MATH 084: INTERMEDIATE ALGEBRA FOUNDATIONS FOR SOC SCI, LIB ARTS AND GENE



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Highline College

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Math 084 – Intermediate Algebra Foundations for Soc Sci, Lib Arts and GenEd

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Glossary

Glossary

Detailed Licensing

Licensing

*A detailed breakdown of this resource's licensing can be found in **Back Matter/Detailed Licensing**.*

CHAPTER OVERVIEW

1: Introduction to Graphing - By Hand and Calculator

1.1: Graphing Equations by Hand

1.2: The Graphing Calculator

1.3: Introduction to Graphing (optional practice)

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1.1: Graphing Equations by Hand

We begin with the definition of an *ordered pair*.

Ordered Pair

The construct (x, y), where x and y are any real numbers, is called an ordered pair of real numbers.

$(4, 3)$, $(-3, 4)$, $(-2, -3)$, and $(3, -1)$ are examples of ordered pairs.

Order Matters

Pay particular attention to the phrase "ordered pairs." Order matters. Consequently, the ordered pair (x, y) is not the same as the ordered pair (y, x), because the numbers are presented in a different order.

The Cartesian Coordinate System

Pictured in Figure 1.1.1 is a Cartesian Coordinate System. On a grid, we've created two real lines, one horizontal labeled x (we'll refer to this one as the x-axis), and the other vertical labeled y (we'll refer to this one as the y-axis).

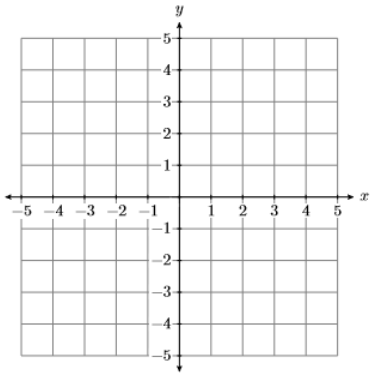


Figure 1.1.1: The Cartesian Coordinate System.

Two Important Points:

Here are two important points to be made about the horizontal and vertical axes in Figure 1.1.1.

1. As you move from left to right along the horizontal axis (the x-axis in Figure 1.1.1), the numbers grow larger. The positive direction is to the right, the negative direction is to the left.
2. As you move from bottom to top along the vertical axis (the y-axis in Figure 1.1.1), the numbers grow larger. The positive direction is upward, the negative direction is downward.

Additional Comments:

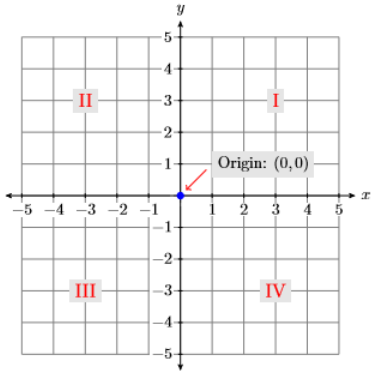


Figure 1.1.2: Numbering the quadrants and indicating the coordinates of the origin.

Two additional comments are in order:

3. The point where the horizontal and vertical axes intersect in Figure 1.1.2 is called the origin of the coordinate system. The origin has coordinates $(0, 0)$.
4. The horizontal and vertical axes divide the plane into four quadrants, numbered $\mathrm{I}, \mathrm{II}, \mathrm{MI}$, and IV (roman numerals for one, two, three, and four), as shown in Figure 1.1.2. Note that the quadrants are numbered in a counter-clockwise order.

Note

Rene Descartes (1596-1650) was a French philosopher and mathematician who is well known for the famous phrase"cogito ergo sum" (I think, therefore I am), which appears in his Discours de la methode pour bien conduire sa raison, et chercher la verite dans les sciences (Discourse on the Method of Rightly Conducting the Reason, and Seeking Truth in the Sciences). In that same treatise, Descartes introduces his coordinate system, a method for representing points in the plane via pairs of real numbers. Indeed, the Cartesian plane of modern day is so named in honor of Rene Descartes, who some call the "Father of Modern Mathematics"

Plotting Ordered Pairs

Before we can plot any points or draw any graphs, we first need to set up a Cartesian Coordinate System on a sheet of graph paper? How do we do this? What is required?

how to setup cartesian coordinate system

Draw and label each axis.

If we are going to plot points (x,y), then, on a sheet of graph paper, perform each of the following initial tasks.

1. Use a ruler to draw the horizontal and vertical axes.
2. Label the horizontal axis as the x-axis and the vertical axis as the y-axis.

We don't always label the horizontal axis as the x-axis and the vertical axis as the y-axis. For example, if we want to plot the velocity of an object as a function of time, then we would be plotting points (t, v). In that case, we would label the horizontal axis as the t-axis and the vertical axis as the v-axis.

Indicate the scale on each axis.

3. Label at least one vertical gridline with its numerical value.
4. Label at least one horizontal gridline with its numerical value.

The scales on the horizontal and vertical axes may differ. However, on each axis, the scale must remain consistent. That is, as you count to the right from the origin on the x-axis, if each gridline represents one unit, then as you count to the left from the origin on the x-axis, each gridline must also represent one unit. Similar comments are in order for the y-axis, where the scale must also be consistent, whether you are counting up or down.

The result of this first step is shown in Figure 1.1.3.

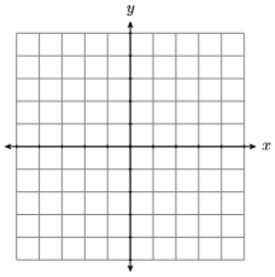


Figure 1.1.3: Draw and label each axis.

An example is shown in Figure 1.1.4. Note that the scale indicated on the x-axis indicates that each gridline counts as 1-unit as we count from left-to-right. The scale on the y-axis indicates that each gridlines counts as 2-units as we count from bottom-to-top.

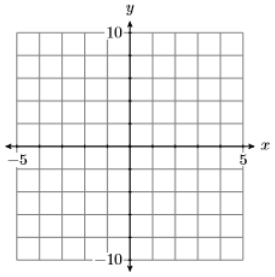


Figure 1.1.4: Indicate the scale on each axis.

Now that we know how to set up a Cartesian Coordinate System on a sheet of graph paper, here are two examples of how we plot points on our coordinate system.

To plot the ordered pair $(4, 3)$, start at the origin and move 4 units to the right along the horizontal axis, then 3 units upward in the direction of the vertical axis.

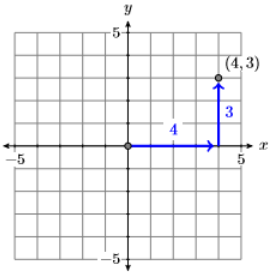


Figure 1.1.5: Plot of $(4, 3)$.

To plot the ordered pair $(-2, -3)$, start at the origin and move 2 units to the left along the horizontal axis, then 3 units downward in the direction of the vertical axis.

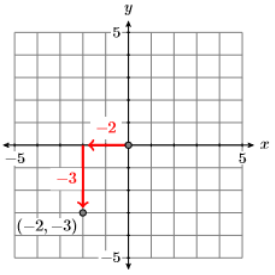


Figure 1.1.6: Plot of $(-2, -3)$

Continuing in this manner, each ordered pair (x, y) of real numbers is associated with a unique point in the Cartesian plane. Vice-versa, each point in the Cartesian point is associated with a unique ordered pair of real numbers. Because of this association, we begin to use the words "point" and "ordered pair" as equivalent expressions, sometimes referring to the "point" (x, y) and other times to the "ordered pair" (x, y).

Example 1.1.1

Identify the coordinates of the point P in Figure 1.1.7.

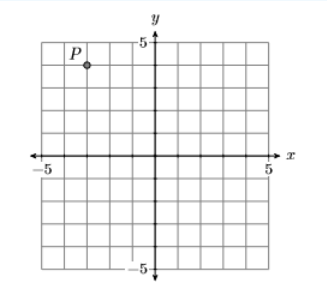


Figure 1.1.7: Identify the coordinates of the point P.

Solution

In Figure 1.1.8, start at the origin, move 3 units to the left and 4 units up to reach the point P. This indicates that the coordinates of the point P are $(-3, 4)$.

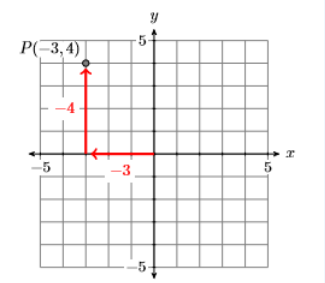
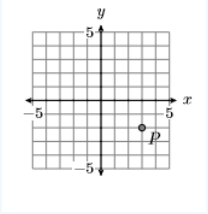


Figure 1.1.8: Start at the origin, move 3 units left and 4 units up.

Exercise 1.1.1

Identify the coordinates of the point P in the graph below.



Answer

$(3, -2)$

Equations in Two Variables

Note

The variables do not have to always be x and y. For example, the equation $v = 2 + 3.2t$ is an equation in two variables, v and t.

The equation $y = x + 1$ is an equation in two variables, in this case x and y. Consider the point $(x, y) = (2, 3)$. If we substitute 2 for x and 3 for y in the equation $y = x + 1$, we get the following result:

$$\begin{aligned} y &= x + 1 && \text{Original equation.} \\ 3 &= 2 + 1 && \text{Substitute: 2 for } x, 3 \text{ for } y \\ 3 &= 3 && \text{Simplify both sides.} \end{aligned}$$

Because the last line is a true statement, we say that $(2, 3)$ is a solution of the equation $y = x + 1$. Alternately, we say that $(2, 3)$ satisfies the equation $y = x + 1$. On the other hand, consider the point $(x, y) = (-3, 1)$. If we substitute -3 for x and 1 for y in the equation $y = x + 1$, we get the following result.

$$\begin{aligned} y &= x + 1 && \text{Original equation.} \\ 1 &= -3 + 1 && \text{Substitute: } -3 \text{ for } x, 1 \text{ for } y \\ 1 &= -2 && \text{Simplify both sides.} \end{aligned}$$

Because the last line is a false statement, the point $(-3, 1)$ is **not** a solution of the equation $y = x + 1$; that is, the point $(-3, 1)$ does **not** satisfy the equation $y = x + 1$.

Solutions of an equation in two variables

Given an equation in the variables x and y and a point $(x, y) = (a, b)$, if upon subsituting a for x and b for y a true statement results, then the point $(x, y) = (a, b)$ is said to be a solution of the given equation. Alternately, we say that the point $(x, y) = (a, b)$ satisfies the given equation.

Example 1.1.2

Which of the ordered pairs $(0, -3)$ and $(1, 1)$ satisfy the equation $y = 3x - 2$?

Solution

Substituting the ordered pairs $(0, -3)$ and $(1, 1)$ into the equation $y = 3x - 2$ lead to the following results:

Consider $(x, y) = (0, -3)$. Substitute 0 for x and -3 for y:

$$\begin{aligned} y &= 3x - 2 \\ -3 &= 3(0) - 2 \\ -3 &= -2 \end{aligned}$$

The resulting statement is false.

Consider $(x, y) = (1, 1)$. Substitute 1 for x and 1 for y:

$$\begin{aligned} y &= 3x - 2 \\ 1 &= 3(1) - 2 \\ 1 &= 1 \end{aligned}$$

The resulting statement is true.

Thus, the ordered pair $(0, -3)$ does not satisfy the equation $y = 3x - 2$, but the ordered pair $(1, 1)$ does satisfy the equation $y = 3x - 2$.

Exercise 1.1.2

Which of the ordered pairs $(-1, 3)$ and $(2, 1)$ satisfy the equation $y = 2x + 5$?

Answer

$(-1, 3)$

Graphing Equations in Two Variables

Let's first define what is meant by the graph of an equation in two variables.

The graph of an equation

The graph of an equation is the set of all points that satisfy the given equation.

Example 1.1.3

Sketch the graph of the equation $y = x + 1$.

Solution

The definition requires that we plot all points in the Cartesian Coordinate System that satisfy the equation $y = x + 1$. Let's first create a table of points that satisfy the equation. Start by creating three columns with headers x, y, and (x, y), then select some values for x and put them in the first column.

Take the first value of x, namely $x = -3$, and substitute it into the equation $y = x + 1$.

$$\begin{aligned} y &= x + 1 \\ y &= -3 + 1 \\ y &= -2 \end{aligned}$$

x	$y = x + 1$	(x, y)
-3	-2	$(-3, -2)$
-2		
-1		
0		
1		
2		
3		

Thus, when $x = -3$, we have $y = -2$. Enter this value into the table.

Continue substituting each tabular value of x into the equation $y = x + 1$ and use each result to complete the corresponding entries in the table.

$$\begin{aligned} y &= -3 + 1 = -2 \\ y &= -2 + 1 = -1 \\ y &= -1 + 1 = 0 \\ y &= 0 + 1 = 1 \\ y &= 1 + 1 = 2 \\ y &= 2 + 1 = 3 \\ y &= 3 + 1 = 4 \end{aligned}$$

x	$y = x + 1$	(x, y)
-3	-2	$(-3, -2)$
-2	-1	$(-2, -1)$
-1	0	$(-1, 0)$
0	1	$(0, 1)$
1	2	$(1, 2)$
2	3	$(2, 3)$
3	4	$(3, 4)$

The last column of the table now contains seven points that satisfy the equation $y = x + 1$. Plot these points on a Cartesian Coordinate System (see Figure \(\PageIndex{9}\)).

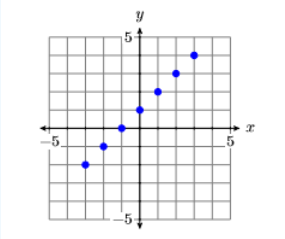


Figure 1.1.9: Seven points that satisfy the equation $y = x + 1$.

In Figure 1.1.9, we have plotted seven points that satisfy the given equation $y = x + 1$. However, the definition requires that we plot all points that satisfy the equation. It appears that a pattern is developing in Figure 1.1.9, but let's calculate and plot a few more points in order to be sure. Add the x-values -2.5, -1.5, -0.5, 0.5, 1.5, and 2.5 to the x-column of the table, then use the equation $y = x + 1$ to evaluate y at each one of these x-values.

$$y = -2.5 + 1 = -1.5$$
$$y = -1.5 + 1 = -0.5$$
$$y = -0.5 + 1 = 0.5$$
$$y = 0.5 + 1 = 1.5$$
$$y = 1.5 + 1 = 2.5$$
$$y = 2.5 + 1 = 3.5$$

x	$y = x + 1$	(x, y)
-2.5	-1.5	$(-2.5, -1.5)$
-1.5	-0.5	$(-1.5, -0.5)$
-0.5	0.5	$(-0.5, 0.5)$
0.5	1.5	$(0.5, 1.5)$
1.5	2.5	$(1.5, 2.5)$
2.5	3.5	$(2.5, 3.5)$

Add these additional points to the graph in Figure 1.1.9 to produce the image shown in Figure 1.1.10.

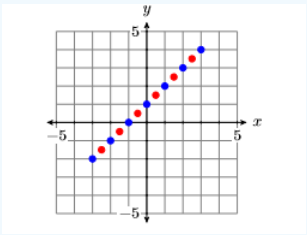


Figure 1.1.10: Adding additional points to the graph of $y = x + 1$.

There are an infinite number of points that satisfy the equation $y = x + 1$. In Figure 1.1.10, we've plotted only 13 points that satisfy the equation. However, the collection of points plotted in Figure 1.1.10 suggest that if we were to plot the remainder of the points that satisfy the equation $y = x + 1$, we would get the graph of the line shown in Figure 1.1.11.

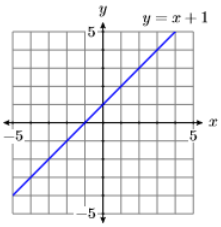
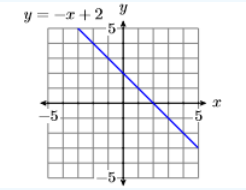


Figure 1.1.11: The graph of $y = x + 1$ is a line.

Exercise 1.1.3

Sketch the graph of the equation $y = -x + 2$.

Answer



Guidelines and Requirements

Example 1.1.3 suggests that we should use the following guidelines when sketching the graph of an equation.

Guidelines for drawing the graph of an equation

When asked to draw the graph of an equation, perform each of the following steps:

1. Set up and calculate a table of points that satisfy the given equation.
2. Set up a Cartesian Coordinate System on graph paper and plot the points in your table on the system. Label each axis (usually x and y) and indicate the scale on each axis.
3. If the number of points plotted are enough to envision what the shape of the final curve will be, then draw the remaining points that satisfy the equation as imagined. Use a ruler if you believe the graph is a line. If the graph appears to be a curve, freehand the graph without the use of a ruler.
4. If the number of plotted points do not provide enough evidence to envision the final shape of the graph, add more points to your table, plot them, and try again to envision the final shape of the graph. If you still cannot predict the eventual shape of the graph, keep adding points to your table and plotting them until you are convinced of the final shape of the graph.

Here are some additional requirements that must be followed when sketching the graph of an equation.

Graph paper, lines, curves, and rulers.

The following are requirements for this class:

5. All graphs are to be drawn on graph paper.
6. All lines are to be drawn with a ruler. This includes the horizontal and vertical axes.
7. If the graph of an equation is a curve instead of a line, then the graph should be drawn freehand, without the aid of a ruler.

Using the TABLE Feature of the Graphing Calculator

As the equations become more complicated, it can become quite tedious to create tables of points that satisfy the equation. Fortunately, the graphing calculator has a TABLE feature that enables you to easily construct tables of points that satisfy the given equation.

Example 1.1.4

Use the graphing calculator to help create a table of points that satisfy the equation $y = x^2 - 7$. Plot the points in your table. If you don't feel that there is enough evidence to envision what the final shape of the graph will be, use the calculator to add more points to your table and plot them. Continue this process until your are convinced of the final shape of the graph.

Solution



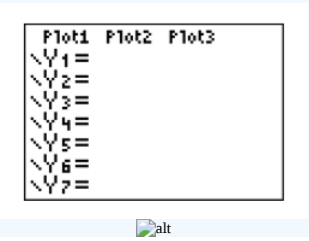
Figure 1.1.12: The graphing calculator.

The first step is to load the equation $y = x^2 - 7$ into the **Y=** menu of the graphing calculator. The topmost row of buttons on your calculator (see Figure 1.1.12) have the following appearance:



Figure 1.1.13. Use the up-and-down arrow keys (see Figure 1.1.15) to move the cursor after **Y1**= in the **Y**= menu, then use the following keystrokes to enter the equation $y = x^2 - 7$. The result is shown in Figure 1.1.14.





alt

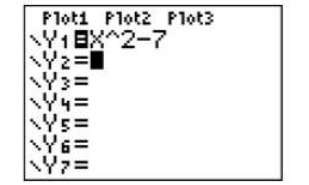


Figure 1.1.13: Opening the **Y**= menu. ***Figure* 1.1.14:** Entering the equation $y = x^2 - 7$.



Figure 1.1.15: Arrow keys move the cursor.

The next step is to “set up” the table. First, note that the calculator has symbolism printed on its case above each of its buttons. Above the WINDOW button you’ll note the phrase TBLSET. Note that it is in the same color as the 2ND button. Thus, to open the setup window for the table, enter the following keystrokes.

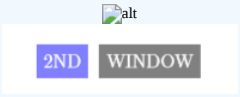


Figure 1.1.16) equal to the first x-value you want to see in your table. In this case, enter -4 after TblStart. SetTbl to the increment you want for your x-values. In this case, set Tbl equal to 1. Finally, set both the independent and dependent variables to “automatic.” In each case, use the arrow keys to highlight the word AUTO and press ENTER. The result is shown in Figure 1.1.16.

Next, note the word TABLE above the GRAPH button is in the same color as the 2ND key. To open the TABLE, enter the following keystrokes.



Figure 1.1.17. Note that you can use the up-and-down arrow keys to scroll through the table.



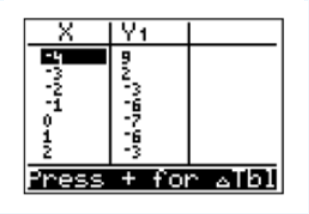


Figure 1.1.16: Opening the TBLSET menu. **Figure 1.1.17:** Table of points satisfying $y = x^2 - 7$.

Next, enter the results from your calculator’s table into a table on a sheet of graph paper, then plot the points in the table. The results are shown in Figure 1.1.18.

x	$y = x^2 - 7$	(x, y)
-4	9	$(-4, 9)$
-3	2	$(-3, 2)$
-2	-3	$(-2, -3)$
-1	-6	$(-1, -6)$
0	-7	$(0, -7)$
1	-6	$(1, -6)$
2	-3	$(2, -3)$
3	2	$(3, 2)$
4	9	$(4, 9)$

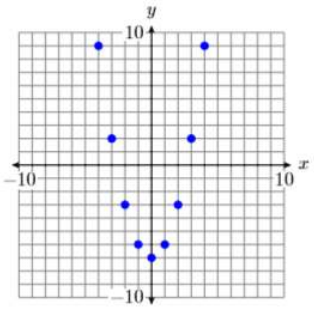


Figure 1.1.18: Plotting nine points that satisfy the equation $y = x^2 - 7$.

In Figure 1.1.18, the eventual shape of the graph of $y = x^2 - 7$ may be evident already, but let’s add a few more points to our table and plot them. Open the table “setup” window again by pressing 2ND WINDOW. Set TblStart to -4 again, then set the increment Tbl to 0.5. The result is shown in Figure 1.1.19.



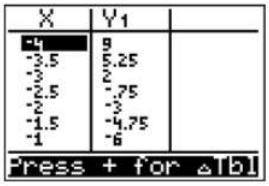


Figure 1.1.19: SetTbl equal to 0.5. **Figure 1.1.20:** More points satisfying $y = x^2 - 7$.

Add these new points to the table on your graph paper and plot them (see Figure 1.1.21).

x	$y = x^2 - 7$	(x, y)
−3.5	5.25	(−3.5, 5.25)
−2.5	−0.75	(−2.5, −0.75)
−1.5	−4.75	(−1.5, −4.75)
−0.5	−6.75	(−0.5, −6.75)
0.5	−6.75	(0.5, −6.75)
1.5	−4.75	(1.5, −4.75)
2.5	−0.75	(2.5, −0.75)
3.5	5.25	(3.5, 5.25)

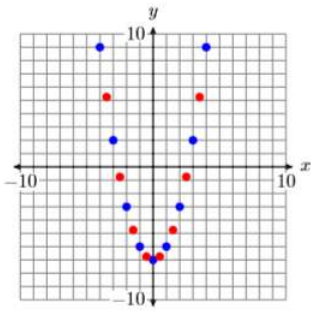


Figure 1.1.21: Adding additional points to the graph of $y = x^2 - 7$.

There are an infinite number of points that satisfy the equation $y = x^2 - 7$. In Figure 1.1.21, we've plotted only 17 points that satisfy the equation $y = x^2 - 7$. However, the collection of points in Figure 1.1.21 suggest that if we were to plot the remainder of the points that satisfy the equation $y = x^2 - 7$, the result would be the curve (called a parabola) shown in Figure 1.1.22

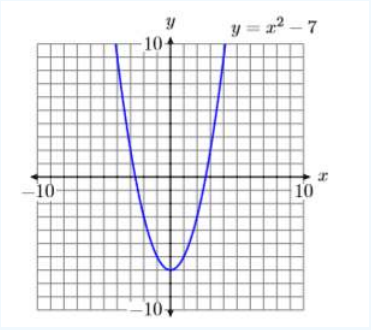


Figure 1.1.22: The graph of $y = x^2 - 7$ is a curve called a parabola.

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1.2: The Graphing Calculator

It's time to learn how to use a graphing calculator to sketch the graph of an equation. We will use the TI-84 graphing calculator in this section, but the skills we introduce will work equally well on the ancient TI-82 and the less ancient TI-83 graphing calculators.



Figure 1.2.1: The Graphing Calculator.

In this introduction to the graphing calculator, you will need to use several keys on the upper half of the graphing calculator (see Figure 1.2.2). The upand-down and left-and-right arrow keys are located in the upper right corner of Figure 1.2.2. These are used for moving the cursor on the calculator view screen and various menus. Immediately below these arrow keys is the CLEAR button, used to clear the view screen and equations in the **Y**= menu.

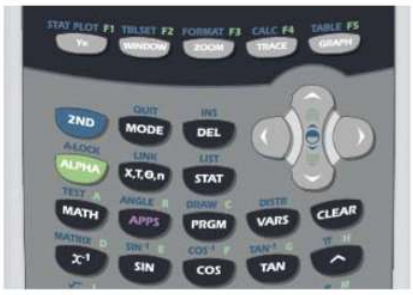


Figure 1.2.2: Top half of calculator.

It is not uncommon that people share their graphing calculators with friends who make changes to the settings in the calculator. Let's take a moment to make sure we have some common settings on our calculators.

Note

Above each button are one or more commands located on the calculator's case. Press the 2ND key to access a command having the same color as the 2ND key. Press the ALPHA key to access a command having the same color as the ALPHA key.

Locate and push the MODE in the first row of Figure 1.2.2. This will open the window shown in Figure 1.2.3. Make sure that the mode settings on your calculator are identical to the ones shown in Figure 1.2.3. If not, use the up-and-down arrow keys to move to the non-matching line item. This should place a blinking cursor over the first item on the line. Press the ENTER button on the lower-right corner of your calculator to make the selection permanent. Once you've completed your changes, press 2ND MODE again to quit the MODE menu.



Figure 1.2.3: Settings in the MODE window.

Note

The QUIT command is located above the MODE button on the calculator case and is used to exit the current menu.

Next, note the buttons across the first row of the calculator, located immediately below the view screen.



Figure 1.2.2). Press the 2ND button, then the **Y**= button. This opens the stat plot menu shown in Figure 1.2.4.

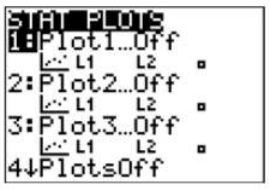


Figure 1.2.4: The STAT PLOT menu.

We need all of the stat plots to be "off." If any of the three stat plots are "on," select **4:PlotsOff** (press the number 4 on your keyboard), then press the ENTER key on the lower right corner of your calculator. That's it! Your calculator should now be ready for the upcoming exercises.

Example 1.2.1

Use the graphing calculator to sketch the graph of $y = x + 1$.

Solution

Recall that we drew the graph of $y = x + 1$ by hand in Example 3.1.1 of Section 3.1 (see Figure 3..1.11). In this example, we will use the graphing calculator to produce the same result.

Press the **Y**= button. The window shown in Figure 1.2.5 appears. If any equations appear in the **Y**= menu of Figure 1.2.5, use the up-and-down arrow keys and the CLEAR button (located below the up-and-down and left-and-right arrow keys) to delete them.

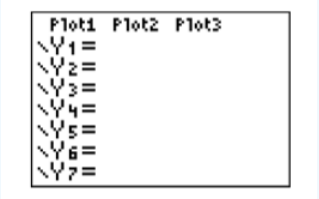


Figure 1.2.5: Press the Y= button to open the Y= menu.

Next, move the cursor to **Y1**=, then enter the equation $y = x + 1$ in **Y1** with the following button keystrokes. The result is shown in Figure 1.2.6.

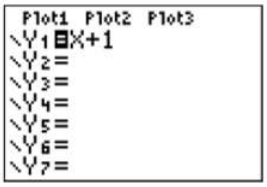


Figure 1.2.6: Enter $y = x + 1$ in **Y1**=.



Figure 1.2.7. From the ZOOM menu, select **6:ZStandard**.



Figure 1.2.7: Select **6:ZStandard** from the **ZOOM** menu.

There are two ways to make this selection:

1. Use the down-arrow key to move downward in the **ZOOM** menu until **6:ZStandard** is highlighted, then press the **ENTER** key.
2. A quicker alternative is to simply press the number 6 on the calculator keyboard to select **6:ZStandard**.

The resulting graph of $y = x + 1$ is shown in Figure 1.2.8. Note that the result is identical to the graph of $y = x + 1$ drawn by hand in Example 3.1.3 of Section 3.1 (see Figure 3..1.11).

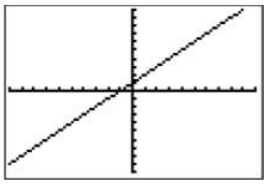
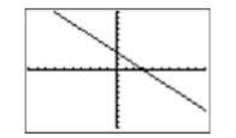


Figure 1.2.8: The graph of $y = x + 1$ is a line.

Exercise 1.2.1

Use the graphing calculator to sketch the graph of $y = -x + 3$.

Answer



Example 1.2.2

Use the graphing calculator to sketch the graph of $y = x^2 - 7$.

Solution

Recall that we drew the graph of $y = x^2 - 7$ by hand in Example 4 of Section 3.1 (see Figure 3.1.22). In this example, we will use the graphing calculator to produce the same result. Press the **Y**= button and enter the equation $y = x^2 - 7$ into **Y1** (see

Figure 1.2.9) with the following keystrokes:





Figure 1.2.9: Enter $y = x^2 - 7$ in **Y1**.

The caret (∧) symbol (see Figure 1.2.2) is located in the last column of buttons on the calculator, just underneath the CLEAR button, and means "raised to." The caret button is used for entering exponents. For example, x^2 is entered as $\mathrm{X} \wedge 2$, x^3 is entered as $\mathrm{X} \wedge 3$, and so on.

Press the **ZOOM** button, then select **6:ZStandard** from the **ZOOM** menu to produce the graph of $y = x^2 - 7$ shown in Figure 1.2.10. Note that the result in Figure 1.2.10 agrees with the graph of $y = x^2 - 7$ we drew by hand in Example 4 of Section 3.1 (see Figure 3.1.22).

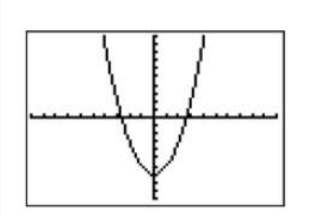
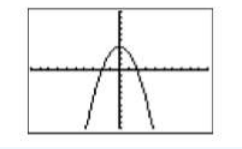


Figure 1.2.10: The graph of $y = x^2 - 7$ is a parabola.

Exercise 1.2.2

Use the graphing calculator to sketch the graph of $y = -x^2 + 4$.

Answer



Reproducing Calculator Results on Homework Paper

In this section we delineate recommendations and requirements when reproducing graphing calculator results on your homework paper.

Consider again the final result of Example 1.2.2 shown in Figure 1.2.10. To determine the scale at each end of each axis, press the WINDOW button on the topmost row of buttons on your calculator. The WINDOW settings for Figure 1.2.10 are shown in Figure 1.2.11. Figure 1.2.12 presents a visual explanation of each of the entries Xmin, Xmax, Ymin and Ymax.

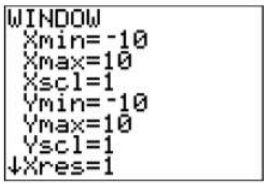


Figure 1.2.11: The **WINDOW** parameters.

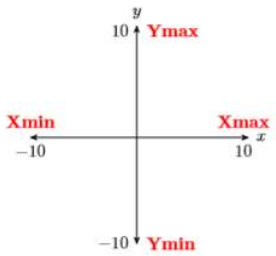


Figure 1.2.12: Xmin, Xmax, Ymin and Ymax contain the scale at the end of each axis.

Xmin and Xmax indicate the scale at the left- and right-hand ends of the x-axis, respectively, while Ymin and Ymax indicate the scale at the bottomand top-ends of the y-axis. Finally, as we shall see shortly, Xscl and Yscl control the spacing between tick marks on the x- and y-axes, respectively.

When reproducing the graph in your calculator viewing window on your homework paper, follow the *Calculator Submission Guidelines*.

Calculator Submission Guidelines

1. All lines (including the horizontal and vertical axes) should be drawn with a ruler. All curves should be drawn freehand.
2. Set up a coordinate system on your homework paper that mimics closely the coordinate system in your calculator's view screen. Label your axes (usually with x and y).
3. Indicate the scale at each end of each axis. Use Xmin, Xmax, Ymin and Ymax in the **WINDOW** menu for this purpose.
4. Copy the graph from your calculator's viewing window onto your coordinate system. Label the graph with its equation.

For example, to report the results of Figure 1.2.10, draw the axes with a ruler, label the horizontal axis with x, the vertical axis with y, then place the values of Xmin, Xmax, Ymin and Ymax at the appropriate end of each axis (see Figure 1.2.13).

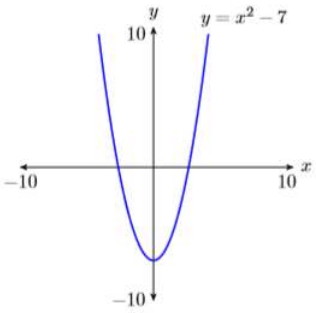


Figure 1.2.13: Reporting the graph of $y = x^2 - 7$ on your homework paper.

Because the graph is a curve, make a freehand copy that closely mimics the graph shown in Figure 1.2.10, then label it with its equation, as shown in Figure 1.2.13.

Adjusting the Viewing Window

Note that Figure 1.2.12gives us some sense of the meaning of the "standard viewing window" produced by selecting **6:ZStandard** from the ZOOM menu. Each time you select **6:ZStandard** from the ZOOM menu, Xmin is set to -10, Xmax to 10, and Xscl to

1 (distance between the tick marks on the x-axis). Similarly, Ymin is set to -10, Ymax to 10, and Yscl to 1 (distance between the tick marks on the y-axis). You can, however, override these settings as we shall see in the next example.

Example 1.2.3

Sketch the graphs of the following equations on the same viewing screen.

$$y = \frac{5}{4}x - 3 \qquad \text{and} \qquad y = \frac{2}{3}x + 4$$

Adjust the viewing window so that the point at which the graphs intersect (where the graphs cross one another) is visible in the viewing window, then use the TRACE button to approximate the coordinates of the point of intersection.

Solution

We must first decide on the proper syntax to use when entering the equations. In the case of the first equation, note that

$$y = \frac{5}{4}x - 3$$

is pronounced "y equals five-fourths x minus three," and means "y equals five-fourths times x minus three." Enter this equation into **Y1** in the **Y**= menu as $5/4 * \mathbf{X} - 3$ (see Figure 1.2.14), using the button keystrokes:





Figure 1.2.14: Enter equations.

The division and times buttons are located in the rightmost column of the calculator. Similarly, enter the second equation into **Y2** as $2/3 * X + 4$ using the button keystrokes:



Figure 1.2.15.

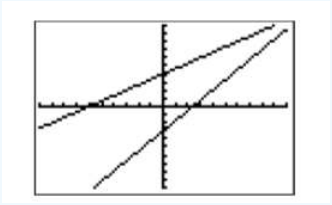


Figure 1.2.15: The graphs.

When we examine the resulting graphs in Figure 1.2.15, it appears that their point of intersection will occur off the screen above and to the right of the upper right-corner of the current viewing window. With this thought in mind, let's extend the x-axis to the right by increasing the value of Xmax to 20. Further, let's extend the y-axis upward by increasing the value of Ymin to 20. Press the WINDOW button on the top row of the calculator, then make the adjustments shown in Figure 1.2.16.

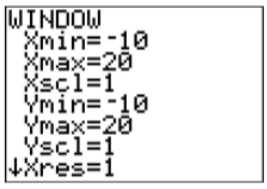


Figure 1.2.16: Adjusting the viewing window.

If we select **6:ZStandard** from the ZOOM menu, our changes to the WINDOW parameters will be discarded and the viewing window will be returned to the "standard viewing window." If we want to keep our changes to the WINDOW parameters, the correct approach at this point is to push the GRAPH button on the top row of the calculator. The resulting graph is shown in Figure 1.2.17. Note that the point of intersection of the two lines is now visible in the viewing window.

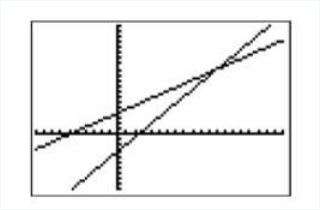


Figure 1.2.17: The point of intersection is visible in the new viewing window.

The image in Figure 1.2.17 is ready for recording onto your homework. However, we think we would have a better picture if we made a couple more changes:

1. It would be nicer if the point of intersection were more centered in the viewing window.
2. There are far too many tick marks.

With these thoughts in mind, make the changes to the WINDOW parameters shown in Figure 1.2.18, then push the GRAPH button to produce the image in Figure 1.2.19.

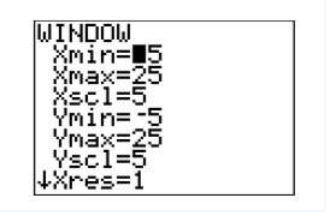


Figure 1.2.18: A final adjustment of the viewing window.

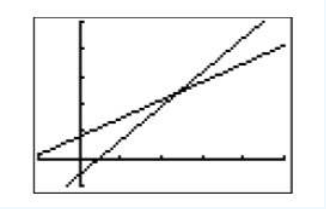


Figure 1.2.19: A centered point of intersection and fewer tick marks.

Finally, push the TRACE button on the top row of buttons, then use the left- and right-arrow keys to move the cursor atop the point of intersection. An approximation of the coordinates of the point of intersection are reported at the bottom of the viewing window (see Figure 1.2.20).

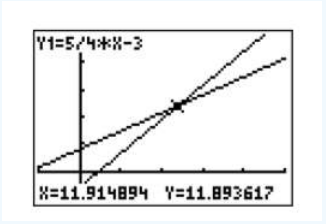


Figure 1.2.20: Approximate coordinates of the point of intersection reported by the TRACE button.

Note

The TRACE button is only capable of providing a very rough approximation of the point of intersection. In Chapter 4, we'll introduce the intersection utility on the CALC menu, which will report a much more accurate result.

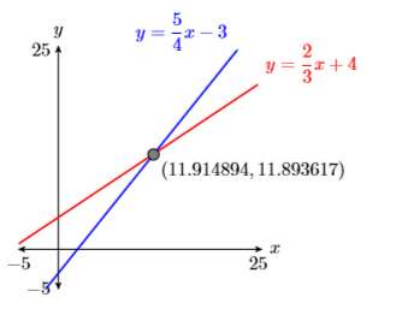


Figure 1.2.21: Reporting the answer on your homework.

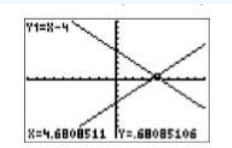
In reporting the answer on our homework paper, note that we followed the *Calculator Submission Guidelines*.

Exercise 1.2.3

Approximate the point of intersection of the graphs of $y = x - 4$ and $y = 5 - x$ using the TRACE button.

Answer

(4.68,0.68)



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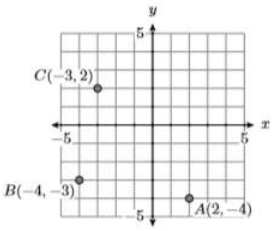
1.3: Introduction to Graphing (optional practice)

3.1: Graphing Equations by Hand

In Exercises 1-6, set up a Cartesian Coordinate system on a sheet of graph paper, then plot the given points.

1) $A(2,-4), B(-4,-3)$, and $C(-3,2)$

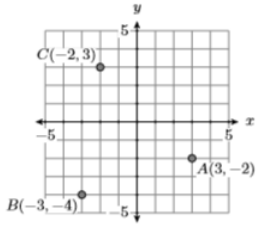
Answer



2) $A(3,-4), B(-3,-2)$, and $C(-4,4)$

3) $A(3,-2), B(-3,-4)$, and $C(-2,3)$

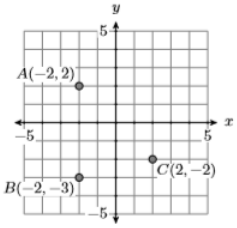
Answer



4) $A(3,-4), B(-3,-4)$, and $C(-4,2)$

5) $A(-2,2), B(-2,-3)$, and $C(2,-2)$

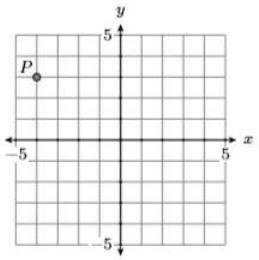
Answer



6) $A(-3,2), B(2,-4)$, and $C(-3,-4)$

In Exercises 7-10, identify the coordinates of point P in each of the given coordinate systems.

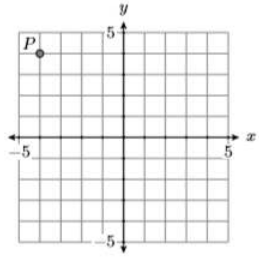
7)



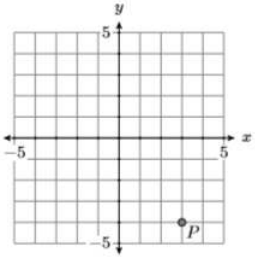
Answer

$(-4, 3)$

8)



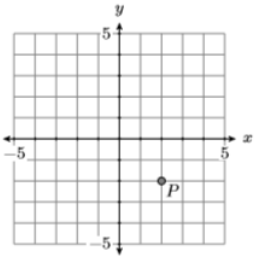
9)



Answer

$(3, -4)$

10)



11) Which of the points $(8, 36)$, $(5, 20)$, $(4, 13)$, and $(-2, -12)$ is a solution of the equation $y = 5x - 5$?

Answer

$(5, 20)$

12) Which of the points $(0, -9)$, $(6, -46)$, $(1, -14)$, and $(-6, 36)$ is a solution of the equation $y = -7x - 7$?

13) Which of the points $(-9, 49)$, $(1, 1)$, $(-6, 39)$, and $(2, -3)$ is a solution of the equation $y = -5x + 6$?

Answer

$(1, 1)$

14) Which of the points $(7, -15)$, $(-9, 30)$, $(-6, 19)$, and $(5, -11)$ is a solution of the equation $y = -3x + 3$?

15) Which of the points $(1, 12)$, $(7, 395)$, $(-7, 398)$, and $(0, 1)$ is a solution of the equation $y = 8x^2 + 3$?

Answer

$(7, 395)$

16) Which of the points $(-7, 154)$, $(7, 153)$, $(-2, 21)$, and $(2, 16)$ is a solution of the equation $y = 3x^2 + 6$?

17) Which of the points $(8, 400)$, $(5, 158)$, $(0, 3)$, and $(2, 29)$ is a solution of the equation $y = 6x^2 + 2x$?

Answer

$(8, 400)$

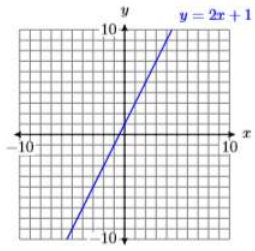
18) Which of the points $(-6, -114)$, $(6, -29)$, $(-7, -149)$, and $(4, -1)$ is a solution of the equation $y = -2x^2 + 7x$?

In Exercises 19-26, complete the table of points that satisfy the given equation. Set up a coordinate system on a sheet of graph paper, plot the points from the completed table, then draw the graph of the given equation.

19)

x	$y = 2x + 1$	(x, y)
-3		
-2		
-1		
0		
1		
2		
3		

Answer



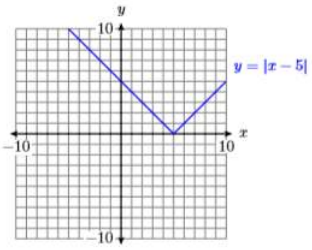
20)

x	$y = x - 4$	(x, y)
-3		
-2		
-1		
0		
1		
2		
3		

21)

x	$y = \|x - 5\|$	(x, y)
2		
3		
4		
5		
6		
7		
8		

Answer



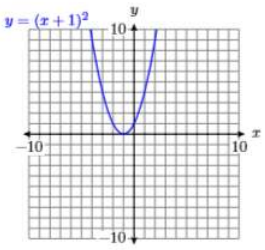
22)

x	$y = \|x + 2\|$	(x, y)
-5		
-4		
-3		
-2		
-1		
0		
1		

23)

x	$y = (x+1)^2$	(x, y)
-4		
-3		
-2		
-1		
0		
1		
2		

Answer



24)

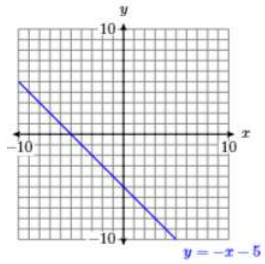
x	$y = (x+5)^2$	(x, y)
-8		
-7		
-6		
-5		
-4		
-3		
-2		

25)

x	$y = -x - 5$	(x, y)
-3		
-2		
-1		
0		
1		
2		

x	$y=-x-5$	(x,y)
3		

Answer

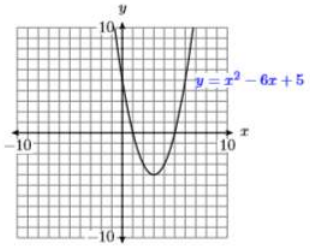


26)

x	$y=-2x+3$	(x,y)
-3		
-2		
-1		
0		
1		
2		
3		

27) Use a graphing calculator to complete a table of points satisfying the equation $y=x^2-6x+5$. Use integer values of x, starting at 0 and ending at 6. After completing the table, set up a coordinate system on a sheet of graph paper. Label and scale each axis, then plot the points in your table. Finally, use the plotted points as evidence to draw the graph of $y=x^2-6x+5$.

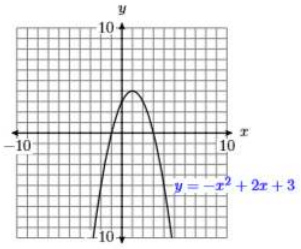
Answer



28) Use a graphing calculator to complete a table of points satisfying the equation $y=x^2-2x-3$. Use integer values of x, starting at $-2$and ending at 4. After completing the table, set up a coordinate system on a sheet of graph paper. Label and scale each axis, then plot the points in your table. Finally, use the plotted points as evidence to draw the graph of $y=x^2-2x-3$.

29) Use a graphing calculator to complete a table of points satisfying the equation $y=-x^2+2x+3$. Use integer values of x, starting at -2 and ending at 4. After completing the table, set up a coordinate system on a sheet of graph paper. Label and scale each axis, then plot the points in your table. Finally, use the plotted points as evidence to draw the graph of $y=-x^2+2x+3$.

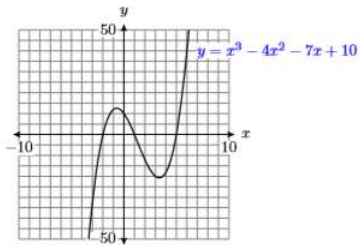
Answer



30) Use a graphing calculator to complete a table of points satisfying the equation $y = -x^2 - 2x + 8$. Use integer values of x, starting at -5 and ending at 3. After completing the table, set up a coordinate system on a sheet of graph paper. Label and scale each axis, then plot the points in your table. Finally, use the plotted points as evidence to draw the graph of $y = -x^2 - 2x + 8$.

31) Use a graphing calculator to complete a table of points satisfying the equation $y = x^3 - 4x^2 - 7x + 10$. Use integer values of x, starting at -3 and ending at 6. After completing the table, set up a coordinate system on a sheet of graph paper. Label and scale each axis, then plot the points in your table. Finally, use the plotted points as evidence to draw the graph of $y = x^3 - 4x^2 - 7x + 10$.

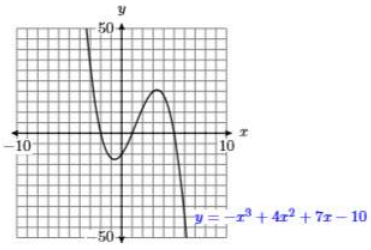
Answer



32) Use a graphing calculator to complete a table of points satisfying the equation $y = x^3 + 3x^2 - 13x - 15$. Use integer values of x, starting at -6 and ending at 4. After completing the table, set up a coordinate system on a sheet of graph paper. Label and scale each axis, then plot the points in your table. Finally, use the plotted points as evidence to draw the graph of $y = x^3 + 3x^2 - 13x - 15$.

33) Use a graphing calculator to complete a table of points satisfying the equation $y = -x^3 + 4x^2 + 7x - 10$. Use integer values of x, starting at -3 and ending at 6. After completing the table, set up a coordinate system on a sheet of graph paper. Label and scale each axis, then plot the points in your table. Finally, use the plotted points as evidence to draw the graph of $y = -x^3 + 4x^2 + 7x - 10$.

Answer

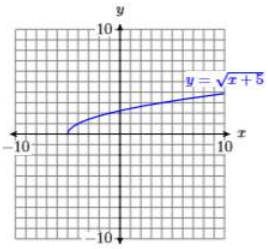


34) Use a graphing calculator to complete a table of points satisfying the equation $y = -x^3 + x^2 + 12x$. Use integer values of x, starting at -4 and ending at 5. After completing the table, set up a coordinate system on a sheet of graph paper. Label and scale each axis, then plot the points in your table. Finally, use the plotted points as evidence to draw the graph of $y = -x^3 + x^2 + 12x$.

35) Use a graphing calculator to complete a table of points satisfying the equation $y = \sqrt{x+5}$. Use integer values of x, starting at -5 and ending at 10. Round your y-values to the nearest tenth. After completing the table, set up a coordinate system on a sheet of

graph paper. Label and scale each axis, then plot the points in your table. Finally, use the plotted points as evidence to draw the graph of $y = \sqrt{x+5}$.

Answer



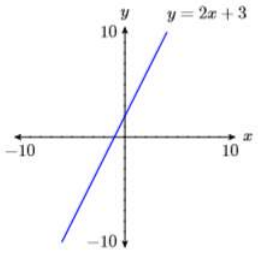
36) Use a graphing calculator to complete a table of points satisfying the equation $y = \sqrt{4-x}$. Use integer values of x, starting at -10 and ending at 4. Round your y-values to the nearest tenth. After completing the table, set up a coordinate system on a sheet of graph paper. Label and scale each axis, then plot the points in your table. Finally, use the plotted points as evidence to draw the graph of $y = \sqrt{4-x}$.

3.2: The Graphing Calculator

In Exercises 1-16, enter the given equation in the Y= menu of your calculator, then select 6:ZStandard from the ZOOM menu to produce its graph. Follow the Calculator Submission Guidelines outlined in the subsection Reproducing Calculator Results on Homework Paper when submitting your result on your homework paper.

1) $y = 2x + 3$

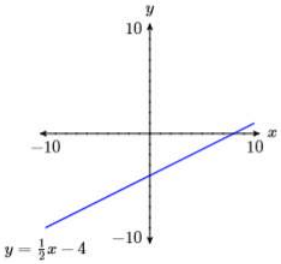
Answer



2) $y = 3 - 2x$

3) $y = \frac{1}{2}x - 4$

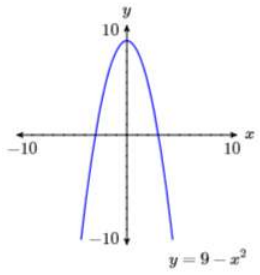
Answer



4) $y = -\frac{2}{3}x + 2$

5) $y = 9 - x^2$

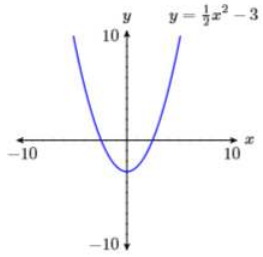
Answer



6) $y = x^2 - 4$

7) $y = \frac{1}{2}x^2 - 3$

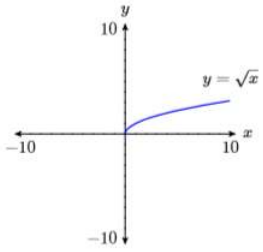
Answer



8) $y = 4 - \frac{1}{3}x^2$

9) $y = \sqrt{x}$

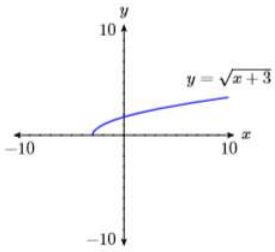
Answer



10) $y = \sqrt{-x}$

11) $y = \sqrt{x+3}$

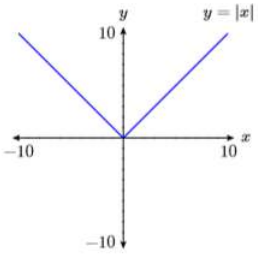
Answer



12) $y = \sqrt{5 - x}$

13) $y = |x|$

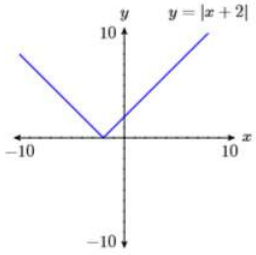
Answer



14) $y = -|x|$

15) $y = |x + 2|$

Answer

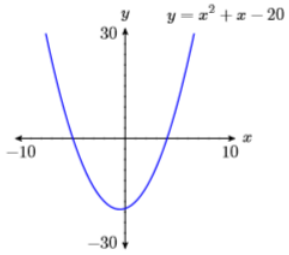


16) $y = -|x - 3|$

In Exercises 17-22, the graph of each given equation is a parabola, having a "u-shape" similar to the graph in Figure 3.2.13 Some of the parabolas "open up" and some "open down." Adjust Ymin and/or Ymax in the WINDOW menu so that the "turning point" of the parabola, called its vertex, is visible in the viewing screen of your graphing calculator. Follow the *Calculator Submission Guidelines* outlined in the subsection Reproducing Calculator Results on Homework Paper when submitting your result on your homework paper.

17) $y = x^2 + x - 20$

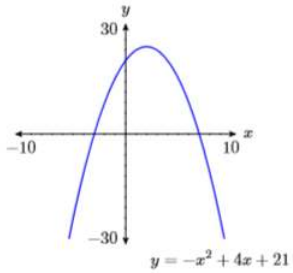
Answer



18) $y = -x^2 - 2x + 24$

19) $y = -x^2 + 4x + 21$

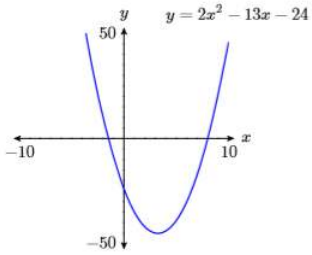
Answer



20) $y = x^2 + 6x - 16$

21) $y = 2x^2 - 13x - 24$

Answer

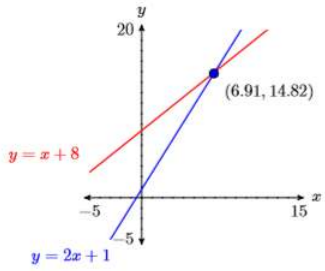


22) $y = -3x^2 - 19x + 14$

In Exercises 23-28, sketch the given lines, then adjust the WINDOW parameters so that the point of intersection of the two lines is visible in the viewing window. Use the TRACE key to estimate the point of intersection. Follow the *Calculator Submission Guidelines* outlined in the subsection Reproducing Calculator Results on Homework Paper when submitting your result on your homework paper. Be sure to also include your estimate of the point of intersection in your sketch.

23) $y = 2x + 1$ and $y = x + 8$

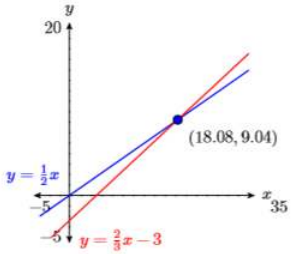
Answer



24) $y = 4 - 2x$ and $y = -6 - x$

25) $y = \frac{1}{2}x$ and $y = \frac{2}{3}x - 3$

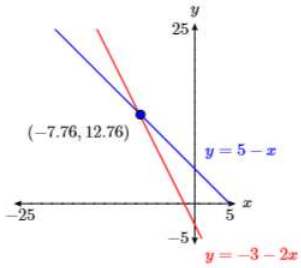
Answer



26) $y = 2x + 5$ and $y = x - 6$

27) $y = 5 - x$ and $y = -3 - 2x$

Answer



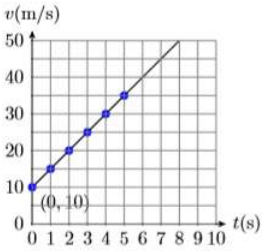
28) $y = 1 - \frac{1}{2}x$ and $y = -4 - \frac{4}{5}x$

3.3: Rates and Slope

1) An object's initial velocity at time $t = 0$ seconds is $v = 10$ meters per second. It then begins to pick up speed (accelerate) at a rate of 5 meters per second per second (5m/s/s or 5m/s^2).

a. Set up a Cartesian Coordinate System on a sheet of graph paper. Label and scale each axis. Include units with your labels.
b. Plot the point representing the initial velocity at time $t = 0$ seconds. Then plot a minimum of 5 additional points using the fact that the object is accelerating at a rate of 5 meters per second per second.
c. Sketch the line representing the object's velocity versus time.
d. Calculate the slope of the line.

Answer



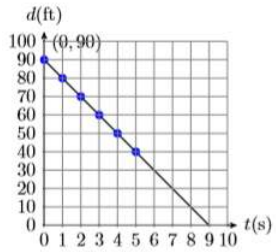
2) An object's initial velocity at time $t = 0$ seconds is $v = 40$ meters per second. It then begins to lose speed at a rate of 5 meters per second per second (5m/s/s or 5m/s^2).

a. Set up a Cartesian Coordinate System on a sheet of graph paper. Label and scale each axis. Include units with your labels.
b. Plot the point representing the initial velocity at time $t = 0$ seconds. Then plot a minimum of 5 additional points using the fact that the object is losing speed at a rate of 5 meters per second per second.
c. Sketch the line representing the object's velocity versus time.
d. Calculate the slope of the line.

3) David first sees his brother when the distance separating them is 90 feet. He begins to run toward his brother, decreasing the distance d between him and his brother at a constant rate of 10 feet per second (10 ft/s).

a. Set up a Cartesian Coordinate System on a sheet of graph paper. Label and scale each axis. Include units with your labels.
b. Plot the point representing David's initial distance from his brother at time t = 0 seconds. Then plot a minimum of 5 additional points using the fact that David's distance from his brother is decreasing at a constant rate of 10 feet per second (10ft/s).
c. Sketch the line representing David's distance from his brother versus time.
d. Find the slope of the line.

Answer:



4) David initially stands 20 feet from his brother when he sees his girl friend Mary in the distance. He begins to run away from his brother and towards Mary, increasing the distance d between him and his brother at a constant rate of 10 feet per second (10ft/s).

a. Set up a Cartesian Coordinate System on a sheet of graph paper. Label and scale each axis. Include units with your labels.
b. Plot the point representing David's initial distance from his brother at time $t = 0$ seconds. Then plot a minimum of 5 additional points using the fact that David's distance from his brother is increasing at a constant rate of 10 feet per second (10ft/s).
c. Sketch the line representing David's distance from his brother versus time.
d. Find the slope of the line.

In Exercises 5-14, calculate the slope of the line passing through the points P and Q. Be sure to reduce your answer to lowest terms.

5) $P(9, 0), Q(-9, 15)$

Answer

$$-\frac{5}{6}$$

6) $P(19, -17), Q(-13, 19)$

7) $P(0, 11), Q(16, -11)$

Answer

$$-\frac{11}{8}$$

8) $P(-10, -8), Q(11, 19)$

9) $P(11, 1), Q(-1, -1)$

Answer

$$\frac{1}{6}$$

10) $P(16, -15), Q(-11, 12)$

11) $P(-18, 8), Q(3, -10)$

Answer

$$-\frac{6}{7}$$

12) $P(9, 9), Q(-6, 3)$

13) $P(-18, 10), Q(-9, 7)$

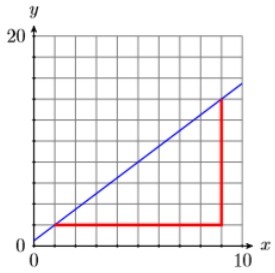
Answer

$$-\frac{1}{3}$$

14) $P(-7, 20), Q(7, 8)$

In Exercises 15-18, use the right triangle provided to help determine the slope of the line. Be sure to pay good attention to the scale provided on each axis when counting boxes to determine the change in y and the change in x.

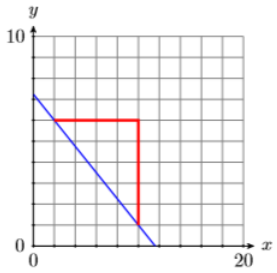
15)



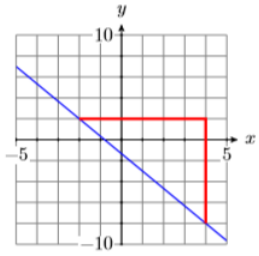
Answer

$$\frac{3}{2}$$

16)



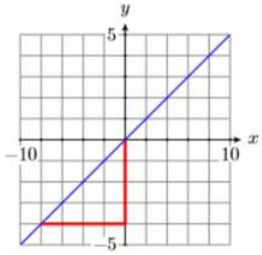
17)



Answer

$-\frac{5}{3}$

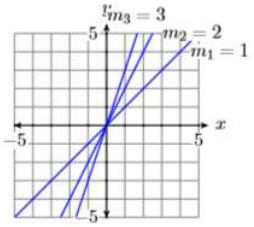
18)



19) On one coordinate system, sketch each of the lines that pass through the following pairs of points. Label each line with its slope, then explain the relationship between the slope found and the steepness of the line drawn.

a. $(0, 0)$ and $(1, 1)$
b. $(0, 0)$ and $(1, 2)$
c. $(0, 0)$ and $(1, 3)$

Answer



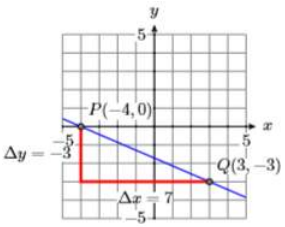
20) On one coordinate system, sketch each of the lines that pass through the following pairs of points. Label each line with its slope, then explain the relationship between the slope found and the steepness of the line drawn.

a. $(0, 0)$ and $(1, -1)$
b. $(0, 0)$ and $(1, -2)$
c. $(0, 0)$ and $(1, -3)$

In Exercises 21-30, setup a coordinate system on graph paper, then sketch the line passing through the point P with the slope m.

21) $P(-4, 0), m = -3/7$

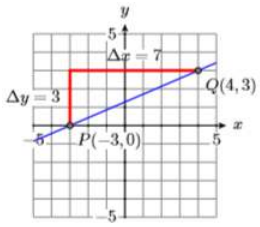
Answer



22) $P(-3, 0), m = -3/7$

23) $P(-3, 0), m = 3/7$

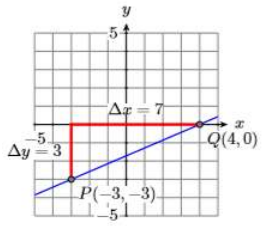
Answer



24) $P(-3, 0), m = 3/4$

25) $P(-3, -3), m = 3/7$

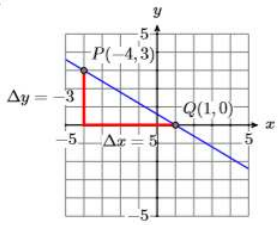
Answer



26) $P(-2,3), m=-3/5$

27) $P(-4,3), m=-3/5$

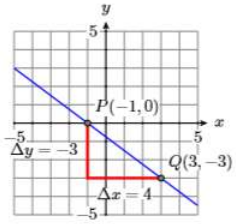
Answer



28) $P(-1,-3), m=3/4$

29) $P(-1,0), m=-3/4$

Answer



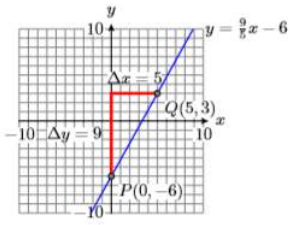
30) $P(-3,3), m=-3/4$

3.4: Slope-Intercept Form of a Line

In Exercises 1-6, setup a coordinate system on graph paper, then sketch the line having the given equation. Label the line with its equation.

1) $y=\frac{9}{5}x-6$

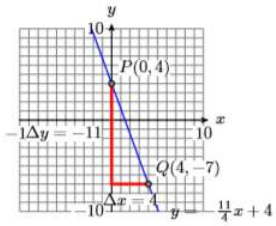
Answer



2) $y=\frac{8}{7}x-1$

3) $y=-\frac{11}{4}x+4$

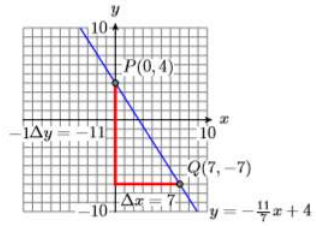
Answer



4) $y = \frac{5}{4}x$

5) $y = -\frac{11}{7}x + 4$

Answer

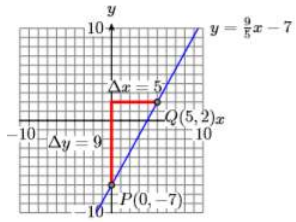


6) $y = -\frac{7}{5}x + 7$

In Exercises 7-12, sketch the line with given y-intercept slope. Label the line with the slope-intercept form of its equation.

7) $(0, -7), 9/5$

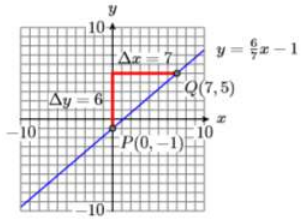
Answer



8) $(0, 7), -4/5$

9) $(0, -1), 6/7$

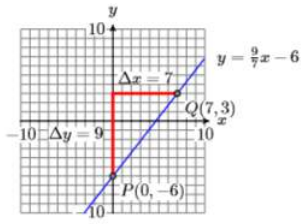
Answer



10) $(0, 1), -7/5$

11) $(0, -6), 9/7$

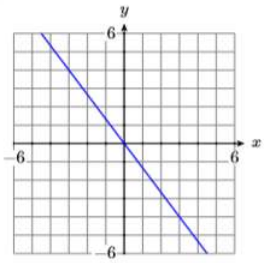
Answer



12) $(0, -5), 7/5$

In Exercises 13-20, determine the equation of the given line in slope-intercept form.

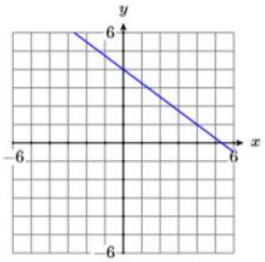
13)



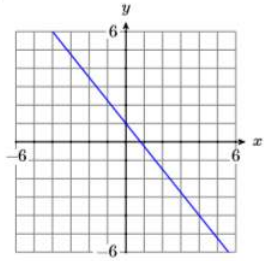
Answer

$$y = -\frac{4}{3}x$$

14)



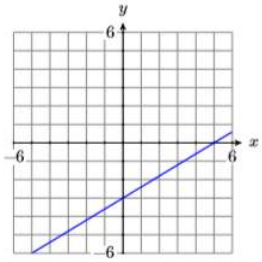
15)



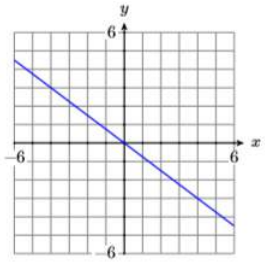
Answer

$$y = -\frac{5}{4}x + 1$$

16)



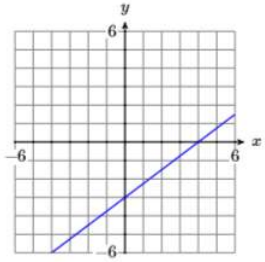
17)



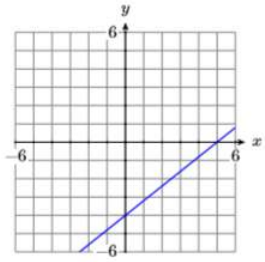
Answer

$y = -\frac{3}{4}x$

18)



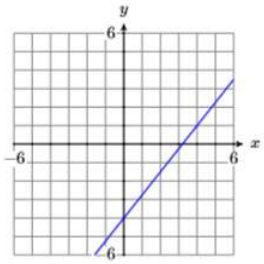
19)



Answer

$y = \frac{4}{5}x - 4$

20)



21) Assume that the relationship between an object's velocity and its time is linear. At $t = 0$ seconds, the object's initial velocity is 20m/s. It then begins to speed up at a constant rate of 5 meters per second per second.

a. Set up a coordinate system, placing the time t on the horizontal axis and the velocity v on the vertical axis. Label and scale each axis. Include units in your labels.
b. Use the initial velocity and the rate at which the object's velocity is increasing to draw the line representing the object's velocity at time t. Use the slope-intercept form to determine the equation of the line.
c. Replace x and y in the equation found in part (b) with t and v, respectively. Use the result to determine the velocity of the object after 14 seconds.

Answer

c) 90m/s

22) Assume that the relationship between an object's velocity and its time is linear. At $t = 0$ seconds, the object's initial velocity is 80m/s. It then begins to lose speed at a constant rate of 4 meters per second per second.

a. Set up a coordinate system, placing the time t on the horizontal axis and the velocity v on the vertical axis. Label and scale each axis. Include units in your labels.
b. Use the initial velocity and the rate at which the object's velocity is increasing to draw the line representing the object's velocity at time t. Use the slope-intercept form to determine the equation of the line.
c. Replace x and y in the equation found in part (b) with t and v, respectively. Use the result to determine the velocity of the object after 13 seconds.

23) A water tank initially (at time $t = 0$ min) contains 100 gallons of water. A pipe is opened and water pours into the tank at a constant rate of 25 gallons per minute. Assume that the relation between the volume V of water in the tank and time t is linear.

a. Set up a coordinate system, placing the time t on the horizontal axis and the volume of water V on the vertical axis. Label and scale each axis. Include units in your labels.
b. Use the initial volume of water in the tank and the rate at which the volume of water is increasing to draw the line representing the volume V of water in the tank at time t. Use the slope-intercept form to determine the equation of the line.
c. Replace x and y in the equation found in part (b) with t and V, respectively. Use the result to predict how much time must pass until the volume of water in the tank reaches 400 gallons.

Answer

c) 12 min

24) A water tank initially (at time $t = 0$ min) contains 800 gallons of water. A spigot is opened at the bottom of the tank and water pours out at a constant rate of 40 gallons per minute. Assume that the relation between the volume V of water in the tank and time t is linear.

a. Set up a coordinate system, placing the time t on the horizontal axis and the volume of water V on the vertical axis. Label and scale each axis. Include units in your labels.
b. Use the initial volume of water in the tank and the rate at which the volume of water is decreasing to draw the line representing the volume V of water in the tank at time t. Use the slope-intercept form to determine the equation of the line.

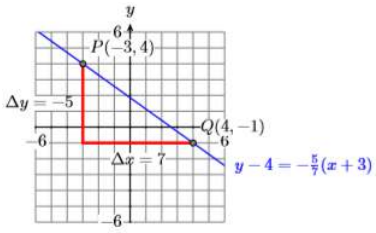
c. Replace x and y in the equation found in part (b) with t and V, respectively. Use the result to predict how much time must pass until the water tank is empty.

3.5: Point-Slope Form of a Line

In Exercises 1-6, set up a coordinate system on a sheet of graph paper, then sketch the line through the given point with the given slope. Label the line with its equation in point-slope form.

1) $m = -5/7, P(-3, 4)$

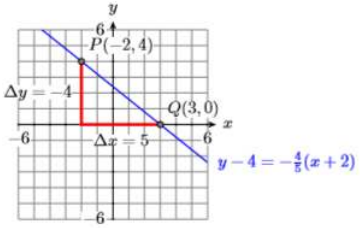
Answer



2) $m = 3/4, P(-2, -4)$

3) $m = -4/5, P(-2, 4)$

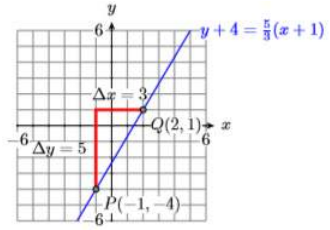
Answer



4) $m = 5/6, P(-3, -1)$

5) $m = 5/3, P(-1, -4)$

Answer

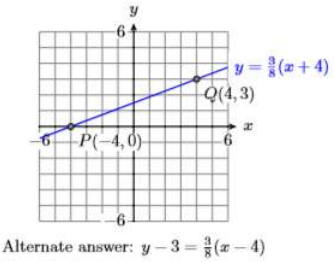


6) $m = -3/8, P(-4, 0)$

In Exercises 7-12, set up a coordinate system on a sheet of graph paper, then sketch the line through the given points. Label the line with the point-slope form of its equation.

7) $P(-4, 0)$ and $Q(4, 3)$

Answer

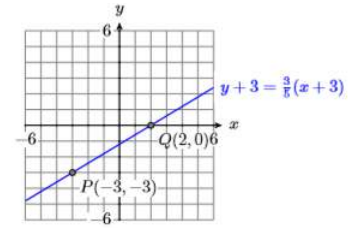


Alternate answer: $y - 3 = \frac{3}{8}(x - 4)$

8) $P(-2, 4)$ and $Q(2, -1)$

9) $P(-3, -3)$ and $Q(2, 0)$

Answer

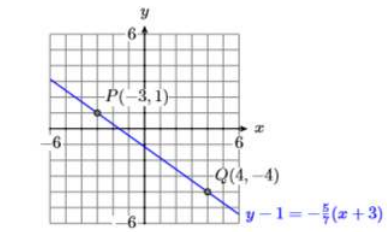


Alternate answer: $y = \frac{3}{5}(x - 2)$

10) $P(-3, 4)$ and $Q(2, 0)$

11) $P(-3, 1)$ and $Q(4, -4)$

Answer



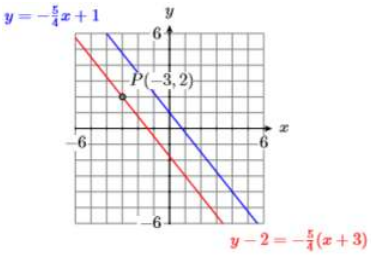
Alternate answer: $y + 4 = -\frac{5}{7}(x - 4)$

12) $P(-1, 0)$ and $Q(4, 3)$

In Exercises 13-18, on a sheet of graph paper, sketch the given line. Plot the point P and draw a line through P that is parallel to the first line. Label this second line with its equation in point-slope form.

13) $y = -\frac{5}{4}x + 1, P(-3, 2)$

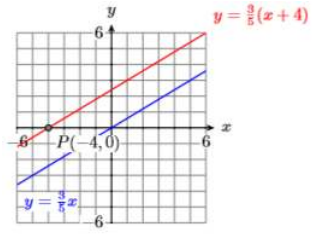
Answer



14) $y = -\frac{3}{5}x, P(-4, 0)$

15) $y = \frac{3}{5}x, P(-4, 0)$

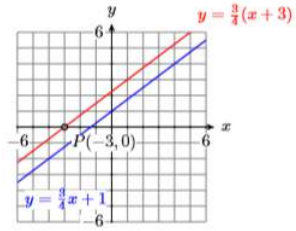
Answer



16) $y = \frac{5}{3}x - 1, P(-2, -2)$

17) $y = \frac{3}{4}x + 1, P(-3, 0)$

Answer

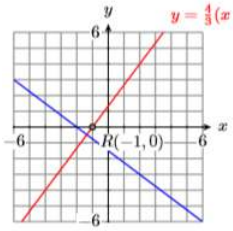


18) $y = \frac{5}{4}x - 4, P(-2, -2)$

In Exercises 19-24, on a sheet of graph paper, sketch the line passing through the points P and Q. Plot the point R and draw a line through R that is perpendicular to the line passing through the points P and Q. Label this second line with its equation in point-slope form.

19) $P(-2, 0), Q(2, -3)$,and $R(-1, 0)$

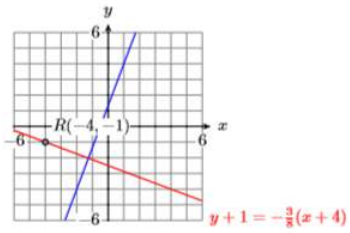
Answer



20) $P(-1, 3), Q(2, -2)$,and $R(-1, 0)$

21) $P(-2, -4), Q(1, 4)$,and $R(-4, -1)$

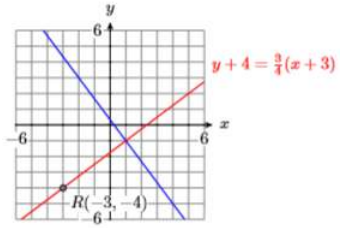
Answer



22) $P(-4,-4), Q(-1,3)$,and $R(-4,2)$

23) $P(-2,3), Q(1,-1)$,and $R(-3,-4)$

Answer



24) $P(-4,4), Q(1,-4)$,and $R(-4,-3)$

25) Assume that the relationship between an object's velocity and its time is linear. At 3 seconds, the object's velocity is 50ft/s. At 14 seconds, the object's velocity is 30ft/s.

a. Set up a coordinate system, placing the time t on the horizontal axis and the velocity v on the vertical axis. Label and scale each axis. Include units in your labels.
b. Plot the points determined by the given data and draw a line through them. Use the point-slope form of the line to determine the equation of the line.
c. Replace x and y in the equation found in part (b) with t and v, respectively, then solve the resulting equation for v.
d. Use the result of part (c) to determine the velocity of the object after 6 seconds.

Answer

44.5ft/s

26) Water freezes at approximately 32°F and 273°K, where F represents the temperature measured on the Fahrenheit scale and K represents the temperature measured on the Kelvin scale. Water boils at approximately 212°F and 373°K. Assume that the relation between the Fahrenheit and Kelvin temperatures is linear.

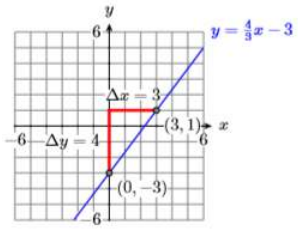
a. Set up a coordinate system, placing the Kelvin temperature K on the horizontal axis and the Fahrenheit temperature F on the vertical axis. Label and scale each axis. Include units in your labels.
b. Plot the points determined by the given data and draw a line through them. Use the point-slope form of the line to determine the equation of the line.
c. Replace x and y in the equation found in part (b) with K and F, respectively, then solve the resulting equation for F.
d. Use the result of part (c) to determine the Fahrenheit temperature of the object if the Kelvin temperature is 212°K.

3.6: Standard Form of a Line

In Exercises 1-6, place the given standard form into slope-intercept form and sketch its graph. Label the graph with its slope-intercept form.

1) $4x - 3y = 9$

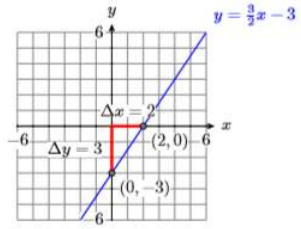
Answer



2) $2x - 3y = 3$

3) $3x - 2y = 6$

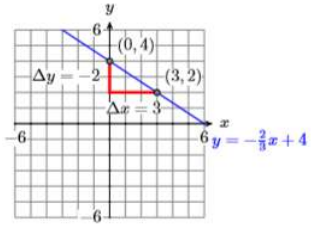
Answer



4) $5x - 3y = 3$

5) $2x + 3y = 12$

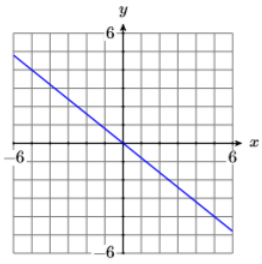
Answer



6) $3x + 4y = 8$

In Exercises 7-10, determine an equation of the given line in standard form.

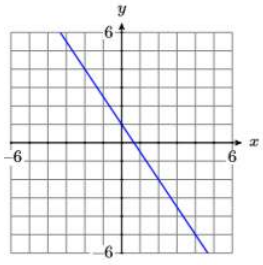
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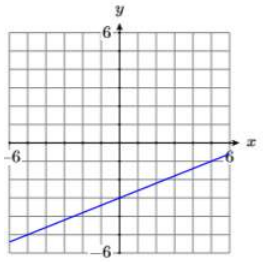
Answer

$4x + 5y = 0$

8)



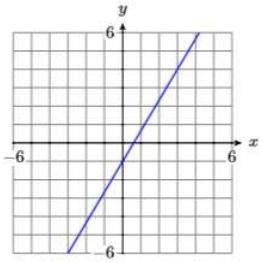
9)



Answer

$2x - 5y = 15$

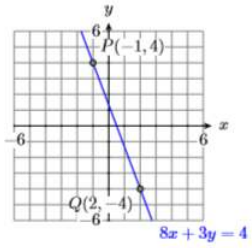
10)



In Exercises 11-16, sketch the line passing through the points P and Q. Label the line with its equation in standard form.

11) $P(-1, 4)$ and $Q(2, -4)$

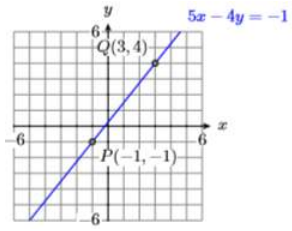
Answer



12) $P(-1, 4)$ and $Q(3, 1)$

13) $P(-1, -1)$ and $Q(3, 4)$

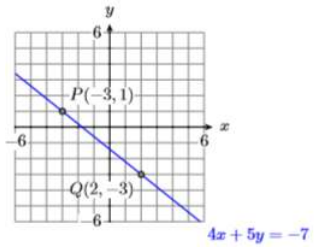
Answer



14) $P(2, -1)$ and $Q(4, 4)$

15) $P(-3, 1)$ and $Q(2, -3)$

Answer

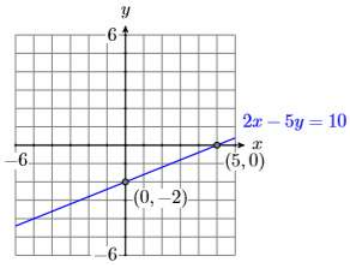


16) $P(-4, 3)$ and $Q(0, 0)$

In Exercises 17-22, plot the x- and y-intercepts of the line having the given equation, then draw the line through the intercepts and label it with its equation.

17) $2x - 5y = 10$

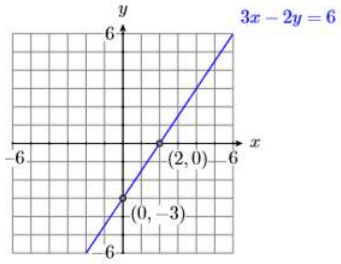
Answer



18) $2x + 3y = -6$

19) $3x - 2y = 6$

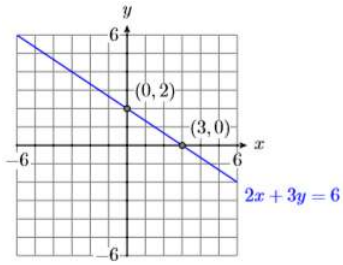
Answer



20) $3x - 4y = 12$

21) $2x + 3y = 6$

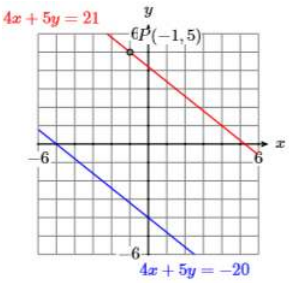
Answer



22) $2x - 3y = -6$

23) Find an equation of the line (in standard form) that passes through the point $P(-1, 5)$ that is parallel to the line $4x + 5y = -20$.

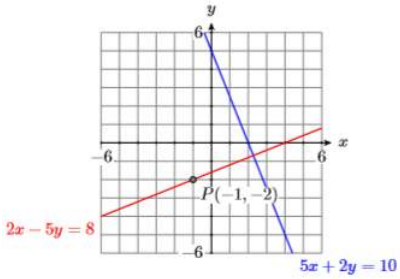
Answer



24) Find an equation of the line (in standard form) that passes through the point $P(-3, 2)$ that is parallel to the line $3x + 5y = -15$.

25) Find an equation of the line (in standard form) that passes through the point $P(-1, -2)$ that is perpendicular to the line $5x + 2y = 10$.

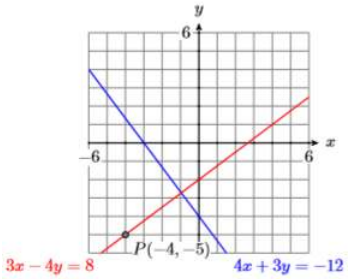
Answer



26) Find an equation of the line (in standard form) that passes through the point $P(-1, -2)$ that is perpendicular to the line $2x + 5y = -10$.

27) Find an equation of the line (in standard form) that passes through the point $P(-4, -5)$ that is perpendicular to the line $4x + 3y = -12$.

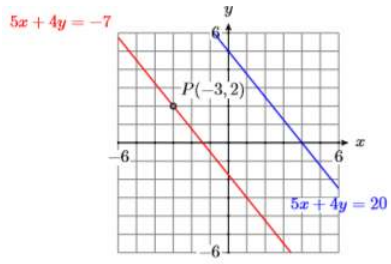
Answer



28) Find an equation of the line (in standard form) that passes through the point $P(-2,-3)$ that is perpendicular to the line $3x+2y=6$.

29) Find an equation of the line (in standard form) that passes through the point $P(-3,2)$ that is parallel to the line $5x+4y=20$.

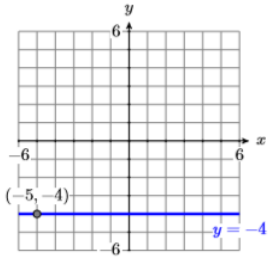
Answer



30) Find an equation of the line (in standard form) that passes through the point $P(-1,3)$ that is parallel to the line $3x+5y=-15$.

31) Sketch the equation of the horizontal line passing through the point $P(-5,-4)$ and label it with its equation.

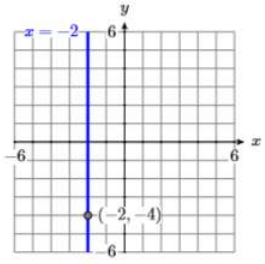
Answer



32) Sketch the equation of the vertical line passing through the point $P(-4,4)$ and label it with its equation.

33) Sketch the equation of the vertical line passing through the point $P(-2,-4)$ and label it with its equation.

Answer



34) Sketch the equation of the vertical line passing through the point $P(1,3)$ and label it with its equation.

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CHAPTER OVERVIEW

2: Solving Linear Equations, Formulas, and Inequalities

In this chapter, you will explore linear equations, develop a strategy for solving them, and relate them to real-world situations.

2.1: Prelude to Solving Linear Equations
2.2: Use a General Strategy to Solve Linear Equations
2.2E: Use a General Strategy to Solve Linear Equations (Exercises)
2.3: Use a Problem Solving Strategy
2.3E: Exercises
2.4: Solve a Formula for a Specific Variable
2.4E: Exercises
2.5: The Pythagorean Theorem
2.6: Solve Mixture and Uniform Motion Applications (optional practice)
2.6E: Exercises
2.7: Solve Applications with Linear Inequalities
2.7E: Exercises
2.8: Solve Linear Inequalities - Algebra Emphasis
2.8E: Exercises
2.9: Solve Compound Inequalities - Union, Intersection
2.9E: Exercises
2.10: Graphs of Linear Inequalities in Two Variables (optional challenge practice)
2.10E: Exercises
2.11: Solve Absolute Value Inequalities (optional challenge practice)
2.11E: Exercises

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2.1: Prelude to Solving Linear Equations

Imagine being a pilot, but not just any pilot—a drone pilot. Drones, or unmanned aerial vehicles, are devices that can be flown remotely. They contain sensors that can relay information to a command center where the pilot is located. Larger drones can also carry cargo. In the near future, several companies hope to use drones to deliver materials and piloting a drone will become an important career. Law enforcement and the military are using drones rather than send personnel into dangerous situations.



Figure 2.1.1: This drone is flying high in the sky while its pilot remains safely on the ground. (credit: “Unsplash” / Pixabay)

Building and piloting a drone requires the ability to program a set of actions, including taking off, turning, and landing. This, in turn, requires the use of linear equations. In this chapter, you will explore linear equations, develop a strategy for solving them, and relate them to real-world situations.

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2.2: Use a General Strategy to Solve Linear Equations

Summary

By the end of this section, you will be able to:

- Use the commutative and associative properties
- Use the properties of identity, inverse, and zero
- Simplify expressions using the Distributive Property

Before you get started, take this readiness quiz.

1. Simplify: $\frac{3}{2}(12x+20)$.
2. Simplify: $5-2(n+1)$.
3. Find the LCD of $\frac{5}{6}$ and $\frac{1}{4}$.

Solve Linear Equations Using a General Strategy

Solving an equation is like discovering the answer to a puzzle. The purpose in solving an equation is to find the value or values of the variable that makes it a true statement. Any value of the variable that makes the equation true is called a **solution** to the equation. It is the answer to the puzzle!

Solution OF AN EQUATION

A **solution** of an equation is a value of a variable that makes a true statement when substituted into the equation.

To determine whether a number is a solution to an equation, we substitute the value for the variable in the equation. If the resulting equation is a true statement, then the number is a solution of the equation.

DETERMINE WHETHER A NUMBER IS A Solution TO AN EQUATION

1. Substitute the number for the variable in the equation.
2. Simplify the expressions on both sides of the equation.
3. Determine whether the resulting equation is true.
   - If it is true, the number is a solution.
   - If it is not true, the number is not a solution.

Example 2.2.1

Determine whether the values are solutions to the equation: $5y+3=10y-4$.

a. $y=\frac{3}{5}$
b. $y=\frac{7}{5}$

Solution

Since a solution to an equation is a value of the variable that makes the equation true, begin by substituting the value of the solution for the variable.

a.

	$5y+3=10y-4$
Substitute $\frac{3}{5}$ for y	$5\left(\frac{3}{5}\right)+3\stackrel{?}{=}10\left(\frac{3}{5}\right)-4$
Multiply.	$3+3\stackrel{?}{=}6-4$
Simplify.	$6\neq 2$

Since $y=\frac{3}{5}$ does not result in a true equation, $y=\frac{3}{5}$ is not a solution to the equation $5y+3=10y-4$.

b.

	$5y+3=10y-4$
Substitute $\frac{7}{5}$ for y	$5\left(\frac{7}{5}\right)+3\stackrel{?}{=}10\left(\frac{7}{5}\right)-4$
Multiply.	$7+3\stackrel{?}{=}14-4$
Simplify.	$10=10\checkmark$

Since $y=\frac{7}{5}$ results in a true equation, $y=\frac{7}{5}$ is a solution to the equation $5y+3=10y-4$.

Exercise 2.2.1A

Determine whether the values are solutions to the equation: $9y+2=6y+3$.

a. $y=\frac{4}{3}$
b. $y=\frac{1}{3}$

Answer a

no

Answer b

yes

Exercise 2.2.1B

Determine whether the values are solutions to the equation: $4x-2=2x+1$.

a. $x=\frac{3}{2}$
b. $x=-\frac{1}{2}$

Answer a

yes

Answer b

no

There are many types of equations that we will learn to solve. In this section we will focus on a **linear equation.**

LINEAR EQUATION

A **linear equation** is an equation in one variable that can be written, where a and b are real numbers and $a\neq 0$, as:

$$ax+b=0 \tag{2.2.1}$$

To solve a linear equation it is a good idea to have an overall strategy that can be used to solve any linear equation. In the next example, we will give the steps of a general strategy for solving any linear equation. Simplifying each side of the equation as much as possible first makes the rest of the steps easier.

EXAMPLE 2.2.2

Solve: $7(n-3)-8=-15$

Answer

Step 1. Simplify each side of the equation as much as possible.	Use the Distributive Property. Notice that each side of the equation is now simplified as much as possible.	$7(n-3)-8=-15$ $7n-21-8=-15$ $7n-29=-15$
Step 2. Collect all variable terms on one side of the equation.	Nothing to do—all n's are on the left side.	
Step 3. Collect constant terms on the other side of the equation.	To get constants only on the right, add 29 to each side. Simplify.	$7n-29+29=-15+29$ $7n=14$
Step 4. Make the coefficient of the variable term equal to 1.	Divide each side by 7. Simplify.	$\frac{7n}{7}=\frac{14}{7}$ $n=2$
Step 5. Check the solution.	Let $n=2$ Subtract.	**Check:** $7(n-3)-8=-15$ $7(2-3)-8\stackrel{?}{=}-15$ $7(-1)-8\stackrel{?}{=}-15$ $-7-8\stackrel{?}{=}-15$ $-15=-15$ ✓

Solve: $2(m-4)+3=-1$.

Answer

$m=2$

? Exercise 2.2.2B

Solve: $5(a-3)+5=-10$.

Answer

$a=0$

These steps are summarized in the **General Strategy for Solving Linear Equations** below.

SOLVE LINEAR EQUATIONS USING A GENERAL STRATEGY

1. Simplify each side of the equation as much as possible.
   Use the Distributive Property to remove any parentheses.
   Combine like terms.
2. Collect all the variable terms on one side of the equation.
   Use the Addition or Subtraction Property of Equality.
3. Collect all the constant terms on the other side of the equation.
   Use the Addition or Subtraction Property of Equality.
4. Make the coefficient of the variable term equal to 1.
   Use the Multiplication or Division Property of Equality.
   State the solution to the equation.
5. Check the solution.
   Substitute the solution into the original equation to make sure the result is a true statement.

EXAMPLE 2.2.3

Solve: $\frac{2}{3}(3m-6)=5-m$.

Answer

	$\frac{2}{3}(3m-6)=5-m$
Distribute.	$2m-4=5-m$
Add m to both sides to get the variables only on the left.	$2m+m-4=5-m+m$
Simplify.	$3m-4=5$
Add 4 to both sides to get constants only on the right.	$3m-4+4=5+4$
Simplify.	$3m=9$
Divide both sides by three.	$\frac{3m}{3}=\frac{9}{3}$
Simplify.	$m=3$

Check:	$\frac{2}{3}(3m-6)=5-m$
Let $m=3$.	$\frac{2}{3}(3\cdot 3-6)\stackrel{?}{=}5-3$
	$\frac{2}{3}(9-6)\stackrel{?}{=}2$
	$\frac{2}{3}(3)\stackrel{?}{=}2$
	$2=2$ ✓

Exercise 2.2.3A

Solve: $\frac{1}{3}(6u+3)=7-u$.

Answer

$u=2$

Exercise 2.2.3B

Solve: $\frac{2}{3}(9x-12)=8+2x$.

Answer

$x=4$

We can solve equations by getting all the **variable** terms to either side of the **equal sign**. By collecting the variable terms on the side where the **coefficient** of the variable is larger, we avoid working with some negatives. This will be a good strategy when we solve inequalities later in this chapter. It also helps us prevent errors with negatives.

EXAMPLE 2.2.4

Solve: $4(x-1)-2=5(2x+3)+6$.

Answer

	$4(x-1)-2=5(2x+3)+6$

Distribute.	$4x-4-2=10x+15+6$
Combine like terms.	$4x-6=10x+21$
Subtract $4x$ from each side to get the variables only on the right since $10>4$.	$4x-4x-6=10x-4x+21$
Simplify.	$-6=6x+21$
Subtract 21 from each side to get the constants on left.	$-6-21=6x+21-21$
Simplify.	$-27=6x$
Divide both sides by 6.	$\frac{-27}{6}=\frac{6x}{6}$
Simplify.	$-\frac{9}{2}=x$
Check:	$4(x-1)-2=5(2x+3)+6 \quad \backslash)$
Let $x=-92$.	$4\left(-\frac{9}{2}-1\right)-2\stackrel{?}{=}5\left(2\left(-\frac{9}{2}\right)+3\right)+6$
	$4\left(-\frac{11}{2}\right)-2\stackrel{?}{=}5(-9+3)+6$
	$-22-2\stackrel{?}{=}5(-6)+6$
	$-24\stackrel{?}{=}-30+6$
	$-24=-24\checkmark$

Exercise 2.2.4A

Solve: $6(p-3)-7=5(4p+3)-12$.

Answer

$p=-2$

Exercise 2.2.4B

Solve: $8(q+1)-5=3(2q-4)-1$.

Answer

$q=-8$

EXAMPLE 2.2.5

Solve: $10[3-8(2s-5)]=15(40-5s)$.

Answer

	$10[3-8(2s-5)]=15(40-5s)$
Simplify from the innermost parentheses first.	$10[3-16s+40]=15(40-5s)$
Combine like terms in the brackets.	$10[43-16s]=15(40-5s)$
Distribute.	$430-160s=600-75s$
Add $160s$ to both sides to get the $160s$ to both sides to get the variables to the right.	$430-160s+160s=600-75s+160s$
Simplify.	$430=600+85s$

Subtract 600 from both sides to get the constants to the left.		$430 - 600 = 600 + 85s - 600$
Simplify.		$-170 = 85s$
Divide both sides by 85.		$\frac{-170}{85} = \frac{85s}{85}$
Simplify.		$-2 = s$, so $s = -2$
Check:	$10[3 - 8(2s - 5)] = 15(40 - 5s)$	
Let $s = -2$.	$10[3 - 8(2(-2) - 5)] \stackrel{?}{=} 15(40 - 5(-2))$	
	$10[3 - 8(-4 - 5)] \stackrel{?}{=} 15(40 + 10)$	
	$10[3 - 8(-9)] \stackrel{?}{=} 15(50)$	
	$10(3 + 72) \stackrel{?}{=} 750$	
	$10(75) \stackrel{?}{=} 750$	
	$750 = 750$ ✓	

Exercise 2.2.5A

Solve: $6[4 - 2(7y - 1)] = 8(13 - 8y)$.

Answer

$y = -\frac{17}{5}$

Exercise 2.2.5B

Solve: $12[1 - 5(4z - 1)] = 3(24 + 11z)$.

Answer

$z = 0$

Classify Equations

Whether or not an equation is true depends on the value of the variable. The equation $7x + 8 = -13$ is true when we replace the variable, *x*, with the value -3, but not true when we replace *x* with any other value. An equation like this is called a **conditional equation**. All the equations we have solved so far are conditional equations.

CONDITIONAL EQUATION

An equation that is true for one or more values of the variable and false for all other values of the variable is a **conditional equation**.

Now let's consider the equation $7y + 14 = 7(y + 2)$. Do you recognize that the left side and the right side are equivalent? Let's see what happens when we solve for *y*.

Solve:

	$7y + 14 = 7(y + 2)$
Distribute.	$7y + 14 = 7y + 14$
Subtract $7y$ to each side to get the y's to one side.	$7y - 7y + 14 = 7y - 7y + 14$

Simplify—the y's are eliminated.	$14 = 14$
	But $14 = 14$ is true.

This means that the equation $7y + 14 = 7(y + 2)$ is true for any value of y. We say the solution to the equation is all of the real numbers. An equation that is true for any value of the variable is called an **identity**.

IDENTITY

An equation that is true for any value of the variable is called an **identity**.

The solution of an identity is valid for all real numbers.

What happens when we solve the equation $-8z = -8z + 9$?

Solve:

	$-8z = -8z + 9$
Add $8z$ to both sides to leave the constant alone on the right.	$-8z + 8z = -8z + 8z + 9$
Simplify—the z's are eliminated.	$0 \neq 9$
	But $0 \neq 9$.

Solving the equation $-8z = -8z + 9$ led to the false statement $0 = 9$. The equation $-8z = -8z + 9$ will not be true for any value of z. It has no solution. An equation that has no solution, or that is false for all values of the variable, is called a contradiction.

CONTRADICTION

An equation that is false for all values of the variable is called a **contradiction**.

A contradiction has no solution.

The next few examples will ask us to classify an equation as conditional, an identity, or as a contradiction.

? EXAMPLE 2.2.6

Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution: $6(2n - 1) + 3 = 2n - 8 + 5(2n + 1)$.

Answer

	$6(2n - 1) + 3 = 2n - 8 + 5(2n + 1)$
Distribute.	$12n - 6 + 3 = 2n - 8 + 10n + 5$
Combine like terms.	$12n - 3 = 12n - 3$
Subtract $12n$ from each side to get the n's to one side.	$12n - 12n - 3 = 12n - 12n - 3$
Simplify.	$-3 = -3$
This is a true statement.	The equation is an identity.
	The solution is all real numbers.

Exercise 2.2.6A

Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution: $4+9(3x-7)=-42x-13+23(3x-2)$.

Answer

identity; all real numbers

Exercise 2.2.6B

Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution: $8(1-3x)+15(2x+7)=2(x+50)+4(x+3)+1$.

Answer

identity; all real numbers

EXAMPLE 2.2.7

Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution: $8+3(a-4)=0$.

Answer

	$8+3(a-4)=0$
Distribute.	$8+3a-12=0$
Combine like terms.	$3a-4=0$
Add 4 to both sides.	$3a-4+4=0+4$
Simplify.	$3a=4$
Divide.	$\frac{3a}{3}=\frac{4}{3}$
Simplify.	$a=\frac{4}{3}$
The equation is true when $a=\frac{4}{3}$.	This is a conditional equation.
	The solution is $a=\frac{4}{3}$.

Exercise 2.2.7A

Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution: $11(q+3)-5=19$.

Answer

conditional equation; $q=-\frac{9}{11}$

Exercise 2.2.7B

Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution: $6+14(k-8)=95$.

Answer

conditional equation; $k=\frac{201}{14}$

? EXAMPLE 2.2.8

Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution: $5m+3(9+3m)=2(7m-11)$.

Answer

	$5m+3(9+3m)=2(7m-11)$
Distribute.	$5m+27+9m=14m-22$
Combine like terms.	$14m+27=14m-22$
Subtract $14m$ from both sides.	$14m+27-14m=14m-22-14m$
Simplify.	$27\neq -22$
But $27\neq -22$.	The equation is a contradiction.
	It has no solution.

Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution: $12c+5(5+3c)=3(9c-4)$.

Answer

contradiction; no solution

? Exercise 2.2.8B

Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution: $4(7d+18)=13(3d-2)-11d$.

Answer

contradiction; no solution

We summarize the methods for classifying equations in the table.

Type of equation	What happens when you solve it?	Solution
Conditional Equation	True for one or more values of the variables and false for all other values	One or more values
Identity	True for any value of the variable	All real numbers
Contradiction	False for all values of the variable	No solution

Solve Equations with Fraction or Decimal Coefficients

We could use the General Strategy to solve the next example. This method would work fine, but many students do not feel very confident when they see all those fractions. So, we are going to show an alternate method to solve equations with fractions. This alternate method eliminates the fractions.

We will apply the Multiplication Property of Equality and multiply both sides of an equation by the **least common denominator** (LCD) of all the fractions in the equation. The result of this operation will be a new equation, equivalent to the first, but without fractions. This process is called *clearing* the equation of fractions.

To clear an equation of decimals, we think of all the decimals in their fraction form and then find the LCD of those denominators.

Exercise 2.2.9A

Solve: $\frac{1}{12}x + \frac{5}{6} = \frac{3}{4}$.

Answer

Step 1. Find the least common denominator of *all* the fractions and decimals in the equation.	What is the LCD of $\frac{1}{12}$, $\frac{5}{6}$, and $\frac{3}{4}$?	$\frac{1}{12}x + \frac{5}{6} = \frac{3}{4}$ LCD = 12
Step 2. Multiply both sides of the equation by that LCD. This clears the fractions and decimals.	Multiply both sides of the equation by the LCD, 12. Use the Distributive Property. Simplify—and notice, no more fractions!	$12\left(\frac{1}{12}x + \frac{5}{6}\right) = 12\left(\frac{3}{4}\right)$ $12 \cdot \frac{1}{12}x + 12 \cdot \frac{5}{6} = 12 \cdot \frac{3}{4}$ $x + 10 = 9$
Step 3. Solve using the General Strategy for Solving Linear Equations.	To isolate the variable term, subtract 10. Simplify.	$x + 10 - 10 = 9 - 10$ $x = -1$ **Check:** $\frac{1}{12}x + \frac{5}{6} = \frac{3}{4}$ $\frac{1}{12}(-1) + \frac{5}{6} \stackrel{?}{=} \frac{3}{4}$ $-\frac{1}{12} + \frac{5}{6} \stackrel{?}{=} \frac{3}{4}$ $-\frac{1}{12} + \frac{10}{12} \stackrel{?}{=} \frac{9}{12}$ $\frac{9}{12} = \frac{9}{12}$ ✓

Exercise 2.2.9B

Solve: $\frac{1}{4}x + \frac{1}{2} = \frac{5}{8}$.

Answer

$x = \frac{1}{2}$

Exercise 2.2.9C

Solve: $\frac{1}{8}x + \frac{1}{2} = \frac{1}{4}$.

Answer

$x = -2$

Notice in the previous example, once we cleared the equation of fractions, the equation was like those we solved earlier in this chapter. We changed the problem to one we already knew how to solve. We then used the **General Strategy for Solving Linear Equations**.

SOLVE EQUATIONS WITH FRACTION OR DECIMAL COEFFICIENTS.

1. Find the least common denominator (LCD) of *all* the fractions and decimals (in fraction form) in the equation.
2. Multiply both sides of the equation by that LCD. This clears the fractions and decimals.
3. Solve using the General Strategy for Solving Linear Equations.

EXAMPLE 2.2.10

Solve: $5 = \frac{1}{2}y + \frac{2}{3}y - \frac{3}{4}y$.

Answer

We want to clear the fractions by multiplying both sides of the equation by the LCD of all the fractions in the equation.

Find the LCD of all fractions in the equation.	$5 = \frac{1}{2}y + \frac{2}{3}y - \frac{3}{4}y$
The LCD is 12.	
Multiply both sides of the equation by 12.	$12(5) = 12 \cdot \left(\frac{1}{2}y + \frac{2}{3}y - \frac{3}{4}y\right)$
Distribute.	$12(5) = 12 \cdot \frac{1}{2}y + 12 \cdot \frac{2}{3}y - 12 \cdot \frac{3}{4}y$
Simplify—notice, no more fractions.	$60 = 6y + 8y - 9y$
Combine like terms.	$60 = 5y$
Divide by five.	$\frac{60}{5} = \frac{5y}{5}$
Simplify.	$12 = y$
Check:	$5 = \frac{1}{2}y + \frac{2}{3}y - \frac{3}{4}y$
Let $y = 12$.	$5 \stackrel{?}{=} \frac{1}{2}(12) + \frac{2}{3}(12) - \frac{3}{4}(12)$
	$5 \stackrel{?}{=} 6 + 8 - 9$
	$5 = 5$ ✓

Exercise 2.2.10A

Solve: $7 = \frac{1}{2}x + \frac{3}{4}x - \frac{2}{3}x$.

Answer

$x = 12$

Exercise 2.2.10B

Solve: $-1 = \frac{1}{2}u + \frac{1}{4}u - \frac{2}{3}u$.

Answer

$u = -12$

In the next example, we'll distribute before we clear the fractions.

EXAMPLE 2.2.11

Solve: $\frac{1}{2}(y-5) = \frac{1}{4}(y-1)$.

Answer

	$\frac{1}{2}(y-5) = \frac{1}{4}(y-1)$
Distribute.	$\frac{1}{2} \cdot y - \frac{1}{2} \cdot 5 = \frac{1}{4} \cdot y - \frac{1}{4} \cdot 1$

Simplify.	$\frac{1}{2}y-\frac{5}{2}=\frac{1}{4}y-\frac{1}{4}$
Multiply by the LCD, four.	$4\left(\frac{1}{2}y-\frac{5}{2}\right)=4\left(\frac{1}{4}y-\frac{1}{4}\right)$
Distribute.	$4\cdot\frac{1}{2}y-4\cdot\frac{5}{2}=4\cdot\frac{1}{4}y-4\cdot\frac{1}{4}$
Simplify.	$2y-10=y-1$
Collect the variables to the left.	$2y-y-10=y-y-1$
Simplify.	$y-10=-1$
Collect the constants to the right.	$y-10+10=-1+10$
Simplify.	$y=9$
An alternate way to solve this equation is to clear the fractions without distributing first. If you multiply the factors correctly, this method will be easier.	
	$\frac{1}{2}(y-5)=\frac{1}{4}(y-1)$
Multiply by the LCD, 4.	$4\cdot\frac{1}{2}(y-5)=4\cdot\frac{1}{4}(y-1)$
Multiply four times the fractions.	$2(y-5)=1(y-1)$
Distribute.	$2y-10=y-1$
Collect the variables to the left.	$2y-y-10=y-y-1$
Simplify.	$y-10=-1$
Collect the constants to the right.	$y-10+10=-1+10$
Simplify.	$y=9$
Check:	$\frac{1}{2}(y-5)=\frac{1}{4}(y-1)$
Let $y=9$.	$\frac{1}{2}(9-5)\stackrel{?}{=}\frac{1}{4}(9-1)$
Finish the check on your own.	

? Exercise 2.2.11A

Solve: $\frac{1}{5}(n+3)=\frac{1}{4}(n+2)$.

Answer

$n=2$

? Exercise 2.2.11B

Solve: $\frac{1}{2}(m-3)=\frac{1}{4}(m-7)$.

Answer

$m=-1$

When you multiply both sides of an equation by the LCD of the fractions, make sure you multiply each term by the LCD—even if it does not contain a fraction.

EXAMPLE 2.2.12

Solve: $\frac{4q+3}{2}+6=\frac{3q+5}{4}$

Answer

	$\frac{4q+3}{2}+6=\frac{3q+5}{4}$
Multiply both sides by the LCD, 4.	$4\left(\frac{4q+3}{2}+6\right)=4\left(\frac{3q+5}{4}\right)$
Distribute.	$4\left(\cdot\frac{4q+3}{2}\right)+4\cdot 6=4\cdot\left(\frac{3q+5}{4}\right)$
Simplify.	$2(4q+3)+24=3q+5$
	$8q+6+24=3q+5$
	$8q+30=3q+5$
Collect the variables to the left.	$8q-3q+30=3q-3q+5$
Simplify.	$5q+30=5$
Collect the constants to the right.	$5q+30-30=5-30$
Simplify.	$5q=-25$
Divide both sides by five.	$\frac{5q}{5}=\frac{-25}{5}$
Simplify.	$q=-5$
Check: $\frac{4q+3}{2}+6=\frac{3q+5}{4}$	
Let $q=-5$.	$\frac{4(-5)+3}{2}+6\stackrel{?}{=}\frac{3(-5)+5}{4}$
Finish the check on your own.	

Solve: $\frac{3r+5}{6}+1=\frac{4r+3}{3}$.

Answer

$r=3$

Exercise 2.2.12B

Solve: $\frac{2s+3}{2}+1=\frac{3s+2}{4}$.

Answer

$s=-8$

Some equations have decimals in them. This kind of equation may occur when we solve problems dealing with money or percentages. But decimals can also be expressed as fractions. For example, $0.7=\frac{7}{10}$ and $0.29=\frac{29}{100}$. So, with an equation with decimals, we can use the same method we used to clear fractions—multiply both sides of the equation by the **least common denominator**.

The next example uses an equation that is typical of the ones we will see in the money applications in a later section. Notice that we will clear all decimals by multiplying by the LCD of their fraction form.

Solve: $0.25x+0.05(x+3)=2.85$.

Answer

Look at the decimals and think of the equivalent fractions:

$$0.25 = \frac{25}{100}, \qquad 0.05 = \frac{5}{100}, \qquad 2.85 = 2\frac{85}{100}. \tag{2.2.2}$$

Notice, the LCD is 100. By multiplying by the LCD we will clear the decimals from the equation.

	$0.25x + 0.05(x + 3) = 2.85$
Distribute first.	$0.25x + 0.05x + 0.15 = 2.85$
Combine like terms.	$0.30x + 0.15 = 2.85$
To clear decimals, multiply by 100.	$100(0.30x + 0.15) = 100(2.85)$
Distribute.	$30x + 15 = 285$
Subtract 15 from both sides.	$30x + 15 - 15 = 285 - 15$
Simplify.	$30x = 270$
Divide by 30.	$\frac{30x}{30} = \frac{270}{30}$
Simplify.	$x = 9$
Check it yourself by substituting $x = 9$ into the original equation.	

? Exercise 2.2.13A

Solve: $0.25n + 0.05(n + 5) = 2.95$.

Answer

$n = 9$

? Exercise 2.2.13B

Solve: $0.10d + 0.05(d - 5) = 2.15$.

Answer

$d = 16$

Key Concepts

- **How to determine whether a number is a solution to an equation**
  1. Substitute the number in for the variable in the equation.
  2. Simplify the expressions on both sides of the equation.
  3. Determine whether the resulting equation is true.

     If it is true, the number is a solution.

     If it is not true, the number is not a solution.
- **How to Solve Linear Equations Using a General Strategy**
  1. Simplify each side of the equation as much as possible.

     Use the Distributive Property to remove any parentheses.

     Combine like terms.
  2. Collect all the variable terms on one side of the equation.

Use the Addition or Subtraction Property of Equality.

3. Collect all the constant terms on the other side of the equation.

   Use the Addition or Subtraction Property of Equality.

4. Make the coefficient of the variable term equal to 1.

   Use the Multiplication or Division Property of Equality.

   State the solution to the equation.

5. Check the solution.

   Substitute the solution into the original equation to make sure the result is a true statement.

- **How to Solve Equations with Fraction or Decimal Coefficients**
  1. Find the least common denominator (LCD) of *all* the fractions and decimals (in fraction form) in the equation.
  2. Multiply both sides of the equation by that LCD. This clears the fractions and decimals.
  3. Solve using the General Strategy for Solving Linear Equations.

Glossary

conditional equation

An equation that is true for one or more values of the variable and false for all other values of the variable is a conditional equation.

contradiction

An equation that is false for all values of the variable is called a contradiction. A contradiction has no solution.

identity

An equation that is true for any value of the variable is called an Identity. The solution of an identity is all real numbers.

linear equation

A linear equation is an equation in one variable that can be written, where a and b are real numbers and $a \neq 0$, as $ax + b = 0$.

solution of an equation

A solution of an equation is a value of a variable that makes a true statement when substituted into the equation.

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2.2E: Use a General Strategy to Solve Linear Equations (Exercises)

Practice Makes Perfect

Solve Equations Using the General Strategy

In the following exercises, determine whether the given values are solutions to the equation.

1. $6y+10=12y$

a. $y=\frac{5}{3}$

b. $y=-\frac{1}{2}$

Answer

a. yes
b. no

2. $4x+9=8x$

a. $x=-\frac{7}{8}$

b. $x=\frac{9}{4}$

3. $8u-1=6u$

a. $u=-\frac{1}{2}$

b. $u=\frac{1}{2}$

Answer

a. no
b. yes

4. $9v-2=3v$

a. $v=-\frac{1}{3}$

b. $v=\frac{1}{3}$

In the following exercises, solve each linear equation.

5. $15(y-9)=-60$

Answer

$y=5$

6. $-16(3n+4)=32$

7. $-(w-12)=30$

Answer

$w=-18$

8. $-(t-19)=28$

9. $51+5(4-q)=56$

Answer

$q=3$

10. $-6+6(5-k)=15$

11. $3(10-2x)+54=0$

Answer

$x=14$

12. $-2(11-7x)+54=4$

13. $\frac{2}{3}(9c-3)=22$

Answer

$c=4$

14. $\frac{3}{5}(10x-5)=27$

15. $\frac{1}{5}(15c+10)=c+7$

Answer

$c=\frac{5}{2}$

16. $\frac{1}{4}(20d+12)=d+7$

17. $3(4n-1)-2=8n+3$

Answer

$n=2$

18. $9(2m-3)-8=4m+7$

19. $12+2(5-3y)=-9(y-1)-2$

Answer

$y=-5$

20. $-15+4(2-5y)=-7(y-4)+4$

21. $5+6(3s-5)=-3+2(8s-1)$

Answer

$s=10$

22. $-12+8(x-5)=-4+3(5x-2)$

23. $4(p-4)-(p+7)=5(p-3)$

Answer

$p=-4$

24. $3(a-2)-(a+6)=4(a-1)$

25. $4[5-8(4c-3)]=12(1-13c)-8$

Answer

$c=-4$

26. $5[9-2(6d-1)]=11(4-10d)-139$

27. $3[-9+8(4h-3)]=2(5-12h)-19$

Answer

$h=\frac{3}{4}$

28. $3[-14+2(15k-6)]=8(3-5k)-24$

29. $5[2(m+4)+8(m-7)]=2[3(5+m)-(21-3m)]$

Answer

$m=6$

30. $10[5(n+1)+4(n-1)]=11[7(5+n)-(25-3n)]$

Classify Equations

In the following exercises, classify each equation as a conditional equation, an identity, or a contradiction and then state the solution.

31. $23z+19=3(5z-9)+8z+46$

Answer

identity; all real numbers

32. $15y+32=2(10y-7)-5y+46$

33. $18(5j-1)+29=47$

Answer

conditional equation; $j=\frac{2}{5}$

34. $24(3d-4)+100=52$

35. $22(3m-4)=8(2m+9)$

Answer

conditional equation; $m = 165$

36. $30(2n-1) = 5(10n+8)$

37. $7v+42 = 11(3v+8)-2(13v-1)$

Answer

contradiction; no solution

38. $18u-51 = 9(4u+5)-6(3u-10)$

39. $45(3y-2) = 9(15y-6)$

Answer

contradiction; no solution

40. $60(2x-1) = 15(8x+5)$

41. $9(14d+9)+4d = 13(10d+6)+3$

Answer

identity; all real numbers

42. $11(8c+5)-8c = 2(40c+25)+5$

Solve Equations with Fraction or Decimal Coefficients

In the following exercises, solve each equation with fraction coefficients.

43. $\frac{1}{4}x - \frac{1}{2} = -\frac{3}{4}$

Answer

$x = -1$

44. $\frac{3}{4}x - \frac{1}{2} = \frac{1}{4}$

45. $\frac{5}{6}y - \frac{2}{3} = -\frac{3}{2}$

Answer

$y = -1$

46. $\frac{5}{6}y - \frac{1}{3} = -\frac{7}{6}$

47. $\frac{1}{2}a + \frac{3}{8} = \frac{3}{4}$

Answer

$a = \frac{3}{4}$

48. $\frac{5}{8}b + \frac{1}{2} = -\frac{3}{4}$

49. $2 = \frac{1}{3}x - \frac{1}{2}x + \frac{2}{3}x$

Answer

$x = 4$

50. $2 = \frac{3}{5}x - \frac{1}{3}x + \frac{2}{5}x$

51. $\frac{1}{3}w + \frac{5}{4} = w - \frac{1}{4}$

Answer

$w = \frac{9}{4}$

52. $\frac{1}{2}a - \frac{1}{4} = \frac{1}{6}a + \frac{1}{12}$

53. $\frac{1}{3}b + \frac{1}{5} = \frac{2}{5}b - \frac{3}{5}$

Answer

$b = 12$

54. $\frac{1}{3}x + \frac{2}{5} = \frac{1}{5}x - \frac{2}{5}$

55. $\frac{1}{4}(p-7) = \frac{1}{3}(p+5)$

Answer

$p = -41$

56. $\frac{1}{5}(q+3) = \frac{1}{2}(q-3)$

57. $\frac{1}{2}(x+4) = \frac{3}{4}$

Answer

$x = -\frac{5}{2}$

58. $\frac{1}{3}(x+5) = \frac{5}{6}$

59. $\frac{4n+8}{4} = \frac{n}{3}$

Answer

$n = -3$

60. $\frac{3p+6}{3} = \frac{p}{2}$

61. $\frac{3x+4}{2} + 1 = \frac{5x+10}{8}$

Answer

$x = -2$

62. $\frac{10y-2}{3}+3=\frac{10y+1}{9}$

63. $\frac{7u-1}{4}-1=\frac{4u+8}{5}$

Answer

$u=3$

64. $\frac{3v-6}{2}+5=\frac{11v-4}{5}$

In the following exercises, solve each equation with decimal coefficients.

65. $0.4x+0.6=0.5x-1.2$

Answer

$x=18$

66. $0.7x+0.4=0.6x+2.4$

67. $0.9x-1.25=0.75x+1.75$

Answer

$x=20$

68. $1.2x-0.91=0.8x+2.29$

69. $0.05n+0.10(n+8)=2.15$

Answer

$n=9$

70. $0.05n+0.10(n+7)=3.55$

71. $0.10d+0.25(d+5)=4.05$

Answer

$d=8$

72. $0.10d+0.25(d+7)=5.25$

Everyday Math

73. **Fencing** Micah has 74 feet of fencing to make a dog run in his yard. He wants the length to be 2.5 feet more than the width. Find the length, L, by solving the equation $2L+2(L-2.5)=74$.

Answer

$L=19.75$ feet

74. Stamps Paula bought $22.82 worth of 49-cent stamps and 21-cent stamps. The number of 21-cent stamps was eight less than the number of 49-cent stamps. Solve the equation $0.49s+0.21(s-8)=22.82$ for s, to find the number of 49-cent stamps Paula bought.

Writing Exercises

75. Using your own words, list the steps in the general strategy for solving linear equations.

Answer

Answers will vary.

76. Explain why you should simplify both sides of an equation as much as possible before collecting the variable terms to one side and the constant terms to the other side.

77. What is the first step you take when solving the equation $3-7(y-4)=38$? Why is this your first step?

Answer

Answers will vary.

78. If an equation has several fractions, how does multiplying both sides by the LCD make it easier to solve?

79. If an equation has fractions only on one side, why do you have to multiply both sides of the equation by the LCD?

Answer

Answers will vary.

80. For the equation $0.35x+2.1=3.85$, how do you clear the decimal?

Self Check

a. After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
solve linear equations using a general strategy.			
classify equations.			
solve equations with fraction or decimal coefficients.			

b. If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them. What did you do to become confident of your ability to do these things? Be specific.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math every topic builds upon previous work. It is important to make sure you have a strong foundation before you move on. Who can you ask for help? Your fellow classmates and instructor are good resources. Is there a place on campus where math tutors are available? Can your study skills be improved?

…no - I don't get it! This is a warning sign and you must not ignore it. You should get help right away or you will quickly be overwhelmed. See your instructor as soon as you can to discuss your situation. Together you can come up with a plan to get you the help you need.

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2.3: Use a Problem Solving Strategy

Learning Objectives

By the end of this section, you will be able to:

- Use a problem solving strategy for word problems
- Solve number word problems
- Solve percent applications
- Solve simple interest applications

Before you get started, take this readiness quiz.

1. Translate "six less than twice x" into an algebraic expression.
   If you missed this problem, review [link].
2. Convert 4.5% to a decimal.
   If you missed this problem, review [link].
3. Convert 0.6 to a percent.
   If you missed this problem, review [link].

Have you ever had any negative experiences in the past with word problems? When we feel we have no control, and continue repeating negative thoughts, we set up barriers to success. Realize that your negative experiences with word problems are in your past. To move forward you need to calm your fears and change your negative feelings.

Start with a fresh slate and begin to think positive thoughts. Repeating some of the following statements may be helpful to turn your thoughts positive. Thinking positive thoughts is a first step towards success.

- I think I can! I think I can!
- While word problems were hard in the past, I think I can try them now.
- I am better prepared now—I think I will begin to understand word problems.
- I am able to solve equations because I practiced many problems and I got help when I needed it—I can try that with word problems.
- It may take time, but I can begin to solve word problems.
- You are now well prepared and you are ready to succeed. If you take control and believe you can be successful, you will be able to master word problems.

Use a Problem Solving Strategy for Word Problems

Now that we can solve equations, we are ready to apply our new skills to word problems. We will develop a strategy we can use to solve any word problem successfully.

EXAMPLE 2.3.1

Normal yearly snowfall at the local ski resort is 12 inches more than twice the amount it received last season. The normal yearly snowfall is 62 inches. What was the snowfall last season at the ski resort?

Solution:

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	What was the snowfall last season?
Step 3. Name what we are looking for and choose a variable to represent it.	Let $s =$ the snowfall last season.
Step 4. Translate. Restate the problem in one sentence with all the important information.	The normal snowfall was twelve more than twice the amount last year

Translate into an equation.	$62 = 2s + 12$
Step 5. Solve the equation.	$62 = 2s + 12$
Subtract 12 from each side.	$62 - 12 = 2s + 12 - 12$
Simplify.	$50 = 2s$
Divide each side by two.	$\frac{50}{2} = \frac{2s}{2}$
Simplify.	$25 = s$
Step 6. Check: First, is our answer reasonable? Yes, having 25 inches of snow seems OK. The problem says the normal snowfall is twelve inches more than twice the number of last season. Twice 25 is 50 and 12 more than that is 62.	
Step 7. Answer the question.	The snowfall last season was 25 inches.

Try It! 2.3.1

Guillermo bought textbooks and notebooks at the bookstore. The number of textbooks was three more than twice the number of notebooks. He bought seven textbooks. How many notebooks did he buy?

Answer

He bought two notebooks

Try It! 2.3.2

Gerry worked Sudoku puzzles and crossword puzzles this week. The number of Sudoku puzzles he completed is eight more than twice the number of crossword puzzles. He completed 22 Sudoku puzzles. How many crossword puzzles did he do?

Answer

He did seven crossword puzzles

We summarize an effective strategy for problem solving.

PROBLEM SOLVING STRATEGY FOR WORD PROBLEMS

1. **Read** the problem. Make sure all the words and ideas are understood.
2. **Identify** what you are looking for.
3. **Name** what you are looking for. Choose a variable to represent that quantity.
4. **Translate** into an equation. It may be helpful to restate the problem in one sentence with all the important information. Then, translate the English sentence into an algebra equation.
5. **Solve** the equation using proper algebra techniques.
6. **Check** the answer in the problem to make sure it makes sense.
7. **Answer** the question with a complete sentence.

Solve Number Word Problems

We will now apply the problem solving strategy to “number word problems.” Number word problems give some clues about one or more numbers and we use these clues to write an equation. Number word problems provide good practice for using the Problem Solving Strategy.

EXAMPLE 2.3.2

The sum of seven times a number and eight is thirty-six. Find the number.

Solution:

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	the number
Step 3. Name what you are looking for and choose a variable to represent it.	Let n = the number.
Step 4. Translate: Restate the problem as one sentence. Translate into an equation.	The sum of seven times a number and 8 is 36 $7n+8 = 36$
Step 5. Solve the equation. Subtract eight from each side and simplify. Divide each side by seven and simplify.	$7n+8=36$ $7n=28$ $n=4$
Step 6. Check. Is the sum of seven times four plus eight equal to 36?	$7\cdot 4+8 \stackrel{?}{=} 36$ $28+8 \stackrel{?}{=} 36$ $36=36\checkmark$
Step 7. Answer the question.	The number is 4.

Did you notice that we left out some of the steps as we solved this equation? If you're not yet ready to leave out these steps, write down as many as you need.

Try It! 2.3.3

The sum of four times a number and two is fourteen. Find the number.

Answer

3

Try It! 2.3.4

The sum of three times a number and seven is twenty-five. Find the number.

Answer

6

Some number word problems ask us to find two or more numbers. It may be tempting to name them all with different variables, but so far, we have only solved equations with one variable. In order to avoid using more than one variable, we will define the numbers in terms of the same variable. Be sure to read the problem carefully to discover how all the numbers relate to each other.

EXAMPLE 2.3.3

The sum of two numbers is negative fifteen. One number is nine less than the other. Find the numbers.

Solution:

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	two numbers
Step 3. Name what you are looking for by choosing a variable to represent the first number. “One number is nine less than the other.”	Let $n = 1^{\text{st}}$ number. $n - 9 = 2^{\text{nd}}$ number
Step 4. Translate. Write as one sentence. Translate into an equation.	The sum of two numbers is negative fifteen. 1st number + 2nd number is negative fifteen $n + n - 9 = -15$
Step 5. Solve the equation. Combine like terms. Add nine to each side and simplify. Simplify.	$n + n - 9 = -15$ $2n = -6$ $-3 - 9$ -12
Step 6. Check. Is -12 nine less than -3? $-3 - 9 \stackrel{?}{=} -12$ $-12 = -12$✓ Is their sum -15? $-3 + (-12) \stackrel{?}{=} -15$ $-15 = -15$✓	
Step 7. Answer the question.	The numbers are -3 and -12.

? Try It! 2.3.5

The sum of two numbers is negative twenty-three. One number is seven less than the other. Find the numbers.

Answer

$-15, -8$

? Try It! 2.3.6

The sum of two numbers is negative eighteen. One number is forty more than the other. Find the numbers.

Answer

$-29, 11$

Some number problems involve **consecutive integers**. Consecutive integers are integers that immediately follow each other. Examples of consecutive integers are:

$$\begin{array}{rrrr} 1, & 2, & 3, & 4 \\ -10, & -9, & -8, & -7 \\ 150, & 151, & 152, & 153 \end{array}$$

Notice that each number is one more than the number preceding it. Therefore, if we define the first integer as n, the next consecutive integer is $n+1$. The one after that is one more than $n+1$, so it is $n+1+1$, which is $n+2$.

$$\begin{array}{ll} n & 1^{\text{st}} \text{integer} \\ n+1 & 2^{\text{nd}} \text{consecutive integer} \\ n+2 & 3^{\text{rd}} \text{consecutive integer} \quad \text{etc.} \end{array}$$

We will use this notation to represent consecutive integers in the next example.

EXAMPLE 2.3.4

Find three consecutive integers whose sum is -54.

Solution:

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	three consecutive integers
Step 3. Name each of the three numbers	Let $n = 1^{\text{st}}$integer. $n+1 = 2^{\text{nd}}$consecutive integer $n+2 = 3^{\text{rd}}$consecutive integer
Step 4. Translate. Restate as one sentence. Translate into an equation.	The sum of the three integers is -54. $n+n+1+n+2=-54$
Step 5. Solve the equation. Combine like terms. Subtract three from each side. Divide each side by three.	$n+n+1+n+2=-54$ $3n+3=-54$ $3n=-57$ $n=-19$ 1st integer $n+1$ 2nd integer -18 $-19+2$ -17
Step 6. Check. $-19+(-18)+(-17) = -54$ $-54=-54$✓	
Step 7. Answer the question.	The three consecutive integers are −17,−18, and −19.

Try It! 2.3.7

Find three consecutive integers whose sum is -96.

Answer

$-33, -32, -31$

Try It! 2.3.8

Find three consecutive integers whose sum is -36.

Answer

$-13, -12, -11$

Now that we have worked with consecutive integers, we will expand our work to include **consecutive even integers** and **consecutive odd integers**. Consecutive even integers are even integers that immediately follow one another. Examples of consecutive even integers are:

$$24, 26, 28$$

$$-12, -10, -8$$

Notice each integer is two more than the number preceding it. If we call the first one n, then the next one is $n+2$. The one after that would be $n+2+2$ or $n+4$.

n 1^{st}integer
$n+1$ 2^{nd}consecutive integer
$n+2$ 3^{rd}consecutive integer etc.

Consecutive odd integers are odd integers that immediately follow one another. Consider the consecutive odd integers 63, 65, and 67.

$$63, 65, 67$$

$$n, n+2, n+4$$

n 1^{st}integer
$n+1$ 2^{nd}consecutive integer
$n+2$ 3^{rd}consecutive integer etc.

Does it seem strange to have to add two (an even number) to get the next odd number? Do we get an odd number or an even number when we add 2 to 3? to 11? to 47?

Whether the problem asks for consecutive even numbers or odd numbers, you do not have to do anything different. The pattern is still the same—to get to the next odd or the next even integer, add two.

EXAMPLE 2.3.5

Find three consecutive even integers whose sum is 120.

Solution:

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	three consecutive even integers
Step 3. Name each of the three numbers	Let $n = 1^{\text{st}}$consecutive even integer. $n+2 = 2^{\text{nd}}$consecutive even integer. $n+4 = 3^{\text{rd}}$consecutive even integer.
Step 4. Translate. Restate as one sentence. Translate into an equation.	The sum of the three even integers is 120 $n+n+2+n+4=120$
Step 5. Solve the equation. Combine like terms. Subtract three from each side. Divide each side by three.	$n+n+2+n+4=120$ $3n+6=120$ $3n=114$ $n=38$ 1^{st}integer $n+2$ 2^{nd}integer $38+2$ 40 $n+2$ 3^{rd}integer $38+4$ 42
Step 6. Check. $38+40+42 \stackrel{?}{=} 120$ $120 = 120$✓	
Step 7. Answer the question.	The three consecutive integers are 38, 40, and 42.

Try It! 2.3.9

Find three consecutive even integers whose sum is 102.

Answer

$32, 34, 36$

Try It! 2.3.10

Find three consecutive even integers whose sum is -24.

Answer

$-10, -8, -6$

When a number problem is in a real life context, we still use the same strategies that we used for the previous examples.

EXAMPLE 2.3.6

A married couple together earns $110,000 a year. The wife earns $16,000 less than twice what her husband earns. What does the husband earn?

Solution:

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	How much does the husband earn?
Step 3. Name each of the three numbers	Let $h =$ the amount the husband earns.
Step 4. Translate. Restate the problem in one sentence with all the important information. Translate into an equation.	$2h - 16,000 =$ the amount the wife earns. Together the husband and wife earn $110,000. $h + 2h - 16,000 = 110,000$
Step 5. Solve the equation. Combine like terms. Add 16,000 to both sides and simplify. Divide each side by three.	$h + 2h - 16,000 = 110,000$ $3h - 16,000 = 110,000$ $3h = 126,000$ $h = 42,000$ amount husband earns $2h - 16,000$ amount wife earns $2(42,000) - 16,000$ $84,000 - 16,000$ $68,000$
Step 6. Check. If the wife earns $68,000 and the husband earns $42,000, is that $110,000? Yes!	
Step 7. Answer the question.	The husband earns $42,000 a year.

According to the National Automobile Dealers Association, the average cost of a car in 2014 was $28,400. This was $1,600 less than six times the cost in 1975. What was the average cost of a car in 1975?

Answer

The average cost was $5,000.

Try It! 2.3.11

US Census data shows that the median price of new home in the U.S. in November 2014 was $280,900. This was $10,700 more than 14 times the price in November 1964. What was the median price of a new home in November 1964?

Answer

The median price was $19,300.

Solve Percent Applications

There are several methods to solve percent equations. In algebra, it is easiest if we just translate English sentences into algebraic equations and then solve the equations. Be sure to change the given percent to a decimal before you use it in the equation.

EXAMPLE 2.3.7

Translate and solve:

a. What number is 45% of 84?
b. 8.5% of what amount is $4.76?
c. 168 is what percent of 112?

Solution:

a.

	What number is 45% of 84?
Translate into algebra. Let n = the number.	$n = 0.45 \cdot 84$
Multiply.	$n = 37.8$
	37.8 is 45% of 84.

b.

	8.5% of what amount is $4.76?
Translate. Let n = the amount.	$0.085 \cdot n = 4.76$
Multiply.	$0.085n = 4.76$
Divide both sides by 0.085 and simplify.	$n = 56$
	8.5% of $56 is $4.76

c.

We are asked to find percent, so we must have our result in percent form.	168 is what percent of 112?
Translate into algebra. Let p = the percent.	$168 = p \cdot 112$
Multiply.	$168 = 112p$
Divide both sides by 112 and simplify.	$1.5 = p$
Convert to percent.	$150\% = p$
	168 is 150% of 112.

Try It! 2.3.12

Translate and solve:

a. What number is 45% of 80?
b. 7.5% of what amount is $1.95?
c. 110 is what percent of 88?

Answer

a. 36 b. $26 c. 125%

Try It! 2.3.13

Translate and solve:

a. What number is 55% of 60?
b. 8.5% of what amount is $3.06?
c. 126 is what percent of 72?

Answer

a. 33 b. $36 c. 175%

Now that we have a problem solving strategy to refer to, and have practiced solving basic percent equations, we are ready to solve percent applications. Be sure to ask yourself if your final answer makes sense—since many of the applications we will solve involve everyday situations, you can rely on your own experience.

EXAMPLE 2.3.8

The label on Audrey's yogurt said that one serving provided 12 grams of protein, which is 24% of the recommended daily amount. What is the total recommended daily amount of protein?

Solution:

What are you asked to find?	What total amount of protein is recommended?
Choose a variable to represent it.	Let a = total amount of protein.
Write a sentence that gives the information to find it.	12 g is 24% of the total amount
Translate into an equation.	$12 = 0.24 \cdot a$
Solve.	$50 = a$
Check: Does this make sense? Yes, 24% is about $\frac{1}{4}$ of the total and 12 is about $\frac{1}{4}$ of 50.	
Write a complete sentence to answer the question.	The amount of protein that is recommended is 50 g.

Try It! 2.3.14

One serving of wheat square cereal has 7 grams of fiber, which is 28% of the recommended daily amount. What is the total recommended daily amount of fiber?

Answer

25 grams

Try It! 2.3.15

One serving of rice cereal has 190 mg of sodium, which is 8% of the recommended daily amount. What is the total recommended daily amount of sodium?

Answer

2,375 mg

Remember to put the answer in the form requested. In the next example we are looking for the percent.

EXAMPLE 2.3.9

Veronica is planning to make muffins from a mix. The package says each muffin will be 240 calories and 60 calories will be from fat. What percent of the total calories is from fat?

Solution:

What are you asked to find?	What percent of the total calories is fat?
Choose a variable to represent it.	Let $p =$ percent of fat.
Write a sentence that gives the information to find it.	What percent of 240 is 60?
Translate the sentence into an equation.	$p \cdot 240 = 60$
Multiply.	$240p = 60$
Divide both sides by 240.	$p = 0.25$
Put in percent form.	$p = 25\%$
Check: does this make sense? Yes, 25% is one-fourth; 60 is one-fourth of 240. So, 25% makes sense.	
Write a complete sentence to answer the question.	Of the total calories in each muffin, 25% is fat.

Try It! 2.3.16

Mitzi received some gourmet brownies as a gift. The wrapper said each 28% brownie was 480 calories, and had 240 calories of fat. What percent of the total calories in each brownie comes from fat? Round the answer to the nearest whole percent.

Answer

50%

Try It! 2.3.17

The mix Ricardo plans to use to make brownies says that each brownie will be 190 calories, and 76 calories are from fat. What percent of the total calories are from fat? Round the answer to the nearest whole percent.

Answer

40%

It is often important in many fields—business, sciences, pop culture—to talk about how much an amount has increased or decreased over a certain period of time. This increase or decrease is generally expressed as a percent and called the **percent change**.

To find the percent change, first we find the amount of change, by finding the difference of the new amount and the original amount. Then we find what percent the amount of change is of the original amount.

FIND PERCENT CHANGE

1. Find the amount of change.

$$\text{change} = \text{new amount} - \text{original amount} \tag{2.3.1}$$

2. Find what percent the amount of change is of the original amount.

change is what percent of the original amount?

EXAMPLE 2.3.10

Recently, the California governor proposed raising community college fees from $36 a unit to $46 a unit. Find the percent change. (Round to the nearest tenth of a percent.)

Solution:

Find the amount of change.	$46 - 36 = 10$
Find the percent.	Change is what percent of the original amount?
Let $p =$ the percent.	10 is what percent of 36?
Translate to an equation.	$10 = p \cdot 36$
Simplify.	$10 = 36p$
Divide both sides by 36.	$0.278 \approx p$
Change to percent form; round to the nearest tenth	$27.8\% \approx p$
Write a complete sentence to answer the question.	The new fees are approximately a 27.8% increase over the old fees.
Remember to round the division to the nearest thousandth in order to round the percent to the nearest tenth.	

Try It! 2.3.18

Find the percent change. (Round to the nearest tenth of a percent.) In 2011, the IRS increased the deductible mileage cost to 55.5 cents from 51 cents.

Answer

8.8%

Try It! 2.3.19

Find the percent change. (Round to the nearest tenth of a percent.) In 1995, the standard bus fare in Chicago was $1.50. In 2008, the standard bus fare was 2.25.

Answer

50

Applications of discount and mark-up are very common in retail settings.

When you buy an item on sale, the original price has been discounted by some dollar amount. The **discount rate**, usually given as a percent, is used to determine the **amount of the discount**. To determine the amount of discount, we multiply the discount rate by the original price.

The price a retailer pays for an item is called the **original cost**. The retailer then adds a **mark-up** to the original cost to get the **list price**, the price he sells the item for. The mark-up is usually calculated as a percent of the original cost. To determine the amount of mark-up, multiply the mark-up rate by the original cost.

DISCOUNT

$$\text{amount of discount} = \text{discount rate} \cdot \text{original price}$$
$$\text{sale price} = \text{original amount} - \text{discount price}$$

The sale price should always be less than the original price.

MARK-UP

$$\text{amount of mark-up} = \text{mark-up rate} \cdot \text{original price}$$
$$\text{list price} = \text{original cost} - \text{mark-up}$$

The list price should always be more than the original cost.

EXAMPLE 2.3.11

Liam's art gallery bought a painting at an original cost of $750. Liam marked the price up 40%. Find

a. the amount of mark-up and
b. the list price of the painting.

Solution:

a.

Identify what you are asked to find, and choose a variable to represent it.	What is the amount of mark-up? Let $m =$ the amount of mark-up.
Write a sentence that gives the information to find it.	The mark-up is 40% of the $750 original cost
Translate into an equation.	$m = 0.40 \times 750$
Solve the equation.	$m = 300$
Write a complete sentence.	The mark-up on the painting was $300.

b.

Identify what you are asked to find, and choose a variable to represent it.	What is the list price? Let $p =$ the list price.
Write a sentence that gives the information to find it.	The list price is original cost plus the mark-up
Translate into an equation.	$p = 750 + 300$
Solve the equation.	$p = 1{,}050$
Check.	Is the list price more than the original cost? Is $1,050 more than $750? Yes.
Write a complete sentence.	The list price of the painting was $1,050.

Try It! 2.3.20

Find a. the amount of mark-up and b. the list price: Jim's music store bought a guitar at original cost $1,200. Jim marked the price up 50%.

Answer

a. $600 b. $1,800

Try It! 2.3.21

Find a. the amount of mark-up and b. the list price: The Auto Resale Store bought Pablo's Toyota for $8,500. They marked the price up 35%.

Answer

a. $2,975 b. $11,475

Solve Simple Interest Applications

Interest is a part of our daily lives. From the interest earned on our savings to the interest we pay on a car loan or credit card debt, we all have some experience with interest in our lives.

The amount of money you initially deposit into a bank is called the **principal**, P, and the bank pays you interest, I. When you take out a loan, you pay interest on the amount you borrow, also called the principal.

In either case, the interest is computed as a certain percent of the principal, called the **rate of interest**, r. The rate of interest is usually expressed as a percent per year, and is calculated by using the decimal equivalent of the percent. The variable t, (for time) represents the number of years the money is saved or borrowed.

Interest is calculated as simple interest or compound interest. Here we will use simple interest.

SIMPLE INTEREST

If an amount of money, P, called the principal, is invested or borrowed for a period of t years at an annual interest rate r, the amount of interest, I, earned or paid is

$$I = Prt \qquad \text{where} \quad \begin{aligned} I &= \text{interest} \\ P &= \text{principal} \\ r &= \text{rate} \\ t &= \text{time} \end{aligned}$$

Interest earned or paid according to this formula is called **simple interest**.

The formula we use to calculate interest is $I = Prt$. To use the formula we substitute in the values for variables that are given, and then solve for the unknown variable. It may be helpful to organize the information in a chart.

EXAMPLE 2.3.12

Areli invested a principal of $950 in her bank account that earned simple interest at an interest rate of 3%. How much interest did she earn in five years?

Solution:

$I = ?$
$P = \$950$
$r = 3\%$
$t = 5$ years

Identify what you are asked to find, and choose a variable to represent it.	What is the simple interest? Let $I =$ interest.
Write the formula.	$I = Prt$
Substitute in the given information.	$I = (950)(0.03)(5)$
Simplify.	$I = 142.5$
Check. Is $142.50 a reasonable amount of interest on $ 950? Yes.	
Write a complete sentence.	The interest is $142.50.

Try It! 2.3.22

Nathaly deposited $12,500 in her bank account where it will earn 4% simple interest. How much interest will Nathaly earn in five years?

Answer

He will earn $2,500.

Try It! 2.3.23

Susana invested a principal of $36,000 in her bank account that earned simple interest at an interest rate of 6.5%. How much interest did she earn in three years?

Answer

She earned $7,020.

There may be times when we know the amount of interest earned on a given principal over a certain length of time, but we do not know the rate.

EXAMPLE 2.3.13

Hang borrowed $7,500 from her parents to pay her tuition. In five years, she paid them $1,500 interest in addition to the $7,500 she borrowed. What was the rate of simple interest?

Solution:

$I = \$1500$
$P = \$7500$
$r = ?$
$t = 5$ years

Identify what you are asked to find, and choose a variable to represent it.	What is rate of simple interest? Let $r =$ rate of interest
Write the formula.	$I = Prt$
Substitute in the given information.	$1,500 = (7,500)r(5)$
Multiply.	$1,500 = 37,500r$
Divide.	$0.04 = r$
Change to percent form	$r = 4\%$

Check.

$$I = Prt$$
$$1,500 \stackrel{?}{=} (7,500)(0.04)(5)$$
$$1,500 = 1,500\checkmark$$

Write a complete sentence. The rate of interest was 4%.

Try It! 2.3.24

Jim lent his sister $5,000 to help her buy a house. In three years, she paid him the $5,000, plus $900 interest. What was the rate of simple interest?

Answer

The rate of simple interest was 6%.

Try It! 2.3.25

Loren lent his brother $3,000 to help him buy a car. In four years, his brother paid him back the $3,000 plus $660 in interest. What was the rate of simple interest?

Answer

The rate of simple interest was 5.5%.

In the next example, we are asked to find the principal—the amount borrowed.

EXAMPLE 2.3.14

Sean's new car loan statement said he would pay $4,866,25 in interest from a simple interest rate of 8.5% over five years. How much did he borrow to buy his new car?

Solution:

$$\begin{aligned} I &= 4,866.25 \\ P &= ? \\ r &= 8.5\% \\ t &= 5 \text{ years} \end{aligned}$$

Identify what you are asked to find, and choose a variable to represent it.	What is the amount borrowed (the principal)? Let P = principal borrowed
Write the formula.	$I = Prt$
Substitute in the given information.	$4,866.25 = P(0.085)(5)$
Multiply.	$4,866.25 = 0.425P$
Divide.	$11,450 = P$

Check.

$$\begin{aligned} I &= Prt \\ 4,866.25 &\stackrel{?}{=} (11,450)(0.085)(5) \\ 4,866.25 &= 4,866.25\checkmark \end{aligned}$$

Write a complete sentence. The principal was $\$11,450$.

Try It! 2.3.26

Eduardo noticed that his new car loan papers stated that with a 7.5% simple interest rate, he would pay $6,596.25 in interest over five years. How much did he borrow to pay for his car?

Answer

He paid $17,590.

Try It! 2.3.27

In five years, Gloria's bank account earned $2,400 interest at 5% simple interest. How much had she deposited in the account?

Answer

She deposited $9,600.

Access this online resource for additional instruction and practice with using a problem solving strategy.

- Begining Arithmetic Problems

Key Concepts

- **How To Use a Problem Solving Strategy for Word Problems**
  1. **Read** the problem. Make sure all the words and ideas are understood.
  2. **Identify** what you are looking for.
  3. **Name** what you are looking for. Choose a variable to represent that quantity.
  4. **Translate** into an equation. It may be helpful to restate the problem in one sentence with all the important information. Then, translate the English sentence into an algebra equation.
  5. **Solve** the equation using proper algebra techniques.
  6. **Check** the answer in the problem to make sure it makes sense.
  7. **Answer** the question with a complete sentence.
- **How To Find Percent Change**
  1. Find the amount of change

     $$\text{change} = \text{new amount} - \text{original amount}$$

  2. Find what percent the amount of change is of the original amount.

     $$\text{change is what percent of the original amount?}$$

- $$\text{amount of discount} = \text{discount rate} \cdot \text{original price}$$
  $$\text{sale price} = \text{original amount} - \text{discount price}$$
- $$\text{amount of mark-up} = \text{mark-up rate} \cdot \text{original price}$$
  $$\text{list price} = \text{original cost} - \text{mark-up}$$
- If an amount of money, P, called the principal, is invested or borrowed for a period of t years at an annual interest rate r, the amount of interest, I, earned or paid is:

$$I = Prt \qquad \begin{array}{l} I = interest \\ \text{where } P = principal \\ r = rate \\ t = time \end{array}$$

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2.3E: Exercises

Practice Makes Perfect

Use a Problem Solving Strategy for Word Problems

1. List five positive thoughts you can say to yourself that will help you approach word problems with a positive attitude. You may want to copy them on a sheet of paper and put it in the front of your notebook, where you can read them often.

Answer

Answers will vary.

2. List five negative thoughts that you have said to yourself in the past that will hinder your progress on word problems. You may want to write each one on a small piece of paper and rip it up to symbolically destroy the negative thoughts.

In the following exercises, solve using the problem solving strategy for word problems. Remember to write a complete sentence to answer each question.

3. There are 16 girls in a school club. The number of girls is four more than twice the number of boys. Find the number of boys.

Answer

six boys

4. There are 18 Cub Scouts in Troop 645. The number of scouts is three more than five times the number of adult leaders. Find the number of adult leaders.

5. Huong is organizing paperback and hardback books for her club's used book sale. The number of paperbacks is 12 less than three times the number of hardbacks. Huong had 162 paperbacks. How many hardback books were there?

Answer

58 hardback books

6. Jeff is lining up children's and adult bicycles at the bike shop where he works. The number of children's bicycles is nine less than three times the number of adult bicycles. There are 42 adult bicycles. How many children's bicycles are there?

Solve Number Word Problems

In the following exercises, solve each number word problem.

7. The difference of a number and 12 is three. Find the number.

Answer

15

8. The difference of a number and eight is four. Find the number.

9. The sum of three times a number and eight is 23. Find the number.

Answer

5

10. The sum of twice a number and six is 14. Find the number.

11. The difference of twice a number and seven is 17. Find the number.

Answer

12

12. The difference of four times a number and seven is 21. Find the number.

13. Three times the sum of a number and nine is 12. Find the number.

Answer

-5

14. Six times the sum of a number and eight is 30. Find the number.

15. One number is six more than the other. Their sum is 42. Find the numbers.

Answer

$18,\ 24$

16. One number is five more than the other. Their sum is 33. Find the numbers.

17. The sum of two numbers is 20. One number is four less than the other. Find the numbers.

Answer

$8,\ 12$

18. The sum of two numbers is 27. One number is seven less than the other. Find the numbers.

19. One number is 14 less than another. If their sum is increased by seven, the result is 85. Find the numbers.

Answer

$32,\ 46$

20. One number is 11 less than another. If their sum is increased by eight, the result is 71. Find the numbers.

21. The sum of two numbers is 14. One number is two less than three times the other. Find the numbers.

Answer

$4,\ 10$

22. The sum of two numbers is zero. One number is nine less than twice the other. Find the numbers.

23. The sum of two consecutive integers is 77. Find the integers.

Answer

$38,\ 39$

24. The sum of two consecutive integers is 89. Find the integers.

25. The sum of three consecutive integers is 78. Find the integers.

Answer

$25,\ 26,\ 27$

26. The sum of three consecutive integers is 60. Find the integers.

27. Find three consecutive integers whose sum is -36.

Answer

$-11,\ -12,\ -13$

28. Find three consecutive integers whose sum is -3.

29. Find three consecutive even integers whose sum is 258.

Answer

$84,\ 86,\ 88$

30. Find three consecutive even integers whose sum is 222.

31. Find three consecutive odd integers whose sum is -213.

Answer

$-69,\ -71,\ -73$

32. Find three consecutive odd integers whose sum is -267.

33. Philip pays $\$1,620$ in rent every month. This amount is $\$120$ more than twice what his brother Paul pays for rent. How much does Paul pay for rent?

Answer

$\$750$

34. Marc just bought an SUV for $\$54,000$. This is $\$7,400$ less than twice what his wife paid for her car last year. How much did his wife pay for her car?

