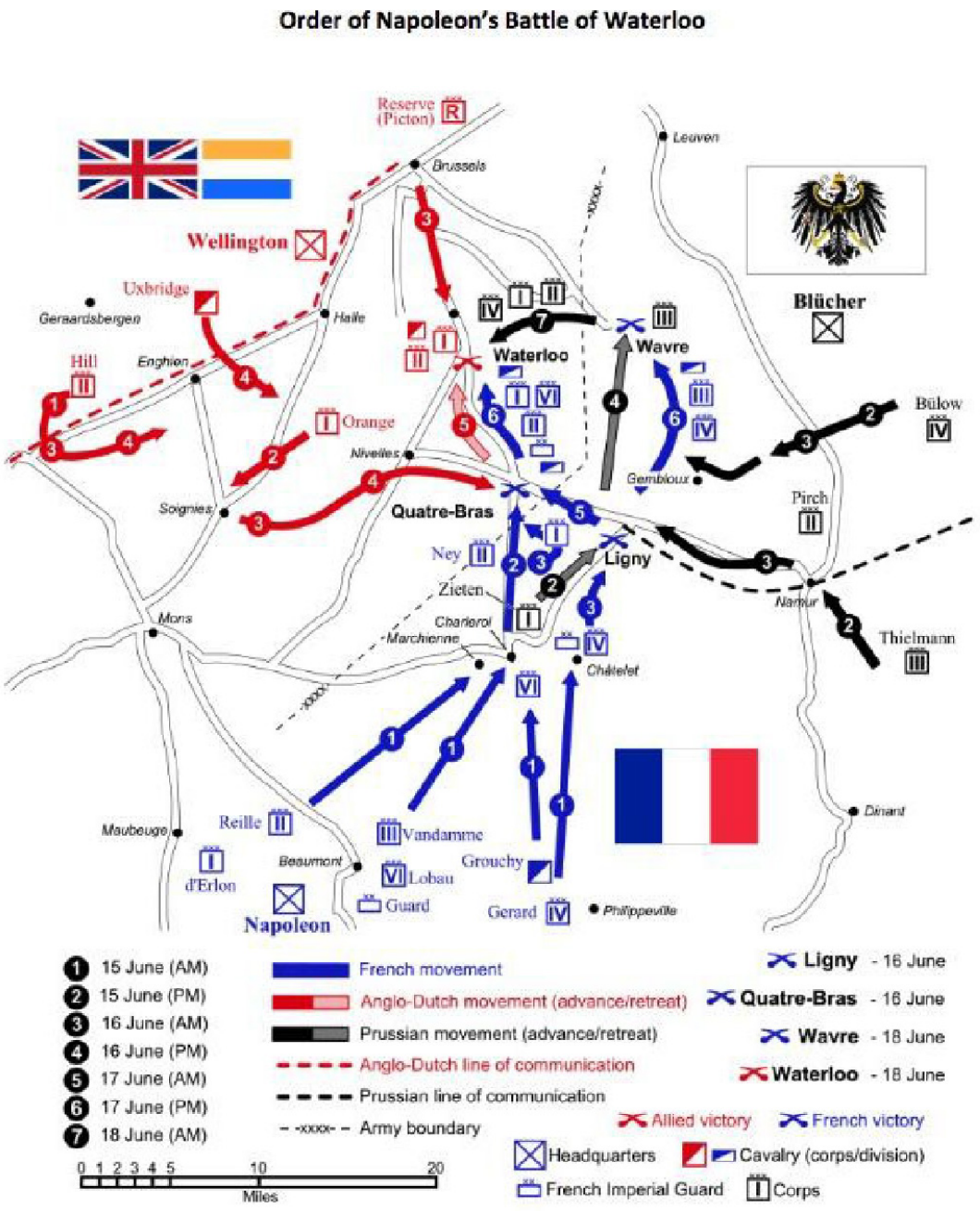
If you use an ereader or ebooks to read texts for class or read articles from the Internet on your laptop or tablet, you can still take effective notes. Depending on the features of your device, you have many choices. Almost all electronic reading platforms allow readers to highlight and underline text. Some devices allow you to add a written text in addition to marking a word or passage that you can collect at the end of your notetaking session. Look into the specific tools for your device and learn how to use the features that allow you to take notes electronically. You can also find apps on devices to help with taking notes, some of which you may automatically have installed when you buy the product. Microsoft's OneNote, Google Keep, and the Notes feature on phones are relatively easy to use, and you may already have free access to those.

Taking Notes on Non-Text Items (i.e., Tables, Maps, Figures, etc.)

You may also encounter situations as you study and read textbooks, primary sources, and other resources for your classes that are not actually texts. You can still take notes on maps, charts, graphs, images, and tables, and your approach to these non-text features is similar to when you prepare to take notes over a passage of text. For example, if you are looking at the following map, you may immediately come up with several questions. Or it may initially appear overwhelming. Start by asking yourself these questions:

What is the main point of this map?

- Who is the intended audience?
- Where is it?
- What time period does it depict?
- What does the map's legend (the explanation of symbols) include?
- What other information do I need to make sense of this map?



Source: Wikipedia Creative Commons, https://en.wikipedia.org/wiki/Order_of_battle_of_the_Waterloo_campaign

Figure 1.5.16 Graphics, charts, graphs, and other visual items are also important to annotate. Not only do they often convey important information, but they may appear on exams or in other situations where you'll need to use or demonstrate knowledge. Credit: "Lpankonin" / Wikipedia Commons / Attribution 3.0 Generic (CC BY 3.0)

You may want to make an extra copy of a graphic or table before you add annotations if you are dealing with a lot of information. Making sense of all the elements will take time, and you don't want to add to the confusion.

Returning to Your Notes

Later, as soon as possible after the class, you can go back to your notes and add in missing parts. Just as you may generate questions as you're reading new material, you may leave a class session or lecture or activities with many questions. Write those

down in a place where they won't get lost in all your other notes.

The exact timing of when you get back to the notes you take in class or while you are reading an assignment will vary depending on how many other classes you have or what other obligations you have in your daily schedule. A good starting place that is also easy to remember is to make every effort to review your notes within 24 hours of first taking them. Longer than that and you are likely to have forgotten some key features you need to include; must less time than that, and you may not think you need to review the information you so recently wrote down, and you may postpone the task too long.

Use your phone or computer to set reminders for all your note review sessions so that it becomes a habit and you keep on top of the schedule.

Your personal notes play a significant role in your test preparation. They should enhance how you understand the lessons, textbooks, lab sessions, and assignments. All the time and effort you put into first taking the notes and then annotating and organizing the notes will be for naught if you do not formulate an effective and efficient way to use them before sectional exams or comprehensive tests.

The whole cycle of reading, notetaking in class, reviewing and enhancing your notes, and preparing for exams is part of a continuum you ideally will carry into your professional life. Don't try to take short cuts; recognize each step in the cycle as a building block. Learning doesn't end, which shouldn't fill you with dread; it should help you recognize that all this work you're doing in the classroom and during your own study and review sessions is ongoing and cumulative. Practicing effective strategies now will help you be a stronger professional.

Activity

What resources can you find about reading and notetaking that will actually help you with these crucial skills? How do you go about deciding what resources are valuable for improving your reading and notetaking skills?

The selection and relative value of study guides and books about notetaking vary dramatically. Ask your instructors for recommendations and see what the library has available on this topic. The following list is not comprehensive, but will give you a starting point for books and articles on notetaking in college.

- *College Rules!: How to Study, Survive, and Succeed in College*, by Sherri Nist-Olejnik and Jodi Patrick Holschuh. More than just notetaking, this book covers many aspects of transitioning into the rigors of college life and studying.
- *Effective Notetaking*, by Fiona McPherson. This small volume has suggestions for using your limited time wisely before, during, and after notetaking sessions.
- *How to Study in College*, by Walter Pauk. This is the book that introduced Pauk's notetaking suggestions we now call the Cornell Method. It is a bit dated (from the 1940s), but still contains some valuable information.
- *Learn to Listen, Listen to Learn 2: Academic Listening and Note-taking*, by Roni S. Lebauer. The main point of this book is to help students get the most from college lectures by watching for clues to lecture organization and adapting this information into strong notes.
- *Study Skills: Do I Really Need this Stuff?*, by Steve Piscitelli. Written in a consistently down-to-earth manner, this book will help you with the foundations of strong study skills, including time management, effective notetaking, and seeing the big picture.
- "What Reading Does for the Mind," by Anne Cunningham and Keith Stanovich, 1998, https://www.aft.org/sites/default/files/periodicals/cunningham.pdf
- Adler, Mortimer J. and Charles Van Doren. *How to Read a Book: The Classic Guide to Intelligent Reading. NY: Simon & Schuster, 1940.*
- Berns, Gregory S., Kristina Blaine, Michael J. Prietula, and Brandon E. Pye. *Brain Connectivity.* Dec 2013.ahead of print http://doi.org/10.1089/brain.2013.0166

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1.6: Enhanced Strategies for Time and Task Management

Questions to consider:

- What strategy helps me prioritize my top tasks?
- How do I make the best use of my time when prioritizing?
- How do I make sure I tackle unpleasant tasks instead of putting them off?
- What's the best way to plan for long-term tasks?
- How do I find time in a busy schedule?

Over the years, people have developed a number of different strategies to manage time and tasks. Some of the strategies have proven to be effective and helpful, while others have been deemed not as useful.

The good news is that the approaches that do not work very well or do not really help in managing time do not get passed along very often. But others, those which people find of value, do. What follows here are three unique strategies that have become staples of time management. While not everyone will find that all three work for them in every situation, enough people have found them beneficial to pass them along with high recommendations.

Daily Top Three

The idea behind the *daily top three* approach is that you determine which three things are the most important to finish that day, and these become the tasks that you complete. It is a very simple technique that is effective because each day you are finishing tasks and removing them from your list. Even if you took one day off a week and completed no tasks on that particular day, a *daily top three* strategy would have you finishing 18 tasks in the course of a single week. That is a good amount of things crossed off your list.

analysis question

Analysis: Think about what would be your top three tasks for today? What would you have on the list tomorrow?

Pomodoro Technique



Figure 1.6.14 The Pomodoro Technique is named after a type of kitchen timer, but you can use any clock or countdown timer. (Marco Verch /Flickr / Attribution 2.0 Generic (CC BY 2.0))

The Pomodoro Technique was developed by Francesco Cirillo. The basic concept is to use a timer to set work intervals that are followed by a short break. The intervals are usually about 25 minutes long and are called *pomodoros*, which comes from the Italian word for tomato because Cirillo used a tomato-shaped kitchen timer to keep track of the intervals.

In the original technique there are six steps:

1. Decide on the task to be done.
2. Set the timer to the desired interval.
3. Work on the task.
4. When the timer goes off, put a check mark on a piece of paper.
5. If you have fewer than four check marks, take a short break (3–5 minutes), then go to Step 1 or 2 (whichever is appropriate).
6. After four pomodoros, take a longer break (15–30 minutes), reset your check mark count to zero, and then go to Step 1 or 2.

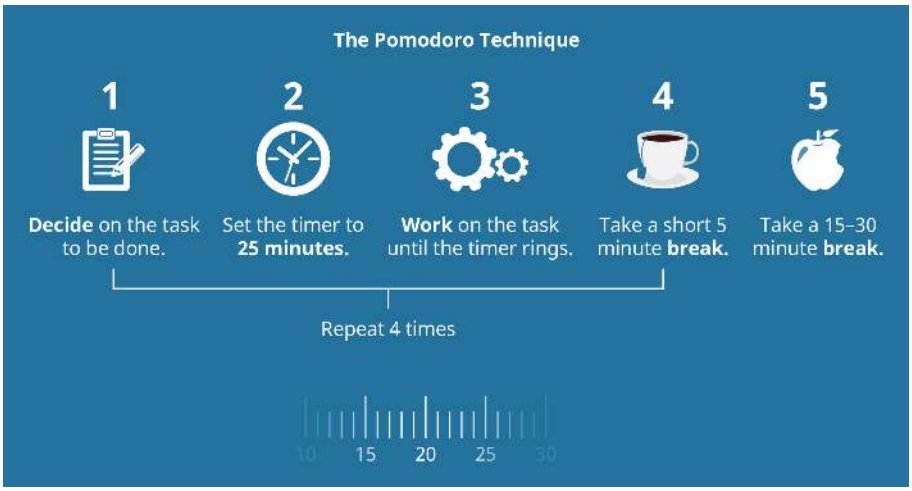


Figure 1.6.15 The Pomodoro Technique contains five defined steps.

There are several reasons this technique is deemed effective for many people. One is the benefit that is derived from quick cycles of work and short breaks. This helps reduce mental fatigue and the lack of productivity caused by it. Another is that it tends to encourage practitioners to break tasks down to things that can be completed in about 25 minutes, which is something that is usually manageable from the perspective of time available. It is much easier to squeeze in three 25-minute sessions of work time during the day than it is to set aside a 75- minute block of time.

Eat the Frog

Of our three quick strategies, *eat the frog* probably has the strangest name and may not sound the most inviting. The name comes from a famous quote, attributed to Mark Twain: “Eat a live frog first thing in the morning and nothing worse will happen to you the rest of the day.” *Eat the Frog* is also the title of a best-selling book by Brian Tracy that deals with time management and avoiding procrastination.

How this applies to time and task management is based on the concept that if a person takes care of the biggest or most unpleasant task first, everything else will be easier after that.

Although stated in a humorous way, there is a good deal of truth in this. First, we greatly underestimate how much worry can impact our performance. If you are continually distracted by anxiety over a task you are dreading, it can affect the task you are working on at the time. Second, not only will you have a sense of accomplishment and relief when the task you are concerned with is finished and out of the way, but other tasks will seem lighter and not as difficult.

Application: Try Three Time Management Strategies

Over the next two weeks, try each of these three methods to see which ones might work for you. Is there one you favor over the others? Might each of these three approaches serve you better in different situations or with different tasks? Do you have a creative alternative or possibly a way to use some combination of these techniques?

In addition to these three strategies, you could also develop whole new approaches from suggestions found earlier in this chapter. For example, you could apply some of the strategies for avoiding procrastination or for setting appropriate priorities and see how they work in combination with these techniques or on their own.

The key is to find which system works best for you.

Breaking Down the Steps and Spreading Them over Shorter Work Periods

Above, you read about several different tried-and-tested strategies for effective time management—approaches that have become staples in the professional world. In this section you will read about two more creative techniques that combine elements from these other methods to handle tasks when time is scarce and long periods of time are a luxury you just do not have.

The concept behind this strategy is to break tasks into smaller, more manageable units that do not require as much time to complete. As an illustration of how this might work, imagine that you are assigned a two-page paper that is to include references. You estimate that to complete the paper—start to finish—would take you between four and a half and five hours. You look at your

calendar over the next week and see that there simply are no open five-hour blocks (unless you decided to only get three hours of sleep one night). Rightly so, you decide that going without sleep is not a good option. While looking at your calendar, you do see that you can squeeze in an hour or so every night. Instead of trying to write the entire paper in one sitting, you break it up into much smaller components as shown in the table below:

Table 3.8: Breaking Down Projects into Manageable-Sized Tasks

Day/Time	Task	Time
Monday, 6:00 p.m.	Write outline; look for references.	60 minutes
Tuesday, 6:00 p.m.	Research references to support outline; look for good quotes.	60 minutes
Wednesday, 7:00 p.m.	Write paper introduction and first page draft.	60 minutes
Thursday, 6:00 p.m.	Write second page and closing draft.	60 minutes
Friday, 5:00 p.m.	Rewrite and polish final draft.	60 minutes
Saturday, 10:00 a.m.	*Only if needed—finish or polish final draft.*	60 minutes?

While this is a simple example, you can see how it would redistribute tasks to fit your available time in a way that would make completing the paper possible. In fact, if your time constraints were even more rigid, it would be possible to break these divided tasks down even further. You could use a variation of the Pomodoro Technique and write for three 20-minute segments each day at different times. The key is to look for ways to break down the entire task into smaller steps and spread them out to fit your schedule.

Table 3.9

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
8:00–10:00		Work		Work			
10:00–12:00	Algebra	Work	Algebra	Work	Algebra	10 a.m.–11 a.m. *Only if needed*	Work
12:00–2:00	Lunch/study	1 p.m. English Comp	Lunch/study	1 p.m. English Comp	Lunch/study	Family picnic	Work
2:00–4:00	History	English Comp	History	English Comp	History	Family picnic	
4:00–6:00	Study for Algebra quiz.	Grocery	Study for History exam.	Study for History exam.	5 p.m.–6 p.m. Rewrite and polish final draft.	Family picnic	Laundry
6:00–7:00	Write outline; look for references.	Research references to support outline; look for good quotes.	Research presentation project.	Write second page and closing draft	Create presentation.	Meet with Darcy.	Prepare school stuff for next week.
7:00–8:00	Free time	Free time	Write paper introduction and first page draft.	Research presentation project.	Create presentation.		Free time

STUDENT PROFILE

"Time management is probably one of the hardest things I had to pick up when I got to college. For starters, I didn't have anyone to come wake me up if I forgot to set an alarm or to tell me to get out of bed so that I wouldn't be late. I had to start placing my phone far away from my bed; so that way, I would have to get out of bed in order to turn the alarm off. Accomplishing work on time can also be difficult. It's tough to find the fine balance between when you have to stay in and work on assignments and when is acceptable to go out and do leisure activities.

"I learned the 8-8-8 rule. Every day you spend eight hours working on school work or going to class, eight hours of free time to do what you want, and then eight hours to sleep at night so that you will get enough rest. Sleep is crucial for time management. I learned very quickly that you cannot focus or be productive if you are struggling to keep your head from falling over because you are so tired. Basically, I've learned that if you want to be successful in college, then you have to be on top of your game when it comes to time. It's something thing you cannot make up once it's gone."

—Preston Allen, University of Central Arkansas

Analyzing Your Schedule and Creating Time to Work

Of all the strategies covered in this chapter, this one may require the most discipline, but it can also be the most beneficial in time management. The fact is most of us waste time throughout the day. Some of it is due to a lack of awareness, but it can also be caused by the constraints of our current schedules. An example of this is when we have 15 to 20 minutes before we must leave to go somewhere. We don't do anything with that time because we are focused on leaving or where we are going, and we might not be organized enough to accomplish something in that short of a time period. In fact, a good deal of our 24- hour days are spent a few minutes at a time waiting for the next thing scheduled to occur. These small units of time add up to a fair amount each day.

The intent of this strategy is to recapture those lost moments and use them to your advantage. This may take careful observation and consideration on your part, but the results of using this as a method of time management are more than worth it.

The first step is to look for those periods of time that are wasted or that can be repurposed. In order to identify them, you will need to pay attention to what you do throughout the day and how much time you spend doing it. The example of waiting for the next thing in your schedule has already been given, but there are many others. How much time do you spend in activities after you have really finished doing them but are still lingering because you have not begun to do something else (e.g., letting the next episode play while binge-watching, reading social media posts or waiting for someone to reply, surfing the Internet, etc.)? You might be surprised to learn how much time you use up each day by just adding a few unproductive minutes here and there.

If you set a limit on how much time you spend on each activity, you might find that you can recapture time to do other things. An example of this would be limiting yourself to reading news for 30 minutes. Instead of reading the main things that interest you and then spending an additional amount of time just looking at things that you are only casually interested in because that is what you are doing at the moment, you could stop after a certain allotted period and use the extra time you have gained on something else.

After you identify periods of lost time, the next step will be to envision how you might restructure your activities to bring those extra minutes together into useful blocks of time. Using the following scenario as an illustration, we will see how this could be accomplished.



Figure 1.6.16 Sarah has to balance a lot of obligations.

On Tuesday nights, Sarah has a routine: After work, she does her shopping for the week (2 hours driving and shopping) and then prepares and eats dinner (1 hour). After dinner, she spends time on homework (1 hour) and catching up with friends, reading the news, and other Internet activities (1 hour), and then she watches television or reads before going to bed (1 hour). While it may seem that there is very little room for improvement in her schedule without cutting out something she enjoys, limiting the amount of time she spends on each activity and rethinking how she goes about each task can make a significant difference.

In this story, Sarah's Tuesday-night routine includes coming home from work, taking stock of which items in her home she might need to purchase, and then driving to the store. While at the store, she spends time picking out and selecting groceries as she plans for meals she will eat during the rest of the week. Then, after making her purchases, she drives home. Instead, if she took the time to make a list and plan for what she needed at the store before she arrived, she would not spend as much time looking for inspiration in each aisle. Also, if she had a prepared list, not only could she quickly pick up each item, but she could stop at the store on the way home from work, thus cutting out the extra travel time. If purchasing what she needed took 30 minutes less because she was more organized and she cut out an additional 20 minutes of travel time by saving the extra trip to the store from her house, she could recapture a significant amount of her Tuesday evening. If she then limited the time she spent catching up with friends and such to 30 minutes or maybe did some of that while she prepared dinner, she would find that she had added almost an extra hour and a half to the time available to her on that evening, without cutting out anything she needed to do or enjoys. If she decided to spend her time on study or homework, this would more than double the time she previously had available in her schedule for homework.

Analysis Question: Reflection

Analysis: Identify areas in the way you spend your day where you may be able to recapture and repurpose time. Are there things you can move around to gain more time? Are there ways you can combine tasks or reduce travel time?

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1.7: How to Manage Time

Questions to consider:

- How can I use time-on-task estimates to improve time management?
- What behaviors can help or hinder when it comes to managing time?

In this next section you will learn about managing time and prioritizing tasks. This is not only a valuable skill for pursuing an education, but it can become an ability that follows you through the rest of your life, especially if your career takes you into a leadership role.



Figure 1.7.1 An online calendar is a very useful tool for keeping track of classes, meetings, and other events. Most learning management systems contain these features, or you can use a calendar application.

ANALYSIS QUESTION

Read each statement in the brief self-evaluation tool below, and check the answer that best applies to you. There are no right or wrong answers.

Table 3.3

	Always	Usually	Sometimes	Rarely	Never
I like to be given strict deadlines for each task. It helps me stay organized and on track.					
I would rather be 15 minutes early than 1 minute late.					
I like to improvise instead of planning everything out ahead of time.					

	Always	Usually	Sometimes	Rarely	Never
I prefer to be able to manage when and how I do each task.					
I have a difficult time estimating how long a task will take.					
I have more motivation when there is an upcoming deadline. It helps me focus.					
I have difficulty keeping priorities in the most beneficial order.					

This exercise is intended to help you recognize some things about your own time management style. The important part is for you to identify any areas where you might be able to improve and to find solutions for them. This chapter will provide some solutions, but there are many others that can be found by researching time management strategies.

After you have decided your best response to each statement, think about what they may mean in regard to potential strengths and/or challenges for you when it comes to time management in college. If you are a person that likes strict deadlines, what would you do if you took a course that only had one large paper due at the end? Would you set yourself a series of mini deadlines that made you more comfortable and that kept things moving along for you? Or, if you have difficulty prioritizing tasks, would it help you to make a list of the tasks to do and order them, so you know which ones must be finished first?

How to Manage Time

The simplest way to manage your time is to accurately plan for how much time it will take to do each task, and then set aside that amount of time. How you divide the time is up to you. If it is going to take you five hours to study for a final exam, you can plan to spread it over five days, with an hour each night, or you can plan on two hours one night and three hours the next. What you would not want to do is plan on studying only a few hours the night before the exam and find that you fell very short on the time you estimated you would need. If that were to happen, you would have run out of time before finishing, with no way to go back and change your decision. In this kind of situation, you might even be tempted to “pull an all-nighter,” which is a phrase that has been used among college students for decades. In essence it means going without sleep for the entire night and using that time to finish an assignment. While this method of trying to make up for poor planning is common enough to have a name, rarely does it produce the best work.

Table 3.4 Sample Time Estimate Table

Activity	Estimated Time	Actual Time
Practice Quiz	5 minutes	15 minutes
Lab Conclusions	20 minutes	35 minutes
Food shopping	45 minutes	30 minutes
Drive to work	20 minutes	20 minutes
Physical Therapy	1 hour	50 minutes

Of all the parts of time management, accurately predicting how long a task will take is usually the most difficult—and the most elusive. Part of the problem comes from the fact that most of us are not very accurate timekeepers, especially when we are busy applying ourselves to a task. The other issue that makes it so difficult to accurately estimate time on task is that our estimations must also account for things like interruptions or unforeseen problems that cause delays.?

When it comes to academic activities, many tasks can be dependent upon the completion of other things first, or the time a task takes can vary from one instance to another, both of which add to the complexity and difficulty of estimating how much time and effort are required.

For example, if an instructor assigned three chapters of reading, you would not really have any idea how long each chapter might take to read until you looked at them. The first chapter might be 30 pages long while the second is 45. The third chapter could be only 20 pages but made up mostly of charts and graphs for you to compare. By page count, it might seem that the third chapter would take the least amount of time, but actually studying charts and graphs to gather information can take longer than regular reading.?

To make matters even more difficult, when it comes to estimating time on task for something as common as reading, not all reading takes the same amount of time. Fiction, for example, is usually a faster read than a technical manual. But something like the novel *Finnegan's Wake* by James Joyce is considered so difficult that most readers never finish it.

ACTIVITY

To better understand how much time different kinds of material can take to read, try this experiment. You will use two examples of famous texts that are very close to being the same number of words: *The Gettysburg Address* and the opening paragraphs from *A Christmas Carol*. Before you begin, estimate how long it will take you to read each, and predict which you think will take longer. When you do the reading, use a stopwatch function on a device such as a phone or some other timer to see how long it actually takes.

Make certain that you are reading for understanding, not just skimming over words. If you must reread a section to better comprehend what is being said, that is appropriate. The goal here is to compare reading of different texts, not to see how fast you can sight-read the words on a page.

After you have finished *The Gettysburg Address*, read and time *A Christmas Carol* and compare both of your times.

The Gettysburg Address
Abraham Lincoln
Gettysburg, Pennsylvania November 19, 1863
Word count: 278

Four score and seven years ago our fathers brought forth on this continent, a new nation, conceived in Liberty, and dedicated to the proposition that all men are created equal.

Now we are engaged in a great civil war, testing whether that nation, or any nation so conceived and so dedicated, can long endure. We are met on a great battle-field of that war. We have come to dedicate a portion of that field, as a final resting place for those who here gave their lives that that nation might live. It is altogether fitting and proper that we should do this.

But, in a larger sense, we can not dedicate -- we can not consecrate -- we can not hallow -- this ground. The brave men, living and dead, who struggled here, have consecrated it, far above our poor power to add or detract. The world will little note, nor long remember what we say here, but it can never forget what they did here. It is for us the living, rather, to be dedicated here to the unfinished work which they who fought here have thus far so nobly advanced. It is rather for us to be here dedicated to the great task remaining before us -- that from these honored dead we take increased devotion to that cause for which they gave the last full measure of devotion -- that we here highly resolve that these dead shall not have died in vain -- that this nation, under God, shall have a new birth of freedom -- and that government of the people, by the people, for the people, shall not perish from the earth.

A Christmas Carol
Charles Dickens
Chapman & Hall, 1843
Word count: 260

Marley was dead: to begin with. There is no doubt whatever about that. The register of his burial was signed by the clergyman, the clerk, the undertaker, and the chief mourner. Scrooge signed it: and Scrooge's name was good upon 'Change, for anything he chose to put his hand to. Old Marley was as dead as a door-nail.

Mind! I don't mean to say that I know, of my own knowledge, what there is particularly dead about a door-nail. I might have been inclined, myself, to regard a coffin-nail as the deadest piece of ironmongery in the trade. But the wisdom of our ancestors is in the simile; and my unhallowed hands shall not disturb it, or the Country's done for. You will therefore permit me to repeat, emphatically, that Marley was as dead as a door-nail.

Scrooge knew he was dead? Of course he did. How could it be otherwise? Scrooge and he were partners for I don't know how many years. Scrooge was his sole executor, his sole administrator, his sole assign, his sole residuary legatee, his sole friend, and sole mourner. And even Scrooge was not so dreadfully cut up by the sad event, but that he was an excellent man of business on the very day of the funeral, and solemnised it with an undoubted bargain.

The mention of Marley's funeral brings me back to the point I started from. There is no doubt that Marley was dead. This must be distinctly understood, or nothing wonderful can come of the story I am going to relate.

In comparing the two, was one or the other easier to understand or faster to read? Was it the piece you predicted you would read faster?

It is important to note that in this case both readings were only three paragraphs long. While there may have only been half a minute or so between the reading of each, that amount of time would multiply greatly over an entire chapter.

Knowing Yourself

While you can find all sorts of estimates online as to how long a certain task may take, it is important to know these are only averages. People read at different speeds, people write at different speeds, and those numbers even change for each individual depending on the environment.

If you are trying to read in surroundings that have distractions (e.g., conversations, phone calls, etc.), reading 10 pages can take you a lot longer than if you are reading in a quiet area. By the same token, you may be reading in a quiet environment (e.g., in bed after everyone in the house has gone to sleep), but if you are tired, your attention and retention may not be what it would be if you were refreshed.

In essence, the only way you are going to be able to manage your time accurately is to know yourself and to know how long it takes you to do each task. But where to begin?

Below, you will find a table of common college academic activities. This list has been compiled from a large number of different sources, including colleges, publishers, and professional educators, to help students estimate their own time on tasks. The purpose of this table is to both give you a place to begin in your estimates and to illustrate how different factors can impact the actual time spent.

You will notice that beside each task there is a column for the *unit*, followed by the average *time on task*, and a column for notes. The *unit* is whatever is being measured (e.g., pages read, pages written, etc.), and the *time on task* is an average time it takes students to do these tasks. It is important to pay attention to the notes column, because there you will find factors that influence the time on task. These factors can dramatically change the amount of time the activity takes.

Table 3.5 Time on task for common college activities.

Time on Task			
Activity	**Unit**	**Time on task**	**Notes**
General academic reading (textbook, professional journals)	1 page	5–7 minutes	Be aware that your personal reading speed may differ and may change over time.
Technical reading (math, charts and data)	1 page	10–15 minutes	Be aware that your personal reading speed may differ and may change over time.

Time on Task			
Simple Quiz or homework question: short answer—oriented toward recall or identification type answers	Per question	1–2 minutes	Complexity of question will greatly influence the time required.
Complex Quiz or homework question: short answer—oriented toward application, evaluation, or synthesis of knowledge	Per question	2–3 minutes	Complexity of question will greatly influence the time required.
Math problem sets, complex	Per question	15 minutes	For example, algebra, complex equations, financial calculations
Writing: short, no research	Per page	60 minutes	Short essays, single-topic writing assignments, summaries, freewriting assignments, journaling—includes drafting, writing, proofing, and finalizing
Writing: research paper	Per page	105 minutes	Includes research time, drafting, editing, proofing, and finalizing (built into per-page calculation)
Study for quiz	Per chapter	60 minutes	45–90 minutes per chapter, depending upon complexity of material
Study for exam	Per exam	90 minutes	1–2 hours, depending upon complexity of material

Again, these are averages, and it does not mean anything if your times are a little slower or a little faster. There is no "right amount of time," only the time that it takes you to do something so you can accurately plan and manage your time.

There is also another element to look for in the table. These are differentiations in the similar activities that will also affect the time you spend. A good example of this can be found in the first four rows. Each of these activities involves reading, but you can see that depending on the material being read and its complexity, the time spent can vary greatly. Not only do these differences in time account for the different types of materials you might read (as you found in the comparative reading exercise earlier in this chapter), but also they also take into consideration the time needed to think about what you are reading to truly understand and comprehend what it is saying.

GET CONNECTED

Which apps help you best prepare for success when managing your time?

Do you have trouble keeping track of multiple tasks over the course of a term?

Trello lets you organize all your obligations in helpful boards. You can share them with others (project collaborators), set alerts as reminders, and mark tasks off as you complete them.

Do you use a particular app to help you manage your time?

Sticky note apps are available for PC, Mac, and mobile devices. They let you post quick reminders, reorganize them as needed, and view them separately or as a full to-do list.

What do you wish you could improve about your time management skills?

> Toggl helps you keep track of how and where you are spending your time so you can budget better and make time management changes that free you up for the really important stuff.

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1.8: Goal Setting and Motivation

Questions to consider:

- How do I set motivational goals?
- What are SMART goals?
- What's the importance of an action plan?
- How do I keep to my plan?

Motivation often means the difference between success and failure. That applies to school, to specific tasks, and to life in general. One of the most effective ways to keep motivated is to set goals.

Goals can be big or small. A goal can range from *I am going to write one extra page tonight*, to *I am going to work to get an A in this course*, all the way to *I am going to graduate in the top of my class so I can start my career with a really good position*. The great thing about goals is that they can include and influence a number of other things that all work toward a much bigger picture. For example, if your goal is to get an A in a certain course, all the reading, studying, and every assignment you do for that course contributes to the larger goal. You have motivation to do each of those things and to do them well.

Setting goals is something that is frequently talked about, but it is often treated as something abstract. Like time management, goal setting is best done with careful thought and planning. This next section will explain how you can apply tested techniques to goal setting and what the benefits of each can be.

Set Goals That Motivate You

The first thing to know about goal setting is that a goal is a specific end result you desire. If the goal is not something you are really interested in, there is little motivational drive to achieve it. Think back to when you were much younger and some well-meaning adult set a goal for you—something that didn't really appeal to you at all. How motivated were you to achieve the goal? More than likely, if you were successful at all in meeting the goal, it was because you were motivated by earning the approval of someone or receiving a possible reward, or you were concerned with avoiding something adverse that might happen if you did not do what you were told. From an honest perspective in that situation, your real goal was based on something else, not the meeting of the goal set for you. To get the most from the goals you set, make sure they are things that you are interested in achieving.

That is not to say you shouldn't set goals that are supported by other motivations (e.g., If I finish studying by Friday, I can go out on Saturday), but the idea is to be intellectually honest with your goals.

Set SMART Goals

Goals should also be SMART. In this case, the word *smart* is not only a clever description of the type of goal, but it is also an acronym that stands for Specific, Measurable, Attainable, Relevant, and Time-bound. The reason these are all desirable traits for your goals is because they not only help you plan how to meet the goal, but they can also contribute to your decision-making processes during the planning stage.

What does it mean to create SMART goals?

- Specific—For a goal to be specific, it must be defined enough to actually determine the goal. A goal of *get a good job when I graduate* is too general. It doesn't define what a good job is. In fact, it doesn't even necessarily include a job in your chosen profession. A more specific goal would be something like *be hired as a nurse in a place of employment where it is enjoyable to work and that has room for promotion.*
- Measurable—The concept of *measurable* is one that is often overlooked when setting goals. What this means is that the goal should have clearly defined outcomes that are detailed enough to measure and can be used for planning of how you will achieve the goal. For example, setting a goal of *doing well in school* is a bit undefined, but making a goal of *graduating with a GPA above 3.0* is measurable and something you can work with. If your goal is measurable, you can know ahead of time how many points you will have to earn on a specific assignment to stay in that range or how many points you will need to make up in the next assignment if you do not do as well as you planned.
- Attainable—*Attainable* or *achievable* goals means they are reasonable and within your ability to accomplish. While a goal of *make an extra one million dollars by the end of the week* is something that would be nice to achieve, the odds that you could make that happen in a single week are not very realistic.

- Relevant—For goal setting, *relevant* means it applies to the situation. In relation to college, a goal of *getting a horse to ride* is not very relevant, but *getting dependable transportation* is something that would contribute to your success in school.
- Time-bound—Time-bound means you set a specific time frame to achieve the goal. *I will get my paper written by Wednesday* is time-bound. You know when you have to meet the goal. *I will get my paper written sometime soon* does not help you plan how and when you will accomplish the goal.

In the following table you can see some examples of goals that do and do not follow the SMART system. As you read each one, think about what elements make them SMART or how you might change those that are not.

Table 3.6

Goal	Is it SMART?	
I am going to be rich someday.	No	There is nothing really specific, measurable, or time-bound in this goal.
I will graduate with my degree, on time.	Yes	The statement calls out specific, measureable, and time-bound details. The other attributes of attainable and relevant are implied.
I am going to save enough money to buy a newer car by June.	Yes	All SMART attributes are covered in this goal.
I would like to do well in all my courses next semester.	No	While this is clearly time-bound and meets most of the SMART goal attributes, it is not specific or measurable without defining what “do well” means.
I am going to start being a nicer person.	No	While most of the SMART attributes are implied, there is nothing really measurable in this goal.
I will earn at least a 3.0 GPA in all my courses next semester.	Yes	All of the SMART attributes are present in this goal.
I am going to start being more organized.	No	While most of the SMART attributes are implied, there is nothing really measurable in this goal.

APPLICATION

Try writing two SMART goals—something with a one-week time frame and something that you will accomplish over the next year. Make certain that you include all the appropriate elements—Specific, Measurable, Attainable, Relevant, and Time-bound.

Make an Action Plan

Like anything else, making a step-by-step action plan of how you will attain your goals is the best way to make certain you achieve them. It doesn’t matter if it is a smaller goal with immediate results (e.g., finish all your homework due by Friday) or something bigger that takes several years to accomplish (graduate with my degree in the proper amount of time).

The planning techniques you use for time management and achieving goals can be similar. In fact, accurate goal setting is very much a part of time management if you treat the completion of each task as a goal.

What follows is an example of a simple action plan that lists the steps for writing a short paper. You can use something like this or modify it in a way that would better suit your own preferences.

Table 3.7

Action Plan		
Task	**Objective**	**When**

Action Plan		
Choose topic.	Select something interesting.	Needs to be done by Monday!
Write outline, look for references.	Create structure of paper and outline each part.	Monday, 6:00 p.m.
Research references to support outline, look for good quotes.	Strengthen paper and resources.	Tuesday, 6:00 p.m.
Write paper introduction and first page draft.	Get main ideas and thesis statement down.	Wednesday, 7:00 p.m.
Write second page and closing draft.	Finish main content and tie it all together.	Thursday, 6:00 p.m.
Rewrite and polish final draft.	Clean up for grammar, writing style, and effective communication.	Friday, 5:00 p.m.

Another useful approach to goal setting is to create SMART goals and then write them down. For most people there is a higher level of commitment when we write something down. If you have your goals written out, you can refer to each component of the SMART acronym and make certain you are on track to achieve it.

Stick with It!

As with anything else, the key to reaching goals is to keep at it, keep yourself motivated, and overcome any obstacles along the way. In the following graphic you will find seven methods that highly successful people use to accomplish this.



Figure 1.8.13 These seven ways to stay motivated are good suggestions from highly successful people. What other strategies would you suggest?

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1.9: Test Taking and Preparation

Students need to develop a mindset that every time they attend class or complete a reading assignment they are preparing for the next exam, because oftentimes so much of their grade rides on those exams. In college, it is not uncommon for your entire course grade to be based on two or three exams with no options for a retake or extra credit. Instead of an exam being over one or two chapters, exams are more likely to be over 5+ chapters of material, and in the case of a comprehensive exam, an entire semester's worth of material. Intentionally working through the process of preparing for, taking, and learning from the testing process will facilitate successful outcomes.



Table 5-4. Effective Technique for Test Preparation and Taking

Before the Test	During the Test	After the Test
Go to all of your classes and take lecture notes and read all of your assigned reading using techniques described in this chapter.	Mark your exam with the necessary identification. Read all instructions on the exam. Scan the exam for question types, point values, etc.	Conduct an error analysis for items missed on the test. Look up the correct answers. Determine the nature of the errors you made.
Develop a study plan and schedule the time to review for an exam over several days. Refer to "Create a Five-Day Study Plan for Exams."	Develop a plan for how you will use your time to complete all questions on the exam.	Talk with your professor about items you missed on the exam. Discuss adjustments in your test preparation methods if necessary.

Schedule study sessions with classmates in advance of the night before the exam.	Read each item carefully and fully before marking an answer. You might find clues to the answer and will be less likely to make a preventable mistake.	Analyze the test questions to see if patterns arise, such as,"Were most of the questions from the lecture, the book, or both?" "What types of questions were asked?" "What levels of thinking were being tested based on Bloom's Taxonomy?" etc.
Be sure you have materials you need for the exam ready at least the night before the exam, such as multiple writing utensils, scantrons, blue books, notebook paper, etc.	Skip difficult questions and come back to them if you are pressed for time. Caution: if using a scantron to record answers be very careful that you do not incorrectly record your answers on the wrong line and get off track.	Review the methods that you used to study for the exam and determine what adjustments you need to make in preparing for the next exam.
Get a good night's rest the night before the exam.	If time is available at the end of the exam, review questions you marked as "difficult" in order to check for errors.	Determine your overall grade in the course based on the new information that this grade has provided you. Talk with your professor and academic advisor if you have concerns about your overall grade.
Eat a good breakfast prior to the exam. Avoid caffeine prior to the exam.	If you have an essay question to write, take the time to briefly outline an answer to ensure that you are answering all aspects of the question and that you provide a well-organized answer for your faculty member to read.	Archive your exam for future reference.
To ensure that you are on time to your exam, set two alarm clocks or arrange to have a friend call you to make sure you are awake.	Try not to leave any answers blank. You might be awarded partial credit or even make a correct guess.	
	Take a moment to take some deep cleansing breaths (or any other quick relaxation techniques) if you feel stressed by a question or the exam as a whole.	

Preparation for an exam should begin the first day of class. Repeated exposure to material over a longer period of time (Baddeley, 1982; Bahrick & Hall, 1991) optimizes the recall of information when it is needed. Every assigned reading, every opportunity to take notes and engage in class activities, and a routine of reviewing notes should be considered activities toward exam preparation. The volume of information that students are responsible for on each exam is much larger than experienced in high school. Careful review of the material prior to each exam is required for optimal performance, and student reports tell us that the habit of "cramming" the night before the exam is not going to lead to positive, long-term results. Research of the brain supports this typical finding. Physical and neurochemical changes in the synapses of the brain occur when information is learned, a process called long-term potentiation (Kandel & Schwartz, 1982). Without proper time and repeated stimulation of the neural circuits, long-term learning of this information will not occur efficiently and may not be accessible when needed for a test. Planning for a review period over five days prior to the exam and healthy eating and sleep habits cannot be overemphasized.

Activity 5-4

Identify when your next substantial exam will be given in a class. Create a five-day study plan using the formula described in "Create a Five-Day Study Plan for Exams" and the "Five-Day Study Plan Worksheet" at the end of this chapter. Once you have created this plan, incorporate the schedule into your time management calendar for implementation.

Oftentimes, students jump right in to taking an exam feeling stressed and rushed with a need to regurgitate information immediately. However, approaching a test in a careful and methodical way can help ensure the best results. You can best utilize your exam time by taking the time to survey the exam directions and point values and planning how to approach sections of the

test. Strategy is particularly important for tests with mixed types of questions (i.e., multiple choice and essay) or tests with multiple essay questions. For ex- ample, if you were to spend too much time on the multiple choice questions and not allocate enough time to complete an essay question, you may have answered the majority of the total number of questions but still do poorly on the exam if the essay question was worth a larger portion of the exam's total points. Plan your time carefully and manage your stress.

Create a Five-Day Study Plan for Exams

Making a plan that will allow for the careful review of all of the assigned and presented material leads to less stress the day of an exam and the best conditions for recall and performance on an exam. Here is a simple method for creating a study plan:

1. Break the material for the exam into four manageable "chunks." If material can be logically divided by chapters, use that method. If not, make up your own chunks based on the content of the material. Note: It is not a good idea to break up information by the method of delivery (i.e., one chunk equals notes, one chunk equals readings, etc.). You want to include all information on specific topics together in one chunk (e.g., information about the topic of osmosis from the lecture notes and the textbook in a chunk).
2. Plan to spend about two hours studying on each of the five days.
3. You should work with the material on the exam in two ways, by preparing that material for review and by reviewing that material. Preparation of material incorporates study techniques discussed earlier in this chapter such as combining and clarifying lecture and textbook notes, creating visual organizers for topics, developing cram cards, solving practice problems, creating self-testing questions using multiple levels of Bloom's Taxonomy, etc. Once you get the hang of preparing material for review, you could do some of that along the way (for instance creating cram cards for readings and notes) to reduce the amount of time needed for preparation at this time, leaving more time for review.

Example of the Schedule for a Five-Day Study Plan

Day One
Prepare 1st Chunk – 2 hours

Day Two
Prepare 2nd Chunk – 2 hours
Review 1st Chunk – 30 minutes

Day Three
Prepare 3rd Chunk – 1.5 hours
Review 2nd Chunk – 30 minutes
Review 1st Chunk – 15 minutes

Day Four
Prepare 4th Chunk – 1 hour
Review 3rd Chunk – 30 minutes
Review 2nd Chunk – 15 minutes
Review 1st Chunk – 10 minutes

Day Five
Review 4th Chunk – 1 hour
Review 3rd Chunk – 30 minutes
Review 2nd Chunk – 30 minutes
Review 1st Chunk – 30 minutes

Taking some time to plan out what material will go into each chunk will ensure that you do not miss anything. Use the "Five-Day Study Plan Worksheet" at the end of this chapter as a guide for your planning.

This method typically works very well for courses that offer two or three tests throughout the semester; however, a similar method could be modified by the number of days or the amount of time spent each day if a course offers more exams with less material on each exam.

Tips for Better Test Taking

The types of test items on a test are as different as the professors who created the tests. "Objective" test questions refer to items in which students have to recognize the correct answer from a list of provided options (e.g., multiple choice, true-false, matching). In

most cases there is one best answer, though it is important to note that a professor could indicate in the directions that you can select more than one answer. “Subjective” test items are the fill-in-the-blank, short answer, or essay questions in which students must recall and produce the answer.

Multiple Choice Questions

- Read the question and all of the answer options first.
- Mark out any answers that you know are not correct.
- Once you have selected an answer, do not change it unless you misread the question and know the new answer is correct. Usually, your first answer is the correct one if you are making an educated guess.
- Many times, the correct answer has more information in it.
- If there is no penalty for guessing, do not leave any items blank—make an educated guess.
- If there is an “All of the Above” option and there are two options that are correct, select “All of the Above.”
- In a question with “All of the Above” and “None of the Above,” if you are certain that one of the answers is true, do not select “None of the Above” and, likewise, if one of the answers is false, then do not select “All of the Above.”

True-False Questions

- Read the statement carefully, but do not read too much into the statement. Base your answer on the information provided.
- With statements that have multiple facts, if any one of the facts is false the entire statement is false.
- Qualifying words like “always,” “never,” and “every” indicate that this would have to be true all of the time. If it is not true all of the time, then you should answer false.
- Qualifying words like “usually,” “sometimes,” and “generally” indicate that it could be true or false depending on the situation. Oftentimes the answer is true.

Essay Questions

- Most Important Rule: Make sure you understand what the question is asking you to do. An essay question is not an opportunity to simply regurgitate everything you know about a particular topic. Revisit Question Cues listed in the “Table 5-1: “Bloom’s Taxonomy: Cognitive Domain” and be sure you know what you are supposed to accomplish when you see those cues. Underline key words and let those words help guide you. If you are not sure, ask your professor for clarification on what he or she wants you to accomplish in your answer.
- Take a minute or so to plan out your answer and jot down a brief outline of key points to guide your writing before you start. A well-organized answer tends to score more points when graded.
- Formula for Essay Questions:
  - Introductory statements that define terms and describe what you will accomplish in your essay. This section should be brief.
  - State your first main idea, and then give supporting facts, examples, statistics, or details. Follow with your next main idea and continue the pattern until complete. The bulk of your time should be spent developing this section of the essay as it would likely include the details your professor wants to see in grading. Typically, each main idea would have its own paragraph. Use transitional works like “first, second, third…,” “next,” “also,” “however,” etc., to aid your reader.
  - End with a summary or final conclusion. This section is also likely to be brief.
- Budget time to proofread and revise if needed.
- Write legibly. Neater papers tend to receive higher marks.

Adapted from Dembo and Seli (2008) and TestTakingTips.com (2003–2009).

Once the exam has been completed, the learning experience is not over. Look over the test after you get it back from your professor and continue your review of the material that may be important for a comprehensive final or information that serves as a foundation for material to be learned in other courses. Even if only a scantron answer sheet is returned to you, professors are willing to give students access to review the test results during their office hours. Comprehensive post-test review not only includes reviewing your correct and incorrect answers to the question but also looking at where questions came from (i.e., lecture notes, readings, or both). Also, consider the study strategies you used and how they worked for you in order to help you prepare for future exams. Overall, it is important to emphasize the benefit of talking with your professor after each exam to gain insights into the information presented in the class as well as your learning and test strategies.

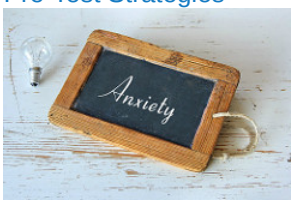
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1.10: Pre- Mid- and Post-Test-Taking Strategies

Phyllis Nissila

Pre-Test Strategies



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Q: When should you start preparing for the first test? Circle…

1. The night before.
2. The week prior.
3. The first day of classes.

If you answered "3. The first day of classes," you are correct. If you circled all three, you are also correct. Preparing to pass tests is something that begins when learning begins and continues all the way through to the final exam.

Many students, however, don't start thinking about test taking, whether weekly exams, mid-terms, or finals, until the day before when they engage in an all-nighter, or cramming. From the previous unit on memory, you might recall that the brain can only process an average of 5-7 new pieces of information at a time. Additionally, unless memory devices are used to aid memory and to cement information into long term memory (or at least until the test is over tomorrow!) chances are slim that students who cram will effectively learn and remember the information.

Additionally, a lot of students are unaware of the many strategies available to help with the test-taking experience before, during, and after. For starters, take a look at what has helped you so far.

EXERCISE 30-1

Pre-Test Taking Strategies

PART A:

Put a check mark next to the pre-test strategies you already employ.

____ Organize your notebook and other class materials the first week of classes.

____ Maintain your organized materials throughout the term.

____ Take notes on key points from lectures and other materials.

____ Make sure you understand the information as you go along.

____ Access your instructor's help and the help of a study group, as needed.

____ Organize a study group, if desired.

____ Create study tools such as flashcards, graphic organizers, etc. as study aids.

____ Complete all homework assignments on time.

____ Review likely test items several times beforehand.

____ Ask your instructor what items are likely to be covered on the test.

____ Ask your instructor if she or he can provide a study guide or practice test.

____ Ask your instructor if he/she gives partial credit for test items such as essays.

____ Maintain an active learner attitude.

____ Schedule extra study time in the days just prior to the test.

____ Gather all notes, handouts, and other materials needed before studying.

____ Review all notes, handouts, and other materials.

____ Organize your study area for maximum concentration and efficiency.

____ Create and use mnemonic devices to aid memory.

____ Put key terms, formulas, etc., on a single study sheet that can be quickly reviewed.

____ Schedule study times short enough (1-2 hours) so you do not get burned out.

____ Get plenty of sleep the night before.

____ Set a back-up alarm in case the first alarm doesn't sound or you sleep through it.

____ Have a good breakfast with complex carbs and protein to see you through.

____ Show up 5-10 minutes early to get completely settled before the test begins.

____ Use the restroom beforehand to minimize distractions.

PART B

By reviewing the pre-test strategies, above, you have likely discovered new ideas to add to what you already use. Make a list of them.

Mid-Test Strategies

Here is a list of the most common–and useful–strategies to survive this ubiquitous college experience.

- Scan the test, first, to get the big picture of how many test items there are, what types there are (multiple choice, matching, essay, etc.), and the point values of each item or group of items.
- Determine which way you want to approach the test: Some students start with the easy questions first, that is, the ones they immediately know the answers to, saving the difficult ones for later, knowing they can spend the remaining time on them. Some students begin with the biggest-point items first, to make sure they get the most points.
- Determine a schedule that takes into consideration how long you have to test, and the types of questions on the test. Essay questions, for example, will require more time than multiple choice, or matching questions.
- Keep your eye on the clock.
- If you can mark on the test, put a check mark next to items you are not sure of just yet. It is easy to go back and find them to answer later on. You might just find help in other test questions covering similar information.
- Sit where you are most comfortable. That said, sitting near the front has a couple of advantages: You may be less distracted by other students. If a classmate comes up with a question for the instructor and there is an important clarification given, you will be better able to hear it and apply it, if needed.
- Wear ear plugs, if noise distracts you.
- You do NOT have to start with #1! It you are unsure of it, mark it to come back to later on.
- Bring water…this helps calm the nerves, for one, and water is also needed for optimum brain function.
- If permitted, get up and stretch (or stretch in your chair) from time to time to relieve tension and assist the blood to the brain!
- Remember to employ strategies to reduce test-taking anxiety (covered in the next lesson)
- If despite all of your best efforts to prepare for a test you just cannot remember the answer to a given item for multiple choice, matching, and/or true/false questions, employ one or more of the following educated guessing (also known as "educated selection") techniques. By using these techniques, you have a *better* chance of selecting the correct answer. It is usually best to avoid selecting an extreme or all-inclusive answer (also known as 100% modifiers) such as "always," and "never". Choose, instead, words such as "usually," "sometimes," etc. (also known as in-between modifiers). If the answers are numbers, choose one of the middle numbers. If you have options such as "all of the above," or "both A and B," make sure each item is true before selecting those options. Choose the longest, or most inclusive, answer. Make sure to match the grammar of question and answer. For example, if the question indicates a plural answer, look for the plural answer. Regarding matching tests: count both sides to be matched. If there are more questions than answers, ask if you can use an answer more than once. Pay close attention to items that ask you to choose the "best" answer. This means one answer is better or more inclusive than a similar answer. Read all of the response options.

Post-Test Strategies

In addition to sighing that big sigh of relief, here are a few suggestions to help with future tests.

- If you don't understand why you did not get an item right, ask the instructor. This is especially useful for quizzes that contain information that may be incorporated into more inclusive exams such as mid-terms and finals.
- Analyze your results to help you in the future. For example, see if most of your incorrect answers were small things such as failing to include the last step in a math item, or neglecting to double-check for simple errors in a short-answer or essay item. See where in the test you made the most errors: beginning, middle, or end. Also analyze which type of questions, true/false, multiple choice, essay, etc. And which topics were missed. This will help you pay closer attention to those sections in the future.

EXERCISE 30-2

Write a letter of advice to Chen incorporating 10 test-taking tips and strategies you think will help him.

Chen believes he is good at organization, and he usually is–for about the first two weeks of classes. He then becomes overwhelmed with all of the handouts and materials and tends to start slipping in the organization department. When it comes to tests, he worries that his notes might not cover all of the right topics and that he will not be able to remember all of the key terms and points–especially for his math class. During tests, he sometimes gets stuck on an item and tends to spend too much time there. He also sometimes changes answers but finds out later that his original selection was correct. Chen is also easily distracted by other students and noises which makes it hard for him to concentrate sometimes, and, unfortunately, he does admit to occasionally "cramming" the night before.

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Adaptions: Changed formatting, removed one exercise.

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1.11: Test-Taking Strategy Specifics

Dave Dillon

"By failing to prepare, you are preparing to fail."

– Benjamin Franklin

Discipline, Preparation and Execution

From what I have seen in my own educational experiences, along with coaching and teaching at the college and university level for over 20 years, test taking (with few exceptions) comes down to discipline, preparation and execution. Students wanting to be successful have to have the self-discipline to schedule time to study well in advance of the exam. They have to actually do the work: the preparation needed in order to have the best opportunity for success on the exam. Then they must execute: they have to be able to apply their preparation accordingly and perform well on the exam.

Preparation for an exam is not glamorous. It's easy to find other things to do that are more interesting and fun. Students need to keep themselves motivated with their "eyes on the prize." Think of it like this: if the most important event of your life was coming up and you wanted to perform to the best of your ability in that event, you would likely spend some time preparing for it, rehearsing for it, practicing it, etc. A student may argue that an exam they will be taking would not be the most important event of their life, but think back to the chapter on Passion. If you're already spending the time, effort, energy and money to attend college, why not do it to the best of your ability?

It would be beneficial to spread this preparation and practice out over time and prepare periodically rather than to wait until the last minute and binge study or cram. Your preparation would not be the same and this may affect your test score. Binge studying and cramming also are not healthy. Staying up late puts stress on our brain and body, and not getting adequate sleep places our bodies at risk for getting sick.

"One of the most important keys to success is having the discipline to do what you know you should do, even when you don't feel like doing it."

– Unknown

"The will to succeed is important, but what's more important is the will to prepare."

– Bobby Knight

Everyone wants to be successful. When the exam is passed out, everyone wants to perform well. But what often separates successful students and less successful students is the preparation time put in.

Studying the right thing is a process and a skill. As you gain more experience, you will learn how to become better at knowing what to study. It can be very frustrating to spend a lot of time preparing and studying and then finding out that what you studied was not on the exam. You will see a lot of variance with exams due to different instructors, classes and types of tests. The better you become at predicting what will be on the exam and study accordingly, the better you will perform on your exams. Try placing yourself in your instructor's shoes and design questions you think your instructor would ask. It's often an eye-opening experience for students and a great study strategy.

Preparation for Exam Strategies

Find out as much about the exam in advance as you can.

Some professors will tell you how many questions there will be, what format the exam will be in, how much time you will have, etc., and others will not. I encourage students to ask questions about the exam if there is not information given. I also encourage students to ask those questions before class, after class, in professors' office hours or via e-mail rather than during class.

Know the test

If you know how many questions, what the format is, and/or how much time you will have, you can start to mentally prepare for the exam much more so than if you are coming in with no information. There are two more important aspects that you may or may not know: a) what will be covered or asked on the exam; b) how the exam will be scored. Obviously, the more you know about what will be covered, the easier it is for you to be able to prepare for the exam. Most exam scoring is standardized, but not always.

Look for opportunities where some areas of the exam are worth more points than others. For example: An exam consists of 21 questions, with 10 being True/False, 10 being multiple choice, and one essay question. The T/F questions are worth 1 point each (10 points), the multiple-choice questions are worth 2 points each (20 points), and the essay question is worth 30 points. We know that the essay question is the most valuable (it is worth half of the value of the exam). And we should allocate our time for it accordingly. I would advise starting with the essay question. Do a quick analysis of time to be able to spend your time on the exam wisely. You want to spend some time with the exam question since it is so valuable, without sacrificing adequate time to ensure the T/F and multiple-choice questions are answered.

Often, the order of the exam in this scenario will be: T/F first, multiple choice second and essay third. Most students will go in the chronological order of the exam, but a savvy student would start with the essay. If an exam were to last for 30 minutes with this format of questions, I might recommend a student spent 15 minutes on the essay question, ten minutes on the multiple choice, three minutes on the T/F and two minutes reviewing their answer.

Also, look for situations where exams penalize students for incorrectly answering a question. This does not occur very often, but is the case with some exams. With the SAT for example, students are awarded one point for a correct answer and ¼ of a point subtracted for an incorrect answer. Points are not awarded nor subtracted for leaving a question blank. Thus, the strategy for a multiple-choice question is: if you can narrow down the potentially correct answer to two rather than four or five, it is statistically advantageous to answer the question and guess between the two answers; however, if a student had no idea if any of the answers were correct or incorrect, it would be best to leave the answer blank. Remember, this is rare, but it is important to understand the strategy when students take these exams.

In conclusion, the more information you have about the exam, the better you can prepare for content, allocation of time spent on aspects of the exam, and the more confident you will be in knowing how and when to attempt to answer questions.

Take care of your body

Before the exam, it is important to prepare your brain and body for optimal performance for your exam. Do not cram the night before. Get a good night's sleep. Make sure you eat (nutritiously) before the exam. I recommend exercising the day before and if possible a few hours before the exam.

Strategies for Specific Exam Formats

True or False Questions

Look for qualifiers. A qualifier is a word that is absolute. Examples are: all, never, no, always, none, every, only, entirely. They are often seen in false statements. This is because it is more difficult to create a true statement using a qualifier like never, no, always, etc. For example, "All cats chase mice." Cats may be known for chasing mice, but not all of them do so. The answer here is false and the qualifier "all" gave us a tip. Qualifiers such as: sometimes, many, some, most, often, and usually are commonly found in true statements. For example: "Most cats chase mice." This is true and the qualifier "most" gave us a tip.

Make sure to read the entire statement. All parts of a sentence must be true if the whole statement is to be true. If one part of it is false, the whole sentence is false. Long sentences are often false for this reason.

Students should guess on True or False questions they do not know the answer to unless there is a penalty for an incorrect answer.

Multiple Choice Questions

Think of multiple choice questions as four (or five) true or false statements in one. One of the statements is true (the correct answer) and the others will be false. Apply the same strategy toward qualifiers. If you see an absolute qualifier in one of the answer choices, it is probably false and not the correct answer. Try to identify the true statement. If you can do this, you have the answer as there is only one. If you cannot do this at first, try eliminating answers you know to be false.

If there is no penalty for incorrect answers, my suggestion is to guess if you are not certain of the answer. If there is a penalty for incorrect answers, common logic is to guess if you can eliminate two of the answers as incorrect (pending what the penalty is). If there's a penalty and you cannot narrow down the answers, it's best to leave it blank. You may wish to ask your instructor for clarification.

Answers that are strange and unrelated to the question are usually false. If two answers have a word that looks or sounds similar, one of those is usually correct. For example: abductor/ adductor. If you see these as two of the four or five choices, one of them is usually correct. Also look for answers that are grammatically incorrect. These are usually incorrect answers. If you have to completely guess, choose B or C. It is statistically proven to be correct more than 25 percent of the time. If there are four answers

for each question, and an exam had standardized the answers, each answer on the exam A, B, C and D would be equal. But most instructors do not standardize their answers, and more correct answers are found in the middle (B and C then the extremes A and D or E). “People writing isolated four-choice questions hide the correct answer in the two middle positions about 70% of the time.”[1] This is 20 percent more correct answers found in B or C than a standardized exam with equal correct answers for each letter.

Matching Questions

Although less common than the other types of exams, you will likely see some matching exams during your time in college. First, read the instructions and take a look at both lists to determine what the items are and their relationship. It is especially important to determine if both lists have the same number of items and if all items are to be used, and used only once.

Matching exams become much more difficult if one list has more items than the other or if items either might not be used or could be used more than once. If your exam instructions do not discern this, you may wish to ask your instructor for further clarification. I advise students to take a look at the whole list before selecting an answer because a more correct answer may be found further into the list. Mark items when you are sure you have a match (pending the number of items in the list this may eliminate answers for the future). Guessing (if needed) should take place once you have selected answers you are certain about.

Short-Answer Questions

Read all of the instructions first. Budget your time and then read all of the questions. Answer the ones you know best or feel the most confident with. Then go back to the other ones. If you do not know the answer and there is no penalty for incorrect answers, guess. Use common sense. Sometimes instructors will award partial credit for a logical answer that is related even if it is not the correct answer.

Essay Questions

Keep in mind that knowing the format of the exam can help you determine how to study. If I know that I am taking a True-False exam, I know that I will need to discern whether a statement is True or False. I will need to know subject content for the course. But if I am studying for short answer and especially for essay questions, I must know a lot more. For essay questions, I must have much greater content knowledge and be able to make a coherent argument that answers the question using information from textbooks, lectures or other course materials. I have to place a lot more time and thought into studying for an essay exam than for True-False or Multiple-Choice exams.

Read the essay question(s) and the instructions first. Plan your time wisely and organize your answer before you start to write. Address the answer to the question in your first or second sentence. It may help to restate the original question. Write clearly and legibly. Instructors have difficulty grading essays that they cannot read. Save some time for review when you have finished writing to check spelling, grammar and coherent thought in your answer. Make sure you have addressed all parts of the essay question.

During the Exam:

Always read the directions first. Read them thoroughly.

Preview the exam to help you allocate proper time for each area.

Skip questions if you do not know the answer but make a mark somewhere to ensure you are able to go back to those questions (you may need to reallocate your prepared time for this depending on how many there are).

Allocate some time to review your answers before submitting your exam or the exam time expiring.

One of the biggest mistakes that students make after they take an exam in a course is that they do not use the exam for the future. The exam contains a lot of information that can be helpful in studying for future exams. Students that perform well on an exam often put it away thinking they do not need it anymore. Students who do poorly on an exam often put it away, not wanting to think about it any further.

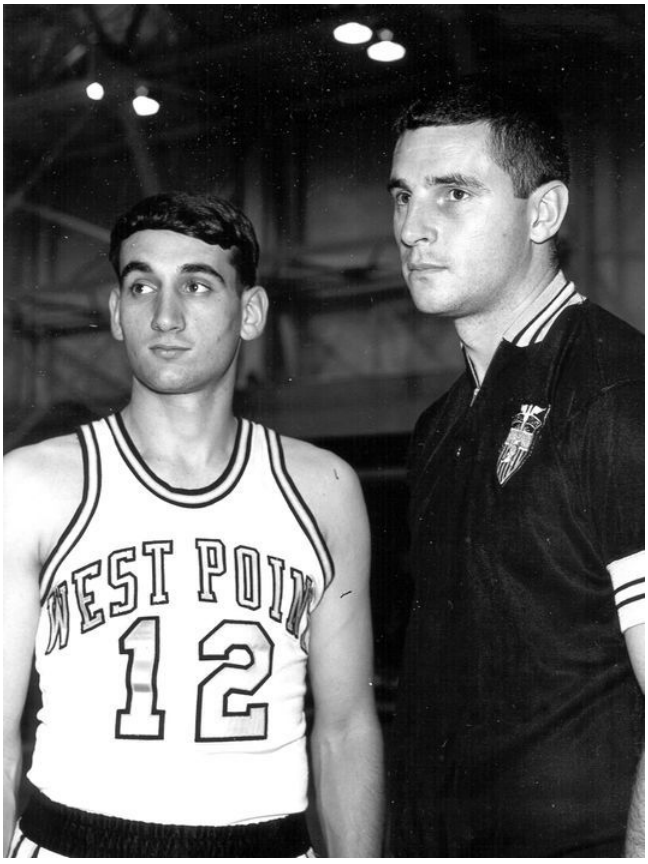
In both cases, students are missing out on the value of reviewing their exams. It is wise to review exams for three reasons: 1) students should review the answers that were correct because they may see those questions on future exams and it is important to reinforce learning; 2) students should review the answers that were incorrect in order to learn what the correct answer was and why. These questions also may appear on a future exam. In addition, occasionally an answer is marked incorrect, when it should have been marked correct. The student would never know this if they didn’t review their exam; and 3) there is value in reviewing the exam to try to predict what questions or what format will be used by a professor for a future exam in the same course.

Author's Story

What does it mean to be a "poor test taker?" Think about that. Does it mean that the student has put effort into studying but has difficulty under pressure? Does it mean that a student studies the wrong material? Is the student prepared but does not execute well? Could the student have a learning disability? Are they missing key strategies for taking tests? It could mean any of those things. And while I believe that it may be true that a student may be a "poor test taker," it does not by any means mean it is permanent. Students willing to work hard and learn can improve their test taking skills and raise their confidence.

In high school chemistry, I earned a D grade. In microeconomics at UC Santa Cruz, I struggled and barely passed. Both of these classes had grading systems that were heavily weighted by exams. I was an above average student in my other classes in both high school and college, and so I was told (and believed for a long time) that I was not a "good test taker." I no longer believe this. I will not make excuses. The responsibility for the grade I earned rests solely with me. I mention this because many students come to my office and complain that their professor grades in a way that is unfair to them because so much emphasis is on exams and that they understand the material but are "not good test takers." It may be the case that my grade or another students' grade would have been better had the course grade been determined with less weight levied to exams. That to me is irrelevant. It does not absolve me from being responsible for knowing what the grading metrics were at the beginning of the course and choosing to continue in the course after learning this knowledge. In the end, I realized I was not a poor test taker and that I needed to spend more time studying and preparing to perform well on exams. I changed my attitude from being afraid of exams to one of believing I was going to do well regardless of the class or the format of the exam. Once I had confidence that I had the necessary tools and was willing to work hard, it changed my entire perspective and experience with exams.

Note: if you think you may have a learning disability and would like to get assessed, contact your college to see what steps to follow to begin the assessment process.



Mike Krzyzewski, point guard at Army with Bob Knight, coach at Army.

Thinking about excuses reminds me of a story about Mike Krzyzewski, the successful Duke University men's basketball coach. Before Krzyzewski coached, he was a student at the United States Military Academy at West Point. One day in his first year, Krzyzweski and his roommate were walking and his roommate stepped in a puddle. Mud splashed onto Krzyzewski's uniform. Immediately, an upperclassman screamed at Krzyzewski about not knowing the rules of wearing a clean uniform. Plebes (first-year

students at military academies) are allowed three answers when asked a question: “Yes sir!” “No sir!” And “No excuse sir!” Krzyzewski repeated himself, “No excuse sir!” Despite wanting to explain what had happened and that it was not his fault, he realized that there was no excuse. It was his responsibility to keep a clean uniform.

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1. Yigal Attali and Maya Bar-Hillel, “Guess Where: The Position of Correct Answers in Multiple-Choice Test Items as a Psychometric Variable,” *Journal of Educational Measurement* 40 no. 2 (2003):109-128. ↵

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CHAPTER OVERVIEW

2: Arithmetic

2.1: Adding Integers
2.2: Subtracting Integers
2.3: Multiplication and Division of Integers
2.4: Order of Operations

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2.1: Adding Integers

Like our work with the whole numbers, addition of integers is best explained through the use of number line diagrams. However, before we start, let's take a moment to discuss the concept of a vector.

Vectors

A *vector* is a mathematical object that possesses two import qualities: (1) magnitude or length, and (2) direction.

Vectors are a fundamental problem solving tool in mathematics, science, and engineering. In physics, vectors are used to represent forces, position, velocity, and acceleration, while engineers use vectors to represent both internal and external forces on structures, such as bridges and buildings. In this course, and in this particular section, we will concentrate on the use of vectors to help explain addition of integers.

Vectors on the Number Line

Consider the number line in Figure 2.1.1.

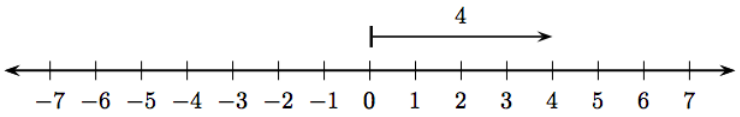


Figure 2.1.1: A vector representing positive four.

Above the line we've drawn a vector with tail starting at the integer 0 and arrowhead finishing at the integer 4. There are two important things to note about this vector:

1. The magnitude (length) of the vector in Figure 2.1.1 is four.
2. The vector in Figure 2.1.1 points to the right.

We will agree that the vector in Figure 2.1.1 represents positive four. It is not important that the vector start at the origin. Consider, for example, the vector pictured in Figure 2.1.2.

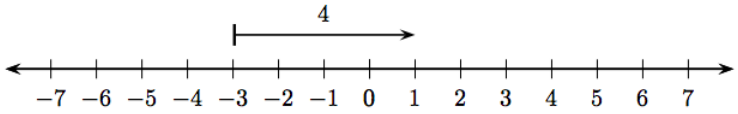


Figure 2.1.2: A vector representing positive four.

Again, there are two important observations to be made:

1. The magnitude (length) of the vector in Figure 2.1.2 is four.
2. The vector in Figure 2.1.2 points to the right.

Hopefully, you have the idea. Any vector that has length 4 and points to the right will represent positive four, regardless of its starting or finishing point. Conversely, consider the vector in Figure 2.1.3, which starts at the integer 4 and finishes at the integer −3.

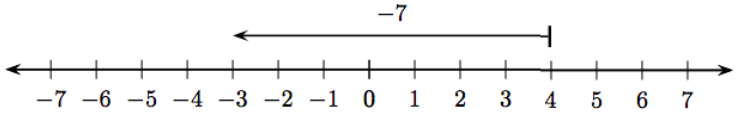


Figure 2.1.3: A vector representing negative seven.

Two observations:

1. The magnitude (length) of the vector in Figure 2.1.3 is seven.
2. The vector in Figure 2.1.3 points to the left.

We will agree that the vector in Figure 2.1.3 represents negative seven. We could select different starting and finishing points for our vector, but as long as the vector has length seven and points to the left, it represents the integer −7.

Important Observation

A vector pointing to the right represents a **positive** number. A vector pointing to the left represents a **negative** number.

Magnitude and Absolute Value

In Figure 2.1.1 and Figure 2.1.2, the vectors pictured represent the integer positive four. Note that the absolute value of four is four; that is, $|4| = 4$. Note also that this absolute value is the magnitude or length of the vectors representing the integer positive four in Figure 2.1.1 and Figure 2.1.2.

In Figure 2.1.3, the vector pictured represents the integer -7. Note that $|-7| = 7$. This shows that the absolute value represents the magnitude or length of the vector representing -7.

Magnitude and Absolute Value

If a is an integer, then $|a|$ gives the magnitude or length of the vector that represents the integer a.

Adding Integers with Like Signs

Because the positive integers are also whole numbers, we've already seen how to add to them in Section 1.2.

Example 1

Find the sum $3 + 4$.

Solution

To add the positive integers 3 and 4, proceed as follows.

1. Start at the integer 0, then draw a vector 3 units in length pointing to the right, as shown in Figure 2.1.4. This arrow has magnitude (length) three and represents the positive integer 3.
2. Draw a second vector of length four that points to the right, starting at the end of the first vector representing the positive integer 3. This arrow has magnitude (length) four and represents the positive integer 4.
3. The sum of the positive integers 3 and 4 could be represented by a vector that starts at the integer 0 and ends at the positive integer 7. However, we prefer to mark this sum on the number line as a solid dot at the positive integer 7. This integer represents the sum of the positive integers 3 and 4.

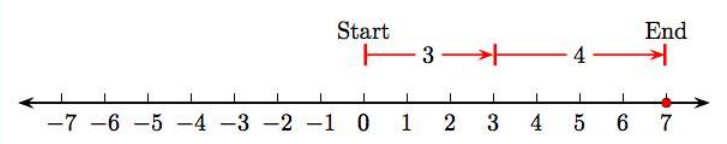


Figure 2.1.4: Adding two positive integers on the number line.

Thus, $3 + 4 = 7$

Exercise

Use a number line diagram to show the sum $5 + 7$.

Answer

12

Negative integers are added in a similar fashion.

Example 2

Find the sum $-3+(-4)$.

Solution

To add the negative integers -3 and -4, proceed as follows.

1. Start at the integer 0, then draw a vector 3 units in length pointing to the left, as shown in Figure 2.1.5. This arrow has magnitude (length) three and represents the negative integer −3.
2. Draw a second vector of length four that points to the left, starting at the end of the first vector representing the negative integer −3. This arrow has magnitude (length) four and represents the negative integer −4.
3. The sum of the negative integers −3 and −4 could be represented by a vector that starts at the integer 0 and ends at the negative integer −7. However, we prefer to mark this sum on the number line as a solid dot at the negative integer −7. This integer represents the sum of the negative integers −3 and −4.

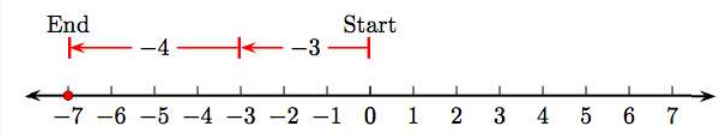


Figure 2.1.5: Adding two positive integers on the number line.

Thus, −3+(−4) = −7.

Drawing on Physical Intuition. Imagine that you are "walking the number line" in Figure 2.1.5. You start at the origin (zero) and take 3 paces *to the left*. Next, you walk an additional four paces *to the left*, landing at the number −7

Exercise

Use a number line diagram to show the sum −7+(−3).

Answer

−10

It should come as no surprise that the procedure used to add two negative integers comprises two steps.

Adding Two Negative Integers

To add two negative integers, proceed as follows:

1. Add the magnitudes of the integers.
2. Prefix the common negative sign

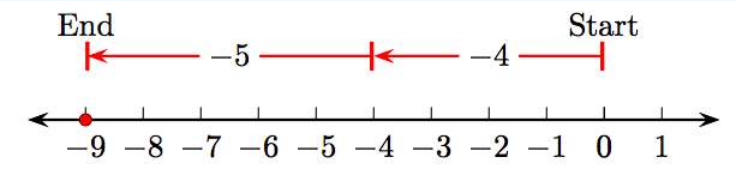
Example 3

Find the sums: (a) −4+(−5), (b) −12 + (−9), and (c) −2+(−16).

Solution

We'll examine three separate but equivalent approaches, as discussed in the narrative above.

a) The number line schematic



shows that (−4) + (−5) = −9.

b) Drawing on physical intuition, start at zero, walk 12 units to the left, then an additional 9 units to the left. You should find yourself 21 units to the left of the origin (zero). Hence, −12 + (−9) = −21.

c) Following the algorithm above in "Adding Two Negative Integers," first add the magnitudes of −2 and −16; that is, 2+16 = 18. Now prefix the common sign. Hence, −2+(−16) = −18.

Exercise

Find the sum: −5+(−9).

Answer

−14

Adding Integers with Unlike Signs

Adding integers with unlike signs is no harder than adding integers with like signs.

Example 4

Find the sum −8 + 4.

Solution

To find the sum −8 + 4, proceed as follows:

1. Start at the integer 0, then draw a vector eight units in length pointing to the left, as shown in Figure 2.1.6. This arrow has magnitude (length) eight and represents the negative integer −8.
2. Draw a second vector of length four that points to the right, starting at the end of the first vector representing the negative integer −8. This arrow (also shown in Figure 2.1.6) has magnitude (length) four and represents the positive integer 4.
3. The sum of the negative integers −8 and 4 could be represented by a vector that starts at the integer 0 and ends at the negative integer −4. However, we prefer to mark this sum on the number line as a solid dot at the negative integer −4. This integer represents the sum of the integers −8 and 4.

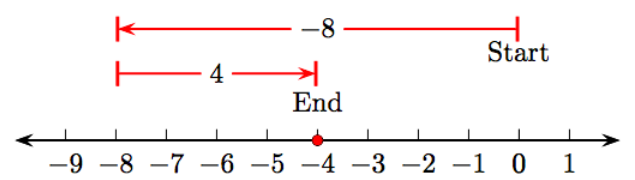


Figure 2.1.6: Adding −8 and 4 on the number line.

Thus, −8+4= −4.

Drawing on Physical Intuition. Imagine you are "walking the number line in Figure 2.1.6. You start at the origin (zero) and walk eight paces *to the left*. Next, turn around and walk four paces *to the right*, landing on the number −4.

Exercise

Use a number line diagram to show the sum −9 + 2.

Answer

−7

Note that adding integers with unlike signs is a *subtractive process*. This is due to the reversal of direction experienced in drawing Figure 2.1.6 in Example 4.

Adding Two Integers with Unlike Signs

To add two integers with unlike signs, proceed as follows:

1. Subtract the smaller magnitude from the larger magnitude.
2. Prefix the sign of the number with the larger magnitude.

For example, to find the sum −8 + 4 of Example 4, we would note that the integers −8 and 4 have magnitudes 8 and 4, respectively. We would then apply the process outlined in "Adding Two Integers with Unlike Signs."

1. Subtract the smaller magnitude from the larger magnitude; that is, 8 − 4 = 4.
2. Prefix the sign of the number with the larger magnitude. Because −8 has the larger magnitude and its sign is negative, we prefix a negative sign to the difference of the magnitudes. Thus, −8+4= −4.

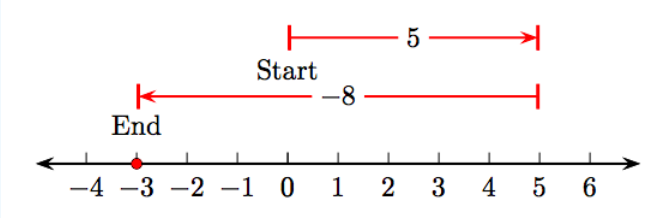
Example 5

Find the sums: (a) 5+ (−8), (b) −12+ 16, and (c) −117+115.

Solution

We'll examine three separate but equivalent approaches, as discussed in the narrative above.

a) The number line schematic



shows that 5 + (−8) = −3.

b) Drawing on physical intuition, start at zero, walk 12 units to the left, then turn around and walk 16 units to the right. You should find yourself 4 units to the right of the origin (zero). Hence, −12 + 16 = 4.

c) Following the algorithm in "Adding Two Integers with Unlike Signs," subtract the smaller magnitude from the larger magnitude, thus 117−115 = 2. Because −117 has the larger magnitude and its sign is negative, we prefix a negative sign to the difference of the magnitudes. Thus, −117 + 115 = −2.

Exercise

Use a number line diagram to show the sum 5 + (−11).

Answer

−6

Properties of Addition of Integers

You will be pleased to learn that the properties of addition for whole numbers also apply to addition of integers.

The Commutative Property of Addition

Let a and b represent two integers. Then,

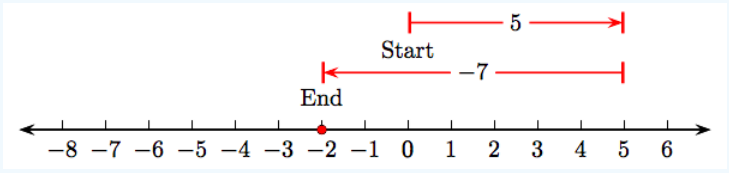
$$a+b=b+a.$$

Example 6

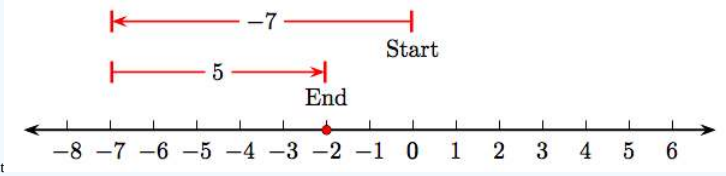
Show that 5 + (−7) = −7 + 5.

Solution

The number line schematic



shows that 5 + (−7) = −2. On the other hand, the number line schematic



shows that −7+5= −2. Therefore, 5 + (−7) = −7 + 5.

Exercise

Use a number line diagram to show that −8 + 6 is the same as 6 + (−8).

Addition of integers is also associative.

The Associative Property of Addition

Let *a*, *b*, and *c* represent integers. Then,

$$(a+b)+c=a+(b+c)$$

.

Example 7

Show that (−9 + 6) + 2 = −9 + (6 + 2).

Solution

On the left, the grouping symbols demand that we add −9 and 6 first. Thus,

$$\begin{aligned}(-9+6)+2 &= -3+2\\ &= -1.\end{aligned}$$

On the right, the grouping symbols demand that we add 6 and 2 first. Thus,

$$\begin{aligned}-9+(6+2) &= -9+8\\ &= -1.\end{aligned}$$

Both sides simplify to −1. Therefore, (−9 + 6) + 2 = −9 + (6 + 2).

Exercise

Show that the expression (−8 + 5) + 3 is the same as −8 + (5 + 3) by simplifying each of the two expressions independently.

The Additive Identity Property

The integer zero is called the *additive identity*. If *a* is any integer, then

a +0= a and 0 + a = a.

Thus, for example, −8+0= −8 and 0 + (−113) = −113.

Finally, every integer has a unique opposite, called its *additive inverse*.

The Additive Inverse Property

Let *a* represent any integer. Then there is a unique integer −*a*, called the **opposite** or **additive inverse** of *a*, such that

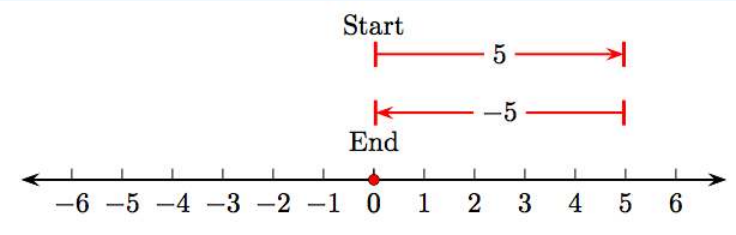
a + (−a) = 0 and −a + a = 0.

Example 8

Show that 5 + (−5) = 0.

Solution

The number line schematic



clearly shows that 5 + (−5) = 0.

Exercise

Use a number line diagram to show that 9 + (−9) = 0.

Important Observation

We have used several equivalent phrases to pronounce the integer $-a$. We've used "the opposite of a," "negative a," and "the additive inverse of a." All are equivalent pronunciations.

Grouping for Efficiency

Order of operations require that we perform all additions as they occur, working from left to right.

Example 9

Simplify −7+8+(−9) + 12.

Solution

We perform the additions as they occur, working left to right.

$$\begin{aligned} &= 7+8+(-9)+12 = 1+(-9)+12 && \text{Working left to right, } -7+8=1. \\ &= -8+12 && \text{Working left to right, } 1+(-9=-8. \\ &= 4 && -8+12=4 \end{aligned}$$

Thus, −7+8+(−9) + 12 = 4.

Exercise

Simplify: −8+9+(−4) + 2.

Answer

−1

The commutative property of addition tells us that changing the order of addition does not change the answer. The associative property of addition tells us that a sum is not affected by regrouping. Let's work Example 9 again, first grouping positive and negative numbers together.

Example 10

Simplify −7+8+(−9) + 12.

Solution

The commutative and associative properties allows us to change the order of addition and regroup.

$$\begin{aligned} -7+8+(-9)+12 &= -7+(-9)+8+12 && \text{Use the commutative property to change the order.} \\ &= [-7+(-9)]+[8+12] && \text{Use the associative property to regroup.} \\ &= -16+20 && \text{Add the negatives. Add the positives.} \\ &= 4 && \text{One final addition.} \end{aligned}$$

Thus, −7+8+(−9) + 12 = 4.

Exercise

Simplify: −11 + 7 + (−12) + 3.

Answer

−13

At first glance, there seems to be no advantage in using the technique in Example 10 over the technique used in Example 9. However, the technique in Example 10 is much quicker in practice, particularly if you eliminate some of the explanatory steps.

Efficient Grouping

When asked to find the sum of a number of integers, it is most efficient to first add all the positive integers, then add the negatives, then add the results.

Example 11

Simplify −7+8+(−9) + 12.

Solution

Add the positive integers first, then the negatives, then add the results.

$$\begin{aligned} -7+8+(-9)+12 &= 20+(-16) && \text{Add the positives: } 8+12=20. \\ & && \text{Add the negatives: } -7+(-9)=-16. \\ &= 4 && \text{Add the results: } 20+(-16)=4. \end{aligned}$$

Thus, −7+8+(−9) + 12 = 4.

Exercise

Simplify: −11 + 3 + (−2) + 7.

Answer

−3

Using Correct Notation

Never write +−! That is, the notation

$$9 + -4 \text{ and } -8 + -6$$

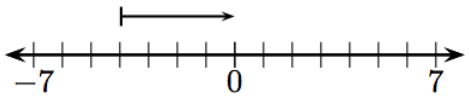
should not be used. Instead, use grouping symbols as follows:

$$9+(-4) \text{ and } -8+(-6)$$

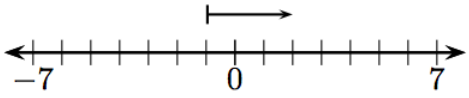
Exercises

In Exercises 1-12, what integer is represented by the given vector?

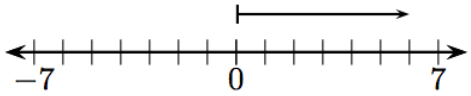
1.



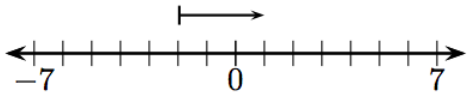
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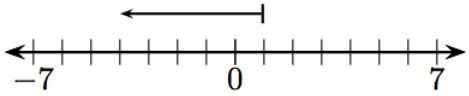
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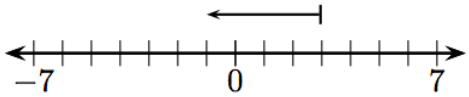
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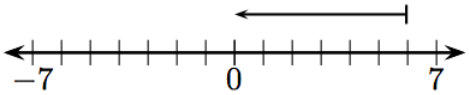
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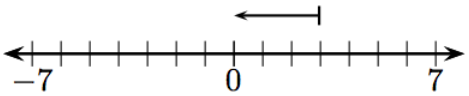
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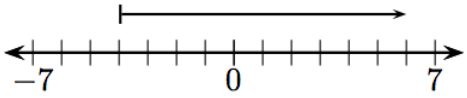
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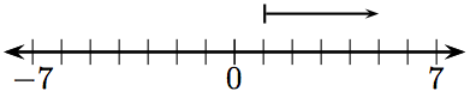
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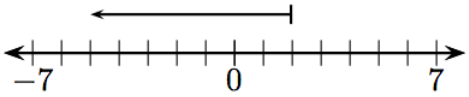
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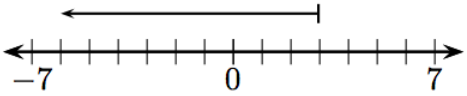
10.



11.



12.



In Exercises 13-36, find the sum of the given integers.

13. −15 + 1
14. −1 + 18
15. 18 + (−10)
16. 2+(−19)
17. −10 + (−12)
18. −1+(−7)
19. 5 + 10
20. 1 + 12
21. 2+5
22. 14 + 1
23. 19 + (−15)
24. 20 + (−17)
25. −2+(−7)
26. −14 + (−6)
27. −6 + 16
28. −2 + 14
29. −11 + (−6)
30. −7+(−8)
31. 14 + (−9)
32. 5+(−15)
33. 10 + 11

34. 14 + 11

35. −13 + 1

36. −8+2

In Exercises 37-52, state the property of addition depicted by the given identity.

37. −1 + (3 + (−8)) = (−1 + 3) + (−8)

38. −4 + (6 + (−5)) = (−4 + 6) + (−5)

39. 7+(−7) = 0

40. 14 + (−14) = 0

41. 15 + (−18) = −18 + 15

42. 14 + (−8) = −8 + 14

43. −15 + 0 = −15

44. −11 + 0 = −11

45. −7 + (1 + (−6)) = (−7 + 1) + (−6)

46. −4 + (8 + (−1)) = (−4 + 8) + (−1)

47. 17 + (−2) = −2 + 17

48. 5+(−13) = −13 + 5

49. −4+0= −4

50. −7+0= −7

51. 19 + (−19) = 0

52. 5+(−5) = 0

In Exercises 53-64, state the additive inverse of the given integer.

