MATH 100 (BASIC MATH)



Kathy Davis
Manchester University

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Licensing

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CHAPTER OVERVIEW

1: Whole Number Review

The sections in this module will help you to review whole numbers. Your goal is to demonstrate mastery on each of the following objectives.

- **Objective 1.1 - Place Value:** Identify digits in a given place value position and identify the place value of given digits for whole numbers and use this knowledge to write the names of numbers.
- **Objective 1.2 - Add Whole Numbers:** Calculate the sum of single digit and multiple digit whole number addition problems, including applications, without the use of a calculator.
- **Objective 1.3 - Subtract Whole Numbers:** Calculate the difference of single digit and multiple digit whole number subtraction problems, including applications, without the use of a calculator.
- **Objective 1.4 - Multiply Whole Numbers:** Calculate the product of single digit and multiple digit whole number multiplication problems, including applications, *using various notations* without the use of a calculator.
- **Objective 1.5 - Divide Whole Numbers:** Calculate the quotient of single digit and multiple digit whole number division problems, including applications, *using various notations* without the use of a calculator.

1.1: Introduction to Whole Numbers
1.2: Add Whole Numbers
1.3: Subtract Whole Numbers
1.4: Multiply Whole Numbers
1.5: Divide Whole Numbers

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1.1: Introduction to Whole Numbers

Learning Objectives

By the end of this section, you will be able to:

- Identify counting numbers and whole numbers
- Model whole numbers
- Identify the place value of a digit
- Use place value to name whole numbers
- Use place value to write whole numbers
- Identify Counting Numbers and Whole Numbers

Learning algebra is similar to learning a language. You start with a basic vocabulary and then add to it as you go along. You need to practice often until the vocabulary becomes easy to you. The more you use the vocabulary, the more familiar it becomes.

Algebra uses numbers and symbols to represent words and ideas. Let's look at the numbers first. The most basic numbers used in algebra are those we use to count objects: $1, 2, 3, 4, 5, \ldots$ and so on. These are called the counting numbers. The notation "…" is called an ellipsis, which is another way to show "and so on", or that the pattern continues endlessly. Counting numbers are also called natural numbers.

Counting Numbers

The counting numbers start with 1 and continue.

$$1, 2, 3, 4, 5 \ldots$$

Counting numbers and whole numbers can be visualized on a number line as shown in Figure 1.2.

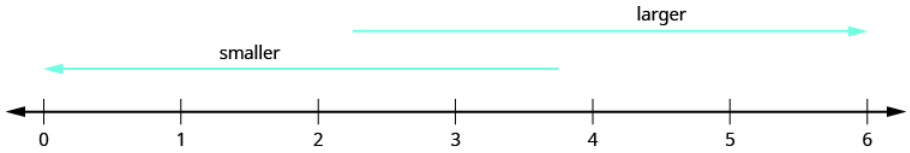


Figure 1.2 The numbers on the number line increase from left to right, and decrease from right to left.

The point labeled 0 is called the origin. The points are equally spaced to the right of 0 and labeled with the counting numbers. When a number is paired with a point, it is called the coordinate of the point.

The discovery of the number zero was a big step in the history of mathematics. Including zero with the counting numbers gives a new set of numbers called the whole numbers.

Whole Numbers

The whole numbers are the counting numbers and zero.

$$0, 1, 2, 3, 4, 5 \ldots$$

We stopped at 5 when listing the first few counting numbers and whole numbers. We could have written more numbers if they were needed to make the patterns clear.

Example 1.1

Which of the following are ⓐ counting numbers? ⓑ whole numbers?

$0, \frac{1}{4}, 3, 5.2, 15, 105$

Answer

- ⓐ The counting numbers start at 1, so 0 is not a counting number. The numbers $3, 15,$ are all counting numbers.
- ⓑ Whole numbers are counting numbers and 0. The numbers $0, 3, 15,$ are whole numbers.

The numbers $\frac{1}{4}$ and 5.2 are neither counting numbers nor whole numbers. We will discuss these numbers later.

Try It 1.1

Which of the following are ⓐ counting numbers ⓑ whole numbers?

$0, \frac{2}{3}, 2, 9, 11.8, 241, 376$

Try It 1.2

Which of the following are ⓐ counting numbers ⓑ whole numbers?

$0, \frac{5}{3}, 7, 8.8, 13, 201$

Model Whole Numbers

Our number system is called a place value system because the value of a digit depends on its position, or place, in a number. The number 537 has a different value than the number 735. Even though they use the same digits, their value is different because of the different placement of the 7 and the 5.

Money gives us a familiar model of place value. Suppose a wallet contains three Math input error



Three $100 bills
3 × $100
$300

Seven $10 bills
7 × $10
$70

Four $1 bills
4 × $1
$4

Figure 1.3

Find the total value of each kind of bill, and then add to find the total. The wallet contains $374.



Base-10 blocks provide another way to model place value, as shown in Figure 1.4. The blocks can be used to represent hundreds, tens, and ones. Notice that the tens rod is made up of 10 ones, and the hundreds square is made of 10 tens, or 100 ones.

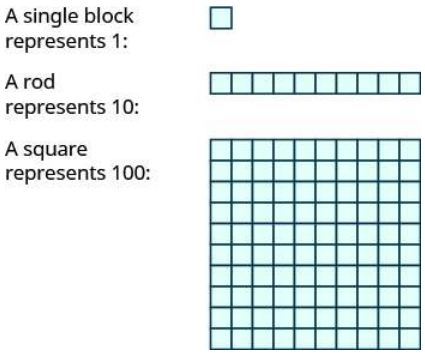


Figure 1.4

Figure 1.5 shows the number 138 modeled with base- blocks.

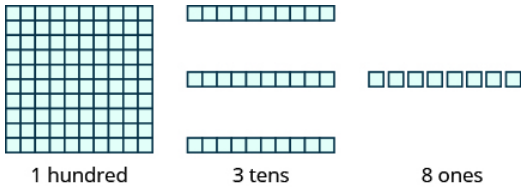
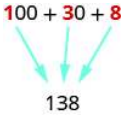


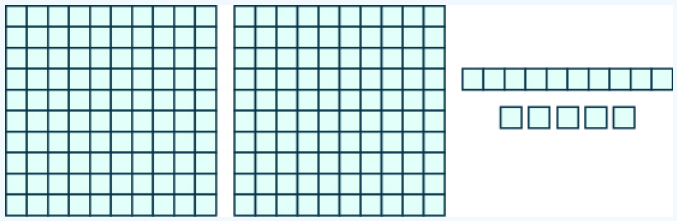
Figure 1.5 We use place value notation to show the value of the number 138.



Digit	Place value	Number	Value	Total value
1	hundreds	1	100	100
3	tens	3	10	30
8	ones	8	1	+ 8
				Sum =

Example 1.2

Use place value notation to find the value of the number modeled by the base- blocks shown.

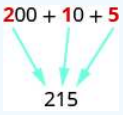


Answer

There are 2 hundreds squares, which is 200.

There is 1 tens rod, which is 10.

There are 5 ones blocks, which is 5.

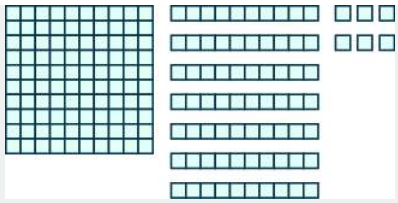


Digit	Place value	Number	Value	Total value
2	hundreds	2	100	200
1	tens	1	10	10
5	ones	5	1	+ 5
				215

The base- blocks model the number 215.

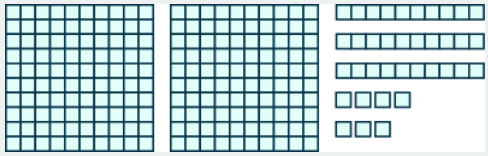
Try It 1.3

Use place value notation to find the value of the number modeled by the base- blocks shown.



Try It 1.4

Use place value notation to find the value of the number modeled by the base- blocks shown.



Manipulative Mathematics

Doing the Manipulative Mathematics activity Number Line-Part 1 will help you develop a better understanding of the counting numbers and the whole numbers.

Identify the Place Value of a Digit

By looking at money and base- blocks, we saw that each place in a number has a different value. A place value chart is a useful way to summarize this information. The place values are separated into groups of three, called periods. The periods are *ones*, *thousands*, *millions*, *billions*, *trillions*, and so on. In a written number, commas separate the periods.

Just as with the base- blocks, where the value of the tens rod is ten times the value of the ones block and the value of the hundreds square is ten times the tens rod, the value of each place in the place-value chart is ten times the value of the place to the right of it.

Figure 1.6 shows how the number $5{,}278{,}194$ is written in a place value chart.

Place Value														
Trillions			Billions			Millions			Thousands			Ones		
Hundred trillions	Ten trillions	Trillions	Hundred billions	Ten billions	Billions	Hundred millions	Ten millions	Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
								5	2	7	8	1	9	4

Figure 1.6

- The digit 5 is in the millions place. Its value is $5{,}000{,}000$.
- The digit 2 is in the hundred thousands place. Its value is $200{,}000$.
- The digit 7 is in the ten thousands place. Its value is $70{,}000$.
- The digit 8 is in the thousands place. Its value is $8{,}000$.
- The digit 1 is in the hundreds place. Its value is 100.
- The digit 9 is in the tens place. Its value is 90.
- The digit 4 is in the ones place. Its value is 4.

Example 1.3

In the number $63{,}407{,}218$; find the place value of each of the following digits:

a. ⓐ 7

b. ⓑ 0
c. ⓒ 1
d. ⓓ 6
e. ⓔ 3

Answer

Write the number in a place value chart, starting at the right.

Trillions			Billions			Millions			Thousands			Ones		
Hundred trillions	Ten trillions	Trillions	Hundred billions	Ten billions	Billions	Hundred millions	Ten millions	Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
							6	3	4	0	7	2	1	8

- ⓐ The 7 is in the thousands place.
- ⓑ The 0 is in the ten thousands place.
- ⓒ The 1 is in the tens place.
- ⓓ The 6 is in the ten millions place.
- ⓔ The 3 is in the millions place.

Try It 1.5

For each number, find the place value of digits listed: $27{,}493{,}615$

a. ⓐ 2
b. ⓑ 1
c. ⓒ 4
d. ⓓ 7
e. ⓔ 5

Try It 1.6

For each number, find the place value of digits listed: $519{,}711{,}641{,}328$

a. ⓐ 9
b. ⓑ 4
c. ⓒ 2
d. ⓓ 6
e. ⓔ 7

Use Place Value to Name Whole Numbers

When you write a check, you write out the number in words as well as in digits. To write a number in words, write the number in each period followed by the name of the period without the ‘s’ at the end. Start with the digit at the left, which has the largest place value. The commas separate the periods, so wherever there is a comma in the number, write a comma between the words. The ones period, which has the smallest place value, is not named.

Additional Online Resources

- Determine Place Value
- Write a Whole Number in Digits from Words

Section 1.1 Exercises

Practice Makes Perfect

Identify Counting Numbers and Whole Numbers

In the following exercises, determine which of the following numbers are ⓐ counting numbers ⓑ whole numbers.

1.

$0, \frac{2}{3}, 5, 8.1, 125$

2.

$0, \frac{7}{10}, 3, 20.5, 300$

3.

$0, \frac{4}{9}, 3.9, 50, 221$

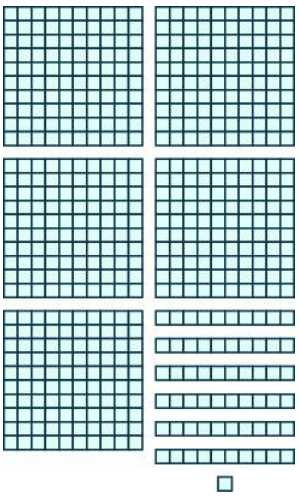
4.

$0, \frac{3}{5}, 10, 303, 422.6$

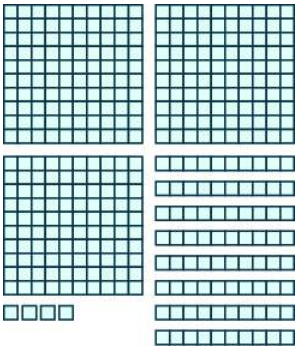
Model Whole Numbers

In the following exercises, use place value notation to find the value of the number modeled by the base- blocks.

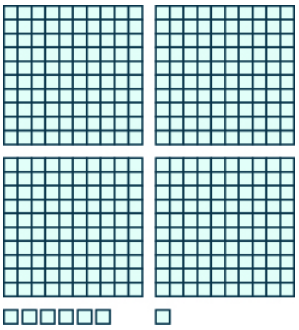
5.



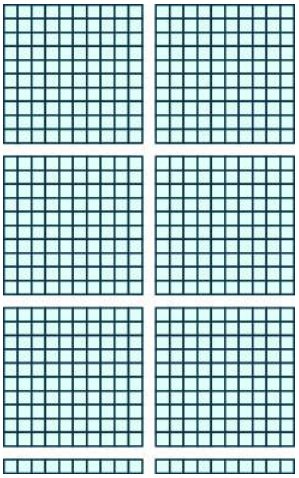
6.



7.



8.



Identify the Place Value of a Digit

In the following exercises, find the place value of the given digits.

9.

579,601

a. ⓐ 9
b. ⓑ 6
c. ⓒ 0
d. ⓓ 7
e. ⓔ 5

10.

398,127

1. ⓐ 9
2. ⓑ 3
3. ⓒ 2
4. ⓓ 8
5. ⓔ 7

11.

56,804,379

1. ⓐ 8
2. ⓑ 6
3. ⓒ 4
4. ⓓ 7
5. ⓔ 0

12.

$78{,}320{,}465$

1. ⓐ 8
2. ⓑ 4
3. ⓒ 2
4. ⓓ 6
5. ⓔ 7

Use Place Value to Name Whole Numbers

In the following exercises, name each number in words.

13.

$1{,}078$

14.

$5{,}902$

15.

$364{,}510$

16.

$146{,}023$

17.

$5{,}846{,}103$

18.

$1{,}458{,}398$

19.

$37{,}889{,}005$

20.

$62{,}008{,}465$

21.

The height of Mount Ranier is $14{,}410$ feet.

22.

The height of Mount Adams is $12{,}276$ feet.

23.

Seventy years is $613{,}200$ hours.

24.

One year is $525{,}600$ minutes.

25.

The U.S. Census estimate of the population of Miami-Dade county was $2{,}617{,}176$.

26.

The population of Chicago was $2{,}718{,}782$.

27.

There are projected to be $23{,}867{,}000$ college and university students in the US in five years.

28.

About twelve years ago there were $20{,}665{,}415$ registered automobiles in California.

29.

The population of China is expected to reach $1{,}377{,}583{,}156$ in 2016.

30.

The population of India is estimated at $1{,}267{,}401{,}849$ as of July $1, 2014$.

Use Place Value to Write Whole Numbers

In the following exercises, write each number as a whole number using digits.

31.

four hundred twelve

32.

two hundred fifty-three

33.

thirty-five thousand, nine hundred seventy-five

34.

sixty-one thousand, four hundred fifteen

35.

eleven million, forty-four thousand, one hundred sixty-seven

36.

eighteen million, one hundred two thousand, seven hundred eighty-three

37.

three billion, two hundred twenty-six million, five hundred twelve thousand, seventeen

38.

eleven billion, four hundred seventy-one million, thirty-six thousand, one hundred six

39.

The population of the world was estimated to be seven billion, one hundred seventy-three million people.

40.

The age of the solar system is estimated to be four billion, five hundred sixty-eight million years.

41.

Lake Tahoe has a capacity of thirty-nine trillion gallons of water.

42.

The federal government budget was three trillion, five hundred billion dollars.

Everyday Math

51.

Writing a Check Jorge bought a car for $\$24{,}493$. He paid for the car with a check. Write the purchase price in words.

52.

Writing a Check Marissa's kitchen remodeling cost $\$18{,}549$. She wrote a check to the contractor. Write the amount paid in words.

Writing Exercises

57.

In your own words, explain the difference between the counting numbers and the whole numbers.

58.

Give an example from your everyday life where it helps to round numbers.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
identify counting numbers and whole numbers.			
model whole numbers.			
identify the place value of a digit.			
use place value to name whole numbers.			
use place value to write whole numbers.			
round whole numbers.			

ⓑ If most of your checks were...

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them. What did you do to become confident of your ability to do these things? Be specific.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is important to make sure you have a strong foundation before you move on. Whom can you ask for help? Your fellow classmates and instructor are good resources. Is there a place on campus where math tutors are available? Can your study skills be improved?

…no—I don't get it! This is a warning sign and you must not ignore it. You should get help right away or you will quickly be overwhelmed. See your instructor as soon as you can to discuss your situation. Together you can come up with a plan to get you the help you need.

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1.2: Add Whole Numbers

Learning Objectives

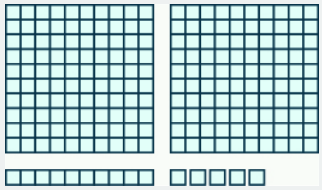
By the end of this section, you will be able to:

- Use addition notation
- Model addition of whole numbers
- Add whole numbers without models
- Translate word phrases to math notation
- Add whole numbers in applications

Be Prepared 1.1

Before you get started, take this readiness quiz.

What is the number modeled by the base- blocks?



If you missed this problem, review [link].

Be Prepared 1.2

Write the number three hundred forty-two thousand six using digits?
If you missed this problem, review [link].

Use Addition Notation

A college student has a part-time job. Last week he worked 3 hours on Monday and 4 hours on Friday. To find the total number of hours he worked last week, he added 3 and 4.

The operation of addition combines numbers to get a sum. The notation we use to find the sum of 3 and 4 is:

$$3+4$$

We read this as *three plus four* and the result is the sum of three and four. The numbers 3 and 4 are called the addends. A math statement that includes numbers and operations is called an expression.

Addition Notation

To describe addition, we can use symbols and words.

Operation	Notation	Expression	Read as	Result
Addition	$+$	$3+4$	three plus four	the sum of 3 and 4

Example 1.11

Translate from math notation to words:

1. ⓐ $7+1$
2. ⓑ $12+14$

Answer

- ⓐ The expression consists of a plus symbol connecting the addends 7 and 1. We read this as *seven plus one*. The result is *the sum of seven and one*.
- ⓑ The expression consists of a plus symbol connecting the addends 12 and 14. We read this as *twelve plus fourteen*. The result is the *sum of twelve and fourteen*.

Try It 1.21

Translate from math notation to words:

1. ⓐ $8 + 4$
2. ⓑ $18 + 11$

Try It 1.22

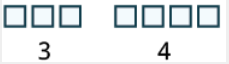
Translate from math notation to words:

1. ⓐ $21 + 16$
2. ⓑ $100 + 200$

Model Addition of Whole Numbers

Addition is really just counting. We will model addition with base-10 blocks. Remember, a block represents 1 and a rod represents 10. Let's start by modeling the addition expression we just considered, $3 + 4$.

Each addend is less than 10, so we can use ones blocks.

We start by modeling the first number with 3 blocks.	
Then we model the second number with 4 blocks.	
Count the total number of blocks.	

There are 7 blocks in all. We use an equal sign $(=)$ to show the sum. A math sentence that shows that two expressions are equal is called an equation. We have shown that. $3 + 4 = 7$.

Manipulative Mathematics

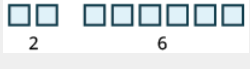
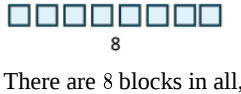
Example 1.12

Model the addition $2 + 6$.

Answer

$2 + 6$ means the sum of 2 and 6

Each addend is less than 10, so we can use ones blocks.

Model the first number with 2 blocks.	
Model the second number with 6 blocks.	
Count the total number of blocks	 There are 8 blocks in all, so $2 + 6 = 8$.

Try It 1.23

Model: $3 + 6$.

Try It 1.24

Model: $5 + 1$.

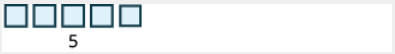
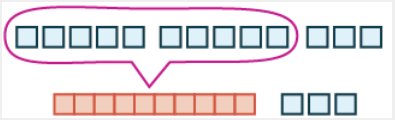
When the result is 10 or more ones blocks, we will exchange the 10 blocks for one rod.

Example 1.13

Model the addition $5 + 8$.

Answer

$5 + 8$ means the sum of 5 and 8.

Each addend is less than 10, se we can use ones blocks.	
Model the first number with 5 blocks.	
Model the second number with 8 blocks.	
Count the result. There are more than 10 blocks so we exchange 10 ones blocks for 1 tens rod.	
Now we have 1 ten and 3 ones, which is 13.	5 + 8 = 13

Notice that we can describe the models as ones blocks and tens rods, or we can simply say *ones* and *tens*. From now on, we will use the shorter version but keep in mind that they mean the same thing.

Try It 1.25

Model the addition: $5 + 7$.

Try It 1.26

Model the addition: $6 + 8$.

Next we will model adding two digit numbers.

Example 1.14

Model the addition: $17 + 26$.

Answer

$17 + 26$ means the sum of 17 and 26.

Model the 17.	1 ten and 7 ones	
Model the 26.	2 tens and 6 ones	

Combine.	3 tens and 13 ones	
Exchange 10 ones for 1 ten.	4 tens and 3 ones $40 + 3 = 43$	
We have shown that $17 + 26 = 43$		

Try It 1.27

Model each addition: $15 + 27$.

Try It 1.28

Model each addition: $16 + 29$.

Add Whole Numbers Without Models

Now that we have used models to add numbers, we can move on to adding without models. Before we do that, make sure you know all the one digit addition facts. You will need to use these number facts when you add larger numbers.

Imagine filling in Table 1.1 by adding each row number along the left side to each column number across the top. Make sure that you get each sum shown. If you have trouble, model it. It is important that you memorize any number facts you do not already know so that you can quickly and reliably use the number facts when you add larger numbers.

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

Table 1.1

Did you notice what happens when you add zero to a number? The sum of any number and zero is the number itself. We call this the Identity Property of Addition. Zero is called the additive identity.

Identity Property of Addition

The sum of any number a and 0 is the number.

$$a + 0 = a$$
$$0 + a = a$$

Example 1.15

Find each sum:

1. ⓐ $0 + 11$
2. ⓑ $42 + 0$

Answer

ⓐ The first addend is zero. The sum of any number and zero is the number.	$0 + 11 = 11$
ⓑ The second addend is zero. The sum of any number and zero is the number.	$42 + 0 = 42$

Try It 1.29

Find each sum:

1. ⓐ $0 + 19$
2. ⓑ $39 + 0$

Try It 1.30

Find each sum:

1. ⓐ $0 + 24$
2. ⓑ $57 + 0$

Look at the pairs of sums.

$2 + 3 = 5$	$3 + 2 = 5$
$4 + 7 = 11$	$7 + 4 = 11$
$8 + 9 = 17$	$9 + 8 = 17$

Notice that when the order of the addends is reversed, the sum does not change. This property is called the Commutative Property of Addition, which states that changing the order of the addends does not change their sum.

Commutative Property of Addition

Changing the order of the addends a and b does not change their sum.

$$a + b = b + a$$

Example 1.16

Add:

1. ⓐ $8 + 7$
2. ⓑ $7 + 8$

Answer

- ⓐ

Add.	$8 + 7$
	15

- ⓑ

Add.	$7+8$
	15

Did you notice that changing the order of the addends did not change their sum? We could have immediately known the sum from part ⓑ just by recognizing that the addends were the same as in part ⓐ, but in the reverse order. As a result, both sums are the same.

Try It 1.31

Add: $9+7$ and $7+9$.

Try It 1.32

Add: $8+6$ and $6+8$.

Example 1.17

Add: $28+61$.

Answer

To add numbers with more than one digit, it is often easier to write the numbers vertically in columns.

Write the numbers so the ones and tens digits line up vertically.	28 $+61$
Then add the digits in each place value. Add the ones: $8+1=9$ Add the tens: $2+6=8$	28 $+61$ 89

Try It 1.33

Add: $32+54$.

Try It 1.34

Add: $25+74$.

In the previous example, the sum of the ones and the sum of the tens were both less than Math input error

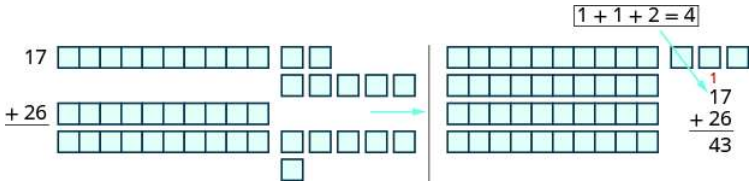


Figure 1.10

When we add the ones, $7+6$, we get 13 ones. Because we have more than 10 ones, we can exchange 10 of the ones for 1 ten. Now we have 4 tens and 3 ones. Without using the model, we show this as a small red 1 above the digits in the tens place.

When the sum in a place value column is greater than 9, we carry over to the next column to the left. Carrying is the same as regrouping by exchanging. For example, 10 ones for 1 ten or 10 tens for 1 hundred.

How To

Add whole numbers.

1. Step 1. Write the numbers so each place value lines up vertically.
2. Step 2. Add the digits in each place value. Work from right to left starting with the ones place. If a sum in a place value is more than 9, carry to the next place value.
3. Step 3. Continue adding each place value from right to left, adding each place value and carrying if needed.

Example 1.18

Add: $43 + 69$.

Answer

Write the numbers so the digits line up vertically.	$\begin{array}{r} 43 \\ +69 \\ \hline \end{array}$
Add the digits in each place. Add the ones: $3 + 9 = 12$	
Write the 2 in the ones place in the sum. Add the 1 ten to the tens place.	$\begin{array}{r} {\scriptstyle 1} \\ 43 \\ +69 \\ \hline 2 \end{array}$
Now add the tens: $1 + 4 + 6 = 11$ Write the 11 in the sum.	$\begin{array}{r} {\scriptstyle 1} \\ 43 \\ +69 \\ \hline 112 \end{array}$

Try It 1.35

Add: $35 + 98$.