35. Laurie has $\$46,000$ invested in stocks and bonds. The amount invested in stocks is $\$8,000$ less than three times the amount invested in bonds. How much does Laurie have invested in bonds?

Answer

$\$13,500$

36. Erica earned a total of $\$50,450$ last year from her two jobs. The amount she earned from her job at the store was $\$1,250$ more than three times the amount she earned from her job at the college. How much did she earn from her job at the college?

Solve Percent Applications

In the following exercises, translate and solve.

37. a. What number is 45% of 120? b. 81 is 75% of what number? c. What percent of 260 is 78?

Answer

a. 54
b. 108
c. 30%

38. a. What number is 65% of 100? b. 93 is 75% of what number? c. What percent of 215 is 86?

39. a. 250% of 65 is what number? b. 8.2% of what amount is $2.87? c. 30 is what percent of 20?

Answer

a. 162.5
b. $35
c. 150%

40. a. 150% of 90 is what number? b. 6.4% of what amount is $2.88? c. 50 is what percent of 40?

In the following exercises, solve.

41. Geneva treated her parents to dinner at their favorite restaurant. The bill was $74.25. Geneva wants to leave 16% of the total bill as a tip. How much should the tip be?

Answer

$11.88

42. When Hiro and his co-workers had lunch at a restaurant near their work, the bill was $90.50. They want to leave 18% of the total bill as a tip. How much should the tip be?

43. One serving of oatmeal has 8 grams of fiber, which is 33% of the recommended daily amount. What is the total recommended daily amount of fiber?

Answer

24.2 g

44. One serving of trail mix has 67 grams of carbohydrates, which is 22% of the recommended daily amount. What is the total recommended daily amount of carbohydrates?

45. A bacon cheeseburger at a popular fast food restaurant contains 2070 milligrams (mg) of sodium, which is 86% of the recommended daily amount. What is the total recommended daily amount of sodium?

Answer

2407 mg

46. A grilled chicken salad at a popular fast food restaurant contains 650 milligrams (mg) of sodium, which is 27% of the recommended daily amount. What is the total recommended daily amount of sodium?

47. The nutrition fact sheet at a fast food restaurant says the fish sandwich has 380 calories, and 171 calories are from fat. What percent of the total calories is from fat?

Answer

45%

48. The nutrition fact sheet at a fast food restaurant says a small portion of chicken nuggets has 190 calories, and 114 calories are from fat. What percent of the total calories is from fat?

49. Emma gets paid $3,000 per month. She pays $750 a month for rent. What percent of her monthly pay goes to rent?

Answer

25%

50. Dimple gets paid $3,200 per month. She pays $960 a month for rent. What percent of her monthly pay goes to rent?

In the following exercises, solve.

51. Tamanika received a raise in her hourly pay, from $15.50 to $17.36. Find the percent change.

Answer

12%

52. Ayodele received a raise in her hourly pay, from $24.50 to $25.48. Find the percent change.

53. Annual student fees at the University of California rose from about $4,000 in 2000 to about $12,000 in 2010. Find the percent change.

Answer

200%

54. The price of a share of one stock rose from $12.50 to $50. Find the percent change.

55. A grocery store reduced the price of a loaf of bread from $2.80 to $2.73. Find the percent change.

Answer

−2.5%

56. The price of a share of one stock fell from $8.75 to $8.54. Find the percent change.

57. Hernando's salary was $49,500 last year. This year his salary was cut to $44,055. Find the percent change.

Answer

−11%

58. In ten years, the population of Detroit fell from 950,000 to about 712,500. Find the percent change.

In the following exercises, find a. the amount of discount and b. the sale price.

59. Janelle bought a beach chair on sale at 60% off. The original price was $44.95.

Answer

a. $26.97
b. $17.98

60. Errol bought a skateboard helmet on sale at 40% off. The original price was $49.95.

In the following exercises, find a. the amount of discount and b. the discount rate (Round to the nearest tenth of a percent if needed.)

61. Larry and Donna bought a sofa at the sale price of $1,344. The original price of the sofa was $1,920.

Answer

a. $576
b. 30%

62. Hiroshi bought a lawnmower at the sale price of $240. The original price of the lawnmower is $300.

In the following exercises, find a. the amount of the mark-up and b. the list price.

63. Daria bought a bracelet at original cost $16 to sell in her handicraft store. She marked the price up 45%. What was the list price of the bracelet?

Answer

a. $7.20
b. $23.20

64. Regina bought a handmade quilt at original cost $120 to sell in her quilt store. She marked the price up 55%. What was the list price of the quilt?

65. Tom paid $0.60 a pound for tomatoes to sell at his produce store. He added a 33% mark-up. What price did he charge his customers for the tomatoes?

Answer

a. $0.20
b. $0.80

66. Flora paid her supplier $0.74 a stem for roses to sell at her flower shop. She added an 85% mark-up. What price did she charge her customers for the roses?

Solve Simple Interest Applications

In the following exercises, solve.

67. Casey deposited $1,450 in a bank account that earned simple interest at an interest rate of 4%. How much interest was earned in two years?

Answer

$116

68. Terrence deposited $5,720 in a bank account that earned simple interest at an interest rate of 6%. How much interest was earned in four years?

69. Robin deposited $31,000 in a bank account that earned simple interest at an interest rate of 5.2%. How much interest was earned in three years?

Answer

$4836

70. Carleen deposited $16,400 in a bank account that earned simple interest at an interest rate of 3.9% How much interest was earned in eight years?

71. Hilaria borrowed $8,000 from her grandfather to pay for college. Five years later, she paid him back the $8,000, plus $1,200 interest. What was the rate of simple interest?

Answer

3%

72. Kenneth lent his niece $1,200 to buy a computer. Two years later, she paid him back the $1,200, plus $96 interest. What was the rate of simple interest?

73. Lebron lent his daughter $20,000 to help her buy a condominium. When she sold the condominium four years later, she paid him the $20,000, plus $3,000 interest. What was the rate of simple interest?

Answer

3.75%

74. Pablo borrowed $50,000 to start a business. Three years later, he repaid the $50,000, plus $9,375 interest. What was the rate of simple interest?

75. In 10 years, a bank account that paid 5.25% simple interest earned $18,375 interest. What was the principal of the account?

Answer

$35,000

76. In 25 years, a bond that paid 4.75% simple interest earned $2,375 interest. What was the principal of the bond?

77. Joshua's computer loan statement said he would pay $1,244.34 in simple interest for a three-year loan at 12.4%. How much did Joshua borrow to buy the computer?

Answer

$3345

78. Margaret's car loan statement said she would pay $7,683.20 in simple interest for a five-year loan at 9.8%. How much did Margaret borrow to buy the car?

Everyday Math

79. **Tipping** At the campus coffee cart, a medium coffee costs $1.65. MaryAnne brings $2.00 with her when she buys a cup of coffee and leaves the change as a tip. What percent tip does she leave?

Answer

17.5%

80. **Tipping** Four friends went out to lunch and the bill came to $53.75 They decided to add enough tip to make a total of $64, so that they could easily split the bill evenly among themselves. What percent tip did they leave?

Writing Exercises

81. What has been your past experience solving word problems? Where do you see yourself moving forward?

Answer

Answers will vary.

82. Without solving the problem “44 is 80% of what number” think about what the solution might be. Should it be a number that is greater than 44 or less than 44? Explain your reasoning.

83. After returning from vacation, Alex said he should have packed 50% fewer shorts and 200% more shirts. Explain what Alex meant.

Answer

Answers will vary.

84. Because of road construction in one city, commuters were advised to plan that their Monday morning commute would take 150% of their usual commuting time. Explain what this means.

Self Check

a. After completing the exercises, use this checklist to evaluate your mastery of the objective of this section.

I can...	Confidently	With some help	No-I don't get it!
use a problem-solving strategy for word problems.			
solve number problems.			
solve percent applications.			
solve simple interest applications.			

b. After reviewing this checklist, what will you do to become confident for all objectives?

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2.4: Solve a Formula for a Specific Variable

Learning Objectives

By the end of this section, you will be able to:

- Solve a formula for a specific variable
- Use formulas to solve geometry applications

Before you get started, take this readiness quiz.

1. Evaluate $2(x+3)$ when $x=5$.
   If you missed this problem, review [link].
2. The length of a rectangle is three less than the width. Let *w* represent the width. Write an expression for the length of the rectangle.
   If you missed this problem, review [link].
3. Evaluate $\frac{1}{2}bh$ when $b=14$ and $h=9$.
   If you missed this problem, review [link].

Solve a Formula for a Specific Variable

We have all probably worked with some geometric formulas in our study of mathematics. Formulas are used in so many fields, it is important to recognize formulas and be able to manipulate them easily.

It is often helpful to solve a formula for a specific variable. If you need to put a formula in a spreadsheet, it is not unusual to have to solve it for a specific variable first. We isolate that variable on one side of the equals sign with a coefficient of one and all other variables and constants are on the other side of the equal sign.

Geometric formulas often need to be solved for another variable, too. The formula $V=\frac{1}{3}\pi r^2 h$ is used to find the **volume** of a right circular cone when given the radius of the base and height. In the next example, we will solve this formula for the height.

EXAMPLE 2.4.1

Solve the formula $V=\frac{1}{3}\pi r^2 h$ for *h*.

Answer

Write the formula.			$V=\frac{1}{3}\pi r^2 h$
Remove the fraction on the right.			$3\cdot V=3\cdot\frac{1}{3}\pi r^2 h$
Simplify.			$3V=\pi r^2 h$
Divide both sides by πr^2.			$\frac{3V}{\pi r^2}=h$

We could now use this formula to find the height of a right circular cone when we know the volume and the radius of the base, by using the formula $h=\frac{3V}{\pi r^2}$.

EXAMPLE 2.4.2:

Use the formula $A=\frac{1}{2}bh$ to solve for *b*.

Answer

$b=\frac{2A}{h}$

? EXAMPLE 2.4.3:

Use the formula $A = \frac{1}{2}bh$ to solve for h.

Answer

$h = \frac{2A}{b}$

In the sciences, we often need to change **temperature** from Fahrenheit to Celsius or vice versa. If you travel in a foreign country, you may want to change the Celsius temperature to the more familiar Fahrenheit temperature.

? EXAMPLE 2.4.4

Solve the formula $C = \frac{5}{9}(F - 32)$ for F.

Answer

Write the formula.			$C = \frac{5}{9}(F - 32)$
Remove the fraction on the right.			$\frac{9}{5}C = \frac{9}{5} \cdot \frac{5}{9}(F - 32)$
Simplify.			$\frac{9}{5}C = (F - 32)$
Add 32 to both sides.			$\frac{9}{5}C + 32 = F$

We can now use the formula $F = \frac{9}{5}C + 32$ to find the Fahrenheit temperature when we know the Celsius temperature.

? EXAMPLE 2.4.5:

Solve the formula $F = \frac{9}{5}C + 32$ for C.

Answer

$C = \frac{5}{9}(F - 32)$

? EXAMPLE 2.4.6:

Solve the formula $A = \frac{1}{2}h(b + B)$ for b.

Answer

$b = \frac{2A - Bh}{h}$

The next example uses the formula for the **surface area** of a right cylinder.

? EXAMPLE 2.4.7

Solve the formula $S = 2\pi r^2 + 2\pi rh$ for h.

Answer

Write the formula.	$S = 2\pi r^2 + 2\pi r h$
Isolate the h term by subtracting $2\pi r^2$ from each side.	$S - 2\pi r^2 = 2\pi r^2 - 2\pi r^2 + 2\pi r h$
Simplify.	$S - 2\pi r^2 = 2\pi r h$

Solve for h by dividing both sides by $2\pi r$.	$\frac{S-2\pi r^2}{2\pi r} = \frac{2\pi r h}{2\pi r}$
Simplify.	$\frac{S-2\pi r^2}{2\pi r} = h$

? EXAMPLE 2.4.8:

Solve the formula $A = P + Prt$ for t.

Answer

$t = \frac{A-P}{Pr}$

? EXAMPLE 2.4.9:

Solve the formula $A = P + Prt$ for r.

Answer

$r = \frac{A-P}{Pt}$

Sometimes we might be given an equation that is solved for y and need to solve it for x, or vice versa. In the following example, we're given an equation with both x and y on the same side and we'll solve it for y.

Solve the formula $8x + 7y = 15$ for y.

Answer

We will isolate y on one side of the equation.	$8x + 7y = 15$
Subtract $6x$ from both sides to isolate the term with y.	$8x - 8x + 7y = 15 - 8x$
Simplify.	$7y = 15 - 8x$
Divide both sides by 7 to make the coefficient of y one.	$\frac{7y}{7} = \frac{15-8x}{7}$
Simplify.	$y = \frac{15-8x}{7}$

Solve the formula $4x + 7y = 9$ for y.

Answer

$y = \frac{9-4x}{7}$

? EXAMPLE 2.4.12

Solve the formula $5x + 8y = 1$ for y.

Answer

$y = \frac{1-5x}{8}$

Use Formulas to Solve Geometry Applications

In this objective we will use some common geometry formulas. We will adapt our problem solving strategy so that we can solve geometry applications. The geometry formula will name the variables and give us the equation to solve.

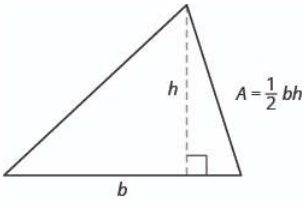
In addition, since these applications will all involve shapes of some sort, most people find it helpful to draw a figure and label it with the given information. We will include this in the first step of the problem solving strategy for geometry applications.

SOLVE GEOMETRY APPLICATIONS.

1. **Read** the problem and make sure all the words and ideas are understood.
2. **Identify** what you are looking for.
3. **Name** what we are looking for by choosing a variable to represent it. Draw the figure and label it with the given information.
4. **Translate** into an equation by writing the appropriate formula or model for the situation. Substitute in the given information.
5. **Solve** the equation using good algebra techniques.
6. **Check** the answer in the problem and make sure it makes sense.
7. **Answer** the question with a complete sentence.

When we solve geometry applications, we often have to use some of the properties of the figures. We will review those properties as needed.

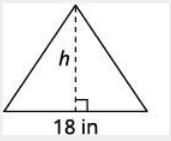
The next example involves the area of a triangle. The **area** of a triangle is one-half the base times the height. We can write this as $A = \frac{1}{2}bh$, where b = length of the base and h = height.



EXAMPLE 2.4.13

The area of a triangular painting is 126 square inches. The base is 18 inches. What is the height?

Answer

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	height of a triangle
Step 3. Name.	
Choose a variable to represent it.	Let $h =$ the height.
Draw the figure and label it with the given information.	Area = 126 sq. in.
	
Step 4. Translate.	
Write the appropriate formula.	$A = \frac{1}{2}bh$
Substitute in the given information.	$126 = \frac{1}{2} \cdot 18 \cdot h$
Step 5. Solve the equation.	$126 = 9h$

Divide both sides by 9.	$14 = h$
Step 6. Check. $A = \frac{1}{2}bh$ $126 \stackrel{?}{=} 12 \cdot 18 \cdot 14$ $126 = 126 \checkmark$	
Step 7. Answer the question.	The height of the triangle is 14 inches.

? EXAMPLE 2.4.14:

The area of a triangular church window is 90 square meters. The base of the window is 15 meters. What is the window's height?

Answer

The window's height is 12 meters.

? EXAMPLE 2.4.15:

A triangular tent door has area 15 square feet. The height is five feet. What is the length of the base?

Answer

The length of the base is 6 feet.

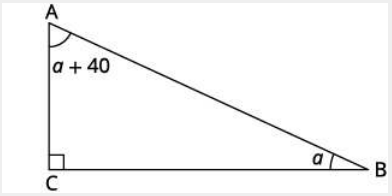
In the next example, we will work with a **right triangle**. To solve for the measure of each angle, we need to use two triangle properties. In any triangle, the sum of the measures of the angles is 180°. We can write this as a formula: $m\angle A + m\angle B + m\angle C = 180$. Also, since the triangle is a right triangle, we remember that a right triangle has one 90° angle.

Here, we will have to define one angle in terms of another. We will wait to draw the figure until we write expressions for all the angles we are looking for.

? EXAMPLE 2.4.16:

The measure of one angle of a right triangle is 40 degrees more than the measure of the smallest angle. Find the measures of all three angles.

Answer

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	the measures of all three angles
Step 3. Name. Choose a variable to represent it.	Let a = 1st angle. $a + 40$ = 2nd angle 90 = 3rd angle (the right angle)
Draw the figure and label it with the given information.	
Step 4. Translate.	
Write the appropriate formula.	$m\angle A + m\angle B + m\angle C = 180$

Substitute into the formula.	$a+(a+40)+90=180$
Step 5. Solve the equation.	$2a+130=180$ $2a=50$ $a=25$ first angle $a+40$ second angle $25+40$ 65 90 third angle
Step 6. Check. $25+65+90 \stackrel{?}{=} 180$ $180=180$✓	
Step 7. Answer the question.	The three angles measure 25°, 65°, and 90°.

? EXAMPLE 2.4.17:

The measure of one angle of a right triangle is 50 more than the measure of the smallest angle. Find the measures of all three angles.

Answer

The measures of the angles are 20°, 70°, and 90°.

? EXAMPLE 2.4.18:

The measure of one angle of a right triangle is 30 more than the measure of the smallest angle. Find the measures of all three angles.

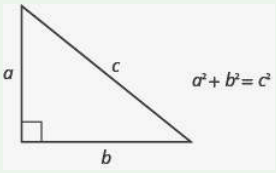
Answer

The measures of the angles are 30°, 60°, and 90°.

The next example uses another important geometry formula. The **Pythagorean Theorem** tells how the lengths of the three sides of a right triangle relate to each other. Writing the formula in every exercise and saying it aloud as you write it may help you memorize the Pythagorean Theorem.

THE PYTHAGOREAN THEOREM

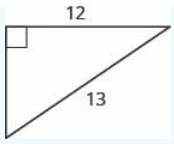
In any right triangle, where a and b are the lengths of the legs, and c is the length of the hypotenuse, the sum of the squares of the lengths of the two legs equals the square of the length of the hypotenuse.



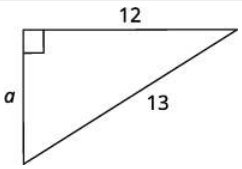
We will use the Pythagorean Theorem in the next example.

? EXAMPLE 2.4.19:

Use the Pythagorean Theorem to find the length of the other leg in

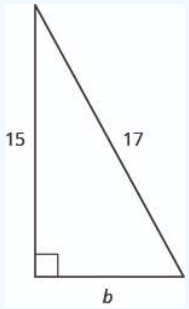


Answer

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	the length of the leg of the triangle
Step 3. Name.	
Choose a variable to represent it.	Let a = the leg of the triangle.
Label side a.	
Step 4. Translate.	
Write the appropriate formula. Substitute.	$a^2+b^2=c^2$ $a^2+12^2=13^2$
Step 5. Solve the equation. Isolate the variable term. Use the definition of square root. Simplify.	$a^2+144=169$ $a^2=25$ $a=\sqrt{25}$ $a=5$
Step 6. Check. $5^2+12^2 \stackrel{?}{=} 13^2$ $25+144 \stackrel{?}{=} 169$ $169=169$ ✓	
Step 7. Answer the question.	The length of the leg is 5.

? EXAMPLE 2.4.20:

Use the Pythagorean Theorem to find the length of the leg in the figure.

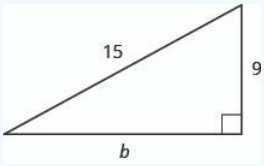


Answer

The length of the leg is 8.

? EXAMPLE 2.4.21:

Use the Pythagorean Theorem to find the length of the leg in the figure.



Answer

The length of the leg is 12.

The next example is about the **perimeter** of a rectangle. Since the perimeter is just the distance around the rectangle, we find the sum of the lengths of its four sides—the sum of two lengths and two widths. We can write is as $P = 2L + 2W$ where L is the length and W is the width. To solve the example, we will need to define the length in terms of the width.

? EXAMPLE 2.4.22:

The length of a rectangle is six centimeters more than twice the width. The perimeter is 96 centimeters. Find the length and width.

Answer

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	the length and the width
Step 3. Name. Choose a variable to represent the width. The length is six more than twice the width.	Let $w =$ width. $2w + 6 =$ length w, 2w + 6 $P = 96$ cm
Step 4. Translate.	
Write the appropriate formula.	$P = 2L + 2W$
Substitute in the given information.	$96 = 2(2w + 6) + 2w$
Step 5. Solve the equation.	$96 = 4w + 12 + 2w$ $96 = 6w + 12$ $84 = 6w$ $14 = w$ (width) $2w + 6$ (length) $2(14) + 6$ 34 The length is 34 cm.
Step 6. Check. 14 cm, 34 cm $P = 2L + 2W$ $96 \stackrel{?}{=} 2 \cdot 34 + 2 \cdot 14$ $96 = 96$✓	
Step 7. Answer the question.	The length is 34 cm and the width is 14 cm

? EXAMPLE 2.4.23:

The length of a rectangle is seven more than twice the width. The perimeter is 110 inches. Find the length and width.

Answer

The length is 16 inches and the width is 39 inches.

? EXAMPLE 2.4.24:

The width of a rectangle is eight yards less than twice the length. The perimeter is 86 yards. Find the length and width.

Answer

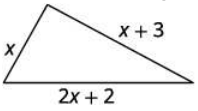
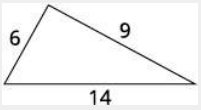
The length is 17 yards and the width is 26 yards.

The next example is about the **perimeter** of a triangle. Since the perimeter is just the distance around the triangle, we find the sum of the lengths of its three sides. We can write this as $P = a + b + c$, where a, b, and c are the lengths of the sides.

? EXAMPLE 2.4.25:

One side of a triangle is three inches more than the first side. The third side is two inches more than twice the first. The perimeter is 29 inches. Find the length of the three sides of the triangle.

Answer

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	the lengths of the three sides of a triangle
Step 3. Name. Choose a variable to represent the length of the first side.	Let $x =$ length of 1st side. $x + 3$ = length of 2nd side $2x + 2$ = length of 3rd side  Perimeter is 29 in.
Step 4. Translate. Write the appropriate formula. Substitute in the given information.	$P = a + b + c$ $29 = x + (x + 3) + (2x + 2)$
Step 5. Solve the equation.	$29 = 4x + 5$ $24 = 4x$ $6 = x$ length of first side $x + 3$ length of second side $6 + 3$ 9 $2x + 2$ length of second side $2 \cdot 6 + 2$ 14
Step 6. Check.  $29 \stackrel{?}{=} 6 + 9 + 14$ $29 = 29$✓	
Step 7. Answer the question.	The lengths of the sides of the triangle are 6, 9, and 14 inches.

? EXAMPLE 2.4.26:

One side of a triangle is seven inches more than the first side. The third side is four inches less than three times the first. The perimeter is 28 inches. Find the length of the three sides of the triangle.

Answer

The lengths of the sides of the triangle are 5, 11 and 12 inches.

? EXAMPLE 2.4.27:

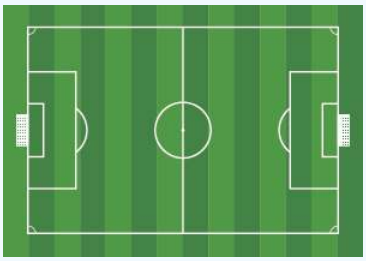
One side of a triangle is three feet less than the first side. The third side is five feet less than twice the first. The perimeter is 20 feet. Find the length of the three sides of the triangle.

Answer

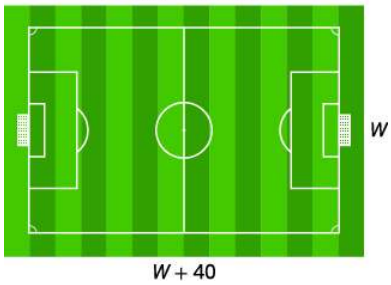
The lengths of the sides of the triangle are 4, 7 and 9 feet.

? EXAMPLE 2.4.28:

The perimeter of a rectangular soccer field is 360 feet. The length is 40 feet more than the width. Find the length and width.



Answer

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	the length and width of the soccer field
Step 3. Name. Choose a variable to represent it. The length is 40 feet more than the width. Draw the figure and label it with the given information.	Let w = width. $w + 40 =$ length Perimeter = 360 feet 
Step 4. Translate. Write the appropriate formula and substitute.	$P = 2L + 2W$ $360 = 2(w + 40) + 2w$
Step 5. Solve the equation.	

Step 6. Check. $P = 2L + 2W$ $360 \stackrel{?}{=} 2(110) + 2(70)$ $360 = 360\checkmark$	$360 = 2w + 80 + 2w$ $360 = 4w + 80$ $280 = 4w$ $70 = w$ the width of the field $w + 40$ the length of the field $70 + 40$ 110
Step 7. Answer the question.	The length of the soccer field is 110 feet and the width is 70 feet.

? EXAMPLE 2.4.29:

The perimeter of a rectangular swimming pool is 200 feet. The length is 40 feet more than the width. Find the length and width.

Answer

The length of the swimming pool is 70 feet and the width is 30 feet.

? EXAMPLE 2.4.30:

The length of a rectangular garden is 30 yards more than the width. The perimeter is 300 yards. Find the length and width.

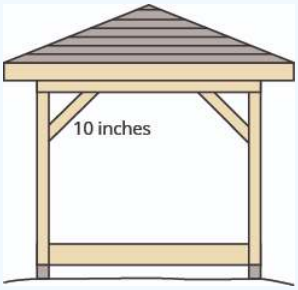
Answer

The length of the garden is 90 yards and the width is 60 yards.

Applications of these geometric properties can be found in many everyday situations as shown in the next example.

? EXAMPLE 2.4.31:

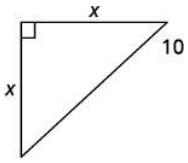
Kelvin is building a gazebo and wants to brace each corner by placing a 10” piece of wood diagonally as shown.



How far from the corner should he fasten the wood if wants the distances from the corner to be equal? Approximate to the nearest tenth of an inch.

Answer

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	the distance from the corner that the bracket should be attached

Step 3. Name. Choose a variable to represent it. Draw the figure and label it with the given information.	Let $x =$ the distance from the corner. 
Step 4. Translate. Write the appropriate formula and substitute.	$a^2+b^2=c^2 \quad x^2+x^2=10^2$
Step 5. Solve the equation. Isolate the variable. Use the definition of square root. Simplify. Approximate to the nearest tenth.	$2x^2=100$ $x^2=50$ $x=\sqrt{50}$ $x\approx 7.1$
Step 6. Check. $a^2+b^2=c^2$ $(7.1)^2+(7.1)^2\approx 10^2$ Yes.	
Step 7. Answer the question.	Kelvin should fasten each piece of wood approximately 7.1" from the corner.

? EXAMPLE 2.4.32:

John puts the base of a 13-foot ladder five feet from the wall of his house as shown in the figure. How far up the wall does the ladder reach?

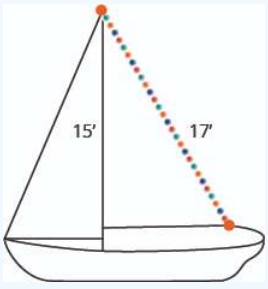


Answer

The ladder reaches 12 feet.

? EXAMPLE 2.4.33:

Randy wants to attach a 17-foot string of lights to the top of the 15 foot mast of his sailboat, as shown in the figure. How far from the base of the mast should he attach the end of the light string?



Answer

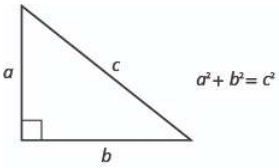
He should attach the lights 8 feet from the base of the mast.

Access this online resource for additional instruction and practice with solving for a variable in literal equations.

- Solving Literal Equations

Key Concepts

- **How To Solve Geometry Applications**
  1. **Read** the problem and make sure all the words and ideas are understood.
  2. **Identify** what you are looking for.
  3. **Name** what you are looking for by choosing a variable to represent it. Draw the figure and label it with the given information.
  4. **Translate** into an equation by writing the appropriate formula or model for the situation. Substitute in the given information.
  5. **Solve** the equation using good algebra techniques.
  6. **Check** the answer in the problem and make sure it makes sense.
  7. **Answer** the question with a complete sentence.
- **The Pythagorean Theorem**
  - In any right triangle, where a and b are the lengths of the legs, and c is the length of the hypotenuse, the sum of the squares of the lengths of the two legs equals the square of the length of the hypotenuse.



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2.4E: Exercises

Practice Makes Perfect

Solve a Formula for a Specific Variable

In the following exercises, solve the given formula for the specified variable.

1. Solve the formula $C = \pi d$ for d.

Answer

$$d = \frac{C}{\pi}$$

2. Solve the formula $C = \pi d$ for π.

3. Solve the formula $V = LWH$ for L.

Answer

$$L = \frac{V}{WH}$$

4. Solve the formula $V = LWH$ for H.

5. Solve the formula $A = \frac{1}{2}bh$ for b.

Answer

$$b = \frac{2A}{h}$$

6. Solve the formula $A = \frac{1}{2}bh$ for h.

7. Solve the formula

$A = \frac{1}{2}d_1d_2$ for d_1.

Answer

$$d_1 = \frac{2A}{d_2}$$

8. Solve the formula

$A = \frac{1}{2}d_1d_2$ for d_2.

9. Solve the formula

$A = \frac{1}{2}h(b_1 + b_2)$ for b_1.

Answer

$$b_1 = \frac{2A}{h} - b_2$$

10. Solve the formula

$A = \frac{1}{2}h(b_1 + b_2)$ for b_2.

11. Solve the formula

$h = 54t + \frac{1}{2}at^2$ for a.

Answer

$$a = \frac{2h - 108t}{t^2}$$

12. Solve the formula

$h = 48t + \frac{1}{2}at^2$ for a.

13. Solve $180 = a + b + c$ for a.

Answer

$$a = 180 - b - c$$

14. Solve $180 = a + b + c$ for c.

15. Solve the formula

$A = \frac{1}{2}pI + B$ for p.

Answer

$$p = \frac{2A - 2B}{I}$$

16. Solve the formula

$A = \frac{1}{2}pI + B$ for I.

17. Solve the formula

$P = 2L + 2W$ for L.

Answer

$$L = \frac{P - 2W}{2}$$

18. Solve the formula

$P = 2L + 2W$ for W.

In the following exercises, solve for the formula for y.

19. Solve the formula

$8x + y = 15$ for y.

Answer

$$y = 15 - 8x$$

20. Solve the formula

$9x + y = 13$ for y.

21. Solve the formula

$-4x + y = -6$ for y.

Answer

$y = -6 + 4x$

22. Solve the formula

$-5x + y = -1$ for y.

23. Solve the formula

$x - y = -4$ for y.

Answer

$y = 4 + x$

24. Solve the formula

$x - y = -3$ for y.

25. Solve the formula

$4x + 3y = 7$ for y.

Answer

$y = \frac{7-4x}{3}$

26. Solve the formula

$3x + 2y = 11$ for y.

27. Solve the formula

$2x + 3y = 12$ for y.

Answer

$y = \frac{12-2x}{3}$

28. Solve the formula

$5x + 2y = 10$ for y.

29. Solve the formula

$3x - 2y = 18$ for y.

Answer

$y = \frac{18-3x}{-2}$

30. Solve the formula

$4x - 3y = 12$ for y.

Use Formulas to Solve Geometry Applications

In the following exercises, solve using a geometry formula.

31. A triangular flag has area 0.75 square feet and height 1.5 foot. What is its base?

Answer

1 foot

32. A triangular window has area 24 square feet and height six feet. What is its base?

33. What is the base of a triangle with area 207 square inches and height 18 inches?

Answer

23 inches

34. What is the height of a triangle with area 893 square inches and base 38 inches?

35. The two smaller angles of a right triangle have equal measures. Find the measures of all three angles.

Answer

$45^\circ, 45^\circ, 90^\circ$

36. The measure of the smallest angle of a right triangle is 20° less than the measure of the next larger angle. Find the measures of all three angles.

37. The angles in a triangle are such that one angle is twice the smallest angle, while the third angle is three times as large as the smallest angle. Find the measures of all three angles.

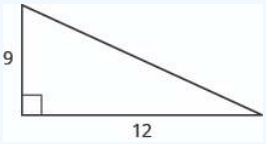
Answer

$30^\circ, 60^\circ, 90^\circ$

38. The angles in a triangle are such that one angle is 20 more than the smallest angle, while the third angle is three times as large as the smallest angle. Find the measures of all three angles.

In the following exercises, use the Pythagorean Theorem to find the length of the hypotenuse.

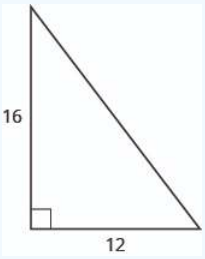
39.



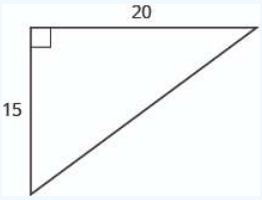
Answer

15

40.



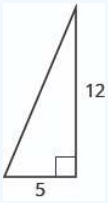
41.



Answer

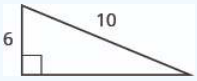
25

42.



In the following exercises, use the Pythagorean Theorem to find the length of the unknown leg. Round to the nearest tenth if necessary.

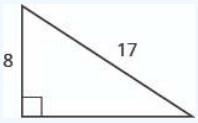
43.



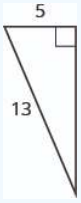
Answer

8

44.



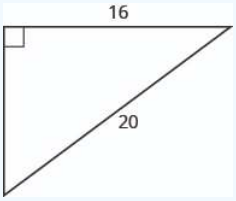
45.



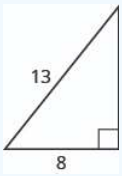
Answer

12

46.



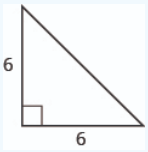
47.



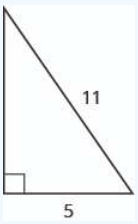
Answer

10.2

48.



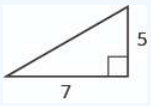
49.



Answer

9.8

50.



In the following exercises, solve using a geometry formula.

51. The width of a rectangle is seven meters less than the length. The perimeter is 58 meters. Find the length and width.

Answer

18 meters, 11 meters

52. The length of a rectangle is eight feet more than the width. The perimeter is 60 feet. Find the length and width.

53. The width of the rectangle is 0.7 meters less than the length. The perimeter of a rectangle is 52.6 meters. Find the dimensions of the rectangle.

Answer

13.5m, 12.8m

54. The length of the rectangle is 1.1 meters less than the width. The perimeter of a rectangle is 49.4 meters. Find the dimensions of the rectangle.

55. The perimeter of a rectangle of 150 feet. The length of the rectangle is twice the width. Find the length and width of the rectangle.

Answer

25 ft, 50 ft

56. The length of the rectangle is three times the width. The perimeter of a rectangle is 72 feet. Find the length and width of the rectangle.

57. The length of the rectangle is three meters less than twice the width. The perimeter of a rectangle is 36 meters. Find the dimensions of the rectangle.

Answer

7 m, 11 m

58. The length of a rectangle is five inches more than twice the width. The perimeter is 34 inches. Find the length and width.

59. The perimeter of a triangle is 39 feet. One side of the triangle is one foot longer than the second side. The third side is two feet longer than the second side. Find the length of each side.

Answer

12 ft, 13 ft, 14 ft

60. The perimeter of a triangle is 35 feet. One side of the triangle is five feet longer than the second side. The third side is three feet longer than the second side. Find the length of each side.

61. One side of a triangle is twice the smallest side. The third side is five feet more than the shortest side. The perimeter is 17 feet. Find the lengths of all three sides.

Answer

3 ft, 6 ft, 8 ft

62. One side of a triangle is three times the smallest side. The third side is three feet more than the shortest side. The perimeter is 13 feet. Find the lengths of all three sides.

63. The perimeter of a rectangular field is 560 yards. The length is 40 yards more than the width. Find the length and width of the field.

Answer

120 yd, 160 yd

64. The perimeter of a rectangular atrium is 160 feet. The length is 16 feet more than the width. Find the length and width of the atrium.

65. A rectangular parking lot has perimeter 250 feet. The length is five feet more than twice the width. Find the length and width of the parking lot.

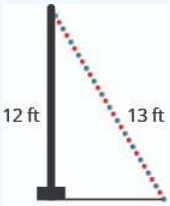
Answer

40 ft, 85 ft

66. A rectangular rug has perimeter 240 inches. The length is 12 inches more than twice the width. Find the length and width of the rug.

In the following exercises, solve. Approximate answers to the nearest tenth, if necessary.

67. A 13-foot string of lights will be attached to the top of a 12-foot pole for a holiday display as shown. How far from the base of the pole should the end of the string of lights be anchored?



Answer

5 feet

68. Pam wants to put a banner across her garage door diagonally, as shown, to congratulate her son for his college graduation. The garage door is 12 feet high and 16 feet wide. How long should the banner be to fit the garage door?



69. Chi is planning to put a diagonal path of paving stones through her flower garden as shown. The flower garden is a square with side 10 feet. What will the length of the path be?



Answer

14.1 feet

70. Brian borrowed a 20-foot extension ladder to use when he paints his house. If he sets the base of the ladder six feet from the house as shown, how far up will the top of the ladder reach?



Everyday Math

71. Converting temperature While on a tour in Greece, Tatyana saw that the temperature was 40° Celsius. Solve for F in the formula $C = \frac{5}{9}(F - 32)$ to find the Fahrenheit temperature.

Answer

104°F

72. Converting temperature Yon was visiting the United States and he saw that the temperature in Seattle one day was 50° Fahrenheit. Solve for C in the formula $F = \frac{9}{5}C + 32$ to find the Celsius temperature

73. Christa wants to put a fence around her triangular flowerbed. The sides of the flowerbed are six feet, eight feet and 10 feet. How many feet of fencing will she need to enclose her flowerbed?

Answer

24 ft

74. Jose just removed the children's play set from his back yard to make room for a rectangular garden. He wants to put a fence around the garden to keep the dog out. He has a 50-foot roll of fence in his garage that he plans to use. To fit in the backyard, the width of the garden must be 10 feet. How long can he make the other side?

Writing Exercises

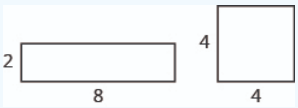
75. If you need to put tile on your kitchen floor, do you need to know the perimeter or the area of the kitchen? Explain your reasoning.

Answer

Answers will vary.

76. If you need to put a fence around your backyard, do you need to know the perimeter or the area of the backyard? Explain your reasoning.

77. Look at the two figures below.



a. Which figure looks like it has the larger area? Which looks like it has the larger perimeter?

b. Now calculate the area and perimeter of each figure. Which has the larger area? Which has the larger perimeter?

c. Were the results of part (b) the same as your answers in part (a)? Is that surprising to you?

Answer

a. Answers will vary. b. The areas are the same. The 2×8 rectangle has a larger perimeter than the 4×4 square.

c. Answers will vary.

78. Write a geometry word problem that relates to your life experience, then solve it and explain all your steps.

Self Check

a. After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
solve a formula for a specific variable.			
use formulas to solve geometry applications.			

b. What does this checklist tell you about your mastery of this section? What steps will you take to improve?

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2.5: The Pythagorean Theorem

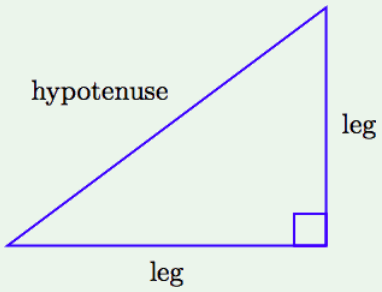
Pythagoras was a Greek mathematician and philosopher, born on the island of Samos (ca. 582 BC). He founded a number of schools, one in particular in a town in southern Italy called Crotone, whose members eventually became known as the Pythagoreans. The inner circle at the school, the Mathematikoi, lived at the school, rid themselves of all personal possessions, were vegetarians, and observed a strict vow of silence. They studied mathematics, philosophy, and music, and held the belief that numbers constitute the true nature of things, giving numbers a mystical or even spiritual quality.

Today, nothing is known of Pythagoras's writings, perhaps due to the secrecy and silence of the Pythagorean society. However, one of the most famous theorems in all of mathematics does bear his name, the Pythagorean Theorem.

Prior to revealing the contents of the Pythagorean Theorem, we pause to provide the definition of a right triangle and its constituent parts.

Right Triangle

A triangle with one right angle (90°) is called a **right triangle**. In the figure below, the right angle is marked with a little square.

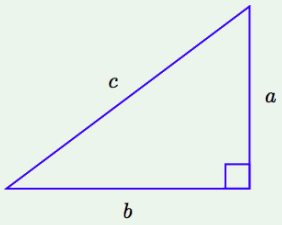


The side of the triangle that is directly opposite the right angle is called the **hypotenuse**. The sides of the triangle that include the right angle are called the *legs* of the right triangle.

Now we can state one of the most ancient theorems of mathematics, the *Pythagorean Theorem*.

Pythagorean Theorem

Let c represent the length of the *hypotenuse* of a right triangle, and let a and b represent the lengths of its legs, as pictured in the image that follows.



The relationship involving the legs and hypotenuse of the right triangle, given by

$$a^2 + b^2 = c^2,$$

is called the **Pythagorean Theorem**.

Here are two important observations.

Observations Regarding the Hypotenuse

Two important facts regarding the hypotenuse of the right triangle are:

1. The hypotenuse is the longest side of the triangle and lies directly opposite the right angle.
2. In the Pythagorean equation $a^2 + b^2 = c^2$, the hypotenuse lies by itself on one side of the equation.

The Pythagorean Theorem can **only** be applied to right triangles.

Let's look at a simple application of the Pythagorean Theorem.

Example 1

The legs of a right triangle measure 3 and 4 meters, respectively. Find the length of the hypotenuse.

Solution

Let's follow the *Requirements for Word Problem Solutions*.

1. *Set up a Variable Dictionary*. Let c represent the length of the hypotenuse, as pictured in the following sketch.

2. *Set up an Equation*. The Pythagorean Theorem says that

$$a^2 + b^2 = c^2.$$

In this example, the legs are known. Substitute 4 for a and 3 for b (3 for a and 4 for b works equally well) into the Pythagorean equation.

$$4^2 + 3^2 = c^2$$

3. *Solve the Equation*.

$$\begin{aligned} 4^2 + 3^2 &= c^2 && \text{The Pythagorean equation.} \\ 16 + 9 &= c^2 && \text{Exponents first: } 4^2 = 16 \text{ and } 3^2 = 9. \\ 25 &= c^2 && \text{Add: } 16 + 9 = 25. \\ 5 &= c && \text{Take the nonnegative square root.} \end{aligned}$$

Technically, there are two answers to $c^2 = 25$, i.e., $c = -5$ or $c = 5$. However, c represents the hypotenuse of the right triangle and must be nonnegative. Hence, we must choose c = 5.

4. *Answer the Question*. The hypotenuse has length 5 meters.

5. *Look Back*. Do the numbers satisfy the Pythagorean Theorem? The sum of the squares of the legs should equal the square of the hypotenuse. Let's check.

$$\begin{aligned} 4^2 + 3^2 &= 5^2 \\ 16 + 9 &= 25 \\ 25 &= 25 \end{aligned}$$

All is well!

Exercise

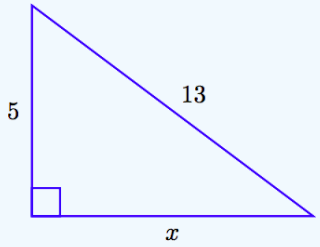
The legs of a right triangle measure 5 and 12 feet, respectively. Find the length of the hypotenuse.

Answer

13 feet

Example 2

Given the following right triangle, find the length of the missing side.



Solution

Note that the hypotenuse (across from the right angle) has length 13. This quantity should lie on one side of the Pythagorean equation all by itself. The sum of the squares of the legs go on the other side. Hence,

$$5^2 + x^2 = 13^2$$

Solve the equation for x.

$$\begin{aligned} 25 + x^2 &= 169 && \text{Exponents first: } 5^2 = 25 \text{ and } 13^2 = 169. \\ 25 + x^2 - 25 &= 169 - 25 && \text{Subtract 25 from both sides.} \\ x^2 &= 144 && \text{Simplify both sides.} \\ x &= 12 && \text{Take the nonnegative square root of 144.} \end{aligned}$$

Exercise

The hypotenuse of a right triangle measures 25 centimeters. One leg of the right triangle measures 24 centimeters. Find the length of the remaining leg.

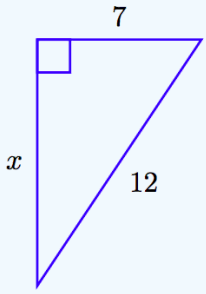
Answer

7 centimeters

Perfect squares are nice, but not required.

Example 3

Given the following right triangle, find the **exact** length of the missing side.



Solution

Note that the hypotenuse (across from the right angle) has length 12. This quantity should lie on one side of the Pythagorean equation all by itself. The sum of the squares of the legs go on the other side. Hence,

$$x^2 + 7^2 = 12^2$$

Solve the equation for x.

$$\begin{aligned} x^2+49 &= 144 && \text{Exponents first: } 7^2=49 \text{ and } 12^2=144. \\ x^2+49-49 &= 144-49 && \text{Subtract 49 from both sides.} \\ x^2 &= 95 && \text{Simplify both sides.} \\ x &= \sqrt{95} && \text{Take the nonnegative square root of 95.} \end{aligned}$$

Hence, the exact length of the missing side is $\sqrt{95}$.

? Exercise

The hypotenuse and one leg of a right triangle measure 9 and 7 inches, respectively. Find the length of the remaining leg.

Answer

Add texts here. Do not delete this text first.

Important Observation

Any attempt to use your calculator to approximate 95 in Example 3 would be an error as the instructions asked for an **exact** answer.

Sometimes an approximate answer is desired, particularly in applications.

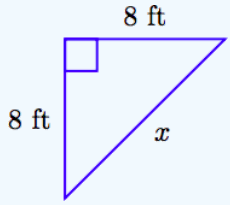
✓ Example 4

Ginny want to create a vegetable garden in the corner of her yard in the shape of a right triangle. She cuts two boards of length 8 feet which will form the legs of her garden. Find the length of board she should cut to form the hypotenuse of her garden, correct to the nearest tenth of a foot.

Solution

We follow the *Requirements for Word Problem Solutions*.

1. *Set up a Variable Dictionary*. We begin with a labeled sketch. Let x represent the length of the unknown hypotenuse.



2. *Set Up an Equation*. The hypotenuse is isolated on one side of the Pythagorean equation.

$$x^2 = 8^2+8^2$$

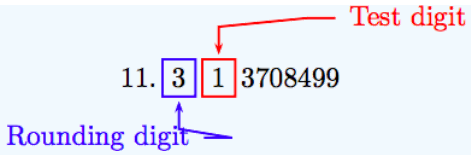
3. *Solve the Equation*.

$$\begin{aligned} x^2 &= 8^2+8^2 && \text{The Pythagorean equation.} \\ x^2 &= 64+64 && \text{Exponents first: } 8^2=64 \text{ and } 8^2=64. \\ x^2 &= 128 && \text{Add: } 64+64=128. \\ x &= \sqrt{128} && \text{Take the nonnegative square root.} \end{aligned}$$

4. *Answer the Question*. The exact length of the hypotenuse is $\sqrt{128}$ feet, but we're asked to find the hypotenuse to the nearest tenth of a foot. Using a calculator, we find an approximation for $\sqrt{128}$.

$$\sqrt{128} \approx 11.313708499$$

To round to the nearest tenth, first identify the rounding and test digits.



The test digit is less than five. So we leave the rounding digit alone and truncate. Therefore, correct to the nearest tenth of a foot, the length of the hypotenuse is approximately 11.3 feet.

5. *Look Back*. The sum of the squares of the legs is

$$\begin{aligned} 8^2+8^2 &= 64+64 \\ &= 128. \end{aligned}$$

The square of the hypotenuse is

$$(11.3)^2 = 127.69$$

These are almost the same, the discrepancy due to the fact that we rounded to find an approximation for the hypotenuse.

? Exercise

A 15 foot ladder leans against the wall of a building. The base of the ladder lies 5 feet from the base of the wall. How high up the wall does the top of the ladder reach? Round your answer to the nearest tenth of a foot.

Answer

14.1 feet

Exercises

In Exercises 1-16, your solutions should include a well-labeled sketch.

1. The length of one leg of a right triangle is 15 meters, and the length of the hypotenuse is 25 meters. Find the exact length of the other leg.

2. The length of one leg of a right triangle is 7 meters, and the length of the hypotenuse is 25 meters. Find the exact length of the other leg.

3. The lengths of two legs of a right triangle are 12 meters and 16 meters. Find the exact length of the hypotenuse.

4. The lengths of two legs of a right triangle are 9 meters and 12 meters. Find the exact length of the hypotenuse.

5. The length of one leg of a right triangle is 13 meters, and the length of the hypotenuse is 22 meters. Find the exact length of the other leg.

6. The length of one leg of a right triangle is 6 meters, and the length of the hypotenuse is 15 meters. Find the exact length of the other leg.

7. The lengths of two legs of a right triangle are 2 meters and 21 meters. Find the exact length of the hypotenuse.

8. The lengths of two legs of a right triangle are 7 meters and 8 meters. Find the exact length of the hypotenuse.

9. The length of one leg of a right triangle is 12 meters, and the length of the hypotenuse is 19 meters. Find the exact length of the other leg.

10. The length of one leg of a right triangle is 5 meters, and the length of the hypotenuse is 10 meters. Find the exact length of the other leg.

11. The lengths of two legs of a right triangle are 6 meters and 8 meters. Find the exact length of the hypotenuse.

12. The lengths of two legs of a right triangle are 5 meters and 12 meters. Find the exact length of the hypotenuse.

13. The length of one leg of a right triangle is 6 meters, and the length of the hypotenuse is 10 meters. Find the exact length of the other leg.

14. The length of one leg of a right triangle is 9 meters, and the length of the hypotenuse is 15 meters. Find the exact length of the other leg.

15. The lengths of two legs of a right triangle are 6 meters and 22 meters. Find the exact length of the hypotenuse.

16. The lengths of two legs of a right triangle are 9 meters and 19 meters. Find the exact length of the hypotenuse.

In Exercises 17-24, your solutions should include a well-labeled sketch.

17. The lengths of two legs of a right triangle are 3 meters and 18 meters. Find the length of the hypotenuse. Round your answer to the nearest hundredth.

18. The lengths of two legs of a right triangle are 10 feet and 16 feet. Find the length of the hypotenuse. Round your answer to the nearest tenth.

19. The length of one leg of a right triangle is 2 meters, and the length of the hypotenuse is 17 meters. Find the length of the other leg. Round your answer to the nearest tenth.

20. The length of one leg of a right triangle is 4 meters, and the length of the hypotenuse is 12 meters. Find the length of the other leg. Round your answer to the nearest hundredth.

21. The lengths of two legs of a right triangle are 15 feet and 18 feet. Find the length of the hypotenuse. Round your answer to the nearest hundredth.

22. The lengths of two legs of a right triangle are 6 feet and 13 feet. Find the length of the hypotenuse. Round your answer to the nearest tenth.

23. The length of one leg of a right triangle is 4 meters, and the length of the hypotenuse is 8 meters. Find the length of the other leg. Round your answer to the nearest hundredth.

24. The length of one leg of a right triangle is 3 meters, and the length of the hypotenuse is 15 meters. Find the length of the other leg. Round your answer to the nearest tenth.

---

25. Greta and Fritz are planting a 13-meter by 18-meter rectangular garden, and are laying it out using string. They would like to know the length of a diagonal to make sure that right angles are formed. Find the length of a diagonal. Round your answer to the nearest hundredth. Your solution should include a well-labeled sketch.

26. Markos and Angelina are planting an 11- meter by 19-meter rectangular garden, and are laying it out using string. They would like to know the length of a diagonal to make sure that right angles are formed. Find the length of a diagonal. Round your answer to the nearest tenth. Your solution should include a well-labeled sketch.

27. The base of a 24-meter long guy wire is located 10 meters from the base of the telephone pole that it is anchoring. How high up the pole does the guy wire reach? Round your answer to the nearest hundredth. Your solution should include a well-labeled sketch.

28. The base of a 30-foot long guy wire is located 9 feet from the base of the telephone pole that it is anchoring. How high up the pole does the guy wire reach? Round your answer to the nearest hundredth. Your solution should include a well-labeled sketch.

---

29. **Hiking Trail.** A hiking trail runs due south for 8 kilometers, then turns west for about 15 kilometers, and then heads northeast on a direct path to the starting point. How long is the entire trail?

30. **Animal Trail.** An animal trail runs due east from a watering hole for 12 kilometers, then goes north for 5 kilometers. Then the trail turns southwest on a direct path back to the watering hole. How long is the entire trail?

31. **Upper Window.** A 10-foot ladder leans against the wall of a house. How close to the wall must the bottom of the ladder be in order to reach a window 8 feet above the ground?

32. **How high?** A 10-foot ladder leans against the wall of a house. How high will the ladder be if the bottom of the ladder is 4 feet from the wall? Round your answer to the nearest tenth.

Answers

1. 20 meters
3. 20 meters
5. $\sqrt{315}$ meters
7. $\sqrt{445}$ meters
9. $\sqrt{217}$ meters
11. 10 meters
13. 8 meters
15. $\sqrt{520}$ meters
17. 18.25 meters
19. 16.9 meters
21. 23.43 feet
23. 6.93 meters
25. 22.20 meters
27. 21.82 meters
29. 40 kilometers
31. 6 ft.

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2.6: Solve Mixture and Uniform Motion Applications (optional practice)

Learning Objectives

By the end of this section, you will be able to:

- Solve coin word problems
- Solve ticket and stamp word problems
- Solve mixture word problems
- Solve uniform motion applications

Before you get started, take this readiness quiz.

1. Simplify: $0.25x + 0.10(x + 4)$.
   If you missed this problem, review [link].
2. The number of adult tickets is three more than twice the number of children tickets. Let $c$represent the number of children tickets. Write an expression for the number of adult tickets.
   If you missed this problem, review [link].
3. Convert 4.2% to a decimal.
   If you missed this problem, review [link].

Solve Coin Word Problems

Using algebra to find the number of nickels and pennies in a piggy bank may seem silly. You may wonder why we just don't open the bank and count them. But this type of problem introduces us to some techniques that will be useful as we move forward in our study of mathematics.



If we have a pile of dimes, how would we determine its value? If we count the number of dimes, we'll know how many we have—the *number* of dimes. But this does not tell us the *value* of all the dimes. Say we counted 23 dimes, how much are they worth? Each dime is worth \$0.10—that is the *value* of one dime. To find the total value of the pile of 23 dimes, multiply 23 by \$0.10 to get \$2.30.

The number of dimes times the value of each dime equals the total value of the dimes.

$$\begin{aligned} number \cdot value &= total\, value \\ 23 \cdot \$0.10 &= \$2.30 \end{aligned}$$

This method leads to the following model.

TOTAL VALUE OF COINS

For the same type of coin, the total value of a number of coins is found by using the model

$$number \cdot value = total\, value$$

- *number* is the number of coins
- *value* is the value of each coin
- *total value* is the total value of all the coins

If we had several types of coins, we could continue this process for each type of coin, and then we would know the total value of each type of coin. To get the total value of *all* the coins, add the total value of each type of coin.

Example 2.6.1

Jesse has $3.02 worth of pennies and nickels in his piggy bank. The number of nickels is three more than eight times the number of pennies. How many nickels and how many pennies does Jesse have?

Answer

Step 1. Read the problem. Determine the types of coins involved. Create a table. Write in the value of each type of coin.	pennies and nickels Pennies are worth $0.10. Nickels are worth $0.05.
Step 2. Identify what we are looking for.	the number of pennies and nickels
Step 3. Name. Represent the number of each type of coin using variables. The number of nickels is defined in terms of the number of pennies, so start with pennies. The number of nickels is three more than eight times the number of pennies.	Let $p =$ number of pennies. $8p + 3 =$ number of nickels
In the chart, multiply the number and the value to get the total value of each type of coin.	

Type	Number	• Value ($)	= Total Value ($)
pennies	p	0.01	$0.01p$
nickels	$8p + 3$	0.05	$0.05(8p + 3)$
			$3.02

Step 4. Translate. Write the equation by adding the total value of all the types of coins.	$0.01p + 0.05(8p + 3) = 3.02$
Step 5. Solve the equation.	$0.01p + 0.40p + 0.15 = 3.02$ $0.41p + 0.15 = 3.02$ $0.41p = 2.87$ $p = 7$ pennies
How many nickels?	$8p + 3$ $8(7) + 3$ 59 nickels
Step 6. Check the answer in the problem and make sure it makes sense. Jesse has 7 pennies and 59 nickels. Is the total value $3.02? $7(0.01) + 59(0.05) \stackrel{?}{=} 3.02$ $3.02 = 3.02$✓	

Example 2.6.2

Jesse has $6.55 worth of quarters and nickels in his pocket. The number of nickels is five more than two times the number of quarters. How many nickels and how many quarters does Jesse have?

Answer

Jess has 41 nickels and 18 quarters.

Example 2.6.3

Elane has $7.00 total in dimes and nickels in her coin jar. The number of dimes that Elane has is seven less than three times the number of nickels. How many of each coin does Elane have?

Answer

Elane has 22 nickels and 59 dimes.

The steps for solving a coin word problem are summarized below.

SOLVE COIN WORD PROBLEMS.

1. **Read** the problem. Make sure all the words and ideas are understood.
   - Determine the types of coins involved.
   - Create a table to organize the information.
     - Label the columns “type,” “number,” “value,” and “total value.”
     - List the types of coins.
     - Write in the value of each type of coin.
     - Write in the total value of all the coins.

Type	Number	• Value ($) =	Total Value ($)

2. **Identify** what you are looking for.
3. **Name** what you are looking for. Choose a variable to represent that quantity.
   - Use variable expressions to represent the number of each type of coin and write them in the table.
   - Multiply the number times the value to get the total value of each type of coin.
4. **Translate** into an equation.
   - It may be helpful to restate the problem in one sentence with all the important information. Then, translate the sentence into an equation.
   - Write the equation by adding the total values of all the types of coins.
5. **Solve** the equation using good algebra techniques.
6. **Check** the answer in the problem and make sure it makes sense.
7. **Answer** the question with a complete sentence.

Solve Ticket and Stamp Word Problems

Problems involving tickets or stamps are very much like coin problems. Each type of ticket and stamp has a value, just like each type of coin does. So to solve these problems, we will follow the same steps we used to solve coin problems.

Example 2.6.4

Danny paid $15.75 for stamps. The number of 49-cent stamps was five less than three times the number of 35-cent stamps. How many 49-cent stamps and how many 35-cent stamps did Danny buy?

Answer

Step 1. Determine the types of stamps involved.	49-cent stamps and 35-cent stamps
Step 2. Identify we are looking for.	the number of 49-cent stamps and the number of 35-cent stamps

Step 3. Write variable expressions to represent the number of each type of stamp.	Let x = number of 35-cent stamps.
"The number of 49-cent stamps was five less than three times the number of 35-cent stamps."	$3x-5=3x-5=$ number of 49-cent stamps

Type	Number	· Value($)	= Total Value ($)
49 cent stamps	$3x-5$	0.49	$0.49(3x-5)$
35 cent stamps	x	0.35	$0.35x$
			15.75

Step 4. Write the equation from the total values.	$0.49(3x-5)+0.35x=15.75$
Step 5. Solve the equation.	$1.47x-2.45+0.35x=15.75$ $1.82x-2.45=15.75$ $1.82x=18.2$
How many 49-cent stamps?	$x=10$ 35-cent stamps $3x-5$ $3(10)-5$ 25 49-cent stamps
Step 6. Check. $10(0.35)+25(0.49)3.50+12.2515.75=?=?$ $=15.7515.7515.75\checkmark 10(0.35)+25(0.49)=?15.753.50+12.25=?$ $15.7515.75=15.75\checkmark$	
Step 7. Answer the question with a complete sentence.	Danny bought ten 35-cent stamps and twenty-five 49-cent stamps.

Example 2.6.5

Eric paid $19.88 for stamps. The number of 49-cent stamps was eight more than twice the number of 35-cent stamps. How many 49-cent stamps and how many 35-cent stamps did Eric buy?

Answer

Eric bought thirty-two 49-cent stamps and twelve 35-cent stamps.

Example 2.6.6

Kailee paid $14.74 for stamps. The number of 49-cent stamps was four less than three times the number of 20-cent stamps. How many 49-cent stamps and how many 20-cent stamps did Kailee buy?

Answer

Kailee bought twenty-six 49-cent stamps and ten 20-cent stamps.

In most of our examples so far, we have been told that one quantity is four more than twice the other, or something similar. In our next example, we have to relate the quantities in a different way.

Suppose Aniket sold a total of 100 tickets. Each ticket was either an adult ticket or a child ticket. If he sold 20 child tickets, how many adult tickets did he sell?

Did you say "80"? How did you figure that out? Did you subtract 20 from 100?

If he sold 45 child tickets, how many adult tickets did he sell?

Did you say "55"? How did you find it? By subtracting 45 from 100?

Now, suppose Aniket sold x child tickets. Then how many adult tickets did he sell? To find out, we would follow the same logic we used above. In each case, we subtracted the number of child tickets from 100 to get the number of adult tickets. We now do the same with x.

We have summarized this in the table.

Child tickets	Adult tickets
20	80
45	55
75	25
x	$100 - x$

We will apply this technique in the next example.

Example 2.6.7

A whale-watching ship had 40 paying passengers on board. The total revenue collected from tickets was $1,196. Full-fare passengers paid $32 each and reduced-fare passengers paid $26 each. How many full-fare passengers and how many reduced-fare passengers were on the ship?

Answer

Step 1. Determine the types of tickets involved.	full-fare tickets and reduced-fare tickets
Step 2. Identify what we are looking for.	the number of full-fare tickets and reduced-fare tickets
Step 3. Name. Represent the number of each type of ticket using variables.	Let f = the number of full-fare tickets. $40-f=40-f=$ the number of reduced-fare tickets
We know the total number of tickets sold was 40. This means the number of reduced-fare tickets is 40 less the number of full-fare tickets. Multiply the number times the value to get the total value of each type of ticket.	
	<table><tr><th>Type</th><th>Number</th><th>·</th><th>Value ($)</th><th>=</th><th>Total Value ($)</th></tr><tr><td>Full-fare</td><td>f</td><td></td><td>32</td><td></td><td>$32f$</td></tr><tr><td>Reduced-fare</td><td>$40-f$</td><td></td><td>26</td><td></td><td>$26(40-f)$</td></tr><tr><td></td><td></td><td></td><td></td><td></td><td>1,196</td></tr></table>
Step 4. Translate. Write the equation by adding the total values of each type of ticket.	$32f + 26(40 - f) = 1{,}196$
Step 5. Solve the equation.	$32f + 1{,}040 - 26f = 1{,}196$ $6f = 156$ $f = 26$ full-fare tickets
How many reduced-fare?	$40 - f$ $40 - 26$ 14 reduced-fare tickets
Step 6. Check the answer. There were 26 full-fare tickets at $32 each and 14 reduced-fare tickets at $26 each. Is the total value $116? $26 \cdot 32 + 14 \cdot 26 = 832 + 364 = 1{,}196$ ✓ $26 \cdot 32 + 83214 \cdot 26 = 364 = 1{,}196$ ✓	
Step 7. Answer the question.	They sold 26 full-fare and 14 reduced-fare tickets.

Example 2.6.8

During her shift at the museum ticket booth, Leah sold 115 tickets for a total of $1,163. Adult tickets cost $12 and student tickets cost $5. How many adult tickets and how many student tickets did Leah sell?

Answer

84 adult tickets, 31 student tickets

Example 2.6.9

Galen sold 810 tickets for his church's carnival for a total revenue of $2,820. Children's tickets cost $3 each and adult tickets cost $5 each. How many children's tickets and how many adult tickets did he sell?

Answer

615 children's tickets and 195 adult tickets

Solve Mixture Word Problems

Now we'll solve some more general applications of the mixture model. In mixture problems, we are often mixing two quantities, such as raisins and nuts, to create a mixture, such as trail mix. In our tables we will have a row for each item to be mixed as well as one for the final mixture.

Example 2.6.10

Henning is mixing raisins and nuts to make 25 pounds of trail mix. Raisins cost $4.50 a pound and nuts cost $8 a pound. If Henning wants his cost for the trail mix to be $6.60 a pound, how many pounds of raisins and how many pounds of nuts should he use?

Answer

Step 1. Determine what is being mixed.	The 25 pounds of trail mix will come from mixing raisins and nuts.
Step 2. Identify what we are looking for.	the number of pounds of raisins and nuts
Step 3. Represent the number of each type of ticket using variables. As before, we fill in a chart to organize our information. We enter the price per pound for each item. We multiply the number times the value to get the total value.	Let x=x= number of pounds of raisins. 25−x=25−x= number of pounds of nuts

Type	Number of pounds	· Price per Pound ($)	= Total Value ($)
Raisins	x	4.50	$4.5x$
Nuts	$25-x$	8	$8(25-x)$
Trail mix	25	6.60	25(6.60)

Notice that the last column in the table gives the information for the total amount of the mixture.	
Step 4. Translate into an equation.	The value of the raisins plus the value of the nuts will be the value of the trail mix.
Step 5. Solve the equation.	$4.5x + 8(25 - x) = 25(6.60)$
	$4.5x + 200 - 8x = 165$ $-3.5x = -35$ $x = 10$ pounds of raisins
Find the number of pounds of nuts.	$25 - x$ $25 - 10$ 15 pounds of nuts

Step 6. Check. 4.5(10)+8(15)45+120165=?=? =25(6.60)165165✓4.5(10)+8(15)=?25(6.60)45+120=? 165165=165✓	
Step 7. Answer the question.	Henning mixed ten pounds of raisins with 15 pounds of nuts.

? Example 2.6.11

Orlando is mixing nuts and cereal squares to make a party mix. Nuts sell for $7 a pound and cereal squares sell for $4 a pound. Orlando wants to make 30 pounds of party mix at a cost of $6.50 a pound, how many pounds of nuts and how many pounds of cereal squares should he use?

Answer

Orlando mixed five pounds of cereal squares and 25 pounds of nuts.

? Example 2.6.12

Becca wants to mix fruit juice and soda to make a punch. She can buy fruit juice for $3 a gallon and soda for $4 a gallon. If she wants to make 28 gallons of punch at a cost of $3.25 a gallon, how many gallons of fruit juice and how many gallons of soda should she buy?

Answer

Becca mixed 21 gallons of fruit punch and seven gallons of soda.

Solve Uniform Motion Applications

When you are driving down the interstate using your cruise control, the speed of your car stays the same—it is uniform. We call a problem in which the speed of an object is constant a **uniform motion** application. We will use the distance, rate, and time formula, $D = rt$, to compare two scenarios, such as two vehicles traveling at different rates or in opposite directions.

Our problem solving strategies will still apply here, but we will add to the first step. The first step will include drawing a diagram that shows what is happening in the example. Drawing the diagram helps us understand what is happening so that we will write an appropriate equation. Then we will make a table to organize the information, like we did for the coin, ticket, and stamp applications.

The steps are listed here for easy reference:

SOLVE A UNIFORM MOTION APPLICATION.

1. **Read** the problem. Make sure all the words and ideas are understood.
   - Draw a diagram to illustrate what is happening.
   - Create a table to organize the information.
     - Label the columns rate, time, distance.
     - List the two scenarios.
     - Write in the information you know.

	Rate	• Time	= Distance

2. **Identify** what you are looking for.
3. **Name** what you are looking for. Choose a variable to represent that quantity.

- Complete the chart.
- Use variable expressions to represent that quantity in each row.
- Multiply the rate times the time to get the distance.

4. **Translate** into an equation.
   - Restate the problem in one sentence with all the important information.
   - Then, translate the sentence into an equation.
5. **Solve** the equation using good algebra techniques.
6. **Check** the answer in the problem and make sure it makes sense.
7. **Answer** the question with a complete sentence.

?

Wayne and Dennis like to ride the bike path from Riverside Park to the beach. Dennis's speed is seven miles per hour faster than Wayne's speed, so it takes Wayne two hours to ride to the beach while it takes Dennis 1.5 hours for the ride. Find the speed of both bikers.

Answer

Step 1. Read the problem. Make sure all the words and ideas are understood.

Draw a diagram to illustrate what it happening. Shown below is a sketch of what is happening in the example.



Create a table to organize the information.

- Label the columns "Rate," "Time," and "Distance."
- List the two scenarios.
- Write in the information you know.

	Rate (mph)	•	Time (hrs)	=	Distance (miles)
Dennis			1.5		
Wayne			2		

Step 2. Identify what you are looking for.

You are asked to find the speed of both bikers.

Notice that the distance formula uses the word "rate," but it is more common to use "speed"

when we talk about vehicles in everyday English.

Step 3. Name what we are looking for. Choose a variable to represent that quantity.

Complete the chart Use variable expressions to represent that quantity in each row.
We are looking for the speed of the bikers. Let's let r represent Wayne's speed. Since Dennis' speed is 7 mph faster, we represent that as $r+7$

$r+7 = \text{Dennis' speed}$

$r = \text{Wayne's speed}$

Fill in the speeds into the chart.

	Rate (mph)	•	Time (hrs)	=	Distance (miles)
Dennis	$r+7$		1.5		
Wayne	r		2		

Multiply the rate times the time to get the distance.

	Rate (mph)	•	Time (hrs)	=	Distance (miles)
Dennis	$r+7$		1.5		$1.5(r+7)$
Wayne	r		2		$2r$

Step 4. Translate into an equation.

Restate the problem in one sentence with all the important information.

Then, translate the sentence into an equation.
The equation to model this situation will come from the relation between the distances. Look at the diagram we drew above. How is the distance travelled by Dennis related to the distance travelled by Wayne?
Since both bikers leave from Riverside and travel to the beach, they travel the same distance. So we write:

	distance traveled by Dennis	=	distance traveled by Wayne
Translate to an equation.	$1.5(r+7)$	=	$2r$

Step 5. Solve the equation using algebra techniques.

Now solve this equation.	$1.5(r+7)=2r$ $1.5r+10.5=2r$ $10.5=0.5r$ $21=r$ So Wayne's speed is 21 mph.
Find Dennis' speed.	$r+7$ $21+7$ 28 Dennis' speed 28 mph.

Step 6. Check the answer in the problem and make sure it makes sense.

$28 \text{ mph}(1.5 \text{ hours}) = 42 \text{ miles}\checkmark$
$21 \text{ mph}(2 \text{ hours}) = 42 \text{ miles}\checkmark$

Step 7. Answer the question with a complete sentence.

Wayne rode at 21 mph and Dennis rode at 28 mph.

Restate the problem in one sentence with all the important information.

Then, translate the sentence into an equation.
The equation to model this situation will come from the relation between the distances. Look at the diagram we drew above. How is the distance travelled by Dennis related to the distance travelled by Wayne?
Since both bikers leave from Riverside and travel to the beach, they travel the same distance. So we write:

	distance traveled by Dennis	=	distance traveled by Wayne
Translate to an equation.	$1.5(r+7)$	=	$2r$

An express train and a local train leave Pittsburgh to travel to Washington, D.C. The express train can make the trip in four hours and the local train takes five hours for the trip. The speed of the express train is 12 miles per hour faster than the speed of the local train. Find the speed of both trains.

Answer

The speed of the local train is 48 mph and the speed of the express train is 60 mph.

Example 2.6.15

Jeromy can drive from his house in Cleveland to his college in Chicago in 4.5 hours. It takes his mother six hours to make the same drive. Jeromy drives 20 miles per hour faster than his mother. Find Jeromy's speed and his mother's speed.

Answer

Jeromy drove at a speed of 80 mph and his mother drove 60 mph.

In Example, we had two bikers traveling the same distance. In the next example, two people drive toward each other until they meet.

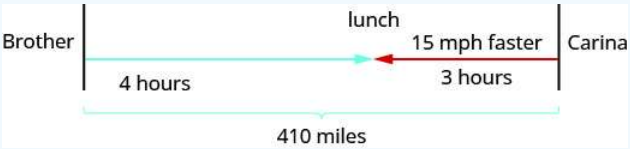
Example 2.6.16

Carina is driving from her home in Anaheim to Berkeley on the same day her brother is driving from Berkeley to Anaheim, so they decide to meet for lunch along the way in Buttonwillow. The distance from Anaheim to Berkeley is 395 miles. It takes Carina three hours to get to Buttonwillow, while her brother drives four hours to get there. Carina's average speed is 15 miles per hour faster than her brother's average speed. Find Carina's and her brother's average speeds.

Answer

Step 1. Read the problem. Make sure all the words and ideas are understood.

Draw a diagram to illustrate what it happening. Below shows a sketch of what is happening in the example.



Create a table to organize the information.

- Label the columns rate, time, distance.
- List the two scenarios.
- Write in the information you know.

	Rate (mph)	•	Time (hrs)	=	Distance (miles)
Carina			3		
Brother			4		
					395

Step 2. Identify what we are looking for.

We are asked to find the average speeds of Carina and her brother.

Step 3. Name what we are looking for. Choose a variable to represent that quantity.

Complete the chart. Use variable expressions to represent that quantity in each row.
We are looking for their average speeds. Let's let r represent the average speed of Carina. Since the brother's speed is 15

mph faster, we represent that as $r+15$.
Fill in the speeds into the chart. Multiply the rate times the time to get the distance.

	Rate (mph)	• Time (hrs)	= Distance (miles)
Carina	$r+15$	3	$3(r+15)$
Brother	r	4	$4r$
			395

Step 4. Translate into an equation.

Restate the problem in one sentence with all the important information. Then, translate the sentence into an equation.
Again, we need to identify a relationship between the distances in order to write an equation. Look at the diagram we created above and notice the relationship between the distance Carina traveled and the distance her brother traveled.
The distance Carina traveled plus the distance her brother travel must add up to 410 miles. So we write:

	distance traveled by Carina	+	distance traveled by her brother	= 395
Translate to an equation.	$3(r+15)$	+	$4r$	$=395$

Step 5. Solve the equation using algebra techniques.

Now solve this equation.	$3(r+15)+4r=395$ $3r+45+4r=395$ $7r+45=395$ $7r=350$ $r=50$ So Carina's brother's speed was 50 mph.
Carina's speed is r+15.r+15.	$r+15$ $50+15$ 65 Her brother's speed was 65 mph.

Step 6. Check the answer in the problem and make sure it makes sense.

$$\begin{aligned} \text{Carina drove} \quad & 65 \text{ mph}(3 \text{ hours}) = \underline{195 \text{ miles}} \\ \text{Her brother drove} \quad & 50 \text{ mph}(4 \text{ hours}) = \underline{200 \text{ miles}} \\ & \qquad\qquad\qquad\quad 395 \text{ miles}\checkmark \end{aligned}$$

Step 7. Answer the question with a complete sentence.

Carina drove 65 mph and her brother 50 mph.

my drove at a speed of 80 mph and his mother drove 60 mph.

? Example 2.6.17

Christopher and his parents live 115 miles apart. They met at a restaurant between their homes to celebrate his mother's birthday. Christopher drove one and a half hours while his parents drove one hour to get to the restaurant. Christopher's average speed was ten miles per hour faster than his parents' average speed. What were the average speeds of Christopher and of his parents as they drove to the restaurant?

Answer

Christopher's speed was 50 mph and his parents' speed was 40 mph.

? Example 2.6.18

Ashley goes to college in Minneapolis, 234 miles from her home in Sioux Falls. She wants her parents to bring her more winter clothes, so they decide to meet at a restaurant on the road between Minneapolis and Sioux Falls. Ashley and her parents both drove two hours to the restaurant. Ashley's average speed was seven miles per hour faster than her parents' average speed. Find Ashley's and her parents' average speed.

Answer

Ashley's parents drove 55 mph and Ashley drove 62 mph.

As you read the next example, think about the relationship of the distances traveled. Which of the previous two examples is more similar to this situation?

? Example 2.6.19

Two truck drivers leave a rest area on the interstate at the same time. One truck travels east and the other one travels west. The truck traveling west travels at 70 mph and the truck traveling east has an average speed of 60 mph. How long will they travel before they are 325 miles apart?

Answer

Step 1. Read the problem. Make all the words and ideas are understood.

Draw a diagram to illustrate what it happening.



Create a table to organize the information.

- Label the columns rate, time, distance.
- List the two scenarios.
- Write in the information you know.

	Rate (mph)	•	Time (hrs)	=	Distance (miles)
West	70				
East	60				
					325

Step 2. Identify what we are looking for.

We are asked to find the amount of time the trucks will travel until they are 325 miles apart.

Step 3. Name what we are looking for. Choose a variable to represent that quantity.

Complete the chart. Use variable expressions to represent that quantity in each row.
We are looking for the time travelled. Both trucks will travel the same amount of time.
Let's call the time t. Since their speeds are different, they will travel different distances.

Multiply the rate times the time to get the distance.

	Rate (mph)	• Time (hrs)	= Distance (miles)
West	70	t	$70t$
East	60	t	$60t$
			325

Step 4. Translate into an equation.

Restate the problem in one sentence with all the important information. Then, translate the sentence into an equation.
We need to find a relation between the distances in order to write an equation. Looking at the diagram, what is the relationship between the distances each of the trucks will travel?
The distance travelled by the truck going west plus the distance travelled by the truck going east must add up to 325 miles. So we write:

	distance traveled by westbound truck	+	distance traveled by eastbound truck	= 325
Translate to an equation.	$70t$	+	$60t$	= 325

Step 5. Solve the equation using algebra techniques.

$$\begin{aligned} \text{Now solve this equation} \qquad 70t+60t &= 325 \\ 130t &= 325 \\ t &= 2.5 \end{aligned}$$

So it will take the trucks 2.5 hours to be 325 miles apart.

Step 6. Check the answer in the problem and make sure it makes sense.

$$\begin{aligned} \text{Truck going West} \quad 70 \text{ mph}(2.5 \text{ hours}) &= 175 \text{ miles} \\ \text{Truck going East} \quad 60 \text{ mph}(2.5 \text{ hours}) &= \underline{150 \text{ miles}} \\ & \quad 325 \text{ miles}\checkmark \end{aligned}$$

Step 7. Answer the question with a complete sentence.
It will take the trucks 2.5 hours to be 325 miles apart.

? Example 2.6.20

Pierre and Monique leave their home in Portland at the same time. Pierre drives north on the turnpike at a speed of 75 miles per hour while Monique drives south at a speed of 68 miles per hour. How long will it take them to be 429 miles apart?

Answer

Pierre and Monique will be 429 miles apart in 3 hours.

? Example 2.6.21

Thanh and Nhat leave their office in Sacramento at the same time. Thanh drives north on I-5 at a speed of 72 miles per hour. Nhat drives south on I-5 at a speed of 76 miles per hour. How long will it take them to be 330 miles apart?

Answer

Thanh and Nhat will be 330 miles apart in 2.2 hours.

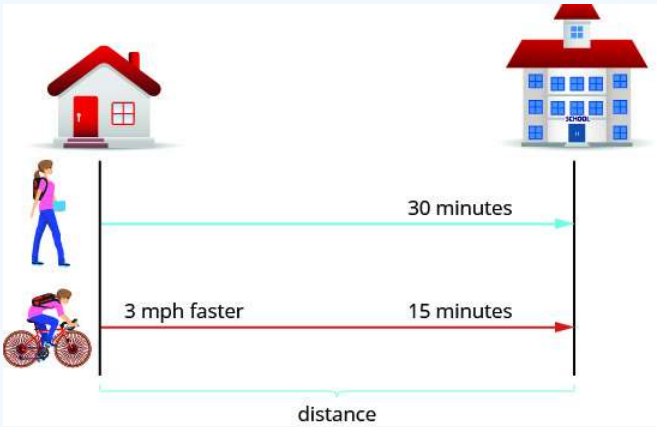
It is important to make sure that the units match when we use the distance rate and time formula. For instance, if the rate is in miles per hour, then the time must be in hours.

Example 2.6.22

When Naoko walks to school, it takes her 30 minutes. If she rides her bike, it takes her 15 minutes. Her speed is three miles per hour faster when she rides her bike than when she walks. What is her speed walking and her speed riding her bike?

Answer

First, we draw a diagram that represents the situation to help us see what is happening.



We are asked to find her speed walking and riding her bike. Let's call her walking speed r. Since her biking speed is three miles per hour faster, we will call that speed $r+3$. We write the speeds in the chart.

The speed is in miles per hour, so we need to express the times in hours, too, in order for the units to be the same. Remember, 1 hour is 60 minutes. So:

$$30 \text{ minutes is } \frac{30}{60} \text{ or } \frac{1}{2} \text{ hour}$$
$$15 \text{ minutes is } \frac{15}{60} \text{ or } \frac{1}{4} \text{ hour}$$

We write the times in the chart.

Next, we multiply rate times time to fill in the distance column.

	Rate (mph)	• Time (hrs)	= Distance (miles)
Walk	r	$\frac{1}{2}$	$\frac{1}{2}r$
Bike	$r+3$	$\frac{1}{4}$	$\frac{1}{4}(r+3)$

The equation will come from the fact that the distance from Naoko's home to her school is the same whether she is walking or riding her bike.
So we say:

	distance walked = distance covered by bike
Translate to an equation.	$\frac{1}{2}r = \frac{1}{4}(r+3)$
Solve this equation.	$\frac{1}{2}r = \frac{1}{4}(r+3)$
Clear the fractions by multiplying by the LCD of all the fractions in the equation.	$8 \cdot \frac{1}{2}r = 8 \cdot \frac{1}{4}(r+3)$

Simplify.	$4r = 2(r + 3)$ $4r = 2r + 6$ $2r = 6$ $r = 3$ mph walking speed $r + 3$ biking speed $3 + 3$
	6

Let's check if this works.

$$\begin{aligned} \text{Walk 3 mph (0.5 hour)} &= 1.5 \text{ miles} \\ \text{Bike 6 mph (0.25 hour)} &= 1.5 \text{ miles} \end{aligned}$$

Yes, either way Naoko travels 1.5 miles to school.

Naoko's walking speed is 3 mph and her speed riding her bike is 6 mph.

Example 2.6.23

Suzy takes 50 minutes to hike uphill from the parking lot to the lookout tower. It takes her 30 minutes to hike back down to the parking lot. Her speed going downhill is 1.2 miles per hour faster than her speed going uphill. Find Suzy's uphill and downhill speeds.

Answer

Suzy's speed uphill is 1.81.8 mph and downhill is three mph.

Example 2.6.24

Llewyn takes 45 minutes to drive his boat upstream from the dock to his favorite fishing spot. It takes him 30 minutes to drive the boat back downstream to the dock. The boat's speed going downstream is four miles per hour faster than its speed going upstream. Find the boat's upstream and downstream speeds.

Answer

The boat's speed upstream is eight mph and downstream is 12 mph.

In the distance, rate and time formula, time represents the actual amount of elapsed time (in hours, minutes, etc.). If a problem gives us starting and ending times as clock times, we must find the elapsed time in order to use the formula.

Example 2.6.25

Cruz is training to compete in a triathlon. He left his house at 6:00 and ran until 7:30. Then he rode his bike until 9:45. He covered a total distance of 51 miles. His speed when biking was 1.6 times his speed when running. Find Cruz's biking and running speeds.

Answer

A diagram will help us model this trip.



Next, we create a table to organize the information. We know the total distance is 51 miles. We are looking for the rate of speed for each part of the trip. The rate while biking is 1.6 times the rate of running. If we let r = the rate running, then the rate biking is $1.6r$.

The times here are given as clock times. Cruz started from home at 6:00 a.m. and started biking at 7:30 a.m. So he spent 1.5 hours running. Then he biked from 7:30 a.m until 9:45 a.m. So he spent 2.25 hours biking.

Now, we multiply the rates by the times.

	Rate (mph)	• Time (hrs)	= Distance (miles)
run	r	1.5	$1.5r$
bike	$1.6r$	2.25	$2.25(1.6r)$
			51

By looking at the diagram, we can see that the sum of the distance running and the distance biking is 255 miles.

	distance running + distance biking = 51
Translate to an equation.	$1.5r + 2.25(1.6r) = 51$
Solve this equation.	$1.5r + 2.25(1.6r) = 51$ $1.5r + 3.6r = 51$ $5.1r = 51$ $r = 10$ mph running $1.6r$ biking speed $1.6(10)$ 16 mph
Check. Run 10 mph (1.5 hour) = 15 mi Bike 16 mph (2.25 hour) = 36 mi 51 mi	

Example 2.6.26

Hamilton loves to travel to Las Vegas, 255 miles from his home in Orange County. On his last trip, he left his house at 2:00 p.m. The first part of his trip was on congested city freeways. At 4:00 pm, the traffic cleared and he was able to drive through the desert at a speed 1.75 times faster than when he drove in the congested area. He arrived in Las Vegas at 6:30 p.m. How fast was he driving during each part of his trip?

Answer

Hamilton drove 40 mph in the city and 70 mph in the desert.

Example 2.6.27

Phuong left home on his bicycle at 10:00. He rode on the flat street until 11:15, then rode uphill until 11:45. He rode a total of 31 miles. His speed riding uphill was 0.6 times his speed on the flat street. Find his speed biking uphill and on the flat street.

Answer

Phuong rode uphill at a speed of 12 mph and on the flat street at 20 mph.

Key Concepts

- **Total Value of Coins**
  For the same type of coin, the total value of a number of coins is found by using the model
  number·value=totalvaluenumber·value=totalvalue
  - *number* is the number of coins
  - *value* is the value of each coin
  - *total value* is the total value of all the coins
- **How to solve coin word problems.**
  1. **Read** the problem. Make sure all the words and ideas are understood.
     Determine the types of coins involved.
     Create a table to organize the information.
     Label the columns “type,” “number,” “value,” “total value.”
     List the types of coins.
     Write in the value of each type of coin.
     Write in the total value of all the coins.

Type	Number • Value ($) =	Total Value ($)

  2. **Identify** what you are looking for.
  3. **Name** what you are looking for. Choose a variable to represent that quantity.
     Use variable expressions to represent the number of each type of coin and write them in the table.
     Multiply the number times the value to get the total value of each type of coin.
  4. **Translate** into an equation.
     It may be helpful to restate the problem in one sentence with all the important information. Then, translate the sentence into an equation.
     Write the equation by adding the total values of all the types of coins.
  5. **Solve** the equation using good algebra techniques.
  6. **Check** the answer in the problem and make sure it makes sense.
  7. **Answer** the question with a complete sentence.
- **How To Solve a Uniform Motion Application**
  1. **Read** the problem. Make sure all the words and ideas are understood.
     Draw a diagram to illustrate what it happening.
     Create a table to organize the information.
     Label the columns rate, time, distance.
     List the two scenarios.
     Write in the information you know.

	Rate • Time =	Distance

  2. **Identify** what you are looking for.
  3. **Name** what you are looking for. Choose a variable to represent that quantity.
     Complete the chart.
     Use variable expressions to represent that quantity in each row.
     Multiply the rate times the time to get the distance.
  4. **Translate** into an equation.
     Restate the problem in one sentence with all the important information.
     Then, translate the sentence into an equation.
  5. **Solve** the equation using good algebra techniques.

6. **Check** the answer in the problem and make sure it makes sense.
7. **Answer** the question with a complete sentence.

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2.6E: Exercises

Practice Makes Perfect

Solve Coin Word Problems

In the following exercises, solve each coin word problem.

1. Michaela has $2.05 in dimes and nickels in her change purse. She has seven more dimes than nickels. How many coins of each type does she have?

Answer

nine nickels, 16 dimes

2. Liliana has $2.10 in nickels and quarters in her backpack. She has 12 more nickels than quarters. How many coins of each type does she have?

3. In a cash drawer there is $125 in $5 and $10 bills. The number of $10 bills is twice the number of $5 bills. How many of each type of bill is in the drawer?

Answer

ten $10 bills, five $5 bills

4. Sumanta has $175 in $5 and $10 bills in his drawer. The number of $5 bills is three times the number of $10 bills. How many of each are in the drawer?

5. Chi has $11.30 in dimes and quarters. The number of dimes is three more than three times the number of quarters. How many of each are there?

Answer

63 dimes, 20 quarters

6. Alison has $9.70 in dimes and quarters. The number of quarters is eight more than four times the number of dimes. How many of each coin does she have?

7. Mukul has $3.75 in quarters, dimes and nickels in his pocket. He has five more dimes than quarters and nine more nickels than quarters. How many of each coin are in his pocket?

Answer

16 nickels, 12 dimes, seven quarters

8. Vina has $4.70 in quarters, dimes and nickels in her purse. She has eight more dimes than quarters and six more nickels than quarters. How many of each coin are in her purse?

Solve Ticket and Stamp Word Problems

In the following exercises, solve each ticket or stamp word problem.

9. The first day of a water polo tournament the total value of tickets sold was $17,610. One-day passes sold for $20 and tournament passes sold for $30. The number of tournament passes sold was 37 more than the number of day passes sold. How many day passes and how many tournament passes were sold?

Answer

330 day passes, 367 tournament passes

10. At the movie theater, the total value of tickets sold was $2,612.50. Adult tickets sold for $10 each and senior/child tickets sold for $7.50 each. The number of senior/child tickets sold was 25 less than twice the number of adult tickets sold. How many senior/child tickets and how many adult tickets were sold?

11. Julie went to the post office and bought both $0.41 stamps and $0.26 postcards. She spent $51.40. The number of stamps was 20 more than twice the number of postcards. How many of each did she buy?

Answer

40 postcards, 100 stamps

12. Jason went to the post office and bought both $0.41 stamps and $0.26 postcards and spent $10.28 The number of stamps was four more than twice the number of postcards. How many of each did he buy?

13. Hilda has $210 worth of $10 and $12 stock shares. The number of $10 shares is five more than twice the number of $12 shares. How many of each type of share does she have?

Answer

15 $10 shares, five $12 shares

14. Mario invested $475 in $45 and $25 stock shares. The number of $25 shares was five less than three times the number of $45 shares. How many of each type of share did he buy?

15. The ice rink sold 95 tickets for the afternoon skating session, for a total of $828. General admission tickets cost $10 each and youth tickets cost $8 each. How many general admission tickets and how many youth tickets were sold?

Answer

34 general, 61 youth

16. For the 7:30 show time, 140 movie tickets were sold. Receipts from the $13 adult tickets and the $10 senior tickets totaled $1,664. How many adult tickets and how many senior tickets were sold?

17. The box office sold 360 tickets to a concert at the college. The total receipts were $4,170. General admission tickets cost $15 and student tickets cost $10. How many of each kind of ticket was sold?

Answer

114 general, 246 student

18. Last Saturday, the museum box office sold 281 tickets for a total of $3,954. Adult tickets cost $15 and student tickets cost $12. How many of each kind of ticket was sold

Solve Mixture Word Problems

In the following exercises, solve each mixture word problem.

19. Macario is making 12 pounds of nut mixture with macadamia nuts and almonds. Macadamia nuts cost $9 per pound and almonds cost $5.25 per pound. How many pounds of macadamia nuts and how many pounds of almonds should Macario use for the mixture to cost $6.50 per pound to make?

Answer

Four pounds of macadamia nuts, eight pounds almonds

20. Carmen wants to tile the floor of his house. He will need 1,000 square feet of tile. He will do most of the floor with a tile that costs $1.50 per square foot,

21. Riley is planning to plant a lawn in his yard. He will need nine pounds of grass seed. He wants to mix Bermuda seed that costs $4.80 per pound with Fescue seed that costs $3.50 per pound. How much of each seed should he buy so that the overall cost will be $4.02 per pound?

Answer

3.63.6 lbs Bermuda seed, 5.45.4 lbs Fescue seed

22. Vartan was paid $25,000 for a cell phone app that he wrote and wants to invest it to save for his son's education. He wants to put some of the money into a bond that pays 4% annual interest and the rest into stocks that pay 9% annual interest. If he wants to earn 7.4% annual interest on the total amount, how much money should he invest in each account?

23. Vern sold his 1964 Ford Mustang for $55,000 and wants to invest the money to earn him 5.8% interest per year. He will put some of the money into Fund A that earns 3% per year and the rest in Fund B that earns 10% per year. How much should he invest into each fund if he wants to earn 5.8% interest per year on the total amount?

Answer

$33,000 in Fund A, $22,000 in Fund B

24. Dominic pays 7% interest on his $15,000 college loan and 12% interest on his $11,000 car loan. What average interest rate does he pay on the total $26,000 he owes? (Round your answer to the nearest tenth of a percent.)

25. Liam borrowed a total of $35,000 to pay for college. He pays his parents 3% interest on the $8,000 he borrowed from them and pays the bank 6.8% on the rest. What average interest rate does he pay on the total $35,000? (Round your answer to the nearest tenth of a percent.)

Answer

5.9

Solve Uniform Motion Applications

In the following exercises, solve.

26. Lilah is moving from Portland to Seattle. It takes her three hours to go by train. Mason leaves the train station in Portland and drives to the train station in Seattle with all Lilah's boxes in his car. It takes him 2.4 hours to get to Seattle, driving at 15 miles per hour faster than the speed of the train. Find Mason's speed and the speed of the train.

27. Kathy and Cheryl are walking in a fundraiser. Kathy completes the course in 4.8 hours and Cheryl completes the course in eight hours. Kathy walks two miles per hour faster than Cheryl. Find Kathy's speed and Cheryl's speed.

Answer

Kathy 5 mph, Cheryl 3 mph

28. Two busses go from Sacramento to San Diego. The express bus makes the trip in 6.8 hours and the local bus takes 10.2 hours for the trip. The speed of the express bus is 25 mph faster than the speed of the local bus. Find the speed of both busses.

29. A commercial jet and a private airplane fly from Denver to Phoenix. It takes the commercial jet 1.6 hours for the flight, and it takes the private airplane 2.6 hours. The speed of the commercial jet is 210 miles per hour faster than the speed of the private airplane. Find the speed of both airplanes to the nearest 10 mph.

Answer

commercial 540 mph, private plane 330 mph

30. Saul drove his truck three hours from Dallas towards Kansas City and stopped at a truck stop to get dinner. At the truck stop he met Erwin, who had driven four hours from Kansas City towards Dallas. The distance between Dallas and Kansas City is 542 miles, and Erwin's speed was eight miles per hour slower than Saul's speed. Find the speed of the two truckers.

31. Charlie and Violet met for lunch at a restaurant between Memphis and New Orleans. Charlie had left Memphis and drove 4.8 hours towards New Orleans. Violet had left New Orleans and drove two hours towards Memphis, at a speed 10 miles per hour faster than Charlie's speed. The distance between Memphis and New Orleans is 394 miles. Find the speed of the two drivers.

Answer

Violet 65 mph, Charlie 55 mph

32. Sisters Helen and Anne live 332 miles apart. For Thanksgiving, they met at their other sister's house partway between their homes. Helen drove 3.2 hours and Anne drove 2.8 hours. Helen's average speed was four miles per hour faster than Anne's. Find Helen's average speed and Anne's average speed.

33. Ethan and Leo start riding their bikes at the opposite ends of a 65-mile bike path. After Ethan has ridden 1.5 hours and Leo has ridden two hours, they meet on the path. Ethan's speed is six miles per hour faster than Leo's speed. Find the speed of the two bikers.

Answer

Ethan 22 mph, Leo 16 mph

34. Elvira and Aletheia live 3.1 miles apart on the same street. They are in a study group that meets at a coffee shop between their houses. It took Elvira half an hour and Aletheia two-thirds of an hour to walk to the coffee shop. Aletheia's speed is 0.6 miles per hour slower than Elvira's speed. Find both women's walking speeds.

35. DaMarcus and Fabian live 23 miles apart and play soccer at a park between their homes. DaMarcus rode his bike for three-quarters of an hour and Fabian rode his bike for half an hour to get to the park. Fabian's speed was six miles per hour faster than DaMarcus' speed. Find the speed of both soccer players.

Answer

DaMarcus 16 mph, Fabian 22 mph

36. Cindy and Richard leave their dorm in Charleston at the same time. Cindy rides her bicycle north at a speed of 18 miles per hour. Richard rides his bicycle south at a speed of 14 miles per hour. How long will it take them to be 96 miles apart?

37. Matt and Chris leave their uncle's house in Phoenix at the same time. Matt drives west on I-60 at a speed of 76 miles per hour. Chris drives east on I-60 at a speed of 82 miles per hour. How many hours will it take them to be 632 miles apart?

Answer

four hours

38. Two busses leave Billings at the same time. The Seattle bus heads west on I-90 at a speed of 73 miles per hour while the Chicago bus heads east at a speed of 79 miles an hour. How many hours will it take them to be 532 miles apart?

39. Two boats leave the same dock in Cairo at the same time. One heads north on the Mississippi River while the other heads south. The northbound boat travels four miles per hour. The southbound boat goes eight miles per hour. How long will it take them to be 54 miles apart?

Answer

4.5 hours

40. Lorena walks the path around the park in 30 minutes. If she jogs, it takes her 20 minutes. Her jogging speed is 1.5 miles per hour faster than her walking speed. Find Lorena's walking speed and jogging speed

41. Julian rides his bike uphill for 45 minutes, then turns around and rides back downhill. It takes him 15 minutes to get back to where he started. His uphill speed is 3.2 miles per hour slower than his downhill speed. Find Julian's uphill and downhill speed.

Answer

uphill 1.6 mph, downhill 4.8 mph

42. Cassius drives his boat upstream for 45 minutes. It takes him 30 minutes to return downstream. His speed going upstream is three miles per hour slower than his speed going downstream. Find his upstream and downstream speeds.

43. It takes Darline 20 minutes to drive to work in light traffic. To come home, when there is heavy traffic, it takes her 36 minutes. Her speed in light traffic is 24 miles per hour faster than her speed in heavy traffic. Find her speed in light traffic and in heavy traffic.

Answer

light traffic 54 mph, heavy traffic 30 mph

44. At 1:30, Marlon left his house to go to the beach, a distance of 7.6 miles. He rode his skateboard until 2:15, and then walked the rest of the way. He arrived at the beach at 3:00. Marlon's speed on his skateboard is 2.5 times his walking speed. Find his speed when skateboarding and when walking.

45. Aaron left at 9:15 to drive to his mountain cabin 108 miles away. He drove on the freeway until 10:45 and then drove on a mountain road. He arrived at 11:05. His speed on the freeway was three times his speed on the mountain road. Find Aaron's speed on the freeway and on the mountain road.

Answer

freeway 72 mph, mountain road 24 mph

46. Marisol left Los Angeles at 2:30 to drive to Santa Barbara, a distance of 95 miles. The traffic was heavy until 3:20. She drove the rest of the way in very light traffic and arrived at 4:20. Her speed in heavy traffic was 40 miles per hour slower than her speed in light traffic. Find her speed in heavy traffic and in light traffic.

47. Lizette is training for a marathon. At 7:00 she left her house and ran until 8:15 then she walked until 11:15. She covered a total distance of 19 miles. Her running speed was five miles per hour faster than her walking speed. Find her running and walking speeds.

Answer

running eight mph, walking three mph

Everyday Math

48. John left his house in Irvine at 8:35 a.m. to drive to a meeting in Los Angeles, 45 miles away. He arrived at the meeting at 9:50 a.m.. At 5:30 p.m. he left the meeting and drove home. He arrived home at 7:18 p.m.

ⓐ What was his average speed on the drive from Irvine to Los Angeles?

ⓑ What was his average speed on the drive from Los Angeles to Irvine?

ⓒ What was the total time he spent driving to and from this meeting?

49. Sarah wants to arrive at her friend's wedding at 3:00. The distance from Sarah's house to the wedding is 95 miles. Based on usual traffic patterns, Sarah predicts she can drive the first 15 miles at 60 miles per hour, the next 10 miles at 30 miles per hour, and the remainder of the drive at 70 miles per hour.

ⓐ How long will it take Sarah to drive the first 15 miles?

ⓑ How long will it take Sarah to drive the next 10 miles?

ⓒ How long will it take Sarah to drive the rest of the trip?

ⓓ What time should Sarah leave her house?

Answer

ⓐ 15 minutes ⓑ 20 minutes ⓒ one hour (d) 1:25

Writing Exercises

50. Suppose you have six quarters, nine dimes, and four pennies. Explain how you find the total value of all the coins.

51. Do you find it helpful to use a table when solving coin problems? Why or why not?

Answer

Answers will vary.

52. In the table used to solve coin problems, one column is labeled "number" and another column is labeled "value." What is the difference between the "number" and the "value"?

53. When solving a uniform motion problem, how does drawing a diagram of the situation help you?

Answer

Answers will vary.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
solve coin word problems.			
solve ticket and stamp word problems.			
solve mixture word problems.			
solve uniform motion applications.			

ⓑ On a scale of 1-10, how would you rate your mastery of this section in light of your responses on the checklist? How can you improve this?

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2.7: Solve Applications with Linear Inequalities

Learning Objectives

By the end of this section, you will be able to:

- Solve applications with linear inequalities

Note

Before you get started, take this readiness quiz.

1. Write as an inequality: x is at least 30.
   If you missed this problem, review Exercise 2.7.34.
2. Solve $8-3y<41$.
   If you missed this problem, review Exercise 2.7.22.

Solve Applications with Linear Inequalities

Many real-life situations require us to solve inequalities. In fact, inequality applications are so common that we often do not even realize we are doing algebra. For example, how many gallons of gas can be put in the car for $20? Is the rent on an apartment affordable? Is there enough time before class to go get lunch, eat it, and return? How much money should each family member's holiday gift cost without going over budget?

The method we will use to solve applications with linear inequalities is very much like the one we used when we solved applications with equations. We will read the problem and make sure all the words are understood. Next, we will identify what we are looking for and assign a variable to represent it. We will restate the problem in one sentence to make it easy to translate into an inequality. Then, we will solve the inequality.

Example 2.7.1

Emma got a new job and will have to move. Her monthly income will be $5,265. To qualify to rent an apartment, Emma's monthly income must be at least three times as much as the rent. What is the highest rent Emma will qualify for?

Solution

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	the highest rent Emma will qualify for
Step 3. Name what we are looking for.	
Choose a variable to represent that quantity.	Let r = rent
Step 4. Translate into an inequality.	
First write a sentence that gives the information to find it.	Emma's monthly income must be at least three times the rent.
Step 5. Solve the inequality.	$5265 \geq 3r$
Remember, $a>x$ has the same meaning as $x<a$	$1755 \geq r$ $r \leq 1755$
Step 6. Check the answer in the problem and make sure it makes sense. A maximum rent of $1,755 seems reasonable for an income of $5,265.	
Step 7. Answer the question with a complete sentence.	The maximum rent is $1,755.

Try It 2.7.2

Alan is loading a pallet with boxes that each weighs 45 pounds. The pallet can safely support no more than 900 pounds. How many boxes can he safely load onto the pallet?

Answer

There can be no more than 20 boxes.

Try It 2.7.3

The elevator in Yehire's apartment building has a sign that says the maximum weight is 2,100 pounds. If the average weight of one person is 150 pounds, how many people can safely ride the elevator?

Answer

A maximum of 14 people can safely ride in the elevator.

Sometimes an application requires the solution to be a whole number, but the algebraic solution to the inequality is not a whole number. In that case, we must round the algebraic solution to a whole number. The context of the application will determine whether we round up or down. To check applications like this, we will round our answer to a number that is easy to compute with and make sure that number makes the inequality true.

Example 2.7.4

Dawn won a mini-grant of $4,000 to buy tablet computers for her classroom. The tablets she would like to buy cost $254.12 each, including tax and delivery. What is the maximum number of tablets Dawn can buy?

Solution

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	the maximum number of tablets Dawn can buy
Step 3. Name what we are looking for. Choose a variable to represent that quantity.	Let n = the number of tablets.
Step 4. Translate. write a sentence that gives the information to find it.	$254.12 times the number of tablets is no more than $4000.
Translate into an inequality.	$254.12n \leq 4000$
Step 5. Solve the inequality.	$n \leq 15.74$
But n must be a whole number of tablets, so round to 15.	$n \leq 15$
Step 6. Check the answer in the problem and make sure it makes sense. Rounding down the price to $250, 15 tablets would cost $3750, while 16 tablets would be $4000. So a maximum of 15 tablets at $254.12 seems reasonable.	
Step 7. Answer the answer in the problem complete sentence.	the question with a Dawn can buy a maximum of 15 tablets.

Try It 2.7.5

Angie has $20 to spend on juice boxes for her son's preschool picnic. Each pack of juice boxes costs $2.63. What is the maximum number of packs she can buy?

Answer

seven packs

Try It 2.7.6

Daniel wants to surprise his girlfriend with a birthday party at her favorite restaurant. It will cost $42.75 per person for dinner, including tip and tax. His budget for the party is $500. What is the maximum number of people Daniel can have at the party?

Answer

11 people

Example 2.7.7

Pete works at a computer store. His weekly pay will be either a fixed amount, $925, or $500 plus 12% of his total sales. How much should his total sales be for his variable pay option to exceed the fixed amount of $925?

Solution

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	the total sales needed for his variable pay option to exceed the fixed amount of $925
Step 3. Name what we are looking for. Choose a variable to represent that quantity.	Let s = the total sales.
Step 4. Translate. write a sentence that gives the information to find it. Translate into an inequality. Remember to convert the percent to a decimal.	$500 plus 12% of total sales is more than $925. $500 + 0.12s > 925$
Step 5. Solve the inequality.	$0.12s > 425$ $s > 3541.\overline{66}$
Step 6. Check the answer in the problem and make sure it makes sense. Rounding down the price to $250, 15 tablets would cost $3750, while If we round the total sales up to $4000, we see that $500+0.12(4000) = 980$, which is more than $925.	
Step 7. Answer the the question with a complete sentence.	The total sales must be more than $3541.67

Try It 2.7.8

Tiffany just graduated from college and her new job will pay her $20000 per year plus 2% of all sales. She wants to earn at least $100000 per year. For what total sales will she be able to achieve her goal?

Answer

at least $4000000

Try It 2.7.9

Christian has been offered a new job that pays $24000 a year plus 3% of sales. For what total sales would this new job pay more than his current job which pays $60000?

Answer

at least $1200000

Example 2.7.10

Sergio and Lizeth have a very tight vacation budget. They plan to rent a car from a company that charges $75 a week plus $0.25 a mile. How many miles can they travel and still keep within their $200 budget?

Solution

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	the number of miles Sergio and Lizeth can travel
Step 3. Name what we are looking for. Choose a variable to represent that quantity.	Let m = the number of miles.
Step 4. Translate. write a sentence that gives the information to find it. Translate into an inequality.	$75 plus 0.25 times the number of miles is less than or equal to $200. $75 + 25m \leq 200$
Step 5. Solve the inequality.	$0.25m \leq 125$ $m \leq 500$ miles
Step 6. Check the answer in the problem and make sure it makes sense. Yes, $75 + 0.25(500) = 200$.	
Step 7. Answer the the question with a complete sentence.	Sergio and Lizeth can travel 500 miles and still stay on budget.

Try It 2.7.11

Taleisha's phone plan costs her $28.80 a month plus $0.20 per text message. How many text messages can she use and keep her monthly phone bill no more than $50?

Answer

no more than 106 text messages

Try It 2.7.12

Rameen's heating bill is $5.42 per month plus $1.08 per therm. How many therms can Rameen use if he wants his heating bill to be a maximum of $87.50?

Answer

no more than 76 therms

A common goal of most businesses is to make a profit. *Profit* is the money that remains when the expenses have been subtracted from the money earned. In the next example, we will find the number of jobs a small businessman needs to do every month in order to make a certain amount of **profit**.

Example 2.7.13

Elliot has a landscape maintenance business. His monthly expenses are $1,100. If he charges $60 per job, how many jobs must he do to earn a profit of at least $4,000 a month?

Solution

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	the number of jobs Elliot needs
Step 3. Name what we are looking for.	
Choose a variable to represent it	Let j = the number of jobs.
Step 4. Translate. write a sentence that	$60 times the number of jobs minus $1,100 is at least $4,000.
gives the information to find it.	less than or equal to $200.
Translate into an inequality.	$60j - 1100 \geq 4000$
Step 5. Solve the inequality.	$60j \geq 5100$
	$j \geq 85$ jobs
Step 6. Check the answer in the problem and make sure it makes sense.	If Elliot did 90 jobs, his profit would be $60(90) - 1{,}100$, or $4,300. This is more than $4,000.
Step 7. Answer the the question with a complete sentence.	Elliot must work at least 85 jobs.

Try It 2.7.14

Caleb has a pet sitting business. He charges $32 per hour. His monthly expenses are $2272. How many hours must he work in order to earn a profit of at least $800 per month?

Answer

at least 96 hours

Try It 2.7.15

Felicity has a calligraphy business. She charges $2.50 per wedding invitation. Her monthly expenses are $650. How many invitations must she write to earn a profit of at least $2800 per month?

Answer

at least 1380 invitations

Sometimes life gets complicated! There are many situations in which several quantities contribute to the total expense. We must make sure to account for all the individual expenses when we solve problems like this.

Example 2.7.16

Brenda's best friend is having a destination wedding and the event will last 3 days. Brenda has $500 in savings and can earn $15 an hour babysitting. She expects to pay $350 airfare, $375 for food and entertainment and $60 a night for her share of a hotel room. How many hours must she babysit to have enough money to pay for the trip?

Solution

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	the number of hours Brenda must babysit
Step 3. Name what we are looking for. Choose a variable to represent that quantity.	Let h = the number of hours.
Step 4. Translate. write a sentence that gives the information to find it.	The expenses must be less than or equal to the income. The cost of airfare plus the cost of food and entertainment and the hotel bill must be less than or equal to the savings plus the amount earned babysitting.
Translate into an inequality.	$\$350 + \$375 + \$60(3) \leq \$500 + \$15h$
Step 5. Solve the inequality.	$905 \leq 500 + 15h$ $405 \leq 15h$ $27 \leq h$ $h \geq 27$
Step 6. Check the answer in the problem and make sure it makes sense.	
We substitute 27 into the inequality.	
$905 \leq 500 + 15h$	
$905 \leq 500 + 15(27)$	
$905 \leq 905$	
Step 7. Answer the the question with a complete sentence.	Brenda must babysit at least 27 hours.

? Try It 2.7.17

Malik is planning a 6-day summer vacation trip. He has $840 in savings, and he earns $45 per hour for tutoring. The trip will cost him $525 for airfare, $780 for food and sightseeing, and $95 per night for the hotel. How many hours must he tutor to have enough money to pay for the trip?

Answer

at least 23 hours

? Try It 2.7.18

Josue wants to go on a 10-day road trip next spring. It will cost him $180 for gas, $450 for food, and $49 per night for a motel. He has $520 in savings and can earn $30 per driveway shoveling snow. How many driveways must he shovel to have enough money to pay for the trip?

Answer

at least 20 driveways

Key Concepts

- **Solving inequalities**
  1. **Read** the problem.
  2. **Identify** what we are looking for.
  3. **Name** what we are looking for. Choose a variable to represent that quantity.
  4. **Translate.** Write a sentence that gives the information to find it. Translate into an inequality.

5. **Solve** the inequality.
6. **Check** the answer in the problem and make sure it makes sense.
7. **Answer** the question with a complete sentence.

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2.7E: Exercises

Practice Makes Perfect

Solve Applications with Linear Inequalities

In the following exercises, solve.

Exercise 2.7E.1

Mona is planning her son's birthday party and has a budget of $285. The Fun Zone charges $19 per child. How many children can she have at the party and stay within her budget?

Answer

15 children

Exercise 2.7E.2

Carlos is looking at apartments with three of his friends. They want the monthly rent to be no more than $2360. If the roommates split the rent evenly among the four of them, what is the maximum rent each will pay?

Exercise 2.7E.3

A water taxi has a maximum load of 1,800 pounds. If the average weight of one person is 150 pounds, how many people can safely ride in the water taxi?

Answer

12 people

Exercise 2.7E.4

Marcela is registering for her college classes, which cost $105 per unit. How many units can she take to have a maximum cost of $1,365?

Exercise 2.7E.5

Arleen got a $20 gift card for the coffee shop. Her favorite iced drink costs $3.79. What is the maximum number of drinks she can buy with the gift card?

Answer

five drinks

Exercise 2.7E.6

Teegan likes to play golf. He has budgeted $60 next month for the driving range. It costs him $10.55 for a bucket of balls each time he goes. What is the maximum number of times he can go to the driving range next month?

Exercise 2.7E.7

Joni sells kitchen aprons online for $32.50 each. How many aprons must she sell next month if she wants to earn at least $1,000?

Answer

31 aprons

Exercise 2.7E.8

Ryan charges his neighbors $17.50 to wash their car. How many cars must he wash next summer if his goal is to earn at least $1,500?

Exercise 2.7E.9

Keshad gets paid $2,400 per month plus 6% of his sales. His brother earns $3,300 per month. For what amount of total sales will Keshad's monthly pay be higher than his brother's monthly pay?

Answer

$15,000

Exercise 2.7E.10

Kimuyen needs to earn $4,150 per month in order to pay all her expenses. Her job pays her $3,475 per month plus 4% of her total sales. What is the minimum Kimuyen's total sales must be in order for her to pay all her expenses?

Exercise 2.7E.11

Andre has been offered an entry-level job. The company offered him $48,000 per year plus 3.5% of his total sales. Andre knows that the average pay for this job is $62,000. What would Andre's total sales need to be for his pay to be at least as high as the average pay for this job?

Answer

$400,000

Exercise 2.7E.12

Nataly is considering two job offers. The first job would pay her $83,000 per year. The second would pay her $66,500 plus 15% of her total sales. What would her total sales need to be for her salary on the second offer be higher than the first?

Exercise 2.7E.13

Jake's water bill is $24.80 per month plus $2.20 per ccf (hundred cubic feet) of water. What is the maximum number of ccf Jake can use if he wants his bill to be no more than $60?

Answer

16 ccf

Exercise 2.7E.14

Kiyoshi's phone plan costs $17.50 per month plus $0.15 per text message. What is the maximum number of text messages Kiyoshi can use so the phone bill is no more than $56.50?

Exercise 2.7E.15

Marlon's TV plan costs $49.99 per month plus $5.49 per first-run movie. How many first-run movies can he watch if he wants to keep his monthly bill to be a maximum of $100?

Answer

nine movies

Exercise 2.7E.16

Kellen wants to rent a banquet room in a restaurant for her cousin's baby shower. The restaurant charges $350 for the banquet room plus $32.50 per person for lunch. How many people can Kellen have at the shower if she wants the maximum cost to be $1,500?

Exercise 2.7E.17

Moshde runs a hairstyling business from her house. She charges $45 for a haircut and style. Her monthly expenses are $960. She wants to be able to put at least $1,200 per month into her savings account order to open her own salon. How many "cut & styles" must she do to save at least $1,200 per month?

Answer

48 cut & styles

Exercise 2.7E.18

Noe installs and configures software on home computers. He charges $125 per job. His monthly expenses are $1,600. How many jobs must he work in order to make a profit of at least $2,400?

Exercise 2.7E.19

Katherine is a personal chef. She charges $115 per four-person meal. Her monthly expenses are $3,150. How many four-person meals must she sell in order to make a profit of at least $1,900?

Answer

44 meals

Exercise 2.7E.20

Melissa makes necklaces and sells them online. She charges $88 per necklace. Her monthly expenses are $3745. How many necklaces must she sell if she wants to make a profit of at least $1,650?

Exercise 2.7E.21

Five student government officers want to go to the state convention. It will cost them $110 for registration, $375 for transportation and food, and $42 per person for the hotel. There is $450 budgeted for the convention in the student government savings account. They can earn the rest of the money they need by having a car wash. If they charge $5 per car, how many cars must they wash in order to have enough money to pay for the trip?

Answer

49 cars

Exercise 2.7E.22

Cesar is planning a 4-day trip to visit his friend at a college in another state. It will cost him $198 for airfare, $56 for local transportation, and $45 per day for food. He has $189 in savings and can earn $35 for each lawn he mows. How many lawns must he mow to have enough money to pay for the trip?

Exercise 2.7E.23

Alonzo works as a car detailer. He charges $175 per car. He is planning to move out of his parents' house and rent his first apartment. He will need to pay $120 for application fees, $950 for security deposit, and first and last months' rent at $1,140 per month. He has $1,810 in savings. How many cars must he detail to have enough money to rent the apartment?

Answer

9 cars

Exercise 2.7E.24

Eun-Kyung works as a tutor and earns $60 per hour. She has $792 in savings. She is planning an anniversary party for her parents. She would like to invite 40 guests. The party will cost her $1,520 for food and drinks and $150 for the photographer. She will also have a favor for each of the guests, and each favor will cost $7.50. How many hours must she tutor to have enough money for the party?

Everyday Math

Exercise 2.7E.25

Maximum Load on a Stage In 2014, a high school stage collapsed in Fullerton, California, when 250 students got on stage for the finale of a musical production. Two dozen students were injured. The stage could support a maximum of 12,750 pounds. If the average weight of a student is assumed to be 140 pounds, what is the maximum number of students who could safely be on the stage?

Answer

91 students

Exercise 2.7E.26

Maximum Weight on a Boat In 2004, a water taxi sank in Baltimore harbor and five people drowned. The water taxi had a maximum capacity of 3,500 pounds (25 people with average weight 140 pounds). The average weight of the 25 people on the water taxi when it sank was 168 pounds per person. What should the maximum number of people of this weight have been?

Exercise 2.7E.27

Wedding Budget Adele and Walter found the perfect venue for their wedding reception. The cost is $9,850 for up to 100 guests, plus $38 for each additional guest. How many guests can attend if Adele and Walter want the total cost to be no more than $12,500?

Answer

169 guests

Exercise 2.7E.28

Shower Budget Penny is planning a baby shower for her daughter-in-law. The restaurant charges $950 for up to 25 guests, plus $31.95 for each additional guest. How many guests can attend if Penny wants the total cost to be no more than $1,500?

Writing Exercises

? Exercise 2.7E. 29

Find your last month's phone bill and the hourly salary you are paid at your job. (If you do not have a job, use the hourly salary you would realistically be paid if you had a job.) Calculate the number of hours of work it would take you to earn at least enough money to pay your phone bill by writing an appropriate inequality and then solving it.

Answer

Answers will vary.

? Exercise 2.7E. 30

Find out how many units you have left, after this term, to achieve your college goal and estimate the number of units you can take each term in college. Calculate the number of terms it will take you to achieve your college goal by writing an appropriate inequality and then solving it.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
solve applications with linear inequalities.			

ⓑ What does this checklist tell you about your mastery of this section? What steps will you take to improve?

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2.8: Solve Linear Inequalities - Algebra Emphasis

Learning Objectives

By the end of this section, you will be able to:

- Graph inequalities on the number line
- Solve linear inequalities
- Translate words to an inequality and solve
- Solve applications with linear inequalities

Before you get started, take this readiness quiz.

1. Translate from algebra to English: $15 > x$.
   If you missed this problem, review [link].
2. Translate to an algebraic expression: 15 is less than x.
   If you missed this problem, review [link].

Graph Inequalities on the Number Line

What number would make the inequality $x > 3$ true? Are you thinking, "x could be four"? That's correct, but x could be 6, too, or 37, or even 3.001. Any number greater than three is a solution to the inequality $x > 3$. We show all the solutions to the inequality $x > 3$ on the number line by shading in all the numbers to the right of three, to show that all numbers greater than three are solutions. Because the number three itself is not a solution, we put an open parenthesis at three.

We can also represent inequalities using **interval notation**. There is no upper end to the solution to this inequality. In interval notation, we express $x > 3$ as $(3, \infty)$. The symbol ∞ is read as "**infinity**." It is not an actual number. Figure 2.8.1 shows both the number line and the interval notation.

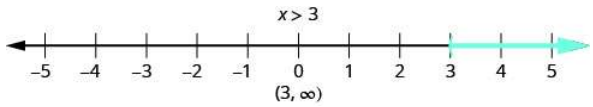


Figure 2.8.1: The inequality $x > 3$ is graphed on this number line and written in interval notation.

We use the left parenthesis symbol, (, to show that the endpoint of the inequality is not included. The left bracket symbol, [, shows that the endpoint is included.

The inequality $x \leq 1$ means all numbers less than or equal to one. Here we need to show that one is a solution, too. We do that by putting a bracket at $x = 1$. We then shade in all the numbers to the left of one, to show that all numbers less than one are solutions (Figure 2.8.2). There is no lower end to those numbers. We write $x \leq 1 x \leq 1$ in interval notation as $(-\infty, 1]$. The symbol $-\infty$ is read as "negative infinity."

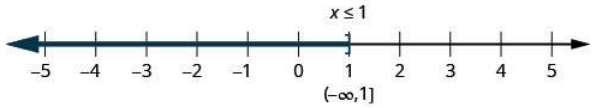
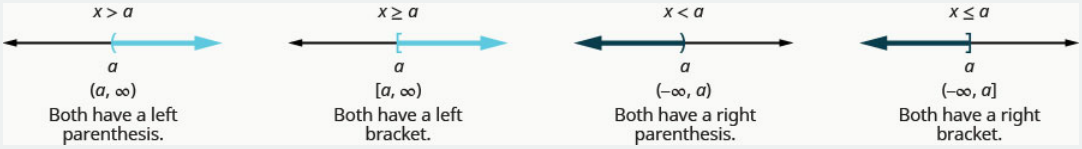


Figure 2.8.2: The inequality $x \leq 1$ is graphed on this number line and written in interval notation.

Figure 2.8.3 shows both the number line and interval notation.

INEQUALITIES, NUMBER LINES, AND INTERVAL NOTATION



The notation for inequalities on a number line and in interval notation use the same symbols to express the endpoints of intervals.

Example 2.8.1

Graph each inequality on the number line and write in interval notation.

a. $x \geq -3$
b. $x < 2.5$
c. $x \leq -\frac{3}{5}$

Answer

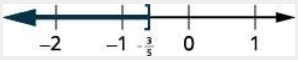
ⓐ

	$x \geq -3$
Shade to the right of -3, and put a bracket at -3.	
Write in interval notation.	$[-3, \infty)$

ⓑ

	$x < 2.5$
Shade to the left of 2.5 and put a parenthesis at 2.5.	
Write in interval notation.	$(-\infty, 2.5)$

ⓒ

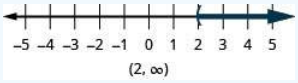
	$x \leq -\frac{3}{5}$
Shade to the left of $-\frac{3}{5}$, and put a bracket at $-\frac{3}{5}$.	
Write in interval notation.	$\left(-\infty, \frac{3}{5}\right]$

Graph each inequality on the number line and write in interval notation:

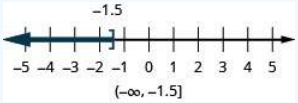
a. $x > 2$
b. $x \leq -1.5$
c. $x \geq \frac{3}{4}$.

Answer

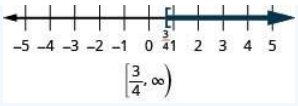
ⓐ



ⓑ



ⓒ



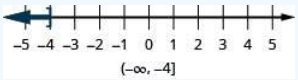
Example 2.8.3

Graph each inequality on the number line and write in interval notation:

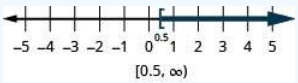
a. $x \leq -4$
b. $x \geq 0.5$
c. $x < -\frac{2}{3}$.

Answer

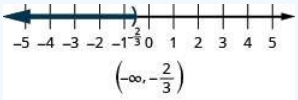
ⓐ



ⓑ



ⓒ



What numbers are greater than two but less than five? Are you thinking say, 2.5, 3, $3\frac{2}{3}$, 4, 4, 99? We can represent all the numbers between two and five with the inequality $2 < x < 5$. We can show $2 < x < 5$ on the number line by shading all the numbers between two and five. Again, we use the parentheses to show the numbers two and five are not included. See Figure.



The interval notation would be (2, 5).

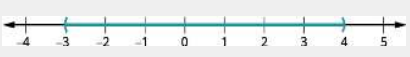
Figure 2.8.3

Graph each inequality on the number line and write in interval notation.

ⓐ $-3 < x < 4$ ⓑ $-6 \leq x < -1$ ⓒ $0 \leq x \leq 2.5$

Answer

ⓐ

		$-3 < x < 4$
Shade between -3 and 4. Put a parentheses at -3 and 4.		
Write in interval notation.		$(-3, 4)$

ⓑ

			$-6 \leq x < -1$

Shade between -6 and -1. Put a bracket at -6, and a parenthesis at -1.			
Write in interval notation.			$[-6, 1)$

ⓒ

			$0 \leq x \leq 2.5$
Shade between 0 and 2.5. Put a bracket at 0 and at 2.5.			
Write in interval notation.			$[0, 2.5]$

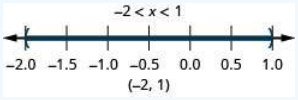
Example 2.8.5

Graph each inequality on the number line and write in interval notation:

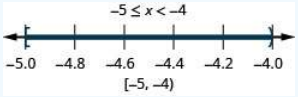
ⓐ $-2 < x < 1$ ⓑ $-5 \leq x < -4$ ⓒ $1 \leq x \leq 4.25$

Answer

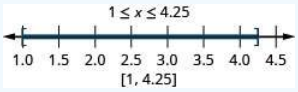
ⓐ



ⓑ



ⓒ



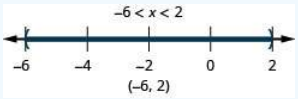
Example 2.8.6

Graph each inequality on the number line and write in interval notation:

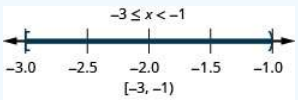
ⓐ $-6 < x < 2$ ⓑ $-3 \leq x < -1$ ⓒ $2.5 \leq x \leq 6$

Answer

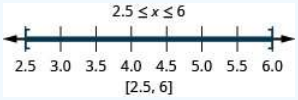
ⓐ



ⓑ



ⓒ



Solve Linear Inequalities

A linear inequality is much like a linear equation—but the equal sign is replaced with an inequality sign. A **linear inequality** is an inequality in one variable that can be written in one of the forms, $ax+b<c$, $ax+b\leq c$, $ax+b>c$, or $ax+b\geq c$.

LINEAR INEQUALITY

A linear inequality is an inequality in one variable that can be written in one of the following forms where a, b, and c are real numbers and $a\neq 0$:

$$ax+b<c,\quad ax+b\leq c,\quad ax+b>c,\quad ax+b\geq c.$$

When we solved linear equations, we were able to use the properties of equality to add, subtract, multiply, or divide both sides and still keep the equality. Similar properties hold true for inequalities.

We can add or subtract the same quantity from both sides of an inequality and still keep the inequality. For example:

$$\begin{array}{ll} -4<2 & -4<2 \\ -4-5<2-5 & -4+7<2+7 \\ -9<-3 \text{ True} & 3<9 \text{ True} \end{array}$$

Notice that the inequality sign stayed the same.

This leads us to the Addition and Subtraction Properties of Inequality.

ADDITION AND SUBTRACTION PROPERTY OF INEQUALITY

For any numbers a, b, and c, if $a<b$, then

$$\begin{array}{ll} a+c<b+c & a-c<b-c \\ a+c>b+c & a-c>b-c \end{array}$$

We can add or subtract the same quantity from both sides of an inequality and still keep the inequality

What happens to an inequality when we divide or multiply both sides by a constant?

Let's first multiply and divide both sides by a positive number.

$$\begin{array}{ll} 10<15 & 10<15 \\ 10(5)<15(5) & \frac{10}{5}<\frac{15}{5} \\ 50<75 \text{ True} & 2<3 \text{ True} \end{array}$$

The inequality signs stayed the same.

Does the inequality stay the same when we divide or multiply by a negative number?

$$\begin{array}{ll} 10<15 & 10<15 \\ 10(-5)\ ?\ 15(-5) & \frac{10}{-5}\ ?\ \frac{15}{-5} \\ -50\ ?\ -75 & -2\ ?\ -3 \\ -50>-75 & -2>-3 \end{array}$$

Notice that when we filled in the inequality signs, the inequality signs reversed their direction.

When we divide or multiply an inequality by a positive number, the inequality sign stays the same. When we divide or multiply an inequality by a negative number, the inequality sign reverses.

This gives us the Multiplication and Division Property of Inequality.

MULTIPLICATION AND DIVISION PROPERTY OF INEQUALITY

For any numbers a, b, and c,

multiply or divide by a positive

if $a < b$ and $c > 0$, then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$.
if $a > b$ and $c > 0$, then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$.

multiply or divide by a negative

if $a < b$ and $c < 0$, then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$.
if $a > b$ and $c < 0$, then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$.

When we divide or multiply an inequality by a:

- positive number, the inequality stays the same.
- negative number, the inequality reverses.

Sometimes when solving an inequality, as in the next example, the variable ends upon the right. We can rewrite the inequality in reverse to get the variable to the left.

$x > a$ has the same meaning as $a < x$

Think about it as "If Xander is taller than Andy, then Andy is shorter than Xander."

Example 2.8.7

Solve each inequality. Graph the solution on the number line, and write the solution in interval notation.

ⓐ $x - \frac{3}{8} \leq \frac{3}{4}$ ⓑ $9y < 54$ ⓒ $-15 < \frac{3}{5}z$

Answer

ⓐ

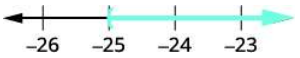
	$x - \frac{3}{8} \leq \frac{3}{4}$
Add 3838 to both sides of the inequality.	$x - \frac{3}{8} + \frac{3}{8} \leq \frac{3}{4} + \frac{3}{8}$
Simplify.	$x \leq \frac{9}{8}$
Graph the solution on the number line.	
Write the solution in interval notation.	$(-\infty, \frac{9}{8}]$

ⓑ

	$9y < 54$
Divide both sides of the inequality by 9; since 9 is positive, the inequality stays the same.	$\frac{9y}{9} < \frac{54}{9}$
Simplify.	$y < 6$

Graph the solution on the number line.	
Write the solution in interval notation.	$(-\infty, 6)$

ⓒ

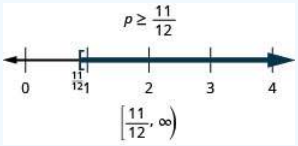
	$-15 < \frac{3}{5}z$
Multiply both sides of the inequality by $\frac{5}{3}$. Since $\frac{5}{3}$ is positive, the inequality stays the same.	$\frac{5}{3}(-15) < \frac{5}{3}\left(\frac{3}{5}z\right)$
Simplify.	$-25 < z$
Rewrite with the variable on the left.	$z > -25$
Graph the solution on the number line.	
Write the solution in interval notation.	$(-25, \infty)$

Solve each inequality, graph the solution on the number line, and write the solution in interval notation:

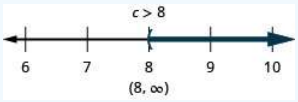
ⓐ $p - \frac{3}{4} \geq \frac{1}{6}$ ⓑ $9c > 72$ ⓒ $24 \leq \frac{3}{8}m$

Answer

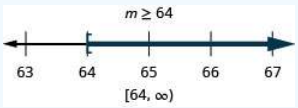
ⓐ



ⓑ



ⓒ



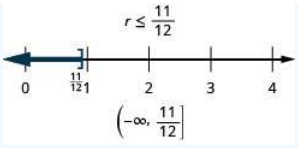
Example 2.8.9

Solve each inequality, graph the solution on the number line, and write the solution in interval notation:

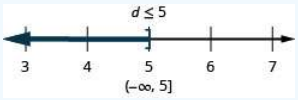
ⓐ $r - \frac{1}{3} \leq \frac{7}{12}$ ⓑ $12d \leq 60$ ⓒ $-24 < \frac{4}{3}n$

Answer

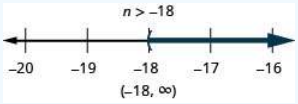
ⓐ



ⓑ



ⓒ



Be careful when you multiply or divide by a negative number—remember to reverse the inequality sign.

Example 2.8.10

Solve each inequality, graph the solution on the number line, and write the solution in interval notation.

ⓐ $-\frac{1}{3}m \ge \frac{6}{5}$ ⓑ $\frac{n}{-2} \ge 8$

Answer

ⓐ

	$-13m \ge 65$
Divide both sides of the inequality by $-\frac{1}{3}$. Since $-\frac{1}{3}$ is a negative, the inequality reverses.	$\frac{-13m}{-13} \le \frac{65}{-13}$
Simplify.	$m \le -5$
Graph the solution on the number line.	
Write the solution in interval notation.	$(-\infty, -5]$

ⓑ

	$\frac{n}{-2} \ge 8$
Multiply both sides of the inequality by -2. Since -2 is a negative, the inequality reverses.	$-2\left(\frac{n}{-2}\right) \le -2(8)$
Simplify.	$n \le -16$
Graph the solution on the number line.	
Write the solution in interval notation.	$(-\infty, -16]$

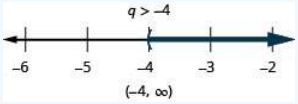
Example 2.8.11

Solve each inequality, graph the solution on the number line, and write the solution in interval notation:

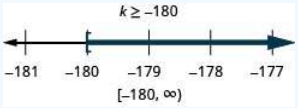
ⓐ $-8q < 32$ ⓑ $\frac{k}{-12} \le 15$.

Answer

ⓐ



ⓑ



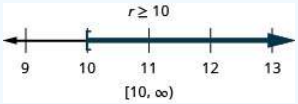
Example 2.8.12

Solve each inequality, graph the solution on the number line, and write the solution in interval notation:

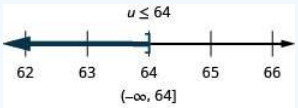
ⓐ $-7r \leq -70$ ⓑ $\frac{u}{-4} \geq -16$.

Answer

ⓐ



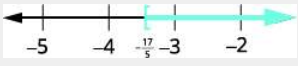
ⓑ



Most inequalities will take more than one step to solve. We follow the same steps we used in the general strategy for solving linear equations, but make sure to pay close attention when we multiply or divide to isolate the variable.

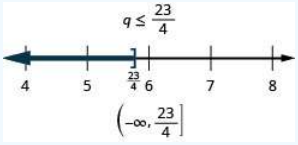
Solve the inequality $6y \leq 11y + 17$, graph the solution on the number line, and write the solution in interval notation.

Answer

	$6y \leq 11y + 17$
Subtract 11y11y from both sides to collect the variables on the left.	$6y - 11y \leq 11y - 11y + 17$
Simplify.	$-5y \leq 17$
Divide both sides of the inequality by $-5,-5$, and reverse the inequality.	$\frac{-5y}{-5} \geq \frac{17}{-5}$
Simplify.	$y \geq \frac{17}{-5}$
Graph the solution on the number line.	
Write the solution in interval notation.	$\left[-\frac{17}{5}, \infty\right)$

Solve the inequality, graph the solution on the number line, and write the solution in interval notation: $3q \geq 7q - 23$.

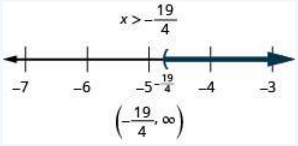
Answer



Example 2.8.15

Solve the inequality, graph the solution on the number line, and write the solution in interval notation: $6x < 10x + 19$.

Answer

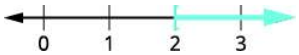


When solving inequalities, it is usually easiest to collect the variables on the side where the coefficient of the variable is largest. This eliminates negative coefficients and so we don't have to multiply or divide by a negative—which means we don't have to remember to reverse the inequality sign.

Example 2.8.16

Solve the inequality $8p + 3(p - 12) > 7p - 28$, graph the solution on the number line, and write the solution in interval notation.

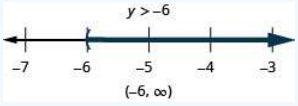
Answer

	$8p + 3(p - 12) > 7p - 28$
Simplify each side as much as possible.	
Distribute.	$8p + 3p - 36 > 7p - 28$
Combine like terms.	$11p - 36 > 7p - 28$
Subtract $7p$ from both sides to collect the variables on the left, since $11 > 7$.	$11p - 36 - 7p > 7p - 28 - 7p$
Simplify.	$4p - 36 > -28$
Add 36 to both sides to collect the constants on the right.	$4p - 36 + 36 > -28 + 36$
Simplify.	$4p > 8$
Divide both sides of the inequality by 4; the inequality stays the same.	$\frac{4p}{4} > \frac{8}{4}$
Simplify.	$p > 2$
Graph the solution on the number line.	
Write the solution in interval notation.	$(2, \infty)$

Example 2.8.17

Solve the inequality $9y+2(y+6)>5y-24$, graph the solution on the number line, and write the solution in interval notation.

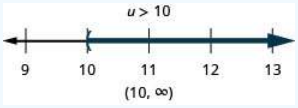
Answer



Example 2.8.18

Solve the inequality $6u+8(u-1)>10u+32$, graph the solution on the number line, and write the solution in interval notation.

Answer



Just like some equations are identities and some are contradictions, inequalities may be identities or contradictions, too. We recognize these forms when we are left with only constants as we solve the inequality. If the result is a true statement, we have an identity. If the result is a false statement, we have a contradiction.

Example 2.8.19

Solve the inequality $8x-2(5-x)<4(x+9)+6x$, graph the solution on the number line, and write the solution in interval notation.

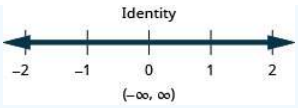
Answer

Simplify each side as much as possible.		$8x-2(5-x)<4(x+9)+6x$
Distribute.		$8x-10+2x<4x+36+6x$
Combine like terms.		$10x-10<10x+36$
Subtract $10x$ from both sides to collect the variables on the left.		$10x-10-10x<10x+36-10x$
Simplify.		$-10<36$
The x's are gone, and we have a true statement.		The inequality is an identity. The solution is all real numbers.
Graph the solution on the number line.		
Write the solution in interval notation.		$(-\infty,\infty)$

Example 2.8.20

Solve the inequality $4b-3(3-b)>5(b-6)+2b$, graph the solution on the number line, and write the solution in interval notation.

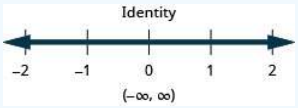
Answer



Example 2.8.21

Solve the inequality $9h-7(2-h)<8(h+11)+8h$, graph the solution on the number line, and write the solution in interval notation.

Answer



We can clear fractions in inequalities much as we did in equations. Again, be careful with the signs when multiplying or dividing by a negative.

Example 2.8.22

Solve the inequality $\frac{1}{3}a-\frac{1}{8}a>\frac{5}{24}a+\frac{3}{4}$, graph the solution on the number line, and write the solution in interval notation.

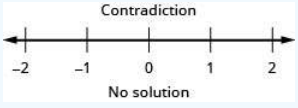
Answer

	$\frac{1}{3}a-\frac{1}{8}a>\frac{5}{24}a+\frac{3}{4}$
Multiply both sides by the LCD, 24, to clear the fractions.	$24\left(\frac{1}{3}a-\frac{1}{8}a\right)>24\left(\frac{5}{24}a+\frac{3}{4}\right)$
Simplify.	$8a-3a>5a+18$
Combine like terms.	$5a>5a+18$
Subtract $5a$ from both sides to collect the variables on the left.	$5a-5a>5a-5a+18$
Simplify.	$0>18$
The statement is false.	The inequality is a contradiction. There is no solution.
Graph the solution on the number line.	
Write the solution in interval notation.	There is no solution.

Example 2.8.23

Solve the inequality $\frac{1}{4}x-\frac{1}{12}x>\frac{1}{6}x+\frac{7}{8}$, graph the solution on the number line, and write the solution in interval notation.

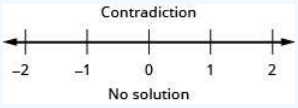
Answer



Example 2.8.24

Solve the inequality $\frac{2}{5}z - \frac{1}{3}z < \frac{1}{15}z - \frac{3}{5}$, graph the solution on the number line, and write the solution in interval notation.

Answer



Translate to an Inequality and Solve

To translate English sentences into inequalities, we need to recognize the phrases that indicate the inequality. Some words are easy, like “more than” and “less than.” But others are not as obvious. Table shows some common phrases that indicate inequalities.

$>$	$\geq$	$<$	$\leq$
is greater than	is greater than or equal to	is less than	is less than or equal to
is more than	is at least	is smaller than	is at most
is larger than	is no less than	has fewer than	is no more than
exceeds	is the minimum	is lower than	is the maximum

Example 2.8.25

Translate and solve. Then graph the solution on the number line, and write the solution in interval notation.

Twenty-seven less than x is at least 48.

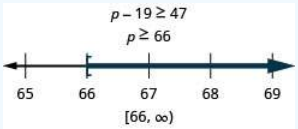
Answer

	Twenty-seven less than x is at least 48.
Translate.	$x - 27 \geq 48$
Solve—add 27 to both sides.	$x - 27 + 27 \geq 48 + 27$
Simplify.	$x \geq 75$
Graph on the number line.	
Write in interval notation.	$[75, \infty)$

Translate and solve. Then graph the solution on the number line, and write the solution in interval notation.

Nineteen less than p is no less than 47.

Answer

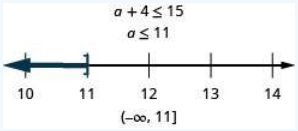


Example 2.8.27

Translate and solve. Then graph the solution on the number line, and write the solution in interval notation.

Four more than a is at most 15.

Answer



Solve Applications with Linear Inequalities

Many real-life situations require us to solve inequalities. The method we will use to solve applications with linear inequalities is very much like the one we used when we solved applications with equations.

We will read the problem and make sure all the words are understood. Next, we will identify what we are looking for and assign a variable to represent it. We will restate the problem in one sentence to make it easy to translate into an inequality. Then, we will solve the inequality.

Sometimes an application requires the solution to be a whole number, but the algebraic solution to the inequality is not a whole number. In that case, we must round the algebraic solution to a whole number. The context of the application will determine whether we round up or down.

Example 2.8.28

Dawn won a mini-grant of $4,000 to buy tablet computers for her classroom. The tablets she would like to buy cost $254.12 each, including tax and delivery. What is the maximum number of tablets Dawn can buy?

Solution

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	the maximum number of tablets Dawn can buy
Step 3. Name what you are looking for. Choose a variable to represent that quantity.	Let n = the number of tablets.
Step 4. Translate Write a sentence that gives the information to find it.	$254.12 times the number of tablets is no more than $4,000.
Translate into an inequality.	$254.12n \leq 4000$
Step 5. Solve the inequality.	$n \leq 15.74$ but n must be a whole number of tablets, so round to 15. $n \leq 15$ tablets
Step 6. Check the answer in the problem and make sure it makes sense.	Rounding down the price to $250, 15 tablets would cost $3,750, while 16 tablets would be $4,000. So a maximum of 15 tablets at $254.12 seems reasonable.
Step 7. Write a sentence that answers the question.	Dawn can buy a maximum of 15 tablets.

Try It 2.8.29

Angie has $20 to spend on juice boxes for her son's preschool picnic. Each pack of juice boxes costs $2.63. What is the maximum number of packs she can buy?

Answer

Angie can buy 7 packs of juice.

Try It 2.8.30

Daniel wants to surprise his girlfriend with a birthday party at her favorite restaurant. It will cost $42.75 per person for dinner, including tip and tax. His budget for the party is $500. What is the maximum number of people Daniel can have at the party?

Answer

Daniel can have 11 people at the party.

Example 2.8.31

Taleisha's phone plan costs her $28.80 a month plus $0.20 per text message. How many text messages can she send/receive and keep her monthly phone bill no more than $50?

Solution

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	the number of text messages Taleisha can make
Step 3. Name what you are looking for. Choose a variable to represent that quantity.	Let $t =$ the number of text messages.
Step 4. Translate Write a sentence that gives the information to find it.	28.80 plus 0.20 times the number of text messages is less than or equal to 50.
Translate into an inequality.	$28.80 + 0.20t \leq 50$
Step 5. Solve the inequality.	$0.2t \leq 21.2$ $t \leq 106$ text messages
Step 6. Check the answer in the problem and make sure it makes sense.	Yes, $28.80 + 0.20(106) = 50$.
Step 7. Write a sentence that answers the question.	Taleisha can send/receive no more than 106 text messages to keep her bill no more than 50.

Try It 2.8.32

Sergio and Lizeth have a very tight vacation budget. They plan to rent a car from a company that charges $75 a week plus $0.25 a mile. How many miles can they travel during the week and still keep within their $200 budget?

Answer

Sergio and Lizeth can travel no more than 500 miles.

Try It 2.8.33

Rameen's heating bill is $5.42 per month plus $1.08 per therm. How many therms can Rameen use if he wants his heating bill to be a maximum of $87.50.

Answer

Rameen's heating bill is $5.42 per month plus $1.08 per therm. How many therms can Rameen use if he wants his heating bill to be a maximum of $87.50.

Profit is the money that remains when the costs have been subtracted from the revenue. In the next example, we will find the number of jobs a small businesswoman needs to do every month in order to make a certain amount of profit.

Example 2.8.34

Felicity has a calligraphy business. She charges $2.50 per wedding invitation. Her monthly expenses are $650. How many invitations must she write to earn a profit of at least $2,800 per month?

Solution

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	the number of invitations Felicity needs to write
Step 3. Name what you are looking for. Choose a variable to represent that quantity.	Let $j =$ the number of invitations.
Step 4. Translate Write a sentence that gives the information to find it.	$2.50 times the number of invitations minus $650 is at least $2,800.
Translate into an inequality.	$2.50j - 650 \geq 2,800$
Step 5. Solve the inequality.	$2.5j \geq 3,450$ $j \geq 1,380$ invitations
Step 6. Check the answer in the problem and make sure it makes sense.	If Felicity wrote 1400 invitations, her profit would be $2.50(1400) - 650$, or $2,850. This is more than $2800.
Step 7. Write a sentence that answers the question.	Felicity must write at least 1,380 invitations.

Try It 2.8.35

Caleb has a pet sitting business. He charges $32 per hour. His monthly expenses are $2,272. How many hours must he work in order to earn a profit of at least $800 per month?

Answer

Caleb must work at least 96 hours.

Try It 2.8.36

Elliot has a landscape maintenance business. His monthly expenses are $1,100. If he charges $60 per job, how many jobs must he do to earn a profit of at least $4,000 a month?

Answer

Elliot must work at least 85 jobs.

There are many situations in which several quantities contribute to the total expense. We must make sure to account for all the individual expenses when we solve problems like this.

Example 2.8.37

Malik is planning a six-day summer vacation trip. He has $840 in savings, and he earns $45 per hour for tutoring. The trip will cost him $525 for airfare, $780 for food and sightseeing, and $95 per night for the hotel. How many hours must he tutor to have enough money to pay for the trip?

Solution

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	the number of hours Malik must tutor
Step 3. Name what you are looking for. Choose a variable to represent that quantity.	Let h = the number of hours.
Step 4. Translate Write a sentence that gives the information to find it.	The expenses must be less than or equal to the income. The cost of airfare plus the cost of food and sightseeing and the hotel bill must be less than the savings plus the amount earned tutoring.
Translate into an inequality.	$525 + 780 + 95(6) \leq 840 + 45h$
Step 5. Solve the inequality.	$1,875 \leq 840 + 45h$ $1,035 \leq 45h$ $h \geq 23$ hours
Step 6. Check the answer in the problem and make sure it makes sense.	We substitute 23 into the inequality. $1,875 \leq 840 + 45h$ $1,875 \leq 840 + 45(23)$ $1,875 \leq 1875$
Step 7. Write a sentence that answers the question.	Malik must tutor at least 23 hours.

Try It 2.8.38

Brenda's best friend is having a destination wedding and the event will last three days. Brenda has $500 in savings and can earn $15 an hour babysitting. She expects to pay $350 airfare, $375 for food and entertainment and $60 a night for her share of a hotel room. How many hours must she babysit to have enough money to pay for the trip?

Answer

Brenda must babysit at least 27 hours.

Try It 2.8.39

Josue wants to go on a 10-night road trip with friends next spring. It will cost him $180 for gas, $450 for food, and $49 per night to share a motel room. He has $520 in savings and can earn $30 per driveway shoveling snow. How many driveways must he shovel to have enough money to pay for the trip?

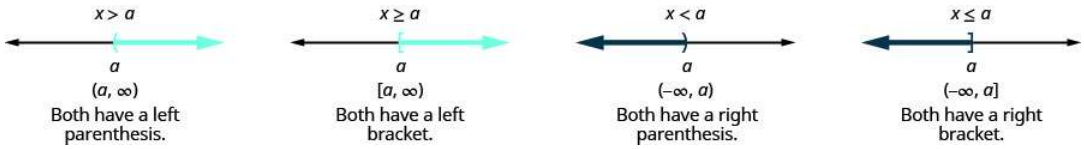
Answer

Josue must shovel at least 20 driveways.

Key Concepts

- **Inequalities, Number Lines, and Interval Notation**

  $x > a \quad x \geq a \quad x < a \quad x \leq a$

- **Linear Inequality**
  - A **linear inequality** is an inequality in one variable that can be written in one of the following forms where a, b, and c are real numbers and $a \neq 0$:

$$ax + b < c, \qquad ax + b \leq c, \qquad ax + b > c, \qquad ax + b \geq c.$$

- **Addition and Subtraction Property of Inequality**
  - For any numbers *a*, *b*, and *c*, if a<b,thena<b,then

$$a+c<b+c \quad a-c<b-c$$
$$a+c>b+c \quad a-c>b-c$$

  - We can add or subtract the same quantity from both sides of an inequality and still keep the inequality.
- **Multiplication and Division Property of Inequality**
  - For any numbers *a*, *b*, and *c*,
    multiply or divide by a **positive**

$$\text{if } a<b \text{ and } c>0, \text{ then } ac<bc \text{ and } \tfrac{a}{c}<\tfrac{b}{c}.$$
$$\text{if } a>b \text{ and } c>0, \text{ then } ac>bc \text{ and } \tfrac{a}{c}>\tfrac{b}{c}.$$

    multiply or divide by a **negative**

$$\text{if } a<b \text{ and } c<0, \text{ then } ac>bc \text{ and } \tfrac{a}{c}>\tfrac{b}{c}.$$
$$\text{if } a>b \text{ and } c<0, \text{ then } ac<bc \text{ and } \tfrac{a}{c}<\tfrac{b}{c}.$$

- **Phrases that indicate inequalities**

$>$	$\geq$	$<$	$\leq$
is greater than	is greater than or equal to	is less than	is less than or equal to
is more than	is at least	is smaller than	is at most
is larger than	is no less than	has fewer than	is no more than
exceeds	is the minimum	is lower than	is the maximum

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2.8E: Exercises

Practice Makes Perfect

Graph Inequalities on the Number Line

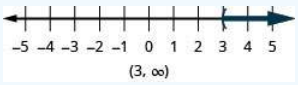
In the following exercises, graph each inequality on the number line and write in interval notation.