53. 18

54. 10

55. 12

56. 15

57. −16

58. −4

59. 11

60. 13

61. −15

62. −19

63. −18

64. −9

In Exercises 65-80, find the sum of the given integers.

65. 6+(−1) + 3 + (−4)

66. 6+(−3) + 2 + (−7)

67. 15 + (−1) + 2

68. 11 + (−16) + 16

69. −17 + 12 + 3

70. −5+(−3) + 2

71. 7 + 20 + 19

72. 14 + (−14) + (−20)

73. 4+(−8) + 2 + (−5)

74. 6+(−3) + 7 + (−2)

75. 7+(−8) + 2 + (−1)

76. 8+(−9) + 5 + (−3)

77. 9+(−3) + 4 + (−1)

78. 1+(−9) + 7 + (−6)

79. 9 + 10 + 2

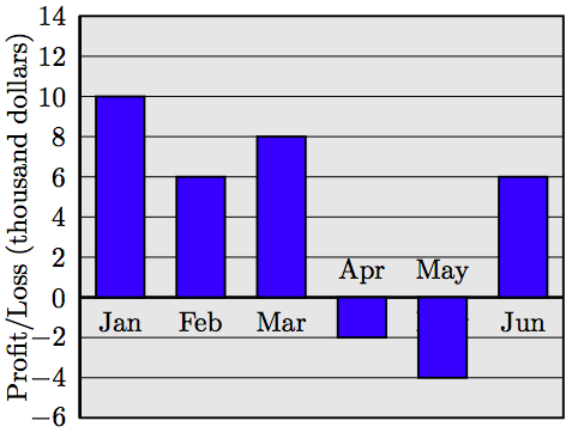
80. −6 + 15 + (−18)

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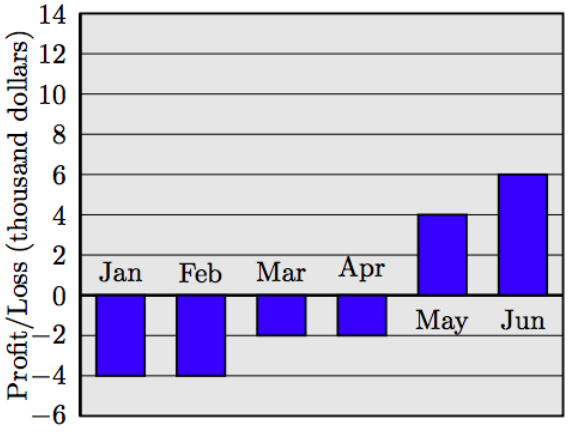
81. **Bank Account.** Gerry opened a new bank account, depositing a check for $215. He then made several withdraws of $40, $75, and $20 before depositing another check for $185. How much is in Gerry's account now?

82. **Dead Sea Sinking.** Due to tectonic plate movement, the Dead Sea is sinking about 1 meter each year. If it's currently −418 meters now, what will Dead Sea elevation be in 5 years? Write an expression that models this situation and compute the result.

83. **Profit and Loss.** Profits and losses for the first six months of the fiscal year for a small business are shown in the following bar chart. Sum the profits and losses from each month. Was there a net profit or loss over the six-month period? How much?



84. **Profit and Loss.** Profits and losses for the first six months of the fiscal year for a small business are shown in the following bar chart. Sum the profits and losses from each month. Was there a net profit or loss over the six-month period? How much?



Answers

1. 4
3. 6
5. −5
7. −6
9. 10
11. −7
13. −14
15. 8
17. −22
19. 15
21. 7
23. 4
25. −9
27. 10
29. −17
31. 5
33. 21
35. −12
37. Associative property of addition
39. Additive inverse property
41. Commutative property of addition
43. Additive identity property
45. Associative property of addition
47. Commutative property of addition
49. Additive identity property
51. Additive inverse property

53. −18

55. −12

57. 16

59. −11

61. 15

63. 18

65. 4

67. 16

69. −2

71. 46

73. −7

75. 0

77. 9

79. 21

81. $265

83. Net Profit: $24,000

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2.2: Subtracting Integers

In Section 1.2, we stated that "Subtraction is the opposite of addition." Thus, to subtract 4 from 7, we walked seven units to the right on the number line, but then walked 4 units in the opposite direction (to the left), as shown in Figure 2.2.1.

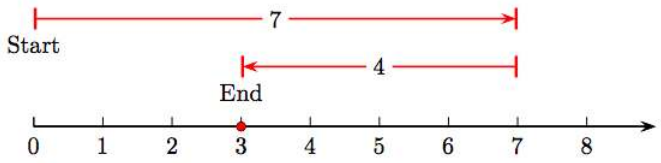


Figure 2.2.1: Subtraction requires that we reverse direction.

Thus, 7 − 4 = 3. The key phrase is "add the opposite." Thus, the subtraction 7 − 4 becomes the addition 7 + (−4), which we would picture on the number line as shown in Figure 2.2.2.

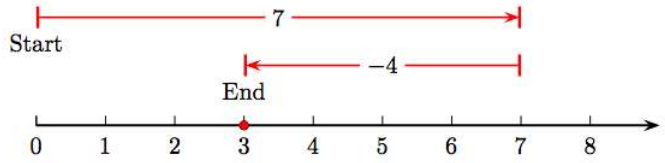


Figure 2.2.2: Subtraction means add the opposite.

Figure 2.2.1 and Figure 2.2.2 provide ample evidence that the subtraction 7−4 is identical to the addition 7+(−4). Again, subtraction means "add the opposite." That is, 7 − 4=7+(−4).

Defining Subtraction

Subtraction means "add the opposite." That is, if a and b are any integers, then

$$a - b = a + (-b).$$

Thus, for example, −123−150 = −123+(−150) and −57−(−91) = −57+91. In each case, subtraction means "add the opposite." In the first case, subtracting 150 is the same as adding −150. In the second case, subtracting−91 is the same as adding 91.

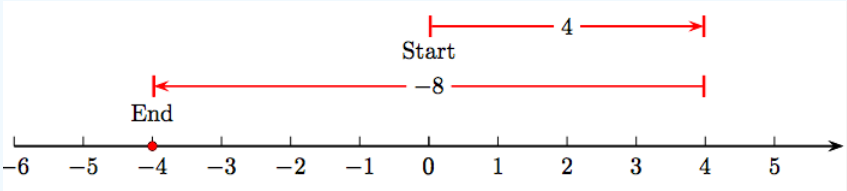
Example 1

Find the differences: (a) 4 − 8, (b) −15 − 13, and (c) −117 − (−115).

Solution

In each case, subtraction means "add the opposite."

a) Change the subtraction to addition with the phrase "subtraction means add the opposite." That is, 4−8 = 4+(−8). We can now perform this addition on the number line.



Thus, 4 − 8=4+(−8) = −4.

b) First change the subtraction into addition by "adding the opposite." That is, −15 − 13 = −15 + (−13). We can now use physical intuition to perform the addition. Start at the origin (zero), walk 15 units to the left, then an additional 13 units to the left, arriving at the answer −28. That is,

$$\begin{aligned} -15 - 13 &= -15 + (-13) \\ &= -28. \end{aligned}$$

c) First change the subtraction into addition by "adding the opposite." That is, −117 − (−115) = −117 + 115. Using "Adding Two Integers with Unlike Signs" from Section 2.2, first subtract the smaller magnitude from the larger magnitude; that is, 117 − 115 = 2. Because −117 has the larger magnitude and its sign is negative, prefix a negative sign to the difference in magnitudes. Thus,

$$\begin{aligned} -117-(-115) &= -117+115 \\ &= -2. \end{aligned}$$

? Exercise

Use each of the techniques in parts (a), (b), and (c) of Example 1 to evaluate the difference −11 − (−9).

Answer

−2

Order of Operations

We will now apply the "Rules Guiding Order of Operations" from Section 1.5 to a number of example exercises.

✔ Example 2

Simplify −5 − (−8) − 7.

Solution

We work from left to right, changing each subtraction by "adding the opposite."

$$\begin{aligned} -5-(-8)-7 &= -5+8+(-7) && \text{Add the opposite of } -8, \text{ which is } 8. \\ & && \text{Add the opposite of } 7, \text{ which is } -7. \\ &= 3+(-7) && \text{Working left to right, } -5+8=3. \\ &= -4 && 3+(-7)=-4. \end{aligned}$$

? Exercise

Simplify: −3 − (−9) − 11.

Answer

−5

Grouping symbols say "do me first."

✔ Example 2.2.1

Simplify −2 − (−2 − 4).

Solution

Parenthetical expressions must be evaluated first.

$$\begin{aligned} -2(-2-4) &= -2-(-2+(-4)) && \text{Simplify the parenthetical expression} \\ & && \text{first. Add the opposite of } 4, \text{ which is } -4. \\ &= -2-(-6) && \text{Inside the parentheses, } -2+(-4)=-6. \\ &= -2+6 && \text{Subtracting a } -6 \text{ is the same as adding a } 6. \\ &= 4 && \text{Add: } -2+6=4. \end{aligned}$$

Exercise

Simplify: $-3-(-3-3)$.

Answer

3

Change as a Difference

Suppose that when I leave my house in the early morning, the temperature outside is 40° Fahrenheit. Later in the day, the temperature measures 60° Fahrenheit. How do I measure the change in the temperature?

The Change in a Quantity

To measure the change in a quantity, always subtract the *former* measurement from the *latter* measurement. That is:

$$\text{Change in a Quantity} = \text{Latter Measurement} - \text{Former Measurement}$$

Thus, to measure the change in temperature, I perform a subtraction as follows:

$$\begin{aligned} \text{Change in Temperature} &= \text{Latter Measurement} && - \text{Former Measurement} \\ &= 60^\circ\text{F} && -40^\circ\text{F} \\ &= 20^\circ\text{F} \end{aligned}$$

Note that the positive answer is in accord with the fact that the temperature has increased.

Example 4

Suppose that in the afternoon, the temperature measures 65° Fahrenheit, then late evening the temperature drops to 44° Fahrenheit. Find the change in temperature.

Solution

To measure the change in temperature, we must subtract the former measurement from the latter measurement.

$$\begin{aligned} \text{Change in Temperature} &= \text{Latter Measurement} && - \text{Former Measurement} \\ &= 44^\circ\text{F} && -65^\circ\text{F} \\ &= -11^\circ\text{F} \end{aligned}$$

Note that the negative answer is in accord with the fact that the temperature has decreased. There has been a "change" of -11° Fahrenheit.

Exercise

Marianne awakes to a morning temperature of 54° Fahrenheit. A storm hits, dropping the temperature to 43° Fahrenheit. Find the change in temperature.

Answer

-11° Fahrenheit

Example 5

Sometimes a bar graph is not the most appropriate visualization for your data. For example, consider the bar graph in Figure 2.2.3 depicting the Dow Industrial Average for seven consecutive days in March of 2009. Because the bars are of almost equal height, it is difficult to detect fluctuation or change in the Dow Industrial Average.

Let's determine the change in the Dow Industrial average on a day-to-day basis. Remember to subtract the latter measurement minus the former (current day minus former day). This gives us the following changes.

Consecutive Days	Change in Dow Industrial Average
Sun-Mon	6900 - 7000 = -100
Mon-Tues	6800 - 6900 = -100
Tues-Wed	6800 - 6800 = 0
Wed-Thu	7000 - 6800 = 200
Thu-Fri	7100 - 7000 = 100
Fri-Sat	7200 - 7100 = 100

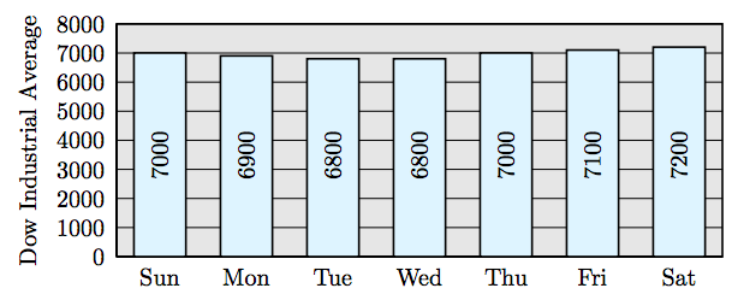


Figure 2.2.3: Profit and loss bar graph.

We will use the data in the table to construct a line graph. On the horizontal axis, we place the pairs of consecutive days (see Figure 2.2.4). On the vertical axis we place the Change in the Industrial Dow Average. At each pair of days we plot a point at a height equal to the change in Dow Industrial Average as calculated in our table.

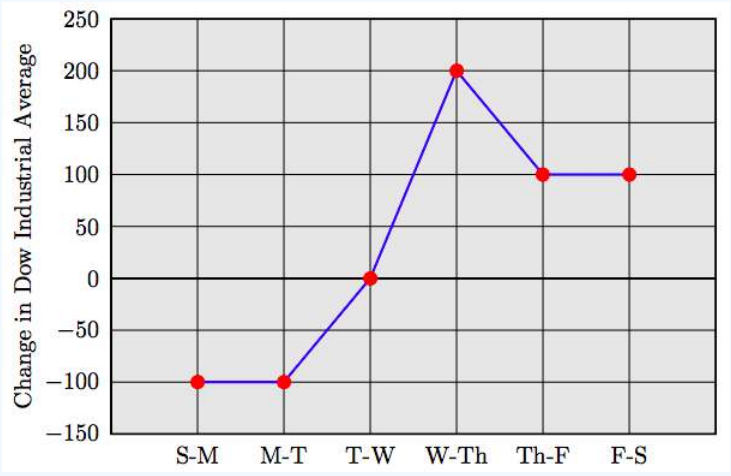


Figure 2.2.4: Plotting change in Dow versus consecutive days.

Note that the data as displayed by Figure 2.2.4 more readily shows the changes in the Dow Industrial Average on a day-to-day basis. For example, it is now easy to pick the day that saw the greatest increase in the Dow (from Wednesday to Thursday, the Dow rose 200 points).

Exercises

In Exercises 1-24, find the difference.

1. 16 − 20
2. 17 − 2

3. $10 - 12$

4. $16 - 8$

5. $14 - 11$

6. $5 - 8$

7. $7 - (-16)$

8. $20 - (-10)$

9. $-4 - (-9)$

10. $-13 - (-3)$

11. $8 - (-3)$

12. $14 - (-20)$

13. $2 - 11$

14. $16 - 2$

15. $-8 - (-10)$

16. $-14 - (-2)$

17. $13 - (-1)$

18. $12 - (-13)$

19. $-4 - (-2)$

20. $-6 - (-8)$

21. $7 - (-8)$

22. $13 - (-14)$

23. $-3 - (-10)$

24. $-13 - (-9)$

In Exercises 25-34, simplify the given expression.

25. $14 - 12 - 2$

26. $-19 - (-7) - 11$

27. $-20 - 11 - 18$

28. $7 - (-13) - (-1)$

29. $5 - (-10) - 20$

30. $-19 - 12 - (-8)$

31. $-14 - 12 - 19$

32. $-15 - 4 - (-6)$

33. $-11 - (-7) - (-6)$

34. $5 - (-5) - (-14)$

In Exercises 35-50, simplify the given expression.

35. $-2 - (-6 - (-5))$

36. $6 - (-14 - 9)$

37. $(-5 - (-8)) - (-3 - (-2))$

38. $(-6 - (-8)) - (-9 - 3)$

39. $(6 - (-9)) - (3 - (-6))$

40. $(-2 - (-3)) - (3 - (-6))$

41. $-1 - (10 - (-9))$

42. $7 - (14 - (-8))$

43. $3 - (-8 - 17)$

44. $1 - (-1 - 4)$

45. $13 - (16 - (-1))$

46. $-7 - (-3 - (-8))$

47. $(7 - (-8)) - (5 - (-2))$

48. $(6 - 5) - (7 - 3)$

49. $(6 - 4) - (-8 - 2)$

50. $(2 - (-6)) - (-9 - (-3))$

---

51. The first recorded temperature is 42°F. Four hours later, the second temperature is 65°F. What is the change in temperature?

52. The first recorded temperature is 79°F. Four hours later, the second temperature is 46°F. What is the change in temperature?

53. The first recorded temperature is 30°F. Four hours later, the second temperature is 51°F. What is the change in temperature?

54. The first recorded temperature is 109°F. Four hours later, the second temperature is 58°F. What is the change in temperature?

55. Typical temperatures in Fairbanks, Alaska in January are −2 degrees Fahrenheit in the daytime and −19 degrees Fahrenheit at night. What is the change in temperature from day to night?

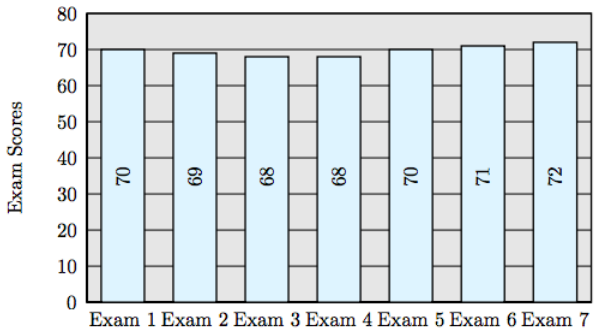
56. Typical summertime temperatures in Fairbanks, Alaska in July are 79 degrees Fahrenheit in the daytime and 53 degrees Fahrenheit at night. What is the change in temperature from day to night?

57. **Communication.** A submarine 1600 feet below sea level communicates with a pilot flying 22,500 feet in the air directly above the submarine. How far is the communique traveling?

58. **Highest to Lowest.** The highest spot on earth is on Mount Everest in Nepal-Tibet at 8,848 meters. The lowest point on the earth's crust is the Mariana's Trench in the North Pacific Ocean at 10,923 meters below sea level. What is the distance between the highest and the lowest points on earth? *Wikipedia* http://en.Wikipedia.org/wiki/Extremes_on_Earth

59. **Lowest Elevation.** The lowest point in North America is Death Valley, California at -282 feet. The lowest point on the entire earth's landmass is on the shores of the Dead Sea along the Israel-Jordan border with an elevation of -1,371 feet. How much lower is the Dead Sea shore from Death Valley?

60. **Exam Scores.** Freida's scores on her first seven mathematics exams are shown in the following bar chart. Calculate the differences between consecutive exams, then create a line graph of the differences on each pair of consecutive exams. Between which two pairs of consecutive exams did Freida show the most improvement?



Answers

1. −4

3. −2

5. 3

7. 23

9. 5 11. 11 13. −9 15. 2 17. 14 19. −2 21. 15 23. 7 25. 0 27. −49 29. −5 31. −45 33. 2 35. −1 37. 4 39. 6 41. −20 43. 28 45. −4 47. 8 49. 12 51. 23◦ F 53. 21◦ F 55. −17 degrees Fahrenheit 57. 24,100 feet 59. 1,089 feet lower

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2.3: Multiplication and Division of Integers

Before we begin, let it be known that the integers satisfy the same properties of multiplication as do the whole numbers.

Integer Properties of Multiplication

Commutative Property. If *a* and *b* are integers, then their product commutes. That is,

$$a \cdot b = b \cdot a.$$

Associative Property. If a, b, and c are integers, then their product is associative. That is,

$$(a \cdot b) \cdot c = a \cdot (b \cdot c).$$

Multiplicative Identity Property. If *a* is any integer, then

$$a \cdot 1 = a \text{ and } 1 \cdot a = a.$$

Because multiplying any integer by 1 returns the identical integer, the integer 1 is called the **multiplicative identity**.

In Section 1.3, we learned that multiplication is equivalent to *repeated addition*. For example,

$$3 \cdot 4 = \underbrace{4+4+4}_{\text{three fours}}$$

On the number line, three sets of four is equivalent to walking three sets of four units to the right, starting from zero, as shown in Figure 2.3.1.

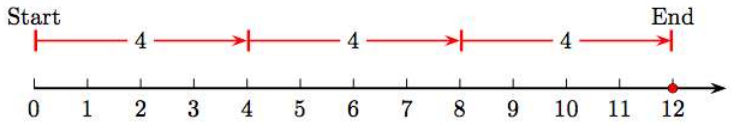


Figure 2.3.1: Note that 3 · 4 = 4 + 4 + 4. That is, 3 · 4 = 12.

This example and a little thought should convince readers that the product of two positive integers will always be a positive integer.

The Product of Two Positive Integers

If *a* and *b* are two positive integers, then their product *ab* is also a positive integer.

For example, 2 · 3 = 6 and 13 · 117 = 1521. In each case, the product of two positive integers is a positive integer.

The Product of a Positive Integer and a Negative Integer

If we continue with the idea that multiplication is equivalent to repeated addition, then it must be that

$$3 \cdot (-4) = \underbrace{-4+(-4)+(-4)}_{\text{three negative fours}}.$$

Pictured on the number line, 3 · (−4) would then be equivalent to walking three sets of negative four units (to the left), starting from zero, as shown in Figure 2.3.2.

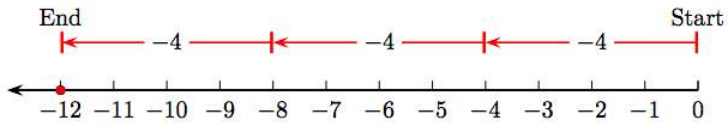


Figure 2.3.2: Note that 3 · (−4) = −4+(−4) + (−4). That is, 3 · (−4) = −12.

Note, at least in this particular case, that the product of a positive integer and a negative integer is a negative integer.

We've shown that 3 · (−4) = −12. However, integer multiplication is commutative, so it must also be true that −4 · 3 = −12. That is, the product of a negative integer and a positive integer is also a negative integer. Although not a proof, this argument motivates the

following fact about integer multiplication.

The Product of a Positive Integer and a Negative Integer

Two facts are true:

1. If a is a positive integer and b is a negative integer, then the product ab is a negative integer.
2. If a is a negative integer and b is a positive integer, then the product ab is a negative integer.

Thus, for example, $5 \cdot (-12) = -60$ and $-13 \cdot 2 = -26$. In each case the answer is negative because we are taking a product where one of the factors is positive and the other is negative.

The Distributive Property

The integers satisfy the distributive property.

The Distributive Property

Let a, b, and c be integers. Then,

$$a \cdot (b + c) = a \cdot b + a \cdot c.$$

We say that "multiplication is distributive with respect to addition."

Note how the a is "distributed." The a is multiplied times each term in the parentheses.

For example, consider the expression $3 \cdot (4 + 5)$. We can evaluate this expression according to the order of operations, simplifying the expression inside the parentheses first.

$$\begin{aligned} 3 \cdot (4+5) &= 3 \cdot 9 \\ &= 27 \end{aligned}$$

But we can also use the distributive property, multiplying each term inside the parentheses by three, then simplifying the result.

$$\begin{aligned} &\text{Distribute the 3: } 3(4+5) \\ &\text{Perform multiplications first: } = 3 \cdot 4 + 3 \cdot 5 \\ &\text{Add: } = 12 + 15 \\ &\text{Sum: } = 27 \end{aligned}$$

Note that evaluating $3 \cdot (4 + 5)$ using the distributive property provides the same result as the evaluation (2.1) using the order of operations.

The Multiplicative Property of Zero

The distributive property can be used to provide proofs of a number of important properties of integers. One important property is the fact that if you multiply an integer by zero, the product is zero. Here is a proof of that fact that uses the distributive property.

Let a be any integer. Then,

$$\begin{aligned} a \cdot 0 &= a \cdot (0+0) && \text{Additive Identity Property: } = 0 + 0 = 0. \\ a \cdot 0 &= a \cdot 0 + a \cdot 0 && \text{Distribute } a \text{ times each zero in the parentheses.} \end{aligned}$$

Next, to "undo" the effect of adding $a \cdot 0$, subtract $a \cdot 0$ from both sides of the equation.

$$\begin{aligned} a \cdot 0 - a \cdot 0 &= a \cdot 0 + a \cdot 0 + a \cdot 0 - a \cdot 0 && \text{Subtract } a \cdot 0 \text{ from both sides.} \\ 0 &= a \cdot 0 && a \cdot 0 - a \cdot 0 = 0 \text{ on each side.} \end{aligned}$$

Multiplicative Property of Zero

Let a represent any integer. Then

$$a \cdot 0 = 0 \text{ and } 0 \cdot a = 0.$$

Thus, for example, $-18 \cdot 0 = 0$ and $0 \cdot 122 = 0$.

Multiplying by Minus One

Here is another useful application of the distributive property.

$$\begin{aligned}(-1)a + a &= (-1)a + 1a && \text{Replace } a \text{ with } 1a.\\ &= (-1+1)a && \text{Use the distributive property to factor out } a.\\ &= 0a && \text{Replace } -1+1 \text{ with } 0.\\ &= 0 && \text{Replace } 0a \text{ with } 0.\end{aligned}$$

Thus, $(-1)a + a = 0$. That is, if you add $(-1)a$ to a you get zero. However, the Additive Inverse Property says that $-a$ is the *unique* number that you add to a to get zero. The conclusion must be that $(-1)a = -a$.

Multiplying by Minus One

If a is any integer, then

$$(-1)a = -a.$$

Thus, for example, $-1(4) = -4$ and $-1(-4) = -(-4) = 4$.

This property is rather important, as we will see in future work. Not only does it tell us that $(-1)a = -a$, but it also tells us that if we see $-a$, then it can be interpreted to mean $(-1)a$.

The Product of Two Negative Integers

We can employ the multiplicative property of -1, that is, $(-1)a = -a$ to find the product of two negative numbers.

$$\begin{aligned}(-4)(-3) &= [(-1)(4)](-3) && \text{Replace } -4 \text{ with } (-1)(4).\\ &= (-1)[(4)(-3)] && \text{Use the associative property to regroup.}\\ &= (-1)(-12) && \text{We know: } (4)(-3) = -12.\\ &= -(-12) && (-1)a = -a. \text{ Here } (-1)(-12) = -(-12).\\ &= 12 && -(-a) = a. \text{ Here } -(-12) = 12.\end{aligned}$$

Thus, at least in the case of $(-4)(-3)$, the product of two negative integers is a positive integer. This is true in general.

The Product of Two Negative Integers

If both a and b are negative integers, then their product ab is a positive integer.

Thus, for example, $(-5)(-7) = 35$ and $(-112)(-6) = 672$. In each case the answer is positive, because the product of two negative integers is a positive integer.

Memory Device

Here's a simple memory device to help remember the rules for finding the product of two integers.

Like and Unlike Signs

There are two cases:

Unlike Signs. The product of two integers with unlike signs is negative. That is:

(+)(−) = −

(−)(+) = −

Like Signs. The product of two integers with like signs is positive. That is:

(+)(+) = +

(−)(−)=+

Example 1

Simplify: (a) (−3)(−2), (b) (4)(−10), and (c) (12)(−3).

Solution

In each example, we use the “like” and “unlike” signs approach.

a) Like signs gives a positive result. Hence, (−3)(−2) = 6.

b) Unlike signs gives a negative result. Hence, (4)(−10) = −40.

c) Unlike signs gives a negative result. Hence, (12)(−3) = −36.

Exercise

Simplify: (a) (−12)(4) and (b) (−3)(−11).

Answer

(a) −48, (b) 33

Example 2

Simplify (−3)(2)(−4)(−2).

Solution

Order of operations demands that we work from left to right.

$$\begin{aligned}(-3)(2)(-4)(-2) &= (-6)(-4)(-2) && \text{Work left to right: } (-3)(2) = -6.\\ &= (24)(-2) && \text{Work left to right: } (-6)(-4) = 24.\\ &= -48 && \text{Multiply: } (24)(-2) = -48.\end{aligned}$$

Hence, (−3)(2)(−4)(−2) = −48.

Exercise

Simplify: (−2)(−3)(4)(−1).

Answer

−24.

Example 3

Simplify: (a)$(-2)^3$ and (c) $(-3)^4$.

Solution

In each example, use

$$a^m = \underbrace{a \cdot a \cdot a \cdot \cdots \cdot a}_{m \text{ times}},$$

then work left to right with the multiplication.

a) Use the definition of an exponent, then order of operations.

$$\begin{aligned}(-2)^3 &= (-2)(-2)(-2) && \text{Write } -2 \text{ as a factor three times.}\\ &= 4(-2) && \text{Work left to right: } (-2)(-2) = 4.\\ &= -8\end{aligned}$$

b) Use the definition of an exponent, then order of operations.

$$\begin{aligned}(-3)^4 &= (-3)(-3)(-3)(-3) && \text{Write } -3 \text{ as a factor four times.}\\ &= 9(-3)(-3) && \text{Work left to right: } (-3)(-3)=9.\\ &= -27(-3) && \text{Work left to right: } 9(-3)=-27.\\ &= 81\end{aligned}$$

Exercise

Simplify: (a) $(-2)^2$ and (b) -2^2.

Answer

(a) 4 and (b) −4

Example 3 motivates the following fact.

Even and Odd

Two facts are apparent.

1. If a product contains an odd number of negative factors, then the product is negative.
2. If a product contains an even number of negative factors, then the product is positive.

Thus, for example,

$$(-2)^5 = (-2)(-2)(-2)(-2)(-2) = -32$$

quickly evaluates as −32 as it has an odd number of negative factors. On the other hand,

$$(-2)^6 = (-2)(-2)(-2)(-2)(-2)(-2) = 64$$

quickly evaluates as 64 as it has an even number of negative factors.

Division of Integers

Consider that

$\frac{12}{3} = 4$ because $3(4) = 12$ and $\frac{-12}{-3} = 4$ because $-3(4) = -12$.

In like manner,

$\frac{12}{-3} = -4$ because -3(-4) = 12 and $\frac{-12}{3} = -4$ because $3(-4) = -12$.

Thus, the rules for dividing integers are the same as the rules for multiplying integers.

Like and Unlike Signs

There are two cases:

Unlike Signs. The quotient of two integers with *unlike* signs is negative. That is,

$$\begin{aligned}(+)(-) &= -\\ \frac{(-)}{(+)} &= -\end{aligned}$$

Like Signs. The quotient of two integers with like signs is positive. That is,

$$\begin{aligned}(+)(+) &= +\\ \frac{(-)}{(-)} &= +\end{aligned}$$

Thus, for example, 12/(−6) = −2 and −44/(−4) = 11. In the first case, unlike signs gives a negative quotient. In the second case, like signs gives a positive quotient.

One final reminder.

Division by Zero is Undefined

If a is any integer, the quotient

$$\frac{a}{0}$$

is undefined. Division by zero is meaningless.

See the discussion in Section 1.3 for a discussion on division by zero.

Example 4

Simplify: (a) −12/(−4), (b) 6/(−3), and (c) −15/0.

Solution

In each example, we use the “like” and “unlike” signs approach.

a) Like signs gives a positive result. Hence,

$$\frac{-12}{-4} = 3.$$

b) Unlike signs gives a negative result. Hence,

$$\frac{6}{-3} = -2.$$

c) Division by zero is undefined. Hence,

$$\frac{-15}{0}$$

is undefined.

Exercise

Simplify: (a) −24/4 and (b) −28/(−7).

Answer

(a) −6, (b) 4

Exercises

In Exercises 1-16, state the property of multiplication depicted by the given identity.

1. (−2) (−16)(13) = (−2)(−16) (13)
2. (10) (−15)(−6) = (10)(−15) (−6)
3. (−17)(−10) = (−10)(−17)
4. (−5)(3) = (3)(−5)
5. (4)(11) = (11)(4)
6. (−5)(−11) = (−11)(−5)
7. 16 · (8 + (−15) = 16 · 8 + 16 · (−15)

8. $1 \cdot (-16 + (-6) = 1 \cdot (-16) + 1 \cdot (-6)$

9. $(17)\ (20)(11) = (17)(20)\ (11)$

10. $(14)\ (-20)(-18) = (14)(-20)\ (-18)$

11. $-19 \cdot 1 = -19$

12. $-17 \cdot 1 = -17$

13. $8 \cdot 1=8$

14. $-20 \cdot 1 = -20$

15. $14 \cdot (-12 + 7 = 14 \cdot (-12) + 14 \cdot 7$

16. $-14 \cdot (-3+6 = -14 \cdot (-3) + (-14) \cdot 6$

In Exercises 17-36, simplify each given expression.

17. $4 \cdot 7$

18. $4 \cdot 2$

19. $3 \cdot (-3)$

20. $7 \cdot (-9)$

21. $-1 \cdot 10$

22. $-1 \cdot 11$

23. $-1 \cdot 0$

24. $-8 \cdot 0$

25. $-1 \cdot (-14)$

26. $-1 \cdot (-13)$

27. $-1 \cdot (-19)$

28. $-1 \cdot (-17)$

29. $2 \cdot 0$

30. $-6 \cdot 0$

31. $-3 \cdot 8$

32. $7 \cdot (-3)$

33. $7 \cdot 9$

34. $6 \cdot 3$

35. $-1 \cdot 5$

36. $-1 \cdot 2$

In Exercises 37-48, simplify each given expression.

37. $(-7)(-1)(3)$

38. $(10)(6)(3)$

39. $(-7)(9)(10)(-10)$

40. $(-8)(-5)(7)(-9)$

41. $(6)(5)(8)$

42. $(7)(-1)(-9)$

43. $(-10)(4)(-3)(8)$

44. $(8)(-2)(-5)(2)$

45. $(6)(-3)(-8)$

46. $(-5)(-4)(1)$

47. $(2)(1)(3)(4)$

48. $(7)(5)(1)(4)$

In Exercises 49-60, compute the exact value.

49. $(-4)^4$

50. $(-3)^4$

51. $(-5)^4$

52. $(-2)^2$

53. $(-5)^2$

54. $(-3)^3$

55. $(-6)^2$

56. $(-6)^4$

57. $(-4)^5$

58. $(-4)^2$

59. $(-5)^3$

60. $(-3)^2$

In Exercises 61-84, simplify each given expression.

61. $-16 \div (-8)$

62. $-33 \div (-3)$

63. $\frac{-8}{1}$

64. $\frac{40}{-20}$

65. $\frac{-1}{0}$

66. \(\frac{2}{0}\

67. $-3 \div 3$

68. $-58 \div 29$

69. $\frac{56}{-28}$

70. $\frac{60}{-12}$

71. $0 \div 15$

72. $0 \div (-4)$

73. $\frac{63}{21}$

74. $\frac{-6}{-1}$

75. $\frac{78}{13}$

76. $\frac{-84}{-14}$

77. $0 \div 5$

78. $0 \div (-16)$

79. $\frac{17}{0}$

80. $\frac{-20}{0}$

81. $-45 \div 15$

82. $-28 \div 28$

83. $12 \div 3$

84. $-22 \div (-22)$

---

85. **Scuba**. A diver goes down 25 feet. A second diver then dives down 5 times further than the first diver. Write the final depth of the second diver as an integer.

86. **Investing Loss**. An investing club of five friends has lost $4400 on a trade. If they share the loss equally, write each members' loss as an integer.

Answers

1. Associative property of multiplication

3. Commutative property of multiplication

5. Commutative property of multiplication

7. Distributive property

9. Associative property of multiplication

11. Multiplicative identity property

13. Multiplicative identity property

15. Distributive property

17. 28

19. −9

21. −10

23. 0

25. 14

27. 19

29. 0

31. −24

33. 63

35. −5

37. 21

39. 6300

41. 240

43. 960

45. 144

47. 24

49. 256

51. 625

53. 25

55. 36

57. −1024

59. −125

61. 2

63. −8

65. Division by zero is undefined.

67. −1

69. −2

71. 0

73. 3

75. 6

77. 0

79. Division by zero is undefined.

81. −3 83. 4

85. −125 feet

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2.4: Order of Operations

The order in which we evaluate expressions can be ambiguous. Take for example, the expression $4 + 3 \cdot 2$. If we do the addition first, then

$$4+3 \cdot 2=7 \cdot 2$$

$$= 14.$$

On the other hand, if we do the multiplication first, then

$$4+3 \cdot 2=4+6$$

$$= 10.$$

So, what are we to do? Of course, grouping symbols can remove the ambiguity

Grouping Symbols

Parentheses, brackets, or curly braces can be used to group parts of an expression. Each of the following are equivalent:

$$(4 + 3) \cdot 2 \text{ or}$$

$$[4 + 3] \cdot 2 \text{ or}$$

$$\{4+3\} \cdot 2$$

In each case, the rule is “evaluate the expression inside the grouping symbols first.” If grouping symbols are nested, evaluate the expression in the innermost pair of grouping symbols first.

For example,

$$(4 + 3) \cdot 2$$

$$7 \cdot 2$$

$$14$$

Note how the expression contained in the parentheses was evaluated first. Another way to avoid ambiguities in evaluating expressions is to establish an order in which operations should be performed. The following guidelines should always be strictly enforced when evaluating expressions.

Rules Guiding Order of Operations

When evaluating expressions, proceed in the following order.

1. Evaluate expressions contained in **grouping symbols** first. If grouping symbols are nested, evaluate the expression in the innermost pair of grouping symbols first.
2. Evaluate all **exponents** that appear in the expression.
3. Perform all **multiplications and divisions** in the order that they appear in the expression, moving left to right.
4. Perform all **additions and subtractions** in the order that they appear in the expression, moving left to right.

Note on Order of Operations

There are many ways in which Order of Operations is taught across the world. In the United States two of the most common mnemonic device is
"PEMDAS" - Parentheses, Exponents, Multiplication & Division, Addition & Subtraction. There is also the phrase "Please excuse my dear Aunt Sally" Which harbors the same meaning, just different words. Ask your classmates how they were taught this. You will find students have a broad range of systems to remember this rule.

Example 1

Evaluate 4 + 3 · 2.

Solution

Because of the established *Rules Guiding Order of Operations*, this expression is no longer ambiguous. There are no grouping symbols or exponents, so we immediately go to rule three, evaluate all multiplications and divisions in the order that they appear, moving left to right. After that we invoke rule four, performing all additions and subtractions in the order that they appear, moving left to right.

$$\begin{aligned} 4+3\cdot 2 \\ 4+6 \\ 10 \end{aligned}$$

Thus, 4 + 3 · 2 simplifies to 10.

Exercise

Simplify: 8 + 2 · 5.

Answer

18

Example 2

Evaluate -18 − 2 + 3.

Solution

Follow the *Rules Guiding Order of Operations*. Addition has no precedence over subtraction, nor does subtraction have precedence over addition. We are to perform additions and subtractions as they occur, moving left to right.

$$\begin{aligned} -18-2+3 & \text{ Subtract: -18 } - \text{ 2 gives -20.} \\ -20+3 & \text{ Add: -20 } + \text{ 3 gives -17.} \\ -17 & \text{ Most Simplified Form} \end{aligned}$$

Thus, -18 − 2 + 3 simplifies to -17.

Exercise

Simplify: 17 − 8 + 2.

Answer

11

Example 3

Evaluate 54 ÷ 9 · 2.

Solution

Follow the *Rules Guiding Order of Operations*. Division has no precedence over multiplication, nor does multiplication have precedence over division. We are to perform divisions and multiplications as they occur, moving left to right.

$$\begin{aligned} 54\div 9\cdot 2 & \text{ Divide: 54 by 9 simplifies to 6} \\ 6\cdot 2 & \text{ Multiply: 6 } \cdot \text{ 2 simplifies to 12.} \\ 12 & \text{ Most Simplified Form} \end{aligned}$$

Thus, 54 ÷ 9 · 2 simplifies to 12.

Exercise

Simplify: $72 \div 9 \cdot 2$.

Answer

16

Example 4

Evaluate $2 \cdot 3^2 - 12$.

Solution

Follow the *Rules Guiding Order of Operations*, exponents first, then multiplication, then subtraction.

$$\begin{aligned} 2 \cdot 3^2 - 12 &\quad \text{There are no group symbols, so evaluate the exponent: } 3^2 \text{ simplifies to } 9. \\ 2 \cdot 9 - 12 &\quad \text{Multiply: } 2 \cdot 9 \text{ simplifies to } 18. \\ 18 - 12 &\quad \text{Subtract: } 18 - 12 \text{ simplifies to } 6. \\ 6 &\quad \text{Most Simplified Form} \end{aligned}$$

Thus, $2 \cdot 3^2 - 12$ simplifies to 6.

Exercise

Simplify: $14 + 3 \cdot 4^2$

Answer

62

Example 5

Evaluate $12 + 2(3 + 2 \cdot 5)^2$.

Solution

Follow the Rules Guiding Order of Operations, evaluate the expression inside the parentheses first, then exponents, then multiplication, then addition.

$$\begin{aligned} 12+2(3+5\cdot 5)^2 &\quad \text{Multiply inside parentheses: } 2 \cdot 5 \text{ to get } 10. \\ 12+2(3+10)^2 &\quad \text{Add inside parentheses/grouping symbols: } 3+10 \text{ to get } 13. \\ 12+2(13)^2 &\quad \text{Exponents are next: } (13)^2 \text{ to get } 169. \\ 12+2(169) &\quad \text{Multiplication is next: } 2(169) \text{ to get } 338. \\ 12+338 &\quad \text{Finally add: } 12+338 \text{ to get } 350. \\ 350 &\quad \text{Most Simplified Form} \end{aligned}$$

Thus, $12 + 2(3 + 2 \cdot 5)$ 2 simplifies to 350.

Exercise

Simplify: $3(2 + 3 \cdot 4)^2 - 11$.

Answer

577

Example 6

Evaluate 2{2 + 2[2 + 2]}.

Solution

When grouping symbols are nested, evaluate the expression between the pair of innermost grouping symbols first.

$$\begin{aligned} 2(2+2[2+2]) &\quad \text{Innermost grouping first: } 2+2 \text{ to get } 4. \\ 2(2+2[4]) &\quad \text{Multiply next: } 2[4] \text{ to get } 8. \\ 2(2+8) &\quad \text{Add inside braces: } 2+8 \text{ to get } 10. \\ 2(10) &\quad \text{Multiply: } 2(10) \text{ to get } 20 \\ 20 &\quad \text{Most Simplified Form} \end{aligned}$$

Thus, 2(2 + 2[2 + 2]) simplifies to 20.

Exercise

Simplify: 2{3 + 2[3 + 2]}.

Answer

26

Fraction Bars

Consider the expression

$$\frac{6^2+8^2}{(2+3)^2}$$

Because a fraction bar means division, the above expression is equivalent to

$$\left(6^2+8^2\right) \div (2+3)^2$$

The position of the grouping symbols signals how we should proceed. We should simplify the numerator, then the denominator, then divide.

Fractional Expressions

If a fractional expression is present, evaluate the numerator and denominator first, then divide.

Example 7

Evaluate the expression

$$\frac{(6)^2+8^2}{(2+3)^2}.$$

Solution

Simplify the numerator and denominator first, then divide.

$$\begin{aligned} \frac{(6)^2+8^2}{(2-7)^2} &\quad \text{Simplify parentheses in denominator first: } 2-7 \text{ simplifies to } -5 \\ \frac{6^2+8^2}{(-5)^2} &\quad \text{Exponents are next: } (6)^2 \text{ simplifies to } 36,\ 8^2 \text{ simplifies to } 64,\ (-5)^2 \text{ simplifies to } 25. \\ \frac{36+64}{25} &\quad \text{Add in numerator: } 36+64=100 \\ \frac{100}{25} &\quad \text{Divide: } 100 \div 25 = 4. \\ 4 &\quad \text{Most Simplified Form} \end{aligned}$$

Thus, $\frac{6^2+8^2}{(2-7)^2}$ simplifies to 4.

Exercise

Simplify: $\frac{12+3\cdot 2}{6}$ **Answer**

3

The Distributive Property

Consider the expression 2 · (3 + 4). If we follow the "Rules Guiding Order of Operations," we would evaluate the expression inside the parentheses first. 2 · (3 + 4) = 2 · 7 Parentheses first: 3 + 4 = 7. = 14 Multiply: 2 · 7 = 14.

However, we could also choose to "distribute" the 2, first multiplying 2 times each addend in the parentheses.

$$\begin{aligned} 2\cdot(3+4) &= 2\cdot 3+2\cdot 4 \quad \text{Multiply 2 times both 3 and 4.} \\ &= 6+8 \quad \text{Multiply: } 2\cdot 3=6 \text{ and } 2\cdot 4=8. \\ &= 14 \quad \text{Add: } 6+8=14. \end{aligned}$$

The fact that we get the same answer in the second approach is an illustration of an important property of whole numbers.[1]

The Distributive Property

Let *a*, *b*, and *c* be any whole numbers. Then,

a · (*b* + *c*) can be distributed to look like this: *a* · *b* + *a* · *c*.

We say that "multiplication is distributive with respect to addition."

Multiplication is distributive with respect to addition. If you are not computing the product of a number and a sum of numbers, the distributive property does not apply.

Caution! Wrong Answer Ahead!

If you are calculating the product of a number and the product of two numbers, the distributive property must not be used. For example, here is a common misapplication of the distributive property.

$$\begin{aligned} 2\cdot(3\cdot 4) &= (2\cdot 3)\cdot(2\cdot 4) \\ &= 6\cdot 8 \\ &= 48 \end{aligned}$$

This result is quite distant from the correct answer, which is found by computing the product within the parentheses first.

$$\begin{aligned} 2\cdot(3\cdot 4) &= 2\cdot 12 \\ &= 24. \end{aligned}$$

In order to apply the distributive property, you must be multiplying times a sum.

Example 8

Use the distributive property to calculate 4 · (5 + 11).

Solution

This is the product of a number and a sum, so the distributive property may be applied.

$$\begin{aligned} 4\cdot(5+11) &\quad \text{Distribute the 4 times addend in the sum.} \\ 4\cdot 5+4\cdot 11 &\quad \text{Multiply: } 4\cdot 5 \text{ gives 20 and } 4\cdot 11 \text{ gives 44.} \\ 20+44 &\quad \text{Add: } 20+44 \text{ gives 64.} \\ 64 &\quad \text{Most Simplified Form} \end{aligned}$$

Readers should check that the same answer is found by computing the sum within the parentheses first.

Exercise

Distribute: $5 \cdot (11 + 8)$.

Answer

95

The distributive property is the underpinning of the multiplication algorithm learned in our childhood years.

Example 9

Multiply: $6 \cdot 43$.

Solution

We'll express 43 as sum, then use the distributive property.

$$\begin{aligned} 6 \cdot 43 & \quad \text{Express 43 as a sum: } 43 = 40 + 3. \text{ Or use standard multiplication as explained below.} \\ 6 \cdot (40 + 3) & \quad \text{Distribute the 6.} \\ 6 \cdot 40 + 6 \cdot 3 & \quad \text{Multiply: } 6 \cdot 40 \text{ to get 240 and } 6 \cdot 3 = 18. \\ 240 + 18 & \quad \text{Add: } 240 + 18 = 258. \\ 258 & \quad \text{Most Simplified Form} \end{aligned}$$

Readers should be able to see this application of the distributive property in the more familiar algorithmic form:

$$\begin{array}{r} 43 \\ \times 6 \\ \hline 18 \\ 240 \\ \hline 258 \end{array}$$

Or in the even more condensed form with "carrying:"

$$\begin{array}{r} {}^{1}43 \\ \times 6 \\ \hline 258 \end{array}$$

Exercise

Use the distributive property to evaluate $8 \cdot 92$.

Answer

736

Multiplication is also distributive with respect to subtraction.

The Distributive Property (Subtraction)

Let a, b, and c be any whole numbers. Then,

$a \cdot (b - c)$ distributing will yield $a \cdot b - a \cdot c$.

We say the multiplication is "distributive with respect to subtraction."

Example 10

Use the distributive property to simplify: $3 \cdot (12 - 8)$.

Solution

This is the product of a number and a difference, so the distributive property may be applied.

$$\begin{aligned} 3\cdot(12-8) &\quad \text{Distribute the 3 times each term in the difference.}\\ 3\cdot 12-3\cdot 8 &\quad \text{Multiply: } 3\cdot 12 \text{ simplifies to 36 and } 3\cdot 8 \text{ simplifies to 24.}\\ 36-24 &\quad \text{Subtract: } 36-24 \text{ simplifies to 12.}\\ 12 &\quad \text{Simplified Form} \end{aligned}$$

Alternate solution

Note what happens if we use the usual "order of operations" to evaluate the expression.

$$\begin{aligned} 3\cdot(12-8) &= 3\cdot 4 \quad \text{Parentheses first: } 12-8=4.\\ &= 12 \quad \text{Multiply: } 3\cdot 4=12. \end{aligned}$$

Same answer.

Exercise

Distribute: $8\cdot(9-2)$.

Answer

56

Exercises

In Exercises 1-12, simplify the given expression.

1. $5+2\cdot 2$
2. $5+2\cdot 8$
3. $23-7\cdot 2$
4. $37-3\cdot 7$
5. $4\cdot 3+2\cdot 5$
6. $2\cdot 5+9\cdot 7$
7. $6\cdot 5+4\cdot 3$
8. $5\cdot 2+9\cdot 8$
9. $9+2\cdot 3$
10. $3+6\cdot 6$
11. $32-8\cdot 2$
12. $24-2\cdot 5$

In Exercises 13-28, simplify the given expression.

13. $45\div 3\cdot 5$
14. $20\div 1\cdot 4$
15. $2\cdot 9\div 3\cdot 18$
16. $19\cdot 20\div 4\cdot 16$
17. $30\div 2\cdot 3$
18. $27\div 3\cdot 3$
19. $8-6+1$
20. $15-5+10$

21. $14 \cdot 16 \div 16 \cdot 19$
22. $20 \cdot 17 \div 17 \cdot 14$
23. $15 \cdot 17 + 10 \div 10 - 12 \cdot 4$
24. $14 \cdot 18 + 9 \div 3 - 7 \cdot 13$
25. $22 - 10 + 7$
26. $29 - 11 + 1$
27. $20 \cdot 10 + 15 \div 5 - 7 \cdot 6$
28. $18 \cdot 19 + 18 \div 18 - 6 \cdot 7$

In Exercises 29-40, simplify the given expression.

29. $9+8 \div \{4+4\}$
30. $10 + 20 \div \{2+2\}$
31. $7 \cdot [8 - 5] - 10$
32. $11 \cdot [12 - 4] - 10$
33. $(18 + 10) \div (2 + 2)$
34. $(14 + 7) \div (2 + 5)$
35. $9 \cdot (10 + 7) - 3 \cdot (4 + 10)$
36. $9 \cdot (7 + 7) - 8 \cdot (3 + 8)$
37. $2 \cdot \{8 + 12\} \div 4$
38. $4 \cdot \{8+7\} \div 3$
39. $9+6 \cdot (12 + 3)$
40. $3+5 \cdot (10 + 12)$

In Exercises 41-56, simplify the given expression.

41. $2+9 \cdot [7 + 3 \cdot (9 + 5)]$
42. $6+3 \cdot [4 + 4 \cdot (5 + 8)]$
43. $7+3 \cdot [8 + 8 \cdot (5 + 9)]$
44. $4+9 \cdot [7 + 6 \cdot (3 + 3)]$
45. $6 - 5[11 - (2 + 8)]$
46. $15 - 1[19 - (7 + 3)]$
47. $11 - 1[19 - (2 + 15)]$
48. $9 - 8[6 - (2 + 3)]$
49. $4\{7[9 + 3] - 2[3 + 2]\}$
50. $4\{8[3 + 9] - 4[6 + 2]\}$
51. $9 \cdot [3 + 4 \cdot (5 + 2)]$
52. $3 \cdot [4 + 9 \cdot (8 + 5)]$
53. $3\{8[6 + 5] - 8[7 + 3]\}$
54. $2\{4[6 + 9] - 2[3 + 4]\}$
55. $3 \cdot [2 + 4 \cdot (9 + 6)]$
56. $8 \cdot [3 + 9 \cdot (5 + 2)]$

In Exercises 57-68, simplfiy the given expression.

57. $(5-2)^2$
58. $(5-3)^4$
59. $(4+2)^2$
60. $(3+5)^2$
61. 2^3+3^3
62. 5^4+2^4
63. 2^3-1^3
64. 3^2-1^2
65. $12\cdot 5^2+8\cdot 9+4$
66. $6\cdot 3^2+7\cdot 5+12$
67. $9-3\cdot 2+12\cdot 10^2$
68. $11-2\cdot 3+12\cdot 4^2$

In Exercises 69-80, simplify the given expression.

69. $4^2-(13+2)$
70. $3^3-(7+6)$
71. $3^3-(7+12)$
72. $4^3-(6+5)$
73. $19+3[12-(2^3+1)]$
74. $13+12[14-(2^2+1)]$
75. $17+7[13-(2^2+6)]$
76. $10+1[16-(2^2+9)]$
77. $4^3-(12+1)$
78. $5^3-(17+15)$
79. $5+7[11-(2^2+1)]$
80. $10+11[20-(2^2+1)]$

In Exercises 81-92, simplify the given expression.

81. $\frac{13+35}{3(4)}$
82. $\frac{35+28}{7(3)}$
83. $\frac{64-(8\cdot 6-3)}{4\cdot 7-9}$
84. $\frac{19-(4\cdot 3-2)}{6\cdot 3-9}$
85. $\frac{2+13}{4-1}$
86. $\frac{7+1}{8-4}$
87. $\frac{17+14}{9-8}$
88. $\frac{16+2}{13-11}$
89. $\frac{37+27}{8(2)}$

90. $\frac{16+38}{6(3)}$

91. $\frac{40-(3\cdot 7-9)}{8\cdot 2-2}$

92. $\frac{60-(8\cdot 6-3)}{5\cdot 4-5}$

In Exercises 93-100, use the distributive property to evaluate the given expression.

93. $5 \cdot (8 + 4)$

94. $8 \cdot (4 + 2)$

95. $7 \cdot (8 - 3)$

96. $8 \cdot (9 - 7)$

97. $6 \cdot (7 - 2)$

98. $4 \cdot (8 - 6)$

99. $4 \cdot (3 + 2)$

100. $4 \cdot (9 + 6)$

In Exercises 101-104, use the distributive property to evaluate the given expression using the technique shown in Example 9.

101. $9 \cdot 62$

102. $3 \cdot 76$

103. $3 \cdot 58$

104. $7 \cdot 57$

Answers

1. 9

3. 9

5. 22

7. 42

9. 15

11. 16

13. 75

15. 108

17. 45

19. 3

21. 266

23. 208

25. 19

27. 161

29. 10

31. 11

33. 7

35. 111

37. 10

39. 99

41. 443

43. 367

45. 1

47. 9

49. 296

51. 279

53. 24

55. 186

57. 9

59. 36

61. 35

63. 7

65. 376

67. 1203

69. 1

71. 8

73. 28

75. 38

77. 51

79. 47

81. 4

83. 1

85. 5

87. 31

89. 4

91. 2

93. 60

95. 35

97. 30

99. 20

101. 558

103. 174

[1]Later, we'll see that this property applies to all numbers, not just whole numbers

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CHAPTER OVERVIEW

3: Fractions

3.1: Visualize Fractions (Part 1)
3.2: Visualize Fractions (Part 2)
3.3: Multiply Fractions
3.4: Add and Subtract Fractions with Common Denominators
3.5: Divide Fractions
3.6: Add and Subtract Fractions with Common Denominators
3.7: Add and Subtract Fractions with Different Denominators (Part 2)

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3.1: Visualize Fractions (Part 1)

Learning Objectives

- Understand the meaning of fractions
- Model improper fractions and mixed numbers
- Convert between improper fractions and mixed numbers
- Model equivalent fractions
- Find equivalent fractions
- Locate fractions and mixed numbers on the number line
- Order fractions and mixed numbers

be prepared!

Before you get started, take this readiness quiz.

1. Simplify: $5 \bullet 2 + 1$. If you missed this problem, review Example 2.1.8.
2. Fill in the blank with $<$ or $>$: $-2 \_\_ -5$. If you missed this problem, review Example 3.1.2.

Understand the Meaning of Fractions

Andy and Bobby love pizza. On Monday night, they share a pizza equally. How much of the pizza does each one get? Are you thinking that each boy gets half of the pizza? That's right. There is one whole pizza, evenly divided into two parts, so each boy gets one of the two equal parts. In math, we write $\frac{1}{2}$ to mean one out of two parts.

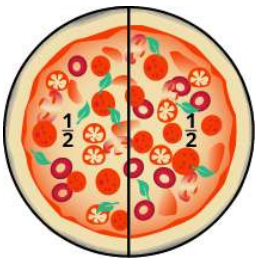


Figure 3.1.1

On Tuesday, Andy and Bobby share a pizza with their parents, Fred and Christy, with each person getting an equal amount of the whole pizza. How much of the pizza does each person get? There is one whole pizza, divided evenly into four equal parts. Each person has one of the four equal parts, so each has $\frac{1}{4}$ of the pizza.



Figure 3.1.2

On Wednesday, the family invites some friends over for a pizza dinner. There are a total of 12 people. If they share the pizza equally, each person would get $\frac{1}{12}$ of the pizza.



Figure 3.1.3

Definition: Fractions

A fraction is written $\frac{a}{b}$, where a and b are integers and $b \neq 0$. In a fraction, a is called the numerator and b is called the denominator.

A fraction is a way to represent parts of a whole. The denominator b represents the number of equal parts the whole has been divided into, and the numerator a represents how many parts are included. The denominator, b, cannot equal zero because division by zero is undefined.

In Figure 3.1.4, the circle has been divided into three parts of equal size. Each part represents $\frac{1}{3}$ of the circle. This type of model is called a fraction circle. Other shapes, such as rectangles, can also be used to model fractions.

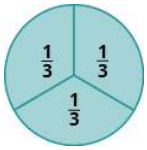


Figure 3.1.4

What does the fraction $\frac{2}{3}$ represent? The fraction $\frac{2}{3}$ means two of three equal parts.

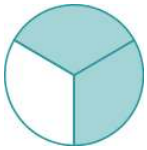
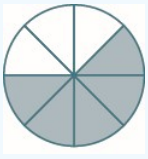
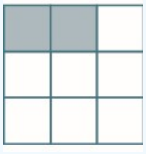


Figure 3.1.5

Example 3.1.1: name the fraction

Name the fraction of the shape that is shaded in each of the figures.

a. 

b. 

Solution

We need to ask two questions. First, how many equal parts are there? This will be the denominator. Second, of these equal parts, how many are shaded? This will be the numerator.

a.

How many equal parts are there?	There are eight equal parts.
How many are shaded?	Five parts are shaded.

Five out of eight parts are shaded. Therefore, the fraction of the circle that is shaded is $\frac{5}{8}$.

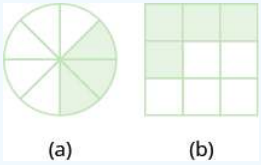
b.

How many equal parts are there?	There are nineequal parts.
How many are shaded?	Two parts are shaded.

Two out of nine parts are shaded. Therefore, the fraction of the square that is shaded is $\frac{2}{9}$.

Exercise 3.1.1

Name the fraction of the shape that is shaded in each figure:



Answer a

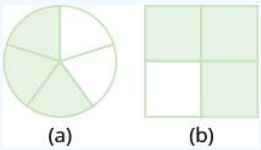
$\frac{3}{8}$

Answer b

$\frac{4}{9}$

Exercise 3.1.2

Name the fraction of the shape that is shaded in each figure:



Answer a

$\frac{3}{5}$

Answer b

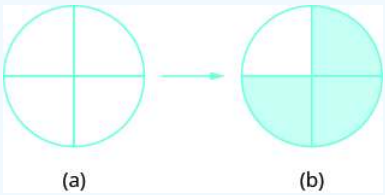
$\frac{3}{4}$

Example 3.1.2:

Shade $\frac{3}{4}$ of the circle.

Solution

The denominator is 4, so we divide the circle into four equal parts (a). The numerator is 3, so we shade three of the four parts (b).



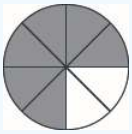
$\frac{3}{4}$ of the circle is shaded.

Exercise 3.1.3

Shade $\frac{6}{8}$ of the circle.



Answer



Exercise 3.1.4

Shade $\frac{2}{5}$ of the rectangle.

Answer



In Example 3.1.1 and Example 3.1.2, we used circles and rectangles to model fractions. Fractions can also be modeled as manipulatives called fraction tiles, as shown in Figure 3.1.6. Here, the whole is modeled as one long, undivided rectangular tile. Beneath it are tiles of equal length divided into different numbers of equally sized parts.

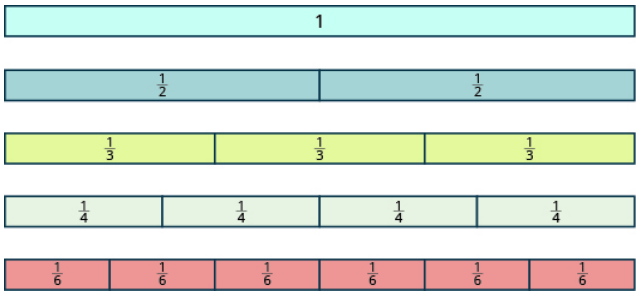


Figure 3.1.6

We'll be using fraction tiles to discover some basic facts about fractions. Refer to Figure 3.1.6 to answer the following questions:

How many $\frac{1}{2}$ tiles does it take to make one whole tile?	It takes two halves to make a whole, so two out of two is $\frac{2}{2} = 1$.
How many $\frac{1}{3}$ tiles does it take to make one whole tile?	It takes three thirds, so three out of three is $\frac{3}{3} = 1$.
How many $\frac{1}{4}$ tiles does it take to make one whole tile?	It takes four fourths, so four out of four is $\frac{4}{4} = 1$.
How many $\frac{1}{5}$ tiles does it take to make one whole tile?	It takes six sixths, so six out of six is $\frac{6}{6} = 1$.
What if the whole were divided into 24 equal parts? (We have not shown fraction tiles to represent this, but try to visualize it in your mind.) How many $\frac{1}{24}$ tiles does it take to make one whole tile?	It takes 24 twenty-fourths, so $\frac{24}{24} = 1$.

It takes 24 twenty-fourths, so $\frac{24}{24} = 1$. This leads us to the Property of One.

Definition: Property of One

Any number, except zero, divided by itself is one.

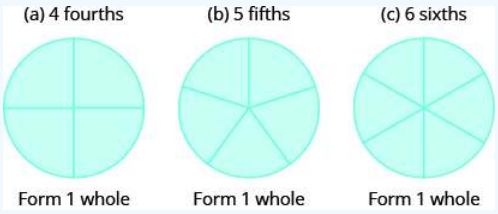
$$\frac{a}{a} = 1 \qquad (a \neq 0) \tag{3.1.1}$$

Example 3.1.3: fraction circles to form wholes

Use fraction circles to make wholes using the following pieces:

a. 4 fourths
b. 5 fifths
c. 6 sixths

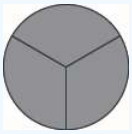
Solution



Exercise 3.1.5

Use fraction circles to make wholes with the following pieces: 3 thirds.

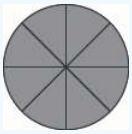
Answer



Exercise 3.1.6

Use fraction circles to make wholes with the following pieces: 8 eighths.

Answer



What if we have more fraction pieces than we need for 1 whole? We'll look at this in the next example.

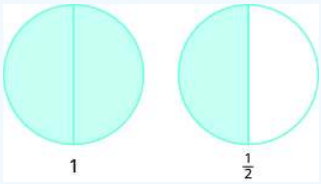
Example 3.1.4: fraction circles to form whole

Use fraction circles to make wholes using the following pieces:

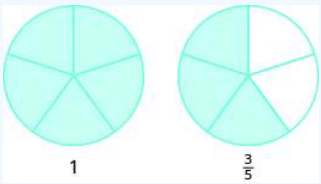
a. 3 halves
b. 8 fifths
c. 7 thirds

Solution

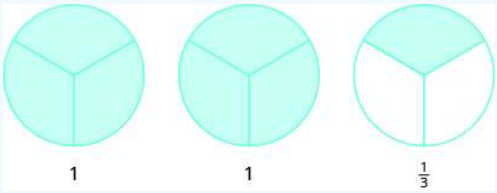
a. 3 halves make 1 whole with 1 half left over.



b. 8 fifths make 1 whole with 3 fifths left over.



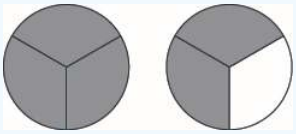
c. 7 thirds make 2 wholes with 1 third left over.



Exercise 3.1.7

Use fraction circles to make wholes with the following pieces: 5 thirds.

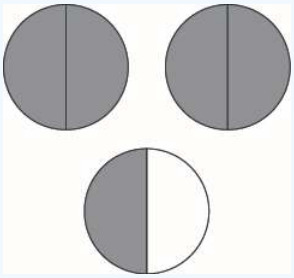
Answer



Exercise 3.1.8

Use fraction circles to make wholes with the following pieces: 5 halves.

Answer



Model Improper Fractions and Mixed Numbers

In Example 3.1.4b, you had eight equal fifth pieces. You used five of them to make one whole, and you had three fifths left over. Let us use fraction notation to show what happened. You had eight pieces, each of them one fifth, $\frac{1}{5}$, so altogether you had eight fifths, which we can write as $\frac{8}{5}$. The fraction $\frac{8}{5}$ is one whole, 1, plus three fifths, $\frac{3}{5}$, or $1\frac{3}{5}$, which is read as *one and three-fifths*. The number $1\frac{3}{5}$ is called a mixed number. A mixed number consists of a whole number and a fraction.

Definition: Mixed Numbers

A mixed number consists of a whole number a and a fraction $\frac{b}{c}$ where $c \neq 0$. It is written as follows.

$$a\frac{b}{c} \qquad c \neq 0 \tag{3.1.2}$$

Fractions such as $\frac{5}{4}$, $\frac{3}{2}$, $\frac{5}{5}$, and $\frac{7}{3}$ are called improper fractions. In an improper fraction, the numerator is greater than or equal to the denominator, so its value is greater than or equal to one. When a fraction has a numerator that is smaller than the denominator, it is called a proper fraction, and its value is less than one. Fractions such as $\frac{1}{2}$, $\frac{3}{7}$, and $\frac{11}{18}$ are proper fractions.

Definition: Proper and Improper Fractions

The fraction $\frac{a}{b}$ is a proper fraction if $a < b$ and an improper fraction if $a \geq b$.

Example 3.1.5: improper fraction

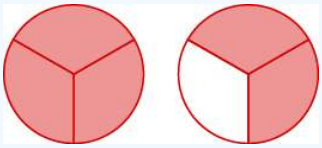
Name the improper fraction modeled. Then write the improper fraction as a mixed number.

Solution

Each circle is divided into three pieces, so each piece is $\frac{1}{3}$ of the circle. There are four pieces shaded, so there are four thirds or $\frac{4}{3}$. The figure shows that we also have one whole circle and one third, which is $1\frac{1}{3}$. So, $\frac{4}{3}=1\frac{1}{3}$.

Exercise 3.1.9

Name the improper fraction. Then write it as a mixed number.

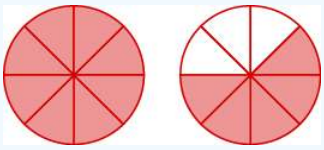


Answer

$\frac{5}{3}=1\frac{2}{3}$

Exercise 3.1.10

Name the improper fraction. Then write it as a mixed number.



Answer

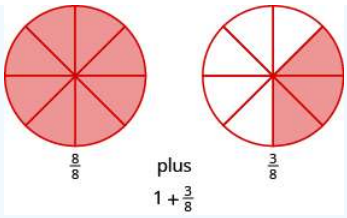
$\frac{13}{8}=1\frac{5}{8}$

Example 3.1.6: model a fraction

Draw a figure to model $\frac{11}{8}$.

Solution

The denominator of the improper fraction is 8. Draw a circle divided into eight pieces and shade all of them. This takes care of eight eighths, but we have 11 eighths. We must shade three of the eight parts of another circle.

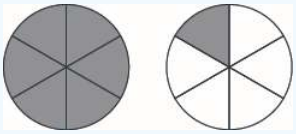


So, $\frac{11}{8}=1\frac{3}{8}$.

? Exercise3.1.11

Draw a figure to model $\frac{7}{6}$.

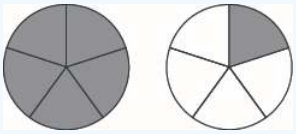
Answer



? Exercise 3.1.12

Draw a figure to model $\frac{6}{5}$.

Answer



✓ Example 3.1.7: model a fraction

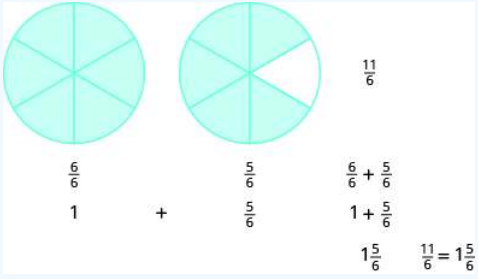
Use a model to rewrite the improper fraction $\frac{11}{6}$ as a mixed number.

Solution

We start with 11 sixths $\left(\frac{11}{6}\right)$. We know that six sixths makes one whole.

$$\frac{6}{6}=1$$

That leaves us with five more sixths, which is $\frac{5}{6}$ (11 sixths minus 6 sixths is 5 sixths). So, $\frac{11}{6}=1\frac{5}{6}$.



Exercise 3.1.13

Use a model to rewrite the improper fraction as a mixed number: $\frac{9}{7}$.

Answer

$1\frac{2}{7}$

Exercise 3.1.14

Use a model to rewrite the improper fraction as a mixed number: $\frac{7}{4}$.

Answer

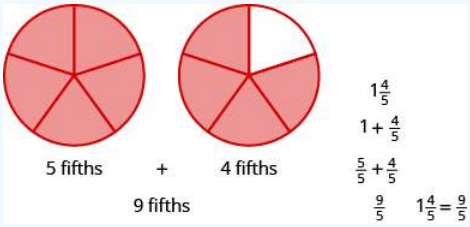
$1\frac{3}{4}$

Example 3.1.8: model a fraction

Use a model to rewrite the mixed number $1\frac{4}{5}$ as an improper fraction.

Solution

The mixed number $1\frac{4}{5}$ means one whole plus four fifths. The denominator is 5, so the whole is $\frac{5}{5}$. Together five fifths and four fifths equals nine fifths. So, $1\frac{4}{5}=\frac{9}{5}$.



Exercise 3.1.15

Use a model to rewrite the mixed number as an improper fraction: $1\frac{3}{8}$.

Answer

$\frac{11}{8}$

Exercise 3.1.16

Use a model to rewrite the mixed number as an improper fraction: $1\frac{5}{6}$.

Answer

$\frac{11}{6}$

Convert between Improper Fractions and Mixed Numbers

In Example 3.1.7, we converted the improper fraction $\frac{11}{6}$ to the mixed number $1\frac{5}{6}$ using fraction circles. We did this by grouping six sixths together to make a whole; then we looked to see how many of the 11 pieces were left. We saw that $\frac{11}{6}$ made one whole group of six sixths plus five more sixths, showing that $\frac{11}{6} = \frac{15}{6}$.

The division expression $\frac{11}{6}$ (which can also be written as $6\overline{)11}$) tells us to find how many groups of 6 are in 11. To convert an improper fraction to a mixed number without fraction circles, we divide.

Example 3.1.9:

Convert $\frac{11}{6}$ to a mixed number.

Solution

Divide the denominator into the numerator.	Remember $\frac{11}{6}$ means $11 \div 6$.
Identify the quotient, remainder and divisor.	$6\overline{)11}$ 1 ← quotient; divisor → 6; 5 ← remainder
Write the mixed number as $quotient\ \frac{remainder}{divisor}$.	$1\frac{5}{6}$

So, $\frac{11}{6} = 1\frac{5}{6}$.

Exercise 3.1.17

Convert the improper fraction to a mixed number: $\frac{13}{7}$.

Answer

$1\frac{6}{7}$

Exercise 3.1.18

Convert the improper fraction to a mixed number: $\frac{14}{9}$.

Answer

$1\frac{5}{9}$

HOW TO: CONVERT AN IMPROPER FRACTION TO A MIXED NUMBER

Step 1. Divide the denominator into the numerator.

Step 2. Identify the quotient, remainder, and divisor.

Step 3. Write the mixed number as $quotient\ \frac{remainder}{divisor}$.

Example 3.1.10:

Convert the improper fraction $\frac{33}{8}$ to a mixed number.

Solution

Divide the denominator into the numerator.	Remember, $\frac{33}{8}$ means $8\overline{)33}$.
Identify the quotient, remainder, and divisor.	$8\overline{)33}$ (4 ← quotient; divisor → 8; 32; 1 ← remainder)
Write the mixed number as $quotient\ \frac{remainder}{divisor}$.	$4\frac{1}{8}$

So, $\frac{33}{8} = 4\frac{1}{8}$.

Exercise 3.1.19

Convert the improper fraction to a mixed number: $\frac{23}{7}$.

Answer

$3\frac{2}{7}$

Exercise 3.1.20

Convert the improper fraction to a mixed number: $\frac{48}{11}$.

Answer

$4\frac{4}{11}$

In Example 3.1.8, we changed $1\frac{4}{5}$ to an improper fraction by first seeing that the whole is a set of five fifths. So we had five fifths and four more fifths.

$$\frac{5}{5} + \frac{4}{5} = \frac{9}{5}$$

Where did the nine come from? There are nine fifths—one whole (five fifths) plus four fifths. Let us use this idea to see how to convert a mixed number to an improper fraction.

Example 3.1.11: convert

Convert the mixed number $4\frac{2}{3}$ to an improper fraction.

Multiply the whole number by the denominator.	$4\frac{2}{3}$
The whole number is 4 and the denominator is 3.	$\frac{4 \cdot 3 + \square}{\square}$

Simplify.	$\frac{12+\square}{\square}$
Add the numerator to the product.	
The numerator of the mixed number is 2.	$\frac{12+2}{\square}$
Simplify.	$\frac{14}{\square}$
Write the final sum over the original denominator.	
The denominator is 3.	$\frac{14}{3}$

Exercise 3.1.21

Convert the mixed number to an improper fraction: $3\frac{5}{7}$.

Answer

$\frac{26}{7}$

Exercise 3.1.22

Convert the mixed number to an improper fraction: $2\frac{7}{8}$.

Answer

$\frac{23}{8}$

HOW TO: CONVERT A MIXED NUMBER TO AN IMPROPER FRACTION

Step 1. Multiply the whole number by the denominator.

Step 2. Add the numerator to the product found in Step 1.

Step 3. Write the final sum over the original denominator.

Example 3.1.12:

Convert the mixed number $10\frac{2}{7}$ to an improper fraction.

Multiply the whole number by the denominator.	$10\frac{2}{7}$
The whole number is 10 and the denominator is 7.	$\frac{10\cdot 7+\square}{\square}$
Simplify.	$\frac{70+\square}{\square}$
Add the numerator to the product.	
The numerator of the mixed number is 2.	$\frac{70+2}{\square}$

Simplify.	$\frac{72}{\square}$
Write the final sum over the original denominator.	
The denominator is 7.	$\frac{72}{7}$

Exercise 3.1.23

Convert the mixed number to an improper fraction: $4\frac{6}{11}$.

Answer

$\frac{50}{11}$

Exercise 3.1.24

Convert the mixed number to an improper fraction: $11\frac{1}{3}$.

Answer

$\frac{34}{3}$

Contributors and Attributions

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3.2: Visualize Fractions (Part 2)

Model Equivalent Fractions

Let's think about Andy and Bobby and their favorite food again. If Andy eats $\frac{1}{2}$ of a pizza and Bobby eats $\frac{2}{4}$ of the pizza, have they eaten the same amount of pizza? In other words, does $\frac{1}{2} = \frac{2}{4}$? We can use fraction tiles to find out whether Andy and Bobby have eaten *equivalent* parts of the pizza.

Definition: Equivalent Fractions

Equivalent fractions are fractions that have the same value.

Fraction tiles serve as a useful model of equivalent fractions. You may want to use fraction tiles to do the following activity. Or you might make a copy of Figure 4.3 and extend it to include eighths, tenths, and twelfths.

Start with a $\frac{1}{2}$ tile. How many fourths equal one-half? How many of the $\frac{1}{4}$ tiles exactly cover the $\frac{1}{2}$ tile?

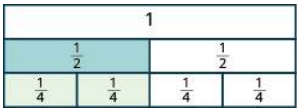


Figure 3.2.7

Since two $\frac{1}{4}$ tiles cover the $\frac{1}{2}$ tile, we see that $\frac{2}{4}$ is the same as $\frac{1}{2}$, or $\frac{2}{4} = \frac{1}{2}$.

How many of the $\frac{1}{6}$ tiles cover the $\frac{1}{2}$ tile?

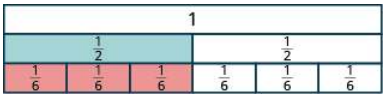


Figure 3.2.8

Since three $\frac{1}{6}$ tiles cover the $\frac{1}{2}$ tile, we see that $\frac{3}{6}$ is the same as $\frac{1}{2}$. So, $\frac{3}{6} = \frac{1}{2}$. The fractions are equivalent fractions.

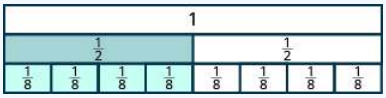
Example 3.2.13: equivalent fractions

Use fraction tiles to find equivalent fractions. Show your result with a figure.

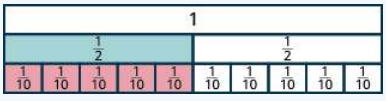
a. How many eighths equal one-half?
b. How many tenths equal one-half?
c. How many twelfths equal one-half?

Solution

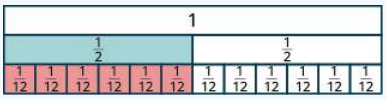
a. It takes four $\frac{1}{8}$ tiles to exactly cover the $\frac{1}{2}$ tile, so $\frac{4}{8} = \frac{1}{2}$.



b. It takes five $\frac{1}{10}$ tiles to exactly cover the $\frac{1}{2}$ tile, so $\frac{5}{10} = \frac{1}{2}$.



c. It takes six $\frac{1}{12}$ tiles to exactly cover the $\frac{1}{2}$ tile, so $\frac{6}{12} = \frac{1}{2}$.



Suppose you had tiles marked $\frac{1}{20}$. How many of them would it take to equal $\frac{1}{2}$? Are you thinking ten tiles? If you are, you're right, because $\frac{10}{20} = \frac{1}{2}$.

We have shown that $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}, \frac{6}{12}$, and $\frac{10}{20}$ are all equivalent fractions.

? Exercise 3.2.25

Use fraction tiles to find equivalent fractions: How many eighths equal one-fourth?

Answer

2

? Exercise 3.2.26

Use fraction tiles to find equivalent fractions: How many twelfths equal one-fourth?

Answer

3

Find Equivalent Fractions

We used fraction tiles to show that there are many fractions equivalent to $\frac{1}{2}$. For example, $\frac{2}{4}, \frac{3}{6}$, and $\frac{4}{8}$ are all equivalent to $\frac{1}{2}$. When we lined up the fraction tiles, it took four of the $\frac{1}{8}$ tiles to make the same length as a $\frac{1}{2}$ tile. This showed that $\frac{4}{8} = \frac{1}{2}$. See Example 3.2.13.

We can show this with pizzas, too. Figure $3.2.9a$ shows a single pizza, cut into two equal pieces with $\frac{1}{2}$ shaded. Figure $3.2.9b$ shows a second pizza of the same size, cut into eight pieces with $\frac{4}{8}$ shaded.

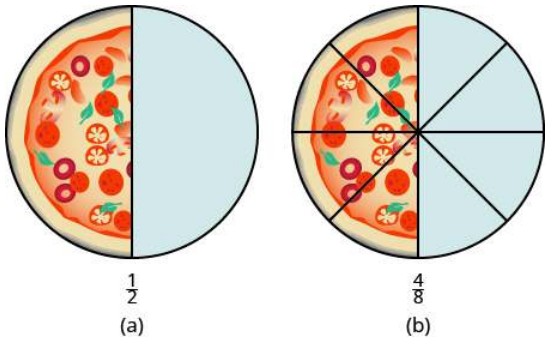


Figure 3.2.9

This is another way to show that $\frac{1}{2}$ is equivalent to $\frac{4}{8}$. How can we use mathematics to change $\frac{1}{2}$ into $frac48$? How could you take a pizza that is cut into two pieces and cut it into eight pieces? You could cut each of the two larger pieces into four smaller

pieces! The whole pizza would then be cut into eight pieces instead of just two. Mathematically, what we've described could be written as:

$$\frac{1 \cdot 4}{2 \cdot 4} = \frac{4}{8}$$

These models lead to the Equivalent Fractions Property, which states that if we multiply the numerator and denominator of a fraction by the same number, the value of the fraction does not change.

Definition: Equivalent Fractions Property

If a, b, and c are numbers where $b \neq 0$ and $c \neq 0$, then

$$\frac{a}{b} = \frac{a \cdot c}{b \cdot c} \tag{3.2.1}$$

When working with fractions, it is often necessary to express the same fraction in different forms. To find equivalent forms of a fraction, we can use the Equivalent Fractions Property. For example, consider the fraction one-half.

$$\frac{1 \cdot 3}{2 \cdot 3} = \frac{3}{6} \text{ so } \frac{1}{2} = \frac{3}{6}$$
$$\frac{1 \cdot 2}{2 \cdot 2} = \frac{2}{4} \text{ so } \frac{1}{2} = \frac{2}{4}$$
$$\frac{1 \cdot 10}{2 \cdot 10} = \frac{10}{20} \text{ so } \frac{1}{2} = \frac{10}{20}$$

So, we say that $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, and $\frac{10}{20}$ are equivalent fractions.

Example 3.2.14: equivalent fractions

Find three fractions equivalent to $\frac{2}{5}$.

Solution

To find a fraction equivalent to $\frac{2}{5}$, we multiply the numerator and denominator by the same number (but not zero). Let us multiply them by 2, 3, and 5.

$$\frac{2 \cdot 2}{5 \cdot 2} = \frac{4}{10} \qquad \frac{2 \cdot 3}{5 \cdot 3} = \frac{6}{15} \qquad \frac{2 \cdot 5}{5 \cdot 5} = \frac{10}{25}$$

So, $\frac{4}{10}$, $\frac{6}{15}$, and $\frac{10}{25}$ are equivalent to $\frac{2}{5}$.

Exercise 3.2.27

Find three fractions equivalent to $\frac{3}{5}$.

Answer

Correct answers include $\frac{6}{10}$, $\frac{9}{15}$ and $\frac{12}{20}$

Exercise 3.2.28

Find three fractions equivalent to $\frac{4}{5}$.

Answer

Correct answers include $\frac{8}{10}$, $\frac{12}{15}$ and $\frac{16}{20}$

Example 3.2.15: equivalent fractions

Find a fraction with a denominator of 21 that is equivalent to $\frac{2}{7}$.

Solution

To find equivalent fractions, we multiply the numerator and denominator by the same number. In this case, we need to multiply the denominator by a number that will result in 21.

Since we can multiply 7 by 3 to get 21, we can find the equivalent fraction by multiplying both the numerator and denominator by 3.

$$\frac{2}{7} = \frac{2 \cdot 3}{7 \cdot 3} = \frac{6}{21}$$

Exercise 3.2.29

Find a fraction with a denominator of 21 that is equivalent to $\frac{6}{7}$.

Answer

$\frac{18}{21}$

Exercise 3.2.30

Find a fraction with a denominator of 100 that is equivalent to $\frac{3}{10}$.

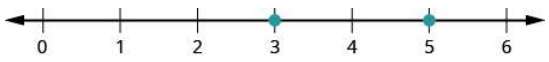
Answer

$\frac{30}{100}$

Locate Fractions and Mixed Numbers on the Number Line

Now we are ready to plot fractions on a number line. This will help us visualize fractions and understand their values.

Let us locate $\frac{1}{5}, \frac{4}{5}, 3, 3\frac{1}{3}, \frac{7}{4}, \frac{9}{2}, 5$, and $\frac{8}{3}$ on the number line. We will start with the whole numbers 3 and 5 because they are the easiest to plot.



The proper fractions listed are $\frac{1}{5}$ and $\frac{4}{5}$. We know proper fractions have values less than one, so $\frac{1}{5}$ and $\frac{1}{5}$ are located between the whole numbers 0 and 1. The denominators are both 5, so we need to divide the segment of the number line between 0 and 1 into five equal parts. We can do this by drawing four equally spaced marks on the number line, which we can then label as $\frac{1}{5}, \frac{2}{5}, \frac{3}{5}$, and $\frac{4}{5}$. Now plot points at $\frac{1}{5}$ and $\frac{4}{5}$.



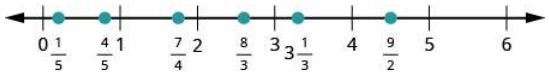
The only mixed number to plot is $3\frac{1}{3}$. Between what two whole numbers is $3\frac{1}{3}$? Remember that a mixed number is a whole number plus a proper fraction, so $3\frac{1}{3} > 3$. Since it is greater than 3, but not a whole unit greater, $3\frac{1}{3}$ is between 3 and 4. We need to divide the portion of the number line between 3 and 4 into three equal pieces (thirds) and plot $3\frac{1}{3}$ at the first mark.



Finally, look at the improper fractions $\frac{7}{4}$, $\frac{9}{2}$, and $\frac{8}{3}$. Locating these points will be easier if you change each of them to a mixed number.

$$\frac{7}{4} = 1\frac{3}{4}, \qquad \frac{9}{2} = 4\frac{1}{2}, \qquad \frac{8}{3} = 2\frac{2}{3}$$

Here is the number line with all the points plotted.

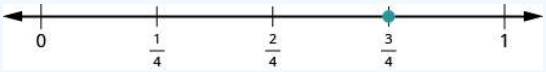


Example 3.2.16: locate and label

Locate and label the following on a number line: $\frac{3}{4}$, $\frac{4}{3}$, $\frac{5}{3}$, $4\frac{1}{5}$, and $\frac{7}{2}$.

Solution

Start by locating the proper fraction $\frac{3}{4}$. It is between 0 and 1. To do this, divide the distance between 0 and 1 into four equal parts. Then plot $\frac{3}{4}$.



Next, locate the mixed number $4\frac{1}{5}$. It is between 4 and 5 on the number line. Divide the number line between 4 and 5 into five equal parts, and then plot $4\frac{1}{5}$ one-fifth of the way between 4 and 5.



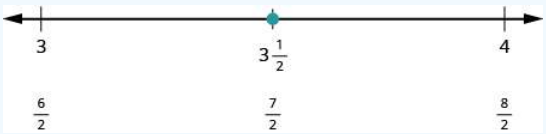
Now locate the improper fractions $\frac{4}{3}$ and $\frac{5}{3}$. It is easier to plot them if we convert them to mixed numbers first.

$$\frac{4}{3} = 1\frac{1}{3}, \qquad \frac{5}{3} = 1\frac{2}{3}$$

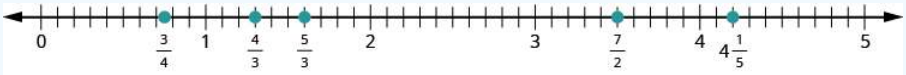
Divide the distance between 1 and 2 into thirds.



Next let us plot $\frac{7}{2}$. We write it as a mixed number, $\frac{7}{2} = 3\frac{1}{2}$. Plot it between 3 and 4.



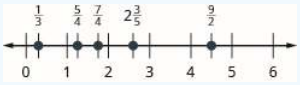
The number line shows all the numbers located on the number line.



Exercise 3.2.31

Locate and label the following on a number line: $\frac{1}{3}, \frac{5}{4}, \frac{7}{4}, 2\frac{3}{5}, \frac{9}{2}$.

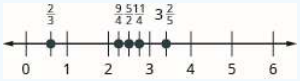
Answer



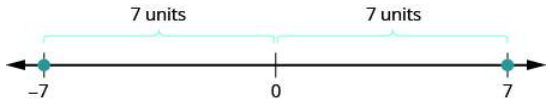
Exercise 3.2.32

Locate and label the following on a number line: $\frac{2}{3}, \frac{5}{2}, \frac{9}{4}, \frac{11}{4}, 3\frac{2}{5}$.

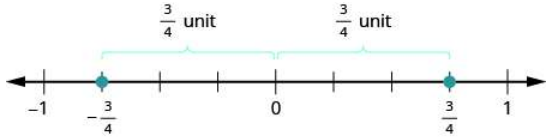
Answer



In Introduction to Integers, we defined the opposite of a number. It is the number that is the same distance from zero on the number line but on the opposite side of zero. We saw, for example, that the opposite of 7 is -7 and the opposite of -7 is 7.



Fractions have opposites, too. The opposite of $\frac{3}{4}$ is $-\frac{3}{4}$. It is the same distance from 0 on the number line, but on the opposite side of 0.



Thinking of negative fractions as the opposite of positive fractions will help us locate them on the number line. To locate $-\frac{15}{8}$ on the number line, first think of where $\frac{15}{8}$ is located. It is an improper fraction, so we first convert it to the mixed number $1\frac{7}{8}$ and see that it will be between 1 and 2 on the number line. So its opposite, $-\frac{15}{8}$, will be between -1 and -2 on the number line.



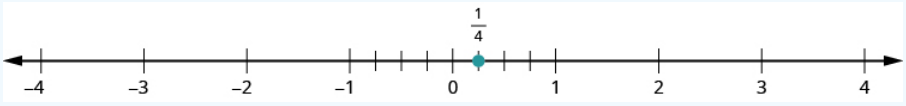
Example 3.2.17: locate and label

Locate and label the following on the number line: $\frac{1}{4}, -\frac{1}{4}, 1\frac{1}{3}, -1\frac{1}{3}, \frac{5}{2}$, and $-\frac{5}{2}$.

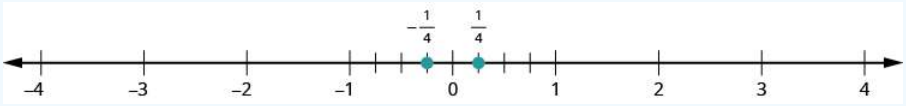
Solution

Draw a number line. Mark 0 in the middle and then mark several units to the left and right.

