MATH 091: ESSENTIALS OF INTERMEDIATE ALGEBRA



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Math 091 - Essentials of Intermediate Algebr

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Glossary

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Licensing

*A detailed breakdown of this resource's licensing can be found in **Back Matter/Detailed Licensing**.*

CHAPTER OVERVIEW

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1.1: It's All in the Mindset

Questions to consider

- What is a growth mindset, and how does it affect my learning?
- What are performance goals versus learning goals?

In the previous sections of this chapter you have focused on a number of concepts and models about learning. One of the things they all have in common is that they utilize different approaches to education by presenting new ways to think about learning. In each of these, the common element has been a better understanding of yourself as a learner and how to apply what you know about yourself to your own learning experience. If you were to distill all that you have learned in this chapter so far down to a single factor, it would be about using your mindset to your best advantage. In this next section, you will examine how all of this works in a broader sense by learning about the significance of certain mindsets and how they can hinder or promote your own learning efforts.



Figure 1.1.7: Many fields of study and work create intersections of growth and fixed mindset. People may feel great ability to grow and learn in some areas, like art and communication, but feel more limited in others, such as planning and financials. Recognizing these intersections will help you approach new topics and tasks. (Credit: mentatdgt / Pexels)

Performance vs. Learning Goals

As you have discovered in this chapter, much of our ability to learn is governed by our motivations and goals. What has not yet been covered in detail has been how sometimes hidden goals or mindsets can impact the learning process. In truth, we all have goals that we might not be fully aware of, or if we are aware of them, we might not understand how they help or restrict our ability to learn. An illustration of this can be seen in a comparison of a student that has *performance*-based goals with a student that has *learning*-based goals.

If you are a student with strict performance goals, your primary psychological concern might be to appear intelligent to others. At first, this might not seem to be a bad thing for college, but it can truly limit your ability to move forward in your own learning. Instead, you would tend to play it safe without even realizing it. For example, a student who is strictly performance-goal-oriented will often only says things in a classroom discussion when they think it will make them look knowledgeable to the instructor or their classmates. For example, a performance-oriented student might ask a question that she knows is beyond the topic being covered (e.g., asking about the economics of Japanese whaling while discussing the book *Moby Dick* in an American literature course). Rarely will they ask a question in class because they actually do not understand a concept. Instead they will ask questions that make them look intelligent to others or in an effort to “stump the teacher.” When they do finally ask an honest question, it may be because they are more afraid that their lack of understanding will result in a poor performance on an exam rather than simply wanting to learn.

If you are a student who is driven by learning goals, your interactions in classroom discussions are usually quite different. You see the opportunity to share ideas and ask questions as a way to gain knowledge quickly. In a classroom discussion you can ask for

clarification immediately if you don't quite understand what is being discussed. If you are a person guided by learning goals, you are less worried about what others think since you are there to learn and you see that as the most important goal.

Another example where the difference between the two mindsets is clear can be found in assignments and other coursework. If you are a student who is more concerned about performance, you may avoid work that is challenging. You will take the "easy A" route by relying on what you already know. You will not step out of your comfort zone because your psychological goals are based on approval of your performance instead of being motivated by learning.

This is very different from a student with a learning-based psychology. If you are a student who is motivated by learning goals, you may actively seek challenging assignments, and you will put a great deal of effort into using the assignment to expand on what you already know. While getting a good grade is important to you, what is even more important is the learning itself.

If you find that you sometimes lean toward performance-based goals, do not feel discouraged. Many of the best students tend to initially focus on performance until they begin to see the ways it can restrict their learning. The key to switching to learning-based goals is often simply a matter of first recognizing the difference and seeing how making a change can positively impact your own learning.

What follows in this section is a more in-depth look at the difference between performance- and learning-based goals. This is followed by an exercise that will give you the opportunity to identify, analyze, and determine a positive course of action in a situation where you believe you could improve in this area.

WHAT STUDENTS SAY

1. In the past, did you feel like you had control over your own learning?
    a. No. Someone has always dictated how and what I learned.
    b. Yes. I always look for ways to take control of what and how I learned.
    c. I am uncertain. I never thought about it before.
2. Have you ever heard of learning styles or do you know your own learning style?
    a. No. I have never heard of learning styles.
    b. Yes. I have heard of learning styles and know my own.
    c. Yes. I have heard of learning styles, but I don't think they're accurate or relate to me.
3. Which factors other than intelligence do you think have the greatest influence on learning?
    a. Motivation
    b. Perseverance
    c. Understanding how I learn
    d. Good teachers and support

You can also take the anonymous What Students Say surveys to add your voice to this textbook. Your responses will be included in updates.

Students offered their views on these questions, and the results are displayed in the graphs below.

In the past, did you feel like you had control over your own learning?

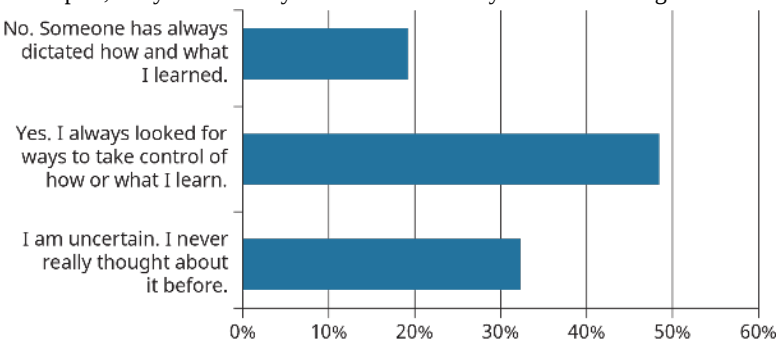


Figure 1.1.8

Have you ever heard of learning styles or do you know your own learning style?

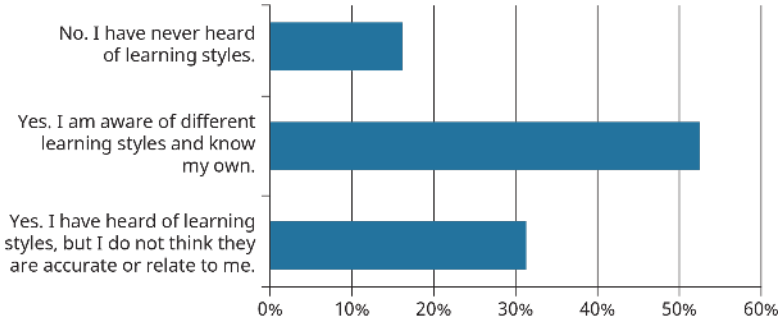


Figure 1.1.9

Which factors other than intelligence do you think have the greatest influence on learning?

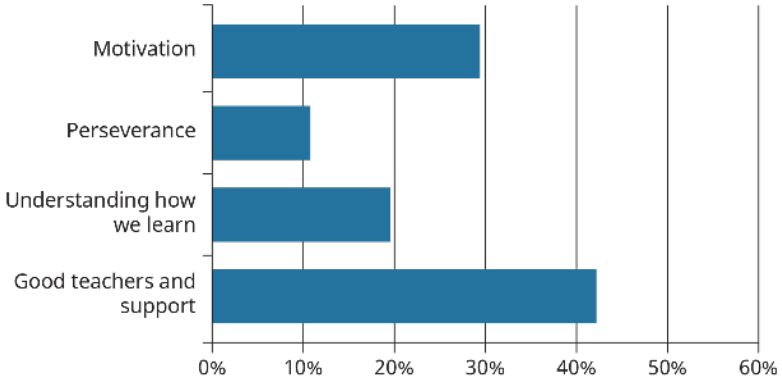


Figure 1.1.10

Fixed vs. Growth Mindset

The research-based model of these two mindsets and their influence on learning was presented in 1988 by Carol Dweck.[7] In Dr. Dweck's work, she determined that a student's perception about their own learning accompanied by a broader goal of learning had a significant influence on their ability to overcome challenges and grow in knowledge and ability. This has become known as the Fixed vs. Growth Mindset model. In this model, the *performance*-goal-oriented student is represented by the *fixed* mindset, while the *learning*-goal-oriented student is represented by the *growth* mindset.

In the following graphic, based on Dr. Dweck's research, you can see how many of the components associated with learning are impacted by these two mindsets.

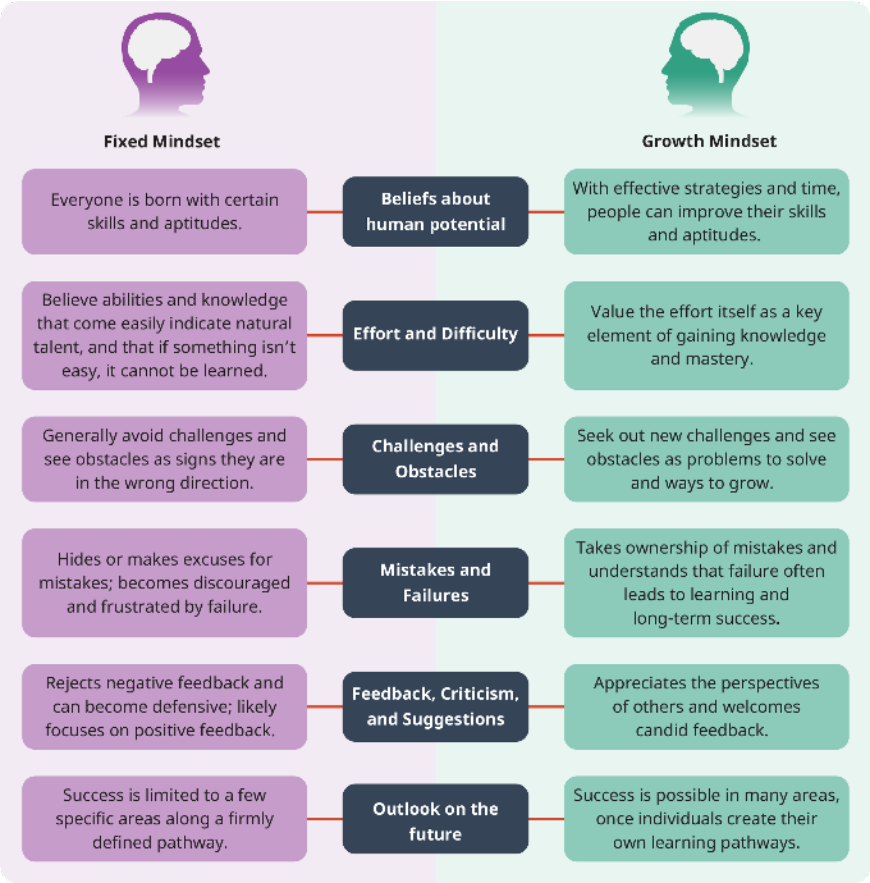


Figure 1.1.11: The differences between fixed and growth mindset are clear when aligned to key elements of learning and personality. (Credit: Based on work by Dr. Carol Dweck)

The Growth Mindset and Lessons About Failing

Something you may have noticed is that a growth mindset would tend to give a learner grit and persistence. If you had learning as your major goal, you would normally keep trying to attain that goal even if it took you multiple attempts. Not only that, but if you learned a little bit more with each try you would see each attempt as a success, even if you had not achieved complete mastery of whatever it was you were working to learn.

With that in mind, it should come as no surprise that Dr. Dweck found that those people who believed their abilities could change through learning (growth vs. a fixed mindset) readily accepted learning challenges and persisted despite early failures.

Improving Your Ability to Learn

As strange as it may seem, research into fixed vs. growth mindsets has shown that if you believe you can learn something new, you greatly improve your ability to learn. At first, this may seem like the sort of feel-good advice we often encounter in social media posts or quotes that are intended to inspire or motivate us (e.g., *believe in yourself!*), but in looking at the differences outlined between a fixed and a growth mindset, you can see how each part of the growth mindset path would increase your probability of success when it came to learning.

ACTIVITY

Very few people have a strict fixed or growth mindset all of the time. Often we tend to lean one way or another in certain situations. For example, a person trying to improve their ability in a sport they enjoy may exhibit all of the growth mindset traits and characteristics, but they find themselves blocked in a fixed mindset when they try to learn something in another area like computer programming or arithmetic.

In this exercise, do a little self-analysis and think of some areas where you may find yourself hindered by a fixed mindset. Using the outline presented below, in the far right column, write down how you can change your own behavior for each of the parts of the learning process. What will you do to move from a fixed to a growth mindset? For example, say you were trying to

learn to play a musical instrument. In the *Challenges* row, you might pursue a growth path by trying to play increasingly more difficult songs rather than sticking to the easy ones you have already mastered. In the *Criticism* row, you might take someone's comment about a weakness in timing as a motivation for you to practice with a metronome. For *Success of others* you could take inspiration from a famous musician that is considered a master and study their techniques.

Whatever it is that you decide you want to use for your analysis, apply each of the Growth characteristics to determine a course of action to improve.

Table 2.4

Parts of the learning process	Growth characteristic	What will you do to adopt a growth mindset?
Challenges	Embraces challenges	
Obstacles	Persists despite setbacks	
Effort	Sees effort as a path to success	
Criticism	Learns from criticism	
Success of Others	Finds learning and inspiration in the success of others	

Footnotes

- Dweck, C.S. & Leggett, E.L. (1988). A Social-Cognitive Approach to Motivation and Personality

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1.2: Failure Is Not an Option

Failure Is Not an Option

Nathan Wallace

In the movie Apollo 13, Ed Harris portrays NASA flight director Gene Kranz as he successfully guides the crew of a damaged spacecraft to safety. In a famous scene during which Kranz and his staff are attempting to overcome some extremely daunting challenges, Harris shouts, “Failure is not an option!” This singular statement perfectly articulated the determination of Kranz to bring the Apollo astronauts back to Earth.

This “failure is not an option” credo was perfect for the life and death situation that NASA was facing. Failure meant that the astronauts on Apollo 13 would never come home, and that outcome was unacceptable. Attending college, on the other hand, shouldn’t be a life or death experience, though it sometimes might feel like one. Failure, though never the intended outcome, can and sometimes does happen. Sometimes failure manifests itself in election results for a student government post, in a test score, or even in a final grade.

Throughout my life I have had many failures. In high school I drove my parents and teachers crazy because of my lack of academic achievement. I even managed to get an F- in Spanish on my report card. When I told my mom that it was a typo she responded, “So you didn’t get an F?” “No,” I said, “I definitely earned the F, but there’s no such thing as an F-.” To this day I’m not so sure that my reply was accurate. I might have earned that minus after all.

My failures in high school led to only one acceptance from of all the colleges I applied to attend. Furthermore, I was not accepted to the school’s main campus, but to their branch campus. During my first semester there my effort wasn’t much better than in high school, but since my parents were now paying for my education I did enough work to avoid academic probation. It wasn’t until my second semester that I found my niche as a Religious Studies major and started getting good grades, moved to the main campus, and eventually graduated with honors.

Since graduating from college, my career path has taken me into higher education as a Student Affairs administrator. This career has exposed me to many great theories regarding student success, and many of them gave me insight into my own college experience. But it was Stanford psychologist Carol Dweck who appeared to be thinking of me when she wrote the following about fixed mindsets in the introduction to her book titled *Mindset: The New Psychology of Success*:

Believing that your qualities are carved in stone—the fixed mindset—creates an urgency to prove yourself over and over. If you have only a certain amount of intelligence, a certain personality, and a certain moral character—well, then you’d better prove that you have a healthy dose of them. It simply wouldn’t do to look or feel deficient in these most basic characteristics. (Dweck, 2006)

This statement was a revelation to me. I finally understood my problem throughout high school and even in college. I earned good grades because I liked Religious Studies but never really challenged myself inside or outside of the classroom. My problem was that I had a fixed mindset about academic success. I believed that a person is either smart or they’re not, and nothing could be done to significantly change that. I also believed that I was one of the fortunate ones to be “gifted” with an abundance of intelligence.

One might think that having confidence in your intelligence is a whole lot better than thinking that you’re stupid, but the result was the same. My fixed mindset was holding me back because it led to a paralyzing fear of failure. Since as far back as I could remember, my family, friends, and teachers were always telling me how smart I was, and I believed them. But that belief was a double-edged sword. High school and college offered many occasions when self-confidence in my inherent intelligence could be threatened. If I fail on this test or in this course it means that I’m not the smart person I thought I was. If I fail, my family and friends will find out that they were wrong about me.

However, there was a way to avoid all of the risks of academic rigor. I could just not try. If I don’t try I’ll get bad marks on my report card, but those won’t be true indicators of my intelligence. By not putting forth any effort, my intelligence would never be disproven. I would always be able to say to myself and others that, “I could do the work and be a straight A student, but I’m just not interested.” Looking back on this time in my life, it is clear to me that this wasn’t a conscious decision to save face. It was fear, not logic, which was guiding my behavior.

After reading *Mindset* I have made a conscious effort to identify and thwart any remaining fixed mindset thoughts that I continue to hold. Dweck’s book acts as a manual for rooting out fixed mindset thoughts, because she explains that the idea of fixed mindsets is only half of her mindset theory. There is another kind of mindset, and she calls it growth mindset. Dweck writes that, “This growth

mindset is based on the belief that your basic qualities are things you can cultivate through your efforts" (Dweck, 2006). Dweck goes on to explain that we can choose to have a growth mindset about any type of ability, whether it's math, art, athletics, or any other skill that one wishes to cultivate.

I put this theory to the test not long after reading the book. A few years ago I attended a meeting only to find out that it wasn't any ordinary meeting. During this meeting we would be brainstorming solutions to a specific problem. This was going to be a true brainstorming session, led by a facilitator trained in the science of soliciting uninhibited ideas from an audience. As soon as I heard the word *brainstorming* I froze. I have always hated brainstorming. I'm the type of person that likes to think things through two or three times before expressing an opinion. My fear of failing at this task in front of my coworkers paralyzed my mind. I couldn't think.

That's when it hit me. This was fixed mindset thinking. My belief in my brainstorming inadequacies was preventing me from even trying. So I flipped this thinking on its head and decided the best way to improve my brainstorming abilities was to clear my mind and start firing out ideas. I gave it a shot, and though the ideas didn't come out at the prolific rate of some of my colleagues, I had never before had such a positive outcome and experience while brainstorming. Through this experience I found that I really could choose to have a growth mindset, and that this choice produces a greater chance of success. With a greater chance of success comes a smaller chance of failure.

Nevertheless, when it comes to academic success and success in all phases of life, failure is always an option. Though it can be painful, failure can lead to great learning and progress when a specific failure is analyzed through the lens of a growth mindset. By focusing more on effort than on outcomes anyone can learn and grow, regardless of their skill level. Therefore, to make the most of their time in college, students must seek out challenges that will stretch their abilities. These challenges can take many forms and they can occur in a variety of settings, both inside and outside of the classroom. When seeking out challenges there is always the possibility of agonizing defeat, but out of that defeat can be the seeds of great success in the future.

Reference

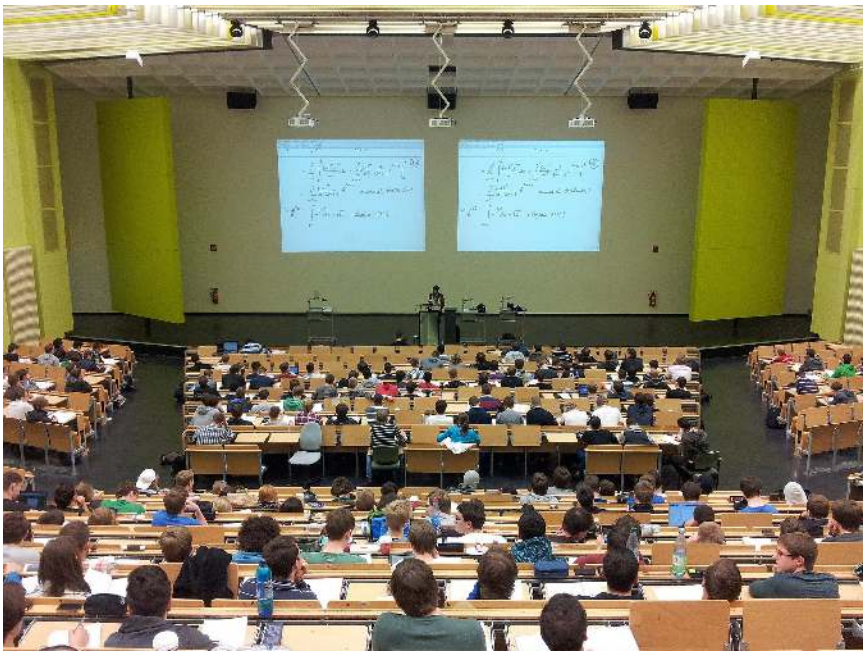
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1.3: Active Learning from Lectures

The learning of information and skills presented in class lectures or discussions should be viewed as a process: preparation to take in the new information, the act of taking in the new information, and then reviewing the information so that it is later accessible (recalled from memory) to use for a project, paper, or test.



The activities suggested for “Before Class” in Table 5-2 help students develop a mental framework into which this new information can be “filed” or organized which aids information recall (Bower, Clark, Lesgold, & Winzenz, 1969). It also makes note taking easier because you are familiar with the material and feel less panicked about getting everything noted in full detail.

Attending class and taking the notes is important primarily because the professor may be presenting information that is not presented in the text. It should also be seen as a structured opportunity to engage with the material that will further aid recall when needed. Oftentimes, students can also get clues as to what a professor perceives as important material from attending class that will aid in test preparation.

Table 5-2: Effective Techniques for Learning from Class

Before Class	During Class	After Class
Complete assigned readings to prepare for class. Take notes or mark your text for easy reference.	Take notes. Active engagement with the material (like writing down what you are hearing) helps improve memory and concentration and material to later review. Have a positive mindset about taking notes you are writing down answers to questions on the test.”	Review your notes and handouts within 24 hours of taking them. Fill in details that you remember but did not have time to write down. Write questions for the notes that you can use for self-testing later. Write a summary of the day’s notes.
Prepare questions that you may have about the reading.	Put a date on your note paper. It helps to also include the text chapter or general topic that might be related to these notes as well.	Compare your notes with a reliable classmate who may help give you ideas on what you missed that might be important or how to better organize the information. Discuss the notes to add another-layer of processing.

Before Class	During Class	After Class
Review notes from the previous class period to get a sense of where you may have left off.	Use active listening techniques. Refer to the informational box called “Ten Tips for Active Listening” for more information.	Follow up on questions that you have written down with your professor, the teaching assistant, or another reliable classmate.
Be sure you have your materials ready for class: text, three-ring notebook with paper, writing utensils, highlighters, charged laptop, etc.	Have your textbook accessible in case your faculty member refers to specific information. Write down that reference in your notes as it is probably important information.	Integrate your class notes with your text notes or markings. If you highlighted in your book, add some of that information to your notes for the “whole” picture.
Get a good night’s sleep. It is easier to concentrate on the lecture when you are alert.	Write down the main points a speaker is making along with any supple-mental details like examples or experiments. Do not expect to be able to write down every word the speaker says.	Develop tools that swill serve to help you review the material now, as well as later. Create visual diagrams* of concepts, develop ram cards* of concepts, reorganize your notes, etc. *Refer to informational boxes on “Visualize Your Information” and “Cram Cards for Long-Term Review.”
Make it a habit to attend every class. Put your classes on your schedule and treat them as a limited opportunity to engage with your professor and this material.	Use abbreviations for commonly used words to increase your speed in note taking. Create some abbreviations of your own.	Self-test your comprehension of the information on an ongoing basis. Do not just “look over” or read what you have written. Use the questions you have created. Cover up the “answer” and see if you can explain what is in the notes. Use the cram cards or visual diagrams to test yourself.
If you cannot attend class due to illness or an-other extraordinary reason, contact a classmate to arrange for notes. It is always a good idea to send an e-mail or call your professor as well.	Develop a system of organization for your notes that works for you. Common methods include the Cornell, Outlining, Mapping, Text-Class, and Sentence Methods. Refer to informational box called “Note Taking Methods: What Is Right for You?”	
	Leave space between main points so that you can add in new information that you may have missed or from the text.	
	If the professor writes it down or puts it on a PowerPoint slide, it is probably important, so write it down.	
	If the professor repeats it, it is important, so write it down.	
	Listen for cues like “very important,” “in summary,” or sequence words like “first,” “second,” etc. This information provides cues for what is important and possible note organization.	
	Ask questions you may have. If you are not comfortable talking in class, write them down and talk with your professor later.	

Adapted from Dembo Sell (2008) and Downing (2008).

Ten Tips for Active Listening

Effective note taking in class requires the use of active listening techniques for optimal results.

1. Accept responsibility as a listener.
2. Adopt a positive attitude toward listening. Listening is a choice.
3. Sit somewhere in the classroom that will minimize distractions and allow you to focus on the professor.
4. Maintain eye contact with your professor.
5. Focus on the content being presented rather than the delivery.
6. Ask questions in class.
7. Ask mental questions and search for the answer in the lecture or discussion.
8. Avoid emotional involvement that may impair your ability to concentrate or filter important information.
9. Clue in on non-verbal communication, such as exaggerated movements, excitement, etc., as it may serve as a cue to important information.
10. Monitor your concentration throughout the class period and continually refocus.

Adapted from Kline (2002) and Treuer (2006).



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1.4: Active Learning from Reading

As with learning from lectures and discussions, the learning of information and skills presented in readings, texts, and written text from a Web site should be viewed as a process: preparation to take in the new information, the act of taking in the new information, and then reviewing the information so that it is later accessible (recalled from memory) to use for a project, paper, or test.



Students who simply start and finish a reading with no further actions taken are really wasting their time. Reading a text needs to be approached properly in order to ensure comprehension and retention of the information. Planning for an active reading session by engaging in the "before reading" activities facilitates an active mindset to the reading session and discourages "auto-pilot reading" that results in scanning of the words but no real careful thought about what is being said.

Preparing for class by reading assigned texts related to the lecture material is extremely important to optimize the lecture experience. If you have a professor who tends to lecture at a fast pace, you have the foundational information and only need to focus on writing down unfamiliar information. If your professor expects class participation in discussions or utilizes active classroom techniques, such as case studies, small group activities and discussions, simulations, or demonstrations, you will understand what is happening and be able to contribute to the learning experience. Marking or highlighting your text for main ideas while reading not only keeps your mind focused on the reading but also serves to prepare you for easier exam review.

Table 5-3. Effective Techniques for Reading

Before Reading	During Reading	After Reading
Preview the reading headings to get a "big picture" of the outline of the reading. Look at the pictures and figures. Check out the bold and italicized words.	As you are reading, seek out the answers to the questions that the reading or you generated rather than passively scanning the words.	Take reading notes from the highlighted material from the text. Utilize visual organizers and summaries to capture information. Revisit the informational box on "Note-Taking Methods: What Is Right for You?" and visit "Visualize Your information and "Cram Cards for Long-term Review."
Preview any questions that the chapter may offer so that you can actively seek answers to those questions.	Monitor your concentration and comprehension. After each paragraph, ask yourself, "What was the main point of that paragraph?" After each section, summarize what you have read.	Answer the questions that a the reading may have provided or that you developed. Reread sections in which you remember little information.
Turn text headings into questions so that you can actively seek answers to those questions. Jot those questions down in the margin of the text or in your notes if you plan to take reading notes.	Mark your text. Highlight the main idea of a paragraph or write down the main idea of a paragraph in your notes. If you cannot mark your text, take reading notes on the main ideas. Refer to the informational box called "In Search of Main Ideas" for more information	Integrate your reading notes and lecture/discussion notes into one location for easier review for an exam.
Plan for a high energy time of the day to read. Consider breaking up a long reading assignment into multiple, shorter reading sessions.	Look up the definitions to words that you do not understand to help build your vocabulary and facilitate understanding of the topic.	Discuss readings with classmates on a regular basis. Take turns explaining to each other sections of the reading. Ask and answer reading questions. Jot down questions that are unclear to the study group to ask the professor in class or during office hours.
Be sure you have your materials ready for reading: text, notebook paper, writing utensils, high-lighters, charged laptop, etc.		

Adapted from Dembo and Seli (2008) and Downing (2008).

In Search of Main Ideas

Some students do not actively take notes or mark their readings and texts because they determine it is a lot of extra work to write down so much information. However, the problem is that students are usually writing down too much information and not really cluing in to the main ideas. Here are some tips for identifying the main ideas when reading:

1. The Table of Contents and chapter outlines provide a broad view of the main points that will be covered in a reading. Flesh out the outlines that are already provided for you.
2. Oftentimes in a textbook, the main idea is the first or last sentence of a paragraph. If it is not the first or the last sentence, then look back at the entire paragraph to see what the overall issue seems to be. Look for the overall pat- terns of your textbooks.
3. Titles, headings, and subheadings announce the major subject. Make these headings into questions, and the answers to the questions will likely be the main ideas.
4. Bold and italic words point to a main idea or key concept that you need to understand.
5. Repetition of key words or phrases throughout the text point to a main idea.
6. Chapter questions at the end of the chapter are about the main ideas of the text. Answer those questions and you will identify your main ideas.
7. Summaries presented at the end of the chapter also tend to restate the main ideas briefly. Flesh these ideas out with some supporting ideas, and you would have a good view of the entire chapter.
8. Stop and look at the visuals—pictures, diagrams, tables, etc. Oftentimes, the message depicted in the graph or picture is typically a main idea.
9. Detailed statistics, several examples in a row, and other details often signal that a main idea is being clarified, proven, supported, etc. Track back or ahead to find the main idea they are trying to illustrate.
10. Text that includes bullet points, numbering, or sequences is often a main idea.
11. Look for organizational patterns in the reading that might highlight the main ideas. For instance, are two issues being compared or contrasted? What was the effect of a certain event? Are problems and various solutions being presented? Is there a timeline of events that is important?
12. Be intentional about searching for the main ideas. Ask yourself at the end of each section or paragraph, "What is the point?" or "What is it that the author wants me to know?"
13. If you are reading a narrative, ask yourself questions like, "Who are the main characters?" "Why is this character important to this story?" "Why did the author chose to tell this part of the story?"

Finally, taking the time to think critically about the reading you have done further solidifies that information in your memory and can serve to prepare you for exam review. Summarizing material using visual organizers (refer to informational box "Visualize Your Information" and Figure 5-1) moves your thinking from low levels of thinking to high levels of thinking on Bloom's Taxonomy. Integrating text and lecture notes and discussing readings with your classmates and professor promotes memory storage of that information and thinking at higher levels as well.

Visualize Your Information

Summarizing main ideas of the readings or lectures can be difficult without the right tools. Visual organizers can help students condense important information into visual diagrams that can be easily reviewed for exams or help organize a writing assignment. Muskingum College's Center for Advancement and Learning (n.d.) hosts a useful Web site that summarizes and provides examples of various visual organizers (a.k.a. Information Organization Strategies) at

muskingum.edu/~cal/database/g...anization.html (Web site wants username and password now.)

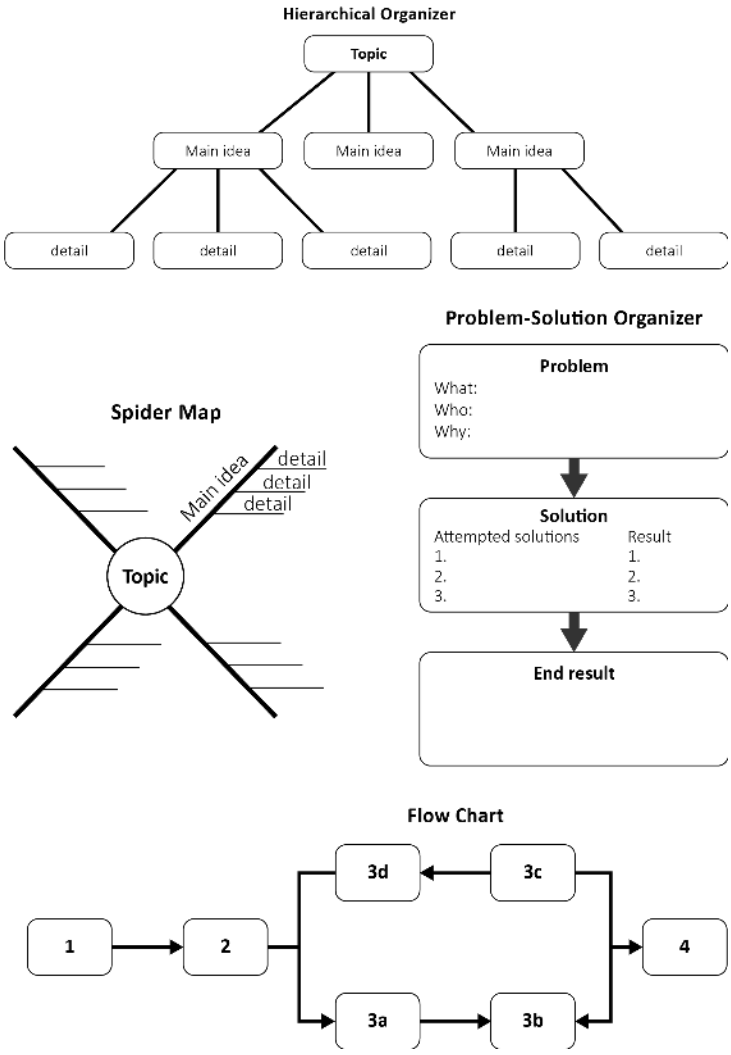
Consider how you might be able to apply several pages of text to a simple flowchart, hierarchical organizer, comparison-contrast organizer, spider map, etc. (See Figure 5-1.)

An additional benefit of using visual organizers is that you are thinking about the material at a deeper level as you identify the proper visual tool and apply the information. While at first it may take more time to determine the proper tool, it will get easier with practice and become second nature.

Activity 5-2

Utilizing what you have learned from the informational boxes "In Search of Main Ideas" and "Visualize Your Information," select a text chapter or a week of class notes and create some visual organizers that summarize the main ideas of that material. Ask a classmate to share in the same exercise and compare your work in a critical manner. What have you learned from this exercise and each other?

Figure 5-1. Examples of visual organizers



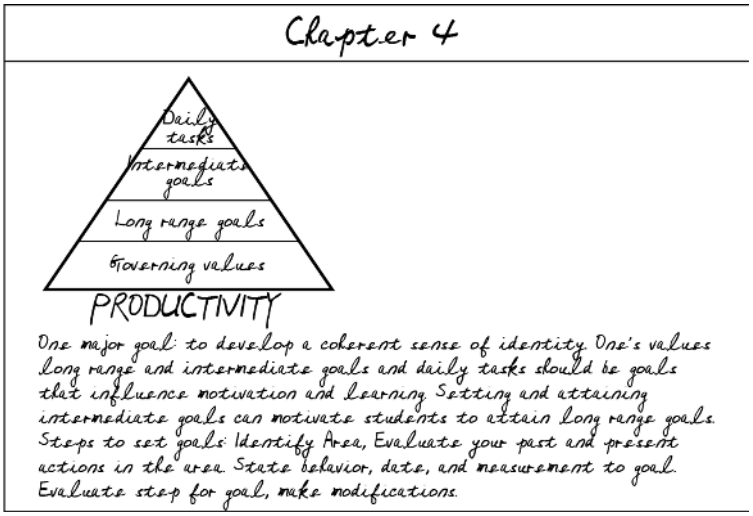
Source: Muskingum College's Center for Advancement and Learning (n.d)

Cram Cards for Long-term Review

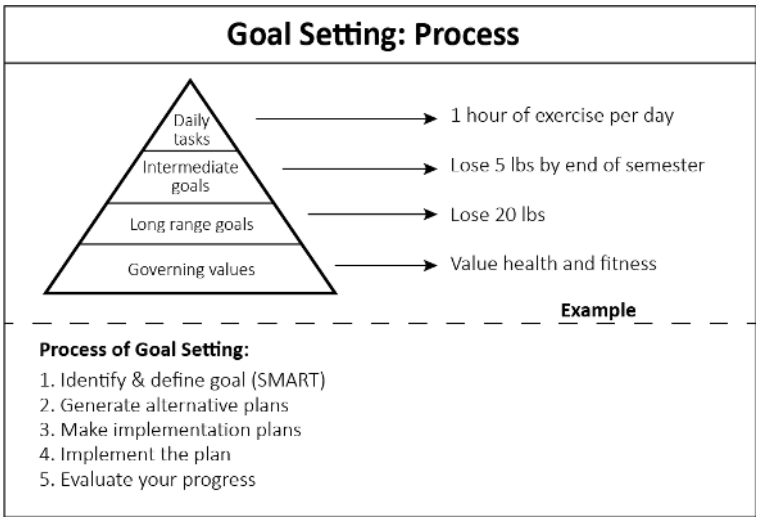
The use of "cram" in cram cards is deceiving. Cram cards are a study tool that promote the active learning of reading or lecture notes and allows for the easy and portable review of material over a longer period of time, therefore promoting later recall. It is a technique that will take some practice to hone your skills in selecting main ideas, but students who use cram cards indicate it is well worth their time.

Tips for Writing Cram Cards:

- Use 4 x 6 index cards (3 x 5 size tends to be too small)—some enthusiasts will even use the colored index cards for different sources of information (i.e., notes versus text reading) or topics (e.g., blue for English and green for Biology).
- Make one cram card per major concept or main idea.
- Do not simply take notes on the card; capture a summary of the main ideas maintaining the proper relationships of the concepts.
- Write the information in your own words.
- Include book examples or provide real-life examples for difficult concepts.
- Include key words.
- Be sure to write legibly.



Example of a Bad Cram Card



Example of a Good Cram Card

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1.5: Taking Notes

Questions to consider

- How can you prepare to take notes to maximize the effectiveness of the experience?
- What are some specific strategies you can employ for better notetaking?
- Why is annotating your notes after the notetaking session a critical step to follow?

Beyond providing a record of the information you are reading or hearing, notes help you organize the ideas and help you make meaning out of something about which you may not be familiar, so notetaking and reading are two compatible skill sets. Taking notes also helps you stay focused on the question at hand. Nanami often takes notes during presentations or class lectures so she can follow the speaker's main points and condense the material into a more readily usable format. Strong notes build on your prior knowledge of a subject, help you discuss trends or patterns present in the information, and direct you toward areas needing further research or reading.



Figure 1.5.9 Strong notes build on your prior knowledge of a subject, help you discuss trends or patterns present in the information, and direct you toward areas needing further research or reading.

It is not a good habit to transcribe every single word a speaker utters—even if you have an amazing ability to do that. Most of us don't have that court-reporter-esque skill level anyway, and if we try, we would end up missing valuable information. Learn to listen for main ideas and distinguish between these main ideas and details that typically support the ideas. Include examples that explain the main ideas, but do so using understandable abbreviations.

Think of all notes as potential study guides. In fact, if you only take notes without actively working on them after the initial notetaking session, the likelihood of the notes helping you is slim. Research on this topic concludes that without active engagement after taking notes, most students forget 60–75 percent of material over which they took the notes—within two days! That sort of defeats the purpose, don't you think? This information about memory loss was first brought to light by 19th-century German psychologist Hermann Ebbinghaus. Fortunately, you do have the power to thwart what is sometimes called the Ebbinghaus Forgetting Curve by reinforcing what you learned through review at intervals shortly after you take in the material and frequently thereafter.

If you are a musician, you'll understand this phenomenon well. When you first attempt a difficult piece of music, you may not remember the chords and notes well at all, but after frequent practice and review, you generate a certain muscle memory and cognitive recall that allows you to play the music more easily.

Notetaking may not be the most glamorous aspect of your higher-education journey, but it is a study practice you will carry throughout college and into your professional life. Setting yourself up for successful notetaking is almost as important as the actual taking of notes, and what you do after your notetaking session is equally significant. Well-written notes help you organize your thoughts, enhance your memory, and participate in class discussion, and they prepare you to respond successfully on exams. With all that riding on your notes, it would behoove you to learn how to take notes properly and continue to improve your notetaking skills.

Analysis Question

Do you currently have a preferred way to take notes? When did you start using it? Has it been effective? What other strategy might work for you?

Preparing to Take Notes

Preparing to take notes means more than just getting out your laptop or making sure you bring pen and paper to class. You'll do a much better job with your notes if you understand why we take notes, have a strong grasp on your preferred notetaking system, determine your specific priorities depending on your situation, and engage in some version of efficient shorthand.

Like handwriting and fingerprints, we all have unique and fiercely independent notetaking habits. These understandably and reasonably vary from one situation to the next, but you can only improve your skills by learning more about ways to take effective notes and trying different methods to find a good fit.

The very best notes are the ones you take in an organized manner that encourages frequent review and use as you progress through a topic or course of study. For this reason, you need to develop a way to organize all your notes for each class so they remain together and organized. As old-fashioned as it sounds, a clunky three-ring binder is an excellent organizational container for class notes. You can easily add to previous notes, insert handouts you may receive in class, and maintain a running collection of materials for each separate course. If the idea of carrying around a heavy binder has you rolling your eyes, then transfer that same structure into your computer files. If you don't organize your many documents into some semblance of order on your computer, you will waste significant time searching for improperly named or saved files.

You may be interested in relatively new research on what is the more effective notetaking strategy: handwriting versus typing directly into a computer. While individuals have strong personal opinions on this subject, most researchers agree that the format of student notes is less important than what students do with the notes they take afterwards. Both handwriting notes and using a computer for notetaking have pros and cons.

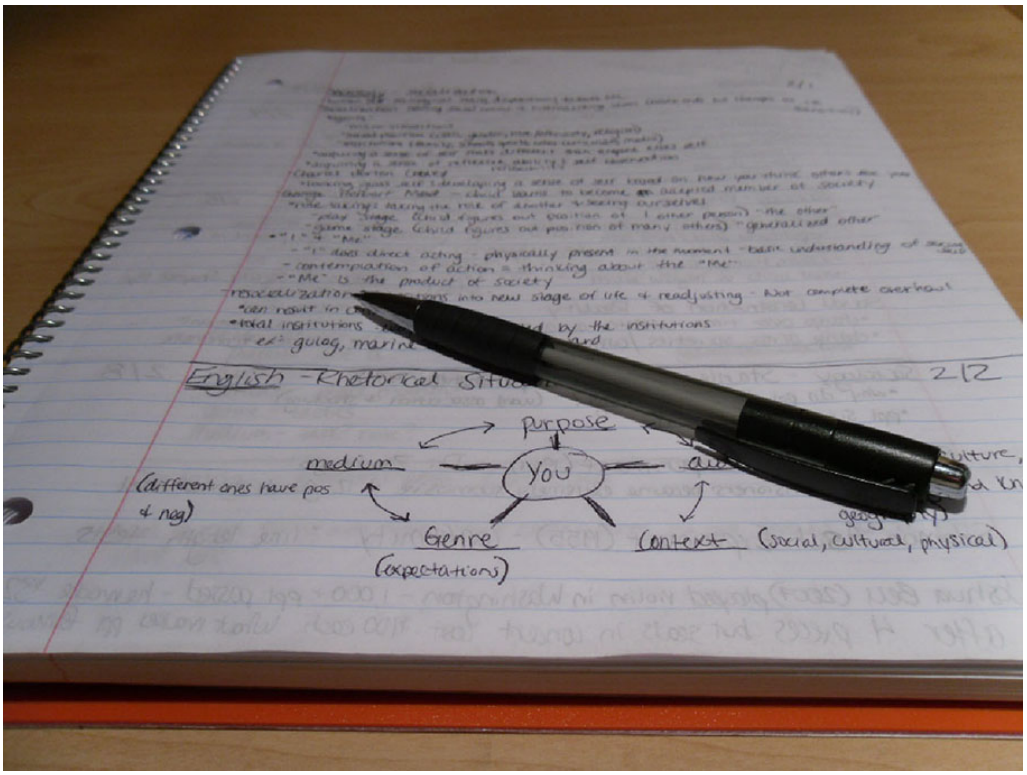


Figure 1.5.10 The best notes are the ones you take in an organized manner. Frequent review and further annotation are important to build a deep and useful understanding of the material. (Credit: English106 / Flickr / Attribution 2.0 Generic (CC-BY 2.0))

Managing Notetaking Systems (Computer, Paper/Pen, Note Cards, Textbook)

Whichever of the many notetaking systems you choose (and new ones seem to come out almost daily), the very best one is the one that you will use consistently. The skill and art of notetaking is not automatic for anyone; it takes a great deal of practice, patience, and continuous attention to detail. Add to that the fact that you may need to master multiple notetaking techniques for different classes, and you have some work to do. Unless you are specifically directed by your instructor, you are free to combine the best parts of different systems if you are most comfortable with that hybrid system.

Just to keep yourself organized, all your notes should start off with an identifier, including at the very least the date, the course name, the topic of the lecture/presentation, and any other information you think will help you when you return to use the notes for further study, test preparation, or assignment completion. Additional, optional information may be the number of notetaking sessions about this topic or reminders to cross-reference class handouts, textbook pages, or other course materials. It's also always a good idea to leave some blank space in your notes so you can insert additions and questions you may have as you review the material later.

Notetaking Strategies

You may have a standard way you take all your notes for all your classes. When you were in high school, this one-size-fits-all approach may have worked. Now that you're in college, reading and studying more advanced topics, your general method may still work some of the time, but you should have some different strategies in place if you find that your method isn't working as well with college content. You probably will need to adopt different notetaking strategies for different subjects. The strategies in this section represent various ways to take notes in such a way that you are able to study after the initial notetaking session.

Cornell Method

One of the most recognizable notetaking systems is called the *Cornell Method*, a relatively simple way to take effective notes devised by Cornell University education professor Dr. Walter Pauk in the 1940s. In this system, you take a standard piece of note paper and divide it into three sections by drawing a horizontal line across your paper about one to two inches from the bottom of the page (the summary area) and then drawing a vertical line to separate the rest of the page above this bottom area, making the left side about two inches (the recall column) and leaving the biggest area to the right of your vertical line (the notes column). You may want to make one page and then copy as many pages as you think you'll need for any particular class, but one advantage of this

system is that you can generate the sections quickly. Because you have divided up your page, you may end up using more paper than you would if you were writing on the entire page, but the point is not to keep your notes to as few pages as possible. The Cornell Method provides you with a well-organized set of notes that will help you study and review your notes as you move through the course. If you are taking notes on your computer, you can still use the Cornell Method in Word or Excel on your own or by using a template someone else created.

Topic/Objective:	Name:
	Class/Period:
	Date:
Essential Question:	

Questions:	Notes:

Summary:

Figure 1.5.11: The Cornell Method provides a straightforward, organized, and flexible approach

Now that you have the notetaking format generated, the beauty of the Cornell Method is its organized simplicity. Just write on one side of the page (the right-hand notes column)—this will help later when you are reviewing and revising your notes. During your notetaking session, use the notes column to record information over the main points and concepts of the lecture; try to put the ideas into your own words, which will help you not transcribe the speaker's words verbatim. Skip lines between each idea in this column. Practice the shortcut abbreviations covered in the next section and avoid writing in complete sentences. Don't make your notes too cryptic, but you can use bullet points or phrases equally well to convey meaning—we do it all the time in conversation. If you know you will need to expand the notes you are taking in class but don't have time, you can put reminders directly in the notes by adding and underlining the word *expand* by the ideas you need to develop more fully.

As soon as possible after your notetaking session, preferably within eight hours but no more than twenty-four hours, read over your notes column and fill in any details you missed in class, including the places where you indicated you wanted to expand your notes. Then in the recall column, write any key ideas from the corresponding notes column—you can't stuff this smaller recall column as if you're explaining or defining key ideas. Just add the one- or two-word main ideas; these words in the recall column serve as cues to help you remember the detailed information you recorded in the notes column.

Once you are satisfied with your notes and recall columns, summarize this page of notes in two or three sentences using the summary area at the bottom of the sheet. This is an excellent time to get with another classmate or a group of students who all heard the same lecture to make sure you all understood the key points. Now, before you move onto something else, cover the large notes column, and quiz yourself over the key ideas you recorded in the recall column. Repeat this step often as you go along, not just immediately before an exam, and you will help your memory make the connections between your notes, your textbook reading, your in-class work, and assignments that you need to succeed on any quizzes and exams.

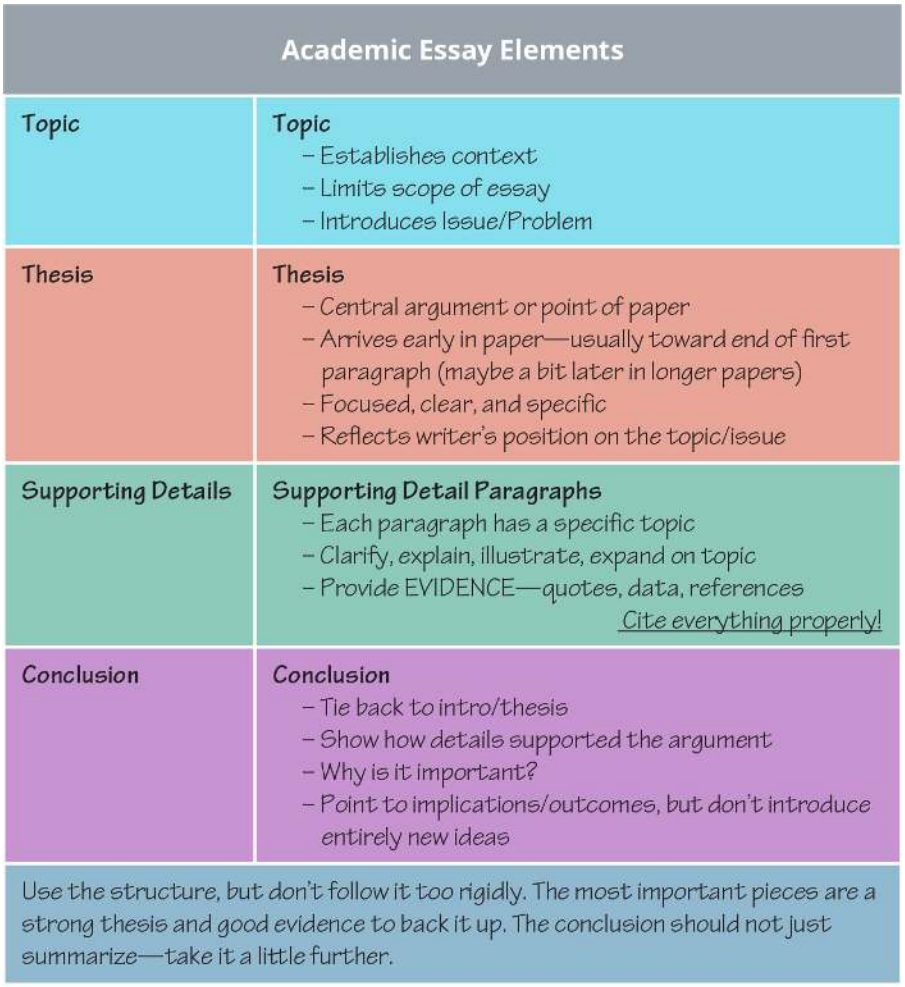


Figure 1.5.12 This sample set of notes in the Cornell Method is designed to make sense of a large amount of information. The process of organizing the notes can help you retain the information more effectively than less consistent methods.

The main advantage of the Cornell Method is that you are setting yourself up to have organized, workable notes. The neat format helps you move into study-mode without needing to re-copy less organized notes or making sense of a large mass of information you aren't sure how to process because you can't remember key ideas or what you meant. If you write notes in your classes without any sort of system and later come across something like "Napoleon—short" in the middle of a glob of notes, what can you do at this point? Is that important? Did it connect with something relevant from the lecture? How would you possibly know? You are your best advocate for setting yourself up for success in college.

Outlining

Other note organizing systems may help you in different disciplines. You can take notes in a formal outline if you prefer, using Roman numerals for each new topic, moving down a line to capital letters indented a few spaces to the right for concepts related to the previous topic, then adding details to support the concepts indented a few more spaces over and denoted by an Arabic numeral. You can continue to add to a formal outline by following these rules.

You don't absolutely have to use the formal numerals and letter, but you have to then be careful to indent so you can tell when you move from a higher level topic to the related concepts and then to the supporting information. The main benefit of an outline is how

organized it is. You have to be on your toes when you are taking notes in class to ensure you keep up the organizational format of the outline, which can be tricky if the lecture or presentation is moving quickly or covering many diverse topics.

The following formal outline example shows the basic pattern:

I. Dogs *(main topic–usually general)*
   A. German Shepherd *(concept related to main topic)*
      1. Protection *(supporting info about the concept)*
      2. Assertive
      3. Loyal
   B. Weimaraner *(concept related to main topic)*
      1. Family-friendly *(supporting info about the concept)*
      2. Active
      3. Healthy
II. Siamese

You would just continue on with this sort of numbering and indenting format to show the connections between main ideas, concepts, and supporting details. Whatever details you do not capture in your notetaking session, you can add after the lecture as you review your outline.

Chart or table

Similar to creating an outline, you can develop a chart to compare and contrast main ideas in a notetaking session. Divide your paper into four or five columns with headings that include either the main topics covered in the lecture or categories such as How?, What?, When used?, Advantages/Pros, Disadvantages/Cons, or other divisions of the information. You write your notes into the appropriate columns as that information comes to light in the presentation.

Example of a Chart to Organize Ideas and Categories

	Structure	Types	Functions in Body	Additional Notes
Carbohydrates				
Lipids				
Proteins				
Nucleic Acid				

This format helps you pull out the salient ideas and establishes an organized set of notes to study later. (If you haven't noticed that this *reviewing later* idea is a constant across all notetaking systems, you should…take note of that.) Notes by themselves that you never reference again are little more than scribblings. That would be a bit like compiling an extensive grocery list so you stay on budget when you shop, work all week on it, and then just throw it away before you get to the store. You may be able to recall a few items, but likely won't be as efficient as you could be if you had the notes to reference. Just as you cannot read all the many books, articles, and documents you need to peruse for your college classes, you cannot remember the most important ideas of all the notes you will take as part of your courses, so you must review.

Concept Mapping and Visual Notetaking

One final notetaking method that appeals to learners who prefer a visual representation of notes is called *mapping* or sometimes *mind mapping* or *concept mapping*, although each of these names can have slightly different uses. Variations of this method abound, so you may want to look for more versions online, but the basic principles are that you are making connections between main ideas through a graphic depiction; some can get rather elaborate with colors and shapes, but a simple version may be more useful at least to begin. Main ideas can be circled or placed in a box with supporting concepts radiating off these ideas shown with a connecting line and possibly details of the support further radiating off the concepts. You can present your main ideas vertically or horizontally, but turning your paper long-ways, or in landscape mode, may prove helpful as you add more main ideas.

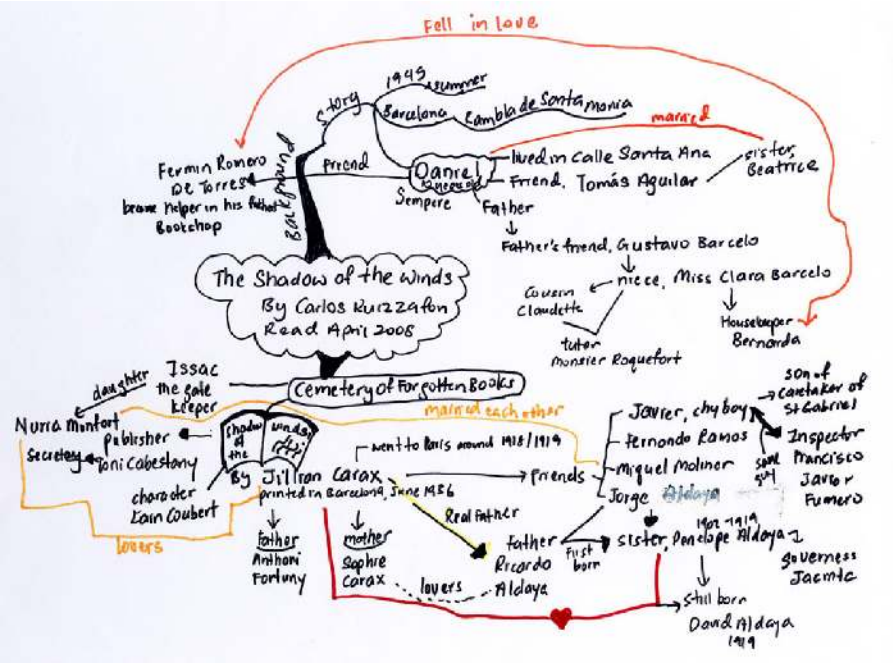


Figure 1.5.13 Concept mapping, sometimes referred to as mind mapping, can be an effective and very personalized approach to capturing information. (Credit: ArtistIvanChew / Flickr / Attribution 2.0 Generic (CC-BY 2.0))

You may be interested in trying visual notetaking or adding pictures to your notes for clarity. Sometimes when you can't come up with the exact wording to explain something or you're trying to add information for complex ideas in your notes, sketching a rough image of the idea can help you remember. According to educator Sherrill Knezel in an article entitled "The Power of Visual Notetaking," this strategy is effective because "When students use images and text in notetaking, it gives them two different ways to pull up the information, doubling their chances of recall." Don't shy away from this creative approach to notetaking just because you believe you aren't an artist; the images don't need to be perfect. You may want to watch Rachel Smith's TEDx Talk called "Drawing in Class" to learn more about visual notetaking.

You can play with different types of notetaking suggestions and find the method(s) you like best, but once you find what works for you, stick with it. You will become more efficient with the method the more you use it, and your notetaking, review, and test prep will become, if not easier, certainly more organized, which can delete decrease your anxiety.

Practicing Decipherable Shorthand

Most college students don't take a class in shorthand, once the domain of secretaries and executive assistants, but maybe they should. That almost-lost art in the age of computers could come in very handy during intense notetaking sessions. Elaborate shorthand systems do exist, but you would be better served in your college notetaking adventures to hone a more familiar, personalized form of shorthand to help you write more in a shorter amount of time. Seemingly insignificant shortcuts can add up to ease the stress notetaking can induce—especially if you ever encounter an "I'm not going to repeat this" kind of presenter! Become familiar with these useful abbreviations:

Shortcut symbol	Meaning
w/, w/o, w/in	with, without, within
&	and
#	number
b/c	because
X, √	incorrect, correct
Diff	different, difference
etc.	and so on

ASAP	as soon as possible
US, UK	United States, United Kingdom
info	information
Measurements: ft, in, k, m	foot, inch, thousand, million
¶	paragraph or new paragraph
Math symbols: =, +, >, <, ÷	equal, plus, greater, less, divided by
WWI, WWII	World Wars I and II
impt	important
?, !, **	denote something is very significant; don't over use

Do you have any other shortcuts or symbols that you use in your notes? Ask your parents if they remember any that you may be able to learn.

Annotating Notes After Initial Notetaking Session

Annotating notes after the initial notetaking session may be one of the most valuable study skills you can master. Whether you are highlighting, underlining, or adding additional notes, you are reinforcing the material in your mind and memory.

Admit it—who can resist highlighting markers? Gone are the days when yellow was the star of the show, and you had to be very careful not to press too firmly for fear of obliterating the words you were attempting to emphasize. Students now have a veritable rainbow of highlighting options and can color-code notes and text passages to their hearts' content. Technological advances may be important, but highlighter color choice is monumental! Maybe.

The only reason to highlight anything is to draw attention to it, so you can easily pick out that ever-so-important information later for further study or reflection. One problem many students have is not knowing when to stop. If what you need to recall from the passage is a particularly apt and succinct definition of the term important to your discipline, highlighting the entire paragraph is less effective than highlighting just the actual term. And if you don't rein in this tendency to color long passages (possibly in multiple colors) you can end up with a whole page of highlighted text. Ironically, that is no different from a page that is not highlighted at all, so you have wasted your time. Your mantra for highlighting text should be *less is more*. Always read your text selection first before you start highlighting anything. You need to know what the overall message is before you start placing emphasis in the text with highlighting.

Another way to annotate notes after initial notetaking is underlying significant words or passages. Albeit not quite as much fun as its colorful cousin highlighting, underlining provides precision to your emphasis.

Some people think of annotations as only using a colored highlighter to mark certain words or phrases for emphasis. Actually, annotations can refer to anything you do with a text to enhance it for your particular use (either a printed text, handwritten notes, or other sort of document you are using to learn concepts). The annotations may include highlighting passages or vocabulary, defining those unfamiliar terms once you look them up, writing questions in the margin of a book, underlining or circling key terms, or otherwise marking a text for future reference. You can also annotate some electronic texts.

Realistically, you may end up doing all of these types of annotations at different times. We know that repetition in studying and reviewing is critical to learning, so you may come back to the same passage and annotate it separately. These various markings can be invaluable to you as a study guide and as a way to see the evolution of your learning about a topic. If you regularly begin a reading session writing down any questions you may have about the topic of that chapter or section and also write out answers to those questions at the end of the reading selection, you will have a good start to what that chapter covered when you eventually need to study for an exam. At that point, you likely will not have time to reread the entire selection especially if it is a long reading selection, but with strong annotations in conjunction with your class notes, you won't need to do that. With experience in reading discipline-specific texts and writing essays or taking exams in that field, you will know better what sort of questions to ask in your annotations

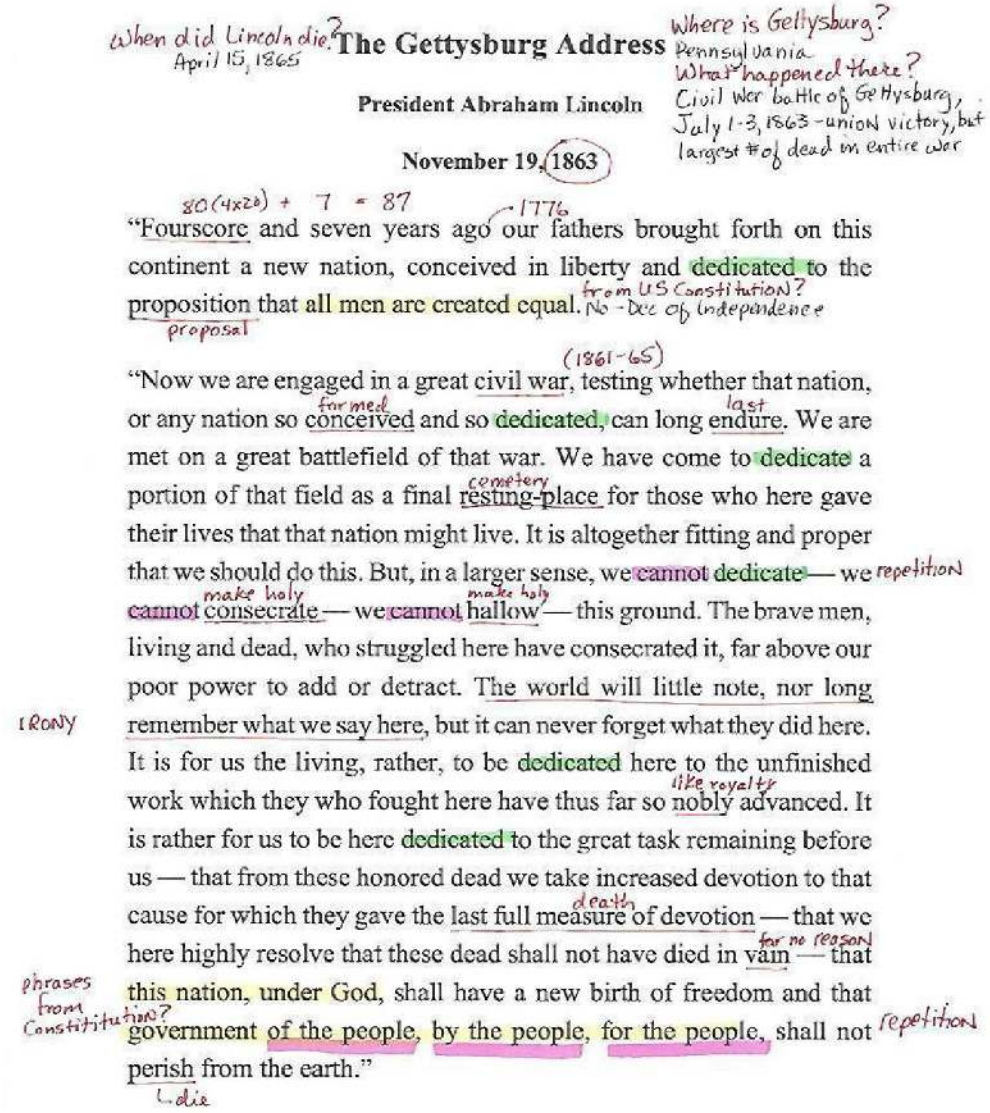


Figure 1.5.14 Annotations may include highlighting important topics, defining unfamiliar terms, writing questions in, underlining or circling key terms, or otherwise marking a text for future reference. Whichever approach you choose, try not to overdo it; neat, organized, and efficient notes are more effective than crowded or overdone notes.

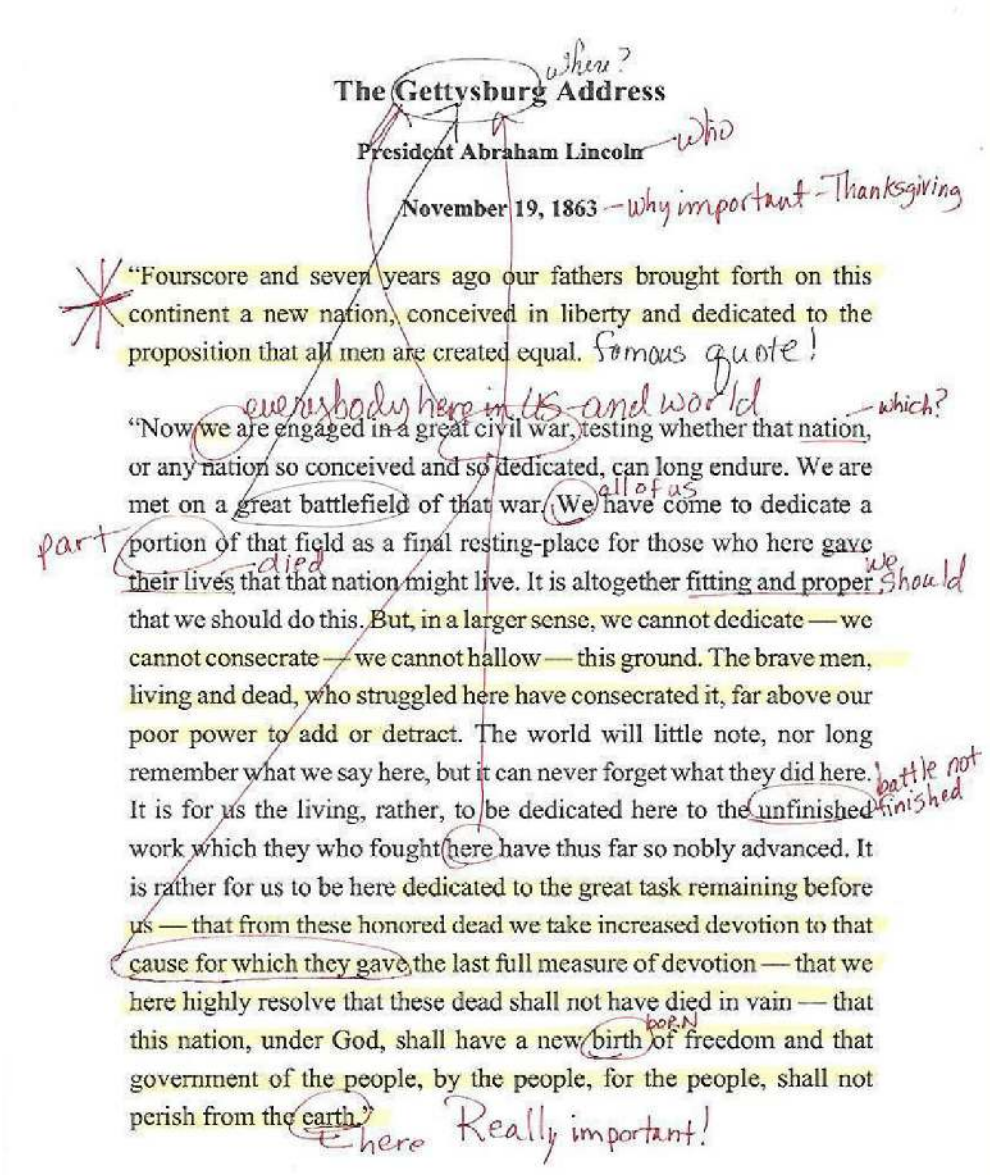


Figure 1.5.15 While these notes may be meaningful to the person who took them, they are neither organized nor consistent. For example, note that some of the more commonly used terms, like "we" and "unfinished," are defined, but less common ones -- "consecrate" and "hallow" -- are not.

What you have to keep in the front of your mind while you are annotating, especially if you are going to conduct multiple annotation sessions, is to not overdo whatever method you use. Be judicious about what you annotate and how you do it on the page, which means you must be neat about it. Otherwise, you end up with a mess of either color or symbols combined with some cryptic notes that probably took you quite a long time to create, but won't be worth as much to you as a study aid as they could be. This is simply a waste of time and effort.

You cannot eat up every smidgen of white space on the page writing out questions or summaries and still have a way to read the original text. If you are lucky enough to have a blank page next to the beginning of the chapter or section you are annotating, use this, but keep in mind that when you start writing notes, you aren't exactly sure how much space you'll need. Use a decipherable shorthand and write only what you need to convey the meaning in very small print. If you are annotating your own notes, you can make a habit of using only one side of the paper in class, so that if you need to add more notes later, you could use the other side. You can also add a blank page to your notes before beginning the next class date in your notebook so you'll end up with extra paper for annotations when you study.

Professional resources may come with annotations that can be helpful to you as you work through the various documentation requirements you'll encounter in college as well. Purdue University's Online Writing Lab (OWL) provides an annotated sample for

how to format a college paper according to guidelines in the Modern Language Association (MLA) manual that you can see, along with other annotations.

Adding Needed Additional Explanations to Notes

Marlon was totally organized and ready to take notes in a designated course notebook at the beginning of every philosophy class session. He always dated his page and indicated what the topic of discussion was. He had various colored highlighters ready to denote the different note purposes he had defined: vocabulary in pink, confusing concepts in green, and note sections that would need additional explanations later in yellow. He also used his own shorthand and an impressive array of symbols to indicate questions (red question mark), highly probable test material (he used a tiny bomb exploding here), additional reading suggestions, and specific topics he would ask his instructor before the next class. Doing everything so precisely, Marlon's methods seemed like a perfect example of how to take notes for success. Inevitably though, by the end of the hour-and-a-half class session, Marlon was frantically switching between writing tools, near to tears, and scouring his notes as waves of yellow teased him with uncertainty. What went wrong?

As with many of us who try diligently to do everything we know how to do for success or what we think we know because we read books and articles on success in between our course work, Marlon is suffering from trying to do too much simultaneously. It's an honest mistake we can make when we are trying to save a little time or think we can multitask and kill two birds with one stone.

Unfortunately, this particular error in judgement can add to your stress level exponentially if you don't step back and see it for what it is. Marlon attempted to take notes in class as well as annotate his notes to get them ready for his test preparation. It was too much to do at one time, but even if he could have done all those things during class, he's missing one critical point about notetaking.

As much as we may want to hurry and get it over with, notetaking in class is just the beginning. Your instructor likely gave you a pre-class assignment to read or complete before coming to that session. The intention of that preparatory lesson is for you to come in with some level of familiarity for the topic under consideration and questions of your own. Once you're in class, you may also need to participate in a group discussion, work with your classmates, or perform some other sort of lesson-directed activity that would necessarily take you away from taking notes. Does that mean you should ignore taking notes for that day? Most likely not. You may just need to indicate in your notes that you worked on a project or whatever other in-class event you experienced that date.

Very rarely in a college classroom will you engage in an activity that is not directly related to what you are studying in that course. Even if you enjoyed every minute of the class session and it was an unusual format for that course, you still need to take some notes. Maybe your first note could be to ask yourself why you think the instructor used that unique teaching strategy for the class that day. Was it effective? Was it worth using the whole class time? How will that experience enhance what you are learning in that course?

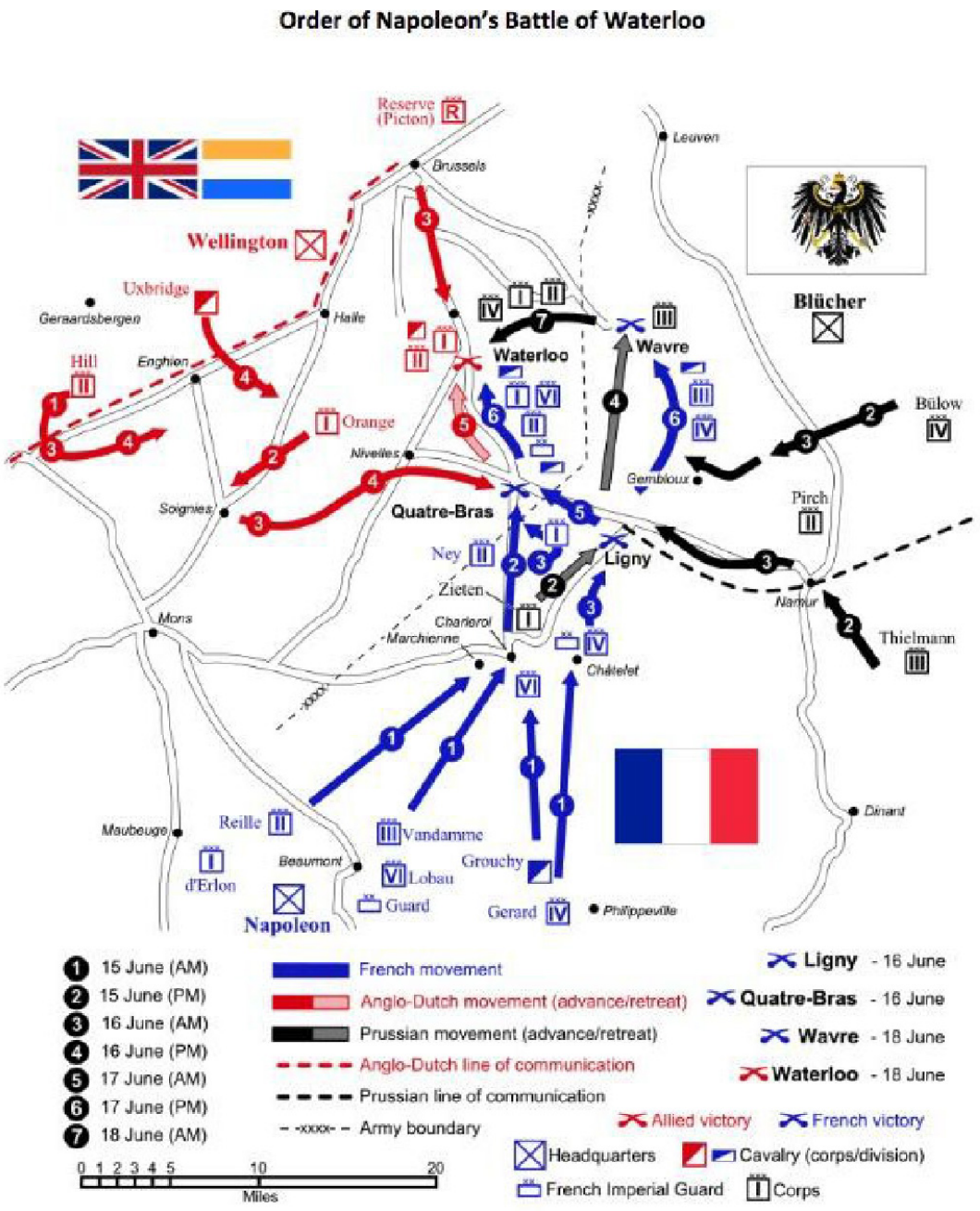
If you use an ereader or ebooks to read texts for class or read articles from the Internet on your laptop or tablet, you can still take effective notes. Depending on the features of your device, you have many choices. Almost all electronic reading platforms allow readers to highlight and underline text. Some devices allow you to add a written text in addition to marking a word or passage that you can collect at the end of your notetaking session. Look into the specific tools for your device and learn how to use the features that allow you to take notes electronically. You can also find apps on devices to help with taking notes, some of which you may automatically have installed when you buy the product. Microsoft's OneNote, Google Keep, and the Notes feature on phones are relatively easy to use, and you may already have free access to those.

Taking Notes on Non-Text Items (i.e., Tables, Maps, Figures, etc.)

You may also encounter situations as you study and read textbooks, primary sources, and other resources for your classes that are not actually texts. You can still take notes on maps, charts, graphs, images, and tables, and your approach to these non-text features is similar to when you prepare to take notes over a passage of text. For example, if you are looking at the following map, you may immediately come up with several questions. Or it may initially appear overwhelming. Start by asking yourself these questions:

What is the main point of this map?

- Who is the intended audience?
- Where is it?
- What time period does it depict?
- What does the map's legend (the explanation of symbols) include?
- What other information do I need to make sense of this map?



Source: Wikipedia Creative Commons, https://en.wikipedia.org/wiki/Order_of_battle_of_the_Waterloo_campaign

Figure 1.5.16 Graphics, charts, graphs, and other visual items are also important to annotate. Not only do they often convey important information, but they may appear on exams or in other situations where you'll need to use or demonstrate knowledge. Credit: "Lpankonin" / Wikipedia Commons / Attribution 3.0 Generic (CC BY 3.0)

You may want to make an extra copy of a graphic or table before you add annotations if you are dealing with a lot of information. Making sense of all the elements will take time, and you don't want to add to the confusion.

Returning to Your Notes

Later, as soon as possible after the class, you can go back to your notes and add in missing parts. Just as you may generate questions as you're reading new material, you may leave a class session or lecture or activities with many questions. Write those

down in a place where they won't get lost in all your other notes.

The exact timing of when you get back to the notes you take in class or while you are reading an assignment will vary depending on how many other classes you have or what other obligations you have in your daily schedule. A good starting place that is also easy to remember is to make every effort to review your notes within 24 hours of first taking them. Longer than that and you are likely to have forgotten some key features you need to include; must less time than that, and you may not think you need to review the information you so recently wrote down, and you may postpone the task too long.

Use your phone or computer to set reminders for all your note review sessions so that it becomes a habit and you keep on top of the schedule.

Your personal notes play a significant role in your test preparation. They should enhance how you understand the lessons, textbooks, lab sessions, and assignments. All the time and effort you put into first taking the notes and then annotating and organizing the notes will be for naught if you do not formulate an effective and efficient way to use them before sectional exams or comprehensive tests.

The whole cycle of reading, notetaking in class, reviewing and enhancing your notes, and preparing for exams is part of a continuum you ideally will carry into your professional life. Don't try to take short cuts; recognize each step in the cycle as a building block. Learning doesn't end, which shouldn't fill you with dread; it should help you recognize that all this work you're doing in the classroom and during your own study and review sessions is ongoing and cumulative. Practicing effective strategies now will help you be a stronger professional.

Activity

What resources can you find about reading and notetaking that will actually help you with these crucial skills? How do you go about deciding what resources are valuable for improving your reading and notetaking skills?

The selection and relative value of study guides and books about notetaking vary dramatically. Ask your instructors for recommendations and see what the library has available on this topic. The following list is not comprehensive, but will give you a starting point for books and articles on notetaking in college.

- *College Rules!: How to Study, Survive, and Succeed in College*, by Sherri Nist-Olejnik and Jodi Patrick Holschuh. More than just notetaking, this book covers many aspects of transitioning into the rigors of college life and studying.
- *Effective Notetaking*, by Fiona McPherson. This small volume has suggestions for using your limited time wisely before, during, and after notetaking sessions.
- *How to Study in College*, by Walter Pauk. This is the book that introduced Pauk's notetaking suggestions we now call the Cornell Method. It is a bit dated (from the 1940s), but still contains some valuable information.
- *Learn to Listen, Listen to Learn 2: Academic Listening and Note-taking*, by Roni S. Lebauer. The main point of this book is to help students get the most from college lectures by watching for clues to lecture organization and adapting this information into strong notes.
- *Study Skills: Do I Really Need this Stuff?*, by Steve Piscitelli. Written in a consistently down-to-earth manner, this book will help you with the foundations of strong study skills, including time management, effective notetaking, and seeing the big picture.
- "What Reading Does for the Mind," by Anne Cunningham and Keith Stanovich, 1998, https://www.aft.org/sites/default/files/periodicals/cunningham.pdf
- Adler, Mortimer J. and Charles Van Doren. *How to Read a Book: The Classic Guide to Intelligent Reading. NY: Simon & Schuster, 1940.*
- Berns, Gregory S., Kristina Blaine, Michael J. Prietula, and Brandon E. Pye. *Brain Connectivity.* Dec 2013.ahead of print http://doi.org/10.1089/brain.2013.0166

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1.6: How to Manage Time

Questions to consider:

- How can I use time-on-task estimates to improve time management?
- What behaviors can help or hinder when it comes to managing time?

In this next section you will learn about managing time and prioritizing tasks. This is not only a valuable skill for pursuing an education, but it can become an ability that follows you through the rest of your life, especially if your career takes you into a leadership role.



Figure 1.6.7 An online calendar is a very useful tool for keeping track of classes, meetings, and other events. Most learning management systems contain these features, or you can use a calendar application.

ANALYSIS QUESTION

Read each statement in the brief self-evaluation tool below, and check the answer that best applies to you. There are no right or wrong answers.

Table 3.3

	Always	Usually	Sometimes	Rarely	Never
I like to be given strict deadlines for each task. It helps me stay organized and on track.					
I would rather be 15 minutes early than 1 minute late.					
I like to improvise instead of planning everything out ahead of time.					

	Always	Usually	Sometimes	Rarely	Never
I prefer to be able to manage when and how I do each task.					
I have a difficult time estimating how long a task will take.					
I have more motivation when there is an upcoming deadline. It helps me focus.					
I have difficulty keeping priorities in the most beneficial order.					

This exercise is intended to help you recognize some things about your own time management style. The important part is for you to identify any areas where you might be able to improve and to find solutions for them. This chapter will provide some solutions, but there are many others that can be found by researching time management strategies.

After you have decided your best response to each statement, think about what they may mean in regard to potential strengths and/or challenges for you when it comes to time management in college. If you are a person that likes strict deadlines, what would you do if you took a course that only had one large paper due at the end? Would you set yourself a series of mini deadlines that made you more comfortable and that kept things moving along for you? Or, if you have difficulty prioritizing tasks, would it help you to make a list of the tasks to do and order them, so you know which ones must be finished first?

How to Manage Time

The simplest way to manage your time is to accurately plan for how much time it will take to do each task, and then set aside that amount of time. How you divide the time is up to you. If it is going to take you five hours to study for a final exam, you can plan to spread it over five days, with an hour each night, or you can plan on two hours one night and three hours the next. What you would not want to do is plan on studying only a few hours the night before the exam and find that you fell very short on the time you estimated you would need. If that were to happen, you would have run out of time before finishing, with no way to go back and change your decision. In this kind of situation, you might even be tempted to “pull an all-nighter,” which is a phrase that has been used among college students for decades. In essence it means going without sleep for the entire night and using that time to finish an assignment. While this method of trying to make up for poor planning is common enough to have a name, rarely does it produce the best work.

Table 3.4 Sample Time Estimate Table

Activity	Estimated Time	Actual Time
Practice Quiz	5 minutes	15 minutes
Lab Conclusions	20 minutes	35 minutes
Food shopping	45 minutes	30 minutes
Drive to work	20 minutes	20 minutes
Physical Therapy	1 hour	50 minutes

Of all the parts of time management, accurately predicting how long a task will take is usually the most difficult—and the most elusive. Part of the problem comes from the fact that most of us are not very accurate timekeepers, especially when we are busy applying ourselves to a task. The other issue that makes it so difficult to accurately estimate time on task is that our estimations must also account for things like interruptions or unforeseen problems that cause delays.?

When it comes to academic activities, many tasks can be dependent upon the completion of other things first, or the time a task takes can vary from one instance to another, both of which add to the complexity and difficulty of estimating how much time and effort are required.

For example, if an instructor assigned three chapters of reading, you would not really have any idea how long each chapter might take to read until you looked at them. The first chapter might be 30 pages long while the second is 45. The third chapter could be only 20 pages but made up mostly of charts and graphs for you to compare. By page count, it might seem that the third chapter would take the least amount of time, but actually studying charts and graphs to gather information can take longer than regular reading.?

To make matters even more difficult, when it comes to estimating time on task for something as common as reading, not all reading takes the same amount of time. Fiction, for example, is usually a faster read than a technical manual. But something like the novel *Finnegan's Wake* by James Joyce is considered so difficult that most readers never finish it.

ACTIVITY

To better understand how much time different kinds of material can take to read, try this experiment. You will use two examples of famous texts that are very close to being the same number of words: *The Gettysburg Address* and the opening paragraphs from *A Christmas Carol*. Before you begin, estimate how long it will take you to read each, and predict which you think will take longer. When you do the reading, use a stopwatch function on a device such as a phone or some other timer to see how long it actually takes.

Make certain that you are reading for understanding, not just skimming over words. If you must reread a section to better comprehend what is being said, that is appropriate. The goal here is to compare reading of different texts, not to see how fast you can sight-read the words on a page.

After you have finished *The Gettysburg Address*, read and time *A Christmas Carol* and compare both of your times.

The Gettysburg Address
Abraham Lincoln
Gettysburg, Pennsylvania November 19, 1863
Word count: 278

Four score and seven years ago our fathers brought forth on this continent, a new nation, conceived in Liberty, and dedicated to the proposition that all men are created equal.

Now we are engaged in a great civil war, testing whether that nation, or any nation so conceived and so dedicated, can long endure. We are met on a great battle-field of that war. We have come to dedicate a portion of that field, as a final resting place for those who here gave their lives that that nation might live. It is altogether fitting and proper that we should do this.

But, in a larger sense, we can not dedicate -- we can not consecrate -- we can not hallow -- this ground. The brave men, living and dead, who struggled here, have consecrated it, far above our poor power to add or detract. The world will little note, nor long remember what we say here, but it can never forget what they did here. It is for us the living, rather, to be dedicated here to the unfinished work which they who fought here have thus far so nobly advanced. It is rather for us to be here dedicated to the great task remaining before us -- that from these honored dead we take increased devotion to that cause for which they gave the last full measure of devotion -- that we here highly resolve that these dead shall not have died in vain -- that this nation, under God, shall have a new birth of freedom -- and that government of the people, by the people, for the people, shall not perish from the earth.

A Christmas Carol
Charles Dickens
Chapman & Hall, 1843
Word count: 260

Marley was dead: to begin with. There is no doubt whatever about that. The register of his burial was signed by the clergyman, the clerk, the undertaker, and the chief mourner. Scrooge signed it: and Scrooge's name was good upon 'Change, for anything he chose to put his hand to. Old Marley was as dead as a door-nail.

Mind! I don't mean to say that I know, of my own knowledge, what there is particularly dead about a door-nail. I might have been inclined, myself, to regard a coffin-nail as the deadest piece of ironmongery in the trade. But the wisdom of our ancestors is in the simile; and my unhallowed hands shall not disturb it, or the Country's done for. You will therefore permit me to repeat, emphatically, that Marley was as dead as a door-nail.

Scrooge knew he was dead? Of course he did. How could it be otherwise? Scrooge and he were partners for I don't know how many years. Scrooge was his sole executor, his sole administrator, his sole assign, his sole residuary legatee, his sole friend, and sole mourner. And even Scrooge was not so dreadfully cut up by the sad event, but that he was an excellent man of business on the very day of the funeral, and solemnised it with an undoubted bargain.

The mention of Marley's funeral brings me back to the point I started from. There is no doubt that Marley was dead. This must be distinctly understood, or nothing wonderful can come of the story I am going to relate.

In comparing the two, was one or the other easier to understand or faster to read? Was it the piece you predicted you would read faster?

It is important to note that in this case both readings were only three paragraphs long. While there may have only been half a minute or so between the reading of each, that amount of time would multiply greatly over an entire chapter.

Knowing Yourself

While you can find all sorts of estimates online as to how long a certain task may take, it is important to know these are only averages. People read at different speeds, people write at different speeds, and those numbers even change for each individual depending on the environment.

If you are trying to read in surroundings that have distractions (e.g., conversations, phone calls, etc.), reading 10 pages can take you a lot longer than if you are reading in a quiet area. By the same token, you may be reading in a quiet environment (e.g., in bed after everyone in the house has gone to sleep), but if you are tired, your attention and retention may not be what it would be if you were refreshed.

In essence, the only way you are going to be able to manage your time accurately is to know yourself and to know how long it takes you to do each task. But where to begin?

Below, you will find a table of common college academic activities. This list has been compiled from a large number of different sources, including colleges, publishers, and professional educators, to help students estimate their own time on tasks. The purpose of this table is to both give you a place to begin in your estimates and to illustrate how different factors can impact the actual time spent.

You will notice that beside each task there is a column for the *unit*, followed by the average *time on task*, and a column for notes. The *unit* is whatever is being measured (e.g., pages read, pages written, etc.), and the *time on task* is an average time it takes students to do these tasks. It is important to pay attention to the notes column, because there you will find factors that influence the time on task. These factors can dramatically change the amount of time the activity takes.

Table 3.5 Time on task for common college activities.

Time on Task			
Activity	**Unit**	**Time on task**	**Notes**
General academic reading (textbook, professional journals)	1 page	5–7 minutes	Be aware that your personal reading speed may differ and may change over time.
Technical reading (math, charts and data)	1 page	10–15 minutes	Be aware that your personal reading speed may differ and may change over time.

Time on Task			
Simple Quiz or homework question: short answer—oriented toward recall or identification type answers	Per question	1–2 minutes	Complexity of question will greatly influence the time required.
Complex Quiz or homework question: short answer—oriented toward application, evaluation, or synthesis of knowledge	Per question	2–3 minutes	Complexity of question will greatly influence the time required.
Math problem sets, complex	Per question	15 minutes	For example, algebra, complex equations, financial calculations
Writing: short, no research	Per page	60 minutes	Short essays, single-topic writing assignments, summaries, freewriting assignments, journaling—includes drafting, writing, proofing, and finalizing
Writing: research paper	Per page	105 minutes	Includes research time, drafting, editing, proofing, and finalizing (built into per-page calculation)
Study for quiz	Per chapter	60 minutes	45–90 minutes per chapter, depending upon complexity of material
Study for exam	Per exam	90 minutes	1–2 hours, depending upon complexity of material

Again, these are averages, and it does not mean anything if your times are a little slower or a little faster. There is no "right amount of time," only the time that it takes you to do something so you can accurately plan and manage your time.

There is also another element to look for in the table. These are differentiations in the similar activities that will also affect the time you spend. A good example of this can be found in the first four rows. Each of these activities involves reading, but you can see that depending on the material being read and its complexity, the time spent can vary greatly. Not only do these differences in time account for the different types of materials you might read (as you found in the comparative reading exercise earlier in this chapter), but also they also take into consideration the time needed to think about what you are reading to truly understand and comprehend what it is saying.

GET CONNECTED

Which apps help you best prepare for success when managing your time?

Do you have trouble keeping track of multiple tasks over the course of a term?

Trello lets you organize all your obligations in helpful boards. You can share them with others (project collaborators), set alerts as reminders, and mark tasks off as you complete them.

Do you use a particular app to help you manage your time?

Sticky note apps are available for PC, Mac, and mobile devices. They let you post quick reminders, reorganize them as needed, and view them separately or as a full to-do list.

What do you wish you could improve about your time management skills?

> Toggl helps you keep track of how and where you are spending your time so you can budget better and make time management changes that free you up for the really important stuff.

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1.7: Enhanced Strategies for Time and Task Management

Questions to consider:

- What strategy helps me prioritize my top tasks?
- How do I make the best use of my time when prioritizing?
- How do I make sure I tackle unpleasant tasks instead of putting them off?
- What's the best way to plan for long-term tasks?
- How do I find time in a busy schedule?

Over the years, people have developed a number of different strategies to manage time and tasks. Some of the strategies have proven to be effective and helpful, while others have been deemed not as useful.

The good news is that the approaches that do not work very well or do not really help in managing time do not get passed along very often. But others, those which people find of value, do. What follows here are three unique strategies that have become staples of time management. While not everyone will find that all three work for them in every situation, enough people have found them beneficial to pass them along with high recommendations.

Daily Top Three

The idea behind the *daily top three* approach is that you determine which three things are the most important to finish that day, and these become the tasks that you complete. It is a very simple technique that is effective because each day you are finishing tasks and removing them from your list. Even if you took one day off a week and completed no tasks on that particular day, a *daily top three* strategy would have you finishing 18 tasks in the course of a single week. That is a good amount of things crossed off your list.

analysis question

Analysis: Think about what would be your top three tasks for today? What would you have on the list tomorrow?

Pomodoro Technique



Figure 1.7.14 The Pomodoro Technique is named after a type of kitchen timer, but you can use any clock or countdown timer. (Marco Verch /Flickr / Attribution 2.0 Generic (CC BY 2.0))

The Pomodoro Technique was developed by Francesco Cirillo. The basic concept is to use a timer to set work intervals that are followed by a short break. The intervals are usually about 25 minutes long and are called *pomodoros*, which comes from the Italian word for tomato because Cirillo used a tomato-shaped kitchen timer to keep track of the intervals.

In the original technique there are six steps:

1. Decide on the task to be done.
2. Set the timer to the desired interval.
3. Work on the task.
4. When the timer goes off, put a check mark on a piece of paper.
5. If you have fewer than four check marks, take a short break (3–5 minutes), then go to Step 1 or 2 (whichever is appropriate).
6. After four pomodoros, take a longer break (15–30 minutes), reset your check mark count to zero, and then go to Step 1 or 2.

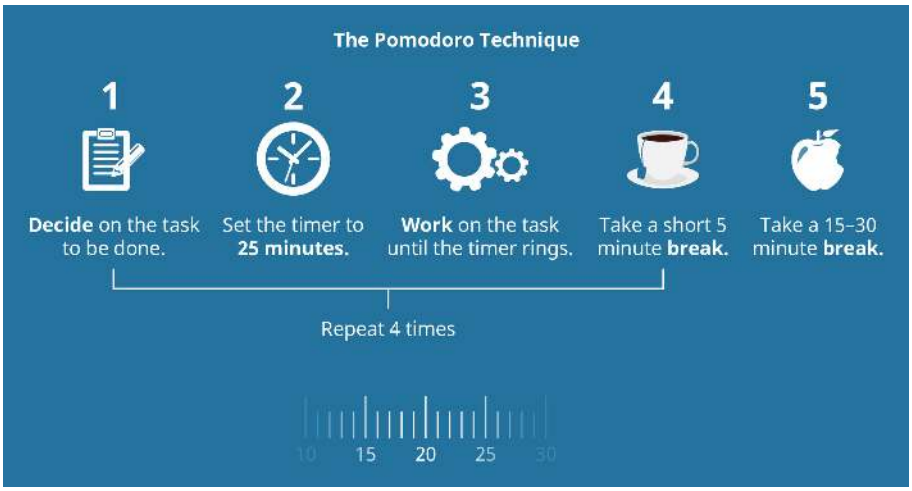


Figure 1.7.15 The Pomodoro Technique contains five defined steps.

There are several reasons this technique is deemed effective for many people. One is the benefit that is derived from quick cycles of work and short breaks. This helps reduce mental fatigue and the lack of productivity caused by it. Another is that it tends to encourage practitioners to break tasks down to things that can be completed in about 25 minutes, which is something that is usually manageable from the perspective of time available. It is much easier to squeeze in three 25-minute sessions of work time during the day than it is to set aside a 75- minute block of time.

Eat the Frog

Of our three quick strategies, *eat the frog* probably has the strangest name and may not sound the most inviting. The name comes from a famous quote, attributed to Mark Twain: “Eat a live frog first thing in the morning and nothing worse will happen to you the rest of the day.” *Eat the Frog* is also the title of a best-selling book by Brian Tracy that deals with time management and avoiding procrastination.

How this applies to time and task management is based on the concept that if a person takes care of the biggest or most unpleasant task first, everything else will be easier after that.

Although stated in a humorous way, there is a good deal of truth in this. First, we greatly underestimate how much worry can impact our performance. If you are continually distracted by anxiety over a task you are dreading, it can affect the task you are working on at the time. Second, not only will you have a sense of accomplishment and relief when the task you are concerned with is finished and out of the way, but other tasks will seem lighter and not as difficult.

Application: Try Three Time Management Strategies

Over the next two weeks, try each of these three methods to see which ones might work for you. Is there one you favor over the others? Might each of these three approaches serve you better in different situations or with different tasks? Do you have a creative alternative or possibly a way to use some combination of these techniques?

In addition to these three strategies, you could also develop whole new approaches from suggestions found earlier in this chapter. For example, you could apply some of the strategies for avoiding procrastination or for setting appropriate priorities and see how they work in combination with these techniques or on their own.

The key is to find which system works best for you.

Breaking Down the Steps and Spreading Them over Shorter Work Periods

Above, you read about several different tried-and-tested strategies for effective time management—approaches that have become staples in the professional world. In this section you will read about two more creative techniques that combine elements from these other methods to handle tasks when time is scarce and long periods of time are a luxury you just do not have.

The concept behind this strategy is to break tasks into smaller, more manageable units that do not require as much time to complete. As an illustration of how this might work, imagine that you are assigned a two-page paper that is to include references. You estimate that to complete the paper—start to finish—would take you between four and a half and five hours. You look at your

calendar over the next week and see that there simply are no open five-hour blocks (unless you decided to only get three hours of sleep one night). Rightly so, you decide that going without sleep is not a good option. While looking at your calendar, you do see that you can squeeze in an hour or so every night. Instead of trying to write the entire paper in one sitting, you break it up into much smaller components as shown in the table below:

Table 3.8: Breaking Down Projects into Manageable-Sized Tasks

Day/Time	Task	Time
Monday, 6:00 p.m.	Write outline; look for references.	60 minutes
Tuesday, 6:00 p.m.	Research references to support outline; look for good quotes.	60 minutes
Wednesday, 7:00 p.m.	Write paper introduction and first page draft.	60 minutes
Thursday, 6:00 p.m.	Write second page and closing draft.	60 minutes
Friday, 5:00 p.m.	Rewrite and polish final draft.	60 minutes
Saturday, 10:00 a.m.	*Only if needed—finish or polish final draft.*	60 minutes?

While this is a simple example, you can see how it would redistribute tasks to fit your available time in a way that would make completing the paper possible. In fact, if your time constraints were even more rigid, it would be possible to break these divided tasks down even further. You could use a variation of the Pomodoro Technique and write for three 20-minute segments each day at different times. The key is to look for ways to break down the entire task into smaller steps and spread them out to fit your schedule.

Table 3.9

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
8:00–10:00		Work		Work			
10:00–12:00	Algebra	Work	Algebra	Work	Algebra	10 a.m.–11 a.m. *Only if needed*	Work
12:00–2:00	Lunch/study	1 p.m. English Comp	Lunch/study	1 p.m. English Comp	Lunch/study	Family picnic	Work
2:00–4:00	History	English Comp	History	English Comp	History	Family picnic	
4:00–6:00	Study for Algebra quiz.	Grocery	Study for History exam.	Study for History exam.	5 p.m.–6 p.m. Rewrite and polish final draft.	Family picnic	Laundry
6:00–7:00	Write outline; look for references.	Research references to support outline; look for good quotes.	Research presentation project.	Write second page and closing draft	Create presentation.	Meet with Darcy.	Prepare school stuff for next week.
7:00–8:00	Free time	Free time	Write paper introduction and first page draft.	Research presentation project.	Create presentation.		Free time

STUDENT PROFILE

"Time management is probably one of the hardest things I had to pick up when I got to college. For starters, I didn't have anyone to come wake me up if I forgot to set an alarm or to tell me to get out of bed so that I wouldn't be late. I had to start placing my phone far away from my bed; so that way, I would have to get out of bed in order to turn the alarm off. Accomplishing work on time can also be difficult. It's tough to find the fine balance between when you have to stay in and work on assignments and when is acceptable to go out and do leisure activities.

"I learned the 8-8-8 rule. Every day you spend eight hours working on school work or going to class, eight hours of free time to do what you want, and then eight hours to sleep at night so that you will get enough rest. Sleep is crucial for time management. I learned very quickly that you cannot focus or be productive if you are struggling to keep your head from falling over because you are so tired. Basically, I've learned that if you want to be successful in college, then you have to be on top of your game when it comes to time. It's something thing you cannot make up once it's gone."

—Preston Allen, University of Central Arkansas

Analyzing Your Schedule and Creating Time to Work

Of all the strategies covered in this chapter, this one may require the most discipline, but it can also be the most beneficial in time management. The fact is most of us waste time throughout the day. Some of it is due to a lack of awareness, but it can also be caused by the constraints of our current schedules. An example of this is when we have 15 to 20 minutes before we must leave to go somewhere. We don't do anything with that time because we are focused on leaving or where we are going, and we might not be organized enough to accomplish something in that short of a time period. In fact, a good deal of our 24- hour days are spent a few minutes at a time waiting for the next thing scheduled to occur. These small units of time add up to a fair amount each day.

The intent of this strategy is to recapture those lost moments and use them to your advantage. This may take careful observation and consideration on your part, but the results of using this as a method of time management are more than worth it.

The first step is to look for those periods of time that are wasted or that can be repurposed. In order to identify them, you will need to pay attention to what you do throughout the day and how much time you spend doing it. The example of waiting for the next thing in your schedule has already been given, but there are many others. How much time do you spend in activities after you have really finished doing them but are still lingering because you have not begun to do something else (e.g., letting the next episode play while binge-watching, reading social media posts or waiting for someone to reply, surfing the Internet, etc.)? You might be surprised to learn how much time you use up each day by just adding a few unproductive minutes here and there.

If you set a limit on how much time you spend on each activity, you might find that you can recapture time to do other things. An example of this would be limiting yourself to reading news for 30 minutes. Instead of reading the main things that interest you and then spending an additional amount of time just looking at things that you are only casually interested in because that is what you are doing at the moment, you could stop after a certain allotted period and use the extra time you have gained on something else.

After you identify periods of lost time, the next step will be to envision how you might restructure your activities to bring those extra minutes together into useful blocks of time. Using the following scenario as an illustration, we will see how this could be accomplished.



Figure 1.7.16 Sarah has to balance a lot of obligations.

On Tuesday nights, Sarah has a routine: After work, she does her shopping for the week (2 hours driving and shopping) and then prepares and eats dinner (1 hour). After dinner, she spends time on homework (1 hour) and catching up with friends, reading the news, and other Internet activities (1 hour), and then she watches television or reads before going to bed (1 hour). While it may seem that there is very little room for improvement in her schedule without cutting out something she enjoys, limiting the amount of time she spends on each activity and rethinking how she goes about each task can make a significant difference.

In this story, Sarah's Tuesday-night routine includes coming home from work, taking stock of which items in her home she might need to purchase, and then driving to the store. While at the store, she spends time picking out and selecting groceries as she plans for meals she will eat during the rest of the week. Then, after making her purchases, she drives home. Instead, if she took the time to make a list and plan for what she needed at the store before she arrived, she would not spend as much time looking for inspiration in each aisle. Also, if she had a prepared list, not only could she quickly pick up each item, but she could stop at the store on the way home from work, thus cutting out the extra travel time. If purchasing what she needed took 30 minutes less because she was more organized and she cut out an additional 20 minutes of travel time by saving the extra trip to the store from her house, she could recapture a significant amount of her Tuesday evening. If she then limited the time she spent catching up with friends and such to 30 minutes or maybe did some of that while she prepared dinner, she would find that she had added almost an extra hour and a half to the time available to her on that evening, without cutting out anything she needed to do or enjoys. If she decided to spend her time on study or homework, this would more than double the time she previously had available in her schedule for homework.

Analysis Question: Reflection

Analysis: Identify areas in the way you spend your day where you may be able to recapture and repurpose time. Are there things you can move around to gain more time? Are there ways you can combine tasks or reduce travel time?

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1.8: Goal Setting and Motivation

Questions to consider:

- How do I set motivational goals?
- What are SMART goals?
- What's the importance of an action plan?
- How do I keep to my plan?

Motivation often means the difference between success and failure. That applies to school, to specific tasks, and to life in general. One of the most effective ways to keep motivated is to set goals.

Goals can be big or small. A goal can range from *I am going to write one extra page tonight*, to *I am going to work to get an A in this course*, all the way to *I am going to graduate in the top of my class so I can start my career with a really good position*. The great thing about goals is that they can include and influence a number of other things that all work toward a much bigger picture. For example, if your goal is to get an A in a certain course, all the reading, studying, and every assignment you do for that course contributes to the larger goal. You have motivation to do each of those things and to do them well.

Setting goals is something that is frequently talked about, but it is often treated as something abstract. Like time management, goal setting is best done with careful thought and planning. This next section will explain how you can apply tested techniques to goal setting and what the benefits of each can be.

Set Goals That Motivate You

The first thing to know about goal setting is that a goal is a specific end result you desire. If the goal is not something you are really interested in, there is little motivational drive to achieve it. Think back to when you were much younger and some well-meaning adult set a goal for you—something that didn't really appeal to you at all. How motivated were you to achieve the goal? More than likely, if you were successful at all in meeting the goal, it was because you were motivated by earning the approval of someone or receiving a possible reward, or you were concerned with avoiding something adverse that might happen if you did not do what you were told. From an honest perspective in that situation, your real goal was based on something else, not the meeting of the goal set for you. To get the most from the goals you set, make sure they are things that you are interested in achieving.

That is not to say you shouldn't set goals that are supported by other motivations (e.g., If I finish studying by Friday, I can go out on Saturday), but the idea is to be intellectually honest with your goals.

Set SMART Goals

Goals should also be SMART. In this case, the word *smart* is not only a clever description of the type of goal, but it is also an acronym that stands for Specific, Measurable, Attainable, Relevant, and Time-bound. The reason these are all desirable traits for your goals is because they not only help you plan how to meet the goal, but they can also contribute to your decision-making processes during the planning stage.

What does it mean to create SMART goals?

- Specific—For a goal to be specific, it must be defined enough to actually determine the goal. A goal of *get a good job when I graduate* is too general. It doesn't define what a good job is. In fact, it doesn't even necessarily include a job in your chosen profession. A more specific goal would be something like *be hired as a nurse in a place of employment where it is enjoyable to work and that has room for promotion.*
- Measurable—The concept of *measurable* is one that is often overlooked when setting goals. What this means is that the goal should have clearly defined outcomes that are detailed enough to measure and can be used for planning of how you will achieve the goal. For example, setting a goal of *doing well in school* is a bit undefined, but making a goal of *graduating with a GPA above 3.0* is measurable and something you can work with. If your goal is measurable, you can know ahead of time how many points you will have to earn on a specific assignment to stay in that range or how many points you will need to make up in the next assignment if you do not do as well as you planned.
- Attainable—*Attainable* or *achievable* goals means they are reasonable and within your ability to accomplish. While a goal of *make an extra one million dollars by the end of the week* is something that would be nice to achieve, the odds that you could make that happen in a single week are not very realistic.

- Relevant—For goal setting, *relevant* means it applies to the situation. In relation to college, a goal of *getting a horse to ride* is not very relevant, but *getting dependable transportation* is something that would contribute to your success in school.
- Time-bound—Time-bound means you set a specific time frame to achieve the goal. *I will get my paper written by Wednesday* is time-bound. You know when you have to meet the goal. *I will get my paper written sometime soon* does not help you plan how and when you will accomplish the goal.

In the following table you can see some examples of goals that do and do not follow the SMART system. As you read each one, think about what elements make them SMART or how you might change those that are not.

Table 3.6

Goal	Is it SMART?	
I am going to be rich someday.	No	There is nothing really specific, measurable, or time-bound in this goal.
I will graduate with my degree, on time.	Yes	The statement calls out specific, measureable, and time-bound details. The other attributes of attainable and relevant are implied.
I am going to save enough money to buy a newer car by June.	Yes	All SMART attributes are covered in this goal.
I would like to do well in all my courses next semester.	No	While this is clearly time-bound and meets most of the SMART goal attributes, it is not specific or measurable without defining what “do well” means.
I am going to start being a nicer person.	No	While most of the SMART attributes are implied, there is nothing really measurable in this goal.
I will earn at least a 3.0 GPA in all my courses next semester.	Yes	All of the SMART attributes are present in this goal.
I am going to start being more organized.	No	While most of the SMART attributes are implied, there is nothing really measurable in this goal.

APPLICATION

Try writing two SMART goals—something with a one-week time frame and something that you will accomplish over the next year. Make certain that you include all the appropriate elements—Specific, Measurable, Attainable, Relevant, and Time-bound.

Make an Action Plan

Like anything else, making a step-by-step action plan of how you will attain your goals is the best way to make certain you achieve them. It doesn’t matter if it is a smaller goal with immediate results (e.g., finish all your homework due by Friday) or something bigger that takes several years to accomplish (graduate with my degree in the proper amount of time).

The planning techniques you use for time management and achieving goals can be similar. In fact, accurate goal setting is very much a part of time management if you treat the completion of each task as a goal.

What follows is an example of a simple action plan that lists the steps for writing a short paper. You can use something like this or modify it in a way that would better suit your own preferences.

Table 3.7

Action Plan		
Task	**Objective**	**When**

Action Plan		
Choose topic.	Select something interesting.	Needs to be done by Monday!
Write outline, look for references.	Create structure of paper and outline each part.	Monday, 6:00 p.m.
Research references to support outline, look for good quotes.	Strengthen paper and resources.	Tuesday, 6:00 p.m.
Write paper introduction and first page draft.	Get main ideas and thesis statement down.	Wednesday, 7:00 p.m.
Write second page and closing draft.	Finish main content and tie it all together.	Thursday, 6:00 p.m.
Rewrite and polish final draft.	Clean up for grammar, writing style, and effective communication.	Friday, 5:00 p.m.

Another useful approach to goal setting is to create SMART goals and then write them down. For most people there is a higher level of commitment when we write something down. If you have your goals written out, you can refer to each component of the SMART acronym and make certain you are on track to achieve it.

Stick with It!

As with anything else, the key to reaching goals is to keep at it, keep yourself motivated, and overcome any obstacles along the way. In the following graphic you will find seven methods that highly successful people use to accomplish this.



Figure 1.8.13 These seven ways to stay motivated are good suggestions from highly successful people. What other strategies would you suggest?

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1.9: Test Taking and Preparation

Students need to develop a mindset that every time they attend class or complete a reading assignment they are preparing for the next exam, because oftentimes so much of their grade rides on those exams. In college, it is not uncommon for your entire course grade to be based on two or three exams with no options for a retake or extra credit. Instead of an exam being over one or two chapters, exams are more likely to be over 5+ chapters of material, and in the case of a comprehensive exam, an entire semester's worth of material. Intentionally working through the process of preparing for, taking, and learning from the testing process will facilitate successful outcomes.



Table 5-4. Effective Technique for Test Preparation and Taking

Before the Test	During the Test	After the Test
Go to all of your classes and take lecture notes and read all of your assigned reading using techniques described in this chapter.	Mark your exam with the necessary identification. Read all instructions on the exam. Scan the exam for question types, point values, etc.	Conduct an error analysis for items missed on the test. Look up the correct answers. Determine the nature of the errors you made.
Develop a study plan and schedule the time to review for an exam over several days. Refer to "Create a Five-Day Study Plan for Exams."	Develop a plan for how you will use your time to complete all questions on the exam.	Talk with your professor about items you missed on the exam. Discuss adjustments in your test preparation methods if necessary.

Schedule study sessions with classmates in advance of the night before the exam.	Read each item carefully and fully before marking an answer. You might find clues to the answer and will be less likely to make a preventable mistake.	Analyze the test questions to see if patterns arise, such as,"Were most of the questions from the lecture, the book, or both?" "What types of questions were asked?" "What levels of thinking were being tested based on Bloom's Taxonomy?" etc.
Be sure you have materials you need for the exam ready at least the night before the exam, such as multiple writing utensils, scantrons, blue books, notebook paper, etc.	Skip difficult questions and come back to them if you are pressed for time. Caution: if using a scantron to record answers be very careful that you do not incorrectly record your answers on the wrong line and get off track.	Review the methods that you used to study for the exam and determine what adjustments you need to make in preparing for the next exam.
Get a good night's rest the night before the exam.	If time is available at the end of the exam, review questions you marked as "difficult" in order to check for errors.	Determine your overall grade in the course based on the new information that this grade has provided you. Talk with your professor and academic advisor if you have concerns about your overall grade.
Eat a good breakfast prior to the exam. Avoid caffeine prior to the exam.	If you have an essay question to write, take the time to briefly outline an answer to ensure that you are answering all aspects of the question and that you provide a well-organized answer for your faculty member to read.	Archive your exam for future reference.
To ensure that you are on time to your exam, set two alarm clocks or arrange to have a friend call you to make sure you are awake.	Try not to leave any answers blank. You might be awarded partial credit or even make a correct guess.	
	Take a moment to take some deep cleansing breaths (or any other quick relaxation techniques) if you feel stressed by a question or the exam as a whole.	

Preparation for an exam should begin the first day of class. Repeated exposure to material over a longer period of time (Baddeley, 1982; Bahrick & Hall, 1991) optimizes the recall of information when it is needed. Every assigned reading, every opportunity to take notes and engage in class activities, and a routine of reviewing notes should be considered activities toward exam preparation. The volume of information that students are responsible for on each exam is much larger than experienced in high school. Careful review of the material prior to each exam is required for optimal performance, and student reports tell us that the habit of "cramming" the night before the exam is not going to lead to positive, long-term results. Research of the brain supports this typical finding. Physical and neurochemical changes in the synapses of the brain occur when information is learned, a process called long-term potentiation (Kandel & Schwartz, 1982). Without proper time and repeated stimulation of the neural circuits, long-term learning of this information will not occur efficiently and may not be accessible when needed for a test. Planning for a review period over five days prior to the exam and healthy eating and sleep habits cannot be overemphasized.

Activity 5-4

Identify when your next substantial exam will be given in a class. Create a five-day study plan using the formula described in "Create a Five-Day Study Plan for Exams" and the "Five-Day Study Plan Worksheet" at the end of this chapter. Once you have created this plan, incorporate the schedule into your time management calendar for implementation.

Oftentimes, students jump right in to taking an exam feeling stressed and rushed with a need to regurgitate information immediately. However, approaching a test in a careful and methodical way can help ensure the best results. You can best utilize your exam time by taking the time to survey the exam directions and point values and planning how to approach sections of the

test. Strategy is particularly important for tests with mixed types of questions (i.e., multiple choice and essay) or tests with multiple essay questions. For ex- ample, if you were to spend too much time on the multiple choice questions and not allocate enough time to complete an essay question, you may have answered the majority of the total number of questions but still do poorly on the exam if the essay question was worth a larger portion of the exam's total points. Plan your time carefully and manage your stress.

Create a Five-Day Study Plan for Exams

Making a plan that will allow for the careful review of all of the assigned and presented material leads to less stress the day of an exam and the best conditions for recall and performance on an exam. Here is a simple method for creating a study plan:

1. Break the material for the exam into four manageable "chunks." If material can be logically divided by chapters, use that method. If not, make up your own chunks based on the content of the material. Note: It is not a good idea to break up information by the method of delivery (i.e., one chunk equals notes, one chunk equals readings, etc.). You want to include all information on specific topics together in one chunk (e.g., information about the topic of osmosis from the lecture notes and the textbook in a chunk).
2. Plan to spend about two hours studying on each of the five days.
3. You should work with the material on the exam in two ways, by preparing that material for review and by reviewing that material. Preparation of material incorporates study techniques discussed earlier in this chapter such as combining and clarifying lecture and textbook notes, creating visual organizers for topics, developing cram cards, solving practice problems, creating self-testing questions using multiple levels of Bloom's Taxonomy, etc. Once you get the hang of preparing material for review, you could do some of that along the way (for instance creating cram cards for readings and notes) to reduce the amount of time needed for preparation at this time, leaving more time for review.

Example of the Schedule for a Five-Day Study Plan

Day One
Prepare 1st Chunk – 2 hours

Day Two
Prepare 2nd Chunk – 2 hours
Review 1st Chunk – 30 minutes

Day Three
Prepare 3rd Chunk – 1.5 hours
Review 2nd Chunk – 30 minutes
Review 1st Chunk – 15 minutes

Day Four
Prepare 4th Chunk – 1 hour
Review 3rd Chunk – 30 minutes
Review 2nd Chunk – 15 minutes
Review 1st Chunk – 10 minutes

Day Five
Review 4th Chunk – 1 hour
Review 3rd Chunk – 30 minutes
Review 2nd Chunk – 30 minutes
Review 1st Chunk – 30 minutes

Taking some time to plan out what material will go into each chunk will ensure that you do not miss anything. Use the "Five-Day Study Plan Worksheet" at the end of this chapter as a guide for your planning.

This method typically works very well for courses that offer two or three tests throughout the semester; however, a similar method could be modified by the number of days or the amount of time spent each day if a course offers more exams with less material on each exam.

Tips for Better Test Taking

The types of test items on a test are as different as the professors who created the tests. "Objective" test questions refer to items in which students have to recognize the correct answer from a list of provided options (e.g., multiple choice, true-false, matching). In

most cases there is one best answer, though it is important to note that a professor could indicate in the directions that you can select more than one answer. “Subjective” test items are the fill-in-the-blank, short answer, or essay questions in which students must recall and produce the answer.

Multiple Choice Questions

- Read the question and all of the answer options first.
- Mark out any answers that you know are not correct.
- Once you have selected an answer, do not change it unless you misread the question and know the new answer is correct. Usually, your first answer is the correct one if you are making an educated guess.
- Many times, the correct answer has more information in it.
- If there is no penalty for guessing, do not leave any items blank—make an educated guess.
- If there is an “All of the Above” option and there are two options that are correct, select “All of the Above.”
- In a question with “All of the Above” and “None of the Above,” if you are certain that one of the answers is true, do not select “None of the Above” and, likewise, if one of the answers is false, then do not select “All of the Above.”

True-False Questions

- Read the statement carefully, but do not read too much into the statement. Base your answer on the information provided.
- With statements that have multiple facts, if any one of the facts is false the entire statement is false.
- Qualifying words like “always,” “never,” and “every” indicate that this would have to be true all of the time. If it is not true all of the time, then you should answer false.
- Qualifying words like “usually,” “sometimes,” and “generally” indicate that it could be true or false depending on the situation. Oftentimes the answer is true.

Essay Questions

- Most Important Rule: Make sure you understand what the question is asking you to do. An essay question is not an opportunity to simply regurgitate everything you know about a particular topic. Revisit Question Cues listed in the “Table 5-1: “Bloom’s Taxonomy: Cognitive Domain” and be sure you know what you are supposed to accomplish when you see those cues. Underline key words and let those words help guide you. If you are not sure, ask your professor for clarification on what he or she wants you to accomplish in your answer.
- Take a minute or so to plan out your answer and jot down a brief outline of key points to guide your writing before you start. A well-organized answer tends to score more points when graded.
- Formula for Essay Questions:
  - Introductory statements that define terms and describe what you will accomplish in your essay. This section should be brief.
  - State your first main idea, and then give supporting facts, examples, statistics, or details. Follow with your next main idea and continue the pattern until complete. The bulk of your time should be spent developing this section of the essay as it would likely include the details your professor wants to see in grading. Typically, each main idea would have its own paragraph. Use transitional works like “first, second, third…,” “next,” “also,” “however,” etc., to aid your reader.
  - End with a summary or final conclusion. This section is also likely to be brief.
- Budget time to proofread and revise if needed.
- Write legibly. Neater papers tend to receive higher marks.

Adapted from Dembo and Seli (2008) and TestTakingTips.com (2003–2009).

Once the exam has been completed, the learning experience is not over. Look over the test after you get it back from your professor and continue your review of the material that may be important for a comprehensive final or information that serves as a foundation for material to be learned in other courses. Even if only a scantron answer sheet is returned to you, professors are willing to give students access to review the test results during their office hours. Comprehensive post-test review not only includes reviewing your correct and incorrect answers to the question but also looking at where questions came from (i.e., lecture notes, readings, or both). Also, consider the study strategies you used and how they worked for you in order to help you prepare for future exams. Overall, it is important to emphasize the benefit of talking with your professor after each exam to gain insights into the information presented in the class as well as your learning and test strategies.

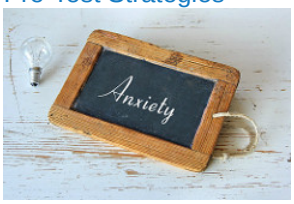
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1.10: Pre- Mid- and Post-Test-Taking Strategies

Phyllis Nissila

Pre-Test Strategies



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Q: When should you start preparing for the first test? Circle…

1. The night before.
2. The week prior.
3. The first day of classes.

If you answered "3. The first day of classes," you are correct. If you circled all three, you are also correct. Preparing to pass tests is something that begins when learning begins and continues all the way through to the final exam.

Many students, however, don't start thinking about test taking, whether weekly exams, mid-terms, or finals, until the day before when they engage in an all-nighter, or cramming. From the previous unit on memory, you might recall that the brain can only process an average of 5-7 new pieces of information at a time. Additionally, unless memory devices are used to aid memory and to cement information into long term memory (or at least until the test is over tomorrow!) chances are slim that students who cram will effectively learn and remember the information.

Additionally, a lot of students are unaware of the many strategies available to help with the test-taking experience before, during, and after. For starters, take a look at what has helped you so far.

EXERCISE 30-1

Pre-Test Taking Strategies

PART A:

Put a check mark next to the pre-test strategies you already employ.

____ Organize your notebook and other class materials the first week of classes.

____ Maintain your organized materials throughout the term.

____ Take notes on key points from lectures and other materials.

____ Make sure you understand the information as you go along.

____ Access your instructor's help and the help of a study group, as needed.

____ Organize a study group, if desired.

____ Create study tools such as flashcards, graphic organizers, etc. as study aids.

____ Complete all homework assignments on time.

____ Review likely test items several times beforehand.

____ Ask your instructor what items are likely to be covered on the test.

____ Ask your instructor if she or he can provide a study guide or practice test.

____ Ask your instructor if he/she gives partial credit for test items such as essays.

____ Maintain an active learner attitude.

____ Schedule extra study time in the days just prior to the test.

____ Gather all notes, handouts, and other materials needed before studying.

____ Review all notes, handouts, and other materials.

____ Organize your study area for maximum concentration and efficiency.

____ Create and use mnemonic devices to aid memory.

____ Put key terms, formulas, etc., on a single study sheet that can be quickly reviewed.

____ Schedule study times short enough (1-2 hours) so you do not get burned out.

____ Get plenty of sleep the night before.

____ Set a back-up alarm in case the first alarm doesn't sound or you sleep through it.

____ Have a good breakfast with complex carbs and protein to see you through.

____ Show up 5-10 minutes early to get completely settled before the test begins.

____ Use the restroom beforehand to minimize distractions.

PART B

By reviewing the pre-test strategies, above, you have likely discovered new ideas to add to what you already use. Make a list of them.

Mid-Test Strategies

Here is a list of the most common–and useful–strategies to survive this ubiquitous college experience.

- Scan the test, first, to get the big picture of how many test items there are, what types there are (multiple choice, matching, essay, etc.), and the point values of each item or group of items.
- Determine which way you want to approach the test: Some students start with the easy questions first, that is, the ones they immediately know the answers to, saving the difficult ones for later, knowing they can spend the remaining time on them. Some students begin with the biggest-point items first, to make sure they get the most points.
- Determine a schedule that takes into consideration how long you have to test, and the types of questions on the test. Essay questions, for example, will require more time than multiple choice, or matching questions.
- Keep your eye on the clock.
- If you can mark on the test, put a check mark next to items you are not sure of just yet. It is easy to go back and find them to answer later on. You might just find help in other test questions covering similar information.
- Sit where you are most comfortable. That said, sitting near the front has a couple of advantages: You may be less distracted by other students. If a classmate comes up with a question for the instructor and there is an important clarification given, you will be better able to hear it and apply it, if needed.
- Wear ear plugs, if noise distracts you.
- You do NOT have to start with #1! It you are unsure of it, mark it to come back to later on.
- Bring water…this helps calm the nerves, for one, and water is also needed for optimum brain function.
- If permitted, get up and stretch (or stretch in your chair) from time to time to relieve tension and assist the blood to the brain!
- Remember to employ strategies to reduce test-taking anxiety (covered in the next lesson)
- If despite all of your best efforts to prepare for a test you just cannot remember the answer to a given item for multiple choice, matching, and/or true/false questions, employ one or more of the following educated guessing (also known as "educated selection") techniques. By using these techniques, you have a *better* chance of selecting the correct answer. It is usually best to avoid selecting an extreme or all-inclusive answer (also known as 100% modifiers) such as "always," and "never". Choose, instead, words such as "usually," "sometimes," etc. (also known as in-between modifiers). If the answers are numbers, choose one of the middle numbers. If you have options such as "all of the above," or "both A and B," make sure each item is true before selecting those options. Choose the longest, or most inclusive, answer. Make sure to match the grammar of question and answer. For example, if the question indicates a plural answer, look for the plural answer. Regarding matching tests: count both sides to be matched. If there are more questions than answers, ask if you can use an answer more than once. Pay close attention to items that ask you to choose the "best" answer. This means one answer is better or more inclusive than a similar answer. Read all of the response options.

Post-Test Strategies

In addition to sighing that big sigh of relief, here are a few suggestions to help with future tests.

- If you don't understand why you did not get an item right, ask the instructor. This is especially useful for quizzes that contain information that may be incorporated into more inclusive exams such as mid-terms and finals.
- Analyze your results to help you in the future. For example, see if most of your incorrect answers were small things such as failing to include the last step in a math item, or neglecting to double-check for simple errors in a short-answer or essay item. See where in the test you made the most errors: beginning, middle, or end. Also analyze which type of questions, true/false, multiple choice, essay, etc. And which topics were missed. This will help you pay closer attention to those sections in the future.

EXERCISE 30-2

Write a letter of advice to Chen incorporating 10 test-taking tips and strategies you think will help him.

Chen believes he is good at organization, and he usually is–for about the first two weeks of classes. He then becomes overwhelmed with all of the handouts and materials and tends to start slipping in the organization department. When it comes to tests, he worries that his notes might not cover all of the right topics and that he will not be able to remember all of the key terms and points–especially for his math class. During tests, he sometimes gets stuck on an item and tends to spend too much time there. He also sometimes changes answers but finds out later that his original selection was correct. Chen is also easily distracted by other students and noises which makes it hard for him to concentrate sometimes, and, unfortunately, he does admit to occasionally "cramming" the night before.

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Adaptions: Changed formatting, removed one exercise.

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1.11: Test-Taking Strategy Specifics

Dave Dillon

"By failing to prepare, you are preparing to fail."

– Benjamin Franklin

Discipline, Preparation and Execution

From what I have seen in my own educational experiences, along with coaching and teaching at the college and university level for over 20 years, test taking (with few exceptions) comes down to discipline, preparation and execution. Students wanting to be successful have to have the self-discipline to schedule time to study well in advance of the exam. They have to actually do the work: the preparation needed in order to have the best opportunity for success on the exam. Then they must execute: they have to be able to apply their preparation accordingly and perform well on the exam.

Preparation for an exam is not glamorous. It's easy to find other things to do that are more interesting and fun. Students need to keep themselves motivated with their "eyes on the prize." Think of it like this: if the most important event of your life was coming up and you wanted to perform to the best of your ability in that event, you would likely spend some time preparing for it, rehearsing for it, practicing it, etc. A student may argue that an exam they will be taking would not be the most important event of their life, but think back to the chapter on Passion. If you're already spending the time, effort, energy and money to attend college, why not do it to the best of your ability?

It would be beneficial to spread this preparation and practice out over time and prepare periodically rather than to wait until the last minute and binge study or cram. Your preparation would not be the same and this may affect your test score. Binge studying and cramming also are not healthy. Staying up late puts stress on our brain and body, and not getting adequate sleep places our bodies at risk for getting sick.

"One of the most important keys to success is having the discipline to do what you know you should do, even when you don't feel like doing it."

– Unknown

"The will to succeed is important, but what's more important is the will to prepare."

– Bobby Knight

Everyone wants to be successful. When the exam is passed out, everyone wants to perform well. But what often separates successful students and less successful students is the preparation time put in.

Studying the right thing is a process and a skill. As you gain more experience, you will learn how to become better at knowing what to study. It can be very frustrating to spend a lot of time preparing and studying and then finding out that what you studied was not on the exam. You will see a lot of variance with exams due to different instructors, classes and types of tests. The better you become at predicting what will be on the exam and study accordingly, the better you will perform on your exams. Try placing yourself in your instructor's shoes and design questions you think your instructor would ask. It's often an eye-opening experience for students and a great study strategy.

Preparation for Exam Strategies

Find out as much about the exam in advance as you can.

Some professors will tell you how many questions there will be, what format the exam will be in, how much time you will have, etc., and others will not. I encourage students to ask questions about the exam if there is not information given. I also encourage students to ask those questions before class, after class, in professors' office hours or via e-mail rather than during class.

Know the test

If you know how many questions, what the format is, and/or how much time you will have, you can start to mentally prepare for the exam much more so than if you are coming in with no information. There are two more important aspects that you may or may not know: a) what will be covered or asked on the exam; b) how the exam will be scored. Obviously, the more you know about what will be covered, the easier it is for you to be able to prepare for the exam. Most exam scoring is standardized, but not always.

Look for opportunities where some areas of the exam are worth more points than others. For example: An exam consists of 21 questions, with 10 being True/False, 10 being multiple choice, and one essay question. The T/F questions are worth 1 point each (10 points), the multiple-choice questions are worth 2 points each (20 points), and the essay question is worth 30 points. We know that the essay question is the most valuable (it is worth half of the value of the exam). And we should allocate our time for it accordingly. I would advise starting with the essay question. Do a quick analysis of time to be able to spend your time on the exam wisely. You want to spend some time with the exam question since it is so valuable, without sacrificing adequate time to ensure the T/F and multiple-choice questions are answered.

Often, the order of the exam in this scenario will be: T/F first, multiple choice second and essay third. Most students will go in the chronological order of the exam, but a savvy student would start with the essay. If an exam were to last for 30 minutes with this format of questions, I might recommend a student spent 15 minutes on the essay question, ten minutes on the multiple choice, three minutes on the T/F and two minutes reviewing their answer.

Also, look for situations where exams penalize students for incorrectly answering a question. This does not occur very often, but is the case with some exams. With the SAT for example, students are awarded one point for a correct answer and ¼ of a point subtracted for an incorrect answer. Points are not awarded nor subtracted for leaving a question blank. Thus, the strategy for a multiple-choice question is: if you can narrow down the potentially correct answer to two rather than four or five, it is statistically advantageous to answer the question and guess between the two answers; however, if a student had no idea if any of the answers were correct or incorrect, it would be best to leave the answer blank. Remember, this is rare, but it is important to understand the strategy when students take these exams.

In conclusion, the more information you have about the exam, the better you can prepare for content, allocation of time spent on aspects of the exam, and the more confident you will be in knowing how and when to attempt to answer questions.

Take care of your body

Before the exam, it is important to prepare your brain and body for optimal performance for your exam. Do not cram the night before. Get a good night's sleep. Make sure you eat (nutritiously) before the exam. I recommend exercising the day before and if possible a few hours before the exam.

Strategies for Specific Exam Formats

True or False Questions

Look for qualifiers. A qualifier is a word that is absolute. Examples are: all, never, no, always, none, every, only, entirely. They are often seen in false statements. This is because it is more difficult to create a true statement using a qualifier like never, no, always, etc. For example, "All cats chase mice." Cats may be known for chasing mice, but not all of them do so. The answer here is false and the qualifier "all" gave us a tip. Qualifiers such as: sometimes, many, some, most, often, and usually are commonly found in true statements. For example: "Most cats chase mice." This is true and the qualifier "most" gave us a tip.

Make sure to read the entire statement. All parts of a sentence must be true if the whole statement is to be true. If one part of it is false, the whole sentence is false. Long sentences are often false for this reason.

Students should guess on True or False questions they do not know the answer to unless there is a penalty for an incorrect answer.

Multiple Choice Questions

Think of multiple choice questions as four (or five) true or false statements in one. One of the statements is true (the correct answer) and the others will be false. Apply the same strategy toward qualifiers. If you see an absolute qualifier in one of the answer choices, it is probably false and not the correct answer. Try to identify the true statement. If you can do this, you have the answer as there is only one. If you cannot do this at first, try eliminating answers you know to be false.

If there is no penalty for incorrect answers, my suggestion is to guess if you are not certain of the answer. If there is a penalty for incorrect answers, common logic is to guess if you can eliminate two of the answers as incorrect (pending what the penalty is). If there's a penalty and you cannot narrow down the answers, it's best to leave it blank. You may wish to ask your instructor for clarification.

Answers that are strange and unrelated to the question are usually false. If two answers have a word that looks or sounds similar, one of those is usually correct. For example: abductor/ adductor. If you see these as two of the four or five choices, one of them is usually correct. Also look for answers that are grammatically incorrect. These are usually incorrect answers. If you have to completely guess, choose B or C. It is statistically proven to be correct more than 25 percent of the time. If there are four answers

for each question, and an exam had standardized the answers, each answer on the exam A, B, C and D would be equal. But most instructors do not standardize their answers, and more correct answers are found in the middle (B and C then the extremes A and D or E). “People writing isolated four-choice questions hide the correct answer in the two middle positions about 70% of the time.”[1] This is 20 percent more correct answers found in B or C than a standardized exam with equal correct answers for each letter.

Matching Questions

Although less common than the other types of exams, you will likely see some matching exams during your time in college. First, read the instructions and take a look at both lists to determine what the items are and their relationship. It is especially important to determine if both lists have the same number of items and if all items are to be used, and used only once.

Matching exams become much more difficult if one list has more items than the other or if items either might not be used or could be used more than once. If your exam instructions do not discern this, you may wish to ask your instructor for further clarification. I advise students to take a look at the whole list before selecting an answer because a more correct answer may be found further into the list. Mark items when you are sure you have a match (pending the number of items in the list this may eliminate answers for the future). Guessing (if needed) should take place once you have selected answers you are certain about.

Short-Answer Questions

Read all of the instructions first. Budget your time and then read all of the questions. Answer the ones you know best or feel the most confident with. Then go back to the other ones. If you do not know the answer and there is no penalty for incorrect answers, guess. Use common sense. Sometimes instructors will award partial credit for a logical answer that is related even if it is not the correct answer.

Essay Questions

Keep in mind that knowing the format of the exam can help you determine how to study. If I know that I am taking a True-False exam, I know that I will need to discern whether a statement is True or False. I will need to know subject content for the course. But if I am studying for short answer and especially for essay questions, I must know a lot more. For essay questions, I must have much greater content knowledge and be able to make a coherent argument that answers the question using information from textbooks, lectures or other course materials. I have to place a lot more time and thought into studying for an essay exam than for True-False or Multiple-Choice exams.

Read the essay question(s) and the instructions first. Plan your time wisely and organize your answer before you start to write. Address the answer to the question in your first or second sentence. It may help to restate the original question. Write clearly and legibly. Instructors have difficulty grading essays that they cannot read. Save some time for review when you have finished writing to check spelling, grammar and coherent thought in your answer. Make sure you have addressed all parts of the essay question.

During the Exam:

Always read the directions first. Read them thoroughly.

Preview the exam to help you allocate proper time for each area.

Skip questions if you do not know the answer but make a mark somewhere to ensure you are able to go back to those questions (you may need to reallocate your prepared time for this depending on how many there are).

Allocate some time to review your answers before submitting your exam or the exam time expiring.

One of the biggest mistakes that students make after they take an exam in a course is that they do not use the exam for the future. The exam contains a lot of information that can be helpful in studying for future exams. Students that perform well on an exam often put it away thinking they do not need it anymore. Students who do poorly on an exam often put it away, not wanting to think about it any further.

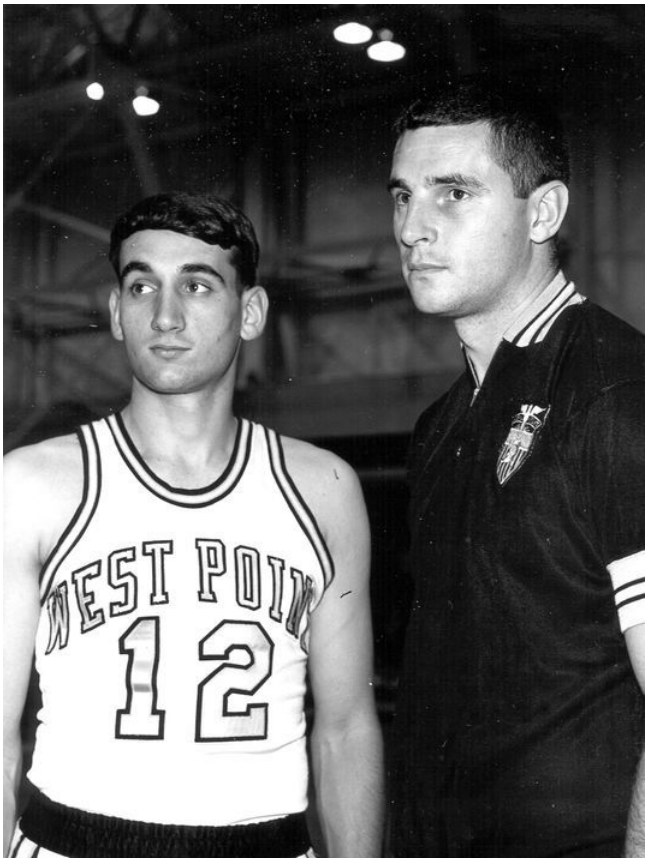
In both cases, students are missing out on the value of reviewing their exams. It is wise to review exams for three reasons: 1) students should review the answers that were correct because they may see those questions on future exams and it is important to reinforce learning; 2) students should review the answers that were incorrect in order to learn what the correct answer was and why. These questions also may appear on a future exam. In addition, occasionally an answer is marked incorrect, when it should have been marked correct. The student would never know this if they didn’t review their exam; and 3) there is value in reviewing the exam to try to predict what questions or what format will be used by a professor for a future exam in the same course.

Author's Story

What does it mean to be a "poor test taker?" Think about that. Does it mean that the student has put effort into studying but has difficulty under pressure? Does it mean that a student studies the wrong material? Is the student prepared but does not execute well? Could the student have a learning disability? Are they missing key strategies for taking tests? It could mean any of those things. And while I believe that it may be true that a student may be a "poor test taker," it does not by any means mean it is permanent. Students willing to work hard and learn can improve their test taking skills and raise their confidence.

In high school chemistry, I earned a D grade. In microeconomics at UC Santa Cruz, I struggled and barely passed. Both of these classes had grading systems that were heavily weighted by exams. I was an above average student in my other classes in both high school and college, and so I was told (and believed for a long time) that I was not a "good test taker." I no longer believe this. I will not make excuses. The responsibility for the grade I earned rests solely with me. I mention this because many students come to my office and complain that their professor grades in a way that is unfair to them because so much emphasis is on exams and that they understand the material but are "not good test takers." It may be the case that my grade or another students' grade would have been better had the course grade been determined with less weight levied to exams. That to me is irrelevant. It does not absolve me from being responsible for knowing what the grading metrics were at the beginning of the course and choosing to continue in the course after learning this knowledge. In the end, I realized I was not a poor test taker and that I needed to spend more time studying and preparing to perform well on exams. I changed my attitude from being afraid of exams to one of believing I was going to do well regardless of the class or the format of the exam. Once I had confidence that I had the necessary tools and was willing to work hard, it changed my entire perspective and experience with exams.

Note: if you think you may have a learning disability and would like to get assessed, contact your college to see what steps to follow to begin the assessment process.



Mike Krzyzewski, point guard at Army with Bob Knight, coach at Army.

Thinking about excuses reminds me of a story about Mike Krzyzewski, the successful Duke University men's basketball coach. Before Krzyzewski coached, he was a student at the United States Military Academy at West Point. One day in his first year, Krzyzweski and his roommate were walking and his roommate stepped in a puddle. Mud splashed onto Krzyzewski's uniform. Immediately, an upperclassman screamed at Krzyzewski about not knowing the rules of wearing a clean uniform. Plebes (first-year

students at military academies) are allowed three answers when asked a question: “Yes sir!” “No sir!” And “No excuse sir!” Krzyzewski repeated himself, “No excuse sir!” Despite wanting to explain what had happened and that it was not his fault, he realized that there was no excuse. It was his responsibility to keep a clean uniform.

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1. Yigal Attali and Maya Bar-Hillel, “Guess Where: The Position of Correct Answers in Multiple-Choice Test Items as a Psychometric Variable,” *Journal of Educational Measurement* 40 no. 2 (2003):109-128. ↵

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CHAPTER OVERVIEW

2: Linear Equations

2.1: Linear Functions - Applications
2.2: Prelude to Solving Linear Equations
2.3: Use a Problem Solving Strategy
2.3E: Exercises
2.4: Solve a Formula for a Specific Variable
2.4E: Exercises
2.5: Use the Rectangular Coordinate System
2.5E: Exercises
2.6: Graph with Intercepts
2.6.1: Exercises
2.7: Understanding the Slope of a Line
2.7E: Exercises
2.8: Use the Slope–Intercept Form of an Equation of a Line
2.8.1: Exercises
2.9: Find the Equation of a Line
2.9E: Exercises
2.10: Chapter Review Exercises

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2.1: Linear Functions - Applications

Learning Objectives

- Represent a linear function.
- Determine whether a linear function is increasing, decreasing, or constant.
- Interpret slope as a rate of change.
- Write and interpret an equation for a linear function.
- Graph linear functions.
- Determine whether lines are parallel or perpendicular.
- Write the equation of a line parallel or perpendicular to a given line.

Just as with the growth of a bamboo plant, there are many situations that involve constant change over time. Consider, for example, the first commercial maglev train in the world, the Shanghai MagLev Train (maglev train in the world, the Shanghai MagLev Train (Figure 2.1.1). It carries passengers comfortably for a 30-kilometer trip from the airport to the subway station in only eight minutes.



Figure 2.1.1: Shanghai MagLev Train (credit: “kanegen”/Flickr)

Suppose a maglev train were to travel a long distance, and that the train maintains a constant speed of 83 meters per second for a period of time once it is 250 meters from the station. How can we analyze the train’s distance from the station as a function of time? In this section, we will investigate a kind of function that is useful for this purpose, and use it to investigate real-world situations such as the train’s distance from the station at a given point in time.maglev train were to travel a long distance, and that the train maintains a constant speed of 83 meters per second for a period of time once it is 250 meters from the station. How can we analyze the train’s distance from the station as a function of time? In this section, we will investigate a kind of function that is useful for this purpose, and use it to investigate real-world situations such as the train’s distance from the station at a given point in time.

Representing Linear Functions

The function describing the train’s motion is a **linear function**, which is defined as a function with a constant rate of change, that is, a polynomial of degree 1. There are several ways to represent a linear function, including word form, function notation, tabular form, and graphical form. We will describe the train’s motion as a function using each method.

Representing a Linear Function in Word Form

Let’s begin by describing the linear function in words. For the train problem we just considered, the following word sentence may be used to describe the function relationship.

- *The train’s distance from the station is a function of the time during which the train moves at a constant speed plus its original distance from the station when it began moving at constant speed.*

The speed is the rate of change. Recall that a rate of change is a measure of how quickly the dependent variable changes with respect to the independent variable. The rate of change for this example is constant, which means that it is the same for each input value. As the time (input) increases by 1 second, the corresponding distance (output) increases by 83 meters. The train began moving at this constant speed at a distance of 250 meters from the station.

Representing a Linear Function in Function Notation

Another approach to representing linear functions is by using function notation. One example of function notation is an equation written in the form known as the slope-intercept form of a line, where x is the input value, m is the rate of change, and b is the initial value of the dependent variable.

$$\begin{aligned} &\text{Equation form} && y = mx + b \\ &\text{Equation notation} && f(x) = mx + b \end{aligned}$$

In the example of the train, we might use the notation $D(t)$ in which the total distance D is a function of the time t. The rate, m, is 83 meters per second. The initial value of the dependent variable b is the original distance from the station, 250 meters. We can write a generalized equation to represent the motion of the train.

$$D(t) = 83t + 250 \tag{2.1.1}$$

Representing a Linear Function in Tabular Form

A third method of representing a linear function is through the use of a table. The relationship between the distance from the station and the time is represented in Figure 2.1.2. From the table, we can see that the distance changes by 83 meters for every 1 second increase in time.

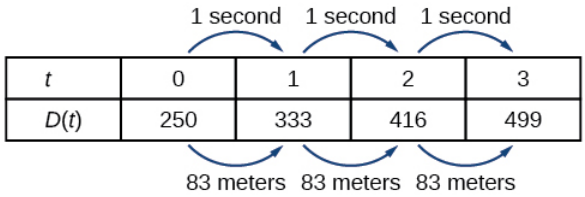


Figure 2.1.2: Tabular representation of the function D showing selected input and output values

Q/A? Can the input in the previous example be any real number?

No. The input represents time, so while nonnegative rational and irrational numbers are possible, negative real numbers are not possible for this example. The input consists of non-negative real numbers.

Representing a Linear Function in Graphical Form

Another way to represent linear functions is visually, using a graph. We can use the function relationship from above, $D(t) = 83t + 250$, to draw a graph, represented in Figure 2.1.3. Notice the graph is a line. When we plot a linear function, the graph is always a line.

The rate of change, which is constant, determines the slant, or **slope** of the line. The point at which the input value is zero is the vertical intercept, or **y-intercept**, of the line. We can see from the graph in Figure 2.1.3 that the y-intercept in the train example we just saw is $(0, 250)$ and represents the distance of the train from the station when it began moving at a constant speed.

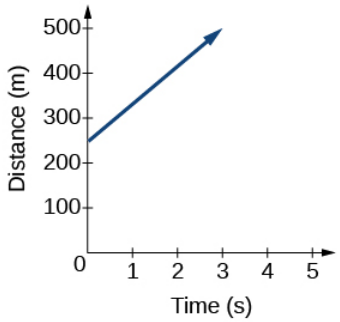


Figure 2.1.3: The graph of $D(t) = 83t + 250$. Graphs of linear functions are lines because the rate of change is constant.

Notice that the graph of the train example is restricted, but this is not always the case. Consider the graph of the line $f(x) = 2x + 1$. Ask yourself what numbers can be input to the function, that is, what is the domain of the function? The domain is comprised of all real numbers because any number may be doubled, and then have one added to the product.

Definition: Linear Function

A **linear function** is a function whose graph is a line. Linear functions can be written in the slope-intercept form of a line

$$f(x) = mx + b \tag{2.1.2}$$

where b is the initial or starting value of the function (when input, $x = 0$), and m is the constant rate of change, or **slope** of the function. The **y-intercept** is at $(0, b)$.

Example 2.1.1: Using a Linear Function to Find the Pressure on a Diver

The pressure, P, in pounds per square inch (PSI) on the diver in Figure 2.1.4 depends upon her depth below the water surface, d, in feet. This relationship may be modeled by the equation, $P(d) = 0.434d + 14.696$. Restate this function in words.



Figure 2.1.4: (credit: Ilse Reijs and Jan-Noud Hutten)Hutten)

To restate the function in words, we need to describe each part of the equation. The pressure as a function of depth equals four hundred thirty-four thousandths times depth plus fourteen and six hundred ninety-six thousandths.

Analysis

The initial value, 14.696, is the pressure in PSI on the diver at a depth of 0 feet, which is the surface of the water. The rate of change, or slope, is 0.434 PSI per foot. This tells us that the pressure on the diver increases 0.434 PSI for each foot her depth increases.

Determining whether a Linear Function Is Increasing, Decreasing, or Constant

The linear functions we used in the two previous examples increased over time, but not every linear function does. A linear function may be increasing, decreasing, or constant. For an increasing function, as with the train example, the output values increase as the input values increase. The graph of an **increasing function** has a positive slope. A line with a positive slope slants upward from left to right as in Figure 2.1.5(a). For a **decreasing function**, the slope is negative. The output values decrease as the input values increase. A line with a negative slope slants downward from left to right as in Figure 2.1.5(b). If the function is constant, the output values are the same for all input values so the slope is zero. A line with a slope of zero is horizontal as in Figure 2.1.5(c).

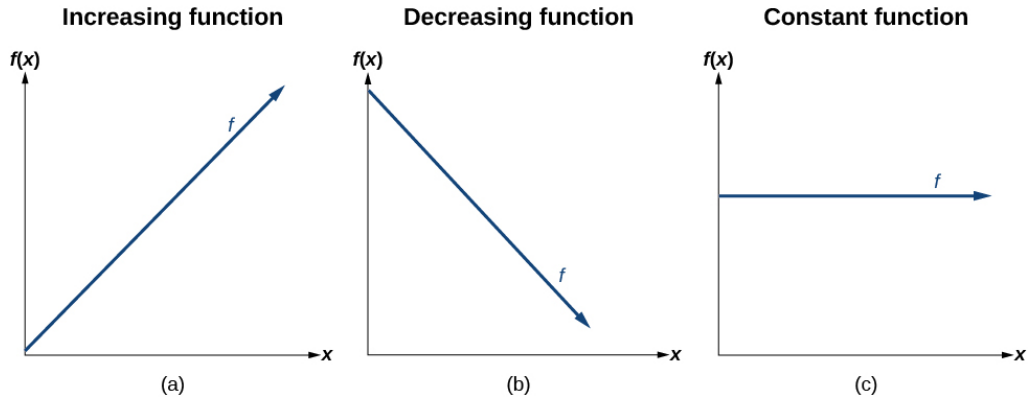


Figure 2.1.5: Three graphs depicting an increasing function, a decreasing function, and a constant function.]
Increasing and Decreasing Functions

Increasing and Decreasing Functions

The slope determines if the function is an **increasing linear function**, a **decreasing linear function**, or a constant function.

- $f(x) = mx + b$ is an increasing function if $m > 0$.
- $f(x) = mx + b$ is an decreasing function if $m < 0$.
- $f(x) = mx + b$ is a constant function if $m = 0$.

Example 2.1.2: decide whether a Function is Increasing, Decreasing, or Constant

Some recent studies suggest that a teenager sends an average of 60 texts per day. For each of the following scenarios, find the linear function that describes the relationship between the input value and the output value. Then, determine whether the graph of the function is increasing, decreasing, or constant.

a. The total number of texts a teen sends is considered a function of time in days. The input is the number of days, and output is the total number of texts sent.
b. A teen has a limit of 500 texts per month in his or her data plan. The input is the number of days, and output is the total number of texts remaining for the month.
c. A teen has an unlimited number of texts in his or her data plan for a cost of $50 per month. The input is the number of days, and output is the total cost of texting each month.

Solution

Analyze each function.

a. The function can be represented as $f(x) = 60x$ where x is the number of days. The slope, 60, is positive so the function is increasing. This makes sense because the total number of texts increases with each day.
b. The function can be represented as $f(x) = 500 - 60x$ where x is the number of days. In this case, the slope is negative so the function is decreasing. This makes sense because the number of texts remaining decreases each day and this function represents the number of texts remaining in the data plan after x days.
c. The cost function can be represented as $f(x) = 50$ because the number of days does not affect the total cost. The slope is 0 so the function is constant.

Calculating and Interpreting Slope

In the examples we have seen so far, we have had the slope provided for us. However, we often need to calculate the slope given input and output values. Given two values for the input, x_1 and x_2, and two corresponding values for the output, y_1 and y_2—which can be represented by a set of points, (x_1, y_1) and (x_2, y_2)—we can calculate the slope m, as follows

$$m = \frac{\text{change in output (rise)}}{\text{change in input (run)}}$$

$$= \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

where Δy is the vertical displacement and Δx is the horizontal displacement. Note in function notation two corresponding values for the output y_1 and y_2 for the function f, $y_1 = f(x_1)$ and $y_2 = f(x_2)$, so we could equivalently write

$$m = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Figure 2.1.6 indicates how the slope of the line between the points, (x_1, y_1) and (x_2, y_2), is calculated. Recall that the slope measures steepness. The greater the absolute value of the slope, the steeper the line is.

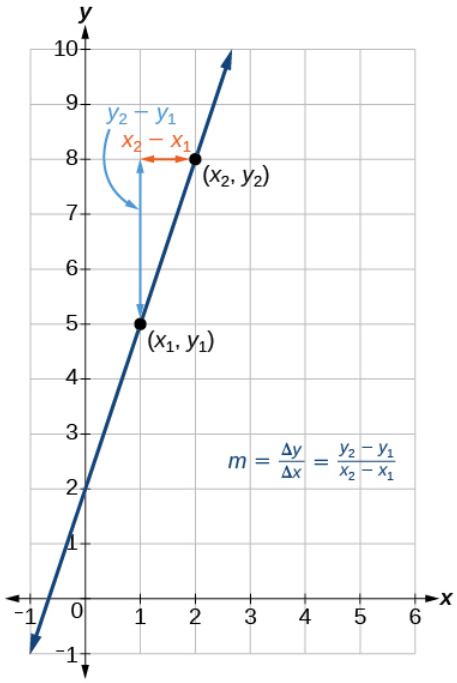


Figure 2.1.6: The slope of a function is calculated by the change in y divided by the change in x. It does not matter which coordinate is used as the (x_2, y_2) and which is the (x_1, y_1),as long as each calculation is started with the elements from the same coordinate pair.

Q & A

Are the units for slope always $\frac{\text{units for the output}}{\text{units for the input}}$?

Yes. Think of the units as the change of output value for each unit of change in input value. An example of slope could be miles per hour or dollars per day. Notice the units appear as a ratio of units for the output per units for the input.