1. ⓐ $x < -2$
ⓑ $x \geq -3.5$
ⓒ $x \leq \frac{2}{3}$

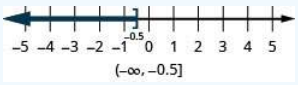
2. ⓐ $x > 3$
ⓑ $x \leq -0.5$
ⓒ $x \geq \frac{1}{3}$

Answer

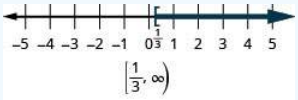
ⓐ



ⓑ



ⓒ

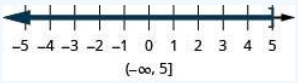


3. ⓐ $x \geq -4$
ⓑ $x < 2.5$
ⓒ $x > -\frac{3}{2}$

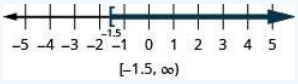
4. ⓐ $x \leq 5$
ⓑ $x \geq -1.5$
ⓒ $x < -\frac{7}{3}$

Answer

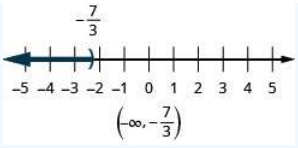
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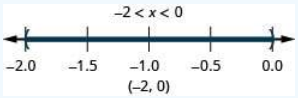


5. ⓐ $-5 < x < 2$
ⓑ $-3 \leq x < -1$
ⓒ $0 \leq x \leq 1.5$

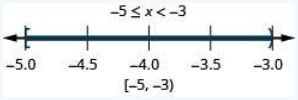
6. ⓐ $-2 < x < 0$
ⓑ $-5 \leq x < -3$
ⓒ $0 \leq x \leq 3.5$

Answer

ⓐ



ⓑ



ⓒ

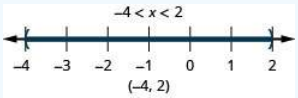


7. ⓐ $-1 < x < 3$
ⓑ $-3 < x \leq -2$
ⓒ $-1.25 \leq x \leq 0$

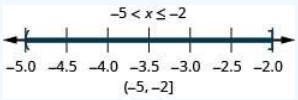
8. ⓐ $-4 < x < 2$
ⓑ $-5 < x \leq -2$
ⓒ $-3.75 \leq x \leq 0$

Answer

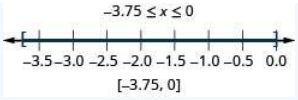
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Solve Linear Inequalities

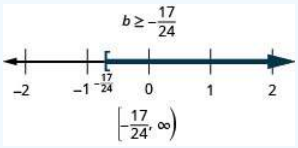
In the following exercises, solve each inequality, graph the solution on the number line, and write the solution in interval notation.

9. ⓐ $a+\frac{3}{4}\geq\frac{7}{10}$
ⓑ $8x>72$
ⓒ $20>\frac{2}{5}h$

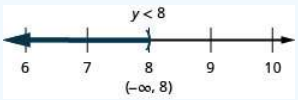
10. ⓐ $b+\frac{7}{8}\geq\frac{1}{6}$
ⓑ $6y<48$
ⓒ $40<\frac{5}{8}k$

Answer

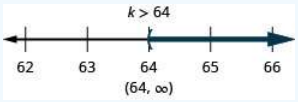
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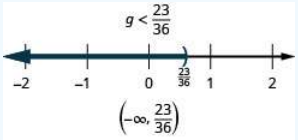


11. ⓐ $f-\frac{13}{20}<-\frac{5}{12}$
ⓑ $9t\geq-27$
ⓒ $\frac{7}{6}j\geq42$

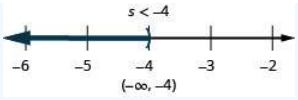
12. ⓐ $g-\frac{11}{12}<-\frac{5}{18}$
ⓑ $7s<-28$
ⓒ $\frac{9}{4}g\leq36$

Answer

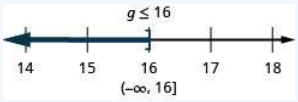
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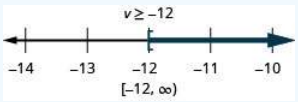


13. ⓐ $-5u \geq 65$
ⓑ $\frac{a}{-3} \leq 9$

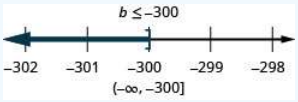
14. ⓐ $-8v \leq 96$
ⓑ $\frac{b}{-10} \geq 30$

Answer

ⓐ



ⓑ

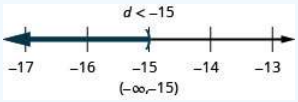


15. ⓐ $-9c < 126$
ⓑ $-25 < \frac{p}{-5}$

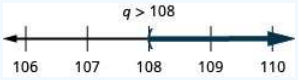
16. ⓐ $-7d > 105$
ⓑ $-18 > \frac{q}{-6}$

Answer

ⓐ



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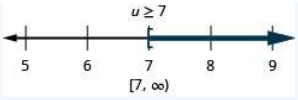
In the following exercises, solve each inequality, graph the solution on the number line, and write the solution in interval notation.

17. $4v \geq 9v - 40$

18. $5u \leq 8u - 21$

Answer

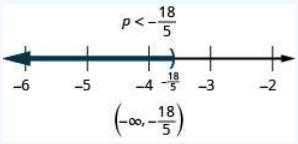
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19. $13q < 7q - 29$

20. $9p > 14p - 18$

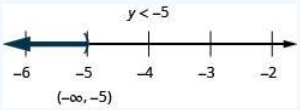
Answer



21. $12x + 3(x + 7) > 10x - 24$

22. $9y + 5(y + 3) < 4y - 35$

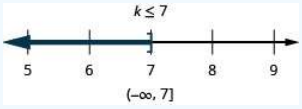
Answer



23. $6h - 4(h - 1) \leq 7h - 11$

24. $4k - (k - 2) \geq 7k - 26$

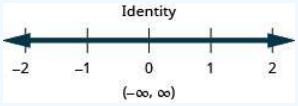
Answer



25. $8m - 2(14 - m) \geq 7(m - 4) + 3m$

26. $6n - 12(3 - n) \leq 9(n - 4) + 9n$

Answer

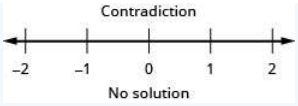


27. $\frac{3}{4}b - \frac{1}{3}b < \frac{5}{12}b - \frac{1}{2}$

28. $9u + 5(2u - 5) \geq 12(u - 1) + 7u$

Answer

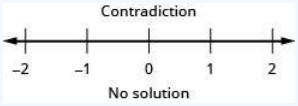
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29. $\frac{2}{3}g-\frac{1}{2}(g-14)\leq\frac{1}{6}(g+42)$

30. $\frac{4}{5}h-\frac{2}{3}(h-9)\geq\frac{1}{15}(2h+90)$

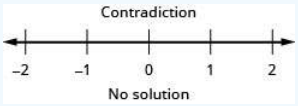
Answer



31. $\frac{5}{6}a-\frac{1}{4}a>\frac{7}{12}a+\frac{2}{3}$

32. $12v+3(4v-1)\leq19(v-2)+5v$

Answer

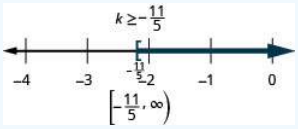


In the following exercises, solve each inequality, graph the solution on the number line, and write the solution in interval notation.

33. $15k\leq-40$

34. $35k\geq-77$

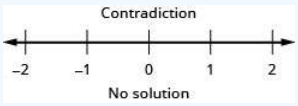
Answer



35. $23p-2(6-5p)>3(11p-4)$

36. $18q-4(10-3q)<5(6q-8)$

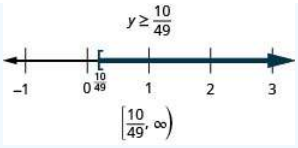
Answer



37. $-\frac{9}{4}x\geq-\frac{5}{12}$

38. $-\frac{21}{8}y\leq-\frac{15}{28}$

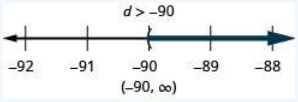
Answer



39. $c + 34 < -99$

40. $d + 29 > -61$

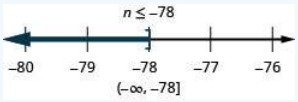
Answer



41. $\frac{m}{18} \geq -4$

42. $\frac{n}{13} \leq -6$

Answer



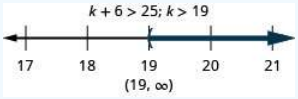
Translate to an Inequality and Solve

In the following exercises, translate and solve. Then graph the solution on the number line and write the solution in interval notation.

43. Three more than h is no less than 25.

44. Six more than k exceeds 25.

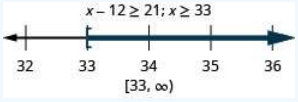
Answer



45. Ten less than w is at least 39.

46. Twelve less than x is no less than 21.

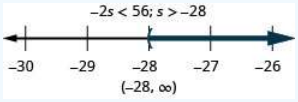
Answer



47. Negative five times r is no more than 95.

48. Negative two times s is lower than 56.

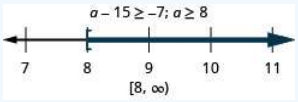
Answer



49. Nineteen less than b is at most -22.

50. Fifteen less than a is at least -7.

Answer



Solve Applications with Linear Inequalities

In the following exercises, solve.

51. Alan is loading a pallet with boxes that each weighs 45 pounds. The pallet can safely support no more than 900 pounds. How many boxes can he safely load onto the pallet?

52. The elevator in Yehire's apartment building has a sign that says the maximum weight is 2100 pounds. If the average weight of one person is 150 pounds, how many people can safely ride the elevator?

Answer

A maximum of 14 people can safely ride in the elevator.

53. Andre is looking at apartments with three of his friends. They want the monthly rent to be no more than $2,360. If the roommates split the rent evenly among the four of them, what is the maximum rent each will pay?

54. Arleen got a $20 gift card for the coffee shop. Her favorite iced drink costs $3.79. What is the maximum number of drinks she can buy with the gift card?

Answer

five drinks

55. Teegan likes to play golf. He has budgeted $60 next month for the driving range. It costs him $10.55 for a bucket of balls each time he goes. What is the maximum number of times he can go to the driving range next month?

56. Ryan charges his neighbors $17.50 to wash their car. How many cars must he wash next summer if his goal is to earn at least $1,500?

Answer

86 cars

57. Keshad gets paid $2,400 per month plus 6% of his sales. His brother earns $3,300 per month. For what amount of total sales will Keshad's monthly pay be higher than his brother's monthly pay?

58. Kimuyen needs to earn $4,150 per month in order to pay all her expenses. Her job pays her $3,475 per month plus 4% of her total sales. What is the minimum Kimuyen's total sales must be in order for her to pay all her expenses?

Answer

$16,875

59. Andre has been offered an entry-level job. The company offered him $48,000 per year plus 3.5% of his total sales. Andre knows that the average pay for this job is $62,000. What would Andre's total sales need to be for his pay to be at least as high as the average pay for this job?

60. Nataly is considering two job offers. The first job would pay her $83,000 per year. The second would pay her $66,500 plus 15% of her total sales. What would her total sales need to be for her salary on the second offer be higher than the first?

Answer

$110,000

61. Jake's water bill is $24.80 per month plus $2.20 per ccf (hundred cubic feet) of water. What is the maximum number of ccf Jake can use if he wants his bill to be no more than $60?

62. Kiyoshi's phone plan costs $17.50 per month plus $0.15 per text message. What is the maximum number of text messages Kiyoshi can use so the phone bill is no more than $56.60?

Answer

260 messages

63. Marlon's TV plan costs $49.99 per month plus $5.49 per first-run movie. How many first-run movies can he watch if he wants to keep his monthly bill to be a maximum of $100?

64. Kellen wants to rent a banquet room in a restaurant for her cousin's baby shower. The restaurant charges $350 for the banquet room plus $32.50 per person for lunch. How many people can Kellen have at the shower if she wants the maximum cost to be $1,500?

Answer

35 people

65. Moshde runs a hairstyling business from her house. She charges $45 for a haircut and style. Her monthly expenses are $960. She wants to be able to put at least $1,200 per month into her savings account order to open her own salon. How many "cut & styles" must she do to save at least $1,200 per month?

66. Noe installs and configures software on home computers. He charges $125 per job. His monthly expenses are $1,600. How many jobs must he work in order to make a profit of at least $2,400?

Answer

32 jobs

67. Katherine is a personal chef. She charges $115 per four-person meal. Her monthly expenses are $3,150. How many four-person meals must she sell in order to make a profit of at least $1,900?

68. Melissa makes necklaces and sells them online. She charges $88 per necklace. Her monthly expenses are $3,745. How many necklaces must she sell if she wants to make a profit of at least $1,650?

Answer

62 necklaces

69. Five student government officers want to go to the state convention. It will cost them $110 for registration, $375 for transportation and food, and $42 per person for the hotel. There is $450 budgeted for the convention in the student government savings account. They can earn the rest of the money they need by having a car wash. If they charge $5 per car, how many cars must they wash in order to have enough money to pay for the trip?

70. Cesar is planning a four-day trip to visit his friend at a college in another state. It will cost him $198 for airfare, $56 for local transportation, and $45 per day for food. He has $189 in savings and can earn $35 for each lawn he mows. How many lawns must he mow to have enough money to pay for the trip?

Answer

seven lawns

71. Alonzo works as a car detailer. He charges $175 per car. He is planning to move out of his parents' house and rent his first apartment. He will need to pay $120 for application fees, $950 for security deposit, and first and last months' rent at $1,140 per month. He has $1,810 in savings. How many cars must he detail to have enough money to rent the apartment?

72. Eun-Kyung works as a tutor and earns $60 per hour. She has $792 in savings. She is planning an anniversary party for her parents. She would like to invite 40 guests. The party will cost her $1,520 for food and drinks and $150 for the photographer. She will also have a favor for each of the guests, and each favor will cost $7.50. How many hours must she tutor to have enough money for the party?

Answer

20 hours

Everyday Math

73. Maximum load on a stage In 2014, a high school stage collapsed in Fullerton, California, when 250 students got on stage for the finale of a musical production. Two dozen students were injured. The stage could support a maximum of 12,750 pounds. If the average weight of a student is assumed to be 140 pounds, what is the maximum number of students who could safely be on the stage?

74. Maximum weight on a boat In 2004, a water taxi sank in Baltimore harbor and five people drowned. The water taxi had a maximum capacity of 3,500 pounds (25 people with average weight 140 pounds). The average weight of the 25 people on the water taxi when it sank was 168 pounds per person. What should the maximum number of people of this weight have been?

Answer

20 people

75. Wedding budget Adele and Walter found the perfect venue for their wedding reception. The cost is $9850 for up to 100 guests, plus $38 for each additional guest. How many guests can attend if Adele and Walter want the total cost to be no more than $12,500?

76. Shower budget Penny is planning a baby shower for her daughter-in-law. The restaurant charges $950 for up to 25 guests, plus $31.95 for each additional guest. How many guests can attend if Penny wants the total cost to be no more than $1,500?

Answer

42 guests

Writing Exercises

77. Explain why it is necessary to reverse the inequality when solving $-5x > 10$.

78. Explain why it is necessary to reverse the inequality when solving $n - 3 < 12$.

Answer

Answers will vary.

79. Find your last month's phone bill and the hourly salary you are paid at your job. Calculate the number of hours of work it would take you to earn at least enough money to pay your phone bill by writing an appropriate inequality and then solving it. Do you feel this is an appropriate number of hours? Is this the appropriate phone plan for you?

80. Find out how many units you have left, after this term, to achieve your college goal and estimate the number of units you can take each term in college. Calculate the number of terms it will take you to achieve your college goal by writing an appropriate inequality and then solving it. Is this an acceptable number of terms until you meet your goal? What are some ways you could accelerate this process?

Answer

Answers will vary.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
graph inequalities on the number line.			
solve linear inequalities.			
translate words to an inequality and solve.			
solve applications with linear inequalities.			

ⓑ After looking at the checklist, do you think you are well-prepared for the next section? Why or why not?

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2.9: Solve Compound Inequalities - Union, Intersection

Learning Objectives

By the end of this section, you will be able to:

- Solve compound inequalities with “and”
- Solve compound inequalities with “or”
- Solve applications with compound inequalities

Before you get started, take this readiness quiz.

1. Simplify: $\frac{2}{5}(x+10)$.
   If you missed this problem, review [link].
2. Simplify: $-(x-4)$.
   If you missed this problem, review [link].

Solve Compound Inequalities with “and”

Now that we know how to solve linear inequalities, the next step is to look at compound inequalities. A **compound inequality** is made up of two inequalities connected by the word “and” or the word “or.” For example, the following are compound inequalities.

$$\begin{array}{lll} x+3>-4 & \text{and} & 4x-5\leq 3 \\ 2(y+1)<0 & \text{or} & y-5\geq -2 \end{array}$$

COMPOUND INEQUALITY

A **compound inequality** is made up of two inequalities connected by the word “and” or the word “or.”

To solve a compound inequality means to find all values of the variable that make the compound inequality a true statement. We solve compound inequalities using the same techniques we used to solve linear inequalities. We solve each inequality separately and then consider the two solutions.

To solve a compound inequality with the word “and,” we look for all numbers that make *both* inequalities true. To solve a compound inequality with the word “or,” we look for all numbers that make *either* inequality true.

Let’s start with the compound inequalities with “and.” Our solution will be the numbers that are solutions to *both* inequalities known as the intersection of the two inequalities. Consider the intersection of two streets—the part where the streets overlap—belongs to both streets.



To find the solution of an "and" compound inequality, we look at the graphs of each inequality and then find the numbers that belong to both graphs—where the graphs overlap.

For the compound inequality $x>-3$ and $x\leq 2$, we graph each inequality. We then look for where the graphs “overlap”. The numbers that are shaded on both graphs, will be shaded on the graph of the solution of the compound inequality. See Figure 2.9.1.

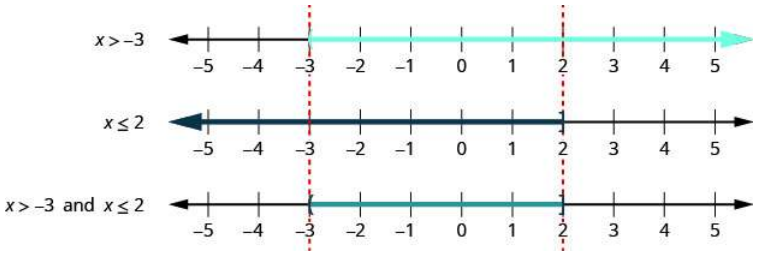


Figure 2.9.1

We can see that the numbers between -3 and 2 are shaded on both of the first two graphs. They will then be shaded on the solution graph.

The number -3 is not shaded on the first graph and so since it is not shaded on both graphs, it is not included on the solution graph.

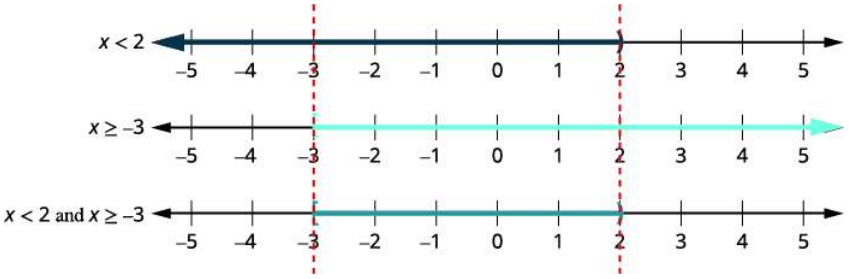
The number two is shaded on both the first and second graphs. Therefore, it is be shaded on the solution graph.

This is how we will show our solution in the next examples.

Example 2.9.1

Solve $6x-3<9$ and $2x+7\geq 3$. Graph the solution and write the solution in interval notation.

Solution

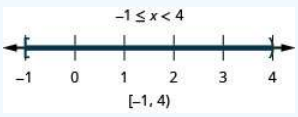
	$6x-3<9$	and	$2x+9\geq 3$
Step 1. Solve each inequality.	$6x-3<9$		$2x+9\geq 3$
	$6x<12$		$2x\geq -6$
	$x<2$	and	$x\geq -3$
Step 2. Graph each solution. Then graph the numbers that make both inequalities true. The final graph will show all the numbers that make both inequalities true—the numbers shaded on *both* of the first two graphs.			
Step 3. Write the solution in interval notation.	$[-3, 2)$		

All the numbers that make both inequalities true are the solution to the compound inequality.

Try It 2.9.2

Solve the compound inequality. Graph the solution and write the solution in interval notation: $4x-7<9$ and $5x+8\geq 3$.

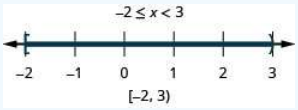
Answer



Try It 2.9.3

Solve the compound inequality. Graph the solution and write the solution in interval notation: $3x-4<5$ and $4x+9\geq 1$.

Answer



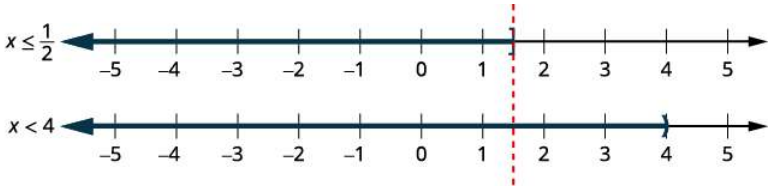
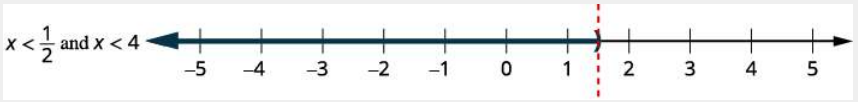
SOLVE A COMPOUND INEQUALITY WITH "AND."

1. Solve each inequality.
2. Graph each solution. Then graph the numbers that make *both* inequalities true.
   This graph shows the solution to the compound inequality.
3. Write the solution in interval notation.

Example 2.9.4

Solve $3(2x+5)\leq 18$ and $2(x-7)<-6$. Graph the solution and write the solution in interval notation.

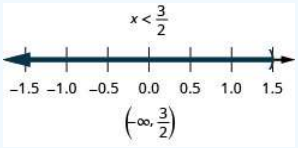
Solution

	$3(2x+5)\leq 18$	and	$2(x-7)<-6$
Solve each inequality.	$6x+15\leq 18$		$2x-14<-6$
	$6x\leq 3$		$2x<8$
	$x\leq \frac{1}{2}$	and	$x<4$
Graph each solution.			
Graph the numbers that make both inequalities true.			
Write the solution in interval notation.	$(-\infty, \frac{1}{2}]$		

Try It 2.9.5

Solve the compound inequality. Graph the solution and write the solution in interval notation: $2(3x+1)\leq 20$ and $4(x-1)<2$.

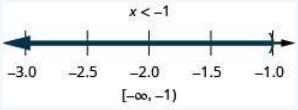
Answer



Try It 2.9.6

Solve the compound inequality. Graph the solution and write the solution in interval notation: $5(3x-1) \leq 10$ and $4(x+3) < 8$.

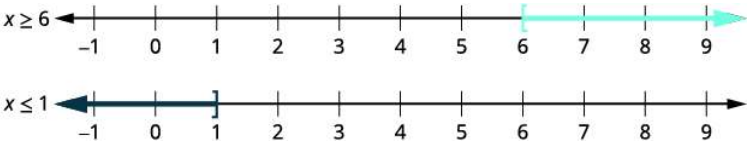
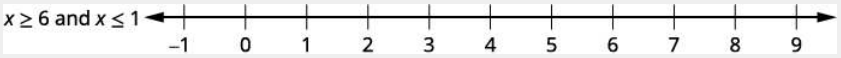
Answer



Example 2.9.7

Solve $\frac{1}{3}x - 4 \geq -2$ and $-2(x-3) \geq 4$. Graph the solution and write the solution in interval notation.

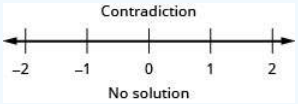
Solution

	$\frac{1}{3}x - 4 \geq -2$	and	$-2(x-3) \geq 4$
Solve each inequality.	$\frac{1}{3}x - 4 \geq -2$		$-2x + 6 \geq 4$
	$\frac{1}{3}x \geq 2$		$-2x \geq -2$
	$x \geq 6$	and	$x \leq 1$
Graph each solution.			
Graph the numbers that make both inequalities true.			
	This is a contradiction. There are no numbers that make both inequalities true, so there is no solution.		

Try It 2.9.8

Solve the compound inequality. Graph the solution and write the solution in interval notation: $\frac{1}{4}x - 3 \geq -1$ and $-3(x-2) \geq 2$.

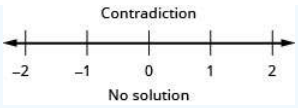
Answer



Try It 2.9.9

Solve the compound inequality. Graph the solution and write the solution in interval notation: $\frac{1}{5}x - 5 \geq -3$ and $-4(x-1) \geq -2$.

Answer



Sometimes we have a compound inequality that can be written more concisely. For example, $a < x$ and $x < b$ can be written simply as $a < x < b$ and then we call it a **double inequality**. The two forms are equivalent.

DOUBLE INEQUALITY

A double inequality is a compound inequality such as $a < x < b$. It is equivalent to $a < x$ and $x < b$.

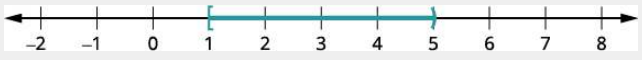
$$\text{Other forms:} \quad \begin{array}{lllll} a < x < b & \text{is equivalent to} & a < x & \text{and} & x < b \\ a \leq x \leq b & \text{is equivalent to} & a \leq x & \text{and} & x \leq b \\ a > x > b & \text{is equivalent to} & a > x & \text{and} & x > b \\ a \geq x \geq b & \text{is equivalent to} & a \geq x & \text{and} & x \geq b \end{array}$$

To solve a double inequality we perform the same operation on all three "parts" of the double inequality with the goal of isolating the variable in the center.

Example 2.9.10

Solve $-4 \leq 3x - 7 < 8$. Graph the solution and write the solution in interval notation.

Solution

	$-4 \leq 3x - 7 < 8$
Add 7 to all three parts.	$-4+7 \leq 3x - 7+7 < 8+7$
Simplify.	$3 \leq 3x < 15$
Divide each part by three.	$\frac{3}{3} \leq \frac{3x}{3} < \frac{15}{3}$
Simplify.	$1 \leq x < 5$
Graph the solution.	
Write the solution in interval notation.	$[1, 5)$

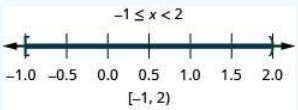
When written as a double inequality, $1 \leq x < 5$, it is easy to see that the solutions are the numbers caught between one and five, including one, but not five. We can then graph the solution immediately as we did above.

Another way to graph the solution of $1 \leq x < 5$ is to graph both the solution of $x \geq 1$ and the solution of $x < 5$. We would then find the numbers that make both inequalities true as we did in previous examples.

Try It 2.9.11

Solve the compound inequality. Graph the solution and write the solution in interval notation: $-5 \leq 4x - 1 < 7$.

Answer

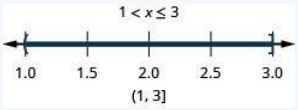


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Try It 2.9.12

Solve the compound inequality. Graph the solution and write the solution in interval notation: $-3 < 2x - 5 \leq 1$.

Answer



Solve Compound Inequalities with "or"

To solve a **compound inequality** with "or", we start out just as we did with the compound inequalities with "and"—we solve the two inequalities. Then we find all the numbers that make *either* inequality true.

Just as the United States is the union of all of the 50 states, the solution will be the union of all the numbers that make either inequality true. To find the solution of the compound inequality, we look at the graphs of each inequality, find the numbers that belong to either graph and put all those numbers together.

To write the solution in **interval notation**, we will often use the **union symbol**, $\cup$, to show the union of the solutions shown in the graphs.

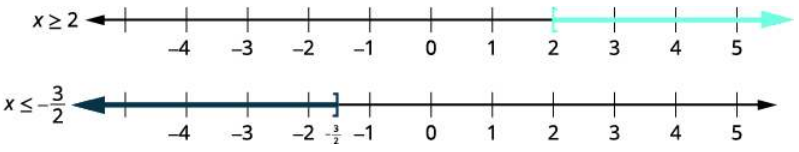
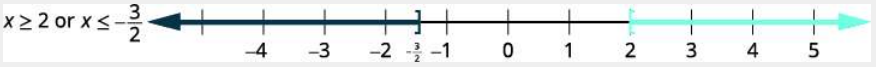
SOLVE A COMPOUND INEQUALITY WITH "OR."

1. Solve each inequality.
2. Graph each solution. Then graph the numbers that make either inequality true.
3. Write the solution in interval notation.

Example 2.9.13

Solve $5 - 3x \leq -1$ or $8 + 2x \leq 5$. Graph the solution and write the solution in interval notation.

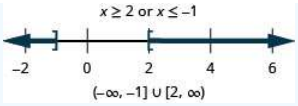
Solution

	$5 - 3x \leq -1$	or	$8 + 2x \leq 5$
Solve each inequality.	$5 - 3x \leq -1$		$8 + 2x \leq 5$
	$-3x \leq -6$		$2x \leq -3$
	$x \geq 2$	or	$x \leq -\frac{3}{2}$
Graph each solution.			
Graph numbers that make either inequality true.			
	$(-\infty, -\frac{3}{2}] \cup [2, \infty)$		

Try It 2.9.14

Solve the compound inequality. Graph the solution and write the solution in interval notation: $1 - 2x \leq -3$ or $7 + 3x \leq 4$.

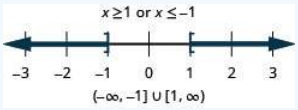
Answer



Try It 2.9.15

Solve the compound inequality. Graph the solution and write the solution in interval notation: $2-5x \leq -3$ or $5+2x \leq 3$.

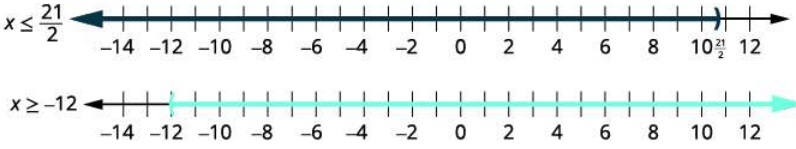
Answer



Example 2.9.16

Solve $\frac{2}{3}x-4 \leq 3$ or $\frac{1}{4}(x+8) \geq -1$. Graph the solution and write the solution in interval notation.

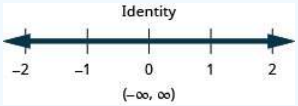
Solution

	$\frac{2}{3}x-4 \leq 3$	or	$\frac{1}{4}(x+8) \geq -1$
Solve each inequality.	$3(\frac{2}{3}x-4) \leq 3(3)$		$4 \cdot \frac{1}{4}(x+8) \geq 4 \cdot (-1)$
	$2x-12 \leq 9$		$x+8 \geq -4$
	$2x \leq 21$		$x \geq -12$
	$x \leq \frac{21}{2}$		
	$x \leq \frac{21}{2}$	or	$x \geq -12$
Graph each solution.			
Graph numbers that make either inequality true.			
	The solution covers all real numbers.		
	$(-\infty, \infty)$		

Try It 2.9.17

Solve the compound inequality. Graph the solution and write the solution in interval notation: $\frac{3}{5}x-7 \leq -1$ or $\frac{1}{3}(x+6) \geq -2$.

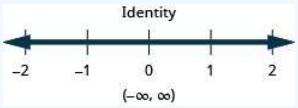
Answer



Try It 2.9.18

Solve the compound inequality. Graph the solution and write the solution in interval notation: $\frac{3}{4}x-3\leq 3$ or $\frac{2}{5}(x+10)\geq 0$.

Answer



Solve Applications with Compound Inequalities

Situations in the real world also involve compound inequalities. We will use the same problem solving strategy that we used to solve linear equation and inequality applications.

Recall the problem solving strategies are to first read the problem and make sure all the words are understood. Then, identify what we are looking for and assign a variable to represent it. Next, restate the problem in one sentence to make it easy to translate into a **compound inequality**. Last, we will solve the compound inequality.

Example 2.9.19

Due to the drought in California, many communities have tiered water rates. There are different rates for Conservation Usage, Normal Usage and Excessive Usage. The usage is measured in the number of hundred cubic feet (hcf) the property owner uses.

During the summer, a property owner will pay $24.72 plus $1.54 per hcf for Normal Usage. The bill for Normal Usage would be between or equal to $57.06 and $171.02. How many hcf can the owner use if he wants his usage to stay in the normal range?

Solution

Identify what we are looking for.	The number of hcf he can use and stay in the “normal usage” billing range.
Name what we are looking for.	Let x= the number of hcf he can use.
Translate to an inequality.	Bill is $24.72 plus $1.54 times the number of hcf he uses or $24.72+1.54x$.
	$\underbrace{\text{His bill will be between or equal to \$57.06 and \$171.02.}}$ $57.06\leq 24.72+1.54x\leq 171.02$
Solve the inequality.	$57.06\leq 24.72+1.54x\leq 171.02$ $57.06-24.72\leq 24.72-24.72+1.54x\leq 171.02-24.72$ $32.34\leq 1.54x\leq 146.3$ $\frac{32.34}{1.54}\leq\frac{1.54x}{1.54}\leq\frac{146.3}{1.54}$ $21\leq x\leq 95$
Answer the question.	The property owner can use $21-95$ hcf and still fall within the “normal usage” billing range.

Try It 2.9.20

Due to the drought in California, many communities now have tiered water rates. There are different rates for Conservation Usage, Normal Usage and Excessive Usage. The usage is measured in the number of hundred cubic feet (hcf) the property owner uses.

During the summer, a property owner will pay $24.72 plus $1.32 per hcf for Conservation Usage. The bill for Conservation Usage would be between or equal to $31.32 and $51.12. How many hcf can the owner use if she wants her usage to stay in the

conservation range?

Answer

The homeowner can use $5-20$ hcf and still fall within the “conservation usage” billing range.

? Try It 2.9.21

Due to the drought in California, many communities have tiered water rates. There are different rates for Conservation Usage, Normal Usage and Excessive Usage. The usage is measured in the number of hundred cubic feet (hcf) the property owner uses.

During the winter, a property owner will pay $24.72 plus $1.54 per hcf for Normal Usage. The bill for Normal Usage would be between or equal to $49.36 and $86.32. How many hcf will he be allowed to use if he wants his usage to stay in the normal range?

Answer

The homeowner can use $16-40$ hcf and still fall within the “normal usage” billing range.

Access this online resource for additional instruction and practice with solving compound inequalities.

- Compound inequalities

Key Concepts

- **How to solve a compound inequality with “and”**
  1. Solve each inequality.
  2. Graph each solution. Then graph the numbers that make *both* inequalities true. This graph shows the solution to the compound inequality.
  3. Write the solution in interval notation.
- **Double Inequality**
  - A **double inequality** is a compound inequality such as $a < x < b$. It is equivalent to $a < x$ and $x < b$.

    Other forms:

$$\begin{array}{lll} a < x < b & \text{is equivalent to} & a < x \text{ and } x < b \\ a \leq x \leq b & \text{is equivalent to} & a \leq x \text{ and } x \leq b \\ a > x > b & \text{is equivalent to} & a > x \text{ and } x > b \\ a \geq x \geq b & \text{is equivalent to} & a \geq x \text{ and } x \geq b \end{array}$$

- **How to solve a compound inequality with “or”**
  1. Solve each inequality.
  2. Graph each solution. Then graph the numbers that make either inequality true.
  3. Write the solution in interval notation.

Glossary

compound inequality

A compound inequality is made up of two inequalities connected by the word “and” or the word “or.”

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2.9E: Exercises

Practice Makes Perfect

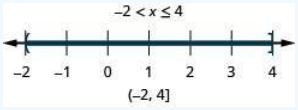
Solve Compound Inequalities with "and"

In the following exercises, solve each inequality, graph the solution, and write the solution in interval notation.

1. $x < 3$ and $x \geq 1$

2. $x \leq 4$ and $x > -2$

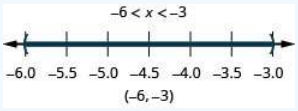
Answer



3. $x \geq -4$ and $x \leq -1$

4. $x > -6$ and $x < -3$

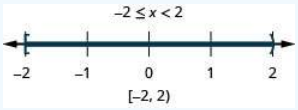
Answer



5. $5x - 2 < 8$ and $6x + 9 \geq 3$

6. $4x - 1 < 7$ and $2x + 8 \geq 4$

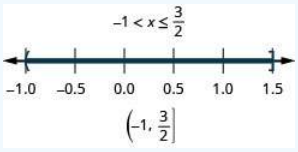
Answer



7. $4x + 6 \leq 2$ and $2x + 1 \geq -5$

8. $4x - 2 \leq 4$ and $7x - 1 > -8$

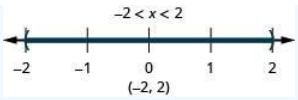
Answer



9. $2x - 11 < 5$ and $3x - 8 > -5$

10. $7x - 8 < 6$ and $5x + 7 > -3$

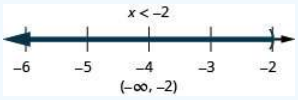
Answer



11. $4(2x-1) \leq 12$ and $2(x+1) < 4$

12. $5(3x-2) \leq 5$ and $3(x+3) < 3$

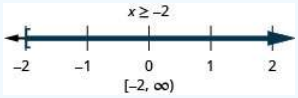
Answer



13. $3(2x-3) > 3$ and $4(x+5) \geq 4$

14. $-3(x+4) < 0$ and $-1(3x-1) \leq 7$

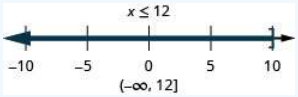
Answer



15. $\frac{1}{2}(3x-4) \leq 1$ and $\frac{1}{3}(x+6) \leq 4$

16. $\frac{3}{4}(x-8) \leq 3$ and $\frac{1}{5}(x-5) \leq 3$

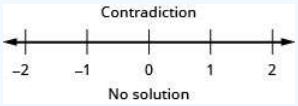
Answer



17. $5x-2 \leq 3x+4$ and $3x-4 \geq 2x+1$

18. $\frac{3}{4}x-5 \geq -2$ and $-3(x+1) \geq 6$

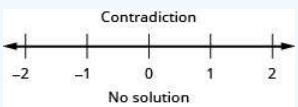
Answer



19. $\frac{2}{3}x-6 \geq -4$ and $-4(x+2) \geq 0$

20. $\frac{1}{2}(x-6)+2 < -5$ and $4-\frac{2}{3}x < 6$

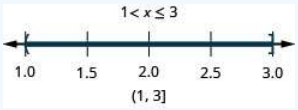
Answer



21. $-5 \leq 4x - 1 < 7$

22. $-3 < 2x - 5 \leq 1$

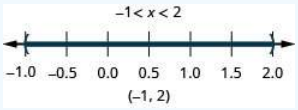
Answer



23. $5 < 4x + 1 < 9$

24. $-1 < 3x + 2 < 8$

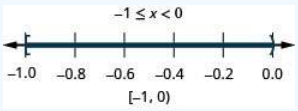
Answer



25. $-8 < 5x + 2 \leq -3$

26. $-6 \leq 4x - 2 < -2$

Answer



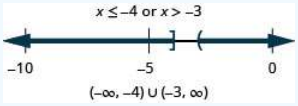
Solve Compound Inequalities with "or"

In the following exercises, solve each inequality, graph the solution on the number line, and write the solution in interval notation.

27. $x \leq -2$ or $x > 3$

28. $x \leq -4$ or $x > -3$

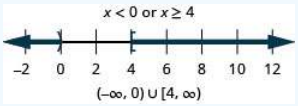
Answer



29. $x < 2$ or $x \geq 5$

30. $x < 0$ or $x \geq 4$

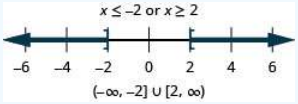
Answer



31. $2+3x \leq 4$ or $5-2x \leq -1$

32. $4-3x \leq -2$ or $2x-1 \leq -5$

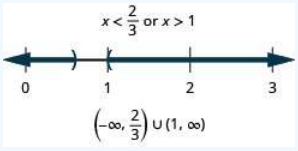
Answer



33. $2(3x-1) < 4$ or $3x-5 > 1$

34. $3(2x-3) < -5$ or
$4x-1 > 3$

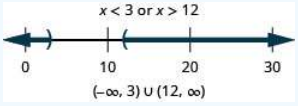
Answer



35. $\frac{3}{4}x-2 > 4$ or $4(2-x) > 0$

36. $\frac{2}{3}x-3 > 5$ or $3(5-x) > 6$

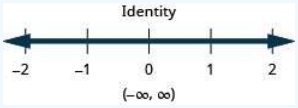
Answer



37. $3x-2 > 4$ or $5x-3 \leq 7$

38. $2(x+3) \geq 0$ or $3(x+4) \leq 6$

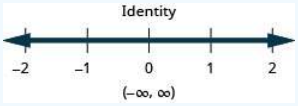
Answer



39. $\frac{1}{2}x-3 \leq 4$ or $\frac{1}{3}(x-6) \geq -2$

40. $\frac{3}{4}x+2 \leq -1$ or $\frac{1}{2}(x+8) \geq -3$

Answer



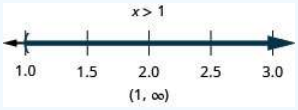
Mixed practice

In the following exercises, solve each inequality, graph the solution on the number line, and write the solution in interval notation.

41. $3x+7 \leq 1$ and $2x+3 \geq -5$

42. $6(2x-1) > 6$ and $5(x+2) \geq 0$

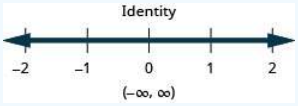
Answer



43. $4-7x \geq -3$ or $5(x-3)+8 > 3$

44. $\frac{1}{2}x-5 \leq 3$ or $\frac{1}{4}(x-8) \geq -3$

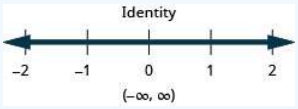
Answer



45. $-5 \leq 2x-1 < 7$

46. $\frac{1}{5}(x-5)+6 < 4$ and $3-\frac{2}{3}x < 5$

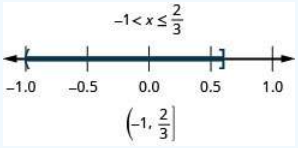
Answer



47. $4x-2 > 6$ or $3x-1 \leq -2$

48. $6x-3 \leq 1$ and $5x-1 > -6$

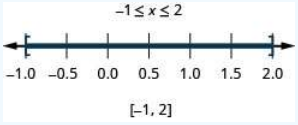
Answer



49. $-2(3x-4) \leq 2$ and $-4(x-1) < 2$

50. $-5 \leq 3x-2 \leq 4$

Answer



Solve Applications with Compound Inequalities

In the following exercises, solve.

51. Penelope is playing a number game with her sister June. Penelope is thinking of a number and wants June to guess it. Five more than three times her number is between 2 and 32. Write a compound inequality that shows the range of numbers that Penelope might be thinking of.

52. Gregory is thinking of a number and he wants his sister Lauren to guess the number. His first clue is that six less than twice his number is between four and forty-two. Write a compound inequality that shows the range of numbers that Gregory might be thinking of.

Answer

$5 \leq n \leq 24$

53. Andrew is creating a rectangular dog run in his back yard. The length of the dog run is 18 feet. The perimeter of the dog run must be at least 42 feet and no more than 72 feet. Use a compound inequality to find the range of values for the width of the dog run.

54. Elouise is creating a rectangular garden in her back yard. The length of the garden is 12 feet. The perimeter of the garden must be at least 36 feet and no more than 48 feet. Use a compound inequality to find the range of values for the width of the garden.

Answer

$6 \leq w \leq 12$

Everyday Math

55. Blood Pressure A person's blood pressure is measured with two numbers. The systolic blood pressure measures the pressure of the blood on the arteries as the heart beats. The diastolic blood pressure measures the pressure while the heart is resting.

ⓐ Let x be your systolic blood pressure. Research and then write the compound inequality that shows you what a normal systolic blood pressure should be for someone your age.

ⓑ Let y be your diastolic blood pressure. Research and then write the compound inequality that shows you what a normal diastolic blood pressure should be for someone your age.

56. Body Mass Index (BMI) is a measure of body fat is determined using your height and weight.

ⓐ Let x be your BMI. Research and then write the compound inequality to show the BMI range for you to be considered normal weight.

ⓑ Research a BMI calculator and determine your BMI. Is it a solution to the inequality in part (a)?

Answer

ⓐ answers vary ⓑ answers vary

Writing Exercises

57. In your own words, explain the difference between the properties of equality and the properties of inequality.

58. Explain the steps for solving the compound inequality $2-7x \geq -5$ or $4(x-3)+7>3$.

Answer

Answers will vary.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
solve compound inequalities with "and".			
solve compound inequalities with "or".			
solve applications with compound inequalities.			

ⓑ What does this checklist tell you about your mastery of this section? What steps will you take to improve?

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2.10: Graphs of Linear Inequalities in Two Variables (optional challenge practice)

Learning Objectives

By the end of this section, you will be able to:

- Verify solutions to an inequality in two variables
- Recognize the relation between the solutions of an inequality and its graph
- Graph linear inequalities

Note

Before you get started, take this readiness quiz.

1. Solve: $4x+3>23$.
   If you missed this problem, review Exercise 2.7.22.
2. Translate from algebra to English: $x<5$.
   If you missed this problem, review Exercise 1.3.1.
3. Evaluate $3x-2y$ when $x=1,\ y=-2$.
   If you missed this problem, review Exercise 1.5.28.

Verify Solutions to an Inequality in Two Variables

We have learned how to solve inequalities in one variable. Now, we will look at inequalities in two variables. Inequalities in two variables have many applications. If you ran a business, for example, you would want your revenue to be greater than your costs—so that your business would make a profit.

LINEAR INEQUALITY

A **linear inequality** is an inequality that can be written in one of the following forms:

$$Ax+By>C \quad Ax+By\geq C \quad Ax+By<C \quad Ax+By\leq C$$

where A and B are not both zero.

Do you remember that an inequality with one variable had many solutions? The solution to the inequality $x>3$ is any number greater than 3. We showed this on the number line by shading in the number line to the right of 3, and putting an open parenthesis at 3. See Figure 2.10.1.

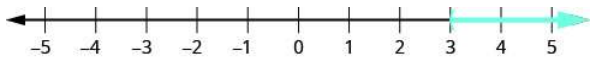


Figure 2.10.1

Similarly, inequalities in two variables have many solutions. Any ordered pair (x,y) that makes the inequality true when we substitute in the values is a solution of the inequality.

Solution OF A LINEAR INEQUALITY

An ordered pair (x,y) is a **solution of a linear inequality** if the inequality is true when we substitute the values of x and y.

Example 2.10.1

Determine whether each ordered pair is a solution to the inequality $y>x+4$:

1. $(0,0)$
2. $(1,6)$
3. $(2,6)$
4. $(-5,-15)$

5. $(-8, 12)$

Solution

1.

$(0, 0)$	$y > x + 4$
Substitute 0 for x and 0 for y.	$0 \stackrel{?}{>} 0 + 4$
Simplify.	$0 \not> 4$ So, $(0, 0)$ is not a solution to $y > x + 4$.

2.

$(1, 6)$	$y > x + 4$
Substitute 1 for x and 6 for y.	$6 \stackrel{?}{>} 1 + 4$
Simplify.	$6 > 5$ So, $(1, 6)$ is a solution to $y > x + 4$.

3.

$(2, 6)$	$y > x + 4$
Substitute 2 for x and 6 for y.	$6 \stackrel{?}{>} 2 + 4$
Simplify.	$6 \not> 6$ So, $(2, 6)$ is not a solution to $y > x + 4$.

4.

$(-5, -15)$	$y > x + 4$
Substitute –5 for x and –15 for y.	$-15 \stackrel{?}{>} -5 + 4$
Simplify.	$-15 \not> -1$ So, $(-5, -15)$ is not a solution to $y > x + 4$.

5.

$(-8,12)$	$y > x + 4$
Substitute –8 for x and 12 for y.	$12 \stackrel{?}{>} -8 + 4$
Simplify.	$12 > -4$ So, $(-8, 12)$ is a solution to $y > x + 4$.

? Try It 2.10.2

Determine whether each ordered pair is a solution to the inequality $y > x - 3$:

1. $(0, 0)$
2. $(4, 9)$
3. $(-2, 1)$
4. $(-5, -3)$
5. $(5, 1)$

Answer

1. yes

2. yes
3. yes
4. yes
5. no

Try It 2.10.3

Determine whether each ordered pair is a solution to the inequality $y < x + 1$:

1. $(0, 0)$
2. $(8, 6)$
3. $(-2, -1)$
4. $(3, 4)$
5. $(-1, -4)$

Answer

1. yes
2. yes
3. no
4. no
5. yes

Recognize the Relation Between the Solutions of an Inequality and its Graph

Now, we will look at how the solutions of an inequality relate to its graph.

Let's think about the number line in Figure 2.10.1 again. The point $x = 3$ separated that number line into two parts. On one side of 3 are all the numbers less than 3. On the other side of 3 all the numbers are greater than 3. See Figure 2.10.2

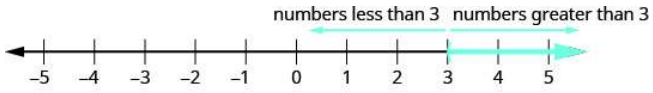


Figure 2.10.2

The solution to $x > 3$ is the shaded part of the number line to the right of $x = 3$.

Similarly, the line $y = x + 4$ separates the plane into two regions. On one side of the line are points with $y < x + 4$. On the other side of the line are the points with $y > x + 4$. We call the line $y = x + 4$ a boundary line.

BOUNDARY LINE

The line with equation $Ax + By = C$ is the **boundary line** that separates the region where $Ax + By > C$ from the region where $Ax + By < C$.

For an inequality in one variable, the endpoint is shown with a parenthesis or a bracket depending on whether or not aa is included in the solution:



Similarly, for an inequality in two variables, the boundary line is shown with a solid or dashed line to indicate whether or not it the line is included in the solution. This is summarized in Table 2.10.1.

Table 2.10.1

$Ax + By < C$	$Ax + By \leq C$
$Ax + By > C$	$Ax + By \geq C$
Boundary line is not included in solution.	Boundary line is included in solution.

Boundary line is dashed.	Boundary line is solid.

Now, let's take a look at what we found in Example 2.10.1. We'll start by graphing the line $y = x + 4$, and then we'll plot the five points we tested. See Figure 2.10.3

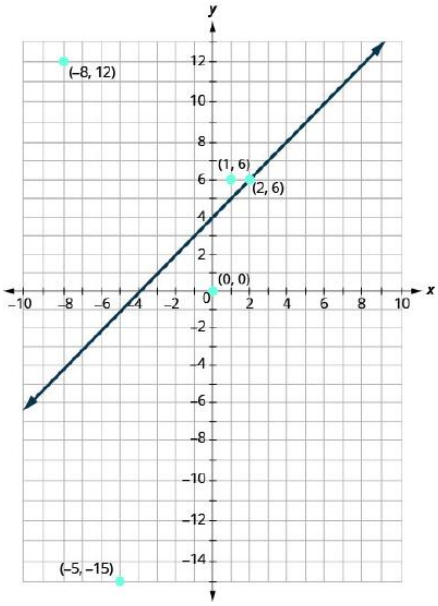


Figure 2.10.3

In Example 2.10.1 we found that some of the points were solutions to the inequality $y > x + 4$ and some were not.

Which of the points we plotted are solutions to the inequality $y > x + 4$? The points $(1, 6)$ and $(-8, 12)$ are solutions to the inequality $y > x + 4$. Notice that they are both on the same side of the boundary line $y = x + 4$.

The two points $(0, 0)$ and $(-5, -15)$ are on the other side of the **boundary line** $y = x + 4$, and they are not solutions to the inequality $y > x + 4$. For those two points, $y < x + 4$.

What about the point $(2, 6)$? Because $6 = 2 + 4$, the point is a solution to the equation $y = x + 4$. So the point $(2, 6)$ is on the boundary line.

Let's take another point on the left side of the boundary line and test whether or not it is a solution to the inequality $y > x + 4$. The point $(0, 10)$ clearly looks to be to the left of the boundary line, doesn't it? Is it a solution to the inequality?

$$\begin{aligned} &y > x + 4 \\ &10 \overset{?}{>} 0 + 4 \\ &10 > 4 \qquad \text{So, } (0, 10) \text{ is a solution to } y > x + 4. \end{aligned} \tag{2.10.1}$$

Any point you choose on the left side of the boundary line is a solution to the inequality $y > x + 4$. All points on the left are solutions.

Similarly, all points on the right side of the boundary line, the side with $(0, 0)$ and $(-5, -15)$, are not solutions to $y > x + 4$. See Figure 2.10.4

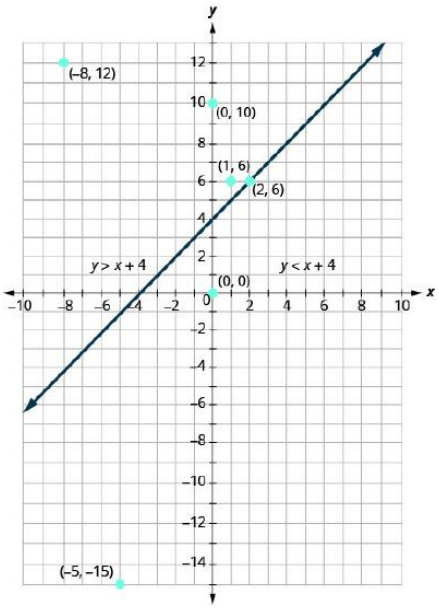


Figure 2.10.4

The graph of the inequality $y > x + 4$ is shown in Figure $2.10.5$ below. The line $y = x + 4$ divides the plane into two regions. The shaded side shows the solutions to the inequality $y > x + 4$.

The points on the boundary line, those where $y = x + 4$, are not solutions to the inequality $y > x + 4$, so the line itself is not part of the solution. We show that by making the line dashed, not solid.

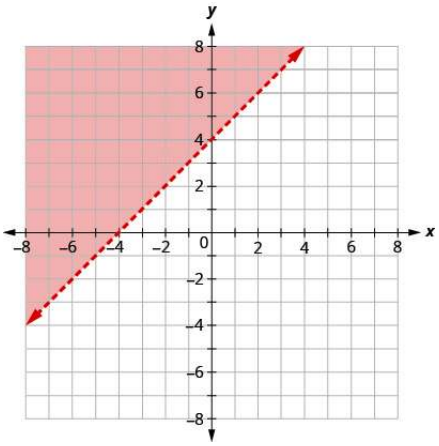
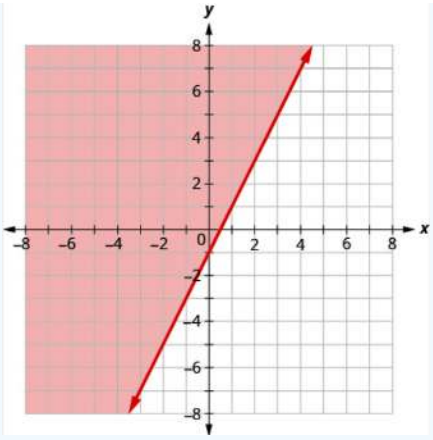


Figure 2.10.5: The graph of the inequality y>x+4.

Example 2.10.4

The boundary line shown is $y = 2x - 1$. Write the inequality shown by the graph.



Solution

The line $y = 2x - 1$ is the boundary line. On one side of the line are the points with $y > 2x - 1$ and on the other side of the line are the points with $y < 2x - 1$.

Let's test the point $(0, 0)$ and see which inequality describes its side of the boundary line.

At $(0, 0)$, which inequality is true:

$$\begin{array}{lll} y > 2x - 1 & \text{or} & y < 2x - 1? \\ y > 2x - 1 & & y < 2x - 1 \\ 0 > 2 \cdot 0 - 1 & & 0 < 2 \cdot 0 - 1 \\ 0 > -1 \text{ True} & & 0 < -1 \text{ False} \end{array} \tag{2.10.2}$$

Since $y > 2x - 1$ is true, the side of the line with $(0, 0)$, is the solution. The shaded region shows the solution of the inequality $y > 2x - 1$.

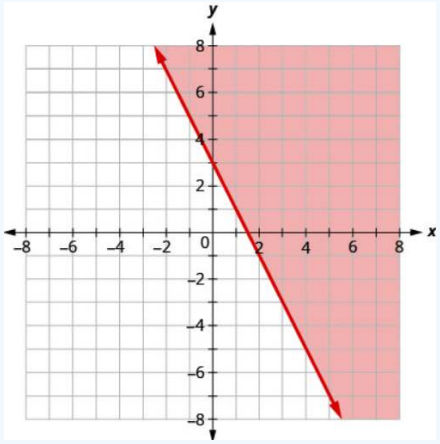
Since the boundary line is graphed with a solid line, the inequality includes the equal sign.

The graph shows the inequality $y \geq 2x - 1$.

We could use any point as a test point, provided it is not on the line. Why did we choose $(0, 0)$? Because it's the easiest to evaluate. You may want to pick a point on the other side of the boundary line and check that $y < 2x - 1$.

? Try It 2.10.5

Write the inequality shown by the graph with the boundary line $y = -2x + 3$.



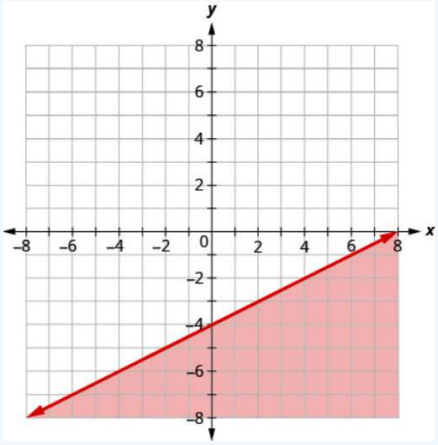
Answer

$y \geq -2x + 3$

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Try It 2.10.6

Write the inequality shown by the graph with the boundary line $y = \frac{1}{2}x - 4$.

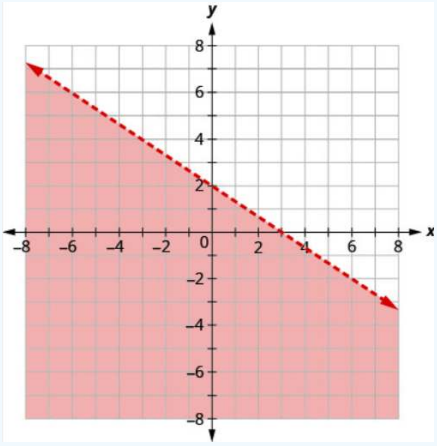


Answer

$y \leq \frac{1}{2}x - 4$

Example 2.10.7

The boundary line shown is $2x + 3y = 6$. Write the inequality shown by the graph.



Solution

The line $2x + 3y = 6$ is the boundary line. On one side of the line are the points with $2x + 3y > 6$ and on the other side of the line are the points with $2x + 3y < 6$.

Let's test the point $(0, 0)$ and see which inequality describes its side of the boundary line.

At $(0, 0)$, which inequality is true:

$$\begin{aligned} 2x + 3y &> 6 \quad \text{or} \quad 2x + 3y < 6? \\ 2x + 3y &> 6 \qquad\quad 2x + 3y < 6 \\ 2(0) + 3(0) &> 6 \qquad\quad 2(0) + 3(0) < 6 \\ 0 &> 6 \quad \text{False} \qquad\quad 0 < 6 \quad \text{True} \end{aligned} \tag{2.10.3}$$

So the side with $(0, 0)$ is the side where $2x + 3y < 6$.

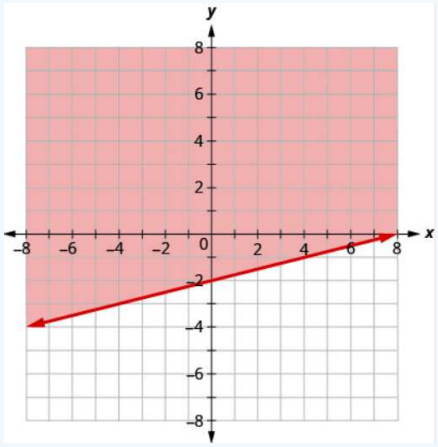
(You may want to pick a point on the other side of the boundary line and check that $2x + 3y > 6$.)

Since the boundary line is graphed as a dashed line, the inequality does not include an equal sign.

The graph shows the solution to the inequality $2x + 3y < 6$.

Try It 2.10.8

Write the inequality shown by the shaded region in the graph with the boundary line $x - 4y = 8$.

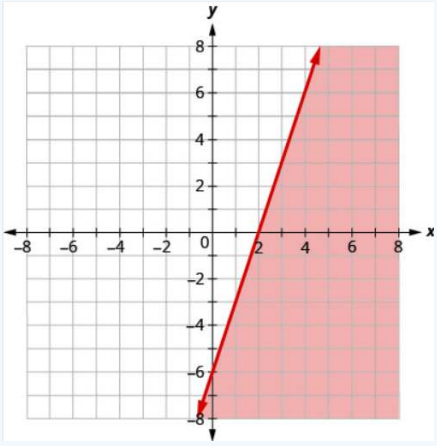


Answer

$x - 4y \leq 8$

Try It 2.10.9

Write the inequality shown by the shaded region in the graph with the boundary line $3x - y = 6$.



Answer

$3x - y \leq 6$

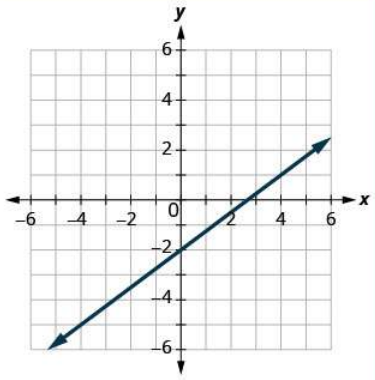
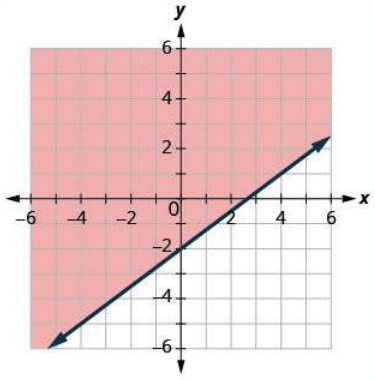
Graph Linear Inequalities

Now, we're ready to put all this together to graph linear inequalities.

Example 2.10.10: How to Graph Linear Inequalities

Graph the **linear inequality** $y \geq \frac{3}{4}x - 2$.

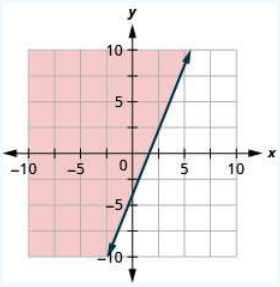
Solution

Step 1. Identify and graph the boundary line. • If the inequality is ≤ or ≥, the boundary line is solid. • If the inequality is < or >, the boundary line is dashed.	Replace the inequality sign with an equal sign to find the boundary line. Graph the boundary line $y = \frac{3}{4}x - 2$. The inequality sign is ≥, so we draw a solid line.	
Step 2. Test a point that is not on the boundary line. Is it a solution of the inequality?	We'll test (0, 0). Is it a solution of the inequality?	At (0, 0), is $y \geq \frac{3}{4}x - 2$? $0 \stackrel{?}{\geq} \frac{3}{4}(0) - 2$ $0 \geq -2$ So, (0, 0) is a solution.
Step 3. Shade in one side of the boundary line. • If the test point is a solution, shade in the side that includes the point. • If the test point is not a solution, shade in the opposite side.	The test point (0, 0), is a solution to $y \geq \frac{3}{4}x - 2$. So we shade in that side.	

Try It 2.10.11

Graph the linear inequality $y \geq \frac{5}{2}x - 4$.

Answer

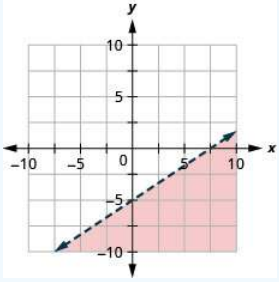


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Try It 2.10.12

Graph the linear inequality $y < \frac{2}{3}x - 5$.

Answer



The steps we take to graph a linear inequality are summarized here.

GRAPH A LINEAR INEQUALITY.

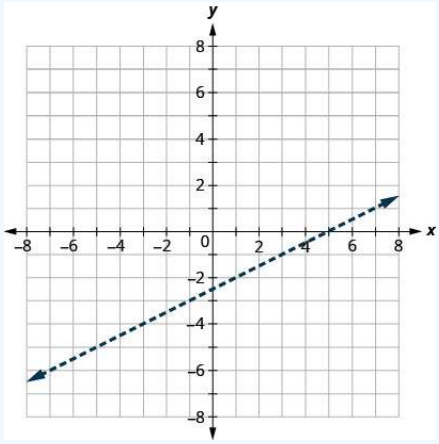
1. Identify and graph the boundary line.
   - If the inequality is $\leq$ or $\geq$, the boundary line is solid.
   - If the inequality is $<$ or $>$, the boundary line is dashed.
2. Test a point that is not on the boundary line. Is it a solution of the inequality?
3. Shade in one side of the boundary line.
   - If the test point is a solution, shade in the side that includes the point.
   - If the test point is not a solution, shade in the opposite side.

Example 2.10.13

Graph the linear inequality $x - 2y < 5$.

Solution

First we graph the boundary line $x - 2y = 5$. The inequality is $<$ so we draw a dashed line.



Then we test a point. We'll use $(0, 0)$ again because it is easy to evaluate and it is not on the boundary line.

Is $(0, 0)$ a solution of $x - 2y < 5$?

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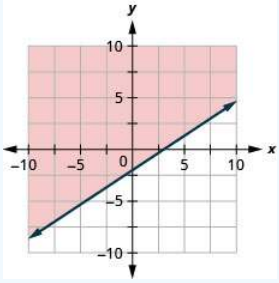
$$0-2(0) \stackrel{?}{<} 5$$
$$0-0 \stackrel{?}{<} 5$$
$$0<5$$

The point $(0,0)$ is a solution of $x-2y<5$, so we shade in that side of the boundary line.

Try It 2.10.14

Graph the linear inequality $2x-3y \leq 6$.

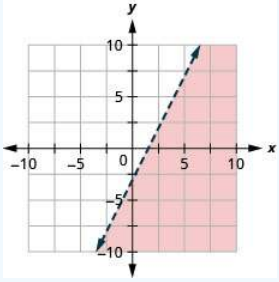
Answer



Try It 2.10.15

Graph the linear inequality $2x-y>3$.

Answer



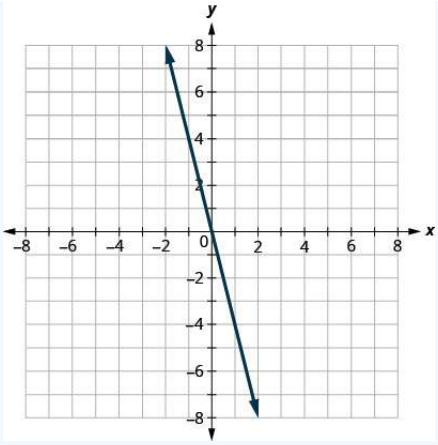
What if the boundary line goes through the origin? Then we won't be able to use $(0,0)$ as a test point. No problem—we'll just choose some other point that is not on the boundary line.

Example 2.10.16

Graph the linear inequality $y \leq -4x$.

Solution

First we graph the boundary line $y=-4x$. It is in slope–intercept form, with $m=-4$ and $b=0$. The inequality is $\leq$ so we draw a solid line.



Now, we need a test point. We can see that the point $(1, 0)$ is not on the boundary line.

Is $(1, 0)$ a solution of $y \leq -4x$?

$0 \stackrel{?}{\leq} -4(1)$

$0 \not\leq -4$

The point $(1, 0)$ is not a solution to $y \leq -4x$, so we shade in the opposite side of the boundary line. See Figure 2.10.6

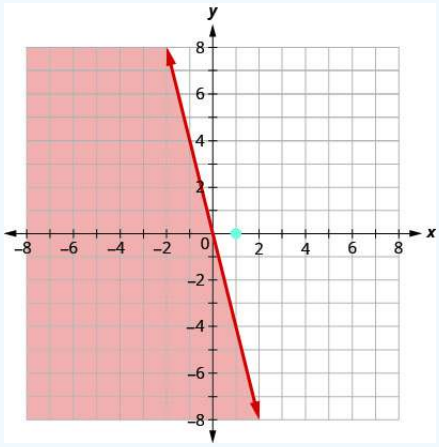
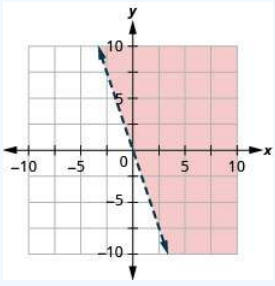


Figure 2.10.6

? Try It 2.10.17

Graph the linear inequality $y > -3x$.

Answer

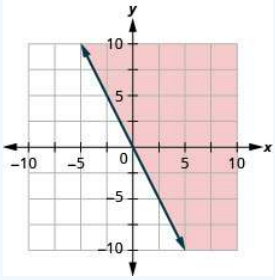


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Try It 2.10.18

Graph the linear inequality $y \geq -2x$.

Answer



Some linear inequalities have only one variable. They may have an x but no y, or a y but no x. In these cases, the boundary line will be either a vertical or a horizontal line. Do you remember?

$x = a$ vertical line
$y = b$ horizontal line

Example 2.10.19

Graph the linear inequality $y > 3$.

Solution

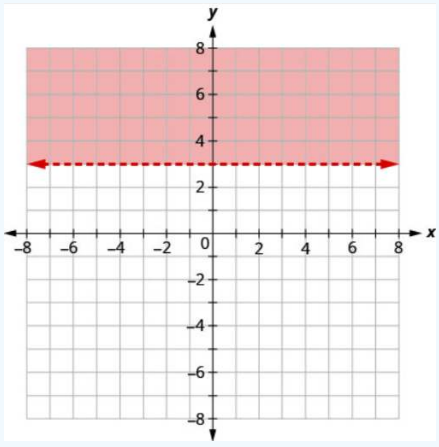
First we graph the boundary line $y = 3$. It is a horizontal line. The inequality is $>$ so we draw a dashed line.

We test the point $(0, 0)$.

$$\begin{aligned} y &> 3 \\ 0 &\not> 3 \end{aligned} \tag{2.10.4}$$

$(0, 0)$ is not a solution to $y > 3$.

So we shade the side that does not include $(0, 0)$.

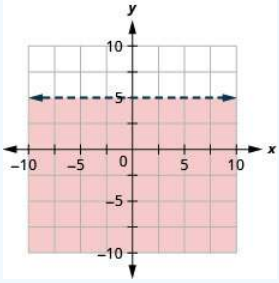


Try It 2.10.20

Graph the linear inequality $y < 5$.

Answer

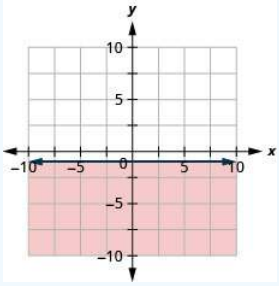
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Try It 2.10.21

Graph the linear inequality $y \leq -1$.

Answer



Key Concepts

- **To Graph a Linear Inequality**
  1. Identify and graph the boundary line.
     If the inequality is $\leq$ or $\geq$, the boundary line is solid.
     If the inequality is $<$ or $>$, the boundary line is dashed.
  2. Test a point that is not on the boundary line. Is it a solution of the inequality?
  3. Shade in one side of the boundary line.
     If the test point is a solution, shade in the side that includes the point.
     If the test point is not a solution, shade in the opposite side.

Glossary

boundary line

The line with equation $Ax + By = C$ that separates the region where $Ax + By > C$ from the region where $Ax + By < C$.

linear inequality

An inequality that can be written in one of the following forms:

$$Ax + By > C \quad Ax + By \geq C \quad Ax + By < C \quad Ax + By \leq C \tag{2.10.5}$$

where A and B are not both zero.

solution of a linear inequality

An ordered pair $(x,\ y)$ is a solution to a linear inequality the inequality is true when we substitute the values of x and y.

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2.10E: Exercises

Practice Makes Perfect

Verify Solutions to an Inequality in Two Variables

In the following exercises, determine whether each ordered pair is a solution to the given inequality.

Exercise 2.10.1

Determine whether each ordered pair is a solution to the inequality $y > x - 1$:

1. $(0, 1)$
2. $(-4, -1)$
3. $(4, 2)$
4. $(3, 0)$
5. $(-2, -3)$

Exercise 2.10.2

Determine whether each ordered pair is a solution to the inequality $y > x - 3$:

1. $(0, 0)$
2. $(2, 1)$
3. $(-1, -5)$
4. $(-6, -3)$
5. $(1, 0)$

Answer

1. yes
2. no
3. no
4. yes
5. no

Exercise 2.10.3

Determine whether each ordered pair is a solution to the inequality $y < x + 2$:

1. $(0, 3)$
2. $(-3, -2)$
3. $(-2, 0)$
4. $(0, 0)$
5. $(-1, 4)$

Exercise 2.10.4

Determine whether each ordered pair is a solution to the inequality $y < x + 5$:

1. $(-3, 0)$
2. $(1, 6)$
3. $(-6, -2)$
4. $(0, 1)$
5. $(5, -4)$

Answer

1. yes
2. no
3. no
4. yes
5. yes

Exercise 2.10.5

Determine whether each ordered pair is a solution to the inequality $x+y>4$:

1. $(5,1)$
2. $(-2,6)$
3. $(3,2)$
4. $(10,-5)$
5. $(0,0)$

Exercise 2.10.6

Determine whether each ordered pair is a solution to the inequality $x+y>2$:

1. $(1,1)$
2. $(4,-3)$
3. $(0,0)$
4. $(-8,12)$
5. $(3,0)$

Answer

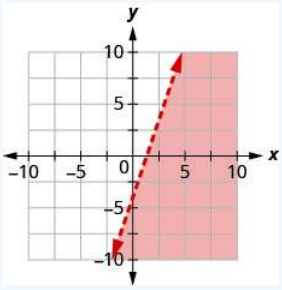
1. no
2. no
3. no
4. yes
5. yes

Recognize the Relation Between the Solutions of an Inequality and its Graph

In the following exercises, write the inequality shown by the shaded region.

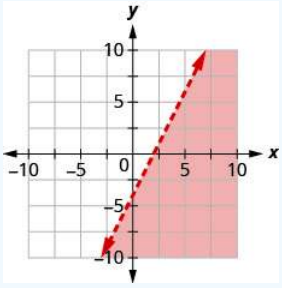
Exercise 2.10.7

Write the inequality shown by the graph with the boundary line $y=3x-4$.



Exercise 2.10.8

Write the inequality shown by the graph with the boundary line $y = 2x - 4$.

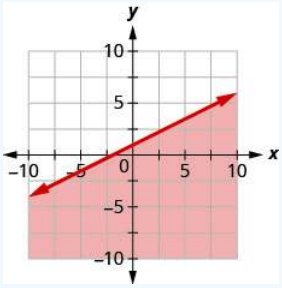


Answer

$y < 2x - 4$

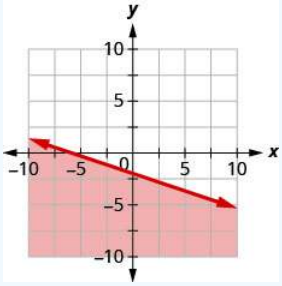
Exercise 2.10.9

Write the inequality shown by the graph with the boundary line $y = \frac{1}{2}x + 1$



Exercise 2.10.10

Write the inequality shown by the graph with the boundary line $y = -\frac{1}{3}x - 2$



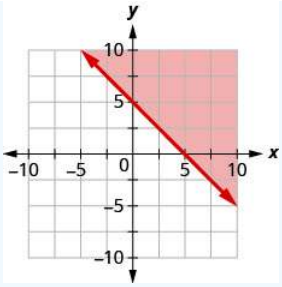
Answer

$y \leq -\frac{1}{3}x - 2$

Exercise 2.10.11

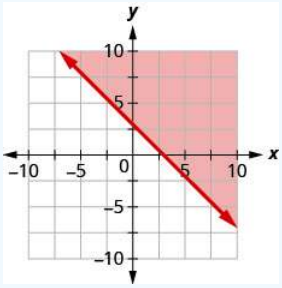
Write the inequality shown by the shaded region in the graph with the boundary line $x + y = 5$.

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Exercise 2.10.12

Write the inequality shown by the shaded region in the graph with the boundary line $x + y = 3$.

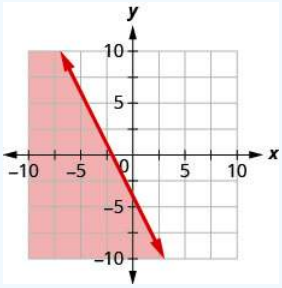


Answer

$x + y \geq 3$

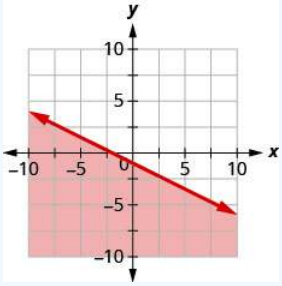
Exercise 2.10.13

Write the inequality shown by the shaded region in the graph with the boundary line $2x + y = -4$.



Exercise 2.10.14

Write the inequality shown by the shaded region in the graph with the boundary line $x + 2y = -2$.



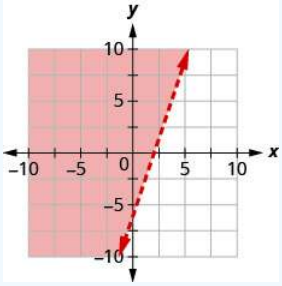
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Answer

$x + 2y \geq -2$

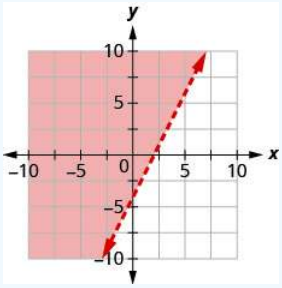
Exercise 2.10.15

Write the inequality shown by the shaded region in the graph with the boundary line $3x - y = 6$.



Exercise 2.10.16

Write the inequality shown by the shaded region in the graph with the boundary line $2x - y = 4$.

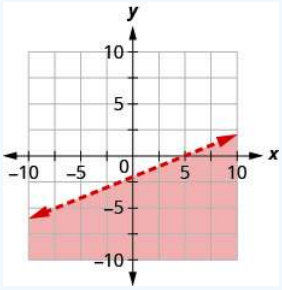


Answer

$2x - y < 4$

Exercise 2.10.17

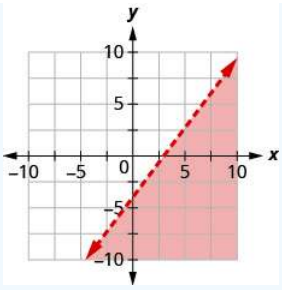
Write the inequality shown by the shaded region in the graph with the boundary line $2x - 5y = 10$.



Exercise 2.10.18

Write the inequality shown by the shaded region in the graph with the boundary line $4x - 3y = 12$.

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Answer

$4x - 3y > 12$

Graph Linear Inequalities

In the following exercises, graph each linear inequality.

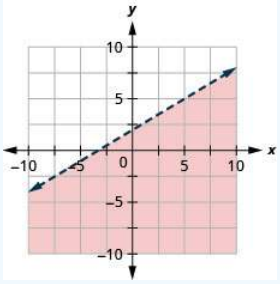
Exercise 2.10.19

Graph the linear inequality $y > \frac{2}{3}x - 1$

Exercise 2.10.20

Graph the linear inequality $y < \frac{3}{5}x + 2$

Answer



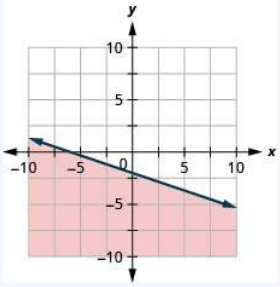
Exercise 2.10.21

Graph the linear inequality $y \leq -\frac{1}{2}x + 4$

Exercise 2.10.22

Graph the linear inequality $y \geq -\frac{1}{3}x - 2$

Answer



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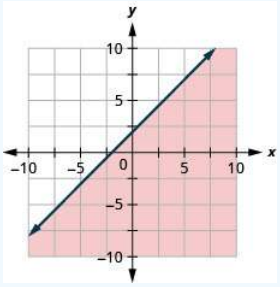
Exercise 2.10.23

Graph the linear inequality $x - y \leq 3$

Exercise 2.10.24

Graph the linear inequality $x - y \geq -2$

Answer



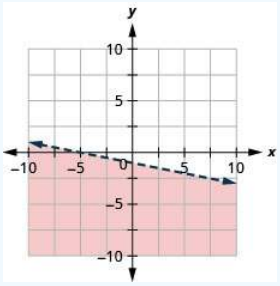
Exercise 2.10.25

Graph the linear inequality $4x + y > -4$

Exercise 2.10.26

Graph the linear inequality $x + 5y < -5$

Answer



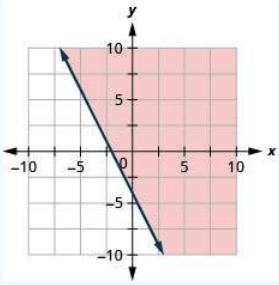
Exercise 2.10.27

Graph the linear inequality $3x + 2y \geq -6$

Exercise 2.10.28

Graph the linear inequality $4x + 2y \geq -8$

Answer



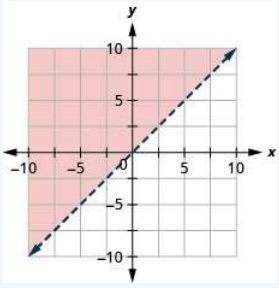
Exercise 2.10.29

Graph the linear inequality $y > 4x$

Exercise 2.10.30

Graph the linear inequality $y > x$

Answer



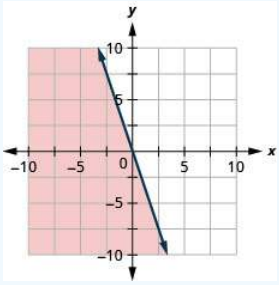
Exercise 2.10.31

Graph the linear inequality $y \leq -x$

Exercise 2.10.32

Graph the linear inequality $y \leq -3x$

Answer



Exercise 2.10.33

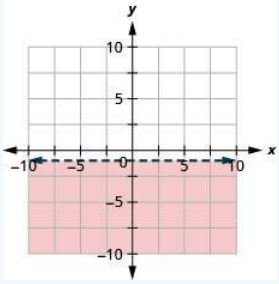
Graph the linear inequality $y \geq -2$

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Exercise 2.10.34

Graph the linear inequality $y < -1$

Answer



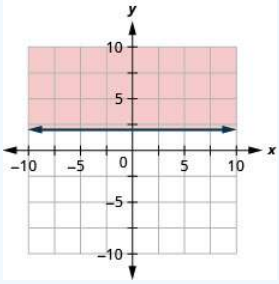
Exercise 2.10.35

Graph the linear inequality $y < 4$

Exercise 2.10.36

Graph the linear inequality $y \geq 2$

Answer



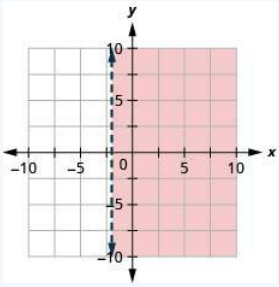
Exercise 2.10.37

Graph the linear inequality $x \leq 5$

Exercise 2.10.38

Graph the linear inequality $x > -2$

Answer



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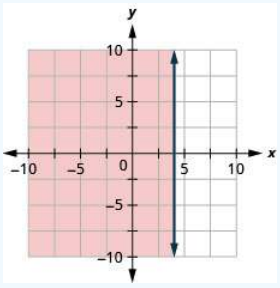
Exercise 2.10.39

Graph the linear inequality $x > -3$

Exercise 2.10.40

Graph the linear inequality $x \leq 4$

Answer



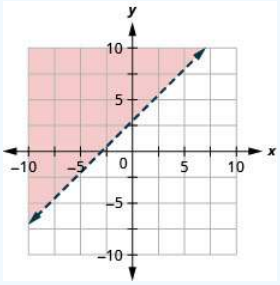
Exercise 2.10.41

Graph the linear inequality $x - y < 4$

Exercise 2.10.42

Graph the linear inequality $x - y < -3$

Answer



Exercise 2.10.43

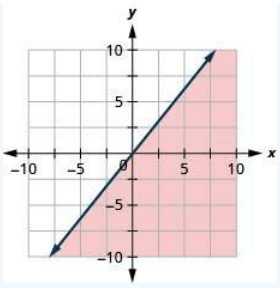
Graph the linear inequality $y \geq \frac{3}{2}x$

Exercise 2.10.44

Graph the linear inequality $y \leq \frac{5}{4}x$

Answer

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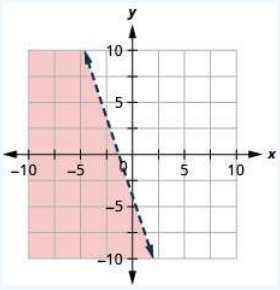
Exercise 2.10.45

Graph the linear inequality $y > -2x + 1$

Exercise 2.10.46

Graph the linear inequality $y < -3x - 4$

Answer



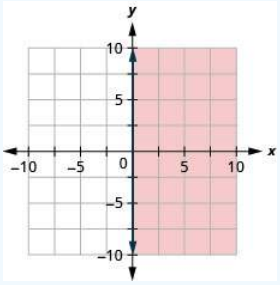
Exercise 2.10.47

Graph the linear inequality $x \leq -1$

Exercise 2.10.48

Graph the linear inequality $x \geq 0$

Answer



Everyday Math

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Exercise 2.10.49

Money. Gerry wants to have a maximum of \$100 cash at the ticket booth when his church carnival opens. He will have \$1 bills and \$5 bills. If x is the number of \$1 bills and y is the number of \$5 bills, the inequality $x+5y \leq 100$ models the situation.

1. Graph the inequality.
2. List three solutions to the inequality $x+5y \leq 100$ where both x and y are integers.

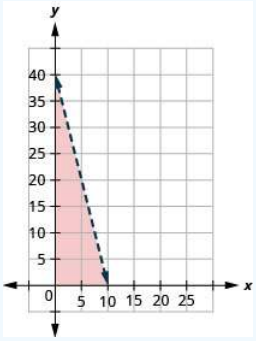
Exercise 2.10.50

Shopping. Tula has \$20 to spend at the used book sale. Hardcover books cost \$2 each and paperback books cost \$0.50 each. If x is the number of hardcover books Tula can buy and y is the number of paperback books she can buy, the inequality $2x+\frac{1}{2}y \leq 20$ models the situation.

1. Graph the inequality.
2. List three solutions to the inequality $2x+\frac{1}{2}y \leq 20$ where both x and y are whole numbers.

Answer

1.



2. Answers will vary.

Writing Exercises

Exercise 2.10.51

Lester thinks that the solution of any inequality with a $>$ sign is the region above the line and the solution of any inequality with a $<$ sign is the region below the line. Is Lester correct? Explain why or why not.

Exercise 2.10.52

Explain why in some graphs of linear inequalities the boundary line is solid but in other graphs it is dashed.

Self Check

a. After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
verify solutions to an inequality in two variables.			
recognize the relation between the solutions of an inequality and its graph.			
graph linear inequalities.			

b. What does this checklist tell you about your mastery of this section? What steps will you take to improve?

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2.11: Solve Absolute Value Inequalities (optional challenge practice)

Learning Objectives

By the end of this section, you will be able to:

- Solve absolute value equations
- Solve absolute value inequalities with “less than”
- Solve absolute value inequalities with “greater than”
- Solve applications with absolute value

Before you get started, take this readiness quiz.

1. Evaluate: $-|7|$.
   If you missed this problem, review [link].
2. Fill in $<, >, <, >$, or $=$ for each of the following pairs of numbers.
   ⓐ $|-8|\_\_\_-|-8|$ ⓑ $12\_\_\_-|-12|$ ⓒ $|-6|\_\_\_-6$ ⓓ $-(-15)\_\_\_-|-15|$
   If you missed this problem, review [link].
3. Simplify: $14-2|8-3(4-1)|$.
   If you missed this problem, review [link].

Solve Absolute Value Equations

As we prepare to solve absolute value equations, we review our definition of **absolute value**.

ABSOLUTE VALUE

The absolute value of a number is its distance from zero on the number line.

The absolute value of a number n is written as $|n|$ and $|n| \geq 0$ for all numbers.

Absolute values are always greater than or equal to zero.

We learned that both a number and its opposite are the same distance from zero on the number line. Since they have the same distance from zero, they have the same absolute value. For example:

- -5 is 5 units away from 0, so $|-5| = 5$.
- 5 is 5 units away from 0, so $|5| = 5$.

Figure $2.11.1$ illustrates this idea.

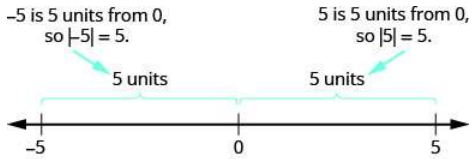


Figure 2.11.1: The numbers 5 and -5 are both five units away from zero.

For the equation |x|=5,|x|=5, we are looking for all numbers that make this a true statement. We are looking for the numbers whose distance from zero is 5. We just saw that both 5 and −5−5 are five units from zero on the number line. They are the solutions to the equation.

$$\begin{array}{ll} \text{If} & |x| = 5 \\ \text{then} & x = -5 \text{ or } x = 5 \end{array}$$

The solution can be simplified to a single statement by writing $x = \pm 5$. This is read, “x is equal to positive or negative 5”.

We can generalize this to the following property for absolute value equations.

ABSOLUTE VALUE EQUATIONS

For any algebraic expression, u, and any positive real number, a,

$$\begin{aligned} &\text{if} && |u| = a \\ &\text{then} && u = -a \text{ or } u = a \end{aligned}$$

Remember that an absolute value cannot be a negative number.

Example 2.11.1

Solve:

a. $|x| = 8$
b. $|y| = -6$
c. $|z| = 0$

Solution a

$$\begin{aligned} & && |x| = 8 \\ &\text{Write the equivalent equations.} && x = -8 \text{ or } x = 8 \\ & && x = \pm 8 \end{aligned}$$

Solution b

$$|y| = -6$$

No solution

Since an absolute value is always positive, there are no solutions to this equation.

Solution c

$$\begin{aligned} & && |z| = 0 \\ &\text{Write the equivalent equations.} && z = -0 \text{ or } z = 0 \\ &\text{Since } -0 = 0, && z = 0 \end{aligned}$$

Both equations tell us that z=0z=0 and so there is only one solution.

EXERCISE 2.11.2

Solve:

a. $|x| = 2$
b. $|y| = -4$
c. $|z| = 0$

Answer a

± 2

Answer b

no solution

Answer c

0

EXERCISE 2.11.3

Solve:

a. $|x| = 11$
b. $|y| = -5$

c. $|z| = 0$

Answer a

± 11

Answer b

no solution

Answer c

0

To solve an **absolute value equation**, we first isolate the absolute value expression using the same procedures we used to solve linear equations. Once we isolate the absolute value expression we rewrite it as the two equivalent equations.

How to Solve Absolute Value Equations

? Example 2.11.4

Solve $|5x - 4| - 3 = 8$.

Solution

Step 1. Isolate the absolute value expression.	Add 3 to both sides.	$\|5x-4\|-3=8$ $\|5x-4\|=11$
Step 2. Write the equivalent equations.		$5x-4=-11$ or $5x-4=11$
Step 3. Solve each equation.	Add 4 to each side. Divide each side by 5.	$5x=-7$ $\quad$ $5x=15$ $x=-\frac{7}{5}$ or $x=3$
Step 4. Check each solution.	Substitute 3 and $-\frac{7}{5}$ into the original equation.	$\|5x-4\|-3=8$ $x=3$ $\|5\cdot 3-4\|-3\stackrel{?}{=}8$ $\|15-4\|-3\stackrel{?}{=}8$ $\|11\|-3\stackrel{?}{=}8$ $11-3\stackrel{?}{=}8$ $8=8$ ✓ $\|5x-4\|-3=8$ $x=-\frac{7}{5}$ $\left\|5\left(-\frac{7}{5}\right)-4\right\|-3\stackrel{?}{=}8$ $\|-7-4\|-3\stackrel{?}{=}8$ $\|-11\|-3\stackrel{?}{=}8$ $11-3\stackrel{?}{=}8$ $8=8$ ✓

? EXERCISE 2.11.5

Solve: $|3x - 5| - 1 = 6$.

Answer

$x = 4,\ x = -\frac{2}{3}$

EXERCISE 2.11.6

Solve: $|4x - 3| - 5 = 2$.

Answer

$x = -1,\ x = \frac{5}{2}$

The steps for solving an absolute value equation are summarized here.

SOLVE ABSOLUTE VALUE EQUATIONS.

1. Isolate the absolute value expression.
2. Write the equivalent equations.
3. Solve each equation.
4. Check each solution.

Example 2.11.7

Solve $2|x - 7| + 5 = 9$.

Solution

	$2\|x - 7\| + 5 = 9$
Isolate the absolute value expression.	$2\|x - 7\| = 4$
	$\|x - 7\| = 2$
Write the equivalent equations.	$x - 7 = -2$ or $x - 7 = 2$
Solve each equation.	$x = 5$ or $x = 9$
Check: $2\|x-7\|+5=9$ $2\|5-7\|+5\stackrel{?}{=}9$ $2\|-2\|+5\stackrel{?}{=}9$ $2\cdot2+5\stackrel{?}{=}9$ $4+5\stackrel{?}{=}9$ $9=9$ ✓ $2\|x-7\|+5=9$ $2\|9-7\|+5\stackrel{?}{=}9$ $2\|2\|+5\stackrel{?}{=}9$ $2\cdot2+5\stackrel{?}{=}9$ $4+5\stackrel{?}{=}9$ $9=9$ ✓	

Exercise 2.11.8

Solve: $3|x - 4| - 4 = 8$.

Answer

$x = 8,\ x = 0$

Exercise 2.11.9

Solve: $2|x - 5| + 3 = 9$.

Answer

$x = 8,\ x = 2$

Remember, an absolute value is always positive!

Example 2.11.10

Solve: $|\frac{2}{3}x - 4| + 11 = 3$.

Solution

	$\|\frac{2}{3}x - 4\| = -8$
Isolate the absolute value term.	$\|\frac{2}{3}x - 4\| = -8$
An absolute value cannot be negative.	No solution

Exercise 2.11.11

Solve: $|\frac{3}{4}x - 5| + 9 = 4$.

Answer

No solution

Exercise 2.11.12

Solve: $|\frac{5}{6}x + 3| + 8 = 6$.

Answer

No solution

Some of our absolute value equations could be of the form $|u| = |v|$ where u and v are algebraic expressions. For example, $|x - 3| = |2x + 1|$.

How would we solve them? If two algebraic expressions are equal in absolute value, then they are either equal to each other or negatives of each other. The property for absolute value equations says that for any algebraic expression, u, and a positive real number, a, if $|u| = a$, then $u = -a$ or $u = a$.

This tell us that

if	$\|u\| = \|v\|$		
then	$\|u\| = v$	or	$\|u\| = -v$
and so	$u = v$ or $u = -v$	or	$u = -v$ or $u = -(-v)$

This leads us to the following property for equations with two absolute values.

EQUATIONS WITH TWO ABSOLUTE VALUES

For any algebraic expressions, u and v,

if	$\|u\| = \|v\|$
then	$u = -v$ or $u = v$

When we take the opposite of a quantity, we must be careful with the signs and to add parentheses where needed.

Example 2.11.13

Solve: $|5x-1|=|2x+3|$.

Solution

$$|5x-1|=|2x+3|$$

Write the equivalent equations.	$5x-1=-(2x+3)$	or	$5x-1=2x+3$
	$5x-1=-2x-3$	or	$3x-1=3$
Solve each equation.	$7x-1=-3$		$3x=4$
	$7x=-2$		$x=43$
	$x=-27$	or	$x=43$

Check.

We leave the check to you.

Exercise 2.11.14

Solve: $|7x-3|=|3x+7|$.

Answer

$x=-\frac{2}{5},\ x=\frac{5}{2}$

Exercise 2.11.15

Solve: $|6x-5|=|3x+4|$.

Answer

$x=3, x=19$

Solve Absolute Value Inequalities with "Less Than"

Let's look now at what happens when we have an **absolute value inequality**. Everything we've learned about solving inequalities still holds, but we must consider how the absolute value impacts our work. Again we will look at our definition of absolute value. The absolute value of a number is its distance from zero on the number line. For the equation $|x|=5$, we saw that both 5 and -5 are five units from zero on the number line. They are the solutions to the equation.

$$|x|=5$$
$$x=-5 \quad \text{or} \quad x=5$$

What about the inequality $|x|\leq 5$? Where are the numbers whose distance is less than or equal to 5? We know -5 and 5 are both five units from zero. All the numbers between -5 and 5 are less than five units from zero (Figure 2.11.2).

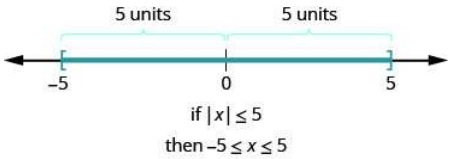


Figure 2.11.2.

In a more general way, we can see that if $|u|\leq a$, then $-a\leq u\leq a$ (Figure 2.11.3).

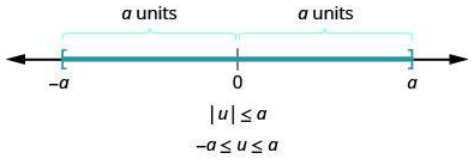


Figure 2.11.3.

This result is summarized here.

ABSOLUTE VALUE INEQUALITIES WITH $<$ OR $\leq$

For any algebraic expression, u, and any positive real number, a,

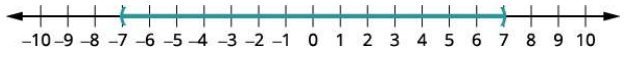
$$\text{if} \quad |u| < a, \quad \text{then} \quad -a < u < a$$
$$\text{if} \quad |u| \leq a, \quad \text{then} \quad -a \leq u \leq a$$

After solving an inequality, it is often helpful to check some points to see if the solution makes sense. The graph of the solution divides the number line into three sections. Choose a value in each section and substitute it in the original inequality to see if it makes the inequality true or not. While this is not a complete check, it often helps verify the solution.

Example 2.11.16

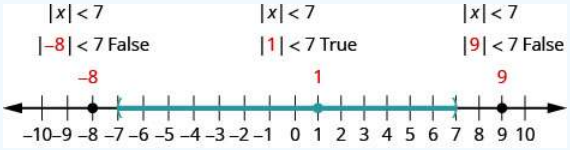
Solve $|x| < 7$. Graph the solution and write the solution in interval notation.

Solution

	$\lvert x\rvert < 7$
Write the equivalent inequality.	$-7 < x < 7$
Graph the solution.	
Write the solution using interval notation.	$(-7, 7)$

Check:

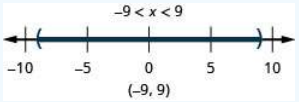
To verify, check a value in each section of the number line showing the solution. Choose numbers such as −8,−8, 1, and 9.



EXERCISE 2.11.17

Graph the solution and write the solution in interval notation: $|x| < 9$.

Answer

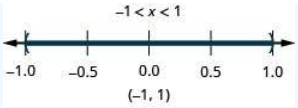


EXERCISE 2.11.18

Graph the solution and write the solution in interval notation: $|x| < 1$.

Answer

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Example 2.11.19

Solve $|5x-6| \leq 4$. Graph the solution and write the solution in interval notation.

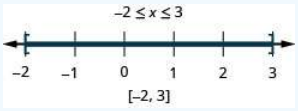
Solution

Step 1. Isolate the absolute value expression. It is isolated.	$\lvert 5x-6 \rvert \leq 4$
Step 2. Write the equivalent compound inequality.	$-4 \leq 5x-6 \leq 4$
Step 3. Solve the compound inequality.	$2 \leq 5x \leq 10$ $\frac{2}{5} \leq x \leq 2$
Step 4. Graph the solution.	
Step 5. Write the solution using interval notation.	$[\frac{2}{5}, 2]$
Check: The check is left to you.	

EXERCISE 2.11.20

Solve $|2x-1| \leq 5$. Graph the solution and write the solution in interval notation:

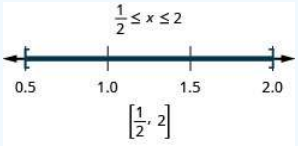
Answer



EXERCISE 2.11.21

Solve $|4x-5| \leq 3$. Graph the solution and write the solution in interval notation:

Answer



SOLVE ABSOLUTE VALUE INEQUALITIES WITH $<$ OR $\leq$

1. Isolate the absolute value expression.
2. Write the equivalent compound inequality.

$$|u| < a \quad \text{is equivalent to} \quad -a < u < a$$
$$|u| \leq a \quad \text{is equivalent to} \quad -a \leq u \leq a$$

3. Solve the compound inequality.
4. Graph the solution

5. Write the solution using interval notation.

Solve Absolute Value Inequalities with “Greater Than”

What happens for absolute value inequalities that have “greater than”? Again we will look at our definition of absolute value. The absolute value of a number is its distance from zero on the number line.

We started with the inequality $|x| \leq 5$. We saw that the numbers whose distance is less than or equal to five from zero on the number line were -5 and 5 and all the numbers between -5 and 5 (Figure 2.11.4).

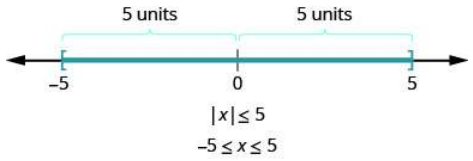


Figure 2.11.4.

Now we want to look at the inequality $|x| \geq 5$. Where are the numbers whose distance from zero is greater than or equal to five?

Again both -5 and 5 are five units from zero and so are included in the solution. Numbers whose distance from zero is greater than five units would be less than -5 and greater than 5 on the number line (Figure 2.11.5).

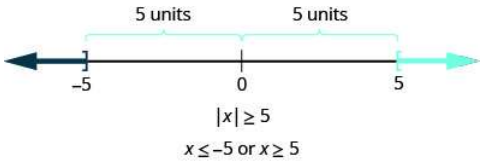


Figure 2.11.5.

In a more general way, we can see that if $|u| \geq a$, then $u \leq -a$ or $u \leq a$. See Figure.

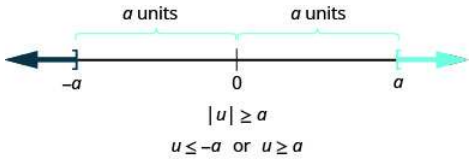


Figure 2.11.6.

This result is summarized here.

ABSOLUTE VALUE INEQUALITIES WITH $>$ OR $\geq$

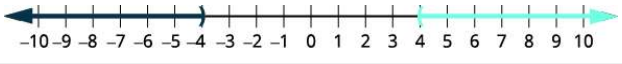
For any algebraic expression, *u*, and any positive real number, *a*,

$$\begin{array}{lll} \text{if} & |u| > a, & \text{then } u < -a \text{ or } u > a \\ \text{if} & |u| \geq a, & \text{then } u \leq -a \text{ or } u \geq a \end{array}$$

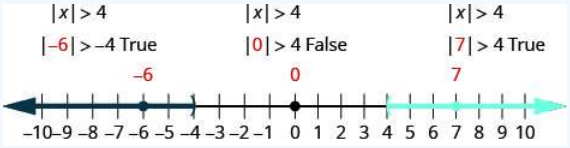
Example 2.11.22

Solve $|x| > 4$. Graph the solution and write the solution in interval notation.

Solution

	$\lvert x\rvert > 4$
Write the equivalent inequality.	$x < -4$ or $x > 4$
Graph the solution.	
Write the solution using interval notation.	$(-\inf, -4) \cup (4, \inf)$
Check:	

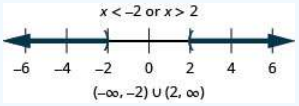
To verify, check a value in each section of the number line showing the solution. Choose numbers such as −6,−6, 0, and 7.



EXERCISE 2.11.23

Solve $|x| > 2$. Graph the solution and write the solution in interval notation.

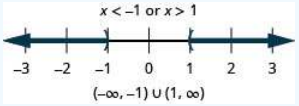
Answer



EXERCISE 2.11.24

Solve $|x| > 1$. Graph the solution and write the solution in interval notation.

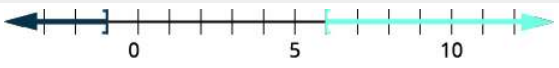
Answer



Example 2.11.25

Solve $|2x-3| \geq 5$. Graph the solution and write the solution in interval notation.

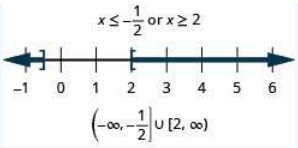
Solution

	$\lvert 2x-3\rvert \geq 5$
Step 1. Isolate the absolute value expression. It is isolated.	
Step 2. Write the equivalent compound inequality.	$2x-3 \leq -5$ or $2x-3 \geq 5$
Step 3. Solve the compound inequality.	$2x \leq -2$ or $2x \geq 8$ $x \leq -1$ or $x \geq 4$
Step 4. Graph the solution.	
Step 5. Write the solution using interval notation.	$(-\inf, -1] \cup [4, \inf)$
Check: The check is left to you.	

EXERCISE 2.11.26

Solve $|4x-3| \geq 5$. Graph the solution and write the solution in interval notation.

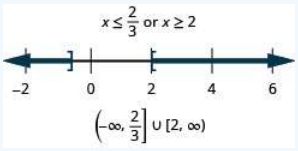
Answer



? EXERCISE 2.11.27

Solve $|3x - 4| \ge 2$. Graph the solution and write the solution in interval notation.

Answer



SOLVE ABSOLUTE VALUE INEQUALITIES WITH $>$ OR $\ge$.

1. Isolate the absolute value expression.
2. Write the equivalent compound inequality.

$$\begin{array}{lllll} |u| > a & \text{is equivalent to} & u < -a & \text{or} & u > a \\ |u| \ge a & \text{is equivalent to} & u \le -a & \text{or} & u \ge a \\ |u| > a & \text{is equivalent to} & u < -a & \text{or} & u > a \\ |u| \ge a & \text{is equivalent to} & u \le -a & \text{or} & u \ge a \end{array}$$

3. Solve the compound inequality.
4. Graph the solution
5. Write the solution using interval notation.

Solve Applications with Absolute Value

Absolute value inequalities are often used in the manufacturing process. An item must be made with near perfect specifications. Usually there is a certain *tolerance* of the difference from the specifications that is allowed. If the difference from the specifications exceeds the tolerance, the item is rejected.

$$|\text{actual-ideal}| \le \text{tolerance}$$

? Example 2.11.28

The ideal diameter of a rod needed for a machine is 60 mm. The actual diameter can vary from the ideal diameter by 0.075 mm. What range of diameters will be acceptable to the customer without causing the rod to be rejected?

Solution

	Let $x =$ the actual measurement
Use an absolute value inequality to express this situation.	$\|\text{actual-ideal}\| \le \text{tolerance}$
	$\|x - 60\| \le 0.075$
Rewrite as a compound inequality.	$-0.075 \le x - 60 \le 0.075$
Solve the inequality.	$59.925 \le x \le 60.075$
Answer the question.	The diameter of the rod can be between $59.925mm$ and $60.075mm$.

EXERCISE 2.11.29

The ideal diameter of a rod needed for a machine is 80 mm. The actual diameter can vary from the ideal diameter by 0.009 mm. What range of diameters will be acceptable to the customer without causing the rod to be rejected?

Answer

The diameter of the rod can be between 79.991 and 80.009 mm.

EXERCISE 2.11.30

The ideal diameter of a rod needed for a machine is 75 mm. The actual diameter can vary from the ideal diameter by 0.05 mm. What range of diameters will be acceptable to the customer without causing the rod to be rejected?

Answer

The diameter of the rod can be between 74.95 and 75.05 mm.

Access this online resource for additional instruction and practice with solving linear absolute value equations and inequalities.

- Solving Linear Absolute Value Equations and Inequalities

Key Concepts

- **Absolute Value**
  The absolute value of a number is its distance from 0 on the number line.
  The absolute value of a number *n* is written as $|n|$ and $|n| \geq 0$ for all numbers.
  Absolute values are always greater than or equal to zero.
- **Absolute Value Equations**
  For any algebraic expression, *u*, and any positive real number, *a*,
  if $|u| = a$
  then $u = -a$ or $u = a$
  Remember that an absolute value cannot be a negative number.
- **How to Solve Absolute Value Equations**
  1. Isolate the absolute value expression.
  2. Write the equivalent equations.
  3. Solve each equation.
  4. Check each solution.
- **Equations with Two Absolute Values**
  For any algebraic expressions, *u* and *v*,
  if $|u| = |v|$
  then $u = -v$ or $u = v$
- **Absolute Value Inequalities with** $<$ or $\leq$
  For any algebraic expression, *u*, and any positive real number, *a*,
  if $|u| = a$ then $-a < u < a$
  if $|u| \leq a$ then $-a \leq u \leq a$
- **How To Solve Absolute Value Inequalities with** $<$ or $\leq$
  1. Isolate the absolute value expression.
  2. Write the equivalent compound inequality.
     $|u| < a$ is equivalent to $-a < u < a$
     $|u| \leq a$ is equivalent to $-a \leq u \leq a$
  3. Solve the compound inequality.
  4. Graph the solution
  5. Write the solution using interval notation

- **Absolute Value Inequalities with $>$ or $\geq$**
  For any algebraic expression, u, and any positive real number, a,
  if $|u| > a$, then $u < -a$ or $u > a$
  if $|u| \geq a$, then $u \leq -a$ or $u \geq a$
- **How To Solve Absolute Value Inequalities with $>$ or $\geq$**
  1. Isolate the absolute value expression.
  2. Write the equivalent compound inequality.
     $|u| > a$ is equivalent to $u < -a$ or $u > a$
     $|u| \geq a$ is equivalent to $u \leq -a$ or $u \geq a$
  3. Solve the compound inequality.
  4. Graph the solution
  5. Write the solution using interval notation

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2.11E: Exercises

Practice Makes Perfect

Solve Absolute Value Equations

In the following exercises, solve.

1. a. $|x| = 6$ b. $|y| = -3$ c. $|z| = 0$

2. a. $|x| = 4$ b. $|y| = -5$ c. $|z| = 0$

Answer

a. $x = 4, x = -4$ b. no solution c. $z = 0$

3. a. $|x| = 7$ b. $|y| = -11$ c. $|z| = 0$

4. a. $|x| = 3$ b. $|y| = -1$ c. $|z| = 0$

Answer

a. $x = 3, x = -3$ b. no solution c. $z = 0$

5. $|2x - 3| - 4 = 1$

6. $|4x - 1| - 3 = 0$

Answer

$x = 1,\ x = -\frac{1}{2}$

7. $|3x - 4| + 5 = 7$

8. $|4x + 7| + 2 = 5$

Answer

$x = -1,\ x = -\frac{5}{2}$

9. $4|x - 1| + 2 = 10$

10. $3|x - 4| + 2 = 11$

Answer

$x = 7,\ x = 1$

11. $3|4x - 5| - 4 = 11$

12. $3|x + 2| - 5 = 4$

Answer

$x = 1,\ x = -5$

13. $-2|x-3|+8=-4$

14. $-3|x-4|+4=-5$

Answer

$x=7,\ x=1$

15. $|34x-3|+7=2$

16. $|35x-2|+5=2$

Answer

no solution

17. $|12x+5|+4=1$

18. $|14x+3|+3=1$

Answer

no solution

19. $|3x-2|=|2x-3|$

20. $|4x+3|=|2x+1|$

Answer

$x=-1,\ x=-\frac{2}{3}$

21. $|6x-5|=|2x+3|$

22. $|6-x|=|3-2x|$

Answer

$x=-3,\ x=3$

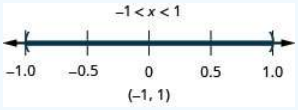
Solve Absolute Value Inequalities with "less than"

In the following exercises, solve each inequality. Graph the solution and write the solution in interval notation.

23. $|x|<5$

24. $|x|<1$

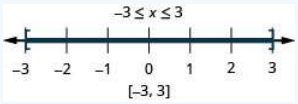
Answer



25. $|x|\leq 8$

26. $|x| \leq 3$

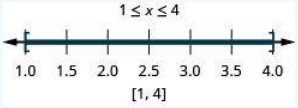
Answer



27. $|3x - 3| \leq 6$

28. $|2x - 5| \leq 3$

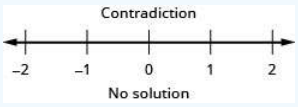
Answer



29. $|2x + 3| + 5 < 4$

30. $|3x - 7| + 3 < 1$

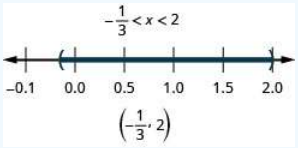
Answer



31. $|4x - 3| < 1$

32. $|6x - 5| < 7$

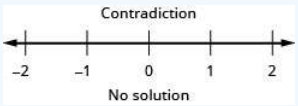
Answer



33. $|x - 4| \leq -1$

34. $|5x + 1| \leq -2$

Answer



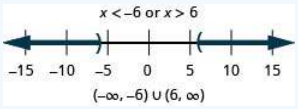
Solve Absolute Value Inequalities with "greater than"

In the following exercises, solve each inequality. Graph the solution and write the solution in interval notation.

35. $|x| > 3$

36. $|x| > 6$

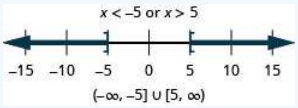
Answer



37. $|x| \geq 2$

38. $|x| \geq 5$

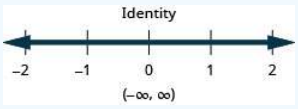
Answer



39. $|3x - 8| > -1$

40. $|x - 5| > -2$

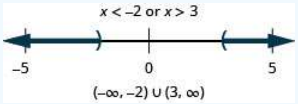
Answer



41. $|3x - 2| > 4$

42. $|2x - 1| > 5$

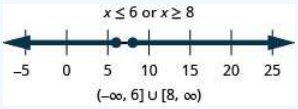
Answer



43. $|x + 3| \geq 5$

44. $|x - 7| \geq 1$

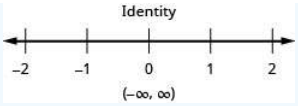
Answer



45. $3|x| + 4 \geq 1$

46. $5|x| + 6 \geq 1$

Answer



In the following exercises, solve. For each inequality, also graph the solution and write the solution in interval notation.

47. $2|x+6|+4=8$

48. $|3x-4| \geq 2$

Answer

$x=4, x=27$

49. $|6x-5|=|2x+3|$

50. $|4x-3|<5$

Answer

$x=3, x=2$

51. $|2x-5|+2=3$

52. $|3x+1|-3=7$

Answer

$x=3, x=-\frac{11}{3}$

53. $|7x+2|+8<4$

54. $5|2x-1|-3=7$

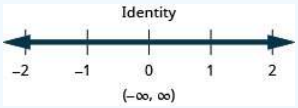
Answer

$x=\frac{3}{2}, x=-\frac{1}{2}$

55. $|x-7|>-3$

56. $|8-x|=|4-3x|$

Answer



Solve Applications with Absolute Value

In the following exercises, solve.

57. A chicken farm ideally produces 200,000 eggs per day. But this total can vary by as much as 25,000 eggs. What is the maximum and minimum expected production at the farm?

58. An organic juice bottler ideally produces 215,000 bottle per day. But this total can vary by as much as 7,500 bottles. What is the maximum and minimum expected production at the bottling company?

Answer

The minimum to maximum expected production is 207,500 to 2,225,000 bottles

59. In order to insure compliance with the law, Miguel routinely overshoots the weight of his tortillas by 0.5 gram. He just received a report that told him that he could be losing as much as $100,000 per year using this practice. He now plans to buy new equipment that guarantees the thickness of the tortilla within 0.005 inches. If the ideal thickness of the tortilla is 0.04 inches, what thickness of tortillas will be guaranteed?

60. At Lilly's Bakery, the ideal weight of a loaf of bread is 24 ounces. By law, the actual weight can vary from the ideal by 1.5 ounces. What range of weight will be acceptable to the inspector without causing the bakery being fined?

Answer

The acceptable weight is 22.5 to 25.5 ounces.

Writing Exercises

61. Write a graphical description of the absolute value of a number

62. In your own words, explain how to solve the absolute value inequality, $|3x-2| \geq 4$.

Answer

Answers will vary.

Self Check

a. After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
solve absolute value equations.			
solve absolute value inequalities with "less than".			
solve absolute value inequalities with "greater than".			
solve applications with absolute value.			

b. What does this checklist tell you about your mastery of this section? What steps will you take to improve?

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CHAPTER OVERVIEW

3: Graphing Lines in Two Variables

3.1: Use the Rectangular Coordinate System

3.1E: Exercises

3.2: Graph Linear Equations in Two Variables

3.2E: Exercises

3.3: Graph with Intercepts

3.3E: Exercises

3.4: Understanding the Slope of a Line

3.4E: Exercises

3.5: Use the Slope–Intercept Form of an Equation of a Line

3.5E: Exercises

3.6: Find the Equation of a Line

3.6E: Exercises

3.7: Chapter Review Exercises

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3.1: Use the Rectangular Coordinate System

Learning Objectives

By the end of this section, you will be able to:

- Plot points in a rectangular coordinate system
- Verify solutions to an equation in two variables
- Complete a table of solutions to a linear equation
- Find solutions to a linear equation in two variables

Note

Before you get started, take this readiness quiz.

1. Evaluate $x+3$ when $x=-1$.
   If you missed this problem, review Example 1.5.25.
2. Evaluate $2x-5y$ when $x=3$ and y=−2.
   If you missed this problem, review Example 1.5.28.
3. Solve for y: $40-4y=20$
   If you missed this problem, review Example 2.3.1.

Plot Points on a Rectangular Coordinate System

Just like maps use a grid system to identify locations, a grid system is used in algebra to show a relationship between two variables in a **rectangular coordinate system**. The rectangular coordinate system is also called the *xy*-plane or the 'coordinate plane'.

The horizontal number line is called the *x-axis*. The vertical number line is called the *y-axis*. The *x*-axis and the *y*-axis together form the rectangular coordinate system. These axes divide a plane into four regions, called **quadrants**. The quadrants are identified by Roman numerals, beginning on the upper right and proceeding counterclockwise. See Figure \(\PageIndex{1}\).

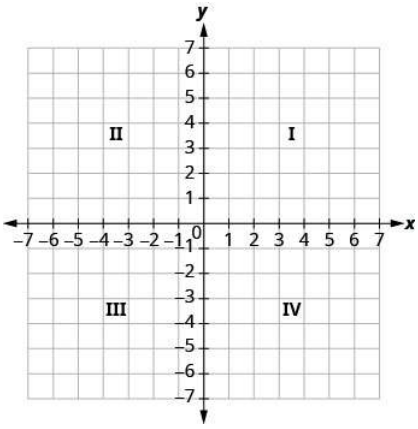


Figure 3.1.1: 'Quadrant' has the root 'quad,' which means 'four.'

In the **rectangular coordinate system**, every point is represented by an *ordered pair*. The first number in the ordered pair is the ***x*-coordinate** of the point, and the second number is the ***y*-coordinate** of the point.

ORDERED PAIR

An **ordered pair**, (x,y)(x,y), gives the coordinates of a point in a rectangular coordinate system.

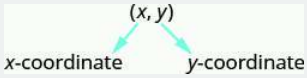


Figure 3.1.2

The first number is the *x*-coordinate.

The second number is the *y*-coordinate.

The phrase 'ordered pair' means the order is important. What is the ordered pair of the point where the axes cross? At that point both coordinates are zero, so its ordered pair is $(0, 0)$. The point $(0, 0)$ has a special name. It is called the **origin**.

THE ORIGIN

The point $(0, 0)$ is called the **origin**. It is the point where the *x*-axis and *y*-axis intersect.

We use the coordinates to locate a point on the *xy*-plane. Let's plot the point $(1, 3)$ as an example. First, locate 1 on the *x*-axis and lightly sketch a vertical line through $x = 1$. Then, locate 3 on the *y*-axis and sketch a horizontal line through $y = 3$. Now, find the point where these two lines meet — that is the point with coordinates $(1, 3)$.

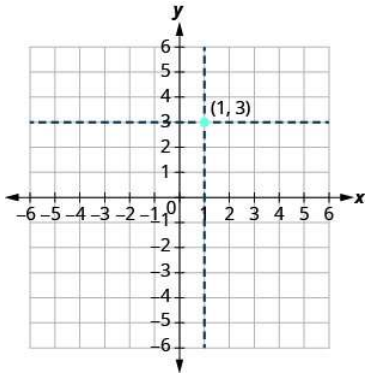


Figure 3.1.3

Notice that the vertical line through $x = 1$ and the horizontal line through $y = 3$ are not part of the graph. We just used them to help us locate the point $(1, 3)$.

Example 3.1.1

Plot each point in the rectangular coordinate system and identify the **quadrant** in which the point is located:

a. (−5,4)
b. (−3,−4)
c. (2,−3)
d. (−2,3)
e. $(3, \frac{5}{2})$

Solution

The first number of the coordinate pair is the *x*-coordinate, and the second number is the *y*-coordinate.

a. Since x=−5, the point is to the left of the *y*-axis. Also, since y=4, the point is above the *x*-axis. The point (−5,4) is in Quadrant II.
b. Since x=−3, the point is to the left of the *y*-axis. Also, since y=−4, the point is below the *x*-axis. The point (−3,−4) is in Quadrant III.
c. Since x=2, the point is to the right of the *y*-axis. Since y=−3, the point is below the *x*-axis. The point (2,−3) is in Quadrant IV.
d. Since x=−2, the point is to the left of the *y*-axis. Since y=3, the point is above the *x*-axis. The point (−2,3) is in Quadrant II.
e. Since x=3, the point is to the right of the *y*-axis. Since $y = \frac{5}{2}$, the point is above the *x*-axis. (It may be helpful to write $\frac{5}{2}$ as a mixed number or decimal.) The point $(3, \frac{5}{2})$ is in Quadrant I.

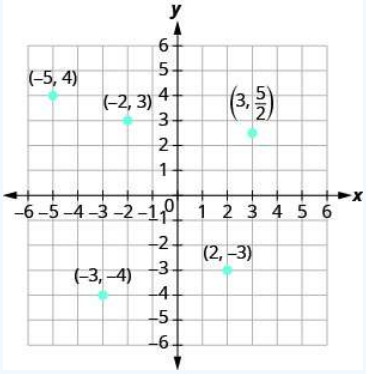


Figure 3.1.4

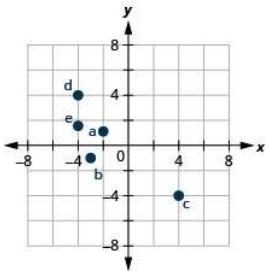
Try It 3.1.2

Plot each point in a rectangular coordinate system and identify the quadrant in which the point is located:

a. (−2,1)
b. (−3,−1)
c. (4,−4)
d. (−4,4)
e. $(-4, \frac{3}{2})$

Answer

The point a (-2,1) is in Quadrant II.
The point b (-3,-1) is in Quadrant III.
The point c (4,-4) is in Quadrant IV.
The point d (-4,4) is in Quadrant II.
The point e $(-4, \frac{3}{2})$ is in Quadrant II.



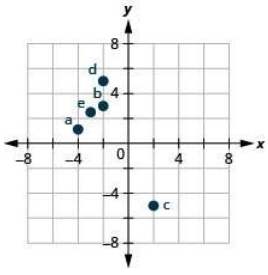
Try It 3.1.3

Plot each point in a rectangular coordinate system and identify the quadrant in which the point is located:

a. (−4,1)
b. (−2,3)
c. (2,−5)
d. (−2,5)
e. $(-3, \frac{5}{2})$

Answer

The point a (-4, 1) is in Quadrant II.
The point b (-2, 3) is in Quadrant II.
The point c (2, -5) is in Quadrant IV.
The point d (-2, 5) is in Quadrant II.
The point e $(-3, \frac{5}{2})$ is in Quadrant II.



How do the signs affect the location of the points? You may have noticed some patterns as you graphed the points in the previous example.

For the point in Figure 3.1.4 in Quadrant IV, what do you notice about the signs of the coordinates? What about the signs of the coordinates of points in the third quadrant? The second quadrant? The first quadrant?

Can you tell just by looking at the coordinates in which quadrant the point (−2,5) is located? In which quadrant is (2,−5) located?

QUADRANTS

We can summarize sign patterns of the quadrants in this way.

Quadrant I	Quadrant II	Quadrant III	Quadrant IV	
(x, y)	(x, y)	(x, y)	(x, y)	(3.1.1)
$(+, +)$	$(-, +)$	$(-, -)$	$(+, -)$	

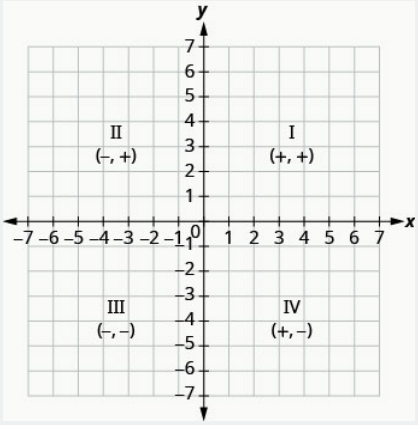


Figure 3.1.5

What if one coordinate is zero as shown in Figure 3.1.6? Where is the point (0,4) located? Where is the point (−2,0) located?

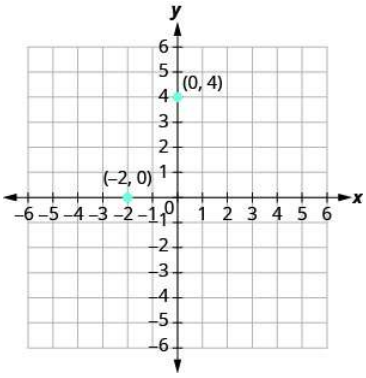


Figure 3.1.6

The point (0,4) is on the *y*-axis and the point (−2,0) is on the *x*-axis.

POINTS ON THE AXES

Points with a ***y*-coordinate** equal to 0 are on the *x*-axis, and have coordinates (a,0).

Points with an ***x*-coordinate** equal to 0 are on the *y*-axis, and have coordinates (0,b).

Example 3.1.4

Plot each point:

1. (0,5)
2. (4,0)
3. (−3,0)
4. (0,0)
5. (0,−1)

Solution

1. Since x=0, the point whose coordinates are (0,5) is on the *y*-axis.
2. Since y=0, the point whose coordinates are (4,0) is on the *x*-axis.
3. Since y=0, the point whose coordinates are (−3,0) is on the *x*-axis.
4. Since x=0 and y=0, the point whose coordinates are (0,0) is the origin.
5. Since x=0, the point whose coordinates are (0,−1) is on the *y*-axis.

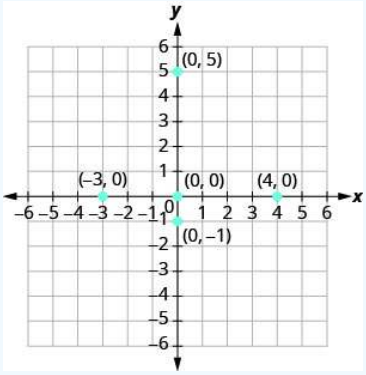


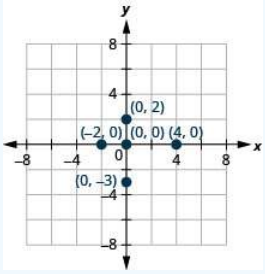
Figure 3.1.7

Try It 3.1.5

Plot each point:

1. (4,0)
2. (−2,0)
3. (0,0)
4. (0,2)
5. (0,−3).

Answer



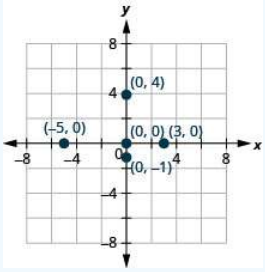
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Try It 3.1.6

Plot each point:

1. (−5,0)
2. (3,0)
3. (0,0)
4. (0,−1)
5. (0,4).

Answer



In algebra, being able to identify the coordinates of a point shown on a graph is just as important as being able to plot points. To identify the *x*-coordinate of a point on a graph, read the number on the *x*-axis directly above or below the point. To identify the *y*-coordinate of a point, read the number on the *y*-axis directly to the left or right of the point. Remember, when you write the **ordered pair** use the correct order, (x,y).

Example 3.1.7

Name the ordered pair of each point shown in the rectangular coordinate system.

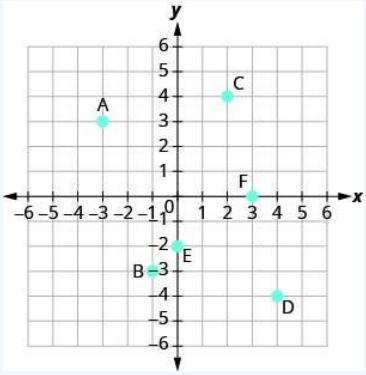


Figure 3.1.8

Solution

Point A is above −3 on the *x*-axis, so the *x*-coordinate of the point is −3.

The point is to the left of 3 on the *y*-axis, so the *y*-coordinate of the point is 3.

The coordinates of the point are (−3,3).

Point B is below −1 on the *x*-axis, so the *x*-coordinate of the point is −1.

The point is to the left of −3 on the *y*-axis, so the *y*-coordinate of the point is −3.

The coordinates of the point are (−1,−3).

Point C is above 2 on the *x*-axis, so the *x*-coordinate of the point is 2.

The point is to the right of 4 on the *y*-axis, so the *y*-coordinate of the point is 4.

The coordinates of the point are (2,4).

Point D is below 4 on the *x*-axis, so the *x*-coordinate of the point is 4.

The point is to the right of −4 on the *y*-axis, so the *y*-coordinate of the point is −4.

The coordinates of the point are (4,−4).

Point E is on the *y*-axis at y=−2. The coordinates of point E are (0,−2).

Point F is on the *x*-axis at x=3. The coordinates of point F are (3,0).

Try It 3.1.8

Name the ordered pair of each point shown in the rectangular coordinate system.

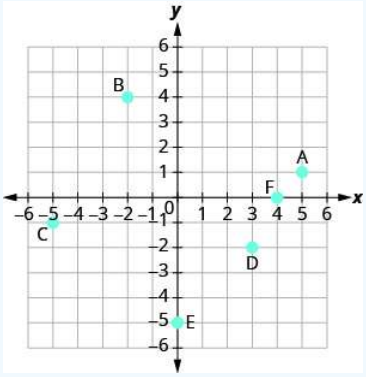


Figure 3.1.9

Answer

A: (5,1) B: (−2,4) C: (−5,−1) D: (3,−2) E: (0,−5) F: (4,0)

Try It 3.1.9

Name the ordered pair of each point shown in the rectangular coordinate system.

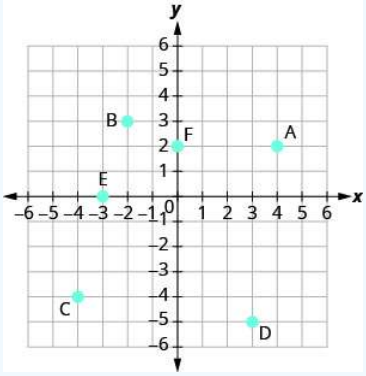


Figure 3.1.10

Answer

A: (4,2) B: (−2,3) C: (−4,−4) D: (3,−5) E: (−3,0) F: (0,2)

Verify Solutions to an Equation in Two Variables

Up to now, all the equations you have solved were equations with just one variable. In almost every case, when you solved the equation you got exactly one solution. The process of solving an equation ended with a statement like x=4. (Then, you checked the solution by substituting back into the equation.)

Here's an example of an equation in one variable, and its one solution.

$$\begin{aligned} 3x+5 &= 17 \\ 3x &= 12 \\ x &= 4 \end{aligned}$$

But equations can have more than one variable. Equations with two variables may be of the form Ax+By=C. Equations of this form are called **linear equations in two variables**.

LINEAR EQUATION

An equation of the form Ax+By=C, where A and B are not both zero, is called a **linear equation in two variables**.

Notice the word *line* in **linear**. Here is an example of a linear equation in two variables, x and y.

$$Ax + By = C$$
$$x + 4y = 8$$
$$A = 1,\ B = 4,\ C = 8$$

Figure 3.1.11

The equation y=−3x+5 is also a **linear equation**. But it does not appear to be in the form Ax+By=C. We can use the Addition Property of Equality and rewrite it in Ax+By=C form.

	y	$=$	$-3x+5$
Add to both sides.	$y+3x$	$=$	$-3x+5+3x$
Simplify.	$y+3x$	$=$	5
Use the Commutative Property to put it in $Ax+By=C$ form.	$3x+y$	$=$	5

By rewriting y=−3x+5 as 3x+y=5, we can easily see that it is a linear equation in two variables because it is of the form Ax+By=C. When an equation is in the form Ax+By=C, we say it is in *standard form*.

STANDARD FORM OF LINEAR EQUATION

A linear equation is in standard form when it is written Ax+By=C.

Most people prefer to have A, B, and C be integers and $A \geq 0$ when writing a linear equation in standard form, although it is not strictly necessary.

Linear equations have infinitely many solutions. For every number that is substituted for x there is a corresponding y value. This pair of values is a *solution* to the linear equation and is represented by the ordered pair (x,y). When we substitute these values of x and y into the equation, the result is a true statement, because the value on the left side is equal to the value on the right side.

Solution OF A LINEAR EQUATION IN TWO VARIABLES

An **ordered pair** (x,y) is a **solution** of the linear equation Ax+By=C, if the equation is a true statement when the *x*- and *y*-values of the ordered pair are substituted into the equation.

Example 3.1.10

Determine which ordered pairs are solutions to the equation x+4y=8.

(a) (0,2)

(b) (2,−4)

(c) (−4,3)

Solution

Substitute the *x*- and *y*-values from each ordered pair into the equation and determine if the result is a true statement.

(a)	(b)	(c)
(0, 2)	(2, –4)	(–4, 3)
$x = 0, y = 2$	$x = 2, y = -4$	$x = -4, y = 3$
$x + 4y = 8$	$x + 4y = 8$	$x + 4y = 8$
$0 + 4 \cdot 2 \stackrel{?}{=} 8$	$2 + 4(-4) \stackrel{?}{=} 8$	$-4 + 4 \cdot 3 \stackrel{?}{=} 8$
$0 + 8 \stackrel{?}{=} 8$	$2 + (-16) \stackrel{?}{=} 8$	$-4 + 12 \stackrel{?}{=} 8$
$8 = 8$ ✓	$-14 \neq 8$	$8 = 8$ ✓
(0, 2) is a solution.	(2, –4) is not a solution.	(–4, 3) is a solution.

Try It 3.1.11

Which of the following ordered pairs are solutions to 2x+3y=6?

1. (3,0)
2. (2,0)
3. (6,−2)

Answer

1, 3

Try It 3.1.12

Which of the following ordered pairs are solutions to the equation 4x−y=8?

1. (0,8)
2. (2,0)
3. (1,−4)

Answer

2, 3

Example 3.1.13

Which of the following ordered pairs are solutions to the equation y=5x−1?

(a) (0,−1)

(b) (1,4)

(c) (−2,−7)

Solution

Substitute the *x*- and *y*-values from each **ordered pair** into the equation and determine if it results in a true statement.

(a)	(b)	(c)
(0, –1)	(1, 4)	(–2, –7)
$x = 0, y = -1$	$x = 1, y = 4$	$x = -2, y = -7$
$y = 5x - 1$	$y = 5x - 1$	$y = 5x - 1$
$-1 \stackrel{?}{=} 5(0) - 1$	$4 \stackrel{?}{=} 5(1) - 1$	$-7 \stackrel{?}{=} 5(-2) - 1$
$-1 \stackrel{?}{=} 0 - 1$	$4 \stackrel{?}{=} 5 - 1$	$-7 \stackrel{?}{=} -10 - 1$
$-1 = -1$ ✓	$4 = 4$ ✓	$-7 \neq -11$
(0, –1) is a solution.	(1, 4) is a solution.	(–2, –7) is not a solution.

Try It 3.1.14

Which of the following ordered pairs are solutions to the equation y=4x−3?

1. (0,3)
2. (1,1)
3. (−1,−1)

Answer

2

Try It 3.1.15

Which of the following ordered pairs are solutions to the equation y=−2x+6?

1. (0,6)
2. (1,4)
3. (−2,−2)

Answer

1, 2

Complete a Table of Solutions to a Linear Equation in Two Variables

In the examples above, we substituted the x- and y-values of a given ordered pair to determine whether or not it was a solution to a linear equation. But how do you find the ordered pairs if they are not given? It's easier than you might think—you can just pick a value for x and then solve the equation for y. Or, pick a value for y and then solve for x.

We'll start by looking at the solutions to the equation y = 5x−1 that we found in Example 3.1.13. We can summarize this information in a table of solutions, as shown in Table 3.1.1.

Table 3.1.1

y=5x−1		
x	**y**	**(x,y)**
0	−1	(0,−1)
1	4	(1,4)

To find a third solution, we'll let x=2 and solve for y.

	$y = 5x - 1$
Substitute $x = 2$.	$y = 5(2) - 1$
Multiply.	$y = 10 - 1$
Simplify.	$y = 9$

Figure 3.1.12

The ordered pair (2,9) is a solution to y=5x−1. We will add it to Table 3.1.2.

Table 3.1.2

y=5x−1		
x	**y**	**(x,y)**
0	−1	(0,−1)
1	4	(1,4)
2	9	(2,9)

We can find more solutions to the equation by substituting in any value of x or any value of y and solving the resulting equation to get another ordered pair that is a solution. There are infinitely many solutions of this equation.

Example 3.1.16

Complete the table to find three solutions to the equation $y=4x-2$.

Table 3.1.3

y=4x−2		
x	**y**	**(x,y)**
0		
−1		
2		

Solution

Substitute $x=0$, $x=-1$, and $x=2$ into $y=4x-2$.

$x=0$	$x=-1$	$x=2$
$y=4x-2$	$y=4x-2$	$y=4x-2$
$y=4\cdot 0-2$	$y=4(-1)-2$	$y=4\cdot 2-2$
$y=0-2$	$y=-4-2$	$y=8-2$
$y=-2$	$y=-6$	$y=6$
$(0,-2)$	$(-1,-6)$	$(2,6)$

The results are summarized in Table 3.1.4

Table 3.1.4

y=4x−2		
x	**y**	**(x,y)**
0	−2	(0,−2)
−1	−6	(−1,−6)
2	6	(2,6)

Try It 3.1.17

Complete the table to find three solutions to this equation: $y=3x-1$.

Table 3.1.5

y=3x−1		
x	**y**	**(x,y)**
0		
−1		
2		

Answer

Table 3.1.6

y=3x−1

x	y	(x,y)
0	-1	(0, -1)
−1	-4	(-1, -4)
2	5	(2, 5)

Try It 3.1.18

Complete the table to find three solutions to this equation: y=6x+1.

Table 3.1.7

y=6x+1		
x	**y**	**(x,y)**
-2		

Answer

Table 3.1.8

y=6x+1		
x	**y**	**(x,y)**
0	1	(0,1)
1	7	(1,7)
−2	−11	(−2,−11)

Example 3.1.19

Complete Table 3.1.9 to find three solutions to the equation 5x−4y=20.

Table 3.1.9

5x−4y=20		
x	**y**	**(x,y)**
	0	
	5	

Solution

Substitute the given value into the equation 5x−4y=20 and solve for the other variable. Then, fill in the values in the table.

$x = 0$	$y = 0$	$y = 5$
$5x - 4y = 20$	$5x - 4y = 20$	$5x - 4y = 20$
$5 \cdot 0 - 4y = 20$	$5x - 4 \cdot 0 = 20$	$5x - 4 \cdot 5 = 20$
$0 - 4y = 20$	$5x - 0 = 20$	$5x - 20 = 20$
$-4y = 20$	$5x = 20$	$5x = 40$
$y = -5$	$x = 4$	$x = 8$
$(0, -5)$	$(4, 0)$	$(8, 5)$

The results are summarized in Table 3.1.10

Table 3.1.10

5x−4y=20		
x	**y**	**(x,y)**
0	−5	(0,−5)
4	0	(4,0)
8	5	(8,5)

Try It 3.1.20

Complete the table to find three solutions to this equation: 2x−5y=20.

Table 3.1.11

2x−5y=20		
x	**y**	**(x,y)**
-5		

Answer

Table 3.1.12

2x−5y=20		
x	**y**	**(x,y)**
0	−4	(0,−4)
10	0	(10,0)
−5	−6	(−5,−6)

Try It 3.1.21

Complete the table to find three solutions to this equation: 3x−4y=12.

Table 3.1.13

3x−4y=12		
x	**y**	**(x,y)**

-4		

Answer

Table 3.1.14

3x−4y=12		
x	**y**	**(x,y)**
0	−3	(0,−3)
4	0	(4,0)
−4	−6	(−4,−6)

Find Solutions to a Linear Equation

To find a solution to a linear equation, you really can pick *any* number you want to substitute into the equation for x or y. But since you'll need to use that number to solve for the other variable it's a good idea to choose a number that's easy to work with.

When the equation is in *y*-form, with the *y* by itself on one side of the equation, it is usually easier to choose values of x and then solve for y.

Example 3.1.22

Find three solutions to the equation y=−3x+2.

Solution

We can substitute any value we want for x or any value for y. Since the equation is in *y*-form, it will be easier to substitute in values of x. Let's pick x=0, x=1, and x=−1.

			$x=0$	$x=1$	$x=-1$
			$y=-3x+2$	$y=-3x+2$	$y=-3x+2$
Substitute the value into the equation.			$y=-3\cdot 0+2$	$y=-3\cdot 1+2$	$y=-3(-1)+2$
Simplify.			$y=0+2$	$y=-3+2$	$y=3+2$
Simplify.			$y=2$	$y=-1$	$y=5$
Write the ordered pair.			(0, 2)	(1, -1)	(-1, 5)
Check.					
y=−3x+2	y=−3x+2	y=−3x+2			
$2\stackrel{?}{=}-3\cdot 0+2$	$-1\stackrel{?}{=}-3\cdot 1+2$	$5\stackrel{?}{=}-3(-1)+2$			
$2\stackrel{?}{=}0+2$	$-1\stackrel{?}{=}-3+2$	$5\stackrel{?}{=}-3+2$			
$2=2\checkmark$	$-1=-1\checkmark$	$5=5\checkmark$			

Table 3.1.15

So, (0,2), (1,−1) and (−1,5) are all solutions to y=−3x+2. We show them in Table 3.1.16

Table 3.1.16

y=−3x+2

x	y	(x,y)
0	2	(0,2)
1	−1	(1,−1)
−1	5	(−1,5)

Try It 3.1.23

Find three solutions to this equation: y=−2x+3.

Answer

Answers will vary. Answers might include (0,3), (1,1) and (−1,5).

Try It 3.1.24

Find three solutions to this equation: y=−4x+1.

Answer

Answers will vary. Answers might include (0,1), (1,−3) and (−1,5).

We have seen how using zero as one value of x makes finding the value of y easy. When an equation is in standard form, with both the x and y on the same side of the equation, it is usually easier to first find one solution when x=0 find a second solution when y=0, and then find a third solution.

Example 3.1.25

Find three solutions to the equation 3x+2y=6.

Solution

We can substitute any value we want for x or any value for y. Since the equation is in standard form, let's pick first x=0, then y=0, and then find a third point.

Table 3.1.17

$x=0$	$y=0$	$x=1$			
			$3x+2y=6$	$3x+2y=6$	$3x+2y=6$
Substitute the value into the equation.			$3(0)+2y=6$	$3x+2(0)=6$	$3(1)+2y=6$
Simplify.			$0+2y=6$	$3x+0=6$	$3+2y=6$
Solve.			$2y=6$	$3x=6$	$2y=3$
			$y=3$	$x=2$	$y=\frac{3}{2}$
Write the ordered pair.			(0, 3)	(2, 0)	$(1, \frac{3}{2})$
Check.					
3x+2y=6	3x+2y=6	3x+2y=6			
$3\cdot 0+2\cdot 3\stackrel{?}{=}6$	$3\cdot 2+2\cdot 0\stackrel{?}{=}6$	$3\cdot 1+2\cdot\frac{3}{2}\stackrel{?}{=}6$			
$0+6\stackrel{?}{=}6$	$6+0\stackrel{?}{=}6$	$3+3\stackrel{?}{=}6$			
$6=6$✓	$6=6$✓	$6=6$✓			

So (0,3), (2,0), and $(1, \frac{3}{2})$ are all solutions to the equation 3x+2y=6. We can list these three solutions in Table 3.1.18.

Table 3.1.18

3x+2y=6		
x	**y**	**(x,y)**
0	3	(0,3)
2	0	(2,0)
1	$\frac{3}{2}$	$(1, \frac{3}{2})$

? Try It 3.1.26

Find three solutions to the equation 2x+3y=6.

Answer

Answers will vary. Answers might include (0,2), (3,0) and (6, −2).

? Try It 3.1.27

Find three solutions to the equation 4x+2y=8.

Answer

Answers will vary. Answers might include (0,4), (2,0) and (−1,6).

Key Concepts

- **Sign Patterns of the Quadrants**

Quadrant I	Quadrant II	Quadrant III	Quadrant IV
(x, y)	(x, y)	(x, y)	(x, y)
$(+, +)$	$(-, +)$	$(-, -)$	$(+, -)$

- **Points on the Axes**
  - On the *x*-axis, y=0. Points with a *y*-coordinate equal to 0 are on the *x*-axis, and have coordinates (a,0).
  - On the *y*-axis, x=0. Points with an *x*-coordinate equal to 0 are on the *y*-axis, and have coordinates (0,b).
- **Solution of a Linear Equation**
  - An ordered pair (x,y) is a solution of the linear equation Ax+By=C, if the equation is a true statement when the *x*- and *y*-values of the ordered pair are substituted into the equation.

Glossary

linear equation

A linear equation is of the form Ax+By=C, where A and B are not both zero, is called a linear equation in two variables.

ordered pair

An ordered pair (x,y) gives the coordinates of a point in a rectangular coordinate system.

origin

The point (0,0) is called the origin. It is the point where the *x*-axis and *y*-axis intersect.

quadrant

The *x*-axis and the *y*-axis divide a plane into four regions, called quadrants.

rectangular coordinate system

A grid system is used in algebra to show a relationship between two variables; also called the *xy*-plane or the ‘coordinate plane’.

***x*-coordinate**

The first number in an ordered pair (x,y).

***y*-coordinate**

The second number in an ordered pair (x,y).

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3.1E: Exercises

Practice Makes Perfect

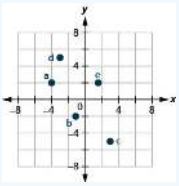
Plot Points in a Rectangular Coordinate System

In the following exercises, plot each point in a rectangular coordinate system and identify the quadrant in which the point is located.

Exercise 3.1E.1

1. (−4,2)
2. (−1,−2)
3. (3,−5)
4. (−3,5)
5. $(\frac{5}{3}, 2)$

Answer



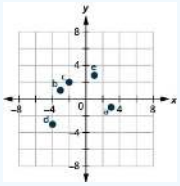
Exercise 3.1E.2

1. (−2,−3)
2. (3,−3)
3. (−4,1)
4. (4,−1)
5. $(\frac{3}{2}, 1)$

Exercise 3.1E.3

1. (3,−1)
2. (−3,1)
3. (−2,2)
4. (−4,−3)
5. $(1, \frac{14}{5})$

Answer



Exercise 3.1E.4

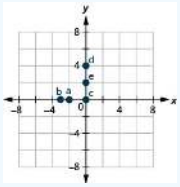
1. (−1,1)
2. (−2,−1)
3. (2,1)
4. (1,−4)
5. $(3, \frac{7}{2})$

In the following exercises, plot each point in a rectangular coordinate system.

Exercise 3.1E.5

1. (−2,0)
2. (−3,0)
3. (0,0)
4. (0,4)
5. (0,2)

Answer



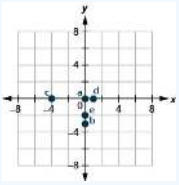
Exercise 3.1E.6

1. (0,1)
2. (0,−4)
3. (−1,0)
4. (0,0)
5. (5,0)

Exercise 3.1E.7

1. (0,0)
2. (0,−3)
3. (−4,0)
4. (1,0)
5. (0,−2)

Answer

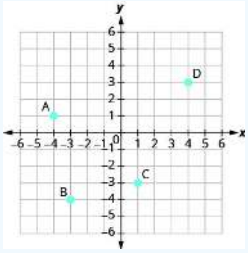


Exercise 3.1E.8

1. (−3,0)
2. (0,5)
3. (0,−2)
4. (2,0)
5. (0,0)

In the following exercises, name the ordered pair of each point shown in the rectangular coordinate system.

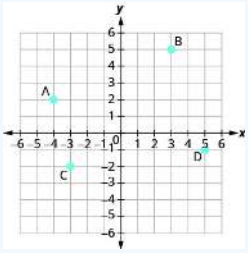
Exercise 3.1E.9



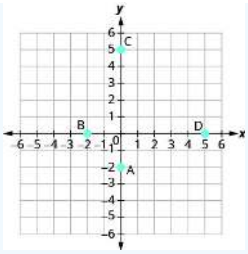
Answer

$A:(-4,1)\quad B:(-3,-4)\quad C:(1,-3)\quad D:(4,3)$

Exercise 3.1E.10



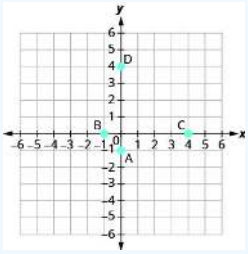
Exercise 3.1E.11



Answer

$A:(0,-2)\quad B:(-2,0)\quad C:(0,5)\quad D:(5,0)$

Exercise 3.1E.12



Verify Solutions to an Equation in Two Variables

In the following exercises, which ordered pairs are solutions to the given equations?

Exercise 3.1E.13

2x+y=6

1. (1,4)
2. (3,0)
3. (2,3)

Answer

1, 2

Exercise 3.1E.14

1. x+3y=9
2. (0,3)
3. (6,1)
4. (−3,−3)

Exercise 3.1E.15

4x−2y=8

1. (3,2)
2. (1,4)
3. (0,−4)

Answer

1, 3

Exercise 3.1E.16

3x−2y=12

1. (4,0)
2. (2,−3)
3. (1,6)

Exercise 3.1E.17

y=4x+3

1. (4,3)
2. (−1,−1)
3. $\left(\frac{1}{2},5\right)$

Answer

2, 3

Exercise 3.1E.18

y=2x−5

1. (0,−5)
2. (2,1)
3. $\left(\frac{1}{2}, -4\right)$

Exercise 3.1E.19

$y = \frac{1}{2}x - 1$

1. (2,0)
2. (−6,−4)
3. (−4,−1)

Answer

1, 2

Exercise 3.1E.20

$y = \frac{1}{3}x + 1$

1. (−3,0)
2. (9,4)
3. (−6,−1)

Complete a Table of Solutions to a Linear Equation

In the following exercises, complete the table to find solutions to each linear equation.