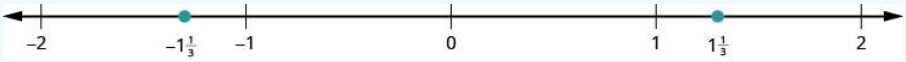
To locate $\frac{1}{4}$, divide the interval between 0 and 1 into four equal parts. Each part represents one-quarter of the distance. So plot $\frac{1}{4}$ at the first mark.



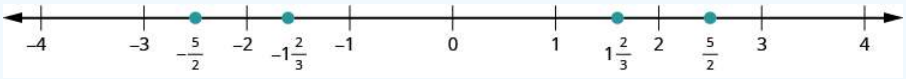
To locate $-\frac{1}{4}$, divide the interval between 0 and -1 into four equal parts. Plot $-\frac{1}{4}$ at the first mark to the left of 0.



Since $1\frac{1}{3}$ is between 1 and 2, divide the interval between 1 and 2 into three equal parts. Plot $1\frac{1}{3}$ at the first mark to the right of 1. Then since $-1\frac{1}{3}$ is the opposite of $1\frac{1}{3}$ it is between -1 and -2. Divide the interval between -1 and -2 into three equal parts. Plot $-1\frac{1}{3}$ at the first mark to the left of -1.



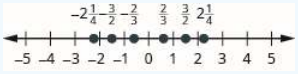
To locate $\frac{5}{2}$ and $-\frac{5}{2}$, it may be helpful to rewrite them as the mixed numbers $2\frac{1}{2}$ and $-2\frac{1}{2}$. Since $2\frac{1}{2}$ is between 2 and 3, divide the interval between 2 and 3 into two equal parts. Plot $\frac{5}{2}$ at the mark. Then since $-2\frac{1}{2}$ is between -2 and -3, divide the interval between -2 and -3 into two equal parts. Plot $-\frac{5}{2}$ at the mark.



Exercise 3.2.33

Locate and label each of the given fractions on a number line: $\frac{2}{3}, -\frac{2}{3}, 2\frac{1}{4}, -2\frac{1}{4}, \frac{3}{2}, -\frac{3}{2}$

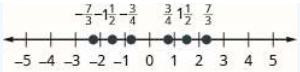
Answer



Exercise 3.2.34

Locate and label each of the given fractions on a number line: $\frac{3}{4}, -\frac{3}{4}, 1\frac{1}{2}, -1\frac{1}{2}, \frac{7}{3}, -\frac{7}{3}$

Answer



Order Fractions and Mixed Numbers

We can use the inequality symbols to order fractions. Remember that $a > b$ means that a is to the right of b on the number line. As we move from left to right on a number line, the values increase.

Example 3.2.18: order

Order each of the following pairs of numbers, using $<$ or $>$:

a. $-\frac{2}{3}\_\_\_\_-1$

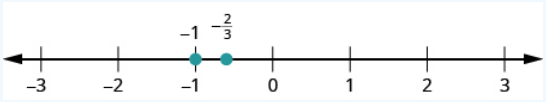
b. $-3\frac{1}{2}\_\_\_\_-3$

c. $-\frac{3}{7}\_\_\_\_-\frac{3}{8}$

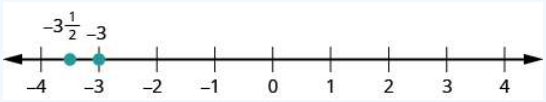
d. $-2\_\_\_\_-\frac{16}{9}$

Solution

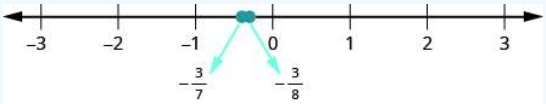
a. $-\frac{2}{3} > -1$



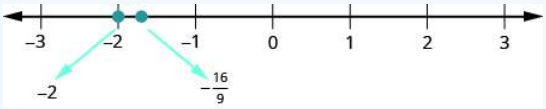
b. $-3\frac{1}{2} < -3$



c. $-\frac{3}{7} < -\frac{3}{8}$



d. $-2 < -\frac{16}{9}$



Exercise 3.2.35

Order each of the following pairs of numbers, using $<$ or $>$:

a. $-\frac{1}{3}\_\_-1$

b. $-1\frac{1}{2}\_\_-2$

c. $-\frac{2}{3}\_\_-\frac{1}{3}$

d. $-3 \_\_ -\frac{7}{3}$

Answer a

$>$

Answer b

$>$

Answer c

$<$

Answer d

$<$

Exercise 3.2.36

Order each of the following pairs of numbers, using $<$ or $>$:

a. $-3 \_\_ -\frac{17}{5}$

b. $-2\frac{1}{4} \_\_ -2$

c. $-\frac{3}{5} \_\_ -\frac{4}{5}$

d. $-4 \_\_ -\frac{10}{3}$

Answer a

$>$

Answer b

$<$

Answer c

$>$

Answer d

$<$

Access Additional Online Resources

- Introduction to Fractions
- Identify Fractions Using Pattern Blocks

Key Concepts

- Property of One
  - Any number, except zero, divided by itself is one.
    $\frac{a}{a} = 1$, where $a \neq 0$.
- Mixed Numbers
  - A mixed number consists of a whole number a and a fraction $\frac{b}{c}$ where $c \neq 0$.
  - It is written as follows: $a\frac{b}{c}$ $c \neq 0$
- Proper and Improper Fractions

- The fraction $\frac{a}{b}$ is a proper fraction if $a < b$ and an improper fraction if $a \geq b$.

- Convert an improper fraction to a mixed number.
  1. Divide the denominator into the numerator.
  2. Identify the quotient, remainder, and divisor.
  3. Write the mixed number as $\frac{\text{remainder}}{\text{divisor}}$.
- Convert a mixed number to an improper fraction.
  1. Multiply the whole number by the denominator.
  2. Add the numerator to the product found in Step 1.
  3. Write the final sum over the original denominator.
- Equivalent Fractions Property
  - If a, b and c are numbers where $b \neq 0$, $c \neq 0$, then \(\dfrac{a}{b} = \dfrac{a\cdot c}{b\cdot c}\]).

Glossary

equivalent fractions

Equivalent fractions are two or more fractions that have the same value.

fraction

A fraction is written $\frac{a}{b}$. in a fraction, a is the numerator and b is the denominator. A fraction represents parts of a whole. The denominator b is the number of equal parts the whole has been divided into, and the numerator a indicates how many parts are included.

mixed number

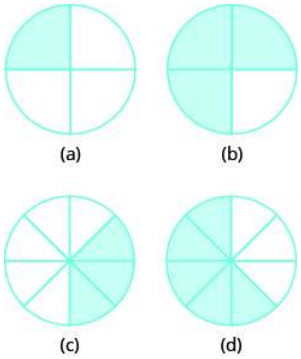
A mixed number consists of a whole number a and a fraction $\frac{b}{c}$ where $c \neq 0$. It is written as $a\frac{b}{c}$, where $c \neq 0$.

proper and improper fractions

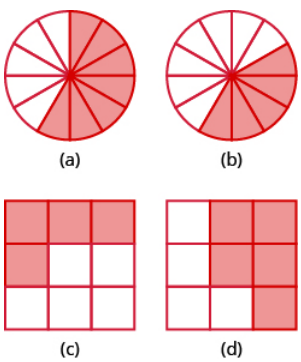
The fraction $\frac{a}{b}$ is proper if $a < b$ and improper if $a > b$.

Practice Makes Perfect

1.



2.



In the following exercises, shade parts of circles or squares to model the following fractions.

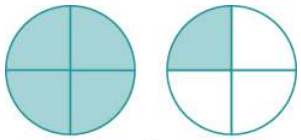
3. $\frac{1}{2}$
4. $\frac{1}{3}$
5. $\frac{3}{4}$
6. $\frac{2}{5}$
7. $\frac{5}{6}$
8. $\frac{7}{8}$
9. $\frac{5}{8}$
10. $\frac{7}{10}$

In the following exercises, use fraction circles to make wholes, if possible, with the following pieces.

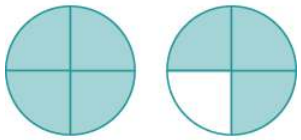
11. 3 thirds
12. 8 eighths
13. 7 sixths
14. 4 thirds
15. 7 fifths
16. 7 fourths

In the following exercises, name the improper fractions. Then write each improper fraction as a mixed number.

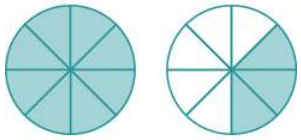
17.



(a)

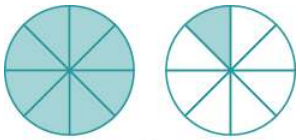


(b)

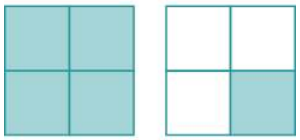


(c)

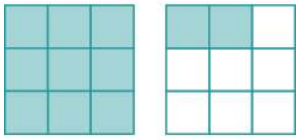
18.



(a)

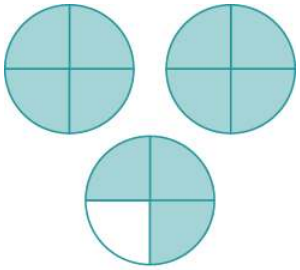


(b)

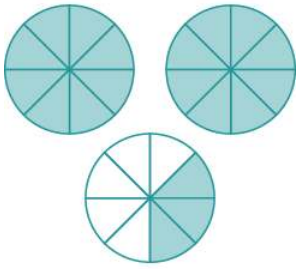


(c)

19.



(a)



(b)

In the following exercises, draw fraction circles to model the given fraction.

20. $\frac{3}{3}$
21. $\frac{4}{4}$
22. $\frac{7}{4}$
23. $\frac{5}{3}$
24. $\frac{11}{6}$
25. $\frac{13}{8}$
26. $\frac{10}{3}$
27. $\frac{9}{4}$

In the following exercises, rewrite the improper fraction as a mixed number.

28. $\frac{3}{2}$
29. $\frac{5}{3}$
30. $\frac{11}{4}$
31. $\frac{13}{5}$
32. $\frac{25}{6}$
33. $\frac{28}{9}$
34. $\frac{42}{13}$
35. $\frac{47}{15}$

In the following exercises, rewrite the mixed number as an improper fraction.

36. $1\frac{2}{3}$
37. $1\frac{2}{5}$
38. $2\frac{1}{4}$
39. $2\frac{5}{6}$
40. $2\frac{7}{9}$
41. $2\frac{5}{7}$
42. $3\frac{4}{7}$
43. $3\frac{5}{9}$

In the following exercises, use fraction tiles or draw a figure to find equivalent fractions.

44. How many sixths equal one-third?
45. How many twelfths equal one-third?
46. How many eighths equal three-fourths?
47. How many twelfths equal three-fourths?
48. How many fourths equal three-halves?
49. How many sixths equal three-halves?

In the following exercises, find three fractions equivalent to the given fraction. Show your work, using figures or algebra.

50. $\frac{1}{4}$
51. $\frac{1}{3}$
52. $\frac{3}{8}$
53. $\frac{5}{6}$
54. $\frac{2}{7}$
55. $\frac{5}{9}$

In the following exercises, plot the numbers on a number line.

56. $\frac{2}{3}, \frac{5}{4}, \frac{12}{5}$

57. $\frac{1}{3}, \frac{7}{4}, \frac{13}{5}$
58. $\frac{1}{4}, \frac{9}{5}, \frac{11}{3}$
59. $\frac{7}{10}, \frac{5}{2}, \frac{13}{8}, 3$
60. $2\frac{1}{3}, -2\frac{1}{3}$
61. $1\frac{3}{4}, -1\frac{3}{5}$
62. $\frac{3}{4}, -\frac{3}{4}, 1\frac{2}{3}, -1\frac{2}{3}, \frac{5}{2}, -\frac{5}{2}$
63. $\frac{2}{5}, -\frac{2}{5}, 1\frac{3}{4}, -1\frac{3}{4}, \frac{8}{3}, -\frac{8}{3}$

In the following exercises, order each of the following pairs of numbers, using < or >.

64. $-1 \_\_ -\frac{1}{4}$
65. $-1 \_\_ -\frac{1}{3}$
66. $-2\frac{1}{2} \_\_ -3$
67. $-1\frac{3}{4} \_\_ -2$
68. $-\frac{5}{12} \_\_ -\frac{7}{12}$
69. $-\frac{9}{10} \_\_ -\frac{3}{10}$
70. $-3 \_\_ -\frac{13}{5}$
71. $-4 \_\_ -\frac{23}{6}$

Everyday Math

72. **Music Measures** A choreographed dance is broken into counts. A $\frac{1}{1}$ count has one step in a count, a $\frac{1}{2}$ count has two steps in a count and a 1 3 count has three steps in a count. How many steps would be in a $\frac{1}{5}$ count? What type of count has four steps in it?
73. **Music Measures** Fractions are used often in music. In 4 4 time, there are four quarter notes in one measure.
    a. How many measures would eight quarter notes make?
    b. The song “Happy Birthday to You” has 25 quarter notes. How many measures are there in “Happy Birthday to You?”
74. **Baking** Nina is making five pans of fudge to serve after a music recital. For each pan, she needs 1 2 cup of walnuts.
    a. How many cups of walnuts does she need for five pans of fudge?
    b. Do you think it is easier to measure this amount when you use an improper fraction or a mixed number? Why?

Writing Exercises

75. Give an example from your life experience (outside of school) where it was important to understand fractions.
76. Explain how you locate the improper fraction $\frac{21}{4}$ on a number line on which only the whole numbers from 0 through 10 are marked.

Self Check

(a) After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
understand the meaning of fractions.			
model improper fractions and mixed numbers.			
convert between improper fractions and mixed numbers.			
model equivalent fractions.			
find equivalent fractions.			
locate fractions and mixed numbers on the number line.			
order fractions and mixed numbers.			

(b) If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them. What did you do to become confident of your ability to do these things? Be specific.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is important to make sure you have a strong foundation before you move on. Who can you ask for help? Your fellow classmates and instructor are good resources. Is there a place on campus where math tutors are available? Can your study skills be improved?

…no—I don't get it! This is a warning sign and you must not ignore it. You should get help right away or you will quickly be overwhelmed. See your instructor as soon as you can to discuss your situation. Together you can come up with a plan to get you the help you need.

Contributors and Attributions

- Lynn Marecek (Santa Ana College) and MaryAnne Anthony-Smith (Formerly of Santa Ana College). This content is licensed under Creative Commons Attribution License v4.0 "Download for free at http://cnx.org/contents/fd53eae1-fa2...49835c3c@5.191."

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3.3: Multiply Fractions

Learning Objectives

- Simplify fractions
- Multiply fractions
- Find reciprocals
- Divide fractions

be prepared!

Before you get started, take this readiness quiz.

1. Find the prime factorization of 48. If you missed this problem, review Example 2.5.1.
2. Draw a model of the fraction $\frac{3}{4}$. If you missed this problem, review Example 4.1.2.
3. Find two fractions equivalent to $\frac{5}{6}$. Answers may vary. Acceptable answers include $\frac{10}{12}$, $\frac{15}{18}$, $\frac{50}{60}$, etc. If you missed this problem, review Example 4.1.14.

Simplify Fractions

In working with equivalent fractions, you saw that there are many ways to write fractions that have the same value, or represent the same part of the whole. How do you know which one to use? Often, we'll use the fraction that is in *simplified* form.

A fraction is considered simplified if there are no common factors, other than 1, in the numerator and denominator. If a fraction does have common factors in the numerator and denominator, we can reduce the fraction to its simplified form by removing the common factors.

Definition: Simplified Fraction

A fraction is considered simplified if there are no common factors in the numerator and denominator.

For example,

- $\frac{2}{3}$ is simplified because there are no common factors of 2 and 3.
- $\frac{10}{15}$ is not simplified because 5 is a common factor of 10 and 15.

The process of simplifying a fraction is often called *reducing the fraction*. In the previous section, we used the Equivalent Fractions Property to find equivalent fractions. We can also use the Equivalent Fractions Property in reverse to simplify fractions. We rewrite the property to show both forms together.

Definition: Equivalent Fractions Property

If a, b, c are numbers where $b \neq 0, c \neq 0$, then $\frac{a}{b} = \frac{a \cdot c}{b \cdot c}$ and $\frac{a \cdot c}{b \cdot c} = \frac{a}{b}$.

Notice that c is a common factor in the numerator and denominator. Anytime we have a common factor in the numerator and denominator, it can be removed.

HOW TO: SIMPLIFY A FRACTION

Step 1. Rewrite the numerator and denominator to show the common factors. If needed, factor the numerator and denominator into prime numbers.

Step 2. Simplify, using the equivalent fractions property, by removing common factors.

Step 3. Multiply any remaining factors.

Example 3.3.1: simplify

Simplify: $\frac{10}{15}$.

Solution

To simplify the fraction, we look for any common factors in the numerator and the denominator.

Notice that 5 is a factor of both 10 and 15.	$\frac{10}{15}$
Factor the numerator and denominator.	$\frac{2 \cdot 5}{3 \cdot 5}$
Remove the common factors.	$\frac{2 \cdot \cancel{5}}{3 \cdot \cancel{5}}$
Simplify.	$\frac{2}{3}$

Exercise 3.3.1

Simplify: $\frac{8}{12}$.

Answer

$\frac{2}{3}$

Exercise 3.3.2

Simplify: $\frac{12}{16}$.

Answer

$\frac{3}{4}$

To simplify a negative fraction, we use the same process as in Example 3.3.1. Remember to keep the negative sign.

Example 3.3.2: simplify

Simplify: $-\frac{18}{24}$.

Solution

We notice that 18 and 24 both have factors of 6.	$-\frac{18}{24}$
Rewrite the numerator and denominator showing the common factor.	$-\frac{3 \cdot 6}{4 \cdot 6}$
Remove common factors.	$-\frac{3 \cdot \cancel{6}}{4 \cdot \cancel{6}}$
Simplify.	$-\frac{3}{4}$

Exercise 3.3.3

Simplify: $-\frac{21}{28}$.

Answer

$-\frac{3}{4}$

Exercise 3.3.4

Simplify: $-\frac{16}{24}$.

Answer

$-\frac{2}{3}$

After simplifying a fraction, it is always important to check the result to make sure that the numerator and denominator do not have any more factors in common. Remember, the definition of a simplified fraction: *a fraction is considered simplified if there are no common factors in the numerator and denominator*.

When we simplify an improper fraction, there is no need to change it to a mixed number.

Example 3.3.3:

Simplify: $-\frac{56}{32}$.

Solution

	$-\frac{56}{32}$
Rewrite the numerator and denominator, showing the common factors, 8.	$-\frac{7\cdot 8}{4\cdot 8}$
Remove common factors.	$-\frac{7\cdot \cancel{8}}{4\cdot \cancel{8}}$
Simplify.	$-\frac{7}{4}$

Exercise 3.3.5

Simplify: $-\frac{54}{42}$.

Answer

$-\frac{9}{7}$

Exercise 3.3.6

Simplify: $-\frac{81}{45}$.

Answer

$-\frac{9}{5}$

HOW TO: SIMPLIFY A FRACTION

Step 1. Rewrite the numerator and denominator to show the common factors. If needed, factor the numerator and denominator into prime numbers.

Step 2. Simplify, using the equivalent fractions property, by removing common factors.

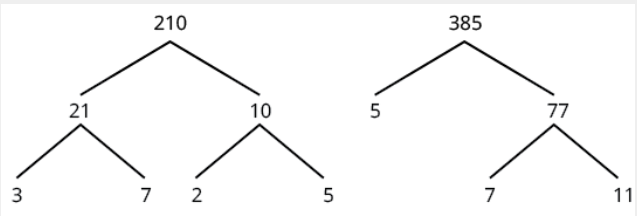
Step 3. Multiply any remaining factors.

Sometimes it may not be easy to find common factors of the numerator and denominator. A good idea, then, is to factor the numerator and the denominator into prime numbers. (You may want to use the factor tree method to identify the prime factors.) Then divide out the common factors using the Equivalent Fractions Property.

Example 3.3.4: simplify

Simplify: $\frac{210}{385}$.

Solution

	$\frac{210}{385}$
Use factor trees to factor the numerator and denominator.	
Rewrite the numerator and denominator as the product of the primes.	$\frac{210}{385} = \frac{2 \cdot 3 \cdot 5 \cdot 7}{5 \cdot 7 \cdot 11}$
Remove the common factors.	$\frac{2 \cdot 3 \cdot \cancel{5} \cdot \cancel{7}}{\cancel{5} \cdot \cancel{7} \cdot 11}$
Simplify.	$\frac{2 \cdot 3}{11}$
Multiply any remaining factors.	$\frac{6}{11}$

Exercise 3.3.7

Simplify: $\frac{69}{120}$.

Answer

$\frac{23}{40}$

Exercise 3.3.8

Simplify: $\frac{120}{192}$.

Answer

$\frac{5}{8}$

We can also simplify fractions containing variables. If a variable is a common factor in the numerator and denominator, we remove it just as we do with an integer factor.

Example 3.3.5: simplify

Simplify: $\frac{5xy}{15x}$.

Solution

	$\frac{5xy}{15x}$
Rewrite numerator and denominator showing common factors.	$\frac{5 \cdot x \cdot y}{3 \cdot 5 \cdot x}$
Remove common factors.	$\frac{\cancel{5} \cdot \cancel{x} \cdot y}{3 \cdot \cancel{5} \cdot \cancel{x}}$
Simplify.	$\frac{y}{3}$

Exercise 3.3.9

Simplify: $\frac{7x}{7y}$.

Answer

$\frac{x}{y}$

Exercise 3.3.10

Simplify: $\frac{9a}{9b}$.

Answer

$\frac{a}{b}$

Multiply Fractions

A model may help you understand multiplication of fractions. We will use fraction tiles to model $\frac{1}{2} \cdot \frac{3}{4}$. To multiply $\frac{1}{2}$ and $\frac{3}{4}$, think $\frac{1}{2}$ of $\frac{3}{4}$.

Start with fraction tiles for three-fourths. To find one-half of three-fourths, we need to divide them into two equal groups. Since we cannot divide the three $\frac{1}{4}$ tiles evenly into two parts, we exchange them for smaller tiles.

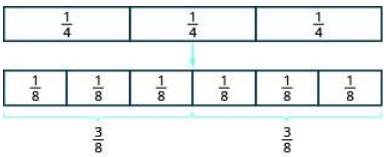


Figure 3.3.1

We see $\frac{6}{8}$ is equivalent to $\frac{3}{4}$. Taking half of the six $\frac{1}{8}$ tiles gives us three $\frac{1}{8}$ tiles, which is $\frac{3}{8}$. Therefore,

$$\frac{1}{2} \cdot \frac{3}{4} = \frac{3}{8}$$

Example 3.3.6: model a fraction

Use a diagram to model $\frac{1}{2} \cdot \frac{3}{4}$.

Solution

First shade in $\frac{3}{4}$ of the rectangle.

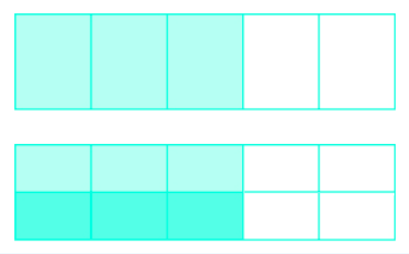
We will take $\frac{1}{2}$ of this $\frac{3}{4}$, so we heavily shade $\frac{1}{2}$ of the shaded region.

Notice that 3 out of the 8 pieces are heavily shaded. This means that $\frac{3}{8}$ of the rectangle is heavily shaded. Therefore, $\frac{1}{2}$ of $\frac{3}{4}$ is $\frac{3}{4}$, or $\frac{1}{2} \cdot \frac{3}{4} = \frac{3}{8}$.

Exercise 3.3.11

Use a diagram to model: $\frac{1}{2} \cdot \frac{3}{5}$.

Answer

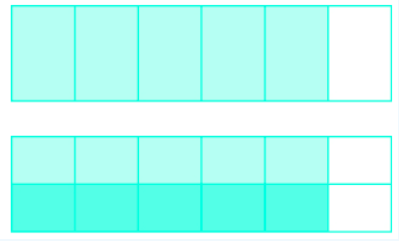


$\frac{3}{10}$

Exercise 3.3.12

Use a diagram to model: $\frac{1}{2} \cdot \frac{5}{6}$.

Answer



$\frac{5}{12}$

Look at the result we got from the model in Example 3.3.6. We found that $\frac{1}{2} \cdot \frac{3}{4} = \frac{3}{8}$. Do you notice that we could have gotten the same answer by multiplying the numerators and multiplying the denominators?

	$\frac{1}{2} \cdot \frac{3}{4}$
Multiply the numerators, and multiply the denominators.	$\frac{1}{2} \cdot \frac{3}{4}$
Simplify.	$\frac{3}{8}$

This leads to the definition of fraction multiplication. To multiply fractions, we multiply the numerators and multiply the denominators. Then we write the fraction in simplified form.

Definition: Fraction Multiplication

If a, b, c, and d are numbers where $b \neq 0$ and $d \neq 0$, then

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd} \tag{3.3.1}$$

Example 3.3.7: multiply

Multiply, and write the answer in simplified form: $\frac{3}{4} \cdot \frac{1}{5}$.

Solution

	$\frac{3}{4} \cdot \frac{1}{5}$
Multiply the numerators, and multiply the denominators.	$\frac{3 \cdot 1}{4 \cdot 5}$
Simplify.	$\frac{3}{20}$

There are no common factors, so the fraction is simplified.

Exercise 3.3.13

Multiply, and write the answer in simplified form: $\frac{1}{3} \cdot \frac{2}{5}$.

Answer

$\frac{2}{15}$

Exercise 3.3.14

Multiply, and write the answer in simplified form: $\frac{3}{5} \cdot \frac{7}{8}$.

Answer

$\frac{21}{40}$

When multiplying fractions, the properties of positive and negative numbers still apply. It is a good idea to determine the sign of the product as the first step. In Example 3.3.8 we will multiply two negatives, so the product will be positive.

Example 3.3.8: multiply

Multiply, and write the answer in simplified form: $-\frac{5}{8}\left(-\frac{2}{3}\right)$.

Solution

	$-\frac{5}{8}\left(-\frac{2}{3}\right)$
The signs are the same, so the product is positive. Multiply the numerators, multiply the denominators.	$\frac{5 \cdot 2}{8 \cdot 3}$
Simplify.	$\frac{10}{24}$
Look for common factors in the numerator and denominator. Rewrite showing common factors.	$\frac{5 \cdot \cancel{2}}{12 \cdot \cancel{2}}$
Remove common factors.	$\frac{5}{12}$

Another way to find this product involves removing common factors earlier.

	$-\frac{5}{8}\left(-\frac{2}{3}\right)$
Determine the sign of the product. Multiply.	$\frac{5 \cdot 2}{8 \cdot 3}$
Show common factors and then remove them.	$\frac{5 \cdot \cancel{2}}{12 \cdot \cancel{2}}$
Multiply remaining factors.	$\frac{5}{12}$

We get the same result.

Exercise 3.3.15

Multiply, and write the answer in simplified form: $-\frac{4}{7}\left(-\frac{5}{8}\right)$.

Answer

$\frac{5}{14}$

Exercise 3.3.16

Multiply, and write the answer in simplified form: $-\frac{7}{12}\left(-\frac{8}{9}\right)$.

Answer

$\frac{14}{27}$

Example 3.3.9: multiply

Multiply, and write the answer in simplified form: $-\frac{14}{15}\cdot\frac{20}{21}$.

Solution

	$-\frac{14}{15}\cdot\frac{20}{21}$
Determine the sign of the product; multiply.	$-\frac{14}{15}\cdot\frac{20}{21}$
Are there any common factors in the numerator and the denominator? We know that 7 is a factor of 14 and 21, and 5 is a factor of 20 and 15.	
Rewrite showing common factors.	$-\frac{2\cdot\cancel{7}\cdot 4\cdot\cancel{5}}{3\cdot\cancel{5}\cdot 3\cdot\cancel{7}}$
Remove the common factors.	$-\frac{2\cdot 4}{3\cdot 3}$
Multiply the remaining factors.	$-\frac{8}{9}$

Exercise 3.3.17

Multiply, and write the answer in simplified form: $-\frac{10}{28}\cdot\frac{8}{15}$.

Answer

$-\frac{4}{21}$

Exercise 3.3.18

Multiply, and write the answer in simplified form: $-\frac{9}{20}\cdot\frac{5}{12}$.

Answer

$-\frac{3}{16}$

When multiplying a fraction by an integer, it may be helpful to write the integer as a fraction. Any integer, a, can be written as $\frac{a}{1}$. So, $3=\frac{3}{1}$, for example.

Example 3.3.10:

Multiply, and write the answer in simplified form:

a. $\frac{1}{7} \cdot 56$

b. $\frac{12}{5}(-20x)$

Solution

a.

	$\frac{1}{7} \cdot 56$
Write 56 as a fraction.	$\frac{1}{7} \cdot \frac{56}{1}$
Determine the sign of the product; multiply.	$\frac{56}{7}$
Simplify.	8

b.

	$\frac{12}{5}(-20x)$
Write −20x as a fraction.	$\frac{12}{5}\left(\frac{-20x}{1}\right)$
Determine the sign of the product; multiply.	$-\frac{12 \cdot 20 \cdot x}{5 \cdot 1}$
Show common factors and then remove them.	$-\frac{12 \cdot 4 \cdot \cancel{5}x}{\cancel{5} \cdot 1}$
Multiply remaining factors; simplify.	$-48x$

Exercise 3.3.19

Multiply, and write the answer in simplified form:

a. $\frac{1}{8} \bullet 72$

b. $\frac{11}{3}(-9a)$

Answer a

9

Answer b

$-33a$

Exercise 3.3.20

Multiply, and write the answer in simplified form:

a. $\frac{3}{8} \bullet 64$

b. $16x \bullet \frac{11}{12}$

Answer a

24

Answer b

$\frac{44x}{3}$

Contributors and Attributions

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3.4: Add and Subtract Fractions with Common Denominators

Learning Objectives

- Model fraction addition
- Add fractions with a common denominator
- Model fraction subtraction
- Subtract fractions with a common denominator

be prepared!

Before you get started, take this readiness quiz.

1. Simplify: $2x+9+3x-4$. If you missed this problem, review Example 2.2.10.
2. Draw a model of the fraction $\frac{3}{4}$. If you missed this problem, review Example 4.1.2.
3. Simplify: $\frac{3+2}{6}$. If you missed this problem, review Example 4.3.12.

Model Fraction Addition

How many quarters are pictured? One quarter plus 2 quarters equals 3 quarters.

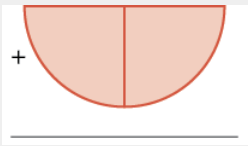
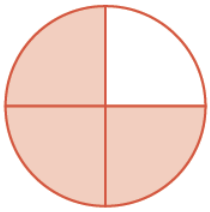


Figure 3.4.1

Remember, quarters are really fractions of a dollar. Quarters are another way to say fourths. So the picture of the coins shows that

$$\begin{matrix} \frac{1}{4} & \frac{2}{4} & \frac{3}{4} \\ one\ quarter + two\ quarters & & = three\ quarters \end{matrix}$$

Let's use fraction circles to model the same example, $\frac{1}{4}+\frac{2}{4}$.

Start with one $\frac{1}{4}$ piece.		$\frac{1}{4}$
Add two more $\frac{1}{4}$ pieces.		$+\frac{2}{4}$
The result is $\frac{3}{4}$.		$\frac{3}{4}$

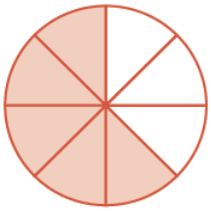
So again, we see that

$$\frac{1}{4}+\frac{2}{4}=\frac{3}{4}$$

Example 3.4.1: addition

Use a model to find the sum $\frac{3}{8}+\frac{2}{8}$.

Solution

Start with three $\frac{1}{8}$ pieces.		$\frac{3}{8}$
Add two $\frac{1}{8}$ pieces.		$+\frac{2}{8}$
How many $\frac{1}{8}$ pieces are there?		$\frac{5}{8}$

There are five $\frac{1}{8}$ pieces, or five-eighths. The model shows that $\frac{3}{8}+\frac{2}{8}=\frac{5}{8}$.

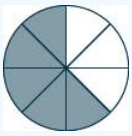
Exercise 3.4.1

Use a model to find each sum. Show a diagram to illustrate your model.

$$\frac{1}{8}+\frac{4}{8}$$

Answer

$\frac{5}{8}$



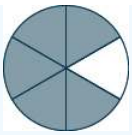
Exercise 3.4.2

Use a model to find each sum. Show a diagram to illustrate your model.

$$\frac{1}{6}+\frac{4}{6}$$

Answer

$\frac{5}{6}$



Add Fractions with a Common Denominator

Example 3.4.1 shows that to add the same-size pieces—meaning that the fractions have the same denominator—we just add the number of pieces.

Definition: Fraction Addition

If a, b, and c are numbers where $c \neq 0$, then

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c} \tag{3.4.1}$$

To add fractions with a common denominator, add the numerators and place the sum over the common denominator.

Example 3.4.2: addition

Find the sum: $\frac{3}{5} + \frac{1}{5}$.

Solution

Add the numerators and place the sum over the common denominator.	$\frac{3+1}{5}$
Simplify.	$\frac{4}{5}$

Exercise 3.4.3

Find each sum: $\frac{3}{6} + \frac{2}{6}$.

Answer

$\frac{5}{6}$

Exercise 3.4.4

Find each sum: $\frac{3}{10} + \frac{7}{10}$.

Answer

1

Example 3.4.3: addition

Find the sum: $\frac{x}{3} + \frac{2}{3}$.

Solution

Add the numerators and place the sum over the common denominator.	$\frac{x+2}{3}$

Note that we cannot simplify this fraction any more. Since x and 2 are not like terms, we cannot combine them.

Exercise 3.4.5

Find the sum: $\frac{x}{4}+\frac{3}{4}$.

Answer

$\frac{x+3}{4}$

Exercise 3.4.6

Find the sum: $\frac{y}{8}+\frac{5}{8}$.

Answer

$\frac{y+5}{8}$

Example 3.4.4: addition

Find the sum: $-\frac{9}{d}+\frac{3}{d}$.

Solution

We will begin by rewriting the first fraction with the negative sign in the numerator.

$$-\frac{a}{b}=\frac{-a}{b}$$

Rewrite the first fraction with the negative in the numerator.	$\frac{-9}{d}+\frac{3}{d}$
Add the numerators and place the sum over the common denominator.	$\frac{-9+3}{d}$
Simplify the numerator.	$\frac{-6}{d}$
Rewrite with negative sign in front of the fraction.	$-\frac{6}{d}$

Exercise 3.4.7

Find the sum: $-\frac{7}{d}+\frac{8}{d}$.

Answer

$\frac{1}{d}$

Exercise 3.4.8

Find the sum: $-\frac{6}{m}+\frac{9}{m}$.

Answer

$\frac{3}{m}$

Example 3.4.5: addition

Find the sum: $\frac{2n}{11}+\frac{5n}{11}$.

Solution

Add the numerators and place the sum over the common denominator.	$\frac{2n+5n}{11}$
Combine like terms.	$\frac{7n}{11}$

Exercise 3.4.9

Find the sum: $\frac{3p}{8}+\frac{6p}{8}$.

Answer

$\frac{9p}{8}$

Exercise 3.4.10

Find the sum: $\frac{2q}{5}+\frac{7q}{5}$.

Answer

$\frac{9q}{5}$

Example 3.4.6: addition

Find the sum: $-\frac{3}{12}+\left(-\frac{5}{12}\right)$.

Solution

Add the numerators and place the sum over the common denominator.	$\frac{-3+(-5)}{12}$
Add.	$\frac{-8}{12}$
Simplify the fraction.	$-\frac{2}{3}$

Exercise 3.4.11

Find each sum: $-\frac{4}{15}+\left(-\frac{6}{15}\right)$.

Answer

$-\frac{2}{3}$

Exercise 3.4.12

Find each sum: $-\frac{5}{21}+\left(-\frac{9}{21}\right)$.

Answer

$-\frac{2}{3}$

Model Fraction Subtraction

Subtracting two fractions with common denominators is much like adding fractions. Think of a pizza that was cut into 12 slices. Suppose five pieces are eaten for dinner. This means that, after dinner, there are seven pieces (or $\frac{7}{12}$ of the pizza) left in the box. If Leonardo eats 2 of these remaining pieces (or $\frac{2}{12}$ of the pizza), how much is left? There would be 5 pieces left (or $\frac{5}{12}$ of the pizza).

$$\frac{7}{12}-\frac{2}{12}=\frac{5}{12}$$

Let's use fraction circles to model the same example, $\frac{7}{12}-\frac{2}{12}$. Start with seven $\frac{1}{12}$ pieces. Take away two $\frac{1}{12}$ pieces. How many twelfths are left?

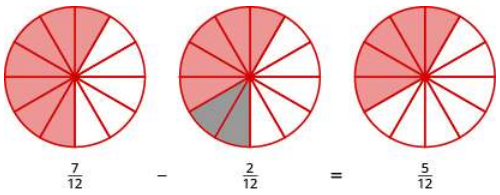


Figure 3.4.2

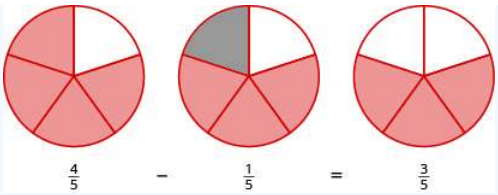
Again, we have five twelfths, $\frac{5}{12}$.

Example 3.4.7: difference

Use fraction circles to find the difference: $\frac{4}{5}-\frac{1}{5}$.

Solution

Start with four $\frac{1}{5}$ pieces. Take away one $\frac{1}{5}$ piece. Count how many fifths are left. There are three $\frac{1}{5}$ pieces left.



Exercise 3.4.13

Use a model to find each difference. Show a diagram to illustrate your model. $\frac{7}{8}-\frac{4}{8}$

Answer

$\frac{3}{8}$, models may differ.

Exercise 3.4.14

Use a model to find each difference. Show a diagram to illustrate your model. $\frac{5}{6}-\frac{4}{6}$

Answer

$\frac{1}{6}$, models may differ.

Subtract Fractions with a Common Denominator

We subtract fractions with a common denominator in much the same way as we add fractions with a common denominator.

Definition: Fraction Subtraction

If a, b, and c are numbers where $c \neq 0$, then

$$\frac{a}{c}-\frac{b}{c}=\frac{a-b}{c} \tag{3.4.2}$$

To subtract fractions with a common denominator, we subtract the numerators and place the difference over the common denominator.

Example 3.4.8: difference

Find the difference: $\frac{23}{24}-\frac{14}{24}$.

Solution

Subtract the numerators and place the difference over the common denominator.	$\frac{23-14}{24}$
Simplify the numerator.	$\frac{9}{24}$
Simplify the fraction by removing common factors.	$\frac{3}{8}$

Exercise 3.4.15

Find the difference: $\frac{19}{28}-\frac{7}{28}$.

Answer

$\frac{3}{7}$

Exercise 3.4.16

Find the difference: $\frac{27}{32}-\frac{11}{32}$.

Answer

$\frac{1}{2}$

Example 3.4.9: difference

Find the difference: $\frac{y}{6}-\frac{1}{6}$.

Solution

Subtract the numerators and place the difference over the common denominator.	$\frac{y-1}{6}$

The fraction is simplified because we cannot combine the terms in the numerator.

Exercise 3.4.17

Find the difference: $\frac{x}{7}-\frac{2}{7}$.

Answer

$\frac{x-2}{7}$

Exercise 3.4.18

Find the difference: $\frac{y}{14}-\frac{13}{14}$.

Answer

$\frac{y-13}{14}$

Example 3.4.10: difference

Find the difference: $-\frac{10}{x}-\frac{4}{x}$.

Solution

Remember, the fraction $-\frac{10}{x}$ can be written as $\frac{-10}{x}$.

Subtract the numerators.	$\frac{-10-4}{x}$
Simplify.	$\frac{-14}{x}$
Rewrite with the negative sign in front of the fraction.	$-\frac{14}{x}$

Exercise 3.4.19

Find the difference: $-\frac{9}{x}-\frac{7}{x}$.

Answer

$-\frac{16}{x}$

Exercise 3.4.20

Find the difference: $-\frac{17}{a}-\frac{5}{a}$.

Answer

$-\frac{22}{a}$

Now lets do an example that involves both addition and subtraction.

Example 3.4.11: simplify

Simplify: $\frac{3}{8}+\left(-\frac{5}{8}\right)-\frac{1}{8}$.

Solution

Combine the numerators over the common denominator.	$\frac{3+(-5)-1}{8}$
Simplify the numerator, working left to right.	$\frac{-2-1}{8}$
Subtract the terms in the numerator.	$\frac{-3}{8}$
Rewrite with the negative sign in front of the fraction.	$-\frac{3}{8}$

Exercise 3.4.21

Simplify: $\frac{2}{5}+\left(-\frac{4}{5}\right)-\frac{3}{5}$.

Answer

-1

Exercise 3.4.22

Simplify: $\frac{5}{9}+\left(-\frac{4}{9}\right)-\frac{7}{9}$.

Answer

$-\frac{2}{3}$

Access Additional Online Resources

- Adding Fractions With Pattern Blocks
- Adding Fractions With Like Denominators
- Subtracting Fractions With Like Denominators

Key Concepts

- Fraction Addition
  - If $a, b,$, and c are numbers where $c \neq 0$, then $\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c}$
  - To add fractions, add the numerators and place the sum over the common denominator.
- Fraction Subtraction
  - If $a, b,$, and c are numbers where $c \neq 0$, then $\frac{a}{c}-\frac{b}{c}=\frac{a-b}{c}$
  - To subtract fractions, subtract the numerators and place the difference over the common denominator.

Practice Makes Perfect

Model Fraction Addition

In the following exercises, use a model to add the fractions. Show a diagram to illustrate your model.

254. $\frac{2}{5}+\frac{1}{5}$
255. $\frac{3}{10}+\frac{4}{10}$
256. $\frac{1}{6}+\frac{3}{6}$
257. $\frac{3}{8}+\frac{3}{8}$

Add Fractions with a Common Denominator

In the following exercises, find each sum.

258. $\frac{4}{9}+\frac{1}{9}$
259. $\frac{2}{9}+\frac{5}{9}$
260. $\frac{6}{13}+\frac{7}{13}$
261. $\frac{9}{15}+\frac{7}{15}$
262. $\frac{x}{4}+\frac{3}{4}$
263. $\frac{y}{3}+\frac{2}{3}$
264. $\frac{7}{p}+\frac{9}{p}$

265. $\frac{8}{q}+\frac{6}{q}$
266. $\frac{8b}{9}+\frac{3b}{9}$
267. $\frac{5a}{7}+\frac{4a}{7}$
268. $\frac{-12y}{8}+\frac{3y}{8}$
269. $\frac{-11x}{5}+\frac{7x}{5}$
270. $-\frac{1}{8}+\left(-\frac{3}{8}\right)$
271. $-\frac{1}{8}+\left(-\frac{5}{8}\right)$
272. $-\frac{3}{16}+\left(-\frac{7}{16}\right)$
273. $-\frac{5}{16}+\left(-\frac{9}{16}\right)$
274. $-\frac{8}{17}+\frac{15}{17}$
275. $-\frac{9}{19}+\frac{17}{19}$
276. $-\frac{6}{13}+\left(-\frac{10}{13}\right)+\left(-\frac{12}{13}\right)$
277. $-\frac{5}{12}+\left(-\frac{7}{12}\right)+\left(-\frac{11}{12}\right)$

Model Fraction Subtraction

In the following exercises, use a model to subtract the fractions. Show a diagram to illustrate your model.

278. $\frac{5}{8}-\frac{2}{8}$
279. $\frac{5}{6}-\frac{2}{6}$

Subtract Fractions with a Common Denominator

In the following exercises, find the difference.

280. $\frac{4}{5}-\frac{1}{5}$
281. $\frac{4}{5}-\frac{3}{5}$
282. $\frac{11}{15}-\frac{7}{15}$
283. $\frac{9}{13}-\frac{4}{13}$
284. $\frac{11}{12}-\frac{5}{12}$
285. $\frac{7}{12}-\frac{5}{12}$
286. $\frac{4}{21}-\frac{19}{21}$
287. $-\frac{8}{9}-\frac{16}{9}$
288. $\frac{y}{17}-\frac{9}{17}$
289. $\frac{x}{19}-\frac{8}{19}$
290. $\frac{5y}{8}-\frac{7}{8}$

291. $\frac{11z}{13} - \frac{8}{13}$
292. $-\frac{8}{d} - \frac{3}{d}$
293. $-\frac{7}{c} - \frac{7}{c}$
294. $-\frac{23}{u} - \frac{15}{u}$
295. $-\frac{29}{v} - \frac{26}{v}$
296. $-\frac{6c}{7} - \frac{5c}{7}$
297. $-\frac{12d}{11} - \frac{9d}{11}$
298. $\frac{-4r}{13} - \frac{5r}{13}$
299. $\frac{-7s}{3} - \frac{7s}{3}$
300. $-\frac{3}{5} - \left(-\frac{4}{5}\right)$
301. $-\frac{3}{7} - \left(-\frac{5}{7}\right)$
302. $-\frac{7}{9} - \left(-\frac{5}{9}\right)$
303. $-\frac{8}{11} - \left(-\frac{5}{11}\right)$

Mixed Practice

In the following exercises, perform the indicated operation and write your answers in simplified form.

304. $-\frac{5}{18} \cdot \frac{9}{10}$
305. $-\frac{3}{14} \cdot \frac{7}{12}$
306. $\frac{n}{5} - \frac{4}{5}$
307. $\frac{6}{11} - \frac{s}{11}$
308. $-\frac{7}{24} - \frac{2}{24}$
309. $-\frac{5}{18} - \frac{1}{18}$
310. $\frac{8}{15} \div \frac{12}{5}$
311. $\frac{7}{12} \div \frac{9}{28}$

Everyday Math

312. **Trail Mix** Jacob is mixing together nuts and raisins to make trail mix. He has $\frac{6}{10}$ of a pound of nuts and $\frac{3}{10}$ of a pound of raisins. How much trail mix can he make?
313. **Baking** Janet needs $\frac{5}{8}$ of a cup of flour for a recipe she is making. She only has $\frac{3}{8}$ of a cup of flour and will ask to borrow the rest from her next-door neighbor. How much flour does she have to borrow?

Writing Exercises

314. Greg dropped his case of drill bits and three of the bits fell out. The case has slots for the drill bits, and the slots are arranged in order from smallest to largest. Greg needs to put the bits that fell out back in the case in the empty slots. Where do the three bits go? Explain how you know.

Bits in case: $\frac{1}{16}, \frac{1}{8}, \_\_\_, \_\_\_, \frac{5}{16}, \frac{3}{8}, \_\_\_, \frac{1}{2}, \frac{9}{16}, \frac{5}{8}$.

Bits that fell out: $\frac{7}{16}, \frac{3}{16}, \frac{1}{4}$.

315. After a party, Lupe has $\frac{5}{12}$ of a cheese pizza, $\frac{4}{12}$ of a pepperoni pizza, and $\frac{4}{12}$ of a veggie pizza left. Will all the slices fit into 1 pizza box? Explain your reasoning.

Self Check

(a) After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
model fraction addition.			
add fractions with a common denominator.			
model fraction subtraction.			
subtract fractions with a common denominator.			
find the least common denominator (LCD).			
convert fractions to equivalent fractions with the LCD.			

(b) On a scale of 1–10, how would you rate your mastery of this section in light of your responses on the checklist? How can you improve this?

Contributors and Attributions

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3.5: Divide Fractions

Find Reciprocals

The fractions $\frac{2}{3}$ and $\frac{3}{2}$ are related to each other in a special way. So are $-\frac{10}{7}$ and $-\frac{7}{10}$. Do you see how? Besides looking like upside-down versions of one another, if we were to multiply these pairs of fractions, the product would be 1.

$$\frac{2}{3}\cdot\frac{3}{2}=1 \quad and \quad -\frac{10}{7}\left(-\frac{7}{10}\right)=1$$

Such pairs of numbers are called reciprocals.

Definition: Reciprocal

The reciprocal of the fraction $\frac{a}{b}$ is $\frac{b}{a}$, where $a\neq 0$ and $b\neq 0$.

A number and its reciprocal have a product of 1.

$$\frac{a}{b}\cdot\frac{b}{a}=1 \tag{4.2.54}$$

To find the reciprocal of a fraction, we invert the fraction. This means that we place the numerator in the denominator and the denominator in the numerator.

To get a positive result when multiplying two numbers, the numbers must have the same sign. So reciprocals must have the same sign.

$$\frac{a}{b}\cdot\frac{b}{a}=1 \text{ positive}$$

$$3\cdot\frac{1}{3}=1 \quad \text{and} \quad -3\cdot\left(-\frac{1}{3}\right)=1$$

both positive both negative

To find the reciprocal, keep the same sign and invert the fraction. The number zero does not have a reciprocal. Why? A number and its reciprocal multiply to 1. Is there any number r so that $0\cdot r=1$? No. So, the number 0 does not have a reciprocal.

Example 3.5.11: reciprocal

Find the reciprocal of each number. Then check that the product of each number and its reciprocal is 1.

a. $\frac{4}{9}$
b. $-\frac{1}{6}$
c. $-\frac{14}{5}$
d. 7

Solution

To find the reciprocals, we keep the sign and invert the fractions.

a.

Find the reciprocal of $\frac{4}{9}$.	The reciprocal of $\frac{4}{9}$ is $\frac{9}{4}$.

Check:

Multiply the number and its reciprocal.	$\frac{4}{9}\cdot\frac{9}{4}$

Multiply numerators and denominators.	$\frac{36}{36}$
Simplify.	1 ✓

b.

Find the reciprocal of $-\frac{1}{6}$.	The reciprocal of $-\frac{1}{6}$ is $\frac{6}{1}$.
Simplify.	-6
Check.	$-\frac{1}{6} \cdot (-6) = 1$ ✓

c.

Find the reciprocal of $-\frac{14}{5}$.	$-\frac{5}{14}$
Check.	$-\frac{14}{5} \cdot \left(-\frac{5}{14}\right) = \frac{70}{70} = 1$ ✓

d.

Find the reciprocal of 7.	
Write 7 as a fraction.	$\frac{7}{1}$
Write the reciprocal of $\frac{7}{1}$.	$\frac{1}{7}$
Check.	$7 \cdot \left(\frac{1}{7}\right) = 1$ ✓

Exercise 3.5.21

Find the reciprocal:

a. $\frac{5}{7}$
b. $-\frac{1}{8}$
c. $-\frac{11}{4}$
d. 14

Answer a

$\frac{7}{5}$

Answer b

-8

Answer c

$-\frac{4}{11}$

Answer d

$\frac{1}{14}$

Exercise 3.5.22

Find the reciprocal:

a. $\frac{3}{7}$

b. $-\frac{1}{12}$

c. $-\frac{14}{9}$

d. 21

Answer a

$\frac{7}{3}$

Answer b

-12

Answer c

$-\frac{9}{14}$

Answer d

$\frac{1}{21}$

In a previous chapter, we worked with opposites and absolute values. Table 3.5.1 compares opposites, absolute values, and reciprocals.

Table 3.5.1

Opposite	Absolute Value	Reciprocal
has opposite sign	is never negative	has same sign, fraction inverts

Example 3.5.12: fractions

Fill in the chart for each fraction in the left column:

Number	Opposite	Absolute Value	Reciprocal
$-\frac{3}{8}$			
$\frac{1}{2}$			
$\frac{9}{5}$			
-5			

Solution

To find the opposite, change the sign. To find the absolute value, leave the positive numbers the same, but take the opposite of the negative numbers. To find the reciprocal, keep the sign the same and invert the fraction.

Number	Opposite	Absolute Value	Reciprocal

Number	Opposite	Absolute Value	Reciprocal
$-\frac{3}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$-\frac{8}{3}$
$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	2
$\frac{9}{5}$	$-\frac{9}{5}$	$\frac{9}{5}$	$\frac{5}{9}$
-5	5	5	$-\frac{1}{5}$

Exercise 3.5.23

Fill in the chart for each number given:

Number	Opposite	Absolute Value	Reciprocal
$-\frac{5}{8}$			
$\frac{1}{4}$			
$\frac{8}{3}$			
-8			

Answer

Number	Opposite	Absolute Value	Reciprocal
$-\frac{5}{8}$	$\frac{5}{8}$	$\frac{5}{8}$	$-\frac{8}{5}$
$\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	4
$\frac{8}{3}$	$-\frac{8}{3}$	$\frac{8}{3}$	$\frac{3}{8}$
-8	8	8	$-\frac{1}{8}$

Exercise 3.5.24

Fill in the chart for each number given:

Number	Opposite	Absolute Value	Reciprocal
$-\frac{4}{7}$			
$\frac{1}{8}$			
$\frac{9}{4}$			
-1			

Answer

Number	Opposite	Absolute Value	Reciprocal
$-\frac{4}{7}$	$\frac{4}{7}$	$\frac{4}{7}$	$-\frac{7}{4}$
$\frac{1}{8}$	$-\frac{1}{8}$	$\frac{1}{8}$	8
$\frac{9}{4}$	$-\frac{9}{4}$	$\frac{9}{4}$	$\frac{4}{9}$
-1	1	1	$-\frac{1}{1}$

Divide Fractions

Why is $12 \div 3 = 4$? We previously modeled this with counters. How many groups of 3 counters can be made from a group of 12 counters?

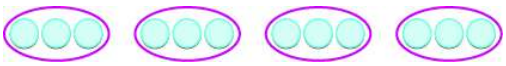


Figure 3.5.2

There are 4 groups of 3 counters. In other words, there are four 3s in 12. So, $12 \div 3 = 4$.

What about dividing fractions? Suppose we want to find the quotient: $\frac{1}{2} \div \frac{1}{6}$. We need to figure out how many $\frac{1}{6}$s there are in $\frac{1}{2}$. We can use fraction tiles to model this division. We start by lining up the half and sixth fraction tiles as shown in Figure 3.5.3. Notice, there are three $\frac{1}{6}$ tiles in $\frac{1}{2}$, so $\frac{1}{2} \div \frac{1}{6} = 3$.

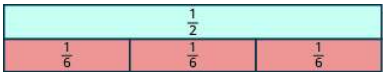


Figure 3.5.3

Example 3.5.13: model

Model: $\frac{1}{4} \div \frac{1}{8}$.

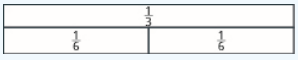
Solution

We want to determine how many $\frac{1}{8}$s are in $\frac{1}{4}$. Start with one $\frac{1}{4}$ tile. Line up $\frac{1}{8}$ tiles underneath the $\frac{1}{4}$ tile.

Exercise 3.5.25

Model: $\frac{1}{3} \div \frac{1}{6}$.

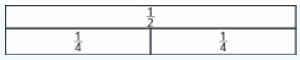
Answer



Exercise 3.5.26

Model: $\frac{1}{2} \div \frac{1}{4}$.

Answer

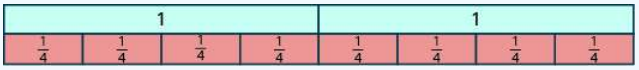


Example 3.5.14: model

Model: $2 \div \frac{1}{4}$.

Solution

We are trying to determine how many $\frac{1}{4}$s there are in 2. We can model this as shown.

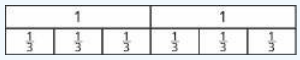


Because there are eight $\frac{1}{4}$s in 2, $2 \div \frac{1}{4} = 8$.

Exercise 3.5.27

Model: $2 \div \frac{1}{3}$

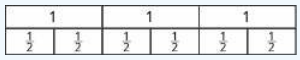
Answer



Exercise 3.5.28

Model: $3 \div \frac{1}{2}$

Answer



Let's use money to model $2 \div \frac{1}{4}$ in another way. We often read $\frac{1}{4}$ as a 'quarter', and we know that a quarter is one-fourth of a dollar as shown in Figure 3.5.4. So we can think of $2 \div \frac{1}{4}$ as, "How many quarters are there in two dollars?" One dollar is 4 quarters, so 2 dollars would be 8 quarters. So again, $2 \div \frac{1}{4} = 8$.



Figure 3.5.4: The U.S. coin called a quarter is worth one-fourth of a dollar.

Using fraction tiles, we showed that $\frac{1}{2} \div \frac{1}{6} = 3$. Notice that $\frac{1}{2} \cdot \frac{6}{1} = 3$ also. How are $\frac{1}{6}$ and $\frac{6}{1}$ related? They are reciprocals. This leads us to the procedure for fraction division.

Definition: Fraction Division

If a, b, c, and d are numbers where $b \neq 0$, $c \neq 0$, and $d \neq 0$, then

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} \tag{3.5.1}$$

To divide fractions, multiply the first fraction by the reciprocal of the second.

We need to say $b \neq 0$, $c \neq 0$ and $d \neq 0$ to be sure we don't divide by zero.

Example 3.5.15: divide

Divide, and write the answer in simplified form: \(\dfrac{2}{5} \div \left(- \dfrac{3}{7}\right).

Solution

Multiply the first fraction by the reciprocal of the second.	$\frac{2}{5}\left(-\frac{7}{3}\right)$
Multiply. The product is negative.	$-\frac{14}{15}$

Exercise 3.5.29

Divide, and write the answer in simplified form: $\frac{3}{7} \div \left(-\frac{2}{3}\right)$.

Answer

$-\frac{9}{14}$

Exercise 3.5.30

Divide, and write the answer in simplified form: $\frac{2}{3} \div \left(-\frac{7}{5}\right)$.

Answer

$-\frac{10}{21}$

Example 3.5.16: divide

Divide, and write the answer in simplified form: $\frac{2}{3} \div \frac{n}{5}$.

Solution

Multiply the first fraction by the reciprocal of the second.	$\frac{2}{3} \div \frac{5}{n}$
Multiply.	$\frac{10}{3n}$

Exercise 3.5.31

Divide, and write the answer in simplified form: $\frac{3}{5} \div \frac{p}{7}$.

Answer

$\frac{21}{5p}$

Exercise 3.5.32

Divide, and write the answer in simplified form: $\frac{5}{8} \div \frac{q}{3}$.

Answer

$\frac{15}{8q}$

Example 3.5.17: divide

Divide, and write the answer in simplified form: $-\frac{3}{4} \div \left(-\frac{7}{8}\right)$.

Solution

Multiply the first fraction by the reciprocal of the second.	$-\frac{3}{4} \cdot \left(-\frac{8}{7}\right)$
Multiply. Remember to determine the sign first.	$\frac{3 \cdot 8}{4 \cdot 7}$
Rewrite to show common factors.	$\frac{3 \cdot \cancel{4} \cdot 2}{\cancel{4} \cdot 7}$
Remove common factors and simplify.	$\frac{6}{7}$

Exercise 3.5.33

Divide, and write the answer in simplified form: $-\frac{2}{3} \div \left(-\frac{5}{6}\right)$.

Answer

$\frac{4}{5}$

Exercise 3.5.34

Divide, and write the answer in simplified form: $-\frac{5}{6} \div \left(-\frac{2}{3}\right)$.

Answer

$\frac{5}{4}$

Example 3.5.18: divide

Divide, and write the answer in simplified form: $\frac{7}{18} \div \frac{14}{27}$.

Solution

Multiply the first fraction by the reciprocal of the second.	$\frac{7}{18} \cdot \frac{27}{14}$
Multiply.	$\frac{7 \cdot 27}{18 \cdot 14}$

Rewrite showing common factors.	$\frac{\cancel{7}\cdot\cancel{9}\cdot 3}{\cancel{9}\cdot\cancel{7}\cdot 2}$
Remove common factors.	$\frac{3}{2\cdot 2}$
Simplify.	$\frac{3}{4}$

Exercise 3.5.35

Divide, and write the answer in simplified form: $\frac{7}{27} \div \frac{35}{36}$.

Answer

$\frac{4}{15}$

Exercise 3.5.36

Divide, and write the answer in simplified form: $\frac{5}{14} \div \frac{15}{28}$.

Answer

$\frac{2}{3}$

Access Additional Online Resources

- Simplifying Fractions
- Multiplying Fractions (Positive Only)
- Multiplying Signed Fractions
- Dividing Fractions (Positive Only)
- Dividing Signed Fractions

Key Concepts

- **Equivalent Fractions Property**
- If a, b, c are numbers where $b \neq 0, c \neq 0$, then $\frac{a}{b} = \frac{a \cdot c}{b \cdot c}$ and $\frac{a \cdot c}{b \cdot c} = \frac{a}{b}$
- **Simplify a fraction.**
  1. Rewrite the numerator and denominator to show the common factors. If needed, factor the numerator and denominator into prime numbers.
  2. Simplify, using the equivalent fractions property, by removing common factors.
  3. Multiply any remaining factors.
- **Fraction Multiplication**
  - If a, b, c, and dd are numbers where $b \neq 0$ and $d \neq 0$, then $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$
- **Reciprocal**
  - A number and its reciprocal have a product of 1. $\frac{a}{b} \cdot \frac{b}{a} = 1$
Opposite	Absolute Value	Reciprocal
has opposite sign	is never negative	has same sign, fraction inverts

- **Fraction Division**
  - If a, b, c, and d are numbers where $b \neq 0$, $c \neq 0$, and $d \neq 0$, then $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$
  - To divide fractions, multiply the first fraction by the reciprocal of the second.

Glossary

reciprocal

The reciprocal of the fraction $\frac{a}{b}$ is $\frac{b}{a}$ where $a \neq 0$ and $b \neq 0$.

simplified fraction

A fraction is considered simplified if there are no common factors in the numerator and denominator.

Practice Makes Perfect

Simplify Fractions

In the following exercises, simplify each fraction. Do not convert any improper fractions to mixed numbers.

77. $\frac{7}{21}$
78. $\frac{8}{24}$
79. $\frac{15}{20}$
80. $\frac{12}{18}$
81. $-\frac{40}{88}$
82. $-\frac{63}{99}$
83. $-\frac{108}{63}$
84. $-\frac{104}{48}$
85. $\frac{120}{252}$
86. $\frac{182}{294}$
87. $-\frac{168}{192}$
88. $-\frac{140}{224}$
89. $\frac{11x}{11y}$
90. $\frac{15a}{15b}$
91. $-\frac{3x}{12y}$
92. $-\frac{4x}{32y}$
93. $\frac{14x^2}{21y}$
94. $\frac{24a}{32b^2}$

Multiply Fractions

In the following exercises, use a diagram to model.

95. $\frac{1}{2} \cdot \frac{2}{3}$
96. $\frac{1}{2} \cdot \frac{5}{8}$
97. $\frac{1}{3} \cdot \frac{5}{6}$
98. $\frac{1}{3} \cdot \frac{2}{5}$

In the following exercises, multiply, and write the answer in simplified form.

99. $\frac{2}{5} \cdot \frac{1}{3}$
100. $\frac{1}{2} \cdot \frac{3}{8}$
101. $\frac{3}{4} \cdot \frac{9}{10}$
102. $\frac{4}{5} \cdot \frac{2}{7}$
103. $-\frac{2}{3}\left(-\frac{3}{8}\right)$
104. $-\frac{3}{4}\left(-\frac{4}{9}\right)$
105. $-\frac{5}{9} \cdot \frac{3}{10}$
106. $-\frac{3}{8} \cdot \frac{4}{15}$
107. $-\frac{7}{12}\left(-\frac{8}{21}\right)$
108. $\frac{5}{12}\left(-\frac{8}{15}\right)$
109. $\left(-\frac{14}{15}\right)\left(\frac{9}{20}\right)$
110. $\left(-\frac{9}{10}\right)\left(\frac{25}{33}\right)$
111. $\left(-\frac{63}{84}\right)\left(-\frac{44}{90}\right)$
112. $\left(-\frac{33}{60}\right)\left(-\frac{40}{88}\right)$
113. $4 \cdot \frac{5}{11}$
114. $5 \cdot \frac{8}{3}$
115. $\frac{3}{7} \cdot 21n$
116. $\frac{5}{6} \cdot 30m$
117. $-28p\left(-\frac{1}{4}\right)$
118. $-51q\left(-\frac{1}{3}\right)$
119. $-8\left(\frac{17}{4}\right)$
120. $\frac{14}{5}(-15)$
121. $-1\left(-\frac{3}{8}\right)$
122. $(-1)\left(-\frac{6}{7}\right)$

123. $\left(\frac{2}{3}\right)^3$
124. $\left(\frac{4}{5}\right)^2$
125. $\left(\frac{6}{5}\right)^4$
126. $\left(\frac{4}{7}\right)^4$

Find Reciprocals In the following exercises, find the reciprocal.

127. $\frac{3}{4}$
128. $\frac{2}{3}$
129. $-\frac{5}{17}$
130. $-\frac{6}{19}$
131. $\frac{11}{8}$
132. -13
133. -19
134. -1
135. 1
136. Fill in the chart.

	Opposite	Absolute Value	Reciprocal
$-\frac{7}{11}$			
$\frac{4}{5}$			
$\frac{10}{7}$			
-8			

137. Fill in the chart.

	Opposite	Absolute Value	Reciprocal
$-\frac{3}{13}$			
$\frac{9}{14}$			
$\frac{15}{7}$			
-9			

Divide Fractions

In the following exercises, model each fraction division.

138. $\frac{1}{2} \div \frac{1}{4}$
139. $\frac{1}{2} \div \frac{1}{8}$
140. $2 \div \frac{1}{5}$

141. $3 \div \frac{1}{4}$

In the following exercises, divide, and write the answer in simplified form.

142. $\frac{1}{2} \div \frac{1}{4}$
143. $\frac{1}{2} \div \frac{1}{8}$
144. $\frac{3}{4} \div \frac{2}{3}$
145. $\frac{4}{5} \div \frac{3}{4}$
146. $-\frac{4}{5} \div \frac{4}{7}$
147. $-\frac{3}{4} \div \frac{3}{5}$
148. $-\frac{7}{9} \div \left(-\frac{7}{9}\right)$
149. $-\frac{5}{6} \div \left(-\frac{5}{6}\right)$
150. $\frac{3}{4} \div \frac{x}{11}$
151. $\frac{2}{5} \div \frac{y}{9}$
152. $\frac{5}{8} \div \frac{a}{10}$
153. $\frac{5}{6} \div \frac{c}{15}$
154. $\frac{5}{18} \div \left(-\frac{15}{24}\right)$
155. $\frac{7}{18} \div \left(-\frac{14}{27}\right)$
156. $\frac{7p}{12} \div \frac{21p}{8}$
157. $\frac{5q}{12} \div \frac{15q}{8}$
158. $\frac{8u}{15} \div \frac{12v}{25}$
159. $\frac{12r}{25} \div \frac{18s}{35}$
160. $-5 \div \frac{1}{2}$
161. $-3 \div \frac{1}{4}$
162. $\frac{3}{4} \div (-12)$
163. $\frac{2}{5} \div (-10)$
164. $-18 \div \left(-\frac{9}{2}\right)$
165. $-15 \div \left(-\frac{5}{3}\right)$
166. $\frac{1}{2} \div \left(-\frac{3}{4}\right) \div \frac{7}{8}$
167. $\frac{11}{2} \div \frac{7}{8} \cdot \frac{2}{11}$

Everyday Math

168. **Baking** A recipe for chocolate chip cookies calls for 3 4 cup brown sugar. Imelda wants to double the recipe.
    a. How much brown sugar will Imelda need? Show your calculation. Write your result as an improper fraction and as a mixed number.
    b. Measuring cups usually come in sets of $\frac{1}{8}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}$, and 1 cup. Draw a diagram to show two different ways that Imelda could measure the brown sugar needed to double the recipe.
169. **Baking** Nina is making 4 pans of fudge to serve after a music recital. For each pan, she needs 2 3 cup of condensed milk.
    a. How much condensed milk will Nina need? Show your calculation. Write your result as an improper fraction and as a mixed number.
    b. Measuring cups usually come in sets of $\frac{1}{8}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}$, and 1 cup. Draw a diagram to show two different ways that Nina could measure the condensed milk she needs.
170. **Portions** Don purchased a bulk package of candy that weighs 5 pounds. He wants to sell the candy in little bags that hold $\frac{1}{4}$ pound. How many little bags of candy can he fill from the bulk package?
171. **Portions** Kristen has $\frac{3}{4}$ yards of ribbon. She wants to cut it into equal parts to make hair ribbons for her daughter's 6 dolls. How long will each doll's hair ribbon be?

Writing Exercises

172. Explain how you find the reciprocal of a fraction.
173. Explain how you find the reciprocal of a negative fraction.
174. Rafael wanted to order half a medium pizza at a restaurant. The waiter told him that a medium pizza could be cut into 6 or 8 slices. Would he prefer 3 out of 6 slices or 4 out of 8 slices? Rafael replied that since he wasn't very hungry, he would prefer 3 out of 6 slices. Explain what is wrong with Rafael's reasoning.
175. Give an example from everyday life that demonstrates how $\frac{1}{2} \cdot \frac{2}{3}$ is $\frac{1}{3}$.

Self Check

(a) After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
simplify fractions.			
multiply fractions.			
find reciprocals.			
divide fractions.			

(b) After reviewing this checklist, what will you do to become confident for all objectives?

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3.6: Add and Subtract Fractions with Common Denominators

Learning Objectives

- Model fraction addition
- Add fractions with a common denominator
- Model fraction subtraction
- Subtract fractions with a common denominator

be prepared!

Before you get started, take this readiness quiz.

1. Simplify: $2x+9+3x-4$. If you missed this problem, review Example 2.2.10.
2. Draw a model of the fraction $\frac{3}{4}$. If you missed this problem, review Example 4.1.2.
3. Simplify: $\frac{3+2}{6}$. If you missed this problem, review Example 4.3.12.

Model Fraction Addition

How many quarters are pictured? One quarter plus 2 quarters equals 3 quarters.

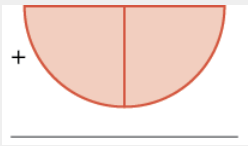
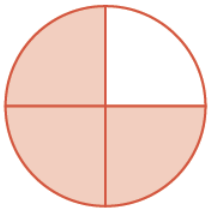


Figure 3.6.1

Remember, quarters are really fractions of a dollar. Quarters are another way to say fourths. So the picture of the coins shows that

$$\begin{matrix} \frac{1}{4} & \frac{2}{4} & \frac{3}{4} \\ one\ quarter + two\ quarters & = & three\ quarters \end{matrix}$$

Let's use fraction circles to model the same example, $\frac{1}{4}+\frac{2}{4}$.

Start with one $\frac{1}{4}$ piece.		$\frac{1}{4}$
Add two more $\frac{1}{4}$ pieces.		$+\frac{2}{4}$
The result is $\frac{3}{4}$.		$\frac{3}{4}$

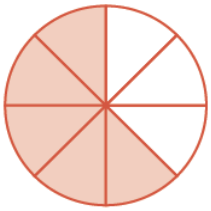
So again, we see that

$$\frac{1}{4}+\frac{2}{4}=\frac{3}{4}$$

Example 3.6.1: addition

Use a model to find the sum $\frac{3}{8}+\frac{2}{8}$.

Solution

Start with three $\frac{1}{8}$ pieces.		$\frac{3}{8}$
Add two $\frac{1}{8}$ pieces.		$+\frac{2}{8}$
How many $\frac{1}{8}$ pieces are there?		$\frac{5}{8}$

There are five $\frac{1}{8}$ pieces, or five-eighths. The model shows that $\frac{3}{8}+\frac{2}{8}=\frac{5}{8}$.

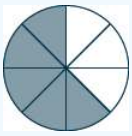
Exercise 3.6.1

Use a model to find each sum. Show a diagram to illustrate your model.

$$\frac{1}{8}+\frac{4}{8}$$

Answer

$\frac{5}{8}$



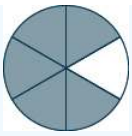
Exercise 3.6.2

Use a model to find each sum. Show a diagram to illustrate your model.

$$\frac{1}{6}+\frac{4}{6}$$

Answer

$\frac{5}{6}$



Add Fractions with a Common Denominator

Example 3.6.1 shows that to add the same-size pieces—meaning that the fractions have the same denominator—we just add the number of pieces.

Definition: Fraction Addition

If a, b, and c are numbers where $c \neq 0$, then

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c} \tag{3.6.1}$$

To add fractions with a common denominator, add the numerators and place the sum over the common denominator.

Example 3.6.2: addition

Find the sum: $\frac{3}{5} + \frac{1}{5}$.

Solution

Add the numerators and place the sum over the common denominator.	$\frac{3+1}{5}$
Simplify.	$\frac{4}{5}$

Exercise 3.6.3

Find each sum: $\frac{3}{6} + \frac{2}{6}$.

Answer

$\frac{5}{6}$

Exercise 3.6.4

Find each sum: $\frac{3}{10} + \frac{7}{10}$.

Answer

1

Example 3.6.3: addition

Find the sum: $\frac{x}{3} + \frac{2}{3}$.

Solution

Add the numerators and place the sum over the common denominator.	$\frac{x+2}{3}$

Note that we cannot simplify this fraction any more. Since x and 2 are not like terms, we cannot combine them.

? Exercise 3.6.5

Find the sum: $\frac{x}{4}+\frac{3}{4}$.

Answer

$\frac{x+3}{4}$

? Exercise 3.6.6

Find the sum: $\frac{y}{8}+\frac{5}{8}$.

Answer

$\frac{y+5}{8}$

✓ Example 3.6.4: addition

Find the sum: $-\frac{9}{d}+\frac{3}{d}$.

Solution

We will begin by rewriting the first fraction with the negative sign in the numerator.

$$-\frac{a}{b}=\frac{-a}{b}$$

Rewrite the first fraction with the negative in the numerator.	$\frac{-9}{d}+\frac{3}{d}$
Add the numerators and place the sum over the common denominator.	$\frac{-9+3}{d}$
Simplify the numerator.	$\frac{-6}{d}$
Rewrite with negative sign in front of the fraction.	$-\frac{6}{d}$

? Exercise 3.6.7

Find the sum: $-\frac{7}{d}+\frac{8}{d}$.

Answer

$\frac{1}{d}$

Exercise 3.6.8

Find the sum: $-\frac{6}{m}+\frac{9}{m}$.

Answer

$\frac{3}{m}$

Example 3.6.5: addition

Find the sum: $\frac{2n}{11}+\frac{5n}{11}$.

Solution

Add the numerators and place the sum over the common denominator.	$\frac{2n+5n}{11}$
Combine like terms.	$\frac{7n}{11}$

Exercise 3.6.9

Find the sum: $\frac{3p}{8}+\frac{6p}{8}$.

Answer

$\frac{9p}{8}$

Exercise 3.6.10

Find the sum: $\frac{2q}{5}+\frac{7q}{5}$.

Answer

$\frac{9q}{5}$

Example 3.6.6: addition

Find the sum: $-\frac{3}{12}+\left(-\frac{5}{12}\right)$.

Solution

Add the numerators and place the sum over the common denominator.	$\frac{-3+(-5)}{12}$
Add.	$\frac{-8}{12}$
Simplify the fraction.	$-\frac{2}{3}$

Exercise 3.6.11

Find each sum: $-\frac{4}{15}+\left(-\frac{6}{15}\right)$.

Answer

$-\frac{2}{3}$

Exercise 3.6.12

Find each sum: $-\frac{5}{21}+\left(-\frac{9}{21}\right)$.

Answer

$-\frac{2}{3}$

Model Fraction Subtraction

Subtracting two fractions with common denominators is much like adding fractions. Think of a pizza that was cut into 12 slices. Suppose five pieces are eaten for dinner. This means that, after dinner, there are seven pieces (or $\frac{7}{12}$ of the pizza) left in the box. If Leonardo eats 2 of these remaining pieces (or $\frac{2}{12}$ of the pizza), how much is left? There would be 5 pieces left (or $\frac{5}{12}$ of the pizza).

$$\frac{7}{12}-\frac{2}{12}=\frac{5}{12}$$

Let's use fraction circles to model the same example, $\frac{7}{12}-\frac{2}{12}$. Start with seven $\frac{1}{12}$ pieces. Take away two $\frac{1}{12}$ pieces. How many twelfths are left?

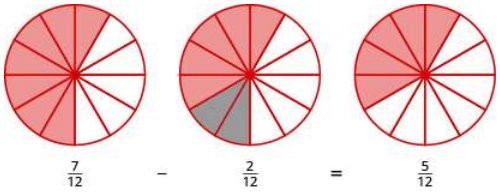


Figure 3.6.2

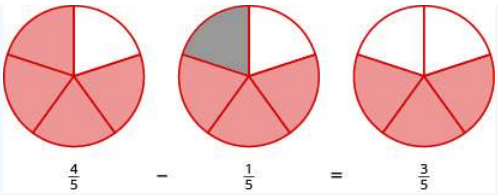
Again, we have five twelfths, $\frac{5}{12}$.

Example 3.6.7: difference

Use fraction circles to find the difference: $\frac{4}{5}-\frac{1}{5}$.

Solution

Start with four $\frac{1}{5}$ pieces. Take away one $\frac{1}{5}$ piece. Count how many fifths are left. There are three $\frac{1}{5}$ pieces left.



Exercise 3.6.13

Use a model to find each difference. Show a diagram to illustrate your model. $\frac{7}{8}-\frac{4}{8}$

Answer

$\frac{3}{8}$, models may differ.

Exercise 3.6.14

Use a model to find each difference. Show a diagram to illustrate your model. $\frac{5}{6}-\frac{4}{6}$

Answer

$\frac{1}{6}$, models may differ.

Subtract Fractions with a Common Denominator

We subtract fractions with a common denominator in much the same way as we add fractions with a common denominator.

Definition: Fraction Subtraction

If a, b, and c are numbers where $c \neq 0$, then

$$\frac{a}{c}-\frac{b}{c}=\frac{a-b}{c} \tag{3.6.2}$$

To subtract fractions with a common denominator, we subtract the numerators and place the difference over the common denominator.

Example 3.6.8: difference

Find the difference: $\frac{23}{24}-\frac{14}{24}$.

Solution

Subtract the numerators and place the difference over the common denominator.	$\frac{23-14}{24}$
Simplify the numerator.	$\frac{9}{24}$
Simplify the fraction by removing common factors.	$\frac{3}{8}$

Exercise 3.6.15

Find the difference: $\frac{19}{28}-\frac{7}{28}$.

Answer

$\frac{3}{7}$

Exercise 3.6.16

Find the difference: $\frac{27}{32}-\frac{11}{32}$.

Answer

$\frac{1}{2}$

Example 3.6.9: difference

Find the difference: $\frac{y}{6}-\frac{1}{6}$.

Solution

Subtract the numerators and place the difference over the common denominator.	$\frac{y-1}{6}$

The fraction is simplified because we cannot combine the terms in the numerator.

Exercise 3.6.17

Find the difference: $\frac{x}{7}-\frac{2}{7}$.

Answer

$\frac{x-2}{7}$

Exercise 3.6.18

Find the difference: $\frac{y}{14}-\frac{13}{14}$.

Answer

$\frac{y-13}{14}$

Example 3.6.10: difference

Find the difference: $-\frac{10}{x}-\frac{4}{x}$.

Solution

Remember, the fraction $-\frac{10}{x}$ can be written as $\frac{-10}{x}$.

Subtract the numerators.	$\frac{-10-4}{x}$
Simplify.	$\frac{-14}{x}$
Rewrite with the negative sign in front of the fraction.	$-\frac{14}{x}$

Exercise 3.6.19

Find the difference: $-\frac{9}{x}-\frac{7}{x}$.

Answer

$-\frac{16}{x}$

Exercise 3.6.20

Find the difference: $-\frac{17}{a}-\frac{5}{a}$.

Answer

$-\frac{22}{a}$

Now lets do an example that involves both addition and subtraction.

Example 3.6.11: simplify

Simplify: $\frac{3}{8}+\left(-\frac{5}{8}\right)-\frac{1}{8}$.

Solution

Combine the numerators over the common denominator.	$\frac{3+(-5)-1}{8}$
Simplify the numerator, working left to right.	$\frac{-2-1}{8}$
Subtract the terms in the numerator.	$\frac{-3}{8}$
Rewrite with the negative sign in front of the fraction.	$-\frac{3}{8}$

Exercise 3.6.21

Simplify: $\frac{2}{5}+\left(-\frac{4}{5}\right)-\frac{3}{5}$.

Answer

-1

Exercise 3.6.22

Simplify: $\frac{5}{9}+\left(-\frac{4}{9}\right)-\frac{7}{9}$.

Answer

$-\frac{2}{3}$

Access Additional Online Resources

- Adding Fractions With Pattern Blocks
- Adding Fractions With Like Denominators
- Subtracting Fractions With Like Denominators

Key Concepts

- Fraction Addition
  - If $a, b,$, and c are numbers where $c \neq 0$, then $\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c}$
  - To add fractions, add the numerators and place the sum over the common denominator.
- Fraction Subtraction
  - If $a, b,$, and c are numbers where $c \neq 0$, then $\frac{a}{c}-\frac{b}{c}=\frac{a-b}{c}$
  - To subtract fractions, subtract the numerators and place the difference over the common denominator.

Practice Makes Perfect

Model Fraction Addition

In the following exercises, use a model to add the fractions. Show a diagram to illustrate your model.

254. $\frac{2}{5}+\frac{1}{5}$
255. $\frac{3}{10}+\frac{4}{10}$
256. $\frac{1}{6}+\frac{3}{6}$
257. $\frac{3}{8}+\frac{3}{8}$

Add Fractions with a Common Denominator

In the following exercises, find each sum.

258. $\frac{4}{9}+\frac{1}{9}$
259. $\frac{2}{9}+\frac{5}{9}$
260. $\frac{6}{13}+\frac{7}{13}$
261. $\frac{9}{15}+\frac{7}{15}$
262. $\frac{x}{4}+\frac{3}{4}$
263. $\frac{y}{3}+\frac{2}{3}$
264. $\frac{7}{p}+\frac{9}{p}$

265. $\frac{8}{q}+\frac{6}{q}$
266. $\frac{8b}{9}+\frac{3b}{9}$
267. $\frac{5a}{7}+\frac{4a}{7}$
268. $\frac{-12y}{8}+\frac{3y}{8}$
269. $\frac{-11x}{5}+\frac{7x}{5}$
270. $-\frac{1}{8}+\left(-\frac{3}{8}\right)$
271. $-\frac{1}{8}+\left(-\frac{5}{8}\right)$
272. $-\frac{3}{16}+\left(-\frac{7}{16}\right)$
273. $-\frac{5}{16}+\left(-\frac{9}{16}\right)$
274. $-\frac{8}{17}+\frac{15}{17}$
275. $-\frac{9}{19}+\frac{17}{19}$
276. $-\frac{6}{13}+\left(-\frac{10}{13}\right)+\left(-\frac{12}{13}\right)$
277. $-\frac{5}{12}+\left(-\frac{7}{12}\right)+\left(-\frac{11}{12}\right)$

Model Fraction Subtraction

In the following exercises, use a model to subtract the fractions. Show a diagram to illustrate your model.

278. $\frac{5}{8}-\frac{2}{8}$
279. $\frac{5}{6}-\frac{2}{6}$

Subtract Fractions with a Common Denominator

In the following exercises, find the difference.

280. $\frac{4}{5}-\frac{1}{5}$
281. $\frac{4}{5}-\frac{3}{5}$
282. $\frac{11}{15}-\frac{7}{15}$
283. $\frac{9}{13}-\frac{4}{13}$
284. $\frac{11}{12}-\frac{5}{12}$
285. $\frac{7}{12}-\frac{5}{12}$
286. $\frac{4}{21}-\frac{19}{21}$
287. $-\frac{8}{9}-\frac{16}{9}$
288. $\frac{y}{17}-\frac{9}{17}$
289. $\frac{x}{19}-\frac{8}{19}$
290. $\frac{5y}{8}-\frac{7}{8}$

291. $\frac{11z}{13} - \frac{8}{13}$
292. $-\frac{8}{d} - \frac{3}{d}$
293. $-\frac{7}{c} - \frac{7}{c}$
294. $-\frac{23}{u} - \frac{15}{u}$
295. $-\frac{29}{v} - \frac{26}{v}$
296. $-\frac{6c}{7} - \frac{5c}{7}$
297. $-\frac{12d}{11} - \frac{9d}{11}$
298. $\frac{-4r}{13} - \frac{5r}{13}$
299. $\frac{-7s}{3} - \frac{7s}{3}$
300. $-\frac{3}{5} - \left(-\frac{4}{5}\right)$
301. $-\frac{3}{7} - \left(-\frac{5}{7}\right)$
302. $-\frac{7}{9} - \left(-\frac{5}{9}\right)$
303. $-\frac{8}{11} - \left(-\frac{5}{11}\right)$

Mixed Practice

In the following exercises, perform the indicated operation and write your answers in simplified form.

304. $-\frac{5}{18} \cdot \frac{9}{10}$
305. $-\frac{3}{14} \cdot \frac{7}{12}$
306. $\frac{n}{5} - \frac{4}{5}$
307. $\frac{6}{11} - \frac{s}{11}$
308. $-\frac{7}{24} - \frac{2}{24}$
309. $-\frac{5}{18} - \frac{1}{18}$
310. $\frac{8}{15} \div \frac{12}{5}$
311. $\frac{7}{12} \div \frac{9}{28}$

Everyday Math

312. **Trail Mix** Jacob is mixing together nuts and raisins to make trail mix. He has $\frac{6}{10}$ of a pound of nuts and $\frac{3}{10}$ of a pound of raisins. How much trail mix can he make?
313. **Baking** Janet needs $\frac{5}{8}$ of a cup of flour for a recipe she is making. She only has $\frac{3}{8}$ of a cup of flour and will ask to borrow the rest from her next-door neighbor. How much flour does she have to borrow?

Writing Exercises

314. Greg dropped his case of drill bits and three of the bits fell out. The case has slots for the drill bits, and the slots are arranged in order from smallest to largest. Greg needs to put the bits that fell out back in the case in the empty slots. Where do the three bits go? Explain how you know.

Bits in case: $\frac{1}{16}, \frac{1}{8}, \_\_\_, \_\_\_, \frac{5}{16}, \frac{3}{8}, \_\_\_, \frac{1}{2}, \frac{9}{16}, \frac{5}{8}$.

Bits that fell out: $\frac{7}{16}, \frac{3}{16}, \frac{1}{4}$.

315. After a party, Lupe has $\frac{5}{12}$ of a cheese pizza, $\frac{4}{12}$ of a pepperoni pizza, and $\frac{4}{12}$ of a veggie pizza left. Will all the slices fit into 1 pizza box? Explain your reasoning.

Self Check

(a) After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
model fraction addition.			
add fractions with a common denominator.			
model fraction subtraction.			
subtract fractions with a common denominator.			
find the least common denominator (LCD).			
convert fractions to equivalent fractions with the LCD.			

(b) On a scale of 1–10, how would you rate your mastery of this section in light of your responses on the checklist? How can you improve this?

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3.7: Add and Subtract Fractions with Different Denominators (Part 2)

Identify and Use Fraction Operations

By now in this chapter, you have practiced multiplying, dividing, adding, and subtracting fractions. The following table summarizes these four fraction operations. Remember: You need a common denominator to add or subtract fractions, but not to multiply or divide fractions.

Summary of Fraction Operations

Fraction multiplication: Multiply the numerators and multiply the denominators.

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd} \tag{3.7.1}$$

Fraction division: Multiply the first fraction by the reciprocal of the second.

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} \tag{3.7.2}$$

Fraction addition: Add the numerators and place the sum over the common denominator. If the fractions have different denominators, first convert them to equivalent forms with the LCD.

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c} \tag{3.7.3}$$

Fraction subtraction: Subtract the numerators and place the difference over the common denominator. If the fractions have different denominators, first convert them to equivalent forms with the LCD.

$$\frac{a}{c} - \frac{a}{c} = \frac{a-b}{c} \tag{3.7.4}$$

Example 3.7.11: simplify

Simplify:

a. $-\frac{1}{4} + \frac{1}{6}$

b. $-\frac{1}{4} \div \frac{1}{6}$

Solution

First we ask ourselves, “What is the operation?”

a. The operation is addition. Do the fractions have a common denominator? No.

Find the LCD.	$\begin{array}{l} 4 = 2 \cdot 2 \\ 6 = 2 \cdot \quad 3 \\ \hline \text{LCD} = 2 \cdot 2 \cdot 3 \\ \text{LCD} = 12 \end{array}$
Rewrite each fraction as an equivalent fraction with the LCD.	$-\frac{1 \cdot 3}{4 \cdot 3} + \frac{1 \cdot 2}{6 \cdot 2}$
Simplify the numerators and denominators.	$-\frac{3}{12} + \frac{2}{12}$
Add the numerators and place the sum over the common denominator.	$-\frac{1}{12}$
Check to see if the answer can be simplified. It cannot.	

b. The operation is division. We do not need a common denominator.

To divide fractions, multiply the first fraction by the reciprocal of the second.	$-\frac{1}{4} \cdot \frac{6}{1}$
Multiply.	$-\frac{6}{4}$
Simplify.	$-\frac{3}{2}$

? Exercise 3.7.21

Simplify:

a. $-\frac{3}{4}-\frac{1}{6}$

b. $-\frac{3}{4} \cdot \frac{1}{6}$

Answer a

$-\frac{11}{12}$

Answer b

$-\frac{1}{8}$

? Exercise 3.7.22

Simplify:

a. $\frac{5}{6} \div\left(-\frac{1}{4}\right)$

b. $\frac{5}{6}-\left(-\frac{1}{4}\right)$

Answer a

$-\frac{10}{3}$

Answer b

$\frac{13}{12}$

✓ Example 3.7.12: simplify

Simplify:

a. $\frac{5x}{6}-\frac{3}{10}$

b. $\frac{5x}{6} \cdot \frac{3}{10}$

Solution

a. The operation is subtraction. The fractions do not have a common denominator.