Note: Calculate Slope

The slope or rate of change of a function, m, can be calculated according to the following:

$$m = \frac{\text{change in output (rise)}}{\text{change in input (run)}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} \tag{2.1.3}$$

where x_1 and x_2 are input values, y_1 and y_2 are output values.

How to: Given two points from a linear function, calculate and interpret the slope.

1. Determine the units for output and input values.
2. Calculate the change of output values and change of input values.
3. Interpret the slope as the change in output values per unit of the input value.

Example 2.1.3: Finding the Slope of a Linear Function

If $f(x)$ is a linear function, and $(3,-2)$ and $(8,1)$ are points on the line, find the slope. Is this function increasing or decreasing?

Solution

The coordinate pairs are $(3,-2)$ and $(8,1)$. To find the rate of change, we divide the change in output by the change in input.

$$m=\frac{\text{change in output (rise)}}{\text{change in input (run)}}=\frac{1-(-2)}{8-3}=\frac{3}{5} \tag{2.1.4}$$

We could also write the slope as $m=0.6$. The function is increasing because $m>0$.

Analysis

As noted earlier, the order in which we write the points does not matter when we compute the slope of the line as long as the first output value, or *y*-coordinate, used corresponds with the first input value, or *x*-coordinate, used.

Try it! 2.1.1

If $f(x)$ is a linear function, and $(2,3)$ and $(0,4)$ are points on the line, find the slope. Is this function increasing or decreasing?

Answer

$m=\frac{4-3}{0-2}=\frac{1}{-2}=-\frac{1}{2}$; decreasing because $m<0$.

Example 2.1.4: Finding the Population Change from a Linear Function

The population of a city increased from 23,400 to 27,800 between 2008 and 2012. Find the change of population per year if we assume the change was constant from 2008 to 2012.

The rate of change relates the change in population to the change in time. The population increased by $27,800-23,400=4400$ people over the four-year time interval. To find the rate of change, divide the change in the number of people by the number of years.

$$\frac{4,400 \text{ people}}{4 \text{ years}}=1,100\frac{\text{people}}{\text{year}} \tag{2.1.5}$$

So the population increased by 1,100 people per year.

Analysis

Because we are told that the population increased, we would expect the slope to be positive. This positive slope we calculated is therefore reasonable.

Try it! 2.1.2

The population of a small town increased from 1,442 to 1,868 between 2009 and 2012. Find the change of population per year if we assume the change was constant from 2009 to 2012.

Answer

$m = \frac{1{,}868-1{,}442}{2{,}012-2{,}009} = \frac{426}{3} = 142$ people per year

Writing the Point-Slope Form of a Linear Equation

Up until now, we have been using the slope-intercept form of a linear equation to describe linear functions. Here, we will learn another way to write a linear function, the **point-slope form**.

$$y - y_1 = m(x - x_1) \tag{2.1.6}$$

The point-slope form is derived from the slope formula.

$$\begin{aligned} m &= \frac{y-y_1}{x-x_1} && \text{assuming } x \neq x_1 \\ m(x-x_1) &= \frac{y-y_1}{x-x_1}(x-x_1) && \text{Multiply both sides by } (x-x_1). \\ m(x-x_1) &= y-y_1 && \text{Simplify} \\ y-y_1 &= m(x-x_1) && \text{Rearrange} \end{aligned}$$

Keep in mind that the slope-intercept form and the point-slope form can be used to describe the same function. We can move from one form to another using basic algebra. For example, suppose we are given an equation in point-slope form, $y-4 = -\frac{1}{2}(x-6)$. We can convert it to the slope-intercept form as shown.

$$\begin{aligned} y-4 &= -\frac{1}{2}(x-6) \\ y-4 &= -\frac{1}{2}x+3 && \text{Distribute the } -\frac{1}{2}. \\ y &= -\frac{1}{2}x+7 && \text{Add 4 to each side.} \end{aligned}$$

Therefore, the same line can be described in slope-intercept form as $y = \frac{1}{2}x + 7$.

Note: Point-Slope Form of a Linear Equation

The **point-slope** form of a linear equation takes the form

$$y - y_1 = m(x - x_1) \tag{2.1.7}$$

where m is the slope, x_1 and y_1 are the x and y coordinates of a specific point through which the line passes.

Writing the Equation of a Line Using a Point and the Slope

The point-slope form is particularly useful if we know one point and the slope of a line. Suppose, for example, we are told that a line has a slope of 2 and passes through the point $(4, 1)$. We know that $m = 2$ and that $x_1 = 4$ and $y_1 = 1$. We can substitute these values into the general point-slope equation.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 1 &= 2(x - 4) \end{aligned}$$

If we wanted to then rewrite the equation in slope-intercept form, we apply algebraic techniques.

$$\begin{aligned} y-1 &= 2(x-4) \\ y-1 &= 2x-8 && \text{Distribute the 2.} \\ y &= 2x-7 && \text{Add 1 to each side.} \end{aligned}$$

Both equations, $y-1=2(x-4)$ and $y=2x-7$, describe the same line. See Figure 2.1.7.

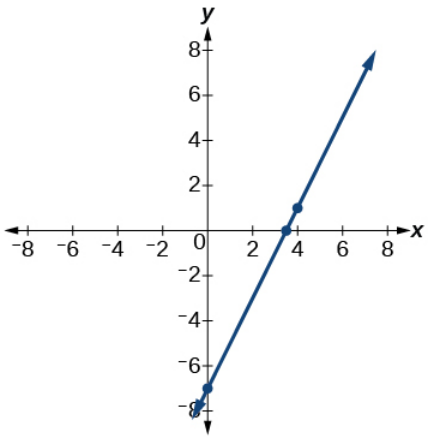


Figure 2.1.7

Example 2.1.5: Writing Linear Equations Using a Point and the Slope

Write the point-slope form of an equation of a line with a slope of 3 that passes through the point $(6,-1)$. Then rewrite it in the slope-intercept form.

Solution

Let's figure out what we know from the given information. The slope is 3, so $m=3$. We also know one point, so we know $x_1=6$ and $y_1=-1$. Now we can substitute these values into the general point-slope equation.

$$\begin{aligned} y-y_1 &= m(x-x_1) \\ y-(-1) &= 3(x-6) && \text{Substitute known values.} \\ y+1 &= 3(x-6) && \text{Distribute } -1 \text{ to find point-slope form.} \end{aligned}$$

Then we use algebra to find the slope-intercept form.

$$\begin{aligned} y+1 &= 3(x-6) \\ y+1 &= 3x-18 && \text{Distribute 3.} \\ y &= 3x-19 && \text{Simplify to slope-intercept form.} \end{aligned}$$

? Try it! 2.1.3

Write the point-slope form of an equation of a line with a slope of –2 that passes through the point $(-2,2)$. Then rewrite it in the slope-intercept form.

Answer

$y-2=-2(x+2)$; $y=-2x-2$

Writing the Equation of a Line Using Two Points

The point-slope form of an equation is also useful if we know any two points through which a line passes. Suppose, for example, we know that a line passes through the points $(0,1)$ and $(3,2)$. We can use the coordinates of the two points to find the slope.

$$\begin{aligned} m &= \frac{y_2-y_1}{x_2-x_1} \\ &= \frac{2-1}{3-0} \\ &= \frac{1}{3} \end{aligned}$$

Now we can use the slope we found and the coordinates of one of the points to find the equation for the line. Let use $(0, 1)$ for our point.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 1 &= \frac{1}{3}(x - 0) \end{aligned}$$

As before, we can use algebra to rewrite the equation in the slope-intercept form.

$$\begin{aligned} y - 1 &= \frac{1}{3}(x - 0) && \\ y - 1 &= \frac{1}{3}x && \text{Distribute the } \frac{1}{3}. \\ y &= \frac{1}{3}x + 1 && \text{Add 1 to each side.} \end{aligned}$$

Both equations describe the line shown in Figure 2.1.8.

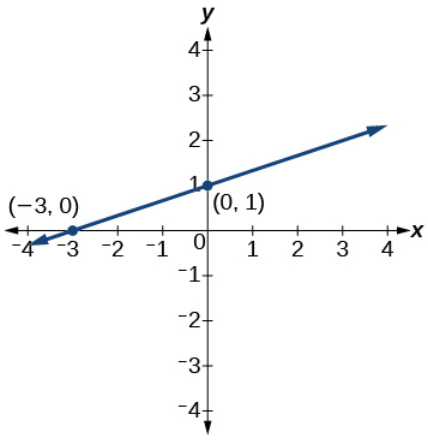


Figure 2.1.8

Example 2.1.6: Writing Linear Equations Using Two Points

Write the point-slope form of an equation of a line that passes through the points $(5, 1)$ and $(8, 7)$. Then rewrite it in the slope-intercept form.

Let's begin by finding the slope.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{7 - 1}{8 - 5} \\ &= \frac{6}{3} \\ &= 2 \end{aligned}$$

So $m = 2$. Next, we substitute the slope and the coordinates for one of the points into the general point-slope equation. We can choose either point, but we will use $(5, 1)$.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 1 &= 2(x - 5) \end{aligned}$$

The point-slope equation of the line is $y_2 - 1 = 2(x_2 - 5)$. To rewrite the equation in slope-intercept form, we use algebra.

$$\begin{aligned} y - 1 &= 2(x - 5) \\ y - 1 &= 2x - 10 \\ y &= 2x - 9 \end{aligned}$$

The slope-intercept equation of the line is $y = 2x - 9$.

Try It! 2.1.4

Write the point-slope form of an equation of a line that passes through the points $(-1, 3)$ and $(0, 0)$. Then rewrite it in the slope-intercept form.

Answer

$y - 0 = -3(x - 0)$; $y = -3x$.

Writing and Interpreting an Equation for a Linear Function

Now that we have written equations for linear functions in both the slope-intercept form and the point-slope form, we can choose which method to use based on the information we are given. That information may be provided in the form of a graph, a point and a slope, two points, and so on. Look at the graph of the function f in Figure 2.1.9.

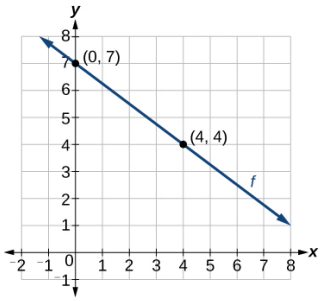


Figure 2.1.9: Graph depicting how to calculate the slope of a line.

We are not given the slope of the line, but we can choose any two points on the line to find the slope. Let's choose $(0, 7)$ and $(4, 4)$. We can use these points to calculate the slope.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{4 - 7}{4 - 0} \\ &= -\frac{3}{4} \end{aligned}$$

Now we can substitute the slope and the coordinates of one of the points into the point-slope form.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 4 &= -\frac{3}{4}(x - 4) \end{aligned}$$

If we want to rewrite the equation in the slope-intercept form, we would find

$$\begin{aligned} y - 4 &= -\frac{3}{4}(x - 4) \\ y - 4 &= -\frac{3}{4}x + 3 \\ y &= -\frac{3}{4}x + 7 \end{aligned}$$

If we wanted to find the slope-intercept form without first writing the point-slope form, we could have recognized that the line crosses the y-axis when the output value is 7. Therefore, $b = 7$. We now have the initial value b and the slope m so we can substitute m and b into the slope-intercept form of a line.

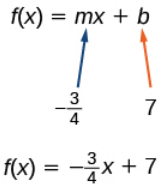


Figure 2.1.10

So the function is $f(x) = -\frac{3}{4}x + 7$, and the linear equation would be $y = -\frac{3}{4}x + 7$.

How to: Given the graph of a linear function, write an equation to represent the function.

1. Identify two points on the line.
2. Use the two points to calculate the slope.
3. Determine where the line crosses the y-axis to identify the y-intercept by visual inspection.
4. Substitute the slope and y-intercept into the slope-intercept form of a line equation.

Example 2.1.7: Writing an Equation for a Linear Function

Write an equation for a linear function given a graph of f shown in Figure 2.1.11.

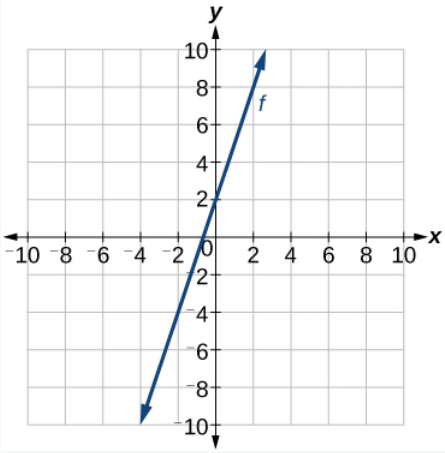


Figure 2.1.11

Solution

Identify two points on the line, such as $(0, 2)$ and $(-2, -4)$. Use the points to calculate the slope.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{4 - 2}{-2 - 0} \\ &= \frac{-6}{-2} \\ &= 3 \end{aligned}$$

Substitute the slope and the coordinates of one of the points into the point-slope form.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - (-4) &= 3(x - (-2)) \\ y + 4 &= 3(x + 2) \end{aligned}$$

We can use algebra to rewrite the equation in the slope-intercept form.

$$\begin{aligned} y+4 &= 3(x+2) \\ y+4 &= 3x+6 \\ y &= 3x+2 \end{aligned}$$

Analysis

This makes sense because we can see from Figure 2.1.12 that the line crosses the y-axis at the point $(0, 2)$, which is the y-intercept, so $b = 2$.

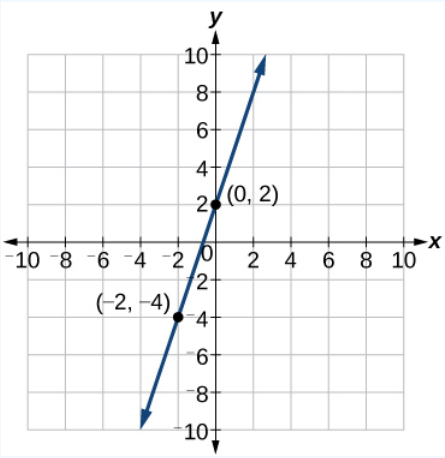


Figure 2.1.12: Graph of an increasing line with points at $(0, 2)$ and $(-2, -4)$.

Example 2.1.8: Writing an Equation for a Linear Cost Function

Suppose Ben starts a company in which he incurs a fixed cost of $1,250 per month for the overhead, which includes his office rent. His production costs are $37.50 per item. Write a linear function C where $C(x)$ is the cost for x items produced in a given month.

Solution

The fixed cost is present every month, $1,250. The costs that can vary include the cost to produce each item, which is $37.50 for Ben. The variable cost, called the marginal cost, is represented by 37.5. The cost Ben incurs is the sum of these two costs, represented by $C(x) = 1250 + 37.5x$.

Analysis

If Ben produces 100 items in a month, his monthly cost is represented by

$$\begin{aligned} C(100) &= 1250 + 37.5(100) \\ &= 5000 \end{aligned}$$

So his monthly cost would be $5,000.

Example 2.1.9: Writing an Equation for a Linear Function Given Two Points

If f is a linear function, with $f(3) = -2$, and $f(8) = 1$, find an equation for the function in slope-intercept form.

Solution

We can write the given points using coordinates.

$$\begin{aligned} f(3) = -2 &\rightarrow (3, 2) \\ f(8) = 1 &\rightarrow (8, 1) \end{aligned}$$

We can then use the points to calculate the slope.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{1-(-2)}{8-3} \\ &= \frac{3}{5} \end{aligned}$$

Substitute the slope and the coordinates of one of the points into the point-slope form.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y-(-2) &= \frac{3}{5}(x-3) \end{aligned}$$

We can use algebra to rewrite the equation in the slope-intercept form.

$$\begin{aligned} y+2 &= \frac{3}{5}(x-3) \\ y+2 &= \frac{3}{5}x - \frac{9}{5} \\ y &= \frac{3}{5}x - \frac{19}{5} \end{aligned}$$

? Try It! 2.1.5

If $f(x)$ is a linear function, with $f(2) = -11$, and $f(4) = -25$, find an equation for the function in slope-intercept form.

Answer

$y = -7x + 3$

Modeling Real-World Problems with Linear Functions

In the real world, problems are not always explicitly stated in terms of a function or represented with a graph. Fortunately, we can analyze the problem by first representing it as a linear function and then interpreting the components of the function. As long as we know, or can figure out, the initial value and the rate of change of a linear function, we can solve many different kinds of real-world problems.

How To: How to: Given a linear function f and the initial value and rate of change, evaluate $f(c)$.

1. Determine the initial value and the rate of change (slope).
2. Substitute the values into $f(x) = mx + b$.
3. Evaluate the function at $x = c$.

Example 2.1.10: Using a Linear Function to Determine the Number of Songs in a Music Collection

Marcus currently has 200 songs in his music collection. Every month, he adds 15 new songs. Write a formula for the number of songs, N, in his collection as a function of time, t, the number of months. How many songs will he own in a year?

Solution

The initial value for this function is 200 because he currently owns 200 songs, so $N(0) = 200$, which means that $b = 200$.

The number of songs increases by 15 songs per month, so the rate of change is 15 songs per month. Therefore we know that $m = 15$. We can substitute the initial value and the rate of change into the slope-intercept form of a line.

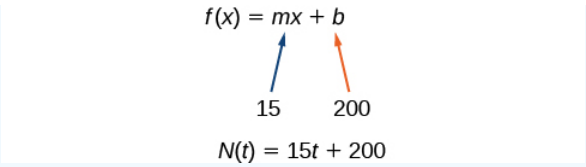


Figure 2.1.13

We can write the formula $N(t) = 15t + 200$.

With this formula, we can then predict how many songs Marcus will have in 1 year (12 months). In other words, we can evaluate the function at $t = 12$.

$$\begin{aligned} N(12) &= 15(12) + 200 \\ &= 180 + 200 \\ &= 380 \end{aligned}$$

Marcus will have 380 songs in 12 months.

Analysis

Notice that N is an increasing linear function. As the input (the number of months) increases, the output (number of songs) increases as well.

Example 2.1.11: Using a Linear Function to Calculate Salary Plus Commission

Working as an insurance salesperson, Ilya earns a base salary plus a commission on each new policy. Therefore, Ilya's weekly income, I,depends on the number of new policies, n, he sells during the week. Last week he sold 3 new policies, and earned \$760 for the week. The week before, he sold 5 new policies and earned \$920. Find an equation for $I(n)$, and interpret the meaning of the components of the equation.

Solution

The given information gives us two input-output pairs: $(3, 760)$ and $(5, 920)$. We start by finding the rate of change.

$$\begin{aligned} m &= \frac{920 - 760}{5 - 3} \\ &= \frac{\$160}{2 \text{ policies}} \\ &= \$80 \text{ per policy} \end{aligned}$$

Keeping track of units can help us interpret this quantity. Income increased by \$160 when the number of policies increased by 2, so the rate of change is \$80 per policy. Therefore, Ilya earns a commission of \$80 for each policy sold during the week.

We can then solve for the initial value.

$$\begin{aligned} I(n) &= 80n + b \\ 760 &= 80(3) + b \text{ when } n = 3, I(3) = 760 \\ 760 - 80(3) &= b \\ 520 &= b \end{aligned}$$

The value of b is the starting value for the function and represents Ilya's income when $n = 0$, or when no new policies are sold. We can interpret this as Ilya's base salary for the week, which does not depend upon the number of policies sold.

We can now write the final equation.

$$I(n) = 80n + 520$$

Our final interpretation is that Ilya's base salary is \$520 per week and he earns an additional \$80 commission for each policy sold.

Example 2.1.12: Using Tabular Form to Write an Equation for a Linear Function

Table 2.1.1 relates the number of rats in a population to time, in weeks. Use the table to write a linear equation.

Table 2.1.1

w, number of weeks	0	2	4	6
P(w), number of rats	1000	1080	1160	1240

Solution

We can see from the table that the initial value for the number of rats is 1000, so $b = 1000$.

Rather than solving for m, we can tell from looking at the table that the population increases by 80 for every 2 weeks that pass. This means that the rate of change is 80 rats per 2 weeks, which can be simplified to 40 rats per week.

$$P(w) = 40w + 1000$$

If we did not notice the rate of change from the table we could still solve for the slope using any two points from the table. For example, using $(2, 1080)$ and $(6, 1240)$

$$\begin{aligned} m &= \frac{1240 - 1080}{6 - 2} \\ &= \frac{160}{4} \\ &= 40 \end{aligned}$$

Is the initial value always provided in a table of values?

No. Sometimes the initial value is provided in a table of values, but sometimes it is not. If you see an input of 0, then the initial value would be the corresponding output. If the initial value is not provided because there is no value of input on the table equal to 0, find the slope, substitute one coordinate pair and the slope into $f(x) = mx + b$, and solve for b.

Try It 2.1.6

A new plant food was introduced to a young tree to test its effect on the height of the tree. Table 2.1.2 shows the height of the tree, in feet, x months since the measurements began. Write a linear function, $H(x)$, where x is the number of months since the start of the experiment.

Table 2.1.2

x	0	2	4	8	12
$H(x)$	12.5	13.5	14.5	16.5	18.5

Answer

$H(x) = 0.5x + 12.5$

Key Equations

- slope-intercept form of a line: $f(x) = mx + b$

- slope: $m = \dfrac{\text{change in output (rise)}}{\text{change in input (run)}} = \dfrac{\Delta y}{\Delta x} = \dfrac{y_2 - y_1}{x_2 - x_1}$
- point-slope form of a line: $y - y_1 = m(x - x_1)$

Key Concepts

- The ordered pairs given by a linear function represent points on a line.
- Linear functions can be represented in words, function notation, tabular form, and graphical form.
- The rate of change of a linear function is also known as the slope.
- An equation in the slope-intercept form of a line includes the slope and the initial value of the function.
- The initial value, or y-intercept, is the output value when the input of a linear function is zero. It is the y-value of the point at which the line crosses the y-axis.
- An increasing linear function results in a graph that slants upward from left to right and has a positive slope.
- A decreasing linear function results in a graph that slants downward from left to right and has a negative slope.
- A constant linear function results in a graph that is a horizontal line.
- Analyzing the slope within the context of a problem indicates whether a linear function is increasing, decreasing, or constant.
- The slope of a linear function can be calculated by dividing the difference between y-values by the difference in corresponding x-values of any two points on the line.
- The slope and initial value can be determined given a graph or any two points on the line.
- One type of function notation is the slope-intercept form of an equation.
- The point-slope form is useful for finding a linear equation when given the slope of a line and one point.
- The point-slope form is also convenient for finding a linear equation when given two points through which a line passes.
- The equation for a linear function can be written if the slope m and initial value b are known.
- A linear function can be used to solve real-world problems.
- A linear function can be written from tabular form.

Footnotes

1 www.chinahighlights.com/shang...glev-train.htm
2 www.cbsnews.com/8301-501465_1...ay-study-says/

Glossary

decreasing linear function

a function with a negative slope: If $f(x) = mx + b$, then $m < 0$.

increasing linear function

a function with a positive slope: If $f(x) = mx + b$, then $m > 0$.

linear function

a function with a constant rate of change that is a polynomial of degree 1, and whose graph is a straight line

point-slope form

the equation for a line that represents a linear function of the form \(y−y_1=m(x−x_1)

slope

the ratio of the change in output values to the change in input values; a measure of the steepness of a line

slope-intercept form

the equation for a line that represents a linear function in the form $f(x) = mx + b$

y-intercept

the value of a function when the input value is zero; also known as initial value

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2.2: Prelude to Solving Linear Equations

Imagine being a pilot, but not just any pilot—a drone pilot. Drones, or unmanned aerial vehicles, are devices that can be flown remotely. They contain sensors that can relay information to a command center where the pilot is located. Larger drones can also carry cargo. In the near future, several companies hope to use drones to deliver materials and piloting a drone will become an important career. Law enforcement and the military are using drones rather than send personnel into dangerous situations.



Figure 2.2.1: This drone is flying high in the sky while its pilot remains safely on the ground. (credit: “Unsplash” / Pixabay)

Building and piloting a drone requires the ability to program a set of actions, including taking off, turning, and landing. This, in turn, requires the use of linear equations. In this chapter, you will explore linear equations, develop a strategy for solving them, and relate them to real-world situations.

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2.3: Use a Problem Solving Strategy

Learning Objectives

By the end of this section, you will be able to:

- Use a problem solving strategy for word problems
- Solve number word problems
- Solve percent applications
- Solve simple interest applications

Before you get started, take this readiness quiz.

1. Translate "six less than twice x" into an algebraic expression.
   If you missed this problem, review [link].
2. Convert 4.5% to a decimal.
   If you missed this problem, review [link].
3. Convert 0.6 to a percent.
   If you missed this problem, review [link].

Have you ever had any negative experiences in the past with word problems? When we feel we have no control, and continue repeating negative thoughts, we set up barriers to success. Realize that your negative experiences with word problems are in your past. To move forward you need to calm your fears and change your negative feelings.

Start with a fresh slate and begin to think positive thoughts. Repeating some of the following statements may be helpful to turn your thoughts positive. Thinking positive thoughts is a first step towards success.

- I think I can! I think I can!
- While word problems were hard in the past, I think I can try them now.
- I am better prepared now—I think I will begin to understand word problems.
- I am able to solve equations because I practiced many problems and I got help when I needed it—I can try that with word problems.
- It may take time, but I can begin to solve word problems.
- You are now well prepared and you are ready to succeed. If you take control and believe you can be successful, you will be able to master word problems.

Use a Problem Solving Strategy for Word Problems

Now that we can solve equations, we are ready to apply our new skills to word problems. We will develop a strategy we can use to solve any word problem successfully.

EXAMPLE 2.3.1

Normal yearly snowfall at the local ski resort is 12 inches more than twice the amount it received last season. The normal yearly snowfall is 62 inches. What was the snowfall last season at the ski resort?

Solution:

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	What was the snowfall last season?
Step 3. Name what we are looking for and choose a variable to represent it.	Let $s =$ the snowfall last season.
Step 4. Translate. Restate the problem in one sentence with all the important information.	The normal snowfall was twelve more than twice the amount last year

Translate into an equation.	$62 = 2s + 12$
Step 5. Solve the equation.	$62 = 2s + 12$
Subtract 12 from each side.	$62 - 12 = 2s + 12 - 12$
Simplify.	$50 = 2s$
Divide each side by two.	$\frac{50}{2} = \frac{2s}{2}$
Simplify.	$25 = s$
Step 6. Check: First, is our answer reasonable? Yes, having 25 inches of snow seems OK. The problem says the normal snowfall is twelve inches more than twice the number of last season. Twice 25 is 50 and 12 more than that is 62.	
Step 7. Answer the question.	The snowfall last season was 25 inches.

Try It! 2.3.1

Guillermo bought textbooks and notebooks at the bookstore. The number of textbooks was three more than twice the number of notebooks. He bought seven textbooks. How many notebooks did he buy?

Answer

He bought two notebooks

Try It! 2.3.2

Gerry worked Sudoku puzzles and crossword puzzles this week. The number of Sudoku puzzles he completed is eight more than twice the number of crossword puzzles. He completed 22 Sudoku puzzles. How many crossword puzzles did he do?

Answer

He did seven crossword puzzles

We summarize an effective strategy for problem solving.

PROBLEM SOLVING STRATEGY FOR WORD PROBLEMS

1. **Read** the problem. Make sure all the words and ideas are understood.
2. **Identify** what you are looking for.
3. **Name** what you are looking for. Choose a variable to represent that quantity.
4. **Translate** into an equation. It may be helpful to restate the problem in one sentence with all the important information. Then, translate the English sentence into an algebra equation.
5. **Solve** the equation using proper algebra techniques.
6. **Check** the answer in the problem to make sure it makes sense.
7. **Answer** the question with a complete sentence.

Solve Number Word Problems

We will now apply the problem solving strategy to “number word problems.” Number word problems give some clues about one or more numbers and we use these clues to write an equation. Number word problems provide good practice for using the Problem Solving Strategy.

EXAMPLE 2.3.2

The sum of seven times a number and eight is thirty-six. Find the number.

Solution:

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	the number
Step 3. Name what you are looking for and choose a variable to represent it.	Let n = the number.
Step 4. Translate: Restate the problem as one sentence. Translate into an equation.	The sum of seven times a number and 8 is 36 $7n+8 = 36$
Step 5. Solve the equation. Subtract eight from each side and simplify. Divide each side by seven and simplify.	$7n+8=36$ $7n=28$ $n=4$
Step 6. Check. Is the sum of seven times four plus eight equal to 36? $7\cdot 4+8 \stackrel{?}{=} 36$ $28+8 \stackrel{?}{=} 36$ $36=36$✓	
Step 7. Answer the question.	The number is 4.

Did you notice that we left out some of the steps as we solved this equation? If you're not yet ready to leave out these steps, write down as many as you need.

Try It! 2.3.3

The sum of four times a number and two is fourteen. Find the number.

Answer

3

Try It! 2.3.4

The sum of three times a number and seven is twenty-five. Find the number.

Answer

6

Some number word problems ask us to find two or more numbers. It may be tempting to name them all with different variables, but so far, we have only solved equations with one variable. In order to avoid using more than one variable, we will define the numbers in terms of the same variable. Be sure to read the problem carefully to discover how all the numbers relate to each other.

EXAMPLE 2.3.3

The sum of two numbers is negative fifteen. One number is nine less than the other. Find the numbers.

Solution:

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	two numbers
Step 3. Name what you are looking for by choosing a variable to represent the first number. “One number is nine less than the other.”	Let $n = 1^{\text{st}}$ number. $n - 9 = 2^{\text{nd}}$ number
Step 4. Translate. Write as one sentence. Translate into an equation.	The sum of two numbers is negative fifteen. 1st number + 2nd number is negative fifteen $n + n - 9 = -15$
Step 5. Solve the equation. Combine like terms. Add nine to each side and simplify. Simplify.	$n + n - 9 = -15$ $2n = -6$ $-3 - 9$ -12
Step 6. Check. Is -12 nine less than -3? $-3 - 9 \stackrel{?}{=} -12$ $-12 = -12$✓ Is their sum -15? $-3 + (-12) \stackrel{?}{=} -15$ $-15 = -15$✓	
Step 7. Answer the question.	The numbers are -3 and -12.

? Try It! 2.3.5

The sum of two numbers is negative twenty-three. One number is seven less than the other. Find the numbers.

Answer

$-15, -8$

? Try It! 2.3.6

The sum of two numbers is negative eighteen. One number is forty more than the other. Find the numbers.

Answer

$-29, 11$

Some number problems involve **consecutive integers**. Consecutive integers are integers that immediately follow each other. Examples of consecutive integers are:

$$\begin{array}{rrrr} 1, & 2, & 3, & 4 \\ -10, & -9, & -8, & -7 \\ 150, & 151, & 152, & 153 \end{array}$$

Notice that each number is one more than the number preceding it. Therefore, if we define the first integer as n, the next consecutive integer is $n+1$. The one after that is one more than $n+1$, so it is $n+1+1$, which is $n+2$.

$$\begin{array}{ll} n & 1^{\text{st}} \text{integer} \\ n+1 & 2^{\text{nd}} \text{consecutive integer} \\ n+2 & 3^{\text{rd}} \text{consecutive integer} \quad \text{etc.} \end{array}$$

We will use this notation to represent consecutive integers in the next example.

EXAMPLE 2.3.4

Find three consecutive integers whose sum is -54.

Solution:

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	three consecutive integers
Step 3. Name each of the three numbers	Let $n = 1^{\text{st}}$integer. $n+1 = 2^{\text{nd}}$consecutive integer $n+2 = 3^{\text{rd}}$consecutive integer
Step 4. Translate. Restate as one sentence. Translate into an equation.	The sum of the three integers is -54. $n+n+1+n+2=-54$
Step 5. Solve the equation. Combine like terms. Subtract three from each side. Divide each side by three.	$n+n+1+n+2=-54$ $3n+3=-54$ $3n=-57$ $n=-19$ 1st integer $n+1$ 2nd integer -18 $-19+2$ -17
Step 6. Check. $-19+(-18)+(-17) = -54$ $-54=-54$✓	
Step 7. Answer the question.	The three consecutive integers are −17,−18, and −19.

Try It! 2.3.7

Find three consecutive integers whose sum is -96.

Answer

$-33, -32, -31$

Try It! 2.3.8

Find three consecutive integers whose sum is -36.

Answer

$-13, -12, -11$

Now that we have worked with consecutive integers, we will expand our work to include **consecutive even integers** and **consecutive odd integers**. Consecutive even integers are even integers that immediately follow one another. Examples of consecutive even integers are:

$$24, 26, 28$$

$$-12, -10, -8$$

Notice each integer is two more than the number preceding it. If we call the first one n, then the next one is $n+2$. The one after that would be $n+2+2$ or $n+4$.

n 1^{st}integer
$n+1$ 2^{nd}consecutive integer
$n+2$ 3^{rd}consecutive integer etc.

Consecutive odd integers are odd integers that immediately follow one another. Consider the consecutive odd integers 63, 65, and 67.

$$63, 65, 67$$

$$n, n+2, n+4$$

n 1^{st}integer
$n+1$ 2^{nd}consecutive integer
$n+2$ 3^{rd}consecutive integer etc.

Does it seem strange to have to add two (an even number) to get the next odd number? Do we get an odd number or an even number when we add 2 to 3? to 11? to 47?

Whether the problem asks for consecutive even numbers or odd numbers, you do not have to do anything different. The pattern is still the same—to get to the next odd or the next even integer, add two.

EXAMPLE 2.3.5

Find three consecutive even integers whose sum is 120.

Solution:

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	three consecutive even integers
Step 3. Name each of the three numbers	Let $n = 1^{\text{st}}$consecutive even integer. $n+2 = 2^{\text{nd}}$consecutive even integer. $n+4 = 3^{\text{rd}}$consecutive even integer.
Step 4. Translate. Restate as one sentence. Translate into an equation.	The sum of the three even integers is 120 $n+n+2+n+4=120$
Step 5. Solve the equation. Combine like terms. Subtract three from each side. Divide each side by three.	$n+n+2+n+4=120$ $3n+6=120$ $3n=114$ $n=38$ 1^{st}integer $n+2$ 2^{nd}integer $38+2$ 40 $n+2$ 3^{rd}integer $38+4$ 42
Step 6. Check. $38+40+42 \stackrel{?}{=} 120$ $120 = 120$✓	
Step 7. Answer the question.	The three consecutive integers are 38, 40, and 42.

Try It! 2.3.9

Find three consecutive even integers whose sum is 102.

Answer

$32, 34, 36$

Try It! 2.3.10

Find three consecutive even integers whose sum is -24.

Answer

$-10, -8, -6$

When a number problem is in a real life context, we still use the same strategies that we used for the previous examples.

EXAMPLE 2.3.6

A married couple together earns $110,000 a year. The wife earns $16,000 less than twice what her husband earns. What does the husband earn?

Solution:

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	How much does the husband earn?
Step 3. Name each of the three numbers	Let $h =$ the amount the husband earns.
Step 4. Translate. Restate the problem in one sentence with all the important information. Translate into an equation.	$2h - 16,000 =$ the amount the wife earns. Together the husband and wife earn $110,000. $h + 2h - 16,000 = 110,000$
Step 5. Solve the equation. Combine like terms. Add 16,000 to both sides and simplify. Divide each side by three.	$h + 2h - 16,000 = 110,000$ $3h - 16,000 = 110,000$ $3h = 126,000$ $h = 42,000$ amount husband earns $2h - 16,000$ amount wife earns $2(42,000) - 16,000$ $84,000 - 16,000$ $68,000$
Step 6. Check. If the wife earns $68,000 and the husband earns $42,000, is that $110,000? Yes!	
Step 7. Answer the question.	The husband earns $42,000 a year.

According to the National Automobile Dealers Association, the average cost of a car in 2014 was $28,400. This was $1,600 less than six times the cost in 1975. What was the average cost of a car in 1975?

Answer

The average cost was $5,000.

Try It! 2.3.11

US Census data shows that the median price of new home in the U.S. in November 2014 was $280,900. This was $10,700 more than 14 times the price in November 1964. What was the median price of a new home in November 1964?

Answer

The median price was $19,300.

Solve Percent Applications

There are several methods to solve percent equations. In algebra, it is easiest if we just translate English sentences into algebraic equations and then solve the equations. Be sure to change the given percent to a decimal before you use it in the equation.

EXAMPLE 2.3.7

Translate and solve:

a. What number is 45% of 84?
b. 8.5% of what amount is $4.76?
c. 168 is what percent of 112?

Solution:

a.

	What number is 45% of 84?
Translate into algebra. Let n = the number.	$n = 0.45 \cdot 84$
Multiply.	$n = 37.8$
	37.8 is 45% of 84.

b.

	8.5% of what amount is $4.76?
Translate. Let n = the amount.	$0.085 \cdot n = 4.76$
Multiply.	$0.085n = 4.76$
Divide both sides by 0.085 and simplify.	$n = 56$
	8.5% of $56 is $4.76

c.

We are asked to find percent, so we must have our result in percent form.	168 is what percent of 112?
Translate into algebra. Let p = the percent.	$168 = p \cdot 112$
Multiply.	$168 = 112p$
Divide both sides by 112 and simplify.	$1.5 = p$
Convert to percent.	$150\% = p$
	168 is 150% of 112.

Try It! 2.3.12

Translate and solve:

a. What number is 45% of 80?
b. 7.5% of what amount is $1.95?
c. 110 is what percent of 88?

Answer

a. 36 b. $26 c. 125%

Try It! 2.3.13

Translate and solve:

a. What number is 55% of 60?
b. 8.5% of what amount is $3.06?
c. 126 is what percent of 72?

Answer

a. 33 b. $36 c. 175%

Now that we have a problem solving strategy to refer to, and have practiced solving basic percent equations, we are ready to solve percent applications. Be sure to ask yourself if your final answer makes sense—since many of the applications we will solve involve everyday situations, you can rely on your own experience.

EXAMPLE 2.3.8

The label on Audrey's yogurt said that one serving provided 12 grams of protein, which is 24% of the recommended daily amount. What is the total recommended daily amount of protein?

Solution:

What are you asked to find?	What total amount of protein is recommended?
Choose a variable to represent it.	Let a = total amount of protein.
Write a sentence that gives the information to find it.	12 g is 24% of the total amount
Translate into an equation.	$12 = 0.24 \cdot a$
Solve.	$50 = a$
Check: Does this make sense? Yes, 24% is about $\frac{1}{4}$ of the total and 12 is about $\frac{1}{4}$ of 50.	
Write a complete sentence to answer the question.	The amount of protein that is recommended is 50 g.

Try It! 2.3.14

One serving of wheat square cereal has 7 grams of fiber, which is 28% of the recommended daily amount. What is the total recommended daily amount of fiber?

Answer

25 grams

Try It! 2.3.15

One serving of rice cereal has 190 mg of sodium, which is 8% of the recommended daily amount. What is the total recommended daily amount of sodium?

Answer

2,375 mg

Remember to put the answer in the form requested. In the next example we are looking for the percent.

EXAMPLE 2.3.9

Veronica is planning to make muffins from a mix. The package says each muffin will be 240 calories and 60 calories will be from fat. What percent of the total calories is from fat?

Solution:

What are you asked to find?	What percent of the total calories is fat?
Choose a variable to represent it.	Let $p =$ percent of fat.
Write a sentence that gives the information to find it.	What percent of 240 is 60?
Translate the sentence into an equation.	$p \cdot 240 = 60$
Multiply.	$240p = 60$
Divide both sides by 240.	$p = 0.25$
Put in percent form.	$p = 25\%$
Check: does this make sense? Yes, 25% is one-fourth; 60 is one-fourth of 240. So, 25% makes sense.	
Write a complete sentence to answer the question.	Of the total calories in each muffin, 25% is fat.

Try It! 2.3.16

Mitzi received some gourmet brownies as a gift. The wrapper said each 28% brownie was 480 calories, and had 240 calories of fat. What percent of the total calories in each brownie comes from fat? Round the answer to the nearest whole percent.

Answer

50%

Try It! 2.3.17

The mix Ricardo plans to use to make brownies says that each brownie will be 190 calories, and 76 calories are from fat. What percent of the total calories are from fat? Round the answer to the nearest whole percent.

Answer

40%

It is often important in many fields—business, sciences, pop culture—to talk about how much an amount has increased or decreased over a certain period of time. This increase or decrease is generally expressed as a percent and called the **percent change**.

To find the percent change, first we find the amount of change, by finding the difference of the new amount and the original amount. Then we find what percent the amount of change is of the original amount.

FIND PERCENT CHANGE

1. Find the amount of change.

$$\text{change} = \text{new amount} - \text{original amount} \tag{2.3.1}$$

2. Find what percent the amount of change is of the original amount.

change is what percent of the original amount?

EXAMPLE 2.3.10

Recently, the California governor proposed raising community college fees from $36 a unit to $46 a unit. Find the percent change. (Round to the nearest tenth of a percent.)

Solution:

Find the amount of change.	$46 - 36 = 10$
Find the percent.	Change is what percent of the original amount?
Let $p =$ the percent.	10 is what percent of 36?
Translate to an equation.	$10 = p \cdot 36$
Simplify.	$10 = 36p$
Divide both sides by 36.	$0.278 \approx p$
Change to percent form; round to the nearest tenth	$27.8\% \approx p$
Write a complete sentence to answer the question.	The new fees are approximately a 27.8% increase over the old fees.
Remember to round the division to the nearest thousandth in order to round the percent to the nearest tenth.	

Try It! 2.3.18

Find the percent change. (Round to the nearest tenth of a percent.) In 2011, the IRS increased the deductible mileage cost to 55.5 cents from 51 cents.

Answer

8.8%

Try It! 2.3.19

Find the percent change. (Round to the nearest tenth of a percent.) In 1995, the standard bus fare in Chicago was $1.50. In 2008, the standard bus fare was 2.25.

Answer

50

Applications of discount and mark-up are very common in retail settings.

When you buy an item on sale, the original price has been discounted by some dollar amount. The **discount rate**, usually given as a percent, is used to determine the **amount of the discount**. To determine the amount of discount, we multiply the discount rate by the original price.

The price a retailer pays for an item is called the **original cost**. The retailer then adds a **mark-up** to the original cost to get the **list price**, the price he sells the item for. The mark-up is usually calculated as a percent of the original cost. To determine the amount of mark-up, multiply the mark-up rate by the original cost.

DISCOUNT

$$\text{amount of discount} = \text{discount rate} \cdot \text{original price}$$
$$\text{sale price} = \text{original amount} - \text{discount price}$$

The sale price should always be less than the original price.

MARK-UP

$$\text{amount of mark-up} = \text{mark-up rate} \cdot \text{original price}$$
$$\text{list price} = \text{original cost} - \text{mark-up}$$

The list price should always be more than the original cost.

EXAMPLE 2.3.11

Liam's art gallery bought a painting at an original cost of $750. Liam marked the price up 40%. Find

a. the amount of mark-up and
b. the list price of the painting.

Solution:

a.

Identify what you are asked to find, and choose a variable to represent it.	What is the amount of mark-up? Let $m =$ the amount of mark-up.
Write a sentence that gives the information to find it.	The mark-up is 40% of the $750 original cost
Translate into an equation.	$m = 0.40 \times 750$
Solve the equation.	$m = 300$
Write a complete sentence.	The mark-up on the painting was $300.

b.

Identify what you are asked to find, and choose a variable to represent it.	What is the list price? Let $p =$ the list price.
Write a sentence that gives the information to find it.	The list price is original cost plus the mark-up
Translate into an equation.	$p = 750 + 300$
Solve the equation.	$p = 1{,}050$
Check.	Is the list price more than the original cost? Is $1,050 more than $750? Yes.
Write a complete sentence.	The list price of the painting was $1,050.

Try It! 2.3.20

Find a. the amount of mark-up and b. the list price: Jim's music store bought a guitar at original cost $1,200. Jim marked the price up 50%.

Answer

a. $600 b. $1,800

Try It! 2.3.21

Find a. the amount of mark-up and b. the list price: The Auto Resale Store bought Pablo's Toyota for $8,500. They marked the price up 35%.

Answer

a. $2,975 b. $11,475

Solve Simple Interest Applications

Interest is a part of our daily lives. From the interest earned on our savings to the interest we pay on a car loan or credit card debt, we all have some experience with interest in our lives.

The amount of money you initially deposit into a bank is called the **principal**, P, and the bank pays you interest, I. When you take out a loan, you pay interest on the amount you borrow, also called the principal.

In either case, the interest is computed as a certain percent of the principal, called the **rate of interest**, r. The rate of interest is usually expressed as a percent per year, and is calculated by using the decimal equivalent of the percent. The variable t, (for time) represents the number of years the money is saved or borrowed.

Interest is calculated as simple interest or compound interest. Here we will use simple interest.

SIMPLE INTEREST

If an amount of money, P, called the principal, is invested or borrowed for a period of t years at an annual interest rate r, the amount of interest, I, earned or paid is

$$I = Prt \qquad \text{where} \quad \begin{aligned} I &= \text{interest} \\ P &= \text{principal} \\ r &= \text{rate} \\ t &= \text{time} \end{aligned}$$

Interest earned or paid according to this formula is called **simple interest**.

The formula we use to calculate interest is $I = Prt$. To use the formula we substitute in the values for variables that are given, and then solve for the unknown variable. It may be helpful to organize the information in a chart.

EXAMPLE 2.3.12

Areli invested a principal of $950 in her bank account that earned simple interest at an interest rate of 3%. How much interest did she earn in five years?

Solution:

$I = ?$
$P = \$950$
$r = 3\%$
$t = 5$ years

Identify what you are asked to find, and choose a variable to represent it.	What is the simple interest? Let $I =$ interest.
Write the formula.	$I = Prt$
Substitute in the given information.	$I = (950)(0.03)(5)$
Simplify.	$I = 142.5$
Check. Is $142.50 a reasonable amount of interest on $ 950? Yes.	
Write a complete sentence.	The interest is $142.50.

Try It! 2.3.22

Nathaly deposited $12,500 in her bank account where it will earn 4% simple interest. How much interest will Nathaly earn in five years?

Answer

He will earn $2,500.

Try It! 2.3.23

Susana invested a principal of $36,000 in her bank account that earned simple interest at an interest rate of 6.5%. How much interest did she earn in three years?

Answer

She earned $7,020.

There may be times when we know the amount of interest earned on a given principal over a certain length of time, but we do not know the rate.

EXAMPLE 2.3.13

Hang borrowed $7,500 from her parents to pay her tuition. In five years, she paid them $1,500 interest in addition to the $7,500 she borrowed. What was the rate of simple interest?

Solution:

$I = \$1500$
$P = \$7500$
$r = ?$
$t = 5$ years

Identify what you are asked to find, and choose a variable to represent it.	What is rate of simple interest? Let r = rate of interest
Write the formula.	$I = Prt$
Substitute in the given information.	$1,500 = (7,500)r(5)$
Multiply.	$1,500 = 37,500r$
Divide.	$0.04 = r$
Change to percent form	$r = 4\%$

Check.

$$I = Prt$$
$$1,500 \stackrel{?}{=} (7,500)(0.04)(5)$$
$$1,500 = 1,500\checkmark$$

Write a complete sentence. The rate of interest was 4%.

Try It! 2.3.24

Jim lent his sister $5,000 to help her buy a house. In three years, she paid him the $5,000, plus $900 interest. What was the rate of simple interest?

Answer

The rate of simple interest was 6%.

Try It! 2.3.25

Loren lent his brother $3,000 to help him buy a car. In four years, his brother paid him back the $3,000 plus $660 in interest. What was the rate of simple interest?

Answer

The rate of simple interest was 5.5%.

In the next example, we are asked to find the principal—the amount borrowed.

EXAMPLE 2.3.14

Sean's new car loan statement said he would pay $4,866,25 in interest from a simple interest rate of 8.5% over five years. How much did he borrow to buy his new car?

Solution:

$$\begin{aligned} I &= 4,866.25 \\ P &= ? \\ r &= 8.5\% \\ t &= 5 \text{ years} \end{aligned}$$

Identify what you are asked to find, and choose a variable to represent it.	What is the amount borrowed (the principal)? Let P = principal borrowed
Write the formula.	$I = Prt$
Substitute in the given information.	$4,866.25 = P(0.085)(5)$
Multiply.	$4,866.25 = 0.425P$
Divide.	$11,450 = P$

Check.

$$\begin{aligned} I &= Prt \\ 4,866.25 &\stackrel{?}{=} (11,450)(0.085)(5) \\ 4,866.25 &= 4,866.25\checkmark \end{aligned}$$

Write a complete sentence. The principal was $\$11,450$.

Try It! 2.3.26

Eduardo noticed that his new car loan papers stated that with a 7.5% simple interest rate, he would pay $6,596.25 in interest over five years. How much did he borrow to pay for his car?

Answer

He paid $17,590.

Try It! 2.3.27

In five years, Gloria's bank account earned $2,400 interest at 5% simple interest. How much had she deposited in the account?

Answer

She deposited $9,600.

Access this online resource for additional instruction and practice with using a problem solving strategy.

- Begining Arithmetic Problems

Key Concepts

- **How To Use a Problem Solving Strategy for Word Problems**
  1. **Read** the problem. Make sure all the words and ideas are understood.
  2. **Identify** what you are looking for.
  3. **Name** what you are looking for. Choose a variable to represent that quantity.
  4. **Translate** into an equation. It may be helpful to restate the problem in one sentence with all the important information. Then, translate the English sentence into an algebra equation.
  5. **Solve** the equation using proper algebra techniques.
  6. **Check** the answer in the problem to make sure it makes sense.
  7. **Answer** the question with a complete sentence.
- **How To Find Percent Change**
  1. Find the amount of change

     $$\text{change} = \text{new amount} - \text{original amount}$$

  2. Find what percent the amount of change is of the original amount.

     $$\text{change is what percent of the original amount?}$$

- $$\text{amount of discount} = \text{discount rate} \cdot \text{original price}$$
  $$\text{sale price} = \text{original amount} - \text{discount price}$$
- $$\text{amount of mark-up} = \text{mark-up rate} \cdot \text{original price}$$
  $$\text{list price} = \text{original cost} - \text{mark-up}$$
- If an amount of money, P, called the principal, is invested or borrowed for a period of t years at an annual interest rate r, the amount of interest, I, earned or paid is:

$$I = Prt \qquad \begin{array}{l} I = interest \\ \text{where } P = principal \\ r = rate \\ t = time \end{array}$$

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2.3E: Exercises

Practice Makes Perfect

Use a Problem Solving Strategy for Word Problems

1. List five positive thoughts you can say to yourself that will help you approach word problems with a positive attitude. You may want to copy them on a sheet of paper and put it in the front of your notebook, where you can read them often.

Answer

Answers will vary.

2. List five negative thoughts that you have said to yourself in the past that will hinder your progress on word problems. You may want to write each one on a small piece of paper and rip it up to symbolically destroy the negative thoughts.

In the following exercises, solve using the problem solving strategy for word problems. Remember to write a complete sentence to answer each question.

3. There are 16 girls in a school club. The number of girls is four more than twice the number of boys. Find the number of boys.

Answer

six boys

4. There are 18 Cub Scouts in Troop 645. The number of scouts is three more than five times the number of adult leaders. Find the number of adult leaders.

5. Huong is organizing paperback and hardback books for her club's used book sale. The number of paperbacks is 12 less than three times the number of hardbacks. Huong had 162 paperbacks. How many hardback books were there?

Answer

58 hardback books

6. Jeff is lining up children's and adult bicycles at the bike shop where he works. The number of children's bicycles is nine less than three times the number of adult bicycles. There are 42 adult bicycles. How many children's bicycles are there?

Solve Number Word Problems

In the following exercises, solve each number word problem.

7. The difference of a number and 12 is three. Find the number.

Answer

15

8. The difference of a number and eight is four. Find the number.

9. The sum of three times a number and eight is 23. Find the number.

Answer

5

10. The sum of twice a number and six is 14. Find the number.

11. The difference of twice a number and seven is 17. Find the number.

Answer

12

12. The difference of four times a number and seven is 21. Find the number.

13. Three times the sum of a number and nine is 12. Find the number.

Answer

-5

14. Six times the sum of a number and eight is 30. Find the number.

15. One number is six more than the other. Their sum is 42. Find the numbers.

Answer

$18,\ 24$

16. One number is five more than the other. Their sum is 33. Find the numbers.

17. The sum of two numbers is 20. One number is four less than the other. Find the numbers.

Answer

$8,\ 12$

18. The sum of two numbers is 27. One number is seven less than the other. Find the numbers.

19. One number is 14 less than another. If their sum is increased by seven, the result is 85. Find the numbers.

Answer

$32,\ 46$

20. One number is 11 less than another. If their sum is increased by eight, the result is 71. Find the numbers.

21. The sum of two numbers is 14. One number is two less than three times the other. Find the numbers.

Answer

$4,\ 10$

22. The sum of two numbers is zero. One number is nine less than twice the other. Find the numbers.

23. The sum of two consecutive integers is 77. Find the integers.

Answer

$38,\ 39$

24. The sum of two consecutive integers is 89. Find the integers.

25. The sum of three consecutive integers is 78. Find the integers.

Answer

$25,\ 26,\ 27$

26. The sum of three consecutive integers is 60. Find the integers.

27. Find three consecutive integers whose sum is -36.

Answer

$-11,\ -12,\ -13$

28. Find three consecutive integers whose sum is -3.

29. Find three consecutive even integers whose sum is 258.

Answer

$84,\ 86,\ 88$

30. Find three consecutive even integers whose sum is 222.

31. Find three consecutive odd integers whose sum is -213.

Answer

$-69,\ -71,\ -73$

32. Find three consecutive odd integers whose sum is -267.

33. Philip pays $\$1,620$ in rent every month. This amount is $\$120$ more than twice what his brother Paul pays for rent. How much does Paul pay for rent?

Answer

$\$750$

34. Marc just bought an SUV for $\$54,000$. This is $\$7,400$ less than twice what his wife paid for her car last year. How much did his wife pay for her car?

35. Laurie has $\$46,000$ invested in stocks and bonds. The amount invested in stocks is $\$8,000$ less than three times the amount invested in bonds. How much does Laurie have invested in bonds?

Answer

$\$13,500$

36. Erica earned a total of $\$50,450$ last year from her two jobs. The amount she earned from her job at the store was $\$1,250$ more than three times the amount she earned from her job at the college. How much did she earn from her job at the college?

Solve Percent Applications

In the following exercises, translate and solve.

37. a. What number is 45% of 120? b. 81 is 75% of what number? c. What percent of 260 is 78?

Answer

a. 54
b. 108
c. 30%

38. a. What number is 65% of 100? b. 93 is 75% of what number? c. What percent of 215 is 86?

39. a. 250% of 65 is what number? b. 8.2% of what amount is $2.87? c. 30 is what percent of 20?

Answer

a. 162.5
b. $35
c. 150%

40. a. 150% of 90 is what number? b. 6.4% of what amount is $2.88? c. 50 is what percent of 40?

In the following exercises, solve.

41. Geneva treated her parents to dinner at their favorite restaurant. The bill was $74.25. Geneva wants to leave 16% of the total bill as a tip. How much should the tip be?

Answer

$11.88

42. When Hiro and his co-workers had lunch at a restaurant near their work, the bill was $90.50. They want to leave 18% of the total bill as a tip. How much should the tip be?

43. One serving of oatmeal has 8 grams of fiber, which is 33% of the recommended daily amount. What is the total recommended daily amount of fiber?

Answer

24.2 g

44. One serving of trail mix has 67 grams of carbohydrates, which is 22% of the recommended daily amount. What is the total recommended daily amount of carbohydrates?

45. A bacon cheeseburger at a popular fast food restaurant contains 2070 milligrams (mg) of sodium, which is 86% of the recommended daily amount. What is the total recommended daily amount of sodium?

Answer

2407 mg

46. A grilled chicken salad at a popular fast food restaurant contains 650 milligrams (mg) of sodium, which is 27% of the recommended daily amount. What is the total recommended daily amount of sodium?

47. The nutrition fact sheet at a fast food restaurant says the fish sandwich has 380 calories, and 171 calories are from fat. What percent of the total calories is from fat?

Answer

45%

48. The nutrition fact sheet at a fast food restaurant says a small portion of chicken nuggets has 190 calories, and 114 calories are from fat. What percent of the total calories is from fat?

49. Emma gets paid $3,000 per month. She pays $750 a month for rent. What percent of her monthly pay goes to rent?

Answer

25%

50. Dimple gets paid $3,200 per month. She pays $960 a month for rent. What percent of her monthly pay goes to rent?

In the following exercises, solve.

51. Tamanika received a raise in her hourly pay, from $15.50 to $17.36. Find the percent change.

Answer

12%

52. Ayodele received a raise in her hourly pay, from $24.50 to $25.48. Find the percent change.

53. Annual student fees at the University of California rose from about $4,000 in 2000 to about $12,000 in 2010. Find the percent change.

Answer

200%

54. The price of a share of one stock rose from $12.50 to $50. Find the percent change.

55. A grocery store reduced the price of a loaf of bread from $2.80 to $2.73. Find the percent change.

Answer

−2.5%

56. The price of a share of one stock fell from $8.75 to $8.54. Find the percent change.

57. Hernando's salary was $49,500 last year. This year his salary was cut to $44,055. Find the percent change.

Answer

−11%

58. In ten years, the population of Detroit fell from 950,000 to about 712,500. Find the percent change.

In the following exercises, find a. the amount of discount and b. the sale price.

59. Janelle bought a beach chair on sale at 60% off. The original price was $44.95.

Answer

a. $26.97
b. $17.98

60. Errol bought a skateboard helmet on sale at 40% off. The original price was $49.95.

In the following exercises, find a. the amount of discount and b. the discount rate (Round to the nearest tenth of a percent if needed.)

61. Larry and Donna bought a sofa at the sale price of $1,344. The original price of the sofa was $1,920.

Answer

a. $576
b. 30%

62. Hiroshi bought a lawnmower at the sale price of $240. The original price of the lawnmower is $300.

In the following exercises, find a. the amount of the mark-up and b. the list price.

63. Daria bought a bracelet at original cost $16 to sell in her handicraft store. She marked the price up 45%. What was the list price of the bracelet?

Answer

a. $7.20
b. $23.20

64. Regina bought a handmade quilt at original cost $120 to sell in her quilt store. She marked the price up 55%. What was the list price of the quilt?

65. Tom paid $0.60 a pound for tomatoes to sell at his produce store. He added a 33% mark-up. What price did he charge his customers for the tomatoes?

Answer

a. $0.20
b. $0.80

66. Flora paid her supplier $0.74 a stem for roses to sell at her flower shop. She added an 85% mark-up. What price did she charge her customers for the roses?

Solve Simple Interest Applications

In the following exercises, solve.

67. Casey deposited $1,450 in a bank account that earned simple interest at an interest rate of 4%. How much interest was earned in two years?

Answer

$116

68. Terrence deposited $5,720 in a bank account that earned simple interest at an interest rate of 6%. How much interest was earned in four years?

69. Robin deposited $31,000 in a bank account that earned simple interest at an interest rate of 5.2%. How much interest was earned in three years?

Answer

$4836

70. Carleen deposited $16,400 in a bank account that earned simple interest at an interest rate of 3.9% How much interest was earned in eight years?

71. Hilaria borrowed $8,000 from her grandfather to pay for college. Five years later, she paid him back the $8,000, plus $1,200 interest. What was the rate of simple interest?

Answer

3%

72. Kenneth lent his niece $1,200 to buy a computer. Two years later, she paid him back the $1,200, plus $96 interest. What was the rate of simple interest?

73. Lebron lent his daughter $20,000 to help her buy a condominium. When she sold the condominium four years later, she paid him the $20,000, plus $3,000 interest. What was the rate of simple interest?

Answer

3.75%

74. Pablo borrowed $50,000 to start a business. Three years later, he repaid the $50,000, plus $9,375 interest. What was the rate of simple interest?

75. In 10 years, a bank account that paid 5.25% simple interest earned $18,375 interest. What was the principal of the account?

Answer

$35,000

76. In 25 years, a bond that paid 4.75% simple interest earned $2,375 interest. What was the principal of the bond?

77. Joshua's computer loan statement said he would pay $1,244.34 in simple interest for a three-year loan at 12.4%. How much did Joshua borrow to buy the computer?

Answer

$3345

78. Margaret's car loan statement said she would pay $7,683.20 in simple interest for a five-year loan at 9.8%. How much did Margaret borrow to buy the car?

Everyday Math

79. **Tipping** At the campus coffee cart, a medium coffee costs $1.65. MaryAnne brings $2.00 with her when she buys a cup of coffee and leaves the change as a tip. What percent tip does she leave?

Answer

17.5%

80. **Tipping** Four friends went out to lunch and the bill came to $53.75 They decided to add enough tip to make a total of $64, so that they could easily split the bill evenly among themselves. What percent tip did they leave?

Writing Exercises

81. What has been your past experience solving word problems? Where do you see yourself moving forward?

Answer

Answers will vary.

82. Without solving the problem “44 is 80% of what number” think about what the solution might be. Should it be a number that is greater than 44 or less than 44? Explain your reasoning.

83. After returning from vacation, Alex said he should have packed 50% fewer shorts and 200% more shirts. Explain what Alex meant.

Answer

Answers will vary.

84. Because of road construction in one city, commuters were advised to plan that their Monday morning commute would take 150% of their usual commuting time. Explain what this means.

Self Check

a. After completing the exercises, use this checklist to evaluate your mastery of the objective of this section.

I can...	Confidently	With some help	No-I don't get it!
use a problem-solving strategy for word problems.			
solve number problems.			
solve percent applications.			
solve simple interest applications.			

b. After reviewing this checklist, what will you do to become confident for all objectives?

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2.4: Solve a Formula for a Specific Variable

Learning Objectives

By the end of this section, you will be able to:

- Solve a formula for a specific variable
- Use formulas to solve geometry applications

Before you get started, take this readiness quiz.

1. Evaluate $2(x+3)$ when $x=5$.
   If you missed this problem, review [link].
2. The length of a rectangle is three less than the width. Let *w* represent the width. Write an expression for the length of the rectangle.
   If you missed this problem, review [link].
3. Evaluate $\frac{1}{2}bh$ when $b=14$ and $h=9$.
   If you missed this problem, review [link].

Solve a Formula for a Specific Variable

We have all probably worked with some geometric formulas in our study of mathematics. Formulas are used in so many fields, it is important to recognize formulas and be able to manipulate them easily.

It is often helpful to solve a formula for a specific variable. If you need to put a formula in a spreadsheet, it is not unusual to have to solve it for a specific variable first. We isolate that variable on one side of the equals sign with a coefficient of one and all other variables and constants are on the other side of the equal sign.

Geometric formulas often need to be solved for another variable, too. The formula $V=\frac{1}{3}\pi r^2 h$ is used to find the **volume** of a right circular cone when given the radius of the base and height. In the next example, we will solve this formula for the height.

EXAMPLE 2.4.1

Solve the formula $V=\frac{1}{3}\pi r^2 h$ for *h*.

Answer

Write the formula.			$V=\frac{1}{3}\pi r^2 h$
Remove the fraction on the right.			$3\cdot V=3\cdot\frac{1}{3}\pi r^2 h$
Simplify.			$3V=\pi r^2 h$
Divide both sides by πr^2.			$\frac{3V}{\pi r^2}=h$

We could now use this formula to find the height of a right circular cone when we know the volume and the radius of the base, by using the formula $h=\frac{3V}{\pi r^2}$.

EXAMPLE 2.4.2:

Use the formula $A=\frac{1}{2}bh$ to solve for *b*.

Answer

$b=\frac{2A}{h}$

? EXAMPLE 2.4.3:

Use the formula $A = \frac{1}{2}bh$ to solve for h.

Answer

$h = \frac{2A}{b}$

In the sciences, we often need to change **temperature** from Fahrenheit to Celsius or vice versa. If you travel in a foreign country, you may want to change the Celsius temperature to the more familiar Fahrenheit temperature.

? EXAMPLE 2.4.4

Solve the formula $C = \frac{5}{9}(F - 32)$ for F.

Answer

Write the formula.			$C = \frac{5}{9}(F - 32)$
Remove the fraction on the right.			$\frac{9}{5}C = \frac{9}{5} \cdot \frac{5}{9}(F - 32)$
Simplify.			$\frac{9}{5}C = (F - 32)$
Add 32 to both sides.			$\frac{9}{5}C + 32 = F$

We can now use the formula $F = \frac{9}{5}C + 32$ to find the Fahrenheit temperature when we know the Celsius temperature.

? EXAMPLE 2.4.5:

Solve the formula $F = \frac{9}{5}C + 32$ for C.

Answer

$C = \frac{5}{9}(F - 32)$

? EXAMPLE 2.4.6:

Solve the formula $A = \frac{1}{2}h(b + B)$ for b.

Answer

$b = \frac{2A - Bh}{h}$

The next example uses the formula for the **surface area** of a right cylinder.

? EXAMPLE 2.4.7

Solve the formula $S = 2\pi r^2 + 2\pi rh$ for h.

Answer

Write the formula.	$S = 2\pi r^2 + 2\pi r h$
Isolate the h term by subtracting $2\pi r^2$ from each side.	$S - 2\pi r^2 = 2\pi r^2 - 2\pi r^2 + 2\pi r h$
Simplify.	$S - 2\pi r^2 = 2\pi r h$

Solve for h by dividing both sides by $2\pi r$.	$\frac{S-2\pi r^2}{2\pi r} = \frac{2\pi r h}{2\pi r}$
Simplify.	$\frac{S-2\pi r^2}{2\pi r} = h$

? EXAMPLE 2.4.8:

Solve the formula $A = P + Prt$ for t.

Answer

$t = \frac{A-P}{Pr}$

? EXAMPLE 2.4.9:

Solve the formula $A = P + Prt$ for r.

Answer

$r = \frac{A-P}{Pt}$

Sometimes we might be given an equation that is solved for y and need to solve it for x, or vice versa. In the following example, we're given an equation with both x and y on the same side and we'll solve it for y.

Solve the formula $8x + 7y = 15$ for y.

Answer

We will isolate y on one side of the equation.	$8x + 7y = 15$
Subtract $6x$ from both sides to isolate the term with y.	$8x - 8x + 7y = 15 - 8x$
Simplify.	$7y = 15 - 8x$
Divide both sides by 7 to make the coefficient of y one.	$\frac{7y}{7} = \frac{15-8x}{7}$
Simplify.	$y = \frac{15-8x}{7}$

Solve the formula $4x + 7y = 9$ for y.

Answer

$y = \frac{9-4x}{7}$

? EXAMPLE 2.4.12

Solve the formula $5x + 8y = 1$ for y.

Answer

$y = \frac{1-5x}{8}$

Use Formulas to Solve Geometry Applications

In this objective we will use some common geometry formulas. We will adapt our problem solving strategy so that we can solve geometry applications. The geometry formula will name the variables and give us the equation to solve.

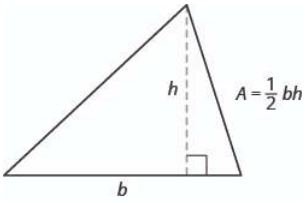
In addition, since these applications will all involve shapes of some sort, most people find it helpful to draw a figure and label it with the given information. We will include this in the first step of the problem solving strategy for geometry applications.

SOLVE GEOMETRY APPLICATIONS.

1. **Read** the problem and make sure all the words and ideas are understood.
2. **Identify** what you are looking for.
3. **Name** what we are looking for by choosing a variable to represent it. Draw the figure and label it with the given information.
4. **Translate** into an equation by writing the appropriate formula or model for the situation. Substitute in the given information.
5. **Solve** the equation using good algebra techniques.
6. **Check** the answer in the problem and make sure it makes sense.
7. **Answer** the question with a complete sentence.

When we solve geometry applications, we often have to use some of the properties of the figures. We will review those properties as needed.

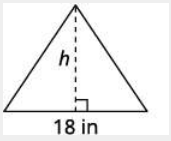
The next example involves the area of a triangle. The **area** of a triangle is one-half the base times the height. We can write this as $A = \frac{1}{2}bh$, where b = length of the base and h = height.



EXAMPLE 2.4.13

The area of a triangular painting is 126 square inches. The base is 18 inches. What is the height?

Answer

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	height of a triangle
Step 3. Name.	
Choose a variable to represent it.	Let $h =$ the height.
Draw the figure and label it with the given information.	Area = 126 sq. in.
	
Step 4. Translate.	
Write the appropriate formula.	$A = \frac{1}{2}bh$
Substitute in the given information.	$126 = \frac{1}{2} \cdot 18 \cdot h$
Step 5. Solve the equation.	$126 = 9h$

Divide both sides by 9.	$14 = h$
Step 6. Check. $A = \frac{1}{2}bh$ $126 \stackrel{?}{=} 12 \cdot 18 \cdot 14$ $126 = 126 \checkmark$	
Step 7. Answer the question.	The height of the triangle is 14 inches.

? EXAMPLE 2.4.14:

The area of a triangular church window is 90 square meters. The base of the window is 15 meters. What is the window's height?

Answer

The window's height is 12 meters.

? EXAMPLE 2.4.15:

A triangular tent door has area 15 square feet. The height is five feet. What is the length of the base?

Answer

The length of the base is 6 feet.

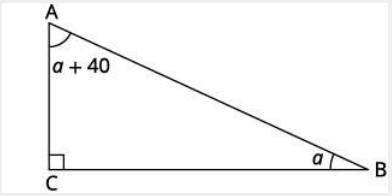
In the next example, we will work with a **right triangle**. To solve for the measure of each angle, we need to use two triangle properties. In any triangle, the sum of the measures of the angles is 180°. We can write this as a formula: $m\angle A + m\angle B + m\angle C = 180$. Also, since the triangle is a right triangle, we remember that a right triangle has one 90° angle.

Here, we will have to define one angle in terms of another. We will wait to draw the figure until we write expressions for all the angles we are looking for.

? EXAMPLE 2.4.16:

The measure of one angle of a right triangle is 40 degrees more than the measure of the smallest angle. Find the measures of all three angles.

Answer

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	the measures of all three angles
Step 3. Name. Choose a variable to represent it.	Let a = 1st angle. $a + 40$ = 2nd angle 90 = 3rd angle (the right angle)
Draw the figure and label it with the given information.	
Step 4. Translate.	
Write the appropriate formula.	$m\angle A + m\angle B + m\angle C = 180$

Substitute into the formula.	$a+(a+40)+90=180$
Step 5. Solve the equation.	$2a+130=180$ $2a=50$ $a=25$ first angle $a+40$ second angle $25+40$ 65 90 third angle
Step 6. Check. $25+65+90 \stackrel{?}{=} 180$ $180=180$✓	
Step 7. Answer the question.	The three angles measure 25°, 65°, and 90°.

? EXAMPLE 2.4.17:

The measure of one angle of a right triangle is 50 more than the measure of the smallest angle. Find the measures of all three angles.

Answer

The measures of the angles are 20°, 70°, and 90°.

? EXAMPLE 2.4.18:

The measure of one angle of a right triangle is 30 more than the measure of the smallest angle. Find the measures of all three angles.

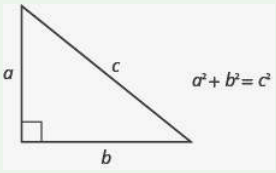
Answer

The measures of the angles are 30°, 60°, and 90°.

The next example uses another important geometry formula. The **Pythagorean Theorem** tells how the lengths of the three sides of a right triangle relate to each other. Writing the formula in every exercise and saying it aloud as you write it may help you memorize the Pythagorean Theorem.

THE PYTHAGOREAN THEOREM

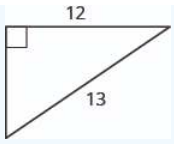
In any right triangle, where a and b are the lengths of the legs, and c is the length of the hypotenuse, the sum of the squares of the lengths of the two legs equals the square of the length of the hypotenuse.



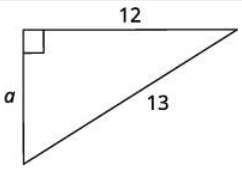
We will use the Pythagorean Theorem in the next example.

? EXAMPLE 2.4.19:

Use the Pythagorean Theorem to find the length of the other leg in

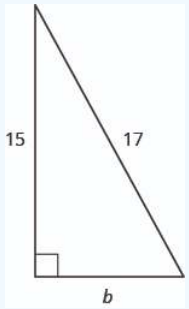


Answer

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	the length of the leg of the triangle
Step 3. Name.	
Choose a variable to represent it.	Let a = the leg of the triangle.
Label side a.	
Step 4. Translate.	
Write the appropriate formula. Substitute.	$a^2+b^2=c^2$ $a^2+12^2=13^2$
Step 5. Solve the equation. Isolate the variable term. Use the definition of square root. Simplify.	$a^2+144=169$ $a^2=25$ $a=\sqrt{25}$ $a=5$
Step 6. Check. $5^2+12^2 \stackrel{?}{=} 13^2$ $25+144 \stackrel{?}{=} 169$ $169=169$ ✓	
Step 7. Answer the question.	The length of the leg is 5.

? EXAMPLE 2.4.20:

Use the Pythagorean Theorem to find the length of the leg in the figure.

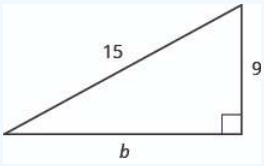


Answer

The length of the leg is 8.

? EXAMPLE 2.4.21:

Use the Pythagorean Theorem to find the length of the leg in the figure.



Answer

The length of the leg is 12.

The next example is about the **perimeter** of a rectangle. Since the perimeter is just the distance around the rectangle, we find the sum of the lengths of its four sides—the sum of two lengths and two widths. We can write is as $P = 2L + 2W$ where L is the length and W is the width. To solve the example, we will need to define the length in terms of the width.

? EXAMPLE 2.4.22:

The length of a rectangle is six centimeters more than twice the width. The perimeter is 96 centimeters. Find the length and width.

Answer

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	the length and the width
Step 3. Name. Choose a variable to represent the width. The length is six more than twice the width.	Let $w =$ width. $2w + 6 =$ length w, 2w + 6 $P = 96$ cm
Step 4. Translate.	
Write the appropriate formula.	$P = 2L + 2W$
Substitute in the given information.	$96 = 2(2w + 6) + 2w$
Step 5. Solve the equation.	$96 = 4w + 12 + 2w$ $96 = 6w + 12$ $84 = 6w$ $14 = w$ (width) $2w + 6$ (length) $2(14) + 6$ 34 The length is 34 cm.
Step 6. Check. 14 cm, 34 cm $P = 2L + 2W$ $96 \stackrel{?}{=} 2 \cdot 34 + 2 \cdot 14$ $96 = 96$✓	
Step 7. Answer the question.	The length is 34 cm and the width is 14 cm

? EXAMPLE 2.4.23:

The length of a rectangle is seven more than twice the width. The perimeter is 110 inches. Find the length and width.

Answer

The length is 16 inches and the width is 39 inches.

? EXAMPLE 2.4.24:

The width of a rectangle is eight yards less than twice the length. The perimeter is 86 yards. Find the length and width.

Answer

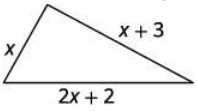
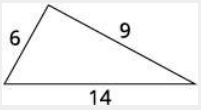
The length is 17 yards and the width is 26 yards.

The next example is about the **perimeter** of a triangle. Since the perimeter is just the distance around the triangle, we find the sum of the lengths of its three sides. We can write this as $P = a + b + c$, where a, b, and c are the lengths of the sides.

? EXAMPLE 2.4.25:

One side of a triangle is three inches more than the first side. The third side is two inches more than twice the first. The perimeter is 29 inches. Find the length of the three sides of the triangle.

Answer

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	the lengths of the three sides of a triangle
Step 3. Name. Choose a variable to represent the length of the first side.	Let $x =$ length of 1st side. $x+3$ = length of 2nd side $2x+2$ = length of 3rd side  Perimeter is 29 in.
Step 4. Translate. Write the appropriate formula. Substitute in the given information.	$P = a + b + c$ $29 = x + (x + 3) + (2x + 2)$
Step 5. Solve the equation.	$29 = 4x + 5$ $24 = 4x$ $6 = x$ length of first side $x + 3$ length of second side $6 + 3$ 9 $2x + 2$ length of second side $2 \cdot 6 + 2$ 14
Step 6. Check.  $29 \stackrel{?}{=} 6 + 9 + 14$ $29 = 29$✓	
Step 7. Answer the question.	The lengths of the sides of the triangle are 6, 9, and 14 inches.

? EXAMPLE 2.4.26:

One side of a triangle is seven inches more than the first side. The third side is four inches less than three times the first. The perimeter is 28 inches. Find the length of the three sides of the triangle.

Answer

The lengths of the sides of the triangle are 5, 11 and 12 inches.

? EXAMPLE 2.4.27:

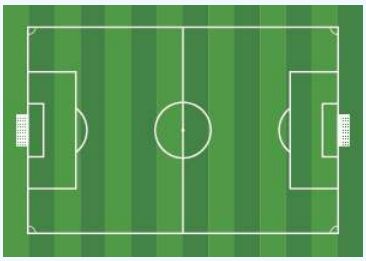
One side of a triangle is three feet less than the first side. The third side is five feet less than twice the first. The perimeter is 20 feet. Find the length of the three sides of the triangle.

Answer

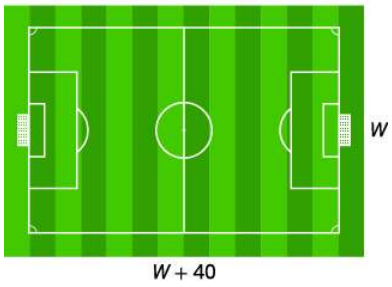
The lengths of the sides of the triangle are 4, 7 and 9 feet.

? EXAMPLE 2.4.28:

The perimeter of a rectangular soccer field is 360 feet. The length is 40 feet more than the width. Find the length and width.



Answer

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	the length and width of the soccer field
Step 3. Name. Choose a variable to represent it. The length is 40 feet more than the width. Draw the figure and label it with the given information.	Let w = width. $w + 40 =$ length Perimeter = 360 feet 
Step 4. Translate. Write the appropriate formula and substitute.	$P = 2L + 2W$ $360 = 2(w + 40) + 2w$
Step 5. Solve the equation.	

Step 6. Check. $P = 2L + 2W$ $360 \stackrel{?}{=} 2(110) + 2(70)$ $360 = 360\checkmark$	$360 = 2w + 80 + 2w$ $360 = 4w + 80$ $280 = 4w$ $70 = w$ the width of the field $w + 40$ the length of the field $70 + 40$ 110
Step 7. Answer the question.	The length of the soccer field is 110 feet and the width is 70 feet.

? EXAMPLE 2.4.29:

The perimeter of a rectangular swimming pool is 200 feet. The length is 40 feet more than the width. Find the length and width.

Answer

The length of the swimming pool is 70 feet and the width is 30 feet.

? EXAMPLE 2.4.30:

The length of a rectangular garden is 30 yards more than the width. The perimeter is 300 yards. Find the length and width.

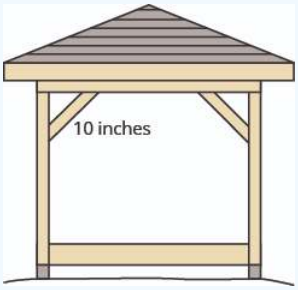
Answer

The length of the garden is 90 yards and the width is 60 yards.

Applications of these geometric properties can be found in many everyday situations as shown in the next example.

? EXAMPLE 2.4.31:

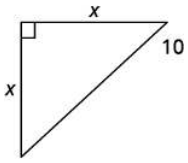
Kelvin is building a gazebo and wants to brace each corner by placing a 10” piece of wood diagonally as shown.



How far from the corner should he fasten the wood if wants the distances from the corner to be equal? Approximate to the nearest tenth of an inch.

Answer

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	the distance from the corner that the bracket should be attached

Step 3. Name. Choose a variable to represent it. Draw the figure and label it with the given information.	Let $x =$ the distance from the corner. 
Step 4. Translate. Write the appropriate formula and substitute.	$a^2+b^2=c^2 \quad x^2+x^2=10^2$
Step 5. Solve the equation. Isolate the variable. Use the definition of square root. Simplify. Approximate to the nearest tenth.	$2x^2=100$ $x^2=50$ $x=\sqrt{50}$ $x\approx 7.1$
Step 6. Check. $a^2+b^2=c^2$ $(7.1)^2+(7.1)^2\approx 10^2$ Yes.	
Step 7. Answer the question.	Kelvin should fasten each piece of wood approximately 7.1" from the corner.

? EXAMPLE 2.4.32:

John puts the base of a 13-foot ladder five feet from the wall of his house as shown in the figure. How far up the wall does the ladder reach?

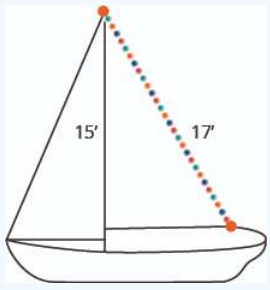


Answer

The ladder reaches 12 feet.

? EXAMPLE 2.4.33:

Randy wants to attach a 17-foot string of lights to the top of the 15 foot mast of his sailboat, as shown in the figure. How far from the base of the mast should he attach the end of the light string?



Answer

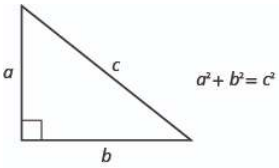
He should attach the lights 8 feet from the base of the mast.

Access this online resource for additional instruction and practice with solving for a variable in literal equations.

- Solving Literal Equations

Key Concepts

- **How To Solve Geometry Applications**
  1. **Read** the problem and make sure all the words and ideas are understood.
  2. **Identify** what you are looking for.
  3. **Name** what you are looking for by choosing a variable to represent it. Draw the figure and label it with the given information.
  4. **Translate** into an equation by writing the appropriate formula or model for the situation. Substitute in the given information.
  5. **Solve** the equation using good algebra techniques.
  6. **Check** the answer in the problem and make sure it makes sense.
  7. **Answer** the question with a complete sentence.
- **The Pythagorean Theorem**
  - In any right triangle, where a and b are the lengths of the legs, and c is the length of the hypotenuse, the sum of the squares of the lengths of the two legs equals the square of the length of the hypotenuse.



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2.4E: Exercises

Practice Makes Perfect

Solve a Formula for a Specific Variable

In the following exercises, solve the given formula for the specified variable.

1. Solve the formula $C = \pi d$ for d.

Answer

$$d = \frac{C}{\pi}$$

2. Solve the formula $C = \pi d$ for π.

3. Solve the formula $V = LWH$ for L.

Answer

$$L = \frac{V}{WH}$$

4. Solve the formula $V = LWH$ for H.

5. Solve the formula $A = \frac{1}{2}bh$ for b.

Answer

$$b = \frac{2A}{h}$$

6. Solve the formula $A = \frac{1}{2}bh$ for h.

7. Solve the formula

$A = \frac{1}{2}d_1d_2$ for d_1.

Answer

$$d_1 = \frac{2A}{d_2}$$

8. Solve the formula

$A = \frac{1}{2}d_1d_2$ for d_2.

9. Solve the formula

$A = \frac{1}{2}h(b_1 + b_2)$ for b_1.

Answer

$$b_1 = \frac{2A}{h} - b_2$$

10. Solve the formula

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$A = \frac{1}{2}h(b_1 + b_2)$ for b_2.

11. Solve the formula

$h = 54t + \frac{1}{2}at^2$ for a.

Answer

$$a = \frac{2h - 108t}{t^2}$$

12. Solve the formula

$h = 48t + \frac{1}{2}at^2$ for a.

13. Solve $180 = a + b + c$ for a.

Answer

$$a = 180 - b - c$$

14. Solve $180 = a + b + c$ for c.

15. Solve the formula

$A = \frac{1}{2}pI + B$ for p.

Answer

$$p = \frac{2A - 2B}{I}$$

16. Solve the formula

$A = \frac{1}{2}pI + B$ for I.

17. Solve the formula

$P = 2L + 2W$ for L.

Answer

$$L = \frac{P - 2W}{2}$$

18. Solve the formula

$P = 2L + 2W$ for W.

In the following exercises, solve for the formula for y.

19. Solve the formula

$8x + y = 15$ for y.

Answer

$$y = 15 - 8x$$

20. Solve the formula

$9x + y = 13$ for y.

21. Solve the formula

$-4x + y = -6$ for y.

Answer

$y = -6 + 4x$

22. Solve the formula

$-5x + y = -1$ for y.

23. Solve the formula

$x - y = -4$ for y.

Answer

$y = 4 + x$

24. Solve the formula

$x - y = -3$ for y.

25. Solve the formula

$4x + 3y = 7$ for y.

Answer

$y = \frac{7-4x}{3}$

26. Solve the formula

$3x + 2y = 11$ for y.

27. Solve the formula

$2x + 3y = 12$ for y.

Answer

$y = \frac{12-2x}{3}$

28. Solve the formula

$5x + 2y = 10$ for y.

29. Solve the formula

$3x - 2y = 18$ for y.

Answer

$y = \frac{18-3x}{-2}$

30. Solve the formula

$4x - 3y = 12$ for y.

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Use Formulas to Solve Geometry Applications

In the following exercises, solve using a geometry formula.

31. A triangular flag has area 0.75 square feet and height 1.5 foot. What is its base?

Answer

1 foot

32. A triangular window has area 24 square feet and height six feet. What is its base?

33. What is the base of a triangle with area 207 square inches and height 18 inches?

Answer

23 inches

34. What is the height of a triangle with area 893 square inches and base 38 inches?

35. The two smaller angles of a right triangle have equal measures. Find the measures of all three angles.

Answer

$45^\circ, 45^\circ, 90^\circ$

36. The measure of the smallest angle of a right triangle is 20° less than the measure of the next larger angle. Find the measures of all three angles.

37. The angles in a triangle are such that one angle is twice the smallest angle, while the third angle is three times as large as the smallest angle. Find the measures of all three angles.

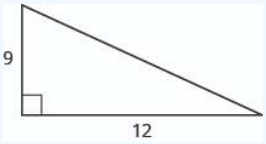
Answer

$30^\circ, 60^\circ, 90^\circ$

38. The angles in a triangle are such that one angle is 20 more than the smallest angle, while the third angle is three times as large as the smallest angle. Find the measures of all three angles.

In the following exercises, use the Pythagorean Theorem to find the length of the hypotenuse.

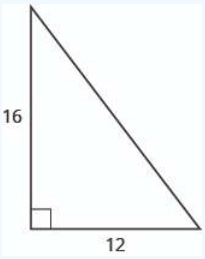
39.



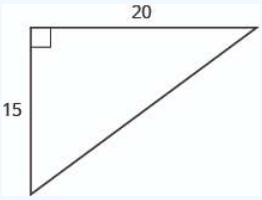
Answer

15

40.



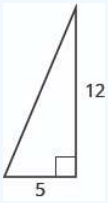
41.



Answer

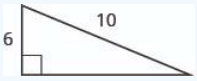
25

42.



In the following exercises, use the Pythagorean Theorem to find the length of the unknown leg. Round to the nearest tenth if necessary.

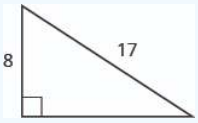
43.



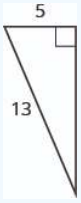
Answer

8

44.



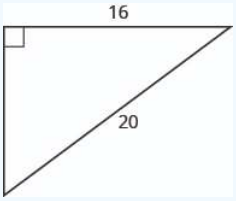
45.



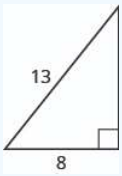
Answer

12

46.



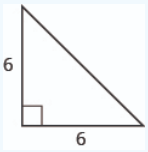
47.



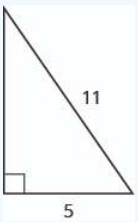
Answer

10.2

48.



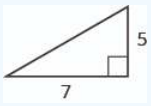
49.



Answer

9.8

50.



In the following exercises, solve using a geometry formula.

51. The width of a rectangle is seven meters less than the length. The perimeter is 58 meters. Find the length and width.

Answer

18 meters, 11 meters

52. The length of a rectangle is eight feet more than the width. The perimeter is 60 feet. Find the length and width.

53. The width of the rectangle is 0.7 meters less than the length. The perimeter of a rectangle is 52.6 meters. Find the dimensions of the rectangle.

Answer

13.5m, 12.8m

54. The length of the rectangle is 1.1 meters less than the width. The perimeter of a rectangle is 49.4 meters. Find the dimensions of the rectangle.

55. The perimeter of a rectangle of 150 feet. The length of the rectangle is twice the width. Find the length and width of the rectangle.

Answer

25 ft, 50 ft

56. The length of the rectangle is three times the width. The perimeter of a rectangle is 72 feet. Find the length and width of the rectangle.

57. The length of the rectangle is three meters less than twice the width. The perimeter of a rectangle is 36 meters. Find the dimensions of the rectangle.

Answer

7 m, 11 m

58. The length of a rectangle is five inches more than twice the width. The perimeter is 34 inches. Find the length and width.

59. The perimeter of a triangle is 39 feet. One side of the triangle is one foot longer than the second side. The third side is two feet longer than the second side. Find the length of each side.

Answer

12 ft, 13 ft, 14 ft

60. The perimeter of a triangle is 35 feet. One side of the triangle is five feet longer than the second side. The third side is three feet longer than the second side. Find the length of each side.

61. One side of a triangle is twice the smallest side. The third side is five feet more than the shortest side. The perimeter is 17 feet. Find the lengths of all three sides.

Answer

3 ft, 6 ft, 8 ft

62. One side of a triangle is three times the smallest side. The third side is three feet more than the shortest side. The perimeter is 13 feet. Find the lengths of all three sides.

63. The perimeter of a rectangular field is 560 yards. The length is 40 yards more than the width. Find the length and width of the field.

Answer

120 yd, 160 yd

64. The perimeter of a rectangular atrium is 160 feet. The length is 16 feet more than the width. Find the length and width of the atrium.

65. A rectangular parking lot has perimeter 250 feet. The length is five feet more than twice the width. Find the length and width of the parking lot.

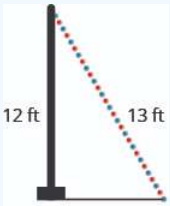
Answer

40 ft, 85 ft

66. A rectangular rug has perimeter 240 inches. The length is 12 inches more than twice the width. Find the length and width of the rug.

In the following exercises, solve. Approximate answers to the nearest tenth, if necessary.

67. A 13-foot string of lights will be attached to the top of a 12-foot pole for a holiday display as shown. How far from the base of the pole should the end of the string of lights be anchored?



Answer

5 feet

68. Pam wants to put a banner across her garage door diagonally, as shown, to congratulate her son for his college graduation. The garage door is 12 feet high and 16 feet wide. How long should the banner be to fit the garage door?



69. Chi is planning to put a diagonal path of paving stones through her flower garden as shown. The flower garden is a square with side 10 feet. What will the length of the path be?



Answer

14.1 feet

70. Brian borrowed a 20-foot extension ladder to use when he paints his house. If he sets the base of the ladder six feet from the house as shown, how far up will the top of the ladder reach?



Everyday Math

71. Converting temperature While on a tour in Greece, Tatyana saw that the temperature was 40° Celsius. Solve for F in the formula $C = \frac{5}{9}(F - 32)$ to find the Fahrenheit temperature.

Answer

104°F

72. Converting temperature Yon was visiting the United States and he saw that the temperature in Seattle one day was 50° Fahrenheit. Solve for C in the formula $F = \frac{9}{5}C + 32$ to find the Celsius temperature

73. Christa wants to put a fence around her triangular flowerbed. The sides of the flowerbed are six feet, eight feet and 10 feet. How many feet of fencing will she need to enclose her flowerbed?

Answer

24 ft

74. Jose just removed the children's play set from his back yard to make room for a rectangular garden. He wants to put a fence around the garden to keep the dog out. He has a 50-foot roll of fence in his garage that he plans to use. To fit in the backyard, the width of the garden must be 10 feet. How long can he make the other side?

Writing Exercises

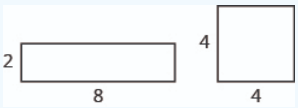
75. If you need to put tile on your kitchen floor, do you need to know the perimeter or the area of the kitchen? Explain your reasoning.

Answer

Answers will vary.

76. If you need to put a fence around your backyard, do you need to know the perimeter or the area of the backyard? Explain your reasoning.

77. Look at the two figures below.



a. Which figure looks like it has the larger area? Which looks like it has the larger perimeter?

b. Now calculate the area and perimeter of each figure. Which has the larger area? Which has the larger perimeter?

c. Were the results of part (b) the same as your answers in part (a)? Is that surprising to you?

Answer

a. Answers will vary. b. The areas are the same. The 2×8 rectangle has a larger perimeter than the 4×4 square.

c. Answers will vary.

78. Write a geometry word problem that relates to your life experience, then solve it and explain all your steps.

Self Check

a. After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
solve a formula for a specific variable.			
use formulas to solve geometry applications.			

b. What does this checklist tell you about your mastery of this section? What steps will you take to improve?

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2.5: Use the Rectangular Coordinate System

Learning Objectives

By the end of this section, you will be able to:

- Plot points in a rectangular coordinate system
- Verify solutions to an equation in two variables
- Complete a table of solutions to a linear equation
- Find solutions to a linear equation in two variables

Note

Before you get started, take this readiness quiz.

1. Evaluate $x+3$ when $x=-1$.
   If you missed this problem, review Example 1.5.25.
2. Evaluate $2x-5y$ when $x=3$ and y=−2.
   If you missed this problem, review Example 1.5.28.
3. Solve for y: $40-4y=20$
   If you missed this problem, review Example 2.3.1.

Plot Points on a Rectangular Coordinate System

Just like maps use a grid system to identify locations, a grid system is used in algebra to show a relationship between two variables in a **rectangular coordinate system**. The rectangular coordinate system is also called the *xy*-plane or the 'coordinate plane'.

The horizontal number line is called the *x-axis*. The vertical number line is called the *y-axis*. The *x*-axis and the *y*-axis together form the rectangular coordinate system. These axes divide a plane into four regions, called **quadrants**. The quadrants are identified by Roman numerals, beginning on the upper right and proceeding counterclockwise. See Figure \(\PageIndex{1}\).

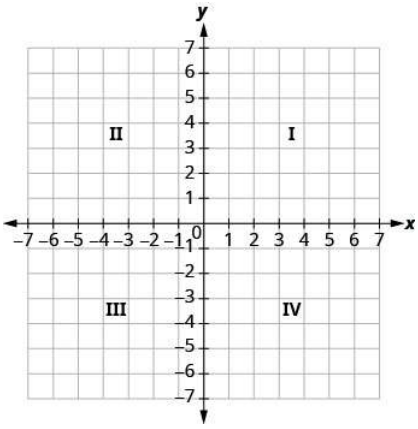


Figure 2.5.1: 'Quadrant' has the root 'quad,' which means 'four.'

In the **rectangular coordinate system**, every point is represented by an *ordered pair*. The first number in the ordered pair is the ***x*-coordinate** of the point, and the second number is the ***y*-coordinate** of the point.

ORDERED PAIR

An **ordered pair**, (x,y)(x,y), gives the coordinates of a point in a rectangular coordinate system.

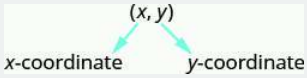


Figure 2.5.2

The first number is the *x*-coordinate.

The second number is the *y*-coordinate.

The phrase 'ordered pair' means the order is important. What is the ordered pair of the point where the axes cross? At that point both coordinates are zero, so its ordered pair is $(0, 0)$. The point $(0, 0)$ has a special name. It is called the **origin**.

THE ORIGIN

The point $(0, 0)$ is called the **origin**. It is the point where the *x*-axis and *y*-axis intersect.

We use the coordinates to locate a point on the *xy*-plane. Let's plot the point $(1, 3)$ as an example. First, locate 1 on the *x*-axis and lightly sketch a vertical line through $x = 1$. Then, locate 3 on the *y*-axis and sketch a horizontal line through $y = 3$. Now, find the point where these two lines meet — that is the point with coordinates $(1, 3)$.

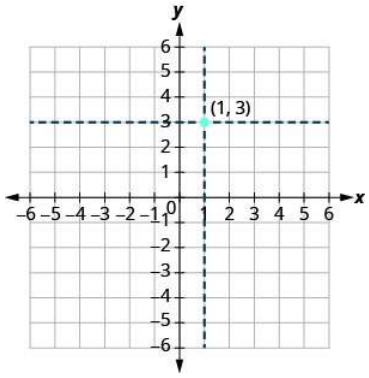


Figure 2.5.3

Notice that the vertical line through $x = 1$ and the horizontal line through $y = 3$ are not part of the graph. We just used them to help us locate the point $(1, 3)$.

Example 2.5.1

Plot each point in the rectangular coordinate system and identify the **quadrant** in which the point is located:

a. (−5,4)
b. (−3,−4)
c. (2,−3)
d. (−2,3)
e. $(3, \frac{5}{2})$

Solution

The first number of the coordinate pair is the *x*-coordinate, and the second number is the *y*-coordinate.

a. Since x=−5, the point is to the left of the *y*-axis. Also, since y=4, the point is above the *x*-axis. The point (−5,4) is in Quadrant II.
b. Since x=−3, the point is to the left of the *y*-axis. Also, since y=−4, the point is below the *x*-axis. The point (−3,−4) is in Quadrant III.
c. Since x=2, the point is to the right of the *y*-axis. Since y=−3, the point is below the *x*-axis. The point (2,−3) is in Quadrant IV.
d. Since x=−2, the point is to the left of the *y*-axis. Since y=3, the point is above the *x*-axis. The point (−2,3) is in Quadrant II.
e. Since x=3, the point is to the right of the *y*-axis. Since $y = \frac{5}{2}$, the point is above the *x*-axis. (It may be helpful to write $\frac{5}{2}$ as a mixed number or decimal.) The point $(3, \frac{5}{2})$ is in Quadrant I.

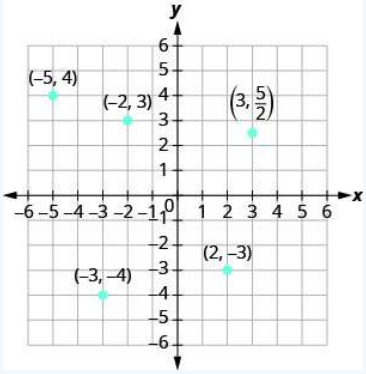


Figure 2.5.4

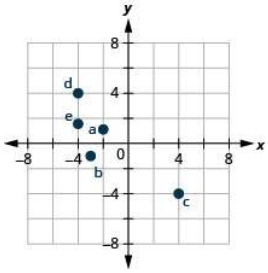
Try It 2.5.2

Plot each point in a rectangular coordinate system and identify the quadrant in which the point is located:

a. (−2,1)
b. (−3,−1)
c. (4,−4)
d. (−4,4)
e. $(-4, \frac{3}{2})$

Answer

The point a (-2,1) is in Quadrant II.
The point b (-3,-1) is in Quadrant III.
The point c (4,-4) is in Quadrant IV.
The point d (-4,4) is in Quadrant II.
The point e $(-4, \frac{3}{2})$ is in Quadrant II.



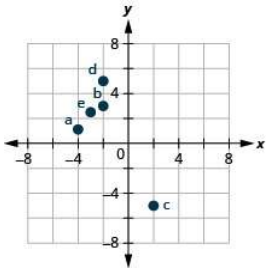
Try It 2.5.3

Plot each point in a rectangular coordinate system and identify the quadrant in which the point is located:

a. (−4,1)
b. (−2,3)
c. (2,−5)
d. (−2,5)
e. $(-3, \frac{5}{2})$

Answer

The point a (-4, 1) is in Quadrant II.
The point b (-2, 3) is in Quadrant II.
The point c (2, -5) is in Quadrant IV.
The point d (-2, 5) is in Quadrant II.
The point e $(-3, \frac{5}{2})$ is in Quadrant II.



How do the signs affect the location of the points? You may have noticed some patterns as you graphed the points in the previous example.

For the point in Figure 2.5.4 in Quadrant IV, what do you notice about the signs of the coordinates? What about the signs of the coordinates of points in the third quadrant? The second quadrant? The first quadrant?

Can you tell just by looking at the coordinates in which quadrant the point (−2,5) is located? In which quadrant is (2,−5) located?

QUADRANTS

We can summarize sign patterns of the quadrants in this way.

Quadrant I	Quadrant II	Quadrant III	Quadrant IV	
(x, y)	(x, y)	(x, y)	(x, y)	(2.5.1)
$(+, +)$	$(-, +)$	$(-, -)$	$(+, -)$	

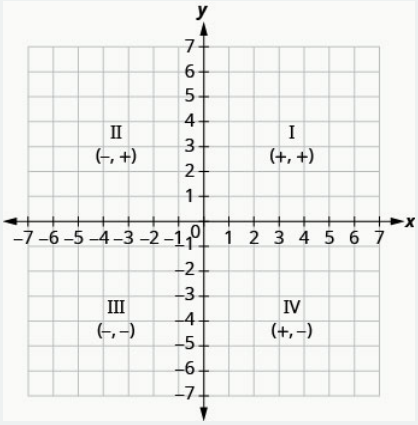


Figure 2.5.5

What if one coordinate is zero as shown in Figure 2.5.6? Where is the point (0,4) located? Where is the point (−2,0) located?

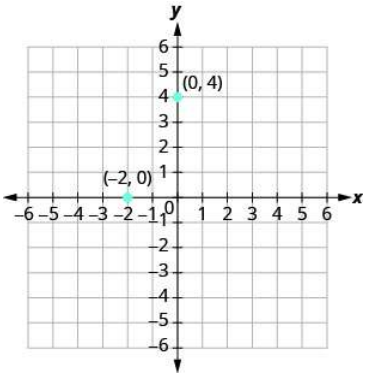


Figure 2.5.6

The point (0,4) is on the *y*-axis and the point (−2,0) is on the *x*-axis.

POINTS ON THE AXES

Points with a ***y*-coordinate** equal to 0 are on the *x*-axis, and have coordinates (a,0).

Points with an ***x*-coordinate** equal to 0 are on the *y*-axis, and have coordinates (0,b).

Example 2.5.4

Plot each point:

1. (0,5)
2. (4,0)
3. (−3,0)
4. (0,0)
5. (0,−1)

Solution

1. Since x=0, the point whose coordinates are (0,5) is on the *y*-axis.
2. Since y=0, the point whose coordinates are (4,0) is on the *x*-axis.
3. Since y=0, the point whose coordinates are (−3,0) is on the *x*-axis.
4. Since x=0 and y=0, the point whose coordinates are (0,0) is the origin.
5. Since x=0, the point whose coordinates are (0,−1) is on the *y*-axis.

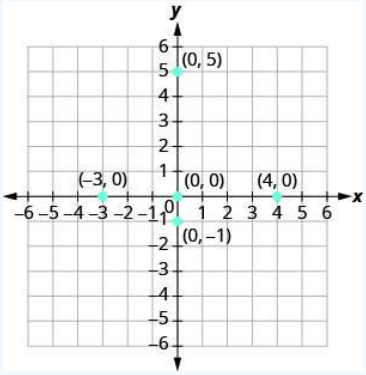


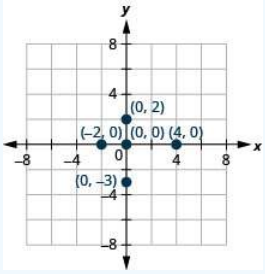
Figure 2.5.7

Try It 2.5.5

Plot each point:

1. (4,0)
2. (−2,0)
3. (0,0)
4. (0,2)
5. (0,−3).

Answer

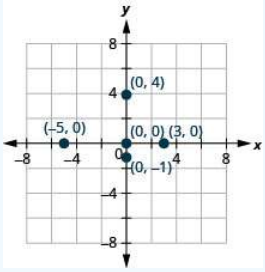


Try It 2.5.6

Plot each point:

1. (−5,0)
2. (3,0)
3. (0,0)
4. (0,−1)
5. (0,4).

Answer



In algebra, being able to identify the coordinates of a point shown on a graph is just as important as being able to plot points. To identify the *x*-coordinate of a point on a graph, read the number on the *x*-axis directly above or below the point. To identify the *y*-coordinate of a point, read the number on the *y*-axis directly to the left or right of the point. Remember, when you write the **ordered pair** use the correct order, (x,y).

Example 2.5.7

Name the ordered pair of each point shown in the rectangular coordinate system.

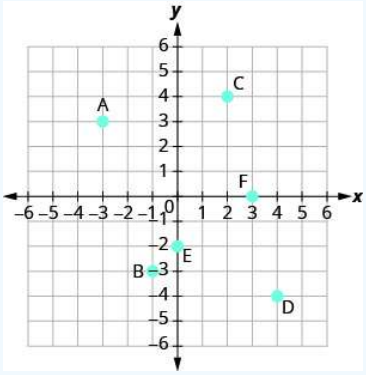


Figure 2.5.8

Solution

Point A is above −3 on the *x*-axis, so the *x*-coordinate of the point is −3.

The point is to the left of 3 on the *y*-axis, so the *y*-coordinate of the point is 3.

The coordinates of the point are (−3,3).

Point B is below −1 on the *x*-axis, so the *x*-coordinate of the point is −1.

The point is to the left of −3 on the *y*-axis, so the *y*-coordinate of the point is −3.

The coordinates of the point are (−1,−3).

Point C is above 2 on the *x*-axis, so the *x*-coordinate of the point is 2.

The point is to the right of 4 on the *y*-axis, so the *y*-coordinate of the point is 4.

The coordinates of the point are (2,4).

Point D is below 4 on the *x*-axis, so the *x*-coordinate of the point is 4.

The point is to the right of −4 on the *y*-axis, so the *y*-coordinate of the point is −4.

The coordinates of the point are (4,−4).

Point E is on the *y*-axis at y=−2. The coordinates of point E are (0,−2).

Point F is on the *x*-axis at x=3. The coordinates of point F are (3,0).

Try It 2.5.8

Name the ordered pair of each point shown in the rectangular coordinate system.

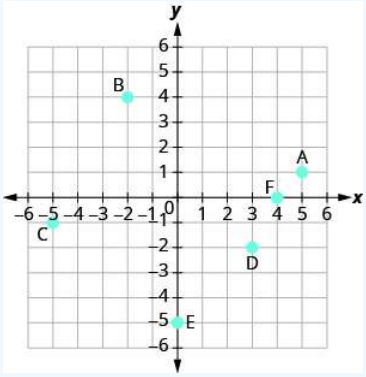


Figure 2.5.9

Answer

A: (5,1) B: (−2,4) C: (−5,−1) D: (3,−2) E: (0,−5) F: (4,0)

Try It 2.5.9

Name the ordered pair of each point shown in the rectangular coordinate system.

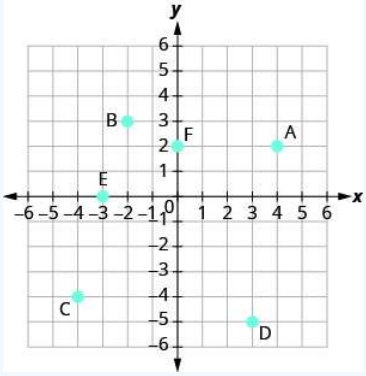


Figure 2.5.10

Answer

A: (4,2) B: (−2,3) C: (−4,−4) D: (3,−5) E: (−3,0) F: (0,2)

Verify Solutions to an Equation in Two Variables

Up to now, all the equations you have solved were equations with just one variable. In almost every case, when you solved the equation you got exactly one solution. The process of solving an equation ended with a statement like x=4. (Then, you checked the solution by substituting back into the equation.)

Here's an example of an equation in one variable, and its one solution.

$$\begin{aligned} 3x+5 &= 17 \\ 3x &= 12 \\ x &= 4 \end{aligned}$$

But equations can have more than one variable. Equations with two variables may be of the form Ax+By=C. Equations of this form are called **linear equations in two variables**.

LINEAR EQUATION

An equation of the form Ax+By=C, where A and B are not both zero, is called a **linear equation in two variables**.

Notice the word *line* in **linear**. Here is an example of a linear equation in two variables, x and y.

$$\begin{aligned} Ax+By&=C \\ x+4y&=8 \\ A=1,\ B&=4,\ C=8 \end{aligned}$$

Figure 2.5.11

The equation y=−3x+5 is also a **linear equation**. But it does not appear to be in the form Ax+By=C. We can use the Addition Property of Equality and rewrite it in Ax+By=C form.

	y	$=$	$-3x+5$
Add to both sides.	$y+3x$	$=$	$-3x+5+3x$
Simplify.	$y+3x$	$=$	5
Use the Commutative Property to put it in $Ax+By=C$ form.	$3x+y$	$=$	5

By rewriting y=−3x+5 as 3x+y=5, we can easily see that it is a linear equation in two variables because it is of the form Ax+By=C. When an equation is in the form Ax+By=C, we say it is in *standard form*.

STANDARD FORM OF LINEAR EQUATION

A linear equation is in standard form when it is written Ax+By=C.

Most people prefer to have A, B, and C be integers and $A \geq 0$ when writing a linear equation in standard form, although it is not strictly necessary.

Linear equations have infinitely many solutions. For every number that is substituted for x there is a corresponding y value. This pair of values is a *solution* to the linear equation and is represented by the ordered pair (x,y). When we substitute these values of x and y into the equation, the result is a true statement, because the value on the left side is equal to the value on the right side.

Solution OF A LINEAR EQUATION IN TWO VARIABLES

An **ordered pair** (x,y) is a **solution** of the linear equation Ax+By=C, if the equation is a true statement when the *x*- and *y*-values of the ordered pair are substituted into the equation.

Example 2.5.10

Determine which ordered pairs are solutions to the equation x+4y=8.

(a) (0,2)

(b) (2,−4)

(c) (−4,3)

Solution

Substitute the *x*- and *y*-values from each ordered pair into the equation and determine if the result is a true statement.

(a)	(b)	(c)
(0, 2)	(2, –4)	(–4, 3)
$x = 0, y = 2$	$x = 2, y = -4$	$x = -4, y = 3$
$x + 4y = 8$	$x + 4y = 8$	$x + 4y = 8$
$0 + 4 \cdot 2 \stackrel{?}{=} 8$	$2 + 4(-4) \stackrel{?}{=} 8$	$-4 + 4 \cdot 3 \stackrel{?}{=} 8$
$0 + 8 \stackrel{?}{=} 8$	$2 + (-16) \stackrel{?}{=} 8$	$-4 + 12 \stackrel{?}{=} 8$
$8 = 8$ ✓	$-14 \neq 8$	$8 = 8$ ✓
(0, 2) is a solution.	(2, –4) is not a solution.	(–4, 3) is a solution.

Try It 2.5.11

Which of the following ordered pairs are solutions to 2x+3y=6?

1. (3,0)
2. (2,0)
3. (6,−2)

Answer

1, 3

Try It 2.5.12

Which of the following ordered pairs are solutions to the equation 4x−y=8?

1. (0,8)
2. (2,0)
3. (1,−4)

Answer

2, 3

Example 2.5.13

Which of the following ordered pairs are solutions to the equation y=5x−1?

(a) (0,−1)

(b) (1,4)

(c) (−2,−7)

Solution

Substitute the *x*- and *y*-values from each **ordered pair** into the equation and determine if it results in a true statement.

(a)	(b)	(c)
(0, –1)	(1, 4)	(–2, –7)
$x = 0, y = -1$	$x = 1, y = 4$	$x = -2, y = -7$
$y = 5x - 1$	$y = 5x - 1$	$y = 5x - 1$
$-1 \stackrel{?}{=} 5(0) - 1$	$4 \stackrel{?}{=} 5(1) - 1$	$-7 \stackrel{?}{=} 5(-2) - 1$
$-1 \stackrel{?}{=} 0 - 1$	$4 \stackrel{?}{=} 5 - 1$	$-7 \stackrel{?}{=} -10 - 1$
$-1 = -1$ ✓	$4 = 4$ ✓	$-7 \neq -11$
(0, –1) is a solution.	(1, 4) is a solution.	(–2, –7) is not a solution.

Try It 2.5.14

Which of the following ordered pairs are solutions to the equation y=4x−3?

1. (0,3)
2. (1,1)
3. (−1,−1)

Answer

2

Try It 2.5.15

Which of the following ordered pairs are solutions to the equation y=−2x+6?

1. (0,6)
2. (1,4)
3. (−2,−2)

Answer

1, 2

Complete a Table of Solutions to a Linear Equation in Two Variables

In the examples above, we substituted the x- and y-values of a given ordered pair to determine whether or not it was a solution to a linear equation. But how do you find the ordered pairs if they are not given? It's easier than you might think—you can just pick a value for x and then solve the equation for y. Or, pick a value for y and then solve for x.

We'll start by looking at the solutions to the equation y = 5x−1 that we found in Example 2.5.13. We can summarize this information in a table of solutions, as shown in Table 2.5.1.

Table 2.5.1

y=5x−1		
x	**y**	**(x,y)**
0	−1	(0,−1)
1	4	(1,4)

To find a third solution, we'll let x=2 and solve for y.

	$y = 5x - 1$
Substitute $x = 2$.	$y = 5(2) - 1$
Multiply.	$y = 10 - 1$
Simplify.	$y = 9$

Figure 2.5.12

The ordered pair (2,9) is a solution to y=5x−1. We will add it to Table 2.5.2.

Table 2.5.2

y=5x−1		
x	**y**	**(x,y)**
0	−1	(0,−1)
1	4	(1,4)
2	9	(2,9)

We can find more solutions to the equation by substituting in any value of x or any value of y and solving the resulting equation to get another ordered pair that is a solution. There are infinitely many solutions of this equation.

Example 2.5.16

Complete the table to find three solutions to the equation $y=4x-2$.

Table 2.5.3

y=4x−2		
x	**y**	**(x,y)**
0		
−1		
2		

Solution

Substitute $x=0$, $x=-1$, and $x=2$ into $y=4x-2$.

$x=0$	$x=-1$	$x=2$
$y=4x-2$	$y=4x-2$	$y=4x-2$
$y=4\cdot 0-2$	$y=4(-1)-2$	$y=4\cdot 2-2$
$y=0-2$	$y=-4-2$	$y=8-2$
$y=-2$	$y=-6$	$y=6$
$(0,-2)$	$(-1,-6)$	$(2,6)$

The results are summarized in Table 2.5.4

Table 2.5.4

y=4x−2		
x	**y**	**(x,y)**
0	−2	(0,−2)
−1	−6	(−1,−6)
2	6	(2,6)

Try It 2.5.17

Complete the table to find three solutions to this equation: $y=3x-1$.

Table 2.5.5

y=3x−1		
x	**y**	**(x,y)**
0		
−1		
2		

Answer

Table 2.5.6

y=3x−1

x	y	(x,y)
0	-1	(0, -1)
−1	-4	(-1, -4)
2	5	(2, 5)

Try It 2.5.18

Complete the table to find three solutions to this equation: y=6x+1.

Table 2.5.7

y=6x+1		
x	**y**	**(x,y)**
-2		

Answer

Table 2.5.8

y=6x+1		
x	**y**	**(x,y)**
0	1	(0,1)
1	7	(1,7)
−2	−11	(−2,−11)

Example 2.5.19

Complete Table 2.5.9 to find three solutions to the equation 5x−4y=20.

Table 2.5.9

5x−4y=20		
x	**y**	**(x,y)**
	0	
	5	

Solution

Substitute the given value into the equation 5x−4y=20 and solve for the other variable. Then, fill in the values in the table.

$x = 0$	$y = 0$	$y = 5$
$5x - 4y = 20$	$5x - 4y = 20$	$5x - 4y = 20$
$5 \cdot 0 - 4y = 20$	$5x - 4 \cdot 0 = 20$	$5x - 4 \cdot 5 = 20$
$0 - 4y = 20$	$5x - 0 = 20$	$5x - 20 = 20$
$-4y = 20$	$5x = 20$	$5x = 40$
$y = -5$	$x = 4$	$x = 8$
$(0, -5)$	$(4, 0)$	$(8, 5)$

The results are summarized in Table 2.5.10

Table 2.5.10

5x−4y=20		
x	**y**	**(x,y)**
0	−5	(0,−5)
4	0	(4,0)
8	5	(8,5)

Try It 2.5.20

Complete the table to find three solutions to this equation: 2x−5y=20.

Table 2.5.11

2x−5y=20		
x	**y**	**(x,y)**
-5		

Answer

Table 2.5.12

2x−5y=20		
x	**y**	**(x,y)**
0	−4	(0,−4)
10	0	(10,0)
−5	−6	(−5,−6)

Try It 2.5.21

Complete the table to find three solutions to this equation: 3x−4y=12.

Table 2.5.13

3x−4y=12		
x	**y**	**(x,y)**

-4		

Answer

Table 2.5.14

3x−4y=12		
x	**y**	**(x,y)**
0	−3	(0,−3)
4	0	(4,0)
−4	−6	(−4,−6)

Find Solutions to a Linear Equation

To find a solution to a linear equation, you really can pick *any* number you want to substitute into the equation for x or y. But since you'll need to use that number to solve for the other variable it's a good idea to choose a number that's easy to work with.

When the equation is in *y*-form, with the *y* by itself on one side of the equation, it is usually easier to choose values of x and then solve for y.

Example 2.5.22

Find three solutions to the equation y=−3x+2.

Solution

We can substitute any value we want for x or any value for y. Since the equation is in *y*-form, it will be easier to substitute in values of x. Let's pick x=0, x=1, and x=−1.

			$x=0$	$x=1$	$x=-1$
			$y=-3x+2$	$y=-3x+2$	$y=-3x+2$
Substitute the value into the equation.			$y=-3\cdot 0+2$	$y=-3\cdot 1+2$	$y=-3(-1)+2$
Simplify.			$y=0+2$	$y=-3+2$	$y=3+2$
Simplify.			$y=2$	$y=-1$	$y=5$
Write the ordered pair.			(0, 2)	(1, -1)	(-1, 5)
Check.					
y=−3x+2	y=−3x+2	y=−3x+2			
$2\stackrel{?}{=}-3\cdot 0+2$	$-1\stackrel{?}{=}-3\cdot 1+2$	$5\stackrel{?}{=}-3(-1)+2$			
$2\stackrel{?}{=}0+2$	$-1\stackrel{?}{=}-3+2$	$5\stackrel{?}{=}-3+2$			
$2=2\checkmark$	$-1=-1\checkmark$	$5=5\checkmark$			

Table 2.5.15

So, (0,2), (1,−1) and (−1,5) are all solutions to y=−3x+2. We show them in Table 2.5.16

Table 2.5.16

y=−3x+2

x	y	(x,y)
0	2	(0,2)
1	−1	(1,−1)
−1	5	(−1,5)

Try It 2.5.23

Find three solutions to this equation: y=−2x+3.

Answer

Answers will vary. Answers might include (0,3), (1,1) and (−1,5).

Try It 2.5.24

Find three solutions to this equation: y=−4x+1.

Answer

Answers will vary. Answers might include (0,1), (1,−3) and (−1,5).

We have seen how using zero as one value of x makes finding the value of y easy. When an equation is in standard form, with both the x and y on the same side of the equation, it is usually easier to first find one solution when x=0 find a second solution when y=0, and then find a third solution.

Example 2.5.25

Find three solutions to the equation 3x+2y=6.

Solution

We can substitute any value we want for x or any value for y. Since the equation is in standard form, let's pick first x=0, then y=0, and then find a third point.

Table 2.5.17

$x=0$	$y=0$	$x=1$			
			$3x+2y=6$	$3x+2y=6$	$3x+2y=6$
Substitute the value into the equation.			$3(0)+2y=6$	$3x+2(0)=6$	$3(1)+2y=6$
Simplify.			$0+2y=6$	$3x+0=6$	$3+2y=6$
Solve.			$2y=6$	$3x=6$	$2y=3$
			$y=3$	$x=2$	$y=\frac{3}{2}$
Write the ordered pair.			(0, 3)	(2, 0)	$(1, \frac{3}{2})$
Check.					
3x+2y=6	3x+2y=6	3x+2y=6			
$3\cdot 0+2\cdot 3\stackrel{?}{=}6$	$3\cdot 2+2\cdot 0\stackrel{?}{=}6$	$3\cdot 1+2\cdot\frac{3}{2}\stackrel{?}{=}6$			
$0+6\stackrel{?}{=}6$	$6+0\stackrel{?}{=}6$	$3+3\stackrel{?}{=}6$			
$6=6$✓	$6=6$✓	$6=6$✓			

So (0,3), (2,0), and $(1, \frac{3}{2})$ are all solutions to the equation 3x+2y=6. We can list these three solutions in Table 2.5.18

Table 2.5.18

3x+2y=6		
x	**y**	**(x,y)**
0	3	(0,3)
2	0	(2,0)
1	$\frac{3}{2}$	$(1, \frac{3}{2})$

? Try It 2.5.26

Find three solutions to the equation 2x+3y=6.

Answer

Answers will vary. Answers might include (0,2), (3,0) and (6, −2).

? Try It 2.5.27

Find three solutions to the equation 4x+2y=8.

Answer

Answers will vary. Answers might include (0,4), (2,0) and (−1,6).

Key Concepts

- **Sign Patterns of the Quadrants**

Quadrant I	Quadrant II	Quadrant III	Quadrant IV
(x, y)	(x, y)	(x, y)	(x, y)
$(+, +)$	$(-, +)$	$(-, -)$	$(+, -)$

- **Points on the Axes**
  - On the *x*-axis, y=0. Points with a *y*-coordinate equal to 0 are on the *x*-axis, and have coordinates (a,0).
  - On the *y*-axis, x=0. Points with an *x*-coordinate equal to 0 are on the *y*-axis, and have coordinates (0,b).
- **Solution of a Linear Equation**
  - An ordered pair (x,y) is a solution of the linear equation Ax+By=C, if the equation is a true statement when the *x*- and *y*-values of the ordered pair are substituted into the equation.

Glossary

linear equation

A linear equation is of the form Ax+By=C, where A and B are not both zero, is called a linear equation in two variables.

ordered pair

An ordered pair (x,y) gives the coordinates of a point in a rectangular coordinate system.

origin

The point (0,0) is called the origin. It is the point where the *x*-axis and *y*-axis intersect.

quadrant

The *x*-axis and the *y*-axis divide a plane into four regions, called quadrants.

rectangular coordinate system

A grid system is used in algebra to show a relationship between two variables; also called the *xy*-plane or the ‘coordinate plane’.

***x*-coordinate**

The first number in an ordered pair (x,y).

***y*-coordinate**

The second number in an ordered pair (x,y).

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2.5E: Exercises

Practice Makes Perfect

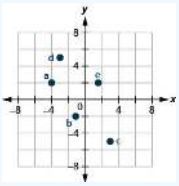
Plot Points in a Rectangular Coordinate System

In the following exercises, plot each point in a rectangular coordinate system and identify the quadrant in which the point is located.

Exercise 2.5E.1

1. (−4,2)
2. (−1,−2)
3. (3,−5)
4. (−3,5)
5. $(\frac{5}{3}, 2)$

Answer



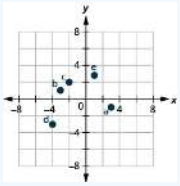
Exercise 2.5E.2

1. (−2,−3)
2. (3,−3)
3. (−4,1)
4. (4,−1)
5. $(\frac{3}{2}, 1)$

Exercise 2.5E.3

1. (3,−1)
2. (−3,1)
3. (−2,2)
4. (−4,−3)
5. $\left(1, \frac{14}{5}\right)$

Answer



Exercise 2.5E.4

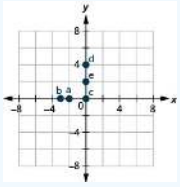
1. (−1,1)
2. (−2,−1)
3. (2,1)
4. (1,−4)
5. $\left(3, \frac{7}{2}\right)$

In the following exercises, plot each point in a rectangular coordinate system.

Exercise 2.5E.5

1. (−2,0)
2. (−3,0)
3. (0,0)
4. (0,4)
5. (0,2)

Answer



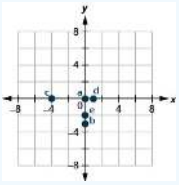
Exercise 2.5E.6

1. (0,1)
2. (0,−4)
3. (−1,0)
4. (0,0)
5. (5,0)

Exercise 2.5E.7

1. (0,0)
2. (0,−3)
3. (−4,0)
4. (1,0)
5. (0,−2)

Answer

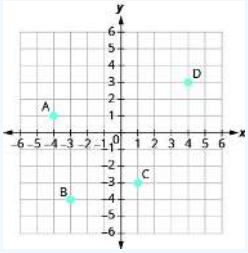


Exercise 2.5E.8

1. (−3,0)
2. (0,5)
3. (0,−2)
4. (2,0)
5. (0,0)

In the following exercises, name the ordered pair of each point shown in the rectangular coordinate system.

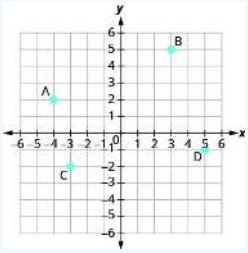
Exercise 2.5E.9



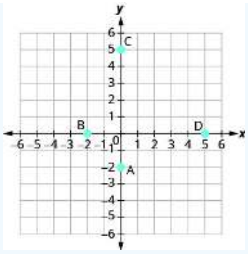
Answer

$A:(-4,1)\quad B:(-3,-4)\quad C:(1,-3)\quad D:(4,3)$

Exercise 2.5E.10



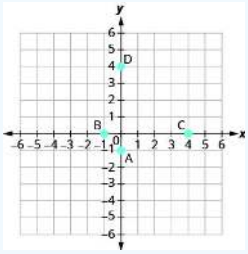
Exercise 2.5E.11



Answer

$A:(0,-2)\quad B:(-2,0)\quad C:(0,5)\quad D:(5,0)$

Exercise 2.5E.12



Verify Solutions to an Equation in Two Variables

In the following exercises, which ordered pairs are solutions to the given equations?

Exercise 2.5E.13

2x+y=6

1. (1,4)
2. (3,0)
3. (2,3)

Answer

1, 2

Exercise 2.5E.14

1. x+3y=9
2. (0,3)
3. (6,1)
4. (−3,−3)

Exercise 2.5E.15

4x−2y=8

1. (3,2)
2. (1,4)
3. (0,−4)

Answer

1, 3

Exercise 2.5E.16

3x−2y=12

1. (4,0)
2. (2,−3)
3. (1,6)

Exercise 2.5E.17

y=4x+3

1. (4,3)
2. (−1,−1)
3. $\left(\frac{1}{2},5\right)$

Answer

2, 3

Exercise 2.5E.18

y=2x−5

1. (0,−5)
2. (2,1)
3. $\left(\frac{1}{2}, -4\right)$

Exercise 2.5E.19

$y = \frac{1}{2}x - 1$

1. (2,0)
2. (−6,−4)
3. (−4,−1)

Answer

1, 2

Exercise 2.5E.20

$y = \frac{1}{3}x + 1$

1. (−3,0)
2. (9,4)
3. (−6,−1)

Complete a Table of Solutions to a Linear Equation

In the following exercises, complete the table to find solutions to each linear equation.

Exercise 2.5E.21

$y = 2x - 4$

x	y	(x,y)
0		
2		
-1		

Answer

x	y	(x,y)
0	−4	(0,−4)
2	0	(2,0)
−1	−6	(−1,−6)

Exercise 2.5E.22

$y = 3x - 1$

x	y	(x,y)
0		
2		
-1		

Exercise 2.5E.23

$y = -x + 5$

x	y	(x,y)
0		
3		
-2		

Answer

x	y	(x,y)
0	5	(0,5)
3	2	(3,2)
−2	7	(−2,7)

Exercise 2.5E.24

$y = -x + 2$

x	y	(x,y)
0		
3		
-2		

Exercise 2.5E.25

$y = \frac{1}{3}x + 1$

x	y	(x,y)
0		
3		
6		

Answer

x	y	(x,y)
0	1	(0,1)
3	2	(3,2)
6	3	(6,3)

Exercise 2.5E.26

$y = \frac{1}{2}x + 4$

x	y	(x,y)
0		
2		
4		

Exercise 2.5E.27

$y = -\frac{3}{2}x - 2$

x	y	(x,y)
0		
2		
-2		

Answer

x	y	(x,y)
0	−2	(0,−2)
2	−5	(2,−5)
−2	1	(−2,1)

Exercise 2.5E.28

$y = -\frac{2}{3}x - 1$

x	y	(x,y)
0		
3		
-3		

Exercise 2.5E.29

$x + 3y = 6$

x	y	(x,y)
0		
3		

	0	

Answer

x	y	(x,y)
0	2	(0,2)
3	4	(3,1)
6	0	(6,0)

Exercise 2.5E.30

x+2y=8

x	y	(x,y)
0		
4		
	0	

Exercise 2.5E.31

$2x-5y=10$

x	y	(x,y)
0		
10		
	0	

Answer

x	y	(x,y)
0	−2	(0,−2)
10	2	(10,2)
5	0	(5,0)

Exercise 2.5E.32

x	y	(x,y)
0		
8		
	0	

Find Solutions to a Linear Equation

In the following exercises, find three solutions to each linear equation.

Exercise 2.5E.33

$y=5x-8$

Answer

Answers will vary.

Exercise 2.5E.34

$y=3x-9$

Exercise 2.5E.35

$y=-4x+5$

Answer

Answers will vary.

Exercise 2.5E.36

$y=-2x+7$

Exercise 2.5E.37

$x+y=8$

Answer

Answers will vary.

Exercise 2.5E.38

$x+y=6$

Exercise 2.5E.39

$x+y=-2$

Answer

Answers will vary.

Exercise 2.5E.40

$x+y=-1$

Exercise 2.5E.41

$3x+y=5$

Answer

Answers will vary.

Exercise 2.5E.42

$2x+y=3$

Exercise 2.5E.43

$4x-y=8$

Answer

Answers will vary.

Exercise 2.5E.44

$5x-y=10$

Exercise 2.5E.45

$2x+4y=8$

Answer

Answers will vary.

Exercise 2.5E.46

$3x+2y=6$

Exercise 2.5E.47

$5x-2y=10$

Answer

Answers will vary.

Exercise 2.5E.48

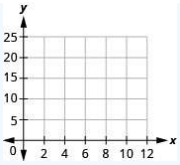
$4x-3y=12$

Everyday Math

Exercise 2.5E.49

Weight of a baby. Mackenzie recorded her baby's weight every two months. The baby's age, in months, and weight, in pounds, are listed in the table below, and shown as an ordered pair in the third column.

1. Plot the points on a coordinate plane.

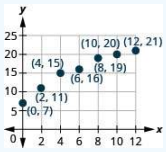


2. Why is only Quadrant I needed?

Age x	Weight y	(x,y)
0	7	(0, 7)
2	11	(2, 11)
4	15	(4, 15)
6	16	(6, 16)
8	19	(8, 19)
10	20	(10, 20)
12	21	(12, 21)

Answer

1.

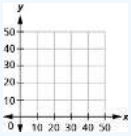


2. Age and weight are only positive.

Exercise 2.5E.50

Weight of a child. Latresha recorded her son's height and weight every year. His height, in inches, and weight, in pounds, are listed in the table below, and shown as an ordered pair in the third column.

1. Plot the points on a coordinate plane.



2. Why is only Quadrant I needed?

Height x	Weight y	(x,y)
28	22	(28, 22)
31	27	(31, 27)
33	33	(33, 33)
37	35	(37, 35)
40	41	(40, 41)
42	45	(42, 45)

Writing Exercises

Exercise 2.5E.51

Explain in words how you plot the point (4,−2) in a rectangular coordinate system?

Answer

Answers will vary.

Exercise 2.5E.52

How do you determine if an ordered pair is a solution to a given equation?

Exercise 2.5E.53

Is the point (−3,0) on the *x*-axis or *y*-axis? How do you know?

Answer

Answers will vary.

? Exercise 2.5E.54

Is the point (0,8) on the *x*-axis or *y*-axis? How do you know?

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
plot points in a rectangular coordinate system.			
identify points on a graph.			
verify solutions to an equation in two variables.			
complete a table of solutions to a linear equation.			
find solutions to a linear equation.			

ⓑ If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them. What did you do to become confident of your ability to do these things? Be specific.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math every topic builds upon previous work. It is important to make sure you have a strong foundation before you move on. Who can you ask for help? Your fellow classmates and instructor are good resources. Is there a place on campus where math tutors are available? Can your study skills be improved?

…no, I don't get it. This is a warning sign and you must not ignore it. You should get help right away or you will quickly be overwhelmed. See your instructor as soon as you can to discuss your situation. Together you can come up with a plan to get you the help you need.

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2.6: Graph with Intercepts

Learning Objectives

By the end of this section, you will be able to:

- Identify the x- and y- intercepts on a graph
- Find the x- and y- intercepts from an equation of a line
- Graph a line using the intercepts

Note

Before you get started, take this readiness quiz.

1. Solve: $3 \cdot 0 + 4y = -2$.
   If you missed this problem, review Exercise 2.2.13.

Identify the *x*- and *y*- Intercepts on a Graph

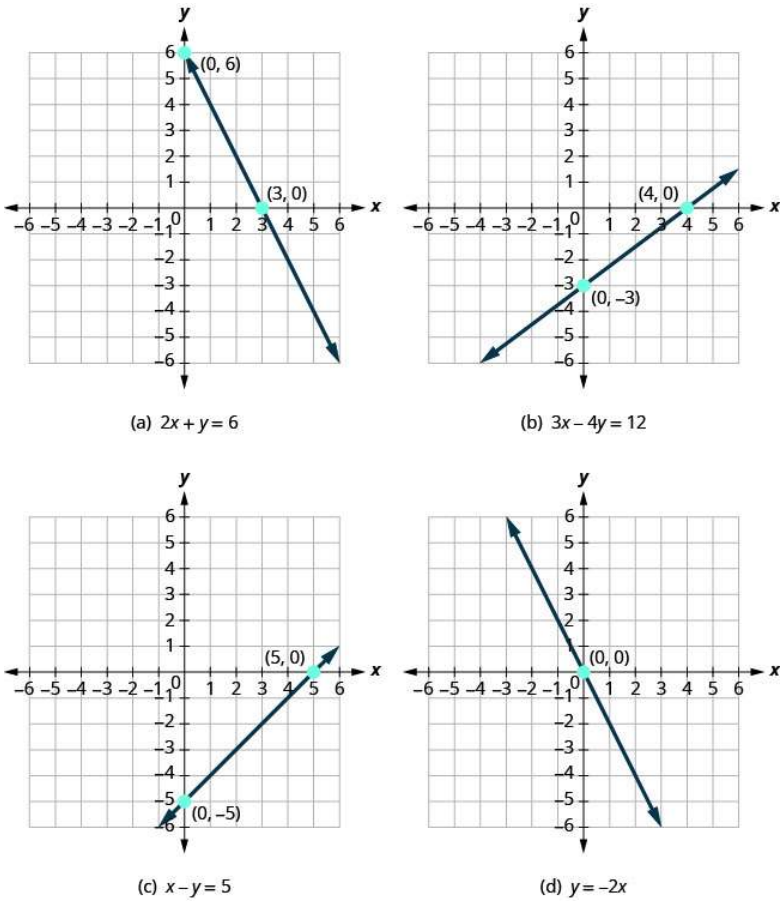
Every linear equation can be represented by a unique line that shows all the solutions of the equation. We have seen that when graphing a line by plotting points, you can use any three solutions to graph. This means that two people graphing the line might use different sets of three points.

At first glance, their two lines might not appear to be the same, since they would have different points labeled. But if all the work was done correctly, the lines should be exactly the same. One way to recognize that they are indeed the same line is to look at where the line crosses the *x*- axis and the *y*- axis. These points are called the *intercepts* of the line.

INTERCEPTS OF A LINE

The points where a line crosses the *x*- axis and the *y*- axis are called the **intercepts of a line**.

Let's look at the graphs of the lines in Figure 2.6.1.



(a) $2x + y = 6$ (b) $3x - 4y = 12$

(c) $x - y = 5$ (d) $y = -2x$

Figure 2.6.1: Examples of graphs crossing the x-negative axis.

First, notice where each of these lines crosses the x axis. See Figure 2.6.1.

Table 2.6.1

Figure	The line crosses the x-axis at:	Ordered pair of this point
Figure (a)	3	(3,0)
Figure (b)	4	(4,0)
Figure (c)	5	(5,0)
Figure (d)	0	(0,0)

Do you see a pattern?

For each row, the *y*- coordinate of the point where the line crosses the *x*- axis is zero. The point where the line crosses the *x*- axis has the form (a,0) and is called the ***x*- intercept of a line**. The *x*- intercept occurs when y is zero. Now, let's look at the points where these lines cross the *y*- axis. See Table 2.6.2.

Table 2.6.2

Figure	The line crosses the x-axis at:	Ordered pair of this point
Figure (a)	6	(0,6)
Figure (b)	−3	(0,−3)
Figure (c)	−5	(0,5)
Figure (d)	0	(0,0)

What is the pattern here?

In each row, the *x*- coordinate of the point where the line crosses the *y*- axis is zero. The point where the line crosses the *y*- axis has the form (0,b) and is called the *y*- *intercept* of the line. The *y*- intercept occurs when x is zero.

X- INTERCEPT AND Y- INTERCEPT OF A LINE

The *x*- intercept is the point (a,0) where the line crosses the *x*- axis.

The *y*- intercept is the point (0,b) where the line crosses the *y*- axis.

	x	*y*
• The *x*-intercept occurs when *y* is zero.	*a*	0
• The *y*-intercept occurs when *x* is zero.	0	*b*

Figure 2.6.2

Example 2.6.1

Find the *x*- and *y*- intercepts on each graph.

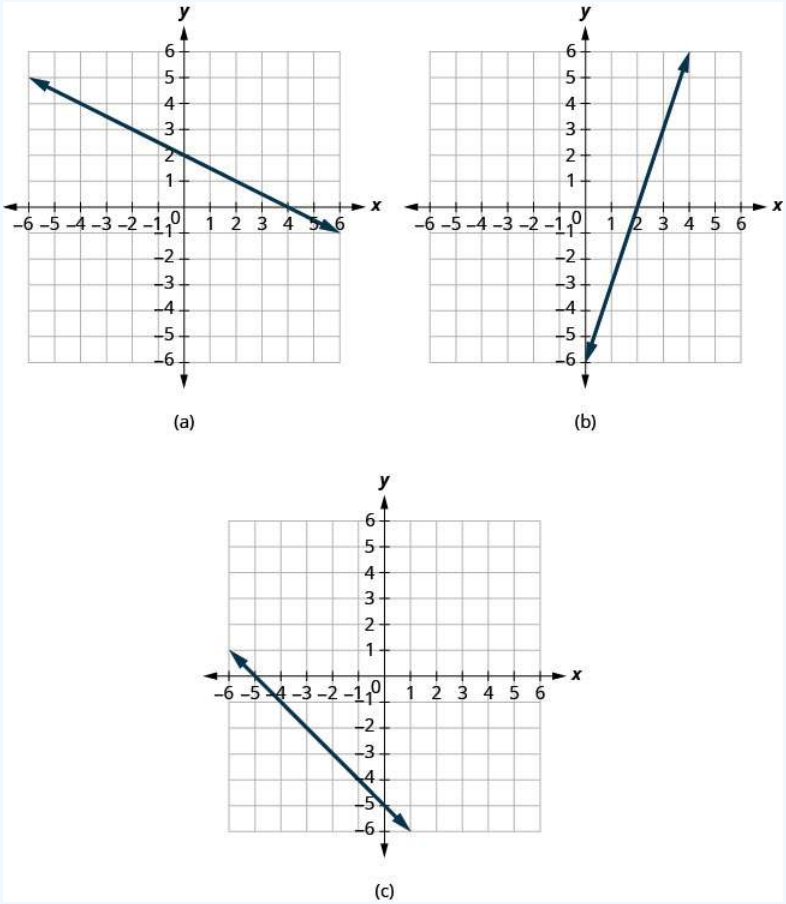


Figure 2.6.3

Solution

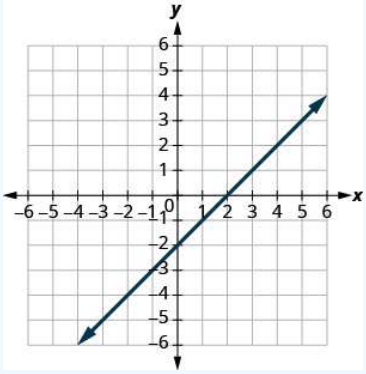
(a) The graph crosses the *x*- axis at the point (4,0). The *x*- intercept is (4,0).
The graph crosses the *y*- axis at the point (0,2). The *y*- intercept is (0,2).

(b) The graph crosses the x- axis at the point (2,0). The x- intercept is (2,0)
The graph crosses the y- axis at the point (0,−6). The y- intercept is (0,−6).

(c) The graph crosses the x- axis at the point (−5,0). The x- intercept is (−5,0).
The graph crosses the y- axis at the point (0,−5). The y- intercept is (0,−5).

Try It 2.6.2

Find the x- and y- intercepts on the graph.

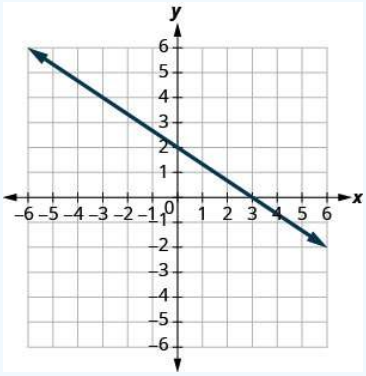


Answer

x- intercept: (2,0); y- intercept: (0,−2)

Try It 2.6.3

Find the x- and y- intercepts on the graph.



Answer

x- intercept: (3,0), y- intercept: (0,2)

Find the x- and y- Intercepts from an Equation of a Line

Recognizing that the ***x*- intercept** occurs when y is zero and that the y- intercept occurs when x is zero, gives us a method to find the intercepts of a line from its equation. To find the x- intercept, let y=0 and solve for x. To find the ***y*- intercept**, let x=0 and solve for y.

X- AND Y- INTERCEPTS FROM THE EQUATION OF A LINE

Use the equation of the line. To find:

- the *x*- intercept of the line, let y=0 and solve for x.
- the *y*- intercept of the line, let x=0 and solve for y.

Example 2.6.4

Find the intercepts of 2x+y=6.

Solution

We will let y=0 to find the *x*- intercept, and let x=0 to find the *y*- intercept. We will fill in the table, which reminds us of what we need to find.

$2x+y=6$	
x	y
	0
0	

x-intercept

y-intercept

To find the *x*- intercept, let y=0.

Table 2.6.3

	$2x+y=6$
Let $y = 0$.	$2x+0=6$
Simplify.	$2x=6$
	$x=3$
The *x*-intercept is	(3, 0)
To find the *y*-intercept, let $x = 0$.	
	$2x+y=6$
Let $x = 0$.	$2\cdot 0+y=6$
Simplify.	$0+y=6$
	$y=6$
The *y*-intercept is	(0, 6)

The intercepts are the points (3,0) and (0,6) as shown in Table 2.6.4

Table 2.6.4

2x+y=6	
x	**y**
3	0
0	6

Try It 2.6.5

Find the intercepts of 3x+y=12.

Answer

x- intercept: (4,0), *y*- intercept: (0,12)

Try It 2.6.6

Find the intercepts of x+4y=8.

Answer

x- intercept: (8,0), *y*- intercept: (0,2)

Example 2.6.7

Find the intercepts of 4x–3y=12.

Solution

To find the *x*-intercept, let $y = 0$.	
	$4x - 3y = 12$
Let $y = 0$.	$4x - 3 \cdot 0 = 12$
Simplify.	$4x - 0 = 12$
	$4x = 12$
	$x = 3$
The *x*-intercept is	(3, 0)
To find the *y*-intercept, let $x = 0$.	
	$4x - 3y = 12$
Let $x = 0$.	$4 \cdot 0 - 3y = 12$
Simplify.	$0 - 3y = 12$
	$-3y = 12$
	$y = -4$
The *y*-intercept is	(0, −4)

Table 2.6.5

The intercepts are the points (3, 0) and (0, −4) as shown in the following table.

Table 2.6.6

4x−3y=12	
x	**y**
3	0
0	−4

Try It 2.6.8

Find the intercepts of 3x–4y=12.

Answer

x- intercept: (4,0), *y*- intercept: (0,−3)

Try It 2.6.9

Find the intercepts of 2x–4y=8.

Answer

x- intercept: (4,0), *y*- intercept: (0,−2)

Graph a Line Using the Intercepts

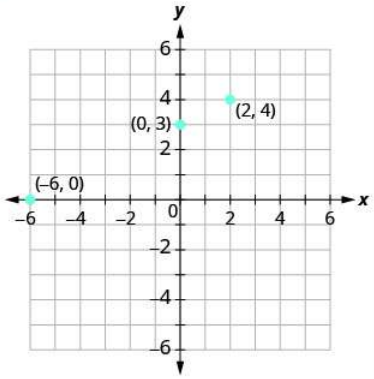
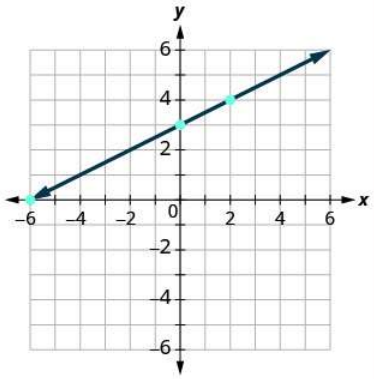
To graph a linear equation by plotting points, you need to find three points whose coordinates are solutions to the equation. You can use the *x*- and *y*- intercepts as two of your three points. Find the intercepts, and then find a third point to ensure accuracy. Make sure the points line up—then draw the line. This method is often the quickest way to graph a line.

Example 2.6.10: How to Graph a Line Using Intercepts

Graph –x+2y=6 using the intercepts.

Solution

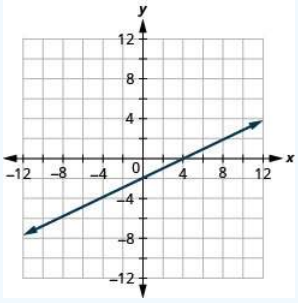
Step 1. Find the *x*- and *y*-intercepts of the line. Let $y = 0$ and solve for x. Let $x = 0$ and solve for y.	Find the *x*-intercept.	Let $y = 0$ $-x + 2y = 6$ $-x + 2(0) = 6$ $-x = 6$ $x = -6$ The *x*-intercept is (–6, 0).
	Find the *y*-intercept.	Let $x = 0$ $-x + 2y = 6$ $-0 + 2y = 6$ $2y = 6$ $y = 3$ The *y*-intercept is (0, 3).
Step 2. Find another solution to the equation.	We'll use $x = 2$.	Let $x = 2$ $-x + 2y = 6$ $-2 + 2y = 6$ $2y = 8$ $y = 4$ A third point is (2, 4).

Step 3. Plot the three points. Check that the points line up.	x | y | (x, y) -6 | 0 | $(-6, 0)$ 0 | 3 | $(0, 3)$ 2 | 4 | $(2, 4)$	
Step 4. Draw the line.	See the graph.	

Try It 2.6.11

Graph x–2y=4 using the intercepts.

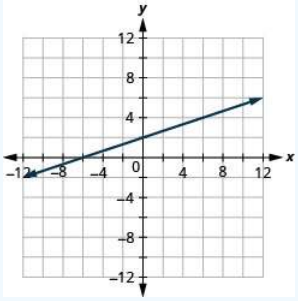
Answer



Try It 2.6.12

Graph –x+3y=6 using the intercepts.

Answer



The steps to graph a linear equation using the intercepts are summarized below.

GRAPH A LINEAR EQUATION USING THE INTERCEPTS.

1. Find the *x*- and *y*- intercepts of the line.
   - Let y=0 and solve for x
   - Let x=0 and solve for y.
2. Find a third solution to the equation.
3. Plot the three points and check that they line up.
4. Draw the line.

Example 2.6.13

Graph 4x–3y=12 using the intercepts.

Solution

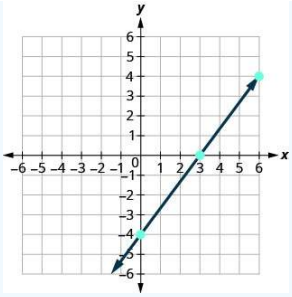
Find the intercepts and a third point.

x-intercept, let $y = 0$	*y*-intercept, let $x = 0$	third point, let $y = 4$
$4x - 3y = 12$	$4x - 3y = 12$	$4x - 3y = 12$
$4x - 3(0) = 12$	$4(0) - 3y = 12$	$4x - 3(4) = 12$
$4x = 12$	$-3y = 12$	$4x - 12 = 12$
$x = 3$	$y = -4$	$4x = 24$
		$x = 6$

We list the points in Table 2.6.7 and show the graph below.

4x−3y=12		
x	**y**	**(x,y)**
3	0	(3,0)
0	−4	(0,−4)
6	4	(6,4)

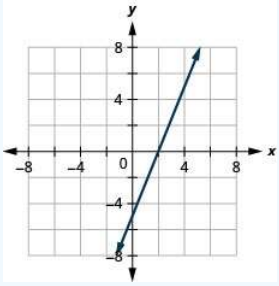
Table 2.6.7



Try It 2.6.14

Graph 5x–2y=10 using the intercepts.

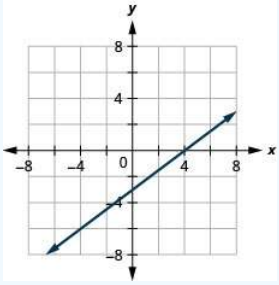
Answer



Try It 2.6.15

Graph 3x–4y=12 using the intercepts.

Answer



Example 2.6.16

Graph y=5x using the intercepts.

Solution

x-intercept	*y*-intercept
Let $y = 0$.	Let $x = 0$.
$y = 5x$	$y = 5x$
$0 = 5x$	$y = 5 \cdot 0$
$0 = x$	$y = 0$
(0, 0)	(0, 0)

This line has only one intercept. It is the point (0,0).

To ensure accuracy we need to plot three points. Since the x- and y- intercepts are the same point, we need *two* more points to graph the line.

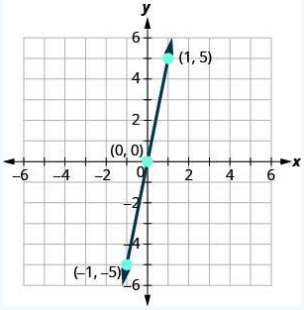
Let $x = 1$.	Let $x = -1$.
$y = 5x$	$y = 5x$
$y = 5 \cdot 1$	$y = 5(-1)$
$y = 5$	$y = -5$

See Table 2.6.8.

y=5x		
x	**y**	**(x,y)**
0	0	(0,0)
1	5	(1,5)
−1	−5	(−1,−5)

Table 2.6.8

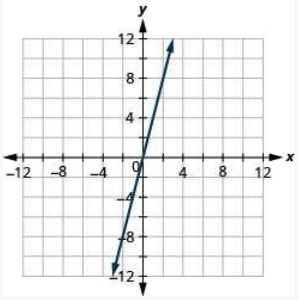
Plot the three points, check that they line up, and draw the line.



Try It 2.6.17

Graph y=4x using the intercepts.

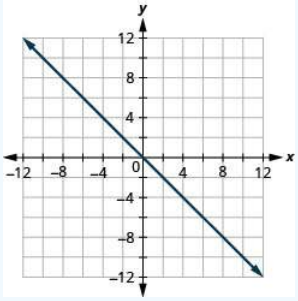
Answer



Try It 2.6.18

Graph y=−x the intercepts.

Answer



Key Concepts

- **Find the *x*- and *y*- Intercepts from the Equation of a Line**
  - Use the equation of the line to find the *x*- intercept of the line, let y=0 and solve for *x*.
  - Use the equation of the line to find the *y*- intercept of the line, let x=0 and solve for *y*.
- **Graph a Linear Equation using the Intercepts**
  1. Find the *x*- and *y*- intercepts of the line.
     Let y=0 and solve for *x*.
     Let x=0 and solve for *y*.
  2. Find a third solution to the equation.
  3. Plot the three points and then check that they line up.
  4. Draw the line.
- **Strategy for Choosing the Most Convenient Method to Graph a Line:**
  - Consider the form of the equation.
  - If it only has one variable, it is a vertical or horizontal line.
    x=a is a vertical line passing through the *x*- axis at a
    y=b is a horizontal line passing through the *y*- axis at b.
  - If *y* is isolated on one side of the equation, graph by plotting points.
  - Choose any three values for *x* and then solve for the corresponding *y*- values.
  - If the equation is of the form ax+by=c, find the intercepts. Find the *x*- and *y*- intercepts and then a third point.

Glossary

intercepts of a line

The points where a line crosses the *x*- axis and the *y*- axis are called the intercepts of the line.

***x*- intercept**

The point (a,0) where the line crosses the *x*- axis; the *x*- intercept occurs when y is zero.

***y*-intercept**

The point (0,b) where the line crosses the *y*- axis; the *y*- intercept occurs when x is zero.

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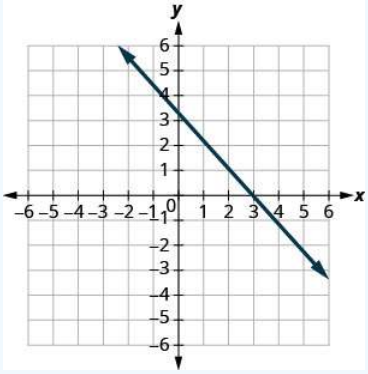
2.6.1: Exercises

Practice Makes Perfect

Identify the *x*- and *y*- Intercepts on a Graph

In the following exercises, find the *x*- and *y*- intercepts on each graph.