Try It 1.36

Add: $72 + 89$.

Example 1.19

Add: $324 + 586$.

Answer

Write the numbers so the digits line up vertically.	$\begin{array}{r} 324 \\ +\ 586 \\ \hline \end{array}$
Add the digits in each place value. Add the ones: $4 + 6 = 10$ Write the 0 in the ones place in the sum and carry the 1 ten to the tens place.	$\begin{array}{r} {\scriptstyle 1} \\ 324 \\ +\ 586 \\ \hline 0 \end{array}$
Add the tens: $1 + 2 + 8 = 11$ Write the 1 in the tens place in the sum and carry the 1 hundred to the hundreds	$\begin{array}{r} {\scriptstyle 1}{\scriptstyle 1} \\ 324 \\ +\ 586 \\ \hline 10 \end{array}$
Add the hundreds: $1 + 3 + 5 = 9$ Write the 9 in the hundreds place.	$\begin{array}{r} {\scriptstyle 1}{\scriptstyle 1} \\ 324 \\ +\ 586 \\ \hline 910 \end{array}$

Try It 1.37

Add: $456 + 376$.

Try It 1.38

Add: $269 + 578$.

Example 1.20

Add: $1{,}683 + 479$.

Answer

Write the numbers so the digits line up vertically.	$\begin{array}{r} 1{,}683 \\ +\ 479 \end{array}$
Add the digits in each place value.	
Add the ones: $3 + 9 = 12$. Write the 2 in the ones place of the sum and carry the 1 ten to the tens place.	$\begin{array}{r} 1{,}6\overset{1}{8}3 \\ +\ 479 \\ \hline 2 \end{array}$
Add the tens: $1 + 7 + 8 = 16$ Write the 6 in the tens place and carry the 1 hundred to the hundreds place.	$\begin{array}{r} 1{,}\overset{1\,1}{6}83 \\ +\ 479 \\ \hline 62 \end{array}$
Add the hundreds: $1 + 6 + 4 = 11$ Write the 1 in the hundreds place and carry the 1 thousand to the thousands place.	$\begin{array}{r} 1{,}\overset{1\,1}{6}83 \\ +\ 479 \\ \hline 162 \end{array}$
Add the thousands $1 + 1 = 2$. Write the 2 in the thousands place of the sum.	$\begin{array}{r} \overset{1\,1\,1}{1{,}}683 \\ +\ 479 \\ \hline 2{,}162 \end{array}$

When the addends have different numbers of digits, be careful to line up the corresponding place values starting with the ones and moving toward the left.

Try It 1.39

Add: $4{,}597 + 685$.

Try It 1.40

Add: $5{,}837 + 695$.

Example 1.21

Add: $21{,}357 + 861 + 8{,}596$.

Answer

Write the numbers so the place values line up vertically.	$\begin{array}{r} 21{,}357 \\ 861 \\ +\ 8{,}596 \end{array}$
Add the digits in each place value.	

Add the ones: $7+1+6=14$ Write the 4 in the ones place of the sum and carry the 1 to the tens place.	$\begin{array}{r} {\scriptstyle 1}\\ 21{,}357\\ 861\\ +\ 8{,}596\\ \hline 4 \end{array}$
Add the tens: $1+5+6+9=21$ Write the 1 in the tens place and carry the 2 to the hundreds place.	$\begin{array}{r} {\scriptstyle 2\,1}\\ 21{,}357\\ 861\\ +\ 8{,}596\\ \hline 14 \end{array}$
Add the hundreds: $2+3+8+5=18$ Write the 8 in the hundreds place and carry the 1 to the thousands place.	$\begin{array}{r} {\scriptstyle 1\,2\,1}\\ 21{,}357\\ 861\\ +\ 8{,}596\\ \hline 814 \end{array}$
Add the thousands $1+1+8=10$. Write the 0 in the thousands place and carry the 1 to the ten thousands place.	$\begin{array}{r} {\scriptstyle 1\,1\,2\,1}\\ 21{,}357\\ 861\\ +\ 8{,}596\\ \hline 0814 \end{array}$
Add the ten-thousands $1+2=3$. Write the 3 in the ten thousands place in the sum.	$\begin{array}{r} {\scriptstyle 1\,1\,2\,1}\\ 21{,}357\\ 861\\ +\ 8{,}596\\ \hline 30{,}814 \end{array}$

This example had three addends. We can add any number of addends using the same process as long as we are careful to line up the place values correctly.

Try It 1.41

Add: $46{,}195+397+6{,}281$.

Try It 1.42

Add: $53{,}762+196+7{,}458$.

Translate Word Phrases to Math Notation

Earlier in this section, we translated math notation into words. Now we'll reverse the process. We'll translate word phrases into math notation. Some of the word phrases that indicate addition are listed in Table 1.2.

Operation	Words	Example	Expression
Addition	plus	1 plus 2	$1+2$
	sum	the sum of 3 and 4	$3+4$
	increased by	5 increased by 6	$5+6$
	more than	8 more than 7	$7+8$
	total of	the total of 9 and 5	$9+5$
	added to	6 added to 4	$4+6$

Table 1.2

Example 1.22

Translate and simplify: the sum of 19 and 23.

Answer

The word *sum* tells us to add. The words *of* 19 *and* 23 tell us the addends.

	The sum of 19 and 23
Translate.	$19 + 23$
Add.	42
	The sum of 19 and 23 is 42.

Try It 1.43

Translate and simplify: the sum of 17 and 26.

Try It 1.44

Translate and simplify: the sum of 28 and 14.

Example 1.23

Translate and simplify: 28 increased by 31.

Answer

The words *increased by* tell us to add. The numbers given are the addends.

	28 increased by 31.
Translate.	$28 + 31$
Add.	59
	So 28 increased by 31 is 59.

Try It 1.45

Translate and simplify: 29 increased by 76.

Try It 1.46

Translate and simplify: 37 increased by 69.

Add Whole Numbers in Applications

Now that we have practiced adding whole numbers, let's use what we've learned to solve real-world problems. We'll start by outlining a plan. First, we need to read the problem to determine what we are looking for. Then we write a word phrase that gives the information to find it. Next we translate the word phrase into math notation and then simplify. Finally, we write a sentence to answer the question.

Example 1.24

Hao earned grades of $87, 93, 68, 95,$ and 89 on the five tests of the semester. What is the total number of points he earned on the five tests?

Answer

We are asked to find the total number of points on the tests.

Write a phrase.	the sum of points on the tests
Translate to math notation.	$87 + 93 + 68 + 95 + 89$

Then we simplify by adding.	
Since there are several numbers, we will write them vertically.	$\begin{array}{r} {}^{3} \\ 87 \\ 93 \\ 68 \\ 95 \\ +89 \\ \hline 432 \end{array}$
Write a sentence to answer the question.	Hao earned a total of 432 points.

Notice that we added *points*, so the sum is 432 *points*. It is important to include the appropriate units in all answers to applications problems.

Try It 1.47

Mark is training for a bicycle race. Last week he rode 18 miles on Monday, 15 miles on Wednesday, 26 miles on Friday, 49 miles on Saturday, and 32 miles on Sunday. What is the total number of miles he rode last week?

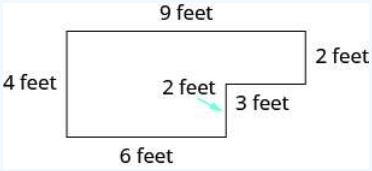
Try It 1.48

Lincoln Middle School has three grades. The number of students in each grade is $230, 165, \text{and } 325$. What is the total number of students?

Some application problems involve shapes. For example, a person might need to know the distance around a garden to put up a fence or around a picture to frame it. The perimeter is the distance around a geometric figure. The perimeter of a figure is the sum of the lengths of its sides.

Example 1.25

Find the perimeter of the patio shown.

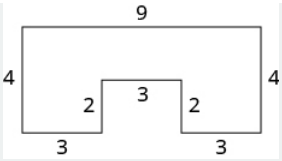


Answer

We are asked to find the perimeter.	
Write a phrase.	the sum of the sides
Translate to math notation.	$4+6+2+3+2+9$
Simplify by adding.	26
Write a sentence to answer the question.	
We added feet, so the sum is 26 feet.	The perimeter of the patio is 26 feet.

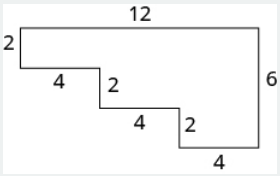
Try It 1.49

Find the perimeter of each figure. All lengths are in inches.



Try It 1.50

Find the perimeter of each figure. All lengths are in inches.



Media

ACCESS ADDITIONAL ONLINE RESOURCES

- Adding Two-Digit Numbers with base-10 blocks
- Adding Three-Digit Numbers with base-10 blocks
- Adding Whole Numbers

Section 1.2 Exercises

Practice Makes Perfect

Use Addition Notation

In the following exercises, translate the following from math expressions to words.

59.

$5 + 2$

60.

$6 + 3$

61.

$13 + 18$

62.

$15 + 16$

63.

$214 + 642$

64.

$438 + 113$

Model Addition of Whole Numbers

In the following exercises, model the addition.

65.

$2 + 4$

66.

$5 + 3$

67.

$8 + 4$

68.

$5 + 9$

69.

$14 + 75$

70.

$15 + 63$

71.

$16 + 25$

72.

$14 + 27$

Add Whole Numbers

In the following exercises, fill in the missing values in each chart.

73.

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2		4	5	6	7		9
1	1	2	3	4			7	8	9	
2		3	4	5	6		8			11
3	3		5		7	8		10		12
4	4	5			8	9		11	12	
5	5	6	7	8			11		13	
6	6	7	8		10			13		15
7			9	10		12			15	16
8	8	9		11			14		16	
9	9	10	11		13	14			17	

74.

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4		6		8	9
1	1	2	3		5	6		8		10
2	2		4		6	7		9	10	
3		4		6			9		11	
4	4	5	6	7			10	11		13
5	5	6		8	9		11	12	13	
6			8	9			12	13		15
7	7	8		10		12			15	16
8	8	9	10		12		14		16	17
9			11	12	13			16		

75.

+	3	4	5	6	7	8	9
6							
7							
8							
9							

76.

+	6	7	8	9
3				
4				
5				
6				
7				
8				
9				

77.

+	5	6	7	8	9
5					
6					
7					
8					
9					

78.

+	6	7	8	9
6				
7				
8				
9				

In the following exercises, add.

79.

1. ⓐ $0 + 13$
2. ⓑ $13 + 0$

80.

1. ⓐ $0 + 5{,}280$
2. ⓑ $5{,}280 + 0$

81.

1. ⓐ $8 + 3$
2. ⓑ $3 + 8$

82.

1. ⓐ $7 + 5$
2. ⓑ $5 + 7$

83.

$45 + 33$

84.

$37 + 22$

85.

$71 + 28$

86.

$43 + 53$

87.

$26 + 59$

88.

$38 + 17$

89.

$64 + 78$

90.

$92 + 39$

91.

$168 + 325$

92.

$247 + 149$

93.

$584 + 277$

94.

$175 + 648$

95.

$832 + 199$

96.

$775 + 369$

97.

$6{,}358 + 492$

98.

$9{,}184 + 578$

99.

$3{,}740 + 18{,}593$

100.

$6{,}118 + 15{,}990$

101.

$485{,}012 + 619{,}848$

102.

$368{,}911 + 857{,}289$

103.

$24{,}731 + 592 + 3{,}868$

104.

$28{,}925 + 817 + 4{,}593$

105.

$8{,}015 + 76{,}946 + 16{,}570$

106.

$6{,}291 + 54{,}107 + 28{,}635$

Translate Word Phrases to Math Notation

In the following exercises, translate each phrase into math notation and then simplify.

107.

the sum of 13 and 18

108.

the sum of 12 and 19

109.

the sum of 90 and 65

110.

the sum of 70 and 38

111.

33 increased by 49

112.

68 increased by 25

113.

250 more than 599

114.

115 more than 286

115.

the total of 628 and 77

116.

the total of 593 and 79

117.

$1{,}482$ added to 915

118.

$2{,}719$ added to 682

Add Whole Numbers in Applications

In the following exercises, solve the problem.

119.

Home remodeling Sophia remodeled her kitchen and bought a new range, microwave, and dishwasher. The range cost $\$1{,}100$, the microwave cost $\$250$, and the dishwasher cost $\$525$. What was the total cost of these three appliances?

120.

Sports equipment Aiden bought a baseball bat, helmet, and glove. The bat cost $\$299$, the helmet cost $\$35$, and the glove cost $\$68$. What was the total cost of Aiden's sports equipment?

121.

Bike riding Ethan rode his bike 14 miles on Monday, 19 miles on Tuesday, 12 miles on Wednesday, 25 miles on Friday, and 68 miles on Saturday. What was the total number of miles Ethan rode?

122.

Business Chloe has a flower shop. Last week she made 19 floral arrangements on Monday, 12 on Tuesday, 23 on Wednesday, 29 on Thursday, and 44 on Friday. What was the total number of floral arrangements Chloe made?

123.

Apartment size Jackson lives in a 7 room apartment. The number of square feet in each room is $238, 120, 156, 196, 100, 132,$ and 225. What is the total number of square feet in all 7 rooms?

124.

Weight Seven men rented a fishing boat. The weights of the men were $175, 192, 148, 169, 205, 181,$ and 225 pounds. What was the total weight of the seven men?

125.

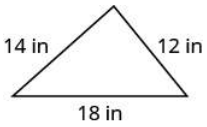
Salary Last year Natalie's salary was $\$82{,}572$. Two years ago, her salary was $\$79{,}316$, and three years ago it was $\$75{,}298$. What is the total amount of Natalie's salary for the past three years?

126.

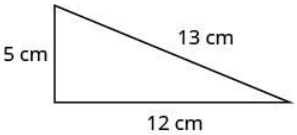
Home sales Emma is a realtor. Last month, she sold three houses. The selling prices of the houses were $\$292{,}540, \$505{,}875,$ and $\$423{,}699$. What was the total of the three selling prices?

In the following exercises, find the perimeter of each figure.

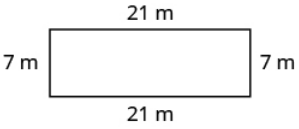
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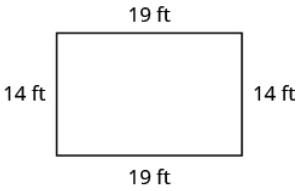
128.



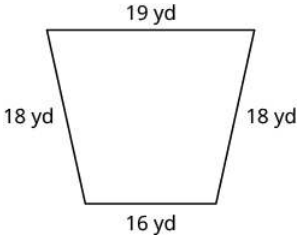
129.



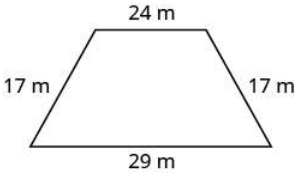
130.



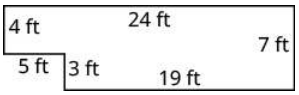
131.



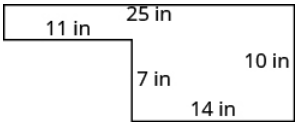
132.



133.



134.



Everyday Math

135.

Calories Paulette had a grilled chicken salad, ranch dressing, and a 16-ounce drink for lunch. On the restaurant's nutrition chart, she saw that each item had the following number of calories:

Grilled chicken salad – 320 calories
Ranch dressing – 170 calories
16-ounce drink – 150 calories

What was the total number of calories of Paulette's lunch?

136.

Calories Fred had a grilled chicken sandwich, a small order of fries, and a 12-oz chocolate shake for dinner. The restaurant's nutrition chart lists the following calories for each item:

Grilled chicken sandwich – 420 calories
Small fries – 230 calories
12-oz chocolate shake – 580 calories

What was the total number of calories of Fred's dinner?

137.

Test scores A student needs a total of 400 points on five tests to pass a course. The student scored 82, 91, 75, 88, and 70. Did the student pass the course?

138.

Elevators The maximum weight capacity of an elevator is 1150 pounds. Six men are in the elevator. Their weights are 210, 145, 183, 230, 159, and 164 pounds. Is the total weight below the elevator's maximum capacity?

Writing Exercises

139.

How confident do you feel about your knowledge of the addition facts? If you are not fully confident, what will you do to improve your skills?

140.

How have you used models to help you learn the addition facts?

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
use addition notation.			
model addition of whole numbers.			
add whole numbers without models.			
translate word phrases to math notation.			
add whole numbers in applications.			

ⓑ After reviewing this checklist, what will you do to become confident for all objectives?

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1.3: Subtract Whole Numbers

Learning Objectives

By the end of this section, you will be able to:

- Use subtraction notation
- Model subtraction of whole numbers
- Subtract whole numbers
- Translate word phrases to math notation
- Subtract whole numbers in applications

Be Prepared 1.3

Before you get started, take this readiness quiz.

Model $3+4$ using base-ten blocks.
If you missed this problem, review [link].

Be Prepared 1.4

Add: $324+586$.
If you missed this problem, review [link].

Use Subtraction Notation

Suppose there are seven bananas in a bowl. Elana uses three of them to make a smoothie. How many bananas are left in the bowl? To answer the question, we subtract three from seven. When we subtract, we take one number away from another to find the difference. The notation we use to subtract 3 from 7 is

$$7-3$$

We read $7-3$ as *seven minus three* and the result is *the difference of seven and three*.

Subtraction Notation

To describe subtraction, we can use symbols and words.

Operation	Notation	Expression	Read as	Result
Subtraction	$-$	$7-3$	seven minus three	the difference of 7 and 3

Example 1.26

Translate from math notation to words: ⓐ $8-1$ ⓑ $26-14$.

Answer

- ⓐ We read this as *eight minus one*. The result is *the difference of eight and one*.
- ⓑ We read this as *twenty-six minus fourteen*. The resuilt is *the difference of twenty-six and fourteen*.

Try It 1.51

Translate from math notation to words:

1. ⓐ $12-4$
2. ⓑ $29-11$

Try It 1.52

Translate from math notation to words:

1. ⓐ $11 - 2$
2. ⓑ $29 - 12$

Model Subtraction of Whole Numbers

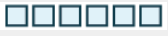
A model can help us visualize the process of subtraction much as it did with addition. Again, we will use base-10 blocks. Remember a block represents 1 and a rod represents 10. Let's start by modeling the subtraction expression we just considered, $7 - 3$.

We start by modeling the first number, 7.	 7
Now take away the second number, 3. We'll circle 3 blocks to show that we are taking them away.	
Count the number of blocks remaining.	
There are 4 ones blocks left.	We have shown that $7 - 3 = 4$.

Example 1.27

Model the subtraction: $8 - 2$.

Answer

$8 - 2$ means the difference of 8 and 2.	
Model the first, 8.	 8
Take away the second number, 2.	
Count the number of blocks remaining.	
There are 6 ones blocks left.	We have shown that $8 - 2 = 6$.

Try It 1.53

Model: $9 - 6$.

Try It 1.54

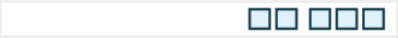
Model: $6 - 1$.

Example 1.28

Model the subtraction: $13 - 8$.

Answer

Model the first number, 13. We use 1 ten and 3 ones.	
Take away the second number, 8. However, there are not 8 ones, so we will exchange the 1 ten for 10 ones.	 10 3

Now we can take away 8 ones.	
Count the blocks remaining.	
There are five ones left.	We have shown that $13 - 8 = 5$.

As we did with addition, we can describe the models as ones blocks and tens rods, or we can simply say ones and tens.

Try It 1.55

Model the subtraction: $12 - 7$.

Try It 1.56

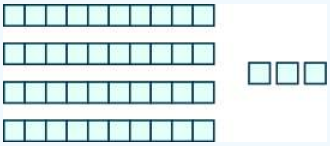
Model the subtraction: $14 - 8$.

Example 1.29

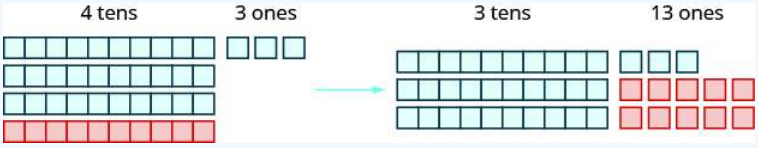
Model the subtraction: $43 - 26$.

Answer

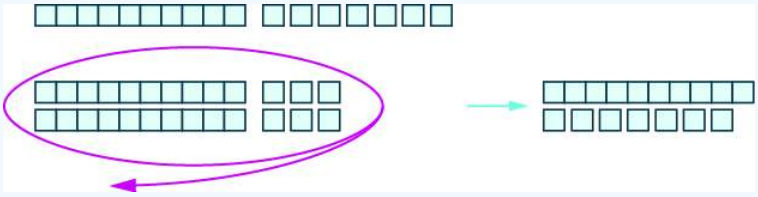
Because $43 - 26$ means 43 take away 26, we begin by modeling the 43.



Now, we need to take away 26, which is 2 tens and 6 ones. We cannot take away 6 ones from 3 ones. So, we exchange 1 ten for 10 ones.



Now we can take away 2 tens and 6 ones.



Count the number of blocks remaining. There is 1 ten and 7 ones, which is 17.

$43 - 26 = 17$

Try It 1.57

Model the subtraction: $42 - 27$.

Try It 1.58

Model the subtraction: $45 - 29$.

Subtract Whole Numbers

Addition and subtraction are inverse operations. Addition undoes subtraction, and subtraction undoes addition.

We know $7 - 3 = 4$ because $4 + 3 = 7$. Knowing all the addition number facts will help with subtraction. Then we can check subtraction by adding. In the examples above, our subtractions can be checked by addition.

$$\begin{array}{lll} 7 - 3 = 4 & \text{because} & 4 + 3 = 7 \\ 13 - 8 = 5 & \text{because} & 5 + 8 = 13 \\ 43 - 26 = 17 & \text{because} & 17 + 26 = 43 \end{array}$$

Example 1.30

Subtract and then check by adding:

1. ⓐ $9 - 7$
2. ⓑ $8 - 3$.

Answer

ⓐ	
	$9 - 7$
Subtract 7 from 9.	2
Check with addition. $2 + 7 = 9$✓	

ⓑ	
	$8 - 3$
Subtract 3 from 8.	5
Check with addition. $5 + 3 = 8$✓	

Try It 1.59

Subtract and then check by adding:

$7 - 0$

Try It 1.60

Subtract and then check by adding:

$6 - 2$

To subtract numbers with more than one digit, it is usually easier to write the numbers vertically in columns just as we did for addition. Align the digits by place value, and then subtract each column starting with the ones and then working to the left.

Example 1.31

Subtract and then check by adding: $89 - 61$.

Answer

Write the numbers so the ones and tens digits line up vertically.	$\begin{array}{r} 89 \\ -61 \end{array}$
Subtract the digits in each place value. Subtract the ones: $9-1=8$ Subtract the tens: $8-6=2$	$\begin{array}{r} 89 \\ -61 \\ \hline 28 \end{array}$
Check using addition. $\begin{array}{r} 28 \\ +61 \\ \hline 89 \end{array}$	

Our answer is correct.

Try It 1.61

Subtract and then check by adding: $86-54$.

Try It 1.62

Subtract and then check by adding: $99-74$.

When we modeled subtracting 26 from 43, we exchanged 1 ten for 10 ones. When we do this without the model, we say we borrow 1 from the tens place and add 10 to the ones place.

How To

Find the difference of whole numbers.

1. Step 1. Write the numbers so each place value lines up vertically.
2. Step 2. Subtract the digits in each place value. Work from right to left starting with the ones place. If the digit on top is less than the digit below, borrow as needed.
3. Step 3. Continue subtracting each place value from right to left, borrowing if needed.
4. Step 4. Check by adding.

Example 1.32

Subtract: $43-26$.

Answer

Write the numbers so each place value lines up vertically.	$\begin{array}{r} 43 \\ -26 \\ \hline \end{array}$
Subtract the ones. We cannot subtract 6 from 3, so we borrow 1 ten. This makes 3 tens and 13 ones. We write these numbers above each place and cross out the original digits.	$\begin{array}{r} {\scriptstyle 3\ 13} \\ \not{4}\not{3} \\ -26 \\ \hline \end{array}$
Now we can subtract the ones. $13-6=7$. We write the 7 in the ones place in the difference.	$\begin{array}{r} {\scriptstyle 3\ 13} \\ \not{4}\not{3} \\ -26 \\ \hline 7 \end{array}$
Now we subtract the tens. $3-2=1$. We write the 1 in the tens place in the difference.	$\begin{array}{r} {\scriptstyle 3\ 13} \\ \not{4}\not{3} \\ -26 \\ \hline 17 \end{array}$

Check by adding. $\begin{array}{r} 17 \\ +26 \\ \hline 43 \checkmark \end{array}$ Our answer is correct.	

Try It 1.63

Subtract and then check by adding: $93 - 58$.

Try It 1.64

Subtract and then check by adding: $81 - 39$.

Example 1.33

Subtract and then check by adding: $207 - 64$.

Answer

Write the numbers so each place value lines up vertically.	$\begin{array}{r} 207 \\ -\ 64 \\ \hline \end{array}$
Subtract the ones. $7 - 4 = 3$. Write the 3 in the ones place in the difference.	$\begin{array}{r} 207 \\ -\ 64 \\ \hline 3 \end{array}$
Subtract the tens. We cannot subtract 6 from 0 so we borrow 1 hundred and add 10 tens to the 0 tens we had. This makes a total of 10 tens. We write 10 above the tens place and cross out the 0. Then we cross out the 2 in the hundreds place and write 1 above it.	$\begin{array}{r} {\scriptstyle 1\ 10} \\ 2\not{0}7 \\ -\ 64 \\ \hline 3 \end{array}$
Now we subtract the tens. $10 - 6 = 4$. We write the 4 in the tens place in the difference.	$\begin{array}{r} {\scriptstyle 1\ 10} \\ 2\not{0}7 \\ -\ 64 \\ \hline 43 \end{array}$
Finally, subtract the hundreds. There is no digit in the hundreds place in the bottom number so we can imagine a 0 in that place. Since $1 - 0 = 1$, we write 1 in the hundreds place in the difference.	$\begin{array}{r} {\scriptstyle 1\ 10} \\ 2\not{0}7 \\ -\ 64 \\ \hline 143 \end{array}$
Check by adding. $\begin{array}{r} {\scriptstyle 1} \\ 143 \\ +\ 64 \\ \hline 207 \checkmark \end{array}$ Our answer is correct.	