Rewrite each fraction as an equivalent fraction with the LCD, 30.	$\frac{5x \cdot 5}{6 \cdot 5}-\frac{3 \cdot 3}{10 \cdot 3}$

	$\frac{25x}{30}-\frac{9}{30}$
Subtract the numerators and place the difference over the common denominator.	$\frac{25x-9}{30}$

b. The operation is multiplication; no need for a common denominator.

To multiply fractions, multiply the numerators and multiply the denominators.	$\frac{5x\cdot 3}{6\cdot 10}$
Rewrite, showing common factors.	$\frac{\cancel{5}\cdot x\cdot \cancel{3}}{2\cdot \cancel{3}\cdot 2\cdot \cancel{5}}$
Remove common factors to simplify.	$\frac{x}{4}$

? Exercise 3.7.23

Simplify:

a. $\frac{3a}{4}-\frac{8}{9}$

b. $\frac{3a}{4}\cdot\frac{8}{9}$

Answer a

$\frac{27a-32}{36}$

Answer b

$\frac{2a}{3}$

? Exercise 3.7.24

Simplify:

a. $\frac{4k}{5}+\frac{5}{6}$

b. $\frac{4k}{5}\div\frac{5}{6}$

Answer a

$\frac{24k+25}{30}$

Answer b

$\frac{24k}{25}$

Use the Order of Operations to Simplify Complex Fractions

In Multiply and Divide Mixed Numbers and Complex Fractions, we saw that a complex fraction is a fraction in which the numerator or denominator contains a fraction. We simplified complex fractions by rewriting them as division problems. For example,

$$\frac{\frac{3}{4}}{\frac{5}{8}} = \frac{3}{4} \div \frac{5}{8}$$

Now we will look at complex fractions in which the numerator or denominator can be simplified. To follow the order of operations, we simplify the numerator and denominator separately first. Then we divide the numerator by the denominator.

HOW TO: SIMPLIFY COMPLEX FRACTIONS

Step 1. Simplify the numerator.

Step 2. Simplify the denominator.

Step 3. Divide the numerator by the denominator.

Step 4. Simplify if possible.

Example 3.7.13: simplify

Simplify: $\frac{\left(\frac{1}{2}\right)^2}{4+3^2}$.

Solution

Simplify the numerator.	$\frac{\frac{1}{4}}{4+3^2}$
Simplify the term with the exponent in the denominator.	$\frac{\frac{1}{4}}{4+9}$
Add the terms in the denominator.	$\frac{\frac{1}{4}}{13}$
Divide the numerator by the denominator.	$\frac{1}{4} \div 13$
Rewrite as multiplication by the reciprocal.	$\frac{1}{4} \cdot \frac{1}{13}$
Multiply.	$\frac{1}{52}$

Exercise 3.7.25

Simplify: $\frac{\left(\frac{1}{3}\right)^2}{2^3+2}$.

Answer

$\frac{1}{90}$

Exercise 3.7.26

Simplify: $\dfrac{1+4^2}{\left(\frac{1}{4}\right)^2}$.

Answer

272

Example 3.7.14: simplify

Simplify: $\dfrac{\frac{1}{2}+\frac{2}{3}}{\frac{3}{4}-\frac{1}{6}}$.

Solution

Rewrite numerator with the LCD of 6 and denominator with LCD of 12.	$\dfrac{\frac{3}{6}+\frac{4}{6}}{\frac{9}{12}-\frac{2}{12}}$
Add in the numerator. Subtract in the denominator.	$\dfrac{\frac{7}{6}}{\frac{7}{12}}$
Divide the numerator by the denominator.	$\frac{7}{6} \div \frac{7}{12}$
Rewrite as multiplication by the reciprocal.	$\frac{7}{6} \cdot \frac{12}{7}$
Rewrite, showing common factors.	$\dfrac{\cancel{7} \cdot \cancel{6} \cdot 2}{\cancel{6}\cancel{7} \cdot 1}$
Simplify.	2

Exercise 3.7.27

Simplify: $\dfrac{\frac{1}{3}+\frac{1}{2}}{\frac{3}{4}-\frac{1}{3}}$.

Answer

2

Exercise 3.7.28

Simplify: $\dfrac{\frac{2}{3}-\frac{1}{2}}{\frac{1}{4}+\frac{1}{3}}$.

Answer

$\frac{2}{7}$

Evaluate Variable Expressions with Fractions

We have evaluated expressions before, but now we can also evaluate expressions with fractions. Remember, to evaluate an expression, we substitute the value of the variable into the expression and then simplify.

Example 3.7.15: evaluate

Evaluate $x+\frac{1}{3}$ when

a. $x=-\frac{1}{3}$

b. $x=-\frac{3}{4}$

Solution

a. To evaluate $x+\frac{1}{3}$ when $x=-\frac{1}{3}$, substitute $-\frac{1}{3}$ for x in the expression.

Substitute $-\frac{1}{3}$ for x.	$-\frac{1}{3}+\frac{1}{3}$
Simplify.	0

b. To evaluate $x+\frac{1}{3}$ when $x=-\frac{3}{4}$, we substitute $-\frac{3}{4}$ for x in the expression.

Substitute $-\frac{3}{4}$ for x.	$-\frac{1}{3}+\frac{1}{3}$
Rewrite as equivalent fractions with the LCD, 12.	$-\frac{3\cdot 3}{4\cdot 3}+\frac{1\cdot 4}{3\cdot 4}$
Simplify the numerators and denominators.	$-\frac{9}{12}+\frac{4}{12}$
Add.	$-\frac{5}{12}$

Exercise 3.7.29

Evaluate $x+\frac{3}{4}$ when:

a. $x=-\frac{7}{4}$

b. $x=-\frac{5}{4}$

Answer a

-1

Answer b

$-\frac{1}{2}$

Exercise 3.7.30

Evaluate $y+\frac{1}{2}$ when:

a. $y=\frac{2}{3}$

b. $y=-\frac{3}{4}$

Answer a

$\frac{7}{6}$

Answer b

$-\frac{1}{4}$

Example 3.7.16: evaluate

Evaluate $y-\frac{5}{6}$ when $y=-\frac{2}{3}$.

Solution

We substitute $-\frac{2}{3}$ for y in the expression.

Substitute $-\frac{2}{3}$ for y.	$-\frac{2}{3}-\frac{5}{6}$
Rewrite as equivalent fractions with the LCD, 6.	$-\frac{4}{6}-\frac{5}{6}$
Subtract.	$-\frac{9}{6}$
Simplify.	$-\frac{3}{2}$

Exercise 3.7.31

Evaluate $y-\frac{1}{2}$ when $y=-\frac{1}{4}$.

Answer

$-\frac{3}{4}$

Exercise 3.7.32

Evaluate $x-\frac{3}{8}$ when $x=-\frac{5}{2}$.

Answer

$-\frac{23}{8}$

Example 3.7.17:

Evaluate $2x^2y$ when $x = \frac{1}{4}$ and $y = -\frac{2}{3}$.

Solution

Substitute the values into the expression. In $2x^2y$, the exponent applies only to x.

Substitute $\frac{1}{4}$ for x and $-\frac{2}{3}$ for y.	$2\left(\frac{1}{4}\right)^2\left(-\frac{2}{3}\right)$
Simplify exponents first.	$2\left(\frac{1}{16}\right)\left(-\frac{2}{3}\right)$
Multiply. The product will be negative.	$-\frac{2}{1}\cdot\frac{1}{16}\cdot\frac{2}{3}$
Simplify.	$-\frac{4}{48}$
Remove the common factors.	$-\frac{1\cdot\cancel{4}}{\cancel{4}\cdot 12}$
Simplify.	$-\frac{1}{12}$

Exercise 3.7.33

Evaluate: $3ab^2$ when $a = -\frac{2}{3}$ and $b = -\frac{1}{2}$.

Answer

$-\frac{1}{2}$

Exercise 3.7.34

Evaluate: $4c^3d$ when $c = -\frac{1}{2}$ and $d = -\frac{4}{3}$.

Answer

$\frac{2}{3}$

Example 3.7.18: evaluate

Evaluate: $\frac{p+q}{r}$ when $p = -4$, $q = -2$, and $r = 8$.

Solution

We substitute the values into the expression and simplify.

Substitute -4 for p, -2 for q and 8 for r.	$\frac{-4+(-2)}{8}$
Add in the numerator first.	$-\frac{6}{8}$
Simplify.	$-\frac{3}{4}$

Exercise 3.7.35

Evaluate: $\frac{a+b}{c}$ when $a=-8$, $b=-7$, and $c=6$.

Answer

$-\frac{5}{2}$

Exercise 3.7.36

Evaluate: $\frac{x+y}{z}$ when $x=9$, $y=-18$, and $z=-6$.

Answer

$\frac{3}{2}$

Practice Makes Perfect

Find the Least Common Denominator (LCD)

In the following exercises, find the least common denominator (LCD) for each set of fractions.

316. $\frac{2}{3}$ and $\frac{3}{4}$
317. $\frac{3}{4}$ and $\frac{2}{5}$
318. $\frac{7}{12}$ and $\frac{5}{8}$
319. $\frac{9}{16}$ and $\frac{7}{12}$
320. $\frac{13}{30}$ and $\frac{25}{42}$
321. $\frac{23}{30}$ and $\frac{5}{48}$
322. $\frac{21}{35}$ and $\frac{39}{56}$
323. $\frac{18}{35}$ and $\frac{33}{49}$
324. $\frac{2}{3}$, $\frac{1}{6}$ and $\frac{3}{4}$
325. $\frac{2}{3}$, $\frac{1}{4}$ and $\frac{3}{5}$

Convert Fractions to Equivalent Fractions with the LCD

In the following exercises, convert to equivalent fractions using the LCD.

326. $\frac{1}{3}$ and $\frac{1}{4}$, LCD = 12
327. $\frac{1}{4}$ and $\frac{1}{5}$, LCD = 20
328. $\frac{5}{12}$ and $\frac{7}{8}$, LCD = 24
329. $\frac{7}{12}$ and $\frac{5}{8}$, LCD = 24
330. $\frac{13}{16}$ and $-\frac{11}{12}$, LCD = 48
331. $\frac{11}{16}$ and $-\frac{5}{12}$, LCD = 48

332. $\frac{1}{3}$, $\frac{5}{6}$, and $\frac{3}{4}$, LCD = 12
333. $\frac{1}{3}$, $\frac{3}{4}$, and $\frac{3}{5}$, LCD = 60

Add and Subtract Fractions with Different Denominators

In the following exercises, add or subtract. Write the result in simplified form.

334. $\frac{1}{3}+\frac{1}{5}$
335. $\frac{1}{4}+\frac{1}{5}$
336. $\frac{1}{2}+\frac{1}{7}$
337. $\frac{1}{3}+\frac{1}{8}$
338. $\frac{1}{3}-\left(-\frac{1}{9}\right)$
339. $\frac{1}{4}-\left(-\frac{1}{8}\right)$
340. $\frac{1}{5}-\left(-\frac{1}{10}\right)$
341. $\frac{1}{2}-\left(-\frac{1}{6}\right)$
342. $\frac{2}{3}+\frac{3}{4}$
343. $\frac{3}{4}+\frac{2}{5}$
344. $\frac{7}{12}+\frac{5}{8}$
345. $\frac{5}{12}+\frac{3}{8}$
346. $\frac{7}{12}-\frac{9}{16}$
347. $\frac{7}{16}-\frac{5}{12}$
348. $\frac{11}{12}-\frac{3}{8}$
349. $\frac{5}{8}-\frac{7}{12}$
350. $\frac{2}{3}-\frac{3}{8}$
351. $\frac{5}{6}-\frac{3}{4}$
352. $-\frac{11}{30}+\frac{27}{40}$
353. $-\frac{9}{20}+\frac{17}{30}$
354. $-\frac{13}{30}+\frac{25}{42}$
355. $-\frac{23}{30}+\frac{5}{48}$
356. $-\frac{39}{56}-\frac{22}{35}$
357. $-\frac{33}{49}-\frac{18}{35}$
358. $-\frac{2}{3}-\left(-\frac{3}{4}\right)$
359. $-\frac{3}{4}-\left(-\frac{4}{5}\right)$

360. $-\frac{9}{16}-\left(-\frac{4}{5}\right)$
361. $-\frac{7}{20}-\left(-\frac{5}{8}\right)$
362. $1+\frac{7}{8}$
363. $1+\frac{5}{6}$
364. $1-\frac{5}{9}$
365. $1-\frac{3}{10}$
366. $\frac{x}{3}+\frac{1}{4}$
367. $\frac{y}{2}+\frac{2}{3}$
368. $\frac{y}{4}-\frac{3}{5}$
369. $\frac{x}{5}-\frac{1}{4}$

Identify and Use Fraction Operations

In the following exercises, perform the indicated operations. Write your answers in simplified form.

370. (a) $\frac{3}{4}+\frac{1}{6}$ (b) $\frac{3}{4}\div\frac{1}{6}$
371. (a) $\frac{2}{3}+\frac{1}{6}$ (b) $\frac{2}{3}\div\frac{1}{6}$
372. (a) $-\frac{2}{5}-\frac{1}{8}$ (b) $-\frac{2}{5}\cdot\frac{1}{8}$
373. (a) $-\frac{4}{5}-\frac{1}{8}$ (b) $-\frac{4}{5}\cdot\frac{1}{8}$
374. (a) $\frac{5n}{6}\div\frac{8}{15}$ (b) $\frac{5n}{6}-\frac{8}{15}$
375. (a) $\frac{3a}{8}\div\frac{7}{12}$ (b) $\frac{3a}{8}-\frac{7}{12}$
376. (a) $\frac{9}{10}\cdot\left(-\frac{11d}{12}\right)$ (b) $\frac{9}{10}+\left(-\frac{11d}{12}\right)$
377. (a) $\frac{4}{15}\cdot\left(-\frac{5}{q}\right)$ (b) $\frac{4}{15}+\left(-\frac{5}{q}\right)$
378. $-\frac{3}{8}\div\left(-\frac{3}{10}\right)$
379. $-\frac{5}{12}\div\left(-\frac{5}{9}\right)$
380. $-\frac{3}{8}+\frac{5}{12}$
381. $-\frac{1}{8}+\frac{7}{12}$
382. $\frac{5}{6}-\frac{1}{9}$
383. $\frac{5}{9}-\frac{1}{6}$
384. $\frac{3}{8}\cdot\left(-\frac{10}{21}\right)$
385. $\frac{7}{12}\cdot\left(-\frac{8}{35}\right)$
386. $-\frac{7}{15}-\frac{y}{4}$
387. $-\frac{3}{8}-\frac{x}{11}$

388. $\frac{11}{12a} \cdot \frac{9a}{16}$

389. $\frac{10y}{13} \cdot \frac{8}{15y}$

Use the Order of Operations to Simplify Complex Fractions

In the following exercises, simplify.

390. $\frac{\left(\frac{1}{5}\right)^2}{2+3^2}$

391. $\frac{\left(\frac{1}{3}\right)^2}{5+2^2}$

392. $\frac{2^3+4^2}{\left(\frac{2}{3}\right)^2}$

393. $\frac{3^3-3^2}{\left(\frac{3}{4}\right)^2}$

394. $\frac{\left(\frac{3}{5}\right)^2}{\left(\frac{3}{7}\right)^2}$

395. $\frac{\left(\frac{3}{4}\right)^2}{\left(\frac{5}{8}\right)^2}$

396. $\frac{2}{\frac{1}{3}+\frac{1}{5}}$

397. $\frac{5}{\frac{1}{4}+\frac{1}{3}}$

398. $\frac{\frac{2}{3}+\frac{1}{2}}{\frac{3}{4}-\frac{2}{3}}$

399. $\frac{\frac{3}{4}+\frac{1}{2}}{\frac{5}{6}-\frac{2}{3}}$

400. $\frac{\frac{7}{8}-\frac{2}{3}}{\frac{1}{2}+\frac{3}{8}}$

401. $\frac{\frac{3}{4}-\frac{3}{5}}{\frac{1}{4}+\frac{2}{5}}$

Mixed Practice

In the following exercises, simplify.

402. $\frac{1}{2}+\frac{2}{3}\cdot\frac{5}{12}$
403. $\frac{1}{3}+\frac{2}{5}\cdot\frac{3}{4}$
404. $1-\frac{3}{5}\div\frac{1}{10}$
405. $1-\frac{5}{6}\div\frac{1}{12}$
406. $\frac{2}{3}+\frac{1}{6}+\frac{3}{4}$
407. $\frac{2}{3}+\frac{1}{4}+\frac{3}{5}$
408. $\frac{3}{8}-\frac{1}{6}+\frac{3}{4}$
409. $\frac{2}{5}+\frac{5}{8}-\frac{3}{4}$
410. $12\left(\frac{9}{20}-\frac{4}{15}\right)$
411. $8\left(\frac{15}{16}-\frac{5}{6}\right)$
412. $\dfrac{\frac{5}{8}+\frac{1}{6}}{\frac{19}{24}}$
413. $\dfrac{\frac{1}{6}+\frac{3}{10}}{\frac{14}{30}}$
414. $\left(\frac{5}{9}+\frac{1}{6}\right)\div\left(\frac{2}{3}-\frac{1}{2}\right)$
415. $\left(\frac{3}{4}+\frac{1}{6}\right)\div\left(\frac{5}{8}-\frac{1}{3}\right)$

In the following exercises, evaluate the given expression. Express your answers in simplified form, using improper fractions if necessary.

416. $x+\frac{1}{2}$ when
    a. $x=-\frac{1}{8}$
    b. $x=-\frac{1}{2}$
417. $x+\frac{2}{3}$ when
    a. $x=-\frac{1}{6}$
    b. $x=-\frac{5}{3}$
418. $x+\left(-\frac{5}{6}\right)$ when
    a. $x=\frac{1}{3}$
    b. $x=-\frac{1}{6}$
419. $x+\left(-\frac{11}{12}\right)$ when
    a. $x=\frac{11}{12}$

b. $x = \frac{3}{4}$

420. $x - \frac{2}{5}$ when

a. $x = \frac{3}{5}$

b. $x = -\frac{3}{5}$

421. $x - \frac{1}{3}$ when

a. $x = \frac{2}{3}$

b. $x = -\frac{2}{3}$

422. $\frac{7}{10} - w$ when

a. $w = \frac{1}{2}$

b. $w = -\frac{1}{2}$

423. $\frac{5}{12} - w$ when

a. $w = \frac{1}{4}$

b. $w = -\frac{1}{4}$

424. $4p^2q$ when $p = -\frac{1}{2}$ and $q = \frac{5}{9}$

425. $5m^2n$ when $m = -\frac{2}{5}$ and $n = \frac{1}{3}$

426. $2x^2y^3$ when $x = -\frac{2}{3}$ and $y = -\frac{1}{2}$

427. $8u^2v^3$ when $u = -\frac{3}{4}$ and $v = -\frac{1}{2}$

428. $\frac{u+v}{w}$ when $u = -4$, $v = -8$, $w = 2$

429. $\frac{m+n}{p}$ when $m = -6$, $n = -2$, $p = 4$

430. $\frac{a+b}{a-b}$ when $a = -3$, $b = 8$

431. $\frac{r-s}{r+s}$ when $r = 10$, $s = -5$

Everyday Math

432. **Decorating** Laronda is making covers for the throw pillows on her sofa. For each pillow cover, she needs $\frac{3}{16}$ yard of print fabric and $\frac{3}{8}$ yard of solid fabric. What is the total amount of fabric Laronda needs for each pillow cover?

433. **Baking** Vanessa is baking chocolate chip cookies and oatmeal cookies. She needs $1\frac{1}{4}$ cups of sugar for the chocolate chip cookies, and $1\frac{1}{8}$ cups for the oatmeal cookies How much sugar does she need altogether?

Writing Exercises

434. Explain why it is necessary to have a common denominator to add or subtract fractions.

435. Explain how to find the LCD of two fractions.

Self Check

(a) After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
add and subtract fractions with different denominators.			
identify and use fraction operations.			
use the order of operations to simplify complex fractions.			
evaluate variable expressions with fractions.			

(b) After looking at the checklist, do you think you are well prepared for the next section? Why or why not?

Contributors and Attributions

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CHAPTER OVERVIEW

4: Percentages and Decimals

4.1: Understand Percent
4.2: Solve General Applications of Percent
4.3: Decimals (Part 1)
4.4: Decimals (Part 2)
4.5: Decimal Operations (Part 1)
4.6: Decimal Operations (Part 2)

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4.1: Understand Percent

Learning Objectives

- Use the definition of percent
- Convert percents to fractions and decimals
- Convert decimals and fractions to percents

be prepared!

Before you get started, take this readiness quiz.

1. Translate “the ratio of 33 to 5” into an algebraic expression. If you missed this problem, review Table 2.4.3.
2. Write 3 5 as a decimal. If you missed this problem, review Example 5.5.1.
3. Write 0.62 as a fraction. If you missed this problem, review Example 5.1.4.

Use the Definition of Percent

How many cents are in one dollar? There are 100 cents in a dollar. How many years are in a century? There are 100 years in a century. Does this give you a clue about what the word “per cent” means? It is really two words, “per cent,” and means per one hundred. A **percent** is a ratio whose denominator is 100. We use the percent symbol %, to show percent.

Definition: Percent

A percent is a ratio whose denominator is 100.

According to data from the American Association of Community Colleges (2015), about 57% of community college students are female. This means 57 out of every 100 community college students are female, as Figure 4.1.1 shows. Out of the 100 squares on the grid, 57 are shaded, which we write as the ratio $\frac{57}{100}$.

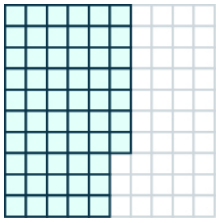


Figure 4.1.1 - Among every 100 community college students, 57 are female.

Similarly, 25% means a ratio of $\frac{25}{100}$, 3% means a ratio of $\frac{3}{100}$ and 100% means a ratio of $\frac{100}{100}$. In words, "one hundred percent" means the total 100% is $\frac{100}{100}$, and since $\frac{100}{100} = 1$, we see that 100% means 1 whole.

Example 4.1.1:

According to the Public Policy Institute of California (2010), 44% of parents of public school children would like their youngest child to earn a graduate degree. Write this percent as a ratio.

Solution

The amount we want to convert is 44%.	44%	
Write the percent as a ratio. Remember that percent means per 100.	$\frac{44}{100}$	(4.1.1)

Exercise 4.1.1:

Write the percent as a ratio. According to a survey, 89% of college students have a smartphone.

Answer

$\frac{89}{100}$

Exercise 4.1.2:

Write the percent as a ratio. A study found that 72% of U.S. teens send text messages regularly.

Answer

$\frac{72}{100}$

Example 4.1.2:

In 2007, according to a U.S. Department of Education report, 21 out of every 100 first-time freshmen college students at 4-year public institutions took at least one remedial course. Write this as a ratio and then as a percent.

Solution

The amount we want to convert is 21 out of 100.	21 out of 100	
Write as a ratio.	$\frac{21}{100}$	(4.1.2)
Convert the 21 per 100 to percent.	21%	

Exercise 4.1.3:

Write as a ratio and then as a percent: The American Association of Community Colleges reported that 62 out of 100 full-time community college students balance their studies with full-time or part time employment.

Answer

$\frac{62}{100}$, 62%

Exercise 4.1.4:

Write as a ratio and then as a percent: In response to a student survey, 41 out of 100 Santa Ana College students expressed a goal of earning an Associate's degree or transferring to a four-year college.

Answer

$\frac{41}{100}$, 41%

Convert Percents to Fractions and Decimals

Since percents are ratios, they can easily be expressed as fractions. Remember that percent means per 100, so the denominator of the fraction is 100.

HOW TO: CONVERT A PERCENT TO A FRACTION

Step 1. Write the percent as a ratio with the denominator 100.

Step 2. Simplify the fraction if possible.

Example 4.1.3:

Convert each percent to a fraction: (a) 36% (b) 125%

Solution

(a) 36%

Write as a ratio with denominator 100.	$\frac{36}{100}$	(4.1.3)
Simplify.	$\frac{9}{25}$	(4.1.4)

(b) 125%

Write as a ratio with denominator 100.	$\frac{125}{100}$	(4.1.5)
Simplify.	$\frac{5}{4}$	(4.1.6)

Exercise 4.1.5:

Convert each percent to a fraction: (a) 48% (b) 110%

Answer a

$\frac{12}{25}$

Answer b

$\frac{11}{10}$

Exercise 4.1.6:

Convert each percent to a fraction: (a) 64% (b) 150%

Answer a

$\frac{16}{25}$

Answer b

$\frac{3}{2}$

The previous example shows that a percent can be greater than 1. We saw that 125% means $\frac{125}{100}$, or $\frac{5}{4}$. These are improper fractions, and their values are greater than one.

Example 4.1.4:

Convert each percent to a fraction: (a) 24.5% (b) $33\frac{1}{3}\%$

Solution

(a) 24.5%

Write as a ratio with denominator 100.	$\frac{24.5}{100}$	(4.1.7)
Clear the decimal by multiplying numerator and denominator by 10.	$\frac{24.5(10)}{100(10)}$	(4.1.8)
Multiply.	$\frac{245}{1000}$	(4.1.9)
Rewrite showing common factors.	$\frac{5 \cdot 49}{5 \cdot 200}$	(4.1.10)
Simplify.	$\frac{49}{200}$	(4.1.11)

(b) $33\frac{1}{3}\%$

Write as a ratio with denominator 100.	$\frac{33\frac{1}{3}}{100}$	(4.1.12)
Write the numerator as an improper fraction.	$\frac{\frac{100}{3}}{100}$	(4.1.13)
Rewrite as fraction division, replacing 100 with $\frac{100}{1}$.	$\frac{100}{3} \div \frac{100}{1}$	(4.1.14)
Multiply by the reciprocal.	$\frac{100}{3} \cdot \frac{1}{100}$	(4.1.15)
Simplify.	$\frac{1}{3}$	(4.1.16)

Exercise 4.1.7:

Convert each percent to a fraction: (a) 64.4% (b) $66\frac{2}{3}\%$

Answer a

$\frac{161}{250}$

Answer b

$\frac{2}{3}$

Exercise 4.1.8:

Convert each percent to a fraction: (a) 42.5% (b) $8\frac{3}{4}\%$

Answer a

$\frac{113}{250}$

Answer b

$\frac{7}{80}$

In Decimals, we learned how to convert fractions to decimals. To convert a percent to a decimal, we first convert it to a fraction and then change the fraction to a decimal.

HOW TO: CONVERT A PERCENT TO A DECIMAL

Step 1. Write the percent as a ratio with the denominator 100.

Step 2. Convert the fraction to a decimal by dividing the numerator by the denominator.

Example 4.1.5:

Convert each percent to a decimal: (a) 6% (b) 78%

Solution

Because we want to change to a decimal, we will leave the fractions with denominator 100 instead of removing common factors.

(a) 6%

Write as a ratio with denominator 100.	$\frac{6}{100}$	(4.1.17)
Change the fraction to a decimal by dividing the numerator by the denominator.	0.06	

(b) 78%

Write as a ratio with denominator 100.	$\frac{78}{100}$	(4.1.18)
Change the fraction to a decimal by dividing the numerator by the denominator.	0.78	

Exercise 4.1.9:

Convert each percent to a decimal: (a) 9% (b) 87%

Answer a

0.09

Answer b

0.87

Exercise 4.1.10:

Convert each percent to a decimal: (a) 3% (b) 91%

Answer a

0.03

Answer b

0.91

Example 4.1.6:

Convert each percent to a decimal: (a) 135% (b) 12.5%

Solution

Write as a ratio with denominator 100.	$\frac{135}{100}$	(4.1.19)
Change the fraction to a decimal by dividing the numerator by the denominator.	1.35	

(b) 12.5%

Write as a ratio with denominator 100.	$\frac{12.5}{100}$	(4.1.20)
Change the fraction to a decimal by dividing the numerator by the denominator.	0.125	

Exercise 4.1.11:

Convert each percent to a decimal: (a) 115% (b) 23.5%

Answer a

1.15

Answer b

0.235

Exercise 4.1.12:

Convert each percent to a decimal: (a) 123% (b) 16.8%

Answer a

1.23

Answer b

0.168

Let's summarize the results from the previous examples in Table 4.1.1, and look for a pattern we could use to quickly convert a percent number to a decimal number.

Table 4.1.1

Percent	Decimal
6%	0.06
78%	0.78
135%	1.35
12.5%	0.125

Do you see the pattern? To convert a percent number to a decimal number, we move the decimal point two places to the left and remove the % sign. (Sometimes the decimal point does not appear in the percent number, but just like we can think of the integer 6 as 6.0, we can think of 6% as 6.0%.) Notice that we may need to add zeros in front of the number when moving the decimal to the left.

Figure 4.1.2 uses the percents in Table 4.1.1 and shows visually how to convert them to decimals by moving the decimal point two places to the left.

Percent	Decimal
006.%	0.06
078.%	0.78
135.%	1.35
012.5%	0.125

Figure 4.1.2

Example 4.1.7:

Among a group of business leaders, 77% believe that poor math and science education in the U.S. will lead to higher unemployment rates. Convert the percent to: (a) a fraction (b) a decimal

Solution

(a)

Write as a ratio with denominator 100.	$\frac{77}{100}$	(4.1.21)

(b)

Change the fraction to a decimal by dividing the numerator by the denominator.	0.77

Exercise 4.1.13:

Twitter's share of web traffic jumped 24% when one celebrity tweeted live on air. Convert the percent to: (a) a fraction and (b) a decimal.

Answer a

$\frac{6}{25}$

Answer b

0.24

Exercise 4.1.14:

The U.S. Census estimated that in 2013, 44% of the population of Boston age 25 or older have a bachelor's or higher degrees. Convert the percent to: (a) a fraction and (b) a decimal.

Answer a

$\frac{22}{50}$

Answer b

0.44

Example 4.1.8:

There are four suits of cards in a deck of cards—hearts, diamonds, clubs, and spades. The probability of randomly choosing a heart from a shuffled deck of cards is 25%. Convert the percent to: (a) a fraction (b) a decimal.



Figure 4.1.3 - (credit: Riles32807, Wikimedia Commons)

Solution

(a)

Write as a ratio with denominator 100.	$\frac{25}{100}$	(4.1.22)
Simplify.	$\frac{1}{4}$	(4.1.23)

(b)

Change the fraction to a decimal by dividing the numerator by the denominator.	0.25

Exercise 4.1.15:

The probability that it will rain Monday is 30%. Convert the percent to: (a) a fraction, and (b) a decimal.

Answer a

$\frac{3}{10}$

Answer b

0.3

Exercise 4.1.16:

The probability of getting heads three times when tossing a coin three times is 12.5%. Convert the percent to: (a) a fraction, and (b) a decimal.

Answer a

$\frac{12.5}{100}$

Answer b

0.125

Convert Decimals and Fractions to Percents

To convert a decimal to a percent, remember that percent means per hundred. If we change the decimal to a fraction whose denominator is 100, it is easy to change that fraction to a percent.

HOW TO: CONVERT A DECIMAL TO A PERCENT

Step 1. Write the decimal as a fraction.

Step 2. If the denominator of the fraction is not 100, rewrite it as an equivalent fraction with denominator 100.

Step 3. Write this ratio as a percent.

Example 4.1.9:

Convert each decimal to a percent: (a) 0.05 (b) 0.83

Solution

(a) 0.05

Write as a fraction. The denominator is 100.	$\frac{5}{100}$	(4.1.24)
Write this ratio as a percent.	5%	

(b)

The denominator is 100.	$\frac{83}{100}$	(4.1.25)
Write this ratio as a percent.	83%	

Exercise 4.1.17:

Convert each decimal to a percent: (a) 0.01 (b) 0.17.

Answer a

1%

Answer b

17%

Exercise 4.1.18:

Convert each decimal to a percent: (a) 0.04 (b) 0.41.

Answer a

4%

Answer b

41%

To convert a mixed number to a percent, we first write it as an improper fraction.

Example 4.1.10:

Convert each decimal to a percent: (a) 1.05 (b) 0.075

Solution

(a) 1.05

Write as a fraction.	$1\frac{5}{100}$	(4.1.26)
Write as an improper fraction. The denominator is 100.	$\frac{105}{100}$	(4.1.27)
Write this ratio as a percent.	105%	

Notice that since 1.05 > 1, the result is more than 100%.

(b) 0.075

Write as a fraction.	$\frac{75}{1,000}$	(4.1.28)
Divide the numerator and denominator by 10, so that the denominator is 100.	$\frac{7.5}{100}$	(4.1.29)
Write this ratio as a percent.	7.5%	

Exercise 4.1.19:

Convert each decimal to a percent: (a) 1.75 (b) 0.0825

Answer a

175%

Answer b

8.25%

Exercise 4.1.20:

Convert each decimal to a percent: (a) 2.25 (b) 0.0925

Answer a

225%

Answer b

9.25%

Let's summarize the results from the previous examples in Table 6.20 so we can look for a pattern.

Table 4.1.2

Decimal	Percent
0.05	5%
0.83	83%
1.05	105%
0.075	7.5%

Do you see the pattern? To convert a decimal to a percent, we move the decimal point two places to the right and then add the percent sign.

Figure 4.1.4 uses the decimal numbers in Table 4.1.2 and shows visually to convert them to percents by moving the decimal point two places to the right and then writing the % sign.

Percent	Decimal
006.%	0.06
078.%	0.78
135.%	1.35
012.5%	0.125

Figure 4.1.4

In Decimals, we learned how to convert fractions to decimals. Now we also know how to change decimals to percents. So to convert a fraction to a percent, we first change it to a decimal and then convert that decimal to a percent.

HOW TO: CONVERT A FRACTION TO A PERCENT

Step 1. Convert the fraction to a decimal.

Step 2. Convert the decimal to a percent.

Example 4.1.11:

Convert each fraction or mixed number to a percent: (a) $\frac{3}{4}$ (b) $\frac{11}{8}$ (c) $2\frac{1}{5}$

Solution

To convert a fraction to a decimal, divide the numerator by the denominator.

(a)

Change to a decimal.	$\frac{3}{4}$	(4.1.30)

Write as a percent by moving the decimal two places.	0.75	
	75%	

(b)

Change to a decimal.	$\frac{11}{8}$	(4.1.31)
Write as a percent by moving the decimal two places.	1.375	
	137.5%	

(c)

Write as an improper fraction.	$2\frac{1}{5}$	(4.1.32)
Change to a decimal.	$\frac{11}{5}$	(4.1.33)
Write as a percent.	2.20	
	220%	

Notice that we needed to add zeros at the end of the number when moving the decimal two places to the right.

? Exercise 4.1.21:

Convert each fraction or mixed number to a percent: (a) $\frac{5}{8}$ (b) $\frac{11}{4}$ (c) $3\frac{2}{5}$

Answer a

62.5%

Answer b

275%

Answer c

340%

? Exercise 4.1.22:

Convert each fraction or mixed number to a percent: (a) $\frac{7}{8}$ (b) $\frac{9}{4}$ (c) $1\frac{3}{5}$

Answer a

87.5%

Answer b

225%

Answer c

160%

Sometimes when changing a fraction to a decimal, the division continues for many decimal places and we will round off the quotient. The number of decimal places we round to will depend on the situation. If the decimal involves money, we round to the hundredths place. For most other cases in this book we will round the number to the nearest thousandth, so the percent will be rounded to the nearest tenth.

Example 4.1.12:

Convert $\frac{5}{7}$ to a percent.

Solution

To change a fraction to a decimal, we divide the numerator by the denominator.

Change to a decimal—rounding to the nearest thousandth.	0.714
Write as a percent.	71.4%

Exercise 4.1.23:

Convert the fraction to a percent: $\frac{3}{7}$

Answer

42.9%

Exercise 4.1.24:

Convert the fraction to a percent: $\frac{4}{7}$

Answer

57.1%

When we first looked at fractions and decimals, we saw that fractions converted to a repeating decimal. When we converted the fraction $\frac{4}{3}$ to a decimal, we wrote the answer as $1.\overline{3}$. We will use this same notation, as well as fraction notation, when we convert fractions to percents in the next example.

Example 4.1.13:

An article in a medical journal claimed that approximately $\frac{1}{3}$ of American adults are obese. Convert the fraction $\frac{1}{3}$ to a percent.

Solution

Change to a decimal.	$3\overline{)1.00}$ $0.33\ldots$ 9 10 9 1
Write as a repeating decimal.	0.333 …
Write as a percent.	$33\frac{1}{3}\%$

We could also write the percent as $33.\overline{3}\%$.

Exercise 4.1.25:

According to the U.S. Census Bureau, about $\frac{1}{9}$ of United States housing units have just 1 bedroom. Convert the fraction to a percent.

Answer

$11.\overline{1}\%$, or $11\frac{1}{9}\%$

Exercise 4.1.26:

According to the U.S. Census Bureau, about $\frac{1}{6}$ of Colorado residents speak a language other than English at home. Convert the fraction to a percent.

Answer

$16.\overline{6}\%$, or $16\frac{2}{3}\%$

Practice Makes Perfect

Use the Definition of Percents

In the following exercises, write each percent as a ratio.

1. In 2014, the unemployment rate for those with only a high school degree was 6.0%.
2. In 2015, among the unemployed, 29% were long-term unemployed.
3. The unemployment rate for those with Bachelor's degrees was 3.2% in 2014.
4. The unemployment rate in Michigan in 2014 was 7.3%.

In the following exercises, write as (a) a ratio and (b) a percent.

5. 57 out of 100 nursing candidates received their degree at a community college.
6. 80 out of 100 firefighters and law enforcement officers were educated at a community college.
7. 42 out of 100 first-time freshmen students attend a community college.
8. 71 out of 100 full-time community college faculty have a master's degree.

Convert Percents to Fractions and Decimals

In the following exercises, convert each percent to a fraction and simplify all fractions.

9. 4%
10. 8%
11. 17%
12. 19%
13. 52%
14. 78%
15. 125%
16. 135%
17. 37.5%
18. 42.5%
19. 18.4%
20. 46.4%
21. $9\frac{1}{2}\%$

22. $8\frac{1}{2}$ %
23. $5\frac{1}{3}$ %
24. $6\frac{2}{3}$ %

In the following exercises, convert each percent to a decimal.

25. 5%
26. 9%
27. 1%
28. 2%
29. 63%
30. 71%
31. 40%
32. 50%
33. 115%
34. 125%
35. 150%
36. 250%
37. 21.4%
38. 39.3%
39. 7.8%
40. 6.4%

In the following exercises, convert each percent to (a) a simplified fraction and (b) a decimal.

41. In 2010, 1.5% of home sales had owner financing. (Source: Bloomberg Businessweek, 5/ 23–29/2011)
42. In 2000, 4.2% of the United States population was of Asian descent. (Source: www.census.gov)
43. According to government data, in 2013 the number of cell phones in India was 70.23% of the population.
44. According to the U.S. Census Bureau, among Americans age 25 or older who had doctorate degrees in 2014, 37.1% are women.
45. A couple plans to have two children. The probability they will have two girls is 25%.
46. Javier will choose one digit at random from 0 through 9. The probability he will choose 3 is 10%.
47. According to the local weather report, the probability of thunderstorms in New York City on July 15 is 60%.
48. A club sells 50 tickets to a raffle. Osbaldo bought one ticket. The probability he will win the raffle is 2%.

Convert Decimals and Fractions to Percents

In the following exercises, convert each decimal to a percent.

49. 0.01
50. 0.03
51. 0.18
52. 0.15
53. 1.35
54. 1.56
55. 3
56. 4
57. 0.009
58. 0.008
59. 0.0875
60. 0.0625
61. 1.5
62. 2.2
63. 2.254
64. 2.317

In the following exercises, convert each fraction to a percent.

65. $\frac{1}{4}$
66. $\frac{1}{5}$
67. $\frac{3}{8}$
68. $\frac{5}{8}$
69. $\frac{7}{4}$
70. $\frac{9}{8}$
71. $6\frac{4}{5}$
72. $5\frac{1}{4}$
73. $\frac{5}{12}$
74. $\frac{11}{12}$
75. $2\frac{2}{3}$
76. $1\frac{2}{3}$
77. $\frac{3}{7}$
78. $\frac{6}{7}$
79. $\frac{5}{9}$
80. $\frac{4}{9}$

In the following exercises, convert each fraction to a percent.

81. $\frac{1}{4}$ of washing machines needed repair.
82. $\frac{1}{5}$ of dishwashers needed repair.

In the following exercises, convert each fraction to a percent.

83. According to the National Center for Health Statistics, in 2012, 7 20 of American adults were obese.
84. The U.S. Census Bureau estimated that in 2013, 85% of Americans lived in the same house as they did 1 year before.

In the following exercises, complete the table.

85. Table 6.26

Fraction	Decimal	Percent
$\frac{1}{2}$ (4.1.34)		
	0.45	
		18%
$\frac{1}{3}$ (4.1.35)		
	0.0008	

Fraction	Decimal	Percent
2		

86. Table 6.27

Fraction	Decimal	Percent
$\frac{1}{4}$ (4.1.36)		
	0.65	
		22%
$\frac{2}{3}$ (4.1.37)		
	0.0004	
3		

Everyday Math

87. Sales tax Felipa says she has an easy way to estimate the sales tax when she makes a purchase. The sales tax in her city is 9.05%. She knows this is a little less than 10%.
    a. Convert 10% to a fraction.
    b. Use your answer from (a) to estimate the sales tax Felipa would pay on a $95 dress.
88. Savings Ryan has 25% of each paycheck automatically deposited in his savings account.
    a. Write 25% as a fraction.
    b. Use your answer from (a) to find the amount that goes to savings from Ryan's $2,400 paycheck.
89. Amelio is shopping for textbooks online. He found three sellers that are offering a book he needs for the same price, including shipping. To decide which seller to buy from he is comparing their customer satisfaction ratings. The ratings are given in the chart.

Seller	Rating
A	4/5
B	3.5/4
C	85%

90. Write seller C's rating as a fraction and a decimal.
91. Write seller B's rating as a percent and a decimal.
92. Write seller A's rating as a percent and a decimal.
93. Which seller should Amelio buy from and why?

Writing Exercises

94. Convert 25%, 50%, 75%, and 100% to fractions. Do you notice a pattern? Explain what the pattern is.
95. Convert $\frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \frac{4}{10}, \frac{5}{10}, \frac{6}{10}, \frac{7}{10}, \frac{8}{10}$, and $\frac{9}{10}$ to percents. Do you notice a pattern? Explain what the pattern is.
96. When the Szetos sold their home, the selling price was 500% of what they had paid for the house 30 years ago. Explain what 500% means in this context.
97. According to cnn.com, cell phone use in 2008 was 600% of what it had been in 2001. Explain what 600% means in this context.

Self Check

(a) After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
use the definition of percent.			
convert percents to fractions and decimals.			
convert decimals and fractions to percents.			

(b) If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them. What did you do to become confident of your ability to do these things? Be specific.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is important to make sure you have a strong foundation before you move on. Who can you ask for help? Your fellow classmates and instructor are good resources. Is there a place on campus where math tutors are available? Can your study skills be improved?

…no—I don't get it! This is a warning sign and you must not ignore it. You should get help right away or you will quickly be overwhelmed. See your instructor as soon as you can to discuss your situation. Together you can come up with a plan to get you the help you need.

Contributors and Attributions

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4.2: Solve General Applications of Percent

Learning Objectives

- Translate and solve basic percent equations
- Solve applications of percent
- Find percent increase and percent decrease

be prepared!

Before you get started, take this readiness quiz.

1. Translate and solve: $\frac{3}{4}$ of x is 24. If you missed this problem, review Example 4.13.11.
2. Simplify: (4.5)(2.38). If you missed this problem, review Example 5.3.5.
3. Solve: 3.5 = 0.7n. If you missed this problem, review Example 5.7.4.

Translate and Solve Basic Percent Equations

We will solve percent equations by using the methods we used to solve equations with fractions or decimals. In the past, you may have solved percent problems by setting them up as proportions. That was the best method available when you did not have the tools of algebra. Now as a prealgebra student, you can translate word sentences into algebraic equations, and then solve the equations.

We'll look at a common application of percent—tips to a server at a restaurant—to see how to set up a basic percent application.

When Aolani and her friends ate dinner at a restaurant, the bill came to $80. They wanted to leave a 20% tip. What amount would the tip be? To solve this, we want to find what *amount* is 20% of $80. The $80 is called the *base*. The amount of the tip would be 0.20(80), or $16 See Figure 4.2.1. To find the amount of the tip, we multiplied the percent by the base.

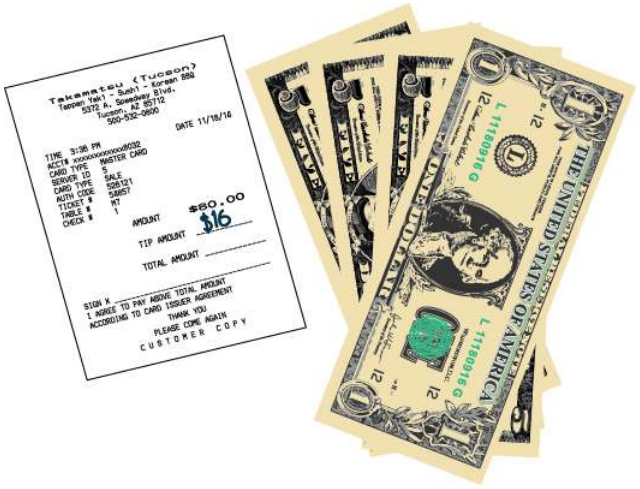


Figure 4.2.1 - A 20% tip for an $80 restaurant bill comes out to $16.

In the next examples, we will find the amount. We must be sure to change the given percent to a decimal when we translate the words into an equation.

Example 4.2.1:

What number is 35% of 90?

Solution

Translate into algebra. Let n = the number.	$\underbrace{\text{What number}}_{n} \quad \underbrace{\text{is}}_{=} \quad \underbrace{35\%}_{0.35} \quad \underbrace{\text{of}}_{\cdot} \quad \underbrace{90?}_{90}$
Multiply.	n = 31.5

31.5 is 35% of 90

Exercise 4.2.1:

What number is 45% of 80?

Answer

36

Exercise 4.2.2:

What number is 55% of 60?

Answer

33

Example 4.2.2:

125% of 28 is what number?

Solution

Translate into algebra. Let a = the number.	$\underbrace{125\%}_{1.25} \quad \underbrace{\text{of}}_{\cdot} \quad \underbrace{28}_{28} \quad \underbrace{\text{is}}_{=} \quad \underbrace{\text{what number?}}_{a}$
Multiply.	35 = a

125% of 28 is 35

Remember that a percent over 100 is a number greater than 1. We found that 125% of 28 is 35, which is greater than 28.

Exercise 4.2.3:

150% of 78 is what number?

Answer

117

Exercise 4.2.4:

175% of 72 is what number?

Answer

126

In the next examples, we are asked to find the base.

Example 4.2.3:

Translate and solve: 36 is 75% of what number?

Solution

Translate. Let b = the number.	$\underbrace{36}_{36}\ \underbrace{\text{is}}_{=}\ \underbrace{75\%}_{0.75}\ \underbrace{\text{of}}_{.}\ \underbrace{\text{what number?}}_{b}$
Divide both sides by 0.75.	$\frac{36}{0.75}=\frac{0.75b}{0.75}$ (4.2.1)
Simplify.	48 = b

36 is 75% of 48

Exercise 4.2.5:

17 is 25% of what number?

Answer

68

Exercise 4.2.6:

40 is 62.5% of what number?

Answer

64

Example 4.2.4:

6.5% of what number is $1.17?

Solution

Translate. Let b = the number.	$\underbrace{6.5\%}_{0.065}\ \underbrace{\text{of}}_{.}\ \underbrace{\text{what number}}_{b}\ \underbrace{\text{is}}_{=}\ \underbrace{\$1.17?}_{1.17}$
Divide both sides by 0.065.	$\frac{0.065n}{0.065}=\frac{1.17}{0.065}$ (4.2.2)
Simplify.	n = 18

6.5% of $18 is $1.17.

Exercise 4.2.7:

7.5% of what number is $1.95?

Answer

$26

Exercise 4.2.8:

8.5% of what number is $3.06?

Answer

$36

In the next examples, we will solve for the percent.

Example 4.2.5:

What percent of 36 is 9?

Solution

Translate into algebra. Let p = the percent.	$\underbrace{p}_{\text{What percent}} \underbrace{\cdot}_{\text{of}} \underbrace{36}_{36} \underbrace{=}_{\text{is}} \underbrace{9}_{9?}$
Divide by 36.	$\frac{36p}{36} = \frac{9}{36}$ (4.2.3)
Simplify.	$p = \frac{1}{4}$ (4.2.4)
Convert to decimal form.	p = 0.25
Convert to percent.	p = 25%

25% of 36 is 9.

Exercise 4.2.9:

What percent of 76 is 57?

Answer

75%

Exercise 4.2.10:

What percent of 120 is 96?

Answer

80%

Example 4.2.6:

144 is what percent of 96?

Solution

Translate into algebra. Let p = the percent.	$\underbrace{144}_{144} \underbrace{=}_{\text{is}} \underbrace{p}_{\text{what percent}} \underbrace{\cdot}_{\text{of}} \underbrace{96}_{96?}$
Divide by 96.	$\frac{144}{96} = \frac{96p}{96}$ (4.2.5)

Simplify.	1.5 = p
Convert to percent.	150% = p

144 is 150% of 96.

Exercise 4.2.11:

110 is what percent of 88?

Answer

125%

Exercise 4.2.12:

126 is what percent of 72?

Answer

175%

Solve Applications of Percent

Many applications of percent occur in our daily lives, such as tips, sales tax, discount, and interest. To solve these applications we'll translate to a basic percent equation, just like those we solved in the previous examples in this section. Once you translate the sentence into a percent equation, you know how to solve it.

We will update the strategy we used in our earlier applications to include equations now. Notice that we will translate a sentence into an equation.

HOW TO: SOLVE AN APPLICATION

- Step 1. Identify what you are asked to find and choose a variable to represent it.
- Step 2. Write a sentence that gives the information to find it.
- Step 3. Translate the sentence into an equation.
- Step 4. Solve the equation using good algebra techniques.
- Step 5. Check the answer in the problem and make sure it makes sense.
- Step 6. Write a complete sentence that answers the question.

Now that we have the strategy to refer to, and have practiced solving basic percent equations, we are ready to solve percent applications. Be sure to ask yourself if your final answer makes sense—since many of the applications we'll solve involve everyday situations, you can rely on your own experience.

Example 4.2.7:

Dezohn and his girlfriend enjoyed a dinner at a restaurant, and the bill was $68.50. They want to leave an 18% tip. If the tip will be 18% of the total bill, how much should the tip be?

Solution

What are you asked to find?	the amount of the tip
Choose a variable to represent it.	Let t = amount of tip.
Write a sentence that give the information to find it.	The tip is 18% of the total bill.

Translate the sentence into an equation.	$\underbrace{t}_{\text{The tip}} = \underbrace{0.18}_{\text{is 18\%}} \cdot \underbrace{\$68.50}_{\text{of \$68.50}}$
Multiply.	t = 12.33
Check. Is this answer reasonable?	If we approximate the bill to \$70 and the percent to 20%, we would have a tip of \$14. So a tip of \$12.33 seems reasonable.
Write a complete sentence that answers the question.	The couple should leave a tip of \$12.33.

? Exercise 4.2.13:

Cierra and her sister enjoyed a special dinner in a restaurant, and the bill was \$81.50. If she wants to leave 18% of the total bill as her tip, how much should she leave?

Answer

\$14.67

? Exercise 4.2.14:

Kimngoc had lunch at her favorite restaurant. She wants to leave 15% of the total bill as her tip. If her bill was \$14.40, how much will she leave for the tip?

Answer

\$2.16

✔ Example 4.2.8:

The label on Masao's breakfast cereal said that one serving of cereal provides 85 milligrams (mg) of potassium, which is 2% of the recommended daily amount. What is the total recommended daily amount of potassium?

Nutrition Facts

Serving Size: 1 cup (47g)
Servings Per Container: About 7

Amount Per Serving	Cereal	With Milk
Calories	180	230
Calories from Fat	10	20
	% Daily Value*	
Total Fat 1g	**2%**	**2%**
Saturated Fat 0g	**0%**	**0%**
Trans Fat 0g		
Polyunsaturated Fat 0.5g		
Monounsaturated Fat 0.5g		
Cholesterol 0mg	**0%**	**0%**
Sodium 190mg	**8%**	**11%**
Potassium 85mg	**2%**	**8%**
Total Carbohydrate 40g	**13%**	**15%**
Dietary Fiber 1g	**4%**	**4%**
Sugars 8g		
Protein 3g		

Solution

What are you asked to find?	the total amount of potassium recommended
Choose a variable to represent it.	Let a = total amount of potassium.

Write a sentence that gives the information to find it.	85 mg is 2% of the total amount.
Translate the sentence into an equation.	$\underbrace{85}_{85}\ \underbrace{\text{mg}\ \text{is}}_{=}\ \underbrace{2\%}_{0.02}\ \underbrace{\text{of}}_{\cdot}\ \underbrace{a}_{a}\ ?$
Divide both sides by 0.02.	$\frac{85}{0.02}=\frac{0.02a}{0.02}$ (4.2.6)
Simplify.	4,250 =a
Check: Is this answer reasonable?	Yes. 2% is a small percent and 85 is a small part of 4,250.
Write a complete sentence that answers the question.	The amount of potassium that is recommended is 4250 mg.

? Exercise 4.2.15:

One serving of wheat square cereal has 7 grams of fiber, which is 29% of the recommended daily amount. What is the total recommended daily amount of fiber?

Answer

24.1 grams

? Exercise 4.2.16:

One serving of rice cereal has 190 mg of sodium, which is 8% of the recommended daily amount. What is the total recommended daily amount of sodium?

Answer

2,375 mg

✓ Example 4.2.9:

Mitzi received some gourmet brownies as a gift. The wrapper said each brownie was 480 calories, and had 240 calories of fat. What percent of the total calories in each brownie comes from fat?

Solution

What are you asked to find?	the percent of the total calories from fat
Choose a variable to represent it.	Let p = percent from fat.
Write a sentence that gives the information to find it.	What percent of 480 is 240?
Translate the sentence into an equation.	$\underbrace{\text{What percent}}_{p}\ \underbrace{\text{of}}_{\cdot}\ \underbrace{480}_{480}\ \underbrace{\text{is}}_{=}\ \underbrace{240?}_{240}$
Divide both sides by 480.	$\frac{p\cdot 480}{480}=\frac{240}{480}$ (4.2.7)
Simplify.	p = 0.5
Convert to percent form.	p = 50%
Check. Is this answer reasonable?	Yes. 240 is half of 480, so 50% makes sense.
Write a complete sentence that answers the question.	Of the total calories in each brownie, 50% is fat.

Exercise 4.2.17:

Veronica is planning to make muffins from a mix. The package says each muffin will be 230 calories and 60 calories will be from fat. What percent of the total calories is from fat? (Round to the nearest whole percent.)

Answer

26%

Exercise 4.2.18:

The brownie mix Ricardo plans to use says that each brownie will be 190 calories, and 70 calories are from fat. What percent of the total calories are from fat?

Answer

37%

Find Percent Increase and Percent Decrease

People in the media often talk about how much an amount has increased or decreased over a certain period of time. They usually express this increase or decrease as a percent.

To find the **percent increase**, first we find the amount of increase, which is the difference between the new amount and the original amount. Then we find what percent the amount of increase is of the original amount.

how to: find percent increase

Step 1. Find the amount of increase.

- increase = new amount − original amount

Step 2. Find the percent increase as a percent of the original amount.

Example 4.2.10:

In 2011, the California governor proposed raising community college fees from $26 per unit to $36 per unit. Find the percent increase. (Round to the nearest tenth of a percent.)

Solution

What are you asked to find?	the percent increase
Choose a variable to represent it.	Let p = percent.
Find the amount of increase.	$\underbrace{36}_{\text{new amount}} - \underbrace{26}_{\text{original amount}} = \underbrace{10}_{\text{increase}}$
Find the percent increase.	The increase is what percent of the original amount?
Translate to an equation.	$\underbrace{10}_{10} \underbrace{\text{ is }}_{=} \underbrace{\text{ what percent }}_{p} \underbrace{\text{ of }}_{\cdot} \underbrace{26?}_{26}$
Divide both sides by 26.	$\frac{10}{26} = \frac{26p}{26}$ (4.2.8)
Round to the nearest thousandth.	0.384 = p
Convert to percent form.	38.4% = p

Write a complete sentence.	The new fees represent a 38.4% increase over the old fees.

Exercise 4.2.19:

In 2011, the IRS increased the deductible mileage cost to 55.5 cents from 51 cents. Find the percent increase. (Round to the nearest tenth of a percent.)

Answer

8.8%

Exercise 4.2.20:

In 1995, the standard bus fare in Chicago was $1.50. In 2008, the standard bus fare was $2.25. Find the percent increase. (Round to the nearest tenth of a percent.)

Answer

50%

Finding the **percent decrease** is very similar to finding the percent increase, but now the amount of decrease is the difference between the original amount and the final amount. Then we find what percent the amount of decrease is of the original amount.

HOW TO: FIND PERCENT DECREASE

Step 1. Find the amount of decrease.

- decrease = original amount − new amount

Step 2. Find the percent decrease as a percent of the original amount.

Example 4.2.11:

The average price of a gallon of gas in one city in June 2014 was $3.71. The average price in that city in July was $3.64. Find the percent decrease.

Solution

The average price of a gallon of gas in one city in June 2014 was $3.71. The average price in that city in July was $3.64. Find the percent decrease.

What are you asked to find?	the percent decrease
Choose a variable to represent it.	Let p = percent.
Find the amount of decrease.	$\underbrace{3.71}_{\text{original amount}} - \underbrace{3.64}_{\text{new amount}} = \underbrace{0.07}_{\text{increase}}$
Find the percent of decrease.	The decrease is what percent of the original amount?
Translate to an equation.	$\underbrace{0.07}_{0.07} \underbrace{\text{is}}_{=} \underbrace{\text{what percent}}_{p} \underbrace{\text{of}}_{\cdot} \underbrace{3.71}_{3.71}$
Divide both sides by 3.71.	$\frac{0.07}{3.71} = \frac{3.71p}{3.71}$ (4.2.9)
Round to the nearest thousandth.	0.019 = p

Convert to percent form.	1.9% = p
Write a complete sentence.	The price of gas decreased 1.9%.

Exercise 4.2.21:

The population of one city was about 672,000 in 2010. The population of the city is projected to be about 630,000 in 2020. Find the percent decrease. (Round to the nearest tenth of a percent.)

Answer

6.3%

Exercise 4.2.22:

Last year Sheila's salary was $42,000. Because of furlough days, this year her salary was $37,800. Find the percent decrease. (Round to the nearest tenth of a percent.)

Answer

10%

ACCESS ADDITIONAL ONLINE RESOURCES

Percent Increase and Percent Decrease Visualization

Practice Makes Perfect

Translate and Solve Basic Percent Equations

In the following exercises, translate and solve.

98. What number is 45% of 120?
99. What number is 65% of 100?
100. What number is 24% of 112?
101. What number is 36% of 124?
102. 250% of 65 is what number?
103. 150% of 90 is what number?
104. 800% of 2,250 is what number?
105. 600% of 1,740 is what number?
106. 28 is 25% of what number?
107. 36 is 25% of what number?
108. 81 is 75% of what number?
109. 93 is 75% of what number?
110. 8.2% of what number is $2.87?
111. 6.4% of what number is $2.88?
112. 11.5% of what number is $108.10?
113. 12.3% of what number is $92.25?
114. What percent of 260 is 78?
115. What percent of 215 is 86?
116. What percent of 1,500 is 540?
117. What percent of 1,800 is 846?
118. 30 is what percent of 20?
119. 50 is what percent of 40?
120. 840 is what percent of 480?

121. 790 is what percent of 395?

Solve Applications of Percents

In the following exercises, solve the applications of percents.

122. Geneva treated her parents to dinner at their favorite restaurant. The bill was $74.25. She wants to leave 16% of the total bill as a tip. How much should the tip be?
123. When Hiro and his co-workers had lunch at a restaurant the bill was $90.50. They want to leave 18% of the total bill as a tip. How much should the tip be?
124. Trong has 12% of each paycheck automatically deposited to his savings account. His last paycheck was $2,165. How much money was deposited to Trong's savings account?
125. Cherise deposits 8% of each paycheck into her retirement account. Her last paycheck was $1,485. How much did Cherise deposit into her retirement account?
126. One serving of oatmeal has 8 grams of fiber, which is 33% of the recommended daily amount. What is the total recommended daily amount of fiber?
127. One serving of trail mix has 67 grams of carbohydrates, which is 22% of the recommended daily amount. What is the total recommended daily amount of carbohydrates?
128. A bacon cheeseburger at a popular fast food restaurant contains 2,070 milligrams (mg) of sodium, which is 86% of the recommended daily amount. What is the total recommended daily amount of sodium?
129. A grilled chicken salad at a popular fast food restaurant contains 650 milligrams (mg) of sodium, which is 27% of the recommended daily amount. What is the total recommended daily amount of sodium?
130. The nutrition fact sheet at a fast food restaurant says the fish sandwich has 380 calories, and 171 calories are from fat. What percent of the total calories is from fat?
131. The nutrition fact sheet at a fast food restaurant says a small portion of chicken nuggets has 190 calories, and 114 calories are from fat. What percent of the total calories is from fat?
132. Emma gets paid $3,000 per month. She pays $750 a month for rent. What percent of her monthly pay goes to rent?
133. Dimple gets paid $3,200 per month. She pays $960 a month for rent. What percent of her monthly pay goes to rent?

Find Percent Increase and Percent Decrease

In the following exercises, find the percent increase or percent decrease.

134. Tamanika got a raise in her hourly pay, from $15.50 to $17.55. Find the percent increase.
135. Ayodele got a raise in her hourly pay, from $24.50 to $25.48. Find the percent increase.
136. Annual student fees at the University of California rose from about $4,000 in 2000 to about $9,000 in 2014. Find the percent increase.
137. The price of a share of one stock rose from $12.50 to $50. Find the percent increase.
138. According to Time magazine (7/19/2011) annual global seafood consumption rose from 22 pounds per person in 1960 to 38 pounds per person today. Find the percent increase. (Round to the nearest tenth of a percent.)
139. In one month, the median home price in the Northeast rose from $225,400 to $241,500. Find the percent increase. (Round to the nearest tenth of a percent.)
140. A grocery store reduced the price of a loaf of bread from $2.80 to $2.73. Find the percent decrease.
141. The price of a share of one stock fell from $8.75 to $8.54. Find the percent decrease.
142. Hernando's salary was $49,500 last year. This year his salary was cut to $44,055. Find the percent decrease.
143. From 2000 to 2010, the population of Detroit fell from about 951,000 to about 714,000. Find the percent decrease. (Round to the nearest tenth of a percent.)
144. In one month, the median home price in the West fell from $203,400 to $192,300. Find the percent decrease. (Round to the nearest tenth of a percent.)
145. Sales of video games and consoles fell from $1,150 million to $1,030 million in one year. Find the percent decrease. (Round to the nearest tenth of a percent.)

Everyday Math

146. **Tipping** At the campus coffee cart, a medium coffee costs $1.65. MaryAnne brings $2.00 with her when she buys a cup of coffee and leaves the change as a tip. What percent tip does she leave?

147. **Late Fees** Alison was late paying her credit card bill of $249. She was charged a 5% late fee. What was the amount of the late fee?

Writing Exercises

148. Without solving the problem “44 is 80% of what number”, think about what the solution might be. Should it be a number that is greater than 44 or less than 44? Explain your reasoning.
149. Without solving the problem “What is 20% of 300?” think about what the solution might be. Should it be a number that is greater than 300 or less than 300? Explain your reasoning.
150. After returning from vacation, Alex said he should have packed 50% fewer shorts and 200% more shirts. Explain what Alex meant.
151. Because of road construction in one city, commuters were advised to plan their Monday morning commute to take 150% of their usual commuting time. Explain what this means.

Self Check

(a) After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
translate and solve basic percent equations.			
solve applications of percent.			
find percent increase and percent decrease.			

(b) After reviewing this checklist, what will you do to become confident for all objectives?

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4.3: Decimals (Part 1)

Learning Objectives

- Name decimals
- Write decimals
- Convert decimals to fractions or mixed numbers
- Locate decimals on the number line
- Order decimals
- Round decimals

be prepared!

Before you get started, take this readiness quiz.

1. Name the number 4,926,015 in words. If you missed this problem, review Example 1.1.4.
2. Round 748 to the nearest ten. If you missed this problem, review Example 1.2.9.
3. Locate $\frac{3}{10}$ on a number line. If you missed this problem, review Example 4.2.16.

Name Decimals

You probably already know quite a bit about decimals based on your experience with money. Suppose you buy a sandwich and a bottle of water for lunch. If the sandwich costs \$3.45, the bottle of water costs \$1.25, and the total sales tax is \$0.33, what is the total cost of your lunch?

$$\begin{array}{rl} \$3.45 & Sandwich \\ \$1.25 & Water \\ +\$0.33 & Tax \\ \hline \$5.03 & Total \end{array}$$

The total is \$5.03. Suppose you pay with a \$5 bill and 3 pennies. Should you wait for change? No, \$5 and 3 pennies is the same as \$5.03.

Because 100 pennies = \$1, each penny is worth $\frac{1}{100}$ of a dollar. We write the value of one penny as \$0.01, since 0.01 = $\frac{1}{100}$.

Writing a number with a decimal is known as decimal notation. It is a way of showing parts of a whole when the whole is a power of ten. In other words, decimals are another way of writing fractions whose denominators are powers of ten. Just as the counting numbers are based on powers of ten, decimals are based on powers of ten. Table 4.3.1 shows the counting numbers.

Table 4.3.1

Counting number	Name
1	One
10 = 10	Ten
10 • 10 = 100	One hundred
10 • 10 • 10 = 1000	One thousand
10 • 10 • 10 • 10 = 10,000	Ten thousand

How are decimals related to fractions? Table 4.3.2 shows the relation.

Table 4.3.2

Decimal	Fraction	Name

Decimal	Fraction	Name
0.1	$\frac{1}{10}$	One tenth
0.01	$\frac{1}{100}$	One hundredth
0.001	$\frac{1}{1000}$	One thousandth
0.0001	$\frac{1}{10,000}$	One ten-thousandth

When we name a whole number, the name corresponds to the place value based on the powers of ten. In Whole Numbers, we learned to read 10,000 as *ten thousand*. Likewise, the names of the decimal places correspond to their fraction values. Notice how the place value names in Figure 4.3.1 relate to the names of the fractions from Table 4.3.2.

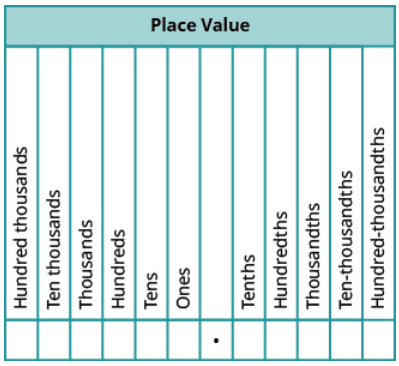


Figure 4.3.1 - This chart illustrates place values to the left and right of the decimal point.

Notice two important facts shown in Figure 4.3.1.

- The “th” at the end of the name means the number is a fraction. “One thousand” is a number larger than one, but “one thousandth” is a number smaller than one.
- The tenths place is the first place to the right of the decimal, but the tens place is two places to the left of the decimal.

Remember that $5.03 lunch? We read $5.03 as *five dollars and three cents*. Naming decimals (those that don’t represent money) is done in a similar way. We read the number 5.03 as *five and three hundredths*.

We sometimes need to translate a number written in decimal notation into words. As shown in Figure 4.3.2, we write the amount on a check in both words and numbers.



Figure 4.3.2 - When we write a check, we write the amount as a decimal number as well as in words. The bank looks at the check to make sure both numbers match. This helps prevent errors.

Let’s try naming a decimal, such as 15.68.	
We start by naming the number to the left of the decimal.	fifteen_____
We use the word “and” to indicate the decimal point.	fifteen and_____

Then we name the number to the right of the decimal point as if it were a whole number.	fifteen and sixty-eight_____
Last, name the decimal place of the last digit.	fifteen and sixty-eight hundredths

The number 15.68 is read *fifteen and sixty-eight hundredths*.

HOW TO: NAME A DECIMAL NUMBER

- Name the number to the left of the decimal point.
- Write “and” for the decimal point.
- Name the “number” part to the right of the decimal point as if it were a whole number.
- Name the decimal place of the last digit.

Example 4.3.1:

Name each decimal: (a) 4.3 (b) 2.45 (c) 0.009 (d) −15.571.

Solution

(a) 4.3

Name the number to the left of the decimal point.	four_____
Write "and" for the decimal point.	four and_____
Name the number to the right of the decimal point as if it were a whole number.	four and three_____
Name the decimal place of the last digit.	four and three tenths

(b) 2.45

Name the number to the left of the decimal point.	two_____
Write "and" for the decimal point.	two and_____
Name the number to the right of the decimal point as if it were a whole number.	two and forty-five_____
Name the decimal place of the last digit.	two and forty-five hundredths

(c) 0.009

Name the number to the left of the decimal point.	Zero is the number to the left of the decimal; it is not included in the name.
Name the number to the right of the decimal point as if it were a whole number.	nine_____
Name the decimal place of the last digit.	nine thousandths

(d) −15.571

Name the number to the left of the decimal point.	negative fifteen
Write "and" for the decimal point.	negative fifteen and_____
Name the number to the right of the decimal point as if it were a whole number.	negative fifteen and five hundred seventyone_____

Name the decimal place of the last digit.	negative fifteen and five hundred seventyone thousandths

Exercise 4.3.1:

Name each decimal: (a) 6.7 (b) 19.58 (c) 0.018 (d) −2.053

Answer a

six and seven tenths

Answer b

nineteen and fifty-eight hundredths

Answer c

eighteen thousandths

Answer d

negative two and fifty-three thousandths

Exercise 4.3.2:

Name each decimal: (a) 5.8 (b) 3.57 (c) 0.005 (d) −13.461

Answer a

five and eight tenths three and fifty-seven hundredths

Answer b

three and fifty-seven hundredths

Answer c

five thousandths

Answer d

negative thirteen and four hundred sixty-one thousandths

Write Decimals

Now we will translate the name of a decimal number into decimal notation. We will reverse the procedure we just used. Let's start by writing the number six and seventeen hundredths:

The word *and* tells us to place a decimal point.	___.___
The word before *and* is the whole number; write it to the left of the decimal point.	6._____
The decimal part is seventeen hundredths. Mark two places to the right of the decimal point for hundredths.	6._ _
Write the numerals for seventeen in the places marked.	6.17

Example 4.3.2:

Write fourteen and thirty-seven hundredths as a decimal.

Solution

Place a decimal point under the word ‘and’.	_____. _____
Translate the words before ‘and’ into the whole number and place it to the left of the decimal point.	14. _____
Mark two places to the right of the decimal point for “hundredths”.	14.__ __
Translate the words after “and” and write the number to the right of the decimal point.	14.37

Fourteen and thirty-seven hundredths is written 14.37.

Exercise 4.3.3:

Write as a decimal: thirteen and sixty-eight hundredths.

Answer

13.68

Exercise 4.3.4:

Write as a decimal: five and eight hundred ninety-four thousandths.

Answer

5.894

HOW TO: WRITE A DECIMAL NUMBER FROM ITS NAME

Step 1. Look for the word “and”—it locates the decimal point.

Step 2. Mark the number of decimal places needed to the right of the decimal point by noting the place value indicated by the last word.

- Place a decimal point under the word “and.” Translate the words before “and” into the whole number and place it to the left of the decimal point.
- If there is no “and,” write a “0” with a decimal point to its right.

Step 3. Translate the words after “and” into the number to the right of the decimal point. Write the number in the spaces—putting the final digit in the last place.

Step 4. Fill in zeros for place holders as needed.

The second bullet in Step 2 is needed for decimals that have no whole number part, like ‘nine thousandths’. We recognize them by the words that indicate the place value after the decimal – such as ‘tenths’ or ‘hundredths.’ Since there is no whole number, there is no ‘and.’ We start by placing a zero to the left of the decimal and continue by filling in the numbers to the right, as we did above.

Example 4.3.3:

Write twenty-four thousandths as a decimal.

Solution

Look for the word "and".	There is no "and" so start with 0.
To the right of the decimal point, put three decimal places for thousandths.	0. _ _ _ tenths hundredths thousandths
Write the number 24 with the 4 in the thousandths place.	0. _ 2 4 tenths hundredths thousandths
Put zeros as placeholders in the remaining decimal places.	0.024

So, twenty-four thousandths is written 0.024

Exercise 4.3.5:

Write as a decimal: fifty-eight thousandths.

Answer

0.058

Exercise 4.3.6:

Write as a decimal: sixty-seven thousandths.

Answer

0.067

Before we move on to our next objective, think about money again. We know that $1 is the same as $1.00. The way we write $1 (or $1.00) depends on the context. In the same way, integers can be written as decimals with as many zeros as needed to the right of the decimal.

$$\begin{aligned} 5 &= 5.0 \qquad -2 = -2.0 \\ 5 &= 5.00 \qquad -2 = -2.00 \\ 5 &= 5.000 \qquad -2 = -2.000 \end{aligned}$$

$$and\ so\ on\ldots$$

Convert Decimals to Fractions or Mixed Numbers

We often need to rewrite decimals as fractions or mixed numbers. Let's go back to our lunch order to see how we can convert decimal numbers to fractions. We know that $5.03 means 5 dollars and 3 cents. Since there are 100 cents in one dollar, 3 cents means $\frac{3}{100}$ of a dollar, so 0.03 = $\frac{3}{100}$.

We convert decimals to fractions by identifying the place value of the farthest right digit. In the decimal 0.03, the 3 is in the hundredths place, so 100 is the denominator of the fraction equivalent to 0.03.

$$0.03 = \frac{3}{100}$$

For our $5.03 lunch, we can write the decimal 5.03 as a mixed number.

$$5.03 = 5\frac{3}{100}$$

Notice that when the number to the left of the decimal is zero, we get a proper fraction. When the number to the left of the decimal is not zero, we get a mixed number.

HOW TO: CONVERT A DECIMAL NUMBER TO A FRACTION OR MIXED NUMBER

Step 1. Look at the number to the left of the decimal.

- If it is zero, the decimal converts to a proper fraction.
- If it is not zero, the decimal converts to a mixed number.
  - Write the whole number.

Step 2. Determine the place value of the final digit.

Step 3. Write the fraction.

- numerator—the 'numbers' to the right of the decimal point
- denominator—the place value corresponding to the final digit

Step 4. Simplify the fraction, if possible.

Example 4.3.4:

Write each of the following decimal numbers as a fraction or a mixed number: (a) 4.09 (b) 3.7 (c) −0.286

Solution

(a) 4.09

There is a 4 to the left of the decimal point. Write "4" as the whole number part of the mixed number.	$4\frac{\square}{\square}$
Determine the place value of the final digit.	4. 0 9 tenths hundredths
Write the fraction. Write 9 in the numerator as it is the number to the right of the decimal point.	$4\frac{9}{\square}$
Write 100 in the denominator as the place value of the final digit, 9, is hundredth.	$4\frac{9}{100}$
The fraction is in simplest form.	So, $4.09 = 4\frac{9}{100}$

Did you notice that the number of zeros in the denominator is the same as the number of decimal places?

(b) 3.7

There is a 3 to the left of the decimal point. Write "3" as the whole number part of the mixed number.	$3\frac{\square}{\square}$
Determine the place value of the final digit.	3. 7 tenths
Write the fraction. Write 7 in the numerator as it is the number to the right of the decimal point.	$3\frac{7}{\square}$
Write 10 in the denominator as the place value of the final digit, 7, is tenths.	$3\frac{7}{10}$
The fraction is in simplest form.	So, $3.7 = 3\frac{7}{10}$

(c) −0.286

There is a 0 to the left of the decimal point. Write a negative sign before the fraction.	$-\frac{\square}{\square}$
Determine the place value of the final digit and write it in the denominator.	$-0.\underset{\text{tenths}}{2}\underset{\text{hundredths}}{8}\underset{\text{thousandths}}{6}$
Write the fraction. Write 286 in the numerator as it is the number to the right of the decimal point. Write 1,000 in the denominator as the place value of the final digit, 6, is thousandths.	$-\frac{286}{1000}$
We remove a common factor of 2 to simplify the fraction.	$-\frac{143}{500}$

? Exercise 4.3.7:

Write as a fraction or mixed number. Simplify the answer if possible. (a) 5.3 (b) 6.07 (c) −0.234

Answer a

$5\frac{3}{10}$

Answer b

$6\frac{7}{100}$

Answer c

$-\frac{234}{1000}$

? Exercise 4.3.8:

Write as a fraction or mixed number. Simplify the answer if possible. (a) 8.7 (b) 1.03 (c) −0.024

Answer a

$8\frac{7}{10}$

Answer b

$1\frac{3}{100}$

Answer c

$-\frac{24}{1000}$

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4.4: Decimals (Part 2)

Locate Decimals on the Number Line

Since decimals are forms of fractions, locating decimals on the number line is similar to locating fractions on the number line.

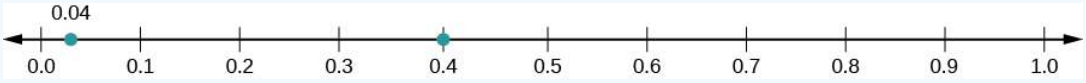
Example 4.4.5:

Locate 0.4 on a number line.

Solution

The decimal 0.4 is equivalent to $\frac{4}{10}$, so 0.4 is located between 0 and 1. On a number line, divide the interval between 0 and 1 into 10 equal parts and place marks to separate the parts.

Label the marks 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0. We write 0 as 0.0 and 1 as 1.0, so that the numbers are consistently in tenths. Finally, mark 0.4 on the number line.



Exercise 4.4.9:

Locate 0.6 on a number line.

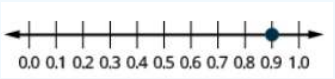
Answer



Exercise 4.4.10:

Locate 0.9 on a number line.

Answer

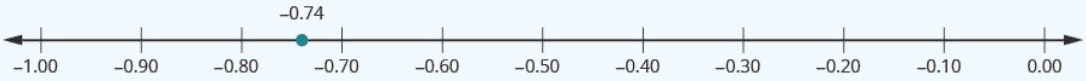


Example 4.4.6:

Locate −0.74 on a number line.

Solution

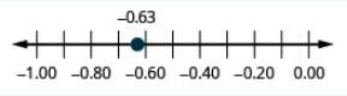
The decimal −0.74 is equivalent to $-\frac{74}{100}$, so it is located between 0 and −1. On a number line, mark off and label the multiples of -0.10 in the interval between 0 and −1 (−0.10, −0.20, etc.) and mark −0.74 between −0.70 and −0.80, a little closer to −0.70.



Exercise 4.4.11:

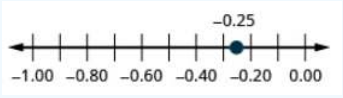
Locate −0.63 on a number line.

Answer



Exercise 4.4.12:

Locate −0.25 on a number line.



Answer

Order Decimals

Which is larger, 0.04 or 0.40?

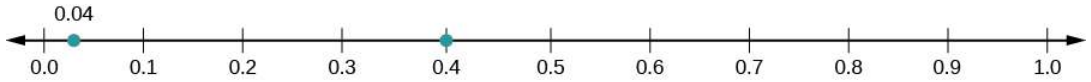
If you think of this as money, you know that $0.40 (forty cents) is greater than $0.04 (four cents). So, 0.40 > 0.04.

In previous chapters, we used the number line to order numbers.

a < b: 'a is less than b' when a is to the left of b on the number line

a > b: 'a is greater than b' when a is to the right of b on the number line

Where are 0.04 and 0.40 located on the number line?



We see that 0.40 is to the right of 0.04. So we know 0.40 > 0.04.

How does 0.31 compare to 0.308? This doesn't translate into money to make the comparison easy. But if we convert 0.31 and 0.308 to fractions, we can tell which is larger.

	0.31		0.308	
Convert to fractions.	$\frac{31}{100}$	(4.4.1)	$\frac{308}{1000}$	(4.4.2)
We need a common denominator to compare them.	$\frac{31 \cdot 10}{100 \cdot 10}$	(4.4.3)	$\frac{308}{1000}$	(4.4.4)
	$\frac{310}{1000}$	(4.4.5)	$\frac{308}{1000}$	(4.4.6)

Because 310 > 308, we know that $\frac{310}{1000} > \frac{308}{1000}$. Therefore, 0.31 > 0.308.

Notice what we did in converting 0.31 to a fraction—we started with the fraction $\frac{31}{100}$ and ended with the equivalent fraction $\frac{310}{1000}$. Converting $\frac{310}{1000}$ back to a decimal gives 0.310. So 0.31 is equivalent to 0.310. Writing zeros at the end of a decimal does

not change its value.

$$\frac{31}{100} = \frac{310}{1000} \quad and \quad 0.31 = 0.310 \tag{4.4.7}$$

If two decimals have the same value, they are said to be equivalent decimals.

$$0.31 = 0.310 \tag{4.4.8}$$

We say 0.31 and 0.310 are equivalent decimals.

HOW TO: ORDER DECIMALS

Step 1. Check to see if both numbers have the same number of decimal places. If not, write zeros at the end of the one with fewer digits to make them match.

Step 2. Compare the numbers to the right of the decimal point as if they were whole numbers.

Step 3. Order the numbers using the appropriate inequality sign.

Example 4.4.7:

Order the following decimals using < or >: (a) 0.64 __0.6 (b) 0.83 __0.803

Solution

(a) 0.64 __0.6

Check to see if both numbers have the same number of decimal places. They do not, so write one zero at the right of 0.6.	0.64 __0.60
Compare the numbers to the right of the decimal point as if they were whole numbers.	64 > 60
Order the numbers using the appropriate inequality sign.	0.64 > 0.60 0.64 > 0.6

(b) 0.83 __0.803

Check to see if both numbers have the same number of decimal places. They do not, so write one zero at the right of 0.83.	0.830 __0.803
Compare the numbers to the right of the decimal point as if they were whole numbers.	830 > 803
Order the numbers using the appropriate inequality sign.	0.830 > 0.803 0.83 > 0.803

Exercise 4.4.13:

Order each of the following pairs of numbers, using < or >: (a) 0.42__0.4 (b) 0.76__0.706

Answer a

>

Answer b

>

Exercise 4.4.14:

Order each of the following pairs of numbers, using < or >: (a) 0.1__0.18 (b) 0.305__0.35

Answer a

<

Answer b

<

When we order negative decimals, it is important to remember how to order negative integers. Recall that larger numbers are to the right on the number line. For example, because −2 lies to the right of −3 on the number line, we know that −2 > −3. Similarly, smaller numbers lie to the left on the number line. For example, because −9 lies to the left of −6 on the number line, we know that −9 < −6.



If we zoomed in on the interval between 0 and −1, we would see in the same way that −0.2 > −0.3 and −0.9 < −0.6.

Example 4.4.8:

Use < or > to order. −0.1__−0.8.

Solution

Write the numbers one under the other, lining up the decimal points.	−0.1 −0.8
They have the same number of digits.	
Since −1 > −8, −1 tenth is greater than −8 tenths.	−0.1 > −0.8

Exercise 4.4.15:

Order each of the following pairs of numbers, using < or >: −0.3___−0.5

Answer

>

Exercise 4.4.16:

Order each of the following pairs of numbers, using < or >: −0.6___−0.7

Answer

>

Round Decimals

In the United States, gasoline prices are usually written with the decimal part as thousandths of a dollar. For example, a gas station might post the price of unleaded gas at $3.279 per gallon. But if you were to buy exactly one gallon of gas at this price, you would pay $3.28 , because the final price would be rounded to the nearest cent. In Whole Numbers, we saw that we round numbers to get an approximate value when the exact value is not needed. Suppose we wanted to round $2.72 to the nearest dollar. Is it closer to $2 or to $3? What if we wanted to round $2.72 to the nearest ten cents; is it closer to $2.70 or to $2.80? The number lines in Figure 4.4.3 can help us answer those questions.

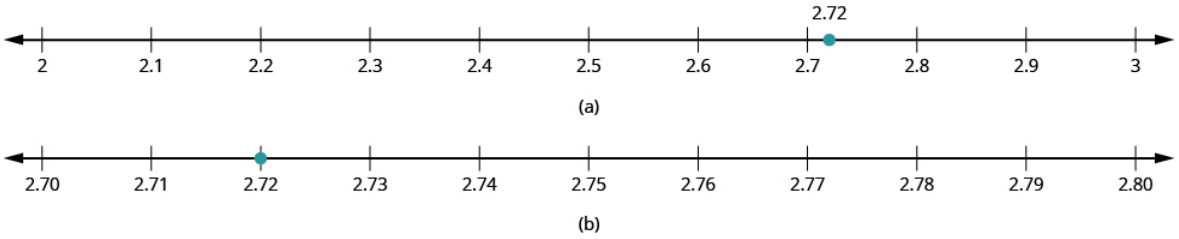


Figure 4.4.3 - (b) We see that 2.72 is closer to 3 than to 2. So, 2.72 rounded to the nearest whole number is 3. (b) We see that 2.72 is closer to 2.70 than 2.80. So we say that 2.72 rounded to the nearest tenth is 2.7.

Can we round decimals without number lines? Yes! We use a method based on the one we used to round whole numbers.

HOW TO: ROUND A DECIMAL

Step 1. Locate the given place value and mark it with an arrow.

Step 2. Underline the digit to the right of the given place value.

Step 3. Is this digit greater than or equal to 5?

- Yes - add 1 to the digit in the given place value.
- No - do not change the digit in the given place value

Step 4. Rewrite the number, removing all digits to the right of the given place value.

Example 4.4.9:

Round 18.379 to the nearest hundredth.

Solution

Locate the hundredths place and mark it with an arrow.	hundredths place ↓ 18.379
Underline the digit to the right of the 7.	hundredths place ↓ 18.379
Because 9 is greater than or equal to 5, add 1 to the 7.	18.379 (add 1, delete)
Rewrite the number, deleting all digits to the right of the hundredths place.	18.38

18.38 is 18.379 rounded to the nearest hundredth.

Exercise 4.4.17:

Round to the nearest hundredth: 1.047.

Answer

1.05

Exercise 4.4.18:

Round to the nearest hundredth: 9.173.

Answer

9.17

Example 4.4.10:

Round 18.379 to the nearest (a) tenth (b) whole number.

Solution

(a) Round 18.379 to the nearest tenth.

Locate the tenths place and mark it with an arrow.	
Underline the digit to the right of the tenths digit.	
Because 7 is greater than or equal to 5, add 1 to the 3.	
Rewrite the number, deleting all digits to the right of the tenths place.	18.4

So, 18.379 rounded to the nearest tenth is 18.4.

(b) Round 18.379 to the nearest whole number.

Locate the ones place and mark it with an arrow.	
Underline the digit to the right of the ones place.	
Since 3 is not greater than or equal to 5, do not add 1 to the 8.	
Rewrite the number, deleting all digits to the right of the ones place.	18

So 18.379 rounded to the nearest whole number is 18.

Exercise 4.4.19:

Round 6.582 to the nearest (a) hundredth (b) tenth (c) whole number.

Answer a

6.58

Answer b

6.6

Answer c

7

Exercise 4.4.20:

Round 15.2175 to the nearest (a) thousandth (b) hundredth (c) tenth.

Answer a

15.218

Answer b

15.22

Answer c

15.2

ACCESS ADDITIONAL ONLINE RESOURCES

Introduction to Decimal Notation

Write a Number in Decimal Notation from Words

Identify Decimals on the Number Line

Rounding Decimals

Writing a Decimal as a Simplified Fraction

Practice Makes Perfect

Name Decimals In the following exercises, name each decimal.

1. 5.5
2. 7.8
3. 5.01
4. 14.02
5. 8.71
6. 2.64
7. 0.002
8. 0.005
9. 0.381
10. 0.479
11. −17.9
12. −31.4

Write Decimals

In the following exercises, translate the name into a decimal number.

13. Eight and three hundredths
14. Nine and seven hundredths
15. Twenty-nine and eighty-one hundredths
16. Sixty-one and seventy-four hundredths
17. Seven tenths
18. Six tenths

19. One thousandth
20. Nine thousandths
21. Twenty-nine thousandths
22. Thirty-five thousandths
23. Negative eleven and nine ten-thousandths
24. Negative fifty-nine and two ten-thousandths
25. Thirteen and three hundred ninety-five ten thousandths
26. Thirty and two hundred seventy-nine thousandths

Convert Decimals to Fractions or Mixed Numbers

In the following exercises, convert each decimal to a fraction or mixed number.

27. 1.99
28. 5.83
29. 15.7
30. 18.1
31. 0.239
32. 0.373
33. 0.13
34. 0.19
35. 0.011
36. 0.049
37. −0.00007
38. −0.00003
39. 6.4
40. 5.2
41. 7.05
42. 9.04
43. 4.006
44. 2.008
45. 10.25
46. 12.75
47. 1.324
48. 2.482
49. 14.125
50. 20.375

Locate Decimals on the Number Line

In the following exercises, locate each number on a number line.

51. 0.8
52. 0.3
53. −0.2
54. −0.9
55. 3.1
56. 2.7
57. −2.5
58. −1.6

Order Decimals

In the following exercises, order each of the following pairs of numbers, using < or >.

59. 0.9__0.6
60. 0.7__0.8
61. 0.37__0.63

62. 0.86__0.69
63. 0.6__0.59
64. 0.27__0.3
65. 0.91__0.901
66. 0.415__0.41
67. −0.5__−0.3
68. −0.1_−0.4
69. −0.62_−0.619
70. −7.31_−7.3

Round Decimals

In the following exercises, round each number to the nearest tenth.

71. 0.67
72. 0.49
73. 2.84
74. 4.63

In the following exercises, round each number to the nearest hundredth.