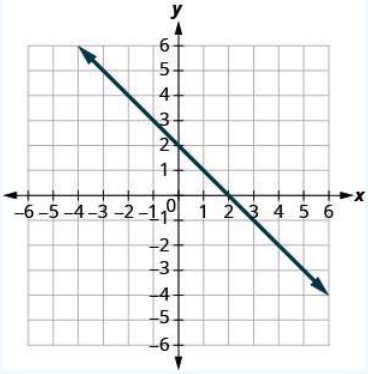
Exercise 2.6.1.1



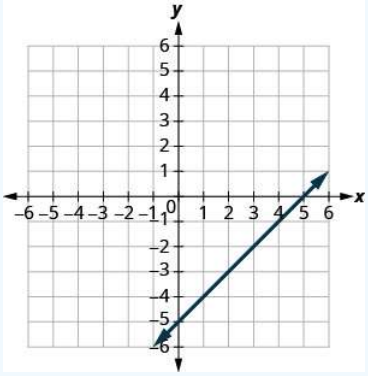
Answer

(3,0),(0,3)

Exercise 2.6.1.2



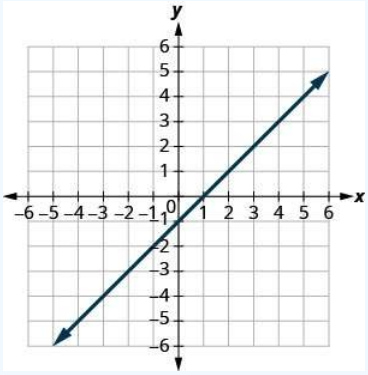
Exercise 2.6.1.3



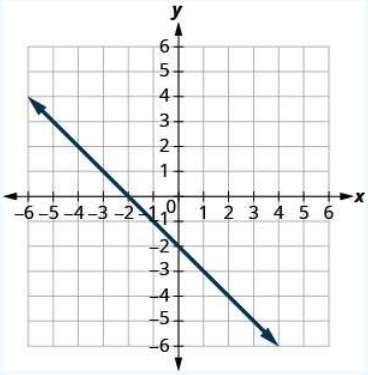
Answer

(5,0),(0,−5)

Exercise 2.6.1.4



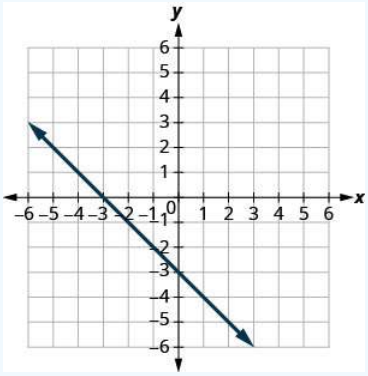
Exercise 2.6.1.5



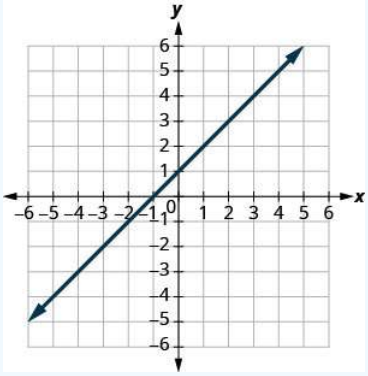
Answer

(−2,0),(0,−2)

Exercise 2.6.1.6



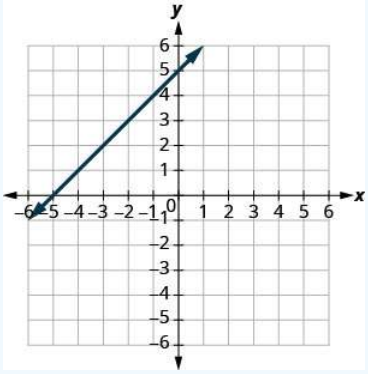
Exercise 2.6.1.7



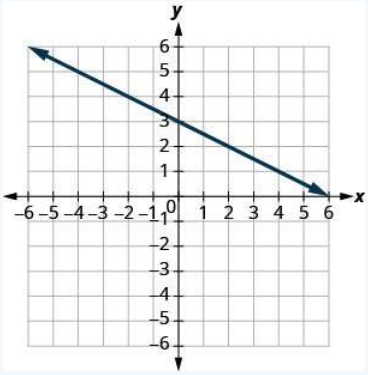
Answer

$(-1,0),(0,1)$

Exercise 2.6.1.8



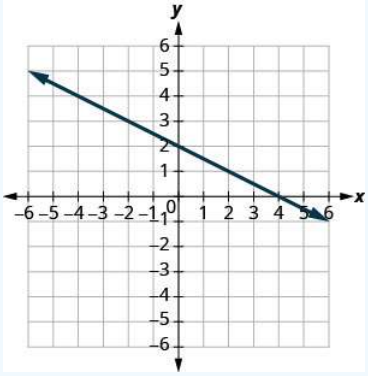
Exercise 2.6.1.9



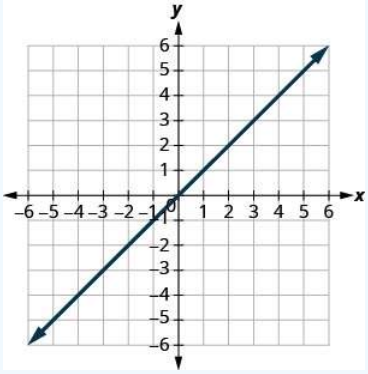
Answer

$(6,0),(0,3)$

Exercise 2.6.1.10



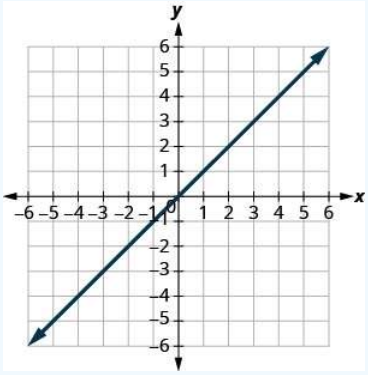
Exercise 2.6.1.11



Answer

(0,0)

Exercise 2.6.1.12



Find the *x*- and *y*- Intercepts from an Equation of a Line

In the following exercises, find the intercepts for each equation.

Exercise 2.6.1.13

$x+y=4$

Answer

$(4,0),(0,4)$

Exercise 2.6.1.14

$x+y=3$

Exercise 2.6.1.15

$x+y=-2$

Answer

$(-2,0),(0,-2)$

Exercise 2.6.1.16

$x+y=-5$

Exercise 2.6.1.17

$x-y=5$

Answer

$(5,0),(0,-5)$

Exercise 2.6.1.18

$x-y=1$

Exercise 2.6.1.19

$x-y=-3$

Answer

$(-3,0),(0,3)$

Exercise 2.6.1.20

$x-y=-4$

Exercise 2.6.1.21

$x+2y=8$

Answer

$(8,0),(0,4)$

Exercise 2.6.1.22

$x+2y=10$

Exercise 2.6.1.23

$3x+y=6$

Answer

$(2,0),(0,6)$

Exercise 2.6.1.24

$3x+y=9$

Exercise 2.6.1.25

$x-3y=12$

Answer

$(12,0),(0,-4)$

Exercise 2.6.1.26

$x-2y=8$

Exercise 2.6.1.27

$4x-y=8$

Answer

$(2,0),(0,-8)$

Exercise 2.6.1.28

$5x-y=5$

Exercise 2.6.1.29

$2x+5y=10$

Answer

$(5,0),(0,2)$

Exercise 2.6.1.30

$2x+3y=6$

Exercise 2.6.1.31

3x–2y=12

Answer

(4,0),(0,−6)

Exercise 2.6.1.32

3x–5y=30

Exercise 2.6.1.33

$y = \frac{1}{3}x + 1$

Answer

(-3,0),(0,1)

Exercise 2.6.1.34

$y = \frac{1}{4}x - 1$

Exercise 2.6.1.35

$y = \frac{1}{5}x + 2$

Answer

(−10,0),(0,2)

Exercise 2.6.1.36

$y = \frac{1}{3}x + 4$

Exercise 2.6.1.37

y=3x

Answer

(0,0)

Exercise 2.6.1.38

y=-2x

Exercise 2.6.1.39

y=-4x

Answer

(0,0)

Exercise 2.6.1.40

y=5x

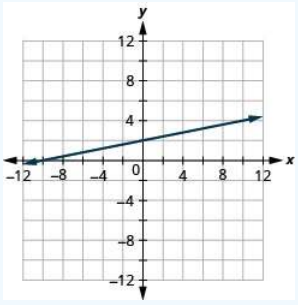
Graph a Line Using the Intercepts

In the following exercises, graph using the intercepts.

Exercise 2.6.1.41

$-x+5y=10$

Answer



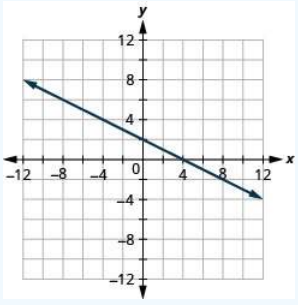
Exercise 2.6.1.42

$-x+4y=8$

Exercise 2.6.1.43

$x+2y=4$

Answer



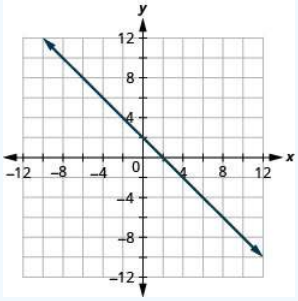
Exercise 2.6.1.44

$x+2y=6$

Exercise 2.6.1.45

$x+y=2$

Answer



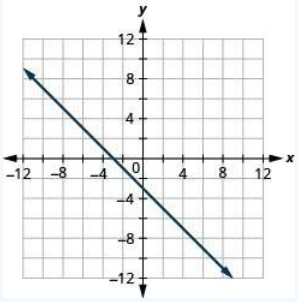
Exercise 2.6.1.46

$x + y = 5$

Exercise 2.6.1.47

$x + y = -3$

Answer



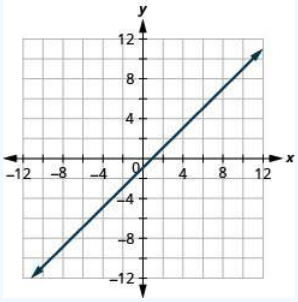
Exercise 2.6.1.48

$x + y = -1$

Exercise 2.6.1.49

$x - y = 1$

Answer



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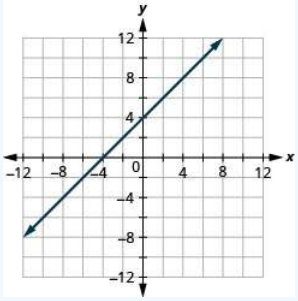
Exercise 2.6.1.50

$x - y = 2$

Exercise 2.6.1.51

$x - y = -4$

Answer



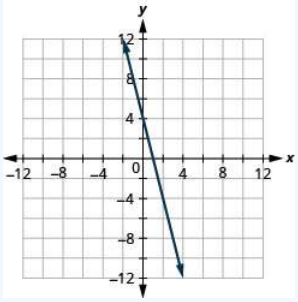
Exercise 2.6.1.52

$x - y = -3$

Exercise 2.6.1.53

$4x + y = 4$

Answer



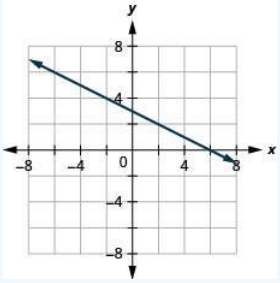
Exercise 2.6.1.54

$3x + y = 3$

Exercise 2.6.1.55

$2x + 4y = 12$

Answer



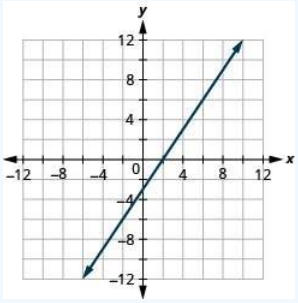
Exercise 2.6.1.56

$3x + 2y = 12$

Exercise 2.6.1.57

$3x - 2y = 6$

Answer



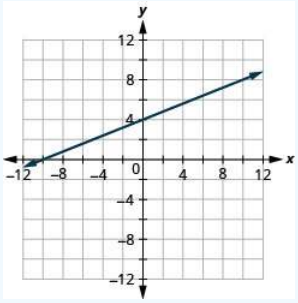
Exercise 2.6.1.58

$5x - 2y = 10$

Exercise 2.6.1.59

$2x - 5y = -20$

Answer



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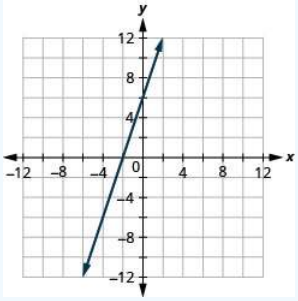
Exercise 2.6.1.60

$3x - 4y = -12$

Exercise 2.6.1.61

$3x - y = -6$

Answer



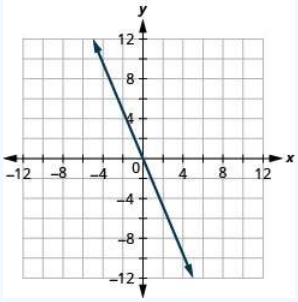
Exercise 2.6.1.62

$2x - y = -8$

Exercise 2.6.1.63

$y = -2x$

Answer



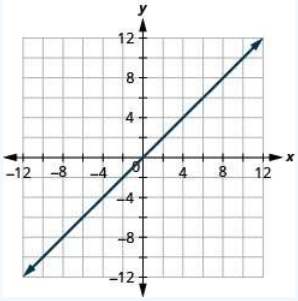
Exercise 2.6.1.64

$y = -4x$

Exercise 2.6.1.65

$y = x$

Answer



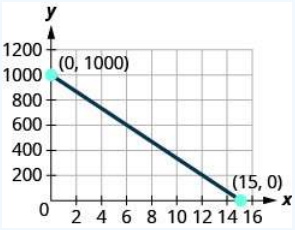
? Exercise 2.6.1.66

y=3x

Everyday Math

? Exercise 2.6.1.67

Road trip. Damien is driving from Chicago to Denver, a distance of 1000 miles. The *x*- axis on the graph below shows the time in hours since Damien left Chicago. The *y*- axis represents the distance he has left to drive.



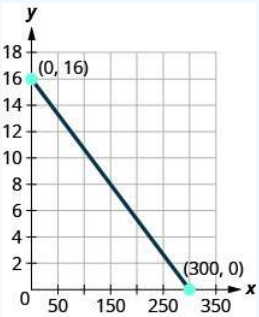
1. Find the *x*- and *y*- intercepts.
2. Explain what the *x*- and *y*- intercepts mean for Damien.

Answer

1. (0,1000),(15,0)
2. At (0,1000), he has been gone 0 hours and has 1000 miles left. At (15,0), he has been gone 15 hours and has 0 miles left to go.

? Exercise 2.6.1.68

Road trip. Ozzie filled up the gas tank of his truck and headed out on a road trip. The *x*- axis on the graph below shows the number of miles Ozzie drove since filling up. The *y*- axis represents the number of gallons of gas in the truck's gas tank.



1. Find the *x*- and *y*- intercepts.
2. Explain what the *x*- and *y*- intercepts mean for Ozzie.

Writing Exercises

Exercise 2.6.1.69

How do you find the x -intercept of the graph of $3x - 2y = 6$?

Answer

Answers will vary.

Exercise 2.6.1.70

Do you prefer to use the method of plotting points or the method using the intercepts to graph the equation 4x+y=−4? Why?

Exercise 2.6.1.71

Do you prefer to use the method of plotting points or the method using the intercepts to graph the equation $y = \frac{2}{3}x - 2$? Why?

Answer

Answers will vary.

Exercise 2.6.1.72

Do you prefer to use the method of plotting points or the method using the intercepts to graph the equation y=6? Why?

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
identify the *x* and *y* intercepts on a graph.			
find the *x* and *y* intercepts from an equation of a line.			
graph a line using the intercepts.			

ⓑ What does this checklist tell you about your mastery of this section? What steps will you take to improve?

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2.7: Understanding the Slope of a Line

Learning Objectives

- By the end of this section, you will be able to:
- Use geoboards to model slope
- Use $m = \frac{\text{rise}}{\text{run}}$ to find the slope of a line from its graph
- Find the slope of horizontal and vertical lines
- Use the slope formula to find the slope of a line between two points
- Graph a line given a point and the slope
- Solve slope applications

Note

Before you get started, take this readiness quiz.

1. Simplify: $\frac{1-4}{8-2}$.
   If you missed this problem, review Exercise 1.6.31
2. Divide: $\frac{0}{4}, \frac{4}{0}$.
   If you missed this problem, review Exercise 1.10.16.
3. Simplify: $\frac{15}{-3}, \frac{-15}{3}, \frac{-15}{-3}$.
   If you missed this problem, review Exercise 1.6.4.

When you graph linear equations, you may notice that some lines tilt up as they go from left to right and some lines tilt down. Some lines are very steep and some lines are flatter. What determines whether a line tilts up or down or if it is steep or flat?

In mathematics, the 'tilt' of a line is called the *slope* of the line. The concept of slope has many applications in the real world. The pitch of a roof, grade of a highway, and a ramp for a wheelchair are some examples where you literally see slopes. And when you ride a bicycle, you feel the slope as you pump uphill or coast downhill.

In this section, we will explore the concept of slope.

Use Geoboards to Model Slope

A **geoboard** is a board with a grid of pegs on it. Using rubber bands on a geoboard gives us a concrete way to model lines on a coordinate grid. By stretching a rubber band between two pegs on a geoboard, we can discover how to find the slope of a line.

Doing the Manipulative Mathematics activity "Exploring Slope" will help you develop a better understanding of the slope of a line. (Graph paper can be used instead of a geoboard, if needed.)

We'll start by stretching a rubber band between two pegs as shown in Figure 2.7.1.

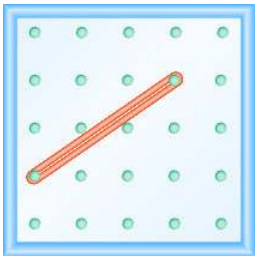


Figure 2.7.1

Doesn't it look like a line?

Now we stretch one part of the rubber band straight up from the left peg and around a third peg to make the sides of a right triangle, as shown in Figure 2.7.2

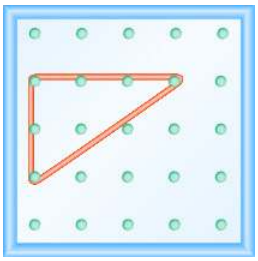


Figure 2.7.2

We carefully make a 90º angle around the third peg, so one of the newly formed lines is vertical and the other is horizontal.

To find the slope of the line, we measure the distance along the vertical and horizontal sides of the triangle. The vertical distance is called the **rise** and the horizontal distance is called the **run**, as shown in Figure 2.7.3.

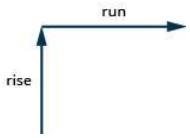


Figure 2.7.3

If our **geoboard** and rubber band look just like the one shown in Figure 2.7.4, the rise is 2. The rubber band goes up 2 units. (Each space is one unit.)

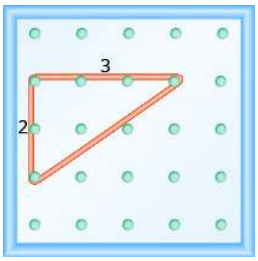


Figure 2.7.4

The rise on this geoboard is 2, as the rubber band goes up two units.

What is the run?

The rubber band goes across 3 units. The run is 3 (see Figure 2.7.4).

The slope of a line is the ratio of the rise to the run. In mathematics, it is always referred to with the letter m.

SLOPE OF A LINE

The **slope of a line** of a line is $m = \frac{\text{rise}}{\text{run}}$.

The **rise** measures the vertical change and the **run** measures the horizontal change between two points on the line.

What is the slope of the line on the geoboard in Figure 2.7.4?

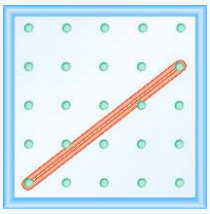
$$m = \frac{\text{rise}}{\text{run}}$$
$$m = \frac{2}{3}$$

The line has slope $\frac{2}{3}$. This means that the line rises 2 units for every 3 units of run.

When we work with geoboards, it is a good idea to get in the habit of starting at a peg on the left and connecting to a peg to the right. If the rise goes up it is positive and if it goes down it is negative. The run will go from left to right and be positive.

Example 2.7.1

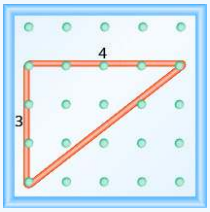
What is the slope of the line on the geoboard shown?



Solution

Use the definition of slope: $m = \frac{\text{rise}}{\text{run}}$.

Start at the left peg and count the spaces up and to the right to reach the second peg.

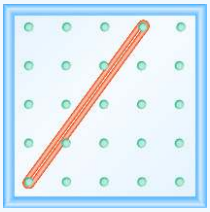


$$\begin{aligned} &\text{The rise is } 3. && m = \tfrac{3}{\text{run}} \\ &\text{The run is } 4. && m = \tfrac{3}{4} \\ & && \text{The slope is } \tfrac{3}{4}. \end{aligned} \tag{2.7.1}$$

This means that the line rises 3 units for every 4 units of run.

Try It 2.7.2

What is the slope of the line on the geoboard shown?

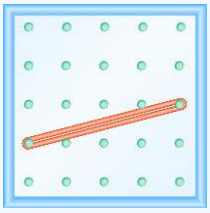


Answer

$\frac{4}{3}$

Try It 2.7.3

What is the slope of the line on the geoboard shown?

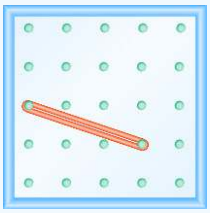


Answer

$\frac{1}{4}$

Example 2.7.4

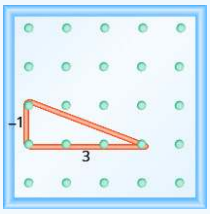
What is the slope of the line on the geoboard shown?



Solution

Use the definition of slope: $m = \frac{\text{rise}}{\text{run}}$.

Start at the left peg and count the units down and to the right to reach the second peg.

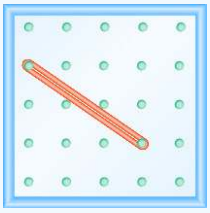


$$\begin{aligned} &\text{The rise is } -1. && m = \tfrac{-1}{\text{run}} \\ &\text{The run is } 3. && m = \tfrac{-1}{3} \\ & && m = -\tfrac{1}{3} \\ & && \text{The slope is } -\tfrac{1}{3} \end{aligned} \qquad (2.7.2)$$

This means that the line drops 1 unit for every 3 units of run.

Try It 2.7.5

What is the slope of the line on the geoboard?

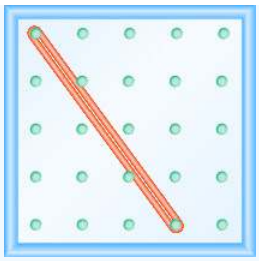


Answer

$-\frac{2}{3}$

Try It 2.7.6

What is the slope of the line on the geoboard?



Answer

$-\frac{4}{3}$

Notice that in Exercise 2.7.1 the slope is positive and in Exercise 2.7.4 the slope is negative. Do you notice any difference in the two lines shown in Figure(a) and Figure(b)?

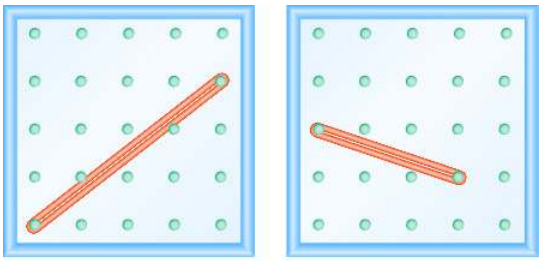


Figure 2.7.5

We 'read' a line from left to right just like we read words in English. As you read from left to right, the line in Figure(a) is going up; it has **positive slope**. The line in Figure(b) is going down; it has **negative slope**.

POSITIVE AND NEGATIVE SLOPES

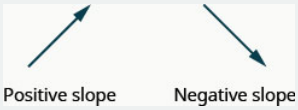


Figure 2.7.6

Example 2.7.7

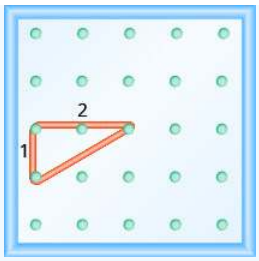
Use a geoboard to model a line with slope $\frac{1}{2}$.

Solution

To model a line on a geoboard, we need the rise and the run.

Use the slope formula.	$m = \frac{\text{rise}}{\text{run}}$
Replace m with $\frac{1}{2}$.	$\frac{1}{2} = \frac{\text{rise}}{\text{run}}$

So, the rise is 1 and the run is 2 .
Start at a peg in the lower left of the geoboard.
Stretch the rubber band up 1 unit, and then right 2 units.

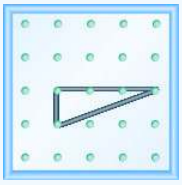


The hypotenuse of the right triangle formed by the rubber band represents a line whose slope is $\frac{1}{2}$.

Try It 2.7.8

Model the slope $m = \frac{1}{3}$. Draw a picture to show your results.

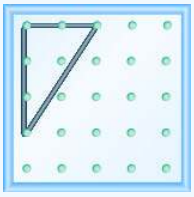
Answer



Try It 2.7.9

Model the slope $m = \frac{3}{2}$. Draw a picture to show your results.

Answer

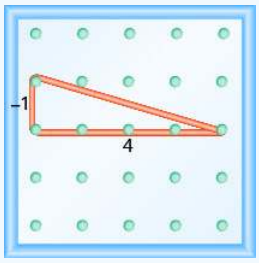


Example 2.7.10

Use a geoboard to model a line with slope $\frac{-1}{4}$.

Solution

Use the slope formula. m = $\frac{\text{rise}}{\text{run}}$.
Replace m with $\frac{-1}{4}$.
So, the rise is -1 and the run is 4.
Since the rise is negative, we choose a starting peg on the upper left that will give us room to count down.
We stretch the rubber band down 1 unit, and then right 4 units.

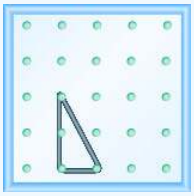


The hypotenuse of the right triangle formed by the rubber band represents a line whose slope is $\frac{-1}{4}$.

Try It 2.7.11

Model the slope $m = -2 = \frac{-2}{1}$. Draw a picture to show your results.

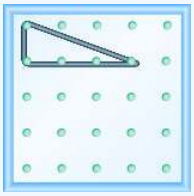
Answer



Try It 2.7.12

Model the slope $m = \frac{-1}{3}$. Draw a picture to show your results.

Answer



Use $m = \frac{\text{rise}}{\text{run}}$ to Find the Slope of a Line from its Graph

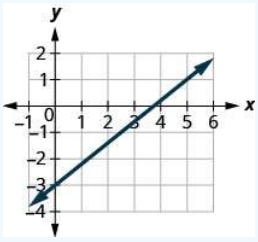
Now, we'll look at some graphs on the xy-coordinate plane and see how to find their slopes. The method will be very similar to what we just modeled on our geoboards.

To find the slope, we must count out the rise and the run. But where do we start?

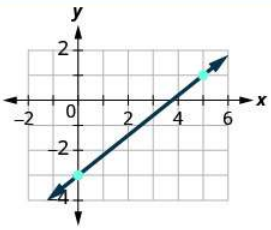
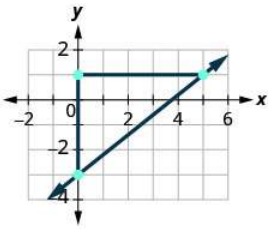
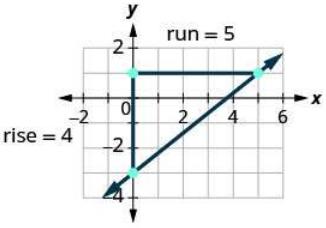
We locate two points on the line whose coordinates are integers. We then start with the point on the left and sketch a right triangle, so we can count the rise and run.

Example 2.7.13:

Find the slope of the line shown.

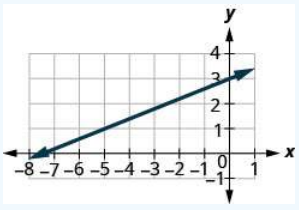


Solution

Step 1. Locate two points on the graph whose coordinates are integers.	Mark (0, –3) and (5, 1).	
Step 2. Starting with the point on the left, sketch a right triangle, going from the first point to the second point.	Starting at (0, –3), sketch a right triangle to (5, 1).	
Step 3. Count the rise and the run on the legs of the triangle.	Count the rise. Count the run.	 The rise is 4. The run is 5.
Step 4. Take the ratio of rise to run to find the slope. $m = \frac{\text{rise}}{\text{run}}$	Use the slope formula. Substitute the values of the rise and run.	$m = \frac{\text{rise}}{\text{run}}$ $m = \frac{4}{5}$ The slope of the line is $\frac{4}{5}$. This means that y increases 4 units as x increases 5 units.

Try It 2.7.14

Find the slope of the line shown.

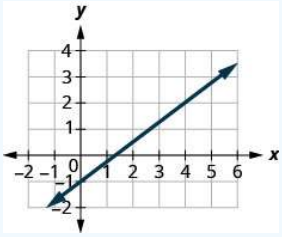


Answer

$\frac{2}{5}$

Try It 2.7.15

Find the slope of the line shown.



Answer

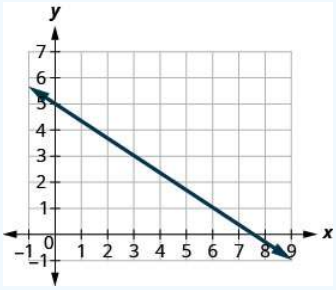
$\frac{3}{4}$

FIND THE SLOPE OF A LINE FROM ITS GRAPH

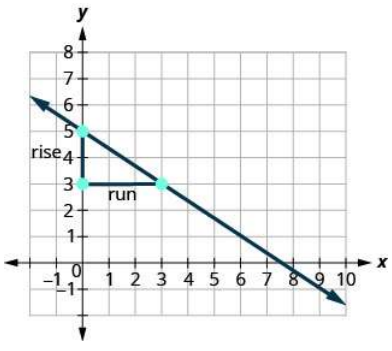
1. Locate two points on the line whose coordinates are integers.
2. Starting with the point on the left, sketch a right triangle, going from the first point to the second point.
3. Count the rise and the run on the legs of the triangle.
4. Take the ratio of rise to run to find the slope, $m = \frac{\text{rise}}{\text{run}}$.

Example 2.7.16

Find the slope of the line shown.



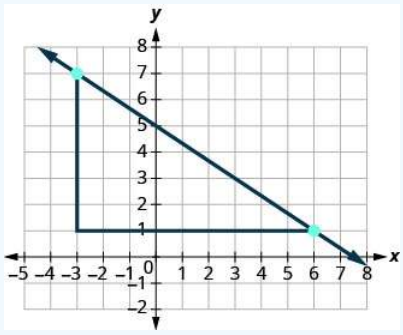
Solution

Locate two points on the graph whose coordinates are integers.	(0,5) and (3,3)
Which point is on the left?	(0,5)
Starting at (0,5), sketch a right triangle to (3,3).	
Count the rise—it is negative.	The rise is −2.
Count the run.	The run is 3.

Use the slope formula.	$m = \frac{\text{rise}}{\text{run}}$
Substitute the values of the rise and run.	$m = \frac{-2}{3}$
Simplify.	$m = -\frac{2}{3}$
	The slope of the line is $-\frac{2}{3}$.

So y increases by 3 units as x decreases by 2 units.

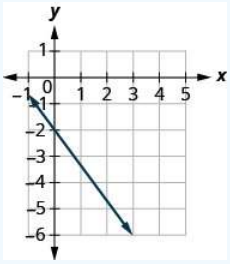
What if we used the points (−3,7) and (6,1) to find the slope of the line?



The rise would be −6 and the run would be 9. Then $m = \frac{-6}{9}$, and that simplifies to $m = -\frac{2}{3}$. Remember, it does not matter which points you use—the slope of the line is always the same.

Try It 2.7.17

Find the slope of the line shown.

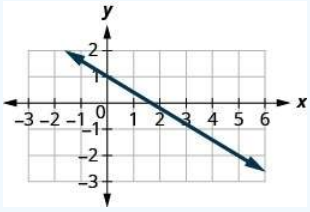


Answer

$-\frac{4}{3}$

Try It 2.7.18

Find the slope of the line shown.



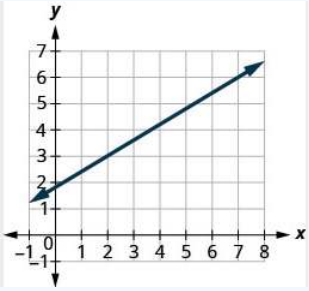
Answer

$-\frac{3}{5}$

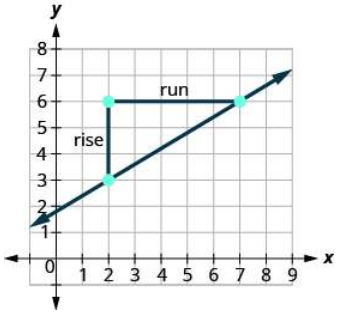
In the last two examples, the lines had *y*-intercepts with integer values, so it was convenient to use the *y*-intercept as one of the points to find the slope. In the next example, the *y*-intercept is a fraction. Instead of using that point, we'll look for two other points whose coordinates are integers. This will make the slope calculations easier.

Example 2.7.19

Find the slope of the line shown.



Solution

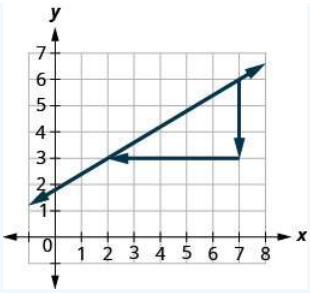
Locate two points on the graph whose coordinates are integers.	(2,3) and (7,6)
Which point is on the left?	(2,3)
Starting at (2,3), sketch a right triangle to (7,6).	
Count the rise.	The rise is 3.
Count the run.	The run is 5.
Use the slope formula.	$m = \frac{\text{rise}}{\text{run}}$
Substitute the values of the rise and run.	$m = \frac{3}{5}$
	The slope of the line is $\frac{3}{5}$.

This means that y increases 5 units as x increases 3 units.

When we used geoboards to introduce the concept of slope, we said that we would always start with the point on the left and count the rise and the run to get to the point on the right. That way the run was always positive and the rise determined whether the slope was positive or negative.

What would happen if we started with the point on the right?

Let's use the points (2,3) and (7,6) again, but now we'll start at (7,6).

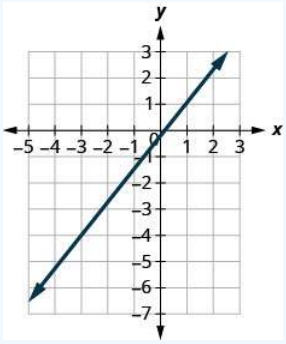


Count the rise.	The rise is -3.
Count the run. It goes from right to left, so it is negative.	The run is -5.
Use the slope formula.	$m = \frac{\text{rise}}{\text{run}}$
Substitute the values of the rise and run.	$m = \frac{-3}{-5}$
	The slope of the line is $\frac{3}{5}$

It does not matter where you start—the slope of the line is always the same.

Try It 2.7.20

Find the slope of the line shown.

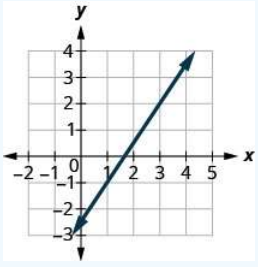


Answer

$\frac{5}{4}$

Try It 2.7.21

Find the slope of the line shown.



Answer

$\frac{3}{2}$

Find the Slope of Horizontal and Vertical Lines

Do you remember what was special about horizontal and vertical lines? Their equations had just one variable.

Horizontal line $y = b$	**Vertical line** $x = a$
y -coordinates are the same.	x -coordinates are the same.

(2.7.3)

So how do we find the slope of the horizontal line y=4? One approach would be to graph the horizontal line, find two points on it, and count the rise and the run. Let's see what happens when we do this.

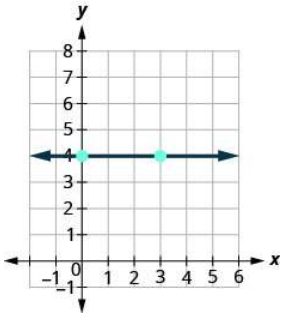


Figure 2.7.7

What is the rise?	The rise is 0.
What is the run?	The run is 3.
	$m = \frac{\text{rise}}{\text{run}}$
	$m = \frac{0}{3}$
What is the slope?	$m = 0$
	The slope of the horizontal line y = 4 is 0.

All horizontal lines have slope 0. When the *y*-coordinates are the same, the rise is 0.

SLOPE OF A HORIZONTAL LINE

The slope of a horizontal line, y=b, is 0.

The floor of your room is horizontal. Its slope is 0. If you carefully placed a ball on the floor, it would not roll away.

Now, we'll consider a vertical line, the line.

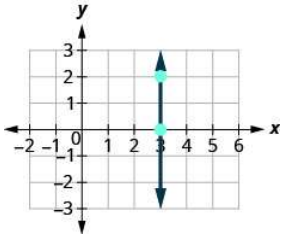


Figure 2.7.8

What is the rise?	The rise is 2.
What is the run?	The run is 0.
	$m = \frac{\text{rise}}{\text{run}}$
What is the slope?	$m = \frac{2}{0}$

But we can't divide by 0. Division by 0 is not defined. So we say that the slope of the vertical line x=3 is undefined.

The slope of any vertical line is undefined. When the *x*-coordinates of a line are all the same, the run is 0.

SLOPE OF A VERTICAL LINE

The slope of a vertical line, x=a, is undefined.

Example 2.7.22

Find the slope of each line:

(a) x=8 (b) y=−5.

Solution

(a) x=8
This is a vertical line.
Its slope is undefined.

(b) y=−5
This is a horizontal line.
It has slope 0.

Try It 2.7.23

Find the slope of the line: x=−4.

Answer

undefined

Try It 2.7.24

Find the slope of the line: y=7.

Answer

0

QUICK GUIDE TO THE SLOPES OF LINES

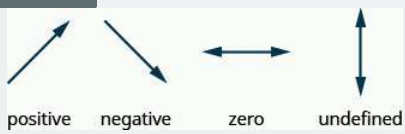


Figure 2.7.9

Remember, we ‘read’ a line from left to right, just like we read written words in English.

Use the Slope Formula to find the Slope of a Line Between Two Points

Doing the Manipulative Mathematics activity “Slope of Lines Between Two Points” will help you develop a better understanding of how to find the slope of a line between two points.

Sometimes we’ll need to find the slope of a line between two points when we don’t have a graph to count out the rise and the run. We could plot the points on grid paper, then count out the rise and the run, but as we’ll see, there is a way to find the slope without graphing. Before we get to it, we need to introduce some algebraic notation.

We have seen that an ordered pair (x,y) gives the coordinates of a point. But when we work with slopes, we use two points. How can the same symbol (x,y) be used to represent two different points? Mathematicians use subscripts to distinguish the points.

$$\begin{aligned}(x_1, y_1) &\quad \text{read } `x \text{ sub } 1, y \text{ sub } 1'\\ (x_2, y_2) &\quad \text{read } `x \text{ sub } 2, y \text{ sub } 2'\end{aligned} \tag{2.7.4}$$

The use of subscripts in math is very much like the use of last name initials in elementary school. Maybe you remember Laura C. and Laura M. in your third grade class?

We will use (x_1, y_1) to identify the first point and (x_2, y_2) to identify the second point.

If we had more than two points, we could use (x_3, y_3), (x_4, y_4), and so on.

Let's see how the rise and run relate to the coordinates of the two points by taking another look at the slope of the line between the points (2,3) and (7,6).

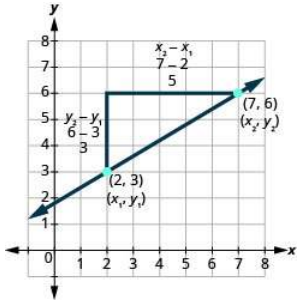


Figure 2.7.10

Since we have two points, we will use subscript notation, $\begin{pmatrix} x_1, y_1 \\ 2, 3 \end{pmatrix} \begin{pmatrix} x_2, y_2 \\ 7, 6 \end{pmatrix}$.

On the graph, we counted the rise of 3 and the run of 5.

Notice that the rise of 3 can be found by subtracting the *y*-coordinates 6 and 3.

$$3 = 6 - 3 \tag{2.7.5}$$

And the run of 5 can be found by subtracting the *x*-coordinates 7 and 2.

$$5 = 7 - 2 \tag{2.7.6}$$

We know $m = \frac{\text{rise}}{\text{run}}$. So $m = \frac{3}{5}$.

We rewrite the rise and run by putting in the coordinates $m = \frac{6-3}{7-2}$

But 6 is y_2, the *y*-coordinate of the second point and 3 is y_1, the *y*-coordinate of the first point.

So we can rewrite the slope using subscript notation. $m = \frac{y2-y1}{7-2}$

Also, 7 is x_2, the *x*-coordinate of the second point and 2 is x_1, the *x*-coordinate of the first point.

So, again, we rewrite the slope using subscript notation. $m = \frac{y2-y1}{x2-x1}$

We've shown that $m = \frac{y2-y1}{x2-x1}$ is really another version of $m = \frac{\text{rise}}{\text{run}}$. We can use this formula to find the slope of a line when we have two points on the line.

SLOPE FORMULA

The slope of the line between two points (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1} \tag{2.7.7}$$

This is the **slope formula**.

The slope is:

$$\begin{gathered} y \text{ of the second point minus } y \text{ of the first point} \\ \text{over} \\ x \text{ of the second point minus } x \text{ of the first point.} \end{gathered} \tag{2.7.8}$$

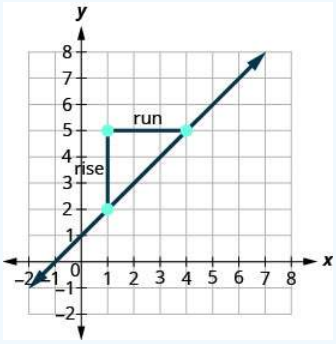
Example 2.7.25

Use the **slope formula** to find the slope of the line between the points (1,2) and (4,5).

Solution

We'll call (1,2) point #1 and (4,5) point #2.	$\begin{pmatrix} x_1, y_1 \\ 1, 2 \end{pmatrix} \begin{pmatrix} x_2, y_2 \\ 4, 5 \end{pmatrix}$
Use the slope formula.	$m = \frac{y_2 - y_1}{x_2 - x_1}$
Substitute the values.	
y of the second point minus y of the first point	$m = \frac{5-2}{x_2 - x_1}$
x of the second point minus x of the first point	$m = \frac{5-2}{4-1}$
Simplify the numerator and the denominator.	$m = \frac{3}{3}$
Simplify.	$m = 1$

Let's confirm this by counting out the slope on a graph using $m = \frac{\text{rise}}{\text{run}}$.



It doesn't matter which point you call point #1 and which one you call point #2. The slope will be the same. Try the calculation yourself.

Try It 2.7.26

Use the slope formula to find the slope of the line through the points: (8,5) and (6,3).

Answer

1

Try It 2.7.27

Use the slope formula to find the slope of the line through the points: (1,5) and (5,9).

Answer

1

Example 2.7.28

Use the slope formula to find the slope of the line through the points (−2,−3) and (−7,4).

Solution

We'll call (-2, -3) point #1 and (-7,4) point #2. $\begin{pmatrix} x_1, y_1 \\ -2, -3 \end{pmatrix} \begin{pmatrix} x_2, y_2 \\ -7, 4 \end{pmatrix}$

Use the slope formula. $m = \frac{y_2 - y_1}{x_2 - x_1}$

Substitute the values.

y of the second point minus y of the first point $m = \frac{4-(-3)}{x_2 - x_1}$

x of the second point minus x of the first point $m = \frac{4-(-3)}{-7-(-2)}$

Simplify the numerator and the denominator. $m = \frac{7}{-5}$

Simplify. $m = -\frac{7}{5}$

Let's verify this slope on the graph shown.

$$m = \frac{\text{rise}}{\text{run}}$$
$$m = \frac{-7}{5}$$
$$m = -\frac{7}{5}$$

Try It 2.7.29

Use the slope formula to find the slope of the line through the points: (−3,4) and (2,−1).

Answer

-1

Try It 2.7.30

Use the slope formula to find the slope of the line through the pair of points: (−2,6) and (−3,−4).

Answer

10

Graph a Line Given a Point and the Slope

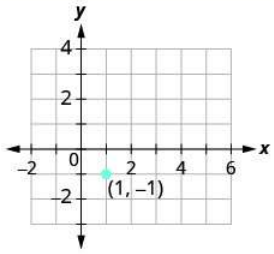
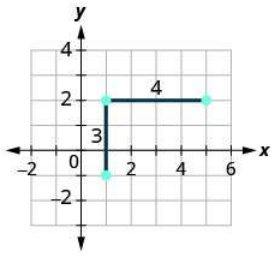
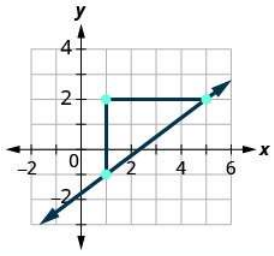
Up to now, in this chapter, we have graphed lines by plotting points, by using intercepts, and by recognizing horizontal and vertical lines.

One other method we can use to graph lines is called the **point–slope method**. We will use this method when we know one point and the slope of the line. We will start by plotting the point and then use the definition of slope to draw the graph of the line.

Example 2.7.31

Graph the line passing through the point (1,−1) whose slope is $m = \frac{3}{4}$.

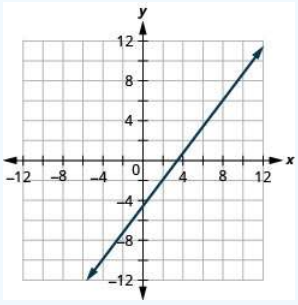
Solution

Step 1. Plot the given point.	Plot (1, –1).	
Step 2. Use the slope formula $m = \frac{\text{rise}}{\text{run}}$ to identify the rise and the run.	Identify the rise and the run.	$m = \frac{3}{4}$ $\frac{\text{rise}}{\text{run}} = \frac{3}{4}$ rise = 3 run = 4
Step 3. Starting at the given point, count out the rise and run to mark the second point.	Start at (1, –1) and count the rise and the run. Up 3 units, right 4 units.	
Step 4. Connect the points with a line.	Connect the two points with a line.	

? Try It 2.7.32

Graph the line passing through the point (2,−2) with the slope $m = \frac{4}{3}$.

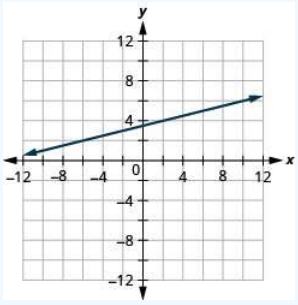
Answer



Try It 2.7.33

Graph the line passing through the point $(-2,3)$ with the slope $m = \frac{1}{4}$.

Answer



GRAPH A LINE GIVEN A POINT AND THE SLOPE.

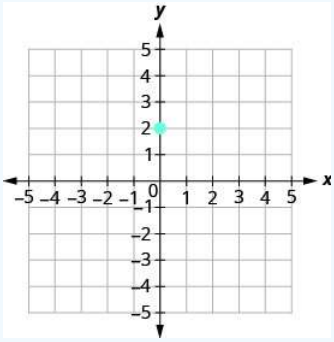
1. Plot the given point.
2. Use the slope formula $m = \frac{\text{rise}}{\text{rise}}$ to identify the rise and the run.
3. Starting at the given point, count out the rise and run to mark the second point.
4. Connect the points with a line.

Example 2.7.34

Graph the line with *y*-intercept 2 whose slope is $m = -\frac{2}{3}$.

Solution

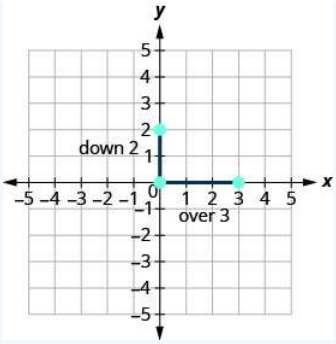
Plot the given point, the *y*-intercept, (0,2).



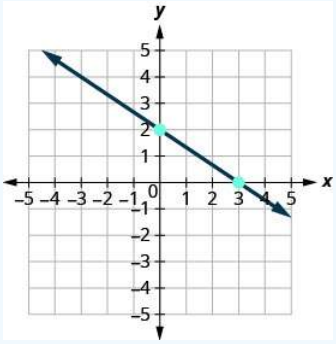
Identify the rise and the run.

$$m = -\frac{2}{3}$$
$$\frac{\text{rise}}{\text{run}} = \frac{-2}{3}$$
$$\text{rise} = -2$$
$$\text{run} = 3$$

Count the rise and the run. Mark the second point.



Connect the two points with a line.

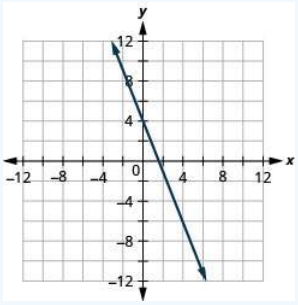


You can check your work by finding a third point. Since the slope is $m = -\frac{2}{3}$, it can be written as $m = \frac{2}{-3}$. Go back to (0,2) and count out the rise, 2, and the run, −3.

Try It 2.7.35

Graph the line with the *y*-intercept 4 and slope $m = -\frac{5}{2}$.

Answer

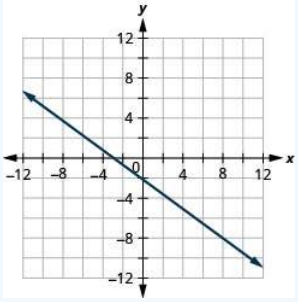


Try It 2.7.36

Graph the line with the *x*-intercept −3 and slope $m = -\frac{3}{4}$.

Answer

https://math.libretexts.org/@go/page/56717

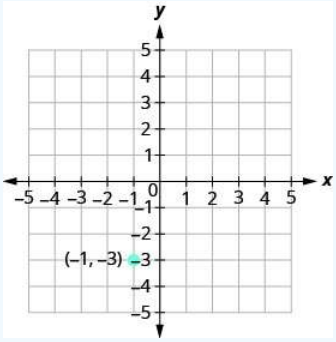


Example 2.7.37

Graph the line passing through the point $(-1,-3)$ whose slope is $m=4$.

Solution

Plot the given point.

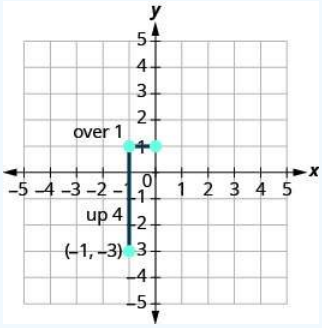


Identify the rise and the run. $m = 4$

Write 4 as a fraction. $\frac{\text{rise}}{\text{run}} = \frac{4}{1}$

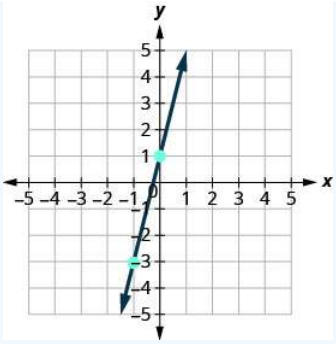
$\text{rise} = 4 \quad \text{run} = 1$

Count the rise and run and mark the second point.



Connect the two points with a line.

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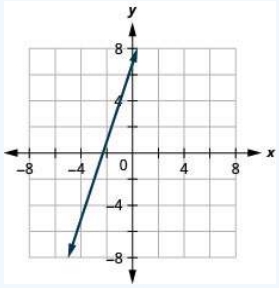


You can check your work by finding a third point. Since the slope is m=4, it can be written as $m = \frac{-4}{-1}$. Go back to (−1,−3) and count out the rise, −4, and the run, −1.

Try It 2.7.38

Graph the line with the point (−2,1) and slope m=3.

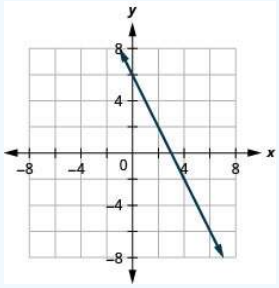
Answer



Try It 2.7.39

Graph the line with the point (4,−2) and slope m=−2.

Answer

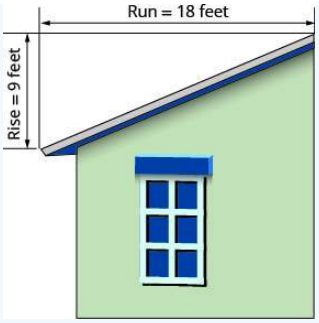


Solve Slope Applications

At the beginning of this section, we said there are many applications of slope in the real world. Let's look at a few now.

Example 2.7.40

The 'pitch' of a building's roof is the slope of the roof. Knowing the pitch is important in climates where there is heavy snowfall. If the roof is too flat, the weight of the snow may cause it to collapse. What is the slope of the roof shown?



Solution

Use the slope formula.	$m = \frac{\text{rise}}{\text{rise}}$
Substitute the values for rise and run.	$m = \frac{9}{18}$
Simplify.	$m = \frac{1}{2}$

The slope of the roof is $\frac{1}{2}$.

The roof rises 1 foot for every 2 feet of horizontal run.

Try It 2.7.41

Use Exercise 2.7.40, substituting the rise = 14 and run = 24.

Answer

$\frac{7}{12}$

Try It 2.7.42

Use Exercise 2.7.40, substituting rise = 15 and run = 36.

Answer

$\frac{5}{12}$

Example 2.7.43

Have you ever thought about the sewage pipes going from your house to the street? They must slope down $\frac{1}{4}$ inch per foot in order to drain properly. What is the required slope?



Solution

Use the slope formula.	$m = \frac{\text{rise}}{\text{run}}$
	$m = \frac{-\frac{1}{4}\text{inch}}{1\text{ foot}}$
	$m = \frac{-\frac{1}{4}\text{ inch}}{12\text{ inches}}$
Simplify.	$m = -\frac{1}{48}$

The slope of the pipe is $-\frac{1}{48}$

The pipe drops 1 inch for every 48 inches of horizontal run.

? Try It 2.7.44

Find the slope of a pipe that slopes down $\frac{1}{3}$ inch per foot.

Answer

$-\frac{1}{36}$

? Try It 2.7.45

Find the slope of a pipe that slopes down $\frac{3}{4}$ inch per yard.

Answer

$-\frac{1}{48}$

Access these online resources for additional instruction and practice with understanding slope of a line.

- Practice Slope with a Virtual Geoboard
- Small, Medium, and Large Virtual Geoboards
- Explore Area and Perimeter with a Geoboard

Key Concepts

- **Find the Slope of a Line from its Graph using $m = \frac{\text{rise}}{\text{run}}$**
  1. Locate two points on the line whose coordinates are integers.
  2. Starting with the point on the left, sketch a right triangle, going from the first point to the second point.
  3. Count the rise and the run on the legs of the triangle.
  4. Take the ratio of rise to run to find the slope.
- **Graph a Line Given a Point and the Slope**
  1. Plot the given point.
  2. Use the slope formula $m = \frac{\text{rise}}{\text{run}}$ to identify the rise and the run.
  3. Starting at the given point, count out the rise and run to mark the second point.
  4. Connect the points with a line.
- **Slope of a Horizontal Line**
  - The slope of a horizontal line, y=b, is 0.
- **Slope of a vertical line**
  - The slope of a vertical line, x=a, is undefined

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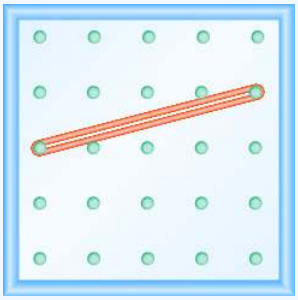
2.7E: Exercises

Practice Makes Perfect

Use Geoboards to Model Slope

In the following exercises, find the slope modeled on each geoboard.

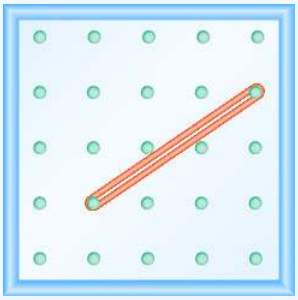
Exercise 2.7E.1



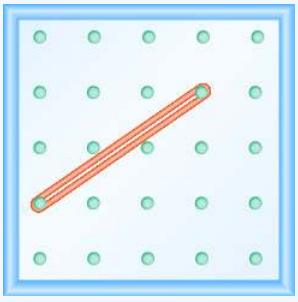
Answer

$\frac{1}{4}$

Exercise 2.7E.2



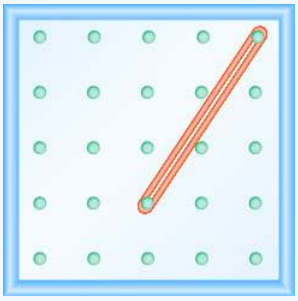
Exercise 2.7E.3



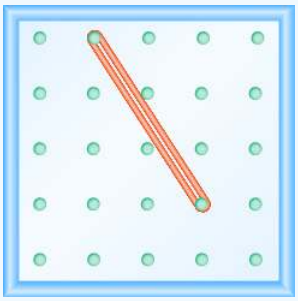
Answer

$\frac{2}{3}$

Exercise 2.7E.4



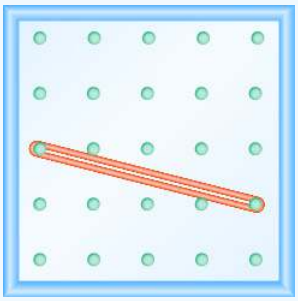
Exercise 2.7E.5



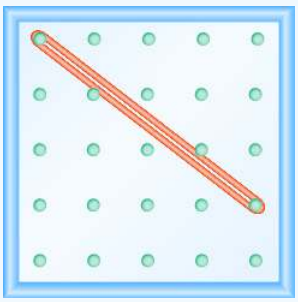
Answer

$\frac{-3}{2} = -\frac{3}{2}$

Exercise 2.7E.6



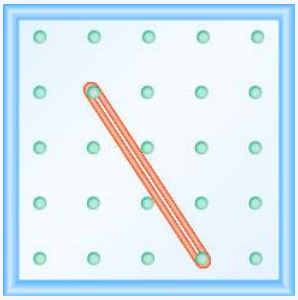
Exercise 2.7E.7



Answer

$-\frac{2}{3}$

Exercise 2.7E.8

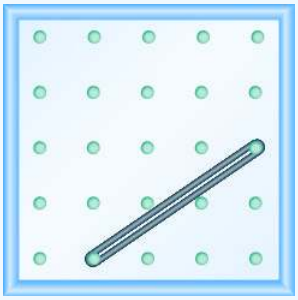


In the following exercises, model each slope. Draw a picture to show your results.

Exercise 2.7E.9

$\frac{2}{3}$

Answer



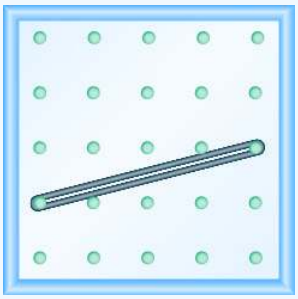
Exercise 2.7E.10

$\frac{3}{4}$

Exercise 2.7E.11

$\frac{1}{4}$

Answer



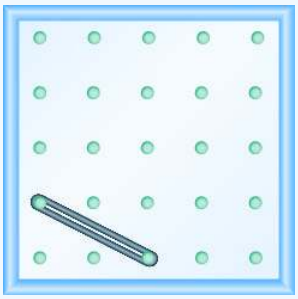
Exercise 2.7E.12

$\frac{4}{3}$

Exercise 2.7E.13

$-\frac{1}{2}$

Answer



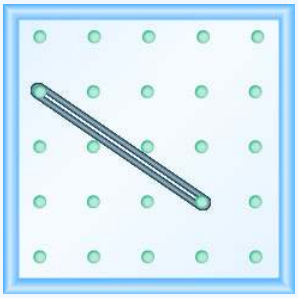
Exercise 2.7E.14

$-\frac{3}{4}$

Exercise 2.7E.15

$-\frac{2}{3}$

Answer



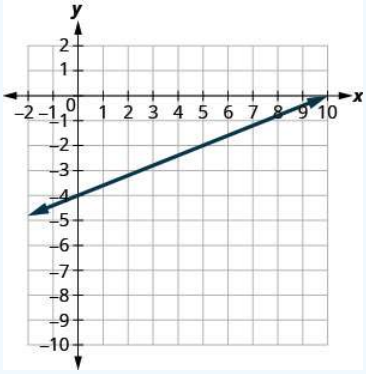
Exercise 2.7E.16

$-\frac{3}{2}$

Use $m = \frac{rise}{run}$ to find the Slope of a Line from its Graph

In the following exercises, find the slope of each line shown.

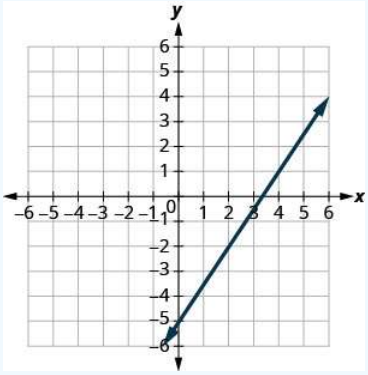
Exercise 2.7E.17



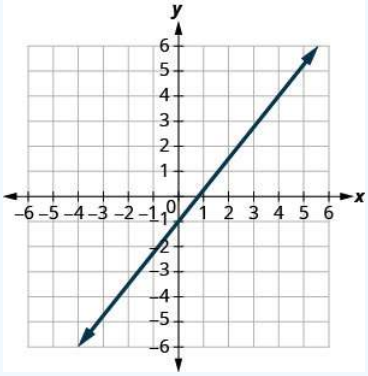
Answer

$\frac{2}{5}$

Exercise 2.7E.18



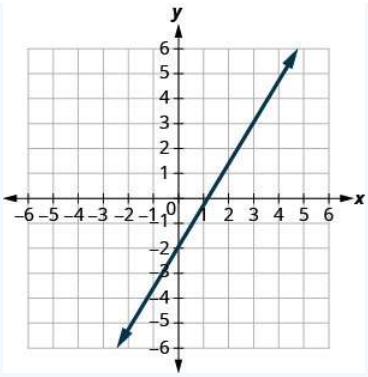
Exercise 2.7E.19



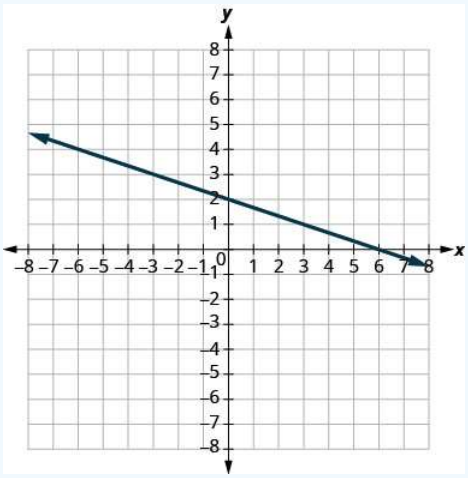
Answer

$\frac{5}{4}$

Exercise 2.7E.20



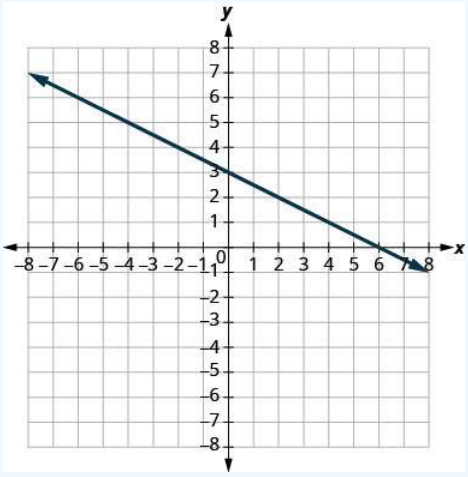
? Exercise 2.7E. 21



Answer

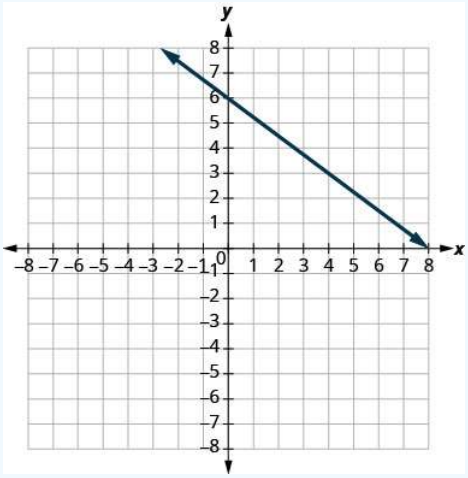
$-\frac{1}{3}$

? Exercise 2.7E. 22



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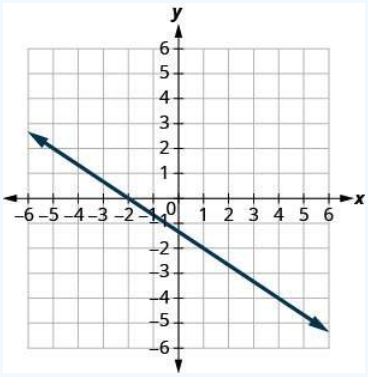
Exercise $2.7E.23$



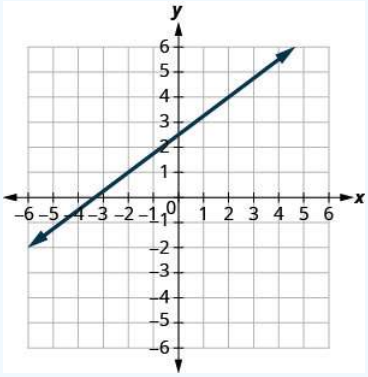
Answer

$-\frac{3}{4}$

Exercise $2.7E.24$



Exercise $2.7E.25$

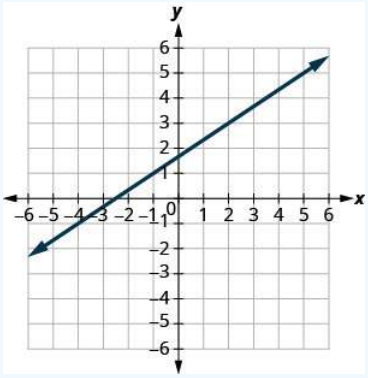


Answer

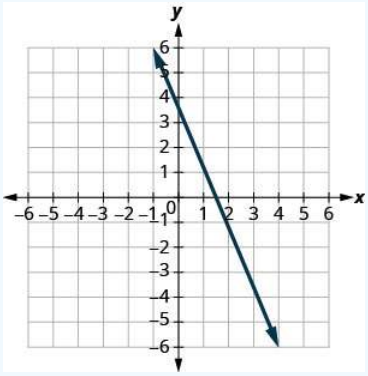
$\frac{3}{4}$

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Exercise $2.7E.26$



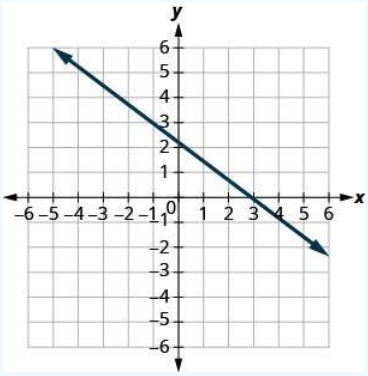
Exercise $2.7E.27$



Answer

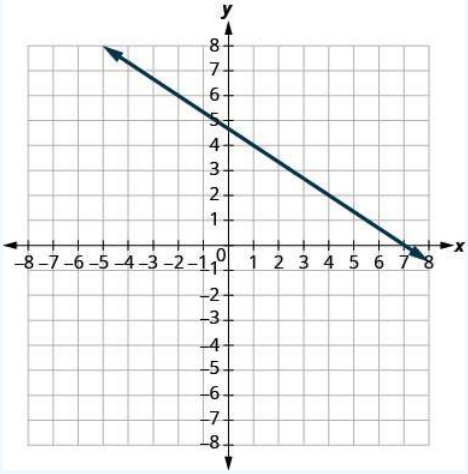
$-\frac{5}{2}$

Exercise $2.7E.28$



Exercise $2.7E.29$

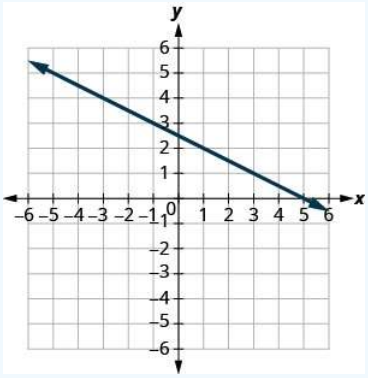
https://math.libretexts.org/@go/page/56718



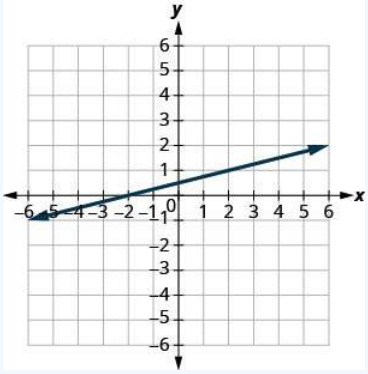
Answer

$-\frac{2}{3}$

Exercise 2.7E.30



Exercise 2.7E.31

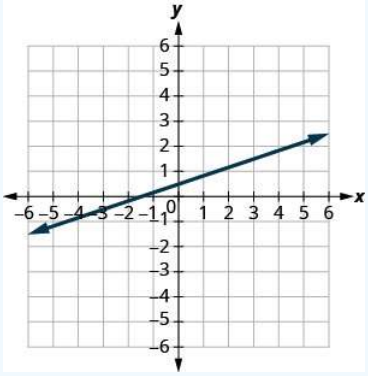


Answer

$\frac{1}{4}$

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Exercise 2.7E.32



Find the Slope of Horizontal and Vertical Lines

In the following exercises, find the slope of each line.

Exercise 2.7E.33

y=3

Answer

0

Exercise 2.7E.34

y=1

Exercise 2.7E.35

x=4

Answer

undefined

Exercise 2.7E.36

x=2

Exercise 2.7E.37

y=−2

Answer

0

Exercise 2.7E.38

y=−3

Exercise 2.7E.39

x=−5

Answer

undefined

Exercise 2.7E.40

x=−4

Use the Slope Formula to find the Slope of a Line between Two Points

In the following exercises, use the slope formula to find the slope of the line between each pair of points.

Exercise 2.7E.41

(1,4),(3,9)

Answer

$\frac{5}{2}$

Exercise 2.7E.42

(2,3),(5,7)

Exercise 2.7E.43

(0,3),(4,6)

Answer

$\frac{3}{4}$

Exercise 2.7E.44

(0,1),(5,4)

Exercise 2.7E.45

(2,5),(4,0)

Answer

$-\frac{5}{2}$

Exercise 2.7E.46

(3,6),(8,0)

Exercise 2.7E.47

(−3,3),(4,−5)

Answer

$-\frac{8}{7}$

Exercise 2.7E.48

$(-2,4),(3,-1)$

Exercise 2.7E.49

$(-1,-2),(2,5)$

Answer

$\frac{7}{3}$

Exercise 2.7E.50

$(-2,-1),(6,5)$

Exercise 2.7E.51

$(4,-5),(1,-2)$

Answer

-1

Exercise 2.7E.52

$(3,-6),(2,-2)$

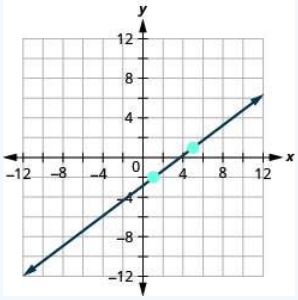
Graph a Line Given a Point and the Slope

In the following exercises, graph each line with the given point and slope.

Exercise 2.7E.53

$(1,-2); m=\frac{3}{4}$

Answer



Exercise 2.7E.54

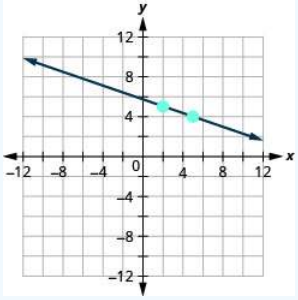
$(1,-1); m=\frac{2}{3}$

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Exercise 2.7E.55

$(2,5); m = -\frac{1}{3}$

Answer



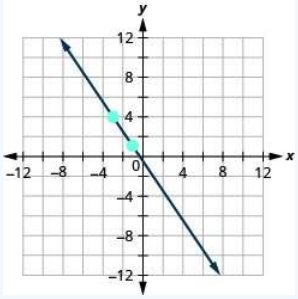
Exercise 2.7E.56

$(1,4); m = -\frac{1}{2}$

Exercise 2.7E.57

$(-3,4); m = -\frac{3}{2}$

Answer



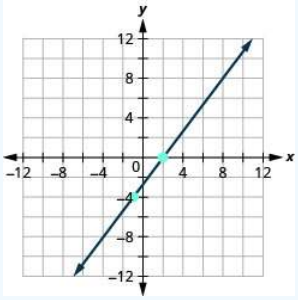
Exercise 2.7E.58

$(-2,5); m = -\frac{5}{4}$

Exercise 2.7E.59

$(-1,-4); m = \frac{4}{3}$

Answer



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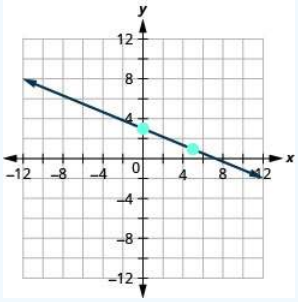
Exercise 2.7E.60

$(-3, -5); m = \frac{3}{2}$

Exercise 2.7E.61

y -intercept $3; m = -\frac{2}{5}$

Answer



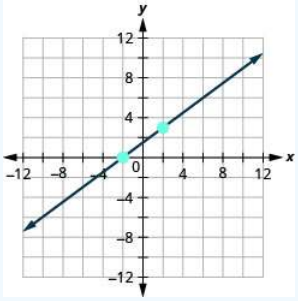
Exercise 2.7E.62

y -intercept $5; m = -\frac{4}{3}$

Exercise 2.7E.63

x -intercept $-2; m = \frac{3}{4}$

Answer



Exercise 2.7E.64

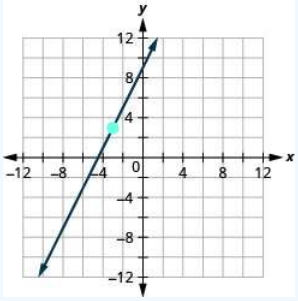
x -intercept $-1; m = \frac{1}{5}$

Exercise 2.7E.65

$(-3, 3); m = 2$

Answer

https://math.libretexts.org/@go/page/56718



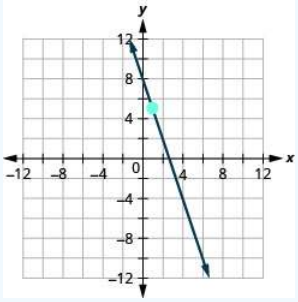
Exercise 2.7E.66

$(-4, 2); m = 4$

Exercise 2.7E.67

$(1, 5); m = -3$

Answer



Exercise 2.7E.68

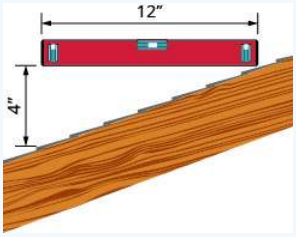
$(2, 3); m = -1$

Everyday Math

Exercise 2.7E.69

Slope of a roof. An easy way to determine the slope of a roof is to set one end of a 12 inch level on the roof surface and hold it level. Then take a tape measure or ruler and measure from the other end of the level down to the roof surface. This will give you the slope of the roof. Builders, sometimes, refer to this as pitch and state it as an "x 12 pitch" meaning $\frac{x}{12}$, where x is the measurement from the roof to the level—the rise. It is also sometimes stated as an "x-in-12 pitch".

1. What is the slope of the roof in this picture?
2. What is the pitch in construction terms?

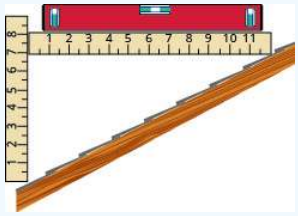


Answer

1. $\frac{1}{3}$
2. 4 12 pitch or 4-in-12 pitch

Exercise 2.7E.70

The slope of the roof shown here is measured with a 12” level and a ruler. What is the slope of this roof?



Exercise 2.7E.71

Road grade. A local road has a grade of 6%. The grade of a road is its slope expressed as a percent. Find the slope of the road as a fraction and then simplify. What rise and run would reflect this slope or grade?

Answer

$\frac{3}{50}$; rise $= 3$, run $= 50$

Exercise 2.7E.72

Highway grade. A local road rises 2 feet for every 50 feet of highway.

1. What is the slope of the highway?
2. The grade of a highway is its slope expressed as a percent. What is the grade of this highway?

Exercise 2.7E.73

Wheelchair ramp. The rules for wheelchair ramps require a maximum 1-inch rise for a 12-inch run.

1. How long must the ramp be to accommodate a 24-inch rise to the door?
2. Create a model of this ramp.

Answer

1. 288 inches (24 feet)
2. Models will vary.

Exercise 2.7E.74

Wheelchair ramp. A 1-inch rise for a 16-inch run makes it easier for the wheelchair rider to ascend a ramp.

1. How long must a ramp be to easily accommodate a 24-inch rise to the door?
2. Create a model of this ramp.

Writing Exercises

Exercise 2.7E.75

What does the sign of the slope tell you about a line?

Answer

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When the slope is a positive number the line goes up from left to right. When the slope is a negative number the line goes down from left to right.

Exercise 2.7E.76

How does the graph of a line with slope $m = \frac{1}{2}$ differ from the graph of a line with slope $m = 2$?

Exercise 2.7E.77

Why is the slope of a vertical line "undefined"?

Answer

A vertical line has 0 run and since division by 0 is undefined the slope is undefined.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it
use geoboards to model slope.			
use $m = \frac{rise}{run}$ to find the slope of a line from its graph.			
find the slope of horizontal and vertical lines.			
use the slope formula to find the slope of a line between two points.			
graph a line given a point and the slope.			
solve slope applications.			

ⓑ On a scale of 1–10, how would you rate your mastery of this section in light of your responses on the checklist? How can you improve this?

Glossary

geoboard

A geoboard is a board with a grid of pegs on it.

negative slope

A negative slope of a line goes down as you read from left to right.

positive slope

A positive slope of a line goes up as you read from left to right.

rise

The rise of a line is its vertical change.

run

The run of a line is its horizontal change.

slope formula

The slope of the line between two points (x_1, y_1) and (x_2, y_2) is $m = \frac{y_2 - y_1}{x_2 - x_1}$.

slope of a line

The slope of a line is $m = \frac{\text{rise}}{\text{run}}$. The rise measures the vertical change and the run measures the horizontal change.

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2.8: Use the Slope–Intercept Form of an Equation of a Line

Learning Objectives

By the end of this section, you will be able to:

- Recognize the relation between the graph and the slope–intercept form of an equation of a line
- Identify the slope and y-intercept form of an equation of a line
- Graph a line using its slope and intercept
- Choose the most convenient method to graph a line
- Graph and interpret applications of slope–intercept
- Use slopes to identify parallel lines
- Use slopes to identify perpendicular lines

Note

Before you get started, take this readiness quiz.

1. Add: $\frac{x}{4}+\frac{1}{4}$.
   If you missed this problem, review Exercise 1.7.1.
2. Find the reciprocal of $\frac{3}{7}$.
   If you missed this problem, review Exercise 1.6.19.
3. Solve $2x-3y=12$ for y.
   If you missed this problem, review Exercise 2.6.16.

Recognize the Relation Between the Graph and the Slope–Intercept Form of an Equation of a Line

We have graphed linear equations by plotting points, using intercepts, recognizing horizontal and vertical lines, and using the point–slope method. Once we see how an equation in slope–intercept form and its graph are related, we'll have one more method we can use to graph lines.

In **Graph Linear Equations in Two Variables**, we graphed the line of the equation $y=12x+3$ by plotting points. See Figure 2.8.1. Let's find the slope of this line.

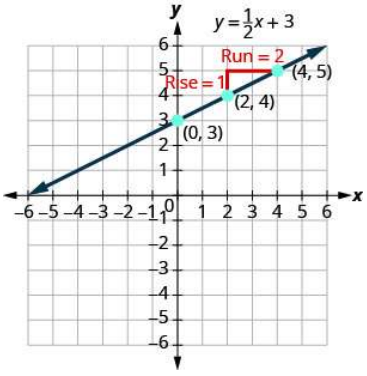


Figure 2.8.1

The red lines show us the rise is 1 and the run is 2. Substituting into the slope formula:

$$m=\frac{\text{rise}}{\text{run}}$$
$$m=\frac{1}{2}$$

What is the y-intercept of the line? The y-intercept is where the line crosses the y-axis, so y-intercept is $(0,3)$. The equation of this line is:

$y = \frac{1}{2}x + 3$

Notice, the line has:

slope $m = \frac{1}{2}$ and y-intercept $(0, 3)$

When a linear equation is solved for y, the coefficient of the x-term is the slope and the constant term is the y-coordinate of the y-intercept. We say that the equation $y = \frac{1}{2}x + 3$ is in slope–intercept form.

$m = \frac{1}{2}$; y-intercept is $(0, 3)$

$y = \frac{1}{2}x + 3$

$y = mx + b$

SLOPE-INTERCEPT FORM OF AN EQUATION OF A LINE

The **slope–intercept form** of an equation of a line with slope m and y-intercept, $(0, b)$ is,

$$y = mx + b \tag{2.8.1}$$

Sometimes the slope–intercept form is called the "y-form."

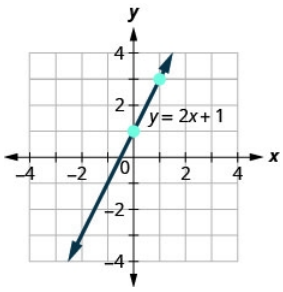
Example 2.8.1

Use the graph to find the slope and y-intercept of the line, $y = 2x + 1$.

Compare these values to the equation $y = mx + b$.

Solution

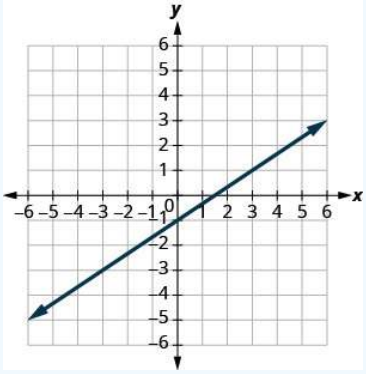
To find the slope of the line, we need to choose two points on the line. We'll use the points $(0, 1)$ and $(1, 3)$.

	
Find the rise and run.	$m = \frac{\text{rise}}{\text{run}}$
	$m = \frac{2}{1}$
	$m = 2$
Find the y-intercept of the line.	The y-intercept is the point $(0, 1)$.
We found slope $m = 2$ and y-intercept $(0, 1)$.	$y = 2x + 1$ $y = mx + b$

The slope is the same as the coefficient of x and the y-coordinate of the y-intercept is the same as the constant term.

Try It 2.8.2

Use the graph to find the slope and y-intercept of the line $y = \frac{2}{3}x - 1$. Compare these values to the equation $y = mx + b$.

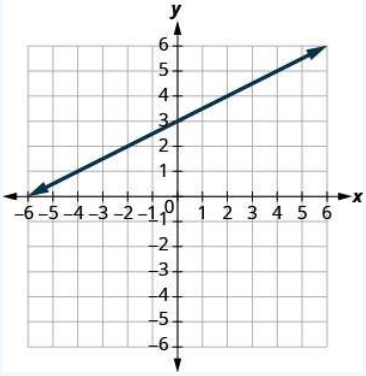


Answer

slope $m = \frac{2}{3}$ and y-intercept $(0, -1)$

Try It 2.8.3

Use the graph to find the slope and y-intercept of the line $y = \frac{1}{2}x + 3$. Compare these values to the equation $y = mx + b$.



Answer

slope $m = \frac{1}{2}$ and y-intercept $(0, 3)$

Identify the Slope and y-Intercept From an Equation of a Line

In **Understand Slope of a Line**, we graphed a line using the slope and a point. When we are given an equation in slope–intercept form, we can use the y-intercept as the point, and then count out the slope from there. Let's practice finding the values of the slope and y-intercept from the equation of a line.

Example 2.8.4

Identify the slope and y-intercept of the line with equation $y = -3x + 5$.

Solution

We compare our equation to the slope–intercept form of the equation.

	$y = mx + b$
Write the equation of the line.	$y = -3x + 5$
Identify the slope.	$m = -3$
Identify the y-intercept.	y-intercept is $(0, 5)$

Try It 2.8.5

Identify the slope and y-intercept of the line $y = \frac{2}{5}x - 1$.

Answer

slope $\frac{2}{5}$; y-intercept $(0,-1)$

Try It 2.8.6

Identify the slope and y-intercept of the line $y = -\frac{4}{3}x + 1$.

Answer

slope $-\frac{4}{3}$; y-intercept $(0,1)$

When an equation of a line is not given in slope–intercept form, our first step will be to solve the equation for y.

Example 2.8.7

Identify the slope and y-intercept of the line with equation $x + 2y = 6$.

Solution

This equation is not in slope–intercept form. In order to compare it to the slope–intercept form we must first solve the equation for y.

Solve for y.	$x + 2y = 6$
Subtract x from each side.	$2y = -x + 6$
Divide both sides by 2.	$\frac{2y}{2} = \frac{-x+6}{2}$
Simplify.	$\frac{2y}{2} = \frac{-x}{2} + \frac{6}{2}$
(Remember: $\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$)	
Simplify.	$y = -\frac{1}{2}x + 3$
Write the slope–intercept form of the equation of the line.	$y = mx + b$
Write the equation of the line.	$y = -\frac{1}{2}x + 3$
Identify the slope.	$m = -\frac{1}{2}$
Identify the y-intercept.	y-intercept is $(0, 3)$

Try It 2.8.8

Identify the slope and y-intercept of the line $x + 4y = 8$.

Answer

slope $-\frac{1}{4}$; y-intercept $(0,2)$

Try It 2.8.9

Identify the slope and y-intercept of the line $3x + 2y = 12$.

Answer

slope $-\frac{3}{2}$; y-intercept (0,6)

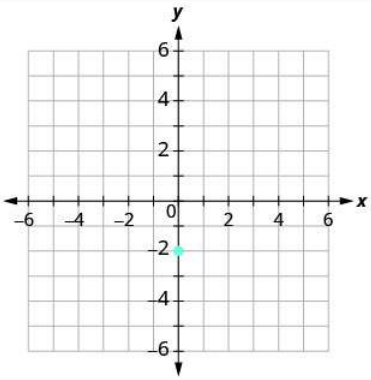
Graph a Line Using its Slope and Intercept

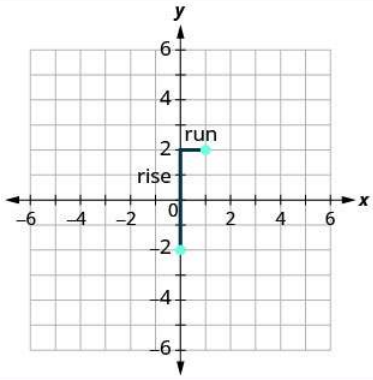
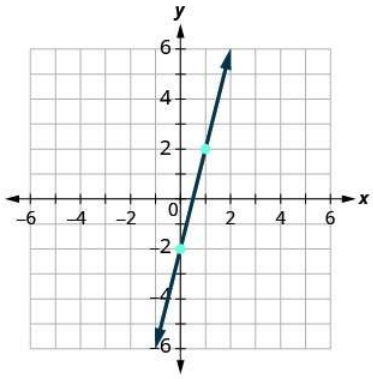
Now that we know how to find the slope and y-intercept of a line from its equation, we can graph the line by plotting the y-intercept and then using the slope to find another point.

Example 2.8.10: How to Graph a Line Using its Slope and Intercept

Graph the line of the equation $y = 4x - 2$ using its slope and y-intercept.

Solution

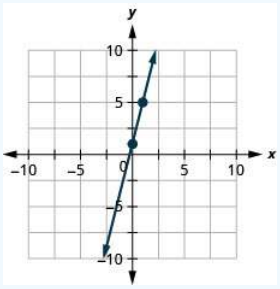
Step 1. Find the slope–intercept form of the equation.	This equation is in slope–intercept form.	$y = 4x - 2$
Step 2. Identify the slope and y-intercept.	Use $y = mx + b$ Find the slope. Find the y-intercept.	$y = mx + b$ $y = 4x + (-2)$ $m = 4$ $b = -2$, $(0, -2)$
Step 3. Plot the y-intercept.	Plot $(0, -2)$.	
Step 4. Use the slope formula $m = \frac{\text{rise}}{\text{run}}$ to identify the rise and the run.	Identify the rise and the run.	$m = 4$ $\frac{\text{rise}}{\text{run}} = \frac{4}{1}$ rise $= 4$ run $= 1$

Step 5. Starting at the *y*-intercept, count out the rise and run to mark the second point.	Start at (0, –2) and count the rise and the run. Up 4, right 1.	
Step 6. Connect the points with a line.	Connect the two points with a line.	

? Try It 2.8.11

Graph the line of the equation $y = 4x + 1$ using its slope and y-intercept.

Answer

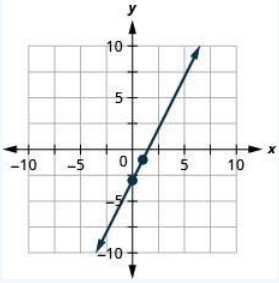


? Try It 2.8.12

Graph the line of the equation $y = 2x - 3$ using its slope and y-intercept.

Answer

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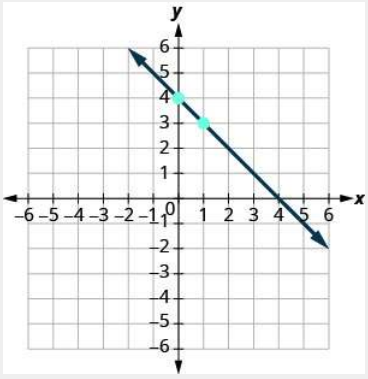
GRAPH A LINE USING ITS SLOPE AND y-INTERCEPT.

1. Find the slope-intercept form of the equation of the line.
2. Identify the slope and y-intercept.
3. Plot the y-intercept.
4. Use the slope formula $\frac{\text{rise}}{\text{run}}$ to identify the rise and the run.
5. Starting at the y-intercept, count out the rise and run to mark the second point.
6. Connect the points with a line.

Example 2.8.13

Graph the line of the equation $y = -x + 4$ using its slope and y-intercept.

Solution

	$y = mx + b$
The equation is in slope–intercept form.	$y = -x + 4$
Identify the slope and y-intercept.	$m = -1$
	y-intercept is $(0, 4)$
Plot the y-intercept.	See graph below.
Identify the rise and the run.	$m = \frac{-1}{1}$
Count out the rise and run to mark the second point.	rise -1, run 1
Draw the line.	
To check your work, you can find another point on the line and make sure it is a solution of the equation. In the graph we see the line goes through $(4, 0)$.	

check.

$y = -x + 4$

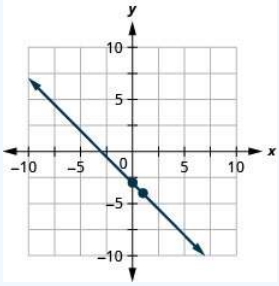
$0 \stackrel{?}{=} -4 + 4$

$0 = 0 \checkmark$

Try It 2.8.14

Graph the line of the equation $y = -x - 3$ using its slope and y-intercept.

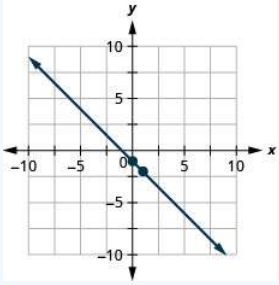
Answer



Try It 2.8.15

Graph the line of the equation $y = -x - 1$ using its slope and y-intercept.

Answer

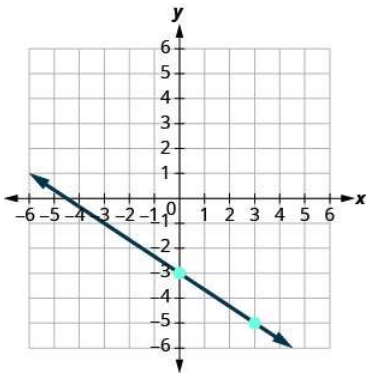


Example 2.8.16

Graph the line of the equation $y = -\frac{2}{3}x - 3$ using its slope and y-intercept.

Solution

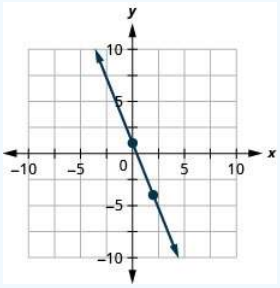
	$y = mx + b$
The equation is in slope–intercept form.	$y = -\frac{2}{3}x - 3$
Identify the slope and y-intercept.	$m = -\frac{2}{3}$; y-intercept is $(0, -3)$
Plot the y-intercept.	See graph below.
Identify the rise and the run.	
Count out the rise and run to mark the second point.	

Draw the line.	

Try It 2.8.17

Graph the line of the equation $y = -\frac{5}{2}x + 1$ using its slope and y-intercept.

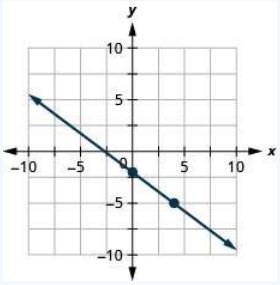
Answer



Try It 2.8.18

Graph the line of the equation $y = -\frac{3}{4}x - 2$ using its slope and y-intercept.

Answer

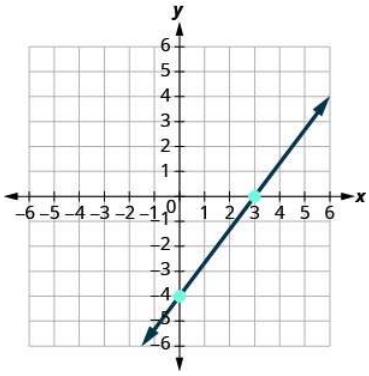


Example 2.8.19

Graph the line of the equation $4x - 3y = 12$ using its slope and y-intercept.

Solution

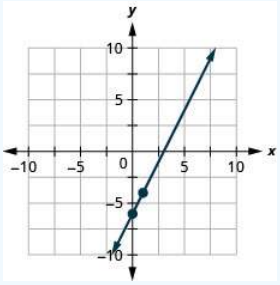
	$4x - 3y = 12$
Find the slope–intercept form of the equation.	$-3y = -4x + 12$
	$-\frac{3y}{3} = \frac{-4x+12}{-3}$

The equation is now in slope–intercept form.	$y = \frac{4}{3}x - 4$
Identify the slope and y-intercept.	$m = \frac{4}{3}$
	y-ntercept is $(0, -4)$
Plot the y-intercept.	See graph below.
Identify the rise and the run; count out the rise and run to mark the second point.	
Draw the line.	

Try It 2.8.20

Graph the line of the equation $2x - y = 6$ using its slope and y-intercept.

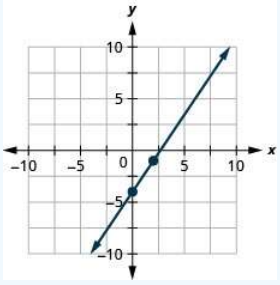
Answer



Try It 2.8.21

Graph the line of the equation $3x - 2y = 8$ using its slope and y-intercept.

Answer



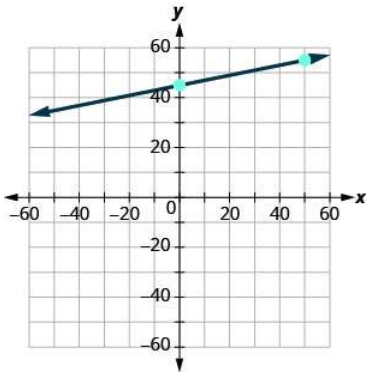
We have used a grid with x and y both going from about -10 to 10 for all the equations we've graphed so far. Not all linear equations can be graphed on this small grid. Often, especially in applications with real-world data, we'll need to extend the axes to bigger positive or smaller negative numbers.

Example 2.8.22

Graph the line of the equation $y = 0.2x + 45$ using its slope and y-intercept.

Solution

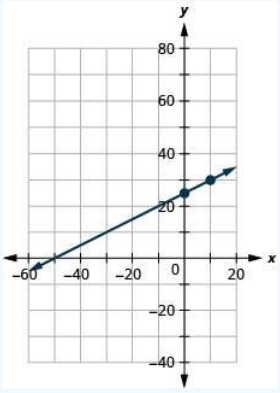
We'll use a grid with the axes going from about -80 to 80.

	$y = mx + b$
The equation is in slope–intercept form.	$y = 0.2x + 45$
Identify the slope and y-intercept.	$m = 0.2$
	The y-intercept is $(0, 45)$
Plot the y-intercept.	See graph below.
Count out the rise and run to mark the second point. The slope is $m = 0.2$; in fraction form this means $m = \frac{2}{10}$. Given the scale of our graph, it would be easier to use the equivalent fraction $m = \frac{10}{50}$.	
Draw the line.	

Try It 2.8.23

Graph the line of the equation $y = 0.5x + 25$ using its slope and y-intercept.

Answer

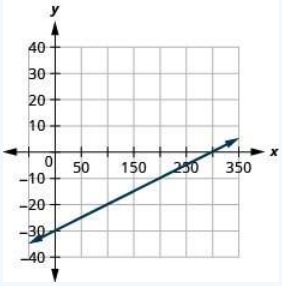


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? Try It 2.8.24

Graph the line of the equation $y = 0.1x - 30$ using its slope and y-intercept.

Answer



Now that we have graphed lines by using the slope and y-intercept, let's summarize all the methods we have used to graph lines. See Figure 2.8.2.

Methods to Graph Lines

Point Plotting	Slope–Intercept	Intercepts	Recognize Vertical and Horizontal Lines
x | y	$y = mx + b$	x | y 0 | | 0	
Find three points. Plot the points, make sure they line up, then draw the line.	Find the slope and y-intercept. Start at the y-intercept, then count the slope to get a second point.	Find the intercepts and a third point. Plot the points, make sure they line up, then draw the line.	The equation has only one variable. $x = a$ vertical $y = b$ horizontal

Figure 2.8.2

Choose the Most Convenient Method to Graph a Line

Now that we have seen several methods we can use to graph lines, how do we know which method to use for a given equation?

While we could plot points, use the slope–intercept form, or find the intercepts for *any* equation, if we recognize the most convenient way to graph a certain type of equation, our work will be easier. Generally, plotting points is not the most efficient way to graph a line. We saw better methods in sections 4.3, 4.4, and earlier in this section. Let's look for some patterns to help determine the most convenient method to graph a line.

Here are six equations we graphed in this chapter, and the method we used to graph each of them.

$$\begin{array}{lll} \#1 & \text{Equation} & \text{Method} \\ \#2 & x = 2 & \text{Vertical line} \\ \#3 & y = 4 & \text{Horizontal line} \\ \#4 & -x + 2y = 6 & \text{Intercepts} \\ \#5 & 4x - 3y = 12 & \text{Intercepts} \\ \#6 & y = 4x - 2 & \text{Slope-intercept} \\ \#7 & y = -x + 4 & \text{Slope-intercept} \end{array} \tag{2.8.2}$$

Equations #1 and #2 each have just one variable. Remember, in equations of this form the value of that one variable is constant; it does not depend on the value of the other variable. Equations of this form have graphs that are vertical or horizontal lines.

In equations #3 and #4, both x and y are on the same side of the equation. These two equations are of the form $Ax + By = C$. We substituted $y = 0$ to find the x-intercept and $x = 0$ to find the y-intercept, and then found a third point by choosing another value for x or y.

Equations #5 and #6 are written in slope–intercept form. After identifying the slope and y-intercept from the equation we used them to graph the line.

This leads to the following strategy.

STRATEGY FOR CHOOSING THE MOST CONVENIENT METHOD TO GRAPH A LINE

Consider the form of the equation.

- If it only has one variable, it is a vertical or horizontal line.
  - $x = a$ is a vertical line passing through the x-axis at a.
  - $y = b$ is a horizontal line passing through the y-axis at b.
- If y is isolated on one side of the equation, in the form $y = mx + b$, graph by using the slope and y-intercept.
  - Identify the slope and y-intercept and then graph.
- If the equation is of the form $Ax + By = C$, find the intercepts.
  - Find the x- and y-intercepts, a third point, and then graph.

Example 2.8.25

Determine the most convenient method to graph each line.

1. $y = -6$
2. $5x - 3y = 15$
3. $x = 7$
4. $y = \frac{2}{5}x - 1$.

Solution

1. $y = -6$
This equation has only one variable, y. Its graph is a horizontal line crossing the y-axis at -6.

2. $5x - 3y = 15$
This equation is of the form $Ax + By = C$. The easiest way to graph it will be to find the intercepts and one more point.

3. $x = 7$
There is only one variable, x. The graph is a vertical line crossing the x-axis at 7.

4. $y = \frac{2}{5}x - 1$
Since this equation is in $y = mx + b$ form, it will be easiest to graph this line by using the slope and y-intercept.

Try It 2.8.26

Determine the most convenient method to graph each line:

1. $3x + 2y = 12$
2. $y = 4$
3. $y = \frac{1}{5}x - 4$
4. $x = -7$

Answer

1. intercepts
2. horizontal line
3. slope–intercept
4. vertical line

Try It 2.8.27

Determine the most convenient method to graph each line:

1. $x = 6$
2. $y = -\frac{3}{4}x + 1$
3. $y = -8$
4. $4x - 3y = -1$

Answer

1. vertical line
2. slope–intercept
3. horizontal line
4. intercepts

Graph and Interpret Applications of Slope–Intercept

Many real-world applications are modeled by linear equations. We will take a look at a few applications here so you can see how equations written in slope–intercept form relate to real-world situations.

Usually when a linear equation models a real-world situation, different letters are used for the variables, instead of x and y. The variable names remind us of what quantities are being measured.

Example 2.8.28

The equation $F = \frac{9}{5}C + 32$ is used to convert temperatures, C, on the Celsius scale to temperatures, F, on the Fahrenheit scale.

1. Find the Fahrenheit temperature for a Celsius temperature of 0.
2. Find the Fahrenheit temperature for a Celsius temperature of 20.
3. Interpret the slope and F-intercept of the equation.
4. Graph the equation.

Solution

1. Find the Fahrenheit temperature for a Celsius temperature of 0.

	$F = \frac{9}{5}C + 32$
Find F when $C = 0$.	$F = \frac{9}{5}(0) + 32$
Simplify.	$F = 32$

2.

Find the Fahrenheit temperature for a Celsius temperature of 20.	$F = \frac{9}{5}C + 32$
Find F when $C = 20$.	$F = \frac{9}{5}(20) + 32$
Simplify.	$F = 36 + 32$
Simplify.	$F = 68$

(2.8.3)

3. Interpret the slope and F-intercept of the equation.

Even though this equation uses F and C, it is still in slope–intercept form.

$y = mx + b$

$F = mC + b$

$F = \frac{9}{5}C + 32$

The slope, $\frac{9}{5}$, means that the temperature Fahrenheit (F) increases 9 degrees when the temperature Celsius (C) increases 5 degrees.

The F-intercept means that when the temperature is 0° on the Celsius scale, it is 32° on the Fahrenheit scale.

4. Graph the equation.

We'll need to use a larger scale than our usual. Start at the F-intercept $(0, 32)$ then count out the rise of 9 and the run of 5 to get a second point. See Figure $2.8.3$.

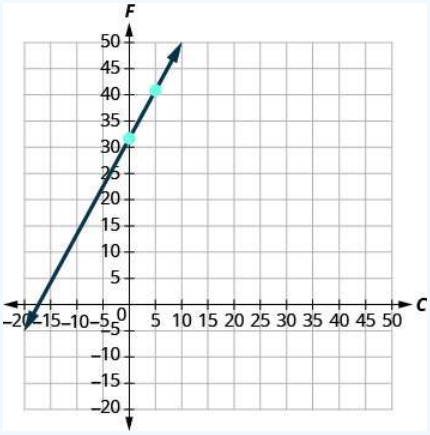


Figure 2.8.3

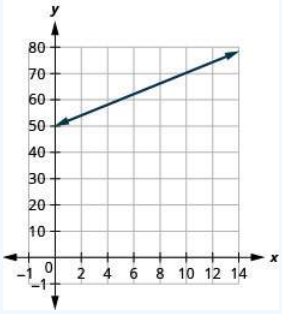
? Try It 2.8.29

The equation $h = 2s + 50$ is used to estimate a woman's height in inches, h, based on her shoe size, s.

1. Estimate the height of a child who wears women's shoe size 0.
2. Estimate the height of a woman with shoe size 8.
3. Interpret the slope and h-intercept of the equation.
4. Graph the equation.

Answer

1. 50 inches
2. 66 inches
3. The slope, 2, means that the height, h, increases by 2 inches when the shoe size, s, increases by 1. The h-intercept means that when the shoe size is 0, the height is 50 inches.
4.



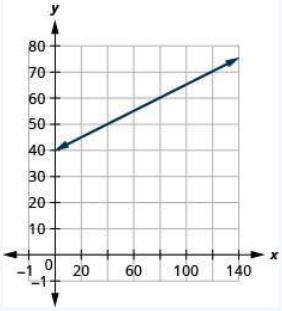
? Try It 2.8.30

The equation $T = \frac{1}{4}n + 40$ is used to estimate the temperature in degrees Fahrenheit, T, based on the number of cricket chirps, n, in one minute.

1. Estimate the temperature when there are no chirps.
2. Estimate the temperature when the number of chirps in one minute is 100.
3. Interpret the slope and T-intercept of the equation.
4. Graph the equation.

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Answer

1. 40 degrees
2. 65 degrees
3. The slope, $\frac{1}{4}$, means that the temperature Fahrenheit (F) increases 1 degree when the number of chirps, n, increases by 4. The T-intercept means that when the number of chirps is 0, the temperature is 40°.
4. 

The cost of running some types business has two components—a *fixed cost* and a *variable cost*. The fixed cost is always the same regardless of how many units are produced. This is the cost of rent, insurance, equipment, advertising, and other items that must be paid regularly. The variable cost depends on the number of units produced. It is for the material and labor needed to produce each item.

Example 2.8.31

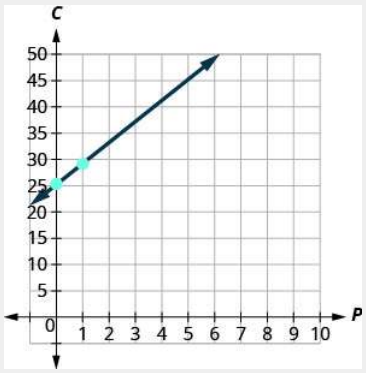
Stella has a home business selling gourmet pizzas. The equation $C = 4p + 25$ models the relation between her weekly cost, C, in dollars and the number of pizzas, p, that she sells.

1. Find Stella's cost for a week when she sells no pizzas.
2. Find the cost for a week when she sells 15 pizzas.
3. Interpret the slope and C-intercept of the equation.
4. Graph the equation.

Solution

1. Find Stella's cost for a week when she sells no pizzas.	$C = 4p + 25$
Find C when $p = 0$.	$C = 4(0) + 25$
Simplify.	$C = 25$
	Stella's fixed cost is \$25 when she sells no pizzas.
2. Find the cost for a week when she sells 15 pizzas.	$C = 4p + 25$
Find C when $p = 15$.	$C = 4(15) + 25$
Simplify.	$C = 60 + 25$
	$C = 85$
	Stella's costs are \$85 when she sells 15 pizzas.
3. Interpret the slope and C-intercept of the equation.	$y = mx + b$ $C = 4p + 25$
	The slope, 4, means that the cost increases by \$4 for each pizza Stella sells. The C-intercept means that even when Stella sells no pizzas, her costs for the week are \$25.

4. Graph the equation. We'll need to use a larger scale than our usual. Start at the C-intercept $(0, 25)$ then count out the rise of 4 and the run of 1 to get a second point.



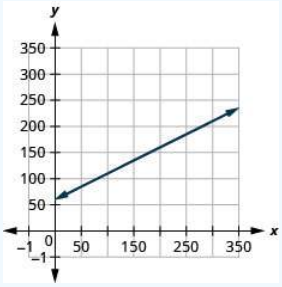
Try It 2.8.32

Sam drives a delivery van. The equation $C = 0.5m + 60$ models the relation between his weekly cost, C, in dollars and the number of miles, m, that he drives.

1. Find Sam's cost for a week when he drives 0 miles.
2. Find the cost for a week when he drives 250 miles.
3. Interpret the slope and C-intercept of the equation.
4. Graph the equation.

Answer

1. $\$60$
2. $\$185$
3. The slope, 0.5, means that the weekly cost, C, increases by $\$0.50$ when the number of miles driven, n, increases by 1. The C-intercept means that when the number of miles driven is 0, the weekly cost is $\$60$.
4.



Try It 2.8.33

Loreen has a calligraphy business. The equation $C = 1.8n + 35$ models the relation between her weekly cost, C, in dollars and the number of wedding invitations, n, that she writes.

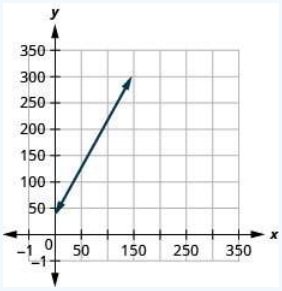
1. Find Loreen's cost for a week when she writes no invitations.
2. Find the cost for a week when she writes 75 invitations.
3. Interpret the slope and C-intercept of the equation.
4. Graph the equation.

Answer

1. $\$35$
2. $\$170$
3. The slope, 1.8, means that the weekly cost, C, increases by $\$1.80$ when the number of invitations, n, increases by 1. The C-intercept means that when the number of invitations is 0, the weekly cost is $\$35$.

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4.



Use Slopes to Identify Parallel Lines

The slope of a line indicates how steep the line is and whether it rises or falls as we read it from left to right. Two lines that have the same slope are called parallel lines. Parallel lines never intersect.

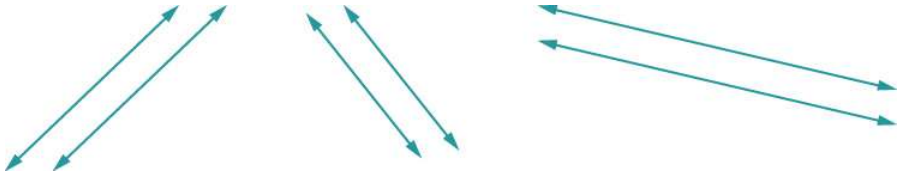


Figure 2.8.4.

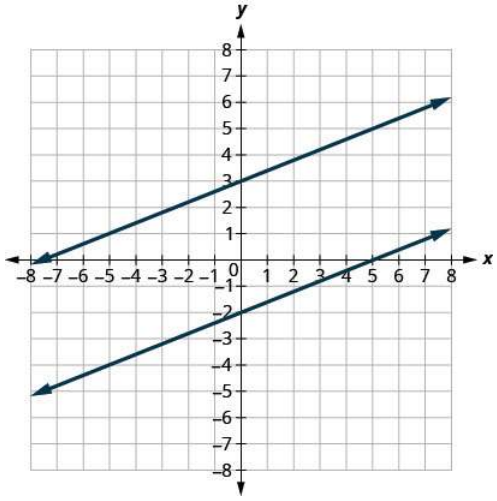


Figure 2.8.4: Verify that both lines have the same slope, $m = \frac{2}{5}$, and different y-intercepts.

What about vertical lines? The slope of a vertical line is undefined, so vertical lines don't fit in the definition above. We say that vertical lines that have different x-intercepts are parallel. See Figure 2.8.5.

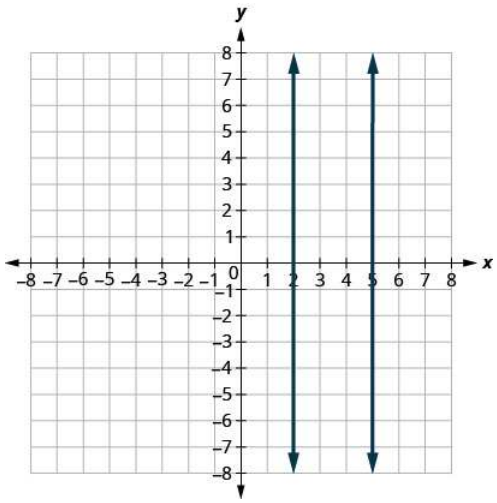


Figure 2.8.5: Vertical lines with different x-intercepts are parallel.

PARALLEL LINES

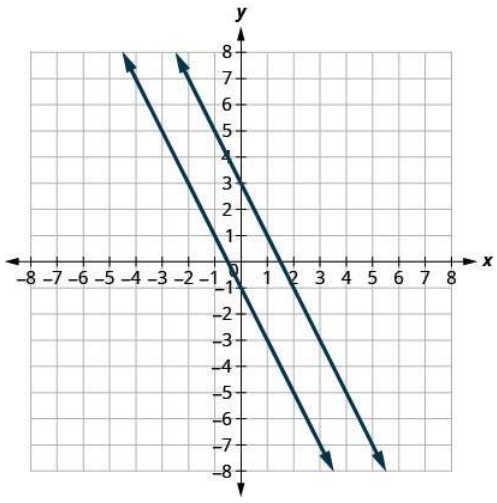
Parallel lines are lines in the same plane that do not intersect.

- Parallel lines have the same slope and different y-intercepts.
- If m_1 and m_2 are the slopes of two parallel lines then $m_1 = m_2$.
- Parallel vertical lines have different x-intercepts.

Let's graph the equations $y = -2x + 3$ and $2x + y = -1$ on the same grid. The first equation is already in slope–intercept form: $y = -2x + 3$. We solve the second equation for y:

$$\begin{aligned} 2x + y &= -1 \\ y &= -2x - 1 \end{aligned}$$

Graph the lines.



Notice the lines look parallel. What is the slope of each line? What is the y-intercept of each line?

$$\begin{array}{lll} y & = mx + b & y = mx + b \\ y & = -2x + 3 & y = -2x - 1 \\ m & = -2 & m = -2 \\ b & = 3, (0, 3) & b = -1, (0, -1) \end{array} \tag{2.8.4}$$

The slopes of the lines are the same and the y-intercept of each line is different. So we know these lines are parallel.

Since parallel lines have the same slope and different y-intercepts, we can now just look at the slope–intercept form of the equations of lines and decide if the lines are parallel.

Example 2.8.34

Use slopes and y-intercepts to determine if the lines $3x-2y=6$ and $y=\frac{3}{2}x+1$ are parallel.

Solution

Solve the first equation for y.

$$\begin{aligned} 3x-2y &= 6 \\ -2y &= -3x+6 \\ \frac{-2y}{-2} &= \frac{-3x+6}{-2} \\ y &= \frac{3}{2}x-3 \end{aligned}$$

The equation is now in slope–intercept form.

The equation of the second line is already in slope–intercept form.

Identify the slope and y-intercept of both lines.

$y=\frac{3}{2}x+1$	$y=\frac{3}{2}x-3$
$y=mx+b$	$y=mx+b$
$m=\frac{3}{2}$	$m=\frac{3}{2}$
y-intercept is $(0,1)$	y-intercept is $(0,-3)$

The lines have the same slope and different y-intercepts and so they are parallel. You may want to graph the lines to confirm whether they are parallel.

Try It 2.8.35

Use slopes and y-intercepts to determine if the lines $2x+5y=5$ and $y=-\frac{2}{5}x-4$ are parallel.

Answer

parallel

Try It 2.8.36

Use slopes and y-intercepts to determine if the lines $4x-3y=6$ and $y=\frac{4}{3}x-1$ are parallel.

Answer

parallel

Example 2.8.37

Use slopes and y-intercepts to determine if the lines $y=-4$ and $y=3$ are parallel.

Solution

Write each equation in slope-intercept form.	$y=-4$	and	$y=3$
Since there is no x term we write $0x$.	$y=0x-4$		$y=0x+3$
Identify the slope and y-intercept of both lines.	$y=mx+b$		$y=mx+b$
	$m=0$		$m=0$
	y-intercept is $(0,4)$		y-intercept is $(0,3)$

The lines have the same slope and different y-intercepts and so they are parallel.

There is another way you can look at this example. If you recognize right away from the equations that these are horizontal lines, you know their slopes are both 0. Since the horizontal lines cross the y-axis at $y=-4$ and at $y=3$, we know the y-intercepts are $(0,-4)$ and $(0,3)$. The lines have the same slope and different y-intercepts and so they are parallel.

? Try It 2.8.38

Use slopes and y-intercepts to determine if the lines $y=8$ and $y=-6$ are parallel.

Answer

parallel

? Try It 2.8.39

Use slopes and y-intercepts to determine if the lines $y=1$ and $y=-5$ are parallel.

Answer

parallel

✓ Example 2.8.40

Use slopes and y-intercepts to determine if the lines $x=-2$ and $x=-5$ are parallel.

Solution

$$x=-2 \text{ and } x=-5 \tag{2.8.5}$$

Since there is no y, the equations cannot be put in slope–intercept form. But we recognize them as equations of vertical lines. Their x-intercepts are -2 and -5. Since their x-intercepts are different, the vertical lines are parallel.

? Try It 2.8.41

Use slopes and y-intercepts to determine if the lines $x=1$ and $x=-5$ are parallel.

Answer

parallel

? Try It 2.8.42

Use slopes and y-intercepts to determine if the lines $x=8$ and $x=-6$ are parallel.

Answer

parallel

✓ Example 2.8.43

Use slopes and y-intercepts to determine if the lines $y=2x-3$ and $-6x+3y=-9$ are parallel. You may want to graph these lines, too, to see what they look like.

Solution

The first equation is already in slope-intercept form. $y = 2x - 3$

Solve the second equation for y

$$\begin{aligned} -6x + 3y &= -9 \\ 3y &= 6x - 9 \\ \tfrac{3y}{3} &= \tfrac{6x-9}{3} \\ y &= 2x - 3 \end{aligned}$$

The second equation is now in slope–intercept form as well.

Identify the slope and y-intercept of both lines.

$y = 2x - 3$	$y = 2x - 3$
$y = mx + b$	$y = mx + b$
$m = 2$	$m = 2$
The y-intercept is $(0, -3)$	The y-intercept is $(0, -3)$

The lines have the same slope, but they also have the same y-intercepts. Their equations represent the same line. They are not parallel; they are the same line.

Try It 2.8.44

Use slopes and y-intercepts to determine if the lines $y = -\frac{1}{2}x + 1$ and $x + 2y = 2$ are parallel.

Answer

not parallel; same line

Try It 2.8.45

Use slopes and y-intercepts to determine if the lines $y = \frac{3}{4}x - 3$ and $3x - 4y = 12$ are parallel.

Answer

not parallel; same line

Use Slopes to Identify Perpendicular Lines

Let's look at the lines whose equations are $y = \frac{1}{4}x - 1$ and $y = -4x + 2$, shown in Figure 2.8.5.

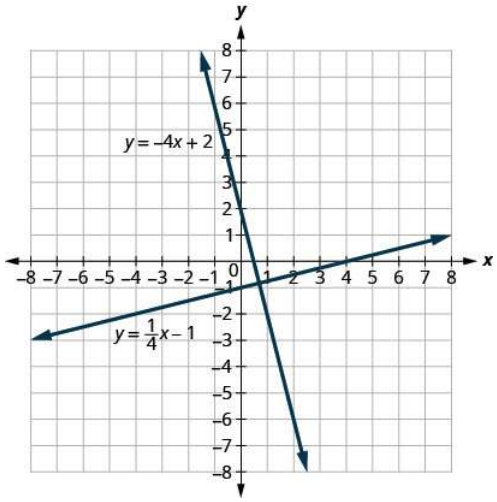


Figure 2.8.5

These lines lie in the same plane and intersect in right angles. We call these lines **perpendicular**.

What do you notice about the slopes of these two lines? As we read from left to right, the line $y = \frac{1}{4}x - 1$ rises, so its slope is positive. The line $y = -4x + 2$ drops from left to right, so it has a negative slope. Does it make sense to you that the slopes of two perpendicular lines will have opposite signs?

If we look at the slope of the first line, $m_1 = \frac{1}{4}$, and the slope of the second line, $m_2 = -4$, we can see that they are *negative reciprocals* of each other. If we multiply them, their product is -1.

$$\begin{array}{c} m_1 \cdot m_2 \\ \frac{1}{4}(-4) \\ -1 \end{array} \tag{2.8.6}$$

This is always true for **perpendicular lines** and leads us to this definition.

PERPENDICULAR LINES

Perpendicular lines are lines in the same plane that form a right angle.

If m_1 and m_2 are the slopes of two perpendicular lines, then:

$$m_1 \cdot m_2 = -1 \text{ and } m_1 = \frac{-1}{m_2} \tag{2.8.7}$$

Vertical lines and horizontal lines are always perpendicular to each other.

We were able to look at the slope–intercept form of linear equations and determine whether or not the lines were parallel. We can do the same thing for perpendicular lines.

We find the slope–intercept form of the equation, and then see if the slopes are negative reciprocals. If the product of the slopes is -1, the lines are perpendicular. Perpendicular lines may have the same y-intercepts.

Example 2.8.46

Use slopes to determine if the lines, $y = -5x - 4$ and $x - 5y = 5$ are perpendicular.

Solution

The first equation is already in slope–intercept form: $y = -5x - 4$

Solve the second equation for y.

$$\begin{array}{rcl} x - 5y & = & 5 \\ -5y & = & -x + 5 \\ \frac{-5y}{-5} & = & \frac{-x+5}{-5} \\ y & = & \frac{1}{5}x - 1 \end{array}$$

The second equation is now in slope-intercept form as well.

Identify the slope of each line.

$$\begin{array}{rclrcl} y & = & -5x - 4 & y & = & \frac{1}{5}x - 1 \\ y & = & mx + b & y & = & mx + b \\ m_1 & = & -5 & m_2 & = & \frac{1}{5} \end{array}$$

The slopes are negative reciprocals of each other, so the lines are perpendicular. We check by multiplying the slopes,

$$\begin{array}{c} m_1 \cdot m_2 \\ -5\left(\frac{1}{5}\right) \\ -1\checkmark \end{array} \tag{2.8.8}$$

Try It 2.8.47

Use slopes to determine if the lines $y = -3x + 2$ and $x - 3y = 4$ are perpendicular.

Answer

perpendicular

Try It 2.8.48

Use slopes to determine if the lines $y = 2x - 5$ and $x + 2y = -6$ are perpendicular.

Answer

perpendicular

Example 2.8.49

Use slopes to determine if the lines, $7x + 2y = 3$ and $2x + 7y = 5$ are perpendicular.

Solution

Solve the equations for y.

$$\begin{aligned} 7x + 2y &= 3 \\ 2y &= -7x + 3 \\ \frac{2y}{2} &= \frac{-7x+3}{2} \\ y &= -\frac{7}{2}x + \frac{3}{2} \end{aligned} \qquad \begin{aligned} 2x + 7y &= 5 \\ 7y &= -2x + 5 \\ \frac{7y}{7} &= \frac{-2x+5}{7} \\ y &= \frac{-2}{7}x + \frac{5}{7} \end{aligned}$$

Identify the slope of each line.

$$\begin{aligned} y &= mx + b \\ m_1 &= -\frac{7}{2} \end{aligned} \qquad \begin{aligned} y &= mx + b \\ m_2 &= -\frac{2}{7} \end{aligned}$$

The slopes are reciprocals of each other, but they have the same sign. Since they are not negative reciprocals, the lines are not perpendicular.

Try It 2.8.50

Use slopes to determine if the lines $5x + 4y = 1$ and $4x + 5y = 3$ are perpendicular.

Answer

not perpendicular

Try It 2.8.51

Use slopes to determine if the lines $2x - 9y = 3$ and $9x - 2y = 1$ are perpendicular.

Answer

not perpendicular

Note

Access this online resource for additional instruction and practice with graphs.

- Explore the Relation Between a Graph and the Slope–Intercept Form of an Equation of a Line

Key Concepts

- The slope–intercept form of an equation of a line with slope mm and y-intercept, $(0, b)$ is, $y = mx + b$.
- **Graph a Line Using its Slope and y-Intercept**
  1. Find the slope-intercept form of the equation of the line.
  2. Identify the slope and y-intercept.
  3. Plot the y-intercept.
  4. Use the slope formula $m = \frac{\text{rise}}{\text{run}}$ to identify the rise and the run.
  5. Starting at the y-intercept, count out the rise and run to mark the second point.
  6. Connect the points with a line.
- **Strategy for Choosing the Most Convenient Method to Graph a Line:** Consider the form of the equation.
  - If it only has one variable, it is a vertical or horizontal line.
    $x = a$ is a vertical line passing through the x-axis at a.
    $y = b$ is a horizontal line passing through the y-axis at b.
  - If y is isolated on one side of the equation, in the form $y = mx + b$, graph by using the slope and y-intercept.
    Identify the slope and y-intercept and then graph.
  - If the equation is of the form $Ax + By = C$, find the intercepts.
    Find the x- and y-intercepts, a third point, and then graph.
- Parallel lines are lines in the same plane that do not intersect.
  - Parallel lines have the same slope and different y-intercepts.
  - If m_1 and m_2 are the slopes of two parallel lines then $m_1 = m_2$.
  - Parallel vertical lines have different x-intercepts.
- Perpendicular lines are lines in the same plane that form a right angle.
  - If m_1 and m_2 are the slopes of two perpendicular lines, then $m_1 \cdot m_2 = -1$ and $m_1 = \frac{-1}{m_2}$.
  - Vertical lines and horizontal lines are always perpendicular to each other.

Glossary

parallel lines

Lines in the same plane that do not intersect.

perpendicular lines

Lines in the same plane that form a right angle.

slope-intercept form of an equation of a line

The slope–intercept form of an equation of a line with slope mm and y-intercept, $(0, b)$ is, $y = mx + b$.

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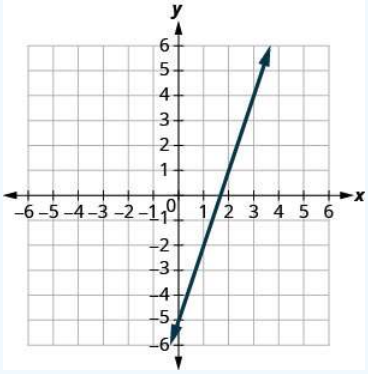
2.8.1: Exercises

Practice Makes Perfect

Recognize the Relation Between the Graph and the Slope–Intercept Form of an Equation of a Line

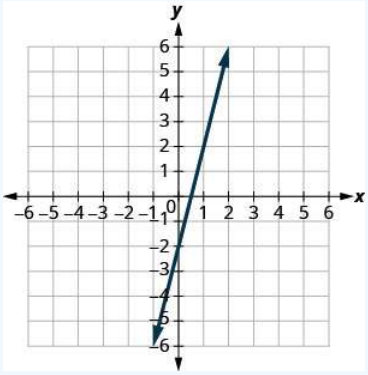
In the following exercises, use the graph to find the slope and y-intercept of each line. Compare the values to the equation $y = mx + b$.

Exercise 2.8.1.1



$y = 3x - 5$

Exercise 2.8.1.2

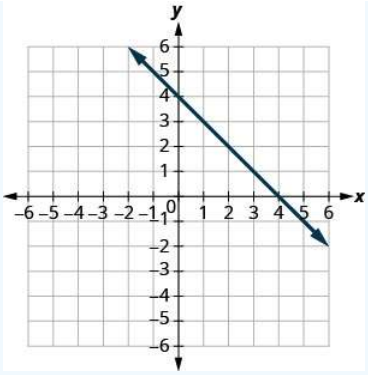


$y = 4x - 2$

Answer

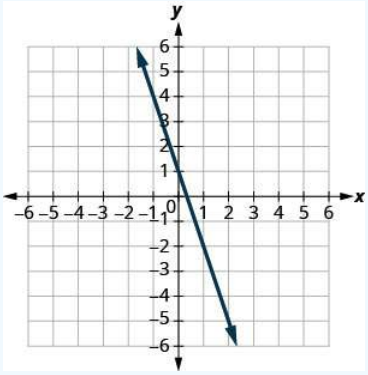
slope $m = 4$ and y-intercept $(0, -2)$

Exercise 2.8.1.3



$y = -x + 4$

Exercise 2.8.1.4

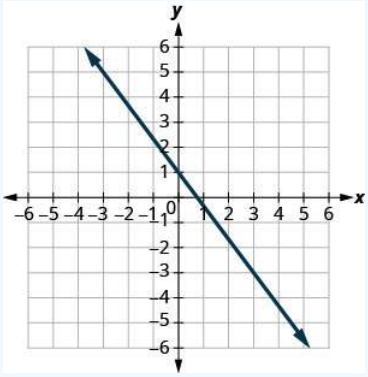


$y = -3x + 1$

Answer

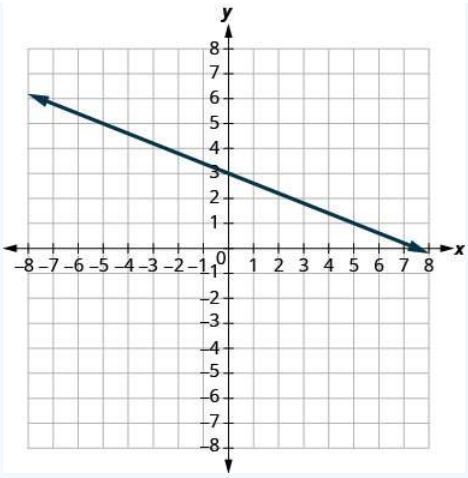
slope $m = -3$ and y-intercept $(0, 1)$

Exercise 2.8.1.5



$y = -\frac{4}{3}x + 1$

Exercise 2.8.1.6



$y = -\frac{2}{5}x + 3$

Answer

slope $m = -\frac{2}{5}$ and y -intercept $(0, 3)$

Identify the Slope and y-Intercept From an Equation of a Line

In the following exercises, identify the slope and y-intercept of each line.

Exercise 2.8.1.7

$y = -7x + 3$

Exercise 2.8.1.8

$y = -9x + 7$

Answer

$m = -9$; y-intercept: $(0, 7)$

Exercise 2.8.1.9

$y = 6x - 8$

Exercise 2.8.1.10

$y = 4x - 10$

Answer

$m = 4$; y-intercept: $(0, -10)$

Exercise 2.8.1.11

$3x + y = 5$

Exercise 2.8.1.12

$4x + y = 8$

Answer

$m = -4 \backslash 0$; \(y-intercept: $(0, 8)$

Exercise 2.8.1.13

$6x + 4y = 12$

Exercise 2.8.1.14

$8x + 3y = 12$

Answer

$m = -\frac{8}{3}$; y-intercept: $(0, 4)$

Exercise 2.8.1.15

$5x - 2y = 6$

Exercise 2.8.1.16

$7x - 3y = 9$

Answer

$m = \frac{7}{3}$; y-intercept: $(0, -3)$

Graph a Line Using Its Slope and Intercept

In the following exercises, graph the line of each equation using its slope and y-intercept.

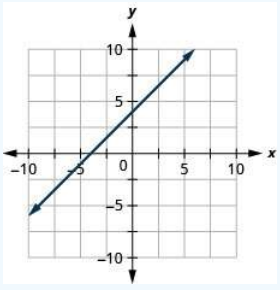
Exercise 2.8.1.17

$y = x + 3$

Exercise 2.8.1.18

$y = x + 4$

Answer



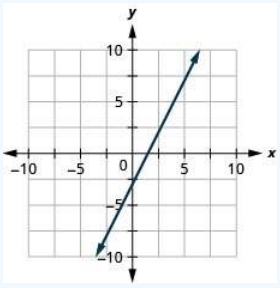
Exercise 2.8.1.19

$y = 3x - 1$

Exercise 2.8.1.20

$y = 2x - 3$

Answer



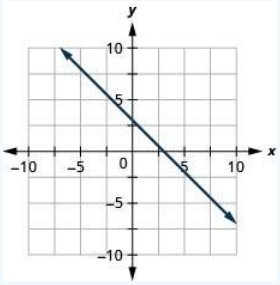
Exercise 2.8.1.21

$y = -x + 2$

Exercise 2.8.1.22

$y = -x + 3$

Answer



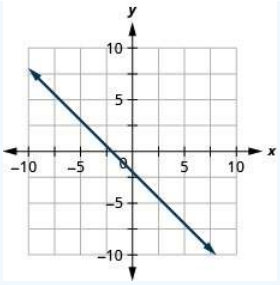
Exercise 2.8.1.23

$y = -x - 4$

Exercise 2.8.1.24

$y = -x - 2$

Answer



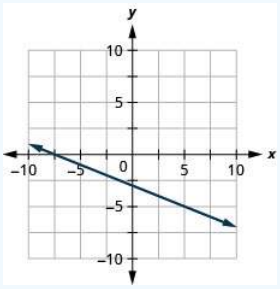
Exercise 2.8.1.25

$y = -\frac{3}{4}x - 1$

Exercise 2.8.1.26

$y = -\frac{2}{5}x - 3$

Answer



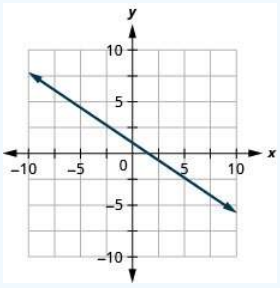
Exercise 2.8.1.27

$y = -\frac{3}{5}x + 2$

Exercise 2.8.1.28

$y = -\frac{2}{3}x + 1$

Answer



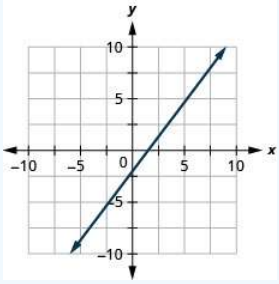
Exercise 2.8.1.29

$3x - 4y = 8$

Exercise 2.8.1.30

$4x - 3y = 6$

Answer



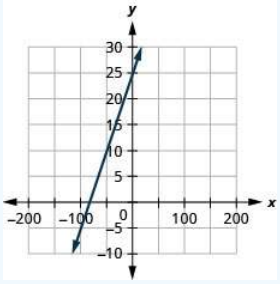
Exercise 2.8.1.31

$y = 0.1x + 15$

Exercise 2.8.1.32

$y = 0.3x + 25$

Answer



Choose the Most Convenient Method to Graph a Line

In the following exercises, determine the most convenient method to graph each line.

Exercise 2.8.1.33

$x = 2$

Exercise 2.8.1.34

$y = 4$

Answer

horizontal line

Exercise 2.8.1.35

$y = 5$

Exercise 2.8.1.36

$x = -3$

Answer

vertical line

Exercise 2.8.1.37

$y = -3x + 4$

Exercise 2.8.1.38

$y = -5x + 2$

Answer

slope–intercept

Exercise 2.8.1.39

$x - y = 5$

Exercise 2.8.1.40

$x - y = 1$

Answer

intercepts

Exercise 2.8.1.41

$y = \frac{2}{3}x - 1$

Exercise 2.8.1.42

$y = \frac{4}{5}x - 3$

Answer

slope–intercept

Exercise 2.8.1.43

$y = -3$

Exercise 2.8.1.44

$y = -1$

Answer

horizontal line

Exercise 2.8.1.45

$3x - 2y = -12$

Exercise 2.8.1.46

$2x - 5y = -10$

Answer

intercepts

Exercise 2.8.1.47

$y = -\frac{1}{4}x + 3$

Exercise 2.8.1.48

$y = -\frac{1}{3}x + 5$

Answer

slope–intercept

Graph and Interpret Applications of Slope–Intercept

Exercise 2.8.1.49

The equation $P = 31 + 1.75w$ models the relation between the amount of Tuyet's monthly water bill payment, P, in dollars, and the number of units of water, w, used.

1. Find Tuyet's payment for a month when 0 units of water are used.
2. Find Tuyet's payment for a month when 12 units of water are used.
3. Interpret the slope and P-intercept of the equation.
4. Graph the equation.

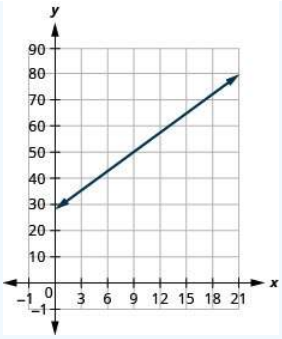
Exercise 2.8.1.50

The equation $P = 28 + 2.54w$ models the relation between the amount of Randy's monthly water bill payment, P, in dollars, and the number of units of water, w, used.

1. Find the payment for a month when Randy used 0 units of water.
2. Find the payment for a month when Randy used 15 units of water.
3. Interpret the slope and P-intercept of the equation.
4. Graph the equation.

Answer

1. $28
2. $66.10
3. The slope, 2.54, means that Randy's payment, P, increases by $2.54 when the number of units of water he used, w, ncreases by 1. The P-intercept means that if the number units of water Randy used was 0, the payment would be $28
4.



Exercise 2.8.1.51

Bruce drives his car for his job. The equation $R = 0.575m + 42$ models the relation between the amount in dollars, R, that he is reimbursed and the number of miles, m, he drives in one day.

1. Find the amount Bruce is reimbursed on a day when he drives 0 miles.
2. Find the amount Bruce is reimbursed on a day when he drives 220 miles.
3. Interpret the slope and R-intercept of the equation.
4. Graph the equation.

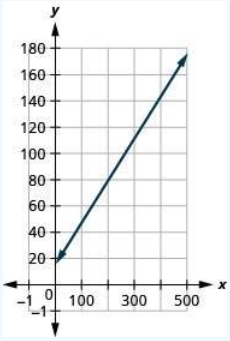
Exercise 2.8.1.52

Janelle is planning to rent a car while on vacation. The equation $C = 0.32m + 15$ models the relation between the cost in dollars, C, per day and the number of miles, m, she drives in one day.

1. Find the cost if Janelle drives the car 0 miles one day.
2. Find the cost on a day when Janelle drives the car 400 miles.
3. Interpret the slope and C-intercept of the equation.
4. Graph the equation.

Answer

1. $15
2. $143
3. The slope, 0.32, means that the cost, C, increases by $0.32 when the number of miles driven, m, increases by 1. The C-intercept means that if Janelle drives 0 miles one day, the cost would be $15.
4.



Exercise 2.8.1.53

Cherie works in retail and her weekly salary includes commission for the amount she sells. The equation $S = 400 + 0.15c$ models the relation between her weekly salary, S, in dollars and the amount of her sales, c, in dollars.

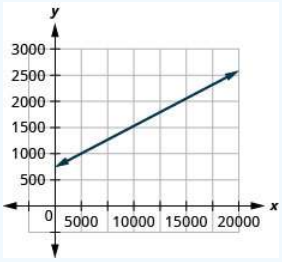
1. Find Cherie's salary for a week when her sales were 0.
2. Find Cherie's salary for a week when her sales were 3600.
3. Interpret the slope and S-intercept of the equation.
4. Graph the equation.

Exercise 2.8.1.54

Patel's weekly salary includes a base pay plus commission on his sales. The equation $S = 750 + 0.09c$ models the relation between his weekly salary, S, in dollars and the amount of his sales, c, in dollars.

1. Find Patel's salary for a week when his sales were 0.
2. Find Patel's salary for a week when his sales were $18,540$.
3. Interpret the slope and S-intercept of the equation.
4. Graph the equation.

Answer

1. $\$750$
2. $\$2418.60$
3. The slope, 0.09, means that Patel's salary, S, increases by $\$0.09$ for every $\$1$ increase in his sales. The S-intercept means that when his sales are $\$0$, his salary is $\$750$
4. 

Exercise 2.8.1.55

Costa is planning a lunch banquet. The equation $C = 450 + 28g$ models the relation between the cost in dollars, C, of the banquet and the number of guests, g.

1. Find the cost if the number of guests is 40.
2. Find the cost if the number of guests is 80.
3. Interpret the slope and C-intercept of the equation.
4. Graph the equation.

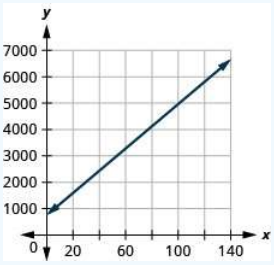
Exercise 2.8.1.56

Margie is planning a dinner banquet. The equation $C = 750 + 42g$ models the relation between the cost in dollars, C, of the banquet and the number of guests, g.

1. Find the cost if the number of guests is 50.
2. Find the cost if the number of guests is 100.
3. Interpret the slope and C-intercept of the equation.
4. Graph the equation.

Answer

1. $\$2850$
2. $\$4950$

3. The slope, 42, means that the cost, C, increases by $\$42$ for when the number of guests increases by 1. The C-intercept means that when the number of guests is 0, the cost would be $\$750$
4. 

Use Slopes to Identify Parallel Lines

In the following exercises, use slopes and y-intercepts to determine if the lines are parallel.

Exercise 2.8.1.57

$y = \frac{3}{4}x - 3; \quad 3x - 4y = -2$

Exercise 2.8.1.58

$y = \frac{2}{3}x - 1; \quad 2x - 3y = -2$

Answer

parallel

Exercise 2.8.1.59

$2x - 5y = -3; \quad y = \frac{2}{5}x + 1$

Exercise 2.8.1.60

$3x - 4y = -2; \quad y = \frac{3}{4}x - 3$

Answer

parallel

Exercise 2.8.1.61

$2x - 4y = 6; \quad x - 2y = 3$

Exercise 2.8.1.62

$6x - 3y = 9; \quad 2x - y = 3$

Answer

not parallel

Exercise 2.8.1.63

$4x+2y=6; \quad 6x+3y=3$

Exercise 2.8.1.64

$8x+6y=6; \quad 12x+9y=12$

Answer

parallel

Exercise 2.8.1.65

$x=5; \quad x=-6$

Exercise 2.8.1.66

$x=7; \quad x=-8$

Answer

parallel

Exercise 2.8.1.67

$x=-4; \quad x=-1$

Exercise 2.8.1.68

$x=-3; \quad x=-2$

Answer

parallel

Exercise 2.8.1.69

$y=2; \quad y=6$

Exercise 2.8.1.70

$y=5; \quad y=1$

Answer

parallel

Exercise 2.8.1.71

$y=-4; \quad y=3$

Exercise 2.8.1.72

$y = -1; \quad y = 2$

Answer

parallel

Exercise 2.8.1.73

$x - y = 2; \quad 2x - 2y = 4$

Exercise 2.8.1.74

$4x + 4y = 8; \quad x + y = 2$

Answer

not parallel

Exercise 2.8.1.75

$x - 3y = 6; \quad 2x - 6y = 12$

Exercise 2.8.1.76

$5x - 2y = 11; \quad 5x - y = 7$

Answer

not parallel

Exercise 2.8.1.77

$3x - 6y = 12; \quad 6x - 3y = 3$

Exercise 2.8.1.78

$4x - 8y = 16; \quad x - 2y = 4$

Answer

not parallel

Exercise 2.8.1.79

$9x - 3y = 6; \quad 3x - y = 2$

Exercise 2.8.1.80

$x - 5y = 10; \quad 5x - y = -10$

Answer

not parallel

Exercise 2.8.1.81

$7x-4y=8; \quad 4x+7y=14$

Exercise 2.8.1.82

$9x-5y=4; \quad 5x+9y=-1$

Answer

not parallel

Use Slopes to Identify Perpendicular Lines

In the following exercises, use slopes and y-intercepts to determine if the lines are perpendicular.

Exercise 2.8.1.83

$3x-2y=8; \quad 2x+3y=6$

Exercise 2.8.1.84

$x-4y=8; \quad 4x+y=2$

Answer

perpendicular

Exercise 2.8.1.85

$2x+5y=3; \quad 5x-2y=6$

Exercise 2.8.1.86

$2x+3y=5; \quad 3x-2y=7$

Answer

perpendicular

Exercise 2.8.1.87

$3x-2y=1; \quad 2x-3y=2$

Exercise 2.8.1.88

$3x-4y=8; \quad 4x-3y=6$

Answer

not perpendicular

Exercise 2.8.1.89

$5x+2y=6; \quad 2x+5y=8$

Exercise 2.8.1.90

$2x+4y=3; \quad 6x+3y=2$

Answer

not perpendicular

Exercise 2.8.1.91

$4x-2y=5; \quad 3x+6y=8$

Exercise 2.8.1.92

$2x-6y=4; \quad 12x+4y=9$

Answer

perpendicular

Exercise 2.8.1.93

$6x-4y=5; \quad 8x+12y=3$

Exercise 2.8.1.94

$8x-2y=7; \quad 3x+12y=9$

Answer

perpendicular

Everyday Math

Exercise 2.8.1.95

The equation $C=\frac{5}{9}F-17.8$ can be used to convert temperatures, F, on the Fahrenheit scale to temperatures, C, on the Celsius scale.

1. Explain what the slope of the equation means.
2. Explain what the C-intercept of the equation means.

Exercise 2.8.1.96

The equation $n=4T-160$ is used to estimate the number of cricket chirps, n, in one minute based on the temperature in degrees Fahrenheit, T.

1. Explain what the slope of the equation means.
2. Explain what the n-intercept of the equation means. Is this a realistic situation?

Answer

1. For every increase of one degree Fahrenheit, the number of chirps increases by four.
2. There would be -160 chirps when the Fahrenheit temperature is 0°. (Notice that this does not make sense; this model cannot be used for all possible temperatures.)

Writing Exercises

Exercise 2.8.1.97

Explain in your own words how to decide which method to use to graph a line.

Exercise 2.8.1.98

Why are all horizontal lines parallel?

Answer

Answers will vary.

Self Check

a. After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
recognize the relation between the graph and the slope-intercept form of an equation of a line.			
identify the slope and *y*-intercept from an equation of a line.			
graph a line using its slope and intercept.			
choose the most convenient method to graph a line.			
graph and interpret applications of slope-intercept.			
use slopes to identify parallel lines.			

b. After looking at the checklist, do you think you are well-prepared for the next section? Why or why not?

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2.9: Find the Equation of a Line

Learning Objectives

By the end of this section, you will be able to:

- Find an equation of the line given the slope and y-intercept
- Find an equation of the line given the slope and a point
- Find an equation of the line given two points
- Find an equation of a line parallel to a given line
- Find an equation of a line perpendicular to a given line

Note

Before you get started, take this readiness quiz.

1. Solve: $\frac{2}{3} = \frac{x}{5}$.
   If you missed this problem, review Exercise 2.2.4.
2. Simplify: $-\frac{2}{5}(x - 15)$.
   If you missed this problem, review Exercise 1.10.34.

How do online retailers know that 'you may also like' a particular item based on something you just ordered? How can economists know how a rise in the minimum wage will affect the unemployment rate? How do medical researchers create drugs to target cancer cells? How can traffic engineers predict the effect on your commuting time of an increase or decrease in gas prices? It's all mathematics.

You are at an exciting point in your mathematical journey as the mathematics you are studying has interesting applications in the real world.

The physical sciences, social sciences, and the business world are full of situations that can be modeled with linear equations relating two variables. Data is collected and graphed. If the data points appear to form a straight line, an equation of that line can be used to predict the value of one variable based on the value of the other variable.

To create a mathematical model of a linear relation between two variables, we must be able to find the equation of the line. In this section we will look at several ways to write the equation of a line. The specific method we use will be determined by what information we are given.

Find an Equation of the Line Given the Slope and *y*-Intercept

We can easily determine the slope and intercept of a line if the equation was written in slope–intercept form, y=mx+b. Now, we will do the reverse—we will start with the slope and *y*-intercept and use them to find the equation of the line.

Example 2.9.1

Find an equation of a line with slope −7 and *y*-intercept (0,−1).

Solution

Since we are given the slope and *y*-intercept of the line, we can substitute the needed values into the slope–intercept form, y=mx+b.

Name the slope.	$m = -7$
Name the *y*-intercept.	*y*-intercept $(0, -1)$
Substitute the values into y=mx+b.	$y = mx + b$
	$y = -7x + (-1)$
	$y = -7x + -1$

Try It 2.9.2

Find an equation of a line with slope $\frac{2}{5}$ and y-intercept (0,4).

Answer

$y = \frac{2}{5}x + 4$

Try It 2.9.3

Find an equation of a line with slope −1 and y-intercept (0,−3).

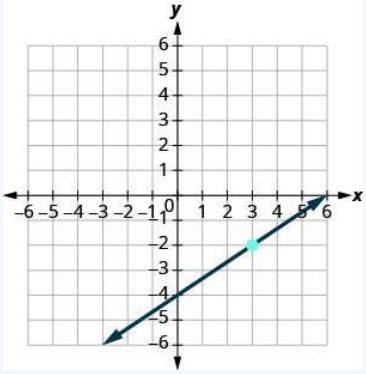
Answer

$y = -x - 3$

Sometimes, the slope and intercept need to be determined from the graph.

Example 2.9.4

Find the equation of the line shown.



Solution

We need to find the slope and y-intercept of the line from the graph so we can substitute the needed values into the slope–intercept form, $y = mx + b$.

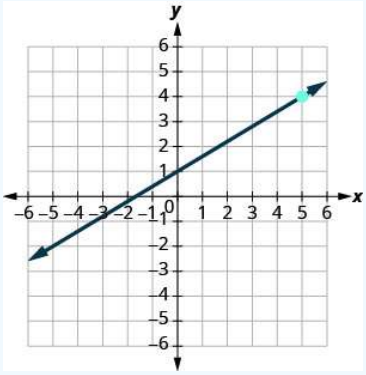
To find the slope, we choose two points on the graph.

The y-intercept is (0,−4) and the graph passes through (3,−2).

Find the slope by counting the rise and run.	$m = \frac{\text{rise}}{\text{run}}$
	$m = \frac{2}{3}$
Find the y-intercept.	y-intercept (0, −4)
Substitute the values into y=mx+b.	$y = mx + b$
	$y = \frac{2}{3}x - 4$

Try It 2.9.5

Find the equation of the line shown in the graph.

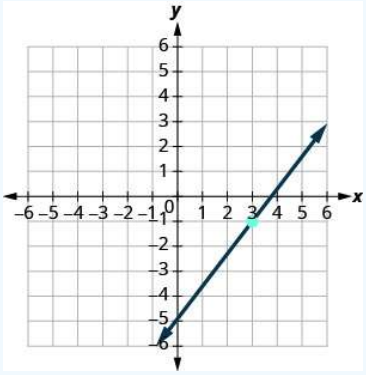


Answer

$y = \frac{3}{5}x + 1$

Try It 2.9.6

Find the equation of the line shown in the graph.



Answer

$y = \frac{4}{3}x - 5$

Find an Equation of the Line Given the Slope and a Point

Finding an equation of a line using the slope–intercept form of the equation works well when you are given the slope and *y*-intercept or when you read them off a graph. But what happens when you have another point instead of the *y*-intercept?

We are going to use the slope formula to derive another form of an equation of the line. Suppose we have a line that has slope mm and that contains some specific point (x_1, y_1) and some other point, which we will just call (x,y). We can write the slope of this line and then change it to a different form.

	m	$= \frac{y-y_1}{x-x_1}$
Multiply both sides of the equation by $x - x_1$.	$m(x-x_1)$	$= \left(\frac{y-y_1}{x-x_1}\right)(x-x_1)$
Simplify.	$m(x-x_1)$	$= y - y_1$
Rewrite the equation with the y terms on the left.	$y - y_1$	$= m(x-x_1)$

This format is called the point–slope form of an equation of a line.

POINT–SLOPE FORM OF AN EQUATION OF A LINE

The **point–slope form** of an equation of a line with slope m and containing the point (x_1, y_1) is

$$y - y_1 = m(x - x_1)$$

We can use the point–slope form of an equation to find an equation of a line when we are given the slope and one point. Then we will rewrite the equation in slope–intercept form. Most applications of linear equations use the the slope–intercept form.

Example 2.9.7: Find an Equation of a Line Given the Slope and a Point

Find an equation of a line with slope $m = \frac{2}{5}$ that contains the point (10,3). Write the equation in slope–intercept form.

Solution

Step 1. Identify the slope.	The slope is given.	$m = \frac{2}{5}$
Step 2. Identify the point.	The point is given.	(x_1, y_1) $(10, 3)$
Step 3. Substitute the values into the point–slope form, $y - y_1 = m(x - x_1)$.		$y - y_1 = m(x - x_1)$
		$y - 3 = \frac{2}{5}(x - 10)$
	Simplify.	$y - 3 = \frac{2}{5}x - 4$
Step 4. Write the equation in slope–intercept form.		$y = \frac{2}{5}x - 1$

Try It 2.9.8

Find an equation of a line with slope $m = \frac{5}{6}$ and containing the point (6,3).

Answer

$y = \frac{5}{6}x - 2$

Try It 2.9.9

Find an equation of a line with slope $m = \frac{2}{3}$ and containing the point (9,2).

Answer

$y = \frac{2}{3}x - 4$

FIND AN EQUATION OF A LINE GIVEN THE SLOPE AND A POINT.

