

MATH 140: COLLEGE ALGEBRA WORKBOOK WITH DESMOS APPLICATIONS



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Math 140: College Algebra Workbook with Desmos Applications

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1: PREREQUISITES

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- 1.4: Linear Functions
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1.1: Appendix A- Review Factoring by Grouping

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Factor the following expressions:

[(a)] $8x^2 + 14x - 15$

[(b)] $3x^2 - 11x + 6$

[(c)] $6x^2 - 25x - 9$

[(d)] $-5x^2 + 7x - 2$

[(e)] $4x^2 - 9$

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1.2: Appendix B Solving Equations and Inequalities

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1. Use Desmos to solve the equation $4l - 0.5l = 7$. On Desmos, enter the two expressions as separate functions: $4l - 0.5l$ and $y = 7$.

Where do the graphs intersect?

2. Solve for z . Assuming D is a constant, what value of D could give us problems? $6z + 2 = D(z - 4)$

3. Solve the following equation for p . Assume $q \neq 0$. $3pq - 14 = 4w$

4. Are the following equations true? Justify your answer.

(a) $(3y + x) + (2x + 4) + (2 - y) = 6 + 4y + 3x$

(b) $2x(xy) = 2x^2y$

(c) $(a + b)^2 = a^2 + b^2$

(d) $16m(4m^2 - 3m + 7) = 64m^3 - 48m^2 + 112m$

5. Solve the following inequalities:

(a) $3x \geq 6$

(b) $6 < 4c - 6 \leq 26$

(c) $12 \leq -3m \leq 15$

(d) $2 - 4w > 18$

(e) $15 \leq 3h + 1 \leq 21$

6. Local zoning regulations in Chicago require that a property be no less than 6 feet from a street. You want to build a fence around your building which stands at most 20 feet away from the street. Write an inequality expressing the possible distances from the street (in feet) your fence can stand.

1.3: Reordering, Regrouping, and the Distributive Law

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1. Simplify each expression:

(a) $10p - 14 - 3p + 11$

(b) $(4 - x) + 2(8y + 2x) - (5y - 3)$

(c) $12wz - 2w - 4wz - 3z$

2. For each part, determine if the two expressions are equivalent. Justify your answer.

(a) $x(2x)$ and $3x$

(b) $x(2x)$ and $2x$

(c) $(4r-3)/7$ and $-3/7 + 4/7r$

(d) $\sqrt{4+2p}$ and $\sqrt{4} + \sqrt{2p}$

3. During a normal month, a bike shop sells q bicycles at p dollars each, so it earns pq dollars (recall that Revenue = Price $\times$ Quantity). During a sale month the shop halves the price but sells three times as many bicycles. Write an expression for the money the shop earns during a sale month. Does the amount of money they earn change, and if so, how many times larger is it?

4. If $x + y + z + w + q + r + s = 1.75$, evaluate the following expression: $(x - 1) + (s - 1) + (y + 2) - (3 - w) + (1 + r) + (0.5 + q) + (z + 3)$

1.4: Linear Functions

Stalactite Growth (Linear Model)

The length of a stalactite is modeled by the formula:

$L(t) = 17.75 + \left(\frac{1}{250}\right)t$, where $L(t)$ represents the length (in inches) and t is the time in years since measurement.

[a)] Enter $L(t) = 17.75 + \left(\frac{1}{250}\right)t$ into Desmos.

[b)] Identify the vertical intercept and explain what it represents in context.

[c)] Use the graph to estimate the stalactite's length after 500 years. Compare with the algebraic calculation.

Slope from Two Points

[a)] Plot the points (3, 5) and (9, 23) in Desmos.

[b)] Use Desmos to draw the line through the points and confirm the slope visually.

[c)] Describe how the steepness of the line matches your calculation.

Car Depreciation

A new Toyota RAV4 costs \$23,500. The car's value depreciates linearly to \$18,823 in three years.

[a)] Write the equation for the car's value.

[b)] Graph the function in Desmos.

[c)] Use the graph to estimate the car's value after 5 years.

[d)] Compare this with your algebraic calculation.

Advertising and Units Sold

A company sells 300 units with no advertising. For each additional \$5000 spent, 20 more units are sold.

[a)] Write a linear equation for units sold.

[b)] Enter your equation into Desmos.

[c)] Use the graph to estimate sales at \$40,000 spent.

[d)] Zoom out and describe how the slope represents rate of change in sales.

Graphing Nonlinear Function}

Consider $y = -x \left(\frac{x - 1000}{900} \right)$.

[a)] Enter the function into Desmos.

[b)] Adjust the viewing window to $-500 \leq x \leq 500$ and $-500 \leq y \leq 500$.

[c)] Compare how the graph looks zoomed in vs. zoomed out. Explain why it appears linear at first and nonlinear at a larger scale.

1.5: Formulas for Linear Functions

1. Find formulas for the linear functions

[(a)] Passes through the points $(-1, 5)$ and $(2, -1)$

[(b)] Slope $2/3$ and passes through the point $(5, 7)$

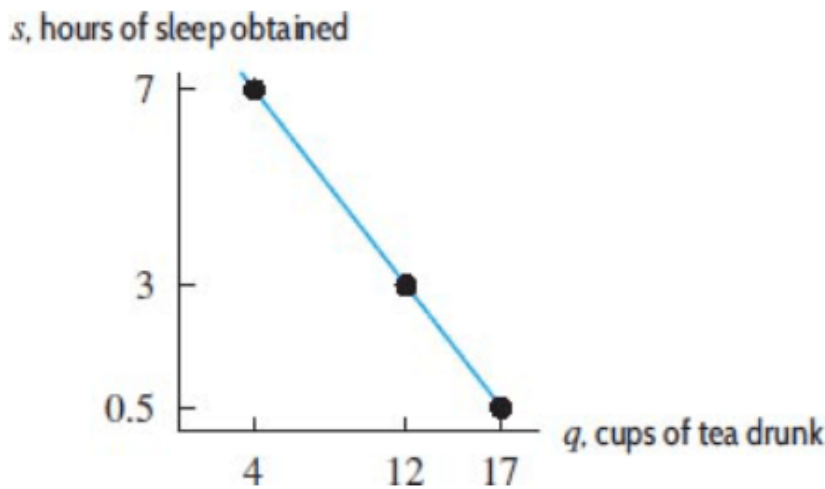
[(c)] Has x -intercept 3 and y -intercept -5

[(d)] The line intersects the graph of $f(x) = \frac{50}{x^2 + 1}$ at $x = 1$ and $x = 7$.

2. Find a formula for the function $q = f(p)$

price per bottle $p(\text{\$})$	0.5	0.75	1
number of bottles sold $q=f(p)$	1500	100	500

3. Find a formula for the linear function shown on the graph $s(q)$



4. Find the equations of the lines parallel to and perpendicular to the line $y + 4x = 7$, and through the point $(1, 5)$.

5. Table below gives the cost, $C(n)$, of producing a certain good as a linear function of n , the number of units produced.

n (units)	100	125	150	175
$C(n)$ (dollars)	11000	11125	11250	11375

a) $C(175)$

b) $C(175) - C(150)$

c) $\frac{C(175) - C(150)}{175 - 150}$

d) Estimate $C(0)$. What is the economic significance of this value?

e) The fixed cost of production is the cost incurred before any goods are produced. The unit cost is the cost of producing an additional unit. Find a formula for $C(n)$ in terms of n , given that

$$\text{Total cost} = \text{Fixed cost} + \text{Unit cost} \cdot \text{Number of units}$$

6. The demand for gasoline can be modeled as a linear function of price. If the price of gasoline is $p = \$3.10$ per gallon, the quantity demanded in a fixed period is $q = 65$ gallons. If the price rises to $\$3.50$ per gallon, the quantity demanded falls to 45 gallons in that period.

a) Find a formula for q in terms of p .

b) Explain the economic significance of the slope of your formula.

c) Explain the economic significance of the q -axis and p -axis intercepts.

7. In a college meal plan, you pay a membership fee; then 30 meals cost 137.50 dollars and 60 meals cost 235 dollars. Find the cost of the membership fee, and write the formula for the cost of a meal plan, C , in terms of the number of meals, n .

1.6: Expressions and Equations

1. Write an expression for the following sequences of operations:

- (a) Add 3 to x and double the result.
- (b) Subtract 1 from x , then divide the result by A .
- (c) Divide x by 2, then add 4, then take the square root of the result.

2. To calculate the balance after investing P dollars for two years at 5% interest, Sharif adds 5% of P to P , and then adds 5% of the result of this calculation to itself. Jared multiplies P by 1.05, and then multiplies the result of this by 1.05 again.

- (a) Write an expression for each calculation.
- (b) Are Jared and Sharif's calculations equivalent? Justify your answer.

3. Eric plans to spend \$20 on ice cream cones at \$1.25 each. Write an equation whose solution is the number of ice cream cones he can buy, and then solve that equation.

4. A town's population P , in thousands, t years after it was founded is given by the function $P = 30 + 2t$

- (a) Write an equation whose solution is when the town's population is 50,000.
- (b) When is the town's population 50,000?

1.7: Review Rules of Exponents

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1. Recall that, by definition, b^n means you multiply b by itself n times: $\underbrace{b \times b \times \dots \times b}_{n \text{ factors of } b}$.

[(a)] Simplify $5^3 \times 5^6$ by following these step:

[i.] Rewrite with each exponential written explicitly (so 5^3 becomes $5 \times 5 \times 5$).

[ii.] You should have just a bunch of 5's multiplied together. Express as a single exponential.

[(b)] Simplify $(2^3)^4$ by following these step:

[i.] Explicitly write this as 2^3 times itself four times.

[ii.] Rewrite, replacing each 2^3 with $2 \times 2 \times 2$.

[iii.] You should have just a bunch of 2's multiplied together. Express as a single exponential.

[(c)] For each of (a) and (b), discuss a general rule of exponents.

2. Show that $\frac{1}{5^3} \times 5^7 = 5^4$. Use this and the rules above to explain why people might write 5^{-3} to mean $\frac{1}{5^3}$. More generally, people write a^{-n} to mean $\frac{1}{a^n}$.

[3.] Since $2^3 = 8$, explain why the exponent rules above would suggest that $8^{\frac{1}{3}} = 2$. What is the value of $8^{\frac{4}{3}}$? (Hint: $8^{\frac{4}{3}} = 8^{\left(\frac{1}{3} \times 4\right)}$.)

[4.] Based on the ideas in the previous question, people write $a^{\frac{m}{n}}$ to mean $(\sqrt[n]{a})^m$ or $\sqrt[n]{a^m}$ (these expressions give the same result). The way to remember this is "power over root": $a^{\frac{m}{n}} = a^{\text{power}}$. Use this rule to evaluate the following without a calculator. (Hint: you can take the power and the root in either order: choose the order that is easier!)

[(a)] $9^{\frac{3}{2}} =$

$$[(b)] 16^{-\frac{3}{4}} =$$

$$[(c)] \frac{1}{2^{-3}} =$$

$$[(d)] \frac{2^{\frac{2}{3}}}{\frac{-4}{2^{\frac{3}{4}}}} =$$

5. Use rules of exponents to write each of the following in the form $f(x) = a \times b^x$, or state that it cannot be done (that is, the function is not exponential).

$$[(a)] f(x) = (3 \times 2^x)^5$$

$$[(b)] g(x) = \frac{2}{3x}$$

$$[(c)] h(x) = 8 \times 4^{x-1}$$

$$[(d)] l(x) = 6 \times 4^x + 7$$

$$[(e)] b(x) = 12 \times 3^{-2x}$$

$$[(f)] r(x) = (8 \times 27^x)^{\frac{1}{3}}$$

6. Solve the equation by combining each side into a single exponential.

$$[(a)] 2^3 2^x = 2^7$$

$$[(b)] \left(9 \times 3^{4x}\right)^{\frac{1}{2}} = 3^{14}$$

2: EQUATIONS AND INEQUALITIES

2.1: Linear Expressions and Equations

2.2: Linear Equations, Equations In the Plane

2.3: Solve Linear Inequalities

2.4: Solving Quadratic Equations

2.5: Systems of Equations

2.6: Introduction to Quadratic Functions

2.1: Linear Expressions and Equations

Warm-up:

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1. Write a formula for the linear equation that satisfies the following properties:

- a. $f(2) = 3$ and f has slope of -4 .
- b. g has slope 2 and passes the y -axis at $g(t) = 13$.
- c. f passes through the points $(-4, 5)$ and $(8, 10)$
- d. $h(-4) = 5$ and $h(8) = 10$

e. $f(-6) = 7$ and $f(3) = -6$

f. d passes through the points $(-18, -290)$ and $(20, 312)$

2. Use the following table to answer the questions below.

Year t	0	1	2
Value of the company $V = f(t)$	2000	1500	1000

a. What does this table represent in practical terms?

b. Write a formula for $f(t)$.

c. Use your formula to find when the value of the computer is \$250.

3. The solid waste generated each year in the cities of the US is increasing. The solid waste generated, in millions of tons, was 88.1 in 1960 and 249.9 in 2010. The trend appears linear during this time.

a. Construct a formula for the amount of municipal solid waste generated in the US by finding the equation of the line through these two points.

b. Use this formula to predict the amount of municipal solid waste generated in the US, in millions of tons, in the year 2020.

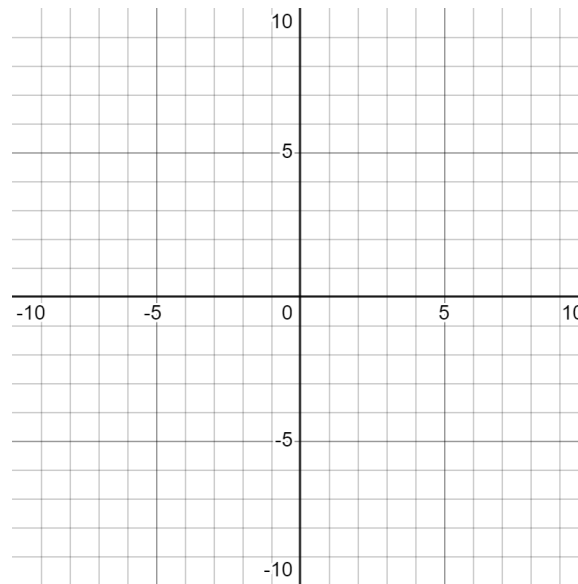
c. Your formula might seem a little weird. What is the "initial amount" of this formula? To fix this, create a new formula that describes the amount of municipal solid waste generated since 1980. (In other words, treat the year 1980 as $t = 0$.) Then, verify that your formula is also correct by finding the waste generated in 2020.

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2.2: Linear Equations, Equations In the Plane

1. Graph the following functions using Desmos:

- $x = 3$
- $y = -4$
- $2x - \frac{8}{7}y = -8$
- $-7x - 2y = 14$



2. How many solutions do the following equations have?

- $3x + 1 = 2x - 3$
- $2x - 1 = 2x + 3$
- $193x - 8762 = 193x - 8762$
- $17\pi x + e^{\pi^{25}} = 4x + 14$

3. For each part, are the lines parallel, perpendicular, or neither? Verify your answer by graphing them on Demos

- $y = 3x + 1$ and $y = 2x - 3$
- $y = 2x - 1$ and $y = 2x + 3$
- $y = \frac{-5}{4}x - 4$ and $y = \frac{4}{5}x + 3$
- $3x + 2y = 20$ and $12x + 8y = 5$

4. Find the equation of the line parallel to the graph $y = 14 - 3x$ with a y -intercept of 6.

5. Find the equation of the line parallel to the graph $6x - 3y = 18$ containing the point $(4, 12)$

6. Find the equation of the line perpendicular to the line $y = 10x - 6$ that intersects the graph at the point $(2, 14)$

7. Find the equation of the line intersecting the function $f(x) = x^3 - 2x^2 + 1$ at $x = 1$ and $x = 0$.

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2.3: Solve Linear Inequalities

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1. Solve the following inequalities:

(a) $6 < 4x - 6 \leq 26$

(b) $12 \leq -3m < 15$

2. Local zoning regulations near Olive Harvey College (OHC) require that a property be no less than 6 feet from a street. OHC personnel want to build a fence around the building which stands 20 feet away from the street. Write an inequality expressing the possible distances from the street in feet your fence can stand.

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2.4: Solving Quadratic Equations

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1. Solve the following equations: (Hint: Use factored form to find the zeroes. You may need to put it into factored form!)

a. $(x - 1)(x + 1) = 0$

b. $x(x + 3) = 0$

c. $x^2 - 4x - 5 = 0$

d. $2(x - 17)(x + 3) = 0$

e. $5x^2 + 35x + 50 = 0$

f. $(2x - 2)(3x + 6) = 0$

g. $(x - 4)^2 = 0$

h. $2(x - 4)^2 - 18 = 0$

i. $x^2 - 9 = 0$

j. $x^2 + 1 = 0$

2. The length of a rectangular garden is 5 times the width of the garden plus 20 feet. If the area of the garden is 700ft^2 , what are the dimensions of the garden?

2.5: Systems of Equations

1. Solve the following system of equations using substitution. Then solve it using elimination. Then solve it system graphically using Desmos.

$$\begin{cases} 3x + y = 9 \\ -2x + y = 4 \end{cases}$$

2. Solve the following systems of equations using elimination:

a.

$$\begin{cases} -4R - 7T = 14 \\ 3R + 14T = 2 \end{cases}$$

b.

$$\begin{cases} 6x + 5y = 12 \\ 2x - 7y = 6 \end{cases}$$

3. Solve the following systems of equations using substitution:

a.

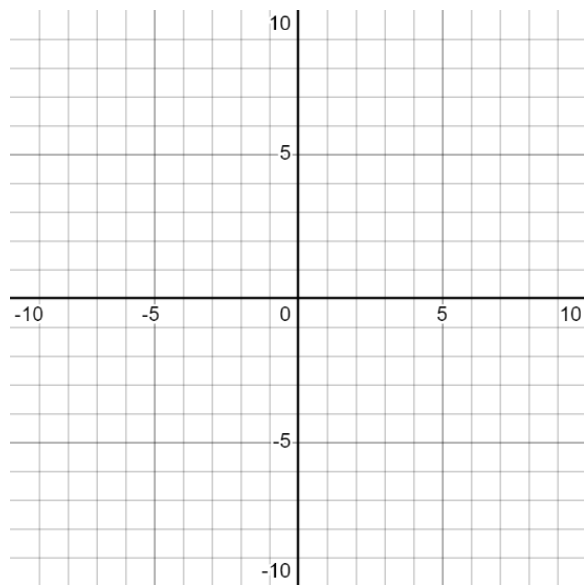
$$\begin{cases} 2x + 6y = 8 \\ x - 7y = 14 \end{cases}$$

b.

$$\begin{cases} 2A - 4B = 10 \\ 4A + 5B = -6 \end{cases}$$

4. Solve the following system of equations graphically:

$$\begin{cases} 3x + 6y = 18 \\ -x + 2y = 4 \end{cases}$$



2.6: Introduction to Quadratic Functions

1. Are the functions quadratic? If so, write the function in the form $f(x) = ax^2 + bx + c$. Then, graph your function in Desmos to verify your result and describe the shape of the graph.

[(a)] $f(x) = (2x - 3)(5 - x)$

[(b)] $w(n) = n(n - 3)(n - 2) - n^2(n - 8)$

2. Find the zeros of $Q(r) = 2r^2 - 6r - 36$ by factoring. Then, graph $y = 2r^2 - 6r - 36$ in Desmos and check whether your x -intercepts match your algebraic solutions.

3. Find the zeros of $Q(x) = 5x - x^2 + 3$ using the quadratic formula. Afterward, plot $y = 5x - x^2 + 3$ in Desmos. Compare the x -intercepts from your graph to those found using the quadratic formula.

4. Find the zeros (if any) of each function algebraically. Then, use Desmos to check whether the graph crosses or just touches the x -axis for each case.

[(a)] $y = (2 - x)(3 - 2x)$

[(b)] $y = 9x^2 + 6x + 1$

[(c)] $N(t) = t^2 - 7t + 10$

5. Find a formula for the quadratic function whose graph has the given properties. Then, graph your function in Desmos to confirm that it meets the given conditions.