Try It 1.65

Subtract and then check by adding: $439 - 52$.

Try It 1.66

Subtract and then check by adding: $318 - 75$.

Example 1.34

Subtract and then check by adding: $910 - 586$.

Answer

Write the numbers so each place value lines up vertically.	$\begin{array}{r} 910 \\ -\ 586 \\ \hline \end{array}$
Subtract the ones. We cannot subtract 6 from 0, so we borrow 1 ten and add 10 ones to the 0 ones we had. This makes 10 ones. We write a 0 above the tens place and cross out the 1. We write the 10 above the ones place and cross out the 0. Now we can subtract the ones. $10 - 6 = 4$.	$\begin{array}{r} {\overset{0}{\cancel{9}}}{\overset{10}{\cancel{1}}}{\overset{10}{\cancel{0}}} \\ -\ 586 \\ \hline \end{array}$
Write the 4 in the ones place of the difference.	$\begin{array}{r} {\overset{0}{\cancel{9}}}{\overset{10}{\cancel{1}}}{\overset{10}{\cancel{0}}} \\ -\ 586 \\ \hline 4 \end{array}$
Subtract the tens. We cannot subtract 8 from 0, so we borrow 1 hundred and add 10 tens to the 0 tens we had, which gives us 10 tens. Write 8 above the hundreds place and cross out the 9. Write 10 above the tens place.	$\begin{array}{r} {\overset{8}{\cancel{9}}}{\overset{10}{\cancel{1}}}{\overset{10}{\cancel{0}}} \\ -\ 586 \\ \hline 4 \end{array}$
Now we can subtract the tens. $10 - 8 = 2$.	$\begin{array}{r} {\overset{8}{\cancel{9}}}{\overset{10}{\cancel{1}}}{\overset{10}{\cancel{0}}} \\ -\ 586 \\ \hline 24 \end{array}$
Subtract the hundreds place. $8 - 5 = 3$ Write the 3 in the hundreds place in the difference.	$\begin{array}{r} {\overset{8}{\cancel{9}}}{\overset{10}{\cancel{1}}}{\overset{10}{\cancel{0}}} \\ -\ 586 \\ \hline 324 \end{array}$
Check by adding. $\begin{array}{r} {\overset{1}{3}}{\overset{1}{2}}4 \\ +\ 586 \\ \hline 910 \checkmark \end{array}$ Our answer is correct.	

Try It 1.67

Subtract and then check by adding: $832 - 376$.

Try It 1.68

Subtract and then check by adding: $847 - 578$.

Example 1.35

Subtract and then check by adding: $2{,}162 - 479$.

Answer

Write the numbers so each place value lines up vertically.	$\begin{array}{r} 2,162 \\ -\ 479 \\ \hline \end{array}$

Subtract the ones. Since we cannot subtract 9 from 2, borrow 1 ten and add 10 ones to the 2 ones to make 12 ones. Write 5 above the tens place and cross out the 6. Write 12 above the ones place and cross out the 2.	$\begin{array}{r} {\scriptstyle 5\ 12} \\ 2,1\cancel{6}\cancel{2} \\ -\ 4\,7\,9 \\ \hline \end{array}$
Now we can subtract the ones.	$12-9=3$
Write 3 in the ones place in the difference.	$\begin{array}{r} {\scriptstyle 5\ 12} \\ 2,1\cancel{6}\cancel{2} \\ -\ 4\,7\,9 \\ \hline 3 \end{array}$
Subtract the tens. Since we cannot subtract 7 from 5, borrow 1 hundred and add 10 tens to the 5 tens to make 15 tens. Write 0 above the hundreds place and cross out the 1. Write 15 above the tens place.	$\begin{array}{r} {\scriptstyle 15} \\ {\scriptstyle 0\ \cancel{1}\ 12} \\ 2,\cancel{1}\,\cancel{6}\,\cancel{2} \\ -\ 4\,7\,9 \\ \hline 3 \end{array}$
Now we can subtract the tens.	$15-7=8$
Write 8 in the tens place in the difference.	$\begin{array}{r} {\scriptstyle 0\ 15\ 12} \\ 2,\cancel{1}\,\cancel{6}\,\cancel{2} \\ -\ 4\,7\,9 \\ \hline 8\,3 \end{array}$
Now we can subtract the hundreds.	$\begin{array}{r} {\scriptstyle 10} \\ {\scriptstyle 1\ \cancel{0}\ 15\ 12} \\ \cancel{2},\cancel{1}\,\cancel{6}\,\cancel{2} \\ -\ 4\,7\,9 \\ \hline 8\,3 \end{array}$
Write 6 in the hundreds place in the difference.	$\begin{array}{r} {\scriptstyle 1\ \ 10\ 15\ 12} \\ \cancel{2},\cancel{1}\,\cancel{6}\,\cancel{2} \\ -\ 4\,7\,9 \\ \hline 6\,8\,3 \end{array}$
Subtract the thousands. There is no digit in the thousands place of the bottom number, so we imagine a 0. $1-0=1$. Write 1 in the thousands place of the difference.	$\begin{array}{r} {\scriptstyle 1\ \ 10\ \cancel{15}\ \cancel{12}} \\ \cancel{2},\cancel{1}\,\cancel{6}\,\cancel{2} \\ -\ 4\,7\,9 \\ \hline 1,6\,8\,3 \end{array}$
Check by adding. $\begin{array}{r} {\scriptstyle 1\ \ 1\ 1} \\ 1,683 \\ +\ \ 479 \\ \hline 2,162\checkmark \end{array}$	

Our answer is correct.

Try It 1.69

Subtract and then check by adding: $4{,}585-697$.

Try It 1.70

Subtract and then check by adding: $5{,}637-899$.

Translate Word Phrases to Math Notation

As with addition, word phrases can tell us to operate on two numbers using subtraction. To translate from a word phrase to math notation, we look for key words that indicate subtraction. Some of the words that indicate subtraction are listed in Table 1.3.

Operation	Word Phrase	Example	Expression

Operation	Word Phrase	Example	Expression
Subtraction	minus	5 minus 1	$5-1$
	difference	the difference of 9 and 4	$9-4$
	decreased by	7 decreased by 3	$7-3$
	less than	5 less than 8	$8-5$
	subtracted from	1 subtracted from 6	$6-1$

Table 1.3

Example 1.36

Translate and then simplify:

1. ⓐ the difference of 13 and 8
2. ⓑ subtract 24 from 43

Answer

- The word *difference* tells us to subtract the two numbers. The numbers stay in the same order as in the phrase.

	the difference of 13 and 8
Translate.	$13-8$
Simplify.	5

- The words *subtract from* tells us to take the first number away from the second. We must be careful to get the order correct.

	subtract 24 from 43
Translate.	$43-24$
Simplify.	19

Try It 1.71

Translate and simplify:

1. ⓐ the difference of 14 and 9
2. ⓑ subtract 21 from 37

Try It 1.72

Translate and simplify:

1. ⓐ 11 decreased by 6
2. ⓑ 18 less than 67

Subtract Whole Numbers in Applications

To solve applications with subtraction, we will use the same plan that we used with addition. First, we need to determine what we are asked to find. Then we write a phrase that gives the information to find it. We translate the phrase into math notation and then simplify to get the answer. Finally, we write a sentence to answer the question, using the appropriate units.

Example 1.37

The temperature in Chicago one morning was 73 degrees Fahrenheit. A cold front arrived and by noon the temperature was 27 degrees Fahrenheit. What was the difference between the temperature in the morning and the temperature at noon?

Answer

We are asked to find the difference between the morning temperature and the noon temperature.

Write a phrase.	the difference of 73 and 27
Translate to math notation. *Difference* tells us to subtract.	$73 - 27$
Then we do the subtraction.	$\begin{array}{r} {}^{6}\not{7}\,{}^{13}\not{3} \\ -\ 2\,7 \\ \hline 4\,6 \end{array}$
Write a sentence to answer the question.	The difference in temperatures was 46 degrees Fahrenheit.

Try It 1.73

The high temperature on June 1^{st} in Boston was 77 degrees Fahrenheit, and the low temperature was 58 degrees Fahrenheit. What was the difference between the high and low temperatures?

Try It 1.74

The weather forecast for June 2 in St Louis predicts a high temperature of 90 degrees Fahrenheit and a low of 73 degrees Fahrenheit. What is the difference between the predicted high and low temperatures?

Example 1.38

A washing machine is on sale for $399. Its regular price is $588. What is the difference between the regular price and the sale price?

Answer

We are asked to find the difference between the regular price and the sale price.

Write a phrase.	the difference between 588 and 399
Translate to math notation.	$588 - 399$
Subtract.	$\begin{array}{r} {}^{4}\not{5}\,{}^{17}\not{8}\,{}^{18}\not{8} \\ -\ 3\,9\,9 \\ \hline 1\,8\,9 \end{array}$
Write a sentence to answer the question.	The difference between the regular price and the sale price is $189.

Try It 1.75

A television set is on sale for $499. Its regular price is $648. What is the difference between the regular price and the sale price?

Try It 1.76

A patio set is on sale for $149. Its regular price is $285. What is the difference between the regular price and the sale price?

Media

ACCESS ADDITIONAL ONLINE RESOURCES

- Model subtraction of two-digit whole numbers
- Model subtraction of three-digit whole numbers
- Subtract Whole Numbers

Section 1.3 Exercises

Practice Makes Perfect

Use Subtraction Notation

In the following exercises, translate from math notation to words.

141.

$15 - 9$

142.

$18 - 16$

143.

$42 - 35$

144.

$83 - 64$

145.

$675 - 350$

146.

$790 - 525$

Model Subtraction of Whole Numbers

In the following exercises, model the subtraction.

147.

$5 - 2$

148.

$8 - 4$

149.

$6 - 3$

150.

$7 - 5$

151.

$18 - 5$

152.

$19 - 8$

153.

$17 - 8$

154.

$17 - 9$

155.

$35 - 13$

156.

$32 - 11$

157.

$61 - 47$

158.

$55 - 36$

Subtract Whole Numbers

In the following exercises, subtract and then check by adding.

159.

$9 - 4$

160.

$9 - 3$

161.

$8 - 0$

162.

$2 - 0$

163.

$38 - 16$

164.

$45 - 21$

165.

$85 - 52$

166.

$99 - 47$

167.

$493 - 370$

168.

$268 - 106$

169.

$5{,}946 - 4{,}625$

170.

$7{,}775 - 3{,}251$

171.

$75 - 47$

172.

$63 - 59$

173.

$461 - 239$

174.

$486 - 257$

175.

$525 - 179$

176.

$542 - 288$

177.

$6{,}318 - 2{,}799$

178.

$8{,}153 - 3{,}978$

179.

$2{,}150 - 964$

180.

$4{,}245 - 899$

181.

$43{,}650 - 8{,}982$

182.

$35{,}162 - 7{,}885$

Translate Word Phrases to Algebraic Expressions

In the following exercises, translate and simplify.

183.

The difference of 10 and 3

184.

The difference of 12 and 8

185.

The difference of 15 and 4

186.

The difference of 18 and 7

187.

Subtract 6 from 9

188.

Subtract 8 from 9

189.

Subtract 28 from 75

190.

Subtract 59 from 81

191.

45 decreased by 20

192.

37 decreased by 24

193.

92 decreased by 67

194.

75 decreased by 49

195.

12 less than 16

196.

15 less than 19

197.

38 less than 61

198.

47 less than 62

Mixed Practice

In the following exercises, simplify.

199.

$76 - 47$

200.

$91 - 53$

201.

$256 - 184$

202.

$305 - 262$

203.

$719 + 341$

204.

$647 + 528$

205.

$2{,}015 - 1{,}993$

206.

$2{,}020 - 1{,}984$

In the following exercises, translate and simplify.

207.

Seventy-five more than thirty-five

208.

Sixty more than ninety-three

209.

13 less than 41

210.

28 less than 36

211.

The difference of 100 and 76

212.

The difference of $1{,}000$ and 945

Subtract Whole Numbers in Applications

In the following exercises, solve.

213.

Temperature The high temperature on June 2 in Las Vegas was 80 degrees and the low temperature was 63 degrees. What was the difference between the high and low temperatures?

214.

Temperature The high temperature on June 1 in Phoenix was 97 degrees and the low was 73 degrees. What was the difference between the high and low temperatures?

215.

Class size Olivia's third grade class has 35 children. Last year, her second grade class had 22 children. What is the difference between the number of children in Olivia's third grade class and her second grade class?

216.

Class size There are 82 students in the school band and 46 in the school orchestra. What is the difference between the number of students in the band and the orchestra?

217.

Shopping A mountain bike is on sale for $\$399$. Its regular price is $\$650$. What is the difference between the regular price and the sale price?

218.

Shopping A mattress set is on sale for $\$755$. Its regular price is $\$1{,}600$. What is the difference between the regular price and the sale price?

219.

Savings John wants to buy a laptop that costs $\$840$. He has $\$685$ in his savings account. How much more does he need to save in order to buy the laptop?

220.

Banking Mason had $\$1{,}125$ in his checking account. He spent $\$892$. How much money does he have left?

Everyday Math

221.

Road trip Noah was driving from Philadelphia to Cincinnati, a distance of 502 miles. He drove 115 miles, stopped for gas, and then drove another 230 miles before lunch. How many more miles did he have to travel?

222.

Test Scores Sara needs 350 points to pass her course. She scored $75, 50, 70,$ and 80 on her first four tests. How many more points does Sara need to pass the course?

Writing Exercises

223.

Explain how subtraction and addition are related.

224.

How does knowing addition facts help you to subtract numbers?

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
use subtraction notation.			
model subtraction of whole numbers.			
subtract whole numbers.			
translate word phrases to math notation.			
subtract whole numbers in applications.			

ⓑ What does this checklist tell you about your mastery of this section? What steps will you take to improve?

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1.4: Multiply Whole Numbers

Learning Objectives

By the end of this section, you will be able to:

- Use multiplication notation
- Model multiplication of whole numbers
- Multiply whole numbers
- Translate word phrases to math notation
- Multiply whole numbers in applications

Be Prepared 1.5

Before you get started, take this readiness quiz.

Add: $1{,}683 + 479$.
If you missed this problem, review [link].

Be Prepared 1.6

Subtract: $605 - 321$.
If you missed this problem, review Example 1.33.

Use Multiplication Notation

Suppose you were asked to count all these pennies shown in Figure 1.11.



Figure 1.11

Would you count the pennies individually? Or would you count the number of pennies in each row and add that number 3 times.

$$8 + 8 + 8$$

Multiplication is a way to represent repeated addition. So instead of adding 8 three times, we could write a multiplication expression.

$$3 \times 8$$

We call each number being multiplied a factor and the result the product. We read 3×8 as *three times eight*, and the result as *the product of three and eight*.

There are several symbols that represent multiplication. These include the symbol $\times$ as well as the dot, $\cdot$, and parentheses $( )$.

Operation Symbols for Multiplication

To describe multiplication, we can use symbols and words.

Operation	Notation	Expression	Read as	Result
Multiplication	$\times$ $\cdot$ $( )$	3×8 $3 \cdot 8$ $3(8)$	three times eight	the product of 3 and 8

Example 1.39

Translate from math notation to words:

1. ⓐ 7×6
2. ⓑ $12 \cdot 14$
3. ⓒ $6(13)$

Answer

- ⓐ We read this as *seven times six* and the result is *the product of seven and six*.
- ⓑ We read this as *twelve times fourteen* and the result is *the product of twelve and fourteen*.
- ⓒ We read this as *six times thirteen* and the result is *the product of six and thirteen*.

Try It 1.77

Translate from math notation to words:

1. ⓐ 8×7
2. ⓑ $18 \cdot 11$

Try It 1.78

Translate from math notation to words:

1. ⓐ $(13)(7)$
2. ⓑ $5(16)$

Model Multiplication of Whole Numbers

There are many ways to model multiplication. Unlike in the previous sections where we used base-10 blocks, here we will use counters to help us understand the meaning of multiplication. A counter is any object that can be used for counting. We will use round blue counters.

Example 1.40

Model: 3×8.

Answer

To model the product 3×8, we'll start with a row of 8 counters.

The other factor is 3, so we'll make 3 rows of 8 counters.



Now we can count the result. There are 24 counters in all.

$3 \times 8 = 24$

If you look at the counters sideways, you'll see that we could have also made 8 rows of 3 counters. The product would have been the same. We'll get back to this idea later.

Try It 1.79

Model each multiplication: 4×6.

Try It 1.80

Model each multiplication: 5×7.

Multiply Whole Numbers

In order to multiply without using models, you need to know all the one digit multiplication facts. Make sure you know them fluently before proceeding in this section.

Table 1.4 shows the multiplication facts. Each box shows the product of the number down the left column and the number across the top row. If you are unsure about a product, model it. It is important that you memorize any number facts you do not already know so you will be ready to multiply larger numbers.

×	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

Table 1.4

What happens when you multiply a number by zero? You can see that the product of any number and zero is zero. This is called the Multiplication Property of Zero.

Multiplication Property of Zero

The product of any number and 0 is 0.

$$a \cdot 0 = 0$$
$$0 \cdot a = 0$$

Example 1.41

Multiply:

1. ⓐ $0 \cdot 11$
2. ⓑ $(42)0$

Answer

ⓐ	$0 \cdot 11$
The product of any number and zero is zero.	0
ⓑ	$(42)0$
Multiplying by zero results in zero.	0

Try It 1.81

Find each product:

1. ⓐ $0 \cdot 19$
2. ⓑ $(39)0$

Try It 1.82

Find each product:

1. ⓐ $0 \cdot 24$
2. ⓑ $(57)0$

What happens when you multiply a number by one? Multiplying a number by one does not change its value. We call this fact the Identity Property of Multiplication, and 1 is called the multiplicative identity.

Identity Property of Multiplication

The product of any number and 1 is the number.

$$1 \cdot a = a$$
$$a \cdot 1 = a$$

Example 1.42

Multiply:

1. ⓐ $(11)1$
2. ⓑ $1 \cdot 42$

Answer

ⓐ	$(11)1$
The product of any number and one is the number.	11
ⓑ	$1 \cdot 42$
Multiplying by one does not change the value.	42

Try It 1.83

Find each product:

1. ⓐ $(19)1$
2. ⓑ $1 \cdot 39$

Try It 1.84

Find each product:

1. ⓐ $(24)(1)$
2. ⓑ 1×57

Earlier in this chapter, we learned that the Commutative Property of Addition states that changing the order of addition does not change the sum. We saw that $8 + 9 = 17$ is the same as $9 + 8 = 17$.

Is this also true for multiplication? Let's look at a few pairs of factors.

$$4 \cdot 7 = 28 \qquad 7 \cdot 4 = 28$$

$$9 \cdot 7 = 63 \qquad 7 \cdot 9 = 63$$

$$8 \cdot 9 = 72 \qquad 9 \cdot 8 = 72$$

When the order of the factors is reversed, the product does not change. This is called the Commutative Property of Multiplication.

Commutative Property of Multiplication

Changing the order of the factors does not change their product.

$$a \cdot b = b \cdot a$$

Example 1.43

Multiply:

1. ⓐ $8 \cdot 7$
2. ⓑ $7 \cdot 8$

Answer

ⓐ	$8 \cdot 7$
Multiply.	56
ⓑ	$7 \cdot 8$
Multiply.	56

Changing the order of the factors does not change the product.

Try It 1.85

Multiply:

1. ⓐ $9 \cdot 6$
2. ⓑ $6 \cdot 9$

Try It 1.86

Multiply:

1. ⓐ $8 \cdot 6$
2. ⓑ $6 \cdot 8$

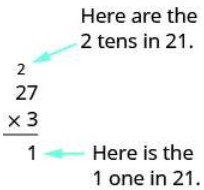
To multiply numbers with more than one digit, it is usually easier to write the numbers vertically in columns just as we did for addition and subtraction.

$$\begin{array}{r} 27 \\ \times 3 \\ \hline \end{array}$$

We start by multiplying 3 by 7.

$$3 \times 7 = 21$$

We write the 1 in the ones place of the product. We carry the 2 tens by writing 2 above the tens place.



Then we multiply the 3 by the 2, and add the 2 above the tens place to the product. So $3 \times 2 = 6$, and $6 + 2 = 8$. Write the 8 in the tens place of the product.

$$\begin{array}{r} {\scriptstyle 2} \\ 27 \\ \times\ 3 \\ \hline 81 \end{array}$$ ← This comes from 3×2 plus the 2 we carried.

The product is 81.

When we multiply two numbers with a different number of digits, it's usually easier to write the smaller number on the bottom. You could write it the other way, too, but this way is easier to work with.

Example 1.44

Multiply: $15 \cdot 4$.

Answer

Write the numbers so the digits 5 and 4 line up vertically.	$\begin{array}{r} 15 \\ \times\ 4 \\ \hline \end{array}$
Multiply 4 by the digit in the ones place of 15. $4 \cdot 5 = 20$.	
Write 0 in the ones place of the product and carry the 2 tens.	$\begin{array}{r} {\scriptstyle 2} \\ 15 \\ \times\ 4 \\ \hline 0 \end{array}$
Multiply 4 by the digit in the tens place of 15. $4 \cdot 1 = 4$. Add the 2 tens we carried. $4 + 2 = 6$.	
Write the 6 in the tens place of the product.	$\begin{array}{r} {\scriptstyle 2} \\ 15 \\ \times\ 4 \\ \hline 60 \end{array}$

Try It 1.87

Multiply: $64 \cdot 8$.

Try It 1.88

Multiply: $57 \cdot 6$.

Example 1.45

Multiply: $286 \cdot 5$.

Answer

Write the numbers so the digits 5 and 6 line up vertically.	$\begin{array}{r} 286 \\ \times\ 5 \\ \hline \end{array}$
Multiply 5 by the digit in the ones place of 286. $5 \cdot 6 = 30$.	
Write the 0 in the ones place of the product and carry the 3 to the tens place.Multiply 5 by the digit in the tens place of 286. $5 \cdot 8 = 40$.	$\begin{array}{r} {\scriptstyle 3} \\ 286 \\ \times\ 5 \\ \hline 0 \end{array}$
Add the 3 tens we carried to get $40 + 3 = 43$. Write the 3 in the tens place of the product and carry the 4 to the hundreds place.	$\begin{array}{r} {\scriptstyle 4\ 3} \\ 286 \\ \times\ 5 \\ \hline 30 \end{array}$

Multiply 5 by the digit in the hundreds place of 286. $5 \cdot 2 = 10$. Add the 4 hundreds we carried to get $10 + 4 = 14$. Write the 4 in the hundreds place of the product and the 1 to the thousands place.	$\overset{4\ 3}{286}$ $\times\ 5$ $1{,}430$

Try It 1.89

Multiply: $347 \cdot 5$.

Try It 1.90

Multiply: $462 \cdot 7$.

When we multiply by a number with two or more digits, we multiply by each of the digits separately, working from right to left. Each separate product of the digits is called a partial product. When we write partial products, we must make sure to line up the place values.

How To

Multiply two whole numbers to find the product.

1. Step 1. Write the numbers so each place value lines up vertically.
2. Step 2. Multiply the digits in each place value.
   - Work from right to left, starting with the ones place in the bottom number.
     - Multiply the ones digit of the bottom number by the ones digit in the top number, then by the tens digit, and so on.
     - If a product in a place value is more than 9, carry to the next place value.
     - Write the partial products, lining up the digits in the place values with the numbers above.
   - Repeat for the tens place in the bottom number, the hundreds place, and so on.
   - Insert a zero as a placeholder with each additional partial product.
3. Step 3. Add the partial products.

Example 1.46

Multiply: $62(87)$.

Answer

Write the numbers so each place lines up vertically.	62 $\times 87$
Start by multiplying 7 by 62. Multiply 7 by the digit in the ones place of 62. $7 \cdot 2 = 14$. Write the 4 in the ones place of the product and carry the 1 to the tens place.	$\overset{1}{62}$ $\times 87$ 4
Multiply 7 by the digit in the tens place of 62. $7 \cdot 6 = 42$. Add the 1 ten we carried. $42 + 1 = 43$. Write the 3 in the tens place of the product and the 4 in the hundreds place.	$\overset{1}{62}$ $\times 87$ 434
The first partial product is 434.	
Now, write a 0 under the 4 in the ones place of the next partial product as a placeholder since we now multiply the digit in the tens place of 87 by 62. Multiply 8 by the digit in the ones place of 62. $8 \cdot 2 = 16$. Write the 6 in the next place of the product, which is the tens place. Carry the 1 to the tens place.	$\overset{1}{\not{x}}$ 62 $\times 87$ 434 60

Multiply 8 by 6, the digit in the tens place of 62, then add the 1 ten we carried to get 49. Write the 9 in the hundreds place of the product and the 4 in the thousands place.	$\begin{array}{r} {\scriptstyle 1\ \ \ } \\ 62 \\ \times\ 87 \\ \hline 434 \\ 4960 \end{array}$
The second partial product is 4960. Add the partial products.	$\begin{array}{r} {\scriptstyle 1\ \ \ } \\ 62 \\ \times\ 87 \\ \hline 434 \\ 4960 \\ \hline 5394 \end{array}$

The product is $5{,}394$.

Try It 1.91

Multiply: $43(78)$.

Try It 1.92

Multiply: $64(59)$.

Example 1.47

Multiply:

1. ⓐ $47 \cdot 10$
2. ⓑ $47 \cdot 100$.