1. Identify the slope.
2. Identify the point.
3. Substitute the values into the point-slope form, $y - y_1 = m(x - x_1)$.
4. Write the equation in slope–intercept form.

Example 2.9.10

Find an equation of a line with slope $m = -\frac{1}{3}$ that contains the point (6,−4). Write the equation in slope–intercept form.

Solution

Since we are given a point and the slope of the line, we can substitute the needed values into the point–slope form, $y - y_1 = m(x - x_1)$.

Identify the slope.	$m = -\frac{1}{3}$
Identify the point.	$\begin{matrix} x_1, & y_1 \\ 6, & -4 \end{matrix}$
Substitute the values into $y - y_1 = m(x - x_1)$.	$y - y_1 = m(x - x_1)$
	$y - (-4) = -\frac{1}{3}(x - 6)$
Simplify.	$y + 4 = -\frac{1}{3}x + 2$
Write in slope–intercept form.	$y = -\frac{1}{3}x - 2$

Try It 2.9.11

Find an equation of a line with slope $m = -\frac{2}{5}$ and containing the point (10,−5).

Answer

$y = -\frac{2}{5}x - 1$

Try It 2.9.12

Find an equation of a line with slope $m = -\frac{3}{4}$, and containing the point (4,−7).

Answer

$y = -\frac{3}{4}x - 4$

Example 2.9.13

Find an equation of a horizontal line that contains the point (−1,2). Write the equation in slope–intercept form.

Solution

Every horizontal line has slope 0. We can substitute the slope and points into the point–slope form, $y - y_1 = m(x - x_1)$.

Identify the slope.	$m = 0$
Identify the point.	$\begin{matrix} x_1, & y_1 \\ -1, & 2 \end{matrix}$
Substitute the values into $y - y_1 = m(x - x_1)$.	$y - y_1 = m(x - x_1)$
	$y - 2 = 0(x - (-1))$
Simplify.	$y - 2 = 0(x + 1)$
	$y - 2 = 0$
	$y = 2$

Write in slope–intercept form.	It is in *y*-form, but could be written y=0x+2.

Did we end up with the form of a horizontal line, y=a?

? Try It 2.9.14

Find an equation of a horizontal line containing the point (−3,8).

Answer

y = 8

? Try It 2.9.15

Find an equation of a horizontal line containing the point (−1,4).

Answer

y = 4

Find an Equation of the Line Given Two Points

When real-world data is collected, a linear model can be created from two data points. In the next example we'll see how to find an equation of a line when just two points are given.

We have two options so far for finding an equation of a line: slope–intercept or point–slope. Since we will know two points, it will make more sense to use the point–slope form.

But then we need the slope. Can we find the slope with just two points? Yes. Then, once we have the slope, we can use it and one of the given points to find the equation.

✓ Example 2.9.16: Find an Equation of a Line Given Two Points

Find an equation of a line that contains the points (5,4) and (3,6). Write the equation in slope–intercept form.

Solution

Step 1. Find the slope using the given points.	To use the point-slope form, we first find the slope.	$m=\frac{y_2-y_1}{x_2-x_1}$ $m=\frac{6-4}{3-5}$ $m=\frac{2}{-2}$ $m=-1$
Step 2. Choose one point.	Choose either point.	$(\overset{x_1}{5},\ \overset{y_1}{4})$
Step 3. Substitute the values into the point-slope form, $y-y_1=m(x-x_1)$.	Simplify.	$y-y_1=m(x-x_1)$ $y-4=-1(x-5)$ $y-4=-1x+5$
Step 4. Write the equation in slope–intercept form.		$y=-1x+9$

Use the point (3,6) and see that you get the same equation.

Try It 2.9.17

Find an equation of a line containing the points (3,1) and (5,6).

Answer

$y = \frac{5}{2}x - \frac{13}{2}$

Try It 2.9.18

Find an equation of a line containing the points (1,4) and (6,2).

Answer

$y = -\frac{2}{5}x + \frac{22}{5}$

FIND AN EQUATION OF A LINE GIVEN TWO POINTS.

1. Find the slope using the given points.
2. Choose one point.
3. Substitute the values into the point-slope form, $y - y_1 = m(x - x_1)$.
4. Write the equation in slope–intercept form.

Example 2.9.19

Find an equation of a line that contains the points (−3,−1) and (2,−2). Write the equation in slope–intercept form.

Solution

Since we have two points, we will find an equation of the line using the point–slope form. The first step will be to find the slope.

Find the slope of the line through (−3, −1) and (2, −2).	$m = \frac{y_2 - y_1}{x_2 - x_1}$
	$m = \frac{-2-(-1)}{2-(-3)}$
	$m = \frac{-1}{5}$
	$m = -\frac{1}{5}$
Choose either point.	$(2, -2)$
Substitute the values into $y - y_1 = m(x - x_1)$.	$y - y_1 = m(x - x_1)$
	$y-(-2) = -\frac{1}{5}(x-2)$
	$y + 2 = -\frac{1}{5}x + \frac{2}{5}$
Write in slope–intercept form.	$y = -\frac{1}{5}x - \frac{8}{5}$

Try It 2.9.20

Find an equation of a line containing the points (−2,−4) and (1,−3).

Answer

$y = \frac{1}{3}x - \frac{10}{3}$

Try It 2.9.21

Find an equation of a line containing the points (−4,−3) and (1,−5).

Answer

$y = -\frac{2}{5}x - \frac{23}{5}$

Example 2.9.22

Find an equation of a line that contains the points (−2,4) and (−2,−3). Write the equation in slope–intercept form.

Solution

Again, the first step will be to find the slope.

Find the slope of the line through $(-2, 4)$ and $(-2, -3)$

$$\begin{aligned} m &= \frac{y_2 - x_1}{x_2 - x_1} \\ m &= \frac{-3-4}{-2-(-2)} \\ m &= \frac{-7}{0} \end{aligned}$$

The slope is undefined.

This tells us it is a vertical line. Both of our points have an *x*-coordinate of −2. So our equation of the line is x=−2. Since there is no yy, we cannot write it in slope–intercept form.

You may want to sketch a graph using the two given points. Does the graph agree with our conclusion that this is a vertical line?

Try It 2.9.23

Find an equation of a line containing the points (5,1) and (5,−4).

Answer

x = 5

Try It 2.9.24

Find an equation of a line containing the points (−4,4) and (−4,3).

Answer

x=−4

We have seen that we can use either the slope–intercept form or the point–slope form to find an equation of a line. Which form we use will depend on the information we are given. This is summarized in Table 2.9.1.

Table 2.9.1

To Write an Equation of a Line		
If given:	Use:	Form:
Slope and *y*-intercept	slope–intercept	y=mx+b

Slope and a point	point–slope	$y - y_1 = m(x - x_1)$
Two points	point–slope	$y - y_1 = m(x - x_1)$

Find an Equation of a Line Parallel to a Given Line

Suppose we need to find an equation of a line that passes through a specific point and is parallel to a given line. We can use the fact that parallel lines have the same slope. So we will have a point and the slope—just what we need to use the point–slope equation.

First let's look at this graphically.

The graph shows the graph of y=2x−3. We want to graph a line parallel to this line and passing through the point (−2,1).

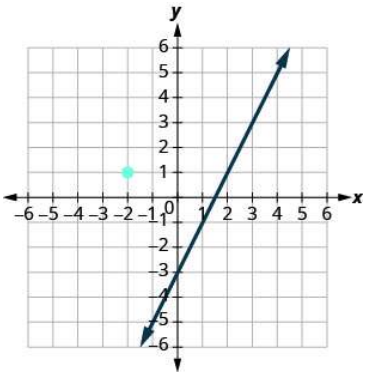


Figure 2.9.1

We know that parallel lines have the same slope. So the second line will have the same slope as y=2x−3. That slope is $m_{\parallel} = 2$. We'll use the notation $m_{\parallel}$ to represent the slope of a line parallel to a line with slope m. (Notice that the subscript ∥ looks like two parallel lines.)

The second line will pass through (−2,1) and have m=2. To graph the line, we start at (−2,1) and count out the rise and run. With m=2 (or $m = \frac{2}{1}$), we count out the rise 2 and the run 1. We draw the line.

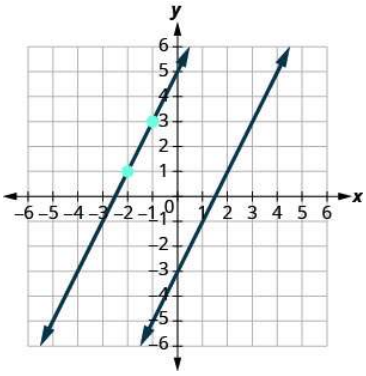


Figure 2.9.2

Do the lines appear parallel? Does the second line pass through (−2,1)?

Now, let's see how to do this algebraically.

We can use either the slope–intercept form or the point–slope form to find an equation of a line. Here we know one point and can find the slope. So we will use the point–slope form.

Example 2.9.25: How to Find an Equation of a Line Parallel to a Given Line

Find an equation of a line parallel to y=2x−3 that contains the point (−2,1). Write the equation in slope–intercept form.

Solution

Step 1. Find the slope of the given line.	The line is in slope–intercept form, $y = 2x - 3$.	$m = 2$
Step 2. Find the slope of the parallel line.	Parallel lines have the same slope.	$m_{\parallel} = 2$
Step 3. Identify the point.	The given point is, (–2, 1).	$\begin{pmatrix} x_1 & y_1 \\ -2, & 1 \end{pmatrix}$
Step 4. Substitute the values into the point–slope form, $y - y_1 = m(x - x_1)$.	Simplify.	$y - y_1 = m(x - x_1)$ $y - 1 = 2(x - (-2))$ $y - 1 = 2(x + 2)$ $y - 1 = 2x + 4$
Step 5. Write the equation in slope–intercept form.		$y = 2x + 5$

Does this equation make sense? What is the *y*-intercept of the line? What is the slope?

? Try It 2.9.26

Find an equation of a line parallel to the line y=3x+1 that contains the point (4,2). Write the equation in slope–intercept form.

Answer

y=3x−10

? Try It 2.9.27

Find an equation of a line parallel to the line $y = \frac{1}{2}x - 3$ that contains the point (6,4).

Answer

$y = \frac{1}{2}x + 1$

FIND AN EQUATION OF A LINE PARALLEL TO A GIVEN LINE.

1. Find the slope of the given line.
2. Find the slope of the parallel line.
3. Identify the point.
4. Substitute the values into the point–slope form, $y - y_1 = m(x - x_1)$.
5. Write the equation in slope–intercept form.

Find an Equation of a Line Perpendicular to a Given Line

Now, let's consider perpendicular lines. Suppose we need to find a line passing through a specific point and which is perpendicular to a given line. We can use the fact that perpendicular lines have slopes that are negative reciprocals. We will again use the point–slope equation, like we did with parallel lines.

The graph shows the graph of y=2x−3. Now, we want to graph a line perpendicular to this line and passing through (−2,1).

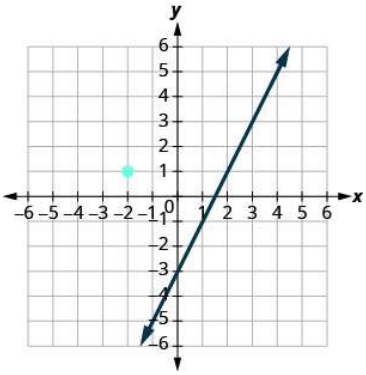


Figure 2.9.3

We know that perpendicular lines have slopes that are negative reciprocals. We'll use the notation $m_{\perp}$ to represent the slope of a line perpendicular to a line with slope m. (Notice that the subscript $_{\perp}$ looks like the right angles made by two perpendicular lines.)

$$\begin{aligned} y &= 2x - 3 \quad \text{perpendicular line} \\ m &= 2 \qquad m_{\perp} = -\tfrac{1}{2} \end{aligned} \tag{2.9.1}$$

We now know the perpendicular line will pass through (−2,1) with $m_{\perp} = -\frac{1}{2}$.

To graph the line, we will start at (−2,1) and count out the rise −1 and the run 2. Then we draw the line.

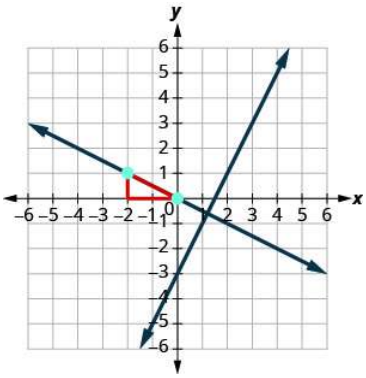


Figure 2.9.4

Do the lines appear perpendicular? Does the second line pass through (−2,1)?

Now, let's see how to do this algebraically. We can use either the slope–intercept form or the point–slope form to find an equation of a line. In this example we know one point, and can find the slope, so we will use the point–slope form.

Example 2.9.28

Find an equation of a line perpendicular to y=2x−3 that contains the point (−2,1). Write the equation in slope–intercept form.

Solution

Step 1. Find the slope of the given line.	The line is in slope–intercept form, $y = 2x - 3$.	$m = 2$
Step 2. Find the slope of the perpendicular line.	The slopes of perpendicular lines are negative reciprocals.	$m_{\perp} = -\frac{1}{2}$
Step 3. Identify the point.	The given point is, (–2, 1)	(x_1, y_1) $(-2, 1)$

Step 4. Substitute the values into the point–slope form, $y-y_1=m(x-x_1)$.	Simplify.	$y-y_1=m(x-x_1)$ $y-1=-\frac{1}{2}(x-(-2))$ $y-1=-\frac{1}{2}(x+2)$ $y-1=-\frac{1}{2}x-1$
Step 5. Write the equation in slope–intercept form.		$y=-\frac{1}{2}x$

Try It 2.9.29

Find an equation of a line perpendicular to the line y=3x+1 that contains the point (4,2). Write the equation in slope–intercept form.

Answer

$y=-\frac{1}{3}x+\frac{10}{3}$

Try It 2.9.30

Find an equation of a line perpendicular to the line $y=\frac{1}{2}x-3$ that contains the point (6,4).

Answer

y=−2x+16

FIND AN EQUATION OF A LINE PERPENDICULAR TO A GIVEN LINE.

1. Find the slope of the given line.
2. Find the slope of the perpendicular line.
3. Identify the point.
4. Substitute the values into the point–slope form, $y-y_1=m(x-x_1)$.
5. Write the equation in slope–intercept form.

Example 2.9.31

Find an equation of a line perpendicular to x=5 that contains the point (3,−2). Write the equation in slope–intercept form.

Solution

Again, since we know one point, the point–slope option seems more promising than the slope–intercept option. We need the slope to use this form, and we know the new line will be perpendicular to x=5. This line is vertical, so its perpendicular will be horizontal. This tells us the $m_\perp=0$.

Identify the point.	$(3$	,	$-2)$
Identify the slope of the perpendicular line.	$m_\perp$	$=$	0
Substitute the values into $y-y_1=m\,(x-x_1)$.	$y-y_1$	$=$	$m\,(x-x_1)$
	$y-(-2)$	$=$	$0(x-3)$
Simplify.	$y+2$	$=$	0
	y	$=$	-2

Sketch the graph of both lines. Do they appear to be perpendicular?

Try It 2.9.32

Find an equation of a line that is perpendicular to the line $x=4$ that contains the point $(4,-5)$. Write the equation in slope–intercept form.

Answer

$y=-5$

Try It 2.9.33

Find an equation of a line that is perpendicular to the line $x=2$ that contains the point $(2,-1)$. Write the equation in slope–intercept form.

Answer

$y=-1$

In Example 2.9.31, we used the point–slope form to find the equation. We could have looked at this in a different way.

We want to find a line that is perpendicular to $x=5$ that contains the point $(3,-2)$. The graph shows us the line $x=5$ and the point $(3,-2)$.

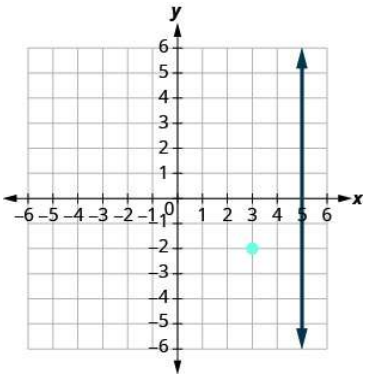


Figure 2.9.5

We know every line perpendicular to a vertical line is horizontal, so we will sketch the horizontal line through $(3,-2)$.

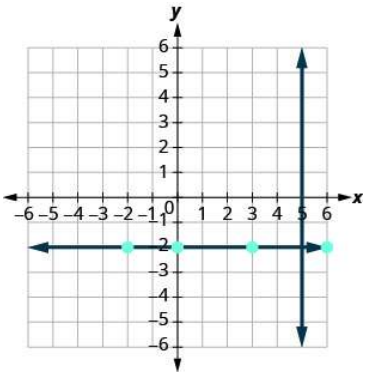


Figure 2.9.6

Do the lines appear perpendicular?

If we look at a few points on this horizontal line, we notice they all have *y*-coordinates of -2. So, the equation of the line perpendicular to the vertical line $x=5$ is $y=-2$.

Example 2.9.34

Find an equation of a line that is perpendicular to y=−4 that contains the point (−4,2). Write the equation in slope–intercept form.

Solution

The line y=−4 is a horizontal line. Any line perpendicular to it must be vertical, in the form x=a. Since the perpendicular line is vertical and passes through (−4,2), every point on it has an *x*-coordinate of −4. The equation of the perpendicular line is x=−4. You may want to sketch the lines. Do they appear perpendicular?

Try It 2.9.35

Find an equation of a line that is perpendicular to the line y=1 that contains the point (−5,1). Write the equation in slope–intercept form.

Answer

x=−5

Try It 2.9.36

Find an equation of a line that is perpendicular to the line y=−5 that contains the point (−4,−5).

Answer

x=−4

Note

Access this online resource for additional instruction and practice with finding the equation of a line.

- Use the Point-Slope Form of an Equation of a Line

Key Concepts

- **To Find an Equation of a Line Given the Slope and a Point**
  1. Identify the slope.
  2. Identify the point.
  3. Substitute the values into the point-slope form, $y-y_1=m(x-x_1)$.
  4. Write the equation in slope-intercept form.
- **To Find an Equation of a Line Given Two Points**
  1. Find the slope using the given points.
  2. Choose one point.
  3. Substitute the values into the point-slope form, $y-y_1=m(x-x_1)$.
  4. Write the equation in slope-intercept form.
- **To Write an Equation of a Line**
  - If given slope and y-intercept, use slope–intercept form $y=mx+b$.
  - If given slope and a point, use point–slope form $y-y_1=m(x-x_1)$.
  - If given two points, use point–slope form $y-y_1=m(x-x_1)$.
- **To Find an Equation of a Line Parallel to a Given Line**
  1. Find the slope of the given line.
  2. Find the slope of the parallel line.
  3. Identify the point.
  4. Substitute the values into the point-slope form, $y-y_1=m(x-x_1)$.

5. Write the equation in slope-intercept form.

- **To Find an Equation of a Line Perpendicular to a Given Line**
  1. Find the slope of the given line.
  2. Find the slope of the perpendicular line.
  3. Identify the point.
  4. Substitute the values into the point-slope form, $y - y_1 = m(x - x_1)$.
  5. Write the equation in slope-intercept form.

Glossary

point–slope form

The point–slope form of an equation of a line with slope mm and containing the point (x_1, y_1) is $y - y_1 = m\,(x - x_1)$.

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2.9E: Exercises

Practice Makes Perfect

Find an Equation of the Line Given the Slope and y-Intercept

In the following exercises, find the equation of a line with given slope and y-intercept. Write the equation in slope–intercept form.

Exercise 2.9E.1

slope 3 and y-intercept $(0, 5)$

Exercise 2.9E.2

slope 4 and y-intercept $(0, 1)$

Answer

$y = 4x + 1$

Exercise 2.9E.3

slope 6 and y-intercept $(0, -4)$

Exercise 2.9E.4

slope 8 and y-intercept $(0, -6)$

Answer

$y = 8x - 6$

Exercise 2.9E.5

slope -1 and y-intercept $(0, 3)$

Exercise 2.9E.6

slope -1 and y-intercept $(0, 7)$

Answer

$y = -x + 7$

Exercise 2.9E.7

slope -2 and y-intercept $(0, -3)$

Exercise 2.9E.8

slope -3 and y-intercept $(0, -1)$

Answer

$y = -3x - 1$

Exercise 2.9E.9

slope $\frac{3}{5}$ and y-intercept $(0, -1)$

Exercise 2.9E.10

slope $\frac{1}{5}$ and y-intercept $(0, -5)$

Answer

$y = \frac{1}{5}x - 5$

Exercise 2.9E.11

slope $-\frac{3}{4}$ and y-intercept $(0, -2)$

Exercise 2.9E.12

slope $-\frac{2}{3}$ and y-intercept $(0, -3)$

Answer

$y = -\frac{2}{3}x - 3$

Exercise 2.9E.13

slope 0 and y-intercept $(0, -1)$

Exercise 2.9E.14

slope 0 and y-intercept $(0, 2)$

Answer

$y = 2$

Exercise 2.9E.15

slope -3 and y-intercept $(0, 0)$

Exercise 2.9E.16

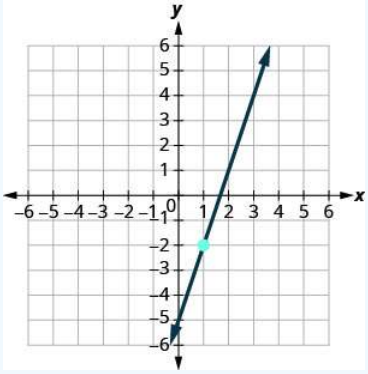
slope -4 and y-intercept $(0, 0)$

Answer

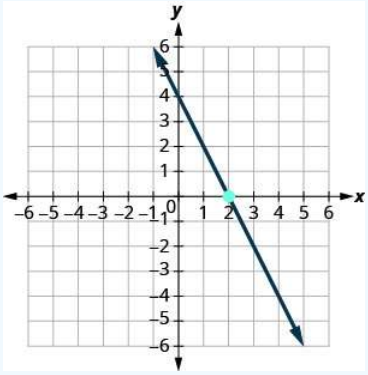
$y = -4x$

In the following exercises, find the equation of the line shown in each graph. Write the equation in slope–intercept form.

Exercise 2.9E.17



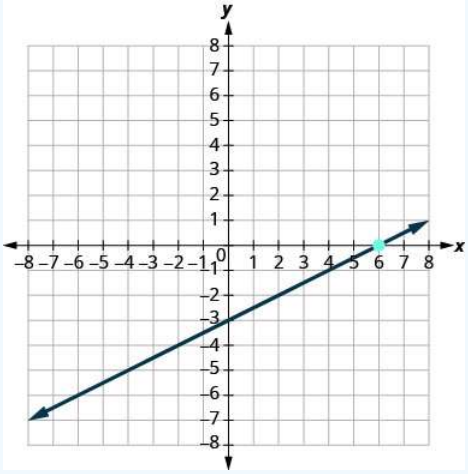
Exercise 2.9E. 18



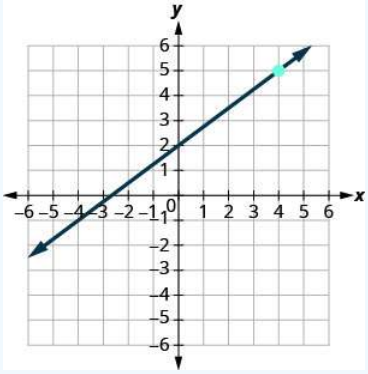
Answer

$y = -2x + 4$

Exercise 2.9E. 19



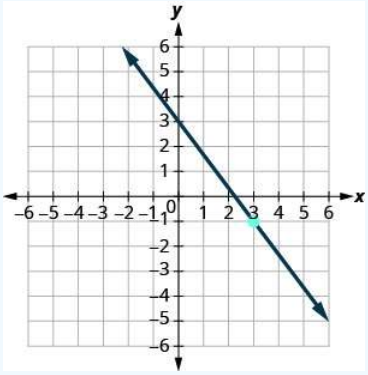
Exercise 2.9E. 20



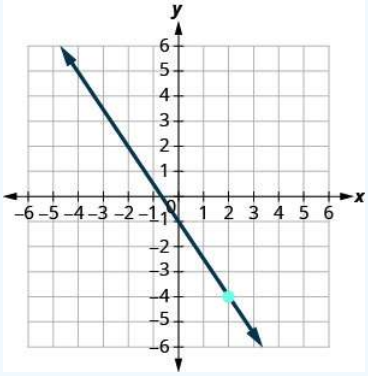
Answer

$y = \frac{3}{4}x + 2$

Exercise $2.9E.21$



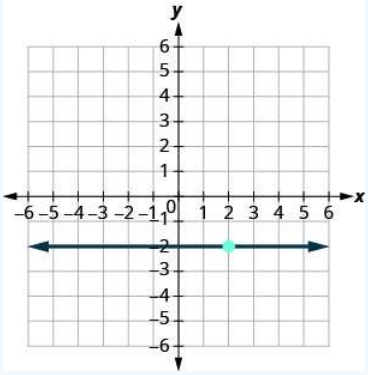
Exercise $2.9E.22$



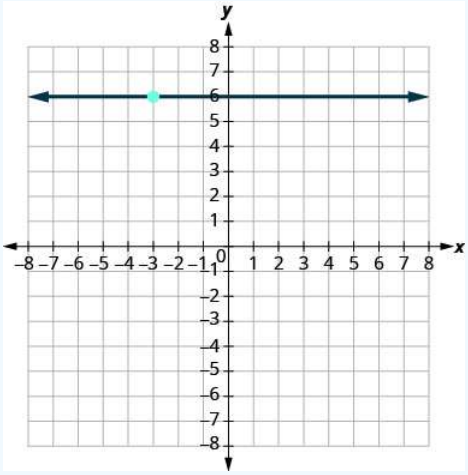
Answer

$y = -\frac{3}{2}x - 1$

Exercise $2.9E.23$



Exercise 2.9E.24



Answer

$y = 6$

Find an Equation of the Line Given the Slope and a Point

In the following exercises, find the equation of a line with given slope and containing the given point. Write the equation in slope–intercept form.

Exercise 2.9E.25

$m = \frac{5}{8}$, point $(8, 3)$

Exercise 2.9E.26

$m = \frac{3}{8}$, point $(8, 2)$

Answer

$y = \frac{3}{8}x - 1$

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Exercise 2.9E.27

$m = \frac{1}{6}$, point $(6, 1)$

Exercise 2.9E.28

$m = \frac{5}{6}$, point $(6, 7)$

Answer

$y = \frac{5}{6}x + 2$

Exercise 2.9E.29

$m = -\frac{3}{4}$, point $(8, -5)$

Exercise 2.9E.30

$m = -\frac{3}{5}$, point $(10, -5)$

Answer

$y = -\frac{3}{5}x + 1$

Exercise 2.9E.31

$m = -\frac{1}{4}$, point $(-12, -6)$

Exercise 2.9E.32

$m = -\frac{1}{3}$, point $(-9, -8)$

Answer

$y = -\frac{1}{3}x - 11$

Exercise 2.9E.33

Horizontal line containing $(-2, 5)$

Exercise 2.9E.34

Horizontal line containing $(-1, 4)$

Answer

$y = 4$

Exercise 2.9E.35

Horizontal line containing $(-2, -3)$

Exercise 2.9E.36

Horizontal line containing $(-1, -7)$

Answer

$y = -7$

Exercise 2.9E.37

$m = -\frac{3}{2}$, point $(-4, -3)$

Exercise 2.9E.38

$m = -\frac{5}{2}$, point $(-8, -2)$

Answer

$y = -\frac{5}{2}x - 22$

Exercise 2.9E.39

$m = -7$, point $(-1, -3)$

Exercise 2.9E.40

$m = -4$, point $(-2, -3)$

Answer

$y = -4x - 11$

Exercise 2.9E.41

Horizontal line containing $(2, -3)$

Exercise 2.9E.42

Horizontal line containing $(4, -8)$

Answer

$y = -8$

Find an Equation of the Line Given Two Points

In the following exercises, find the equation of a line containing the given points. Write the equation in slope–intercept form.

Exercise 2.9E.43

$(2, 6)$ and $(5, 3)$

Exercise 2.9E.44

$(3, 1)$ and $(2, 5)$

Answer

$y = -4x + 13$

Exercise 2.9E.45

$(4, 3)$ and $(8, 1)$

Exercise 2.9E.46

$(2, 7)$ and $(3, 8)$

Answer

$y = x + 5$

Exercise 2.9E.47

$(-3, -4)$ and $(5, -2)$

Exercise 2.9E.48

$(-5, -3)$ and $(4, -6)$

Answer

$y = -\frac{1}{3}x - \frac{14}{3}$

Exercise 2.9E.49

$(-1, 3)$ and $(-6, -7)$

Exercise 2.9E.50

$(-2, 8)$ and $(-4, -6)$

Answer

$y = 7x + 22$

Exercise 2.9E.51

$(6, -4)$ and $(-2, 5)$

Exercise 2.9E.52

$(3, -2)$ and $(-4, 4)$

Answer

$y = -\frac{6}{7}x + \frac{4}{7}$

Exercise 2.9E.53

$(0, 4)$ and $(2, -3)$

Exercise 2.9E.54

$(0, -2)$ and $(-5, -3)$

Answer

$y = \frac{1}{5}x - 2$

Exercise 2.9E.55

$(7, 2)$ and $(7, -2)$

Exercise 2.9E.56

$(4, 2)$ and $(4, -3)$

Answer

$x = 4$

Exercise 2.9E.57

$(-7, -1)$ and $(-7, -4)$

Exercise 2.9E.58

$(-2, 1)$ and $(-2, -4)$

Answer

$x = -2$

Exercise 2.9E.59

$(6, 1)$ and $(0, 1)$

Exercise 2.9E.60

$(6, 2)$ and $(-3, 2)$

Answer

$y = 2$

Exercise 2.9E.61

$(3, -4)$ and $(5, -4)$

Exercise 2.9E.62

$(-6, -3)$ and $(-1, -3)$

Answer

$y = -3$

Exercise 2.9E.63

$(4, 3)$ and $(8, 0)$

Exercise 2.9E.64

$(0, 0)$ and $(1, 4)$

Answer

$y = 4x$

Exercise 2.9E.65

$(-2, -3)$ and $(-5, -6)$

Exercise 2.9E.66

$(-3, 0)$ and $(-7, -2)$

Answer

$y = \frac{1}{2}x + \frac{3}{2}$

Exercise 2.9E.67

$(8, -1)$ and $(8, -5)$

Exercise 2.9E.68

$(3, 5)$ and $(-7, 5)$

Answer

$y = 5$

Find an Equation of a Line Parallel to a Given Line

In the following exercises, find an equation of a line parallel to the given line and contains the given point. Write the equation in slope–intercept form.

Exercise 2.9E.69

line $y = 4x + 2$, point $(1, 2)$

Exercise 2.9E.70

line $y = 3x + 4$, point $(2, 5)$

Answer

$y = 3x - 1$

Exercise 2.9E.71

line $y = -2x - 3$, point $(-1, 3)$

Exercise 2.9E.72

line $y = -3x - 1$, point $(2, -3)$

Answer

$y = -3x + 3$

Exercise 2.9E.73

line $3x - y = 4$, point $(3, 1)$

Exercise 2.9E.74

line $2x - y = 6$, point $(3, 0)$

Answer

$y = 2x - 6$

Exercise 2.9E.75

line $4x + 3y = 6$, point $(0, -3)$

Exercise 2.9E.76

line $2x + 3y = 6$, point $(0, 5)$

Answer

$y = -\frac{2}{3}x + 5$

Exercise 2.9E.77

line $x = -3$, point $(-2, -1)$

Exercise 2.9E.78

line $x = -4$, point $(-3, -5)$

Answer

$x = -3$

Exercise 2.9E.79

line $x - 2 = 0$, point $(1, -2)$

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Exercise 2.9E.80

line $x-6=0$, point $(4,-3)$

Answer

$x=4$

Exercise 2.9E.81

line $y=5$, point $(2,-2)$

Exercise 2.9E.82

line $y=1$, point $(3,-4)$

Answer

$y=-4$

Exercise 2.9E.83

line $y+2=0$, point $(3,-3)$

Exercise 2.9E.84

line $y+7=0$, point $(1,-1)$

Answer

$y=-1$

Find an Equation of a Line Perpendicular to a Given Line

In the following exercises, find an equation of a line perpendicular to the given line and contains the given point. Write the equation in slope–intercept form.

Exercise 2.9E.85

line $y=-2x+3$, point $(2,2)$

Exercise 2.9E.86

line $y=-x+5$, point $(3,3)$

Answer

$y=x$

Exercise 2.9E.87

line $y=\frac{3}{4}x-2$, point $(-3,4)$

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Exercise 2.9E.88

line $y = \frac{2}{3}x - 4$, point $(2, -4)$

Answer

$y = -\frac{3}{2}x - 1$

Exercise 2.9E.89

line $2x - 3y = 8$, point $(4, -1)$

Exercise 2.9E.90

line $4x - 3y = 5$, point $(-3, 2)$

Answer

$y = -\frac{3}{4}x - \frac{1}{4}$

Exercise 2.9E.91

line $2x + 5y = 6$, point $(0, 0)$

Exercise 2.9E.92

line $4x + 5y = -3$, point $(0, 0)$

Answer

$y = \frac{5}{4}x$

Exercise 2.9E.93

line $y - 3 = 0$, point $(-2, -4)$

Exercise 2.9E.94

line $y - 6 = 0$, point $(-5, -3)$

Answer

$x = -5$

Exercise 2.9E.95

line y-axis, point $(3, 4)$

Exercise 2.9E.96

line y-axis, point $(2, 1)$

Answer

$y = 1$

Mixed Practice

In the following exercises, find the equation of each line. Write the equation in slope–intercept form.

Exercise 2.9E.97

Containing the points $(4, 3)$ and $(8, 1)$

Exercise 2.9E.98

Containing the points $(2, 7)$ and $(3, 8)$

Answer

$y = x + 5$

Exercise 2.9E.99

$m = \frac{1}{6}$, containing point $(6, 1)$

Exercise 2.9E.100

$m = \frac{5}{6}$, containing point $(6, 7)$

Answer

$y = \frac{5}{6}x + 2$

Exercise 2.9E.101

Parallel to the line $4x + 3y = 6$, containing point $(0, -3)$

Exercise 2.9E.102

Parallel to the line $2x + 3y = 6$, containing point $(0, 5)$

Answer

$y = -\frac{2}{3}x + 5$

Exercise 2.9E.103

$m = -\frac{3}{4}$, containing point $(8, -5)$

Exercise 2.9E.104

$m = -\frac{3}{5}$, containing point $(10, -5)$

Answer

$y = -\frac{3}{5}x + 1$

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Exercise 2.9E.105

Perpendicular to the line $y-1=0,$ point $(-2,6)$

Exercise 2.9E.106

Perpendicular to the line y-axis, point $(-6,2)$

Answer

$y=2$

Exercise 2.9E.107

Containing the points $(4,3)$ and $(8,1)$

Exercise 2.9E.108

Containing the points $(-2,0)$ and $(-3,-2)$

Answer

$y=x+2$

Exercise 2.9E.109

Parallel to the line $x=-3,$ containing point $(-2,-1)$

Exercise 2.9E.110

Parallel to the line $x=-4,$ containing point $(-3,-5)$

Answer

$x=-3$

Exercise 2.9E.111

Containing the points $(-3,-4)$ and $(2,-5)$

Exercise 2.9E.112

Containing the points $(-5,-3)$ and $(4,-6)$

Answer

$y=-\frac{1}{3}x-\frac{14}{3}$

Exercise 2.9E.113

Perpendicular to the line $x-2y=5,$ containing point $(-2,2)$

Exercise 2.9E.114

Perpendicular to the line $4x+3y=1$, containing point $(0,0)$

Answer

$y=\frac{3}{4}x$

Everyday Math

Exercise 2.9E.115

Cholesterol. The age, x, and LDL cholesterol evel, y, of two men are given by the points $(18,68)$ and $(27,122)$. Find a linear equation that models the relationship between age and LDL cholesterol level.

Exercise 2.9E.116

Fuel consumption. The city mpg, x, and highway mpg, y, of two cars are given by the points $(29,40)$ and $(19,28)$. Find a linear equation that models the relationship between city mpg and highway mpg.

Answer

$y=1.2x+5.2$

Writing Exercises

Exercise 2.9E.117

Why are all horizontal lines parallel?

Exercise 2.9E.118

Explain in your own words why the slopes of two perpendicular lines must have opposite signs.

Answer

Answers will vary.

Self Check

a. After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
find the equation of the line given the slope and *y*-intercept.			
find an equation of the line given the slope and a point.			
find an equation of the line given two points.			
find an equation of a line parallel to a given line.			
find an equation of a line perpendicular to a given line.			

b. On a scale of 1-10, how would you rate your mastery of this section in light of your responses on the checklist? How can you improve this?

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2.10: Chapter Review Exercises

Chapter 4 Review Exercises

Rectangular Coordinate System

Plot Points in a Rectangular Coordinate System

In the following exercises, plot each point in a rectangular coordinate system.

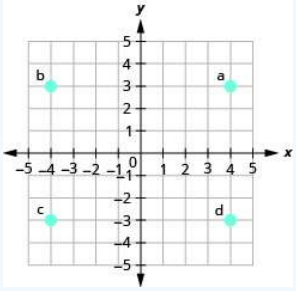
Exercise 2.10.1

1. $(-1,-5)$
2. $(-3,4)$
3. $(2,-3)$
4. $\left(1,\frac{5}{2}\right)$

Exercise 2.10.2

1. $(4,3)$
2. $(-4,3)$
3. $(-4,-3)$
4. $(4,-3)$

Answer



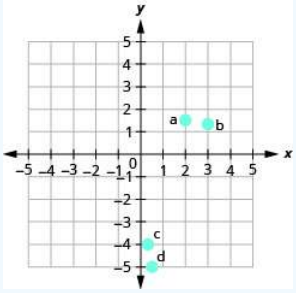
Exercise 2.10.3

1. $(-2,0)$
2. $(0,-4)$
3. $(0,5)$
4. $(3,0)$

Exercise 2.10.4

1. $\left(2,\frac{3}{2}\right)$
2. $\left(3,\frac{4}{3}\right)$
3. $\left(\frac{1}{3},-4\right)$
4. $\left(\frac{1}{2},-5\right)$

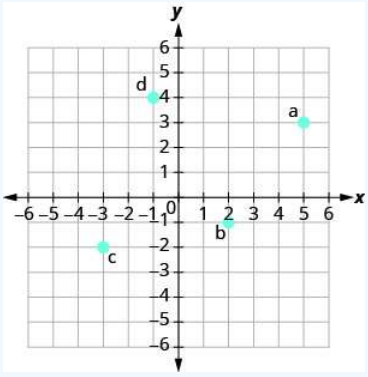
Answer



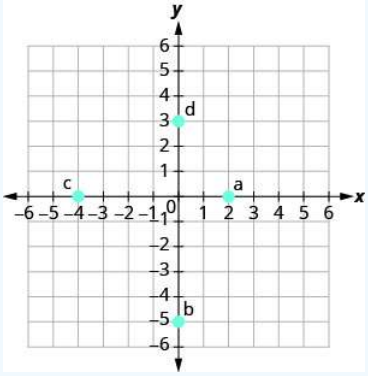
Identify Points on a Graph

In the following exercises, name the ordered pair of each point shown in the rectangular coordinate system.

Exercise 2.10.5



Exercise 2.10.6



Answer

a. (2,0)

b (0,−5)

c (−4.0)

d (0,3)

Verify Solutions to an Equation in Two Variables

In the following exercises, which ordered pairs are solutions to the given equations?

Exercise 2.10.7

$5x + y = 10$

1. (5,1)
2. (2,0)
3. (4,−10)

Exercise 2.10.8

$y = 6x - 2$

1. (1,4)
2. $\left(\frac{1}{3}, 0\right)$
3. (6,−2)

Answer

1, 2

Complete a Table of Solutions to a Linear Equation in Two Variables

In the following exercises, complete the table to find solutions to each linear equation.

Exercise 2.10.9

$y = 4x - 1$

x	y	(x,y)
0		
1		
-2		

Exercise 2.10.10

$y = -\frac{1}{2}x + 3$

x	y	(x,y)
0		
4		
-2		

Answer

x	y	(x,y)
0	3	(0,3)
4	1	(4, 1)
−2	4	(−2,4)

Exercise 2.10.11

$x + 2y = 5$

x	y	(x,y)
	0	
1		
-1		

Exercise 2.10.12

$3x + 2y = 6$

x	y	(x,y)
0		
	0	
-2		

Answer

x	y	(x,y)
0	−3	(0,−3)
2	0	(2,0)
−2	−6	(−2,−6)

Find Solutions to a Linear Equation in Two Variables

In the following exercises, find three solutions to each linear equation.

Exercise 2.10.13

$x + y = 3$

Exercise 2.10.14

$x + y = -4$

Answer

Answers will vary.

Exercise 2.10.15

$y = 3x + 1$

Exercise 2.10.16

$y = -x - 1$

Answer

Answers will vary.

Graphing Linear Equations

Recognize the Relation Between the Solutions of an Equation and its Graph

In the following exercises, for each ordered pair, decide:

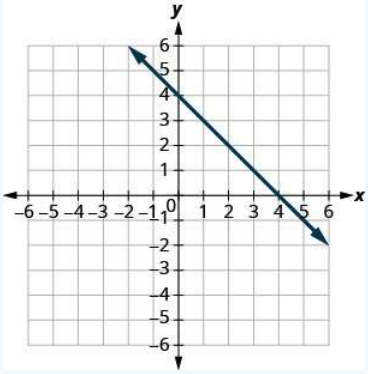
1. Is the ordered pair a solution to the equation?
2. Is the point on the line?

Exercise 2.10.17

$y = -x + 4$

(0,4) (−1,3)

(2,2) (−2,6)

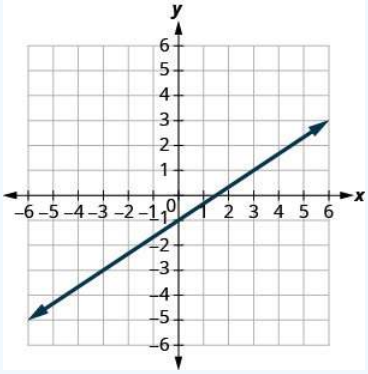


Exercise 2.10.18

$y = \frac{2}{3}x - 1$

$(0, -1)(3, 1)$

$(-3, -3)(6, 4)$



Answer

1. yes; yes
2. yes; no

Graph a Linear Equation by Plotting Points

In the following exercises, graph by plotting points.

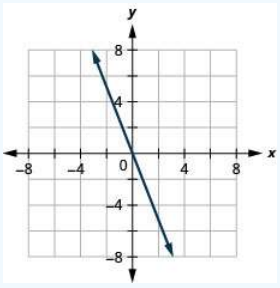
Exercise 2.10.19

$y = 4x - 3$

Exercise 2.10.20

$y = -3x$

Answer



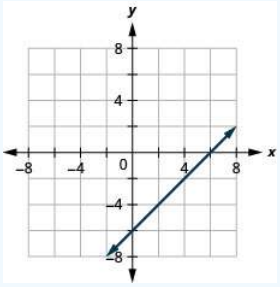
Exercise 2.10.21

$y = \frac{1}{2}x + 3$

Exercise 2.10.22

$x - y = 6$

Answer



Exercise 2.10.23

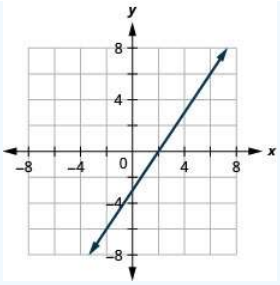
$2x + y = 7$

Exercise 2.10.24

$3x - 2y = 6$

Answer

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Graph Vertical and Horizontal lines

In the following exercises, graph each equation.

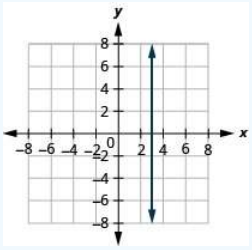
Exercise 2.10.25

$y = -2$

Exercise 2.10.26

$x = 3$

Answer



In the following exercises, graph each pair of equations in the same rectangular coordinate system.

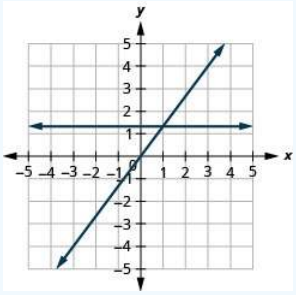
Exercise 2.10.27

$y = -2x$ and $y = -2$

Exercise 2.10.28

$y = \frac{4}{3}x$ and $y = \frac{4}{3}$

Answer

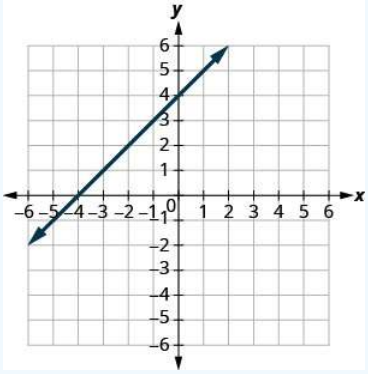


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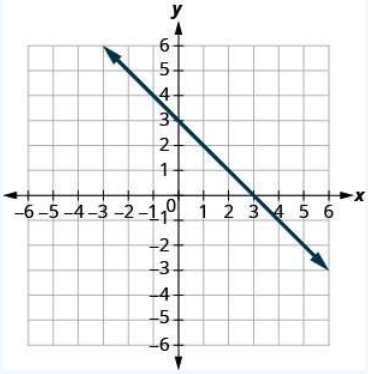
Identify the x- and y-Intercepts on a Graph

In the following exercises, find the x- and y-intercepts.

Exercise 2.10.29



Exercise 2.10.30



Answer

$(3, 0)$ and $(0, 3)$

Find the$x$- and y-Intercepts from an Equation of a Line

In the following exercises, find the intercepts of each equation.

Exercise 2.10.31

$x + y = 5$

Exercise 2.10.32

$x - y = -1$

Answer

$(-1, 0), (0, 1)$

Exercise 2.10.33

$x + 2y = 6$

Exercise 2.10.34

$2x + 3y = 12$

Answer

$(6, 0), (0, 4)$

Exercise 2.10.35

$y = \frac{3}{4}x - 12$

Exercise 2.10.36

$y = 3x$

Answer

$(0, 0)$

Graph a Line Using the Intercepts

In the following exercises, graph using the intercepts.

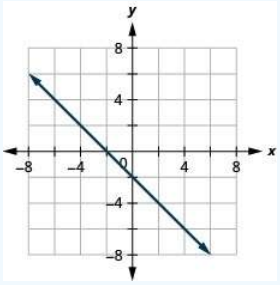
Exercise 2.10.37

$-x + 3y = 3$

Exercise 2.10.38

$x + y = -2$

Answer



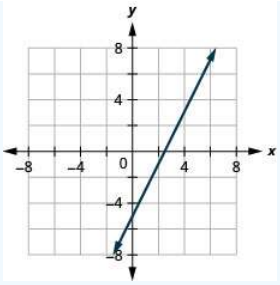
Exercise 2.10.39

$x - y = 4$

Exercise 2.10.40

$2x - y = 5$

Answer



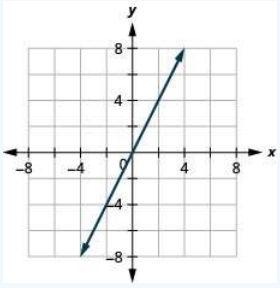
Exercise 2.10.41

$2x - 4y = 8$

Exercise 2.10.42

$y = 2x$

Answer

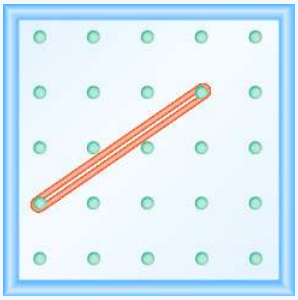


Slope of a Line

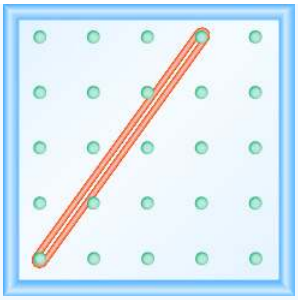
Use Geoboards to Model Slope

In the following exercises, find the slope modeled on each geoboard.

Exercise 2.10.43



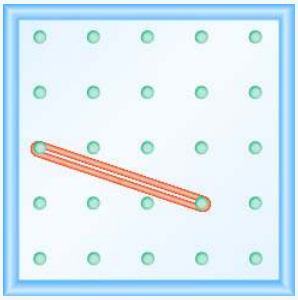
Exercise 2.10.44



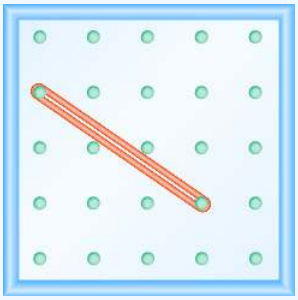
Answer

$\frac{4}{3}$

Exercise 2.10.45



Exercise 2.10.46



Answer

$-\frac{2}{3}$

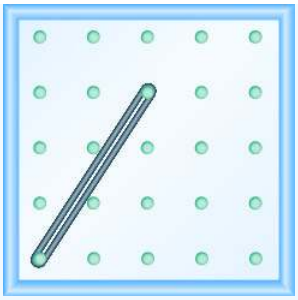
Exercise 2.10.47

$\frac{1}{3}$

Exercise 2.10.48

$\frac{3}{2}$

Answer



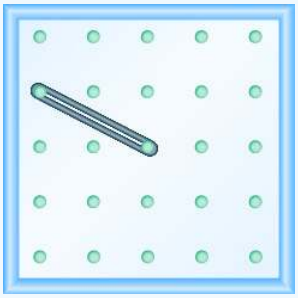
Exercise 2.10.49

$-\frac{2}{3}$

Exercise 2.10.50

$-\frac{1}{2}$

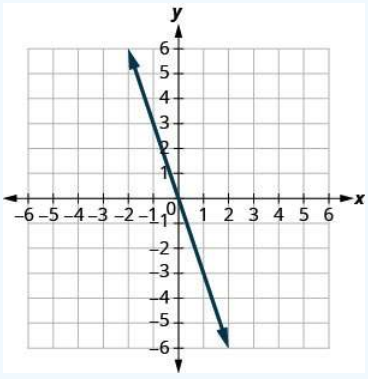
Answer



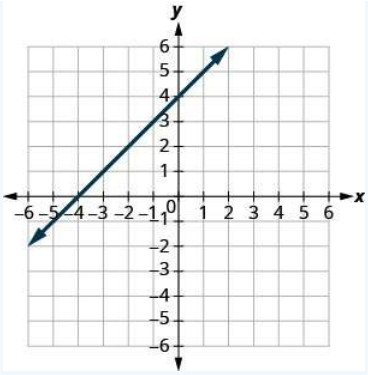
Use $m = \frac{\text{rise}}{\text{run}}$ to find the Slope of a Line from its Graph

In the following exercises, find the slope of each line shown.

Exercise 2.10.51



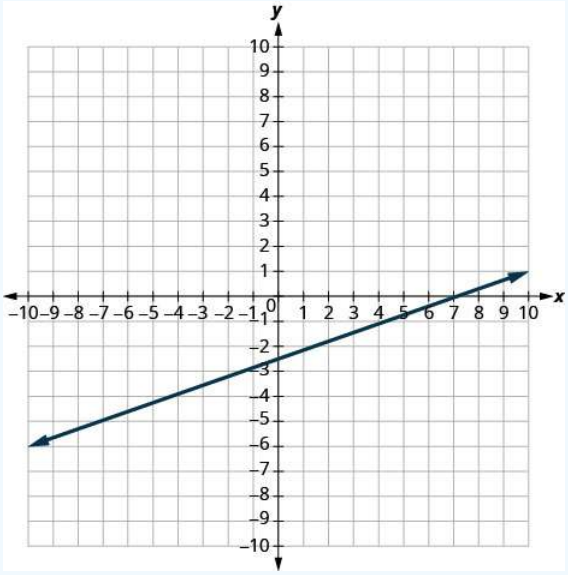
Exercise 2.10.52



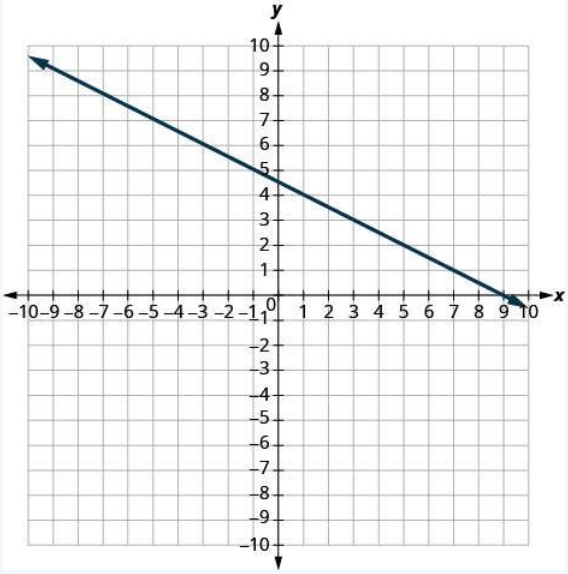
Answer

1

Exercise 2.10.53



Exercise 2.10.54



Answer

$-\frac{1}{2}$

Find the Slope of Horizontal and Vertical Lines

In the following exercises, find the slope of each line.

Exercise 2.10.55

$y = 2$

Exercise 2.10.56

$x = 5$

Answer

undefined

Exercise 2.10.57

$x = -3$

Exercise 2.10.58

$y = -1$

Answer

0

Use the Slope Formula to find the Slope of a Line between Two Points

In the following exercises, use the slope formula to find the slope of the line between each pair of points.

Exercise 2.10.59

$(-1,-1),(0,5)$

Exercise 2.10.60

$(3,5),(4,-1)$

Answer

-6

Exercise 2.10.61

$(-5,-2),(3,2)$

Exercise 2.10.62

$(2,1),(4,6)$

Answer

$\frac{5}{2}$

Graph a Line Given a Point and the Slope

In the following exercises, graph each line with the given point and slope.

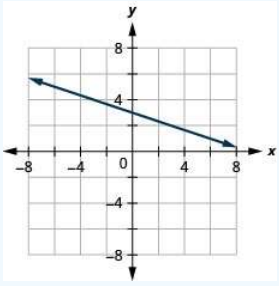
Exercise 2.10.63

$(2,-2);\quad m=\frac{5}{2}$

Exercise 2.10.64

$(-3,4);\quad m=-\frac{1}{3}$

Answer



Exercise 2.10.65

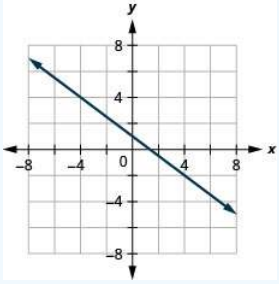
x -intercept $-4;\quad m=3$

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Exercise 2.10.66

y -intercept 1; $m = -\frac{3}{4}$

Answer

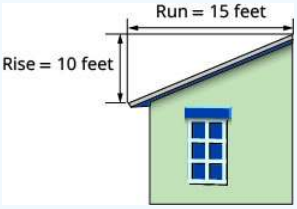


Solve Slope Applications

In the following exercises, solve these slope applications.

Exercise 2.10.67

The roof pictured below has a rise of 10 feet and a run of 15 feet. What is its slope?



Exercise 2.10.68

A mountain road rises 50 feet for a 500-foot run. What is its slope?

Answer

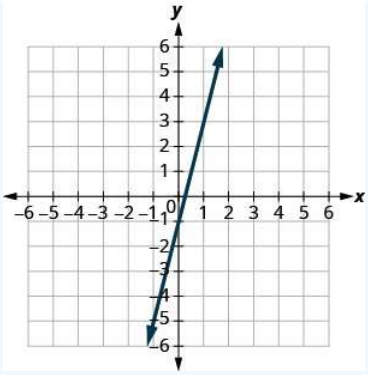
$\frac{1}{10}$

Intercept Form of an Equation of a Line

Recognize the Relation Between the Graph and the Slope–Intercept Form of an Equation of a Line

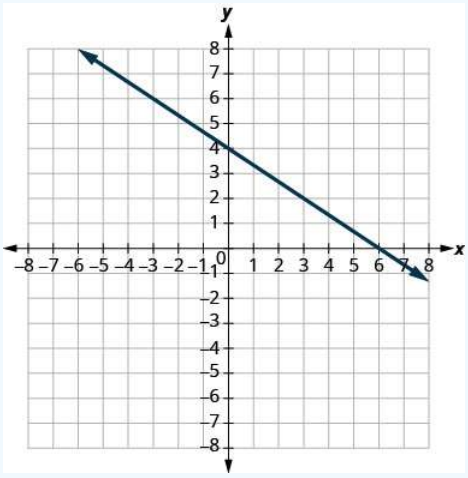
In the following exercises, use the graph to find the slope and y-intercept of each line. Compare the values to the equation $y = mx + b$.

Exercise 2.10.69



$y = 4x - 1$

Exercise 2.10.70



$y = -\frac{2}{3}x + 4$

Answer

slope $m = -\frac{2}{3}$ and y-intercept $(0, 4)$

Identify the Slope and y-Intercept from an Equation of a Line

In the following exercises, identify the slope and y-intercept of each line.

Exercise 2.10.71

$y = -4x + 9$

Exercise 2.10.72

$y = \frac{5}{3}x - 6$

Answer

$\frac{5}{3}; (0, -6)$

Exercise 2.10.73

$5x + y = 10$

Exercise 2.10.74

$4x - 5y = 8$

Answer

$\frac{4}{5}; \quad \left(0, -\frac{8}{5}\right)$

Graph a Line Using Its Slope and Intercept

In the following exercises, graph the line of each equation using its slope and y-intercept.

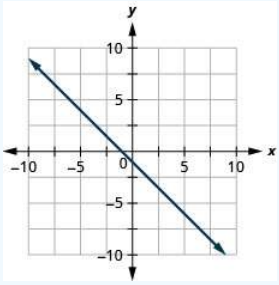
Exercise 2.10.75

$y = 2x + 3$

Exercise 2.10.76

$y = -x - 1$

Answer



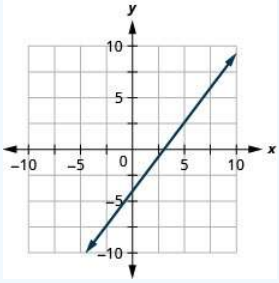
Exercise 2.10.77

$y = -\frac{2}{5}x + 3$

Exercise 2.10.78

$4x - 3y = 12$

Answer



In the following exercises, determine the most convenient method to graph each line.

Exercise 2.10.79

$x = 5$

Exercise 2.10.80

$y = -3$

Answer

horizontal line

Exercise 2.10.81

$2x + y = 5$

Exercise 2.10.82

$x - y = 2$

Answer

intercepts

Exercise 2.10.83

$y = x + 2$

Exercise 2.10.84

$y = \frac{3}{4}x - 1$

Answer

plotting points

Graph and Interpret Applications of Slope–Intercept

Exercise 2.10.85

Katherine is a private chef. The equation $C = 6.5m + 42$ models the relation between her weekly cost, C, in dollars and the number of meals, m, that she serves.

1. Find Katherine's cost for a week when she serves no meals.
2. Find the cost for a week when she serves 14 meals.
3. Interpret the slope and C-intercept of the equation.
4. Graph the equation.

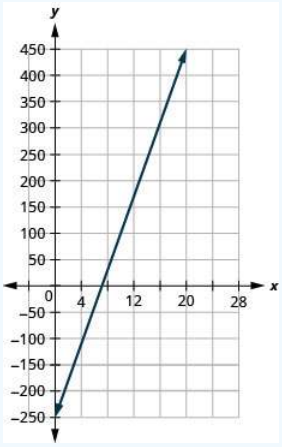
Exercise 2.10.86

Marjorie teaches piano. The equation $P = 35h - 250$ models the relation between her weekly profit, P, in dollars and the number of student lessons, s, that she teaches.

1. Find Marjorie's profit for a week when she teaches no student lessons.
2. Find the profit for a week when she teaches 20 student lessons.

3. Interpret the slope and P-intercept of the equation.
4. Graph the equation.

Answer

1. $-\$250$
2. $\$450$
3. The slope, 35, means that Marjorie's weekly profit, P, increases by $\$35$ for each additional student lesson she teaches. The P-intercept means that when the number of lessons is 0, Marjorie loses $\$250$
4. 

Use Slopes to Identify Parallel Lines

In the following exercises, use slopes and y-intercepts to determine if the lines are parallel.

Exercise 2.10.87

$4x-3y=-1; \quad y=\frac{4}{3}x-3$

Exercise 2.10.88

$2x-y=8; \quad x-2y=4$

Answer

not parallel

Use Slopes to Identify Perpendicular Lines

In the following exercises, use slopes and y-intercepts to determine if the lines are perpendicular.

Exercise 2.10.89

$y=5x-1; \quad 10x+2y=0$

Exercise 2.10.90

$3x-2y=5; \quad 2x+3y=6$

Answer

perpendicular

Find an Equation of the Line Given the Slope and *y*-Intercept

In the following exercises, find the equation of a line with given slope and y-intercept. Write the equation in slope–intercept form.

Exercise 2.10.91

slope $\frac{1}{3}$ and y-intercept $(0, -6)$

Exercise 2.10.92

slope -5 and y-intercept $(0, -3)$

Answer

$y = -5x - 3$

Exercise 2.10.93

slope 0 and y-intercept $(0, 4)$

Exercise 2.10.94

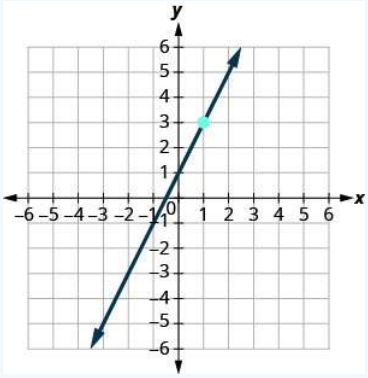
slope -2 and y-intercept $(0, 0)$

Answer

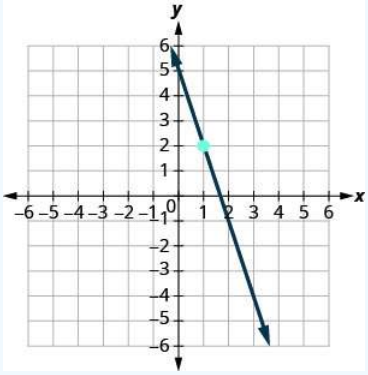
$y = -2x$

In the following exercises, find the equation of the line shown in each graph. Write the equation in slope–intercept form.

Exercise 2.10.95



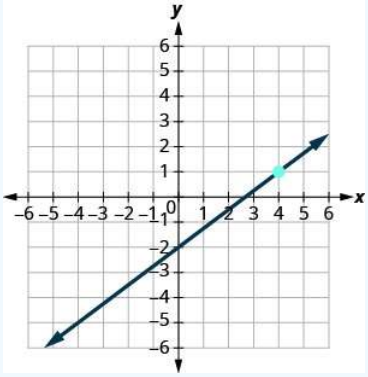
Exercise 2.10.96



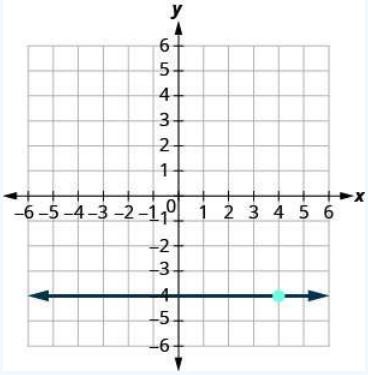
Answer

$y = -3x + 5$

Exercise 2.10.97



Exercise 2.10.98



Answer

$y = -4$

Find an Equation of the Line Given the Slope and a Point

In the following exercises, find the equation of a line with given slope and containing the given point. Write the equation in slope–intercept form.

Exercise 2.10.99

$m = -\frac{1}{4}$, point $(-8, 3)$

Exercise 2.10.100

$m = \frac{3}{5}$, point $(10, 6)$

Answer

$y = \frac{3}{5}x$

Exercise 2.10.101

Horizontal line containing $(-2, 7)$

Exercise 2.10.102

$m = -2$, point $(-1, -3)$

Answer

$y = -2x - 5$

Find an Equation of the Line Given Two Points

In the following exercises, find the equation of a line containing the given points. Write the equation in slope–intercept form.

Exercise 2.10.103

$(2, 10)$ and $(-2, -2)$

Exercise 2.10.104

$(7, 1)$ and $(5, 0)$

Answer

$y = \frac{1}{2}x - \frac{5}{2}$

Exercise 2.10.105

$(3, 8)$ and $(3, -4)$

Exercise 2.10.106

$(5, 2)$ and $(-1, 2)$

Answer

$y = 2$

Find an Equation of a Line Parallel to a Given Line

In the following exercises, find an equation of a line parallel to the given line and contains the given point. Write the equation in slope–intercept form.

Exercise 2.10.107

line $y = -3x + 6$, point $(1, -5)$

Exercise 2.10.108

line $2x + 5y = -10$, point $(10, 4)$

Answer

$y = -\frac{2}{5}x + 8$

Exercise 2.10.109

line $x = 4$, point $(-2, -1)$

Exercise 2.10.110

line $y = -5$, point $(-4, 3)$

Answer

$y = 3$

Find an Equation of a Line Perpendicular to a Given Line

In the following exercises, find an equation of a line perpendicular to the given line and contains the given point. Write the equation in slope–intercept form.

Exercise 2.10.111

line $y = -\frac{4}{5}x + 2$, point $(8, 9)$

Exercise 2.10.112

line $2x - 3y = 9$, point $(-4, 0)$

Answer

$y = -\frac{3}{2}x - 6$

Exercise 2.10.113

line $y = 3$, point $(-1, -3)$

Exercise 2.10.114

line $x = -5$ point $(2, 1)$

Answer

$y = 1$

Graph Linear Inequalities

Verify Solutions to an Inequality in Two Variables

In the following exercises, determine whether each ordered pair is a solution to the given inequality.

Exercise 2.10.115

Determine whether each ordered pair is a solution to the inequality $y < x - 3$:

1. $(0, 1)$
2. $(-2, -4)$
3. $(5, 2)$
4. $(3, -1)$
5. $(-1, -5)$

Exercise 2.10.116

Determine whether each ordered pair is a solution to the inequality $x + y > 4$:

1. $(6, 1)$
2. $(-3, 6)$
3. $(3, 2)$
4. $(-5, 10)$
5. $(0, 0)$

Answer

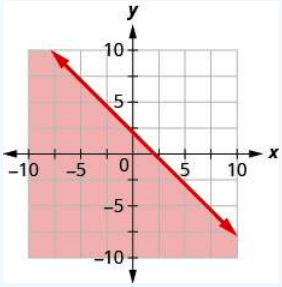
1. yes
2. no
3. yes
4. yes
5. no

Recognize the Relation Between the Solutions of an Inequality and its Graph

In the following exercises, write the inequality shown by the shaded region.

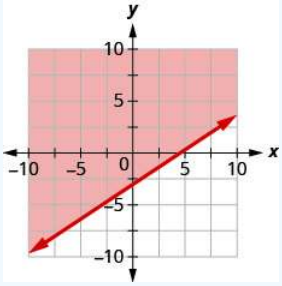
Exercise 2.10.117

Write the inequality shown by the graph with the boundary line $y = -x + 2$.



Exercise 2.10.118

Write the inequality shown by the graph with the boundary line $y = \frac{2}{3}x - 3$

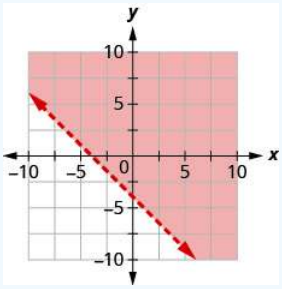


Answer

$y > \frac{2}{3}x - 3$

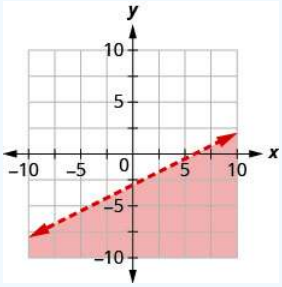
Exercise 2.10.119

Write the inequality shown by the shaded region in the graph with the boundary line $x + y = -4$.



Exercise 2.10.120

Write the inequality shown by the shaded region in the graph with the boundary line $x - 2y = 6$.



Answer

$x - 2y \geq 6$

Graph Linear Inequalities

In the following exercises, graph each linear inequality.

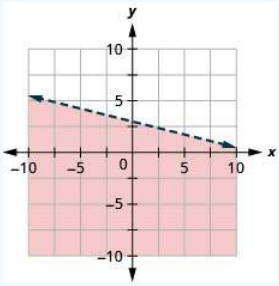
Exercise 2.10.121

Graph the linear inequality $y > \frac{2}{5}x - 4$

Exercise 2.10.122

Graph the linear inequality $y \leq -\frac{1}{4}x + 3$

Answer



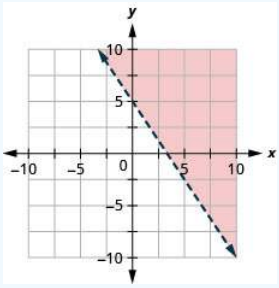
Exercise 2.10.123

Graph the linear inequality $x - y \leq 5$

Exercise 2.10.124

Graph the linear inequality $3x + 2y > 10$

Answer



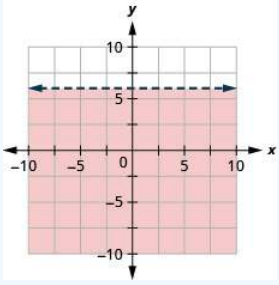
Exercise 2.10.125

Graph the linear inequality $y \leq -3x$

Exercise 2.10.126

Graph the linear inequality $y < 6$

Answer



Practice Test

Exercise 2.10.1

Plot each point in a rectangular coordinate system.

1. $(2, 5)$
2. $(-1, -3)$
3. $(0, 2)$
4. $\left(-4, \frac{3}{2}\right)$
5. $(5, 0)$

Exercise 2.10.2

Which of the given ordered pairs are solutions to the equation $3x - y = 6$?

1. $(3, 3)$
2. $(2, 0)$
3. $(4, -6)$

Answer

1. yes
2. yes
3. no

Exercise 2.10.3

Find three solutions to the linear equation $y = -2x - 4$

Exercise 2.10.4

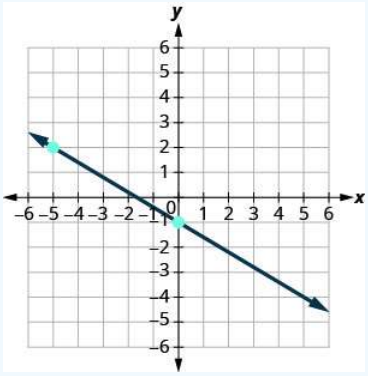
Find the x- and y-intercepts of the equation $4x - 3y = 12$

Answer

$(3, 0), (0, -4)$

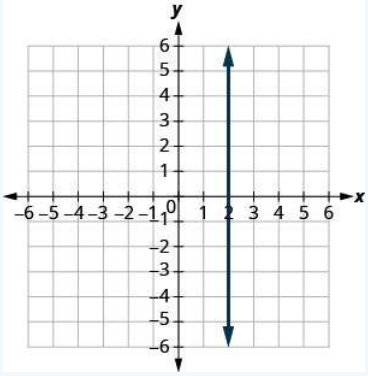
Find the slope of each line shown.

Exercise 2.10.5



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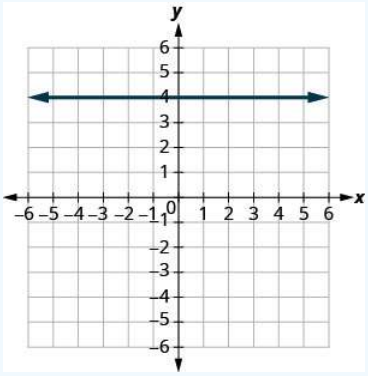
Exercise 2.10.6



Answer

undefined

Exercise 2.10.7



Exercise 2.10.8

Find the slope of the line between the points $(5, 2)$ and $(-1, -4)$

Answer

1

Exercise 2.10.9

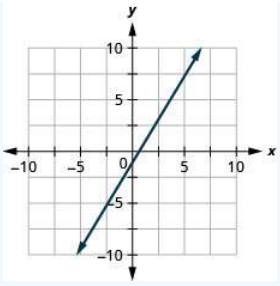
Graph the line with slope $\frac{1}{2}$ containing the point $(-3, -4)$

Graph the line for each of the following equations.

Exercise 2.10.10

$y = \frac{5}{3}x - 1$

Answer



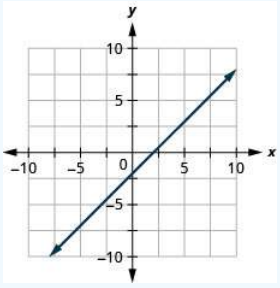
Exercise 2.10.11

$y = -x$

Exercise 2.10.12

$x - y = 2$

Answer



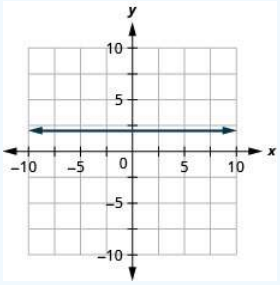
Exercise 2.10.13

$4x + 2y = -8$

Exercise 2.10.14

$y = 2$

Answer



Exercise 2.10.15

$x = -3$

Find the equation of each line. Write the equation in slope–intercept form.

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Exercise 2.10.16

slope $-\frac{3}{4}$ and y-intercept $(0, -2)$

Answer

$y = -\frac{3}{4}x - 2$

Exercise 2.10.17

$m = 2$, point $(-3, -1)$

Exercise 2.10.18

containing $(10, 1)$ and $(6, -1)$

Answer

$y = \frac{1}{2}x - 4$

Exercise 2.10.19

parallel to the line $y = -\frac{2}{3}x - 1$, containing the point $(-3, 8)$

Exercise 2.10.20

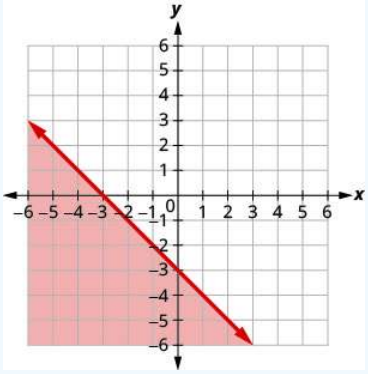
perpendicular to the line $y = \frac{5}{4}x + 2$, containing the point $(-10, 3)$

Answer

$y = -\frac{4}{5}x - 5$

Exercise 2.10.21

Write the inequality shown by the graph with the boundary line $y = -x - 3$.



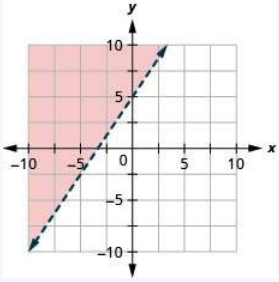
Graph each linear inequality.

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Exercise 2.10.22

$y > \frac{3}{2}x + 5$

Answer



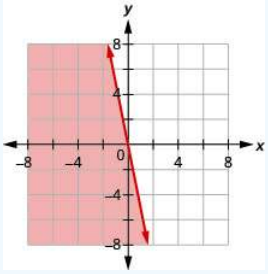
Exercise 2.10.23

$x - y \geq -4$

Exercise 2.10.24

$y \leq -5x$

Answer



Exercise 2.10.1

$y < 3$

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CHAPTER OVERVIEW

3: Functions

3.1: Introduction to Functions
3.2: Relations and Functions
3.2E: Exercises
3.3: Interpreting the Graph of a Function
3.4: The Toolbox Functions
3.4E: Exercises
3.5: Absolute Value Functions

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3.1: Introduction to Functions

Our development of the function concept is a modern one, but quite quick, particularly in light of the fact that today's definition took over 300 years to reach its present state. We begin with the definition of a relation.

Relations

We use the notation (2, 4) to denote what is called an ordered pair. If you think of the positions taken by the ordered pairs (4, 2) and (2, 4) in the coordinate plane (see Figure 3.1.1), then it is immediately apparent why order is important. The ordered pair (4, 2) is simply not the same as the ordered pair (2, 4).

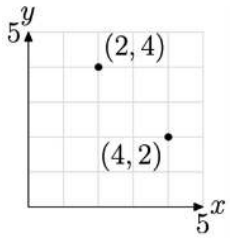


Figure 3.1.1

The first element of an ordered pair is called its abscissa. The second element of an ordered pair is called its ordinate. Thus, for example, the abscissa of (4, 2) is 4, while the ordinate of (4, 2) is 2.

Definition

A collection of ordered pairs is called a **relation**. (2)

For example, the collection of ordered pairs

$$R = \{(0, 1), (0, 2), (3, 4)\}$$

is a relation.

Definition

The **domain** of a relation is the collection of all abscissas of each ordered pair.

Thus, the domain of the relation R in (2) is

$$\text{Domain } = \{0, 3\}$$

Note that we list each abscissa only once.

Definition

The **range** of a relation is the collection of all ordinates of each ordered pair.

Thus, the range of the relation R in (2) is

$$\text{Range } = \{1, 2, 4\}$$

Let's look at an example.

Example 3.1.1

Consider the relation T defined by

$$T = \{(1, 2), (3, 2), (4, 5)\}$$

What are the domain and range of this relation?

Solution

The domain is the collection of abscissas of each ordered pair. Hence, the domain of T is

$$\text{Domain } = \{1, 3, 4\}$$

The range is the collection of ordinates of each ordered pair. Hence, the range of T is

$$\text{Range } = \{2, 5\}$$

Note that we list each ordinate in the range only once.

In Example 3.1.1, the relation is described by listing the ordered pairs. This is not the only way that one can describe a relation. For example, a graph certainly represents a collection of ordered pairs.

Example 3.1.2

Consider the graph of the relation S shown in Figure 3.1.2.

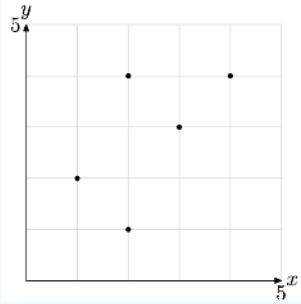


Figure 3.1.2 The graph of a relation.

What are the domain and range of the relation S?

Solution

There are five ordered pairs (points) plotted in Figure 3.1.2. They are

$$S = \{(1, 2), (2, 1), (2, 4), (3, 3), (4, 4)\}$$

Therefore, the relation S has Domain = {1, 2, 3, 4} and Range = {1, 2, 3, 4}. In the case of the range, note how we've sorted the ordinates of each ordered pair in ascending order, taking care not to list any ordinate more than once.

Functions

A function is a very special type of relation. We begin with a formal definition.

Definition

A relation is a function if and only if each object in its domain is paired with one and only one object in its range.

This is not an easy definition, so let's take our time and consider a few examples. Consider, if you will, the relation R in (2), repeated here again for convenience.

$$R = \{(0, 1), (0, 2), (3, 4)\}$$

The domain is {0, 3} and the range is {1, 2, 4}. Note that the number 0 in the domain of R is paired with two numbers from the range, namely, 1 and 2. Therefore, R is not a function.

There is a construct, called a mapping diagram, which can be helpful in determining whether a relation is a function. To craft a mapping diagram, first list the domain on the left, then the range on the right, then use arrows to indicate the ordered pairs in your relation, as shown in Figure 3.1.3.

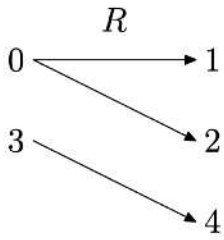


Figure 3.1.3 A mapping diagram for R.

It's clear from the mapping diagram in Figure 3.1.3 that the number 0 in the domain is being paired (mapped) with two different range objects, namely, 1 and 2. Thus, R is not a function.

Let's look at another example.

Example 3.1.3

Is the relation described in Example 3.1.1 a function?

Solution

First, let's repeat the listing of the relation T here for convenience.

$$T = \{(1, 2), (3, 2), (4, 5)\}$$

Next, construct a mapping diagram for the relation T. List the domain on the left, the range on the right, then use arrows to indicate the pairings, as shown in Figure 3.1.4.

From the mapping diagram in Figure 3.1.4, we can see that each domain object on the left is paired (mapped) with exactly one range object on the right. Hence, the relation T is a function.

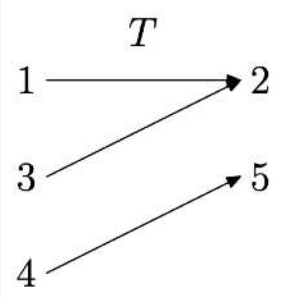


Figure 3.1.4. A mapping diagram for T.

Let's look at another example.

Example 3.1.4

Is the relation of Example 3.1.2, pictured in Figure 3.1.2, a function?

Solution

First, we repeat the graph of the relation from Example 3.1.2 here for convenience. This is shown in Figure 3.1.5(a). Next, we list the ordered pairs of the relation S.

$$S = \{(1, 2), (2, 1), (2, 4), (3, 3), (4, 4)\}$$

Then we create a mapping diagram by first listing the domain on the left, the range on the right, then using arrows to indicate the pairings, as shown in Figure 3.1.5(b).

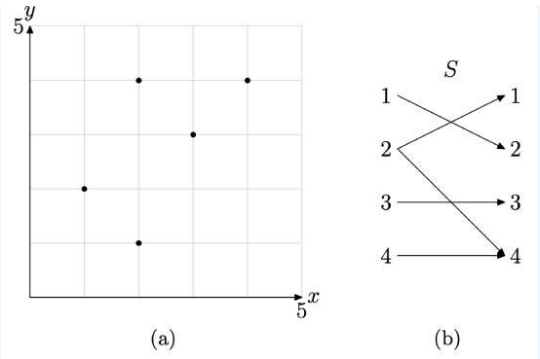


Figure 3.1.5 A graph of the relation S and its corresponding mapping diagram

Each object in the domain of S gets mapped to exactly one range object with one exception. The domain object 2 is paired with two range objects, namely, 1 and 4. Consequently, S is not a function.

This is a good point to summarize what we've learned about functions thus far.

Summary

A function consists of three parts:

1. a set of objects which mathematicians call the **domain**,
2. a second set of objects which mathematicians call the **range**,
3. and a **rule** that describes how to assign a unique range object to each object in the domain.

The rule can take many forms. For example, we can use sets of ordered pairs, graphs, and mapping diagrams to describe the function. In the sections that follow, we will explore other ways of describing a function, for example, through the use of equations and simple word descriptions.

Function Notation

We've used the word "mapping" several times in the previous examples. This is not a word to be taken lightly; it is an important concept. In the case of the mapping diagram in Figure 3.1.5(b), we would say that the number 1 in the domain of S is "mapped" (or "sent") to the number 2 in the range of S.

There are a number of different notations we could use to indicate that the number 1 in the domain is "mapped" or "sent" to the number 2 in the range. One possible notation is

$$S : 1 \longrightarrow 2$$

which we would read as follows: "The relation S maps (sends) 1 to 2." In a similar vein, we see in Figure 3.1.5(b) that the domain objects 3 and 4 are mapped (sent) to the range objects 3 and 4, respectively. In symbols, we would write

$$S : 3 \longrightarrow 3, \text{ and}$$
$$S : 4 \longrightarrow 4$$

A difficulty arises when we examine what happens to the domain object 2. There are two possibilities, either

$$S : 2 \longrightarrow 1$$

or

$$S : 2 \longrightarrow 4$$

Which should we choose? The 1? Or the 4? Thus, S is not well-defined and is not a function, since we don't know which range object to pair with the domain object 1.

The idea of mapping gives us an alternative way to describe a function. We could say that a function is a rule that assigns a unique object in its range to each object in its domain. Take for example, the function that maps each real number to its square. If we name

the function f, then f maps 5 to 25, 6 to 36, −7 to 49, and so on. In symbols, we would write

$$f : 5 \longrightarrow 25, \quad f : 6 \longrightarrow 36, \quad \text{and} \quad f : -7 \longrightarrow 49$$

In general, we could write

$$f : x \longrightarrow x^2$$

Note that each real number x gets mapped to a unique number in the range of f, namely, x^2. Consequently, the function f is well defined. We've succeeded in writing a rule that completely defines the function f.

As another example, let's define a function that takes a real number, doubles it, then adds 3. If we name the function g, then g would take the number 7, double it, then add 3. That is,

$$g : 7 \longrightarrow 2(7) + 3$$

Simplifying, $g : 7 \longrightarrow 17$. Similarly, g would take the number 9, double it, then add 3. That is,

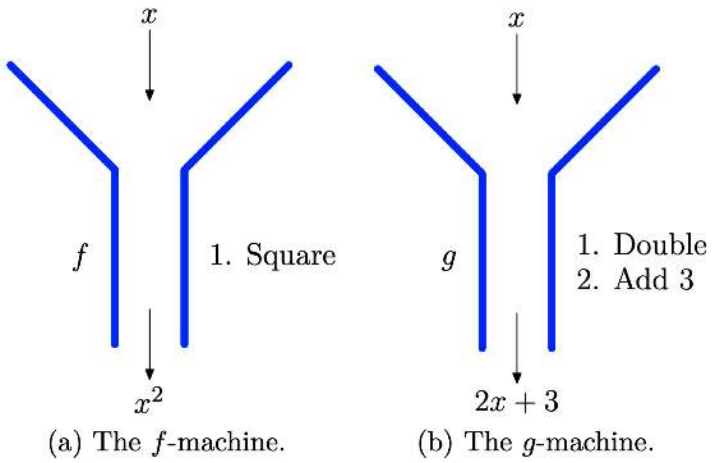
$$g : 9 \longrightarrow 2(9) + 3$$

Simplifying, $g : 9 \longrightarrow 21$. In general, g takes a real number x, doubles it, then adds three. In symbols, we would write

$$g : x \longrightarrow 2x + 3$$

Notice that each real number x is mapped by g to a unique number in its range. Therefore, we've again defined a rule that completely defines the function g.

It is helpful to think of a function as a machine. The machine receives input, processes it according to some rule, then outputs a result. Something goes in (input), then something comes out (output). In the case of the function described by the rule $f : x \longrightarrow x^2$, the "f-machine" receives input x, then applies its "square rule" to the input and outputs x^2, as shown in Figure 3.1.6(a). In the case of the function described by the rule $g : x \longrightarrow 2x + 3$, the "g-machine" receives input x, then applies the rules "double," then "add 3," in that order, then outputs $2x + 3$, as shown in Figure 3.1.6(b).



(a) The f-machine. (b) The g-machine.

Figure 3.1.6 Function machines.

Let's look at another example.

Example 3.1.5

Suppose that f is defined by the following rule. For each real number x,

$$f : x \longrightarrow x^2 - 2x - 3$$

Where does f map the number −3? Is f a function?

Solution

We substitute −3 for x in the rule for f and obtain

$$f : -3 \longrightarrow (-3)^2 - 2(-3) - 3$$

Simplifying,

$$f : -3 \longrightarrow 9 + 6 - 3$$

or,

$$f : -3 \longrightarrow 12$$

Thus, f maps (sends) the number −3 to the number 12. It should be clear that each real number x will be mapped (sent) to a unique real number, as defined by the rule $f : x \longrightarrow x^2 - 2x - 3$. Therefore, f is a function.

Let's look at another example.

Example 3.1.6

Suppose that g is defined by the following rule. For each real number x that is greater than or equal to zero,

$$g : x \longrightarrow \pm\sqrt{x}$$

Where does g map the number 4? Is g a function?

Solution

Again, we substitute 4 for x in the rule for g and obtain

$$g : 4 \longrightarrow \pm\sqrt{4}$$

Simplifying,

$$g : 4 \longrightarrow \pm 2$$

Thus, g maps (sends) the number 4 to two different objects in its range, namely, 2 and −2. Consequently, g is not well-defined and is not a function.

Let's look at another example

Example 3.1.7

Suppose that we have functions f and g, defined by

$$f : x \longrightarrow x^4 + 11 \quad \text{and} \quad g : x \longrightarrow (x+2)^2$$

Where does g send 5?

Solution

In this example, we see a clear advantage of function notation. Because our functions have distinct names, we can simply reference the name of the function we want our readers to use. In this case, we are asked where the function g sends the number 5, so we substitute 5 for x in

$$g : x \longrightarrow (x+2)^2$$

That is,

$$g : 5 \longrightarrow (5+2)^2$$

Simplifying, $g : 5 \longrightarrow 49$.

Modern Notation

Function notation is relatively new, with some of the earliest symbolism first occurring in the 17th century. In a letter to Leibniz (1698), Johann Bernoulli wrote "For denoting any function of a variable quantity x, I rather prefer to use the capital letter having the same name X or the Greek ξ, for it appears at once of what variable it is a function; this relieves the memory."

Mathematicians are fond of the notation

$$f : x \longrightarrow x^2 - 2x$$

because it conveys a sense of what a function does; namely, it "maps" or "sends" the number x to the number $x^2 - 2x$. This is what functions do, they pair each object in their domain with a unique object in their range. Equivalently, functions "send" each object in their domain to a unique object in their range.

However, in common computational situations, the "arrow" notation can be a bit clumsy, so mathematicians tend to favor a slightly different notation. Instead of writing

$$f : x \longrightarrow x^2 - 2x$$

mathematicians tend to favor the notation

$$f(x) = x^2 - 2x$$

It is important to understand from the outset that these two different notations are equivalent; they represent the same function f, one that pairs each real number x in its domain with the real number $x^2 - 2x$ in its range.

The first notation, $f : x \longrightarrow x^2 - 2x$, conveys the sense that the function f is a mapping. If we read this notation aloud, we should pronounce it as "f sends (or maps) x to $x^2 - 2x$." The second notation, $f(x) = x^2 - 2x$, is pronounced "f of x equals $x^2 - 2x$."

Note

The phrase "f of x" is unfortunate, as our readers might recall being trained from a very early age to pair the word "of" with the operation of multiplication. For example, 1/2 of 12 is 6, as in $1/2 \times 12 = 6$. However, in the context of function notation, even though f(x) is read aloud as "f of x," it does **not** mean "f times x." Indeed, if we remind ourselves that the notation $f(x) = x^2 - 2x$ is equivalent to the notation $f : x \longrightarrow x^2 - 2x$, then even though we might say "f of x," we should be thinking "f sends x" or "f maps x." We should **not** be thinking "f times x."

Now, let's see how each of these notations operates on the number 5. In the first case, using the "arrow" notation,

$$f : x \longrightarrow x^2 - 2x$$

To find where f sends 5, we substitute 5 for x as follows.

$$f : 5 \longrightarrow (5)^2 - 2(5)$$

Simplifying, $f : 5 \longrightarrow 15$. Now, because both notations are equivalent, to compute f(5), we again substitute 5 for x in

$$f(x) = x^2 - 2x$$

Thus,

$$f(5) = (5)^2 - 2(5)$$

Simplifying, $f(5) = 15$. This result is read aloud as "f of 5 equals 15," but we want to be thinking "f sends 5 to 15."

Let's look at examples that use this modern notation.

Example 3.1.8

Given $f(x) = x^3 + 3x^2 - 5$, determine $f(-2)$

Solution

Simply substitute −2 for x. That is,

$$\begin{aligned} f(-2) &= (-2)^3 + 3(-2)^2 - 5 \\ &= -8 + 3(4) - 5 \\ &= -8 + 12 - 5 \\ &= -1 \end{aligned}$$

Thus, $f(-2) = -1$. Again, even though this is pronounced "f of −2 equals −1," we still should be thinking "f sends −2 to −1."

Example 3.1.9

Given

$$f(x) = \frac{x+3}{2x-5}$$

determine f(6).

Solution

Simply substitute 6 for x. That is,

$$\begin{aligned} f(6) &= \frac{6+3}{2(6)-5} \\ &= \frac{9}{12-5} \\ &= \frac{9}{7} \end{aligned}$$

Thus, $f(6) = 9/7$. Again, even though this is pronounced "f of 6 equals 9/7," we should still be thinking "f sends 6 to 9/7."

Example 3.1.10

Given $f(x) = 5x - 3$, determine $f(a+2)$.

Solution

If we're thinking in terms of mapping notation, then

$$f : x \longrightarrow 5x - 3$$

Think of this mapping as a "machine." Whatever we put into the machine, it first is multiplied by 5, then 3 is subtracted from the result, as shown in Figure 3.1.7. For example, if we put a 4 into the machine, then the function rule requires that we multiply input 4 by 5, then subtract 3 from the result. That is,

$$f : 4 \longrightarrow 5(4) - 3$$

Simplifying, $f : 4 \longrightarrow 17$

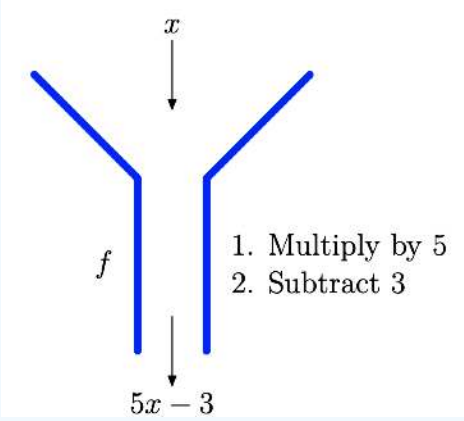


Figure 3.1.7. The multiply by 5 then subtract 3 machine.

Similarly, if we put an a + 2 into the machine, then the function rule requires that we multiply the input a + 2 by 5, then subtract 3 from the result. That is,

$$f : a+2 \longrightarrow 5(a+2) - 3$$

Using modern function notation, we would write

$$f(a+2) = 5(a+2) - 3$$

Note that this is again a simple substitution, where we replace each occurrence of x in the formula $f(x) = 5x - 3$ with the expression a + 2. Finally, use the distributive property to first multiply by 5, then subtract 3.

$$\begin{aligned} f(a+2) &= 5a + 10 - 3 \\ &= 5a + 7 \end{aligned}$$

We will often need to substitute the result of one function evaluation into a second function for evaluation. Let's look at an example.

Example 3.1.11

Given two functions defined by $f(x) = 3x + 2$ and $g(x) = 5 - 4x$, find f(g(2)).

Solution

The nested parentheses in the expression f(g(2)) work in the same manner that they do with nested expressions. The rule is to work the innermost grouping symbols first, proceeding outward as you work. We'll first evaluate g(2), then evaluate f at the result.

We begin. First, evaluate g(2) by substituting 2 for x in the defining equation $g(x) = 5 - 4x$. Note that $g(2) = 5 - 4(2)$, then simplify.

$$f(g(2)) = f(5 - 4(2)) = f(5 - 8) = f(-3)$$

To complete the evaluation, we substitute −3 for x in the defining equation $f(x) = 3x + 2$, then simplify.

$$f(-3) = 3(-3) + 2 = -9 + 2 = -7$$

Hence, $f(g(2)) = -7$.

It is conventional to arrange the work in one contiguous block, as follows.

$$\begin{aligned} f(g(2)) &= f(5 - 4(2)) \\ &= f(-3) \\ &= 3(-3) + 2 \\ &= -7 \end{aligned}$$

You can shorten the task even further if you are willing to do the function substitution and simplification in your head. First, evaluate g at 2, then f at the result.

$$f(g(2)) = f(-3) = -7$$

Let's look at another example of this unique way of combining functions.

Example 3.1.12

Given $f(x) = 5x + 2$ and $g(x) = 3 - 2x$, evaluate $g(f(a))$ and simplify the result.

Solution

We work the inner function evaluation in the expression $g(f(a))$ first. Thus, to evaluate f(a), we substitute a for x in the definition $f(x) = 5x + 2$ to get

$$g(f(a)) = g(5a + 2)$$

Now we need to evaluate $g(5a + 2)$. To do this, we substitute $5a + 2$ for x in the definition $g(x) = 3 - 2x$ to get

$$g(5a + 2) = 3 - 2(5a + 2)$$

We can expand this last result and simplify. Thus,

$$g(f(a)) = 3 - 10a - 4 = -10a - 1$$

Again, it is conventional to arrange the work in one continuous block, as follows.

$$\begin{aligned} g(f(a)) &= g(5a+2) \\ &= 3 - 2(5a+2) \\ &= 3 - 10a - 4 \\ &= -10a - 1 \end{aligned}$$

Hence, $g(f(a)) = -10a - 1$.

Extracting the Domain of a Function

We've seen that the domain of a relation or function is the set of all the first coordinates of its ordered pairs. However, if a functional relationship is defined by an equation such as $f(x) = 3x - 4$, then it is not practical to list all ordered pairs defined by this relationship. For any real x-value, you get an ordered pair. For example, if x = 5, then $f(5) = 3(5) - 4 = 11$, leading to the ordered pair (5, f(5)) or (5, 11). As you can see, the number of such ordered pairs is infinite. For each new x-value, we get another function value and another ordered pair.

Therefore, it is easier to turn our attention to the values of x that yield real number responses in the equation $f(x) = 3x - 4$. This leads to the following key idea.

Definition

If a function is defined by an equation, then the domain of the function is the set of "permissible x-values," the values that produce a real number response defined by the equation.

We sometimes like to say that the domain of a function is the set of "OK x-values to use in the equation." For example, if we define a function with the rule $f(x) = 3x - 4$, it is immediately apparent that we can use any value we want for x in the rule $f(x) = 3x - 4$. Thus, the domain of f is all real numbers. We can write that the domain $D = \mathbb{R}$, or we can use interval notation and write that the domain $D = (-\infty, \infty)$.

It is not the case that x can be any real number in the function defined by the rule $f(x) = \sqrt{x}$. It is not possible to take the square root of a negative number.2 Therefore, x must either be zero or a positive real number. In set-builder notation, we can describe the domain with $D = \{x : x \geq 0\}$. In interval notation, we write $D = [0, \infty)$.

We must also be aware of the fact that we cannot divide by zero. If we define a function with the rule $f(x) = x/(x-3)$, we immediately see that x = 3 will put a zero in the denominator. Division by zero is not defined. Therefore, 3 is not in the domain of f. No other x-value will cause a problem. The domain of f is best described with set-builder notation as $D = \{x : x \neq 3\}$.

Functions Without Formulae

In the previous section, we defined functions by means of a formula, for example, as in

$$f(x) = \frac{x+3}{2-3x}$$

Euler would be pleased with this definition, for as we have said previously, Euler thought of functions as analytic expressions.

However, it really isn't necessary to provide an expression or formula to define a function. There are other forms we can use to express a functional relationship: a graph, a table, or even a narrative description. The only thing that is really important is the requirement that the function be well-defined, and by "well-defined," we mean that each object in the function's domain is paired with one and only one object in its range.

As an example, let's look at a special function π on the natural numbers,3 which returns the number of primes less than or equal to a given natural number. For example, the primes less than or equal to the number 23 are 2, 3, 5, 7, 11, 13, 17, 19, and 23, nine numbers in all. Therefore, the number of primes less than or equal to 23 is nine. In symbols, we would write

$$\pi(23) = 9$$

Note the absence of a formula in the definition of this function. Indeed, the definition is descriptive in nature, so we might write

$$\pi(n) = \text{ number of primes less than or equal to } n$$

The important thing is not how we define this special function π, but the fact that it is well-defined; that is, for each natural number n, there are a fixed number of primes less than or equal to n. Thus, each natural number in the domain of π is paired with one and only one number in its range.

Now, just because our function doesn't provide an expression for calculating the number of primes less than or equal to a given natural number n, it doesn't stop mathematicians from seeking such a formula. Euclid of Alexandria (325-265 BC), a Greek mathematician, proved that the number of primes is infinite, but it was the German mathematician and scientist, Johann Carl Friedrich Gauss (1777-1855), who first proposed that the number of primes less than or equal to n can be approximated by the formula

$$\pi(n) \approx \frac{n}{\ln n}$$

where ln n is the "natural logarithm" of n (to be explained in Chapter 9). This approximation gets better and better with larger and larger values of n. The formula was refined by Gauss, who did not provide a proof, and the problem became known as the Prime Number Theorem. It was not until 1896 that Jacques Salomon Hadamard (1865-1963) and Charles Jean Gustave Nicolas Baron de la Vallee Poussin (1866-1962), working independently, provided a proof of the Prime Number Theorem.

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3.2: Relations and Functions

Learning Objectives

By the end of this section, you will be able to:

- Find the domain and range of a relation
- Determine if a relation is a function
- Find the value of a function

Before you get started, take this readiness quiz.

1. Evaluate $3x-5$ when $x=-2$.
   If you missed this problem, review [link].
2. Evaluate $2x^2-x-3$ when $x=a$.
   If you missed this problem, review [link].
3. Simplify: $7x-1-4x+5$.
   If you missed this problem, review [link].

Find the Domain and Range of a Relation

As we go about our daily lives, we have many data items or quantities that are paired to our names. Our social security number, student ID number, email address, phone number and our birthday are matched to our name. There is a relationship between our name and each of those items.

When your professor gets her class roster, the names of all the students in the class are listed in one column and then the student ID number is likely to be in the next column. If we think of the correspondence as a set of ordered pairs, where the first element is a student name and the second element is that student's ID number, we call this a **relation**.

$$(\text{Student name},\ \text{Student ID }\#)$$

The set of all the names of the students in the class is called the **domain** of the relation and the set of all student ID numbers paired with these students is the range of the relation.

There are many similar situations where one variable is paired or matched with another. The set of ordered pairs that records this matching is a relation.

Definition: Relation

A **relation** is any set of ordered pairs, (x,y). All the x-values in the ordered pairs together make up the **domain**. All the y-values in the ordered pairs together make up the **range**.

Example 3.2.1

For the relation $(1,1),(2,4),(3,9),(4,16),(5,25)$:

a. Find the domain of the relation.
b. Find the range of the relation.

Answer

$\{(1,1),(2,4),(3,9),(4,16),(5,25)\}$

ⓐ The domain is the set of all x-values of the relation. $\{1,2,3,4,5\}$

ⓑ The range is the set of all y-values of the relation. $\{1,4,9,16,25\}$

Example 3.2.2

For the relation $\{(1,1),(2,8),(3,27),(4,64),(5,125)\}$

a. Find the domain of the relation.
b. Find the range of the relation.

Answer a

$\{1,2,3,4,5\}$

Answer b

$\{1,8,27,64,125\}$

Example 3.2.3

For the relation $\{(1,3),(2,6),(3,9),(4,12),(5,15)\}$

a. Find the domain of the relation.
b. Find the range of the relation.

Answer a

$\{1,2,3,4,5\}$

Answer b

$\{3,6,9,12,15\}$

MAPPING

A **mapping** is sometimes used to show a relation. The arrows show the pairing of the elements of the domain with the elements of the range.

Example 3.2.4

Use the **mapping** of the relation shown to

a. list the ordered pairs of the relation,
b. find the domain of the relation, and
c. find the range of the relation.



Answer

ⓐ The arrow shows the matching of the person to their birthday. We create ordered pairs with the person's name as the *x*-value and their birthday as the *y*-value.

{(Alison, April 25), (Penelope, May 23), (June, August 2), (Gregory, September 15), (Geoffrey, January 12), (Lauren, May 10), (Stephen, July 24), (Alice, February 3), (Liz, August 2), (Danny, July 24)}

ⓑ The domain is the set of all x-values of the relation.

{Alison, Penelope, June, Gregory, Geoffrey, Lauren, Stephen, Alice, Liz, Danny}

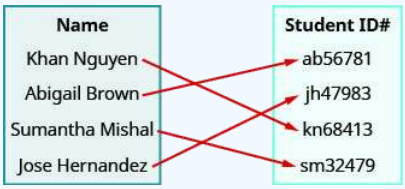
ⓒ The range is the set of all y-values of the relation.

{January 12, February 3, April 25, May 10, May 23, July 24, August 2, September 15}

Example 3.2.5

Use the mapping of the relation shown to

a. list the ordered pairs of the relation
b. find the domain of the relation
c. find the range of the relation.



Answer

ⓐ (Khanh Nguyen, kn68413), (Abigail Brown, ab56781), (Sumantha Mishal, sm32479), (Jose Hern and ez, jh47983)

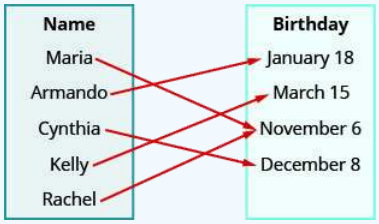
ⓑ {Khanh Nguyen, Abigail Brown, Sumantha Mishal, Jose Hern and ez}

ⓒ {kn68413, ab56781, sm32479, jh47983}

Example 3.2.6

Use the mapping of the relation shown to

a. list the ordered pairs of the relation
b. find the domain of the relation
c. find the range of the relation.



Answer

ⓐ (Maria, November 6), (Arm and o, January 18), (Cynthia, December 8), (Kelly, March 15), (Rachel, November 6)

ⓑ {Maria, Arm and o, Cynthia, Kelly, Rachel}

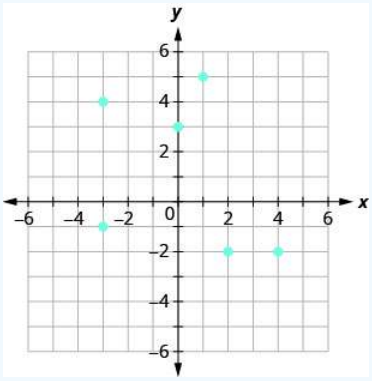
ⓒ{November 6, January 18, December 8, March 15}

A graph is yet another way that a relation can be represented. The set of ordered pairs of all the points plotted is the relation. The set of all x-coordinates is the domain of the relation and the set of all y-coordinates is the range. Generally we write the numbers in ascending order for both the domain and range.

Example 3.2.7

Use the graph of the relation to

a. list the ordered pairs of the relation
b. find the domain of the relation
c. find the range of the relation.



Answer

ⓐ The ordered pairs of the relation are:

$$\{(1,5),(-3,-1),(4,-2),(0,3),(2,-2),(-3,4)\}.$$

ⓑ The domain is the set of all x-values of the relation: $\{-3,0,1,2,4\}$

Notice that while -3 repeats, it is only listed once.

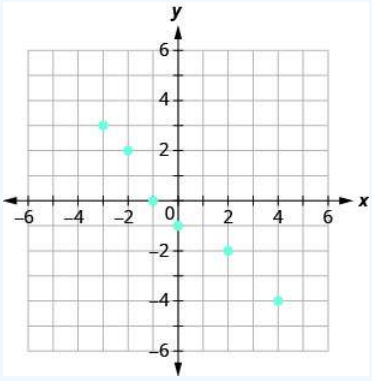
ⓒ The range is the set of all y-values of the relation: $\{-2,-1,3,4,5\}$

Notice that while -2 repeats, it is only listed once.

Example 3.2.8

Use the graph of the relation to

a. list the ordered pairs of the relation
b. find the domain of the relation
c. find the range of the relation.



Answer

a. $(-3,3),(-2,2),(-1,0),(0,-1),(2,-2),(4,-4)$

b. $\{-3,-2,-1,0,2,4\}$

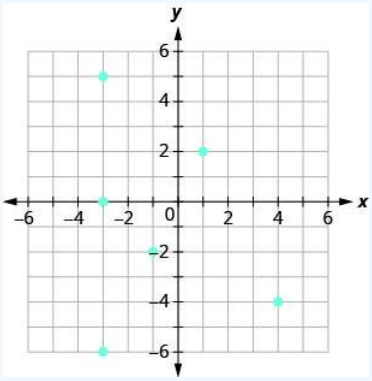
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c. $\{3, 2, 0, -1, -2, -4\}$

Example 3.2.9

Use the graph of the relation to

a. list the ordered pairs of the relation
b. find the domain of the relation
c. find the range of the relation.



Answer

ⓐ $(-3, 0), (-3, 5), (-3, -6), (-1, -2), (1, 2), (4, -4)$
ⓑ $\{-3, -1, 1, 4\}$
ⓒ $\{-6, 0, 5, -2, 2, -4\}$

Determine if a Relation is a Function

A special type of relation, called a **function**, occurs extensively in mathematics. A function is a relation that assigns to each element in its domain exactly one element in the range. For each ordered pair in the relation, each *x*-value is matched with only one *y*-value.

Definition: Function

A **function** is a relation that assigns to each element in its domain exactly one element in the range.

The birthday example from Example helps us understand this definition. Every person has a birthday but no one has two birthdays. It is okay for two people to share a birthday. It is okay that Danny and Stephen share July 24th as their birthday and that June and Liz share August 2nd. Since each person has exactly one birthday, the relation in Example is a function.

The relation shown by the graph in Example includes the ordered pairs $(-3, -1)$ and $(-3, 4)$. Is that okay in a function? No, as this is like one person having two different birthdays.

Example 3.2.10

Use the set of ordered pairs to (i) determine whether the relation is a function (ii) find the domain of the relation (iii) find the range of the relation.

a. $\{(-3, 27), (-2, 8), (-1, 1), (0, 0), (1, 1), (2, 8), (3, 27)\}$
b. $\{(9, -3), (4, -2), (1, -1), (0, 0), (1, 1), (4, 2), (9, 3)\}$

Answer

ⓐ $\{(-3, 27), (-2, 8), (-1, 1), (0, 0), (1, 1), (2, 8), (3, 27)\}$

(i) Each x-value is matched with only one y-value. So this relation is a function.

(ii) The domain is the set of all x-values in the relation.
The domain is: $\{-3, -2, -1, 0, 1, 2, 3\}$

(iii) The range is the set of all y-values in the relation. Notice we do not list range values twice.
The range is: $\{27, 8, 1, 0\}$

ⓑ $\{(9, -3), (4, -2), (1, -1), (0, 0), (1, 1), (4, 2), (9, 3)\}$

(i) The x-value 9 is matched with two y-values, both 3 and -3. So this relation is not a function.

(ii) The domain is the set of all x-values in the relation. Notice we do not list domain values twice.
The domain is: $\{0, 1, 2, 4, 9\}$

(iii) The range is the set of all y-values in the relation.
The range is: $\{-3, -2, -1, 0, 1, 2, 3\}$

Example 3.2.11

Use the set of ordered pairs to (i) determine whether the relation is a function (ii) find the domain of the relation (iii) find the range of the function.

a. $\{(-3, -6), (-2, -4), (-1, -2), (0, 0), (1, 2), (2, 4), (3, 6)\}$
b. $\{(8, -4), (4, -2), (2, -1), (0, 0), (2, 1), (4, 2), (8, 4)\}$

Answer

ⓐ Yes; $\{-3, -2, -1, 0, 1, 2, 3\}$
$\{-6, -4, -2, 0, 2, 4, 6\}$
ⓑ No; $\{0, 2, 4, 8\}$
$\{-4, -2, -1, 0, 1, 2, 4\}$

Example 3.2.12

Use the set of ordered pairs to (i) determine whether the relation is a function (ii) find the domain of the relation (iii) find the range of the relation.

a. $\{(27, -3), (8, -2), (1, -1), (0, 0), (1, 1), (8, 2), (27, 3)\}$
b. $\{(7, -3), (-5, -4), (8, -0), (0, 0), (-6, 4), (-2, 2), (-1, 3)\}$

Answer

ⓐ No; $\{0, 1, 8, 27\}$
$\{-3, -2, -1, 0, 2, 2, 3\}$
ⓑ Yes; $\{7, -5, 8, 0, -6, -2, -1\}$
$\{-3, -4, 0, 4, 2, 3\}$

Example 3.2.13

Use the mapping to

a. determine whether the relation is a function
b. find the domain of the relation
c. find the range of the relation.

NAME	PHONE

Lydia	321-549-3327 427-658-2314
Eugene	321-964-7324
Janet	684-358-7961
Rick	
Marty	684-369-7231 798-367-8541

Answer

ⓐ Both Lydia and Marty have two phone numbers. So each x-value is not matched with only one y-value. So this relation is not a function.

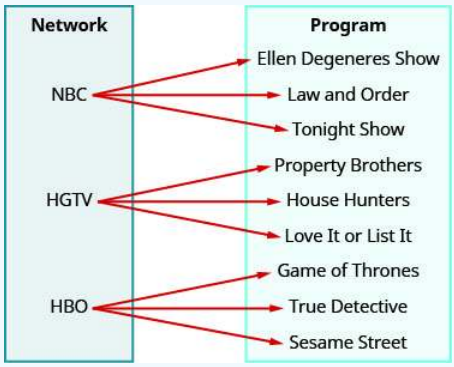
ⓑ The domain is the set of all x-values in the relation. The domain is: {Lydia, Eugene, Janet, Rick, Marty}

ⓒ The range is the set of all y-values in the relation. The range is:

$\{321-549-3327, 427-658-2314, 321-964-7324, 684-358-7961, 684-369-7231, 798-367-8541\}$

Example 3.2.14

Use the mapping to ⓐ determine whether the relation is a function ⓑ find the domain of the relation ⓒ find the range of the relation.



Answer

ⓐ no ⓑ {NBC, HGTV, HBO} ⓒ {Ellen Degeneres Show, Law and Order, Tonight Show, Property Brothers, House Hunters, Love it or List it, Game of Thrones, True Detective, Sesame Street}

Example 3.2.15

Use the mapping to

a. determine whether the relation is a function
b. find the domain of the relation
c. find the range of the relation.

Neal	123-567-4839 work 231-378-5941 cell
Krystal	743-469-9731 cell
Kelvin	567-534-2970 work
George	684-369-7231 cell
Christa	684-369-7231 cell
Mike	639-847-6971 cell

Answer

ⓐ No ⓑ {Neal, Krystal, Kelvin, George, Christa, Mike} ⓒ {123-567-4839 work, 231-378-5941 cell, 743-469-9731 cell, 567-534-2970 work, 684-369-7231 cell, 798-367-8541 cell, 639-847-6971 cell}

In algebra, more often than not, functions will be represented by an equation. It is easiest to see if the equation is a function when it is solved for *y*. If each value of *x* results in only one value of *y*, then the equation defines a function.

? Example 3.2.16

Determine whether each equation is a function.

a. $2x+y=7$
b. $y=x^2+1$
c. $x+y^2=3$

Answer

ⓐ $2x+y=7$

For each value of *x*, we multiply it by -2 and then add 7 to get the *y*-value

	$y=-2x+7$
For example, if $x=3$:	$y=-2\cdot 3+7$
	$y=1$

We have that when $x=3$, then $y=1$. It would work similarly for any value of *x*. Since each value of *x*, corresponds to only one value of *y* the equation defines a function.

ⓑ $y=x^2+1$

For each value of *x*, we square it and then add 1 to get the *y*-value.

	$y=x^2+1$
For example, if $x=2$:	$y=2^2+1$
	$y=5$

We have that when $x=2$, then $y=5$. It would work similarly for any value of *x*. Since each value of *x*, corresponds to only one value of *y* the equation defines a function.

ⓒ

	$x+y^2=3$
Isolate the y term.	$y^2=-x+3$
Let's substitute $x=2$.	$y^2=-2+3$
	$y^2=1$
This give us two values for y.	$y=1\ y=-1$

We have shown that when $x=2$, then $y=1$ and $y=-1$. It would work similarly for any value of x. Since each value of x does not corresponds to only one value of y the equation does not define a function.

Example 3.2.17

Determine whether each equation is a function.

a. $4x+y=-3$
b. $x+y^2=1$
c. $y-x^2=2$

Answer

ⓐ yes ⓑ no ⓒ yes

Example 3.2.18

Determine whether each equation is a function.

a. $x+y^2=4$
b. $y=x^2-7$
c. $y=5x-4$

Answer

ⓐ no ⓑ yes ⓒ yes

Find the Value of a Function

It is very convenient to name a function and most often we name it f, g, h, F, G, or H. In any function, for each x-value from the domain we get a corresponding y-value in the range. For the function f, we write this range value y as $f(x)$. This is called **function notation** and is read f of x or the value of f at x. In this case the parentheses does not indicate multiplication.

Definition: Function Notation

For the function $y=f(x)$

f is the name of the function
x is the domain value
$f(x)$ is the range value y corresponding to the value x

We read $f(x)$ as f of x or the value of f at x.

We call x the independent variable as it can be any value in the domain. We call y the dependent variable as its value depends on x.

INDEPENDENT AND DEPENDENT VARIABLES

For the function $y = f(x)$,

x is the independent variable as it can be any value in the domain
y the dependent variable as its value depends on x

Much as when you first encountered the variable *x*, function notation may be rather unsettling. It seems strange because it is new. You will feel more comfortable with the notation as you use it.

Let's look at the equation $y = 4x - 5$. To find the value of *y* when $x = 2$, we know to substitute $x = 2$ into the equation and then simplify.

	$y = 4x - 5$
Let x=2.	$y = 4 \cdot 2 - 5$
	$y = 3$

The value of the function at $x = 2$ is 3.

We do the same thing using function notation, the equation $y = 4x - 5$ can be written as $f(x) = 4x - 5$. To find the value when $x = 2$, we write:

	$f(x) = 4x - 5$
Let x=2.	$f(2) = 4 \cdot 2 - 5$
	$f(2) = 3$

The value of the function at $x = 2$ is 3.

This process of finding the value of $f(x)$ for a given value of *x* is called *evaluating the function.*

Example 3.2.19

For the function $f(x) = 2x^2 + 3x - 1$, evaluate the function.

a. $f(3)$
b. $f(-2)$
c. $f(a)$

Answer

ⓐ

	$f(x) = 2x^2 + 3x - 1$
To evaluate $f(3)$, substitute 3 for *x*.	$f(3) = 2(3)^2 + 3 \cdot 3 - 1$
Simplify.	$f(3) = 2 \cdot 9 + 3 \cdot 3 - 1$
	$f(3) = 18 + 9 - 1$
	$f(3) = 26$

ⓑ

		$f(x) = 2x^2 + 3x - 1$
To evaluate $f(-2)$, substitute -2 for *x*.		$f(-2) = 2(-2)^2 + 3(-2) - 1$

Simplify.		$f(-2) = 2 \cdot 4 + (-6) - 1$
		$f(-2) = 8 + (-6) - 1$
		$f(-2) = 1$

ⓒ

	$f(x) = 2x^2 + 3x - 1$
To evaluate f(a),f(a), substitute a for x.	$f(a) = 2(a)^2 + 3 \cdot a - 1$
Simplify.	$f(a) = 2a^2 + 3a - 1$

Example 3.2.20

For the function $f(x) = 3x^2 - 2x + 1$, evaluate the function.

a. $f(3)$
b. $f(-1)$
c. $f(t)$

Answer

ⓐ $f(3) = 22$ ⓑ $f(-1) = 6$ ⓒ $f(t) = 3t^2 - 2t - 1$

Example 3.2.21

For the function $f(x) = 2x^2 + 4x - 3$, evaluate the function.

a. $f(2)$
b. $f(-3)$
c. $f(h)$

Answer

ⓐ $(2) = 13$ ⓑ $f(-3) = 3$
ⓒ $f(h) = 2h2 + 4h - 3$

In the last example, we found $f(x)$ for a constant value of x. In the next example, we are asked to find $g(x)$ with values of x that are variables. We still follow the same procedure and substitute the variables in for the x.

Example 3.2.22

For the function $g(x) = 3x - 5$, evaluate the function.

a. $g(h^2)$
b. $g(x + 2)$
c. $g(x) + g(2)$

Answer

ⓐ

			$g(x) = 3x - 5$
To evaluate $g(h^2)$, substitute h^2 for x.			$g(h^2) = 3h^2 - 5$

			$g(h^2) = 3h^2 - 5$

ⓑ

	$g(x) = 3x - 5$
To evaluate $g(x+2)$, substitute $x+2$ for x.	$g(x+2) = 3(x+2) - 5$
Simplify.	$g(x+2) = 3x + 6 - 5$
	$g(x+2) = 3x + 1$

ⓒ

		$g(x) = 3x - 5$
To evaluate $g(x) + g(2)$, first find $g(2)$.		$g(2) = 3 \cdot 2 - 5$
		$g(2) = 1$
Now find $g(x) + g(2)$		$g(x) + g(2) = 3x - 5 + 1$ $g(x) \quad g(2)$
Simplify.		$g(x) + g(2) = 3x - 5 + 1$
		$g(x) + g(2) = 3x - 4$

Notice the difference between part ⓑ and ⓒ. We get $g(x+2) = 3x+1$ and $g(x)+g(2) = 3x-4$. So we see that $g(x+2) \neq g(x)+g(2)$.

Example 3.2.23

For the function $g(x) = 4x - 7$, evaluate the function.

a. $g(m^2)$
b. $g(x-3)$
c. $g(x) - g(3)$

Answer

ⓐ $4m^2 - 7$ ⓑ $4x - 19$
ⓒ $x - 12$

Example 3.2.24

For the function $h(x) = 2x + 1$, evaluate the function.

a. $h(k^2)$
b. $h(x+1)$
c. $h(x) + h(1)$

Answer

ⓐ $2k^2 + 1$ ⓑ $2x + 3$
ⓒ $2x + 4$

Many everyday situations can be modeled using functions.

Example 3.2.25

The number of unread emails in Sylvia's account is 75. This number grows by 10 unread emails a day. The function $N(t) = 75 + 10t$ represents the relation between the number of emails, *N*, and the time, *t*, measured in days.

a. Determine the independent and dependent variable.
b. Find $N(5)$. Explain what this result means.

Answer

ⓐ The number of unread emails is a function of the number of days. The number of unread emails, *N*, depends on the number of days, *t*. Therefore, the variable *N*, is the dependent variable and the variable tt is the independent variable.

ⓑ Find $N(5)$. Explain what this result means.

	$N(t) = 75 + 10t$
Substitute in t=5.t=5.	$N(5) = 75 + 10 \cdot 5$
Simplify.	$N(5) = 75 + 50$
	$N(5) = 125$

Since 5 is the number of days, $N(5)$, is the number of unread emails after 5 days. After 5 days, there are 125 unread emails in the account.

Example 3.2.26

The number of unread emails in Bryan's account is 100. This number grows by 15 unread emails a day. The function $N(t) = 100 + 15t$ represents the relation between the number of emails, *N*, and the time, *t*, measured in days.

a. Determine the independent and dependent variable.
b. Find $N(7)]$. Explain what this result means.

Answer

ⓐ *t* IND; *N* DEP ⓑ 205; the number of unread emails in Bryan's account on the seventh day.

Example 3.2.27

The number of unread emails in Anthony's account is 110. This number grows by 25 unread emails a day. The function $N(t) = 110 + 25t$ represents the relation between the number of emails, *N*, and the time, *t*, measured in days.

a. Determine the independent and dependent variable.
b. Find $N(14)$. Explain what this result means.

Answer

ⓐ *t* IND; *N* DEP ⓑ 460; the number of unread emails in Anthony's account on the fourteenth day

Access this online resource for additional instruction and practice with relations and functions.

- Introduction to Functions

Key Concepts

- **Function Notation:** For the function $y = f(x)$
  - f is the name of the function
  - x is the domain value

- $f(x)$ is the range value y corresponding to the value x
  We read $f(x)$ as f of x or the value of f at x.

- **Independent and Dependent Variables:** For the function $y = f(x)$,
  - x is the independent variable as it can be any value in the domain
  - y is the dependent variable as its value depends on x

Glossary

domain of a relation

The domain of a relation is all the x-values in the ordered pairs of the relation.

function

A function is a relation that assigns to each element in its domain exactly one element in the range.

mapping

A mapping is sometimes used to show a relation. The arrows show the pairing of the elements of the domain with the elements of the range.

range of a relation

The range of a relation is all the y-values in the ordered pairs of the relation.

relation

A relation is any set of ordered pairs,(x,y).(x,y). All the x-values in the ordered pairs together make up the domain. All the y-values in the ordered pairs together make up the range.

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3.2E: Exercises

Practice Makes Perfect

Find the Domain and Range of a Relation

In the following exercises, for each relation a. find the domain of the relation b. find the range of the relation.

1. $\{(1,4),(2,8),(3,12),(4,16),(5,20)\}$

Answer

a. $\{1,2,3,4,5\}$b. $\{4,8,12,16,20\}$

2. $\{(1,-2),(2,-4),(3,-6),(4,-8),(5,-10)\}$

3. $\{(1,7),(5,3),(7,9),(-2,-3),(-2,8)\}$

Answer

a. $\{1,5,7,-2\}$b. $\{7,3,9,-3,8\}$

4. $\{(11,3),(-2,-7),(4,-8),(4,17),(-6,9)\}$

In the following exercises, use the mapping of the relation to a. list the ordered pairs of the relation, b. find the domain of the relation, and c. find the range of the relation.

5.



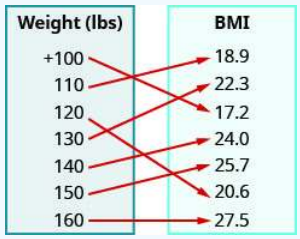
Answer

a. (Rebecca, January 18), (Jennifer, April 1), (John, January 18), (Hector, June 23), (Luis, February 15), (Ebony, April 7), (Raphael, November 6), (Meredith, August 19), (Karen, August 19), (Joseph, July 30)
b. {Rebecca, Jennifer, John, Hector, Luis, Ebony, Raphael, Meredith, Karen, Joseph}
c. {January 18, April 1, June 23, February 15, April 7, November 6, August 19, July 30}

6.



7. For a woman of height $5'4''$ the mapping below shows the corresponding Body Mass Index (BMI). The body mass index is a measurement of body fat based on height and weight. A BMI of $18.5–24.9$is considered healthy.



Answer

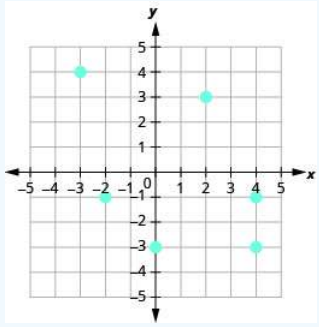
a. $(+100, 17.2), (110, 18.9), (120, 20.6), (130, 22.3), (140, 24.0), (150, 25.7), (160, 27.5)$ b. $\{+100, 110, 120, 130, 140, 150, 160\}$c. $\{17.2, 18.9, 20.6, 22.3, 24.0, 25.7, 27.5\}$

8. For a man of height $5'11''$ the mapping below shows the corresponding Body Mass Index (BMI). The body mass index is a measurement of body fat based on height and weight. A BMI of $18.5–24.9$is considered healthy.



In the following exercises, use the graph of the relation to a. list the ordered pairs of the relation b. find the domain of the relation c. find the range of the relation.

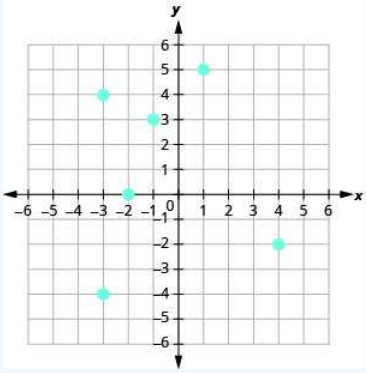
9.



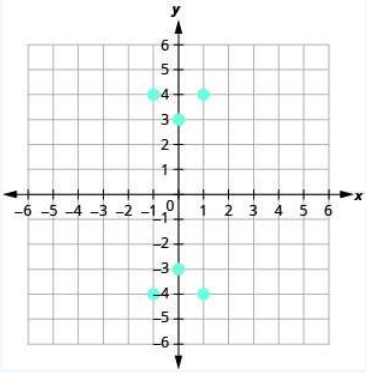
Answer

a. $(2, 3), (4, -3), (-2, -1), (-3, 4), (4, -1), (0, -3)$b. $\{-3, -2, 0, 2, 4\}$
c. $\{-3, -1, 3, 4\}$

10.



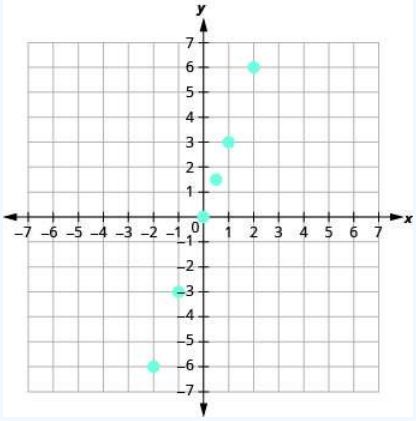
11.



Answer

a. $(1,4),(1,-4),(-1,4),(-1,-4),(0,3),(0,-3)$b. $\{-1,0,1\}$c. $\{-4,-3,3,4\}$

12.



Determine if a Relation is a Function

In the following exercises, use the set of ordered pairs to a. determine whether the relation is a function, b. find the domain of the relation, and c. find the range of the relation.

13. $\{(-3,9),(-2,4),(-1,1),(0,0),(1,1),(2,4),(3,9)\}$

Answer

a. yes b. $\{-3,-2,-1,0,1,2,3\}$c. $\{9,4,1,0\}$

14. $\{(9,-3),(4,-2),(1,-1),(0,0),(1,1),(4,2),(9,3)\}$

15. $\{(-3,27),(-2,8),(-1,1),(0,0),(1,1),(2,8),(3,27)\}$

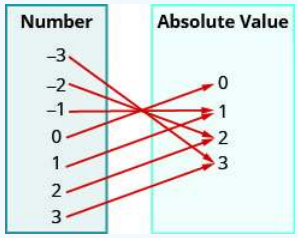
Answer

a. yes b. $\{-3,-2,-1,0,1,2,3\}$c. $\{0,1,8,27\}$

16. $\{(-3,-27),(-2,-8),(-1,-1),(0,0),(1,1),(2,8),(3,27)\}$

In the following exercises, use the mapping to a. determine whether the relation is a function, b. find the domain of the function, and c. find the range of the function.

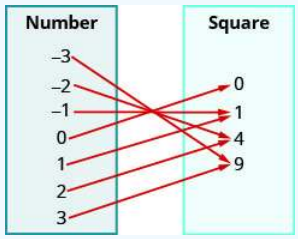
17.



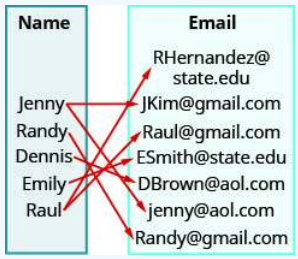
Answer

a. yes b. $\{-3,-2,-1,0,1,2,3\}$c. $\{0,1,2,3\}$

18.



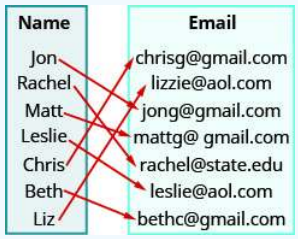
19.



Answer

a. no b. {Jenny, R and y, Dennis, Emily, Raul} c. {RHern and ez@state.edu, JKim@gmail.com, Raul@gmail.com, ESmith@state.edu, DBroen@aol.com, jenny@aol.cvom, R and y@gmail.com}

20.



In the following exercises, determine whether each equation is a function.

21. a. $2x+y=-3$
b. $y=x^2$
c. $x+y^2=-5$

Answer

a. yes b. yes c. no

22. a. $y=3x-5$
b. $y=x^3$
c. $2x+y^2=4$

23. a. $y-3x^3=2$
b. $x+y^2=3$
c. $3x-2y=6$

Answer

a. yes b. no c. yes

24. a. $2x-4y=8$
b. $-4=x^2-y$
c. $y^2=-x+5$

Find the Value of a Function

In the following exercises, evaluate the function: a. $f(2)$ b. $f(-1)$ c. $f(a)$.

25. $f(x)=5x-3$

Answer

a. $f(2)=7$ b. $f(-1)=-8$ c. $f(a)=5a-3$

26. $f(x)=3x+4$

27. $f(x)=-4x+2$

Answer

a. $f(2)=-6$ b. $f(-1)=6$ c. $f(a)=-4a+2$

28. $f(x)=-6x-3$

29. $f(x)=x^2-x+3$

Answer

a. $f(2)=5$ b. $f(-1)=5$
c. $f(a)=a^2-a+3$

30. $f(x)=x^2+x-2$

31. $f(x)=2x^2-x+3$

Answer

a. $f(2)=9$ b. $f(-1)=6$
c. $f(a)=2a^2-a+3$

32. $f(x)=3x^2+x-2$

In the following exercises, evaluate the function: a. $g(h^2)$ b. $g(x+2)$ c. $g(x)+g(2)$.

33. $g(x)=2x+1$

Answer

a. $g(h^2)=2h^2+1$
b. $g(x+2)=4x+5$
c. $g(x)+g(2)=2x+6$

34. $g(x)=5x-8$

35. $g(x)=-3x-2$

Answer

a. $g(h^2)=-3h^2-2$
b. $g(x+2)=-3x-8$
c. $g(x)+g(2)=-3x-10$

36. $g(x)=-8x+2$

37. $g(x)=3-x$

Answer

a. $g(h^2)=3-h^2$
b. $g(x+2)=1-x$
c. $g(x)+g(2)=4-x$

38. $g(x)=7-5x$

In the following exercises, evaluate the function.

39. $f(x)=3x^2-5x;\ f(2)$

Answer

2

40. $g(x) = 4x^2 - 3x$; $g(3)$

41. $F(x) = 2x^2 - 3x + 1$; $F(-1)$

Answer

6

42. $G(x) = 3x^2 - 5x + 2$; $G(-2)$

43. $h(t) = 2|t - 5| + 4$; $f(-4)$

Answer

22

44. $h(y) = 3|y - 1| - 3$; $h(-4)$

45. $f(x) = x + 2x - 1$; $f(2)$

Answer

4

46. $g(x) = x - 2x + 2$; $g(4)$

In the following exercises, solve.

47. The number of unwatched shows in Sylvia's DVR is 85. This number grows by 20 unwatched shows per week. The function $N(t) = 85 + 20t$ represents the relation between the number of unwatched shows, *N*, and the time, *t*, measured in weeks.

a. Determine the independent and dependent variable.

b. Find $N(4)$. Explain what this result means

Answer

a. *t* IND; *N* DEP
b. $N(4) = 165$ the number of unwatched shows in Sylvia's DVR at the fourth week.

48. Every day a new puzzle is downloaded into Ken's account. Right now he has 43 puzzles in his account. The function $N(t) = 43 + t$ represents the relation between the number of puzzles, *N*, and the time, *t*, measured in days.

a. Determine the independent and dependent variable.

b. Find $N(30)$. Explain what this result means.

49. The daily cost to the printing company to print a book is modeled by the function $C(x) = 3.25x + 1500$ where *C* is the total daily cost and *x* is the number of books printed.

a. Determine the independent and dependent variable.

b. Find $N(0)$. Explain what this result means.

c. Find $N(1000)$. Explain what this result means.

Answer

a. x IND; C DEP
b. $N(0) = 1500$ the daily cost if no books are printed
c. $N(1000) = 4750$the daily cost of printing 1000 books

50. The daily cost to the manufacturing company is modeled by the function $C(x) = 7.25x + 2500$ where $C(x)$ is the total daily cost and x is the number of items manufactured.

a. Determine the independent and dependent variable.

b. Find $C(0)$. Explain what this result means.

c. Find $C(1000)$. Explain what this result means.

Writing Exercises

51. In your own words, explain the difference between a relation and a function.

52. In your own words, explain what is meant by domain and range.

53. Is every relation a function? Is every function a relation?

54. How do you find the value of a function?

Self Check

a. After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
find the domain and range of a relation.			
determine if a relation is a function.			
find the value of a function.			

b. After looking at the checklist, do you think you are well-prepared for the next section? Why or why not?

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3.3: Interpreting the Graph of a Function

In the previous section, we began with a function and then drew the graph of the given function. In this section, we will start with the graph of a function, then make a number of interpretations based on the given graph: function evaluations, the domain and range of the function, and solving equations and inequalities.

The Vertical Line Test

Consider the graph of the relation R shown in Figure 3.3.1(a). Recall that we earlier defined a relation as a set of ordered pairs. Surely, the graph shown in Figure 3.3.1(a) is a set of ordered pairs. Indeed, it is an infinite set of ordered pairs, so many that the graph is a solid curve.

In Figure 3.3.1(b), note that we can draw a vertical line that cuts the graph more than once. In Figure 3.3.1(b), we've drawn a vertical line that cuts the graph in two places, once at (x, y_1), then again at (x, y_2), as shown in Figure 3.3.1(c). This means that the domain object x is paired with two different range objects, namely y_1 and y_2, so relation R is not a function.

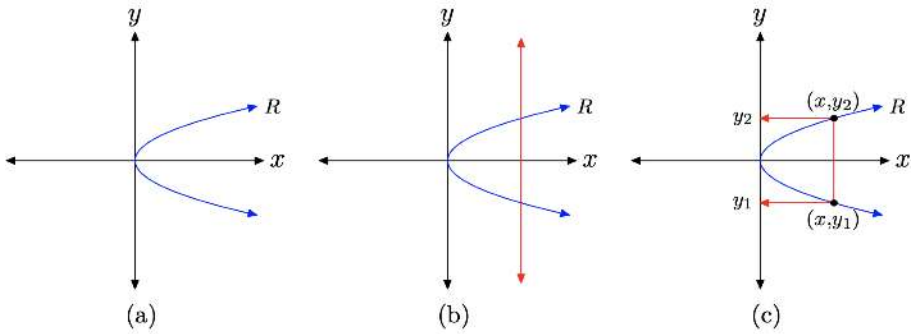


Figure 3.3.1. Explaining the vertical line test for functions.

Recall the definition of a function.

Definition

A relation is a function if and only if each object in its domain is paired with one and only one object in its range.

Consider the mapping diagram in Figure 3.3.2, where we've used arrows to indicate the ordered pairs (x, y_1) and (x, y_2) in Figure 3.3.1(c). Note that x, an object in the domain of R, is mapped to two objects in the range of R, namely y_1 and y_2. Hence, the relation R is not a function.

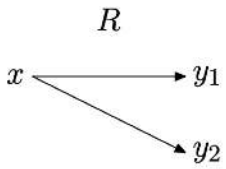


Figure 3.3.2. A mapping diagram representing the points (x, y_1) and (x, y_2) in Figure 3.3.1(c).

This discussion leads to the following result, called the vertical line test for functions.

The Vertical Line Test

If any vertical line cuts the graph of a relation more than once, then the relation is NOT a function.

Hence, the circle pictured in Figure 3.3.3(a) is a relation, but it is not the graph of a function. It is possible to cut the graph of the circle more than once with a vertical line, as shown in Figure 3.3.3(a). On the other hand, the parabola shown in Figure 3.3.3(b) is the graph of a function, because no vertical line will cut the graph more than once.

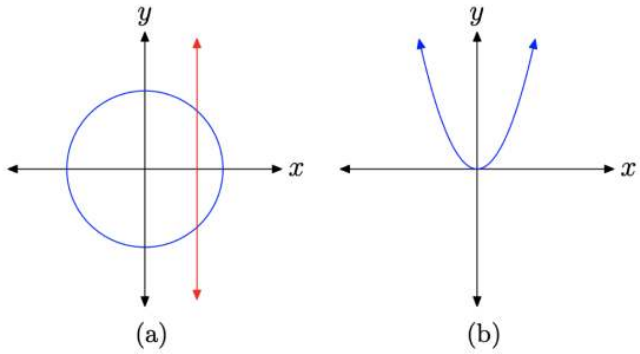


Figure 3.3.3. Use the vertical line test to determine if the graph is the graph of a function.

Reading the Graph for Function Values

We know that the graph of f pictured in Figure 3.3.4 is the graph of a function. We know this because no vertical line will cut the graph of f more than once.

We earlier defined the graph of f as the set of all ordered pairs $(x, f(x))$, so that x is in the domain of f. Consequently, if we select a point P on the graph of f, as in Figure 3.3.4(a), we label the point P(x, f(x)). However, we can also label this point as $P(x, y)$, as shown in Figure 3.3.4(b). This leads to a new interpretation of f(x) as the y-value of the point P. That is, f(x) is the y-value that is paired with x.

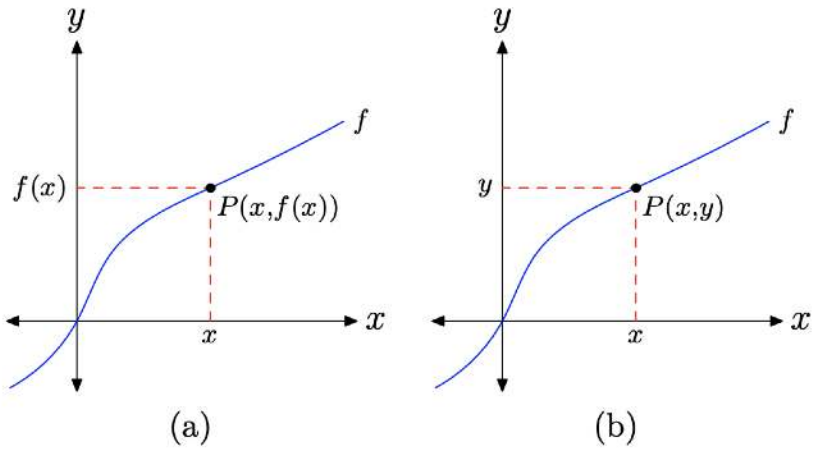


Figure 3.3.4. Reading the graph of a function.

Definition

f(x) is the y-value that is paired with x.

Two more comments are in order. In Figure 3.3.4(a), we select a point P on the graph of f.

1. To find the x-value of the point P, we must project the point P onto the x-axis.
2. To find f(x), the value of y that is paired with x, we must project the point P onto the y-axis.

Let's look at an example.

Example 3.3.1

Given the graph of f in Figure 3.3.5(a), find f(4).

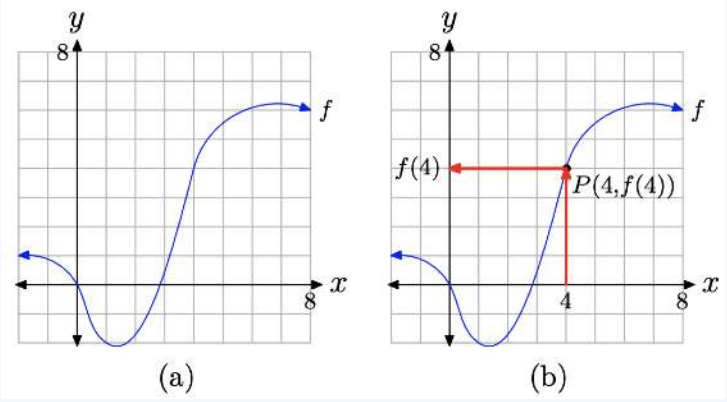


Figure 3.3.5. Finding the value of f(4).

Solution

First, note that the graph of f represents a function. No vertical line will cut the graph of f more than once.

Because f(4) represents the y-value that is paired with an x-value of 4, we first locate 4 on the x-axis, as shown in Figure (\PageIndex{5}\)(b). We then draw a vertical arrow until we intercept the graph of f at the point P(4, f(4)). Finally, we draw a horizontal arrow from the point P until we intercept the y-axis. The projection of the point P onto the y-axis is the value of f(4).

Because we have a grid that shows a scale on each axis, we can approximate the value of f(4). It would appear that the y-value of point P is approximately 4. Thus, $f(4) \approx 4$.

Let's look at another example.

Example 3.3.2

Given the graph of f in Figure 3.3.6(a), find f(5).

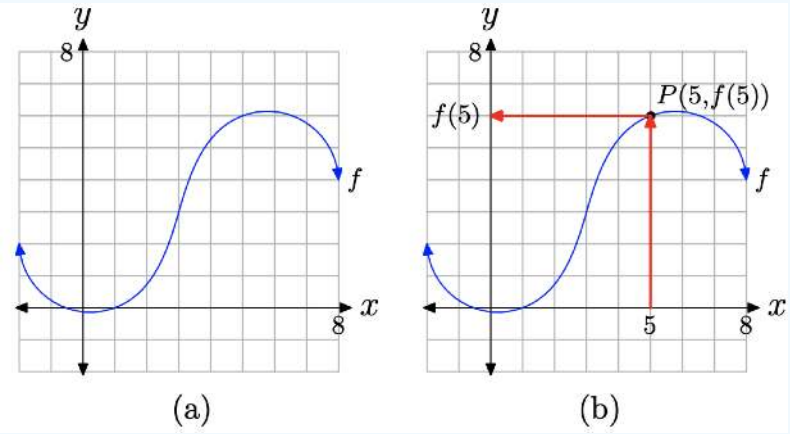


Figure 3.3.6. Finding the value of f(5).

Solution

First, note that the graph of f represents a function. No vertical line will cut the graph of f more than once.

Because f(5) represents the y-value that is paired with an x-value of 5, we first locate 5 on the x-axis, as shown in Figure 3.3.6(b). We then draw a vertical arrow until we intercept the graph of f at the point P(5, f(5)). Finally, we draw a horizontal arrow from the point P until we intercept the y-axis. The projection of the point P onto the y-axis is the value of f(5).

Because we have a grid that shows a scale on each axis, we can approximate the value of f(5). It would appear that the y-value of point P is approximately 6. Thus, $f(5) \approx 6$.

Let's reverse the interpretation in another example.

Example 3.3.3

Given the graph of f in Figure 3.3.7(a), for what value of x does f(x) = −4?

Solution

Again, the graph in Figure 3.3.7 passes the vertical line test and represents the graph of a function.

This time, in the equation $f(x) = -4$, we're given a y-value equal to −4. Consequently, we must reverse the process used in Example 3.3.1 and Example 3.3.2. We first locate the y-value −4 on the y-axis, then draw a horizontal arrow until we intercept

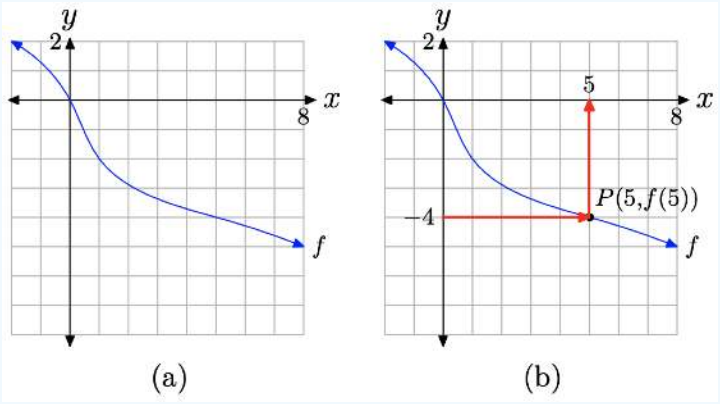


Figure 3.3.7. Finding x so that $f(x) = -4$.

the graph of f at P, as shown in Figure 3.3.7(b). Finally, we draw a vertical arrow from the point P until we intercept the x-axis. The projection of the point P onto the x-axis is the solution of $f(x) = -4$.

Because we have a grid that shows a scale on each axis, we can approximate the x-value of the point P. It seems that $x \approx 5$. Thus, we label the point P(5, f(5)), and the solution of $f(x) = -4$ is approximately $x \approx 5$.

This solution can easily be checked by computing f(5). Simply start with 5 on the x-axis, then reverse the order of the arrows shown in Figure 3.3.7(b). You should wind up at −4 on the y-axis, demonstrating that $f(5) = -4$.

The Domain and Range of a Function

We can use the graph of a function to determine its domain and range. For example, consider the graph of the function shown in Figure 3.3.8(a).

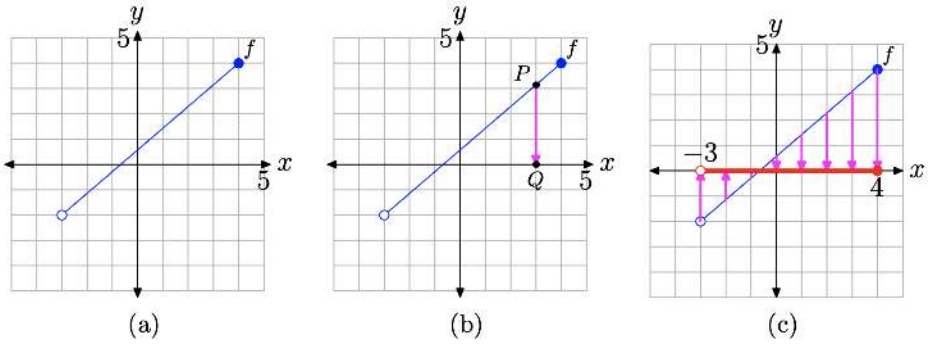


Figure 3.3.8. Determining the domain of a function from its graph.

Note that no vertical line will cut the graph of f more than once, so the graph of f represents a function.

To determine the domain, we must collect the x-values (first coordinates) of every point on the graph of f. In Figure 3.3.8(b), we've selected a point P on the graph of f, which we then project onto the x-axis. The image of this projection is the point Q, and the x-value of the point Q is an element in the domain of f.

Think of the projection shown in Figure 3.3.8(b) in the following manner. Imagine a light source above the point P. The point P blocks out the light and its shadow falls onto the x-axis at the point Q. That is, think of point Q as the "shadow" that the point P

produces when it is projected vertically onto the x-axis.

Now, to find the domain of the function f, we must project each point on the graph of f onto the x-axis. Here's the question: if we project each point on the graph of f onto the x-axis, what part of the x-axis will "lie in shadow" when the process is complete? The answer is shown in Figure 3.3.8(c).

In Figure 3.3.8(c), note that the "shadow" created by projecting each point on the graph of f onto the x-axis is shaded in red (a thicker line if you are viewing this in black and white). This collection of x-values is the domain of the function f. There are three critical points that we need to make about the "shadow" on the x-axis in Figure 3.3.8(c).

1. All points lying between $x = -3$ and $x = 4$ have been shaded on the x-axis in red.
2. The left endpoint of the graph of f is an open circle. This indicates that there is no point plotted at this endpoint. Consequently, there is no point to project onto the x-axis, and this explains the open circle at the left end of our "shadow" on the x-axis.
3. On the other hand, the right endpoint of the graph of f is a filled endpoint. This indicates that this is a plotted point and part of the graph of f. Consequently, when this point is projected onto the x-axis, a shadow falls at x = 4. This explains the filled endpoint at the right end of our "shadow" on the x-axis.

We can describe the x-values of the "shadow" on the x-axis using set-builder notation.

$$\text{Domain of } f = \{x : -3 < x \leq 4\}$$

Note that we don't include −3 in this description because the left end of the shadow on the x-axis is an empty circle. Note that we do include 4 in this description because the right end of the shadow on the x-axis is a filled circle.

We can also describe the x-values of the "shadow" on the x-axis using interval notation.

$$\text{Domain of } f = (-3, 4]$$

We remind our readers that the parenthesis on the left means that we are not including −3, while the bracket on the right means that we are including 4.

To find the range of the function, picture again the graph of f shown in Figure 3.3.9(a). Proceed in a similar manner, only this time project points on the graph of f onto the y-axis, as shown in Figures 3.3.9(b) and (c).

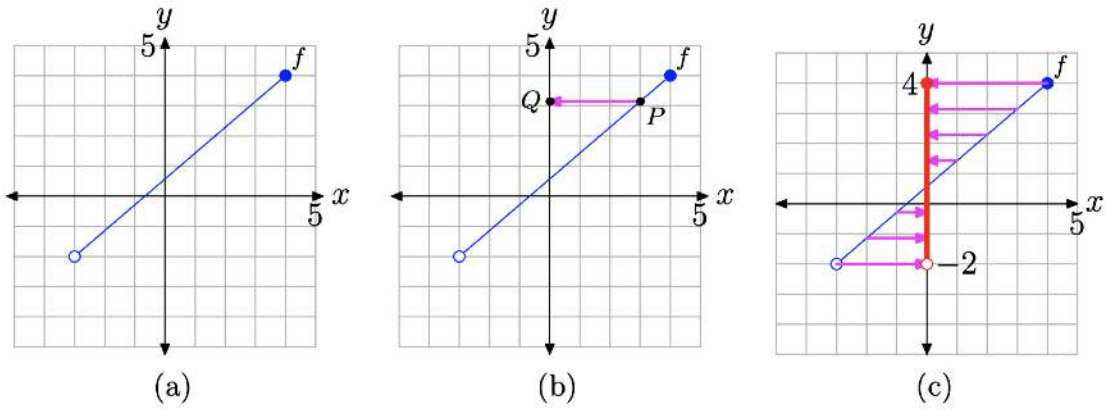


Figure 3.3.9. Determining the range of a function from its graph.

Note which part of the y-axis "lies in shadow" once we've projected all points on the graph of f onto the y-axis.

1. All points lying between $y = -2$ and $y = 4$ have been shaded on the y-axis in red (a thicker line style if you are viewing this in black and white).
2. The left endpoint of the graph of f is an empty circle, so there is no point to project onto the y-axis. Consequently, there is no "shadow" at $y = -2$ on the y-axis and the point is left unshaded (an empty circle).
3. The right endpoint of the graph of f is a filled circle, so there is a "shadow" at $y = 4$ on the y-axis and this point is shaded (a filled circle).

We can now easily describe the range in both set-builder and interval notation.

$$\text{Range of } f = (-2, 4] = \{y : -2 < y \leq 4\}$$

Let's look at another example.

Example 3.3.4

Use set-builder and interval notation to describe the domain and range of the function represented by the graph in Figure 3.3.10(a).