[(a)] A y -intercept of $y = 7$ and the only zero at $x = -2$.

[(b)] A y -intercept of $y = 7$ and x -intercepts at $x = 1, x = 4$.

6. Using the factored form, find the formula for the parabola whose zeros are $x = -1$ and $x = 5$, and which passes through the point $(-2, 6)$. Use Desmos to test different values of a in $y = a(x + 1)(x - 5)$ until the parabola passes through the point $(-2, 6)$.

7. A baseball is "popped" straight up in the air by the batter. The height of the ball above the ground is given by the function $[y = f(t) = -16t^2 + 31t + 2,]$ where t is the time in seconds after the ball is caught.

8. Find a formula for the parabola represented by the graph below. Once you have your equation, graph it in Desmos and verify that your model matches the shape and position of the parabola.

3: FUNCTIONS

3.1: Functions and Function Notation

3.2: Domain and range

3.3: Function composition

3.4: Preview of transformations- shifts

3.5: Piecewise-defined functions

3.6: Average Rates of Change and Behavior of Graphs

3.7: Inverse Functions

3.8: More Functions Notation

3.1: Functions and Function Notation

Warm-up:

ADAPT 3.1.1

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1. Write the following relationships using function notation (ex: y as a function of x is written as $y = f(x)$)
 - a. The amount of revenue R in dollars is a function of the number of sales S .
 - b. Distance traveled d as a function of time t .
2. Identify the dependent and independent variables for each function, then evaluate each function at the given input.
 - a. $g(w) = H = 13w^2 - 4w$ evaluated at $w = 3$
 - b. $f(n) = G = 4n + \frac{3}{n}$ evaluated at $n = 2$

c. $h(k) = Q = 2k - 17$ evaluated at $k = 8 - 2p$

3. Evaluate the following function at the given inputs:

$$h(y) = \frac{y^2 - 2}{2y - 3}$$

a. $h(3)$

b. $h(0)$

c. $h(1.5)$

d. $h(\sqrt{5})$

4. The number of gallons left in a gas tank after driving d miles is given by $G(d) = 17 - 0.05d$.

a. Which is larger: $G(50)$ or $G(100)$?

b. Explain your answer in terms of the expression for $G(d)$ and give a practical interpretation.

5. If $f(a) = 4 - 3(a - 2) - a$, evaluate and simplify $f(1 - 2a)$.

3.2: Domain and range

Warm-up:

ADAPT 3.2.1

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ADAPT 3.2.1

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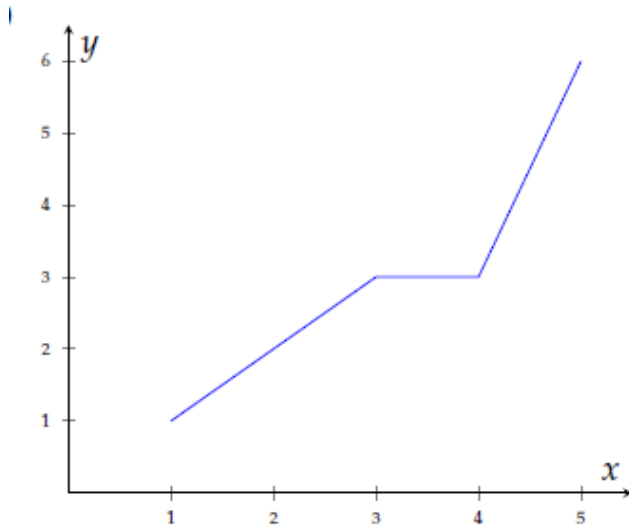


1. For what value(s), if any, is each function undefined? Explain in a sentence what causes the trouble.

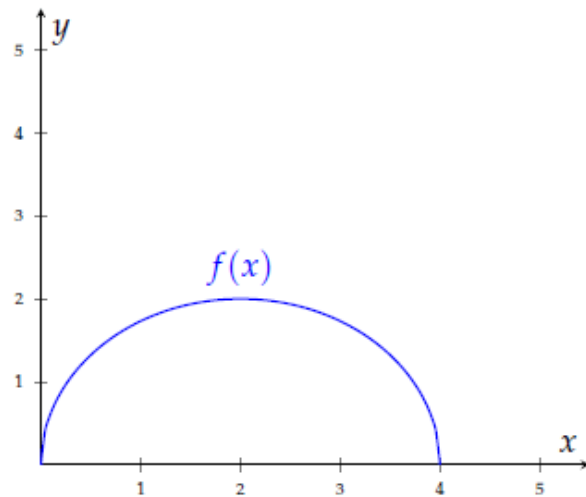
- $f(x) = \frac{x-2}{x-3}$
- $h(x) = \sqrt{x-15}$
- $g(x) = \frac{1}{\sqrt{x-15}}$
- What is the difference in your answer between part (b) and part (c)?

2. Estimate the domain and range of the function. Write your answer as an inequality. Assume the entire graph is shown.

a



b



3. Find the domain of each function algebraically.

- $f(x) = \frac{1}{x+3}$
- $g(t) = \frac{1}{t^2-4}$
- $h(s) = \frac{1}{s^4+2}$
- $p(r) = \sqrt{r^2-16}$
- $q(a) = \frac{a-2}{\sqrt{a-5}}$

4. Give a formula for a function that meets the criteria given.

- The domain is all values of x except $x = -5$.
- The function is undefined for $x < 6$.
- The function is undefined for $x = -2$ and for $x < -4$, but is defined everywhere else.

5. A movie theater seats 200 people. For any particular show, the amount of money the theater makes is a function of the number of people, n , in attendance. If a ticket costs \$4.00, find the domain and range of this function. Sketch its graph.

6. In month $t = 0$, a small group of rabbits escapes from a ship onto an island where there are no rabbits. The island rabbit population, $p(t)$, in month t is given by

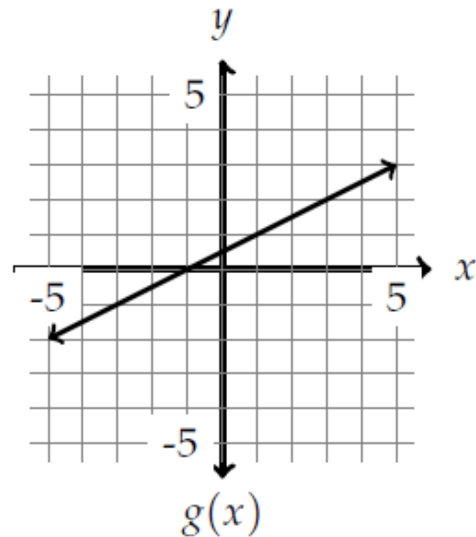
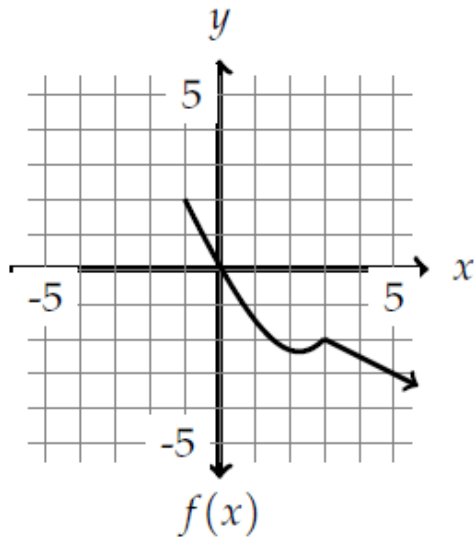
$$p(t) = \frac{1000}{1 + 19(0.9)^t}, \quad t \geq 0$$

- Evaluate $p(0)$, $p(10)$, $p(50)$, and explain their meaning in terms of rabbits.
- Graph $p(t)$ for $0 \leq t \leq 100$. Describe the graph in words. Does it suggest the growth in population you would expect among rabbits on an island?
- Estimate the range of $p(t)$. What does this tell you about the rabbit population?
- Explain how you can find the range of $p(t)$ from its formula.

3.3: Function composition

Warm-up:

1. Below are three functions: f and g are defined by graphs and h is defined by a table. Use them to answer the following questions.



x	-1	0	1	2	3
$h(x)$	2	1	3	4	1

a. Estimate $f \circ g(3)$. Remember, this means $f(g(3))$.

b. Estimate $g \circ f(3)$.

c. Estimate $g(h(1))$.

d. Find $h(h(-1))$.

e. What can you say about $h(h(h(-1)))$?

2. Consider the functions $f(x) = x^2 + x - 3$ and $g(x) = x - 1$.

a. Calculate $f \circ g(4)$.

b. Calculate $f \circ g(-1)$.

c. Give an expression of the form $ax^2 + bx + c$ for the function $f \circ g(x)$. Hint: You need to substitute the x in the expression for

$f(x)$ with the expression for $g(x)$, and then simplify.

d. Check your answer to (c) by plugging in -1 and 4 for x and comparing with (a) and (b).

3. Write each of the following functions as a composition of two simpler functions. Be sure to give your simpler functions names, and write down the order in which they should be composed.

a. $h(x) = \frac{1}{x^2-1}$

b. $v(x) = \sqrt{3x+10}$

4. Olive-Harvey College Cafeteria is preparing a big order for an event. The population (in thousands) of the order of hot dog is given by the function $P = f(t)$ in t years, and the yearly demand (in thousands) for hot dogs as a function of the population is: $D = g(P)$. Write a sentence which gives the meaning of each the following expressions.

a. $f(12) = 2.4$

b. $g(3.5) = 18.4$

c. $g(f(4)) = 12.9$

3.4: Preview of transformations- shifts

1. Use a graphing calculator, Desmos to answer the following questions.

a. Graph the following functions on the same coordinate plane.

$$\begin{aligned}f(x) &= x^2 \\g(x) &= x^2 + 3 \\h(x) &= x^2 - 3\end{aligned}$$

Describe what you notice.

b. Graph the following functions on the same coordinate plane.

$$\begin{aligned}f(x) &= x^2 \\f(x) &= (x + 3)^2 \\f(x) &= (x - 3)^2\end{aligned}$$

Describe what you notice.

c. What is different about the functions in part (??) and part (??)? What effect do those differences have on the resulting graphs?

d. Predict what the graph of $f(x) = (x + 3)^2 - 3$ looks like by sketching it before using a calculator. Then graph the function on a calculator to check your work. Was your graph accurate? Why or why not?

2. The function $P(t)$ gives the number of people in a certain population in year t . Interpret each statement in terms of population.

- a. $P(t) + 100$
- b. $P(t + 100)$

3. Let $s(t)$ denote the average weight (in pounds) of a baby at age t months. The weight, W , of a baby named Ken is related to $s(t)$ by the equation $W = s(t + 4)$.

- a. What can you say about Ken's weight at age $t = 3$ months?
- b. At $t = 6$ months?
- c. Assuming that babies increase in weight over the first year of life, decide if Ken is of average weight for his age, above average, or below average.

4. Figure 2.28 shows a graph of $f(x)$.

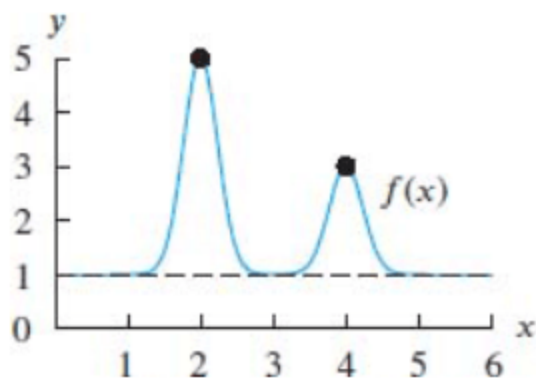


Figure 2.28

Sketch a graph of each of the following transformations of $f(x)$.

- $y = f(x + 2)$
- $y = f(x) + 2$
- $y = f(x + 6) - 4$

5. A graph of a function $g(x)$ contains the point $(-2, 3)$. What point must be on the graph of:

- $g(x) - 4$
- $g(x - 4)$
- $g(x - 2) - 2$

6. Figure 2.31 shows $y = m(r)$. Each graph in parts (a)-(d) is a translation of the graph of $y = m(r)$. Give a formula for each of these functions in terms of m .

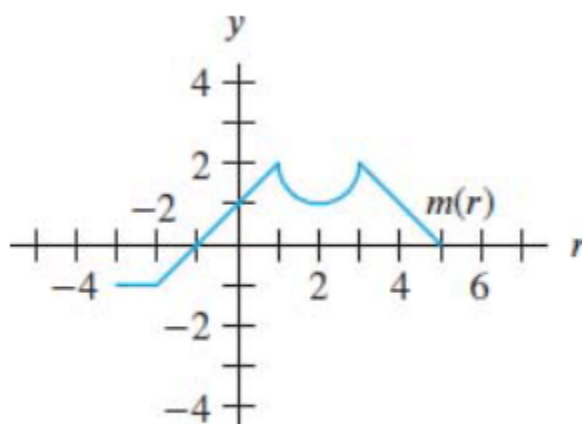
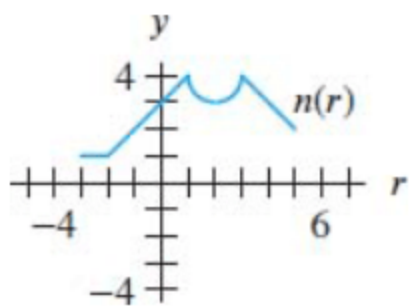
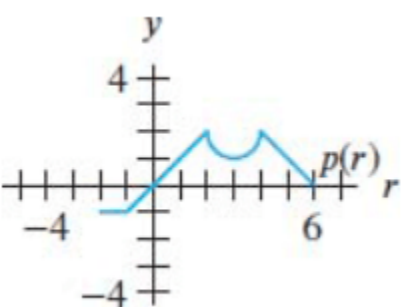


Figure 2.31

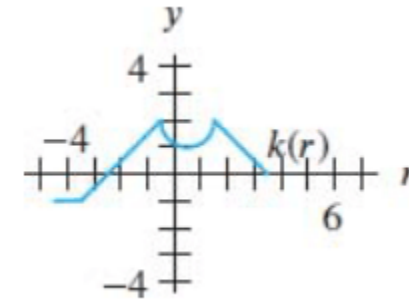
(a)



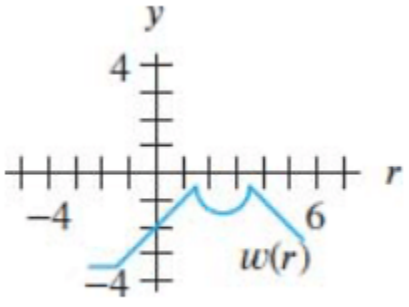
(b)



(c)



(d)



7. The tables below give values of functions v and w .

x	-2	-1	0	1	2		x	3	4	5	6	7
$v(x)$	11	17	20	17	11		$w(x)$	4	10	13	10	4

- Given that $w(x) = v(x - h) + k$, find the constants h and k .
- Use your formula to show that $w(5) = v(0)$.

3.5: Piecewise-defined functions

1. A floor-refinishing company charges \$1.83 per square foot to strip and refinish a tile floor for up to 1000 square feet. There is an additional charge of \$350 for toxic waste disposal for any job that includes more than 150 square feet of tile.

- Express the cost, y , of refinishing a floor as a piecewise-defined function of the number of square feet, x , to be refinished.
- Sketch a graph of the function. Give the domain and range.

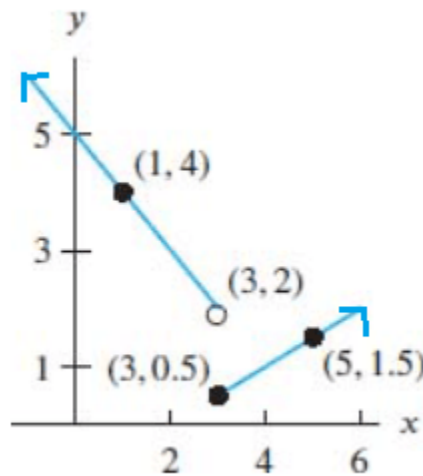
2. The Ironman Triathlon is a race that consists of three parts: a 2.4-mile swim followed by a 112-mile bike race and then a 26.2-mile marathon. A participant swims steadily at 2mph, cycles steadily at 20 mph, and then runs steadily at 9mph. Assuming that no time is lost during the transition from one stage to the next, write a piecewise-defined formula for the distance covered, d , in miles, as a function of the elapsed time t in hours, from the beginning of the race. Draw a graph of the function.

3. Draw a graph of

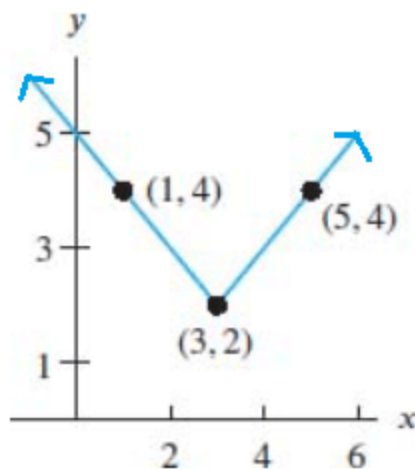
$$f(x) = \begin{cases} x + 1 & \text{for } -2 \leq x < 0 \\ x - 1 & \text{for } 0 \leq x < 2 \\ x - 3 & \text{for } 2 \leq x < 4 \end{cases}$$

4. Write formulas for each of the following functions.

a.



b.



5. The function f is given by

$$f(x) = \begin{cases} 3x & \text{for } -1 \leq x \leq 1 \\ -x+4 & \text{for } x > 1 \end{cases}$$

Use it to answer the following questions.

- Find $f(0)$ and $f(3)$.
- Find the domain and range of $f(x)$.
- Sketch a graph of $f(x)$ using Desmos.

6. Let $f(x) = 10|x - 5|$.

- What are the domain and range of $f(x)$?
- Find all values of x such that $f(x) = 20$.

7. Let $f(x) = 6|x + 5| - 7$

- What are the domain and range of $f(x)$?
- Find all values of x such that $f(x) = 11$.

8. Given that $f(x) = |x^2 - 4|$, use the definition of absolute value to write a piecewise formula for f . Sketch a graph of f .

3.6: Average Rates of Change and Behavior of Graphs

1. Let $g(y) = 2y^2 + 3y - 1$.

- Find the average rate of change of g between $y = 1$ and $y = 3$. Is the function increasing or decreasing?
- Find the average rate of change of g between $y = 0$ and $y = 10$. Is the function increasing or decreasing?

2. Let $f(x) = 4 - x^2$.

- Find $f(0)$ and $f(2)$. What is the average rate of change of $f(x)$ on the interval $0 \leq x \leq 2$? Is the function increasing or decreasing?
- Find the average rate of change of $f(x)$ on the interval $2 \leq x \leq 4$. Is the function increasing or decreasing?
- Find the average rate of change of $f(x)$ on the interval $b \leq x \leq 2b$. Is the function increasing or decreasing?

3. Consider the following function table:

t	-2	-1	0	1	2	3
$f(t)$	-3	-10	-3	3	18	2

- What is $f(-1)$?
- What is $f(3)$?
- What value of t gives us $f(t) = -3$?
- How does f change from $t = -2$ to $t = 2$? (i.e. what is Δf ?)
- What is the average rate of change from $t = -2$ to $t = 2$?

- f. Graph the function using Desmos.

4. Taylor Fritz is competing in the finals of the 2024 US Open. When Fritz returns a serve, the speed of his ball can be modeled by the following function:

$$s(t) = 100t - 21t^2 \quad (3.6.1)$$

where t is the time in seconds after he hits the ball, and $s(t)$ is the speed of the ball (in miles per hour).

- a. Evaluate and interpret $s(3)$ within the context of this problem.
- b. What is the average speed of the ball within the first 3 seconds of the ball being hit? (In other words: what is the average speed between $t = 0$ and $t = 3$?) (Be sure to include units!)
- c. Is the ball's average speed faster in the first 2 seconds after being hit, or between 2 and 4 seconds?
- d. Evaluate and interpret $s(5)$ Does it make sense to ask the same questions as parts (b.) and (c.) if we consider $t = 5$?
- e. Graph the function using Desmos.

3.7: Inverse Functions

1. For each of the functions below, use the "solve for y " method we discussed to find the inverse function and graph the function and its inverse in Desmos.