Exercise 3.1E.21

$y = 2x - 4$

x	y	(x,y)
0		
2		
-1		

Answer

x	y	(x,y)
0	−4	(0,−4)
2	0	(2,0)
−1	−6	(−1,−6)

Exercise 3.1E.22

$y = 3x - 1$

x	y	(x,y)
0		
2		
-1		

Exercise 3.1E.23

$y = -x + 5$

x	y	(x,y)
0		
3		
-2		

Answer

x	y	(x,y)
0	5	(0,5)
3	2	(3,2)
−2	7	(−2,7)

Exercise 3.1E.24

$y = -x + 2$

x	y	(x,y)
0		
3		
-2		

Exercise 3.1E.25

$y = \frac{1}{3}x + 1$

x	y	(x,y)
0		
3		
6		

Answer

x	y	(x,y)
0	1	(0,1)
3	2	(3,2)
6	3	(6,3)

Exercise 3.1E.26

$y = \frac{1}{2}x + 4$

x	y	(x,
0		
2		
4		

Exercise 3.1E.27

$y = -\frac{3}{2}x - 2$

x	y	(x,y)
0		
2		
-2		

Answer

x	y	(x,y)
0	−2	(0,−2)
2	−5	(2,−5)
−2	1	(−2,1)

Exercise 3.1E.28

$y = -\frac{2}{3}x - 1$

x	y	(x,y)
0		
3		
-3		

Exercise 3.1E.29

$x + 3y = 6$

x	y	(x,y)
0		
3		

	0	

Answer

x	y	(x,y)
0	2	(0,2)
3	4	(3,1)
6	0	(6,0)

Exercise 3.1E.30

x+2y=8

x	y	(x,y)
0		
4		
	0	

Exercise 3.1E.31

$2x-5y=10$

x	y	(x,y)
0		
10		
	0	

Answer

x	y	(x,y)
0	−2	(0,−2)
10	2	(10,2)
5	0	(5,0)

Exercise 3.1E.32

x	y	(x,y)
0		
8		
	0	

Find Solutions to a Linear Equation

In the following exercises, find three solutions to each linear equation.

Exercise 3.1E.33

$y=5x-8$

Answer

Answers will vary.

Exercise 3.1E.34

$y=3x-9$

Exercise 3.1E.35

$y=-4x+5$

Answer

Answers will vary.

Exercise 3.1E.36

$y=-2x+7$

Exercise 3.1E.37

$x+y=8$

Answer

Answers will vary.

Exercise 3.1E.38

$x+y=6$

Exercise 3.1E.39

$x+y=-2$

Answer

Answers will vary.

Exercise 3.1E.40

$x+y=-1$

Exercise 3.1E.41

$3x+y=5$

Answer

Answers will vary.

Exercise 3.1E.42

$2x+y=3$

Exercise 3.1E.43

$4x-y=8$

Answer

Answers will vary.

Exercise 3.1E.44

$5x-y=10$

Exercise 3.1E.45

$2x+4y=8$

Answer

Answers will vary.

Exercise 3.1E.46

$3x+2y=6$

Exercise 3.1E.47

$5x-2y=10$

Answer

Answers will vary.

Exercise 3.1E.48

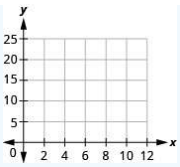
$4x-3y=12$

Everyday Math

Exercise 3.1E.49

Weight of a baby. Mackenzie recorded her baby's weight every two months. The baby's age, in months, and weight, in pounds, are listed in the table below, and shown as an ordered pair in the third column.

1. Plot the points on a coordinate plane.

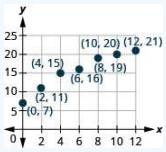


2. Why is only Quadrant I needed?

Age x	Weight y	(x,y)
0	7	(0, 7)
2	11	(2, 11)
4	15	(4, 15)
6	16	(6, 16)
8	19	(8, 19)
10	20	(10, 20)
12	21	(12, 21)

Answer

1.

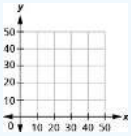


2. Age and weight are only positive.

Exercise 3.1E.50

Weight of a child. Latresha recorded her son's height and weight every year. His height, in inches, and weight, in pounds, are listed in the table below, and shown as an ordered pair in the third column.

1. Plot the points on a coordinate plane.



2. Why is only Quadrant I needed?

Height x	Weight y	(x,y)
28	22	(28, 22)
31	27	(31, 27)
33	33	(33, 33)
37	35	(37, 35)
40	41	(40, 41)
42	45	(42, 45)

Writing Exercises

Exercise 3.1E.51

Explain in words how you plot the point (4,−2) in a rectangular coordinate system?

Answer

Answers will vary.

Exercise 3.1E.52

How do you determine if an ordered pair is a solution to a given equation?

Exercise 3.1E.53

Is the point (−3,0) on the *x*-axis or *y*-axis? How do you know?

Answer

Answers will vary.

? Exercise 3.1E.54

Is the point (0,8) on the *x*-axis or *y*-axis? How do you know?

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
plot points in a rectangular coordinate system.			
identify points on a graph.			
verify solutions to an equation in two variables.			
complete a table of solutions to a linear equation.			
find solutions to a linear equation.			

ⓑ If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them. What did you do to become confident of your ability to do these things? Be specific.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math every topic builds upon previous work. It is important to make sure you have a strong foundation before you move on. Who can you ask for help? Your fellow classmates and instructor are good resources. Is there a place on campus where math tutors are available? Can your study skills be improved?

…no, I don't get it. This is a warning sign and you must not ignore it. You should get help right away or you will quickly be overwhelmed. See your instructor as soon as you can to discuss your situation. Together you can come up with a plan to get you the help you need.

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3.2: Graph Linear Equations in Two Variables

Learning Objectives

By the end of this section, you will be able to:

- Recognize the relationship between the solutions of an equation and its graph.
- Graph a linear equation by plotting points.
- Graph vertical and horizontal lines.

Note

Before you get started, take this readiness quiz.

1. Evaluate $3x+2$ when $x=-1$.
   If you missed this problem, review Exercise 1.5.34.
2. Solve $3x+2y=12$ for y in general.
   If you missed this problem, review Exercise 2.6.16.

Recognize the Relationship Between the Solutions of an Equation and its Graph

In the previous section, we found several solutions to the equation $3x+2y=6$. They are listed in Table 3.2.1. So, the ordered pairs (0,3), (2,0), and $(1, \frac{3}{2})$ are some solutions to the equation $3x+2y=6$. We can plot these solutions in the rectangular coordinate system as shown in Figure 3.2.1.

Table 3.2.1

3x+2y=6		
x	**y**	**(x,y)**
0	3	(0,3)
2	0	(2,0)
1	$\frac{3}{2}$	$(1, \frac{3}{2})$

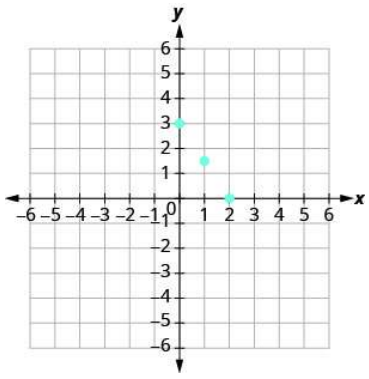


Figure 3.2.1

Notice how the points line up perfectly? We connect the points with a line to get the graph of the equation 3x+2y=6. See Figure 3.2.2. Notice the arrows on the ends of each side of the line. These arrows indicate the line continues.

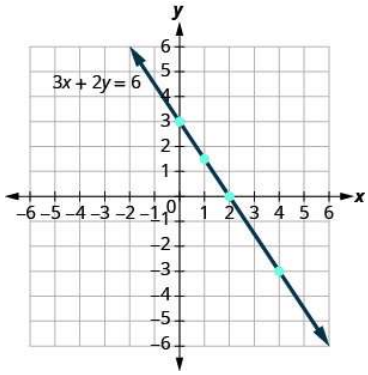


Figure 3.2.2

Every point on the line is a solution of the equation. Also, every solution of this equation is a point on this line. Points *not* on the line are not solutions.

Notice that the point whose coordinates are (−2,6) is on the line shown in Figure 3.2.3. If you substitute x=−2 and y=6 into the equation, you find that it is a solution to the equation.

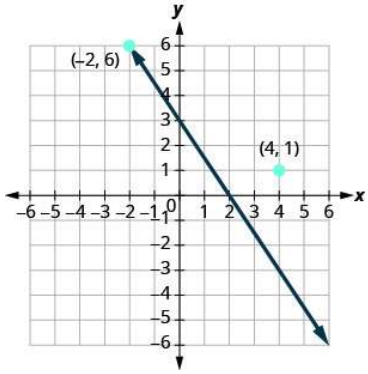


Figure 3.2.3

Test (−2, 6)

$$3x + 2y = 6$$

$$3(-2) + 2(6) = 6$$

$$-6 + 12 = 6$$

$$6 = 6 \checkmark$$

So the point (−2,6) is a solution to the equation $3x + 2y = 6$. (The phrase "the point whose coordinates are (−2,6)" is often shortened to "the point (−2,6).")

What about (4, 1) ?

$$3x + 2y = 6$$

$$3 \cdot 4 + 2 \cdot 1 = 6$$

$$12 + 2 \stackrel{?}{=} 6$$

$$14 \neq 6$$

Figure 3.2.3. This is an example of the saying, "A picture is worth a thousand words." The line shows you *all* the solutions to the equation. Every point on the line is a solution of the equation. And, every solution of this equation is on this line. This line is called the *graph* of the equation $3x + 2y = 6$.

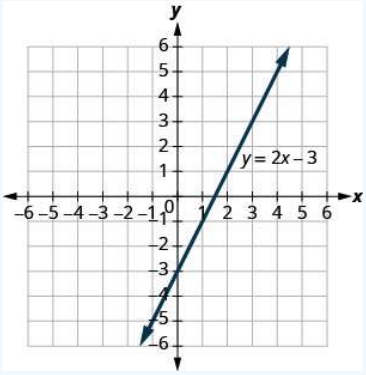
GRAPH OF A LINEAR EQUATION

The **graph of a linear equation** Ax+By=C is a line.

- Every point on the line is a solution of the equation.
- Every solution of this equation is a point on this line.

Example 3.2.1

The graph of y=2x−3 is shown.



For each ordered pair, decide:

1. Is the ordered pair a solution to the equation?
2. Is the point on the line?

A (0,−3) B (3,3) C (2,−3) D (−1,−5)

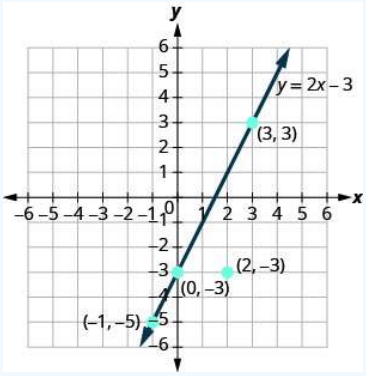
Solution

Substitute the *x*- and *y*- values into the equation to check if the ordered pair is a solution to the equation.

1.

A: (0, –3)	B: (3, 3)	C: (2, –3)	D: (–1, –5)
$y = 2x - 3$	$y = 2x - 3$	$y = 2x - 3$	$y = 2x - 3$
$-3 \stackrel{?}{=} 2(0) - 3$	$3 \stackrel{?}{=} 2(3) - 3$	$-3 \stackrel{?}{=} 2(2) - 3$	$-5 \stackrel{?}{=} 2(-1) - 3$
$-3 = -3$ ✓	$3 = 3$ ✓	$-3 \neq 1$	$-5 = -5$ ✓
(0, –3) is a solution.	(3, 3) is a solution.	(2, –3) is not a solution.	(–1, –5) is a solution.

2. Plot the points A (0,−3), B (3,3), C (2,−3), and D (−1,−5).



The points that are solutions to y=2x−3 are on the line, but the point that is not a solution is not on the line.

The points (0,−3), (3,3), and (−1,−5) are on the line y=2x−3, and the point (2,−3) is not on the line.

Try It 3.2.2

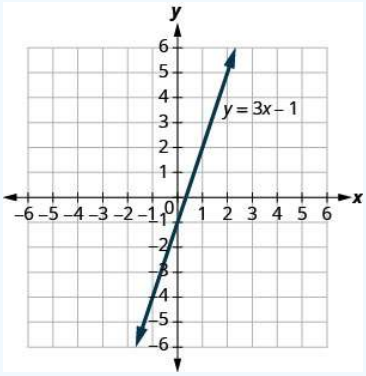
Use the graph of y=3x−1 to decide whether each ordered pair is:

- a solution to the equation.

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- on the line.

1. (0,−1)
2. (2,5)



Answer

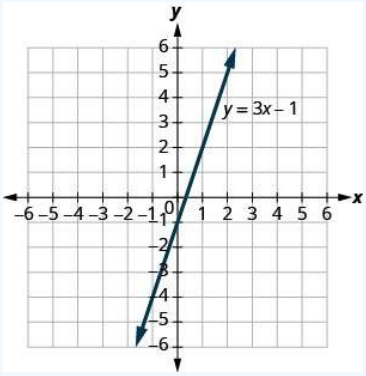
1. yes, yes
2. yes, yes

? Try It 3.2.3

Use graph of y=3x−1 to decide whether each ordered pair is:

- a solution to the equation
- on the line

1. (3,−1)
2. (−1,−4)



Answer

1. no, no
2. yes, yes

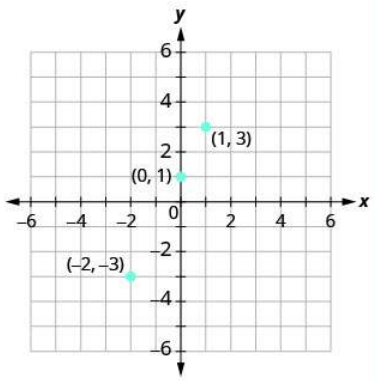
Graph a Linear Equation by Plotting Points

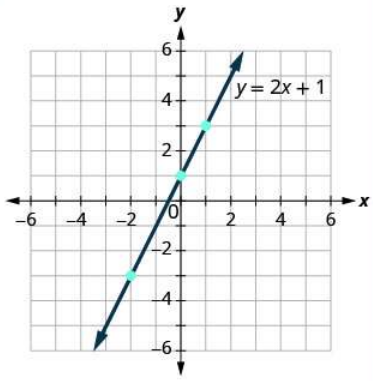
There are several methods that can be used to graph a linear equation. The method we used to graph 3x+2y=6 is called plotting points, or the Point–Plotting Method.

Example 3.2.4: How To Graph an Equation By Plotting Points

Graph the equation y=2x+1 by plotting points.

Solution

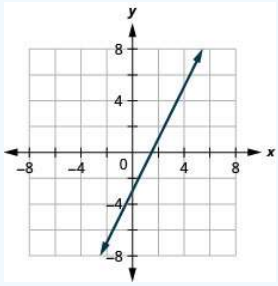
Step 1. Find three points whose coordinates are solutions to the equation.	You can choose any values for x or y. In this case, since y is isolated on the left side of the equation, it is easier to choose values for x.	$y = 2x + 1$ $x = 0$ $y = 2x + 1$ $y = 2 \cdot 0 + 1$ $y = 0 + 1$ $y = 1$ $x = 1$ $y = 2x + 1$ $y = 2 \cdot 1 + 1$ $y = 2 + 1$ $y = 3$ $x = -2$ $y = 2x + 1$ $y = 2(-2) + 1$ $y = -4 + 1$ $y = -3$
Organize the solutions in a table.	Put the three solutions in a table.	$y = 2x + 1$ x | y | (x, y) 0 | 1 | (0, 1) 1 | 3 | (1, 3) –2 | –3 | (–2, –3)
Step 2. Plot the points in a rectangular coordinate system. Check that the points line up. If they do not, carefully check your work!	Plot: (0, 1), (1, 3), (–2, –3). Do the points line up? Yes, the points line up.	

Step 3. Draw the line through the three points. Extend the line to fill the grid and put arrows on both ends of the line.	This line is the graph of $y = 2x + 1$.	

Try It 3.2.5

Graph the equation by plotting points: y=2x−3.

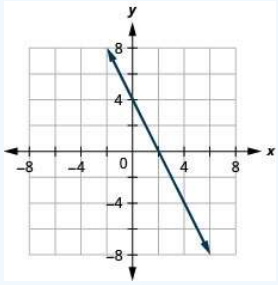
Answer



Try It 3.2.6

Graph the equation by plotting points: y=−2x+4.

Answer



The steps to take when graphing a linear equation by plotting points are summarized below.

GRAPH A LINEAR EQUATION BY PLOTTING POINTS

1. Find three points whose coordinates are solutions to the equation. Organize them in a table.
2. Plot the points in a rectangular coordinate system. Check that the points line up. If they do not, carefully check your work.
3. Draw the line through the three points. Extend the line to fill the grid and put arrows on both ends of the line.

It is true that it only takes two points to determine a line, but it is a good habit to use three points. If you only plot two points and one of them is incorrect, you can still draw a line but it will not represent the solutions to the equation. It will be the wrong line.

If you use three points, and one is incorrect, the points will not line up. This tells you something is wrong and you need to check your work. Look at the difference between part (a) and part (b) in Figure 3.2.4.

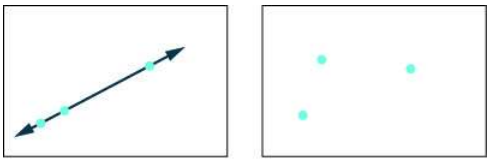


Figure 3.2.4

Let's do another example. This time, we'll show the last two steps all on one grid.

Example 3.2.7

Graph the equation y=−3x.

Solution

Find three points that are solutions to the equation. Here, again, it's easier to choose values for x. Do you see why?

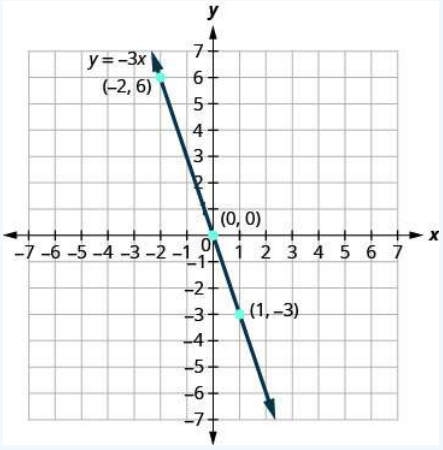
$x = 0$	$x = 1$	$x = -2$
$y = -3x$	$y = -3x$	$y = -3x$
$y = -3 \cdot 0$	$y = -3 \cdot 1$	$y = -3(-2)$
$y = 0$	$y = -3$	$y = 6$

We list the points in Table 3.2.2.

Table 3.2.2

y=−3x		
x	**y**	**(x,y)**
0	0	(0,0)
1	−3	(1,−3)
−2	6	(−2,6)

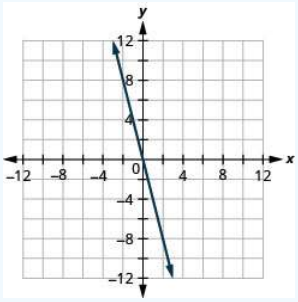
Plot the points, check that they line up, and draw the line.



Try It 3.2.8

Graph the equation by plotting points: y=−4x.

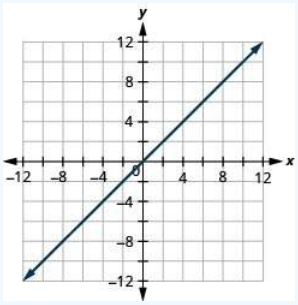
Answer



Try It 3.2.9

Graph the equation by plotting points: y=x.

Answer



When an equation includes a fraction as the coefficient of x, we can still substitute any numbers for x. But the math is easier if we make ‘good’ choices for the values of x. This way we will avoid fraction answers, which are hard to graph precisely.

Example 3.2.10

Graph the equation $y = \frac{1}{2}x + 3$.

Solution

Find three points that are solutions to the equation. Since this equation has the fraction $\frac{1}{2}$ as a coefficient of x, we will choose values of x carefully. We will use zero as one choice and multiples of 2 for the other choices. Why are multiples of 2 a good choice for values of x?

$x = 0$	$x = 2$	$x = 4$
$y = \frac{1}{2}x + 3$	$y = \frac{1}{2}x + 3$	$y = \frac{1}{2}x + 3$
$y = \frac{1}{2}(0) + 3$	$y = \frac{1}{2}(2) + 3$	$y = \frac{1}{2}(4) + 3$
$y = 0 + 3$	$y = 1 + 3$	$y = 2 + 3$
$y = 3$	$y = 4$	$y = 5$

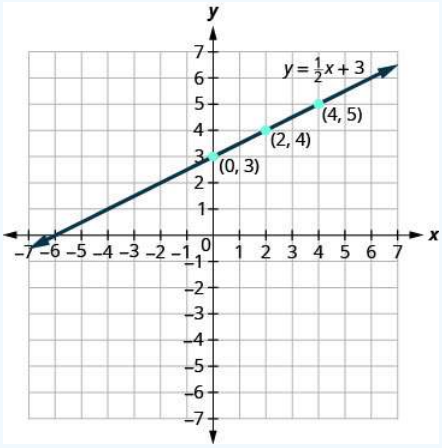
The points are shown in Table 3.2.3.

Table 3.2.3

y=12x+3

x	y	(x,y)
0	3	(0,3)
2	4	(2,4)
4	5	(4,5)

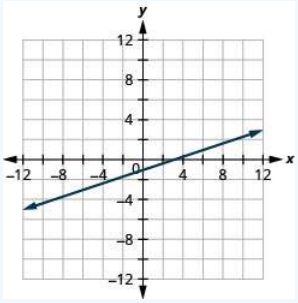
Plot the points, check that they line up, and draw the line.



Try It 3.2.11

Graph the equation $y = \frac{1}{3}x - 1$.

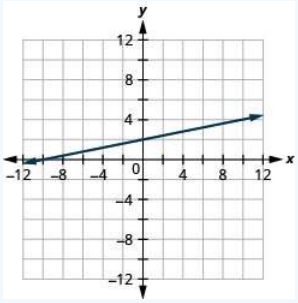
Answer



Try It 3.2.12

Graph the equation $y = \frac{1}{5}x + 2$.

Answer



So far, all the equations we graphed had y given in terms of x. Now we'll graph an equation with x and y on the same side. Let's see what happens in the equation 2x+y=3. If y=0 what is the value of x?

$$\begin{aligned} y &= 0 \\ 2x + y &= 3 \\ 2x + 0 &= 3 \\ 2x &= 3 \\ x &= \frac{3}{2} \\ \left(\frac{3}{2}, 0\right) & \end{aligned}$$

This point has a fraction for the *x*- coordinate and, while we could graph this point, it is hard to be precise graphing fractions. Remember in the example y=12x+3, we carefully chose values for x so as not to graph fractions at all. If we solve the equation 2x+y=3 for y, it will be easier to find three solutions to the equation.

$$\begin{aligned} 2x + y &= 3 \\ y &= -2x + 3 \end{aligned}$$

The solutions for x=0, x=1, and x=−1 are shown in the Table 3.2.4. The graph is shown in Figure 3.2.5.

Table 3.2.4

2x+y=3		
x	**y**	**(x,y)**
0	3	(0,3)
1	1	(1,1)
−1	5	(−1,5)

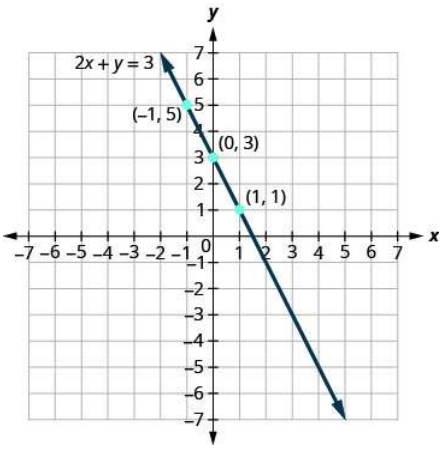


Figure 3.2.5

Can you locate the point $\left(\frac{3}{2}, 0\right)$ which we found by letting y=0, on the line?

Example 3.2.13

Graph the equation 3x+y=−1.

Solution

Find three points that are solutions to the equation.	$3x + y$	$=$	-1
First solve the equation for y.	y	$=$	$-3x - 1$

We'll let x be 0, 1, and −1 to find 3 points. The ordered pairs are shown in Table 3.2.5. Plot the points, check that they line up, and draw the line. See Figure 3.2.6.

Table 3.2.5

3x+y=−1		
x	**y**	**(x,y)**
0	−1	(0,−1)
1	−4	(1,−4)
−1	2	(−1,2)

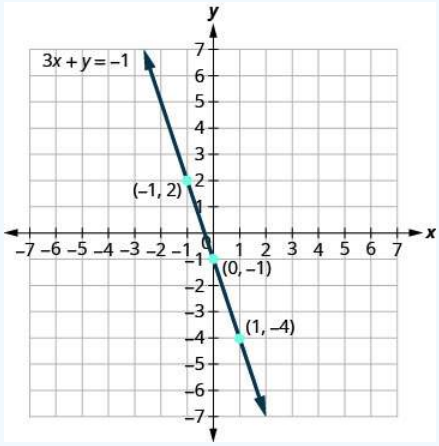
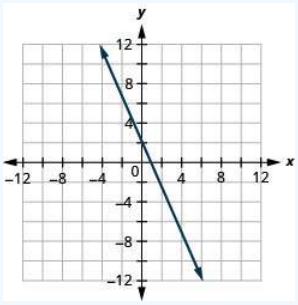


Figure 3.2.6

Try It 3.2.14

Graph the equation 2x+y=2.

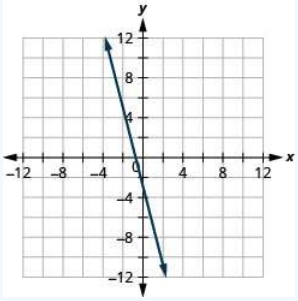
Answer



Try It 3.2.15

Graph the equation 4x+y=−3.

Answer



If you can choose any three points to graph a line, how will you know if your graph matches the one shown in the answers in the book? If the points where the graphs cross the *x*- and *y*-axis are the same, the graphs match!

The equation in Example 3.2.13 was written in standard form, with both x and y on the same side. We solved that equation for y in just one step. But for other equations in standard form it is not that easy to solve for y, so we will leave them in standard form. We can still find a first point to plot by letting x=0 and solving for y. We can plot a second point by letting y=0 and then solving for x. Then we will plot a third point by using some other value for x or y.

Example 3.2.16

Graph the equation $2x-3y=6$.

Solution

Find three points that are solutions to the equation.	$2x-3y = 6$
First let $x=0$.	$2x-3y = 6$
Solve for y.	$2(0)-3y = 6$
	$-3y = 6$
	$y = -2$
Now let $y=0$.	$2x-3(0) = 6$
Solve for x.	$2x = 6$
	$x = 3$
We need a third point. Remember, we can choose any value for x or y. We'll let x = 6.	$2(6)-3y = 6$
	$12-3y = 6$
Solve for y.	$-3y = -6$
	$y = 2$

We list the ordered pairs in Table 3.2.6. Plot the points, check that they line up, and draw the line. See Figure 3.2.7.

Table 3.2.6

2x−3y=6		
x	**yT**	**(x,y)**
0	−2	(0,−2)
3	0	(3,0)
6	2	(6,2)

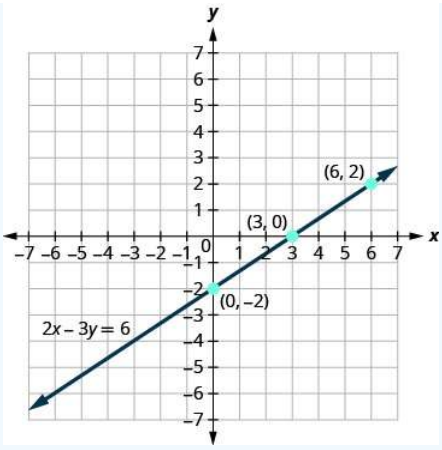
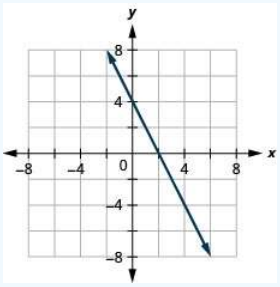


Figure 3.2.7

Try It 3.2.17

Graph the equation $4x + 2y = 8$.

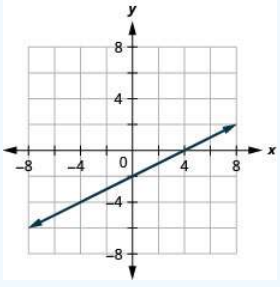
Answer



Try It 3.2.18

Graph the equation $2x - 4y = 8$.

Answer



Graph Vertical and Horizontal Lines

Can we graph an equation with only one variable? Just x and no y, or just y without an x? How will we make a table of values to get the points to plot?

Let's consider the equation x=−3. This equation has only one variable, x. The equation says that x is *always* equal to −3, so its value does not depend on y. No matter what y is, the value of x is always −3.

So to make a table of values, write −3 in for all the x values. Then choose any values for y. Since x does not depend on y, you can choose any numbers you like. But to fit the points on our coordinate graph, we'll use 1, 2, and 3 for the *y*-coordinates. See Table 3.2.7

Table 3.2.7

x=−3		
x	**y**	**(x,y)**
−3	1	(−3,1)
−3	2	(−3,2)
−3	3	(−3,3)

Plot the points from Table 3.2.7 and connect them with a straight line. Notice in Figure 3.2.8 that we have graphed a *vertical line*.

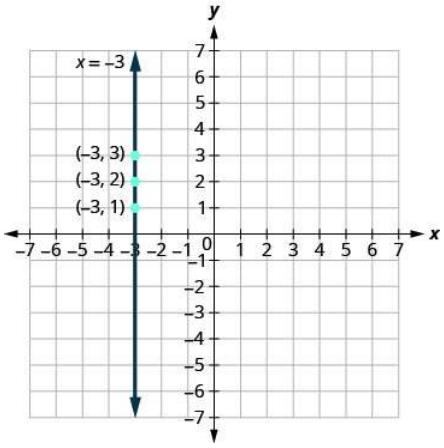


Figure 3.2.8

VERTICAL LINE

A **vertical line** is the graph of an equation of the form x=a.

The line passes through the *x*-axis at (a,0).

Example 3.2.19

Graph the equation x=2.

Solution

The equation has only one variable, x, and x is always equal to 2. We create Table 3.2.8 where x is always 2 and then put in any values for y. The graph is a vertical line passing through the *x*-axis at 2. See Figure 3.2.9.

Table 3.2.8

x=2		
x	**y**	**(x,y)**
2	1	(2,1)
2	2	(2,2)
2	3	(2,3)

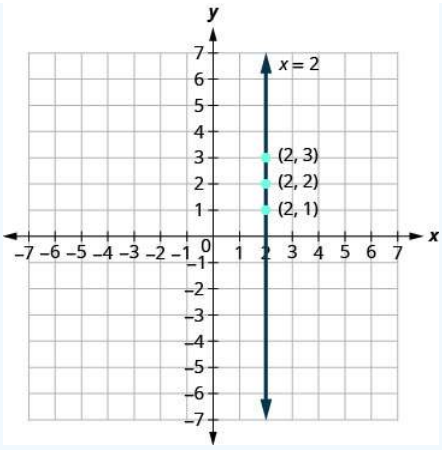
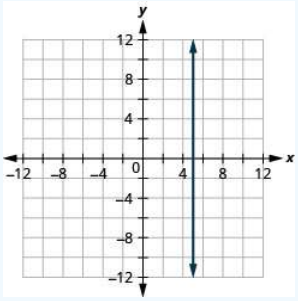


Figure 3.2.9

Try It 3.2.20

Graph the equation x=5.

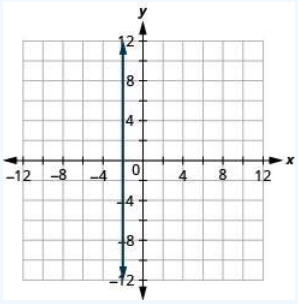
Answer



Try It 3.2.21

Graph the equation x=−2.

Answer



What if the equation has y but no x? Let's graph the equation y=4. This time the *y*- value is a constant, so in this equation, y does not depend on x. Fill in 4 for all the y's in Table 3.2.9 and then choose any values for x. We'll use 0, 2, and 4 for the *x*-coordinates.

Table 3.2.9

y=4		
x	**y**	**(x,y)**

0	4	(0,4)
2	4	(2,4)
4	4	(4,4)

The graph is a horizontal line passing through the *y*-axis at 4. See Figure 3.2.10.

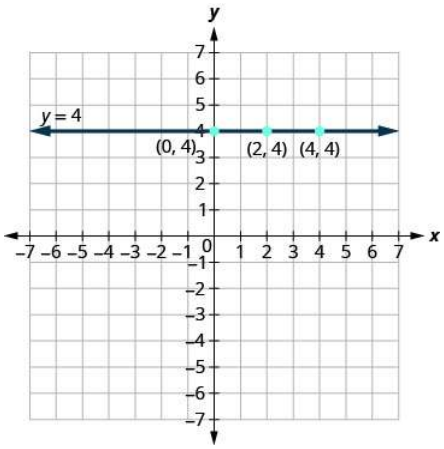


Figure 3.2.10

HORIZONTAL LINE

A **horizontal line** is the graph of an equation of the form y=b.

The line passes through the *y*-axis at (0,b).

Example 3.2.22

Graph the equation y=−1.

Solution

The equation y=−1 has only one variable, y. The value of y is constant. All the ordered pairs in Table 3.2.10 have the same *y*-coordinate. The graph is a horizontal line passing through the *y*-axis at −1, as shown in Figure 3.2.11.

Table 3.2.10

y=−1		
x	**y**	**(x,y)**
	−1	(0,−1)
	−1	(3,−1)
−3	−1	(−3,−1)

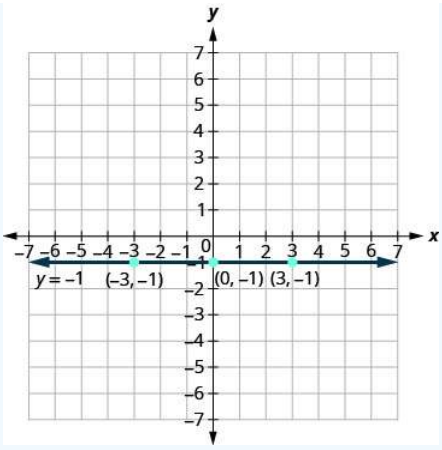
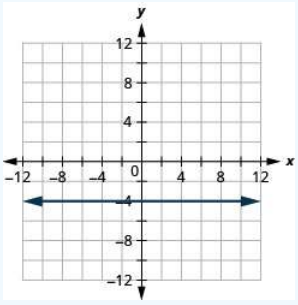


Figure 3.2.11

Try It 3.2.23

Graph the equation y=−4.

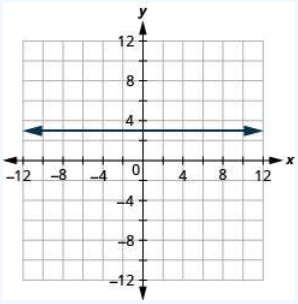
Answer



Try It 3.2.24

Graph the equation y=3.

Answer



The equations for vertical and horizontal lines look very similar to equations like y=4x. What is the difference between the equations y=4x and y=4?

The equation y=4x has both x and y. The value of y depends on the value of x. The *y*-coordinate changes according to the value of x. The equation y=4 has only one variable. The value of y is constant. The *y*-coordinate is always 4. It does not depend on the value of x. See Table 3.2.11.

Table 3.2.11

y=4x				y=4		
x	**y**	**(x,y)**		**x**	**y**	**(x,y)**
0	0	(0,0)		0	4	(0,4)
1	4	(1,4)		1	4	(1,4)
2	8	(2,8)		2	4	(2,4)

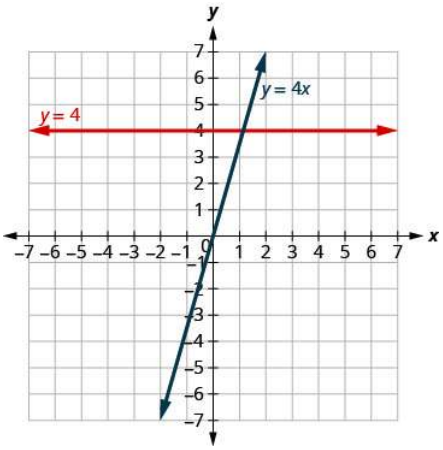


Figure 3.2.12

Notice, in Figure 3.2.12, the equation y=4x gives a slanted line, while y=4 gives a horizontal line.

Example 3.2.25

Graph y=−3x and y=−3 in the same rectangular coordinate system.

Solution

Notice that the first equation has the variable x, while the second does not. See Table 3.2.12 The two graphs are shown in Figure 3.2.13

Table 3.2.12

y=−3x				y=−3		
x	**y**	**(x,y)**		**x**	**y**	**(x,y)**
		(0,0)			−3	(0,−3)
	−3	(1,−3)			−3	(1,−3)
	−6	(2,−6)			−3	(2,−3)

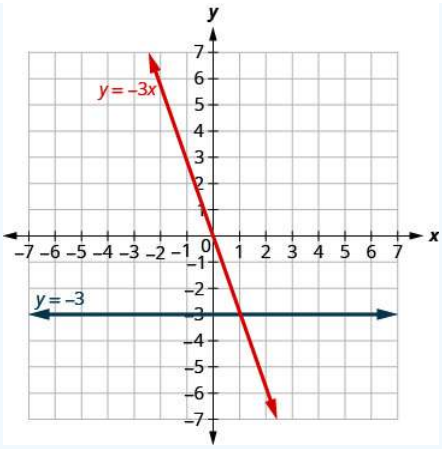
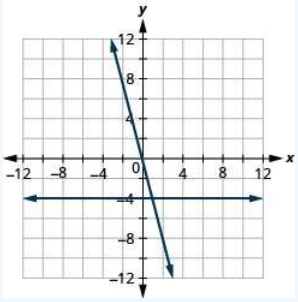


Figure 3.2.13

Try It 3.2.26

Graph y=−4x and y=−4 in the same rectangular coordinate system.

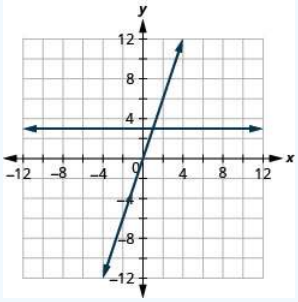
Answer



Try It 3.2.27

Graph y=3 and y=3x in the same rectangular coordinate system.

Answer



Key Concepts

- **Graph a Linear Equation by Plotting Points**
  1. Find three points whose coordinates are solutions to the equation. Organize them in a table.
  2. Plot the points in a rectangular coordinate system. Check that the points line up. If they do not, carefully check your work!
  3. Draw the line through the three points. Extend the line to fill the grid and put arrows on both ends of the line.

Glossary

graph of a linear equation

The graph of a linear equation $Ax+By=C$ is a straight line. Every point on the line is a solution of the equation. Every solution of this equation is a point on this line.

horizontal line

A horizontal line is the graph of an equation of the form $y=b$. The line passes through the y-axis at $(0,b)$.

vertical line

A vertical line is the graph of an equation of the form $x=a$. The line passes through the x-axis at $(a,0)$.

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3.2E: Exercises

Practice Makes Perfect

Recognize the Relationship Between the Solutions of an Equation and its Graph

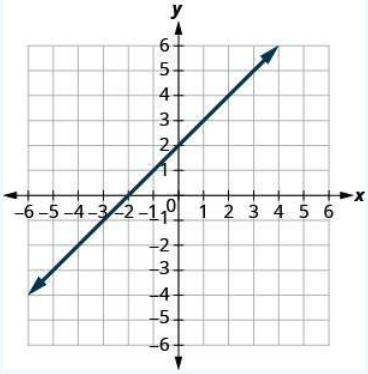
In the following exercises, for each ordered pair, decide:

1. Is the ordered pair a solution to the equation?
2. Is the point on the line?

Exercise 3.2E.1

y=x+2

1. (0,2)
2. (1,2)
3. (−1,1)
4. (−3,−1)



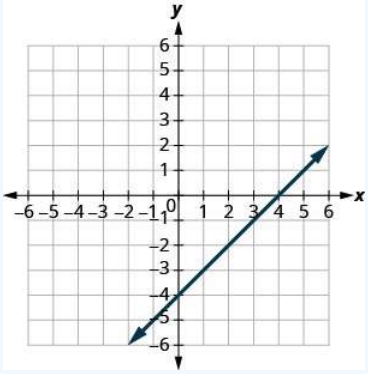
Answer

1. yes; no
2. no; no
3. yes; yes
4. yes; yes

Exercise 3.2E.2

y=x−4

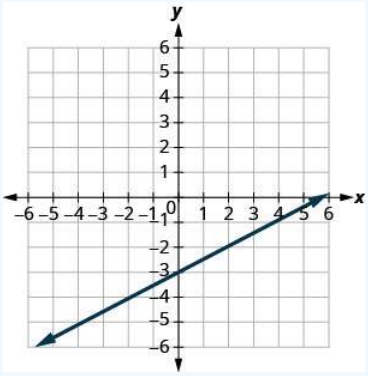
1. (0,−4)
2. (3,−1)
3. (2,2)
4. (1,−5)



Exercise $3.2E.3$

$y = \frac{1}{2}x - 3$

1. (0,−3)
2. (2,−2)
3. (−2,−4)
4. (4,1)



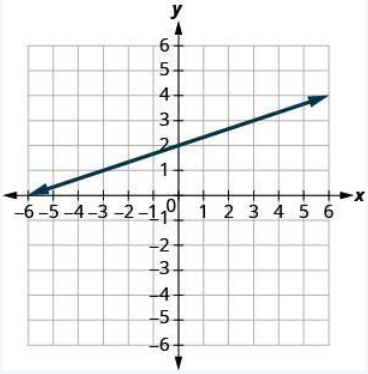
Answer

1. yes; yes
2. yes; yes
3. yes; yes
4. no; no

Exercise $3.2E.4$

$y = \frac{1}{3}x + 2$

1. (0,2)
2. (3,3)
3. (−3,2)
4. (−6,0)



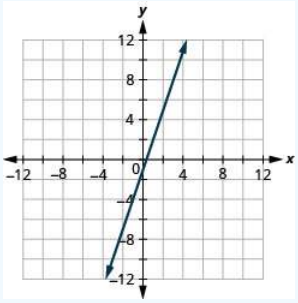
Graph a Linear Equation by Plotting Points

In the following exercises, graph by plotting points.

Exercise 3.2E.5

$y = 3x - 1$

Answer



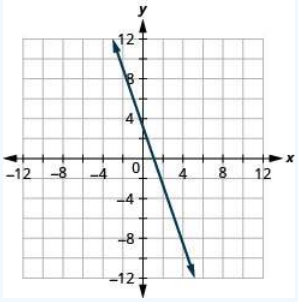
Exercise 3.2E.6

$y = 2x + 3$

Exercise 3.2E.7

$y = -2x + 2$

Answer



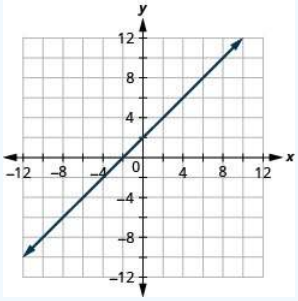
Exercise 3.2E.8

$y = -3x + 1$

Exercise 3.2E.9

$y = x + 2$

Answer



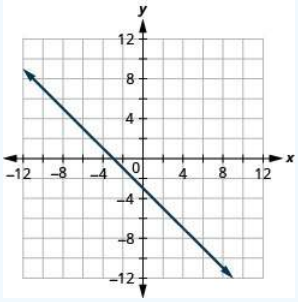
Exercise 3.2E.10

$y = x - 3$

Exercise 3.2E.11

$y = -x - 3$

Answer



Exercise 3.2E.12

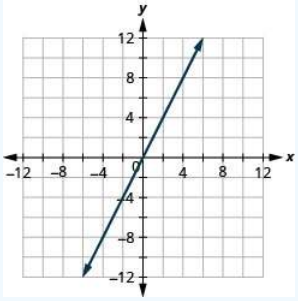
$y = -x - 2$

Exercise 3.2E.13

$y = 2x$

Answer

https://math.libretexts.org/@go/page/54976



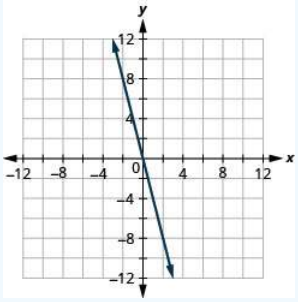
Exercise 3.2E.14

$y = 3x$

Exercise 3.2E.15

$y = -4x$

Answer



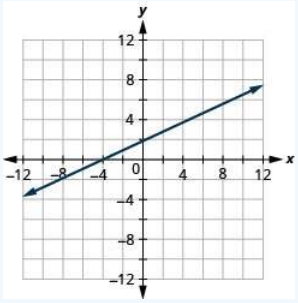
Exercise 3.2E.16

$y = -2x$

Exercise 3.2E.17

$y = \frac{1}{2}x + 2$

Answer



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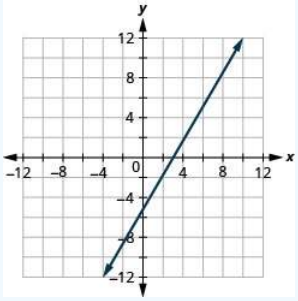
Exercise 3.2E.18

$y = \frac{1}{3}x - 1$

Exercise 3.2E.19

$y = \frac{4}{3}x - 5$

Answer



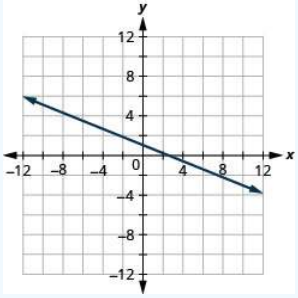
Exercise 3.2E.20

$y = \frac{3}{2}x - 3$

Exercise 3.2E.21

$y = -\frac{2}{5}x + 1$

Answer



Exercise 3.2E.22

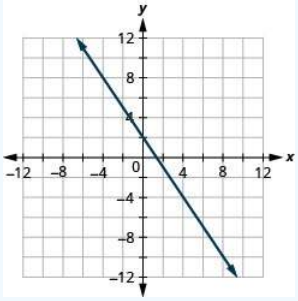
$y = -\frac{4}{5}x - 1$

Exercise 3.2E.23

$y = -\frac{3}{2}x + 2$

Answer

https://math.libretexts.org/@go/page/54976



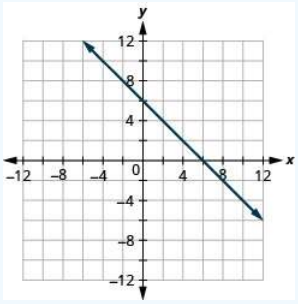
Exercise 3.2E.24

$y = -\frac{5}{3}x + 4$

Exercise 3.2E.25

$x + y = 6$

Answer



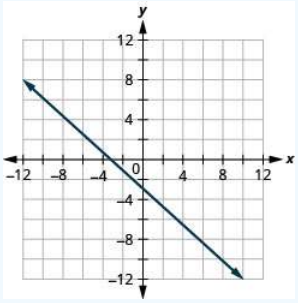
Exercise 3.2E.26

$x + y = 4$

Exercise 3.2E.27

$x + y = -3$

Answer



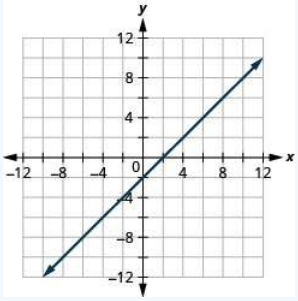
Exercise 3.2E.28

$x + y = -2$

Exercise 3.2E.29

$x - y = 2$

Answer



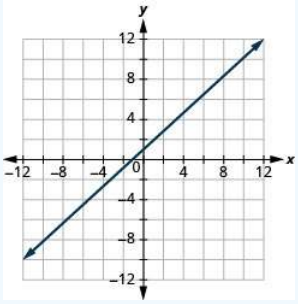
Exercise 3.2E.30

$x - y = 1$

Exercise 3.2E.31

$x - y = -1$

Answer



Exercise 3.2E.32

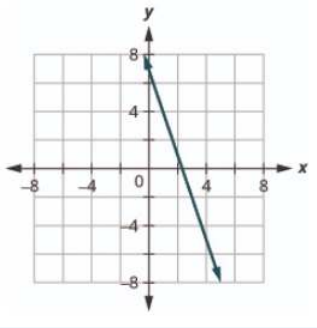
$x - y = -3$

Exercise 3.2E.33

$3x + y = 7$

Answer

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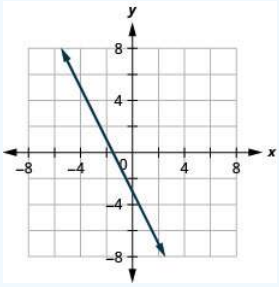
Exercise 3.2E.34

$5x + y = 6$

Exercise 3.2E.35

2x+y=−3

Answer



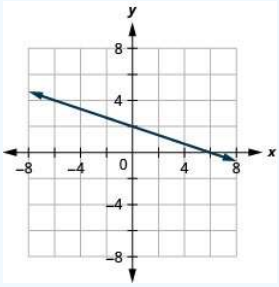
Exercise 3.2E.36

$4x + y = -5$

Exercise 3.2E.37

$\frac{1}{3}x + y = 2$

Answer



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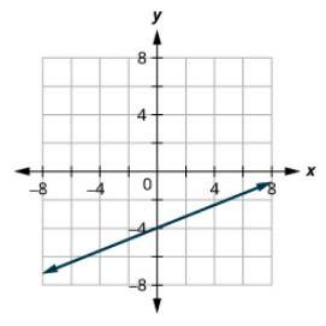
Exercise 3.2E.38

$\frac{1}{2}x + y = 3$

Exercise 3.2E.39

$\frac{2}{5}x - y = 4$

Answer



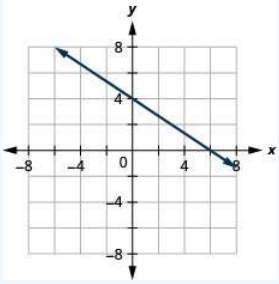
Exercise 3.2E.40

$\frac{3}{4}x - y = 6$

Exercise 3.2E.41

$2x + 3y = 12$

Answer



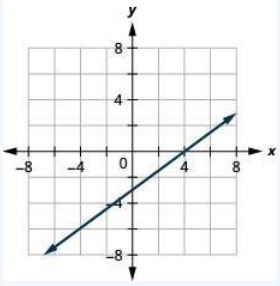
Exercise 3.2E.42

4x+2y=12

Exercise 3.2E.43

3x−4y=12

Answer



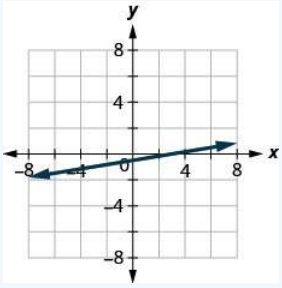
Exercise 3.2E.44

2x−5y=10

Exercise 3.2E.45

x−6y=3

Answer



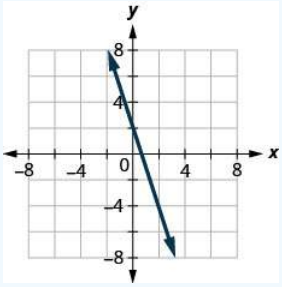
Exercise 3.2E.46

x−4y=2

Exercise 3.2E.47

5x+2y=4

Answer



Exercise 3.2E.48

3x+5y=5

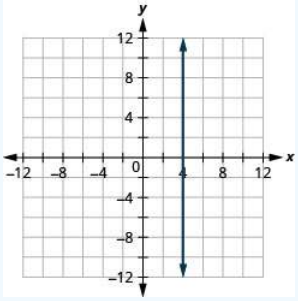
Graph Vertical and Horizontal Lines

In the following exercises, graph each equation.

Exercise 3.2E.49

x=4

Answer



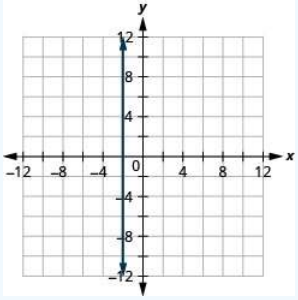
Exercise 3.2E.50

x=3

Exercise 3.2E.51

x=−2

Answer



Exercise 3.2E.52

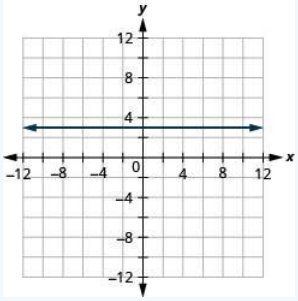
x=−5

Exercise 3.2E.53

y=3

Answer

https://math.libretexts.org/@go/page/54976



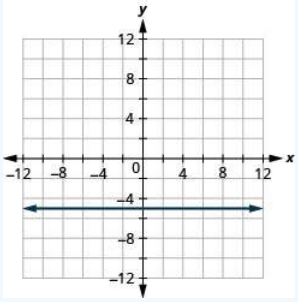
Exercise 3.2E.54

y=1

Exercise 3.2E.55

y=−5

Answer



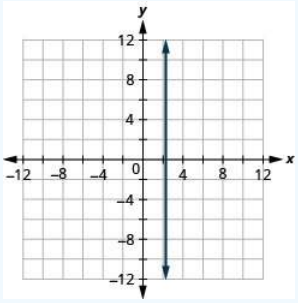
Exercise 3.2E.56

y=−2

Exercise 3.2E.57

$x = \frac{7}{3}$

Answer



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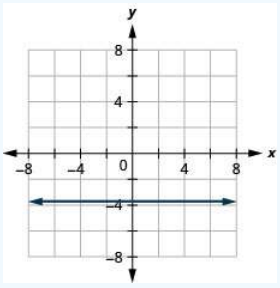
Exercise 3.2E.58

$x = \frac{5}{4}$

Exercise 3.2E.59

$y = -\frac{15}{4}$

Answer



Exercise 3.2E.60

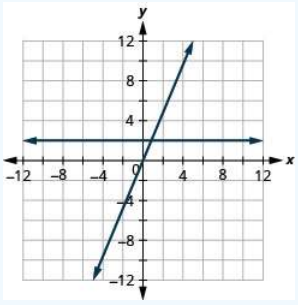
$y = -\frac{5}{3}$

In the following exercises, graph each pair of equations in the same rectangular coordinate system.

Exercise 3.2E.61

y=2x and y=2

Answer



Exercise 3.2E.62

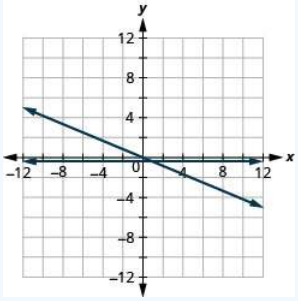
y=5x and y=5

Exercise 3.2E.63

$y = -\frac{1}{2}x$ and $y = -\frac{1}{2}$

Answer

https://math.libretexts.org/@go/page/54976



Exercise 3.2E.64

$y = -\frac{1}{3}x$ and $y = -\frac{1}{3}$

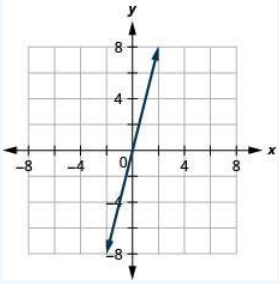
Mixed Practice

In the following exercises, graph each equation.

Exercise 3.2E.65

y=4x

Answer



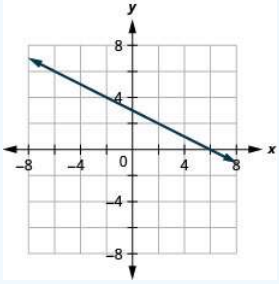
Exercise 3.2E.66

y=2x

Exercise 3.2E.67

$y = -\frac{1}{2}x + 3$

Answer



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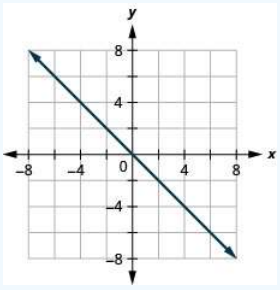
Exercise 3.2E.68

$y = \frac{1}{4}x - 2$

Exercise 3.2E.69

y=−x

Answer



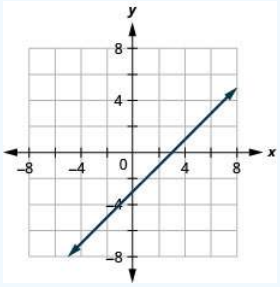
Exercise 3.2E.70

y=x

Exercise 3.2E.71

x−y=3

Answer



Exercise 3.2E.72

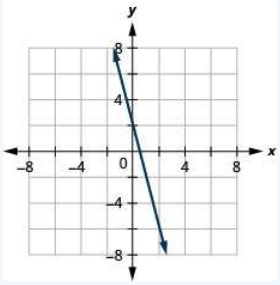
x+y=−5

Exercise 3.2E.73

4x+y=2

Answer

https://math.libretexts.org/@go/page/54976



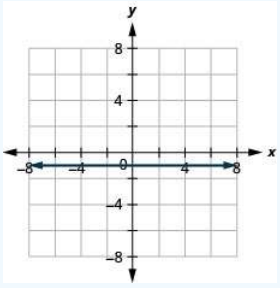
Exercise 3.2E.74

2x+y=6

Exercise 3.2E.75

y=−1

Answer



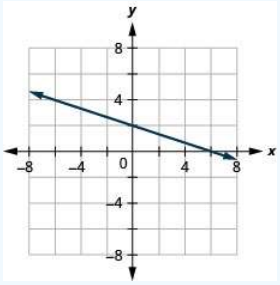
Exercise 3.2E.76

y=5

Exercise 3.2E.77

2x+6y=12

Answer



Exercise 3.2E.78

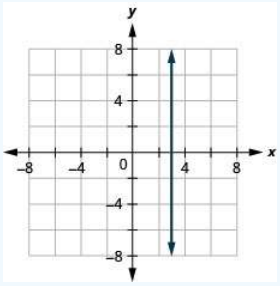
5x+2y=10

https://math.libretexts.org/@go/page/54976

Exercise 3.2E.79

x=3

Answer



Exercise 3.2E.80

x=−4

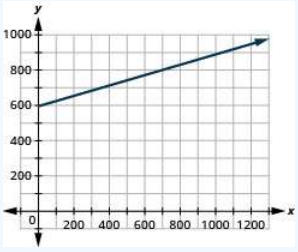
Everyday Math

Exercise 3.2E.81

Motor home cost. The Robinsons rented a motor home for one week to go on vacation. It cost them $594 plus $0.32 per mile to rent the motor home, so the linear equation y=594+0.32x gives the cost, yy, for driving xx miles. Calculate the rental cost for driving 400, 800, and 1200 miles, and then graph the line.

Answer

$722, $850, $978



Exercise 3.2E.82

Weekly earnings. At the art gallery where he works, Salvador gets paid $200 per week plus 15% of the sales he makes, so the equation y=200+0.15x gives the amount, yy, he earns for selling x dollars of artwork. Calculate the amount Salvador earns for selling $900, $1600, and $2000, and then graph the line.

Writing Exercises

Exercise 3.2E.83

Explain how you would choose three x - values to make a table to graph the line $y = \frac{1}{5}x - 2$

Answer

Answers will vary.

? Exercise $3.2E.84$

What is the difference between the equations of a vertical and a horizontal line?

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
recognize the relation between the solutions of an equation and its graph.			
graph a linear equation by plotting points.			
graph vertical and horizontal lines.			

ⓑ After reviewing this checklist, what will you do to become confident for all goals?

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3.3: Graph with Intercepts

Learning Objectives

By the end of this section, you will be able to:

- Identify the x- and y- intercepts on a graph
- Find the x- and y- intercepts from an equation of a line
- Graph a line using the intercepts

Note

Before you get started, take this readiness quiz.

1. Solve: $3 \cdot 0 + 4y = -2$.
   If you missed this problem, review Exercise 2.2.13.

Identify the *x*- and *y*- Intercepts on a Graph

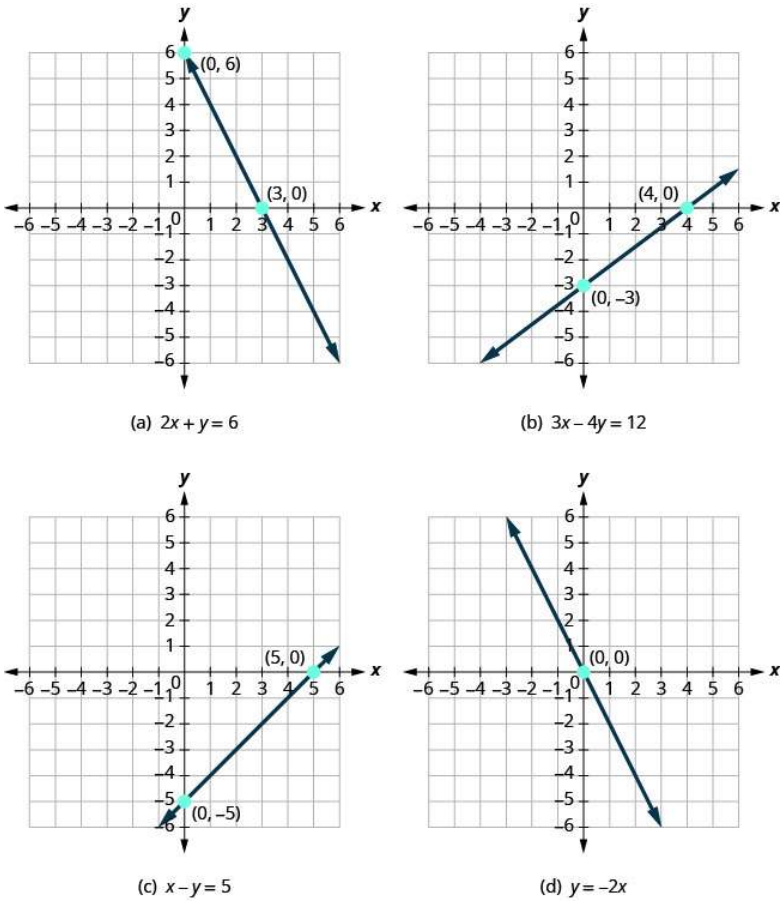
Every linear equation can be represented by a unique line that shows all the solutions of the equation. We have seen that when graphing a line by plotting points, you can use any three solutions to graph. This means that two people graphing the line might use different sets of three points.

At first glance, their two lines might not appear to be the same, since they would have different points labeled. But if all the work was done correctly, the lines should be exactly the same. One way to recognize that they are indeed the same line is to look at where the line crosses the *x*- axis and the *y*- axis. These points are called the *intercepts* of the line.

INTERCEPTS OF A LINE

The points where a line crosses the *x*- axis and the *y*- axis are called the **intercepts of a line**.

Let's look at the graphs of the lines in Figure 3.3.1.



(a) $2x + y = 6$ (b) $3x - 4y = 12$

(c) $x - y = 5$ (d) $y = -2x$

Figure 3.3.1: Examples of graphs crossing the x-negative axis.

First, notice where each of these lines crosses the x axis. See Figure 3.3.1.

Table 3.3.1

Figure	The line crosses the x-axis at:	Ordered pair of this point
Figure (a)	3	(3,0)
Figure (b)	4	(4,0)
Figure (c)	5	(5,0)
Figure (d)	0	(0,0)

Do you see a pattern?

For each row, the *y*- coordinate of the point where the line crosses the *x*- axis is zero. The point where the line crosses the *x*- axis has the form (a,0) and is called the ***x*- intercept of a line**. The *x*- intercept occurs when y is zero. Now, let's look at the points where these lines cross the *y*- axis. See Table 3.3.2.

Table 3.3.2

Figure	The line crosses the x-axis at:	Ordered pair of this point
Figure (a)	6	(0,6)
Figure (b)	−3	(0,−3)
Figure (c)	−5	(0,5)
Figure (d)	0	(0,0)

What is the pattern here?

In each row, the *x*- coordinate of the point where the line crosses the *y*- axis is zero. The point where the line crosses the *y*- axis has the form (0,b) and is called the *y*- *intercept* of the line. The *y*- intercept occurs when x is zero.

X- INTERCEPT AND Y- INTERCEPT OF A LINE

The *x*- intercept is the point (a,0) where the line crosses the *x*- axis.

The *y*- intercept is the point (0,b) where the line crosses the *y*- axis.

	x	*y*
• The *x*-intercept occurs when *y* is zero.	*a*	0
• The *y*-intercept occurs when *x* is zero.	0	*b*

Figure 3.3.2

Example 3.3.1

Find the *x*- and *y*- intercepts on each graph.

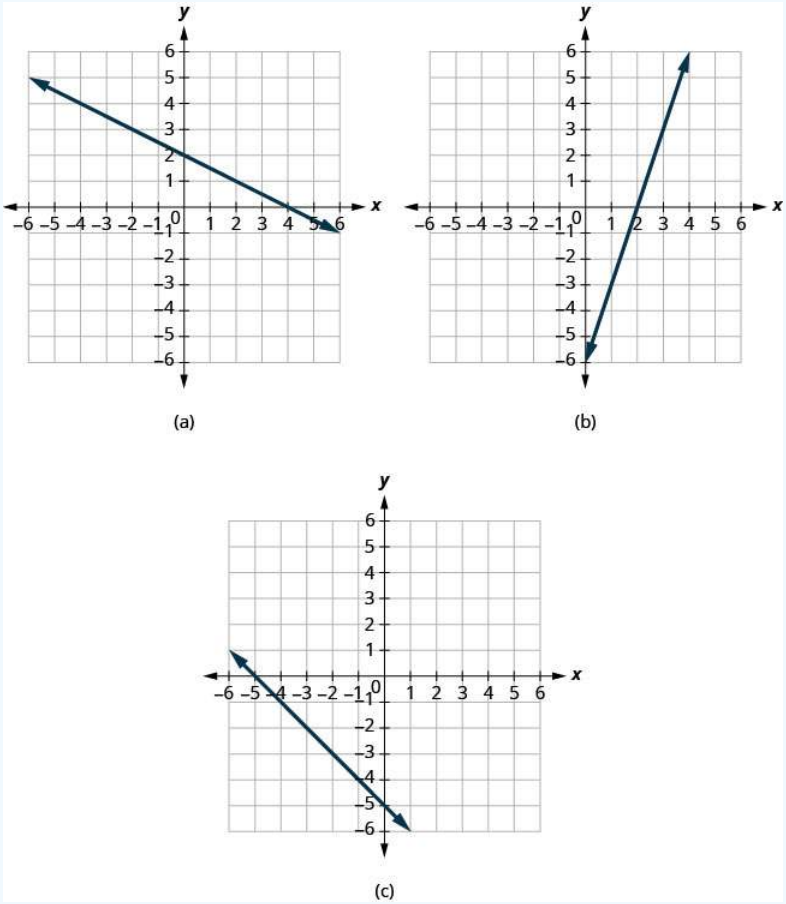


Figure 3.3.3

Solution

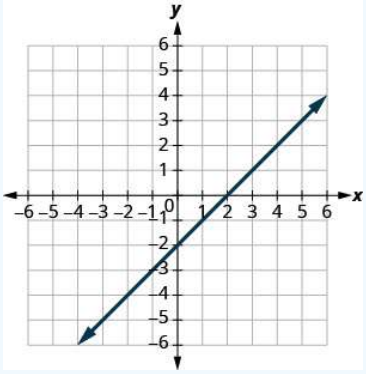
(a) The graph crosses the *x*- axis at the point (4,0). The *x*- intercept is (4,0).
The graph crosses the *y*- axis at the point (0,2). The *y*- intercept is (0,2).

(b) The graph crosses the x- axis at the point (2,0). The x- intercept is (2,0)
The graph crosses the y- axis at the point (0,−6). The y- intercept is (0,−6).

(c) The graph crosses the x- axis at the point (−5,0). The x- intercept is (−5,0).
The graph crosses the y- axis at the point (0,−5). The y- intercept is (0,−5).

Try It 3.3.2

Find the x- and y- intercepts on the graph.

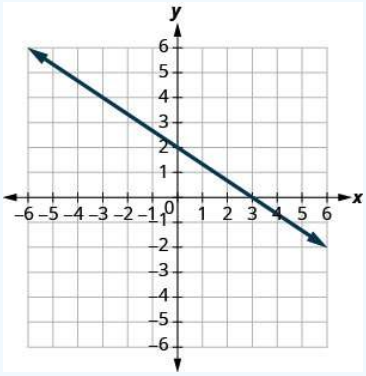


Answer

x- intercept: (2,0); y- intercept: (0,−2)

Try It 3.3.3

Find the x- and y- intercepts on the graph.



Answer

x- intercept: (3,0), y- intercept: (0,2)

Find the x- and y- Intercepts from an Equation of a Line

Recognizing that the ***x*- intercept** occurs when y is zero and that the y- intercept occurs when x is zero, gives us a method to find the intercepts of a line from its equation. To find the x- intercept, let y=0 and solve for x. To find the ***y*- intercept**, let x=0 and solve for y.

X- AND Y- INTERCEPTS FROM THE EQUATION OF A LINE

Use the equation of the line. To find:

- the *x*- intercept of the line, let y=0 and solve for x.
- the *y*- intercept of the line, let x=0 and solve for y.

Example 3.3.4

Find the intercepts of 2x+y=6.

Solution

We will let y=0 to find the *x*- intercept, and let x=0 to find the *y*- intercept. We will fill in the table, which reminds us of what we need to find.

$2x+y=6$	
x	y
	0
0	

x-intercept

y-intercept

To find the *x*- intercept, let y=0.

Table 3.3.3

	$2x+y=6$
Let $y = 0$.	$2x+0=6$
Simplify.	$2x=6$
	$x=3$
The *x*-intercept is	(3, 0)
To find the *y*-intercept, let $x = 0$.	
	$2x+y=6$
Let $x = 0$.	$2\cdot 0+y=6$
Simplify.	$0+y=6$
	$y=6$
The *y*-intercept is	(0, 6)

The intercepts are the points (3,0) and (0,6) as shown in Table 3.3.4

Table 3.3.4

2x+y=6	
x	**y**
3	0
0	6

Try It 3.3.5

Find the intercepts of 3x+y=12.

Answer

x- intercept: (4,0), *y*- intercept: (0,12)

Try It 3.3.6

Find the intercepts of x+4y=8.

Answer

x- intercept: (8,0), *y*- intercept: (0,2)

Example 3.3.7

Find the intercepts of 4x–3y=12.

Solution

To find the *x*-intercept, let $y = 0$.	
	$4x - 3y = 12$
Let $y = 0$.	$4x - 3 \cdot 0 = 12$
Simplify.	$4x - 0 = 12$
	$4x = 12$
	$x = 3$
The *x*-intercept is	(3, 0)
To find the *y*-intercept, let $x = 0$.	
	$4x - 3y = 12$
Let $x = 0$.	$4 \cdot 0 - 3y = 12$
Simplify.	$0 - 3y = 12$
	$-3y = 12$
	$y = -4$
The *y*-intercept is	(0, −4)

Table 3.3.5

The intercepts are the points (3, 0) and (0, −4) as shown in the following table.

Table 3.3.6

4x−3y=12	
x	**y**
3	0
0	−4

Try It 3.3.8

Find the intercepts of 3x–4y=12.

Answer

x- intercept: (4,0), *y*- intercept: (0,−3)

Try It 3.3.9

Find the intercepts of 2x–4y=8.

Answer

x- intercept: (4,0), *y*- intercept: (0,−2)

Graph a Line Using the Intercepts

To graph a linear equation by plotting points, you need to find three points whose coordinates are solutions to the equation. You can use the *x*- and *y*- intercepts as two of your three points. Find the intercepts, and then find a third point to ensure accuracy. Make sure the points line up—then draw the line. This method is often the quickest way to graph a line.

Example 3.3.10: How to Graph a Line Using Intercepts

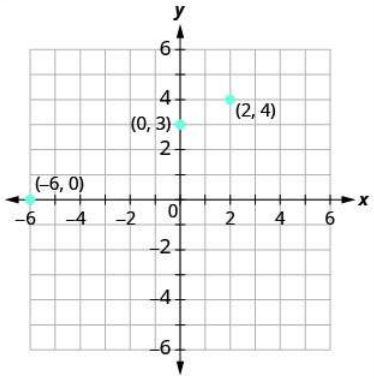
Graph –x+2y=6 using the intercepts.

Solution

Step 1. Find the *x*- and *y*-intercepts of the line. Let $y = 0$ and solve for x. Let $x = 0$ and solve for y.	Find the *x*-intercept.	Let $y = 0$ $-x + 2y = 6$ $-x + 2(0) = 6$ $-x = 6$ $x = -6$ The *x*-intercept is (–6, 0).
	Find the *y*-intercept.	Let $x = 0$ $-x + 2y = 6$ $-0 + 2y = 6$ $2y = 6$ $y = 3$ The *y*-intercept is (0, 3).
Step 2. Find another solution to the equation.	We'll use $x = 2$.	Let $x = 2$ $-x + 2y = 6$ $-2 + 2y = 6$ $2y = 8$ $y = 4$ A third point is (2, 4).

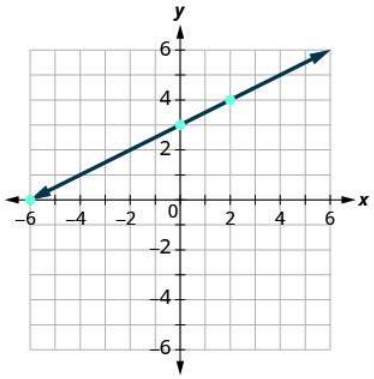
Step 3. Plot the three points. Check that the points line up.

x	y	(x, y)
–6	0	(–6, 0)
0	3	(0, 3)
2	4	(2, 4)



Step 4. Draw the line.

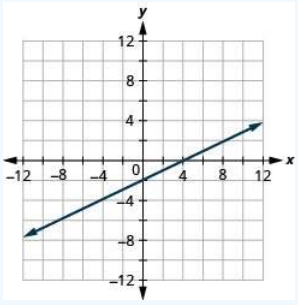
See the graph.



Try It 3.3.11

Graph x–2y=4 using the intercepts.

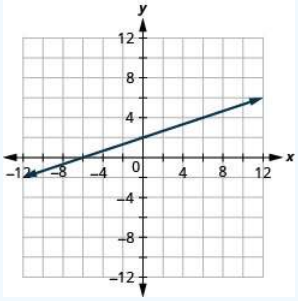
Answer



Try It 3.3.12

Graph –x+3y=6 using the intercepts.

Answer



The steps to graph a linear equation using the intercepts are summarized below.

GRAPH A LINEAR EQUATION USING THE INTERCEPTS.

1. Find the *x*- and *y*- intercepts of the line.
   - Let y=0 and solve for x
   - Let x=0 and solve for y.
2. Find a third solution to the equation.
3. Plot the three points and check that they line up.
4. Draw the line.

Example 3.3.13

Graph 4x–3y=12 using the intercepts.

Solution

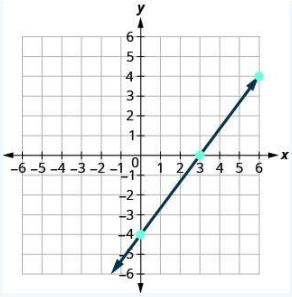
Find the intercepts and a third point.

x-intercept, let $y = 0$	*y*-intercept, let $x = 0$	third point, let $y = 4$
$4x - 3y = 12$	$4x - 3y = 12$	$4x - 3y = 12$
$4x - 3(0) = 12$	$4(0) - 3y = 12$	$4x - 3(4) = 12$
$4x = 12$	$-3y = 12$	$4x - 12 = 12$
$x = 3$	$y = -4$	$4x = 24$
		$x = 6$

We list the points in Table 3.3.7 and show the graph below.

4x−3y=12		
x	**y**	**(x,y)**
3	0	(3,0)
0	−4	(0,−4)
6	4	(6,4)

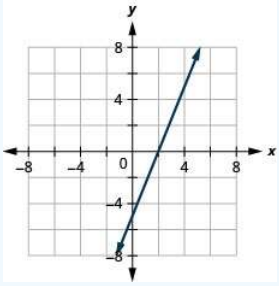
Table 3.3.7



Try It 3.3.14

Graph 5x–2y=10 using the intercepts.

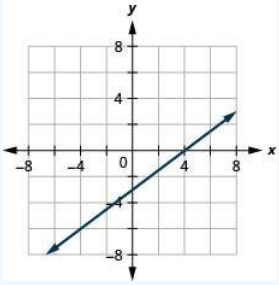
Answer



Try It 3.3.15

Graph 3x–4y=12 using the intercepts.

Answer



Example 3.3.16

Graph y=5x using the intercepts.

Solution

x-intercept	*y*-intercept
Let $y = 0$.	Let $x = 0$.
$y = 5x$	$y = 5x$
$0 = 5x$	$y = 5 \cdot 0$
$0 = x$	$y = 0$
(0, 0)	(0, 0)

This line has only one intercept. It is the point (0,0).

To ensure accuracy we need to plot three points. Since the x- and y- intercepts are the same point, we need *two* more points to graph the line.

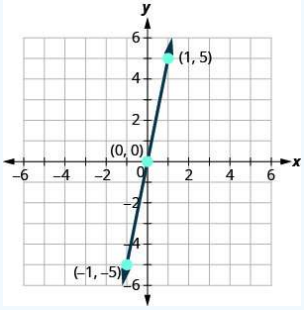
Let $x = 1$.	Let $x = -1$.
$y = 5x$	$y = 5x$
$y = 5 \cdot 1$	$y = 5(-1)$
$y = 5$	$y = -5$

See Table 3.3.8.

y=5x		
x	**y**	**(x,y)**
0	0	(0,0)
1	5	(1,5)
−1	−5	(−1,−5)

Table 3.3.8

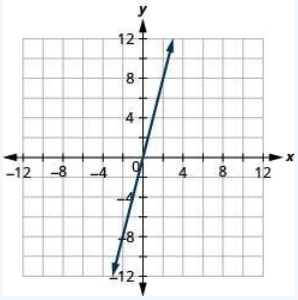
Plot the three points, check that they line up, and draw the line.



Try It 3.3.17

Graph y=4x using the intercepts.

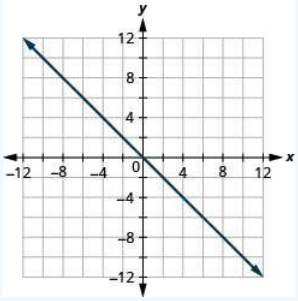
Answer



Try It 3.3.18

Graph y=−x the intercepts.

Answer



Key Concepts

- **Find the *x*- and *y*- Intercepts from the Equation of a Line**
  - Use the equation of the line to find the *x*- intercept of the line, let y=0 and solve for *x*.
  - Use the equation of the line to find the *y*- intercept of the line, let x=0 and solve for *y*.
- **Graph a Linear Equation using the Intercepts**
  1. Find the *x*- and *y*- intercepts of the line.
     Let y=0 and solve for *x*.
     Let x=0 and solve for *y*.
  2. Find a third solution to the equation.
  3. Plot the three points and then check that they line up.
  4. Draw the line.
- **Strategy for Choosing the Most Convenient Method to Graph a Line:**
  - Consider the form of the equation.
  - If it only has one variable, it is a vertical or horizontal line.
    x=a is a vertical line passing through the *x*- axis at a
    y=b is a horizontal line passing through the *y*- axis at b.
  - If *y* is isolated on one side of the equation, graph by plotting points.
  - Choose any three values for *x* and then solve for the corresponding *y*- values.
  - If the equation is of the form ax+by=c, find the intercepts. Find the *x*- and *y*- intercepts and then a third point.

Glossary

intercepts of a line

The points where a line crosses the *x*- axis and the *y*- axis are called the intercepts of the line.

***x*- intercept**

The point (a,0) where the line crosses the *x*- axis; the *x*- intercept occurs when y is zero.

***y*-intercept**

The point (0,b) where the line crosses the *y*- axis; the *y*- intercept occurs when x is zero.

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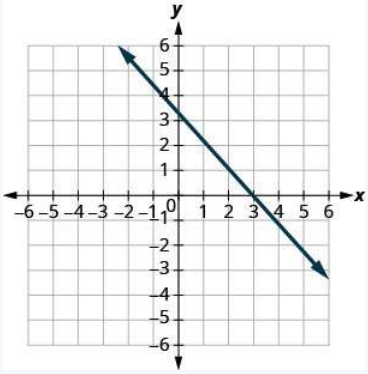
3.3E: Exercises

Practice Makes Perfect

Identify the *x*- and *y*- Intercepts on a Graph

In the following exercises, find the *x*- and *y*- intercepts on each graph.

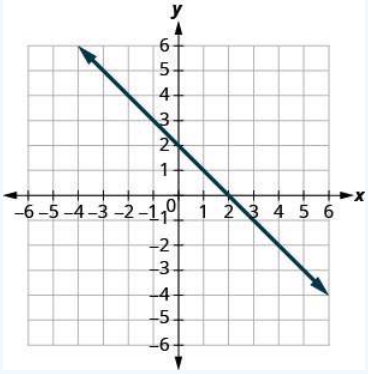
Exercise 3.3E.1



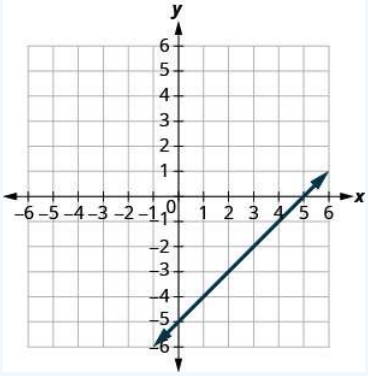
Answer

(3,0),(0,3)

Exercise 3.3E.2



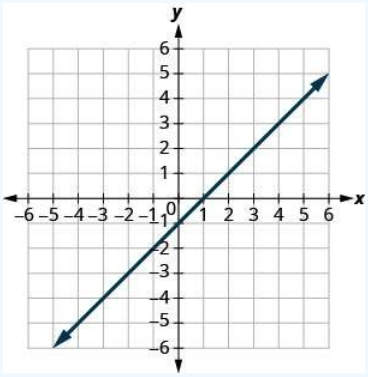
Exercise 3.3E.3



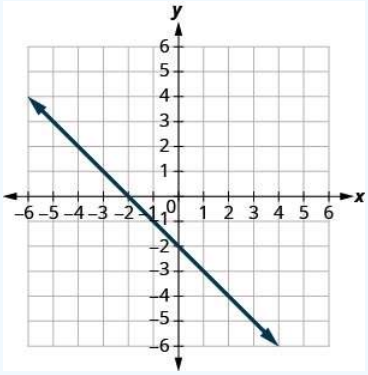
Answer

(5,0),(0,−5)

Exercise 3.3E.4



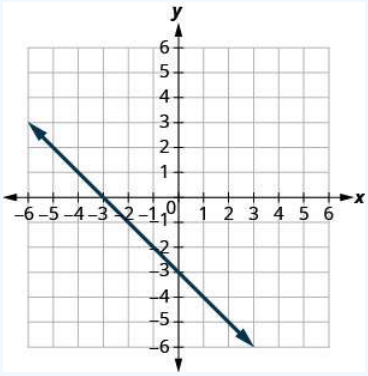
Exercise 3.3E.5



Answer

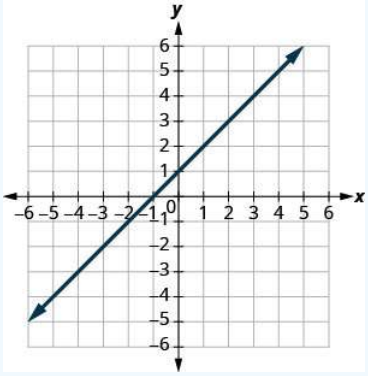
(−2,0),(0,−2)

Exercise 3.3E.6



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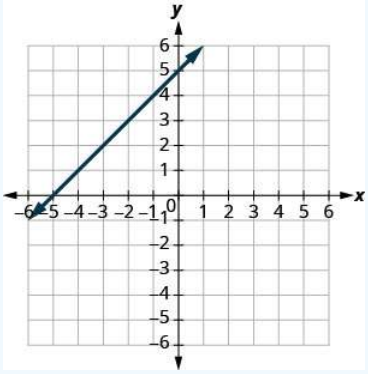
Exercise $3.3E.7$



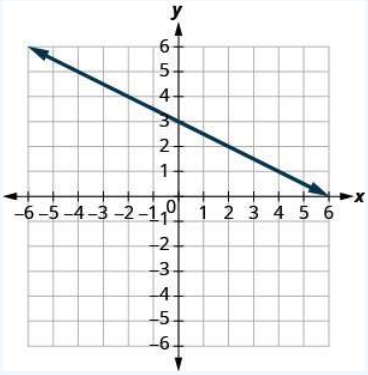
Answer

$(-1,0),(0,1)$

Exercise $3.3E.8$



Exercise $3.3E.9$

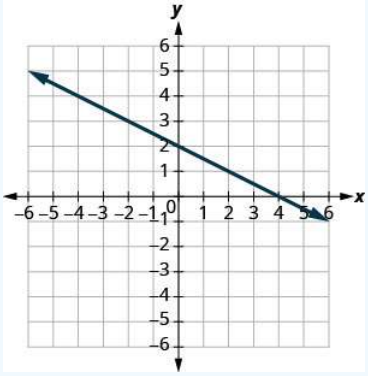


Answer

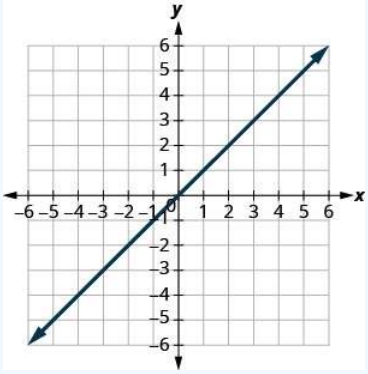
$(6,0),(0,3)$

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Exercise 3.3E. 10



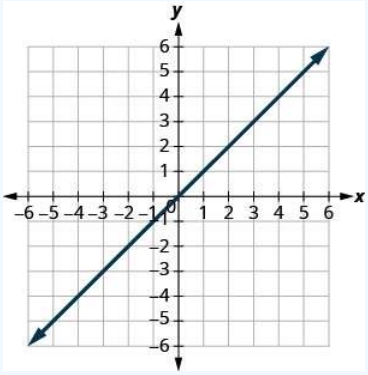
Exercise 3.3E. 11



Answer

(0,0)

Exercise 3.3E. 12



Find the *x*- and *y*- Intercepts from an Equation of a Line

In the following exercises, find the intercepts for each equation.

https://math.libretexts.org/@go/page/54978

Exercise 3.3E.13

x+y=4

Answer

(4,0),(0,4)

Exercise 3.3E.14

x+y=3

Exercise 3.3E.15

x+y=−2

Answer

(−2,0),(0,−2)

Exercise 3.3E.16

x+y=−5

Exercise 3.3E.17

x–y=5

Answer

(5,0),(0,−5)

Exercise 3.3E.18

x–y=1

Exercise 3.3E.19

x–y=−3

Answer

(−3,0),(0,3)

Exercise 3.3E.20

x–y=−4

Exercise 3.3E.21

x+2y=8

Answer

(8,0),(0,4)

Exercise 3.3E.22

x+2y=10

Exercise 3.3E.23

3x+y=6

Answer

(2,0),(0,6)

Exercise 3.3E.24

3x+y=9

Exercise 3.3E.25

x–3y=12

Answer

(12,0),(0,−4)

Exercise 3.3E.26

x–2y=8

Exercise 3.3E.27

4x–y=8

Answer

(2,0),(0,−8)

Exercise 3.3E.28

5x–y=5

Exercise 3.3E.29

2x+5y=10

Answer

(5,0),(0,2)

Exercise 3.3E.30

2x+3y=6

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Exercise 3.3E.31

3x–2y=12

Answer

(4,0),(0,−6)

Exercise 3.3E.32

3x–5y=30

Exercise 3.3E.33

$y = \frac{1}{3}x + 1$

Answer

(-3,0),(0,1)

Exercise 3.3E.34

$y = \frac{1}{4}x - 1$

Exercise 3.3E.35

$y = \frac{1}{5}x + 2$

Answer

(−10,0),(0,2)

Exercise 3.3E.36

$y = \frac{1}{3}x + 4$

Exercise 3.3E.37

y=3x

Answer

(0,0)

Exercise 3.3E.38

y=-2x

Exercise 3.3E.39

y=-4x

Answer

(0,0)

Exercise 3.3E. 40

y=5x

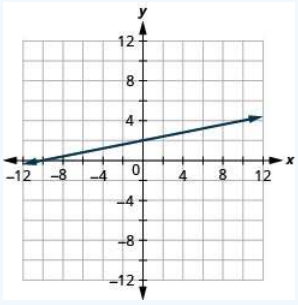
Graph a Line Using the Intercepts

In the following exercises, graph using the intercepts.

Exercise 3.3E. 41

$-x+5y=10$

Answer



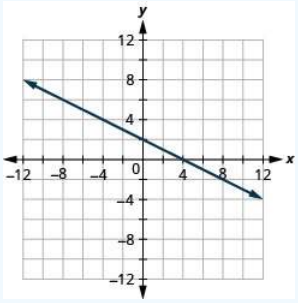
Exercise 3.3E. 42

$-x+4y=8$

Exercise 3.3E. 43

$x+2y=4$

Answer



Exercise 3.3E. 44

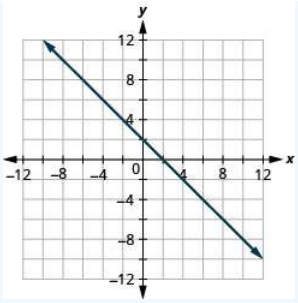
$x+2y=6$

Exercise 3.3E. 45

$x+y=2$

Answer

https://math.libretexts.org/@go/page/54978



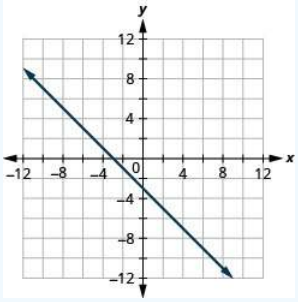
Exercise 3.3E. 46

$x + y = 5$

Exercise 3.3E. 47

$x + y = -3$

Answer



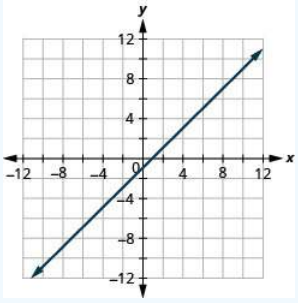
Exercise 3.3E. 48

$x + y = -1$

Exercise 3.3E. 49

$x - y = 1$

Answer



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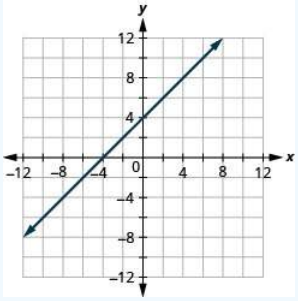
Exercise 3.3E.50

$x - y = 2$

Exercise 3.3E.51

$x - y = -4$

Answer



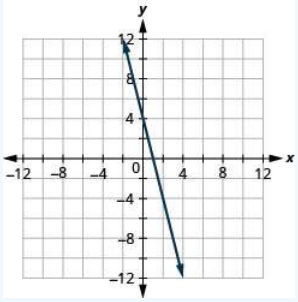
Exercise 3.3E.52

$x - y = -3$

Exercise 3.3E.53

$4x + y = 4$

Answer



Exercise 3.3E.54

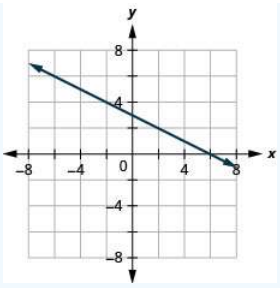
$3x + y = 3$

Exercise 3.3E.55

$2x + 4y = 12$

Answer

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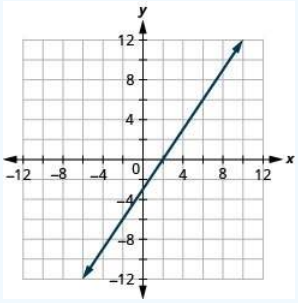
Exercise 3.3E.56

$3x + 2y = 12$

Exercise 3.3E.57

$3x - 2y = 6$

Answer



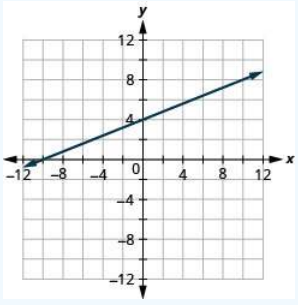
Exercise 3.3E.58

$5x - 2y = 10$

Exercise 3.3E.59

$2x - 5y = -20$

Answer



https://math.libretexts.org/@go/page/54978

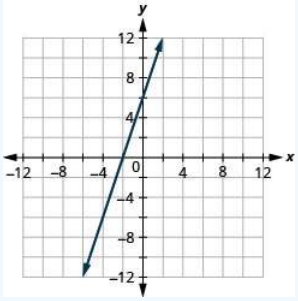
Exercise 3.3E. 60

$3x - 4y = -12$

Exercise 3.3E. 61

$3x - y = -6$

Answer



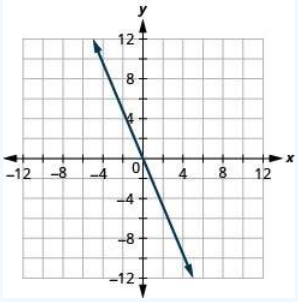
Exercise 3.3E. 62

$2x - y = -8$

Exercise 3.3E. 63

$y = -2x$

Answer



Exercise 3.3E. 64

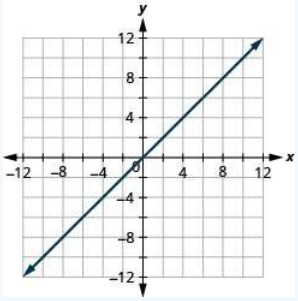
$y = -4x$

Exercise 3.3E. 65

$y = x$

Answer

https://math.libretexts.org/@go/page/54978



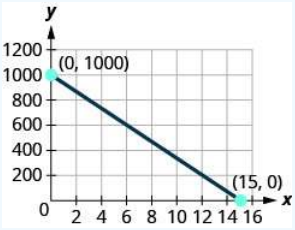
Exercise 3.3E.66

y=3x

Everyday Math

Exercise 3.3E.67

Road trip. Damien is driving from Chicago to Denver, a distance of 1000 miles. The *x*- axis on the graph below shows the time in hours since Damien left Chicago. The *y*- axis represents the distance he has left to drive.



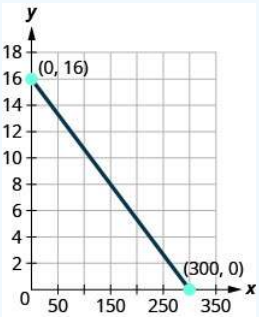
1. Find the *x*- and *y*- intercepts.
2. Explain what the *x*- and *y*- intercepts mean for Damien.

Answer

1. (0,1000),(15,0)
2. At (0,1000), he has been gone 0 hours and has 1000 miles left. At (15,0), he has been gone 15 hours and has 0 miles left to go.

Exercise 3.3E.68

Road trip. Ozzie filled up the gas tank of his truck and headed out on a road trip. The *x*- axis on the graph below shows the number of miles Ozzie drove since filling up. The *y*- axis represents the number of gallons of gas in the truck's gas tank.



1. Find the *x*- and *y*- intercepts.
2. Explain what the *x*- and *y*- intercepts mean for Ozzie.

Writing Exercises

Exercise 3.3E. 69

How do you find the x -intercept of the graph of $3x - 2y = 6$?

Answer

Answers will vary.

Exercise 3.3E. 70

Do you prefer to use the method of plotting points or the method using the intercepts to graph the equation 4x+y=−4? Why?

Exercise 3.3E. 71

Do you prefer to use the method of plotting points or the method using the intercepts to graph the equation $y = \frac{2}{3}x - 2$? Why?

Answer

Answers will vary.

Exercise 3.3E. 72

Do you prefer to use the method of plotting points or the method using the intercepts to graph the equation y=6? Why?

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
identify the *x* and *y* intercepts on a graph.			
find the *x* and *y* intercepts from an equation of a line.			
graph a line using the intercepts.			

ⓑ What does this checklist tell you about your mastery of this section? What steps will you take to improve?

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3.4: Understanding the Slope of a Line

Learning Objectives

- By the end of this section, you will be able to:
- Use geoboards to model slope
- Use $m = \frac{\text{rise}}{\text{run}}$ to find the slope of a line from its graph
- Find the slope of horizontal and vertical lines
- Use the slope formula to find the slope of a line between two points
- Graph a line given a point and the slope
- Solve slope applications

Note

Before you get started, take this readiness quiz.

1. Simplify: $\frac{1-4}{8-2}$.
   If you missed this problem, review Exercise 1.6.31
2. Divide: $\frac{0}{4}, \frac{4}{0}$.
   If you missed this problem, review Exercise 1.10.16.
3. Simplify: $\frac{15}{-3}, \frac{-15}{3}, \frac{-15}{-3}$.
   If you missed this problem, review Exercise 1.6.4.

When you graph linear equations, you may notice that some lines tilt up as they go from left to right and some lines tilt down. Some lines are very steep and some lines are flatter. What determines whether a line tilts up or down or if it is steep or flat?

In mathematics, the 'tilt' of a line is called the *slope* of the line. The concept of slope has many applications in the real world. The pitch of a roof, grade of a highway, and a ramp for a wheelchair are some examples where you literally see slopes. And when you ride a bicycle, you feel the slope as you pump uphill or coast downhill.

In this section, we will explore the concept of slope.

Use Geoboards to Model Slope

A **geoboard** is a board with a grid of pegs on it. Using rubber bands on a geoboard gives us a concrete way to model lines on a coordinate grid. By stretching a rubber band between two pegs on a geoboard, we can discover how to find the slope of a line.

Doing the Manipulative Mathematics activity "Exploring Slope" will help you develop a better understanding of the slope of a line. (Graph paper can be used instead of a geoboard, if needed.)

We'll start by stretching a rubber band between two pegs as shown in Figure 3.4.1.

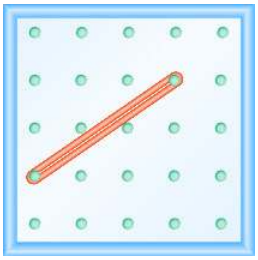


Figure 3.4.1

Doesn't it look like a line?

Now we stretch one part of the rubber band straight up from the left peg and around a third peg to make the sides of a right triangle, as shown in Figure 3.4.2

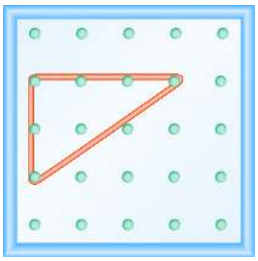


Figure 3.4.2

We carefully make a 90º angle around the third peg, so one of the newly formed lines is vertical and the other is horizontal.

To find the slope of the line, we measure the distance along the vertical and horizontal sides of the triangle. The vertical distance is called the **rise** and the horizontal distance is called the **run**, as shown in Figure 3.4.3.

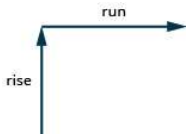


Figure 3.4.3

If our **geoboard** and rubber band look just like the one shown in Figure 3.4.4, the rise is 2. The rubber band goes up 2 units. (Each space is one unit.)

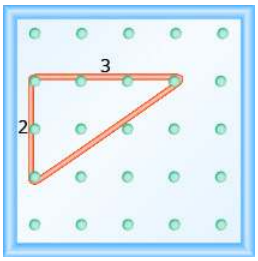


Figure 3.4.4

The rise on this geoboard is 2, as the rubber band goes up two units.

What is the run?

The rubber band goes across 3 units. The run is 3 (see Figure 3.4.4).

The slope of a line is the ratio of the rise to the run. In mathematics, it is always referred to with the letter m.

SLOPE OF A LINE

The **slope of a line** of a line is $m = \frac{\text{rise}}{\text{run}}$.

The **rise** measures the vertical change and the **run** measures the horizontal change between two points on the line.

What is the slope of the line on the geoboard in Figure 3.4.4?

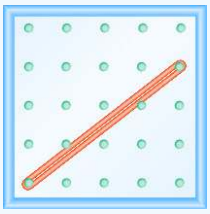
$$m = \frac{\text{rise}}{\text{run}}$$
$$m = \frac{2}{3}$$

The line has slope $\frac{2}{3}$. This means that the line rises 2 units for every 3 units of run.

When we work with geoboards, it is a good idea to get in the habit of starting at a peg on the left and connecting to a peg to the right. If the rise goes up it is positive and if it goes down it is negative. The run will go from left to right and be positive.

Example 3.4.1

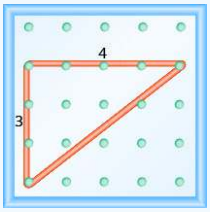
What is the slope of the line on the geoboard shown?



Solution

Use the definition of slope: $m = \frac{\text{rise}}{\text{run}}$.

Start at the left peg and count the spaces up and to the right to reach the second peg.

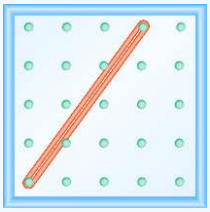


$$\begin{aligned} &\text{The rise is } 3. && m = \tfrac{3}{\text{run}} \\ &\text{The run is } 4. && m = \tfrac{3}{4} \\ & && \text{The slope is } \tfrac{3}{4}. \end{aligned} \tag{3.4.1}$$

This means that the line rises 3 units for every 4 units of run.

Try It 3.4.2

What is the slope of the line on the geoboard shown?

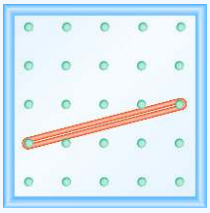


Answer

$\frac{4}{3}$

Try It 3.4.3

What is the slope of the line on the geoboard shown?

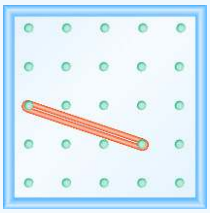


Answer

$\frac{1}{4}$

Example 3.4.4

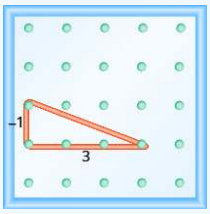
What is the slope of the line on the geoboard shown?



Solution

Use the definition of slope: $m = \frac{\text{rise}}{\text{run}}$.

Start at the left peg and count the units down and to the right to reach the second peg.

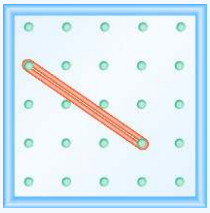


$$\begin{aligned} &\text{The rise is } -1. && m = \tfrac{-1}{\text{run}} \\ &\text{The run is } 3. && m = \tfrac{-1}{3} \\ & && m = -\tfrac{1}{3} \\ & && \text{The slope is } -\tfrac{1}{3} \end{aligned} \tag{3.4.2}$$

This means that the line drops 1 unit for every 3 units of run.

Try It 3.4.5

What is the slope of the line on the geoboard?

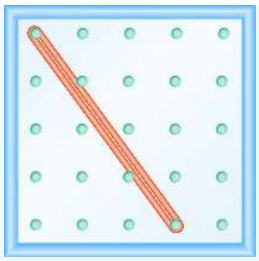


Answer

$-\frac{2}{3}$

Try It 3.4.6

What is the slope of the line on the geoboard?



Answer

$-\frac{4}{3}$

Notice that in Exercise 3.4.1 the slope is positive and in Exercise 3.4.4 the slope is negative. Do you notice any difference in the two lines shown in Figure(a) and Figure(b)?

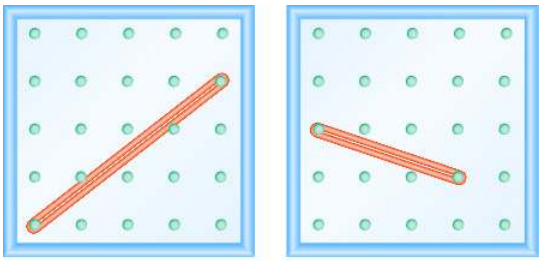


Figure 3.4.5

We 'read' a line from left to right just like we read words in English. As you read from left to right, the line in Figure(a) is going up; it has **positive slope**. The line in Figure(b) is going down; it has **negative slope**.

POSITIVE AND NEGATIVE SLOPES

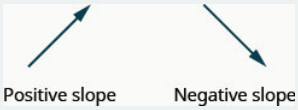


Figure 3.4.6

Example 3.4.7

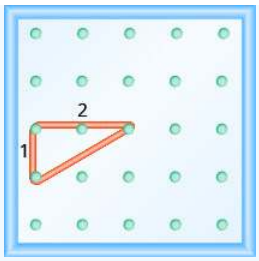
Use a geoboard to model a line with slope $\frac{1}{2}$.

Solution

To model a line on a geoboard, we need the rise and the run.

Use the slope formula.	$m = \frac{\text{rise}}{\text{run}}$
Replace m with $\frac{1}{2}$.	$\frac{1}{2} = \frac{\text{rise}}{\text{run}}$

So, the rise is 1 and the run is 2 .
Start at a peg in the lower left of the geoboard.
Stretch the rubber band up 1 unit, and then right 2 units.

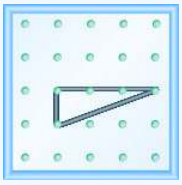


The hypotenuse of the right triangle formed by the rubber band represents a line whose slope is $\frac{1}{2}$.

? Try It 3.4.8

Model the slope $m = \frac{1}{3}$. Draw a picture to show your results.

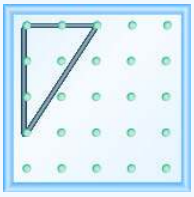
Answer



? Try It 3.4.9

Model the slope $m = \frac{3}{2}$. Draw a picture to show your results.

Answer

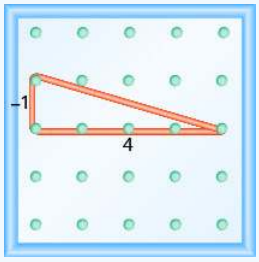


✓ Example 3.4.10

Use a geoboard to model a line with slope $\frac{-1}{4}$.

Solution

Use the slope formula. m = $\frac{\text{rise}}{\text{run}}$.
Replace m with $\frac{-1}{4}$.
So, the rise is -1 and the run is 4.
Since the rise is negative, we choose a starting peg on the upper left that will give us room to count down.
We stretch the rubber band down 1 unit, and then right 4 units.

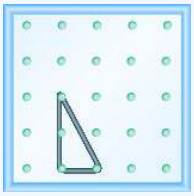


The hypotenuse of the right triangle formed by the rubber band represents a line whose slope is $\frac{-1}{4}$.

Try It 3.4.11

Model the slope $m = -2 = \frac{-2}{1}$. Draw a picture to show your results.

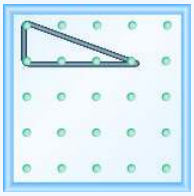
Answer



Try It 3.4.12

Model the slope $m = \frac{-1}{3}$. Draw a picture to show your results.

Answer



Use $m = \frac{\text{rise}}{\text{run}}$ to Find the Slope of a Line from its Graph

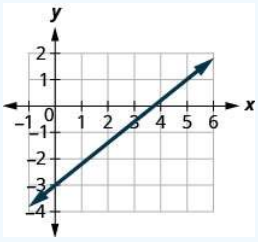
Now, we'll look at some graphs on the xy-coordinate plane and see how to find their slopes. The method will be very similar to what we just modeled on our geoboards.

To find the slope, we must count out the rise and the run. But where do we start?

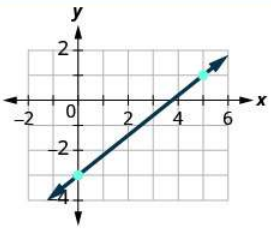
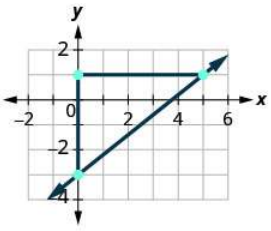
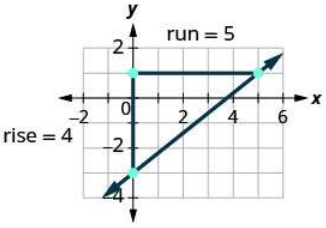
We locate two points on the line whose coordinates are integers. We then start with the point on the left and sketch a right triangle, so we can count the rise and run.

Example 3.4.13:

Find the slope of the line shown.

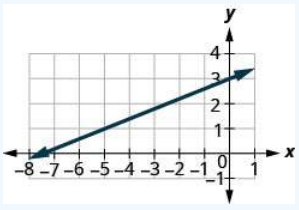


Solution

Step 1. Locate two points on the graph whose coordinates are integers.	Mark (0, –3) and (5, 1).	
Step 2. Starting with the point on the left, sketch a right triangle, going from the first point to the second point.	Starting at (0, –3), sketch a right triangle to (5, 1).	
Step 3. Count the rise and the run on the legs of the triangle.	Count the rise. Count the run.	 The rise is 4. The run is 5.
Step 4. Take the ratio of rise to run to find the slope. $m = \frac{\text{rise}}{\text{run}}$	Use the slope formula. Substitute the values of the rise and run.	$m = \frac{\text{rise}}{\text{run}}$ $m = \frac{4}{5}$ The slope of the line is $\frac{4}{5}$. This means that y increases 4 units as x increases 5 units.

Try It 3.4.14

Find the slope of the line shown.



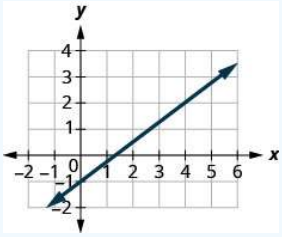
Answer

$\frac{2}{5}$

https://math.libretexts.org/@go/page/54979

Try It 3.4.15

Find the slope of the line shown.



Answer

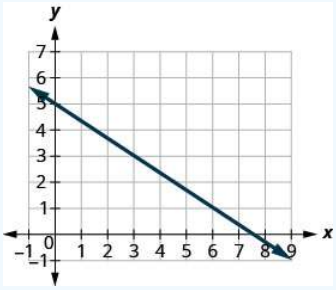
$\frac{3}{4}$

FIND THE SLOPE OF A LINE FROM ITS GRAPH

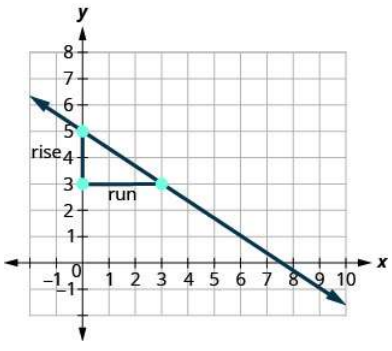
1. Locate two points on the line whose coordinates are integers.
2. Starting with the point on the left, sketch a right triangle, going from the first point to the second point.
3. Count the rise and the run on the legs of the triangle.
4. Take the ratio of rise to run to find the slope, $m = \frac{\text{rise}}{\text{run}}$.

Example 3.4.16

Find the slope of the line shown.



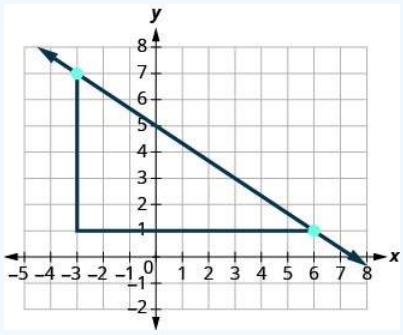
Solution

Locate two points on the graph whose coordinates are integers.	(0,5) and (3,3)
Which point is on the left?	(0,5)
Starting at (0,5), sketch a right triangle to (3,3).	
Count the rise—it is negative.	The rise is −2.
Count the run.	The run is 3.

Use the slope formula.	$m = \frac{\text{rise}}{\text{run}}$
Substitute the values of the rise and run.	$m = \frac{-2}{3}$
Simplify.	$m = -\frac{2}{3}$
	The slope of the line is $-\frac{2}{3}$.

So y increases by 3 units as x decreases by 2 units.

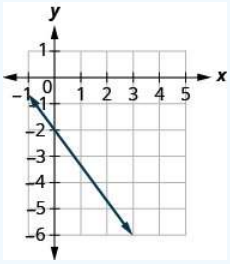
What if we used the points (−3,7) and (6,1) to find the slope of the line?



The rise would be −6 and the run would be 9. Then $m = \frac{-6}{9}$, and that simplifies to $m = -\frac{2}{3}$. Remember, it does not matter which points you use—the slope of the line is always the same.

Try It 3.4.17

Find the slope of the line shown.

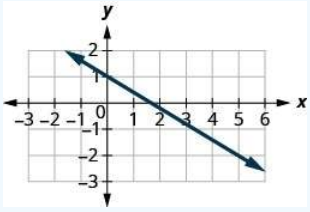


Answer

$-\frac{4}{3}$

Try It 3.4.18

Find the slope of the line shown.



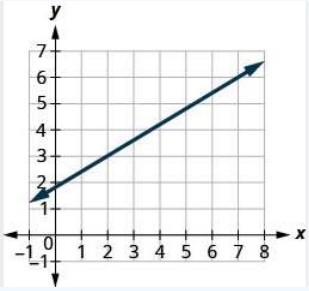
Answer

$-\frac{3}{5}$

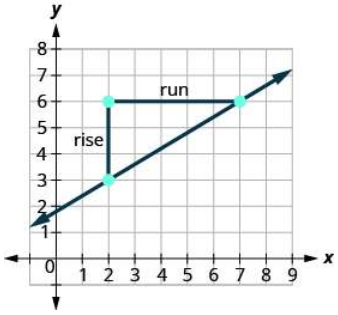
In the last two examples, the lines had *y*-intercepts with integer values, so it was convenient to use the *y*-intercept as one of the points to find the slope. In the next example, the *y*-intercept is a fraction. Instead of using that point, we'll look for two other points whose coordinates are integers. This will make the slope calculations easier.

Example 3.4.19

Find the slope of the line shown.



Solution

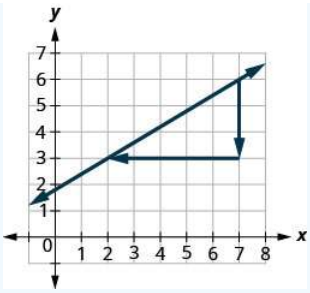
Locate two points on the graph whose coordinates are integers.	(2,3) and (7,6)
Which point is on the left?	(2,3)
Starting at (2,3), sketch a right triangle to (7,6).	
Count the rise.	The rise is 3.
Count the run.	The run is 5.
Use the slope formula.	$m = \frac{\text{rise}}{\text{run}}$
Substitute the values of the rise and run.	$m = \frac{3}{5}$
	The slope of the line is $\frac{3}{5}$.

This means that y increases 5 units as x increases 3 units.

When we used geoboards to introduce the concept of slope, we said that we would always start with the point on the left and count the rise and the run to get to the point on the right. That way the run was always positive and the rise determined whether the slope was positive or negative.

What would happen if we started with the point on the right?

Let's use the points (2,3) and (7,6) again, but now we'll start at (7,6).

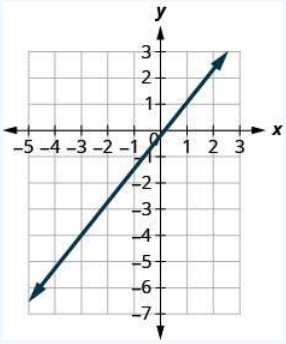


Count the rise.	The rise is -3.
Count the run. It goes from right to left, so it is negative.	The run is-5.
Use the slope formula.	$m = \frac{\text{rise}}{\text{run}}$
Substitute the values of the rise and run.	$m = \frac{-3}{-5}$
	The slope of the line is $\frac{3}{5}$

It does not matter where you start—the slope of the line is always the same.

? Try It 3.4.20

Find the slope of the line shown.

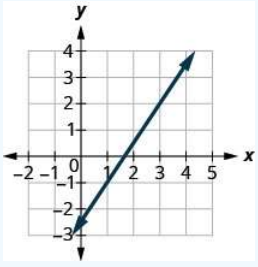


Answer

$\frac{5}{4}$

? Try It 3.4.21

Find the slope of the line shown.



Answer

$\frac{3}{2}$

Find the Slope of Horizontal and Vertical Lines

Do you remember what was special about horizontal and vertical lines? Their equations had just one variable.

Horizontal line $y = b$	**Vertical line** $x = a$
y -coordinates are the same.	x -coordinates are the same.

(3.4.3)

So how do we find the slope of the horizontal line y=4? One approach would be to graph the horizontal line, find two points on it, and count the rise and the run. Let's see what happens when we do this.

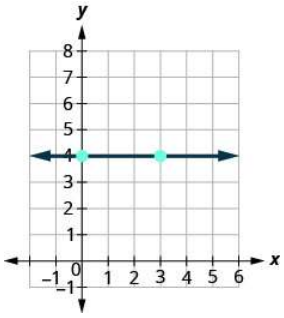


Figure 3.4.7

What is the rise?	The rise is 0.
What is the run?	The run is 3.
	$m = \frac{\text{rise}}{\text{run}}$
	$m = \frac{0}{3}$
What is the slope?	$m = 0$
	The slope of the horizontal line y = 4 is 0.

All horizontal lines have slope 0. When the *y*-coordinates are the same, the rise is 0.

SLOPE OF A HORIZONTAL LINE

The slope of a horizontal line, y=b, is 0.

The floor of your room is horizontal. Its slope is 0. If you carefully placed a ball on the floor, it would not roll away.

Now, we'll consider a vertical line, the line.

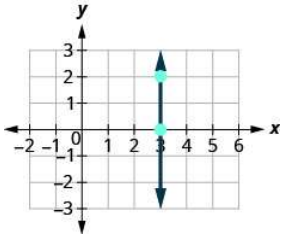


Figure 3.4.8

What is the rise?	The rise is 2.
What is the run?	The run is 0.
	$m = \frac{\text{rise}}{\text{run}}$
What is the slope?	$m = \frac{2}{0}$

But we can't divide by 0. Division by 0 is not defined. So we say that the slope of the vertical line x=3 is undefined.

The slope of any vertical line is undefined. When the *x*-coordinates of a line are all the same, the run is 0.

SLOPE OF A VERTICAL LINE

The slope of a vertical line, x=a, is undefined.

Example 3.4.22

Find the slope of each line:

(a) x=8 (b) y=−5.

Solution

(a) x=8
This is a vertical line.
Its slope is undefined.

(b) y=−5
This is a horizontal line.
It has slope 0.

Try It 3.4.23

Find the slope of the line: x=−4.

Answer

undefined

Try It 3.4.24

Find the slope of the line: y=7.

Answer

0

QUICK GUIDE TO THE SLOPES OF LINES

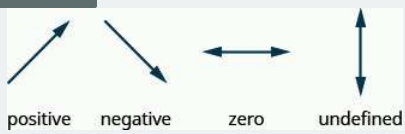


Figure 3.4.9

Remember, we ‘read’ a line from left to right, just like we read written words in English.

Use the Slope Formula to find the Slope of a Line Between Two Points

Doing the Manipulative Mathematics activity “Slope of Lines Between Two Points” will help you develop a better understanding of how to find the slope of a line between two points.

Sometimes we’ll need to find the slope of a line between two points when we don’t have a graph to count out the rise and the run. We could plot the points on grid paper, then count out the rise and the run, but as we’ll see, there is a way to find the slope without graphing. Before we get to it, we need to introduce some algebraic notation.

We have seen that an ordered pair (x,y) gives the coordinates of a point. But when we work with slopes, we use two points. How can the same symbol (x,y) be used to represent two different points? Mathematicians use subscripts to distinguish the points.

$$\begin{aligned}(x_1, y_1) &\quad \text{read } `x \text{ sub } 1, y \text{ sub } 1'\\ (x_2, y_2) &\quad \text{read } `x \text{ sub } 2, y \text{ sub } 2'\end{aligned} \tag{3.4.4}$$

The use of subscripts in math is very much like the use of last name initials in elementary school. Maybe you remember Laura C. and Laura M. in your third grade class?

We will use (x_1, y_1) to identify the first point and (x_2, y_2) to identify the second point.

If we had more than two points, we could use (x_3, y_3), (x_4, y_4), and so on.

Let's see how the rise and run relate to the coordinates of the two points by taking another look at the slope of the line between the points (2,3) and (7,6).

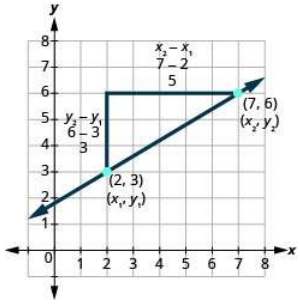


Figure 3.4.10

Since we have two points, we will use subscript notation, $\begin{pmatrix} x_1, y_1 \\ 2, 3 \end{pmatrix} \begin{pmatrix} x_2, y_2 \\ 7, 6 \end{pmatrix}$.

On the graph, we counted the rise of 3 and the run of 5.

Notice that the rise of 3 can be found by subtracting the *y*-coordinates 6 and 3.

$$3 = 6 - 3 \tag{3.4.5}$$

And the run of 5 can be found by subtracting the *x*-coordinates 7 and 2.

$$5 = 7 - 2 \tag{3.4.6}$$

We know $m = \frac{\text{rise}}{\text{run}}$. So $m = \frac{3}{5}$.

We rewrite the rise and run by putting in the coordinates $m = \frac{6-3}{7-2}$

But 6 is y_2, the *y*-coordinate of the second point and 3 is y_1, the *y*-coordinate of the first point.

So we can rewrite the slope using subscript notation. $m = \frac{y2-y1}{7-2}$

Also, 7 is x_2, the *x*-coordinate of the second point and 2 is x_1, the *x*-coordinate of the first point.

So, again, we rewrite the slope using subscript notation. $m = \frac{y2-y1}{x2-x1}$

We've shown that $m = \frac{y2-y1}{x2-x1}$ is really another version of $m = \frac{\text{rise}}{\text{run}}$. We can use this formula to find the slope of a line when we have two points on the line.

SLOPE FORMULA

The slope of the line between two points (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1} \tag{3.4.7}$$

This is the **slope formula**.

The slope is:

$$\begin{array}{c} y \text{ of the second point minus } y \text{ of the first point} \\ \text{over} \\ x \text{ of the second point minus } x \text{ of the first point.} \end{array} \tag{3.4.8}$$

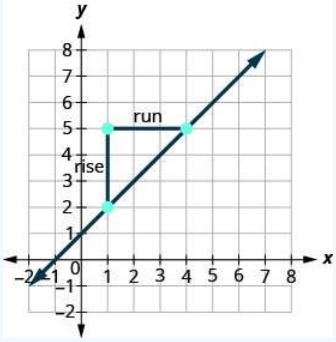
Example 3.4.25

Use the **slope formula** to find the slope of the line between the points (1,2) and (4,5).

Solution

We'll call (1,2) point #1 and (4,5) point #2.	$\begin{pmatrix} x_1, y_1 \\ 1, 2 \end{pmatrix} \begin{pmatrix} x_2, y_2 \\ 4, 5 \end{pmatrix}$
Use the slope formula.	$m = \frac{y_2 - y_1}{x_2 - x_1}$
Substitute the values.	
y of the second point minus y of the first point	$m = \frac{5-2}{x_2 - x_1}$
x of the second point minus x of the first point	$m = \frac{5-2}{4-1}$
Simplify the numerator and the denominator.	$m = \frac{3}{3}$
Simplify.	$m = 1$

Let's confirm this by counting out the slope on a graph using $m = \frac{\text{rise}}{\text{run}}$.



It doesn't matter which point you call point #1 and which one you call point #2. The slope will be the same. Try the calculation yourself.

Try It 3.4.26

Use the slope formula to find the slope of the line through the points: (8,5) and (6,3).

Answer

1

Try It 3.4.27

Use the slope formula to find the slope of the line through the points: (1,5) and (5,9).

Answer

1

Example 3.4.28

Use the slope formula to find the slope of the line through the points (−2,−3) and (−7,4).

Solution

We'll call (-2, -3) point #1 and (-7,4) point #2. $\begin{pmatrix} x_1, y_1 \\ -2, -3 \end{pmatrix} \begin{pmatrix} x_2, y_2 \\ -7, 4 \end{pmatrix}$

Use the slope formula. $m = \frac{y_2 - y_1}{x_2 - x_1}$

Substitute the values.

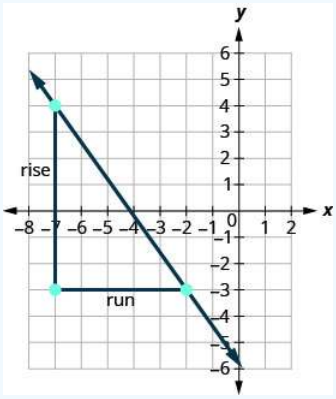
y of the second point minus y of the first point $m = \frac{4-(-3)}{x_2 - x_1}$

x of the second point minus x of the first point $m = \frac{4-(-3)}{-7-(-2)}$

Simplify the numerator and the denominator. $m = \frac{7}{-5}$

Simplify. $m = -\frac{7}{5}$

Let's verify this slope on the graph shown.



$$m = \frac{\text{rise}}{\text{run}}$$
$$m = \frac{-7}{5}$$
$$m = -\frac{7}{5}$$

Try It 3.4.29

Use the slope formula to find the slope of the line through the points: (−3,4) and (2,−1).

Answer

-1

Try It 3.4.30

Use the slope formula to find the slope of the line through the pair of points: (−2,6) and (−3,−4).

Answer

10

Graph a Line Given a Point and the Slope

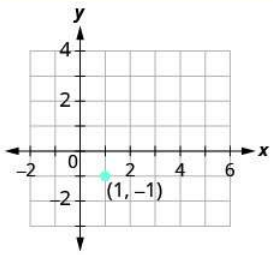
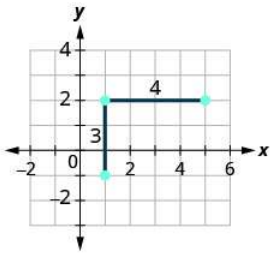
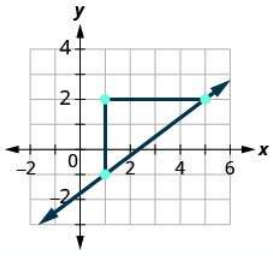
Up to now, in this chapter, we have graphed lines by plotting points, by using intercepts, and by recognizing horizontal and vertical lines.

One other method we can use to graph lines is called the **point–slope method**. We will use this method when we know one point and the slope of the line. We will start by plotting the point and then use the definition of slope to draw the graph of the line.

Example 3.4.31

Graph the line passing through the point $(1,-1)$ whose slope is $m = \frac{3}{4}$.

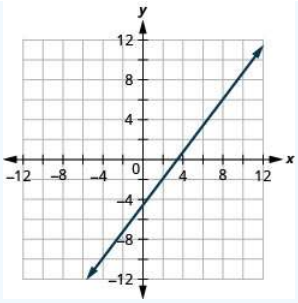
Solution

Step 1. Plot the given point.	Plot (1, –1).	
Step 2. Use the slope formula $m = \frac{\text{rise}}{\text{run}}$ to identify the rise and the run.	Identify the rise and the run.	$m = \frac{3}{4}$ $\frac{\text{rise}}{\text{run}} = \frac{3}{4}$ rise = 3 run = 4
Step 3. Starting at the given point, count out the rise and run to mark the second point.	Start at (1, –1) and count the rise and the run. Up 3 units, right 4 units.	
Step 4. Connect the points with a line.	Connect the two points with a line.	

Try It 3.4.32

Graph the line passing through the point $(2,-2)$ with the slope $m = \frac{4}{3}$.

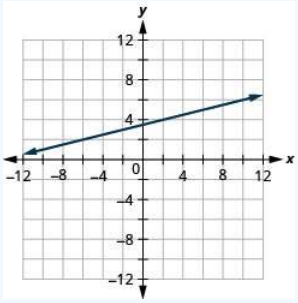
Answer



Try It 3.4.33

Graph the line passing through the point (−2,3) with the slope $m = \frac{1}{4}$.

Answer



GRAPH A LINE GIVEN A POINT AND THE SLOPE.

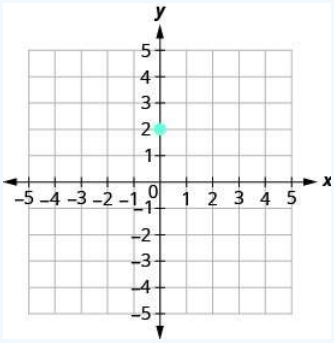
1. Plot the given point.
2. Use the slope formula $m = \frac{\text{rise}}{\text{rise}}$ to identify the rise and the run.
3. Starting at the given point, count out the rise and run to mark the second point.
4. Connect the points with a line.

Example 3.4.34

Graph the line with *y*-intercept 2 whose slope is $m = -\frac{2}{3}$.

Solution

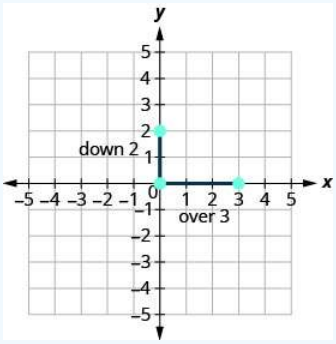
Plot the given point, the *y*-intercept, (0,2).



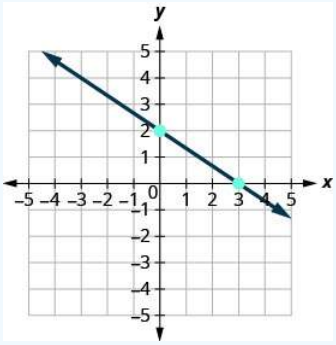
Identify the rise and the run. $m = -\frac{2}{3}$

$$\frac{\text{rise}}{\text{run}} = \frac{-2}{3}$$
$$\text{rise} = -2$$
$$\text{run} = 3$$

Count the rise and the run. Mark the second point.



Connect the two points with a line.

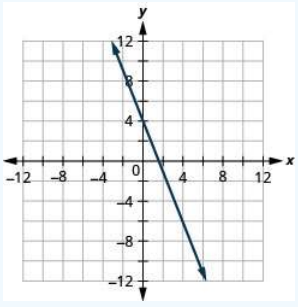


You can check your work by finding a third point. Since the slope is $m = -\frac{2}{3}$, it can be written as $m = \frac{2}{-3}$. Go back to (0,2) and count out the rise, 2, and the run, −3.

? Try It 3.4.35

Graph the line with the *y*-intercept 4 and slope $m = -\frac{5}{2}$.

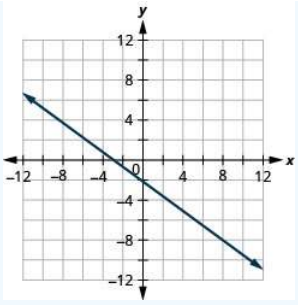
Answer



? Try It 3.4.36

Graph the line with the *x*-intercept −3 and slope $m = -\frac{3}{4}$.

Answer

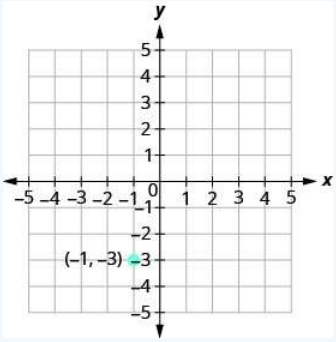


Example 3.4.37

Graph the line passing through the point $(-1,-3)$ whose slope is $m=4$.

Solution

Plot the given point.

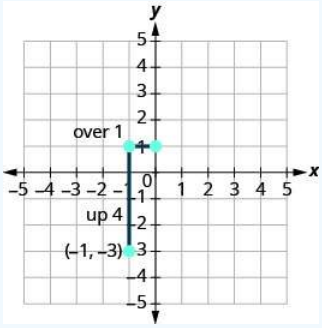


Identify the rise and the run. $m = 4$

Write 4 as a fraction. $\frac{\text{rise}}{\text{run}} = \frac{4}{1}$

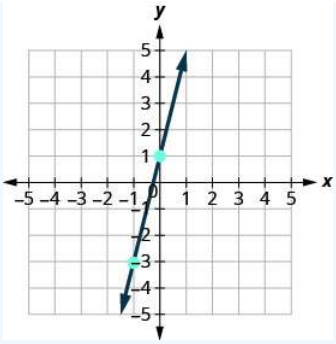
$\text{rise} = 4 \quad \text{run} = 1$

Count the rise and run and mark the second point.



Connect the two points with a line.

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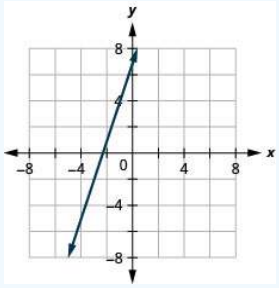


You can check your work by finding a third point. Since the slope is m=4, it can be written as $m = \frac{-4}{-1}$. Go back to (−1,−3) and count out the rise, −4, and the run, −1.

Try It 3.4.38

Graph the line with the point (−2,1) and slope m=3.

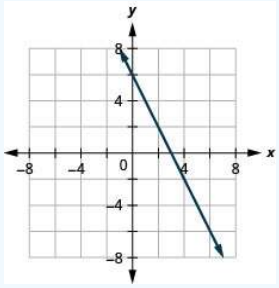
Answer



Try It 3.4.39

Graph the line with the point (4,−2) and slope m=−2.

Answer

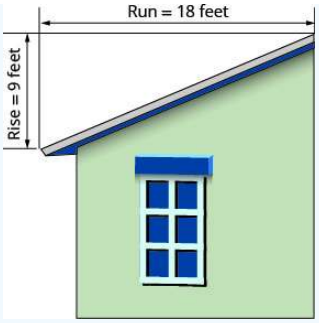


Solve Slope Applications

At the beginning of this section, we said there are many applications of slope in the real world. Let's look at a few now.

Example 3.4.40

The 'pitch' of a building's roof is the slope of the roof. Knowing the pitch is important in climates where there is heavy snowfall. If the roof is too flat, the weight of the snow may cause it to collapse. What is the slope of the roof shown?



Solution

Use the slope formula.	$m = \frac{\text{rise}}{\text{rise}}$
Substitute the values for rise and run.	$m = \frac{9}{18}$
Simplify.	$m = \frac{1}{2}$

The slope of the roof is $\frac{1}{2}$.

The roof rises 1 foot for every 2 feet of horizontal run.

Try It 3.4.41

Use Exercise 3.4.40, substituting the rise = 14 and run = 24.

Answer

$\frac{7}{12}$

Try It 3.4.42

Use Exercise 3.4.40, substituting rise = 15 and run = 36.

Answer

$\frac{5}{12}$

Example 3.4.43

Have you ever thought about the sewage pipes going from your house to the street? They must slope down $\frac{1}{4}$ inch per foot in order to drain properly. What is the required slope?



Solution

Use the slope formula.	$m = \frac{\text{rise}}{\text{run}}$
	$m = \frac{-\frac{1}{4}\text{inch}}{1\text{ foot}}$
	$m = \frac{-\frac{1}{4}\text{ inch}}{12\text{ inches}}$
Simplify.	$m = -\frac{1}{48}$
	The slope of the pipe is $-\frac{1}{48}$

The pipe drops 1 inch for every 48 inches of horizontal run.

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? Try It 3.4.44

Find the slope of a pipe that slopes down $\frac{1}{3}$ inch per foot.

Answer

$-\frac{1}{36}$

? Try It 3.4.45

Find the slope of a pipe that slopes down $\frac{3}{4}$ inch per yard.

Answer

$-\frac{1}{48}$

Access these online resources for additional instruction and practice with understanding slope of a line.

- Practice Slope with a Virtual Geoboard
- Small, Medium, and Large Virtual Geoboards
- Explore Area and Perimeter with a Geoboard

Key Concepts

- **Find the Slope of a Line from its Graph using $m = \frac{\text{rise}}{\text{run}}$**
  1. Locate two points on the line whose coordinates are integers.
  2. Starting with the point on the left, sketch a right triangle, going from the first point to the second point.
  3. Count the rise and the run on the legs of the triangle.
  4. Take the ratio of rise to run to find the slope.
- **Graph a Line Given a Point and the Slope**
  1. Plot the given point.
  2. Use the slope formula $m = \frac{\text{rise}}{\text{run}}$ to identify the rise and the run.
  3. Starting at the given point, count out the rise and run to mark the second point.
  4. Connect the points with a line.
- **Slope of a Horizontal Line**
  - The slope of a horizontal line, y=b, is 0.
- **Slope of a vertical line**
  - The slope of a vertical line, x=a, is undefined

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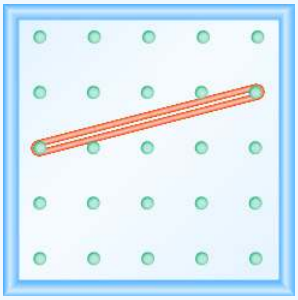
3.4E: Exercises

Practice Makes Perfect

Use Geoboards to Model Slope

In the following exercises, find the slope modeled on each geoboard.

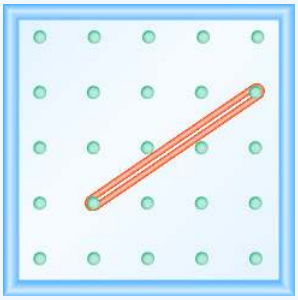
Exercise 3.4E.1



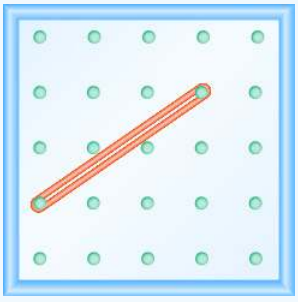
Answer

$\frac{1}{4}$

Exercise 3.4E.2



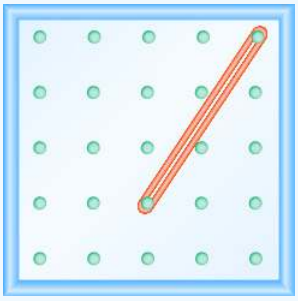
Exercise 3.4E.3



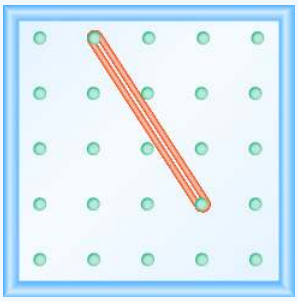
Answer

$\frac{2}{3}$

Exercise 3.4E.4



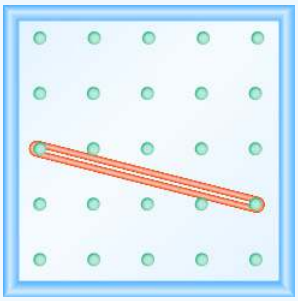
Exercise 3.4E.5



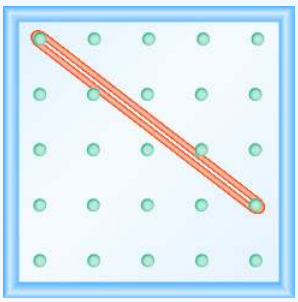
Answer

$\frac{-3}{2} = -\frac{3}{2}$

Exercise 3.4E.6



Exercise 3.4E.7

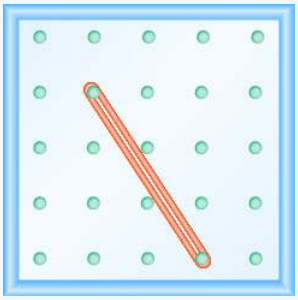


Answer

$-\frac{2}{3}$

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Exercise 3.4E.8

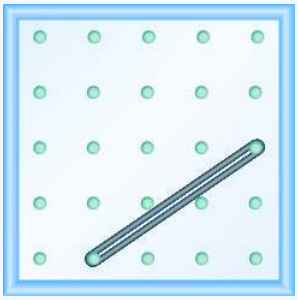


In the following exercises, model each slope. Draw a picture to show your results.

Exercise 3.4E.9

$\frac{2}{3}$

Answer



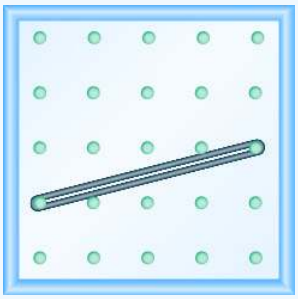
Exercise 3.4E.10

$\frac{3}{4}$

Exercise 3.4E.11

$\frac{1}{4}$

Answer



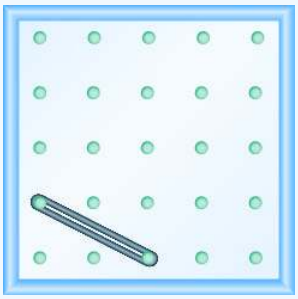
Exercise 3.4E.12

$\frac{4}{3}$

Exercise 3.4E.13

$-\frac{1}{2}$

Answer



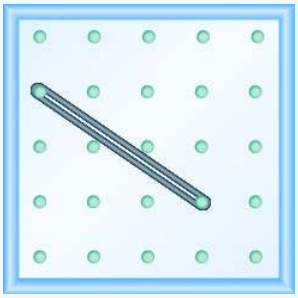
Exercise 3.4E.14

$-\frac{3}{4}$

Exercise 3.4E.15

$-\frac{2}{3}$

Answer



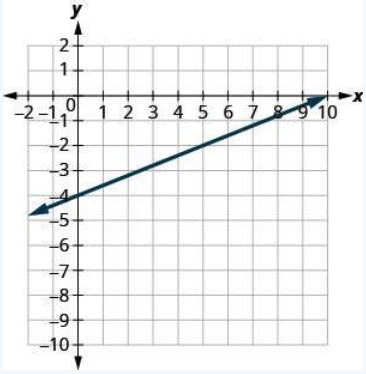
Exercise 3.4E.16

$-\frac{3}{2}$

Use $m = \frac{rise}{run}$ to find the Slope of a Line from its Graph

In the following exercises, find the slope of each line shown.

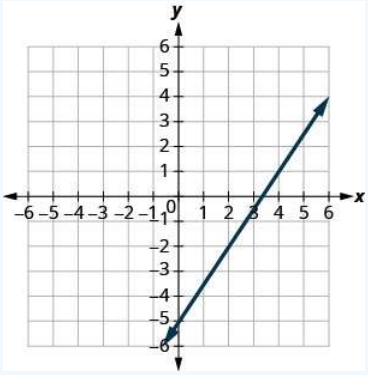
Exercise 3.4E.17



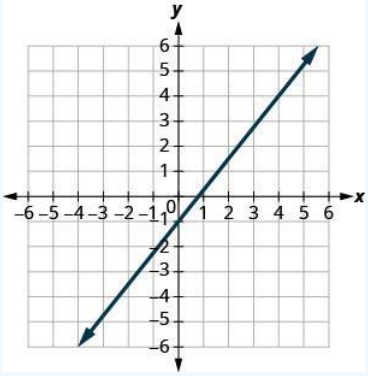
Answer

$\frac{2}{5}$

Exercise 3.4E.18



Exercise 3.4E.19

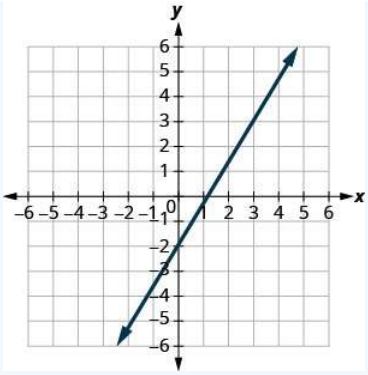


Answer

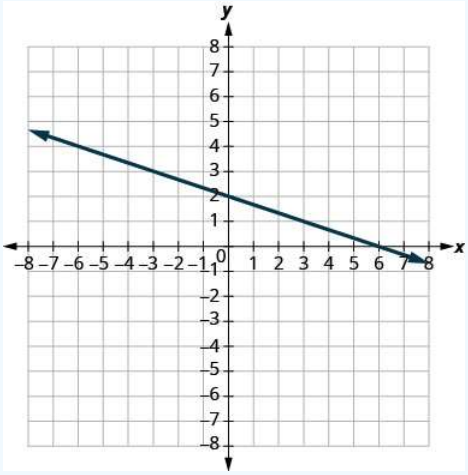
$\frac{5}{4}$

Exercise 3.4E.20

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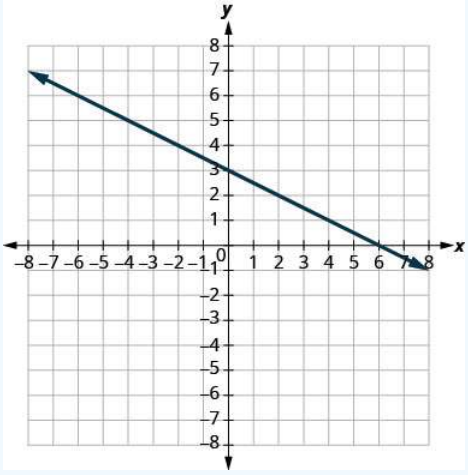
Exercise 3.4E.21



Answer

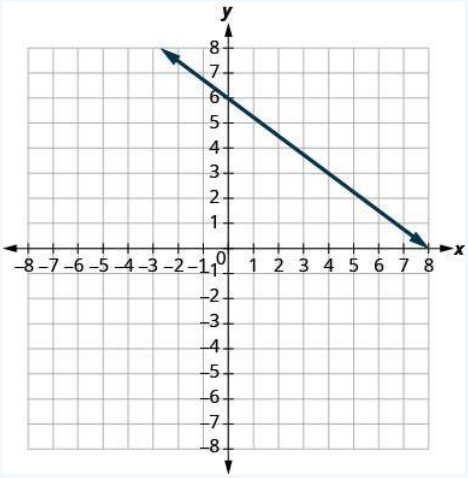
$-\frac{1}{3}$

Exercise 3.4E.22



https://math.libretexts.org/@go/page/54980

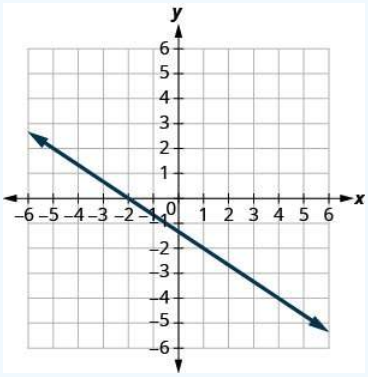
Exercise 3.4E. 23



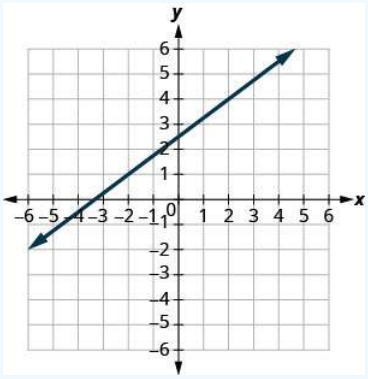
Answer

$-\frac{3}{4}$

Exercise 3.4E. 24



Exercise 3.4E. 25

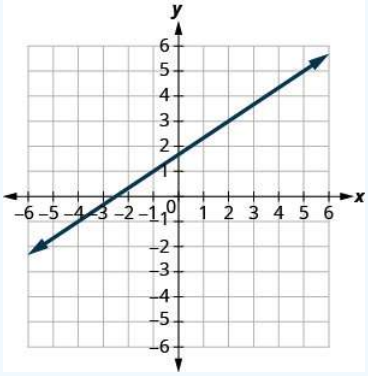


Answer

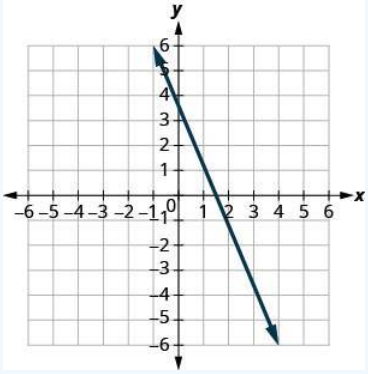
$\frac{3}{4}$

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Exercise 3.4E. 26



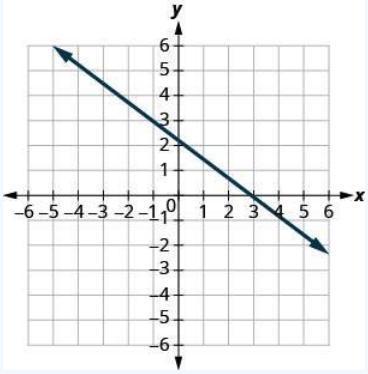
Exercise 3.4E. 27



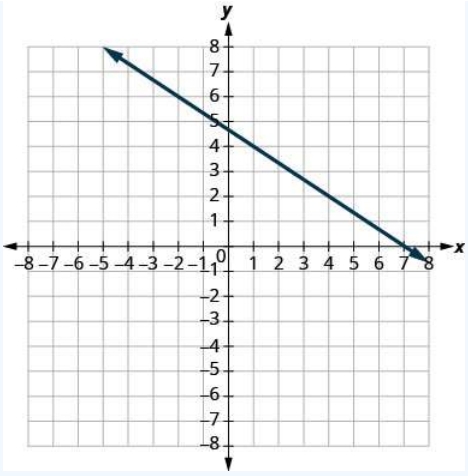
Answer

$-\frac{5}{2}$

Exercise 3.4E. 28



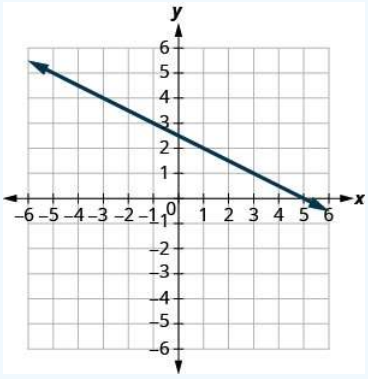
Exercise 3.4E. 29



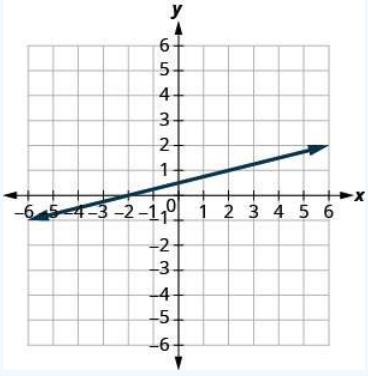
Answer

$-\frac{2}{3}$

? Exercise $3.4E.30$



? Exercise $3.4E.31$

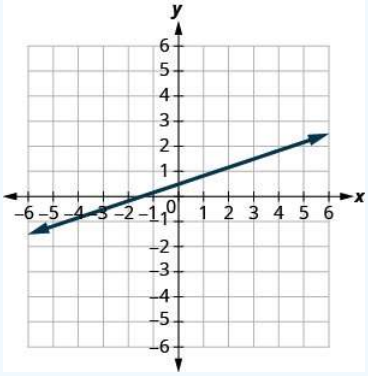


Answer

$\frac{1}{4}$

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Exercise 3.4E.32



Find the Slope of Horizontal and Vertical Lines

In the following exercises, find the slope of each line.