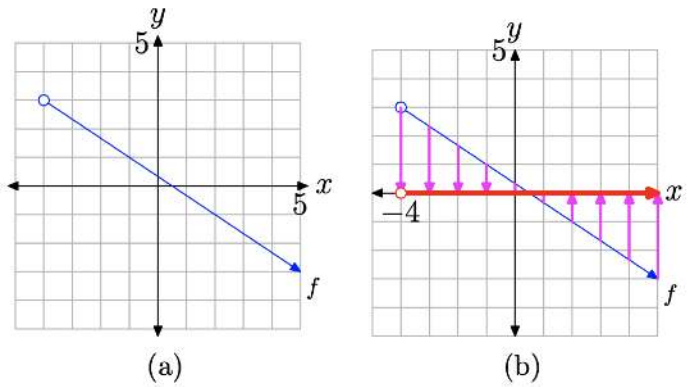


Figure 3.3.10. Determining the domain from the graph of f.

Solution

To determine the domain of f, project each point on the graph of f onto the x-axis. This projection is indicated by the "shadow" on the x-axis in Figure 3.3.10(b). Two important points need to be made about this "shadow" or projection.

1. The left endpoint of the graph of f is empty (indicated by the open circle), so it has no projection onto the x-axis. This is indicated by an open circle at the left end (at $x = -4$) of the "shadow" or projection on the x-axis.

2. The arrowhead on the right end of the graph of f indicates that the graph of f continues downward and to the right indefinitely. Consequently, the projection onto the x-axis is a shadow that moves indefinitely to the right. This is indicated by an arrowhead at the right end of the "shadow" or projection on the x-axis.

Consequently, the domain of f is the collection of x-values represented by the "shadow" or projection onto the x-axis. Note that all x-values to the right of $x = -4$ are shaded on the x-axis. Consequently,

$$\text{Domain of } f = (-4, \infty) = \{x : x > -4\}$$

To find the range, we must project each point on the graph of f (redrawn in Figure 3.3.11(a)) onto the y-axis. The projection is indicated by a "shadow" or projection on the y-axis, as seen in Figure 3.3.11(b). Two important points need to be made about this "shadow" or projection.

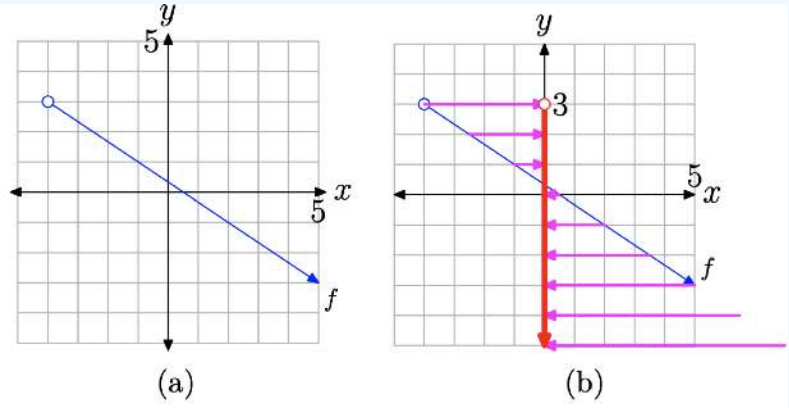


Figure 3.3.11. Determining the range from the graph of f.

1. The left endpoint of the graph of f is empty (indicated by an open circle), so it has no projection onto the y-axis. This is indicated by an open circle at the top end (at $y = 3$) of the "shadow" on the y-axis.

2. The arrowhead on the right end of the graph of f indicates that the graph of f continues downward and to the right indefinitely. Consequently, the projection of the graph of f onto the y-axis is a shadow that moves indefinitely downward. In Figure 3.3.11(b), note how projections of points on the graph of f not visible in the viewing window come in from the lower right corner and cast "shadows" on the y-axis.

Consequently, the range of f is the collection of y-values shaded on the y-axis of the coordinate system shown in Figure 3.3.11(b). Note that all y-values lower than $y = 3$ are shaded on the y-axis. Thus, the range of f is

$$\text{Range of } f = (-\infty, 3) = \{y : y < 3\}$$

Let's look at another example.

Example 3.3.5

Use set-builder and interval notation to describe the domain and range of the function represented by the graph in Figure 3.3.12(a).

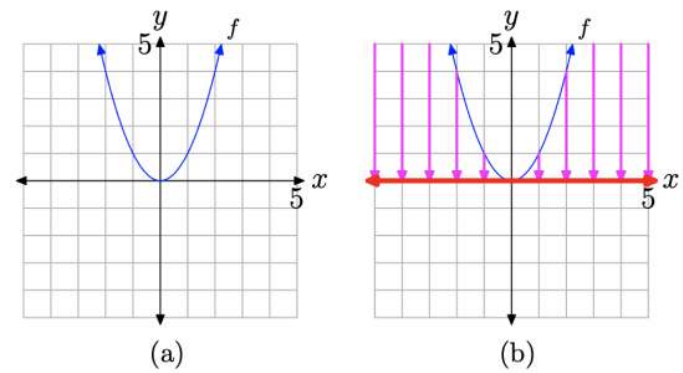


Figure 3.3.12 Determining the domain from the graph of f.

Solution

To determine the domain of f, we must project all points on the graph of f onto the x-axis. This projection is indicated by the red "shadow" (or thicker line style if you are viewing this in black and white) shown on the x-axis in Figure 3.3.12(b). Two important points need to be made about this "shadow" or projection.

1. The arrow at the end of the left half of the graph of f in Figure 3.3.12(a) indicates that this half of the graph of f opens indefinitely to the left and upward. Consequently, when the points on the left half of the graph of f are projected onto the x-axis, the "shadow" or projection extends indefinitely to the left. Note how points on the graph that fall outside the viewing window come in from the upper left corner and cast "shadows" on the x-axis.
2. The arrow at the end of the right half of the graph of f in Figure 3.3.12(a) indicates that this half of the graph of f opens indefinitely to the right and upward. Consequently, when the points on this half of the graph of f are projected onto the x-axis, the "shadow" or projection extends indefinitely to the right.

Consequently, the entire x-axis lies in "shadow," making the domain of f to be

$$\text{Domain of } f = (-\infty, \infty) = \{x : x \in \mathbb{R}\}$$

To determine the range of f, we must project all points on the graph of f onto the y-axis. This projection is indicated by the red "shadow" (or thicker line if you are viewing this in black and white) shown on the y-axis in Figure 3.3.13(b). Two important points need to made about this "shadow" or projection.

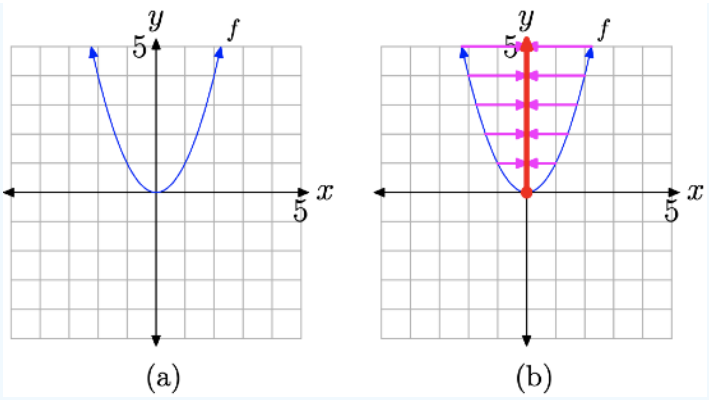


Figure 3.3.13 Determining the range from the graph of f.

1. The graph of f passes through the origin (the point (0, 0)). This is the lowest point on the graph and hence its shadow is the endpoint on the low end of the shaded region on the y-axis.
2. The arrows at the end of each half of the graph of f indicate that the graph opens upward indefinitely. Hence, when points on the graph of f are projected onto the y-axis, the "shadow" or projection extends upward indefinitely. This is indicated by an arrow on the upper end of the "shadow" on the y-axis.

Consequently, all points on the y-axis above and including the point at the origin "lie in shadow." Thus, the range of f is

$$\text{Range of } f = [0, \infty) = \{y : y \geq 0\}$$

Using a Graphing Calculator to Determine Domain and Range

We've learned how to find the domain and range of a function by looking at its graph. Therefore, if we define a function by means of an expression, such as $f(x) = \sqrt{4 - x}$, then we should be able to capture the domain and range of f from its graph, provided, of course, that we can draw the graph of f. We'll find the graphing calculator will be a handy tool for this exercise.

Example 3.3.6

Use set-builder and interval notation to describe the domain and range of the function defined by the rule

$$f(x) = \sqrt{4 - x}$$

Solution

Load the expression defining f into the Y= menu, as shown in Figure 3.3.14(a). Select 6:ZStandard from the ZOOM menu to produce the graph of f shown in Figure 3.3.14(b).

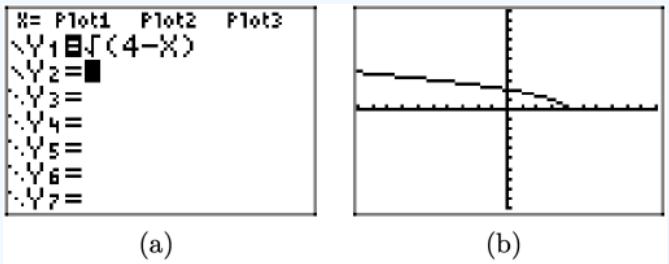


Figure 3.3.14 Sketching the graph of $f(x) = \sqrt{4 - x}$.

Copy the image in Figure 3.3.14(b) onto a sheet of graph paper. Label and scale each axis with the WINDOW parameters xmin, xmax, ymin, and ymax, as shown in Figure 3.3.15(a).

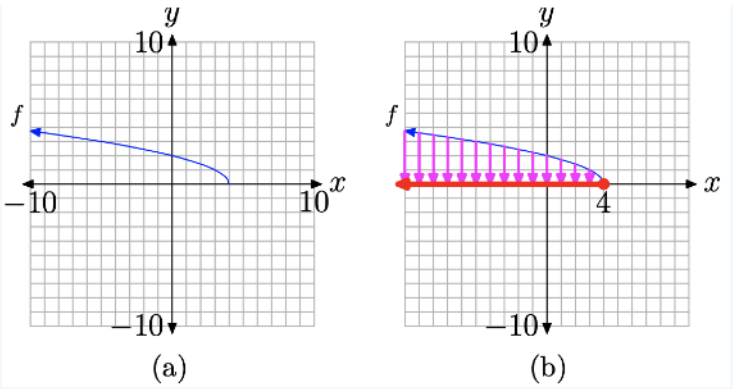


Figure 3.3.15. Capturing the domain of $f(x) = \sqrt{4-x}$ from its graph.

Next, project each point on the graph of f onto the x-axis, as shown in Figure 3.3.15(b). Note that we've made two assumptions about the graph of f.

1. At the left end of the graph in Figures 3.3.14(b) and 3.3.15(b), we assume that the graph of f continues upward and to the left indefinitely. Hence, the "shadow" or projection onto the x-axis will move indefinitely to the left. This is indicated by attaching an arrowhead to the left-hand end of the region that "lies in shadow" on the x-axis, as shown in Figure 3.3.15(b).
2. We also assume that the right end of the graph ends at the point $(4, 0)$. This accounts for the "filled dot" when this point on the graph of f is projected onto the x-axis.

Note that the "shadow" or projection onto the x axis in Figure 3.3.15(b) includes all values of x less than or equal to 4. Thus, the domain of f is

$$\text{Domain of } f = (-\infty, 4] = \{x : x \leq 4\}$$

We can intuit this result by considering the expression that defines f. That is, consider the rule or definition

$$f(x) = \sqrt{4-x}$$

Recall that we earlier defined the domain of f as the set of "permissible" x-values. In this case, it is impossible to take the square root of a negative number, so we must be careful selecting the x-values we use in this rule. Note that $x = 4$ is allowable, as

$$f(0) = \sqrt{4-4} = \sqrt{0} = 0$$

However, numbers larger than 4 cannot be used in this rule. For example, consider what happens when we attempt to use $x = 5$.

$$f(x) = \sqrt{4-5} = \sqrt{-1}$$

We'll leave it to our readers to test other values of x that are less than 4. They will also produce real answers when they are input into the rule $f(x) = \sqrt{4-x}$. Note that this also verifies our earlier conjecture that the "shadow" or projection shown in Figure 3.3.15(b) continues indefinitely to the left.

Instead of "guessing and checking," we can speed up the analysis of the domain of $f(x) = \sqrt{4-x}$ by noting that the expression under the radical must not be a negative number. Hence, $4-x$ must either be greater than or equal to zero. This argument produces an inequality that is easily solved for x.

$$\begin{aligned} 4-x &\geq 0 \\ -x &\geq -4 \\ x &\leq 4 \end{aligned}$$

This last result verifies that the domain of f is all values of x that are less than or equal to 4, which is in complete agreement with the "shadow" or projection onto the x-axis shown in Figure 3.3.15(b).

To determine the range of f, we must project each point on the graph of f onto the y-axis, as shown in Figure 3.3.16(b).

Again, we make two assumptions about the graph of f.

1. At the left-end of the graph of $f(x) = \sqrt{4-x}$ in Figures 3.3.14(b) and 3.3.16(b), we assume that the graph of f continues upward and to the left indefinitely. Thus, when points on the graph of f are projected onto the y-axis, there will be projections coming from the upper left from points on the graph of f that are not visible in the viewing window selected in Figure 3.3.14(b). Hence, the "shadow" or projection on the y-axis shown in Figure 3.3.16(b) continues upward indefinitely. This is indicated with a arrowhead at the upper end of the "shadow" on the y-axis in Figure 3.3.16(b).

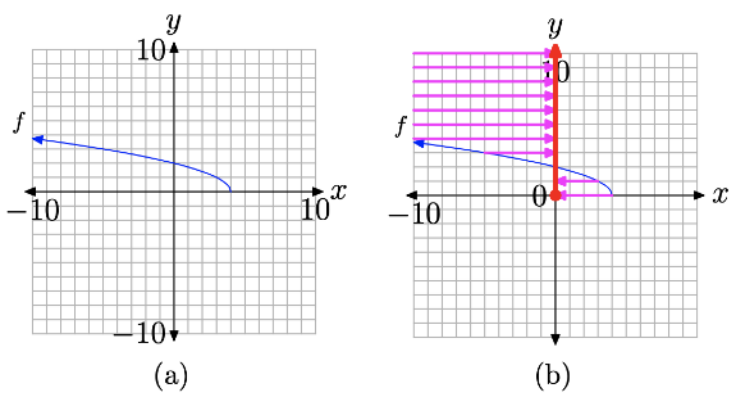


Figure 3.3.16. Determining the range of $f(x) = \sqrt{4-x}$ from its graph.

2. Again, we assume that the right end of the graph of f ends at the point $(4, 0)$. The projection of this point onto the y-axis produces the "filled" endpoint at the origin shown in Figure 3.3.16(b).

Note that the "shadow" or projection onto the y-axis in Figure 3.3.16(b) includes all values of y that are greater than or equal to zero. Hence,

$$\text{Range of } f = [0, \infty) = \{y : y \geq 0\}$$

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3.4: The Toolbox Functions

Learning Objectives

By the end of this section, you will be able to:

- Use the vertical line test
- Identify graphs of basic functions
- Read information from a graph of a function

Before you get started, take this readiness quiz.

1. Evaluate: ⓐ 2^3 ⓑ 3^2.
   If you missed this problem, review [link].
2. Evaluate: ⓐ $|7|$ ⓑ $|-3|$.
   If you missed this problem, review [link].
3. Evaluate: ⓐ $\sqrt{4}$ ⓑ $\sqrt{16}$.
   If you missed this problem, review [link].

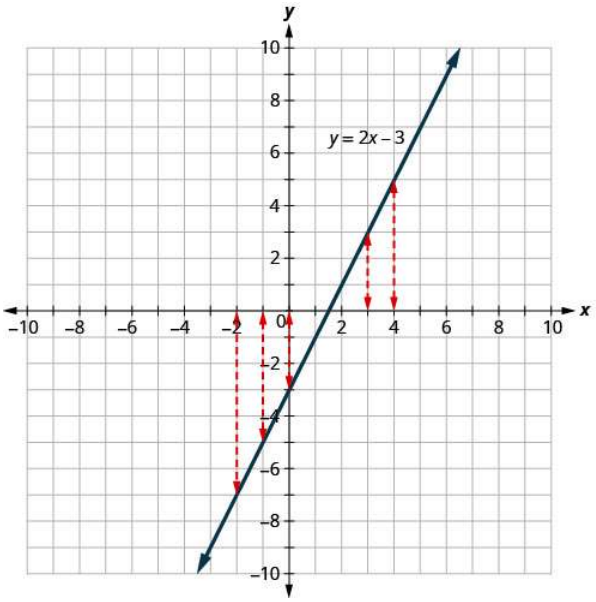
Use the Vertical Line Test

In the last section we learned how to determine if a relation is a function. The relations we looked at were expressed as a set of ordered pairs, a mapping or an equation. We will now look at how to tell if a graph is that of a function.

An ordered pair (x, y) is a solution of a linear equation, if the equation is a true statement when the *x*- and *y*-values of the ordered pair are substituted into the equation.

The graph of a linear equation is a straight line where every point on the line is a solution of the equation and every solution of this equation is a point on this line.

In Figure, we can see that, in graph of the equation $y = 2x - 3$, for every *x*-value there is only one *y*-value, as shown in the accompanying table.



y = 2x – 3		
x	y	(x, y)
–2	–7	(–2, –7)
–1	–5	(–1, –5)
0	–3	(0, –3)
3	3	(3, 3)
4	5	(4, 5)

Figure 3.4.1

A relation is a function if every element of the domain has exactly one value in the range. So the relation defined by the equation $y = 2x - 3$ is a function.

If we look at the graph, each vertical dashed line only intersects the line at one point. This makes sense as in a function, for every *x*-value there is only one *y*-value.

If the vertical line hit the graph twice, the x-value would be mapped to two y-values, and so the graph would not represent a function.

This leads us to the vertical line test. A set of points in a rectangular coordinate system is the graph of a function if every vertical line intersects the graph in at most one point. If any vertical line intersects the graph in more than one point, the graph does not represent a function.

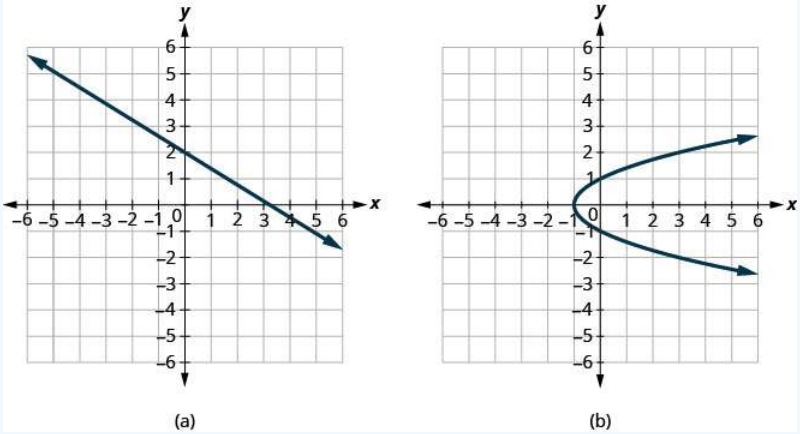
VERTICAL LINE TEST

A set of points in a rectangular coordinate system is the graph of a function if every vertical line intersects the graph in at most one point.

If any vertical line intersects the graph in more than one point, the graph does not represent a function.

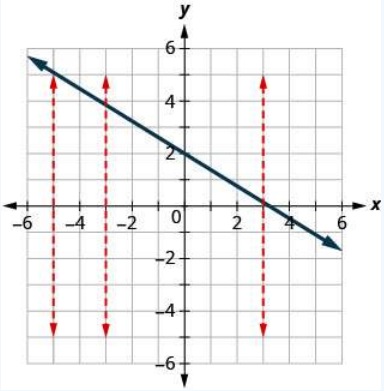
Example 3.4.1

Determine whether each graph is the graph of a function.

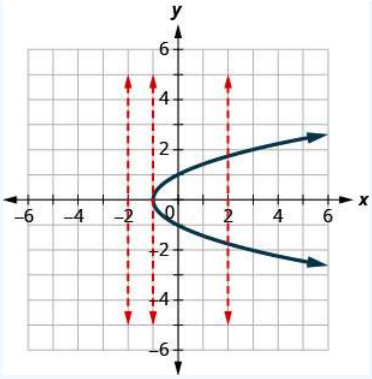


Answer

ⓐ Since any vertical line intersects the graph in at most one point, the graph is the graph of a function.

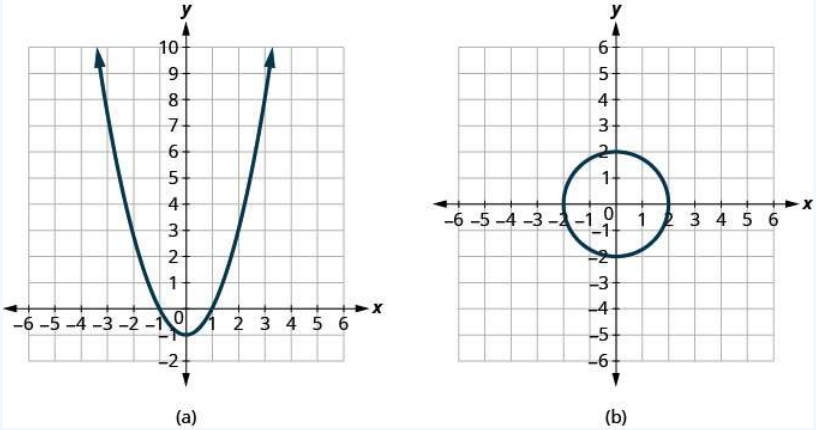


ⓑ One of the vertical lines shown on the graph, intersects it in two points. This graph does not represent a function.



Example 3.4.2

Determine whether each graph is the graph of a function.

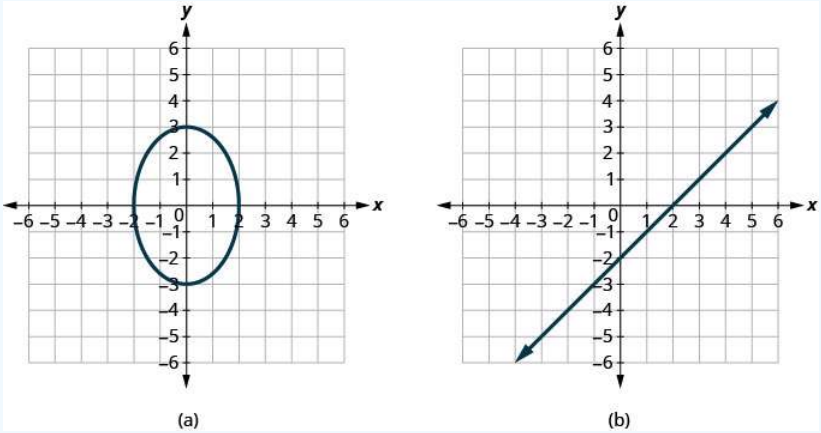


Answer

ⓐ yes ⓑ no

Example 3.4.3

Determine whether each graph is the graph of a function.



Answer

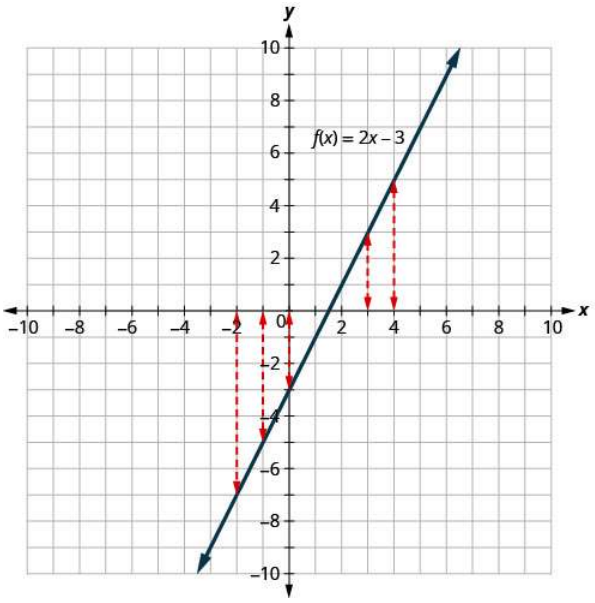
ⓐ no ⓑ yes

Identify Graphs of Basic Functions

We used the equation $y = 2x - 3$ and its graph as we developed the vertical line test. We said that the relation defined by the equation $y = 2x - 3$ is a function.

We can write this as in function notation as $f(x) = 2x - 3$. It still means the same thing. The graph of the function is the graph of all ordered pairs (x, y) where $y = f(x)$. So we can write the ordered pairs as $(x, f(x))$. It looks different but the graph will be the same.

Compare the graph of $y = 2x - 3$ previously shown in Figure with the graph of $f(x) = 2x - 3$ shown in Figure. Nothing has changed but the notation.



f(x) = 2x – 3		
x	f(x)	(x, f(x))
–2	–7	(–2, –7)
–1	–5	(–1, –5)
0	–3	(0, –3)
3	3	(3, 3)
4	5	(4, 5)

Figure 3.4.2

GRAPH OF A FUNCTION

The graph of a function is the graph of all its ordered pairs, (x,y)(x,y) or using function notation, (x,f(x))(x,f(x)) where y=f(x).y=f(x).

f	name of function
x	x-coordinate of the ordered pair
$f(x)$	y-coordinate of the ordered pair

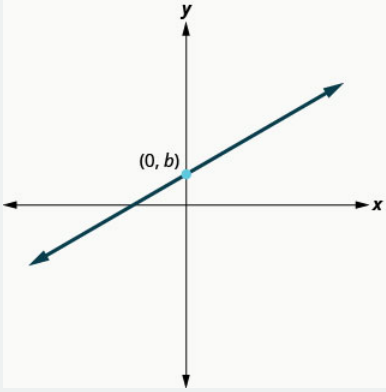
As we move forward in our study, it is helpful to be familiar with the graphs of several basic functions and be able to identify them.

Through our earlier work, we are familiar with the graphs of linear equations. The process we used to decide if $y = 2x - 3$ is a function would apply to all linear equations. All non-vertical linear equations are functions. Vertical lines are not functions as the *x*-value has infinitely many *y*-values.

We wrote linear equations in several forms, but it will be most helpful for us here to use the slope-intercept form of the linear equation. The slope-intercept form of a linear equation is $y = mx + b$. In function notation, this linear function becomes $f(x) = mx + b$ where *m* is the slope of the line and *b* is the *y*-intercept.

The domain is the set of all real numbers, and the range is also the set of all real numbers.

LINEAR FUNCTION



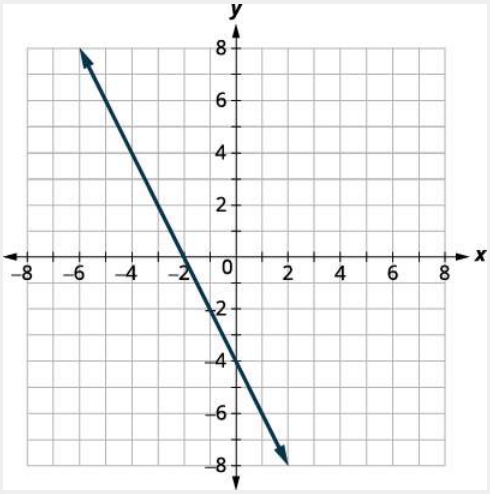
$f(x) = mx + b$
m, b: all real numbers
m: slope of the line
b: y-intercept
Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$

We will use the graphing techniques we used earlier, to graph the basic functions.

Example 3.4.4

Graph: $f(x) = -2x - 4$.

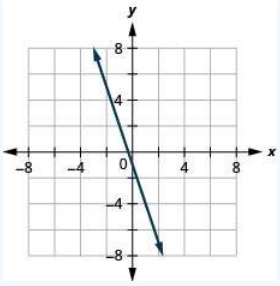
Answer

	$f(x) = -2x - 4$
We recognize this as a linear function.	
Find the slope and y-intercept.	$m = -2$ $b = -4$
Graph using the slope intercept.	

Example 3.4.5

Graph: $f(x) = -3x - 1$

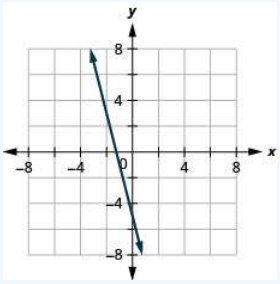
Answer



Example 3.4.6

Graph: $f(x) = -4x - 5$

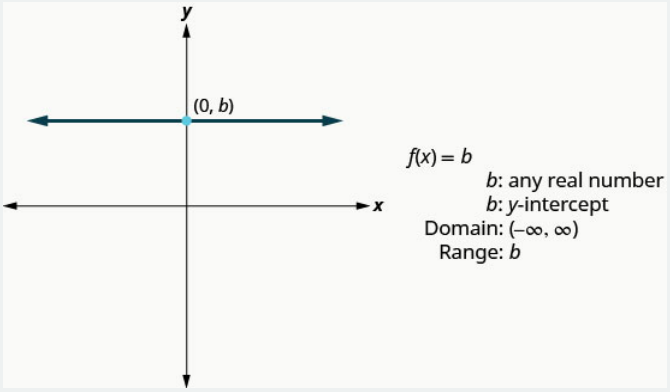
Answer



The next function whose graph we will look at is called the constant function and its equation is of the form $f(x) = b$, where b is any real number. If we replace the $f(x)$ with y, we get $y = b$. We recognize this as the horizontal line whose y-intercept is b. The graph of the function $f(x) = b$, is also the horizontal line whose y-intercept is b.

Notice that for any real number we put in the function, the function value will be b. This tells us the range has only one value, b.

CONSTANT FUNCTION



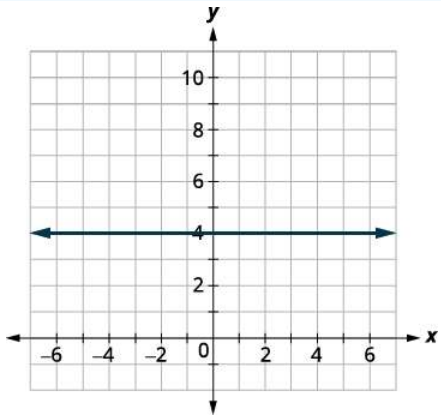
Example 3.4.7

Graph: $f(x) = 4$.

Answer

	$f(x) = 4$
We recognize this as a constant function.	

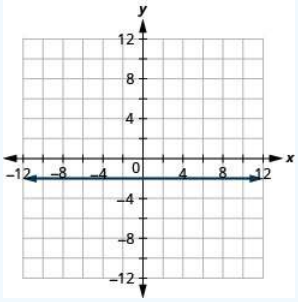
The graph will be a horizontal line through $(0, 4)$.



Example 3.4.8

Graph: $f(x) = -2$.

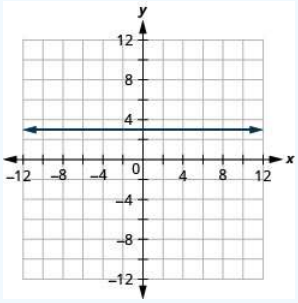
Answer



Example 3.4.9

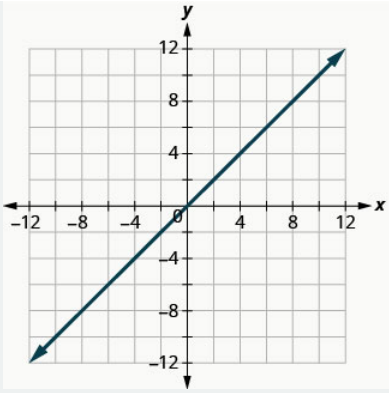
Graph: $f(x) = 3$.

Answer



The identity function, $f(x) = x$ is a special case of the linear function. If we write it in linear function form, $f(x) = 1x + 0$, we see the slope is 1 and the *y*-intercept is 0.

IDENTITY FUNCTION



$f(x) = x$
m: 1
b: 0
Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$

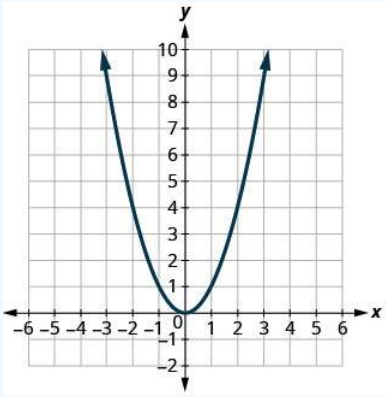
The next function we will look at is not a linear function. So the graph will not be a line. The only method we have to graph this function is point plotting. Because this is an unfamiliar function, we make sure to choose several positive and negative values as well as 0 for our x-values.

?

Graph: $f(x) = x^2$.

Answer

We choose x-values. We substitute them in and then create a chart as shown.

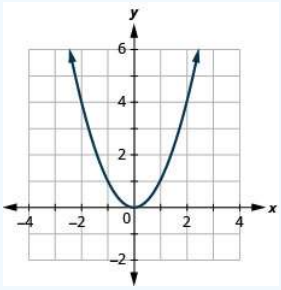


x	$f(x) = x^2$	$(x, f(x))$
−3	9	(−3, 9)
−2	4	(−2, 4)
−1	1	(−1, 1)
0	0	(0, 0)
1	1	(1, 1)
2	4	(2, 4)
3	9	(3, 9)

? Example 3.4.11

Graph: $f(x) = x^2$.

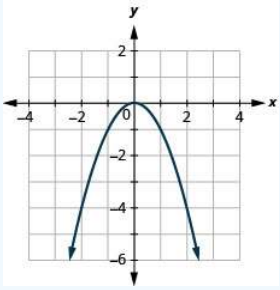
Answer



Example 3.4.12

$f(x) = -x^2$

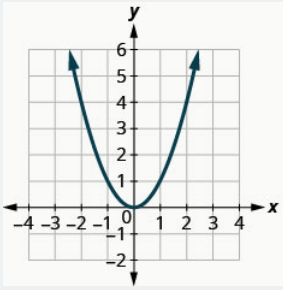
Answer



Looking at the result in Example, we can summarize the features of the square function. We call this graph a parabola. As we consider the domain, notice any real number can be used as an *x*-value. The domain is all real numbers.

The range is not all real numbers. Notice the graph consists of values of *y* never go below zero. This makes sense as the square of any number cannot be negative. So, the range of the square function is all non-negative real numbers.

SQUARE FUNCTION



$f(x) = x^2$
Domain: $(-\infty, \infty)$
Range: $[0, \infty)$

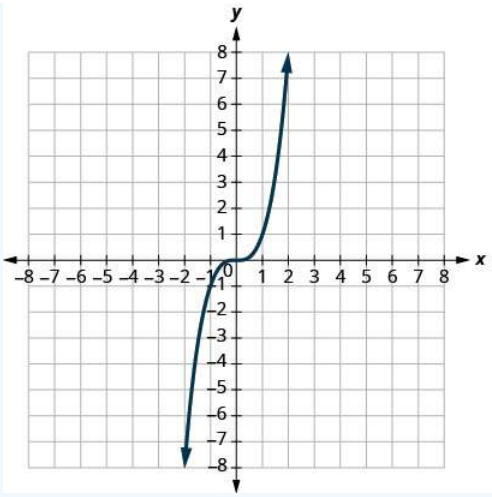
The next function we will look at is also not a linear function so the graph will not be a line. Again we will use point plotting, and make sure to choose several positive and negative values as well as 0 for our *x*-values.

?

Graph: $f(x) = x^3$.

Answer

We choose *x*-values. We substitute them in and then create a chart.

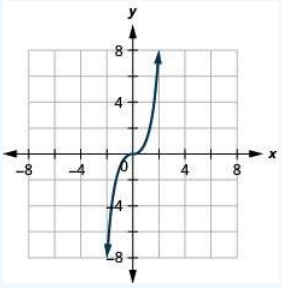


x	$f(x) = x^3$	$(x, f(x))$
–2	–8	(–2, –8)
–1	–1	(–1, –1)
0	0	(0, 0)
1	1	(1, 1)
2	8	(2, 8)

? Example 3.4.14

Graph: $f(x) = x^3$.

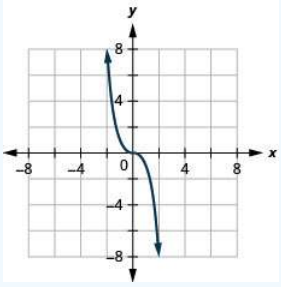
Answer



? Example 3.4.15

Graph: $f(x) = -x^3$.

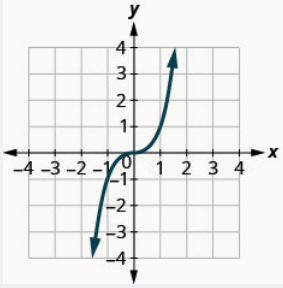
Answer



Looking at the result in Example, we can summarize the features of the cube function. As we consider the domain, notice any real number can be used as an x-value. The domain is all real numbers.

The range is all real numbers. This makes sense as the cube of any non-zero number can be positive or negative. So, the range of the cube function is all real numbers.

CUBE FUNCTION



$f(x) = x^3$
Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$

The next function we will look at does not square or cube the input values, but rather takes the square root of those values.

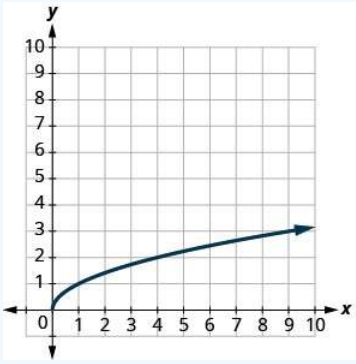
Let's graph the function $f(x) = \sqrt{x}$ and then summarize the features of the function. Remember, we can only take the square root of non-negative real numbers, so our domain will be the non-negative real numbers.

Example 3.4.16

$f(x) = \sqrt{x}$

Answer

We choose *x*-values. Since we will be taking the square root, we choose numbers that are perfect squares, to make our work easier. We substitute them in and then create a chart.

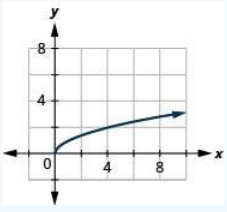


x	$f(x) = \sqrt{x}$	$(x, f(x))$
0	0	(0, 0)
1	1	(1, 1)
4	2	(4, 2)
9	3	(9, 3)

Example 3.4.17

Graph: $f(x) = x$.

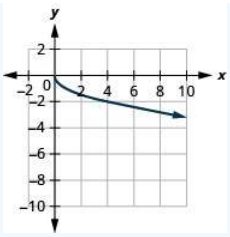
Answer



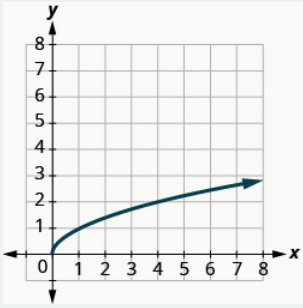
Example 3.4.18

Graph: $f(x) = -\sqrt{x}$.

Answer



SQUARE ROOT FUNCTION



$f(x) = \sqrt{x}$
Domain: $[0, \infty)$
Range: $[0, \infty)$

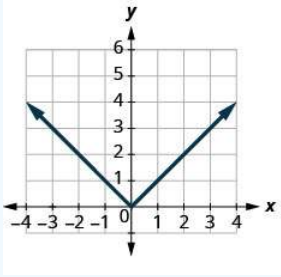
Our last basic function is the absolute value function, $f(x) = |x|$. Keep in mind that the absolute value of a number is its distance from zero. Since we never measure distance as a negative number, we will never get a negative number in the range.

?

Graph: $f(x) = |x|$.

Answer

We choose *x*-values. We substitute them in and then create a chart.

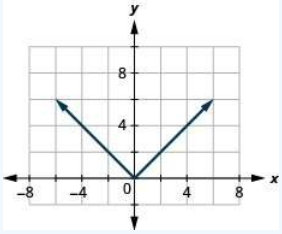


x	$f(x) = \|x\|$	$(x, f(x))$
–3	3	(–3, 3)
–2	2	(–2, 2)
–1	1	(–1, 1)
0	0	(0, 0)
1	1	(1, 1)
2	2	(2, 2)
3	3	(3, 3)

? Example 3.4.20

Graph: $f(x) = |x|$.

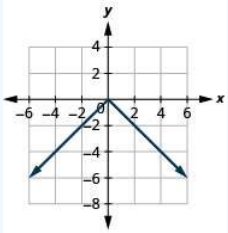
Answer



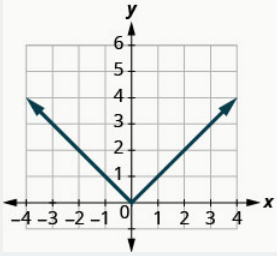
Example 3.4.21

Graph: $f(x) = -|x|$.

Answer



ABSOLUTE VALUE FUNCTION



$f(x) = |x|$
Domain: $(-\infty, \infty)$
Range: $[0, \infty)$

Read Information from a Graph of a Function

In the sciences and business, data is often collected and then graphed. The graph is analyzed, information is obtained from the graph and then often predictions are made from the data.

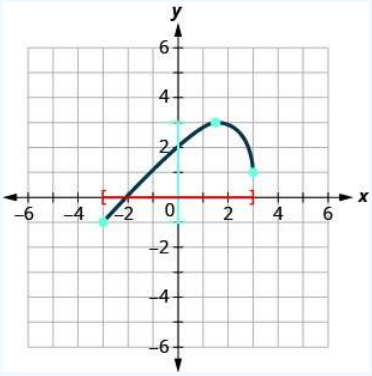
We will start by reading the domain and range of a function from its graph.

Remember the domain is the set of all the *x*-values in the ordered pairs in the function. To find the domain we look at the graph and find all the values of *x* that have a corresponding value on the graph. Follow the value *x* up or down vertically. If you hit the graph of the function then *x* is in the domain.

Remember the range is the set of all the *y*-values in the ordered pairs in the function. To find the range we look at the graph and find all the values of *y* that have a corresponding value on the graph. Follow the value *y* left or right horizontally. If you hit the graph of the function then *y* is in the range.

Example 3.4.22

Use the graph of the function to find its domain and range. Write the domain and range in interval notation.



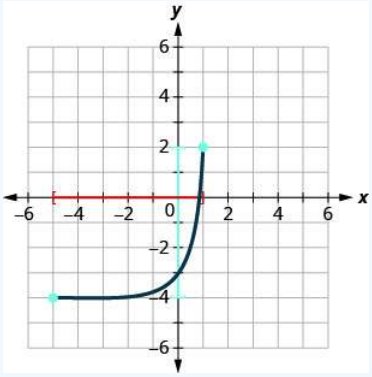
Answer

To find the domain we look at the graph and find all the values of x that correspond to a point on the graph. The domain is highlighted in red on the graph. The domain is $[-3, 3]$.

To find the range we look at the graph and find all the values of y that correspond to a point on the graph. The range is highlighted in blue on the graph. The range is $[-1, 3]$.

Example 3.4.23

Use the graph of the function to find its domain and range. Write the domain and range in interval notation.

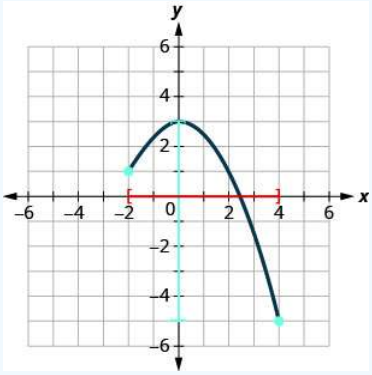


Answer

The domain is $[-5, 1]$. The range is $[-4, 2]$.

Example 3.4.24

Use the graph of the function to find its domain and range. Write the domain and range in interval notation.



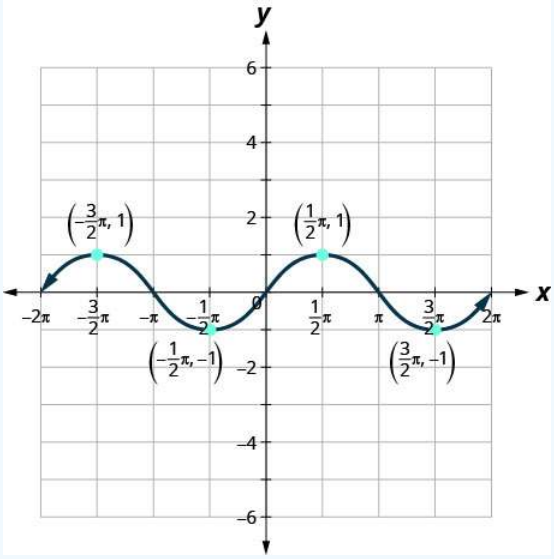
Answer

The domain is $[-2, 4]$. The range is $[-5, 3]$.

We are now going to read information from the graph that you may see in future math classes.

Example 3.4.25

Use the graph of the function to find the indicated values.



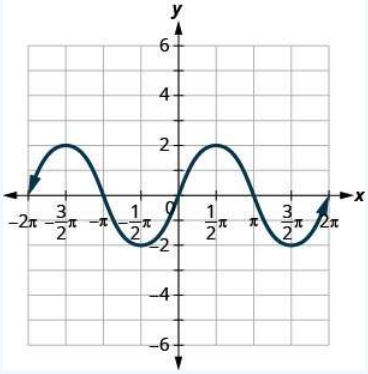
ⓐ Find: $f(0)$.
ⓑ Find: $f(32\pi)$.
ⓒ Find: $f(-12\pi)$.
ⓓ Find the values for x when $f(x) = 0$.
ⓔ Find the x-intercepts.
ⓕ Find the y-intercepts.
ⓖ Find the domain. Write it in interval notation.
ⓗ Find the range. Write it in interval notation.

Answer

ⓐ When $x = 0$, the function crosses the y-axis at 0. So, $f(0) = 0$.
ⓑ When $x = 32\pi$, the y-value of the function is -1. So, $f(32\pi) = -1$.
ⓒ When $x = -12\pi$, the y-value of the function is -1. So, $f(-12\pi) = -1$.
ⓓ The function is 0 at the points, $(-2\pi, 0), (-\pi, 0), (0, 0), (\pi, 0), (2\pi, 0)$. The x-values when $f(x) = 0$ are $-2\pi, -\pi, 0, \pi, 2\pi$.
ⓔ The x-intercepts occur when $y = 0$. So the x-intercepts occur when $f(x) = 0$. The x-intercepts are $(-2\pi, 0), (-\pi, 0), (0, 0), (\pi, 0), (2\pi, 0)$.
ⓕ The y-intercepts occur when x=0.x=0. So the y-intercepts occur at $f(0)$. The y-intercept is $(0, 0)$.
ⓖ This function has a value when x is from -2π to 2π. Therefore, the domain in interval notation is $[-2\pi, 2\pi]$.
ⓗ This function values, or y-values go from -1 to 1. Therefore, the range, in interval notation, is $[-1, 1]$.

? Example 3.4.26

Use the graph of the function to find the indicated values.



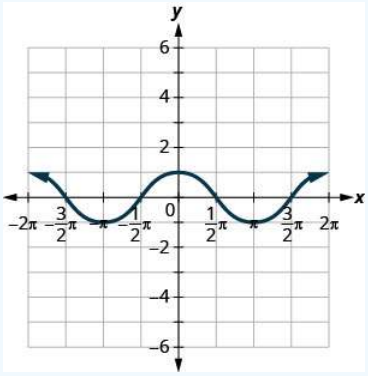
ⓐ Find: f(0).f(0).
ⓑ Find: f(12\pi).f(12\pi).
ⓒ Find: f(−32\pi).f(−32\pi).
ⓓ Find the values for *x* when f(x)=0.f(x)=0.
ⓔ Find the *x*-intercepts.
ⓕ Find the *y*-intercepts.
ⓖ Find the domain. Write it in interval notation.
ⓗ Find the range. Write it in interval notation.

Answer

ⓐ $f(0)=0$ ⓑ $f=(\pi 2)=2$ ⓒ $f=(-3\pi 2)=2$ ⓓ $f(x)=0$ for $x=-2\pi,-\pi,0,\pi,2\pi$ ⓔ $(-2\pi,0),(-\pi,0),(0,0),(\pi,0),(2\pi,0)$ⓕ (0,0)(0,0) ⓖ $[-2\pi,2\pi]$ ⓗ $[-2,2]$

Example 3.4.27

Use the graph of the function to find the indicated values.



ⓐ Find: $f(0)$.
ⓑ Find: $f(\pi)$.
ⓒ Find: $f(-\pi)$.
ⓓ Find the values for *x* when $f(x)=0$.
ⓔ Find the *x*-intercepts.
ⓕ Find the *y*-intercepts.
ⓖ Find the domain. Write it in interval notation.
ⓗ Find the range. Write it in interval notation.

Answer

ⓐ $f(0)=1$ ⓑ $f(\pi)=-1$ ⓒ $f(-\pi)=-1$ ⓓ $f(x)=0$ for $x=-3\pi 2,-\pi 2,\pi 2,3\pi 2$ ⓔ $(-2pi,0),(-pi,0),(0,0),(pi,0),(2pi,0)$ⓕ $(0,1)$ ⓖ $[-2pi,2pi]$ⓗ $[-1,1]$

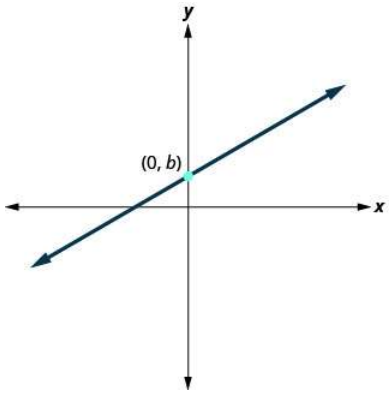
Access this online resource for additional instruction and practice with graphs of functions.

- Find Domain and Range

Key Concepts

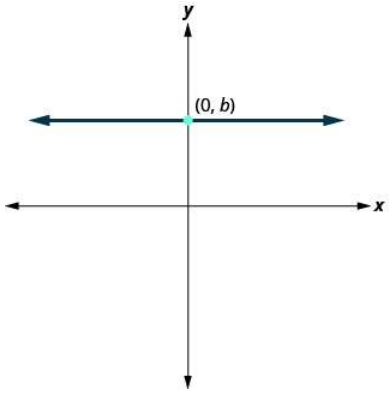
- **Vertical Line Test**
  - A set of points in a rectangular coordinate system is the graph of a function if every vertical line intersects the graph in at most one point.
  - If any vertical line intersects the graph in more than one point, the graph does not represent a function.
- **Graph of a Function**
  - The graph of a function is the graph of all its ordered pairs, (x,y)(x,y) or using function notation, (x,f(x))(x,f(x)) where y=f(x).y=f(x).

    fxf(x)name of functionx-coordinate of the ordered pairy-coordinate of the ordered pairfname of functionxx-coordinate of the ordered pairf(x)y-coordinate of the ordered pair
- **Linear Function**



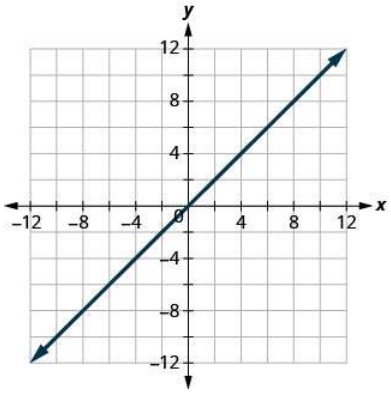
$f(x) = mx + b$
m, b: all real numbers
m: slope of the line
b: y-intercept
Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$

- **Constant Function**



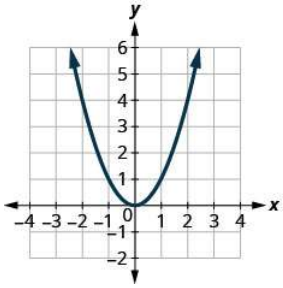
$f(x) = b$
b: any real number
b: y-intercept
Domain: $(-\infty, \infty)$
Range: b

- **Identity Function**



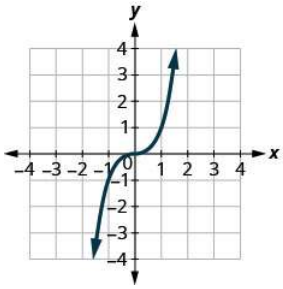
$f(x) = x$
m: 1
b: 0
Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$

- **Square Function**



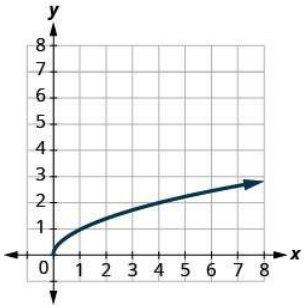
$f(x) = x^2$
Domain: $(-\infty, \infty)$
Range: $[0, \infty)$

- **Cube Function**



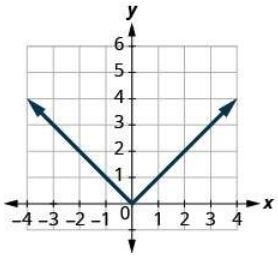
$f(x) = x^3$
Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$

- **Square Root Function**



$f(x) = \sqrt{x}$
Domain: $[0, \infty)$
Range: $[0, \infty)$

- **Absolute Value Function**



$f(x) = |x|$
Domain: $(-\infty, \infty)$
Range: $[0, \infty)$

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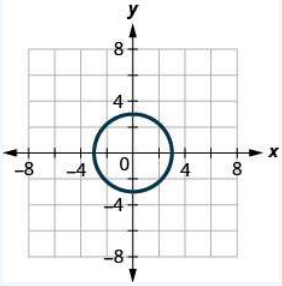
3.4E: Exercises

Practice Makes Perfect

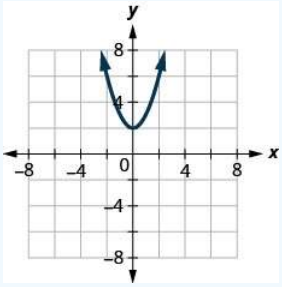
Use the Vertical Line Test

In the following exercises, determine whether each graph is the graph of a function.

1. ⓐ



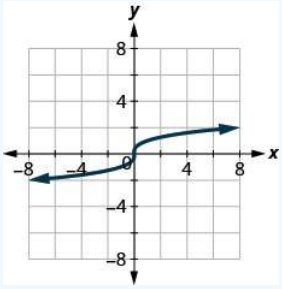
ⓑ



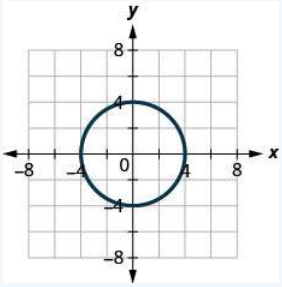
Answer

ⓐ no ⓑ yes

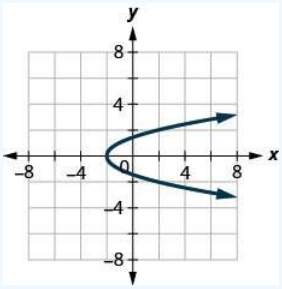
2. ⓐ



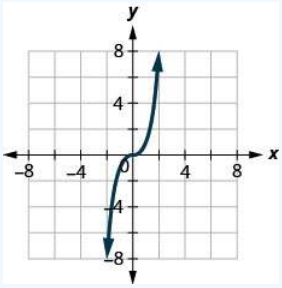
ⓑ



3. ⓐ



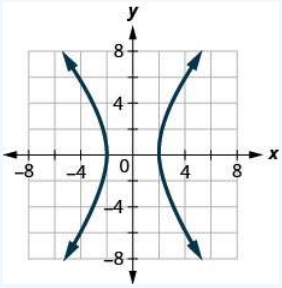
ⓑ



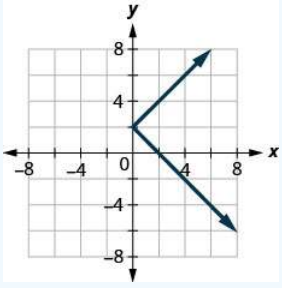
Answer

ⓐ no ⓑ yes

4. ⓐ



ⓑ



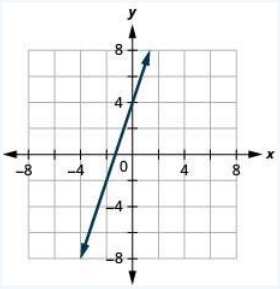
Identify Graphs of Basic Functions

In the following exercises, ⓐ graph each function ⓑ state its domain and range. Write the domain and range in interval notation.

5. $f(x) = 3x + 4$

Answer

ⓐ



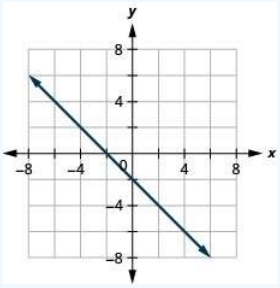
ⓑ $D: (-\text{inf}, \text{inf}),\ R: (-\text{inf}, \text{inf})$

6. $f(x) = 2x + 5$

7. $f(x) = -x - 2$

Answer

ⓐ



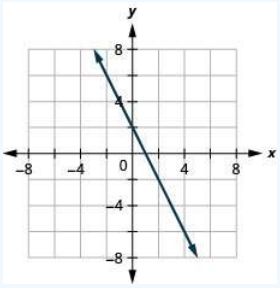
ⓑ $D: (-\text{inf}, \text{inf}),\ R: (-\text{inf}, \text{inf})$

8. $f(x) = -4x - 3$

9. $f(x) = -2x + 2$

Answer

ⓐ



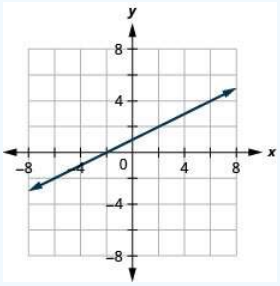
ⓑ $D: (-\text{inf}, \text{inf}),\ R: (-\text{inf}, \text{inf})$

10. $f(x) = -3x + 3$

11. $f(x) = \frac{1}{2}x + 1$

Answer

ⓐ



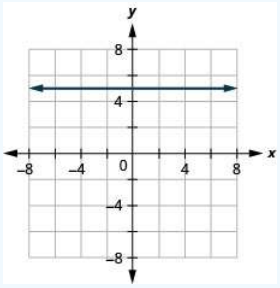
ⓑ $D : (-\text{inf}, \text{inf}),\ R : (-\text{inf}, \text{inf})$

12. $f(x) = \frac{2}{3}x - 2$

13. $f(x) = 5$

Answer

ⓐ



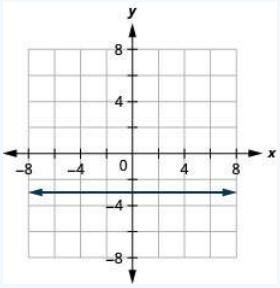
ⓑ $D : (-\text{inf}, \text{inf}), R : 5$

14. $f(x) = 2$

15. $f(x) = -3$

Answer

ⓐ



ⓑ $D : (-\text{inf}, \text{inf}),\ R : -3$

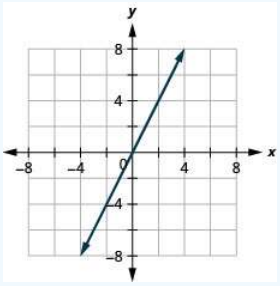
16. $f(x) = -1$

17. $f(x) = 2x$

https://math.libretexts.org/@go/page/56730

Answer

ⓐ



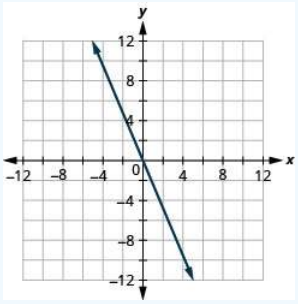
ⓑ $D: (-\text{inf}, \text{inf}),\ R: (-\text{inf}, \text{inf})$

18. $f(x) = 3x$

19. $f(x) = -2x$

Answer

ⓐ



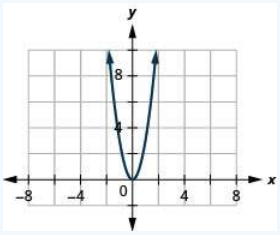
ⓑ $D: (-\text{inf}, \text{inf}), R: (-\text{inf}, \text{inf})$

20. $f(x) = -3x$

21. $f(x) = 3x^2$

Answer

ⓐ



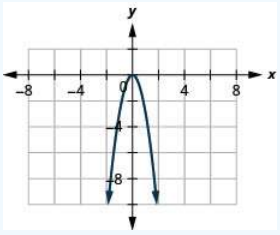
ⓑ $D: (-\text{inf}, \text{inf}),\ R: [0, \text{inf})$

22. $f(x) = 2x^2$

23. $f(x) = -3x^2$

Answer

ⓐ



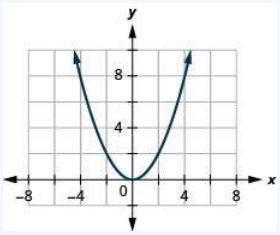
ⓑ $D: (-\text{inf}, \text{inf}),\ R: (-\text{inf}, 0]$

24. $f(x) = -2x^2$

25. $f(x) = 12x^2$

Answer

ⓐ



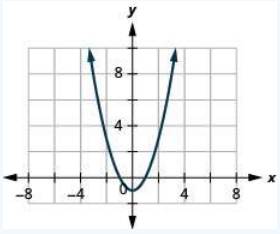
ⓑ $D: (-\text{inf}, \text{inf}),\ R: [-\text{inf}, 0)$

26. $f(x) = \frac{1}{3}x^2$

27. $f(x) = x^2 - 1$

Answer

ⓐ



ⓑ $D: (-\text{inf}, \text{inf}),\ R: [-1, \text{inf})$

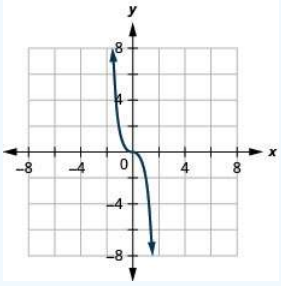
28. $f(x) = x^2 + 1$

29. $f(x) = -2x^3$

Answer

ⓐ

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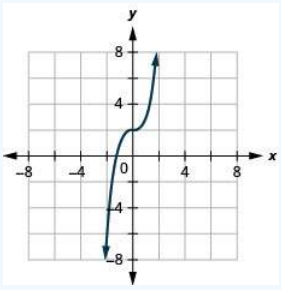
ⓑ $D: (-\text{inf}, \text{inf}),\ R: (-\text{inf}, \text{inf})$

30. $f(x) = 2x^3$

31. $f(x) = x^3 + 2$

Answer

ⓐ



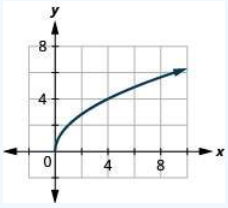
ⓑ $D: (-\text{inf}, \text{inf}), R: (-\text{inf}, \text{inf})$

32. $f(x) = x^3 - 2$

33. $f(x) = 2\sqrt{x}$

Answer

ⓐ



ⓑ $D: [0, \text{inf}), R: [0, \text{inf})$

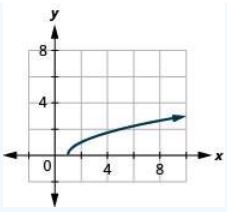
34. $f(x) = -2\sqrt{x}$

35. $f(x) = \sqrt{x-1}$

Answer

ⓐ

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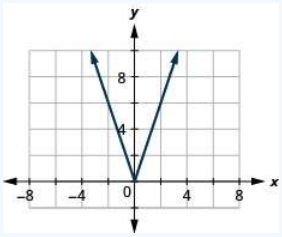
ⓑ $D: [1, \text{inf}), R: [0, \text{inf})$

36. $f(x) = \sqrt{x+1}$

37. $f(x) = 3|x|$

Answer

ⓐ



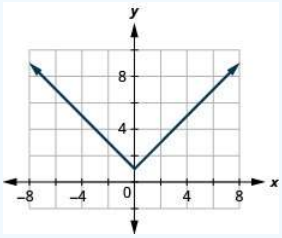
ⓑ $D: [-1, -1, \text{inf}), R: [-\text{inf}, \text{inf})$

38. $f(x) = -2|x|$

39. $f(x) = |x| + 1$

Answer

ⓐ



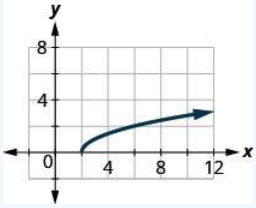
ⓑ $D: (-\text{inf}, \text{inf}), R: [1, \text{inf})$

40. $f(x) = |x| - 1$

Read Information from a Graph of a Function

In the following exercises, use the graph of the function to find its domain and range. Write the domain and range in interval notation.

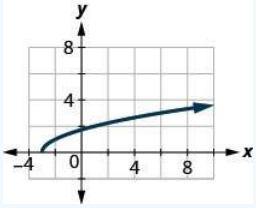
41.



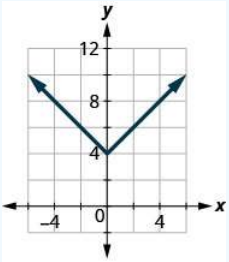
Answer

$D: [2, \text{inf}),\ R: [0, \text{inf})$

42.



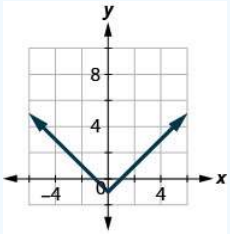
43.



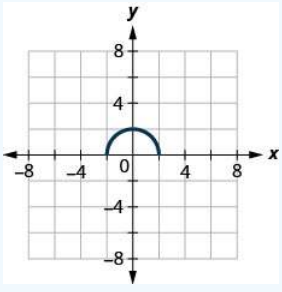
Answer

$D: (-\text{inf}, \text{inf}),\ R: [4, \text{inf})$

44.



45.

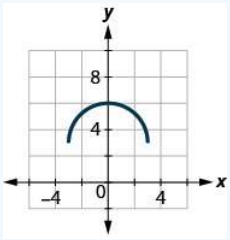


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Answer

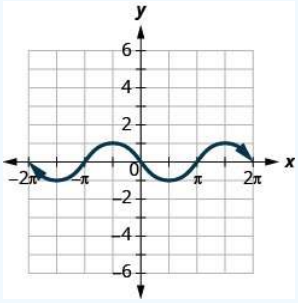
$D: [-2, 2],\ R: [0, 2]$

46.



In the following exercises, use the graph of the function to find the indicated values.

47.

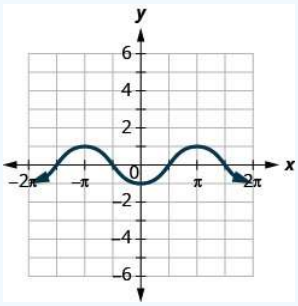


ⓐ Find: $f(0)$.
ⓑ Find: $f(12\pi)$.
ⓒ Find: $f(-32\pi)$.
ⓓ Find the values for x when $f(x) = 0$.
ⓔ Find the x-intercepts.
ⓕ Find the y-intercepts.
ⓖ Find the domain. Write it in interval notation.
ⓗ Find the range. Write it in interval notation.

Answer

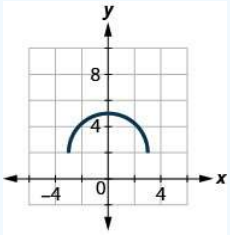
ⓐ $f(0) = 0$ ⓑ $(\pi/2) = -1$
ⓒ $f(-3\pi/2) = -1$ ⓓ $f(x) = 0$ for $x = -2\pi, -\pi, 0, \pi, 2\pi$
ⓔ $(-2\pi, 0), (-\pi, 0), (0, 0), (\pi, 0), (2\pi, 0)(f)(0, 0)$
ⓖ $[-2\pi, 2\pi]$ ⓗ $[-1, 1]$

48.



ⓐ Find: $f(0)$.
ⓑ Find: $f(\pi)$.
ⓒ Find: $f(-\pi)$.
ⓓ Find the values for x when $f(x) = 0$.
ⓔ Find the x-intercepts.
ⓕ Find the y-intercepts.
ⓖ Find the domain. Write it in interval notation.
ⓗ Find the range. Write it in interval notation

49.

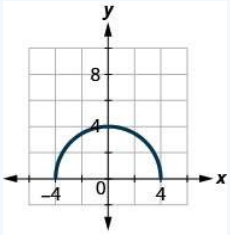


ⓐ Find: $f(0)$.
ⓑ Find: $f(-3)$.
ⓒ Find: $f(3)$.
ⓓ Find the values for x when $f(x) = 0$.
ⓔ Find the x-intercepts.
ⓕ Find the y-intercepts.
ⓖ Find the domain. Write it in interval notation.
ⓗ Find the range. Write it in interval notation.

Answer

ⓐ $f(0) = -6$ ⓑ $f(-3) = 3$ ⓒ $f(3) = 3$ ⓓ $f(x) = 0$ for no x ⓔ none ⓕ $y = 6$ ⓖ $[-3, 3]$ ⓗ $[-3, 6]$

50.



ⓐ Find: $f(0)$.
ⓑ Find the values for x when $f(x) = 0$.
ⓒ Find the x-intercepts.
ⓓ Find the y-intercepts.
ⓔ Find the domain. Write it in interval notation.
ⓕ Find the range. Write it in interval notation

Writing Exercises

51. Explain in your own words how to find the domain from a graph.

52. Explain in your own words how to find the range from a graph.

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53. Explain in your own words how to use the vertical line test.

54. Draw a sketch of the square and cube functions. What are the similarities and differences in the graphs?

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
use the vertical line test.			
identify graphs of basic functions.			
read information from a graph.			

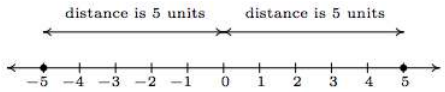
ⓑ After reviewing this checklist, what will you do to become confident for all objectives?

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3.5: Absolute Value Functions

There are a few ways to describe what is meant by the absolute value $|x|$ of a real number x. You may have been taught that $|x|$ is the distance from the real number x to 0 on the number line. So, for example, $|5| = 5$ and $|-5| = 5$, since each is 5 units from 0 on the number line.



Another way to define absolute value is by the equation $|x| = \sqrt{x^2}$. Using this definition, we have $|5| = \sqrt{(5)^2} = \sqrt{25} = 5$ and $|-5| = \sqrt{(-5)^2} = \sqrt{25} = 5$. The long and short of both of these procedures is that $|x|$ takes negative real numbers and assigns them to their positive counterparts while it leaves positive numbers alone. This last description is the one we shall adopt, and is summarized in the following definition.

Definition 2.4: Absolute Value

The **absolute value** of a real number x, denoted $|x|$, is given by

$$|x| = \begin{cases} -x, & \text{if} \quad x < 0 \\ x, & \text{if} \quad x \geq 0 \end{cases} \tag{3.5.1}$$

In Definition 2.4, we define $|x|$ using a piecewise-defined function (Section 1.4). To check that this definition agrees with what we previously understood as absolute value, note that since $5 \geq 0$, to find $|5|$ we use the rule $|x| = x$, so $|5| = 5$. Similarly, since $-5 < 0$, we use the rule $|x| = -x$, so that $|-5| = -(-5) = 5$. This is one of the times when it's best to interpret the expression '$-x$' as 'the opposite of x' as opposed to 'negative x'. Before we begin studying absolute value functions, we remind ourselves of the properties of absolute value.

Theorem 2.1: Properties of Absolute Value

Let a, b and x be real numbers and let n be an integer, then:

- **Product Rule:** $|ab| = |a||b|$
- **Power Rule:** $|a^n| = |a|^n$ whenever a^n is defined
- **Quotient Rule:** $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$, provided $b \neq 0$

Equality Properties:

- $|x| = 0$ if and only if $x = 0$.
- For $c > 0$, $|x| = c$ if and only if $x = c$ or $x = -c$.
- For $c < 0$, $|x| = c$ has no solution.

The proofs of the Product and Quotient Rules in Theorem 2.1 boil down to checking four cases:

- when both a and b are positive;
- when they are both negative;
- when one is positive and the other is negative;
- and when one or both are zero.

For example, suppose we wish to show that $|ab| = |a||b|$. We need to show that this equation is true for all real numbers a and b. If a and b are both positive, then so is ab. Hence, $|a| = a$, $|b| = b$ and $|ab| = ab$. Hence, the equation $|ab| = |a||b|$ is the same as $ab = ab$ which is true. If both a and b are negative, then ab is positive. Hence, $|a| = -a$, $|b| = -b$ and $|ab| = ab$. The equation $|ab| = |a||b|$ becomes $ab = (-a)(-b)$, which is true. Suppose a is positive and b is negative. Then ab is negative, and we have $|ab| = -ab$, $|a| = a$ and $|b| = -b$. The equation $|ab| = |a||b|$ reduces to $-ab = a(-b)$ which is true. A symmetric argument shows the equation $|ab| = |a||b|$ holds when a is negative and b is positive. Finally, if either a or b (or both) are zero, then both sides of $|ab| = |a||b|$ are zero, so the equation holds in this case, too. All of this rhetoric has shown that the equation $|ab| = |a||b|$ holds true in all cases.

The proof of the Quotient Rule is very similar, with the exception that $b \neq 0$. The Power Rule can be shown by repeated application of the Product Rule. The 'Equality Properties' can be proved using Definition 2.4 and by looking at the cases when $x \geq 0$, in which case $|x| = x$, or when $x < 0$, in which case $|x| = -x$. For example, if $c > 0$, and $|x| = c$, then if $x \geq 0$, we have $x = |x| = c$. If, on the other hand, $x < 0$, then $-x = |x| = c$, so $x = -c$. The remaining properties are proved similarly and are left for the Exercises. Our first example reviews how to solve basic equations involving absolute value using the properties listed in Theorem 2.1.

Example 3.5.1: Absolute Value

Solve each of the following equations.

1. $|3x-1| = 6$
2. $3 - |x+5| = 1$
3. $3|2x+1| - 5 = 0$
4. $4 - |5x+3| = 5$
5. $|x| = x^2 - 6$
6. $|x-2| + 1 = x$

Solution

1. The equation $|3x-1| = 6$ is of the form $|x| = c$ for $c > 0$, so by the Equality Properties, $|3x-1| = 6$ is equivalent to $3x - 1 = 6$ or $3x - 1 = -6$. Solving the former, we arrive at $x = \frac{7}{3}$, and solving the latter, we get $x = -\frac{5}{3}$. We may check both of these solutions by substituting them into the original equation and showing that the arithmetic works out.

To use the Equality Properties to solve $3 - |x+5| = 1$, we first isolate the absolute value.

$$\begin{aligned} 3 - |x+5| &= 1 \\ -|x+5| &= -2 \qquad \text{subtract 3} \\ |x+5| &= 2 \qquad \text{divide by } -1 \end{aligned} \tag{3.5.2}$$

From the Equality Properties, we have $x + 5 = 2$ or $x + 5 = -2$, and get our solutions to be $x = -3$ or $x = -7$. We leave it to the reader to check both answers in the original equation.

1. As in the previous example, we first isolate the absolute value in the equation $3|2x+1| - 5 = 0$ and get $|2x+1| = \frac{5}{3}$. Using the Equality Properties, we have $2x + 1 = \frac{5}{3}$ or $2x + 1 = -\frac{5}{3}$. Solving the former gives $x = \frac{1}{3}$ and solving the latter gives $x = -\frac{4}{3}$. As usual, we may substitute both answers in the original equation to check.
2. Upon isolating the absolute value in the equation $4 - |5x+3| = 5$, we get $|5x+3| = -1$. At this point, we know there cannot be any real solution, since, by definition, the absolute value of *anything* is never negative. We are done.
3. The equation $|x| = x^2 - 6$ presents us with some difficulty, since x appears both inside and outside of the absolute value. Moreover, there are values of x for which $x^2 - 6$ is positive, negative and zero, so we cannot use the Equality Properties without the risk of introducing extraneous solutions, or worse, losing solutions. For this reason, we break equations like this into cases by rewriting the term in absolute values, $|x|$, using Definition 2.4. For $x < 0$, $|x| = -x$, so for $x < 0$, the equation $|x| = x^2 - 6$ is equivalent to $-x = x^2 - 6$. Rearranging this gives us $x^2 + x - 6 = 0$, or $(x+3)(x-2) = 0$. We get $x = -3$ or $x = 2$. Since only $x = -3$ satisfies $x < 0$, this is the answer we keep. For $x \geq 0$, $|x| = x$, so the equation $|x| = x^2 - 6$ becomes $x = x^2 - 6$. From this, we get $x^2 - x - 6 = 0$ or $(x-3)(x+2) = 0$. Our solutions are $x = 3$ or $x = -2$, and since only $x = 3$ satisfies $x \geq 0$, this is the one we keep. Hence, our two solutions to $|x| = x^2 - 6$ are $x = -3$ and $x = 3$.
4. To solve $|x-2| + 1 = x$, we first isolate the absolute value and get $|x-2| = x - 1$. Since we see x both inside and outside of the absolute value, we break the equation into cases. The term with absolute values here is $|x-2|$, so we replace 'x' with the quantity '$(x-2)$' in Definition 2.4 to get

$$|x-2| = \begin{cases} -(x-2), & \text{if} \quad (x-2) < 0 \\ (x-2), & \text{if} \quad (x-2) \geq 0 \end{cases} \tag{3.5.3}$$

Simplifying yields

$$|x-2| = \begin{cases} -x+2, & \text{if} \quad x < 2 \\ x-2, & \text{if} \quad x \geq 2 \end{cases} \tag{3.5.4}$$

So, for $x < 2$, $|x-2| = -x+2$ and our equation $|x-2| = x-1$ becomes $-x+2 = x-1$, which gives $x = \frac{3}{2}$. Since this solution satisfies $x < 2$, we keep it. Next, for $x \geq 2$, $|x-2| = x-2$, so the equation $|x-2| = x-1$ becomes $x-2 = x-1$. Here, the equation reduces to $-2 = -1$, which signifies we have no solutions here. Hence, our only solution is $x = \frac{3}{2}$.

Next, we turn our attention to graphing absolute value functions. Our strategy in the next example is to make liberal use of Definition 2.4 along with what we know about graphing linear functions (from Section 2.1) and piecewise-defined functions (from Section 1.4).

Example 3.5.2:

Graph each of the following functions.

1. $f(x) = |x|$
2. $g(x) = |x-3|$
3. $h(x) = |x| - 3$
4. $i(x) = 4 - 2|3x+1|$

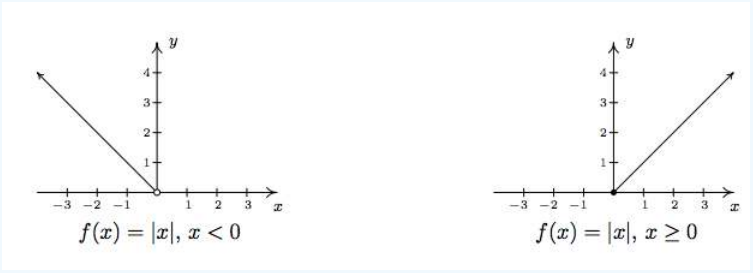
Find the zeros of each function and the x- and y-intercepts of each graph, if any exist. From the graph, determine the domain and range of each function, list the intervals on which the function is increasing, decreasing or constant, and find the relative and absolute extrema, if they exist.

Solution

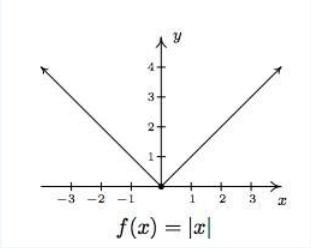
1. To find the zeros of f, we set $f(x) = 0$. We get $|x| = 0$, which, by Theorem 3.5.1 gives us $x = 0$. Since the zeros of f are the x-coordinates of the x-intercepts of the graph of $y = f(x)$, we get $(0, 0)$ as our only x-intercept. To find the y-intercept, we set $x = 0$, and find $y = f(0) = 0$, so that $(0, 0)$ is our y-intercept as well.\footnote{Actually, since functions can have at most one y-intercept (Do you know why?), as soon as we found $(0, 0)$ as the x-intercept, we knew this was also the y-intercept.} Using Definition 3.5.1, we get

$$f(x) = |x| = \begin{cases} -x, & \text{if} \quad x < 0 \\ x, & \text{if} \quad x \geq 0 \end{cases} \tag{3.5.5}$$

Hence, for $x < 0$, we are graphing the line $y = -x$; for $x \geq 0$, we have the line $y = x$. Proceeding as we did in Section 1.6, we get



Notice that we have an 'open circle' at $(0, 0)$ in the graph when $x < 0$. As we have seen before, this is due to the fact that the points on $y = -x$ approach $(0, 0)$ as the x-values approach 0. Since x is required to be strictly less than zero on this stretch, the open circle is drawn at the origin. However, notice that when $x \geq 0$, we get to fill in the point at $(0, 0)$, which effectively 'plugs' the hole indicated by the open circle. Thus we get,



By projecting the graph to the x-axis, we see that the domain is $(-\infty, \infty)$. Projecting to the y-axis gives us the range $[0, \infty)$. The function is increasing on $[0, \infty)$ and decreasing on $(-\infty, 0]$. The relative minimum value of f is the same as the absolute minimum, namely 0 which occurs at $(0, 0)$. There is no relative maximum value of f. There is also no absolute maximum value of f, since the y values on the graph extend infinitely upwards.

2. To find the zeros of g, we set $g(x) = |x-3| = 0$. By Theorem 2.1, we get $x - 3 = 0$ so that $x = 3$. Hence, the x-intercept is $(3, 0)$. To find our y-intercept, we set $x = 0$ so that $y = g(0) = |0-3| = 3$, which yields $(0, 3)$ as our y-intercept. To graph $g(x) = |x-3|$, we use Definition 2.4 to rewrite g as

$$g(x) = |x-3| = \begin{cases} -(x-3), & \text{if} \quad (x-3) < 0 \\ (x-3), & \text{if} \quad (x-3) \geq 0 \end{cases} \tag{3.5.6}$$

Simplifying, we get

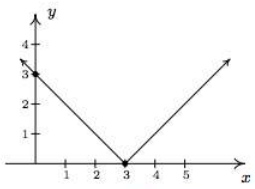
$$g(x) = \begin{cases} -x+3, & \text{if} \quad x < 3 \\ x-3, & \text{if} \quad x \geq 3 \end{cases} \tag{3.5.7}$$

As before, the open circle we introduce at $(3, 0)$ from the graph of $y = -x + 3$ is filled by the point $(3, 0)$ from the line $y = x - 3$. We determine the domain as $(-\infty, \infty)$ and the range as $[0, \infty)$. The function g is increasing on $[3, \infty)$ and decreasing on $(-\infty, 3]$. The relative and absolute minimum value of g is 0 which occurs at $(3, 0)$. As before, there is no relative or absolute maximum value of g.

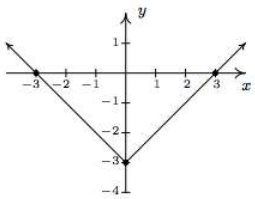
3. Setting $h(x) = 0$ to look for zeros gives $|x| - 3 = 0$. As in Example 3.5.1, we isolate the absolutre value to get $|x| = 3$ so that $x = 3$ or $x = -3$. As a result, we have a pair of x-intercepts: $(-3, 0)$ and $(3, 0)$. Setting $x = 0$ gives $y = h(0) = |0| - 3 = -3$, so our y-intercept is $(0, -3)$. As before, we rewrite the absolute value in h to get

$$h(x) = \begin{cases} -x-3, & \text{if} \quad x < 0 \\ x-3, & \text{if} \quad x \geq 0 \end{cases} \tag{3.5.8}$$

Once again, the open circle at $(0, -3)$ from one piece of the graph of h is filled by the point $(0, -3)$ from the other piece of h. From the graph, we determine the domain of h is $(-\infty, \infty)$ and the range is $[-3, \infty)$. On $[0, \infty)$, h is increasing; on $(-\infty, 0]$ it is decreasing. The relative minimum occurs at the point $(0, -3)$ on the graph, and we see -3 is both the relative and absolute minimum value of h. Also, h has no relative or absolute maximum value.



$g(x) = |x-3|$

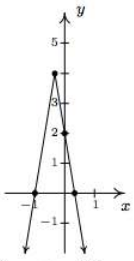


$h(x) = |x| - 3$

4. As before, we set $i(x) = 0$ to find the zeros of i and get $4 - 2|3x+1| = 0$. Isolating the absolute value term gives $|3x+1| = 2$, so either $3x + 1 = 2$ or $3x + 1 = -2$. We get $x = \frac{1}{3}$ or $x = -1$, so our x-intercepts are $\left(\frac{1}{3}, 0\right)$ and $(-1, 0)$. Substituting $x = 0$ gives $y = i(0) = 4 - 2|3(0)+1| = 2$, for a y-intercept of $(0, 2)$. Rewriting the formula for $i(x)$ without absolute values gives

$$i(x) = \begin{cases} 4-2(-(3x+1)), & \text{if} \quad (3x+1) < 0 \\ 4-2(3x+1), & \text{if} \quad (3x+1) \geq 0 \end{cases} = \begin{cases} 6x+6, & \text{if} \quad x < -\frac{1}{3} \\ -6x+2, & \text{if} \quad x \geq -\frac{1}{3} \end{cases} \tag{3.5.9}$$

The usual analysis near the trouble spot $x = -\frac{1}{3}$ gives the 'corner' of this graph is $\left(-\frac{1}{3}, 4\right)$, and we get the distinctive '$\vee$' shape:



$i(x) = 4 - 2|3x + 1|$

The domain of i is $(-\infty, \infty)$ while the range is $(-\infty, 4]$. The function i is increasing on $\left(-\infty, -\frac{1}{3}\right]$ and decreasing on $\left[-\frac{1}{3}, \infty\right)$. The relative maximum occurs at the point $\left(-\frac{1}{3}, 4\right)$ and the relative and absolute maximum value of i is 4. Since the graph of i extends downwards forever more, there is no absolute minimum value. As we can see from the graph, there is no relative minimum, either.

Note that all of the functions in the previous example bear the characteristic '$\vee$' shape of the graph of $y = |x|$. We could have graphed the functions g, h and i in Example 3.5.2 starting with the graph of $f(x) = |x|$ and applying transformations as in Section 1.7 as our next example illustrates.

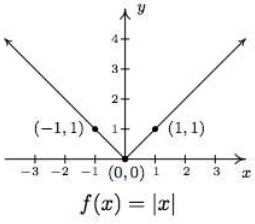
Example 3.5.3:

Graph the following functions starting with the graph of $f(x) = |x|$ and using transformations.

1. $g(x) = |x - 3|$
2. $h(x) = |x| - 3$
3. $i(x) = 4 - 2|3x + 1|$

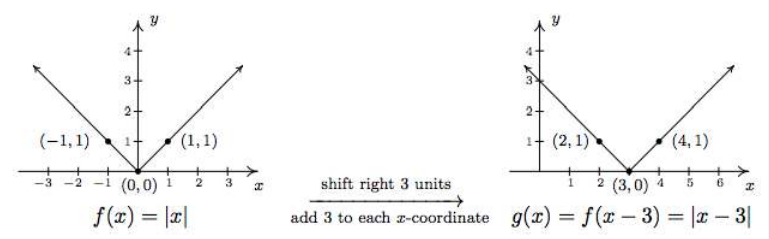
Solution

We begin by graphing $f(x) = |x|$ and labeling three points, $(-1, 1)$, $(0, 0)$ and $(1, 1)$.

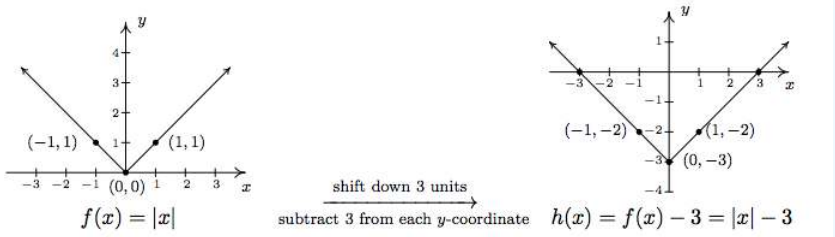


$f(x) = |x|$

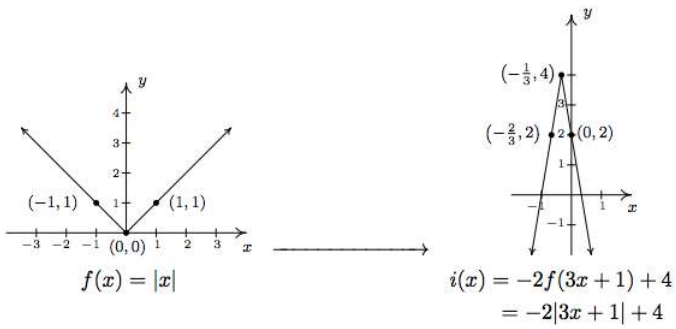
1. Since $g(x) = |x - 3| = f(x - 3)$, Theorem 1.7 tells us to *add* 3 to each of the x-values of the points on the graph of $y = f(x)$ to obtain the graph of $y = g(x)$. This shifts the graph of $y = f(x)$ to the *right* 3 units and moves the point $(-1, 1)$ to $(2, 1)$, $(0, 0)$ to $(3, 0)$ and $(1, 1)$ to $(4, 1)$. Connecting these points in the classic '$\vee$' fashion produces the graph of $y = g(x)$.



2. For $h(x) = |x| - 3 = f(x) - 3$, Theorem 1.7 tells us to *subtract* 3 from each of the y-values of the points on the graph of $y = f(x)$ to obtain the graph of $y = h(x)$. This shifts the graph of $y = f(x)$ *down* 3 units and moves $(-1, 1)$ to $(-1, -2)$, $(0, 0)$ to $(0, -3)$ and $(1, 1)$ to $(1, -2)$. Connecting these points with the '$\vee$' shape produces our graph of $y = h(x)$.



3. We re-write $i(x) = 4 - 2|3x+1| = 4 - 2f(3x+1) = -2f(3x+1)+4$ and apply Theorem 1.7. First, we take care of the changes on the 'inside' of the absolute value. Instead of $|x|$, we have $|3x+1|$, so, in accordance with Theorem 1.7, we first *subtract* 1 from each of the x-values of points on the graph of $y = f(x)$, then *divide* each of those new values by 3. This effects a horizontal shift *left* 1 unit followed by a horizontal *shrink* by a factor of 3. These transformations move $(-1, 1)$ to $\left(-\frac{2}{3}, 1\right)$, $(0, 0)$ to $\left(-\frac{1}{3}, 0\right)$ and $(1, 1)$ to $(0, 1)$. Next, we take care of what's happening 'outside of' the absolute value. Theorem 1.7 instructs us to first *multiply* each y-value of these new points by -2 then *add* 4. Geometrically, this corresponds to a vertical *stretch* by a factor of 2, a reflection across the x-axis and finally, a vertical shift *up* 4 units. These transformations move $\left(-\frac{2}{3}, 1\right)$ to $\left(-\frac{2}{3}, 2\right)$, $\left(-\frac{1}{3}, 0\right)$ to $\left(-\frac{1}{3}, 4\right)$, and $(0, 1)$ to $(0, 2)$. Connecting these points with the usual '$\vee$' shape produces our graph of $y = i(x)$.



□

While the methods in Section 1.7 can be used to graph an entire family of absolute value functions, not all functions involving absolute values posses the characteristic '$\vee$' shape. As the next example illustrates, often there is no substitute for appealing directly to the definition.

Example 3.5.4:

Graph each of the following functions. Find the zeros of each function and the x- and y-intercepts of each graph, if any exist. From the graph, determine the domain and range of each function, list the intervals on which the function is increasing, decreasing or constant, and find the relative and absolute extrema, if they exist.

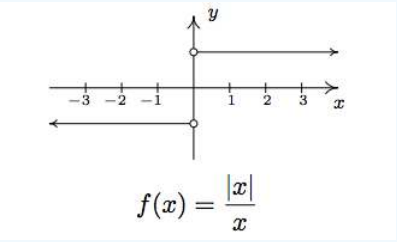
1. $f(x) = \dfrac{|x|}{x}$
2. $g(x) = |x+2| - |x-3| + 1$

Solution

1. We first note that, due to the fraction in the formula of $f(x)$, $x \neq 0$. Thus the domain is $(-\infty, 0) \cup (0, \infty)$. To find the zeros of f, we set $f(x) = \frac{|x|}{x} = 0$. This last equation implies $|x| = 0$, which, from Theorem 3.5.1, implies $x = 0$. However, $x = 0$ is not in the domain of f, which means we have, in fact, no x-intercepts. We have no y-intercepts either, since $f(0)$ is undefined. Re-writing the absolute value in the function gives

$$f(x)=\begin{cases}\dfrac{-x}{x}, & \text{if} \quad x<0 \\ [10pt]\dfrac{x}{x}, & \text{if} \quad x>0\end{cases}=\begin{cases}-1, & \text{if} \quad x<0 \\ 1, & \text{if} \quad x>0\end{cases} \tag{3.5.10}$$

To graph this function, we graph two horizontal lines: $y=-1$ for $x<0$ and $y=1$ for $x>0$. We have open circles at $(0,-1)$ and $(0,1)$ (Can you explain why?) so we get



As we found earlier, the domain is $(-\infty,0)\cup(0,\infty)$. The range consists of just two y-values: $\{-1,1\}$. The function f is constant on $(-\infty,0)$ and $(0,\infty)$. The local minimum value of f is the absolute minimum value of f, namely -1; the local maximum and absolute maximum values for f also coincide – they both are 1. Every point on the graph of f is simultaneously a relative maximum and a relative minimum. (Can you remember why in light of Definition 1.11? This was explored in the Exercises in Section 1.6 .)

2. To find the zeros of g, we set $g(x)=0$. The result is $|x+2|-|x-3|+1=0$. Attempting to isolate the absolute value term is complicated by the fact that there are \textbf{two} terms with absolute values. In this case, it easier to proceed using cases by re-writing the function g with two separate applications of Definition 2.4 to remove each instance of the absolute values, one at a time. In the first round we get

$$g(x)=\begin{cases}-(x+2)-|x-3|+1, & \text{if} \quad (x+2)<0 \\ (x+2)-|x-3|+1, & \text{if} \quad (x+2)\geq 0\end{cases}=\begin{cases}-x-1-|x-3|, & \text{if} \quad x<-2 \\ x+3-|x-3|, & \text{if} \quad x\geq -2\end{cases} \tag{3.5.11}$$

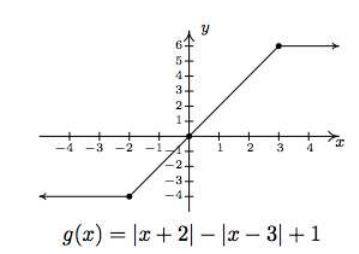
Given that

$$|x-3|=\begin{cases}-(x-3), & \text{if} \quad (x-3)<0 \\ x-3, & \text{if} \quad (x-3)\geq 0\end{cases}=\begin{cases}-x+3, & \text{if} \quad x<3 \\ x-3, & \text{if} \quad x\geq 3\end{cases}, \tag{3.5.12}$$

we need to break up the domain again at $x=3$. Note that if $x<-2$, then $x<3$, so we replace $|x-3|$ with $-x+3$ for that part of the domain, too. Our completed revision of the form of g yields

$$g(x)=\begin{cases}-x-1-(-x+3), & \text{if} \quad x<-2 \\ x+3-(-x+3), & \text{if} \quad x\geq -2 \text{ and } x<3 \\ x+3-(x-3), & \text{if} \quad x\geq 3\end{cases}=\begin{cases}-4, & \text{if} \quad x<-2 \\ 2x, & \text{if} \quad -2\leq x<3 \\ 6, & \text{if} \quad x\geq 3\end{cases} \tag{3.5.13}$$

To solve $g(x)=0$, we see that the only piece which contains a variable is $g(x)=2x$ for $-2\leq x<3$. Solving $2x=0$ gives $x=0$. Since $x=0$ is in the interval $[-2,3)$, we keep this solution and have $(0,0)$ as our only x-intercept. Accordingly, the y-intercept is also $(0,0)$. To graph g, we start with $x<-2$ and graph the horizontal line $y=-4$ with an open circle at $(-2,-4)$. For $-2\leq x<3$, we graph the line $y=2x$ and the point $(-2,-4)$ patches the hole left by the previous piece. An open circle at $(3,6)$ completes the graph of this part. Finally, we graph the horizontal line $y=6$ for $x\geq 3$, and the point $(3,6)$ fills in the open circle left by the previous part of the graph. The finished graph is



The domain of g is all real numbers, $(-\infty, \infty)$, and the range of g is all real numbers between -4 and 6 inclusive, $[-4, 6]$. The function is increasing on $[-2, 3]$ and constant on $(-\infty, -2]$ and $[3, \infty)$. The relative minimum value of f is -4 which matches the absolute minimum. The relative and absolute maximum values also coincide at 6. Every point on the graph of $y = g(x)$ for $x < -2$ and $x > 3$ yields both a relative minimum and relative maximum. The point $(-2, -4)$, however, gives only a relative minimum and the point $(3, 6)$ yields only a relative maximum. (Recall the Exercises in Section 1.6 which dealt with constant functions.)

Many of the applications that the authors are aware of involving absolute values also involve absolute value inequalities. For that reason, we save our discussion of applications for Section 2.4.

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CHAPTER OVERVIEW

4: Systems of Linear Equations

4.1: Systems of Linear Equations - Two Variables

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4.1: Systems of Linear Equations - Two Variables

Learning Objectives

- Solve systems of equations by graphing.
- Solve systems of equations by substitution.
- Solve systems of equations by addition.
- Identify inconsistent systems of equations containing two variables.
- Express the solution of a system of dependent equations containing two variables.

A skateboard manufacturer introduces a new line of boards. The manufacturer tracks its costs, which is the amount it spends to produce the boards, and its revenue, which is the amount it earns through sales of its boards. How can the company determine if it is making a profit with its new line? How many skateboards must be produced and sold before a profit is possible? In this section, we will consider linear equations with two variables to answer these and similar questions.



Figure 4.1.1

Introduction to Systems of Equations

In order to investigate situations such as that of the skateboard manufacturer, we need to recognize that we are dealing with more than one variable and likely more than one equation. **A system of linear equations** consists of two or more linear equations made up of two or more variables such that all equations in the system are considered simultaneously. To find the unique solution to a system of linear equations, we must find a numerical value for each variable in the system that will satisfy all equations in the system at the same time. Some linear systems may not have a solution and others may have an infinite number of solutions. In order for a linear system to have a unique solution, there must be at least as many equations as there are variables. Even so, this does not guarantee a unique solution.

In this section, we will look at systems of linear equations in two variables, which consist of two equations that contain two different variables. For example, consider the following system of linear equations in two variables.

$$\begin{aligned} 2x+y &= 15 \\ 3x-y &= 5 \end{aligned}$$

The *solution* to a system of linear equations in two variables is any ordered pair that satisfies each equation independently. In this example, the ordered pair $(4,7)$ is the solution to the system of linear equations. We can verify the solution by substituting the values into each equation to see if the ordered pair satisfies both equations. Shortly we will investigate methods of finding such a solution if it exists.

$$\begin{aligned} 2(4)+(7) &= 15 \text{ True} \\ 3(4)-(7) &= 5 \text{ True} \end{aligned}$$

In addition to considering the number of equations and variables, we can categorize systems of linear equations by the number of solutions. A consistent system of equations has at least one solution. A consistent system is considered to be an **independent system** if it has a single solution, such as the example we just explored. The two lines have different slopes and intersect at one point in the plane. A consistent system is considered to be a **dependent system** if the equations have the same slope and the same y-intercepts. In other words, the lines coincide so the equations represent the same line. Every point on the line represents a coordinate pair that satisfies the system. Thus, there are an infinite number of solutions.

Another type of system of linear equations is an **inconsistent system**, which is one in which the equations represent two parallel lines. The lines have the same slope and different y-intercepts. There are no points common to both lines; hence, there is no solution to the system.

TYPES OF LINEAR SYSTEMS

There are three types of systems of linear equations in two variables, and three types of solutions.

- An independent system has exactly one solution pair (x, y). The point where the two lines intersect is the only solution.
- An inconsistent system has no solution. Notice that the two lines are parallel and will never intersect.
- A dependent system has infinitely many solutions. The lines are coincident. They are the same line, so every coordinate pair on the line is a solution to both equations.

Figure $4.1.2$ compares graphical representations of each type of system.

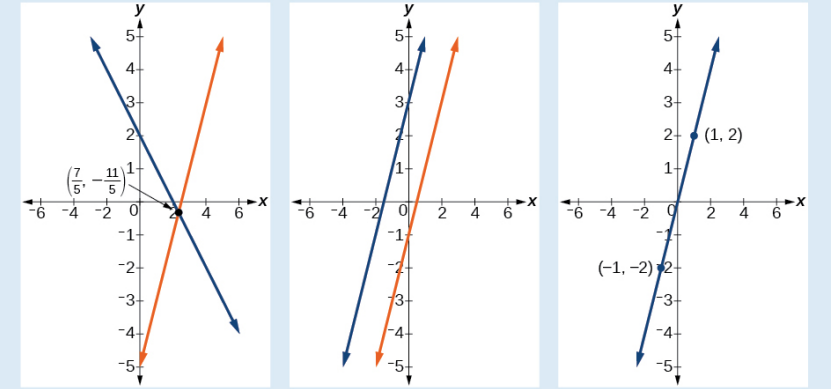


Figure 4.1.2

Given a system of linear equations and an ordered pair, determine whether the ordered pair is a solution

1. Substitute the ordered pair into each equation in the system.
2. Determine whether true statements result from the substitution in both equations; if so, the ordered pair is a solution.

Example 4.1.1: Determining Whether an Ordered Pair Is a Solution to a System of Equations

Determine whether the ordered pair $(5, 1)$ is a solution to the given system of equations.

$$\begin{aligned} x + 3y &= 8 \\ 2x - 9 &= y \end{aligned}$$

Solution

Substitute the ordered pair $(5, 1)$ into both equations.

$$\begin{aligned} (5) + 3(1) &= 8 \\ 8 &= 8 \text{ True} \\ 2(5) - 9 &= (1) \\ 1 &= 1 \text{ True} \end{aligned}$$

The ordered pair $(5, 1)$ satisfies both equations, so it is the solution to the system.

Analysis

We can see the solution clearly by plotting the graph of each equation. Since the solution is an ordered pair that satisfies both equations, it is a point on both of the lines and thus the point of intersection of the two lines. See Figure $4.1.3$.

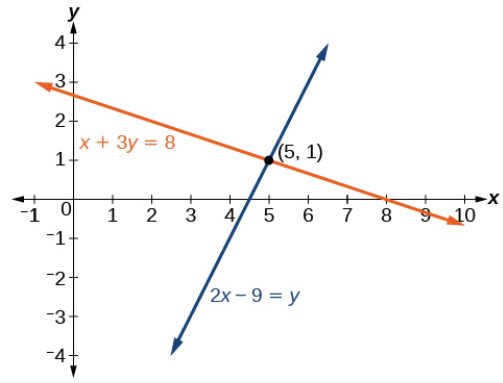


Figure 4.1.3

? Exercise 4.1.1

Determine whether the ordered pair $(8, 5)$ is a solution to the following system.

$$\begin{aligned} 5x - 4y &= 20 \\ 2x + 1 &= 3y \end{aligned}$$

Answer

Not a solution.

Solving Systems of Equations by Graphing

There are multiple methods of solving systems of linear equations. For a system of linear equations in two variables, we can determine both the type of system and the solution by graphing the system of equations on the same set of axes.

✓ Example 4.1.2: Solving a System of Equations in Two Variables by Graphing

Solve the following system of equations by graphing. Identify the type of system.

$$\begin{aligned} 2x + y &= -8 \\ x - y &= -1 \end{aligned}$$

Solution

Solve the first equation for y.

$$\begin{aligned} 2x + y &= -8 \\ y &= -2x - 8 \end{aligned}$$

Solve the second equation for y.

$$\begin{aligned} x - y &= -1 \\ y &= x + 1 \end{aligned}$$

Graph both equations on the same set of axes as in Figure 4.1.4.

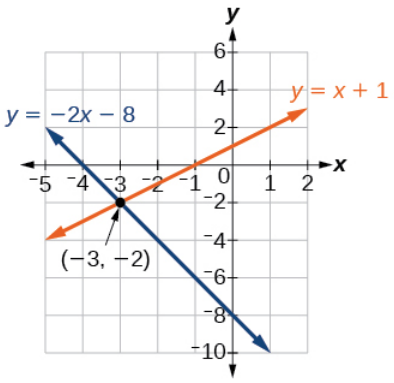


Figure 4.1.4

The lines appear to intersect at the point $(-3,-2)$. We can check to make sure that this is the solution to the system by substituting the ordered pair into both equations.

$$\begin{aligned} 2(-3)+(-2) &= -8 \\ -8 &= -8 \text{ True} \\ (-3)-(-2) &= -1 \\ -1 &= -1 \text{ True} \end{aligned}$$

The solution to the system is the ordered pair $(-3,-2)$,so the system is independent.

Exercise 4.1.2

Solve the following system of equations by graphing.

$$\begin{aligned} 2x-5y &= -25 \\ -4x+5y &= 35 \end{aligned}$$

Answer

The solution to the system is the ordered pair $(-5,3)$.

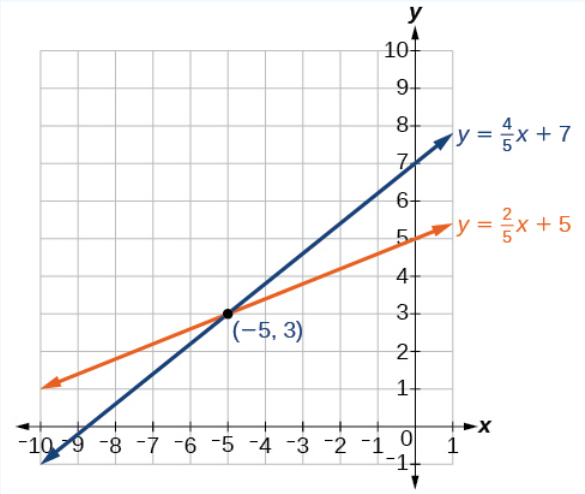


Figure 4.1.5

Q&A

Can graphing be used if the system is inconsistent or dependent?

Yes, in both cases we can still graph the system to determine the type of system and solution. If the two lines are parallel, the system has no solution and is inconsistent. If the two lines are identical, the system has infinite solutions and is a dependent

system.

Solving Systems of Equations by Substitution

Solving a linear system in two variables by graphing works well when the solution consists of integer values, but if our solution contains decimals or fractions, it is not the most precise method. We will consider two more methods of solving a **system of linear equations** that are more precise than graphing. One such method is solving a system of equations by the **substitution method**, in which we solve one of the equations for one variable and then substitute the result into the second equation to solve for the second variable. Recall that we can solve for only one variable at a time, which is the reason the substitution method is both valuable and practical.

How to: Given a system of two equations in two variables, solve using the substitution method.

1. Solve one of the two equations for one of the variables in terms of the other.
2. Substitute the expression for this variable into the second equation, then solve for the remaining variable.
3. Substitute that solution into either of the original equations to find the value of the first variable. If possible, write the solution as an ordered pair.
4. Check the solution in both equations.

Example 4.1.3: Solving a System of Equations in Two Variables by Substitution

Solve the following system of equations by substitution.

$$\begin{aligned} -x+y &= -5 \\ 2x-5y &= 1 \end{aligned}$$

Solution

First, we will solve the first equation for y.

$$\begin{aligned} -x+y &= -5 \\ y &= x-5 \end{aligned}$$

Now we can substitute the expression $x-5$ for y in the second equation.

$$\begin{aligned} 2x-5y &= 1 \\ 2x-5(x-5) &= 1 \\ 2x-5x+25 &= 1 \\ -3x &= -24 \\ x &= 8 \end{aligned}$$

Now, we substitute $x=8$ into the first equation and solve for y.

$$\begin{aligned} -(8)+y &= -5 \\ y &= 3 \end{aligned}$$

Our solution is $(8,3)$.

Check the solution by substituting $(8,3)$ into both equations.

$$\begin{aligned} -x+y &= -5 \\ -(8)+(3) &= -5 \text{ True} \\ 2x-5y &= 1 \\ 2(8)-5(3) &= 1 \text{ True} \end{aligned}$$

Exercise 4.1.3

Solve the following system of equations by substitution.

$$x = y + 3$$
$$4 = 3x - 2y$$

Answer

$(-2, -5)$

Q&A

Can the substitution method be used to solve any linear system in two variables?

Yes, but the method works best if one of the equations contains a coefficient of 1 *or* -1 *so that we do not have to deal with fractions.*

Solving Systems of Equations in Two Variables by the Addition Method

A third method of solving systems of linear equations is the addition method. In this method, we add two terms with the same variable, but opposite coefficients, so that the sum is zero. Of course, not all systems are set up with the two terms of one variable having opposite coefficients. Often we must adjust one or both of the equations by multiplication so that one variable will be eliminated by addition.

How to: Given a system of equations, solve using the addition method.

1. Write both equations with *x*- and *y*-variables on the left side of the equal sign and constants on the right.
2. Write one equation above the other, lining up corresponding variables. If one of the variables in the top equation has the opposite coefficient of the same variable in the bottom equation, add the equations together, eliminating one variable. If not, use multiplication by a nonzero number so that one of the variables in the top equation has the opposite coefficient of the same variable in the bottom equation, then add the equations to eliminate the variable.
3. Solve the resulting equation for the remaining variable.
4. Substitute that value into one of the original equations and solve for the second variable.
5. Check the solution by substituting the values into the other equation.

Example 4.1.4: Solving a System by the Addition Method

Solve the given system of equations by addition.

$$x + 2y = -1$$
$$-x + y = 3$$

Solution

Both equations are already set equal to a constant. Notice that the coefficient of x in the second equation, -1, is the opposite of the coefficient of x in the first equation, 1. We can add the two equations to eliminate x without needing to multiply by a constant.

$$\begin{aligned} x + 2y &= -1 \\ \underline{-x + y} &= \underline{3} \\ 3y &= 2 \end{aligned}$$

Now that we have eliminated x, we can solve the resulting equation for y.

$$3y = 2$$
$$y = \frac{2}{3}$$

Then, we substitute this value for y into one of the original equations and solve for x.

$$\begin{aligned} -x+y &= 3 \\ -x+\frac{2}{3} &= 3 \\ -x &= 3-\frac{2}{3} \\ -x &= \frac{7}{3} \\ x &= -\frac{7}{3} \end{aligned}$$

The solution to this system is $\left(-\frac{7}{3}, \frac{2}{3}\right)$.

Check the solution in the first equation.

$$\begin{aligned} x+2y &= -1 \\ \left(-\frac{7}{3}\right)+2\left(\frac{2}{3}\right) &= \\ -\frac{7}{3}+\frac{4}{3} &= -\frac{3}{3} \\ -1 &= -1 \qquad \text{True} \end{aligned}$$

Analysis

We gain an important perspective on systems of equations by looking at the graphical representation. See Figure 4.1.6 to find that the equations intersect at the solution. We do not need to ask whether there may be a second solution because observing the graph confirms that the system has exactly one solution.

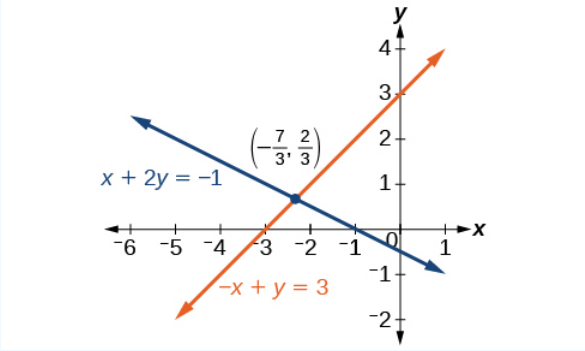


Figure 4.1.6

Example 4.1.5: Using the Addition Method When Multiplication of One Equation Is Required

Solve the given system of equations by the addition method.

$$\begin{aligned} 3x+5y &= -11 \\ x-2y &= 11 \end{aligned}$$

Solution

Adding these equations as presented will not eliminate a variable. However, we see that the first equation has $3x$ in it and the second equation has x. So if we multiply the second equation by -3,the x-terms will add to zero.

$$\begin{aligned} x-2y &= 11 \\ -3(x-2y) &= -3(11) \qquad \text{Multiply both sides by } -3. \\ -3x+6y &= -33 \qquad \text{Use the distributive property.} \end{aligned}$$

Now, let's add them.

$$\begin{aligned} 3x+5y &= -11 \\ \underline{-3x+6y} &= \underline{-33} \\ 11y &= -44 \\ y &= -4 \end{aligned}$$

For the last step, we substitute $y=-4$ into one of the original equations and solve for x.

$$\begin{aligned} 3x+5y &= -11 \\ 3x+5(-4) &= -11 \\ 3x-20 &= -11 \\ 3x &= 9 \\ x &= 3 \end{aligned}$$

Our solution is the ordered pair $(3,-4)$. See Figure 4.1.7. Check the solution in the original second equation.

$$\begin{aligned} x-2y &= 11 \\ (3)-2(-4) &= 3+8 \\ &= 11 \qquad \text{True} \end{aligned}$$

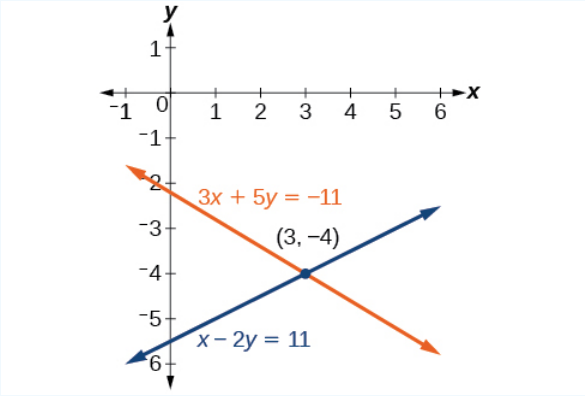


Figure 4.1.7

Exercise 4.1.4

Solve the system of equations by addition.

$$\begin{aligned} 2x-7y &= 2 \\ 3x+y &= -20 \end{aligned}$$

Answer

$(-6,-2)$

Example 4.1.6: Using the Addition Method When Multiplication of Both Equations Is Required

Solve the given system of equations in two variables by addition.

$$\begin{aligned} 2x+3y &= -16 \\ 5x-10y &= 30 \end{aligned}$$

Solution

One equation has $2x$ and the other has $5x$. The least common multiple is $10x$ so we will have to multiply both equations by a constant in order to eliminate one variable. Let's eliminate x by multiplying the first equation by -5 and the second equation by 2.

$$\begin{aligned} -5(2x+3y) &= -5(-16) \\ -10x-15y &= 80 \\ 2(5x-10y) &= 2(30) \\ 10x-20y &= 60 \end{aligned}$$

Then, we add the two equations together.

$$\begin{aligned} -10x-15y &= 80 \\ \underline{10x-20y} &= \underline{60} \\ -35y &= 140 \\ y &= -4 \end{aligned}$$

Substitute $y=-4$ into the original first equation.

$$\begin{aligned} 2x+3(-4) &= -16 \\ 2x-12 &= -16 \\ 2x &= -4 \\ x &= -2 \end{aligned}$$

The solution is $(-2,-4)$. Check it in the other equation.

$$\begin{aligned} 5x-10y &= 30 \\ 5(-2)-10(-4) &= 30 \\ -10+40 &= 30 \\ 30 &= 30 \end{aligned}$$

See Figure 4.1.8.

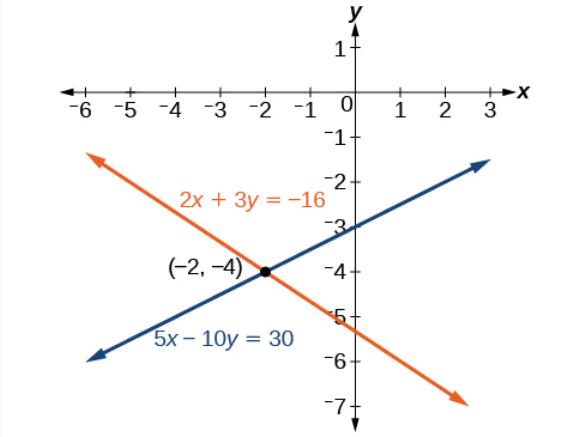


Figure 4.1.8

Example 4.1.7: Using the Addition Method in Systems of Equations Containing Fractions

Solve the given system of equations in two variables by addition.

$$\begin{aligned} \frac{x}{3}+\frac{y}{6} &= 3 \\ \frac{x}{2}-\frac{y}{4} &= 1 \end{aligned}$$

Solution

First clear each equation of fractions by multiplying both sides of the equation by the least common denominator.

$$6\left(\frac{x}{3}+\frac{y}{6}\right)=6(3)$$
$$2x+y=18$$
$$4\left(\frac{x}{2}-\frac{y}{4}\right)=4(1)$$
$$2x-y=4$$

Now multiply the second equation by -1 so that we can eliminate the x-variable.

$$-1(2x-y)=-1(4)$$
$$-2x+y=-4$$

Add the two equations to eliminate the x-variable and solve the resulting equation.

$$2x+y=18$$
$$-2x+y=-4$$
$$2y=14$$
$$y=7$$

Substitute $y=7$ into the first equation.

$$2x+(7)=18$$
$$2x=11$$
$$x=\frac{11}{2}$$
$$=7.5$$

The solution is $\left(\frac{11}{2},7\right)$. Check it in the other equation.

$$\frac{x}{2}-\frac{y}{4}=1$$
$$\frac{\frac{11}{2}}{2}-\frac{7}{4}=1$$
$$\frac{11}{4}-\frac{7}{4}=1$$
$$\frac{4}{4}=1$$

? Exercise 4.1.5

Solve the system of equations by addition.

$$2x+3y=8$$
$$3x+5y=10$$

Answer

$(10,-4)$

Identifying Inconsistent Systems of Equations Containing Two Variables

Now that we have several methods for solving systems of equations, we can use the methods to identify inconsistent systems. Recall that an inconsistent system consists of parallel lines that have the same slope but different y-intercepts. They will never intersect. When searching for a solution to an inconsistent system, we will come up with a false statement, such as $12=0$.

Example 4.1.8: Solving an Inconsistent System of Equations

Solve the following system of equations.

$$\begin{aligned} x &= 9-2y \\ x+2y &= 13 \end{aligned}$$

Solution

We can approach this problem in two ways. Because one equation is already solved for x, the most obvious step is to use substitution.

$$\begin{aligned} x+2y &= 13 \\ (9-2y)+2y &= 13 \\ 9+0y &= 13 \\ 9 &= 13 \end{aligned}$$

Clearly, this statement is a contradiction because $9 \neq 13$. Therefore, the system has no solution.

The second approach would be to first manipulate the equations so that they are both in slope-intercept form. We manipulate the first equation as follows.

$$\begin{aligned} x &= 9-2y \\ 2y &= -x+9 \\ y &= -\frac{1}{2}x+\frac{9}{2} \end{aligned}$$

We then convert the second equation expressed to slope-intercept form.

$$\begin{aligned} x+2y &= 13 \\ 2y &= -x+13 \\ y &= -\frac{1}{2}x+\frac{13}{2} \end{aligned}$$

Comparing the equations, we see that they have the same slope but different y-intercepts. Therefore, the lines are parallel and do not intersect.

$$\begin{aligned} y &= -\frac{1}{2}x+\frac{9}{2} \\ y &= -\frac{1}{2}x+\frac{13}{2} \end{aligned}$$

Analysis

Writing the equations in slope-intercept form confirms that the system is inconsistent because all lines will intersect eventually unless they are parallel. Parallel lines will never intersect; thus, the two lines have no points in common. The graphs of the equations in this example are shown in Figure 4.1.9.

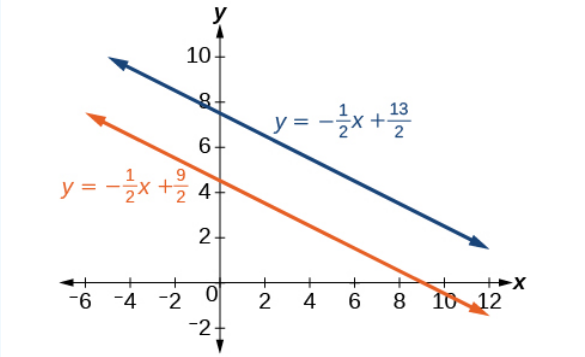


Figure 4.1.9

? Exercise 4.1.6

Solve the following system of equations in two variables.

$$\begin{aligned} 2y-2x &= 2 \\ 2y-2x &= 6 \end{aligned}$$

Answer

No solution. It is an inconsistent system.

Expressing the Solution of a System of Dependent Equations Containing Two Variables

Recall that a dependent system of equations in two variables is a system in which the two equations represent the same line. Dependent systems have an infinite number of solutions because all of the points on one line are also on the other line. After using substitution or addition, the resulting equation will be an identity, such as $0=0$.

✓ Example 4.1.9: Finding a Solution to a Dependent System of Linear Equations

Find a solution to the system of equations using the **addition method**.

$$\begin{aligned} x+3y &= 2 \\ 3x+9y &= 6 \end{aligned}$$

Solution

With the addition method, we want to eliminate one of the variables by adding the equations. In this case, let's focus on eliminating x. If we multiply both sides of the first equation by -3, then we will be able to eliminate the x-variable.

$$\begin{aligned} x+3y &= 2 \\ (-3)(x+3y) &= (-3)(2) \\ -3x-9y &= -6 \end{aligned}$$

Now add the equations.

$$\begin{aligned} -3x-9y &= -6 \\ \underline{+\,3x+9y} &= \underline{6} \\ 0 &= 0 \end{aligned}$$

We can see that there will be an infinite number of solutions that satisfy both equations.

Analysis

If we rewrote both equations in the slope-intercept form, we might know what the solution would look like before adding. Let's look at what happens when we convert the system to slope-intercept form.

$$\begin{aligned} x+3y &= 2 \\ 3y &= -x+2 \\ y &= -\frac{1}{3}x+\frac{2}{3} \\ 3x+9y &= 6 \\ 9y &= -3x+6 \\ y &= -\frac{3}{9}x+\frac{6}{9} \\ y &= -\frac{1}{3}x+\frac{2}{3} \end{aligned}$$

See Figure 4.1.10. Notice the results are the same. The general solution to the system is $\left(x,-\frac{1}{3}x+\frac{2}{3}\right)$.

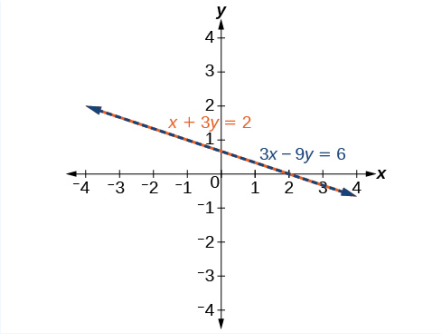


Figure 4.1.10

? Exercise 4.1.7

Solve the following system of equations in two variables.

$$\begin{aligned} y-2x &= 5 \\ -3y+6x &= -15 \end{aligned}$$

Answer

The system is dependent so there are infinite solutions of the form $(x, 2x+5)$.

Using Systems of Equations to Investigate Profits

Using what we have learned about systems of equations, we can return to the skateboard manufacturing problem at the beginning of the section. The skateboard manufacturer's revenue function is the function used to calculate the amount of money that comes into the business. It can be represented by the equation $R = xp$, where x=quantity and p=price. The revenue function is shown in orange in Figure 4.1.11.

The cost function is the function used to calculate the costs of doing business. It includes fixed costs, such as rent and salaries, and variable costs, such as utilities. The cost function is shown in blue in Figure 4.1.11. The x-axis represents quantity in hundreds of units. The y-axis represents either cost or revenue in hundreds of dollars.

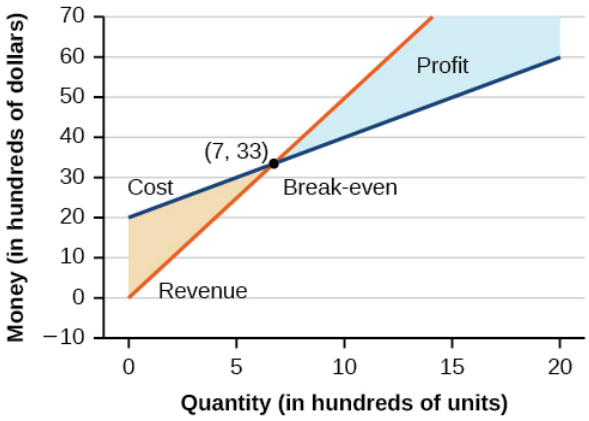


Figure 4.1.11

The point at which the two lines intersect is called the break-even point. We can see from the graph that if 700 units are produced, the cost is $\$3,300$ and the revenue is also $\$3,300$. In other words, the company breaks even if they produce and sell 700 units. They neither make money nor lose money.

The shaded region to the right of the break-even point represents quantities for which the company makes a profit. The shaded region to the left represents quantities for which the company suffers a loss. The profit function is the revenue function minus the cost function, written as $P(x) = R(x) - C(x)$. Clearly, knowing the quantity for which the cost equals the revenue is of great importance to businesses.

Example 4.1.10: Finding the Break-Even Point and the Profit Function Using Substitution

Given the cost function $C(x) = 0.85x + 35,000$ and the revenue function $R(x) = 1.55x$, find the break-even point and the profit function.

Solution

Write the system of equations using y to replace function notation.

$$\begin{aligned} y &= 0.85x + 35,000 \\ y &= 1.55x \end{aligned}$$

Substitute the expression $0.85x + 35,000$ from the first equation into the second equation and solve for x.

$$\begin{aligned} 0.85x + 35,000 &= 1.55x \\ 35,000 &= 0.7x \\ 50,000 &= x \end{aligned}$$

Then, we substitute $x = 50,000$ into either the cost function or the revenue function.

$$1.55(50,000) = 77,500$$

The break-even point is $(50,000, 77,500)$.

The profit function is found using the formula $P(x) = R(x) - C(x)$.

$$\begin{aligned} P(x) &= 1.55x - (0.85x + 35,000) \\ &= 0.7x - 35,000 \end{aligned}$$

The profit function is $P(x) = 0.7x - 35,000$.

Analysis

The cost to produce $50,000$ units is $\$77,500$, and the revenue from the sales of $50,000$ units is also $\$77,500$. To make a profit, the business must produce and sell more than $50,000$ units. See Figure 4.1.12.

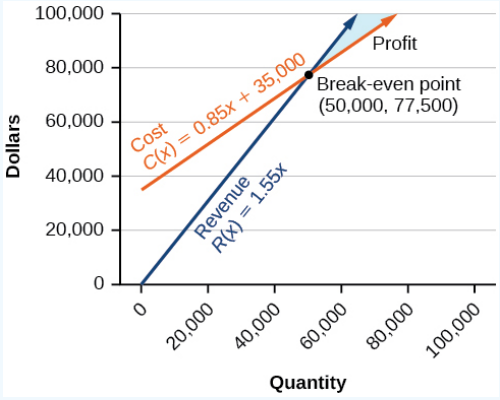


Figure 4.1.12

We see from the graph in Figure 4.1.13 that the profit function has a negative value until $x = 50,000$, when the graph crosses the x-axis. Then, the graph emerges into positive y-values and continues on this path as the profit function is a straight line. This illustrates that the break-even point for businesses occurs when the profit function is 0. The area to the left of the break-even point represents operating at a loss.

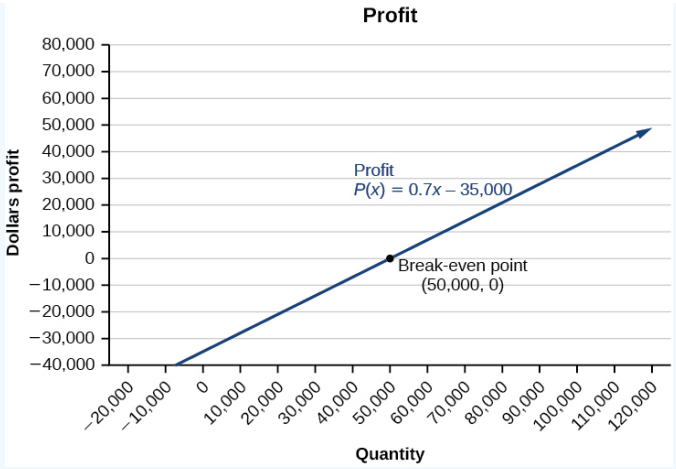


Figure 4.1.13

Example 4.1.11: Writing and Solving a System of Equations in Two Variables

The cost of a ticket to the circus is $\$25.00$ for children and $\$50.00$ for adults. On a certain day, attendance at the circus is $2,000$ and the total gate revenue is $\$70,000$. How many children and how many adults bought tickets?

Solution

Let c = the number of children and a = the number of adults in attendance.

The total number of people is $2,000$. We can use this to write an equation for the number of people at the circus that day.

$$c+a=2,000$$

The revenue from all children can be found by multiplying $\$25.00$ by the number of children, $25c$. The revenue from all adults can be found by multiplying $\$50.00$ by the number of adults, $50a$. The total revenue is $\$70,000$. We can use this to write an equation for the revenue.

$$25c+50a=70,000$$

We now have a system of linear equations in two variables.

$$c+a=2,000$$
$$25c+50a=70,000$$

In the first equation, the coefficient of both variables is 1. We can quickly solve the first equation for either c or a. We will solve for a.

$$\begin{aligned} c+a &= 2,000 \\ a &= 2,000-c \end{aligned}$$

Substitute the expression $2,000-c$ in the second equation fora a and solve for c.

$$\begin{aligned} 25c+50(2,000-c) &= 70,000 \\ 25c+100,000-50c &= 70,000 \\ -25c &= -30,000 \\ c &= 1,200 \end{aligned}$$

Substitute $c=1,200$ into the first equation to solve for a.

$$\begin{aligned} 1,200+a &= 2,000 \\ a &= 800 \end{aligned}$$

We find that $1,200$ children and 800 adults bought tickets to the circus that day.

Exercise 4.1.8

Meal tickets at the circus cost $\$4.00$ for children and $\$12.00$ for adults. If $1,650$ meal tickets were bought for a total of, $\$14,200$, how many children and how many adults bought meal tickets?

Answer

700 children, 950 adults

Media

Access these online resources for additional instruction and practice with systems of linear equations.

- Solving Systems of Equations Using Substitution
- Solving Systems of Equations Using Elimination
- Applications of Systems of Equations

Key Concepts

- A system of linear equations consists of two or more equations made up of two or more variables such that all equations in the system are considered simultaneously.
- The solution to a system of linear equations in two variables is any ordered pair that satisfies each equation independently. See Example 4.1.1.
- Systems of equations are classified as independent with one solution, dependent with an infinite number of solutions, or inconsistent with no solution.
- One method of solving a system of linear equations in two variables is by graphing. In this method, we graph the equations on the same set of axes. See Example 4.1.2.
- Another method of solving a system of linear equations is by substitution. In this method, we solve for one variable in one equation and substitute the result into the second equation. See Example 4.1.3.
- A third method of solving a system of linear equations is by addition, in which we can eliminate a variable by adding opposite coefficients of corresponding variables. See Example 4.1.4.
- It is often necessary to multiply one or both equations by a constant to facilitate elimination of a variable when adding the two equations together. See Example 4.1.5, Example 4.1.6, and Example 4.1.7.
- Either method of solving a system of equations results in a false statement for inconsistent systems because they are made up of parallel lines that never intersect. See Example 4.1.8.
- The solution to a system of dependent equations will always be true because both equations describe the same line. See Example 4.1.9.
- Systems of equations can be used to solve real-world problems that involve more than one variable, such as those relating to revenue, cost, and profit. See Example 4.1.10 and Example 4.1.11.

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CHAPTER OVERVIEW

5: Inequalities

5.1: Solve Applications with Linear Inequalities

5.1E: Exercises

5.2: Solve Linear Inequalities - Algebra Emphasis

5.2E: Exercises

5.3: Solve Compound Inequalities - Union, Intersection

5.3E: Exercises

5.4: Graphs of Linear Inequalities in Two Variables (optional challenge practice)

5.4E: Exercises

5.5: Solve Absolute Value Inequalities (optional challenge practice)

5.5E: Exercises

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5.1: Solve Applications with Linear Inequalities

Learning Objectives

By the end of this section, you will be able to:

- Solve applications with linear inequalities

Note

Before you get started, take this readiness quiz.

1. Write as an inequality: x is at least 30.
   If you missed this problem, review Exercise 2.7.34.
2. Solve $8-3y<41$.
   If you missed this problem, review Exercise 2.7.22.

Solve Applications with Linear Inequalities

Many real-life situations require us to solve inequalities. In fact, inequality applications are so common that we often do not even realize we are doing algebra. For example, how many gallons of gas can be put in the car for $20? Is the rent on an apartment affordable? Is there enough time before class to go get lunch, eat it, and return? How much money should each family member's holiday gift cost without going over budget?

The method we will use to solve applications with linear inequalities is very much like the one we used when we solved applications with equations. We will read the problem and make sure all the words are understood. Next, we will identify what we are looking for and assign a variable to represent it. We will restate the problem in one sentence to make it easy to translate into an inequality. Then, we will solve the inequality.

Example 5.1.1

Emma got a new job and will have to move. Her monthly income will be $5,265. To qualify to rent an apartment, Emma's monthly income must be at least three times as much as the rent. What is the highest rent Emma will qualify for?

Solution

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	the highest rent Emma will qualify for
Step 3. Name what we are looking for. Choose a variable to represent that quantity.	Let r = rent
Step 4. Translate into an inequality. First write a sentence that gives the information to find it.	Emma's monthly income must be at least three times the rent.
Step 5. Solve the inequality. Remember, $a>x$ has the same meaning as $x<a$	$5265 \geq 3r$ $1755 \geq r$ $r \leq 1755$
Step 6. Check the answer in the problem and make sure it makes sense. A maximum rent of $1,755 seems reasonable for an income of $5,265.	
Step 7. Answer the answer in the problem complete sentence.	the question with a The maximum rent is $1,755.

Try It 5.1.2

Alan is loading a pallet with boxes that each weighs 45 pounds. The pallet can safely support no more than 900 pounds. How many boxes can he safely load onto the pallet?

Answer

There can be no more than 20 boxes.

Try It 5.1.3

The elevator in Yehire's apartment building has a sign that says the maximum weight is 2,100 pounds. If the average weight of one person is 150 pounds, how many people can safely ride the elevator?

Answer

A maximum of 14 people can safely ride in the elevator.

Sometimes an application requires the solution to be a whole number, but the algebraic solution to the inequality is not a whole number. In that case, we must round the algebraic solution to a whole number. The context of the application will determine whether we round up or down. To check applications like this, we will round our answer to a number that is easy to compute with and make sure that number makes the inequality true.

Example 5.1.4

Dawn won a mini-grant of $4,000 to buy tablet computers for her classroom. The tablets she would like to buy cost $254.12 each, including tax and delivery. What is the maximum number of tablets Dawn can buy?

Solution

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	the maximum number of tablets Dawn can buy
Step 3. Name what we are looking for. Choose a variable to represent that quantity.	Let n = the number of tablets.
Step 4. Translate. write a sentence that gives the information to find it.	$254.12 times the number of tablets is no more than $4000.
Translate into an inequality.	$254.12n \leq 4000$
Step 5. Solve the inequality.	$n \leq 15.74$
But n must be a whole number of tablets, so round to 15.	$n \leq 15$
Step 6. Check the answer in the problem and make sure it makes sense. Rounding down the price to $250, 15 tablets would cost $3750, while 16 tablets would be $4000. So a maximum of 15 tablets at $254.12 seems reasonable.	
Step 7. Answer the answer in the problem complete sentence.	the question with a Dawn can buy a maximum of 15 tablets.

Try It 5.1.5

Angie has $20 to spend on juice boxes for her son's preschool picnic. Each pack of juice boxes costs $2.63. What is the maximum number of packs she can buy?

Answer

seven packs

Try It 5.1.6

Daniel wants to surprise his girlfriend with a birthday party at her favorite restaurant. It will cost $42.75 per person for dinner, including tip and tax. His budget for the party is $500. What is the maximum number of people Daniel can have at the party?

Answer

11 people

Example 5.1.7

Pete works at a computer store. His weekly pay will be either a fixed amount, $925, or $500 plus 12% of his total sales. How much should his total sales be for his variable pay option to exceed the fixed amount of $925?

Solution

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	the total sales needed for his variable pay option to exceed the fixed amount of $925
Step 3. Name what we are looking for. Choose a variable to represent that quantity.	Let s = the total sales.
Step 4. Translate. write a sentence that gives the information to find it. Translate into an inequality. Remember to convert the percent to a decimal.	$500 plus 12% of total sales is more than $925. $500 + 0.12s > 925$
Step 5. Solve the inequality.	$0.12s > 425$ $s > 3541.\overline{66}$
Step 6. Check the answer in the problem and make sure it makes sense. Rounding down the price to $250, 15 tablets would cost $3750, while If we round the total sales up to $4000, we see that $500+0.12(4000) = 980$, which is more than $925.	
Step 7. Answer the the question with a complete sentence.	The total sales must be more than $3541.67

Try It 5.1.8

Tiffany just graduated from college and her new job will pay her $20000 per year plus 2% of all sales. She wants to earn at least $100000 per year. For what total sales will she be able to achieve her goal?

Answer

at least $4000000

Try It 5.1.9

Christian has been offered a new job that pays $24000 a year plus 3% of sales. For what total sales would this new job pay more than his current job which pays $60000?

Answer

at least $1200000

Example 5.1.10

Sergio and Lizeth have a very tight vacation budget. They plan to rent a car from a company that charges $75 a week plus $0.25 a mile. How many miles can they travel and still keep within their $200 budget?

Solution

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	the number of miles Sergio and Lizeth can travel
Step 3. Name what we are looking for. Choose a variable to represent that quantity.	Let m = the number of miles.
Step 4. Translate. write a sentence that gives the information to find it. Translate into an inequality.	$75 plus 0.25 times the number of miles is less than or equal to $200. $75 + 25m \leq 200$
Step 5. Solve the inequality.	$0.25m \leq 125$ $m \leq 500$ miles
Step 6. Check the answer in the problem and make sure it makes sense. Yes, $75 + 0.25(500) = 200$.	
Step 7. Answer the the question with a complete sentence.	Sergio and Lizeth can travel 500 miles and still stay on budget.

Try It 5.1.11

Taleisha's phone plan costs her $28.80 a month plus $0.20 per text message. How many text messages can she use and keep her monthly phone bill no more than $50?

Answer

no more than 106 text messages

Try It 5.1.12

Rameen's heating bill is $5.42 per month plus $1.08 per therm. How many therms can Rameen use if he wants his heating bill to be a maximum of $87.50?

Answer

no more than 76 therms

A common goal of most businesses is to make a profit. *Profit* is the money that remains when the expenses have been subtracted from the money earned. In the next example, we will find the number of jobs a small businessman needs to do every month in order to make a certain amount of **profit**.

Example 5.1.13

Elliot has a landscape maintenance business. His monthly expenses are $1,100. If he charges $60 per job, how many jobs must he do to earn a profit of at least $4,000 a month?

Solution

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	the number of jobs Elliot needs
Step 3. Name what we are looking for.	
Choose a variable to represent it	Let j = the number of jobs.
Step 4. Translate. write a sentence that	$60 times the number of jobs minus $1,100 is at least $4,000.
gives the information to find it.	less than or equal to $200.
Translate into an inequality.	$60j - 1100 \geq 4000$
Step 5. Solve the inequality.	$60j \geq 5100$
	$j \geq 85$ jobs
Step 6. Check the answer in the problem and make sure it makes sense.	If Elliot did 90 jobs, his profit would be $60(90) - 1{,}100$, or $4,300. This is more than $4,000.
Step 7. Answer the the question with a complete sentence.	Elliot must work at least 85 jobs.

Try It 5.1.14

Caleb has a pet sitting business. He charges $32 per hour. His monthly expenses are $2272. How many hours must he work in order to earn a profit of at least $800 per month?

Answer

at least 96 hours

Try It 5.1.15

Felicity has a calligraphy business. She charges $2.50 per wedding invitation. Her monthly expenses are $650. How many invitations must she write to earn a profit of at least $2800 per month?

Answer

at least 1380 invitations

Sometimes life gets complicated! There are many situations in which several quantities contribute to the total expense. We must make sure to account for all the individual expenses when we solve problems like this.

Example 5.1.16

Brenda's best friend is having a destination wedding and the event will last 3 days. Brenda has $500 in savings and can earn $15 an hour babysitting. She expects to pay $350 airfare, $375 for food and entertainment and $60 a night for her share of a hotel room. How many hours must she babysit to have enough money to pay for the trip?

Solution

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	the number of hours Brenda must babysit
Step 3. Name what we are looking for. Choose a variable to represent that quantity.	Let h = the number of hours.
Step 4. Translate. write a sentence that gives the information to find it.	The expenses must be less than or equal to the income. The cost of airfare plus the cost of food and entertainment and the hotel bill must be less than or equal to the savings plus the amount earned babysitting.
Translate into an inequality.	$\$350 + \$375 + \$60(3) \leq \$500 + \$15h$
Step 5. Solve the inequality.	$905 \leq 500 + 15h$ $405 \leq 15h$ $27 \leq h$ $h \geq 27$
Step 6. Check the answer in the problem and make sure it makes sense.	
We substitute 27 into the inequality.	
$905 \leq 500 + 15h$	
$905 \leq 500 + 15(27)$	
$905 \leq 905$	
Step 7. Answer the the question with a complete sentence.	Brenda must babysit at least 27 hours.

? Try It 5.1.17

Malik is planning a 6-day summer vacation trip. He has $840 in savings, and he earns $45 per hour for tutoring. The trip will cost him $525 for airfare, $780 for food and sightseeing, and $95 per night for the hotel. How many hours must he tutor to have enough money to pay for the trip?

Answer

at least 23 hours

? Try It 5.1.18

Josue wants to go on a 10-day road trip next spring. It will cost him $180 for gas, $450 for food, and $49 per night for a motel. He has $520 in savings and can earn $30 per driveway shoveling snow. How many driveways must he shovel to have enough money to pay for the trip?

Answer

at least 20 driveways

Key Concepts

- **Solving inequalities**
  1. **Read** the problem.
  2. **Identify** what we are looking for.
  3. **Name** what we are looking for. Choose a variable to represent that quantity.
  4. **Translate.** Write a sentence that gives the information to find it. Translate into an inequality.

5. **Solve** the inequality.
6. **Check** the answer in the problem and make sure it makes sense.
7. **Answer** the question with a complete sentence.

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5.1E: Exercises

Practice Makes Perfect

Solve Applications with Linear Inequalities

In the following exercises, solve.

Exercise 5.1E.1

Mona is planning her son's birthday party and has a budget of $285. The Fun Zone charges $19 per child. How many children can she have at the party and stay within her budget?

Answer

15 children

Exercise 5.1E.2

Carlos is looking at apartments with three of his friends. They want the monthly rent to be no more than $2360. If the roommates split the rent evenly among the four of them, what is the maximum rent each will pay?

Exercise 5.1E.3

A water taxi has a maximum load of 1,800 pounds. If the average weight of one person is 150 pounds, how many people can safely ride in the water taxi?

Answer

12 people

Exercise 5.1E.4

Marcela is registering for her college classes, which cost $105 per unit. How many units can she take to have a maximum cost of $1,365?

Exercise 5.1E.5

Arleen got a $20 gift card for the coffee shop. Her favorite iced drink costs $3.79. What is the maximum number of drinks she can buy with the gift card?

Answer

five drinks

Exercise 5.1E.6

Teegan likes to play golf. He has budgeted $60 next month for the driving range. It costs him $10.55 for a bucket of balls each time he goes. What is the maximum number of times he can go to the driving range next month?

Exercise 5.1E.7

Joni sells kitchen aprons online for $32.50 each. How many aprons must she sell next month if she wants to earn at least $1,000?

Answer

31 aprons

Exercise 5.1E.8

Ryan charges his neighbors $17.50 to wash their car. How many cars must he wash next summer if his goal is to earn at least $1,500?

Exercise 5.1E.9

Keshad gets paid $2,400 per month plus 6% of his sales. His brother earns $3,300 per month. For what amount of total sales will Keshad's monthly pay be higher than his brother's monthly pay?

Answer

$15,000

Exercise 5.1E.10

Kimuyen needs to earn $4,150 per month in order to pay all her expenses. Her job pays her $3,475 per month plus 4% of her total sales. What is the minimum Kimuyen's total sales must be in order for her to pay all her expenses?

Exercise 5.1E.11

Andre has been offered an entry-level job. The company offered him $48,000 per year plus 3.5% of his total sales. Andre knows that the average pay for this job is $62,000. What would Andre's total sales need to be for his pay to be at least as high as the average pay for this job?

Answer

$400,000

Exercise 5.1E.12

Nataly is considering two job offers. The first job would pay her $83,000 per year. The second would pay her $66,500 plus 15% of her total sales. What would her total sales need to be for her salary on the second offer be higher than the first?

Exercise 5.1E.13

Jake's water bill is $24.80 per month plus $2.20 per ccf (hundred cubic feet) of water. What is the maximum number of ccf Jake can use if he wants his bill to be no more than $60?

Answer

16 ccf

Exercise 5.1E.14

Kiyoshi's phone plan costs $17.50 per month plus $0.15 per text message. What is the maximum number of text messages Kiyoshi can use so the phone bill is no more than $56.50?

Exercise 5.1E.15

Marlon's TV plan costs $49.99 per month plus $5.49 per first-run movie. How many first-run movies can he watch if he wants to keep his monthly bill to be a maximum of $100?

Answer

nine movies

Exercise 5.1E.16

Kellen wants to rent a banquet room in a restaurant for her cousin's baby shower. The restaurant charges $350 for the banquet room plus $32.50 per person for lunch. How many people can Kellen have at the shower if she wants the maximum cost to be $1,500?

Exercise 5.1E.17

Moshde runs a hairstyling business from her house. She charges $45 for a haircut and style. Her monthly expenses are $960. She wants to be able to put at least $1,200 per month into her savings account order to open her own salon. How many "cut & styles" must she do to save at least $1,200 per month?

Answer

48 cut & styles

Exercise 5.1E.18

Noe installs and configures software on home computers. He charges $125 per job. His monthly expenses are $1,600. How many jobs must he work in order to make a profit of at least $2,400?

Exercise 5.1E.19

Katherine is a personal chef. She charges $115 per four-person meal. Her monthly expenses are $3,150. How many four-person meals must she sell in order to make a profit of at least $1,900?

Answer

44 meals

Exercise 5.1E.20

Melissa makes necklaces and sells them online. She charges $88 per necklace. Her monthly expenses are $3745. How many necklaces must she sell if she wants to make a profit of at least $1,650?

Exercise 5.1E.21

Five student government officers want to go to the state convention. It will cost them $110 for registration, $375 for transportation and food, and $42 per person for the hotel. There is $450 budgeted for the convention in the student government savings account. They can earn the rest of the money they need by having a car wash. If they charge $5 per car, how many cars must they wash in order to have enough money to pay for the trip?

Answer

49 cars

Exercise 5.1E.22

Cesar is planning a 4-day trip to visit his friend at a college in another state. It will cost him $198 for airfare, $56 for local transportation, and $45 per day for food. He has $189 in savings and can earn $35 for each lawn he mows. How many lawns must he mow to have enough money to pay for the trip?

Exercise 5.1E. 23

Alonzo works as a car detailer. He charges $175 per car. He is planning to move out of his parents' house and rent his first apartment. He will need to pay $120 for application fees, $950 for security deposit, and first and last months' rent at $1,140 per month. He has $1,810 in savings. How many cars must he detail to have enough money to rent the apartment?

Answer

9 cars

Exercise 5.1E. 24

Eun-Kyung works as a tutor and earns $60 per hour. She has $792 in savings. She is planning an anniversary party for her parents. She would like to invite 40 guests. The party will cost her $1,520 for food and drinks and $150 for the photographer. She will also have a favor for each of the guests, and each favor will cost $7.50. How many hours must she tutor to have enough money for the party?

Everyday Math

Exercise 5.1E. 25

Maximum Load on a Stage In 2014, a high school stage collapsed in Fullerton, California, when 250 students got on stage for the finale of a musical production. Two dozen students were injured. The stage could support a maximum of 12,750 pounds. If the average weight of a student is assumed to be 140 pounds, what is the maximum number of students who could safely be on the stage?

Answer

91 students

Exercise 5.1E. 26

Maximum Weight on a Boat In 2004, a water taxi sank in Baltimore harbor and five people drowned. The water taxi had a maximum capacity of 3,500 pounds (25 people with average weight 140 pounds). The average weight of the 25 people on the water taxi when it sank was 168 pounds per person. What should the maximum number of people of this weight have been?

Exercise 5.1E. 27

Wedding Budget Adele and Walter found the perfect venue for their wedding reception. The cost is $9,850 for up to 100 guests, plus $38 for each additional guest. How many guests can attend if Adele and Walter want the total cost to be no more than $12,500?

Answer

169 guests

Exercise 5.1E. 28

Shower Budget Penny is planning a baby shower for her daughter-in-law. The restaurant charges $950 for up to 25 guests, plus $31.95 for each additional guest. How many guests can attend if Penny wants the total cost to be no more than $1,500?

Writing Exercises

Exercise 5.1E. 29

Find your last month's phone bill and the hourly salary you are paid at your job. (If you do not have a job, use the hourly salary you would realistically be paid if you had a job.) Calculate the number of hours of work it would take you to earn at least enough money to pay your phone bill by writing an appropriate inequality and then solving it.

Answer

Answers will vary.

Exercise 5.1E. 30

Find out how many units you have left, after this term, to achieve your college goal and estimate the number of units you can take each term in college. Calculate the number of terms it will take you to achieve your college goal by writing an appropriate inequality and then solving it.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
solve applications with linear inequalities.			

ⓑ What does this checklist tell you about your mastery of this section? What steps will you take to improve?

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5.2: Solve Linear Inequalities - Algebra Emphasis

Learning Objectives

By the end of this section, you will be able to:

- Graph inequalities on the number line
- Solve linear inequalities
- Translate words to an inequality and solve
- Solve applications with linear inequalities

Before you get started, take this readiness quiz.

1. Translate from algebra to English: $15 > x$.
   If you missed this problem, review [link].
2. Translate to an algebraic expression: 15 is less than x.
   If you missed this problem, review [link].

Graph Inequalities on the Number Line

What number would make the inequality $x > 3$ true? Are you thinking, "x could be four"? That's correct, but x could be 6, too, or 37, or even 3.001. Any number greater than three is a solution to the inequality $x > 3$. We show all the solutions to the inequality $x > 3$ on the number line by shading in all the numbers to the right of three, to show that all numbers greater than three are solutions. Because the number three itself is not a solution, we put an open parenthesis at three.

We can also represent inequalities using **interval notation**. There is no upper end to the solution to this inequality. In interval notation, we express $x > 3$ as $(3, \infty)$. The symbol ∞ is read as "**infinity**." It is not an actual number. Figure $5.2.1$ shows both the number line and the interval notation.

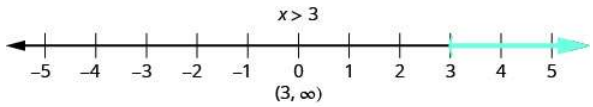


Figure 5.2.1: The inequality $x > 3$ is graphed on this number line and written in interval notation.

We use the left parenthesis symbol, (, to show that the endpoint of the inequality is not included. The left bracket symbol, [, shows that the endpoint is included.

The inequality $x \leq 1$ means all numbers less than or equal to one. Here we need to show that one is a solution, too. We do that by putting a bracket at $x = 1$. We then shade in all the numbers to the left of one, to show that all numbers less than one are solutions (Figure $5.2.2$). There is no lower end to those numbers. We write $x \leq 1 x \leq 1$ in interval notation as $(-\infty, 1]$. The symbol $-\infty$ is read as "negative infinity."

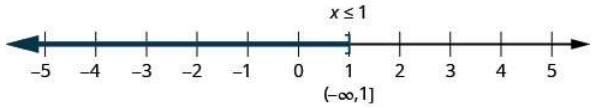
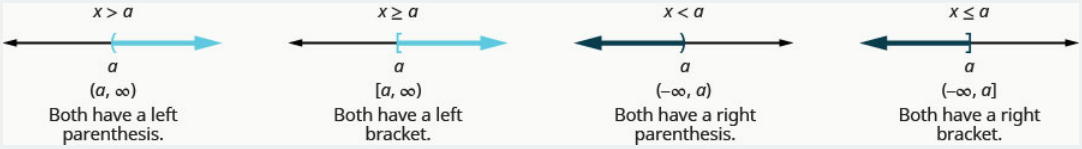


Figure 5.2.2: The inequality $x \leq 1$ is graphed on this number line and written in interval notation.

Figure $5.2.3$ shows both the number line and interval notation.

INEQUALITIES, NUMBER LINES, AND INTERVAL NOTATION



The notation for inequalities on a number line and in interval notation use the same symbols to express the endpoints of intervals.

Example 5.2.1

Graph each inequality on the number line and write in interval notation.

a. $x \geq -3$
b. $x < 2.5$
c. $x \leq -\frac{3}{5}$

Answer

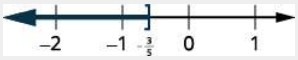
ⓐ

	$x \geq -3$
Shade to the right of -3, and put a bracket at -3.	
Write in interval notation.	$[-3, \infty)$

ⓑ

	$x < 2.5$
Shade to the left of 2.5 and put a parenthesis at 2.5.	
Write in interval notation.	$(-\infty, 2.5)$

ⓒ

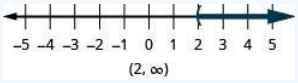
	$x \leq -\frac{3}{5}$
Shade to the left of $-\frac{3}{5}$, and put a bracket at $-\frac{3}{5}$.	
Write in interval notation.	$\left(-\infty, \frac{3}{5}\right]$

Graph each inequality on the number line and write in interval notation:

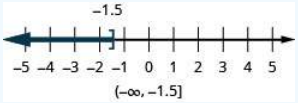
a. $x > 2$
b. $x \leq -1.5$
c. $x \geq \frac{3}{4}$.