[(a)] $f(x) = 2x - 5$.

[(b)] $f(x) = (x + 2)^2 - 1$. Hint: do not expand the square!

[(c)] $h(t) = \frac{1}{3-t}$

[(d)] $b(x) = 4 + e^{3t-8}$

2. For one of your answers above, simplify expressions for $f^{-1} \circ f(x)$ and $f \circ f^{-1}(x)$. For any one you choose, the simplified answer should be the same. What is it?

3. Consider $f(x) = 4x^2 - 1$, $g(x) = 2x - 1$. Find simplified formulas for

[(a)] $f(x) + g(x)$

[(b)] $\frac{f(x)}{g(x)}$

[(c)] $f(x)g(x)$

[(d)] $g(f(x))$

[(e)] $f(g(x))$

4. A car company's total cost is $100000000 + 20000n$, where n is the number of cars they sell. They also estimate that n , the number of cars they sell, is a linear function of the price p , with the property that

- if they sell cars for \$28,000, they can sell 500, 000; and
- for each \$1,000 they increase the price, they will lose 10,000 customers.

[(a)] Write down a combination of functions which gives the company's profit as a function of the price p they decide to sell cars at.

[(b)] What should the company charge for their cars?

3.8: More Functions Notation

1. Consider the table below.

t = time (in years) after the year 2000
 w = number of Chicago '49er victories (3.8.1)

t	0	1	2	3	4	5	6	7	9
w	6	12	10	7	2	4	7	5	7

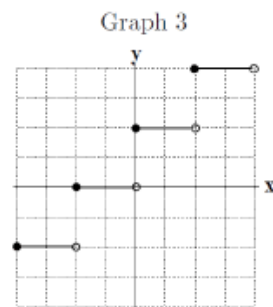
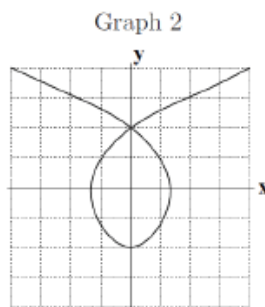
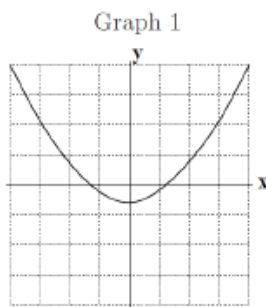
[(a)] Does this table represent w as a function of t ? How do you know?

[(b)] In what other ways could you represent this relationship? Explain.

[(c)] What is $f(4)$?

[(d)] Explain the meaning of $f(4)$ in a sentence or two. Include units.

2. Which of the graphs below represent y as a function of x ? Explain how you know in a sentence.



3. Refer to Graph 1 above. Fill in the missing values:

[(a)] What is the y -value when $x = 0$?

[(b)] $f(0) = ?$

[(c)] What are the x -values that produce a y -value of 0?

[(d)] $f(?) = 0$

4. Suppose you graph a function $p = f(w)$.

[(a)] Which variable goes on the horizontal axis?

[(b)] If $10 = f(-4)$, give the coordinates of a point on your graph.

[(c)] If 6 is a solution of the equation $f(w) = 1$, give a point on your graph.

5. The combined sales tax rate for Chicago, IL is 10.25%.

[(a)] If the price of a Olive Harvey t-shirt is \$20.00, what is the total cost you would pay at the register, including tax?

[(b)] If the price of an item in Chicago is P dollars, express the total cost (including tax), C , as a function of P . Can you simplify your answer?

[(c)] You notice that you can purchase a Chicago t-shirt tax-free at the airport. The shirt, however, costs \$24.00. It is on sale for 15% off. What is the total cost of the t-shirt, after the discount?

[(d)] If the price of an item is still P dollars, express the cost, C , of the item with a 15% discount as a function of P . Can you simplify your answer?

6. There are x male job-applicants at a certain company and y female applicants. Suppose that 15% of the men are accepted and 18% of the women are accepted. Write an expression in terms of x and y representing each of the following quantities:

[(a)] The total number of applicants to the company.

[(b)] The total number of applicants accepted.

[(c)] The percentage of all applicants accepted.

4: LINEAR FUNCTIONS

4.1: Introduction to Linear Functions

4.2: Modeling with linear functions

4.3: Another Introduction to Linear Functions

4.1: Introduction to Linear Functions

Warm-up:

ADAPT 4.1.1

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1. A stalactite (a type of formation hanging like an icicle from the roof of a cave) grows according to the formula $L(t) = 17.75 + \frac{1}{250}t$, where $L(t)$ represents the length of the stalactite, in inches, and t represents the time, in years, since the stalactite was first measured.

- What is the vertical intercept?
- Explain the meaning of the vertical intercept in practical terms.
- What is the slope?
- Explain the meaning of the slope in practical terms.

2. A new Toyota RAV4 costs \$23,500. The car's value depreciates linearly to \$18,823 in three years time. Write a formula which expresses its value, V , in terms of its age, t , in years.

Year	1950	1960	1970	1980	1990	2000
Population of country A (millions)	8.2	9.8	12.4	15.1	14.7	23.9
Population of country B (millions)	7.5	9.9	12.5	14.9	17.2	19.2

3. Chicago experienced approximately linear population growth from 1950 to 2000. On the other hand, Indiana was torn by warfare in the 1980s and did not experience linear or near-linear growth.

- The table below gives the population of these two countries, in millions. Which of these two countries is A and which is B? Explain.
- What is the approximate rate of change of the linear function? What does the rate of change represent in practical terms?
- Estimate the population of Chicago in 1988.

4. A company finds that there is a linear relationship between the amount of money that it spends on advertising and the number of units it sells. If it spends no money on advertising, it sells 300 units. For each additional \$5000 spent, an additional 20 units are sold.

- If x is the amount of money that the company spends on advertising, find a formula for y , the number of units sold as a function of x .
- How many units does the firm sell if it spends \$25,000 on advertising? \$50,000?
- How much advertising money must be spent to sell 700 units?
- What is the slope of the line you found in part (a)? Give an interpretation of the slope that relates units sold and advertising costs.

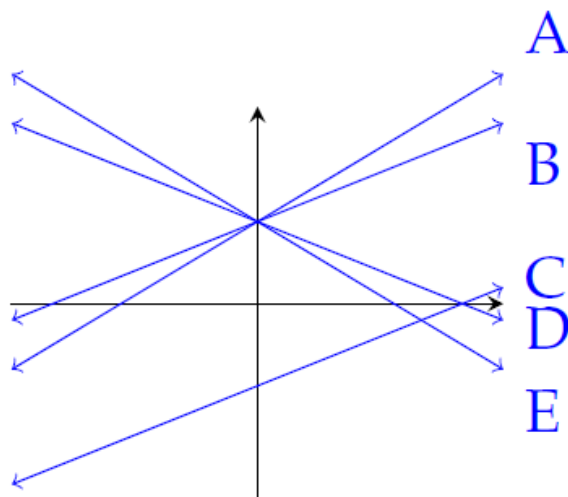
5. Graph the following function using Desmos:

$$y = -x \left(\frac{x-1000}{900} \right)$$

- Is this graph linear? Explain your reasoning.
- Expand the viewing window such that $-500 \leq x \leq 500$ and $-500 \leq y \leq 500$. Is this graph linear?
- Using what you know about rate of change, explain your answers to part (a) and (b).

4.2: Modeling with linear functions

1. Without a calculator, match the following functions to the lines in the figure below.



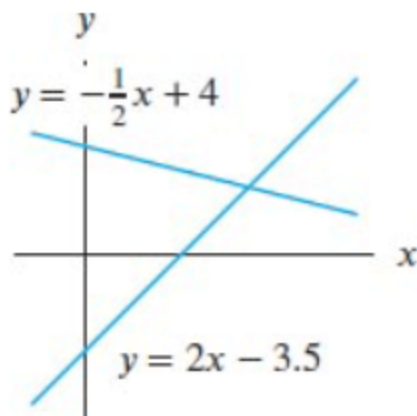
- $f(x) = 5 + 2x$
- $g(x) = -5 + 2x$
- $h(x) = 5 + 3x$
- $j(x) = 5 - 2x$
- $k(x) = 5 - 3x$

2. The cost, in dollars, of renting a car for a day from three different rental agencies and driving it d miles is given by the following functions:

$$\begin{aligned} C_1 &= 0.50d \\ C_2 &= 30 + 0.20d \\ C_3 &= 50 + 0.10d \end{aligned} \tag{4.2.1}$$

- For each agency, describe the rental agreement in words.
- Graph the cost function for each agency on one set of axes.
- For what driving distance does Agency 1 cost the same as Agency 2?
- If you drive 50 miles or less, which agency is cheapest?
- How do you determine which agency provides the cheapest car rental for any given driving distance?

3. Determine the point of intersection.



4. Line l in Figure 1.52 below is parallel to the line $y = 2x + 1$. Find the coordinates of the point P .

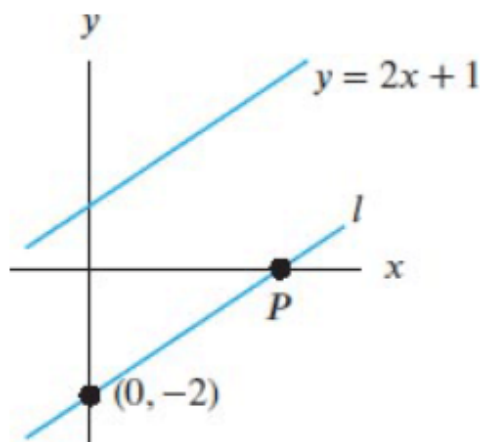


Figure 1.52

a. \

5. In economics, the demand for a product is the amount of that product that consumers are willing to buy at a given price. The quantity demanded of a product usually decreases if the price of that product increases. Suppose that a company believes there is a linear relationship between the demand for its product and its price. The company knows that when the price of its product was \$3 per unit, the quantity demanded weekly was 500 units, and that when the unit price was raised to \$4, the quantity demanded weekly dropped to 300 units. Let D represent the quantity demanded weekly at a unit price of p dollars.

- Calculate D when $p = 5$. Interpret your result.
- Find a formula for D in terms of p .
- The company raises the price of the good and the new quantity demanded weekly is 50 units. What is the new price?
- Give an economic interpretation of the slope of the function you found in part (b).
- Find D when $p = 0$. Find p when $D = 0$. Give economic interpretations of both these results.

4.3: Another Introduction to Linear Functions

1. Are the following functions linear? If so, identify the slope and initial value of the function. If not, explain why not. (Recall that a function is linear if it can be written in the form $f(x) = mx + b$.)

[(a)] $f(x) = 7x + 5$

[(b)] $f(x) = \frac{7}{x} + 5$

[(c)] $f(x) = 3 - 2(x + 4)$

[(d)] $f(x) = 3(7 - x) - (x - 2)$

[(e)] $f(x) = 3x^2 + 4$

[(f)] $y - 3 = \frac{x - 4}{2}$

[(g)] $f(x) = (2 - a)x - 4a$

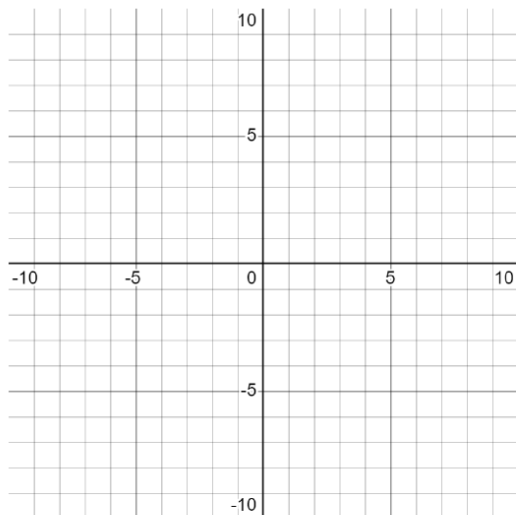
[(h)] $4y - 7x = 4$

2. Graph each of the following functions using Desmos. Don't forget to label them!

[(a)] $y = 3x - 6$

[(b)] $f(x) = 2 - \frac{1}{2}x$

[(c)] $y = -9$ (Hint: Compare this function to the general form of a linear function)



[3.] Cara wants to lose weight. She improves her diet and exercises more regularly. She begins her weight loss journey weighing 227 pounds, and on average she loses about 3 pounds per month.

[(a)] Write a function modeling Carol's current weight W in pounds t months since beginning her weight loss journey. Is this function linear?

[(b)] Approximately how long will it take for Carol to reach her weight target of 160 pounds? check your answer by graphing the function from part a on Desmos.

[(c)] Is this model realistic? Evaluate your function at $t = 100$ and discuss what this means about our model.

[4.] Solve $f(t) = g(t)$, where $f(t) = 40 + 5t$ and $g(t) = -3t + 800$. Confirm your answer by graphing the two functions on Desmos.

5: QUADRATIC FUNCTIONS

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5.1: Introduction to quadratic functions

ADAPT 5.1.1

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ADAPT 5.1.1

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1. Find the zeros of $Q(s) = 3s^2 - 3s - 60$ by factoring. Show your work.
2. Find the zeros of $Q(x) = 5x - x^2 + 3$ using the quadratic formula. Show your work.
3. First, sketch a graph of a quadratic function which has all the following properties: y -intercept equal to -6 , zeros at $x = -2$ and $x = 3$. Then, write a formula for this quadratic function.
4. Without a calculator, graph $y = 3x^2 - 16x - 12$ by factoring and plotting zeros. Then check your answer using Desmos.
5. Find a formula for the quadratic functions described below.
 - a. A y -intercept of $y = 7$ and the only zero at $x = -2$.
 - b.
6. A ball is thrown into the air. Its height (in feet) t seconds later is given by $h(t) = 80t - 16t^2$.
 - a. Evaluate and interpret $h(2)$.
 - b. Solve the equation $h(t) = 80$. Interpret your solutions and illustrate them on a graph of $h(t)$.

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5.2: Vertex of a parabola

1. Let $f(x) = x^2$ and $g(x) = 3(x - 1)^2 + 2$.

- What is the vertex of $f(x)$?
- Since $g(x)$ is a translation of $f(x)$, use your knowledge of horizontal and vertical shifts to identify the vertex of $g(x)$.
- What is the formula for the vertex form of a quadratic function?
- According to the formula, was the vertex you identified in part (b) correct? Why or why not?

2. Let $g(x) = (x + 3)^2 - 4$.

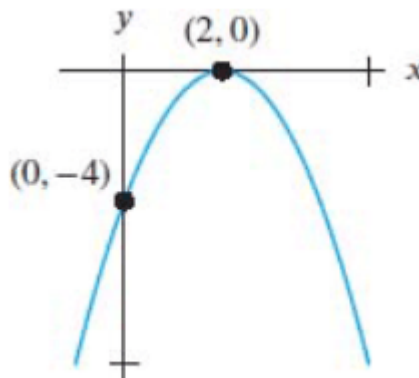
- What is the vertex of $g(x)$?
- Graph $g(x)$ using Desmos. What is the axis of symmetry of $g(x)$?
- How can you find the axis of symmetry for any given quadratic function?
- Write the function $g(x)$ in standard form.
- Explain how the vertical intercept can be found from the standard form.
- Find the zeros of $g(x)$.
- Explain how the axis of symmetry of the parabola is related to the zeros.

3. Let $f(x) = 3x^2 - 6x + 3$.

- What is the vertical intercept of $f(x)$?
- Find the zeros of $f(x)$.
- Write the function $f(x)$ in vertex form.
- What is the vertex of $f(x)$?
- What is the axis of symmetry of $f(x)$?

4. Find a formula for the quadratic function based on the information given.

a.



- Vertex at $(4, 2)$ and y -intercept of $y = 6$.
- Vertex at $(4, 2)$ and zeros at $x = -3$ and $x = 11$.

5. Let $m(n) = \frac{1}{2}n^2$. Write a formula in standard form for each of the transformations listed below. Graph the new function.

- $g(n) = m(n) + 1$
- $h(n) = m(n + 1)$

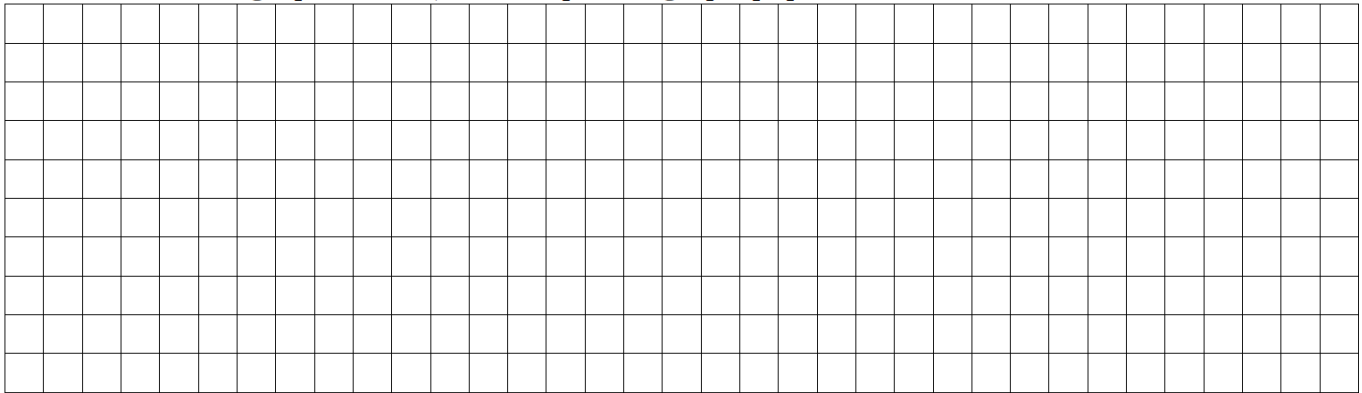
6. Let $f(x) = 2x^2 + 4x + 4$ and $g(x) = -2x^2 + 4x + 4$.

- a. Does $f(x)$ have an absolute maximum or an absolute minimum? How do you know? What is its coordinate?
- b. Does $g(x)$ have an absolute maximum or an absolute minimum? How do you know? What is its coordinate?

7. If you have a string of length 50 cm, what are the dimensions of the rectangle of maximum area that you can enclose with your string? Hint: let one side of your string rectangle have a length of x cm. Then write an expression for the length of the other side, and use the area formula for a rectangle.

5.3: Power functions

1. Use a graphing calculator, Desmos, graph $y = x^2$, $y = x^3$, $y = x^4$, and $y = x^5$ on the same coordinate plane. Draw these graphs below, or on separate graph paper.

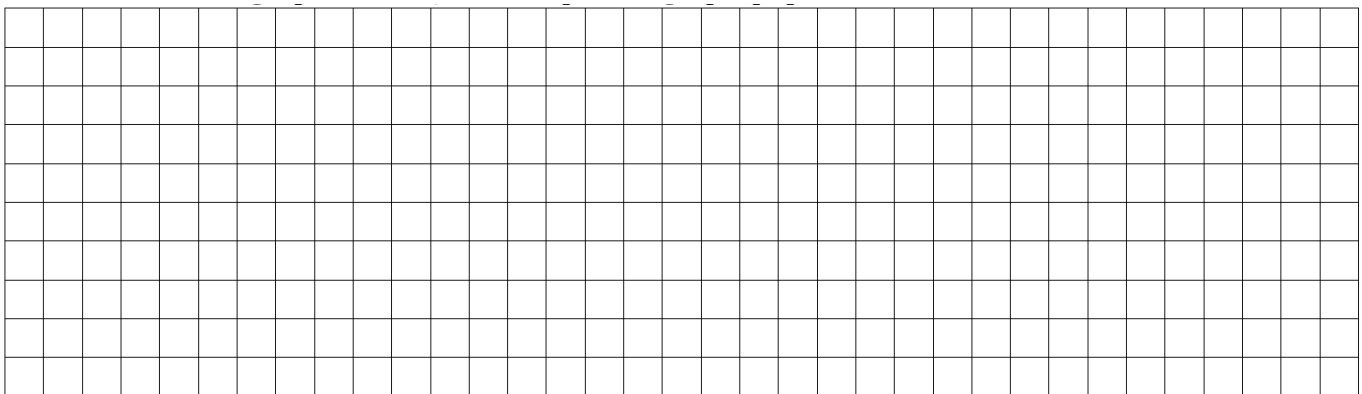


Discuss with your group and write a "rule" in a sentence or two.