Exercise 3.4E.33

y=3

Answer

0

Exercise 3.4E.34

y=1

Exercise 3.4E.35

x=4

Answer

undefined

Exercise 3.4E.36

x=2

Exercise 3.4E.37

y=−2

Answer

0

Exercise 3.4E.38

y=−3

Exercise 3.4E. 39

x=−5

Answer

undefined

Exercise 3.4E. 40

x=−4

Use the Slope Formula to find the Slope of a Line between Two Points

In the following exercises, use the slope formula to find the slope of the line between each pair of points.

Exercise 3.4E. 41

(1,4),(3,9)

Answer

$\frac{5}{2}$

Exercise 3.4E. 42

(2,3),(5,7)

Exercise 3.4E. 43

(0,3),(4,6)

Answer

$\frac{3}{4}$

Exercise 3.4E. 44

(0,1),(5,4)

Exercise 3.4E. 45

(2,5),(4,0)

Answer

$-\frac{5}{2}$

Exercise 3.4E. 46

(3,6),(8,0)

Exercise 3.4E. 47

(−3,3),(4,−5)

Answer

$-\frac{8}{7}$

Exercise 3.4E.48

(−2,4),(3,−1)

Exercise 3.4E.49

(−1,−2),(2,5)

Answer

$\frac{7}{3}$

Exercise 3.4E.50

(−2,−1),(6,5)

Exercise 3.4E.51

(4,−5),(1,−2)

Answer

−1

Exercise 3.4E.52

(3,−6),(2,−2)

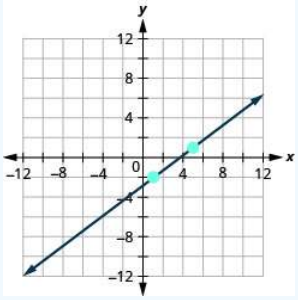
Graph a Line Given a Point and the Slope

In the following exercises, graph each line with the given point and slope.

Exercise 3.4E.53

$(1,-2); m=\frac{3}{4}$

Answer



Exercise 3.4E.54

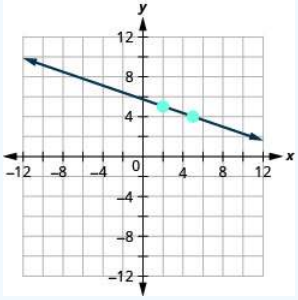
$(1,-1); m=\frac{2}{3}$

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Exercise 3.4E.55

$(2,5); m = -\frac{1}{3}$

Answer



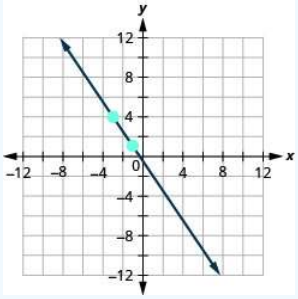
Exercise 3.4E.56

$(1,4); m = -\frac{1}{2}$

Exercise 3.4E.57

$(-3,4); m = -\frac{3}{2}$

Answer



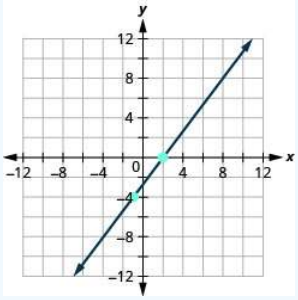
Exercise 3.4E.58

$(-2,5); m = -\frac{5}{4}$

Exercise 3.4E.59

$(-1,-4); m = \frac{4}{3}$

Answer



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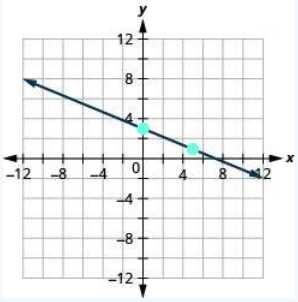
Exercise 3.4E. 60

$(-3, -5); m = \frac{3}{2}$

Exercise 3.4E. 61

y -intercept $3; m = -\frac{2}{5}$

Answer



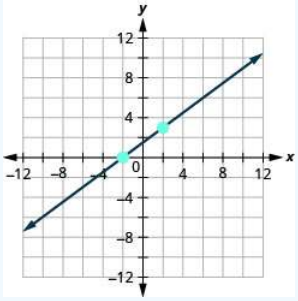
Exercise 3.4E. 62

y -intercept $5; m = -\frac{4}{3}$

Exercise 3.4E. 63

x -intercept $-2; m = \frac{3}{4}$

Answer



Exercise 3.4E. 64

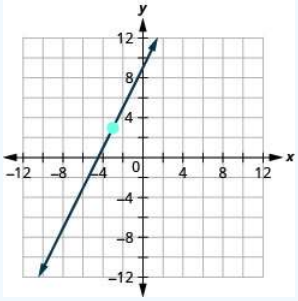
x -intercept $-1; m = \frac{1}{5}$

Exercise 3.4E. 65

$(-3, 3); m = 2$

Answer

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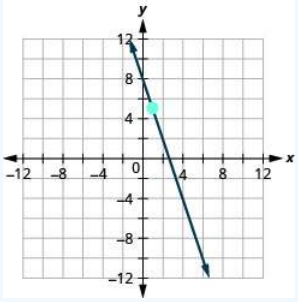
Exercise 3.4E.66

$(-4, 2); m = 4$

Exercise 3.4E.67

$(1, 5); m = -3$

Answer



Exercise 3.4E.68

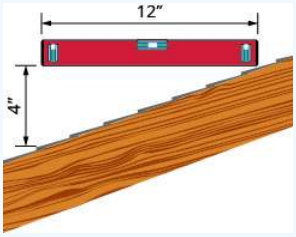
$(2, 3); m = -1$

Everyday Math

Exercise 3.4E.69

Slope of a roof. An easy way to determine the slope of a roof is to set one end of a 12 inch level on the roof surface and hold it level. Then take a tape measure or ruler and measure from the other end of the level down to the roof surface. This will give you the slope of the roof. Builders, sometimes, refer to this as pitch and state it as an "x 12 pitch" meaning $\frac{x}{12}$, where x is the measurement from the roof to the level—the rise. It is also sometimes stated as an "x-in-12 pitch".

1. What is the slope of the roof in this picture?
2. What is the pitch in construction terms?

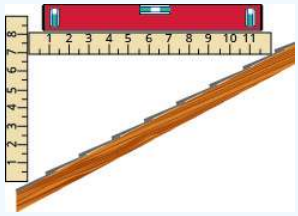


Answer

1. $\frac{1}{3}$
2. 4 12 pitch or 4-in-12 pitch

Exercise 3.4E.70

The slope of the roof shown here is measured with a 12” level and a ruler. What is the slope of this roof?



Exercise 3.4E.71

Road grade. A local road has a grade of 6%. The grade of a road is its slope expressed as a percent. Find the slope of the road as a fraction and then simplify. What rise and run would reflect this slope or grade?

Answer

$\frac{3}{50}$; rise $= 3$, run $= 50$

Exercise 3.4E.72

Highway grade. A local road rises 2 feet for every 50 feet of highway.

1. What is the slope of the highway?
2. The grade of a highway is its slope expressed as a percent. What is the grade of this highway?

Exercise 3.4E.73

Wheelchair ramp. The rules for wheelchair ramps require a maximum 1-inch rise for a 12-inch run.

1. How long must the ramp be to accommodate a 24-inch rise to the door?
2. Create a model of this ramp.

Answer

1. 288 inches (24 feet)
2. Models will vary.

Exercise 3.4E.74

Wheelchair ramp. A 1-inch rise for a 16-inch run makes it easier for the wheelchair rider to ascend a ramp.

1. How long must a ramp be to easily accommodate a 24-inch rise to the door?
2. Create a model of this ramp.

Writing Exercises

Exercise 3.4E.75

What does the sign of the slope tell you about a line?

Answer

When the slope is a positive number the line goes up from left to right. When the slope is a negative number the line goes down from left to right.

Exercise 3.4E.76

How does the graph of a line with slope $m = \frac{1}{2}$ differ from the graph of a line with slope $m = 2$?

Exercise 3.4E.77

Why is the slope of a vertical line "undefined"?

Answer

A vertical line has 0 run and since division by 0 is undefined the slope is undefined.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it
use geoboards to model slope.			
use $m = \frac{rise}{run}$ to find the slope of a line from its graph.			
find the slope of horizontal and vertical lines.			
use the slope formula to find the slope of a line between two points.			
graph a line given a point and the slope.			
solve slope applications.			

ⓑ On a scale of 1–10, how would you rate your mastery of this section in light of your responses on the checklist? How can you improve this?

Glossary

geoboard

A geoboard is a board with a grid of pegs on it.

negative slope

A negative slope of a line goes down as you read from left to right.

positive slope

A positive slope of a line goes up as you read from left to right.

rise

The rise of a line is its vertical change.

run

The run of a line is its horizontal change.

slope formula

The slope of the line between two points (x_1, y_1) and (x_2, y_2) is $m = \frac{y_2 - y_1}{x_2 - x_1}$.

slope of a line

The slope of a line is $m = \frac{\text{rise}}{\text{run}}$. The rise measures the vertical change and the run measures the horizontal change.

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3.5: Use the Slope–Intercept Form of an Equation of a Line

Learning Objectives

By the end of this section, you will be able to:

- Recognize the relation between the graph and the slope–intercept form of an equation of a line
- Identify the slope and y-intercept form of an equation of a line
- Graph a line using its slope and intercept
- Choose the most convenient method to graph a line
- Graph and interpret applications of slope–intercept
- Use slopes to identify parallel lines
- Use slopes to identify perpendicular lines

Note

Before you get started, take this readiness quiz.

1. Add: $\frac{x}{4}+\frac{1}{4}$.
   If you missed this problem, review Exercise 1.7.1.
2. Find the reciprocal of $\frac{3}{7}$.
   If you missed this problem, review Exercise 1.6.19.
3. Solve $2x-3y=12$ for y.
   If you missed this problem, review Exercise 2.6.16.

Recognize the Relation Between the Graph and the Slope–Intercept Form of an Equation of a Line

We have graphed linear equations by plotting points, using intercepts, recognizing horizontal and vertical lines, and using the point–slope method. Once we see how an equation in slope–intercept form and its graph are related, we'll have one more method we can use to graph lines.

In **Graph Linear Equations in Two Variables**, we graphed the line of the equation $y=12x+3$ by plotting points. See Figure 3.5.1. Let's find the slope of this line.

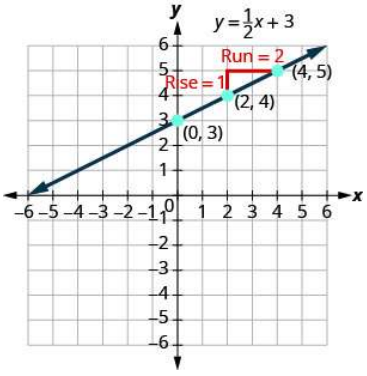


Figure 3.5.1

The red lines show us the rise is 1 and the run is 2. Substituting into the slope formula:

$$m=\frac{\text{rise}}{\text{run}}$$
$$m=\frac{1}{2}$$

What is the y-intercept of the line? The y-intercept is where the line crosses the y-axis, so y-intercept is $(0,3)$. The equation of this line is:

$y = \frac{1}{2}x + 3$

Notice, the line has:

slope $m = \frac{1}{2}$ and y-intercept $(0, 3)$

When a linear equation is solved for y, the coefficient of the x-term is the slope and the constant term is the y-coordinate of the y-intercept. We say that the equation $y = \frac{1}{2}x + 3$ is in slope–intercept form.

$m = \frac{1}{2}$; y-intercept is $(0, 3)$

$y = \frac{1}{2}x + 3$

$y = mx + b$

SLOPE-INTERCEPT FORM OF AN EQUATION OF A LINE

The **slope–intercept form** of an equation of a line with slope m and y-intercept, $(0, b)$ is,

$$y = mx + b \tag{3.5.1}$$

Sometimes the slope–intercept form is called the "y-form."

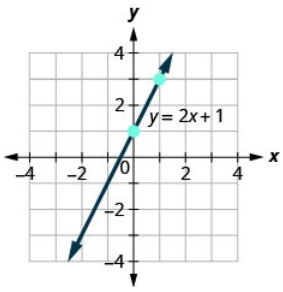
Example 3.5.1

Use the graph to find the slope and y-intercept of the line, $y = 2x + 1$.

Compare these values to the equation $y = mx + b$.

Solution

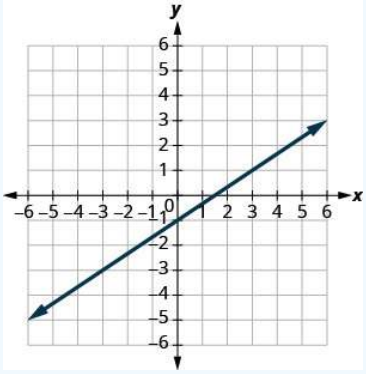
To find the slope of the line, we need to choose two points on the line. We'll use the points $(0, 1)$ and $(1, 3)$.

	
Find the rise and run.	$m = \frac{\text{rise}}{\text{run}}$
	$m = \frac{2}{1}$
	$m = 2$
Find the y-intercept of the line.	The y-intercept is the point $(0, 1)$.
We found slope $m = 2$ and y-intercept $(0, 1)$.	$y = 2x + 1$ $y = mx + b$

The slope is the same as the coefficient of x and the y-coordinate of the y-intercept is the same as the constant term.

Try It 3.5.2

Use the graph to find the slope and y-intercept of the line $y = \frac{2}{3}x - 1$. Compare these values to the equation $y = mx + b$.

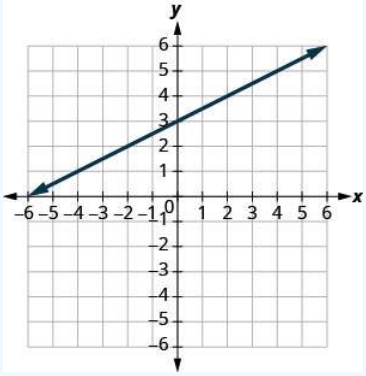


Answer

slope $m = \frac{2}{3}$ and y-intercept $(0, -1)$

Try It 3.5.3

Use the graph to find the slope and y-intercept of the line $y = \frac{1}{2}x + 3$. Compare these values to the equation $y = mx + b$.



Answer

slope $m = \frac{1}{2}$ and y-intercept $(0, 3)$

Identify the Slope and y-Intercept From an Equation of a Line

In **Understand Slope of a Line**, we graphed a line using the slope and a point. When we are given an equation in slope–intercept form, we can use the y-intercept as the point, and then count out the slope from there. Let's practice finding the values of the slope and y-intercept from the equation of a line.

Example 3.5.4

Identify the slope and y-intercept of the line with equation $y = -3x + 5$.

Solution

We compare our equation to the slope–intercept form of the equation.

	$y = mx + b$
Write the equation of the line.	$y = -3x + 5$
Identify the slope.	$m = -3$
Identify the y-intercept.	y-intercept is $(0, 5)$

Try It 3.5.5

Identify the slope and y-intercept of the line $y = \frac{2}{5}x - 1$.

Answer

slope $\frac{2}{5}$; y-intercept $(0,-1)$

Try It 3.5.6

Identify the slope and y-intercept of the line $y = -\frac{4}{3}x + 1$.

Answer

slope $-\frac{4}{3}$; y-intercept $(0,1)$

When an equation of a line is not given in slope–intercept form, our first step will be to solve the equation for y.

Example 3.5.7

Identify the slope and y-intercept of the line with equation $x + 2y = 6$.

Solution

This equation is not in slope–intercept form. In order to compare it to the slope–intercept form we must first solve the equation for y.

Solve for y.	$x + 2y = 6$
Subtract x from each side.	$2y = -x + 6$
Divide both sides by 2.	$\frac{2y}{2} = \frac{-x+6}{2}$
Simplify.	$\frac{2y}{2} = \frac{-x}{2} + \frac{6}{2}$
(Remember: $\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$)	
Simplify.	$y = -\frac{1}{2}x + 3$
Write the slope–intercept form of the equation of the line.	$y = mx + b$
Write the equation of the line.	$y = -\frac{1}{2}x + 3$
Identify the slope.	$m = -\frac{1}{2}$
Identify the y-intercept.	y-intercept is $(0, 3)$

Try It 3.5.8

Identify the slope and y-intercept of the line $x + 4y = 8$.

Answer

slope $-\frac{1}{4}$; y-intercept $(0,2)$

Try It 3.5.9

Identify the slope and y-intercept of the line $3x + 2y = 12$.

Answer

slope $-\frac{3}{2}$; y-intercept (0,6)

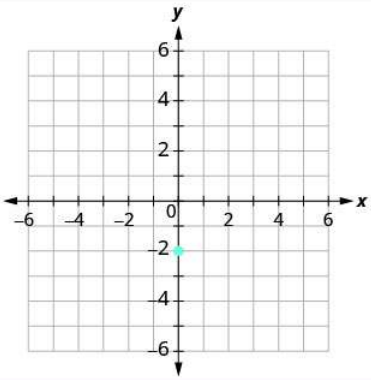
Graph a Line Using its Slope and Intercept

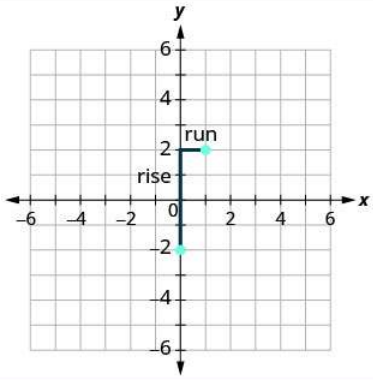
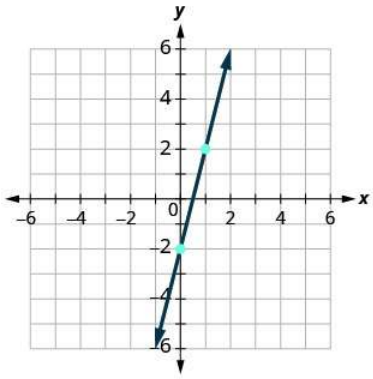
Now that we know how to find the slope and y-intercept of a line from its equation, we can graph the line by plotting the y-intercept and then using the slope to find another point.

Example 3.5.10: How to Graph a Line Using its Slope and Intercept

Graph the line of the equation $y = 4x - 2$ using its slope and y-intercept.

Solution

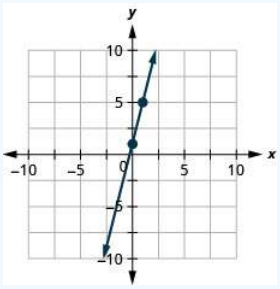
Step 1. Find the slope–intercept form of the equation.	This equation is in slope–intercept form.	$y = 4x - 2$
Step 2. Identify the slope and y-intercept.	Use $y = mx + b$ Find the slope. Find the y-intercept.	$y = mx + b$ $y = 4x + (-2)$ $m = 4$ $b = -2$, $(0, -2)$
Step 3. Plot the y-intercept.	Plot $(0, -2)$.	
Step 4. Use the slope formula $m = \frac{\text{rise}}{\text{run}}$ to identify the rise and the run.	Identify the rise and the run.	$m = 4$ $\frac{\text{rise}}{\text{run}} = \frac{4}{1}$ rise $= 4$ run $= 1$

Step 5. Starting at the *y*-intercept, count out the rise and run to mark the second point.	Start at (0, –2) and count the rise and the run. Up 4, right 1.	
Step 6. Connect the points with a line.	Connect the two points with a line.	

? Try It 3.5.11

Graph the line of the equation $y = 4x + 1$ using its slope and y-intercept.

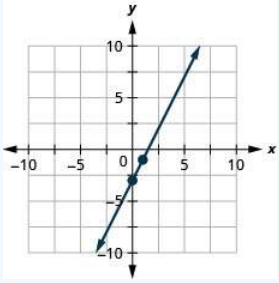
Answer



? Try It 3.5.12

Graph the line of the equation $y = 2x - 3$ using its slope and y-intercept.

Answer



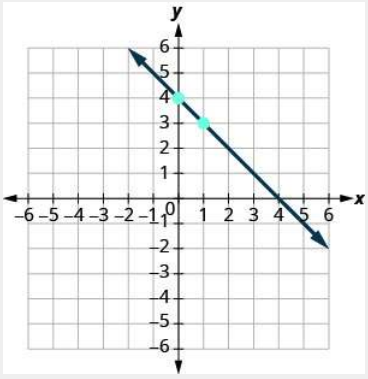
GRAPH A LINE USING ITS SLOPE AND y-INTERCEPT.

1. Find the slope-intercept form of the equation of the line.
2. Identify the slope and y-intercept.
3. Plot the y-intercept.
4. Use the slope formula $\frac{\text{rise}}{\text{run}}$ to identify the rise and the run.
5. Starting at the y-intercept, count out the rise and run to mark the second point.
6. Connect the points with a line.

Example 3.5.13

Graph the line of the equation $y = -x + 4$ using its slope and y-intercept.

Solution

	$y = mx + b$
The equation is in slope–intercept form.	$y = -x + 4$
Identify the slope and y-intercept.	$m = -1$
	y-intercept is $(0, 4)$
Plot the y-intercept.	See graph below.
Identify the rise and the run.	$m = \frac{-1}{1}$
Count out the rise and run to mark the second point.	rise -1, run 1
Draw the line.	
To check your work, you can find another point on the line and make sure it is a solution of the equation. In the graph we see the line goes through $(4, 0)$.	

check.

$y = -x + 4$

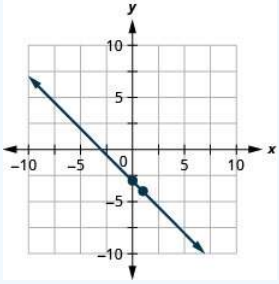
$0 \stackrel{?}{=} -4 + 4$

$0 = 0 \checkmark$

? Try It 3.5.14

Graph the line of the equation $y = -x - 3$ using its slope and y-intercept.

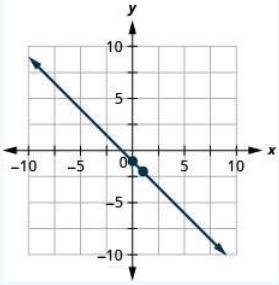
Answer



? Try It 3.5.15

Graph the line of the equation $y = -x - 1$ using its slope and y-intercept.

Answer

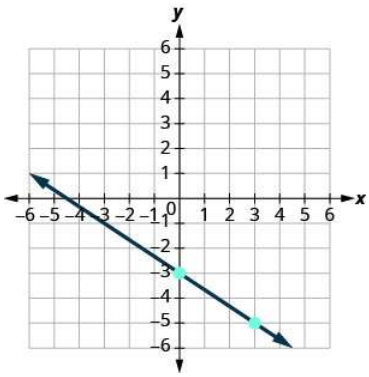


✓ Example 3.5.16

Graph the line of the equation $y = -\frac{2}{3}x - 3$ using its slope and y-intercept.

Solution

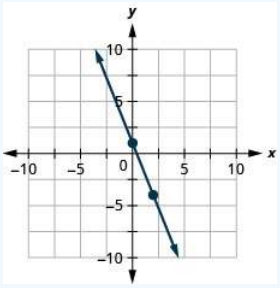
	$y = mx + b$
The equation is in slope–intercept form.	$y = -\frac{2}{3}x - 3$
Identify the slope and y-intercept.	$m = -\frac{2}{3}$; y-intercept is $(0, -3)$
Plot the y-intercept.	See graph below.
Identify the rise and the run.	
Count out the rise and run to mark the second point.	

Draw the line.	

Try It 3.5.17

Graph the line of the equation $y = -\frac{5}{2}x + 1$ using its slope and y-intercept.

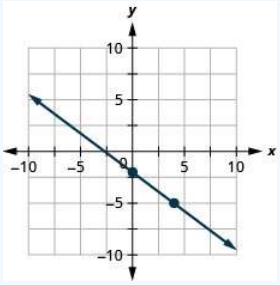
Answer



Try It 3.5.18

Graph the line of the equation $y = -\frac{3}{4}x - 2$ using its slope and y-intercept.

Answer

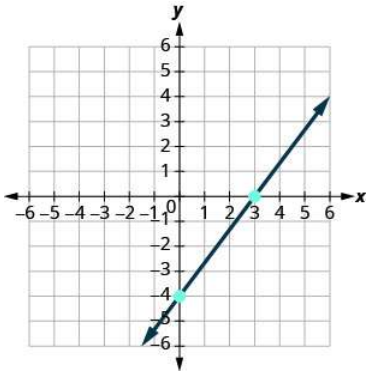


Example 3.5.19

Graph the line of the equation $4x - 3y = 12$ using its slope and y-intercept.

Solution

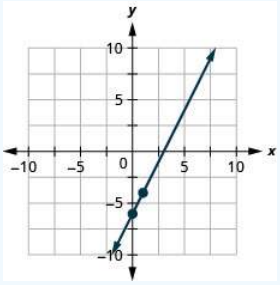
	$4x - 3y = 12$
Find the slope–intercept form of the equation.	$-3y = -4x + 12$
	$-\frac{3y}{3} = \frac{-4x+12}{-3}$

The equation is now in slope–intercept form.	$y = \frac{4}{3}x - 4$
Identify the slope and y-intercept.	$m = \frac{4}{3}$
	y-ntercept is $(0, -4)$
Plot the y-intercept.	See graph below.
Identify the rise and the run; count out the rise and run to mark the second point.	
Draw the line.	

Try It 3.5.20

Graph the line of the equation $2x - y = 6$ using its slope and y-intercept.

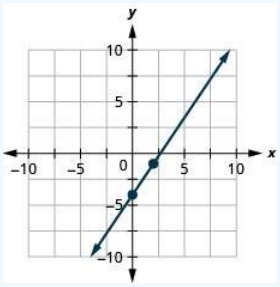
Answer



Try It 3.5.21

Graph the line of the equation $3x - 2y = 8$ using its slope and y-intercept.

Answer



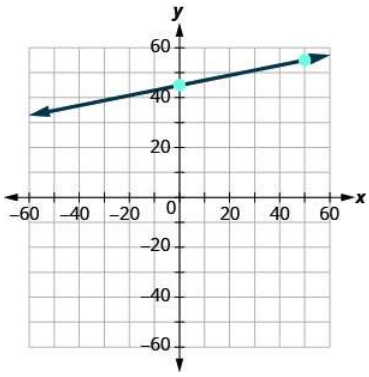
We have used a grid with x and y both going from about -10 to 10 for all the equations we've graphed so far. Not all linear equations can be graphed on this small grid. Often, especially in applications with real-world data, we'll need to extend the axes to bigger positive or smaller negative numbers.

Example 3.5.22

Graph the line of the equation $y = 0.2x + 45$ using its slope and y-intercept.

Solution

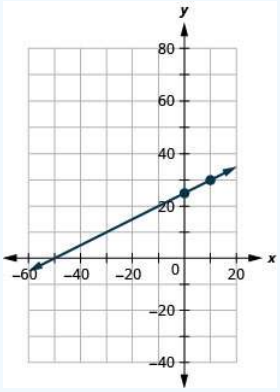
We'll use a grid with the axes going from about -80 to 80.

	$y = mx + b$
The equation is in slope–intercept form.	$y = 0.2x + 45$
Identify the slope and y-intercept.	$m = 0.2$
	The y-intercept is $(0, 45)$
Plot the y-intercept.	See graph below.
Count out the rise and run to mark the second point. The slope is $m = 0.2$; in fraction form this means $m = \frac{2}{10}$. Given the scale of our graph, it would be easier to use the equivalent fraction $m = \frac{10}{50}$.	
Draw the line.	

Try It 3.5.23

Graph the line of the equation $y = 0.5x + 25$ using its slope and y-intercept.

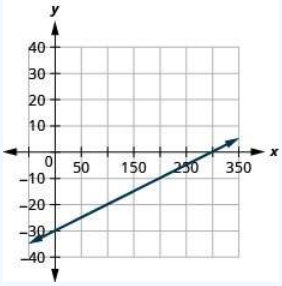
Answer



? Try It 3.5.24

Graph the line of the equation $y = 0.1x - 30$ using its slope and y-intercept.

Answer



Now that we have graphed lines by using the slope and y-intercept, let's summarize all the methods we have used to graph lines. See Figure 3.5.2.

Methods to Graph Lines

Point Plotting	Slope–Intercept	Intercepts	Recognize Vertical and Horizontal Lines
x | y	$y = mx + b$	x | y 0 | | 0	
Find three points. Plot the points, make sure they line up, then draw the line.	Find the slope and y-intercept. Start at the y-intercept, then count the slope to get a second point.	Find the intercepts and a third point. Plot the points, make sure they line up, then draw the line.	The equation has only one variable. $x = a$ vertical $y = b$ horizontal

Figure 3.5.2

Choose the Most Convenient Method to Graph a Line

Now that we have seen several methods we can use to graph lines, how do we know which method to use for a given equation?

While we could plot points, use the slope–intercept form, or find the intercepts for *any* equation, if we recognize the most convenient way to graph a certain type of equation, our work will be easier. Generally, plotting points is not the most efficient way to graph a line. We saw better methods in sections 4.3, 4.4, and earlier in this section. Let's look for some patterns to help determine the most convenient method to graph a line.

Here are six equations we graphed in this chapter, and the method we used to graph each of them.

$$\begin{array}{lll} \#1 & \text{Equation} & \text{Method} \\ \#2 & x = 2 & \text{Vertical line} \\ \#3 & y = 4 & \text{Horizontal line} \\ \#4 & -x + 2y = 6 & \text{Intercepts} \\ \#5 & 4x - 3y = 12 & \text{Intercepts} \\ \#6 & y = 4x - 2 & \text{Slope-intercept} \\ \#7 & y = -x + 4 & \text{Slope-intercept} \end{array} \tag{3.5.2}$$

Equations #1 and #2 each have just one variable. Remember, in equations of this form the value of that one variable is constant; it does not depend on the value of the other variable. Equations of this form have graphs that are vertical or horizontal lines.

In equations #3 and #4, both x and y are on the same side of the equation. These two equations are of the form $Ax + By = C$. We substituted $y = 0$ to find the x-intercept and $x = 0$ to find the y-intercept, and then found a third point by choosing another value for x or y.

Equations #5 and #6 are written in slope–intercept form. After identifying the slope and y-intercept from the equation we used them to graph the line.

This leads to the following strategy.

STRATEGY FOR CHOOSING THE MOST CONVENIENT METHOD TO GRAPH A LINE

Consider the form of the equation.

- If it only has one variable, it is a vertical or horizontal line.
  - $x = a$ is a vertical line passing through the x-axis at a.
  - $y = b$ is a horizontal line passing through the y-axis at b.
- If y is isolated on one side of the equation, in the form $y = mx + b$, graph by using the slope and y-intercept.
  - Identify the slope and y-intercept and then graph.
- If the equation is of the form $Ax + By = C$, find the intercepts.
  - Find the x- and y-intercepts, a third point, and then graph.

Example 3.5.25

Determine the most convenient method to graph each line.

1. $y = -6$
2. $5x - 3y = 15$
3. $x = 7$
4. $y = \frac{2}{5}x - 1$.

Solution

1. $y = -6$
This equation has only one variable, y. Its graph is a horizontal line crossing the y-axis at -6.

2. $5x - 3y = 15$
This equation is of the form $Ax + By = C$. The easiest way to graph it will be to find the intercepts and one more point.

3. $x = 7$
There is only one variable, x. The graph is a vertical line crossing the x-axis at 7.

4. $y = \frac{2}{5}x - 1$
Since this equation is in $y = mx + b$ form, it will be easiest to graph this line by using the slope and y-intercept.

Try It 3.5.26

Determine the most convenient method to graph each line:

1. $3x + 2y = 12$
2. $y = 4$
3. $y = \frac{1}{5}x - 4$
4. $x = -7$

Answer

1. intercepts
2. horizontal line
3. slope–intercept
4. vertical line

Try It 3.5.27

Determine the most convenient method to graph each line:

1. $x = 6$
2. $y = -\frac{3}{4}x + 1$
3. $y = -8$
4. $4x - 3y = -1$

Answer

1. vertical line
2. slope–intercept
3. horizontal line
4. intercepts

Graph and Interpret Applications of Slope–Intercept

Many real-world applications are modeled by linear equations. We will take a look at a few applications here so you can see how equations written in slope–intercept form relate to real-world situations.

Usually when a linear equation models a real-world situation, different letters are used for the variables, instead of x and y. The variable names remind us of what quantities are being measured.

Example 3.5.28

The equation $F = \frac{9}{5}C + 32$ is used to convert temperatures, C, on the Celsius scale to temperatures, F, on the Fahrenheit scale.

1. Find the Fahrenheit temperature for a Celsius temperature of 0.
2. Find the Fahrenheit temperature for a Celsius temperature of 20.
3. Interpret the slope and F-intercept of the equation.
4. Graph the equation.

Solution

1. Find the Fahrenheit temperature for a Celsius temperature of 0.

	$F = \frac{9}{5}C + 32$
Find F when $C = 0$.	$F = \frac{9}{5}(0) + 32$
Simplify.	$F = 32$

2.

Find the Fahrenheit temperature for a Celsius temperature of 20.	$F = \frac{9}{5}C + 32$
Find F when $C = 20$.	$F = \frac{9}{5}(20) + 32$
Simplify.	$F = 36 + 32$
Simplify.	$F = 68$

(3.5.3)

3. Interpret the slope and F-intercept of the equation.

Even though this equation uses F and C, it is still in slope–intercept form.

$y = mx + b$

$F = mC + b$

$F = \frac{9}{5}C + 32$

The slope, $\frac{9}{5}$, means that the temperature Fahrenheit (F) increases 9 degrees when the temperature Celsius (C) increases 5 degrees.

The F-intercept means that when the temperature is 0° on the Celsius scale, it is 32° on the Fahrenheit scale.

4. Graph the equation.

We'll need to use a larger scale than our usual. Start at the F-intercept $(0, 32)$ then count out the rise of 9 and the run of 5 to get a second point. See Figure $3.5.3$.

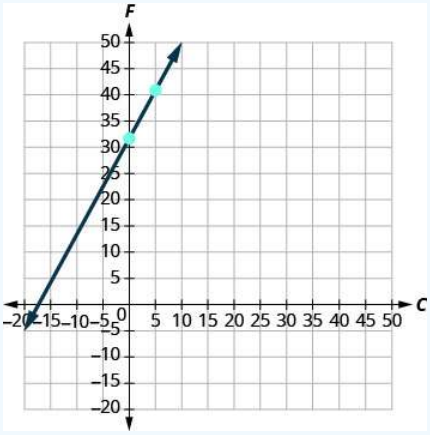


Figure 3.5.3

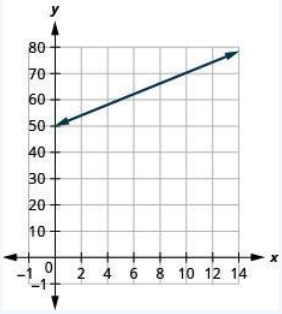
Try It 3.5.29

The equation $h = 2s + 50$ is used to estimate a woman's height in inches, h, based on her shoe size, s.

1. Estimate the height of a child who wears women's shoe size 0.
2. Estimate the height of a woman with shoe size 8.
3. Interpret the slope and h-intercept of the equation.
4. Graph the equation.

Answer

1. 50 inches
2. 66 inches
3. The slope, 2, means that the height, h, increases by 2 inches when the shoe size, s, increases by 1. The h-intercept means that when the shoe size is 0, the height is 50 inches.
4.

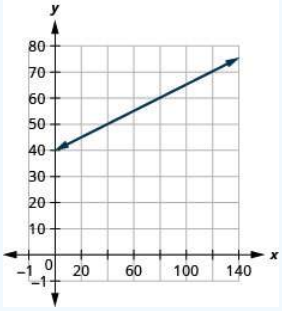


Try It 3.5.30

The equation $T = \frac{1}{4}n + 40$ is used to estimate the temperature in degrees Fahrenheit, T, based on the number of cricket chirps, n, in one minute.

1. Estimate the temperature when there are no chirps.
2. Estimate the temperature when the number of chirps in one minute is 100.
3. Interpret the slope and T-intercept of the equation.
4. Graph the equation.

Answer

1. 40 degrees
2. 65 degrees
3. The slope, $\frac{1}{4}$, means that the temperature Fahrenheit (F) increases 1 degree when the number of chirps, n, increases by 4. The T-intercept means that when the number of chirps is 0, the temperature is 40°.
4. 

The cost of running some types business has two components—a *fixed cost* and a *variable cost*. The fixed cost is always the same regardless of how many units are produced. This is the cost of rent, insurance, equipment, advertising, and other items that must be paid regularly. The variable cost depends on the number of units produced. It is for the material and labor needed to produce each item.

Example 3.5.31

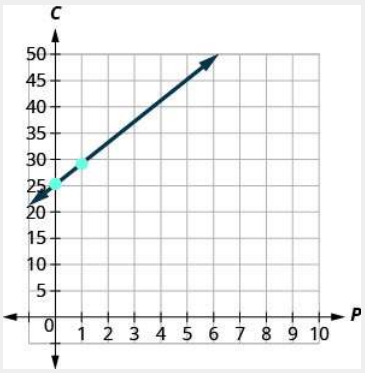
Stella has a home business selling gourmet pizzas. The equation $C = 4p + 25$ models the relation between her weekly cost, C, in dollars and the number of pizzas, p, that she sells.

1. Find Stella's cost for a week when she sells no pizzas.
2. Find the cost for a week when she sells 15 pizzas.
3. Interpret the slope and C-intercept of the equation.
4. Graph the equation.

Solution

1. Find Stella's cost for a week when she sells no pizzas.	$C = 4p + 25$
Find C when $p = 0$.	$C = 4(0) + 25$
Simplify.	$C = 25$
	Stella's fixed cost is \$25 when she sells no pizzas.
2. Find the cost for a week when she sells 15 pizzas.	$C = 4p + 25$
Find C when $p = 15$.	$C = 4(15) + 25$
Simplify.	$C = 60 + 25$
	$C = 85$
	Stella's costs are \$85 when she sells 15 pizzas.
3. Interpret the slope and C-intercept of the equation.	$y = mx + b$ $C = 4p + 25$
	The slope, 4, means that the cost increases by \$4 for each pizza Stella sells. The C-intercept means that even when Stella sells no pizzas, her costs for the week are \$25.

4. Graph the equation. We'll need to use a larger scale than our usual. Start at the C-intercept $(0, 25)$ then count out the rise of 4 and the run of 1 to get a second point.



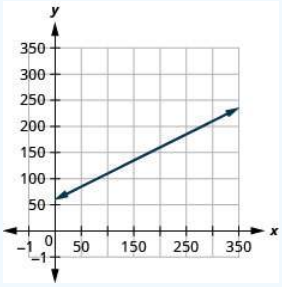
Try It 3.5.32

Sam drives a delivery van. The equation $C = 0.5m + 60$ models the relation between his weekly cost, C, in dollars and the number of miles, m, that he drives.

1. Find Sam's cost for a week when he drives 0 miles.
2. Find the cost for a week when he drives 250 miles.
3. Interpret the slope and C-intercept of the equation.
4. Graph the equation.

Answer

1. $\$60$
2. $\$185$
3. The slope, 0.5, means that the weekly cost, C, increases by $\$0.50$ when the number of miles driven, n, increases by 1. The C-intercept means that when the number of miles driven is 0, the weekly cost is $\$60$.
4.



Try It 3.5.33

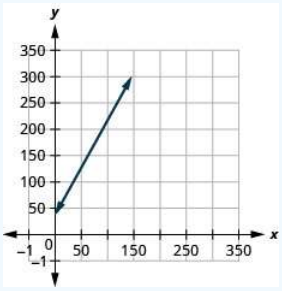
Loreen has a calligraphy business. The equation $C = 1.8n + 35$ models the relation between her weekly cost, C, in dollars and the number of wedding invitations, n, that she writes.

1. Find Loreen's cost for a week when she writes no invitations.
2. Find the cost for a week when she writes 75 invitations.
3. Interpret the slope and C-intercept of the equation.
4. Graph the equation.

Answer

1. $\$35$
2. $\$170$
3. The slope, 1.8, means that the weekly cost, C, increases by $\$1.80$ when the number of invitations, n, increases by 1. The C-intercept means that when the number of invitations is 0, the weekly cost is $\$35$.

4.



Use Slopes to Identify Parallel Lines

The slope of a line indicates how steep the line is and whether it rises or falls as we read it from left to right. Two lines that have the same slope are called parallel lines. Parallel lines never intersect.

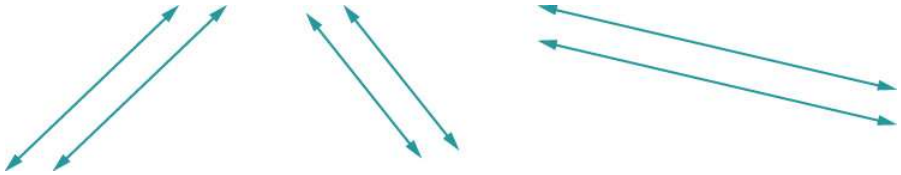


Figure 3.5.4.

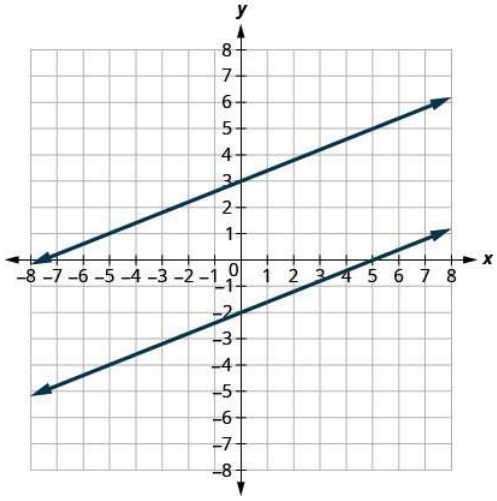


Figure 3.5.4: Verify that both lines have the same slope, $m = \frac{2}{5}$, and different y-intercepts.

What about vertical lines? The slope of a vertical line is undefined, so vertical lines don't fit in the definition above. We say that vertical lines that have different x-intercepts are parallel. See Figure 3.5.5.

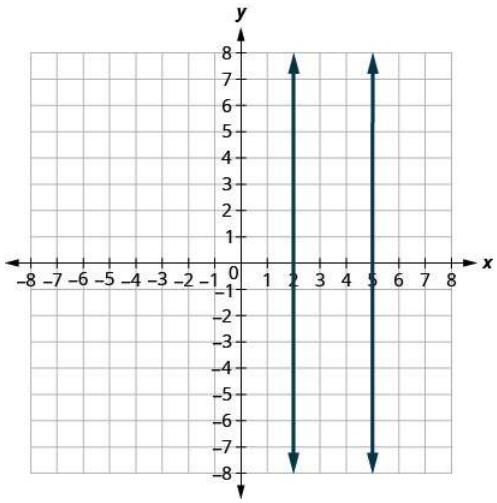


Figure 3.5.5: Vertical lines with different x-intercepts are parallel.

PARALLEL LINES

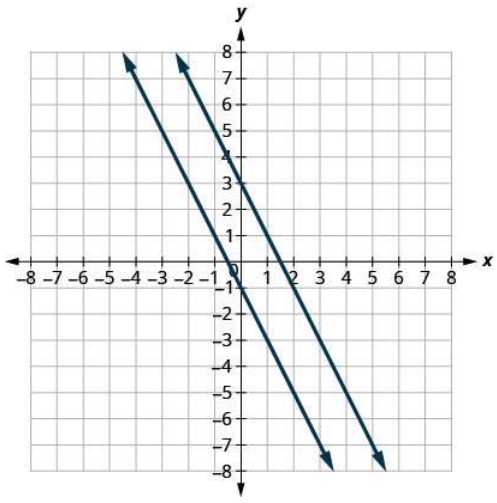
Parallel lines are lines in the same plane that do not intersect.

- Parallel lines have the same slope and different y-intercepts.
- If m_1 and m_2 are the slopes of two parallel lines then $m_1 = m_2$.
- Parallel vertical lines have different x-intercepts.

Let's graph the equations $y = -2x + 3$ and $2x + y = -1$ on the same grid. The first equation is already in slope–intercept form: $y = -2x + 3$. We solve the second equation for y:

$$\begin{aligned} 2x + y &= -1 \\ y &= -2x - 1 \end{aligned}$$

Graph the lines.



Notice the lines look parallel. What is the slope of each line? What is the y-intercept of each line?

$$\begin{array}{lll} y & = mx + b & y = mx + b \\ y & = -2x + 3 & y = -2x - 1 \\ m & = -2 & m = -2 \\ b & = 3, (0, 3) & b = -1, (0, -1) \end{array} \tag{3.5.4}$$

The slopes of the lines are the same and the y-intercept of each line is different. So we know these lines are parallel.

Since parallel lines have the same slope and different y-intercepts, we can now just look at the slope–intercept form of the equations of lines and decide if the lines are parallel.

Example 3.5.34

Use slopes and y-intercepts to determine if the lines $3x-2y=6$ and $y=\frac{3}{2}x+1$ are parallel.

Solution

Solve the first equation for y.

$$\begin{aligned} 3x-2y &= 6 \\ -2y &= -3x+6 \\ \frac{-2y}{-2} &= \frac{-3x+6}{-2} \\ y &= \frac{3}{2}x-3 \end{aligned}$$

The equation is now in slope–intercept form.

The equation of the second line is already in slope–intercept form.

Identify the slope and y-intercept of both lines.

$y=\frac{3}{2}x+1$	$y=\frac{3}{2}x-3$
$y=mx+b$	$y=mx+b$
$m=\frac{3}{2}$	$m=\frac{3}{2}$
y-intercept is $(0,1)$	y-intercept is $(0,-3)$

The lines have the same slope and different y-intercepts and so they are parallel. You may want to graph the lines to confirm whether they are parallel.

Try It 3.5.35

Use slopes and y-intercepts to determine if the lines $2x+5y=5$ and $y=-\frac{2}{5}x-4$ are parallel.

Answer

parallel

Try It 3.5.36

Use slopes and y-intercepts to determine if the lines $4x-3y=6$ and $y=\frac{4}{3}x-1$ are parallel.

Answer

parallel

Example 3.5.37

Use slopes and y-intercepts to determine if the lines $y=-4$ and $y=3$ are parallel.

Solution

Write each equation in slope-intercept form.	$y=-4$	and	$y=3$
Since there is no x term we write $0x$.	$y=0x-4$		$y=0x+3$
Identify the slope and y-intercept of both lines.	$y=mx+b$		$y=mx+b$
	$m=0$		$m=0$
	y-intercept is $(0,4)$		y-intercept is $(0,3)$

The lines have the same slope and different y-intercepts and so they are parallel.

There is another way you can look at this example. If you recognize right away from the equations that these are horizontal lines, you know their slopes are both 0. Since the horizontal lines cross the y-axis at $y=-4$ and at $y=3$, we know the y-intercepts are $(0,-4)$ and $(0,3)$. The lines have the same slope and different y-intercepts and so they are parallel.

? Try It 3.5.38

Use slopes and y-intercepts to determine if the lines $y=8$ and $y=-6$ are parallel.

Answer

parallel

? Try It 3.5.39

Use slopes and y-intercepts to determine if the lines $y=1$ and $y=-5$ are parallel.

Answer

parallel

✓ Example 3.5.40

Use slopes and y-intercepts to determine if the lines $x=-2$ and $x=-5$ are parallel.

Solution

$$x=-2 \text{ and } x=-5 \tag{3.5.5}$$

Since there is no y, the equations cannot be put in slope–intercept form. But we recognize them as equations of vertical lines. Their x-intercepts are -2 and -5. Since their x-intercepts are different, the vertical lines are parallel.

? Try It 3.5.41

Use slopes and y-intercepts to determine if the lines $x=1$ and $x=-5$ are parallel.

Answer

parallel

? Try It 3.5.42

Use slopes and y-intercepts to determine if the lines $x=8$ and $x=-6$ are parallel.

Answer

parallel

✓ Example 3.5.43

Use slopes and y-intercepts to determine if the lines $y=2x-3$ and $-6x+3y=-9$ are parallel. You may want to graph these lines, too, to see what they look like.

Solution

The first equation is already in slope-intercept form. $y = 2x - 3$

Solve the second equation for y

$$\begin{aligned} -6x + 3y &= -9 \\ 3y &= 6x - 9 \\ \tfrac{3y}{3} &= \tfrac{6x-9}{3} \\ y &= 2x - 3 \end{aligned}$$

The second equation is now in slope–intercept form as well.

Identify the slope and y-intercept of both lines.

$y = 2x - 3$	$y = 2x - 3$
$y = mx + b$	$y = mx + b$
$m = 2$	$m = 2$
The y-intercept is $(0, -3)$	The y-intercept is $(0, -3)$

The lines have the same slope, but they also have the same y-intercepts. Their equations represent the same line. They are not parallel; they are the same line.

Try It 3.5.44

Use slopes and y-intercepts to determine if the lines $y = -\frac{1}{2}x + 1$ and $x + 2y = 2$ are parallel.

Answer

not parallel; same line

Try It 3.5.45

Use slopes and y-intercepts to determine if the lines $y = \frac{3}{4}x - 3$ and $3x - 4y = 12$ are parallel.

Answer

not parallel; same line

Use Slopes to Identify Perpendicular Lines

Let's look at the lines whose equations are $y = \frac{1}{4}x - 1$ and $y = -4x + 2$, shown in Figure 3.5.5.

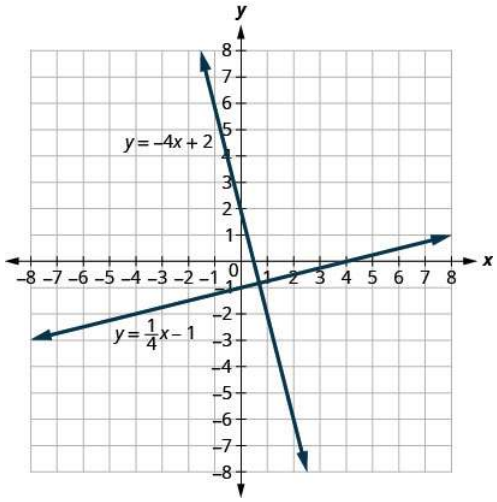


Figure 3.5.5

These lines lie in the same plane and intersect in right angles. We call these lines **perpendicular**.

What do you notice about the slopes of these two lines? As we read from left to right, the line $y = \frac{1}{4}x - 1$ rises, so its slope is positive. The line $y = -4x + 2$ drops from left to right, so it has a negative slope. Does it make sense to you that the slopes of two perpendicular lines will have opposite signs?

If we look at the slope of the first line, $m_1 = \frac{1}{4}$, and the slope of the second line, $m_2 = -4$, we can see that they are *negative reciprocals* of each other. If we multiply them, their product is -1.

$$\begin{array}{c} m_1 \cdot m_2 \\ \frac{1}{4}(-4) \\ -1 \end{array} \tag{3.5.6}$$

This is always true for **perpendicular lines** and leads us to this definition.

PERPENDICULAR LINES

Perpendicular lines are lines in the same plane that form a right angle.

If m_1 and m_2 are the slopes of two perpendicular lines, then:

$$m_1 \cdot m_2 = -1 \text{ and } m_1 = \frac{-1}{m_2} \tag{3.5.7}$$

Vertical lines and horizontal lines are always perpendicular to each other.

We were able to look at the slope–intercept form of linear equations and determine whether or not the lines were parallel. We can do the same thing for perpendicular lines.

We find the slope–intercept form of the equation, and then see if the slopes are negative reciprocals. If the product of the slopes is -1, the lines are perpendicular. Perpendicular lines may have the same y-intercepts.

Example 3.5.46

Use slopes to determine if the lines, $y = -5x - 4$ and $x - 5y = 5$ are perpendicular.

Solution

The first equation is already in slope–intercept form: $y = -5x - 4$

Solve the second equation for y.

$$\begin{array}{rcl} x - 5y & = & 5 \\ -5y & = & -x + 5 \\ \frac{-5y}{-5} & = & \frac{-x+5}{-5} \\ y & = & \frac{1}{5}x - 1 \end{array}$$

The second equation is now in slope-intercept form as well.

Identify the slope of each line.

$$\begin{array}{rclrcl} y & = & -5x - 4 & y & = & \frac{1}{5}x - 1 \\ y & = & mx + b & y & = & mx + b \\ m_1 & = & -5 & m_2 & = & \frac{1}{5} \end{array}$$

The slopes are negative reciprocals of each other, so the lines are perpendicular. We check by multiplying the slopes,

$$\begin{array}{c} m_1 \cdot m_2 \\ -5\left(\frac{1}{5}\right) \\ -1\checkmark \end{array} \tag{3.5.8}$$

Try It 3.5.47

Use slopes to determine if the lines $y = -3x + 2$ and $x - 3y = 4$ are perpendicular.

Answer

perpendicular

Try It 3.5.48

Use slopes to determine if the lines $y = 2x - 5$ and $x + 2y = -6$ are perpendicular.

Answer

perpendicular

Example 3.5.49

Use slopes to determine if the lines, $7x + 2y = 3$ and $2x + 7y = 5$ are perpendicular.

Solution

Solve the equations for y.

$$\begin{aligned} 7x + 2y &= 3 \\ 2y &= -7x + 3 \\ \frac{2y}{2} &= \frac{-7x+3}{2} \\ y &= -\frac{7}{2}x + \frac{3}{2} \end{aligned} \qquad \begin{aligned} 2x + 7y &= 5 \\ 7y &= -2x + 5 \\ \frac{7y}{7} &= \frac{-2x+5}{7} \\ y &= \frac{-2}{7}x + \frac{5}{7} \end{aligned}$$

Identify the slope of each line.

$$\begin{aligned} y &= mx + b \\ m_1 &= -\frac{7}{2} \end{aligned} \qquad \begin{aligned} y &= mx + b \\ m_2 &= -\frac{2}{7} \end{aligned}$$

The slopes are reciprocals of each other, but they have the same sign. Since they are not negative reciprocals, the lines are not perpendicular.

Try It 3.5.50

Use slopes to determine if the lines $5x + 4y = 1$ and $4x + 5y = 3$ are perpendicular.

Answer

not perpendicular

Try It 3.5.51

Use slopes to determine if the lines $2x - 9y = 3$ and $9x - 2y = 1$ are perpendicular.

Answer

not perpendicular

Note

Access this online resource for additional instruction and practice with graphs.

- Explore the Relation Between a Graph and the Slope–Intercept Form of an Equation of a Line

Key Concepts

- The slope–intercept form of an equation of a line with slope mm and y-intercept, $(0, b)$ is, $y = mx + b$.
- **Graph a Line Using its Slope and y-Intercept**
  1. Find the slope-intercept form of the equation of the line.
  2. Identify the slope and y-intercept.
  3. Plot the y-intercept.
  4. Use the slope formula $m = \frac{\text{rise}}{\text{run}}$ to identify the rise and the run.
  5. Starting at the y-intercept, count out the rise and run to mark the second point.
  6. Connect the points with a line.
- **Strategy for Choosing the Most Convenient Method to Graph a Line:** Consider the form of the equation.
  - If it only has one variable, it is a vertical or horizontal line.
    $x = a$ is a vertical line passing through the x-axis at a.
    $y = b$ is a horizontal line passing through the y-axis at b.
  - If y is isolated on one side of the equation, in the form $y = mx + b$, graph by using the slope and y-intercept.
    Identify the slope and y-intercept and then graph.
  - If the equation is of the form $Ax + By = C$, find the intercepts.
    Find the x- and y-intercepts, a third point, and then graph.
- Parallel lines are lines in the same plane that do not intersect.
  - Parallel lines have the same slope and different y-intercepts.
  - If m_1 and m_2 are the slopes of two parallel lines then $m_1 = m_2$.
  - Parallel vertical lines have different x-intercepts.
- Perpendicular lines are lines in the same plane that form a right angle.
  - If m_1 and m_2 are the slopes of two perpendicular lines, then $m_1 \cdot m_2 = -1$ and $m_1 = \frac{-1}{m_2}$.
  - Vertical lines and horizontal lines are always perpendicular to each other.

Glossary

parallel lines

Lines in the same plane that do not intersect.

perpendicular lines

Lines in the same plane that form a right angle.

slope-intercept form of an equation of a line

The slope–intercept form of an equation of a line with slope mm and y-intercept, $(0, b)$ is, $y = mx + b$.

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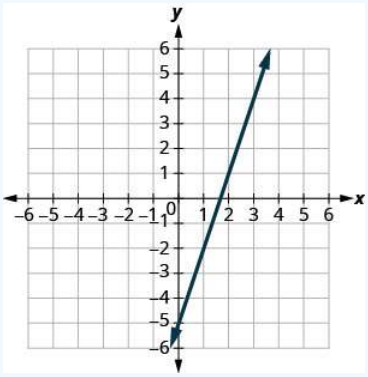
3.5E: Exercises

Practice Makes Perfect

Recognize the Relation Between the Graph and the Slope–Intercept Form of an Equation of a Line

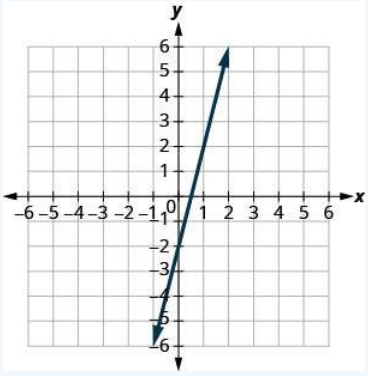
In the following exercises, use the graph to find the slope and y-intercept of each line. Compare the values to the equation $y = mx + b$.

Exercise 3.5E.1



$y = 3x - 5$

Exercise 3.5E.2

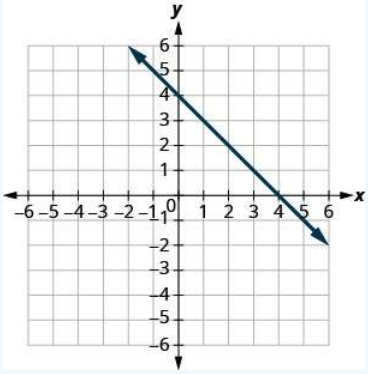


$y = 4x - 2$

Answer

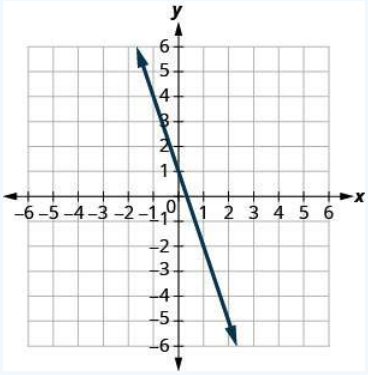
slope $m = 4$ and y-intercept $(0, -2)$

Exercise 3.5E.3



$y = -x + 4$

Exercise $3.5E.4$

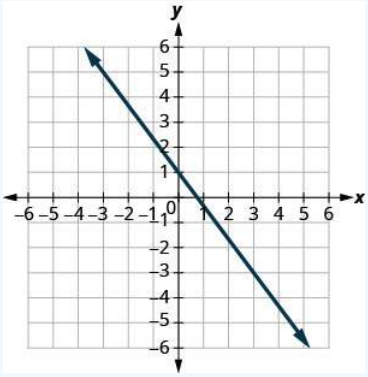


$y = -3x + 1$

Answer

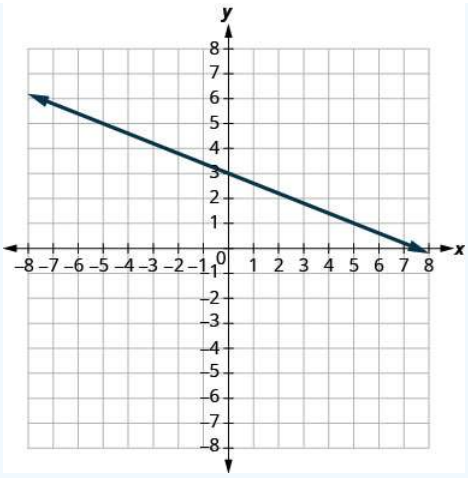
slope $m = -3$ and y-intercept $(0, 1)$

Exercise $3.5E.5$



$y = -\frac{4}{3}x + 1$

Exercise $3.5E.6$



$y = -\frac{2}{5}x + 3$

Answer

slope $m = -\frac{2}{5}$ and y -intercept $(0, 3)$

Identify the Slope and y-Intercept From an Equation of a Line

In the following exercises, identify the slope and y-intercept of each line.

Exercise $3.5E.7$

$y = -7x + 3$

Exercise $3.5E.8$

$y = -9x + 7$

Answer

$m = -9$; y-intercept: $(0, 7)$

Exercise $3.5E.9$

$y = 6x - 8$

Exercise $3.5E.10$

$y = 4x - 10$

Answer

$m = 4$; y-intercept: $(0, -10)$

Exercise $3.5E.11$

$3x + y = 5$

Exercise 3.5E.12

$4x + y = 8$

Answer

$m = -4 \backslash 0$; \(y-intercept: $(0, 8)$

Exercise 3.5E.13

$6x + 4y = 12$

Exercise 3.5E.14

$8x + 3y = 12$

Answer

$m = -\frac{8}{3}$; y-intercept: $(0, 4)$

Exercise 3.5E.15

$5x - 2y = 6$

Exercise 3.5E.16

$7x - 3y = 9$

Answer

$m = \frac{7}{3}$; y-intercept: $(0, -3)$

Graph a Line Using Its Slope and Intercept

In the following exercises, graph the line of each equation using its slope and y-intercept.

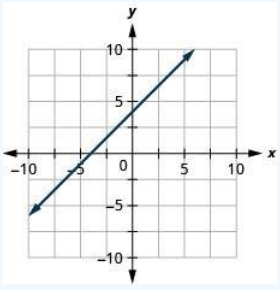
Exercise 3.5E.17

$y = x + 3$

Exercise 3.5E.18

$y = x + 4$

Answer



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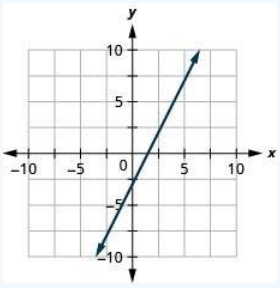
Exercise 3.5E.19

$y = 3x - 1$

Exercise 3.5E.20

$y = 2x - 3$

Answer



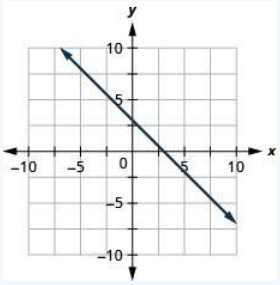
Exercise 3.5E.21

$y = -x + 2$

Exercise 3.5E.22

$y = -x + 3$

Answer



Exercise 3.5E.23

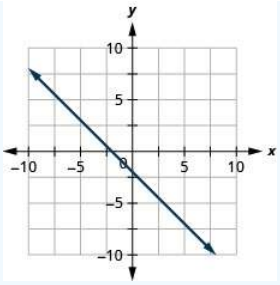
$y = -x - 4$

Exercise 3.5E.24

$y = -x - 2$

Answer

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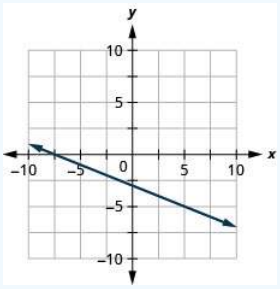
Exercise 3.5E. 25

$y = -\frac{3}{4}x - 1$

Exercise 3.5E. 26

$y = -\frac{2}{5}x - 3$

Answer



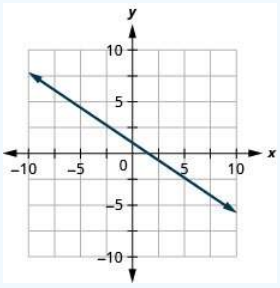
Exercise 3.5E. 27

$y = -\frac{3}{5}x + 2$

Exercise 3.5E. 28

$y = -\frac{2}{3}x + 1$

Answer



Exercise 3.5E. 29

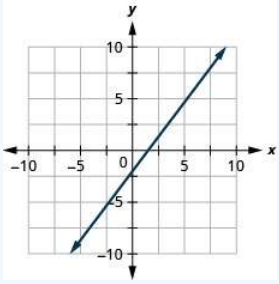
$3x - 4y = 8$

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Exercise 3.5E.30

$4x - 3y = 6$

Answer



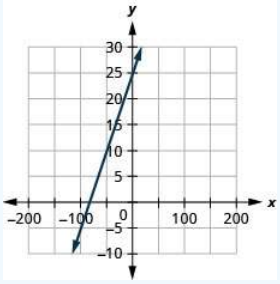
Exercise 3.5E.31

$y = 0.1x + 15$

Exercise 3.5E.32

$y = 0.3x + 25$

Answer



Choose the Most Convenient Method to Graph a Line

In the following exercises, determine the most convenient method to graph each line.

Exercise 3.5E.33

$x = 2$

Exercise 3.5E.34

$y = 4$

Answer

horizontal line

Exercise 3.5E.35

$y = 5$

Exercise 3.5E.36

$x = -3$

Answer

vertical line

Exercise 3.5E.37

$y = -3x + 4$

Exercise 3.5E.38

$y = -5x + 2$

Answer

slope–intercept

Exercise 3.5E.39

$x - y = 5$

Exercise 3.5E.40

$x - y = 1$

Answer

intercepts

Exercise 3.5E.41

$y = \frac{2}{3}x - 1$

Exercise 3.5E.42

$y = \frac{4}{5}x - 3$

Answer

slope–intercept

Exercise 3.5E.43

$y = -3$

Exercise 3.5E.44

$y = -1$

Answer

horizontal line

Exercise 3.5E.45

$3x - 2y = -12$

Exercise 3.5E.46

$2x - 5y = -10$

Answer

intercepts

Exercise 3.5E.47

$y = -\frac{1}{4}x + 3$

Exercise 3.5E.48

$y = -\frac{1}{3}x + 5$

Answer

slope–intercept

Graph and Interpret Applications of Slope–Intercept

Exercise 3.5E.49

The equation $P = 31 + 1.75w$ models the relation between the amount of Tuyet's monthly water bill payment, P, in dollars, and the number of units of water, w, used.

1. Find Tuyet's payment for a month when 0 units of water are used.
2. Find Tuyet's payment for a month when 12 units of water are used.
3. Interpret the slope and P-intercept of the equation.
4. Graph the equation.

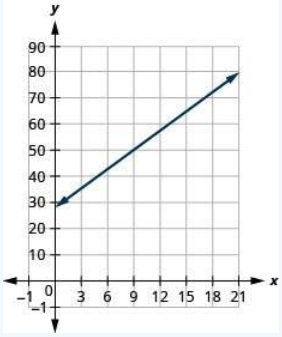
Exercise 3.5E.50

The equation $P = 28 + 2.54w$ models the relation between the amount of Randy's monthly water bill payment, P, in dollars, and the number of units of water, w, used.

1. Find the payment for a month when Randy used 0 units of water.
2. Find the payment for a month when Randy used 15 units of water.
3. Interpret the slope and P-intercept of the equation.
4. Graph the equation.

Answer

1. $28
2. $66.10
3. The slope, 2.54, means that Randy's payment, P, increases by $2.54 when the number of units of water he used, w, ncreases by 1. The P-intercept means that if the number units of water Randy used was 0, the payment would be $28
4.



Exercise $3.5E.51$

Bruce drives his car for his job. The equation $R = 0.575m + 42$ models the relation between the amount in dollars, R, that he is reimbursed and the number of miles, m, he drives in one day.

1. Find the amount Bruce is reimbursed on a day when he drives 0 miles.
2. Find the amount Bruce is reimbursed on a day when he drives 220 miles.
3. Interpret the slope and R-intercept of the equation.
4. Graph the equation.

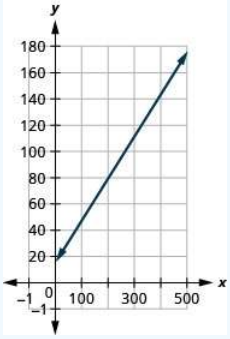
Exercise $3.5E.52$

Janelle is planning to rent a car while on vacation. The equation $C = 0.32m + 15$ models the relation between the cost in dollars, C, per day and the number of miles, m, she drives in one day.

1. Find the cost if Janelle drives the car 0 miles one day.
2. Find the cost on a day when Janelle drives the car 400 miles.
3. Interpret the slope and C-intercept of the equation.
4. Graph the equation.

Answer

1. $\$15$
2. $\$143$
3. The slope, 0.32, means that the cost, C, increases by $\$0.32$ when the number of miles driven, m, increases by 1. The C-intercept means that if Janelle drives 0 miles one day, the cost would be $\$15$.
4.



Exercise $3.5E.53$

Cherie works in retail and her weekly salary includes commission for the amount she sells. The equation $S = 400 + 0.15c$ models the relation between her weekly salary, S, in dollars and the amount of her sales, c, in dollars.

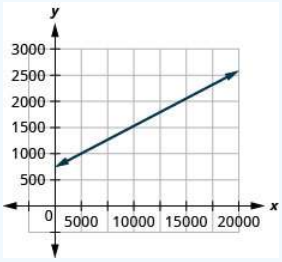
1. Find Cherie's salary for a week when her sales were 0.
2. Find Cherie's salary for a week when her sales were 3600.
3. Interpret the slope and S-intercept of the equation.
4. Graph the equation.

Exercise $3.5E.54$

Patel's weekly salary includes a base pay plus commission on his sales. The equation $S = 750 + 0.09c$ models the relation between his weekly salary, S, in dollars and the amount of his sales, c, in dollars.

1. Find Patel's salary for a week when his sales were 0.
2. Find Patel's salary for a week when his sales were $18,540$.
3. Interpret the slope and S-intercept of the equation.
4. Graph the equation.

Answer

1. $\$750$
2. $\$2418.60$
3. The slope, 0.09, means that Patel's salary, S, increases by $\$0.09$ for every $\$1$ increase in his sales. The S-intercept means that when his sales are $\$0$, his salary is $\$750$
4. 

Exercise $3.5E.55$

Costa is planning a lunch banquet. The equation $C = 450 + 28g$ models the relation between the cost in dollars, C, of the banquet and the number of guests, g.

1. Find the cost if the number of guests is 40.
2. Find the cost if the number of guests is 80.
3. Interpret the slope and C-intercept of the equation.
4. Graph the equation.

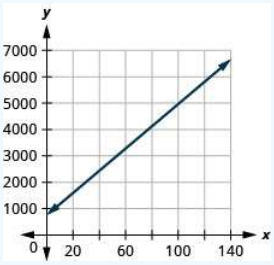
Exercise $3.5E.56$

Margie is planning a dinner banquet. The equation $C = 750 + 42g$ models the relation between the cost in dollars, C, of the banquet and the number of guests, g.

1. Find the cost if the number of guests is 50.
2. Find the cost if the number of guests is 100.
3. Interpret the slope and C-intercept of the equation.
4. Graph the equation.

Answer

1. $\$2850$
2. $\$4950$

3. The slope, 42, means that the cost, C, increases by $\$42$ for when the number of guests increases by 1. The C-intercept means that when the number of guests is 0, the cost would be $\$750$
4. 

Use Slopes to Identify Parallel Lines

In the following exercises, use slopes and y-intercepts to determine if the lines are parallel.

Exercise $3.5E.57$

$y = \frac{3}{4}x - 3; \quad 3x - 4y = -2$

Exercise $3.5E.58$

$y = \frac{2}{3}x - 1; \quad 2x - 3y = -2$

Answer

parallel

Exercise $3.5E.59$

$2x - 5y = -3; \quad y = \frac{2}{5}x + 1$

Exercise $3.5E.60$

$3x - 4y = -2; \quad y = \frac{3}{4}x - 3$

Answer

parallel

Exercise $3.5E.61$

$2x - 4y = 6; \quad x - 2y = 3$

Exercise $3.5E.62$

$6x - 3y = 9; \quad 2x - y = 3$

Answer

not parallel

Exercise 3.5E.63

$4x+2y=6; \quad 6x+3y=3$

Exercise 3.5E.64

$8x+6y=6; \quad 12x+9y=12$

Answer

parallel

Exercise 3.5E.65

$x=5; \quad x=-6$

Exercise 3.5E.66

$x=7; \quad x=-8$

Answer

parallel

Exercise 3.5E.67

$x=-4; \quad x=-1$

Exercise 3.5E.68

$x=-3; \quad x=-2$

Answer

parallel

Exercise 3.5E.69

$y=2; \quad y=6$

Exercise 3.5E.70

$y=5; \quad y=1$

Answer

parallel

Exercise 3.5E.71

$y=-4; \quad y=3$

Exercise 3.5E.72

$y = -1; \quad y = 2$

Answer

parallel

Exercise 3.5E.73

$x - y = 2; \quad 2x - 2y = 4$

Exercise 3.5E.74

$4x + 4y = 8; \quad x + y = 2$

Answer

not parallel

Exercise 3.5E.75

$x - 3y = 6; \quad 2x - 6y = 12$

Exercise 3.5E.76

$5x - 2y = 11; \quad 5x - y = 7$

Answer

not parallel

Exercise 3.5E.77

$3x - 6y = 12; \quad 6x - 3y = 3$

Exercise 3.5E.78

$4x - 8y = 16; \quad x - 2y = 4$

Answer

not parallel

Exercise 3.5E.79

$9x - 3y = 6; \quad 3x - y = 2$

Exercise 3.5E.80

$x - 5y = 10; \quad 5x - y = -10$

Answer

not parallel

Exercise 3.5E.81

$7x-4y=8; \quad 4x+7y=14$

Exercise 3.5E.82

$9x-5y=4; \quad 5x+9y=-1$

Answer

not parallel

Use Slopes to Identify Perpendicular Lines

In the following exercises, use slopes and y-intercepts to determine if the lines are perpendicular.

Exercise 3.5E.83

$3x-2y=8; \quad 2x+3y=6$

Exercise 3.5E.84

$x-4y=8; \quad 4x+y=2$

Answer

perpendicular

Exercise 3.5E.85

$2x+5y=3; \quad 5x-2y=6$

Exercise 3.5E.86

$2x+3y=5; \quad 3x-2y=7$

Answer

perpendicular

Exercise 3.5E.87

$3x-2y=1; \quad 2x-3y=2$

Exercise 3.5E.88

$3x-4y=8; \quad 4x-3y=6$

Answer

not perpendicular

Exercise 3.5E.89

$5x+2y=6; \quad 2x+5y=8$

Exercise 3.5E.90

$2x+4y=3; \quad 6x+3y=2$

Answer

not perpendicular

Exercise 3.5E.91

$4x-2y=5; \quad 3x+6y=8$

Exercise 3.5E.92

$2x-6y=4; \quad 12x+4y=9$

Answer

perpendicular

Exercise 3.5E.93

$6x-4y=5; \quad 8x+12y=3$

Exercise 3.5E.94

$8x-2y=7; \quad 3x+12y=9$

Answer

perpendicular

Everyday Math

Exercise 3.5E.95

The equation $C=\frac{5}{9}F-17.8$ can be used to convert temperatures, F, on the Fahrenheit scale to temperatures, C, on the Celsius scale.

1. Explain what the slope of the equation means.
2. Explain what the C-intercept of the equation means.

Exercise 3.5E.96

The equation $n=4T-160$ is used to estimate the number of cricket chirps, n, in one minute based on the temperature in degrees Fahrenheit, T.

1. Explain what the slope of the equation means.
2. Explain what the n-intercept of the equation means. Is this a realistic situation?

Answer

1. For every increase of one degree Fahrenheit, the number of chirps increases by four.
2. There would be -160 chirps when the Fahrenheit temperature is 0°. (Notice that this does not make sense; this model cannot be used for all possible temperatures.)

Writing Exercises

Exercise 3.5E.97

Explain in your own words how to decide which method to use to graph a line.

Exercise 3.5E.98

Why are all horizontal lines parallel?

Answer

Answers will vary.

Self Check

a. After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
recognize the relation between the graph and the slope-intercept form of an equation of a line.			
identify the slope and *y*-intercept from an equation of a line.			
graph a line using its slope and intercept.			
choose the most convenient method to graph a line.			
graph and interpret applications of slope-intercept.			
use slopes to identify parallel lines.			

b. After looking at the checklist, do you think you are well-prepared for the next section? Why or why not?

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3.6: Find the Equation of a Line

Learning Objectives

By the end of this section, you will be able to:

- Find an equation of the line given the slope and y-intercept
- Find an equation of the line given the slope and a point
- Find an equation of the line given two points
- Find an equation of a line parallel to a given line
- Find an equation of a line perpendicular to a given line

Note

Before you get started, take this readiness quiz.

1. Solve: $\frac{2}{3} = \frac{x}{5}$.
   If you missed this problem, review Exercise 2.2.4.
2. Simplify: $-\frac{2}{5}(x - 15)$.
   If you missed this problem, review Exercise 1.10.34.

How do online retailers know that 'you may also like' a particular item based on something you just ordered? How can economists know how a rise in the minimum wage will affect the unemployment rate? How do medical researchers create drugs to target cancer cells? How can traffic engineers predict the effect on your commuting time of an increase or decrease in gas prices? It's all mathematics.

You are at an exciting point in your mathematical journey as the mathematics you are studying has interesting applications in the real world.

The physical sciences, social sciences, and the business world are full of situations that can be modeled with linear equations relating two variables. Data is collected and graphed. If the data points appear to form a straight line, an equation of that line can be used to predict the value of one variable based on the value of the other variable.

To create a mathematical model of a linear relation between two variables, we must be able to find the equation of the line. In this section we will look at several ways to write the equation of a line. The specific method we use will be determined by what information we are given.

Find an Equation of the Line Given the Slope and *y*-Intercept

We can easily determine the slope and intercept of a line if the equation was written in slope–intercept form, y=mx+b. Now, we will do the reverse—we will start with the slope and *y*-intercept and use them to find the equation of the line.

Example 3.6.1

Find an equation of a line with slope −7 and *y*-intercept (0,−1).

Solution

Since we are given the slope and *y*-intercept of the line, we can substitute the needed values into the slope–intercept form, y=mx+b.

Name the slope.	$m = -7$
Name the *y*-intercept.	*y*-intercept $(0, -1)$
Substitute the values into y=mx+b.	$y = mx + b$
	$y = -7x + (-1)$
	$y = -7x + -1$

Try It 3.6.2

Find an equation of a line with slope $\frac{2}{5}$ and y-intercept (0,4).

Answer

$y = \frac{2}{5}x + 4$

Try It 3.6.3

Find an equation of a line with slope −1 and y-intercept (0,−3).

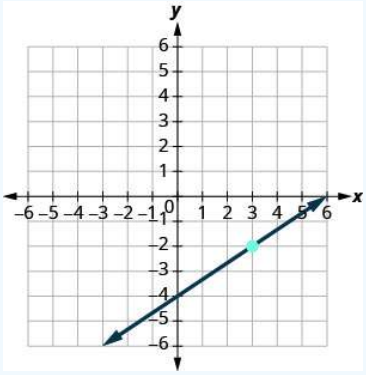
Answer

$y = -x - 3$

Sometimes, the slope and intercept need to be determined from the graph.

Example 3.6.4

Find the equation of the line shown.



Solution

We need to find the slope and y-intercept of the line from the graph so we can substitute the needed values into the slope–intercept form, $y = mx + b$.

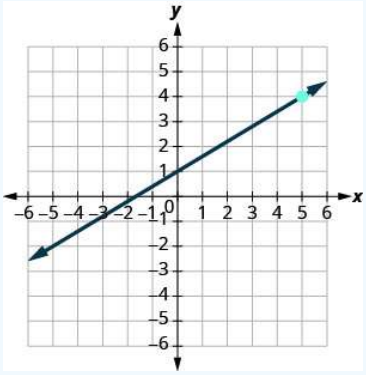
To find the slope, we choose two points on the graph.

The y-intercept is (0,−4) and the graph passes through (3,−2).

Find the slope by counting the rise and run.	$m = \frac{\text{rise}}{\text{run}}$
	$m = \frac{2}{3}$
Find the y-intercept.	y-intercept $(0, -4)$
Substitute the values into y=mx+b.	$y = mx + b$
	$y = \frac{2}{3}x - 4$

Try It 3.6.5

Find the equation of the line shown in the graph.

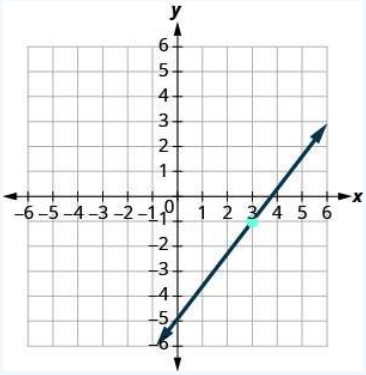


Answer

$y = \frac{3}{5}x + 1$

Try It 3.6.6

Find the equation of the line shown in the graph.



Answer

$y = \frac{4}{3}x - 5$

Find an Equation of the Line Given the Slope and a Point

Finding an equation of a line using the slope–intercept form of the equation works well when you are given the slope and *y*-intercept or when you read them off a graph. But what happens when you have another point instead of the *y*-intercept?

We are going to use the slope formula to derive another form of an equation of the line. Suppose we have a line that has slope mm and that contains some specific point (x_1, y_1) and some other point, which we will just call (x,y). We can write the slope of this line and then change it to a different form.

	m	$= \frac{y-y_1}{x-x_1}$
Multiply both sides of the equation by $x - x_1$.	$m(x-x_1)$	$= \left(\frac{y-y_1}{x-x_1}\right)(x-x_1)$
Simplify.	$m(x-x_1)$	$= y - y_1$
Rewrite the equation with the y terms on the left.	$y - y_1$	$= m(x-x_1)$

This format is called the point–slope form of an equation of a line.

POINT–SLOPE FORM OF AN EQUATION OF A LINE

The **point–slope form** of an equation of a line with slope m and containing the point (x_1, y_1) is

$$y - y_1 = m(x - x_1)$$

We can use the point–slope form of an equation to find an equation of a line when we are given the slope and one point. Then we will rewrite the equation in slope–intercept form. Most applications of linear equations use the the slope–intercept form.

Example 3.6.7: Find an Equation of a Line Given the Slope and a Point

Find an equation of a line with slope $m = \frac{2}{5}$ that contains the point (10,3). Write the equation in slope–intercept form.

Solution

Step 1. Identify the slope.	The slope is given.	$m = \frac{2}{5}$
Step 2. Identify the point.	The point is given.	(x_1, y_1) $(10, 3)$
Step 3. Substitute the values into the point–slope form, $y - y_1 = m(x - x_1)$.		$y - y_1 = m(x - x_1)$
		$y - 3 = \frac{2}{5}(x - 10)$
	Simplify.	$y - 3 = \frac{2}{5}x - 4$
Step 4. Write the equation in slope–intercept form.		$y = \frac{2}{5}x - 1$

Try It 3.6.8

Find an equation of a line with slope $m = \frac{5}{6}$ and containing the point (6,3).

Answer

$y = \frac{5}{6}x - 2$

Try It 3.6.9

Find an equation of a line with slope $m = \frac{2}{3}$ and containing the point (9,2).

Answer

$y = \frac{2}{3}x - 4$

FIND AN EQUATION OF A LINE GIVEN THE SLOPE AND A POINT.

1. Identify the slope.
2. Identify the point.
3. Substitute the values into the point-slope form, $y - y_1 = m(x - x_1)$.
4. Write the equation in slope–intercept form.

Example 3.6.10

Find an equation of a line with slope $m = -\frac{1}{3}$ that contains the point (6,−4). Write the equation in slope–intercept form.

Solution

Since we are given a point and the slope of the line, we can substitute the needed values into the point–slope form, $y - y_1 = m(x - x_1)$.

Identify the slope.	$m = -\frac{1}{3}$
Identify the point.	$\begin{matrix} x_1, & y_1 \\ 6, & -4 \end{matrix}$
Substitute the values into $y - y_1 = m(x - x_1)$.	$y - y_1 = m(x - x_1)$
	$y - (-4) = -\frac{1}{3}(x - 6)$
Simplify.	$y + 4 = -\frac{1}{3}x + 2$
Write in slope–intercept form.	$y = -\frac{1}{3}x - 2$

Try It 3.6.11

Find an equation of a line with slope $m = -\frac{2}{5}$ and containing the point (10,−5).

Answer

$y = -\frac{2}{5}x - 1$

Try It 3.6.12

Find an equation of a line with slope $m = -\frac{3}{4}$, and containing the point (4,−7).

Answer

$y = -\frac{3}{4}x - 4$

Example 3.6.13

Find an equation of a horizontal line that contains the point (−1,2). Write the equation in slope–intercept form.

Solution

Every horizontal line has slope 0. We can substitute the slope and points into the point–slope form, $y - y_1 = m(x - x_1)$.

Identify the slope.	$m = 0$
Identify the point.	$\begin{matrix} x_1, & y_1 \\ -1, & 2 \end{matrix}$
Substitute the values into $y - y_1 = m(x - x_1)$.	$y - y_1 = m(x - x_1)$
	$y - 2 = 0(x - (-1))$
Simplify.	$y - 2 = 0(x + 1)$
	$y - 2 = 0$
	$y = 2$

Write in slope–intercept form.	It is in *y*-form, but could be written y=0x+2.

Did we end up with the form of a horizontal line, y=a?

? Try It 3.6.14

Find an equation of a horizontal line containing the point (−3,8).

Answer

y = 8

? Try It 3.6.15

Find an equation of a horizontal line containing the point (−1,4).

Answer

y = 4

Find an Equation of the Line Given Two Points

When real-world data is collected, a linear model can be created from two data points. In the next example we'll see how to find an equation of a line when just two points are given.

We have two options so far for finding an equation of a line: slope–intercept or point–slope. Since we will know two points, it will make more sense to use the point–slope form.

But then we need the slope. Can we find the slope with just two points? Yes. Then, once we have the slope, we can use it and one of the given points to find the equation.

✓ Example 3.6.16: Find an Equation of a Line Given Two Points

Find an equation of a line that contains the points (5,4) and (3,6). Write the equation in slope–intercept form.

Solution

Step 1. Find the slope using the given points.	To use the point-slope form, we first find the slope.	$m=\frac{y_2-y_1}{x_2-x_1}$ $m=\frac{6-4}{3-5}$ $m=\frac{2}{-2}$ $m=-1$
Step 2. Choose one point.	Choose either point.	$(\overset{x_1}{5},\ \overset{y_1}{4})$
Step 3. Substitute the values into the point-slope form, $y-y_1=m(x-x_1)$.	Simplify.	$y-y_1=m(x-x_1)$ $y-4=-1(x-5)$ $y-4=-1x+5$
Step 4. Write the equation in slope–intercept form.		$y=-1x+9$

Use the point (3,6) and see that you get the same equation.

Try It 3.6.17

Find an equation of a line containing the points (3,1) and (5,6).

Answer

$y = \frac{5}{2}x - \frac{13}{2}$

Try It 3.6.18

Find an equation of a line containing the points (1,4) and (6,2).

Answer

$y = -\frac{2}{5}x + \frac{22}{5}$

FIND AN EQUATION OF A LINE GIVEN TWO POINTS.

1. Find the slope using the given points.
2. Choose one point.
3. Substitute the values into the point-slope form, $y - y_1 = m(x - x_1)$.
4. Write the equation in slope–intercept form.

Example 3.6.19

Find an equation of a line that contains the points (−3,−1) and (2,−2). Write the equation in slope–intercept form.

Solution

Since we have two points, we will find an equation of the line using the point–slope form. The first step will be to find the slope.

Find the slope of the line through (−3, −1) and (2, −2).	$m = \frac{y_2 - y_1}{x_2 - x_1}$
	$m = \frac{-2-(-1)}{2-(-3)}$
	$m = \frac{-1}{5}$
	$m = -\frac{1}{5}$
Choose either point.	$\begin{matrix} x_1 & y_1 \\ 2, & -2 \end{matrix}$
Substitute the values into $y - y_1 = m(x - x_1)$.	$y - y_1 = m(x - x_1)$
	$y-(-2) = -\frac{1}{5}(x-2)$
	$y + 2 = -\frac{1}{5}x + \frac{2}{5}$
Write in slope–intercept form.	$y = -\frac{1}{5}x - \frac{8}{5}$

Try It 3.6.20

Find an equation of a line containing the points (−2,−4) and (1,−3).

Answer

$y = \frac{1}{3}x - \frac{10}{3}$

Try It 3.6.21

Find an equation of a line containing the points (−4,−3) and (1,−5).

Answer

$y = -\frac{2}{5}x - \frac{23}{5}$

Example 3.6.22

Find an equation of a line that contains the points (−2,4) and (−2,−3). Write the equation in slope–intercept form.

Solution

Again, the first step will be to find the slope.

Find the slope of the line through $(-2, 4)$ and $(-2, -3)$

$$\begin{aligned} m &= \frac{y_2 - x_1}{x_2 - x_1} \\ m &= \frac{-3-4}{-2-(-2)} \\ m &= \frac{-7}{0} \end{aligned}$$

The slope is undefined.

This tells us it is a vertical line. Both of our points have an *x*-coordinate of −2. So our equation of the line is x=−2. Since there is no yy, we cannot write it in slope–intercept form.

You may want to sketch a graph using the two given points. Does the graph agree with our conclusion that this is a vertical line?

Try It 3.6.23

Find an equation of a line containing the points (5,1) and (5,−4).

Answer

x = 5

Try It 3.6.24

Find an equation of a line containing the points (−4,4) and (−4,3).

Answer

x=−4

We have seen that we can use either the slope–intercept form or the point–slope form to find an equation of a line. Which form we use will depend on the information we are given. This is summarized in Table 3.6.1.

Table 3.6.1

To Write an Equation of a Line		
If given:	Use:	Form:
Slope and *y*-intercept	slope–intercept	y=mx+b

Slope and a point	point–slope	$y - y_1 = m(x - x_1)$
Two points	point–slope	$y - y_1 = m(x - x_1)$

Find an Equation of a Line Parallel to a Given Line

Suppose we need to find an equation of a line that passes through a specific point and is parallel to a given line. We can use the fact that parallel lines have the same slope. So we will have a point and the slope—just what we need to use the point–slope equation.

First let's look at this graphically.

The graph shows the graph of y=2x−3. We want to graph a line parallel to this line and passing through the point (−2,1).

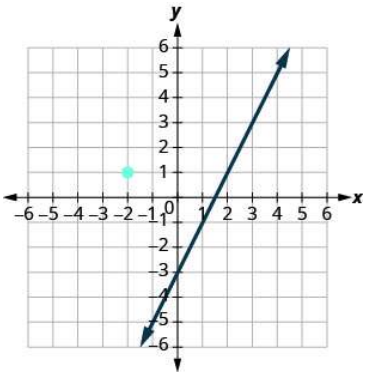


Figure 3.6.1

We know that parallel lines have the same slope. So the second line will have the same slope as y=2x−3. That slope is $m_{\parallel} = 2$. We'll use the notation $m_{\parallel}$ to represent the slope of a line parallel to a line with slope m. (Notice that the subscript ∥ looks like two parallel lines.)

The second line will pass through (−2,1) and have m=2. To graph the line, we start at (−2,1) and count out the rise and run. With m=2 (or $m = \frac{2}{1}$), we count out the rise 2 and the run 1. We draw the line.

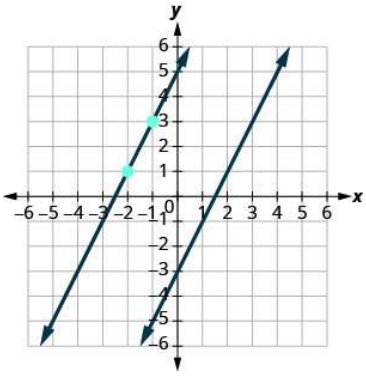


Figure 3.6.2

Do the lines appear parallel? Does the second line pass through (−2,1)?

Now, let's see how to do this algebraically.

We can use either the slope–intercept form or the point–slope form to find an equation of a line. Here we know one point and can find the slope. So we will use the point–slope form.

Example 3.6.25: How to Find an Equation of a Line Parallel to a Given Line

Find an equation of a line parallel to y=2x−3 that contains the point (−2,1). Write the equation in slope–intercept form.

Solution

Step 1. Find the slope of the given line.	The line is in slope–intercept form, $y = 2x - 3$.	$m = 2$
Step 2. Find the slope of the parallel line.	Parallel lines have the same slope.	$m_{\parallel} = 2$
Step 3. Identify the point.	The given point is, (–2, 1).	$\begin{pmatrix} x_1 & y_1 \\ -2, & 1 \end{pmatrix}$
Step 4. Substitute the values into the point–slope form, $y - y_1 = m(x - x_1)$.	Simplify.	$y - y_1 = m(x - x_1)$ $y - 1 = 2(x - (-2))$ $y - 1 = 2(x + 2)$ $y - 1 = 2x + 4$
Step 5. Write the equation in slope–intercept form.		$y = 2x + 5$

Does this equation make sense? What is the *y*-intercept of the line? What is the slope?

? Try It 3.6.26

Find an equation of a line parallel to the line y=3x+1 that contains the point (4,2). Write the equation in slope–intercept form.

Answer

y=3x−10

? Try It 3.6.27

Find an equation of a line parallel to the line $y = \frac{1}{2}x - 3$ that contains the point (6,4).

Answer

$y = \frac{1}{2}x + 1$

FIND AN EQUATION OF A LINE PARALLEL TO A GIVEN LINE.

1. Find the slope of the given line.
2. Find the slope of the parallel line.
3. Identify the point.
4. Substitute the values into the point–slope form, $y - y_1 = m(x - x_1)$.
5. Write the equation in slope–intercept form.

Find an Equation of a Line Perpendicular to a Given Line

Now, let's consider perpendicular lines. Suppose we need to find a line passing through a specific point and which is perpendicular to a given line. We can use the fact that perpendicular lines have slopes that are negative reciprocals. We will again use the point–slope equation, like we did with parallel lines.

The graph shows the graph of y=2x−3. Now, we want to graph a line perpendicular to this line and passing through (−2,1).

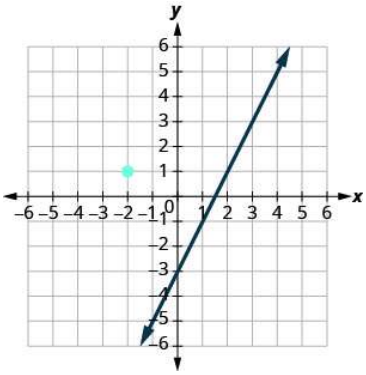


Figure 3.6.3

We know that perpendicular lines have slopes that are negative reciprocals. We'll use the notation $m_{\perp}$ to represent the slope of a line perpendicular to a line with slope m. (Notice that the subscript $_{\perp}$ looks like the right angles made by two perpendicular lines.)

$$\begin{aligned} y &= 2x - 3 \quad \text{perpendicular line} \\ m &= 2 \qquad m_{\perp} = -\tfrac{1}{2} \end{aligned} \tag{3.6.1}$$

We now know the perpendicular line will pass through (−2,1) with $m_{\perp} = -\frac{1}{2}$.

To graph the line, we will start at (−2,1) and count out the rise −1 and the run 2. Then we draw the line.

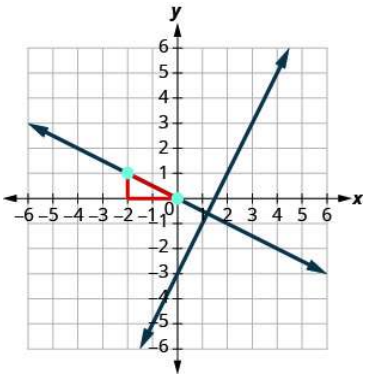


Figure 3.6.4

Do the lines appear perpendicular? Does the second line pass through (−2,1)?

Now, let's see how to do this algebraically. We can use either the slope–intercept form or the point–slope form to find an equation of a line. In this example we know one point, and can find the slope, so we will use the point–slope form.

Example 3.6.28

Find an equation of a line perpendicular to y=2x−3 that contains the point (−2,1). Write the equation in slope–intercept form.

Solution

Step 1. Find the slope of the given line.	The line is in slope–intercept form, $y = 2x - 3$.	$m = 2$
Step 2. Find the slope of the perpendicular line.	The slopes of perpendicular lines are negative reciprocals.	$m_{\perp} = -\frac{1}{2}$
Step 3. Identify the point.	The given point is, (–2, 1)	(x_1, y_1) $(-2, 1)$

Step 4. Substitute the values into the point–slope form, $y-y_1=m(x-x_1)$.	Simplify.	$y-y_1=m(x-x_1)$ $y-1=-\frac{1}{2}(x-(-2))$ $y-1=-\frac{1}{2}(x+2)$ $y-1=-\frac{1}{2}x-1$
Step 5. Write the equation in slope–intercept form.		$y=-\frac{1}{2}x$

Try It 3.6.29

Find an equation of a line perpendicular to the line y=3x+1 that contains the point (4,2). Write the equation in slope–intercept form.

Answer

$y=-\frac{1}{3}x+\frac{10}{3}$

Try It 3.6.30

Find an equation of a line perpendicular to the line $y=\frac{1}{2}x-3$ that contains the point (6,4).

Answer

y=−2x+16

FIND AN EQUATION OF A LINE PERPENDICULAR TO A GIVEN LINE.

1. Find the slope of the given line.
2. Find the slope of the perpendicular line.
3. Identify the point.
4. Substitute the values into the point–slope form, $y-y_1=m(x-x_1)$.
5. Write the equation in slope–intercept form.

Example 3.6.31

Find an equation of a line perpendicular to x=5 that contains the point (3,−2). Write the equation in slope–intercept form.

Solution

Again, since we know one point, the point–slope option seems more promising than the slope–intercept option. We need the slope to use this form, and we know the new line will be perpendicular to x=5. This line is vertical, so its perpendicular will be horizontal. This tells us the $m_\perp=0$.

Identify the point.	$(3$	,	$-2)$
Identify the slope of the perpendicular line.	$m_\perp$	$=$	0
Substitute the values into $y-y_1=m\,(x-x_1)$.	$y-y_1$	$=$	$m\,(x-x_1)$
	$y-(-2)$	$=$	$0(x-3)$
Simplify.	$y+2$	$=$	0
	y	$=$	-2

Sketch the graph of both lines. Do they appear to be perpendicular?

Try It 3.6.32

Find an equation of a line that is perpendicular to the line x=4 that contains the point (4,−5). Write the equation in slope–intercept form.

Answer

y=−5

Try It 3.6.33

Find an equation of a line that is perpendicular to the line x=2 that contains the point (2,−1). Write the equation in slope–intercept form.

Answer

y=−1

In Example 3.6.31, we used the point–slope form to find the equation. We could have looked at this in a different way.

We want to find a line that is perpendicular to x=5 that contains the point (3,−2). The graph shows us the line x=5 and the point (3,−2).

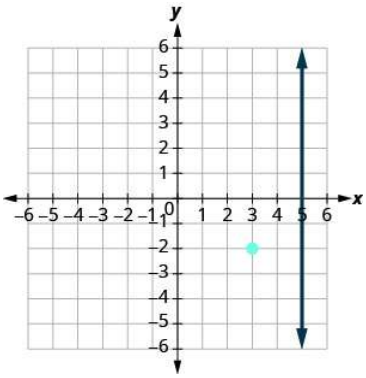


Figure 3.6.5

We know every line perpendicular to a vertical line is horizontal, so we will sketch the horizontal line through (3,−2).

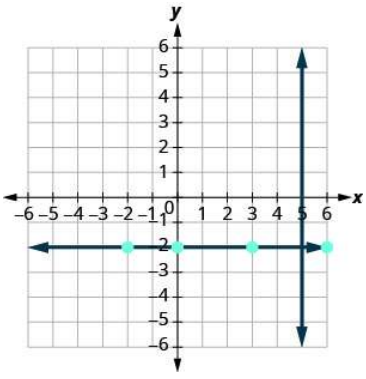


Figure 3.6.6

Do the lines appear perpendicular?

If we look at a few points on this horizontal line, we notice they all have *y*-coordinates of −2. So, the equation of the line perpendicular to the vertical line x=5 is y=−2.

Example 3.6.34

Find an equation of a line that is perpendicular to y=−4 that contains the point (−4,2). Write the equation in slope–intercept form.

Solution

The line y=−4 is a horizontal line. Any line perpendicular to it must be vertical, in the form x=a. Since the perpendicular line is vertical and passes through (−4,2), every point on it has an *x*-coordinate of −4. The equation of the perpendicular line is x=−4. You may want to sketch the lines. Do they appear perpendicular?

Try It 3.6.35

Find an equation of a line that is perpendicular to the line y=1 that contains the point (−5,1). Write the equation in slope–intercept form.

Answer

x=−5

Try It 3.6.36

Find an equation of a line that is perpendicular to the line y=−5 that contains the point (−4,−5).

Answer

x=−4

Note

Access this online resource for additional instruction and practice with finding the equation of a line.

- Use the Point-Slope Form of an Equation of a Line

Key Concepts

- **To Find an Equation of a Line Given the Slope and a Point**
  1. Identify the slope.
  2. Identify the point.
  3. Substitute the values into the point-slope form, $y-y_1=m(x-x_1)$.
  4. Write the equation in slope-intercept form.
- **To Find an Equation of a Line Given Two Points**
  1. Find the slope using the given points.
  2. Choose one point.
  3. Substitute the values into the point-slope form, $y-y_1=m(x-x_1)$.
  4. Write the equation in slope-intercept form.
- **To Write an Equation of a Line**
  - If given slope and y-intercept, use slope–intercept form $y=mx+b$.
  - If given slope and a point, use point–slope form $y-y_1=m(x-x_1)$.
  - If given two points, use point–slope form $y-y_1=m(x-x_1)$.
- **To Find an Equation of a Line Parallel to a Given Line**
  1. Find the slope of the given line.
  2. Find the slope of the parallel line.
  3. Identify the point.
  4. Substitute the values into the point-slope form, $y-y_1=m(x-x_1)$.

5. Write the equation in slope-intercept form.

- **To Find an Equation of a Line Perpendicular to a Given Line**
  1. Find the slope of the given line.
  2. Find the slope of the perpendicular line.
  3. Identify the point.
  4. Substitute the values into the point-slope form, $y - y_1 = m(x - x_1)$.
  5. Write the equation in slope-intercept form.

Glossary

point–slope form

The point–slope form of an equation of a line with slope mm and containing the point (x_1, y_1) is $y - y_1 = m\,(x - x_1)$.

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3.6E: Exercises

Practice Makes Perfect

Find an Equation of the Line Given the Slope and y-Intercept

In the following exercises, find the equation of a line with given slope and y-intercept. Write the equation in slope–intercept form.

Exercise 3.6E.1

slope 3 and y-intercept $(0, 5)$

Exercise 3.6E.2

slope 4 and y-intercept $(0, 1)$

Answer

$y = 4x + 1$

Exercise 3.6E.3

slope 6 and y-intercept $(0, -4)$

Exercise 3.6E.4

slope 8 and y-intercept $(0, -6)$

Answer

$y = 8x - 6$

Exercise 3.6E.5

slope -1 and y-intercept $(0, 3)$

Exercise 3.6E.6

slope -1 and y-intercept $(0, 7)$

Answer

$y = -x + 7$

Exercise 3.6E.7

slope -2 and y-intercept $(0, -3)$

Exercise 3.6E.8

slope -3 and y-intercept $(0, -1)$

Answer

$y = -3x - 1$

Exercise 3.6E.9

slope $\frac{3}{5}$ and y-intercept $(0, -1)$

Exercise 3.6E.10

slope $\frac{1}{5}$ and y-intercept $(0, -5)$

Answer

$y = \frac{1}{5}x - 5$

Exercise 3.6E.11

slope $-\frac{3}{4}$ and y-intercept $(0, -2)$

Exercise 3.6E.12

slope $-\frac{2}{3}$ and y-intercept $(0, -3)$

Answer

$y = -\frac{2}{3}x - 3$

Exercise 3.6E.13

slope 0 and y-intercept $(0, -1)$

Exercise 3.6E.14

slope 0 and y-intercept $(0, 2)$

Answer

$y = 2$

Exercise 3.6E.15

slope -3 and y-intercept $(0, 0)$

Exercise 3.6E.16

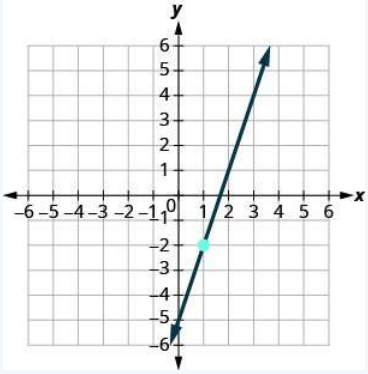
slope -4 and y-intercept $(0, 0)$

Answer

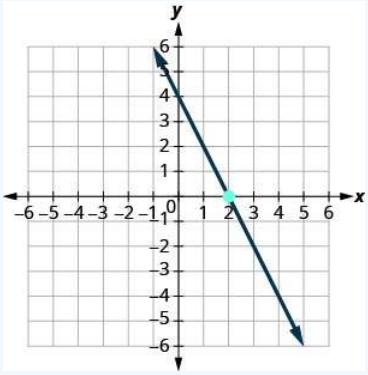
$y = -4x$

In the following exercises, find the equation of the line shown in each graph. Write the equation in slope–intercept form.

Exercise 3.6E.17



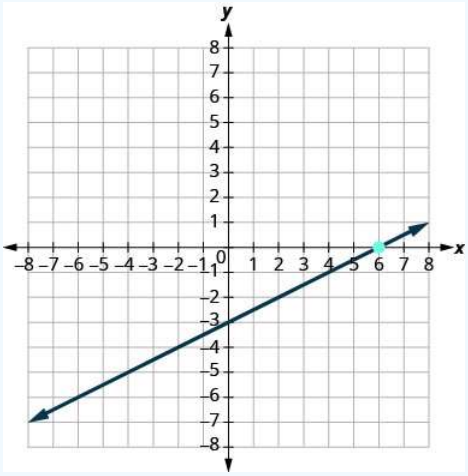
Exercise 3.6E.18



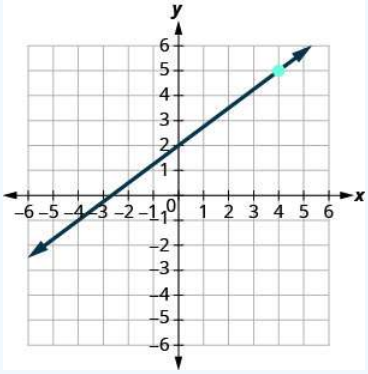
Answer

$y = -2x + 4$

Exercise 3.6E.19



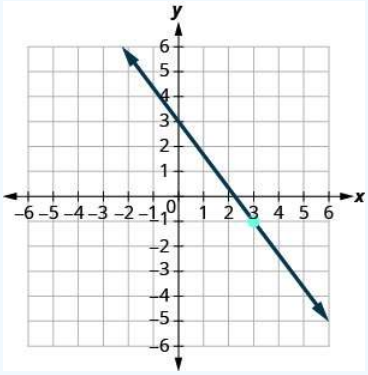
Exercise 3.6E.20



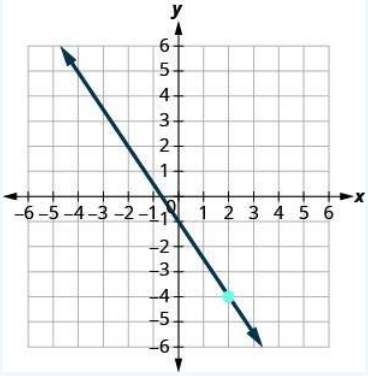
Answer

$y = \frac{3}{4}x + 2$

Exercise $3.6E.21$



Exercise $3.6E.22$

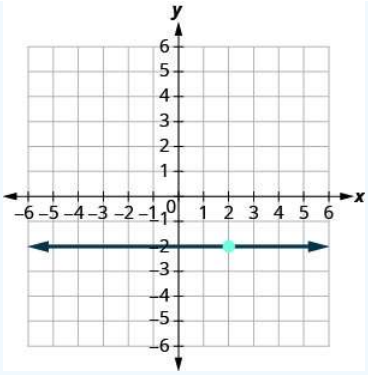


Answer

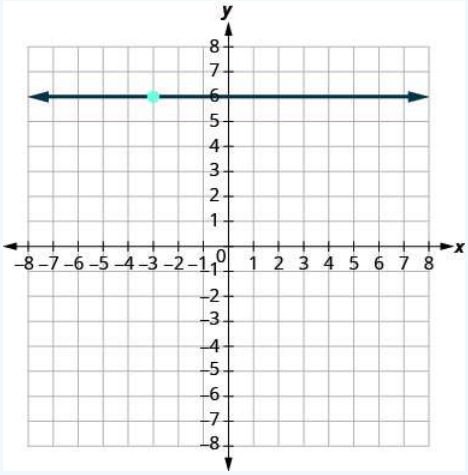
$y = -\frac{3}{2}x - 1$

Exercise $3.6E.23$

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Exercise 3.6E.24



Answer

$y = 6$

Find an Equation of the Line Given the Slope and a Point

In the following exercises, find the equation of a line with given slope and containing the given point. Write the equation in slope–intercept form.

Exercise 3.6E.25

$m = \frac{5}{8}$, point $(8, 3)$

Exercise 3.6E.26

$m = \frac{3}{8}$, point $(8, 2)$

Answer

$y = \frac{3}{8}x - 1$

https://math.libretexts.org/@go/page/54984

Exercise 3.6E.27

$m = \frac{1}{6}$, point $(6, 1)$

Exercise 3.6E.28

$m = \frac{5}{6}$, point $(6, 7)$

Answer

$y = \frac{5}{6}x + 2$

Exercise 3.6E.29

$m = -\frac{3}{4}$, point $(8, -5)$

Exercise 3.6E.30

$m = -\frac{3}{5}$, point $(10, -5)$

Answer

$y = -\frac{3}{5}x + 1$

Exercise 3.6E.31

$m = -\frac{1}{4}$, point $(-12, -6)$

Exercise 3.6E.32

$m = -\frac{1}{3}$, point $(-9, -8)$

Answer

$y = -\frac{1}{3}x - 11$

Exercise 3.6E.33

Horizontal line containing $(-2, 5)$

Exercise 3.6E.34

Horizontal line containing $(-1, 4)$

Answer

$y = 4$

Exercise 3.6E.35

Horizontal line containing $(-2, -3)$

Exercise 3.6E.36

Horizontal line containing $(-1, -7)$

Answer

$y = -7$

Exercise 3.6E.37

$m = -\frac{3}{2}$, point $(-4, -3)$

Exercise 3.6E.38

$m = -\frac{5}{2}$, point $(-8, -2)$

Answer

$y = -\frac{5}{2}x - 22$

Exercise 3.6E.39

$m = -7$, point $(-1, -3)$

Exercise 3.6E.40

$m = -4$, point $(-2, -3)$

Answer

$y = -4x - 11$

Exercise 3.6E.41

Horizontal line containing $(2, -3)$

Exercise 3.6E.42

Horizontal line containing $(4, -8)$

Answer

$y = -8$

Find an Equation of the Line Given Two Points

In the following exercises, find the equation of a line containing the given points. Write the equation in slope–intercept form.

Exercise 3.6E.43

$(2, 6)$ and $(5, 3)$

Exercise 3.6E.44

$(3, 1)$ and $(2, 5)$

Answer

$y = -4x + 13$

Exercise 3.6E.45

$(4, 3)$ and $(8, 1)$

Exercise 3.6E.46

$(2, 7)$ and $(3, 8)$

Answer

$y = x + 5$

Exercise 3.6E.47

$(-3, -4)$ and $(5, -2)$

Exercise 3.6E.48

$(-5, -3)$ and $(4, -6)$

Answer

$y = -\frac{1}{3}x - \frac{14}{3}$

Exercise 3.6E.49

$(-1, 3)$ and $(-6, -7)$

Exercise 3.6E.50

$(-2, 8)$ and $(-4, -6)$

Answer

$y = 7x + 22$

Exercise 3.6E.51

$(6, -4)$ and $(-2, 5)$

Exercise 3.6E.52

$(3, -2)$ and $(-4, 4)$

Answer

$y = -\frac{6}{7}x + \frac{4}{7}$

Exercise 3.6E.53

$(0, 4)$ and $(2, -3)$

Exercise 3.6E.54

$(0, -2)$ and $(-5, -3)$

Answer

$y = \frac{1}{5}x - 2$

Exercise 3.6E.55

$(7, 2)$ and $(7, -2)$

Exercise 3.6E.56

$(4, 2)$ and $(4, -3)$

Answer

$x = 4$

Exercise 3.6E.57

$(-7, -1)$ and $(-7, -4)$

Exercise 3.6E.58

$(-2, 1)$ and $(-2, -4)$

Answer

$x = -2$

Exercise 3.6E.59

$(6, 1)$ and $(0, 1)$

Exercise 3.6E.60

$(6, 2)$ and $(-3, 2)$

Answer

$y = 2$

Exercise 3.6E.61

$(3, -4)$ and $(5, -4)$

Exercise 3.6E.62

$(-6, -3)$ and $(-1, -3)$

Answer

$y = -3$

Exercise 3.6E.63

$(4, 3)$ and $(8, 0)$

Exercise 3.6E.64

$(0, 0)$ and $(1, 4)$

Answer

$y = 4x$

Exercise 3.6E.65

$(-2, -3)$ and $(-5, -6)$

Exercise 3.6E.66

$(-3, 0)$ and $(-7, -2)$

Answer

$y = \frac{1}{2}x + \frac{3}{2}$

Exercise 3.6E.67

$(8, -1)$ and $(8, -5)$

Exercise 3.6E.68

$(3, 5)$ and $(-7, 5)$

Answer

$y = 5$

Find an Equation of a Line Parallel to a Given Line

In the following exercises, find an equation of a line parallel to the given line and contains the given point. Write the equation in slope–intercept form.

Exercise 3.6E.69

line $y = 4x + 2$, point $(1, 2)$

Exercise 3.6E.70

line $y = 3x + 4$, point $(2, 5)$

Answer

$y = 3x - 1$

Exercise 3.6E.71

line $y = -2x - 3$, point $(-1, 3)$

Exercise 3.6E.72

line $y = -3x - 1$, point $(2, -3)$

Answer

$y = -3x + 3$

Exercise 3.6E.73

line $3x - y = 4$, point $(3, 1)$

Exercise 3.6E.74

line $2x - y = 6$, point $(3, 0)$

Answer

$y = 2x - 6$

Exercise 3.6E.75

line $4x + 3y = 6$, point $(0, -3)$

Exercise 3.6E.76

line $2x + 3y = 6$, point $(0, 5)$

Answer

$y = -\frac{2}{3}x + 5$

Exercise 3.6E.77

line $x = -3$, point $(-2, -1)$

Exercise 3.6E.78

line $x = -4$, point $(-3, -5)$

Answer

$x = -3$

Exercise 3.6E.79

line $x - 2 = 0$, point $(1, -2)$

Exercise 3.6E.80

line $x-6=0$, point $(4,-3)$

Answer

$x=4$

Exercise 3.6E.81

line $y=5$, point $(2,-2)$

Exercise 3.6E.82

line $y=1$, point $(3,-4)$

Answer

$y=-4$

Exercise 3.6E.83

line $y+2=0$, point $(3,-3)$

Exercise 3.6E.84

line $y+7=0$, point $(1,-1)$

Answer

$y=-1$

Find an Equation of a Line Perpendicular to a Given Line

In the following exercises, find an equation of a line perpendicular to the given line and contains the given point. Write the equation in slope–intercept form.

Exercise 3.6E.85

line $y=-2x+3$, point $(2,2)$

Exercise 3.6E.86

line $y=-x+5$, point $(3,3)$

Answer

$y=x$

Exercise 3.6E.87

line $y=\frac{3}{4}x-2$, point $(-3,4)$

Exercise 3.6E.88

line $y = \frac{2}{3}x - 4$, point $(2, -4)$

Answer

$y = -\frac{3}{2}x - 1$

Exercise 3.6E.89

line $2x - 3y = 8$, point $(4, -1)$

Exercise 3.6E.90

line $4x - 3y = 5$, point $(-3, 2)$

Answer

$y = -\frac{3}{4}x - \frac{1}{4}$

Exercise 3.6E.91

line $2x + 5y = 6$, point $(0, 0)$

Exercise 3.6E.92

line $4x + 5y = -3$, point $(0, 0)$

Answer

$y = \frac{5}{4}x$

Exercise 3.6E.93

line $y - 3 = 0$, point $(-2, -4)$

Exercise 3.6E.94

line $y - 6 = 0$, point $(-5, -3)$

Answer

$x = -5$

Exercise 3.6E.95

line y-axis, point $(3, 4)$

Exercise 3.6E.96

line y-axis, point $(2, 1)$

Answer

$y = 1$

Mixed Practice

In the following exercises, find the equation of each line. Write the equation in slope–intercept form.

Exercise 3.6E.97

Containing the points $(4, 3)$ and $(8, 1)$

Exercise 3.6E.98

Containing the points $(2, 7)$ and $(3, 8)$

Answer

$y = x + 5$

Exercise 3.6E.99

$m = \frac{1}{6}$, containing point $(6, 1)$

Exercise 3.6E.100

$m = \frac{5}{6}$, containing point $(6, 7)$

Answer

$y = \frac{5}{6}x + 2$

Exercise 3.6E.101

Parallel to the line $4x + 3y = 6$, containing point $(0, -3)$

Exercise 3.6E.102

Parallel to the line $2x + 3y = 6$, containing point $(0, 5)$

Answer

$y = -\frac{2}{3}x + 5$

Exercise 3.6E.103

$m = -\frac{3}{4}$, containing point $(8, -5)$

Exercise 3.6E.104

$m = -\frac{3}{5}$, containing point $(10, -5)$

Answer

$y = -\frac{3}{5}x + 1$

https://math.libretexts.org/@go/page/54984

Exercise 3.6E.105

Perpendicular to the line $y-1=0,$ point $(-2,6)$

Exercise 3.6E.106

Perpendicular to the line y-axis, point $(-6,2)$

Answer

$y=2$

Exercise 3.6E.107

Containing the points $(4,3)$ and $(8,1)$

Exercise 3.6E.108

Containing the points $(-2,0)$ and $(-3,-2)$

Answer

$y=x+2$

Exercise 3.6E.109

Parallel to the line $x=-3,$ containing point $(-2,-1)$

Exercise 3.6E.110

Parallel to the line $x=-4,$ containing point $(-3,-5)$

Answer

$x=-3$

Exercise 3.6E.111

Containing the points $(-3,-4)$ and $(2,-5)$

Exercise 3.6E.112

Containing the points $(-5,-3)$ and $(4,-6)$

Answer

$y=-\frac{1}{3}x-\frac{14}{3}$

Exercise 3.6E.113

Perpendicular to the line $x-2y=5,$ containing point $(-2,2)$

Exercise 3.6E.114

Perpendicular to the line $4x+3y=1$, containing point $(0,0)$

Answer

$y=\frac{3}{4}x$

Everyday Math

Exercise 3.6E.115

Cholesterol. The age, x, and LDL cholesterol evel, y, of two men are given by the points $(18,68)$ and $(27,122)$. Find a linear equation that models the relationship between age and LDL cholesterol level.

Exercise 3.6E.116

Fuel consumption. The city mpg, x, and highway mpg, y, of two cars are given by the points $(29,40)$ and $(19,28)$. Find a linear equation that models the relationship between city mpg and highway mpg.

Answer

$y=1.2x+5.2$

Writing Exercises

Exercise 3.6E.117

Why are all horizontal lines parallel?

Exercise 3.6E.118

Explain in your own words why the slopes of two perpendicular lines must have opposite signs.

Answer

Answers will vary.

Self Check

a. After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
find the equation of the line given the slope and *y*-intercept.			
find an equation of the line given the slope and a point.			
find an equation of the line given two points.			
find an equation of a line parallel to a given line.			
find an equation of a line perpendicular to a given line.			

b. On a scale of 1-10, how would you rate your mastery of this section in light of your responses on the checklist? How can you improve this?

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3.7: Chapter Review Exercises

Chapter 4 Review Exercises

Rectangular Coordinate System

Plot Points in a Rectangular Coordinate System

In the following exercises, plot each point in a rectangular coordinate system.

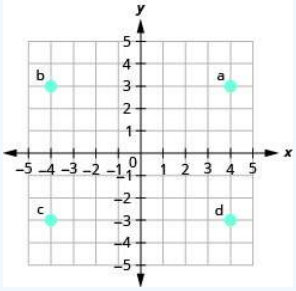
Exercise 3.7.1

1. $(-1,-5)$
2. $(-3,4)$
3. $(2,-3)$
4. $\left(1,\frac{5}{2}\right)$

Exercise 3.7.2

1. $(4,3)$
2. $(-4,3)$
3. $(-4,-3)$
4. $(4,-3)$

Answer



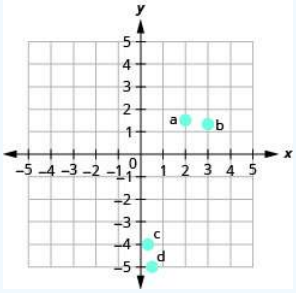
Exercise 3.7.3

1. $(-2,0)$
2. $(0,-4)$
3. $(0,5)$
4. $(3,0)$

Exercise 3.7.4

1. $\left(2,\frac{3}{2}\right)$
2. $\left(3,\frac{4}{3}\right)$
3. $\left(\frac{1}{3},-4\right)$
4. $\left(\frac{1}{2},-5\right)$

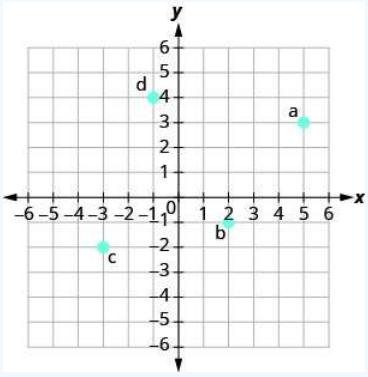
Answer



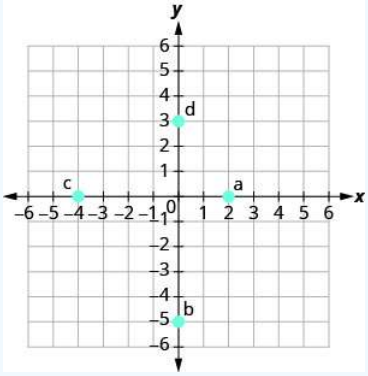
Identify Points on a Graph

In the following exercises, name the ordered pair of each point shown in the rectangular coordinate system.

Exercise 3.7.5



Exercise 3.7.6



Answer

a. (2,0)

b (0,−5)

c (−4.0)

d (0,3)

Verify Solutions to an Equation in Two Variables

In the following exercises, which ordered pairs are solutions to the given equations?

Exercise 3.7.7

$5x + y = 10$

1. (5,1)
2. (2,0)
3. (4,−10)

Exercise 3.7.8

$y = 6x - 2$

1. (1,4)
2. $\left(\frac{1}{3}, 0\right)$
3. (6,−2)

Answer

1, 2

Complete a Table of Solutions to a Linear Equation in Two Variables

In the following exercises, complete the table to find solutions to each linear equation.

Exercise 3.7.9

$y = 4x - 1$

x	y	(x,y)
0		
1		
-2		

Exercise 3.7.10

$y = -\frac{1}{2}x + 3$

x	y	(x,y)
0		
4		
-2		

Answer

x	y	(x,y)
0	3	(0,3)
4	1	(4, 1)
−2	4	(−2,4)

Exercise 3.7.11

$x + 2y = 5$

x	y	(x,y)
	0	
1		
-1		

Exercise 3.7.12

$3x + 2y = 6$

x	y	(x,y)
0		
	0	
-2		

Answer

x	y	(x,y)
0	−3	(0,−3)
2	0	(2,0)
−2	−6	(−2,−6)

Find Solutions to a Linear Equation in Two Variables

In the following exercises, find three solutions to each linear equation.

Exercise 3.7.13

$x + y = 3$

Exercise 3.7.14

$x + y = -4$

Answer

Answers will vary.

Exercise 3.7.15

$y = 3x + 1$

Exercise 3.7.16

$y = -x - 1$

Answer

Answers will vary.

Graphing Linear Equations

Recognize the Relation Between the Solutions of an Equation and its Graph

In the following exercises, for each ordered pair, decide:

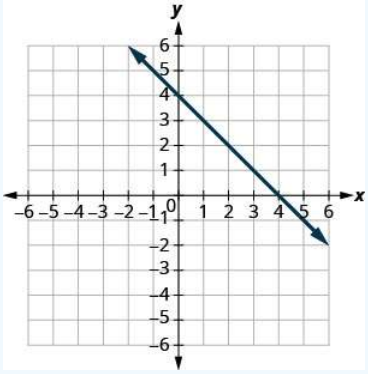
1. Is the ordered pair a solution to the equation?
2. Is the point on the line?

Exercise 3.7.17

$y = -x + 4$

(0,4) (−1,3)

(2,2) (−2,6)

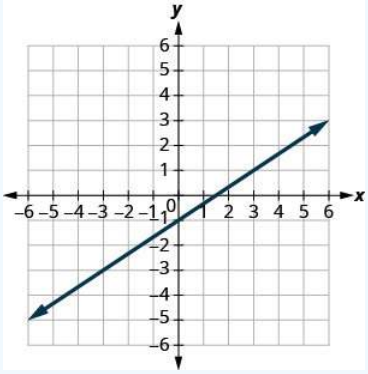


Exercise 3.7.18

$y = \frac{2}{3}x - 1$

$(0, -1)(3, 1)$

$(-3, -3)(6, 4)$



Answer

1. yes; yes
2. yes; no

Graph a Linear Equation by Plotting Points

In the following exercises, graph by plotting points.

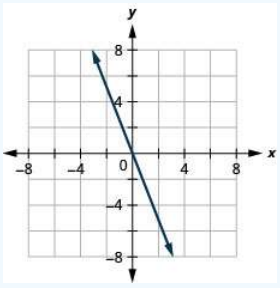
Exercise 3.7.19

$y = 4x - 3$

Exercise 3.7.20

$y = -3x$

Answer



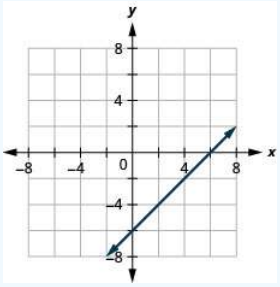
Exercise 3.7.21

$y = \frac{1}{2}x + 3$

Exercise 3.7.22

$x - y = 6$

Answer



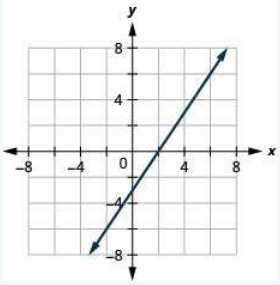
Exercise 3.7.23

$2x + y = 7$

Exercise 3.7.24

$3x - 2y = 6$

Answer



Graph Vertical and Horizontal lines

In the following exercises, graph each equation.

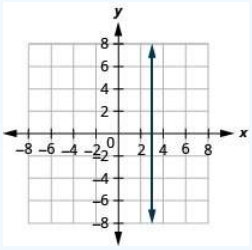
Exercise 3.7.25

$y = -2$

Exercise 3.7.26

$x = 3$

Answer



In the following exercises, graph each pair of equations in the same rectangular coordinate system.

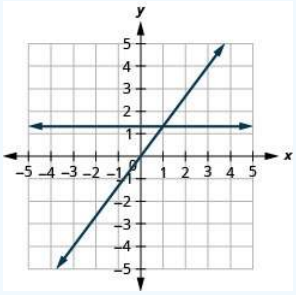
Exercise 3.7.27

$y = -2x$ and $y = -2$

Exercise 3.7.28

$y = \frac{4}{3}x$ and $y = \frac{4}{3}$

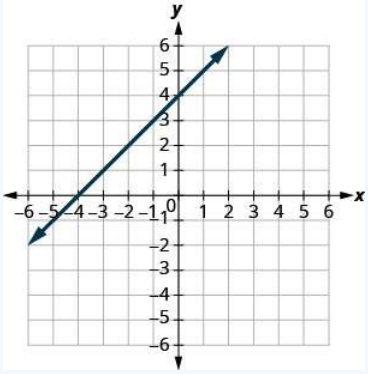
Answer



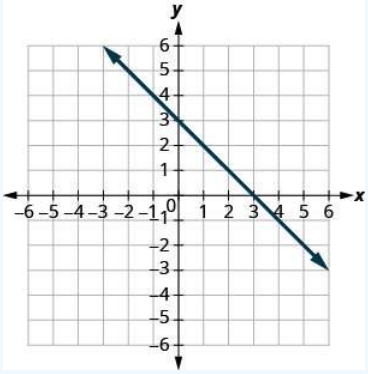
Identify the x- and y-Intercepts on a Graph

In the following exercises, find the x- and y-intercepts.

Exercise 3.7.29



Exercise 3.7.30



Answer

$(3, 0)$ and $(0, 3)$

Find the$x$- and y-Intercepts from an Equation of a Line

In the following exercises, find the intercepts of each equation.

Exercise 3.7.31

$x + y = 5$

Exercise 3.7.32

$x - y = -1$

Answer

$(-1, 0), (0, 1)$

Exercise 3.7.33

$x+2y=6$

Exercise 3.7.34

$2x+3y=12$

Answer

$(6,0),(0,4)$

Exercise 3.7.35

$y=\frac{3}{4}x-12$

Exercise 3.7.36

$y=3x$

Answer

$(0,0)$

Graph a Line Using the Intercepts

In the following exercises, graph using the intercepts.

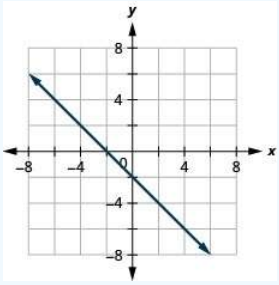
Exercise 3.7.37

$-x+3y=3$

Exercise 3.7.38

$x+y=-2$

Answer



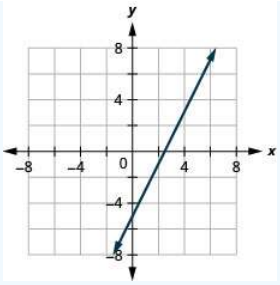
Exercise 3.7.39

$x-y=4$

Exercise 3.7.40

$2x-y=5$

Answer



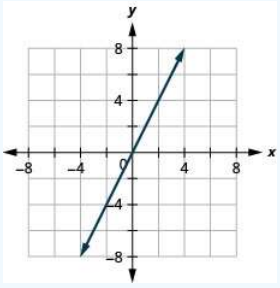
Exercise 3.7.41

$2x - 4y = 8$

Exercise 3.7.42

$y = 2x$

Answer

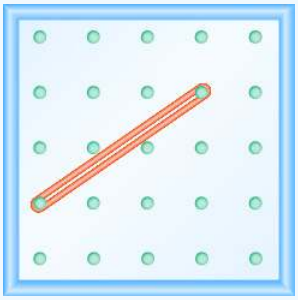


Slope of a Line

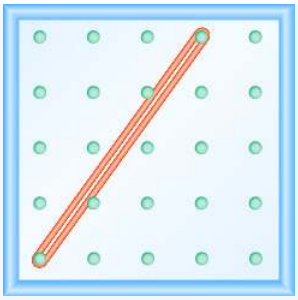
Use Geoboards to Model Slope

In the following exercises, find the slope modeled on each geoboard.

Exercise 3.7.43



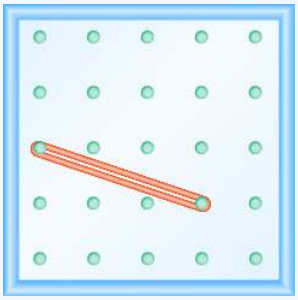
Exercise 3.7.44



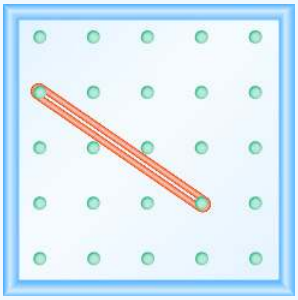
Answer

$\frac{4}{3}$

Exercise 3.7.45



Exercise 3.7.46



Answer

$-\frac{2}{3}$

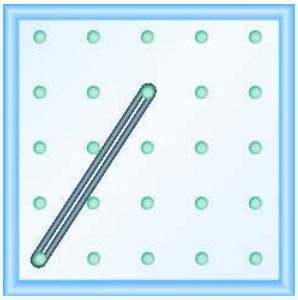
Exercise 3.7.47

$\frac{1}{3}$

Exercise 3.7.48

$\frac{3}{2}$

Answer



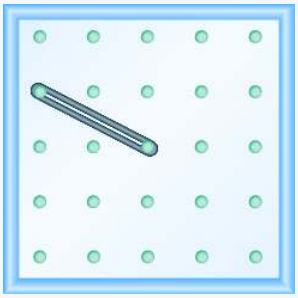
Exercise 3.7.49

$-\frac{2}{3}$

Exercise 3.7.50

$-\frac{1}{2}$

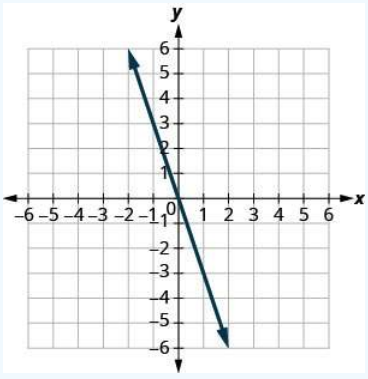
Answer



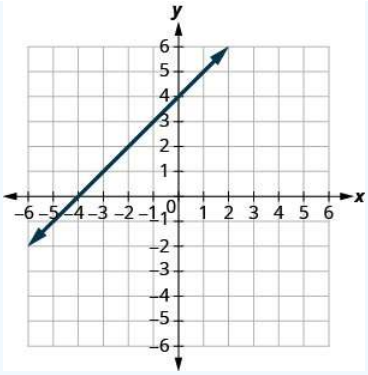
Use $m = \frac{\text{rise}}{\text{run}}$ to find the Slope of a Line from its Graph

In the following exercises, find the slope of each line shown.

Exercise 3.7.51



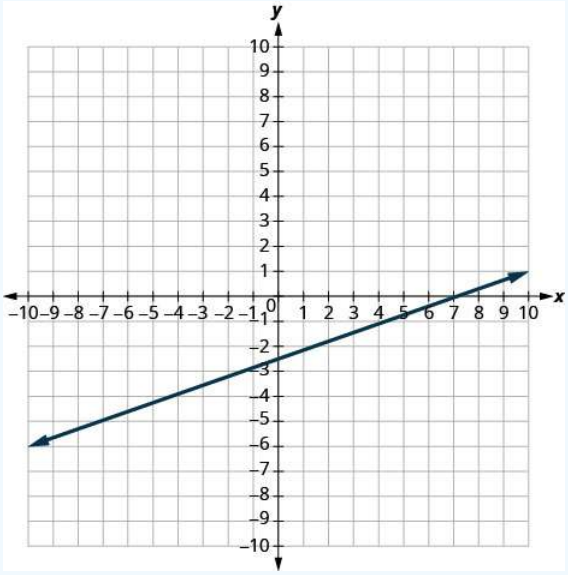
Exercise 3.7.52



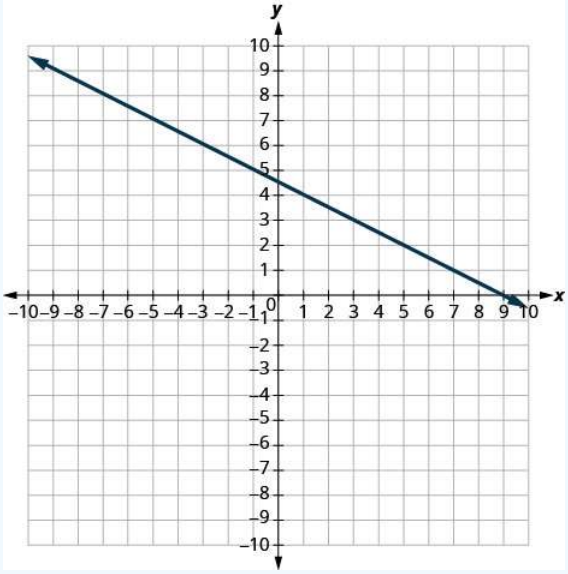
Answer

1

Exercise 3.7.53



Exercise 3.7.54



Answer

$-\frac{1}{2}$

Find the Slope of Horizontal and Vertical Lines

In the following exercises, find the slope of each line.

Exercise 3.7.55

$y = 2$

Exercise 3.7.56

$x = 5$

Answer

undefined

Exercise 3.7.57

$x = -3$

Exercise 3.7.58

$y = -1$

Answer

0

Use the Slope Formula to find the Slope of a Line between Two Points

In the following exercises, use the slope formula to find the slope of the line between each pair of points.

Exercise 3.7.59

$(-1,-1),(0,5)$

Exercise 3.7.60

$(3,5),(4,-1)$

Answer

-6

Exercise 3.7.61

$(-5,-2),(3,2)$

Exercise 3.7.62

$(2,1),(4,6)$

Answer

$\frac{5}{2}$

Graph a Line Given a Point and the Slope

In the following exercises, graph each line with the given point and slope.

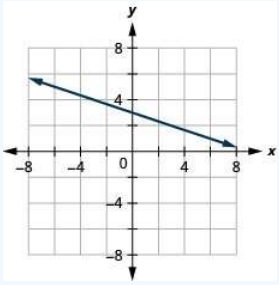
Exercise 3.7.63

$(2,-2)$; $m=\frac{5}{2}$

Exercise 3.7.64

$(-3,4)$; $m=-\frac{1}{3}$

Answer



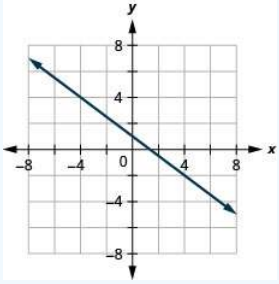
Exercise 3.7.65

x -intercept -4; $m=3$

Exercise 3.7.66

y -intercept 1; $m = -\frac{3}{4}$

Answer

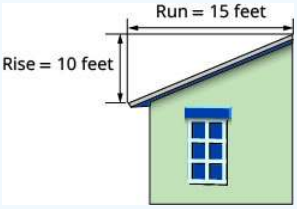


Solve Slope Applications

In the following exercises, solve these slope applications.

Exercise 3.7.67

The roof pictured below has a rise of 10 feet and a run of 15 feet. What is its slope?



Exercise 3.7.68

A mountain road rises 50 feet for a 500-foot run. What is its slope?

Answer

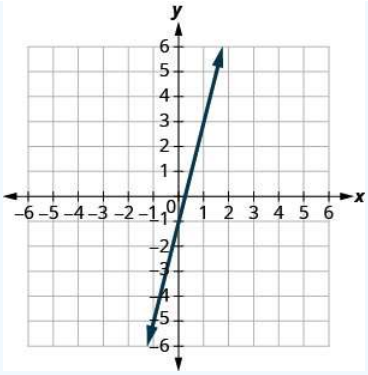
$\frac{1}{10}$

Intercept Form of an Equation of a Line

Recognize the Relation Between the Graph and the Slope–Intercept Form of an Equation of a Line

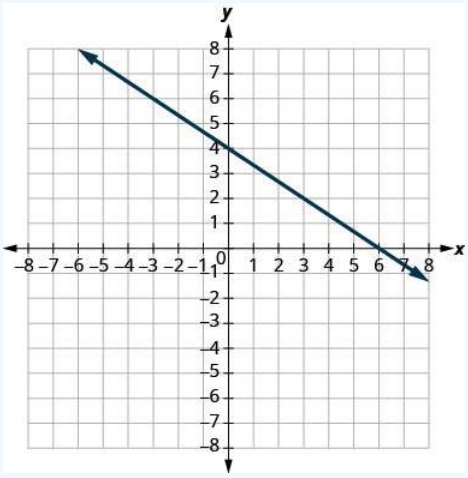
In the following exercises, use the graph to find the slope and y-intercept of each line. Compare the values to the equation $y = mx + b$.

Exercise 3.7.69



$y = 4x - 1$

Exercise 3.7.70



$y = -\frac{2}{3}x + 4$

Answer

slope $m = -\frac{2}{3}$ and y-intercept $(0, 4)$

Identify the Slope and y-Intercept from an Equation of a Line

In the following exercises, identify the slope and y-intercept of each line.

Exercise 3.7.71

$y = -4x + 9$

Exercise 3.7.72

$y = \frac{5}{3}x - 6$

Answer

$\frac{5}{3}; (0, -6)$

Exercise 3.7.73

$5x + y = 10$

Exercise 3.7.74

$4x - 5y = 8$

Answer

$\frac{4}{5}; \quad \left(0, -\frac{8}{5}\right)$

Graph a Line Using Its Slope and Intercept

In the following exercises, graph the line of each equation using its slope and y-intercept.

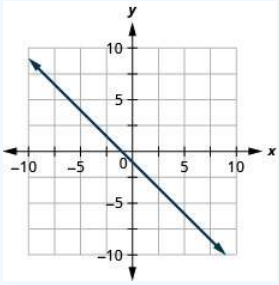
Exercise 3.7.75

$y = 2x + 3$

Exercise 3.7.76

$y = -x - 1$

Answer



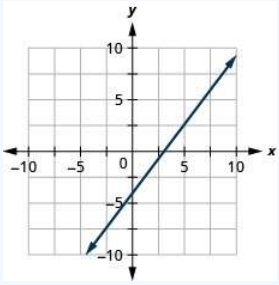
Exercise 3.7.77

$y = -\frac{2}{5}x + 3$

Exercise 3.7.78

$4x - 3y = 12$

Answer



In the following exercises, determine the most convenient method to graph each line.

Exercise 3.7.79

$x = 5$

Exercise 3.7.80

$y = -3$

Answer

horizontal line

Exercise 3.7.81

$2x + y = 5$

Exercise 3.7.82

$x - y = 2$

Answer

intercepts

Exercise 3.7.83

$y = x + 2$

Exercise 3.7.84

$y = \frac{3}{4}x - 1$

Answer

plotting points

Graph and Interpret Applications of Slope–Intercept

Exercise 3.7.85

Katherine is a private chef. The equation $C = 6.5m + 42$ models the relation between her weekly cost, C, in dollars and the number of meals, m, that she serves.

1. Find Katherine's cost for a week when she serves no meals.
2. Find the cost for a week when she serves 14 meals.
3. Interpret the slope and C-intercept of the equation.
4. Graph the equation.

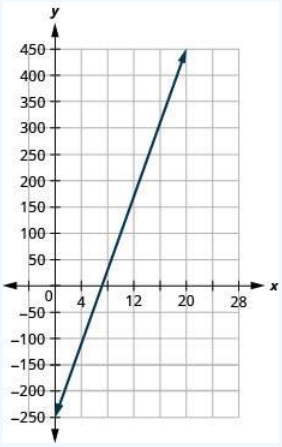
Exercise 3.7.86

Marjorie teaches piano. The equation $P = 35h - 250$ models the relation between her weekly profit, P, in dollars and the number of student lessons, s, that she teaches.

1. Find Marjorie's profit for a week when she teaches no student lessons.
2. Find the profit for a week when she teaches 20 student lessons.

3. Interpret the slope and P-intercept of the equation.
4. Graph the equation.

Answer

1. $-\$250$
2. $\$450$
3. The slope, 35, means that Marjorie's weekly profit, P, increases by $\$35$ for each additional student lesson she teaches. The P-intercept means that when the number of lessons is 0, Marjorie loses $\$250$
4. 

Use Slopes to Identify Parallel Lines

In the following exercises, use slopes and y-intercepts to determine if the lines are parallel.

Exercise 3.7.87

$4x-3y=-1; \quad y=\frac{4}{3}x-3$

Exercise 3.7.88

$2x-y=8; \quad x-2y=4$

Answer

not parallel

Use Slopes to Identify Perpendicular Lines

In the following exercises, use slopes and y-intercepts to determine if the lines are perpendicular.

Exercise 3.7.89

$y=5x-1; \quad 10x+2y=0$

Exercise 3.7.90

$3x-2y=5; \quad 2x+3y=6$

Answer

perpendicular

Find an Equation of the Line Given the Slope and *y*-Intercept

In the following exercises, find the equation of a line with given slope and y-intercept. Write the equation in slope–intercept form.

Exercise 3.7.91

slope $\frac{1}{3}$ and y-intercept $(0, -6)$

Exercise 3.7.92

slope -5 and y-intercept $(0, -3)$

Answer

$y = -5x - 3$

Exercise 3.7.93

slope 0 and y-intercept $(0, 4)$

Exercise 3.7.94

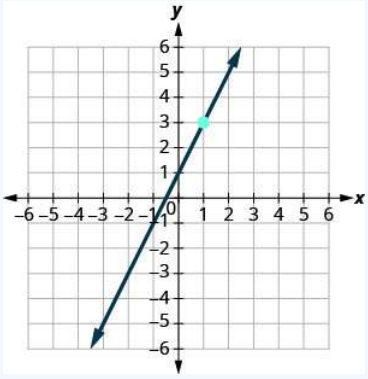
slope -2 and y-intercept $(0, 0)$

Answer

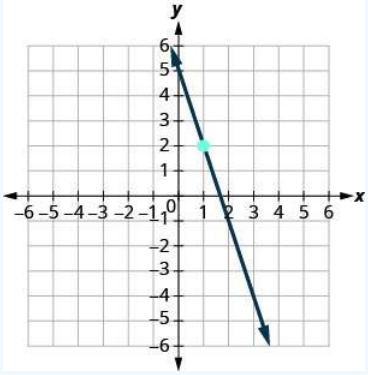
$y = -2x$

In the following exercises, find the equation of the line shown in each graph. Write the equation in slope–intercept form.

Exercise 3.7.95



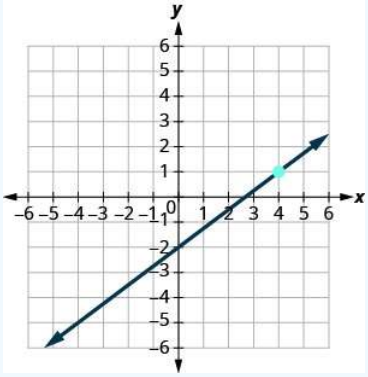
Exercise 3.7.96



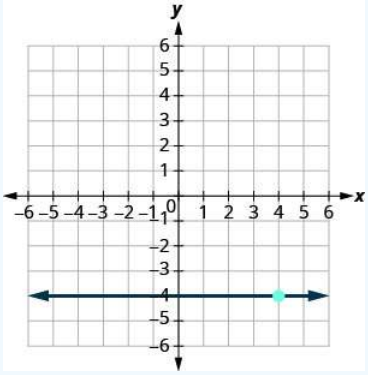
Answer

$y = -3x + 5$

Exercise 3.7.97



Exercise 3.7.98



Answer

$y = -4$

Find an Equation of the Line Given the Slope and a Point

In the following exercises, find the equation of a line with given slope and containing the given point. Write the equation in slope–intercept form.

Exercise 3.7.99

$m = -\frac{1}{4}$, point $(-8, 3)$

Exercise 3.7.100

$m = \frac{3}{5}$, point $(10, 6)$

Answer

$y = \frac{3}{5}x$

Exercise 3.7.101

Horizontal line containing $(-2, 7)$

Exercise 3.7.102

$m = -2$, point $(-1, -3)$

Answer

$y = -2x - 5$

Find an Equation of the Line Given Two Points

In the following exercises, find the equation of a line containing the given points. Write the equation in slope–intercept form.

Exercise 3.7.103

$(2, 10)$ and $(-2, -2)$

Exercise 3.7.104

$(7, 1)$ and $(5, 0)$

Answer

$y = \frac{1}{2}x - \frac{5}{2}$

Exercise 3.7.105

$(3, 8)$ and $(3, -4)$

Exercise 3.7.106

$(5, 2)$ and $(-1, 2)$

Answer

$y = 2$

Find an Equation of a Line Parallel to a Given Line

In the following exercises, find an equation of a line parallel to the given line and contains the given point. Write the equation in slope–intercept form.

Exercise 3.7.107

line $y = -3x + 6$, point $(1, -5)$

Exercise 3.7.108

line $2x + 5y = -10$, point $(10, 4)$

Answer

$y = -\frac{2}{5}x + 8$

Exercise 3.7.109

line $x = 4$, point $(-2, -1)$

Exercise 3.7.110

line $y = -5$, point $(-4, 3)$

Answer

$y = 3$

Find an Equation of a Line Perpendicular to a Given Line

In the following exercises, find an equation of a line perpendicular to the given line and contains the given point. Write the equation in slope–intercept form.

Exercise 3.7.111

line $y = -\frac{4}{5}x + 2$, point $(8, 9)$

Exercise 3.7.112

line $2x - 3y = 9$, point $(-4, 0)$

Answer

$y = -\frac{3}{2}x - 6$

Exercise 3.7.113

line $y = 3$, point $(-1, -3)$

Exercise 3.7.114

line $x = -5$ point $(2, 1)$

Answer

$y = 1$

Graph Linear Inequalities

Verify Solutions to an Inequality in Two Variables

In the following exercises, determine whether each ordered pair is a solution to the given inequality.

Exercise 3.7.115

Determine whether each ordered pair is a solution to the inequality $y < x - 3$:

1. $(0, 1)$
2. $(-2, -4)$
3. $(5, 2)$
4. $(3, -1)$
5. $(-1, -5)$

Exercise 3.7.116

Determine whether each ordered pair is a solution to the inequality $x + y > 4$:

1. $(6, 1)$
2. $(-3, 6)$
3. $(3, 2)$
4. $(-5, 10)$
5. $(0, 0)$

Answer

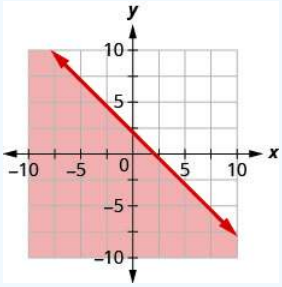
1. yes
2. no
3. yes
4. yes
5. no

Recognize the Relation Between the Solutions of an Inequality and its Graph

In the following exercises, write the inequality shown by the shaded region.

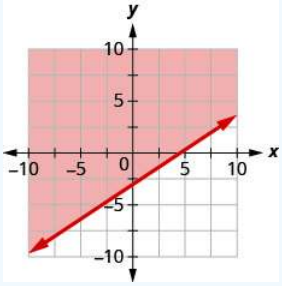
Exercise 3.7.117

Write the inequality shown by the graph with the boundary line $y = -x + 2$.