Answer

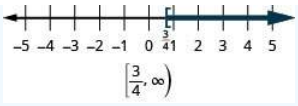
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ⓑ



ⓒ



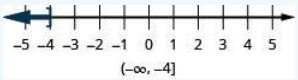
Example 5.2.3

Graph each inequality on the number line and write in interval notation:

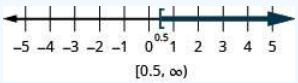
a. $x \leq -4$
b. $x \geq 0.5$
c. $x < -\frac{2}{3}$.

Answer

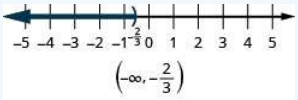
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ⓒ



What numbers are greater than two but less than five? Are you thinking say, 2.5, 3, $3\frac{2}{3}$, 4, 4, 99? We can represent all the numbers between two and five with the inequality $2 < x < 5$. We can show $2 < x < 5$ on the number line by shading all the numbers between two and five. Again, we use the parentheses to show the numbers two and five are not included. See Figure.



The interval notation would be (2, 5).

Figure 5.2.3

Graph each inequality on the number line and write in interval notation.

ⓐ $-3 < x < 4$ ⓑ $-6 \leq x < -1$ ⓒ $0 \leq x \leq 2.5$

Answer

ⓐ

		$-3 < x < 4$
Shade between -3 and 4. Put a parentheses at -3 and 4.		
Write in interval notation.		$(-3, 4)$

ⓑ

			$-6 \leq x < -1$

Shade between -6 and -1. Put a bracket at -6, and a parenthesis at -1.			
Write in interval notation.			$[-6, 1)$

ⓒ

			$0 \leq x \leq 2.5$
Shade between 0 and 2.5. Put a bracket at 0 and at 2.5.			
Write in interval notation.			$[0, 2.5]$

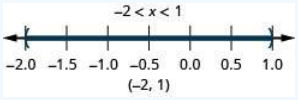
Example 5.2.5

Graph each inequality on the number line and write in interval notation:

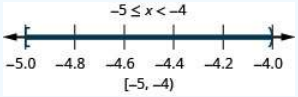
ⓐ $-2 < x < 1$ ⓑ $-5 \leq x < -4$ ⓒ $1 \leq x \leq 4.25$

Answer

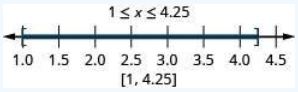
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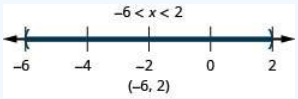
Example 5.2.6

Graph each inequality on the number line and write in interval notation:

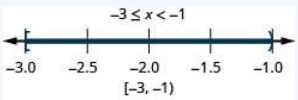
ⓐ $-6 < x < 2$ ⓑ $-3 \leq x < -1$ ⓒ $2.5 \leq x \leq 6$

Answer

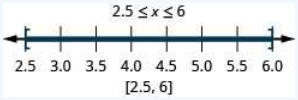
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Solve Linear Inequalities

A linear inequality is much like a linear equation—but the equal sign is replaced with an inequality sign. A **linear inequality** is an inequality in one variable that can be written in one of the forms, $ax+b<c$, $ax+b\leq c$, $ax+b>c$, or $ax+b\geq c$.

LINEAR INEQUALITY

A linear inequality is an inequality in one variable that can be written in one of the following forms where a, b, and c are real numbers and $a\neq 0$:

$$ax+b<c,\quad ax+b\leq c,\quad ax+b>c,\quad ax+b\geq c.$$

When we solved linear equations, we were able to use the properties of equality to add, subtract, multiply, or divide both sides and still keep the equality. Similar properties hold true for inequalities.

We can add or subtract the same quantity from both sides of an inequality and still keep the inequality. For example:

$$\begin{array}{ll} -4<2 & -4<2 \\ -4-5<2-5 & -4+7<2+7 \\ -9<-3 \text{ True} & 3<9 \text{ True} \end{array}$$

Notice that the inequality sign stayed the same.

This leads us to the Addition and Subtraction Properties of Inequality.

ADDITION AND SUBTRACTION PROPERTY OF INEQUALITY

For any numbers a, b, and c, if $a<b$, then

$$\begin{array}{ll} a+c<b+c & a-c<b-c \\ a+c>b+c & a-c>b-c \end{array}$$

We can add or subtract the same quantity from both sides of an inequality and still keep the inequality

What happens to an inequality when we divide or multiply both sides by a constant?

Let's first multiply and divide both sides by a positive number.

$$\begin{array}{ll} 10<15 & 10<15 \\ 10(5)<15(5) & \frac{10}{5}<\frac{15}{5} \\ 50<75 \text{ True} & 2<3 \text{ True} \end{array}$$

The inequality signs stayed the same.

Does the inequality stay the same when we divide or multiply by a negative number?

$$\begin{array}{ll} 10<15 & 10<15 \\ 10(-5)\ ?\ 15(-5) & \frac{10}{-5}\ ?\ \frac{15}{-5} \\ -50\ ?\ -75 & -2\ ?\ -3 \\ -50>-75 & -2>-3 \end{array}$$

Notice that when we filled in the inequality signs, the inequality signs reversed their direction.

When we divide or multiply an inequality by a positive number, the inequality sign stays the same. When we divide or multiply an inequality by a negative number, the inequality sign reverses.

This gives us the Multiplication and Division Property of Inequality.

MULTIPLICATION AND DIVISION PROPERTY OF INEQUALITY

For any numbers a, b, and c,

multiply or divide by a positive

if $a < b$ and $c > 0$, then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$.
if $a > b$ and $c > 0$, then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$.

multiply or divide by a negative

if $a < b$ and $c < 0$, then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$.
if $a > b$ and $c < 0$, then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$.

When we divide or multiply an inequality by a:

- positive number, the inequality stays the same.
- negative number, the inequality reverses.

Sometimes when solving an inequality, as in the next example, the variable ends upon the right. We can rewrite the inequality in reverse to get the variable to the left.

$x > a$ has the same meaning as $a < x$

Think about it as "If Xander is taller than Andy, then Andy is shorter than Xander."

Example 5.2.7

Solve each inequality. Graph the solution on the number line, and write the solution in interval notation.

ⓐ $x - \frac{3}{8} \leq \frac{3}{4}$ ⓑ $9y < 54$ ⓒ $-15 < \frac{3}{5}z$

Answer

ⓐ

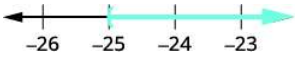
	$x - \frac{3}{8} \leq \frac{3}{4}$
Add 3838 to both sides of the inequality.	$x - \frac{3}{8} + \frac{3}{8} \leq \frac{3}{4} + \frac{3}{8}$
Simplify.	$x \leq \frac{9}{8}$
Graph the solution on the number line.	
Write the solution in interval notation.	$(-\infty, \frac{9}{8}]$

ⓑ

	$9y < 54$
Divide both sides of the inequality by 9; since 9 is positive, the inequality stays the same.	$\frac{9y}{9} < \frac{54}{9}$
Simplify.	$y < 6$

Graph the solution on the number line.	
Write the solution in interval notation.	$(-\infty, 6)$

ⓒ

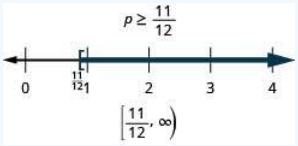
	$-15 < \frac{3}{5}z$
Multiply both sides of the inequality by $\frac{5}{3}$. Since $\frac{5}{3}$ is positive, the inequality stays the same.	$\frac{5}{3}(-15) < \frac{5}{3}\left(\frac{3}{5}z\right)$
Simplify.	$-25 < z$
Rewrite with the variable on the left.	$z > -25$
Graph the solution on the number line.	
Write the solution in interval notation.	$(-25, \infty)$

Solve each inequality, graph the solution on the number line, and write the solution in interval notation:

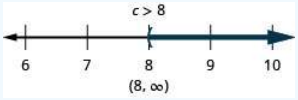
ⓐ $p - \frac{3}{4} \geq \frac{1}{6}$ ⓑ $9c > 72$ ⓒ $24 \leq \frac{3}{8}m$

Answer

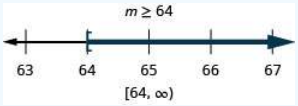
ⓐ



ⓑ



ⓒ



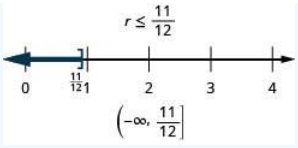
Example 5.2.9

Solve each inequality, graph the solution on the number line, and write the solution in interval notation:

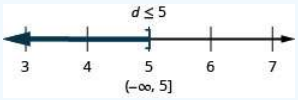
ⓐ $r - \frac{1}{3} \leq \frac{7}{12}$ ⓑ $12d \leq 60$ ⓒ $-24 < \frac{4}{3}n$

Answer

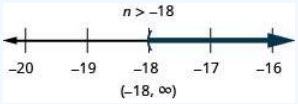
ⓐ



ⓑ



ⓒ



Be careful when you multiply or divide by a negative number—remember to reverse the inequality sign.

Example 5.2.10

Solve each inequality, graph the solution on the number line, and write the solution in interval notation.

ⓐ $-\frac{1}{3}m \ge \frac{6}{5}$ ⓑ $\frac{n}{-2} \ge 8$

Answer

ⓐ

	$-13m \ge 65$
Divide both sides of the inequality by $-\frac{1}{3}$. Since $-\frac{1}{3}$ is a negative, the inequality reverses.	$\frac{-13m}{-13} \le \frac{65}{-13}$
Simplify.	$m \le -5$
Graph the solution on the number line.	
Write the solution in interval notation.	$(-\infty, -5]$

ⓑ

	$\frac{n}{-2} \ge 8$
Multiply both sides of the inequality by -2. Since -2 is a negative, the inequality reverses.	$-2\left(\frac{n}{-2}\right) \le -2(8)$
Simplify.	$n \le -16$
Graph the solution on the number line.	
Write the solution in interval notation.	$(-\infty, -16]$

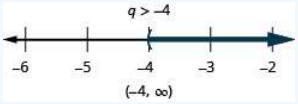
Example 5.2.11

Solve each inequality, graph the solution on the number line, and write the solution in interval notation:

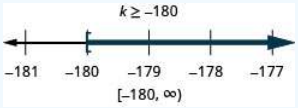
ⓐ $-8q < 32$ ⓑ $\frac{k}{-12} \le 15$.

Answer

ⓐ



ⓑ



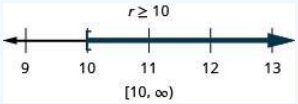
Example 5.2.12

Solve each inequality, graph the solution on the number line, and write the solution in interval notation:

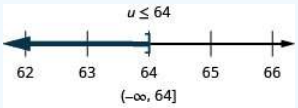
ⓐ $-7r \leq -70$ ⓑ $\frac{u}{-4} \geq -16$.

Answer

ⓐ



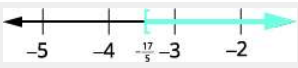
ⓑ



Most inequalities will take more than one step to solve. We follow the same steps we used in the general strategy for solving linear equations, but make sure to pay close attention when we multiply or divide to isolate the variable.

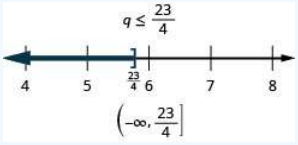
Solve the inequality $6y \leq 11y + 17$, graph the solution on the number line, and write the solution in interval notation.

Answer

	$6y \leq 11y + 17$
Subtract 11y11y from both sides to collect the variables on the left.	$6y - 11y \leq 11y - 11y + 17$
Simplify.	$-5y \leq 17$
Divide both sides of the inequality by $-5,-5$, and reverse the inequality.	$\frac{-5y}{-5} \geq \frac{17}{-5}$
Simplify.	$y \geq \frac{17}{-5}$
Graph the solution on the number line.	
Write the solution in interval notation.	$\left[-\frac{17}{5}, \infty\right)$

Solve the inequality, graph the solution on the number line, and write the solution in interval notation: $3q \geq 7q - 23$.

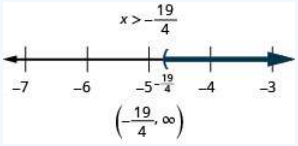
Answer



Example 5.2.15

Solve the inequality, graph the solution on the number line, and write the solution in interval notation: $6x < 10x + 19$.

Answer

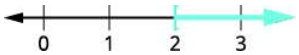


When solving inequalities, it is usually easiest to collect the variables on the side where the coefficient of the variable is largest. This eliminates negative coefficients and so we don't have to multiply or divide by a negative—which means we don't have to remember to reverse the inequality sign.

Example 5.2.16

Solve the inequality $8p + 3(p - 12) > 7p - 28$, graph the solution on the number line, and write the solution in interval notation.

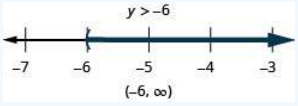
Answer

	$8p + 3(p - 12) > 7p - 28$
Simplify each side as much as possible.	
Distribute.	$8p + 3p - 36 > 7p - 28$
Combine like terms.	$11p - 36 > 7p - 28$
Subtract $7p$ from both sides to collect the variables on the left, since $11 > 7$.	$11p - 36 - 7p > 7p - 28 - 7p$
Simplify.	$4p - 36 > -28$
Add 36 to both sides to collect the constants on the right.	$4p - 36 + 36 > -28 + 36$
Simplify.	$4p > 8$
Divide both sides of the inequality by 4; the inequality stays the same.	$\frac{4p}{4} > \frac{8}{4}$
Simplify.	$p > 2$
Graph the solution on the number line.	
Write the solution in interval notation.	$(2, \infty)$

Example 5.2.17

Solve the inequality $9y+2(y+6)>5y-24$, graph the solution on the number line, and write the solution in interval notation.

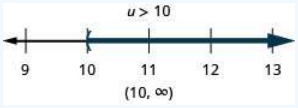
Answer



Example 5.2.18

Solve the inequality $6u+8(u-1)>10u+32$, graph the solution on the number line, and write the solution in interval notation.

Answer



Just like some equations are identities and some are contradictions, inequalities may be identities or contradictions, too. We recognize these forms when we are left with only constants as we solve the inequality. If the result is a true statement, we have an identity. If the result is a false statement, we have a contradiction.

Example 5.2.19

Solve the inequality $8x-2(5-x)<4(x+9)+6x$, graph the solution on the number line, and write the solution in interval notation.

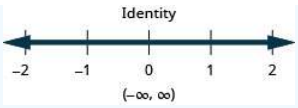
Answer

Simplify each side as much as possible.		$8x-2(5-x)<4(x+9)+6x$
Distribute.		$8x-10+2x<4x+36+6x$
Combine like terms.		$10x-10<10x+36$
Subtract $10x$ from both sides to collect the variables on the left.		$10x-10-10x<10x+36-10x$
Simplify.		$-10<36$
The x's are gone, and we have a true statement.		The inequality is an identity. The solution is all real numbers.
Graph the solution on the number line.		
Write the solution in interval notation.		$(-\infty,\infty)$

Example 5.2.20

Solve the inequality $4b-3(3-b)>5(b-6)+2b$, graph the solution on the number line, and write the solution in interval notation.

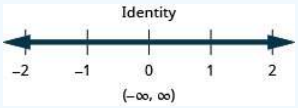
Answer



Example 5.2.21

Solve the inequality $9h-7(2-h)<8(h+11)+8h$, graph the solution on the number line, and write the solution in interval notation.

Answer



We can clear fractions in inequalities much as we did in equations. Again, be careful with the signs when multiplying or dividing by a negative.

Example 5.2.22

Solve the inequality $\frac{1}{3}a-\frac{1}{8}a>\frac{5}{24}a+\frac{3}{4}$, graph the solution on the number line, and write the solution in interval notation.

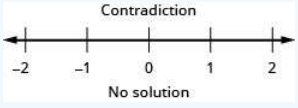
Answer

	$\frac{1}{3}a-\frac{1}{8}a>\frac{5}{24}a+\frac{3}{4}$
Multiply both sides by the LCD, 24, to clear the fractions.	$24\left(\frac{1}{3}a-\frac{1}{8}a\right)>24\left(\frac{5}{24}a+\frac{3}{4}\right)$
Simplify.	$8a-3a>5a+18$
Combine like terms.	$5a>5a+18$
Subtract $5a$ from both sides to collect the variables on the left.	$5a-5a>5a-5a+18$
Simplify.	$0>18$
The statement is false.	The inequality is a contradiction. There is no solution.
Graph the solution on the number line.	
Write the solution in interval notation.	There is no solution.

Example 5.2.23

Solve the inequality $\frac{1}{4}x-\frac{1}{12}x>\frac{1}{6}x+\frac{7}{8}$, graph the solution on the number line, and write the solution in interval notation.

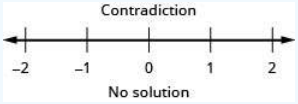
Answer



Example 5.2.24

Solve the inequality $\frac{2}{5}z - \frac{1}{3}z < \frac{1}{15}z - \frac{3}{5}$, graph the solution on the number line, and write the solution in interval notation.

Answer



Translate to an Inequality and Solve

To translate English sentences into inequalities, we need to recognize the phrases that indicate the inequality. Some words are easy, like “more than” and “less than.” But others are not as obvious. Table shows some common phrases that indicate inequalities.

$>$	$\geq$	$<$	$\leq$
is greater than	is greater than or equal to	is less than	is less than or equal to
is more than	is at least	is smaller than	is at most
is larger than	is no less than	has fewer than	is no more than
exceeds	is the minimum	is lower than	is the maximum

Example 5.2.25

Translate and solve. Then graph the solution on the number line, and write the solution in interval notation.

Twenty-seven less than x is at least 48.

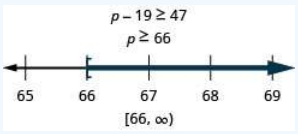
Answer

	Twenty-seven less than x is at least 48.
Translate.	$x - 27 \geq 48$
Solve—add 27 to both sides.	$x - 27 + 27 \geq 48 + 27$
Simplify.	$x \geq 75$
Graph on the number line.	
Write in interval notation.	$[75, \infty)$

Translate and solve. Then graph the solution on the number line, and write the solution in interval notation.

Nineteen less than p is no less than 47.

Answer

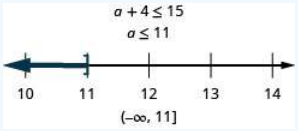


Example 5.2.27

Translate and solve. Then graph the solution on the number line, and write the solution in interval notation.

Four more than a is at most 15.

Answer



Solve Applications with Linear Inequalities

Many real-life situations require us to solve inequalities. The method we will use to solve applications with linear inequalities is very much like the one we used when we solved applications with equations.

We will read the problem and make sure all the words are understood. Next, we will identify what we are looking for and assign a variable to represent it. We will restate the problem in one sentence to make it easy to translate into an inequality. Then, we will solve the inequality.

Sometimes an application requires the solution to be a whole number, but the algebraic solution to the inequality is not a whole number. In that case, we must round the algebraic solution to a whole number. The context of the application will determine whether we round up or down.

Example 5.2.28

Dawn won a mini-grant of $4,000 to buy tablet computers for her classroom. The tablets she would like to buy cost $254.12 each, including tax and delivery. What is the maximum number of tablets Dawn can buy?

Solution

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	the maximum number of tablets Dawn can buy
Step 3. Name what you are looking for. Choose a variable to represent that quantity.	Let n = the number of tablets.
Step 4. Translate Write a sentence that gives the information to find it.	$254.12 times the number of tablets is no more than $4,000.
Translate into an inequality.	$254.12n \leq 4000$
Step 5. Solve the inequality.	$n \leq 15.74$ but n must be a whole number of tablets, so round to 15. $n \leq 15$ tablets
Step 6. Check the answer in the problem and make sure it makes sense.	Rounding down the price to $250, 15 tablets would cost $3,750, while 16 tablets would be $4,000. So a maximum of 15 tablets at $254.12 seems reasonable.
Step 7. Write a sentence that answers the question.	Dawn can buy a maximum of 15 tablets.

Try It 5.2.29

Angie has $20 to spend on juice boxes for her son's preschool picnic. Each pack of juice boxes costs $2.63. What is the maximum number of packs she can buy?

Answer

Angie can buy 7 packs of juice.

Try It 5.2.30

Daniel wants to surprise his girlfriend with a birthday party at her favorite restaurant. It will cost $42.75 per person for dinner, including tip and tax. His budget for the party is $500. What is the maximum number of people Daniel can have at the party?

Answer

Daniel can have 11 people at the party.

Example 5.2.31

Taleisha's phone plan costs her $28.80 a month plus $0.20 per text message. How many text messages can she send/receive and keep her monthly phone bill no more than $50?

Solution

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	the number of text messages Taleisha can make
Step 3. Name what you are looking for. Choose a variable to represent that quantity.	Let $t =$ the number of text messages.
Step 4. Translate Write a sentence that gives the information to find it.	28.80 plus 0.20 times the number of text messages is less than or equal to 50.
Translate into an inequality.	$28.80 + 0.20t \leq 50$
Step 5. Solve the inequality.	$0.2t \leq 21.2$ $t \leq 106$ text messages
Step 6. Check the answer in the problem and make sure it makes sense.	Yes, $28.80 + 0.20(106) = 50$.
Step 7. Write a sentence that answers the question.	Taleisha can send/receive no more than 106 text messages to keep her bill no more than 50.

Try It 5.2.32

Sergio and Lizeth have a very tight vacation budget. They plan to rent a car from a company that charges $75 a week plus $0.25 a mile. How many miles can they travel during the week and still keep within their $200 budget?

Answer

Sergio and Lizeth can travel no more than 500 miles.

Try It 5.2.33

Rameen's heating bill is $5.42 per month plus $1.08 per therm. How many therms can Rameen use if he wants his heating bill to be a maximum of $87.50.

Answer

Rameen's heating bill is $5.42 per month plus $1.08 per therm. How many therms can Rameen use if he wants his heating bill to be a maximum of $87.50.

Profit is the money that remains when the costs have been subtracted from the revenue. In the next example, we will find the number of jobs a small businesswoman needs to do every month in order to make a certain amount of profit.

Example 5.2.34

Felicity has a calligraphy business. She charges $2.50 per wedding invitation. Her monthly expenses are $650. How many invitations must she write to earn a profit of at least $2,800 per month?

Solution

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	the number of invitations Felicity needs to write
Step 3. Name what you are looking for. Choose a variable to represent that quantity.	Let $j =$ the number of invitations.
Step 4. Translate Write a sentence that gives the information to find it.	$2.50 times the number of invitations minus $650 is at least $2,800.
Translate into an inequality.	$2.50j - 650 \geq 2,800$
Step 5. Solve the inequality.	$2.5j \geq 3,450$ $j \geq 1,380$ invitations
Step 6. Check the answer in the problem and make sure it makes sense.	If Felicity wrote 1400 invitations, her profit would be $2.50(1400) - 650$, or $2,850. This is more than $2800.
Step 7. Write a sentence that answers the question.	Felicity must write at least 1,380 invitations.

Try It 5.2.35

Caleb has a pet sitting business. He charges $32 per hour. His monthly expenses are $2,272. How many hours must he work in order to earn a profit of at least $800 per month?

Answer

Caleb must work at least 96 hours.

Try It 5.2.36

Elliot has a landscape maintenance business. His monthly expenses are $1,100. If he charges $60 per job, how many jobs must he do to earn a profit of at least $4,000 a month?

Answer

Elliot must work at least 85 jobs.

There are many situations in which several quantities contribute to the total expense. We must make sure to account for all the individual expenses when we solve problems like this.

Example 5.2.37

Malik is planning a six-day summer vacation trip. He has $840 in savings, and he earns $45 per hour for tutoring. The trip will cost him $525 for airfare, $780 for food and sightseeing, and $95 per night for the hotel. How many hours must he tutor to have enough money to pay for the trip?

Solution

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	the number of hours Malik must tutor
Step 3. Name what you are looking for. Choose a variable to represent that quantity.	Let h = the number of hours.
Step 4. Translate Write a sentence that gives the information to find it.	The expenses must be less than or equal to the income. The cost of airfare plus the cost of food and sightseeing and the hotel bill must be less than the savings plus the amount earned tutoring.
Translate into an inequality.	$525 + 780 + 95(6) \leq 840 + 45h$
Step 5. Solve the inequality.	$1,875 \leq 840 + 45h$ $1,035 \leq 45h$ $h \geq 23$ hours
Step 6. Check the answer in the problem and make sure it makes sense.	We substitute 23 into the inequality. $1,875 \leq 840 + 45h$ $1,875 \leq 840 + 45(23)$ $1,875 \leq 1875$
Step 7. Write a sentence that answers the question.	Malik must tutor at least 23 hours.

Try It 5.2.38

Brenda's best friend is having a destination wedding and the event will last three days. Brenda has $500 in savings and can earn $15 an hour babysitting. She expects to pay $350 airfare, $375 for food and entertainment and $60 a night for her share of a hotel room. How many hours must she babysit to have enough money to pay for the trip?

Answer

Brenda must babysit at least 27 hours.

Try It 5.2.39

Josue wants to go on a 10-night road trip with friends next spring. It will cost him $180 for gas, $450 for food, and $49 per night to share a motel room. He has $520 in savings and can earn $30 per driveway shoveling snow. How many driveways must he shovel to have enough money to pay for the trip?

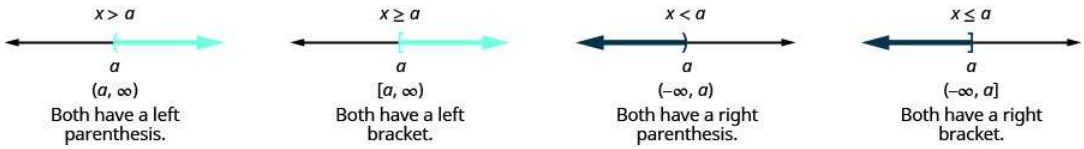
Answer

Josue must shovel at least 20 driveways.

Key Concepts

- **Inequalities, Number Lines, and Interval Notation**

  $x > a \quad x \geq a \quad x < a \quad x \leq a$



- **Linear Inequality**
  - A **linear inequality** is an inequality in one variable that can be written in one of the following forms where a, b, and c are real numbers and $a \neq 0$:

$$ax + b < c, \qquad ax + b \leq c, \qquad ax + b > c, \qquad ax + b \geq c.$$

- **Addition and Subtraction Property of Inequality**
  - For any numbers *a*, *b*, and *c*, if a<b,thena<b,then

$$a+c<b+c \quad a-c<b-c$$
$$a+c>b+c \quad a-c>b-c$$

  - We can add or subtract the same quantity from both sides of an inequality and still keep the inequality.
- **Multiplication and Division Property of Inequality**
  - For any numbers *a*, *b*, and *c*,
    multiply or divide by a **positive**

$$\text{if } a<b \text{ and } c>0, \text{ then } ac<bc \text{ and } \tfrac{a}{c}<\tfrac{b}{c}.$$
$$\text{if } a>b \text{ and } c>0, \text{ then } ac>bc \text{ and } \tfrac{a}{c}>\tfrac{b}{c}.$$

    multiply or divide by a **negative**

$$\text{if } a<b \text{ and } c<0, \text{ then } ac>bc \text{ and } \tfrac{a}{c}>\tfrac{b}{c}.$$
$$\text{if } a>b \text{ and } c<0, \text{ then } ac<bc \text{ and } \tfrac{a}{c}<\tfrac{b}{c}.$$

- **Phrases that indicate inequalities**

$>$	$\geq$	$<$	$\leq$
is greater than	is greater than or equal to	is less than	is less than or equal to
is more than	is at least	is smaller than	is at most
is larger than	is no less than	has fewer than	is no more than
exceeds	is the minimum	is lower than	is the maximum

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5.2E: Exercises

Practice Makes Perfect

Graph Inequalities on the Number Line

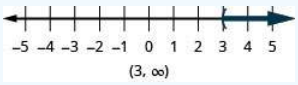
In the following exercises, graph each inequality on the number line and write in interval notation.

1. ⓐ $x < -2$
ⓑ $x \geq -3.5$
ⓒ $x \leq \frac{2}{3}$

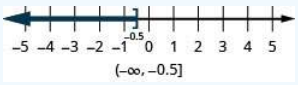
2. ⓐ $x > 3$
ⓑ $x \leq -0.5$
ⓒ $x \geq \frac{1}{3}$

Answer

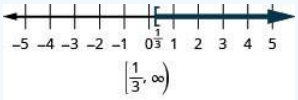
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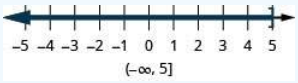


3. ⓐ $x \geq -4$
ⓑ $x < 2.5$
ⓒ $x > -\frac{3}{2}$

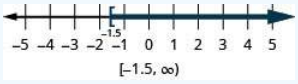
4. ⓐ $x \leq 5$
ⓑ $x \geq -1.5$
ⓒ $x < -\frac{7}{3}$

Answer

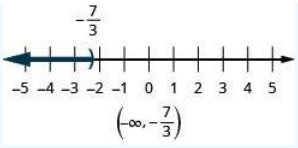
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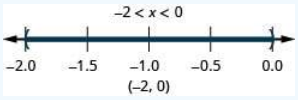


5. ⓐ $-5 < x < 2$
ⓑ $-3 \leq x < -1$
ⓒ $0 \leq x \leq 1.5$

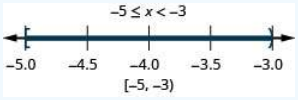
6. ⓐ $-2 < x < 0$
ⓑ $-5 \leq x < -3$
ⓒ $0 \leq x \leq 3.5$

Answer

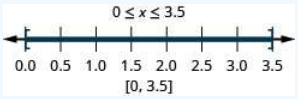
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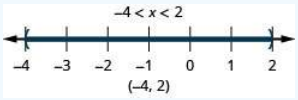


7. ⓐ $-1 < x < 3$
ⓑ $-3 < x \leq -2$
ⓒ $-1.25 \leq x \leq 0$

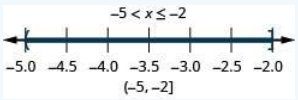
8. ⓐ $-4 < x < 2$
ⓑ $-5 < x \leq -2$
ⓒ $-3.75 \leq x \leq 0$

Answer

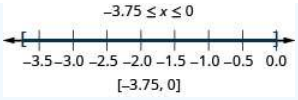
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Solve Linear Inequalities

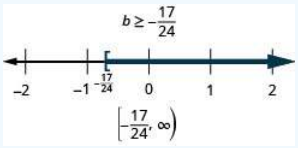
In the following exercises, solve each inequality, graph the solution on the number line, and write the solution in interval notation.

9. ⓐ $a+\frac{3}{4}\geq\frac{7}{10}$
ⓑ $8x>72$
ⓒ $20>\frac{2}{5}h$

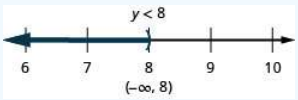
10. ⓐ $b+\frac{7}{8}\geq\frac{1}{6}$
ⓑ $6y<48$
ⓒ $40<\frac{5}{8}k$

Answer

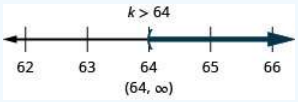
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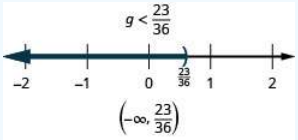


11. ⓐ $f-\frac{13}{20}<-\frac{5}{12}$
ⓑ $9t\geq-27$
ⓒ $\frac{7}{6}j\geq42$

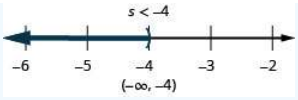
12. ⓐ $g-\frac{11}{12}<-\frac{5}{18}$
ⓑ $7s<-28$
ⓒ $\frac{9}{4}g\leq36$

Answer

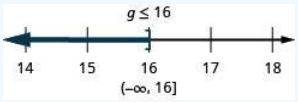
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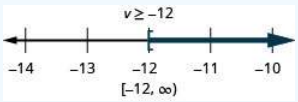


13. ⓐ $-5u \geq 65$
ⓑ $\frac{a}{-3} \leq 9$

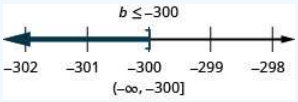
14. ⓐ $-8v \leq 96$
ⓑ $\frac{b}{-10} \geq 30$

Answer

ⓐ



ⓑ

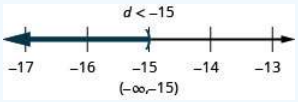


15. ⓐ $-9c < 126$
ⓑ $-25 < \frac{p}{-5}$

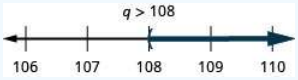
16. ⓐ $-7d > 105$
ⓑ $-18 > \frac{q}{-6}$

Answer

ⓐ



ⓑ



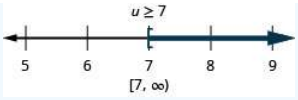
In the following exercises, solve each inequality, graph the solution on the number line, and write the solution in interval notation.

17. $4v \geq 9v - 40$

18. $5u \leq 8u - 21$

Answer

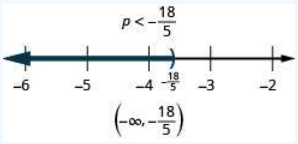
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19. $13q < 7q - 29$

20. $9p > 14p - 18$

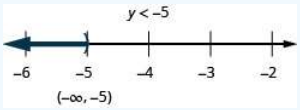
Answer



21. $12x + 3(x + 7) > 10x - 24$

22. $9y + 5(y + 3) < 4y - 35$

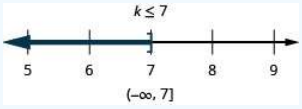
Answer



23. $6h - 4(h - 1) \leq 7h - 11$

24. $4k - (k - 2) \geq 7k - 26$

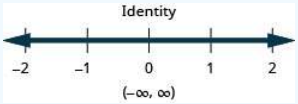
Answer



25. $8m - 2(14 - m) \geq 7(m - 4) + 3m$

26. $6n - 12(3 - n) \leq 9(n - 4) + 9n$

Answer

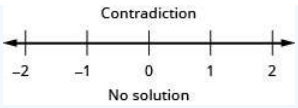


27. $\frac{3}{4}b - \frac{1}{3}b < \frac{5}{12}b - \frac{1}{2}$

28. $9u + 5(2u - 5) \geq 12(u - 1) + 7u$

Answer

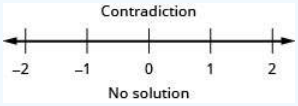
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29. $\frac{2}{3}g-\frac{1}{2}(g-14)\leq\frac{1}{6}(g+42)$

30. $\frac{4}{5}h-\frac{2}{3}(h-9)\geq\frac{1}{15}(2h+90)$

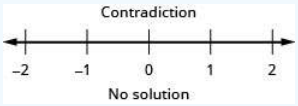
Answer



31. $\frac{5}{6}a-\frac{1}{4}a>\frac{7}{12}a+\frac{2}{3}$

32. $12v+3(4v-1)\leq19(v-2)+5v$

Answer

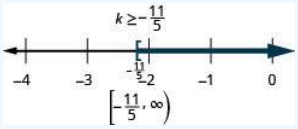


In the following exercises, solve each inequality, graph the solution on the number line, and write the solution in interval notation.

33. $15k\leq-40$

34. $35k\geq-77$

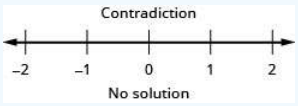
Answer



35. $23p-2(6-5p)>3(11p-4)$

36. $18q-4(10-3q)<5(6q-8)$

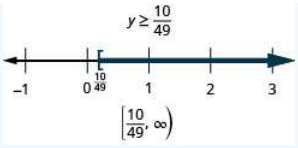
Answer



37. $-\frac{9}{4}x\geq-\frac{5}{12}$

38. $-\frac{21}{8}y\leq-\frac{15}{28}$

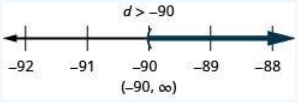
Answer



39. $c + 34 < -99$

40. $d + 29 > -61$

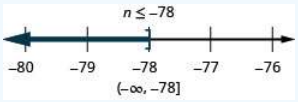
Answer



41. $\frac{m}{18} \geq -4$

42. $\frac{n}{13} \leq -6$

Answer



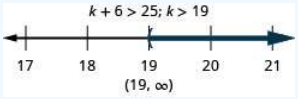
Translate to an Inequality and Solve

In the following exercises, translate and solve. Then graph the solution on the number line and write the solution in interval notation.

43. Three more than h is no less than 25.

44. Six more than k exceeds 25.

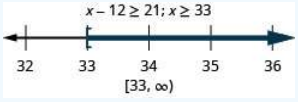
Answer



45. Ten less than w is at least 39.

46. Twelve less than x is no less than 21.

Answer

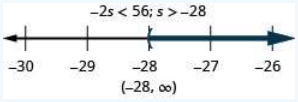


47. Negative five times r is no more than 95.

48. Negative two times s is lower than 56.

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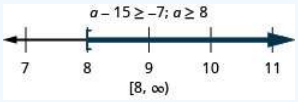
Answer



49. Nineteen less than b is at most -22.

50. Fifteen less than a is at least -7.

Answer



Solve Applications with Linear Inequalities

In the following exercises, solve.

51. Alan is loading a pallet with boxes that each weighs 45 pounds. The pallet can safely support no more than 900 pounds. How many boxes can he safely load onto the pallet?

52. The elevator in Yehire's apartment building has a sign that says the maximum weight is 2100 pounds. If the average weight of one person is 150 pounds, how many people can safely ride the elevator?

Answer

A maximum of 14 people can safely ride in the elevator.

53. Andre is looking at apartments with three of his friends. They want the monthly rent to be no more than $2,360. If the roommates split the rent evenly among the four of them, what is the maximum rent each will pay?

54. Arleen got a $20 gift card for the coffee shop. Her favorite iced drink costs $3.79. What is the maximum number of drinks she can buy with the gift card?

Answer

five drinks

55. Teegan likes to play golf. He has budgeted $60 next month for the driving range. It costs him $10.55 for a bucket of balls each time he goes. What is the maximum number of times he can go to the driving range next month?

56. Ryan charges his neighbors $17.50 to wash their car. How many cars must he wash next summer if his goal is to earn at least $1,500?

Answer

86 cars

57. Keshad gets paid $2,400 per month plus 6% of his sales. His brother earns $3,300 per month. For what amount of total sales will Keshad's monthly pay be higher than his brother's monthly pay?

58. Kimuyen needs to earn $4,150 per month in order to pay all her expenses. Her job pays her $3,475 per month plus 4% of her total sales. What is the minimum Kimuyen's total sales must be in order for her to pay all her expenses?

Answer

$16,875

59. Andre has been offered an entry-level job. The company offered him $48,000 per year plus 3.5% of his total sales. Andre knows that the average pay for this job is $62,000. What would Andre's total sales need to be for his pay to be at least as high as the average pay for this job?

60. Nataly is considering two job offers. The first job would pay her $83,000 per year. The second would pay her $66,500 plus 15% of her total sales. What would her total sales need to be for her salary on the second offer be higher than the first?

Answer

$110,000

61. Jake's water bill is $24.80 per month plus $2.20 per ccf (hundred cubic feet) of water. What is the maximum number of ccf Jake can use if he wants his bill to be no more than $60?

62. Kiyoshi's phone plan costs $17.50 per month plus $0.15 per text message. What is the maximum number of text messages Kiyoshi can use so the phone bill is no more than $56.60?

Answer

260 messages

63. Marlon's TV plan costs $49.99 per month plus $5.49 per first-run movie. How many first-run movies can he watch if he wants to keep his monthly bill to be a maximum of $100?

64. Kellen wants to rent a banquet room in a restaurant for her cousin's baby shower. The restaurant charges $350 for the banquet room plus $32.50 per person for lunch. How many people can Kellen have at the shower if she wants the maximum cost to be $1,500?

Answer

35 people

65. Moshde runs a hairstyling business from her house. She charges $45 for a haircut and style. Her monthly expenses are $960. She wants to be able to put at least $1,200 per month into her savings account order to open her own salon. How many "cut & styles" must she do to save at least $1,200 per month?

66. Noe installs and configures software on home computers. He charges $125 per job. His monthly expenses are $1,600. How many jobs must he work in order to make a profit of at least $2,400?

Answer

32 jobs

67. Katherine is a personal chef. She charges $115 per four-person meal. Her monthly expenses are $3,150. How many four-person meals must she sell in order to make a profit of at least $1,900?

68. Melissa makes necklaces and sells them online. She charges $88 per necklace. Her monthly expenses are $3,745. How many necklaces must she sell if she wants to make a profit of at least $1,650?

Answer

62 necklaces

69. Five student government officers want to go to the state convention. It will cost them $110 for registration, $375 for transportation and food, and $42 per person for the hotel. There is $450 budgeted for the convention in the student government savings account. They can earn the rest of the money they need by having a car wash. If they charge $5 per car, how many cars must they wash in order to have enough money to pay for the trip?

70. Cesar is planning a four-day trip to visit his friend at a college in another state. It will cost him $198 for airfare, $56 for local transportation, and $45 per day for food. He has $189 in savings and can earn $35 for each lawn he mows. How many lawns must he mow to have enough money to pay for the trip?

Answer

seven lawns

71. Alonzo works as a car detailer. He charges $175 per car. He is planning to move out of his parents' house and rent his first apartment. He will need to pay $120 for application fees, $950 for security deposit, and first and last months' rent at $1,140 per month. He has $1,810 in savings. How many cars must he detail to have enough money to rent the apartment?

72. Eun-Kyung works as a tutor and earns $60 per hour. She has $792 in savings. She is planning an anniversary party for her parents. She would like to invite 40 guests. The party will cost her $1,520 for food and drinks and $150 for the photographer. She will also have a favor for each of the guests, and each favor will cost $7.50. How many hours must she tutor to have enough money for the party?

Answer

20 hours

Everyday Math

73. Maximum load on a stage In 2014, a high school stage collapsed in Fullerton, California, when 250 students got on stage for the finale of a musical production. Two dozen students were injured. The stage could support a maximum of 12,750 pounds. If the average weight of a student is assumed to be 140 pounds, what is the maximum number of students who could safely be on the stage?

74. Maximum weight on a boat In 2004, a water taxi sank in Baltimore harbor and five people drowned. The water taxi had a maximum capacity of 3,500 pounds (25 people with average weight 140 pounds). The average weight of the 25 people on the water taxi when it sank was 168 pounds per person. What should the maximum number of people of this weight have been?

Answer

20 people

75. Wedding budget Adele and Walter found the perfect venue for their wedding reception. The cost is $9850 for up to 100 guests, plus $38 for each additional guest. How many guests can attend if Adele and Walter want the total cost to be no more than $12,500?

76. Shower budget Penny is planning a baby shower for her daughter-in-law. The restaurant charges $950 for up to 25 guests, plus $31.95 for each additional guest. How many guests can attend if Penny wants the total cost to be no more than $1,500?

Answer

42 guests

Writing Exercises

77. Explain why it is necessary to reverse the inequality when solving $-5x > 10$.

78. Explain why it is necessary to reverse the inequality when solving $n - 3 < 12$.

Answer

Answers will vary.

79. Find your last month's phone bill and the hourly salary you are paid at your job. Calculate the number of hours of work it would take you to earn at least enough money to pay your phone bill by writing an appropriate inequality and then solving it. Do you feel this is an appropriate number of hours? Is this the appropriate phone plan for you?

80. Find out how many units you have left, after this term, to achieve your college goal and estimate the number of units you can take each term in college. Calculate the number of terms it will take you to achieve your college goal by writing an appropriate inequality and then solving it. Is this an acceptable number of terms until you meet your goal? What are some ways you could accelerate this process?

Answer

Answers will vary.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
graph inequalities on the number line.			
solve linear inequalities.			
translate words to an inequality and solve.			
solve applications with linear inequalities.			

ⓑ After looking at the checklist, do you think you are well-prepared for the next section? Why or why not?

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5.3: Solve Compound Inequalities - Union, Intersection

Learning Objectives

By the end of this section, you will be able to:

- Solve compound inequalities with “and”
- Solve compound inequalities with “or”
- Solve applications with compound inequalities

Before you get started, take this readiness quiz.

1. Simplify: $\frac{2}{5}(x+10)$.
   If you missed this problem, review [link].
2. Simplify: $-(x-4)$.
   If you missed this problem, review [link].

Solve Compound Inequalities with “and”

Now that we know how to solve linear inequalities, the next step is to look at compound inequalities. A **compound inequality** is made up of two inequalities connected by the word “and” or the word “or.” For example, the following are compound inequalities.

$$\begin{array}{lll} x+3>-4 & \text{and} & 4x-5\leq 3 \\ 2(y+1)<0 & \text{or} & y-5\geq -2 \end{array}$$

COMPOUND INEQUALITY

A **compound inequality** is made up of two inequalities connected by the word “and” or the word “or.”

To solve a compound inequality means to find all values of the variable that make the compound inequality a true statement. We solve compound inequalities using the same techniques we used to solve linear inequalities. We solve each inequality separately and then consider the two solutions.

To solve a compound inequality with the word “and,” we look for all numbers that make *both* inequalities true. To solve a compound inequality with the word “or,” we look for all numbers that make *either* inequality true.

Let’s start with the compound inequalities with “and.” Our solution will be the numbers that are solutions to *both* inequalities known as the intersection of the two inequalities. Consider the intersection of two streets—the part where the streets overlap—belongs to both streets.



To find the solution of an "and" compound inequality, we look at the graphs of each inequality and then find the numbers that belong to both graphs—where the graphs overlap.

For the compound inequality $x>-3$ and $x\leq 2$, we graph each inequality. We then look for where the graphs “overlap”. The numbers that are shaded on both graphs, will be shaded on the graph of the solution of the compound inequality. See Figure 5.3.1.

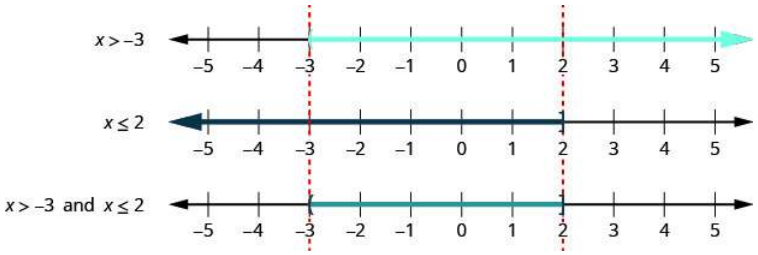


Figure 5.3.1

We can see that the numbers between -3 and 2 are shaded on both of the first two graphs. They will then be shaded on the solution graph.

The number -3 is not shaded on the first graph and so since it is not shaded on both graphs, it is not included on the solution graph.

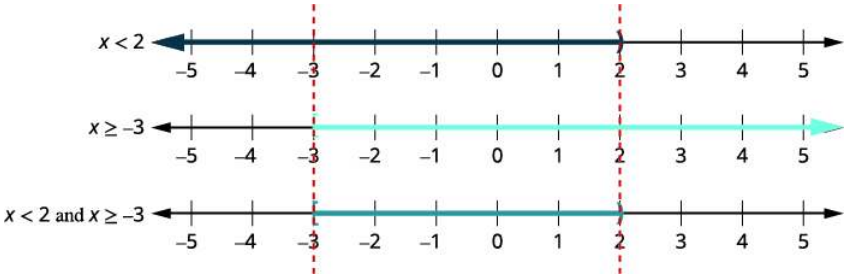
The number two is shaded on both the first and second graphs. Therefore, it is be shaded on the solution graph.

This is how we will show our solution in the next examples.

Example 5.3.1

Solve $6x-3<9$ and $2x+7\geq 3$. Graph the solution and write the solution in interval notation.

Solution

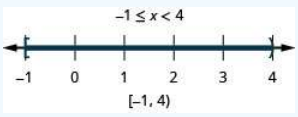
	$6x-3<9$	and	$2x+9\geq 3$
Step 1. Solve each inequality.	$6x-3<9$		$2x+9\geq 3$
	$6x<12$		$2x\geq -6$
	$x<2$	and	$x\geq -3$
Step 2. Graph each solution. Then graph the numbers that make both inequalities true. The final graph will show all the numbers that make both inequalities true—the numbers shaded on *both* of the first two graphs.			
Step 3. Write the solution in interval notation.	$[-3, 2)$		

All the numbers that make both inequalities true are the solution to the compound inequality.

Try It 5.3.2

Solve the compound inequality. Graph the solution and write the solution in interval notation: $4x-7<9$ and $5x+8\geq 3$.

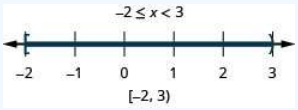
Answer



Try It 5.3.3

Solve the compound inequality. Graph the solution and write the solution in interval notation: $3x-4<5$ and $4x+9\geq 1$.

Answer



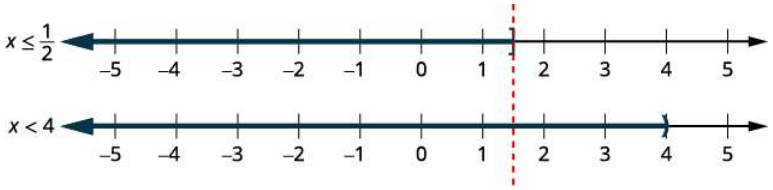
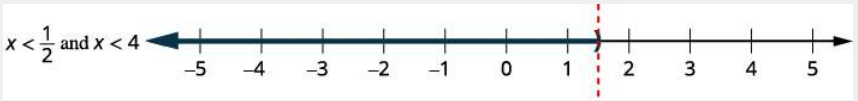
SOLVE A COMPOUND INEQUALITY WITH "AND."

1. Solve each inequality.
2. Graph each solution. Then graph the numbers that make *both* inequalities true.
   This graph shows the solution to the compound inequality.
3. Write the solution in interval notation.

Example 5.3.4

Solve $3(2x+5)\leq 18$ and $2(x-7)<-6$. Graph the solution and write the solution in interval notation.

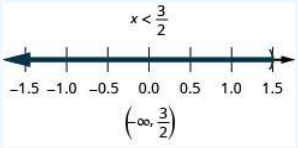
Solution

	$3(2x+5)\leq 18$	and	$2(x-7)<-6$
Solve each inequality.	$6x+15\leq 18$		$2x-14<-6$
	$6x\leq 3$		$2x<8$
	$x\leq \frac{1}{2}$	and	$x<4$
Graph each solution.			
Graph the numbers that make both inequalities true.			
Write the solution in interval notation.	$(-\infty, \frac{1}{2}]$		

Try It 5.3.5

Solve the compound inequality. Graph the solution and write the solution in interval notation: $2(3x+1)\leq 20$ and $4(x-1)<2$.

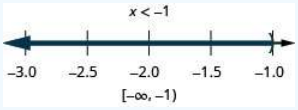
Answer



Try It 5.3.6

Solve the compound inequality. Graph the solution and write the solution in interval notation: $5(3x-1) \leq 10$ and $4(x+3) < 8$.

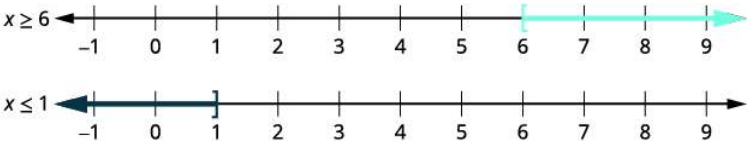
Answer



Example 5.3.7

Solve $\frac{1}{3}x - 4 \geq -2$ and $-2(x-3) \geq 4$. Graph the solution and write the solution in interval notation.

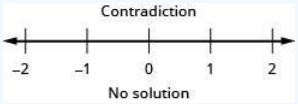
Solution

	$\frac{1}{3}x - 4 \geq -2$	and	$-2(x-3) \geq 4$
Solve each inequality.	$\frac{1}{3}x - 4 \geq -2$		$-2x + 6 \geq 4$
	$\frac{1}{3}x \geq 2$		$-2x \geq -2$
	$x \geq 6$	and	$x \leq 1$
Graph each solution.			
Graph the numbers that make both inequalities true.			
	This is a contradiction. There are no numbers that make both inequalities true, so there is no solution.		

Try It 5.3.8

Solve the compound inequality. Graph the solution and write the solution in interval notation: $\frac{1}{4}x - 3 \geq -1$ and $-3(x-2) \geq 2$.

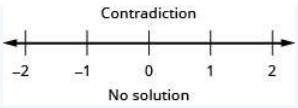
Answer



Try It 5.3.9

Solve the compound inequality. Graph the solution and write the solution in interval notation: $\frac{1}{5}x - 5 \geq -3$ and $-4(x-1) \geq -2$.

Answer



Sometimes we have a compound inequality that can be written more concisely. For example, $a < x$ and $x < b$ can be written simply as $a < x < b$ and then we call it a **double inequality**. The two forms are equivalent.

DOUBLE INEQUALITY

A double inequality is a compound inequality such as $a < x < b$. It is equivalent to $a < x$ and $x < b$.

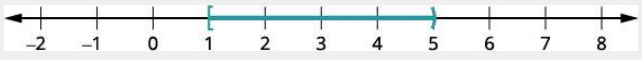
$$\text{Other forms:} \quad \begin{array}{lllll} a < x < b & \text{is equivalent to} & a < x & \text{and} & x < b \\ a \leq x \leq b & \text{is equivalent to} & a \leq x & \text{and} & x \leq b \\ a > x > b & \text{is equivalent to} & a > x & \text{and} & x > b \\ a \geq x \geq b & \text{is equivalent to} & a \geq x & \text{and} & x \geq b \end{array}$$

To solve a double inequality we perform the same operation on all three "parts" of the double inequality with the goal of isolating the variable in the center.

Example 5.3.10

Solve $-4 \leq 3x - 7 < 8$. Graph the solution and write the solution in interval notation.

Solution

	$-4 \leq 3x - 7 < 8$
Add 7 to all three parts.	$-4+7 \leq 3x - 7+7 < 8+7$
Simplify.	$3 \leq 3x < 15$
Divide each part by three.	$\frac{3}{3} \leq \frac{3x}{3} < \frac{15}{3}$
Simplify.	$1 \leq x < 5$
Graph the solution.	
Write the solution in interval notation.	$[1, 5)$

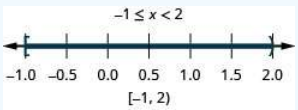
When written as a double inequality, $1 \leq x < 5$, it is easy to see that the solutions are the numbers caught between one and five, including one, but not five. We can then graph the solution immediately as we did above.

Another way to graph the solution of $1 \leq x < 5$ is to graph both the solution of $x \geq 1$ and the solution of $x < 5$. We would then find the numbers that make both inequalities true as we did in previous examples.

Try It 5.3.11

Solve the compound inequality. Graph the solution and write the solution in interval notation: $-5 \leq 4x - 1 < 7$.

Answer

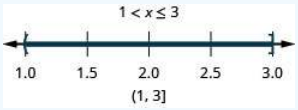


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Try It 5.3.12

Solve the compound inequality. Graph the solution and write the solution in interval notation: $-3 < 2x - 5 \leq 1$.

Answer



Solve Compound Inequalities with "or"

To solve a **compound inequality** with "or", we start out just as we did with the compound inequalities with "and"—we solve the two inequalities. Then we find all the numbers that make *either* inequality true.

Just as the United States is the union of all of the 50 states, the solution will be the union of all the numbers that make either inequality true. To find the solution of the compound inequality, we look at the graphs of each inequality, find the numbers that belong to either graph and put all those numbers together.

To write the solution in **interval notation**, we will often use the **union symbol**, $\cup$, to show the union of the solutions shown in the graphs.

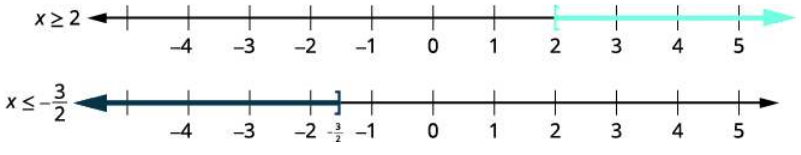
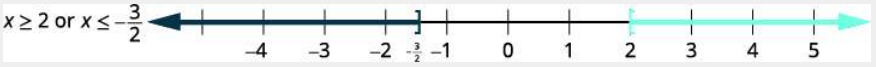
SOLVE A COMPOUND INEQUALITY WITH "OR."

1. Solve each inequality.
2. Graph each solution. Then graph the numbers that make either inequality true.
3. Write the solution in interval notation.

Example 5.3.13

Solve $5 - 3x \leq -1$ or $8 + 2x \leq 5$. Graph the solution and write the solution in interval notation.

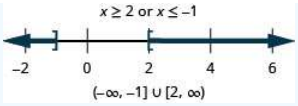
Solution

	$5 - 3x \leq -1$	or	$8 + 2x \leq 5$
Solve each inequality.	$5 - 3x \leq -1$		$8 + 2x \leq 5$
	$-3x \leq -6$		$2x \leq -3$
	$x \geq 2$	or	$x \leq -\frac{3}{2}$
Graph each solution.			
Graph numbers that make either inequality true.			
	$(-\infty, -\frac{3}{2}] \cup [2, \infty)$		

Try It 5.3.14

Solve the compound inequality. Graph the solution and write the solution in interval notation: $1 - 2x \leq -3$ or $7 + 3x \leq 4$.

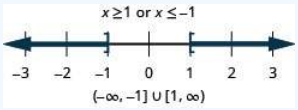
Answer



Try It 5.3.15

Solve the compound inequality. Graph the solution and write the solution in interval notation: $2-5x \leq -3$ or $5+2x \leq 3$.

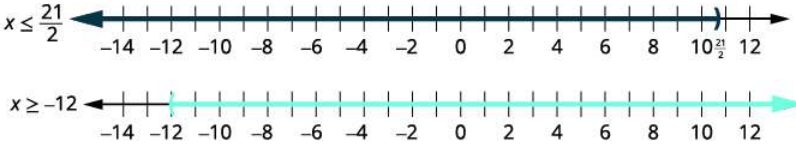
Answer



Example 5.3.16

Solve $\frac{2}{3}x-4 \leq 3$ or $\frac{1}{4}(x+8) \geq -1$. Graph the solution and write the solution in interval notation.

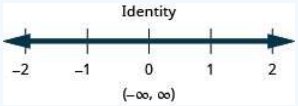
Solution

	$\frac{2}{3}x-4 \leq 3$	or	$\frac{1}{4}(x+8) \geq -1$
Solve each inequality.	$3(\frac{2}{3}x-4) \leq 3(3)$		$4 \cdot \frac{1}{4}(x+8) \geq 4 \cdot (-1)$
	$2x-12 \leq 9$		$x+8 \geq -4$
	$2x \leq 21$		$x \geq -12$
	$x \leq \frac{21}{2}$		
	$x \leq \frac{21}{2}$	or	$x \geq -12$
Graph each solution.			
Graph numbers that make either inequality true.			
	The solution covers all real numbers.		
	$(-\infty, \infty)$		

Try It 5.3.17

Solve the compound inequality. Graph the solution and write the solution in interval notation: $\frac{3}{5}x-7 \leq -1$ or $\frac{1}{3}(x+6) \geq -2$.

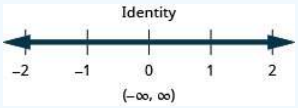
Answer



Try It 5.3.18

Solve the compound inequality. Graph the solution and write the solution in interval notation: $\frac{3}{4}x-3\leq 3$ or $\frac{2}{5}(x+10)\geq 0$.

Answer



Solve Applications with Compound Inequalities

Situations in the real world also involve compound inequalities. We will use the same problem solving strategy that we used to solve linear equation and inequality applications.

Recall the problem solving strategies are to first read the problem and make sure all the words are understood. Then, identify what we are looking for and assign a variable to represent it. Next, restate the problem in one sentence to make it easy to translate into a **compound inequality**. Last, we will solve the compound inequality.

Example 5.3.19

Due to the drought in California, many communities have tiered water rates. There are different rates for Conservation Usage, Normal Usage and Excessive Usage. The usage is measured in the number of hundred cubic feet (hcf) the property owner uses.

During the summer, a property owner will pay $24.72 plus $1.54 per hcf for Normal Usage. The bill for Normal Usage would be between or equal to $57.06 and $171.02. How many hcf can the owner use if he wants his usage to stay in the normal range?

Solution

Identify what we are looking for.	The number of hcf he can use and stay in the “normal usage” billing range.
Name what we are looking for.	Let x= the number of hcf he can use.
Translate to an inequality.	Bill is $24.72 plus $1.54 times the number of hcf he uses or $24.72+1.54x$.
	$\underbrace{\text{His bill will be between or equal to \$57.06 and \$171.02.}}$ $57.06\leq 24.72+1.54x\leq 171.02$
Solve the inequality.	$57.06\leq 24.72+1.54x\leq 171.02$ $57.06-24.72\leq 24.72-24.72+1.54x\leq 171.02-24.72$ $32.34\leq 1.54x\leq 146.3$ $\frac{32.34}{1.54}\leq \frac{1.54x}{1.54}\leq \frac{146.3}{1.54}$ $21\leq x\leq 95$
Answer the question.	The property owner can use $21-95$ hcf and still fall within the “normal usage” billing range.

Try It 5.3.20

Due to the drought in California, many communities now have tiered water rates. There are different rates for Conservation Usage, Normal Usage and Excessive Usage. The usage is measured in the number of hundred cubic feet (hcf) the property owner uses.

During the summer, a property owner will pay $24.72 plus $1.32 per hcf for Conservation Usage. The bill for Conservation Usage would be between or equal to $31.32 and $51.12. How many hcf can the owner use if she wants her usage to stay in the

conservation range?

Answer

The homeowner can use $5-20$ hcf and still fall within the “conservation usage” billing range.

? Try It 5.3.21

Due to the drought in California, many communities have tiered water rates. There are different rates for Conservation Usage, Normal Usage and Excessive Usage. The usage is measured in the number of hundred cubic feet (hcf) the property owner uses.

During the winter, a property owner will pay $24.72 plus $1.54 per hcf for Normal Usage. The bill for Normal Usage would be between or equal to $49.36 and $86.32. How many hcf will he be allowed to use if he wants his usage to stay in the normal range?

Answer

The homeowner can use $16-40$ hcf and still fall within the “normal usage” billing range.

Access this online resource for additional instruction and practice with solving compound inequalities.

- Compound inequalities

Key Concepts

- **How to solve a compound inequality with “and”**
  1. Solve each inequality.
  2. Graph each solution. Then graph the numbers that make *both* inequalities true. This graph shows the solution to the compound inequality.
  3. Write the solution in interval notation.
- **Double Inequality**
  - A **double inequality** is a compound inequality such as $a < x < b$. It is equivalent to $a < x$ and $x < b$.

    Other forms:

$$\begin{array}{lll} a < x < b & \text{is equivalent to} & a < x \text{ and } x < b \\ a \leq x \leq b & \text{is equivalent to} & a \leq x \text{ and } x \leq b \\ a > x > b & \text{is equivalent to} & a > x \text{ and } x > b \\ a \geq x \geq b & \text{is equivalent to} & a \geq x \text{ and } x \geq b \end{array}$$

- **How to solve a compound inequality with “or”**
  1. Solve each inequality.
  2. Graph each solution. Then graph the numbers that make either inequality true.
  3. Write the solution in interval notation.

Glossary

compound inequality

A compound inequality is made up of two inequalities connected by the word “and” or the word “or.”

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5.3E: Exercises

Practice Makes Perfect

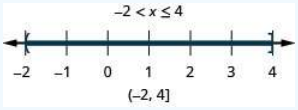
Solve Compound Inequalities with "and"

In the following exercises, solve each inequality, graph the solution, and write the solution in interval notation.

1. $x < 3$ and $x \geq 1$

2. $x \leq 4$ and $x > -2$

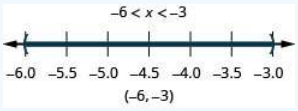
Answer



3. $x \geq -4$ and $x \leq -1$

4. $x > -6$ and $x < -3$

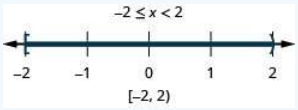
Answer



5. $5x - 2 < 8$ and $6x + 9 \geq 3$

6. $4x - 1 < 7$ and $2x + 8 \geq 4$

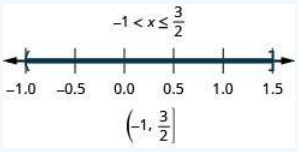
Answer



7. $4x + 6 \leq 2$ and $2x + 1 \geq -5$

8. $4x - 2 \leq 4$ and $7x - 1 > -8$

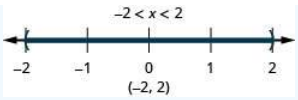
Answer



9. $2x - 11 < 5$ and $3x - 8 > -5$

10. $7x - 8 < 6$ and $5x + 7 > -3$

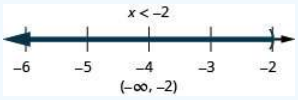
Answer



11. $4(2x-1) \leq 12$ and $2(x+1) < 4$

12. $5(3x-2) \leq 5$ and $3(x+3) < 3$

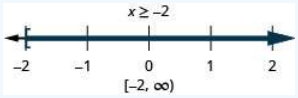
Answer



13. $3(2x-3) > 3$ and $4(x+5) \geq 4$

14. $-3(x+4) < 0$ and $-1(3x-1) \leq 7$

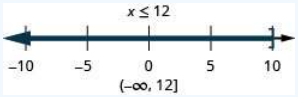
Answer



15. $\frac{1}{2}(3x-4) \leq 1$ and $\frac{1}{3}(x+6) \leq 4$

16. $\frac{3}{4}(x-8) \leq 3$ and $\frac{1}{5}(x-5) \leq 3$

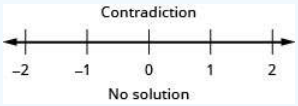
Answer



17. $5x-2 \leq 3x+4$ and $3x-4 \geq 2x+1$

18. $\frac{3}{4}x-5 \geq -2$ and $-3(x+1) \geq 6$

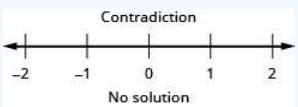
Answer



19. $\frac{2}{3}x-6 \geq -4$ and $-4(x+2) \geq 0$

20. $\frac{1}{2}(x-6)+2 < -5$ and $4-\frac{2}{3}x < 6$

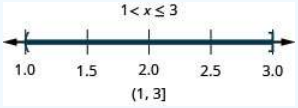
Answer



21. $-5 \leq 4x - 1 < 7$

22. $-3 < 2x - 5 \leq 1$

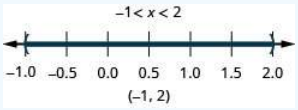
Answer



23. $5 < 4x + 1 < 9$

24. $-1 < 3x + 2 < 8$

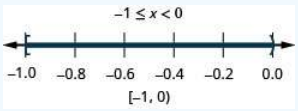
Answer



25. $-8 < 5x + 2 \leq -3$

26. $-6 \leq 4x - 2 < -2$

Answer



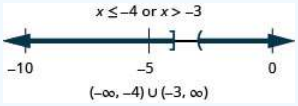
Solve Compound Inequalities with "or"

In the following exercises, solve each inequality, graph the solution on the number line, and write the solution in interval notation.

27. $x \leq -2$ or $x > 3$

28. $x \leq -4$ or $x > -3$

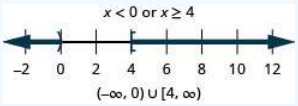
Answer



29. $x < 2$ or $x \geq 5$

30. $x < 0$ or $x \geq 4$

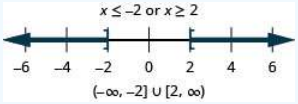
Answer



31. $2+3x \leq 4$ or $5-2x \leq -1$

32. $4-3x \leq -2$ or $2x-1 \leq -5$

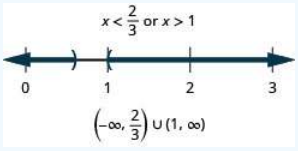
Answer



33. $2(3x-1) < 4$ or $3x-5 > 1$

34. $3(2x-3) < -5$ or
$4x-1 > 3$

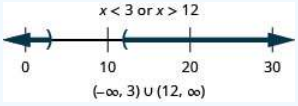
Answer



35. $\frac{3}{4}x-2 > 4$ or $4(2-x) > 0$

36. $\frac{2}{3}x-3 > 5$ or $3(5-x) > 6$

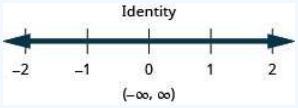
Answer



37. $3x-2 > 4$ or $5x-3 \leq 7$

38. $2(x+3) \geq 0$ or $3(x+4) \leq 6$

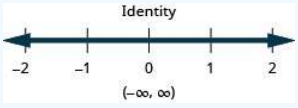
Answer



39. $\frac{1}{2}x-3 \leq 4$ or $\frac{1}{3}(x-6) \geq -2$

40. $\frac{3}{4}x+2 \leq -1$ or $\frac{1}{2}(x+8) \geq -3$

Answer



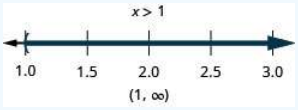
Mixed practice

In the following exercises, solve each inequality, graph the solution on the number line, and write the solution in interval notation.

41. $3x+7 \leq 1$ and $2x+3 \geq -5$

42. $6(2x-1) > 6$ and $5(x+2) \geq 0$

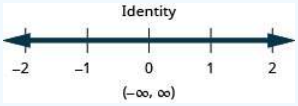
Answer



43. $4-7x \geq -3$ or $5(x-3)+8 > 3$

44. $\frac{1}{2}x-5 \leq 3$ or $\frac{1}{4}(x-8) \geq -3$

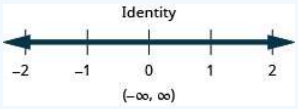
Answer



45. $-5 \leq 2x-1 < 7$

46. $\frac{1}{5}(x-5)+6 < 4$ and $3-\frac{2}{3}x < 5$

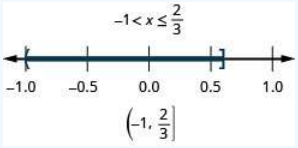
Answer



47. $4x-2 > 6$ or $3x-1 \leq -2$

48. $6x-3 \leq 1$ and $5x-1 > -6$

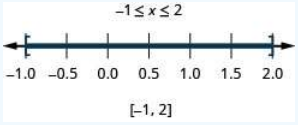
Answer



49. $-2(3x-4) \leq 2$ and $-4(x-1) < 2$

50. $-5 \leq 3x-2 \leq 4$

Answer



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Solve Applications with Compound Inequalities

In the following exercises, solve.

51. Penelope is playing a number game with her sister June. Penelope is thinking of a number and wants June to guess it. Five more than three times her number is between 2 and 32. Write a compound inequality that shows the range of numbers that Penelope might be thinking of.

52. Gregory is thinking of a number and he wants his sister Lauren to guess the number. His first clue is that six less than twice his number is between four and forty-two. Write a compound inequality that shows the range of numbers that Gregory might be thinking of.

Answer

$5 \leq n \leq 24$

53. Andrew is creating a rectangular dog run in his back yard. The length of the dog run is 18 feet. The perimeter of the dog run must be at least 42 feet and no more than 72 feet. Use a compound inequality to find the range of values for the width of the dog run.

54. Elouise is creating a rectangular garden in her back yard. The length of the garden is 12 feet. The perimeter of the garden must be at least 36 feet and no more than 48 feet. Use a compound inequality to find the range of values for the width of the garden.

Answer

$6 \leq w \leq 12$

Everyday Math

55. Blood Pressure A person's blood pressure is measured with two numbers. The systolic blood pressure measures the pressure of the blood on the arteries as the heart beats. The diastolic blood pressure measures the pressure while the heart is resting.

ⓐ Let x be your systolic blood pressure. Research and then write the compound inequality that shows you what a normal systolic blood pressure should be for someone your age.

ⓑ Let y be your diastolic blood pressure. Research and then write the compound inequality that shows you what a normal diastolic blood pressure should be for someone your age.

56. Body Mass Index (BMI) is a measure of body fat is determined using your height and weight.

ⓐ Let x be your BMI. Research and then write the compound inequality to show the BMI range for you to be considered normal weight.

ⓑ Research a BMI calculator and determine your BMI. Is it a solution to the inequality in part (a)?

Answer

ⓐ answers vary ⓑ answers vary

Writing Exercises

57. In your own words, explain the difference between the properties of equality and the properties of inequality.

58. Explain the steps for solving the compound inequality $2-7x \geq -5$ or $4(x-3)+7>3$.

Answer

Answers will vary.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
solve compound inequalities with "and".			
solve compound inequalities with "or".			
solve applications with compound inequalities.			

ⓑ What does this checklist tell you about your mastery of this section? What steps will you take to improve?

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5.4: Graphs of Linear Inequalities in Two Variables (optional challenge practice)

Learning Objectives

By the end of this section, you will be able to:

- Verify solutions to an inequality in two variables
- Recognize the relation between the solutions of an inequality and its graph
- Graph linear inequalities

Note

Before you get started, take this readiness quiz.

1. Solve: $4x+3>23$.
   If you missed this problem, review Exercise 2.7.22.
2. Translate from algebra to English: $x<5$.
   If you missed this problem, review Exercise 1.3.1.
3. Evaluate $3x-2y$ when $x=1,\ y=-2$.
   If you missed this problem, review Exercise 1.5.28.

Verify Solutions to an Inequality in Two Variables

We have learned how to solve inequalities in one variable. Now, we will look at inequalities in two variables. Inequalities in two variables have many applications. If you ran a business, for example, you would want your revenue to be greater than your costs—so that your business would make a profit.

LINEAR INEQUALITY

A **linear inequality** is an inequality that can be written in one of the following forms:

$$Ax+By>C \quad Ax+By\geq C \quad Ax+By<C \quad Ax+By\leq C$$

where A and B are not both zero.

Do you remember that an inequality with one variable had many solutions? The solution to the inequality $x>3$ is any number greater than 3. We showed this on the number line by shading in the number line to the right of 3, and putting an open parenthesis at 3. See Figure 5.4.1.

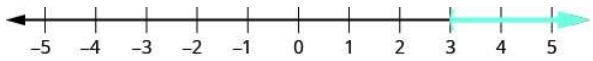


Figure 5.4.1

Similarly, inequalities in two variables have many solutions. Any ordered pair (x,y) that makes the inequality true when we substitute in the values is a solution of the inequality.

Solution OF A LINEAR INEQUALITY

An ordered pair (x,y) is a **solution of a linear inequality** if the inequality is true when we substitute the values of x and y.

Example 5.4.1

Determine whether each ordered pair is a solution to the inequality $y>x+4$:

1. $(0,0)$
2. $(1,6)$
3. $(2,6)$
4. $(-5,-15)$

5. $(-8, 12)$

Solution

1.

$(0, 0)$	$y > x + 4$
Substitute 0 for x and 0 for y.	$0 \stackrel{?}{>} 0 + 4$
Simplify.	$0 \not> 4$ So, $(0, 0)$ is not a solution to $y > x + 4$.

2.

$(1, 6)$	$y > x + 4$
Substitute 1 for x and 6 for y.	$6 \stackrel{?}{>} 1 + 4$
Simplify.	$6 > 5$ So, $(1, 6)$ is a solution to $y > x + 4$.

3.

$(2, 6)$	$y > x + 4$
Substitute 2 for x and 6 for y.	$6 \stackrel{?}{>} 2 + 4$
Simplify.	$6 \not> 6$ So, $(2, 6)$ is not a solution to $y > x + 4$.

4.

$(-5, -15)$	$y > x + 4$
Substitute –5 for x and –15 for y.	$-15 \stackrel{?}{>} -5 + 4$
Simplify.	$-15 \not> -1$ So, $(-5, -15)$ is not a solution to $y > x + 4$.

5.

$(-8,12)$	$y > x + 4$
Substitute –8 for x and 12 for y.	$12 \stackrel{?}{>} -8 + 4$
Simplify.	$12 > -4$ So, $(-8, 12)$ is a solution to $y > x + 4$.

? Try It 5.4.2

Determine whether each ordered pair is a solution to the inequality $y > x - 3$:

1. $(0, 0)$
2. $(4, 9)$
3. $(-2, 1)$
4. $(-5, -3)$
5. $(5, 1)$

Answer

1. yes

2. yes
3. yes
4. yes
5. no

Try It 5.4.3

Determine whether each ordered pair is a solution to the inequality $y < x + 1$:

1. $(0, 0)$
2. $(8, 6)$
3. $(-2, -1)$
4. $(3, 4)$
5. $(-1, -4)$

Answer

1. yes
2. yes
3. no
4. no
5. yes

Recognize the Relation Between the Solutions of an Inequality and its Graph

Now, we will look at how the solutions of an inequality relate to its graph.

Let's think about the number line in Figure 5.4.1 again. The point $x = 3$ separated that number line into two parts. On one side of 3 are all the numbers less than 3. On the other side of 3 all the numbers are greater than 3. See Figure 5.4.2.

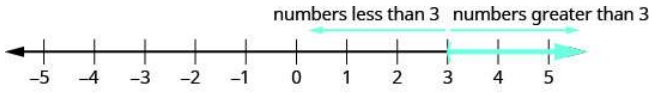


Figure 5.4.2

The solution to $x > 3$ is the shaded part of the number line to the right of $x = 3$.

Similarly, the line $y = x + 4$ separates the plane into two regions. On one side of the line are points with $y < x + 4$. On the other side of the line are the points with $y > x + 4$. We call the line $y = x + 4$ a boundary line.

BOUNDARY LINE

The line with equation $Ax + By = C$ is the **boundary line** that separates the region where $Ax + By > C$ from the region where $Ax + By < C$.

For an inequality in one variable, the endpoint is shown with a parenthesis or a bracket depending on whether or not aa is included in the solution:



Similarly, for an inequality in two variables, the boundary line is shown with a solid or dashed line to indicate whether or not it the line is included in the solution. This is summarized in Table 5.4.1.

Table 5.4.1

$Ax + By < C$	$Ax + By \leq C$
$Ax + By > C$	$Ax + By \geq C$
Boundary line is not included in solution.	Boundary line is included in solution.

Boundary line is dashed.	Boundary line is solid.

Now, let's take a look at what we found in Example 5.4.1. We'll start by graphing the line $y = x + 4$, and then we'll plot the five points we tested. See Figure 5.4.3.

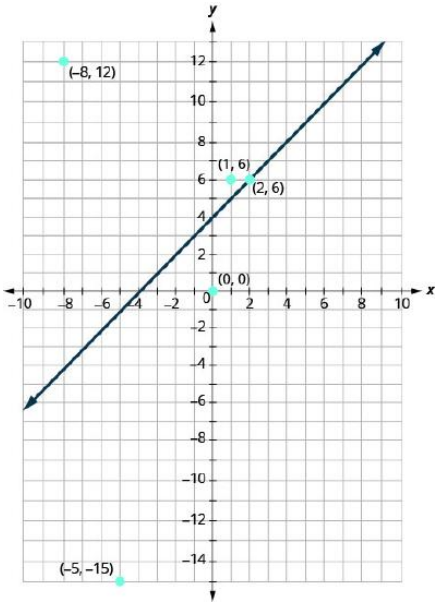


Figure 5.4.3

In Example 5.4.1 we found that some of the points were solutions to the inequality $y > x + 4$ and some were not.

Which of the points we plotted are solutions to the inequality $y > x + 4$? The points $(1, 6)$ and $(-8, 12)$ are solutions to the inequality $y > x + 4$. Notice that they are both on the same side of the boundary line $y = x + 4$.

The two points $(0, 0)$ and $(-5, -15)$ are on the other side of the **boundary line** $y = x + 4$, and they are not solutions to the inequality $y > x + 4$. For those two points, $y < x + 4$.

What about the point $(2, 6)$? Because $6 = 2 + 4$, the point is a solution to the equation $y = x + 4$. So the point $(2, 6)$ is on the boundary line.

Let's take another point on the left side of the boundary line and test whether or not it is a solution to the inequality $y > x + 4$. The point $(0, 10)$ clearly looks to be to the left of the boundary line, doesn't it? Is it a solution to the inequality?

$$\begin{aligned} &y > x + 4 \\ &10 \overset{?}{>} 0 + 4 \\ &10 > 4 \qquad \text{So, } (0, 10) \text{ is a solution to } y > x + 4. \end{aligned} \tag{5.4.1}$$

Any point you choose on the left side of the boundary line is a solution to the inequality $y > x + 4$. All points on the left are solutions.

Similarly, all points on the right side of the boundary line, the side with $(0, 0)$ and $(-5, -15)$, are not solutions to $y > x + 4$. See Figure 5.4.4.

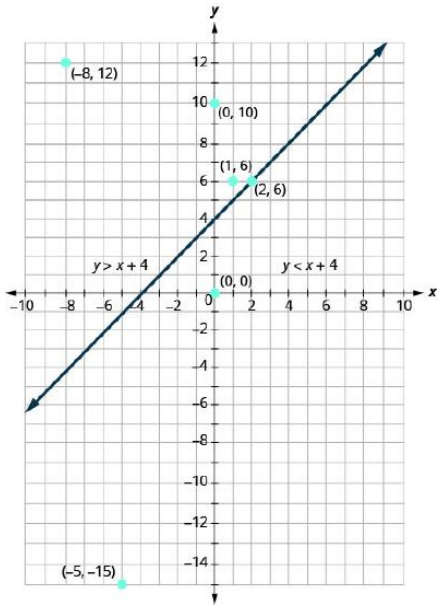


Figure 5.4.4

The graph of the inequality $y > x + 4$ is shown in Figure 5.4.5 below. The line $y = x + 4$ divides the plane into two regions. The shaded side shows the solutions to the inequality $y > x + 4$.

The points on the boundary line, those where $y = x + 4$, are not solutions to the inequality $y > x + 4$, so the line itself is not part of the solution. We show that by making the line dashed, not solid.

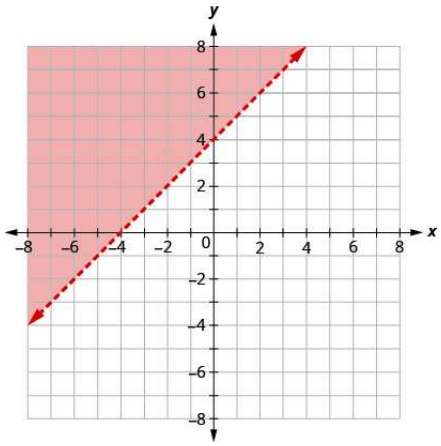
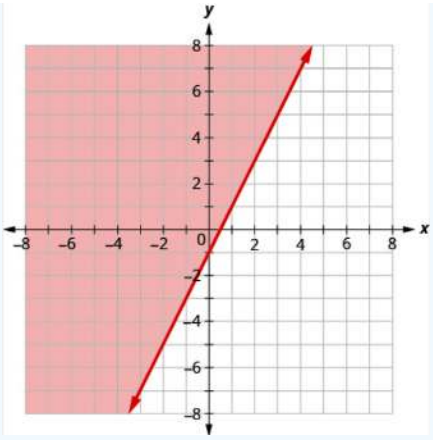


Figure 5.4.5: The graph of the inequality y>x+4.

Example 5.4.4

The boundary line shown is $y = 2x - 1$. Write the inequality shown by the graph.



Solution

The line $y = 2x - 1$ is the boundary line. On one side of the line are the points with $y > 2x - 1$ and on the other side of the line are the points with $y < 2x - 1$.

Let's test the point $(0, 0)$ and see which inequality describes its side of the boundary line.

At $(0, 0)$, which inequality is true:

$$\begin{array}{lll} y > 2x - 1 & \text{or} & y < 2x - 1? \\ y > 2x - 1 & & y < 2x - 1 \\ 0 > 2 \cdot 0 - 1 & & 0 < 2 \cdot 0 - 1 \\ 0 > -1 \text{ True} & & 0 < -1 \text{ False} \end{array} \tag{5.4.2}$$

Since $y > 2x - 1$ is true, the side of the line with $(0, 0)$, is the solution. The shaded region shows the solution of the inequality $y > 2x - 1$.

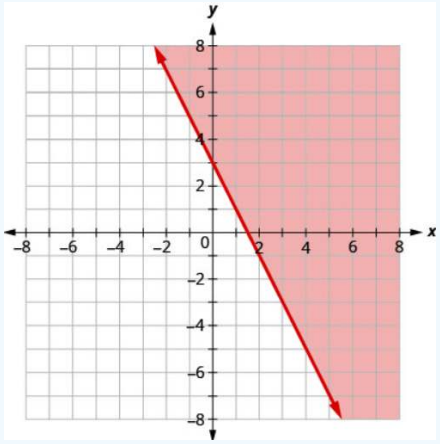
Since the boundary line is graphed with a solid line, the inequality includes the equal sign.

The graph shows the inequality $y \geq 2x - 1$.

We could use any point as a test point, provided it is not on the line. Why did we choose $(0, 0)$? Because it's the easiest to evaluate. You may want to pick a point on the other side of the boundary line and check that $y < 2x - 1$.

? Try It 5.4.5

Write the inequality shown by the graph with the boundary line $y = -2x + 3$.

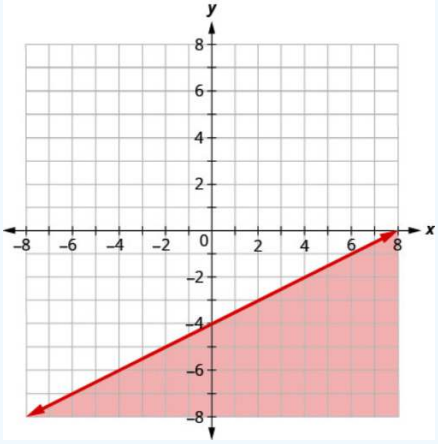


Answer

$y \geq -2x + 3$

Try It 5.4.6

Write the inequality shown by the graph with the boundary line $y = \frac{1}{2}x - 4$.

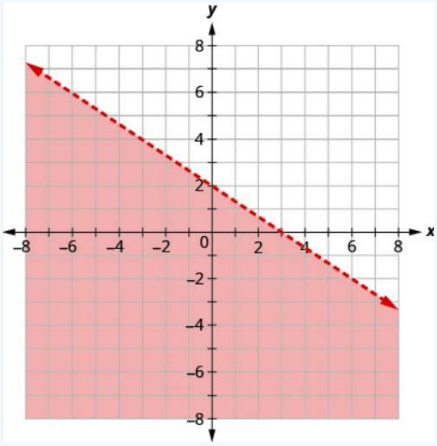


Answer

$y \leq \frac{1}{2}x - 4$

Example 5.4.7

The boundary line shown is $2x + 3y = 6$. Write the inequality shown by the graph.



Solution

The line $2x + 3y = 6$ is the boundary line. On one side of the line are the points with $2x + 3y > 6$ and on the other side of the line are the points with $2x + 3y < 6$.

Let's test the point $(0, 0)$ and see which inequality describes its side of the boundary line.

At $(0, 0)$, which inequality is true:

$$\begin{aligned} 2x + 3y &> 6 \quad \text{or} \quad 2x + 3y < 6? \\ 2x + 3y &> 6 \qquad\quad 2x + 3y < 6 \\ 2(0) + 3(0) &> 6 \qquad\quad 2(0) + 3(0) < 6 \\ 0 &> 6 \quad \text{False} \qquad\quad 0 < 6 \quad \text{True} \end{aligned} \tag{5.4.3}$$

So the side with $(0, 0)$ is the side where $2x + 3y < 6$.

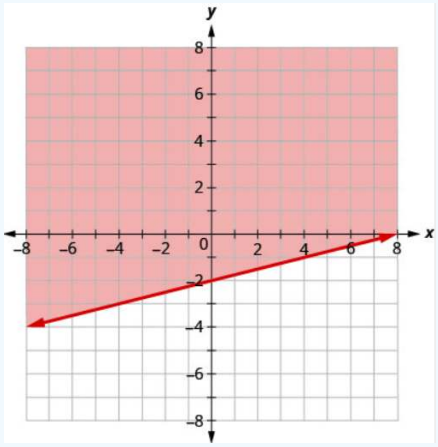
(You may want to pick a point on the other side of the boundary line and check that $2x + 3y > 6$.)

Since the boundary line is graphed as a dashed line, the inequality does not include an equal sign.

The graph shows the solution to the inequality $2x + 3y < 6$.

Try It 5.4.8

Write the inequality shown by the shaded region in the graph with the boundary line $x - 4y = 8$.

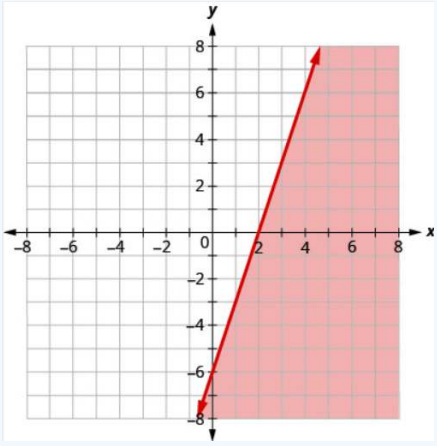


Answer

$x - 4y \leq 8$

Try It 5.4.9

Write the inequality shown by the shaded region in the graph with the boundary line $3x - y = 6$.



Answer

$3x - y \leq 6$

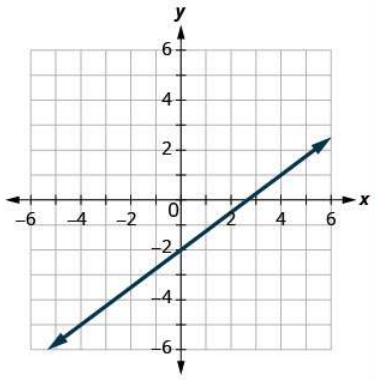
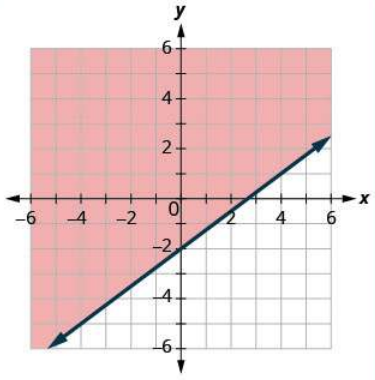
Graph Linear Inequalities

Now, we're ready to put all this together to graph linear inequalities.

Example 5.4.10: How to Graph Linear Inequalities

Graph the **linear inequality** $y \geq \frac{3}{4}x - 2$.

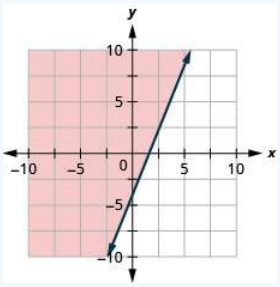
Solution

Step 1. Identify and graph the boundary line. • If the inequality is ≤ or ≥, the boundary line is solid. • If the inequality is < or >, the boundary line is dashed.	Replace the inequality sign with an equal sign to find the boundary line. Graph the boundary line $y = \frac{3}{4}x - 2$. The inequality sign is ≥, so we draw a solid line.	
Step 2. Test a point that is not on the boundary line. Is it a solution of the inequality?	We'll test (0, 0). Is it a solution of the inequality?	At (0, 0), is $y \geq \frac{3}{4}x - 2$? $0 \stackrel{?}{\geq} \frac{3}{4}(0) - 2$ $0 \geq -2$ So, (0, 0) is a solution.
Step 3. Shade in one side of the boundary line. • If the test point is a solution, shade in the side that includes the point. • If the test point is not a solution, shade in the opposite side.	The test point (0, 0), is a solution to $y \geq \frac{3}{4}x - 2$. So we shade in that side.	

Try It 5.4.11

Graph the linear inequality $y \geq \frac{5}{2}x - 4$.

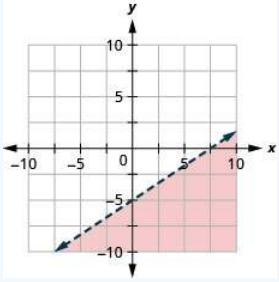
Answer



Try It 5.4.12

Graph the linear inequality $y < \frac{2}{3}x - 5$.

Answer



The steps we take to graph a linear inequality are summarized here.

GRAPH A LINEAR INEQUALITY.

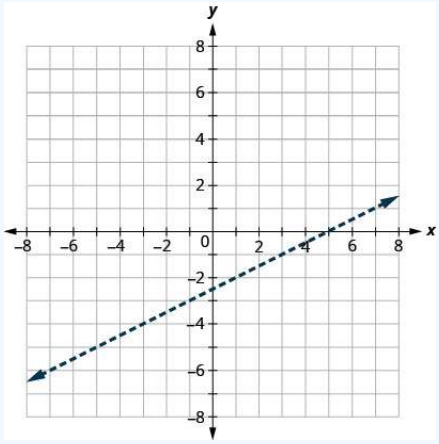
1. Identify and graph the boundary line.
   - If the inequality is $\leq$ or $\geq$, the boundary line is solid.
   - If the inequality is $<$ or $>$, the boundary line is dashed.
2. Test a point that is not on the boundary line. Is it a solution of the inequality?
3. Shade in one side of the boundary line.
   - If the test point is a solution, shade in the side that includes the point.
   - If the test point is not a solution, shade in the opposite side.

Example 5.4.13

Graph the linear inequality $x - 2y < 5$.

Solution

First we graph the boundary line $x - 2y = 5$. The inequality is $<$ so we draw a dashed line.



Then we test a point. We'll use $(0, 0)$ again because it is easy to evaluate and it is not on the boundary line.

Is $(0, 0)$ a solution of $x - 2y < 5$?

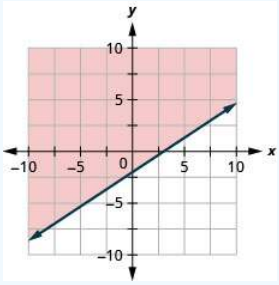
$$0 - 2(0) \stackrel{?}{<} 5$$
$$0 - 0 \stackrel{?}{<} 5$$
$$0 < 5$$

The point $(0, 0)$ is a solution of $x - 2y < 5$, so we shade in that side of the boundary line.

Try It 5.4.14

Graph the linear inequality $2x - 3y \leq 6$.

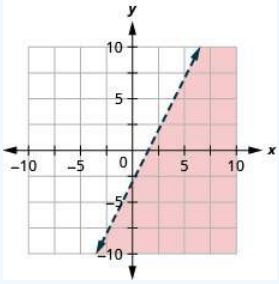
Answer



Try It 5.4.15

Graph the linear inequality $2x - y > 3$.

Answer



What if the boundary line goes through the origin? Then we won't be able to use $(0, 0)$ as a test point. No problem—we'll just choose some other point that is not on the boundary line.

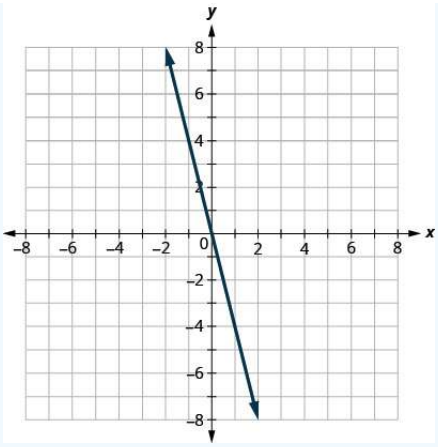
Example 5.4.16

Graph the linear inequality $y \leq -4x$.

Solution

First we graph the boundary line $y = -4x$. It is in slope–intercept form, with $m = -4$ and $b = 0$. The inequality is $\leq$ so we draw a solid line.

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Now, we need a test point. We can see that the point $(1, 0)$ is not on the boundary line.

Is $(1, 0)$ a solution of $y \leq -4x$?

$0 \stackrel{?}{\leq} -4(1)$

$0 \not\leq -4$

The point $(1, 0)$ is not a solution to $y \leq -4x$, so we shade in the opposite side of the boundary line. See Figure 5.4.6.

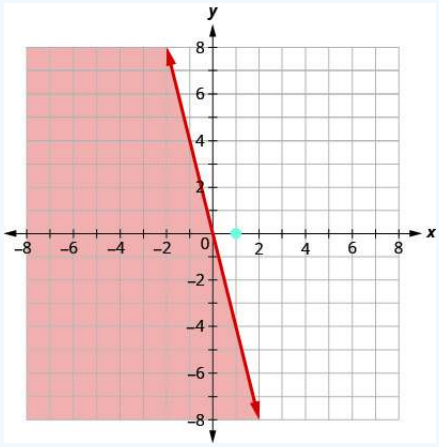
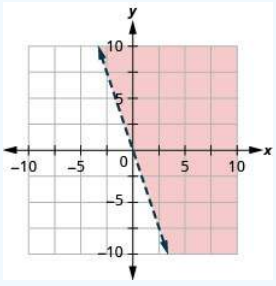


Figure 5.4.6

? Try It 5.4.17

Graph the linear inequality $y > -3x$.

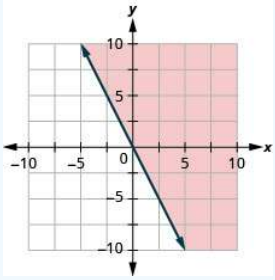
Answer



Try It 5.4.18

Graph the linear inequality $y \geq -2x$.

Answer



Some linear inequalities have only one variable. They may have an x but no y, or a y but no x. In these cases, the boundary line will be either a vertical or a horizontal line. Do you remember?

$x = a$ vertical line
$y = b$ horizontal line

Example 5.4.19

Graph the linear inequality $y > 3$.

Solution

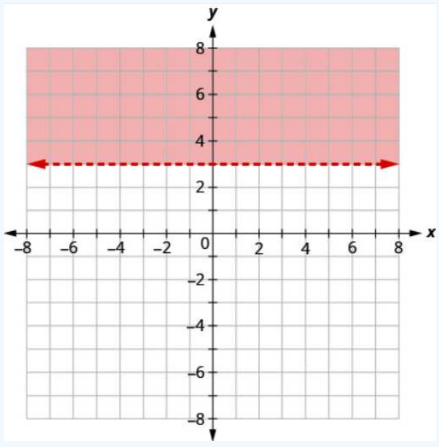
First we graph the boundary line $y = 3$. It is a horizontal line. The inequality is $>$ so we draw a dashed line.

We test the point $(0, 0)$.

$$\begin{aligned} y &> 3 \\ 0 &\not> 3 \end{aligned} \tag{5.4.4}$$

$(0, 0)$ is not a solution to $y > 3$.

So we shade the side that does not include $(0, 0)$.

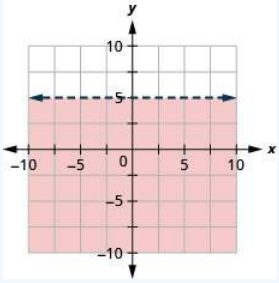


Try It 5.4.20

Graph the linear inequality $y < 5$.

Answer

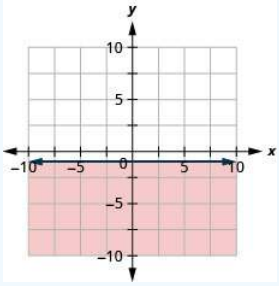
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Try It 5.4.21

Graph the linear inequality $y \leq -1$.

Answer



Key Concepts

- **To Graph a Linear Inequality**
  1. Identify and graph the boundary line.
     If the inequality is $\leq$ or $\geq$, the boundary line is solid.
     If the inequality is $<$ or $>$, the boundary line is dashed.
  2. Test a point that is not on the boundary line. Is it a solution of the inequality?
  3. Shade in one side of the boundary line.
     If the test point is a solution, shade in the side that includes the point.
     If the test point is not a solution, shade in the opposite side.

Glossary

boundary line

The line with equation $Ax + By = C$ that separates the region where $Ax + By > C$ from the region where $Ax + By < C$.

linear inequality

An inequality that can be written in one of the following forms:

$$Ax + By > C \quad Ax + By \geq C \quad Ax + By < C \quad Ax + By \leq C \tag{5.4.5}$$

where A and B are not both zero.

solution of a linear inequality

An ordered pair $(x,\ y)$ is a solution to a linear inequality the inequality is true when we substitute the values of x and y.

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5.4E: Exercises

Practice Makes Perfect

Verify Solutions to an Inequality in Two Variables

In the following exercises, determine whether each ordered pair is a solution to the given inequality.

Exercise 5.4E.1

Determine whether each ordered pair is a solution to the inequality $y > x - 1$:

1. $(0, 1)$
2. $(-4, -1)$
3. $(4, 2)$
4. $(3, 0)$
5. $(-2, -3)$

Exercise 5.4E.2

Determine whether each ordered pair is a solution to the inequality $y > x - 3$:

1. $(0, 0)$
2. $(2, 1)$
3. $(-1, -5)$
4. $(-6, -3)$
5. $(1, 0)$

Answer

1. yes
2. no
3. no
4. yes
5. no

Exercise 5.4E.3

Determine whether each ordered pair is a solution to the inequality $y < x + 2$:

1. $(0, 3)$
2. $(-3, -2)$
3. $(-2, 0)$
4. $(0, 0)$
5. $(-1, 4)$

Exercise 5.4E.4

Determine whether each ordered pair is a solution to the inequality $y < x + 5$:

1. $(-3, 0)$
2. $(1, 6)$
3. $(-6, -2)$
4. $(0, 1)$
5. $(5, -4)$

Answer

1. yes
2. no
3. no
4. yes
5. yes

Exercise 5.4E.5

Determine whether each ordered pair is a solution to the inequality $x+y>4$:

1. $(5,1)$
2. $(-2,6)$
3. $(3,2)$
4. $(10,-5)$
5. $(0,0)$

Exercise 5.4E.6

Determine whether each ordered pair is a solution to the inequality $x+y>2$:

1. $(1,1)$
2. $(4,-3)$
3. $(0,0)$
4. $(-8,12)$
5. $(3,0)$

Answer

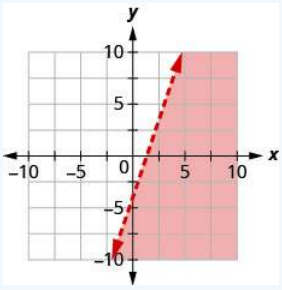
1. no
2. no
3. no
4. yes
5. yes

Recognize the Relation Between the Solutions of an Inequality and its Graph

In the following exercises, write the inequality shown by the shaded region.

Exercise 5.4E.7

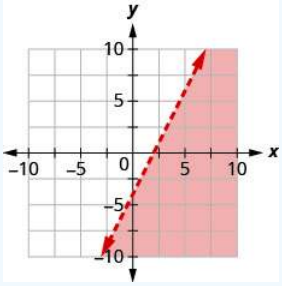
Write the inequality shown by the graph with the boundary line $y=3x-4$.



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Exercise 5.4E.8

Write the inequality shown by the graph with the boundary line $y = 2x - 4$.

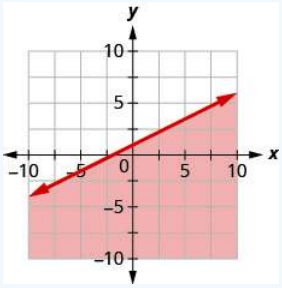


Answer

$y < 2x - 4$

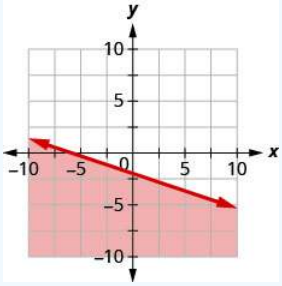
Exercise 5.4E.9

Write the inequality shown by the graph with the boundary line $y = \frac{1}{2}x + 1$



Exercise 5.4E.10

Write the inequality shown by the graph with the boundary line $y = -\frac{1}{3}x - 2$



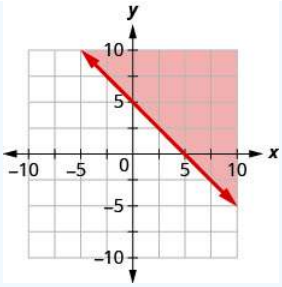
Answer

$y \leq -\frac{1}{3}x - 2$

Exercise 5.4E.11

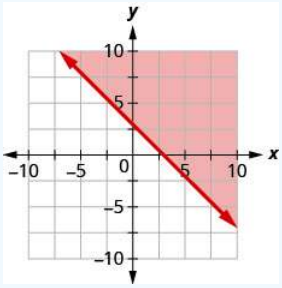
Write the inequality shown by the shaded region in the graph with the boundary line $x + y = 5$.

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Exercise 5.4E. 12

Write the inequality shown by the shaded region in the graph with the boundary line $x + y = 3$.

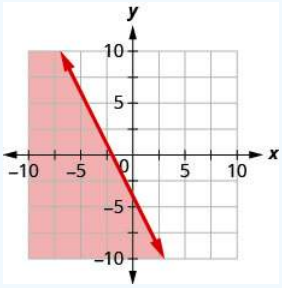


Answer

$x + y \geq 3$

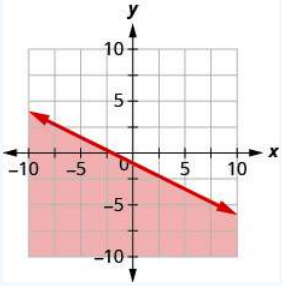
Exercise 5.4E. 13

Write the inequality shown by the shaded region in the graph with the boundary line $2x + y = -4$.



Exercise 5.4E. 14

Write the inequality shown by the shaded region in the graph with the boundary line $x + 2y = -2$.



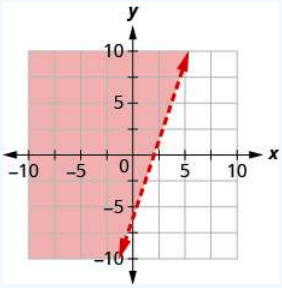
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Answer

$x + 2y \geq -2$

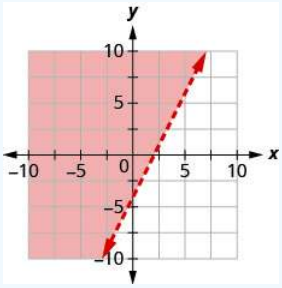
Exercise 5.4E. 15

Write the inequality shown by the shaded region in the graph with the boundary line $3x - y = 6$.



Exercise 5.4E. 16

Write the inequality shown by the shaded region in the graph with the boundary line $2x - y = 4$.

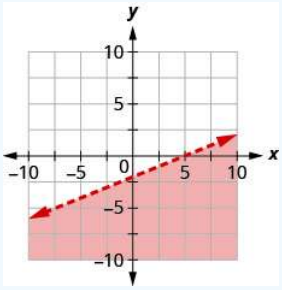


Answer

$2x - y < 4$

Exercise 5.4E. 17

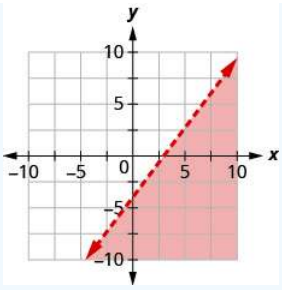
Write the inequality shown by the shaded region in the graph with the boundary line $2x - 5y = 10$.



Exercise 5.4E. 18

Write the inequality shown by the shaded region in the graph with the boundary line $4x - 3y = 12$.

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Answer

$4x - 3y > 12$

Graph Linear Inequalities

In the following exercises, graph each linear inequality.

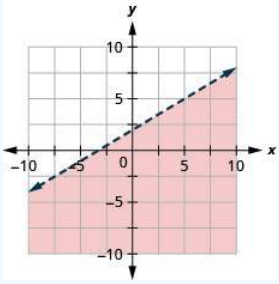
Exercise 5.4E. 19

Graph the linear inequality $y > \frac{2}{3}x - 1$

Exercise 5.4E. 20

Graph the linear inequality $y < \frac{3}{5}x + 2$

Answer



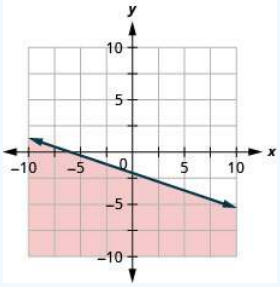
Exercise 5.4E. 21

Graph the linear inequality $y \leq -\frac{1}{2}x + 4$

Exercise 5.4E. 22

Graph the linear inequality $y \geq -\frac{1}{3}x - 2$

Answer



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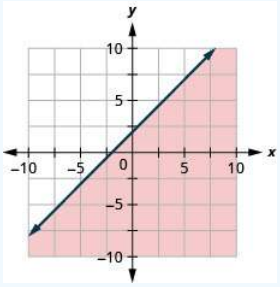
Exercise 5.4E.23

Graph the linear inequality $x - y \leq 3$

Exercise 5.4E.24

Graph the linear inequality $x - y \geq -2$

Answer



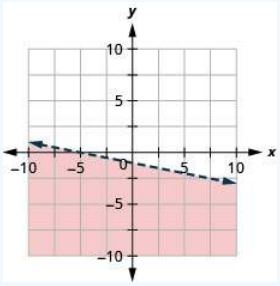
Exercise 5.4E.25

Graph the linear inequality $4x + y > -4$

Exercise 5.4E.26

Graph the linear inequality $x + 5y < -5$

Answer



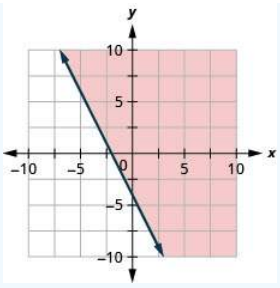
Exercise 5.4E.27

Graph the linear inequality $3x + 2y \geq -6$

Exercise 5.4E.28

Graph the linear inequality $4x + 2y \geq -8$

Answer



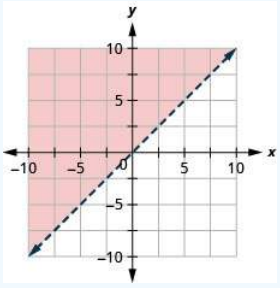
Exercise 5.4E.29

Graph the linear inequality $y > 4x$

Exercise 5.4E.30

Graph the linear inequality $y > x$

Answer



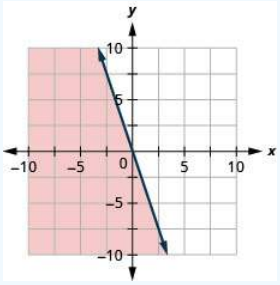
Exercise 5.4E.31

Graph the linear inequality $y \leq -x$

Exercise 5.4E.32

Graph the linear inequality $y \leq -3x$

Answer



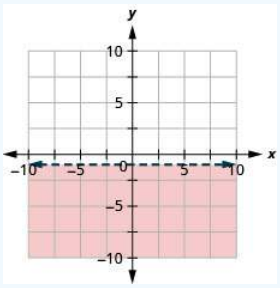
Exercise 5.4E.33

Graph the linear inequality $y \geq -2$

Exercise 5.4E.34

Graph the linear inequality $y < -1$

Answer



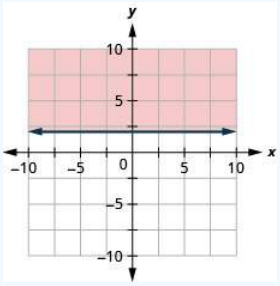
Exercise 5.4E.35

Graph the linear inequality $y < 4$

Exercise 5.4E.36

Graph the linear inequality $y \geq 2$

Answer



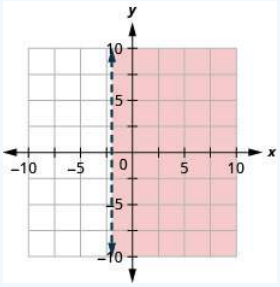
Exercise 5.4E.37

Graph the linear inequality $x \leq 5$

Exercise 5.4E.38

Graph the linear inequality $x > -2$

Answer



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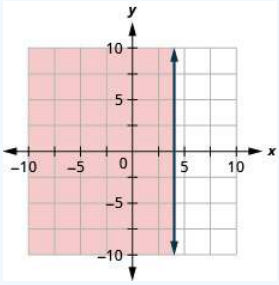
Exercise 5.4E. 39

Graph the linear inequality $x > -3$

Exercise 5.4E. 40

Graph the linear inequality $x \leq 4$

Answer



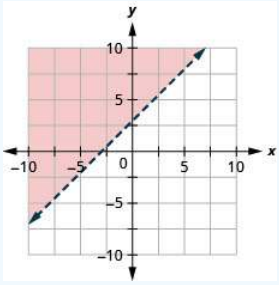
Exercise 5.4E. 41

Graph the linear inequality $x - y < 4$

Exercise 5.4E. 42

Graph the linear inequality $x - y < -3$

Answer



Exercise 5.4E. 43

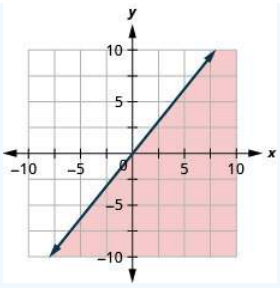
Graph the linear inequality $y \geq \frac{3}{2}x$

Exercise 5.4E. 44

Graph the linear inequality $y \leq \frac{5}{4}x$

Answer

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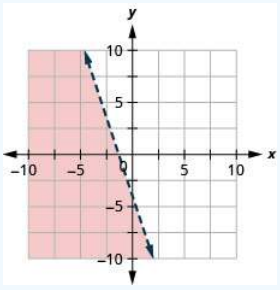
Exercise 5.4E.45

Graph the linear inequality $y > -2x + 1$

Exercise 5.4E.46

Graph the linear inequality $y < -3x - 4$

Answer



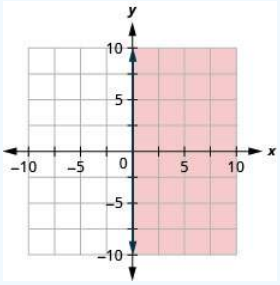
Exercise 5.4E.47

Graph the linear inequality $x \leq -1$

Exercise 5.4E.48

Graph the linear inequality $x \geq 0$

Answer



Everyday Math

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Exercise 5.4E.49

Money. Gerry wants to have a maximum of \$100 cash at the ticket booth when his church carnival opens. He will have \$1 bills and \$5 bills. If x is the number of \$1 bills and y is the number of \$5 bills, the inequality $x+5y \leq 100$ models the situation.

1. Graph the inequality.
2. List three solutions to the inequality $x+5y \leq 100$ where both x and y are integers.

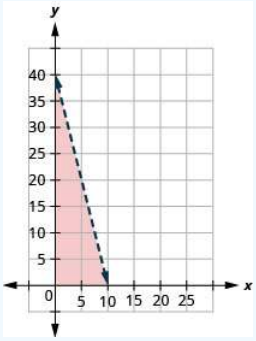
Exercise 5.4E.50

Shopping. Tula has \$20 to spend at the used book sale. Hardcover books cost \$2 each and paperback books cost \$0.50 each. If x is the number of hardcover books Tula can buy and y is the number of paperback books she can buy, the inequality $2x+\frac{1}{2}y \leq 20$ models the situation.

1. Graph the inequality.
2. List three solutions to the inequality $2x+\frac{1}{2}y \leq 20$ where both x and y are whole numbers.

Answer

1.



2. Answers will vary.

Writing Exercises

Exercise 5.4E.51

Lester thinks that the solution of any inequality with a $>$ sign is the region above the line and the solution of any inequality with a $<$ sign is the region below the line. Is Lester correct? Explain why or why not.

Exercise 5.4E.52

Explain why in some graphs of linear inequalities the boundary line is solid but in other graphs it is dashed.

Self Check

a. After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
verify solutions to an inequality in two variables.			
recognize the relation between the solutions of an inequality and its graph.			
graph linear inequalities.			

b. What does this checklist tell you about your mastery of this section? What steps will you take to improve?

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5.5: Solve Absolute Value Inequalities (optional challenge practice)

Learning Objectives

By the end of this section, you will be able to:

- Solve absolute value equations
- Solve absolute value inequalities with “less than”
- Solve absolute value inequalities with “greater than”
- Solve applications with absolute value

Before you get started, take this readiness quiz.

1. Evaluate: $-|7|$.
   If you missed this problem, review [link].
2. Fill in $<, >, <, >$, or $=$ for each of the following pairs of numbers.
   ⓐ $|-8|\_\_\_-|-8|$ ⓑ $12\_\_\_-|-12|$ ⓒ $|-6|\_\_\_-6$ ⓓ $-(-15)\_\_\_-|-15|$
   If you missed this problem, review [link].
3. Simplify: $14-2|8-3(4-1)|$.
   If you missed this problem, review [link].

Solve Absolute Value Equations

As we prepare to solve absolute value equations, we review our definition of **absolute value**.

ABSOLUTE VALUE

The absolute value of a number is its distance from zero on the number line.

The absolute value of a number *n* is written as $|n|$ and $|n| \geq 0$ for all numbers.

Absolute values are always greater than or equal to zero.

We learned that both a number and its opposite are the same distance from zero on the number line. Since they have the same distance from zero, they have the same absolute value. For example:

- -5 is 5 units away from 0, so $|-5| = 5$.
- 5 is 5 units away from 0, so $|5| = 5$.

Figure $5.5.1$ illustrates this idea.

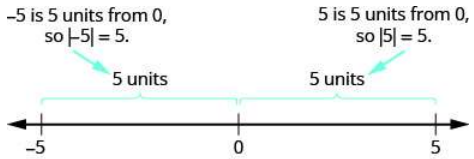


Figure 5.5.1: The numbers 5 and -5 are both five units away from zero.

For the equation |x|=5,|x|=5, we are looking for all numbers that make this a true statement. We are looking for the numbers whose distance from zero is 5. We just saw that both 5 and −5−5 are five units from zero on the number line. They are the solutions to the equation.

If $|x| = 5$
then $x = -5$ or $x = 5$

The solution can be simplified to a single statement by writing $x = \pm 5$. This is read, “*x* is equal to positive or negative 5”.

We can generalize this to the following property for absolute value equations.

ABSOLUTE VALUE EQUATIONS

For any algebraic expression, u, and any positive real number, a,

$$\begin{aligned} &\text{if} && |u| = a \\ &\text{then} && u = -a \text{ or } u = a \end{aligned}$$

Remember that an absolute value cannot be a negative number.

Example 5.5.1

Solve:

a. $|x| = 8$
b. $|y| = -6$
c. $|z| = 0$

Solution a

$$\begin{aligned} & && |x| = 8 \\ &\text{Write the equivalent equations.} && x = -8 \text{ or } x = 8 \\ & && x = \pm 8 \end{aligned}$$

Solution b

$$|y| = -6$$

No solution

Since an absolute value is always positive, there are no solutions to this equation.

Solution c

$$\begin{aligned} & && |z| = 0 \\ &\text{Write the equivalent equations.} && z = -0 \text{ or } z = 0 \\ &\text{Since } -0 = 0, && z = 0 \end{aligned}$$

Both equations tell us that z=0z=0 and so there is only one solution.

EXERCISE 5.5.2

Solve:

a. $|x| = 2$
b. $|y| = -4$
c. $|z| = 0$

Answer a

± 2

Answer b

no solution

Answer c

0

EXERCISE 5.5.3

Solve:

a. $|x| = 11$
b. $|y| = -5$

c. $|z| = 0$

Answer a

± 11

Answer b

no solution

Answer c

0

To solve an **absolute value equation**, we first isolate the absolute value expression using the same procedures we used to solve linear equations. Once we isolate the absolute value expression we rewrite it as the two equivalent equations.

How to Solve Absolute Value Equations

? Example 5.5.4

Solve $|5x - 4| - 3 = 8$.

Solution

Step 1. Isolate the absolute value expression.	Add 3 to both sides.	$\|5x-4\|-3=8$ $\|5x-4\|=11$
Step 2. Write the equivalent equations.		$5x-4=-11$ or $5x-4=11$
Step 3. Solve each equation.	Add 4 to each side. Divide each side by 5.	$5x=-7$ $\quad 5x=15$ $x=-\frac{7}{5}$ or $x=3$
Step 4. Check each solution.	Substitute 3 and $-\frac{7}{5}$ into the original equation.	$\|5x-4\|-3=8$ $x=3$ $\|5\cdot 3-4\|-3\stackrel{?}{=}8$ $\|15-4\|-3\stackrel{?}{=}8$ $\|11\|-3\stackrel{?}{=}8$ $11-3\stackrel{?}{=}8$ $8=8$ ✓ $\|5x-4\|-3=8$ $x=-\frac{7}{5}$ $\left\|5\left(-\frac{7}{5}\right)-4\right\|-3\stackrel{?}{=}8$ $\|-7-4\|-3\stackrel{?}{=}8$ $\|-11\|-3\stackrel{?}{=}8$ $11-3\stackrel{?}{=}8$ $8=8$ ✓

? EXERCISE 5.5.5

Solve: $|3x - 5| - 1 = 6$.

Answer

$x = 4,\ x = -\frac{2}{3}$

EXERCISE 5.5.6

Solve: $|4x - 3| - 5 = 2$.

Answer

$x = -1,\ x = \frac{5}{2}$

The steps for solving an absolute value equation are summarized here.

SOLVE ABSOLUTE VALUE EQUATIONS.

1. Isolate the absolute value expression.
2. Write the equivalent equations.
3. Solve each equation.
4. Check each solution.

Example 5.5.7

Solve $2|x - 7| + 5 = 9$.

Solution

	$2\|x - 7\| + 5 = 9$
Isolate the absolute value expression.	$2\|x - 7\| = 4$
	$\|x - 7\| = 2$
Write the equivalent equations.	$x - 7 = -2$ or $x - 7 = 2$
Solve each equation.	$x = 5$ or $x = 9$
Check: $2\|x-7\|+5=9$ $2\|5-7\|+5\stackrel{?}{=}9$ $2\|-2\|+5\stackrel{?}{=}9$ $2\cdot2+5\stackrel{?}{=}9$ $4+5\stackrel{?}{=}9$ $9=9$ ✓ $2\|x-7\|+5=9$ $2\|9-7\|+5\stackrel{?}{=}9$ $2\|2\|+5\stackrel{?}{=}9$ $2\cdot2+5\stackrel{?}{=}9$ $4+5\stackrel{?}{=}9$ $9=9$ ✓	

Exercise 5.5.8

Solve: $3|x - 4| - 4 = 8$.

Answer

$x = 8,\ x = 0$

Exercise 5.5.9

Solve: $2|x - 5| + 3 = 9$.

Answer

$x = 8,\ x = 2$

Remember, an absolute value is always positive!

Example 5.5.10

Solve: $|\frac{2}{3}x - 4| + 11 = 3$.

Solution

	$|\frac{2}{3}x - 4| = -8$
Isolate the absolute value term.	$|\frac{2}{3}x - 4| = -8$
An absolute value cannot be negative.	No solution

Exercise 5.5.11

Solve: $|\frac{3}{4}x - 5| + 9 = 4$.

Answer

No solution

Exercise 5.5.12

Solve: $|\frac{5}{6}x + 3| + 8 = 6$.

Answer

No solution

Some of our absolute value equations could be of the form $|u| = |v|$ where u and v are algebraic expressions. For example, $|x - 3| = |2x + 1|$.

How would we solve them? If two algebraic expressions are equal in absolute value, then they are either equal to each other or negatives of each other. The property for absolute value equations says that for any algebraic expression, u, and a positive real number, a, if $|u| = a$, then $u = -a$ or $u = a$.

This tell us that

if	$|u| = |v|$		
then	$|u| = v$	or	$|u| = -v$
and so	$u = v$ or $u = -v$	or	$u = -v$ or $u = -(-v)$

This leads us to the following property for equations with two absolute values.

EQUATIONS WITH TWO ABSOLUTE VALUES

For any algebraic expressions, u and v,

if	$|u| = |v|$
then	$u = -v$ or $u = v$

When we take the opposite of a quantity, we must be careful with the signs and to add parentheses where needed.

Example 5.5.13

Solve: $|5x-1|=|2x+3|$.

Solution

$$|5x-1|=|2x+3|$$

Write the equivalent equations.	$5x-1=-(2x+3)$	or	$5x-1=2x+3$
	$5x-1=-2x-3$	or	$3x-1=3$
Solve each equation.	$7x-1=-3$		$3x=4$
	$7x=-2$		$x=43$
	$x=-27$	or	$x=43$

Check.

We leave the check to you.

Exercise 5.5.14

Solve: $|7x-3|=|3x+7|$.

Answer

$x=-\frac{2}{5},\ x=\frac{5}{2}$

Exercise 5.5.15

Solve: $|6x-5|=|3x+4|$.

Answer

$x=3, x=19$

Solve Absolute Value Inequalities with "Less Than"

Let's look now at what happens when we have an **absolute value inequality**. Everything we've learned about solving inequalities still holds, but we must consider how the absolute value impacts our work. Again we will look at our definition of absolute value. The absolute value of a number is its distance from zero on the number line. For the equation $|x|=5$, we saw that both 5 and -5 are five units from zero on the number line. They are the solutions to the equation.

$$|x|=5$$
$$x=-5 \quad \text{or} \quad x=5$$

What about the inequality $|x|\leq 5$? Where are the numbers whose distance is less than or equal to 5? We know -5 and 5 are both five units from zero. All the numbers between -5 and 5 are less than five units from zero (Figure 5.5.2).

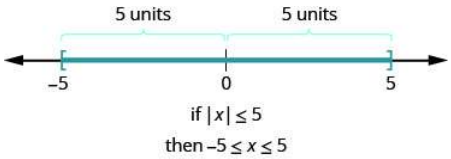


Figure 5.5.2.

In a more general way, we can see that if $|u|\leq a$, then $-a\leq u\leq a$ (Figure 5.5.3).

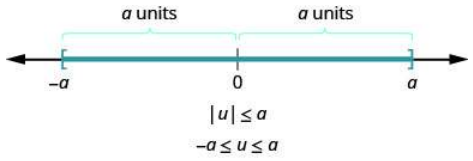


Figure 5.5.3.

This result is summarized here.

ABSOLUTE VALUE INEQUALITIES WITH $<$ OR $\leq$

For any algebraic expression, u, and any positive real number, a,

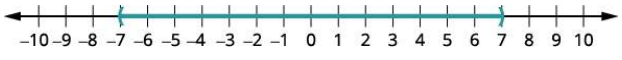
$$\text{if} \quad |u| < a, \quad \text{then} \; -a < u < a$$
$$\text{if} \quad |u| \leq a, \quad \text{then} \; -a \leq u \leq a$$

After solving an inequality, it is often helpful to check some points to see if the solution makes sense. The graph of the solution divides the number line into three sections. Choose a value in each section and substitute it in the original inequality to see if it makes the inequality true or not. While this is not a complete check, it often helps verify the solution.

Example 5.5.16

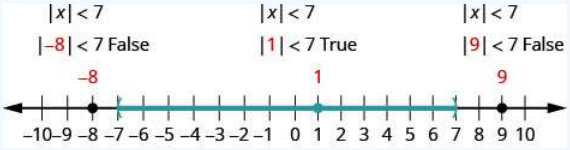
Solve $|x| < 7$. Graph the solution and write the solution in interval notation.

Solution

	$\lvert x\rvert < 7$
Write the equivalent inequality.	$-7 < x < 7$
Graph the solution.	
Write the solution using interval notation.	$(-7, 7)$

Check:

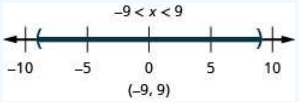
To verify, check a value in each section of the number line showing the solution. Choose numbers such as −8,−8, 1, and 9.



EXERCISE 5.5.17

Graph the solution and write the solution in interval notation: $|x| < 9$.

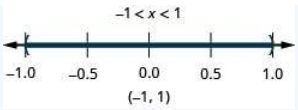
Answer



EXERCISE 5.5.18

Graph the solution and write the solution in interval notation: $|x| < 1$.

Answer



Example 5.5.19

Solve $|5x-6| \leq 4$. Graph the solution and write the solution in interval notation.

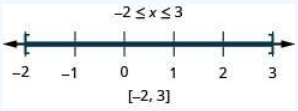
Solution

Step 1. Isolate the absolute value expression. It is isolated.	$\|5x-6\| \leq 4$
Step 2. Write the equivalent compound inequality.	$-4 \leq 5x-6 \leq 4$
Step 3. Solve the compound inequality.	$2 \leq 5x \leq 10$ $\frac{2}{5} \leq x \leq 2$
Step 4. Graph the solution.	
Step 5. Write the solution using interval notation.	$[\frac{2}{5}, 2]$
Check: The check is left to you.	

EXERCISE 5.5.20

Solve $|2x-1| \leq 5$. Graph the solution and write the solution in interval notation:

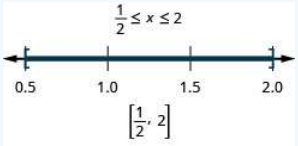
Answer



EXERCISE 5.5.21

Solve $|4x-5| \leq 3$. Graph the solution and write the solution in interval notation:

Answer



SOLVE ABSOLUTE VALUE INEQUALITIES WITH $<$ OR $\leq$

1. Isolate the absolute value expression.
2. Write the equivalent compound inequality.

$$|u| < a \quad \text{is equivalent to} \quad -a < u < a$$
$$|u| \leq a \quad \text{is equivalent to} \quad -a \leq u \leq a$$

3. Solve the compound inequality.
4. Graph the solution

5. Write the solution using interval notation.

Solve Absolute Value Inequalities with “Greater Than”

What happens for absolute value inequalities that have “greater than”? Again we will look at our definition of absolute value. The absolute value of a number is its distance from zero on the number line.

We started with the inequality $|x| \leq 5$. We saw that the numbers whose distance is less than or equal to five from zero on the number line were -5 and 5 and all the numbers between -5 and 5 (Figure 5.5.4).

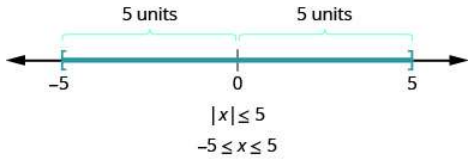


Figure 5.5.4.

Now we want to look at the inequality $|x| \geq 5$. Where are the numbers whose distance from zero is greater than or equal to five?

Again both -5 and 5 are five units from zero and so are included in the solution. Numbers whose distance from zero is greater than five units would be less than -5 and greater than 5 on the number line (Figure 5.5.5).

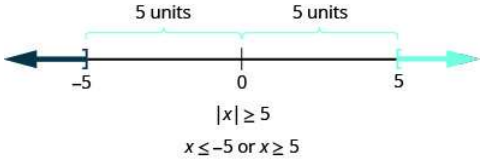


Figure 5.5.5.

In a more general way, we can see that if $|u| \geq a$, then $u \leq -a$ or $u \leq a$. See Figure.

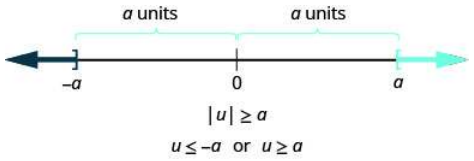


Figure 5.5.6.

This result is summarized here.

ABSOLUTE VALUE INEQUALITIES WITH $>$ OR $\geq$

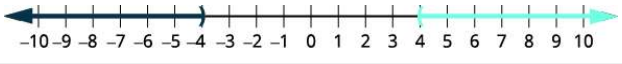
For any algebraic expression, *u*, and any positive real number, *a*,

$$\begin{aligned} &\text{if} \quad |u| > a, \quad &&\text{then } u < -a \text{ or } u > a \\ &\text{if} \quad |u| \geq a, \quad &&\text{then } u \leq -a \text{ or } u \geq a \end{aligned}$$

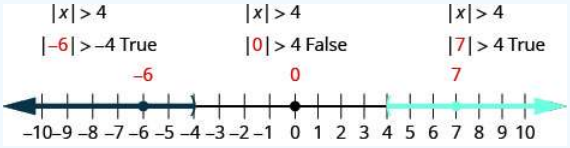
Example 5.5.22

Solve $|x| > 4$. Graph the solution and write the solution in interval notation.

Solution

	$\lvert x\rvert > 4$
Write the equivalent inequality.	$x < -4$ or $x > 4$
Graph the solution.	
Write the solution using interval notation.	$(-\inf, -4) \cup (4, \inf)$
Check:	

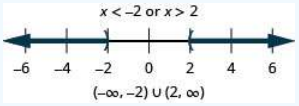
To verify, check a value in each section of the number line showing the solution. Choose numbers such as −6,−6, 0, and 7.



EXERCISE 5.5.23

Solve $|x| > 2$. Graph the solution and write the solution in interval notation.

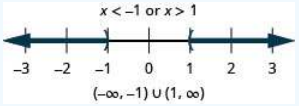
Answer



EXERCISE 5.5.24

Solve $|x| > 1$. Graph the solution and write the solution in interval notation.

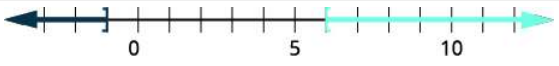
Answer



Example 5.5.25

Solve $|2x-3| \geq 5$. Graph the solution and write the solution in interval notation.

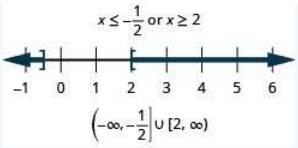
Solution

	$\lvert 2x-3\rvert \geq 5$
Step 1. Isolate the absolute value expression. It is isolated.	
Step 2. Write the equivalent compound inequality.	$2x-3 \leq -5$ or $2x-3 \geq 5$
Step 3. Solve the compound inequality.	$2x \leq -2$ or $2x \geq 8$ $x \leq -1$ or $x \geq 4$
Step 4. Graph the solution.	
Step 5. Write the solution using interval notation.	$(-\inf, -1] \cup [4, \inf)$
Check: The check is left to you.	

EXERCISE 5.5.26

Solve $|4x-3| \geq 5$. Graph the solution and write the solution in interval notation.

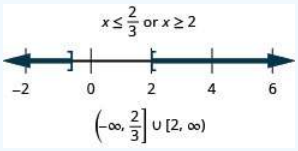
Answer



EXERCISE 5.5.27

Solve $|3x - 4| \geq 2$. Graph the solution and write the solution in interval notation.

Answer



SOLVE ABSOLUTE VALUE INEQUALITIES WITH $>$ OR $\geq$.

1. Isolate the absolute value expression.
2. Write the equivalent compound inequality.

$$
\begin{array}{llllll}
|u| > a & \text{is equivalent to} & u < -a & \text{or} & u > a \\
|u| \geq a & \text{is equivalent to} & u \leq -a & \text{or} & u \geq a \\
|u| > a & \text{is equivalent to} & u < -a & \text{or} & u > a \\
|u| \geq a & \text{is equivalent to} & u \leq -a & \text{or} & u \geq a
\end{array}
$$

3. Solve the compound inequality.
4. Graph the solution
5. Write the solution using interval notation.

Solve Applications with Absolute Value

Absolute value inequalities are often used in the manufacturing process. An item must be made with near perfect specifications. Usually there is a certain *tolerance* of the difference from the specifications that is allowed. If the difference from the specifications exceeds the tolerance, the item is rejected.

$$|\text{actual-ideal}| \leq \text{tolerance}$$

Example 5.5.28

The ideal diameter of a rod needed for a machine is 60 mm. The actual diameter can vary from the ideal diameter by 0.075 mm. What range of diameters will be acceptable to the customer without causing the rod to be rejected?

Solution

	Let $x =$ the actual measurement
Use an absolute value inequality to express this situation.	$\|\text{actual-ideal}\| \leq \text{tolerance}$
	$\|x - 60\| \leq 0.075$
Rewrite as a compound inequality.	$-0.075 \leq x - 60 \leq 0.075$
Solve the inequality.	$59.925 \leq x \leq 60.075$
Answer the question.	The diameter of the rod can be between $59.925mm$ and $60.075mm$.

? ExERCISE 5.5.29

The ideal diameter of a rod needed for a machine is 80 mm. The actual diameter can vary from the ideal diameter by 0.009 mm. What range of diameters will be acceptable to the customer without causing the rod to be rejected?

Answer

The diameter of the rod can be between 79.991 and 80.009 mm.

? ExERCISE 5.5.30

The ideal diameter of a rod needed for a machine is 75 mm. The actual diameter can vary from the ideal diameter by 0.05 mm. What range of diameters will be acceptable to the customer without causing the rod to be rejected?

Answer

The diameter of the rod can be between 74.95 and 75.05 mm.

Access this online resource for additional instruction and practice with solving linear absolute value equations and inequalities.

- Solving Linear Absolute Value Equations and Inequalities

Key Concepts

- **Absolute Value**
  The absolute value of a number is its distance from 0 on the number line.
  The absolute value of a number *n* is written as $|n|$ and $|n| \geq 0$ for all numbers.
  Absolute values are always greater than or equal to zero.
- **Absolute Value Equations**
  For any algebraic expression, *u*, and any positive real number, *a*,
  if $|u| = a$
  then $u = -a$ or $u = a$
  Remember that an absolute value cannot be a negative number.
- **How to Solve Absolute Value Equations**
  1. Isolate the absolute value expression.
  2. Write the equivalent equations.
  3. Solve each equation.
  4. Check each solution.
- **Equations with Two Absolute Values**
  For any algebraic expressions, *u* and *v*,
  if $|u| = |v|$
  then $u = -v$ or $u = v$
- **Absolute Value Inequalities with** $<$ or $\leq$
  For any algebraic expression, *u*, and any positive real number, *a*,
  if $|u| = a$ then $-a < u < a$
  if $|u| \leq a$ then $-a \leq u \leq a$
- **How To Solve Absolute Value Inequalities with** $<$ or $\leq$
  1. Isolate the absolute value expression.
  2. Write the equivalent compound inequality.
     $|u| < a$ is equivalent to $-a < u < a$
     $|u| \leq a$ is equivalent to $-a \leq u \leq a$
  3. Solve the compound inequality.
  4. Graph the solution
  5. Write the solution using interval notation

- **Absolute Value Inequalities with $>$ or $\geq$**
  For any algebraic expression, u, and any positive real number, a,
  if $|u| > a$, then $u < -a$ or $u > a$
  if $|u| \geq a$, then $u \leq -a$ or $u \geq a$
- **How To Solve Absolute Value Inequalities with $>$ or $\geq$**
  1. Isolate the absolute value expression.
  2. Write the equivalent compound inequality.
     $|u| > a$ is equivalent to $u < -a$ or $u > a$
     $|u| \geq a$ is equivalent to $u \leq -a$ or $u \geq a$
  3. Solve the compound inequality.
  4. Graph the solution
  5. Write the solution using interval notation

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5.5E: Exercises

Practice Makes Perfect

Solve Absolute Value Equations

In the following exercises, solve.

1. a. $|x| = 6$ b. $|y| = -3$ c. $|z| = 0$

2. a. $|x| = 4$ b. $|y| = -5$ c. $|z| = 0$

Answer

a. $x = 4, x = -4$ b. no solution c. $z = 0$

3. a. $|x| = 7$ b. $|y| = -11$ c. $|z| = 0$

4. a. $|x| = 3$ b. $|y| = -1$ c. $|z| = 0$

Answer

a. $x = 3, x = -3$ b. no solution c. $z = 0$

5. $|2x - 3| - 4 = 1$

6. $|4x - 1| - 3 = 0$

Answer

$x = 1,\ x = -\frac{1}{2}$

7. $|3x - 4| + 5 = 7$

8. $|4x + 7| + 2 = 5$

Answer

$x = -1,\ x = -\frac{5}{2}$

9. $4|x - 1| + 2 = 10$

10. $3|x - 4| + 2 = 11$

Answer

$x = 7,\ x = 1$

11. $3|4x - 5| - 4 = 11$

12. $3|x + 2| - 5 = 4$

Answer

$x = 1,\ x = -5$

13. $-2|x-3|+8=-4$

14. $-3|x-4|+4=-5$

Answer

$x=7,\ x=1$

15. $|34x-3|+7=2$

16. $|35x-2|+5=2$

Answer

no solution

17. $|12x+5|+4=1$

18. $|14x+3|+3=1$

Answer

no solution

19. $|3x-2|=|2x-3|$

20. $|4x+3|=|2x+1|$

Answer

$x=-1,\ x=-\frac{2}{3}$

21. $|6x-5|=|2x+3|$

22. $|6-x|=|3-2x|$

Answer

$x=-3,\ x=3$

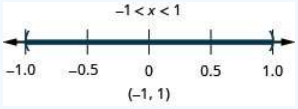
Solve Absolute Value Inequalities with "less than"

In the following exercises, solve each inequality. Graph the solution and write the solution in interval notation.

23. $|x|<5$

24. $|x|<1$

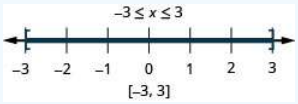
Answer



25. $|x|\leq 8$

26. $|x| \leq 3$

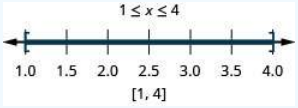
Answer



27. $|3x - 3| \leq 6$

28. $|2x - 5| \leq 3$

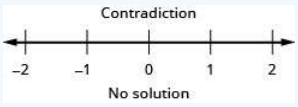
Answer



29. $|2x + 3| + 5 < 4$

30. $|3x - 7| + 3 < 1$

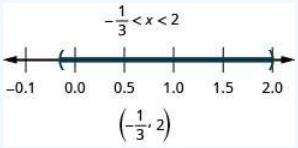
Answer



31. $|4x - 3| < 1$

32. $|6x - 5| < 7$

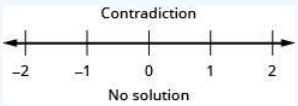
Answer



33. $|x - 4| \leq -1$

34. $|5x + 1| \leq -2$

Answer



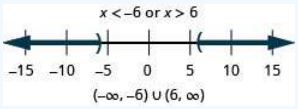
Solve Absolute Value Inequalities with "greater than"

In the following exercises, solve each inequality. Graph the solution and write the solution in interval notation.

35. $|x| > 3$

36. $|x| > 6$

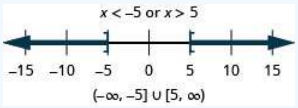
Answer



37. $|x| \geq 2$

38. $|x| \geq 5$

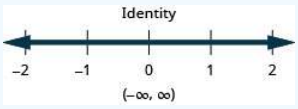
Answer



39. $|3x - 8| > -1$

40. $|x - 5| > -2$

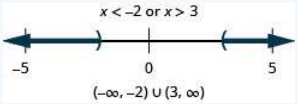
Answer



41. $|3x - 2| > 4$

42. $|2x - 1| > 5$

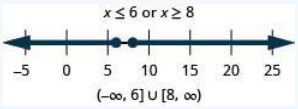
Answer



43. $|x + 3| \geq 5$

44. $|x - 7| \geq 1$

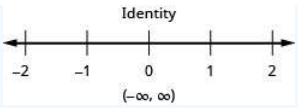
Answer



45. $3|x| + 4 \geq 1$

46. $5|x| + 6 \geq 1$

Answer



In the following exercises, solve. For each inequality, also graph the solution and write the solution in interval notation.

47. $2|x+6|+4=8$

48. $|3x-4| \geq 2$

Answer

$x=4, x=27$

49. $|6x-5|=|2x+3|$

50. $|4x-3|<5$

Answer

$x=3, x=2$

51. $|2x-5|+2=3$

52. $|3x+1|-3=7$

Answer

$x=3, x=-\frac{11}{3}$

53. $|7x+2|+8<4$

54. $5|2x-1|-3=7$

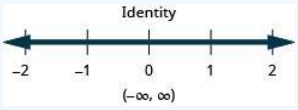
Answer

$x=\frac{3}{2}, x=-\frac{1}{2}$

55. $|x-7|>-3$

56. $|8-x|=|4-3x|$

Answer



Solve Applications with Absolute Value

In the following exercises, solve.

57. A chicken farm ideally produces 200,000 eggs per day. But this total can vary by as much as 25,000 eggs. What is the maximum and minimum expected production at the farm?

58. An organic juice bottler ideally produces 215,000 bottle per day. But this total can vary by as much as 7,500 bottles. What is the maximum and minimum expected production at the bottling company?

Answer

The minimum to maximum expected production is 207,500 to 2,225,000 bottles

59. In order to insure compliance with the law, Miguel routinely overshoots the weight of his tortillas by 0.5 gram. He just received a report that told him that he could be losing as much as $100,000 per year using this practice. He now plans to buy new equipment that guarantees the thickness of the tortilla within 0.005 inches. If the ideal thickness of the tortilla is 0.04 inches, what thickness of tortillas will be guaranteed?

60. At Lilly's Bakery, the ideal weight of a loaf of bread is 24 ounces. By law, the actual weight can vary from the ideal by 1.5 ounces. What range of weight will be acceptable to the inspector without causing the bakery being fined?

Answer

The acceptable weight is 22.5 to 25.5 ounces.

Writing Exercises

61. Write a graphical description of the absolute value of a number

62. In your own words, explain how to solve the absolute value inequality, $|3x-2| \geq 4$.

Answer

Answers will vary.

Self Check

a. After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
solve absolute value equations.			
solve absolute value inequalities with "less than".			
solve absolute value inequalities with "greater than".			
solve applications with absolute value.			

b. What does this checklist tell you about your mastery of this section? What steps will you take to improve?

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CHAPTER OVERVIEW

6: Exponential Functions

6.1: Integer Exponents and Scientific Notation

6.1.1: Exercises

6.2: Exponential Functions

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6.1: Integer Exponents and Scientific Notation

Learning Objectives

By the end of this section, you will be able to:

- Use the definition of a negative exponent
- Simplify expressions with integer exponents
- Convert from decimal notation to scientific notation
- Convert scientific notation to decimal form
- Multiply and divide using scientific notation

Note

Before you get started, take this readiness quiz.

1. What is the place value of the 6 in the number 64891?
   If you missed this problem, review Example 1.2.1.
2. Name the decimal: 0.0012.
   If you missed this problem, review Example 1.8.1.
3. Subtract: 5−(−3).
   If you missed this problem, review Example 1.4.33.

Use the Definition of a Negative Exponent

We saw that the Quotient Property for Exponents introduced earlier in this chapter, has two forms depending on whether the exponent is larger in the numerator or the denominator.

QUOTIENT PROPERTY FOR EXPONENTS

If a is a real number, $a \neq 0$, and m and n are whole numbers, then

$$\frac{a^m}{a^n} = a^{m-n}, m > n \tag{6.1.1}$$

and

$$\frac{a^m}{a^n} = \frac{1}{a^{n-m}}, n > m \tag{6.1.2}$$

What if we just subtract exponents regardless of which is larger?

Let's consider $\frac{x^2}{x^5}$.

We subtract the exponent in the denominator from the exponent in the numerator.

$$\begin{array}{c} \frac{x^2}{x^5} \\ x^{2-5} \\ x^{-3} \end{array} \tag{6.1.3}$$

We can also simplify $\frac{x^2}{x^5}$ by dividing out common factors:

$$\begin{array}{c} \frac{x \cdot x}{x \cdot x \cdot x \cdot x \cdot x} \\ \frac{1}{x^3} \end{array}$$

This implies that $x^{-3} = \frac{1}{x^3}$ and it leads us to the definition of a *negative exponent*.

Definition: NEGATIVE EXPONENT

If n is an integer and $a \neq 0$, then $a^{-n} = \frac{1}{a^n}$

The **negative exponent** tells us we can re-write the expression by taking the reciprocal of the base and then changing the sign of the exponent.

Any expression that has negative exponents is not considered to be in simplest form. We will use the definition of a **negative exponent** and other properties of exponents to write the expression with only positive exponents.

For example, if after simplifying an expression we end up with the expression x^{-3}, we will take one more step and write $\frac{1}{x^3}$. The answer is considered to be in simplest form when it has only positive exponents.

Example 6.1.1

Simplify:

1. 4^{-2}
2. 10^{-3}

Solution

1.

	4^{-2}
Use the definition of a negative exponent, $a^{-n} = \frac{1}{a^n}$,	$\frac{1}{4^2}$
Simplify.	$\frac{1}{16}$

2.

	10^{-3}
Use the definition of a negative exponent, $a^{-n} = \frac{1}{a^n}$,	$\frac{1}{10^3}$
Simplify.	$\frac{1}{1000}$

Try It 6.1.2

Simplify:

1. 2^{-3}
2. 10^{-7}

Answer

1. $\frac{1}{8}$
2. $\frac{1}{10^7}$

Try It 6.1.3

Simplify:

1. 3^{-2}
2. 10^{-4}

Answer

1. $\frac{1}{9}$
2. $\frac{1}{10,000}$

In Example 6.1.1 we raised an integer to a negative exponent. What happens when we raise a fraction to a negative exponent? We'll start by looking at what happens to a fraction whose numerator is one and whose denominator is an integer raised to a negative exponent.

	$\frac{1}{a^{-n}}$
Use the definition of a negative exponent, $a^{-n}=\frac{1}{a^n}$	$\frac{1}{\frac{1}{a^n}}$
Simplify the complex fraction.	$1\cdot\frac{a^n}{1}$
Multiply.	a^n

This leads to the Property of Negative Exponents.

PROPERTY OF NEGATIVE EXPONENTS

If n is an integer and $a\neq 0$, then $\frac{1}{a^{-n}}=a^n$.

Example 6.1.4

Simplify:

1. $\frac{1}{y^{-4}}$
2. $\frac{1}{3^{-2}}$

Solution

1.

	$\frac{1}{y^{-4}}$
Use the property of a negative exponent, $\frac{1}{a^{-n}}=a^n$.	y^4

2.

	$\frac{1}{3^{-2}}$
Use the property of a negative exponent, $\frac{1}{a^{-n}}=a^n$.	3^2
Simplify.	9

Try It 6.1.5

Simplify:

1. $\frac{1}{p^{-8}}$
2. $\frac{1}{4^{-3}}$

Answer

1. p^8
2. 64

Try It 6.1.6

Simplify:

1. $\dfrac{1}{q^{-7}}$
2. $\dfrac{1}{2^{-4}}$

Answer

1. q^7
2. 16

Suppose now we have a fraction raised to a negative exponent. Let's use our definition of negative exponents to lead us to a new property.

	$\left(\frac{3}{4}\right)^{-2}$
Use the definition of a negative exponent, $a^{-n}=\frac{1}{a^n}$	$\frac{1}{\left(\frac{3}{4}\right)^{2}}$
Simplify the denominator.	$\frac{1}{\frac{9}{16}}$
Simplify the complex fraction.	$\frac{16}{9}$
But we know that $\frac{16}{9}$ is $\left(\frac{4}{3}\right)^2$	
This tells us that:	$\left(\frac{3}{4}\right)^{-2}=\left(\frac{4}{3}\right)^{2}$

To get from the original fraction raised to a negative exponent to the final result, we took the reciprocal of the base—the fraction—and changed the sign of the exponent.

This leads us to the *Quotient to a Negative Power Property*.

QUOTIENT TO A NEGATIVE EXPONENT PROPERTY

If a and b are real numbers, $a\neq 0, b\neq 0,$ and n is an integer, then $\left(\frac{a}{b}\right)^{-n}=\left(\frac{b}{a}\right)^{n}$

Example 6.1.7

Simplify:

1. $\left(\frac{5}{7}\right)^{-2}$
2. $\left(-\frac{2x}{y}\right)^{-3}$

Solution

1. $\left(\frac{5}{7}\right)^{-2}$

   Use the Quotient to a Negative Exponent Property, $\left(\frac{a}{b}\right)^{-n}=\left(\frac{b}{a}\right)^{n}$. Take the reciprocal of the fraction and change the sign of the exponent. . $\left(\frac{7}{5}\right)^{2}$

   Simplify. $\frac{49}{25}$

2. $\left(-\frac{2x}{y}\right)^{-3}$

   Use the Quotient to a Negative Exponent Property, $\left(\frac{a}{b}\right)^{-n}=\left(\frac{b}{a}\right)^{n}$. Take the reciprocal of the fraction and change the sign of the exponent. $\left(-\frac{y}{2x}\right)^{3}$

   Simplify. $-\frac{y^3}{8x^3}$

? Try It 6.1.8

Simplify:

1. $\left(\frac{2}{3}\right)^{-4}$
2. $\left(-\frac{6m}{n}\right)^{-2}$

Answer

1. $\frac{81}{16}$
2. $\frac{n^2}{36m^2}$

? Try It 6.1.9

Simplify:

1. $\left(\frac{3}{5}\right)^{-3}$
2. $\left(-\frac{a}{2b}\right)^{-4}$

Answer

1. $\frac{125}{27}$
2. $\frac{16b^4}{a^4}$

When simplifying an expression with exponents, we must be careful to correctly identify the base.

Example 6.1.10

Simplify:

1. $(-3)^{-2}$
2. -3^{-2}
3. $\left(-\frac{1}{3}\right)^{-2}$
4. $-\left(\frac{1}{3}\right)^{-2}$

Solution

1. Here the exponent applies to the base -3.

	$(-3)^{-2}$
Take the reciprocal of the base and change the sign of the exponent.	$\frac{1}{(-3)^2}$
Simplify.	$\frac{1}{9}$

2. The expression -3^{-2} means "find the opposite of 3^{-2}". Here the exponent applies to the base 3.

	-3^{-2}
Rewrite as a product with -1	$-1 \cdot 3^{-2}$
Take the reciprocal of the base and change the sign of the exponent.	$-1 \cdot \frac{1}{3^2}$
Simplify.	$-\frac{1}{9}$

3. Here the exponent applies to the base $\left(-\frac{1}{3}\right)$.

	$\left(-\frac{1}{3}\right)^{-2}$
Take the reciprocal of the base and change the sign of the exponent.	$\left(-\frac{3}{1}\right)^{2}$
Simplify.	9

4. The expression $-\left(\frac{1}{3}\right)^{-2}$ means "find the opposite of $\left(\frac{1}{3}\right)^{-2}$". Here the exponent applies to the base $\left(\frac{1}{3}\right)$.