- If we can write the general form of a power function as $f(x) = kx^p$, what can you say about all graphs that have a positive, even integer p ?
- What can you say about all graphs that have a positive, odd integer p ?

2. Rewrite $y = x^{-2}$, $y = x^{-3}$, and $y = x^{-p}$ as functions without negative integers. Describe in a sentence how to rewrite a negative-integer power function with only positive powers.

3. Use a graphing calculator, Desmos, graph $y = x^{-2}$, $y = x^{-3}$, $y = x^{-4}$, and $y = x^{-5}$ on the same coordinate plane. Draw these graphs below, or on separate graph paper.



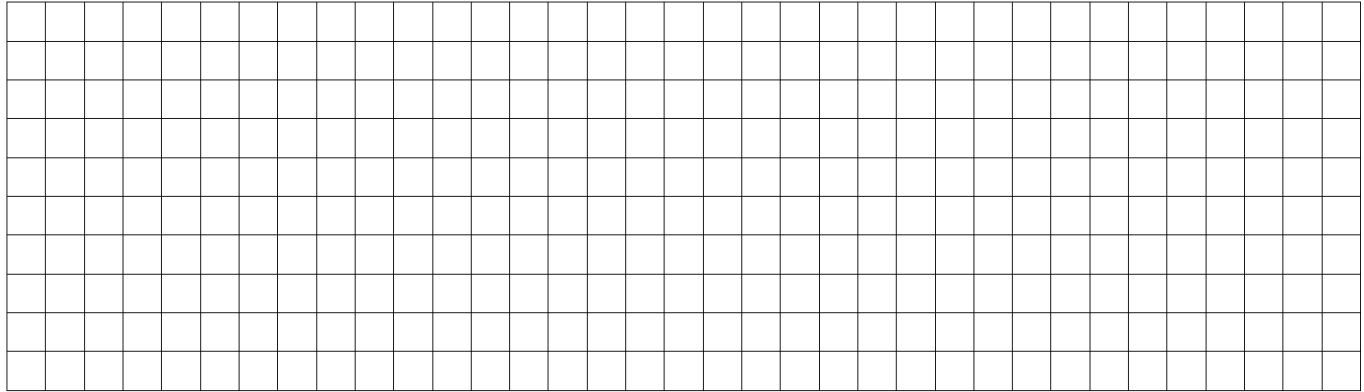
Discuss with your group and write a "rule" in a sentence or two.

- What can you say about all graphs that have a negative, even integer p ?
- What can you say about all graphs that have a negative, odd integer p ?

4. Rewrite $y = x^{\frac{1}{3}}$ and $y = 2x^{\frac{1}{2}}$ without fractional exponents.

5. Use a graphing calculator, Desmos, graph $y = x^{\frac{1}{2}}$, $y = x^{\frac{1}{3}}$, $y = x^{\frac{1}{4}}$, and $y = x^{\frac{1}{5}}$ on the same coordinate plane. Draw these graphs

below, or on separate graph paper.



Discuss with your group and write a "rule" in a sentence or two.

- a. What can you say about all graphs that have a fractional power with an even denominator?
- b. What can you say about all graphs that have a fractional power with an odd denominator?

6. Each part of this problem will explore a function's long-run behavior, that is, what happens to the value of $f(x)$ as x becomes infinitely large.

- a. For the functions $f(x) = x^2$ and $g(x) = x^3$, make a table of values for $x = 10, 20, 30, 40$, and 50 . What value does each function appear to be approaching as x increases?
- b. For the functions $f(x) = x^{-2}$ and $g(x) = x^{-3}$, make a table of values for $x = 10, 20, 30, 40$, and 50 . What value does each function appear to be approaching as x increases?
- c. Use a similar method to test the long-run behavior of $y = x^{\frac{1}{2}}$ and $y = x^{\frac{1}{4}}$.
- d. Discuss with your group and make a general rule about the long-run behavior of a power function $y = x^p$. Make sure you clearly indicate any different cases depending on the value of p .

7. For the functions $f(x) = x^{-2}$ and $g(x) = x^{-3}$, make a table of values for $x = 0.1, 0.05, 0.01, 0.001, 0.0001$, and 0 . What value does each function appear to be approaching as x approaches zero from the right?

5.4: Power functions and proportionality

1. If we have a function $y = 3x$, we say that y is directly proportional to x .
 - a. What is the constant of proportionality, k , in this function?
 - b. Write a new function where y is directly proportional to x , but has a constant of proportionality equal to 6 .
 - c. Now let's say we have a function where y is directly proportional to the square of x . The constant of proportionality is 4 . Write the formula for this new function.

2. If we have a function $y = \frac{3}{x}$, we say that y is inversely proportional to x .
 - a. What is the constant of proportionality, k , in this function?
 - b. Write a new function where y is inversely proportional to x , but has a constant of proportionality equal to 6 .
 - c. Now let's say we have a function where y is inversely proportional to the square of x . The constant of proportionality is 4 . Write the formula for this new function.

3. Suppose c is directly proportional to the square of d . Let $c = 8$ when $d = 2$.
 - a. Find the constant of proportionality, k .
 - b. Write the formula for c as a function of d .
 - c. Check to make sure your formula is correct.

4. Two oil tankers crash in the Pacific Ocean. The spreading oil slick has a circular shape, and the radius of the circle is increasing at 200 meters per hour.
 - a. Express the radius of the spill, r , as a function of time, t , in hours since the crash.
 - b. The area of the spill A is proportional to the square of the radius r . When the radius is 400 meters, the area of the spill is 502,654 square meters. Find a formula for A as a function of t .
 - c. Cleanup efforts begin 7 hours after the spill. How large an area is covered by oil at that time?

5. In a microwave oven, cooking time is inversely proportional to the amount of power used. It takes 6.5 minutes to heat a frozen dinner at 750 watts.
 - a. Write a formula for the cooking time, t , as a function of power level, w .
 - b. Fill in Table 11.4 with the cooking times needed to heat the frozen dinner at various power levels.
 - c. Graph the function $t = f(w)$.
 - d. If it takes 3 minutes to heat a rhubarb crumble at 250 watts, how long will it take at 500 watts?

6. If y is inversely proportional to x , and $y = 6$ when $x = 4$:
 - a. Find the constant of proportionality, k .
 - b. Write a formula for y in terms of x .
 - c. Check to make sure your formula is correct.
 - d. Using your formula, find x when $y = 8$.

7. In a given power function, $f(2) = 16$ and $f(3) = 54$. Your goal will be to find a possible formula for the power function.
 - a. Write two equations that you know, by substituting the values above in the general form of the power function $f(x) = kx^p$.
 - b. Write the ratio of the two equations you found in the form $\frac{f(3)}{f(2)}$.
 - c. Simplify both sides of the equation and solve for p .

- d. How do you think you can solve for k ? Try your strategy to find the value of k .
- e. Using your values of p and k , write the possible formula for the power function.
- f. How can you check to make sure your formula is correct?

8. A new power function includes the points $(1, \frac{3}{2})$ and $(2, \frac{3}{8})$. Using the same process as above, find a possible formula for the power function.

- a. Write two equations that you know, by substituting the values above in the general form of the power function $f(x) = kx^p$.
- b. Use the ratio of $\frac{f(2)}{f(1)}$ to solve for p .
- c. How do you think you can solve for k ?
- d. Write the possible formula for the power function.
- e. Check to make sure your formula is correct.

5.5: Graphs of Polynomial Functions

1. Let $u(x) = -\frac{1}{5}(x-3)(x+1)(x+5)$ and $v(x) = -\frac{1}{5}x^2(x-5)$.

- Find the zeros for $u(x)$ and $v(x)$.
- Graph u and v using a graphing calculator or other graphing tool. Did you identify the zeros correctly? How do you know?
- How are the graphs similar? How are they different?
- Using Desmos, zoom out to a much larger viewing window. What do you notice about the graphs now? How are they similar, and how are they different?

2. Let $y = 1 - 2x^4 + x^3$.

- How many terms does this polynomial have?
- What is the degree of this polynomial?
- What is the leading coefficient?
- What is the constant term?
- If $x = 100$, what is the value of each term? If $x = 100$, what is the value of the function?
- As x becomes really large, which term can be used as an approximation of the value of the function?
- Describe the end behavior of the function as $x \rightarrow \infty$, and describe the end behavior of the function as $x \rightarrow -\infty$.

3. Let $f(x) = -\frac{1}{5}(x-3)(x+1)(x+5)$.

- What is the degree of this polynomial?
- What is the leading coefficient?
- As x becomes really large, which term can be used as an approximation of the value of the function?

Describe the end behavior of the function as $x \rightarrow \infty$, and describe the end behavior of the function as $x \rightarrow -\infty$.

4. The town of Liddleville was founded in 1890. Its population y (in hundreds) was modeled, for t years since 1890, by

$$y = 1 - 0.58t + 4.89t^2 - 1.872t^3 + 0.247t^4 - 0.0111t^5. \quad (5.5.1)$$

- Graph the function using Desmos. Use an appropriate viewing window to make sure you understand what is happening with the function.
- What was the population of Liddleville when it was founded?
- When did the population of Liddleville reach zero? Give the year and the month.
- What was the largest population of Liddleville after 1890? When did Liddleville reach that population? Give the month and year.
- What population was predicted for 1898? Explain what this means.

5. The volume, V , in milliliters, of 1 kg of water as a function of temperature T is given, for $0 \leq T \leq 30$ degrees C, by

$$V = 999.87 - 0.06426T + 0.0085143T^2 - 0.0000679T^3. \quad (5.5.2)$$

- a. Graph V using Desmos.
- b. Describe the shape of your graph. Does V increase or decrease as T increases? Does the graph curve upward or downward? What does the graph tell us about how the volume varies with temperature?
- c. At what temperature does water have the maximum density? How does that appear on your graph? (Density=Mass/Volume. In this problem, the mass of the water is 1 kg .)

5.6: Rational functions

1. Bronze is an alloy, or mixture, of copper and tin. The alloy initially contains 3 kg of copper and 9 kg tin. You add x kg of copper to this 12 kg of alloy. The concentration of copper in the alloy is a function of x :

$$(\text{Concentration of copper}) = f(x) = \frac{\text{Total amount of copper}}{\text{Total amount of alloy}}.$$

- Find a formula for f in terms of x , the amount of copper added.
- Evaluate the following expressions and explain their significance for the alloy:
 - $f\left(\frac{1}{2}\right)$
 - $f(0)$
 - $f(-1)$
 - $f^{-1}\left(\frac{1}{2}\right)$
 - $f^{-1}(0)$
- Graph $f(x)$ using Desmos. Interpret the intercepts in the context of the alloy.
- Zoom out so you can see a larger viewing window. Does the appearance of the graph agree with what you expect to happen when large amounts of copper are added to the alloy?

2. The total cost $C(n)$ for a producer to manufacture n units of a good is given by

$$C(n) = 5000 + 50n.$$

The average cost of producing n units is $a(n) = \frac{C(n)}{n}$.

- Evaluate $C(1)$, $C(100)$, $C(1000)$, and $C(10000)$. Explain in a sentence the economic significance of these values.
- Evaluate: $a(1)$, $a(100)$, $a(1000)$, and $a(10000)$. Explain in a sentence the economic significance of these values.
- Based on part (b), what trend do you notice in the values of $a(n)$ as n gets large? Explain this trend in economic terms.

3. Knowing that we can express a rational function as $r(x) = \frac{p(x)}{q(x)}$, answer the questions below.

- If $f(x) = \frac{x^2+1}{-3x^2+5}$, what is $p(x)$ and what is $q(x)$?
- What are the leading terms of p and q for this rational function?
- To determine the long-run behavior of a rational function, we can simply examine the function $y = \frac{\text{Leading term of } p}{\text{Leading term of } q}$.
- What is the new rational function we can use to examine the long-run behavior of $f(x)$?
- What does this new function tell us about the long-run behavior of $f(x)$?
- Graph $f(x)$ using Desmos. Does the graph match your answer to part (f)?

4. Using the same steps from problem 1, determine the long-run behaviors of the following:

- $f(x) = \frac{2x^2+1}{x^2+5}$
- $g(x) = \frac{x^3+1}{x^2+5}$
- $h(x) = \frac{x+1}{x^2+5}$
- Check your work on each part above by using Desmos.
- Compare the long-run behavior of $f(x)$, $g(x)$, and $h(x)$.
- What do you notice? Discuss.

5. Let $f(x) = \frac{3x^2+x}{2x^2+5x^3}$.

- Graph $f(x)$ using Desmos.
- What does the function approaches when x goes to positive/negative infinity?

6. It costs a company \$30,000 to begin production of a good, plus \$3 for every unit of the good produced. Let x be the number of units produced by the company.

- Find a formula for $C(x)$, the total cost for the production of x units of the good.
- Find a formula for the company's average cost per unit, $a(x)$.
- Graph $y = a(x)$. What is the horizontal asymptote?
- Explain in economic terms why the graph of a has the long-run behavior that it does.
- Explain in economic terms why the graph of a has the vertical asymptote that it does.
- Find a formula for $a^{-1}(y)$. Give an economic interpretation of $a^{-1}(y)$.
- The company makes a profit if the average cost of its good is less than \$5 per unit. Find the minimum number of units the company can produce and make a profit.

5.7: Quadratic Equation Vertex Form

1. Find the vertex and axis of symmetry of the graph of $v(t) = t^2 + 11t - 4$. Graph using Desmos
2. Find the vertex and axis of symmetry for the parabola whose equation is $y = 3x^2 - 6x + 5$. Graph using Desmos
3. Convert the quadratic functions to vertex form. Identify the vertex and the axis of symmetry.
[(a)] $f(x) = x^2 + 8x + 3$
[(b)] $g(x) = -2x^2 + 12x + 4$
4. Find the value of k so that the graph of $y = (x - 3)^2 + k$ passes through the point $(6, 13)$.
5. The parabola $y = ax^2 + k$ has vertex $(0, -2)$ and passes through the point $(3, 4)$. Find its equation.
6. If $f(x) = x^2$ and $g(3) = 16$, give a possible formula for $g(x)$, which is a result of applying a horizontal shift 2 units to the right and appropriate vertical shift to $f(x) = x^2$.
7. Find a formula for the quadratic function whose graph has the given properties
[(a)] Vertex at $(4, 2)$ and y -intercept of $y = -4$
[(b)] Vertex at $(4, 2)$ and zeros at $x = -3, 11$.
[(c)] A single x -intercept at $x = 1/2$ and a y -intercept at 3.
[(d)] Graph part a, b, and c using Desmos

5.8: Dividing Polynomials

For problems 1-6 use long division to perform the indicated division.

[1.] Divide $7x^2 + 4x - 9$ by $x - 1$

[2.] Divide $8x^3 - 4x + 1$ by $x + 6$

[3.] Divide $x^4 - 2x^2 + 7x$ by $x - 4$

[4.] Divide $2x^4 - 9x^3 + 2x + 8$ by $x + 3$

[5.] Divide $8x^4 + x^3 - 3x^2 + 1$ by $x^2 - 2$

[6.] Divide $4x^5 - 7x^3 + x^2 - 4x + 2$ by $2x^2 - 3x - 6$

For problems 7-11 use synthetic division to perform the indicated division.

[7.] Divide $-x^3 - 8x^2 + x + 10$ by $x + 2$

[8.] Divide $10x^3 - 9x$ by $x - 10$

[9.] Divide $3x^4 + 5x^3 + x - 2$ by $x + 7$

[10.] Divide $x^4 + 2x^3 - 9x + 11$ by $x + 3$

[11.] Divide $5x^4 - 4x^3 + 3x^2 - 2x + 1$ by $x - 1$

5.9: Dividing Polynomials

For problems 1-6 use long division to perform the indicated division.

1. Divide

$$\frac{7x^2 + 4x - 9}{x - 1}. \quad (5.9.1)$$

2. Divide

$$\frac{8x^3 - 4x + 1}{x + 6}. \quad (5.9.2)$$

3. Divide

$$\frac{x^4 - 2x^2 + 7x}{x - 4}. \quad (5.9.3)$$

4. Divide

$$\frac{2x^4 - 9x^3 + 2x + 8}{x + 3}. \quad (5.9.4)$$

5. Divide

$$\frac{8x^4 + x^3 - 3x^2 + 1}{x^2 - 2}. \quad (5.9.5)$$

6. Divide

$$\frac{4x^5 - 7x^3 + x^2 - 4x + 2}{2x^2 - 3x - 6}. \quad (5.9.6)$$

For problems 7-11 use synthetic division to perform the indicated division.

7. Divide

$$\frac{-x^3 - 8x^2 + x + 10}{x + 2}. \quad (5.9.7)$$

8. Divide

$$\frac{10x^3 - 9x}{x - 10}. \quad (5.9.8)$$

9. Divide

$$\frac{3x^4 + 5x^3 + x - 2}{x + 7}. \quad (5.9.9)$$

10. Divide

$$\frac{x^4 + 2x^3 - 9x + 11}{x + 3}. \quad (5.9.10)$$

11. Divide

$$\frac{5x^4 - 4x^3 + 3x^2 - 2x + 1}{x - 1}. \quad (5.9.11)$$

12. Desmos Exploration Choose one of Problems 7-11. Enter the polynomial into Desmos and evaluate the function at the value associated with the divisor $x - c$.

- Record the value of $f(c)$.
- Compare this value to the remainder from your synthetic division.
- What relationship do you notice between $f(c)$ and the remainder?

5.10: Zeros of Polynomial Functions

1. Using Desmos graphing calculator, graph the following functions:

$$\begin{aligned} f(x) &= x^4 - 4x^3 + 16x - 16 \\ g(x) &= x^4 - 4x^3 - 4x^2 + 16x \\ h(x) &= x^4 + x^3 - 8x^2 - 12x \end{aligned} \tag{5.10.1}$$

[(a)] Begin by zooming out to a very large viewing window. What do you notice about each function?

[(b)] Zoom back in to a standard viewing window. What is similar about each function? What is different about each function?

[(c)] How many zeros does each function have? List the zeros for each function.

2. Factor $f(x) = 8x^3 - 4x^2 - 60x$ completely, and determine the zeros of f .

3. Use a graph to determine the zeros of $f(x) = x^4 + x^3 - 11x^2 - 9x + 18$. Write $f(x)$ in factored form.

4. Sketch a graph of $g(x) = 5(x - 4)(x^2 - 25)$ without using a calculator. Label all x and y -intercepts.

5. Let $r(x) = (x - 4)^2$ and $s(x) = (x - 4)^3$.

[(a)] Knowing that $r(x) = (x - 4)^2 = (x - 4)(x - 4)$, identify the zeros of $r(x)$. How many zeros does this function have? How many times did the zero occur?

[(b)] Knowing that $s(x) = (x - 4)^3 = (x - 4)(x - 4)(x - 4)$, identify the zeros of $s(x)$. How many zeros does this function have? How many times did the zero occur?

[(c)] Using a calculator or graphing tool, graph $r(x)$ and $s(x)$ on the same coordinate plane. What is different about the graphs?

[(d)] Now let $t(x) = (x - 4)^4$. Identify the zeros of $t(x)$. How many zeros does this function have? How many times did the zero occur?

[(e)] Graph $r(x)$ and $t(x)$ on the same coordinate plane. Zoom in to examine the zero(s) closely. What is different about the graphs?

[(f)] Next, let $u(x) = (x - 4)^5$. Identify the zeros of $t(x)$. How many zeros does this function have? How many times did the zero occur?

[(g)] Graph $s(x)$ and $u(x)$ on the same coordinate plane. Zoom in to examine the zero(s) closely. What is different about the graphs?

[(h)] Use what you have learned to make a generalization about a factor $(x - k)^p$ when p is even. First describe the general shape of the function near the zero. Then describe what happens as the power p increases.

[(i)] Use what you have learned to make a generalization about a factor $(x - k)^p$ when p is odd. First describe the general shape of the function near the zero. Then describe what happens as the power p increases.

6. Next, let's examine the function $f(x) = (x - 2)^2(x + 1)^3$.

[(a)] What are the zeros of this function? How many times does each zero occur?