Answer

ⓐ $47 \cdot 10$.	$\begin{array}{r} 47 \\ \times 10 \\ \hline 00 \\ 470 \\ \hline 470 \end{array}$
ⓑ $47 \cdot 100$	$\begin{array}{r} 47 \\ \times 100 \\ \hline 00 \\ 000 \\ 4700 \\ \hline 4{,}700 \end{array}$

When we multiplied 47 times 10, the product was 470. Notice that 10 has one zero, and we put one zero after 47 to get the product. When we multiplied 47 times 100, the product was $4{,}700$. Notice that 100 has two zeros and we put two zeros after 47 to get the product.

Do you see the pattern? If we multiplied 47 times $10{,}000$, which has four zeros, we would put four zeros after 47 to get the product $470{,}000$.

Try It 1.93

Multiply:

1. ⓐ $54 \cdot 10$
2. ⓑ $54 \cdot 100$.

Try It 1.94

Multiply:

1. ⓐ $75 \cdot 10$
2. ⓑ $75 \cdot 100$.

Example 1.48

Multiply: $(354)(438)$.

Answer

There are three digits in the factors so there will be 3 partial products. We do not have to write the 0 as a placeholder as long as we write each partial product in the correct place.



Try It 1.95

Multiply: $(265)(483)$.

Try It 1.96

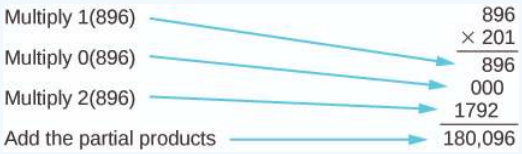
Multiply: $(823)(794)$.

Example 1.49

Multiply: $(896)201$.

Answer

There should be 3 partial products. The second partial product will be the result of multiplying 896 by 0.



Notice that the second partial product of all zeros doesn't really affect the result. We can place a zero as a placeholder in the tens place and then proceed directly to multiplying by the 2 in the hundreds place, as shown.

Multiply by 10, but insert only one zero as a placeholder in the tens place. Multiply by 200, putting the 2 from the 12. $2 \cdot 6 = 12$ in the hundreds place.

$$\begin{array}{r} 896 \\ \times 201 \\ \hline 896 \\ 17920 \\ \hline 180{,}096 \end{array}$$

Try It 1.97

Multiply: $(718)509$.

Try It 1.98

Multiply: $(627)804$.

When there are three or more factors, we multiply the first two and then multiply their product by the next factor. For example:

to multiply	$8 \cdot 3 \cdot 2$
first multiply $8 \cdot 3$	$24 \cdot 2$
then multiply $24 \cdot 2$.	48

Translate Word Phrases to Math Notation

Earlier in this section, we translated math notation into words. Now we'll reverse the process and translate word phrases into math notation. Some of the words that indicate multiplication are given in Table 1.5.

Operation	Word Phrase	Example	Expression
Multiplication	times product twice	3 times 8 the product of 3 and 8 twice 4	$3 \times 8, 3 \cdot 8, (3)(8),$ $(3)8,$ or $3(8)$ $2 \cdot 4$

Table 1.5

Example 1.50

Translate and simplify: the product of 12 and 27.

Answer

The word *product* tells us to multiply. The words *of* 12 *and* 27 tell us the two factors.

	the product of 12 and 27
Translate.	$12 \cdot 27$
Multiply.	324

Try It 1.99

Translate and simplify: the product of 13 and 28.

Try It 1.100

Translate and simplify: the product of 47 and 14.

Example 1.51

Translate and simplify: twice two hundred eleven.

Answer

The word *twice* tells us to multiply by 2.

	twice two hundred eleven
Translate.	2(211)
Multiply.	422

Try It 1.101

Translate and simplify: twice one hundred sixty-seven.

Try It 1.102

Translate and simplify: twice two hundred fifty-eight.

Multiply Whole Numbers in Applications

We will use the same strategy we used previously to solve applications of multiplication. First, we need to determine what we are looking for. Then we write a phrase that gives the information to find it. We then translate the phrase into math notation and simplify to get the answer. Finally, we write a sentence to answer the question.

Example 1.52

Humberto bought 4 sheets of stamps. Each sheet had 20 stamps. How many stamps did Humberto buy?

Answer

We are asked to find the total number of stamps.

Write a phrase for the total.	the product of 4 and 20
Translate to math notation.	$4 \cdot 20$
Multiply.	$\begin{array}{r} 20 \\ \times\, 4 \\ \hline 80 \end{array}$
Write a sentence to answer the question.	Humberto bought 80 stamps.

Try It 1.103

Valia donated water for the snack bar at her son's baseball game. She brought 6 cases of water bottles. Each case had 24 water bottles. How many water bottles did Valia donate?

Try It 1.104

Vanessa brought 8 packs of hot dogs to a family reunion. Each pack has 10 hot dogs. How many hot dogs did Vanessa bring?

Example 1.53

When Rena cooks rice, she uses twice as much water as rice. How much water does she need to cook 4 cups of rice?

Answer

We are asked to find how much water Rena needs.

Write as a phrase.	twice as much as 4 cups
Translate to math notation.	$2 \cdot 4$
Multiply to simplify.	8
Write a sentence to answer the question.	Rena needs 8 cups of water for 4 cups of rice.

Try It 1.105

Erin is planning her flower garden. She wants to plant twice as many dahlias as sunflowers. If she plants 14 sunflowers, how many dahlias does she need?

Try It 1.106

A college choir has twice as many women as men. There are 18 men in the choir. How many women are in the choir?

Example 1.54

Van is planning to build a patio. He will have 8 rows of tiles, with 14 tiles in each row. How many tiles does he need for the patio?

Answer

We are asked to find the total number of tiles.

Write a phrase.	the product of 8 and 14
Translate to math notation.	$8 \cdot 14$
Multiply to simplify.	$\begin{array}{r} {\scriptstyle 3} \\ 14 \\ \times 8 \\ \hline 112 \end{array}$
Write a sentence to answer the question.	Van needs 112 tiles for his patio.

Try It 1.107

Jane is tiling her living room floor. She will need 16 rows of tile, with 20 tiles in each row. How many tiles does she need for the living room floor?

Try It 1.108

Yousef is putting shingles on his garage roof. He will need 24 rows of shingles, with 45 shingles in each row. How many shingles does he need for the garage roof?

If we want to know the size of a wall that needs to be painted or a floor that needs to be carpeted, we will need to find its area. The area is a measure of the amount of surface that is covered by the shape. Area is measured in square units. We often use square inches, square feet, square centimeters, or square miles to measure area. A square centimeter is a square that is one centimeter (cm.) on a side. A square inch is a square that is one inch on each side, and so on.

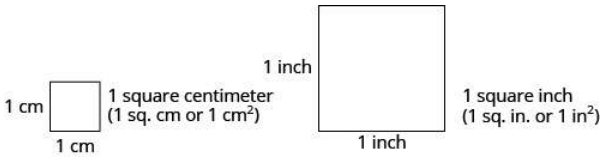


Figure 1.12 The area of a rectangle is the product of its length and its width, or 6 square feet.

Example 1.55

Jen's kitchen ceiling is a rectangle that measures 9 feet long by 12 feet wide. What is the area of Jen's kitchen ceiling?

Answer

We are asked to find the area of the kitchen ceiling.

Write a phrase for the area.	the product of 9 and 12
Translate to math notation.	$9 \cdot 12$

Multiply.	$\begin{array}{r} {}^{1}12 \\ \times 9 \\ \hline 108 \end{array}$
Answer with a sentence.	The area of Jen's kitchen ceiling is 108 square feet.

Try It 1.109

Zoila bought a rectangular rug. The rug is 8 feet long by 5 feet wide. What is the area of the rug?

Try It 1.110

Rene’s driveway is a rectangle 45 feet long by 20 feet wide. What is the area of the driveway?

Media

ACCESS ADDITIONAL ONLINE RESOURCES

- Multiplying Whole Numbers
- Multiplication with Partial Products
- Example of Multiplying by Whole Numbers

Section 1.4 Exercises

Practice Makes Perfect

Use Multiplication Notation

In the following exercises, translate from math notation to words.

225.

4×7

226.

8×6

227.

$5 \cdot 12$

228.

$3 \cdot 9$

229.

$(10)(25)$

230.

$(20)(15)$

231.

$42(33)$

232.

$39(64)$

Model Multiplication of Whole Numbers

In the following exercises, model the multiplication.

233.

3×6

234.

4×5

235.

5×9

236.

3×9

Multiply Whole Numbers

In the following exercises, fill in the missing values in each chart.

237.

×	0	1	2	3	4	5	6	7	8	9
0	0	0	0		0	0	0	0		0
1	0	1	2	3			6	7	8	
2		2	4	6	8		12			18
3	0		6		12	15		21		27
4	0	4			16	20		28	32	
5	0	5	10	15			30		40	
6	0	6	12		24			42		54
7			14	21		35			56	63
8	0	8		24			48		64	
9	0	9	18		36	45			72	

238.

×	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0		0		0	0
1	0	1	2		4	5		7		9
2	0		4		8	10		14	16	
3		3		9			18		24	
4	0	4	8	12			24	28		36
5	0	5		15	20		30	35	40	
6			12	18			36	42		54
7	0	7		21		35			56	63
8	0	8	16		32		48		64	72
9			18	27	36			63		

239.

×	3	4	5	6	7	8	9
4							
5							
6							
7							
8							
9							

240.

×	4	5	6	7	8	9
3						
4						
5						
6						
7						
8						
9						

241.

×	3	4	5	6	7	8	9
6							
7							
8							
9							

242.

×	6	7	8	9
3				
4				
5				
6				
7				
8				
9				

243.

×	5	6	7	8	9
5					
6					
7					
8					
9					

244.

×	6	7	8	9
6				
7				
8				
9				

In the following exercises, multiply.

245.

$0 \cdot 15$

246.

$0 \cdot 41$

247.

$(99)0$

248.

$(77)0$

249.

$1 \cdot 43$

250.

$1 \cdot 34$

251.

$(28)1$

252.

$(65)1$

253.

$1(240{,}055)$

254.

$1(189{,}206)$

255.

1. ⓐ $7 \cdot 6$
2. ⓑ $6 \cdot 7$

256.

1. ⓐ 8×9
2. ⓑ 9×8

257.

$(79)(5)$

258.

$(58)(4)$

259.

$275 \cdot 6$

260.

$638 \cdot 5$

261.

$3{,}421 \times 7$

262.

$9{,}143 \times 3$

263.

$52(38)$

264.

$37(45)$

265.

$96 \cdot 73$

266.

$89 \cdot 56$

267.

27×85

268.

53×98

269.

$23 \cdot 10$

270.

$19 \cdot 10$

271.

$(100)(36)$

272.

$(100)(25)$

273.

$1{,}000(88)$

274.

$1{,}000(46)$

275.

$50 \times 1{,}000{,}000$

276.

$30 \times 1{,}000{,}000$

277.

247×139

278.

156×328

279.

$586(721)$

280.

$472(855)$

281.

$915 \cdot 879$

282.

$968 \cdot 926$

283.

$(104)(256)$

284.

$(103)(497)$

285.

$348(705)$

286.

$485(602)$

287.

$2{,}719 \times 543$

288.

$3{,}581 \times 724$

Translate Word Phrases to Math Notation

In the following exercises, translate and simplify.

289.

the product of 18 and 33

290.

the product of 15 and 22

291.

fifty-one times sixty-seven

292.

forty-eight times seventy-one

293.

twice 249

294.

twice 589

295.

ten times three hundred seventy-five

296.

ten times two hundred fifty-five

Mixed Practice

In the following exercises, simplify.

297.

38×37

298.

86×29

299.

$415 - 267$

300.

$341 - 285$

301.

$6{,}251 + 4{,}749$

302.

$3{,}816 + 8{,}184$

303.

$(56)(204)$

304.

$(77)(801)$

305.

$947 \cdot 0$

306.

$947 + 0$

307.

$15{,}382 + 1$

308.

$15{,}382 \cdot 1$

In the following exercises, translate and simplify.

309.

the difference of 50 and 18

310.

the difference of 90 and 66

311.

twice 35

312.

twice 140

313.

20 more than 980

314.

65 more than 325

315.

the product of 12 and 875

316.

the product of 15 and 905

317.

subtract 74 from 89

318.

subtract 45 from 99

319.

the sum of 3,075 and 95

320.

the sum of 6,308 and 724

321.

366 less than 814

322.

388 less than 925

Multiply Whole Numbers in Applications

In the following exercises, solve.

323.

Party supplies Tim brought 9 six-packs of soda to a club party. How many cans of soda did Tim bring?

324.

Sewing Kanisha is making a quilt. She bought 6 cards of buttons. Each card had four buttons on it. How many buttons did Kanisha buy?

325.

Field trip Seven school busses let off their students in front of a museum in Washington, DC. Each school bus had 44 students. How many students were there?

326.

Gardening Kathryn bought 8 flats of impatiens for her flower bed. Each flat has 24 flowers. How many flowers did Kathryn buy?

327.

Charity Rey donated 15 twelve-packs of t-shirts to a homeless shelter. How many t-shirts did he donate?

328.

School There are 28 classrooms at Anna C. Scott elementary school. Each classroom has 26 student desks. What is the total number of student desks?

329.

Recipe Stephanie is making punch for a party. The recipe calls for twice as much fruit juice as club soda. If she uses 10 cups of club soda, how much fruit juice should she use?

330.

Gardening Hiroko is putting in a vegetable garden. He wants to have twice as many lettuce plants as tomato plants. If he buys 12 tomato plants, how many lettuce plants should he get?

331.

Government The United States Senate has twice as many senators as there are states in the United States. There are 50 states. How many senators are there in the United States Senate?

332.

Recipe Andrea is making potato salad for a buffet luncheon. The recipe says the number of servings of potato salad will be twice the number of pounds of potatoes. If she buys 30 pounds of potatoes, how many servings of potato salad will there be?

333.

Painting Jane is painting one wall of her living room. The wall is rectangular, 13 feet wide by 9 feet high. What is the area of the wall?

334.

Home décor Shawnte bought a rug for the hall of her apartment. The rug is 3 feet wide by 18 feet long. What is the area of the rug?

335.

Room size The meeting room in a senior center is rectangular, with length 42 feet and width 34 feet. What is the area of the meeting room?

336.

Gardening June has a vegetable garden in her yard. The garden is rectangular, with length 23 feet and width 28 feet. What is the area of the garden?

337.

NCAA basketball According to NCAA regulations, the dimensions of a rectangular basketball court must be 94 feet by 50 feet. What is the area of the basketball court?

338.

NCAA football According to NCAA regulations, the dimensions of a rectangular football field must be 360 feet by 160 feet. What is the area of the football field?

Everyday Math

339.

Stock market Javier owns 300 shares of stock in one company. On Tuesday, the stock price rose $12 per share. How much money did Javier's portfolio gain?

340.

Salary Carlton got a $200 raise in each paycheck. He gets paid 24 times a year. How much higher is his new annual salary?

Writing Exercises

341.

How confident do you feel about your knowledge of the multiplication facts? If you are not fully confident, what will you do to improve your skills?

342.

How have you used models to help you learn the multiplication facts?

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
use multiplication notation.			
model multiplication of whole numbers.			
multiply whole numbers.			
translate word phrases to math notation.			
multiply whole numbers in applications.			

ⓑ On a scale of 1–10, how would you rate your mastery of this section in light of your responses on the checklist? How can you improve this?

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1.5: Divide Whole Numbers

Learning Objectives

By the end of this section, you will be able to:

- Use division notation
- Model division of whole numbers
- Divide whole numbers
- Translate word phrases to math notation
- Divide whole numbers in applications

Be Prepared 1.7

Before you get started, take this readiness quiz.

Multiply: $27 \cdot 3$.
If you missed this problem, review Example 1.44.

Be Prepared 1.8

Subtract: $43 - 26$.
If you missed this problem, review Example 1.32

Be Prepared 1.9

Multiply: $62(87)$.
If you missed this problem, review Example 1.45.

Use Division Notation

So far we have explored addition, subtraction, and multiplication. Now let's consider division. Suppose you have the Math input error



Figure 1.13

You might put 4 cookies in first bag, 4 in the second bag, and so on until you run out of cookies. Doing it this way, you would fill 3 bags.



In other words, starting with the 12 cookies, you would take away, or subtract, 4 cookies at a time. Division is a way to represent repeated subtraction just as multiplication represents repeated addition.

Instead of subtracting 4 repeatedly, we can write

$$12 \div 4$$

We read this as *twelve divided by four* and the result is the quotient of 12 and 4. The quotient is 3 because we can subtract 4 from 12 exactly 3 times. We call the number being divided the dividend and the number dividing it the divisor. In this case, the dividend is 12 and the divisor is 4.

In the past you may have used the notation $4\overline{)12}$, but this division also can be written as $12 \div 4$, $12/4$, $\frac{12}{4}$. In each case the 12 is the dividend and the 4 is the divisor.

Operation Symbols for Division

To represent and describe division, we can use symbols and words.

Operation	Notation	Expression	Read as	Result
Division	$\div$ $\frac{a}{b}$ $b\ a$ a/b	$12 \div 4$ $\frac{12}{4}$ $4\ 12$ $12/4$	Twelve divided by four	the quotient of 12 and 4

Division is performed on two numbers at a time. When translating from math notation to English words, or English words to math notation, look for the words *of* and *and* to identify the numbers.

Example 1.56

Translate from math notation to words.

ⓐ $64 \div 8$ ⓑ $\frac{42}{7}$ ⓒ $4\ 28$

Answer

- ⓐ We read this as *sixty-four divided by eight* and the result is *the quotient of sixty-four and eight*.
- ⓑ We read this as *forty-two divided by seven* and the result is *the quotient of forty-two and seven*.
- ⓒ We read this as *twenty-eight divided by four* and the result is *the quotient of twenty-eight and four*.

Try It 1.111

Translate from math notation to words:

ⓐ $84 \div 7$ ⓑ $\frac{18}{6}$ ⓒ $8\ 24$

Try It 1.112

Translate from math notation to words:

ⓐ $72 \div 9$ ⓑ $\frac{21}{3}$ ⓒ $6\ 54$

Model Division of Whole Numbers

As we did with multiplication, we will model division using counters. The operation of division helps us organize items into equal groups as we start with the number of items in the dividend and subtract the number in the divisor repeatedly.

Manipulative Mathematics

Doing the Manipulative Math Worksheets activity "Model Division of Whole Numbers" will help you develop a better understanding of dividing whole numbers.

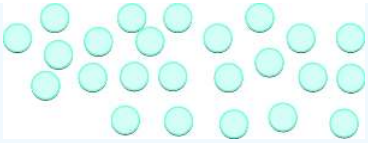
Example 1.57

Model the division: $24 \div 8$.

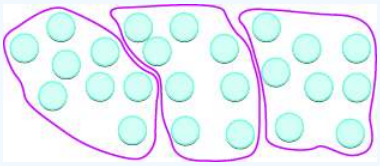
Answer

To find the quotient $24 \div 8$, we want to know how many groups of 8 are in 24.

Model the dividend. Start with 24 counters.



The divisor tell us the number of counters we want in each group. Form groups of 8 counters.



Count the number of groups. There are 3 groups.

$24 \div 8 = 3$

Try It 1.113

Model: $24 \div 6$.

Try It 1.114

Model: $42 \div 7$.

Divide Whole Numbers

We said that addition and subtraction are inverse operations because one undoes the other. Similarly, division is the inverse operation of multiplication. We know $12 \div 4 = 3$ because $3 \cdot 4 = 12$. Knowing all the multiplication number facts is very important when doing division.

We check our answer to division by multiplying the quotient by the divisor to determine if it equals the dividend. In Example 1.57, we know $24 \div 8 = 3$ is correct because $3 \cdot 8 = 24$.

Example 1.58

Divide. Then check by multiplying. ⓐ $42 \div 6$ ⓑ $\frac{72}{9}$ ⓒ $7\ 63$

Answer

- ⓐ

	$42 \div 6$
Divide 42 by 6.	7
Check by multiplying. $7 \cdot 6$	
42✓	

- ⓑ

	$\frac{72}{9}$
Divide 72 by 9.	8
Check by multiplying. $8 \cdot 9$	
72✓	

ⓒ	
	$7\ 63$
Divide 63 by 7.	9
Check by multiplying. $9 \cdot 7$	
63✓	

Try It 1.115

Divide. Then check by multiplying:

ⓐ $54 \div 6$ ⓑ $\frac{27}{9}$

Try It 1.116

Divide. Then check by multiplying:

ⓐ $\frac{36}{9}$ ⓑ $8\ 40$

What is the quotient when you divide a number by itself?

Division Properties of One

Any number (except 0) divided by itself is one.	$a \div a = 1$
Any number divided by one is the same number.	$a \div 1 = a$

Table 1.6

Example 1.59

Divide. Then check by multiplying:

1. ⓐ $11 \div 11$
2. ⓑ $\frac{19}{1}$
3. ⓒ $1\ 7$

Answer

ⓐ	
	$11 \div 11$
A number divided by itself is 1.	1
Check by multiplying. $1 \cdot 11$	
11✓	

ⓑ	
	$\frac{19}{1}$
A number divided by 1 equals itself.	19
Check by multiplying. $19 \cdot 1$	

19✓

ⓒ	
	$1\ 7$
A number divided by 1 equals itself.	7
Check by multiplying. $7 \cdot 1$	
7✓	

Try It 1.117

Divide. Then check by multiplying:

ⓐ $14 \div 14$ ⓑ $\frac{27}{1}$

Try It 1.118

Divide. Then check by multiplying:

ⓐ $\frac{16}{1}$ ⓑ $1\ 4$

Suppose we have $\$0$, and want to divide it among 3 people. How much would each person get? Each person would get $\$0$. Zero divided by any number is 0.

Now suppose that we want to divide $\$10$ by 0. That means we would want to find a number that we multiply by 0 to get 10. This cannot happen because 0 times any number is 0. Division by zero is said to be *undefined*.

These two ideas make up the Division Properties of Zero.

Division Properties of Zero

Zero divided by any number is 0.	$0 \div a = 0$
Dividing a number by zero is undefined.	$a \div 0$ undefined

Table 1.7

Another way to explain why division by zero is undefined is to remember that division is really repeated subtraction. How many times can we take away 0 from 10? Because subtracting 0 will never change the total, we will never get an answer. So we cannot divide a number by 0.

Example 1.60

Divide. Check by multiplying: ⓐ $0 \div 3$ ⓑ $10/0$.

Answer

ⓐ	
	$0 \div 3$
Zero divided by any number is zero.	0
Check by multiplying. $0 \cdot 3$	
0✓	

- ⓑ

	$10/0$
Division by zero is undefined.	undefined

Try It 1.119

Divide. Then check by multiplying:

ⓐ $0 \div 2$ ⓑ $17/0$

Try It 1.120

Divide. Then check by multiplying:

ⓐ $0 \div 6$ ⓑ $13/0$

When the divisor or the dividend has more than one digit, it is usually easier to use the $4\ \ 12$ notation. This process is called long division. Let's work through the process by dividing 78 by 3.

Divide the first digit of the dividend, 7, by the divisor, 3.	
The divisor 3 can go into 7 two times since $2 \times 3 = 6$. Write the 2 above the 7 in the quotient.	$\begin{array}{r} 2 \\ 3\overline{)78} \end{array}$
Multiply the 2 in the quotient by 3 and write the product, 6, under the 7.	$\begin{array}{r} 2 \\ 3\overline{)78} \\ 6 \end{array}$
Subtract that product from the first digit in the dividend. Subtract $7 - 6$. Write the difference, 1, under the first digit in the dividend.	$\begin{array}{r} 2 \\ 3\overline{)78} \\ \underline{6} \\ 1 \end{array}$
Bring down the next digit of the dividend. Bring down the 8.	$\begin{array}{r} 2 \\ 3\overline{)78} \\ \underline{6} \\ 18 \end{array}$
Divide 18 by the divisor, 3. The divisor 3 goes into 18 six times.	$\begin{array}{r} 26 \\ 3\overline{)78} \\ \underline{6} \\ 18 \end{array}$
Write 6 in the quotient above the 8.	
Multiply the 6 in the quotient by the divisor and write the product, 18, under the dividend. Subtract 18 from 18.	$\begin{array}{r} 26 \\ 3\overline{)78} \\ \underline{6} \\ 18 \\ \underline{18} \\ 0 \end{array}$

We would repeat the process until there are no more digits in the dividend to bring down. In this problem, there are no more digits to bring down, so the division is finished.

$$\text{So } 78 \div 3 = 26.$$

Check by multiplying the quotient times the divisor to get the dividend. Multiply 26×3 to make sure that product equals the dividend, 78.

$$\begin{array}{r} {}^{1}26 \\ \times 3 \\ \hline 78\checkmark \end{array}$$

It does, so our answer is correct.

How To

Divide whole numbers.

1. Step 1. Divide the first digit of the dividend by the divisor.
   If the divisor is larger than the first digit of the dividend, divide the first two digits of the dividend by the divisor, and so on.
2. Step 2. Write the quotient above the dividend.
3. Step 3. Multiply the quotient by the divisor and write the product under the dividend.
4. Step 4. Subtract that product from the dividend.
5. Step 5. Bring down the next digit of the dividend.
6. Step 6. Repeat from Step 1 until there are no more digits in the dividend to bring down.
7. Step 7. Check by multiplying the quotient times the divisor.