	$-\left(\frac{1}{3}\right)^{-2}$
Rewrite as a product with -1	$-1 \cdot \left(\frac{1}{3}\right)^{-2}$
Take the reciprocal of the base and change the sign of the exponent.	$-1 \cdot \left(\frac{3}{1}\right)^{2}$
Simplify.	-9

Try It 6.1.11

Simplify:

1. $(-5)^{-2}$
2. -5^{-2}
3. $\left(-\frac{1}{5}\right)^{-2}$

4. $-\left(\frac{1}{5}\right)^{-2}$

Answer

1. $\frac{1}{25}$
2. $-\frac{1}{25}$
3. 25
4. -25

Try It 6.1.12

Simplify:

1. $(-7)^{-2}$
2. -7^{-2}
3. $\left(-\frac{1}{7}\right)^{-2}$
4. $-\left(\frac{1}{7}\right)^{-2}$

Answer

1. $\frac{1}{49}$
2. $-\frac{1}{49}$
3. 49
4. -49

We must be careful to follow the Order of Operations. In the next example, parts (a) and (b) look similar, but the results are different.

Example 6.1.13

Simplify:

1. $4 \cdot 2^{-1}$
2. $(4 \cdot 2)^{-1}$

Solution

1.

Do exponents before multiplication.	$4 \cdot 2^{-1}$
Use $a^{-n} = \frac{1}{a^n}$	$4 \cdot \frac{1}{2^1}$
Simplify.	2

2.

	$(4 \cdot 2)^{-1}$
Simplify inside the parentheses first.	$(8)^{-1}$
Use $a^{-n} = \frac{1}{a^n}$	$\frac{1}{8^1}$
Simplify.	$\frac{1}{8}$

Try It 6.1.14

Simplify:

1. $6 \cdot 3^{-1}$
2. $(6 \cdot 3)^{-1}$

Answer

1. 2
2. $\frac{1}{18}$

Try It 6.1.15

Simplify:

1. $8 \cdot 2^{-2}$
2. $(8 \cdot 2)^{-2}$

Answer

1. 2
2. $\frac{1}{256}$

When a variable is raised to a negative exponent, we apply the definition the same way we did with numbers. We will assume all variables are non-zero.

Example 6.1.16

Simplify:

1. x^{-6}
2. $\left(u^4\right)^{-3}$

Solution

1. Use the definition of a negative exponent, $a^{-n} = \frac{1}{a^n}$ $\quad x^{-6}$ $\quad \frac{1}{x^6}$

2. $\left(u^4\right)^{-3}$

   Use the definition of a negative exponent, $a^{-n} = \frac{1}{a^n}$ $\quad \frac{1}{\left(u^4\right)^3}$

   Simplify. $\quad \frac{1}{u^{12}}$

Try It 6.1.17

Simplify:

1. y^{-7}
2. $\left(z^3\right)^{-5}$

Answer

1. $\frac{1}{y^7}$

2. $\dfrac{1}{z^{15}}$

Try It 6.1.18

Simplify:

1. p^{-9}
2. $\left(q^{4}\right)^{-6}$

Answer

1. $\dfrac{1}{p^{9}}$
2. $\dfrac{1}{q^{24}}$

When there is a product and an exponent we have to be careful to apply the exponent to the correct quantity. According to the Order of Operations, we simplify expressions in parentheses before applying exponents. We'll see how this works in the next example.

Example 6.1.19

Simplify:

1. $5y^{-1}$
2. $(5y)^{-1}$
3. $(-5y)^{-1}$

Solution

1. Notice the exponent applies to just the base y.

	$5y^{-1}$
Take the reciprocal of the base y, and change the sign of the exponent.	$5 \cdot \dfrac{1}{y^{1}}$
Simplify.	$\dfrac{5}{y}$

2. Here the parentheses make the exponent apply to the base $5y$

	$(5y)^{-1}$
Take the reciprocal of the base $5y$, and change the sign of the exponent.	$\dfrac{1}{(5y)^{1}}$
Simplify.	$\dfrac{1}{5y}$

3. The base here is $-5y$

	$(-5y)^{-1}$
Take the reciprocal of the base $-5y$ and change the sign of the exponent.	$\dfrac{1}{(-5y)^{1}}$
Simplify.	$\dfrac{1}{-5y}$
Use $\dfrac{a}{-b}=-\dfrac{a}{b}$	$-\dfrac{1}{5y}$

Try It 6.1.20

Simplify:

1. $8p^{-1}$
2. $(8p)^{-1}$
3. $(-8p)^{-1}$

Answer

1. $\frac{8}{p}$
2. $\frac{1}{8p}$
3. $-\frac{1}{8p}$

Try It 6.1.21

Simplify:

1. $11q^{-1}$
2. $(11q)^{-1}$
3. $(-11q)^{-1}$

Answer

1. $\frac{11}{q}$
2. $\frac{1}{11q}$
3. $-\frac{1}{11q}$

With negative exponents, the Quotient Rule needs only one form $\frac{a^m}{a^n} = a^{m-n}$, for $a \neq 0$. When the exponent in the denominator is larger than the exponent in the numerator, the exponent of the quotient will be negative.

Simplify Expressions with Integer Exponents

All of the exponent properties we developed earlier in the chapter with whole number exponents apply to integer exponents, too. We restate them here for reference.

SUMMARY OF EXPONENT PROPERTIES

If a and b are real numbers, and m and n are integers, then

Product Property	$a^m \cdot a^n$	$=$	a^{m+n}
Power Property	$(a^m)^n$	$=$	$a^{m \cdot n}$
Product to a Power	$(ab)^m$	$=$	$a^m b^m$
Quotient Property	$\frac{a^m}{a^n}$	$=$	$a^{m-n}, \quad a \neq 0$
Zero Exponent Property	a^0	$=$	$1, \quad a \neq 0$
Quotient to a Power Property	$\left(\frac{a}{b}\right)^m$	$=$	$\frac{a^m}{b^m}, \quad b \neq 0$
Properties of Negative Exponents	a^{-n}	$=$	$\frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$
Quotient to a Negative Exponents	$\left(\frac{a}{b}\right)^{-n}$	$=$	$\left(\frac{b}{a}\right)^n$

Example 6.1.22

Simplify:

1. $x^{-4} \cdot x^6$
2. $y^{-6} \cdot y^4$
3. $z^{-5} \cdot z^{-3}$

Solution

1.

	$x^{-4} \cdot x^6$
Use the Product Property, $a^m \cdot a^n = a^{m+n}$	x^{-4+6}
Simplify.	x^2

2.

	$y^{-6} \cdot y^4$
Notice the same bases, so add the exponents.	y^{-6+4}
Simplify.	y^{-2}
Use the definition of a negative exponent, $a^{-n} = \frac{1}{a^n}$	$\frac{1}{y^2}$

3.

	$z^{-5} \cdot z^{-3}$
Add the exponents, since the bases are the same.	z^{-5-3}
Simplify.	z^{-8}
Use the definition of a negative exponent to take the reciprocal of the base and change the sign of the exponent.	$\frac{1}{z^8}$

Try It 6.1.23

Simplify:

1. $x^{-3} \cdot x^7$
2. $y^{-7} \cdot y^2$
3. $z^{-4} \cdot z^{-5}$

Answer

1. x^4
2. $\frac{1}{y^5}$

3. $\frac{1}{z^9}$

Try It 6.1.24

Simplify:

1. $a^{-1} \cdot a^6$
2. $b^{-8} \cdot b^4$
3. $c^{-8} \cdot c^{-7}$

Answer

1. a^5
2. $\frac{1}{b^4}$
3. $\frac{1}{c^{15}}$

In the next two examples, we'll start by using the Commutative Property to group the same variables together. This makes it easier to identify the like bases before using the Product Property.

Example 6.1.25

Simplify: $\left(m^4 n^{-3}\right)\left(m^{-5} n^{-2}\right)$

Solution

	$\left(m^4 n^{-3}\right)\left(m^{-5} n^{-2}\right)$
Use the Commutative Property to get like bases together.	$m^4 m^{-5} \cdot n^{-2} n^{-3}$
Add the exponents for each base.	$m^{-1} \cdot n^{-5}$
Take reciprocals and change the signs of the exponents.	$\frac{1}{m^1} \cdot \frac{1}{n^5}$
Simplify.	$\frac{1}{mn^5}$

Try It 6.1.26

Simplify: $\left(p^6 q^{-2}\right)\left(p^{-9} q^{-1}\right)$

Answer

$\frac{1}{p^3 q^3}$

Try It 6.1.27

Simplify: $\left(r^5 s^{-3}\right)\left(r^{-7} s^{-5}\right)$

Answer

$\frac{1}{r^2 s^8}$

If the monomials have numerical coefficients, we multiply the coefficients, just like we did earlier.

Example 6.1.28

Simplify: $\left(2x^{-6}y^{8}\right)\left(-5x^{5}y^{-3}\right)$

Solution

	$\left(2x^{-6}y^{8}\right)\left(-5x^{5}y^{-3}\right)$
Rewrite with the like bases together.	$2(-5)\cdot\left(x^{-6}x^{5}\right)\cdot\left(y^{8}y^{-3}\right)$
Multiply the coefficients and add the exponents of each variable.	$-10\cdot x^{-1}\cdot y^{5}$
Use the definition of a negative exponent, $a^{-n}=\frac{1}{a^{n}}$	$-10\cdot\frac{1}{x^{1}}\cdot y^{5}$
Simplify.	$\frac{-10y^{5}}{x}$

Try It 6.1.29

Simplify: $\left(3u^{-5}v^{7}\right)\left(-4u^{4}v^{-2}\right)$

Answer

$-\frac{12v^{5}}{u}$

Try It 6.1.30

Simplify: $\left(-6c^{-6}d^{4}\right)\left(-5c^{-2}d^{-1}\right)$

Answer

$\frac{30d^{3}}{c^{8}}$

In the next two examples, we'll use the Power Property and the Product to a Power Property.

Example 6.1.31

Simplify: $\left(6k^{3}\right)^{-2}$

Solution

	$\left(6k^{3}\right)^{-2}$
Use the Product to a Power Property, $(ab)^{m}=a^{n}b^{m}$	$(6)^{-2}\left(k^{3}\right)^{-2}$
Use the Power Property, $(a^{m})^{n}=a^{m\cdot n}$	$6^{-2}k^{-6}$
Use the definition of a negative exponent, $a^{-n}=\frac{1}{a^{n}}$	$\frac{1}{6^{2}}\cdot\frac{1}{k^{6}}$
Simplify.	$\frac{1}{36k^{6}}$

Try It 6.1.32

Simplify: $\left(-4x^{4}\right)^{-2}$

Answer

$\frac{1}{16x^{8}}$

Try It 6.1.33

Simplify: $\left(2b^3\right)^{-4}$

Answer

$$\frac{1}{16b^{12}}$$

Example 6.1.34

Simplify: $\left(5x^{-3}\right)^2$

Solution

	$\left(5x^{-3}\right)^2$
Use the Product to a Power Property, $(ab)^m = a^n b^m$	$5^2\left(x^{-3}\right)^2$
Simplify 5^2 and multiply the exponents of x using the Power Property, $(a^m)^n = a^{m \cdot n}$.	$25 \cdot x^{-6}$
Rewrite x^{-6} by using the Definition of a Negative Exponent, $a^{-n} = \frac{1}{a^n}$	$25 \cdot \frac{1}{x^6}$
Simplify.	$\frac{25}{x^6}$

Try It 6.1.35

Simplify: $\left(8a^{-4}\right)^2$

Answer

$$\frac{64}{a^8}$$

Try It 6.1.36

Simplify: $\left(2c^{-4}\right)^3$

Answer

$$\frac{8}{c^{12}}$$

To simplify a fraction, we use the Quotient Property and subtract the exponents.

Example 6.1.37

Simplify: $\frac{r^5}{r^{-4}}$

Solution

	$\frac{r^5}{r^{-4}}$
Use the Quotient Property, $\frac{a^n}{a^n} = a^{m-n}$	$r^{5-(-4)}$
Simplify.	r^9

Try It 6.1.38

Simplify: $\frac{x^8}{x^{-3}}$

Answer

x^{11}

Try It 6.1.39

Simplify: $\frac{y^8}{y^{-6}}$

Answer

y^{14}

Convert from Decimal Notation to Scientific Notation

Remember working with place value for whole numbers and decimals? Our number system is based on powers of 10. We use tens, hundreds, thousands, and so on. Our decimal numbers are also based on powers of tens—tenths, hundredths, thousandths, and so on. Consider the numbers 4,000 and 0.004. We know that 4,000 means $4 \times 1,000$ and 0.004 means $4 \times \frac{1}{1,000}$.

If we write the 1000 as a power of ten in exponential form, we can rewrite these numbers in this way:

$$\begin{array}{ll} 4,000 & 0.004 \\ 4 \times 1,000 & 4 \times \frac{1}{1,000} \\ 4 \times 10^3 & 4 \times \frac{1}{10^3} \\ & 4 \times 10^{-3} \end{array} \tag{6.1.4}$$

When a number is written as a product of two numbers, where the first factor is a number greater than or equal to one but less than 10, and the second factor is a power of 10 written in exponential form, it is said to be in *scientific notation.*

SCIENTIFIC NOTATION

A number is expressed in **scientific notation** when it is of the form

$$a \times 10^n \text{ where } 1 \leq a < 10 \text{ and } n \text{ is an integer} \tag{6.1.5}$$

It is customary in scientific notation to use as the $\times$ multiplication sign, even though we avoid using this sign elsewhere in algebra.

If we look at what happened to the decimal point, we can see a method to easily convert from decimal notation to scientific notation.

$4000. = 4 \times 10^3$	$0.004 = 4 \times 10^{-3}$
$4000. = 4 \times 10^3$	$0.004 = 4 \times 10^{-3}$
Moved the decimal point 3 places to the left.	Moved the decimal point 3 places to the right.

In both cases, the decimal was moved 3 places to get the first factor between 1 and 10.

The power of 10 is positive when the number is larger than 1 : $4,000 = 4 \times 10^3$

The power of 10 is negative when the number is between 0 and 1 : $0.004 = 4 \times 10^{-3}$

Example 6.1.40: HOW TO CONVERT FROM DECIMAL NOTATION TO SCIENTIFIC NOTATION

Write in scientific notation: 37000.

Solution

Step 1. Move the decimal point so that the first factor is greater than or equal to 1 but less than 10.	Remember, there is a decimal at the end of 37,000. Move the decimal after the 3. 3.700 is between 1 and 10.	37,000.
Step 2. Count the number of decimal places, *n*, that the decimal point was moved.	The decimal point was moved 4 places to the left.	37000.
Step 3. Write the number as a product with a power of 10. If the original number is: Greater than 1, the power of 10 will be 10^n. Between 0 and 1, the power of 10 will be 10^{-n}.	37,000 is greater than 1 so the power of 10 will have exponent 4.	3.7×10^4
Step 4. Check.	Check to see if your answer makes sense.	10^4 is 10,000 and 10,000 times 3.7 will be 37,000. $37{,}000 = 3.7 \times 10^4$

Try It 6.1.41

Write in scientific notation: 96000.

Answer

9.6×10^4

Try It 6.1.42

Write in scientific notation: 48300.

Answer

4.83×10^4

HOW TO: Convert from decimal notation to scientific notation

1. Step 1. Move the decimal point so that the first factor is greater than or equal to 1 but less than 10.
2. Step 2. Count the number of decimal places, *n*, that the decimal point was moved.
3. Step 3. Write the number as a product with a power of 10.
   If the original number is:
   - greater than 1, the power of 10 will be 10^n.
   - between 0 and 1, the power of 10 will be 10^{-n}.
4. Step 4. Check.

Example 6.1.43

Write in scientific notation: 0.0052.

Solution

The original number, 0.0052, is between 0 and 1 so we will have a negative power of 10.

	0.0052
Move the decimal point to get 5.2, a number between 1 and 10.	0.0052
Count the number of decimal places the point was moved.	3 places
Write as a product with a power of 10.	5.2×10^{-3}
Check.	
5.2×10^{-3} $5.2 \times \frac{1}{10^3}$ $5.2 \times \frac{1}{1000}$ 5.2×0.001	
0.0052	$0.0052 = 5.2 \times 10^{-3}$

Try It 6.1.44

Write in scientific notation: 0.0078

Answer

7.8×10^{-3}

Try It 6.1.45

Write in scientific notation: 0.0129

Answer

1.29×10^{-2}

Convert Scientific Notation to Decimal Form

How can we convert from scientific notation to decimal form? Let's look at two numbers written in scientific notation and see.

$$\begin{array}{cc} 9.12 \times 10^{4} & 9.12 \times 10^{-4} \\ 9.12 \times 10,000 & 9.12 \times 0.0001 \\ 91,200 & 0.000912 \end{array} \tag{6.1.6}$$

If we look at the location of the decimal point, we can see an easy method to convert a number from scientific notation to decimal form.

$$9.12 \times 10^{4} = 91,200 \quad 9.12 \times 10^{-4} = 0.000912 \tag{6.1.7}$$

$9.12 \times 10^{4} = 91{,}200$	$9.12 \times 10^{-4} = 0.000912$
$9.12\_\_\_\times 10^{4} = 91{,}200$	$\_\_\_9.12 \times 10^{-4} = 0.000912$
Move the decimal point 4 places to the right.	Move the decimal point 4 places to the left.

In both cases the decimal point moved 4 places. When the exponent was positive, the decimal moved to the right. When the exponent was negative, the decimal point moved to the left.

Example 6.1.46

Convert to decimal form: 6.2×10^{3}

Solution

Step 1. Determine the exponent, *n*, on the factor 10.	The exponent is 3.	6.2×10^{3}
Step 2. Move the decimal *n* places, adding zeros if needed. If the exponent is positive, move the decimal point *n* places to the right. If the exponent is negative, move the decimal point $\lvert n \rvert$ places to the left.	The exponent is positive, so move the decimal point 3 places to the right. We need to add 2 zeros as placeholders.	6.200. 6,200
Step 3. Check to see if your answer makes sense.		10^{3} is 1000 and 1000 times 6.2 will be 6,200. $6.2 \times 10^{3} = 6{,}200$

Try It 6.1.47

Convert to decimal form: 1.3×10^{3}

Answer

$1,300$

Try It 6.1.48

Convert to decimal form: 9.25×10^{4}

Answer

$92,500$

The steps are summarized below.

HOW TO

Convert scientific notation to decimal form.

To convert scientific notation to decimal form:

1. Step 1. Determine the exponent, n, on the factor 10.
2. Step 2. Move the decimal n places, adding zeros if needed.

- If the exponent is positive, move the decimal point n places to the right.
- If the exponent is negative, move the decimal point $|n|$ places to the left.

3. Step 3. Check.

Example 6.1.49

Convert to decimal form: 8.9×10^{-2}

Solution

	8.9×10^{-2}
Determine the exponent, n, on the factor 10.	The exponent is –2.
Since the exponent is negative, move the decimal point 2 places to the left.	8.9
Add zeros as needed for placeholders.	$8.9 \times 10^{-2} = 0.089$

Try It 6.1.50

Convert to decimal form: 1.2×10^{-4}

Answer

0.00012

Try It 6.1.51

Convert to decimal form: 7.5×10^{-2}

Answer

0.075

Multiply and Divide Using Scientific Notation

Astronomers use very large numbers to describe distances in the universe and ages of stars and planets. Chemists use very small numbers to describe the size of an atom or the charge on an electron. When scientists perform calculations with very large or very small numbers, they use scientific notation. Scientific notation provides a way for the calculations to be done without writing a lot of zeros. We will see how the Properties of Exponents are used to multiply and divide numbers in scientific notation.

Example 6.1.52

Multiply. Write answers in decimal form: $\left(4 \times 10^{5}\right)\left(2 \times 10^{-7}\right)$

Solution

	$\left(4 \times 10^{5}\right)\left(2 \times 10^{-7}\right)$
Use the Commutative Property to rearrange the factors.	$4 \cdot 2 \cdot 10^{5} \cdot 10^{-7}$
Multiply.	8×10^{-2}
Change to decimal form by moving the decimal two places left.	0.08

Try It 6.1.53

Multiply $(3 \times 10^{6})(2 \times 10^{-8})$. Write answers in decimal form.

Answer

0.06

Try It 6.1.54

Multiply $\left(3 \times 10^{-2}\right)\left(3 \times 10^{-1}\right)$. Write answers in decimal form.

Answer

0.009

Example 6.1.55

Divide. Write answers in decimal form: $\dfrac{9 \times 10^{3}}{3 \times 10^{-2}}$

Solution

	$\dfrac{9 \times 10^{3}}{3 \times 10^{-2}}$
Separate the factors, rewriting as the product of two fractions.	$\dfrac{9}{3} \times \dfrac{10^{3}}{10^{-2}}$
Divide.	3×10^{5}
Change to decimal form by moving the decimal five places right.	300000

Try It 6.1.56

Divide $\dfrac{8 \times 10^{4}}{2 \times 10^{-1}}$. Write answers in decimal form.

Answer

$400,000$

Try It 6.1.57

Divide $\dfrac{8 \times 10^{2}}{4 \times 10^{-2}}$. Write answers in decimal form.

Answer

$20,000$

MEDIA ACCESS ADDITIONAL ONLINE RESOURCES

Access these online resources for additional instruction and practice with integer exponents and scientific notation:

- Negative Exponents
- Scientific Notation
- Scientific Notation 2

Key Concepts

- **Property of Negative Exponents**
  - If n is a positive integer and $a \neq 0$, then $\frac{1}{a^{-n}} = a^n$
- **Quotient to a Negative Exponent**
  - If a and b are real numbers, $b \neq 0$ and n is an integer , then $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$
- **To convert scientific notation to decimal form:**
  1. Determine the exponent, n on the factor 10.
  2. Move the decimal n places, adding zeros if needed.
     - If the exponent is positive, move the decimal point n places to the right.
     - If the exponent is negative, move the decimal point $|n|$ places to the left.
  3. Check.
- **To convert a decimal to scientific notation:**
  1. Move the decimal point so that the first factor is greater than or equal to 1 but less than 10.
  2. Count the number of decimal places, n that the decimal point was moved.
  3. Write the number as a product with a power of 10. If the original number is:
     - greater than 1, the power of 10 will be 10^n
     - between 0 and 1, the power of 10 will be 10^{-n}
  4. Check.

Glossary

negative exponent

If n is a positive integer and $a \neq 0$, then $a^{-n} = \frac{1}{a^n}$.

scientific notation

A number is expressed in scientific notation when it is of the form $a \times 10^n$ where $a \geq 1$ and a<10 and n is an integer.

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6.1.1: Exercises

Practice Makes Perfect

Use the Definition of a Negative Exponent

In the following exercises, simplify.

Exercise 1

1. 4^{-2}
2. 10^{-3}

Exercise 2

1. 3^{-4}
2. 10^{-2}

Answer

1. $\frac{1}{81}$
2. $\frac{1}{100}$

Exercise 3

1. 5^{-3}
2. 10^{-5}

Exercise 4

1. 2^{-8}
2. 10^{-2}

Answer

1. $\frac{1}{256}$
2. $\frac{1}{100}$

Exercise 5

1. $\frac{1}{c^{-5}}$
2. $\frac{1}{3^{-2}}$

Exercise 6

1. $\frac{1}{c^{-5}}$
2. $\frac{1}{5^{-2}}$

Answer

1. c^5
2. 25

Exercise 7

1. $\frac{1}{q^{-10}}$
2. $\frac{1}{10^{-3}}$

Exercise 8

1. $\frac{1}{t^{-9}}$
2. $\frac{1}{10^{-4}}$

Answer

1. t^9
2. 10000

Exercise 9

1. $\left(\frac{5}{8}\right)^{-2}$
2. $\left(-\frac{3m}{n}\right)^{-2}$

Exercise 10

1. $\left(\frac{3}{10}\right)^{-2}$
2. $\left(-\frac{2c}{d}\right)^{-3}$

Answer

1. $\frac{100}{9}$
2. $-\frac{d^3}{8c^3}$

Exercise 11

1. $\left(\frac{4}{9}\right)^{-3}$
2. $\left(-\frac{u^2}{2v}\right)^{-5}$

Exercise 12

1. $\left(\frac{7}{2}\right)^{-3}$
2. $\left(-\frac{3}{xy^2}\right)^{-3}$

Answer

1. $\frac{8}{343}$
2. $-\frac{x^3y^6}{27}$

Exercise 13

1. $(-5)^{-2}$
2. -5^{-2}
3. $\left(-\frac{1}{5}\right)^{-2}$
4. $-\left(\frac{1}{5}\right)^{-2}$

Exercise 14

1. $(-7)^{-2}$
2. -7^{-2}
3. $\left(-\frac{1}{7}\right)^{-2}$
4. $-\left(\frac{1}{7}\right)^{-2}$

Answer

1. $\frac{1}{49}$
2. $-\frac{1}{49}$
3. 49
4. −49

Exercise 15

1. -3^{-3}
2. $\left(-\frac{1}{3}\right)^{-3}$
3. $-\left(\frac{1}{3}\right)^{-3}$
4. $(-3)^{-3}$

Exercise 16

1. -5^{-3}
2. $\left(-\frac{1}{5}\right)^{-3}$
3. $-\left(\frac{1}{5}\right)^{-3}$
4. $(-5)^{-3}$

Answer

1. $-\frac{1}{125}$
2. −125
3. −125
4. $-\frac{1}{125}$

Exercise 17

1. $3 \cdot 5^{-1}$
2. $(3 \cdot 5)^{-1}$

Exercise 18

1. $2 \cdot 5^{-1}$
2. $(2 \cdot 5)^{-1}$

Answer

1. $\frac{2}{5}$
2. $\frac{1}{10}$

Exercise 19

1. $4 \cdot 5^{-2}$
2. $(4 \cdot 5)^{-2}$

Exercise 20

1. $3 \cdot 4^{-2}$
2. $(3 \cdot 4)^{-2}$

Answer

1. $\frac{3}{16}$
2. $\frac{1}{144}$

Exercise 21

1. m^{-4}
2. $(x^3)^{-4}$

Exercise 22

1. b^{-5}
2. $(k^2)^{-5}$

Answer

1. $\frac{1}{b^5}$
2. $\frac{1}{k^{10}}$

Exercise 23

1. p^{-10}
2. $(q^6)^{-8}$

Exercise 24

1. s^{-8}
2. $(a^9)^{-10}$

Answer

1. $\frac{1}{s^8}$
2. $\frac{1}{a^{90}}$

Exercise 25

1. $7n^{-1}$
2. $(7n)^{-1}$
3. $(-7n)^{-1}$

Exercise 26

1. $6r^{-1}$
2. $(6r)^{-1}$
3. $(-6r)^{-1}$

Answer

1. $\frac{6}{r}$
2. $\frac{1}{6r}$
3. $-\frac{1}{6r}$

Exercise 27

1. $(3p)^{-2}$
2. $3p^{-2}$
3. $-3p^{-2}$

Exercise 28

1. $(2q)^{-4}$
2. $2q^{-4}$
3. $-2q^{-4}$

Answer

1. $\frac{1}{16q^4}$
2. $\frac{2}{q^4}$
3. $-\frac{2}{q^4}$

Simplify Expressions with Integer Exponents

In the following exercises, simplify.

Exercise 29

1. b^4b^{-8}
2. $r^{-2}r^5$
3. $x^{-7}x^{-3}$

Exercise 30

1. $s^3 \cdot s^{-7}$
2. $q^{-8} \cdot q^3$
3. $y^{-2} \cdot y^{-5}$

Answer

1. $\frac{1}{s^4}$
2. $\frac{1}{q^5}$
3. $\frac{1}{y^7}$

? Exercise 31

1. $a^3 \cdot a^{-3}$
2. $a \cdot a^3$
3. $a \cdot a^{-3}$

? Exercise 32

1. $y^5 \cdot y^{-5}$
2. $y \cdot y^5$
3. $y \cdot y^{-5}$

Answer

1. 1
2. y^6
3. $\frac{1}{y^4}$

? Exercise 33

$p^5 \cdot p^{-2} \cdot p^{-4}$

? Exercise 34

$x^4 \cdot x^{-2} \cdot x^{-3}$

Answer

$\frac{1}{x}$

? Exercise 35

$(w^4 x^{-5})(w^{-2} x^{-4})$

? Exercise 36

$(m^3 n^{-3})(m^{-5} n^{-1})$

Answer

$\frac{1}{m^2 n^4}$

Exercise 37

$(uv^{-2})(u^{-5}v^{-3})$

Exercise 38

$(pq^{-4})(p^{-6}q^{-3})$

Answer

$$\frac{1}{p^5q^7}$$

Exercise 39

$(-6c^{-3}d^9)(2c^4d^{-5})$

Exercise 40

$(-2j^{-5}k^8)(7j^2k^{-3})$

Answer

$$-\frac{14k^5}{j^3}$$

Exercise 41

$(-4r^{-2}s^{-8})(9r^4s^3)$

Exercise 42

$(-5m^4n^6)(8m^{-5}n^{-3})$

Answer

$$-\frac{40n^3}{m}$$

Exercise 43

$(5x^2)^{-2}$

Exercise 44

$(4y^3)^{-3}$

Answer

$$\frac{1}{64y^9}$$

Exercise 45

$(3z^{-3})^2$

Exercise 46

$(2p^{-5})^2$

Answer

$$\frac{4}{p^{10}}$$

Exercise 47

$$\frac{t^9}{t^{-3}}$$

Exercise 48

$$\frac{n^5}{n^{-2}}$$

Answer

n^7

Exercise 49

$$\frac{x^{-7}}{x^{-3}}$$

Exercise 50

$$\frac{y^{-5}}{y^{-10}}$$

Answer

y^5

Convert from Decimal Notation to Scientific Notation

In the following exercises, write each number in scientific notation.

Exercise 51

57,000

Exercise 52

340,000

Answer

3.4×10^5

Exercise 53

8,750,000

Exercise 54

1,290,000

Answer

1.29×10^6

Exercise 55

0.026

Exercise 56

0.041

Answer

4.1×10^{-2}

Exercise 57

0.00000871

Exercise 58

0.00000103

Answer

1.03×10^{-6}

Convert Scientific Notation to Decimal Form

In the following exercises, convert each number to decimal form.

Exercise 59

5.2×10^2

Exercise 60

8.3×10^2

Answer

830

Exercise 61

7.5×10^6

Exercise 62

1.6×10^{10}

Answer

16,000,000,000

Exercise 63

2.5×10^{-2}

Exercise 64

3.8×10^{-2}

Answer

0.038

Exercise 65

4.13×10^{-5}

Exercise 66

1.93×10^{-5}

Answer

0.0000193

Multiply and Divide Using Scientific Notation

In the following exercises, multiply. Write your answer in decimal form.

Exercise 67

$\left(3 \times 10^{-5}\right)\left(3 \times 10^{9}\right)$

Exercise 68

$\left(2 \times 10^{2}\right)\left(1 \times 10^{-4}\right)$

Answer

0.02

Exercise 69

$\left(7.1 \times 10^{-2}\right)\left(2.4 \times 10^{-4}\right)$

Exercise 70

$\left(3.5 \times 10^{-4}\right)\left(1.6 \times 10^{-2}\right)$

Answer

5.6×10^{-6}

In the following exercises, divide. Write your answer in decimal form.

Exercise 71

$$\frac{7 \times 10^{-3}}{1 \times 10^{-7}}$$

Exercise 72

$$\frac{5 \times 10^{-2}}{1 \times 10^{-10}}$$

Answer

500,000,000

Exercise 73

$$\frac{6 \times 10^{4}}{3 \times 10^{-2}}$$

Exercise 74

$$\frac{8 \times 10^{6}}{4 \times 10^{-1}}$$

Answer

20,000,000

Everyday Math

Exercise 75

The population of the United States on July 4, 2010 was almost 310,000,000. Write the number in scientific notation.

Exercise 76

The population of the world on July 4, 2010 was more than 6,850,000,000. Write the number in scientific notation

Answer

6.85×10^{9}

Exercise 77

The average width of a human hair is 0.0018 centimeters. Write the number in scientific notation.

Exercise 78

The probability of winning the 2010 Megamillions lottery was about 0.0000000057. Write the number in scientific notation.

Answer

5.7×10^{-9}

Exercise 79

In 2010, the number of Facebook users each day who changed their status to 'engaged" was 2×10^4. Convert this number to decimal form.

Exercise 80

At the start of 2012, the US federal budget had a deficit of more than $\$1.5 \times 10^{13}$. Convert this number to decimal form.

Answer

15,000,000,000,000

Exercise 81

The concentration of carbon dioxide in the atmosphere is 3.9×10^{-4}. Convert this number to decimal form.

Exercise 82

The width of a proton is 1×10^{-5} of the width of an atom. Convert this number to decimal form.

Answer

0.00001

Exercise 83

Health care costs The Centers for Medicare and Medicaid projects that consumers will spend more than $4 trillion on health care by 2017.

1. Write 4 trillion in decimal notation.
2. Write 4 trillion in scientific notation.

Exercise 84

Coin production In 1942, the U.S. Mint produced 154,500,000 nickels. Write 154,500,000 in scientific notation.

Answer

1.545×10^8

Exercise 85

Distance The distance between Earth and one of the brightest stars in the night star is 33.7 light years. One light year is about 6,000,000,000,000 (6 trillion), miles.

1. Write the number of miles in one light year in scientific notation.
2. Use scientific notation to find the distance between Earth and the star in miles. Write the answer in scientific notation.

Exercise 86

Debt At the end of fiscal year 2015 the gross United States federal government debt was estimated to be approximately $18,600,000,000,000 ($18.6 trillion), according to the Federal Budget. The population of the United States was approximately 300,000,000 people at the end of fiscal year 2015.

1. Write the debt in scientific notation.
2. Write the population in scientific notation.
3. Find the amount of debt per person by using scientific notation to divide the debt by the population. Write the answer in scientific notation.

Answer

1. 1.86×10^{13}
2. 3×10^{8}
3. 6.2×10^{4}

Writing Exercises

Exercise 87

1. Explain the meaning of the exponent in the expression 2^3.
2. Explain the meaning of the exponent in the expression 2^{-3}.

Exercise 88

When you convert a number from decimal notation to scientific notation, how do you know if the exponent will be positive or negative?

Answer

answers will vary

Self Check

a. After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
use the definition of a negative exponent.			
simplify expressions with integer exponents.			
convert from decimal notation to scientific notation.			
convert scientific notation to decimal form.			
multiply and divide using scientific notation.			

b. Overall, after looking at the checklist, do you think you are well-prepared for the next section? Why or why not?

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6.2: Exponential Functions

Let's suppose that the current population of the city of Pleasantville is 10000 and that the population is growing at a rate of 2% per year. In order to analyze the population growth over a period of years, we'll try to develop a formula for the population as a function of time, and then graph the result.

First, note that at the end of one year, the population increase is 2% of 10000, or 200 people. We would now have 10,200 people in Pleasantville. At the end of the second year, take another 2% of 10200, which is an increase of 204 people, for a total of 10404. Because the increase each year is not constant, the graph of population versus time cannot be a line. Hence, our eventual population function will not be linear.

To develop our population formula, we start by letting the function P(t) represent the population of Pleasantville at time t, where we measure t in years. We will start time at t = 0 when the initial population of Pleasantville is 10000. In other words, P(0) = 10000. The key to understanding this example is the fact that the population increases by 2% each year. We are making an assumption here that this overall growth accounts for births, deaths, and people coming into and leaving Pleasantville. That is, at the end of the first year, the population of Pleasantville will be 102% of the initial population. Thus,

$$P(1) = 1.02P(0) = 1.02(10000). \tag{6.2.1}$$

We could multiply out the right side of this equation, but it will actually be more useful to leave it in its current form.

Now each year the population increases by 2%. Therefore, at the end of the second year, the population will be 102% of the population at the end of the first year. In other words,

$$P(2) = 1.02P(1). \tag{6.2.2}$$

If we replace $P(1)$ in Equation 6.2.2 with the result found in Equation 6.2.1, then

$$P(2) = (1.02)(1.02)(10000) = (1.02)^2(10000). \tag{6.2.3}$$

Let's iterate one more year. At the end of the third year, the population will be 102% of the population at the end of the second year, so

$$P(3) = 1.02P(2). \tag{6.2.4}$$

However, if we replace $P(2)$ in Equation 6.2.4 with the result found in Equation 6.2.3, we obtain

$$P(3) = (1.02)(1.02)^2(10000) = (1.02)^3(10000). \tag{6.2.5}$$

The pattern should now be clear. The population at the end of t years is given by the function.

$$P(t) = (1.02)^t(10000)$$

It is traditional in mathematics and science to place the initial population in front in this formula, writing instead

$$P(t) = 10000(1.02)^t. \tag{6.2.6}$$

Our function $P(t)$ is defined by Equation 6.2.6 for all positive integers {1, 2, 3, . . .}, and $P(0) = 10000$, the initial population. **Figure 1** shows a plot of our function. Although points are plotted only at integer values of t from 0 to 40, that's enough to show the trend of the population over time. The population starts at 10000, increases over time, and the yearly increase (the difference in population from one year to the next) also gets larger as time passes.

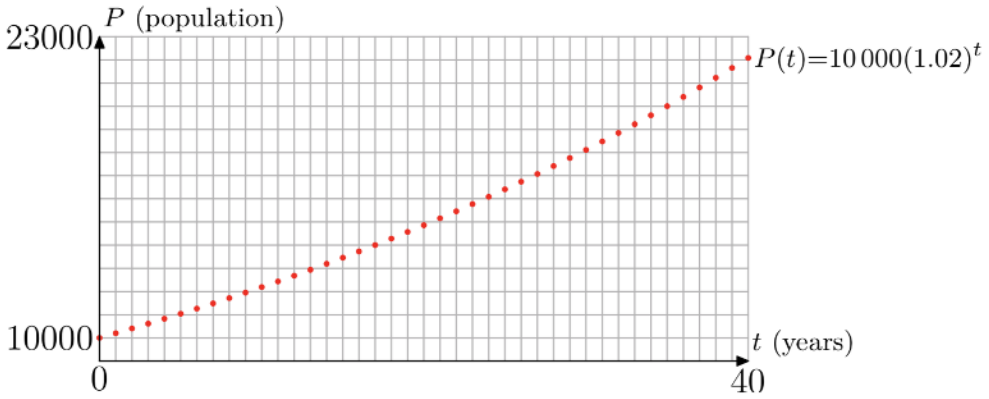


Figure 1. Graph of population P(t) of Pleasantville for t = 0, 1, 2, 3,...

? Exercise 6.2.7

We can now use the function P(t) to predict the population in later years. Assuming that the growth rate of 2% continues, what will the population of Pleasantville be after 40 years? What will it be after 100 years?

Solution

Substitute $t = 40$ and $t = 100$ into Equation 6.2.6. The population in 40 years will be

$$P(40) = 10000(1.02)^{40} \approx 22080$$

and the population in 100 years will be

$$P(100) = 10000(1.02)^{100} \approx 72446.$$

What would be different if we had started with a population of 12000? By tracing over our previous steps, it should be easy to see that the new formula would be

$$P(t) = 12000(1.02)^t.$$

Similarly, if the growth rate had been 3% per year instead of 2%, then we would have ended up with the formula

$$P(t) = 10000(1.03)^t.$$

Thus, by letting P_0 represent the initial population, and r represent the growth rate (in decimal form), we can generalize the formula to

$$P(t) = P_0(1+r)^t. \tag{6.2.7}$$

Note that our formula for the function $P(t)$ is different from the previous functions that we've studied so far, in that the input variable t is part of the exponent in the formula. Thus, this is a new type of function.

Now let's contrast the situation in Pleasantville with the population dynamics of Ghosttown. Ghosttown also starts with a population of 10000, but several factories have closed, so some people are leaving for better opportunities. In this case, the population of Ghosttown is decreasing at a rate of 2% per year. We'll again develop a formula for the population as a function of time, and then graph the result.

First, note that at the end of one year, the population decrease is 2% of 10000, or 200 people. We would now have 9800 people left in Ghosttown. At the end of the second year, take another 2% of 9800, which is a decrease of 196 people, for a total of 9604. As before, because the decrease each year is not constant, the graph of population versus time cannot be a line, so our eventual population function will not be linear.

Now let the function $P(t)$ represent the population of Ghosttown at time t, where we measure t in years. The initial population of Ghosttown at $t = 0$ is 10000, so $P(0) = 10,000$. Since the population decreases by 2% each year, at the end of the first year the population of Ghosttown will be 98% of the initial population. Thus,

$$P(1) = 0.98P(0) = 0.98(10000). \tag{6.2.8}$$

Each year the population decreases by 2%. Therefore, at the end of the second year, the population will be 98% of the population at the end of the first year. In other words,

$$P(2) = 0.98P(1). \tag{6.2.9}$$

If we replace $P(1)$ in Equation 6.2.9 with the result found in Equation 6.2.8, then

$$P(2) = (0.98)(0.98)(10000) = (0.98)^2(10000). \tag{6.2.10}$$

Let's iterate one more year. At the end of the third year, the population will be 98% of the population at the end of the second year, so

$$P(3) = 0.98P(2). \tag{6.2.11}$$

However, if we replace $P(2)$ in Equation 6.2.11 with the result found in Equation \label{11}, we obtain

$$P(3) = (0.98)(0.98)^2(10000) = (0.98)^3(10000). \tag{6.2.12}$$

The pattern should now be clear. The population at the end of t years is given by the function

$$P(t) = (0.98)^t(10000),$$

Or equivalently,

$$P(t) = 10000(0.98)^t \). \tag{6.2.13}$$

Our function $P(t)$ is defined by Equation 6.2.13 for all positive integers {1, 2, 3, . . .}, and P(0) = 10000, the initial population. **Figure 2** shows a plot of our function. Although points are plotted only at integer values of t from 0 to 40, that's enough to show the trend of the population over time. The population starts at 10 000, decreases over time, and the yearly decrease (the difference in population from one year to the next) also gets smaller as time passes.

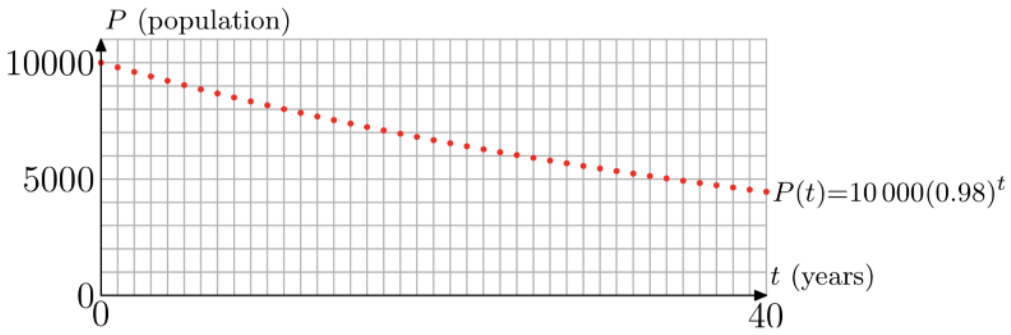


Figure 2. Graph of population P(t) of Ghosttown for t = 0, 1, 2, 3,...

? Exercise 6.2.15

Assuming that the rate of decrease continues at 2%, predict the population of Ghosttown after 40 years and after 100 years.

Solution

Substitute $t = 40$ and $t = 100$ into Equation 6.2.13. The population in 40 years will be

$$P(40) = 10000(0.98)^{40} \approx 4457,$$

and the population in 100 years will be

$$P(100) = 10000(0.98)^{100} \approx 1326$$

Note that if we had instead started with a population of 9000, for example, then the new formula would be

$$P(t) = 9000(0.98)^t.$$

Similarly, if the rate of decrease had been 5% per year instead of 2%, then we would have ended up with the formula

$$P(t) = 10000(0.95)^t.$$

Thus, by letting P_0 represent the initial population, and r represent the growth rate (in decimal form), we can generalize the formula to

$$P(t) = P_0(1-r)^t. \tag{6.2.14}$$

Definition

As noted before, our functions $P(t)$ in our Pleasantville and Ghosttown examples are a new type of function, because the input variable t is part of the exponent in the formula.

Definition 6.2.17: Exponential Function

An **exponential function** is a function of the form

$$f(t) = b^t$$

where b > 0 and $b \neq 1$. b is called the **base** of the exponential function. More generally, a function of the form

$$f(t) = Ab^t,$$

where b > 0, $b \neq 1$, and $A \neq 0$, is also referred to as an exponential function. In this case, the value of the function when t = 0 is f(0) = A, so A is the **initial amount**.

In applications, you will almost always encounter exponential functions in the more general form Ab^t. In fact, note that in the previous population examples, the function P(t) has this form $P(t) = Ab^t$, with $A = P_0$, $b = 1 + r$ in Pleasantville, and $b = 1 - r$ in Ghosttown. In particular, $A = P_0$ is the initial population.

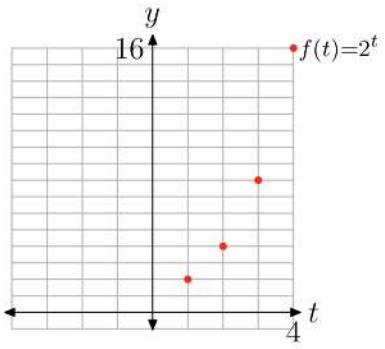
Since exponential functions are often used to model processes that vary with time, we usually use the input variable t (although of course any variable can be used). Also, you may be curious why the definition says $b \neq 1$, since 1^t just equals 1. We'll explain this curiosity at the end of this section.

Graphs of Exponential Functions

We'll develop the properties for the basic exponential function b^t first, and then note the minor changes for the more general form Ab^t. For a working example, let's use base b = 2, and let's compute some values of $f(t) = 2^t$ and plot the result (**Figure 3**).

t	$f(t) = 2^t$
1	2
2	4
3	8
4	16

(a)



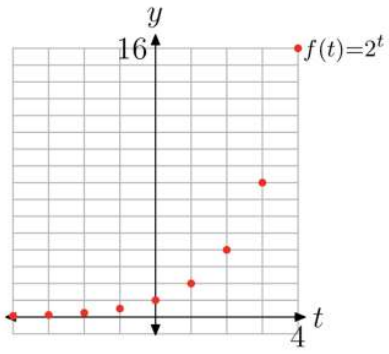
(b)

Figure 3. Plotting points (t, f(t)) defined by the function $f(t) = 2^t$, with t = 1, 2, 3, 4,....

Recall from the previous section that 2t is also defined for negative exponents t and the 0 exponent. Thus, the exponential function $f(t) = 2^t$ is defined for all integers. **Figure 4** shows a new table and plot with points added at 0 and negative integer values.

t	$f(t) = 2^t$
-4	$1/16$
-3	$1/8$
-2	$1/4$
-1	$1/2$
0	0
1	2
2	4
3	8
4	16

(a)



(b)

Figure 4. plotting points (t, f(t)) defined by the function $f(t) = 2^t$, with t = ..., −3, −2, −1, 0, 1, 2, 3....

However, the previous section showed that 2^t is also defined for rational and irrational exponents. Therefore, the domain of the exponential function $f(t) = 2^t$ is the set of all real numbers. When we add in the values of the function at all rational and irrational values of t, we obtain a final continuous curve as shown in **Figure 5**.

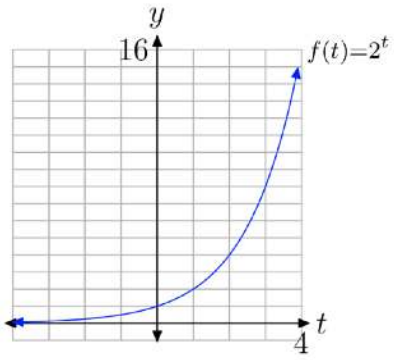


Figure 5.

Note several properties of the graph in **Figure 5**:

1. Moving from left to right, the curve rises, which means that the function increases as t increases. In fact, the function increases rapidly for positive t.
2. The graph lies above the t-axis, so the values of the function are always positive. Therefore, the range of the function is $(0, \infty)$.
3. The graph has a horizontal asymptote y = 0 (the t-axis) on the left side. This means that the function almost "dies out" (the values get closer and closer to 0) as t approaches $-\infty$.

What about the graphs of other exponential functions with different bases? We'll use the calculator to explore several of these.

First, use your calculator to compare $y_1(x) = 2^x$ and $y_2(x) = 3^x$. As can be seen in **Figure 6**(a), the graph of 3^x rises faster that 2^x for x > 0, and dies out faster for x < 0.

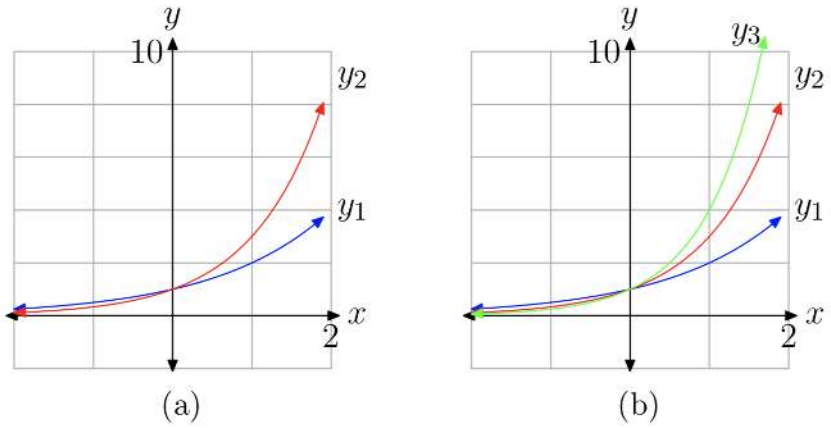
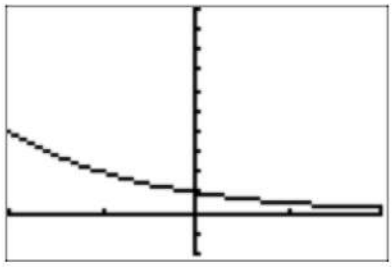


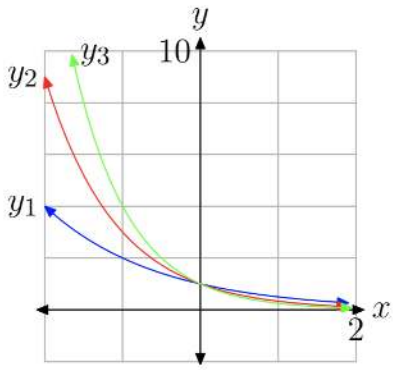
Figure 6. Comparing functions $y_1(x) = 2^x$, $y_2(x) = 3^x$, and $y_3(x) = 4^x$

Next, add in $y_3(x) = 4^x$. The result is shown in **Figure 6**(b). Again, increasing the size of the base to b = 4 results in a function which rises even faster on the right and likewise dies out faster on the left. If you continue to increase the size of the base b, you'll see that this trend continues. That's not terribly surprising because, if we compute the value of these functions at a fixed positive x, for example at x = 2, then the values increase: $2^2 < 3^2 < 4^2 < \ldots$. Similarly, at x = −2, the values decrease: $2^{-2} > 3^{-2} > 4^{-2} > \ldots$.

All of the functions in our experiments so far share the properties listed in (a)–(c) above: the function increases, the range is $(0, \infty)$, and the graph has a horizontal asymptote y = 0 on the left side. Now let's try smaller values of the base b. First use the calculator to plot the graph of $y_1(x) = (\frac{1}{2})^x$ (see **Figure 7**(a)).



(a) Graph of $y_1(x) = (1/2)^x$



(b) Comparing functions $y_1(x) = (1/2)^x$, $y_2(x) = (1/3)^x$, and $y_3(x) = (1/4)^x$

Figure 7.

This graph is much different. It rises rapidly to the left, and almost dies out on the right. Compare this with $y_2(x) = (\frac{1}{3})^x$ and $y_3(x) = (\frac{1}{4})^x$ (see **Figure 7**(b)). As the base gets smaller, the graph rises faster on the left, and dies out faster on the right.

Using reflection properties, it's easy to understand the appearance of these last three graphs. Note that

$$(\tfrac{1}{2})^x = (2^{-1})^x = 2^{-x}$$

so it follows that the graph of $(\frac{1}{2})^x$ is just a reflection in the *y*-axis of the graph of 2^x (see **Figure 8**).

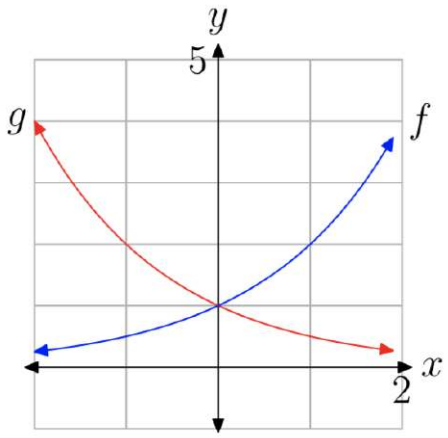


Figure 8. Comparing functions $f(x) = 2^x$ and $g(x) = (\frac{1}{2})^x = 2^{-x}$

Thus, we seem to have two different types of graphs, and therefore two types of exponential functions: one type is increasing, and the other decreasing. Our experiments above, along with a little more experimentation, should convince you that b^x is increasing for b > 1, and decreasing for 0 < b < 1. The first type of functions are called exponential growth functions, and the second type are exponential decay functions.

PROPERTIES OF EXPONENTIAL GROWTH FUNCTIONS: $F(X) = B^X$ WITH B > 1

- The domain is the set of all real numbers.
- Moving from left to right, the graph rises, which means that the function increases as x increases. The function increases rapidly for positive x.
- The graph lies above the x-axis, so the values of the function are always positive. Therefore, the range is $(0, \infty)$.
- The graph has a horizontal asymptote y = 0 (the x-axis) on the left side. This means that the function almost "dies out" (the values get closer and closer to 0) as x approaches $-\infty$.

The second property above deserves some additional explanation. Looking at **Figure 6**(b), it appears that y_2 and y_3 increase rapidly as x increases, but y_1 appears to increase slowly. However, this is due to the fact that the graph of $y_1(x) = 2^x$ is only shown on the interval $[-2, 2]$. In **Figure 5**, the same function is graphed on the interval $[-4, 4]$, and it certainly appears to increase rapidly in that graph. The point here is that exponential growth functions eventually increase rapidly as *x* increases. If you graph the function on a large enough interval, the function will eventually become very steep on the right side of the graph. This is an important property of the exponential growth functions, and will be explored further in the exercises.

PROPERTIES OF EXPONENTIAL DECAY FUNCTIONS: $F(X) = B^X$ WITH 0 < B < 1

- The domain is the set of all real numbers.
- Moving from left to right, the graph falls, which means that the function de- creases as x increases. The function decreases rapidly for negative x.
- The graph lies above the x-axis, so the values of the function are always positive. Therefore, the range is $(0, \infty)$.
- The graph has a horizontal asymptote y = 0 (the x-axis) on the right side. This means that the function almost "dies out" (the values get closer and closer to 0) as x approaches ∞.

Why do we refrain from using the base *b* = 1? After all, 1^x is certainly defined: it has the value 1 for all *x*. But that means that $f(x) = 1^x$ is just a constant linear function – its graph is a horizontal line. Therefore, this function doesn't share the same properties as the other exponential functions, and we've already classified it as a linear function. Thus, 1^x is not considered to be an exponential function.

Example 6.2.19

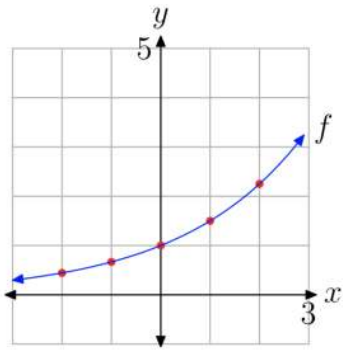
Plot the graph of the function $f(x) = (1.5)^x$. Identify the range of the function and the horizontal asymptote.

Solution

Since the base 1.5 is larger than 1, this is an exponential growth function. Therefore, its graph will have a shape similar to the graphs in **Figure 6**. The graph rises, there will be a horizontal asymptote $y = 0$ on the left side, and the range of the function is $(0, \infty)$. The graph can then be plotted by hand by using this knowledge along with approximate values at $x = -2, -1, 0, 1, 2$. See **Figure 9**.

x	$f(x) = (1.5)^x$
-2	0.44
-1	0.67
0	1
1	1.5
2	2.25

(a)



(b)

Figure 9. Graph of $f(x) = (1.5)^x$

Example 6.2.20

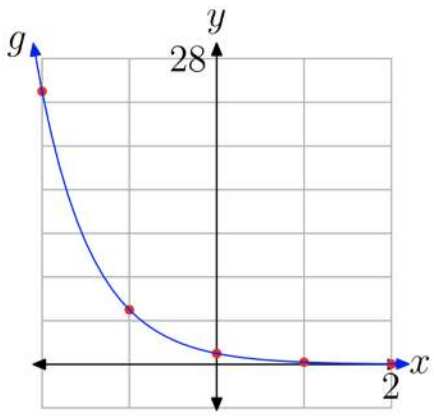
Plot the graph of the function $g(x) = (0.2)^x$. Identify the range of the function and the horizontal asymptote.

Solution

Since the base 0.2 is smaller than 1, this is an exponential decay function. Therefore, its graph will have a shape similar to the graphs in **Figure 7**. The graph falls, there will be a horizontal asymptote y = 0 on the right side, and the range of the function is \((0, \infty)\). The graph can then be plotted by hand by using this knowledge along with approximate values at x = −2, −1, 0, 1, 2. See **Figure 10**.

x	$g(x) = (0.2)^x$
-2	25
-1	5
0	1
1	0.2
2	0.04

(a)



(b)

Figure 10. Graph of $f(x) = (0.2)^x$

Example 6.2.21

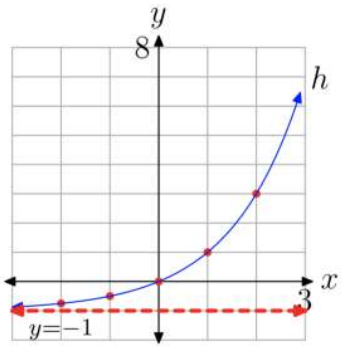
Plot the graph of the function $f(x) = 2^x - 1$. Identify the range of the function and the horizontal asymptote.

Solution

The graph of h can be obtained from the graph of $f(x) = 2^x$ (see **Figure 5**) by a vertical shift down 1 unit. Therefore, the horizontal asymptote y = 0 of the graph of f will also be shifted down 1 unit, so the graph of h has a horizontal asymptote $y = -1$. Similarly, the range of *f* will be shifted down to \((−1, \infty)\) = Range(h). The graph can then be plotted by hand by using this knowledge along with approximate values at x = −2, −1, 0, 1, 2. See **Figure 11**.

x	$h(x) = 2^x - 1$
-2	-0.75
-1	-0.5
0	0
1	1
2	3

(a)



(b)

Figure 11. Graph of $f(x) = 2^x - 1$

In later sections of this chapter, we will also see more general exponential functions of the form $f(x) = Ab^x$ (in fact, the Pleasantville and Ghosttown functions at the beginning of this section are of this form). If A is positive, then the graphs of these functions can be obtained from the basic exponential graphs by vertical scaling, so the graphs will have the same general shape as either the exponential growth curves (if b > 1) or the exponential decay curves (if $0 < b < 1$) we plotted earlier.

Exercise

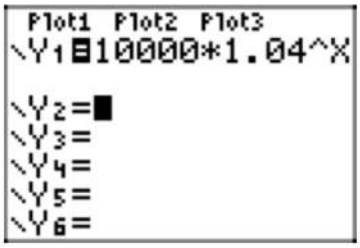
Exercise 6.2.1

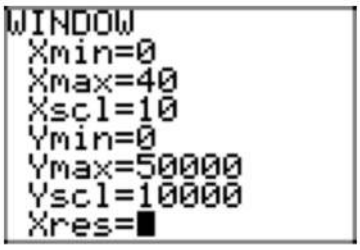
The current population of Fortuna is 10,000 hearty souls. It is known that the population is growing at a rate of 4% per year. Assuming this rate remains constant, perform each of the following tasks.

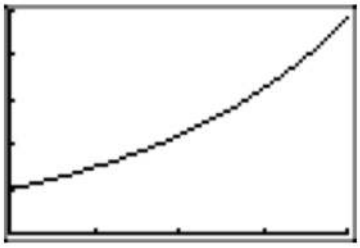
1. Set up an equation that models the population P(t) as a function of time t.
2. Use the model in the previous part to predict the population 40 years from now.
3. Use your calculator to sketch the graph of the population over the next 40 years.

Answer

1. $P(t) = 10000(1.04)^t$
2. $P(40) \approx 48101$
3.







? Exercise 6.2.2

The population of the town of Imagination currently numbers 12,000 people. It is known that the population is growing at a rate of 6% per year. Assuming this rate remains constant, perform each of the following tasks.

1. Set up an equation that models the population P(t) as a function of time t.
2. Use the model in the previous part to predict the population 30 years from now.
3. Use your calculator to sketch the graph of the population over the next 30 years.

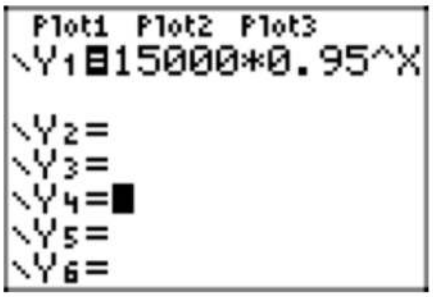
? Exercise 6.2.3

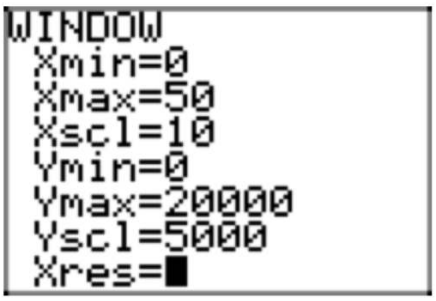
The population of the town of Despairia currently numbers 15,000 individuals. It is known that the population is decaying at a rate of 5% per year. Assuming this rate remains constant, perform each of the following tasks.

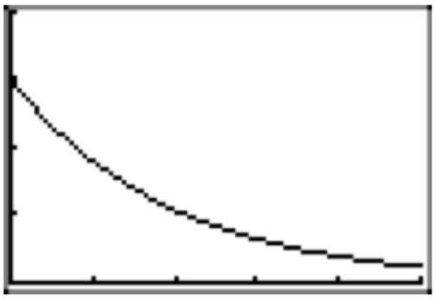
1. Set up an equation that models the population P(t) as a function of time t.
2. Use the model in the previous part to predict the population 50 years from now.
3. Use your calculator to sketch the graph of the population over the next 50 years.

Answer

1. $p(t) = 15000(0.95)^t$
2. $P(50) \approx 1154$
3.







? Exercise 6.2.4

The population of the town of Hopeless currently numbers 25,000 individuals. It is known that the population is decaying at a rate of 6% per year. Assuming this rate remains constant, perform each of the following tasks.

1. Set up an equation that models the population P(t) as a function of time t.
2. Use the model in the previous part to predict the population 40 years from now.
3. Use your calculator to sketch the graph of the population over the next 40 years.

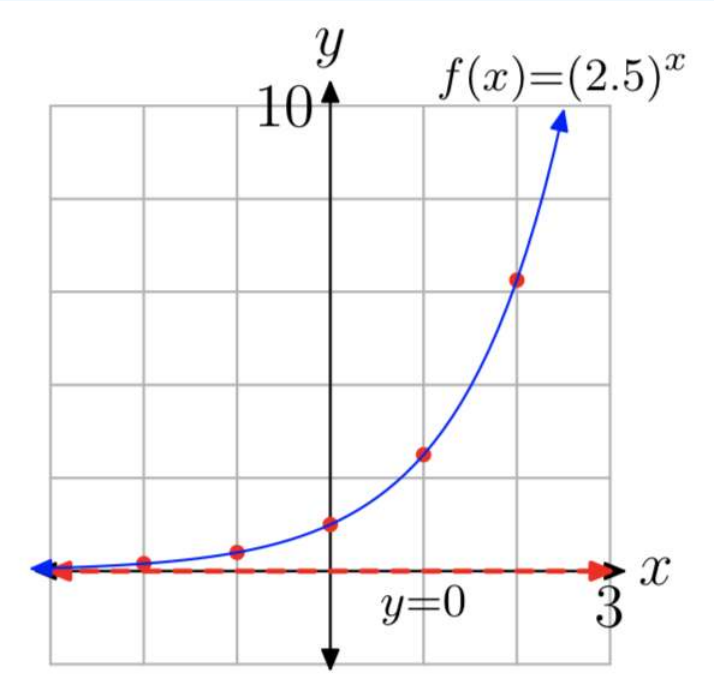
In **Exercises 5-12**, perform each of the following tasks for the given function.

1. Find the y-intercept of the graph of the function. Also, use your calculator to find two points on the graph to the right of the y-axis, and two points to the left.
2. Using your five points from (a) as a guide, set up a coordinate system on graph paper. Choose and label appropriate scales for each axis. Plot the five points, and any additional points you feel are necessary to discern the shape of the graph.
3. Draw the horizontal asymptote with a dashed line, and label it with its equation.
4. Sketch the graph of the function.
5. Use interval notation to describe both the domain and range of the function.

Exercise 6.2.5

$f(x) = (2.5)^x$

Answer

1. The y-intercept is (0, 1). Evaluating the function at x = 1, 2, −1, −2 to obtain the points (1, 2.5), (2, 6.25), (−1, 0.4), (−2, 0.16) (other answers are possible).
2. See the graph in part (4).
3. The horizontal asymptote is y = 0. See the graph in part (4).
4. 

5. Domain = $(-\infty, \infty)$, Range = $(0, \infty)$

Exercise 6.2.6

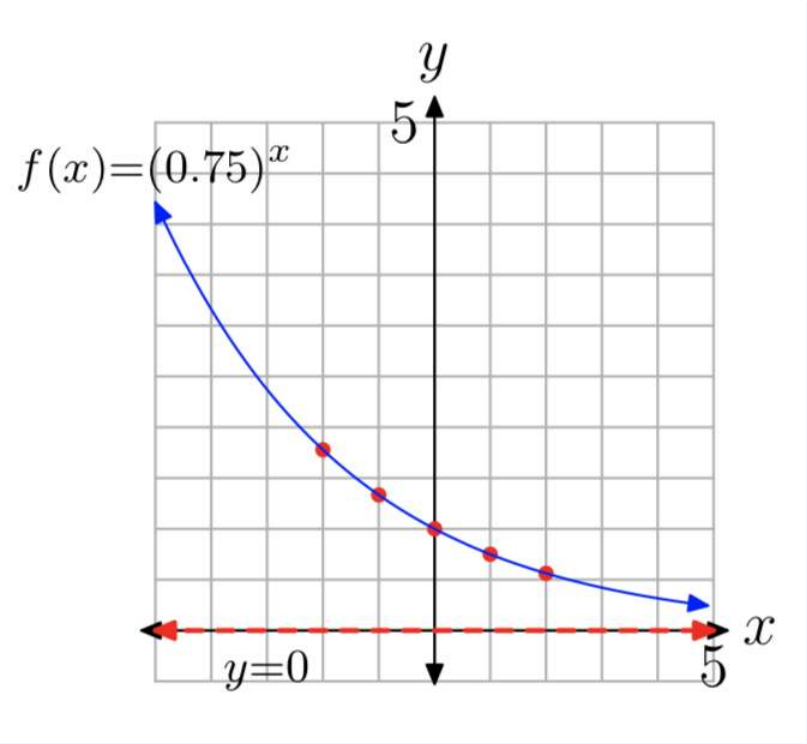
$f(x) = (0.1)^x$

Exercise 6.2.7

$f(x) = (0.75)^x$

Answer

1. The y-intercept is (0, 1). Evaluate the function at x = 1, 2, −1, −2 to obtain the points (1, 0.75), (2, 0.56), (−1, 1.34), (−2, 1.78) (other answers are possible).
2. See the graph in part (4).
3. The horizontal asymptote is y = 0. See the graph in part (4).

4. 

5. Domain = $(-\infty, \infty)$, Range = $(0, \infty)$

? Exercise 6.2.8

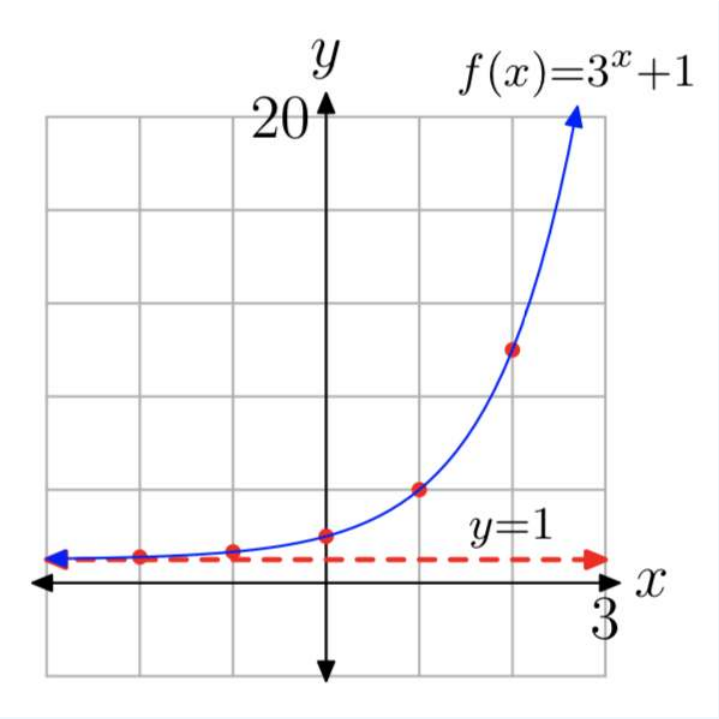
$f(x) = (1.1)^x$

? Exercise 6.2.9

$f(x) = 3^x + 1$

Answer

1. The y-intercept is (0, 2). Evaluate the function at x = 1, 2, −1, −2 to obtain the points (1, 4), (2, 10), (−1, 1.34), (−2, 1.11) (other answers are possible).
2. See the graph in part (4).
3. The horizontal asymptote is y = 1. See the graph in part (4).

4. 

5. Domain = $(-\infty, \infty)$, Range = $(1, \infty)$

? Exercise 6.2.10

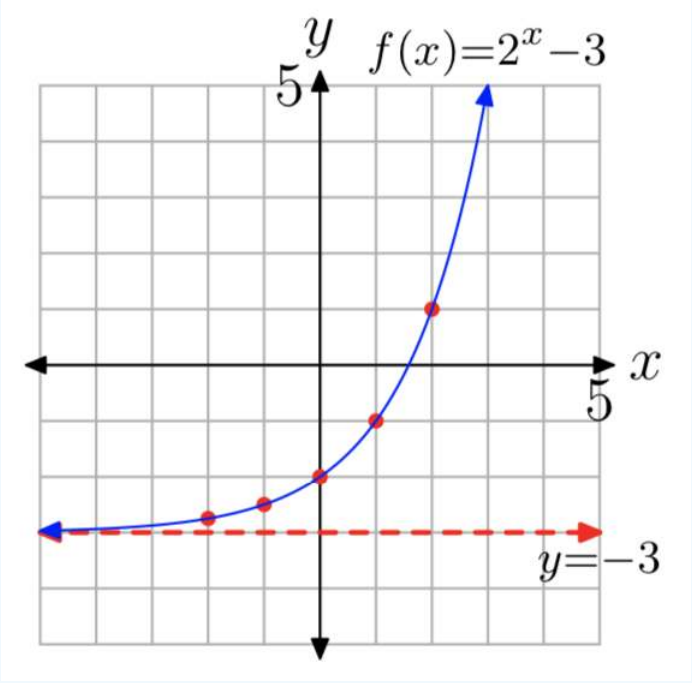
$f(x) = 4^x - 5$

? Exercise 6.2.11

$f(x) = 2^x - 3$

Answer

1. The y-intercept is (0, −2). Evaluate the function at x = 1, 2, −1, −2 to obtain the points (1, −1), (2, 1), (−1, −2.5), (−2, −2.75) (other answers are possible).
2. See the graph in part (4).
3. The horizontal asymptote is y = −3. See the graph in part (4).



4.

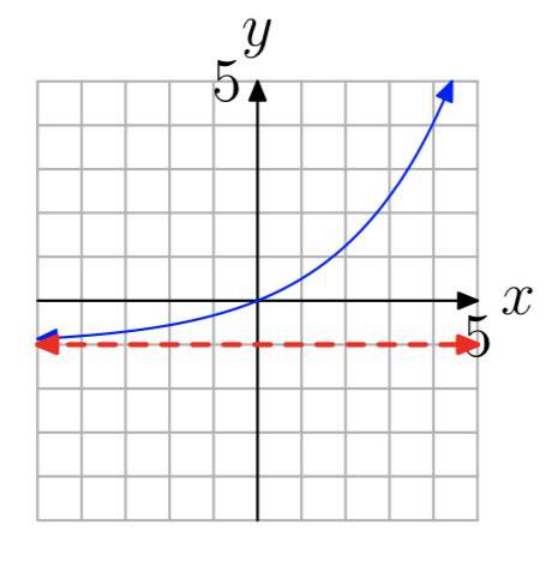
5. Domain = $(-\infty, \infty)$, Range = $(-3, \infty)$

? Exercise 6.2.12

$f(x) = 5^x + 2$

In **Exercises 13-20**, the graph of an exponential function of the form $f(x) = b^x + c$ is shown. The dashed red line is a horizontal asymptote. Determine the range of the function. Express your answer in interval notation.

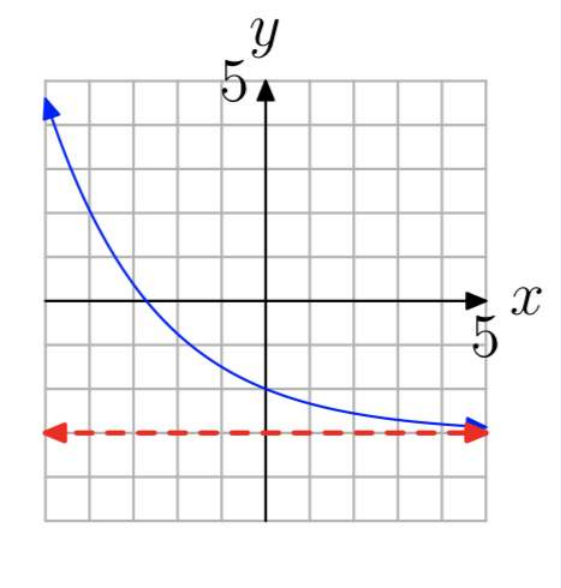
? Exercise 6.2.13



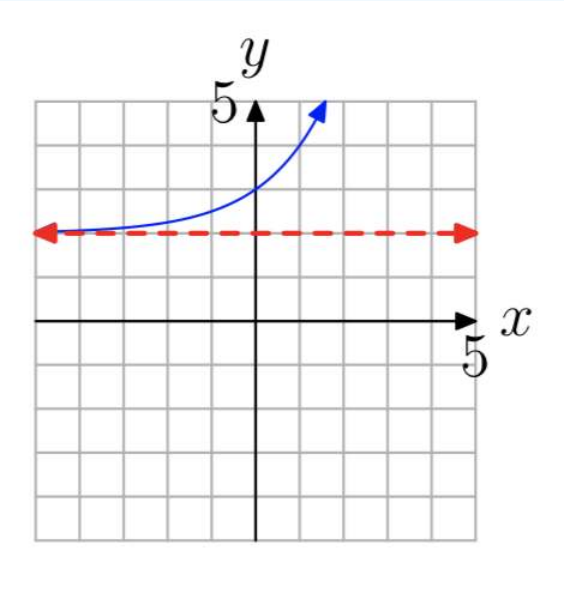
Answer

$(-1, \infty)$

Exercise 6.2.14



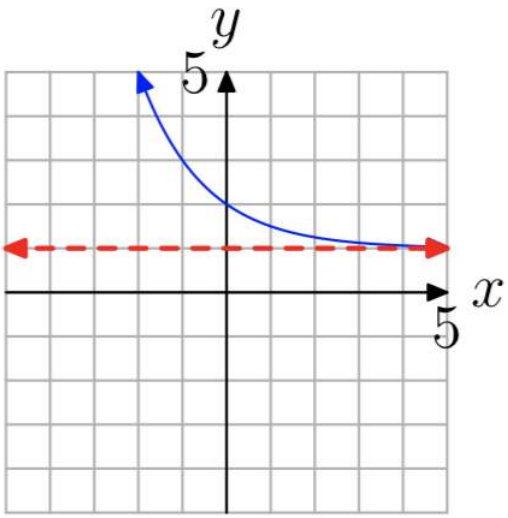
Exercise 6.2.15



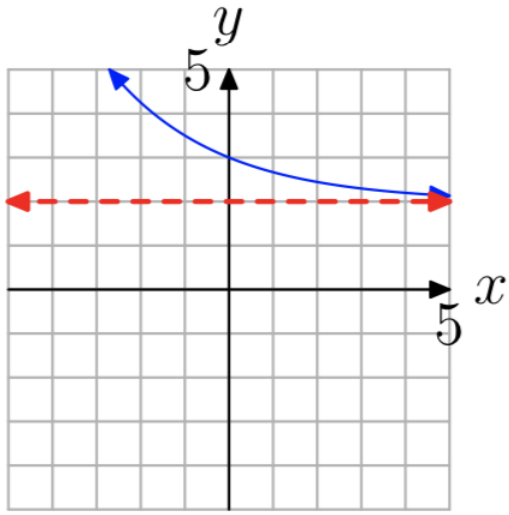
Answer

$(2, \infty)$

Exercise 6.2.16



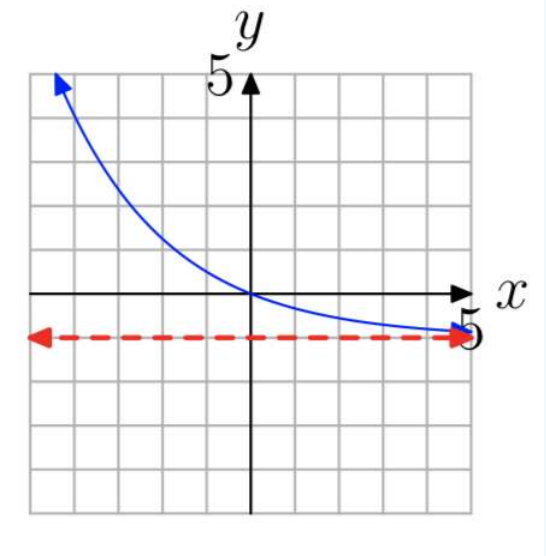
Exercise 6.2.17



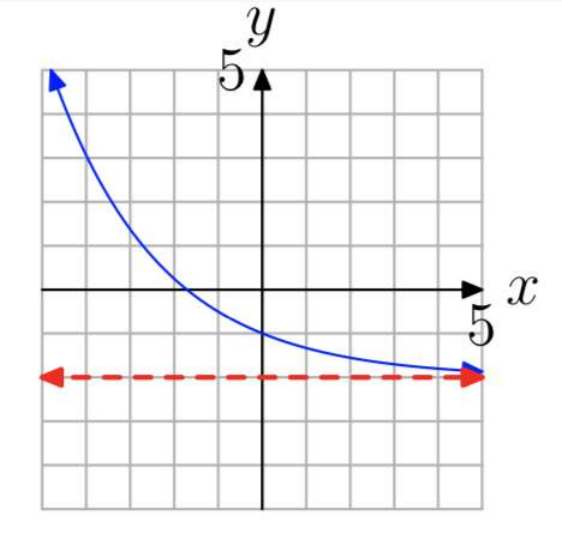
Answer

$(2, \infty)$

Exercise 6.2.18



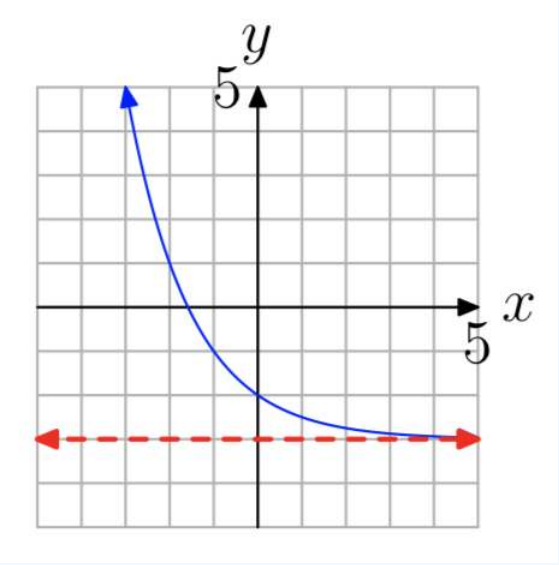
Exercise 6.2.19



Answer

$(-2, \infty)$

Exercise 6.2.20



In **Exercises 21-32**, compute f(p) at the given value p.

Exercise 6.2.21

$f(x) = (\frac{1}{3})^x$; p = −4

Answer

81

Exercise 6.2.22

$f(x) = (\frac{3}{4})^x$; p = 1

Exercise 6.2.23

$f(x) = 5^x$; p = 5

Answer

3125

Exercise 6.2.24

$f(x) = (\frac{1}{3})^x$; p = 4

Exercise 6.2.25

$f(x) = 4^x$; p = −4

Answer

$\frac{1}{256}$

Exercise 6.2.26

$f(x) = 5^x$; p = −3

Exercise 6.2.27

$f(x) = (\frac{5}{2})^x$; p = −3

Answer

$\frac{8}{125}$

Exercise 6.2.28

$f(x) = 9^x$; p = 3

Exercise 6.2.29

$f(x) = 5^x$; p = −4

Answer

$\frac{1}{625}$

Exercise 6.2.30

$f(x) = 9^x$; p = 0

Exercise 6.2.31

$f(x) = (\frac{6}{5})^x$; p = −4

Answer

$\frac{625}{1296}$

Exercise 6.2.32

$f(x) = (\frac{3}{5})^x$; p = 0

In **Exercises 33-40**, use your calculator to evaluate the function at the given value p. Round your answer to the nearest hundredth.

Exercise 6.2.33

$f(x) = 10^x$; p = −0.7.

Answer

0.20

Exercise 6.2.34

$f(x) = 10^x$; p = −1.6.

Exercise 6.2.35

$f(x) = (\frac{2}{5})^x$; p = 3.67.

Answer

0.03

Exercise 6.2.36

$f(x) = 2^x; p = -\frac{3}{4}$.

Exercise 6.2.37

$f(x) = 10^x$; p = 2.07.

Answer

117.49

Exercise 6.2.38

$f(x) = 7^x; p = \frac{4}{3}$.

Exercise 6.2.39

$f(x) = 10^x; p = -\frac{1}{5}$.

Answer

0.63

Exercise 6.2.40

$f(x) = (\frac{4}{3})^x$; p = 1.15

Exercise 6.2.41

This exercise explores the property that exponential growth functions eventually increase rapidly as x increases. Let $f(x) = 1.05^x$. Use your graphing calculator to graph f on the intervals

(a) [0, 10] and (b) [0, 100].

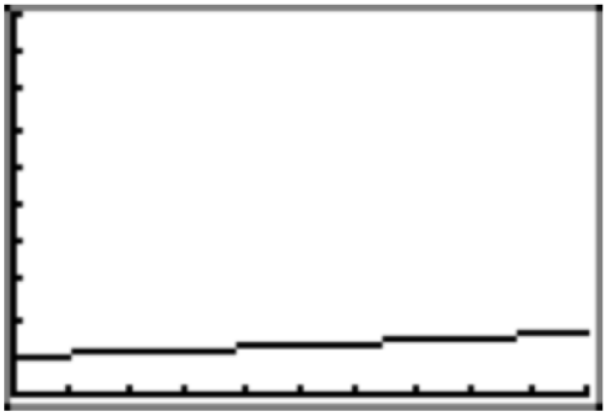
For (a), use Ymin = 0 and Ymax = 10.

For (b), use Ymin = 0 and Ymax = 100.

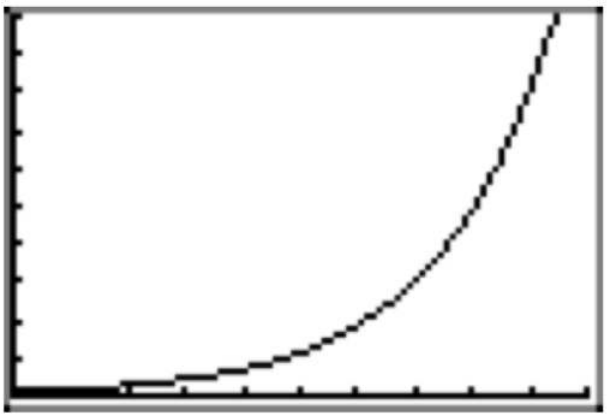
Make accurate copies of the images in your viewing window on your homework paper. What do you observe when you compare the two graphs?

Answer

(a) The graph on the interval [0, 10] increases very slowly. In fact, the graph looks almost linear.



(b) The graph on the interval [0, 100] increases slowly at first, but then increases very rapidly on the second half of the interval.



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CHAPTER OVERVIEW

7: Quadratic Functions

7.1: Zeros of the Quadratic

7.2: Solve Quadratic Equations Using the Square Root Property

7.2E: Exercises

7.3: Solve Quadratic Equations Using the Quadratic Formula

7.3E: Exercises

7.4: Solve Applications of Quadratic Equations

7.4E: Exercises

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7.1: Zeros of the Quadratic

We've seen how vertex form and intelligent use of the axis of symmetry can help to draw an accurate graph of the quadratic function defined by the equation $f(x) = ax^2 + bx + c$. When drawing the graph of the parabola it is helpful to know where the graph of the parabola crosses the x-axis. That is the primary goal of this section, to find the zero crossings or x-intercepts of the parabola.

Before we begin, you'll need to review the techniques that will enable you to factor the quadratic expression $ax^2 + bx + c$.

Factoring $ax^2 + bx + c$ when a = 1

Our intent in this section is to provide a quick review of techniques used to factor quadratic trinomials. We begin by showing how to factor trinomials having the form $ax^2 + bx + c$, where the leading coefficient is a = 1; that is, trinomials having the form $x^2 + bx + c$. In the next section, we will address the technique used to factor $ax^2 + bx + c$ when $a \neq 1$.

Let's begin with an example.

Example 7.1.1

Factor $x^2 + 16x - 36$

Solution

Note that the leading coefficient, the coefficient of x^2, is a 1. This is an important observation, because the technique presented here will not work when the leading coefficient does not equal 1.

Note the constant term of the trinomial $x^2 + 16x - 36$ is −36. List all integer pairs whose product equals −36.

$1, -36$	$-1, 36$
$2, -18$	$\boxed{-2, 18}$
$3, -12$	$-3, 12$
$4, -9$	$-4, 9$
$6, -6$	$-6, 6$

Note that we've framed the pair −2, 18. We've done this because the sum of this pair of integers equals the coefficient of x in the trinomial expression $x^2 + 16x - 36$. Use this framed pair to factor the trinomial.

$$x^2 + 16x - 36 = (x - 2)(x + 18)$$

It is important that you check your result. Use the distributive property to multiply.

$$\begin{aligned}(x - 2)(x + 18) &= x(x + 18) - 2(x + 18)\\ &= x^2 + 18x - 2x - 36\\ &= x^2 + 16x - 36\end{aligned}$$

Thus, our factorization is correct.

Let's summarize the technique.

Algorithm

To factor the quadratic $x^2 + bx + c$, proceed as follows:

1. List all the integer pairs whose product equals c.
2. Circle or frame the pair whose sum equals the coefficient of x, namely b. Use this pair to factor the trinomial.

Let's look at another example.

Example 7.1.2

Factor the trinomial $x^2 - 25x - 84$

Solution

List all the integer pairs whose product is −84.

$1, -84$	$-1, 84$
$2, -42$	$-2, 42$
$\boxed{3, -28}$	$-3, 28$
$4, -21$	$-4, 21$
$6, -14$	$-6, 14$
$7, -12$	$-7, 12$

We've framed the pair whose sum equals the coefficient of x, namely −25. Use this pair to factor the trinomial.

$$x^2 - 25x - 84 = (x+3)(x-28)$$

Check

$$\begin{aligned}(x+3)(x-28) &= x(x-28)+3(x-28)\\ &= x^2-28x+3x-84\\ &= x^2-25x-84\end{aligned}$$

With experience, there are a number of ideas that will quicken the process.

- As you are listing the integer pairs, should you happen to note that the current pair has the appropriate sum, there is no need to list the remaining integer pairs. Simply halt the process of listing the integer pairs and use the current pair to factor the trinomial.
- Some students are perfectly happy being asked "Can you think of an integer pair whose product is c and whose sum is b (where b and c refer to the coefficients of x^2+bx+c)?" If you can pick the pair "out of the air" like this, all is well and good.

Use the integer pair to factor the trinomial and don't bother listing any integer pairs.

Now, let's investigate how to proceed when the leading coefficient is not 1.

Factoring $ax^2 + bx + c$ when $a \neq 1$

When $a \neq 1$, we use a technique called the ac-test to factor the trinomial ax^2+bx+c . The process is best explained with an example.

Example 7.1.3

Factor $2x^2+13x-24$.

Solution

Note that the leading coefficient does not equal 1. Indeed, the coefficient of x^2 in this example is a 2. Therefore, the technique of the previous examples will not work. Thus, we turn to a similar technique called the ac-test.

First, compare

$$2x^2+13x-24 \qquad \text{and} \qquad ax^2+bx+c$$

and note that a = 2, b = 13, and c = −24. Compute the product of a and c. This is how the technique earns its name "ac-test."

$$ac = (2)(-24) = -48$$

List all integer pairs whose product is ac = −48.

1, −48	−1, 48
2, −24	−2, 24
3, −16	−3, 16
4, −12	−4, 12
6, −8	−6, 8

We've framed the pair whose sum is b = 13. The next step is to rewrite the trinomial $2x^2+13x-24$, splitting the middle term into a sum, using our framed integer pair.

$$2x^2+13x-24=2x^2-3x+16x-24$$

We factor an x out of the first two terms, then an 8 out of the last two terms. This process is called factoring by grouping.

$$2x^2-3x+16x-24=x(2x-3)+8(2x-3)$$

We now factor out a common factor of 2x − 3.

$$x(2x-3)+8(2x-3)=(x+8)(2x-3)$$

It's helpful to see the complete process as a coherent unit.

$$\begin{aligned}2x^2+13x-24 &= 2x^2-3x+16x-24\\ &= x(2x-3)+8(2x-3)\\ &= (x+8)(2x-3)\end{aligned}$$

Check

Again, it is important to check the answer by multiplication.

$$\begin{aligned}(x+8)(2x-3) &= x(2x-3)+8(2x-3)\\ &= 2x^2-3x+16x-24\\ &= 2x^2+13x-24\end{aligned}$$

Because this is the original trinomial, our solution checks.

Let's summarize this process.

Algorithm: ac-Test

To factor the quadratic ax^2+bx+c, proceed as follows:

1. List all integer pairs whose product equals ac.
2. Circle or frame the pair whose sum equals the coefficient of x, namely b.
3. Use the circled pair to express the middle term bx as a sum.
4. Factor by "grouping."

Let's look at another example.

Example 7.1.4

Factor $3x^2+34x-24$

Solution

Compare

$$3x^2+34x-24 \quad \text{and} \quad ax^2+bx+c$$

and note that a = 3, b = 34 and c = −24. List all integer pairs whose product equals ac = (3)(−24) = −72.

$1, -72$		$-1, 72$
$2, -36$		$\boxed{-2, 36}$
$3, -24$		$-3, 24$
$4, -18$		$-4, 18$
$6, -12$		$-6, 12$
$8, -9$		$-8, 9$

We've framed the pair whose sum is the same as b = 34, the coefficient of x in $3x^2 + 34x - 24$. Again, possible shortcuts are possible. If you can "think" of a pair whose product is ac = −72 and whose sum is b = 34, then it is not necessary to list any integer pairs. Alternatively, if you come across the needed pair as you are listing them, then you can halt the process. There is no need to list the remaining pairs if you have the one you need.

Use the framed pair to express the middle term as a sum, then factor by grouping.

$$\begin{aligned} 3x^2 + 34x - 24 &= 3x^2 - 2x + 36x - 24 \\ &= x(3x - 2) + 12(3x - 2) \\ &= (x + 12)(3x - 2) \end{aligned}$$

We leave it to the reader to check this result.

Intercepts

The points where the graph of a function crosses the x-axis are called the x-intercepts of graph of the function. Consider the graph of the quadratic function f in Figure 7.1.1.

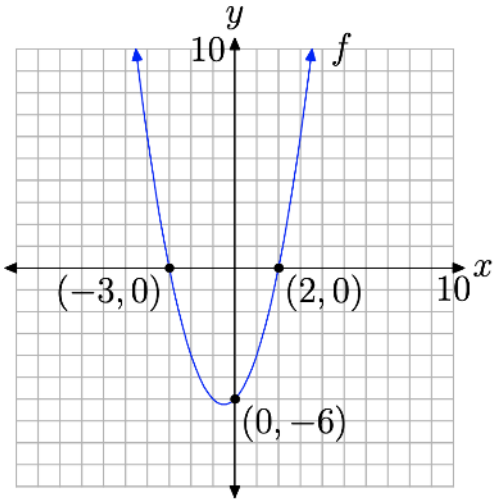


Figure 7.1.1: The x- and y-intercepts are key features of any graph.

Note that the graph of the f crosses the x-axis at (−3, 0) and (2, 0). These are the x-intercepts of the parabola. Note that the y-coordinate of each x-intercept is zero.

In function notation, the solutions of f(x) = 0 (note the similarity to y = 0) are the x-coordinates of the points where the graph of f crosses the x-axis. Analyzing the graph of f in Figure 7.1.1, we see that both −3 and 2 are solutions of f(x) = 0.

Thus, the process for finding the x-intercepts is clear.

Finding x-intercepts

To find the x-intercepts of the graph of any function, set y = 0 and solve for x. Alternatively, if function notation is used, set f(x) = 0 and solve for x.

Let's look at an example.

Example 7.1.5

Find the x-intercepts of the graph of the quadratic function defined by $y = x^2 + 2x - 48$.

Solution

To find the x-intercepts, first set y = 0.

$$0 = x^2 + 2x - 48$$

Next, factor the trinomial on the right. Note that the coefficient of x^2 is 1. We need only think of two integers whose product equals the constant term −48 and whose sum equals the coefficient of x, namely 2. The numbers 8 and −6 come to mind, so the trinomial factors as follows (readers should check this result).

$$0 = (x + 8)(x - 6)$$

To complete the solution, we need to use an important property of the real numbers called the zero product property.

Zero Product Property

If a and b are any real numbers such that

$$ab = 0$$

, then either a = 0 or b = 0.

In our case, we have 0 = (x + 8)(x − 6). Therefore, it must be the case that either

$$x + 8 = 0 \qquad \text{or} \qquad x - 6 = 0$$

These equations can be solved independently to produce

$$x = -8 \quad \text{or} \quad x = 6$$

Thus, the x-intercepts of the graph of $y = x^2 + 2x - 48$ are located at (−8, 0) and (6, 0).

Let's look at another example.

Example 7.1.6

Find the x-intercepts of the graph of the quadratic function $f(x) = 2x^2 - 7x - 15$.

Solution

To find the x-intercepts of the graph of the quadratic function f, we begin by setting

$$f(x) = 0$$

Of course, $f(x) = 2x^2 - 7x - 15$, so we can substitute to obtain

$$2x^2 - 7x - 15 = 0$$

We will now use the ac-test to factor the trinomial on the left. Note that ac = (2)(−15) = −30. List the integer pairs whose products equal −30.

$1, -30$		$-1, 20$
$2, -15$		$-2, 15$
$\boxed{3, -10}$		$-3, 10$
$5, -6$		$-5, 6$

Note that the framed pair sum to the coefficient of x in $2x^2 - 7x - 15$. Use the framed pair to express the middle term as a sum, then factor by grouping

$$\begin{aligned} 2x^2 - 7x - 15 &= 0 \\ 2x^2 + 3x - 10x - 15 &= 0 \\ x(2x+3) - 5(2x+3) &= 0 \\ (x-5)(2x+3) &= 0 \end{aligned}$$

Now we can use the zero product property. Either

$$x - 5 = 0 \qquad \text{or} \qquad 2x + 3 = 0$$

Each of these can be solved independently to obtain

$$x = 5 \qquad \text{or} \qquad x = -3/2$$

Thus, the x-intercepts of the graph of the quadratic function $f(x) = 2x^2 - 7x - 15$ are located at (−3/2, 0) and (5, 0).

One more definition is in order.

Definition 7: Zeros of a Function

he solutions of f(x) = 0 are called the **zeros** of the function f.

Thus, in the last example, both −3/2 and 5 are zeros of the quadratic function $f(x) = 2x^2 - 7x - 15$. Note the intimate relationship between the zeros of the quadratic function and the x-intercepts of the graph. Note that −3/2 is a zero and (−3/2, 0) is an x-intercept. Similarly, 5 is a zero and (5, 0) is an x-intercept.

The graphing calculator can be used to find the zeros of a function.

Example 7.1.7

Use the graphing calculator to find the zeros of the function $f(x) = 2x^2 - 7x - 15$.

Solution

Enter the function $f(x) = 2x^2 - 7x - 15$ into Y1 in the Y= menu; then adjust the window parameters as shown in Figure 7.1.2(b). Push the GRAPH button to produce the parabola shown in Figure 7.1.2(c).

To find a zero of the function, proceed as follows:

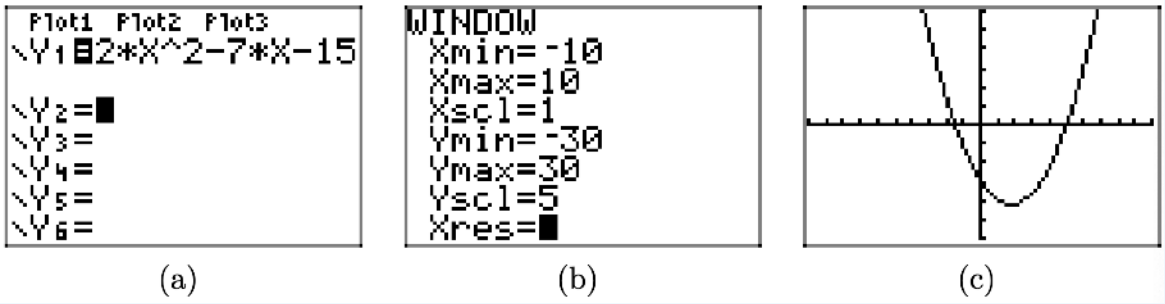
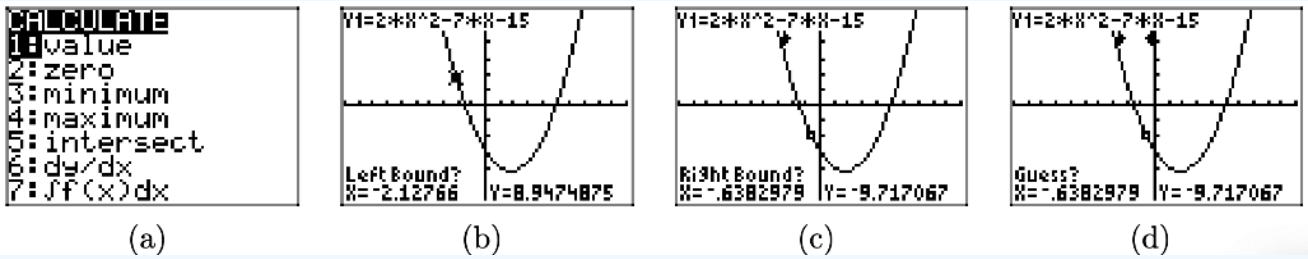


Figure 7.1.2. Plotting the quadratic function $f(x) = 2x^2 - 7x - 15$.

- Press 2nd TRACE to open the CALCULATE window shown in Figure 7.1.3(a). From this menu, select 2:zero.
- The calculator responds by asking for a “Left bound.” Use the arrow keys to move the cursor slightly to the left of the leftmost x-intercept, as shown in Figure 7.1.3(b). Press the ENTER key.
- The calculator responds by asking for a “Right bound.” Use the arrow keys to move the cursor slightly to the right of the leftmost x-intercept, as shown in Figure 7.1.3(c). Press the ENTER key.

- The calculator responds by asking for a “Guess.” You may use the arrow keys to select a starting x-value any where between the left- and right-bounds you selected (note that the calculator marks these on the screen in Figure 7.1.3(d)). However, the cursor already lies between these marks, so we typically just hit ENTER at this point. We suggest you do so also.



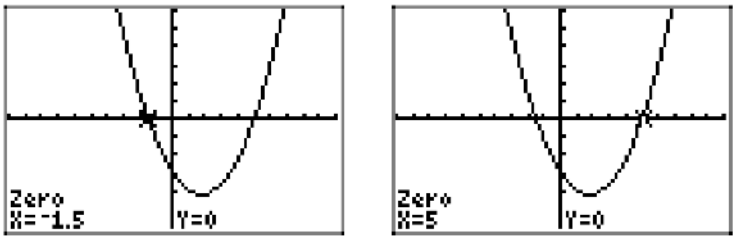
(a) (b) (c) (d)

Figure 7.1.3. Using the zero utility to find an x-intercept.

The calculator responds by marking the x-intercept and reporting its x-value at the bottom of the screen, as shown in Figure 7.1.4(a). This is one of the zeros of the function. Note that this value of −1.5 agrees nicely with our hand calculated result −3/2 in Example 7.1.6. We followed precisely the same procedure outlined above to find the second x-intercept shown in Figure 7.1.4(b). Note that it also agrees with the hand calculated solution of Example 7.1.6.

In a similar vein, the point where the graph of a function crosses the y-axis is called the y-intercept of the graph of the function. In Figure 7.1.1 the y-intercept of the parabola is (0, −6). Note that the x-coordinate of this y-intercept is zero.

Thus, the process for finding y-intercepts should be clear.



(a) (b)

Figure 7.1.4: The zeros of $f(x) - 2x^2 - 7x - 15$.

Finding Y-Intercepts

To find the y-intercepts of the graph of any function, set $x = 0$ and solve for y. Alternatively, if function notation is used, simply evaluate f(0).

Example 7.1.8

Find the y-intercept of the quadratic function defined by $f(x) = x^2 - 3x - 11$.

Solution

Evaluate the function at x = 0.

$$f(0) = (0)^2 - 3(0) - 11 = -11$$

The coordinates of the y-intercept are (0, −11).

Putting it All Together

We will find both x- and y-intercepts extremely useful when drawing the graph of a quadratic function.

Example 7.1.9

Place the quadratic function $y = x^2 + 2x - 24$ in vertex form. Plot the vertex and axis of symmetry and label them with their coordinates and equation, respectively. Find and plot the x- and y-intercepts of the parabola and label them with their coordinates.

Solution

Take half of the coefficient of x, square, then add and subtract this amount to balance the equation. Factor and combine coefficients.

$$y = x^2 + 2x + 1 - 1 - 24$$
$$y = (x+1)^2 - 25$$

The graph is a parabola that opens upward; it is shifted 1 unit to the left and 25 units downward. This information is enough to plot and label the vertex, then plot and label the axis of symmetry, as shown in Figure 7.1.5(a).

To find the x-intercepts, let y = 0 in $y = x^2 + 2x - 24$.

$$0 = x^2 + 2x - 24$$

The leading coefficient is a 1. The integer pair −4 and 6 has product −24 and sum 2. Thus, the right-hand side factors as follows.

$$0 = (x+6)(x-4)$$

In order that this product equals zero, either

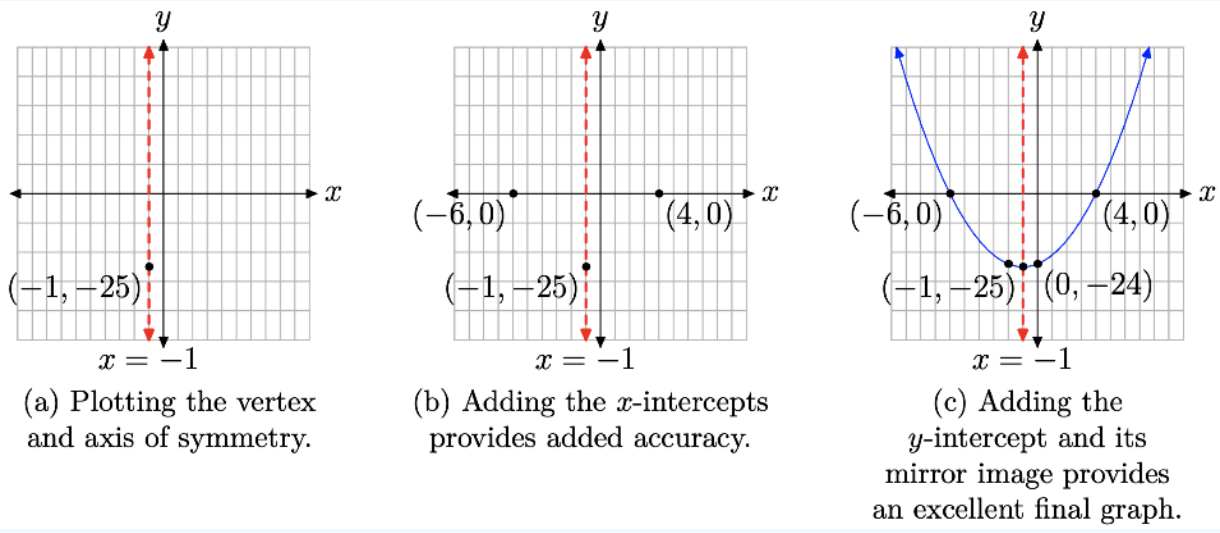
$$x + 6 = 0 \qquad \text{or} \qquad x - 4 = 0$$

Solve each of these linear equations independently.

$$x = -6 \qquad \text{or} \qquad x = 4$$

Recall that we let y = 0. We've found two solutions, x = −6 and x = 4. Thus, we have x-intercepts at (−6, 0) and (4, 0), as pictured in Figure 7.1.5(b).

Finally, to find the y-intercept, let x = 0 in $y = x^2 + 2x - 24$. With this substitution, y = −24. Thus, the y-intercept is (0, −24), as pictured in Figure 7.1.5(c). Note that we've also included the mirror image of the y-intercept across the axis of symmetry.



(a) Plotting the vertex and axis of symmetry.

(b) Adding the x-intercepts provides added accuracy.

(c) Adding the y-intercept and its mirror image provides an excellent final graph.

Figure 7.1.5

Let's look at one final example.

https://math.libretexts.org/@go/page/56815

Example 7.1.10

Plot the parabola represented by the equation $f(x) = -2x^2 - 7x + 15$. Plot and label the vertex, axis of symmetry, and the x- and y-intercepts.

Solution

First, factor out a −2.

$$f(x) = -2\left[x^2 + \frac{7}{2}x - \frac{15}{2}\right]$$

Half of 7/2 is 7/4. Squared, this amounts to 49/16. Add and subtract this last amount to keep the equation balanced.

$$f(x) = -2\left[x^2 + \frac{7}{2}x + \frac{49}{16} - \frac{49}{16} - \frac{15}{2}\right]$$

The first three terms inside the parentheses form a perfect square trinomial. The last two constants are combined with a common denominator.

$$f(x) = -2\left[\left(x^2 + \tfrac{7}{2}x + \tfrac{49}{16}\right) - \tfrac{49}{16} - \tfrac{120}{16}\right]$$
$$f(x) = -2\left[\left(x + \tfrac{7}{4}\right)^2 - \tfrac{169}{16}\right]$$

Finally, redistribute the −2.

$$f(x) = -2\left(x + \frac{7}{4}\right)^2 + \frac{169}{8}$$

The graph of this last equation is a parabola that opens downward, translated 7/4 units to the left and 169/8 units upward. This is enough information to plot and label the vertex and axis of symmetry, as shown in Figure 7.1.6(a).

To find the y-intercepts, set f(x) = 0 in $f(x) = -2x^2 - 7x + 15$. We will also multiply both sides of the resulting equation by −1.

$$0 = -2x^2 - 7x + 15$$
$$0 = 2x^2 + 7x - 15$$

After comparing $2x^2 + 7x - 15$ with $ax^2 + bx + c$, we note that the integer pair −3 and 10 have product equal to ac = −30 and sum equal to b = 7. Use this pair to express the middle term of $2x^2 + 7x - 15$ as a sum and then factor by grouping.

$$0 = 2x^2 - 3x + 10x - 15$$
$$0 = x(2x - 3) + 5(2x - 3)$$
$$0 = (x + 5)(2x - 3)$$

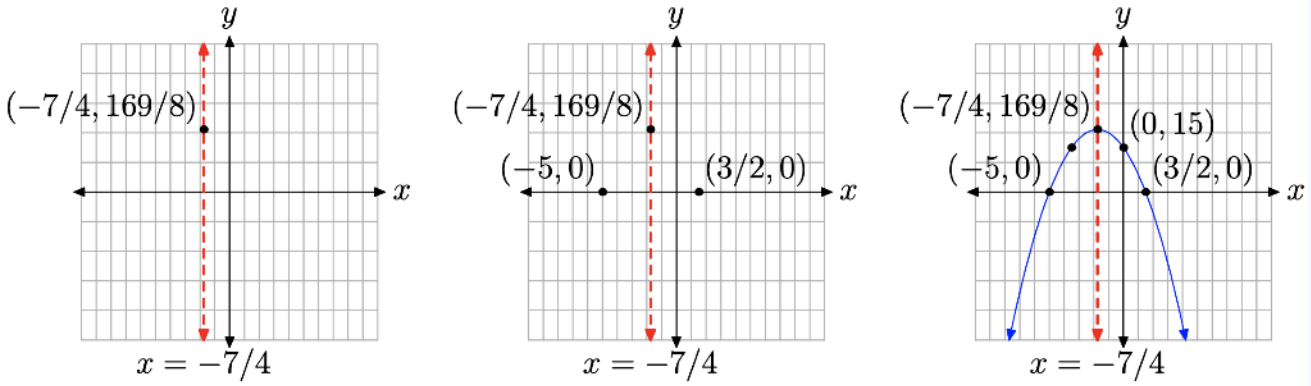
By the zero product property, either

$$x + 5 = 0 \qquad \text{or} \qquad 2x - 3 = 0$$

Solve these linear equations independently.

$$x = -5 \qquad \text{or} \qquad x = \frac{3}{2}$$

These x-values are the zeros of f (they make f(x) = 0), so we have x-intercepts at (−5, 0) and (3/2, 0), as shown in Figure 7.1.6(b).



(a) Plotting the vertex and axis of symmetry.

(b) Adding the x-intercepts provides added accuracy.

(c) Adding the y-intercept and its mirror image provide an excellent final graph.

Figure 7.1.6

Finally, to find the y-intercept, set x = 0 in $f(x) = -2x^2 - 7x + 15$ to get f(0) = 15. Note the positioning of the y-intercept (0, 15) and its mirror image across the axis of symmetry in Figure 7.1.6(c).

Exercise

In **Exercises 1-8**, factor the given quadratic polynomial.

? Exercise 7.1.1

$x^2 + 9x + 14$

Answer

(x+2)(x+7)

? Exercise 7.1.2

$x^2 + 6x + 5$

? Exercise 7.1.3

$x^2 + 10x + 9$

Answer

(x+9)(x+1)

? Exercise 7.1.4

$x^2 + 4x - 21$

? Exercise 7.1.5

$x^2 - 4x - 5$

Answer

(x−5)(x+1)

Exercise 7.1.6

$x^2 + 7x - 8$

Exercise 7.1.7

$x^2 - 7x + 12$

Answer

(x−4)(x−3)

Exercise 7.1.8

$x^2 + 5x - 24$

In **Exercises 9-16**, find the zeros of the given quadratic function.

Exercise 7.1.9

$f(x) = x^2 - 2x - 15$

Answer

Zeros: x = −3, x = 5

Exercise 7.1.10

$f(x) = x^2 + 4x - 32$

Exercise 7.1.11

$f(x) = x^2 + 10x - 39$

Answer

Zeros: x = −13, x = 3

Exercise 7.1.12

$f(x) = x^2 + 4x - 45$

Exercise 7.1.13

$f(x) = x^2 - 14x + 40$

Answer

Zeros: x = 4, x = 10

Exercise 7.1.14

$f(x) = x^2 - 5x - 14$

Exercise 7.1.15

$f(x) = x^2 + 9x - 36$

Answer

Zeros: x = −12, x = 3

Exercise 7.1.16

$f(x) = x^2 + 11x - 26$

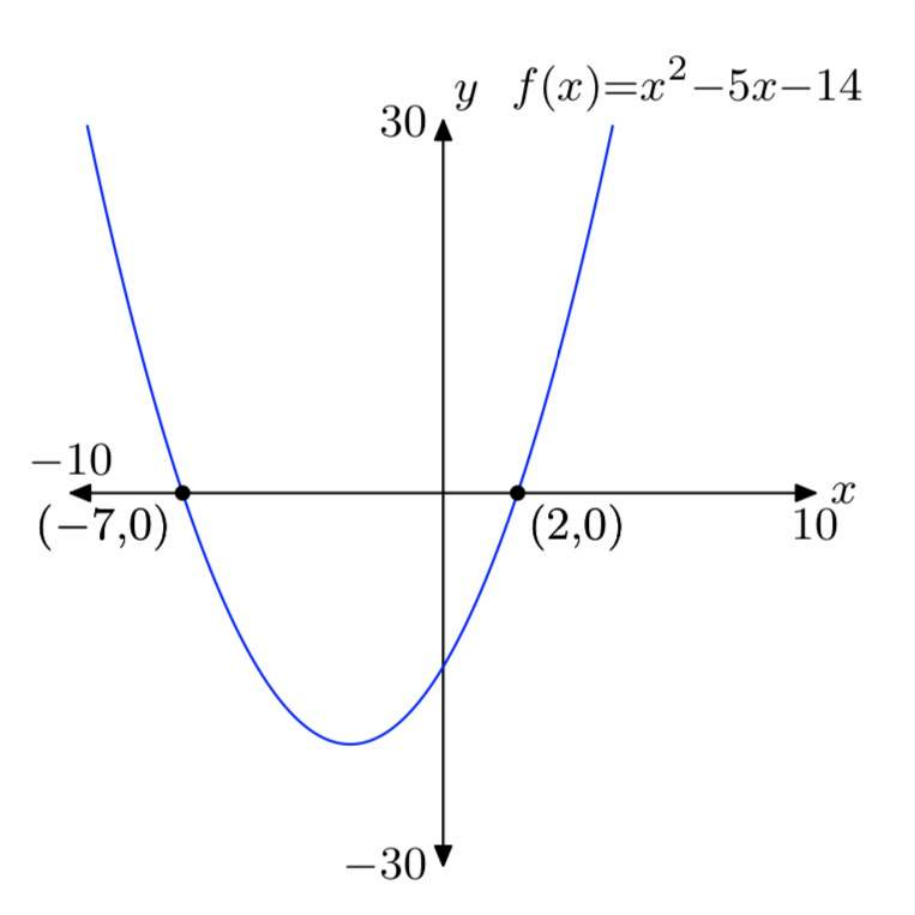
In **Exercises 17-22**, perform each of the following tasks for the quadratic functions.

1. Load the function into Y1 of the Y= of your graphing calculator. Adjust the window parameters so that the vertex is visible in the viewing window.
2. Set up a coordinate system on your homework paper. Label and scale each axis with xmin, xmax, ymin, and ymax. Make a reasonable copy of the image in the viewing window of your calculator on this coordinate system and label it with its equation.
3. Use the zero utility on your graphing calculator to find the zeros of the function. Use these results to plot the x-intercepts on your coordinate system and label them with their coordinates.
4. Use a strictly algebraic technique (no calculator) to find the zeros of the given quadratic function. Show your work next to your coordinate system. Be stubborn! Work the problem until your algebraic and graphically zeros are a reasonable match.

Exercise 7.1.17

$f(x) = x^2 + 5x - 14$

Answer



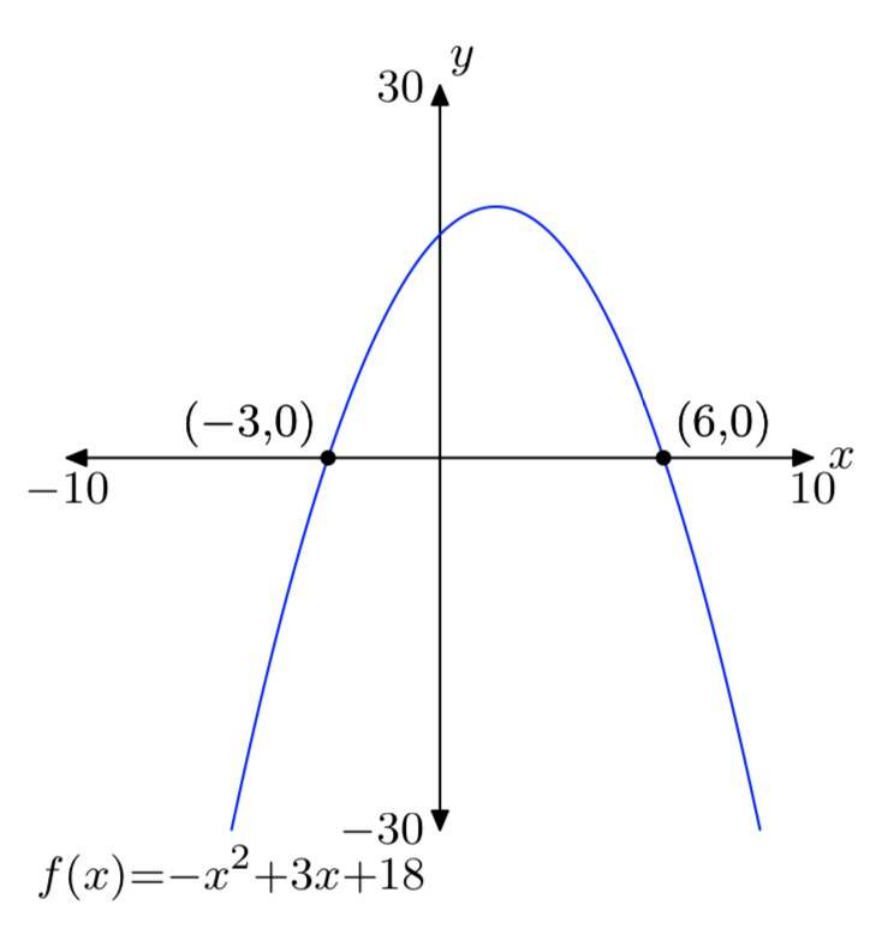
? Exercise 7.1.18

$f(x) = x^2 + x - 20$

? Exercise 7.1.19

$f(x) = -x^2 + 3x + 18$

Answer



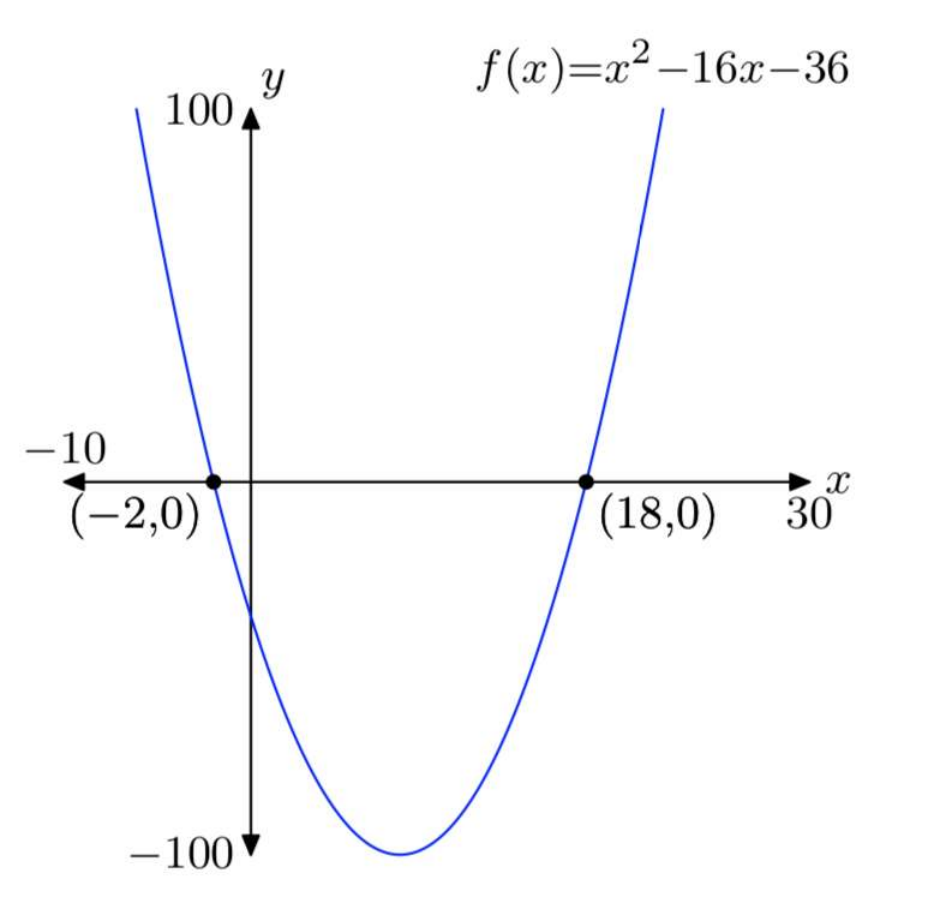
? Exercise 7.1.20

$f(x) = -x^2 + 3x + 40$

? Exercise 7.1.21

$f(x) = x^2 - 16x - 36$

Answer



? Exercise 7.1.22

$f(x) = x^2 + 4x - 96$

In **Exercises 23-30**, perform each of the following tasks for the given quadratic function.

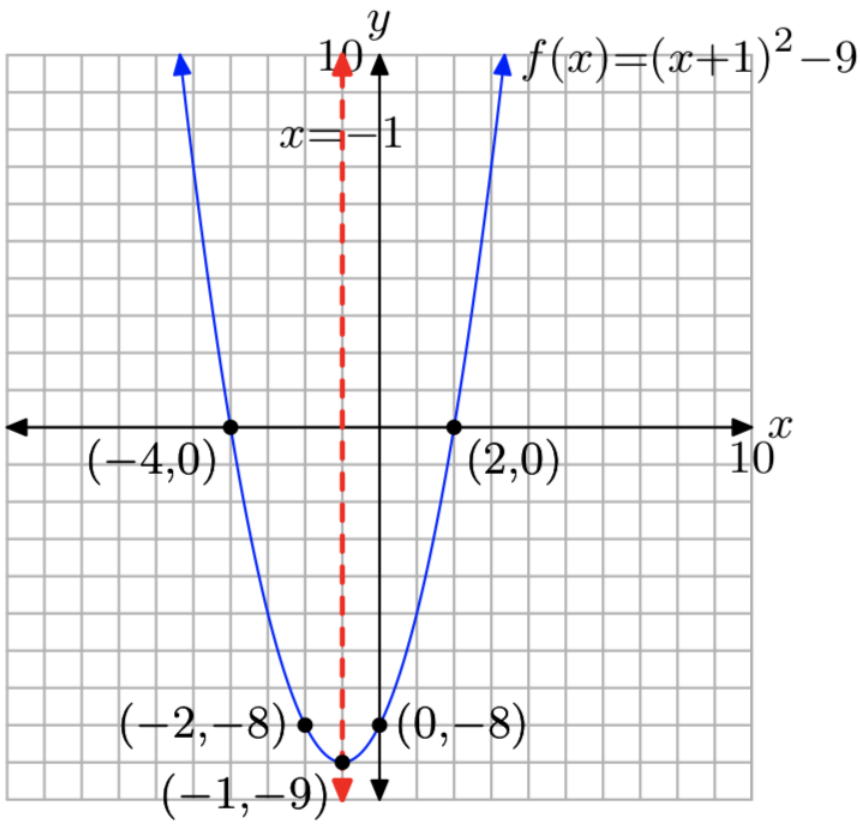
1. Set up a coordinate system on graph paper. Label and scale each axis. Remember to draw all lines with a ruler.
2. Use the technique of completing the square to place the quadratic function in vertex form. Plot the vertex on your coordinate system and label it with its coordinates. Draw the axis of symmetry on your coordinate system and label it with its equation.
3. Use a strictly algebraic technique (no calculators) to find the x-intercepts of the graph of the given quadratic function. Plot them on your coordinate system and label them with their coordinates.
4. Find the y-intercept of the graph of the quadratic function. Plot the y-intercept on your coordinate system and its mirror image across the axis of symmetry, then label these points with their coordinates.
5. Using all the information plotted, draw the graph of the quadratic function and label it with the vertex form of its equation. Use interval notation to describe the domain and range of the quadratic function.

? Exercise 7.1.23

$f(x) = x^2 + 2x - 8$

Answer

Domain = $(-\infty, \infty)$, Range = $[-9, \infty)$



Exercise 7.1.24

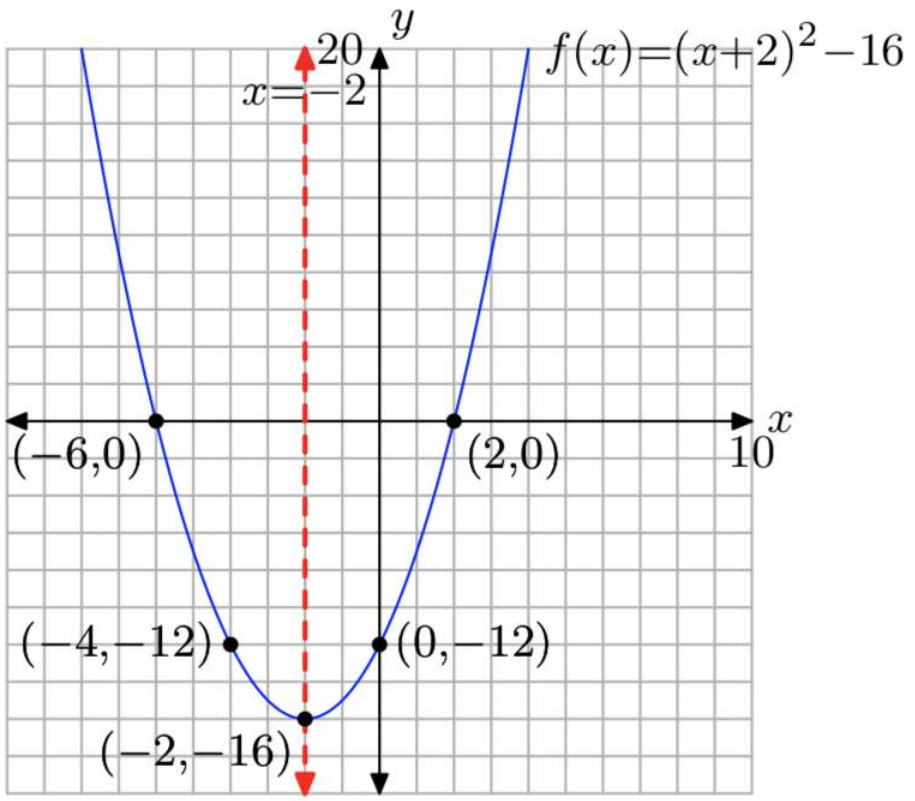
$f(x)=x^2-6x+8$

Exercise 7.1.25

$f(x)=x^2+4x-12$

Answer

Domain = $(-\infty, \infty)$, Range = $[-16, \infty)$



Exercise 7.1.26

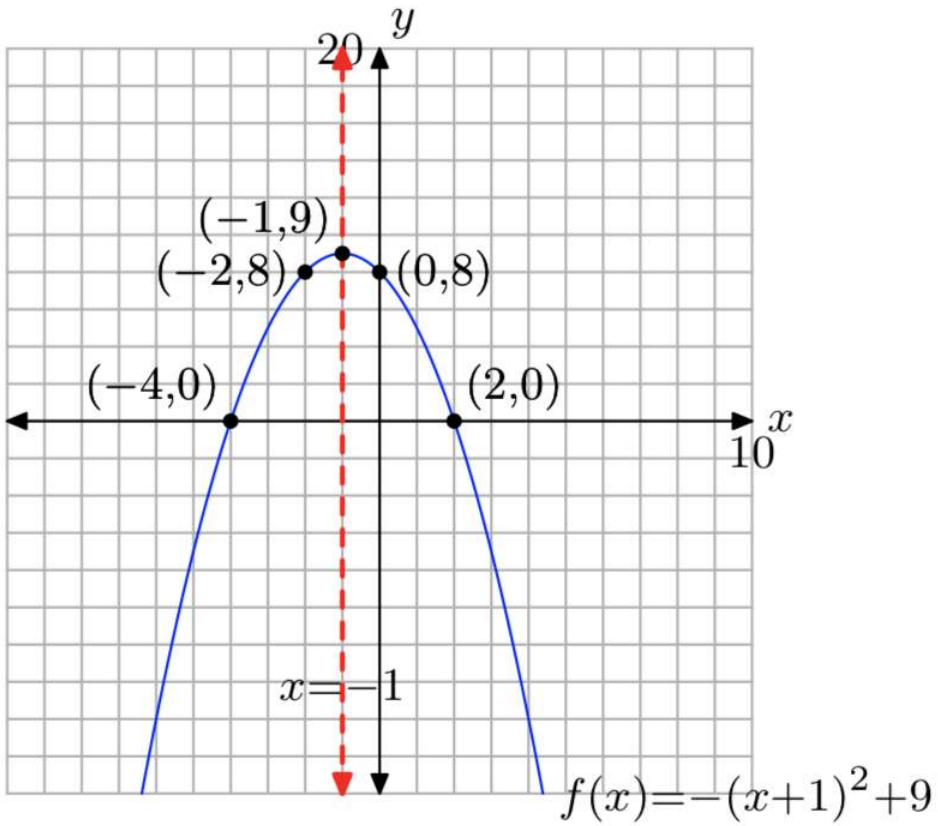
$f(x) = x^2 + 8x + 12$

Exercise 7.1.27

$f(x) = -x^2 - 2x + 8$

Answer

Domain = $(-\infty, \infty)$, Range = $(-\infty, 9]$



Exercise 7.1.28

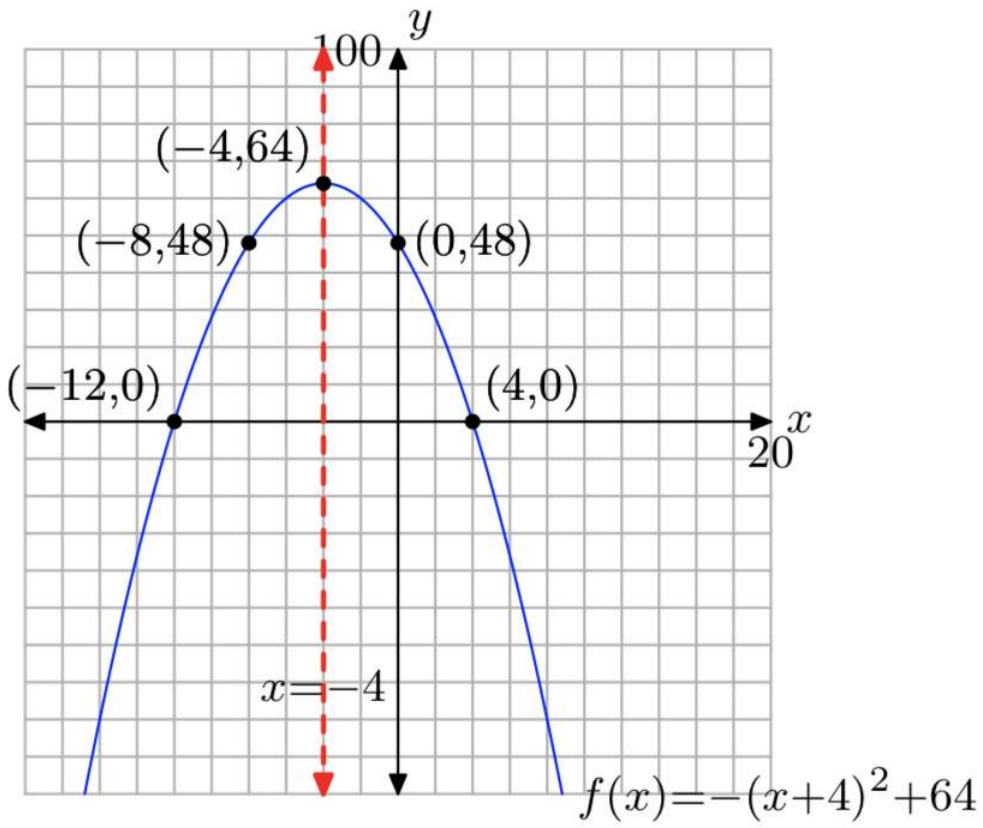
$f(x) = -x^2 - 2x + 24$

Exercise 7.1.29

$f(x) = -x^2 - 8x + 48$

Answer

Domain = $(-\infty, \infty)$, Range = $(-\infty, 64]$



Exercise 7.1.30

$f(x) = -x^2 - 8x + 20$

In **Exercises 31-38**, factor the given quadratic polynomial.

Exercise 7.1.31

$42x^2 + 5x - 2$

Answer

(7x+2)(6x−1)

Exercise 7.1.32

$3x^2 + 7x - 20$

Exercise 7.1.33

$5x^2 - 19x + 12$

Answer

(x−3)(5x−4)

Exercise 7.1.34

$54x^2 - 3x - 1$

Exercise 7.1.35

$-4x^2 + 9x - 5$

Answer

(4x−5)(−x+1)

Exercise 7.1.36

$3x^2 - 5x - 12$

Exercise 7.1.37

$2x^2 - 3x - 35$

Answer

(2x+7)(x−5)

Exercise 7.1.38

$-6x^2 + 25x + 9$

In **Exercises 39-46**, find the zeros of the given quadratic functions.

Exercise 7.1.39

$f(x) = 2x^2 - 3x - 20$

Answer

Zeros: $x = -\frac{5}{2}$, x = 4

Exercise 7.1.40

$f(x) = 2x^2 - 7x - 30$

Exercise 7.1.41

$f(x) = -2x^2 + x + 28$

Answer

Zeros: $x = -\frac{7}{2}$, x = 4

Exercise 7.1.42

$f(x) = -2x^2 + 15x - 22$

Exercise 7.1.43

$f(x) = 3x^2 - 20x + 12$

Answer

Zeros: $x = \frac{2}{3}$, x = 6

Exercise 7.1.44

$f(x) = 4x^2 + 11x - 20$

Exercise 7.1.45

$f(x) = -4x^2 + 4x + 15$

Answer

Zeros: $x = -\frac{3}{2}$, $x = \frac{5}{2}$

Exercise 7.1.46

$f(x) = -6x^2 - x + 12$

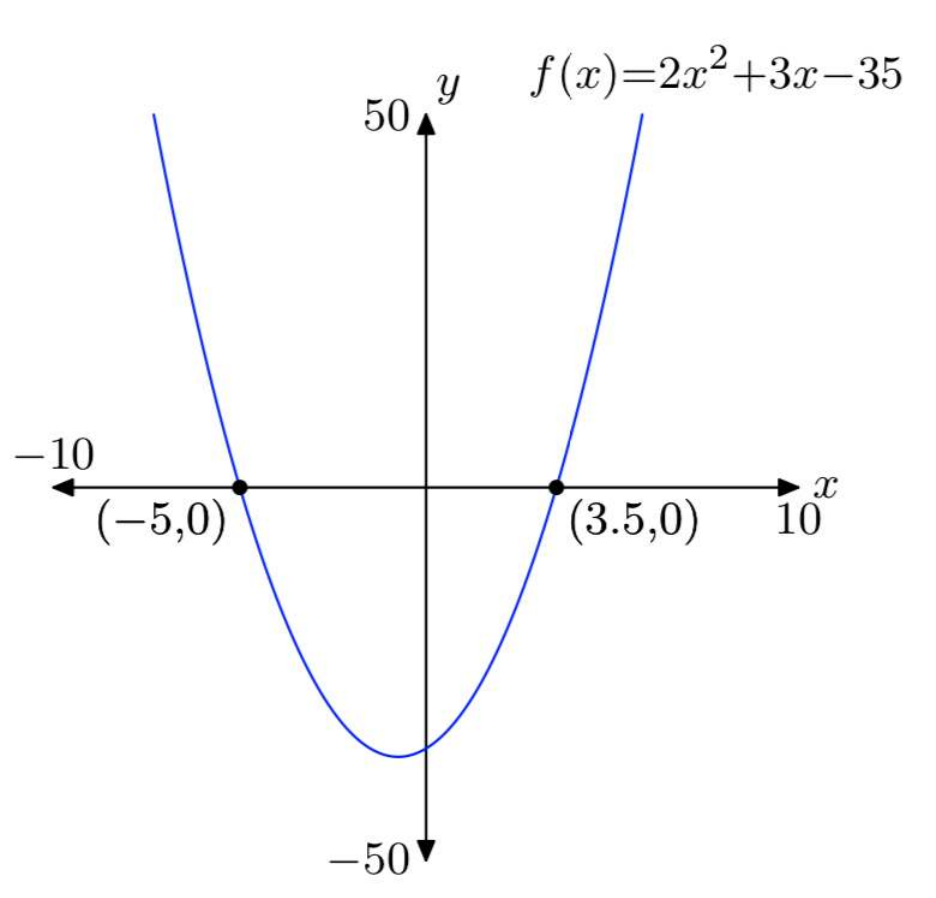
In **Exercises 47-52**, perform each of the following tasks for the given quadratic functions.

1. Load the function into Y1 of the Y= of your graphing calculator. Adjust the window parameters so that the vertex is visible in the viewing window.
2. Set up a coordinate system on your homework paper. Label and scale each axis with xmin, xmax, ymin, and ymax. Make a reasonable copy of the image in the viewing window of your calculator on this coordinate system and label it with its equation.
3. Use the zero utility on your graphing calculator to find the zeros of the function. Use these results to plot the x-intercepts on your coordinate system and label them with their co- ordinates.
4. Use a strictly algebraic technique (no calculator) to find the zeros of the given quadratic function. Show your work next to your coordinate system. Be stubborn! Work the problem until your algebraic and graphically zeros are a reasonable match.

Exercise 7.1.47

$f(x) = 2x^2 + 3x - 35$

Answer



Exercise 7.1.48

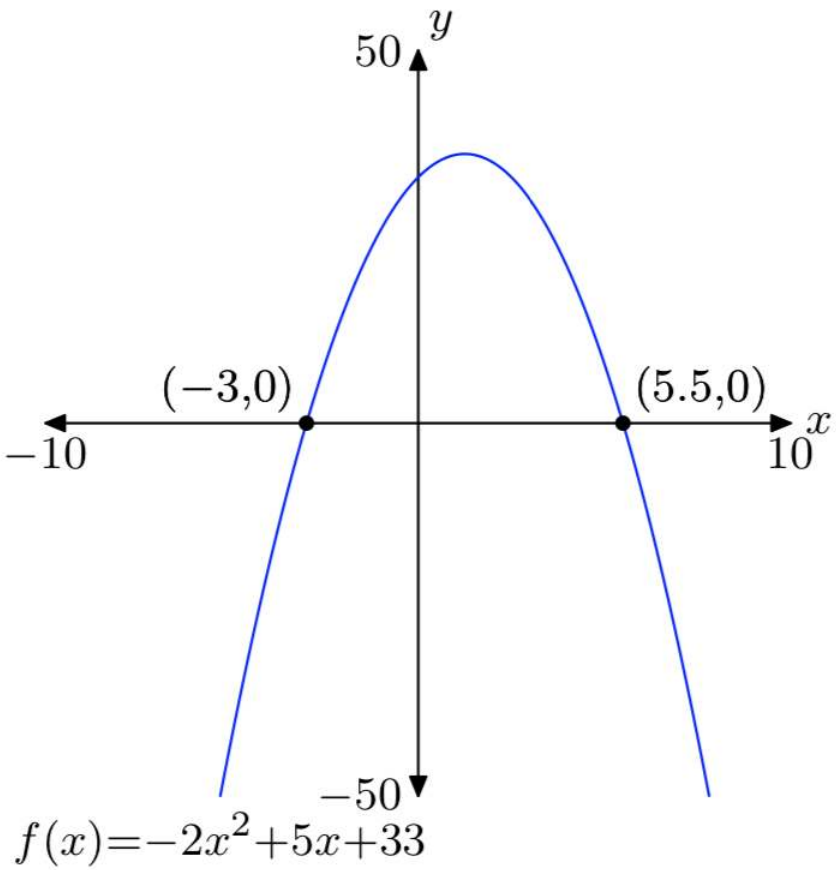
$f(x) = 2x^2 - 5x - 42$

Exercise 7.1.49

$f(x) = -2x^2 + 5x + 33$

Answer

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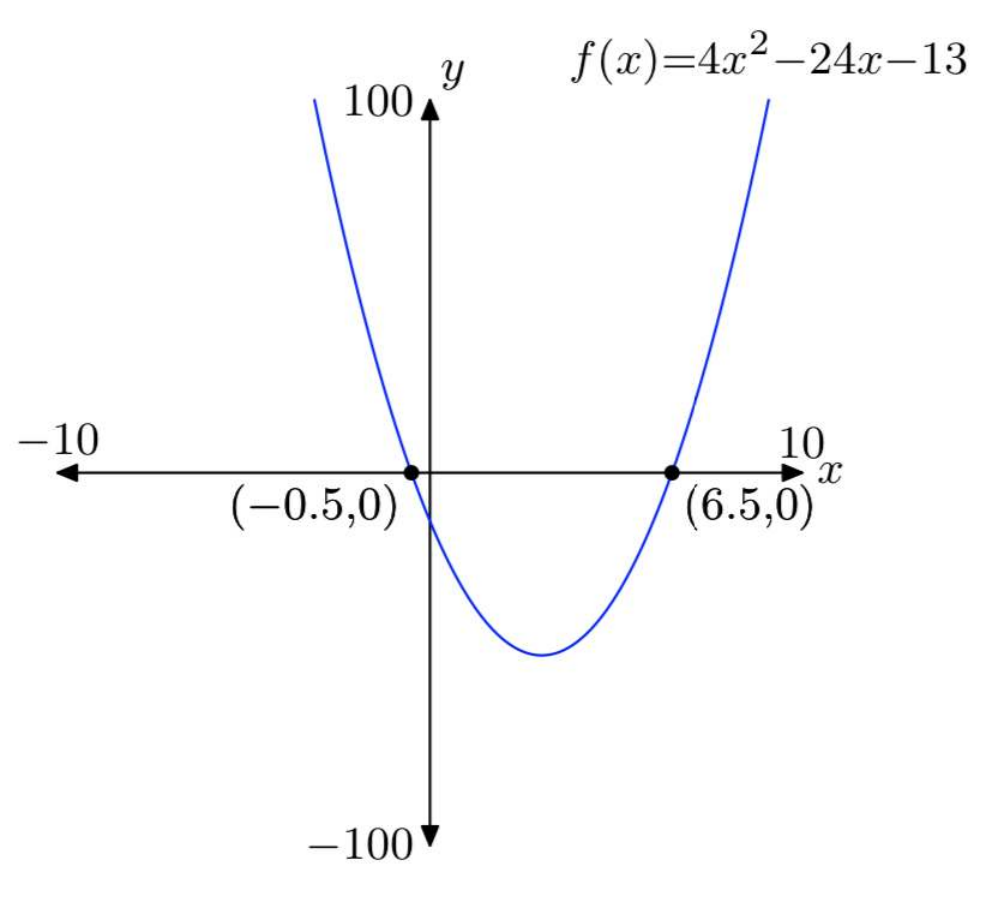
? Exercise 7.1.50

$f(x) = -2x^2 - 5x + 52$

? Exercise 7.1.51

$f(x) = 4x^2 - 24x - 13$

Answer



? Exercise 7.1.52

$f(x) = 4x^2 + 24x - 45$

In **Exercises 53-60**, perform each of the following tasks for the given quadratic functions.

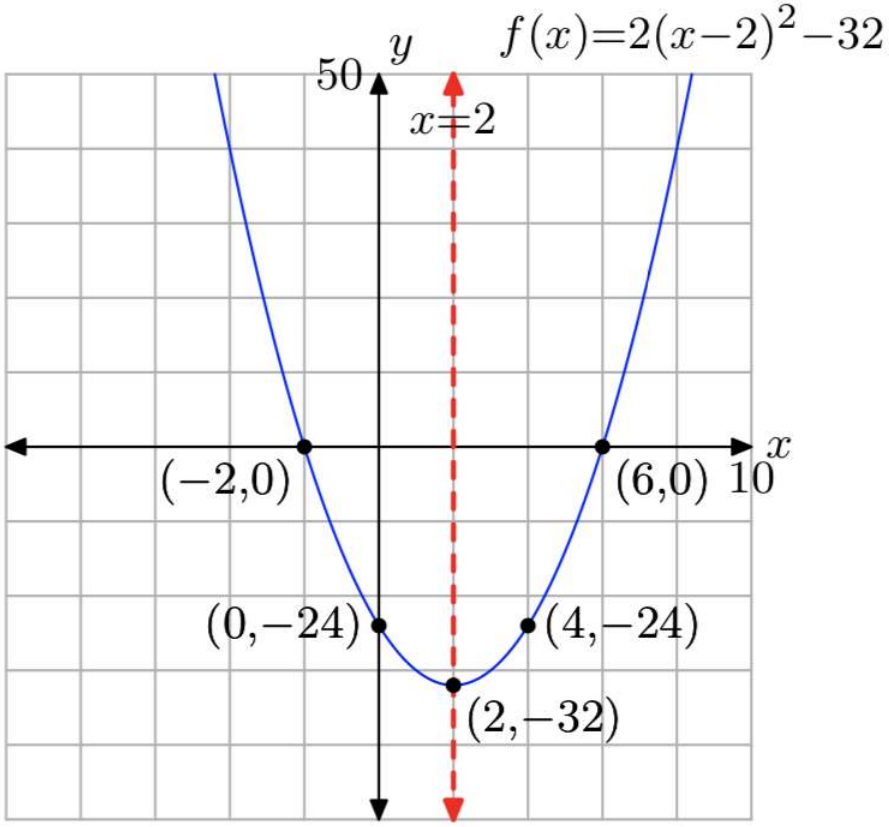
1. Set up a coordinate system on graph paper. Label and scale each axis. Remember to draw all lines with a ruler.
2. Use the technique of completing the square to place the quadratic function in vertex form. Plot the vertex on your coordinate system and label it with its coordinates. Draw the axis of symmetry on your coordinate system and label it with its equation.
3. Use a strictly algebraic method (no calculators) to find the x-intercepts of the graph of the quadratic function. Plot them on your coordinate system and label them with their coordinates.
4. Find the y-intercept of the graph of the quadratic function. Plot the y-intercept on your coordinate system and its mirror image across the axis of symmetry, then label these points with their coordinates.
5. Using all the information plotted, draw the graph of the quadratic function and label it with the vertex form of its equation. Use interval notation to describe the domain and range of the quadratic function.

? Exercise 7.1.53

$f(x) = 2x^2 - 8x - 24$

Answer

Domain = $(-\infty, \infty)$, Range = $[-32, \infty)$



Exercise 7.1.54

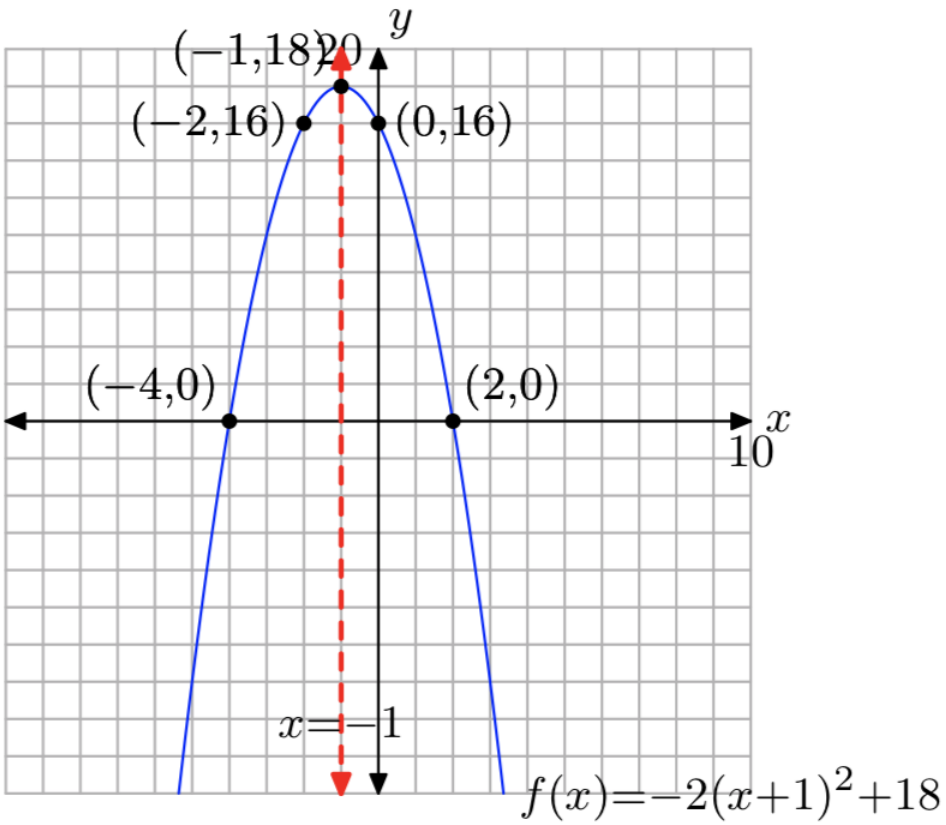
$f(x) = 2x^2 - 4x - 6$

Exercise 7.1.55

$f(x) = -2x^2 - 4x + 16$

Answer

Domain = $(-\infty, \infty)$, Range = $(-\infty, 18]$



? Exercise 7.1.56

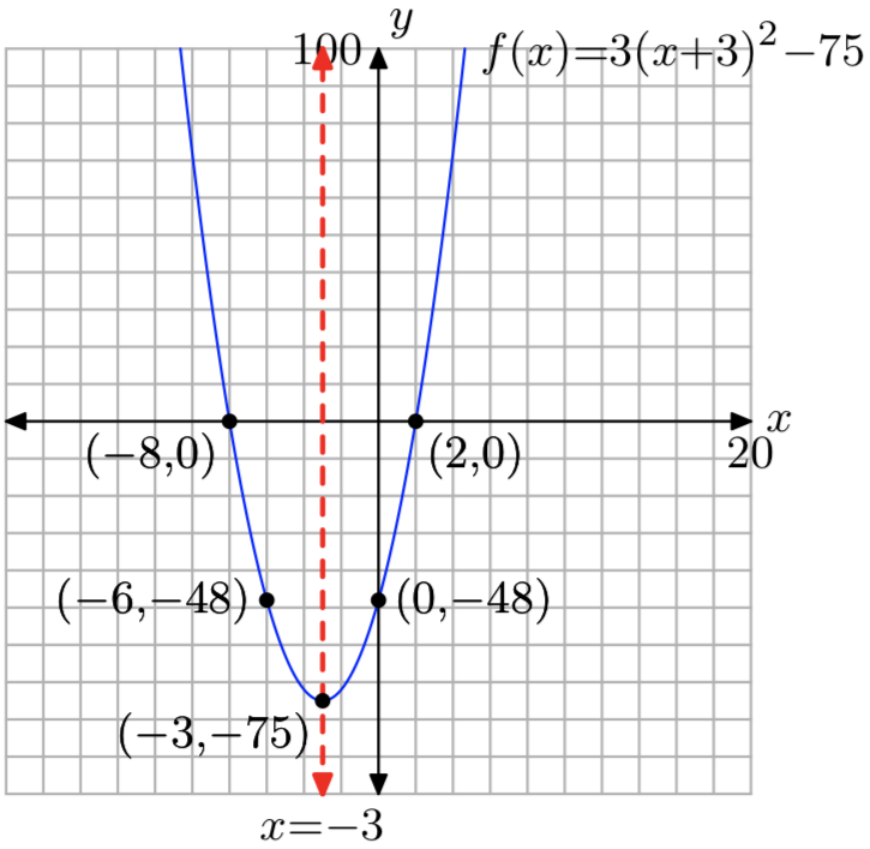
$f(x) = -2x^2 - 16x + 40$

? Exercise 7.1.57

$f(x) = 3x^2 + 18x - 48$

Answer

Domain = $(-\infty, \infty)$, Range = $[-75, \infty)$



Exercise 7.1.58

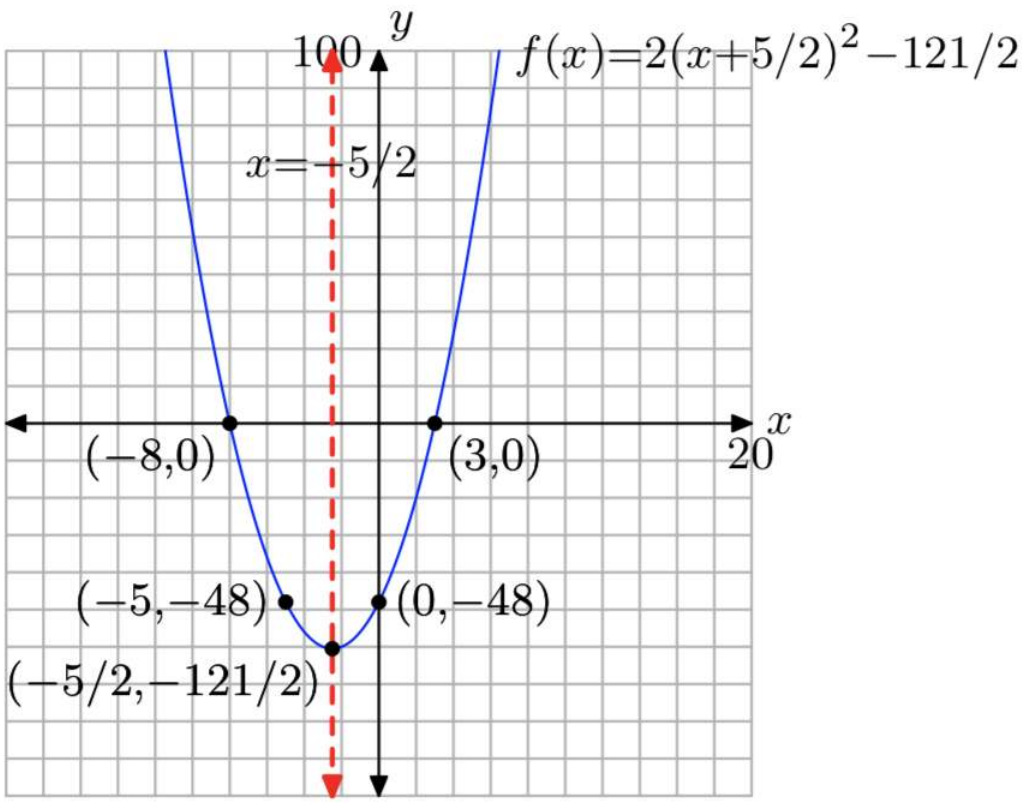
$f(x) = 3x^2 + 18x - 216$

Exercise 7.1.59

$f(x) = 2x^2 + 10x - 48$

Answer

Domain = $(-\infty, \infty)$, Range = $[-\frac{121}{2}, \infty)$

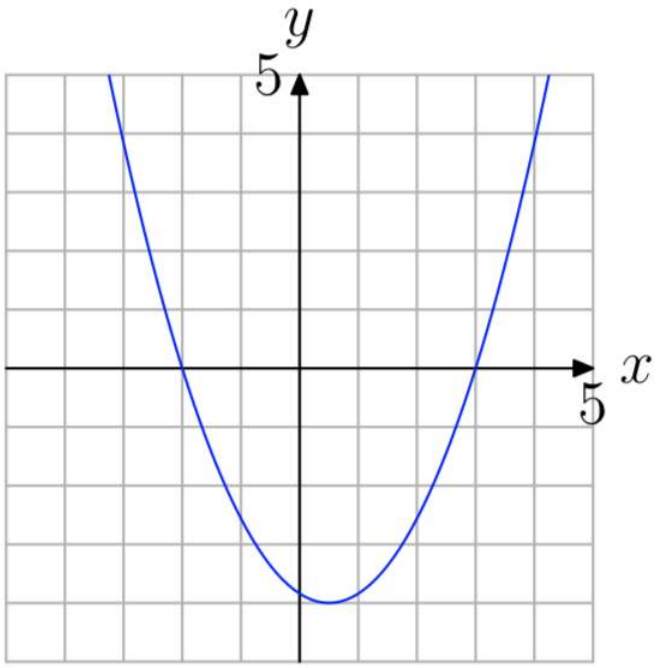


? Exercise 7.1.60

$f(x) = 2x^2 - 10x - 100$

In **Exercises 61-66**, Use the graph of $f(x) = ax^2 + bx + c$ shown to find all solutions of the equation f(x) = 0. (Note: Every solution is an integer.)