75. 0.845
76. 0.761
77. 5.7932
78. 3.6284
79. 0.299
80. 0.697
81. 4.098
82. 7.096

In the following exercises, round each number to the nearest (a) hundredth (b) tenth (c) whole number.

83. 5.781
84. 1.638
85. 63.479
86. 84.281

Everyday Math

87. **Salary Increase** Danny got a raise and now makes $58,965.95 a year. Round this number to the nearest: (a) dollar (b) thousand dollars (c) ten thousand dollars.
88. **New Car Purchase** Selena's new car cost $23,795.95. Round this number to the nearest: (a) dollar (b) thousand dollars (c) ten thousand dollars.
89. **Sales Tax** Hyo Jin lives in San Diego. She bought a refrigerator for $1624.99 and when the clerk calculated the sales tax it came out to exactly $142.186625. Round the sales tax to the nearest (a) penny (b) dollar.
90. **Sales Tax** Jennifer bought a $1,038.99 dining room set for her home in Cincinnati. She calculated the sales tax to be exactly $67.53435. Round the sales tax to the nearest (a) penny (b) dollar.

Writing Exercises

91. How does your knowledge of money help you learn about decimals?
92. Explain how you write "three and nine hundredths" as a decimal.
93. Jim ran a 100-meter race in 12.32 seconds. Tim ran the same race in 12.3 seconds. Who had the faster time, Jim or Tim? How do you know?
94. Gerry saw a sign advertising postcards marked for sale at "10 for 0.99¢." What is wrong with the advertised price?

Self Check

(a) After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
name decimals.			
write decimals.			
convert decimals to fractions or mixed numbers.			
locate decimals on the number line.			
order decimals.			
round decimals.			

(b) If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them. What did you do to become confident of your ability to do these things? Be specific.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is important to make sure you have a strong foundation before you move on. Who can you ask for help? Your fellow classmates and instructor are good resources. Is there a place on campus where math tutors are available? Can your study skills be improved?

…no—I don't get it! This is a warning sign and you must not ignore it. You should get help right away or you will quickly be overwhelmed. See your instructor as soon as you can to discuss your situation. Together you can come up with a plan to get you the help you need.

Contributors and Attributions

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4.5: Decimal Operations (Part 1)

Learning Objectives

- Add and subtract decimals
- Multiply decimals
- Divide decimals
- Use decimals in money applications

be prepared!

Before you get started, take this readiness quiz.

1. Simplify $\frac{70}{100}$. If you missed this problem, review Example 4.3.1.
2. Multiply $\frac{3}{10} \cdot \frac{9}{10}$. If you missed this problem, review Example 4.3.7.
3. Divide $-36 \div (-9)$. If you missed this problem, review Example 3.7.3.

Add and Subtract Decimals

Let's take one more look at the lunch order from the start of Decimals, this time noticing how the numbers were added together.

$$\begin{array}{rl} \$3.45 & Sandwich \\ \$1.25 & Water \\ +\$0.33 & Tax \\ \hline \$5.03 & Total \end{array}$$

All three items (sandwich, water, tax) were priced in dollars and cents, so we lined up the dollars under the dollars and the cents under the cents, with the decimal points lined up between them. Then we just added each column, as if we were adding whole numbers. By lining up decimals this way, we can add or subtract the corresponding place values just as we did with whole numbers.

HOW TO: ADD OR SUBTRACT DECIMALS

Step 1. Write the numbers vertically so the decimal points line up.

Step 2. Use zeros as place holders, as needed.

Step 3. Add or subtract the numbers as if they were whole numbers. Then place the decimal in the answer under the decimal points in the given numbers.

Example 4.5.1:

Add: 3.7 + 12.4.

Solution

Write the numbers vertically so the decimal points line up.	$\begin{array}{r} 3.7 \\ +12.4 \\ \hline \end{array}$
Place holders are not needed since both numbers have the same number of decimal places.	

Add the numbers as if they were whole numbers. Then place the decimal in the answer under the decimal points in the given numbers.	$\begin{array}{r} {\scriptstyle 1} \\ 3.7 \\ +12.4 \\ \hline 16.1 \end{array}$

Exercise 4.5.1:

Add: 5.7 + 11.9.

Answer

17.6

Exercise 4.5.2:

Add: 18.32 + 14.79.

Answer

13.11

Example 4.5.2:

Add: 23.5 + 41.38.

Solution

Write the numbers vertically so the decimal points line up.	$\begin{array}{r} 23.5 \\ +41.38 \\ \hline \end{array}$
Place 0 as a place holder after the 5 in 23.5, so that both numbers have two decimal places.	$\begin{array}{r} 23.50 \\ +41.38 \\ \hline \end{array}$
Add the numbers as if they were whole numbers. Then place the decimal in the answer under the decimal points in the given numbers.	$\begin{array}{r} 23.50 \\ +41.38 \\ \hline 64.88 \end{array}$

Exercise 4.5.3:

Add: 4.8 + 11.69.

Answer

16.49

Exercise 4.5.4:

Add: 5.123 + 18.47.

Answer

23.593

How much change would you get if you handed the cashier a $20 bill for a $14.65 purchase? We will show the steps to calculate this in the next example.

Example 4.5.3:

Subtract: $20 - 14.65$.

Solution

Write the numbers vertically so the decimal points line up. Remember 20 is a whole number, so place the decimal point after the 0.	$\begin{array}{r} 20. \\ -14.65 \\ \hline \end{array}$
Place two zeros after the decimal point in 20, as place holders so that both numbers have two decimal places.	$\begin{array}{r} 20.00 \\ -14.65 \\ \hline \end{array}$
Subtract the numbers as if they were whole numbers. Then place the decimal in the answer under the decimal points in the given numbers.	$\begin{array}{r} {\scriptstyle 9}\ {\scriptstyle 9}\ {\scriptstyle 9} \\ 1\ \cancel{10}\ \cancel{10}\ \cancel{10} \\ \cancel{2}\ \cancel{0}.\ \cancel{0}\ \cancel{0} \\ -1\ 4\ .\ 6\ 5 \\ \hline 5\ .\ 3\ 5 \end{array}$

Exercise 4.5.5:

Subtract: $10 - 9.58$.

Answer

0.42

Exercise 4.5.6:

Subtract: $50 - 37.42$.

Answer

12.58

Example 4.5.4:

Subtract: $2.51 - 7.4$.

Solution

If we subtract 7.4 from 2.51, the answer will be negative since $7.4 > 2.51$. To subtract easily, we can subtract 2.51 from 7.4. Then we will place the negative sign in the result.

Write the numbers vertically so the decimal points line up.	$\begin{array}{r} 7.4 \\ -2.51 \\ \hline \end{array}$
Place zero after the 4 in 7.4 as a place holder, so that both numbers have two decimal places.	$\begin{array}{r} 7.40 \\ -2.51 \\ \hline \end{array}$

Subtract and place the decimal in the answer.	$\begin{array}{r} 7.40 \\ -2.51 \\ \hline 4.89 \end{array}$
Remember that we are really subtracting 2.51 − 7.4 so the answer is negative.	2.51 − 7.4 = − 4.89

? Exercise 4.5.7:

Subtract: 4.77 − 6.3.

Answer

-1.53

? Exercise 4.5.8:

Subtract: 8.12 − 11.7.

Answer

-3.58

Multiply Decimals

Multiplying decimals is very much like multiplying whole numbers—we just have to determine where to place the decimal point. The procedure for multiplying decimals will make sense if we first review multiplying fractions.

Do you remember how to multiply fractions? To multiply fractions, you multiply the numerators and then multiply the denominators. So let's see what we would get as the product of decimals by converting them to fractions first. We will do two examples side-by-side in Table 5.22. Look for a pattern.

Table 4.5.1

	A	B
	(0.3)(0.7)	(0.2)(0.46)
Convert to fractions.	$\left(\frac{3}{10}\right)\left(\frac{7}{10}\right)$ (4.5.1)	$\left(\frac{2}{10}\right)\left(\frac{46}{100}\right)$ (4.5.2)
Multiply.	$\frac{21}{100}$ (4.5.3)	$\frac{92}{1000}$ (4.5.4)
Convert back to decimals	0.21	0.092

There is a pattern that we can use. In A, we multiplied two numbers that each had one decimal place, and the product had two decimal places. In B, we multiplied a number with one decimal place by a number with two decimal places, and the product had three decimal places.

How many decimal places would you expect for the product of (0.01)(0.004)? If you said "five", you recognized the pattern. When we multiply two numbers with decimals, we count all the decimal places in the factors—in this case two plus three—to get the number of decimal places in the product—in this case five.

$$(0.01)\ (0.004) = 0.00004$$
$$\text{2 places}\quad \text{3 places}\quad \text{5 places}$$

$$\left(\frac{1}{100}\right)\left(\frac{4}{1000}\right) = \frac{4}{100{,}000}$$

Once we know how to determine the number of digits after the decimal point, we can multiply decimal numbers without converting them to fractions first. The number of decimal places in the product is the sum of the number of decimal places in the factors.

The rules for multiplying positive and negative numbers apply to decimals, too, of course.

Definition: Multiplying Two Numbers

When multiplying two numbers,

- if their signs are the same, the product is positive.
- if their signs are different, the product is negative.

When you multiply signed decimals, first determine the sign of the product and then multiply as if the numbers were both positive. Finally, write the product with the appropriate sign.

HOW TO: MULTIPLY DECIMAL NUMBERS

Step 1. Determine the sign of the product.

Step 2. Write the numbers in vertical format, lining up the numbers on the right.

Step 3. Multiply the numbers as if they were whole numbers, temporarily ignoring the decimal points.

Step 4. Place the decimal point. The number of decimal places in the product is the sum of the number of decimal places in the factors. If needed, use zeros as placeholders.

Step 5. Write the product with the appropriate sign.

Example 4.5.5:

Multiply: (3.9)(4.075).

Solution

Determine the sign of the product. The signs are the same.	The product will be positive.
Write the numbers in vertical format, lining up the numbers on the right.	$\begin{array}{r} 4.075 \\ \times 3.9 \\ \hline \end{array}$
Multiply the numbers as if they were whole numbers, temporarily ignoring the decimal points.	$\begin{array}{r} 4.075 \\ \times 3.9 \\ \hline 36675 \\ 12225 \\ \hline 158925 \end{array}$
Place the decimal point. Add the number of decimal places in the factors (1 + 3). Place the decimal point 4 places from the right.	$\begin{array}{rl} 4.075 & 3\ places \\ \times 3.9 & 1\ place \\ \hline 36675 & \\ 12225 & \\ \hline 158925 & 4\ places \end{array}$
The product is positive.	(3.9)(4.075) = 15.8925

Exercise 4.5.9:

Multiply: 4.5(6.107).

Answer

27.4815

Exercise 4.5.10:

Multiply: 10.79(8.12).

Answer

87.6148

Example 4.5.6:

Multiply: (−8.2)(5.19).

Solution

The signs are different.	The product will be negative.
Write in vertical format, lining up the numbers on the right.	$\begin{array}{r} 5.19 \\ \times 8.2 \\ \hline \end{array}$
Multiply.	$\begin{array}{r} 5.19 \\ \times 8.2 \\ \hline 1038 \\ 4152 \\ \hline 42558 \end{array}$
Place the decimal point 3 places from the right. (−8.2)(5.19) 1 place 2 places	$\begin{array}{r} 5.19 \\ \times 8.2 \\ \hline 1038 \\ 4152 \\ \hline 42.558 \end{array}$
The product is negative.	(−8.2)(5.19) = −42.558

Exercise 4.5.11:

Multiply: (4.63)(−2.9).

Answer

−13.427

Exercise 4.5.12:

Multiply: (−7.78)(4.9).

Answer

-38.122

In the next example, we'll need to add several placeholder zeros to properly place the decimal point.

Example 4.5.7:

Multiply: (0.03)(0.045).

Solution

The product is positive.	(0.03)(0.045)
Write in vertical format, lining up the numbers on the right.	$\begin{array}{r} 0.045 \\ \times 0.03 \\ \hline \end{array}$
Multiply.	$\begin{array}{r} 0.045 \\ \times 0.03 \\ \hline 135 \end{array}$
The decimal point must be 5 places from the right. (0.03)(0.045) 2 places 3 places Add zeros as needed to get the 5 places.	$\begin{array}{r} 0.045 \\ \times\ 0.03 \\ \hline 0.00135 \end{array}$
The product is positive.	(0.03)(0.045) = 0.00135

Exercise 4.5.13:

Multiply: (0.04)(0.087).

Answer

0.00348

Exercise 4.5.14:

Multiply: (0.09)(0.067).

Answer

0.00603

Multiply by Powers of 10

In many fields, especially in the sciences, it is common to multiply decimals by powers of 10. Let's see what happens when we multiply 1.9436 by some powers of 10.

1.9436(10)	1.9436(100)	1.9436(1000)
$\begin{array}{r} 1.9436 \\ \times\ \ 10 \\ \hline 19.4360 \end{array}$	$\begin{array}{r} 1.9436 \\ \times\ 100 \\ \hline 194.3600 \end{array}$	$\begin{array}{r} 1.9436 \\ \times\ 1000 \\ \hline 1943.6000 \end{array}$

Look at the results without the final zeros. Do you notice a pattern?

$$1.9436(10) = 19.436$$
$$1.9436(100) = 194.36$$
$$1.9436(1000) = 1943.6$$

The number of places that the decimal point moved is the same as the number of zeros in the power of ten. Table 5.26 summarizes the results.

Table 4.5.2

Multiply by	Number of zeros	Number of places decimal point moves
10	1	1 place to the right
100	2	2 places to the right
1,000	3	3 places to the right
10,000	4	4 places to the right

We can use this pattern as a shortcut to multiply by powers of ten instead of multiplying using the vertical format. We can count the zeros in the power of 10 and then move the decimal point that same of places to the right. So, for example, to multiply 45.86 by 100, move the decimal point 2 places to the right.

$$45.86 \times 100 = 4586.$$

Sometimes when we need to move the decimal point, there are not enough decimal places. In that case, we use zeros as placeholders. For example, let's multiply 2.4 by 100. We need to move the decimal point 2 places to the right. Since there is only one digit to the right of the decimal point, we must write a 0 in the hundredths place.

$$2.4 \times 100 = 240.$$

HOW TO: MULTIPLY A DECIMAL BY A POWER OF 10

Step 1. Move the decimal point to the right the same number of places as the number of zeros in the power of 10.

Step 2. Write zeros at the end of the number as placeholders if needed.

Example 4.5.8:

Multiply 5.63 by factors of (a) 10 (b) 100 (c) 1000.

Solution

By looking at the number of zeros in the multiple of ten, we see the number of places we need to move the decimal to the right.

(a) 5.63(10)

There is 1 zero in 10, so move the decimal point 1 place to the right.	5.63
	56.3

(b) 5.63(100)

There are 2 zeros in 100, so move the decimal point 2 places to the right.	5.63
	563

(c) 5.63(1000)

There are 3 zeros in 1000, so move the decimal point 3 places to the right.	5.63

A zero must be added at the end.	5,630

Exercise 4.5.15:

Multiply 2.58 by factors of (a) 10 (b) 100 (c) 1000.

Answer a

25.8

Answer b

258

Answer c

$2,580$

Exercise 4.5.16:

Multiply 14.2 by factors of (a) 10 (b) 100 (c) 1000.

Answer a

142

Answer b

$1,420$

Answer c

$14,200$

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4.6: Decimal Operations (Part 2)

Divide Decimals

Just as with multiplication, division of decimals is very much like dividing whole numbers. We just have to figure out where the decimal point must be placed.

To understand decimal division, let's consider the multiplication problem

$$(0.2)(4) = 0.8 \tag{4.6.1}$$

Remember, a multiplication problem can be rephrased as a division problem. So we can write 0.8 ÷ 4 = 0.2 We can think of this as "If we divide 8 tenths into four groups, how many are in each group?" Figure 4.6.1 shows that there are four groups of two-tenths in eight-tenths. So 0.8 ÷ 4 = 0.2.

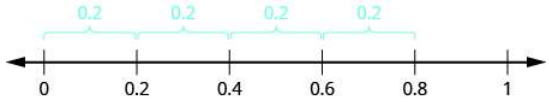


Figure 4.6.1

Using long division notation, we would write

$$4\overline{)0.8}\quad 0.2$$

Notice that the decimal point in the quotient is directly above the decimal point in the dividend.

To divide a decimal by a whole number, we place the decimal point in the quotient above the decimal point in the dividend and then divide as usual. Sometimes we need to use extra zeros at the end of the dividend to keep dividing until there is no remainder.

HOW TO: DIVIDE A DECIMAL BY A WHOLE NUMBER

Step 1. Write as long division, placing the decimal point in the quotient above the decimal point in the dividend.

Step 2. Divide as usual.

Example 4.6.9:

Divide: 0.12 ÷ 3.

Solution

Write as long division, placing the decimal point in the quotient above the decimal point in the dividend.	$3\overline{)0.12}$
Divide as usual. Since 3 does not go into 0 or 1 we use zeros as placeholders.	$3\overline{)0.12}\quad 0.04$ 12 0

0.12 ÷ 3 = 0.04

Exercise 4.6.17:

Divide: 0.28 ÷ 4.

Answer

0.07

Exercise 4.6.18:

Divide: 0.56 ÷ 7.

Answer

0.08

In everyday life, we divide whole numbers into decimals—money—to find the price of one item. For example, suppose a case of 24 water bottles cost $3.99. To find the price per water bottle, we would divide $3.99 by 24, and round the answer to the nearest cent (hundredth).

Example 4.6.10:

Divide: $3.99 ÷ 24.

Solution

Place the decimal point in the quotient above the decimal point in the dividend.	$24\overline{)3.99}$
Divide as usual. When do we stop? Since this division involves money, we round it to the nearest cent (hundredth). To do this, we must carry the division to the thousandths place.	$\begin{array}{r} 0.166 \\ 24\overline{)3.990} \\ \underline{24} \\ 159 \\ \underline{144} \\ 150 \\ \underline{144} \\ 6 \end{array}$
Round to the nearest cent.	$\$0.166 \approx \$0.17$ (4.6.2)

$3.99 ÷ 24 ≈ $0.17

This means the price per bottle is 17 cents.

Exercise 4.6.19:

Divide: $6.99 ÷ 36.

Answer

$0.19

Exercise 4.6.20:

Divide: $4.99 ÷ 12.

Answer

$0.42

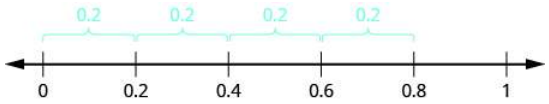
Divide a Decimal by Another Decimal

So far, we have divided a decimal by a whole number. What happens when we divide a decimal by another decimal? Let's look at the same multiplication problem we looked at earlier, but in a different way.

$$(0.2)(4) = 0.8 \quad (4.6.3)$$

Remember, again, that a multiplication problem can be rephrased as a division problem. This time we ask, “Ho w many times does 0.2 go into 0.8?” Because (0.2)(4) = 0.8, we can say that 0.2 goes into 0.8 four times. This means that 0.8 divided by 0.2 is 4.

$$0.8 \div 0.2 = 4 \tag{4.6.4}$$



We would get the same answer, 4, if we divide 8 by 2, both whole numbers. Why is this so? Let’s think about the division problem as a fraction.

$$\frac{0.8}{0.2} \tag{4.6.5}$$

$$\frac{(0.8)10}{(0.2)10} \tag{4.6.6}$$

$$\frac{8}{2} \tag{4.6.7}$$

$$4 \tag{4.6.8}$$

We multiplied the numerator and denominator by 10 and ended up just dividing 8 by 2. To divide decimals, we multiply both the numerator and denominator by the same power of 10 to make the denominator a whole number. Because of the Equivalent Fractions Property, we haven’t changed the value of the fraction. The effect is to move the decimal points in the numerator and denominator the same number of places to the right.

We use the rules for dividing positive and negative numbers with decimals, too. When dividing signed decimals, first determine the sign of the quotient and then divide as if the numbers were both positive. Finally, write the quotient with the appropriate sign. It may help to review the vocabulary for division:

$a \div b$ — a dividend, b divisor

$\frac{a}{b}$ — a dividend, b divisor

$b\overline{)a}$ — b divisor, a dividend

HOW TO: DIVIDE DECIMAL NUMBERS

Step 1. Determine the sign of the quotient.

Step 2. Make the divisor a whole number by moving the decimal point all the way to the right. Move the decimal point in the dividend the same number of places to the right, writing zeros as needed.

Step 3. Divide. Place the decimal point in the quotient above the decimal point in the dividend.

Step 4. Write the quotient with the appropriate sign.

Example 4.6.11:

Divide: −2.89 ÷ (3.4).

Solution

Determine the sign of the quotient.	The quotient will be negative.
Make the divisor the whole number by 'moving' the decimal point all the way to the right. 'Move' the decimal point in the dividend the same number of places to the right.	$3.4\overline{)2.89}$
Divide. Place the decimal point in the quotient above the decimal point in the dividend. Add zeros as needed until the remainder is zero.	$34.\overline{)28.90}$ → 0.85; 272; 170; 170; 0

Write the quotient with the appropriate sign.	$-2.89 \div (3.4) = -0.85$

Exercise 4.6.21:

Divide: $-1.989 \div 5.1$.

Answer

-0.39

Exercise 4.6.22:

Divide: $-2.04 \div 5.1$.

Answer

-0.4

Example 4.6.12:

Divide: $-25.65 \div (-0.06)$.

Solution

The signs are the same.	The quotient is positive.
Make the divisor a whole number by 'moving' the decimal point all the way to the right. 'Move' the decimal point in the dividend the same number of places.	$0.06\overline{)25.65}$
Divide. Place the decimal point in the quotient above the decimal point in the dividend.	$\begin{array}{r} 427.5 \\ 006.\overline{)2565.0} \\ -24 \\ \hline 16 \\ -12 \\ \hline 45 \\ -42 \\ \hline 30 \\ 30 \\ \hline \end{array}$
Write the quotient with the appropriate sign.	$-25.65 \div (-0.06) = 427.5$

Exercise 4.6.23:

Divide: $-23.492 \div (-0.04)$.

Answer

587.3

Exercise 4.6.24:

Divide: $-4.11 \div (-0.12)$.

Answer

34.25

Now we will divide a whole number by a decimal number.

Example 4.6.13

Divide: 4 ÷ (0.05).

Solution

The signs are the same.	The quotient is positive.
Make the divisor a whole number by 'moving' the decimal point all the way to the right. Move the decimal point in the dividend the same number of places, adding zeros as needed.	$0.05\overline{)4.00}$
Divide. Place the decimal point in the quotient above the decimal point in the dividend.	$\begin{array}{r} 80. \\ 5\overline{)400.} \\ \underline{40} \\ 00 \\ \underline{00} \end{array}$
Write the quotient with the appropriate sign.	4 ÷ 0.05 = 80

We can relate this example to money. How many nickels are there in four dollars? Because 4 ÷ 0.05 = 80, there are 80 nickels in $4.

Exercise 4.6.25:

Divide: 6 ÷ 0.03.

Answer

200

Exercise 4.6.26:

Divide: 7 ÷ 0.02

Answer

350

Use Decimals in Money Applications

We often apply decimals in real life, and most of the applications involving money. The Strategy for Applications we used in The Language of Algebra gives us a plan to follow to help find the answer. Take a moment to review that strategy now.

Strategy for Applications

1. Identify what you are asked to find.
2. Write a phrase that gives the information to find it.
3. Translate the phrase to an expression.
4. Simplify the expression.
5. Answer the question with a complete sentence.

Example 4.6.14:

Paul received $50 for his birthday. He spent $31.64 on a video game. How much of Paul's birthday money was left?

Solution

What are you asked to find?	How much did Paul have left?
Write a phrase.	$50 less $31.64
Translate.	$50 - 31.64$
Simplify.	18.36
Write a sentence.	Paul has $18.36 left.

Exercise 4.6.27:

Nicole earned $35 for babysitting her cousins, then went to the bookstore and spent $18.48 on books and coffee. How much of her babysitting money was left?

Answer

$16.52

Exercise 4.6.28:

Amber bought a pair of shoes for $24.75 and a purse for $36.90. The sales tax was $4.32. How much did Amber spend?

Answer

$65.97

Example 4.6.15:

Jessie put 8 gallons of gas in her car. One gallon of gas costs $3.529. How much does Jessie owe for the gas? (Round the answer to the nearest cent.)

Solution

What are you asked to find?	How much did Jessie owe for all the gas?
Write a phrase.	8 times the cost of one gallon of gas
Translate.	8($3.529)
Simplify.	$28.232
Round to the nearest cent.	$28.23
Write a sentence.	Jessie owes $28.23 for her gas purchase.

Exercise 4.6.29:

Hector put 13 gallons of gas into his car. One gallon of gas costs $3.175. How much did Hector owe for the gas? Round to the nearest cent.

Answer

$41.28

Exercise 4.6.30:

Christopher bought 5 pizzas for the team. Each pizza cost $9.75. How much did all the pizzas cost?

Answer

$48.75

Example 4.6.16:

Four friends went out for dinner. They shared a large pizza and a pitcher of soda. The total cost of their dinner was $31.76. If they divide the cost equally, how much should each friend pay?

Solution

What are you asked to find?	How much should each friend pay?
Write a phrase.	$31.76 divided equally among the four friends.
Translate to an expression.	$31.76 ÷ 4
Simplify.	$7.94
Write a sentence.	Each friend should pay $7.94 for his share of the dinner.

Exercise 4.6.31:

Six friends went out for dinner. The total cost of their dinner was $92.82. If they divide the bill equally, how much should each friend pay?

Answer

$15.47

Exercise 4.6.32:

Chad worked 40 hours last week and his paycheck was $570. How much does he earn per hour?

Answer

$14.25

Be careful to follow the order of operations in the next example. Remember to multiply before you add.

Example 4.6.17:

Marla buys 6 bananas that cost $0.22 each and 4 oranges that cost $0.49 each. How much is the total cost of the fruit?

Solution

What are you asked to find?	How much is the total cost of the fruit?
Write a phrase.	6 times the cost of each banana plus 4 times the cost of each orange
Translate to an expression.	6($0.22) + 4($0.49)
Simplify.	$1.32 + $1.96
Add.	$3.28

Write a sentence.	Marla's total cost for the fruit is $3.28.

Exercise 4.6.33:

Suzanne buys 3 cans of beans that cost $0.75 each and 6 cans of corn that cost $0.62 each. How much is the total cost of these groceries?

Answer

$5.97

Exercise 4.6.34:

Lydia bought movie tickets for the family. She bought two adult tickets for $9.50 each and four children's tickets for $6.00 each. How much did the tickets cost Lydia in all?

Answer

$43.00

ACCESS ADDITIONAL ONLINE RESOURCES

Adding and Subtracting Decimals

Multiplying Decimals

Multiplying by Powers of Ten

Dividing Decimals

Dividing by Powers of Ten

Practice Makes Perfect

Add and Subtract Decimals

In the following exercises, add or subtract.

95. 16.92 + 7.56
96. 18.37 + 9.36
97. 256.37 − 85.49
98. 248.25 − 91.29
99. 21.76 − 30.99
100. 15.35 − 20.88
101. 37.5 + 12.23
102. 38.6 + 13.67
103. −16.53 − 24.38
104. −19.47 − 32.58
105. −38.69 + 31.47
106. −29.83 + 19.76
107. −4.2 + (− 9.3)
108. −8.6 + (− 8.6)
109. 100 − 64.2
110. 100 − 65.83
111. 72.5 − 100
112. 86.2 − 100
113. 15 + 0.73

114. 27 + 0.87
115. 2.51 + 40
116. 9.38 + 60
117. 91.75 − (− 10.462)
118. 94.69 − (− 12.678)
119. 55.01 − 3.7
120. 59.08 − 4.6
121. 2.51 − 7.4
122. 3.84 − 6.1

Multiply Decimals

In the following exercises, multiply.

123. (0.3)(0.4)
124. (0.6)(0.7)
125. (0.24)(0.6)
126. (0.81)(0.3)
127. (5.9)(7.12)
128. (2.3)(9.41)
129. (8.52)(3.14)
130. (5.32)(4.86)
131. (−4.3)(2.71)
132. (− 8.5)(1.69)
133. (−5.18)(− 65.23)
134. (− 9.16)(− 68.34)
135. (0.09)(24.78)
136. (0.04)(36.89)
137. (0.06)(21.75)
138. (0.08)(52.45)
139. (9.24)(10)
140. (6.531)(10)
141. (55.2)(1,000)
142. (99.4)(1,000)

Divide Decimals

In the following exercises, divide.

143. 0.15 ÷ 5
144. 0.27 ÷ 3
145. 4.75 ÷ 25
146. 12.04 ÷ 43
147. $8.49 ÷ 12
148. $16.99 ÷ 9
149. $117.25 ÷ 48
150. $109.24 ÷ 36
151. 0.6 ÷ 0.2
152. 0.8 ÷ 0.4
153. 1.44 ÷ (− 0.3)
154. 1.25 ÷ (− 0.5)
155. −1.75 ÷ (− 0.05)
156. −1.15 ÷ (− 0.05)
157. 5.2 ÷ 2.5
158. 6.5 ÷ 3.25
159. 12 ÷ 0.08

160. 5 ÷ 0.04
161. 11 ÷ 0.55
162. 14 ÷ 0.35

Mixed Practice

In the following exercises, simplify.

163. 6(12.4 − 9.2)
164. 3(15.7 − 8.6)
165. $24(0.5) + (0.3)^2$
166. $35(0.2) + (0.9)^2$
167. 1.15(26.83 + 1.61)
168. 1.18(46.22 + 3.71)
169. \$45 + 0.08(\$45)
170. \$63 + 0.18(\$63)
171. 18 ÷ (0.75 + 0.15)
172. 27 ÷ (0.55 + 0.35)
173. (1.43 + 0.27) ÷ (0.9 − 0.05)
174. (1.5 − 0.06) ÷ (0.12 + 0.24)
175. [\$75.42 + 0.18(\$75.42)] ÷ 5
176. [\$56.31 + 0.22(\$56.31)] ÷ 4

Use Decimals in Money Applications

In the following exercises, use the strategy for applications to solve.

177. **Spending money** Brenda got \$40 from the ATM. She spent \$15.11 on a pair of earrings. How much money did she have left?
178. **Spending money** Marissa found \$20 in her pocket. She spent \$4.82 on a smoothie. How much of the \$20 did she have left?
179. **Shopping** Adam bought a t-shirt for \$18.49 and a book for \$8.92 The sales tax was \$1.65. How much did Adam spend?
180. **Restaurant** Roberto's restaurant bill was \$20.45 for the entrée and \$3.15 for the drink. He left a \$4.40 tip. How much did Roberto spend?
181. **Coupon** Emily bought a box of cereal that cost \$4.29. She had a coupon for \$0.55 off, and the store doubled the coupon. How much did she pay for the box of cereal?
182. **Coupon** Diana bought a can of coffee that cost \$7.99. She had a coupon for \$0.75 off, and the store doubled the coupon. How much did she pay for the can of coffee?
183. **Diet** Leo took part in a diet program. He weighed 190 pounds at the start of the program. During the first week, he lost 4.3 pounds. During the second week, he had lost 2.8 pounds. The third week, he gained 0.7 pounds. The fourth week, he lost 1.9 pounds. What did Leo weigh at the end of the fourth week?
184. **Snowpack** On April 1, the snowpack at the ski resort was 4 meters deep, but the next few days were very warm. By April 5, the snow depth was 1.6 meters less. On April 8, it snowed and added 2.1 meters of snow. What was the total depth of the snow?
185. **Coffee** Noriko bought 4 coffees for herself and her coworkers. Each coffee was \$3.75. How much did she pay for all the coffees?
186. **Subway Fare** Arianna spends \$4.50 per day on subway fare. Last week she rode the subway 6 days. How much did she spend for the subway fares? 187. Income Mayra earns \$9.25 per hour. Last week she worked 32 hours. How much did she earn?
187. **Income** Peter earns \$8.75 per hour. Last week he worked 19 hours. How much did he earn?
188. **Hourly Wage** Alan got his first paycheck from his new job. He worked 30 hours and earned \$382.50. How much does he earn per hour?
189. **Hourly Wage** Maria got her first paycheck from her new job. She worked 25 hours and earned \$362.50. How much does she earn per hour?
190. **Restaurant** Jeannette and her friends love to order mud pie at their favorite restaurant. They always share just one piece of pie among themselves. With tax and tip, the total cost is \$6.00. How much does each girl pay if the total number sharing the mud pie is (a) 2? (b) 3? (c) 4? (d) 5? (e) 6?
191. **Pizza** Alex and his friends go out for pizza and video games once a week. They share the cost of a \$15.60 pizza equally. How much does each person pay if the total number sharing the pizza is (a) 2? (b) 3? (c) 4? (d) 5? (e) 6?

192. **Fast Food** At their favorite fast food restaurant, the Carlson family orders 4 burgers that cost $3.29 each and 2 orders of fries at $2.74 each. What is the total cost of the order?
193. **Home Goods** Chelsea needs towels to take with her to college. She buys 2 bath towels that cost $9.99 each and 6 washcloths that cost $2.99 each. What is the total cost for the bath towels and washcloths?
194. **Zoo** The Lewis and Chousmith families are planning to go to the zoo together. Adult tickets cost $29.95 and children's tickets cost $19.95. What will the total cost be for 4 adults and 7 children?
195. **Ice Skating** Jasmine wants to have her birthday party at the local ice skating rink. It will cost $8.25 per child and $12.95 per adult. What will the total cost be for 12 children and 3 adults?

Everyday Math

197. **Paycheck** Annie has two jobs. She gets paid $14.04 per hour for tutoring at City College and $8.75 per hour at a coffee shop. Last week she tutored for 8 hours and worked at the coffee shop for 15 hours. (a) How much did she earn? (b) If she had worked all 23 hours as a tutor instead of working both jobs, how much more would she have earned?
198. **Paycheck** Jake has two jobs. He gets paid $7.95 per hour at the college cafeteria and $20.25 at the art gallery. Last week he worked 12 hours at the cafeteria and 5 hours at the art gallery. (a) How much did he earn? (b) If he had worked all 17 hours at the art gallery instead of working both jobs, how much more would he have earned?

Writing Exercises

199. In the 2010 winter Olympics, two skiers took the silver and bronze medals in the Men's Super-G ski event. The silver medalist's time was 1 minute 30.62 seconds and bronze medalist's time was 1 minute 30.65 seconds. Whose time was faster? Find the difference in their times and then write the name of that decimal.
200. Find the quotient of 0.12 ÷ 0.04 and explain in words all the steps taken.

Self Check

(a) After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
add and subtract decimals.			
multiply decimals.			
divide decimals.			
use decimals in money applications.			

(b) After reviewing this checklist, what will you do to become confident for all objectives?

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CHAPTER OVERVIEW

5: Geometry

5.1: Unit Conversion - American System
5.2: Unit Conversion- Metric System
5.3: American Units to Metric Units and Vice-Versa
5.4: Perimeter and Area
5.5: Volume
5.6: Solve a Formula for a Specific Variable
5.6E: Exercises

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5.1: Unit Conversion - American System

In this section we will develop a technique for converting units used in the American system. We begin with a discussion of common measurements of length in the United States.

Units of Length

The most common units of length are the inch, foot, yard, and mile. Our focus will be on the technique used to convert from one unit of length to another.

American Units of Length

Facts relating common units of length.

- 1 foot (ft) = 12 inches (in)
- 1 yard (yd) = 3 feet (ft)
- 1 mile (mi) = 5280 feet (ft)

Take for example, the fact that there are 3 feet in 1 yard, which can be stated as an equation, using the common abbreviations for feet (ft) and yards (yd).

$$3 \text{ ft} = 1 \text{ yd}$$

If we divide both sides of the equation by 3 ft,

$$\frac{3 \text{ ft}}{3 \text{ ft}} = \frac{1 \text{ yd}}{3 \text{ ft}},$$

or equivalently,

$$1 = \frac{1 \text{ yd}}{3 \text{ ft}}.$$

The key observation is the fact that the ratio 1 yd/3 ft equals the number 1. Consequently, multiplying by the "conversion factor" 1 yd/3 ft is equivalent to multiplying by 1. This can be used to change a measurement in feet to yards.

Example 1

Change 36 feet to yards.

Solution

Multiply by the conversion factor 1 yd/3 ft.

$$\begin{aligned} 36 \text{ ft} &= 36 \text{ ft} \cdot 1 && \text{Multiplicative Identity Property.} && (5.1.1)\\ &= 36 \text{ ft} \cdot \frac{1 \text{ yd}}{3 \text{ ft}} && \text{Replace 1 with 1 yd/3 ft.} && (5.1.2)\\ &= 36 \ \cancel{\text{ft}} \cdot \frac{1 \text{ yd}}{3 \ \cancel{\text{ft}}} && \text{Cancel common unit.} && (5.1.3)\\ &= \frac{36 \cdot 1}{3} \text{ yd} && \text{Multiply fractions.} && (5.1.4)\\ &= \frac{36}{3} \text{ yd} && \text{Simplify.} && (5.1.5)\\ &= 12 \text{ yd} && \text{Divide.} && (5.1.6) \end{aligned}$$

Hence, 36 feet equals 12 yards.

Exercise

Change 81 feet to yards.

Answer

27 yards

On the other hand, we can start again with

$$3 \text{ ft} = 1 \text{ yd}$$

and divide both sides of the equation by 1 yd.

$$\frac{3 \text{ ft}}{1 \text{ yd}} = \frac{1 \text{ yd}}{1 \text{ yd}}$$

This gives the conversion factor

$$\frac{3 \text{ ft}}{1 \text{ yd}} = 1.$$

The key observation is the fact that the ratio 3 ft/1 yd equals the number 1. Consequently, multiplying by the "conversion factor" 3 ft/1 yd is equivalent to multiplying by 1. This can be used to change a measurement in yards to feet.

Example 2

Change 18 yards to feet.

Solution

Multiply by the conversion factor 3 ft/1 yd.

$$18 \text{ yd} = 18 \text{ yd} \cdot 1 \quad \text{Multiplicative Identity Property.} \tag{5.1.7}$$

$$= 18 \text{ yd} \cdot \frac{3 \text{ ft}}{1 \text{ yd}} \quad \text{Replace 1 with 3 ft/1 yd.} \tag{5.1.8}$$

$$= 18 \ \cancel{\text{yd}} \cdot \frac{3 \text{ ft}}{1 \ \cancel{\text{yd}}} \quad \text{Cancel common unit.} \tag{5.1.9}$$

$$= \frac{18 \cdot 3}{1} \text{ ft} \quad \text{Multiply fractions.} \tag{5.1.10}$$

$$= 54 \text{ ft} \quad \text{Simplify.} \tag{5.1.11}$$

Hence, 18 yards equals 54 feet.

Exercise

Change 15 yards to feet.

Answer

45 feet

Another common comparison is the fact that there are 12 inches in 1 foot. This can be represented as an equation using the common abbreviation for inches (in) and feet (ft).

$$12 \text{ in} = 1 \text{ ft}$$

Dividing both sides by 12 in

$$\frac{12 \text{ in}}{12 \text{ in}} = \frac{1 \text{ ft}}{12 \text{ in}},$$

yields the conversion factor

$$1 = \frac{1 \text{ ft}}{12 \text{ in}}.$$

The key observation is the fact that the ratio 1 ft/12 in equals the number 1. Consequently, multiplying by the "conversion factor" 1 ft/12 in is equivalent to multiplying by 1. This can be used to change a measurement in inches to feet.

Example 3

Change 24 inches to feet.

Solution

Multiply by the conversion factor 1 ft/12 in.

$$\begin{aligned} 24\text{ in} &= 24\text{ in}\cdot 1 && \text{Multiplicative Identity Property.}\\ &= 24\text{ in}\cdot\frac{1\text{ ft}}{12\text{ in}} && \text{Replace 1 with 1 ft/12 in.}\\ &= 24\ \cancel{\text{in}}\cdot\frac{1\text{ in}}{12\ \cancel{\text{in}}} && \text{Cancel common unit.}\\ &= \frac{24\cdot 1}{12}\text{ ft} && \text{Multiply fractions.}\\ &= 2\text{ ft} && \text{Simplify.} \end{aligned}$$

Hence, 24 inches equals 2 feet.

Exercise

Change 48 inches to feet.

Answer

4 feet

We provide a summary of conversion factors for units of length in Table 6.1.

Table 6.1: Conversion factors for units of length.

Convert	Conversion Factor	Convert	Conversion Factor
feet to inches	12 in/1 ft	inches to feet	1 ft/12 in
yards to feet	3 ft/1 yd	feet to yards	1 yd/3 ft
miles to feet	5280 ft/1 mi	feet to miles	1 mi/5280 ft

Some conversions require more than one application of a conversion factor.

Example 4

Change 4 yards to inches.

Solution

We multiply by a chain of conversion factors, the first to change yards to feet, the second to change feet to inches.

$$\begin{aligned} 4\text{ yd} &= 4\text{ yd}\cdot\frac{3\text{ ft}}{1\text{ yd}}\cdot\frac{12\text{ in}}{1\text{ ft}} && \text{Multiply by conversion factors.}\\ &= 4\ \cancel{\text{yd}}\cdot\frac{3\ \cancel{\text{ft}}}{1\ \cancel{\text{yd}}}\cdot\frac{12\text{ in}}{1\ \cancel{\text{ft}}} && \text{Cancel common units.}\\ &= \frac{4\cdot 3\cdot 12}{1\cdot 1}\text{ in} && \text{Multiply fractions.}\\ &= 144\text{ in} && \text{Simplify.} \end{aligned}$$

Hence, 4 yards equals 144 inches.

Exercise

Change 8 yards to inches.

Answer

288 inches

Example 5

Change 2 miles to yards.

Solution

We multiply by a chain of conversion factors, the first to change miles to feet, the second to change feet to yards.

$$2\text{ mi} = 2\text{ mi} \cdot \frac{5280\text{ ft}}{1\text{ mi}} \cdot \frac{1\text{ yd}}{3\text{ ft}} \quad \text{Multiply by conversion factors.}$$

$$= 2\ \cancel{\text{mi}} \cdot \frac{5280\ \cancel{\text{ft}}}{1\ \cancel{\text{mi}}} \cdot \frac{1\text{ yd}}{3\ \cancel{\text{ft}}} \quad \text{Cancel common units.}$$

$$= \frac{2 \cdot 5280 \cdot 1}{1 \cdot 3}\text{ yd} \quad \text{Multiply fractions.}$$

$$= 3520\text{ yd} \quad \text{Simplify.}$$

Hence, 2 miles equals 3,520 yards.

Exercise

Change 5 miles to yards.

Answer

8,800 yards

Units of Weight

The most common units of weight are the ounce, pound, and ton. Our focus will remain on how to convert from one unit to another.

American Units of Weight

Facts relating common units of weight.

- 1 pound (lb) = 16 ounces (oz)
- 1 ton = 2000 pounds (lb)

The above facts lead to the conversion factors in Table 6.2.

Table 6.2: Conversion factors for units of weight.

Convert	Conversion Factor	Convert	Conversion Factor
pounds to ounces	16 oz/1 lb	ounces to pounds	1 lb/16 oz
tons to pounds	2000 lb/1 ton	pounds to tons	1 ton/2000 lb

Example 6

Change $2\frac{1}{2}$ pounds to ounces.

Solution

Multiply by the appropriate conversion factor.

$$
\begin{aligned}
2\frac{1}{2}\text{ lb} &= 2\frac{1}{2}\text{ lb}\cdot\frac{16\text{ oz}}{1\text{ lb}} && \text{Multiply by conversion factor.}\\
&= 2\frac{1}{2}\ \cancel{\text{lb}}\cdot\frac{16\text{ oz}}{1\ \cancel{\text{lb}}} && \text{Cancel common units.}\\
&= \left(2\frac{1}{2}\cdot 16\right)\text{ oz} && \text{Multiply fractions.}\\
&= \left(\frac{5}{2}\cdot 16\right)\text{ oz} && \text{Mixed to improper fraction.}\\
&= \frac{80}{2}\text{ oz} && \text{Multiply.}\\
&= 40\text{ oz} && \text{Divide.}
\end{aligned}
$$

Hence, $2\frac{1}{2}$ pounds equals 40 ounces.

Exercise

Change $6\frac{1}{4}$ pounds to ounces.

Answer

100 ounces

Example 7

Change 3.2 tons to ounces.

Solution

This exercise requires multiplying by a chain of conversion factors.

$$
\begin{aligned}
3.2\text{ ton} &= 3.2\text{ ton}\cdot\frac{2000\text{ lb}}{1\text{ ton}}\cdot\frac{16\text{ oz}}{1\text{ lb}} && \text{Multiply by conversion factors.}\\
&= 3.2\ \cancel{\text{ton}}\cdot\frac{2000\ \cancel{\text{lb}}}{1\ \cancel{\text{ton}}}\cdot\frac{16\text{ oz}}{1\ \cancel{\text{lb}}} && \text{Cancel common units.}\\
&= \frac{3.2\cdot 2000\cdot 16}{1\cdot 1}\text{ oz} && \text{Multiply fractions.}\\
&= 102,400\text{ oz} && \text{Simplify.}
\end{aligned}
$$

Hence, 3.2 tons equals 102,400 ounces.

Exercise

Change 4.1 tons to ounces.

Answer

128,000 ounces

Units of Volume

The most common units of volume are fluid ounces, cups, pints, quarts, and gallons. We will focus on converting from one unit to another.

American Units of Volume

Facts relating common units of volume.

- 1 cup (c) = 8 fluid ounces (fl oz)
- 1 pint (pt) = 2 cups (c)

- 1 quart (qt) = 2 pints (pt)
- 1 gallon (gal) = 4 quarts (qt)

These facts lead to the conversion factors listed in Table 6.3.

Table 6.3: Conversion factors for units of volume.

Convert	Conversion Factor	Convert	Conversion Factor
cups to ounces	8 fl oz/1 c	ounces to cups	1c/8 fl oz
pints to cups	2 c/1 pt	cups to pints	1 pt/2 c
quarts to pints	2 pt/1 qt	pints to quarts	1 qt/2 pt
gallons to quarts	4 qt/1 gal	quarts to gallons	1 gal/4 qt

Example 8

Change 5.6 gallons to pints.

Solution

This exercise requires multiplying by a chain of conversion factors.

$$5.6\ \text{gal} = 5.6\ \text{gal} \cdot \frac{4\ \text{qt}}{1\ \text{gal}} \cdot \frac{2\ \text{pt}}{1\ \text{qt}} \quad \text{Multiply by conversion factors.}$$

$$= 5.6\ \cancel{\text{gal}} \cdot \frac{4\ \cancel{\text{qt}}}{1\ \cancel{\text{gal}}} \cdot \frac{2\ \text{pt}}{1\ \cancel{\text{qt}}} \quad \text{Cancel common units.}$$

$$= \frac{5.6 \cdot 4 \cdot 2}{1 \cdot 1}\ \text{pt} \quad \text{Multiply fractions.}$$

$$= 44.8\ \text{pt} \quad \text{Simplify.}$$

Hence, 5.6 gallons equals 44.8 pints.

Exercise

Change 3.2 gallons to pints.

Answer

25.6 pints

Units of Time

The most common units of time are seconds, minutes, hours, days, and years.

American Units of Time

Facts relating common units of time.

- 1 minute (min) = 60 seconds (s)
- 1 hour (hr) = 60 minutes (min)
- 1 day (day) = 24 hours (hr)
- 1 year (yr) = 365 days (day)

These facts lead to the conversion factors in Table 6.4.

Table 6.4: Conversion factors for units of time.

Convert	Conversion Factor	Convert	Conversion Factor

Convert	Conversion Factor	Convert	Conversion Factor
minutes to seconds	60 s/1 min	seconds to minutes	1 min/60 s
hours to minutes	60 min/1 hr	minutes to hours	1 hr/60 min
days to hours	24 hr/1 day	hours to days	1 day/24 hr
years to days	365 day/1 yr	days to years	1 yr/365 day

Example 9

How many seconds in a year?

Solution

A chain of conversion factors is needed.

$$1\text{ yr} = 1\text{ yr} \cdot \frac{365\text{ day}}{1\text{ yr}} \cdot \frac{24\text{ hr}}{1\text{ day}} \cdot \frac{60\text{ min}}{1\text{ hr}} \cdot \frac{60\text{ s}}{1\text{ min}} \quad \text{Conversion factors.}$$

$$= 1\ \cancel{\text{yr}} \cdot \frac{365\ \cancel{\text{day}}}{1\ \cancel{\text{yr}}} \cdot \frac{24\ \cancel{\text{hr}}}{1\ \cancel{\text{day}}} \cdot \frac{60\ \cancel{\text{min}}}{1\ \cancel{\text{hr}}} \cdot \frac{60\text{ s}}{1\ \cancel{\text{min}}} \quad \text{Cancel common units.}$$

$$= \frac{1 \cdot 365 \cdot 24 \cdot 60 \cdot 60}{1 \cdot 1 \cdot 1 \cdot 1}\text{ s} \quad \text{Multiply fractions.}$$

$$= 31,536,000\text{ s} \quad \text{Simplify.}$$

Thus, 1 year equals 31,536,000 seconds.

Exercise

How many seconds in a day?

Answer

86,400 seconds

Converting Units of Speed

Ever wonder how fast a baseball is moving?

Example 10

A professional pitcher can throw a baseball at 95 miles per hour. How fast is this in feet per second? Round your answer to the nearest foot per second.

Solution

There are 5280 feet in a mile, 60 minutes in an hour, and 60 seconds in a minute.

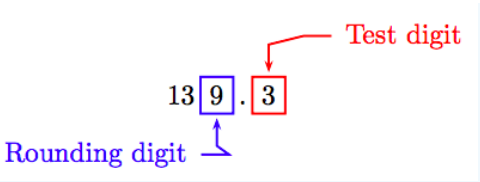
$$95\frac{\text{mi}}{\text{h}} \approx 95\frac{\text{mi}}{\text{h}} \cdot \frac{5280\text{ ft}}{1\text{ mi}} \cdot \frac{1\text{ h}}{60\text{ min}} \cdot \frac{1\text{ min}}{60\text{ s}} \quad \text{Conversion factors.}$$

$$\approx 95\frac{\cancel{\text{mi}}}{\cancel{\text{h}}} \cdot \frac{5280\text{ ft}}{1\ \cancel{\text{mi}}} \cdot \frac{1\ \cancel{\text{h}}}{60\ \cancel{\text{min}}} \cdot \frac{1\ \cancel{\text{min}}}{60\text{ s}} \quad \text{Cancel common units.}$$

$$\approx \frac{95 \cdot 5280 \cdot 1 \cdot 1}{1 \cdot 60 \cdot 60}\frac{\text{ft}}{\text{s}} \quad \text{Multiply fractions.}$$

$$\approx 139.3\frac{\text{ft}}{\text{s}} \quad \text{Multiply and divide.}$$

To round to the nearest foot per second, identify the rounding and test digits.



Because the test digit is less than 5, leave the rounding digit alone and truncate. Thus, to the nearest foot per second, the speed is approximately 139 feet per second.

Whew! Since the batter stands at home plate, which is about 60 feet from where the pitch is delivered, the batter has less than 1/2 a second to react to the pitch!

Exercise

A women's softball pitcher can throw her fastball at 60 miles per hour. How fast is this in feet per second? Round your answer to the nearest foot per second.

Answer

88 feet per second

Exercises

1. Change 8 yards to feet.
2. Change 60 yards to feet.
3. Change 261 feet to yards.
4. Change 126 feet to yards.
5. Change 235 inches to yards. Round your answer to the nearest tenth of a yard.
6. Change 244 inches to yards. Round your answer to the nearest tenth of a yard.
7. Change 141 feet to yards.
8. Change 78 feet to yards.
9. Change 2.8 miles to feet.
10. Change 4.9 miles to feet.
11. Change 104 inches to yards. Round your answer to the nearest tenth of a yard.
12. Change 101 inches to yards. Round your answer to the nearest tenth of a yard.
13. Change 168,372 inches to miles, correct to the nearest tenth of a mile.
14. Change 198,550 inches to miles, correct to the nearest tenth of a mile.
15. Change 82 feet to inches.
16. Change 80 feet to inches.
17. Change 2.9 yards to inches. Round your answer to the nearest inch.
18. Change 4.5 yards to inches. Round your answer to the nearest inch.
19. Change 25,756 feet to miles. Round your answer to the nearest tenth of a mile.
20. Change 19,257 feet to miles. Round your answer to the nearest tenth of a mile.
21. Change 5 yards to feet.
22. Change 20 yards to feet.
23. Change 169,312 inches to miles, correct to the nearest tenth of a mile.

24. Change 162,211 inches to miles, correct to the nearest tenth of a mile.
25. Change 1.5 yards to inches. Round your answer to the nearest inch.
26. Change 2.1 yards to inches. Round your answer to the nearest inch.
27. Change 360 inches to feet.
28. Change 768 inches to feet.
29. Change 48 inches to feet.
30. Change 528 inches to feet.
31. Change 15,363 feet to miles. Round your answer to the nearest tenth of a mile.
32. Change 8,540 feet to miles. Round your answer to the nearest tenth of a mile.
33. Change 1.7 miles to inches.
34. Change 4.7 miles to inches.
35. Change 3.1 miles to inches.
36. Change 1.8 miles to inches.
37. Change 3.6 miles to feet.
38. Change 3.1 miles to feet.
39. Change 18 feet to inches.
40. Change 33 feet to inches.

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41. Change $5\frac{1}{8}$ pounds to ounces.
42. Change $3\frac{1}{16}$ pounds to ounces.
43. Change 2.4 tons to ounces.
44. Change 3.4 tons to ounces.
45. Change 34 ounces to pounds. Express your answer as a fraction reduced to lowest terms.
46. Change 78 ounces to pounds. Express your answer as a fraction reduced to lowest terms.
47. Change 2.2 tons to pounds.
48. Change 4.8 tons to pounds.
49. Change 70 ounces to pounds. Express your answer as a fraction reduced to lowest terms.
50. Change 20 ounces to pounds. Express your answer as a fraction reduced to lowest terms.
51. Change 9,560 pounds to tons. Round your answer to the nearest tenth of a ton.
52. Change 9,499 pounds to tons. Round your answer to the nearest tenth of a ton.
53. Change $2\frac{1}{2}$ pounds to ounces.
54. Change $7\frac{1}{16}$ pounds to ounces.
55. Change 5.9 tons to pounds.
56. Change 2.1 tons to pounds.
57. Change 2.5 tons to ounces.
58. Change 5.3 tons to ounces.
59. Change 8,111 pounds to tons. Round your answer to the nearest tenth of a ton.
60. Change 8,273 pounds to tons. Round your answer to the nearest tenth of a ton.

---

61. Change 4.5625 pints to fluid ounces.
62. Change 2.9375 pints to fluid ounces.
63. Change 32 fluid ounces to pints.
64. Change 160 fluid ounces to pints.
65. Change 3.7 gallons to pints.
66. Change 2.4 gallons to pints.
67. Change 216 pints to gallons.
68. Change 96 pints to gallons.
69. Change 544 fluid ounces to pints.
70. Change 432 fluid ounces to pints.
71. Change 112 pints to gallons.
72. Change 200 pints to gallons.
73. Change 7.7 gallons to pints.
74. Change 5.7 gallons to pints.
75. Change 3.875 pints to fluid ounces.
76. Change 3 pints to fluid ounces.

---

77. Change 7.8 years to hours.
78. Change 4.7 years to hours.
79. Change 7.6 years to hours.
80. Change 6.6 years to hours.
81. Change 4,025,005 seconds to days. Round your answer to the nearest tenth of a day.
82. Change 4,672,133 seconds to days. Round your answer to the nearest tenth of a day.
83. Change 37,668 hours to years.
84. Change 40,296 hours to years.
85. Change 22,776 hours to years.
86. Change 29,784 hours to years.
87. Change 96 days to seconds.
88. Change 50 days to seconds.
89. Change 40 days to seconds.
90. Change 10 days to seconds.
91. Change 3,750,580 seconds to days. Round your answer to the nearest tenth of a day.
92. Change 4,493,469 seconds to days. Round your answer to the nearest tenth of a day.

---

93. Change 367 feet per second to miles per hour. Round your answer to the nearest mile per hour.
94. Change 354 feet per second to miles per hour. Round your answer to the nearest mile per hour.
95. Change 442 feet per second to miles per hour. Round your answer to the nearest mile per hour.
96. Change 388 feet per second to miles per hour. Round your answer to the nearest mile per hour.
97. Change 30 miles per hour to feet per second. Round your answer to the nearest foot per second.
98. Change 99 miles per hour to feet per second. Round your answer to the nearest foot per second.

99. Change 106 miles per hour to feet per second. Round your answer to the nearest foot per second.

100. Change 119 miles per hour to feet per second. Round your answer to the nearest foot per second.

---

101. **Strong man.** Famed strongman Joe Rollino, who was still bending quarters with his fingers at age 104, once lifted 3, 200 pounds at Coney Island Amusement Park. How many tons did Joe lift that day? *Associated Press-Times-Standard 01/12/10 NYC amusement park strongman, 104, killed by van.*

102. **Earth day.** The amount of time it takes the Earth to rotate once around its axis is one day. How many seconds is that?

103. **Water break.** "The average age of Washington, DC's water pipes is 76 years, and they are not alone. Every two minutes, somewhere in the country, a pipe breaks." How many pipes break each year in the US? *New York Times 03/14/10 Saving U.S. water and sewer systems could be costly.*

Answers

1. 24 feet

3. 87 yards

5. 6.5 yards

7. 47 yards

9. 14,784 feet

11. 2.9 yards

13. 2.7 miles

15. 984 inches

17. 104 inches

19. 4.9 miles

21. 15 feet

23. 2.7 miles

25. 54 inches

27. 30 feet

29. 4 feet

31. 2.9 miles

33. 107,712 inches

35. 196,416 inches

37. 19,008 feet

39. 216 inches

41. 82 ounces

43. 76,800 ounces

45. $2\frac{1}{8}$ pounds

47. 4,400 pounds

49. $4\frac{3}{8}$ pounds

51. 4.8 tons

53. 40 ounces

55. 11,800 pounds

57. 80,000 ounces

59. 4.1 tons

61. 73 fluid ounces

63. 2 pints

65. 29.6 pints

67. 27 gallons

69. 34 pints

71. 14 gallons

73. 61.6 pints

75. 62 fluid ounces

77. 68,328 hours

79. 66,576 hours

81. 46.6 days

83. 4.3 years

85. 2.6 years

87. 8,294,400 seconds

89. 3,456,000 seconds

91. 43.4 days

93. 250 mi/hr

95. 301 mi/hr

97. 44 ft/s

99. 155 ft/s

101. 1.6 tons

103. 262, 800

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5.2: Unit Conversion- Metric System

The metric system of units is the standard system of units preferred by scientists. It is based on the base ten number system and its decimal format is more friendly to users of this system. There is a common set of prefixes adopted by the metric system to indicate a power of ten to apply to the base unit.

Metric System Preferences

This is a list of standard prefixes for the metric system and their meanings.

$$\begin{array}{ll} \text{deka} = 10 & \text{deci} = 1/10 \\ \text{hecto} = 100 & \text{centi} = 1/100 \\ \text{kilo} = 1000 & \text{milli} = 1/1000 \end{array}$$

Thus, for example, a decameter is 10 meters, a hectoliter is 100 liters, and a kilogram is 1000 grams. Similarly, a decimeter is 1/10 of a meter, a centiliter is 1/100 of a liter, and a milligram is 1/1000 of a gram.

Units of Length

The standard measure of length in the metric system is the meter.

> *Historically, the meter was defined by the French Academy of Sciences as the length between two marks on a platinum-iridium bar, which was designed to represent 110,000,000 of the distance from the equator to the north pole through Paris. In 1983, it was redefined by the International Bureau of Weights and Measures (BIPM) as the distance travelled by light in free space in 1299,792,458 of a second. (Wikipedia)*

We can apply the standard prefixes to get the following result.

Metric Units of Length

These units of length are used in the metric system.

Unit Length	Unit Abbreviation
1 kilometer = 1000 meters	km
1 hectometer = 100 meters	hm
1 dekameter = 10 meters	dam
1 meter	m
1 decimeter = $\frac{1}{10}$ meter	dm
1 centimeter = $\frac{1}{100}$ meter	cm
1 millimeter = $\frac{1}{1000}$ meter	mm

We can use these facts to build conversion factors as we did in Section 6.3. For example, because

$$1 \text{ km} = 1000 \text{ m},$$

we can divide both sides by 1000 m to produce the conversion factor

$$1 = \frac{1 \text{ km}}{1000 \text{ m}}.$$

This conversion factor can help change meters into kilometers.

Before using this conversion factor in an example, we repeat here the rules for multiplying and dividing by powers of ten. We will be making heavy use of these rules in this section.

Multiplying and Dividing by Powers of Ten

- Multiplying a decimal number by 10^n will move the decimal point n places to the right. For example, $3.2567 \cdot 10^2 = 3.2567 \cdot 100 = 325.67$.
- Dividing a decimal number by 10^n will move the decimal point n places to the left. For example, $3.2567/10^2 = 3.2567/100 = 0.032567$.

And now the example.

Example 1

Change 2,326 meters to kilometers.

Solution

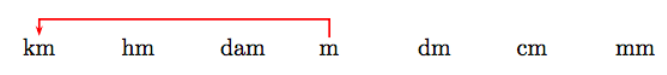
Multiply by the conversion factor 1 km/1000m.

$$\begin{aligned} 2326\ \text{m} &= 2326\ \text{m} \cdot \frac{1\ \text{km}}{1000\ \text{m}} && \text{Apply conversion factor.} \\ &= 2326\ \cancel{\text{m}} \cdot \frac{1\ \text{km}}{1000\ \cancel{\text{m}}} && \text{Cancel common units.} \\ &= \frac{2326 \cdot 1}{1000}\ \text{km} && \text{Multiply fractions.} \\ &= 2.326\ \text{km} && \text{Simplify.} \end{aligned}$$

In the last step, note that dividing by 1000 moves the decimal point three places to the left. Thus, 2326 meters is equal to 2.326 kilometers.

Alternate Solution

A second solution depends upon the fact that multiplying or dividing by a power of ten will move the decimal point right or left a number of places equal to the number of zeros present in the multiplier or divisor. Thus, as we saw above, dividing by 1000 moved the decimal point 3 places to the left. Suppose that we arrange the metric units of length in order, from largest to smallest, as shown below.



Note that we must move 3 places left to move from the meters (m) abbreviation to the kilometers (km) abbreviation. In like manner, if we write 2,326 meters as 2,326.0meters, then we can convert to kilometers by moving the decimal 3 places to the left.



Exercise

Change 1,156 meters to kilometers

Answer

1.156 kilometers

Example 2

Change 537 centimeters to meters.

Solution

We know that

$$1\text{ cm} = \frac{1}{100}\text{ m},$$

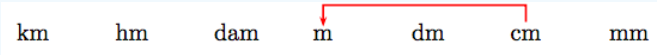
or multiplying both sides of this result by 100,

$$100\text{ cm} = 1\text{ m}.$$

Dividing both sides of this last result by 100 cm, we obtain the conversion factor 1 m/100 cm.

$$\begin{aligned} 537\text{ cm} &= 537\text{ cm} \cdot \frac{1\text{ m}}{100\text{ cm}} && \text{Apply conversion factor.} \\ &= 537\ \cancel{\text{cm}} \cdot \frac{1\text{ m}}{100\ \cancel{\text{cm}}} && \text{Cancel common units.} \\ &= \frac{537 \cdot 1}{100}\text{ m} && \text{Multiply fractions.} \\ &= 5.37\text{ m} \end{aligned}$$

In the last step, note that dividing by 100 moves the decimal point two places to the left. Alternately, we can set up our ordered list of units.



Note that we must move 2 places left to move from the centimeters (cm) abbreviation to the meters (m) abbreviation. In like manner, if we write 537 centimeters as 537.0 centimeters, then we can convert to meters by moving the decimal 2 places to the left.



Sometimes more than one conversion factor is needed.

Exercise

Change 276 centimeters to meters.

Answer

2.76 meters

Example 3

Change 10.2 dekameters to centimeters.

Solution

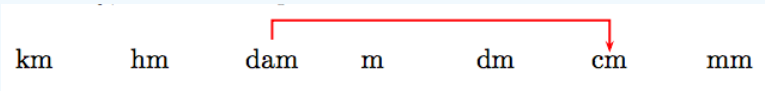
We have two facts:

- 1 dam=10 m, which yields the conversion factor 10 m/1 dam.
- 1 cm=(1/100) m or 100 cm=1 m, which yields the conversion factor 100 cm/1 m.

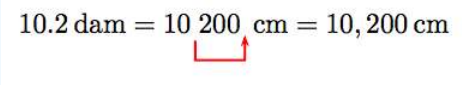
$$
\begin{aligned}
10.2\ \text{dam} &= 10.2\ \text{dam} \cdot \frac{10\ \text{m}}{1\ \text{dam}} \cdot \frac{100\ \text{cm}}{1\ \text{m}} && \text{Apply conversion factor.} \\
&= 10.2\ \cancel{\text{dam}} \cdot \frac{10\ \cancel{\text{m}}}{1\ \cancel{\text{dam}}} \cdot \frac{100\ \text{cm}}{1\ \cancel{\text{m}}} && \text{Cancel common units.} \\
&= \frac{10.2 \cdot 10 \cdot 100}{1}\ \text{cm} && \text{Multiply fractions.} \\
&= 10,200\ \text{cm}
\end{aligned}
$$

In the last step, note that multiplying by 10, then by 100, moves the decimal point three places to the right.

Alternately, we can set up our ordered list of units.



Note that we must move 3 places right to move from the dekameters (dam) abbreviation to the centimeters (cm) abbreviation. In like manner, we can convert 10.2 dekameters to centimeters by moving the decimal 3 places to the right.



Exercise

Change 13.5 dekameters to centimeters

Answer

13,500 centimeters

Units of Mass

The fundamental unit of mass in the metric system is called a gram. Originally, it was defined to be equal to one cubic centimeter of water measured at the temperature of melting ice. Now it is simply defined as 1/1000 of a kilogram, which is defined by a physical prototype preserved by the International Bureau of Weights and Measures (Wikipedia). The mass of an object is not the same as an object's weight, but rather a resistance to motion when an external force is applied.

The same metric system prefixes apply.

Metric Units of Mass

These units of mass are used in the metric system.

Units of Mass	Unit Abbreviation
1 kilogram = 1000 grams	kg
1 hectogram = 100 grams	hg
1 dekagram = 10 grams	dag
1 gram	g
1 decigram = $\frac{1}{10}$ gram	dg
1 centigram = $\frac{1}{100}$ gram	cg
1 milligram = $\frac{1}{1000}$ gram	mg

Example 4

Convert 0.025 dekagrams to milligrams.

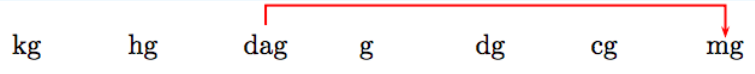
Solution

We'll use two conversion factors:

- 1 dag=10 g, which yields the conversion factor 10 g/1 dag.
- 1 mg=(1/1000) g, which yields the conversion factor 1000 mg/1 g.

$$\begin{aligned} 0.025\text{ dag} &= 0.025\text{ dag} \cdot \frac{10\text{ g}}{1\text{ dag}} \cdot \frac{1000\text{ mg}}{1\text{ g}} && \text{Apply conversion factor.} \\ &= 0.025\ \cancel{\text{dag}} \cdot \frac{10\ \cancel{\text{g}}}{1\ \cancel{\text{dag}}} \cdot \frac{1000\text{ mg}}{1\ \cancel{\text{g}}} && \text{Cancel common units.} \\ &= \frac{0.025 \cdot 10 \cdot 1000}{1}\text{ mg} && \text{Multiply fractions.} \\ &= 250\text{ mg} \end{aligned}$$

Alternately, we can set up our ordered list of units.



Note that we must move 4 places right to move from the dekagrams (dag) abbreviation to the milligrams (mg) abbreviation. In like manner, we can convert 0.025 dekagrams to milligrams by moving the decimal 4 places to the right.

$$0.0250\text{ dag} = 0\ 0250.\text{ mg} = 250\text{ mg}$$

Exercise

Convert 0.05 dekagrams to milligrams.

Answer

500 milligrams

Units of Volume

The fundamental unit of volume in the metric system is called a litre. Originally, one litre was defined as the volume of one kilogram of water measured at 4° C at 760 millimeters of mercury (Wikipedia). Currently, 1 litre is defined as 1 cubic decimeter (imagine a cube with each edge 1/10 of a meter).

The same metric system prefixes apply.

Metric Units of Volume

These units of volume are used in the metric system.

Unit of Volume	Unit Abbreviation
1 kilolitre = 1000 litres	kL
1 hectolitre = 100 litres	hL
1 dekalitre = 10 litres	daL
1 litre	L
1 decilitre = $\frac{1}{10}$ litre	dL

1 centilitre = $\frac{1}{100}$ litre	cL
1 millilitre = $\frac{1}{1000}$ litre	mL

Example 5

Convert 11,725 millilitres to dekalitres.

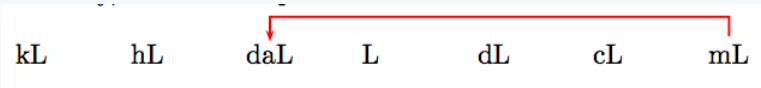
Solution

We'll use two conversion factors:

- 1 daL=10 L, which yields the conversion factor 1 daL/10 L.
- 1 mL=(1/1000) L, which yields the conversion factor 1 L/1000 mL.

$$\begin{aligned} 11,725\ \text{mL} &= 11,725\ \text{mL} \cdot \frac{1\ \text{L}}{1000\ \text{mL}} \cdot \frac{1\ \text{daL}}{10\ \text{L}} && \text{Apply conversion factors.} \\ &= 11,725\ \cancel{\text{mL}} \cdot \frac{1\ \cancel{\text{L}}}{1000\ \cancel{\text{mL}}} \cdot \frac{1\ \text{daL}}{10\ \cancel{\text{L}}} && \text{Cancel common units.} \\ &= \frac{11,725 \cdot 1 \cdot 1}{1000 \cdot 10}\ \text{daL} && \text{Multiply fractions.} \\ &= 1.1725\ \text{daL} \end{aligned}$$

Alternately, we can set up our ordered list of units.



Note that we must move 4 places left to move from the millitres (mL) abbreviation to the dekalitres (daL) abbreviation. In like manner, we can convert 11,725 millilitres to dekalitres by moving the decimal 4 places to the left.

$$11,725.0\ \text{mL} = 1.1725\ 0\ \text{daL} = 1.1725\ \text{daL}$$

Exercise

Convert 5,763 millilitres to dekalitres.

Answer

0.5763 dekalitres

Exercises

1. What is the meaning of the metric system prefix centi?
2. What is the meaning of the metric system prefix deka?
3. What is the meaning of the metric system prefix hecto?
4. What is the meaning of the metric system prefix kilo?
5. What is the meaning of the metric system prefix deci?
6. What is the meaning of the metric system prefix milli?

---

7. What is the meaning of the metric system abbreviation mg?
8. What is the meaning of the metric system abbreviation g?
9. What is the meaning of the metric system abbreviation m?

10. What is the meaning of the metric system abbreviation km?
11. What is the meaning of the metric system abbreviation kL?
12. What is the meaning of the metric system abbreviation daL?
13. What is the meaning of the metric system abbreviation hm?
14. What is the meaning of the metric system abbreviation dm?
15. What is the meaning of the metric system abbreviation dam?
16. What is the meaning of the metric system abbreviation cm?
17. What is the meaning of the metric system abbreviation dL?
18. What is the meaning of the metric system abbreviation L?
19. What is the meaning of the metric system abbreviation hg?
20. What is the meaning of the metric system abbreviation kg?
21. What is the meaning of the metric system abbreviation dg?
22. What is the meaning of the metric system abbreviation dag?
23. What is the meaning of the metric system abbreviation hL?
24. What is the meaning of the metric system abbreviation cL?

---

25. Change 5,490 millimeters to meters.
26. Change 8,528 millimeters to meters.
27. Change 64 meters to millimeters.
28. Change 65 meters to millimeters.
29. Change 4,571 millimeters to meters.
30. Change 8,209 millimeters to meters.
31. Change 15 meters to centimeters.
32. Change 12 meters to centimeters.
33. Change 569 centimeters to meters.
34. Change 380 centimeters to meters.
35. Change 79 meters to centimeters.
36. Change 60 meters to centimeters.
37. Change 7.6 kilometers to meters.
38. Change 4.9 kilometers to meters.
39. Change 861 centimeters to meters.
40. Change 427 centimeters to meters.
41. Change 4,826 meters to kilometers.
42. Change 1,929 meters to kilometers.
43. Change 4,724 meters to kilometers.
44. Change 1,629 meters to kilometers.
45. Change 6.5 kilometers to meters.
46. Change 7.9 kilometers to meters.
47. Change 17 meters to millimeters.

48. Change 53 meters to millimeters.

---

49. Change 512 milligrams to centigrams.
50. Change 516 milligrams to centigrams.
51. Change 541 milligrams to centigrams.
52. Change 223 milligrams to centigrams.
53. Change 70 grams to centigrams.
54. Change 76 grams to centigrams.
55. Change 53 centigrams to milligrams.
56. Change 30 centigrams to milligrams.
57. Change 83 kilograms to grams.
58. Change 70 kilograms to grams.
59. Change 8,196 grams to kilograms.
60. Change 6,693 grams to kilograms.
61. Change 564 centigrams to grams.
62. Change 884 centigrams to grams.
63. Change 38 grams to centigrams.
64. Change 88 grams to centigrams.
65. Change 77 centigrams to milligrams.
66. Change 61 centigrams to milligrams.
67. Change 5,337 grams to kilograms.
68. Change 4,002 grams to kilograms.
69. Change 15 kilograms to grams.
70. Change 45 kilograms to grams.
71. Change 833 centigrams to grams.
72. Change 247 centigrams to grams.

---

73. Change 619,560 centilitres to kilolitres.
74. Change 678,962 centilitres to kilolitres.
75. Change 15.2 litres to millilitres.
76. Change 9.7 litres to millilitres.
77. Change 10,850 centilitres to litres.
78. Change 15,198 centilitres to litres.
79. Change 10.7 litres to millilitres.
80. Change 17.3 litres to millilitres.
81. Change 15,665 millilitres to litres.
82. Change 12,157 millilitres to litres.
83. Change 6.3 kilolitres to centilitres.
84. Change 8.3 kilolitres to centilitres.
85. Change 4.5 kilolitres to centilitres.

86. Change 6.2 kilolitres to centilitres.
87. Change 10.6 litres to centilitres.
88. Change 16.6 litres to centilitres.
89. Change 14,383 centilitres to litres.
90. Change 11,557 centilitres to litres.
91. Change 9.9 litres to centilitres.
92. Change 19.5 litres to centilitres.
93. Change 407,331 centilitres to kilolitres.
94. Change 827,348 centilitres to kilolitres.
95. Change 14,968 millilitres to litres.
96. Change 18,439 millilitres to litres.

Answers

1. 1/100
3. 100
5. 1/10
7. milligram
9. meter
11. kilolitre or kiloliter
13. hectometer
15. dekameter
17. decilitre or deciliter
19. hectogram
21. decigram
23. hectolitre or hectoliter
25. 5.49 meters
27. 64,000 millimeters
29. 4.571 meters
31. 1,500 centimeters
33. 5.69 meters
35. 7,900 centimeters
37. 7,600 meters
39. 8.61 meters
41. 4.826 kilometers
43. 4.724 kilometers
45. 6,500 meters
47. 17,000 millimeters
49. 51.2 centigrams
51. 54.1 centigrams

53. 7,000 centigrams
55. 530 milligrams
57. 83,000 grams
59. 8.196 kilograms
61. 5.64 grams
63. 3,800 centigrams
65. 770 milligrams
67. 5.337 kilograms
69. 15,000 grams
71. 8.33 grams
73. 6.1956 kilolitres
75. 15,200 millilitres
77. 108.5 litres
79. 10,700 millilitres
81. 15.665 litres
83. 630,000 centilitres
85. 450,000 centilitres
87. 1,060 centilitres
89. 143.83 litres
91. 990 centilitres
93. 4.07331 kilolitres
95. 14.968 litres

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5.3: American Units to Metric Units and Vice-Versa

We often need to convert from the American system of units to the metric system of units or vice-versa (imagine traveling to a European country using the metric system). That will be our focus in this section.

Converting Units of Length

One meter is slightly longer than one yard. Indeed,

$$1\text{ m} \approx 1.0936\text{ yd}. \tag{5.3.1}$$

If we divide both sides of this equation by 1.0936, then

$$\frac{1\text{ m}}{1.0936} \approx \frac{1.0936\text{ yd}}{1.0936}$$
$$0.9144\text{ m} \approx 1\text{ yd}$$

Further conversions can be made. For example, to change meters to feet, we make the following conversions.

$$\begin{aligned} 1\text{ m} &\approx 1\text{ m} \cdot 1\frac{\text{yd}}{0.9144\text{ m}} \cdot \frac{3\text{ ft}}{1\text{ yd}} && \text{Apply conversion factors.} \\ &\approx 1\ \cancel{\text{m}} \cdot \frac{1\ \cancel{\text{yd}}}{0.9144\ \cancel{\text{m}}} \cdot \frac{3\text{ ft}}{1\ \cancel{\text{yd}}} && \text{Cancel common units.} \\ &\approx \frac{1 \cdot 1 \cdot 3}{0.9144 \cdot 1}\text{ ft} && \text{Multiply fractions.} \\ &\approx 3.2808\text{ ft} \end{aligned}$$

Table 6.5 shows some of the most common conversions between American units of length and metric units of length.

Table 6.5: Length conversions: American — Metric.

American to Metric	Metric to American
1 in ≈ 2.54 cm	1 cm ≈ 0.3937 in
1 ft ≈ 0.3048 m	1 m ≈ 3.2808 ft
1 yd ≈ 0.9144 m	1 m ≈ 1.0936 yd
1 mi ≈ 1.6093 km	1 km ≈ 0.6214 mi

Example 1

A car's speedometer shows that a family has currently traveled 154 miles in route to their vacation destination. Convert this distance to kilometers.

Solution

Choose 1 mi = 1.6093 km from Table 6.5.

$$\begin{aligned} 154\text{ mi} &\approx 154\text{ mi} \cdot \frac{1.6093\text{ km}}{1\text{ mi}} && \text{Apply conversion factor.} \\ &\approx 154\ \cancel{\text{mi}} \cdot \frac{1.6093\text{ km}}{1\ \cancel{\text{mi}}} && \text{Cancel common units.} \\ &\approx \frac{154 \cdot 1.6093}{1}\text{ km} && \text{Multiply fractions.} \\ &\approx 247.8\text{ km} \end{aligned}$$

Hence, 154 miles is approximately 247.8 kilometers.

Alternate Solution

Note that this would work equally well if we chose 1 km ≈ 0.6214 mi from Table 6.5.

$$\begin{aligned} 154\text{ mi} &\approx 154\text{ mi} \cdot \frac{1\text{ km}}{0.6214\text{ mi}} && \text{Apply conversion factor.} \\ &\approx 154\ \cancel{\text{mi}} \cdot \frac{1\text{ km}}{0.6214\ \cancel{\text{mi}}} && \text{Cancel common units.} \\ &\approx \frac{154 \cdot 1}{0.6214}\text{ km} && \text{Multiply fractions.} \\ &\approx 247.8\text{ km} \end{aligned}$$

? Exercise

Change 227 miles to kilometers. Round to the nearest tenth of a kilometer.

Answer

365.3 kilometers

Converting Units of Weight and Mass

It is known that

$$1\text{ kg} \approx 2.2\text{ lb}. \tag{5.3.2}$$

Dividing both sides of this equation by 2.2,

$$\frac{1\text{ kg}}{2.2} \approx \frac{2.2\text{ lb}}{2.2}$$

$$0.454\text{ kg} \approx 1\text{lb}$$

A summary of the more common conversion factors regarding mass and weight are given in Table 6.6.

Table 6.6: Mass—Weight conversions: American — Metric.

American to Metric	Metric to American
1 oz ≈ 28.35 g	1 g ≈ 0.035 oz
1 lb ≈ 0.454 kg	1 kg ≈ 2.2 lb

✔ Example 2

Change 2.3 kilograms to ounces.

Solution

One kilogram weighs 2.2 pounds and there are 16 ounces in a pound.

$$\begin{aligned} 2.3\text{ kg} &\approx 2.3\text{ kg} \cdot \frac{2.2\text{ lb}}{1\text{ kg}} \cdot \frac{16\text{ oz}}{1\text{ lb}} && \text{Apply conversion factors.} \\ &\approx 2.3\ \cancel{\text{kg}} \cdot \frac{2.2\ \cancel{\text{lb}}}{1\ \cancel{\text{kg}}} \cdot \frac{16\text{ oz}}{1\ \cancel{\text{lb}}} && \text{Cancel common units.} \\ &\approx \frac{2.3 \cdot 2.2 \cdot 16}{1 \cdot 1}\text{ oz} && \text{Multiply fractions.} \\ &\approx 80.96\text{ oz} \end{aligned}$$

Hence, 2.3 kilograms weighs 80.96 ounces.

Alternate Solution

Another approach uses the facts that 1 kilogram equals 1000 grams and 1 ounce equals 28.35 grams.

$$\begin{aligned} 2.3\text{ kg} &\approx 2.3\text{ kg} \cdot \frac{1000\text{ g}}{1\text{ kg}} \cdot \frac{1\text{ oz}}{28.35\text{ g}} && \text{Apply conversion factors.} \\ &\approx 2.3\ \cancel{\text{kg}} \cdot \frac{1000\ \cancel{\text{g}}}{1\ \cancel{\text{kg}}} \cdot \frac{1\text{ oz}}{28.35\ \cancel{\text{g}}} && \text{Cancel common units.} \\ &\approx \frac{2.3 \cdot 1000 \cdot 1}{1 \cdot 28.35}\text{ oz} && \text{Multiply fractions.} \\ &\approx 81.13\text{ oz} \end{aligned}$$

Roundoff Error

Why the discrepancy in answers? This difference in approximations is due to something called *round-off error*. Indeed, in the first calculation, we used a conversion factor that is rounded to the nearest tenth; i.e., 1 kg ≈ 2.2 lb. If we use a more accurate conversion factor to change kilograms to pounds, namely 1 kg ≈ 2.2046 lb, we get the following result.

$$\begin{aligned} 2.3\text{ kg} &\approx 2.3\text{ kg} \cdot \frac{2.2046\text{ lb}}{1\text{ kg}} \cdot \frac{16\text{ oz}}{1\text{ lb}} && \text{Apply conversion factors.} \\ &\approx 2.3\ \cancel{\text{kg}} \cdot \frac{2.2046\ \cancel{\text{lb}}}{1\ \cancel{\text{kg}}} \cdot \frac{16\text{oz}}{1\ \cancel{\text{lb}}} && \text{Cancel common units.} \\ &\approx \frac{2.3 \cdot 2.2046 \cdot 16}{1 \cdot 1}\text{ oz} && \text{Multiply fractions.} \\ &\approx 81.13\text{ oz} \end{aligned}$$

Note that this result is in better agreement with the second result above.

? Exercise

Change 5.7 kilograms to ounces. Round to the nearest ounce.

Answer

201 ounces

Important Observation

To obtain better approximations, you need to use a conversion factor that is more accurate. Any time you feel you need more accuracy, you might try an online conversion utility, such as the one at: www.france-property-and-information.com/metric_conversion_table.htm

Converting Units of Volume

It is known that

$$1\text{ qt} \approx 0.946\text{ L}.$$

Dividing both sides of this equation by 0.946,

$$1\text{ L} \approx 1.06\text{ qt}.$$

Again, these conversion factors for volume have been well worked out by scientists. The more common conversion factors for volume are shown in Table 6.7.

American to Metric	Metric to American
1 fl oz ≈ 0.030 L	1 L ≈ 33.8 fl oz
1 pt ≈ 0.473 L	1 L ≈ 2.1 pt
1 qt ≈ 0.946 L	1 L ≈ 1.06 qt
1 gal ≈ 3.785 L	1 L ≈ 0.264 gal

Table 6.7: Volume conversions: American — Metric.

Example 3

Change 2.5 dekalitres to gallons.

Solution

Recall that 1 daL = 10 L and 1 L = 0.264 gal.

$$\begin{aligned} 2.5\ \text{daL} &\approx 2.5\ \text{daL} \cdot \frac{10\ \text{L}}{1\ \text{daL}} \cdot \frac{0.264\ \text{gal}}{1\ \text{L}} && \text{Apply conversion factors.} \\ &\approx 2.5\ \cancel{\text{daL}} \cdot \frac{10\ \cancel{\text{L}}}{1\ \cancel{\text{daL}}} \cdot \frac{0.264\ \text{gal}}{1\ \cancel{\text{L}}} && \text{Cancel common units.} \\ &\approx \frac{2.5 \cdot 10 \cdot 0.264}{1 \cdot 1}\ \text{gal} && \text{Multiply fractions.} \\ &\approx 6.6\ \text{gal} \end{aligned}$$

Hence, 2.5 dekalitres is approximately equal to 6.6 gallons.

Exercise

Change 3.2 dekalitres to gallons.Round your answer to the nearest tenth of a gallon.

Answer

0.8 gallons

Converting Units of Speed

Modern speedometers often show a car's speed in both miles per hour and kilometers per hour.

Example 4

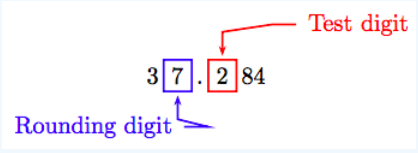
A car's speedometer is showing it speeding along at 60 kilometers per hour. How fast is it traveling in miles per hour? Round your answer to the nearest mile per hour.

Solution

From Table 6.5, 1 km ≈ 0.6214 mi.

$$\begin{aligned} 60\frac{\text{km}}{\text{h}} &\approx 60\frac{\text{km}}{\text{h}} \cdot \frac{0.6214\ \text{mi}}{1\ \text{km}} && \text{Apply conversion factor.} \\ &\approx 60\frac{\cancel{\text{km}}}{\text{h}} \cdot \frac{0.6214\ \text{mi}}{1\ \cancel{\text{km}}} && \text{Cancel common units.} \\ &\approx \frac{60 \cdot 0.6214}{1}\frac{\text{mi}}{\text{h}} && \text{Multiply fractions.} \\ &\approx 37.284\frac{\text{mi}}{\text{h}} \end{aligned}$$

To round to the nearest mile per hour, identify the rounding and test digits.



Because the test digit is less than 5, leave the rounding digit alone and truncate. Thus, to the nearest mile per hour, the speed is approximately 37 miles per hour.

Exercise

A car's speedometer registers 45 kilometers per hour. Change this speed to miles per hour. Round your answer to the nearest mile per hour.

Answer

28 miles per hour

Exercises

1. Use the conversion 1 in = 2.54 cm to convert 68 inches to centimeters, rounded to the nearest tenth of a centimeter.
2. Use the conversion 1 in = 2.54 cm to convert 42 inches to centimeters, rounded to the nearest tenth of a centimeter.
3. Use the conversion 1 in = 2.54 cm to convert 44 centimeters to inches, rounded to the nearest tenth of a inch.
4. Use the conversion 1 in = 2.54 cm to convert 22 centimeters to inches, rounded to the nearest tenth of a inch.
5. Use the conversion 1 mi = 1.6093 km to convert 79 miles to kilometers, rounded to the nearest tenth of a kilometer.
6. Use the conversion 1 mi = 1.6093 km to convert 39 miles to kilometers, rounded to the nearest tenth of a kilometer.
7. Use the conversion 1 yd = 0.9144 m to convert 1489 centimeters to yards, rounded to the nearest tenth of a yard.
8. Use the conversion 1 yd = 0.9144 m to convert 1522 centimeters to yards, rounded to the nearest tenth of a yard.
9. Use the conversion 1 yd = 0.9144 m to convert 28 yards to centimeters, rounded to the nearest tenth of a centimeter.
10. Use the conversion 1 yd = 0.9144 m to convert 34 yards to centimeters, rounded to the nearest tenth of a centimeter.
11. Use the conversion 1 m = 3.2808 ft to convert 8.6 meters to inches, rounded to the nearest tenth of a inch.
12. Use the conversion 1 m = 3.2808 ft to convert 8.3 meters to inches, rounded to the nearest tenth of a inch.
13. Use the conversion 1 in = 2.54 cm to convert 60 inches to centimeters, rounded to the nearest tenth of a centimeter.
14. Use the conversion 1 in = 2.54 cm to convert 75 inches to centimeters, rounded to the nearest tenth of a centimeter.
15. Use the conversion 1 m = 3.2808 ft to convert 208 inches to meters, rounded to the nearest tenth of a inch.
16. Use the conversion 1 m = 3.2808 ft to convert 228 inches to meters, rounded to the nearest tenth of a inch.
17. Use the conversion 1 m = 1.0936 yd to convert 20 yards to meters, rounded to the nearest tenth of a meter.
18. Use the conversion 1 m = 1.0936 yd to convert 44 yards to meters, rounded to the nearest tenth of a meter.
19. Use the conversion 1 mi = 1.6093 km to convert 29 miles to kilometers, rounded to the nearest tenth of a kilometer.
20. Use the conversion 1 mi = 1.6093 km to convert 15 miles to kilometers, rounded to the nearest tenth of a kilometer.
21. Use the conversion 1 m = 1.0936 yd to convert 8.2 meters to yards, rounded to the nearest tenth of a yard.
22. Use the conversion 1 m = 1.0936 yd to convert 6.9 meters to yards, rounded to the nearest tenth of a yard.
23. Use the conversion 1 mi = 1.6093 km to convert 4.9 kilometers to miles, rounded to the nearest tenth of a mile.
24. Use the conversion 1 mi = 1.6093 km to convert 4.2 kilometers to miles, rounded to the nearest tenth of a mile.
25. Use the conversion 1 m = 1.0936 yd to convert 25 yards to meters, rounded to the nearest tenth of a meter.
26. Use the conversion 1 m = 1.0936 yd to convert 2 yards to meters, rounded to the nearest tenth of a meter.
27. Use the conversion 1 in = 2.54 cm to convert 47 centimeters to inches, rounded to the nearest tenth of a inch.
28. Use the conversion 1 in = 2.54 cm to convert 19 centimeters to inches, rounded to the nearest tenth of a inch.
29. Use the conversion 1 mi = 1.6093 km to convert 8.3 kilometers to miles, rounded to the nearest tenth of a mile.
30. Use the conversion 1 mi = 1.6093 km to convert 4.8 kilometers to miles, rounded to the nearest tenth of a mile.