Example 1.61

Divide $2{,}596 \div 4$. Check by multiplying:

Answer

Let's rewrite the problem to set it up for long division.	$4\overline{)2596}$
Divide the first digit of the dividend, 2, by the divisor, 4.	$4\overline{)2596}$
Since 4 does not go into 2, we use the first two digits of the dividend and divide 25 by 4. The divisor 4 goes into 25 six times.	
We write the 6 in the quotient above the 5.	$\begin{array}{r} 6 \\ 4\overline{)2596} \end{array}$
Multiply the 6 in the quotient by the divisor 4 and write the product, 24, under the first two digits in the dividend.	$\begin{array}{r} 6 \\ 4\overline{)2596} \\ 24 \end{array}$
Subtract that product from the first two digits in the dividend. Subtract $25 - 24$. Write the difference, 1, under the second digit in the dividend.	$\begin{array}{r} 6 \\ 4\overline{)2596} \\ \underline{24} \\ 1 \end{array}$
Now bring down the 9 and repeat these steps. There are 4 fours in 19. Write the 4 over the 9. Multiply the 4 by 4 and subtract this product from 19.	$\begin{array}{r} 64 \\ 4\overline{)2596} \\ \underline{24} \\ 19 \\ \underline{16} \\ 3 \end{array}$
Bring down the 6 and repeat these steps. There are 9 fours in 36. Write the 9 over the 6. Multiply the 9 by 4 and subtract this product from 36.	$\begin{array}{r} 649 \\ 4\overline{)2596} \\ \underline{24} \\ 19 \\ \underline{16} \\ 36 \\ \underline{36} \\ 0 \end{array}$
So $2{,}596 \div 4 = 649$.	

Check by multiplying. $\begin{array}{r} {}^{1}{}^{3} \\ 649 \\ \times \quad 4 \\ \hline 2{,}596 \end{array}$ ✓	

It equals the dividend, so our answer is correct.

Try It 1.121

Divide. Then check by multiplying: $2{,}636 \div 4$

Try It 1.122

Divide. Then check by multiplying: $2{,}716 \div 4$

Example 1.62

Divide $4{,}506 \div 6$. Check by multiplying:

Answer

Let's rewrite the problem to set it up for long division.	$6\overline{)4506}$
First we try to divide 6 into 4.	$6\overline{)4506}$
Since that won't work, we try 6 into 45. There are 7 sixes in 45. We write the 7 over the 5.	$\begin{array}{r} 7 \\ 6\overline{)4506} \end{array}$
Multiply the 7 by 6 and subtract this product from 45.	$\begin{array}{r} 7 \\ 6\overline{)4506} \\ 42 \\ \hline 3 \end{array}$
Now bring down the 0 and repeat these steps. There are 5 sixes in 30. Write the 5 over the 0. Multiply the 5 by 6 and subtract this product from 30.	$\begin{array}{r} 75 \\ 6\overline{)4506} \\ 42 \\ \hline 30 \\ 30 \\ \hline 0 \end{array}$
Now bring down the 6 and repeat these steps. There is 1 six in 6. Write the 1 over the 6. Multiply 1 by 6 and subtract this product from 6.	$\begin{array}{r} 751 \\ 6\overline{)4506} \\ 42 \\ \hline 30 \\ 30 \\ \hline 06 \\ 6 \\ \hline 0 \end{array}$
Check by multiplying. $\begin{array}{r} {}^{3} \\ 751 \\ \times \quad 6 \\ \hline 4{,}506 \end{array}$ ✓	

It equals the dividend, so our answer is correct.

Try It 1.123

Divide. Then check by multiplying: $4{,}305 \div 5$.

Try It 1.124

Divide. Then check by multiplying: $3{,}906 \div 6$.

Example 1.63

Divide $7{,}263 \div 9$. Check by multiplying.

Answer

Let's rewrite the problem to set it up for long division.	$9\overline{)7263}$
First we try to divide 9 into 7.	$9\overline{)7263}$
Since that won't work, we try 9 into 72. There are 8 nines in 72. We write the 8 over the 2.	$\begin{array}{r} 8 \\ 9\overline{)7263} \end{array}$
Multiply the 8 by 9 and subtract this product from 72.	$\begin{array}{r} 8 \\ 9\overline{)7263} \\ 72 \\ \hline 0 \end{array}$
Now bring down the 6 and repeat these steps. There are 0 nines in 6. Write the 0 over the 6. Multiply the 0 by 9 and subtract this product from 6.	$\begin{array}{r} 80 \\ 9\overline{)7263} \\ 72 \\ \hline 06 \\ 0 \\ \hline 6 \end{array}$
Now bring down the 3 and repeat these steps. There are 7 nines in 63. Write the 7 over the 3. Multiply the 7 by 9 and subtract this product from 63.	$\begin{array}{r} 807 \\ 9\overline{)7263} \\ 72 \\ \hline 06 \\ 0 \\ \hline 63 \\ 63 \\ \hline 0 \end{array}$
Check by multiplying. $\begin{array}{r} {\scriptstyle 6}\,807 \\ \times \quad 9 \\ \hline 7{,}263 \checkmark \end{array}$	

It equals the dividend, so our answer is correct.

Try It 1.125

Divide. Then check by multiplying: $4{,}928 \div 7$.

Try It 1.126

Divide. Then check by multiplying: $5{,}663 \div 7$.

So far all the division problems have worked out evenly. For example, if we had Math input error



Figure 1.14

Try to put the cookies in groups of eight as in Figure 1.15.

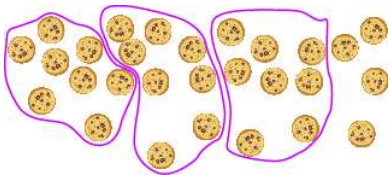


Figure 1.15

There are 3 groups of eight cookies, and 4 cookies left over. We call the 4 cookies that are left over the remainder and show it by writing R4 next to the 3. (The R stands for remainder.)

To check this division we multiply 3 times 8 to get 24, and then add the remainder of 4.

$$\begin{array}{r} 3 \\ \times 8 \\ \hline 24 \\ +4 \\ \hline 28 \end{array}$$

Example 1.64

Divide $1{,}439 \div 4$. Check by multiplying.

Answer

Let's rewrite the problem to set it up for long division.	$4\overline{)1439}$
First we try to divide 4 into 1. Since that won't work, we try 4 into 14. There are 3 fours in 14. We write the 3 over the 4.	$\begin{array}{r} 3 \\ 4\overline{)1439} \end{array}$
Multiply the 3 by 4 and subtract this product from 14.	$\begin{array}{r} 3 \\ 4\overline{)1439} \\ \underline{12} \\ 2 \end{array}$
Now bring down the 3 and repeat these steps. There are 5 fours in 23. Write the 5 over the 3. Multiply the 5 by 4 and subtract this product from 23.	$\begin{array}{r} 35 \\ 4\overline{)1439} \\ \underline{12}\downarrow \\ 23 \\ \underline{20} \\ 3 \end{array}$
Now bring down the 9 and repeat these steps. There are 9 fours in 39. Write the 9 over the 9. Multiply the 9 by 4 and subtract this product from 39. There are no more numbers to bring down, so we are done. The remainder is 3.	$\begin{array}{r} 359\text{R}3 \\ 4\overline{)1439} \\ \underline{12}\vert \\ 23 \\ \underline{20}\downarrow \\ 39 \\ \underline{36} \\ 3 \end{array}$
Check by multiplying. $\begin{array}{rl} {\scriptstyle 2\ 3}\ 359 & \text{quotient} \\ \times\ \ 4 & \text{divisor} \\ \hline 1{,}436 & \\ +\ \ 3 & \text{remainder} \\ \hline 1{,}439\ \checkmark & \end{array}$	

So $1{,}439 \div 4$ is 359 with a remainder of 3. Our answer is correct.

Try It 1.127

Divide. Then check by multiplying: $3{,}812 \div 8$.

Try It 1.128

Divide. Then check by multiplying: $4{,}319 \div 8$.

Example 1.65

Divide and then check by multiplying: $1{,}461 \div 13$.

Answer

Let's rewrite the problem to set it up for long division.	$13\overline{)1{,}461}$
First we try to divide 13 into 1. Since that won't work, we try 13 into 14. There is 1 thirteen in 14. We write the 1 over the 4.	$\begin{array}{r} 1 \\ 13\overline{)1461} \end{array}$
Multiply the 1 by 13 and subtract this product from 14.	$\begin{array}{r} 1 \\ 13\overline{)1461} \\ \underline{13} \\ 1 \end{array}$
Now bring down the 6 and repeat these steps. There is 1 thirteen in 16. Write the 1 over the 6. Multiply the 1 by 13 and subtract this product from 16.	$\begin{array}{r} 11 \\ 13\overline{)1461} \\ \underline{13\downarrow} \\ 16 \\ \underline{13} \\ 3 \end{array}$
Now bring down the 1 and repeat these steps. There are 2 thirteens in 31. Write the 2 over the 1. Multiply the 2 by 13 and subtract this product from 31. There are no more numbers to bring down, so we are done. The remainder is 5. $1{,}462 \div 13$ is 112 with a remainder of 5.	$\begin{array}{r} 112\text{R}5 \\ 13\overline{)1461} \\ \underline{13} \\ 16 \\ \underline{13\downarrow} \\ 31 \\ \underline{26} \\ 5 \end{array}$
Check by multiplying. $\begin{array}{rl} 112 & \text{quotient} \\ \times\ 13 & \text{divisor} \\ \hline 336 & \\ 1{,}120 & \\ +\ \ 5 & \text{remainder} \\ \hline 1{,}461 & \checkmark \end{array}$	

Our answer is correct.

Try It 1.129

Divide. Then check by multiplying: $1{,}493 \div 13$.

Try It 1.130

Divide. Then check by multiplying: $1{,}461 \div 12$.

Example 1.66

Divide and check by multiplying: $74{,}521 \div 241$.

Answer

Let's rewrite the problem to set it up for long division.	$241\overline{)74{,}521}$
First we try to divide 241 into 7. Since that won't work, we try 241 into 74. That still won't work, so we try 241 into 745. Since 2 divides into 7 three times, we try 3. Since $3 \times 241 = 723$, we write the 3 over the 5 in 745. Note that 4 would be too large because $4 \times 241 = 964$, which is greater than 745.	
Multiply the 3 by 241 and subtract this product from 745.	$\begin{array}{r} 3 \\ 241\overline{)74521} \\ 723 \\ \hline 22 \end{array}$
Now bring down the 2 and repeat these steps. 241 does not divide into 222. We write a 0 over the 2 as a placeholder and then continue.	$\begin{array}{r} 30 \\ 241\overline{)74521} \\ 723 \\ \hline 222 \end{array}$
Now bring down the 1 and repeat these steps. Try 9. Since $9 \times 241 = 2{,}169$, we write the 9 over the 1. Multiply the 9 by 241 and subtract this product from 2,221.	$\begin{array}{r} 309\text{ R}52 \\ 241\overline{)74521} \\ 723 \\ \hline 2221 \\ 2169 \\ \hline 52 \end{array}$
There are no more numbers to bring down, so we are finished. The remainder is 52. So $74{,}521 \div 241$ is 309 with a remainder of 52.	
Check by multiplying. $\begin{array}{rl} 3 & \\ 309 & \text{quotient} \\ \times\ 241 & \text{divisor} \\ \hline 309 & \\ 12{,}360 & \\ 61{,}800 & \\ \hline 72{,}469 & \\ +\ \ 52 & \text{remainder} \\ \hline 74{,}521 & \checkmark \end{array}$	

Sometimes it might not be obvious how many times the divisor goes into digits of the dividend. We will have to guess and check numbers to find the greatest number that goes into the digits without exceeding them.

Try It 1.131

Divide. Then check by multiplying: $78{,}641 \div 256$.

Try It 1.132

Divide. Then check by multiplying: $76{,}461 \div 248$.

Translate Word Phrases to Math Notation

Earlier in this section, we translated math notation for division into words. Now we'll translate word phrases into math notation. Some of the words that indicate division are given in Table 1.8.

Operation	Word Phrase	Example	Expression

Operation	Word Phrase	Example	Expression
Division	divided by quotient of divided into	12 divided by 4 the quotient of 12 and 4 4 divided into 12	$12 \div 4$ $\frac{12}{4}$ $12/4$ $4\overline{)12}$

Table 1.8

Example 1.67

Translate and simplify: the quotient of 51 and 17.

Answer

The word *quotient* tells us to divide.

	the quotient of 51 and 17
Translate.	$51 \div 17$
Divide.	3

We could just as correctly have translated *the quotient of* 51 *and* 17 using the notation $17\overline{)51}$ or $\frac{51}{17}$.

Try It 1.133

Translate and simplify: the quotient of 91 and 13.

Try It 1.134

Translate and simplify: the quotient of 52 and 13.

Divide Whole Numbers in Applications

We will use the same strategy we used in previous sections to solve applications. First, we determine what we are looking for. Then we write a phrase that gives the information to find it. We then translate the phrase into math notation and simplify it to get the answer. Finally, we write a sentence to answer the question.

Example 1.68

Cecelia bought a 160-ounce box of oatmeal at the big box store. She wants to divide the 160 ounces of oatmeal into 8-ounce servings. She will put each serving into a plastic bag so she can take one bag to work each day. How many servings will she get from the big box?

Answer

We are asked to find the how many servings she will get from the big box.

Write a phrase.	160 ounces divided by 8 ounces
Translate to math notation.	$160 \div 8$
Simplify by dividing.	20
Write a sentence to answer the question.	Cecelia will get 20 servings from the big box.

Try It 1.135

Marcus is setting out animal crackers for snacks at the preschool. He wants to put 9 crackers in each cup. One box of animal crackers contains 135 crackers. How many cups can he fill from one box of crackers?

Try It 1.136

Andrea is making bows for the girls in her dance class to wear at the recital. Each bow takes 4 feet of ribbon, and 36 feet of ribbon are on one spool. How many bows can Andrea make from one spool of ribbon?

Media

ACCESS ADDITIONAL ONLINE RESOURCES

- Dividing Whole Numbers
- Dividing Whole Numbers No Remainder
- Dividing Whole Numbers With Remainder

Section 1.5 Exercises

Practice Makes Perfect

Use Division Notation

In the following exercises, translate from math notation to words.

343.

$54 \div 9$

344.

$\frac{56}{7}$

345.

$\frac{32}{8}$

346.

$6\ \ 42$

347.

$48 \div 6$

348.

$\frac{63}{9}$

349.

$7\ \ 63$

350.

$72 \div 8$

Model Division of Whole Numbers

In the following exercises, model the division.

351.

$15 \div 5$

352.

$10 \div 5$

353.

$\frac{14}{7}$

354.

$\frac{18}{6}$

355.

$4\ \overline{)\ 20}$

356.

$3\ \overline{)\ 15}$

357.

$24 \div 6$

358.

$16 \div 4$

Divide Whole Numbers

In the following exercises, divide. Then check by multiplying.

359.

$18 \div 2$

360.

$14 \div 2$

361.

$\frac{27}{3}$

362.

$\frac{30}{3}$

363.

$4\ \overline{)\ 28}$

364.

$4\ \overline{)\ 36}$

365.

$\frac{45}{5}$

366.

$\frac{35}{5}$

367.

$72/8$

368.

$8\ \overline{)\ 64}$

369.

$\frac{35}{7}$

370.

$42 \div 7$

371.

$15\ \overline{)\ 15}$

372.

$12\ \overline{)\ 12}$

373.

$43 \div 43$

374.

$37 \div 37$

375.

$\frac{23}{1}$

376.

$\frac{29}{1}$

377.

$19 \div 1$

378.

$17 \div 1$

379.

$0 \div 4$

380.

$0 \div 8$

381.

$\frac{5}{0}$

382.

$\frac{9}{0}$

383.

$\frac{26}{0}$

384.

$\frac{32}{0}$

385.

$12\ \ 0$

386.

$16\ \ 0$

387.

$72 \div 3$

388.

$57 \div 3$

389.

$\frac{96}{8}$

390.

$\frac{78}{6}$

391.

$5\ \ 465$

392.

$4\ \ 528$

393.

$924 \div 7$

394.

https://math.libretexts.org/@go/page/229532

$861 \div 7$

395.

$\frac{5,226}{6}$

396.

$\frac{3,776}{8}$

397.

$4\overline{)\,31,324}$

398.

$5\overline{)\,46,855}$

399.

$7,209 \div 3$

400.

$4,806 \div 3$

401.

$5,406 \div 6$

402.

$3,208 \div 4$

403.

$4\overline{)\,2,816}$

404.

$6\overline{)\,3,624}$

405.

$\frac{91,881}{9}$

406.

$\frac{83,256}{8}$

407.

$2,470 \div 7$

408.

$3,741 \div 7$

409.

$8\overline{)\,55,305}$

410.

$9\overline{)\,51,492}$

411.

$\frac{431,174}{5}$

412.

$\frac{297,277}{4}$

413.

$130,016 \div 3$

414.

$105{,}609 \div 2$

415.

$15\overline{)\,5{,}735}$

416.

$\frac{4{,}933}{21}$

417.

$56{,}883 \div 67$

418.

$43{,}725/75$

419.

$\frac{30{,}144}{314}$

420.

$26{,}145 \div 415$

421.

$273\overline{)\,542{,}195}$

422.

$816{,}243 \div 462$

Mixed Practice

In the following exercises, simplify.

423.

$15\,(204)$

424.

$74 \cdot 391$

425.

$256 - 184$

426.

$305 - 262$

427.

$719 + 341$

428.

$647 + 528$

429.

$25\overline{)\,875}$

430.

$1104 \div 23$

Translate Word Phrases to Algebraic Expressions

In the following exercises, translate and simplify.

431.

the quotient of 45 and 15

432.

the quotient of 64 and 16

433.

the quotient of 288 and 24

434.

the quotient of 256 and 32

Divide Whole Numbers in Applications

In the following exercises, solve.

435.

Trail mix Ric bought 64 ounces of trail mix. He wants to divide it into small bags, with 2 ounces of trail mix in each bag. How many bags can Ric fill?

436.

Crackers Evie bought a 42 ounce box of crackers. She wants to divide it into bags with 3 ounces of crackers in each bag. How many bags can Evie fill?

437.

Astronomy class There are 125 students in an astronomy class. The professor assigns them into groups of 5. How many groups of students are there?

438.

Flower shop Melissa's flower shop got a shipment of 152 roses. She wants to make bouquets of 8 roses each. How many bouquets can Melissa make?

439.

Baking One roll of plastic wrap is 48 feet long. Marta uses 3 feet of plastic wrap to wrap each cake she bakes. How many cakes can she wrap from one roll?

440.

Dental floss One package of dental floss is 54 feet long. Brian uses 2 feet of dental floss every day. How many days will one package of dental floss last Brian?

Mixed Practice

In the following exercises, solve.

441.

Miles per gallon Susana's hybrid car gets 45 miles per gallon. Her son's truck gets 17 miles per gallon. What is the difference in miles per gallon between Susana's car and her son's truck?

442.

Distance Mayra lives 53 miles from her mother's house and 71 miles from her mother-in-law's house. How much farther is Mayra from her mother-in-law's house than from her mother's house?

443.

Field trip The 45 students in a Geology class will go on a field trip, using the college's vans. Each van can hold 9 students. How many vans will they need for the field trip?

444.

Potting soil Aki bought a 128 ounce bag of potting soil. How many 4 ounce pots can he fill from the bag?

445.

Hiking Bill hiked 8 miles on the first day of his backpacking trip, 14 miles the second day, 11 miles the third day, and 17 miles the fourth day. What is the total number of miles Bill hiked?

446.

Reading Last night Emily read 6 pages in her Business textbook, 26 pages in her History text, 15 pages in her Psychology text, and 9 pages in her math text. What is the total number of pages Emily read?

447.

Patients LaVonne treats 12 patients each day in her dental office. Last week she worked 4 days. How many patients did she treat last week?

448.

Scouts There are 14 boys in Dave's scout troop. At summer camp, each boy earned 5 merit badges. What was the total number of merit badges earned by Dave's scout troop at summer camp?

Writing Exercises

449.

Contact lenses Jenna puts in a new pair of contact lenses every 14 days. How many pairs of contact lenses does she need for 365 days?

450.

Cat food One bag of cat food feeds Lara's cat for 25 days. How many bags of cat food does Lara need for 365 days?

Everyday Math

451.

Explain how you use the multiplication facts to help with division.

452.

Oswaldo divided 300 by 8 and said his answer was 37 with a remainder of 4. How can you check to make sure he is correct?

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
use division notation.			
model division of whole numbers.			
divide whole numbers.			
translate word phrases to algebraic expressions.			
divide whole numbers in applications.			

ⓑ Overall, after looking at the checklist, do you think you are well-prepared for the next Chapter? Why or why not?

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CHAPTER OVERVIEW

2: Making Sense of Mathematical Expressions

The sections in this module will help you to learn about make sense of mathematical expressions and to use rounding to approximate the answers to mathematical expressions. Your goal is to demonstrate mastery on each of the following objectives.

- **Objective 2.1 - Explonents with Whole Numbers. C**alculate values involving exponents for whole numbers with values up to 1000, including applications, without the use of a calculator.
- **Objective 2.2 - Expressions with Order of Operations.** Simplify expressions using the standard order of operations.
- **Objective 2.3 - Rounding Whole Numbers:** Round a given number to the given place value using standard techniques and benchmark numbers.
- **Objective 2.4 - Approximating Basic Operations:** Estimate answers to whole number addition, subtraction, multiplication, and division problems using rounding and clustering.

2.1: Use the Language of Algebra
2.2: Introduction to Whole Numbers
2.3: Estimation by Rounding
2.4: Estimation by Clustering

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2.1: Use the Language of Algebra

Learning Objectives

By the end of this section, you will be able to:

- Use variables and algebraic symbols
- Identify expressions and equations
- Simplify expressions with exponents
- Simplify expressions using the order of operations

Be Prepared 2.1

Before you get started, take this readiness quiz.

Add: $43 + 69$.
If you missed this problem, review Example 1.19.

Be Prepared 2.2

Multiply: $(896)201$.
If you missed this problem, review Example 1.48.

Be Prepared 2.3

Divide: $7{,}263 \div 9$.
If you missed this problem, review Example 1.64.

Use Variables and Algebraic Symbols

Greg and Alex have the same birthday, but they were born in different years. This year Greg is 20 years old and Alex is 23, so Alex is 3 years older than Greg. When Greg was 12, Alex was 15. When Greg is 35, Alex will be 38. No matter what Greg's age is, Alex's age will always be 3 years more, right?

In the language of algebra, we say that Greg's age and Alex's age are variable and the three is a constant. The ages change, or vary, so age is a variable. The 3 years between them always stays the same, so the age difference is the constant.

In algebra, letters of the alphabet are used to represent variables. Suppose we call Greg's age Math input error

Greg's age	Alex's age
12	15
20	23
35	38
g	$g + 3$

Table 2.1

Letters are used to represent variables. Letters often used for variables are $x, y, a, b,$ and c.

Variables and Constants

A variable is a letter that represents a number or quantity whose value may change.

A constant is a number whose value always stays the same.

To write algebraically, we need some symbols as well as numbers and variables. There are several types of symbols we will be using. In Whole Numbers, we introduced the symbols for the four basic arithmetic operations: addition, subtraction, multiplication,

and division. We will summarize them here, along with words we use for the operations and the result.

Operation	Notation	Say:	The result is…
Addition	$a+b$	a plus b	the sum of a and b
Subtraction	$a-b$	a minus b	the difference of a and b
Multiplication	$a \cdot b, (a)(b), (a)b, a(b)$	a times b	The product of a and b
Division	$a \div b, a/b, \frac{a}{b}, b\ a$	a divided by b	The quotient of a and b

In algebra, the cross symbol, $\times$, is not used to show multiplication because that symbol may cause confusion. Does $3xy$ mean $3 \times y$ (three times y) or $3 \cdot x \cdot y$ (three times x times y)? To make it clear, use • or parentheses for multiplication.

We perform these operations on two numbers. When translating from symbolic form to words, or from words to symbolic form, pay attention to the words *of* or *and* to help you find the numbers.

- The *sum* ***of*** 5 ***and*** 3 means add 5 plus 3, which we write as $5+3$.
- The *difference* ***of*** 9 ***and*** 2 means subtract 9 minus 2, which we write as $9-2$.
- The *product* ***of*** 4 ***and*** 8 means multiply 4 times 8, which we can write as $4 \cdot 8$.
- The *quotient* ***of*** 20 ***and*** 5 means divide 20 by 5, which we can write as $20 \div 5$.

Example 2.1

Translate from algebra to words:

1. ⓐ $12+14$
2. ⓑ $(30)(5)$
3. ⓒ $64 \div 8$
4. ⓓ $x-y$

Answer

ⓐ

$12+14$

12 plus 14

the sum of twelve and fourteen

ⓑ

$(30)\,(5)$

30 times 5

the product of thirty and five

ⓒ

$64 \div 8$

64 divided by 8

the quotient of sixty-four and eight

ⓓ

$x-y$

x minus y

the difference of x and y

Try It 2.1

Translate from algebra to words.

1. ⓐ $18 + 11$
2. ⓑ $(27)(9)$
3. ⓒ $84 \div 7$
4. ⓓ $p - q$

Try It 2.2

Translate from algebra to words.

1. ⓐ $47 - 19$
2. ⓑ $72 \div 9$
3. ⓒ $m + n$
4. ⓓ $(13)(7)$

When two quantities have the same value, we say they are equal and connect them with an *equal sign*.

Equality Symbol

$a = b$ is read a is equal to b

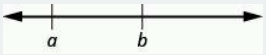
The symbol $=$ is called the equal sign.

An inequality is used in algebra to compare two quantities that may have different values. The number line can help you understand inequalities. Remember that on the number line the numbers get larger as they go from left to right. So if we know that b is greater than a, it means that b is to the right of a on the number line. We use the symbols "$<$" and "$>$" for inequalities.

Inequality

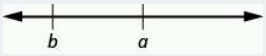
$a < b$ is read a is less than b

a is to the left of b on the number line



$a > b$ is read a is greater than b

a is to the right of b on the number line



The expressions $a < b$ and $a > b$ can be read from left-to-right or right-to-left, though in English we usually read from left-to-right. In general,

$a < b$ is equivalent to $b > a$.	For example, $7 < 11$ is equivalent to $11 > 7$.
$a > b$ is equivalent to $b < a$.	For example, $17 > 4$ is equivalent to $4 < 17$.

When we write an inequality symbol with a line under it, such as $a \leq b$, it means $a < b$ or $a = b$. We read this a is less than or equal to b. Also, if we put a slash through an equal sign, $\neq$, it means not equal.