[(b)] Predict the shape of the graph around each zero by drawing a sketch.

[(c)] Graph the function $f(x)$ using a calculator or graphing tool. Did your prediction match the graph? Why or why not?

[(d)] Now consider the function $g(x) = (x - 2)^4(x + 1)^5$. Predict the shape of the graph around each zero by drawing a sketch.

[(e)] Graph the function $g(x)$ using a calculator or graphing tool. Did your prediction match the graph? Why or why not?

7. Graph $h(x) = x^4 + x^3 - 8x^2 - 12x$, and use the graph to determine the factored form of $h(x)$. Be sure to consider the shape of the graph around each zero.

8. Find a possible formula when the polynomial $f(x)$ is third degree with $f(-3) = 0$, $f(1) = 0$, $f(4) = 0$, and $f(2) = 5$.

9. Find at least two different third-degree polynomials having zeros at $x = -1$ and $x = 2$ (and nowhere else), and y -intercept at $y = 3$.

10. An open-top box is to be constructed from a 6 -in by 8 -in rectangular sheet of tin by cutting out squares of equal size at each corner, then folding up the resulting flaps. Let x denote the length of the side of each cut-out square. Assume negligible thickness.

[(a)] Draw a sketch of this problem in three stages: sketch the original sheet of tin, sketch the tin with the squares cut out of each corner, and sketch the final box. Each time, label each side.

[(b)] Find a formula for the volume of the box as a function of x . You can leave your answer in factored form.

[(c)] Sketch a graph of the volume function.

[(d)] For what values of x does the formula from part (a) make sense in the context of the problem?

[(e)] How can you determine the maximum volume of the box?

[(f)] What, approximately, is the maximum volume of the box?

5.11: Rational functions

1. Bronze is an alloy, or mixture, of copper and tin. The alloy initially contains 3 kg of copper and 9 kg tin. You add x kg of copper to this 12 kg of alloy. The concentration of copper in the alloy is a function of x :

$$(\text{Concentration of copper}) = f(x) = \frac{\text{Total amount of copper}}{\text{Total amount of alloy}}. \quad (5.11.1)$$

[(a)] Find a formula for f in terms of x , the amount of copper added.

[(b)] Evaluate the following expressions and explain their significance for the alloy:

[i.] $f\left(\frac{1}{2}\right)$

[ii.] $f(0)$

[iii.] $f(-1)$

[iv.] $f^{-1}\left(\frac{1}{2}\right)$

[v.] $f^{-1}(0)$

[(c)] Graph $f(x)$ using Desmos. Interpret the intercepts in the context of the alloy.

[(d)] Zoom out so you can see a larger viewing window. Does the appearance of the graph agree with what you expect to happen when large amounts of copper are added to the alloy?

2. The total cost $C(n)$ for a producer to manufacture n units of a good is given by

$$C(n) = 5000 + 50n. \quad (5.11.2)$$

The average cost of producing n units is $a(n) = \frac{C(n)}{n}$.

[(a)] Evaluate $C(1)$, $C(100)$, $C(1000)$, and $C(10000)$. Explain in a sentence the economic significance of these values.

[(b)] Evaluate: $a(1)$, $a(100)$, $a(1000)$, and $a(10000)$. Explain in a sentence the economic significance of these values.

[(c)] Based on part (b), what trend do you notice in the values of $a(n)$ as n gets large? Explain this trend in economic terms.

3. Knowing that we can express a rational function as $r(x) = \frac{p(x)}{q(x)}$, answer the questions below.

[(a)] If $f(x) = \frac{x^2+1}{-3x^2+5}$, what is $p(x)$ and what is $q(x)$?

[(b)] What are the leading terms of p and q for this rational function?

[(c)] To determine the long-run behavior of a rational function, we can simply examine the function $y = \frac{\text{Leading term of } p}{\text{Leading term of } q}$. What is the new rational function we can use to examine the long-run behavior of $f(x)$?

[(d)] What does this new function tell us about the long-run behavior of $f(x)$?

[(e)] Graph $f(x)$ using Desmos. Does the graph match your answer to part (f)?

[4.] Using the same steps from problem 1, determine the long-run behaviors of the following:

[(a)] $f(x) = \frac{2x^2+1}{x^2+5}$

[(b)] $g(x) = \frac{x^3+1}{x^2+5}$

[(c)] $h(x) = \frac{x+1}{x^2+5}$

[(d)] Check your work on each part above by using a calculator.

[(e)] Compare the long-run behavior of $f(x)$, $g(x)$, and $h(x)$. What do you notice? Discuss.

[5.] Let $m(x) = \frac{-x}{x+1} + \frac{x}{x+2}$.

[(a)] Write $m(x)$ as a rational function.

[(b)] Determine the long-run behavior of $m(x)$.

[6.] Let $f(x) = \frac{3x^2+x}{2x^2+5x^3}$

[(a)] Graph $f(x)$ using Desmos.

(b)] Determine the long-run behavior of $f(x)$?

[7.] Let $g(x) = \frac{1}{1+\frac{1}{x}}$.

[(a)] Rewrite $g(x)$ as a standard rational function.

[(b)] Using what you have learned in previous questions, find the horizontal asymptote, if it exists, of $g(x)$.

[8.] It costs a company \$30,000 to begin production of a good, plus \$3 for every unit of the good produced. Let x be the number of units produced by the company.

[(a)] Find a formula for $C(x)$, the total cost for the production of x units of the good.

[(b)] Find a formula for the company's average cost per unit, $a(x)$.

[(c)] Graph $y = a(x)$. What is the horizontal asymptote?

[(d)] Explain in economic terms why the graph of a has the long-run behavior that it does.

[(e)] Explain in economic terms why the graph of a has the vertical asymptote that it does.

[(f)] Find a formula for $a^{-1}(y)$. Give an economic interpretation of $a^{-1}(y)$.

[(g)] The company makes a profit if the average cost of its good is less than \$5 per unit. Find the minimum number of units the company can produce and make a profit.

6: EXPONENTIAL FUNCTIONS

- 6.1: Introduction to exponential functions
- 6.2: The logarithm function and its applications
- 6.3: Logarithms and exponential models
- 6.4: Logarithmic Functions and Applications
- 6.5: Comparing power, exponential and log functions
- 6.6: Comparing linear and exponential functions Graphs of exponential functions
- 6.7: Compound interest and the number e

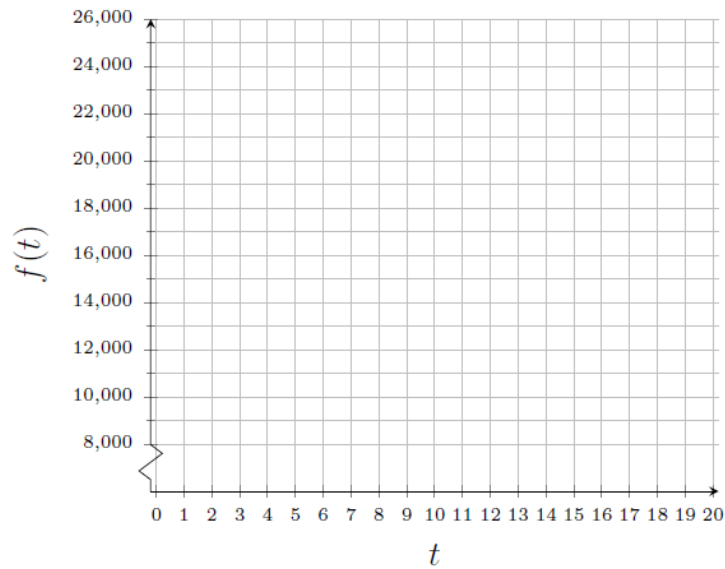
6.1: Introduction to exponential functions

1. Suppose \$8000 is invested in an account such that the balance of the account increases by 6% each year.

a. Fill in the following table giving the account balance $f(t)$ after t years

t (years)	0	1	2	5	10	15	20
$f(t)$	8000	8480					

b. Draw a graph for the account balance.

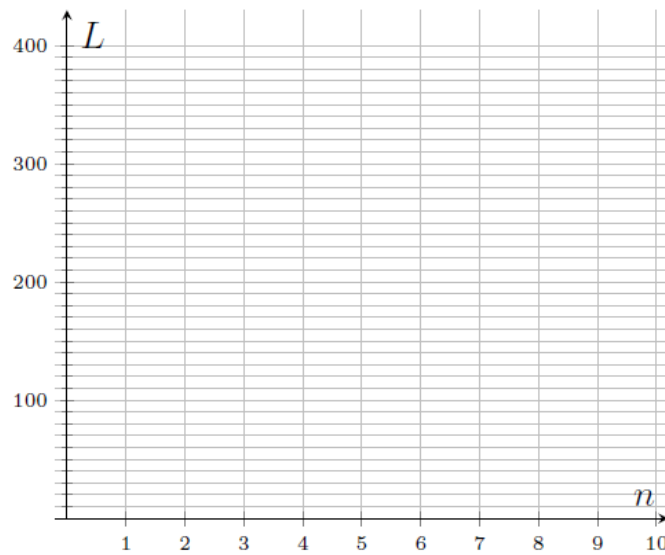


c. Write an expression for $f(t)$

2. A lake is polluted with toxic substance X. Each year, 55% of the remaining substance X is washed downstream and replaced by clean water. Initially, the lake water contains substance X at a level of 420 parts per million (ppm).

n	0	1	2	3	5	10
L	420	189				

b. Draw the graph of L



c. Write an expression for L as a function of n .

3. The population (in thousands) after t years of Alphaville is given by the function $a(t)$, and the population (in thousands) of Betatown after t years is given by $b(t)$:

$$\begin{aligned} a(t) &= 2.4 \times 1.033^t \\ b(t) &= 4.3 \times 0.985^t \end{aligned} \quad (6.1.1)$$

- Find the initial (i.e. "year zero") population of each town.
- One town has an increasing population, and one town has a decreasing population. Which is which? Explain in a sentence how you know.
- For each town, what is the percent increase or decrease for the population each year?
- Will there be a time when the towns' populations are the same? If so, estimate that time, or explain why it won't ever happen.

6.2: The logarithm function and its applications

1. For each log statement below, write an equivalent exponential expression, and state whether your expressions are true or false.

Hint: use Desmos: <https://www.desmos.com/scientific>

[(a)] $\log(31) = 3.43$.

[(b)] $\ln(150) = 5.01$.

2. For each exponential statement below, write an equivalent log expression, and state whether your expressions are true or false.

[(a)] $10^6 = 1000000$.

[(b)] $e^{10} = 22026.5$.

3. Evaluate without a calculator:

[(a)] $\log(100)$

[(b)] $\log(1000)$

[(c)] $\log(\sqrt{10})$.

4. Estimate without using a calculator. Then check with your calculator and think about how your answers compare.

[(a)] $\log(106372)$

[(b)] $\log(3120)$

5. Solve the following equations. Give an exact answer.

[(a)] $10^x = 71$ (Hint: take a log on both sides.)

[(b)] $5 \times 2^t = 700$

[(c)] $\frac{1}{20} = 0.7^{5t}$

[(d)] $b^{7.5} = \frac{1}{3}$

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6.3: Logarithms and exponential models

1. The population of a region (in thousands of people) is $P = 3.4 \times 1.023^t$ years after a census taken in 2000.
 - a. What is the annual percent growth rate of the population?
 - b. How long does it take for the population to double? This is called, naturally, the doubling time.
 - c. Is the doubling time constant? That is, does the population take the same time to double from any starting point? Discuss with your group, and write an explanation of your answer in a sentence or two.
2. The radioactive isotope cobalt-60 has a half-life of 5.27 years.
 - a. Use the half-life to write an expression for $f(t)$, the amount of cobalt-60 left after t years. (Hint: set up an equation $\frac{q_0}{2} = Q_0 \times b^{5.27}$ and solve.)
 - b. If a sample contains 20 g of cobalt-60 today and is placed in storage for 30 years, how much cobalt-60 should be in the sample at the end of that time? Write an expression and evaluate with a calculator.
 - c. A sample containing cobalt-60 has been in storage for 10 years, and is found to contain 12g of the isotope. How much did it have before being placed in storage? You will need to set this up as an equation.
 - d. How long does it take for 90% of the isotope in a sample to decay?
3. Consider the exponential function $g(t) = 12 \times 1.247^t$.

- a. Rewrite the expression for $g(t)$ in the form $g(t) = ae^{kt}$. (Hint: you can figure out the value of a without an equation, then solve an equation to find k .)
- b. Give the growth rate and the continuous growth rate for g .

4. Suppose the quantity of a pollutant in a lake is decreasing exponentially at a continuous rate of 3.4\% per day. Find the non-continuous rate of decay, giving your answer as a percentage.

5. According to Newton's law of cooling, the temperature of an object that has been cooling for an amount of time t is given by the formula

$$T(t) = T_a + (T_0 - T_a)e^{-kt} \quad (6.3.1)$$

where T_0 is the initial temperature, T_a is the ambient temperature of its surroundings, and k is a constant that depends on the object and where it is.

The same formula works for warming.

A pie is taken out of the oven, and its temperature is $210^\circ F$. It is set out on the counter in a room where the air is 70° degrees F.

- a. After 10 minutes the pie has cooled to 200° . Use this to find the constant k for this situation.
- b. How long will it take for the pie to cool to $150^\circ F$?

6.4: Logarithmic Functions and Applications

1. For these questions, you need to know what the graph of $y = e^x$ and $y = \ln x$ look like. You should know these graphs, but if you need a reminder, use a calculator or online graphing tool.

[(a)] What is the equation of the asymptote of the graph of $y = e^x$?

[(b)] What is the equation of the asymptote of the graph of $y = \ln x$?

[(c)] Explain in a sentence the connection between these two asymptotes.

2. The pH of a solution is a measure of its acidity or basicity. It is computed from the solution's hydrogen ion concentration $[H^+]$, in moles per liter, by

$$\text{pH} = -\log [H^+] . \quad (6.4.1)$$

[(a)] Use a calculator to find, the hydrogen ion concentration of each of the following. How much precision do you need for reasonable answers?

[i.] ... hydrochloric acid, with a pH of 0.

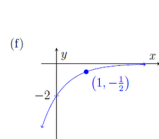
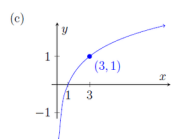
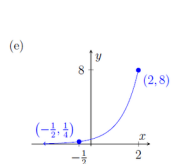
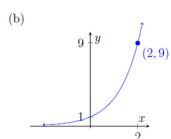
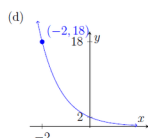
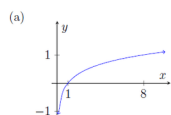
[ii.] ...lemon juice, with a pH of 2.

[iii.] ... urine, with a pH of 6.5.

[iv.] ... household bleach, with a pH of 13.5.

[(b)] Considering your answers to the previous questions, why is the logarithm used in the definition of pH? Explain in a sentence or two.

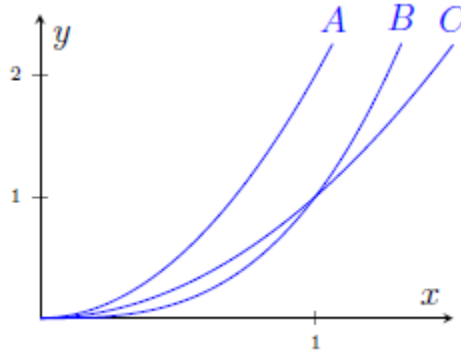
3. Each of the following graphs is an exponential or a logarithmic function. Give an expression for each function.



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6.5: Comparing power, exponential and log functions

1. (a) Match the functions $f(x) = x^2$, $g(x) = 2x^2$ and $h(x) = x^3$ to their graphs below.



(b) Do graphs A and B intersect for $x > 0$? If so, for what value(s) of x ? If not, explain how you know.

(c) Do graphs A and C intersect for $x > 0$? If so, for what value(s) of x ? If not, explain how you know.

2. Data from four functions are in the tables below. One function is linear, one is logarithmic, and the other two are power functions - one cubic (power of 3) and one quadratic (power of 2). Find a formula for each function, and explain how you made your choices.

x	$f(x)$	x	$k(x)$	x	$m(x)$	x	$z(x)$
1.8	1.75	2.8	0.36	0.2	-0.699	-4	6.4
2.2	3.19	3	0.6	0.6	-0.222	-2	1.6
2.6	5.27	3.2	0.84	1	0	0	0
3	8.1	3.4	1.08	1.4	0.146	2	1.6
3.4	11.79	3.6	1.32	1.8	0.255	4	6.4

3. The functions $y = x^{\frac{1}{2}}$ and $y = \ln x$ both approach ∞ as $x \rightarrow \infty$. Which one grows faster? Support your conclusion numerically by computing values of the two functions for some large values of x .

4. The functions $y = x^{\frac{1}{2}}$ and $y = x^2$ both approach ∞ as $x \rightarrow \infty$. Which one grows faster? Support your conclusion numerically by computing values of the two functions for some large values of x .

5. What is the end behavior of each of the following functions? That is, what value (a number, ∞ , or $-\infty$) does the function approach as $x \rightarrow \infty$ and as $x \rightarrow -\infty$? Support your answer numerically.

(a) $y = \frac{3x^2 + x}{x^{\frac{1}{3}}}$

(b) $y = \frac{x^5}{e^x}$

(c) $y = \frac{\ln x}{x^{\frac{1}{3}}}$

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6.6: Comparing linear and exponential functions Graphs of exponential functions

[1.] In 2020, a small grocery store on Chicago's South Side earned a revenue of 191 thousand dollars in 2020. In 2030, it will have a revenue of 210 thousand dollars

[(a)] Suppose the growth in the business's revenue is linear:

[i.] By how much does the revenue increase each year?

[ii.] Find the revenue in 2050 and the revenue in 2070.

[iii.] Find a linear formula for the business's revenue in thousands of dollars as a function of time t in years since 2020. That is, a formula that looks like $N = at + b$.

[iv.] Test your formula by plugging in $t = 0$ and $t = 10$ and comparing to the question.

[(b)] Suppose the growth in the business's revenue is exponential.

[i.] By what factor is the revenue multiplied each year?

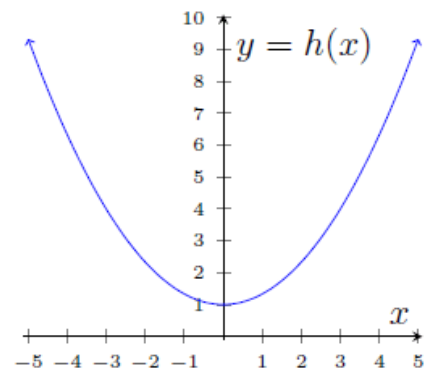
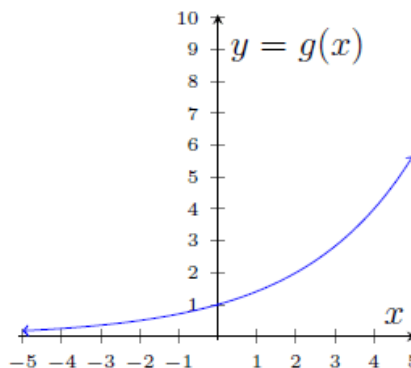
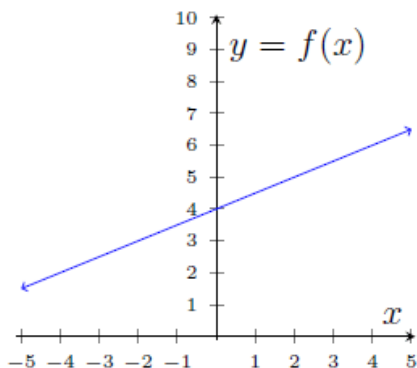
[ii.] Find your revenue in 2050 and the revenue in 2070. How do these values compare to the ones you got when you assumed linear growth?

[iii.] Find an exponential formula for your business's revenue in thousands of dollars as a function of time t in years since 2020. That is, a formula that looks like $N = a \times b^t$.

[iv.] Test your formula by plugging in $t = 0$ and $t = 10$ and comparing to the question.