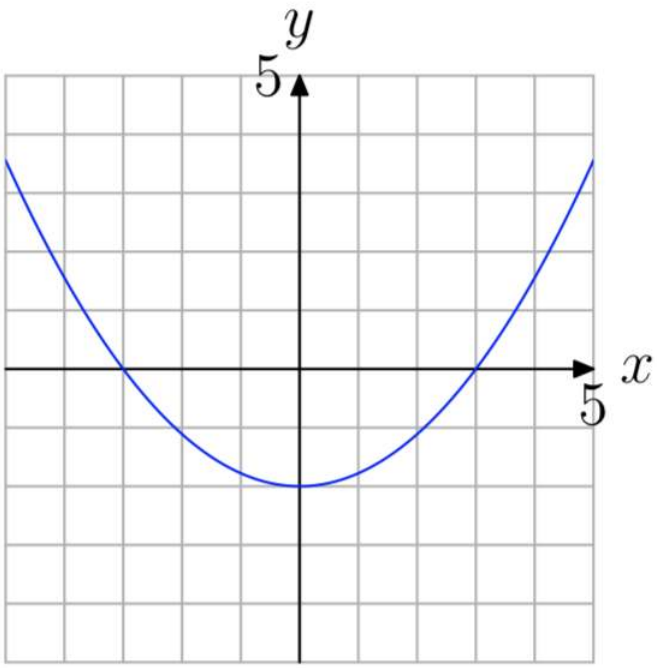
? Exercise 7.1.61



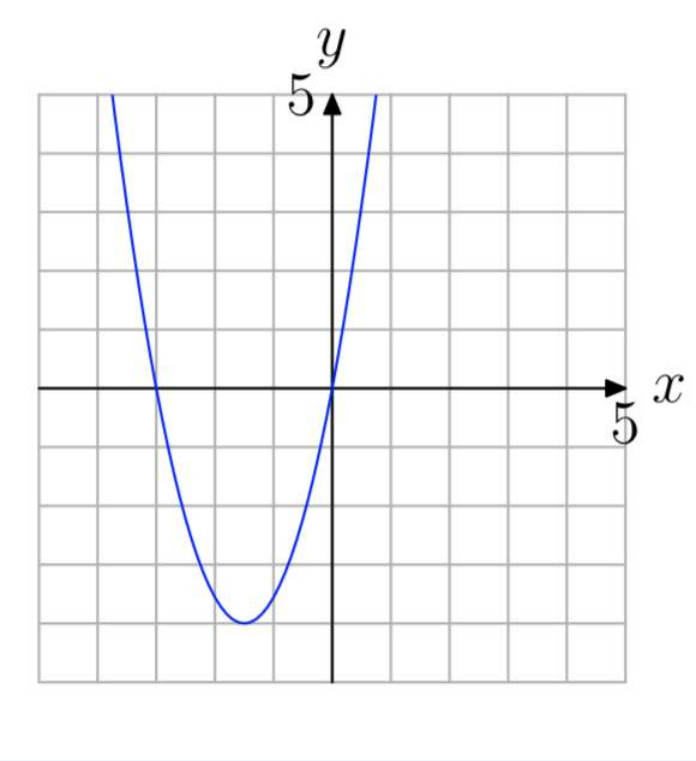
Answer

−2, 3

? Exercise 7.1.62



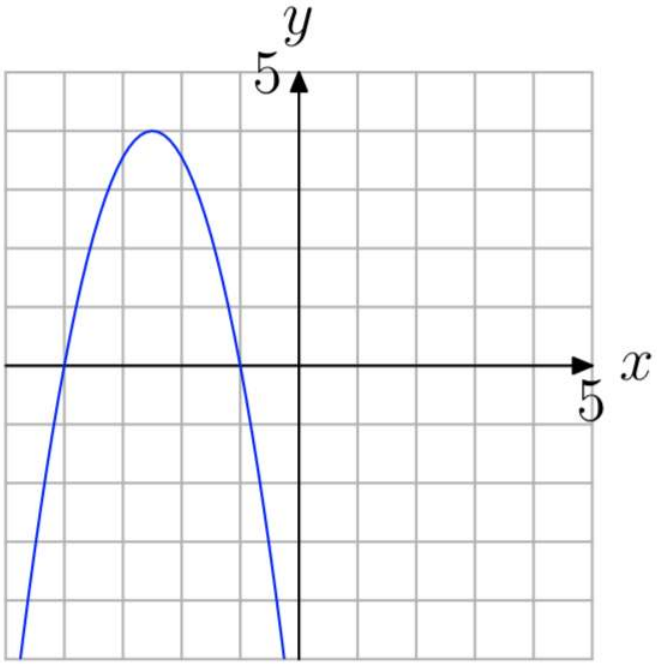
Exercise 7.1.63



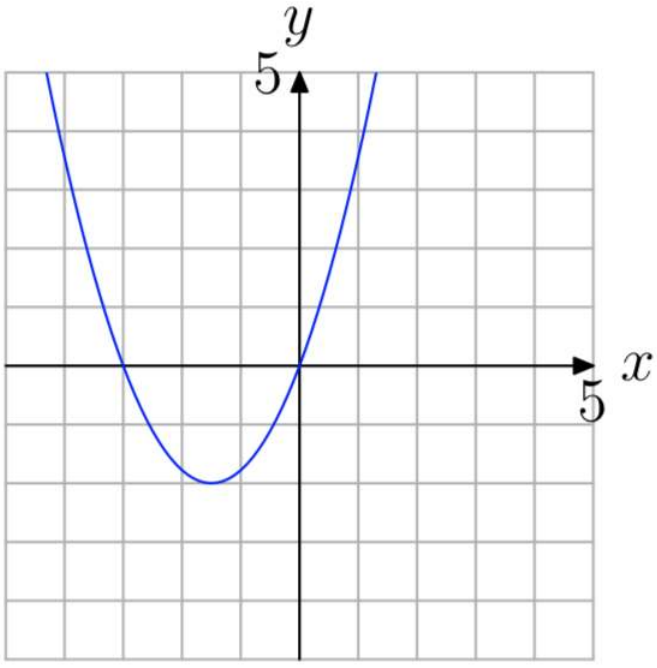
Answer

−3, 0

Exercise 7.1.64



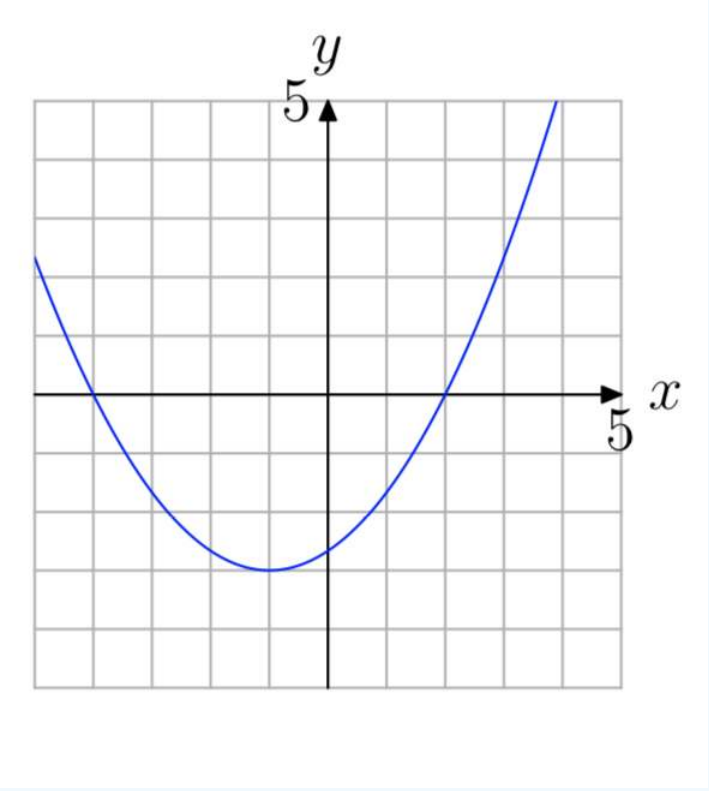
Exercise 7.1.65



Answer

−3, 0

Exercise 7.1.66



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7.2: Solve Quadratic Equations Using the Square Root Property

Learning Objectives

By the end of this section, you will be able to:

- Solve quadratic equations of the form $ax^2 = k$ using the Square Root Property
- Solve quadratic equations of the form $a(x-h)^2 = k$ using the Square Root Property

Before you get started, take this readiness quiz.

1. Simplify: $\sqrt{128}$.
   If you missed this problem, review Example 8.13.
2. Simplify: $\sqrt{\frac{32}{5}}$.
   If you missed this problem, review Example 8.50.
3. Factor: $9x^2 - 12x + 4$.
   If you missed this problem, review Example 6.23.

A quadratic equation is an equation of the form $ax^2 + bx + c = 0$, where $a \neq 0$. Quadratic equations differ from linear equations by including a quadratic term with the variable raised to the second power of the form ax^2. We use different methods to solve quadratic equations than linear equations, because just adding, subtracting, multiplying, and dividing terms will not isolate the variable.

We have seen that some quadratic equations can be solved by factoring. In this chapter, we will learn three other methods to use in case a quadratic equation cannot be factored.

Solve Quadratic Equations of the Form $ax^2 = k$ using the Square Root Property

We have already solved some quadratic equations by factoring. Let's review how we used factoring to solve the quadratic equation $x^2 = 9$.

$$x^2 = 9$$

Put the equation in standard form.

$$x^2 - 9 = 0$$

Factor the difference of squares.

$$(x-3)(x+3) = 0$$

Use the Zero Produce Property.

$$x - 3 = 0 \quad x - 3 = 0$$

Solve each equation.

$$x = 3 \quad x = -3$$

We can easily use factoring to find the solutions of similar equations, like $x^2 = 16$ and $x^2 = 25$, because 16 and 25 are perfect squares. In each case, we would get two solutions, $x = 4, x = -4$ and $x = 5, x = -5$

But what happens when we have an equation like $x^2 = 7$? Since 7 is not a perfect square, we cannot solve the equation by factoring.

Previously we learned that since 169 is the square of 13, we can also say that 13 is a *square root* of 169. Also, $(-13)^2 = 169$, so -13 is also a square root of 169. Therefore, both 13 and -13 are square roots of 169. So, every positive number has two square roots—one positive and one negative. We earlier defined the square root of a number in this way:

If $n^2 = m$, then n is a square root of m.

Since these equations are all of the form $x^2 = k$, the square root definition tells us the solutions are the two square roots of k. This leads to the **Square Root Property**.

Definition 7.2.1

Square Root Property

If $x^2 = k$, then

$$x = \sqrt{k} \quad \text{or} \quad x = -\sqrt{k} \quad \text{or} \quad x = \pm\sqrt{k}$$

Notice that the Square Root Property gives two solutions to an equation of the form $x^2 = k$, the principal square root of k and its opposite. We could also write the solution as $x = \pm\sqrt{k}$. We read this as x equals positive or negative the square root of k.

Now we will solve the equation $x^2 = 9$ again, this time using the Square Root Property.

$$\begin{aligned} & & x^2 &= 9 \\ &\text{Use the Square Root Property.} & x &= \pm\sqrt{9} \\ & & x &= \pm 3 \end{aligned}$$

So $x = 3$ or $x = -3$

What happens when the constant is not a perfect square? Let's use the Square Root Property to solve the equation $x^2 = 7$.

$$x^2 = 7$$

Use the Square Root Property. $x = \sqrt{7}, \quad x = -\sqrt{7}$

We cannot simplify $\sqrt{7}$, so we leave the answer as a radical.

Example 7.2.1 How to Solve a Quadratic Equation of the form $ax^2 - k$ Using the Square Root Property

Solve: $x^2 - 50 = 0$.

Solution:

Table 9.1.1

Step 1: Isolate the quadratic term and make its coefficient one.	Add 50 to both sides to get x^2 by itself.	$x^2 - 50 = 0$ $x^2 = 50$
Step 2: Use the Square Root Property.	Remember to write the $\pm$ symbol.	$x = \pm\sqrt{50}$
Step 3: Simplify the radical.	Rewrite to show two solutions.	$x = \pm\sqrt{25}\cdot\sqrt{2}$ $x = \pm 5\sqrt{2}$ $x = 5\sqrt{2},\ x = -5\sqrt{2}$
Step 4: Check the solutions.	Substitute in $x = 5\sqrt{2}$ and $x = -5\sqrt{2}$	$x^2 - 50 = 0$ $(5\sqrt{2})^2 - 50 \stackrel{?}{=} 0$ $25\cdot 2 - 50 \stackrel{?}{=} 0$ $0 = 0$ $x^2 - 50 = 0$ $(-5\sqrt{2})^2 - 50 \stackrel{?}{=} 0$ $25\cdot 2 - 50 \stackrel{?}{=} 0$ $0 = 0$

Exercise 7.2.1

Solve: $x^2 - 48 = 0$.

Answer

$x = 4\sqrt{3}, x = -4\sqrt{3}$

Exercise 7.2.2

Solve: $y^2 - 27 = 0$.

Answer

$y = 3\sqrt{3}, y = -3\sqrt{3}$

The steps to take to use the **Square Root Property** to solve a quadratic equation are listed here.

Solve a Quadratic Equation Using the Square Root Property

1. Isolate the quadratic term and make its coefficient one.
2. Use Square Root Property.
3. Simplify the radical.
4. Check the solutions.

In order to use the Square Root Property, the coefficient of the variable term must equal one. In the next example, we must divide both sides of the equation by the coefficient 3 before using the Square Root Property.

Example 7.2.2

Solve: $3z^2 = 108$.

Solution:

Table 9.1.2

	$3z^2 = 108$
The quadratic term is isolated. Divide by 3 to make its coefficient 1.	$\frac{3z^2}{3} = \frac{108}{3}$
Simplify.	$z^2 = 36$
Use the Square Root Property.	$z = \pm\sqrt{36}$
Simplify the radical.	$z = \pm 6$
Rewrite to show two solutions.	$z = 6, \quad z = -6$
Check the solutions: $3z^2 = 108$ $\quad$ $3z^2 = 108$ $3(6)^2 \stackrel{?}{=} 108$ $\quad$ $3(-6)^2 \stackrel{?}{=} 108$ $3(36) \stackrel{?}{=} 108$ $\quad$ $3(36) \stackrel{?}{=} 108$ $108 = 108$ ✓ $\quad$ $108 = 108$ ✓ Figure 9.1.1	

Exercise 7.2.3

Solve: $2x^2 = 98$.

Answer

$x = 7, x = -7$

Exercise 7.2.4

Solve: $5m^2 = 80$.

Answer

$m = 4, m = -4$

The Square Root Property states 'If $x^2 = k$,' What will happen if $k < 0$? This will be the case in the next example.

Example 7.2.3

Solve: $x^2 + 72 = 0$.

Solution:

Table 9.1.3

	$x^2 + 72 = 0$
Isolate the quadratic term.	$x^2 = -72$
Use the Square Root Property.	$x = \pm\sqrt{-72}$
Simplify using complex numbers.	$x = \pm\sqrt{72}i$
Simplify the radical.	$x = \pm 6\sqrt{2}i$
Rewrite to show two solutions	$x = 6\sqrt{2}i, x = -6\sqrt{2}i$
Check the solutions: $x^2 + 72 = 0$ $(6\sqrt{2}i)^2 + 72 \stackrel{?}{=} 0$ $(6)^2 \cdot (\sqrt{2})^2 \cdot i^2 + 72 \stackrel{?}{=} 0$ $36 \cdot 2 \cdot (-1) + 72 \stackrel{?}{=} 0$ $0 = 0$ ✓	$x^2 + 72 = 0$ $(-6\sqrt{2}i)^2 + 72 \stackrel{?}{=} 0$ $(-6)^2 \cdot (\sqrt{2})^2 \cdot i^2 + 72 \stackrel{?}{=} 0$ $36 \cdot 2 \cdot (-1) + 72 \stackrel{?}{=} 0$ $0 = 0$ ✓

Exercise 7.2.5

Solve: $c^2 + 12 = 0$.

Answer

$c = 2\sqrt{3}i, \quad c = -2\sqrt{3}i$

Exercise 7.2.6

Solve: $q^2 + 24 = 0$.

Answer

$c = 2\sqrt{6}i, \quad c = -2\sqrt{6}i$

Our method also works when fractions occur in the equation, we solve as any equation with fractions. In the next example, we first isolate the quadratic term, and then make the coefficient equal to one.

Example 7.2.4

Solve: $\frac{2}{3}u^2 + 5 = 17$.

Solution:

Table 9.1.4

	$\frac{2}{3}u^2+5=17$
Isolate the quadratic term.	$\frac{2}{3}u^2=12$
Multiply by $\frac{3}{2}$ to make the coefficient 1.	$\frac{3}{2}\cdot\frac{2}{3}u^2=\frac{3}{2}\cdot 12$
Simplify.	$u^2=18$
Use the Square Root Property.	$u=\pm\sqrt{18}$
Simplify the radical.	$u=\pm\sqrt{9\cdot 2}$
Simplify.	$u=\pm 3\sqrt{2}$
Rewrite to show two solutions.	$u=3\sqrt{2},\quad u=-3\sqrt{2}$
Check: $\frac{2}{3}u^2+5=17$ $\frac{2}{3}(3\sqrt{2})^2+5\stackrel{?}{=}17$ $\frac{2}{3}\cdot 18+5\stackrel{?}{=}17$ $12+5\stackrel{?}{=}17$ $17=17\checkmark$	$\frac{2}{3}u^2+5=17$ $\frac{2}{3}(-3\sqrt{2})^2+5\stackrel{?}{=}17$ $\frac{2}{3}\cdot 18+5\stackrel{?}{=}17$ $12+5\stackrel{?}{=}17$ $17=17\checkmark$

? Exercise 7.2.7

Solve: $\frac{1}{2}x^2+4=24$.

Answer

$x=2\sqrt{10}, x=-2\sqrt{10}$

? Exercise 7.2.8

Solve: $\frac{3}{4}y^2-3=18$.

Answer

$y=2\sqrt{7}, y=-2\sqrt{7}$

The solutions to some equations may have fractions inside the radicals. When this happens, we must **rationalize the denominator**.

✓ Example 7.2.5

Solve: $2x^2-8=41$.

Solution:

Table 9.1.5

	$2x^2-8=41$
Isolate the quadratic term.	$2x^2=49$
Divide by 2 to make the coefficient 1.	$\frac{2x^2}{2}=\frac{49}{2}$
Simplify.	$x^2=\frac{49}{2}$

Use the Square Root Property.	$x=\pm\sqrt{\frac{49}{2}}$
Rewrite the radical as a fraction of square roots.	$x=\pm\frac{\sqrt{49}}{\sqrt{2}}$
Rationalize the denominator.	$x=\pm\frac{\sqrt{49}\cdot\sqrt{2}}{\sqrt{2}\cdot\sqrt{2}}$
Simplify.	$x=\pm\frac{7\sqrt{2}}{2}$
Rewrite to show two solutions.	$x=\frac{7\sqrt{2}}{2},\quad x=-\frac{7\sqrt{2}}{2}$
Check: We leave te check for you.	

? Exercise 7.2.9

Solve: $5r^2-2=34$.

Answer

$r=\frac{6\sqrt{5}}{5},\quad r=-\frac{6\sqrt{5}}{5}$

? Exercise 7.2.10

Solve: $3t^2+6=70$.

Answer

$t=\frac{8\sqrt{3}}{3},\quad t=-\frac{8\sqrt{3}}{3}$

Solve Quadratic Equation of the Form $a(x-h)^2=k$ Using the Square Root Property

We can use the **Square Root Property** to solve an equation of the form $a(x-h)^2=k$ as well. Notice that the quadratic term, x, in the original form $ax^2=k$ is replaced with $(x-h)$.

$ax^2=k$ $\quad$ $a(x-h)^2=k$

Figure 9.1.20

The first step, like before, is to isolate the term that has the variable squared. In this case, a binomial is being squared. Once the binomial is isolated, by dividing each side by the coefficient of a, then the Square Root Property can be used on $(x-h)^2$.

✓ Example 7.2.6

Solve: $4(y-7)^2=48$.

Solution:

Table 9.1.6

	$4(y-7)^2=48$
Divide both sides by the coefficient 4.	$(y-7)^2=12$
Use the Square Root Property on the binomial.	$y-7=\pm\sqrt{12}$
Simplify the radical.	$y-7=\pm2\sqrt{3}$
Solve for y.	$y=7\pm2\sqrt{3}$

Rewrite to show two solutions.	$y = 7 + 2\sqrt{3}$ $y = 7 - 2\sqrt{3}$

Check:

$4(y-7)^2 = 48$ | $4(y-7)^2 = 48$

$4(7+2\sqrt{3}-7)^2 \stackrel{?}{=} 48$ | $4(7-2\sqrt{3}-7)^2 \stackrel{?}{=} 48$

$4(2\sqrt{3})^2 \stackrel{?}{=} 48$ | $4(-2\sqrt{3})^2 \stackrel{?}{=} 48$

$4(12) \stackrel{?}{=} 48$ | $4(12) \stackrel{?}{=} 48$

$48 = 48$ ✓ | $48 = 48$ ✓

Figure 9.1.21

? Exercise 7.2.11

Solve: $3(a-3)^2 = 54$.

Answer

$a = 3 + 3\sqrt{2}, \quad a = 3 - 3\sqrt{2}$

? Exercise 7.2.12

Solve: $2(b+2)^2 = 80$.

Answer

$b = -2 + 2\sqrt{10}, \quad b = -2 - 2\sqrt{10}$

Remember when we take the square root of a fraction, we can take the square root of the numerator and denominator separately.

✓ Example 7.2.7

Solve: $\left(x - \frac{1}{3}\right)^2 = \frac{5}{9}$.

Solution:

$$\left(x - \tfrac{1}{3}\right)^2 = \tfrac{5}{9}$$

Use the Square Root Property.

$$x - \tfrac{1}{3} = \pm\sqrt{\tfrac{5}{9}}$$

Rewrite the radical as a fraction of square roots.

$$x - \tfrac{1}{3} = \pm\tfrac{\sqrt{5}}{\sqrt{9}}$$

Simplify the radical.

$$x - \tfrac{1}{3} = \pm\tfrac{\sqrt{5}}{3}$$

Solve for x.

$$x = \tfrac{1}{3} \pm \tfrac{\sqrt{5}}{3}$$

Rewrite to show two solutions.

$$x = \tfrac{1}{3} + \tfrac{\sqrt{5}}{3}, x = \tfrac{1}{3} - \tfrac{\sqrt{5}}{3}$$

Check:

We leave the check for you.

Exercise 7.2.13

Solve: $\left(x-\frac{1}{2}\right)^2=\frac{5}{4}$.

Answer

$x=\frac{1}{2}+\frac{\sqrt{5}}{2}, x=\frac{1}{2}-\frac{\sqrt{5}}{2}$

Exercise 7.2.14

Solve: $\left(y+\frac{3}{4}\right)^2=\frac{7}{16}$.

Answer

$y=-\frac{3}{4}+\frac{\sqrt{7}}{4}, y=-\frac{3}{4}-\frac{\sqrt{7}}{4}$

We will start the solution to the next example by isolating the binomial term.

Example 7.2.8

Solve: $2(x-2)^2+3=57$.

Solution:

$$2(x-2)^2+3=57$$

Subtract 3 from both sides to isolate the binomial term.

$$2(x-2)^2=54$$

Divide both sides by 2.

$$(x-2)^2=27$$

Use the Square Root Property.

$$x-2=\pm\sqrt{27}$$

Simplify the radical.

$$x-2=\pm 3\sqrt{3}$$

Solve for x.

$$x=2\pm 3\sqrt{3}$$

Rewrite to show two solutions.

$$x=2+3\sqrt{3}, x=2-3\sqrt{3}$$

Check:

We leave the check for you.

Exercise 7.2.15

Solve: $5(a-5)^2+4=104$.

Answer

$a=5+2\sqrt{5}, a=5-2\sqrt{5}$

Exercise 7.2.16

Solve: $3(b+3)^2-8=88$.

Answer

$b=-3+4\sqrt{2}, \quad b=-3-4\sqrt{2}$

Sometimes the solutions are complex numbers.

Example 7.2.9

Solve: $(2x-3)^2=-12$.

Solution:

$$(2x-3)^2=-12$$

Use the Square Root Property.

$$2x-3=\pm\sqrt{-12}$$

Simplify the radical.

$$2x-3=\pm 2\sqrt{3}i$$

Add 3 to both sides.

$$2x=3\pm 2\sqrt{3}i$$

Divide both sides by 2.

$$x=\frac{3\pm 2\sqrt{3}i}{2}$$

Rewrite in standard form.

$$x=\frac{3}{2}\pm\frac{2\sqrt{3}i}{2}$$

Simplify.

$$x=\frac{3}{2}\pm\sqrt{3}i$$

Rewrite to show two solutions.

$$x=\frac{3}{2}+\sqrt{3}i, x=\frac{3}{2}-\sqrt{3}i$$

Check:

We leave the check for you.

Exercise 7.2.17

Solve: $(3r+4)^2=-8$.

Answer

$r=-\frac{4}{3}+\frac{2\sqrt{2}i}{3}, r=-\frac{4}{3}-\frac{2\sqrt{2}i}{3}$

Exercise 7.2.18

Solve: $(2t-8)^2=-10$.

Answer

$t=4+\frac{\sqrt{10}i}{2}, t=4-\frac{\sqrt{10}i}{2}$

The left sides of the equations in the next two examples do not seem to be of the form $a(x-h)^2$. But they are perfect square trinomials, so we will factor to put them in the form we need.

Example 7.2.10

Solve: $4n^2+4n+1=16$.

Solution:

We notice the left side of the equation is a perfect square trinomial. We will factor it first.

Table 9.1.7

	$4n^2+4n+1=16$
Factor the perfect square trinomial.	$(2n+1)^2=16$
Use the Square Root Property.	$2n+1=\pm\sqrt{16}$
Simplify the radical.	$2n+1=\pm 4$
Solve for n.	$2n=-1\pm 4$
Divide each side by 2.	$\frac{2n}{2}=\frac{-1\pm 4}{2}$ $n=\frac{-1\pm 4}{2}$
Rewrite to show two solutions.	$n=\frac{-1+4}{2}, n=\frac{-1-4}{2}$
Simplify each equation.	$n=\frac{3}{2}, \quad n=-\frac{5}{2}$
Check: $4n^2+4n+1=16$ $4\left(\frac{3}{2}\right)^2+4\left(\frac{3}{2}\right)+1\stackrel{?}{=}16$ $4\left(\frac{9}{4}\right)+4\left(\frac{3}{2}\right)+1\stackrel{?}{=}16$ $9+6+1\stackrel{?}{=}16$ $16=16$ ✓ $4n^2+4n+1=16$ $4\left(-\frac{5}{2}\right)^2+4\left(-\frac{5}{2}\right)+1\stackrel{?}{=}16$ $4\left(\frac{25}{4}\right)+4\left(-\frac{5}{2}\right)+1\stackrel{?}{=}16$ $25-10+1\stackrel{?}{=}16$ $16=16$ ✓	

Figure 9.1.22

Exercise 7.2.19

Solve: $9m^2-12m+4=25$.

Answer

$m=\frac{7}{3}, \quad m=-1$

Exercise 7.2.20

Solve: $16n^2+40n+25=4$.

Answer

$n=-\frac{3}{4}, \quad n=-\frac{7}{4}$

Access this online resource for additional instruction and practice with using the Square Root Property to solve quadratic equations.

- Solving Quadratic Equations: The Square Root Property

- Using the Square Root Property to Solve Quadratic Equations

Key Concepts

- Square Root Property
  - If $x^2 = k$, then $x = \sqrt{k}$ or $x = -\sqrt{k}$or $x = \pm\sqrt{k}$

  How to solve a quadratic equation using the square root property.

  1. Isolate the quadratic term and make its coefficient one.
  2. Use Square Root Property.
  3. Simplify the radical.
  4. Check the solutions.

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7.2E: Exercises

Practice Makes Perfect

ExerciseS 1 - 22: Solve Quadratic Equations of the Form $ax^2 = k$ Using the Square Root Property

In the following exercises, solve each equation.

1. $a^2 = 49$
2. $b^2 = 144$
3. $r^2 - 24 = 0$
4. $t^2 - 75 = 0$
5. $u^2 - 300 = 0$
6. $v^2 - 80 = 0$
7. $4m^2 = 36$
8. $3n^2 = 48$
9. $\frac{4}{3}x^2 = 48$
10. $\frac{5}{3}y^2 = 60$
11. $x^2 + 25 = 0$
12. $y^2 + 64 = 0$
13. $x^2 + 63 = 0$
14. $y^2 + 45 = 0$
15. $\frac{4}{3}x^2 + 2 = 110$
16. $\frac{2}{3}y^2 - 8 = -2$
17. $\frac{2}{5}a^2 + 3 = 11$
18. $\frac{3}{2}b^2 - 7 = 41$
19. $7p^2 + 10 = 26$
20. $2q^2 + 5 = 30$
21. $5y^2 - 7 = 25$
22. $3x^2 - 8 = 46$

Answer

1. $a = \pm 7$
3. $r = \pm 2\sqrt{6}$
5. $u = \pm 10\sqrt{3}$
7. $m = \pm 3$
9. $x = \pm 6$
11. $x = \pm 5i$
13. $x = \pm 3\sqrt{7}i$
15. $x = \pm 9$
17. $a = \pm 2\sqrt{5}$

19. $p = \pm\frac{4\sqrt{7}}{7}$

21. $y = \pm\frac{4\sqrt{10}}{5}$

Exercises 23 - 46: Solve Quadratic Equations of the Form $a(x-h)^2 = k$ Using the Square Root Property

In the following exercises, solve each equation.

23. $(u-6)^2 = 64$

24. $(v+10)^2 = 121$

25. $(m-6)^2 = 20$

26. $(n+5)^2 = 32$

27. $\left(r-\frac{1}{2}\right)^2 = \frac{3}{4}$

28. $\left(x+\frac{1}{5}\right)^2 = \frac{7}{25}$

29. $\left(y+\frac{2}{3}\right)^2 = \frac{8}{81}$

30. $\left(t-\frac{5}{6}\right)^2 = \frac{11}{25}$

31. $(a-7)^2+5 = 55$

32. $(b-1)^2-9 = 39$

33. $4(x+3)^2-5 = 27$

34. $5(x+3)^2-7 = 68$

35. $(5c+1)^2 = -27$

36. $(8d-6)^2 = -24$

37. $(4x-3)^2+11 = -17$

38. $(2y+1)^2-5 = -23$

39. $m^2-4m+4 = 8$

40. $n^2+8n+16 = 27$

41. $x^2-6x+9 = 12$

42. $y^2+12y+36 = 32$

43. $25x^2-30x+9 = 36$

44. $9y^2+12y+4 = 9$

45. $36x^2-24x+4 = 81$

46. $64x^2+144x+81 = 25$

Answer

23. $u = 14, u = -2$

25. $m = 6 \pm 2\sqrt{5}$

27. $r = \frac{1}{2} \pm \frac{\sqrt{3}}{2}$

29. $y = -\frac{2}{3} \pm \frac{2\sqrt{2}}{9}$

31. $a = 7 \pm 5\sqrt{2}$

33. $x = -3 \pm 2\sqrt{2}$

35. $c = -\frac{1}{5} \pm \frac{3\sqrt{3}}{5}i$

37. $x = \frac{3}{4} \pm \frac{\sqrt{7}}{2}i$

39. $m = 2 \pm 2\sqrt{2}$

41. $x = 3 + 2\sqrt{3}, x = 3 - 2\sqrt{3}$

43. $x = -\frac{3}{5}, x = \frac{9}{5}$

45. $x = -\frac{7}{6}, x = \frac{11}{6}$

ExerciseS 47 - 68: Mixed Practice

In the following exercises, solve using the Square Root Property.

47. $2r^2 = 32$

48. $4t^2 = 16$

49. $(a-4)^2 = 28$

50. $(b+7)^2 = 8$

51. $9w^2 - 24w + 16 = 1$

52. $4z^2 + 4z + 1 = 49$

53. $a^2 - 18 = 0$

54. $b^2 - 108 = 0$

55. $\left(p - \frac{1}{3}\right)^2 = \frac{7}{9}$

56. $\left(q - \frac{3}{5}\right)^2 = \frac{3}{4}$

57. $m^2 + 12 = 0$

58. $n^2 + 48 = 0$

59. $u^2 - 14u + 49 = 72$

60. $v^2 + 18v + 81 = 50$

61. $(m-4)^2 + 3 = 15$

62. $(n-7)^2 - 8 = 64$

63. $(x+5)^2 = 4$

64. $(y-4)^2 = 64$

65. $6c^2 + 4 = 29$

66. $2d^2 - 4 = 77$

67. $(x-6)^2 + 7 = 3$

68. $(y-4)^2 + 10 = 9$

Answer

47. $r = \pm 4$

49. $a = 4 \pm 2\sqrt{7}$

51. $w = 1, w = \frac{5}{3}$

53. $a = \pm 3\sqrt{2}$

55. $p = \frac{1}{3} \pm \frac{\sqrt{7}}{3}$

57. $m = \pm 2\sqrt{2i}$

59. $u = 7 \pm 6\sqrt{2}$

61. $m = 4 \pm 2\sqrt{3}$

63. $x = -3, x = -7$

65. $c = \pm \frac{5\sqrt{6}}{6}$

67. $x = 6 \pm 2i$

? ExerciseS 69 - 70: Writing exercises

69. In your own words, explain the Square Root Property.

70. In your own words, explain how to use the Square Root Property to solve the quadratic equation $(x+2)^2 = 16$.

Answer

69. Answers will vary.

Self Check

a. After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
solve quadratic equations of the form $ax^2 = k$ using the square root property.			
solve quadratic equations of the form $a(x-h)^2 = k$ using the square root property.			

Figure 9.1.23

Choose how would you respond to the statement "I can solve quadratic equations of the form a times the square of x minus h equals k using the Square Root Property." "Confidently," "with some help," or "No, I don't get it."

b. If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them. What did you do to become confident of your ability to do these things? Be specific.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math every topic builds upon previous work. It is important to make sure you have a strong foundation before you move on. Who can you ask for help? Your fellow classmates and instructor are good resources. Is there a place on campus where math tutors are available? Can your study skills be improved?

…no - I don't get it! This is a warning sign and you must not ignore it. You should get help right away or you will quickly be overwhelmed. See your instructor as soon as you can to discuss your situation. Together you can come up with a plan to get you the help you need.

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7.3: Solve Quadratic Equations Using the Quadratic Formula

Learning Objectives

By the end of this section, you will be able to:

- Solve quadratic equations using the Quadratic Formula
- Use the discriminant to predict the number and type of solutions of a quadratic equation
- Identify the most appropriate method to use to solve a quadratic equation

Before you get started, take this readiness quiz.

1. Evaluate b^2-4ab when $a=3$ and $b=-2$.
2. Simplify $\sqrt{108}$.
3. Simplify $\sqrt{50}$.

Solve Quadratic Equations Using the Quadratic Formula

When we solved quadratic equations in the last section by completing the square, we took the same steps every time. By the end of the exercise set, you may have been wondering 'isn't there an easier way to do this?' The answer is 'yes'. Mathematicians look for patterns when they do things over and over in order to make their work easier. In this section we will derive and use a formula to find the solution of a quadratic equation.

We have already seen how to solve a formula for a specific variable 'in general', so that we would do the algebraic steps only once, and then use the new formula to find the value of the specific variable. Now we will go through the steps of completing the square using the general form of a quadratic equation to solve a quadratic equation for x.

We start with the standard form of a quadratic equation and solve it for x by completing the square.

	$ax^2+bx+c=0, \quad a\neq 0$
Isolate the variable terms on one side.	$ax^2+bx \quad =-c$
Make the coefficient of x^2 equal to 1, by dividing by a.	$\frac{ax^2}{a}+\frac{b}{a}x \quad =-\frac{c}{a}$
Simplify.	$x^2+\frac{b}{a}x \quad =-\frac{c}{a}$
To complete the square, find $\left(\frac{1}{2}\cdot\frac{b}{a}\right)^2$ and add it to both sides of the equation.	
$\left(\frac{1}{2}\frac{b}{a}\right)^2=\frac{b^2}{4a^2}$	$x^2+\frac{b}{a}x+\frac{b^2}{4a^2}=-\frac{c}{a}+\frac{b^2}{4a^2}$
The left side is a perfect square, factor it.	$\left(x+\frac{b}{2a}\right)^2=-\frac{c}{a}+\frac{b^2}{4a^2}$
Find the common denominator of the right side and write equivalent fractions with the common denominator.	$\left(x+\frac{b}{2a}\right)^2=\frac{b^2}{4a^2}-\frac{c\cdot 4a}{a\cdot 4a}$
Simplify.	$\left(x+\frac{b}{2a}\right)^2=\frac{b^2}{4a^2}-\frac{4ac}{4a^2}$
Combine to one fraction.	$\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2}$
Use the square root property.	$x+\frac{b}{2a}=\pm\sqrt{\frac{b^2-4ac}{4a^2}}$

Simplify the radical.	$x+\frac{b}{2a}=\pm\frac{\sqrt{b^2-4ac}}{2a}$
Add $-\frac{b}{2a}$ to both sides of the equation.	$x=-\frac{b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$
Combine the terms on the right side.	$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

The final equation is called the "Quadratic Formula."

Definition 7.3.1: Quadratic Formula

The solutions to a **quadratic equation** of the form $ax^2+bx+c=0$, where $a\neq 0$ are given by the formula:

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a} \tag{7.3.1}$$

To use the **Quadratic Formula**, we substitute the values of a, b, and c from the standard form into the expression on the right side of the formula. Then we simplify the expression. The result is the pair of solutions to the quadratic equation.

Notice the Quadratic Formula (Equation 7.3.1) is an equation. Make sure you use both sides of the equation.

Example 7.3.1 How to Solve a Quadratic Equation Using the Quadratic Formula

Solve by using the Quadratic Formula: $2x^2+9x-5=0$.

Solution:

Step 1: Write the quadratic equation in standard form. Identify the a, b, c values.	This equation is in standard form.	$ax^2+bx+c=0$ $2x^2+9x-5=0$ $a=2, b=9, c=-5$
Step 2: Write the quadratic formula. Then substitute in the values of a, b, c.	Substitute in $a=2, b=9, c=-5$	$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $x=\frac{-9\pm\sqrt{9^2-4\cdot 2\cdot(-5)}}{2\cdot 2}$
Step 3: Simplify the fraction, and solve for x.		$x=\frac{-9\pm\sqrt{81-(-40)}}{4}$ $x=\frac{-9\pm\sqrt{121}}{4}$ $x=\frac{-9\pm 11}{4}$ $x=\frac{-9+11}{4}$ $x=\frac{-9-11}{4}$ $x=\frac{2}{4}$ $x=\frac{-20}{4}$ $x=\frac{1}{2}$ $x=-5$

Step 4: Check the solutions.	Put each answer in the original equation to check. Substitute $x=\frac{1}{2}$ and $x=-5$.	$2x^2+9x-5=0$ $2\left(\frac{1}{2}\right)^2+9\cdot\frac{1}{2}-5\stackrel{?}{=}0$ $2\cdot\frac{1}{4}+0\cdot\frac{1}{2}-5\stackrel{?}{=}0$ $2\cdot\frac{1}{4}+9\cdot\frac{1}{2}-5\stackrel{?}{=}0$ $\frac{1}{2}+\frac{9}{2}-5\stackrel{?}{=}0$ $\frac{10}{2}-5\stackrel{?}{=}0$ $5-5\stackrel{?}{=}0$ $0=0$ $2x^2+9x-5=0$ $2(-5)^2+9(-5)-5\stackrel{?}{=}0$ $2\cdot25-45-5\stackrel{?}{=}0$ $50-45-5\stackrel{?}{=}0$ $0=0$

Exercise 7.3.1

Solve by using the Quadratic Formula: $3y^2-5y+2=0$.

Answer

$y=1, y=\frac{2}{3}$

Exercise 7.3.2

Solve by using the Quadratic Formula: $4z^2+2z-6=0$.

Answer

$z=1, z=-\frac{3}{2}$

HowTo: Solve a Quadratic Equation Using the Quadratic Formula

1. Write the quadratic equation in standard form, $ax^2+bx+c=0$. Identify the values of a, b, and c.
2. Write the Quadratic Formula. Then substitute in the values of a, b, and c.
3. Simplify.
4. Check the solutions.

If you say the formula as you write it in each problem, you'll have it memorized in no time! And remember, the Quadratic Formula is an EQUATION. Be sure you start with "$x=$".

Example 7.3.2

Solve by using the Quadratic Formula: $x^2-6x=-5$.

Solution:

	$x^2-6x=-5$

Write the equation in standard form by adding 5 to each side.	$x^2-6x+5=0$
This equation is now in standard form.	$ax^2+bx+c=0$ $x^2-6x+5=0$
Identify the values of a, b, c.	$a=1, b=-6, c=5$
Write the Quadratic Formula.	$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$
Then substitute in the values of a, b, c.	$x=\frac{-(-6)\pm\sqrt{(-6)^2-4\cdot 1\cdot(5)}}{2\cdot 1}$
Simplify.	$x=\frac{6\pm\sqrt{36-20}}{2}$ $x=\frac{6\pm\sqrt{16}}{2}$ $x=\frac{6\pm 4}{2}$
Rewrite to show two solutions.	$x=\frac{6+4}{2}, \quad x=\frac{6-4}{2}$
Simplify.	$x=\frac{10}{2}, \quad x=\frac{2}{2}$
	$x=5, \quad x=1$
Check: $x^2-6x+5=0$ $\quad$ $x^2-6x+5=0$ $5^2-6\cdot 5+5\stackrel{?}{=}0$ $\quad$ $1^2-6\cdot 1+5\stackrel{?}{=}0$ $25-30+5\stackrel{?}{=}0$ $\quad$ $1-6+5\stackrel{?}{=}0$ $0=0$ ✓ $\quad$ $0=0$ ✓	

Exercise 7.3.3

Solve by using the Quadratic Formula: $a^2-2a=15$.

Answer

$a=-3, a=5$

Exercise 7.3.4

Solve by using the Quadratic Formula: $b^2+24=-10b$.

Answer

$b=-6, b=-4$

When we solved quadratic equations by using the Square Root Property, we sometimes got answers that had radicals. That can happen, too, when using the **Quadratic Formula**. If we get a **radical** as a solution, the final answer must have the radical in its simplified form.

Example 7.3.3

Solve by using the Quadratic Formula: $2x^2+10x+11=0$.

Solution:

	$2x^2+10x+11=0$
This equation is in standard form.	$ax^2+bx+c=0$ $2x^2+10x+11=0$

Identify the values of a, b and c.	$a = 2,\ b = 10,\ c = 11$
Write the Quadratic Formula.	$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Then substitute in the values of a, b, and c.	$x = \dfrac{-(10) \pm \sqrt{(10)^2 - 4 \cdot 2 \cdot (11)}}{2 \cdot 2}$
Simplify.	$x = \dfrac{-10 \pm \sqrt{100 - 88}}{4}$
	$x = \dfrac{-10 \pm \sqrt{12}}{4}$
Simplify the radical.	$x = \dfrac{-10 \pm 2\sqrt{3}}{4}$
Factor out the common factor in the numerator.	$x = \dfrac{2(-5 \pm \sqrt{3})}{4}$ $x = \dfrac{2(-5 \pm \sqrt{3})}{4}$
Remove the common factors.	$x = \dfrac{-5 \pm \sqrt{3}}{2}$
Rewrite to show two solutions.	$x = \dfrac{-5 + \sqrt{3}}{2}, \quad x = \dfrac{-5 - \sqrt{3}}{2}$
Check: We leave the check for you!	

? Exercise 7.3.5

Solve by using the Quadratic Formula: $3m^2 + 12m + 7 = 0$.

Answer

$$m = \frac{-6 + \sqrt{15}}{3}, m = \frac{-6 - \sqrt{15}}{3}$$

? Exercise 7.3.6

Solve by using the Quadratic Formula: $5n^2 + 4n - 4 = 0$.

Answer

$$n = \frac{-2 + 2\sqrt{6}}{5}, n = \frac{-2 - 2\sqrt{6}}{5}$$

When we substitute a, b, and c into the Quadratic Formula and the **radicand** is negative, the quadratic equation will have imaginary or complex solutions. We will see this in the next example.

✔ Example 7.3.4

Solve by using the Quadratic Formula: $3p^2 + 2p + 9 = 0$.

Solution:

Table 9.3.5

	$3p^2 + 2p + 9 = 0$
This equation is in standard form.	$ax^2 + bx + c = 0$ $3p^2 + 2p + 9 = 0$

Identify the values of a, b, c.	$a=3,\ b=2,\ c=9$
Write the Quadratic Formula.	$p=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$
Then substitute in the values of a, b, c.	$p=\frac{-(2)\pm\sqrt{(2)^2-4\cdot 3\cdot(9)}}{2\cdot 3}$
Simplify.	$p=\frac{-2\pm\sqrt{4-108}}{6}$
	$p=\frac{-2\pm\sqrt{-104}}{6}$
Simplify the radical using complex numbers.	$p=\frac{-2\pm\sqrt{104}\,i}{6}$
Simplify the radical.	$p=\frac{-2\pm 2\sqrt{26}\,i}{6}$
Factor the common factor in the numerator.	$p=\frac{2\left(-1\pm\sqrt{26}\,i\right)}{6}$
Remove the common factors.	$p=\frac{-1\pm\sqrt{26}\,i}{3}$
Rewrite in standard $a+bi$ form.	$p=-\frac{1}{3}\pm\frac{\sqrt{26}\,i}{3}$
Write as two solutions.	$p=-\frac{1}{3}+\frac{\sqrt{26}\,i}{3},\quad p=-\frac{1}{3}-\frac{\sqrt{26}\,i}{3}$

? Exercise 7.3.7

Solve by using the Quadratic Formula: $4a^2-2a+8=0$.

Answer

$$a=\frac{1}{4}+\frac{\sqrt{31}}{4}i,\quad a=\frac{1}{4}-\frac{\sqrt{31}}{4}i$$

? Exercise 7.3.8

Solve by using the Quadratic Formula: $5b^2+2b+4=0$.

Answer

$$b=-\frac{1}{5}+\frac{\sqrt{19}}{5}i,\quad b=-\frac{1}{5}-\frac{\sqrt{19}}{5}i$$

Remember, to use the Quadratic Formula, the equation must be written in standard form, $ax^2+bx+c=0$. Sometimes, we will need to do some algebra to get the equation into standard form before we can use the Quadratic Formula.

✓ Example 7.3.5

Solve by using the Quadratic Formula: $x(x+6)+4=0$.

Solution:

Our first step is to get the equation in standard form.

Table 9.3.6

	$x(x+6)+4=0$
Distribute to get the equation in standard form.	$x^2+6x+4=0$

This equation is now in standard form.	$ax^2+bx+c=0$ $x^2+6x+4=0$
Identify the values of a, b, c.	$a=1,\ b=6,\ c=4$
Write the Quadratic Formula.	$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$
Then substitute in the values of a, b, c.	$x=\frac{-(6)\pm\sqrt{(6)^2-4\cdot 1\cdot(4)}}{2\cdot 1}$
Simplify.	$x=\frac{-6\pm\sqrt{36-16}}{2}$
	$x=\frac{-6\pm\sqrt{20}}{2}$
Simplify the radical.	$x=\frac{-6\pm 2\sqrt{5}}{2}$
Factor the common factor in the numerator.	$x=\frac{2(-3\pm 2\sqrt{5})}{2}$
Remove the common factors.	$x=-3\pm 2\sqrt{5}$
Write as two solutions.	$x=-3+2\sqrt{5},\ x=-3-2\sqrt{5}$
Check: We leave the check for you!	

Exercise 7.3.9

Solve by using the Quadratic Formula: $x(x+2)-5=0$.

Answer

$x=-1+\sqrt{6}, x=-1-\sqrt{6}$

Exercise 7.3.10

Solve by using the Quadratic Formula: $3y(y-2)-3=0$.

Answer

$y=1+\sqrt{2}, y=1-\sqrt{2}$

When we solved linear equations, if an equation had too many fractions we cleared the fractions by multiplying both sides of the equation by the LCD. This gave us an equivalent equation—without fractions— to solve. We can use the same strategy with quadratic equations.

Example 7.3.6

Solve by using the Quadratic Formula: $\frac{1}{2}u^2+\frac{2}{3}u=\frac{1}{3}$.

Solution:

Our first step is to clear the fractions.

Table 9.3.7

	$\frac{1}{2}u^2+\frac{2}{3}u=\frac{1}{3}$
Multiply both sides by the LCD, 6, to clear the fractions.	$6\left(\frac{1}{2}u^2+\frac{2}{3}u\right)=6\left(\frac{1}{3}\right)$

Multiply.	$3u^2 + 4u = 2$
Subtract 2 to get the equation in standard form.	$ax^2 + bx + c = 0$ $3u^2 + 4u - 2 = 0$
Identify the values of a, b, and c.	$a = 3,\ b = 4,\ c = -2$
Write the Quadratic Formula.	$u = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Then substitute in the values of a, b, and c.	$u = \frac{-(4) \pm \sqrt{(4)^2 - 4 \cdot 3 \cdot (-2)}}{2 \cdot 3}$
Simplify.	$u = \frac{-4 \pm \sqrt{16 + 24}}{6}$
	$u = \frac{-4 \pm \sqrt{40}}{6}$
Simplify the radical.	$u = \frac{-4 \pm 2\sqrt{10}}{6}$
Factor the common factor in the numerator.	$u = \frac{2(-2 \pm \sqrt{10})}{6}$
Remove the common factors.	$u = \frac{-2 \pm \sqrt{10}}{3}$
Rewrite to show two solutions.	$u = \frac{-2 + \sqrt{10}}{3}, \quad u = \frac{-2 - \sqrt{10}}{3}$
Check: We leave the check for you!	

? Exercise 7.3.11

Solve by using the Quadratic Formula: $\frac{1}{4}c^2 - \frac{1}{3}c = \frac{1}{12}$.

Answer

$c = \frac{2+\sqrt{7}}{3}, \quad c = \frac{2-\sqrt{7}}{3}$

? Exercise 7.3.12

Solve by using the Quadratic Formula: $\frac{1}{9}d^2 - \frac{1}{2}d = -\frac{1}{3}$.

Answer

$d = \frac{9+\sqrt{33}}{4}, d = \frac{9-\sqrt{33}}{4}$

Think about the equation $(x-3)^2 = 0$. We know from the **Zero Product Property** that this equation has only one solution, $x = 3$.

We will see in the next example how using the **Quadratic Formula** to solve an equation whose standard form is a perfect square **trinomial** equal to 0 gives just one solution. Notice that once the radicand is simplified it becomes 0, which leads to only one solution.

Example 7.3.7

Solve by using the Quadratic Formula: $4x^2 - 20x = -25$.

Solution:

Table 9.3.8

	$4x^2 - 20x = -25$
Add 25 to get the equation in standard form.	$ax^2 + bx + c = 0$ $4x^2 - 20x + 25 = 0$
Identify the values of a, b, and c.	$a = 4,\ b = -20,\ c = 25$
Write the quadratic formula.	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Then substitute in the values of a, b, and c.	$x = \frac{-(-20) \pm \sqrt{(-20)^2 - 4 \cdot 4 \cdot (25)}}{2 \cdot 4}$
Simplify.	$x = \frac{20 \pm \sqrt{400 - 400}}{8}$
	$x = \frac{20 \pm \sqrt{0}}{8}$
Simplify the radical.	$x = \frac{20}{8}$
Simplify the fraction.	$x = \frac{5}{2}$
Check: We leave the check for you!	

Did you recognize that $4x^2 - 20x + 25$ is a perfect square trinomial. It is equivalent to $(2x - 5)^2$? If you solve $4x^2 - 20x + 25 = 0$ by factoring and then using the Square Root Property, do you get the same result?

Exercise 7.3.13

Solve by using the Quadratic Formula: $r^2 + 10r + 25 = 0$.

Answer

$r = -5$

Exercise 7.3.14

Solve by using the Quadratic Formula: $25t^2 - 40t = -16$.

Answer

$t = \frac{4}{5}$

Use the Discriminant to Predict the Number and Type of Solutions of a Quadratic Equation

When we solved the quadratic equations in the previous examples, sometimes we got two real solutions, one real solution, and sometimes two complex solutions. Is there a way to predict the number and type of solutions to a quadratic equation without actually solving the equation?

Yes, the expression under the radical of the Quadratic Formula makes it easy for us to determine the number and type of solutions. This expression is called the **discriminant**.

Definition 7.3.2

Discriminant

In the Quadratic Formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, the quantity $b^2 - 4ac$ is called the discriminant.

Figure 9.3.85

Let's look at the discriminant of the equations in some of the examples and the number and type of solutions to those quadratic equations.

Table 9.3.9

Quadratic Equation (in standard form)	Discriminat $b^2 - 4ac$	Value of the Discriminant	Number and Type of Solutions
$2x^2 + 9x - 5 = 0$	$9^2 - 4 \cdot 2(-5)$ 121	$+$	2 real
$4x^2 - 20x + 25 = 0$	$(-20)^2 - 4 \cdot 4 \cdot 25$ 0	0	1 real
$3p^2 + 2p + 9 = 0$	$2^2 - 4 \cdot 3 \cdot 9$ -104	$-$	2 complex

When the discriminant is positive, the quadratic equation has **2 real solutions**. $x = \frac{-b \pm \sqrt{+}}{2a}$

When the discriminant is zero, the quadratic equation has **1 real solution**. $x = \frac{-b \pm \sqrt{0}}{2a}$

When the discriminant is negative, the quadratic equation has **2 complex solutions**. $x = \frac{-b \pm \sqrt{-}}{2a}$

Figure 9.3.86

Using the Discriminant $b^2 - 4ac$, to Determine the Number and Type of Solutions of a Quadratic Equation

For a quadratic equation of the form $ax^2 + bx + c = 0$, $a \neq 0$,

- If $b^2 - 4ac > 0$, the equation has 2 real solutions.
- if $b^2 - 4ac = 0$, the equation has 1 real solution.
- if $b^2 - 4ac < 0$, the equation has 2 complex solutions.

Example 7.3.8

Determine the number of solutions to each quadratic equation.

a. $3x^2 + 7x - 9 = 0$
b. $5n^2 + n + 4 = 0$
c. $9y^2 - 6y + 1 = 0$

Solution:

To determine the number of solutions of each quadratic equation, we will look at its discriminant.

a.

$$3x^2 + 7x - 9 = 0$$

The equation is in standard form, identify a, b, and c.

$$a = 3, \quad b = 7, \quad c = -9$$

Write the discriminant.

$$b^2-4ac$$

Substitute in the values of a, b, and c.

$$(7)^2-4\cdot 3\cdot(-9)$$

Simplify.

$$\begin{array}{c} 49+108 \\ 157 \end{array}$$

Since the discriminant is positive, there are 2 real solutions to the equation.

b.

$$5n^2+n+4=0$$

The equation is in standard form, identify a, b, and c.

$$a=5,\quad b=1,\quad c=4$$

Write the discriminant.

$$b^2-4ac$$

Substitute in the values of a, b, and c.

$$(1)^2-4\cdot 5\cdot 4$$

Simplify.

$$\begin{array}{c} 1-80 \\ -79 \end{array}$$

Since the discriminant is negative, there are 2 complex solutions to the equation.

c.

$$9y^2-6y+1=0$$

The equation is in standard form, identify a, b, and c.

$$a=9,\quad b=-6,\quad c=1$$

Write the discriminant.

$$b^2-4ac$$

Substitute in the values of a, b, and c.

$$(-6)^2-4\cdot 9\cdot 1$$

Simplify.

$$\begin{array}{c} 36-36 \\ 0 \end{array}$$

Since the discriminant is 0, there is 1 real solution to the equation.

? Exercise 7.3.15

Determine the number and type of solutions to each quadratic equation.

a. $8m^2-3m+6=0$
b. $5z^2+6z-2=0$
c. $9w^2+24w+16=0$

Answer

a. 2 complex solutions
b. 2 real solutions

c. 1 real solution

Exercise 7.3.16

Determine the number and type of solutions to each quadratic equation.

a. $b^2+7b-13=0$
b. $5a^2-6a+10=0$
c. $4r^2-20r+25=0$

Answer

a. 2 real solutions
b. 2 complex solutions
c. 1 real solution

Identify the Most Appropriate Method to Use to Solve a Quadratic Equation

We summarize the four methods that we have used to solve quadratic equations below.

Methods for Solving Quadratic Equations

1. Factoring
2. Square Root Property
3. Completing the Square
4. Quadratic Formula

Given that we have four methods to use to solve a quadratic equation, how do you decide which one to use? Factoring is often the quickest method and so we try it first. If the equation is $ax^2=k$ or $a(x-h)^2=k$ we use the Square Root Property. For any other equation, it is probably best to use the Quadratic Formula. Remember, you can solve any quadratic equation by using the Quadratic Formula, but that is not always the easiest method.

What about the method of Completing the Square? Most people find that method cumbersome and prefer not to use it. We needed to include it in the list of methods because we completed the square in general to derive the Quadratic Formula. You will also use the process of Completing the Square in other areas of algebra.

Identify the Most Appropriate Method to Solve a Quadratic Equation

1. Try **Factoring** first. If the quadratic factors easily, this method is very quick.
2. Try the **Square Root Property** next. If the equation fits the form $ax^2=k$ or $a(x-h)^2=k$, it can easily be solved by using the Square Root Property.
3. Use the **Quadratic Formula**. Any other quadratic equation is best solved by using the Quadratic Formula.

The next example uses this strategy to decide how to solve each quadratic equation.

Example 7.3.9

Identify the most appropriate method to use to solve each quadratic equation.

a. $5z^2=17$
b. $4x^2-12x+9=0$
c. $8u^2+6u=11$

Solution:

a.

$$5z^2=17$$

Since the equation is in the $ax^2=k$, the most appropriate method is to use the Square Root Property.

b.

$$4x^2 - 12x + 9 = 0$$

We recognize that the left side of the equation is a perfect square trinomial, and so factoring will be the most appropriate method.

c.

$$8u^2 + 6u = 11$$

Put the equation in standard form.

$$8u^2 + 6u - 11 = 0$$

While our first thought may be to try factoring, thinking about all the possibilities for trial and error method leads us to choose the Quadratic Formula as the most appropriate method.

Exercise 7.3.17

Identify the most appropriate method to use to solve each quadratic equation.

a. $x^2 + 6x + 8 = 0$
b. $(n-3)^2 = 16$
c. $5p^2 - 6p = 9$

Answer

a. Factoring
b. Square Root Property
c. Quadratic Formula

Exercise 7.3.18

Identify the most appropriate method to use to solve each quadratic equation.

a. $8a^2 + 3a - 9 = 0$
b. $4b^2 + 4b + 1 = 0$
c. $5c^2 = 125$

Answer

a. Quadratic Formula
b. Factoring or Square Root Property
c. Square Root Property

Access these online resources for additional instruction and practice with using the Quadratic Formula.

- Using the Quadratic Formula
- Solve a Quadratic Equation Using the Quadratic Formula with Complex Solutions
- Discriminant in Quadratic Formula

Key Concepts

- Quadratic Formula
  - The solutions to a quadratic equation of the form $ax^2 + bx + c = 0, a \neq 0$ are given by the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- How to solve a quadratic equation using the Quadratic Formula.
  1. Write the quadratic equation in standard form, $ax^2 + bx + c = 0$. Identify the values of a, b, c.
  2. Write the Quadratic Formula. Then substitute in the values of a, b, c.
  3. Simplify.

4. Check the solutions.

- Using the Discriminant, b^2-4ac, to Determine the Number and Type of Solutions of a Quadratic Equation
  - For a quadratic equation of the form $ax^2+bx+c=0, a\neq 0$,
    - If $b^2-4ac>0$, the equation has 2 real solutions.
    - If $b^2-4ac=0$, the equation has 1 real solution.
    - If $b^2-4ac<0$, the equation has 2 complex solutions.
- Methods to Solve Quadratic Equations:
  - Factoring
  - Square Root Property
  - Completing the Square
  - Quadratic Formula
- How to identify the most appropriate method to solve a quadratic equation.
  1. Try Factoring first. If the quadratic factors easily, this method is very quick.
  2. Try the **Square Root Property** next. If the equation fits the form $ax^2=k$ or $a(x-h)^2=k$, it can easily be solved by using the Square Root Property.
  3. Use the **Quadratic Formula.** Any other quadratic equation is best solved by using the Quadratic Formula.

Glossary

discriminant

In the Quadratic Formula, $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$, the quantity b^2-4ac is called the discriminant.

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7.3E: Exercises

Practice Makes Perfect

? ExerciseS 1 - 32: Solve Quadratic Equations Using the Quadratic Formula

In the following exercises, solve by using the Quadratic Formula.

1. $4m^2+m-3=0$
2. $4n^2-9n+5=0$
3. $2p^2-7p+3=0$
4. $3q^2+8q-3=0$
5. $p^2+7p+12=0$
6. $q^2+3q-18=0$
7. $r^2-8r=33$
8. $t^2+13t=-40$
9. $3u^2+7u-2=0$
10. $2p^2+8p+5=0$
11. $2a^2-6a+3=0$
12. $5b^2+2b-4=0$
13. $x^2+8x-4=0$
14. $y^2+4y-4=0$
15. $3y^2+5y-2=0$
16. $6x^2+2x-20=0$
17. $2x^2+3x+3=0$
18. $2x^2-x+1=0$
19. $8x^2-6x+2=0$
20. $8x^2-4x+1=0$
21. $(v+1)(v-5)-4=0$
22. $(x+1)(x-3)=2$
23. $(y+4)(y-7)=18$
24. $(x+2)(x+6)=21$
25. $\frac{1}{4}m^2+\frac{1}{12}m=\frac{1}{3}$
26. $\frac{1}{3}n^2+n=-\frac{1}{2}$
27. $\frac{3}{4}b^2+\frac{1}{2}b=\frac{3}{8}$
28. $\frac{1}{9}c^2+\frac{2}{3}c=3$
29. $16c^2+24c+9=0$
30. $25d^2-60d+36=0$
31. $25q^2+30q+9=0$

32. $16y^2+8y+1=0$

Answer

1. $m=-1, m=\frac{3}{4}$
3. $p=\frac{1}{3}, p=2$
5. $p=-4, p=-3$
7. $r=-3, r=11$
9. $u=\frac{-7\pm\sqrt{73}}{6}$
11. $a=\frac{3\pm\sqrt{3}}{2}$
13. $x=-4\pm2\sqrt{5}$
15. $y=-\frac{2}{3}, y=-1$
17. $x=-\frac{3}{4}\pm\frac{\sqrt{15}}{4}i$
19. $x=\frac{3}{8}\pm\frac{\sqrt{7}}{8}i$
21. $v=2\pm2\sqrt{2}$
23. $y=-4, y=7$
25. $m=1, m=\frac{-4}{3}$
27. $b=\frac{-2\pm\sqrt{22}}{6}$
29. $c=-\frac{3}{4}$
31. $q=-\frac{3}{5}$

? ExerciseS 33 - 36 Use the Discriminant to Predict the Number of Real Solutions of a Quadratic Equation

In the following exercises, determine the number of real solutions for each quadratic equation.

33. a. $4x^2-5x+16=0$
    b. $36y^2+36y+9=0$
    c. $6m^2+3m-5=0$
34. a. $9v^2-15v+25=0$
    b. $100w^2+60w+9=0$
    c. $5c^2+7c-10=0$
35. a. $r^2+12r+36=0$
    b. $8t^2-11t+5=0$
    c. $3v^2-5v-1=0$
36. a. $25p^2+10p+1=0$
    b. $7q^2-3q-6=0$
    c. $7y^2+2y+8=0$

Answer

33. a. no real solutions b. 1 c. 2

35. a. 1 b. no real solutions c. 2

? ExerciseS 37 - 40: Identify the Most Appropriate Method to Use to Solve a Quadratic Equation

In the following exercises, identify the most appropriate method (Factoring, Square Root, or Quadratic Formula) to use to solve each quadratic equation. Do not solve.

37. a. $x^2-5x-24=0$
    b. $(y+5)^2=12$
    c. $14m^2+3m=11$
38. a. $(8v+3)^2=81$
    b. $w^2-9w-22=0$
    c. $4n^2-10=6$
39. a. $6a^2+14=20$
    b. $\left(x-\frac{1}{4}\right)^2=\frac{5}{16}$
    c. $y^2-2y=8$
40. a. $8b^2+15b=4$
    b. $\frac{5}{9}v^2-\frac{2}{3}v=1$
    c. $\left(w+\frac{4}{3}\right)^2=\frac{2}{9}$

Answer

37. a. Factor b. Square Root c. Quadratic Formula

39. a. Quadratic Formula b. Square Root c. Factor

? ExerciseS 41 - 42: Writing Exercises

41. Solve the equation $x^2+10x=120$
    a. by completing the square
    b. using the Quadratic Formula
    c. Which method do you prefer? Why?
42. Solve the equation $12y^2+23y=24$
    a. by completing the square
    b. using the Quadratic Formula
    c. Which method do you prefer? Why?

Answer

41. Answers will vary

Self Check

a. After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
solve quadratic equations using the quadratic formula.			
use the discriminant to predict the number of solutions of a quadratic equation.			
identify the most appropriate method to use to solve a quadratic equation.			

Figure 9.3.87

b. What does this checklist tell you about your mastery of this section? What steps will you take to improve?

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7.4: Solve Applications of Quadratic Equations

Learning Objectives

By the end of this section, you will be able to:

- Solve applications modeled by quadratic equations

Before you get started, take this readiness quiz.

1. The sum of two consecutive odd numbers is -100. Find the numbers.
   If you missed this problem, review Example 2.18.
2. Solve: $\frac{2}{x+1}+\frac{1}{x-1}=\frac{1}{x^2-1}$.
   If you missed this problem, review Example 7.35.
3. Find the length of the hypotenuse of a right triangle with legs 5 inches and 12 inches.
   If you missed this problem, review Example 2.34.

Solve Applications Modeled by Quadratic Equations

We solved some applications that are modeled by quadratic equations earlier, when the only method we had to solve them was factoring. Now that we have more methods to solve quadratic equations, we will take another look at applications.

Let's first summarize the methods we now have to solve quadratic equations.

Methods to Solve Quadratic Equations

1. Factoring
2. Square Root Property
3. Completing the Square
4. Quadratic Formula

As you solve each equation, choose the method that is most convenient for you to work the problem. As a reminder, we will copy our usual Problem-Solving Strategy here so we can follow the steps.

Use a Problem-Solving Strategy

1. **Read** the problem. Make sure all the words and ideas are understood.
2. **Identify** what we are looking for.
3. **Name** what we are looking for. Choose a variable to represent that quantity.
4. **Translate** into an equation. It may be helpful to restate the problem in one sentence with all the important information. Then, translate the English sentence into an algebraic equation.
5. **Solve** the equation using algebra techniques.
6. **Check** the answer in the problem and make sure it makes sense.
7. **Answer** the question with a complete sentence.

We have solved number applications that involved consecutive even and odd integers, by modeling the situation with linear equations. Remember, we noticed each even integer is 2 more than the number preceding it. If we call the first one n, then the next one is $n+2$. The next one would be $n+2+2$ or $n+4$. This is also true when we use odd integers. One set of even integers and one set of odd integers are shown below.

	Consecutive even integers
	$64, 66, 68$
n	1st even integer
$n+2$	2nd consecutive even integer
$n+4$	3rd consecutive even integer

	Consecutive odd integers
	$77, 79, 81$
n	1st odd integer
$n+2$	2nd consecutive odd integer
$n+4$	3rd consecutive odd integer

Some applications of odd or even consecutive integers are modeled by quadratic equations. The notation above will be helpful as you name the variables.

Example 7.4.1

The product of two consecutive odd integers is 195. Find the integers.

Solution:

Step 1: **Read** the problem

Step 2: **Identify** what we are looking for.

We are looking for two consecutive odd integers.

Step 3: **Name** what we are looking for.

Let $n =$ the first odd integer.

$n+2 =$ the next odd integer.

Step 4: **Translate** into an equation. State the problem in one sentence.

"The product of two consecutive odd integers is 195." The product of the first odd integer and the second odd integer is 195.

Translate into an equation.

$$n(n+2) = 195$$

Step 5: **Solve** the equation. Distribute.

$$n^2 + 2n = 195$$

Write the equation in standard form.

$$n^2 + 2n - 195 = 0$$

Factor.

$$(n+15)(n-13) = 0$$

Use the Zero Product Property.

$$n+15 = 0 \quad n-13 = 0$$

Solve each equation.

$$n = -15, \quad n = 13$$

There are two values of n that are solutions. This will give us two pairs of consecutive odd integers for our solution.

First odd integer $n = 13$	First odd integer $n = -15$
next odd integer $n+2$	next odd integer $n+2$
$13+2$	$-15+2$
15	-13

Step 6: **Check** the answer.

Do these pairs work? Are they consecutive odd integers?

$13, 15$ yes
$-13, -15$ yes

Is their product 195?

$13 \cdot 15 = 195$ yes

$-13(-15) = 195$ yes

Step 7: **Answer** the question.

Two consecutive odd integers whose product is 195 are $13, 15$ and $-13, -15$.

Exercise 7.4.1

The product of two consecutive odd integers is 99. Find the integers.

Answer

The two consecutive odd integers whose product is 99 are $9, 11$, and $-9, -11$.

Exercise 7.4.2

The product of two consecutive even integers is 168. Find the integers.

Answer

The two consecutive even integers whose product is 128 are $12, 14$ and $-12, -14$.

We will use the formula for the area of a triangle to solve the next example.

Definition 7.4.1

Area of a Triangle

For a triangle with base, b, and height, h, the area, A, is given by the formula $A = \frac{1}{2}bh$.

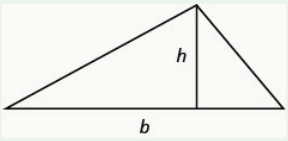


Figure 9.5.1

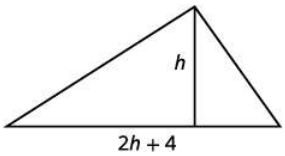
Recall that when we solve geometric applications, it is helpful to draw the figure.

Example 7.4.2

An architect is designing the entryway of a restaurant. She wants to put a triangular window above the doorway. Due to energy restrictions, the window can only have an area of 120 square feet and the architect wants the base to be 4 feet more than twice the height. Find the base and height of the window.

Solution:

Table 9.5.1

Step 1: **Read** the problem. Draw a picture.	
Step 2: **Identify** what we are looking for.	We are looking for the base and height.
Step 3: **Name** what we are looking for.	Let $h =$ the height of the triangle. $2h + 4 =$ the base of the triangle.

Step 4: **Translate** into an equation. We know the area. Write the formula for the area of a triangle.	$A=\frac{1}{2}bh$
Step 5: **Solve** the equation. Substitute in the values.	$120=\frac{1}{2}(2h+4)h$
Distribute.	$120=h^2+2h$
This is a quadratic equation, rewrite it in standard form.	$h^2+2h-120=0$
Factor.	$(h-10)(h+12)=0$
Use the Zero Product Property.	$h-10=0 \quad h+12=0$
Simplify.	$h=10, \quad h=\cancel{-12}$
Since h is the height of a window, a value of $h=-12$ does not make sense.	
The height of the triangle $h=10$.	
The base of the triangle $2h+4$. $2\cdot 10+4$ 24	
Step 6: **Check** the answer. Does a triangle with height 10 and base 24 have area 120? Yes.	
Step 7: **Answer** the question.	The height of the triangular window is 10 feet and the base is 24 feet.

? Exercise 7.4.3

Find the base and height of a triangle whose base is four inches more than six times its height and has an area of 456 square inches.

Answer

The height of the triangle is 12 inches and the base is 76 inches.

? Exercise 7.4.4

If a triangle that has an area of 110 square feet has a base that is two feet less than twice the height, what is the length of its base and height?

Answer

The height of the triangle is 11 feet and the base is 20 feet.

In the two preceding examples, the number in the radical in the **Quadratic Formula** was a perfect square and so the solutions were rational numbers. If we get an irrational number as a solution to an application problem, we will use a calculator to get an approximate value.

We will use the formula for the area of a rectangle to solve the next example.

Definition 7.4.2

Area of a Rectangle

For a rectangle with length, L, and width, W, the area, A, is given by the formula $A=LW$.

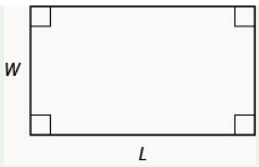


Figure 9.5.3

Example 7.4.3

Mike wants to put 150 square feet of artificial turf in his front yard. This is the maximum area of artificial turf allowed by his homeowners association. He wants to have a rectangular area of turf with length one foot less than 3 times the width. Find the length and width. Round to the nearest tenth of a foot.

Solution:

Table 9.5.2

Step 1: **Read** the problem. Draw a picture.	
Step 2: **Identify** what we are looking for.	We are looking for the length and width.
Step 3: **Name** what we are looking for.	Let $w =$ the width of the rectangle. $3w - 1 =$ the length of the rectangle
Step 4: **Translate** into an equation. We know the area. Write the formula for the area of a rectangle.	$A = L \cdot W$
Step 5: **Solve** the equation. Substitute in the values.	$150 = (3w - 1)w$
Distribute.	$150 = 3w^2 - w$
This is a quadratic equation; rewrite it in standard form. Solve the equation using the Quadratic Formula.	$ax^2 + bx + c = 0$ $3w^2 - w - 150 = 0$
Identify the a, b, c values.	$a = 3,\ b = -1,\ c = -150$
Write the Quadratic Formula.	$w = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Then substitute in the values of a, b, c.	$w = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \cdot 3 \cdot (-150)}}{2 \cdot 3}$
Simplify.	$w = \frac{1 \pm \sqrt{1 + 1800}}{6}$ $w = \frac{1 \pm \sqrt{1801}}{6}$ Figure 9.5.13
Rewrite to show two solutions.	$w = \frac{1 + \sqrt{1801}}{6}, \quad w = \frac{1 - \sqrt{1801}}{6}$
Approximate the answers using a calculator. We eliminate the negative solution for the width.	$w \approx 7.2, \quad \cancel{w \approx -6.9}$ Width $w \approx 7.2$ Length $\approx 3w - 1$ $\approx 3(7.2) - 1$ ≈ 20.6
Step 6: **Check** the answer. Make sure that the answers make sense. Since the answers are approximate, the area will not come out exactly to 150.	
Step 7: **Answer** the question.	The width of the rectangle is approximately 7.2 feet and the length is approximately 20.6 feet.

Exercise 7.4.5

The length of a 200 square foot rectangular vegetable garden is four feet less than twice the width. Find the length and width of the garden, to the nearest tenth of a foot.

Answer

The length of the garden is approximately 18 feet and the width 11 feet.

Exercise 7.4.6

A rectangular tablecloth has an area of 80 square feet. The width is 5 feet shorter than the length.What are the length and width of the tablecloth to the nearest tenth of a foot?

Answer

The length of the tablecloth is approximately 11.8 feet and the width 6.8 feet.

The **Pythagorean Theorem** gives the relation between the legs and hypotenuse of a right triangle. We will use the Pythagorean Theorem to solve the next example.

Definition 7.4.3

Pythagorean Theorem

In any right triangle, where a and b are the lengths of the legs, and c is the length of the hypotenuse, $a^2+b^2=c^2$.

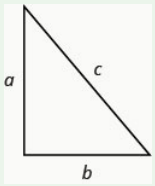


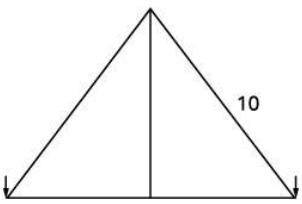
Figure 9.5.16

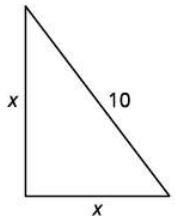
Example 7.4.4

Rene is setting up a holiday light display. He wants to make a 'tree' in the shape of two right triangles, as shown below, and has two 10-foot strings of lights to use for the sides. He will attach the lights to the top of a pole and to two stakes on the ground. He wants the height of the pole to be the same as the distance from the base of the pole to each stake. How tall should the pole be?

Solution:

Table 9.5.3

Step 1: **Read** the problem. Draw a picture.	
Step 2: **Identify** what we are looking for.	We are looking for the height of the pole.

Step 3: **Name** what we are looking for.	The distance from the base of the pole to either stake is the same as the height of the pole. Let $x =$ the height of the pole. $x =$ the distance from pole to stake Each side is a right triangle. We draw a picture of one of them.  Figure 9.5.18
Step 4: **Translate** into an equation. We can use the Pythagorean Theorem to solve for x. Write the Pythagorean Theorem.	$a^2+b^2=c^2$
Step 5: **Solve** the equation. Substitute.	$x^2+x^2=10^2$
Simplify.	$2x^2=100$
Divide by 2 to isolate the variable.	$\frac{2x^2}{2}=\frac{100}{2}$
Simplify.	$x^2=50$
Use the Square Root Property.	$x=\pm\sqrt{50}$
Simplify the radical.	$x=\pm 5\sqrt{2}$
Rewrite to show two solutions.	$x=5\sqrt{2}, \quad x=-5\sqrt{2}$
	If we approximate this number to the nearest tenth with a calculator, we find $x \approx 7.1$.
Step 6: **Check** the answer. Check on your own in the Pythagorean Theorem.	
Step 7: **Answer** the question.	The pole should be about 7.1 feet tall.

? Exercise 7.4.7

The sun casts a shadow from a flag pole. The height of the flag pole is three times the length of its shadow. The distance between the end of the shadow and the top of the flag pole is 20 feet. Find the length of the shadow and the length of the flag pole. Round to the nearest tenth.

Answer

The length of the flag pole's shadow is approximately 6.3 feet and the height of the flag pole is 18.9 feet.

? Exercise 7.4.8

The distance between opposite corners of a rectangular field is four more than the width of the field. The length of the field is twice its width. Find the distance between the opposite corners. Round to the nearest tenth.

Answer

The distance between the opposite corners is approximately 7.2 feet.

The height of a projectile shot upward from the ground is modeled by a quadratic equation. The initial velocity, v_0, propels the object up until gravity causes the object to fall back down.

Definition 7.4.4

The height in feet, h, of an object shot upwards into the air with initial velocity, v_0, after t seconds is given by the formula

$$h = -16t^2 + v_0 t$$

We can use this formula to find how many seconds it will take for a firework to reach a specific height.

Example 7.4.5

A firework is shot upwards with initial velocity 130 feet per second. How many seconds will it take to reach a height of 260 feet? Round to the nearest tenth of a second.

Solution:

Table 9.5.4

Step 1: **Read** the problem.	
Step 2: **Identify** what we are looking for.	We are looking for the number of seconds, which is time.
Step 3: **Name** what we are looking for.	Let t = the number of seconds.
Step 4: **Translate** into an equation. Use the formula.	$h = -16t^2 + v_0 t$
Step 5: **Solve** the equation. We know the velocity v_0 is 130 feet per second. The height is 260 feet. Substitute the values.	$260 = -16t^2 + 130t$
This is a quadratic equation, rewrite it in standard form. Solve the equation using the Quadratic Formula.	$ax^2 + bx + c = 0$ $16t^2 - 130t + 260 = 0$
Identify the values of a, b, c.	$a = 16,\ b = -130,\ c = 260$
Write the Quadratic Formula.	$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Then substitute in the values of a, b, c.	$t = \frac{-(-130) \pm \sqrt{(-130)^2 - 4 \cdot 16 \cdot (260)}}{2 \cdot 16}$
Simplify.	$t = \frac{130 \pm \sqrt{16{,}900 - 16{,}640}}{32}$ $t = \frac{130 \pm \sqrt{260}}{32}$ Figure 9.5.26
Rewrite to show two solutions.	$t = \frac{130 + \sqrt{260}}{32}, \quad t = \frac{130 - \sqrt{260}}{32}$
Approximate the answer with a calculator.	$t \approx 4.6$ seconds, $t \approx 3.6$ seconds
Step 6: **Check** the answer. The check is left to you.	
Step 7: **Answer** the question.	The firework will go up and then fall back down. As the firework goes up, it will reach 260 feet after approximately 3.6 seconds. It will also pass that height on the way down at 4.6 seconds.

Exercise 7.4.9

An arrow is shot from the ground into the air at an initial speed of 108 ft/s. Use the formula $h = -16t^2 + v_0 t$ to determine when the arrow will be 180 feet from the ground. Round the nearest tenth.

Answer

The arrow will reach 180 feet on its way up after 3 seconds and again on its way down after approximately 3.8 seconds.

Exercise 7.4.10

A man throws a ball into the air with a velocity of 96 ft/s. Use the formula $h = -16t^2 + v_0 t$ to determine when the height of the ball will be 48 feet. Round to the nearest tenth.

Answer

The ball will reach 48 feet on its way up after approximately $.6$ second and again on its way down after approximately 5.4 seconds.

We have solved uniform motion problems using the formula $D = rt$ in previous chapters. We used a table like the one below to organize the information and lead us to the equation.

	Rate	• Time	= Distance

Figure 9.5.29

The formula $D = rt$ assumes we know r and t and use them to find D. If we know D and r and need to find t, we would solve the equation for t and get the formula $t = \frac{D}{r}$.

Some uniform motion problems are also modeled by quadratic equations.

Example 7.4.6

Professor Smith just returned from a conference that was $2,000$ miles east of his home. His total time in the airplane for the round trip was 9 hours. If the plane was flying at a rate of 450 miles per hour, what was the speed of the jet stream?

Solution:

This is a uniform motion situation. A diagram will help us visualize the situation.

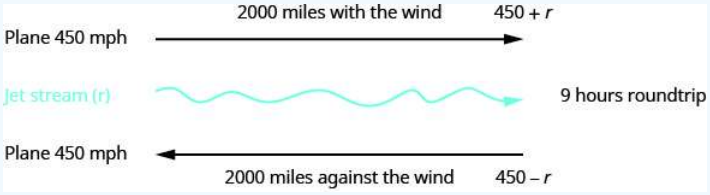


Figure 9.5.30

We fill in the chart to organize the information.

We are looking for the speed of the jet stream. Let $r =$ the speed of the jet stream.

When the plane flies with the wind, the wind increases its speed and so the rate is $450 + r$.

When the plane flies against the wind, the wind decreases its speed and the rate is $450 - r$.

Table 9.5.5

Write in the rates. Write in the distances. Since $D = r \cdot t$, we solve for t and get $t = \frac{D}{r}$. We divide the distance by the rate in each row, and place the expression in the time column.	<table><tr><th>Type</th><th>Rate</th><th>·</th><th>Time</th><th>=</th><th>Distance</th></tr><tr><td>Headwind</td><td>$450 - r$</td><td></td><td>$\frac{2000}{450-r}$</td><td></td><td>2000</td></tr><tr><td>Tailwind</td><td>$450 + r$</td><td></td><td>$\frac{2000}{450+r}$</td><td></td><td>2000</td></tr><tr><td></td><td></td><td></td><td>9</td><td></td><td></td></tr></table>
We know the times add to 9 and so we write our equation.	$\frac{2000}{450-r} + \frac{2000}{450+r} = 9$
We multiply both sides by the LCD.	$(450-r)(450+r)\left(\frac{2000}{450-r} + \frac{2000}{450+r}\right) = 9(450-r)(450+r)$
Simplify.	$2000(450+r) + 2000(450-r) = 9(450-r)(450+r)$
Factor the 2, 000.	$2000(450+r+450-r) = 9\left(450^2 - r^2\right)$
Solve.	$2000(900) = 9\left(450^2 - r^2\right)$
Divide by 9.	$2000(100) = 450^2 - r^2$
Simplify.	$200000 = 202500 - r^2$ $-2500 = -r^2$ \ $50 = r$ The speed of the jet stream is 50 mph.
Check: Is 50 mph a reasonable speed for the jet stream? Yes. If the plane is traveling 450 mph and the wind is 50 mph, Tailwind $450 + 50 = 500\text{mph} \quad \frac{2000}{500} = 4$ hours Headwind $450 - 50 = 400\text{mph} \quad \frac{2000}{400} = 5$ hours The times add to 9 hours, so it checks.	

The speed of the jet stream was 50 mph.

Exercise 7.4.11

MaryAnne just returned from a visit with her grandchildren back east. The trip was 2400 miles from her home and her total time in the airplane for the round trip was 10 hours. If the plane was flying at a rate of 500 miles per hour, what was the speed of the jet stream?

Answer

The speed of the jet stream was 100 mph.

Exercise 7.4.12

Gerry just returned from a cross country trip. The trip was 3000 miles from his home and his total time in the airplane for the round trip was 11 hours. If the plane was flying at a rate of 550 miles per hour, what was the speed of the jet stream?

Answer

The speed of the jet stream was 50 mph.

Work applications can also be modeled by quadratic equations. We will set them up using the same methods we used when we solved them with rational equations.We'll use a similar scenario now.

Example 7.4.7

The weekly gossip magazine has a big story about the presidential election and the editor wants the magazine to be printed as soon as possible. She has asked the printer to run an extra printing press to get the printing done more quickly. Press #1 takes 12 hours more than Press #2 to do the job and when both presses are running they can print the job in 8 hours. How long does it take for each press to print the job alone?

Solution:

This is a work problem. A chart will help us organize the information.

We are looking for how many hours it would take each press separately to complete the job.

Table 9.5.6

Let $x =$ the number of hours for Press #2 to complete the job. Enter the hours per job for Press #1, Press #2, and when they work together.	<table><tr><th></th><th>Number of hours needed to complete the job.</th><th>Part of job completed/hour</th></tr><tr><td>Press #1</td><td>$x+12$</td><td>$\frac{1}{x+12}$</td></tr><tr><td>Press #2</td><td>x</td><td>$\frac{1}{x}$</td></tr><tr><td>Together</td><td>8</td><td>$\frac{1}{8}$</td></tr></table>
The part completed by Press #1 plus the part completed by Press #2 equals the amount completed together. Translate to an equation.	Work completed by Press #1 + Press #2 = Together $\frac{1}{x+12} + \frac{1}{x} = \frac{1}{8}$
Solve.	$\frac{1}{x+12}+\frac{1}{x}=\frac{1}{8}$
Multiply by the LCD, $8x(x+12)$.	$8x(x+12)\left(\frac{1}{x+12}+\frac{1}{x}\right)=\left(\frac{1}{8}\right)8x(x+12)$
Simplify.	$8x+8(x+12)=x(x+12)$ $8x+8x+96=x^2+12x$ Figure 9.5.37 $0=x^2-4x-96$ Figure 9.5.38
Solve.	$0=(x-12)(x+8)$ $x-12=0, x+8=0$ Figure 9.5.40 $x=12, x=-8$ hours Figure 9.5.41
Since the idea of negative hours does not make sense, we use the values $x=12$.	$12+12$ 24 hours 12 12 hours Figure 9.5.43
Write our sentence answer.	Press #1 would take 24 hours and Press #2 would take 12 hours to do the job alone.

Exercise 7.4.13

The weekly news magazine has a big story naming the Person of the Year and the editor wants the magazine to be printed as soon as possible. She has asked the printer to run an extra printing press to get the printing done more quickly. Press #1 takes 6 hours more than Press #2 to do the job and when both presses are running they can print the job in 4 hours. How long does it take for each press to print the job alone?

Answer

Press #1 would take 12 hours, and Press #2 would take 6 hours to do the job alone.

Exercise 7.4.14

Erlinda is having a party and wants to fill her hot tub. If she only uses the red hose it takes 3 hours more than if she only uses the green hose. If she uses both hoses together, the hot tub fills in 2 hours. How long does it take for each hose to fill the hot tub?

Answer

The red hose take 6 hours and the green hose take 3 hours alone.

Access these online resources for additional instruction and practice with solving applications modeled by quadratic equations.

- Word Problems Involving Quadratic Equations
- Quadratic Equation Word Problems
- Applying the Quadratic Formula

Key Concepts

- Methods to Solve Quadratic Equations
  - Factoring
  - Square Root Property
  - Completing the Square
  - Quadratic Formula
- How to use a Problem-Solving Strategy.
  1. **Read** the problem. Make sure all the words and ideas are understood.
  2. **Identify** what we are looking for.
  3. **Name** what we are looking for. Choose a variable to represent that quantity.
  4. **Translate** into an equation. It may be helpful to restate the problem in one sentence with all the important information. Then, translate the English sentence into an algebra equation.
  5. **Solve** the equation using good algebra techniques.
  6. **Check** the answer in the problem and make sure it makes sense.
  7. **Answer** the question with a complete sentence.
- Area of a Triangle
  - For a triangle with base, b, and height, h, the area, A, is given by the formula $A = \frac{1}{2}bh$.

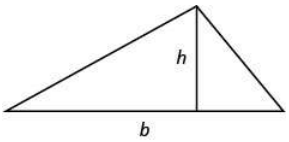


Figure 9.5.1

- Area of a Rectangle
  - For a rectangle with length,L, and width, W, the area, A, is given by the formula $A = LW$.

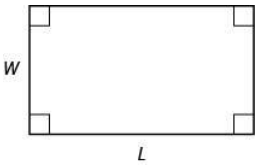


Figure 9.5.3

- Pythagorean Theorem
  - In any right triangle, where a and b are the lengths of the legs, and c is the length of the hypotenuse, $a^2 + b^2 = c^2$.

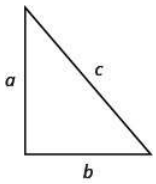


Figure 9.5.16

- Projectile motion
  - The height in feet, h, of an object shot upwards into the air with initial velocity, v_0, after t seconds is given by the formula $h = -16t^2 + v_0 t$.

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7.4E: Exercises

Practice Makes Perfect

Exercise $7.4E.15$ Solve Applications Modeled by Quadratic Equations

In the following exercises, solve using any method.

1. The product of two consecutive odd numbers is 255. Find the numbers.
2. The product of two consecutive even numbers is 360. Find the numbers.
3. The product of two consecutive even numbers is 624. Find the numbers.
4. The product of two consecutive odd numbers is $1,023$. Find the numbers.
5. The product of two consecutive odd numbers is 483. Find the numbers.
6. The product of two consecutive even numbers is 528. Find the numbers.

Answer

1. Two consecutive odd numbers whose product is 255 are 15 and 17, and -15 and -17.

3. The first and second consecutive odd numbers are 24 and 26, and -26 and -24.

5. Two consecutive odd numbers whose product is 483 are 21 and 23, and -21 and -23.

Exercise $7.4E.16$ Solve Applications Modeled by Quadratic Equations

In the following exercises, solve using any method. Round your answers to the nearest tenth, if needed.

1. A triangle with area 45 square inches has a height that is two less than four times the base Find the base and height of the triangle.
2. The base of a triangle is six more than twice the height. The area of the triangle is 88 square yards. Find the base and height of the triangle.
3. The area of a triangular flower bed in the park has an area of 120 square feet. The base is 4 feet longer that twice the height. What are the base and height of the triangle?
4. A triangular banner for the basketball championship hangs in the gym. It has an area of 75 square feet. What is the length of the base and height , if the base is two-thirds of the height?
5. The length of a rectangular driveway is five feet more than three times the width. The area is 50 square feet. Find the length and width of the driveway.
6. A rectangular lawn has area 140 square yards. Its length is six yards less than twice its width. What are the length and width of the lawn?
7. A rectangular table for the dining room has a surface area of 24 square feet. The length is two more feet than twice the width of the table. Find the length and width of the table.
8. The new computer has a surface area of 168 square inches. If the the width is 5.5 inches less that the length, what are the dimensions of the computer?
9. The hypotenuse of a right triangle is twice the length of one of its legs. The length of the other leg is three feet. Find the lengths of the three sides of the triangle.
10. The hypotenuse of a right triangle is 10 cm long. One of the triangle's legs is three times as the length of the other leg . Round to the nearest tenth. Find the lengths of the three sides of the triangle.
11. A rectangular garden will be divided into two plots by fencing it on the diagonal. The diagonal distance from one corner of the garden to the opposite corner is five yards longer than the width of the garden. The length of the garden is three times the width. Find the length of the diagonal of the garden.

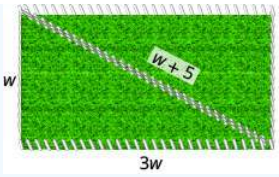


Figure 9.5.44

12. Nautical flags are used to represent letters of the alphabet. The flag for the letter, O consists of a yellow right triangle and a red right triangle which are sewn together along their hypotenuse to form a square. The hypotenuse of the two triangles is three inches longer than a side of the flag. Find the length of the side of the flag.

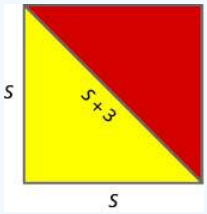


Figure 9.5.45

13. Gerry plans to place a 25-foot ladder against the side of his house to clean his gutters. The bottom of the ladder will be 5 feet from the house.How for up the side of the house will the ladder reach?

14. John has a 10-foot piece of rope that he wants to use to support his 8-foot tree. How far from the base of the tree should he secure the rope?

15. An arrow is shot vertically upward at a rate of $v_0 = 220$ feet per second. Use the projectile formula $h = -16t^2 + v_0 t$, to determine when the height of the arrow will be 400 feet.

16. A firework rocket is shot upward at a rate of 640 ft/sec. Use the projectile formula $h = -16t^2 + v_0 t$ to determine when the height of the firework rocket will be 1200 feet.

17. A bullet is fired straight up from a BB gun with initial velocity 1120 feet per second at an initial height of 8 feet. Use the formula $h = -16t^2 + v_0 t + 8$ to determine how many seconds it will take for the bullet to hit the ground. (That is, when will $h = 0$?)

18. A stone is dropped from a 196-foot platform. Use the formula $h = -16t^2 + v_0 t + 196$ to determine how many seconds it will take for the stone to hit the ground. (Since the stone is dropped, $v_0 = 0$.)

19. The businessman took a small airplane for a quick flight up the coast for a lunch meeting and then returned home. The plane flew a total of 4 hours and each way the trip was 200 miles. What was the speed of the wind that affected the plane which was flying at a speed of 120 mph?

20. The couple took a small airplane for a quick flight up to the wine country for a romantic dinner and then returned home. The plane flew a total of 5 hours and each way the trip was 300 miles. If the plane was flying at 125 mph, what was the speed of the wind that affected the plane?

21. Roy kayaked up the river and then back in a total time of 6 hours. The trip was 4 miles each way and the current was difficult. If Roy kayaked at a speed of 5 mph, what was the speed of the current?

22. Rick paddled up the river, spent the night camping, and then paddled back. He spent 10 hours paddling and the campground was 24 miles away. If Rick kayaked at a speed of 5 mph, what was the speed of the current?

23. Two painters can paint a room in 2 hours if they work together. The less experienced painter takes 3 hours more than the more experienced painter to finish the job. How long does it take for each painter to paint the room individually?

24. Two gardeners can do the weekly yard maintenance in 8 minutes if they work together. The older gardener takes 12 minutes more than the younger gardener to finish the job by himself. How long does it take for each gardener to do the weekly yard maintenance individually?

25. It takes two hours for two machines to manufacture $10,000$ parts. If Machine #1 can do the job alone in one hour less than Machine #2 can do the job, how long does it take for each machine to manufacture $10,000$ parts alone?

26. Sully is having a party and wants to fill his swimming pool. If he only uses his hose it takes 2 hours more than if he only uses his neighbor's hose. If he uses both hoses together, the pool fills in 4 hours. How long does it take for each hose to fill the pool?

Answer

1. The width of the triangle is 5 inches and the height is 18 inches.
3. The base is 24 feet and the height of the triangle is 10 feet.
5. The length of the driveway is 15.0 feet and the width is 3.3 feet.
7. The length of table is 8 feet and the width is 3 feet.
9. The length of the legs of the right triangle are 3.2 and 9.6 cm.
11. The length of the diagonal fencing is 7.3 yards.
13. The ladder will reach 24.5 feet on the side of the house.
15. The arrow will reach 400 feet on its way up in 2.2 seconds and on the way down in 11.6 seconds.
17. The bullet will take 70 seconds to hit the ground.
19. The speed of the wind was 49 mph.
21. The speed of the current was 4.3 mph.
23. The less experienced painter takes 6 hours and the experienced painter takes 3 hours to do the job alone.
25. Machine #1 takes 3.6 hours and Machine #2 takes 4.6 hours to do the job alone.

? Exercise 7.4E. 17 writing exercises

1. Make up a problem involving the product of two consecutive odd integers.
   a. Start by choosing two consecutive odd integers. What are your integers?
   b. What is the product of your integers?
   c. Solve the equation $n(n+2)=p$, where p is the product you found in part (b).
   d. Did you get the numbers you started with?
2. Make up a problem involving the product of two consecutive even integers.
   a. Start by choosing two consecutive even integers. What are your integers?
   b. What is the product of your integers?
   c. Solve the equation $n(n+2)=p$, where p is the product you found in part (b).
   d. Did you get the numbers you started with?

Answer

1. Answers will vary.

Self Check

a. After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
solve applications of the quadratic formula.			

Figure 9.5.46

b. After looking at the checklist, do you think you are well-prepared for the next section? Why or why not?

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CHAPTER OVERVIEW

8: Scatterplots and Regression

8.1: Scatter Plots

8.2: Fitting Linear Models to Data

8.3: Fitting Exponential Models to Data

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8.1: Scatter Plots

Before we take up the discussion of linear regression and correlation, we need to examine a way to display the relation between two variables *x* and *y*. The following example illustrates a scatter plot.

Example 8.1.1

In Europe and Asia, m-commerce is popular. M-commerce users have special mobile phones that work like electronic wallets as well as provide phone and Internet services. Users can do everything from paying for parking to buying a TV set or soda from a machine to banking to checking sports scores on the Internet. For the years 2000 through 2004, was there a relationship between the year and the number of m-commerce users? Construct a scatter plot. Let $x =$ the year and let $y =$ the number of m-commerce users, in millions.

Table* 8.1.1:: Table showing the number of m-commerce users (in millions) by year.*

x (year)	y (# of users)
2000	0.5
2002	20.0
2003	33.0
2004	47.0

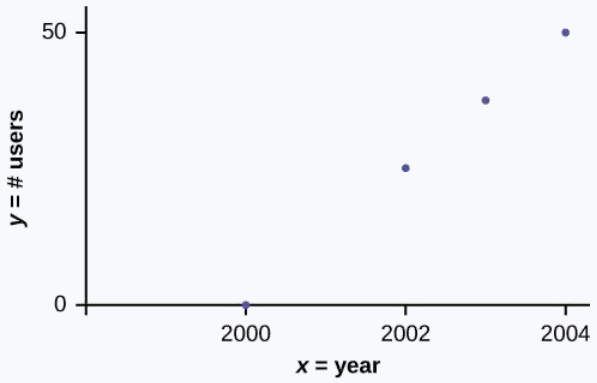


Figure* 8.1.1*: Scatter plot showing the number of m-commerce users (in millions) by year.

To create a scatter plot

a. Enter your X data into list L1 and your Y data into list L2.
b. Press 2nd STATPLOT ENTER to use Plot 1. On the input screen for PLOT 1, highlight On and press ENTER. (Make sure the other plots are OFF.)
c. For TYPE: highlight the very first icon, which is the scatter plot, and press ENTER.
d. For Xlist:, enter L1 ENTER and for Ylist: L2 ENTER.
e. For Mark: it does not matter which symbol you highlight, but the square is the easiest to see. Press ENTER.
f. Make sure there are no other equations that could be plotted. Press Y = and clear any equations out.
g. Press the ZOOM key and then the number 9 (for menu item "ZoomStat") ; the calculator will fit the window to the data. You can press WINDOW to see the scaling of the axes.

Exercise 8.1.1

Amelia plays basketball for her high school. She wants to improve to play at the college level. She notices that the number of points she scores in a game goes up in response to the number of hours she practices her jump shot each week. She records the following data:

X (hours practicing jump shot)	Y (points scored in a game)

X (hours practicing jump shot)	Y (points scored in a game)
5	15
7	22
9	28
10	31
11	33
12	36

Construct a scatter plot and state if what Amelia thinks appears to be true.

Answer

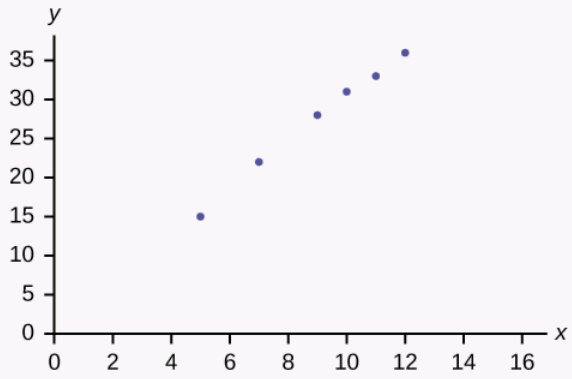


Figure 8.1.2

Yes, Amelia's assumption appears to be correct. The number of points Amelia scores per game goes up when she practices her jump shot more.

A scatter plot shows the *direction of a relationship* between the variables. A clear direction happens when there is either:

- High values of one variable occurring with high values of the other variable or low values of one variable occurring with low values of the other variable.
- High values of one variable occurring with low values of the other variable.

You can determine the *strength of the relationship* by looking at the scatter plot and seeing how close the points are to a line, a power function, an exponential function, or to some other type of function. For a linear relationship there is an exception. Consider a scatter plot where all the points fall on a horizontal line providing a "perfect fit." The horizontal line would in fact show no relationship.

When you look at a scatter plot, you want to notice the *overall pattern* and any *deviations* from the pattern. The following scatterplot examples illustrate these concepts.

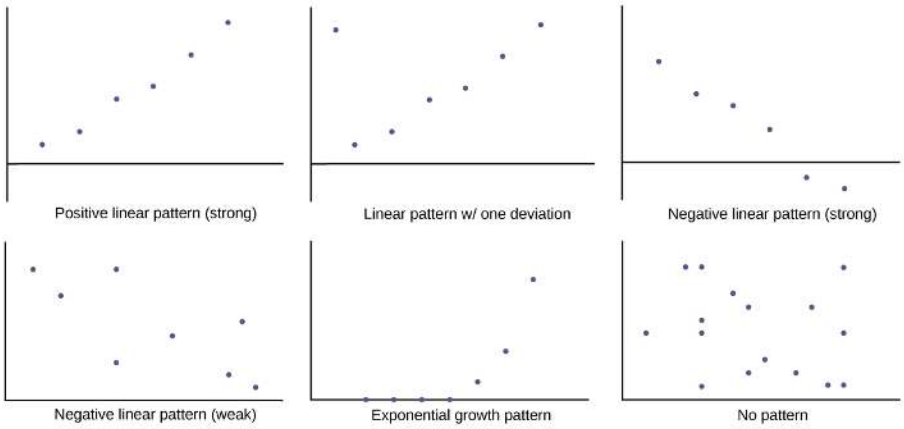


Figure* 8.1.3*:

In this chapter, we are interested in scatter plots that show a linear pattern. Linear patterns are quite common. The linear relationship is strong if the points are close to a straight line, except in the case of a horizontal line where there is no relationship. If we think that the points show a linear relationship, we would like to draw a line on the scatter plot. This line can be calculated through a process called linear regression. However, we only calculate a regression line if one of the variables helps to explain or predict the other variable. If x is the independent variable and y the dependent variable, then we can use a regression line to predict y for a given value of x

Summary

Scatter plots are particularly helpful graphs when we want to see if there is a linear relationship among data points. They indicate both the direction of the relationship between the x variables and the y variables, and the strength of the relationship. We calculate the strength of the relationship between an independent variable and a dependent variable using linear regression.

Contributors

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8.2: Fitting Linear Models to Data

Learning Objectives

- Draw and interpret scatter diagrams.
- Use a graphing utility to find the line of best fit.
- Distinguish between linear and nonlinear relations.
- Fit a regression line to a set of data and use the linear model to make predictions.

A professor is attempting to identify trends among final exam scores. His class has a mixture of students, so he wonders if there is any relationship between age and final exam scores. One way for him to analyze the scores is by creating a diagram that relates the age of each student to the exam score received. In this section, we will examine one such diagram known as a scatter plot.

Drawing and Interpreting Scatter Plots

A scatter plot is a graph of plotted points that may show a relationship between two sets of data. If the relationship is from a **linear model**, or a model that is nearly linear, the professor can draw conclusions using his knowledge of linear functions. Figure 8.2.1 shows a sample scatter plot.

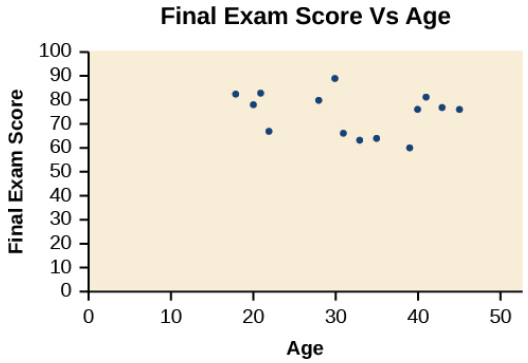


Figure 8.2.1: A scatter plot of age and final exam score variables

Notice this scatter plot does not indicate a **linear relationship**. The points do not appear to follow a trend. In other words, there does not appear to be a relationship between the age of the student and the score on the final exam.

Example 8.2.1: Using a Scatter Plot to Investigate Cricket Chirps

Table shows the number of cricket chirps in 15 seconds, for several different air temperatures, in degrees Fahrenheit[1]. Plot this data, and determine whether the data appears to be linearly related.

Table 8.2.1

Chirps	44	35	20.4	33	31	35	18.5	37	26
Temperature	80.5	70.5	57	66	68	72	52	73.5	53

Solution

Plotting this data, as depicted in Figure 8.2.2 suggests that there may be a trend. We can see from the trend in the data that the number of chirps increases as the temperature increases. The trend appears to be roughly linear, though certainly not perfectly so.

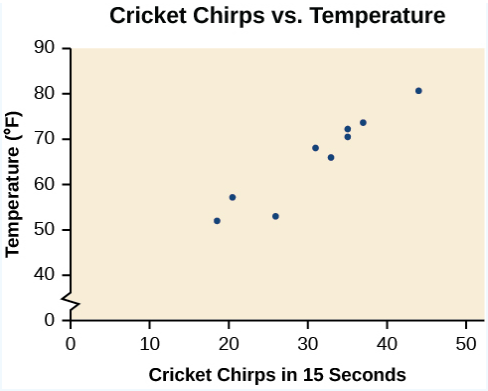


Figure 8.2.2: The line regression is generally positive.

Finding the Line of Best Fit

Once we recognize a need for a linear function to model that data, the natural follow-up question is "what is that linear function?" One way to approximate our linear function is to sketch the line that seems to best fit the data. Then we can extend the line until we can verify the y-intercept. We can approximate the slope of the line by extending it until we can estimate the $\frac{\text{rise}}{\text{run}}$.

Example 8.2.2: Finding a Line of Best Fit

Find a linear function that fits the data in Table 8.2.1 by "eyeballing" a line that seems to fit.

Solution

On a graph, we could try sketching a line.

Using the starting and ending points of our hand drawn line, points $(0, 30)$ and $(50, 90)$, this graph has a slope of

$$m = \frac{60}{50} = 1.2 \tag{8.2.1}$$

and a y-intercept at 30. This gives an equation of

$$T(c) = 1.2c + 30 \tag{8.2.2}$$

where c is the number of chirps in 15 seconds, and $T(c)$ is the temperature in degrees Fahrenheit. The resulting equation is represented in Figure 8.2.3.

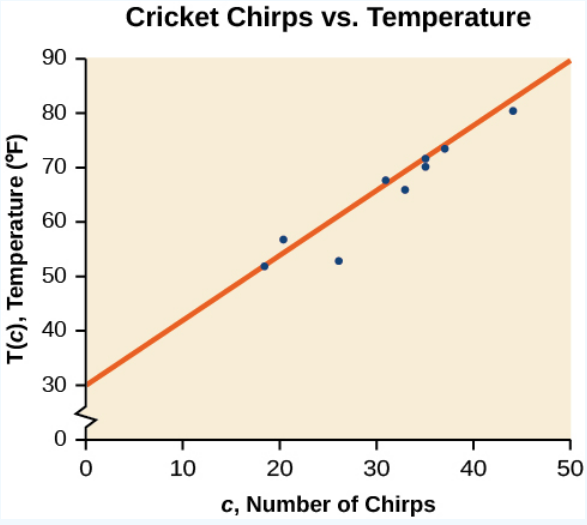


Figure 8.2.3: Scatter plot, showing the line of best fit.

Analysis

This linear equation can then be used to approximate answers to various questions we might ask about the trend.

Recognizing Interpolation or Extrapolation

While the data for most examples does not fall perfectly on the line, the equation is our best guess as to how the relationship will behave outside of the values for which we have data. We use a process known as **interpolation** when we predict a value inside the domain and range of the data. The process of **extrapolation** is used when we predict a value outside the domain and range of the data.

Figure 8.2.4 compares the two processes for the cricket-chirp data addressed in Example 8.2.2. We can see that interpolation would occur if we used our model to predict temperature when the values for chirps are between 18.5 and 44. Extrapolation would occur if we used our model to predict temperature when the values for chirps are less than 18.5 or greater than 44.

There is a difference between making predictions inside the domain and range of values for which we have data and outside that domain and range. Predicting a value outside of the domain and range has its limitations. When our model no longer applies after a certain point, it is sometimes called **model breakdown**. For example, predicting a cost function for a period of two years may involve examining the data where the input is the time in years and the output is the cost. But if we try to extrapolate a cost when $x = 50$, that is in 50 years, the model would not apply because we could not account for factors fifty years in the future.

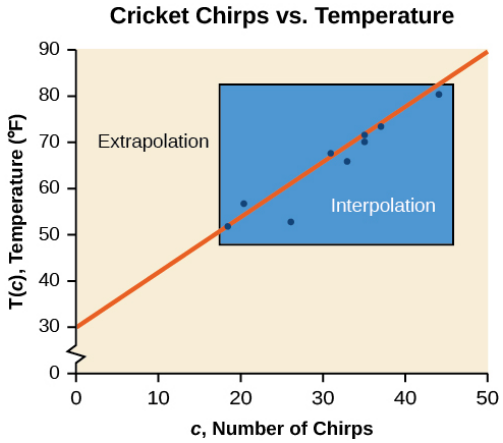


Figure 8.2.4: Interpolation occurs within the domain and range of the provided data whereas extrapolation occurs outside.

Interpolation and Extrapolation

Different methods of making predictions are used to analyze data.

- The method of **extrapolation** involves predicting a value outside the domain and/or range of the data.
- **Model breakdown** occurs at the point when the model no longer applies.

Example 8.2.3: Understanding Interpolation and Extrapolation

Use the cricket data from Table 8.2.1 to answer the following questions:

a. Would predicting the temperature when crickets are chirping 30 times in 15 seconds be interpolation or extrapolation? Make the prediction, and discuss whether it is reasonable.
b. Would predicting the number of chirps crickets will make at 40 degrees be interpolation or extrapolation? Make the prediction, and discuss whether it is reasonable.

Solution

a. The number of chirps in the data provided varied from 18.5 to 44. A prediction at 30 chirps per 15 seconds is inside the domain of our data, so would be interpolation. Using our model:

$$T(30) = 30 + 1.2(30) \tag{8.2.3}$$
$$= 66 \text{ degrees} \tag{8.2.4}$$

Based on the data we have, this value seems reasonable.
b. The temperature values varied from 52 to 80.5. Predicting the number of chirps at 40 degrees is extrapolation because 40 is outside the range of our data. Using our model:

$$40 = 30 + 1.2c \quad (8.2.5)$$
$$10 = 1.2c \quad (8.2.6)$$
$$c \approx 8.33 \quad (8.2.7)$$

We can compare the regions of interpolation and extrapolation using Figure 8.2.5.

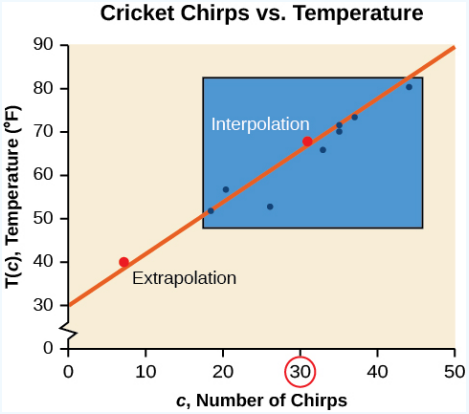


Figure 8.2.5: Scatter plot, showing the line of best fit and where interpolation and extrapolation occurs.

Analysis

Our model predicts the crickets would chirp 8.33 times in 15 seconds. While this might be possible, we have no reason to believe our model is valid outside the domain and range. In fact, generally crickets stop chirping altogether below around 50 degrees.

Exercise 8.2.1

According to the data from Table 8.2.1, what temperature can we predict it is if we counted 20 chirps in 15 seconds?

Solution

54°F

Finding the Line of Best Fit Using a Graphing Utility

While eyeballing a line works reasonably well, there are statistical techniques for fitting a line to data that minimize the differences between the line and data values[2]. One such technique is called **least squares regression** and can be computed by many graphing calculators, spreadsheet software, statistical software, and many web-based calculators[3]. Least squares regression is one means to determine the line that best fits the data, and here we will refer to this method as linear regression.

How To: Given data of input and corresponding outputs from a linear function, find the best fit line using linear regression.

1. Enter the input in List 1 (L1).
2. Enter the output in List 2 (L2).
3. On a graphing utility, select Linear Regression (LinReg).

Example 8.2.4: Finding a Least Squares Regression Line

Find the least squares regression line using the cricket-chirp data in Table 8.2.1.

Solution

Enter the input (chirps) in List 1 (L1).
Enter the output (temperature) in List 2 (L2). See Table 8.2.2.

Table 8.2.2

L1	44	35	20.4	33	31	35	18.5	37	26
L2	80.5	70.5	57	66	68	72	52	73.5	53

On a graphing utility, select Linear Regression (LinReg). Using the cricket chirp data from earlier, with technology we obtain the equation:

$$T(c) = 30.281 + 1.143c \tag{8.2.8}$$

Analysis

Notice that this line is quite similar to the equation we "eyeballed" but should fit the data better. Notice also that using this equation would change our prediction for the temperature when hearing 30 chirps in 15 seconds from 66 degrees to:

$$\begin{aligned} T(30) &= 30.281 + 1.143(30) && (8.2.9)\\ &= 64.571 && (8.2.10)\\ &\approx 64.6 \text{ degrees} && (8.2.11) \end{aligned}$$

The graph of the scatter plot with the least squares regression line is shown in Figure 8.2.6.

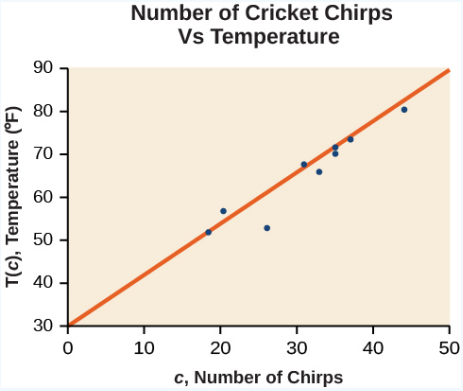


Figure 8.2.6: Scatter plot, showing the line of best fit.

Will there ever be a case where two different lines will serve as the best fit for the data?

No. There is only one best fit line.

Distinguishing Between Linear and Non-Linear Models

As we saw above with the cricket-chirp model, some data exhibit strong linear trends, but other data, like the final exam scores plotted by age, are clearly nonlinear. Most calculators and computer software can also provide us with the **correlation coefficient**, which is a measure of how closely the line fits the data. Many graphing calculators require the user to turn a "diagnostic on" selection to find the correlation coefficient, which mathematicians label as r. The correlation coefficient provides an easy way to get an idea of how close to a line the data falls.

We should compute the correlation coefficient only for data that follows a linear pattern or to determine the degree to which a data set is linear. If the data exhibits a nonlinear pattern, the correlation coefficient for a linear regression is meaningless. To get a sense for the relationship between the value of r and the graph of the data, Figure 8.2.7 shows some large data sets with their correlation coefficients. Remember, for all plots, the horizontal axis shows the input and the vertical axis shows the output.

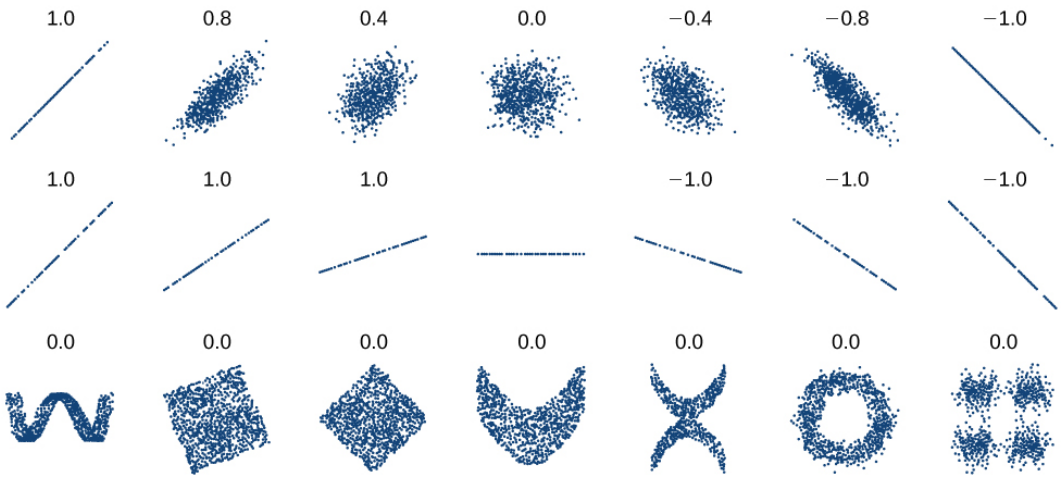


Figure 8.2.7: Plotted data and related correlation coefficients. (credit: “DenisBoigelot,” Wikimedia Commons)

Correlation Coefficient

The **correlation coefficient** is a value, r, between –1 and 1.

- $r > 0$ suggests a positive (increasing) relationship
- $r < 0$ suggests a negative (decreasing) relationship
- The closer the value is to 0, the more scattered the data.
- The closer the value is to 1 or –1, the less scattered the data is.

Example 8.2.5: Finding a Correlation Coefficient

Calculate the correlation coefficient for cricket-chirp data in Table 8.2.1.

Solution

Because the data appear to follow a linear pattern, we can use technology to calculate r. Enter the inputs and corresponding outputs and select the Linear Regression. The calculator will also provide you with the correlation coefficient, $r = 0.9509$. This value is very close to 1, which suggests a strong increasing linear relationship.

Note: For some calculators, the Diagnostics must be turned "on" in order to get the correlation coefficient when linear regression is performed: [2nd]>[0]>[alpha][x–1], then scroll to DIAGNOSTICSON.

Predicting with a Regression Line

Once we determine that a set of data is linear using the correlation coefficient, we can use the regression line to make predictions. As we learned above, a regression line is a line that is closest to the data in the scatter plot, which means that only one such line is a best fit for the data.

Example 8.2.6: Using a Regression Line to Make Predictions

Gasoline consumption in the United States has been steadily increasing. Consumption data from 1994 to 2004 is shown in Table 8.2.3. Determine whether the trend is linear, and if so, find a model for the data. Use the model to predict the consumption in 2008.

Table 8.2.3

Year	'94	'95	'96	'97	'98	'99	'00	'01	'02	'03	'04
Consumption (billions of gallons)	113	116	118	119	123	125	126	128	131	133	136

The scatter plot of the data, including the least squares regression line, is shown in Figure 8.2.8.

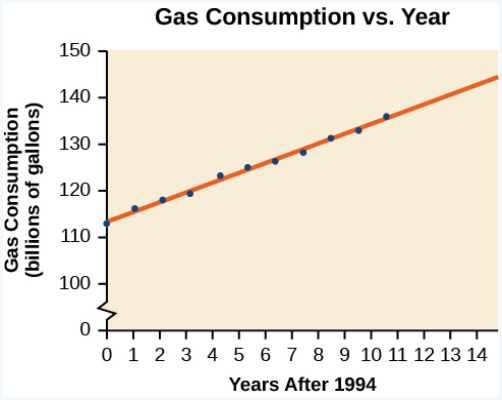


Figure 8.2.8: Scatter plot, showing the line of best fit.

We can introduce new input variable, t,representing years since 1994.

The least squares regression equation is:

$$C(t) = 113.318 + 2.209t \tag{8.2.12}$$

Using technology, the correlation coefficient was calculated to be 0.9965, suggesting a very strong increasing linear trend.

Using this to predict consumption in 2008 $(t = 14)$,

$$C(14) = 113.318 + 2.209(14) \tag{8.2.13}$$
$$= 144.244 \tag{8.2.14}$$

The model predicts 144.244 billion gallons of gasoline consumption in 2008.

Exercise 8.2.1

Use the model we created using technology in Example 8.2.6 to predict the gas consumption in 2011. Is this an interpolation or an extrapolation?

Answer

150.871 billion gallons; extrapolation

Key Concepts

- Scatter plots show the relationship between two sets of data.
- Scatter plots may represent linear or non-linear models.
- The line of best fit may be estimated or calculated, using a calculator or statistical software.
- Interpolation can be used to predict values inside the domain and range of the data, whereas extrapolation can be used to predict values outside the domain and range of the data.
- The correlation coefficient, r, indicates the degree of linear relationship between data.
- A regression line best fits the data.
- The least squares regression line is found by minimizing the squares of the distances of points from a line passing through the data and may be used to make predictions regarding either of the variables.

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8.3: Fitting Exponential Models to Data

Learning Objectives

- Build an exponential model from data.
- Build a logarithmic model from data.
- Build a logistic model from data.

In previous sections of this chapter, we were either given a function explicitly to graph or evaluate, or we were given a set of points that were guaranteed to lie on the curve. Then we used algebra to find the equation that fit the points exactly. In this section, we use a modeling technique called *regression analysis* to find a curve that models data collected from real-world observations. With regression analysis, we don't expect all the points to lie perfectly on the curve. The idea is to find a model that best fits the data. Then we use the model to make predictions about future events.

Do not be confused by the word *model*. In mathematics, we often use the terms *function*, *equation*, and *model* interchangeably, even though they each have their own formal definition. The term *model* is typically used to indicate that the equation or function approximates a real-world situation.

We will concentrate on three types of regression models in this section: exponential, logarithmic, and logistic. Having already worked with each of these functions gives us an advantage. Knowing their formal definitions, the behavior of their graphs, and some of their real-world applications gives us the opportunity to deepen our understanding. As each regression model is presented, key features and definitions of its associated function are included for review. Take a moment to rethink each of these functions, reflect on the work we've done so far, and then explore the ways regression is used to model real-world phenomena.

Building an Exponential Model from Data

As we've learned, there are a multitude of situations that can be modeled by exponential functions, such as investment growth, radioactive decay, atmospheric pressure changes, and temperatures of a cooling object. What do these phenomena have in common? For one thing, all the models either increase or decrease as time moves forward. But that's not the whole story. It's the *way* data increase or decrease that helps us determine whether it is best modeled by an exponential equation. Knowing the behavior of exponential functions in general allows us to recognize when to use exponential regression, so let's review exponential growth and decay.

Recall that exponential functions have the form $y = ab^x$ or $y = A_0e^{kx}$. When performing regression analysis, we use the form most commonly used on graphing utilities, $y = ab^x$. Take a moment to reflect on the characteristics we've already learned about the exponential function $y = ab^x$ (assume $a > 0$):

- b must be greater than zero and not equal to one.
- The initial value of the model is $y = a$.
  - If $b > 1$, the function models exponential growth. As x increases, the outputs of the model increase slowly at first, but then increase more and more rapidly, without bound.
  - If $0 < b < 1$, the function models **exponential decay**. As x increases, the outputs for the model decrease rapidly at first and then level off to become asymptotic to the *x*-axis. In other words, the outputs never become equal to or less than zero.

As part of the results, your calculator will display a number known as the *correlation coefficient*, labeled by the variable r, or r^2. (You may have to change the calculator's settings for these to be shown.) The values are an indication of the "goodness of fit" of the regression equation to the data. We more commonly use the value of r^2 instead of r, but the closer either value is to 1, the better the regression equation approximates the data.

EXPONENTIAL REGRESSION

Exponential regression is used to model situations in which growth begins slowly and then accelerates rapidly without bound, or where decay begins rapidly and then slows down to get closer and closer to zero. We use the command "ExpReg" on a graphing utility to fit an exponential function to a set of data points. This returns an equation of the form,

$$y = ab^x \tag{8.3.1}$$

Note that:

- b must be non-negative.
- when $b > 1$, we have an exponential growth model.
- when $0 < b < 1$, we have an exponential decay model.

How to: Given a set of data, perform exponential regression using a graphing utility

1. Use the STAT then EDIT menu to enter given data.
   i. Clear any existing data from the lists.
   ii. List the input values in the L1 column.
   iii. List the output values in the L2 column.
2. Graph and observe a scatter plot of the data using the STATPLOT feature.
   i. Use ZOOM [9] to adjust axes to fit the data.
   ii. Verify the data follow an exponential pattern.
3. Find the equation that models the data.
   i. Select "ExpReg" from the STAT then CALC menu.
   ii. Use the values returned for a and b to record the model, $y = ab^x$.
4. Graph the model in the same window as the scatterplot to verify it is a good fit for the data.

In 2007, a university study was published investigating the crash risk of alcohol impaired driving. Data from $2,871$ crashes were used to measure the association of a person's blood alcohol level (BAC) with the risk of being in an accident. Table $8.3.1$ shows results from the study. The *relative risk* is a measure of how many times more likely a person is to crash. So, for example, a person with a BAC of 0.09 is 3.54 times as likely to crash as a person who has not been drinking alcohol.

Table 8.3.1

BAC	0	0.01	0.03	0.05	0.07	0.09
Relative Risk of Crashing	1	1.03	1.06	1.38	2.09	3.54
BAC	0.11	0.13	0.15	0.17	0.19	0.21
Relative Risk of Crashing	6.41	12.6	22.1	39.05	65.32	99.78

a. Let x represent the BAC level, and let y represent the corresponding relative risk. Use exponential regression to fit a model to these data.
b. After 6 drinks, a person weighing 160 pounds will have a BAC of about 0.16. How many times more likely is a person with this weight to crash if they drive after having a 6-pack of beer? Round to the nearest hundredth.

Solution

a. Using the STAT then EDIT menu on a graphing utility, list the BAC values in L1 and the relative risk values in L2. Then use the STATPLOT feature to verify that the scatterplot follows the exponential pattern shown in Figure $8.3.1$:

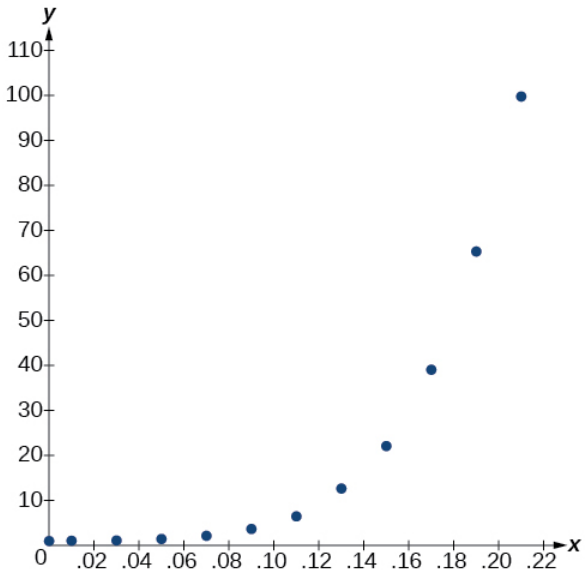


Figure 8.3.1

Use the "ExpReg" command from the STAT then CALC menu to obtain the exponential model,

$$y = 0.58304829(2.20720213E10)^{x}$$

Converting from scientific notation, we have:

$$y = 0.58304829(22,072,021,300)^{x}$$

Notice that $r^2 \approx 0.97$ which indicates the model is a good fit to the data. To see this, graph the model in the same window as the scatterplot to verify it is a good fit as shown in Figure 8.3.2:

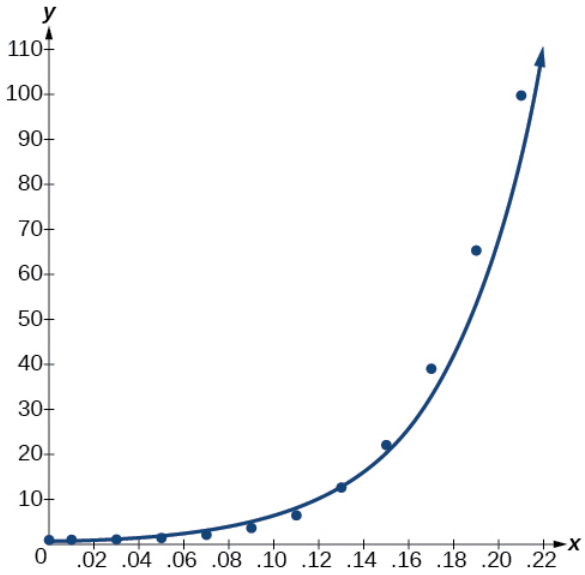


Figure 8.3.2

b. Use the model to estimate the risk associated with a BAC of 0.16. Substitute 0.16 for x in the model and solve for y.

$$\begin{aligned} y &= 0.58304829(22,072,021,300)^{x} && \text{Use the regression model found in part } (a) \\ &= 0.58304829(22,072,021,300)^{0.16} && \text{Substitute } 0.16 \text{ for x} \\ &\approx 26.35 && \text{Round to the nearest hundredth} \end{aligned}$$

If a 160-pound person drives after having 6 drinks, he or she is about 26.35 times more likely to crash than if driving while sober.

Exercise 8.3.1

Table 8.3.2 shows a recent graduate's credit card balance each month after graduation.

Table 8.3.2

Month	1	2	3	4	5	6	7	8
Debt ($)	620.00	761.88	899.80	1039.93	1270.63	1589.04	1851.31	2154.92

a. Use exponential regression to fit a model to these data.
b. If spending continues at this rate, what will the graduate's credit card debt be one year after graduating?

Answer a

The exponential regression model that fits these data is $y = 522.88585984(1.19645256)^x$

Answer b

If spending continues at this rate, the graduate's credit card debt will be $\$4,499.38$ after one year.

Q&A: Is it reasonable to assume that an exponential regression model will represent a situation indefinitely?

No. Remember that models are formed by real-world data gathered for regression. It is usually reasonable to make estimates within the interval of original observation (interpolation). However, when a model is used to make predictions, it is important to use reasoning skills to determine whether the model makes sense for inputs far beyond the original observation interval (extrapolation).

Building a Logarithmic Model from Data

Just as with exponential functions, there are many real-world applications for logarithmic functions: intensity of sound, pH levels of solutions, yields of chemical reactions, production of goods, and growth of infants. As with exponential models, data modeled by logarithmic functions are either always increasing or always decreasing as time moves forward. Again, it is the *way* they increase or decrease that helps us determine whether a **logarithmic model** is best.

Recall that logarithmic functions increase or decrease rapidly at first, but then steadily slow as time moves on. By reflecting on the characteristics we've already learned about this function, we can better analyze real world situations that reflect this type of growth or decay. When performing logarithmic **regression analysis**, we use the form of the logarithmic function most commonly used on graphing utilities, $y = a + b\ln(x)$. For this function

- All input values, x,must be greater than zero.
- The point $(1, a)$ is on the graph of the model.
- If $b > 0$,the model is increasing. Growth increases rapidly at first and then steadily slows over time.
- If $b < 0$,the model is decreasing. Decay occurs rapidly at first and then steadily slows over time.

LOGARITHMIC REGRESSION

Logarithmic regression is used to model situations where growth or decay accelerates rapidly at first and then slows over time. We use the command "LnReg" on a graphing utility to fit a logarithmic function to a set of data points. This returns an equation of the form,

$$y = a + b\ln(x) \tag{8.3.2}$$

Note that

- all input values, x,must be non-negative.
- when $b > 0$, the model is increasing.
- when $b < 0$, the model is decreasing.

How to: Given a set of data, perform logarithmic regression using a graphing utility

1. Use the STAT then EDIT menu to enter given data.
   i. Clear any existing data from the lists.
   ii. List the input values in the L1 column.
   iii. List the output values in the L2 column.
2. Graph and observe a scatter plot of the data using the STATPLOT feature.
   i. Use ZOOM [9] to adjust axes to fit the data.
   ii. Verify the data follow a logarithmic pattern.
3. Find the equation that models the data.
   i. Select "LnReg" from the STAT then CALC menu.
   ii. Use the values returned for a and b to record the model, $y = a + b\ln(x)$.
4. Graph the model in the same window as the scatterplot to verify it is a good fit for the data.

Example 8.3.2: Using Logarithmic Regression to Fit a Model to Data

Due to advances in medicine and higher standards of living, life expectancy has been increasing in most developed countries since the beginning of the 20th century. Table 8.3.3 shows the average life expectancies, in years, of Americans from 1900–2010.

Table 8.3.3

Year	1900	1910	1920	1930	1940	1950
Life Expectancy(Years)	47.3	50.0	54.1	59.7	62.9	68.2
Year	1960	1970	1980	1990	2000	2010
Life Expectancy(Years)	69.7	70.8	73.7	75.4	76.8	78.7

a. Let x represent time in decades starting with $x = 1$ for the year 1900, $x = 2$ for the year 1910, and so on. Let y represent the corresponding life expectancy. Use logarithmic regression to fit a model to these data.
b. Use the model to predict the average American life expectancy for the year 2030.

Solution

a. Using the STAT then EDIT menu on a graphing utility, list the years using values $1-12$ in L1 and the corresponding life expectancy in L2. Then use the STATPLOT feature to verify that the scatterplot follows a logarithmic pattern as shown in Figure 8.3.3:

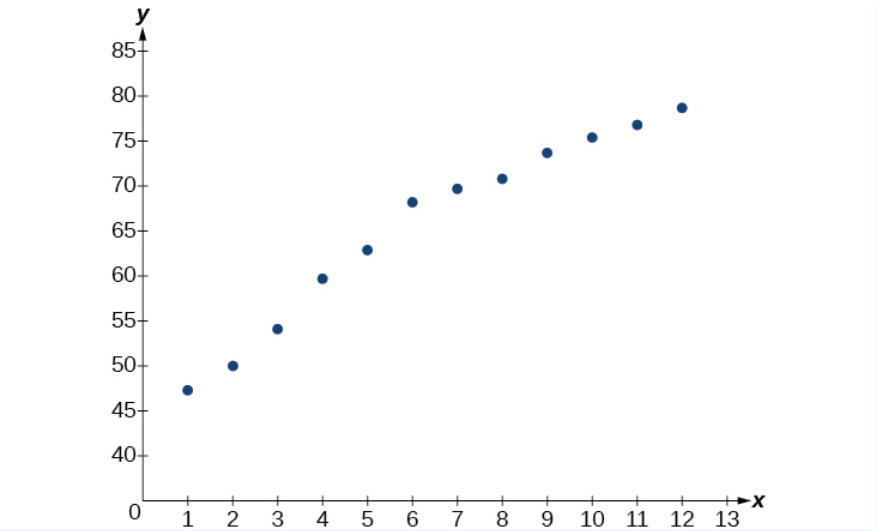


Figure 8.3.3

Use the "LnReg" command from the STAT then CALC menu to obtain the logarithmic model,

$$y = 42.52722583 + 13.85752327 \ln(x)$$

Next, graph the model in the same window as the scatterplot to verify it is a good fit as shown in Figure 8.3.4:

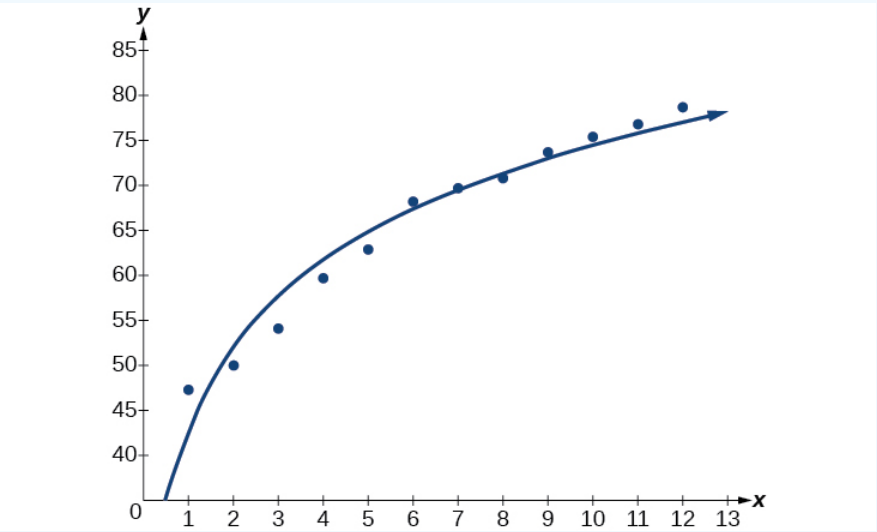


Figure 8.3.4

b. To predict the life expectancy of an American in the year 2030, substitute $x = 14$ for the in the model and solve for y:

$$\begin{aligned} y &= 42.52722583 + 13.85752327 \ln(x) && \text{Use the regression model found in part } (a) \\ &= 42.52722583 + 13.85752327 \ln(14) && \text{Substitute 14 for x} \\ &\approx 79.1 && \text{Round to the nearest tenth} \end{aligned}$$

If life expectancy continues to increase at this pace, the average life expectancy of an American will be 79.1 by the year 2030.

? Exercise 8.3.2

Sales of a video game released in the year 2000 took off at first, but then steadily slowed as time moved on. Table 8.3.4 shows the number of games sold, in thousands, from the years 2000–2010.

Table 8.3.4

Year	2000	2001	2002	2003	2004	2005

Number Sold (thousands)	142	149	154	155	159	161
Year	2006	2007	2008	2009	2010	-
Number Sold (thousands)	163	164	164	166	167	-

Let x represent time in years starting with $x = 1$ for the year 2000. Let y represent the number of games sold in thousands.

a. Use logarithmic regression to fit a model to these data.
b. If games continue to sell at this rate, how many games will sell in 2015? Round to the nearest thousand.

Answer a

The logarithmic regression model that fits these data is $y = 141.91242949 + 10.45366573 \ln(x)$

Answer b

If sales continue at this rate, about $171,000$games will be sold in the year 2015

Building a Logistic Model from Data

Like exponential and logarithmic growth, logistic growth increases over time. One of the most notable differences with logistic growth models is that, at a certain point, growth steadily slows and the function approaches an upper bound, or *limiting value*. Because of this, logistic regression is best for modeling phenomena where there are limits in expansion, such as availability of living space or nutrients.

It is worth pointing out that logistic functions actually model resource-limited exponential growth. There are many examples of this type of growth in real-world situations, including population growth and spread of disease, rumors, and even stains in fabric. When performing logistic regression analysis, we use the form most commonly used on graphing utilities:

$$y = \frac{c}{1 + ae^{-bx}}$$

Recall that:

- $\frac{c}{1+a}$ is the initial value of the model.
- when $b > 0$, the model increases rapidly at first until it reaches its point of maximum growth rate, $(\frac{\ln(a)}{b}, \frac{c}{2})$. At that point, growth steadily slows and the function becomes asymptotic to the upper bound $y = c$.
- c is the limiting value, sometimes called the *carrying capacity*, of the model.

LOGISTIC REGRESSION

Logistic regression is used to model situations where growth accelerates rapidly at first and then steadily slows to an upper limit. We use the command “Logistic” on a graphing utility to fit a logistic function to a set of data points. This returns an equation of the form

$$y = \frac{c}{1 + ae^{-bx}} \tag{8.3.3}$$

Note that

- The initial value of the model is $\frac{c}{1+a}$.
- Output values for the model grow closer and closer to $y = c$ as time increases.

How to: Given a set of data, perform logistic regression using a graphing utility

1. Use the STAT then EDIT menu to enter given data.
   i. Clear any existing data from the lists.
   ii. List the input values in the L1 column.
   iii. List the output values in the L2 column.
2. Graph and observe a scatter plot of the data using the STATPLOT feature.
   i. Use ZOOM [9] to adjust axes to fit the data.
   ii. Verify the data follow a logistic pattern.
3. Find the equation that models the data.
   i. Select "Logistic" from the STAT then CALC menu.
   ii. Use the values returned for a, b, and c to record the model, $y = \dfrac{c}{1 + ae^{-bx}}$.
4. Graph the model in the same window as the scatterplot to verify it is a good fit for the data.

Example 8.3.3: Using Logistic Regression to Fit a Model to Data

Mobile telephone service has increased rapidly in America since the mid 1990s. Today, almost all residents have cellular service. Table 8.3.5 shows the percentage of Americans with cellular service between the years 1995 and 2012.

Table 8.3.5

Year	Americans with Cellular Service (%)	Year	Americans with Cellular Service (%)
1995	12.69	2004	62.852
1996	16.35	2005	68.63
1997	20.29	2006	76.64
1998	25.08	2007	82.47
1999	30.81	2008	85.68
2000	38.75	2009	89.14
2001	45.00	2010	91.86
2002	49.16	2011	95.28
2003	55.15	2012	98.17

a. Let x represent time in years starting with $x = 0$ for the year 1995. Let y represent the corresponding percentage of residents with cellular service. Use logistic regression to fit a model to these data.
b. Use the model to calculate the percentage of Americans with cell service in the year 2013. Round to the nearest tenth of a percent.
c. Discuss the value returned for the upper limit, c. What does this tell you about the model? What would the limiting value be if the model were exact?

Solution

a. Using the STAT then EDIT menu on a graphing utility, list the years using values $0–15$ in L1 and the corresponding percentage in L2. Then use the STATPLOT feature to verify that the scatterplot follows a logistic pattern as shown in Figure 8.3.5:

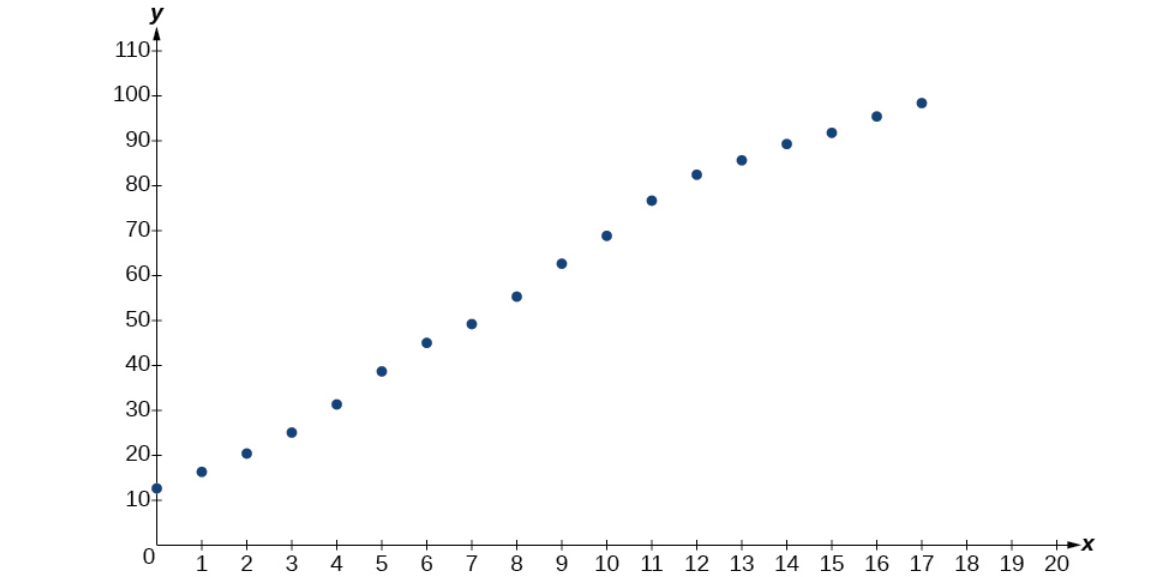


Figure 8.3.5

Use the "Logistic" command from the STAT then CALC menu to obtain the logistic model,

$$y = 105.73795261 + 6.88328979e^{-0.2595440013x} \tag{8.3.4}$$

Next, graph the model in the same window as shown in Figure 8.3.6 the scatterplot to verify it is a good fit:

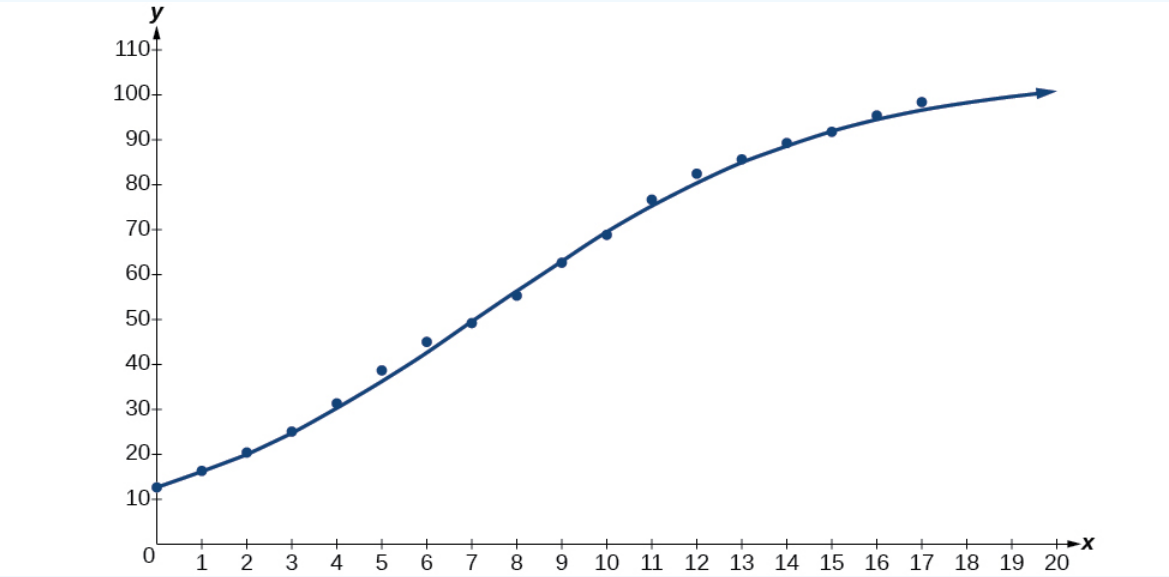


Figure 8.3.6

b. To approximate the percentage of Americans with cellular service in the year 2013, substitute $x = 18$ for the in the model and solve for y:

$$\begin{aligned} y &= \frac{105.7379526}{1 + 6.88328979e^{-0.2595440013x}} && \text{Use the regression model found in part } (a) \\ &= \frac{105.7379526}{1 + 6.88328979e^{-0.2595440013(18)}} && \text{Substitute 18 for x} \\ &\approx 99.3 && \text{Round to the nearest tenth} \end{aligned}$$

According to the model, about 98.8% of Americans had cellular service in 2013.

c. The model gives a limiting value of about 105. This means that the maximum possible percentage of Americans with cellular service would be 105, which is impossible. (How could over 100 of a population have cellular service?) If the model were exact, the limiting value would be $c = 100$ and the model's outputs would get very close to, but never actually reach 100. After all, there will always be someone out there without cellular service!

Exercise 8.3.3

Table 8.3.6 shows the population, in thousands, of harbor seals in the Wadden Sea over the years 1997 to 2012.

Table 8.3.6

Year	Seal Population (Thousands)	Year	Seal Population (Thousands)
1997	3.493	2005	19.590
1998	5.282	2006	21.955
1999	6.357	2007	22.862
2000	9.201	2008	23.869
2001	11.224	2009	24.243
2002	12.964	2010	24.344
2003	16.226	2011	24.919
2004	18.137	2012	25.108

Let x represent time in years starting with $x = 0$ for the year 1997. Let y represent the number of seals in thousands.

a. Use logistic regression to fit a model to these data.
b. Use the model to predict the seal population for the year 2020.
c. To the nearest whole number, what is the limiting value of this model?

Answer a

The logistic regression model that fits these data is $y = \dfrac{25.65665979}{1 + 6.113686306e^{-0.3852149008x}}$.

Answer b

If the population continues to grow at this rate, there will be about $25,634$seals in 2020.

Answer c

To the nearest whole number, the carrying capacity is $25,657$.

Media

Access this online resource for additional instruction and practice with exponential function models.

- Exponential Regression on a Calculator

Visit this website for additional practice questions from Learningpod.

Key Concepts

- Exponential regression is used to model situations where growth begins slowly and then accelerates rapidly without bound, or where decay begins rapidly and then slows down to get closer and closer to zero.
- We use the command "ExpReg" on a graphing utility to fit function of the form $y = ab^x$ to a set of data points. See Example 8.3.1.
- Logarithmic regression is used to model situations where growth or decay accelerates rapidly at first and then slows over time.

- We use the command “LnReg” on a graphing utility to fit a function of the form $y = a + b\ln(x)$ to a set of data points. See Example 8.3.2.
- Logistic regression is used to model situations where growth accelerates rapidly at first and then steadily slows as the function approaches an upper limit.
- We use the command “Logistic” on a graphing utility to fit a function of the form $y = \dfrac{c}{1 + ae^{-bx}}$ to a set of data points. See Example 8.3.3.

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