31. Use the conversion 1 yd = 0.9144 m to convert 41 yards to centimeters, rounded to the nearest tenth of a centimeter.
32. Use the conversion 1 yd = 0.9144 m to convert 20 yards to centimeters, rounded to the nearest tenth of a centimeter.
33. Use the conversion 1 m = 3.2808 ft to convert 3.7 meters to inches, rounded to the nearest tenth of a inch.
34. Use the conversion 1 m = 3.2808 ft to convert 7.9 meters to inches, rounded to the nearest tenth of a inch.
35. Use the conversion 1 yd = 0.9144 m to convert 1323 centimeters to yards, rounded to the nearest tenth of a yard.
36. Use the conversion 1 yd = 0.9144 m to convert 1715 centimeters to yards, rounded to the nearest tenth of a yard.
37. Use the conversion 1 m = 1.0936 yd to convert 8.4 meters to yards, rounded to the nearest tenth of a yard.
38. Use the conversion 1 m = 1.0936 yd to convert 7.3 meters to yards, rounded to the nearest tenth of a yard.
39. Use the conversion 1 m = 3.2808 ft to convert 289 inches to meters, rounded to the nearest tenth of a inch.
40. Use the conversion 1 m = 3.2808 ft to convert 251 inches to meters, rounded to the nearest tenth of a inch.

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41. Use the conversion 1 kg = 2.2 lb to convert 15.8 kilograms to pounds, rounded to the nearest tenth of a pound.
42. Use the conversion 1 kg = 2.2 lb to convert 19.4 kilograms to pounds, rounded to the nearest tenth of a pound.
43. Use the conversion 1 oz = 28.35 g to convert 35 ounces to grams, rounded to the nearest tenth of a gram.
44. Use the conversion 1 oz = 28.35 g to convert 33 ounces to grams, rounded to the nearest tenth of a gram.
45. Use the conversion 1 kg = 2.2 lb to convert 2.48 kilograms to ounces, rounded to the nearest tenth of a ounce.
46. Use the conversion 1 kg = 2.2 lb to convert 3.74 kilograms to ounces, rounded to the nearest tenth of a ounce.
47. Use the conversion 1 kg = 2.2 lb to convert 2.35 kilograms to ounces, rounded to the nearest tenth of a ounce.
48. Use the conversion 1 kg = 2.2 lb to convert 3.57 kilograms to ounces, rounded to the nearest tenth of a ounce.
49. Use the conversion 1 kg = 2.2 lb to convert 15 pounds to kilograms, rounded to the nearest tenth of a kilogram.
50. Use the conversion 1 kg = 2.2 lb to convert 27 pounds to kilograms, rounded to the nearest tenth of a kilogram.
51. Use the conversion 1 kg = 2.2 lb to convert 10.4 kilograms to pounds, rounded to the nearest tenth of a pound.
52. Use the conversion 1 kg = 2.2 lb to convert 17.7 kilograms to pounds, rounded to the nearest tenth of a pound.
53. Use the conversion 1 kg = 2.2 lb to convert 352 ounces to kilograms, rounded to the nearest tenth of a kilogram.
54. Use the conversion 1 kg = 2.2 lb to convert 326 ounces to kilograms, rounded to the nearest tenth of a kilogram.
55. Use the conversion 1 oz = 28.35 g to convert 96 grams to ounces, rounded to the nearest tenth of a ounce.
56. Use the conversion 1 oz = 28.35 g to convert 100 grams to ounces, rounded to the nearest tenth of a ounce.
57. Use the conversion 1 oz = 28.35 g to convert 14 ounces to grams, rounded to the nearest tenth of a gram.
58. Use the conversion 1 oz = 28.35 g to convert 29 ounces to grams, rounded to the nearest tenth of a gram.
59. Use the conversion 1 kg = 2.2 lb to convert 54 pounds to kilograms, rounded to the nearest tenth of a kilogram.
60. Use the conversion 1 kg = 2.2 lb to convert 82 pounds to kilograms, rounded to the nearest tenth of a kilogram.
61. Use the conversion 1 oz = 28.35 g to convert 92 grams to ounces, rounded to the nearest tenth of a ounce.
62. Use the conversion 1 oz = 28.35 g to convert 103 grams to ounces, rounded to the nearest tenth of a ounce.
63. Use the conversion 1 kg = 2.2 lb to convert 388 ounces to kilograms, rounded to the nearest tenth of a kilogram.
64. Use the conversion 1 kg = 2.2 lb to convert 395 ounces to kilograms, rounded to the nearest tenth of a kilogram.

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65. Use the conversion 1 qt = 0.946 L to convert 55.1 litres to quarts, rounded to the nearest tenth of a quart.
66. Use the conversion 1 qt = 0.946 L to convert 50.3 litres to quarts, rounded to the nearest tenth of a quart.
67. Use the conversion 1 L = 33.8 fl oz to convert 72073 fluid ounces to kilolitres, rounded to the nearest tenth of a kilolitre.
68. Use the conversion 1 L = 33.8 fl oz to convert 56279 fluid ounces to kilolitres, rounded to the nearest tenth of a kilolitre.

69. Use the conversion 1 L = 33.8 fl oz to convert 2.5 kilolitres to fluid ounces.

70. Use the conversion 1 L = 33.8 fl oz to convert 4.5 kilolitres to fluid ounces.

71. Use the conversion 1 qt = 0.946 L to convert 24 quarts to litres, rounded to the nearest tenth of a litre.

72. Use the conversion 1 qt = 0.946 L to convert 21 quarts to litres, rounded to the nearest tenth of a litre.

73. Use the conversion 1 qt = 0.946 L to convert 30 quarts to litres, rounded to the nearest tenth of a litre.

74. Use the conversion 1 qt = 0.946 L to convert 22 quarts to litres, rounded to the nearest tenth of a litre.

75. Use the conversion 1 gal = 3.785 L to convert 11.8 gallons to litres, rounded to the nearest tenth of a litre.

76. Use the conversion 1 gal = 3.785 L to convert 13.5 gallons to litres, rounded to the nearest tenth of a litre.

77. Use the conversion 1 gal = 3.785 L to convert 50.5 litres to gallons, rounded to the nearest tenth of a gallon.

78. Use the conversion 1 gal = 3.785 L to convert 55.9 litres to gallons, rounded to the nearest tenth of a gallon.

79. Use the conversion 1 L = 33.8 fl oz to convert 8.3 kilolitres to fluid ounces.

80. Use the conversion 1 L = 33.8 fl oz to convert 5.3 kilolitres to fluid ounces.

81. Use the conversion 1 qt = 0.946 L to convert 42.4 litres to quarts, rounded to the nearest tenth of a quart.

82. Use the conversion 1 qt = 0.946 L to convert 55.4 litres to quarts, rounded to the nearest tenth of a quart.

83. Use the conversion 1 gal = 3.785 L to convert 17.2 gallons to litres, rounded to the nearest tenth of a litre.

84. Use the conversion 1 gal = 3.785 L to convert 19.6 gallons to litres, rounded to the nearest tenth of a litre.

85. Use the conversion 1 L = 33.8 fl oz to convert 51274 fluid ounces to kilolitres, rounded to the nearest tenth of a kilolitre.

86. Use the conversion 1 L = 33.8 fl oz to convert 82164 fluid ounces to kilolitres, rounded to the nearest tenth of a kilolitre.

87. Use the conversion 1 gal = 3.785 L to convert 55.6 litres to gallons, rounded to the nearest tenth of a gallon.

88. Use the conversion 1 gal = 3.785 L to convert 59.2 litres to gallons, rounded to the nearest tenth of a gallon.

---

89. Change 60 miles per hour to kilometers per hour. Round your answer to the nearest kilometer per hour.

90. Change 56 miles per hour to kilometers per hour. Round your answer to the nearest kilometer per hour.

91. Change 77 miles per hour to kilometers per hour. Round your answer to the nearest kilometer per hour.

92. Change 57 miles per hour to kilometers per hour. Round your answer to the nearest kilometer per hour.

93. Change 42 kilometers per hour to miles per hour. Round your answer to the nearest mile per hour.

94. Change 56 kilometers per hour to miles per hour. Round your answer to the nearest mile per hour.

95. Change 62 kilometers per hour to miles per hour. Round your answer to the nearest mile per hour.

96. Change 63 kilometers per hour to miles per hour. Round your answer to the nearest mile per hour.

---

97. **Tallest tower.** The world's tallest tower in Dubai has 160 floors at a height of 2, 717 feet. Convert the height of the tower to the nearest tenth of a meter. *Associated Press-Times-Standard 02/09/10 World's tallest tower closed a month after opening.*

98. **High peaks.** For the first time, foreigners will be allowed to climb nearly 100 high-altitude Himalayan peaks on the Indian side of Kashmir, peaks ranging from 9, 840 feet to nearly 26, 246 feet. Convert the highest of the peaks to the nearest tenth of a meter. *Associated Press-TimesStandard 04/11/10 India opens Himalayan peaks to foreigners.*

99. **Ancient find.** In the southern Egyptian town of Luxor, a 3, 400-year-old 4-meter statue of Thoth, the ancient Egyptian god of Wisdom and Magic, was unearthed. Convert the height of the statue to the nearest tenth of a foot. *Associated Press-Times-Standard 03/17/10 3,400-year-old statues unearthed in Egypt.*

100. **Arctic wind.** Blizzard condition winds in the Arctic blew 80 miles per hour. Find the wind speed to the nearest kilometer per hour. *Associated Press-Times-Standard 12/31/09 After 562-mile ski trek, seven women reach the South Pole.*

101. **Solar plane.** The Solar Impulse lifted off from a military airport at a speed no faster than 28 miles per hour. Convert the speed of the solar-powered plane to the nearest kilometer per hour. *Associated Press-Times-Standard 04/09/10 Solar-powered plane makes successful maiden flight.*

Answers

1. 172.7 centimeters
3. 17.3 inches
5. 127.1 kilometers
7. 16.3 yards
9. 2560.3 centimeters
11. 338.6 inches
13. 152.4 centimeters
15. 5.3 meters
17. 18.3 meters
19. 46.7 kilometers
21. 9.0 yards
23. 3.0 miles
25. 22.9 meters
27. 18.5 inches
29. 5.2 miles
31. 3749.0 centimeters
33. 145.7 inches
35. 14.5 yards
37. 9.2 yards
39. 7.3 meters
41. 34.8 pounds
43. 992.3 grams
45. 87.3 ounces
47. 82.7 ounces
49. 6.8 kilograms
51. 22.9 pounds
53. 10.0 kilograms
55. 3.4 ounces
57. 396.9 grams
59. 24.5 kilograms
61. 3.2 ounces
63. 11.0 kilograms
65. 58.2 quarts
67. 2.1 kilolitres

69. 84500 ounces

71. 22.7 litres

73. 28.4 litres

75. 44.7 litres

77. 13.3 gallons

79. 280540 ounces

81. 44.8 quarts

83. 65.1 litres

85. 1.5 kilolitres

87. 14.7 gallons

89. 97 km/hr

91. 124 km/hr

93. 26 mi/hr

95. 39 mi/hr

97. 828.2 meters

99. 13.1 feet

101. 45 km/hr

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5.4: Perimeter and Area

Perimeter

Perimeter is a one-dimensional measurement that is taken around the outside of a closed geometric shape. Let's start our discussion of the concept of perimeter with an example.

Guided Example

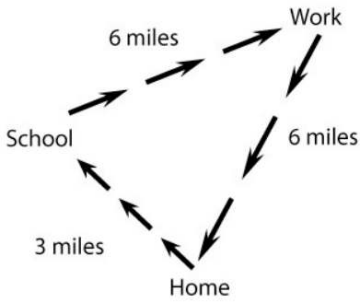


Figure 1.

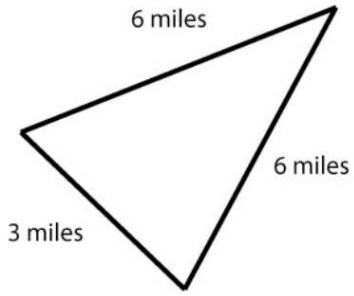


Figure 2.

Joseph does not own a car so must ride the bus or walk everywhere he goes. On Mondays, he must get to school, to work, and back home again. His route is pictured in figure 1.

The obvious question to ask in this situation is, "how many miles does Joseph travel on Mondays"? To compute, we each distance: $3 + 6 + 6 = 15$.

Joseph travels 15 miles on Mondays.

Another way to work with this situation is to draw a shape that represents Joseph's travel route and is labeled with the distance from one spot to another.

Notice that the shape made by Joseph's route is that of a closed geometric figure with three sides (a triangle) (see figure 2). What we can ask about this shape is, "what is the *perimeter* of the triangle"?

Perimeter means "distance around a closed figure or shape" and to compute we add each length: $3 + 6 + 6 = 15$

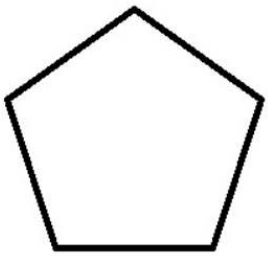
Our conclusion is the same as above: Joseph travels 15 miles on Mondays.

However, what we did was model the situation with a geometric shape and then apply a specific geometric concept (*perimeter*) to computer how far Joseph traveled.

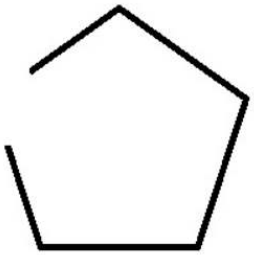
Notes on Perimeter

- Perimeter is a one-dimensional measurement that represents the distance around a closed geometric figure or shape (no gaps).
- To find perimeter, add the lengths of each side of the shape.
- If there are units, include units in your final result. Units will always be of single dimension (i.e. feet, inches, yards, centimeters, etc…)

To compute perimeter, our shapes must be closed. Figure 3 shows the difference between a closed figure and an open figure.



Closed figure:
There is an inside and an outside to the shape. To get from inside to outside, you must cross the boundary of the shape



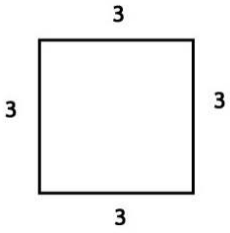
Open figure Not closed:
There isn't an inside or outside. Even a portion that seems enclosed can be reached without crossing the boundary of the shape.

Figure 3.

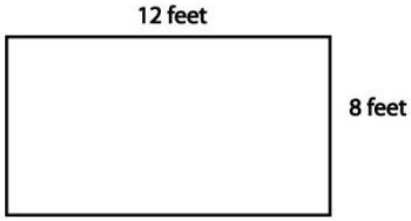
Example 1

Find the perimeter for each of the shapes below.

1. Add the lengths of each side.



2. Sometimes you have to make assumptions if lengths are not labeled.

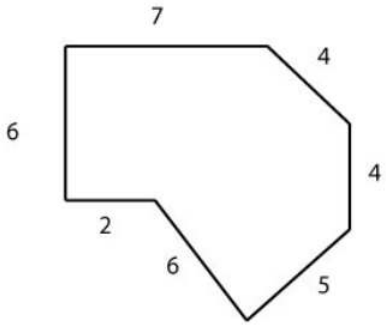


Solutions

1. 12 units
2. 40 feet

Example 2

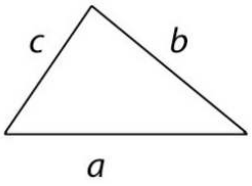
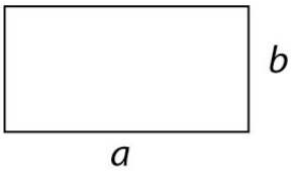
How do we find the perimeter of this more complicated shape?



Solution

Just keep adding those side lengths. 6 + 7 + 4 + 4 + 5 + 6 + 2 = 34 units

If you look closely at the shapes in the previous examples, you might notice some ways to write each perimeter as a more explicit formula. See if the results from what we have done so far match the formulas below.

Shape		Perimeter
Triangle with side lengths: a, b, c		$P = a + b + c$
Square with side length a:		$P = a + a + a + a$ $P = 4a$
Rectangle with side lengths a, b:		$P = a + b + a + b$ $P = a + a + b + b$ $P = 2a + 2b$

Circumference

You may realize that we have not yet discussed the distance around a very important geometric shape: a circle! The distance around a circle has a special name called the **circumference**. To find the circumference of a circle, we use this formula: $C = 2\pi r$

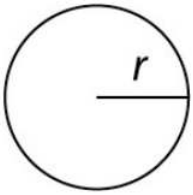


Figure 4.

In this formula, π is pronounced "pi," and is defined as the circumference of a circle divided by its diameter: $\pi = \frac{C}{d}$. We usually replace π with the approximation 3.14. The letter r represents the **radius** of the circle.

Let's see where the formula for circumference comes from. Figure 4 shows a generic circle with radius r.

Notes about $C = 2\pi r$

Remember that in the formula, when computing the circumference $C = 2\pi r$, we multiply as follows **usually** substituting 3.14 in place of π :

$C = 2 \times 3.14 \times r$

Often, the use of ( ) will help make the different parts of the formula easier to see:

$C = (2) \times (3.14) \times (r)$

Origins of $C = 2\pi r$

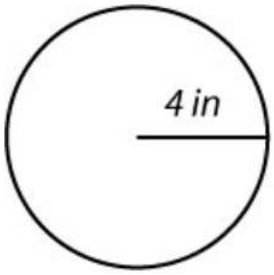
As mentioned earlier, the special number π is defined as the ratio of a circle's circumference to its diameter. We can write this in equation form as: $\pi = \frac{C}{d}$.

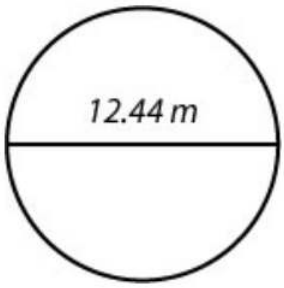
We know from our previous work that to identify the unknown, C, we can move d to the other side of the equation by writing $C = \pi d$. The diameter is all the way across the circle's middle so the diameter is twice the radius. We can update C in terms of the radius as $C = \pi(2r)$. With a little final rearranging of the order our parts are written in, we can say that $C = 2\pi r$.

Let's use the formula to find the circumference of a few circles.

Example 3

Find the circumference of each of the following circles. Leave your answers first in exact form and then in rounded form (to the hundredths place). (Note that when a radius is given, its value is centered above a radius segment. When a diameter is given, its value is centered above a diameter segment.)

1. 

2. 

Solutions

1. Exact 8π in; rounded from exact answer 25.13 in; rounded using 3.14 for π 25.12 in
2. Exact 12.44π m; rounded from exact answer 39.08 m; rounded using 3.14 for π 39.06 m

Exact Form vs. Rounded Form

- π is a number in exact form. It is not rounded.
- 3.14 is a rounded form approximation for π.

Why does it matter which form we use? It matters because when we round, we introduce error into our final result. For this class, that error is usually acceptable. However, you will find in other subjects such as physics or chemistry, that level of accuracy is a concept of great importance. Let's see an example of the difference in forms.

Example 4

The radius of the moon is about 1079 miles. What is the circumference? Let's solve this using both the exact form and the rounded form:

Exact Solution

$C = 2\pi r = 2\pi(1079) = 2158\pi$

To round **from** the exact solution, use the π button on your calculator to get

$2158\pi \approx 6779.56$

Rounded Solution

$C = 2\pi r = 2(3.14)(1079) \approx 6776.12$

Notice that our final results are different. That difference is the error created by using 3.14 as an initial approximation for \pi . When doing homework and tests, read the directions carefully on each problem to see which form to use.

Example 5

Find the circumference or perimeter given in each described situation below. Include a drawing of the shape with the included information. Use the examples to help determine what shapes to draw. Show all work. As in the examples, if units are included then units should be present in your final result. Round to tenths unless indicated otherwise.

1. Find the perimeter of a square with side length 2.17 feet.
2. Find the perimeter of a rectangle with sides of length 4.2 and 3.8.
3. Find the perimeter of a triangle with sides of length 2, 5, 7.
4. Find the circumference of a circle with radius 6 inches. Present answer in exact form and also compute using 3.14 for π . Present rounded form to the nearest tenth.
5. Find the circumference of a circle with diameter 14.8 inches. Present answer in exact form and also compute using 3.14 for π . Present rounded form to the nearest tenth.

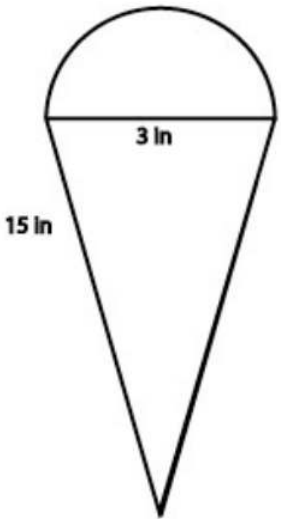
Solutions

1. 8.68 feet
2. 16
3. 14
4. Exact 12π in, Rounded 37.7 in
5. Exact 14.8π in, Rounded 46.5 in

Example 6

Finding the Distance around Non-Standard Shapes

The basic formulas for perimeter of straight-line shapes and the circumference of a circle will help us find the distance around more complicated figures. Find the distance around the following shape. Round final answer to tenths and use 3.14 for π.



Solution

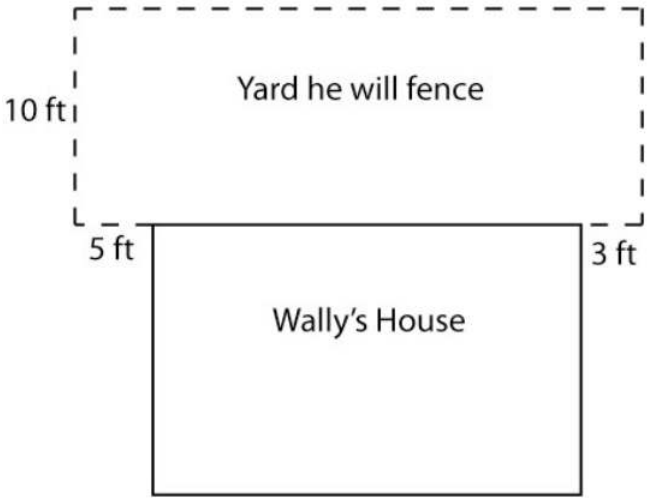
34.7 in

Example 7

Applications of Perimeter and Circumference

Our knowledge of basic geometric shapes can be applied to solve “real-life” problems.

Wally wants to add a fence to the back of his house to make some room for his children to play safely (see diagram below). He began measuring his yard but got distracted and forgot to finish measuring before he went to the store. If he remembers that the back wall of his house is 15 yards long, does he have enough information to buy the fencing he needs? If so, how many feet should he buy?



Solution

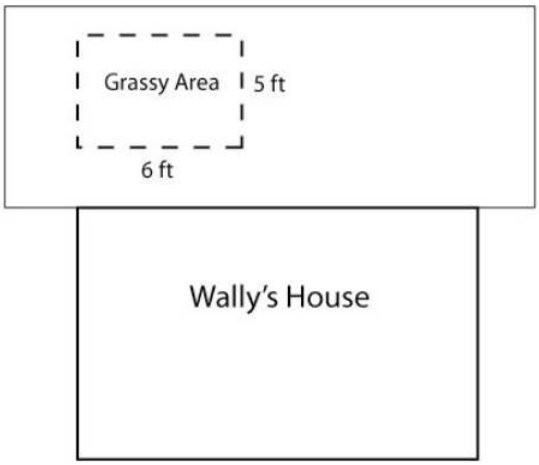
81 feet

Area

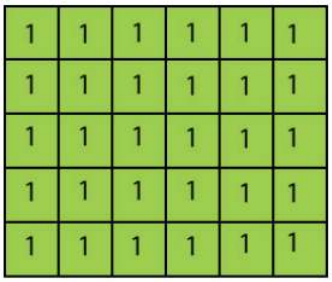
Let's take another look at Wally's backyard from Example 7 in order to introduce the next concept: area.

Guided Example

Wally successfully fenced his yard but now wants to add some landscaping and create a grassy area as shown below.



He heads down to the local lawn store and finds out that in order to determine how much sod he needs, he must figure out the square footage of the area he wants to add grass to. On his way home, he realizes that if he divides the grassy area into sections that are 1 foot by 1 foot and then counts them, he can determine the square footage. Here is the information Wally drew up when he got home.



6 top squares, 5 side squares
Each square is
1 ft x 1 ft = 1 square foot

Total unit squares = 30

Grassy Area = 30 square feet

Wally correctly determined the area of the rectangular grassy section to be 30 square feet.

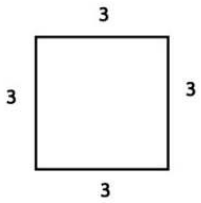
Notes on Area

- **Area** is a two-dimensional measurement that represents the amount of space inside a two-dimensional shape.
- To find the area, count the number of unit squares inside the shape.
- If there are units, include units in your final result. Units will always be two-dimensional (i.e. square feet, square yards, square miles, etc…)

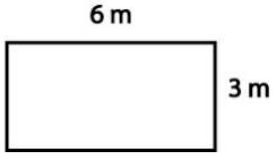
Example 8

Find the area for each of the shapes below.

1. Remember to count the unit squares inside the shape.

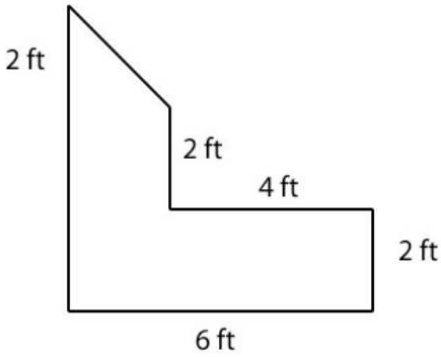


2. Is there a pattern here that would make our work easier?

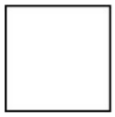


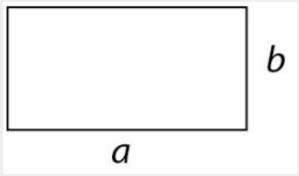
Example 9

How do we find the area for shapes that are more complicated? Break up the areas into shapes that we recognize and add the area values together.

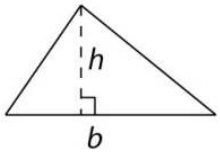
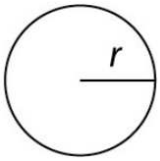
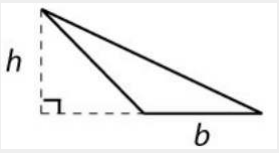


If you look closely at the shapes in the previous examples, you might notice some ways to write each area as a more explicit formula. See if the results from what we have done so far match the formulas below.

Shape		Area
Square with side length a	 a	$A = a \cdot a$ $A = a^2$

Shape		Area
Rectangle with side lengths a, b		$A=a\cdot{b}\\$ (You will also see this as $A=\text{length}\cdot\text{width}\\$)

The area formulas for the shapes below are more complicated to derive so the formulas are listed for you in the table.

Shape	Shape
Triangle with height h and base b  $\displaystyle{A}=\frac{1}{2}bh=\frac{bh}{2}\\$ Read as “one-half base times height” Note that h is the straight-line distance from top of the triangle directly to the other side. The small box next to h indicates this. In math terms the box indicates a 90º (right) angle.	Circle with radius r  $A=\pi{r}^2\\$ Read as “pi times radius squared”
	If your triangle is as pictured at left, then the height is drawn and measured outside the triangle. The area formula is the same.

Example 10

Find the area for each described situation. Create a drawing of the shape with the included information. Show all work. As in the examples, if units are included then units should be present in your final result. Use 3.14 for \pi and round answers to tenths as needed.

1. Find the area of a rectangle whose length is 12.9 meters and height is one-third that amount.
2. Find the area of a triangle with base $\displaystyle{24}\frac{1}{2}\\$ inches and height 7 inches.
3. Find the area of a circle with radius $\displaystyle{2}\frac{1}{3}\\$ inches. Present answer in exact form and also compute rounded form using 3.14 for π . Present rounded form to the nearest tenth.

Solutions

1. 55.5 m^2 or 55.5 square meters (rounded)
2. 85.8 in^2 or 85.8 square inches (rounded)
3. Exact 49/9 π in^2, Rounded 17.1 in^2

Example 11

Find the area given each described situation. Include a drawing of the shape with the included information. Show all work. As in the examples, if units are included then units should be present in your final result. Round answers to tenths unless otherwise indicated.

1. Find the area of a square with side length 4.2 feet.
2. Find the area of a rectangle with sides of length 4.2 and 3.8.
3. Find the area of a triangle with height 7 inches and base 12 inches.

4. Find the area of a circle with radius 6 inches. Present answer in exact form and also compute using 3.14 for π . Present rounded form to the nearest tenth.

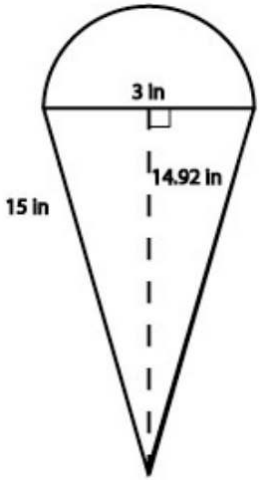
Solutions

1. 17.64 ft^2 or 17.64 square feet
2. 16.0
3. 42 in^2 or 42 square inches
4. Exact 36π in^2 or 36π square inches, Rounded using 3.14 for π 113.0 in^2 or 113.0 square inches

Example 12

Finding the Area of Non-Standard Shapes

The basic formulas for area will help us find the area of more complicated figures. This is the same problem we found the perimeter for earlier. Find the area of the given shape. Compute using 3.14 for π and round to the nearest tenth.



Solution

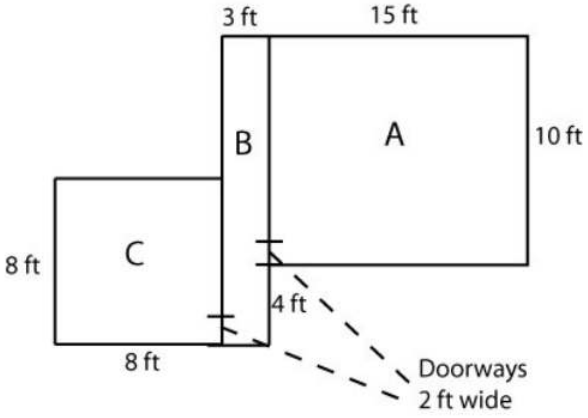
Rounded using 3.14 for π 25.9 in^2

Example 13

Applications of Area and Perimeter

We can combine our knowledge of area/perimeter to solve problems such as this one.

Wally is still fixing up his house and has a flooring project to complete. He wants to buy enough bamboo flooring to cover the floor space in rooms A, C and hallway B and enough bamboo edging for baseboards in all the spaces as well. How many square feet of flooring and how many feet of baseboards should he buy?



Solution

256 ft^2 flooring, 108 ft edging

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5.5: Volume

Let's revisit our friend Wally from "Area and Perimeter" and use another aspect of his yard to introduce the concept of **volume**. Wally is a swimmer and wants to install a lap pool in his backyard. Because he has some extra space, he is going to build a pool that is 25 yards long, 2 yards wide, and 2 yards deep. How many cubic yards of water must be used to fill the pool (assuming right to the top).

Much as we did with area (counting unit squares), with **volume** we will be counting unit cubes. What is the volume of a unit cube? Let's look at figure 1:

Finding the Volume of a Cube

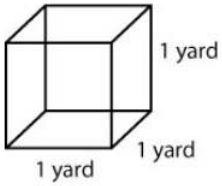


Figure 1. Volume = 1 yd × 1yd × yd = 1 cubic yard

The shape at left is a cube (all sides are equal length). In particular, because all sides are of length 1, this cube is called a unit cube.

We know the area of the base from our previous work (1 yd × 1 yd or 1 square yard). We are going to take that area and extend it vertically through a height of 1 yard so our volume becomes

Volume = 1 yd × 1yd × yd = 1 cubic yard

How does this help Wally? Well, if he can count the number of unit cubes in his pool, he can determine the volume of water needed to fill the pool.

Guided Example

Filling Wally's Pool

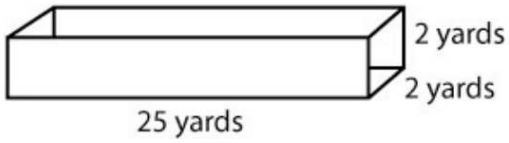
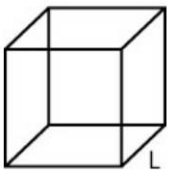
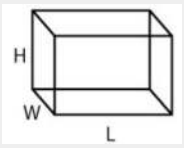


Figure 2. Volume = Length × Width × Height **or** V = LWH

If we fill the pool with unit cubes, we can fill 25 unit cubes along the length, 2 along the width and 2 along the height. That would give us 25 × 2 × 2 = 100 unit cubes or:

Volume = 25 yd × 2 yd × 2 yd = 100 cubic yd

Explicit formulas for the types of rectangular solids used in the previous section are as follows:

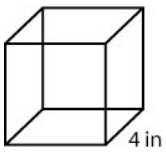
Shape		Volume
Cub of side length L		$V = L \times L \times L$ $V = L^3$
Box with sides of length L, W, H		$V = L \times W \times H$

Notes on Volume

- **Volume** is a three-dimensional measurement that represents the amount of space inside a closed three-dimensional shape.
- To find volume, count the number of unit cubes inside a given shape.
- If there are units, include units in your final result. Units will always be three-dimensional (i.e. cubic feet, cubic yards, cubic miles, etc…)

Example 1

Find the volume of each shape below.

1. 

2. A box with sides of length 2 ft, 3 ft, 2.5 ft.

Solutions

1. 64 in^3
2. 15 ft^3

Volume of A Circular Cylinder

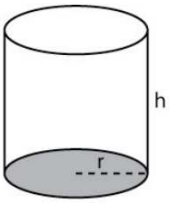


Figure 3.

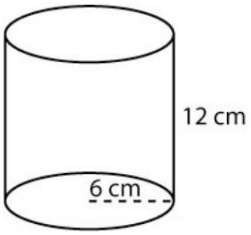
Can we use what we know about the area of a circle to formulate the volume of a can (also called a **cylinder**)? Take a look at figure 3.

The base circle is shaded. If we take the area of that circle ($A = \pi r^2$) and extend it up through the height *h*, then our volume for the can would be:

$V = \pi r^2 h$.

Example 2

Find the volume of the cylinder shown below.



Solution

432π cm^3

Volume of Other Shapes

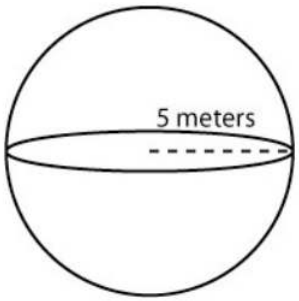
The chart below shows the volumes of some other basic geometric shapes.

Shape	Volume

Shape	Volume
Sphere	$V = \frac{4}{3}\pi r^3$
Cone	$V = \frac{1}{3}\pi r^2 h$
Pyramid	$V = \frac{1}{3}LWH$

Example 3

Find the volume of a sphere with radius 5 meters.



Solution

$\frac{500}{3}\text{cm}^3$

Example 4

Determine the volume of each of the following. Include a drawing of the shape with the included information. Show all work. As in the examples, if units are included then units should be present in your final result. Use 3.14 for π and round answers to tenths as needed.

1. Find the volume of a cube with side 3.25 meters.
2. Find the volume of a box with sides of length 4 feet by 2.5 feet by 6 feet.
3. Find the volume of a can with radius 4.62 cm and height 10 cm.
4. Find the volume of a sphere with diameter 12 yards.

Solution

1. 34.3 m^3 or 34.3 cubic meters
2. 60 ft^3 or 60 cubic feet
3. 670.2 cm^3 or 670.2 cubic centimeters
4. 904.3 yd^3 or 904.3 cubic yards

Example 5

Applications of Volume

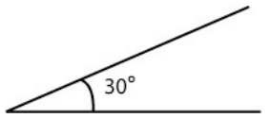
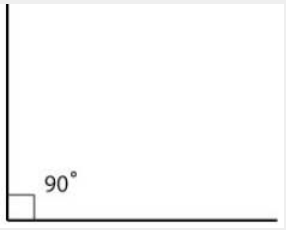
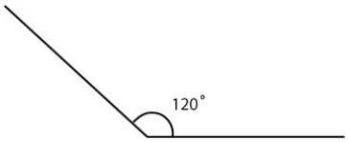
If you drank sodas from 5 cans each of diameter 4 inches and height 5 inches, how many cubic inches of soda did you drink? Use 3.14 for π and round to tenths.

Solution

314.0 in^3 (rounded)

Angles

Angles, often measured in *degrees*, measure the amount of rotation or "arc" between intersecting line segments. We need to have some sense of what an angle is before moving on to the next topic. See some examples and terminology below.

	• This angle measures 30º • This angle measure is less than 90º • Angles less than 90º are called **acute angles**
	• This angle measures 90º • Angles that measure exactly 90º are called **right angles** • The small box in the angle corner denotes a right angle
	• This angle measures 120º • This angle measure is more than 90º • Angles that measure more than 90º but less 180º than are called **obtuse angles**
	• This angle measures 180º • Angles that measure exactly 180º are called **straight angles**

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5.6: Solve a Formula for a Specific Variable

Learning Objectives

By the end of this section, you will be able to:

- Solve a formula for a specific variable
- Use formulas to solve geometry applications

Before you get started, take this readiness quiz.

1. Evaluate $2(x+3)$ when $x=5$.
   If you missed this problem, review [link].
2. The length of a rectangle is three less than the width. Let *w* represent the width. Write an expression for the length of the rectangle.
   If you missed this problem, review [link].
3. Evaluate $\frac{1}{2}bh$ when $b=14$ and $h=9$.
   If you missed this problem, review [link].

Solve a Formula for a Specific Variable

We have all probably worked with some geometric formulas in our study of mathematics. Formulas are used in so many fields, it is important to recognize formulas and be able to manipulate them easily.

It is often helpful to solve a formula for a specific variable. If you need to put a formula in a spreadsheet, it is not unusual to have to solve it for a specific variable first. We isolate that variable on one side of the equals sign with a coefficient of one and all other variables and constants are on the other side of the equal sign.

Geometric formulas often need to be solved for another variable, too. The formula $V=\frac{1}{3}\pi r^2 h$ is used to find the **volume** of a right circular cone when given the radius of the base and height. In the next example, we will solve this formula for the height.

EXAMPLE 5.6.1

Solve the formula $V=\frac{1}{3}\pi r^2 h$ for *h*.

Answer

Write the formula.			$V=\frac{1}{3}\pi r^2 h$
Remove the fraction on the right.			$3\cdot V=3\cdot\frac{1}{3}\pi r^2 h$
Simplify.			$3V=\pi r^2 h$
Divide both sides by πr^2.			$\frac{3V}{\pi r^2}=h$

We could now use this formula to find the height of a right circular cone when we know the volume and the radius of the base, by using the formula $h=\frac{3V}{\pi r^2}$.

EXAMPLE 5.6.2:

Use the formula $A=\frac{1}{2}bh$ to solve for *b*.

Answer

$b=\frac{2A}{h}$

EXAMPLE 5.6.3:

Use the formula $A = \frac{1}{2}bh$ to solve for h.

Answer

$h = \frac{2A}{b}$

In the sciences, we often need to change **temperature** from Fahrenheit to Celsius or vice versa. If you travel in a foreign country, you may want to change the Celsius temperature to the more familiar Fahrenheit temperature.

EXAMPLE 5.6.4

Solve the formula $C = \frac{5}{9}(F - 32)$ for F.

Answer

Write the formula.			$C = \frac{5}{9}(F - 32)$
Remove the fraction on the right.			$\frac{9}{5}C = \frac{9}{5} \cdot \frac{5}{9}(F - 32)$
Simplify.			$\frac{9}{5}C = (F - 32)$
Add 32 to both sides.			$\frac{9}{5}C + 32 = F$

We can now use the formula $F = \frac{9}{5}C + 32$ to find the Fahrenheit temperature when we know the Celsius temperature.

EXAMPLE 5.6.5:

Solve the formula $F = \frac{9}{5}C + 32$ for C.

Answer

$C = \frac{5}{9}(F - 32)$

EXAMPLE 5.6.6:

Solve the formula $A = \frac{1}{2}h(b + B)$ for b.

Answer

$b = \frac{2A - Bh}{h}$

The next example uses the formula for the **surface area** of a right cylinder.

EXAMPLE 5.6.7

Solve the formula $S = 2\pi r^2 + 2\pi rh$ for h.

Answer

Write the formula.	$S = 2\pi r^2 + 2\pi r h$
Isolate the h term by subtracting $2\pi r^2$ from each side.	$S - 2\pi r^2 = 2\pi r^2 - 2\pi r^2 + 2\pi r h$
Simplify.	$S - 2\pi r^2 = 2\pi r h$

Solve for h by dividing both sides by $2\pi r$.	$\frac{S-2\pi r^2}{2\pi r} = \frac{2\pi r h}{2\pi r}$
Simplify.	$\frac{S-2\pi r^2}{2\pi r} = h$

? EXAMPLE 5.6.8:

Solve the formula $A = P + Prt$ for t.

Answer

$t = \frac{A-P}{Pr}$

? EXAMPLE 5.6.9:

Solve the formula $A = P + Prt$ for r.

Answer

$r = \frac{A-P}{Pt}$

Sometimes we might be given an equation that is solved for y and need to solve it for x, or vice versa. In the following example, we're given an equation with both x and y on the same side and we'll solve it for y.

Solve the formula $8x + 7y = 15$ for y.

Answer

We will isolate y on one side of the equation.	$8x + 7y = 15$
Subtract $6x$ from both sides to isolate the term with y.	$8x - 8x + 7y = 15 - 8x$
Simplify.	$7y = 15 - 8x$
Divide both sides by 7 to make the coefficient of y one.	$\frac{7y}{7} = \frac{15-8x}{7}$
Simplify.	$y = \frac{15-8x}{7}$

Solve the formula $4x + 7y = 9$ for y.

Answer

$y = \frac{9-4x}{7}$

? EXAMPLE 5.6.12

Solve the formula $5x + 8y = 1$ for y.

Answer

$y = \frac{1-5x}{8}$

Use Formulas to Solve Geometry Applications

In this objective we will use some common geometry formulas. We will adapt our problem solving strategy so that we can solve geometry applications. The geometry formula will name the variables and give us the equation to solve.

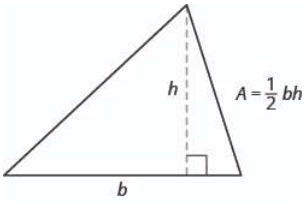
In addition, since these applications will all involve shapes of some sort, most people find it helpful to draw a figure and label it with the given information. We will include this in the first step of the problem solving strategy for geometry applications.

SOLVE GEOMETRY APPLICATIONS.

1. **Read** the problem and make sure all the words and ideas are understood.
2. **Identify** what you are looking for.
3. **Name** what we are looking for by choosing a variable to represent it. Draw the figure and label it with the given information.
4. **Translate** into an equation by writing the appropriate formula or model for the situation. Substitute in the given information.
5. **Solve** the equation using good algebra techniques.
6. **Check** the answer in the problem and make sure it makes sense.
7. **Answer** the question with a complete sentence.

When we solve geometry applications, we often have to use some of the properties of the figures. We will review those properties as needed.

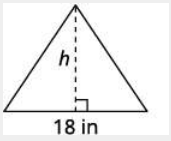
The next example involves the area of a triangle. The **area** of a triangle is one-half the base times the height. We can write this as $A = \frac{1}{2}bh$, where b = length of the base and h = height.



EXAMPLE 5.6.13

The area of a triangular painting is 126 square inches. The base is 18 inches. What is the height?

Answer

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	height of a triangle
Step 3. Name.	
Choose a variable to represent it.	Let $h =$ the height.
Draw the figure and label it with the given information.	Area = 126 sq. in.
	
Step 4. Translate.	
Write the appropriate formula.	$A = \frac{1}{2}bh$
Substitute in the given information.	$126 = \frac{1}{2} \cdot 18 \cdot h$
Step 5. Solve the equation.	$126 = 9h$

Divide both sides by 9.	$14 = h$
Step 6. Check. $A = \frac{1}{2}bh$ $126 \stackrel{?}{=} 12 \cdot 18 \cdot 14$ $126 = 126 \checkmark$	
Step 7. Answer the question.	The height of the triangle is 14 inches.

? EXAMPLE 5.6.14:

The area of a triangular church window is 90 square meters. The base of the window is 15 meters. What is the window's height?

Answer

The window's height is 12 meters.

? EXAMPLE 5.6.15:

A triangular tent door has area 15 square feet. The height is five feet. What is the length of the base?

Answer

The length of the base is 6 feet.

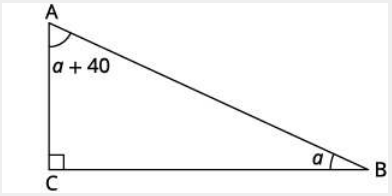
In the next example, we will work with a **right triangle**. To solve for the measure of each angle, we need to use two triangle properties. In any triangle, the sum of the measures of the angles is 180°. We can write this as a formula: $m\angle A + m\angle B + m\angle C = 180$. Also, since the triangle is a right triangle, we remember that a right triangle has one 90° angle.

Here, we will have to define one angle in terms of another. We will wait to draw the figure until we write expressions for all the angles we are looking for.

? EXAMPLE 5.6.16:

The measure of one angle of a right triangle is 40 degrees more than the measure of the smallest angle. Find the measures of all three angles.

Answer

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	the measures of all three angles
Step 3. Name. Choose a variable to represent it.	Let a = 1st angle. $a + 40$ = 2nd angle 90 = 3rd angle (the right angle)
Draw the figure and label it with the given information.	
Step 4. Translate.	
Write the appropriate formula.	$m\angle A + m\angle B + m\angle C = 180$

Substitute into the formula.	$a+(a+40)+90=180$
Step 5. Solve the equation.	$2a+130=180$ $2a=50$ $a=25$ first angle $a+40$ second angle $25+40$ 65 90 third angle
Step 6. Check. $25+65+90 \stackrel{?}{=} 180$ $180=180$✓	
Step 7. Answer the question.	The three angles measure 25°, 65°, and 90°.

? EXAMPLE 5.6.17:

The measure of one angle of a right triangle is 50 more than the measure of the smallest angle. Find the measures of all three angles.

Answer

The measures of the angles are 20°, 70°, and 90°.

? EXAMPLE 5.6.18:

The measure of one angle of a right triangle is 30 more than the measure of the smallest angle. Find the measures of all three angles.

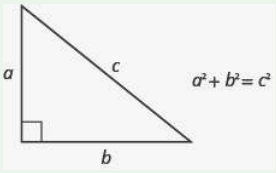
Answer

The measures of the angles are 30°, 60°, and 90°.

The next example uses another important geometry formula. The **Pythagorean Theorem** tells how the lengths of the three sides of a right triangle relate to each other. Writing the formula in every exercise and saying it aloud as you write it may help you memorize the Pythagorean Theorem.

THE PYTHAGOREAN THEOREM

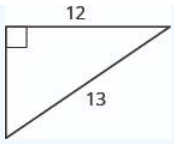
In any right triangle, where a and b are the lengths of the legs, and c is the length of the hypotenuse, the sum of the squares of the lengths of the two legs equals the square of the length of the hypotenuse.



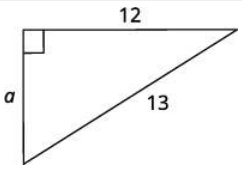
We will use the Pythagorean Theorem in the next example.

? EXAMPLE 5.6.19:

Use the Pythagorean Theorem to find the length of the other leg in

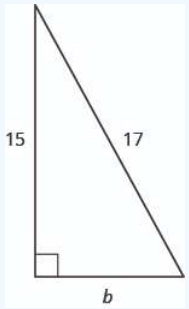


Answer

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	the length of the leg of the triangle
Step 3. Name.	
Choose a variable to represent it.	Let a = the leg of the triangle.
Label side a.	
Step 4. Translate.	
Write the appropriate formula. Substitute.	$a^2+b^2=c^2$ $a^2+12^2=13^2$
Step 5. Solve the equation. Isolate the variable term. Use the definition of square root. Simplify.	$a^2+144=169$ $a^2=25$ $a=\sqrt{25}$ $a=5$
Step 6. Check. $5^2+12^2 \stackrel{?}{=} 13^2$ $25+144 \stackrel{?}{=} 169$ $169=169$ ✓	
Step 7. Answer the question.	The length of the leg is 5.

? EXAMPLE 5.6.20:

Use the Pythagorean Theorem to find the length of the leg in the figure.

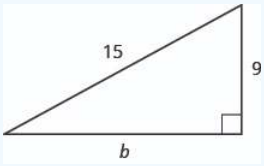


Answer

The length of the leg is 8.

? EXAMPLE 5.6.21:

Use the Pythagorean Theorem to find the length of the leg in the figure.



Answer

The length of the leg is 12.

The next example is about the **perimeter** of a rectangle. Since the perimeter is just the distance around the rectangle, we find the sum of the lengths of its four sides—the sum of two lengths and two widths. We can write is as $P = 2L + 2W$ where L is the length and W is the width. To solve the example, we will need to define the length in terms of the width.

? EXAMPLE 5.6.22:

The length of a rectangle is six centimeters more than twice the width. The perimeter is 96 centimeters. Find the length and width.

Answer

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	the length and the width
Step 3. Name. Choose a variable to represent the width. The length is six more than twice the width.	Let $w =$ width. $2w + 6 =$ length w, 2w + 6 $P = 96$ cm
Step 4. Translate.	
Write the appropriate formula.	$P = 2L + 2W$
Substitute in the given information.	$96 = 2(2w + 6) + 2w$
Step 5. Solve the equation.	$96 = 4w + 12 + 2w$ $96 = 6w + 12$ $84 = 6w$ $14 = w$ (width) $2w + 6$ (length) $2(14) + 6$ 34 The length is 34 cm.
Step 6. Check. 14 cm, 34 cm $P = 2L + 2W$ $96 \stackrel{?}{=} 2 \cdot 34 + 2 \cdot 14$ $96 = 96$✓	
Step 7. Answer the question.	The length is 34 cm and the width is 14 cm

? EXAMPLE 5.6.23:

The length of a rectangle is seven more than twice the width. The perimeter is 110 inches. Find the length and width.

Answer

The length is 16 inches and the width is 39 inches.

? EXAMPLE 5.6.24:

The width of a rectangle is eight yards less than twice the length. The perimeter is 86 yards. Find the length and width.

Answer

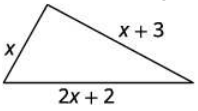
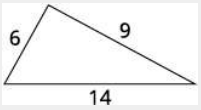
The length is 17 yards and the width is 26 yards.

The next example is about the **perimeter** of a triangle. Since the perimeter is just the distance around the triangle, we find the sum of the lengths of its three sides. We can write this as $P = a + b + c$, where a, b, and c are the lengths of the sides.

? EXAMPLE 5.6.25:

One side of a triangle is three inches more than the first side. The third side is two inches more than twice the first. The perimeter is 29 inches. Find the length of the three sides of the triangle.

Answer

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	the lengths of the three sides of a triangle
Step 3. Name. Choose a variable to represent the length of the first side.	Let $x =$ length of 1st side. $x + 3$ = length of 2nd side $2x + 2$ = length of 3rd side  Perimeter is 29 in.
Step 4. Translate. Write the appropriate formula. Substitute in the given information.	$P = a + b + c$ $29 = x + (x + 3) + (2x + 2)$
Step 5. Solve the equation.	$29 = 4x + 5$ $24 = 4x$ $6 = x$ length of first side $x + 3$ length of second side $6 + 3$ 9 $2x + 2$ length of second side $2 \cdot 6 + 2$ 14
Step 6. Check.  $29 \stackrel{?}{=} 6 + 9 + 14$ $29 = 29$✓	
Step 7. Answer the question.	The lengths of the sides of the triangle are 6, 9, and 14 inches.

? EXAMPLE 5.6.26:

One side of a triangle is seven inches more than the first side. The third side is four inches less than three times the first. The perimeter is 28 inches. Find the length of the three sides of the triangle.

Answer

The lengths of the sides of the triangle are 5, 11 and 12 inches.

? EXAMPLE 5.6.27:

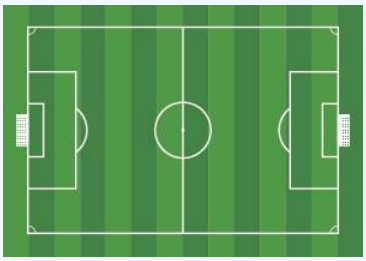
One side of a triangle is three feet less than the first side. The third side is five feet less than twice the first. The perimeter is 20 feet. Find the length of the three sides of the triangle.

Answer

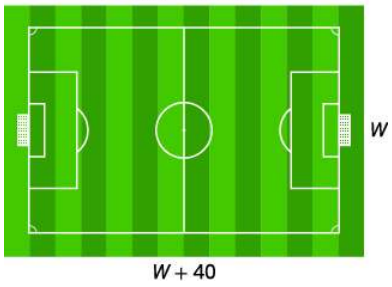
The lengths of the sides of the triangle are 4, 7 and 9 feet.

? EXAMPLE 5.6.28:

The perimeter of a rectangular soccer field is 360 feet. The length is 40 feet more than the width. Find the length and width.



Answer

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	the length and width of the soccer field
Step 3. Name. Choose a variable to represent it. The length is 40 feet more than the width. Draw the figure and label it with the given information.	Let w = width. $w + 40 =$ length Perimeter = 360 feet 
Step 4. Translate. Write the appropriate formula and substitute.	$P = 2L + 2W$ $360 = 2(w + 40) + 2w$
Step 5. Solve the equation.	

Step 6. Check. $P = 2L + 2W$ $360 \stackrel{?}{=} 2(110) + 2(70)$ $360 = 360\checkmark$	$360 = 2w + 80 + 2w$ $360 = 4w + 80$ $280 = 4w$ $70 = w$ the width of the field $w + 40$ the length of the field $70 + 40$ 110
Step 7. Answer the question.	The length of the soccer field is 110 feet and the width is 70 feet.

? EXAMPLE 5.6.29:

The perimeter of a rectangular swimming pool is 200 feet. The length is 40 feet more than the width. Find the length and width.

Answer

The length of the swimming pool is 70 feet and the width is 30 feet.

? EXAMPLE 5.6.30:

The length of a rectangular garden is 30 yards more than the width. The perimeter is 300 yards. Find the length and width.

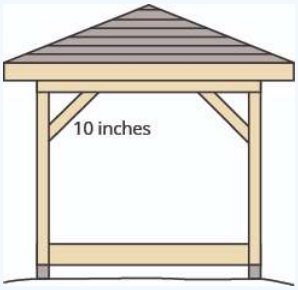
Answer

The length of the garden is 90 yards and the width is 60 yards.

Applications of these geometric properties can be found in many everyday situations as shown in the next example.

? EXAMPLE 5.6.31:

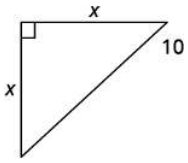
Kelvin is building a gazebo and wants to brace each corner by placing a 10” piece of wood diagonally as shown.



How far from the corner should he fasten the wood if wants the distances from the corner to be equal? Approximate to the nearest tenth of an inch.

Answer

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	the distance from the corner that the bracket should be attached

Step 3. Name. Choose a variable to represent it. Draw the figure and label it with the given information.	Let $x =$ the distance from the corner. 
Step 4. Translate. Write the appropriate formula and substitute.	$a^2+b^2=c^2 \quad x^2+x^2=10^2$
Step 5. Solve the equation. Isolate the variable. Use the definition of square root. Simplify. Approximate to the nearest tenth.	$2x^2=100$ $x^2=50$ $x=\sqrt{50}$ $x\approx 7.1$
Step 6. Check. $a^2+b^2=c^2$ $(7.1)^2+(7.1)^2\approx 10^2$ Yes.	
Step 7. Answer the question.	Kelvin should fasten each piece of wood approximately 7.1" from the corner.

? EXAMPLE 5.6.32:

John puts the base of a 13-foot ladder five feet from the wall of his house as shown in the figure. How far up the wall does the ladder reach?

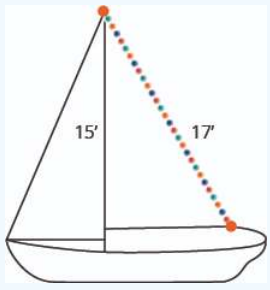


Answer

The ladder reaches 12 feet.

? EXAMPLE 5.6.33:

Randy wants to attach a 17-foot string of lights to the top of the 15 foot mast of his sailboat, as shown in the figure. How far from the base of the mast should he attach the end of the light string?



Answer

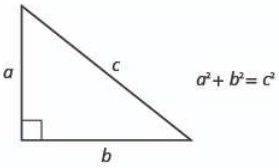
He should attach the lights 8 feet from the base of the mast.

Access this online resource for additional instruction and practice with solving for a variable in literal equations.

- Solving Literal Equations

Key Concepts

- **How To Solve Geometry Applications**
  1. **Read** the problem and make sure all the words and ideas are understood.
  2. **Identify** what you are looking for.
  3. **Name** what you are looking for by choosing a variable to represent it. Draw the figure and label it with the given information.
  4. **Translate** into an equation by writing the appropriate formula or model for the situation. Substitute in the given information.
  5. **Solve** the equation using good algebra techniques.
  6. **Check** the answer in the problem and make sure it makes sense.
  7. **Answer** the question with a complete sentence.
- **The Pythagorean Theorem**
  - In any right triangle, where a and b are the lengths of the legs, and c is the length of the hypotenuse, the sum of the squares of the lengths of the two legs equals the square of the length of the hypotenuse.



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5.6E: Exercises

Practice Makes Perfect

Solve a Formula for a Specific Variable

In the following exercises, solve the given formula for the specified variable.

1. Solve the formula $C = \pi d$ for d.

Answer

$$d = \frac{C}{\pi}$$

2. Solve the formula $C = \pi d$ for π.

3. Solve the formula $V = LWH$ for L.

Answer

$$L = \frac{V}{WH}$$

4. Solve the formula $V = LWH$ for H.

5. Solve the formula $A = \frac{1}{2}bh$ for b.

Answer

$$b = \frac{2A}{h}$$

6. Solve the formula $A = \frac{1}{2}bh$ for h.

7. Solve the formula

$A = \frac{1}{2}d_1d_2$ for d_1.

Answer

$$d_1 = \frac{2A}{d_2}$$

8. Solve the formula

$A = \frac{1}{2}d_1d_2$ for d_2.

9. Solve the formula

$A = \frac{1}{2}h(b_1 + b_2)$ for b_1.

Answer

$$b_1 = \frac{2A}{h} - b_2$$

10. Solve the formula

$A = \frac{1}{2}h(b_1 + b_2)$ for b_2.

11. Solve the formula

$h = 54t + \frac{1}{2}at^2$ for a.

Answer

$$a = \frac{2h - 108t}{t^2}$$

12. Solve the formula

$h = 48t + \frac{1}{2}at^2$ for a.

13. Solve $180 = a + b + c$ for a.

Answer

$$a = 180 - b - c$$

14. Solve $180 = a + b + c$ for c.

15. Solve the formula

$A = \frac{1}{2}pI + B$ for p.

Answer

$$p = \frac{2A - 2B}{I}$$

16. Solve the formula

$A = \frac{1}{2}pI + B$ for I.

17. Solve the formula

$P = 2L + 2W$ for L.

Answer

$$L = \frac{P - 2W}{2}$$

18. Solve the formula

$P = 2L + 2W$ for W.

In the following exercises, solve for the formula for y.

19. Solve the formula

$8x + y = 15$ for y.

Answer

$$y = 15 - 8x$$

20. Solve the formula

$9x + y = 13$ for y.

21. Solve the formula

$-4x + y = -6$ for y.

Answer

$y = -6 + 4x$

22. Solve the formula

$-5x + y = -1$ for y.

23. Solve the formula

$x - y = -4$ for y.

Answer

$y = 4 + x$

24. Solve the formula

$x - y = -3$ for y.

25. Solve the formula

$4x + 3y = 7$ for y.

Answer

$y = \frac{7-4x}{3}$

26. Solve the formula

$3x + 2y = 11$ for y.

27. Solve the formula

$2x + 3y = 12$ for y.

Answer

$y = \frac{12-2x}{3}$

28. Solve the formula

$5x + 2y = 10$ for y.

29. Solve the formula

$3x - 2y = 18$ for y.

Answer

$y = \frac{18-3x}{-2}$

30. Solve the formula

$4x - 3y = 12$ for y.

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Use Formulas to Solve Geometry Applications

In the following exercises, solve using a geometry formula.

31. A triangular flag has area 0.75 square feet and height 1.5 foot. What is its base?

Answer

1 foot

32. A triangular window has area 24 square feet and height six feet. What is its base?

33. What is the base of a triangle with area 207 square inches and height 18 inches?

Answer

23 inches

34. What is the height of a triangle with area 893 square inches and base 38 inches?

35. The two smaller angles of a right triangle have equal measures. Find the measures of all three angles.

Answer

$45^\circ, 45^\circ, 90^\circ$

36. The measure of the smallest angle of a right triangle is 20° less than the measure of the next larger angle. Find the measures of all three angles.

37. The angles in a triangle are such that one angle is twice the smallest angle, while the third angle is three times as large as the smallest angle. Find the measures of all three angles.

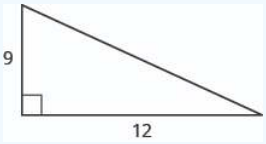
Answer

$30^\circ, 60^\circ, 90^\circ$

38. The angles in a triangle are such that one angle is 20 more than the smallest angle, while the third angle is three times as large as the smallest angle. Find the measures of all three angles.

In the following exercises, use the Pythagorean Theorem to find the length of the hypotenuse.

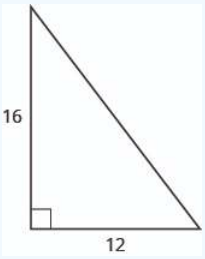
39.



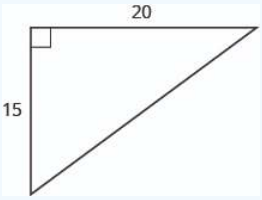
Answer

15

40.



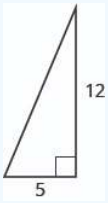
41.



Answer

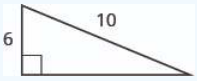
25

42.



In the following exercises, use the Pythagorean Theorem to find the length of the unknown leg. Round to the nearest tenth if necessary.

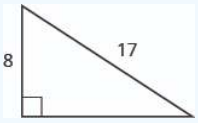
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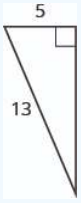
Answer

8

44.



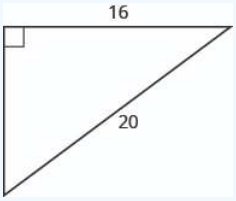
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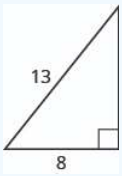
Answer

12

46.



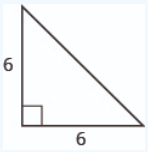
47.



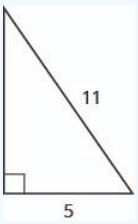
Answer

10.2

48.



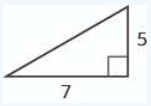
49.



Answer

9.8

50.



In the following exercises, solve using a geometry formula.

51. The width of a rectangle is seven meters less than the length. The perimeter is 58 meters. Find the length and width.

Answer

18 meters, 11 meters

52. The length of a rectangle is eight feet more than the width. The perimeter is 60 feet. Find the length and width.

53. The width of the rectangle is 0.7 meters less than the length. The perimeter of a rectangle is 52.6 meters. Find the dimensions of the rectangle.

Answer

13.5m, 12.8m

54. The length of the rectangle is 1.1 meters less than the width. The perimeter of a rectangle is 49.4 meters. Find the dimensions of the rectangle.

55. The perimeter of a rectangle of 150 feet. The length of the rectangle is twice the width. Find the length and width of the rectangle.

Answer

25 ft, 50 ft

56. The length of the rectangle is three times the width. The perimeter of a rectangle is 72 feet. Find the length and width of the rectangle.

57. The length of the rectangle is three meters less than twice the width. The perimeter of a rectangle is 36 meters. Find the dimensions of the rectangle.

Answer

7 m, 11 m

58. The length of a rectangle is five inches more than twice the width. The perimeter is 34 inches. Find the length and width.

59. The perimeter of a triangle is 39 feet. One side of the triangle is one foot longer than the second side. The third side is two feet longer than the second side. Find the length of each side.

Answer

12 ft, 13 ft, 14 ft

60. The perimeter of a triangle is 35 feet. One side of the triangle is five feet longer than the second side. The third side is three feet longer than the second side. Find the length of each side.

61. One side of a triangle is twice the smallest side. The third side is five feet more than the shortest side. The perimeter is 17 feet. Find the lengths of all three sides.

Answer

3 ft, 6 ft, 8 ft

62. One side of a triangle is three times the smallest side. The third side is three feet more than the shortest side. The perimeter is 13 feet. Find the lengths of all three sides.

63. The perimeter of a rectangular field is 560 yards. The length is 40 yards more than the width. Find the length and width of the field.

Answer

120 yd, 160 yd

64. The perimeter of a rectangular atrium is 160 feet. The length is 16 feet more than the width. Find the length and width of the atrium.

65. A rectangular parking lot has perimeter 250 feet. The length is five feet more than twice the width. Find the length and width of the parking lot.

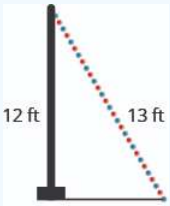
Answer

40 ft, 85 ft

66. A rectangular rug has perimeter 240 inches. The length is 12 inches more than twice the width. Find the length and width of the rug.

In the following exercises, solve. Approximate answers to the nearest tenth, if necessary.

67. A 13-foot string of lights will be attached to the top of a 12-foot pole for a holiday display as shown. How far from the base of the pole should the end of the string of lights be anchored?



Answer

5 feet

68. Pam wants to put a banner across her garage door diagonally, as shown, to congratulate her son for his college graduation. The garage door is 12 feet high and 16 feet wide. How long should the banner be to fit the garage door?



69. Chi is planning to put a diagonal path of paving stones through her flower garden as shown. The flower garden is a square with side 10 feet. What will the length of the path be?



Answer

14.1 feet

70. Brian borrowed a 20-foot extension ladder to use when he paints his house. If he sets the base of the ladder six feet from the house as shown, how far up will the top of the ladder reach?



Everyday Math

71. Converting temperature While on a tour in Greece, Tatyana saw that the temperature was 40° Celsius. Solve for F in the formula $C = \frac{5}{9}(F - 32)$ to find the Fahrenheit temperature.

Answer

104°F

72. Converting temperature Yon was visiting the United States and he saw that the temperature in Seattle one day was 50° Fahrenheit. Solve for C in the formula $F = \frac{9}{5}C + 32$ to find the Celsius temperature

73. Christa wants to put a fence around her triangular flowerbed. The sides of the flowerbed are six feet, eight feet and 10 feet. How many feet of fencing will she need to enclose her flowerbed?

Answer

24 ft

74. Jose just removed the children's play set from his back yard to make room for a rectangular garden. He wants to put a fence around the garden to keep the dog out. He has a 50-foot roll of fence in his garage that he plans to use. To fit in the backyard, the width of the garden must be 10 feet. How long can he make the other side?

Writing Exercises

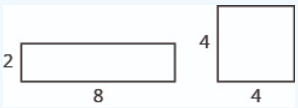
75. If you need to put tile on your kitchen floor, do you need to know the perimeter or the area of the kitchen? Explain your reasoning.

Answer

Answers will vary.

76. If you need to put a fence around your backyard, do you need to know the perimeter or the area of the backyard? Explain your reasoning.

77. Look at the two figures below.



a. Which figure looks like it has the larger area? Which looks like it has the larger perimeter?

b. Now calculate the area and perimeter of each figure. Which has the larger area? Which has the larger perimeter?

c. Were the results of part (b) the same as your answers in part (a)? Is that surprising to you?

Answer

a. Answers will vary. b. The areas are the same. The 2×8 rectangle has a larger perimeter than the 4×4 square.

c. Answers will vary.

78. Write a geometry word problem that relates to your life experience, then solve it and explain all your steps.

Self Check

a. After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
solve a formula for a specific variable.			
use formulas to solve geometry applications.			

b. What does this checklist tell you about your mastery of this section? What steps will you take to improve?

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CHAPTER OVERVIEW

6: Expressions and Equations

6.1: Evaluating Algebraic Expressions
6.2: Combining Like Terms
6.3: Adding and Subtracting Polynomials
6.4: Multiplying Polynomials
6.5: Dividing Polynomials
6.6: Prelude to Solving Linear Equations
6.7: Use a General Strategy to Solve Linear Equations
6.7E: Use a General Strategy to Solve Linear Equations (Exercises)
6.8: Use a Problem Solving Strategy
6.8E: Exercises

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6.1: Evaluating Algebraic Expressions

In this section we will evaluate *algebraic expressions* for given values of the variables contained in the expressions. Here are some simple tips to help you be successful.

Tips for Evaluating Algebraic Expressions

1. Replace all occurrences of variables in the expression with open parentheses. Leave room between the parentheses to substitute the given value of the variable.
2. Substitute the given values of variables in the open parentheses prepared in the first step.
3. Evaluate the resulting expression according to the Rules Guiding Order of Operations.

Let's begin with an example.

Example 1

Evaluate the expression $x^2 - 2xy + y^2$ at $x = -3$ and $y = 2$.

Solution

Following "Tips for Evaluating Algebraic Expressions," first replace all occurrences of variables in the expression $x^2 - 2xy + y^2$ with open parentheses.

$$x^2 - 2xy + y^2 = (\,)^2 - 2(\,)(\,) + (\,)^2$$

Secondly, replace each variable with its given value, and thirdly, follow the "Rules Guiding Order of Operations" to evaluate the resulting expression.

$$\begin{aligned} & x^2 - 2xy + y^2 && \text{Original expression.} \\ = {} & (-3)^2 - 2(-3)(2) + (2)^2 && \text{Substitute } -3 \text{ for } x \text{ and } 2 \text{ for } y. \\ = {} & 9 - 2(-3)(2) + 4 && \text{Evaluate exponents first.} \\ = {} & 9 - (-6)(2) + 4 && \text{Left to right, multiply } 2(-3) = -6. \\ = {} & 9 - (-12) + 4 && \text{Left to right, multiply: } (-6)(2) = -12. \\ = {} & 9 + 12 + 4 && \text{Add the opposite.} \\ = {} & 25 && \text{Add.} \end{aligned}$$

Exercise

If $x = -2$ and $y = -1$, evaluate $x^3 - y^3$.

Answer

-7

Example 2

Evaluate the expression $(a - b)^2$ If $a = 3$ and $b = -5$, at $a = 3$ and $b = -5$.

Solution

Following "Tips for Evaluating Algebraic Expressions," first replace all occurrences of variables in the expression $(a - b)^2$ with open parentheses.

$$(a - b)^2 = ((\,) - (\,))^2$$

Secondly, replace each variable with its given value, and thirdly, follow the "Rules Guiding Order of Operations" to evaluate the resulting expression.

$$\begin{aligned}(a-b)^2 &= ((3)-(-5))^2 && \text{Substitute 3 for } a \text{ and } -5 \text{ for } b.\\ &= (3+5)^2 && \text{Add the opposite: } (3)-(-5)=3+5\\ &= 8^2 && \text{Simplify inside parentheses: } 3+5=8\\ &= 64 && \text{Evaluate exponent: } 8^2=64\end{aligned}$$

? Exercise

If a = 3 and b = −5, evaluate $a^2 - b^2$.

Answer

−16

✓ Example 3

Evaluate the expression |a|−|b| at a = 5 and b = −7.

Solution

Following "Tips for Evaluating Algebraic Expressions," first replace all occurrences of variables in the expression |a|−|b| with open parentheses.

$$|a|-|b| = |(\,)|-|(\,)|$$

Secondly, replace each variable with its given value, and thirdly, follow the "Rules Guiding Order of Operations" to evaluate the resulting expression.

$$\begin{aligned}|a|-|b| = |(5)| &= |(-7)| && \text{Substitute 5 for } a \text{ and } -7 \text{ for } b.\\ &= 5-7 && \text{Absolute values first: } |(5)|=5 \text{ and } |(-7)|=7|\\ &= 5+(-7) && \text{Add the opposites: } 5-7=5+(-7).\\ &= -2 && \text{Add: } 5+(-7)=-2.\end{aligned}$$

? Exercise

If a = 5 and b = −7, evaluate 2|a| − 3|b|.

Answer

−11

✓ Example 4

Evaluate the expression |a − b| at a = 5 and b = −7.

Solution

Following "Tips for Evaluating Algebraic Expressions," first replace all occurrences of variables in the expression |a − b| with open parentheses.

$$|a-b| = |(\,)-(\,)|$$

Secondly, replace each variable with its given value, and thirdly, follow the "Rules Guiding Order of Operations" to evaluate the resulting expression.

$$\begin{aligned}|a-b| &= |(5)-(-7)| && \text{Substitute 5 for } a \text{ and } -7 \text{ for } b.\\ &= |5+7| && \text{Add the opposite: } 5-(-7)=5+7.\\ &= |12| && \text{Add: } 5+7=12.\\ &= 12 && \text{Take the absolute value: } |12|=12.\end{aligned}$$

Exercise

If $a = 5$ and $b = -7$, evaluate $|2a - 3b|$.

Answer

31

Example 5

Evaluate the expression

$$\frac{ad-bc}{a+b}$$

at $a = 5$, $b = -3$, $c = 2$, and $d = -4$.

Solution

Following "Tips for Evaluating Algebraic Expressions," first replace all occurrences of variables in the expression with open parentheses.

$$\frac{ad-bc}{a+b} = \frac{(\,)(\,)-(\,)(\,)}{(\,)+(\,)}$$

Secondly, replace each variable with its given value, and thirdly, follow the "Rules Guiding Order of Operations" to evaluate the resulting expression.

$$\begin{aligned}
\frac{ad-bc}{a+b} &= \frac{(5)-(-3)(2)}{(5)+(-3)} && \text{Substitute: } 5 \text{ for } a,\ -3 \text{ for } b,\ 2 \text{ for } c,\ -4 \text{ for } d.\\
&= \frac{-20-(-6)}{2} && \text{Numerator: } (5)(=4) = -20,\ (-3)(2) = -6.\\
&&& \text{Denominator: } 5+(-3) = 2.\\
&= \frac{-20+6}{2} && \text{Numerator: Add the opposite.}\\
&= \frac{-14}{2} && \text{Numerator: } -20+6 = -14.\\
&= -7 && \text{Divide.}
\end{aligned}$$

Exercise

If a = −7, b = −3, c = −15, 15, and $d = -14$, evaluate:

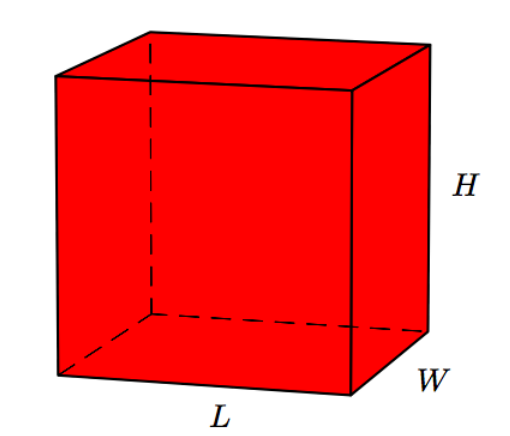
$$\frac{a^2+b^2}{c+d}$$

Answer

−2

Example 6

Pictured below is a rectangular prism.



The volume of the rectangular prism is given by the formula

$$V = LWH,$$

where L is the length, W is the width, and H is the height of the rectangular prism. Find the volume of a rectangular prism having length 12 feet, width 4 feet, and height 6 feet.

Solution

Following "Tips for Evaluating Algebraic Expressions," first replace all occurrences of of L, W, and H in the formula

$$V = LWH$$

with open parentheses.

$$V = (\,)(\,)(\,)$$

Next, substitute 12 ft for L, 4 ft for W, and 6 ft for H and simplify.

$$\begin{aligned} V &= (12\text{ft})(4\text{ft})(6\text{ft}) \\ &= 288\text{ft}^3 \end{aligned}$$

Hence, the volume of the rectangular prism is 288 cubic feet.

? Exercise

The surface area of the prism pictured in this example is given by the following formula:

$$S = 2(WH + LH + LW)$$

If L = 12, W = 4, and H = 6 feet, respectively, calculate the surface area.

Answer

288 square feet

Exercises

In Exercises 1-12, evaluate the expression at the given value of x.

1. $-3x^2 - 6x + 3$ at $x = 7$
2. $7x^2 - 7x + 1$ at $x = -8$
3. $-6x - 6$ at $x = 3$
4. $6x - 1$ at $x = -10$
5. $5x^2 + 2x + 4$ at $x = -1$

6. $4x^2 - 9x + 4$ at $x = -3$

7. $-9x - 5$ at $x = -2$

8. $-9x + 12$ at $x = 5$

9. $4x^2 + 2x + 6$ at $x = -6$

10. $-3x^2 + 7x + 4$ at $x = -7$

11. $12x + 10$ at $x = -12$

12. $-6x + 7$ at $x = 11$

In Exercises 13-28, evaluate the expression at the given values of x and y.

13. $|x|-|y|$ at $x = -5$ and $y = 4$

14. $|x|-|y|$ at $x = -1$ and $y = -2$

15. $-5x^2 + 2y^2$ at $x = 4$ and $y = 2$

16. $-5x^2 - 4y^2$ at $x = -2$ and $y = -5$

17. $|x|-|y|$ at $x = 0$ and $y = 2$

18. $|x|-|y|$ at $x = -2$ and $y = 0$

19. $|x - y|$ at $x = 4$ and $y = 5$

20. $|x - y|$ at $x = -1$ and $y = -4$

21. $5x^2 - 4xy + 3y^2$ at $x = 1$ and $y = -4$

22. $3x^2 + 5xy + 3y^2$ at $x = 2$ and $y = -1$

23. $|x - y|$ at $x = 4$ and $y = 4$

24. $|x - y|$ at $x = 3$ and $y = -5$

25. $-5x^2 - 3xy + 5y^2$ at $x = -1$ and $y = -2$

26. $3x^2 - 2xy - 5y^2$ at $x = 2$ and $y = 5$

27. $5x^2 + 4y^2$ at $x = -2$ and $y = -2$

28. $-4x^2 + 2y^2$ at $x = 4$ and $y = -5$

In Exercises 29-40, evaluate the expression at the given value of x.

29. $\frac{9+9x}{-x}$ at $x = -3$

30. $\frac{9-2x}{-x}$ at $x = -1$

31. $\frac{-8x+9}{-9+x}$ at $x = 10$

32. $\frac{2x+4}{1+x}$ at $x = 0$

33. $\frac{-4+9x}{7x}$ at $x = 2$

34. $\frac{-1-9x}{x}$ at $x = -1$

35. $\frac{-12-7x}{x}$ at $x = -1$

36. $\frac{12+11x}{3x}$ at $x = -6$

37. $\frac{6x-10}{5}$ + x at $x = -6$

38. $\frac{11x+11}{-4}$ + x at $x = 5$

39. $\frac{10x+11}{5}$ + x at $x = -4$

40. $\frac{6x+12}{-3}$ + x at x = 2

41. The formula

$$d = 16t^2$$

gives the distance (in feet) that an object falls from rest in terms of the time t that has elapsed since its release. Find the distance d (in feet) that an object falls in t = 4 seconds.

42. The formula

$$d = 16t^2$$

gives the distance (in feet) that an object falls from rest in terms of the time t that has elapsed since its release. Find the distance d (in feet) that an object falls in t = 24 seconds.

43. The formula

$$C = \frac{5(F-32)}{9}$$

gives the Celcius temperature C in terms of the Fahrenheit temperature F. Use the formula to find the Celsius temperature (◦ C) if the Fahrenheit temperature is F = 230◦ F.

44. The formula

$$C = \frac{5(F-32)}{9}$$

gives the Celcius temperature C in terms of the Fahrenheit temperature F. Use the formula to find the Celsius temperature (°C) if the Fahrenheit temperature is F = 95 °F.

45. The Kelvin scale of temperature is used in chemistry and physics. Absolute zero occurs at 0° K, the temperature at which molecules have zero kinetic energy. Water freezes at 273° K and boils at K = 373° K. To change Kelvin temperature to Fahrenheit temperature, we use the formula

$$F = \frac{9(K-273)}{5} + 32.$$

Use the formula to change 28° K to Fahrenheit.

46. The Kelvin scale of temperature is used in chemistry and physics. Absolute zero occurs at 0° K, the temperature at which molecules have zero kinetic energy. Water freezes at 273° K and boils at K = 373° K. To change Kelvin temperature to Fahrenheit temperature, we use the formula

$$F = \frac{9(K-273)}{5} + 32.$$

Use the formula to change 248° K to Fahrenheit.

47. A ball is thrown vertically upward. Its velocity t seconds after its release is given by the formula

$$v = v0 - gt,$$

where v_0 is its initial velocity, g is the acceleration due to gravity, and v is the velocity of the ball at time t. The acceleration due to gravity is g = 32 feet per second per second. If the initial velocity of the ball is v_0 = 272 feet per second, find the speed of the ball after t = 6 seconds.

48. A ball is thrown vertically upward. Its velocity t seconds after its release is given by the formula

$$v = v_0 - gt,$$

where v_0 is its initial velocity, g is the acceleration due to gravity, and v is the velocity of the ball at time t. The acceleration due to gravity is g = 32 feet per second per second. If the initial velocity of the ball is v_0 = 470 feet per second, find the speed of the ball after t = 4 seconds.

49. **Even numbers**. Evaluate the expression 2n for the following values:

i) n = 1

ii) n = 2

iii) n = 3

iv) n = −4

v) n = −5

vi) Is the result always an even number? Explain.

50. **Odd numbers**. Evaluate the expression 2n + 1 for the following values:

i) n = 1

ii) n = 2

iii) n = 3

iv) n = −4

v) n = −5

vi) Is the result always an odd number? Explain.

Answers

1. −186

3. −24

5. 7

7. 13

9. 138

11. −134

13. 1

15. −72

17. −2

19. 1

21. 69

23. 0

25. 9

27. 36

29. −6

31. −71

33. 1

35. 5

37. 46

39. −29

41. 256 feet

43. 110 degrees

45. −409° F

47. 80 feet per second

49.

i) 2

ii) 4

iii) 6

iv) −8

v) −10

vi) Yes, the result will always be an even number because 2 will always be a factor of the product 2n.

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6.2: Combining Like Terms

We begin our discussion with the definition of a *term*.

Definition: Term

A **term** is a single number or variable, or it can be the product of a number (called its coefficient) and one or more variables (called its variable part). The terms in an algebraic expression are separated by addition symbols.

Example 1

Identify the terms in the algebraic expression

$$3x^2 + 5xy + 9y^2 + 12$$

For each term, identify its coefficient and variable part.

Solution

In tabular form, we list each term of the expression 3x^2 + 5xy + 9y^2 + 12, its coefficient, and its variable part.

Term	Coefficient	Variable Part
$3x^2$	3	x^2
$5xy$	5	xy
$9y^2$	9	y^2
12	12	None

Exercise

How many terms are in the algebraic expression $3x^2 + 2xy - 3y^2$?

Answer

3

Example 2

Identify the terms in the algebraic expression

$$a^3 - 3a^2b + 3ab^2 - b^3$$

For each term, identify its coefficient and variable part.

Solution

The first step is to write each difference as a sum, because the terms of an expression are defined above to be those items separated by addition symbols.

$$a^3 + (-3a^2b) + 3ab^2 + (-b^3)$$

In tabular form, we list each term of the expression $a^3 + (-3a^2b) + 3ab^2 + (-b^3)$, its coefficient, and its variable part.

Term	Coefficient	Variable Part
a^3	1	a^3
$-3a^2b$	-3	a^2b

Term	Coefficient	Variable Part
$3ab^2$	3	ab^2
$-b^3$	-1	b^3

Exercise

How many terms are in the algebraic expression $11 - a^2 - 2ab + 3b^2a$?

Answer

4

Like Terms

We define what is meant by “like terms” and “unlike terms.”

Definition

Like and Unlike Terms. The variable parts of two terms determine whether the terms are like terms or unlike terms.

Like Terms. Two terms are called like terms if they have identical variable parts, which means that the terms must contain the same variables raised to the same exponential powers.

Unlike Terms. Two terms are called unlike terms if their variable parts are different.

Example 3

Classify each of the following pairs as either *like terms* or *unlike terms*: (a) $3x$ and $-7x$, (b) $2y$ and $3y^2$, (c) $-3t$ and $5u$, and (d) $-4a^3$ and $3a^3$.

Solution

Like terms must have *identical* variable parts.

a. $3x$ and $-7x$ have identical variable parts. They are “like terms.”
b. $2y$ and $3y^2$ do not have identical variable parts (the exponents differ). They are “unlike terms.”
c. $-3t$ and $5u$ do not have identical variable parts (different variables). They are “unlike terms.”
d. $-4a^3$ and $3a^3$ have identical variable parts. They are “like terms.”

Exercise

Are $-3xy$ and $11xy$ *like* or *unlike* terms?

Answer

Like terms

Combining Like Terms

When using the distributive property, it makes no difference whether the multiplication is on the left or the right, one still distributes the multiplication times each term in the parentheses.

Distributive Property

If a, b, and c are integers, then

$$a(b + c) = ab + ac \text{ and } (b + c)a = ba + ca.$$

In either case, you distribute a times each term of the sum.

"Like terms" can be combined and simplified. The tool used for combining like terms is the distributive property. For example, consider the expression $3y + 7y$, composed of two "like terms" with a common variable part. We can use the distributive property and write

$$3y + 7y = (3 + 7)y$$

Note that we are using the distributive property in reverse, "factoring out" the common variable part of each term. Checking our work, note that if we redistribute the variable part y times each term in the parentheses, we are returned to the original expression $3y + 7y$.

Example 4

Use the distributive property to combine like terms (if possible) in each of the following expressions: (a) $-5x^2 - 9x^2$, (b) $-5ab + 7ab$, (c) $4y^3 - 7y^2$, and (d) $3xy^2 - 7xy^2$.

Solution

If the terms are "like terms," you can use the distributive property to "factor out" the common variable part.

a) Factor out the common variable part x^2.

$$\begin{aligned} -5x^2 - 9x^2 &= (-5 - 9)x^2 \quad \text{Use the distributive property.} \\ &= -14x^2 \quad \text{Simplify: } -5 - 9 = -5 + (-9) = -14. \end{aligned}$$

b) Factor out the common variable part ab.

$$\begin{aligned} -5ab + 7ab &= (-5 + 7)ab \quad \text{Use the distributive property.} \\ &= 2ab \quad \text{Simplify: } -5 + 7 = 2. \end{aligned}$$

c) The terms in the expression $4y^3 - 7y^2$ have different variable parts (the exponents are different). These are "unlike terms" and cannot be combined.

d) Factor out the common variable part xy^2.

$$\begin{aligned} 3xy^2 - 7xy^2 &= (3 - 7)xy^2 \quad \text{Use the distributive property.} \\ &= -4xy^2 \quad \text{Simplify: } 3 - 7 = 3 + (-7) = -4. \end{aligned}$$

Exercise

Simplify: $-8z - 11z$

Answer

$-19z$

Speeding Things Up a Bit

Once you've written out all the steps for combining like terms, like those shown in Example 4, you can speed things up a bit by following this rule:

Combining Like Terms

To combine like terms, simply add their coefficients and keep the common variable part.

Thus for example, when presented with the sum of two like terms, such as in $5x + 8x$, simply add the coefficients and repeat the common variable part; that is, $5x + 8x = 13x$.

Example 5

Combine like terms:

a. $-9y - 8y$,
b. $-3y^5 + 4y^5$ and
c. $-3u^2 + 2u^2$.

Solution

a) Add the coefficients and repeat the common variable part. Therefore,

$$-9y - 8y = -17y.$$

b) Add the coefficients and repeat the common variable part. Therefore,

$$-3y^5 + 4y^5 = 1y^5.$$

However, note that $1y^5 = y^5$. Following the rule that the final answer should use as few symbols as possible, a better answer is $-3y^5 + 4y^5 = y^5$.

c) Add the coefficients and repeat the common variable part. Therefore,

$$-3u^2 + 2u^2 = (-1)u^2.$$

However, note that $(-1)u^2 = -u^2$. Following the rule that the final answer should use as few symbols as possible, a better answer is $-3u^2 + 2u^2 = -u^2$.

Exercise

Combine: $-3x$, and $-4x^2$

Answer

$-7x^2$

Simplify

A frequently occurring instruction asks the reader to *simplify* an expression.

Simplify

The instruction simplify is a generic term that means "try to write the expression in its most compact form, using the fewest symbols possible."

One way you can accomplish this goal is by combining like terms when they are present.

Example 6

Simplify: $2x + 3y - 5x + 8y$.