Exercise 3.7.118

Write the inequality shown by the graph with the boundary line $y = \frac{2}{3}x - 3$

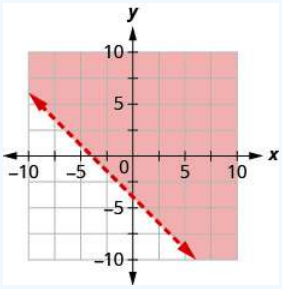


Answer

$y > \frac{2}{3}x - 3$

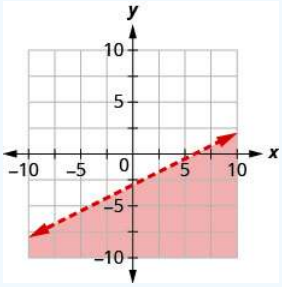
Exercise 3.7.119

Write the inequality shown by the shaded region in the graph with the boundary line $x + y = -4$.



Exercise 3.7.120

Write the inequality shown by the shaded region in the graph with the boundary line $x - 2y = 6$.



Answer

$x - 2y \geq 6$

Graph Linear Inequalities

In the following exercises, graph each linear inequality.

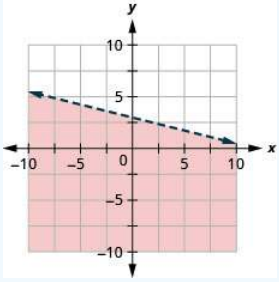
Exercise 3.7.121

Graph the linear inequality $y > \frac{2}{5}x - 4$

Exercise 3.7.122

Graph the linear inequality $y \leq -\frac{1}{4}x + 3$

Answer



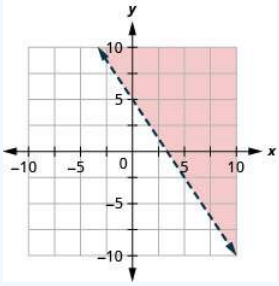
Exercise 3.7.123

Graph the linear inequality $x - y \leq 5$

Exercise 3.7.124

Graph the linear inequality $3x + 2y > 10$

Answer



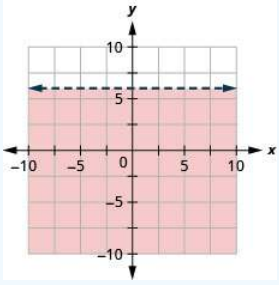
Exercise 3.7.125

Graph the linear inequality $y \leq -3x$

Exercise 3.7.126

Graph the linear inequality $y < 6$

Answer



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Practice Test

Exercise 3.7.1

Plot each point in a rectangular coordinate system.

1. $(2, 5)$
2. $(-1, -3)$
3. $(0, 2)$
4. $\left(-4, \frac{3}{2}\right)$
5. $(5, 0)$

Exercise 3.7.2

Which of the given ordered pairs are solutions to the equation $3x - y = 6$?

1. $(3, 3)$
2. $(2, 0)$
3. $(4, -6)$

Answer

1. yes
2. yes
3. no

Exercise 3.7.3

Find three solutions to the linear equation $y = -2x - 4$

Exercise 3.7.4

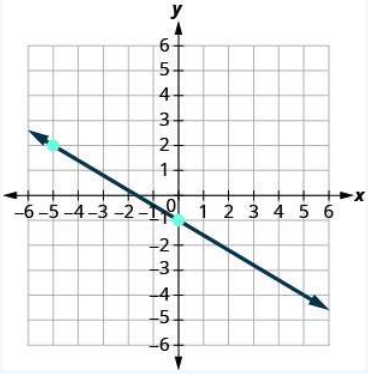
Find the x- and y-intercepts of the equation $4x - 3y = 12$

Answer

$(3, 0), (0, -4)$

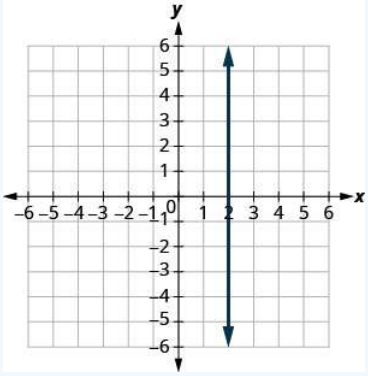
Find the slope of each line shown.

Exercise 3.7.5



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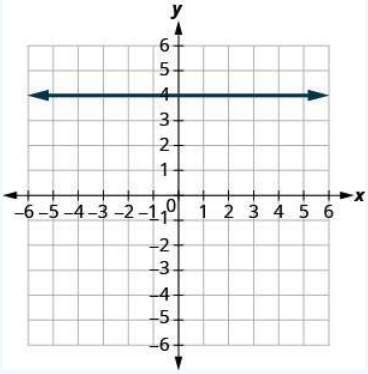
Exercise 3.7.6



Answer

undefined

Exercise 3.7.7



Exercise 3.7.8

Find the slope of the line between the points $(5, 2)$ and $(-1, -4)$

Answer

1

Exercise 3.7.9

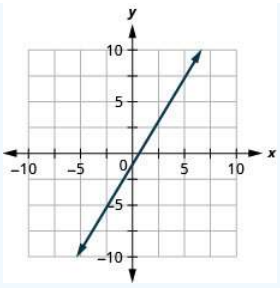
Graph the line with slope $\frac{1}{2}$ containing the point $(-3, -4)$

Graph the line for each of the following equations.

Exercise 3.7.10

$y = \frac{5}{3}x - 1$

Answer



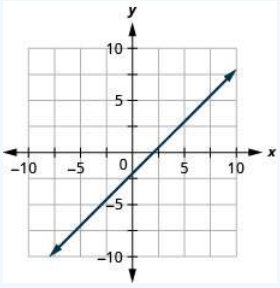
Exercise 3.7.11

$y = -x$

Exercise 3.7.12

$x - y = 2$

Answer



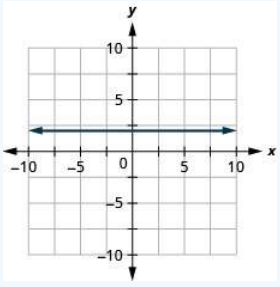
Exercise 3.7.13

$4x + 2y = -8$

Exercise 3.7.14

$y = 2$

Answer



Exercise 3.7.15

$x = -3$

Find the equation of each line. Write the equation in slope–intercept form.

Exercise 3.7.16

slope $-\frac{3}{4}$ and y-intercept $(0, -2)$

Answer

$y = -\frac{3}{4}x - 2$

Exercise 3.7.17

$m = 2$, point $(-3, -1)$

Exercise 3.7.18

containing $(10, 1)$ and $(6, -1)$

Answer

$y = \frac{1}{2}x - 4$

Exercise 3.7.19

parallel to the line $y = -\frac{2}{3}x - 1$, containing the point $(-3, 8)$

Exercise 3.7.20

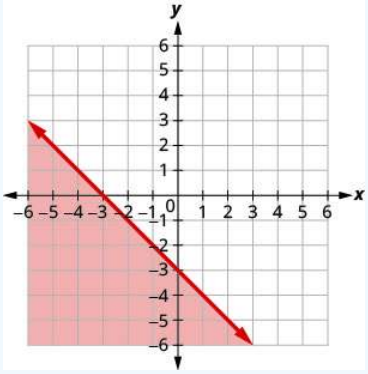
perpendicular to the line $y = \frac{5}{4}x + 2$, containing the point $(-10, 3)$

Answer

$y = -\frac{4}{5}x - 5$

Exercise 3.7.21

Write the inequality shown by the graph with the boundary line $y = -x - 3$.



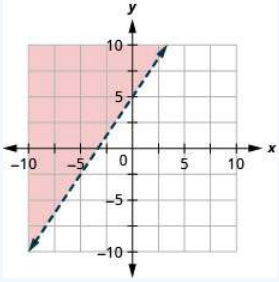
Graph each linear inequality.

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Exercise 3.7.22

$y > \frac{3}{2}x + 5$

Answer



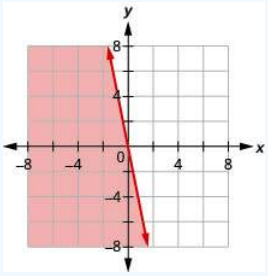
Exercise 3.7.23

$x - y \geq -4$

Exercise 3.7.24

$y \leq -5x$

Answer



Exercise 3.7.1

$y < 3$

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CHAPTER OVERVIEW

4: Functions

4.1: Introduction to Functions
4.2: Relations and Functions
4.2E: Exercises
4.3: Interpreting the Graph of a Function
4.4: The Toolbox Functions
4.4E: Exercises
4.5: Linear Functions - Applications
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4.7: Quadratic Functions
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4.9: Absolute Value Functions
4.10: Exponential Functions
4.11: Regression

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4.1: Introduction to Functions

Our development of the function concept is a modern one, but quite quick, particularly in light of the fact that today's definition took over 300 years to reach its present state. We begin with the definition of a relation.

Relations

We use the notation (2, 4) to denote what is called an ordered pair. If you think of the positions taken by the ordered pairs (4, 2) and (2, 4) in the coordinate plane (see Figure 4.1.1), then it is immediately apparent why order is important. The ordered pair (4, 2) is simply not the same as the ordered pair (2, 4).

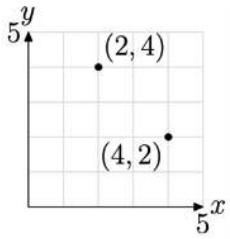


Figure 4.1.1

The first element of an ordered pair is called its abscissa. The second element of an ordered pair is called its ordinate. Thus, for example, the abscissa of (4, 2) is 4, while the ordinate of (4, 2) is 2.

Definition

A collection of ordered pairs is called a **relation**. (2)

For example, the collection of ordered pairs

$$R = \{(0, 1), (0, 2), (3, 4)\}$$

is a relation.

Definition

The **domain** of a relation is the collection of all abscissas of each ordered pair.

Thus, the domain of the relation R in (2) is

$$\text{Domain } = \{0, 3\}$$

Note that we list each abscissa only once.

Definition

The **range** of a relation is the collection of all ordinates of each ordered pair.

Thus, the range of the relation R in (2) is

$$\text{Range } = \{1, 2, 4\}$$

Let's look at an example.

Example 4.1.1

Consider the relation T defined by

$$T = \{(1, 2), (3, 2), (4, 5)\}$$

What are the domain and range of this relation?

Solution

The domain is the collection of abscissas of each ordered pair. Hence, the domain of T is

$$\text{Domain } = \{1, 3, 4\}$$

The range is the collection of ordinates of each ordered pair. Hence, the range of T is

$$\text{Range } = \{2, 5\}$$

Note that we list each ordinate in the range only once.

In Example 4.1.1, the relation is described by listing the ordered pairs. This is not the only way that one can describe a relation. For example, a graph certainly represents a collection of ordered pairs.

Example 4.1.2

Consider the graph of the relation S shown in Figure 4.1.2.

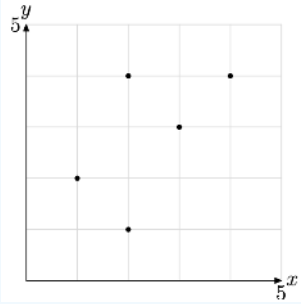


Figure 4.1.2 The graph of a relation.

What are the domain and range of the relation S?

Solution

There are five ordered pairs (points) plotted in Figure 4.1.2. They are

$$S = \{(1, 2), (2, 1), (2, 4), (3, 3), (4, 4)\}$$

Therefore, the relation S has Domain = {1, 2, 3, 4} and Range = {1, 2, 3, 4}. In the case of the range, note how we've sorted the ordinates of each ordered pair in ascending order, taking care not to list any ordinate more than once.

Functions

A function is a very special type of relation. We begin with a formal definition.

Definition

A relation is a function if and only if each object in its domain is paired with one and only one object in its range.

This is not an easy definition, so let's take our time and consider a few examples. Consider, if you will, the relation R in (2), repeated here again for convenience.

$$R = \{(0, 1), (0, 2), (3, 4)\}$$

The domain is {0, 3} and the range is {1, 2, 4}. Note that the number 0 in the domain of R is paired with two numbers from the range, namely, 1 and 2. Therefore, R is not a function.

There is a construct, called a mapping diagram, which can be helpful in determining whether a relation is a function. To craft a mapping diagram, first list the domain on the left, then the range on the right, then use arrows to indicate the ordered pairs in your relation, as shown in Figure 4.1.3.

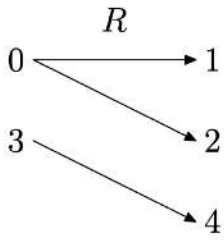


Figure 4.1.3 A mapping diagram for R.

It's clear from the mapping diagram in Figure 4.1.3 that the number 0 in the domain is being paired (mapped) with two different range objects, namely, 1 and 2. Thus, R is not a function.

Let's look at another example.

Example 4.1.3

Is the relation described in Example 4.1.1 a function?

Solution

First, let's repeat the listing of the relation T here for convenience.

$$T = \{(1, 2), (3, 2), (4, 5)\}$$

Next, construct a mapping diagram for the relation T. List the domain on the left, the range on the right, then use arrows to indicate the pairings, as shown in Figure 4.1.4.

From the mapping diagram in Figure 4.1.4, we can see that each domain object on the left is paired (mapped) with exactly one range object on the right. Hence, the relation T is a function.

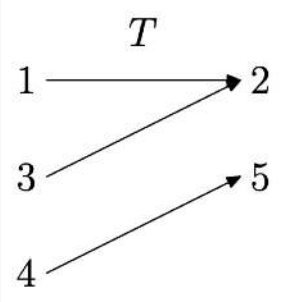


Figure 4.1.4. A mapping diagram for T.

Let's look at another example.

Example 4.1.4

Is the relation of Example 4.1.2, pictured in Figure 4.1.2, a function?

Solution

First, we repeat the graph of the relation from Example 4.1.2 here for convenience. This is shown in Figure 4.1.5(a). Next, we list the ordered pairs of the relation S.

$$S = \{(1, 2), (2, 1), (2, 4), (3, 3), (4, 4)\}$$

Then we create a mapping diagram by first listing the domain on the left, the range on the right, then using arrows to indicate the pairings, as shown in Figure 4.1.5(b).

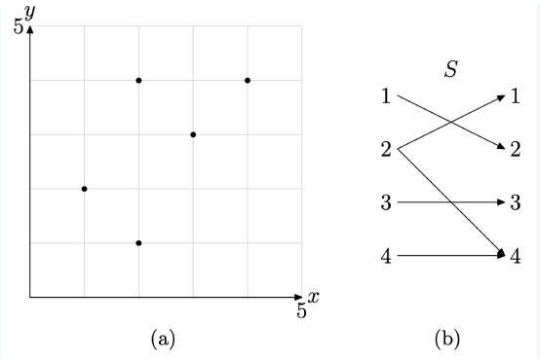


Figure 4.1.5 A graph of the relation S and its corresponding mapping diagram

Each object in the domain of S gets mapped to exactly one range object with one exception. The domain object 2 is paired with two range objects, namely, 1 and 4. Consequently, S is not a function.

This is a good point to summarize what we've learned about functions thus far.

Summary

A function consists of three parts:

1. a set of objects which mathematicians call the **domain**,
2. a second set of objects which mathematicians call the **range**,
3. and a **rule** that describes how to assign a unique range object to each object in the domain.

The rule can take many forms. For example, we can use sets of ordered pairs, graphs, and mapping diagrams to describe the function. In the sections that follow, we will explore other ways of describing a function, for example, through the use of equations and simple word descriptions.

Function Notation

We've used the word "mapping" several times in the previous examples. This is not a word to be taken lightly; it is an important concept. In the case of the mapping diagram in Figure 4.1.5(b), we would say that the number 1 in the domain of S is "mapped" (or "sent") to the number 2 in the range of S.

There are a number of different notations we could use to indicate that the number 1 in the domain is "mapped" or "sent" to the number 2 in the range. One possible notation is

$$S : 1 \longrightarrow 2$$

which we would read as follows: "The relation S maps (sends) 1 to 2." In a similar vein, we see in Figure 4.1.5(b) that the domain objects 3 and 4 are mapped (sent) to the range objects 3 and 4, respectively. In symbols, we would write

$$S : 3 \longrightarrow 3, \text{ and}$$
$$S : 4 \longrightarrow 4$$

A difficulty arises when we examine what happens to the domain object 2. There are two possibilities, either

$$S : 2 \longrightarrow 1$$

or

$$S : 2 \longrightarrow 4$$

Which should we choose? The 1? Or the 4? Thus, S is not well-defined and is not a function, since we don't know which range object to pair with the domain object 1.

The idea of mapping gives us an alternative way to describe a function. We could say that a function is a rule that assigns a unique object in its range to each object in its domain. Take for example, the function that maps each real number to its square. If we name

the function f, then f maps 5 to 25, 6 to 36, −7 to 49, and so on. In symbols, we would write

$$f : 5 \longrightarrow 25, \quad f : 6 \longrightarrow 36, \quad \text{and} \quad f : -7 \longrightarrow 49$$

In general, we could write

$$f : x \longrightarrow x^2$$

Note that each real number x gets mapped to a unique number in the range of f, namely, x^2. Consequently, the function f is well defined. We've succeeded in writing a rule that completely defines the function f.

As another example, let's define a function that takes a real number, doubles it, then adds 3. If we name the function g, then g would take the number 7, double it, then add 3. That is,

$$g : 7 \longrightarrow 2(7) + 3$$

Simplifying, $g : 7 \longrightarrow 17$. Similarly, g would take the number 9, double it, then add 3. That is,

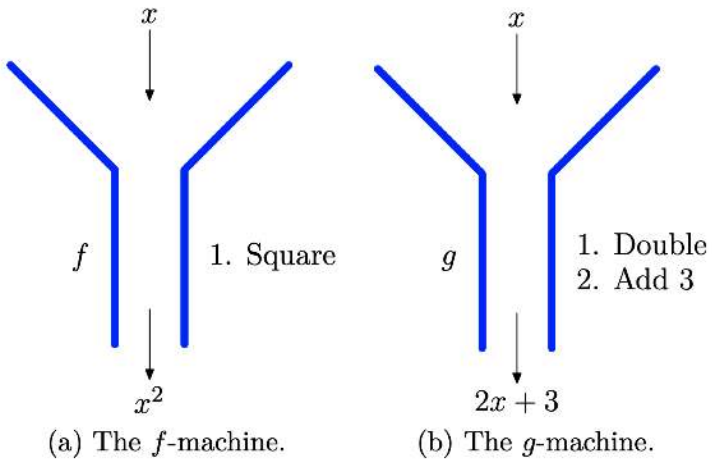
$$g : 9 \longrightarrow 2(9) + 3$$

Simplifying, $g : 9 \longrightarrow 21$. In general, g takes a real number x, doubles it, then adds three. In symbols, we would write

$$g : x \longrightarrow 2x + 3$$

Notice that each real number x is mapped by g to a unique number in its range. Therefore, we've again defined a rule that completely defines the function g.

It is helpful to think of a function as a machine. The machine receives input, processes it according to some rule, then outputs a result. Something goes in (input), then something comes out (output). In the case of the function described by the rule $f : x \longrightarrow x^2$, the "f-machine" receives input x, then applies its "square rule" to the input and outputs x^2, as shown in Figure 4.1.6(a). In the case of the function described by the rule $g : x \longrightarrow 2x + 3$, the "g-machine" receives input x, then applies the rules "double," then "add 3," in that order, then outputs $2x + 3$, as shown in Figure 4.1.6(b).



(a) The f-machine. (b) The g-machine.

Figure 4.1.6 Function machines.

Let's look at another example.

Example 4.1.5

Suppose that f is defined by the following rule. For each real number x,

$$f : x \longrightarrow x^2 - 2x - 3$$

Where does f map the number −3? Is f a function?

Solution

We substitute −3 for x in the rule for f and obtain

$$f : -3 \longrightarrow (-3)^2 - 2(-3) - 3$$

Simplifying,

$$f : -3 \longrightarrow 9 + 6 - 3$$

or,

$$f : -3 \longrightarrow 12$$

Thus, f maps (sends) the number −3 to the number 12. It should be clear that each real number x will be mapped (sent) to a unique real number, as defined by the rule $f : x \longrightarrow x^2 - 2x - 3$. Therefore, f is a function.

Let's look at another example.

Example 4.1.6

Suppose that g is defined by the following rule. For each real number x that is greater than or equal to zero,

$$g : x \longrightarrow \pm\sqrt{x}$$

Where does g map the number 4? Is g a function?

Solution

Again, we substitute 4 for x in the rule for g and obtain

$$g : 4 \longrightarrow \pm\sqrt{4}$$

Simplifying,

$$g : 4 \longrightarrow \pm 2$$

Thus, g maps (sends) the number 4 to two different objects in its range, namely, 2 and −2. Consequently, g is not well-defined and is not a function.

Let's look at another example

Example 4.1.7

Suppose that we have functions f and g, defined by

$$f : x \longrightarrow x^4 + 11 \quad \text{and} \quad g : x \longrightarrow (x+2)^2$$

Where does g send 5?

Solution

In this example, we see a clear advantage of function notation. Because our functions have distinct names, we can simply reference the name of the function we want our readers to use. In this case, we are asked where the function g sends the number 5, so we substitute 5 for x in

$$g : x \longrightarrow (x+2)^2$$

That is,

$$g : 5 \longrightarrow (5+2)^2$$

Simplifying, $g : 5 \longrightarrow 49$.

Modern Notation

Function notation is relatively new, with some of the earliest symbolism first occurring in the 17th century. In a letter to Leibniz (1698), Johann Bernoulli wrote "For denoting any function of a variable quantity x, I rather prefer to use the capital letter having the same name X or the Greek ξ, for it appears at once of what variable it is a function; this relieves the memory."

Mathematicians are fond of the notation

$$f : x \longrightarrow x^2 - 2x$$

because it conveys a sense of what a function does; namely, it "maps" or "sends" the number x to the number $x^2 - 2x$. This is what functions do, they pair each object in their domain with a unique object in their range. Equivalently, functions "send" each object in their domain to a unique object in their range.

However, in common computational situations, the "arrow" notation can be a bit clumsy, so mathematicians tend to favor a slightly different notation. Instead of writing

$$f : x \longrightarrow x^2 - 2x$$

mathematicians tend to favor the notation

$$f(x) = x^2 - 2x$$

It is important to understand from the outset that these two different notations are equivalent; they represent the same function f, one that pairs each real number x in its domain with the real number $x^2 - 2x$ in its range.

The first notation, $f : x \longrightarrow x^2 - 2x$, conveys the sense that the function f is a mapping. If we read this notation aloud, we should pronounce it as "f sends (or maps) x to $x^2 - 2x$." The second notation, $f(x) = x^2 - 2x$, is pronounced "f of x equals $x^2 - 2x$."

Note

The phrase "f of x" is unfortunate, as our readers might recall being trained from a very early age to pair the word "of" with the operation of multiplication. For example, 1/2 of 12 is 6, as in $1/2 \times 12 = 6$. However, in the context of function notation, even though f(x) is read aloud as "f of x," it does **not** mean "f times x." Indeed, if we remind ourselves that the notation $f(x) = x^2 - 2x$ is equivalent to the notation $f : x \longrightarrow x^2 - 2x$, then even though we might say "f of x," we should be thinking "f sends x" or "f maps x." We should **not** be thinking "f times x."

Now, let's see how each of these notations operates on the number 5. In the first case, using the "arrow" notation,

$$f : x \longrightarrow x^2 - 2x$$

To find where f sends 5, we substitute 5 for x as follows.

$$f : 5 \longrightarrow (5)^2 - 2(5)$$

Simplifying, $f : 5 \longrightarrow 15$. Now, because both notations are equivalent, to compute f(5), we again substitute 5 for x in

$$f(x) = x^2 - 2x$$

Thus,

$$f(5) = (5)^2 - 2(5)$$

Simplifying, $f(5) = 15$. This result is read aloud as "f of 5 equals 15," but we want to be thinking "f sends 5 to 15."

Let's look at examples that use this modern notation.

Example 4.1.8

Given $f(x) = x^3 + 3x^2 - 5$, determine $f(-2)$

Solution

Simply substitute −2 for x. That is,

$$\begin{aligned} f(-2) &= (-2)^3 + 3(-2)^2 - 5 \\ &= -8 + 3(4) - 5 \\ &= -8 + 12 - 5 \\ &= -1 \end{aligned}$$

Thus, $f(-2) = -1$. Again, even though this is pronounced "f of −2 equals −1," we still should be thinking "f sends −2 to −1."

Example 4.1.9

Given

$$f(x) = \frac{x+3}{2x-5}$$

determine f(6).

Solution

Simply substitute 6 for x. That is,

$$\begin{aligned} f(6) &= \frac{6+3}{2(6)-5} \\ &= \frac{9}{12-5} \\ &= \frac{9}{7} \end{aligned}$$

Thus, $f(6) = 9/7$. Again, even though this is pronounced "f of 6 equals 9/7," we should still be thinking "f sends 6 to 9/7."

Example 4.1.10

Given $f(x) = 5x - 3$, determine $f(a+2)$.

Solution

If we're thinking in terms of mapping notation, then

$$f : x \longrightarrow 5x - 3$$

Think of this mapping as a "machine." Whatever we put into the machine, it first is multiplied by 5, then 3 is subtracted from the result, as shown in Figure 4.1.7. For example, if we put a 4 into the machine, then the function rule requires that we multiply input 4 by 5, then subtract 3 from the result. That is,

$$f : 4 \longrightarrow 5(4) - 3$$

Simplifying, $f : 4 \longrightarrow 17$

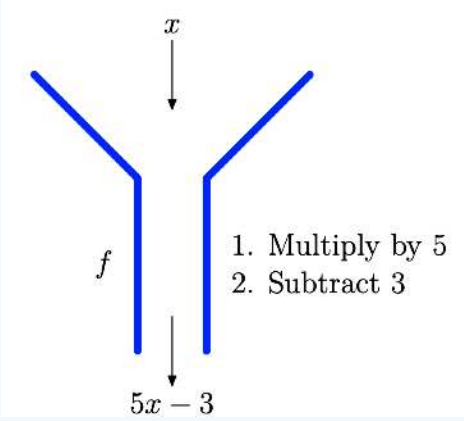


Figure 4.1.7. The multiply by 5 then subtract 3 machine.

Similarly, if we put an a + 2 into the machine, then the function rule requires that we multiply the input a + 2 by 5, then subtract 3 from the result. That is,

$$f : a+2 \longrightarrow 5(a+2) - 3$$

Using modern function notation, we would write

$$f(a+2) = 5(a+2) - 3$$

Note that this is again a simple substitution, where we replace each occurrence of x in the formula $f(x) = 5x - 3$ with the expression a + 2. Finally, use the distributive property to first multiply by 5, then subtract 3.

$$\begin{aligned} f(a+2) &= 5a + 10 - 3 \\ &= 5a + 7 \end{aligned}$$

We will often need to substitute the result of one function evaluation into a second function for evaluation. Let's look at an example.

Example 4.1.11

Given two functions defined by $f(x) = 3x + 2$ and $g(x) = 5 - 4x$, find f(g(2)).

Solution

The nested parentheses in the expression f(g(2)) work in the same manner that they do with nested expressions. The rule is to work the innermost grouping symbols first, proceeding outward as you work. We'll first evaluate g(2), then evaluate f at the result.

We begin. First, evaluate g(2) by substituting 2 for x in the defining equation $g(x) = 5 - 4x$. Note that $g(2) = 5 - 4(2)$, then simplify.

$$f(g(2)) = f(5 - 4(2)) = f(5 - 8) = f(-3)$$

To complete the evaluation, we substitute −3 for x in the defining equation $f(x) = 3x + 2$, then simplify.

$$f(-3) = 3(-3) + 2 = -9 + 2 = -7$$

Hence, $f(g(2)) = -7$.

It is conventional to arrange the work in one contiguous block, as follows.

$$\begin{aligned} f(g(2)) &= f(5 - 4(2)) \\ &= f(-3) \\ &= 3(-3) + 2 \\ &= -7 \end{aligned}$$

You can shorten the task even further if you are willing to do the function substitution and simplification in your head. First, evaluate g at 2, then f at the result.

$$f(g(2)) = f(-3) = -7$$

Let's look at another example of this unique way of combining functions.

Example 4.1.12

Given $f(x) = 5x + 2$ and $g(x) = 3 - 2x$, evaluate $g(f(a))$ and simplify the result.

Solution

We work the inner function evaluation in the expression $g(f(a))$ first. Thus, to evaluate f(a), we substitute a for x in the definition $f(x) = 5x + 2$ to get

$$g(f(a)) = g(5a + 2)$$

Now we need to evaluate $g(5a + 2)$. To do this, we substitute $5a + 2$ for x in the definition $g(x) = 3 - 2x$ to get

$$g(5a + 2) = 3 - 2(5a + 2)$$

We can expand this last result and simplify. Thus,

$$g(f(a)) = 3 - 10a - 4 = -10a - 1$$

Again, it is conventional to arrange the work in one continuous block, as follows.

$$\begin{aligned} g(f(a)) &= g(5a+2) \\ &= 3 - 2(5a+2) \\ &= 3 - 10a - 4 \\ &= -10a - 1 \end{aligned}$$

Hence, $g(f(a)) = -10a - 1$.

Extracting the Domain of a Function

We've seen that the domain of a relation or function is the set of all the first coordinates of its ordered pairs. However, if a functional relationship is defined by an equation such as $f(x) = 3x - 4$, then it is not practical to list all ordered pairs defined by this relationship. For any real x-value, you get an ordered pair. For example, if x = 5, then $f(5) = 3(5) - 4 = 11$, leading to the ordered pair (5, f(5)) or (5, 11). As you can see, the number of such ordered pairs is infinite. For each new x-value, we get another function value and another ordered pair.

Therefore, it is easier to turn our attention to the values of x that yield real number responses in the equation $f(x) = 3x - 4$. This leads to the following key idea.

Definition

If a function is defined by an equation, then the domain of the function is the set of "permissible x-values," the values that produce a real number response defined by the equation.

We sometimes like to say that the domain of a function is the set of "OK x-values to use in the equation." For example, if we define a function with the rule $f(x) = 3x - 4$, it is immediately apparent that we can use any value we want for x in the rule $f(x) = 3x - 4$. Thus, the domain of f is all real numbers. We can write that the domain $D = \mathbb{R}$, or we can use interval notation and write that the domain $D = (-\infty, \infty)$.

It is not the case that x can be any real number in the function defined by the rule $f(x) = \sqrt{x}$. It is not possible to take the square root of a negative number.2 Therefore, x must either be zero or a positive real number. In set-builder notation, we can describe the domain with $D = \{x : x \geq 0\}$. In interval notation, we write $D = [0, \infty)$.

We must also be aware of the fact that we cannot divide by zero. If we define a function with the rule $f(x) = x/(x-3)$, we immediately see that x = 3 will put a zero in the denominator. Division by zero is not defined. Therefore, 3 is not in the domain of f. No other x-value will cause a problem. The domain of f is best described with set-builder notation as $D = \{x : x \neq 3\}$.

Functions Without Formulae

In the previous section, we defined functions by means of a formula, for example, as in

$$f(x) = \frac{x+3}{2-3x}$$

Euler would be pleased with this definition, for as we have said previously, Euler thought of functions as analytic expressions.

However, it really isn't necessary to provide an expression or formula to define a function. There are other forms we can use to express a functional relationship: a graph, a table, or even a narrative description. The only thing that is really important is the requirement that the function be well-defined, and by "well-defined," we mean that each object in the function's domain is paired with one and only one object in its range.

As an example, let's look at a special function π on the natural numbers,3 which returns the number of primes less than or equal to a given natural number. For example, the primes less than or equal to the number 23 are 2, 3, 5, 7, 11, 13, 17, 19, and 23, nine numbers in all. Therefore, the number of primes less than or equal to 23 is nine. In symbols, we would write

$$\pi(23) = 9$$

Note the absence of a formula in the definition of this function. Indeed, the definition is descriptive in nature, so we might write

$$\pi(n) = \text{ number of primes less than or equal to } n$$

The important thing is not how we define this special function π, but the fact that it is well-defined; that is, for each natural number n, there are a fixed number of primes less than or equal to n. Thus, each natural number in the domain of π is paired with one and only one number in its range.

Now, just because our function doesn't provide an expression for calculating the number of primes less than or equal to a given natural number n, it doesn't stop mathematicians from seeking such a formula. Euclid of Alexandria (325-265 BC), a Greek mathematician, proved that the number of primes is infinite, but it was the German mathematician and scientist, Johann Carl Friedrich Gauss (1777-1855), who first proposed that the number of primes less than or equal to n can be approximated by the formula

$$\pi(n) \approx \frac{n}{\ln n}$$

where ln n is the "natural logarithm" of n (to be explained in Chapter 9). This approximation gets better and better with larger and larger values of n. The formula was refined by Gauss, who did not provide a proof, and the problem became known as the Prime Number Theorem. It was not until 1896 that Jacques Salomon Hadamard (1865-1963) and Charles Jean Gustave Nicolas Baron de la Vallee Poussin (1866-1962), working independently, provided a proof of the Prime Number Theorem.

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4.2: Relations and Functions

Learning Objectives

By the end of this section, you will be able to:

- Find the domain and range of a relation
- Determine if a relation is a function
- Find the value of a function

Before you get started, take this readiness quiz.

1. Evaluate $3x-5$ when $x=-2$.
   If you missed this problem, review [link].
2. Evaluate $2x^2-x-3$ when $x=a$.
   If you missed this problem, review [link].
3. Simplify: $7x-1-4x+5$.
   If you missed this problem, review [link].

Find the Domain and Range of a Relation

As we go about our daily lives, we have many data items or quantities that are paired to our names. Our social security number, student ID number, email address, phone number and our birthday are matched to our name. There is a relationship between our name and each of those items.

When your professor gets her class roster, the names of all the students in the class are listed in one column and then the student ID number is likely to be in the next column. If we think of the correspondence as a set of ordered pairs, where the first element is a student name and the second element is that student's ID number, we call this a **relation**.

$$(\text{Student name},\ \text{Student ID }\#)$$

The set of all the names of the students in the class is called the **domain** of the relation and the set of all student ID numbers paired with these students is the range of the relation.

There are many similar situations where one variable is paired or matched with another. The set of ordered pairs that records this matching is a relation.

Definition: Relation

A **relation** is any set of ordered pairs, (x,y). All the x-values in the ordered pairs together make up the **domain**. All the y-values in the ordered pairs together make up the **range**.

Example 4.2.1

For the relation $(1,1),(2,4),(3,9),(4,16),(5,25)$:

a. Find the domain of the relation.
b. Find the range of the relation.

Answer

$\{(1,1),(2,4),(3,9),(4,16),(5,25)\}$

ⓐ The domain is the set of all x-values of the relation. $\{1,2,3,4,5\}$

ⓑ The range is the set of all y-values of the relation. $\{1,4,9,16,25\}$

Example 4.2.2

For the relation $\{(1,1),(2,8),(3,27),(4,64),(5,125)\}$

a. Find the domain of the relation.
b. Find the range of the relation.

Answer a

$\{1,2,3,4,5\}$

Answer b

$\{1,8,27,64,125\}$

Example 4.2.3

For the relation $\{(1,3),(2,6),(3,9),(4,12),(5,15)\}$

a. Find the domain of the relation.
b. Find the range of the relation.

Answer a

$\{1,2,3,4,5\}$

Answer b

$\{3,6,9,12,15\}$

MAPPING

A **mapping** is sometimes used to show a relation. The arrows show the pairing of the elements of the domain with the elements of the range.

Example 4.2.4

Use the **mapping** of the relation shown to

a. list the ordered pairs of the relation,
b. find the domain of the relation, and
c. find the range of the relation.



Answer

ⓐ The arrow shows the matching of the person to their birthday. We create ordered pairs with the person's name as the *x*-value and their birthday as the *y*-value.

{(Alison, April 25), (Penelope, May 23), (June, August 2), (Gregory, September 15), (Geoffrey, January 12), (Lauren, May 10), (Stephen, July 24), (Alice, February 3), (Liz, August 2), (Danny, July 24)}

ⓑ The domain is the set of all x-values of the relation.

{Alison, Penelope, June, Gregory, Geoffrey, Lauren, Stephen, Alice, Liz, Danny}

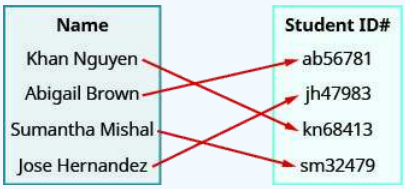
ⓒ The range is the set of all y-values of the relation.

{January 12, February 3, April 25, May 10, May 23, July 24, August 2, September 15}

Example 4.2.5

Use the mapping of the relation shown to

a. list the ordered pairs of the relation
b. find the domain of the relation
c. find the range of the relation.



Answer

ⓐ (Khanh Nguyen, kn68413), (Abigail Brown, ab56781), (Sumantha Mishal, sm32479), (Jose Hern and ez, jh47983)

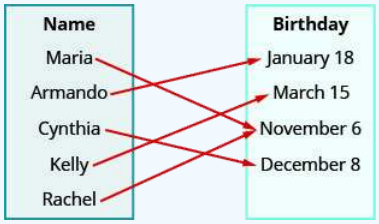
ⓑ {Khanh Nguyen, Abigail Brown, Sumantha Mishal, Jose Hern and ez}

ⓒ {kn68413, ab56781, sm32479, jh47983}

Example 4.2.6

Use the mapping of the relation shown to

a. list the ordered pairs of the relation
b. find the domain of the relation
c. find the range of the relation.



Answer

ⓐ (Maria, November 6), (Arm and o, January 18), (Cynthia, December 8), (Kelly, March 15), (Rachel, November 6)

ⓑ {Maria, Arm and o, Cynthia, Kelly, Rachel}

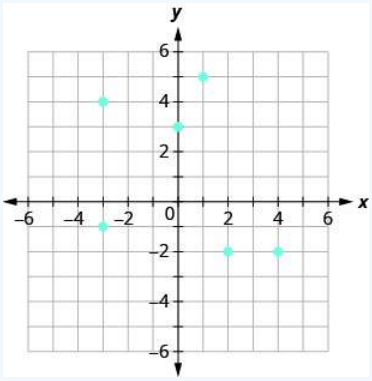
ⓒ{November 6, January 18, December 8, March 15}

A graph is yet another way that a relation can be represented. The set of ordered pairs of all the points plotted is the relation. The set of all x-coordinates is the domain of the relation and the set of all y-coordinates is the range. Generally we write the numbers in ascending order for both the domain and range.

Example 4.2.7

Use the graph of the relation to

a. list the ordered pairs of the relation
b. find the domain of the relation
c. find the range of the relation.



Answer

ⓐ The ordered pairs of the relation are:

$$\{(1,5),(-3,-1),(4,-2),(0,3),(2,-2),(-3,4)\}.$$

ⓑ The domain is the set of all x-values of the relation: $\{-3,0,1,2,4\}$

Notice that while -3 repeats, it is only listed once.

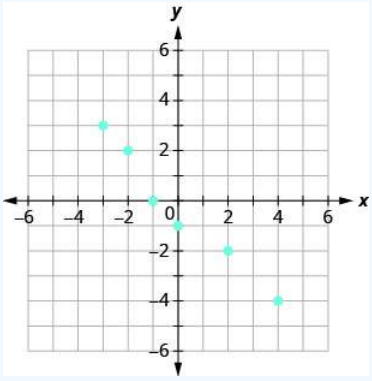
ⓒ The range is the set of all y-values of the relation: $\{-2,-1,3,4,5\}$

Notice that while -2 repeats, it is only listed once.

Example 4.2.8

Use the graph of the relation to

a. list the ordered pairs of the relation
b. find the domain of the relation
c. find the range of the relation.



Answer

a. $(-3,3),(-2,2),(-1,0),(0,-1),(2,-2),(4,-4)$

b. $\{-3,-2,-1,0,2,4\}$

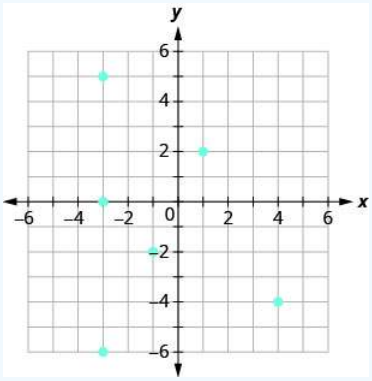
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c. $\{3, 2, 0, -1, -2, -4\}$

Example 4.2.9

Use the graph of the relation to

a. list the ordered pairs of the relation
b. find the domain of the relation
c. find the range of the relation.



Answer

ⓐ $(-3, 0), (-3, 5), (-3, -6), (-1, -2), (1, 2), (4, -4)$
ⓑ $\{-3, -1, 1, 4\}$
ⓒ $\{-6, 0, 5, -2, 2, -4\}$

Determine if a Relation is a Function

A special type of relation, called a **function**, occurs extensively in mathematics. A function is a relation that assigns to each element in its domain exactly one element in the range. For each ordered pair in the relation, each *x*-value is matched with only one *y*-value.

Definition: Function

A **function** is a relation that assigns to each element in its domain exactly one element in the range.

The birthday example from Example helps us understand this definition. Every person has a birthday but no one has two birthdays. It is okay for two people to share a birthday. It is okay that Danny and Stephen share July 24th as their birthday and that June and Liz share August 2nd. Since each person has exactly one birthday, the relation in Example is a function.

The relation shown by the graph in Example includes the ordered pairs $(-3, -1)$ and $(-3, 4)$. Is that okay in a function? No, as this is like one person having two different birthdays.

Example 4.2.10

Use the set of ordered pairs to (i) determine whether the relation is a function (ii) find the domain of the relation (iii) find the range of the relation.

a. $\{(-3, 27), (-2, 8), (-1, 1), (0, 0), (1, 1), (2, 8), (3, 27)\}$
b. $\{(9, -3), (4, -2), (1, -1), (0, 0), (1, 1), (4, 2), (9, 3)\}$

Answer

ⓐ $\{(-3, 27), (-2, 8), (-1, 1), (0, 0), (1, 1), (2, 8), (3, 27)\}$

(i) Each x-value is matched with only one y-value. So this relation is a function.

(ii) The domain is the set of all x-values in the relation.
The domain is: $\{-3, -2, -1, 0, 1, 2, 3\}$

(iii) The range is the set of all y-values in the relation. Notice we do not list range values twice.
The range is: $\{27, 8, 1, 0\}$

ⓑ $\{(9, -3), (4, -2), (1, -1), (0, 0), (1, 1), (4, 2), (9, 3)\}$

(i) The x-value 9 is matched with two y-values, both 3 and -3. So this relation is not a function.

(ii) The domain is the set of all x-values in the relation. Notice we do not list domain values twice.
The domain is: $\{0, 1, 2, 4, 9\}$

(iii) The range is the set of all y-values in the relation.
The range is: $\{-3, -2, -1, 0, 1, 2, 3\}$

Example 4.2.11

Use the set of ordered pairs to (i) determine whether the relation is a function (ii) find the domain of the relation (iii) find the range of the function.

a. $\{(-3, -6), (-2, -4), (-1, -2), (0, 0), (1, 2), (2, 4), (3, 6)\}$
b. $\{(8, -4), (4, -2), (2, -1), (0, 0), (2, 1), (4, 2), (8, 4)\}$

Answer

ⓐ Yes; $\{-3, -2, -1, 0, 1, 2, 3\}$
$\{-6, -4, -2, 0, 2, 4, 6\}$
ⓑ No; $\{0, 2, 4, 8\}$
$\{-4, -2, -1, 0, 1, 2, 4\}$

Example 4.2.12

Use the set of ordered pairs to (i) determine whether the relation is a function (ii) find the domain of the relation (iii) find the range of the relation.

a. $\{(27, -3), (8, -2), (1, -1), (0, 0), (1, 1), (8, 2), (27, 3)\}$
b. $\{(7, -3), (-5, -4), (8, -0), (0, 0), (-6, 4), (-2, 2), (-1, 3)\}$

Answer

ⓐ No; $\{0, 1, 8, 27\}$
$\{-3, -2, -1, 0, 2, 2, 3\}$
ⓑ Yes; $\{7, -5, 8, 0, -6, -2, -1\}$
$\{-3, -4, 0, 4, 2, 3\}$

Example 4.2.13

Use the mapping to

a. determine whether the relation is a function
b. find the domain of the relation
c. find the range of the relation.

NAME	PHONE

Lydia	321-549-3327 427-658-2314
Eugene	321-964-7324
Janet	684-358-7961
Rick	
Marty	684-369-7231 798-367-8541

Answer

ⓐ Both Lydia and Marty have two phone numbers. So each x-value is not matched with only one y-value. So this relation is not a function.

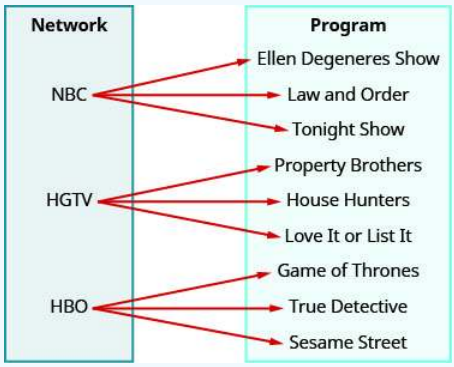
ⓑ The domain is the set of all x-values in the relation. The domain is: {Lydia, Eugene, Janet, Rick, Marty}

ⓒ The range is the set of all y-values in the relation. The range is:

$\{321-549-3327, 427-658-2314, 321-964-7324, 684-358-7961, 684-369-7231, 798-367-8541\}$

Example 4.2.14

Use the mapping to ⓐ determine whether the relation is a function ⓑ find the domain of the relation ⓒ find the range of the relation.



Answer

ⓐ no ⓑ {NBC, HGTV, HBO} ⓒ {Ellen Degeneres Show, Law and Order, Tonight Show, Property Brothers, House Hunters, Love it or List it, Game of Thrones, True Detective, Sesame Street}

Example 4.2.15

Use the mapping to

a. determine whether the relation is a function
b. find the domain of the relation
c. find the range of the relation.

Neal	123-567-4839 work 231-378-5941 cell
Krystal	743-469-9731 cell
Kelvin	567-534-2970 work
George	684-369-7231 cell
Christa	684-369-7231 cell
Mike	639-847-6971 cell

Answer

ⓐ No ⓑ {Neal, Krystal, Kelvin, George, Christa, Mike} ⓒ {123-567-4839 work, 231-378-5941 cell, 743-469-9731 cell, 567-534-2970 work, 684-369-7231 cell, 798-367-8541 cell, 639-847-6971 cell}

In algebra, more often than not, functions will be represented by an equation. It is easiest to see if the equation is a function when it is solved for *y*. If each value of *x* results in only one value of *y*, then the equation defines a function.

? Example 4.2.16

Determine whether each equation is a function.

a. $2x+y=7$
b. $y=x^2+1$
c. $x+y^2=3$

Answer

ⓐ $2x+y=7$

For each value of *x*, we multiply it by -2 and then add 7 to get the *y*-value

	$y=-2x+7$
For example, if $x=3$:	$y=-2\cdot 3+7$
	$y=1$

We have that when $x=3$, then $y=1$. It would work similarly for any value of *x*. Since each value of *x*, corresponds to only one value of *y* the equation defines a function.

ⓑ $y=x^2+1$

For each value of *x*, we square it and then add 1 to get the *y*-value.

	$y=x^2+1$
For example, if $x=2$:	$y=2^2+1$
	$y=5$

We have that when $x=2$, then $y=5$. It would work similarly for any value of *x*. Since each value of *x*, corresponds to only one value of *y* the equation defines a function.

ⓒ

	$x+y^2=3$
Isolate the y term.	$y^2=-x+3$
Let's substitute $x=2$.	$y^2=-2+3$
	$y^2=1$
This give us two values for y.	$y=1\ y=-1$

We have shown that when $x=2$, then $y=1$ and $y=-1$. It would work similarly for any value of x. Since each value of x does not corresponds to only one value of y the equation does not define a function.

Example 4.2.17

Determine whether each equation is a function.

a. $4x+y=-3$
b. $x+y^2=1$
c. $y-x^2=2$

Answer

ⓐ yes ⓑ no ⓒ yes

Example 4.2.18

Determine whether each equation is a function.

a. $x+y^2=4$
b. $y=x^2-7$
c. $y=5x-4$

Answer

ⓐ no ⓑ yes ⓒ yes

Find the Value of a Function

It is very convenient to name a function and most often we name it f, g, h, F, G, or H. In any function, for each x-value from the domain we get a corresponding y-value in the range. For the function f, we write this range value y as $f(x)$. This is called **function notation** and is read f of x or the value of f at x. In this case the parentheses does not indicate multiplication.

Definition: Function Notation

For the function $y=f(x)$

f is the name of the function
x is the domain value
$f(x)$ is the range value y corresponding to the value x

We read $f(x)$ as f of x or the value of f at x.

We call x the independent variable as it can be any value in the domain. We call y the dependent variable as its value depends on x.

INDEPENDENT AND DEPENDENT VARIABLES

For the function $y = f(x)$,

x is the independent variable as it can be any value in the domain
y the dependent variable as its value depends on x

Much as when you first encountered the variable *x*, function notation may be rather unsettling. It seems strange because it is new. You will feel more comfortable with the notation as you use it.

Let's look at the equation $y = 4x - 5$. To find the value of *y* when $x = 2$, we know to substitute $x = 2$ into the equation and then simplify.

	$y = 4x - 5$
Let x=2.	$y = 4 \cdot 2 - 5$
	$y = 3$

The value of the function at $x = 2$ is 3.

We do the same thing using function notation, the equation $y = 4x - 5$ can be written as $f(x) = 4x - 5$. To find the value when $x = 2$, we write:

	$f(x) = 4x - 5$
Let x=2.	$f(2) = 4 \cdot 2 - 5$
	$f(2) = 3$

The value of the function at $x = 2$ is 3.

This process of finding the value of $f(x)$ for a given value of *x* is called *evaluating the function.*

Example 4.2.19

For the function $f(x) = 2x^2 + 3x - 1$, evaluate the function.

a. $f(3)$
b. $f(-2)$
c. $f(a)$

Answer

ⓐ

	$f(x) = 2x^2 + 3x - 1$
To evaluate $f(3)$, substitute 3 for *x*.	$f(3) = 2(3)^2 + 3 \cdot 3 - 1$
Simplify.	$f(3) = 2 \cdot 9 + 3 \cdot 3 - 1$
	$f(3) = 18 + 9 - 1$
	$f(3) = 26$

ⓑ

		$f(x) = 2x^2 + 3x - 1$
To evaluate $f(-2)$, substitute -2 for *x*.		$f(-2) = 2(-2)^2 + 3(-2) - 1$

Simplify.		$f(-2) = 2 \cdot 4 + (-6) - 1$
		$f(-2) = 8 + (-6) - 1$
		$f(-2) = 1$

ⓒ

	$f(x) = 2x^2 + 3x - 1$
To evaluate f(a),f(a), substitute a for x.	$f(a) = 2(a)^2 + 3 \cdot a - 1$
Simplify.	$f(a) = 2a^2 + 3a - 1$

Example 4.2.20

For the function $f(x) = 3x^2 - 2x + 1$, evaluate the function.

a. $f(3)$
b. $f(-1)$
c. $f(t)$

Answer

ⓐ $f(3) = 22$ ⓑ $f(-1) = 6$ ⓒ $f(t) = 3t^2 - 2t - 1$

Example 4.2.21

For the function $f(x) = 2x^2 + 4x - 3$, evaluate the function.

a. $f(2)$
b. $f(-3)$
c. $f(h)$

Answer

ⓐ $(2) = 13$ ⓑ $f(-3) = 3$
ⓒ $f(h) = 2h2 + 4h - 3$

In the last example, we found $f(x)$ for a constant value of x. In the next example, we are asked to find $g(x)$ with values of x that are variables. We still follow the same procedure and substitute the variables in for the x.

Example 4.2.22

For the function $g(x) = 3x - 5$, evaluate the function.

a. $g(h^2)$
b. $g(x + 2)$
c. $g(x) + g(2)$

Answer

ⓐ

			$g(x) = 3x - 5$
To evaluate $g(h^2)$, substitute h^2 for x.			$g(h^2) = 3h^2 - 5$

			$g(h^2) = 3h^2 - 5$

ⓑ

	$g(x) = 3x - 5$
To evaluate $g(x+2)$, substitute $x+2$ for x.	$g(x+2) = 3(x+2) - 5$
Simplify.	$g(x+2) = 3x + 6 - 5$
	$g(x+2) = 3x + 1$

ⓒ

		$g(x) = 3x - 5$
To evaluate $g(x) + g(2)$, first find $g(2)$.		$g(2) = 3 \cdot 2 - 5$
		$g(2) = 1$
Now find $g(x) + g(2)$		$g(x) + g(2) = 3x - 5 + 1$ $g(x) \quad g(2)$
Simplify.		$g(x) + g(2) = 3x - 5 + 1$
		$g(x) + g(2) = 3x - 4$

Notice the difference between part ⓑ and ⓒ. We get $g(x+2) = 3x+1$ and $g(x)+g(2) = 3x-4$. So we see that $g(x+2) \neq g(x)+g(2)$.

Example 4.2.23

For the function $g(x) = 4x - 7$, evaluate the function.

a. $g(m^2)$
b. $g(x-3)$
c. $g(x) - g(3)$

Answer

ⓐ $4m^2 - 7$ ⓑ $4x - 19$
ⓒ $x - 12$

Example 4.2.24

For the function $h(x) = 2x + 1$, evaluate the function.

a. $h(k^2)$
b. $h(x+1)$
c. $h(x) + h(1)$

Answer

ⓐ $2k^2 + 1$ ⓑ $2x + 3$
ⓒ $2x + 4$

Many everyday situations can be modeled using functions.

Example 4.2.25

The number of unread emails in Sylvia's account is 75. This number grows by 10 unread emails a day. The function $N(t) = 75 + 10t$ represents the relation between the number of emails, *N*, and the time, *t*, measured in days.

a. Determine the independent and dependent variable.
b. Find $N(5)$. Explain what this result means.

Answer

ⓐ The number of unread emails is a function of the number of days. The number of unread emails, *N*, depends on the number of days, *t*. Therefore, the variable *N*, is the dependent variable and the variable tt is the independent variable.

ⓑ Find $N(5)$. Explain what this result means.

	$N(t) = 75 + 10t$
Substitute in t=5.t=5.	$N(5) = 75 + 10 \cdot 5$
Simplify.	$N(5) = 75 + 50$
	$N(5) = 125$

Since 5 is the number of days, $N(5)$, is the number of unread emails after 5 days. After 5 days, there are 125 unread emails in the account.

Example 4.2.26

The number of unread emails in Bryan's account is 100. This number grows by 15 unread emails a day. The function $N(t) = 100 + 15t$ represents the relation between the number of emails, *N*, and the time, *t*, measured in days.

a. Determine the independent and dependent variable.
b. Find $N(7)]$. Explain what this result means.

Answer

ⓐ *t* IND; *N* DEP ⓑ 205; the number of unread emails in Bryan's account on the seventh day.

Example 4.2.27

The number of unread emails in Anthony's account is 110. This number grows by 25 unread emails a day. The function $N(t) = 110 + 25t$ represents the relation between the number of emails, *N*, and the time, *t*, measured in days.

a. Determine the independent and dependent variable.
b. Find $N(14)$. Explain what this result means.

Answer

ⓐ *t* IND; *N* DEP ⓑ 460; the number of unread emails in Anthony's account on the fourteenth day

Access this online resource for additional instruction and practice with relations and functions.

- Introduction to Functions

Key Concepts

- **Function Notation:** For the function $y = f(x)$
  - f is the name of the function
  - x is the domain value

- $f(x)$ is the range value y corresponding to the value x
  We read $f(x)$ as f of x or the value of f at x.

- **Independent and Dependent Variables:** For the function $y = f(x)$,
  - x is the independent variable as it can be any value in the domain
  - y is the dependent variable as its value depends on x

Glossary

domain of a relation

The domain of a relation is all the x-values in the ordered pairs of the relation.

function

A function is a relation that assigns to each element in its domain exactly one element in the range.

mapping

A mapping is sometimes used to show a relation. The arrows show the pairing of the elements of the domain with the elements of the range.

range of a relation

The range of a relation is all the y-values in the ordered pairs of the relation.

relation

A relation is any set of ordered pairs,(x,y).(x,y). All the x-values in the ordered pairs together make up the domain. All the y-values in the ordered pairs together make up the range.

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4.2E: Exercises

Practice Makes Perfect

Find the Domain and Range of a Relation

In the following exercises, for each relation a. find the domain of the relation b. find the range of the relation.

1. $\{(1,4),(2,8),(3,12),(4,16),(5,20)\}$

Answer

a. $\{1,2,3,4,5\}$b. $\{4,8,12,16,20\}$

2. $\{(1,-2),(2,-4),(3,-6),(4,-8),(5,-10)\}$

3. $\{(1,7),(5,3),(7,9),(-2,-3),(-2,8)\}$

Answer

a. $\{1,5,7,-2\}$b. $\{7,3,9,-3,8\}$

4. $\{(11,3),(-2,-7),(4,-8),(4,17),(-6,9)\}$

In the following exercises, use the mapping of the relation to a. list the ordered pairs of the relation, b. find the domain of the relation, and c. find the range of the relation.

5.



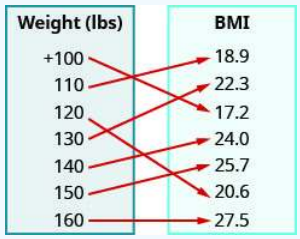
Answer

a. (Rebecca, January 18), (Jennifer, April 1), (John, January 18), (Hector, June 23), (Luis, February 15), (Ebony, April 7), (Raphael, November 6), (Meredith, August 19), (Karen, August 19), (Joseph, July 30)
b. {Rebecca, Jennifer, John, Hector, Luis, Ebony, Raphael, Meredith, Karen, Joseph}
c. {January 18, April 1, June 23, February 15, April 7, November 6, August 19, July 30}

6.



7. For a woman of height $5'4''$ the mapping below shows the corresponding Body Mass Index (BMI). The body mass index is a measurement of body fat based on height and weight. A BMI of $18.5–24.9$is considered healthy.



Answer

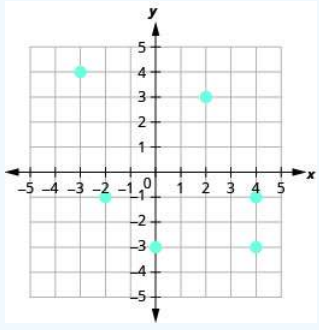
a. $(+100, 17.2), (110, 18.9), (120, 20.6), (130, 22.3), (140, 24.0), (150, 25.7), (160, 27.5)$ b. $\{+100, 110, 120, 130, 140, 150, 160\}$c. $\{17.2, 18.9, 20.6, 22.3, 24.0, 25.7, 27.5\}$

8. For a man of height $5'11''$ the mapping below shows the corresponding Body Mass Index (BMI). The body mass index is a measurement of body fat based on height and weight. A BMI of $18.5–24.9$is considered healthy.



In the following exercises, use the graph of the relation to a. list the ordered pairs of the relation b. find the domain of the relation c. find the range of the relation.

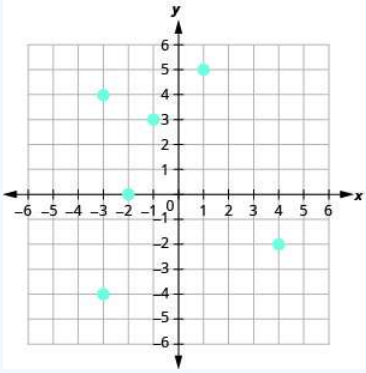
9.



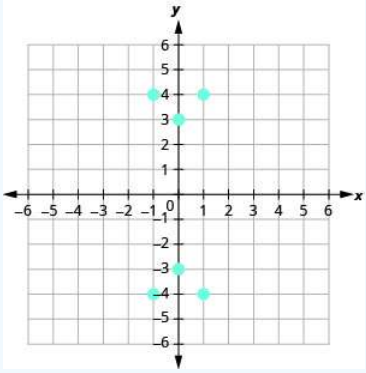
Answer

a. $(2, 3), (4, -3), (-2, -1), (-3, 4), (4, -1), (0, -3)$b. $\{-3, -2, 0, 2, 4\}$
c. $\{-3, -1, 3, 4\}$

10.



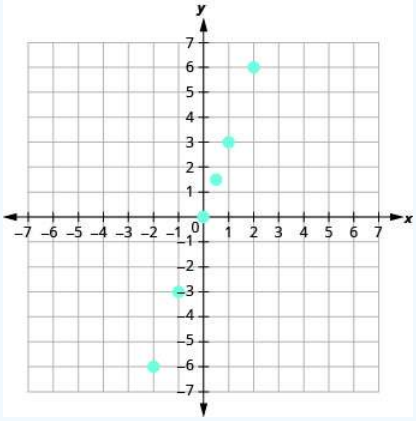
11.



Answer

a. $(1,4),(1,-4),(-1,4),(-1,-4),(0,3),(0,-3)$b. $\{-1,0,1\}$c. $\{-4,-3,3,4\}$

12.



Determine if a Relation is a Function

In the following exercises, use the set of ordered pairs to a. determine whether the relation is a function, b. find the domain of the relation, and c. find the range of the relation.

13. $\{(-3,9),(-2,4),(-1,1),(0,0),(1,1),(2,4),(3,9)\}$

Answer

a. yes b. $\{-3,-2,-1,0,1,2,3\}$c. $\{9,4,1,0\}$

14. $\{(9,-3),(4,-2),(1,-1),(0,0),(1,1),(4,2),(9,3)\}$

15. $\{(-3,27),(-2,8),(-1,1),(0,0),(1,1),(2,8),(3,27)\}$

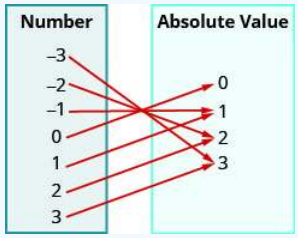
Answer

a. yes b. $\{-3,-2,-1,0,1,2,3\}$c. $\{0,1,8,27\}$

16. $\{(-3,-27),(-2,-8),(-1,-1),(0,0),(1,1),(2,8),(3,27)\}$

In the following exercises, use the mapping to a. determine whether the relation is a function, b. find the domain of the function, and c. find the range of the function.

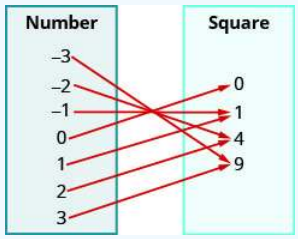
17.



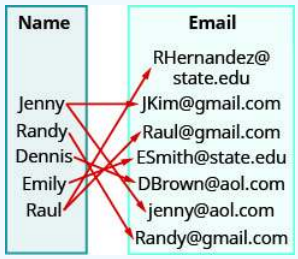
Answer

a. yes b. $\{-3,-2,-1,0,1,2,3\}$c. $\{0,1,2,3\}$

18.



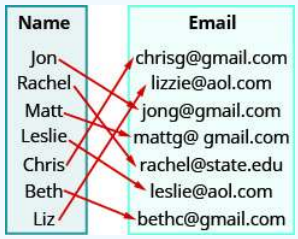
19.



Answer

a. no b. {Jenny, R and y, Dennis, Emily, Raul} c. {RHern and ez@state.edu, JKim@gmail.com, Raul@gmail.com, ESmith@state.edu, DBroen@aol.com, jenny@aol.cvom, R and y@gmail.com}

20.



In the following exercises, determine whether each equation is a function.

21. a. $2x+y=-3$
b. $y=x^2$
c. $x+y^2=-5$

Answer

a. yes b. yes c. no

22. a. $y=3x-5$
b. $y=x^3$
c. $2x+y^2=4$

23. a. $y-3x^3=2$
b. $x+y^2=3$
c. $3x-2y=6$

Answer

a. yes b. no c. yes

24. a. $2x-4y=8$
b. $-4=x^2-y$
c. $y^2=-x+5$

Find the Value of a Function

In the following exercises, evaluate the function: a. $f(2)$ b. $f(-1)$ c. $f(a)$.

25. $f(x)=5x-3$

Answer

a. $f(2)=7$ b. $f(-1)=-8$ c. $f(a)=5a-3$

26. $f(x)=3x+4$

27. $f(x)=-4x+2$

Answer

a. $f(2)=-6$ b. $f(-1)=6$ c. $f(a)=-4a+2$

28. $f(x)=-6x-3$

29. $f(x)=x^2-x+3$

Answer

a. $f(2)=5$ b. $f(-1)=5$
c. $f(a)=a^2-a+3$

30. $f(x)=x^2+x-2$

31. $f(x)=2x^2-x+3$

Answer

a. $f(2)=9$ b. $f(-1)=6$
c. $f(a)=2a^2-a+3$

32. $f(x)=3x^2+x-2$

In the following exercises, evaluate the function: a. $g(h^2)$ b. $g(x+2)$ c. $g(x)+g(2)$.

33. $g(x)=2x+1$

Answer

a. $g(h^2)=2h^2+1$
b. $g(x+2)=4x+5$
c. $g(x)+g(2)=2x+6$

34. $g(x)=5x-8$

35. $g(x)=-3x-2$

Answer

a. $g(h^2)=-3h^2-2$
b. $g(x+2)=-3x-8$
c. $g(x)+g(2)=-3x-10$

36. $g(x)=-8x+2$

37. $g(x)=3-x$

Answer

a. $g(h^2)=3-h^2$
b. $g(x+2)=1-x$
c. $g(x)+g(2)=4-x$

38. $g(x)=7-5x$

In the following exercises, evaluate the function.

39. $f(x)=3x^2-5x;\ f(2)$

Answer

2

40. $g(x) = 4x^2 - 3x$; $g(3)$

41. $F(x) = 2x^2 - 3x + 1$; $F(-1)$

Answer

6

42. $G(x) = 3x^2 - 5x + 2$; $G(-2)$

43. $h(t) = 2|t - 5| + 4$; $f(-4)$

Answer

22

44. $h(y) = 3|y - 1| - 3$; $h(-4)$

45. $f(x) = x + 2x - 1$; $f(2)$

Answer

4

46. $g(x) = x - 2x + 2$; $g(4)$

In the following exercises, solve.

47. The number of unwatched shows in Sylvia's DVR is 85. This number grows by 20 unwatched shows per week. The function $N(t) = 85 + 20t$ represents the relation between the number of unwatched shows, *N*, and the time, *t*, measured in weeks.

a. Determine the independent and dependent variable.

b. Find $N(4)$. Explain what this result means

Answer

a. *t* IND; *N* DEP
b. $N(4) = 165$ the number of unwatched shows in Sylvia's DVR at the fourth week.

48. Every day a new puzzle is downloaded into Ken's account. Right now he has 43 puzzles in his account. The function $N(t) = 43 + t$ represents the relation between the number of puzzles, *N*, and the time, *t*, measured in days.

a. Determine the independent and dependent variable.

b. Find $N(30)$. Explain what this result means.

49. The daily cost to the printing company to print a book is modeled by the function $C(x) = 3.25x + 1500$ where *C* is the total daily cost and *x* is the number of books printed.

a. Determine the independent and dependent variable.

b. Find $N(0)$. Explain what this result means.

c. Find $N(1000)$. Explain what this result means.

Answer

a. x IND; C DEP
b. $N(0) = 1500$ the daily cost if no books are printed
c. $N(1000) = 4750$the daily cost of printing 1000 books

50. The daily cost to the manufacturing company is modeled by the function $C(x) = 7.25x + 2500$ where $C(x)$ is the total daily cost and x is the number of items manufactured.

a. Determine the independent and dependent variable.

b. Find $C(0)$. Explain what this result means.

c. Find $C(1000)$. Explain what this result means.

Writing Exercises

51. In your own words, explain the difference between a relation and a function.

52. In your own words, explain what is meant by domain and range.

53. Is every relation a function? Is every function a relation?

54. How do you find the value of a function?

Self Check

a. After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
find the domain and range of a relation.			
determine if a relation is a function.			
find the value of a function.			

b. After looking at the checklist, do you think you are well-prepared for the next section? Why or why not?

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4.3: Interpreting the Graph of a Function

In the previous section, we began with a function and then drew the graph of the given function. In this section, we will start with the graph of a function, then make a number of interpretations based on the given graph: function evaluations, the domain and range of the function, and solving equations and inequalities.

The Vertical Line Test

Consider the graph of the relation R shown in Figure 4.3.1(a). Recall that we earlier defined a relation as a set of ordered pairs. Surely, the graph shown in Figure 4.3.1(a) is a set of ordered pairs. Indeed, it is an infinite set of ordered pairs, so many that the graph is a solid curve.

In Figure 4.3.1(b), note that we can draw a vertical line that cuts the graph more than once. In Figure 4.3.1(b), we've drawn a vertical line that cuts the graph in two places, once at (x, y_1), then again at (x, y_2), as shown in Figure 4.3.1(c). This means that the domain object x is paired with two different range objects, namely y_1 and y_2, so relation R is not a function.

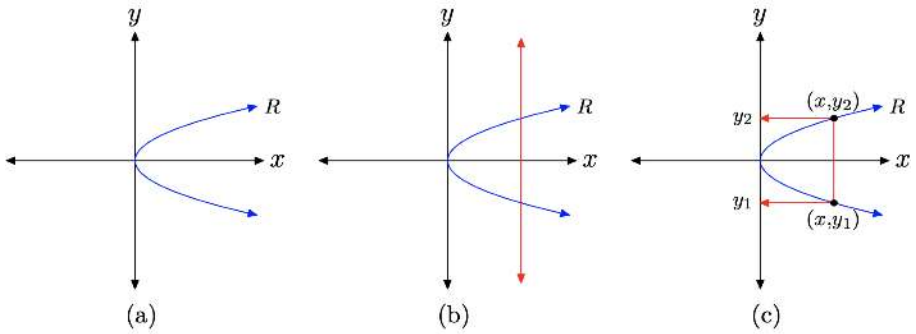


Figure 4.3.1. Explaining the vertical line test for functions.

Recall the definition of a function.

Definition

A relation is a function if and only if each object in its domain is paired with one and only one object in its range.

Consider the mapping diagram in Figure 4.3.2, where we've used arrows to indicate the ordered pairs (x, y_1) and (x, y_2) in Figure 4.3.1(c). Note that x, an object in the domain of R, is mapped to two objects in the range of R, namely y_1 and y_2. Hence, the relation R is not a function.

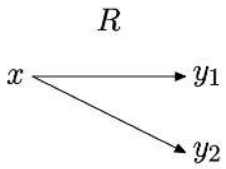


Figure 4.3.2. A mapping diagram representing the points (x, y_1) and (x, y_2) in Figure 4.3.1(c).

This discussion leads to the following result, called the vertical line test for functions.

The Vertical Line Test

If any vertical line cuts the graph of a relation more than once, then the relation is NOT a function.

Hence, the circle pictured in Figure 4.3.3(a) is a relation, but it is not the graph of a function. It is possible to cut the graph of the circle more than once with a vertical line, as shown in Figure 4.3.3(a). On the other hand, the parabola shown in Figure 4.3.3(b) is the graph of a function, because no vertical line will cut the graph more than once.

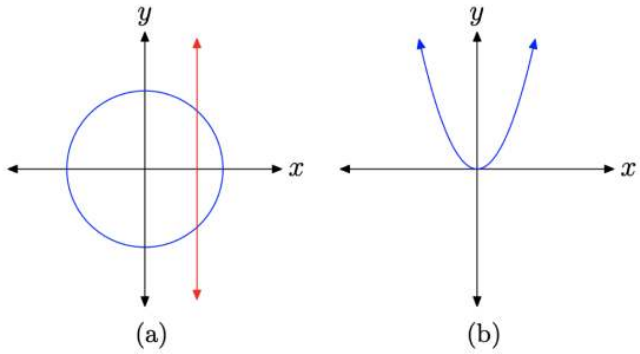


Figure 4.3.3. Use the vertical line test to determine if the graph is the graph of a function.

Reading the Graph for Function Values

We know that the graph of f pictured in Figure 4.3.4 is the graph of a function. We know this because no vertical line will cut the graph of f more than once.

We earlier defined the graph of f as the set of all ordered pairs $(x, f(x))$, so that x is in the domain of f. Consequently, if we select a point P on the graph of f, as in Figure 4.3.4(a), we label the point P(x, f(x)). However, we can also label this point as $P(x, y)$, as shown in Figure 4.3.4(b). This leads to a new interpretation of f(x) as the y-value of the point P. That is, f(x) is the y-value that is paired with x.

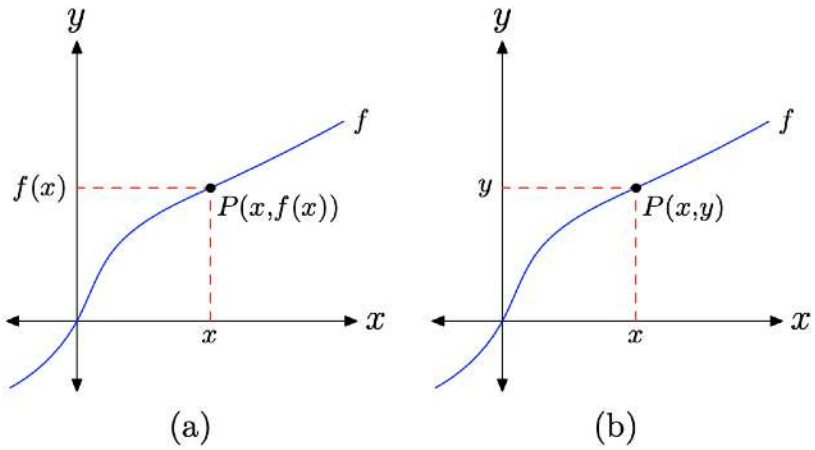


Figure 4.3.4. Reading the graph of a function.

Definition

f(x) is the y-value that is paired with x.

Two more comments are in order. In Figure 4.3.4(a), we select a point P on the graph of f.

1. To find the x-value of the point P, we must project the point P onto the x-axis.
2. To find f(x), the value of y that is paired with x, we must project the point P onto the y-axis.

Let's look at an example.

Example 4.3.1

Given the graph of f in Figure 4.3.5(a), find f(4).

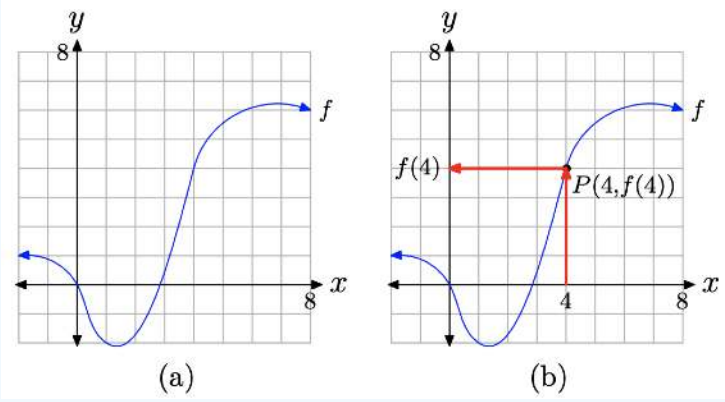


Figure 4.3.5. Finding the value of f(4).

Solution

First, note that the graph of f represents a function. No vertical line will cut the graph of f more than once.

Because f(4) represents the y-value that is paired with an x-value of 4, we first locate 4 on the x-axis, as shown in Figure (\PageIndex{5}\)(b). We then draw a vertical arrow until we intercept the graph of f at the point P(4, f(4)). Finally, we draw a horizontal arrow from the point P until we intercept the y-axis. The projection of the point P onto the y-axis is the value of f(4).

Because we have a grid that shows a scale on each axis, we can approximate the value of f(4). It would appear that the y-value of point P is approximately 4. Thus, $f(4) \approx 4$.

Let's look at another example.

Example 4.3.2

Given the graph of f in Figure 4.3.6(a), find f(5).

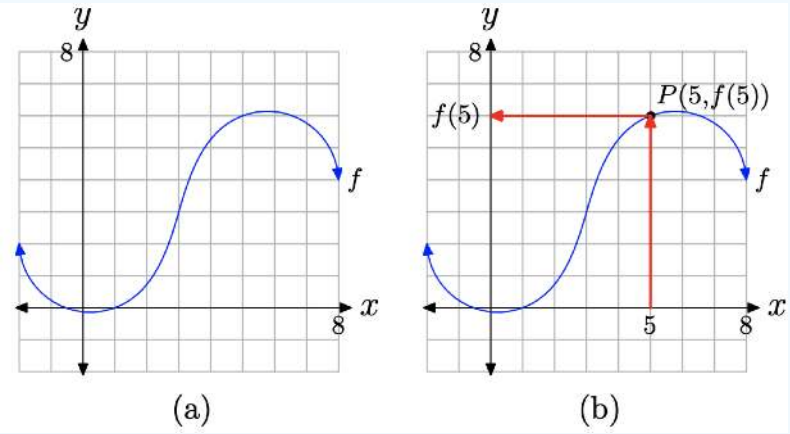


Figure 4.3.6. Finding the value of f(5).

Solution

First, note that the graph of f represents a function. No vertical line will cut the graph of f more than once.

Because f(5) represents the y-value that is paired with an x-value of 5, we first locate 5 on the x-axis, as shown in Figure 4.3.6(b). We then draw a vertical arrow until we intercept the graph of f at the point P(5, f(5)). Finally, we draw a horizontal arrow from the point P until we intercept the y-axis. The projection of the point P onto the y-axis is the value of f(5).

Because we have a grid that shows a scale on each axis, we can approximate the value of f(5). It would appear that the y-value of point P is approximately 6. Thus, $f(5) \approx 6$.

Let's reverse the interpretation in another example.

Example 4.3.3

Given the graph of f in Figure 4.3.7(a), for what value of x does f(x) = −4?

Solution

Again, the graph in Figure 4.3.7 passes the vertical line test and represents the graph of a function.

This time, in the equation $f(x) = -4$, we're given a y-value equal to −4. Consequently, we must reverse the process used in Example 4.3.1 and Example 4.3.2. We first locate the y-value −4 on the y-axis, then draw a horizontal arrow until we intercept

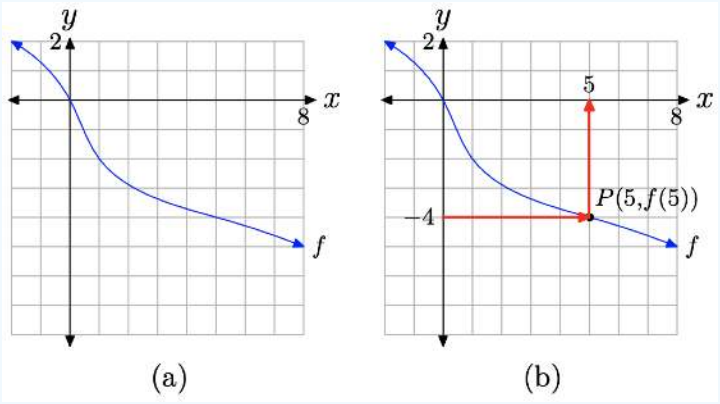


Figure 4.3.7. Finding x so that $f(x) = -4$.

the graph of f at P, as shown in Figure 4.3.7(b). Finally, we draw a vertical arrow from the point P until we intercept the x-axis. The projection of the point P onto the x-axis is the solution of $f(x) = -4$.

Because we have a grid that shows a scale on each axis, we can approximate the x-value of the point P. It seems that $x \approx 5$. Thus, we label the point P(5, f(5)), and the solution of $f(x) = -4$ is approximately $x \approx 5$.

This solution can easily be checked by computing f(5). Simply start with 5 on the x-axis, then reverse the order of the arrows shown in Figure 4.3.7(b). You should wind up at −4 on the y-axis, demonstrating that $f(5) = -4$.

The Domain and Range of a Function

We can use the graph of a function to determine its domain and range. For example, consider the graph of the function shown in Figure 4.3.8(a).

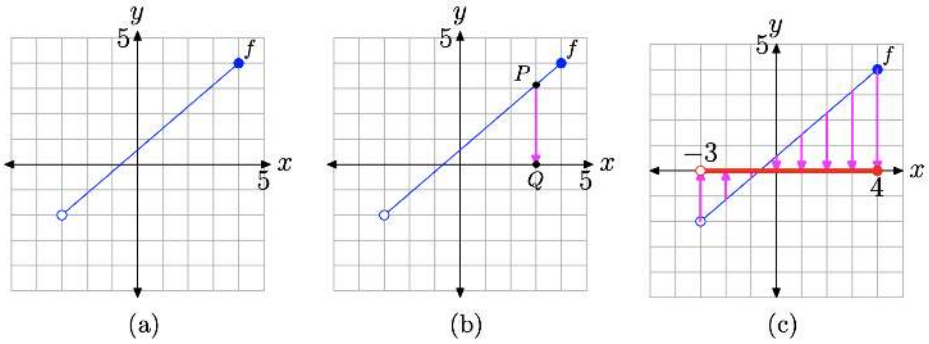


Figure 4.3.8. Determining the domain of a function from its graph.

Note that no vertical line will cut the graph of f more than once, so the graph of f represents a function.

To determine the domain, we must collect the x-values (first coordinates) of every point on the graph of f. In Figure 4.3.8(b), we've selected a point P on the graph of f, which we then project onto the x-axis. The image of this projection is the point Q, and the x-value of the point Q is an element in the domain of f.

Think of the projection shown in Figure 4.3.8(b) in the following manner. Imagine a light source above the point P. The point P blocks out the light and its shadow falls onto the x-axis at the point Q. That is, think of point Q as the "shadow" that the point P

produces when it is projected vertically onto the x-axis.

Now, to find the domain of the function f, we must project each point on the graph of f onto the x-axis. Here's the question: if we project each point on the graph of f onto the x-axis, what part of the x-axis will "lie in shadow" when the process is complete? The answer is shown in Figure 4.3.8(c).

In Figure 4.3.8(c), note that the "shadow" created by projecting each point on the graph of f onto the x-axis is shaded in red (a thicker line if you are viewing this in black and white). This collection of x-values is the domain of the function f. There are three critical points that we need to make about the "shadow" on the x-axis in Figure 4.3.8(c).

1. All points lying between $x = -3$ and $x = 4$ have been shaded on the x-axis in red.
2. The left endpoint of the graph of f is an open circle. This indicates that there is no point plotted at this endpoint. Consequently, there is no point to project onto the x-axis, and this explains the open circle at the left end of our "shadow" on the x-axis.
3. On the other hand, the right endpoint of the graph of f is a filled endpoint. This indicates that this is a plotted point and part of the graph of f. Consequently, when this point is projected onto the x-axis, a shadow falls at x = 4. This explains the filled endpoint at the right end of our "shadow" on the x-axis.

We can describe the x-values of the "shadow" on the x-axis using set-builder notation.

$$\text{Domain of } f = \{x : -3 < x \leq 4\}$$

Note that we don't include −3 in this description because the left end of the shadow on the x-axis is an empty circle. Note that we do include 4 in this description because the right end of the shadow on the x-axis is a filled circle.

We can also describe the x-values of the "shadow" on the x-axis using interval notation.

$$\text{Domain of } f = (-3, 4]$$

We remind our readers that the parenthesis on the left means that we are not including −3, while the bracket on the right means that we are including 4.

To find the range of the function, picture again the graph of f shown in Figure 4.3.9(a). Proceed in a similar manner, only this time project points on the graph of f onto the y-axis, as shown in Figures 4.3.9(b) and (c).

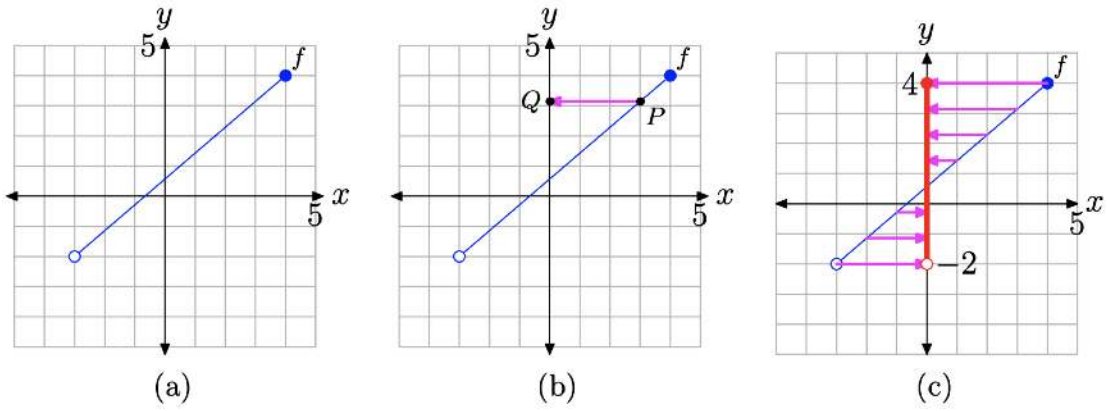


Figure 4.3.9. Determining the range of a function from its graph.

Note which part of the y-axis "lies in shadow" once we've projected all points on the graph of f onto the y-axis.

1. All points lying between $y = -2$ and $y = 4$ have been shaded on the y-axis in red (a thicker line style if you are viewing this in black and white).
2. The left endpoint of the graph of f is an empty circle, so there is no point to project onto the y-axis. Consequently, there is no "shadow" at $y = -2$ on the y-axis and the point is left unshaded (an empty circle).
3. The right endpoint of the graph of f is a filled circle, so there is a "shadow" at $y = 4$ on the y-axis and this point is shaded (a filled circle).

We can now easily describe the range in both set-builder and interval notation.

$$\text{Range of } f = (-2, 4] = \{y : -2 < y \leq 4\}$$

Let's look at another example.

Example 4.3.4

Use set-builder and interval notation to describe the domain and range of the function represented by the graph in Figure 4.3.10(a).

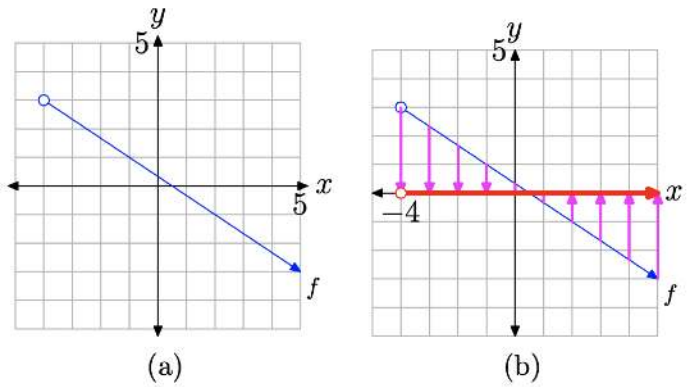


Figure 4.3.10. Determining the domain from the graph of f.

Solution

To determine the domain of f, project each point on the graph of f onto the x-axis. This projection is indicated by the "shadow" on the x-axis in Figure 4.3.10(b). Two important points need to be made about this "shadow" or projection.

1. The left endpoint of the graph of f is empty (indicated by the open circle), so it has no projection onto the x-axis. This is indicated by an open circle at the left end (at $x = -4$) of the "shadow" or projection on the x-axis.

2. The arrowhead on the right end of the graph of f indicates that the graph of f continues downward and to the right indefinitely. Consequently, the projection onto the x-axis is a shadow that moves indefinitely to the right. This is indicated by an arrowhead at the right end of the "shadow" or projection on the x-axis.

Consequently, the domain of f is the collection of x-values represented by the "shadow" or projection onto the x-axis. Note that all x-values to the right of $x = -4$ are shaded on the x-axis. Consequently,

$$\text{Domain of } f = (-4, \infty) = \{x : x > -4\}$$

To find the range, we must project each point on the graph of f (redrawn in Figure 4.3.11(a)) onto the y-axis. The projection is indicated by a "shadow" or projection on the y-axis, as seen in Figure 4.3.11(b). Two important points need to be made about this "shadow" or projection.

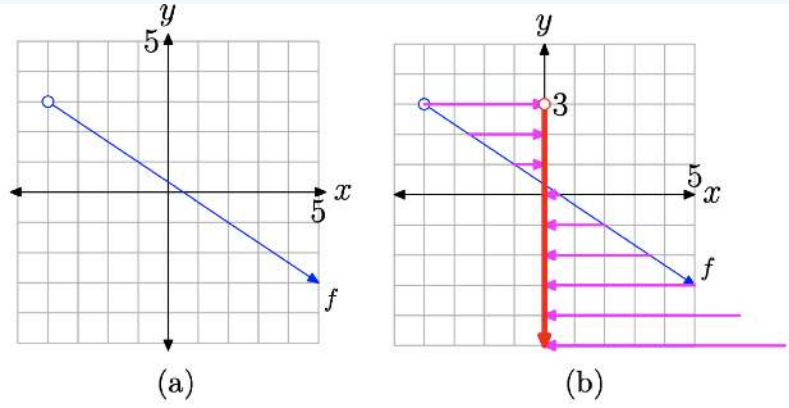


Figure 4.3.11. Determining the range from the graph of f.

1. The left endpoint of the graph of f is empty (indicated by an open circle), so it has no projection onto the y-axis. This is indicated by an open circle at the top end (at $y = 3$) of the "shadow" on the y-axis.

2. The arrowhead on the right end of the graph of f indicates that the graph of f continues downward and to the right indefinitely. Consequently, the projection of the graph of f onto the y-axis is a shadow that moves indefinitely downward. In Figure 4.3.11(b), note how projections of points on the graph of f not visible in the viewing window come in from the lower right corner and cast "shadows" on the y-axis.

Consequently, the range of f is the collection of y-values shaded on the y-axis of the coordinate system shown in Figure 4.3.11(b). Note that all y-values lower than $y = 3$ are shaded on the y-axis. Thus, the range of f is

$$\text{Range of } f = (-\infty, 3) = \{y : y < 3\}$$

Let's look at another example.

Example 4.3.5

Use set-builder and interval notation to describe the domain and range of the function represented by the graph in Figure 4.3.12(a).

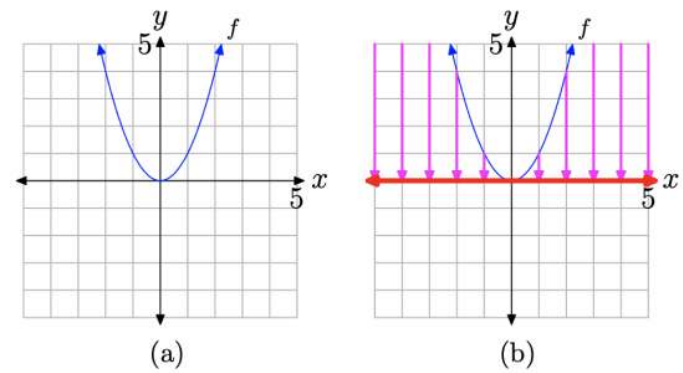


Figure 4.3.12 Determining the domain from the graph of f.

Solution

To determine the domain of f, we must project all points on the graph of f onto the x-axis. This projection is indicated by the red "shadow" (or thicker line style if you are viewing this in black and white) shown on the x-axis in Figure 4.3.12(b). Two important points need to be made about this "shadow" or projection.

1. The arrow at the end of the left half of the graph of f in Figure 4.3.12(a) indicates that this half of the graph of f opens indefinitely to the left and upward. Consequently, when the points on the left half of the graph of f are projected onto the x-axis, the "shadow" or projection extends indefinitely to the left. Note how points on the graph that fall outside the viewing window come in from the upper left corner and cast "shadows" on the x-axis.
2. The arrow at the end of the right half of the graph of f in Figure 4.3.12(a) indicates that this half of the graph of f opens indefinitely to the right and upward. Consequently, when the points on this half of the graph of f are projected onto the x-axis, the "shadow" or projection extends indefinitely to the right.

Consequently, the entire x-axis lies in "shadow," making the domain of f to be

$$\text{Domain of } f = (-\infty, \infty) = \{x : x \in \mathbb{R}\}$$

To determine the range of f, we must project all points on the graph of f onto the y-axis. This projection is indicated by the red "shadow" (or thicker line if you are viewing this in black and white) shown on the y-axis in Figure 4.3.13(b). Two important points need to made about this "shadow" or projection.

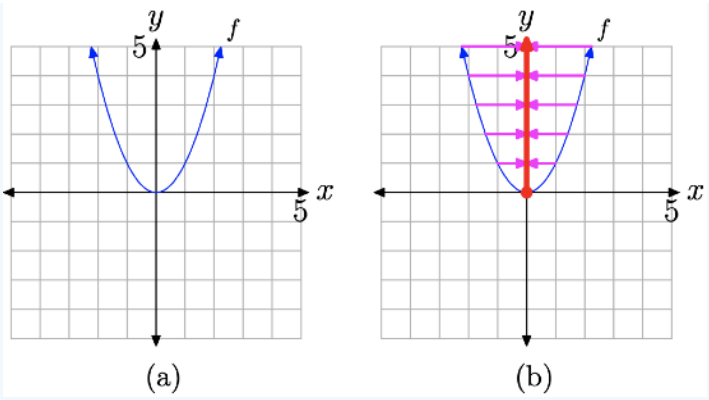


Figure 4.3.13 Determining the range from the graph of f.

1. The graph of f passes through the origin (the point (0, 0)). This is the lowest point on the graph and hence its shadow is the endpoint on the low end of the shaded region on the y-axis.
2. The arrows at the end of each half of the graph of f indicate that the graph opens upward indefinitely. Hence, when points on the graph of f are projected onto the y-axis, the "shadow" or projection extends upward indefinitely. This is indicated by an arrow on the upper end of the "shadow" on the y-axis.

Consequently, all points on the y-axis above and including the point at the origin "lie in shadow." Thus, the range of f is

$$\text{Range of } f = [0, \infty) = \{y : y \geq 0\}$$

Using a Graphing Calculator to Determine Domain and Range

We've learned how to find the domain and range of a function by looking at its graph. Therefore, if we define a function by means of an expression, such as $f(x) = \sqrt{4 - x}$, then we should be able to capture the domain and range of f from its graph, provided, of course, that we can draw the graph of f. We'll find the graphing calculator will be a handy tool for this exercise.

Example 4.3.6

Use set-builder and interval notation to describe the domain and range of the function defined by the rule

$$f(x) = \sqrt{4 - x}$$

Solution

Load the expression defining f into the Y= menu, as shown in Figure 4.3.14(a). Select 6:ZStandard from the ZOOM menu to produce the graph of f shown in Figure 4.3.14(b).

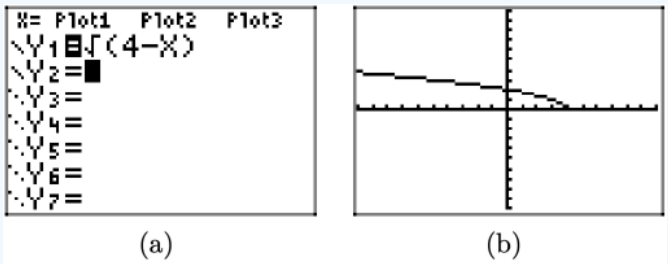


Figure 4.3.14 Sketching the graph of $f(x) = \sqrt{4 - x}$.

Copy the image in Figure 4.3.14(b) onto a sheet of graph paper. Label and scale each axis with the WINDOW parameters xmin, xmax, ymin, and ymax, as shown in Figure 4.3.15(a).

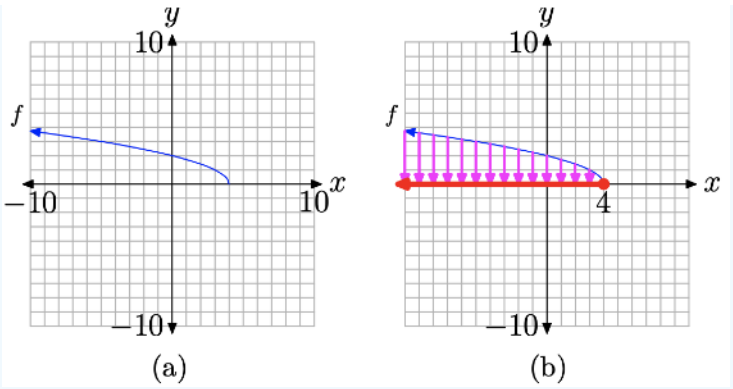


Figure 4.3.15. Capturing the domain of $f(x) = \sqrt{4-x}$ from its graph.

Next, project each point on the graph of f onto the x-axis, as shown in Figure 4.3.15(b). Note that we've made two assumptions about the graph of f.

1. At the left end of the graph in Figures 4.3.14(b) and 4.3.15(b), we assume that the graph of f continues upward and to the left indefinitely. Hence, the "shadow" or projection onto the x-axis will move indefinitely to the left. This is indicated by attaching an arrowhead to the left-hand end of the region that "lies in shadow" on the x-axis, as shown in Figure 4.3.15(b).
2. We also assume that the right end of the graph ends at the point $(4, 0)$. This accounts for the "filled dot" when this point on the graph of f is projected onto the x-axis.

Note that the "shadow" or projection onto the x axis in Figure 4.3.15(b) includes all values of x less than or equal to 4. Thus, the domain of f is

$$\text{Domain of } f = (-\infty, 4] = \{x : x \leq 4\}$$

We can intuit this result by considering the expression that defines f. That is, consider the rule or definition

$$f(x) = \sqrt{4-x}$$

Recall that we earlier defined the domain of f as the set of "permissible" x-values. In this case, it is impossible to take the square root of a negative number, so we must be careful selecting the x-values we use in this rule. Note that $x = 4$ is allowable, as

$$f(0) = \sqrt{4-4} = \sqrt{0} = 0$$

However, numbers larger than 4 cannot be used in this rule. For example, consider what happens when we attempt to use $x = 5$.

$$f(x) = \sqrt{4-5} = \sqrt{-1}$$

We'll leave it to our readers to test other values of x that are less than 4. They will also produce real answers when they are input into the rule $f(x) = \sqrt{4-x}$. Note that this also verifies our earlier conjecture that the "shadow" or projection shown in Figure 4.3.15(b) continues indefinitely to the left.

Instead of "guessing and checking," we can speed up the analysis of the domain of $f(x) = \sqrt{4-x}$ by noting that the expression under the radical must not be a negative number. Hence, $4-x$ must either be greater than or equal to zero. This argument produces an inequality that is easily solved for x.

$$\begin{aligned} 4-x &\geq 0 \\ -x &\geq -4 \\ x &\leq 4 \end{aligned}$$

This last result verifies that the domain of f is all values of x that are less than or equal to 4, which is in complete agreement with the "shadow" or projection onto the x-axis shown in Figure 4.3.15(b).

To determine the range of f, we must project each point on the graph of f onto the y-axis, as shown in Figure 4.3.16(b).

Again, we make two assumptions about the graph of f.

1. At the left-end of the graph of $f(x) = \sqrt{4-x}$ in Figures 4.3.14(b) and 4.3.16(b), we assume that the graph of f continues upward and to the left indefinitely. Thus, when points on the graph of f are projected onto the y-axis, there will be projections coming from the upper left from points on the graph of f that are not visible in the viewing window selected in Figure 4.3.14(b). Hence, the "shadow" or projection on the y-axis shown in Figure 4.3.16(b) continues upward indefinitely. This is indicated with a arrowhead at the upper end of the "shadow" on the y-axis in Figure 4.3.16(b).

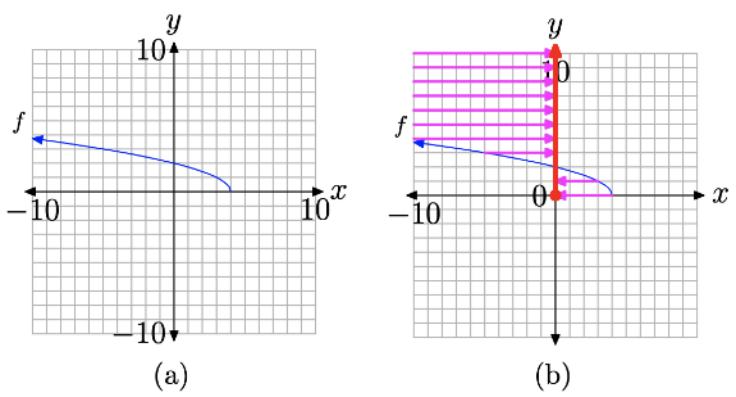


Figure 4.3.16. Determining the range of $f(x) = \sqrt{4-x}$ from its graph.

2. Again, we assume that the right end of the graph of f ends at the point $(4, 0)$. The projection of this point onto the y-axis produces the "filled" endpoint at the origin shown in Figure 4.3.16(b).

Note that the "shadow" or projection onto the y-axis in Figure 4.3.16(b) includes all values of y that are greater than or equal to zero. Hence,

$$\text{Range of } f = [0, \infty) = \{y : y \geq 0\}$$

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4.4: The Toolbox Functions

Learning Objectives

By the end of this section, you will be able to:

- Use the vertical line test
- Identify graphs of basic functions
- Read information from a graph of a function

Before you get started, take this readiness quiz.

1. Evaluate: ⓐ 2^3 ⓑ 3^2.
   If you missed this problem, review [link].
2. Evaluate: ⓐ $|7|$ ⓑ $|-3|$.
   If you missed this problem, review [link].
3. Evaluate: ⓐ $\sqrt{4}$ ⓑ $\sqrt{16}$.
   If you missed this problem, review [link].

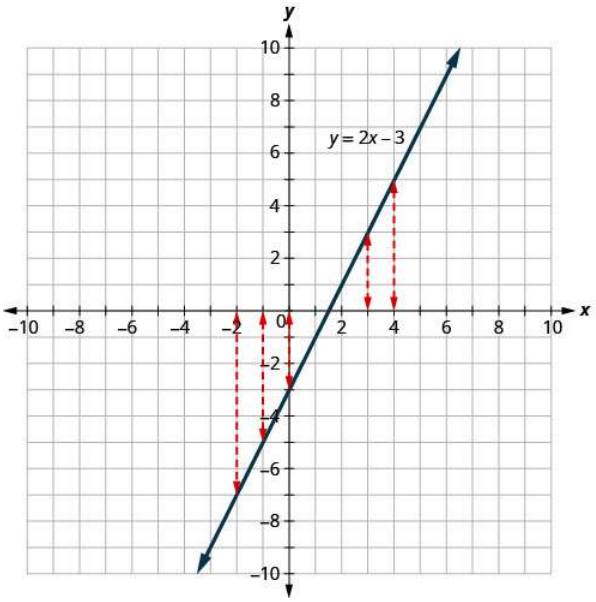
Use the Vertical Line Test

In the last section we learned how to determine if a relation is a function. The relations we looked at were expressed as a set of ordered pairs, a mapping or an equation. We will now look at how to tell if a graph is that of a function.

An ordered pair (x, y) is a solution of a linear equation, if the equation is a true statement when the *x*- and *y*-values of the ordered pair are substituted into the equation.

The graph of a linear equation is a straight line where every point on the line is a solution of the equation and every solution of this equation is a point on this line.

In Figure, we can see that, in graph of the equation $y = 2x - 3$, for every *x*-value there is only one *y*-value, as shown in the accompanying table.



y = 2x – 3		
x	y	(x, y)
–2	–7	(–2, –7)
–1	–5	(–1, –5)
0	–3	(0, –3)
3	3	(3, 3)
4	5	(4, 5)

Figure 4.4.1

A relation is a function if every element of the domain has exactly one value in the range. So the relation defined by the equation $y = 2x - 3$ is a function.

If we look at the graph, each vertical dashed line only intersects the line at one point. This makes sense as in a function, for every *x*-value there is only one *y*-value.

If the vertical line hit the graph twice, the x-value would be mapped to two y-values, and so the graph would not represent a function.

This leads us to the vertical line test. A set of points in a rectangular coordinate system is the graph of a function if every vertical line intersects the graph in at most one point. If any vertical line intersects the graph in more than one point, the graph does not represent a function.

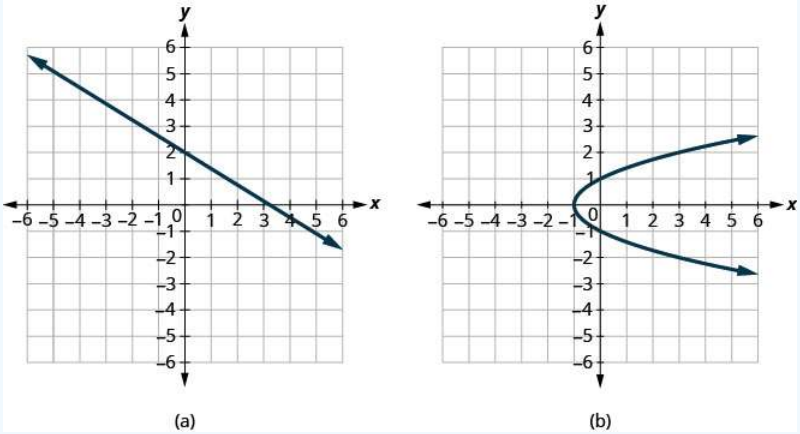
VERTICAL LINE TEST

A set of points in a rectangular coordinate system is the graph of a function if every vertical line intersects the graph in at most one point.

If any vertical line intersects the graph in more than one point, the graph does not represent a function.

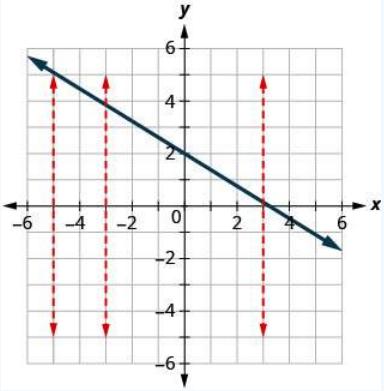
Example 4.4.1

Determine whether each graph is the graph of a function.

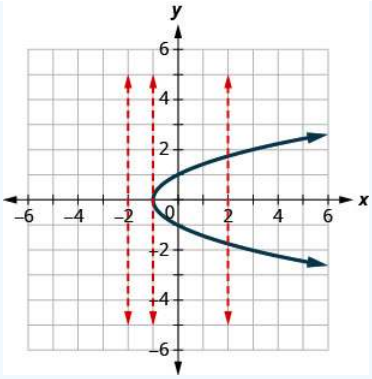


Answer

ⓐ Since any vertical line intersects the graph in at most one point, the graph is the graph of a function.

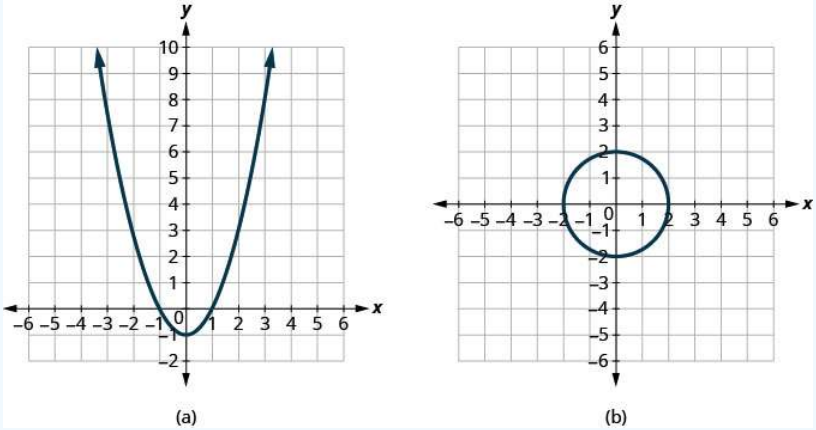


ⓑ One of the vertical lines shown on the graph, intersects it in two points. This graph does not represent a function.



Example 4.4.2

Determine whether each graph is the graph of a function.

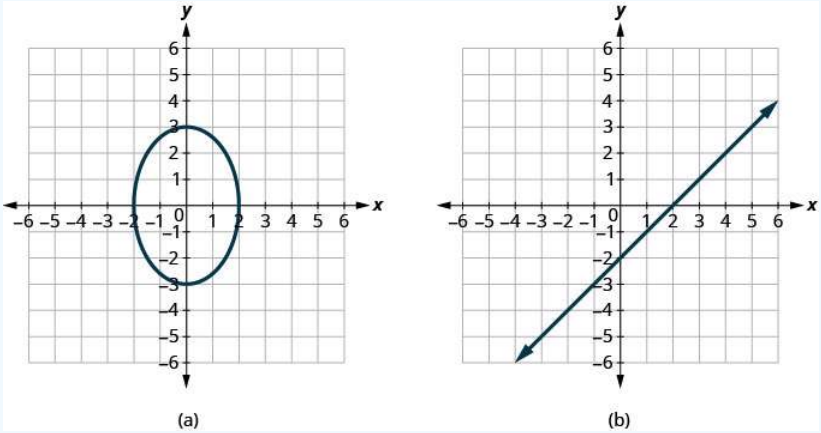


Answer

ⓐ yes ⓑ no

Example 4.4.3

Determine whether each graph is the graph of a function.



Answer

ⓐ no ⓑ yes

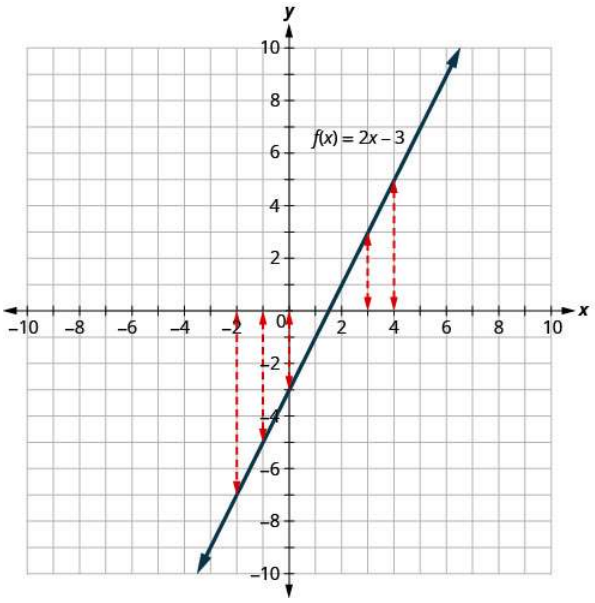
https://math.libretexts.org/@go/page/54991

Identify Graphs of Basic Functions

We used the equation $y = 2x - 3$ and its graph as we developed the vertical line test. We said that the relation defined by the equation $y = 2x - 3$ is a function.

We can write this as in function notation as $f(x) = 2x - 3$. It still means the same thing. The graph of the function is the graph of all ordered pairs (x, y) where $y = f(x)$. So we can write the ordered pairs as $(x, f(x))$. It looks different but the graph will be the same.

Compare the graph of $y = 2x - 3$ previously shown in Figure with the graph of $f(x) = 2x - 3$ shown in Figure. Nothing has changed but the notation.



f(x) = 2x – 3		
x	f(x)	(x, f(x))
–2	–7	(–2, –7)
–1	–5	(–1, –5)
0	–3	(0, –3)
3	3	(3, 3)
4	5	(4, 5)

Figure 4.4.2

GRAPH OF A FUNCTION

The graph of a function is the graph of all its ordered pairs, (x,y)(x,y) or using function notation, (x,f(x))(x,f(x)) where y=f(x).y=f(x).

f	name of function
x	x-coordinate of the ordered pair
$f(x)$	y-coordinate of the ordered pair

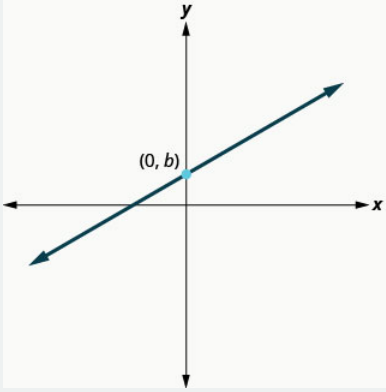
As we move forward in our study, it is helpful to be familiar with the graphs of several basic functions and be able to identify them.

Through our earlier work, we are familiar with the graphs of linear equations. The process we used to decide if $y = 2x - 3$ is a function would apply to all linear equations. All non-vertical linear equations are functions. Vertical lines are not functions as the *x*-value has infinitely many *y*-values.

We wrote linear equations in several forms, but it will be most helpful for us here to use the slope-intercept form of the linear equation. The slope-intercept form of a linear equation is $y = mx + b$. In function notation, this linear function becomes $f(x) = mx + b$ where *m* is the slope of the line and *b* is the *y*-intercept.

The domain is the set of all real numbers, and the range is also the set of all real numbers.

LINEAR FUNCTION



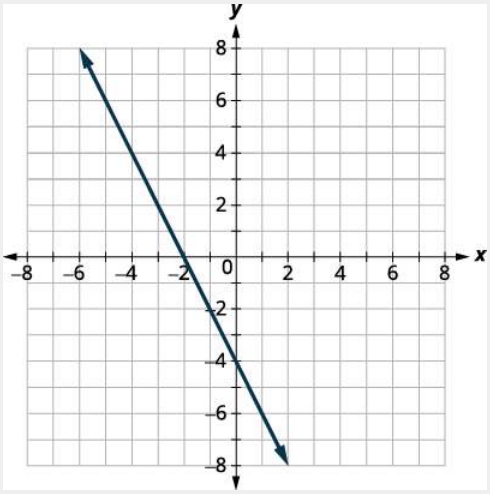
$f(x) = mx + b$
m, b: all real numbers
m: slope of the line
b: y-intercept
Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$

We will use the graphing techniques we used earlier, to graph the basic functions.

Example 4.4.4

Graph: $f(x) = -2x - 4$.

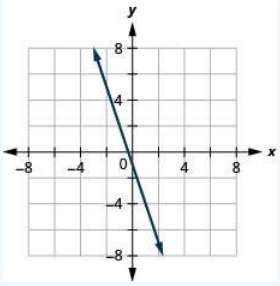
Answer

	$f(x) = -2x - 4$
We recognize this as a linear function.	
Find the slope and y-intercept.	$m = -2$ $b = -4$
Graph using the slope intercept.	

Example 4.4.5

Graph: $f(x) = -3x - 1$

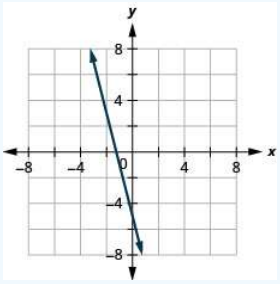
Answer



Example 4.4.6

Graph: $f(x) = -4x - 5$

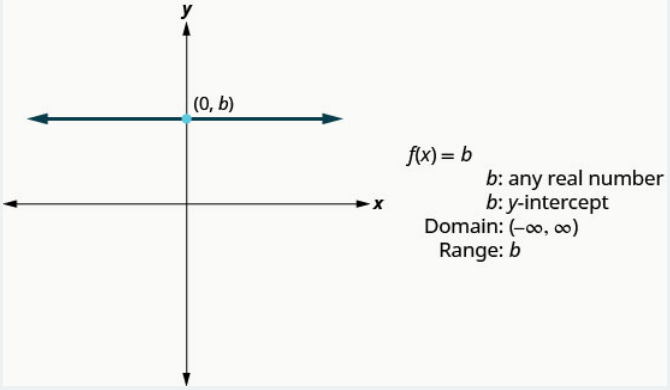
Answer



The next function whose graph we will look at is called the constant function and its equation is of the form $f(x) = b$, where b is any real number. If we replace the $f(x)$ with y, we get $y = b$. We recognize this as the horizontal line whose y-intercept is b. The graph of the function $f(x) = b$, is also the horizontal line whose y-intercept is b.

Notice that for any real number we put in the function, the function value will be b. This tells us the range has only one value, b.

CONSTANT FUNCTION



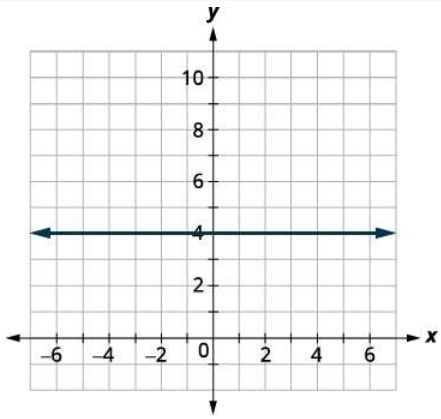
Example 4.4.7

Graph: $f(x) = 4$.

Answer

	$f(x) = 4$
We recognize this as a constant function.	

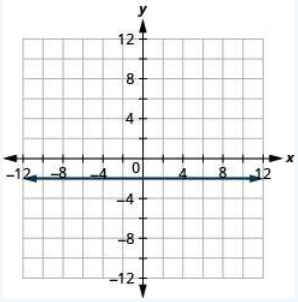
The graph will be a horizontal line through $(0, 4)$.



Example 4.4.8

Graph: $f(x) = -2$.

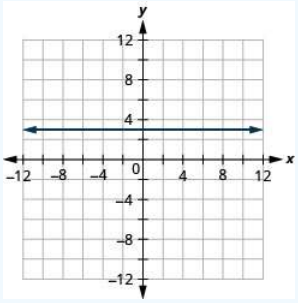
Answer



Example 4.4.9

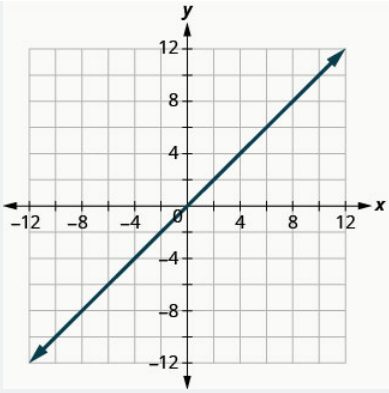
Graph: $f(x) = 3$.

Answer



The identity function, $f(x) = x$ is a special case of the linear function. If we write it in linear function form, $f(x) = 1x + 0$, we see the slope is 1 and the *y*-intercept is 0.

IDENTITY FUNCTION



$f(x) = x$
m: 1
b: 0
Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$

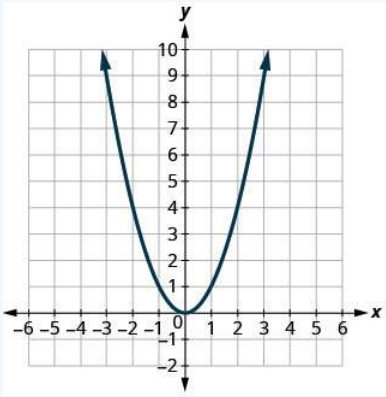
The next function we will look at is not a linear function. So the graph will not be a line. The only method we have to graph this function is point plotting. Because this is an unfamiliar function, we make sure to choose several positive and negative values as well as 0 for our x-values.

?

Graph: $f(x) = x^2$.

Answer

We choose x-values. We substitute them in and then create a chart as shown.

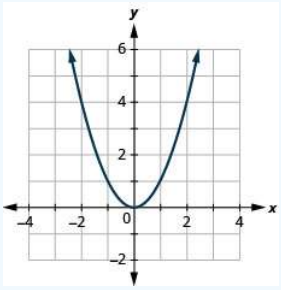


x	$f(x) = x^2$	$(x, f(x))$
−3	9	(−3, 9)
−2	4	(−2, 4)
−1	1	(−1, 1)
0	0	(0, 0)
1	1	(1, 1)
2	4	(2, 4)
3	9	(3, 9)

? Example 4.4.11

Graph: $f(x) = x^2$.

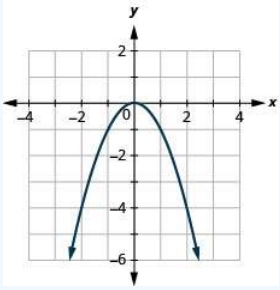
Answer



Example 4.4.12

$f(x) = -x^2$

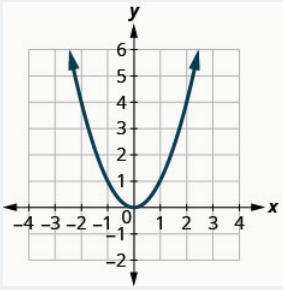
Answer



Looking at the result in Example, we can summarize the features of the square function. We call this graph a parabola. As we consider the domain, notice any real number can be used as an *x*-value. The domain is all real numbers.

The range is not all real numbers. Notice the graph consists of values of *y* never go below zero. This makes sense as the square of any number cannot be negative. So, the range of the square function is all non-negative real numbers.

SQUARE FUNCTION



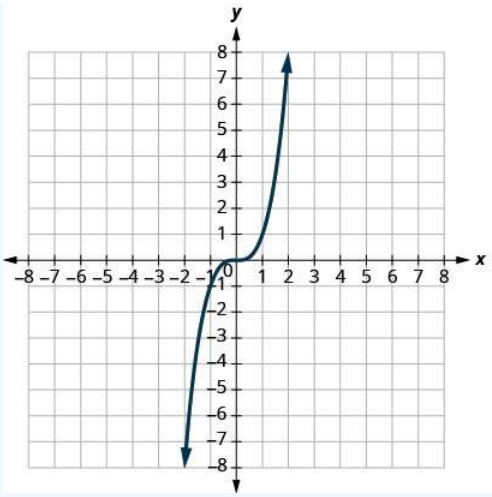
$f(x) = x^2$
Domain: $(-\infty, \infty)$
Range: $[0, \infty)$

The next function we will look at is also not a linear function so the graph will not be a line. Again we will use point plotting, and make sure to choose several positive and negative values as well as 0 for our *x*-values.

Graph: $f(x) = x^3$.

Answer

We choose *x*-values. We substitute them in and then create a chart.

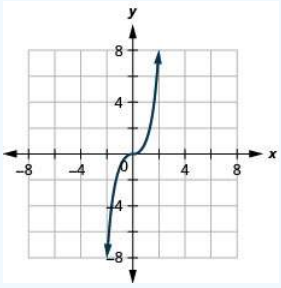


x	$f(x) = x^3$	$(x, f(x))$
–2	–8	(–2, –8)
–1	–1	(–1, –1)
0	0	(0, 0)
1	1	(1, 1)
2	8	(2, 8)

Example 4.4.14

Graph: $f(x) = x^3$.

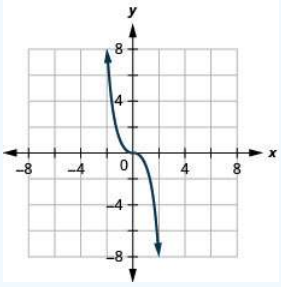
Answer



Example 4.4.15

Graph: $f(x) = -x^3$.

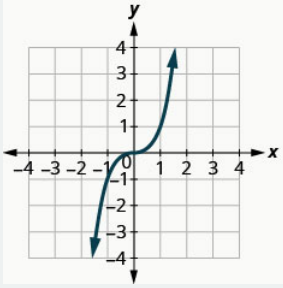
Answer



Looking at the result in Example, we can summarize the features of the cube function. As we consider the domain, notice any real number can be used as an x-value. The domain is all real numbers.

The range is all real numbers. This makes sense as the cube of any non-zero number can be positive or negative. So, the range of the cube function is all real numbers.

CUBE FUNCTION



$f(x) = x^3$
Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$

The next function we will look at does not square or cube the input values, but rather takes the square root of those values.

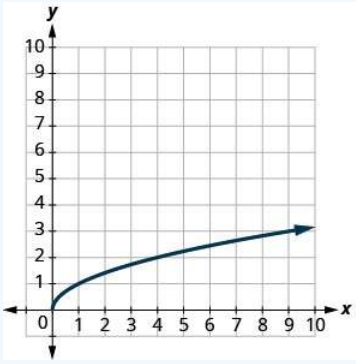
Let's graph the function $f(x) = \sqrt{x}$ and then summarize the features of the function. Remember, we can only take the square root of non-negative real numbers, so our domain will be the non-negative real numbers.

Example 4.4.16

$f(x) = \sqrt{x}$

Answer

We choose *x*-values. Since we will be taking the square root, we choose numbers that are perfect squares, to make our work easier. We substitute them in and then create a chart.

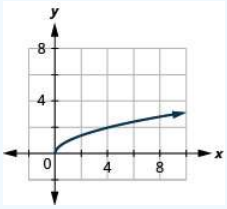


x	$f(x) = \sqrt{x}$	$(x, f(x))$
0	0	(0, 0)
1	1	(1, 1)
4	2	(4, 2)
9	3	(9, 3)

Example 4.4.17

Graph: $f(x) = x$.

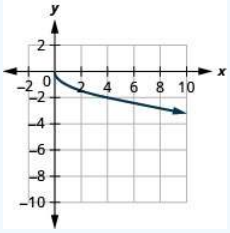
Answer



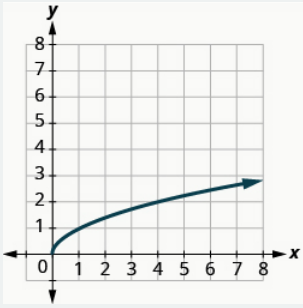
Example 4.4.18

Graph: $f(x) = -\sqrt{x}$.

Answer



SQUARE ROOT FUNCTION



$f(x) = \sqrt{x}$
Domain: $[0, \infty)$
Range: $[0, \infty)$

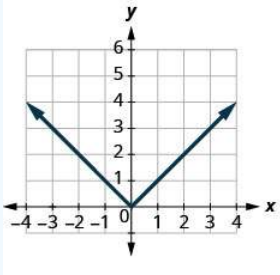
Our last basic function is the absolute value function, $f(x) = |x|$. Keep in mind that the absolute value of a number is its distance from zero. Since we never measure distance as a negative number, we will never get a negative number in the range.

?

Graph: $f(x) = |x|$.

Answer

We choose *x*-values. We substitute them in and then create a chart.

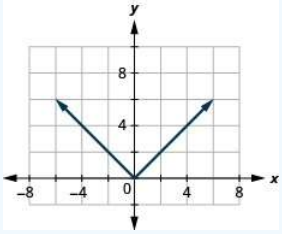


x	$f(x) = \|x\|$	$(x, f(x))$
–3	3	(–3, 3)
–2	2	(–2, 2)
–1	1	(–1, 1)
0	0	(0, 0)
1	1	(1, 1)
2	2	(2, 2)
3	3	(3, 3)

? Example 4.4.20

Graph: $f(x) = |x|$.

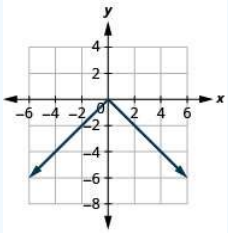
Answer



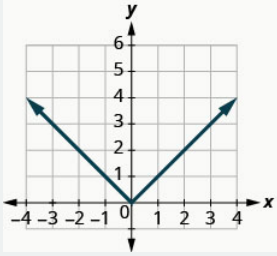
Example 4.4.21

Graph: $f(x) = -|x|$.

Answer



ABSOLUTE VALUE FUNCTION



$f(x) = |x|$
Domain: $(-\infty, \infty)$
Range: $[0, \infty)$

Read Information from a Graph of a Function

In the sciences and business, data is often collected and then graphed. The graph is analyzed, information is obtained from the graph and then often predictions are made from the data.

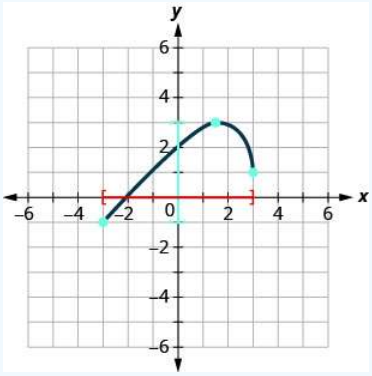
We will start by reading the domain and range of a function from its graph.

Remember the domain is the set of all the *x*-values in the ordered pairs in the function. To find the domain we look at the graph and find all the values of *x* that have a corresponding value on the graph. Follow the value *x* up or down vertically. If you hit the graph of the function then *x* is in the domain.

Remember the range is the set of all the *y*-values in the ordered pairs in the function. To find the range we look at the graph and find all the values of *y* that have a corresponding value on the graph. Follow the value *y* left or right horizontally. If you hit the graph of the function then *y* is in the range.

Example 4.4.22

Use the graph of the function to find its domain and range. Write the domain and range in interval notation.



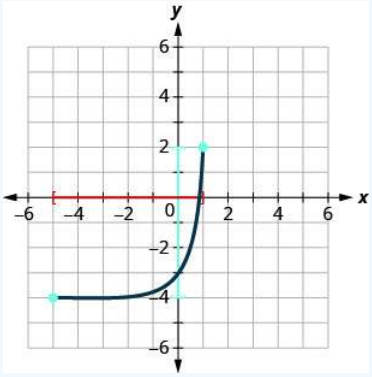
Answer

To find the domain we look at the graph and find all the values of x that correspond to a point on the graph. The domain is highlighted in red on the graph. The domain is $[-3, 3]$.

To find the range we look at the graph and find all the values of y that correspond to a point on the graph. The range is highlighted in blue on the graph. The range is $[-1, 3]$.

Example 4.4.23

Use the graph of the function to find its domain and range. Write the domain and range in interval notation.

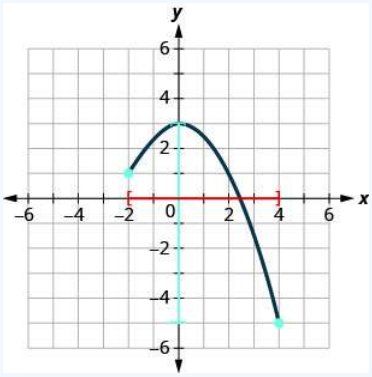


Answer

The domain is $[-5, 1]$. The range is $[-4, 2]$.

Example 4.4.24

Use the graph of the function to find its domain and range. Write the domain and range in interval notation.



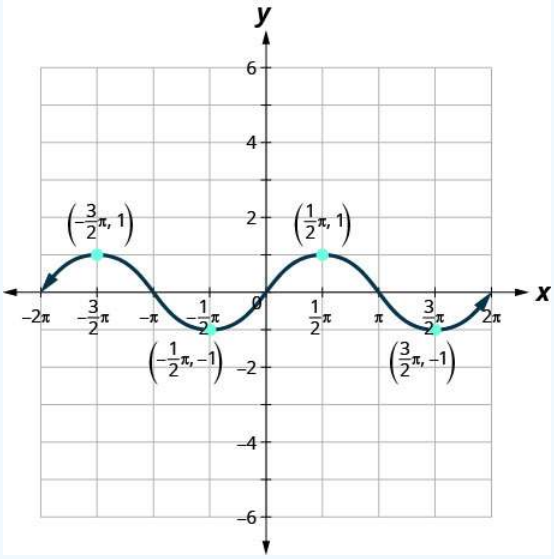
Answer

The domain is $[-2, 4]$. The range is $[-5, 3]$.

We are now going to read information from the graph that you may see in future math classes.

Example 4.4.25

Use the graph of the function to find the indicated values.



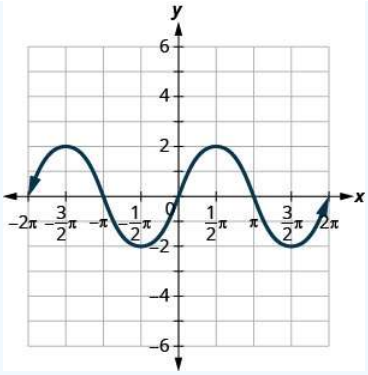
ⓐ Find: $f(0)$.
ⓑ Find: $f(32\pi)$.
ⓒ Find: $f(-12\pi)$.
ⓓ Find the values for x when $f(x) = 0$.
ⓔ Find the x-intercepts.
ⓕ Find the y-intercepts.
ⓖ Find the domain. Write it in interval notation.
ⓗ Find the range. Write it in interval notation.

Answer

ⓐ When $x = 0$, the function crosses the y-axis at 0. So, $f(0) = 0$.
ⓑ When $x = 32\pi$, the y-value of the function is -1. So, $f(32\pi) = -1$.
ⓒ When $x = -12\pi$, the y-value of the function is -1. So, $f(-12\pi) = -1$.
ⓓ The function is 0 at the points, $(-2\pi, 0), (-\pi, 0), (0, 0), (\pi, 0), (2\pi, 0)$. The x-values when $f(x) = 0$ are $-2\pi, -\pi, 0, \pi, 2\pi$.
ⓔ The x-intercepts occur when $y = 0$. So the x-intercepts occur when $f(x) = 0$. The x-intercepts are $(-2\pi, 0), (-\pi, 0), (0, 0), (\pi, 0), (2\pi, 0)$.
ⓕ The y-intercepts occur when x=0.x=0. So the y-intercepts occur at $f(0)$. The y-intercept is $(0, 0)$.
ⓖ This function has a value when x is from -2π to 2π. Therefore, the domain in interval notation is $[-2\pi, 2\pi]$.
ⓗ This function values, or y-values go from -1 to 1. Therefore, the range, in interval notation, is $[-1, 1]$.

? Example 4.4.26

Use the graph of the function to find the indicated values.



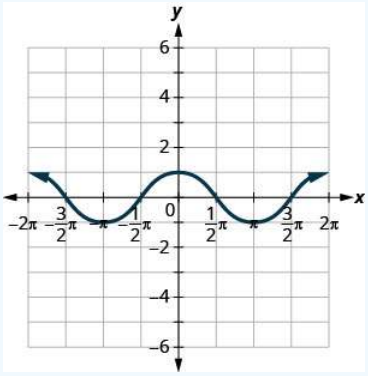
ⓐ Find: f(0).f(0).
ⓑ Find: f(12\pi).f(12\pi).
ⓒ Find: f(−32\pi).f(−32\pi).
ⓓ Find the values for *x* when f(x)=0.f(x)=0.
ⓔ Find the *x*-intercepts.
ⓕ Find the *y*-intercepts.
ⓖ Find the domain. Write it in interval notation.
ⓗ Find the range. Write it in interval notation.

Answer

ⓐ $f(0)=0$ ⓑ $f=(\pi 2)=2$ ⓒ $f=(-3\pi 2)=2$ ⓓ $f(x)=0$ for $x=-2\pi,-\pi,0,\pi,2\pi$ ⓔ $(-2\pi,0),(-\pi,0),(0,0),(\pi,0),(2\pi,0)$ⓕ (0,0)(0,0) ⓖ $[-2\pi,2\pi]$ ⓗ $[-2,2]$

Example 4.4.27

Use the graph of the function to find the indicated values.



ⓐ Find: $f(0)$.
ⓑ Find: $f(\pi)$.
ⓒ Find: $f(-\pi)$.
ⓓ Find the values for *x* when $f(x)=0$.
ⓔ Find the *x*-intercepts.
ⓕ Find the *y*-intercepts.
ⓖ Find the domain. Write it in interval notation.
ⓗ Find the range. Write it in interval notation.

Answer

ⓐ $f(0)=1$ ⓑ $f(\pi)=-1$ ⓒ $f(-\pi)=-1$ ⓓ $f(x)=0$ for $x=-3\pi 2,-\pi 2,\pi 2,3\pi 2$ ⓔ $(-2pi,0),(-pi,0),(0,0),(pi,0),(2pi,0)$ⓕ $(0,1)$ ⓖ $[-2pi,2pi]$ⓗ $[-1,1]$

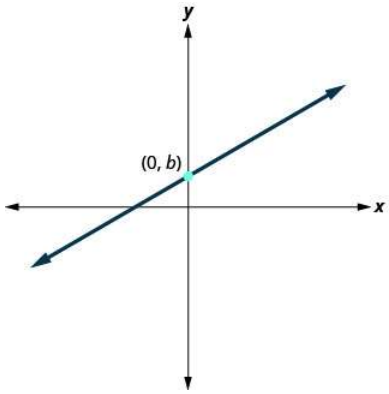
Access this online resource for additional instruction and practice with graphs of functions.

- Find Domain and Range

Key Concepts

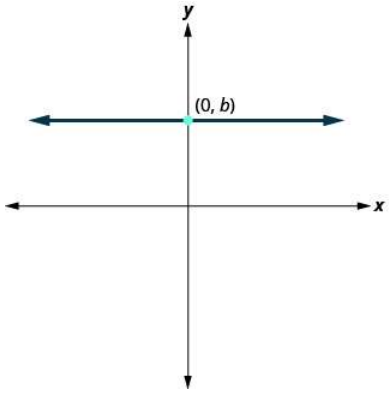
- **Vertical Line Test**
  - A set of points in a rectangular coordinate system is the graph of a function if every vertical line intersects the graph in at most one point.
  - If any vertical line intersects the graph in more than one point, the graph does not represent a function.
- **Graph of a Function**
  - The graph of a function is the graph of all its ordered pairs, (x,y)(x,y) or using function notation, (x,f(x))(x,f(x)) where y=f(x).y=f(x).

    fxf(x)name of functionx-coordinate of the ordered pairy-coordinate of the ordered pairfname of functionxx-coordinate of the ordered pairf(x)y-coordinate of the ordered pair
- **Linear Function**



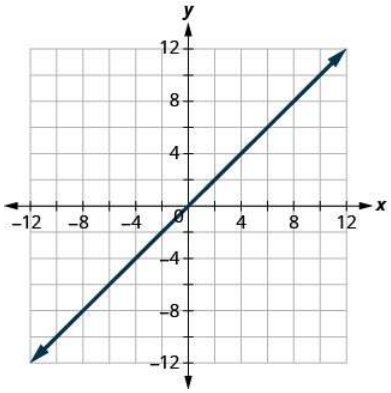
$f(x) = mx + b$
m, b: all real numbers
m: slope of the line
b: y-intercept
Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$

- **Constant Function**



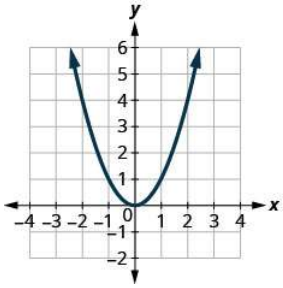
$f(x) = b$
b: any real number
b: y-intercept
Domain: $(-\infty, \infty)$
Range: b

- **Identity Function**



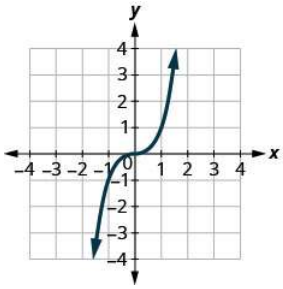
$f(x) = x$
m: 1
b: 0
Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$

- **Square Function**



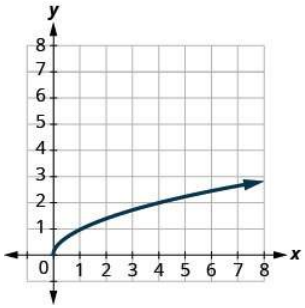
$f(x) = x^2$
Domain: $(-\infty, \infty)$
Range: $[0, \infty)$

- **Cube Function**



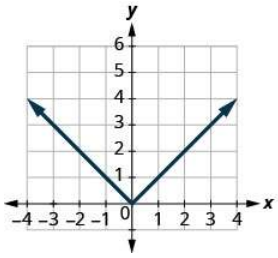
$f(x) = x^3$
Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$

- **Square Root Function**



$f(x) = \sqrt{x}$
Domain: $[0, \infty)$
Range: $[0, \infty)$

- **Absolute Value Function**



$f(x) = |x|$
Domain: $(-\infty, \infty)$
Range: $[0, \infty)$

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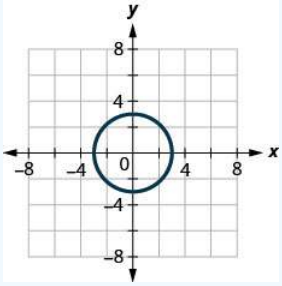
4.4E: Exercises

Practice Makes Perfect

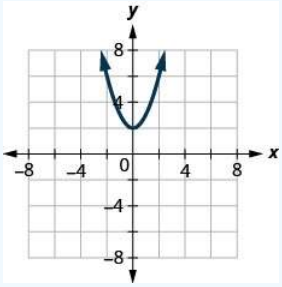
Use the Vertical Line Test

In the following exercises, determine whether each graph is the graph of a function.

1. ⓐ



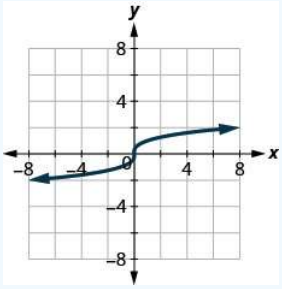
ⓑ



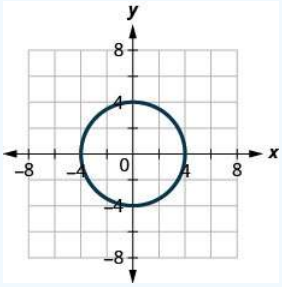
Answer

ⓐ no ⓑ yes

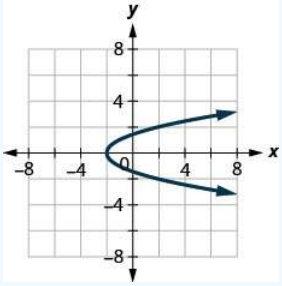
2. ⓐ



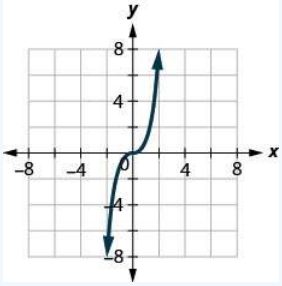
ⓑ



3. ⓐ



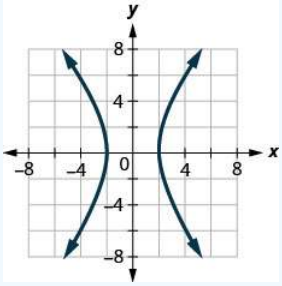
ⓑ



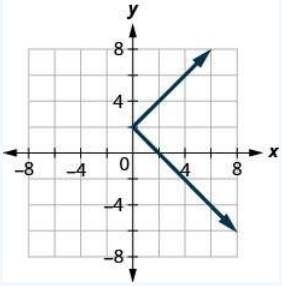
Answer

ⓐ no ⓑ yes

4. ⓐ



ⓑ



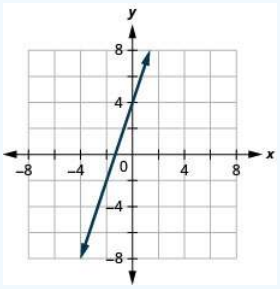
Identify Graphs of Basic Functions

In the following exercises, ⓐ graph each function ⓑ state its domain and range. Write the domain and range in interval notation.

5. $f(x) = 3x + 4$

Answer

ⓐ



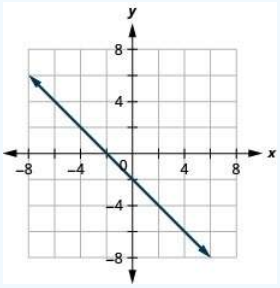
ⓑ $D: (-\text{inf}, \text{inf}),\ R: (-\text{inf}, \text{inf})$

6. $f(x) = 2x + 5$

7. $f(x) = -x - 2$

Answer

ⓐ



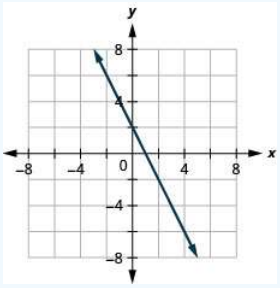
ⓑ $D: (-\text{inf}, \text{inf}),\ R: (-\text{inf}, \text{inf})$

8. $f(x) = -4x - 3$

9. $f(x) = -2x + 2$

Answer

ⓐ



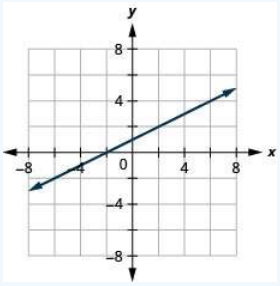
ⓑ $D: (-\text{inf}, \text{inf}),\ R: (-\text{inf}, \text{inf})$

10. $f(x) = -3x + 3$

11. $f(x) = \frac{1}{2}x + 1$

Answer

ⓐ



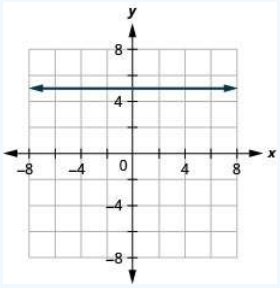
ⓑ $D : (-\text{inf}, \text{inf}),\ R : (-\text{inf}, \text{inf})$

12. $f(x) = \frac{2}{3}x - 2$

13. $f(x) = 5$

Answer

ⓐ



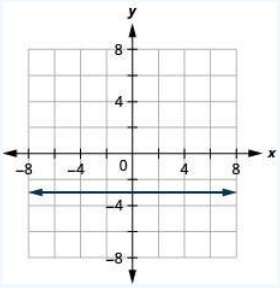
ⓑ $D : (-\text{inf}, \text{inf}), R : 5$

14. $f(x) = 2$

15. $f(x) = -3$

Answer

ⓐ



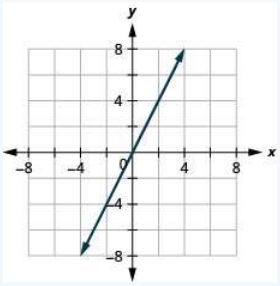
ⓑ $D : (-\text{inf}, \text{inf}),\ R : -3$

16. $f(x) = -1$

17. $f(x) = 2x$

Answer

ⓐ



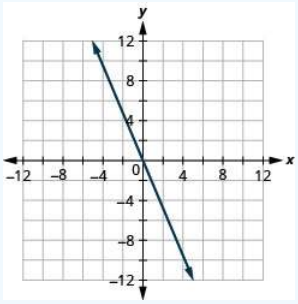
ⓑ $D: (-\text{inf}, \text{inf}),\ R: (-\text{inf}, \text{inf})$

18. $f(x) = 3x$

19. $f(x) = -2x$

Answer

ⓐ



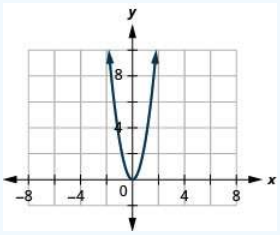
ⓑ $D: (-\text{inf}, \text{inf}), R: (-\text{inf}, \text{inf})$

20. $f(x) = -3x$

21. $f(x) = 3x^2$

Answer

ⓐ



ⓑ $D: (-\text{inf}, \text{inf}),\ R: [0, \text{inf})$

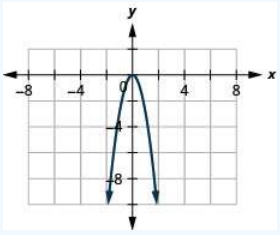
22. $f(x) = 2x^2$

23. $f(x) = -3x^2$

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Answer

ⓐ



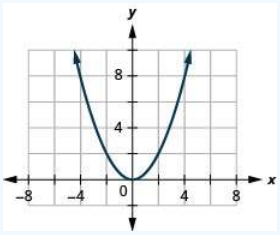
ⓑ $D : (-\text{inf}, \text{inf}),\ R : (-\text{inf}, 0]$

24. $f(x) = -2x^2$

25. $f(x) = 12x^2$

Answer

ⓐ



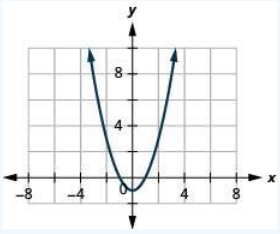
ⓑ $D : (-\text{inf}, \text{inf}),\ R : [-\text{inf}, 0)$

26. $f(x) = \frac{1}{3}x^2$

27. $f(x) = x^2 - 1$

Answer

ⓐ



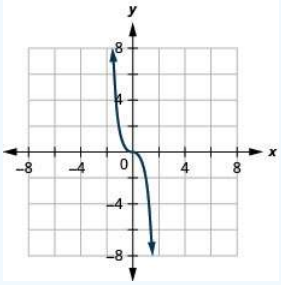
ⓑ $D : (-\text{inf}, \text{inf}),\ R : [-1, \text{inf})$

28. $f(x) = x^2 + 1$

29. $f(x) = -2x^3$

Answer

ⓐ



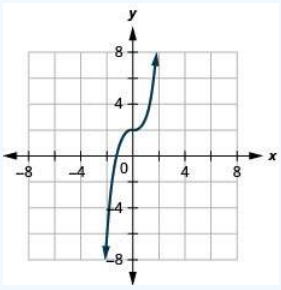
ⓑ $D: (-\text{inf}, \text{inf}),\ R: (-\text{inf}, \text{inf})$

30. $f(x) = 2x^3$

31. $f(x) = x^3 + 2$

Answer

ⓐ



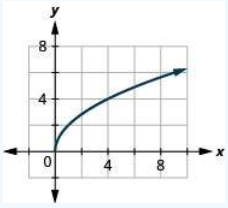
ⓑ $D: (-\text{inf}, \text{inf}), R: (-\text{inf}, \text{inf})$

32. $f(x) = x^3 - 2$

33. $f(x) = 2\sqrt{x}$

Answer

ⓐ



ⓑ $D: [0, \text{inf}), R: [0, \text{inf})$

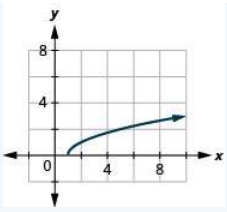
34. $f(x) = -2\sqrt{x}$

35. $f(x) = \sqrt{x-1}$

Answer

ⓐ

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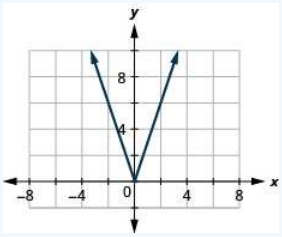
ⓑ $D: [1, \text{inf}), R: [0, \text{inf})$

36. $f(x) = \sqrt{x+1}$

37. $f(x) = 3|x|$

Answer

ⓐ



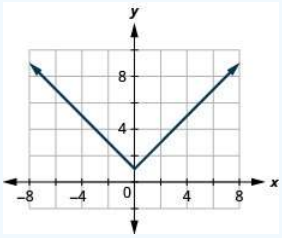
ⓑ $D: [-1, -1, \text{inf}), R: [-\text{inf}, \text{inf})$

38. $f(x) = -2|x|$

39. $f(x) = |x| + 1$

Answer

ⓐ



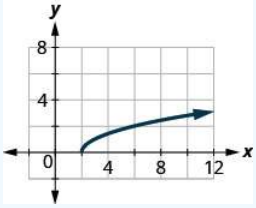
ⓑ $D: (-\text{inf}, \text{inf}), R: [1, \text{inf})$

40. $f(x) = |x| - 1$

Read Information from a Graph of a Function

In the following exercises, use the graph of the function to find its domain and range. Write the domain and range in interval notation.

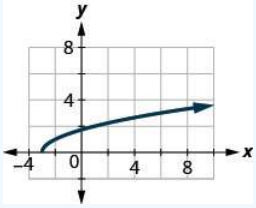
41.



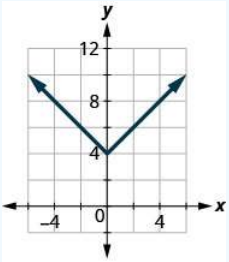
Answer

$D: [2, \text{inf}),\ R: [0, \text{inf})$

42.



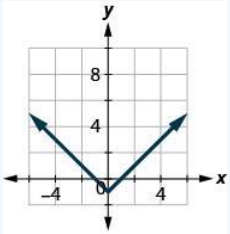
43.