We summarize the symbols of equality and inequality in Table 2.2.

Algebraic Notation	Say
$a = b$	a is equal to b

Algebraic Notation	Say
$a \neq b$	a is not equal to b
$a < b$	a is less than b
$a > b$	a is greater than b
$a \leq b$	a is less than or equal to b
$a \geq b$	a is greater than or equal to b

Table 2.2

<> Symbols and

The symbols $<$ and $>$ each have a smaller side and a larger side.

smaller side $<$ larger side
larger side $>$ smaller side

The smaller side of the symbol faces the smaller number and the larger faces the larger number.

Example 2.2

Translate from algebra to words:

1. ⓐ $20 \leq 35$
2. ⓑ $11 \neq 15 - 3$
3. ⓒ $9 > 10 \div 2$
4. ⓓ $x + 2 < 10$

Answer

ⓐ
$20 \leq 35$
20 is less than or equal to 35

ⓑ
$11 \neq 15 - 3$
11 is not equal to 15 minus 3

ⓒ
$9 > 10 \div 2$
9 is greater than 10 divided by 2

ⓓ
$x + 2 < 10$
x plus 2 is less than 10

Try It 2.3

Translate from algebra to words.

1. ⓐ $14 \leq 27$

2. ⓑ $19 - 2 \neq 8$
3. ⓒ $12 > 4 \div 2$
4. ⓓ $x - 7 < 1$

Try It 2.4

Translate from algebra to words.

1. ⓐ $19 \geq 15$
2. ⓑ $7 = 12 - 5$
3. ⓒ $15 \div 3 < 8$
4. ⓓ $y - 3 > 6$

Example 2.3

The information in Figure 2.2 compares the fuel economy in miles-per-gallon (mpg) of several cars. Write the appropriate symbol $=, <, \text{or} >$ in each expression to compare the fuel economy of the cars.

Car	Prius	Mini Cooper	Toyota Corolla	Versa	Honda Fit
Fuel economy (mpg)	48	27	28	26	27

Figure 2.2 (credit: modification of work by Bernard Goldbach, Wikimedia Commons)

1. ⓐ MPG of Prius_____ MPG of Mini Cooper
2. ⓑ MPG of Versa_____ MPG of Fit
3. ⓒ MPG of Mini Cooper_____ MPG of Fit
4. ⓓ MPG of Corolla_____ MPG of Versa
5. ⓔ MPG of Corolla_____ MPG of Prius

Answer

ⓐ	
	MPG of Prius____MPG of Mini Cooper
Find the values in the chart.	48____27
Compare.	48 > 27
	MPG of Prius > MPG of Mini Cooper

ⓑ	
	MPG of Versa____MPG of Fit
Find the values in the chart.	26____27
Compare.	26 < 27
	MPG of Versa < MPG of Fit

ⓒ	
	MPG of Mini Cooper____MPG of Fit
Find the values in the chart.	27____27
Compare.	27 = 27
	MPG of Mini Cooper = MPG of Fit

ⓓ	
	MPG of Corolla____MPG of Versa
Find the values in the chart.	28____26
Compare.	28 > 26
	MPG of Corolla > MPG of Versa

ⓔ	
	MPG of Corolla____MPG of Prius
Find the values in the chart.	28____48
Compare.	28 < 48
	MPG of Corolla < MPG of Prius

Try It 2.5

Use Figure 2.2 to fill in the appropriate $\text{symbol}, =, <, \text{or} >$.

1. ⓐ MPG of Prius_____MPG of Versa
2. ⓑ MPG of Mini Cooper_____ MPG of Corolla

Try It 2.6

Use Figure 2.2 to fill in the appropriate $\text{symbol}, =, <, \text{or} >$.

1. ⓐ MPG of Fit_____ MPG of Prius
2. ⓑ MPG of Corolla _____ MPG of Fit

Grouping symbols in algebra are much like the commas, colons, and other punctuation marks in written language. They indicate which expressions are to be kept together and separate from other expressions. Table 2.3 lists three of the most commonly used grouping symbols in algebra.

Common Grouping Symbols	
parentheses	( )
brackets	[ ]
braces	{ }

Table 2.3

Here are some examples of expressions that include grouping symbols. We will simplify expressions like these later in this section.

$$8(14-8) \qquad 21-3[2+4(9-8)] \qquad 24\div\{13-2[1(6-5)+4]\}$$

Identify Expressions and Equations

What is the difference in English between a phrase and a sentence? A phrase expresses a single thought that is incomplete by itself, but a sentence makes a complete statement. “Running very fast” is a phrase, but “The football player was running very fast” is a sentence. A sentence has a subject and a verb.

In algebra, we have *expressions* and *equations*. An expression is like a phrase. Here are some examples of expressions and how they relate to word phrases:

Expression	Words	Phrase

Expression	Words	Phrase
$3+5$	3 plus 5	the sum of three and five
$n-1$	n minus one	the difference of n and one
$6 \cdot 7$	6 times 7	the product of six and seven
$\frac{x}{y}$	x divided by y	the quotient of x and y

Notice that the phrases do not form a complete sentence because the phrase does not have a verb. An equation is two expressions linked with an equal sign. When you read the words the symbols represent in an equation, you have a complete sentence in English. The equal sign gives the verb. Here are some examples of equations:

Equation	Sentence
$3+5=8$	The sum of three and five is equal to eight.
$n-1=14$	n minus one equals fourteen.
$6 \cdot 7=42$	The product of six and seven is equal to forty-two.
$x=53$	x is equal to fifty-three.
$y+9=2y-3$	y plus nine is equal to two y minus three.

Expressions and Equations

An expression is a number, a variable, or a combination of numbers and variables and operation symbols.

An equation is made up of two expressions connected by an equal sign.

Example 2.4

Determine if each is an expression or an equation:

1. ⓐ $16-6=10$
2. ⓑ $4 \cdot 2+1$
3. ⓒ $x \div 25$
4. ⓓ $y+8=40$

Answer

ⓐ $16-6=10$	This is an equation—two expressions are connected with an equal sign.
ⓑ $4 \cdot 2+1$	This is an expression—no equal sign.
ⓒ $x \div 25$	This is an expression—no equal sign.
ⓓ $y+8=40$	This is an equation—two expressions are connected with an equal sign.

Try It 2.7

Determine if each is an expression or an equation:

1. ⓐ $23+6=29$
2. ⓑ $7 \cdot 3-7$

Try It 2.8

Determine if each is an expression or an equation:

1. $y \div 14$
2. $x - 6 = 21$

Simplify Expressions with Exponents

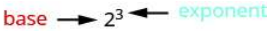
To simplify a numerical expression means to do all the math possible. For example, to simplify $4 \cdot 2 + 1$ we'd first multiply $4 \cdot 2$ to get 8 and then add the 1 to get 9. A good habit to develop is to work down the page, writing each step of the process below the previous step. The example just described would look like this:

$$4 \cdot 2 + 1$$

$$8 + 1$$

$$9$$

Suppose we have the expression $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$. We could write this more compactly using exponential notation. Exponential notation is used in algebra to represent a quantity multiplied by itself several times. We write $2 \cdot 2 \cdot 2$ as 2^3 and $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ as 2^9. In expressions such as 2^3, the 2 is called the base and the 3 is called the exponent. The exponent tells us how many factors of the base we have to multiply.



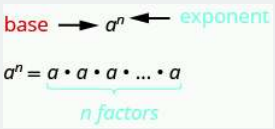
$$\text{means multiply three factors of } 2$$

We say 2^3 is in exponential notation and $2 \cdot 2 \cdot 2$ is in expanded notation.

Exponential Notation

For any expression a^n, a is a factor multiplied by itself n times if n is a positive integer.

$$a^n \text{means multiply } n \text{ factors of } a$$



The expression a^n is read a to the n^{th} power.

For powers of $n = 2$ and $n = 3$, we have special names.

$$a^2 \text{ is read as "}a \text{ squared"}$$
$$a^3 \text{ is read as "}a \text{ cubed"}$$

Table 2.4 lists some examples of expressions written in exponential notation.

Exponential Notation	In Words
7^2	7 to the second power, or 7 squared
5^3	5 to the third power, or 5 cubed
9^4	9 to the fourth power
12^5	12 to the fifth power

Table 2.4

Example 2.5

Write each expression in exponential form:

1. ⓐ $16 \cdot 16 \cdot 16 \cdot 16 \cdot 16 \cdot 16 \cdot 16$
2. ⓑ $9 \cdot 9 \cdot 9 \cdot 9 \cdot 9$
3. ⓒ $x \cdot x \cdot x \cdot x$
4. ⓓ $a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a$

Answer

ⓐ The base 16 is a factor 7 times.	16^7
ⓑ The base 9 is a factor 5 times.	9^5
ⓒ The base x is a factor 4 times.	x^4
ⓓ The base a is a factor 8 times.	a^8

Try It 2.9

Write each expression in exponential form:

$41 \cdot 41 \cdot 41 \cdot 41 \cdot 41$

Try It 2.10

Write each expression in exponential form:

$7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7$

Example 2.6

Write each exponential expression in expanded form:

1. ⓐ 8^6
2. ⓑ x^5

Answer

ⓐ The base is 8 and the exponent is 6, so 8^6 means $8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8$

ⓑ The base is x and the exponent is 5, so x^5 means $x \cdot x \cdot x \cdot x \cdot x$

Try It 2.11

Write each exponential expression in expanded form:

1. ⓐ 4^8
2. ⓑ a^7

Try It 2.12

Write each exponential expression in expanded form:

1. ⓐ 8^8
2. ⓑ b^6

To simplify an exponential expression without using a calculator, we write it in expanded form and then multiply the factors.

Example 2.7

Simplify: 3^4.

Answer

	3^4
Expand the expression.	$3 \cdot 3 \cdot 3 \cdot 3$
Multiply left to right.	$9 \cdot 3 \cdot 3$
	$27 \cdot 3$
Multiply.	81

Try It 2.13

Simplify:

1. ⓐ 5^3
2. ⓑ 1^7

Try It 2.14

Simplify:

1. ⓐ 7^2
2. ⓑ 0^5

Simplify Expressions Using the Order of Operations

We've introduced most of the symbols and notation used in algebra, but now we need to clarify the order of operations. Otherwise, expressions may have different meanings, and they may result in different values.

For example, consider the expression:

$$4 + 3 \cdot 7$$

Some students say it simplifies to 49.		Some students say it simplifies to 25.	
	$4 + 3 \cdot 7$		$4 + 3 \cdot 7$
Since $4 + 3$ gives 7.	$7 \cdot 7$	Since $3 \cdot 7$ is 21.	$4 + 21$
And $7 \cdot 7$ is 49.	49	And $21 + 4$ makes 25.	25

Imagine the confusion that could result if every problem had several different correct answers. The same expression should give the same result. So mathematicians established some guidelines called the order of operations, which outlines the order in which parts of an expression must be simplified.

Order of Operations

When simplifying mathematical expressions perform the operations in the following order:

1. **P**arentheses and other Grouping Symbols
   - Simplify all expressions inside the parentheses or other grouping symbols, working on the innermost parentheses first.
2. **E**xponents
   - Simplify all expressions with exponents.
3. **M**ultiplication and **D**ivision
   - Perform all multiplication and division in order from left to right. These operations have equal priority.
4. **A**ddition and **S**ubtraction
   - Perform all addition and subtraction in order from left to right. These operations have equal priority.

Students often ask, "How will I remember the order?" Here is a way to help you remember: Take the first letter of each key word and substitute the silly phrase. **P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally.

Order of Operations	
Please	**P**arentheses
Excuse	**E**xponents
My **D**ear	**M**ultiplication and **D**ivision
Aunt **S**ally	**A**ddition and **S**ubtraction

It's good that '**M**y **D**ear' goes together, as this reminds us that **m**ultiplication and **d**ivision have equal priority. We do not always do multiplication before division or always do division before multiplication. We do them in order from left to right.

Similarly, '**A**unt **S**ally' goes together and so reminds us that **a**ddition and **s**ubtraction also have equal priority and we do them in order from left to right.

Manipulative Mathematics

Example 2.8

Simplify the expressions:

1. ⓐ $4+3\cdot 7$
2. ⓑ $(4+3)\cdot 7$

Answer

ⓐ	
	$4+3\cdot 7$
Are there any **p**arentheses? No.	
Are there any **e**xponents? No.	
Is there any **m**ultiplication or **d**ivision? Yes.	
Multiply first.	$4+3\cdot 7$
Add.	$4+21$
	25

ⓑ	
	$(4+3)\cdot 7$
Are there any **p**arentheses? Yes.	$(4+3)\cdot 7$
Simplify inside the parentheses.	$(7)7$
Are there any **e**xponents? No.	
Is there any **m**ultiplication or **d**ivision? Yes.	
Multiply.	49

Try It 2.15

Simplify the expressions:

1. ⓐ $12-5\cdot 2$
2. ⓑ $(12-5)\cdot 2$

Try It 2.16

Simplify the expressions:

1. ⓐ $8 + 3 \cdot 9$
2. ⓑ $(8 + 3) \cdot 9$

Example 2.9

Simplify:

1. ⓐ $18 \div 9 \cdot 2$
2. ⓑ $18 \cdot 9 \div 2$

Answer

ⓐ	
	$18 \div 9 \cdot 2$
Are there any **p**arentheses? No.	
Are there any **e**xponents? No.	
Is there any **m**ultiplication or **d**ivision? Yes.	
Multiply and divide from left to right. Divide.	$2 \cdot 2$
Multiply.	4

ⓑ	
	$18 \cdot 9 \div 2$
Are there any **p**arentheses? No.	
Are there any **e**xponents? No.	
Is there any **m**ultiplication or **d**ivision? Yes.	
Multiply and divide from left to right.	
Multiply.	$162 \div 2$
Divide.	81

Try It 2.17

Simplify:

$42 \div 7 \cdot 3$

Try It 2.18

Simplify:

$12 \cdot 3 \div 4$

Example 2.10

Simplify: $18 \div 6 + 4(5 - 2)$.

Answer

	$18 \div 6 + 4(5 - 2)$
Parentheses? Yes, subtract first.	$18 \div 6 + 4(3)$
Exponents? No.	
Multiplication or division? Yes.	
Divide first because we multiply and divide left to right.	$3 + 4(3)$
Any other multiplication or division? Yes.	
Multiply.	$3 + 12$
Any other multiplication or division? No.	
Any addition or subtraction? Yes.	15

Try It 2.19

Simplify:

$30 \div 5 + 10(3 - 2)$

Try It 2.20

Simplify:

$70 \div 10 + 4(6 - 2)$

When there are multiple grouping symbols, we simplify the innermost parentheses first and work outward.

Example 2.11

Simplify: $5 + 2^3 + 3\left[6 - 3(4 - 2)\right]$.

Answer

	$5 + 2^3 + 3[6 - 3(4 - 2)]$
Are there any parentheses (or other grouping symbol)? Yes.	
Focus on the parentheses that are inside the brackets.	$5 + 2^3 + 3[6 - 3(4 - 2)]$
Subtract.	$5 + 2^3 + 3[6 - 3(2)]$
Continue inside the brackets and multiply.	$5 + 2^3 + 3[6 - 6]$
Continue inside the brackets and subtract.	$5 + 2^3 + 3[0]$
The expression inside the brackets requires no further simplification.	
Are there any exponents? Yes.	
Simplify exponents.	$5 + 2^3 + 3[0]$
Is there any multiplication or division? Yes.	
Multiply.	$5 + 8 + 3[0]$
Is there any addition or subtraction? Yes.	
Add.	$5 + 8 + 0$
Add.	$13 + 0$

	13

Try It 2.21

Simplify:

$9 + 5^3 - [4(9 + 3)]$

Try It 2.22

Simplify:

$7^2 - 2\,[4(5 + 1)]$

Example 2.12

Simplify: $2^3 + 3^4 \div 3 - 5^2$.

Answer

	$2^3 + 3^4 \div 3 - 5^2$
If an expression has several exponents, they may be simplified in the same step.	
Simplify exponents.	$2^3 + 3^4 \div 3 - 5^2$
Divide.	$8 + 81 \div 3 - 25$
Add.	$8 + 27 - 25$
Subtract.	$35 - 25$
	10

Try It 2.23

Simplify:

$3^2 + 2^4 \div 2 + 4^3$

Try It 2.24

Simplify:

$6^2 - 5^3 \div 5 + 8^2$

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ACCESS ADDITIONAL ONLINE RESOURCES

- Order of Operations
- Order of Operations – The Basics
- Ex: Evaluate an Expression Using the Order of Operations
- Example 3: Evaluate an Expression Using The Order of Operations

Section 2.1 Exercises

Use Variables and Algebraic Symbols

In the following exercises, translate from algebraic notation to words.

1.

$16 - 9$

2.

$25 - 7$

3.

$5 \cdot 6$

4.

$3 \cdot 9$

5.

$28 \div 4$

6.

$45 \div 5$

7.

$x + 8$

8.

$x + 11$

9.

$(2)(7)$

10.

$(4)(8)$

11.

$14 < 21$

12.

$17 < 35$

13.

$36 \geq 19$

14.

$42 \geq 27$

15.

$3n = 24$

16.

$6n = 36$

17.

$y - 1 > 6$

18.

$y - 4 > 8$

19.

$2 \leq 18 \div 6$

20.

$3 \leq 20 \div 4$

21.

$a \neq 7 \cdot 4$

22.

$a \neq 1 \cdot 12$

Identify Expressions and Equations

In the following exercises, determine if each is an expression or an equation.

23.

$9 \cdot 6 = 54$

24.

$7 \cdot 9 = 63$

25.

$5 \cdot 4 + 3$

26.

$6 \cdot 3 + 5$

27.

$x + 7$

28.

$x + 9$

29.

$y - 5 = 25$

30.

$y - 8 = 32$

Simplify Expressions with Exponents

In the following exercises, write in exponential form.

31.

$3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

32.

$4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$

33.

$x \cdot x \cdot x \cdot x \cdot x$

34.

$y \cdot y \cdot y \cdot y \cdot y \cdot y$

In the following exercises, write in expanded form.

35.

5^3

36.

8^3

37.

2^8

38.

10^5

Simplify Expressions Using the Order of Operations

In the following exercises, simplify.

39.

1. ⓐ $3 + 8 \cdot 5$
2. ⓑ $(3+8) \cdot 5$

40.

1. ⓐ $2 + 6 \cdot 3$
2. ⓑ $(2+6) \cdot 3$

41.

$2^3 - 12 \div (9 - 5)$

42.

$3^2 - 18 \div (11 - 5)$

43.

$3 \cdot 8 + 5 \cdot 2$

44.

$4 \cdot 7 + 3 \cdot 5$

45.

$2 + 8(6 + 1)$

46.

$4 + 6(3 + 6)$

47.

$4 \cdot 12/8$

48.

$2 \cdot 36/6$

49.

$6 + 10/2 + 2$

50.

$9 + 12/3 + 4$

51.

$(6 + 10) \div (2 + 2)$

52.

$(9 + 12) \div (3 + 4)$

53.

$20 \div 4 + 6 \cdot 5$

54.

$33 \div 3 + 8 \cdot 2$

55.

$20 \div (4+6) \cdot 5$

56.

$33 \div (3+8) \cdot 2$

57.

$4^2 + 5^2$

58.

$3^2 + 7^2$

59.

$(4+5)^2$

60.

$(3+7)^2$

61.

$3(1+9 \cdot 6) - 4^2$

62.

$5(2+8 \cdot 4) - 7^2$

63.

$2[1+3(10-2)]$

64.

$5[2+4(3-2)]$

Everyday Math

65.

Basketball In the 2014 NBA playoffs, the San Antonio Spurs beat the Miami Heat. The table below shows the heights of the starters on each team. Use this table to fill in the appropriate symbol $(=, <, >)$.

Spurs	Height		Heat	Height
Tim Duncan	$83''$		Rashard Lewis	$82''$
Boris Diaw	$80''$		LeBron James	$80''$
Kawhi Leonard	$79''$		Chris Bosh	$83''$
Tony Parker	$74''$		Dwyane Wade	$76''$
Danny Green	$78''$		Ray Allen	$77''$

1. ⓐ Height of Tim Duncan____Height of Rashard Lewis
2. ⓑ Height of Boris Diaw____Height of LeBron James
3. ⓒ Height of Kawhi Leonard____Height of Chris Bosh
4. ⓓ Height of Tony Parker____Height of Dwyane Wade
5. ⓔ Height of Danny Green____Height of Ray Allen

66.

Elevation In Colorado there are more than 50 mountains with an elevation of over $14{,}000\,\text{feet}$. The table shows the ten tallest. Use this table to fill in the appropriate inequality symbol.

Mountain	Elevation
Mt. Elbert	$14{,}433'$

Mountain	Elevation
Mt. Massive	14,421′
Mt. Harvard	14,420′
Blanca Peak	14,345′
La Plata Peak	14,336′
Uncompahgre Peak	14,309′
Crestone Peak	14,294′
Mt. Lincoln	14,286′
Grays Peak	14,270′
Mt. Antero	14,269′

1. ⓐ Elevation of La Plata Peak____Elevation of Mt. Antero
2. ⓑ Elevation of Blanca Peak____Elevation of Mt. Elbert
3. ⓒ Elevation of Gray's Peak____Elevation of Mt. Lincoln
4. ⓓ Elevation of Mt. Massive____Elevation of Crestone Peak
5. ⓔ Elevation of Mt. Harvard____Elevation of Uncompahgre Peak

Writing Exercises

67.

Explain the difference between an expression and an equation.

68.

Why is it important to use the order of operations to simplify an expression?

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
use variables and algebraic symbols.			
identify expressions and equations.			
simplify expressions with exponents.			
simplify expressions using the order of operations.			

ⓑ If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them. What did you do to become confident of your ability to do these things? Be specific.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is important to make sure you have a strong foundation before you move on. Whom can you ask for help? Your fellow classmates and instructor are good resources. Is there a place on campus where math tutors are available? Can your study skills be improved?

…no—I don't get it! This is a warning sign and you must not ignore it. You should get help right away or you will quickly be overwhelmed. See your instructor as soon as you can to discuss your situation. Together you can come up with a plan to get you the help you need.

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2.2: Introduction to Whole Numbers

Learning Objectives

By the end of this section, you will be able to:

- Round whole numbers

Rounding Whole Numbers

We round whole numbers to a given decimal place based on what number they are closest to on a number line. For example (Figure 1.8), we see that if we want to round the number 72 to the ones place, it is closest to 70.

Math input error

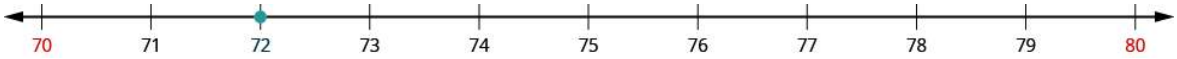


Figure 1.8 We can see that 72 is closer to 70, so 72 rounded to the nearest ten is 70.

However, what happens when the number is halfway between two numbers? For example, how do we round 75 to the ones place when it is halfway between 70 and 80?

Figure 1.9.

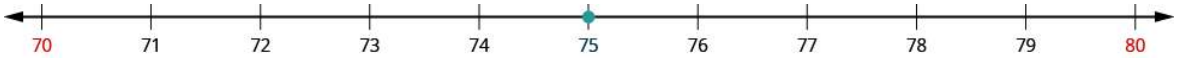
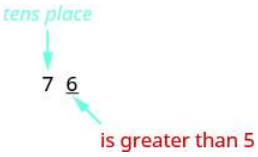


Figure 1.9 The number 75 is exactly midway between 70 and 80.

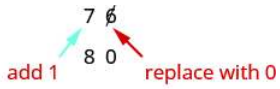
So that everyone rounds the same way in cases like this, mathematicians have agreed to round to the higher number, 80. So, 75 rounded to the nearest ten is 80.

Now that we have looked at this process on the number line, we can introduce a more general procedure. To round a number to a specific place, look at the number to the right of that place. If the number is less than 5, round down. If it is greater than or equal to 5, round up.

So, for example, to round 76 to the nearest ten, we look at the digit in the ones place.

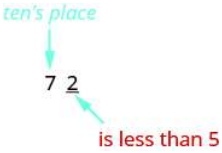


The digit in the ones place is a 6. Because 6 is greater than or equal to 5, we increase the digit in the tens place by one. So the 7 in the tens place becomes an 8. Now, replace any digits to the right of the 8 with zeros. So, 76 rounds to 80.

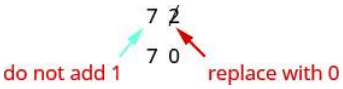


76 rounded to the nearest ten is 80.

Let's look again at rounding 72 to the nearest 10. Again, we look to the ones place.



The digit in the ones place is 2. Because 2 is less than 5, we keep the digit in the tens place the same and replace the digits to the right of it with zero. So 72 rounded to the nearest ten is 70.



How To

Round a whole number to a specific place value.

1. Step 1. Locate the given place value. All digits to the left of that place value do not change unless the digit immediately to the left is 9, in which case it may. (See Step 3.)
2. Step 2. Underline the digit to the right of the given place value.
3. Step 3. Determine if this digit is greater than or equal to 5.
   - Yes—add 1 to the digit in the given place value. If that digit is 9, replace it with 0 and add 1 to the digit immediately to its left. If that digit is also a 9, repeat.
   - No—do not change the digit in the given place value.
4. Step 4. Replace all digits to the right of the given place value with zeros.

Example 1.8

Round 843 to the nearest ten.

Answer

Locate the tens place.	tens place ↓ 843
Underline the digit to the right of the tens place.	84$\underline{3}$
Since 3 is less than 5, do not change the digit in the tens place.	84$\underline{3}$
Replace all digits to the right of the tens place with zeros.	84$\underline{0}$
	Rounding 843 to the nearest ten gives 840.

Try It 1.15

Round to the nearest ten: 157.

Try It 1.16

Round to the nearest ten: 884.