[(c)] Is it better for the business if the growth is linear or exponential? Explain why in a sentence

[2.] (a) The functions f , g , and h are given by the following graphs. For each function, state whether it could be a linear function, an exponential function, or neither, and explain how you know.



[(b)] $C(t)$ is the cost of producing a business's product t years after 2024. The cost doubles every two years. Is $r(t)$ a linear function, an exponential function, or neither? Explain how you know.

[(c)] A car company makes a profit of $\pi = 10000N - 500000000$ at a production level of N cars. Is π a linear function of N , an

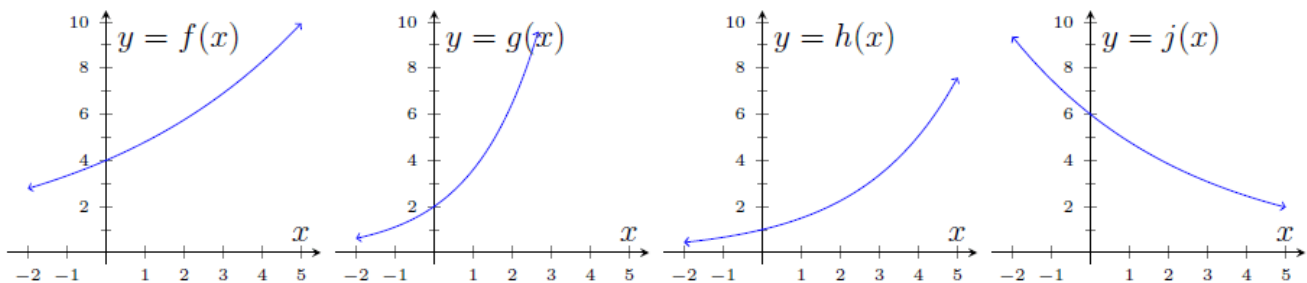
exponential function of N , or neither? Explain how you know.

[(d)] Some values of functions u, v , and w are given in the tables below. The values are not all exact; some are rounded to two decimal places.

t	-1	0	1	2	3	4
$u(t)$	20	8	3.2	1.28	0.51	0.20
$v(t)$	-0.84	1	-0.84	0.41	0.15	-0.67
$w(t)$	8.67	8	7.33	6.67	6	5.33

For each function, state whether it could be a linear function, an exponential function, or neither, and explain how you know.

3. Four exponential functions are graphed below. They all have the form $y = ab^x$.



[(a)] Which function has the largest value of a ? Explain in a sentence which feature of the graph tells you this.

[(b)] Which function has the largest value of b ? Explain in a sentence which feature of the graph tells you this.

[(c)] For each function, determine whether b is greater than 1 or less than 1. Explain in a sentence which feature of the graph tells you this.

[4.] Madeline opens a math tutoring business. The balance the business's bank account is given by $f(t) = 72 + 125 * 0.96^t$, t weeks after she started the business.

[(a)] How much money did Madeline start with?

[(b)] How much money is left in the bank after 1 week?

[(c)] What will the balance of her account be in the long run?

[(d)] What is $\lim_{t \rightarrow \infty} f(t)$? Hint: this is the same question as the last part!

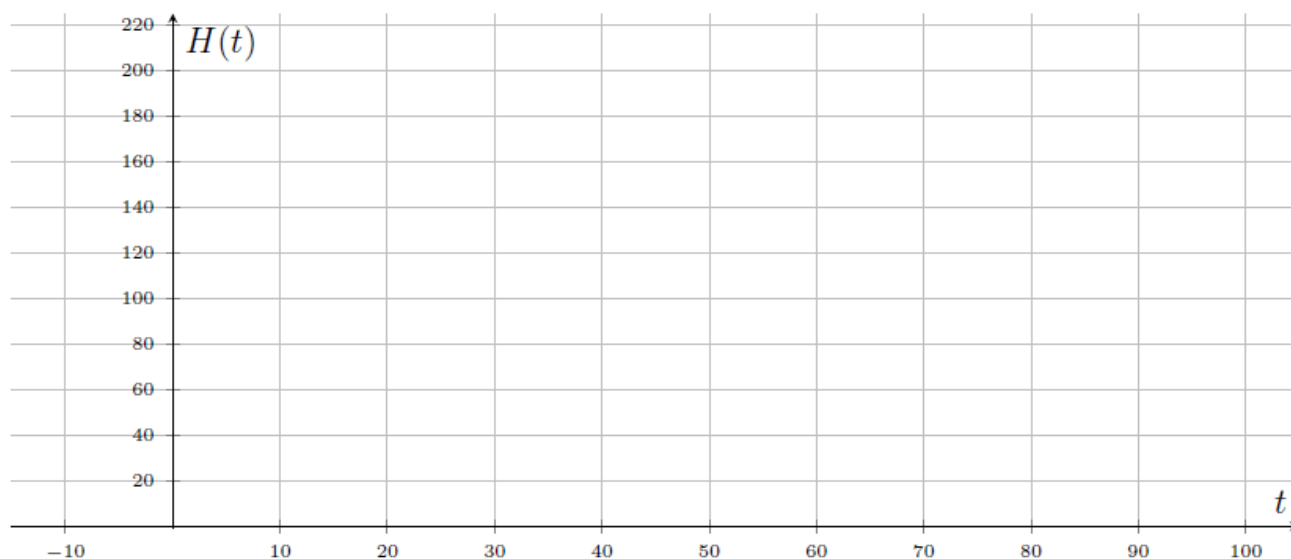
[(e)] Using a calculator as necessary to find values accurate to two decimal places, find the balance of the account after ...

[i.] ... 10 weeks

[ii.] ... 30 weeks

[iii.] ... 60 weeks

[(g)] Sketch a graph of the function $f(t)$ for t from 0 to 120



[(h)] Approximately how long will it take for Madeline's account to decrease to \$75? Answer using your graph

6.7: Compound interest and the number e

[1.] A bank account currently has a balance of \$5,000. It gets interest at a (nominal) rate of 6%. The following parts explore the effects of compounding on the interest earned by this account. Remember that interest compounded n times per year at a (nominal annual) rate of r means the account balance grows exponentially with an annual growth factor of $(1 + \frac{r}{n})^n$.

[(a)] Suppose the interest is compounded annually (once per year). Find the annual growth factor, and the balance after 10 years.

[(b)] Suppose the interest is compounded quarterly (4 times per year). Find the annual growth factor, and the balance after 10 years.

[(c)] Fill in the first two rows of the table below with your work above. Then complete the calculations to fill in the next three rows. Ignore the last (empty) row for now.

Compounding		Nominal Rate	Annual growth factor	Effective annual rate	10 year balance
Annually	1	6%	1.06	6%	
Quarterly	4	6%	1.0614	6.14%	
Monthly					
Daily					
Hourly					

[(d)] Discuss with your group: what meaning can you give to the "effective annual rate"?

[(2)]. (a) Based on your answers to the parts in question 1, give an approximate annual growth factor and balance for interest compounded every minute when the nominal rate is 6% and the initial balance is \$5,000. (Don't compute anything, just look for the trend.)

[(b)] Compute $e^{.06}$ and $5000 \times e^{.06 \times 10}$, where $e \approx 2.718$. (A scientific calculator will usually have an " e " button on it.) Then, add a row to the table where the compounding is "continuous", meaning, roughly, "as frequent as possible". Fill in all the columns for this new row. Where do the values you just computed fit?

[(c)] People say interest is compounded continuously if the account grows according to $A = Pe^{rt}$, where P is the principal and r is the nominal interest rate. So for the account we're discussing, $P = 5000$ and $r = 0.06$.

[i.] Using continuous compounding, calculate the balance after 1 year.

[ii.] What is the effective rate for interest at a nominal rate of 6% compounded continuously? (Hint: what is $e^{.06}$?)
For the remaining questions, use the base- e exponential function of the form

$$Q = ae^{kt} \quad (6.7.1)$$

where Q is the amount as a function of time, a is the original amount, and k is the continuous growth rate (expressed as a decimal, not a percentage).

3. In the initial stages of an epidemic, the number of infected people tends to increase exponentially. Assume that initially there are 10 infected people, and that the number is increasing at a continuous growth rate of 20% per day (i.e. $k = 0.2$).

[(a)] About how many people will be infected after 1 week?

[(b)] About how many people will be infected after one month?

[(c)] If the epidemic has broken out in an isolated city of two million people and nothing is done, how long will it be before a half the population is infected? Estimate your answer.

[4.] The revenue of Fred's online store is increasing exponentially.

(a) What will the revenue be after 50 years if ...

[i.] ... the revenue increases at an annual growth rate of 3%.

[ii.] ... the revenue increases at a continuous growth rate of 3% per year.

[(b)] How could you know which of the answers above was going to be bigger, without doing the calculation? Discuss with your group and answer in a sentence.

[(c)] If the revenue is growing at a continuous growth rate of 3%, by what percentage does it increase each year?

7: SYSTEMS OF EQUATIONS

7.1: Systems of Linear Equations in Two Variables

7.2: System of Equations Application Problems

7.3: Systems of Linear Equations Three Variables

7.4: Mixed Systems of Equations

7.1: Systems of Linear Equations in Two Variables

Warm-up:

ADAPT 7.1.1

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ADAPT 7.1.1

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1. Solve the following system of equations using substitution. Then solve it using elimination. Then solve it system graphically using Desmos.

$$\begin{cases} 3x + y = 9 \\ -2x + y = 4 \end{cases} \quad (7.1.1)$$

2. Solve the following systems of equations using elimination:

(a)

$$\begin{cases} -4R - 7T = 14 \\ 3R + 14T = 2 \end{cases} \quad (7.1.2)$$

(b)

$$\begin{cases} 6x + 5y = 12 \\ 2x - 7y = 6 \end{cases} \quad (7.1.3)$$

3. Solve the following systems of equations using substitution:

(a)

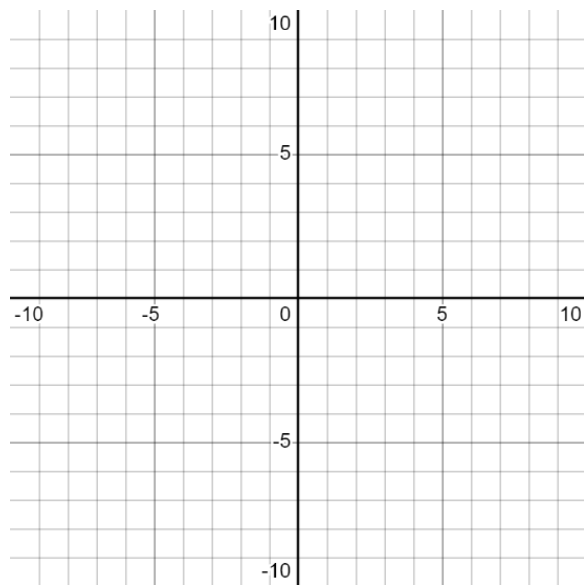
$$\begin{cases} 2x + 6y = 8 \\ x - 7y = 14 \end{cases} \quad (7.1.4)$$

(b)

$$\begin{cases} 2A - 4B = 10 \\ 4A + 5B = -6 \end{cases} \quad (7.1.5)$$

4. Solve the following system of equations graphically:

$$\begin{cases} 3x + 6y = 18 \\ -x + 2y = 4 \end{cases} \quad (7.1.6)$$



This page titled [7.1: Systems of Linear Equations in Two Variables](#) is shared under a [CC BY 4.0](#) license and was authored, remixed, and/or curated by Lynda Zenati.

7.2: System of Equations Application Problems

1. At a sushi restaurant, Table A orders 3 tuna rolls and 2 salmon rolls and spends \$37, while Table B orders 2 tuna rolls and 5 salmon rolls, spending \$43. What are the individual costs of one tuna roll and one salmon roll? Graph the equations in Desmos.
2. Find two numbers with sum 17 and difference 12. Graph the equations in Desmos.
3. Rashod and Camden are renovating their house. They want to place new tile on the walls and new wood panels on the flooring. To renovate the bathroom, they purchased 160 ft^2 of tile and 40 ft^2 of floor panels for \$5200, and to renovate the kitchen they purchased 60 ft^2 of tile and 100 ft^2 of floor panels for \$6200. How much does 1 ft^2 of tile cost? How much does 1 ft^2 of wood panels cost? Graph the equations in Desmos.
4. A farmer raises chickens and pigs. His animals together have a total of 95 heads and a total of 310 legs.
 - a. Formulate a system of linear equations from the information given.
 - b. How many chickens and how many pigs does the farmer have?
5. A motel plans to build small rooms of size 250 sq.ft and large rooms of size 500 sq.ft , for a total area of $16,000 \text{ sq.ft}$. Also, local fire codes limit the legal occupancy of the small rooms to 2 people and of the large rooms to 5 people, and the total occupancy of the entire motel is limited to 150 people.
 - a. Formulate a system of linear equations of the data given to express the constraints imposed by the size of the motel and by the fire code.
 - b. Solve the resulting system of equations. What does your solution tell you about the motel?

7.3: Systems of Linear Equations Three Variables

ADAPT 7.3.1

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[View this Interactive Content online](#)



ADAPT 7.3.1

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ADAPT 7.3.1

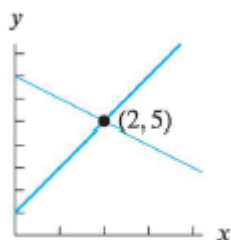
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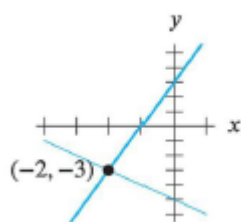


7.4: Mixed Systems of Equations

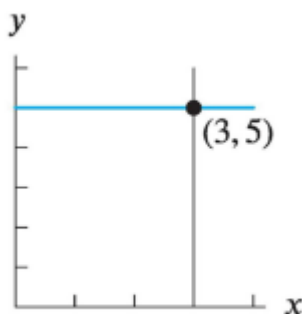
The graphs below represent systems of two linear equations. Write the equations and verify that the coordinates of the intersection satisfy both equations using Desmos.



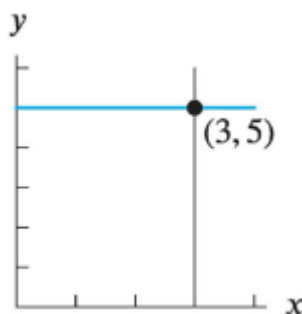
[1.]



[2.]



[3.]



[4.]

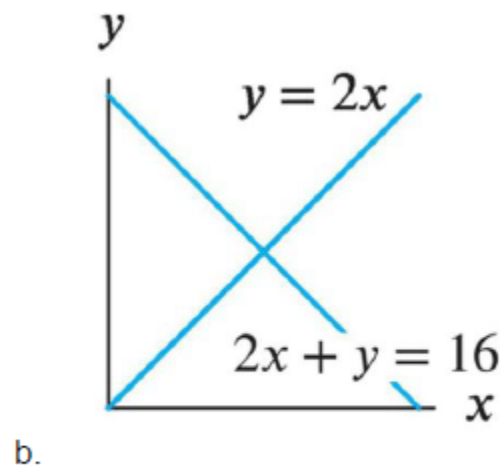
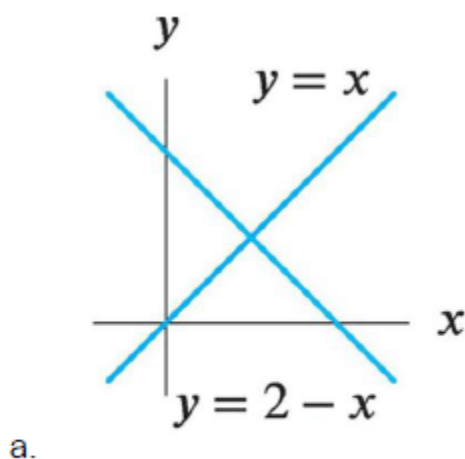
5. Without solving the equations, decide how many solutions the system has. Verify your answer using Desmos.

a.
$$\begin{cases} x - 2y = 7 \\ x + y = 9 \end{cases}$$

b. $\begin{cases} x + 3y = 1 \\ 3x + 9y = 3 \end{cases}$

c. $\begin{cases} 3y = 2 - x \\ x = 2 + 3y \end{cases}$

6. Determine the points of intersection for problems below. Verify your answer using Desmos.



7. Find two numbers with sum 17 and difference 12.8. A farmer raises chickens and pigs. His animals together have a total of 95 heads and a total of 310 legs.

[a.] Formulate a system of linear equations from the information given.

[b.] How many chickens and how many pigs does the farmer have? Draw the two equations in Desmos.

9. A motel plans to build small rooms of size 250 sq.ft and large rooms of size 500 sq.ft , for a total area of 16,000 sq.ft. Also, local fire codes limit the legal occupancy of the small rooms to 2 people and of the large rooms to 5 people, and the total occupancy of the entire motel is limited to 150 people.

[a.] Formulate a system of linear equations of the data given to express the constraints imposed by the size of the motel and by the fire code.

[b.] Solve the resulting system of equations. What does your solution tell you about the motel? Draw the two equations in Desmos.

8: AVERAGE RATE OF CHANGE

8.1: Formulas for Linear Functions

8.2: Average Rate of Change

8.3: Review Rules of exponents

8.4: Equations and Inequalities Appendices B-C

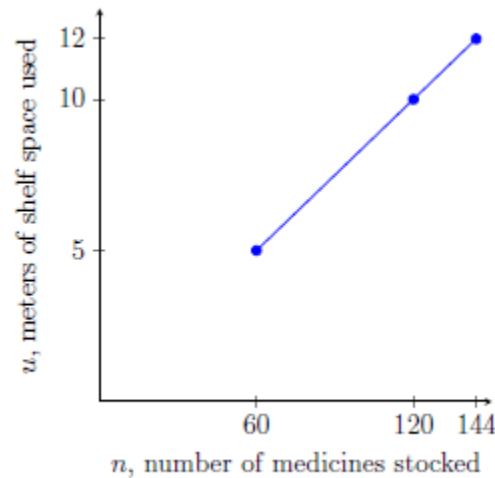
8.1: Formulas for Linear Functions

1. Use the following table to answer the questions below.

Year t	0	1	2
Value of the company $V = f(t)$	2000	1500	1000

- What does this table represent in practical terms?
- What is the rate of change, or slope, for this function?
- What is the vertical intercept?
- Write a formula for this function.

2. Use the following graph to answer the questions below.



- What does this graph represent in practical terms?
- What is the rate of change, or slope, for this function?
- What is the vertical intercept?
- Write a formula for this function.
- When compared to the diagram, does your formula make sense? Explain.

3. Find the slope and the y -intercept for each of the following linear functions.

- $3x + 5y = 20$
- $\frac{x-y}{5} = 2$

4. Find formulas for the linear functions described below.

- Passes through the points $(0, 1)$ and $(1, -4)$
- Passes through the points $(-1, -6)$ and $(2, 3)$
- $g(100) = 2000$ and $g(400) = 3800$

5. Are the following lines parallel? Perpendicular? Neither? Explain how you know.

- a. $y = 4x + 3; y = 13 - \frac{1}{4}x$
- b. $y = 2x + 3; 2y = 4x - 14$
- c. $y = 4x + 7; 4y = x - 2$

6. John wants to buy a half-dozen rolls. The local bakery sells sesame and poppy-seed rolls for the same price.

Make a table of all the possible combinations of rolls if he buys a half-dozen, where s is the number of sesame seed rolls and p is the number of poppy-seed rolls.

Find a formula for p as a function of s .

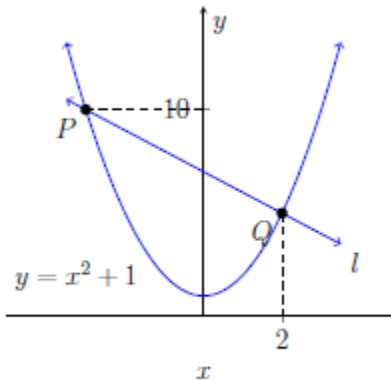
Graph this function.

7. The solid waste generated each year in the cities of the US is increasing. The solid waste generated, in millions of tons, was 88.1 in 1960 and 249.9 in 2010. The trend appears linear during this time.

Construct a formula for the amount of municipal solid waste generated in the US by finding the equation of the line through these two points.