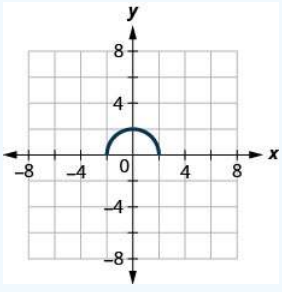
Answer

$D: (-\text{inf}, \text{inf}),\ R: [4, \text{inf})$

44.



45.

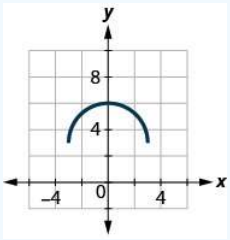


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Answer

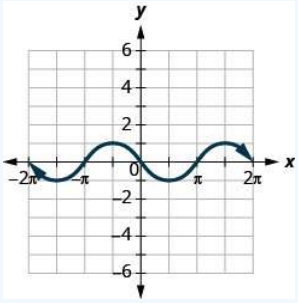
$D: [-2, 2],\ R: [0, 2]$

46.



In the following exercises, use the graph of the function to find the indicated values.

47.

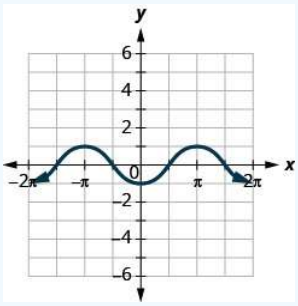


ⓐ Find: $f(0)$.
ⓑ Find: $f(12\pi)$.
ⓒ Find: $f(-32\pi)$.
ⓓ Find the values for x when $f(x) = 0$.
ⓔ Find the x-intercepts.
ⓕ Find the y-intercepts.
ⓖ Find the domain. Write it in interval notation.
ⓗ Find the range. Write it in interval notation.

Answer

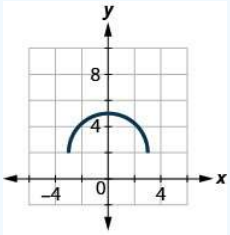
ⓐ $f(0) = 0$ ⓑ $(\pi/2) = -1$
ⓒ $f(-3\pi/2) = -1$ ⓓ $f(x) = 0$ for $x = -2\pi, -\pi, 0, \pi, 2\pi$
ⓔ $(-2\pi, 0), (-\pi, 0), (0, 0), (\pi, 0), (2\pi, 0)(f)(0, 0)$
ⓖ $[-2\pi, 2\pi]$ ⓗ $[-1, 1]$

48.



ⓐ Find: $f(0)$.
ⓑ Find: $f(\pi)$.
ⓒ Find: $f(-\pi)$.
ⓓ Find the values for x when $f(x) = 0$.
ⓔ Find the x-intercepts.
ⓕ Find the y-intercepts.
ⓖ Find the domain. Write it in interval notation.
ⓗ Find the range. Write it in interval notation

49.

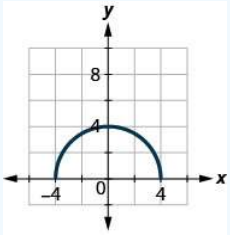


ⓐ Find: $f(0)$.
ⓑ Find: $f(-3)$.
ⓒ Find: $f(3)$.
ⓓ Find the values for x when $f(x) = 0$.
ⓔ Find the x-intercepts.
ⓕ Find the y-intercepts.
ⓖ Find the domain. Write it in interval notation.
ⓗ Find the range. Write it in interval notation.

Answer

ⓐ $f(0) = -6$ ⓑ $f(-3) = 3$ ⓒ $f(3) = 3$ ⓓ $f(x) = 0$ for no x ⓔ none ⓕ $y = 6$ ⓖ $[-3, 3]$ ⓗ $[-3, 6]$

50.



ⓐ Find: $f(0)$.
ⓑ Find the values for x when $f(x) = 0$.
ⓒ Find the x-intercepts.
ⓓ Find the y-intercepts.
ⓔ Find the domain. Write it in interval notation.
ⓕ Find the range. Write it in interval notation

Writing Exercises

51. Explain in your own words how to find the domain from a graph.

52. Explain in your own words how to find the range from a graph.

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53. Explain in your own words how to use the vertical line test.

54. Draw a sketch of the square and cube functions. What are the similarities and differences in the graphs?

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
use the vertical line test.			
identify graphs of basic functions.			
read information from a graph.			

ⓑ After reviewing this checklist, what will you do to become confident for all objectives?

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4.5: Linear Functions - Applications

Learning Objectives

- Represent a linear function.
- Determine whether a linear function is increasing, decreasing, or constant.
- Interpret slope as a rate of change.
- Write and interpret an equation for a linear function.
- Graph linear functions.
- Determine whether lines are parallel or perpendicular.
- Write the equation of a line parallel or perpendicular to a given line.

Just as with the growth of a bamboo plant, there are many situations that involve constant change over time. Consider, for example, the first commercial maglev train in the world, the Shanghai MagLev Train (maglev train in the world, the Shanghai MagLev Train (Figure 4.5.1). It carries passengers comfortably for a 30-kilometer trip from the airport to the subway station in only eight minutes.



Figure 4.5.1: Shanghai MagLev Train (credit: “kanegen”/Flickr)

Suppose a maglev train were to travel a long distance, and that the train maintains a constant speed of 83 meters per second for a period of time once it is 250 meters from the station. How can we analyze the train’s distance from the station as a function of time? In this section, we will investigate a kind of function that is useful for this purpose, and use it to investigate real-world situations such as the train’s distance from the station at a given point in time.maglev train were to travel a long distance, and that the train maintains a constant speed of 83 meters per second for a period of time once it is 250 meters from the station. How can we analyze the train’s distance from the station as a function of time? In this section, we will investigate a kind of function that is useful for this purpose, and use it to investigate real-world situations such as the train’s distance from the station at a given point in time.

Representing Linear Functions

The function describing the train’s motion is a **linear function**, which is defined as a function with a constant rate of change, that is, a polynomial of degree 1. There are several ways to represent a linear function, including word form, function notation, tabular form, and graphical form. We will describe the train’s motion as a function using each method.

Representing a Linear Function in Word Form

Let’s begin by describing the linear function in words. For the train problem we just considered, the following word sentence may be used to describe the function relationship.

- *The train’s distance from the station is a function of the time during which the train moves at a constant speed plus its original distance from the station when it began moving at constant speed.*

The speed is the rate of change. Recall that a rate of change is a measure of how quickly the dependent variable changes with respect to the independent variable. The rate of change for this example is constant, which means that it is the same for each input value. As the time (input) increases by 1 second, the corresponding distance (output) increases by 83 meters. The train began moving at this constant speed at a distance of 250 meters from the station.

Representing a Linear Function in Function Notation

Another approach to representing linear functions is by using function notation. One example of function notation is an equation written in the form known as the slope-intercept form of a line, where x is the input value, m is the rate of change, and b is the initial value of the dependent variable.

$$\begin{aligned} &\text{Equation form} && y = mx + b \\ &\text{Equation notation} && f(x) = mx + b \end{aligned}$$

In the example of the train, we might use the notation $D(t)$ in which the total distance D is a function of the time t. The rate, m, is 83 meters per second. The initial value of the dependent variable b is the original distance from the station, 250 meters. We can write a generalized equation to represent the motion of the train.

$$D(t) = 83t + 250 \tag{4.5.1}$$

Representing a Linear Function in Tabular Form

A third method of representing a linear function is through the use of a table. The relationship between the distance from the station and the time is represented in Figure 4.5.2. From the table, we can see that the distance changes by 83 meters for every 1 second increase in time.

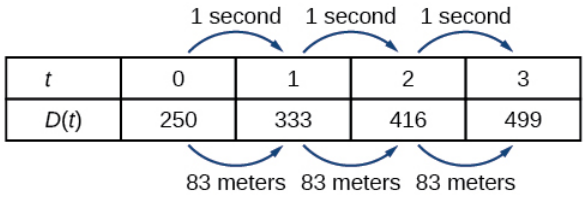


Figure 4.5.2: Tabular representation of the function D showing selected input and output values

Q/A? Can the input in the previous example be any real number?

No. The input represents time, so while nonnegative rational and irrational numbers are possible, negative real numbers are not possible for this example. The input consists of non-negative real numbers.

Representing a Linear Function in Graphical Form

Another way to represent linear functions is visually, using a graph. We can use the function relationship from above, $D(t) = 83t + 250$, to draw a graph, represented in Figure 4.5.3. Notice the graph is a line. When we plot a linear function, the graph is always a line.

The rate of change, which is constant, determines the slant, or **slope** of the line. The point at which the input value is zero is the vertical intercept, or **y-intercept**, of the line. We can see from the graph in Figure 4.5.3 that the y-intercept in the train example we just saw is $(0, 250)$ and represents the distance of the train from the station when it began moving at a constant speed.

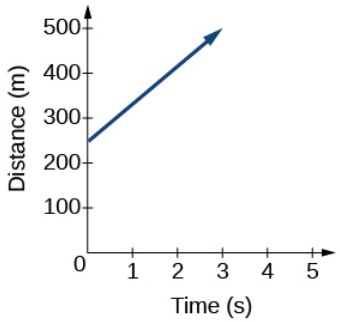


Figure 4.5.3: The graph of $D(t) = 83t + 250$. Graphs of linear functions are lines because the rate of change is constant.

Notice that the graph of the train example is restricted, but this is not always the case. Consider the graph of the line $f(x) = 2x + 1$. Ask yourself what numbers can be input to the function, that is, what is the domain of the function? The domain is comprised of all real numbers because any number may be doubled, and then have one added to the product.

Definition: Linear Function

A **linear function** is a function whose graph is a line. Linear functions can be written in the slope-intercept form of a line

$$f(x) = mx + b \tag{4.5.2}$$

where b is the initial or starting value of the function (when input, $x = 0$), and m is the constant rate of change, or **slope** of the function. The **y-intercept** is at $(0, b)$.

Example 4.5.1: Using a Linear Function to Find the Pressure on a Diver

The pressure, P, in pounds per square inch (PSI) on the diver in Figure 4.5.4 depends upon her depth below the water surface, d, in feet. This relationship may be modeled by the equation, $P(d) = 0.434d + 14.696$. Restate this function in words.



Figure 4.5.4: (credit: Ilse Reijs and Jan-Noud Hutten)Hutten)

To restate the function in words, we need to describe each part of the equation. The pressure as a function of depth equals four hundred thirty-four thousandths times depth plus fourteen and six hundred ninety-six thousandths.

Analysis

The initial value, 14.696, is the pressure in PSI on the diver at a depth of 0 feet, which is the surface of the water. The rate of change, or slope, is 0.434 PSI per foot. This tells us that the pressure on the diver increases 0.434 PSI for each foot her depth increases.

Determining whether a Linear Function Is Increasing, Decreasing, or Constant

The linear functions we used in the two previous examples increased over time, but not every linear function does. A linear function may be increasing, decreasing, or constant. For an increasing function, as with the train example, the output values increase as the input values increase. The graph of an **increasing function** has a positive slope. A line with a positive slope slants upward from left to right as in Figure 4.5.5(a). For a **decreasing function**, the slope is negative. The output values decrease as the input values increase. A line with a negative slope slants downward from left to right as in Figure 4.5.5(b). If the function is constant, the output values are the same for all input values so the slope is zero. A line with a slope of zero is horizontal as in Figure 4.5.5(c).

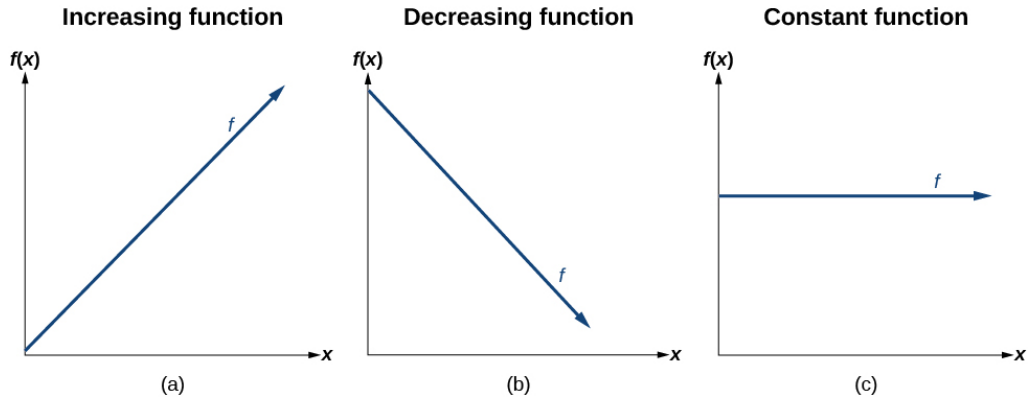


Figure 4.5.5: Three graphs depicting an increasing function, a decreasing function, and a constant function.]
Increasing and Decreasing Functions

Increasing and Decreasing Functions

The slope determines if the function is an **increasing linear function**, a **decreasing linear function**, or a constant function.

- $f(x) = mx + b$ is an increasing function if $m > 0$.
- $f(x) = mx + b$ is an decreasing function if $m < 0$.
- $f(x) = mx + b$ is a constant function if $m = 0$.

Example 4.5.2: decide whether a Function is Increasing, Decreasing, or Constant

Some recent studies suggest that a teenager sends an average of 60 texts per day. For each of the following scenarios, find the linear function that describes the relationship between the input value and the output value. Then, determine whether the graph of the function is increasing, decreasing, or constant.

a. The total number of texts a teen sends is considered a function of time in days. The input is the number of days, and output is the total number of texts sent.
b. A teen has a limit of 500 texts per month in his or her data plan. The input is the number of days, and output is the total number of texts remaining for the month.
c. A teen has an unlimited number of texts in his or her data plan for a cost of $50 per month. The input is the number of days, and output is the total cost of texting each month.

Solution

Analyze each function.

a. The function can be represented as $f(x) = 60x$ where x is the number of days. The slope, 60, is positive so the function is increasing. This makes sense because the total number of texts increases with each day.
b. The function can be represented as $f(x) = 500 - 60x$ where x is the number of days. In this case, the slope is negative so the function is decreasing. This makes sense because the number of texts remaining decreases each day and this function represents the number of texts remaining in the data plan after x days.
c. The cost function can be represented as $f(x) = 50$ because the number of days does not affect the total cost. The slope is 0 so the function is constant.

Calculating and Interpreting Slope

In the examples we have seen so far, we have had the slope provided for us. However, we often need to calculate the slope given input and output values. Given two values for the input, x_1 and x_2, and two corresponding values for the output, y_1 and y_2—which can be represented by a set of points, (x_1, y_1) and (x_2, y_2)—we can calculate the slope m, as follows

$$m = \frac{\text{change in output (rise)}}{\text{change in input (run)}}$$

$$= \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

where Δy is the vertical displacement and Δx is the horizontal displacement. Note in function notation two corresponding values for the output y_1 and y_2 for the function f, $y_1 = f(x_1)$ and $y_2 = f(x_2)$, so we could equivalently write

$$m = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Figure $4.5.6$ indicates how the slope of the line between the points, (x_1, y_1) and (x_2, y_2), is calculated. Recall that the slope measures steepness. The greater the absolute value of the slope, the steeper the line is.

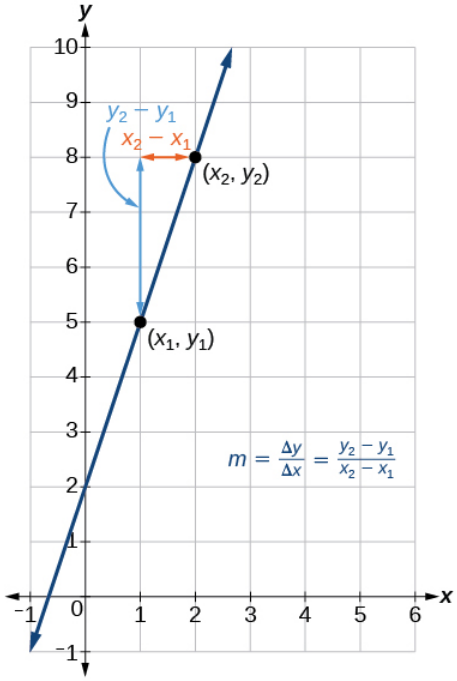


Figure 4.5.6: The slope of a function is calculated by the change in y divided by the change in x. It does not matter which coordinate is used as the (x_2, y_2) and which is the (x_1, y_1),as long as each calculation is started with the elements from the same coordinate pair.

Q & A

Are the units for slope always $\frac{\text{units for the output}}{\text{units for the input}}$?

Yes. Think of the units as the change of output value for each unit of change in input value. An example of slope could be miles per hour or dollars per day. Notice the units appear as a ratio of units for the output per units for the input.

Note: Calculate Slope

The slope or rate of change of a function, m, can be calculated according to the following:

$$m = \frac{\text{change in output (rise)}}{\text{change in input (run)}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} \tag{4.5.3}$$

where x_1 and x_2 are input values, y_1 and y_2 are output values.

How to: Given two points from a linear function, calculate and interpret the slope.

1. Determine the units for output and input values.
2. Calculate the change of output values and change of input values.
3. Interpret the slope as the change in output values per unit of the input value.

Example 4.5.3: Finding the Slope of a Linear Function

If $f(x)$ is a linear function, and $(3,-2)$ and $(8,1)$ are points on the line, find the slope. Is this function increasing or decreasing?

Solution

The coordinate pairs are $(3,-2)$ and $(8,1)$. To find the rate of change, we divide the change in output by the change in input.

$$m=\frac{\text{change in output (rise)}}{\text{change in input (run)}}=\frac{1-(-2)}{8-3}=\frac{3}{5} \tag{4.5.4}$$

We could also write the slope as $m=0.6$. The function is increasing because $m>0$.

Analysis

As noted earlier, the order in which we write the points does not matter when we compute the slope of the line as long as the first output value, or *y*-coordinate, used corresponds with the first input value, or *x*-coordinate, used.

Try it! 4.5.1

If $f(x)$ is a linear function, and $(2,3)$ and $(0,4)$ are points on the line, find the slope. Is this function increasing or decreasing?

Answer

$m=\frac{4-3}{0-2}=\frac{1}{-2}=-\frac{1}{2}$; decreasing because $m<0$.

Example 4.5.4: Finding the Population Change from a Linear Function

The population of a city increased from 23,400 to 27,800 between 2008 and 2012. Find the change of population per year if we assume the change was constant from 2008 to 2012.

The rate of change relates the change in population to the change in time. The population increased by $27,800-23,400=4400$ people over the four-year time interval. To find the rate of change, divide the change in the number of people by the number of years.

$$\frac{4,400 \text{ people}}{4 \text{ years}}=1,100\frac{\text{people}}{\text{year}} \tag{4.5.5}$$

So the population increased by 1,100 people per year.

Analysis

Because we are told that the population increased, we would expect the slope to be positive. This positive slope we calculated is therefore reasonable.

Try it! 4.5.2

The population of a small town increased from 1,442 to 1,868 between 2009 and 2012. Find the change of population per year if we assume the change was constant from 2009 to 2012.

Answer

$m = \frac{1{,}868-1{,}442}{2{,}012-2{,}009} = \frac{426}{3} = 142$ people per year

Writing the Point-Slope Form of a Linear Equation

Up until now, we have been using the slope-intercept form of a linear equation to describe linear functions. Here, we will learn another way to write a linear function, the **point-slope form**.

$$y - y_1 = m(x - x_1) \tag{4.5.6}$$

The point-slope form is derived from the slope formula.

$$\begin{aligned} m &= \frac{y-y_1}{x-x_1} && \text{assuming } x \neq x_1 \\ m(x-x_1) &= \frac{y-y_1}{x-x_1}(x-x_1) && \text{Multiply both sides by } (x-x_1). \\ m(x-x_1) &= y-y_1 && \text{Simplify} \\ y-y_1 &= m(x-x_1) && \text{Rearrange} \end{aligned}$$

Keep in mind that the slope-intercept form and the point-slope form can be used to describe the same function. We can move from one form to another using basic algebra. For example, suppose we are given an equation in point-slope form, $y-4 = -\frac{1}{2}(x-6)$. We can convert it to the slope-intercept form as shown.

$$\begin{aligned} y-4 &= -\frac{1}{2}(x-6) \\ y-4 &= -\frac{1}{2}x+3 && \text{Distribute the } -\frac{1}{2}. \\ y &= -\frac{1}{2}x+7 && \text{Add 4 to each side.} \end{aligned}$$

Therefore, the same line can be described in slope-intercept form as $y = \frac{1}{2}x + 7$.

Note: Point-Slope Form of a Linear Equation

The **point-slope** form of a linear equation takes the form

$$y - y_1 = m(x - x_1) \tag{4.5.7}$$

where m is the slope, x_1 and y_1 are the x and y coordinates of a specific point through which the line passes.

Writing the Equation of a Line Using a Point and the Slope

The point-slope form is particularly useful if we know one point and the slope of a line. Suppose, for example, we are told that a line has a slope of 2 and passes through the point $(4, 1)$. We know that $m = 2$ and that $x_1 = 4$ and $y_1 = 1$. We can substitute these values into the general point-slope equation.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 1 &= 2(x - 4) \end{aligned}$$

If we wanted to then rewrite the equation in slope-intercept form, we apply algebraic techniques.

$$\begin{aligned} y-1 &= 2(x-4) \\ y-1 &= 2x-8 && \text{Distribute the 2.} \\ y &= 2x-7 && \text{Add 1 to each side.} \end{aligned}$$

Both equations, $y-1=2(x-4)$ and $y=2x-7$, describe the same line. See Figure 4.5.7.

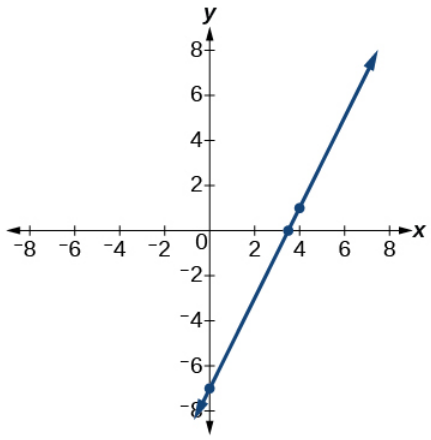


Figure 4.5.7

Example 4.5.5: Writing Linear Equations Using a Point and the Slope

Write the point-slope form of an equation of a line with a slope of 3 that passes through the point $(6,-1)$. Then rewrite it in the slope-intercept form.

Solution

Let's figure out what we know from the given information. The slope is 3, so $m=3$. We also know one point, so we know $x_1=6$ and $y_1=-1$. Now we can substitute these values into the general point-slope equation.

$$\begin{aligned} y-y_1 &= m(x-x_1) \\ y-(-1) &= 3(x-6) && \text{Substitute known values.} \\ y+1 &= 3(x-6) && \text{Distribute } -1 \text{ to find point-slope form.} \end{aligned}$$

Then we use algebra to find the slope-intercept form.

$$\begin{aligned} y+1 &= 3(x-6) \\ y+1 &= 3x-18 && \text{Distribute 3.} \\ y &= 3x-19 && \text{Simplify to slope-intercept form.} \end{aligned}$$

? Try it! 4.5.3

Write the point-slope form of an equation of a line with a slope of –2 that passes through the point $(-2,2)$. Then rewrite it in the slope-intercept form.

Answer

$y-2=-2(x+2)$; $y=-2x-2$

Writing the Equation of a Line Using Two Points

The point-slope form of an equation is also useful if we know any two points through which a line passes. Suppose, for example, we know that a line passes through the points $(0,1)$ and $(3,2)$. We can use the coordinates of the two points to find the slope.

$$\begin{aligned} m &= \frac{y_2-y_1}{x_2-x_1} \\ &= \frac{2-1}{3-0} \\ &= \frac{1}{3} \end{aligned}$$

Now we can use the slope we found and the coordinates of one of the points to find the equation for the line. Let use $(0,1)$ for our point.

$$\begin{aligned} y-y_1 &= m(x-x_1) \\ y-1 &= \frac{1}{3}(x-0) \end{aligned}$$

As before, we can use algebra to rewrite the equation in the slope-intercept form.

$$\begin{aligned} y-1 &= \frac{1}{3}(x-0) && \\ y-1 &= \frac{1}{3}x && \text{Distribute the } \frac{1}{3}. \\ y &= \frac{1}{3}x+1 && \text{Add 1 to each side.} \end{aligned}$$

Both equations describe the line shown in Figure 4.5.8.

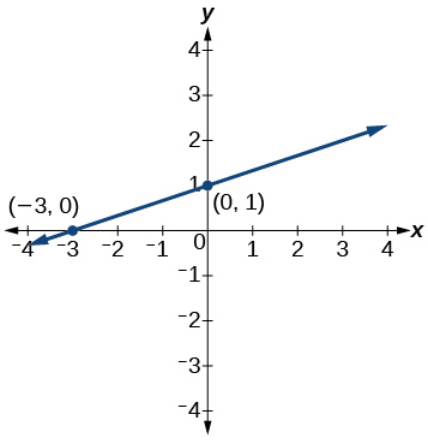


Figure 4.5.8

Example 4.5.6: Writing Linear Equations Using Two Points

Write the point-slope form of an equation of a line that passes through the points $(5,1)$ and $(8,7)$. Then rewrite it in the slope-intercept form.

Let's begin by finding the slope.

$$\begin{aligned} m &= \frac{y_2-y_1}{x_2-x_1} \\ &= \frac{7-1}{8-5} \\ &= \frac{6}{3} \\ &= 2 \end{aligned}$$

So $m=2$. Next, we substitute the slope and the coordinates for one of the points into the general point-slope equation. We can choose either point, but we will use $(5,1)$.

$$\begin{aligned} y-y_1 &= m(x-x_1) \\ y-1 &= 2(x-5) \end{aligned}$$

The point-slope equation of the line is $y_2-1=2(x_2-5)$. To rewrite the equation in slope-intercept form, we use algebra.

$$\begin{aligned} y-1 &= 2(x-5) \\ y-1 &= 2x-10 \\ y &= 2x-9 \end{aligned}$$

The slope-intercept equation of the line is $y = 2x - 9$.

Try It! 4.5.4

Write the point-slope form of an equation of a line that passes through the points $(-1, 3)$ and $(0, 0)$. Then rewrite it in the slope-intercept form.

Answer

$y - 0 = -3(x - 0)$; $y = -3x$.

Writing and Interpreting an Equation for a Linear Function

Now that we have written equations for linear functions in both the slope-intercept form and the point-slope form, we can choose which method to use based on the information we are given. That information may be provided in the form of a graph, a point and a slope, two points, and so on. Look at the graph of the function f in Figure 4.5.9.

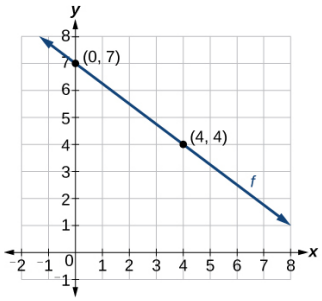


Figure 4.5.9: Graph depicting how to calculate the slope of a line.

We are not given the slope of the line, but we can choose any two points on the line to find the slope. Let's choose $(0, 7)$ and $(4, 4)$. We can use these points to calculate the slope.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{4 - 7}{4 - 0} \\ &= -\frac{3}{4} \end{aligned}$$

Now we can substitute the slope and the coordinates of one of the points into the point-slope form.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 4 &= -\frac{3}{4}(x - 4) \end{aligned}$$

If we want to rewrite the equation in the slope-intercept form, we would find

$$\begin{aligned} y - 4 &= -\frac{3}{4}(x - 4) \\ y - 4 &= -\frac{3}{4}x + 3 \\ y &= -\frac{3}{4}x + 7 \end{aligned}$$

If we wanted to find the slope-intercept form without first writing the point-slope form, we could have recognized that the line crosses the y-axis when the output value is 7. Therefore, $b = 7$. We now have the initial value b and the slope m so we can substitute m and b into the slope-intercept form of a line.

$$f(x) = mx + b$$

$$f(x) = -\frac{3}{4}x + 7$$

Figure 4.5.10

So the function is $f(x) = -\frac{3}{4}x + 7$, and the linear equation would be $y = -\frac{3}{4}x + 7$.

How to: Given the graph of a linear function, write an equation to represent the function.

1. Identify two points on the line.
2. Use the two points to calculate the slope.
3. Determine where the line crosses the y-axis to identify the y-intercept by visual inspection.
4. Substitute the slope and y-intercept into the slope-intercept form of a line equation.

Example 4.5.7: Writing an Equation for a Linear Function

Write an equation for a linear function given a graph of f shown in Figure 4.5.11.

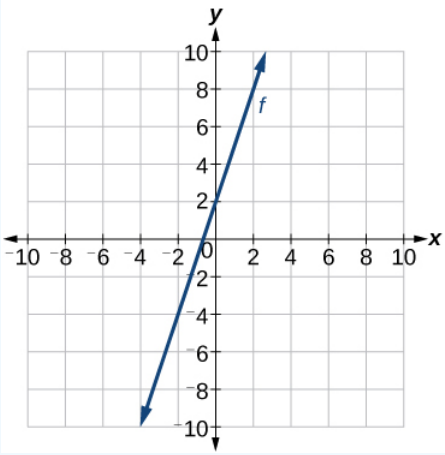


Figure 4.5.11

Solution

Identify two points on the line, such as $(0, 2)$ and $(-2, -4)$. Use the points to calculate the slope.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{4 - 2}{-2 - 0} \\ &= \frac{-6}{-2} \\ &= 3 \end{aligned}$$

Substitute the slope and the coordinates of one of the points into the point-slope form.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - (-4) &= 3(x - (-2)) \\ y + 4 &= 3(x + 2) \end{aligned}$$

We can use algebra to rewrite the equation in the slope-intercept form.

$$\begin{aligned} y+4 &= 3(x+2) \\ y+4 &= 3x+6 \\ y &= 3x+2 \end{aligned}$$

Analysis

This makes sense because we can see from Figure $4.5.12$ that the line crosses the y-axis at the point $(0, 2)$, which is the y-intercept, so $b = 2$.

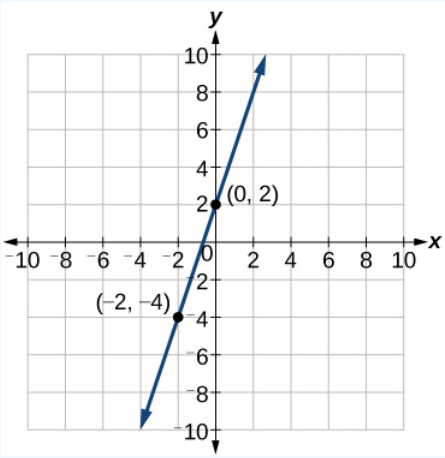


Figure 4.5.12: Graph of an increasing line with points at $(0, 2)$ and $(-2, -4)$.

Example 4.5.8: Writing an Equation for a Linear Cost Function

Suppose Ben starts a company in which he incurs a fixed cost of $1,250 per month for the overhead, which includes his office rent. His production costs are $37.50 per item. Write a linear function C where $C(x)$ is the cost for x items produced in a given month.

Solution

The fixed cost is present every month, $1,250. The costs that can vary include the cost to produce each item, which is $37.50 for Ben. The variable cost, called the marginal cost, is represented by 37.5. The cost Ben incurs is the sum of these two costs, represented by $C(x) = 1250 + 37.5x$.

Analysis

If Ben produces 100 items in a month, his monthly cost is represented by

$$\begin{aligned} C(100) &= 1250 + 37.5(100) \\ &= 5000 \end{aligned}$$

So his monthly cost would be $5,000.

Example 4.5.9: Writing an Equation for a Linear Function Given Two Points

If f is a linear function, with $f(3) = -2$, and $f(8) = 1$, find an equation for the function in slope-intercept form.

Solution

We can write the given points using coordinates.

$$\begin{aligned} f(3) = -2 &\rightarrow (3, 2) \\ f(8) = 1 &\rightarrow (8, 1) \end{aligned}$$

We can then use the points to calculate the slope.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{1-(-2)}{8-3} \\ &= \frac{3}{5} \end{aligned}$$

Substitute the slope and the coordinates of one of the points into the point-slope form.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y-(-2) &= \frac{3}{5}(x-3) \end{aligned}$$

We can use algebra to rewrite the equation in the slope-intercept form.

$$\begin{aligned} y+2 &= \frac{3}{5}(x-3) \\ y+2 &= \frac{3}{5}x - \frac{9}{5} \\ y &= \frac{3}{5}x - \frac{19}{5} \end{aligned}$$

? Try It! 4.5.5

If $f(x)$ is a linear function, with $f(2) = -11$, and $f(4) = -25$, find an equation for the function in slope-intercept form.

Answer

$y = -7x + 3$

Modeling Real-World Problems with Linear Functions

In the real world, problems are not always explicitly stated in terms of a function or represented with a graph. Fortunately, we can analyze the problem by first representing it as a linear function and then interpreting the components of the function. As long as we know, or can figure out, the initial value and the rate of change of a linear function, we can solve many different kinds of real-world problems.

How To: How to: Given a linear function f and the initial value and rate of change, evaluate $f(c)$.

1. Determine the initial value and the rate of change (slope).
2. Substitute the values into $f(x) = mx + b$.
3. Evaluate the function at $x = c$.

Example 4.5.10: Using a Linear Function to Determine the Number of Songs in a Music Collection

Marcus currently has 200 songs in his music collection. Every month, he adds 15 new songs. Write a formula for the number of songs, N, in his collection as a function of time, t, the number of months. How many songs will he own in a year?

Solution

The initial value for this function is 200 because he currently owns 200 songs, so $N(0) = 200$, which means that $b = 200$.

The number of songs increases by 15 songs per month, so the rate of change is 15 songs per month. Therefore we know that $m = 15$. We can substitute the initial value and the rate of change into the slope-intercept form of a line.

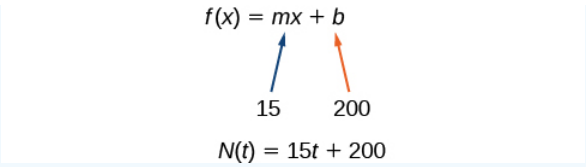


Figure 4.5.13

We can write the formula $N(t) = 15t + 200$.

With this formula, we can then predict how many songs Marcus will have in 1 year (12 months). In other words, we can evaluate the function at $t = 12$.

$$\begin{aligned} N(12) &= 15(12) + 200 \\ &= 180 + 200 \\ &= 380 \end{aligned}$$

Marcus will have 380 songs in 12 months.

Analysis

Notice that N is an increasing linear function. As the input (the number of months) increases, the output (number of songs) increases as well.

Example 4.5.11: Using a Linear Function to Calculate Salary Plus Commission

Working as an insurance salesperson, Ilya earns a base salary plus a commission on each new policy. Therefore, Ilya's weekly income, I,depends on the number of new policies, n, he sells during the week. Last week he sold 3 new policies, and earned \$760 for the week. The week before, he sold 5 new policies and earned \$920. Find an equation for $I(n)$, and interpret the meaning of the components of the equation.

Solution

The given information gives us two input-output pairs: $(3, 760)$ and $(5, 920)$. We start by finding the rate of change.

$$\begin{aligned} m &= \frac{920 - 760}{5 - 3} \\ &= \frac{\$160}{2 \text{ policies}} \\ &= \$80 \text{ per policy} \end{aligned}$$

Keeping track of units can help us interpret this quantity. Income increased by \$160 when the number of policies increased by 2, so the rate of change is \$80 per policy. Therefore, Ilya earns a commission of \$80 for each policy sold during the week.

We can then solve for the initial value.

$$\begin{aligned} I(n) &= 80n + b \\ 760 &= 80(3) + b \text{ when } n = 3, I(3) = 760 \\ 760 - 80(3) &= b \\ 520 &= b \end{aligned}$$

The value of b is the starting value for the function and represents Ilya's income when $n = 0$, or when no new policies are sold. We can interpret this as Ilya's base salary for the week, which does not depend upon the number of policies sold.

We can now write the final equation.

$$I(n) = 80n + 520$$

Our final interpretation is that Ilya's base salary is \$520 per week and he earns an additional \$80 commission for each policy sold.

Example 4.5.12: Using Tabular Form to Write an Equation for a Linear Function

Table 4.5.1 relates the number of rats in a population to time, in weeks. Use the table to write a linear equation.

Table 4.5.1

w, number of weeks	0	2	4	6
P(w), number of rats	1000	1080	1160	1240

Solution

We can see from the table that the initial value for the number of rats is 1000, so $b = 1000$.

Rather than solving for m, we can tell from looking at the table that the population increases by 80 for every 2 weeks that pass. This means that the rate of change is 80 rats per 2 weeks, which can be simplified to 40 rats per week.

$$P(w) = 40w + 1000$$

If we did not notice the rate of change from the table we could still solve for the slope using any two points from the table. For example, using $(2, 1080)$ and $(6, 1240)$

$$\begin{aligned} m &= \frac{1240 - 1080}{6 - 2} \\ &= \frac{160}{4} \\ &= 40 \end{aligned}$$

Is the initial value always provided in a table of values?

No. Sometimes the initial value is provided in a table of values, but sometimes it is not. If you see an input of 0, then the initial value would be the corresponding output. If the initial value is not provided because there is no value of input on the table equal to 0, find the slope, substitute one coordinate pair and the slope into $f(x) = mx + b$, and solve for b.

Try It 4.5.6

A new plant food was introduced to a young tree to test its effect on the height of the tree. Table 4.5.2 shows the height of the tree, in feet, x months since the measurements began. Write a linear function, $H(x)$, where x is the number of months since the start of the experiment.

Table 4.5.2

x	0	2	4	8	12
$H(x)$	12.5	13.5	14.5	16.5	18.5

Answer

$H(x) = 0.5x + 12.5$

Key Equations

- slope-intercept form of a line: $f(x) = mx + b$

- slope: $m = \dfrac{\text{change in output (rise)}}{\text{change in input (run)}} = \dfrac{\Delta y}{\Delta x} = \dfrac{y_2 - y_1}{x_2 - x_1}$
- point-slope form of a line: $y - y_1 = m(x - x_1)$

Key Concepts

- The ordered pairs given by a linear function represent points on a line.
- Linear functions can be represented in words, function notation, tabular form, and graphical form.
- The rate of change of a linear function is also known as the slope.
- An equation in the slope-intercept form of a line includes the slope and the initial value of the function.
- The initial value, or y-intercept, is the output value when the input of a linear function is zero. It is the y-value of the point at which the line crosses the y-axis.
- An increasing linear function results in a graph that slants upward from left to right and has a positive slope.
- A decreasing linear function results in a graph that slants downward from left to right and has a negative slope.
- A constant linear function results in a graph that is a horizontal line.
- Analyzing the slope within the context of a problem indicates whether a linear function is increasing, decreasing, or constant.
- The slope of a linear function can be calculated by dividing the difference between y-values by the difference in corresponding x-values of any two points on the line.
- The slope and initial value can be determined given a graph or any two points on the line.
- One type of function notation is the slope-intercept form of an equation.
- The point-slope form is useful for finding a linear equation when given the slope of a line and one point.
- The point-slope form is also convenient for finding a linear equation when given two points through which a line passes.
- The equation for a linear function can be written if the slope m and initial value b are known.
- A linear function can be used to solve real-world problems.
- A linear function can be written from tabular form.

Footnotes

1 www.chinahighlights.com/shang...glev-train.htm
2 www.cbsnews.com/8301-501465_1...ay-study-says/

Glossary

decreasing linear function

a function with a negative slope: If $f(x) = mx + b$, then $m < 0$.

increasing linear function

a function with a positive slope: If $f(x) = mx + b$, then $m > 0$.

linear function

a function with a constant rate of change that is a polynomial of degree 1, and whose graph is a straight line

point-slope form

the equation for a line that represents a linear function of the form \(y−y_1=m(x−x_1)

slope

the ratio of the change in output values to the change in input values; a measure of the steepness of a line

slope-intercept form

the equation for a line that represents a linear function in the form $f(x) = mx + b$

y-intercept

the value of a function when the input value is zero; also known as initial value

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4.7: Quadratic Functions

You may recall studying quadratic equations in Intermediate Algebra. In this section, we review those equations in the context of our next family of functions: the quadratic functions.

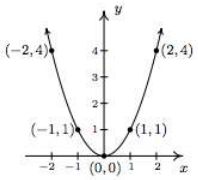
definition 2.5: quadratic function

A **quadratic function** is a function of the form

$$f(x) = ax^2 + bx + c, \tag{4.7.1}$$

where a, b and c are real numbers with $a \neq 0$. The domain of a quadratic function is $(-\infty, \infty)$.

The most basic quadratic function is $f(x) = x^2$, whose graph appears below. Its shape should look familiar from Intermediate Algebra -- it is called a parabola. The point $(0, 0)$ is called the **vertex** of the parabola. In this case, the vertex is a relative minimum and is also the where the absolute minimum value of f can be found.



$f(x) = x^2$

Much like many of the absolute value functions in Section 2.2, knowing the graph of $f(x) = x^2$ enables us to graph an entire family of quadratic functions using transformations.

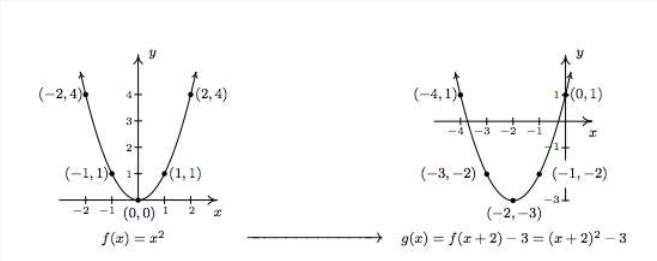
Example 4.7.1:

Graph the following functions starting with the graph of $f(x) = x^2$ and using transformations. Find the vertex, state the range and find the x- and y-intercepts, if any exist.

1. $g(x) = (x+2)^2 - 3$
2. $h(x) = -2(x-3)^2 + 1$

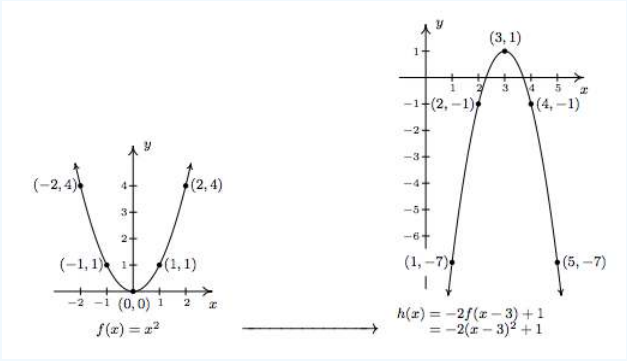
Solution

1. Since $g(x) = (x+2)^2 - 3 = f(x+2) - 3$, Theorem 1.7 instructs us to first *subtract* 2 from each of the x-values of the points on $y = f(x)$. This shifts the graph of $y = f(x)$ to the *left* 2 units and moves $(-2, 4)$ to $(-4, 4)$, $(-1, 1)$ to $(-3, 1)$, $(0, 0)$ to $(-2, 0)$, $(1, 1)$ to $(-1, 1)$ and $(2, 4)$ to $(0, 4)$. Next, we *subtract* 3 from each of the y-values of these new points. This moves the graph *down* 3 units and moves $(-4, 4)$ to $(-4, 1)$, $(-3, 1)$ to $(-3, -2)$, $(-2, 0)$ to $(-2, 3)$, $(-1, 1)$ to $(-1, -2)$ and $(0, 4)$ to $(0, 1)$. We connect the dots in parabolic fashion to get



From the graph, we see that the vertex has moved from $(0, 0)$ on the graph of $y = f(x)$ to $(-2, -3)$ on the graph of $y = g(x)$. This sets $[-3, \infty)$ as the range of g. We see that the graph of $y = g(x)$ crosses the x-axis twice, so we expect two x-intercepts. To find these, we set $y = g(x) = 0$ and solve. Doing so yields the equation $(x+2)^2 - 3 = 0$, or $(x+2)^2 = 3$. Extracting square roots gives $x + 2 = \pm\sqrt{3}$, or $x = -2 \pm \sqrt{3}$. Our x-intercepts are $(-2 - \sqrt{3}, 0) \approx (-3.73, 0)$ and $(-2 + \sqrt{3}, 0) \approx (-0.27, 0)$. The y-intercept of the graph, $(0, 1)$ was one of the points we originally plotted, so we are done.

2. Following Theorem 1.7 once more, to graph $h(x) = -2(x-3)^2 + 1 = -2f(x-3) + 1$, we first start by *adding* 3 to each of the x-values of the points on the graph of $y = f(x)$. This effects a horizontal shift *right* 3 units and moves $(-2, 4)$ to $(1, 4)$, $(-1, 1)$ to $(2, 1)$, $(0, 0)$ to $(3, 0)$, $(1, 1)$ to $(4, 1)$ and $(2, 4)$ to $(5, 4)$. Next, we *multiply* each of our y-values first by -2 and then *add* 1 to that result. Geometrically, this is a vertical *stretch* by a factor of 2, followed by a reflection about the x-axis, followed by a vertical shift *up* 1 unit. This moves $(1, 4)$ to $(1, -7)$, $(2, 1)$ to $(2, -1)$, $(3, 0)$ to $(3, 1)$, $(4, 1)$ to $(4, -1)$ and $(5, 4)$ to $(5, -7)$.



The vertex is $(3, 1)$ which makes the range of h $(-\infty, 1]$. From our graph, we know that there are two x-intercepts, so we set $y = h(x) = 0$ and solve. We get $-2(x-3)^2 + 1 = 0$ which gives $(x-3)^2 = \frac{1}{2}$. Extracting square roots\footnote{and rationalizing denominators!} gives $x - 3 = \pm\frac{\sqrt{2}}{2}$, so that when we add 3 to each side,\footnote{and get common denominators!} we get $x = \frac{6 \pm \sqrt{2}}{2}$. Hence, our x-intercepts are $\left(\frac{6-\sqrt{2}}{2}, 0\right) \approx (2.29, 0)$ and $\left(\frac{6+\sqrt{2}}{2}, 0\right) \approx (3.71, 0)$. Although our graph doesn't show it, there is a y-intercept which can be found by setting $x = 0$. With $h(0) = -2(0-3)^2 + 1 = -17$, we have that our y-intercept is $(0, -17)$. □

A few remarks about Example 4.7.1 are in order. First note that neither the formula given for $g(x)$ nor the one given for $h(x)$ match the form given in Definition 2.5. We could, of course, convert both $g(x)$ and $h(x)$ into that form by expanding and collecting like terms. Doing so, we find $g(x) = (x+2)^2 - 3 = x^2 + 4x + 1$ and $h(x) = -2(x-3)^2 + 1 = -2x^2 + 12x - 17$. While these `simplified' formulas for $g(x)$ and $h(x)$ satisfy Definition 2.5, they do not lend themselves to graphing easily. For that reason, the form of g and h presented in Example 4.7.2 is given a special name, which we list below, along with the form presented in Definition 2.5.

Definition 2.6: Standard and General Form of Quadratic Functions

Suppose f is a quadratic function.

- The **general form** of the quadratic function f is

$$f(x) = ax^2 + bx + c \tag{4.7.2}$$

  where a, b and c are real numbers with $a \neq 0$.
- The **standard form** of the quadratic function f is

$$f(x) = a(x-h)^2 + k \tag{4.7.3}$$

  where a, h and k are real numbers with $a \neq 0$.

It is important to note at this stage that we have no guarantees that *every* quadratic function can be written in standard form. This is actually true, and we prove this later in the exposition, but for now we celebrate the advantages of the standard form, starting with the following theorem.

Theorem 2.2: Vertex Formula for Quadratics in Standard Form

For the quadratic function $f(x) = a(x-h)^2 + k$, where a, h and k are real numbers with $a \neq 0$, the vertex of the graph of $y = f(x)$ is (h, k).

We can readily verify the formula given Theorem 2.2 with the two functions given in Example 4.7.1. After a (slight) rewrite, $g(x) = (x+2)^2 - 3 = (x-(-2))^2 + (-3)$, and we identify $h = -2$ and $k = -3$. Sure enough, we found the vertex of the graph of $y = g(x)$ to be $(-2, -3)$. For $h(x) = -2(x-3)+1$, no rewrite is needed. We can directly identify $h = 3$ and $k = 1$ and, sure enough, we found the vertex of the graph of $y = h(x)$ to be $(3, 1)$.

To see why the formula in Theorem 2.2 produces the vertex, consider the graph of the equation $y = a(x-h)^2 + k$. When we substitute $x = h$, we get $y = k$, so (h, k) is on the graph. If $x \neq h$, then $x - h \neq 0$ so $(x-h)^2$ is a positive number. If $a > 0$, then $a(x-h)^2$ is positive, thus $y = a(x-h)^2 + k$ is always a number larger than k. This means that when $a > 0$, (h, k) is the lowest point on the graph and thus the parabola must open upwards, making (h, k) the vertex. A similar argument shows that if $a < 0$, (h, k) is the highest point on the graph, so the parabola opens downwards, and (h, k) is also the vertex in this case.

Alternatively, we can apply the machinery in Section 1.7. Since the vertex of $y = x^2$ is $(0, 0)$, we can determine the vertex of $y = a(x-h)^2 + k$ by determining the final destination of $(0, 0)$ as it is moved through each transformation. To obtain the formula $f(x) = a(x-h)^2 + k$, we start with $g(x) = x^2$ and first define $g_1(x) = ag(x) = ax^2$. This is results in a vertical scaling and/or reflection.\footnote{Just a scaling if $a > 0$. If $a < 0$, there is a reflection involved.} Since we multiply the output by a, we multiply the y-coordinates on the graph of g by a, so the point $(0, 0)$ remains $(0, 0)$ and remains the vertex. Next, we define $g_{\tiny 2}(x) = g_{\tiny)1)}(x-h) = a(x-h)^2$. This induces a horizontal shift right or left h units\footnote{Right if $h > 0$, left if $h < 0$.} moves the vertex, in either case, to $(h, 0)$. Finally, $f(x) = g_2(x) + k = a(x-h)^2 + k$ which effects a vertical shift up or down k units\footnote{Up if $k > 0$, down if $k < 0$ } resulting in the vertex moving from $(h, 0)$ to (h, k).

In addition to verifying Theorem 2.2, the arguments in the two preceding paragraphs have also shown us the role of the number a in the graphs of quadratic functions. The graph of $y = a(x-h)^2 + k$ is a parabola `opening upwards' if $a > 0$, and `opening downwards' if $a < 0$. Moreover, the symmetry enjoyed by the graph of $y = x^2$ about the y-axis is translated to a symmetry about the vertical line $x = h$ which is the vertical line through the vertex.\footnote{You should use transformations to verify this!} This line is called the **axis of symmetry** of the parabola and is dashed in the figures below.



Graphs of $y = a(x-h)^2 + k$.

Without a doubt, the standard form of a quadratic function, coupled with the machinery in Section 1.7, allows us to list the attributes of the graphs of such functions quickly and elegantly. What remains to be shown, however, is the fact that every quadratic function *can be written* in standard form. To convert a quadratic function given in general form into standard form, we employ the ancient rite of `Completing the Square'. We remind the reader how this is done in our next example.

Example 4.7.2:

Convert the functions below from general form to standard form. Find the vertex, axis of symmetry and any x- or y-intercepts. Graph each function and determine its range.

1. $f(x) = x^2 - 4x + 3$.
2. $g(x) = 6 - x - x^2$

Solution

1. To convert from general form to standard form, we complete the square.\footnote{If you forget why we do what we do to complete the square, start with $a(x-h)^2 + k$, multiply it out, step by step, and then reverse the process.} First, we verify that the coefficient of x^2 is 1. Next, we find the coefficient of x, in this case -4, and take half of it to get $\frac{1}{2}(-4) = -2$. This tells us that our target perfect square quantity is $(x-2)^2$. To get an expression equivalent to $(x-2)^2$, we need to add $(-2)^2 = 4$ to the $x^2 - 4x$ to create a perfect square trinomial, but to keep the balance, we must also subtract it. We collect the terms which create the perfect square and gather the remaining constant terms. Putting it all together, we get

$$\begin{aligned} f(x) &= x^2-4x+3 && (\text{Compute } \tfrac{1}{2}(-4)=-2.)\\ &= \left(x^2-4x+\underline{4}-\underline{4}\right)+3 && (\text{Add and subtract } (-2)^2=4 \text{ to } (x^2+4x).)\\ &= \left(x^2-4x+4\right)-4+3 && (\text{Group the perfect square trinomial.})\\ &= (x-2)^2-1 && (\text{Factor the perfect square trinomial.}) \end{aligned} \tag{4.7.4}$$

Of course, we can always check our answer by multiplying out $f(x)=(x-2)^2-1$ to see that it simplifies to $f(x)=x^2-4x-1$. In the form $f(x)=(x-2)^2-1$, we readily find the vertex to be $(2,-1)$ which makes the axis of symmetry $x=2$. To find the x-intercepts, we set $y=f(x)=0$. We are spoiled for choice, since we have *two* formulas for $f(x)$. Since we recognize $f(x)=x^2-4x+3$ to be easily factorable,\footnote{Experience pays off, here!} we proceed to solve $x^2-4x+3=0$. Factoring gives $(x-3)(x-1)=0$ so that $x=3$ or $x=1$. The x-intercepts are then $(1,0)$ and $(3,0)$. To find the y-intercept, we set $x=0$. Once again, the general form $f(x)=x^2-4x+3$ is easiest to work with here, and we find $y=f(0)=3$. Hence, the y-intercept is $(0,3)$. With the vertex, axis of symmetry and the intercepts, we get a pretty good graph without the need to plot additional points. We see that the range of f is $[-1,\infty)$ and we are done.

2. To get started, we rewrite $g(x)=6-x-x^2=-x^2-x+6$ and note that the coefficient of x^2 is -1, not 1. This means our first step is to factor out the (-1) from both the x^2 and x terms. We then follow the completing the square recipe as above.

$$\begin{aligned} g(x) &= -x^2-x+6\\ &= (-1)\left(x^2+x\right)+6 && (\text{Factor the coefficient of } x^2 \text{ from } x^2 \text{ and } x.)\\ &= (-1)\left(x^2+x+\tfrac{1}{4}-\tfrac{1}{4}\right)+6\\ &= (-1)\left(x^2+x+\tfrac{1}{4}\right)+(-1)\left(-\tfrac{1}{4}\right)+6 && (\text{Group the perfect square trinomial.})\\ &= -\left(x+\tfrac{1}{2}\right)^2+\tfrac{25}{4} \end{aligned} \tag{4.7.5}$$

From $g(x)=-\left(x+\frac{1}{2}\right)^2+\frac{25}{4}$, we get the vertex to be $\left(-\frac{1}{2},\frac{25}{4}\right)$ and the axis of symmetry to be $x=-\frac{1}{2}$. To get the x-intercepts, we opt to set the given formula $g(x)=6-x-x^2=0$. Solving, we get $x=-3$ and $x=2$, so the x-intercepts are $(-3,0)$ and $(2,0)$. Setting $x=0$, we find $g(0)=6$, so the y-intercept is $(0,6)$. Plotting these points gives us the graph below. We see that the range of g is $\left(-\infty,\frac{25}{4}\right]$.

With Example 4.7.2 fresh in our minds, we are now in a position to show that every quadratic function can be written in standard form. We begin with $f(x)=ax^2+bx+c$, assume $a\neq 0$, and complete the square in *complete* generality.

$$\begin{aligned} f(x) &= ax^2+bx+c\\ &= a\left(x^2+\frac{b}{a}x\right)+c && (\text{Factor out coefficient of } x^2 \text{ from } x^2 \text{ and } x.)\\ &= a\left(x^2+\frac{b}{a}x+\underline{\frac{b^2}{4a^2}}-\underline{\frac{b^2}{4a^2}}\right)+c\\ &= a\left(x^2+\frac{b}{a}x+\frac{b^2}{4a^2}\right)-a\left(\frac{b^2}{4a^2}\right)+c && (\text{Group the perfect square trinomial.})\\ &= a\left(x+\frac{b}{2a}\right)^2+\frac{4ac-b^2}{4a} && (\text{Factor and get a common denominator.}) \end{aligned} \tag{4.7.6}$$

Comparing this last expression with the standard form, we identify $(x-h)$ with $\left(x+\frac{b}{2a}\right)$ so that $h=-\frac{b}{2a}$. Instead of memorizing the value $k=\frac{4ac-b^2}{4a}$, we see that $f\left(-\frac{b}{2a}\right)=\frac{4ac-b^2}{4a}$. As such, we have derived a vertex formula for the general form. We summarize both vertex formulas in the box at the top of the next page.

Equation 2.4: Vertex Formulas for Quadratic Functions

Suppose a, b, c, h and k are real numbers with $a\neq 0$.

- If $f(x)=a(x-h)^2+k$, the vertex of the graph of $y=f(x)$ is the point (h,k).
- If $f(x)=ax^2+bx+c$, the vertex of the graph of $y=f(x)$ is the point $\left(-\frac{b}{2a},f\left(-\frac{b}{2a}\right)\right)$.

There are two more results which can be gleaned from the completed-square form of the general form of a quadratic function,

$$f(x) = ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a} \tag{4.7.7}$$

We have seen that the number a in the standard form of a quadratic function determines whether the parabola opens upwards (if $a > 0$) or downwards (if $a < 0$). We see here that this number a is none other than the coefficient of x^2 in the general form of the quadratic function. In other words, it is the coefficient of x^2 alone which determines this behavior -- a result that is generalized in Section 3.1. The second treasure is a re-discovery of the **quadratic formula**.

Equation 2.5: The Quadratic Formula

If a, b and c are real numbers with $a \neq 0$, then the solutions to $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}. \tag{4.7.8}$$

Assuming the conditions of Equation 2.5, the solutions to $ax^2 + bx + c = 0$ are precisely the zeros of $f(x) = ax^2 + bx + c$. Since

$$f(x) = ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a} \tag{4.7.9}$$

the equation $ax^2 + bx + c = 0$ is equivalent to

$$a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a} = 0. \tag{4.7.10}$$

Solving gives

$$\begin{aligned}
a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a} &= 0 \\
[15pt] a\left(x + \frac{b}{2a}\right)^2 &= -\frac{4ac - b^2}{4a} \\
[15pt] \frac{1}{a}\left[a\left(x + \frac{b}{2a}\right)^2\right] &= \frac{1}{a}\left(\frac{b^2 - 4ac}{4a}\right) \\
[15pt] \left(x + \frac{b}{2a}\right)^2 &= \frac{b^2 - 4ac}{4a^2} \\
[15pt] x + \frac{b}{2a} &= \pm\sqrt{\frac{b^2 - 4ac}{4a^2}} \qquad \text{extract square roots} \\
[15pt] x + \frac{b}{2a} &= \pm\frac{\sqrt{b^2 - 4ac}}{2a} \\
[15pt] x &= -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} \\
[15pt] x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\end{aligned} \tag{4.7.11}$$

In our discussions of domain, we were warned against having negative numbers underneath the square root. Given that $\sqrt{b^2 - 4ac}$ is part of the Quadratic Formula, we will need to pay special attention to the radicand $b^2 - 4ac$. It turns out that the quantity $b^2 - 4ac$ plays a critical role in determining the nature of the solutions to a quadratic equation. It is given a special name.

Definition 2.7: discriminant

If a, b and c are real numbers with $a \neq 0$, then the **discriminant** of the quadratic equation $ax^2 + bx + c = 0$ is the quantity

$$b^2 - 4ac. \tag{4.7.12}$$

The discriminant `discriminates' between the kinds of solutions we get from a quadratic equation. These cases, and their relation to the discriminant, are summarized below.

Theorem 2.3: Discriminant Trichotomy

Let a, b and c be real numbers with $a \neq 0$.

- If $b^2 - 4ac < 0$, the equation $ax^2 + bx + c = 0$ has no real solutions.
- If $b^2 - 4ac = 0$, the equation $ax^2 + bx + c = 0$ has exactly one real solution.
- If $b^2 - 4ac > 0$, the equation $ax^2 + bx + c = 0$ has exactly two real solutions.

The proof of Theorem 2.3 stems from the position of the discriminant in the quadratic equation, and is left as a good mental exercise for the reader. The next example exploits the fruits of all of our labor in this section thus far.

Example 4.7.3:

Recall that the profit (Section 1.4) for a product is defined by the equation $\text{Profit} = \text{Revenue} - \text{Cost}$, or $P(x) = R(x) - C(x)$. In Example 2.1.7 the weekly revenue, in dollars, made by selling x PortaBoy Game Systems was found to be $R(x) = -1.5x^2 + 250x$ with the restriction (carried over from the price-demand function) that $0 \leq x \leq 166$. The cost, in dollars, to produce x PortaBoy Game Systems is given in Example 2.1.5 as $C(x) = 80x + 150$ for $x \geq 0$.

1. Determine the weekly profit function $P(x)$.
2. Graph $y = P(x)$. Include the x- and y-intercepts as well as the vertex and axis of symmetry.
3. Interpret the zeros of P.
4. Interpret the vertex of the graph of $y = P(x)$.
5. Recall that the weekly price-demand equation for PortaBoys is $p(x) = -1.5x + 250$, where $p(x)$ is the price per PortaBoy, in dollars, and x is the weekly sales. What should the price per system be in order to maximize profit?

Solution

1. To find the profit function $P(x)$, we subtract

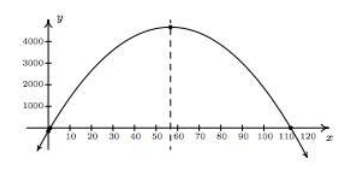
$$P(x) = R(x) - C(x) = \left(-1.5x^2 + 250x\right) - (80x + 150) = -1.5x^2 + 170x - 150. \tag{4.7.13}$$

   Since the revenue function is valid when $0 \leq x \leq 166$, P is also restricted to these values.
2. To find the x-intercepts, we set $P(x) = 0$ and solve $-1.5x^2 + 170x - 150 = 0$. The mere thought of trying to factor the left hand side of this equation could do serious psychological damage, so we resort to the quadratic formula, Equation 2.5. Identifying $a = -1.5$, $b = 170$, and $c = -150$, we obtain

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-170 \pm \sqrt{170^2 - 4(-1.5)(-150)}}{2(-1.5)} \\ &= \frac{-170 \pm \sqrt{28000}}{-3} \\ &= \frac{170 \pm 20\sqrt{70}}{3} \end{aligned} \tag{4.7.14}$$

We get two x-intercepts: $\left(\frac{170-20\sqrt{70}}{3}, 0\right)$ and $\left(\frac{170+20\sqrt{70}}{3}, 0\right)$. To find the y-intercept, we set $x = 0$ and find $y = P(0) = -150$ for a y-intercept of $(0, -150)$. To find the vertex, we use the fact that $P(x) = -1.5x^2 + 170x - 150$ is in the general form of a quadratic function and appeal to Equation 2.4. Substituting $a = -1.5$ and $b = 170$, we get $x = -\frac{170}{2(-1.5)} = \frac{170}{3}$. To find the y-coordinate of the vertex, we compute $P\left(\frac{170}{3}\right) = \frac{14000}{3}$ and find that our vertex is $\left(\frac{170}{3}, \frac{14000}{3}\right)$. The axis of symmetry is the vertical line passing through the vertex so it is the line $x = \frac{170}{3}$. To sketch a reasonable graph, we approximate the x-intercepts, $(0.89, 0)$ and $(112.44, 0)$, and the vertex, $(56.67, 4666.67)$. (Note that in order to get the x-intercepts and the vertex to show up in the same picture, we had to scale the x-axis differently than the y-

axis. This results in the left-hand x-intercept and the y-intercept being uncomfortably close to each other and to the origin in the picture.)



3. The zeros of P are the solutions to $P(x)=0$, which we have found to be approximately 0.89 and 112.44 As we saw in Example 1.5.3, these are the `break-even' points of the profit function, where enough product is sold to recover the cost spent to make the product. More importantly, we see from the graph that as long as x is between 0.89 and 112.44, the graph $y=P(x)$ is above the x-axis, meaning $y=P(x)>0$ there. This means that for these values of x, a profit is being made. Since x represents the weekly sales of PortaBoy Game Systems, we round the zeros to positive integers and have that as long as 1, but no more than 112 game systems are sold weekly, the retailer will make a profit.

4. From the graph, we see that the maximum value of P occurs at the vertex, which is approximately $(56.67, 4666.67)$. As above, x represents the weekly sales of PortaBoy systems, so we can't sell 56.67 game systems. Comparing $P(56)=4666$ and $P(57)=4666.5$, we conclude that we will make a maximum profit of $)4666.50$ if we sell 57 game systems.

5. In the previous part, we found that we need to sell 57 PortaBoys per week to maximize profit. To find the price per PortaBoy, we substitute $x=57$ into the price-demand function to get $p(57)=-1.5(57)+250=164.5$. The price should be set at $)164.50$. □

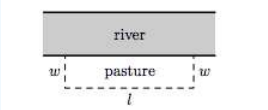
Our next example is another classic application of quadratic functions.

Example 4.7.4:

Much to Donnie's surprise and delight, he inherits a large parcel of land in Ashtabula County from one of his (e)strange(d) relatives. The time is finally right for him to pursue his dream of farming alpaca. He wishes to build a rectangular pasture, and estimates that he has enough money for 200 linear feet of fencing material. If he makes the pasture adjacent to a stream (so no fencing is required on that side), what are the dimensions of the pasture which maximize the area? What is the maximum area? If an average alpaca needs 25 square feet of grazing area, how many alpaca can Donnie keep in his pasture?

Solution

It is always helpful to sketch the problem situation, so we do so below.



We are tasked to find the dimensions of the pasture which would give a maximum area. We let w denote the width of the pasture and we let l denote the length of the pasture. Since the units given to us in the statement of the problem are feet, we assume w and l are measured in feet. The area of the pasture, which we'll call A, is related to w and l by the equation $A=wl$. Since w and l are both measured in feet, A has units of feet^2, or square feet. We are given the total amount of fencing available is 200 feet, which means $w+l+w=200$, or, $l+2w=200$. We now have two equations, $A=wl$ and $l+2w=200$. In order to use the tools given to us in this section to *maximize* A, we need to use the information given to write A as a function of just *one* variable, either w or l. This is where we use the equation $l+2w=200$. Solving for l, we find $l=200-2w$, and we substitute this into our equation for A. We get $A=wl=w(200-2w)=200w-2w^2$. We now have A as a function of w, $A(w)=200w-2w^2=-2w^2+200w$.

Before we go any further, we need to find the applied domain of A so that we know what values of w make sense in this problem situation.\footnote{Donnie would be very upset if, for example, we told him the width of the pasture needs to be -50 feet.} Since w represents the width of the pasture, $w>0$. Likewise, l represents the length of the pasture, so $l=200-2w>0$. Solving this latter inequality, we find $w<100$. Hence, the function we wish to maximize is $A(w)=-2w^2+200w$ for $0<w<100$. Since A is a quadratic function (of w), we know that the graph of $y=A(w)$ is a

parabola. Since the coefficient of w^2 is -2, we know that this parabola opens downwards. This means that there is a maximum value to be found, and we know it occurs at the vertex. Using the vertex formula, we find $w = -\frac{200}{2(-2)} = 50$, and $A(50) = -2(50)^2 + 200(50) = 5000$. Since $w = 50$ lies in the applied domain, $0 < w < 100$, we have that the area of the pasture is maximized when the width is 50 feet. To find the length, we use $l = 200 - 2w$ and find $l = 200 - 2(50) = 100$, so the length of the pasture is 100 feet. The maximum area is $A(50) = 5000$, or 5000 square feet. If an average alpaca requires 25 square feet of pasture, Donnie can raise $\frac{5000}{25} = 200$ average alpaca. □

We conclude this section with the graph of a more complicated absolute value function.

Example 4.7.5:

Graph $f(x) = |x^2 - x - 6|$.

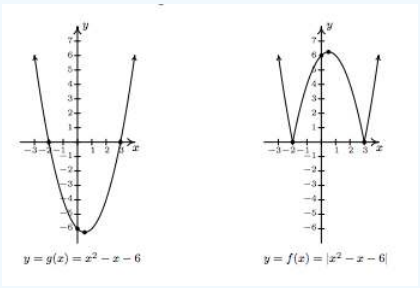
Solution

Using the definition of absolute value, Definition 2.4, we have

$$f(x) = \begin{cases} -\left(x^2 - x - 6\right), & \text{if} \quad x^2 - x - 6 < 0 \\ x^2 - x - 6, & \text{if} \quad x^2 - x - 6 \geq 0 \end{cases} \tag{4.7.15}$$

The trouble is that we have yet to develop any analytic techniques to solve nonlinear inequalities such as $x^2 - x - 6 < 0$. You won't have to wait long; this is one of the main topics of Section 2.4.

Nevertheless, we can attack this problem graphically. To that end, we graph $y = g(x) = x^2 - x - 6$ using the intercepts and the vertex. To find the x-intercepts, we solve $x^2 - x - 6 = 0$. Factoring gives $(x-3)(x+2) = 0$ so $x = -2$ or $x = 3$. Hence, $(-2, 0)$ and $(3, 0)$ are x-intercepts. The y-intercept $(0, -6)$ is found by setting $x = 0$. To plot the vertex, we find $x = -\frac{b}{2a} = -\frac{-1}{2(1)} = \frac{1}{2}$, and $y = \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right) - 6 = -\frac{25}{4} = -6.25$. Plotting, we get the parabola seen below on the left. To obtain points on the graph of $y = f(x) = |x^2 - x - 6|$, we can take points on the graph of $g(x) = x^2 - x - 6$ and apply the absolute value to each of the y values on the parabola. We see from the graph of g that for $x \leq -2$ or $x \geq 3$, the y values on the parabola are greater than or equal to zero (since the graph is on or above the x-axis), so the absolute value leaves these portions of the graph alone. For x between -2 and 3, however, the y values on the parabola are negative. For example, the point $(0, -6)$ on $y = x^2 - x - 6$ would result in the point $(0, |-6|) = (0, -(-6)) = (0, 6)$ on the graph of $f(x) = |x^2 - x - 6|$. Proceeding in this manner for all points with x-coordinates between -2 and 3 results in the graph seen below on the right.



If we take a step back and look at the graphs of g and f in the last example, we notice that to obtain the graph of f from the graph of g, we reflect a *portion* of the graph of g about the x-axis. We can see this analytically by substituting $g(x) = x^2 - x - 6$ into the formula for $f(x)$ and calling to mind Theorem 1.4 from Section 1.7.

$$f(x) = \begin{cases} -g(x), & \text{if} \quad g(x) < 0 \\ g(x), & \text{if} \quad g(x) \geq 0 \end{cases} \tag{4.7.16}$$

The function f is defined so that when $g(x)$ is negative (i.e., when its graph is below the x-axis), the graph of f is its refection across the x-axis. This is a general template to graph functions of the form $f(x) = |g(x)|$. From this perspective, the graph of $f(x) = |x|$ can be obtained by reflection the portion of the line $g(x) = x$ which is below the x-axis back above the x-axis creating the characteristic `\/' shape.

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4.8: The Square Root Function

In this section we turn our attention to the square root function, the function defined by the equation

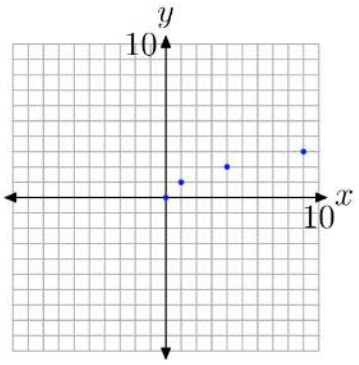
$$f(x) = \sqrt{x}$$

We begin the section by drawing the graph of the function, then we address the domain and range. After that, we'll investigate a number of different transformations of the function.

The Graph of the Square Root Function

Let's create a table of points that satisfy the equation of the function, then plot the points from the table on a Cartesian coordinate system on graph paper. We'll continue creating and plotting points until we are convinced of the eventual shape of the graph.

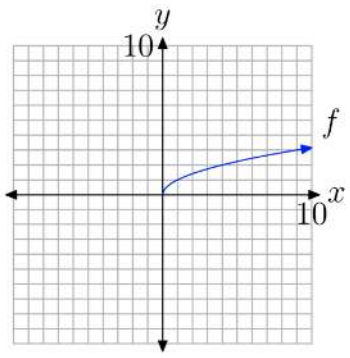
We know we cannot take the square root of a negative number. Therefore, we don't want to put any negative x-values in our table. To further simplify our computations, let's use numbers whose square root is easily calculated. This brings to mind perfect squares such as 0, 1, 4, 9, and so on. We've placed these numbers as x-values in the table in **Figure 1**(b), then calculated the square root of each. In **Figure 1**(a), you see each of the points from the table plotted as a solid dot. If we continue to add points to the table, plot them, the graph will eventually fill in and take the shape of the solid curve shown in **Figure 1**(c).



(a)

x	$f(x) = \sqrt{x}$
0	0
1	1
4	2
9	3

(b)

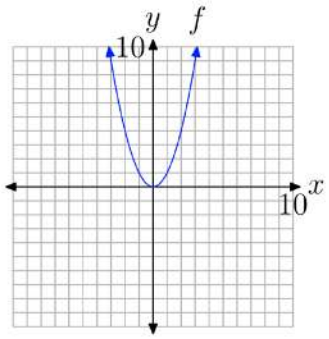


(c)

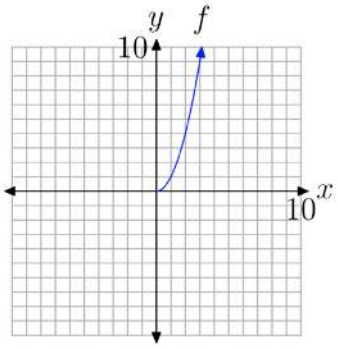
Figure 1.} \text{Creating the graph of} f(x)= \sqrt{x}\\ \nonumber \end{array} \nonumber \]

The point plotting approach used to draw the graph of $f(x) = \sqrt{x}$ in **Figure 1** is a tested and familiar procedure. However, a more sophisticated approach involves the theory of inverses developed in the previous chapter.

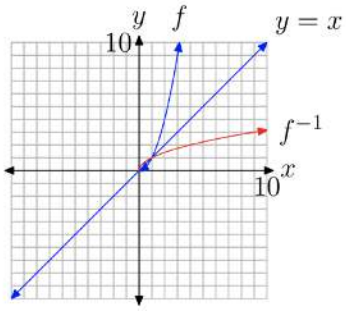
In a sense, taking the square root is the "inverse" of squaring. Well, not quite, as the squaring function $f(x) = x^2$ in **Figure 2**(a) fails the horizontal line test and is not one-to-one. However, if we limit the domain of the squaring function, then the graph of $f(x) = x^2$ in **Figure 2**(b), where $x \geq 0$, does pass the horizontal line test and is one-to-one. Therefore, the graph of $f(x) = x^2$, $x \geq 0$, has an inverse, and the graph of its inverse is found by reflecting the graph of $f(x) = x^2$, $x \geq 0$, across the line $y = x$ (see **Figure 2**(c)).



(a) $f(x) = x^2$.



(b) $f(x) = x^2$, $x \geq 0$.



(c) Reflecting the graph in (b) across the line $y = x$ produces the graph of $f^{-1}(x) = \sqrt{x}$.

Figure 2.} \text{Sketching the inverse of} f(x)= x^2, x \ge 0}\\ \nonumber \end{array} \nonumber \]

To find the equation of the inverse, recall that the procedure requires that we switch the roles of *x* and *y*, then solve the resulting equation for *y*. Thus, first write $f(x) = x^2$, $x \geq 0$, in the form

$$y = x^2, x \geq 0$$

Next, switch *x* and *y*.

$$x = y^2, y \geq 0$$

When we solve this last equation for *y*, we get two solutions,

$$y = \pm\sqrt{x}$$

However, in **equation (2)**, note that *y* must be greater than or equal to zero. Hence, we must choose the nonnegative answer in **equation (3)**, so the inverse of $f(x) = x^2$, $x \geq 0$, has equation

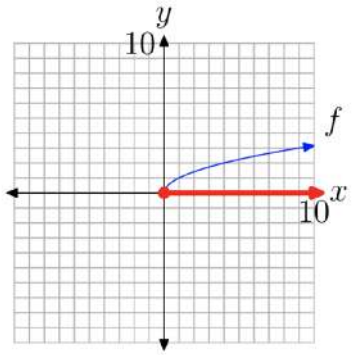
$$f^{-1}(x) = \sqrt{x}$$

This is the equation of the reflection of the graph of $f(x) = x^2$, $x \geq 0$, that is pictured in **Figure 2**(c). Note the exact agreement with the graph of the square root function in **Figure 1**(c).

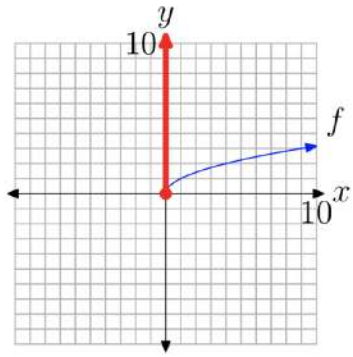
The sequence of graphs in **Figure 2** also help us identify the domain and range of the square root function.

- In **Figure 2**(a), the parabola opens outward indefinitely, both left and right. Consequently, the domain is $D_f = (-\infty, \infty)$, or all real numbers. Also, the graph has vertex at the origin and opens upward indefinitely, so the range is $R_f = [0, \infty)$.
- In **Figure 2**(b), we restricted the domain. Thus, the graph of $f(x) = x^2$, $x \geq 0$, now has domain $D_f = [0, \infty)$. The range is unchanged and is $R_f = [0, \infty)$.
- In **Figure 2**(c), we've reflected the graph of $f(x) = x^2$, $x \geq 0$, across the line y = x to obtain the graph of $f^{-1}(x) = \sqrt{x}$. Because we've interchanged the role of x and y, the domain of the square root function must equal the range of $f(x) = x^2$, $x \geq 0$. That is, $D_{f^{-1}} = [0, \infty)$.Similarly,the range of the square root function must equal the domain of $f(x) = x^2$, $x \geq 0$. Hence, $R_{f^{-1}} = [0, \infty)$.

Of course, we can also determine the domain and range of the square root function by projecting all points on the graph onto the *x*- and *y*-axes, as shown in **Figures 3**(a) and (b), respectively.



(a) Domain $= [0, \infty)$



(b) Range $= [0, \infty)$

Figure 3.} \text{Project on to the axes to find the domain and range}}\\ \nonumber \end{array} \nonumber \]

Some might object to the range, asking "How do we know that the graph of the square root function picture in **Figure 3**(b) rises indefinitely?" Again, the answer lies in the sequence of graphs in **Figure 2**. In **Figure 2**(c), note that the graph of $f(x) = x^2$, $x \geq 0$, opens indefinitely to the right as the graph rises to infinity. Hence, after reflecting this graph across the line $y = x$, the resulting graph must rise upward indefinitely as it moves to the right. Thus, the range of the square root function is $[0, \infty)$.

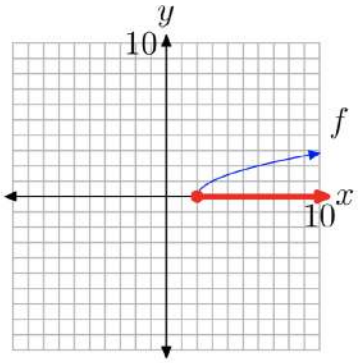
Translations

If we shift the graph of $y = \sqrt{x}$ right and left, or up and down, the domain and/or range are affected.

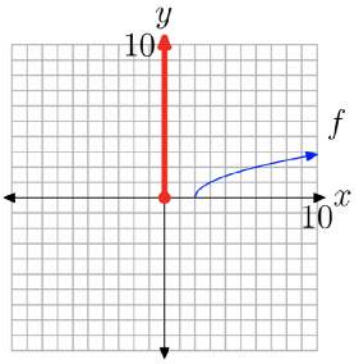
? Example 4.8.4

Sketch the graph of $f(x) = \sqrt{x-2}$. Use your graph to determine the domain and range.

We know that the basic equation $y = \sqrt{x}$ has the graph shown in **Figures 1**(c). If we replace x with $x-2$, the basic equation $y = \sqrt{x}$ becomes $f(x) = \sqrt{x-2}$. From our previous work with geometric transformations, we know that this will shift the graph two units to the right, as shown in **Figures 4** (a) and (b).



(a) Domain $= [2, \infty)$



(b) Range $= [0, \infty)$

Figure 4. To draw the graph of the $f(x) = \sqrt{x-2}$, shift the graph of $y = \sqrt{x}$ two units to the right.

To find the domain, we project each point on the graph of f onto the x-axis, as shown in **Figure 4**(a). Note that all points to the right of or including 2 are shaded on the x-axis. Consequently, the domain of f is

$$\text{Domain} = [2, \infty) = \{\text{x: } x \geq 0\}$$

As there has been no shift in the vertical direction, the range remains the same. To find the range, we project each point on the graph onto the y-axis, as shown in **Figure 4**(b). Note that all points at and above zero are shaded on the y-axis. Thus, the range of f is

$$\text{Range} = [0, \infty) = \{\text{y: } y \geq 0\}.$$

We can find the domain of this function algebraically by examining its defining equation $f(x) = \sqrt{x-2}$. We understand that we cannot take the square root of a negative number. Therefore, the expression under the radical must be nonnegative (positive or zero). That is,

$$x - 2 \geq 0\,.$$

Solving this inequality for *x*,

$$x \geq 2.$$

Thus, the domain of f is Domain = $[2, \infty)$, which matches the graphical solution above.

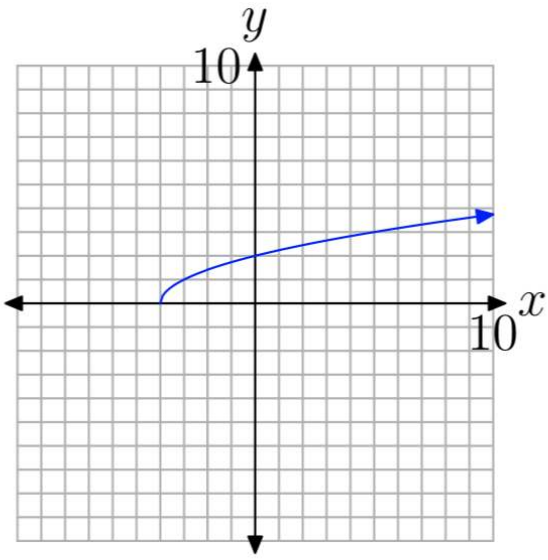
Let's look at another example.

? Example 4.8.5

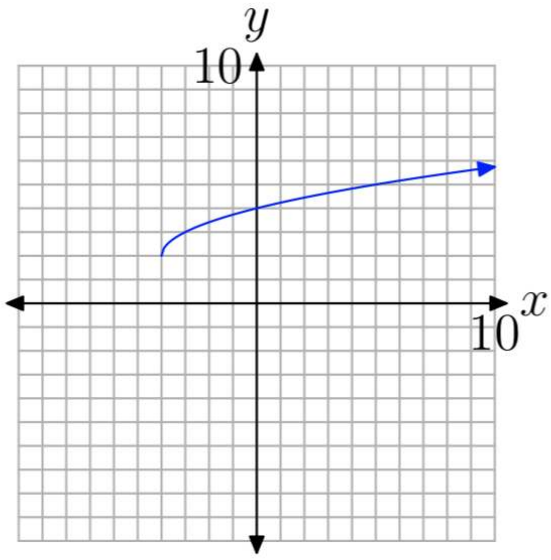
Sketch the graph of $f(x) = \sqrt{x+4} + 2$. Use your graph to determine the domain and range of f.

Again, we know that the basic equation $y = \sqrt{x}$ has the graph shown in **Figure 1**(c). If we replace *x* with *x*+4, the basic equation $y = \sqrt{x}$ becomes $y = \sqrt{x+4}$. From our previous work with geometric transformations, we know that this will shift the graph of $y = \sqrt{x}$ four units to the left, as shown in **Figure 5**(a).

If we know add 2 to the equation $y = \sqrt{x+4}$ to produce the equation $y = \sqrt{x+4} + 2$, this will shift the graph of $y = \sqrt{x+4}$ two units upward, as shown in **Figure 5**(b).



(a) To draw the graph of $y = \sqrt{x+4}$, shift the graph of $y = \sqrt{x}$ four units to the left.



(b) To draw the graph of $y = \sqrt{x+4} + 2$, shift the graph of $y = \sqrt{x+4}$ two units upward.

Figure 5. Translating the original equation $y = \sqrt{x}$ to get the graph of $y = \sqrt{x+4} + 2$

To identify th domain of the $f(x) = \sqrt{x+4} + 2$, we project all points on the graph of f onto the x-axis, as shown in **Figure 6**(a). Note that all points to the right of or including -4 are shaded on the *x*-axis. Thus, the domain of $f(x) = \sqrt{x+4} + 2$ is

$$\text{Domain} = [-4, \infty) = \{x: x \geq -4\}$$

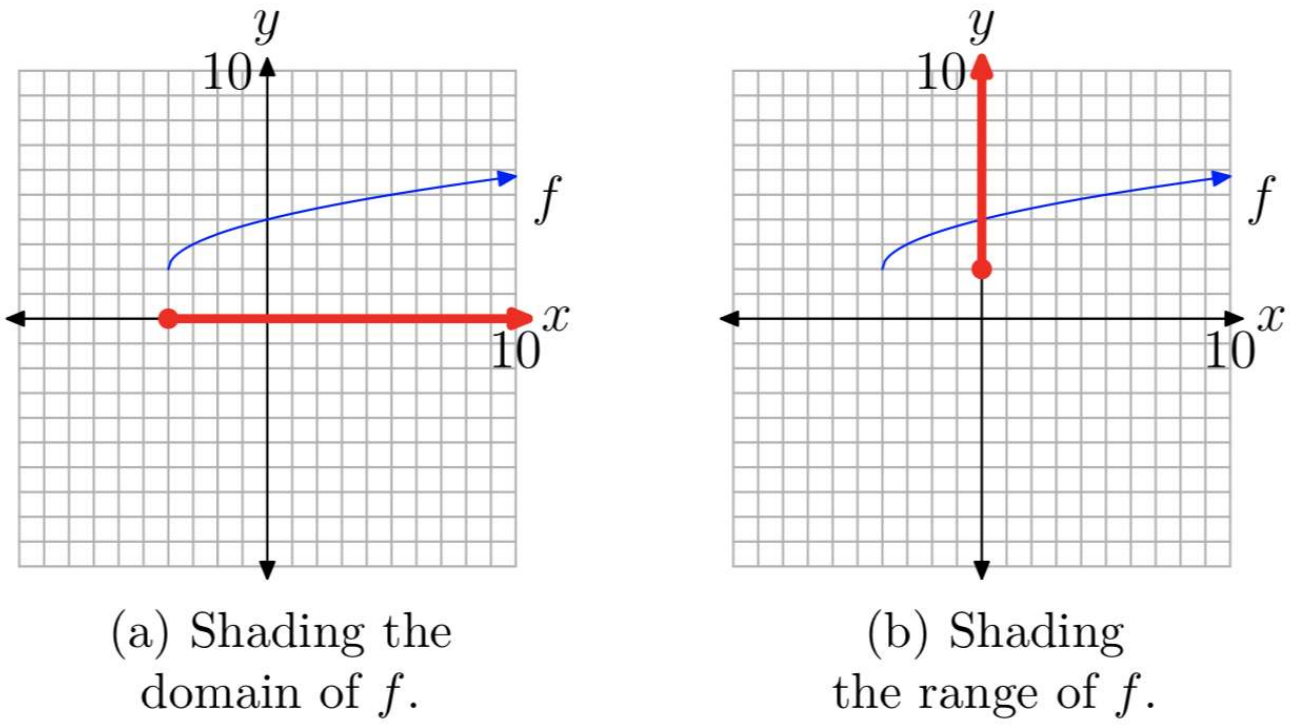


Figure 6. Project points of f onto the axes to determine the domain and range

Similarly, to find the range of *f*, project all points on the graph of *f* onto the *y*-axis, as shown in **Figure 6**(b). Note that all points on the *y*-axis greater than or including 2 are shaded. Consequently, the range of *f* is

$$\text{Range} = [2, \infty) = \{\text{y: } y \geq 2\}$$

We can also find the domain of *f* algebraically by examining the equation $f(x) = \sqrt{x+4} + 2$. We cannot take the square root of a negative number, so the expression under the radical must be nonnegative (zero or positive). Consequently,

$$x + 4 \geq 0\,.$$

Solving this inequality for *x*,

$$x \geq -4.$$

Thus, the domain of f is Domain = $[-4, \infty)$, which matches the graphical solution presented above.

Reflections

If we start with the basic equation $y = \sqrt{x}$, then replace x with −x, then the graph of the resulting equation $y = \sqrt{-x}$ is captured by reflecting the graph of $y = \sqrt{x}$ (see **Figure 1**(c)) horizontally across the y-axis. The graph of $y = \sqrt{-x}$ is shown in **Figure 7**(a).

Similarly, the graph of $y = -\sqrt{x}$ would be a vertical reflection of the graph of $y = \sqrt{x}$ across the x-axis, as shown in **Figure 7**(b).

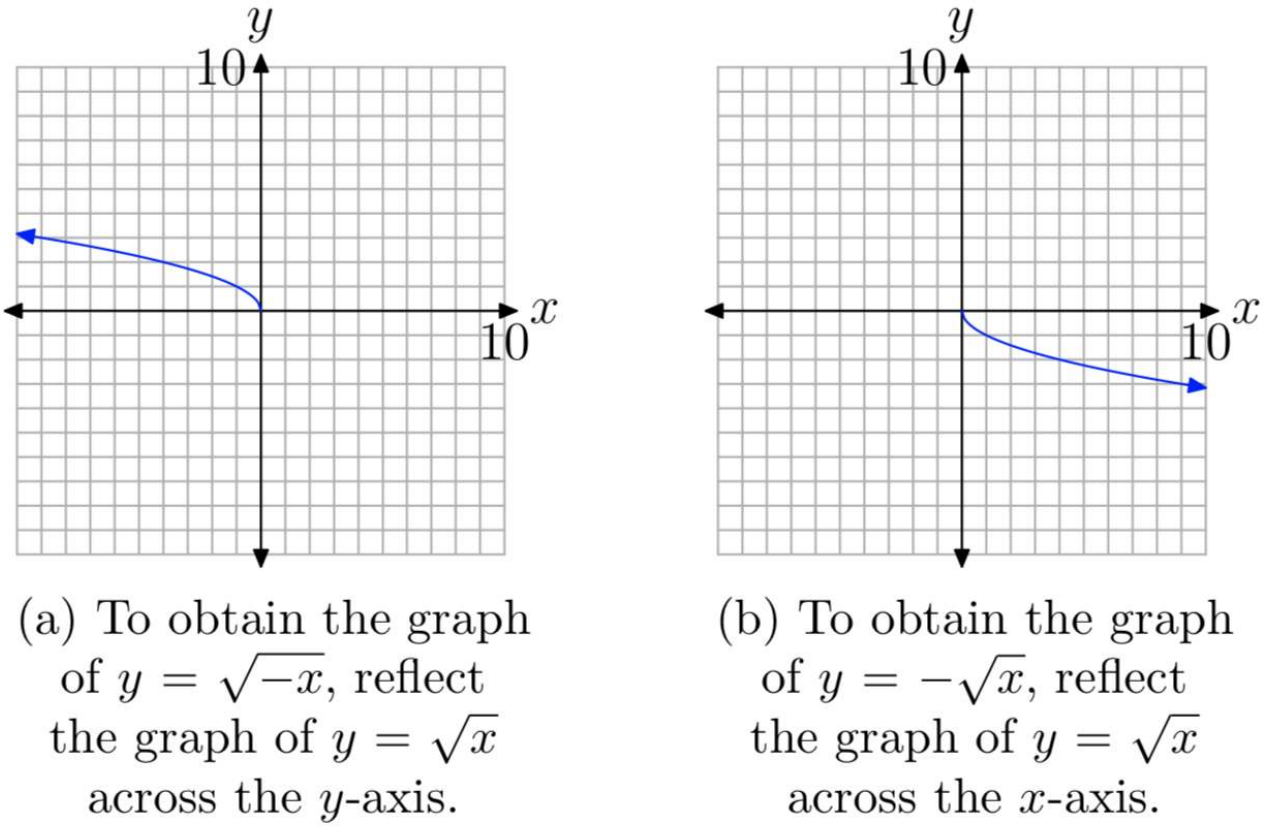


Figure 7. Reflecting the graph of $y = \sqrt{x}$ across the x- and y-axes.

More often than not, you will be asked to perform a reflection **and** a translation.

? Example 4.8.6

Sketch the graph of $f(x) = \sqrt{4-x}$. Use the resulting graph to determine the domain and range of f.

First, rewrite the equation $f(x) = \sqrt{4-x}$ as follows:

$$f(x) = \sqrt{-(x-4)}$$

Definition

Reflections First. It is usually more intuitive to perform reflections before translations.

With this thought in mind, we first sketch the graph of $f(x) = \sqrt{-x}$, which is a reflection of the graph of $f(x) = \sqrt{x}$ across the *y*-axis. This is shown in **Figure 8**(a).

Now, in $f(x) = \sqrt{-x}$ replace *x* with *x*−4 to obtain $f(x) = \sqrt{-(x-4)}$. This shifts the graph of $f(x) = \sqrt{-x}$ four units to the right, as pictured in **Figure 8**(b).

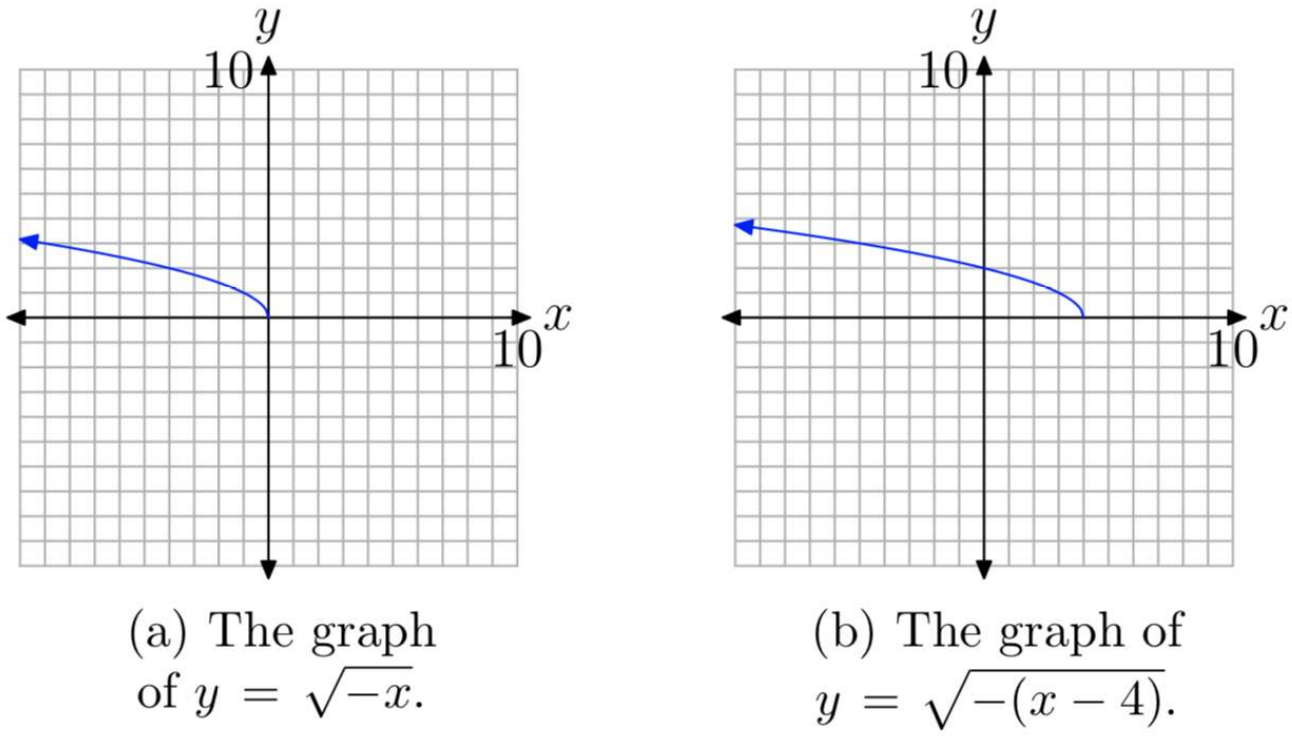


Figure 8. A reflection followed by a translation.

To find the domain of the function $f(x) = \sqrt{-(x-4)}$, or equivalently, $f(x) = \sqrt{4-x}$, project each point on the graph of *f* onto the *x*-axis, as shown in **Figure 9**(a). Note that all real numbers less than or equal to 4 are shaded on the *x*-axis. Hence, the domain of *f* is

$$\text{Domain} = (-\infty, 4] = \{x: x \leq 4\}.$$

Similarly, to obtain the range of f, project each point on the graph of f onto they-axis, as shown in **Figure 9**(b). Note that all real numbers greater than or equal to zero are shaded on the y-axis. Hence, the range of f is

$$\text{Range} = [0, \infty) = \{x: x \geq 0\}.$$

We can also find the domain of the function *f* by examining the equation $f(x) = \sqrt{4-x}$. We cannot take the square root of a negative number, so the expression under the radical must be nonnegative (zero or positive). Consequently,

$$4 - x \geq 0\,.$$

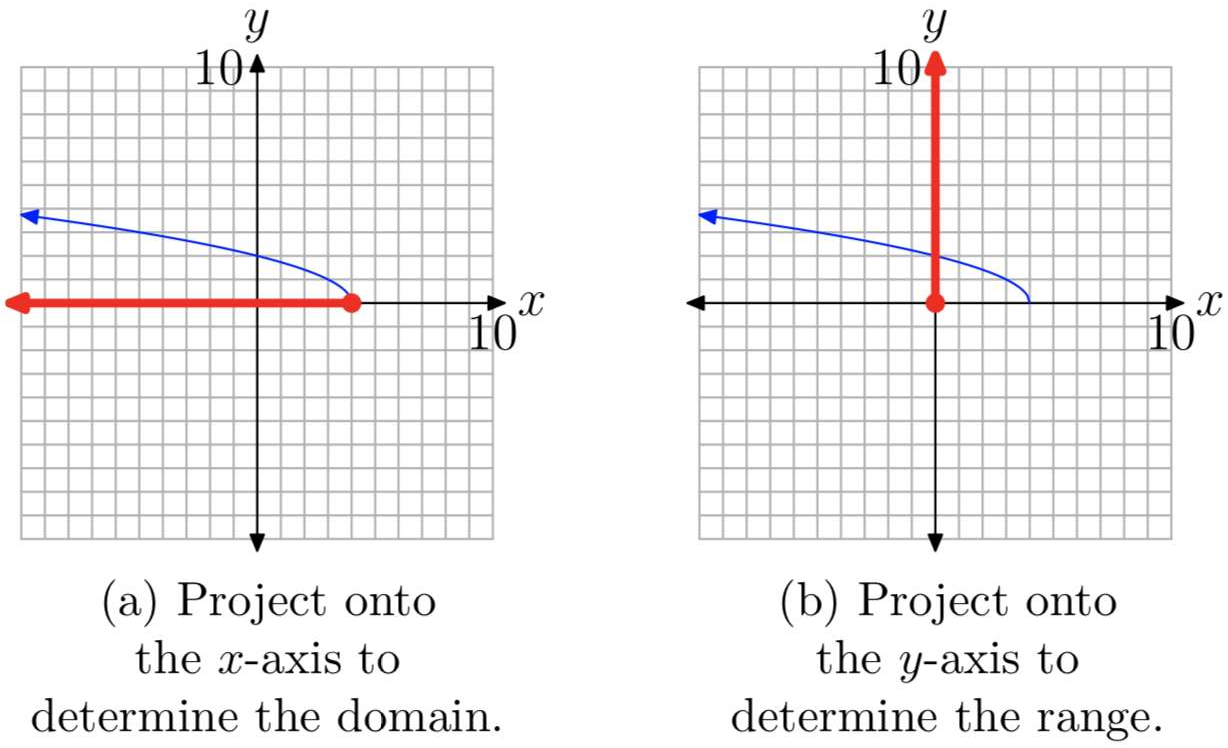


Figure 9. Determining the domain and range of $f(x) = \sqrt{4 - x}$

Solve this last inequality for *x*. First subtract 4 from both sides of the inequality, then multiply both sides of the resulting inequality by −1. Of course, multiplying by a negative number reverses the inequality symbol.

$$-x \geq -4$$

$$x \leq 4$$

Thus, the domain of f is {x: $x \leq 4$}. In interval notation, Domain = $(-\infty, 4]$. This agree nicely with the graphical result found above.

More often than not, it will take a combination of your graphing calculator and a little algebraic manipulation to determine the domain of a square root function.

? Example 4.8.7

Sketch the graph of $f(x) = \sqrt{5 - 2x}$ Use the graph and an algebraic technique to determine the domain of the function.

Load the function into Y1 in the Y= menu of your calculator, as shown in **Figure 10**(a). Select 6:ZStandard from the ZOOM menu to produce the graph shown in **Figure 10**(b).

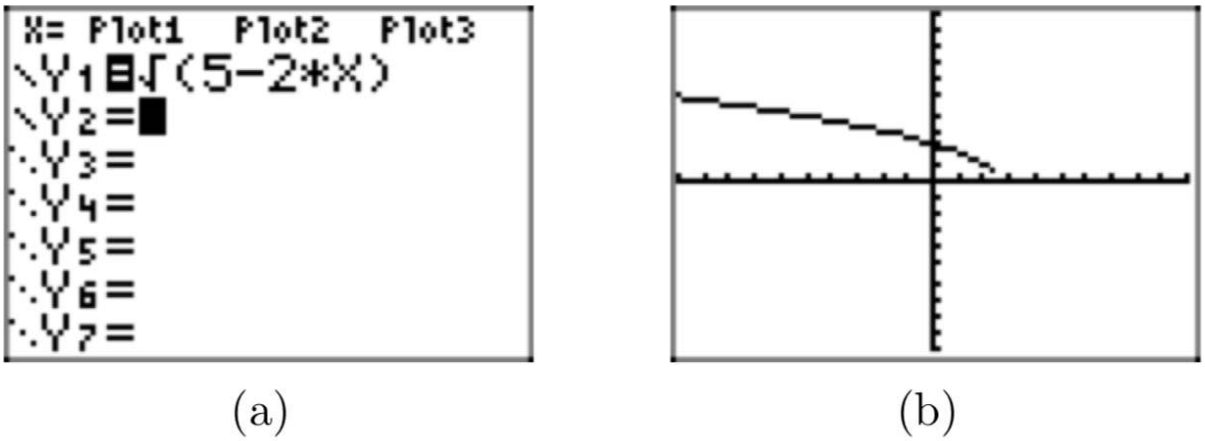


Figure 10. Drawing the graph of f(x) = \sqrt{5−2x} on the graphing calculator.

Look carefully at the graph in **Figure 10**(b) and note that it's difficult to tell if the graph comes all the way down to "touch" the x-axis near $x \approx 2.5$. However, our previous experience with the square root function makes us believe that this is just an artifact of insufficient resolution on the calculator that is preventing the graph from "touching" the x-axis at $x \approx 2.5$.

An algebraic approach will settle the issue. We can determine the domain of f by examine the equation $f(x) = \sqrt{5-2x}$. Consequently, We cannot take the square root of a negative number, so the expression under the radical must be nonnegative (zero or positive).

$$5 - 2x \geq 0\,.$$

Solve this last inequality for *x*. First, subtract 5 from both sides of the inequality.

$$-2x \geq -5.$$

Next, divide both sides of this last inequality by −2. Remember that we must reverse the inequality the moment we divide by a negative number.

$$\frac{-2x}{-2} \leq \frac{-5}{-2}\,.$$

$$x \leq \tfrac{5}{2}.$$

Thus, the domain of f is {x: $x \leq \frac{5}{2}$}. In interval notation, Domain = $(-\infty, \frac{5}{2}]$. This agree nicely with the graphical result found above.

Further introspection reveals that this argument also settles the issue of whether or not the graph "touches" the *x*-axis at $x = \frac{5}{2}$. If you remain unconvinced, then substitute $x = \frac{5}{2}$ in $f(x) = \sqrt{5-2x}$ to see

$$f(\tfrac{5}{2}) = \sqrt{5 - 2(\tfrac{5}{2})} = \sqrt{0} = 0\,.$$

Thus, the graph of f "touches" the x-axis at the point $(\frac{5}{2}, 0)$.

In **Exercise 1-10**, complete each of the following tasks:

1. Set up a coordinate system on a sheet of graph paper. Label and scale each axis.
2. Complete the table of points for the given function. Plot each of the points on your coordinate system, then use them to help draw the graph of the given function.
3. Use different colored pencils to project all points onto the *x*- and *y*-axes to determine the domain and range. Use interval notation to describe the do- main of the given function.

? Exercise 4.8.1

$f(x) = -\sqrt{x}$

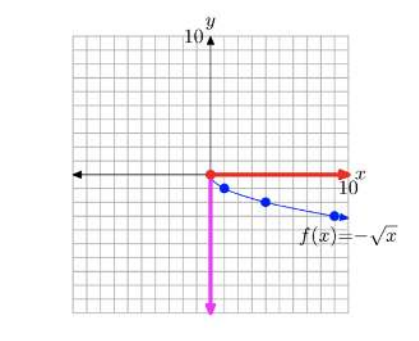
x	0	1	4	9

f (x)				

Answer

x	0	1	4	9
f (x)	0	−1	−2	−3

Plot the points in the table and use them to help draw the graph.



Project all points on the graph onto the x-axis to determine the domain: Domain = $[0, \infty)$. Project all points on the graph onto the y-axis to determine the range: Range = $(-\infty, 0]$.

Exercise 4.8.2

$f(x) = \sqrt{-x}$

x	0	−1	−4	−9
f (x)				

Exercise 4.8.3

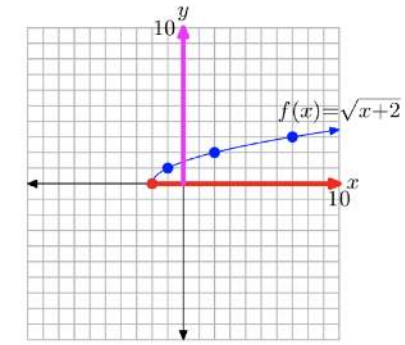
$f(x) = \sqrt{x+2}$

x	−2	−1	2	7
f (x)				

Answer

x	−2	−1	2	7
$f(x)$	0	1	2	3

Plot the points in the table and use them to help draw the graph.



Project all points on the graph onto the x-axis to determine the domain: Domain = \([−2, \infty)\). Project all points on the graph onto the y-axis to determine the range: Range = $[0, \infty)$.

Exercise 4.8.4

$f(x) = \sqrt{5 - x}$

x	−4	1	4	5
f (x)				

Exercise 4.8.5

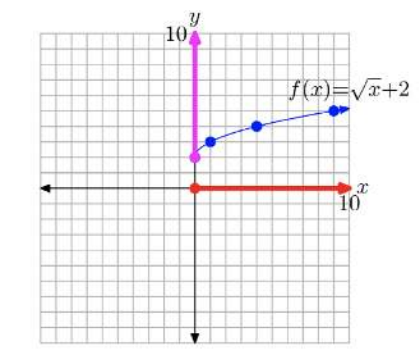
$f(x) = \sqrt{x} + 2$

x	*0*	*1*	*4*	*9*
f (x)				

Answer

x	0	1	4	9
f (x)	2	3	4	5

Plot the points in the table and use them to draw the graph of *f*.



Project all points on the graph onto the x-axis to determine the domain: Domain = $[0, \infty)$. Project all points on the graph onto the y-axis to determine the range: Range = $[2, \infty)$.

Exercise 4.8.6

$f(x) = \sqrt{x} - 1$

x	0	1	4	9
f (x)				

Exercise 4.8.7

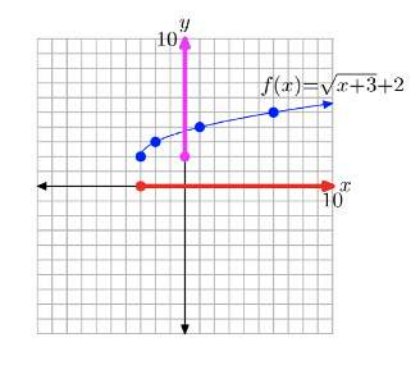
$f(x) = \sqrt{x+3} + 2$

x	−3	−2	1	6
f (x)				

Answer

x	−3	−2	1	6
f (x)	2	3	4	5

Plot the points in the table and use them to draw the graph of *f*.



Project all points on the graph onto the x-axis to determine the domain: Domain = \([−3, \infty)\). Project all points on the graph onto the y-axis to determine the range: Range = $[2, \infty)$.

Exercise 4.8.8

$f(x) = \sqrt{x-1} + 3$

x	1	2	5	10
f (*x*)				

Exercise 4.8.9

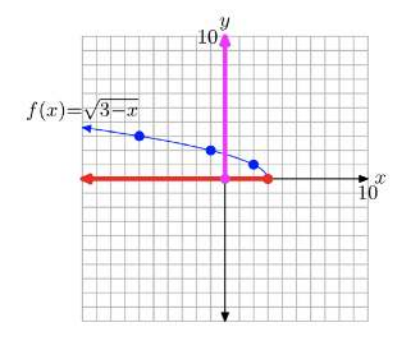
$f(x) = \sqrt{3-x}$

x	−6	−1	2	3
f (*x*)				

Answer

x	-6	-1	2	3
$f(x)$	3	2	1	0

Plot the points in the table and use them to draw the graph of f.



Project all points on the graph onto the x-axis to determine the domain: Domain = \((−\infty, 3]\). Project all points on the graph onto the y-axis to determine the range: Range = $[0, \infty)$.

? Exercise 4.8.10

$f(x) = -\sqrt{x+3}$

x	-3	-2	1	6
$f(x)$				

In **Exercises 11-20**, perform each of the following tasks.

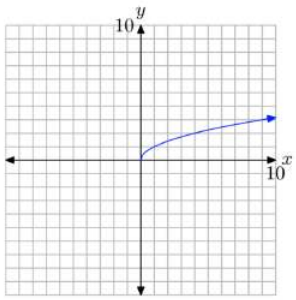
1. Set up a coordinate system on a sheet of graph paper. Label and scale each axis. Remember to draw all lines with a ruler.
2. Use geometric transformations to draw the graph of the given function on your coordinate system without the use of a graphing calculator. Note: You may **check** your solution with your calculator, but you should be able to produce the graph without the use of your calculator.
3. Use different colored pencils to project the points on the graph of the function onto the x- and y-axes. Use interval notation to describe the domain and range of the function.

? Exercise 4.8.11

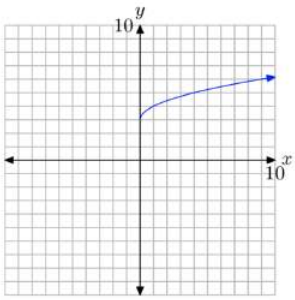
$f(x) = \sqrt{x}+3$

Answer

First, plot the graph of $y = \sqrt{x}$, as shown in (a). Then, add 3 to produce the equation $y = \sqrt{x}+3$. This will shift the graph of of $y = \sqrt{x}$ upward 3 units, as shown in (b).

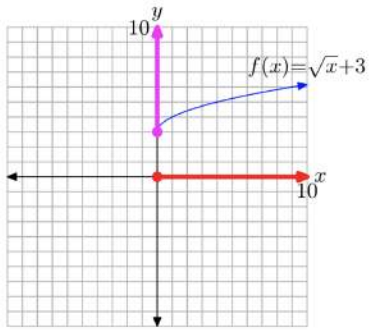


(a) $y = \sqrt{x}$.



(b) $y = \sqrt{x} + 3$.

Project all points on the graph onto the x-axis to determine the domain: Domain = $[0, \infty)$. Project all points on the graph onto the y-axis to determine the range: Range = $[3, \infty)$.



? Exercise 4.8.12

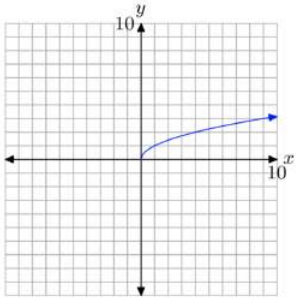
$f(x) = \sqrt{x+3}$

? Exercise 4.8.13

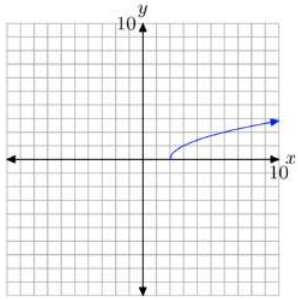
$f(x) = \sqrt{x-2}$

Answer

First, plot the graph of $y = \sqrt{x}$, as shown in (a). Then, replace x with x − 2 to produce the equation $y = \sqrt{x-2}$. This will shift the graph of $y = \sqrt{x}$ to the right 2 units, as shown in (b).

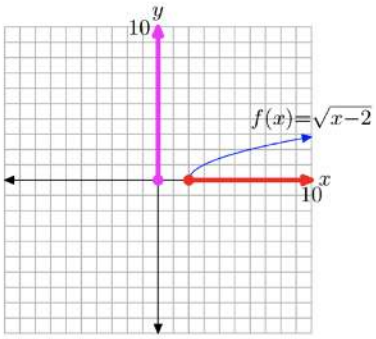


(a) $y = \sqrt{x}$.



(b) $y = \sqrt{x-2}$.

Project all points on the graph onto the x-axis to determine the domain: Domain = $[2, \infty)$. Project all points on the graph onto the y-axis to determine the range: Range = $[0, \infty)$.



Exercise 4.8.14

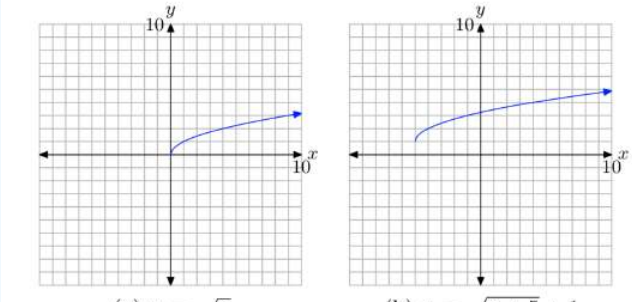
$f(x) = \sqrt{x} - 2$

Exercise 4.8.15

$f(x) = \sqrt{x+5} + 1$

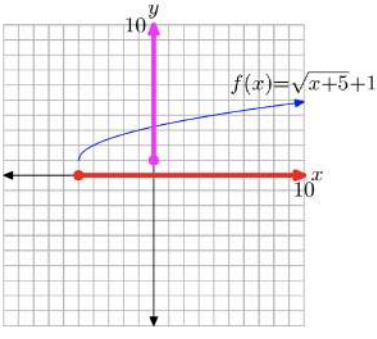
Answer

First, plot the graph of $y = \sqrt{x}$, as shown in (a). Then, replace x with x + 5 to produce the equation $y = \sqrt{x+5}$. Then add 1 to produce the equation $f(x) = \sqrt{x+5} + 1$. This will shift the graph of $y = \sqrt{x}$ to the left 5 units, then upwards 1 unit, as shown in (b).



(a) $y = \sqrt{x}$. (b) $y = \sqrt{x+5} + 1$.

Project all points on the graph onto the x-axis to determine the domain: Domain = $[-5, \infty)$. Project all points on the graph onto the y-axis to determine the range: Range = $[1, \infty)$.



Exercise 4.8.16

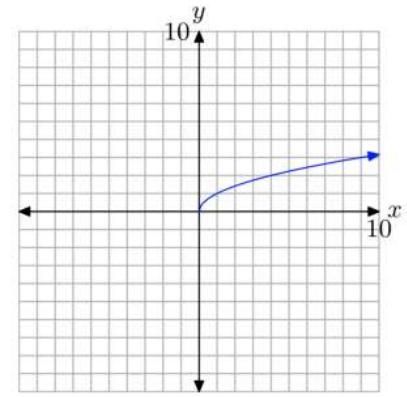
$f(x) = \sqrt{x-2} - 1$

Exercise 4.8.17

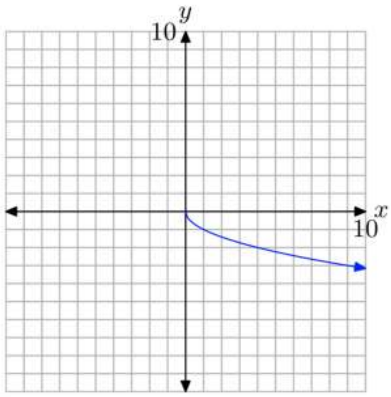
$y = -\sqrt{x+4}$

Answer

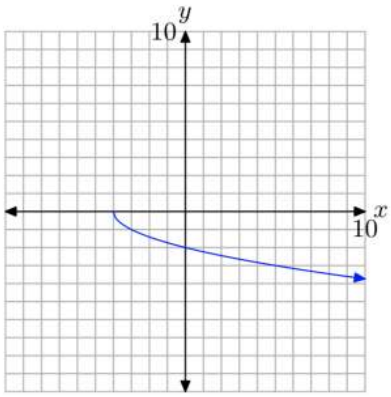
First, plot the graph of $y = \sqrt{x}$, as shown in (a). Then, negate to produce the $y = -\sqrt{x}$. This will reflect the graph of $y = \sqrt{x}$ across the x-axis as shown in (b). Finally, replace x with x + 4 to produce the equation $y = -\sqrt{x+4}$. This will shift the graph of $y = -\sqrt{x}$ four units to the left, as shown in (c).



(a) $y = \sqrt{x}$.

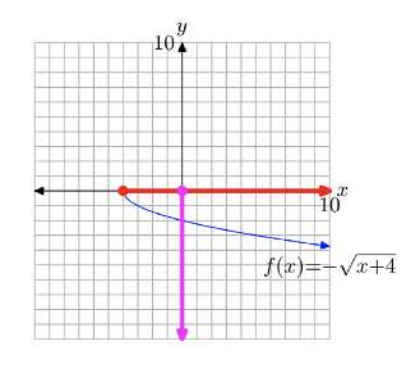


(b) $y = -\sqrt{x}$.



(c) $y = -\sqrt{x+4}$.

Project all points on the graph onto the x-axis to determine the domain: Domain = $[-4, \infty)$. Project all points on the graph onto the y-axis to determine the range: Range = $(-\infty, 0]$.



Exercise 4.8.18

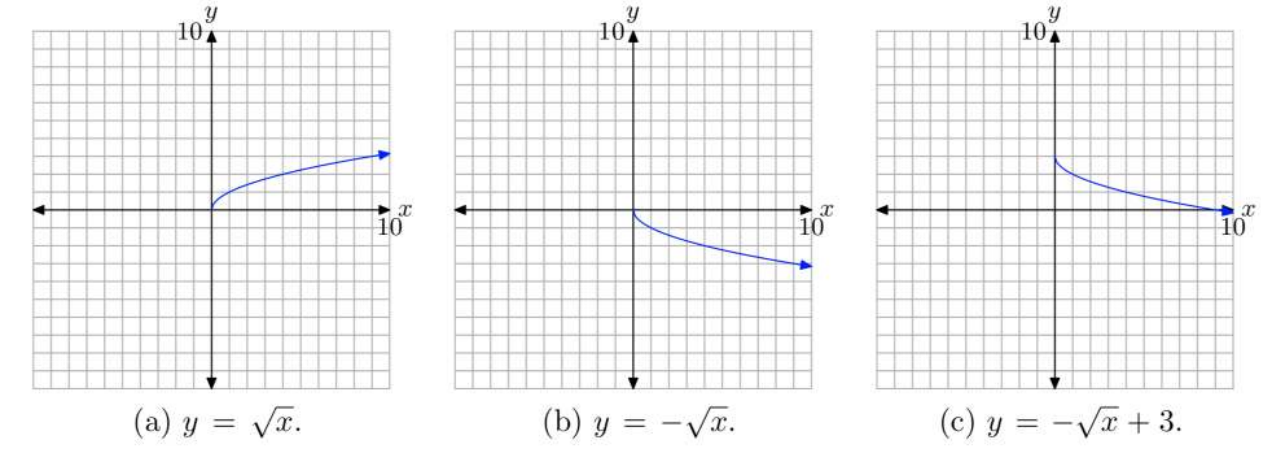
$f(x) = -\sqrt{x} + 4$

Exercise 4.8.19

$f(x) = -\sqrt{x} + 3$

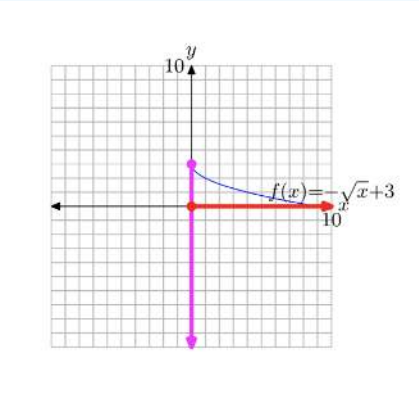
Answer

First, plot the graph of $y = \sqrt{x}$, as shown in (a). Then, negate to produce the $y = -\sqrt{x}$. This will reflect the graph of $y = \sqrt{x}$ across the x-axis as shown in (b). Finally, add 3 to produce the equation $y = -\sqrt{x} + 3$. This will shift the graph of $y = -\sqrt{x}$ three units upward, as shown in (c).



(a) $y = \sqrt{x}$. (b) $y = -\sqrt{x}$. (c) $y = -\sqrt{x} + 3$.

Project all points on the graph onto the x-axis to determine the domain: Domain = $[0, \infty)$. Project all points on the graph onto the y-axis to determine the range: Range = $(-\infty, 3]$.



? Exercise 4.8.20

$f(x) = -\sqrt{x+3}$

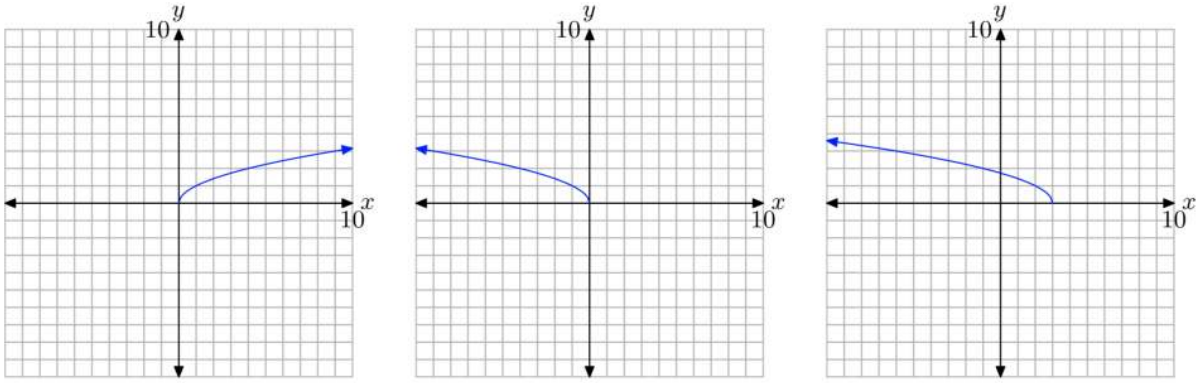
? Exercise 4.8.21

To draw the graph of the function $f(x) = \sqrt{3-x}$, perform each of the following steps in sequence without the aid of a calculator.

1. Set up a coordinate system and sketch the graph of $y = \sqrt{x}$. Label the graph with its equation.
2. Set up a second coordinate system and sketch the graph of $y = \sqrt{-x}$. Label the graph with its equation.
3. Set up a third coordinate system and sketch the graph of $y = \sqrt{-(x-3)}$. Label the graph with its equation. This is the graph of $y = \sqrt{3-x}$. Use interval notation to state the domain and range of this function.

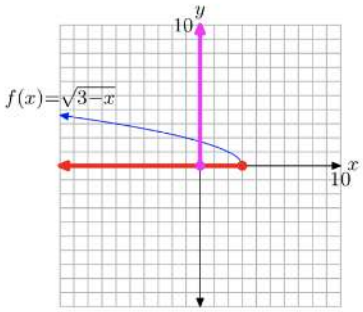
Answer

First, plot the graph of $y = \sqrt{x}$, as shown in (a). Then, replace x with $-x$ to produce the equation $y = \sqrt{-x}$. This will reflect the graph of $y = \sqrt{x}$ across the y-axis, as shown in (b). Finally, replace x with $x - 3$ to produce the equation \(y = \sqrt{-(x - 3)}\). This will shift the graph of $y = \sqrt{-x}$ three units to the right, as shown in (c).



(a) $y = \sqrt{x}$. (b) $y = \sqrt{-x}$. (c) $y = \sqrt{-(x-3)}$.

Project all points on the graph onto the x-axis to determine the domain: Domain = $(-\infty, 3]$. Project all points on the graph onto the y-axis to determine the range: Range = $[0, \infty)$.



Exercise 4.8.22

To draw the graph of the function $f(x) = \sqrt{-x-3}$, perform each of the following steps in sequence.

1. Set up a coordinate system and sketch the graph of $y = \sqrt{x}$. Label the graph with its equation.
2. Set up a second coordinate system and sketch the graph of $y = \sqrt{-x}$. Label the graph with its equation.
3. Set up a third coordinate system and sketch the graph of $y = \sqrt{-(x+3)}$. Label the graph with its equation. This is the graph of $y = \sqrt{-x-3}$. Use interval notation to state the domain and range of this function.

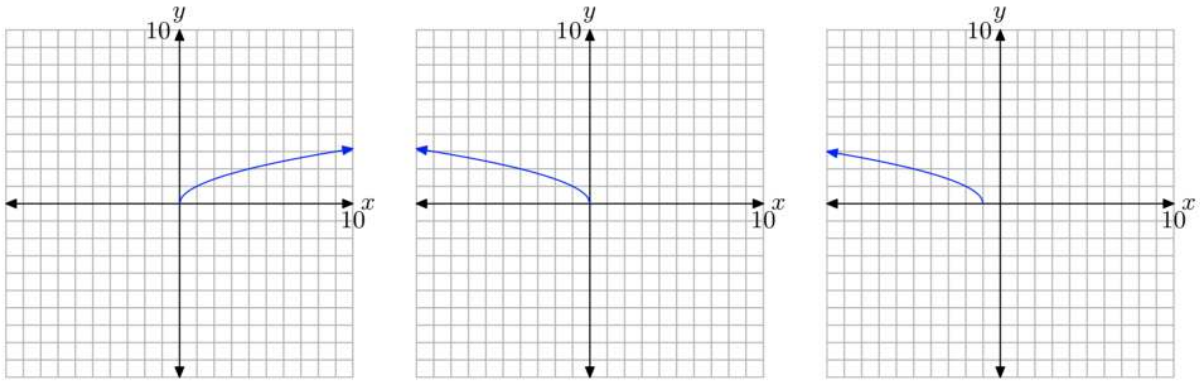
Exercise 4.8.23

To draw the graph of the function $f(x) = \sqrt{-x-3}$, perform each of the following steps in sequence without the aid of a calculator.

1. Set up a coordinate system and sketch the graph of $y = \sqrt{x}$. Label the graph with its equation.
2. Set up a second coordinate system and sketch the graph of $y = \sqrt{-x}$. Label the graph with its equation.
3. Set up a third coordinate system and sketch the graph of $y = \sqrt{-(x+1)}$. Label the graph with its equation. This is the graph of $y = \sqrt{-x-1}$. Use interval notation to state the domain and range of this function.

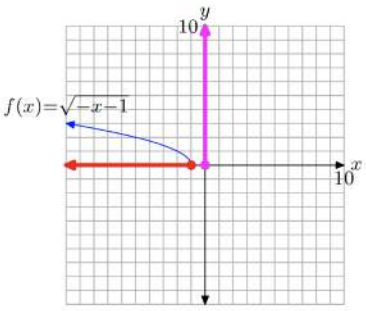
Answer

First, plot the graph of $y = \sqrt{x}$, as shown in (a). Then, replace x with −x to produce the equation $y = \sqrt{-x}$. This will reflect the graph of $y = \sqrt{x}$ across the y-axis, as shown in (b). Finally, replace x with x + 1 to produce the equation $y = \sqrt{-(x+1)}$. This will shift the graph of $y = \sqrt{-x}$ one unit to the left, as shown in (c).



(a) $y = \sqrt{x}$. (b) $y = \sqrt{-x}$. (c) $y = \sqrt{-(x+1)}$.

Project all points on the graph onto the x-axis to determine the domain: Domain = $(-\infty, -1]$. Project all points on the graph onto the y-axis to determine the range: Range = $[0, \infty)$.



? Exercise 4.8.24

To draw the graph of the function $f(x) = \sqrt{1-x}$, perform each of the following steps in sequence.

1. Set up a coordinate system and sketch the graph of $y = \sqrt{x}$. Label the graph with its equation.
2. Set up a second coordinate system and sketch the graph of $y = \sqrt{-x}$. Label the graph with its equation.
3. Set up a third coordinate system and sketch the graph of $y = \sqrt{-(x-1)}$. Label the graph with its equation. This is the graph of $y = \sqrt{1-x}$. Use interval notation to state the domain and range of this function.

In **Exercises 25-28**, perform each of the following tasks.

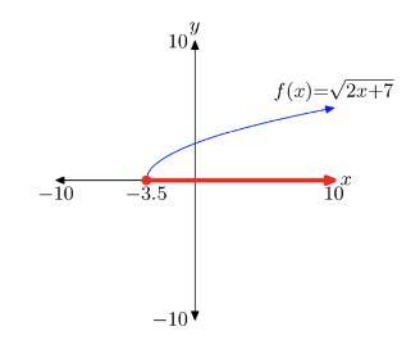
1. Draw the graph of the given function with your graphing calculator. Copy the image in your viewing window onto your homework paper. Label and scale each axis with xmin, xmax, ymin, and ymax. Label your graph with its equation. Use the graph to determine the domain of the function and describe the domain with interval notation.
2. Use a purely algebraic approach to determine the domain of the given function. Use interval notation to de- scribe your result. Does it agree with the graphical result from part 1?

? Exercise 4.8.25

$f(x) = \sqrt{2x+7}$

Answer

We use a graphing calculator to produce the following graph of $f(x) = \sqrt{2x+7}$



We estimate that the domain will consist of all real numbers to the right of approximately -3.5. To find an algebraic solution, note that you cannot take the square root of a negative number. Hence, the expression under the radical in $f(x) = \sqrt{2x+7}$ must be greater than or equal to zero.

$$2x+7 \geq 0$$
$$2x \geq -7$$
$$x \geq -\tfrac{7}{2}$$

Hence, the domain is $[-\frac{7}{2}, \infty)$.

Exercise 4.8.26

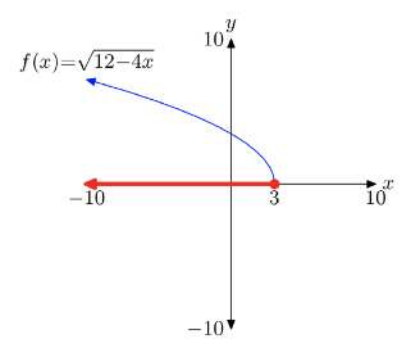
$f(x) = \sqrt{7-2x}$

Exercise 4.8.27

$f(x) = \sqrt{12-4x}$

Answer

We use a graphing calculator to produce the following graph of $f(x) = \sqrt{12-4x}$.



We estimate that the domain will consist of all real numbers to the right of approximately 3. To find an algebraic solution, note that you cannot take the square root of a negative number. Hence, the expression under the radical in $f(x) = \sqrt{12-4x}$ must be greater than or equal to zero.

$$12-4x \geq 0$$
$$-4x \geq -12$$
$$x \leq 3$$

Hence, the domain is $(-\infty, 3]$.

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Exercise 4.8.28

$f(x) = \sqrt{12+2x}$

In **Exercises 29-40**, find the domain of the given function algebraically.

Exercise 4.8.29

$f(x) = \sqrt{2x+9}$

Answer

The even root of a negative number is not defined as a real number. Thus, 2x + 9 must be greater than or equal to zero. Since $2x+9 \geq 0$ implies that $x \geq -\frac{9}{2}$, the domain is the interval $[-\frac{9}{2}, \infty)$.

Exercise 4.8.30

$f(x) = \sqrt{-3x+3}$

Exercise 4.8.31

$f(x) = \sqrt{-8x-3}$

Answer

The even root of a negative number is not defined as a real number. Thus, −8x−3 must be greater than or equal to zero. Since $-8x-3 \geq 0$ implies that $x \leq -\frac{3}{8}$, the domain is the interval $(-\infty, -\frac{3}{8}]$.

Exercise 4.8.32

$f(x) = \sqrt{-3x+6}$

Exercise 4.8.33

$f(x) = \sqrt{-6x-8}$

Answer

The even root of a negative number is not defined as a real number. Thus, −6x−8 must be greater than or equal to zero. Since $-6x-8 \geq 0$ implies that $x \leq -\frac{4}{3}$, the domain is the interval $(-\infty, \frac{4}{3}]$.

Exercise 4.8.34

$f(x) = \sqrt{8x-6}$

Exercise 4.8.35

$f(x) = \sqrt{-7x+2}$

Answer

The even root of a negative number is not defined as a real number. Thus, −7x+2 must be greater than or equal to zero. Since $-7x+2 \geq 0$ implies that $x \leq \frac{2}{7}$, the domain is the interval $(-\infty, \frac{2}{7}]$.

Exercise 4.8.36

$f(x) = \sqrt{8x - 3}$

Exercise 4.8.37

$f(x) = \sqrt{6x + 3}$

Answer

The even root of a negative number is not defined as a real number. Thus, 6x+3 must be greater than or equal to zero. Since $6x + 3 \geq 0$ implies that $x \geq -\frac{1}{2}$, the domain is the interval $[-\frac{1}{2}, \infty)$.

Exercise 4.8.38

$f(x) = \sqrt{x - 5}$

Exercise 4.8.39

$f(x) = \sqrt{-7x - 8}$

Answer

The even root of a negative number is not defined as a real number. Thus, −7x−8 must be greater than or equal to zero. Since $-7x - 8 \geq 0$ implies that $x \leq -\frac{8}{7}$, the domain is the interval $(-\infty, -\frac{8}{7}]$

Exercise 4.8.40

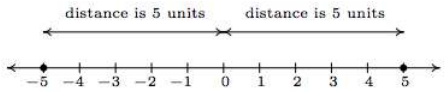
$f(x) = \sqrt{7x + 8}$

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4.9: Absolute Value Functions

There are a few ways to describe what is meant by the absolute value $|x|$ of a real number x. You may have been taught that $|x|$ is the distance from the real number x to 0 on the number line. So, for example, $|5| = 5$ and $|-5| = 5$, since each is 5 units from 0 on the number line.



Another way to define absolute value is by the equation $|x| = \sqrt{x^2}$. Using this definition, we have $|5| = \sqrt{(5)^2} = \sqrt{25} = 5$ and $|-5| = \sqrt{(-5)^2} = \sqrt{25} = 5$. The long and short of both of these procedures is that $|x|$ takes negative real numbers and assigns them to their positive counterparts while it leaves positive numbers alone. This last description is the one we shall adopt, and is summarized in the following definition.

Definition 2.4: Absolute Value

The **absolute value** of a real number x, denoted $|x|$, is given by

$$|x| = \begin{cases} -x, & \text{if} \quad x < 0 \\ x, & \text{if} \quad x \geq 0 \end{cases} \tag{4.9.1}$$

In Definition 2.4, we define $|x|$ using a piecewise-defined function (Section 1.4). To check that this definition agrees with what we previously understood as absolute value, note that since $5 \geq 0$, to find $|5|$ we use the rule $|x| = x$, so $|5| = 5$. Similarly, since $-5 < 0$, we use the rule $|x| = -x$, so that $|-5| = -(-5) = 5$. This is one of the times when it's best to interpret the expression '$-x$' as 'the opposite of x' as opposed to 'negative x'. Before we begin studying absolute value functions, we remind ourselves of the properties of absolute value.

Theorem 2.1: Properties of Absolute Value

Let a, b and x be real numbers and let n be an integer, then:

- **Product Rule:** $|ab| = |a||b|$
- **Power Rule:** $|a^n| = |a|^n$ whenever a^n is defined
- **Quotient Rule:** $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$, provided $b \neq 0$

Equality Properties:

- $|x| = 0$ if and only if $x = 0$.
- For $c > 0$, $|x| = c$ if and only if $x = c$ or $x = -c$.
- For $c < 0$, $|x| = c$ has no solution.

The proofs of the Product and Quotient Rules in Theorem 2.1 boil down to checking four cases:

- when both a and b are positive;
- when they are both negative;
- when one is positive and the other is negative;
- and when one or both are zero.

For example, suppose we wish to show that $|ab| = |a||b|$. We need to show that this equation is true for all real numbers a and b. If a and b are both positive, then so is ab. Hence, $|a| = a$, $|b| = b$ and $|ab| = ab$. Hence, the equation $|ab| = |a||b|$ is the same as $ab = ab$ which is true. If both a and b are negative, then ab is positive. Hence, $|a| = -a$, $|b| = -b$ and $|ab| = ab$. The equation $|ab| = |a||b|$ becomes $ab = (-a)(-b)$, which is true. Suppose a is positive and b is negative. Then ab is negative, and we have $|ab| = -ab$, $|a| = a$ and $|b| = -b$. The equation $|ab| = |a||b|$ reduces to $-ab = a(-b)$ which is true. A symmetric argument shows the equation $|ab| = |a||b|$ holds when a is negative and b is positive. Finally, if either a or b (or both) are zero, then both sides of $|ab| = |a||b|$ are zero, so the equation holds in this case, too. All of this rhetoric has shown that the equation $|ab| = |a||b|$ holds true in all cases.

The proof of the Quotient Rule is very similar, with the exception that $b \neq 0$. The Power Rule can be shown by repeated application of the Product Rule. The 'Equality Properties' can be proved using Definition 2.4 and by looking at the cases when $x \geq 0$, in which case $|x| = x$, or when $x < 0$, in which case $|x| = -x$. For example, if $c > 0$, and $|x| = c$, then if $x \geq 0$, we have $x = |x| = c$. If, on the other hand, $x < 0$, then $-x = |x| = c$, so $x = -c$. The remaining properties are proved similarly and are left for the Exercises. Our first example reviews how to solve basic equations involving absolute value using the properties listed in Theorem 2.1.

Example 4.9.1: Absolute Value

Solve each of the following equations.

1. $|3x - 1| = 6$
2. $3 - |x + 5| = 1$
3. $3|2x + 1| - 5 = 0$
4. $4 - |5x + 3| = 5$
5. $|x| = x^2 - 6$
6. $|x - 2| + 1 = x$

Solution

1. The equation $|3x - 1| = 6$ is of the form $|x| = c$ for $c > 0$, so by the Equality Properties, $|3x - 1| = 6$ is equivalent to $3x - 1 = 6$ or $3x - 1 = -6$. Solving the former, we arrive at $x = \frac{7}{3}$, and solving the latter, we get $x = -\frac{5}{3}$. We may check both of these solutions by substituting them into the original equation and showing that the arithmetic works out.

To use the Equality Properties to solve $3 - |x + 5| = 1$, we first isolate the absolute value.

$$\begin{aligned} 3 - |x+5| &= 1 \\ -|x+5| &= -2 \qquad \text{subtract 3} \\ |x+5| &= 2 \qquad \text{divide by } -1 \end{aligned} \tag{4.9.2}$$

From the Equality Properties, we have $x + 5 = 2$ or $x + 5 = -2$, and get our solutions to be $x = -3$ or $x = -7$. We leave it to the reader to check both answers in the original equation.

1. As in the previous example, we first isolate the absolute value in the equation $3|2x + 1| - 5 = 0$ and get $|2x + 1| = \frac{5}{3}$. Using the Equality Properties, we have $2x + 1 = \frac{5}{3}$ or $2x + 1 = -\frac{5}{3}$. Solving the former gives $x = \frac{1}{3}$ and solving the latter gives $x = -\frac{4}{3}$. As usual, we may substitute both answers in the original equation to check.
2. Upon isolating the absolute value in the equation $4 - |5x + 3| = 5$, we get $|5x + 3| = -1$. At this point, we know there cannot be any real solution, since, by definition, the absolute value of *anything* is never negative. We are done.
3. The equation $|x| = x^2 - 6$ presents us with some difficulty, since x appears both inside and outside of the absolute value. Moreover, there are values of x for which $x^2 - 6$ is positive, negative and zero, so we cannot use the Equality Properties without the risk of introducing extraneous solutions, or worse, losing solutions. For this reason, we break equations like this into cases by rewriting the term in absolute values, $|x|$, using Definition 2.4. For $x < 0$, $|x| = -x$, so for $x < 0$, the equation $|x| = x^2 - 6$ is equivalent to $-x = x^2 - 6$. Rearranging this gives us $x^2 + x - 6 = 0$, or $(x + 3)(x - 2) = 0$. We get $x = -3$ or $x = 2$. Since only $x = -3$ satisfies $x < 0$, this is the answer we keep. For $x \geq 0$, $|x| = x$, so the equation $|x| = x^2 - 6$ becomes $x = x^2 - 6$. From this, we get $x^2 - x - 6 = 0$ or $(x - 3)(x + 2) = 0$. Our solutions are $x = 3$ or $x = -2$, and since only $x = 3$ satisfies $x \geq 0$, this is the one we keep. Hence, our two solutions to $|x| = x^2 - 6$ are $x = -3$ and $x = 3$.
4. To solve $|x - 2| + 1 = x$, we first isolate the absolute value and get $|x - 2| = x - 1$. Since we see x both inside and outside of the absolute value, we break the equation into cases. The term with absolute values here is $|x - 2|$, so we replace 'x' with the quantity '$(x - 2)$' in Definition 2.4 to get

$$|x-2| = \begin{cases} -(x-2), & \text{if} \quad (x-2) < 0 \\ (x-2), & \text{if} \quad (x-2) \geq 0 \end{cases} \tag{4.9.3}$$

Simplifying yields

$$|x-2| = \begin{cases} -x+2, & \text{if} \quad x < 2 \\ x-2, & \text{if} \quad x \geq 2 \end{cases} \tag{4.9.4}$$

So, for $x < 2$, $|x-2| = -x+2$ and our equation $|x-2| = x-1$ becomes $-x+2 = x-1$, which gives $x = \frac{3}{2}$. Since this solution satisfies $x < 2$, we keep it. Next, for $x \geq 2$, $|x-2| = x-2$, so the equation $|x-2| = x-1$ becomes $x-2 = x-1$. Here, the equation reduces to $-2 = -1$, which signifies we have no solutions here. Hence, our only solution is $x = \frac{3}{2}$.

Next, we turn our attention to graphing absolute value functions. Our strategy in the next example is to make liberal use of Definition 2.4 along with what we know about graphing linear functions (from Section 2.1) and piecewise-defined functions (from Section 1.4).

Example 4.9.2:

Graph each of the following functions.

1. $f(x) = |x|$
2. $g(x) = |x-3|$
3. $h(x) = |x| - 3$
4. $i(x) = 4 - 2|3x+1|$

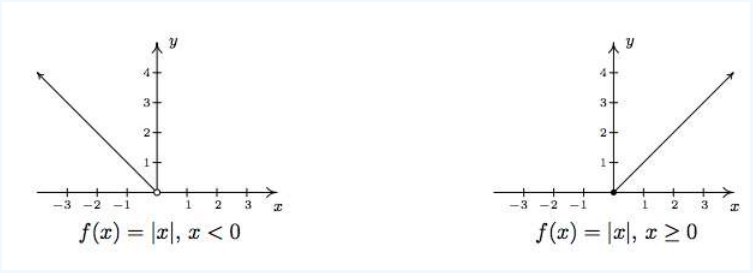
Find the zeros of each function and the x- and y-intercepts of each graph, if any exist. From the graph, determine the domain and range of each function, list the intervals on which the function is increasing, decreasing or constant, and find the relative and absolute extrema, if they exist.

Solution

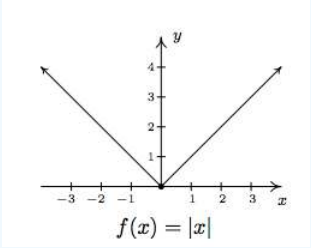
1. To find the zeros of f, we set $f(x) = 0$. We get $|x| = 0$, which, by Theorem 4.9.1 gives us $x = 0$. Since the zeros of f are the x-coordinates of the x-intercepts of the graph of $y = f(x)$, we get $(0, 0)$ as our only x-intercept. To find the y-intercept, we set $x = 0$, and find $y = f(0) = 0$, so that $(0, 0)$ is our y-intercept as well.\footnote{Actually, since functions can have at most one y-intercept (Do you know why?), as soon as we found $(0, 0)$ as the x-intercept, we knew this was also the y-intercept.} Using Definition 4.9.1, we get

$$f(x) = |x| = \begin{cases} -x, & \text{if} \quad x < 0 \\ x, & \text{if} \quad x \geq 0 \end{cases} \tag{4.9.5}$$

Hence, for $x < 0$, we are graphing the line $y = -x$; for $x \geq 0$, we have the line $y = x$. Proceeding as we did in Section 1.6, we get



Notice that we have an 'open circle' at $(0, 0)$ in the graph when $x < 0$. As we have seen before, this is due to the fact that the points on $y = -x$ approach $(0, 0)$ as the x-values approach 0. Since x is required to be strictly less than zero on this stretch, the open circle is drawn at the origin. However, notice that when $x \geq 0$, we get to fill in the point at $(0, 0)$, which effectively 'plugs' the hole indicated by the open circle. Thus we get,



By projecting the graph to the x-axis, we see that the domain is $(-\infty, \infty)$. Projecting to the y-axis gives us the range $[0, \infty)$. The function is increasing on $[0, \infty)$ and decreasing on $(-\infty, 0]$. The relative minimum value of f is the same as the absolute minimum, namely 0 which occurs at $(0, 0)$. There is no relative maximum value of f. There is also no absolute maximum value of f, since the y values on the graph extend infinitely upwards.

2. To find the zeros of g, we set $g(x) = |x-3| = 0$. By Theorem 2.1, we get $x - 3 = 0$ so that $x = 3$. Hence, the x-intercept is $(3, 0)$. To find our y-intercept, we set $x = 0$ so that $y = g(0) = |0-3| = 3$, which yields $(0, 3)$ as our y-intercept. To graph $g(x) = |x-3|$, we use Definition 2.4 to rewrite g as

$$g(x) = |x-3| = \begin{cases} -(x-3), & \text{if} \quad (x-3) < 0 \\ (x-3), & \text{if} \quad (x-3) \geq 0 \end{cases} \tag{4.9.6}$$

Simplifying, we get

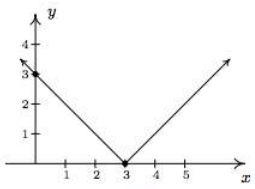
$$g(x) = \begin{cases} -x+3, & \text{if} \quad x < 3 \\ x-3, & \text{if} \quad x \geq 3 \end{cases} \tag{4.9.7}$$

As before, the open circle we introduce at $(3, 0)$ from the graph of $y = -x + 3$ is filled by the point $(3, 0)$ from the line $y = x - 3$. We determine the domain as $(-\infty, \infty)$ and the range as $[0, \infty)$. The function g is increasing on $[3, \infty)$ and decreasing on $(-\infty, 3]$. The relative and absolute minimum value of g is 0 which occurs at $(3, 0)$. As before, there is no relative or absolute maximum value of g.

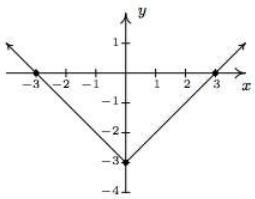
3. Setting $h(x) = 0$ to look for zeros gives $|x| - 3 = 0$. As in Example 4.9.1, we isolate the absolutre value to get $|x| = 3$ so that $x = 3$ or $x = -3$. As a result, we have a pair of x-intercepts: $(-3, 0)$ and $(3, 0)$. Setting $x = 0$ gives $y = h(0) = |0| - 3 = -3$, so our y-intercept is $(0, -3)$. As before, we rewrite the absolute value in h to get

$$h(x) = \begin{cases} -x-3, & \text{if} \quad x < 0 \\ x-3, & \text{if} \quad x \geq 0 \end{cases} \tag{4.9.8}$$

Once again, the open circle at $(0, -3)$ from one piece of the graph of h is filled by the point $(0, -3)$ from the other piece of h. From the graph, we determine the domain of h is $(-\infty, \infty)$ and the range is $[-3, \infty)$. On $[0, \infty)$, h is increasing; on $(-\infty, 0]$ it is decreasing. The relative minimum occurs at the point $(0, -3)$ on the graph, and we see -3 is both the relative and absolute minimum value of h. Also, h has no relative or absolute maximum value.



$g(x) = |x-3|$

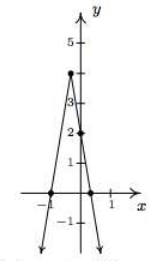


$h(x) = |x| - 3$

4. As before, we set $i(x) = 0$ to find the zeros of i and get $4 - 2|3x+1| = 0$. Isolating the absolute value term gives $|3x+1| = 2$, so either $3x + 1 = 2$ or $3x + 1 = -2$. We get $x = \frac{1}{3}$ or $x = -1$, so our x-intercepts are $\left(\frac{1}{3}, 0\right)$ and $(-1, 0)$. Substituting $x = 0$ gives $y = i(0) = 4 - 2|3(0)+1| = 2$, for a y-intercept of $(0, 2)$. Rewriting the formula for $i(x)$ without absolute values gives

$$i(x) = \begin{cases} 4-2(-(3x+1)), & \text{if} \quad (3x+1) < 0 \\ 4-2(3x+1), & \text{if} \quad (3x+1) \geq 0 \end{cases} = \begin{cases} 6x+6, & \text{if} \quad x < -\frac{1}{3} \\ -6x+2, & \text{if} \quad x \geq -\frac{1}{3} \end{cases} \tag{4.9.9}$$

The usual analysis near the trouble spot $x = -\frac{1}{3}$ gives the 'corner' of this graph is $\left(-\frac{1}{3}, 4\right)$, and we get the distinctive '$\vee$' shape:



$i(x) = 4 - 2|3x + 1|$

The domain of i is $(-\infty, \infty)$ while the range is $(-\infty, 4]$. The function i is increasing on $\left(-\infty, -\frac{1}{3}\right]$ and decreasing on $\left[-\frac{1}{3}, \infty\right)$. The relative maximum occurs at the point $\left(-\frac{1}{3}, 4\right)$ and the relative and absolute maximum value of i is 4. Since the graph of i extends downwards forever more, there is no absolute minimum value. As we can see from the graph, there is no relative minimum, either.

Note that all of the functions in the previous example bear the characteristic '$\vee$' shape of the graph of $y = |x|$. We could have graphed the functions g, h and i in Example 4.9.2 starting with the graph of $f(x) = |x|$ and applying transformations as in Section 1.7 as our next example illustrates.

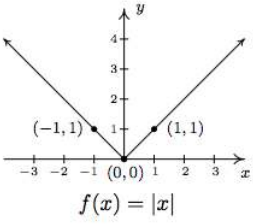
Example 4.9.3:

Graph the following functions starting with the graph of $f(x) = |x|$ and using transformations.