Solution

Use the commutative property to reorder terms and the associative and distributive properties to regroup and combine like terms.

$$\begin{aligned} 2x + 3y - 5x + 8y &= (2x - 5x) + (3y + 8y) && \text{Reorder and regroup.} \\ &= -3x + 11y && \text{Combine like terms:} \\ & && 2x - 5x = -3x \text{ and } 3y + 8y = 11y. \end{aligned}$$

Alternate solution

Of course, you do not need to show the regrouping step. If you are more comfortable combining like terms in your head, you are free to present your work as follows:

$$2x+3y-5x+8y=-3x+11y.$$

Exercise

Simplify: $-3a + 4b - 7a - 9b$

Answer

$-10a - 5b$

Example 7

Simplify: $-2x - 3 - (3x + 4)$.

Solution

First, distribute the negative sign.

$$-2x-3-(3x+4)=-2x-3-3x-4 \quad -(3x+4)=-3x-4.$$

Next, use the commutative property to reorder, then the associative property to regroup. Then combine like terms.

$$\begin{aligned} &=(-2x-3x)+(-3-4) && \text{Reorder and regroup.}\\ &=-5x+(-7) && \text{Combine like terms:}\\ & && -2x-3x=-5x.\\ &=-5x-7 && \text{Simplify:}\\ & && -5x+(-7)=-5x-7. \end{aligned}$$

Alternate solution

You may skip the second step if you wish, simply combining like terms mentally. That is, it is entirely possible to order your work as follows:

$$\begin{aligned} -2x-3-(3x+4)&=-2x-3-3x-4 && \text{Distribute negative sign.}\\ &=-5x-7 && \text{Combine like terms.} \end{aligned}$$

Exercise

Simplify: $-9a - 4 - (4a - 8)$

Answer

$-13a + 4$

Example 8

Simplify: $2(5 - 3x) - 4(x + 3)$.

Solution

Use the distributive property to expand, then use the commutative and associative properties to group the like terms and combine them.

\[\begin{aligned} 2(5-3x)-4(x+3) = 10-6x-4x-12 ~ & \textcolor{red}{ \text{ Use the distributive property.}} \\ =(-6x-4x)+(10-12) ~ & \textcolor{red}{ \text{ Group like terms.}} \\ =-10x-2 ~ & \textcolor{red}{ \text{ Combine like terms: }} \\ ~ & \textcolor{red}{-6x-4x=-10x \text{ and}} \\ ~ & \textcolor{red}{10-12=-2.} \end{aligned}\nonumber \]

Alternate solution

You may skip the second step if you wish, simply combining like terms mentally. That is, it is entirely possible to order your work as follows:

$$\begin{aligned} 2(5-3x)-4(x+3) &= 10-6x-4x-12 \quad \text{Distribute.} \\ &= -10x-2 \quad \text{Combine like terms.} \end{aligned}$$

Exercise

Simplify: $-2(3a-4)-2(5-a)$

Answer

$-4a-2$

Example 9

Simplify: $-8(3x^2y-9xy)-8(-7x^2-8xy)$

Solution

We will proceed a bit quicker with this solution, using the distributive property to expand, then combining like terms mentally.

$$\begin{aligned} -8(3x^2y-9xy)-8(-7x^2y-8xy) &= -24x^2y+72xy+56x^2y+64xy \\ &= 32x^2y+136xy \end{aligned}$$

Exercise

Simplify: $(a^2-2ab)-2(3ab+a^2)$

Answer

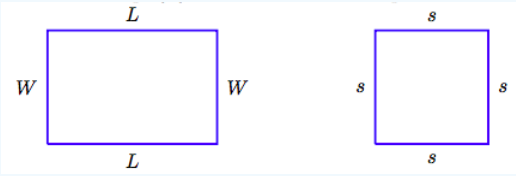
$-a^2-8ab$

Applications

We can simplify a number of useful formulas by combining like terms.

Example 10

Find the perimeter P of the (a) rectangle and (b) square pictured below. Simplify your answer as much as possible.



Solution

The perimeter of any polygonal figure is the sum of the lengths of its sides.

a) To find the perimeter P of the rectangle, sum its four sides.

$$P = L+W+L+W.$$

Combine like terms.

$$P = 2L+2W.$$

b) To find the perimeter P of the square, sum its four sides.

$$P = s + s + s + s.$$

Combine like terms.

$$P = 4s.$$

? Exercise

A regular hexagon has six equal sides, each with length *x*. Find its perimeter in terms of *x*.

Answer

P = 6*x*

Sometimes it is useful to replace a variable with an expression containing another variable.

✔ Example 11

The length of a rectangle is three feet longer than twice its width. Find the perimeter *P* of the rectangle in terms of its width alone.

Solution

From the previous problem, the perimeter of the rectangle is given by

$$P = 2L + 2W,$$

where *L* and *W* are the length and width of the rectangle, respectively. This equation gives the perimeter in terms of its length and width, but we're asked to get the perimeter in terms of the width alone. However, we're also given the fact that the length is three feet longer than twice the width.

$$\begin{array}{rcrcr} cyanLength & \text{is} & \text{Three Feet} & \text{longer than} & \text{Twice the Width} \\ L & = & 3 & + & 2W \end{array}$$

Because *L* = 3+2*W*, we can replace *L* with 3+2*W* in the perimeter equation 3.1.

$$P = 2L + 2W$$

$$P = 2(3 + 2W) + 2W$$

Use the distributive property, then combine like terms.

$$P = 6 + 4W + 2W$$

$$P = 6 + 6W.$$

This last equation gives the perimeter *P* in terms of the width *W* alone.

? Exercise

The length *L* of a rectangle is 5 meters longer than twice its width *W*. Find the perimeter *P* of the rectangle in terms of its width *W*.

Answer

Answer: *P* = 6*W* + 10

Example 12

The width of a rectangle is two feet less than its length. Find the perimeter P of the rectangle in terms of its length alone.

Solution

Again, the perimeter of a rectangle is given by the equation

$$P = 2L + 2W,$$

where L and W are the length and width of the rectangle, respectively. This equation gives the perimeter in terms of its length and width, but we're asked to get the perimeter in terms of the length alone.

However, we're also given the fact that the width is two feet less than the length.

Width	is	Length	minus	Two feet
W	$=$	L	$-$	2

Because $W = L-2$, we can replace W with $L-2$ in the perimeter equation 3.2.

$$P = 2L + 2W$$

$$P = 2L + 2(L - 2)$$

Use the distributive property, then combine like terms.

$$P = 2L + 2L - 4$$

$$P = 4L - 4.$$

This last equation gives the perimeter P in terms of the length L alone.

Exercise

The width W of a rectangle is 5 feet less than twice its width L. Find the perimeter P of the rectangle in terms of its length L.

Answer

$P = 6L - 10$

Exercises

In Exercises 1-16, combine like terms by first using the distributive property to factor out the common variable part, and then simplifying.

1. $17xy^2 + 18xy^2 + 20xy^2$
2. $13xy - 3xy + xy$
3. $-8xy^2 - 3xy^2 - 10xy^2$
4. $-12xy - 2xy + 10xy$
5. $4xy - 20xy$
6. $-7y^3 + 15y^3$
7. $12r - 12r$
8. $16s - 5s$
9. $-11x - 13x + 8x$
10. $-9r - 10r + 3r$
11. $-5q + 7q$
12. $17n + 15n$

13. $r - 13r - 7r$

14. $19m + m + 15m$

15. $3x^3 - 18x^3$

16. $13x^2y + 2x^2y$

In Exercises 17-32, combine like terms by first rearranging the terms, then using the distributive property to factor out the common variable part, and then simplifying.

17. $-8 + 17n + 10 + 8n$

18. $11 + 16s - 14 - 6s$

19. $-2x^3 - 19x^2y - 15x^2y + 11x^3$

20. $-9x^2y - 10y^3 - 10y^3 + 17x^2y$

21. $-14xy - 2x^3 - 2x^3 - 4xy$

22. $-4x^3 + 12xy + 4xy - 12x^3$

23. $-13 + 16m + m + 16$

24. $9 - 11x - 8x + 15$

25. $-14x^2y - 2xy^2 + 8x^2y + 18xy^2$

26. $-19y^2 + 18y^3 - 5y^2 - 17y^3$

27. $-14x^3 + 16xy + 5x^3 + 8xy$

28. $-16xy + 16y^2 + 7xy + 17y^2$

29. $9n + 10 + 7 + 15n$

30. $-12r + 5 + 17 + 17r$

31. $3y + 1 + 6y + 3$

32. $19p + 6 + 8p + 13$

In Exercises 33-56, simplify the expression by first using the distributive property to expand the expression, and then rearranging and combining like terms mentally.

33. $-4(9x^2y + 8) + 6(10x^2y - 6)$

34. $-4(-4xy + 5y^3) + 6(-5xy - 9y^3)$

35. $3(-4x^2 + 10y^2) + 10(4y^2 - x^2)$

36. $-7(-7x^3 + 6x^2) - 7(-10x^2 - 7x^3)$

37. $-s + 7 - (-1 - 3s)$

38. $10y - 6 - (-10 - 10y)$

39. $-10q - 10 - (-3q + 5)$

40. $-2n + 10 - (7n - 1)$

41. $7(8y + 7) - 6(8 - 7y)$

42. $-6(-5n - 4) - 9(3 + 4n)$

43. $7(10x^2 - 8xy^2) - 7(9xy^2 + 9x^2)$

44. $10(8x^2y - 10xy^2) + 3(8xy^2 + 2x^2y)$

45. $-2(6 + 4n) + 4(-n - 7)$

46. $-6(-2 - 6m) + 5(-9m + 7)$

47. $8 - (4 + 8y)$

48. $-1 - (8 + s)$

49. $-8(-n + 4) - 10(-4n + 3)$

50. $3(8r - 7) - 3(2r - 2)$

51. $-5 - (10p + 5)$

52. $-1 - (2p - 8)$

53. $7(1 + 7r) + 2(4 - 5r)$

54. $(5 - s) + 10(9 + 5s)$

55. $-2(-5 - 8x^2) - 6(6)$

56. $8(10y^2 + 3x^3) - 5(-7y^2 - 7x^3)$

---

57. The length L of a rectangle is 2 feet longer than 6 times its width W. Find the perimeter of the rectangle in terms of its width alone.

58. The length L of a rectangle is 7 feet longer than 6 times its width W. Find the perimeter of the rectangle in terms of its width alone.

59. The width W of a rectangle is 8 feet shorter than its length L. Find the perimeter of the rectangle in terms of its length alone.

60. The width W of a rectangle is 9 feet shorter than its length L. Find the perimeter of the rectangle in terms of its length alone.

61. The length L of a rectangle is 9 feet shorter than 4 times its width W. Find the perimeter of the rectangle in terms of its width alone.

62. The length L of a rectangle is 2 feet shorter than 6 times its width W. Find the perimeter of the rectangle in terms of its width alone.

Answers

1. $55xy^2$

3. $-21xy^2$

5. $-16xy$

7. 0

9. $-16x$

11. $2q$

13. $-19r$

15. $-15x^3$

17. $2 + 25n$

19. $9x^3 - 34x^2y$

21. $-18xy - 4x^3$

23. $3 + 17m$

25. $-6x^2y + 16xy^2$

27. $-9x^3 + 24xy$

29. $24n + 17$

31. $9y + 4$

33. $24x^2y - 68$

35. $-22x^2 + 70y^2$

37. $2s + 8$

39. $-7q - 15$

41. $98y + 1$

43. $7x^2 - 119xy^2$

45. $-40 - 12n$

47. $4 - 8y$

49. $48n - 62$

51. $-10 - 10p$

53. $15 + 39r$

55. $-26 + 16x^2$

57. $4 + 14W$

59. $4L - 16$

61. $10W - 18$

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6.3: Adding and Subtracting Polynomials

Learning Objectives

- Add polynomials.
- Subtract polynomials.
- Add and subtract polynomial functions

Adding Polynomials

Recall that we combine like terms, or terms with the same variable part, as a means to simplify expressions. To do this, add the coefficients of the terms to obtain a single term with the same variable part. For example,

$$5x^2+8x^2=13x^2 \tag{6.3.1}$$

Notice that the variable part, x^2, does not change. This, in addition to the commutative and associative properties of addition, allows us to add polynomials.

Example 6.3.1

Add:

$$3x+(4x-5).$$

Solution:

The property $+(a+b)=a+b$, which was derived using the distributive property, allows us to remove the parentheses so that we can add like terms.

Answer:

$7x-5$

Example 6.3.2

Add:

$$(3x^2+3x+5)+(2x^2-x-2)\ .$$

Solution:

Remove the parentheses and then combine like terms.

Answer:

$5x^2+2x+3$

Example 6.3.3

Add:

$$(-5x^2y-2xy^2+7xy)+(4x^2y+7xy^2-3xy)\,.$$

Solution:

Remember that the variable parts have to be exactly the same before we can add the coefficients.

$$\begin{aligned}&(-5x^2y-2xy^2+7xy)+(4x^2y+7xy^2-3xy)\\&=\underline{-5x^2y}\underline{-2xy^2}+\underline{\underline{7xy}}+\underline{4x^2y}+\underline{7xy^2}-\underline{\underline{3xy}}\\&=-x^2y+5xy^2+4xy\end{aligned}$$

Answer:

$$-x^2y+5xy^2+4xy$$

It is common practice to present the terms of the simplified polynomial expression in descending order based on their degree. In other words, we typically present polynomials in standard form, with terms in order from highest to lowest degree.

Example 6.3.4

Add:

$$(a-4a^3+a^5-8)+(-9a^5+a^4-7a+5+a^3) \quad .$$

Solution:

Answer:

$-8a^5+a^4-3a^3-6a-3$

Exercise 6.3.1

Add:

$$(6-5x^3+x^2-x)+(x^2+x+6x^3-1) \quad .$$

Answer

x^3+2x^2+5

Subtracting Polynomials

When subtracting polynomials, we see that the parentheses become very important. Recall that the distributive property allowed us to derive the following:

$$-(a+b)=-a-b \tag{6.3.2}$$

In other words, when subtracting an algebraic expression, we remove the parentheses by subtracting each term.

Example 6.3.5

Subtract:

$$10x-(3x+5).$$

Solution:

Subtract each term within the parentheses and then combine like terms.

Answer:

$7x-5$

Subtracting a quantity is equivalent to multiplying it by -1.

Example 6.3.6

Subtract:

$$(3x^2+3x+5)-(2x^2-x-2) \quad .$$

Solution:

Distribute the -1, remove the parentheses, and then combine like terms.

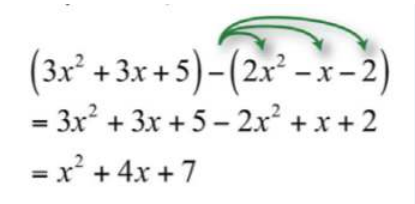


Figure 6.3.1

Answer:

$$x^2+4x+7$$

Multiplying the terms of a polynomial by -1 changes all the signs.

Example 6.3.7

Subtract:

$$(-5x^3-2x^2+7)-(4x^3+7x^2-3x+2) \quad .$$

Solution:

Distribute the -1, remove the parentheses, and then combine like terms.

Answer:

$-9x^3-9x^2+3x+5$

Example 6.3.8

Subtract $6x^2-3x-1$ from $2x^2+5x-2$.

Solution

Since subtraction is not commutative, we must take care to set up the difference correctly. First, write the quantity $(2x^2+5x-2)$; from this, subtract the quantity $(6x^2-3x-1)$.

Answer:

$-4x^2+8x-1$

Example 6.3.9

Simplify:

$$(2x^2-3x+5)-(x^2-3x+1)+(5x^2-4x-8) \quad .$$

Solution:

Apply the distributive property, remove the parentheses, and then combine like terms.

Answer:

$6x^2-4x-4$

Exercise 6.3.2

Subtract:

$$(8x^2y-5xy^2+6)-(x^2y+2xy^2-1) \quad .$$

Answer

$7x^2y-7xy^2+7$

Adding and Subtracting Polynomial Functions

We use function notation to indicate addition and subtraction of functions as follows:

Table 6.3.1

Addition of functions:	$(f+g)(x)=f(x)+g(x)$
Subtraction of functions:	$(f-g)(x)=f(x)-g(x)$

When using function notation, be careful to group the entire function and add or subtract accordingly.

Example 6.3.10

Calculate:

$$(f+g)(x), \text{ given } f(x)=-x^2-3x+5 \text{ and } g(x)=3x^2+2x+1\ .$$

Solution:

The notation $(f+g)(x)$ indicates that you should add the functions $f(x)+g(x)$ and collect like terms.

$$\begin{aligned} f(x)+g(x) &= (-x^2-3x+5)+(3x^2+2x+1) \\ &= -x^2-3x+5+3x^2+2x+1 \\ &= 2x^2-x+6 \end{aligned}$$

Answer:

$$(f+g)(x)=2x^2-x+6$$

Example 6.3.11

Calculate:

$$(f-g)(x), \text{ given } f(x)=2x-3 \text{ and } g(x)=-2x^2+2x+5\ .$$

Solution:

The notation $(f-g)(x)$ indicates that you should subtract the functions $f(x)-g(x)$:

Answer:

$(f-g)(x)=2x^2-8$

We may be asked to evaluate the sum or difference of two functions. We have the option to first find the sum or difference and use the resulting function to evaluate for the given variable, or to first evaluate each function and then find the sum or difference.

Example 6.3.12

Calculate:

$$(f-g)(5), \text{ given } f(x)=x^2+x-7 \text{ and } g(x)=4x+10\,.$$

Solution:

First, find $(f-g)(x)=f(x)-g(x)$.

Therefore,

$(f-g)(x)=x^2-3x-17$

Next, substitute 5 for the variable x.

Answer:

$(f-g)(5)=-7$

Alternate Solution:

Since $(f-g)(5)=f(5)-g(5)$, we can find $f(5)$ and $g(5)$ and then subtract the results.

$$\begin{array}{l|l} f(x)=x^2+x-7 & g(x)=4x+10 \\ f(5)=(5)^2+(5)-7 & g(5)=4(5)+10 \\ \quad =25+5-7 & \quad =20+10 \\ \quad =23 & \quad =30 \end{array}$$

Therefore, we have

Answer:

$(f-g)(5)=-7$

Key Takeaways

- When adding polynomials, remove the associated parentheses and then combine like terms.
- When subtracting polynomials, distribute the -1 and subtract all the terms before removing the parentheses and combining like terms.
- The notation $(f+g)(x)$ indicates that you add the functions.
- The notation $(f-g)(x)$ indicates that you subtract the functions

? Exercise 6.3.3 Addition of Polynomials

Add.

1. $(2x+1)+(-x+7)$
2. $(-6x+5)+(3x-1)$
3. $(\frac{2}{3}x+\frac{1}{2})+(\frac{1}{3}x-2)$
4. $(\frac{1}{3}x-\frac{3}{4})+(\frac{5}{6}x+\frac{1}{8})$
5. $(2x+1)+(x-3)+(5x-2)$
6. $(2x-8)+(-3x^2+7x-5)$
7. $(x^2-3x+7)+(3x^2-8x-5)$
8. $(-5x^2-1+x)+(-x+7x^2-9)$
9. $(\frac{1}{2}x^2-\frac{1}{3}x+\frac{1}{6})+(-\frac{3}{2}x^2+\frac{2}{3}x-1)$
10. $(-\frac{3}{5}x2+\frac{1}{4}x-6)+(2x^2-\frac{3}{8}x+\frac{5}{2})$
11. $(x^2+5)+(3x^2-2x+1)+(x^2+x-3)$
12. $(a^3-a^2+a-8)+(a^3+a^2+6a-2)$
13. $(a^3-8)+(-3a^3+5a^2-2)$
14. $(4a^5+5a^3-a)+(3a^4-2a^2+7)$
15. $(2x^2+5x-12)+(7x-5)$
16. $(3x+5)+(x^2-x+1)+(x^3+2x^2-3x+6)$
17. $(6x^5-7x^3+x^2-15)+(x^4+2x^3-6x+12)$
18. $(1+7x-5x^3+4x^4)+(-3x^3+5-x^2+x)$
19. $(x^2y^2-7xy+7)+(4x^2y^2-3xy-8)$
20. $(x^2+xy-y^2)+(7x^2-5xy+2y^2)$
21. $(2x^2+3xy-7y^2)+(-5x^2-3xy+8y^2)$
22. $(a^2b^2-100)+(2a^2b^2-3ab+20)$
23. $(ab^2-3a^2b+ab-3)+(-2a^2b+ab^2-7ab-1)$
24. $(10a^2b-7ab+8ab^2)+(6a^2b-ab+5ab^2)$
25. Find the sum of $2x+8$ and $7x-1$.
26. Find the sum of $13x-15$ and $16x+110$.
27. Find the sum of $x^2-10x+8$ and $5x^2-2x-6$.
28. Find the sum of $a^2-5a+10$ and $-9a^2+7a-11$.
29. Find the sum of x^2y^2-xy+6 and x^2y^2+xy-7.
30. Find the sum of $x^2-9xy+7y^2$ and $-3x^2-3xy+7y^2$.

Answer

1. $x+8$
3. $x-\frac{3}{2}$
5. $8x-4$
7. $4x^2-11x+2$
9. $-x^2+\frac{1}{3}x-\frac{5}{6}$
11. $5x^2-x+3$
13. $-2a^3+5a^2-10$
15. $2x^2+12x-17$
17. $6x^5+x^4-5x^3+x^2-6x-3$
19. $5x^2y^2-10xy-1$
21. $-3x^2+y^2$
23. $-5a^2b+2ab^2-6ab-4$
25. $9x+7$
27. $6x^2-12x+2$
29. $2x^2y^2-1$

Exercise 6.3.4 Subtraction of Polynomials

Subtract.

1. $(5x-3)-(2x-1)$
2. $(-4x+1)-(7x+10)$
3. $(\frac{1}{4}x-\frac{3}{4})-(\frac{3}{4}x+\frac{1}{8})$
4. $(-\frac{3}{5}x+\frac{3}{7})-(\frac{2}{5}x-\frac{3}{2})$
5. $(x^2+7x-5)-(4x^2-5x+1)$
6. $(-6x^2+3x-12)-(-6x^2+3x-12)$
7. $(-3x^3+4x-8)-(-x^2+4x+10)$
8. $(\frac{1}{2}x^2+\frac{1}{3}x-\frac{3}{4})-(\frac{3}{2}x^2-\frac{1}{6}x+\frac{1}{2})$
9. $(\frac{5}{9}x^2+\frac{1}{5}x-\frac{1}{3})-(\frac{1}{3}x^2+\frac{3}{10}x+\frac{5}{9})$
10. $(a^3-4a^2+3a-7)-(7a^3-2a^2-6a+9)$
11. $(3a^3+5a^2-2)-(a^3-a+8)$
12. $(5x^5+4x^3+x^2-6)-(4x^4-3x^3-x+3)$
13. $(3-5x-x^3+5x^4)-(-5x^3+2-x^2-7x)$
14. $(x^5-6x^3+9x)-(4x^4+2x^2-5)$
15. $(2x^2y^2-4xy+9)-(3x^2y^2-3xy-5)$
16. $(x^2+xy-y^2)-(x^2+xy-y^2)$
17. $(2x^2+3xy-7y^2)-(-5x^2-3xy+8y^2)$
18. $(ab^2-3a^2b+ab-3)-(-2a^2b+ab^2-7ab-1)$
19. $(10a^2b-7ab+8ab^2)-(6a^2b-ab+5ab^2)$
20. $(10a^2b^2+5ab-6)-(5a^2b^2+5ab-6)$
21. Subtract $3x+1$ from $5x-9$.
22. Subtract $x^2-5x+10$ from x^2+5x-5.
23. Find the difference of $3x-7$ and $8x+6$.
24. Find the difference of $2x^2+3x-5$ and x^2-9.

25. The cost in dollars of producing customized coffee mugs with a company logo is given by the formula $C = 150 + 0.10x$, where x is the number of cups produced. The revenue from selling the cups in the company store is given by $R = 10x - 0.05x^2$, where x is the number of units sold.
    a. Find a formula for the profit. (profit = revenue − cost)
    b. Find the profit from producing and selling 100 mugs in the company store.
26. The cost in dollars of producing sweat shirts is given by the formula $C = 10q + 1200$, where C is the cost and q represents the quantity produced. The revenue generated by selling the sweat shirts for $\$37$ each is given by $R = 37q$, where q represents the quantity sold. Determine the profit generated if 125 sweat shirts are produced and sold.
27. The outer radius of a washer is 3 times the radius of the hole.

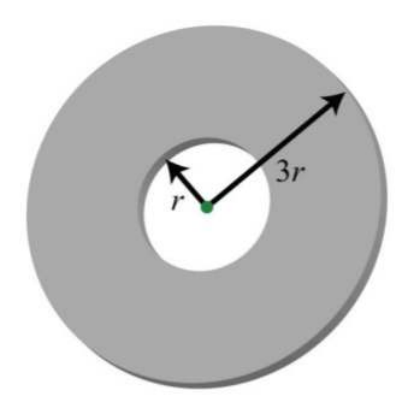


***Figure* 6.3.2**

    a. Derive a formula for the area of the face of the washer.
    b. What is the area of the washer if the hole has a diameter of 10 millimeters?
28. Derive a formula for the surface area of the following rectangular solid.

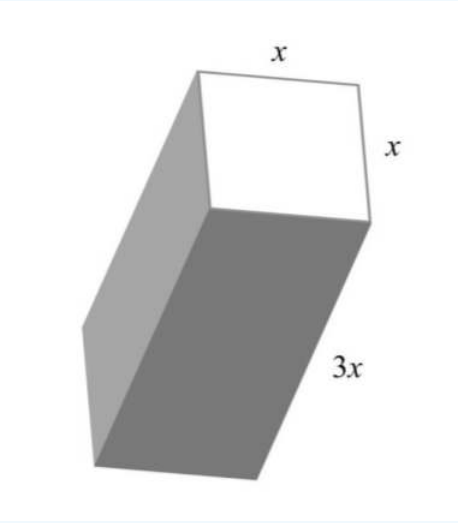


***Figure* 6.3.3**

Answer

1. $3x - 2$
3. $-\frac{1}{2}x - \frac{7}{8}$
5. $-3x^2 + 12x - 6$
7. $-3x^3 + x^2 - 18$
9. $\frac{2}{9}x^2 - \frac{1}{10}x - \frac{8}{9}$
11. $2a^3 + 5a^2 + a - 10$

13. $5x^4+4x^3+x^2+2x+1$
15. $-x^2y^2-xy+14$
17. $7x^2+6xy-15y^2$
19. $4a^2b+3ab^2-6ab$
21. $2x-10$
23. $-5x-13$
25. a. $P=-0.05x^2+9.9x-150$; b. \$340
27. a. $A=8\pi r^2$; b. 628.32square millimeters

Exercise 6.3.5 Addition and Subtraction of Polynomial

Simplify.

1. $(2x+3)-(5x-8)+(x-7)$
2. $(3x-5)-(7x-11)-(5x+2)$
3. $(3x-2)-(4x-1)+(x+7)$
4. $(5x-3)-(2x+1)-(x-1)$
5. $(5x^2-3x+2)-(x^2+x-4)+(7x^2-2x-6)$
6. $(-2x^3+x^2-8)-(3x^2+x-6)-(2x-1)$
7. $(2x-7)-(x^2+3x-7)+(6x-1)$
8. $(6x^2-10x+13)+(4x^2-9)-(9-x^2)$
9. $(a^2-b^2)-(2a^2+3ab-4b^2)+(5ab-1)$
10. $(a^2-3ab+b^2)-(a^2+\ b^2)-(3ab-5)$
11. $(\frac{1}{2}x^2-\frac{3}{4}x+\frac{1}{4})-(\frac{3}{2}x-\frac{3}{4})+(\frac{5}{4}x-\frac{1}{2})$
12. $(\frac{9}{5}x^2-\frac{1}{3}x+2)-(\frac{3}{10}x^2-\frac{4}{5})-(x+\frac{5}{2})$

Answer

1. $-2x+4$
3. 6
5. $11x^2-6x$
7. $-x^2+5x-1$
9. $-a^2+2ab+3b^2-1$
11. $12x^2-x+12$

Exercise 6.3.6 Addition and Subtraction of Polynomial Functions

Find $(f+g)(x)$ and $(f-g)(x)$, given the following functions.

1. $f(x)=4x-1$ and $g(x)=-3x+1$
2. $f(x)=-x+5$ and $g(x)=2x-3$
3. $f(x)=3x^2-5x+7$ and $g(x)=-2x^2+5x-1$
4. $f(x)=x^3+2x^2-6x+2$ and $g(x)=2x^3+2x^2-5x-1$
5. $f(x)=\frac{1}{2}x+\frac{1}{3}$ and $g(x)=\frac{1}{5}x^2-\frac{3}{2}x+\frac{1}{6}$
6. $f(x)=x^2-5x+\frac{1}{3}$ and $g(x)=\frac{2}{3}x^2-x-\frac{1}{2}$

Answer

1. $(f+g)(x)=x$ and $(f-g)(x)=7x-2$
3. $(f+g)(x)=x^2+6$ and $(f-g)(x)=5x^2-10x+8$

5. $(f+g)(x)=\frac{1}{5}x^2-x+\frac{1}{2}$ and $(f-g)(x)=-\frac{1}{5}x^2+2x+\frac{1}{6}$

Exercise 6.3.7 Addition and Subtraction of Polynomial Functions

Given $f(x)=2x-3$ and $g(x)=x^2+3x-1$, find the following.

1. $(f+g)(x)$
2. $(g+f)(x)$
3. $(f-g)(x)$
4. $(g-f)(x)$
5. $(g+g)(x)$
6. $(f+g)(3)$
7. $(f+g)(-2)$
8. $(f+g)(0)$
9. $(f-g)(0)$
10. $(f-g)(-2)$
11. $(g-f)(-2)$
12. $(g-f)(12)$

Answer

1. $(f+g)(x)=x^2+5x-4$
3. $(f-g)(x)=-x^2-x-2$
5. $(g+g)(x)=2x^2+6x-2$
7. $(f+g)(-2)=-10$
9. $(f-g)(0)=-2$
11. $(g-f)(-2)=4$

Exercise 6.3.8 Addition and Subtraction of Polynomial Functions

Given $f(x)=5x^2-3x+2$ and $g(x)=2x^2+6x-4$, find the following.

1. $(f+g)(x)$
2. $(g+f)(x)$
3. $(f-g)(x)$
4. $(g-f)(x)$
5. $(f+g)(-2)$
6. $(f-g)(-2)$
7. $(f+g)(0)$
8. $(f-g)(0)$

Answer

1. $(f+g)(x)=7x^2+3x-2$
3. $(f-g)(x)=3x^2-9x+6$
5. $(f+g)(-2)=20$
7. $(f+g)(0)=-2$

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6.4: Multiplying Polynomials

Learning Objectives

- Multiply a polynomial by a monomial.
- Multiply a polynomial by a binomial.
- Multiply a polynomial by any size polynomial.
- Recognize and calculate special products.
- Multiply polynomial functions.

Multiplying by a Monomial

Recall the product rule for exponents: if m and n are positive integers, then

$$x^m \cdot x^n = x^{m+n} \tag{6.4.1}$$

In other words, when multiplying two expressions with the same base, add the exponents. This rule applies when multiplying a monomial by a monomial. To find the product of monomials, multiply the coefficients and add the exponents of variable factors with the same base. For example,

$$\begin{aligned} 3x \cdot 5x^2 &= 3 \cdot 5 \cdot x^1 \cdot x^2 && \textit{Commutative property} \\ &= 15x^{1+2} && \textit{Product rule for exponents} \\ &= 15x^3 \end{aligned}$$

To multiply a polynomial by a monomial, apply the distributive property and then simplify each term.

Example 6.4.1

Multiply:

$$-5x(4x-2).$$

Solution:

In this case, multiply the monomial, $-5x$, by the binomial, $4x-2$. Apply the distributive property and then simplify.

$$\begin{aligned} &-5x(4x-2) \\ &= (-5x)\cdot 4x + (-5x)\cdot(-2) \\ &= -20x^2 + 10x \end{aligned}$$

Figure 6.4.1

Answer:

$$-20x^2+10x$$

Example 6.4.2

Multiply:

$$2x^2(3x^2-5x+1).$$

Solution:

Apply the distributive property and then simplify.

$$2x^2\left(3x^2-5x+1\right)$$
$$=2x^2\cdot 3x^2-2x^2\cdot 5x+2x^2\cdot 1$$
$$=6x^4-10x^3+2x^2$$

Figure 6.4.2

Answer:

$$6x^4-10x^3+2x^2$$

Example 6.4.3

Multiply:

$$-3ab^2(a^2b^3+2a^3b-6ab-4)\ .$$

Solution:

Answer:

To summarize, multiplying a polynomial by a monomial involves the distributive property and the product rule for exponents. Multiply all of the terms of the polynomial by the monomial. For each term, multiply the coefficients and add exponents of variables where the bases are the same.

Exercise 6.4.1

Multiply:

$$-5x^2y(2xy^2-3xy+6x^2y-1)\,.$$

Answer

$$-10x^3y^3+15x^3y^2-30x^4y^2+5x^2y$$

Multiplying by a Binomial

In the same way that we used the distributive property to find the product of a monomial and a binomial, we will use it to find the product of two binomials.

$$\begin{aligned}(a+b)(c+d)&\\ &=(a+b)\cdot c+(a+b)\cdot d\\ &=ac+bc+ad+bd\\ &=ac+ad+bc+bd\end{aligned}$$

Here we apply the distributive property multiple times to produce the final result. This same result is obtained in one step if we apply the distributive property to a and b separately as follows:

$$(a+b)(c+d)=ac+ad+bc+bd$$

Figure 6.4.3

This is often called the FOIL method. We add the products of the first terms of each binomial ac, the $o$uter terms ad, the $i$nner terms bc, and finally the last terms bd. This mnemonic device only works for products of binomials; hence it is best to just remember that the distributive property applies.

Example 6.4.4

Multiply:

$$(2x+3)(5x-2).$$

Solution:

Distribute $2x$ and then distribute 3.

Simplify by combining like terms.

$=10x^2+11x-6$

Answer:

$$10x^2+11x-6$$

Example 6.4.5

Multiply:

$$\left(\tfrac{1}{2}x-\tfrac{1}{4}\right)\left(\tfrac{1}{2}x+\tfrac{1}{4}\right).$$

Solution:

Distribute $\frac{1}{2}x$ and then distribute $-\frac{1}{4}$.

$$\begin{aligned}\left(\frac{1}{2}x-\frac{1}{4}\right)\left(\frac{1}{2}x+\frac{1}{4}\right) &= \frac{1}{2}x\cdot\frac{1}{2}x+\frac{1}{2}x\cdot\frac{1}{4}+\left(-\frac{1}{4}\right)\cdot\frac{1}{2}x+\left(-\frac{1}{4}\right)\cdot\frac{1}{4}\\ &= \frac{1}{4}x^2+\frac{1}{8}x-\frac{1}{8}x-\frac{1}{16}\\ &= \frac{1}{4}x^2-\frac{1}{16}\end{aligned}$$

Answer:

$$\tfrac{1}{4}x^2-\tfrac{1}{16}$$

Example 6.4.6

Multiply:

$$(3y^2-1)(2y+1).$$

Solution:

Answer:

$6y^3+3y^2-2y-1$

After applying the distributive property, combine any like terms.

Example 6.4.7

Multiply:

$$(x^2-5)(3x^2-2x+2).$$

Solution:

After multiplying each term of the trinomial by x^2 and -5, simplify.

Answer:

$3x^4-2x^3-13x^2+10x-10$

Example 6.4.8

Multiply:

$$(2x-1)^3.$$

Solution:

Perform one product at a time.

$$\begin{aligned}(2x-1)^3 &= (2x-1)\underbrace{(2x-1)(2x-1)}_{\textit{multiply first}}\\ &= (2x-1)\underbrace{(4x^2-2x-2x+1)}_{\textit{simplify}}\\ &= \underbrace{(2x-1)(4x^2-4x+1)}_{\textit{multiply second}}\\ &= 8x^3-8x^2+2x-4x^2+4x-1\\ &= 8x^3-12x^2+6x-1\end{aligned}$$

Figure 6.4.4

Answer:

$$8x^3-12x^2+6x-1$$

At this point, it is worth pointing out a common mistake:

$$(2x-1)^3 \neq (2x)^3-(1)^3$$

The confusion comes from the product to a power rule of exponents, where we apply the power to all factors. Since there are two terms within the parentheses, that rule does not apply. Care should be taken to understand what is different in the following two examples:

$$(xy)^2 = x^2y^2 \quad \checkmark$$
$$(x+y)^2 \neq x^2+y^2 \quad x$$

Exercise 6.4.2

Multiply:

$$(2x-3)(7x^2-5x+4)\,.$$

Answer

$$14x^3-31x^2+23x-12$$

Product of Polynomials

When multiplying polynomials, we apply the distributive property many times. Multiply all of the terms of each polynomial and then combine like terms.

Example 6.4.9

Multiply:

$$(2x^2+x-3)(x^2-2x+5)\ .$$

Solution:

Multiply each term of the first trinomial by each term of the second trinomial and then combine like terms.

Aligning like terms in columns, as we have here, aids in the simplification process

Answer:

$2x^4-3x^3+5x^2+11x-15$

Notice that when multiplying a trinomial by a trinomial, we obtain nine terms before simplifying. In fact, when multiplying an n-term polynomial by an m-term polynomial, we will obtain $n \times m$ terms. In the previous example, we were asked to multiply and found that

$$(2x^2+x-3)(x^2-2x+5)=2x^4-3x^3+5x^2+11x-15$$

Because it is easy to make a small calculation error, it is a good practice to trace through the steps mentally to verify that the operations were performed correctly. Alternatively, we can check by evaluating any value for x in both expressions to verify that the results are the same. Here we choose $x=2$:

$$\begin{aligned}(2x^2+x-3)(x^2-2x+5) &= (2(2)^2+(2)-3)((2)^2-2(2)+5)\\ &= (8+2-3)(4-4+5)\\ &= (7)(5)\\ &= 35\end{aligned}$$

Because the results could coincidentally be the same, a check by evaluating does not necessarily prove that we have multiplied correctly. However, after verifying a few values, we can be fairly confident that the product is correct.

Multiply:

$$(x^2-2x-3)^2\ .$$

Answer

$x^4-4x^3-2x^2+12x+9$

Special Products

In this section, the goal is to recognize certain special products that occur often in our study of algebra. We will develop three formulas that will be very useful as we move along. The three should be memorized. We begin by considering the following two calculations:

$$\begin{aligned}(a+b)^2 &= (a+b)(a+b)\\ &= a^2+ab+ba+b^2\\ &= a^2+ab+ab+b^2\\ &= a^2+2ab+b^2\end{aligned}$$

$$\begin{aligned}(a-b)^2 &= (a-b)(a-b)\\ &= a^2-ab-ba+b^2\\ &= a^2-ab-ab+b^2\\ &= a^2-2ab+b^2\end{aligned}$$

This leads us to two formulas that describe perfect square trinomials:

$$(a+b)^2=a^2+2ab+b^2 \tag{6.4.2}$$

$$(a-b)^2=a^2-2ab+b^2 \tag{6.4.3}$$

We can use these formulas to quickly square a binomial.

Example 6.4.10

Multiply:

$$(3x+5)^2.$$

Solution:

Here $a=3x$ and $b=5$. Apply the formula:

$$\begin{array}{rl} (a+b)^2 = & a^2 \quad +2 \quad a \quad b \quad + \quad b^2 \\ & \downarrow \qquad\quad \downarrow \quad \downarrow \qquad \downarrow \\ (3x+5)^2 & = (3x)^2 + 2\cdot(3x)(5) + (5)^2 \\ & = 9x^2 + 30x + 25 \end{array}$$

Answer:

$$9x^2 + 30x + 25$$

This process should become routine enough to be performed mentally.

Example 6.4.11

Multiply:

$$(x-4)^2.$$

Solution:

Here $a = x$ and $b = 4$. Apply the appropriate formula as follows:

$$\begin{array}{rl} (a-b)^2 = & a^2 \quad -2 \quad a \quad b \quad + b^2 \\ & \downarrow \qquad\quad \downarrow \quad \downarrow \qquad \downarrow \\ (x-4)^2 & = (x)^2 - 2\cdot(x)(4) + (4)^2 \\ & = x^2 - 8x + 16 \end{array}$$

Answer:

$$x^2 - 8x + 16$$

Our third special product follows:

$$\begin{aligned} (a+b)(a-b) &= a^2 - ab + ba - b^2 \\ &= a^2 - ab + ab - b^2 \\ &= a^2 - b^2 \end{aligned}$$

This product is called difference of squares:

$$(a+b)(a-b) = a^2 - b^2 \tag{6.4.4}$$

The binomials $(a+b)$ and $(a-b)$ are called conjugate binomials. Therefore, when conjugate binomials are multiplied, the middle term eliminates, and the product is itself a binomial.

Example 6.4.12

Multiply:

$$(7x+4)(7x-4).$$

Solution:

Answer:

$49x^2 - 16$

Exercise 6.4.4

Multiply:

$$(-5x+2)^2.$$

Answer

$25x^2 - 20x + 4$

Multiplying Polynomial Functions

We use function notation to indicate multiplication as follows:

Table 6.4.1

Multiplication of functions:	$(f \cdot g)(x) = f(x) \cdot g(x)$

Example 6.4.13

Calculate:

$$(f \cdot g)(x), \text{ given } f(x) = 5x^2 \text{ and } g(x) = -x^2 + 2x - 3 \ .$$

Solution:

Multiply all terms of the trinomial by the monomial function $f(x)$.

$$\begin{aligned}(f \cdot g)(x) &= f(x) \cdot g(x) \\ &= 5x^2 \cdot (-x^2 + 2x - 3) \\ &= -5x^4 + 10x^3 - 15x^2\end{aligned}$$

Answer:

$$(f \cdot g)(x) = -5x^4 + 10x^3 - 15x^2$$

Example 6.4.14

Calculate:

$$(f \cdot g)(-1), \text{ given } f(x) = -x + 3 \text{ and } g(x) = 4x^2 - 3x + 6 \ .$$

Solution:

First, determine $(f \cdot g)(x)$.

$$\begin{aligned}(f \cdot g)(x) &= f(x) \cdot g(x) \\ &= (-x + 3)(4x^2 - 3x + 6) \\ &= -4x^3 + 3x^2 - 6x + 12x^2 - 9x + 18 \\ &= -4x^3 + 15x^2 - 15x + 18\end{aligned}$$

We have

\((f\cdot g)(x) = -4x^{3}+15x^{2}-15x+18

Next, substitute -1 for the variable x.

$$\begin{aligned}(f \cdot g)(-1) &= -4(-1)^3 + 15(-1)^2 - 15(-1) + 18 \\ &= -4 \cdot (-1) + 15 \cdot 1 + 15 + 18 \\ &= 4 + 15 + 15 + 18 \\ &= 52\end{aligned}$$

Answer:

$$(f \cdot g)(-1) = 52$$

Because $(f \cdot g)(-1) = f(-1) \cdot g(-1)$, we could alternatively calculate $f(-1)$ and $g(-1)$ separately and then multiply the results (try this as an exercise). However, if we were asked to evaluate multiple values for the function $(f \cdot g)(x)$, it would be best to first determine the general form, as we have in the previous example.

Key Takeaways

- To multiply a polynomial by a monomial, apply the distributive property and then simplify each of the resulting terms.

- To multiply polynomials, multiply each term in the first polynomial with each term in the second polynomial. Then combine like terms.
- The product of an n-term polynomial and an m-term polynomial results in an $m \times n$ term polynomial before like terms are combined.
- Check results by evaluating values in the original expression and in your answer to verify that the results are the same.
- Use the formulas for special products to quickly multiply binomials that occur often in algebra.

? Exercise 6.4.5 Product of a Monomial and a Polynomial

Multiply.

1. $5x(-3x^2y)$
2. $(-2x^3y^2)(-3xy^4)$
3. $\frac{1}{2}(4x-3)$
4. $-\frac{3}{4}(\frac{2}{3}x-6)$
5. $3x(5x-2)$
6. $-4x(2x-1)$
7. $x^2(3x+2)$
8. $-6x^2(5x+3)$
9. $2ab(4a-2b)$
10. $5a^2b(a^2-b^2)$
11. $6x^2y^3(-3x^3y+xy^2)$
12. $3ab^3(-5ab^3+6a^2b)$
13. $-\frac{1}{2}x^2y(4xy-10)$
14. $-3x^4y^2(3x^8y^3)$
15. $2x^2(-5x^3)(3x^4)$
16. $4ab(a^2b^3c)(a^4b^2c^4)$
17. $-2(5x^2-3x+4)$
18. $\frac{4}{5}(25x^2-50xy+5y^2)$
19. $3x(5x^2-2x+3)$
20. $-x(x^2+x-1)$
21. $x^2(3x^2-5x-7)$
22. $x^3(-4x^2-7x+9)$
23. $\frac{1}{4}x^4(8x^3-2x^2+\frac{1}{2}x-5)$
24. $-\frac{1}{3}x^3(\frac{3}{2}x^5-\frac{2}{3}x^3+\frac{9}{2}x-1)$
25. $a^2b(a^2-3ab+b^2)$
26. $6a^2bc^3(2a-3b+c^2)$
27. $\frac{2}{3}xy^2(9x^3y-27xy+3xy^3)$
28. $-3x^2y^2(12x^2-10xy-6y^2)$
29. Find the product of $3x$ and $2x^2-3x+5$.
30. Find the product of $-8y$ and $y^2-2y+12$.
31. Find the product of $-4x$ and $x^4-3x^3+2x^2-7x+8$.
32. Find the product of $3xy^2$ and $-2x^2y+4xy-xy^2$.

Answer

1. $-15x^3y$
3. $2x-\frac{3}{2}$
5. $15x^2-6x$
7. $3x^3+2x^2$
9. $8a^2b-4ab^2$
11. $-18x^5y^4+6x^3y^5$

13. $-2x^3y^2+5x^2y$
15. $-30x^9$
17. $-10x^2+6x-8$
19. $15x^3-6x^2+9x$
21. $3x^4-5x^3-7x^2$
23. $2x^7-\frac{1}{2}x^6+\frac{1}{8}x^5-\frac{5}{4}x^4$
25. $a^4b-3a^3b^2+a^2b^3$
27. $6x^4y^3-18x^2y^3+2x^2y^5$
29. $6x^3-9x^2+15x$
31. $-4x^5+12x^4-8x^3+28x^2-32x$

? Exercise 6.4.6 Product of a Binomial and a Polynomial

Multiply.

1. $(3x-2)(x+4)$
2. $(x+2)(x-3)$
3. $(x-1)(x+1)$
4. $(3x-1)(3x+1)$
5. $(2x-5)(x+3)$
6. $(5x-2)(3x+4)$
7. $(-3x+1)(x-1)$
8. $(x+5)(-x+1)$
9. $(y-\frac{2}{3})(y+\frac{2}{3})$
10. $(\frac{1}{2}x+\frac{1}{3})(\frac{3}{2}x-\frac{2}{3})$
11. $(\frac{3}{4}x+\frac{1}{5})(\frac{1}{4}x+\frac{2}{5})$
12. $(\frac{1}{5}x+\frac{3}{10})(\frac{3}{5}x-\frac{5}{2})$
13. $(y^2-2)(y+2)$
14. $(y^3-1)(y^2+2)$
15. $(a^2-b^2)(a^2+b^2)$
16. $(a^2-3b)^2$
17. $(x-5)(2x^2+3x+4)$
18. $(3x-1)(x^2-4x+7)$
19. $(2x-3)(4x^2+6x+9)$
20. $(5x+1)(25x^2-5x+1)$
21. $(x-\frac{1}{2})(3x^2+4x-1)$
22. $(\frac{1}{3}x-\frac{1}{4})(3x^2+9x-3)$
23. $(x+3)^3$
24. $(x-2)^3$
25. $(3x-1)^3$
26. $(2x+y)^3$
27. $(5x-2)(2x^3-4x^2+3x-2)$
28. $(x^2-2)(x^3-2x^2+x+1)$

Answer

1. $3x^2+10x-8$
3. x^2-1
5. $2x^2+x-15$

7. $-3x^2+4x-1$
9. $y^2-\frac{4}{9}$
11. $\frac{3}{16}x^2+\frac{7}{20}x+\frac{2}{25}$
13. y^3+2y^2-2y-4
15. a^4-b^4
17. $2x^3-7x^2-11x-20$
19. $8x^3-27$
21. $3x^3+\frac{5}{2}x^2-3x+12$
23. $x^3+9x^2+27x+27$
25. $27x^3-27x^2+9x-1$
27. $10x^4-24x^3+23x^2-16x+4$

Exercise 6.4.7 Product of Polynomials

Multiply.

1. $(x^2-x+1)(x^2+2x+1)$
2. $(3x^2-2x-1)(2x^2+3x-4)$
3. $(2x^2-3x+5)(x^2+5x-1)$
4. $(a+b+c)(a-b-c)$
5. $(a+2b-c)^2$
6. $(x+y+z)^2$
7. $(x-3)^4$
8. $(x+y)^4$
9. Find the volume of a rectangular solid with sides measuring x, $x+2$, and $x+4$ units.
10. Find the volume of a cube where each side measures $x-5$ units.

Answer

1. x^4+x^3+x+1
3. $2x^4+7x^3-12x^2+28x-5$
5. $a^2+4ab-2ac+4b^2-4bc+c^2$
7. $x^4-12x^3+54x^2-108x+81$
9. x^3+6x^2+8x

Exercise 6.4.8 Special Products

Multiply.

1. $(x+2)^2$
2. $(x-3)^2$
3. $(2x+5)^2$
4. $(3x-7)^2$
5. $(-x+2)^2$
6. $(-9x+1)^2$
7. $(a+6)^2$
8. $(2a-3b)^2$
9. $(\frac{2}{3}x+\frac{3}{4})^2$

10. $(\frac{1}{2}x-\frac{3}{5})^2$
11. $(x^2+2)^2$
12. $(x^2+y^2)^2$
13. $(x+4)(x-4)$
14. $(2x+1)(2x-1)$
15. $(5x+3)(5x-3)$
16. $(\frac{1}{5}x-\frac{1}{3})(\frac{1}{5}x+\frac{1}{3})$
17. $(\frac{3}{2}x+\frac{2}{5})(\frac{3}{2}x-\frac{2}{5})$
18. $(2x-3y)(2x+3y)$
19. $(4x-y)(4x+y)$
20. $(a^3-b^3)(a^3+b^3)$
21. A box is made by cutting out the corners and folding up the edges of a square piece of cardboard. A template for a cardboard box with a height of 2 inches is given. Find a formula for the volume, if the initial piece of cardboard is a square with sides measuring x inches.

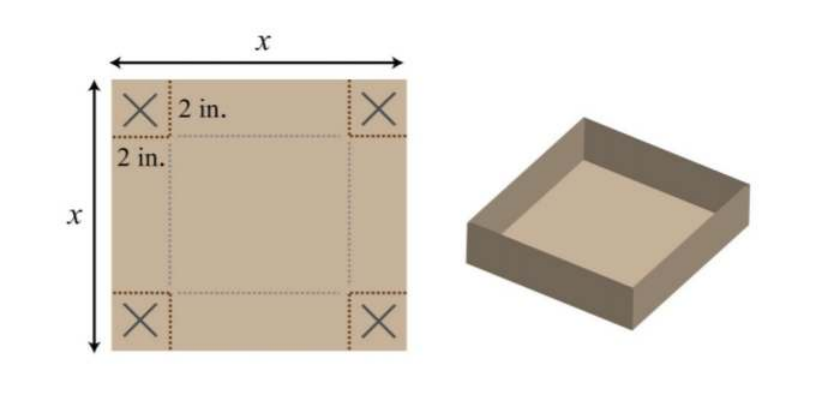


***Figure* 6.4.5**

22. A template for a cardboard box with a height of x inches is given. Find a formula for the volume, if the initial piece of cardboard is a square with sides measuring 12 inches.

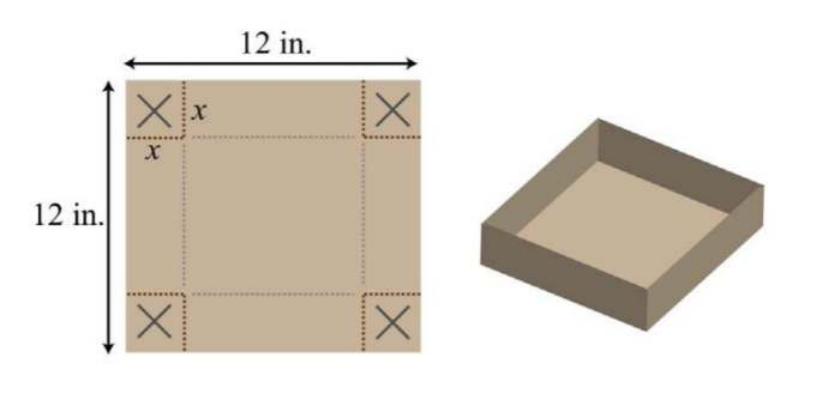


***Figure* 6.4.6**

Answer

1. x^2+4x+4
3. $4x^2+20x+25$
5. x^2-4x+4
7. $a^2+12a+36$
9. $\frac{4}{9}x^2+x+\frac{9}{16}$
11. x^4+4x^2+4

13. $x^2 - 16$
15. $25x^2 - 9$
17. $\frac{9}{4}x^2 - \frac{4}{25}$
19. $16x^2 - y^2$
21. $V = 2x^2 - 16x + 32$ cubic inches

Exercise 6.4.9 Multiplying Polynomial Functions

For each problem, calculate $(f \cdot g)(x)$, given the functions.

1. $f(x) = 8x$ and $g(x) = 3x - 5$
2. $f(x) = x^2$ and $g(x) = -5x + 1$
3. $f(x) = x - 7$ and $g(x) = 6x - 1$
4. $f(x) = 5x + 3$ and $g(x) = x^2 + 2x - 3$
5. $f(x) = x^2 + 6x - 3$ and $g(x) = 2x^2 - 3x + 5$
6. $f(x) = 3x^2 - x + 1$ and $g(x) = -x^2 + 2x - 1$

Answer

1. $(f \cdot g)(x) = 24x^2 - 40x$
3. $(f \cdot g)(x) = 6x^2 - 43x + 7$
5. $(f \cdot g)(x) = 2x^4 + 9x^3 - 19x^2 + 39x - 15$

Exercise 6.4.10 Multiplying Polynomial Functions

Given $f(x) = 2x - 3$ and $g(x) = 3x - 1$, find the following

1. $(f \cdot g)(x)$
2. $(g \cdot f)(x)$
3. $(f \cdot g)(0)$
4. $(f \cdot g)(-1)$
5. $(f \cdot g)(1)$
6. $(f \cdot g)(\frac{1}{2})$

Answer

1. $(f \cdot g)(x) = 6x^2 - 11x + 3$
3. $(f \cdot g)(0) = 3$
5. $(f \cdot g)(1) = -2$

Exercise 6.4.11 Multiplying Polynomial Functions

Given $f(x) = 5x - 1$ and $g(x) = 2x^2 - 4x + 5$, find the following.

1. $(f \cdot g)(x)$
2. $(g \cdot f)(x)$
3. $(f \cdot g)(0)$
4. $(f \cdot g)(-1)$
5. $(f \cdot g)(1)$
6. $(f \cdot g)(\frac{1}{2})$
7. $(f \cdot f)(x)$
8. $(g \cdot g)(x)$

Answer

1. $(f \cdot g)(x) = 10x^3 - 22x^2 + 29x - 5$
3. $(f \cdot g)(0) = -5$
5. $(f \cdot g)(1) = 12$
7. $(f \cdot f)(x) = 25x^2 - 10x + 1$

Exercise 6.4.12 Discussion Board Topics

1. Explain why $(x + y)^2 \neq x^2 + y2$.
2. Explain how to quickly multiply a binomial with its conjugate. Give an example.
3. What are the advantages and disadvantages of using the mnemonic device FOIL?

Answer

1. Answers may vary
3. Answers may vary

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6.5: Dividing Polynomials

Learning Objectives

- Divide by a monomial.
- Divide by a polynomial using the division algorithm.
- Divide polynomial functions.

Dividing by a Polynomial

Recall the quotient rule for exponents: if x is nonzero and m and n are positive integers, then

$$\frac{x^m}{x^n} = x^{m-n} \tag{6.5.1}$$

In other words, when dividing two expressions with the same base, subtract the exponents. This rule applies when dividing a monomial by a monomial. In this section, we will assume that all variables in the denominator are nonzero.

Example 6.5.1

Divide:

$$\frac{28y^3}{7y}.$$

Solution:

Divide the coefficients and subtract the exponents of the variable y.

$$\begin{aligned}\frac{28y^3}{7y} &= \frac{28}{7}y^{3-1} \\ &= 4y^2\end{aligned}$$

Answer:

$$4y^2$$

Example 6.5.2

Divide:

$$\frac{24x^7y^5}{8x^3y^2}.$$

Solution:

Divide the coefficients and apply the quotient rule by subtracting the exponents of the like bases.

$$\begin{aligned}\frac{24x^7y^5}{8x^3y^2} &= \frac{24}{8}x^{7-3}y^{5-2} \\ &= 3x^4y^3\end{aligned}$$

Answer:

$$3x^4y^3$$

When dividing a polynomial by a monomial, we may treat the monomial as a common denominator and break up the fraction using the following property:

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c} \tag{6.5.2}$$

Applying this property results in terms that can be treated as quotients of monomials.

Example 6.5.3

Divide:

$$\frac{-5x^4+25x^3-15x^2}{5x^2}.$$

Solution:

Break up the fraction by dividing each term in the numerator by the monomial in the denominator and then simplify each term.

Answer:

$-x^2+5x-3\cdot 1$

Check your division by multiplying the answer, the quotient, by the monomial in the denominator, the divisor, to see if you obtain the original numerator, the dividend.

$$\frac{dividend}{divisor}=quotient \tag{6.5.3}$$

or

$$dividend=divisor\cdot quotient \tag{6.5.4}$$

$$\begin{aligned} 5x^2\cdot(-x^2+5x-3) &= 5x^2\cdot(-x^2)+5x^2\cdot 5x-5x^2\cdot 3 \\ &= -5x^4+25x^3-15x^2 \quad \checkmark \end{aligned}$$

Example 6.5.4

Divide:

$$\frac{9a^4b-7a^3b^2+3a^2b}{-3a^2b}.$$

Solution:

Answer:

$-3a^2+\frac{7}{3}ab-1$. The check is optional and is left to the reader.

Exercise 6.5.1

$(16x^5-8x^4+5x^3+2x^2)\div(2x^2)$.

Answer

$8x^3-4x^2+\frac{5}{2}x+1$

Dividing by a Polynomial

The same technique outlined for dividing by a monomial ***does not*** work for polynomials with two or more terms in the denominator. In this section, we will outline a process called polynomial long division, which is based on the division algorithm for real numbers. For the sake of clarity, we will assume that all expressions in the denominator are nonzero.

Example 6.5.5

Divide:

$$\frac{x^3+3x^2-8x-4}{x-2}.$$

Solution:

Here $x-2$ is the divisor and x^3+3x^2-8x-4 is the dividend.

Step 1: To determine the first term of the quotient, divide the leading term of the dividend by the leading term of the divisor.

$$x-2\overline{)\,x^3+3x^2-8x-4}\qquad \begin{array}{l}\textit{Begin by dividing the leading terms.}\\ x^3\div x=x^2\end{array}$$

Figure 6.5.1

Step 2: Multiply the first term of the quotient by the divisor, remembering to distribute, and line up like terms with the dividend.

$$\begin{array}{r} x^2 \\ x-2\overline{)\,x^3+3x^2-8x-4} \\ \underline{x^3-2x^2} \end{array}\qquad \begin{array}{l}\textit{Distribute and line up like terms.}\\ x^2\cdot(x-2)=x^3-2x^2\end{array}$$

Figure 6.5.2

Step 3: Subtract the resulting quantity from the dividend. Take care to subtract both terms.

$$\begin{array}{r} x^2 \\ x-2\overline{)\,x^3+3x^2-8x-4} \\ \underline{-(x^3-2x^2)} \\ 5x^2 \end{array}\qquad \begin{array}{l}\textit{Subtract.}\\ -(x^3-2x^2)=-x^3+2x^2\end{array}$$

Figure 6.5.3

Step 4: Bring down the remaining terms and repeat the process from step 1.

$$\begin{array}{r} x^2\quad ? \\ x-2\overline{)\,x^3+3x^2-8x-4} \\ \underline{-(x^3-2x^2)} \\ 5x^2-8x-4 \end{array}\qquad \textit{Bring down the remaining terms.}$$

Figure 6.5.4

Notice that the leading term is eliminated and that the result has a degree that is one less than the dividend. The complete process is illustrated below:

$$
\begin{array}{r}
9x^2-12x+16 \\
3x+4\overline{)\ 27x^3+0x^2+0x+64} \\
-(27x^3+36x^2) \\
\hline
-36x^2+\ 0x+64 \\
-(-36x^2-48x) \\
\hline
48x+64 \\
-(48x+64) \\
\hline
0
\end{array}
$$

Figure 6.5.5

Polynomial long division ends when the degree of the remainder is less than the degree of the divisor. Here the remainder is 0. Therefore, the binomial divides the polynomial evenly and the answer is the quotient shown above the division line.

$$\frac{x^3+3x^2-8x-4}{x-2} = x^2+5x+2$$

To check the answer, multiply the divisor by the quotient to see if you obtain the dividend:

$$x^3+3x^2-8x-4 = (x-2)(x^2+5x+2)$$

Answer:

$$x^2+5x+2$$

Next, we demonstrate the case where there is a nonzero remainder.

$$
\begin{array}{r}
quotient \\
divisor\overline{)dividend} \\
\vdots \\
\hline
remainder
\end{array}
$$

Figure 6.5.6

Just as with real numbers, the final answer adds the fraction where the remainder is the numerator and the divisor is the denominator to the quotient. In general, when dividing we have

$$\frac{dividend}{divisor} = quotient + \frac{remainder}{divisor} \tag{6.5.5}$$

If we multiply both sides by the divisor we obtain

$$dividend = quotient \times divisor + remainder \tag{6.5.6}$$

Example 6.5.6

Divide:

$$\frac{6x^2-5x+3}{2x-1}$$

Solution:

Since the denominator is a binomial, begin by setting up polynomial long division.

$$2x-1\overline{)\,6x^2-5x+3}$$

Figure 6.5.7

To start, determine what monomial times $2x-1$ results in a leading term $6x^2$. This is the quotient of the given leading terms: $(6x^2)\div(2x)=3x$. Multiply $3x$ times the divisor $2x-1$ and line up the result with like terms of the dividend.

$$\begin{array}{r} 3x \\ 2x-1\overline{)\,6x^2-5x+3} \\ 6x^2-3x \end{array}$$

Figure 6.5.8

Subtract the result from the dividend and bring down the constant term $+3$.

$$\begin{array}{r} 3x\ \ ? \\ 2x-1\overline{)\,6x^2-5x+3} \\ -(6x^2-3x) \\ \hline -2x+3 \end{array}$$

Figure 6.5.9

Subtracting eliminates the leading term and $-5x-(-3x)=-5x+3x=-2x$. The quotient of $-2x$ and $2x$ is -1. Multiply $2x-1$ by -1 and line up the result.

$$\begin{array}{r} 3x-1 \\ 2x-1\overline{)\,6x^2-5x+3} \\ -(6x^2-3x) \\ \hline -2x+3 \\ -2x+1 \end{array}$$

Figure 6.5.10

Subtract again and notice that we are left with a remainder.

$$\begin{array}{r} 3x-1 \\ 2x-1\overline{)\,6x^2-5x+3} \\ -(6x^2-3x) \\ \hline -2x+3 \\ -(-2x+1) \\ \hline 2 \end{array}$$

Figure 6.5.11

The constant term 2 has degree 0, and thus the division ends. We may write

$$\frac{6x^2-5x+3}{2x-1}=3x-1+\frac{2}{2x-1}$$

Answer:

$3x-1+\frac{2}{2x-1}$. To check that this result is correct, we multiply as follows:

$$\begin{aligned} quotient \times divisor + remainder &= (3x-1)(2x-1)+2 \\ &= 6x^2-3x-2x+1+2 \\ &= 6x^2-5x+3 = dividend \quad \checkmark \end{aligned}$$

Occasionally, some of the powers of the variables appear to be missing within a polynomial. This can lead to errors when lining up like terms. Therefore, when first learning how to divide polynomials using long division, fill in the missing terms with zero coefficients, called placeholders.

Example 6.5.7

Divide:

$$\frac{27x^3+64}{3x+4}.$$

Solution:

Notice that the binomial in the numerator does not have terms with degree 2 or 1. The division is simplified if we rewrite the expression with placeholders:

$$27x^3+64 = 27x^3+0x^2+0x+64$$

Set up polynomial long division:

$$3x+4\overline{)\,27x^3+0x^2+0x+64}$$

Figure 6.5.12

We begin with $27x^3 \div 3x = 9x^2$ and work the rest of the division algorithm.

$$\begin{array}{r} 9x^2-12x+16 \\ 3x+4\overline{)\;27x^3+0x^2+0x+64} \\ \underline{-\left(27x^3+36x^2\right)} \\ -36x^2+\;\;0x+64 \\ \underline{-\left(-36x^2-48x\right)} \\ 48x+64 \\ \underline{-\left(48x+64\right)} \\ 0 \end{array}$$

Figure 6.5.13

Answer:

$$9x^2-12x+16$$

Example 6.5.8

Divide:

$$\frac{3x^4-2x^3+6x^2+23x-7}{x^2-2x+5}.$$

Solution:

$$x^2-2x+5\overline{)3x^4-2x^3+6x^2+23x-7}$$

Figure 6.5.14

Begin the process by dividing the leading terms to determine the leading term of the quotient $3x^4 \div x^2 = 3x^2$. Take care to distribute and line up the like terms. Continue the process until the remainder has a degree less than 2.

$$\begin{array}{r} 3x^2+4x-1 \\ x^2-2x+5\overline{)3x^4-2x^3+6x^2+23x-7} \\ \underline{-(3x^4-6x^3+15x^2)} \\ 4x^3-9x^2+23x-7 \\ \underline{-(4x^3-8x^2+20x)} \\ -x^2+3x-7 \\ \underline{-(-x^2+2x-5)} \\ x-2 \end{array}$$

Figure 6.5.15

The remainder is $x-2$. Write the answer with the remainder:

$$\frac{3x^4-2x^3+6x^2+23x-7}{x^2-2x+5} = 3x^2+4x-1+\frac{x-2}{x^2-2x+5}$$

Answer:

$$3x^2+4x-1+\frac{x-2}{x^2-2x+5}$$

Polynomial long division takes time and practice to master. Work lots of problems and remember that you may check your answers by multiplying the quotient by the divisor (and adding the remainder if present) to obtain the dividend.

? Exercise 6.5.2

Divide:

$$\frac{20x^4-32x^3+7x^2+8x-10}{5x-3}.$$

Answer

$4x^3-4x^2-x+1-\frac{7}{5}x-3$

Dividing Polynomial Functions

We may use function notation to indicate division as follows:

Table 6.5.1

Division of functions:	$(f/g)(x) = \frac{f(x)}{g(x)}$

The quotient of two polynomial functions does not necessarily have a domain of all real numbers. The values for x that make the function in the denominator 0 are restricted from the domain. This will be discussed in more detail at a later time. For now, assume all functions in the denominator are nonzero.

Example 6.5.9

Calculate:

$$(f/g)(x) \text{ given } f(x) = 6x^5 - 36x^4 + 12x^3 - 6x^2 \text{ and } g(x) = -6x^2 .$$

Solution:

The notation indicates that we should divide:

$$\begin{aligned}(f/g)(x) &= \frac{f(x)}{g(x)} \\ &= \frac{6x^5 - 36x^4 + 12x^3 - 6x^2}{-6x^2} \\ &= \frac{6x^5}{-6x^2} - \frac{36x^4}{-6x^2} + \frac{12x^3}{-6x^2} - \frac{6x^2}{-6x^2} \\ &= -1x^{5-2} + 6x^{4-2} - 2x^{3-2} + 1x^{2-2} \\ &= -x^3 + 6x^2 - 2x + 1\end{aligned}$$

Answer:

$$(f/g)(x) = -x^3 + 6x^2 - 2x + 1$$

Example 6.5.10

Calculate:

$$(f/g)(-1), \text{ given } f(x) = -3x^3 + 7x^2 - 11x - 1 \text{ and } g(x) = 3x - 1 .$$

Solution:

First, determine $(f/g)(x)$.

$$(f/g)(x) = \tfrac{f(x)}{g(x)} = \tfrac{-3x^3+7x^2-11x-1}{3x-1}$$

$$\begin{array}{r} -x^2 + 2x - 3 \\ 3x-1\overline{)\,-3x^3 + 7x^2 - 11x - 1} \\ \underline{-(-3x^3 + x^2)} \\ 6x^2 - 11x - 1 \\ \underline{-(6x^2 - 2x)} \\ -9x - 1 \\ \underline{-(-9x + 3)} \\ -4 \end{array}$$

Figure 6.5.16

Therefore,

$$(f/g)(x) = -x^2 + 2x - 3 - \tfrac{4}{3x-1}$$

Substitute -1 for the variable x.

Answer:

$(f/g)(-1) = -5$

Key Takeaways

- When dividing by a monomial, divide all terms in the numerator by the monomial and then simplify each term. To simplify each term, divide the coefficients and apply the quotient rule for exponents.
- When dividing a polynomial by another polynomial, apply the division algorithm.
- To check the answer after dividing, multiply the divisor by the quotient and add the remainder (if necessary) to obtain the dividend.
- It is a good practice to include placeholders when performing polynomial long division.

? Exercise 6.5.3 Dividing by a Monomial

Divide.

1. $\frac{81y^5}{9y^2}$
2. $\frac{36y^9}{9y^3}$
3. $\frac{52x^2y}{4xy}$
4. $\frac{24xy^5}{2xy^4}$
5. $\frac{25x^2y^5z^3}{5xyz}$
6. $\frac{-77x^4y^9z^2}{2x^3y^3z}$
7. $\frac{125a^3b^2c}{-10abc}$
8. $\frac{36a^2b^3c^5}{-6a^2b^2c^3}$
9. $\frac{9x^2+27x-3}{3}$
10. $\frac{10x^3-5x^2+40x-15}{5}$
11. $\frac{20x^3-10x^2+30x}{2x}$
12. $\frac{10x^4+8x^2-6x}{24x}$
13. $\frac{-6x^5-9x^3+3x}{-3x}$
14. $\frac{36a^{12}-6a^9+12a^5}{-12a^5}$
15. $\frac{-12x^5+18x^3-6x^2}{-6x^2}$
16. $\frac{-49a^8+7a^5-21a^3}{7a^3}$
17. $\frac{9x^7-6x^4+12x^3-x^2}{3x^2}$
18. $\frac{8x^9+16x^7-24x^4+8x^3}{-8x^3}$
19. $\frac{16a^7-32a^6+20a^5-a^4}{4a^4}$
20. $\frac{5a^6+2a^5+6a^3-12a^2}{3a^2}$
21. $\frac{-4x^2y^3+16x^7y^8-8x^2y^5}{-4x^2y^3}$
22. $\frac{100a^{10}b^{30}c^5-50a^{20}b^5c^{40}+20a^5b^{20}c^{10}}{10a^5b^5c^5}$
23. Find the quotient of $-36x^9y^7$ and $2x^8y^5$.
24. Find the quotient of $144x^3y^{10}z^2$ and $-12x^3y^5z$.
25. Find the quotient of $3a^4-18a^3+27a^2$ and $3a^2$.
26. Find the quotient of $64a^2bc^3-16a^5bc^7$ and $4a^2bc^3$.

Answer

1. $9y^3$

3. $13x$

5. $5xy^4z^2$

7. $-\frac{25}{2}a^2b$

9. $3x^2+9x-1$

11. $10x^2-5x+15$

13. $2x^4+3x^2-1$

15. $2x^3-3x+1$

17. $3x^5-2x^2+4x-\frac{1}{3}$

19. $4a^3-8a^2+5a-\frac{1}{4}$

21. $-4x^5y^5+2y^2+1$

23. $-18xy^2$

25. a^2-6a+9

? Exercise 6.5.4 Dividing by a Polynomial

Divide.

1. $(2x^2-5x-3)\div(x-3)$
2. $(3x^2+5x-2)\div(x+2)$
3. $(6x^2+11x+3)\div(3x+1)$
4. $(8x^2-14x+3)\div(2x-3)$
5. $\frac{x^3-x^2-2x-12}{x-3}$
6. $\frac{2x^3+11x^2+4x-5}{x+5}$
7. $\frac{2x^3-x^2-4x+3}{2x+3}$
8. $\frac{-15x^3-14x^2+23x-6}{5x-2}$
9. $\frac{14x^4-9x^3+22x^2+4x-1}{7x-1}$
10. $\frac{8x^5+16x^4-8x^3-5x^2-21x+10}{2x+5}$
11. $\frac{x^2+8x+17}{x+5}$
12. $\frac{2x^2-5x+5}{x-2}$
13. $\frac{6x^2-13x+9}{-2x+1}$
14. $\frac{-12x^2+x+1}{3x+2}$
15. $\frac{x^3+9x^2+19x+1}{x+4}$
16. $\frac{2x^3-13x^2+17x-11}{x-5}$
17. $\frac{9x^3-12x^2+16x-15}{3x-2}$
18. $\frac{3x^4-8x^3+5x^2-5x+9}{x-2}$
19. $(6x^5-13x^4+4x^3-3x^2+13x-2)\div(3x+1)$
20. $(8x^5-22x^4+19x^3-20x^2+23x-3)\div(2x-3)$
21. $\frac{5x^5+12x^4+12x^3-7x^2-19x+3}{x^2+2x+3}$
22. $\frac{6x^5-17x^4+5x^3+16x^2-7x-3}{2x^2-3x-1}$
23. $\frac{x^5+7x^4-x^3-7x^2-49x+9}{x^2+7x-1}$
24. $\frac{5x^6-6x^4-4x^2+x+2}{5x^2-1}$
25. $\frac{x^3-27}{x-3}$
26. $\frac{8x^3+125}{2x+5}$
27. $(15x^5-9x^4-20x^3+12x^2+15x-9)\div(5x-3)$

28. $(2x^6-5x^5-4x^4+10x^3+6x^2-17x+5)\div(2x-5)$
29. $\frac{x^5-2x^3+3x-1}{x-1}$
30. $\frac{x^4-3x^2+5x-1}{3x+2}$
31. $\frac{a^2-4}{a+2}$
32. $\frac{a^5+1}{a^5+1}$
33. $\frac{a^6-1}{a-1}$
34. $\frac{x^5-1}{x-1}$
35. $\frac{x^5+x^4+6x^3+12x^2-4}{x^2+x-1}$
36. $\frac{50x^6-30x^5-5x^4+15x^3-5x+1}{5x^2-3x+2}$
37. $\frac{5x^5-15x^3+25x^2-5}{5x}$
38. $\frac{-36x^6+12x^4-6x^2}{6x^2}$
39. $\frac{150x^5y^2z^{15}-10x^3y^6z^5+4x^3y^2z^4}{10x^3y^2z^5}$
40. $\frac{27m^6+9m^4-81m^2+1}{9m^2}$
41. Divide $3x^6-2x^5+27x^4-18x^3-6x^2+7x-10$ by $3x-2$.
42. Divide $8x^6+4x^5-14x^4-5x^3+x^2-2x-3$ by $2x+1$.

Answer

1. $2x+1$
3. $2x+3$
5. x^2+2x+4
7. x^2-2x+1
9. $2x^3-x^2+3x+1$
11. $x+3+\frac{2}{x+5}$
13. $-3x+5+\frac{4}{-2x+1}$
15. $x^2+5x-1+\frac{5}{x+4}$
17. $3x^2-2x+4-\frac{7}{3x-2}$
19. $2x^4-5x^3+3x^2-2x+5-\frac{7}{3x+1}$
21. $5x^3+2x^2-7x+1$
23. $x^3-7+\frac{2}{x^2+7x-1}$
25. x^2+3x+9
27. $3x^4-4x^2+3$
29. $x^4+x^3-x^2-x+2+\frac{1}{x-1}$
31. $a-2$
33. $a^5+a^4+a^3+a^2+a+1$
35. $x^3+7x+5+\frac{2x+1}{x^2+x-1}$
37. $x^4-3x^2+5x-\frac{1}{x}$
39. $15x^2z^{10}-y^4+\frac{2}{5z}$
41. $x^5+9x^3-2x+1-\frac{8}{3x-2}$

Exercise 6.5.5 Dividing Polynomial Functions

Calculate $(f/g)(x)$, given the functions.

1. $f(x)=40x^8$ and $g(x)=10x^5$
2. $f(x)=54x^5$ and $g(x)=9x^3$
3. $f(x)=12x^2+24x-15$ and $g(x)=2x+5$
4. $f(x)=-8x^2+30x-7$ and $g(x)=2x-7$
5. $f(x)=18x^2-36x+5$ and $g(x)=3x-5$
6. $f(x)=-7x^2+29x-6$ and $g(x)=7x-1$
7. $f(x)=10x^3-9x^2+27x-10$ and $g(x)=5x-2$
8. $f(x)=15x^3+28x^2-11x+56$ and $g(x)=3x+8$
9. $f(x)=2x^4+5x^3-11x^2-19x+20$ and $g(x)=x^2+x-5$
10. $f(x)=4x^4-12x^3-20x^2+26x-3$ and $g(x)=2x^2+2x-3$

Answer

1. $(f/g)(x)=4x^3$
3. $(f/g)(x)=6x-3$
5. $(f/g)(x)=6x-2-\frac{5}{3x-5}$
7. $(f/g)(x)=2x^2-x+5$
9. $(f/g)(x)=2x^2+3x-4$

Exercise 6.5.6 Dividing Polynomial Functions

Given $f(x)=6x^3+4x^2-11x+3$ and $g(x)=3x-1$, find the following.

1. $(f/g)(x)$
2. $(f/g)(-1)$
3. $(f/g)(0)$
4. $(f/g)(1)$

Answer

1. $(f/g)(x)=2x^2+2x-3$
3. $(f/g)(0)=-3$

Exercise 6.5.7 Dividing Polynomial Functions

Given $f(x)=5x^3-13x^2+7x+3$ and $g(x)=x-2$, find the following.

1. $(f/g)(x)$
2. $(f/g)(-3)$
3. $(f/g)(0)$
4. $(f/g)(7)$

Answer

1. $(f/g)(x)=5x^2-3x+1+\frac{5}{x-2}$
3. $(f/g)(0)=-\frac{3}{2}$

Exercise 6.5.8 Discussion Board Topics

1. How do you use the distributive property when dividing a polynomial by a monomial?
2. Compare long division of real numbers with polynomial long division. Provide an example of each.

Answer

1. Answers may vary

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6.6: Prelude to Solving Linear Equations

Imagine being a pilot, but not just any pilot—a drone pilot. Drones, or unmanned aerial vehicles, are devices that can be flown remotely. They contain sensors that can relay information to a command center where the pilot is located. Larger drones can also carry cargo. In the near future, several companies hope to use drones to deliver materials and piloting a drone will become an important career. Law enforcement and the military are using drones rather than send personnel into dangerous situations.



Figure 6.6.1: This drone is flying high in the sky while its pilot remains safely on the ground. (credit: “Unsplash” / Pixabay)

Building and piloting a drone requires the ability to program a set of actions, including taking off, turning, and landing. This, in turn, requires the use of linear equations. In this chapter, you will explore linear equations, develop a strategy for solving them, and relate them to real-world situations.

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6.7: Use a General Strategy to Solve Linear Equations

Summary

By the end of this section, you will be able to:

- Use the commutative and associative properties
- Use the properties of identity, inverse, and zero
- Simplify expressions using the Distributive Property

Before you get started, take this readiness quiz.

1. Simplify: $\frac{3}{2}(12x+20)$.
2. Simplify: $5-2(n+1)$.
3. Find the LCD of $\frac{5}{6}$ and $\frac{1}{4}$.

Solve Linear Equations Using a General Strategy

Solving an equation is like discovering the answer to a puzzle. The purpose in solving an equation is to find the value or values of the variable that makes it a true statement. Any value of the variable that makes the equation true is called a **solution** to the equation. It is the answer to the puzzle!

Solution OF AN EQUATION

A **solution** of an equation is a value of a variable that makes a true statement when substituted into the equation.

To determine whether a number is a solution to an equation, we substitute the value for the variable in the equation. If the resulting equation is a true statement, then the number is a solution of the equation.

DETERMINE WHETHER A NUMBER IS A Solution TO AN EQUATION

1. Substitute the number for the variable in the equation.
2. Simplify the expressions on both sides of the equation.
3. Determine whether the resulting equation is true.
   - If it is true, the number is a solution.
   - If it is not true, the number is not a solution.

Example 6.7.1

Determine whether the values are solutions to the equation: $5y+3=10y-4$.

a. $y=\frac{3}{5}$
b. $y=\frac{7}{5}$

Solution

Since a solution to an equation is a value of the variable that makes the equation true, begin by substituting the value of the solution for the variable.

a.

	$5y+3=10y-4$
Substitute $\frac{3}{5}$ for y	$5\left(\frac{3}{5}\right)+3\stackrel{?}{=}10\left(\frac{3}{5}\right)-4$
Multiply.	$3+3\stackrel{?}{=}6-4$
Simplify.	$6\neq 2$

Since $y=\frac{3}{5}$ does not result in a true equation, $y=\frac{3}{5}$ is not a solution to the equation $5y+3=10y-4$.

b.

	$5y+3=10y-4$
Substitute $\frac{7}{5}$ for y	$5\left(\frac{7}{5}\right)+3\stackrel{?}{=}10\left(\frac{7}{5}\right)-4$
Multiply.	$7+3\stackrel{?}{=}14-4$
Simplify.	$10=10\checkmark$

Since $y=\frac{7}{5}$ results in a true equation, $y=\frac{7}{5}$ is a solution to the equation $5y+3=10y-4$.

Exercise 6.7.1A

Determine whether the values are solutions to the equation: $9y+2=6y+3$.

a. $y=\frac{4}{3}$
b. $y=\frac{1}{3}$

Answer a

no

Answer b

yes

Exercise 6.7.1B

Determine whether the values are solutions to the equation: $4x-2=2x+1$.

a. $x=\frac{3}{2}$
b. $x=-\frac{1}{2}$

Answer a

yes

Answer b

no

There are many types of equations that we will learn to solve. In this section we will focus on a **linear equation.**

LINEAR EQUATION

A **linear equation** is an equation in one variable that can be written, where a and b are real numbers and $a\neq 0$, as:

$$ax+b=0 \tag{6.7.1}$$

To solve a linear equation it is a good idea to have an overall strategy that can be used to solve any linear equation. In the next example, we will give the steps of a general strategy for solving any linear equation. Simplifying each side of the equation as much as possible first makes the rest of the steps easier.

EXAMPLE 6.7.2

Solve: $7(n-3)-8=-15$

Answer

Step 1. Simplify each side of the equation as much as possible.	Use the Distributive Property. Notice that each side of the equation is now simplified as much as possible.	$7(n-3)-8=-15$ $7n-21-8=-15$ $7n-29=-15$
Step 2. Collect all variable terms on one side of the equation.	Nothing to do—all n's are on the left side.	
Step 3. Collect constant terms on the other side of the equation.	To get constants only on the right, add 29 to each side. Simplify.	$7n-29+29=-15+29$ $7n=14$
Step 4. Make the coefficient of the variable term equal to 1.	Divide each side by 7. Simplify.	$\frac{7n}{7}=\frac{14}{7}$ $n=2$
Step 5. Check the solution.	Let $n=2$ Subtract.	**Check:** $7(n-3)-8=-15$ $7(2-3)-8\stackrel{?}{=}-15$ $7(-1)-8\stackrel{?}{=}-15$ $-7-8\stackrel{?}{=}-15$ $-15=-15$ ✓

Solve: $2(m-4)+3=-1$.

Answer

$m=2$

Exercise $6.7.2B$

Solve: $5(a-3)+5=-10$.

Answer

$a=0$

These steps are summarized in the **General Strategy for Solving Linear Equations** below.

SOLVE LINEAR EQUATIONS USING A GENERAL STRATEGY

1. Simplify each side of the equation as much as possible.
   Use the Distributive Property to remove any parentheses.
   Combine like terms.
2. Collect all the variable terms on one side of the equation.
   Use the Addition or Subtraction Property of Equality.
3. Collect all the constant terms on the other side of the equation.
   Use the Addition or Subtraction Property of Equality.
4. Make the coefficient of the variable term equal to 1.
   Use the Multiplication or Division Property of Equality.
   State the solution to the equation.
5. Check the solution.
   Substitute the solution into the original equation to make sure the result is a true statement.

EXAMPLE 6.7.3

Solve: $\frac{2}{3}(3m-6)=5-m$.

Answer

	$\frac{2}{3}(3m-6)=5-m$
Distribute.	$2m-4=5-m$
Add m to both sides to get the variables only on the left.	$2m+m-4=5-m+m$
Simplify.	$3m-4=5$
Add 4 to both sides to get constants only on the right.	$3m-4+4=5+4$
Simplify.	$3m=9$
Divide both sides by three.	$\frac{3m}{3}=\frac{9}{3}$
Simplify.	$m=3$

Check:	$\frac{2}{3}(3m-6)=5-m$
Let $m=3$.	$\frac{2}{3}(3\cdot 3-6)\stackrel{?}{=}5-3$
	$\frac{2}{3}(9-6)\stackrel{?}{=}2$
	$\frac{2}{3}(3)\stackrel{?}{=}2$
	$2=2$ ✓

Exercise 6.7.3A

Solve: $\frac{1}{3}(6u+3)=7-u$.

Answer

$u=2$

Exercise 6.7.3B

Solve: $\frac{2}{3}(9x-12)=8+2x$.

Answer

$x=4$

We can solve equations by getting all the **variable** terms to either side of the **equal sign**. By collecting the variable terms on the side where the **coefficient** of the variable is larger, we avoid working with some negatives. This will be a good strategy when we solve inequalities later in this chapter. It also helps us prevent errors with negatives.

EXAMPLE 6.7.4

Solve: $4(x-1)-2=5(2x+3)+6$.

Answer

	$4(x-1)-2=5(2x+3)+6$

Distribute.	$4x-4-2=10x+15+6$
Combine like terms.	$4x-6=10x+21$
Subtract $4x$ from each side to get the variables only on the right since $10>4$.	$4x-4x-6=10x-4x+21$
Simplify.	$-6=6x+21$
Subtract 21 from each side to get the constants on left.	$-6-21=6x+21-21$
Simplify.	$-27=6x$
Divide both sides by 6.	$\frac{-27}{6}=\frac{6x}{6}$
Simplify.	$-\frac{9}{2}=x$
Check:	$4(x-1)-2=5(2x+3)+6 \quad \backslash)$
Let $x=-92$.	$4\left(-\frac{9}{2}-1\right)-2\stackrel{?}{=}5\left(2\left(-\frac{9}{2}\right)+3\right)+6$
	$4\left(-\frac{11}{2}\right)-2\stackrel{?}{=}5(-9+3)+6$
	$-22-2\stackrel{?}{=}5(-6)+6$
	$-24\stackrel{?}{=}-30+6$
	$-24=-24\checkmark$

Exercise 6.7.4A

Solve: $6(p-3)-7=5(4p+3)-12$.

Answer

$p=-2$

Exercise 6.7.4B

Solve: $8(q+1)-5=3(2q-4)-1$.

Answer

$q=-8$

EXAMPLE 6.7.5

Solve: $10[3-8(2s-5)]=15(40-5s)$.

Answer

	$10[3-8(2s-5)]=15(40-5s)$
Simplify from the innermost parentheses first.	$10[3-16s+40]=15(40-5s)$
Combine like terms in the brackets.	$10[43-16s]=15(40-5s)$
Distribute.	$430-160s=600-75s$
Add $160s$ to both sides to get the $160s$ to both sides to get the variables to the right.	$430-160s+160s=600-75s+160s$
Simplify.	$430=600+85s$

Subtract 600 from both sides to get the constants to the left.		$430 - 600 = 600 + 85s - 600$
Simplify.		$-170 = 85s$
Divide both sides by 85.		$\frac{-170}{85} = \frac{85s}{85}$
Simplify.		$-2 = s$, so $s = -2$
Check:	$10[3 - 8(2s - 5)] = 15(40 - 5s)$	
Let $s = -2$.	$10[3 - 8(2(-2) - 5)] \stackrel{?}{=} 15(40 - 5(-2))$	
	$10[3 - 8(-4 - 5)] \stackrel{?}{=} 15(40 + 10)$	
	$10[3 - 8(-9)] \stackrel{?}{=} 15(50)$	
	$10(3 + 72) \stackrel{?}{=} 750$	
	$10(75) \stackrel{?}{=} 750$	
	$750 = 750$ ✓	

Exercise 6.7.5A

Solve: $6[4 - 2(7y - 1)] = 8(13 - 8y)$.

Answer

$y = -\frac{17}{5}$

Exercise 6.7.5B

Solve: $12[1 - 5(4z - 1)] = 3(24 + 11z)$.

Answer

$z = 0$

Classify Equations

Whether or not an equation is true depends on the value of the variable. The equation $7x + 8 = -13$ is true when we replace the variable, x, with the value -3, but not true when we replace x with any other value. An equation like this is called a **conditional equation**. All the equations we have solved so far are conditional equations.

CONDITIONAL EQUATION

An equation that is true for one or more values of the variable and false for all other values of the variable is a **conditional equation**.

Now let's consider the equation $7y + 14 = 7(y + 2)$. Do you recognize that the left side and the right side are equivalent? Let's see what happens when we solve for y.

Solve:

	$7y + 14 = 7(y + 2)$
Distribute.	$7y + 14 = 7y + 14$
Subtract $7y$ to each side to get the y's to one side.	$7y - 7y + 14 = 7y - 7y + 14$

Simplify—the y's are eliminated.	$14 = 14$
	But $14 = 14$ is true.