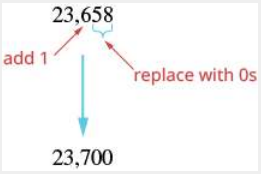
Example 1.9

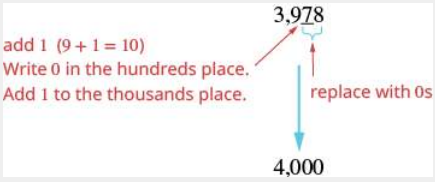
Round each number to the nearest hundred:

a. ⓐ 23,658
b. ⓑ 3,978

Answer

ⓐ	
Locate the hundreds place.	hundreds place ↓ 23,658
The digit to the right of the hundreds place is 5. Underline the digit to the right of the hundreds place.	23,6$\underline{5}$8

Since 5 is greater than or equal to 5, round up by adding 1 to the digit in the hundreds place. Then replace all digits to the right of the hundreds place with zeros.	 So 23,658 rounded to the nearest hundred is 23,700.

ⓑ	
Locate the hundreds place.	
Underline the digit to the right of the hundreds place.	3,978
The digit to the right of the hundreds place is 7. Since 7 is greater than or equal to 5, round up by added 1 to the 9. Then place all digits to the right of the hundreds place with zeros.	 So 3,978 rounded to the nearest hundred is 4,000.

Try It 1.17

Round to the nearest hundred: 17,852.

Try It 1.18

Round to the nearest hundred: 4,951.

Example 1.10

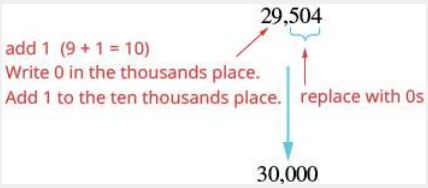
Round each number to the nearest thousand:

a. ⓐ 147,032
b. ⓑ 29,504

Answer

ⓐ	
Locate the thousands place. Underline the digit to the right of the thousands place.	
The digit to the right of the thousands place is 0. Since 0 is less than 5, we do not change the digit in the thousands place.	147,032
We then replace all digits to the right of the thousands pace with zeros.	147,000 So 147,032 rounded to the nearest thousand is 147,000.

ⓑ	
Locate the thousands place.	

Underline the digit to the right of the thousands place.	29,504
The digit to the right of the thousands place is 5. Since 5 is greater than or equal to 5, round up by adding 1 to the 9. Then replace all digits to the right of the thousands place with zeros.	 So 29,504 rounded to the nearest thousand is 30,000.

Notice that in part ⓑ, when we add 1 thousand to the 9 thousands, the total is 10 thousands. We regroup this as 1 ten thousand and 0 thousands. We add the 1 ten thousand to the 2 ten thousands and put a 0 in the thousands place.

Try It 1.19

Round to the nearest thousand: $63{,}921$.

Try It 1.20

Round to the nearest thousand: $156{,}437$.

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ACCESS ADDITIONAL ONLINE RESOURCES

- Determine Place Value
- Write a Whole Number in Digits from Words

Exercises

Practice Makes Perfect

Round Whole Numbers

In the following exercises, round to the indicated place value.

43.

Round to the nearest ten:

1. ⓐ 386
2. ⓑ $2{,}931$

44.

Round to the nearest ten:

1. ⓐ 792
2. ⓑ $5{,}647$

45.

Round to the nearest hundred:

1. ⓐ $13{,}748$
2. ⓑ $391{,}794$

46.

Round to the nearest hundred:

1. ⓐ $28{,}166$
2. ⓑ $481{,}628$

47.

Round to the nearest ten:

1. ⓐ $1{,}492$
2. ⓑ $1{,}497$

48.

Round to the nearest thousand:

1. ⓐ $2{,}391$
2. ⓑ $2{,}795$

49.

Round to the nearest hundred:

1. ⓐ $63{,}994$
2. ⓑ $63{,}949$

50.

Round to the nearest thousand:

1. ⓐ $163{,}584$
2. ⓑ $163{,}246$

Everyday Math

Buying a Car Jorge bought a car for $\$24{,}493$. Round the price to the nearest:

1. ⓐ ten dollars
2. ⓑ hundred dollars
3. ⓒ thousand dollars
4. ⓓ ten-thousand dollars

54.

Remodeling a Kitchen Marissa's kitchen remodeling cost $\$18{,}549$. Round the cost to the nearest:

1. ⓐ ten dollars
2. ⓑ hundred dollars
3. ⓒ thousand dollars
4. ⓓ ten-thousand dollars

55.

Population The population of China was $1{,}355{,}692{,}544$ in 2014. Round the population to the nearest:

1. ⓐ billion people
2. ⓑ hundred-million people
3. ⓒ million people

56.

Astronomy The average distance between Earth and the sun is $149{,}597{,}888$ kilometers. Round the distance to the nearest:

1. ⓐ hundred-million kilometers
2. ⓑ ten-million kilometers
3. ⓒ million kilometers

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2.3: Estimation by Rounding

Learning Objectives

- understand the reason for estimation
- be able to estimate the result of an addition, multiplication, subtraction, or division using the rounding technique

When beginning a computation, it is valuable to have an idea of what value to expect for the result. When a computation is completed, it is valuable to know if the result is reasonable.

In the rounding process, it is important to note two facts:

1. The rounding that is done in estimation does not always follow the rules of rounding discussed in [link] (Rounding Whole Numbers). Since estimation is concerned with the expected value of a computation, rounding is done using *convenience* as the guide rather than using hard-and-fast rounding rules. For example, if we wish to estimate the result of the division $80 \div 26$ *conveniently* divided by 20 than by 30.
2. Since rounding may occur out of convenience, and different people have different ideas of what may be convenient, results of an estimation done by rounding may vary. For a particular computation, different people may get different estimated results. *Results may vary*.

Definition: Estimation

Estimation is the process of determining an expected value of a computation.

Common words used in estimation are *about*, *near*, and *between*.

Estimation by Rounding

The rounding technique estimates the result of a computation by rounding the numbers involved in the computation to one or two nonzero digits.

Sample Set A

Estimate the sum: $2,357 + 6,106$.

Solution

Notice that 2,357 is near $\underbrace{2,400}_{\text{two nonzero digits}}$ and that 6,106 is near $\underbrace{6,100}_{\text{two nonzero digits}}$

The sum can be estimated by $2,400 + 6,100 = 8,500$. (It is quick and easy to add 24 and 61.)

Thus, $2,357 + 6,106$ is about 8,400. In *fact*, $2,357 + 6,106 = 8,463$.

Practice Set A

Estimate the sum: $4,216 + 3,942$.

Answer

$4,216 + 3,942 : 4,200 + 3,900$. About 8,100. In fact, 8,158.

Practice Set A

Estimate the sum: $812 + 514$.

Answer

$812 + 514 : 800 + 500$. About 1,300. In fact, 1,326.

Practice Set A

Estimate the sum: $43,892 + 92,106$.

Answer

$43,892 + 92,106 : 44,000 + 92,000$. About 136,000. In fact, 135,998.

Sample Set B

Estimate the difference: $5,203 - 3,015$.

Solution

Notice that 5,203 is near $\underbrace{5,200}_{\text{two nonzero digits}}$ and that 3,015 is near $\underbrace{3,000}_{\text{one nonzero digit}}$

The difference can be estimated by $5,200 - 3,000 = 2,200$.

Thus, $5,203 - 3,015$ is *about* 2,200. In *fact*, $5,203 - 3,015 = 2,188$.

We could make a less accurate estimation by observing that 5,203 is near 5,000. The number 5,000 has only one nonzero digit rather than two (as does 5,200). This fact makes the estimation quicker (but a little less accurate). We then estimate the difference by $5,000 - 3,000 = 2,000$. and conclude that $5,203 - 3,015$ is about 2,000. This is why we say "answers may vary."

Practice Set B

Estimate the difference: $628 - 413$.

Answer

628 − 413 : 600 − 400. About 200. In fact, 215.

Practice Set B

Estimate the difference: $7,842 - 5,209$.

Answer

7,842 - 5,209 : 7,800 - 5,200. About 2,600. In fact, 2,633.

Practice Set B

Estimate the difference: $73,812 - 28,492$.

Answer

73,812 - 28,492 : 74,000 − 28,000. About 46,000. In fact, 45,320.

Sample Set C

Estimate the product: $73 \cdot 46$.

Solution

Notice that 73 is near $\underbrace{70}_{\text{one nonzero digit}}$ and that 46 is near $\underbrace{50}_{\text{one nonzero digit}}$.

The product can be estimated by $70 \cdot 50 = 3,500$. (Recall that to multiply numbers ending in zeros, we multiply the nonzero digits and affix to this product the total number of ending zeros in the factors. See [link] for a review of this technique.)

Thus, $73 \cdot 46$ is about 3,500. In fact, $73 \cdot 46 = 3,358$.

Sample Set C

Estimate the product: $87 \cdot 4,316$.

Solution

Notice that 87 is near $\underbrace{90}_{\text{one nonzero digit}}$ and that 4,316 is near $\underbrace{4,000}_{\text{one nonzero digit}}$.

The product can be estimated by $90 \cdot 4,000 = 360,000$.

Thus, $87 \cdot 4,316$ is about 360,000. In fact, $87 \cdot 4,316 = 375,492$.

Practice Set C

Estimate the product: $31 \cdot 87$.

Answer

$31 \cdot 87 : 30 \cdot 90$. About 2,700. In fact, 2,697.

Practice Set C

Estimate the product: $18 \cdot 42$.

Answer

$18 \cdot 42 : 20 \cdot 40$. About 800. In fact, 756.

Practice Set C

Estimate the product: $16 \cdot 94$.

Answer

$16 \cdot 94 : 15 \cdot 100$. About 1,500. In fact, 1,504.

Sample Set D

Estimate the quotient: $153 \div 17$.

Solution

Notice that 153 is close to $\underbrace{150}_{\text{two nonzero digits}}$ and that 17 is close to $\underbrace{15}_{\text{two nonzero digits}}$.

The quotient can be estimated by $150 \div 15 = 10$.

Thus, $153 \div 17$ is about 10. In fact, $153 \div 17 = 9$.

Sample Set D

Estimate the quotient: $742,000 \div 2,400$.

Solution

Notice that 742,000 is close to $\underbrace{700,000}_{\text{one nonzero digit}}$ and that 2,400 is close to $\underbrace{2,000}_{\text{one nonzero digit}}$.

The quotient can be estimated by $700,000 \div 2,000 = 350$.

Thus, $742,000 \div 2,400$ is about 350. In fact, $742,000 \div 2,400 = 309.1\overline{6}$.

Practice Set D

Estimate the quotient: $221 \div 18$.

Answer

$221 \div 18 : 200 \div 20$. About 10. In fact, 12.27.

Practice Set D

Estimate the quotient: $4,079 \div 381$.

Answer

$4,079 \div 381 : 4,000 \div 400$. About 10. In fact, 10.70603675...

Practice Set D

Estimate the quotient: $609,000 \div 16,000$.

Answer

$609,000 \div 16,000 : 600,000 \div 15,000$. About 10. In fact, 38.0625.

Sample Set E

Estimate the sum: $53.82 \div 41.6$.

Solution

Notice that 53.82 is close to $\underbrace{54}_{\text{two nonzero digits}}$ and that 41.6 is close to $\underbrace{42}_{\text{two nonzero digits}}$.

The quotient can be estimated by $54 + 42 = 96$.

Thus, $53.82 + 41.6$ is about 96. In fact, $53.82 + 41.6 = 95.42$.

Practice Set E

Estimate the sum: $61.02 + 26.8$.

Answer

$61.02 + 26.8 : 61 + 27$. About 88. In fact, 87.82.

Practice Set E

Estimate the sum: $109.12 + 137.88$.

Answer

$109.12 + 137.88 : 110 + 138$. About 248. In fact, 247. We could have estimated 137.88 with 140. Then $110 + 140$ is an easy mental addition. We would conclude then that $109.12 + 137.88$ is about 250.

Sample Set F

Estimate the product: (31.28)(14.2).

Solution

Notice that 31.28 is close to $\underbrace{30}_{\text{one nonzero digit}}$ and that 14.2 is close to $\underbrace{15}_{\text{two nonzero digits}}$.

The product can be estimated by $30 \cdot 15 = 450$. ($3 \cdot 15 = 45$, then affix one zero.)

Thus, (31.28)(14.2) is about 450. In fact, $31.28)(14.2) = 444.176$.

Sample Set F

Estimate 21% of 5.42.

Solution

Notice that $21\% = .21$ as a decimal, and that .21 is close to $\underbrace{.2}_{\text{one nonzero digit}}$

Notice also that 5.42 is close to $\underbrace{5.}_{\text{one nonzero digit}}$.

Then, 21% of 5.42 can be estimated by $(.2)(5) = 1$.

Thus, 21% of 5.42 is about 1. In fact, 21% of 5.42 is 1.1382.

Practice Set F

Estimate the product: (47.8)(21.1).

Answer

(47.8)(21.1) : (50)(20). About 1,000. In fact, 1,008.58.

Practice Set F

Estimate 32% of 14.88.

Answer

32% of 14.88: (.3)(15). About 4.5. In fact, 4.7616.

Exercises

Estimate each calculation using the method of rounding. After you have made an estimate, find the exact value and compare this to the estimated result to see if your estimated value is reasonable. Results may vary.

Exercise 2.3.1

$1,402 + 2,198$

Answer

about 3,600; in fact 3,600

Exercise 2.3.2

$3,481 + 4,216$

Exercise 2.3.3

$921 + 796$

Answer

about 1,700; in fact 1,717

Exercise 2.3.4

$611 + 806$

Exercise 2.3.5

$4,681 + 9,325$

Answer

about 14,000; in fact 14,006

Exercise 2.3.6

$6,476 + 7,814$

Exercise 2.3.7

$7,805 - 4,266$

Answer

about 3,500; in fact 3,539

Exercise 2.3.8

$8,427 - 5,342$

Exercise 2.3.9

$14,106 - 8,412$

Answer

about 5,700; in fact 5,694

Exercise 2.3.10

$26,486 - 18,931$

Exercise 2.3.11

$32 \cdot 53$

Answer

about 1,500; in fact 1,696

Exercise 2.3.12

$67 \cdot 42$

Exercise 2.3.13

$628 \cdot 891$

Answer

about 540,000; in fact 559,548

Exercise 2.3.14

$426 \cdot 741$

Exercise 2.3.15

$18,012 \cdot 32,416$

Answer

about 583,200,000; in fact 583,876,992

Exercise 2.3.16

$22,481 \cdot 51,076$

Exercise 2.3.17

$287 \div 19$

Answer

about 15; in fact 15.11

Exercise 2.3.18

$884 \div 33$

Exercise 2.3.19

$1,254 \div 57$

Answer

about 20; in fact 22

Exercise 2.3.20

$2,189 \div 42$

Exercise 2.3.21

$8,092 \div 239$

Answer

about 33; in fact 33.86

Exercise 2.3.22

$2,688 \div 48$

Exercise 2.3.23

$72.14 + 21.08$

Answer

about 93.2; in fact 93.22

Exercise 2.3.24

$43.016 + 47.58$

Exercise 2.3.25

$96.53 - 26.91$

Answer

about 70; in fact 69.62

Exercise 2.3.26

$115.0012 - 25.018$

Exercise 2.3.27

$206.19 + 142.38$

Answer

about 348.6; in fact 348.57

Exercise 2.3.28

$592.131 + 211.6$

Exercise 2.3.29

$(32.12)(48.7)$

Answer

about 1,568.0; in fact 1,564.244

Exercise 2.3.30

$(87.013)(21.07)$

Exercise 2.3.31

$(3.003)(16.52)$

Answer

about 49.5; in fact 49.60956

Exercise 2.3.32

$(6.032)(14.091)$

Exercise 2.3.33

$(114.06)(384.3)$

Answer

about 43,776; in fact 43,833.258

Exercise 2.3.34

$(5, 137.118)(263.56)$

Exercise 2.3.35

$(6.92)(0.88)$

Answer

about 6.21; in fact 6.0896

Exercise 2.3.36

$(83.04)(1.03)$

Exercise 2.3.37

$(17.31)(.003)$

Answer

about 0.0519; in fact 0.05193

Exercise 2.3.38

$(14.016)(.016)$

Exercise 2.3.39

93% of 7.01

Answer

about 6.3; in fact 6.5193

Exercise 2.3.40

107% of 12.6

Exercise 2.3.41

32% of 15.3

Answer

about 4.5; in fact 4.896

Exercise 2.3.42

74% of 21.93

Exercise 2.3.43

18% of 4.118

Answer

about 0.8; in fact 0.74124

Exercise 2.3.44

4% of .863

Exercise 2.3.45

2% of .0039

Answer

about 0.00008; in fact 0.000078

Exercises for Review

Exercise 2.3.46

Find the difference: $\frac{7}{10} - \frac{5}{16}$.

Exercise 2.3.47

Find the value $\frac{6 - \frac{1}{4}}{6 + \frac{1}{4}}$.

Answer

$\frac{23}{25}$

Exercise 2.3.48

Convert the complex decimal $1.11\frac{1}{4}$ to a decimal.

Exercise 2.3.49

A woman 5 foot tall casts an 8-foot shadow at a particular time of the day. How tall is a tree that casts a 96-foot shadow at the same time of the day?

Answer

60 feet tall

Exercise 2.3.50

11.62 is 83% of what number?

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2.4: Estimation by Clustering

Learning Objectives

- understand the concept of clustering
- be able to estimate the result of adding more than two numbers when clustering occurs using the clustering technique

Cluster

When more than two numbers are to be added, the sum may be estimated using the clustering technique. The rounding technique could also be used, but if several of the numbers are seen to **cluster** (are seen to be close to) one particular number, the clustering technique provides a quicker estimate. Consider a sum such as

$32 + 68 + 29 + 73$

Notice two things:

There are more than two numbers to be added.

Clustering occurs.

a. Both 68 and 73 cluster around 70, so $68 + 73$ is close to $80 + 70 = 2(70) = 140$.

$\overbrace{32 + \underbrace{68 + 29 + 71}}$

b. Both 32 and 29 cluster around 30, so $32 + 29$ is close to $30 + 30 = 2(30) = 60$.

The sum may be estimated by

$$\begin{aligned}(2 \cdot 30) + (2 \cdot 70) &= 60 + 140 \\ &= 200\end{aligned}$$

In fact, $32 + 68 + 29 + 73 = 202$.

Sample Set A

Estimate each sum. Results may vary.

$27 + 48 + 31 + 52$.

Solution

27 and 31 cluster near 30. Their sum is about $2 \cdot 30 = 60$.

48 and 52 cluster near 50. Their sum is about $2 \cdot 50 = 100$.

Thus, $27 + 48 + 31 + 52$ is about $\begin{aligned}(2 \cdot 30) + (2 \cdot 50) &= 60 + 100 \\ &= 160\end{aligned}$

In fact, $27 + 48 + 31 + 52 = 158$..

Sample Set A

$88 + 21 + 19 + 91$.

Solution

88 and 91 cluster near 90. Their sum is about $2 \cdot 90 = 180$.

21 and 19 cluster near 20. Their sum is about $2 \cdot 20 = 40$.

Thus, $88 + 21 + 19 + 91$ is about $\begin{aligned}(2 \cdot 90) + (2 \cdot 20) &= 180 + 40 \\ &= 220\end{aligned}$

In fact, $88 + 21 + 19 + 91 = 219$.

Sample Set A

$17 + 21 + 48 + 18$.

Solution

17, 21, and 18 cluster near 20. Their sum is about $3 \cdot 20 = 60$.

48 is about 50.

Thus, $17 + 21 + 48 + 18$ is about $\begin{aligned} (3 \cdot 20) + 50 &= 60 + 50 \\ &= 110 \end{aligned}$

In fact, $17 + 21 + 48 + 18 = 104$..

Sample Set A

$61 + 48 + 39 + 57 + 52$.

Solution

61 and 57 cluster near 60. Their sum is about $2 \cdot 60 = 120$.

48, 49, and 52 cluster near 50. Their sum is about $3 \cdot 50 = 150$.

Thus, $61 + 48 + 39 + 57 + 52$ is about $\begin{aligned} (2 \cdot 60) + (3 \cdot 50) &= 120 + 150 \\ &= 270 \end{aligned}$

In fact, $61 + 48 + 39 + 57 + 52 = 267$..

Sample Set A

$706 + 321 + 293 + 684$.

Solution

706 and 684 cluster near 700. Their sum is about $2 \cdot 700 = 1,400$.

321 and 293 cluster near 300. Their sum is about $2 \cdot 300 = 600$.

Thus, $706 + 321 + 293 + 684$ is about $\begin{aligned} (2 \cdot 700) + (2 \cdot 300) &= 1,400 + 600 \\ &= 2,000 \end{aligned}$

In fact, $706 + 321 + 293 + 684 = 2,004$..

Practice Set A

Use the clustering method to estimate each sum.

$28 + 51 + 31 + 47$

Answer

$(2 \cdot 30) + (2 \cdot 50) = 60 + 100 = 160$

Practice Set A

$42 + 39 + 68 + 41$

Answer

$(3 \cdot 40) + 70 = 120 + 70 = 190$

Practice Set A

$37 + 39 + 83 + 42 + 79$

Answer

$(3 \cdot 40) + (2 \cdot 80) = 120 + 160 = 280$

Practice Set A

$612 + 585 + 830 + 794$

Answer

$(2 \cdot 600) + (2 \cdot 800) = 1,200 + 1,600 = 2,800$

Exercises

Use the clustering method to estimate each sum. Results may vary.

Exercise 2.4.1

$28 + 51 + 31 + 47$

Answer

$2(30) + 2(50) = 160(157)$

Exercise 2.4.2

$42 + 19 + 39 + 23$

Exercise 2.4.3

$88 + 62 + 59 + 90$

Answer

$2(90) + 2(60) = 300(299)$

Exercise 2.4.4

$76 + 29 + 33 + 82$

Exercise 2.4.5

$19 + 23 + 87 + 21$

Answer

$3(20) + 90 = 150(150)$

Exercise 2.4.6

$41 + 28 + 42 + 37$

Exercise 2.4.7

$89 + 32 + 89 + 93$

Answer

$3(90) + 30 = 300(303)$

Exercise 2.4.8

$73 + 72 + 27 + 71$

Exercise 2.4.9

$43 + 62 + 61 + 55$

Answer

$40 + 3(60) = 220(221)$

Exercise 2.4.10

$31 + 77 + 31 + 27$

Exercise 2.4.11

$57 + 34 + 28 + 61 + 62$

Answer

$3(60) + 2(30) = 240(242)$

Exercise 2.4.12

$94 + 18 + 23 + 91 + 19$

Exercise 2.4.13

$103 + 72 + 66 + 97 + 99$

Answer

$3(100) + 2(70) = 440(437)$

Exercise 2.4.14

$42 + 121 + 119 + 124 + 41$

Exercise 2.4.15

$19 + 24 + 87 + 23 + 91 + 93$

Answer

$3(20) + 3(90) = 330(337)$

Exercise 2.4.16

$108 + 61 + 63 + 96 + 57 + 99$

Exercise 2.4.17

$518 + 721 + 493 + 689$

Answer

$2(500) + 2(700) = 2,400(2,421)$

Exercise 2.4.18

$981 + 1208 + 1214 + 1006$

Exercise 2.4.19

$23 + 81 + 77 + 79 + 19 + 81$

Answer

$2(20) + 4(80) = 360(360)$

Exercise 2.4.20

$94 + 68 + 66 + 101 + 106 + 71 + 110$

Exercises for Review

Exercise 2.4.21

Specify all the digits greater than 6.

Answer

7, 8, 9

Exercise 2.4.22

Find the product: $\frac{2}{3} \cdot \frac{9}{14} \cdot \frac{7}{12}$.

Exercise 2.4.23

Convert 0.06 to a fraction.

Answer

$\frac{3}{50}$

Exercise 2.4.24

Write the proportion in fractional form: "5 is to 8 as 25 is to 40."

Exercise 2.4.25

Estimate the sum using the method of rounding: $4,882 + 2,704$.

Answer

$4,900 + 2,700 = 7,600 (7,586)$

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CHAPTER OVERVIEW

3: Signed Numbers

The sections in this module will help you to learn about signed numbers. Your goal is to demonstrate mastery on each of the following objectives.

- **Objective 3.1 - Absolute Values and Numberlines:** Identify signed numbers on a numberline and understand that absolute value of a signed number measures the distance from zero on the numberline.
- **Objective 3.2 - Adding and Subtracting Signed Numbers:** Perform addition and subtraction involving signed numbers, including applications, without the use of a calculator.
- **Objective 3.3 - Multiplying and Dividing Signed Numbers:** Perform multiplication, division, and exponentiation involving signed numbers, including applications, without the use of a calculator.

3.1: Introduction to Integers
3.2: Add Integers
3.3: Subtract Integers
3.4: Multiply and Divide Integers

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3.1: Introduction to Integers

Learning Objectives

By the end of this section, you will be able to:

- Locate positive and negative numbers on the number line
- Order positive and negative numbers
- Find opposites
- Simplify expressions with absolute value
- Translate word phrases to expressions with integers

Be Prepared 3.1

Before you get started, take this readiness quiz.

Plot $0, 1,$ on a number line.
If you missed this problem, review Example 1.1.

Be Prepared 3.2

Fill in the appropriate symbol: $(=, <, \text{or } >): 2 \_\_ 4$
If you missed this problem, review Example 2.3.

Locate Positive and Negative Numbers on the Number Line

Do you live in a place that has very cold winters? Have you ever experienced a temperature below zero? If so, you are already familiar with negative numbers. A negative number is a number that is less than Math input error

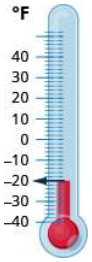


Figure 3.2 Temperatures below zero are described by negative numbers.

Temperatures are not the only negative numbers. A bank overdraft is another example of a negative number. If a person writes a check for more than he has in his account, his balance will be negative.