1. $g(x) = |x - 3|$
2. $h(x) = |x| - 3$
3. $i(x) = 4 - 2|3x + 1|$

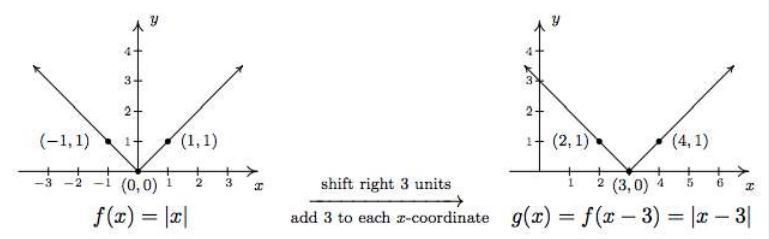
Solution

We begin by graphing $f(x) = |x|$ and labeling three points, $(-1, 1)$, $(0, 0)$ and $(1, 1)$.

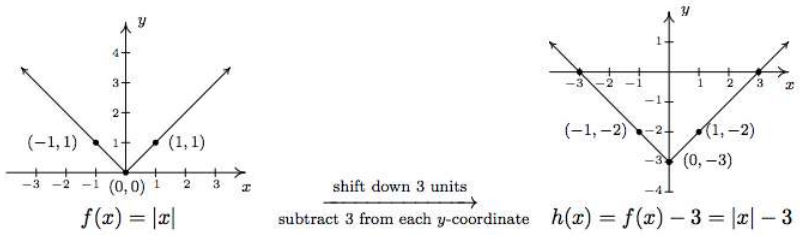


$f(x) = |x|$

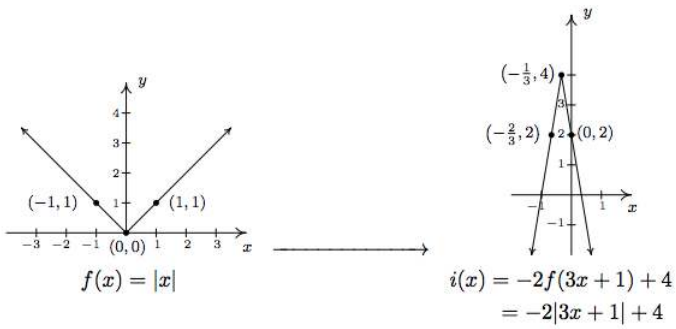
1. Since $g(x) = |x - 3| = f(x - 3)$, Theorem 1.7 tells us to *add* 3 to each of the x-values of the points on the graph of $y = f(x)$ to obtain the graph of $y = g(x)$. This shifts the graph of $y = f(x)$ to the *right* 3 units and moves the point $(-1, 1)$ to $(2, 1)$, $(0, 0)$ to $(3, 0)$ and $(1, 1)$ to $(4, 1)$. Connecting these points in the classic '$\vee$' fashion produces the graph of $y = g(x)$.



2. For $h(x) = |x| - 3 = f(x) - 3$, Theorem 1.7 tells us to *subtract* 3 from each of the y-values of the points on the graph of $y = f(x)$ to obtain the graph of $y = h(x)$. This shifts the graph of $y = f(x)$ *down* 3 units and moves $(-1, 1)$ to $(-1, -2)$, $(0, 0)$ to $(0, -3)$ and $(1, 1)$ to $(1, -2)$. Connecting these points with the '$\vee$' shape produces our graph of $y = h(x)$.



3. We re-write $i(x) = 4 - 2|3x+1| = 4 - 2f(3x+1) = -2f(3x+1) + 4$ and apply Theorem 1.7. First, we take care of the changes on the 'inside' of the absolute value. Instead of $|x|$, we have $|3x+1|$, so, in accordance with Theorem 1.7, we first *subtract* 1 from each of the x-values of points on the graph of $y = f(x)$, then *divide* each of those new values by 3. This effects a horizontal shift *left* 1 unit followed by a horizontal *shrink* by a factor of 3. These transformations move $(-1, 1)$ to $\left(-\frac{2}{3}, 1\right)$, $(0, 0)$ to $\left(-\frac{1}{3}, 0\right)$ and $(1, 1)$ to $(0, 1)$. Next, we take care of what's happening 'outside of' the absolute value. Theorem 1.7 instructs us to first *multiply* each y-value of these new points by -2 then *add* 4. Geometrically, this corresponds to a vertical *stretch* by a factor of 2, a reflection across the x-axis and finally, a vertical shift *up* 4 units. These transformations move $\left(-\frac{2}{3}, 1\right)$ to $\left(-\frac{2}{3}, 2\right)$, $\left(-\frac{1}{3}, 0\right)$ to $\left(-\frac{1}{3}, 4\right)$, and $(0, 1)$ to $(0, 2)$. Connecting these points with the usual '$\vee$' shape produces our graph of $y = i(x)$.



□

While the methods in Section 1.7 can be used to graph an entire family of absolute value functions, not all functions involving absolute values posses the characteristic '$\vee$' shape. As the next example illustrates, often there is no substitute for appealing directly to the definition.

Example 4.9.4:

Graph each of the following functions. Find the zeros of each function and the x- and y-intercepts of each graph, if any exist. From the graph, determine the domain and range of each function, list the intervals on which the function is increasing, decreasing or constant, and find the relative and absolute extrema, if they exist.

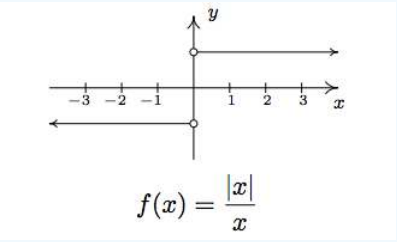
1. $f(x) = \dfrac{|x|}{x}$
2. $g(x) = |x+2| - |x-3| + 1$

Solution

1. We first note that, due to the fraction in the formula of $f(x)$, $x \neq 0$. Thus the domain is $(-\infty, 0) \cup (0, \infty)$. To find the zeros of f, we set $f(x) = \frac{|x|}{x} = 0$. This last equation implies $|x| = 0$, which, from Theorem 4.9.1, implies $x = 0$. However, $x = 0$ is not in the domain of f, which means we have, in fact, no x-intercepts. We have no y-intercepts either, since $f(0)$ is undefined. Re-writing the absolute value in the function gives

$$f(x)=\begin{cases}\dfrac{-x}{x}, & \text{if} \quad x<0 \\ [10pt]\dfrac{x}{x}, & \text{if} \quad x>0\end{cases}=\begin{cases}-1, & \text{if} \quad x<0 \\ 1, & \text{if} \quad x>0\end{cases} \tag{4.9.10}$$

To graph this function, we graph two horizontal lines: $y=-1$ for $x<0$ and $y=1$ for $x>0$. We have open circles at $(0,-1)$ and $(0,1)$ (Can you explain why?) so we get



As we found earlier, the domain is $(-\infty,0)\cup(0,\infty)$. The range consists of just two y-values: $\{-1,1\}$. The function f is constant on $(-\infty,0)$ and $(0,\infty)$. The local minimum value of f is the absolute minimum value of f, namely -1; the local maximum and absolute maximum values for f also coincide – they both are 1. Every point on the graph of f is simultaneously a relative maximum and a relative minimum. (Can you remember why in light of Definition 1.11? This was explored in the Exercises in Section 1.6 .)

2. To find the zeros of g, we set $g(x)=0$. The result is $|x+2|-|x-3|+1=0$. Attempting to isolate the absolute value term is complicated by the fact that there are \textbf{two} terms with absolute values. In this case, it easier to proceed using cases by re-writing the function g with two separate applications of Definition 2.4 to remove each instance of the absolute values, one at a time. In the first round we get

$$g(x)=\begin{cases}-(x+2)-|x-3|+1, & \text{if} \quad (x+2)<0 \\ (x+2)-|x-3|+1, & \text{if} \quad (x+2)\geq 0\end{cases}=\begin{cases}-x-1-|x-3|, & \text{if} \quad x<-2 \\ x+3-|x-3|, & \text{if} \quad x\geq -2\end{cases} \tag{4.9.11}$$

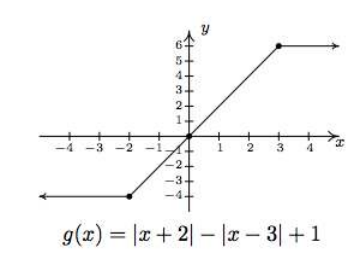
Given that

$$|x-3|=\begin{cases}-(x-3), & \text{if} \quad (x-3)<0 \\ x-3, & \text{if} \quad (x-3)\geq 0\end{cases}=\begin{cases}-x+3, & \text{if} \quad x<3 \\ x-3, & \text{if} \quad x\geq 3\end{cases}, \tag{4.9.12}$$

we need to break up the domain again at $x=3$. Note that if $x<-2$, then $x<3$, so we replace $|x-3|$ with $-x+3$ for that part of the domain, too. Our completed revision of the form of g yields

$$g(x)=\begin{cases}-x-1-(-x+3), & \text{if} \quad x<-2 \\ x+3-(-x+3), & \text{if} \quad x\geq -2 \text{ and } x<3 \\ x+3-(x-3), & \text{if} \quad x\geq 3\end{cases}=\begin{cases}-4, & \text{if} \quad x<-2 \\ 2x, & \text{if} \quad -2\leq x<3 \\ 6, & \text{if} \quad x\geq 3\end{cases} \tag{4.9.13}$$

To solve $g(x)=0$, we see that the only piece which contains a variable is $g(x)=2x$ for $-2\leq x<3$. Solving $2x=0$ gives $x=0$. Since $x=0$ is in the interval $[-2,3)$, we keep this solution and have $(0,0)$ as our only x-intercept. Accordingly, the y-intercept is also $(0,0)$. To graph g, we start with $x<-2$ and graph the horizontal line $y=-4$ with an open circle at $(-2,-4)$. For $-2\leq x<3$, we graph the line $y=2x$ and the point $(-2,-4)$ patches the hole left by the previous piece. An open circle at $(3,6)$ completes the graph of this part. Finally, we graph the horizontal line $y=6$ for $x\geq 3$, and the point $(3,6)$ fills in the open circle left by the previous part of the graph. The finished graph is



The domain of g is all real numbers, $(-\infty, \infty)$, and the range of g is all real numbers between -4 and 6 inclusive, $[-4, 6]$. The function is increasing on $[-2, 3]$ and constant on $(-\infty, -2]$ and $[3, \infty)$. The relative minimum value of f is -4 which matches the absolute minimum. The relative and absolute maximum values also coincide at 6. Every point on the graph of $y = g(x)$ for $x < -2$ and $x > 3$ yields both a relative minimum and relative maximum. The point $(-2, -4)$, however, gives only a relative minimum and the point $(3, 6)$ yields only a relative maximum. (Recall the Exercises in Section 1.6 which dealt with constant functions.)

Many of the applications that the authors are aware of involving absolute values also involve absolute value inequalities. For that reason, we save our discussion of applications for Section 2.4.

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4.10: Exponential Functions

Let's suppose that the current population of the city of Pleasantville is 10000 and that the population is growing at a rate of 2% per year. In order to analyze the population growth over a period of years, we'll try to develop a formula for the population as a function of time, and then graph the result.

First, note that at the end of one year, the population increase is 2% of 10000, or 200 people. We would now have 10,200 people in Pleasantville. At the end of the second year, take another 2% of 10200, which is an increase of 204 people, for a total of 10404. Because the increase each year is not constant, the graph of population versus time cannot be a line. Hence, our eventual population function will not be linear.

To develop our population formula, we start by letting the function P(t) represent the population of Pleasantville at time t, where we measure t in years. We will start time at t = 0 when the initial population of Pleasantville is 10000. In other words, P(0) = 10000. The key to understanding this example is the fact that the population increases by 2% each year. We are making an assumption here that this overall growth accounts for births, deaths, and people coming into and leaving Pleasantville. That is, at the end of the first year, the population of Pleasantville will be 102% of the initial population. Thus,

$$P(1) = 1.02P(0) = 1.02(10000). \tag{4.10.1}$$

We could multiply out the right side of this equation, but it will actually be more useful to leave it in its current form.

Now each year the population increases by 2%. Therefore, at the end of the second year, the population will be 102% of the population at the end of the first year. In other words,

$$P(2) = 1.02P(1). \tag{4.10.2}$$

If we replace $P(1)$ in Equation 4.10.2 with the result found in Equation 4.10.1, then

$$P(2) = (1.02)(1.02)(10000) = (1.02)^2(10000). \tag{4.10.3}$$

Let's iterate one more year. At the end of the third year, the population will be 102% of the population at the end of the second year, so

$$P(3) = 1.02P(2). \tag{4.10.4}$$

However, if we replace $P(2)$ in Equation 4.10.4 with the result found in Equation 4.10.3, we obtain

$$P(3) = (1.02)(1.02)^2(10000) = (1.02)^3(10000). \tag{4.10.5}$$

The pattern should now be clear. The population at the end of t years is given by the function.

$$P(t) = (1.02)^t(10000)$$

It is traditional in mathematics and science to place the initial population in front in this formula, writing instead

$$P(t) = 10000(1.02)^t. \tag{4.10.6}$$

Our function $P(t)$ is defined by Equation 4.10.6 for all positive integers {1, 2, 3, . . .}, and $P(0) = 10000$, the initial population. **Figure 1** shows a plot of our function. Although points are plotted only at integer values of t from 0 to 40, that's enough to show the trend of the population over time. The population starts at 10000, increases over time, and the yearly increase (the difference in population from one year to the next) also gets larger as time passes.

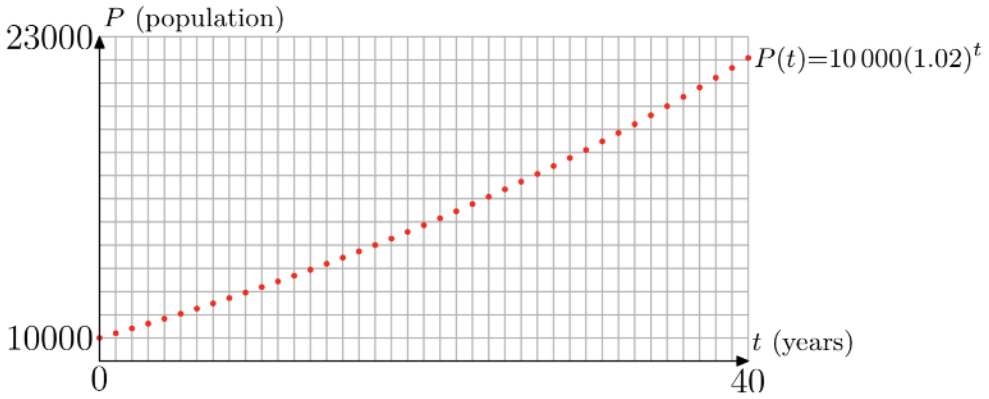


Figure 1. Graph of population P(t) of Pleasantville for t = 0, 1, 2, 3,...

? Exercise 4.10.7

We can now use the function P(t) to predict the population in later years. Assuming that the growth rate of 2% continues, what will the population of Pleasantville be after 40 years? What will it be after 100 years?

Solution

Substitute $t = 40$ and $t = 100$ into Equation 4.10.6. The population in 40 years will be

$$P(40) = 10000(1.02)^{40} \approx 22080$$

and the population in 100 years will be

$$P(100) = 10000(1.02)^{100} \approx 72446.$$

What would be different if we had started with a population of 12000? By tracing over our previous steps, it should be easy to see that the new formula would be

$$P(t) = 12000(1.02)^t.$$

Similarly, if the growth rate had been 3% per year instead of 2%, then we would have ended up with the formula

$$P(t) = 10000(1.03)^t.$$

Thus, by letting P_0 represent the initial population, and r represent the growth rate (in decimal form), we can generalize the formula to

$$P(t) = P_0(1+r)^t. \tag{4.10.7}$$

Note that our formula for the function $P(t)$ is different from the previous functions that we've studied so far, in that the input variable t is part of the exponent in the formula. Thus, this is a new type of function.

Now let's contrast the situation in Pleasantville with the population dynamics of Ghosttown. Ghosttown also starts with a population of 10000, but several factories have closed, so some people are leaving for better opportunities. In this case, the population of Ghosttown is decreasing at a rate of 2% per year. We'll again develop a formula for the population as a function of time, and then graph the result.

First, note that at the end of one year, the population decrease is 2% of 10000, or 200 people. We would now have 9800 people left in Ghosttown. At the end of the second year, take another 2% of 9800, which is a decrease of 196 people, for a total of 9604. As before, because the decrease each year is not constant, the graph of population versus time cannot be a line, so our eventual population function will not be linear.

Now let the function $P(t)$ represent the population of Ghosttown at time t, where we measure t in years. The initial population of Ghosttown at $t = 0$ is 10000, so $P(0) = 10,000$. Since the population decreases by 2% each year, at the end of the first year the population of Ghosttown will be 98% of the initial population. Thus,

$$P(1) = 0.98P(0) = 0.98(10000). \tag{4.10.8}$$

Each year the population decreases by 2%. Therefore, at the end of the second year, the population will be 98% of the population at the end of the first year. In other words,

$$P(2) = 0.98P(1). \tag{4.10.9}$$

If we replace $P(1)$ in Equation 4.10.9 with the result found in Equation 4.10.8, then

$$P(2) = (0.98)(0.98)(10000) = (0.98)^2(10000). \tag{4.10.10}$$

Let's iterate one more year. At the end of the third year, the population will be 98% of the population at the end of the second year, so

$$P(3) = 0.98P(2). \tag{4.10.11}$$

However, if we replace $P(2)$ in Equation 4.10.11 with the result found in Equation \label{11}, we obtain

$$P(3) = (0.98)(0.98)^2(10000) = (0.98)^3(10000). \tag{4.10.12}$$

The pattern should now be clear. The population at the end of t years is given by the function

$$P(t) = (0.98)^t(10000),$$

Or equivalently,

$$P(t) = 10000(0.98)^t \). \tag{4.10.13}$$

Our function $P(t)$ is defined by Equation 4.10.13 for all positive integers {1, 2, 3, . . .}, and P(0) = 10000, the initial population. **Figure 2** shows a plot of our function. Although points are plotted only at integer values of t from 0 to 40, that's enough to show the trend of the population over time. The population starts at 10 000, decreases over time, and the yearly decrease (the difference in population from one year to the next) also gets smaller as time passes.

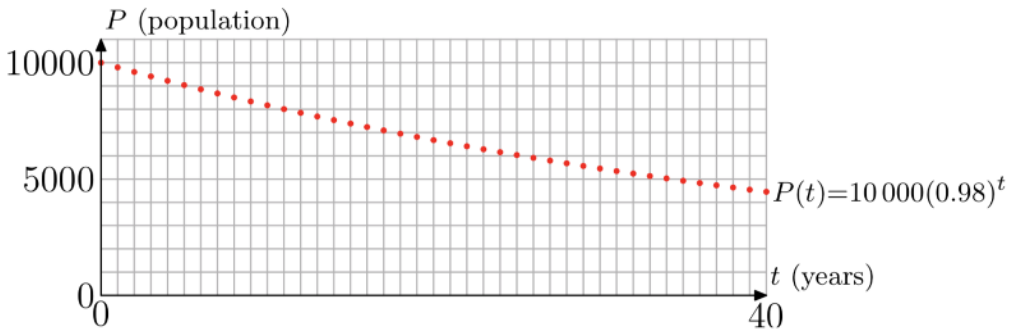


Figure 2. Graph of population P(t) of Ghosttown for t = 0, 1, 2, 3,...

? Exercise 4.10.15

Assuming that the rate of decrease continues at 2%, predict the population of Ghosttown after 40 years and after 100 years.

Solution

Substitute $t = 40$ and $t = 100$ into Equation 4.10.13. The population in 40 years will be

$$P(40) = 10000(0.98)^{40} \approx 4457,$$

and the population in 100 years will be

$$P(100) = 10000(0.98)^{100} \approx 1326$$

Note that if we had instead started with a population of 9000, for example, then the new formula would be

$$P(t) = 9000(0.98)^t.$$

Similarly, if the rate of decrease had been 5% per year instead of 2%, then we would have ended up with the formula

$$P(t) = 10000(0.95)^t.$$

Thus, by letting P_0 represent the initial population, and r represent the growth rate (in decimal form), we can generalize the formula to

$$P(t) = P_0(1-r)^t. \tag{4.10.14}$$

Definition

As noted before, our functions $P(t)$ in our Pleasantville and Ghosttown examples are a new type of function, because the input variable t is part of the exponent in the formula.

Definition 4.10.17: Exponential Function

An **exponential function** is a function of the form

$$f(t) = b^t$$

where b > 0 and $b \neq 1$. b is called the **base** of the exponential function. More generally, a function of the form

$$f(t) = Ab^t,$$

where b > 0, $b \neq 1$, and $A \neq 0$, is also referred to as an exponential function. In this case, the value of the function when t = 0 is f(0) = A, so A is the **initial amount**.

In applications, you will almost always encounter exponential functions in the more general form Ab^t. In fact, note that in the previous population examples, the function P(t) has this form $P(t) = Ab^t$, with $A = P_0$, $b = 1 + r$ in Pleasantville, and $b = 1 - r$ in Ghosttown. In particular, $A = P_0$ is the initial population.

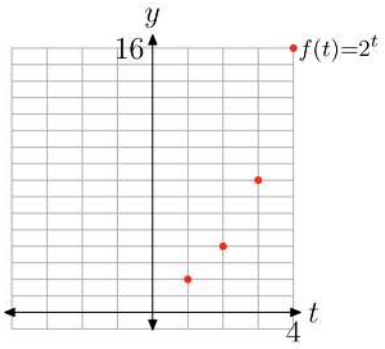
Since exponential functions are often used to model processes that vary with time, we usually use the input variable t (although of course any variable can be used). Also, you may be curious why the definition says $b \neq 1$, since 1^t just equals 1. We'll explain this curiosity at the end of this section.

Graphs of Exponential Functions

We'll develop the properties for the basic exponential function b^t first, and then note the minor changes for the more general form Ab^t. For a working example, let's use base b = 2, and let's compute some values of $f(t) = 2^t$ and plot the result (**Figure 3**).

t	$f(t) = 2^t$
1	2
2	4
3	8
4	16

(a)



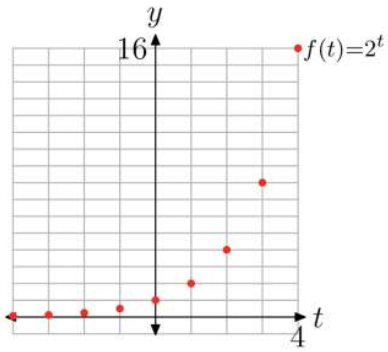
(b)

Figure 3. Plotting points (t, f(t)) defined by the function $f(t) = 2^t$, with t = 1, 2, 3, 4,....

Recall from the previous section that 2t is also defined for negative exponents t and the 0 exponent. Thus, the exponential function $f(t) = 2^t$ is defined for all integers. **Figure 4** shows a new table and plot with points added at 0 and negative integer values.

t	$f(t) = 2^t$
-4	$1/16$
-3	$1/8$
-2	$1/4$
-1	$1/2$
0	0
1	2
2	4
3	8
4	16

(a)



(b)

Figure 4. plotting points (t, f(t)) defined by the function $f(t) = 2^t$, with t = ..., −3, −2, −1, 0, 1, 2, 3....

However, the previous section showed that 2^t is also defined for rational and irrational exponents. Therefore, the domain of the exponential function $f(t) = 2^t$ is the set of all real numbers. When we add in the values of the function at all rational and irrational values of t, we obtain a final continuous curve as shown in **Figure 5**.

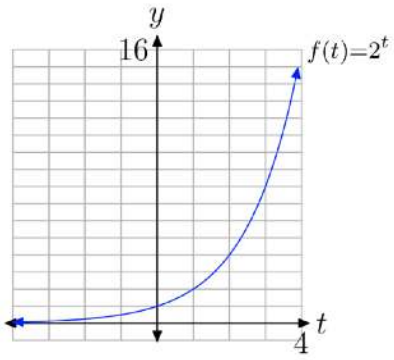


Figure 5.

Note several properties of the graph in **Figure 5**:

1. Moving from left to right, the curve rises, which means that the function increases as t increases. In fact, the function increases rapidly for positive t.
2. The graph lies above the t-axis, so the values of the function are always positive. Therefore, the range of the function is $(0, \infty)$.
3. The graph has a horizontal asymptote y = 0 (the t-axis) on the left side. This means that the function almost "dies out" (the values get closer and closer to 0) as t approaches $-\infty$.

What about the graphs of other exponential functions with different bases? We'll use the calculator to explore several of these.

First, use your calculator to compare $y_1(x) = 2^x$ and $y_2(x) = 3^x$. As can be seen in **Figure 6**(a), the graph of 3^x rises faster that 2^x for x > 0, and dies out faster for x < 0.

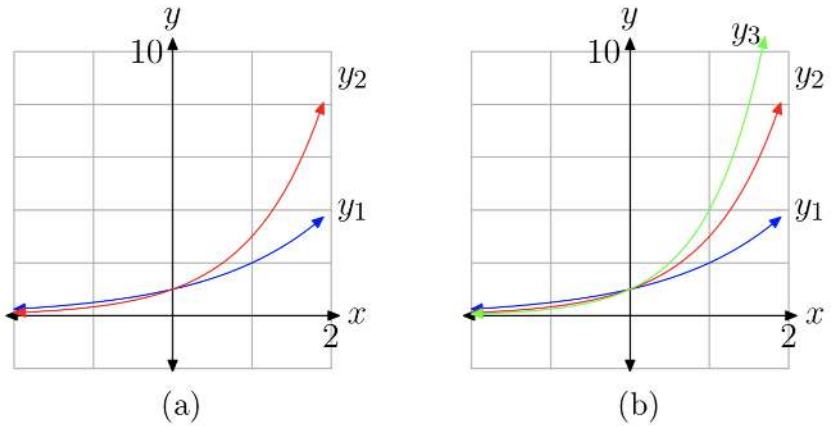
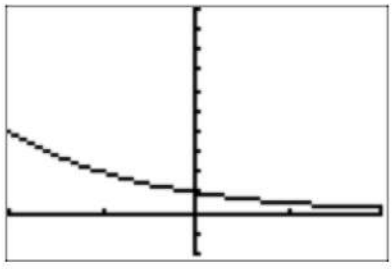


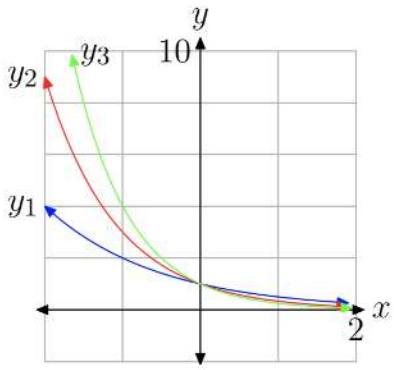
Figure 6. Comparing functions $y_1(x) = 2^x$, $y_2(x) = 3^x$, and $y_3(x) = 4^x$

Next, add in $y_3(x) = 4^x$. The result is shown in **Figure 6**(b). Again, increasing the size of the base to b = 4 results in a function which rises even faster on the right and likewise dies out faster on the left. If you continue to increase the size of the base b, you'll see that this trend continues. That's not terribly surprising because, if we compute the value of these functions at a fixed positive x, for example at x = 2, then the values increase: $2^2 < 3^2 < 4^2 < \ldots$. Similarly, at x = −2, the values decrease: $2^{-2} > 3^{-2} > 4^{-2} > \ldots$.

All of the functions in our experiments so far share the properties listed in (a)–(c) above: the function increases, the range is $(0, \infty)$, and the graph has a horizontal asymptote y = 0 on the left side. Now let's try smaller values of the base b. First use the calculator to plot the graph of $y_1(x) = (\frac{1}{2})^x$ (see **Figure 7**(a)).



(a) Graph of $y_1(x) = (1/2)^x$



(b) Comparing functions $y_1(x) = (1/2)^x$, $y_2(x) = (1/3)^x$, and $y_3(x) = (1/4)^x$

Figure 7.

This graph is much different. It rises rapidly to the left, and almost dies out on the right. Compare this with $y_2(x) = (\frac{1}{3})^x$ and $y_3(x) = (\frac{1}{4})^x$ (see **Figure 7**(b)). As the base gets smaller, the graph rises faster on the left, and dies out faster on the right.

Using reflection properties, it's easy to understand the appearance of these last three graphs. Note that

$$(\tfrac{1}{2})^x = (2^{-1})^x = 2^{-x}$$

so it follows that the graph of $(\frac{1}{2})^x$ is just a reflection in the *y*-axis of the graph of 2^x (see **Figure 8**).

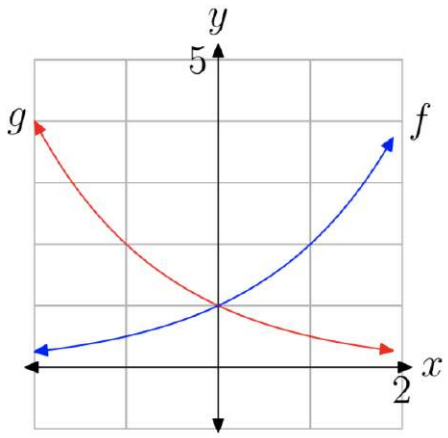


Figure 8. Comparing functions $f(x) = 2^x$ and $g(x) = (\frac{1}{2})^x = 2^{-x}$

Thus, we seem to have two different types of graphs, and therefore two types of exponential functions: one type is increasing, and the other decreasing. Our experiments above, along with a little more experimentation, should convince you that b^x is increasing for b > 1, and decreasing for 0 < b < 1. The first type of functions are called exponential growth functions, and the second type are exponential decay functions.

PROPERTIES OF EXPONENTIAL GROWTH FUNCTIONS: $F(X) = B^X$ WITH B > 1

- The domain is the set of all real numbers.
- Moving from left to right, the graph rises, which means that the function increases as x increases. The function increases rapidly for positive x.
- The graph lies above the x-axis, so the values of the function are always positive. Therefore, the range is $(0, \infty)$.
- The graph has a horizontal asymptote y = 0 (the x-axis) on the left side. This means that the function almost "dies out" (the values get closer and closer to 0) as x approaches $-\infty$.

The second property above deserves some additional explanation. Looking at **Figure 6**(b), it appears that y_2 and y_3 increase rapidly as x increases, but y_1 appears to increase slowly. However, this is due to the fact that the graph of $y_1(x) = 2^x$ is only shown on the interval $[-2, 2]$. In **Figure 5**, the same function is graphed on the interval $[-4, 4]$, and it certainly appears to increase rapidly in that graph. The point here is that exponential growth functions eventually increase rapidly as *x* increases. If you graph the function on a large enough interval, the function will eventually become very steep on the right side of the graph. This is an important property of the exponential growth functions, and will be explored further in the exercises.

PROPERTIES OF EXPONENTIAL DECAY FUNCTIONS: $F(X) = B^X$ WITH 0 < B < 1

- The domain is the set of all real numbers.
- Moving from left to right, the graph falls, which means that the function de- creases as x increases. The function decreases rapidly for negative x.
- The graph lies above the x-axis, so the values of the function are always positive. Therefore, the range is $(0, \infty)$.
- The graph has a horizontal asymptote y = 0 (the x-axis) on the right side. This means that the function almost "dies out" (the values get closer and closer to 0) as x approaches ∞.

Why do we refrain from using the base *b* = 1? After all, 1^x is certainly defined: it has the value 1 for all *x*. But that means that $f(x) = 1^x$ is just a constant linear function – its graph is a horizontal line. Therefore, this function doesn't share the same properties as the other exponential functions, and we've already classified it as a linear function. Thus, 1^x is not considered to be an exponential function.

Example 4.10.19

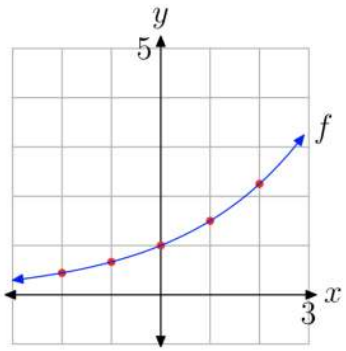
Plot the graph of the function $f(x) = (1.5)^x$. Identify the range of the function and the horizontal asymptote.

Solution

Since the base 1.5 is larger than 1, this is an exponential growth function. Therefore, its graph will have a shape similar to the graphs in **Figure 6**. The graph rises, there will be a horizontal asymptote $y = 0$ on the left side, and the range of the function is $(0, \infty)$. The graph can then be plotted by hand by using this knowledge along with approximate values at $x = -2, -1, 0, 1, 2$. See **Figure 9**.

x	$f(x) = (1.5)^x$
-2	0.44
-1	0.67
0	1
1	1.5
2	2.25

(a)



(b)

Figure 9. Graph of $f(x) = (1.5)^x$

Example 4.10.20

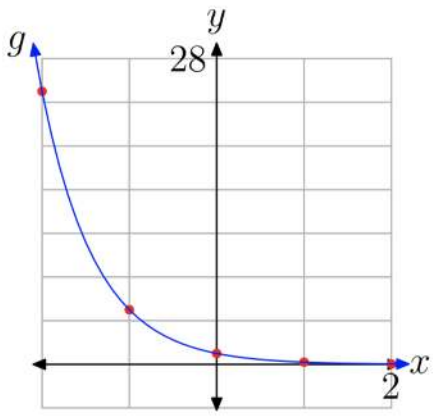
Plot the graph of the function $g(x) = (0.2)^x$. Identify the range of the function and the horizontal asymptote.

Solution

Since the base 0.2 is smaller than 1, this is an exponential decay function. Therefore, its graph will have a shape similar to the graphs in **Figure 7**. The graph falls, there will be a horizontal asymptote y = 0 on the right side, and the range of the function is \((0, \infty)\). The graph can then be plotted by hand by using this knowledge along with approximate values at x = −2, −1, 0, 1, 2. See **Figure 10**.

x	$g(x) = (0.2)^x$
-2	25
-1	5
0	1
1	0.2
2	0.04

(a)



(b)

Figure 10. Graph of $f(x) = (0.2)^x$

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Example 4.10.21

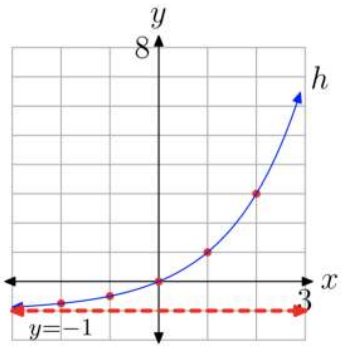
Plot the graph of the function $f(x) = 2^x - 1$. Identify the range of the function and the horizontal asymptote.

Solution

The graph of h can be obtained from the graph of $f(x) = 2^x$ (see **Figure 5**) by a vertical shift down 1 unit. Therefore, the horizontal asymptote y = 0 of the graph of f will also be shifted down 1 unit, so the graph of h has a horizontal asymptote $y = -1$. Similarly, the range of *f* will be shifted down to \((−1, \infty)\) = Range(h). The graph can then be plotted by hand by using this knowledge along with approximate values at x = −2, −1, 0, 1, 2. See **Figure 11**.

x	$h(x) = 2^x - 1$
-2	-0.75
-1	-0.5
0	0
1	1
2	3

(a)



(b)

Figure 11. Graph of $f(x) = 2^x - 1$

In later sections of this chapter, we will also see more general exponential functions of the form $f(x) = Ab^x$ (in fact, the Pleasantville and Ghosttown functions at the beginning of this section are of this form). If A is positive, then the graphs of these functions can be obtained from the basic exponential graphs by vertical scaling, so the graphs will have the same general shape as either the exponential growth curves (if b > 1) or the exponential decay curves (if $0 < b < 1$) we plotted earlier.

Exercise

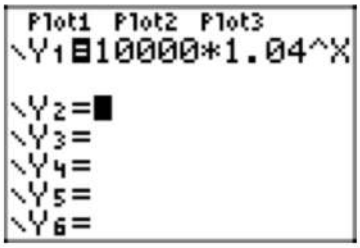
Exercise 4.10.1

The current population of Fortuna is 10,000 hearty souls. It is known that the population is growing at a rate of 4% per year. Assuming this rate remains constant, perform each of the following tasks.

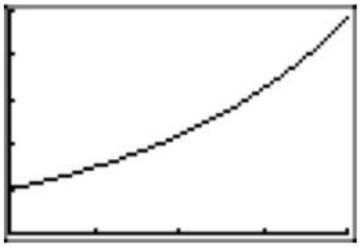
1. Set up an equation that models the population P(t) as a function of time t.
2. Use the model in the previous part to predict the population 40 years from now.
3. Use your calculator to sketch the graph of the population over the next 40 years.

Answer

1. $P(t) = 10000(1.04)^t$
2. $P(40) \approx 48101$
3.







Exercise 4.10.2

The population of the town of Imagination currently numbers 12,000 people. It is known that the population is growing at a rate of 6% per year. Assuming this rate remains constant, perform each of the following tasks.

1. Set up an equation that models the population P(t) as a function of time t.
2. Use the model in the previous part to predict the population 30 years from now.
3. Use your calculator to sketch the graph of the population over the next 30 years.

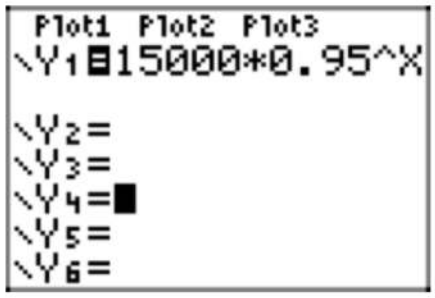
Exercise 4.10.3

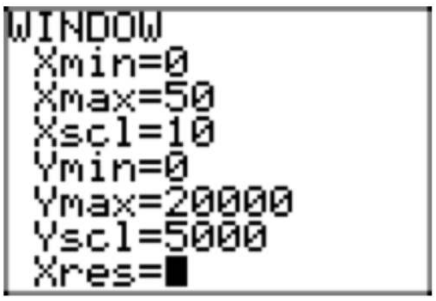
The population of the town of Despairia currently numbers 15,000 individuals. It is known that the population is decaying at a rate of 5% per year. Assuming this rate remains constant, perform each of the following tasks.

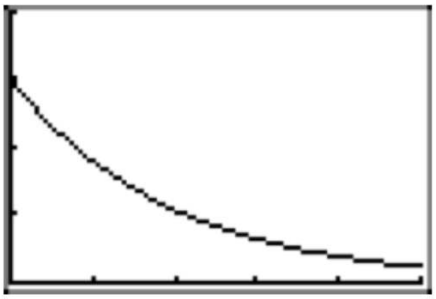
1. Set up an equation that models the population P(t) as a function of time t.
2. Use the model in the previous part to predict the population 50 years from now.
3. Use your calculator to sketch the graph of the population over the next 50 years.

Answer

1. $p(t) = 15000(0.95)^t$
2. $P(50) \approx 1154$
3.







? Exercise 4.10.4

The population of the town of Hopeless currently numbers 25,000 individuals. It is known that the population is decaying at a rate of 6% per year. Assuming this rate remains constant, perform each of the following tasks.

1. Set up an equation that models the population P(t) as a function of time t.
2. Use the model in the previous part to predict the population 40 years from now.
3. Use your calculator to sketch the graph of the population over the next 40 years.

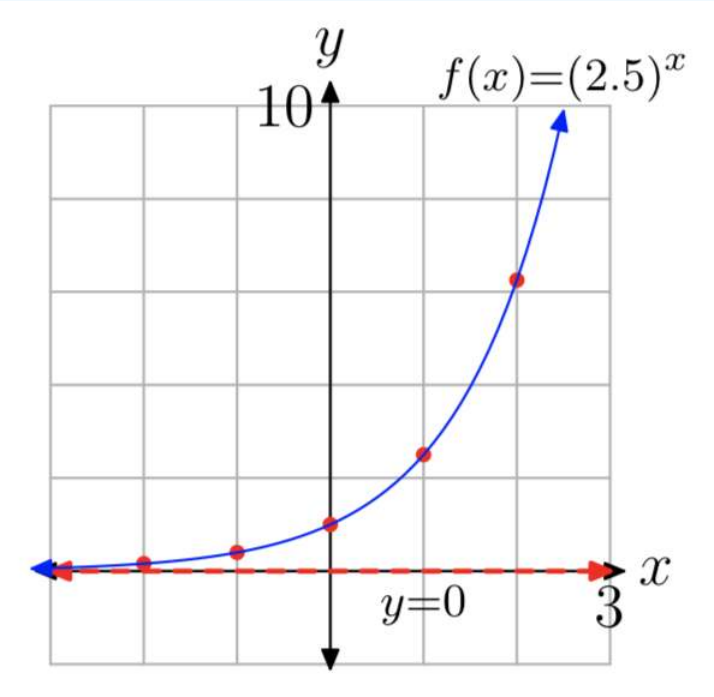
In **Exercises 5-12**, perform each of the following tasks for the given function.

1. Find the y-intercept of the graph of the function. Also, use your calculator to find two points on the graph to the right of the y-axis, and two points to the left.
2. Using your five points from (a) as a guide, set up a coordinate system on graph paper. Choose and label appropriate scales for each axis. Plot the five points, and any additional points you feel are necessary to discern the shape of the graph.
3. Draw the horizontal asymptote with a dashed line, and label it with its equation.
4. Sketch the graph of the function.
5. Use interval notation to describe both the domain and range of the function.

Exercise 4.10.5

$f(x) = (2.5)^x$

Answer

1. The y-intercept is (0, 1). Evaluating the function at x = 1, 2, −1, −2 to obtain the points (1, 2.5), (2, 6.25), (−1, 0.4), (−2, 0.16) (other answers are possible).
2. See the graph in part (4).
3. The horizontal asymptote is y = 0. See the graph in part (4).
4. 

5. Domain = $(-\infty, \infty)$, Range = $(0, \infty)$

Exercise 4.10.6

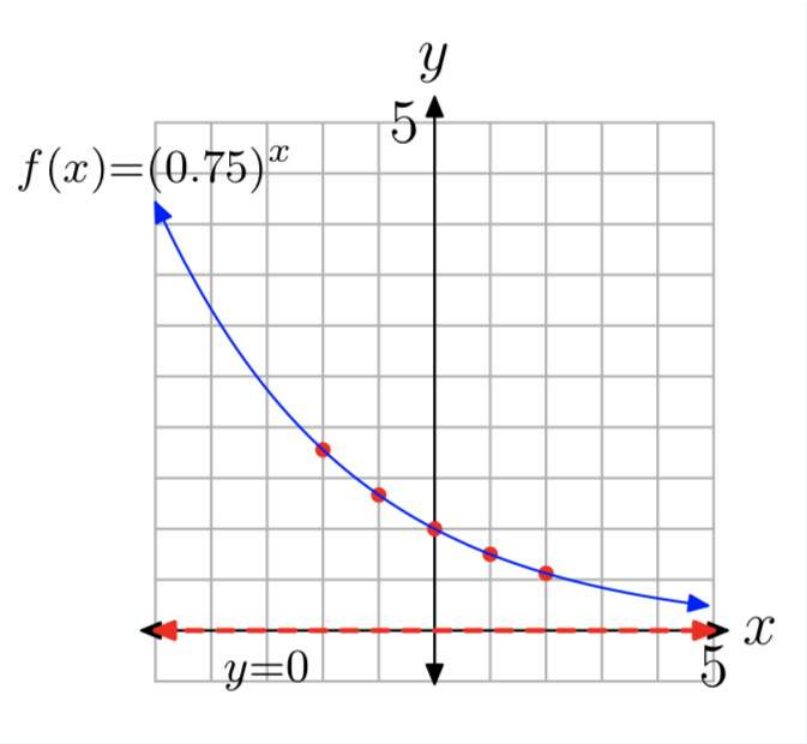
$f(x) = (0.1)^x$

Exercise 4.10.7

$f(x) = (0.75)^x$

Answer

1. The y-intercept is (0, 1). Evaluate the function at x = 1, 2, −1, −2 to obtain the points (1, 0.75), (2, 0.56), (−1, 1.34), (−2, 1.78) (other answers are possible).
2. See the graph in part (4).
3. The horizontal asymptote is y = 0. See the graph in part (4).

4. 

5. Domain = $(-\infty, \infty)$, Range = $(0, \infty)$

? Exercise 4.10.8

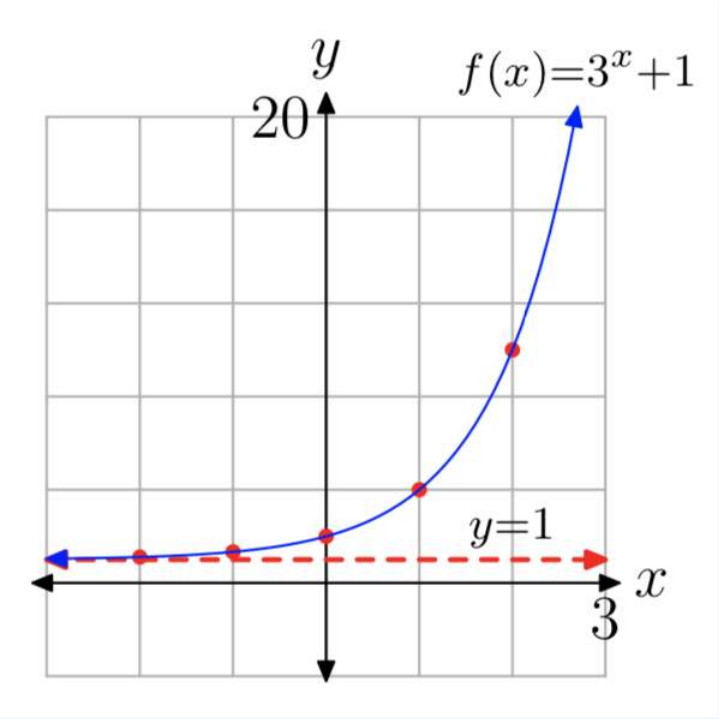
$f(x) = (1.1)^x$

? Exercise 4.10.9

$f(x) = 3^x + 1$

Answer

1. The y-intercept is (0, 2). Evaluate the function at x = 1, 2, −1, −2 to obtain the points (1, 4), (2, 10), (−1, 1.34), (−2, 1.11) (other answers are possible).
2. See the graph in part (4).
3. The horizontal asymptote is y = 1. See the graph in part (4).

4. 

5. Domain = $(-\infty, \infty)$, Range = $(1, \infty)$

? Exercise 4.10.10

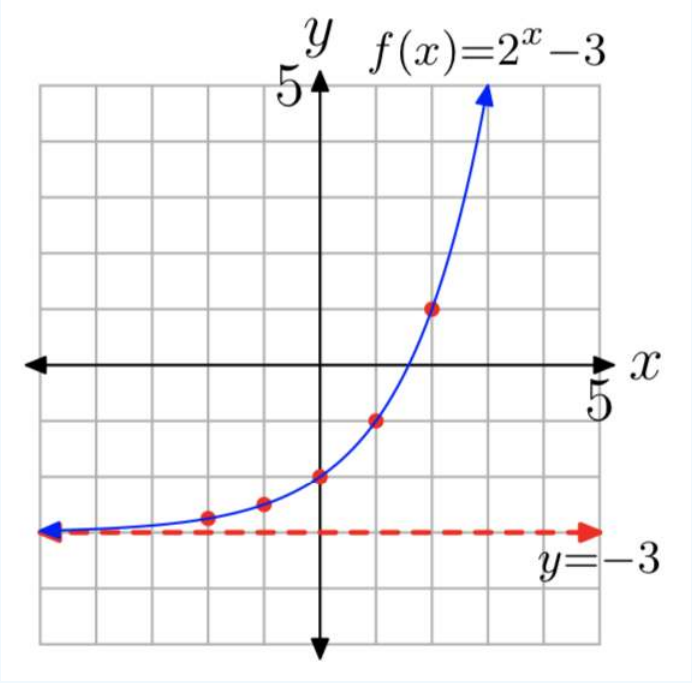
$f(x) = 4^x - 5$

? Exercise 4.10.11

$f(x) = 2^x - 3$

Answer

1. The y-intercept is (0, −2). Evaluate the function at x = 1, 2, −1, −2 to obtain the points (1, −1), (2, 1), (−1, −2.5), (−2, −2.75) (other answers are possible).
2. See the graph in part (4).
3. The horizontal asymptote is y = −3. See the graph in part (4).



4.

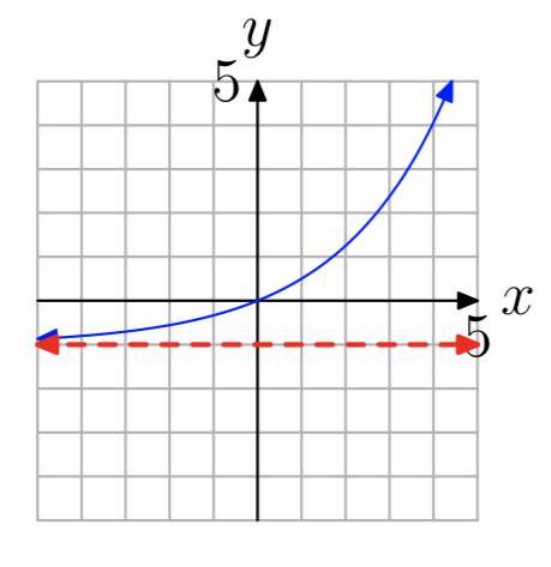
5. Domain = $(-\infty, \infty)$, Range = $(-3, \infty)$

? Exercise 4.10.12

$f(x) = 5^x + 2$

In **Exercises 13-20**, the graph of an exponential function of the form $f(x) = b^x + c$ is shown. The dashed red line is a horizontal asymptote. Determine the range of the function. Express your answer in interval notation.

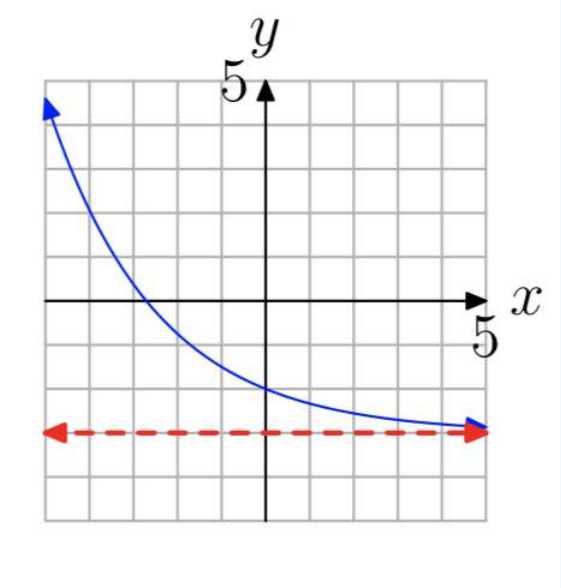
? Exercise 4.10.13



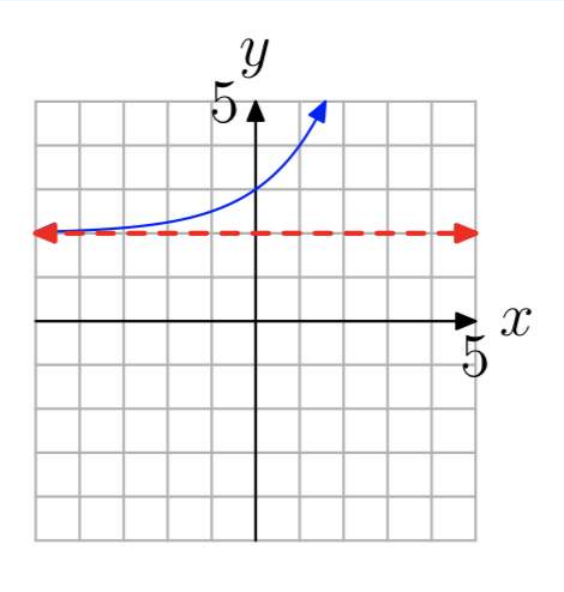
Answer

$(-1, \infty)$

Exercise 4.10.14



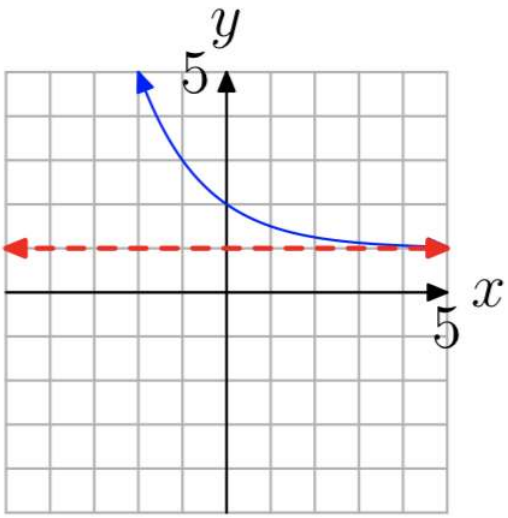
Exercise 4.10.15



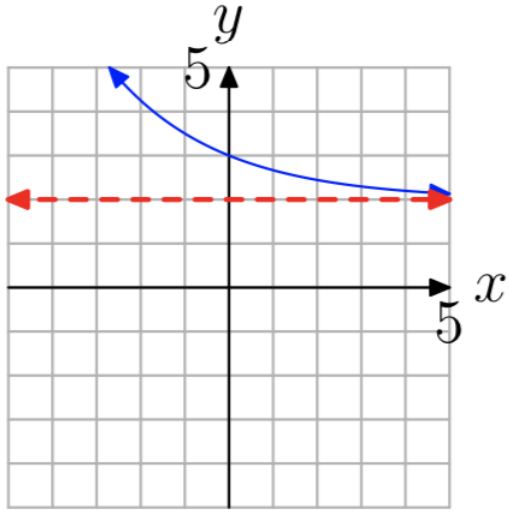
Answer

$(2, \infty)$

Exercise 4.10.16



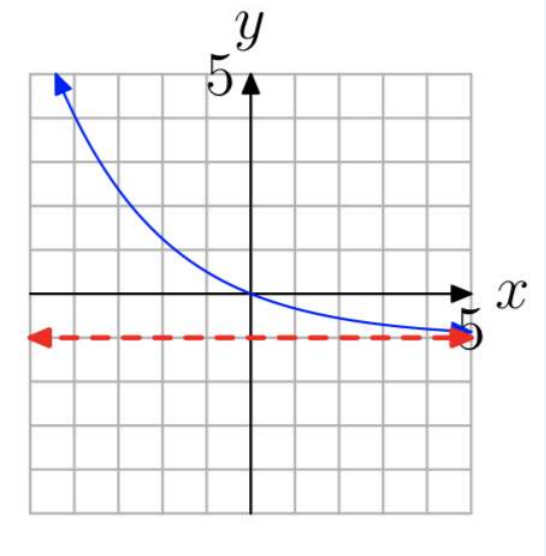
Exercise 4.10.17



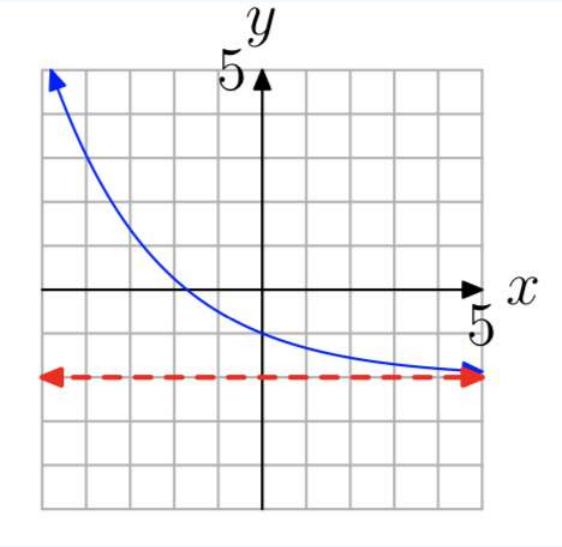
Answer

$(2, \infty)$

Exercise 4.10.18



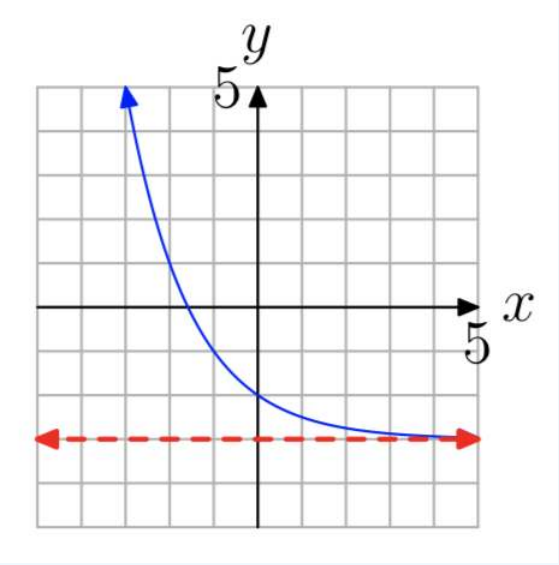
? Exercise 4.10.19



Answer

$(-2, \infty)$

? Exercise 4.10.20



In **Exercises 21-32**, compute f(p) at the given value p.

Exercise 4.10.21

$f(x) = (\frac{1}{3})^x$; p = −4

Answer

81

Exercise 4.10.22

$f(x) = (\frac{3}{4})^x$; p = 1

Exercise 4.10.23

$f(x) = 5^x$; p = 5

Answer

3125

Exercise 4.10.24

$f(x) = (\frac{1}{3})^x$; p = 4

Exercise 4.10.25

$f(x) = 4^x$; p = −4

Answer

$\frac{1}{256}$

Exercise 4.10.26

$f(x) = 5^x$; p = −3

Exercise 4.10.27

$f(x) = (\frac{5}{2})^x$; p = −3

Answer

$\frac{8}{125}$

Exercise 4.10.28

$f(x) = 9^x$; p = 3

Exercise 4.10.29

$f(x) = 5^x$; p = −4

Answer

$\frac{1}{625}$

Exercise 4.10.30

$f(x) = 9^x$; p = 0

Exercise 4.10.31

$f(x) = (\frac{6}{5})^x$; p = −4

Answer

$\frac{625}{1296}$

Exercise 4.10.32

$f(x) = (\frac{3}{5})^x$; p = 0

In **Exercises 33-40**, use your calculator to evaluate the function at the given value p. Round your answer to the nearest hundredth.

Exercise 4.10.33

$f(x) = 10^x$; p = −0.7.

Answer

0.20

Exercise 4.10.34

$f(x) = 10^x$; p = −1.6.

Exercise 4.10.35

$f(x) = (\frac{2}{5})^x$; p = 3.67.

Answer

0.03

Exercise 4.10.36

$f(x) = 2^x; p = -\frac{3}{4}$.

Exercise 4.10.37

$f(x) = 10^x$; p = 2.07.

Answer

117.49

Exercise 4.10.38

$f(x) = 7^x; p = \frac{4}{3}$.

Exercise 4.10.39

$f(x) = 10^x; p = -\frac{1}{5}$.

Answer

0.63

Exercise 4.10.40

$f(x) = (\frac{4}{3})^x$; p = 1.15

Exercise 4.10.41

This exercise explores the property that exponential growth functions eventually increase rapidly as x increases. Let $f(x) = 1.05^x$. Use your graphing calculator to graph f on the intervals

(a) [0, 10] and (b) [0, 100].

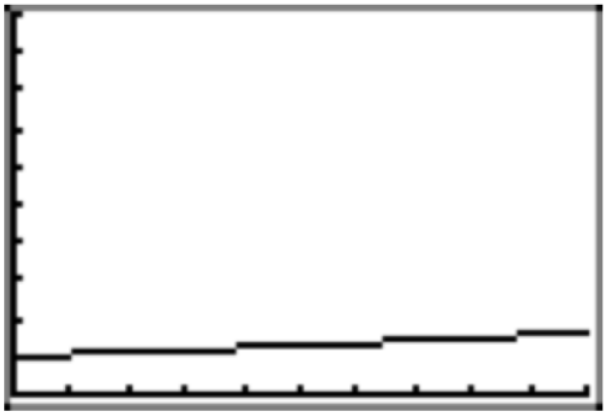
For (a), use Ymin = 0 and Ymax = 10.

For (b), use Ymin = 0 and Ymax = 100.

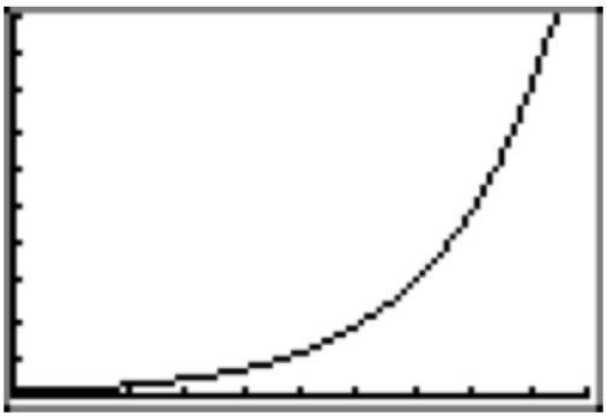
Make accurate copies of the images in your viewing window on your homework paper. What do you observe when you compare the two graphs?

Answer

(a) The graph on the interval [0, 10] increases very slowly. In fact, the graph looks almost linear.



(b) The graph on the interval [0, 100] increases slowly at first, but then increases very rapidly on the second half of the interval.

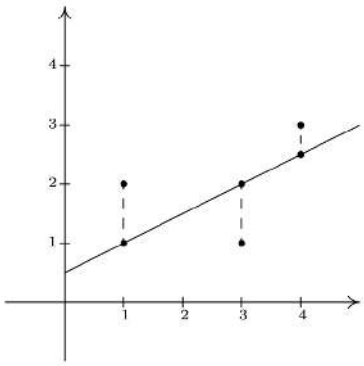


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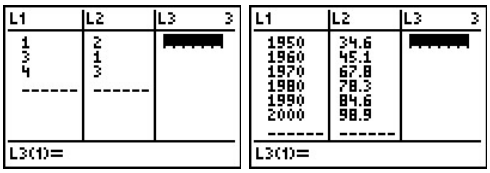
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4.11: Regression

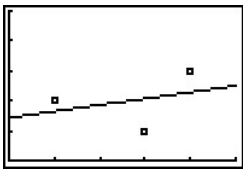
We have seen examples already in the text where linear and quadratic functions are used to model a wide variety of real world phenomena ranging from production costs to the height of a projectile above the ground. In this section, we use some basic tools from statistical analysis to quantify linear and quadratic trends that we may see in real world data in order to generate linear and quadratic models. Our goal is to give the reader an understanding of the basic processes involved, but we are quick to refer the reader to a more advanced course for a complete exposition of this material. Suppose we collected three data points: $\{(1,2),(3,1),(4,3)\}$ By plotting these points, we can clearly see that they do not lie along the same line. If we pick any two of the points, we can find a line containing both which completely misses the third, but our aim is to find a line which is in some sense 'close' to all the points, even though it may go through none of them. The way we measure 'closeness' in this case is to find the **total squared error** between the data points and the line. Consider our three data points and the line $y=\frac{1}{2}x+\frac{1}{2}$. For each of our data points, we find the vertical distance between the point and the line. To accomplish this, we need to find a point on the line directly above or below each data point - in other words, a point on the line with the same x-coordinate as our data point. For example, to find the point on the line directly below $(1,2)$, we plug $x=1$ into $y=\frac{1}{2}x+\frac{1}{2}$ and we get the point $(1,1)$. Similarly, we get $(3,1)$ to correspond to $(3,2)$ and $\left(4,\frac{5}{2}\right)$ for $(4,3)$.



We find the total squared error E by taking the sum of the squares of the differences of the y-coordinates of each data point and its corresponding point on the line. For the data and line above $E=(2-1)^2+(1-2)^2+\left(3-\frac{5}{2}\right)^2=\frac{9}{4}$. Using advanced mathematical machinery, (specifically Calculus and Linear Algebra) it is possible to find the line which results in the lowest value of E. This line is called the **least squares regression line**, or sometimes the 'line of best fit'. The formula for the line of best fit requires notation we won't present until Chapter 9, so we will revisit it then. The graphing calculator can come to our assistance here, since it has a built-in feature to compute the regression line. We enter the data and perform the Linear Regression feature and we get



The calculator tells us that the line of best fit is $y=ax+b$ where the slope is $a\approx 0.214$ and the y-coordinate of the y-intercept is $b\approx 1.428$. (We will stick to using three decimal places for our approximations.) Using this line, we compute the total squared error for our data to be $E\approx 1.786$. The value r is the **correlation coefficient** and is a measure of how close the data is to being on the same line. The closer $|r|$ is to 1, the better the linear fit. Since $r\approx 0.327$, this tells us that the line of best fit doesn't fit all that well - in other words, our data points aren't close to being linear. The value r^2 is called the **coefficient of determination** and is also a measure of the goodness of fit.\footnote{We refer the interested reader to a course in Statistics to explore the significance of r and r^2.} Plotting the data with its regression line results in the picture below.



Our first example looks at energy consumption in the US over the past 50 years.

Year	Energy Usage, in Quads
1950	34.6
1960	45.1
1970	67.8
1980	78.3
1990	84.6
2000	98.9

(4.11.1)

The unit 1 Quad is 1 Quadrillion = 10^{15} BTUs, which is enough heat to raise Lake Erie roughly 1°F}

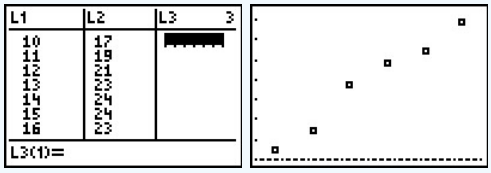
Example 4.11.1: Energy Consumption

Using the energy consumption data given above,

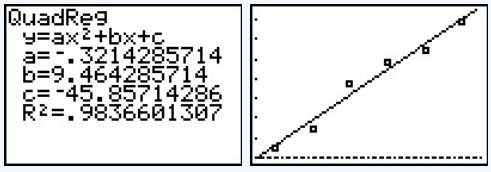
1. Plot the data using a graphing calculator.
2. Find the least squares regression line and comment on the goodness of fit.
3. Interpret the slope of the line of best fit.
4. Use the regression line to predict the annual US energy consumption in the year 2013.
5. Use the regression line to predict when the annual consumption will reach 120 Quads.

Solution

1. Entering the data into the calculator gives



2. Performing a linear regression produces



We can tell both from the correlation coefficient as well as the graph that the regression line is a good fit to the data.

3. The slope of the regression line is $a \approx 1.287$. To interpret this, recall that the slope is the rate of change of the y-coordinates with respect to the x-coordinates. Since the y-coordinates represent the energy usage in Quads, and the x-coordinates represent years, a slope of positive 1.287 indicates an increase in annual energy usage at the rate of 1.287 Quads per year.
4. To predict the energy needs in 2013, we substitute $x = 2013$ into the equation of the line of best fit to get

$$y = 1.287(2013) - 2473.890 \approx 116.841. \tag{4.11.2}$$

The predicted annual energy usage of the US in 2013 is approximately 116.841 Quads.

5. To predict when the annual US energy usage will reach 120 Quads, we substitute $y = 120$ into the equation of the line of best fit to get

$$120 = 1.287x - 2473.908. \tag{4.11.3}$$

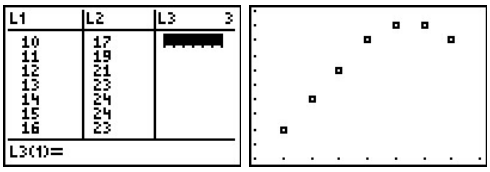
Solving for x yields $x \approx 2015.454$ Since the regression line is increasing, we interpret this result as saying the annual usage in 2015 won't yet be 120 Quads, but that in 2016, the demand will be more than 120 Quads.

Our next example gives us an opportunity to find a nonlinear model to fit the data. According to the National Weather Service, the predicted hourly temperatures for Painesville on March 3, 2009 were given as summarized below.

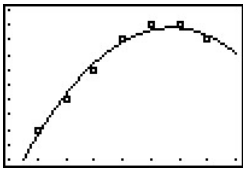
Time	Temperature, °F
10AM	17
11AM	19
12PM	21
1PM	23
2PM	24
3PM	24
4PM	23

(4.11.4)

To enter this data into the calculator, we need to adjust the x values, since just entering the numbers could cause confusion. (Do you see why?) We have a few options available to us. Perhaps the easiest is to convert the times into the 24 hour clock time so that 1 PM is 13, 2 PM is 14, etc.. If we enter these data into the graphing calculator and plot the points we get



While the beginning of the data looks linear, the temperature begins to fall in the afternoon hours. This sort of behavior reminds us of parabolas, and, sure enough, it is possible to find a parabola of best fit in the same way we found a line of best fit. The process is called *quadratic regression* and its goal is to minimize the least square error of the data with their corresponding points on the parabola. The calculator has a built in feature for this as well which yields



The coefficient of determination R^2 seems reasonably close to 1, and the graph visually seems to be a decent fit. We use this model in our next example.

Example 4.11.2: Quadratic Regression

Using the quadratic model for the temperature data above, predict the warmest temperature of the day. When will this occur?

Solution

The maximum temperature will occur at the vertex of the parabola. Recalling the Vertex Formula, Equation 2.4,

$$x = -\frac{b}{2a} \approx -\frac{9.464}{2(-0.321)} \approx 14.741. \tag{4.11.5}$$

This corresponds to roughly 2:45 PM. To find the temperature, we substitute $x = 14.741$ into

$$y = -0.321x^2 + 9.464x - 45.857 \tag{4.11.6}$$

to get $y \approx 23.899$, or 23.899° F.

The results of the last example should remind you that regression models are just that, models. Our predicted warmest temperature was found to be 23.899° F, but our data says it will warm to 24° F. It's all well and good to observe trends and guess at a model, but a more thorough investigation into *why* certain data should be linear or quadratic in nature is usually in order - and that, most often, is the business of scientists.

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CHAPTER OVERVIEW

5: Rules of Exponents and Scientific Notation

5.1: Simplify Monomials

5.1.1: Exercises

5.2: Integer Exponents and Scientific Notation

5.2.1: Exercises

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5.1: Simplify Monomials

Learning Objectives

By the end of this section, you will be able to:

- Simplify expressions using the Quotient Property for Exponents
- Simplify expressions with zero exponents
- Simplify expressions using the quotient to a Power Property
- Simplify expressions by applying several properties
- Divide monomials

Note

Before you get started, take this readiness quiz.

1. Simplify: $\frac{8}{24}$.
   If you missed this problem, review Example 1.6.4.
2. Simplify: $(2m^3)^5$.
   If you missed this problem, review Example 6.2.22.
3. Simplify: $\frac{12x}{12y}$
   If you missed this problem, review Example 1.6.10.

Simplify Expressions Using the Quotient Property for Exponents

Earlier in this chapter, we developed the properties of exponents for multiplication. We summarize these properties below.

SUMMARY OF EXPONENT PROPERTIES FOR MULTIPLICATION

If a and b are real numbers, and m and n are whole numbers, then

$$\begin{aligned} &\textbf{Product Property} && a^m \cdot a^n = a^{m+n} \\ &\textbf{Power Property} && (a^m)^n = a^{mn} \\ &\textbf{Product to a Power} && (ab)^m = a^m b^m \end{aligned} \tag{5.1.1}$$

Now we will look at the exponent properties for division. A quick memory refresher may help before we get started. You have learned to simplify fractions by dividing out common factors from the numerator and denominator using the Equivalent Fractions Property. This property will also help you work with algebraic fractions—which are also quotients.

EQUIVALENT FRACTIONS PROPERTY

If a, b, and c are whole numbers where $b \neq 0, c \neq 0$.

$$\text{then} \quad \frac{a}{b} = \frac{a \cdot c}{b \cdot c} \quad \text{and} \quad \frac{a \cdot c}{b \cdot c} = \frac{a}{b} \tag{5.1.2}$$

As before, we'll try to discover a property by looking at some examples.

Consider	$\frac{x^5}{x^2}$	and	$\frac{x^2}{x^3}$
What do they mean?	$\frac{x \cdot x \cdot x \cdot x \cdot x}{x \cdot x}$		$\frac{x \cdot x}{x \cdot x \cdot x}$
Use the Equivalent Fractions Property.	$\frac{\cancel{x} \cdot \cancel{x} \cdot x \cdot x \cdot x}{\cancel{x} \cdot \cancel{x}}$		$\frac{\cancel{x} \cdot \cancel{x} \cdot 1}{\cancel{x} \cdot \cancel{x} \cdot x}$
Simplify.	x^3		$\frac{1}{x}$

(5.1.3)

Notice, in each case the bases were the same and we subtracted exponents.

When the larger exponent was in the numerator, we were left with factors in the numerator.

When the larger exponent was in the denominator—notice the numerator of 1.

We write:

$\frac{x^5}{x^2}$	$\frac{x^2}{x^3}$
x^{5-2}	$\frac{1}{x^{3-2}}$
x^3	$\frac{1}{x}$

(5.1.4)

This leads to the *Quotient Property for Exponents*.

QUOTIENT PROPERTY FOR EXPONENTS

If a is a real number, $a \neq 0$, and m and n are whole numbers, then

$$\frac{a^m}{a^n} = a^{m-n}, m > n \text{ and } \frac{a^m}{a^n} = \frac{1}{a^{n-m}}, n > m \tag{5.1.5}$$

A couple of examples with numbers may help to verify this property.

$\frac{3^4}{3^2}$	$=$	3^{4-2}	$\frac{5^2}{5^3}$	$=$	$\frac{1}{5^{3-2}}$
$\frac{81}{9}$	$=$	3^2	$\frac{25}{125}$	$=$	$\frac{1}{5^1}$
9	$=$	$9\checkmark$	$\frac{1}{5}$	$=$	$\frac{1}{5}\checkmark$

(5.1.6)

Example 5.1.1

Simplify:

1. $\frac{x^9}{x^7}$
2. $\frac{3^{10}}{3^2}$

Solution

To simplify an expression with a quotient, we need to first compare the exponents in the numerator and denominator.

1.

Since 9 > 7, there are more factors of *x* in the numerator.	$\frac{x^9}{x^7}$
Use the Quotient Property, $\frac{a^m}{a^n} = a^{m-n}$	x^{9-7}

Simplify.	x^2

2.

Since 10 > 2, there are more factors of x in the numerator.	$\frac{3^{10}}{3^2}$
Use the Quotient Property, $\frac{a^m}{a^n} = a^{m-n}$	3^{10-2}
Simplify.	3^8

Notice that when the larger exponent is in the numerator, we are left with factors in the numerator.

Try It 5.1.2

Simplify:

a. $\frac{x^{15}}{x^{10}}$
b. $\frac{6^{14}}{6^5}$

Answer

1. x^5
2. 6^9

Try It 5.1.3

Simplify:

1. $\frac{y^{43}}{y^{37}}$
2. $\frac{10^{15}}{10^7}$

Answer

1. y^6
2. 10^8

Example 5.1.4

Simplify:

1. $\frac{b^8}{b^{12}}$
2. $\frac{7^3}{7^5}$

Solution

To simplify an expression with a quotient, we need to first compare the exponents in the numerator and denominator.

1.

Since 12 > 8, there are more factors of b in the denominator.	$\frac{b^8}{b^{12}}$

Use the Quotient Property, $\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$	$\frac{1}{b^{12-8}}$
Simplify.	$\frac{1}{b^4}$

2.

Since 5 > 3, there are more factors of 3 in the denominator.	$\frac{7^3}{7^5}$
Use the Quotient Property, $\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$	$\frac{1}{7^{5-3}}$
Simplify.	$\frac{1}{7^2}$
Simplify.	$\frac{1}{49}$

Notice that when the larger exponent is in the denominator, we are left with factors in the denominator.

Try It 5.1.5

Simplify:

1. $\frac{x^{18}}{x^{22}}$
2. $\frac{12^{15}}{12^{30}}$

Answer

1. $\frac{1}{x^4}$
2. $\frac{1}{12^{15}}$

Try It 5.1.6

Simplify:

1. $\frac{m^7}{m^{15}}$
2. $\frac{9^8}{9^{19}}$

Answer

1. $\frac{1}{m^8}$
2. $\frac{1}{9^{11}}$

Notice the difference in the two previous examples:

- If we start with more factors in the numerator, we will end up with factors in the numerator.
- If we start with more factors in the denominator, we will end up with factors in the denominator.

The first step in simplifying an expression using the Quotient Property for Exponents is to determine whether the exponent is larger in the numerator or the denominator.

Example 5.1.7

Simplify:

1. $\frac{a^5}{a^9}$
2. $\frac{x^{11}}{x^7}$

Solution

1. Is the exponent of a larger in the numerator or denominator? Since 9 > 5, there are more a's in the denominator and so we will end up with factors in the denominator.

	$\frac{a^5}{a^9}$
Use the Quotient Property, $\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$	$\frac{1}{a^{9-5}}$
Simplify.	$\frac{1}{a^4}$

2. Notice there are more factors of xx in the numerator, since 11 > 7. So we will end up with factors in the numerator.

	$\frac{x^{11}}{x^7}$
Use the Quotient Property, $\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$	x^{11-7}
Simplify.	x^4

Try It 5.1.8

Simplify:

1. $\frac{b^{19}}{b^{11}}$
2. $\frac{z^5}{z^{11}}$

Answer

1. b^8
2. $\frac{1}{z^6}$

Try It 5.1.9

Simplify:

1. $\frac{p^9}{p^{17}}$
2. $\frac{w^{13}}{w^9}$

Answer

1. $\frac{1}{p^8}$
2. w^4

Simplify Expressions with an Exponent of Zero

A special case of the Quotient Property is when the exponents of the numerator and denominator are equal, such as an expression like $\frac{a^m}{a^m}$. From your earlier work with fractions, you know that:

$$\frac{2}{2}=1 \quad \frac{17}{17}=1 \quad \frac{-43}{-43}=1 \tag{5.1.7}$$

In words, a number divided by itself is 1. So, $\frac{x}{x}=1$, for any $x(x\neq 0)$, since any number divided by itself is 1.

The Quotient Property for Exponents shows us how to simplify $\frac{a^m}{a^n}$ when $m>n$ and when $n<m$ by subtracting exponents. What if $m=n$?

Consider $\frac{8}{8}$, which we know is 1.

	$\frac{8}{8}$	$=$	1
Write 8 as 2^3.	$\frac{2^3}{2^3}$	$=$	1
Subtract exponents.	2^{3-3}	$=$	1
Simplify.	2^0	$=$	1

Now we will simplify $\frac{a^m}{a^m}$ in two ways to lead us to the definition of the zero exponent. In general, for $a\neq 0$:

$\frac{a^m}{a^m}$	$\frac{a^m}{a^m}$
a^{m-m}	$\frac{\overbrace{y\cdot y\cdot\cdots\cdot y}^{m \text{ factors}}}{\underbrace{y\cdot y\cdot\cdots\cdot y}_{m \text{ factors}}}$
a^0	1

We see $\frac{a^m}{a^m}$ simplifies to a^0 and to 1. So $a^0=1$.

ZERO EXPONENT

If a is a non-zero number, then $a^0=1$.

Any nonzero number raised to the zero power is 1.

In this text, we assume any variable that we raise to the zero power is not zero.

Example 5.1.10

Simplify:

a. 9^0
b. n^0

Solution

The definition says any non-zero number raised to the zero power is 1.

a.	Use the definition of the zero exponent.	9^0 1
b.	Use the definition of the zero exponent.	n^0 1

Try It 5.1.11

Simplify:

1. 15^0
2. m^0

Answer

1. 1
2. 1

Try It 5.1.12

Simplify:

a. k^0
b. 29^0

Answer

a. 1
b. 1

Now that we have defined the zero exponent, we can expand all the Properties of Exponents to include whole number exponents.

What about raising an expression to the zero power? Let's look at $(2x)^0$. We can use the product to a power rule to rewrite this expression.

	$(2x)^0$
Use the product to a power rule.	2^0x^0
Use the zero exponent property.	$1 \cdot 1$
Simplify.	1

(5.1.8)

This tells us that any nonzero expression raised to the zero power is one.

Example 5.1.13

Simplify:

a. $(5b)^0$
b. $(-4a^2b)^0$.

Solution

a.	Use the definition of the zero exponent.	$(5b)^0$ 1
b.	Use the definition of the zero exponent.	$(-4a^2b)^0$ 1

Try It 5.1.14

Simplify:

a. $(11z)^0$
b. $(-11pq^3)^0$.

Answer

a. 1
b. 1

Try It 5.1.15

Simplify:

a. $(-6d)^0$
b. $(-8m^2n^3)^0$.

Answer

a. 1
b. 1

Simplify Expressions Using the Quotient to a Power Property

Now we will look at an example that will lead us to the Quotient to a Power Property.

$$\begin{array}{ll} & \left(\frac{x}{y}\right)^3 \\ \text{This means:} & \frac{x}{y}\cdot\frac{x}{y}\cdot\frac{x}{y} \\ \text{Multiply the fractions.} & \frac{x\cdot x\cdot x}{y\cdot y\cdot y} \\ \text{Write with exponents.} & \frac{x^3}{y^3} \end{array} \tag{5.1.9}$$

Notice that the exponent applies to both the numerator and the denominator.

$$\begin{array}{ll} \text{We see that } \left(\frac{x}{y}\right)^3 \text{ is } \frac{x^3}{y^3} & \\ \text{We write:} & \left(\frac{x}{y}\right)^3 \\ & \frac{x^3}{y^3} \end{array} \tag{5.1.10}$$

This leads to the *Quotient to a Power Property for Exponents*.

QUOTIENT TO A POWER PROPERTY FOR EXPONENTS

If a and b are real numbers, $b\neq 0$, and m is a counting number, then

$$\left(\frac{a}{b}\right)^m=\frac{a^m}{b^m} \tag{5.1.11}$$

To raise a fraction to a power, raise the numerator and denominator to that power.

An example with numbers may help you understand this property:

$$\left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3}$$
$$\frac{2}{3}\cdot\frac{2}{3}\cdot\frac{2}{3} = \frac{8}{27}$$
$$\frac{8}{27} = \frac{8}{27}\checkmark$$

Example 5.1.16

Simplify:

a. $\left(\frac{3}{7}\right)^2$

b. $\left(\frac{b}{3}\right)^4$

c. $\left(\frac{k}{j}\right)^3$

Solution

1.

	$\left(\frac{3}{7}\right)^2$
Use the Quotient Property, $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	$\frac{3^2}{7^2}$
Simplify.	$\frac{9}{49}$

2.

	$\left(\frac{b}{3}\right)^4$
Use the Quotient Property, $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	$\frac{b^4}{3^4}$
Simplify.	$\frac{b^4}{81}$

3.

	$\left(\frac{k}{j}\right)^3$
Raise the numerator and denominator to the third power.	$\frac{k^3}{j^3}$

Try It 5.1.17

Simplify:

a. $\left(\frac{5}{8}\right)^2$

b. $\left(\frac{p}{10}\right)^4$

c. $\left(\frac{m}{n}\right)^7$

Answer

a. $\frac{25}{64}$
b. $\frac{p^4}{10,000}$
c. $\frac{m^7}{n^7}$

? Try It 5.1.18

Simplify:

a. $\left(\frac{1}{3}\right)^3$
b. $\left(\frac{-2}{q}\right)^3$
c. $\left(\frac{w}{x}\right)^4$

Answer

a. $\frac{1}{27}$
b. $\frac{-8}{q^3}$
c. $\frac{w^4}{x^4}$

Simplify Expressions by Applying Several Properties

We'll now summarize all the properties of exponents so they are all together to refer to as we simplify expressions using several properties. Notice that they are now defined for whole number exponents.

SUMMARY OF EXPONENT PROPERTIES

If a and b are real numbers, and m and n are whole numbers, then

$$\begin{array}{lrcll} \textbf{Product Property} & a^m \cdot a^n & = & a^{m+n} & \\ \textbf{Power Property} & (a^m)^n & = & a^{m \cdot n} & \\ \textbf{Product to a Power} & (ab)^m & = & a^m b^m & \\ \textbf{Quotient Property} & \frac{a^m}{a^n} & = & a^{m-n}, & a \neq 0, m > n \\ & \frac{a^n}{a^n} & = & 1, & a \neq 0, n > m \\ \textbf{Zero Exponent Definition} & a^0 & = & 1, & a \neq 0 \\ \textbf{Quotient to a Power Property} & \left(\frac{a}{b}\right)^m & = & \frac{a^m}{b^m}, & b \neq 0 \end{array} \tag{5.1.12}$$

✓ Example 5.1.19

Simplify: $\frac{\left(y^4\right)^2}{y^6}$

Solution

	$\frac{\left(y^4\right)^2}{y^6}$
Multiply the exponents in the numerator.	$\frac{y^8}{y^6}$
Subtract the exponents.	y^2

? Try It 5.1.20

Simplify: $\frac{\left(m^5\right)^4}{m^7}$

Answer

m^{13}

? Try It 5.1.21

Simplify: $\frac{\left(k^2\right)^6}{k^7}$

Answer

k^5

✓ Example 5.1.22

Simplify: $\frac{b^{12}}{\left(b^2\right)^6}$

Solution

	$\frac{b^{12}}{\left(b^2\right)^6}$
Multiply the exponents in the numerator.	$\frac{b^{12}}{b^{12}}$
Subtract the exponents.	b^0
Simplify	1

Notice that after we simplified the denominator in the first step, the numerator and the denominator were equal. So the final value is equal to 1.

? Try It 5.1.23

Simplify $\frac{n^{12}}{\left(n^3\right)^4}$.

Answer

1

? Try It 5.1.24

Simplify $\frac{x^{15}}{\left(x^3\right)^5}$.

Answer

1

Example 5.1.25

Simplify: $\left(\frac{y^9}{y^4}\right)^2$

Solution

	$\left(\frac{y^9}{y^4}\right)^2$
Remember parentheses come before exponents.	
Notice the bases are the same, so we can simplify inside the parentheses. Subtract the exponents.	$\left(y^5\right)^2$
Multiply the exponents.	y^{10}

Try It 5.1.26

Simplify: $\left(\frac{r^5}{r^3}\right)^4$

Answer

r^8

Try It 5.1.27

Simplify: $\left(\frac{v^6}{v^4}\right)^3$

Answer

v^6

Example 5.1.28

Simplify: $\left(\frac{j^2}{k^3}\right)^4$

Solution

Here we cannot simplify inside the parentheses first because the bases are not the same.	$\left(\frac{j^2}{k^3}\right)^4$
Raise the numerator and denominator to the 4th power using the Quotient to a Power Property, $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	$\frac{\left(j^2\right)^4}{\left(k^3\right)^4}$
Use the Power Property and simplify.	$\frac{j^8}{k^{12}}$

Try It 5.1.29

Simplify: $\left(\frac{a^3}{b^2}\right)^4$

Answer

$\frac{a^{12}}{b^8}$

Try It 5.1.30

Simplify: $\left(\frac{q^7}{r^5}\right)^3$

Answer

$\frac{q^{21}}{r^{15}}$

Example 5.1.31

Simplify: $\left(\frac{2m^2}{5n}\right)^4$

Solution

	$\left(\frac{2m^2}{5n}\right)^4$
Raise the numerator and denominator to the 4th power using the Quotient to a Power Property, $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	$\frac{(2m^2)^4}{(5n)^4}$
Use the Product to a Power Property, $(ab)^m = a^m b^m$	$\frac{2^4(m^2)^4}{5^4 n^4}$
Use the Power Property and simplify.	$\frac{16m^8}{625n^4}$

Try It 5.1.32

Simplify: $\left(\frac{7x^3}{9y}\right)^2$

Answer

$\frac{49x^6}{81y^2}$

Try It 5.1.33

Simplify: $\left(\frac{3x^4}{7y}\right)^2$

Answer

$\frac{9x^8}{49y^2}$

Example 5.1.34

Simplify: $\dfrac{\left(x^3\right)^4\left(x^2\right)^5}{\left(x^6\right)^5}$

Solution

	$\dfrac{\left(x^3\right)^4\left(x^2\right)^5}{\left(x^6\right)^5}$
Use the Power Property, $(a^m)^n = a^{m \cdot n}$	$\dfrac{\left(x^{12}\right)\left(x^{10}\right)}{\left(x^{30}\right)}$
Add the exponents in the numerator.	$\dfrac{x^{22}}{x^{30}}$
Use the Quotient Property, $\dfrac{a^m}{a^n} = \dfrac{1}{a^{n-m}}$	$\dfrac{1}{x^8}$

Try It 5.1.35

Simplify: $\dfrac{\left(a^2\right)^3\left(a^2\right)^4}{\left(a^4\right)^5}$

Answer

$\dfrac{1}{a^6}$

Try It 5.1.36

Simplify: $\dfrac{\left(p^3\right)^4\left(p^5\right)^3}{\left(p^7\right)^6}$

Answer

$\dfrac{1}{p^{15}}$

Example 5.1.37

Simplify: $\dfrac{\left(10p^3\right)^2}{(5p)^3(2p^5)^4}$

Solution

	$\frac{(10p^3)^2}{(5p)^3(2p^5)^4}$
Use the Product to a Power Property, $(ab)^m = a^m b^m$	$\frac{(10)^2(p^3)^2}{(5)^3(p)^3(2)^4(p^5)^4}$
Use the Power Property, $(a^m)^n = a^{m \cdot n}$	$\frac{100p^6}{125p^3 \cdot 16p^{20}}$
Add the exponents in the denominator.	$\frac{100p^6}{125 \cdot 16p^{23}}$
Use the Quotient Property, $\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$	$\frac{100}{125 \cdot 16p^{17}}$
Simplify.	$\frac{1}{20p^{17}}$

? Try It 5.1.38

Simplify: $\frac{(3r^3)^2(r^3)^7}{(r^3)^3}$

Answer

$9r^{18}$

? Try It 5.1.39

Simplify: $\frac{(2x^4)^5}{(4x^3)^2(x^3)^5}$

Answer

$\frac{2}{x}$

Divide Monomials

You have now been introduced to all the properties of exponents and used them to simplify expressions. Next, you'll see how to use these properties to divide monomials. Later, you'll use them to divide polynomials.

✓ Example 5.1.40

Find the quotient: $56x^7 \div 8x^3$

Solution

	$56x^7 \div 8x^3$
Rewrite as a fraction.	$\frac{56x^7}{8x^3}$
Use fraction multiplication.	$\frac{56}{8} \cdot \frac{x^7}{x^3}$
Simplify and use the Quotient Property.	$7x^4$

Try It 5.1.41

Find the quotient: $42y^9 \div 6y^3$

Answer

$7y^6$

Try It 5.1.42

Find the quotient: $48z^8 \div 8z^2$

Answer

$6z^6$

Example 5.1.43

Find the quotient: $\dfrac{45a^2b^3}{-5ab^5}$

Solution

When we divide monomials with more than one variable, we write one fraction for each variable.

	$\dfrac{45a^2b^3}{-5ab^5}$
Use fraction multiplication.	$\dfrac{45}{-5} \cdot \dfrac{a^2}{a} \cdot \dfrac{b^3}{b^5}$
Simplify and use the Quotient Property.	$-9 \cdot a \cdot \dfrac{1}{b^2}$
Multiply.	$-\dfrac{9a}{b^2}$

Try It 5.1.44

Find the quotient: $\dfrac{-72a^7b^3}{8a^{12}b^4}$

Answer

$-\dfrac{9}{a^5b}$

Try It 5.1.45

Find the quotient: $\dfrac{-63c^8d^3}{7c^{12}d^2}$

Answer

$\dfrac{-9d}{c^4}$

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Example 5.1.46

Find the quotient: $\frac{24a^5b^3}{48ab^4}$

Solution

	$\frac{24a^5b^3}{48ab^4}$
Use fraction multiplication.	$\frac{24}{48}\cdot\frac{a^5}{a}\cdot\frac{b^3}{b^4}$
Simplify and use the Quotient Property.	$\frac{1}{2}\cdot a^4\cdot\frac{1}{b}$
Multiply.	$\frac{a^4}{2b}$

Try It 5.1.47

Find the quotient: $\frac{16a^7b^6}{24ab^8}$

Answer

$\frac{2a^6}{3b^2}$

Try It 5.1.48

Find the quotient: $\frac{27p^4q^7}{-45p^{12}q}$

Answer

$-\frac{3q^6}{5p^8}$

Once you become familiar with the process and have practiced it step by step several times, you may be able to simplify a fraction in one step.

Example 5.1.49

Find the quotient: $\frac{14x^7y^{12}}{21x^{11}y^6}$

Solution

Be very careful to simplify $\frac{14}{21}$ by dividing out a common factor, and to simplify the variables by subtracting their exponents.

	$\frac{14x^7y^{12}}{21x^{11}y^6}$
Simplify and use the Quotient Property.	$\frac{2y^6}{3x^4}$

Try It 5.1.50

Find the quotient: $\frac{28x^5y^{14}}{49x^9y^{12}}$

Answer

$\frac{4y^2}{7x^4}$

Try It 5.1.51

Find the quotient: $\frac{30m^5n^{11}}{48m^{10}n^{14}}$

Answer

$\frac{5}{8m^5n^3}$

In all examples so far, there was no work to do in the numerator or denominator before simplifying the fraction. In the next example, we'll first find the product of two monomials in the numerator before we simplify the fraction. This follows the order of operations. Remember, a fraction bar is a grouping symbol.

Example 5.1.52

Find the quotient: $\frac{\left(6x^2y^3\right)\left(5x^3y^2\right)}{\left(3x^4y^5\right)}$

Solution

Answer

	$\frac{\left(6x^2y^3\right)\left(5x^3y^2\right)}{\left(3x^4y^5\right)}$
Simplify the numerator.	$\frac{30x^5y^5}{3x^4y^5}$
Simplify.	$10x$

Try It 5.1.53

Find the quotient: $\frac{\left(6a^4b^5\right)\left(4a^2b^5\right)}{12a^5b^8}$

Answer

$2ab^2$

Try It 5.1.54

Find the quotient: $\frac{\left(-12x^6y^9\right)\left(-4x^5y^8\right)}{-12x^{10}y^{12}}$

Answer

$-4xy^5$

Note

Access these online resources for additional instruction and practice with dividing monomials:

- Rational Expressions
- Dividing Monomials
- Dividing Monomials 2

Key Concepts

- **Quotient Property for Exponents:**
  - If a is a real number, $a \neq 0$, and m,n are whole numbers, then: $\frac{a^m}{a^n} = a^{m-n}, m > n$ and $\frac{a^m}{a^n} = \frac{1}{a^{m-n}}, n > m$
- **Zero Exponent**
  - If a is a non-zero number, then $a^0 = 1$.
- **Quotient to a Power Property for Exponents**:
  - If a and b are real numbers, $b \neq 0$, and mm is a counting number, then: $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$
  - To raise a fraction to a power, raise the numerator and denominator to that power.
- **Summary of Exponent Properties**
  - If a,b are real numbers and m,nm,n are whole numbers, then

Product Property	$a^m \cdot a^n$	$=$	a^{m+n}
Power Property	$(a^m)^n$	$=$	$a^{m \cdot n}$
Product to a Power	$(ab)^m$	$=$	$a^m b^m$
Quotient Property	$\frac{a^m}{a^n}$	$=$	$a^{m-n}, a \neq 0, m > n$
	$\frac{a^n}{a^n}$	$=$	$1, a \neq 0, n > m$
Zero Exponent Definition	a^0	$=$	$1, a \neq 0$
Quotient to a Power Property	$\left(\frac{a}{b}\right)^m$	$=$	$\frac{a^m}{b^m}, b \neq 0$

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5.1.1: Exercises

Practice Makes Perfect

Simplify Expressions Using the Quotient Property for Exponents

In the following exercises, simplify.

Exercise 5.1.1.1

a. $\frac{x^{18}}{x^3}$

b. $\frac{5^{12}}{5^3}$

Exercise 5.1.1.2

a. $\frac{y^{20}}{y^{10}}$

b. $\frac{7^{16}}{7^2}$

Answer

a. y^{10}

b. 7^{14}

Exercise 5.1.1.3

a. $\frac{p^{21}}{p^7}$

b. $\frac{4^{16}}{4^4}$

Exercise 5.1.1.4

a. $\frac{u^{24}}{u^3}$

b. $\frac{9^{15}}{9^5}$

Answer

a. u^{21}

b. 9^{10}

Exercise 5.1.1.5

a. $\frac{q^{18}}{q^{36}}$

b. $\frac{10^2}{10^3}$

Exercise 5.1.1.6

a. $\frac{t^{10}}{t^{40}}$
b. $\frac{8^3}{8^5}$

Answer

a. $\frac{1}{t^{30}}$
b. $\frac{1}{64}$

Exercise 5.1.1.7

a. $\frac{b}{b^9}$
b. $\frac{4}{4^6}$

Exercise 5.1.1.8

a. $\frac{x}{x^7}$
b. $\frac{10}{10^3}$

Answer

a. $\frac{1}{x^6}$
b. $\frac{1}{100}$

Simplify Expressions with Zero Exponents

In the following exercises, simplify.

Exercise 5.1.1.9

a. 20^0
b. b^0

Exercise 5.1.1.10

a. 13^0
b. k^0

Answer

a. 1
b. 1

Exercise 5.1.1.11

a. -27^0
b. $-\left(27^0\right)$

Exercise 5.1.1.12

a. -15^0
b. $-\left(15^0\right)$

Answer

a. -1
b. -1

Exercise 5.1.1.13

a. $(25x)^0$
b. $25x^0$

Exercise 5.1.1.14

a. $(6y)^0$
b. $6y^0$

Answer

a. 1
b. 6

Exercise 5.1.1.15

a. $(12x)^0$
b. $\left(-56p^4q^3\right)^0$

Exercise 5.1.1.16

a. $7y^0(17y)^0$
b. $\left(-93c^7d^{15}\right)^0$

Answer

a. 7
b. 1

Exercise 5.1.1.17

a. $12n^0-18m^0$
b. $(12n)^0-(18m)^0$

Exercise 5.1.1.18

a. $15r^0-22s^0$
b. $(15r)^0-(22s)^0$

Answer

a. -7
b. 0

Simplify Expressions Using the Quotient to a Power Property

In the following exercises, simplify.

Exercise 5.1.1.19

a. $\left(\frac{3}{4}\right)^3$
b. $\left(\frac{p}{2}\right)^5$
c. $\left(\frac{x}{y}\right)^6$

Exercise 5.1.1.20

a. $\left(\frac{2}{5}\right)^2$
b. $\left(\frac{x}{3}\right)^4$
c. $\left(\frac{a}{b}\right)^5$

Answer

a. $\frac{4}{25}$
b. $\frac{x^4}{81}$
c. $\left(\frac{a}{b}\right)^5$

Exercise 5.1.1.21

a. $\left(\frac{a}{3b}\right)^4$
b. $\left(\frac{5}{4m}\right)^2$

Exercise 5.1.1.22

a. $\left(\frac{a}{3b}\right)^4$
b. $\left(\frac{10}{3q}\right)^4$

Answer

a. $\frac{x^3}{8y^3}$
b. $\frac{10,000}{81q^4}$

Simplify Expressions by Applying Several Properties

In the following exercises, simplify.

Exercise 5.1.1.23

$$\frac{\left(a^2\right)^3}{a^4}$$

Exercise 5.1.1.24

$$\frac{\left(p^3\right)^4}{p^5}$$

Answer

p^7

Exercise 5.1.1.25

$$\frac{\left(y^3\right)^4}{y^{10}}$$

Exercise 5.1.1.26

$$\frac{\left(x^4\right)^5}{x^{15}}$$

Answer

x^5

Exercise 5.1.1.27

$$\frac{u^6}{\left(u^3\right)^2}$$

Exercise 5.1.1.28

$$\frac{v^{20}}{\left(v^4\right)^5}$$

Answer

1

Exercise 5.1.1.29

$$\frac{m^{12}}{\left(m^8\right)^3}$$

Exercise 5.1.1.30

$$\frac{n^8}{\left(n^6\right)^4}$$

Answer

$$\frac{1}{n^{16}}$$

Exercise 5.1.1.31

$$\left(\frac{p^9}{p^3}\right)^5$$

Exercise 5.1.1.32

$$\left(\frac{q^8}{q^2}\right)^3$$

Answer

$$q^{18}$$

Exercise 5.1.1.33

$$\left(\frac{r^2}{r^6}\right)^3$$

Exercise 5.1.1.34

$$\left(\frac{m^4}{m^7}\right)^4$$

Answer

$$\frac{1}{m^{12}}$$

Exercise 5.1.1.35

$$\left(\frac{p}{r^{11}}\right)^2$$

Exercise 5.1.1.36

$$\left(\frac{a}{b^6}\right)^3$$

Answer

$$\frac{a^3}{b^{18}}$$

Exercise 5.1.1.37

$$\left(\frac{w^5}{x^3}\right)^8$$

Exercise 5.1.1.38

$$\left(\frac{y^4}{z^{10}}\right)^5$$

Answer

$$\frac{y^{20}}{z^{50}}$$

Exercise 5.1.1.39

$$\left(\frac{2j^3}{3k}\right)^4$$

Exercise 5.1.1.40

$$\left(\frac{3m^5}{5n}\right)^3$$

Answer

$$\frac{27m^{15}}{125n^3}$$

Exercise 5.1.1.41

$$\left(\frac{3c^2}{4d^6}\right)^3$$

Exercise 5.1.1.42

$$\left(\frac{5u^7}{2v^3}\right)^4$$

Answer

$$\frac{625u^{28}}{16v^{12}}$$

Exercise 5.1.1.43

$$\left(\frac{k^2k^8}{k^3}\right)^2$$

Exercise 5.1.1.44

$$\left(\frac{j^2j^5}{j^4}\right)^3$$

Answer

$$j^9$$

Exercise 5.1.1.45

$$\frac{\left(t^2\right)^5\left(t^4\right)^2}{\left(t^3\right)^7}$$

Exercise 5.1.1.46

$$\frac{\left(q^3\right)^6\left(q^2\right)^3}{\left(q^4\right)^8}$$

Answer

$$\frac{1}{q^8}$$

Exercise 5.1.1.47

$$\frac{\left(-2p^2\right)^4\left(3p^4\right)^2}{\left(-6p^3\right)^2}$$

Exercise 5.1.1.48

$$\frac{\left(-2k^3\right)^2\left(6k^2\right)^4}{\left(9k^4\right)^2}$$

Answer

$64k^6$

Exercise 5.1.1.49

$$\frac{\left(-4m^3\right)^2\left(5m^4\right)^3}{\left(-10m^6\right)^3}$$

Exercise 5.1.1.50

$$\frac{\left(-10n^2\right)^3\left(4n^5\right)^2}{\left(2n^8\right)^2}$$

Answer

$-4{,}000$

Divide Monomials

In the following exercises, divide the monomials.

Exercise 5.1.1.51

$56b^8 \div 7b^2$

Exercise 5.1.1.52

$63v^{10} \div 9v^2$

Answer

$7v^8$

Exercise 5.1.1.53

$-88y^{15} \div 8y^3$

Exercise 5.1.1.54

$-72u^{12} \div 12u^4$

Answer

$-6u^8$

Exercise 5.1.1.55

$$\frac{45a^6b^8}{-15a^{10}b^2}$$

Exercise 5.1.1.56

$$\frac{54x^9y^3}{-18x^6y^{15}}$$

Answer

$$-\frac{3x^3}{y^{12}}$$

Exercise 5.1.1.57

$$\frac{15r^4s^9}{18r^9s^2}$$

Exercise 5.1.1.58

$$\frac{20m^8n^4}{30m^5n^9}$$

Answer

$$\frac{2m^3}{3n^5}$$

Exercise 5.1.1.59

$$\frac{18a^4b^8}{-27a^9b^5}$$

Exercise 5.1.1.60

$$\frac{45x^5y^9}{-60x^8y^6}$$

Answer

$$\frac{-3y^3}{4x^3}$$

Exercise 5.1.1.61

$$\frac{64q^{11}r^9s^3}{48q^6r^8s^5}$$

Exercise 5.1.1.62

$$\frac{65a^{10}b^8c^5}{42a^7b^6c^8}$$

Answer

$$\frac{65a^3b^2}{42c^3}$$

Exercise 5.1.1.63

$$\frac{\left(10m^5n^4\right)\left(5m^3n^6\right)}{25m^7n^5}$$

Exercise 5.1.1.64

$$\frac{\left(-18p^4q^7\right)\left(-6p^3q^8\right)}{-36p^{12}q^{10}}$$

Answer

$$\frac{-3q^5}{p^5}$$

Exercise 5.1.1.65

$$\frac{\left(6a^4b^3\right)\left(4ab^5\right)}{\left(12a^2b\right)\left(a^3b\right)}$$

Exercise 5.1.1.66

$$\frac{\left(4u^2v^5\right)\left(15u^3v\right)}{\left(12u^3v\right)\left(u^4v\right)}$$

Answer

$$\frac{5v^4}{u^2}$$

Mixed Practice

Exercise 5.1.1.67

a. $24a^5 + 2a^5$
b. $24a^5 - 2a^5$
c. $24a^5 \cdot 2a^5$
d. $24a^5 \div 2a^5$

Exercise 5.1.1.68

a. $15n^{10} + 3n^{10}$
b. $15n^{10} - 3n^{10}$
c. $15n^{10} \cdot 3n^{10}$
d. $15n^{10} \div 3n^{10}$

Answer

a. $18n^{10}$
b. $12n^{10}$
c. $45n^{20}$
d. 5

Exercise 5.1.1.69

a. $p^4 \cdot p^6$
b. $\left(p^4\right)^6$

Exercise 5.1.1.70

a. $q^5 \cdot q^3$
b. $\left(q^5\right)^3$

Answer

a. q^8
b. q^{15}

Exercise 5.1.1.71

a. $\dfrac{y^3}{y}$
b. $\dfrac{y}{y^3}$

Exercise 5.1.1.72

a. $\dfrac{z^6}{z^5}$
b. $\dfrac{z^5}{z^6}$

Answer

a. z
b. $\dfrac{1}{z}$

Exercise 5.1.1.73

$\left(8x^5\right)(9x) \div 6x^3$

Exercise 5.1.1.74

$(4y)\left(12y^7\right) \div 8y^2$

Answer

$6y^6$

Exercise 5.1.1.75

$\frac{27a^7}{3a^3} + \frac{54a^9}{9a^5}$

Exercise 5.1.1.76

$\frac{32c^{11}}{4c^5} + \frac{42c^9}{6c^3}$

Answer

$15c^6$

Exercise 5.1.1.77

$\frac{32y^5}{8y^2} - \frac{60y^{10}}{5y^7}$

Exercise 5.1.1.78

$\frac{48x^6}{6x^4} - \frac{35x^9}{7x^7}$

Answer

$3x^2$

Exercise 5.1.1.79

$\frac{63r^6s^3}{9r^4s^2} - \frac{72r^2s^2}{6s}$

Exercise 5.1.1.80

$\frac{56y^4z^5}{7y^3z^3} - \frac{45y^2z^2}{5y}$

Answer

$-yz^2$

Everyday Math

Exercise 5.1.1.81

Memory One megabyte is approximately 10^6 bytes. One gigabyte is approximately 10^9 bytes. How many megabytes are in one gigabyte?

Exercise 5.1.1.82

Memory One gigabyte is approximately 10^9 bytes. One terabyte is approximately 10^12 bytes. How many gigabytes are in one terabyte?

Answer

10^3

Writing Exercises

Exercise 5.1.1.83

Jennifer thinks the quotient $\frac{a^{24}}{a^6}$ simplifies to a^4. What is wrong with her reasoning?

Exercise 5.1.1.84

Maurice simplifies the quotient $\frac{d^7}{d}$ by writing $\frac{\cancel{d}^7}{\cancel{d}} = 7$. What is wrong with his reasoning?

Answer

Answers will vary.

Exercise 5.1.1.85

When Drake simplified -3^0 and $(-3)^0$ he got the same answer. Explain how using the Order of Operations correctly gives different answers.

Exercise 5.1.1.86

Robert thinks x^0 simplifies to 0. What would you say to convince Robert he is wrong?

Answer

Answers will vary.

Self Check

a. After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
simplify expressions using the Quotient Property for Exponents.			
simplify expressions with zero exponents.			
simplify expressions using the Quotient to a Power Property.			
simplify expressions by applying several properties.			
divide monomials.			

b. On a scale of 1-10, how would you rate your mastery of this section in light of your responses on the checklist? How can you improve this?

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5.2: Integer Exponents and Scientific Notation

Learning Objectives

By the end of this section, you will be able to:

- Use the definition of a negative exponent
- Simplify expressions with integer exponents
- Convert from decimal notation to scientific notation
- Convert scientific notation to decimal form
- Multiply and divide using scientific notation

Note

Before you get started, take this readiness quiz.

1. What is the place value of the 6 in the number 64891?
   If you missed this problem, review Example 1.2.1.
2. Name the decimal: 0.0012.
   If you missed this problem, review Example 1.8.1.
3. Subtract: 5−(−3).
   If you missed this problem, review Example 1.4.33.

Use the Definition of a Negative Exponent

We saw that the Quotient Property for Exponents introduced earlier in this chapter, has two forms depending on whether the exponent is larger in the numerator or the denominator.

QUOTIENT PROPERTY FOR EXPONENTS

If a is a real number, $a \neq 0$, and m and n are whole numbers, then

$$\frac{a^m}{a^n} = a^{m-n}, m > n \tag{5.2.1}$$

and

$$\frac{a^m}{a^n} = \frac{1}{a^{n-m}}, n > m \tag{5.2.2}$$

What if we just subtract exponents regardless of which is larger?

Let's consider $\frac{x^2}{x^5}$.

We subtract the exponent in the denominator from the exponent in the numerator.

$$\begin{array}{c} \frac{x^2}{x^5} \\ x^{2-5} \\ x^{-3} \end{array} \tag{5.2.3}$$

We can also simplify $\frac{x^2}{x^5}$ by dividing out common factors:

$$\begin{array}{c} \frac{x \cdot x}{x \cdot x \cdot x \cdot x \cdot x} \\ \frac{1}{x^3} \end{array}$$

This implies that $x^{-3} = \frac{1}{x^3}$ and it leads us to the definition of a *negative exponent*.

Definition: NEGATIVE EXPONENT

If n is an integer and $a \neq 0$, then $a^{-n} = \frac{1}{a^n}$

The **negative exponent** tells us we can re-write the expression by taking the reciprocal of the base and then changing the sign of the exponent.

Any expression that has negative exponents is not considered to be in simplest form. We will use the definition of a **negative exponent** and other properties of exponents to write the expression with only positive exponents.

For example, if after simplifying an expression we end up with the expression x^{-3}, we will take one more step and write $\frac{1}{x^3}$. The answer is considered to be in simplest form when it has only positive exponents.

Example 5.2.1

Simplify:

1. 4^{-2}
2. 10^{-3}

Solution

1.

	4^{-2}
Use the definition of a negative exponent, $a^{-n} = \frac{1}{a^n}$,	$\frac{1}{4^2}$
Simplify.	$\frac{1}{16}$

2.

	10^{-3}
Use the definition of a negative exponent, $a^{-n} = \frac{1}{a^n}$,	$\frac{1}{10^3}$
Simplify.	$\frac{1}{1000}$

Try It 5.2.2

Simplify:

1. 2^{-3}
2. 10^{-7}

Answer

1. $\frac{1}{8}$
2. $\frac{1}{10^7}$

Try It 5.2.3

Simplify:

1. 3^{-2}
2. 10^{-4}

Answer

1. $\frac{1}{9}$
2. $\frac{1}{10,000}$

In Example 5.2.1 we raised an integer to a negative exponent. What happens when we raise a fraction to a negative exponent? We'll start by looking at what happens to a fraction whose numerator is one and whose denominator is an integer raised to a negative exponent.

	$\frac{1}{a^{-n}}$
Use the definition of a negative exponent, $a^{-n}=\frac{1}{a^n}$	$\frac{1}{\frac{1}{a^n}}$
Simplify the complex fraction.	$1\cdot\frac{a^n}{1}$
Multiply.	a^n

This leads to the Property of Negative Exponents.

PROPERTY OF NEGATIVE EXPONENTS

If n is an integer and $a\neq 0$, then $\frac{1}{a^{-n}}=a^n$.

Example 5.2.4

Simplify:

1. $\frac{1}{y^{-4}}$
2. $\frac{1}{3^{-2}}$

Solution

1.

	$\frac{1}{y^{-4}}$
Use the property of a negative exponent, $\frac{1}{a^{-n}}=a^n$.	y^4

2.

	$\frac{1}{3^{-2}}$
Use the property of a negative exponent, $\frac{1}{a^{-n}}=a^n$.	3^2
Simplify.	9

Try It 5.2.5

Simplify:

1. $\frac{1}{p^{-8}}$
2. $\frac{1}{4^{-3}}$

Answer

1. p^8
2. 64

Try It 5.2.6

Simplify:

1. $\dfrac{1}{q^{-7}}$
2. $\dfrac{1}{2^{-4}}$

Answer

1. q^7
2. 16

Suppose now we have a fraction raised to a negative exponent. Let's use our definition of negative exponents to lead us to a new property.

	$\left(\frac{3}{4}\right)^{-2}$
Use the definition of a negative exponent, $a^{-n}=\frac{1}{a^n}$	$\frac{1}{\left(\frac{3}{4}\right)^{2}}$
Simplify the denominator.	$\frac{1}{\frac{9}{16}}$
Simplify the complex fraction.	$\frac{16}{9}$
But we know that $\frac{16}{9}$ is $\left(\frac{4}{3}\right)^{2}$	
This tells us that:	$\left(\frac{3}{4}\right)^{-2}=\left(\frac{4}{3}\right)^{2}$

To get from the original fraction raised to a negative exponent to the final result, we took the reciprocal of the base—the fraction—and changed the sign of the exponent.

This leads us to the *Quotient to a Negative Power Property*.

QUOTIENT TO A NEGATIVE EXPONENT PROPERTY

If a and b are real numbers, $a\neq 0, b\neq 0,$ and n is an integer, then $\left(\frac{a}{b}\right)^{-n}=\left(\frac{b}{a}\right)^{n}$

Example 5.2.7

Simplify:

1. $\left(\frac{5}{7}\right)^{-2}$
2. $\left(-\frac{2x}{y}\right)^{-3}$

Solution

1. $\left(\frac{5}{7}\right)^{-2}$

   Use the Quotient to a Negative Exponent Property, $\left(\frac{a}{b}\right)^{-n}=\left(\frac{b}{a}\right)^{n}$. Take the reciprocal of the fraction and change the sign of the exponent. . $\left(\frac{7}{5}\right)^{2}$

   Simplify. $\frac{49}{25}$

2. $\left(-\frac{2x}{y}\right)^{-3}$

   Use the Quotient to a Negative Exponent Property, $\left(\frac{a}{b}\right)^{-n}=\left(\frac{b}{a}\right)^{n}$. Take the reciprocal of the fraction and change the sign of the exponent. $\left(-\frac{y}{2x}\right)^{3}$

   Simplify. $-\frac{y^3}{8x^3}$

? Try It 5.2.8

Simplify:

1. $\left(\frac{2}{3}\right)^{-4}$
2. $\left(-\frac{6m}{n}\right)^{-2}$

Answer

1. $\frac{81}{16}$
2. $\frac{n^2}{36m^2}$

? Try It 5.2.9

Simplify:

1. $\left(\frac{3}{5}\right)^{-3}$
2. $\left(-\frac{a}{2b}\right)^{-4}$

Answer

1. $\frac{125}{27}$
2. $\frac{16b^4}{a^4}$

When simplifying an expression with exponents, we must be careful to correctly identify the base.

Example 5.2.10

Simplify:

1. $(-3)^{-2}$
2. -3^{-2}
3. $\left(-\frac{1}{3}\right)^{-2}$
4. $-\left(\frac{1}{3}\right)^{-2}$

Solution

1. Here the exponent applies to the base -3.

	$(-3)^{-2}$
Take the reciprocal of the base and change the sign of the exponent.	$\frac{1}{(-3)^2}$
Simplify.	$\frac{1}{9}$

2. The expression -3^{-2} means "find the opposite of 3^{-2}". Here the exponent applies to the base 3.

	-3^{-2}
Rewrite as a product with -1	$-1 \cdot 3^{-2}$
Take the reciprocal of the base and change the sign of the exponent.	$-1 \cdot \frac{1}{3^2}$
Simplify.	$-\frac{1}{9}$

3. Here the exponent applies to the base $\left(-\frac{1}{3}\right)$.

	$\left(-\frac{1}{3}\right)^{-2}$
Take the reciprocal of the base and change the sign of the exponent.	$\left(-\frac{3}{1}\right)^{2}$
Simplify.	9

4. The expression $-\left(\frac{1}{3}\right)^{-2}$ means "find the opposite of $\left(\frac{1}{3}\right)^{-2}$". Here the exponent applies to the base $\left(\frac{1}{3}\right)$.

	$-\left(\frac{1}{3}\right)^{-2}$
Rewrite as a product with -1	$-1 \cdot \left(\frac{1}{3}\right)^{-2}$
Take the reciprocal of the base and change the sign of the exponent.	$-1 \cdot \left(\frac{3}{1}\right)^{2}$
Simplify.	-9

Try It 5.2.11

Simplify:

1. $(-5)^{-2}$
2. -5^{-2}
3. $\left(-\frac{1}{5}\right)^{-2}$

4. $-\left(\frac{1}{5}\right)^{-2}$

Answer

1. $\frac{1}{25}$
2. $-\frac{1}{25}$
3. 25
4. -25

Try It 5.2.12

Simplify:

1. $(-7)^{-2}$
2. -7^{-2}
3. $\left(-\frac{1}{7}\right)^{-2}$
4. $-\left(\frac{1}{7}\right)^{-2}$

Answer

1. $\frac{1}{49}$
2. $-\frac{1}{49}$
3. 49
4. -49

We must be careful to follow the Order of Operations. In the next example, parts (a) and (b) look similar, but the results are different.

Example 5.2.13

Simplify:

1. $4 \cdot 2^{-1}$
2. $(4 \cdot 2)^{-1}$

Solution

1.

Do exponents before multiplication.	$4 \cdot 2^{-1}$
Use $a^{-n} = \frac{1}{a^n}$	$4 \cdot \frac{1}{2^1}$
Simplify.	2

2.

	$(4 \cdot 2)^{-1}$
Simplify inside the parentheses first.	$(8)^{-1}$
Use $a^{-n} = \frac{1}{a^n}$	$\frac{1}{8^1}$
Simplify.	$\frac{1}{8}$

Try It 5.2.14

Simplify:

1. $6 \cdot 3^{-1}$
2. $(6 \cdot 3)^{-1}$

Answer

1. 2
2. $\frac{1}{18}$

Try It 5.2.15

Simplify:

1. $8 \cdot 2^{-2}$
2. $(8 \cdot 2)^{-2}$

Answer

1. 2
2. $\frac{1}{256}$

When a variable is raised to a negative exponent, we apply the definition the same way we did with numbers. We will assume all variables are non-zero.

Example 5.2.16

Simplify:

1. x^{-6}
2. $\left(u^4\right)^{-3}$

Solution

1. x^{-6}

   Use the definition of a negative exponent, $a^{-n} = \frac{1}{a^n}$ $\frac{1}{x^6}$

2. $\left(u^4\right)^{-3}$

   Use the definition of a negative exponent, $a^{-n} = \frac{1}{a^n}$ $\frac{1}{\left(u^4\right)^3}$

   Simplify. $\frac{1}{u^{12}}$

Try It 5.2.17

Simplify:

1. y^{-7}
2. $\left(z^3\right)^{-5}$

Answer

1. $\frac{1}{y^7}$

2. $\dfrac{1}{z^{15}}$

Try It 5.2.18

Simplify:

1. p^{-9}
2. $\left(q^{4}\right)^{-6}$

Answer

1. $\dfrac{1}{p^{9}}$
2. $\dfrac{1}{q^{24}}$

When there is a product and an exponent we have to be careful to apply the exponent to the correct quantity. According to the Order of Operations, we simplify expressions in parentheses before applying exponents. We'll see how this works in the next example.

Example 5.2.19

Simplify:

1. $5y^{-1}$
2. $(5y)^{-1}$
3. $(-5y)^{-1}$

Solution

1. Notice the exponent applies to just the base y.

	$5y^{-1}$
Take the reciprocal of the base y, and change the sign of the exponent.	$5 \cdot \dfrac{1}{y^{1}}$
Simplify.	$\dfrac{5}{y}$

2. Here the parentheses make the exponent apply to the base $5y$

	$(5y)^{-1}$
Take the reciprocal of the base $5y$, and change the sign of the exponent.	$\dfrac{1}{(5y)^{1}}$
Simplify.	$\dfrac{1}{5y}$

3. The base here is $-5y$

	$(-5y)^{-1}$
Take the reciprocal of the base $-5y$ and change the sign of the exponent.	$\dfrac{1}{(-5y)^{1}}$
Simplify.	$\dfrac{1}{-5y}$
Use $\dfrac{a}{-b}=-\dfrac{a}{b}$	$-\dfrac{1}{5y}$

Try It 5.2.20

Simplify:

1. $8p^{-1}$
2. $(8p)^{-1}$
3. $(-8p)^{-1}$

Answer

1. $\frac{8}{p}$
2. $\frac{1}{8p}$
3. $-\frac{1}{8p}$

Try It 5.2.21

Simplify:

1. $11q^{-1}$
2. $(11q)^{-1}$
3. $(-11q)^{-1}$

Answer

1. $\frac{11}{q}$
2. $\frac{1}{11q}$
3. $-\frac{1}{11q}$

With negative exponents, the Quotient Rule needs only one form $\frac{a^m}{a^n} = a^{m-n}$, for $a \neq 0$. When the exponent in the denominator is larger than the exponent in the numerator, the exponent of the quotient will be negative.

Simplify Expressions with Integer Exponents

All of the exponent properties we developed earlier in the chapter with whole number exponents apply to integer exponents, too. We restate them here for reference.

SUMMARY OF EXPONENT PROPERTIES

If a and b are real numbers, and m and n are integers, then

Product Property	$a^m \cdot a^n$	$=$	a^{m+n}
Power Property	$(a^m)^n$	$=$	$a^{m \cdot n}$
Product to a Power	$(ab)^m$	$=$	$a^m b^m$
Quotient Property	$\frac{a^m}{a^n}$	$=$	$a^{m-n}, \quad a \neq 0$
Zero Exponent Property	a^0	$=$	$1, \quad a \neq 0$
Quotient to a Power Property	$\left(\frac{a}{b}\right)^m$	$=$	$\frac{a^m}{b^m}, \quad b \neq 0$
Properties of Negative Exponents	a^{-n}	$=$	$\frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$
Quotient to a Negative Exponents	$\left(\frac{a}{b}\right)^{-n}$	$=$	$\left(\frac{b}{a}\right)^n$

Example 5.2.22

Simplify:

1. $x^{-4} \cdot x^6$
2. $y^{-6} \cdot y^4$
3. $z^{-5} \cdot z^{-3}$

Solution

1.

	$x^{-4} \cdot x^6$
Use the Product Property, $a^m \cdot a^n = a^{m+n}$	x^{-4+6}
Simplify.	x^2

2.

	$y^{-6} \cdot y^4$
Notice the same bases, so add the exponents.	y^{-6+4}
Simplify.	y^{-2}
Use the definition of a negative exponent, $a^{-n} = \frac{1}{a^n}$	$\frac{1}{y^2}$

3.

	$z^{-5} \cdot z^{-3}$
Add the exponents, since the bases are the same.	z^{-5-3}
Simplify.	z^{-8}
Use the definition of a negative exponent to take the reciprocal of the base and change the sign of the exponent.	$\frac{1}{z^8}$

Try It 5.2.23

Simplify:

1. $x^{-3} \cdot x^7$
2. $y^{-7} \cdot y^2$
3. $z^{-4} \cdot z^{-5}$

Answer

1. x^4
2. $\frac{1}{y^5}$

3. $\frac{1}{z^9}$

Try It 5.2.24

Simplify:

1. $a^{-1} \cdot a^6$
2. $b^{-8} \cdot b^4$
3. $c^{-8} \cdot c^{-7}$

Answer

1. a^5
2. $\frac{1}{b^4}$
3. $\frac{1}{c^{15}}$

In the next two examples, we'll start by using the Commutative Property to group the same variables together. This makes it easier to identify the like bases before using the Product Property.

Example 5.2.25

Simplify: $\left(m^4 n^{-3}\right)\left(m^{-5} n^{-2}\right)$

Solution

	$\left(m^4 n^{-3}\right)\left(m^{-5} n^{-2}\right)$
Use the Commutative Property to get like bases together.	$m^4 m^{-5} \cdot n^{-2} n^{-3}$
Add the exponents for each base.	$m^{-1} \cdot n^{-5}$
Take reciprocals and change the signs of the exponents.	$\frac{1}{m^1} \cdot \frac{1}{n^5}$
Simplify.	$\frac{1}{mn^5}$

Try It 5.2.26

Simplify: $\left(p^6 q^{-2}\right)\left(p^{-9} q^{-1}\right)$

Answer

$\frac{1}{p^3 q^3}$

Try It 5.2.27

Simplify: $\left(r^5 s^{-3}\right)\left(r^{-7} s^{-5}\right)$

Answer

$\frac{1}{r^2 s^8}$

If the monomials have numerical coefficients, we multiply the coefficients, just like we did earlier.

Example 5.2.28

Simplify: $\left(2x^{-6}y^{8}\right)\left(-5x^{5}y^{-3}\right)$

Solution

	$\left(2x^{-6}y^{8}\right)\left(-5x^{5}y^{-3}\right)$
Rewrite with the like bases together.	$2(-5)\cdot\left(x^{-6}x^{5}\right)\cdot\left(y^{8}y^{-3}\right)$
Multiply the coefficients and add the exponents of each variable.	$-10\cdot x^{-1}\cdot y^{5}$
Use the definition of a negative exponent, $a^{-n}=\frac{1}{a^{n}}$	$-10\cdot\frac{1}{x^{1}}\cdot y^{5}$
Simplify.	$\frac{-10y^{5}}{x}$

Try It 5.2.29

Simplify: $\left(3u^{-5}v^{7}\right)\left(-4u^{4}v^{-2}\right)$

Answer

$-\frac{12v^{5}}{u}$

Try It 5.2.30

Simplify: $\left(-6c^{-6}d^{4}\right)\left(-5c^{-2}d^{-1}\right)$

Answer

$\frac{30d^{3}}{c^{8}}$

In the next two examples, we'll use the Power Property and the Product to a Power Property.

Example 5.2.31

Simplify: $\left(6k^{3}\right)^{-2}$

Solution

	$\left(6k^{3}\right)^{-2}$
Use the Product to a Power Property, $(ab)^{m}=a^{n}b^{m}$	$(6)^{-2}\left(k^{3}\right)^{-2}$
Use the Power Property, $(a^{m})^{n}=a^{m\cdot n}$	$6^{-2}k^{-6}$
Use the definition of a negative exponent, $a^{-n}=\frac{1}{a^{n}}$	$\frac{1}{6^{2}}\cdot\frac{1}{k^{6}}$
Simplify.	$\frac{1}{36k^{6}}$

Try It 5.2.32

Simplify: $\left(-4x^{4}\right)^{-2}$

Answer

$\frac{1}{16x^{8}}$

Try It 5.2.33

Simplify: $\left(2b^3\right)^{-4}$

Answer

$$\frac{1}{16b^{12}}$$

Example 5.2.34

Simplify: $\left(5x^{-3}\right)^2$

Solution

	$\left(5x^{-3}\right)^2$
Use the Product to a Power Property, $(ab)^m = a^n b^m$	$5^2\left(x^{-3}\right)^2$
Simplify 5^2 and multiply the exponents of x using the Power Property, $(a^m)^n = a^{m \cdot n}$.	$25 \cdot x^{-6}$
Rewrite x^{-6} by using the Definition of a Negative Exponent, $a^{-n} = \frac{1}{a^n}$	$25 \cdot \frac{1}{x^6}$
Simplify.	$\frac{25}{x^6}$

Try It 5.2.35

Simplify: $\left(8a^{-4}\right)^2$

Answer

$$\frac{64}{a^8}$$

Try It 5.2.36

Simplify: $\left(2c^{-4}\right)^3$

Answer

$$\frac{8}{c^{12}}$$

To simplify a fraction, we use the Quotient Property and subtract the exponents.

Example 5.2.37

Simplify: $\frac{r^5}{r^{-4}}$

Solution

	$\frac{r^5}{r^{-4}}$
Use the Quotient Property, $\frac{a^n}{a^n} = a^{m-n}$	$r^{5-(-4)}$
Simplify.	r^9

? Try It 5.2.38

Simplify: $\frac{x^8}{x^{-3}}$

Answer

x^{11}

? Try It 5.2.39

Simplify: $\frac{y^8}{y^{-6}}$

Answer

y^{14}

Convert from Decimal Notation to Scientific Notation

Remember working with place value for whole numbers and decimals? Our number system is based on powers of 10. We use tens, hundreds, thousands, and so on. Our decimal numbers are also based on powers of tens—tenths, hundredths, thousandths, and so on. Consider the numbers 4,000 and 0.004. We know that 4,000 means $4 \times 1,000$ and 0.004 means $4 \times \frac{1}{1,000}$.

If we write the 1000 as a power of ten in exponential form, we can rewrite these numbers in this way:

$$\begin{array}{ll} 4,000 & 0.004 \\ 4 \times 1,000 & 4 \times \frac{1}{1,000} \\ 4 \times 10^3 & 4 \times \frac{1}{10^3} \\ & 4 \times 10^{-3} \end{array} \tag{5.2.4}$$

When a number is written as a product of two numbers, where the first factor is a number greater than or equal to one but less than 10, and the second factor is a power of 10 written in exponential form, it is said to be in *scientific notation.*

SCIENTIFIC NOTATION

A number is expressed in **scientific notation** when it is of the form

$$a \times 10^n \text{ where } 1 \leq a < 10 \text{ and } n \text{ is an integer} \tag{5.2.5}$$

It is customary in scientific notation to use as the $\times$ multiplication sign, even though we avoid using this sign elsewhere in algebra.

If we look at what happened to the decimal point, we can see a method to easily convert from decimal notation to scientific notation.

$4000. = 4 \times 10^3$	$0.004 = 4 \times 10^{-3}$
$4000. = 4 \times 10^3$	$0.004 = 4 \times 10^{-3}$
Moved the decimal point 3 places to the left.	Moved the decimal point 3 places to the right.

In both cases, the decimal was moved 3 places to get the first factor between 1 and 10.

The power of 10 is positive when the number is larger than 1 : $4,000 = 4 \times 10^3$

The power of 10 is negative when the number is between 0 and 1 : $0.004 = 4 \times 10^{-3}$

Example 5.2.40: HOW TO CONVERT FROM DECIMAL NOTATION TO SCIENTIFIC NOTATION

Write in scientific notation: 37000.

Solution

Step 1. Move the decimal point so that the first factor is greater than or equal to 1 but less than 10.	Remember, there is a decimal at the end of 37,000. Move the decimal after the 3. 3.700 is between 1 and 10.	37,000.
Step 2. Count the number of decimal places, *n*, that the decimal point was moved.	The decimal point was moved 4 places to the left.	37000.
Step 3. Write the number as a product with a power of 10. If the original number is: Greater than 1, the power of 10 will be 10^n. Between 0 and 1, the power of 10 will be 10^{-n}.	37,000 is greater than 1 so the power of 10 will have exponent 4.	3.7×10^4
Step 4. Check.	Check to see if your answer makes sense.	10^4 is 10,000 and 10,000 times 3.7 will be 37,000. $37{,}000 = 3.7 \times 10^4$

Try It 5.2.41

Write in scientific notation: 96000.

Answer

9.6×10^4

Try It 5.2.42

Write in scientific notation: 48300.

Answer

4.83×10^4

HOW TO: Convert from decimal notation to scientific notation

1. Step 1. Move the decimal point so that the first factor is greater than or equal to 1 but less than 10.
2. Step 2. Count the number of decimal places, *n*, that the decimal point was moved.
3. Step 3. Write the number as a product with a power of 10.
   If the original number is:
   - greater than 1, the power of 10 will be 10^n.
   - between 0 and 1, the power of 10 will be 10^{-n}.
4. Step 4. Check.

Example 5.2.43

Write in scientific notation: 0.0052.

Solution

The original number, 0.0052, is between 0 and 1 so we will have a negative power of 10.

	0.0052
Move the decimal point to get 5.2, a number between 1 and 10.	0.0052
Count the number of decimal places the point was moved.	3 places
Write as a product with a power of 10.	5.2×10^{-3}
Check.	
5.2×10^{-3} $5.2 \times \frac{1}{10^3}$ $5.2 \times \frac{1}{1000}$ 5.2×0.001	
0.0052	$0.0052 = 5.2 \times 10^{-3}$

Try It 5.2.44

Write in scientific notation: 0.0078

Answer

7.8×10^{-3}

Try It 5.2.45

Write in scientific notation: 0.0129

Answer

1.29×10^{-2}

Convert Scientific Notation to Decimal Form

How can we convert from scientific notation to decimal form? Let's look at two numbers written in scientific notation and see.

$$\begin{array}{ll} 9.12 \times 10^{4} & 9.12 \times 10^{-4} \\ 9.12 \times 10,000 & 9.12 \times 0.0001 \\ 91,200 & 0.000912 \end{array} \tag{5.2.6}$$

If we look at the location of the decimal point, we can see an easy method to convert a number from scientific notation to decimal form.

$$9.12 \times 10^{4} = 91,200 \quad 9.12 \times 10^{-4} = 0.000912 \tag{5.2.7}$$

$9.12 \times 10^{4} = 91{,}200$	$9.12 \times 10^{-4} = 0.000912$
$9.12\_\_\_\times 10^{4} = 91{,}200$	$\_\_\_9.12 \times 10^{-4} = 0.000912$
Move the decimal point 4 places to the right.	Move the decimal point 4 places to the left.

In both cases the decimal point moved 4 places. When the exponent was positive, the decimal moved to the right. When the exponent was negative, the decimal point moved to the left.

Example 5.2.46

Convert to decimal form: 6.2×10^{3}

Solution

Step 1. Determine the exponent, *n*, on the factor 10.	The exponent is 3.	6.2×10^{3}
Step 2. Move the decimal *n* places, adding zeros if needed. If the exponent is positive, move the decimal point *n* places to the right. If the exponent is negative, move the decimal point $\lvert n \rvert$ places to the left.	The exponent is positive, so move the decimal point 3 places to the right. We need to add 2 zeros as placeholders.	6.200. 6,200
Step 3. Check to see if your answer makes sense.		10^{3} is 1000 and 1000 times 6.2 will be 6,200. $6.2 \times 10^{3} = 6{,}200$

Try It 5.2.47

Convert to decimal form: 1.3×10^{3}

Answer

$1,300$

Try It 5.2.48

Convert to decimal form: 9.25×10^{4}

Answer

$92,500$

The steps are summarized below.

HOW TO

Convert scientific notation to decimal form.

To convert scientific notation to decimal form:

1. Step 1. Determine the exponent, n, on the factor 10.
2. Step 2. Move the decimal n places, adding zeros if needed.

- If the exponent is positive, move the decimal point n places to the right.
- If the exponent is negative, move the decimal point $|n|$ places to the left.

3. Step 3. Check.

Example 5.2.49

Convert to decimal form: 8.9×10^{-2}

Solution

	8.9×10^{-2}
Determine the exponent, n, on the factor 10.	The exponent is –2.
Since the exponent is negative, move the decimal point 2 places to the left.	8.9
Add zeros as needed for placeholders.	$8.9 \times 10^{-2} = 0.089$

Try It 5.2.50

Convert to decimal form: 1.2×10^{-4}

Answer

0.00012

Try It 5.2.51

Convert to decimal form: 7.5×10^{-2}

Answer

0.075

Multiply and Divide Using Scientific Notation

Astronomers use very large numbers to describe distances in the universe and ages of stars and planets. Chemists use very small numbers to describe the size of an atom or the charge on an electron. When scientists perform calculations with very large or very small numbers, they use scientific notation. Scientific notation provides a way for the calculations to be done without writing a lot of zeros. We will see how the Properties of Exponents are used to multiply and divide numbers in scientific notation.

Example 5.2.52

Multiply. Write answers in decimal form: $\left(4 \times 10^{5}\right)\left(2 \times 10^{-7}\right)$

Solution

	$\left(4 \times 10^{5}\right)\left(2 \times 10^{-7}\right)$
Use the Commutative Property to rearrange the factors.	$4 \cdot 2 \cdot 10^{5} \cdot 10^{-7}$
Multiply.	8×10^{-2}
Change to decimal form by moving the decimal two places left.	0.08

Try It 5.2.53

Multiply $(3 \times 10^{6})(2 \times 10^{-8})$. Write answers in decimal form.

Answer

0.06

Try It 5.2.54

Multiply $\left(3 \times 10^{-2}\right)\left(3 \times 10^{-1}\right)$. Write answers in decimal form.

Answer

0.009

Example 5.2.55

Divide. Write answers in decimal form: $\dfrac{9 \times 10^{3}}{3 \times 10^{-2}}$

Solution

	$\dfrac{9 \times 10^{3}}{3 \times 10^{-2}}$
Separate the factors, rewriting as the product of two fractions.	$\dfrac{9}{3} \times \dfrac{10^{3}}{10^{-2}}$
Divide.	3×10^{5}
Change to decimal form by moving the decimal five places right.	300000

Try It 5.2.56

Divide $\dfrac{8 \times 10^{4}}{2 \times 10^{-1}}$. Write answers in decimal form.

Answer

$400,000$

Try It 5.2.57

Divide $\dfrac{8 \times 10^{2}}{4 \times 10^{-2}}$. Write answers in decimal form.

Answer

$20,000$

MEDIA ACCESS ADDITIONAL ONLINE RESOURCES

Access these online resources for additional instruction and practice with integer exponents and scientific notation:

- Negative Exponents
- Scientific Notation
- Scientific Notation 2

Key Concepts

- **Property of Negative Exponents**
  - If n is a positive integer and $a \neq 0$, then $\frac{1}{a^{-n}} = a^n$
- **Quotient to a Negative Exponent**
  - If a and b are real numbers, $b \neq 0$ and n is an integer , then $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$
- **To convert scientific notation to decimal form:**
  1. Determine the exponent, n on the factor 10.
  2. Move the decimal n places, adding zeros if needed.
     - If the exponent is positive, move the decimal point n places to the right.
     - If the exponent is negative, move the decimal point $|n|$ places to the left.
  3. Check.
- **To convert a decimal to scientific notation:**
  1. Move the decimal point so that the first factor is greater than or equal to 1 but less than 10.
  2. Count the number of decimal places, n that the decimal point was moved.
  3. Write the number as a product with a power of 10. If the original number is:
     - greater than 1, the power of 10 will be 10^n
     - between 0 and 1, the power of 10 will be 10^{-n}
  4. Check.

Glossary

negative exponent

If n is a positive integer and $a \neq 0$, then $a^{-n} = \frac{1}{a^n}$.

scientific notation

A number is expressed in scientific notation when it is of the form $a \times 10^n$ where $a \geq 1$ and a<10 and n is an integer.

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5.2.1: Exercises

Practice Makes Perfect

Use the Definition of a Negative Exponent

In the following exercises, simplify.

Exercise 1

1. 4^{-2}
2. 10^{-3}

Exercise 2

1. 3^{-4}
2. 10^{-2}

Answer

1. $\frac{1}{81}$
2. $\frac{1}{100}$

Exercise 3

1. 5^{-3}
2. 10^{-5}

Exercise 4

1. 2^{-8}
2. 10^{-2}

Answer

1. $\frac{1}{256}$
2. $\frac{1}{100}$

Exercise 5

1. $\frac{1}{c^{-5}}$
2. $\frac{1}{3^{-2}}$

Exercise 6

1. $\frac{1}{c^{-5}}$
2. $\frac{1}{5^{-2}}$

Answer

1. c^5
2. 25

Exercise 7

1. $\frac{1}{q^{-10}}$
2. $\frac{1}{10^{-3}}$

Exercise 8

1. $\frac{1}{t^{-9}}$
2. $\frac{1}{10^{-4}}$

Answer

1. t^{9}
2. 10000

Exercise 9

1. $\left(\frac{5}{8}\right)^{-2}$
2. $\left(-\frac{3m}{n}\right)^{-2}$

Exercise 10

1. $\left(\frac{3}{10}\right)^{-2}$
2. $\left(-\frac{2c}{d}\right)^{-3}$

Answer

1. $\frac{100}{9}$
2. $-\frac{d^{3}}{8c^{3}}$

Exercise 11

1. $\left(\frac{4}{9}\right)^{-3}$
2. $\left(-\frac{u^{2}}{2v}\right)^{-5}$

Exercise 12

1. $\left(\frac{7}{2}\right)^{-3}$
2. $\left(-\frac{3}{xy^{2}}\right)^{-3}$

Answer

1. $\frac{8}{343}$
2. $-\frac{x^3y^6}{27}$

Exercise 13

1. $(-5)^{-2}$
2. -5^{-2}
3. $\left(-\frac{1}{5}\right)^{-2}$
4. $-\left(\frac{1}{5}\right)^{-2}$

Exercise 14

1. $(-7)^{-2}$
2. -7^{-2}
3. $\left(-\frac{1}{7}\right)^{-2}$
4. $-\left(\frac{1}{7}\right)^{-2}$

Answer

1. $\frac{1}{49}$
2. $-\frac{1}{49}$
3. 49
4. −49

Exercise 15

1. -3^{-3}
2. $\left(-\frac{1}{3}\right)^{-3}$
3. $-\left(\frac{1}{3}\right)^{-3}$
4. $(-3)^{-3}$

Exercise 16

1. -5^{-3}
2. $\left(-\frac{1}{5}\right)^{-3}$
3. $-\left(\frac{1}{5}\right)^{-3}$
4. $(-5)^{-3}$

Answer

1. $-\frac{1}{125}$
2. −125
3. −125
4. $-\frac{1}{125}$

Exercise 17

1. $3 \cdot 5^{-1}$
2. $(3 \cdot 5)^{-1}$

Exercise 18

1. $2 \cdot 5^{-1}$
2. $(2 \cdot 5)^{-1}$

Answer

1. $\frac{2}{5}$
2. $\frac{1}{10}$

Exercise 19

1. $4 \cdot 5^{-2}$
2. $(4 \cdot 5)^{-2}$

Exercise 20

1. $3 \cdot 4^{-2}$
2. $(3 \cdot 4)^{-2}$

Answer

1. $\frac{3}{16}$
2. $\frac{1}{144}$

Exercise 21

1. m^{-4}
2. $(x^3)^{-4}$

Exercise 22

1. b^{-5}
2. $(k^2)^{-5}$

Answer

1. $\frac{1}{b^5}$
2. $\frac{1}{k^{10}}$

Exercise 23

1. p^{-10}
2. $(q^6)^{-8}$

Exercise 24

1. s^{-8}
2. $(a^9)^{-10}$

Answer

1. $\frac{1}{s^8}$
2. $\frac{1}{a^{90}}$

Exercise 25

1. $7n^{-1}$
2. $(7n)^{-1}$
3. $(-7n)^{-1}$

Exercise 26

1. $6r^{-1}$
2. $(6r)^{-1}$
3. $(-6r)^{-1}$

Answer

1. $\frac{6}{r}$
2. $\frac{1}{6r}$
3. $-\frac{1}{6r}$

Exercise 27

1. $(3p)^{-2}$
2. $3p^{-2}$
3. $-3p^{-2}$

Exercise 28

1. $(2q)^{-4}$
2. $2q^{-4}$
3. $-2q^{-4}$

Answer

1. $\frac{1}{16q^4}$
2. $\frac{2}{q^4}$
3. $-\frac{2}{q^4}$

Simplify Expressions with Integer Exponents

In the following exercises, simplify.

Exercise 29

1. b^4b^{-8}
2. $r^{-2}r^5$
3. $x^{-7}x^{-3}$

Exercise 30

1. $s^3 \cdot s^{-7}$
2. $q^{-8} \cdot q^3$
3. $y^{-2} \cdot y^{-5}$

Answer

1. $\frac{1}{s^4}$
2. $\frac{1}{q^5}$
3. $\frac{1}{y^7}$

Exercise 31

1. $a^3 \cdot a^{-3}$
2. $a \cdot a^3$
3. $a \cdot a^{-3}$

Exercise 32

1. $y^5 \cdot y^{-5}$
2. $y \cdot y^5$
3. $y \cdot y^{-5}$

Answer

1. 1
2. y^6
3. $\frac{1}{y^4}$

Exercise 33

$p^5 \cdot p^{-2} \cdot p^{-4}$

Exercise 34

$x^4 \cdot x^{-2} \cdot x^{-3}$

Answer

$\frac{1}{x}$

Exercise 35

$(w^4 x^{-5})(w^{-2} x^{-4})$

Exercise 36

$(m^3 n^{-3})(m^{-5} n^{-1})$

Answer

$\frac{1}{m^2 n^4}$

Exercise 37

$(uv^{-2})(u^{-5}v^{-3})$

Exercise 38

$(pq^{-4})(p^{-6}q^{-3})$

Answer

$$\frac{1}{p^5q^7}$$

Exercise 39

$(-6c^{-3}d^9)(2c^4d^{-5})$

Exercise 40

$(-2j^{-5}k^8)(7j^2k^{-3})$

Answer

$$-\frac{14k^5}{j^3}$$

Exercise 41

$(-4r^{-2}s^{-8})(9r^4s^3)$

Exercise 42

$(-5m^4n^6)(8m^{-5}n^{-3})$

Answer

$$-\frac{40n^3}{m}$$

Exercise 43

$(5x^2)^{-2}$

Exercise 44

$(4y^3)^{-3}$

Answer

$$\frac{1}{64y^9}$$

Exercise 45

$(3z^{-3})^2$

Exercise 46

$(2p^{-5})^2$

Answer

$$\frac{4}{p^{10}}$$

Exercise 47

$$\frac{t^9}{t^{-3}}$$

Exercise 48

$$\frac{n^5}{n^{-2}}$$

Answer

n^7

Exercise 49

$$\frac{x^{-7}}{x^{-3}}$$

Exercise 50

$$\frac{y^{-5}}{y^{-10}}$$

Answer

y^5

Convert from Decimal Notation to Scientific Notation

In the following exercises, write each number in scientific notation.

Exercise 51

57,000

Exercise 52

340,000

Answer

3.4×10^5

Exercise 53

8,750,000

Exercise 54

1,290,000

Answer

1.29×10^6

Exercise 55

0.026

Exercise 56

0.041

Answer

4.1×10^{-2}

Exercise 57

0.00000871

Exercise 58

0.00000103

Answer

1.03×10^{-6}

Convert Scientific Notation to Decimal Form

In the following exercises, convert each number to decimal form.

Exercise 59

5.2×10^2

Exercise 60

8.3×10^2

Answer

830

Exercise 61

7.5×10^6

Exercise 62

1.6×10^{10}

Answer

16,000,000,000

Exercise 63

2.5×10^{-2}

Exercise 64

3.8×10^{-2}

Answer

0.038

Exercise 65

4.13×10^{-5}

Exercise 66

1.93×10^{-5}

Answer

0.0000193

Multiply and Divide Using Scientific Notation

In the following exercises, multiply. Write your answer in decimal form.

Exercise 67

$\left(3 \times 10^{-5}\right)\left(3 \times 10^{9}\right)$

Exercise 68

$\left(2 \times 10^{2}\right)\left(1 \times 10^{-4}\right)$

Answer

0.02

Exercise 69

$\left(7.1 \times 10^{-2}\right)\left(2.4 \times 10^{-4}\right)$

Exercise 70

$\left(3.5 \times 10^{-4}\right)\left(1.6 \times 10^{-2}\right)$

Answer

5.6×10^{-6}

In the following exercises, divide. Write your answer in decimal form.

Exercise 71

$$\frac{7 \times 10^{-3}}{1 \times 10^{-7}}$$

Exercise 72

$$\frac{5 \times 10^{-2}}{1 \times 10^{-10}}$$

Answer

500,000,000

Exercise 73

$$\frac{6 \times 10^{4}}{3 \times 10^{-2}}$$

Exercise 74

$$\frac{8 \times 10^{6}}{4 \times 10^{-1}}$$

Answer

20,000,000

Everyday Math

Exercise 75

The population of the United States on July 4, 2010 was almost 310,000,000. Write the number in scientific notation.

Exercise 76

The population of the world on July 4, 2010 was more than 6,850,000,000. Write the number in scientific notation

Answer

6.85×10^{9}

Exercise 77

The average width of a human hair is 0.0018 centimeters. Write the number in scientific notation.

Exercise 78

The probability of winning the 2010 Megamillions lottery was about 0.0000000057. Write the number in scientific notation.

Answer

5.7×10^{-9}

Exercise 79

In 2010, the number of Facebook users each day who changed their status to 'engaged" was 2×10^4. Convert this number to decimal form.

Exercise 80

At the start of 2012, the US federal budget had a deficit of more than $\$1.5 \times 10^{13}$. Convert this number to decimal form.

Answer

15,000,000,000,000

Exercise 81

The concentration of carbon dioxide in the atmosphere is 3.9×10^{-4}. Convert this number to decimal form.

Exercise 82

The width of a proton is 1×10^{-5} of the width of an atom. Convert this number to decimal form.

Answer

0.00001

Exercise 83

Health care costs The Centers for Medicare and Medicaid projects that consumers will spend more than $4 trillion on health care by 2017.

1. Write 4 trillion in decimal notation.
2. Write 4 trillion in scientific notation.

Exercise 84

Coin production In 1942, the U.S. Mint produced 154,500,000 nickels. Write 154,500,000 in scientific notation.

Answer

1.545×10^8

Exercise 85

Distance The distance between Earth and one of the brightest stars in the night star is 33.7 light years. One light year is about 6,000,000,000,000 (6 trillion), miles.

1. Write the number of miles in one light year in scientific notation.
2. Use scientific notation to find the distance between Earth and the star in miles. Write the answer in scientific notation.

Exercise 86

Debt At the end of fiscal year 2015 the gross United States federal government debt was estimated to be approximately $18,600,000,000,000 ($18.6 trillion), according to the Federal Budget. The population of the United States was approximately 300,000,000 people at the end of fiscal year 2015.

1. Write the debt in scientific notation.
2. Write the population in scientific notation.
3. Find the amount of debt per person by using scientific notation to divide the debt by the population. Write the answer in scientific notation.

Answer

1. 1.86×10^{13}
2. 3×10^{8}
3. 6.2×10^{4}

Writing Exercises

Exercise 87

1. Explain the meaning of the exponent in the expression 2^3.
2. Explain the meaning of the exponent in the expression 2^{-3}.

Exercise 88

When you convert a number from decimal notation to scientific notation, how do you know if the exponent will be positive or negative?

Answer

answers will vary

Self Check

a. After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
use the definition of a negative exponent.			
simplify expressions with integer exponents.			
convert from decimal notation to scientific notation.			
convert scientific notation to decimal form.			
multiply and divide using scientific notation.			

b. Overall, after looking at the checklist, do you think you are well-prepared for the next section? Why or why not?

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