This means that the equation $7y + 14 = 7(y + 2)$ is true for any value of y. We say the solution to the equation is all of the real numbers. An equation that is true for any value of the variable is called an **identity**.

IDENTITY

An equation that is true for any value of the variable is called an **identity**.

The solution of an identity is valid for all real numbers.

What happens when we solve the equation $-8z = -8z + 9$?

Solve:

	$-8z = -8z + 9$
Add $8z$ to both sides to leave the constant alone on the right.	$-8z + 8z = -8z + 8z + 9$
Simplify—the z's are eliminated.	$0 \neq 9$
	But $0 \neq 9$.

Solving the equation $-8z = -8z + 9$ led to the false statement $0 = 9$. The equation $-8z = -8z + 9$ will not be true for any value of z. It has no solution. An equation that has no solution, or that is false for all values of the variable, is called a contradiction.

CONTRADICTION

An equation that is false for all values of the variable is called a **contradiction**.

A contradiction has no solution.

The next few examples will ask us to classify an equation as conditional, an identity, or as a contradiction.

? EXAMPLE 6.7.6

Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution: $6(2n - 1) + 3 = 2n - 8 + 5(2n + 1)$.

Answer

	$6(2n - 1) + 3 = 2n - 8 + 5(2n + 1)$
Distribute.	$12n - 6 + 3 = 2n - 8 + 10n + 5$
Combine like terms.	$12n - 3 = 12n - 3$
Subtract $12n$ from each side to get the n's to one side.	$12n - 12n - 3 = 12n - 12n - 3$
Simplify.	$-3 = -3$
This is a true statement.	The equation is an identity.
	The solution is all real numbers.

Exercise 6.7.6A

Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution: $4+9(3x-7)=-42x-13+23(3x-2)$.

Answer

identity; all real numbers

Exercise 6.7.6B

Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution: $8(1-3x)+15(2x+7)=2(x+50)+4(x+3)+1$.

Answer

identity; all real numbers

EXAMPLE 6.7.7

Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution: $8+3(a-4)=0$.

Answer

	$8+3(a-4)=0$
Distribute.	$8+3a-12=0$
Combine like terms.	$3a-4=0$
Add 4 to both sides.	$3a-4+4=0+4$
Simplify.	$3a=4$
Divide.	$\frac{3a}{3}=\frac{4}{3}$
Simplify.	$a=\frac{4}{3}$
The equation is true when $a=\frac{4}{3}$.	This is a conditional equation.
	The solution is $a=\frac{4}{3}$.

Exercise 6.7.7A

Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution: $11(q+3)-5=19$.

Answer

conditional equation; $q=-\frac{9}{11}$

Exercise 6.7.7B

Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution: $6+14(k-8)=95$.

Answer

conditional equation; $k=\frac{201}{14}$

? EXAMPLE 6.7.8

Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution: $5m+3(9+3m)=2(7m-11)$.

Answer

	$5m+3(9+3m)=2(7m-11)$
Distribute.	$5m+27+9m=14m-22$
Combine like terms.	$14m+27=14m-22$
Subtract $14m$ from both sides.	$14m+27-14m=14m-22-14m$
Simplify.	$27\neq -22$
But $27\neq -22$.	The equation is a contradiction.
	It has no solution.

Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution: $12c+5(5+3c)=3(9c-4)$.

Answer

contradiction; no solution

? Exercise 6.7.8B

Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution: $4(7d+18)=13(3d-2)-11d$.

Answer

contradiction; no solution

We summarize the methods for classifying equations in the table.

Type of equation	What happens when you solve it?	Solution
Conditional Equation	True for one or more values of the variables and false for all other values	One or more values
Identity	True for any value of the variable	All real numbers
Contradiction	False for all values of the variable	No solution

Solve Equations with Fraction or Decimal Coefficients

We could use the General Strategy to solve the next example. This method would work fine, but many students do not feel very confident when they see all those fractions. So, we are going to show an alternate method to solve equations with fractions. This alternate method eliminates the fractions.

We will apply the Multiplication Property of Equality and multiply both sides of an equation by the **least common denominator** (LCD) of all the fractions in the equation. The result of this operation will be a new equation, equivalent to the first, but without fractions. This process is called *clearing* the equation of fractions.

To clear an equation of decimals, we think of all the decimals in their fraction form and then find the LCD of those denominators.

Exercise 6.7.9A

Solve: $\frac{1}{12}x + \frac{5}{6} = \frac{3}{4}$.

Answer

Step 1. Find the least common denominator of *all* the fractions and decimals in the equation.	What is the LCD of $\frac{1}{12}$, $\frac{5}{6}$, and $\frac{3}{4}$?	$\frac{1}{12}x + \frac{5}{6} = \frac{3}{4}$ LCD = 12
Step 2. Multiply both sides of the equation by that LCD. This clears the fractions and decimals.	Multiply both sides of the equation by the LCD, 12. Use the Distributive Property. Simplify—and notice, no more fractions!	$12\left(\frac{1}{12}x + \frac{5}{6}\right) = 12\left(\frac{3}{4}\right)$ $12 \cdot \frac{1}{12}x + 12 \cdot \frac{5}{6} = 12 \cdot \frac{3}{4}$ $x + 10 = 9$
Step 3. Solve using the General Strategy for Solving Linear Equations.	To isolate the variable term, subtract 10. Simplify.	$x + 10 - 10 = 9 - 10$ $x = -1$ **Check:** $\frac{1}{12}x + \frac{5}{6} = \frac{3}{4}$ $\frac{1}{12}(-1) + \frac{5}{6} \stackrel{?}{=} \frac{3}{4}$ $-\frac{1}{12} + \frac{5}{6} \stackrel{?}{=} \frac{3}{4}$ $-\frac{1}{12} + \frac{10}{12} \stackrel{?}{=} \frac{9}{12}$ $\frac{9}{12} = \frac{9}{12}$ ✓

Exercise 6.7.9B

Solve: $\frac{1}{4}x + \frac{1}{2} = \frac{5}{8}$.

Answer

$x = \frac{1}{2}$

Exercise 6.7.9C

Solve: $\frac{1}{8}x + \frac{1}{2} = \frac{1}{4}$.

Answer

$x = -2$

Notice in the previous example, once we cleared the equation of fractions, the equation was like those we solved earlier in this chapter. We changed the problem to one we already knew how to solve. We then used the **General Strategy for Solving Linear Equations**.

SOLVE EQUATIONS WITH FRACTION OR DECIMAL COEFFICIENTS.

1. Find the least common denominator (LCD) of *all* the fractions and decimals (in fraction form) in the equation.
2. Multiply both sides of the equation by that LCD. This clears the fractions and decimals.
3. Solve using the General Strategy for Solving Linear Equations.

EXAMPLE 6.7.10

Solve: $5 = \frac{1}{2}y + \frac{2}{3}y - \frac{3}{4}y$.

Answer

We want to clear the fractions by multiplying both sides of the equation by the LCD of all the fractions in the equation.

Find the LCD of all fractions in the equation.	$5 = \frac{1}{2}y + \frac{2}{3}y - \frac{3}{4}y$
The LCD is 12.	
Multiply both sides of the equation by 12.	$12(5) = 12 \cdot \left(\frac{1}{2}y + \frac{2}{3}y - \frac{3}{4}y\right)$
Distribute.	$12(5) = 12 \cdot \frac{1}{2}y + 12 \cdot \frac{2}{3}y - 12 \cdot \frac{3}{4}y$
Simplify—notice, no more fractions.	$60 = 6y + 8y - 9y$
Combine like terms.	$60 = 5y$
Divide by five.	$\frac{60}{5} = \frac{5y}{5}$
Simplify.	$12 = y$
Check:	$5 = \frac{1}{2}y + \frac{2}{3}y - \frac{3}{4}y$
Let $y = 12$.	$5 \stackrel{?}{=} \frac{1}{2}(12) + \frac{2}{3}(12) - \frac{3}{4}(12)$
	$5 \stackrel{?}{=} 6 + 8 - 9$
	$5 = 5$ ✓

Exercise 6.7.10A

Solve: $7 = \frac{1}{2}x + \frac{3}{4}x - \frac{2}{3}x$.

Answer

$x = 12$

Exercise 6.7.10B

Solve: $-1 = \frac{1}{2}u + \frac{1}{4}u - \frac{2}{3}u$.

Answer

$u = -12$

In the next example, we'll distribute before we clear the fractions.

EXAMPLE 6.7.11

Solve: $\frac{1}{2}(y-5) = \frac{1}{4}(y-1)$.

Answer

	$\frac{1}{2}(y-5) = \frac{1}{4}(y-1)$
Distribute.	$\frac{1}{2} \cdot y - \frac{1}{2} \cdot 5 = \frac{1}{4} \cdot y - \frac{1}{4} \cdot 1$

Simplify.	$\frac{1}{2}y-\frac{5}{2}=\frac{1}{4}y-\frac{1}{4}$
Multiply by the LCD, four.	$4\left(\frac{1}{2}y-\frac{5}{2}\right)=4\left(\frac{1}{4}y-\frac{1}{4}\right)$
Distribute.	$4\cdot\frac{1}{2}y-4\cdot\frac{5}{2}=4\cdot\frac{1}{4}y-4\cdot\frac{1}{4}$
Simplify.	$2y-10=y-1$
Collect the variables to the left.	$2y-y-10=y-y-1$
Simplify.	$y-10=-1$
Collect the constants to the right.	$y-10+10=-1+10$
Simplify.	$y=9$
An alternate way to solve this equation is to clear the fractions without distributing first. If you multiply the factors correctly, this method will be easier.	
	$\frac{1}{2}(y-5)=\frac{1}{4}(y-1)$
Multiply by the LCD, 4.	$4\cdot\frac{1}{2}(y-5)=4\cdot\frac{1}{4}(y-1)$
Multiply four times the fractions.	$2(y-5)=1(y-1)$
Distribute.	$2y-10=y-1$
Collect the variables to the left.	$2y-y-10=y-y-1$
Simplify.	$y-10=-1$
Collect the constants to the right.	$y-10+10=-1+10$
Simplify.	$y=9$
Check:	$\frac{1}{2}(y-5)=\frac{1}{4}(y-1)$
Let $y=9$.	$\frac{1}{2}(9-5)\stackrel{?}{=}\frac{1}{4}(9-1)$
Finish the check on your own.	

? Exercise 6.7.11A

Solve: $\frac{1}{5}(n+3)=\frac{1}{4}(n+2)$.

Answer

$n=2$

? Exercise 6.7.11B

Solve: $\frac{1}{2}(m-3)=\frac{1}{4}(m-7)$.

Answer

$m=-1$

When you multiply both sides of an equation by the LCD of the fractions, make sure you multiply each term by the LCD—even if it does not contain a fraction.

EXAMPLE 6.7.12

Solve: $\frac{4q+3}{2}+6=\frac{3q+5}{4}$

Answer

	$\frac{4q+3}{2}+6=\frac{3q+5}{4}$
Multiply both sides by the LCD, 4.	$4\left(\frac{4q+3}{2}+6\right)=4\left(\frac{3q+5}{4}\right)$
Distribute.	$4\left(\cdot\frac{4q+3}{2}\right)+4\cdot 6=4\cdot\left(\frac{3q+5}{4}\right)$
Simplify.	$2(4q+3)+24=3q+5$
	$8q+6+24=3q+5$
	$8q+30=3q+5$
Collect the variables to the left.	$8q-3q+30=3q-3q+5$
Simplify.	$5q+30=5$
Collect the constants to the right.	$5q+30-30=5-30$
Simplify.	$5q=-25$
Divide both sides by five.	$\frac{5q}{5}=\frac{-25}{5}$
Simplify.	$q=-5$
Check: $\frac{4q+3}{2}+6=\frac{3q+5}{4}$	
Let $q=-5$.	$\frac{4(-5)+3}{2}+6\stackrel{?}{=}\frac{3(-5)+5}{4}$
Finish the check on your own.	

Solve: $\frac{3r+5}{6}+1=\frac{4r+3}{3}$.

Answer

$r=3$

Exercise 6.7.12B

Solve: $\frac{2s+3}{2}+1=\frac{3s+2}{4}$.

Answer

$s=-8$

Some equations have decimals in them. This kind of equation may occur when we solve problems dealing with money or percentages. But decimals can also be expressed as fractions. For example, $0.7=\frac{7}{10}$ and $0.29=\frac{29}{100}$. So, with an equation with decimals, we can use the same method we used to clear fractions—multiply both sides of the equation by the **least common denominator**.

The next example uses an equation that is typical of the ones we will see in the money applications in a later section. Notice that we will clear all decimals by multiplying by the LCD of their fraction form.

Solve: $0.25x+0.05(x+3)=2.85$.

Answer

Look at the decimals and think of the equivalent fractions:

$$0.25 = \frac{25}{100}, \qquad 0.05 = \frac{5}{100}, \qquad 2.85 = 2\frac{85}{100}. \tag{6.7.2}$$

Notice, the LCD is 100. By multiplying by the LCD we will clear the decimals from the equation.

	$0.25x + 0.05(x + 3) = 2.85$
Distribute first.	$0.25x + 0.05x + 0.15 = 2.85$
Combine like terms.	$0.30x + 0.15 = 2.85$
To clear decimals, multiply by 100.	$100(0.30x + 0.15) = 100(2.85)$
Distribute.	$30x + 15 = 285$
Subtract 15 from both sides.	$30x + 15 - 15 = 285 - 15$
Simplify.	$30x = 270$
Divide by 30.	$\frac{30x}{30} = \frac{270}{30}$
Simplify.	$x = 9$
Check it yourself by substituting $x = 9$ into the original equation.	

? Exercise 6.7.13A

Solve: $0.25n + 0.05(n + 5) = 2.95$.

Answer

$n = 9$

? Exercise 6.7.13B

Solve: $0.10d + 0.05(d - 5) = 2.15$.

Answer

$d = 16$

Key Concepts

- **How to determine whether a number is a solution to an equation**
  1. Substitute the number in for the variable in the equation.
  2. Simplify the expressions on both sides of the equation.
  3. Determine whether the resulting equation is true.

     If it is true, the number is a solution.

     If it is not true, the number is not a solution.
- **How to Solve Linear Equations Using a General Strategy**
  1. Simplify each side of the equation as much as possible.

     Use the Distributive Property to remove any parentheses.

     Combine like terms.
  2. Collect all the variable terms on one side of the equation.

Use the Addition or Subtraction Property of Equality.

3. Collect all the constant terms on the other side of the equation.

   Use the Addition or Subtraction Property of Equality.

4. Make the coefficient of the variable term equal to 1.

   Use the Multiplication or Division Property of Equality.

   State the solution to the equation.

5. Check the solution.

   Substitute the solution into the original equation to make sure the result is a true statement.

- **How to Solve Equations with Fraction or Decimal Coefficients**
  1. Find the least common denominator (LCD) of *all* the fractions and decimals (in fraction form) in the equation.
  2. Multiply both sides of the equation by that LCD. This clears the fractions and decimals.
  3. Solve using the General Strategy for Solving Linear Equations.

Glossary

conditional equation

An equation that is true for one or more values of the variable and false for all other values of the variable is a conditional equation.

contradiction

An equation that is false for all values of the variable is called a contradiction. A contradiction has no solution.

identity

An equation that is true for any value of the variable is called an Identity. The solution of an identity is all real numbers.

linear equation

A linear equation is an equation in one variable that can be written, where a and b are real numbers and $a \neq 0$, as $ax + b = 0$.

solution of an equation

A solution of an equation is a value of a variable that makes a true statement when substituted into the equation.

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6.7E: Use a General Strategy to Solve Linear Equations (Exercises)

Practice Makes Perfect

Solve Equations Using the General Strategy

In the following exercises, determine whether the given values are solutions to the equation.

1. $6y+10=12y$

a. $y=\frac{5}{3}$

b. $y=-\frac{1}{2}$

Answer

a. yes
b. no

2. $4x+9=8x$

a. $x=-\frac{7}{8}$

b. $x=\frac{9}{4}$

3. $8u-1=6u$

a. $u=-\frac{1}{2}$

b. $u=\frac{1}{2}$

Answer

a. no
b. yes

4. $9v-2=3v$

a. $v=-\frac{1}{3}$

b. $v=\frac{1}{3}$

In the following exercises, solve each linear equation.

5. $15(y-9)=-60$

Answer

$y=5$

6. $-16(3n+4)=32$

7. $-(w-12)=30$

Answer

$w=-18$

8. $-(t-19)=28$

9. $51+5(4-q)=56$

Answer

$q=3$

10. $-6+6(5-k)=15$

11. $3(10-2x)+54=0$

Answer

$x=14$

12. $-2(11-7x)+54=4$

13. $\frac{2}{3}(9c-3)=22$

Answer

$c=4$

14. $\frac{3}{5}(10x-5)=27$

15. $\frac{1}{5}(15c+10)=c+7$

Answer

$c=\frac{5}{2}$

16. $\frac{1}{4}(20d+12)=d+7$

17. $3(4n-1)-2=8n+3$

Answer

$n=2$

18. $9(2m-3)-8=4m+7$

19. $12+2(5-3y)=-9(y-1)-2$

Answer

$y=-5$

20. $-15+4(2-5y)=-7(y-4)+4$

21. $5+6(3s-5)=-3+2(8s-1)$

Answer

$s=10$

22. $-12+8(x-5)=-4+3(5x-2)$

23. $4(p-4)-(p+7)=5(p-3)$

Answer

$p=-4$

24. $3(a-2)-(a+6)=4(a-1)$

25. $4[5-8(4c-3)]=12(1-13c)-8$

Answer

$c=-4$

26. $5[9-2(6d-1)]=11(4-10d)-139$

27. $3[-9+8(4h-3)]=2(5-12h)-19$

Answer

$h=\frac{3}{4}$

28. $3[-14+2(15k-6)]=8(3-5k)-24$

29. $5[2(m+4)+8(m-7)]=2[3(5+m)-(21-3m)]$

Answer

$m=6$

30. $10[5(n+1)+4(n-1)]=11[7(5+n)-(25-3n)]$

Classify Equations

In the following exercises, classify each equation as a conditional equation, an identity, or a contradiction and then state the solution.

31. $23z+19=3(5z-9)+8z+46$

Answer

identity; all real numbers

32. $15y+32=2(10y-7)-5y+46$

33. $18(5j-1)+29=47$

Answer

conditional equation; $j=\frac{2}{5}$

34. $24(3d-4)+100=52$

35. $22(3m-4)=8(2m+9)$

Answer

conditional equation; $m = 165$

36. $30(2n-1) = 5(10n+8)$

37. $7v+42 = 11(3v+8)-2(13v-1)$

Answer

contradiction; no solution

38. $18u-51 = 9(4u+5)-6(3u-10)$

39. $45(3y-2) = 9(15y-6)$

Answer

contradiction; no solution

40. $60(2x-1) = 15(8x+5)$

41. $9(14d+9)+4d = 13(10d+6)+3$

Answer

identity; all real numbers

42. $11(8c+5)-8c = 2(40c+25)+5$

Solve Equations with Fraction or Decimal Coefficients

In the following exercises, solve each equation with fraction coefficients.

43. $\frac{1}{4}x - \frac{1}{2} = -\frac{3}{4}$

Answer

$x = -1$

44. $\frac{3}{4}x - \frac{1}{2} = \frac{1}{4}$

45. $\frac{5}{6}y - \frac{2}{3} = -\frac{3}{2}$

Answer

$y = -1$

46. $\frac{5}{6}y - \frac{1}{3} = -\frac{7}{6}$

47. $\frac{1}{2}a + \frac{3}{8} = \frac{3}{4}$

Answer

$a = \frac{3}{4}$

48. $\frac{5}{8}b + \frac{1}{2} = -\frac{3}{4}$

49. $2 = \frac{1}{3}x - \frac{1}{2}x + \frac{2}{3}x$

Answer

$x = 4$

50. $2 = \frac{3}{5}x - \frac{1}{3}x + \frac{2}{5}x$

51. $\frac{1}{3}w + \frac{5}{4} = w - \frac{1}{4}$

Answer

$w = \frac{9}{4}$

52. $\frac{1}{2}a - \frac{1}{4} = \frac{1}{6}a + \frac{1}{12}$

53. $\frac{1}{3}b + \frac{1}{5} = \frac{2}{5}b - \frac{3}{5}$

Answer

$b = 12$

54. $\frac{1}{3}x + \frac{2}{5} = \frac{1}{5}x - \frac{2}{5}$

55. $\frac{1}{4}(p-7) = \frac{1}{3}(p+5)$

Answer

$p = -41$

56. $\frac{1}{5}(q+3) = \frac{1}{2}(q-3)$

57. $\frac{1}{2}(x+4) = \frac{3}{4}$

Answer

$x = -\frac{5}{2}$

58. $\frac{1}{3}(x+5) = \frac{5}{6}$

59. $\frac{4n+8}{4} = \frac{n}{3}$

Answer

$n = -3$

60. $\frac{3p+6}{3} = \frac{p}{2}$

61. $\frac{3x+4}{2} + 1 = \frac{5x+10}{8}$

Answer

$x = -2$

62. $\frac{10y-2}{3}+3=\frac{10y+1}{9}$

63. $\frac{7u-1}{4}-1=\frac{4u+8}{5}$

Answer

$u=3$

64. $\frac{3v-6}{2}+5=\frac{11v-4}{5}$

In the following exercises, solve each equation with decimal coefficients.

65. $0.4x+0.6=0.5x-1.2$

Answer

$x=18$

66. $0.7x+0.4=0.6x+2.4$

67. $0.9x-1.25=0.75x+1.75$

Answer

$x=20$

68. $1.2x-0.91=0.8x+2.29$

69. $0.05n+0.10(n+8)=2.15$

Answer

$n=9$

70. $0.05n+0.10(n+7)=3.55$

71. $0.10d+0.25(d+5)=4.05$

Answer

$d=8$

72. $0.10d+0.25(d+7)=5.25$

Everyday Math

73. **Fencing** Micah has 74 feet of fencing to make a dog run in his yard. He wants the length to be 2.5 feet more than the width. Find the length, L, by solving the equation $2L+2(L-2.5)=74$.

Answer

$L=19.75$ feet

74. Stamps Paula bought $22.82 worth of 49-cent stamps and 21-cent stamps. The number of 21-cent stamps was eight less than the number of 49-cent stamps. Solve the equation $0.49s+0.21(s-8)=22.82$ for s, to find the number of 49-cent stamps Paula bought.

Writing Exercises

75. Using your own words, list the steps in the general strategy for solving linear equations.

Answer

Answers will vary.

76. Explain why you should simplify both sides of an equation as much as possible before collecting the variable terms to one side and the constant terms to the other side.

77. What is the first step you take when solving the equation $3-7(y-4)=38$? Why is this your first step?

Answer

Answers will vary.

78. If an equation has several fractions, how does multiplying both sides by the LCD make it easier to solve?

79. If an equation has fractions only on one side, why do you have to multiply both sides of the equation by the LCD?

Answer

Answers will vary.

80. For the equation $0.35x+2.1=3.85$, how do you clear the decimal?

Self Check

a. After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
solve linear equations using a general strategy.			
classify equations.			
solve equations with fraction or decimal coefficients.			

b. If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them. What did you do to become confident of your ability to do these things? Be specific.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math every topic builds upon previous work. It is important to make sure you have a strong foundation before you move on. Who can you ask for help? Your fellow classmates and instructor are good resources. Is there a place on campus where math tutors are available? Can your study skills be improved?

…no - I don't get it! This is a warning sign and you must not ignore it. You should get help right away or you will quickly be overwhelmed. See your instructor as soon as you can to discuss your situation. Together you can come up with a plan to get you the help you need.

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6.8: Use a Problem Solving Strategy

Learning Objectives

By the end of this section, you will be able to:

- Use a problem solving strategy for word problems
- Solve number word problems
- Solve percent applications
- Solve simple interest applications

Before you get started, take this readiness quiz.

1. Translate "six less than twice x" into an algebraic expression.
   If you missed this problem, review [link].
2. Convert 4.5% to a decimal.
   If you missed this problem, review [link].
3. Convert 0.6 to a percent.
   If you missed this problem, review [link].

Have you ever had any negative experiences in the past with word problems? When we feel we have no control, and continue repeating negative thoughts, we set up barriers to success. Realize that your negative experiences with word problems are in your past. To move forward you need to calm your fears and change your negative feelings.

Start with a fresh slate and begin to think positive thoughts. Repeating some of the following statements may be helpful to turn your thoughts positive. Thinking positive thoughts is a first step towards success.

- I think I can! I think I can!
- While word problems were hard in the past, I think I can try them now.
- I am better prepared now—I think I will begin to understand word problems.
- I am able to solve equations because I practiced many problems and I got help when I needed it—I can try that with word problems.
- It may take time, but I can begin to solve word problems.
- You are now well prepared and you are ready to succeed. If you take control and believe you can be successful, you will be able to master word problems.

Use a Problem Solving Strategy for Word Problems

Now that we can solve equations, we are ready to apply our new skills to word problems. We will develop a strategy we can use to solve any word problem successfully.

EXAMPLE 6.8.1

Normal yearly snowfall at the local ski resort is 12 inches more than twice the amount it received last season. The normal yearly snowfall is 62 inches. What was the snowfall last season at the ski resort?

Solution:

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	What was the snowfall last season?
Step 3. Name what we are looking for and choose a variable to represent it.	Let $s =$ the snowfall last season.
Step 4. Translate. Restate the problem in one sentence with all the important information.	The normal snowfall was twelve more than twice the amount last year

Translate into an equation.	$62 = 2s + 12$
Step 5. Solve the equation.	$62 = 2s + 12$
Subtract 12 from each side.	$62 - 12 = 2s + 12 - 12$
Simplify.	$50 = 2s$
Divide each side by two.	$\frac{50}{2} = \frac{2s}{2}$
Simplify.	$25 = s$
Step 6. Check: First, is our answer reasonable? Yes, having 25 inches of snow seems OK. The problem says the normal snowfall is twelve inches more than twice the number of last season. Twice 25 is 50 and 12 more than that is 62.	
Step 7. Answer the question.	The snowfall last season was 25 inches.

Try It! 6.8.1

Guillermo bought textbooks and notebooks at the bookstore. The number of textbooks was three more than twice the number of notebooks. He bought seven textbooks. How many notebooks did he buy?

Answer

He bought two notebooks

Try It! 6.8.2

Gerry worked Sudoku puzzles and crossword puzzles this week. The number of Sudoku puzzles he completed is eight more than twice the number of crossword puzzles. He completed 22 Sudoku puzzles. How many crossword puzzles did he do?

Answer

He did seven crossword puzzles

We summarize an effective strategy for problem solving.

PROBLEM SOLVING STRATEGY FOR WORD PROBLEMS

1. **Read** the problem. Make sure all the words and ideas are understood.
2. **Identify** what you are looking for.
3. **Name** what you are looking for. Choose a variable to represent that quantity.
4. **Translate** into an equation. It may be helpful to restate the problem in one sentence with all the important information. Then, translate the English sentence into an algebra equation.
5. **Solve** the equation using proper algebra techniques.
6. **Check** the answer in the problem to make sure it makes sense.
7. **Answer** the question with a complete sentence.

Solve Number Word Problems

We will now apply the problem solving strategy to “number word problems.” Number word problems give some clues about one or more numbers and we use these clues to write an equation. Number word problems provide good practice for using the Problem Solving Strategy.

EXAMPLE 6.8.2

The sum of seven times a number and eight is thirty-six. Find the number.

Solution:

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	the number
Step 3. Name what you are looking for and choose a variable to represent it.	Let n = the number.
Step 4. Translate: Restate the problem as one sentence. Translate into an equation.	The sum of seven times a number and 8 is 36 $7n+8 = 36$
Step 5. Solve the equation. Subtract eight from each side and simplify. Divide each side by seven and simplify.	$7n+8=36$ $7n=28$ $n=4$
Step 6. Check. Is the sum of seven times four plus eight equal to 36? $7\cdot 4+8 \stackrel{?}{=} 36$ $28+8 \stackrel{?}{=} 36$ $36=36$✓	
Step 7. Answer the question.	The number is 4.

Did you notice that we left out some of the steps as we solved this equation? If you're not yet ready to leave out these steps, write down as many as you need.

Try It! 6.8.3

The sum of four times a number and two is fourteen. Find the number.

Answer

3

Try It! 6.8.4

The sum of three times a number and seven is twenty-five. Find the number.

Answer

6

Some number word problems ask us to find two or more numbers. It may be tempting to name them all with different variables, but so far, we have only solved equations with one variable. In order to avoid using more than one variable, we will define the numbers in terms of the same variable. Be sure to read the problem carefully to discover how all the numbers relate to each other.

EXAMPLE 6.8.3

The sum of two numbers is negative fifteen. One number is nine less than the other. Find the numbers.

Solution:

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	two numbers
Step 3. Name what you are looking for by choosing a variable to represent the first number. “One number is nine less than the other.”	Let $n = 1^{\text{st}}$ number. $n - 9 = 2^{\text{nd}}$ number
Step 4. Translate. Write as one sentence. Translate into an equation.	The sum of two numbers is negative fifteen. 1st number + 2nd number is negative fifteen $n + n - 9 = -15$
Step 5. Solve the equation. Combine like terms. Add nine to each side and simplify. Simplify.	$n + n - 9 = -15$ $2n = -6$ $-3 - 9$ -12
Step 6. Check. Is -12 nine less than -3? $-3 - 9 \stackrel{?}{=} -12$ $-12 = -12$✓ Is their sum -15? $-3 + (-12) \stackrel{?}{=} -15$ $-15 = -15$✓	
Step 7. Answer the question.	The numbers are -3 and -12.

? Try It! 6.8.5

The sum of two numbers is negative twenty-three. One number is seven less than the other. Find the numbers.

Answer

$-15, -8$

? Try It! 6.8.6

The sum of two numbers is negative eighteen. One number is forty more than the other. Find the numbers.

Answer

$-29, 11$

Some number problems involve **consecutive integers**. Consecutive integers are integers that immediately follow each other. Examples of consecutive integers are:

$$\begin{array}{rrrr} 1, & 2, & 3, & 4 \\ -10, & -9, & -8, & -7 \\ 150, & 151, & 152, & 153 \end{array}$$

Notice that each number is one more than the number preceding it. Therefore, if we define the first integer as n, the next consecutive integer is $n+1$. The one after that is one more than $n+1$, so it is $n+1+1$, which is $n+2$.

$$\begin{array}{ll} n & 1^{\text{st}}\text{integer} \\ n+1 & 2^{\text{nd}}\text{consecutive integer} \\ n+2 & 3^{\text{rd}}\text{consecutive integer} \qquad \text{etc.} \end{array}$$

We will use this notation to represent consecutive integers in the next example.

EXAMPLE 6.8.4

Find three consecutive integers whose sum is -54.

Solution:

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	three consecutive integers
Step 3. Name each of the three numbers	Let $n = 1^{\text{st}}$integer. $n+1 = 2^{\text{nd}}$consecutive integer $n+2 = 3^{\text{rd}}$consecutive integer
Step 4. Translate. Restate as one sentence. Translate into an equation.	The sum of the three integers is -54. $n+n+1+n+2=-54$
Step 5. Solve the equation. Combine like terms. Subtract three from each side. Divide each side by three.	$n+n+1+n+2=-54$ $3n+3=-54$ $3n=-57$ $n=-19$ 1st integer $n+1$ 2nd integer -18 $-19+2$ -17
Step 6. Check. $-19+(-18)+(-17) = -54$ $-54 = -54$✓	
Step 7. Answer the question.	The three consecutive integers are −17,−18, and −19.

Try It! 6.8.7

Find three consecutive integers whose sum is -96.

Answer

$-33, -32, -31$

Try It! 6.8.8

Find three consecutive integers whose sum is -36.

Answer

$-13, -12, -11$

Now that we have worked with consecutive integers, we will expand our work to include **consecutive even integers** and **consecutive odd integers**. Consecutive even integers are even integers that immediately follow one another. Examples of consecutive even integers are:

$$24, 26, 28$$

$$-12, -10, -8$$

Notice each integer is two more than the number preceding it. If we call the first one n, then the next one is $n+2$. The one after that would be $n+2+2$ or $n+4$.

n 1^{st}integer
$n+1$ 2^{nd}consecutive integer
$n+2$ 3^{rd}consecutive integer etc.

Consecutive odd integers are odd integers that immediately follow one another. Consider the consecutive odd integers 63, 65, and 67.

$$63, 65, 67$$

$$n, n+2, n+4$$

n 1^{st}integer
$n+1$ 2^{nd}consecutive integer
$n+2$ 3^{rd}consecutive integer etc.

Does it seem strange to have to add two (an even number) to get the next odd number? Do we get an odd number or an even number when we add 2 to 3? to 11? to 47?

Whether the problem asks for consecutive even numbers or odd numbers, you do not have to do anything different. The pattern is still the same—to get to the next odd or the next even integer, add two.

EXAMPLE 6.8.5

Find three consecutive even integers whose sum is 120.

Solution:

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	three consecutive even integers
Step 3. Name each of the three numbers	Let $n = 1^{\text{st}}$consecutive even integer. $n+2 = 2^{\text{nd}}$consecutive even integer. $n+4 = 3^{\text{rd}}$consecutive even integer.
Step 4. Translate. Restate as one sentence. Translate into an equation.	The sum of the three even integers is 120 $n+n+2+n+4=120$
Step 5. Solve the equation. Combine like terms. Subtract three from each side. Divide each side by three.	$n+n+2+n+4=120$ $3n+6=120$ $3n=114$ $n=38$ 1^{st}integer $n+2$ 2^{nd}integer $38+2$ 40 $n+2$ 3^{rd}integer $38+4$ 42
Step 6. Check. $38+40+42 \stackrel{?}{=} 120$ $120 = 120$✓	
Step 7. Answer the question.	The three consecutive integers are 38, 40, and 42.

Try It! 6.8.9

Find three consecutive even integers whose sum is 102.

Answer

$32, 34, 36$

Try It! 6.8.10

Find three consecutive even integers whose sum is -24.

Answer

$-10, -8, -6$

When a number problem is in a real life context, we still use the same strategies that we used for the previous examples.

EXAMPLE 6.8.6

A married couple together earns $110,000 a year. The wife earns $16,000 less than twice what her husband earns. What does the husband earn?

Solution:

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	How much does the husband earn?
Step 3. Name each of the three numbers	Let $h =$ the amount the husband earns.
Step 4. Translate. Restate the problem in one sentence with all the important information. Translate into an equation.	$2h - 16,000 =$ the amount the wife earns. Together the husband and wife earn $110,000. $h + 2h - 16,000 = 110,000$
Step 5. Solve the equation. Combine like terms. Add 16,000 to both sides and simplify. Divide each side by three.	$h + 2h - 16,000 = 110,000$ $3h - 16,000 = 110,000$ $3h = 126,000$ $h = 42,000$ amount husband earns $2h - 16,000$ amount wife earns $2(42,000) - 16,000$ $84,000 - 16,000$ $68,000$
Step 6. Check. If the wife earns $68,000 and the husband earns $42,000, is that $110,000? Yes!	
Step 7. Answer the question.	The husband earns $42,000 a year.

According to the National Automobile Dealers Association, the average cost of a car in 2014 was $28,400. This was $1,600 less than six times the cost in 1975. What was the average cost of a car in 1975?

Answer

The average cost was $5,000.

Try It! 6.8.11

US Census data shows that the median price of new home in the U.S. in November 2014 was $280,900. This was $10,700 more than 14 times the price in November 1964. What was the median price of a new home in November 1964?

Answer

The median price was $19,300.

Solve Percent Applications

There are several methods to solve percent equations. In algebra, it is easiest if we just translate English sentences into algebraic equations and then solve the equations. Be sure to change the given percent to a decimal before you use it in the equation.

EXAMPLE 6.8.7

Translate and solve:

a. What number is 45% of 84?
b. 8.5% of what amount is $4.76?
c. 168 is what percent of 112?

Solution:

a.

	What number is 45% of 84?
Translate into algebra. Let n = the number.	$n = 0.45 \cdot 84$
Multiply.	$n = 37.8$
	37.8 is 45% of 84.

b.

	8.5% of what amount is $4.76?
Translate. Let n = the amount.	$0.085 \cdot n = 4.76$
Multiply.	$0.085n = 4.76$
Divide both sides by 0.085 and simplify.	$n = 56$
	8.5% of $56 is $4.76

c.

We are asked to find percent, so we must have our result in percent form.	168 is what percent of 112?
Translate into algebra. Let p = the percent.	$168 = p \cdot 112$
Multiply.	$168 = 112p$
Divide both sides by 112 and simplify.	$1.5 = p$
Convert to percent.	$150\% = p$
	168 is 150% of 112.

Try It! 6.8.12

Translate and solve:

a. What number is 45% of 80?
b. 7.5% of what amount is $1.95?
c. 110 is what percent of 88?

Answer

a. 36 b. $26 c. 125%

Try It! 6.8.13

Translate and solve:

a. What number is 55% of 60?
b. 8.5% of what amount is $3.06?
c. 126 is what percent of 72?

Answer

a. 33 b. $36 c. 175%

Now that we have a problem solving strategy to refer to, and have practiced solving basic percent equations, we are ready to solve percent applications. Be sure to ask yourself if your final answer makes sense—since many of the applications we will solve involve everyday situations, you can rely on your own experience.

EXAMPLE 6.8.8

The label on Audrey's yogurt said that one serving provided 12 grams of protein, which is 24% of the recommended daily amount. What is the total recommended daily amount of protein?

Solution:

What are you asked to find?	What total amount of protein is recommended?
Choose a variable to represent it.	Let a = total amount of protein.
Write a sentence that gives the information to find it.	12 g is 24% of the total amount
Translate into an equation.	$12 = 0.24 \cdot a$
Solve.	$50 = a$
Check: Does this make sense? Yes, 24% is about $\frac{1}{4}$ of the total and 12 is about $\frac{1}{4}$ of 50.	
Write a complete sentence to answer the question.	The amount of protein that is recommended is 50 g.

Try It! 6.8.14

One serving of wheat square cereal has 7 grams of fiber, which is 28% of the recommended daily amount. What is the total recommended daily amount of fiber?

Answer

25 grams

Try It! 6.8.15

One serving of rice cereal has 190 mg of sodium, which is 8% of the recommended daily amount. What is the total recommended daily amount of sodium?

Answer

2,375 mg

Remember to put the answer in the form requested. In the next example we are looking for the percent.

EXAMPLE 6.8.9

Veronica is planning to make muffins from a mix. The package says each muffin will be 240 calories and 60 calories will be from fat. What percent of the total calories is from fat?

Solution:

What are you asked to find?	What percent of the total calories is fat?
Choose a variable to represent it.	Let $p =$ percent of fat.
Write a sentence that gives the information to find it.	What percent of 240 is 60?
Translate the sentence into an equation.	$p \cdot 240 = 60$
Multiply.	$240p = 60$
Divide both sides by 240.	$p = 0.25$
Put in percent form.	$p = 25\%$
Check: does this make sense? Yes, 25% is one-fourth; 60 is one-fourth of 240. So, 25% makes sense.	
Write a complete sentence to answer the question.	Of the total calories in each muffin, 25% is fat.

Try It! 6.8.16

Mitzi received some gourmet brownies as a gift. The wrapper said each 28% brownie was 480 calories, and had 240 calories of fat. What percent of the total calories in each brownie comes from fat? Round the answer to the nearest whole percent.

Answer

50%

Try It! 6.8.17

The mix Ricardo plans to use to make brownies says that each brownie will be 190 calories, and 76 calories are from fat. What percent of the total calories are from fat? Round the answer to the nearest whole percent.

Answer

40%

It is often important in many fields—business, sciences, pop culture—to talk about how much an amount has increased or decreased over a certain period of time. This increase or decrease is generally expressed as a percent and called the **percent change**.

To find the percent change, first we find the amount of change, by finding the difference of the new amount and the original amount. Then we find what percent the amount of change is of the original amount.

FIND PERCENT CHANGE

1. Find the amount of change.

$$\text{change} = \text{new amount} - \text{original amount} \tag{6.8.1}$$

2. Find what percent the amount of change is of the original amount.

change is what percent of the original amount?

EXAMPLE 6.8.10

Recently, the California governor proposed raising community college fees from $36 a unit to $46 a unit. Find the percent change. (Round to the nearest tenth of a percent.)

Solution:

Find the amount of change.	$46 - 36 = 10$
Find the percent.	Change is what percent of the original amount?
Let $p =$ the percent.	10 is what percent of 36?
Translate to an equation.	$10 = p \cdot 36$
Simplify.	$10 = 36p$
Divide both sides by 36.	$0.278 \approx p$
Change to percent form; round to the nearest tenth	$27.8\% \approx p$
Write a complete sentence to answer the question.	The new fees are approximately a 27.8% increase over the old fees.
Remember to round the division to the nearest thousandth in order to round the percent to the nearest tenth.	

Try It! 6.8.18

Find the percent change. (Round to the nearest tenth of a percent.) In 2011, the IRS increased the deductible mileage cost to 55.5 cents from 51 cents.

Answer

8.8%

Try It! 6.8.19

Find the percent change. (Round to the nearest tenth of a percent.) In 1995, the standard bus fare in Chicago was $1.50. In 2008, the standard bus fare was 2.25.

Answer

50

Applications of discount and mark-up are very common in retail settings.

When you buy an item on sale, the original price has been discounted by some dollar amount. The **discount rate**, usually given as a percent, is used to determine the **amount of the discount**. To determine the amount of discount, we multiply the discount rate by the original price.

The price a retailer pays for an item is called the **original cost**. The retailer then adds a **mark-up** to the original cost to get the **list price**, the price he sells the item for. The mark-up is usually calculated as a percent of the original cost. To determine the amount of mark-up, multiply the mark-up rate by the original cost.

DISCOUNT

$$\text{amount of discount} = \text{discount rate} \cdot \text{original price}$$
$$\text{sale price} = \text{original amount} - \text{discount price}$$

The sale price should always be less than the original price.

MARK-UP

$$\text{amount of mark-up} = \text{mark-up rate} \cdot \text{original price}$$
$$\text{list price} = \text{original cost} - \text{mark-up}$$

The list price should always be more than the original cost.

EXAMPLE 6.8.11

Liam's art gallery bought a painting at an original cost of $750. Liam marked the price up 40%. Find

a. the amount of mark-up and
b. the list price of the painting.

Solution:

a.

Identify what you are asked to find, and choose a variable to represent it.	What is the amount of mark-up? Let $m =$ the amount of mark-up.
Write a sentence that gives the information to find it.	The mark-up is 40% of the $750 original cost
Translate into an equation.	$m = 0.40 \times 750$
Solve the equation.	$m = 300$
Write a complete sentence.	The mark-up on the painting was $300.

b.

Identify what you are asked to find, and choose a variable to represent it.	What is the list price? Let $p =$ the list price.
Write a sentence that gives the information to find it.	The list price is original cost plus the mark-up
Translate into an equation.	$p = 750 + 300$
Solve the equation.	$p = 1{,}050$
Check.	Is the list price more than the original cost? Is $1,050 more than $750? Yes.
Write a complete sentence.	The list price of the painting was $1,050.

Try It! 6.8.20

Find a. the amount of mark-up and b. the list price: Jim's music store bought a guitar at original cost $1,200. Jim marked the price up 50%.

Answer

a. $600 b. $1,800

Try It! 6.8.21

Find a. the amount of mark-up and b. the list price: The Auto Resale Store bought Pablo's Toyota for $8,500. They marked the price up 35%.

Answer

a. $2,975 b. $11,475

Solve Simple Interest Applications

Interest is a part of our daily lives. From the interest earned on our savings to the interest we pay on a car loan or credit card debt, we all have some experience with interest in our lives.

The amount of money you initially deposit into a bank is called the **principal**, P, and the bank pays you interest, I. When you take out a loan, you pay interest on the amount you borrow, also called the principal.

In either case, the interest is computed as a certain percent of the principal, called the **rate of interest**, r. The rate of interest is usually expressed as a percent per year, and is calculated by using the decimal equivalent of the percent. The variable t, (for time) represents the number of years the money is saved or borrowed.

Interest is calculated as simple interest or compound interest. Here we will use simple interest.

SIMPLE INTEREST

If an amount of money, P, called the principal, is invested or borrowed for a period of t years at an annual interest rate r, the amount of interest, I, earned or paid is

$$I = Prt \qquad \text{where} \quad \begin{aligned} I &= \text{interest} \\ P &= \text{principal} \\ r &= \text{rate} \\ t &= \text{time} \end{aligned}$$

Interest earned or paid according to this formula is called **simple interest**.

The formula we use to calculate interest is $I = Prt$. To use the formula we substitute in the values for variables that are given, and then solve for the unknown variable. It may be helpful to organize the information in a chart.

EXAMPLE 6.8.12

Areli invested a principal of $950 in her bank account that earned simple interest at an interest rate of 3%. How much interest did she earn in five years?

Solution:

$I = ?$
$P = \$950$
$r = 3\%$
$t = 5$ years

Identify what you are asked to find, and choose a variable to represent it.	What is the simple interest? Let $I =$ interest.
Write the formula.	$I = Prt$
Substitute in the given information.	$I = (950)(0.03)(5)$
Simplify.	$I = 142.5$
Check. Is $142.50 a reasonable amount of interest on $ 950? Yes.	
Write a complete sentence.	The interest is $142.50.

Try It! 6.8.22

Nathaly deposited $12,500 in her bank account where it will earn 4% simple interest. How much interest will Nathaly earn in five years?

Answer

He will earn $2,500.

Try It! 6.8.23

Susana invested a principal of $36,000 in her bank account that earned simple interest at an interest rate of 6.5%. How much interest did she earn in three years?

Answer

She earned $7,020.

There may be times when we know the amount of interest earned on a given principal over a certain length of time, but we do not know the rate.

EXAMPLE 6.8.13

Hang borrowed $7,500 from her parents to pay her tuition. In five years, she paid them $1,500 interest in addition to the $7,500 she borrowed. What was the rate of simple interest?

Solution:

$I = \$1500$
$P = \$7500$
$r = ?$
$t = 5$ years

Identify what you are asked to find, and choose a variable to represent it.	What is rate of simple interest? Let r = rate of interest
Write the formula.	$I = Prt$
Substitute in the given information.	$1,500 = (7,500)r(5)$
Multiply.	$1,500 = 37,500r$
Divide.	$0.04 = r$
Change to percent form	$r = 4\%$

Check.

$$I = Prt$$
$$1,500 \stackrel{?}{=} (7,500)(0.04)(5)$$
$$1,500 = 1,500\checkmark$$

Write a complete sentence. The rate of interest was 4%.

Try It! 6.8.24

Jim lent his sister $5,000 to help her buy a house. In three years, she paid him the $5,000, plus $900 interest. What was the rate of simple interest?

Answer

The rate of simple interest was 6%.

Try It! 6.8.25

Loren lent his brother $3,000 to help him buy a car. In four years, his brother paid him back the $3,000 plus $660 in interest. What was the rate of simple interest?

Answer

The rate of simple interest was 5.5%.

In the next example, we are asked to find the principal—the amount borrowed.

EXAMPLE 6.8.14

Sean's new car loan statement said he would pay $4,866,25 in interest from a simple interest rate of 8.5% over five years. How much did he borrow to buy his new car?

Solution:

$$\begin{aligned} I &= 4,866.25 \\ P &= ? \\ r &= 8.5\% \\ t &= 5 \text{ years} \end{aligned}$$

Identify what you are asked to find, and choose a variable to represent it.	What is the amount borrowed (the principal)? Let P = principal borrowed
Write the formula.	$I = Prt$
Substitute in the given information.	$4,866.25 = P(0.085)(5)$
Multiply.	$4,866.25 = 0.425P$
Divide.	$11,450 = P$

Check.

$$\begin{aligned} I &= Prt \\ 4,866.25 &\stackrel{?}{=} (11,450)(0.085)(5) \\ 4,866.25 &= 4,866.25\checkmark \end{aligned}$$

Write a complete sentence. The principal was $\$11,450$.

Try It! 6.8.26

Eduardo noticed that his new car loan papers stated that with a 7.5% simple interest rate, he would pay $6,596.25 in interest over five years. How much did he borrow to pay for his car?

Answer

He paid $17,590.

Try It! 6.8.27

In five years, Gloria's bank account earned $2,400 interest at 5% simple interest. How much had she deposited in the account?

Answer

She deposited $9,600.

Access this online resource for additional instruction and practice with using a problem solving strategy.

- Begining Arithmetic Problems

Key Concepts

- **How To Use a Problem Solving Strategy for Word Problems**
  1. **Read** the problem. Make sure all the words and ideas are understood.
  2. **Identify** what you are looking for.
  3. **Name** what you are looking for. Choose a variable to represent that quantity.
  4. **Translate** into an equation. It may be helpful to restate the problem in one sentence with all the important information. Then, translate the English sentence into an algebra equation.
  5. **Solve** the equation using proper algebra techniques.
  6. **Check** the answer in the problem to make sure it makes sense.
  7. **Answer** the question with a complete sentence.
- **How To Find Percent Change**
  1. Find the amount of change

     $$\text{change} = \text{new amount} - \text{original amount}$$

  2. Find what percent the amount of change is of the original amount.

     $$\text{change is what percent of the original amount?}$$

- $$\text{amount of discount} = \text{discount rate} \cdot \text{original price}$$
  $$\text{sale price} = \text{original amount} - \text{discount price}$$
- $$\text{amount of mark-up} = \text{mark-up rate} \cdot \text{original price}$$
  $$\text{list price} = \text{original cost} - \text{mark-up}$$
- If an amount of money, P, called the principal, is invested or borrowed for a period of t years at an annual interest rate r, the amount of interest, I, earned or paid is:

$$I = Prt \qquad \begin{array}{l} I = interest \\ \text{where } P = principal \\ r = rate \\ t = time \end{array}$$

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6.8E: Exercises

Practice Makes Perfect

Use a Problem Solving Strategy for Word Problems

1. List five positive thoughts you can say to yourself that will help you approach word problems with a positive attitude. You may want to copy them on a sheet of paper and put it in the front of your notebook, where you can read them often.

Answer

Answers will vary.

2. List five negative thoughts that you have said to yourself in the past that will hinder your progress on word problems. You may want to write each one on a small piece of paper and rip it up to symbolically destroy the negative thoughts.

In the following exercises, solve using the problem solving strategy for word problems. Remember to write a complete sentence to answer each question.

3. There are 16 girls in a school club. The number of girls is four more than twice the number of boys. Find the number of boys.

Answer

six boys

4. There are 18 Cub Scouts in Troop 645. The number of scouts is three more than five times the number of adult leaders. Find the number of adult leaders.

5. Huong is organizing paperback and hardback books for her club's used book sale. The number of paperbacks is 12 less than three times the number of hardbacks. Huong had 162 paperbacks. How many hardback books were there?

Answer

58 hardback books

6. Jeff is lining up children's and adult bicycles at the bike shop where he works. The number of children's bicycles is nine less than three times the number of adult bicycles. There are 42 adult bicycles. How many children's bicycles are there?

Solve Number Word Problems

In the following exercises, solve each number word problem.

7. The difference of a number and 12 is three. Find the number.

Answer

15

8. The difference of a number and eight is four. Find the number.

9. The sum of three times a number and eight is 23. Find the number.

Answer

5

10. The sum of twice a number and six is 14. Find the number.

11. The difference of twice a number and seven is 17. Find the number.

Answer

12

12. The difference of four times a number and seven is 21. Find the number.

13. Three times the sum of a number and nine is 12. Find the number.

Answer

-5

14. Six times the sum of a number and eight is 30. Find the number.

15. One number is six more than the other. Their sum is 42. Find the numbers.

Answer

$18,\ 24$

16. One number is five more than the other. Their sum is 33. Find the numbers.

17. The sum of two numbers is 20. One number is four less than the other. Find the numbers.

Answer

$8,\ 12$

18. The sum of two numbers is 27. One number is seven less than the other. Find the numbers.

19. One number is 14 less than another. If their sum is increased by seven, the result is 85. Find the numbers.

Answer

$32,\ 46$

20. One number is 11 less than another. If their sum is increased by eight, the result is 71. Find the numbers.

21. The sum of two numbers is 14. One number is two less than three times the other. Find the numbers.

Answer

$4,\ 10$

22. The sum of two numbers is zero. One number is nine less than twice the other. Find the numbers.

23. The sum of two consecutive integers is 77. Find the integers.

Answer

$38,\ 39$

24. The sum of two consecutive integers is 89. Find the integers.

25. The sum of three consecutive integers is 78. Find the integers.

Answer

$25,\ 26,\ 27$

26. The sum of three consecutive integers is 60. Find the integers.

27. Find three consecutive integers whose sum is -36.

Answer

$-11,\ -12,\ -13$

28. Find three consecutive integers whose sum is -3.

29. Find three consecutive even integers whose sum is 258.

Answer

$84,\ 86,\ 88$

30. Find three consecutive even integers whose sum is 222.

31. Find three consecutive odd integers whose sum is -213.

Answer

$-69,\ -71,\ -73$

32. Find three consecutive odd integers whose sum is -267.

33. Philip pays $\$1,620$ in rent every month. This amount is $\$120$ more than twice what his brother Paul pays for rent. How much does Paul pay for rent?

Answer

$\$750$

34. Marc just bought an SUV for $\$54,000$. This is $\$7,400$ less than twice what his wife paid for her car last year. How much did his wife pay for her car?

35. Laurie has $\$46,000$ invested in stocks and bonds. The amount invested in stocks is $\$8,000$ less than three times the amount invested in bonds. How much does Laurie have invested in bonds?

Answer

$\$13,500$

36. Erica earned a total of $\$50,450$ last year from her two jobs. The amount she earned from her job at the store was $\$1,250$ more than three times the amount she earned from her job at the college. How much did she earn from her job at the college?

Solve Percent Applications

In the following exercises, translate and solve.

37. a. What number is 45% of 120? b. 81 is 75% of what number? c. What percent of 260 is 78?

Answer

a. 54
b. 108
c. 30%

38. a. What number is 65% of 100? b. 93 is 75% of what number? c. What percent of 215 is 86?

39. a. 250% of 65 is what number? b. 8.2% of what amount is $2.87? c. 30 is what percent of 20?

Answer

a. 162.5
b. $35
c. 150%

40. a. 150% of 90 is what number? b. 6.4% of what amount is $2.88? c. 50 is what percent of 40?

In the following exercises, solve.

41. Geneva treated her parents to dinner at their favorite restaurant. The bill was $74.25. Geneva wants to leave 16% of the total bill as a tip. How much should the tip be?

Answer

$11.88

42. When Hiro and his co-workers had lunch at a restaurant near their work, the bill was $90.50. They want to leave 18% of the total bill as a tip. How much should the tip be?

43. One serving of oatmeal has 8 grams of fiber, which is 33% of the recommended daily amount. What is the total recommended daily amount of fiber?

Answer

24.2 g

44. One serving of trail mix has 67 grams of carbohydrates, which is 22% of the recommended daily amount. What is the total recommended daily amount of carbohydrates?

45. A bacon cheeseburger at a popular fast food restaurant contains 2070 milligrams (mg) of sodium, which is 86% of the recommended daily amount. What is the total recommended daily amount of sodium?

Answer

2407 mg

46. A grilled chicken salad at a popular fast food restaurant contains 650 milligrams (mg) of sodium, which is 27% of the recommended daily amount. What is the total recommended daily amount of sodium?

47. The nutrition fact sheet at a fast food restaurant says the fish sandwich has 380 calories, and 171 calories are from fat. What percent of the total calories is from fat?

Answer

45%

48. The nutrition fact sheet at a fast food restaurant says a small portion of chicken nuggets has 190 calories, and 114 calories are from fat. What percent of the total calories is from fat?

49. Emma gets paid $3,000 per month. She pays $750 a month for rent. What percent of her monthly pay goes to rent?

Answer

25%

50. Dimple gets paid $3,200 per month. She pays $960 a month for rent. What percent of her monthly pay goes to rent?

In the following exercises, solve.

51. Tamanika received a raise in her hourly pay, from $15.50 to $17.36. Find the percent change.

Answer

12%

52. Ayodele received a raise in her hourly pay, from $24.50 to $25.48. Find the percent change.

53. Annual student fees at the University of California rose from about $4,000 in 2000 to about $12,000 in 2010. Find the percent change.

Answer

200%

54. The price of a share of one stock rose from $12.50 to $50. Find the percent change.

55. A grocery store reduced the price of a loaf of bread from $2.80 to $2.73. Find the percent change.

Answer

−2.5%

56. The price of a share of one stock fell from $8.75 to $8.54. Find the percent change.

57. Hernando's salary was $49,500 last year. This year his salary was cut to $44,055. Find the percent change.

Answer

−11%

58. In ten years, the population of Detroit fell from 950,000 to about 712,500. Find the percent change.

In the following exercises, find a. the amount of discount and b. the sale price.

59. Janelle bought a beach chair on sale at 60% off. The original price was $44.95.

Answer

a. $26.97
b. $17.98

60. Errol bought a skateboard helmet on sale at 40% off. The original price was $49.95.

In the following exercises, find a. the amount of discount and b. the discount rate (Round to the nearest tenth of a percent if needed.)

61. Larry and Donna bought a sofa at the sale price of $1,344. The original price of the sofa was $1,920.

Answer

a. $576
b. 30%

62. Hiroshi bought a lawnmower at the sale price of $240. The original price of the lawnmower is $300.

In the following exercises, find a. the amount of the mark-up and b. the list price.

63. Daria bought a bracelet at original cost $16 to sell in her handicraft store. She marked the price up 45%. What was the list price of the bracelet?

Answer

a. $7.20
b. $23.20

64. Regina bought a handmade quilt at original cost $120 to sell in her quilt store. She marked the price up 55%. What was the list price of the quilt?

65. Tom paid $0.60 a pound for tomatoes to sell at his produce store. He added a 33% mark-up. What price did he charge his customers for the tomatoes?

Answer

a. $0.20
b. $0.80

66. Flora paid her supplier $0.74 a stem for roses to sell at her flower shop. She added an 85% mark-up. What price did she charge her customers for the roses?

Solve Simple Interest Applications

In the following exercises, solve.

67. Casey deposited $1,450 in a bank account that earned simple interest at an interest rate of 4%. How much interest was earned in two years?

Answer

$116

68. Terrence deposited $5,720 in a bank account that earned simple interest at an interest rate of 6%. How much interest was earned in four years?

69. Robin deposited $31,000 in a bank account that earned simple interest at an interest rate of 5.2%. How much interest was earned in three years?

Answer

$4836

70. Carleen deposited $16,400 in a bank account that earned simple interest at an interest rate of 3.9% How much interest was earned in eight years?

71. Hilaria borrowed $8,000 from her grandfather to pay for college. Five years later, she paid him back the $8,000, plus $1,200 interest. What was the rate of simple interest?

Answer

3%

72. Kenneth lent his niece $1,200 to buy a computer. Two years later, she paid him back the $1,200, plus $96 interest. What was the rate of simple interest?

73. Lebron lent his daughter $20,000 to help her buy a condominium. When she sold the condominium four years later, she paid him the $20,000, plus $3,000 interest. What was the rate of simple interest?

Answer

3.75%

74. Pablo borrowed $50,000 to start a business. Three years later, he repaid the $50,000, plus $9,375 interest. What was the rate of simple interest?

75. In 10 years, a bank account that paid 5.25% simple interest earned $18,375 interest. What was the principal of the account?

Answer

$35,000

76. In 25 years, a bond that paid 4.75% simple interest earned $2,375 interest. What was the principal of the bond?

77. Joshua's computer loan statement said he would pay $1,244.34 in simple interest for a three-year loan at 12.4%. How much did Joshua borrow to buy the computer?

Answer

$3345

78. Margaret's car loan statement said she would pay $7,683.20 in simple interest for a five-year loan at 9.8%. How much did Margaret borrow to buy the car?

Everyday Math

79. **Tipping** At the campus coffee cart, a medium coffee costs $1.65. MaryAnne brings $2.00 with her when she buys a cup of coffee and leaves the change as a tip. What percent tip does she leave?

Answer

17.5%

80. **Tipping** Four friends went out to lunch and the bill came to $53.75 They decided to add enough tip to make a total of $64, so that they could easily split the bill evenly among themselves. What percent tip did they leave?

Writing Exercises

81. What has been your past experience solving word problems? Where do you see yourself moving forward?

Answer

Answers will vary.

82. Without solving the problem “44 is 80% of what number” think about what the solution might be. Should it be a number that is greater than 44 or less than 44? Explain your reasoning.

83. After returning from vacation, Alex said he should have packed 50% fewer shorts and 200% more shirts. Explain what Alex meant.

Answer

Answers will vary.

84. Because of road construction in one city, commuters were advised to plan that their Monday morning commute would take 150% of their usual commuting time. Explain what this means.

Self Check

a. After completing the exercises, use this checklist to evaluate your mastery of the objective of this section.

I can...	Confidently	With some help	No-I don't get it!
use a problem-solving strategy for word problems.			
solve number problems.			
solve percent applications.			
solve simple interest applications.			

b. After reviewing this checklist, what will you do to become confident for all objectives?

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CHAPTER OVERVIEW

7: Statistics

7.1: Categorizing data
7.2: Presenting Categorical Data Graphically
7.3: Presenting Quantitative Data Graphically
7.4: Measures of Central Tendency

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7.1: Categorizing data

Once we have gathered data, we might wish to classify it. Roughly speaking, data can be classified as **categorical data** or **quantitative data**.

Quantitative and categorical data

- **Categorical (qualitative) data** are pieces of information that allow us to classify the objects under investigation into various categories.
- **Quantitative data** are responses that are numerical in nature and with which we can perform meaningful arithmetic calculations.

Example 3

We might conduct a survey to determine the name of the favorite movie that each person in a math class saw in a movie theater.

When we conduct such a survey, the responses would look like: *Finding Nemo*, *The Hulk*, or *Terminator 3: Rise of the Machines*. We might count the number of people who give each answer, but the answers themselves do not have any numerical values: we cannot perform computations with an answer like "*Finding Nemo*." This would be categorical data.

Example 4

A survey could ask the number of movies you have seen in a movie theater in the past 12 months (0, 1, 2, 3, 4, ...)

This would be quantitative data.

Other examples of quantitative data would be the running time of the movie you saw most recently (104 minutes, 137 minutes, 104 minutes, ...) or the amount of money you paid for a movie ticket the last time you went to a movie theater ($5.50, $7.75, $9, ...).

Sometimes, determining whether or not data is categorical or quantitative can be a bit trickier.

Example 5

Suppose we gather respondents' ZIP codes in a survey to track their geographical location.

ZIP codes are numbers, but we can't do any meaningful mathematical calculations with them (it doesn't make sense to say that 98036 is "twice" 49018 — that's like saying that Lynnwood, WA is "twice" Battle Creek, MI, which doesn't make sense at all), so ZIP codes are really categorical data.

Example 6

A survey about the movie you most recently attended includes the question "How would you rate the movie you just saw?" with these possible answers:

1 - it was awful
2 - it was just OK
3 - I liked it
4 - it was great
5 - best movie ever!

Again, there are numbers associated with the responses, but we can't really do any calculations with them: a movie that rates a 4 is not necessarily twice as good as a movie that rates a 2, whatever that means; if two people see the movie and one of them thinks it stinks and the other thinks it's the best ever it doesn't necessarily make sense to say that "on average they liked it."

As we study movie-going habits and preferences, we shouldn't forget to specify the population under consideration. If we survey 3-7 year-olds the runaway favorite might be *Finding Nemo*. 13-17 year-olds might prefer *Terminator 3*. And 33-37 year-olds might

prefer...well, *Finding Nemo*.

? Try it Now 3

Classify each measurement as categorical or quantitative

a. Eye color of a group of people
b. Daily high temperature of a city over several weeks
c. Annual income

Answer

a. Categorical
b. Quantitative
c. Quantitative

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7.2: Presenting Categorical Data Graphically

Categorical, or qualitative, data are pieces of information that allow us to classify the objects under investigation into various categories. We usually begin working with categorical data by summarizing the data into a **frequency table.**

Frequency Table

A frequency table is a table with two columns. One column lists the categories, and another for the frequencies with which the items in the categories occur (how many items fit into each category).

Example 1

An insurance company determines vehicle insurance premiums based on known risk factors. If a person is considered a higher risk, their premiums will be higher. One potential factor is the color of your car. The insurance company believes that people with some color cars are more likely to get in accidents. To research this, they examine police reports for recent total-loss collisions. The data is summarized in the frequency table below.

Color	Frequency
Blue	25
Green	52
Red	41
White	36
Black	39
Grey	23

Sometimes we need an even more intuitive way of displaying data. This is where charts and graphs come in. There are many, many ways of displaying data graphically, but we will concentrate on one very useful type of graph called a bar graph. In this section we will work with bar graphs that display categorical data; the next section will be devoted to bar graphs that display quantitative data.

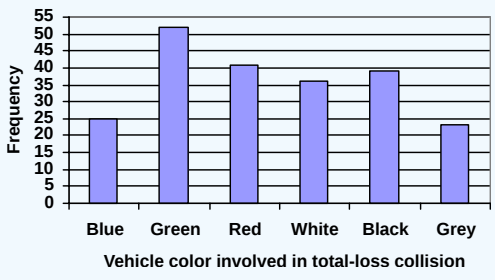
Bar graph

A **bar graph** is a graph that displays a bar for each category with the length of each bar indicating the frequency of that category.

To construct a bar graph, we need to draw a vertical axis and a horizontal axis. The vertical direction will have a scale and measure the frequency of each category; the horizontal axis has no scale in this instance. The construction of a bar chart is most easily described by use of an example.

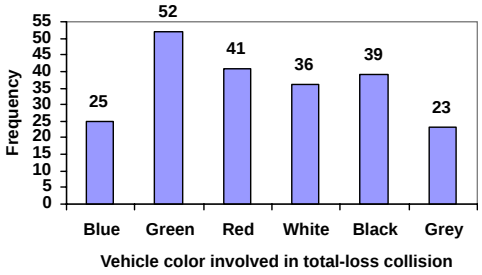
Example 2

Using our car data from above, note the highest frequency is 52, so our vertical axis needs to go from 0 to 52, but we might as well use 0 to 55, so that we can put a hash mark every 5 units:



Notice that the height of each bar is determined by the frequency of the corresponding color. The horizontal gridlines are a nice touch, but not necessary. In practice, you will find it useful to draw bar graphs using graph paper, so the gridlines will already be in

place, or using technology. Instead of gridlines, we might also list the frequencies at the top of each bar, like this:



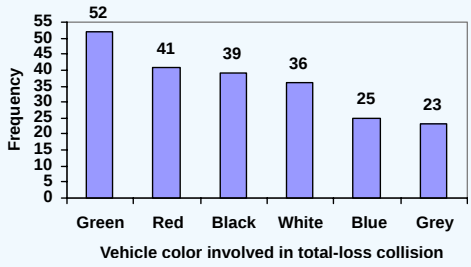
In this case, our chart might benefit from being reordered from largest to smallest frequency values. This arrangement can make it easier to compare similar values in the chart, even without gridlines. When we arrange the categories in decreasing frequency order like this, it is called a **Pareto chart**.

Pareto chart

A **Pareto chart** is a bar graph ordered from highest to lowest frequency

Example 3

Transforming our bar graph from earlier into a Pareto chart, we get:



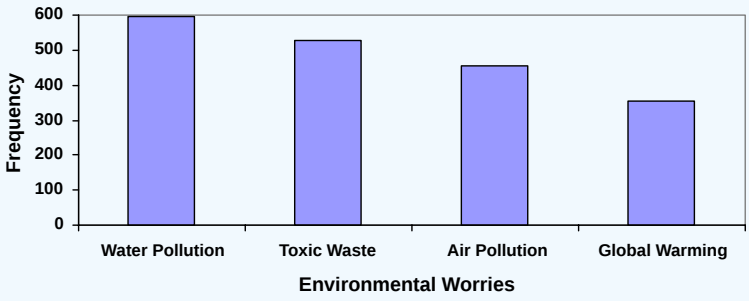
Example 4

In a survey[1], adults were asked whether they personally worried about a variety of environmental concerns. The numbers (out of 1012 surveyed) who indicated that they worried "a great deal" about some selected concerns are summarized below.

Environmental Issue	Frequency
Pollution of drinking water	597
Contamination of soil and water by toxic waste	526
Air pollution	455
Global warming	354

Solution

This data could be shown graphically in a bar graph:



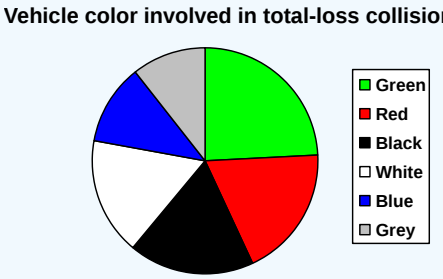
To show relative sizes, it is common to use a pie chart.

Pie Chart

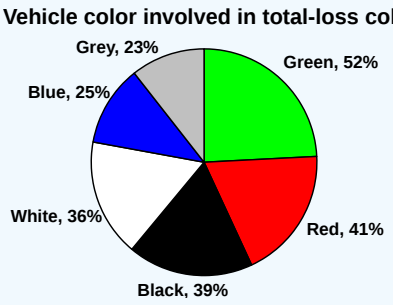
A **pie chart** is a circle with wedges cut of varying sizes marked out like slices of pie or pizza. The relative sizes of the wedges correspond to the relative frequencies of the categories.

Example 5

For our vehicle color data, a pie chart might look like this:



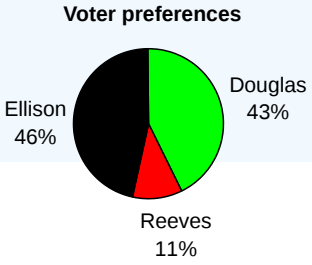
Pie charts can often benefit from including frequencies or relative frequencies (percents) in the chart next to the pie slices. Often having the category names next to the pie slices also makes the chart clearer.



Example 6

The pie chart to the right shows the percentage of voters supporting each candidate running for a local senate seat.

If there are 20,000 voters in the district, the pie chart shows that about 11% of those, about 2,200 voters, support Reeves.



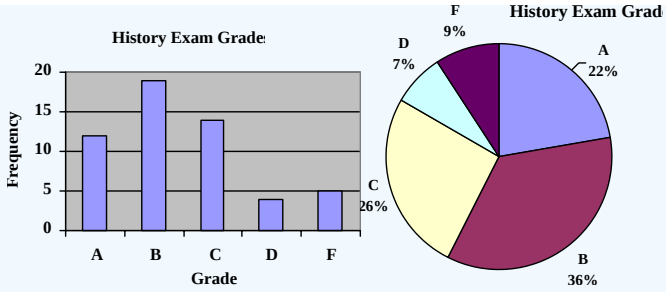
Pie charts look nice, but are harder to draw by hand than bar charts since to draw them accurately we would need to compute the angle each wedge cuts out of the circle, then measure the angle with a protractor. Computers are much better suited to drawing pie charts. Common software programs like Microsoft Word or Excel, OpenOffice.org Write or Calc, or Google Docs are able to create bar graphs, pie charts, and other graph types. There are also numerous online tools that can create graphs[2].

Try it Now 1

Create a bar graph and a pie chart to illustrate the grades on a history exam below.

A: 12 students, B: 19 students, C: 14 students, D: 4 students, F: 5 students

Answer



Don't get fancy with graphs! People sometimes add features to graphs that don't help to convey dd8.svg their information. For example, 3-dimensional bar charts like the one shown below are usually not as effective as their two-dimensional counterparts.

Here is another way that fanciness can lead to trouble. Instead of plain bars, it is tempting to substitute meaningful images. This type of graph is called a **pictogram**.

Pictogram

A **pictogram** is a statistical graphic in which the size of the picture is intended to represent the frequencies or size of the values being represented.

Example 7

A labor union might produce the graph to the right to show the difference between the average manager salary and the average worker salary.

Looking at the picture, it would be reasonable to guess that the manager salaries is 4 times as large as the worker salaries – the area of the bag looks about 4 times as large. However, the manager salaries are in fact only twice as large as worker salaries, which were reflected in the picture by making the manager bag twice as tall.

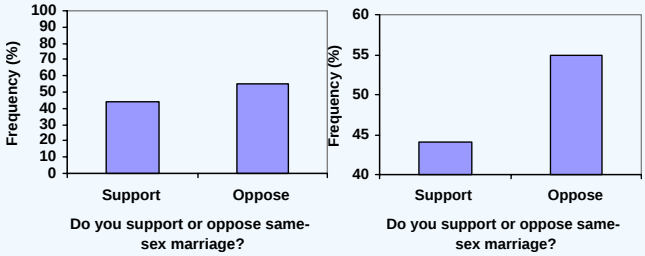


Manager Salaries **Worker Salaries**

Another distortion in bar charts results from setting the baseline to a value other than zero. The baseline is the bottom of the vertical axis, representing the least number of cases that could have occurred in a category. Normally, this number should be zero.

Example 8

Compare the two graphs below showing support for same-sex marriage rights from a poll taken in December 2008[3]. The difference in the vertical scale on the first graph suggests a different story than the true differences in percentages; the second graph makes it look like twice as many people oppose marriage rights as support it.

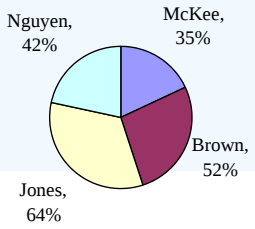


Try it Now 2

A poll was taken asking people if they agreed with the positions of the 4 candidates for a county office. Does the pie chart present a good representation of this data? Explain.

Answer

While the pie chart accurately depicts the relative size of the people agreeing with each candidate, the chart is confusing, since usually percents on a pie chart represent the percentage of the pie the slice represents.



[1] Gallup Poll. March 5-8, 2009. http://www.pollingreport.com/enviro.htm

[2] For example: http://nces.ed.gov/nceskids/createAgraph/ or http://docs.google.com

[3]CNN/Opinion Research Corporation Poll. Dec 19-21, 2008, from http://www.pollingreport.com/civil.htm

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7.3: Presenting Quantitative Data Graphically

Quantitative, or numerical, data can also be summarized into frequency tables.

Example 9

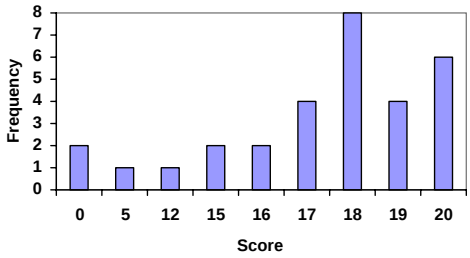
A teacher records scores on a 20-point quiz for the 30 students in his class. The scores are:

19 20 18 18 17 18 19 17 20 18 20 16 20 15 17 12 18 19 18 19 17 20 18 16 15 18 20 5 0 0

These scores could be summarized into a frequency table by grouping like values:

Score	Frequency
0	2
5	1
12	1
15	2
16	2
17	4
18	8
19	4
20	6

Using this table, it would be possible to create a standard bar chart from this summary, like we did for categorical data:



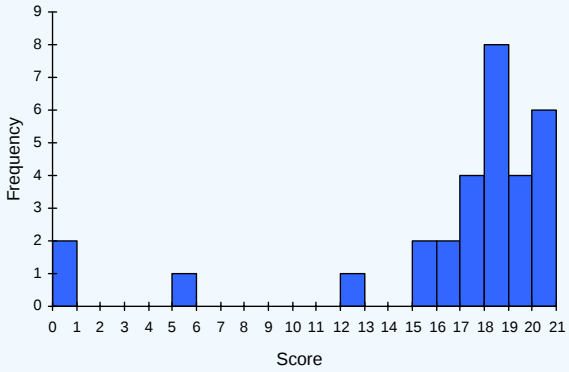
However, since the scores are numerical values, this chart doesn't really make sense; the first and second bars are five values apart, while the later bars are only one value apart. It would be more correct to treat the horizontal axis as a number line. This type of graph is called a **histogram**.

Histogram

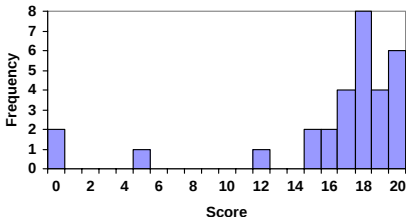
A histogram is like a bar graph, but where the horizontal axis is a number line

Example 10

For the values above, a histogram would look like:



Notice that in the histogram, a bar represents values on the horizontal axis from that on the left hand-side of the bar up to, but not including, the value on the right hand side of the bar. Some people choose to have bars start at ½ values to avoid this ambiguity.



Unfortunately, not a lot of common software packages can correctly graph a histogram. About the best you can do in Excel or Word is a bar graph with no gap between the bars and spacing added to simulate a numerical horizontal axis.

If we have a large number of widely varying data values, creating a frequency table that lists every possible value as a category would lead to an exceptionally long frequency table, and probably would not reveal any patterns. For this reason, it is common with quantitative data to group data into **class intervals**.

Class Intervals

Class intervals are groupings of the data. In general, we define class intervals so that:

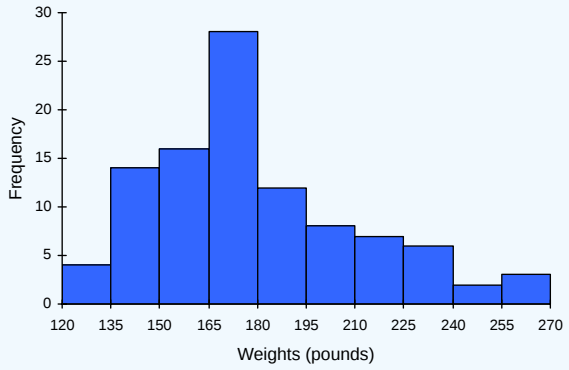
- Each interval is equal in size. For example, if the first class contains values from 120-129, the second class should include values from 130-139.
- We have somewhere between 5 and 20 classes, typically, depending upon the number of data we're working with.

Example 11

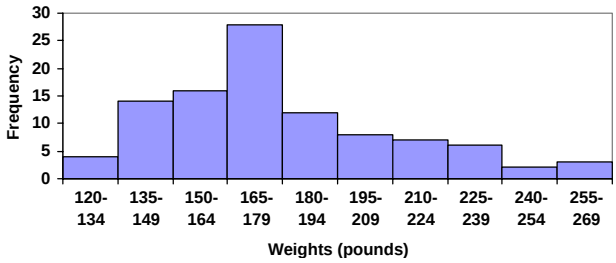
Suppose that we have collected weights from 100 male subjects as part of a nutrition study. For our weight data, we have values ranging from a low of 121 pounds to a high of 263 pounds, giving a total span of 263-121 = 142. We could create 7 intervals with a width of around 20, 14 intervals with a width of around 10, or somewhere in between. Often time we have to experiment with a few possibilities to find something that represents the data well. Let us try using an interval width of 15. We could start at 121, or at 120 since it is a nice round number.

Interval	Frequency
$120-134$	4
$135-149$	14
$150-164$	16
$165-179$	28
$180-194$	12
$195-209$	8
$210-224$	7
$225-239$	6
$240-254$	2
$255-269$	3

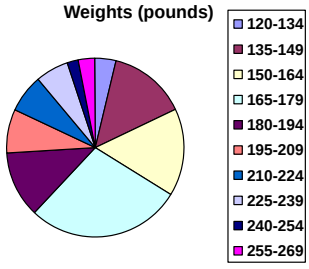
A histogram of this data would look like:



In many software packages, you can create a graph similar to a histogram by putting the class intervals as the labels on a bar chart.



Other graph types such as pie charts are possible for quantitative data. The usefulness of different graph types will vary depending upon the number of intervals and the type of data being represented. For example, a pie chart of our weight data is difficult to read because of the quantity of intervals we used.



Try it Now 3

The total cost of textbooks for the term was collected from 36 students. Create a histogram for this data.

$140 $160 $160 $165 $180 $220 $235 $240 $250 $260 $280 $285

$285 $285 $290 $300 $300 $305 $310 $310 $315 $315 $320 $320

$330 $340 $345 $350 $355 $360 $360 $380 $395 $420 $460 $460

Answer

Using a class intervals of size 55, we can group our data into six intervals:

cost interval	Frequency
$140 – 194	5
$195 – 249	3
$250 – 304	9
$305 – 359	12
$360 – 414	4
$415 – 469	3

We can use the frequency distribution to generate the histogram.

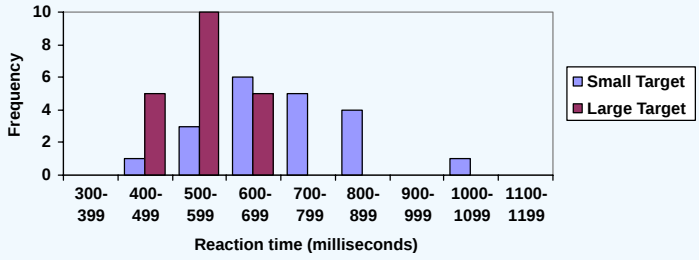
When collecting data to compare two groups, it is desirable to create a graph that compares quantities.

Example 12

The data below came from a task in which the goal is to move a computer mouse to a target on the screen as fast as possible. On 20 of the trials, the target was a small rectangle; on the other 20, the target was a large rectangle. Time to reach the target was recorded on each trial.

Interval (milliseconds)	Frequency small target	Frequency large target
300 – 399	0	0
400 – 499	1	5
500 – 599	3	10
600 – 699	6	5
700 – 799	5	0
800 – 899	4	0
900 – 999	0	0
1000 – 1099	1	0
1100 – 1199	0	0

One option to represent this data would be a comparative histogram or bar chart, in which bars for the small target group and large target group are placed next to each other.

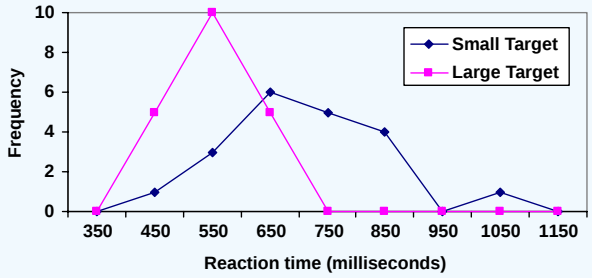


Frequency polygon

An alternative representation is a **frequency polygon**. A frequency polygon starts out like a histogram, but instead of drawing a bar, a point is placed in the midpoint of each interval at height equal to the frequency. Typically the points are connected with straight lines to emphasize the distribution of the data.

Example 13

This graph makes it easier to see that reaction times were generally shorter for the larger target, and that the reaction times for the smaller target were more spread out.



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7.4: Measures of Central Tendency

Let's begin by trying to find the most "typical" value of a data set.

Note that we just used the word "typical" although in many cases you might think of using the word "average." We need to be careful with the word "average" as it means different things to different people in different contexts. One of the most common uses of the word "average" is what mathematicians and statisticians call the **arithmetic mean**, or just plain old **mean** for short. "Arithmetic mean" sounds rather fancy, but you have likely calculated a mean many times without realizing it; the mean is what most people think of when they use the word "average".

Mean

The **mean** of a set of data is the sum of the data values divided by the number of values.

Example 14

Marci's exam scores for her last math class were: 79, 86, 82, 94. The mean of these values would be:

Solution

$$\frac{79+86+82+94}{4} = 85.25.$$

Typically we round means to one more decimal place than the original data had. In this case, we would round 85.25 to 85.3.

Example 15

The number of touchdown (TD) passes thrown by each of the 31 teams in the National Football League in the 2000 season are shown below.

37 33 33 32 29 28 28 23 22 22 22 21 21 21 20

20 19 19 18 18 18 18 16 15 14 14 14 12 12 9 6

Solution

Adding these values, we get 634 total TDs. Dividing by 31, the number of data values, we get $\frac{634}{31} = 20.4516$. It would be appropriate to round this to 20.5.

It would be most correct for us to report that "The mean number of touchdown passes thrown in the NFL in the 2000 season was 20.5 passes," but it is not uncommon to see the more casual word "average" used in place of "mean.

Try it Now 4

The price of a jar of peanut butter at 5 stores was: $3.29, $3.59, $3.79, $3.75, and $3.99. Find the mean price.

Answer

Add texts here. Do not delete this text first.

Example 16

The one hundred families in a particular neighborhood are asked their annual household income, to the nearest $5 thousand dollars. The results are summarized in a frequency table below.

Income (thousands of dollars)	Frequency
15	6
20	8
25	11
30	17
35	19
40	20
45	12
50	7

Solution

Calculating the mean by hand could get tricky if we try to type in all 100 values:

$$\frac{15+\cdots+15+20+\cdots+20+25+\cdots+25+\cdots}{100}$$

We could calculate this more easily by noticing that adding 15 to itself six times is the same as $15 \cdot 6 = 90$. Using this simplification, we get

$$\frac{15 \cdot 6+20 \cdot 8+25 \cdot 11+30 \cdot 17+35 \cdot 19+40 \cdot 20+45 \cdot 12+50 \cdot 7}{100} = \frac{3390}{100} = 33.9$$

The mean household income of our sample is 33.9 thousand dollars ($33,900).

Example 17

Extending off the last example, suppose a new family moves into the neighborhood example that has a household income of $5 million ($5000 thousand). Adding this to our sample, our mean is now:

Solution

$$\frac{15 \cdot 6+20 \cdot 8+25 \cdot 11+30 \cdot 17+35 \cdot 19+40 \cdot 20+45 \cdot 12+50 \cdot 7+5000 \cdot 1}{101} = \frac{8390}{101} = 83.069$$

While 83.1 thousand dollars ($83,069) is the correct mean household income, it no longer represents a “typical” value.

Imagine the data values on a see-saw or balance scale. The mean is the value that keeps the data in balance, like in the picture below.



If we graph our household data, the $5 million data value is so far out to the right that the mean has to adjust up to keep things in balance



For this reason, when working with data that have **outliers** – values far outside the primary grouping – it is common to use a different measure of center, the **median**.

Median

To find the median, begin by listing the data in order from smallest to largest, or largest to smallest.

If the number of data values, N, is odd, then the median is the middle data value. This value can be found by rounding $\frac{N}{2}$ up to the next whole number.

If the number of data values is even, there is no one middle value, so we find the mean of the two middle values (values $\frac{N}{2}$ and $\frac{N}{2}+1$)

Example 18

Returning to the football touchdown data, we would start by listing the data in order. Luckily, it was already in decreasing order, so we can work with it without needing to reorder it first.

37 33 33 32 29 28 28 23 22 22 22 21 21 21 20

20 19 19 18 18 18 18 16 15 14 14 14 12 12 9 6

Solution

Since there are 31 data values, an odd number, the median will be the middle number, the 16th data value (\frac{31}{2} = 15.5\), round up to 16, leaving 15 values below and 15 above). The 16th data value is 20, so the median number of touchdown passes in the 2000 season was 20 passes. Notice that for this data, the median is fairly close to the mean we calculated earlier, 20.5.

Example 19

Find the median of these quiz scores: 5 10 8 6 4 8 2 5 7 7

We start by listing the data in order: 2 4 5 5 6 7 7 8 8 10

Solution

Since there are 10 data values, an even number, there is no one middle number. So we find the mean of the two middle numbers, 6 and 7, and get $\frac{6+7}{2} = 6.5$.

The median quiz score was 6.5.

Try it Now 5

The price of a jar of peanut butter at 5 stores were: $3.29, $3.59, $3.79, $3.75, and $3.99. Find the median price.

Answer

First we put the data in order: $3.29, $3.59, $3.75, $3.79, $3.99. Since there are an odd number of data, the median will be the middle value, $3.75.

Example 20

Let us return now to our original household income data

Income (thousands of dollars)	Frequency
15	6
20	8
25	11
30	17
35	19
40	20
45	12
50	7

Solution

Here we have 100 data values. If we didn't already know that, we could find it by adding the frequencies. Since 100 is an even number, we need to find the mean of the middle two data values - the 50th and 51st data values. To find these, we start counting

up from the bottom:

There are 6 data values of \$15, so	Values 1 to 6 are \$15 thousand
The next 8 data values are \$20, so	Values 7 to $(6+8)=14$ are \$20 thousand
The next 11 data values are \$25, so	Values 15 to $(14+11)=25$ are \$25 thousand
The next 17 data values are \$30, so	Values 26 to $(25+17)=42$ are \$30 thousand
The next 19 data values are \$35, so	Values 43 to $(42+19)=61$ are \$35 thousand

From this we can tell that values 50 and 51 will be \$35 thousand, and the mean of these two values is \$35 thousand. The median income in this neighborhood is \$35 thousand.

Example 21

If we add in the new neighbor with a \$5 million household income, then there will be 101 data values, and the 51st value will be the median. As we discovered in the last example, the 51st value is \$35 thousand. Notice that the new neighbor did not affect the median in this case. The median is not swayed as much by outliers as the mean is.

In addition to the mean and the median, there is one other common measurement of the "typical" value of a data set: the **mode**.

Mode

The **mode** is the element of the data set that occurs most frequently.

The mode is fairly useless with data like weights or heights where there are a large number of possible values. The mode is most commonly used for categorical data, for which median and mean cannot be computed.

Example 22

In our vehicle color survey, we collected the data

Color	Frequency
Blue	3
Green	5
Red	4
White	3
Black	2
Grey	3

For this data, Green is the mode, since it is the data value that occurred the most frequently.

It is possible for a data set to have more than one mode if several categories have the same frequency, or no modes if each every category occurs only once.

Try it Now 6

Reviewers were asked to rate a product on a scale of 1 to 5. Find

a. The mean rating
b. The median rating
c. The mode rating

Rating	Frequency
1	4
2	8
3	7
4	3
5	1

Answer

a. The mean is $\frac{1 \cdot 4 + 2 \cdot 8 + 3 \cdot 7 + 4 \cdot 3 + 5 \cdot 1}{23} \approx 2.5$
b. There are 23 data values, so the median will be the 12th data value. Ratings of 1 are the first 4 values, while a rating of 2 are the next 8 values, so the 12th value will be a rating of 2. The median is 2.
c. The mode is the most frequent rating. The mode rating is 2.

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