Use this formula to predict the amount of municipal solid waste generated in the US, in millions of tons, in the year 2020.

8. In Figure 1.40, what are the coordinates of points P and Q ? Find the equation of line l .

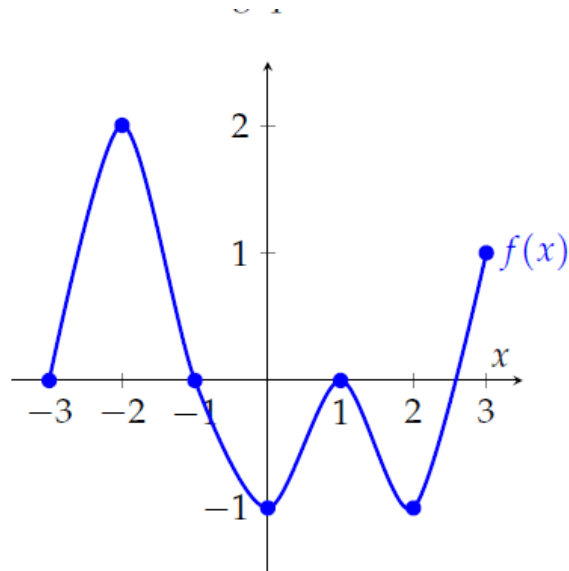


8.2: Average Rate of Change

1. The table below shows the temperature, T , in Chicago, Illinois t hours after midnight. When does the temperature decrease the fastest: between midnight and 3 a.m. or between 3 a.m. and 4 a.m.?

t hours after midnight	0	3	4
T (temp in °F)	85	76	70

2. Use the graph below to answer the following questions.



- Find the average rate of change of f for $1 \leq x \leq 3$.
- Find the average rate of change of f for $-2 \leq x \leq -1$.
- Find the average rate of change of f for $-3 \leq x \leq 1$.
- For this function, on which intervals is the function increasing? Decreasing?

3. Car A drives 10 miles down I-94 in 20 minutes. Car B drives 3 miles down Lake Shore Drive in 7 minutes. Which car has a faster average speed?

4. The table below shows the times achieved every 10 meters by Carl Lewis in the 100-meter final of the World Championship in Rome in 1987.

Time t (seconds)	0	1.94	2.96	3.91	4.78	5.64	6.5	7.36	8.22	9.07	9.93
Distance d (meters)	0	10	20	30	40	50	60	70	80	90	100

a. For each successive time interval, calculate the average rate of change of distance. What is a common name for the average rate of change of distance?

b. Where did Carl Lewis attain his maximum speed during this race? Some runners are running their fastest as they cross the finish line. Does that seem to be true in this case?

8.3: Review Rules of exponents

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8.4: Equations and Inequalities Appendices B-C

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8: APPENDIX

8.1: How to upload an image to Desmos

8.2: Getting set up with Desmos

8.3: Desmos User Guide.

8.4: Using Desmos to Model Mathematics in the Real World

8.5: Activities using Desmos

8.6: Different forms of a quadratic relation using Desmos

8.7: Formative Assessments

8.7.1: Quiz 1

8.7.2: Quiz 3

8.7.3: Quiz 4

8.7.4: Quiz 5

8.7.5: Quiz 6

8.7.6: Quiz 7

8.7.7: Quiz 8

8.7.8: Quiz 2

8.8: Instructor Guide on using the Workbok

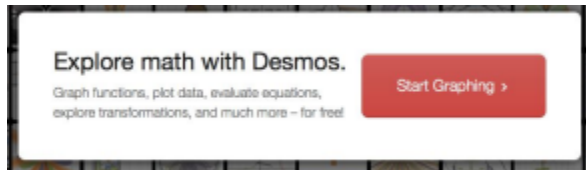
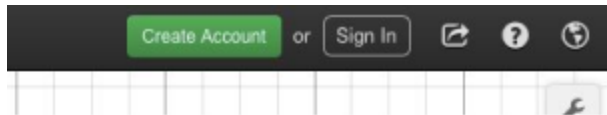
8.1: How to upload an image to Desmos

How to upload an image to



Using a browser:

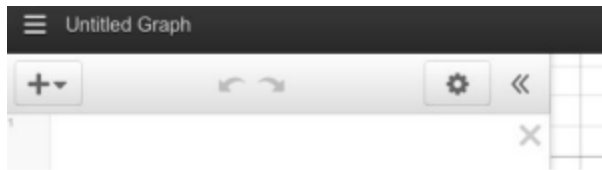
1. Go to www.desmos.com
2. Click on “Start Graphing”
3. In order to save your work, sign in by clicking the option at the top right and use a gmail account.*



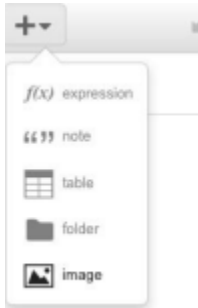
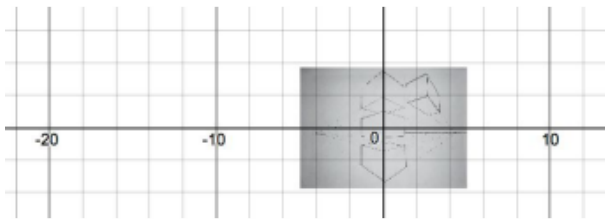
4. Set your axes to your desired range of values by clicking on the wrench in the top right. Projector mode is useful to make the grid lines darker on your image.



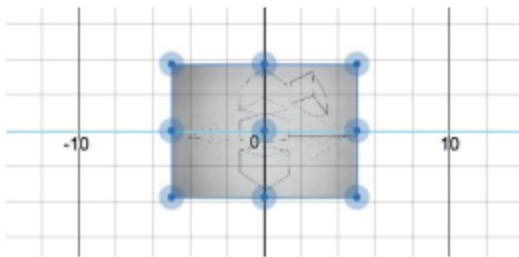
5. Select the drop down menu from the plus sign at the top left of the screen.



6. From the options that appear, select image and find the image on your device. *



7. Your image will be inserted so that the centre is at (0,0).



8. To move your image, click on it and use the centre circle. To resize your image, drag it from the corners.

Using the App:

Open the app and follow steps 2-8. The buttons may be in slightly different locations, but they will look the same.

When you are using your image to analyze math, it will be easier to do so on an iPad, laptop or desktop. The screens on phones are too small to clearly read accurate points.

8.2: Getting set up with Desmos

Desmos is a wonderful free calculator that has statistical capabilities. Not only can we use Desmos to do calculations, but we can also use its platform to engage in activities that help develop a deep understanding of challenging mathematical concepts.

In order to utilize desmos to its fullest, you will need to create an account. This way, any progress you make in an activity will be saved. You can also access feedback that I leave for you when you have an account.

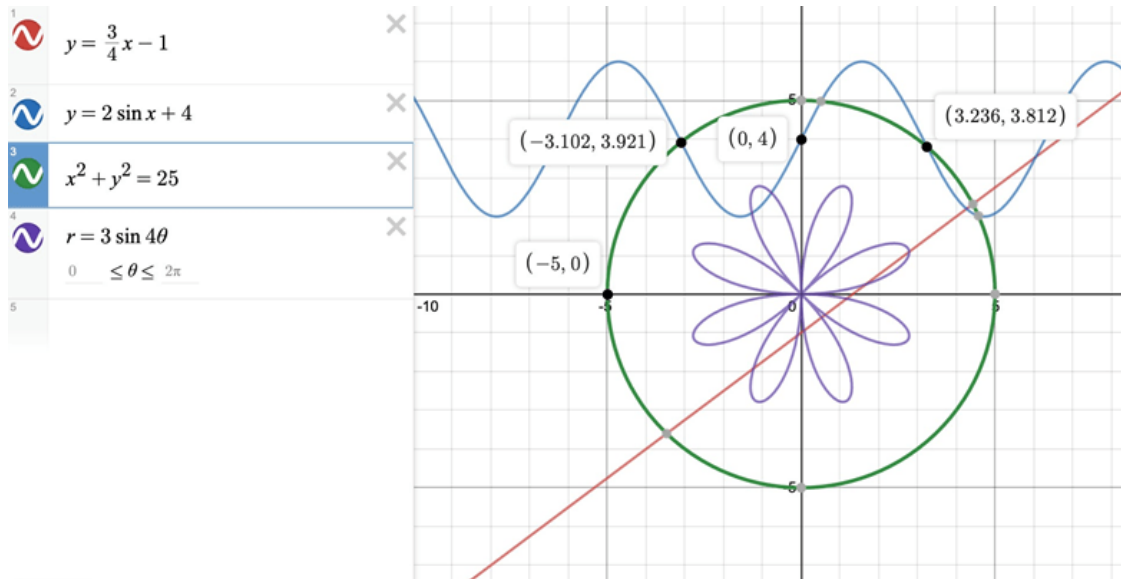
To create an account:

1. Go to student.desmos.com
2. Click "create account"
3. Enter your email, your first AND last name (as it appears on the roster), and create a password. Make your password easy to remember so that you can easily sign in for activities in our class
4. Click the Create Account button

That's it! Now you are ready to complete any Desmos activities.

Example:

- To graph an equation, type the equation into the next line on the left (i.e., $y = \frac{3}{4}x - 1$).
- To restrict the domain, type your restricted domain in set notation after the function (for example: $y = \frac{3}{4}x - 1 \{-2 < x < 4\}$).
- Click along your graph to identify specific points on the function. By doing this, you are identifying and choosing your domain and range for the graph. This will also assist you to fill in the function table, as you create your art instead of waiting until the end.
- Click on the Gear that says "Edit List" above your functions to change the color or line style of each equation.
- Make sure you save your graph!



You can input equations using any form. You are not restricted to $y = f(x)$ functions like graphing calculators. Finding intersection points and intercepts is simple. Just click on the point where the graphs intersect and its coordinates will be displayed!

Clicking the tiny keyboard icon in the bottom left corner allows you to pull up additional symbols and functions. Knowing some basic code to input equations makes this much faster though. For instance, typing "pi" and "theta" and "infinity" will input π , θ , and ∞ . Typing "int" will start a definite integral. Typing "^" allows you to quickly put in an exponent and "sqrt" and "cbrt" start square roots and cube roots.

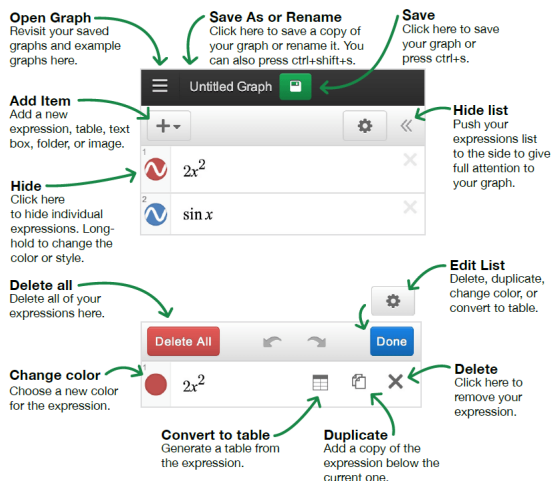
8.3: Desmos User Guide.

Learn more about graphing functions, plotting tables of data, evaluating equations, exploring transformations, and more! If you have questions that aren't answered in here, send us an email at calculator@desmos.com.

Getting Started with Desmos	1
Tables	2
Variables and Sliders	3
Settings, Zoom, and Language	5
Graphable Expressions	6
Supported Functions	8
Keyboard Shortcuts	9

Making a graph

Welcome to Desmos!! To create a new graph, just type your expression in the expression list bar. As you are typing your expression, the calculator will immediately draw your graph on the graph paper.



Open Graph
Revisit your saved graphs and example graphs here.

Save As or Rename
Click here to save a copy of your graph or rename it. You can also press ctrl+shift+s.

Save
Click here to save your graph or press ctrl+s.

Add Item
Add a new expression, table, text box, folder, or image.

Hide
Click here to hide individual expressions. Long-press to change the color or style.

Delete all
Delete all of your expressions here.

Change color
Choose a new color for the expression.

Convert to table
Generate a table from the expression.

Duplicate
Add a copy of the expression below the current one.

Hide list
Push your expressions list to the side to give full attention to your graph.

Edit List
Delete, duplicate, change color, or convert to table.

Delete
Click here to remove your expression.

Dive deeper into data with tables! You can create a new table or convert an existing expression into a table. If your existing expression includes sliders, they will remain functional after the expression is converted into a table.

Convert to Table
In edit mode, click the "convert to table" icon to create a table from your expression.

Add Table
Click the "Add Item" button to create a new table.

Options Menu
Click the icon to change color, connect or hide the points, or insert a new column.

Add Column
Make a new column by hitting the right arrow key or clicking in the top cell of the column.

Add Row
Make a new row by hitting the down arrow key or clicking in the cell.

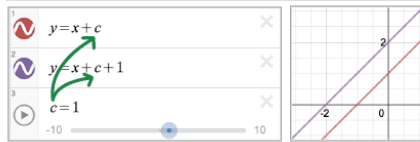
Convert to Table
In edit mode, click the "convert to table" icon to create a table from your expression.

You can graph a single line by entering an expression like $y = 2x + 3$. To make the graph more dynamic, you can use parameters instead of constants: for example, $y = mx + b$. Add sliders for the undefined parameters by clicking the prompt or define the parameters yourself by entering $m=2$ and $b=3$. When you give constant values to parameters like m and b , the calculator will automatically allow you to adjust their values with sliders. Adjusting m with the slider changes the slope of the line, and adjusting b with the slider changes the intercept.

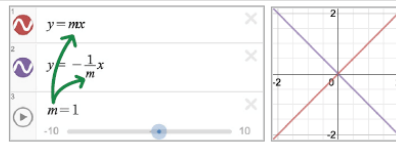
Any time you have free variables in an expression, the calculator will offer to let you define them with sliders:



You can use the same variables in several expressions to plot curves that will change together. For example:

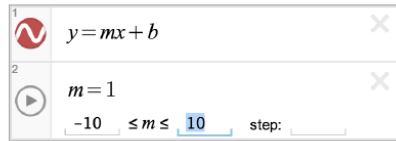


The value of c defines two parallel lines that move up and down together.

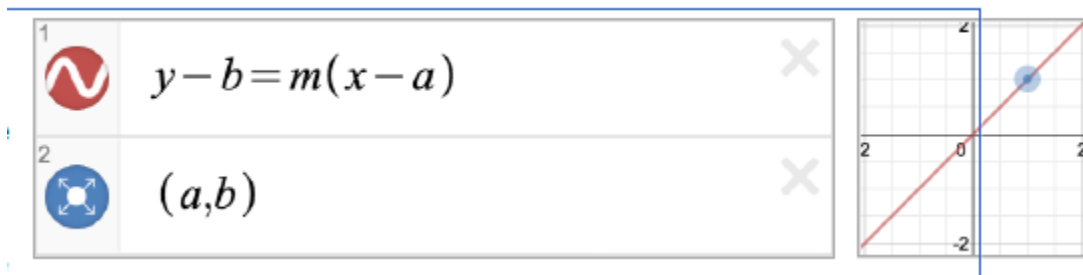


These two lines stay perpendicular to each other for any value of m .

To adjust the limits and interval of your slider, click either of the values at the ends of the slider bar. Input your desired values and click the expression or the graph to complete the adjustment.



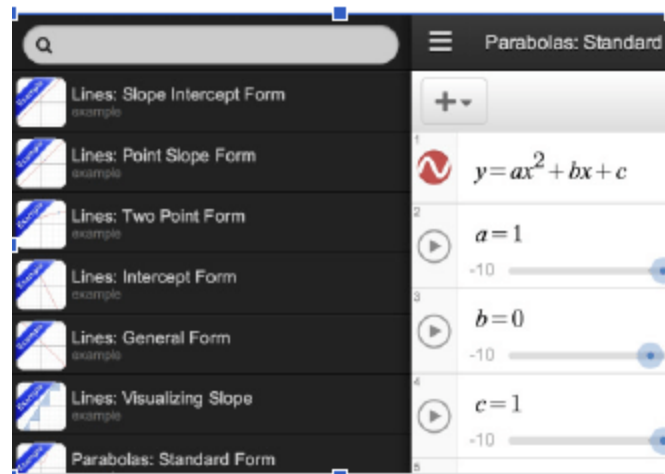
To create a movable point, enter a point with a variable for at least one coordinate. Click and drag the point around the graph to change the value of the parameter(s). To make graphs more interactive, use parameters from your movable point in your expressions. For example, you can graph the line $y - b = m(x - a)$ and plot the movable point (a, b) to see the line move when you drag the point - don't forget to add sliders!



Saving a graph

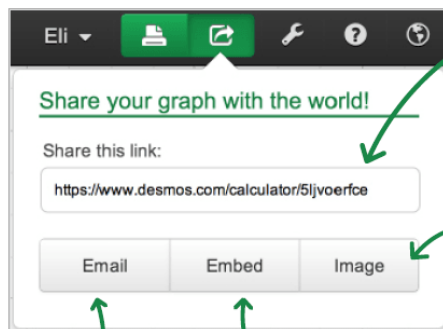
You will need to be signed in to save and open your graphs. Save a graph by pressing the button located to the right of the title bar or pressing ctrl+s on your keyboard.

Access your saved graphs by clicking the my graphs icon:



Sharing a graph

Clicking  in the top toolbar will allow you to share your graphs.



Permalink

Underneath the social sharing options, you will see a permalink for your graph. You can copy this link and share it with anyone. When they open it, they will see your graph and all of the equations.

Image

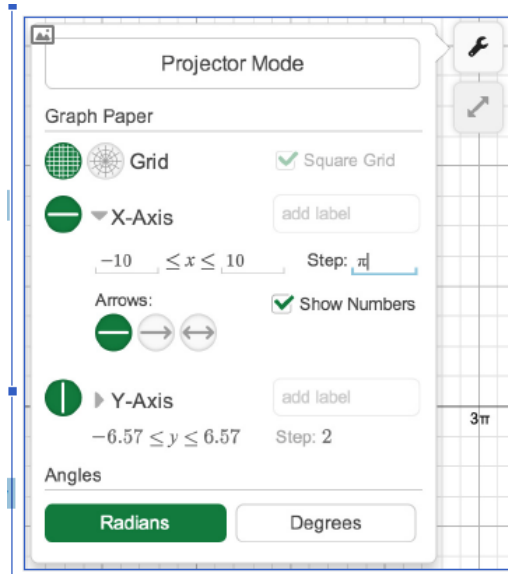
Grab a screenshot of your graph by clicking the Image button. An image of your graph will open in a new window, which you can print or right-click to save as an image.

Email

Email your graph by clicking here. You can send to multiple recipients and even add a custom message.

Embed

Copy the HTML embed code to post your graph in a website or wiki. You can also use the BBCode for your graph to share in any of our partner forums.



Settings

Projector Mode

Using projector mode will make the graph and axes lines thicker, and the number labels larger. This is great for students sitting in the back of the classroom who are looking at Desmos on the projector.

Graph Paper

In the graph paper section you can change the grid and axes. Click the green circular icons to choose between Cartesian and Polar grids and show or hide axes and labels.

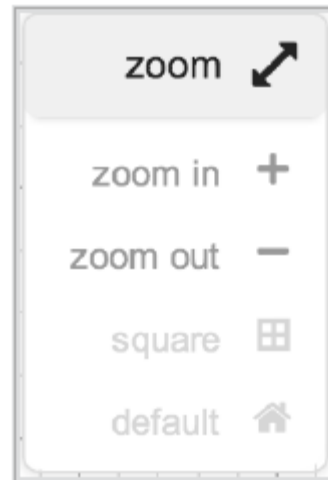
Click the gray arrow next to an axis to adjust the window size, add a step (try “pi”), or add arrows.

Angles

Choose between radians and degrees here.

Zoom

You may zoom in and out of the calculator by using the zoom buttons in the zoom menu. If your axes are no longer square, click



the square option. To return to the default view, click the default option.

Other ways to zoom:

If you are using a touch-enabled device, you may also use the pinch and zoom technique within the graphing space. While using a computer with a mouse, you can use the scroll wheel for zooming. Click and drag within the graphing window to pan your graph within the graphing space.

Language

To change the language of the calculator interface, click the icon and select your language from the list. If you don't see your language and would like to help with translation, email us at translations@desmos.com.

Graphable Expressions

Type of Graph	Regular function
Regular function	$y = 2x + 1$
x in terms of y	$x = \sqrt{1 - y^2}$
Inequalities	$y > \log(x)$ $x \leq 2y + 3$

Polar	$r = \sin(5\theta)$
Piecewise	$\begin{aligned} &y = x \{x < 0\} \setminus \\ &\& x = \sin(y) \{-\pi \end{aligned}$
Point	(1, 0)
Point List	(1,1), (2,2), (3,3)
Movable Point	(a,b)
Parametric	$(\sin(2t), \cos(3t))$
Implicit	$x^2 + y^2 = 1$ $y^2 + \sin(x)y + x = 2$

Exponents & Logs	Hyperbolic Trig Functions	Precalc & Calculus
exp(x)	sinh(x)	d/dx
ln(x)	cosh(x)	$\sum$
log(x)	tanh(x)	$\prod$
log ₁₀ (x)	sech(x)	Define your own function You can also create your own functions, which can be defined with any letter (except for the special ones, like x,y,t, and e). Your custom functions can then be used like any other function on this list. For example, if you type f(x) = x^2, you can then graph y = f(x+1), which is the same parabola but shifted over to the left: 
x^n	csch(x)	
Trig Functions	coth(x)	You can even define functions that take more than one argument. For example, you can enter: g(a,b) = sin(a-b). This won't graph, but you could then graph something like y = g(x,2): 
sin(x)	Stats & Probability	
cos(x)	ceil(x)	
tan(x)	floor(x)	
sec(x)	round(x)	
csc(x)	abs(x)	
cot(x)	min(a,b)	
Inverse Trig Functions	max(a,b)	
arcsin(x)	lcm(a,b)	
arccos(x)	gcd(a,b)	
arctan(x)	nCr(n,r)	
arcsec(x)	nPr(n,r)	
arccsc(x)	! (factorial)	
arccot(x)		

Keyboard Shortcuts

Open Graph: ctrl + o

Save: ctrl + s

Save-As or Rename: ctrl + shift + s

Undo: ctrl + z

Redo: ctrl + y

New expression: Press “enter”

Move up/down: Press the $\uparrow$ and $\downarrow$ arrow keys Move left/right: Press the $\leftarrow$ and $\rightarrow$ arrow keys Add a new text box: Press the quotation keys (“ ”) Delete an expression: Press “delete”

Symbols

$\sum$: Type “sum”

π : Type “pi”

θ : Type “theta”

$\sqrt{\quad}$: Type “sqrt”

$\prod$: Type “prod”

8.4: Using Desmos to Model Mathematics in the Real World

Mathematics is all around us. Objects found in everyday surroundings often exhibit patterns that can be described using algebraic functions. In this project, you will use the free online graphing calculator Desmos to investigate how mathematical functions can model real-world objects and structures. Rather than studying functions only as graphs on a coordinate plane, you will explore how functions can be used to describe features found in photographs that you take yourself. Desmos allows users to import images and overlay graphs directly onto photographs. This technology enables students to connect algebraic equations with physical objects and see how mathematics is used to describe the world around them.

Directions: Find examples of each function listed below in the world around you (physical surroundings).

Use Desmos for this project (make sure you create an account w/your Google account and save your work!). See below for all the equations that must be included in your picture. Each equation will have a domain and/or range restriction as well. You will be graded on using all the required equations, complexity and accuracy of image, and creativity.

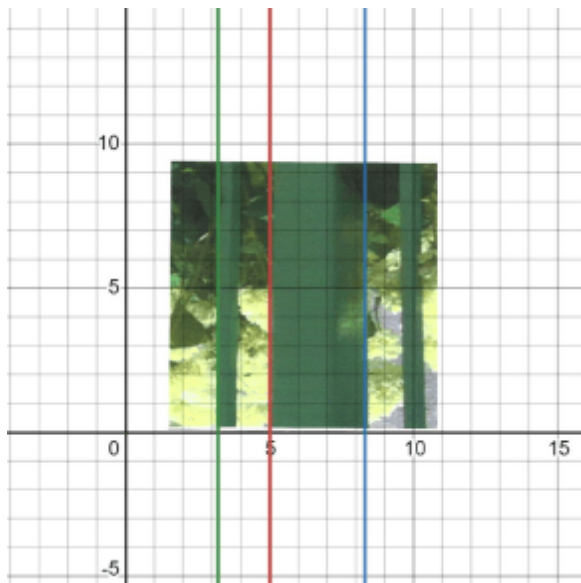
- a) Linear function (two examples)
- b) Quadratic function (two examples)
- c) Power function (one example)

Getting Started in Desmos

- Visit www.desmos.com/calculator.
- Create a free account or sign in using Google.
- Select the + menu and choose Image.
- Upload your photograph.
- Resize and position the image on the coordinate grid.
- Create a table of points that follow the object you wish to model.
- Identify points along the object you wish to model
- Use those points to develop an equation that closely represents the shape or pattern
- Enter equations and adjust parameters until the graph closely matches the image. Make sure to Refine the equation until it provides a reasonable mathematical model.
- Capture a screenshot showing both the photograph and the graph for submission.

See example: Parallel lines on the fence in nature





Background Desmos: Desmos is free and is available at <https://www.desmos.com>. At the website, one needs only click on Start Graphing and a coordinate grid will appear. On the left side will be a button with a plus sign. Clicking on this button will produce a drop-down menu where an item can be added such as an expression, table, or image. When image is selected, the user is allowed to browse saved files, where the chosen photograph can be selected. Desmos will automatically center the image at $(0, 0)$, but both the center and the size of the image can be changed by clicking on the center and entering a different point or size. If your picture is too dark, click on the settings icon, go to edit list, and then select the circular image of the picture. You will see five different options for the opacity of the picture (one of which will be the original version).

EXTENSIONS: When you have created the polynomial, please answer the following questions:

- Where is the function increasing or decreasing?
- Where are the zeros? Where are the local maxima and minima?
- What is the domain and range? What is the domain and range of only the part of the graph in the picture?
- What is the y-intercept?
- What is the end behavior, and how does this relate to your equation?
- Why do some equations fit better than others? What happens when certain components of an equation are removed?

Math 140 Real-World Functions Project Rubric

Criteria	Level 1 (Below 60%)	Level 2 (60-70%)	Level 3 (70-80%)	Level 4 (80-100%)
Mathematical Modeling (60%)	Limited understanding of function types. Equations are missing, incorrect, or poorly related to the photos.	Some understanding of function types. Several equations partially model the pictures but may contain errors.	Good understanding of function types. Most equations appropriately model the pictures with minor errors.	Excellent understanding of function types. All equations accurately model the pictures and demonstrate strong algebraic reasoning.
Graphs and Accuracy (20%)	Graphs are missing, incomplete, or do not match equations.	Some graphs match equations, but several contain errors or missing labels.	Most graphs accurately represent equations and include appropriate scales and labels.	All graphs are accurate, clearly labeled, and effectively demonstrate the modeled functions.

Communication and Organization (10%)	Work is difficult to follow. Photos, equations, graphs, and descriptions are poorly organized.	Some organization is evident, but explanations or formatting are incomplete.	Project is organized and mostly easy to follow, with minor formatting issues.	Project is exceptionally organized, professional, and easy to follow. Photos, equations, graphs, and explanations are clearly connected.
Creativity and Real-World Connections (10%)	Minimal effort to connect mathematics to real-world objects.	Basic examples with limited originality.	Interesting realworld examples that demonstrate thoughtful observation.	Highly creative and connect to realworld

8.5: Activities using Desmos

Prior Knowledge: Graphing points and identifying quadrants

<https://student.desmos.com/> and enter code TE4K8J

Creating and solving equations

<https://student.desmos.com/> and enter code CC9QB

Central Park: writing equations

<https://student.desmos.com/> and enter code 2R3T9W

Graphing linear inequalities

<https://student.desmos.com/> and enter code YWKNH4

Linear and non-linear equations and tables

<https://student.desmos.com/> and enter code 5A4MX

Deeper understanding of domain and range

<https://student.desmos.com/> and enter code NZT3Z

Exploring function families and characteristics of graphs

<https://student.desmos.com/> and enter code 6B4GJ (Needs 2 people)

Exploring functions and relations

<https://student.desmos.com/> and enter code XSAD (Needs 2 people)

Match my line (Writing Linear Equations)

<https://student.desmos.com/> and enter code PASQ4

Linear marbles

<https://student.desmos.com/> and enter code A78BZY

Exploring parallel and perpendicular lines

<https://student.desmos.com/> and enter code PJ3DAF

Exploring solutions to linear equations (points on the line)

<https://student.desmos.com/> and enter code 5CWA

Graphing absolute value functions with transformations

<https://student.desmos.com/> and enter code HBAMS

Introduction to solving systems of equations

<https://student.desmos.com/> and enter code PW6Z4

Exploring elimination method for solving systems of equations

<https://student.desmos.com/> and enter code SJHU

Exploring exponential functions

<https://student.desmos.com/> and enter code JQFVN Unit#5: Writing linear, exponential, and quadratic functions

<https://student.desmos.com/> and enter code EVXZA

Transformation mini-golf

<https://student.desmos.com/> and enter code W65YJ

Desmos graphing project: <https://www.desmos.com/calculator/rk9afhgdvf>

8.6: Different forms of a quadratic relation using Desmos

1. Open a web browser and go to www.desmos.com. Click “Launch Calculator”

2. Graph the equation

$$y=x^2+k$$

. Click on **Add slider: k** to make ‘ k ’ into a slider.

1. Use the slider to describe the parabola when

1. $k = 0$

2. $k = 5$

3. $k = -3$

3. Graph the equation

$$y=ax^2$$

. Click on **Add slider: a** to make ‘ a ’ into a slider.

1. Use the slider to describe the parabola when

1. $a = 1$

2. a is any negative number

3. a is between 0 and 1

4. a is bigger than 1

4. Graph the equation

$$y=(x-h)^2$$

. Click on **Add slider: h** to make ‘ h ’ into a slider.

1. Use the slider to describe the parabola when

1. $h = 0$

2. $h = 5$

3. $h = -3$

5. Graph the equation

$$y=a(x-h)^2+ky$$

. Add sliders for a , h and k .

1. Set the following values on the sliders and write the coordinates of the vertex and the direction of opening :

1. $a = 1, h = 2, k = 4$ Vertex: _____ Direction: _____

2. $a = -0.5, h = -3, k = 5$ Vertex: _____ Direction: _____

3. $a = 3, h = 2.5, k = -4$ Vertex: _____ Direction: _____

2. $y = a(x-h)^2 + k$

is sometimes called **vertex form** of a parabola. Explain why it has this name

3. Write the equation of a parabola in the form

$$y = a(x-h)^2 + k$$

that fits each description :

1. Thinner than

$$y = x^2$$

, opening up, with a vertex at (3, -5) _____

2. Wider than

$$y = x^2$$

, opening down with a vertex at (-2, 4) _____

6. Graph the equation

$$y = a(x-r)(x-s)$$

and make a, r and s sliders.

1. Explain how changing a changes the graph.

2. What key points on the graph correspond to the values of r and s ? [2 marks]

3. Set the following values on the sliders and fill in the descriptions: [3 marks]

1. $a = 1, r = 3, s = 5$ Vertex: _____ Zero(es): _____

2. $a = -1, r = -3, s = -3$ Vertex: _____ Zero(es): _____

3. $a = 2, r = 3, s = 5$ Vertex: _____ Zero(es): _____

7. The height of a pop-fly baseball since the time it was hit is given by the equation

$$y = -5(x+0.2)(x-3)$$

where y is height of the ball in metres and x is the time since it was hit, in seconds.

1. Graph this equation and list the following key points [2 marks]

Vertex: _____ y-intercept: _____

Left zero: _____ Right zero: _____

2. Use the information from a. to answer each of the following questions. Identify which key point was used for each. [5 marks]

1. What is the maximum height the ball reached?

2. How long did it take the ball to reach this height?

3. For how long was the ball in the air?
4. From what height was the ball hit?
5. Does the zero at $x = -0.2$ mean anything for the ball? Explain.
8. Graph the parabola

$$y = -2(x+3)(x-5)$$

. Write the equation for this parabola in vertex form (

$$y = a(x-h)^2 + k$$

9. The form

$$y = a(x-h)^2 + k$$

gives the vertex of a parabola and the form

$$y = a(x-r)(x-s) \quad y = a(x-r)(x-s)$$

gives the zeroes. What key point(s) are given by standard form (

$$y = ax^2 + bx + c$$

)? Does standard form give any other important information?

8.7: Formative Assessments

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8.7.1: Quiz 1

MATH 140 - Quiz 1

Name: _____

Instructions: Show ALL of your work. Read all instructions carefully. Don't forget units. Round answers when appropriate.

Question 1 (4 points)

Simplify the following expression:

$$2(y + x) - 3(x - 4) + y - 8 \quad (8.7.1.1)$$

Question 2 (6 points)

Solve the following equation for w :

$$Aw - 6 = 12 \quad (8.7.1.2)$$

Question 3 (1 point) BONUS

In the previous question, what value for A could give us problems?

In other words, what value makes the equation no longer true?

8.7.2: Quiz 3

MATH 140 - Quiz 3

Name: _____

Show ALL of your work. Read all instructions. Don't forget units. Round answers when appropriate.

Average Rate of Change:

$$\frac{\Delta f}{\Delta t} = \frac{f(b) - f(a)}{b - a}$$

1. (4 points) Aisha and Luis are part of their college's running club. Aisha completes 11 miles during a weekend group run in 61 minutes, while Luis finishes 7 miles in 40 minutes.

Calculate each runner's average speed (in miles per hour) to determine who ran faster.

2. Consider the function $f(x) = x(2 - x) + 1$.

(a) (6 points) Find the average rate of change of f between $1 \leq x \leq 3$.

(b) (3 points) Find the average rate of change of f between $-2 \leq x \leq 0$.

(c) (1 point) BONUS: Is the graph of f steeper on $[-2, 0]$ or on $[1, 3]$? Justify your answer by graphing f using Desmos. 

8.7.3: Quiz 4

MATH 100 - Quiz 4

Name: _____

Show ALL of your work. Read all instructions. Don't forget units. Round answers when appropriate.

1. (6 points) Give the formula for a linear function passing through the points $(-1, 7)$ and $(5, 34)$.

Your answer must be fully simplified.

2. (4 points) Graph the function

$$f(x) = -\frac{1}{2}x - 1$$

Note: You must plot at least two points on the graph to show your work and receive a score. Drawing a graph without any justification will not be awarded points.

3. (1 point) BONUS: Give the average rate of change and initial value of the function

$$f(x) = 2q - 7 + (3 + q)x$$

8.7.4: Quiz 5

MATH 100 - Quiz 5

Name: _____

Show ALL of your work. Read all instructions. Don't forget units. Round answers when appropriate.

1. (5 points) Expand and simplify the following expression:

$$(2x - 3)(x + 4)$$

2. (5 points) Factor the following expression:

$$x^2 - 23x + 90$$

3. (1 point) BONUS: Factor the following expression:

$$x^2 - 25$$

8.7.5: Quiz 6

MATH 100 - Quiz 6

Name: _____

Show ALL of your work. Read all instructions. Don't forget units. Round answers when appropriate.

MATH 100 - Quiz 6

Name: _____

Show ALL of your work. Read all instructions. Don't forget units. Round answers when appropriate.

List of formulas:

Standard form:

$$y = ax^2 + bx + c$$

Factored form:

$$y = a(x - p)(x - q)$$

Vertex form:

$$y = a(x - h)^2 + k$$

1. (4 points) For each function, identify its form and give its important values.

Examples:

- Standard form: identify a , b , and c
- Factored form: identify a , p , and q
- Vertex form: identify a , h , and k

(a) (2 points)

$$y = 6x - 9x^2$$

(b) (2 points)

$$g(z) = -3(z + 6)^2 - 90$$

2. (6 points) Give a possible formula for the quadratic function graphed below. Show all of your work!

Information shown on the graph:

- Vertex: $(4, -3)$
- Point: $(1, 7)$
- Use the graph provided by your instructor.

3. (1 point) BONUS

Find the zeroes of

$$y = 100(x - 345)^2 + 685$$

8.7.6: Quiz 7

MATH 100 - Quiz 7

Name: _____

Show ALL of your work. Read all instructions. Don't forget units. Round answers when appropriate.

List of formulas:

Standard form:

$$y = ax^2 + bx + c$$

Factored form:

$$y = a(x - p)(x - q)$$

Vertex form:

$$y = a(x - h)^2 + k$$

1. (5 points) Convert the following quadratic function to vertex form by completing the square.

$$y = 3x^2 + 15x - 18$$

2. (5 points) Convert the following quadratic function to factored form.

$$y = 5x^2 + 21x - 20$$

3. (1 point) BONUS: Give the exact coordinates of the minimum point on the curve

$$y = x^2 - \sqrt{7}x + \sqrt{98}$$

8.7.7: Quiz 8

MATH 100 - Quiz 8

Name: _____

Show ALL of your work. Read all instructions. Don't forget units. Round answers when appropriate.

List of formulas:

Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad (8.7.7.1)$$

Discriminant:

$$D = b^2 - 4ac \quad (8.7.7.2)$$

1. (6 points) Solve the following equation. Give an exact answer.

$$-6x^2 + 29x - 9 = 0$$

2. (4 points) How many solutions does the following equation have?

Justify your answer. ****DO NOT SOLVE.****

$$52x^2 - 3x + 5 = 0$$

3. (1 point) BONUS: Solve

$$x^4 - 5x^2 + 6 = 0$$

8.7.8: Quiz 2

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8.8: Instructor Guide on using the Workbok

Any instructor is welcome to use these materials as they see fit. The following are some suggestions on how to make use of them to support active learning.

Suggestions for using the Math 140 Workbook

The worksheets provided here are intended to be used as “groupwork activities” enhanced with [Desmos](#) to support active learning in Math 140 classes. Here are a few ideas about how to make use of them.

Before the class session: Before class begins, have a plan for how you’ll divide students into groups. How to do this is up to each instructor. I have found success using randomly assigned groups of three which persist for a few weeks at a time so that students can get to know each other in their groups. When you are planning the class session, decide what students need to know before they begin working, and what you expect them to work out in their groups as they go through the worksheet problems.

Begin with a short lecture: Most of the problem set assumes that the instructor will give a short “mini lecture” of about 20 to 25 minutes at the start of class. This should introduce any necessary new ideas and probably give a few examples. A reading-over and quick solution of a given section worksheet should provide some ideas about what needs to get covered in the minilecture, and what can be left for students to work out in the worksheet activity as a group. It is important to remind students that it is expected that students may struggle with a given problem.

Give about half an hour for working on the worksheet activity in groups

During this time, the instructor can circulate around the room and listen actively as groups are working, provide hints, or answer questions as needed. I found it helpful to guide students in their thinking and not impose a certain strategy or method to solve the problem.

Student buy-in is essential when implementing active learning strategies (Yoshinobu, 2019). If students buy-in into the group work, then they are engaged with it. These are some ideas I used in my classes:

▫	Introduce active learning on day one, so students get used to it.
▫	Communicate and promote sources of support for learning. My role as institutional agent of support for students both inside and outside the classroom.
▫	Cultivate a welcoming and inclusive course climate.
▫	Offers students the opportunity to make corrections based on feedback provided on tests or quizzes.
▫	Being aware of our own biases and our Identity.

Finish with a Wrap-up: This is a very important step. During the last 20 to 25 minutes, instructors may choose to have students present some solutions for the rest of the class. For more computational practice problems, instructors might simply read out answers and see which problems students might want to discuss further. Also, instructors may have planned to present some definition or theorem that formalizes an idea that students have been working with during the group work.

Self -Reflection: Reflection questions encourage instructors to reflect on their success or any adjustments they may implement for the next group work activity. If you find yourself struggling to implement active-learning methods in your courses, I suggest starting small. Here are some actions you can consider:

- Pick one class meeting and introduce some think-pair-share activities; you could also offer multiple ways for students to engage with class and the material, where students work individually and then in groups.
- Please share any other ideas you used in your own classroom.

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