Elevations can also be represented by negative numbers. The elevation at sea level is Math input error

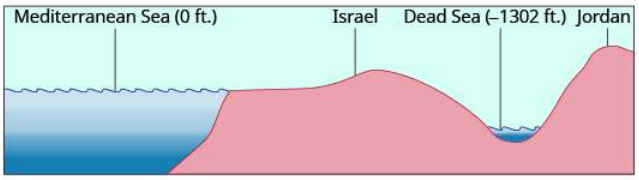


Figure 3.3 The surface of the Mediterranean Sea has an elevation of 0 ft. The diagram shows that nearby mountains have higher (positive) elevations whereas the Dead Sea has a lower (negative) elevation.

Depths below the ocean surface are also described by negative numbers. A submarine, for example, might descend to a depth of Math input error

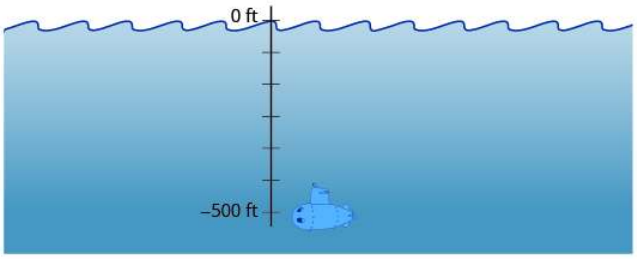


Figure 3.4 Depths below sea level are described by negative numbers. A submarine 500 ft below sea level is at -500 ft.

Both positive and negative numbers can be represented on a number line. Recall that the number line created in Add Whole Numbers started at Math input error

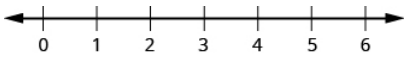


Figure 3.5

Now we need to extend the number line to include negative numbers. We mark several units to the left of zero, keeping the intervals the same width as those on the positive side. We label the marks with negative numbers, starting with Math input error

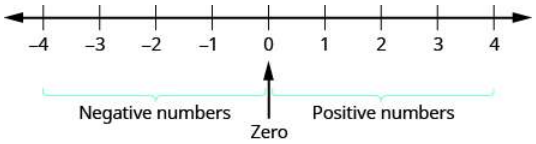


Figure 3.6 On a number line, positive numbers are to the right of zero. Negative numbers are to the left of zero. What about zero? Zero is neither positive nor negative.

The arrows at either end of the line indicate that the number line extends forever in each direction. There is no greatest positive number and there is no smallest negative number.

Manipulative Mathematics

Doing the Manipulative Mathematics activity "Number Line-part 2" will help you develop a better understanding of integers.

Example 3.1

Plot the numbers on a number line:

1. ⓐ 3
2. ⓑ -3
3. ⓒ -2

Answer

Draw a number line. Mark 0 in the center and label several units to the left and right.

1. ⓐ To plot Math input error

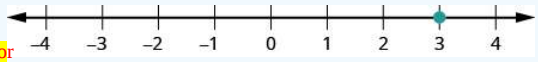


Figure 3.7

- ⓑ To plot Math input error

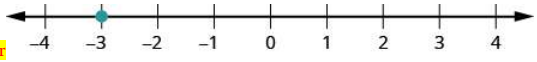


Figure 3.8

- ⓒ To plot Math input error

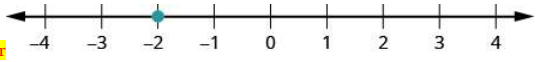


Figure 3.9

Try It 3.1

Plot the numbers on a number line.

1. ⓐ 1
2. ⓑ -1
3. ⓒ -4

Try It 3.2

Plot the numbers on a number line.

1. ⓐ -4
2. ⓑ 4
3. ⓐ -1

Order Positive and Negative Numbers

We can use the number line to compare and order positive and negative numbers. Going from left to right, numbers increase in value. Going from right to left, numbers decrease in value. See Figure 3.10.

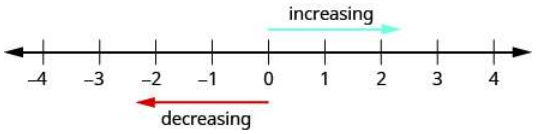


Figure 3.10

Just as we did with positive numbers, we can use inequality symbols to show the ordering of positive and negative numbers. Remember that we use the notation Math input error

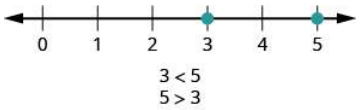
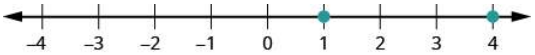


Figure 3.11 The number 3 is to the left of 5 on the number line. So 3 is less than 5, and 5 is greater than 3.

The numbers lines to follow show a few more examples.

ⓐ



4 is to the right of 1 on the number line, so $4 > 1$.

1 is to the left of 4 on the number line, so $1 < 4$.

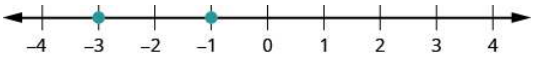
ⓑ



-2 is to the left of 1 on the number line, so $-2 < 1$.

1 is to the right of -2 on the number line, so $1 > -2$.

ⓒ



-1 is to the right of -3 on the number line, so $-1 > -3$.

-3 is to the left of -1 on the number line, so $-3 < -1$.

Example 3.2

Order each of the following pairs of numbers using $<$ or $>$:

1. ⓐ $14 \_\_ 6$
2. ⓑ $-1 \_\_ 9$

3. ⓒ $-1 \_\_ -4$
4. ⓓ $2 \_\_ -20$

Answer

Begin by plotting the numbers on a number line as shown in Figure 3.12.

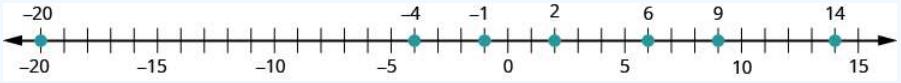


Figure 3.12

ⓐ Compare 14 and 6.	$14 \_\_ 6$
14 is to the right of 6 on the number line.	$14 > 6$

ⓑ Compare −1 and 9.	$-1 \_\_ 9$
−1 is to the left of 9 on the number line.	$-1 < 9$

ⓒ Compare −1 and −4.	$-1 \_\_ -4$
−1 is to the right of −4 on the number line.	$-1 > -4$

ⓓ Compare 2 and −20.	$2 \_\_ -20$
2 is to the right of −20 on the number line.	$2 > -20$

Try It 3.3

Order each of the following pairs of numbers using $<$ or $>$.

1. ⓐ $15 \_\_ 7$
2. ⓑ $-2 \_\_ 5$
3. ⓒ $-3 \_\_ -7$
4. ⓓ $5 \_\_ -17$

Try It 3.4

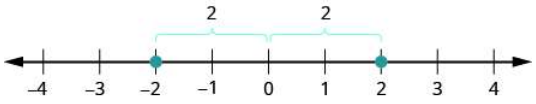
Order each of the following pairs of numbers using $<$ or $>$.

1. ⓐ $8 \_\_ 13$
2. ⓑ $3 \_\_ -4$
3. ⓒ $-5 \_\_ -2$
4. ⓓ $9 \_\_ -21$

Find Opposites

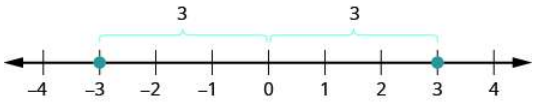
On the number line, the negative numbers are a mirror image of the positive numbers with zero in the middle. Because the numbers

Math input error



The numbers –2 and 2 are opposites.

(a)



The numbers –3 and 3 are opposites.

(b)

Figure 3.13

Opposite

The opposite of a number is the number that is the same distance from zero on the number line, but on the opposite side of zero.

Example 3.3

Find the opposite of each number:

1. ⓐ 7
2. ⓑ -10

Answer

1. ⓐ The number Math input error



Figure 3.14

- ⓑ The number Math input error



Figure 3.15

Try It 3.5

Find the opposite of each number:

1. ⓐ 4
2. ⓑ -3

Try It 3.6

Find the opposite of each number:

1. ⓐ 8
2. ⓑ -5

Opposite Notation

Just as the same word in English can have different meanings, the same symbol in algebra can have different meanings. The specific meaning becomes clear by looking at how it is used. You have seen the symbol "$-$", in three different ways.

$10-4$	Between two numbers, the symbol indicates the operation of subtraction. We read $10-4$ as 10 *minus* 4.
-8	In front of a number, the symbol indicates a negative number. We read -8 as *negative eight*.
$-x$	In front of a variable or a number, it indicates the opposite. We read$-x$ as *the opposite of* x.
$-(-2)$	Here we have two signs. The sign in the parentheses indicates that the number is negative 2. The sign outside the parentheses indicates the opposite. We read $-(-2)$ as *the opposite of* -2.

Opposite Notation

$-a$ means the opposite of the number a

The notation $-a$ is read *the opposite of* a.

Example 3.4

Simplify: $-(-6)$.

Answer

	$-(-6)$
The opposite of -6 is 6.	6

Try It 3.7

Simplify:

$-(-1)$

Try It 3.8

Simplify:

$-(-5)$

Integers

The set of counting numbers, their opposites, and 0 is the set of integers.

Integers

Integers are counting numbers, their opposites, and zero.

$$\ldots -3, -2, -1, 0, 1, 2, 3 \ldots$$

We must be very careful with the signs when evaluating the opposite of a variable.

Example 3.5

Evaluate $-x$:

1. ⓐ when $x = 8$
2. ⓑ when $x = -8$.

Answer

ⓐ To evaluate $-x$ when $x = 8$, substitute 8 for x.	
	$-x$
Substitute 8 for x.	$-(8)$
Simplify.	-8

ⓑ To evaluate $-x$ when $x = -8$, substitute -8 for x.	
	$-x$
Substitute −8 for x.	$-(-8)$
Simplify.	8

Try It 3.9

Evaluate $-n$:

1. ⓐ when $n = 4$
2. ⓑ when $n = -4$

Try It 3.10

Evaluate: $-m$:

1. ⓐ when $m = 11$
2. ⓑ when $m = -11$

Simplify Expressions with Absolute Value

We saw that numbers such as 5 and -5 are opposites because they are the same distance from 0 on the number line. They are both five units from 0. The distance between 0 and any number on the number line is called the absolute value of that number. Because distance is never negative, the absolute value of any number is never negative.

The symbol for absolute value is two vertical lines on either side of a number. So the absolute value of Math input error

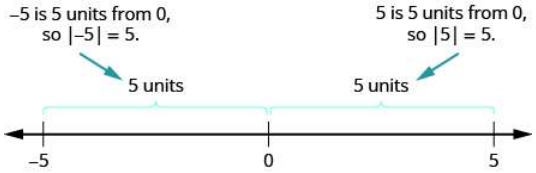


Figure 3.16

Absolute Value

The absolute value of a number is its distance from 0 on the number line.

The absolute value of a number n is written as $|n|$.

$$|n| \geq 0 \text{ for all numbers}$$

Example 3.6

Simplify:

1. ⓐ $|3|$
2. ⓑ $|-44|$

3. ⓒ $|0|$

Answer

ⓐ	
	$|3|$
3 is 3 units from zero.	3

ⓑ	
	$|-44|$
−44 is 44 units from zero.	44

ⓒ	
	$|0|$
0 is already at zero.	0

Try It 3.11

Simplify:

1. ⓐ $|12|$
2. ⓑ $-|-28|$

Try It 3.12

Simplify:

1. ⓐ $|9|$
2. ⓑ $-|37|$

We treat absolute value bars just like we treat parentheses in the order of operations. We simplify the expression inside first.

Example 3.7

Evaluate:

1. ⓐ $|x|$ when $x = -35$
2. ⓑ $|-y|$ when $y = -20$
3. ⓒ $-|u|$ when $u = 12$
4. ⓓ $-|p|$ when $p = -14$

Answer

ⓐ To find $|x|$ when $x = -35$:	
	$|x|$
Substitute −35 for x.	$|-35|$
Take the absolute value.	35

ⓑ To find $|-y|$ when $y = -20$:	
	$|-y|$

Substitute -20 for y.	$\vert-(-20)\vert$
Simplify.	$\vert 20\vert$
Take the absolute value.	20

ⓒ To find $-\vert u\vert$ when $u=12$:	
	$-\vert u\vert$
Substitute 12 for u.	$-\vert 12\vert$
Take the absolute value.	-12

ⓓ To find $-\vert p\vert$ when $p=-14$:	
	$-\vert p\vert$
Substitute -14 for p.	$-\vert -14\vert$
Take the absolute value.	-14

Notice that the result is negative only when there is a negative sign outside the absolute value symbol.

Try It 3.13

Evaluate:

1. ⓐ $\vert x\vert$ when $x=-17$
2. ⓑ $\vert -y\vert$ when $y=-39$
3. ⓒ $-\vert m\vert$ when $m=22$
4. ⓓ $-\vert p\vert$ when $p=-11$

Try It 3.14

1. ⓐ $\vert y\vert$ when $y=-23$
2. ⓑ $\vert -y\vert$ when $y=-21$
3. ⓒ $-\vert n\vert$ when $n=37$
4. ⓓ $-\vert q\vert$ when $q=-49$

Example 3.8

Fill in $<, >, \text{or } =$ for each of the following:

1. ⓐ $\vert -5\vert \_\_ -\vert -5\vert$
2. ⓑ $8 \_\_ -\vert -8\vert$
3. ⓒ $-9 \_\_ -\vert -9\vert$
4. ⓓ $-\vert -7\vert \_\_ -7$

Answer

To compare two expressions, simplify each one first. Then compare.

ⓐ	
	$\vert -5\vert \_\_ -\vert -5\vert$
Simplify.	$5 \_\_ -5$
Order.	$5 > -5$

ⓑ	
	$8 \_\_ -\lvert -8\rvert$
Simplify.	$8 \_\_ -8$
Order.	$8 > -8$

ⓒ	
	$-9 \_\_ -\lvert -9\rvert$
Simplify.	$-9 \_\_ -9$
Order.	$-9 = -9$

ⓓ	
	$-\lvert -7\rvert \_\_ -7$
Simplify.	$-7 \_\_ -7$
Order.	$-7 = -7$

Try It 3.15

Fill in $<, >, \text{or} =$ for each of the following:

1. ⓐ $\lvert -9\rvert \_\_ -\lvert -9\rvert$
2. ⓑ $2 \_\_ -\lvert -2\rvert$
3. ⓒ $-8 \_\_ \lvert -8\rvert$
4. ⓓ $-\lvert -5\rvert \_\_ -5$

Try It 3.16

Fill in $<, >, \text{or} =$ for each of the following:

1. ⓐ $7 \_\_ -\lvert -7\rvert$
2. ⓑ $-\lvert -11\rvert \_\_ -11$
3. ⓒ $\lvert -4\rvert \_\_ -\lvert -4\rvert$
4. ⓓ $-1 \_\_ \lvert -1\rvert$

Absolute value bars act like grouping symbols. First simplify inside the absolute value bars as much as possible. Then take the absolute value of the resulting number, and continue with any operations outside the absolute value symbols.

Example 3.9

Simplify:

1. ⓐ $\lvert 9-3\rvert$
2. ⓑ $4\lvert -2\rvert$

Answer

For each expression, follow the order of operations. Begin inside the absolute value symbols just as with parentheses.

ⓐ	
	$\lvert 9-3\rvert$
Simplify inside the absolute value sign.	$\lvert 6\rvert$

Take the absolute value.	6

ⓑ	
	$4\lvert -2\rvert$
Take the absolute value.	$4 \cdot 2$
Multiply.	8

Try It 3.17

Simplify:

1. ⓐ $\lvert 12-9\rvert$
2. ⓑ $3\lvert -6\rvert$

Try It 3.18

Simplify:

1. ⓐ $\lvert 27-16\rvert$
2. ⓑ $9\lvert -7\rvert$

Example 3.10

Simplify: $\lvert 8+7\rvert - \lvert 5+6\rvert$.

Answer

For each expression, follow the order of operations. Begin inside the absolute value symbols just as with parentheses.

	$\lvert 8+7\rvert - \lvert 5+6\rvert$
Simplify inside each absolute value sign.	$\lvert 15\rvert - \lvert 11\rvert$
Subtract.	4

Try It 3.19

Simplify: $\lvert 1+8\rvert - \lvert 2+5\rvert$

Try It 3.20

Simplify: $\lvert 9-5\rvert - \lvert 7-6\rvert$

Example 3.11

Simplify: $24 - \lvert 19 - 3(6-2)\rvert$.

Answer

We use the order of operations. Remember to simplify grouping symbols first, so parentheses inside absolute value symbols would be first.

	$24 - \lvert 19 - 3\,(6-2)\rvert$
Simplify in the parentheses first.	$24 - \lvert 19 - 3\,(4)\rvert$
Multiply $3\,(4)$.	$24 - \lvert 19 - 12\rvert$

Subtract inside the absolute value sign.	$24-\|7\|$
Take the absolute value.	$24-7$
Subtract.	17

Try It 3.21

Simplify: $19-|11-4(3-1)|$

Try It 3.22

Simplify: $9-|8-4(7-5)|$

Translate Word Phrases into Expressions with Integers

Now we can translate word phrases into expressions with integers. Look for words that indicate a negative sign. For example, the word *negative* in "negative twenty" indicates -20. So does the word *opposite* in "the opposite of 20."

Example 3.12

Translate each phrase into an expression with integers:

1. ⓐ the opposite of positive fourteen
2. ⓑ the opposite of -11
3. ⓒ negative sixteen
4. ⓓ two minus negative seven

Answer

1. ⓐ the opposite of fourteen
   -14
2. ⓑ the opposite of -11
   $-(-11)=11$
3. ⓒ negative sixteen
   -16
4. ⓓ two minus negative seven
   $2-(-7)$

Try It 3.23

Translate each phrase into an expression with integers:

1. ⓐ the opposite of positive nine
2. ⓑ the opposite of -15
3. ⓒ negative twenty
4. ⓓ eleven minus negative four

Try It 3.24

Translate each phrase into an expression with integers:

1. ⓐ the opposite of negative nineteen
2. ⓑ the opposite of twenty-two
3. ⓒ negative nine
4. ⓓ negative eight minus negative five

As we saw at the start of this section, negative numbers are needed to describe many real-world situations. We'll look at some more applications of negative numbers in the next example.

Example 3.13

Translate into an expression with integers:

1. ⓐ The temperature is 12 degrees Fahrenheit below zero.
2. ⓑ The football team had a gain of 3 yards.
3. ⓒ The elevation of the Dead Sea is $1{,}302$ feet below sea level.
4. ⓓ A checking account is overdrawn by $\$40$.

Answer

Look for key phrases in each sentence. Then look for words that indicate negative signs. Don't forget to include units of measurement described in the sentence.

ⓐ	The temperature is 12 degrees Fahrenheit below zero.
Below zero tells us that 12 is a negative number.	-12°F

ⓑ	The football team had a gain of 3 yards.
A *gain* tells us that 3 is a positive number.	3 yards

ⓒ	The elevation of the Dead Sea is 1,302 feet below sea level.
Below sea level tells us that 1,302 is a negative number.	$-1{,}302$ feet

ⓓ	A checking account is overdrawn by $40.
Overdrawn tells us that 40 is a negative number.	$-\$40$

Try It 3.25

Translate into an expression with integers:

The football team had a gain of 5 yards.

Try It 3.26

Translate into an expression with integers:

The scuba diver was 30 feet below the surface of the water.

Media

ACCESS ADDITIONAL ONLINE RESOURCES

- Introduction to Integers
- Simplifying the Opposites of Negative Integers
- Comparing Absolute Value of Integers
- Comparing Integers Using Inequalities

Section 3.1 Exercises

Locate Positive and Negative Numbers on the Number Line

For the following exercises, draw a number line and locate and label the given points on that number line.

1.
   a. 2
   b. -2
   c. -5

2.
   a. 5
   b. -5
   c. -2

3.
   a. -8
   b. 8
   c. -6

4.
   a. -7
   b. 7
   c. -1

Order Positive and Negative Numbers on the Number Line

In the following exercises, order each of the following pairs of numbers, using $<$ or $>$.

5.
   a. 9_4
   b. -3_6
   c. -8_-2
   d. 1_-10

6.
   a. $6\_2;$
   b. $-7\_4;$
   c. $-9\_-1;$
   d. 9_-3

7.
   a. $-5\_1;$
   b. $-4\_-9;$
   c. $6\_10;$
   d. 3_-8

8.
   a. $-7\_3;$
   b. $-10\_-5;$
   c. $2\_-6;$
   d. 8_9

Find Opposites

In the following exercises, find the opposite of each number.

9.
   a. 2
   b. -6

10.

1. ⓐ 9
2. ⓑ -4

11.

1. ⓐ -8
2. ⓑ 1

12.

1. ⓐ -2
2. ⓑ 6

In the following exercises, simplify.

13.

$-(-4)$

14.

$-(-8)$

15.

$-(-15)$

16.

$-(-11)$

In the following exercises, evaluate.

17.

$-m$ when

1. ⓐ $m = 3$
2. ⓑ $m = -3$

18.

$-p$ when

1. ⓐ $p = 6$
2. ⓑ $p = -6$

19.

$-c$ when

1. ⓐ $c = 12$
2. ⓑ $c = -12$

20.

$-d$ when

1. ⓐ $d = 21$
2. ⓑ $d = -21$

Simplify Expressions with Absolute Value

In the following exercises, simplify each absolute value expression.

21.

1. ⓐ $|7|$
2. ⓑ $|-25|$
3. ⓒ $|0|$

22.

1. ⓐ $|5|$
2. ⓑ $|20|$
3. ⓒ $|-19|$

23.

1. ⓐ $|-32|$
2. ⓑ $|-18|$
3. ⓒ $|16|$

24.

1. ⓐ $|-41|$
2. ⓑ $|-40|$
3. ⓒ $|22|$

In the following exercises, evaluate each absolute value expression.

25.

1. ⓐ $|x|$ when $x = -28$
2. ⓑ $|-u|$ when $u = -15$

26.

1. ⓐ $|y|$ when $y = -37$
2. ⓑ $|-z|$ when $z = -24$

27.

1. ⓐ $-|p|$ when $p = 19$
2. ⓑ $-|q|$ when $q = -33$

28.

1. ⓐ $-|a|$ when $a = 60$
2. ⓑ $-|b|$ when $b = -12$

In the following exercises, fill in $<, >, \text{or } =$ to compare each expression.

29.

1. ⓐ $-6\_|-6|$
2. ⓑ $-|-3|\_-3$

30.

1. ⓐ $-8\_|-8|$
2. ⓑ $-|-2|\_-2$

31.

1. ⓐ $|-3|\_-|-3|$
2. ⓑ $4\_-|-4|$

32.

1. ⓐ $|-5|\_-|-5|$
2. ⓑ $9\_-|-9|$

In the following exercises, simplify each expression.

33.

$|8-4|$

34.

$|9-6|$

35.

$8\,|-7|$

36.

$5\,|-5|$

37.

$|15-7|-|14-6|$

38.

$|17-8|-|13-4|$

39.

$18-|2(8-3)|$

40.

$15-|3(8-5)|$

41.

$8(14-2\,|-2|)$

42.

$6(13-4\,|-2|)$

Translate Word Phrases into Expressions with Integers

Translate each phrase into an expression with integers. *Do not simplify*.

43.

1. ⓐ the opposite of 8
2. ⓑ the opposite of -6
3. ⓒ negative three
4. ⓓ 4 minus negative 3

44.

1. ⓐ the opposite of 11
2. ⓑ the opposite of -4
3. ⓒ negative nine
4. ⓓ 8 minus negative 2

45.

1. ⓐ the opposite of 20
2. ⓑ the opposite of -5
3. ⓒ negative twelve
4. ⓓ 18 minus negative 7

46.

1. ⓐ the opposite of 15
2. ⓑ the opposite of -9
3. ⓒ negative sixty
4. ⓓ 12 minus 5

47.

a temperature of 6 degrees below zero

48.

a temperature of 14 degrees below zero

49.

an elevation of 40 feet below sea level

50.

an elevation of 65 feet below sea level

51.

a football play loss of 12 yards

52.

a football play gain of 4 yards

53.

a stock gain of $\$3$

54.

a stock loss of $\$5$

55.

a golf score one above par

56.

a golf score of 3 below par

Everyday Math

57.

Elevation The highest elevation in the United States is Mount McKinley, Alaska, at $20{,}320$ feet above sea level. The lowest elevation is Death Valley, California, at 282 feet below sea level. Use integers to write the elevation of:

1. ⓐ Mount McKinley
2. ⓑ Death Valley

58.

Extreme temperatures The highest recorded temperature on Earth is $58°$ Celsius, recorded in the Sahara Desert in 1922. The lowest recorded temperature is $90°$ below $0°$ Celsius, recorded in Antarctica in 1983. Use integers to write the:

1. ⓐ highest recorded temperature
2. ⓑ lowest recorded temperature

59.

State budgets In June, 2011, the state of Pennsylvania estimated it would have a budget surplus of $\$540$ million. That same month, Texas estimated it would have a budget deficit of $\$27$ billion. Use integers to write the budget:

1. ⓐ surplus
2. ⓑ deficit

60.

College enrollments Across the United States, community college enrollment grew by $1{,}400{,}000$ students from 2007 to 2010. In California, community college enrollment declined by $110{,}171$ students from 2009 to 2010. Use integers to write the change in enrollment:

1. ⓐ growth
2. ⓑ decline

Writing Exercises

61.

Give an example of a negative number from your life experience.

62.

What are the three uses of the "−" sign in algebra? Explain how they differ.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
locate positive and negative numbers on the number line.			
order positive and negative numbers.			
find opposites.			
simplify expressions with absolute value.			
translate word phrases to expressions with integers.			

ⓑ If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them. What did you do to become confident of your ability to do these things? Be specific.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is important to make sure you have a strong foundation before you move on. Whom can you ask for help? Your fellow classmates and instructor are good resources. Is there a place on campus where math tutors are available? Can your study skills be improved?

…no—I don't get it! This is a warning sign and you must not ignore it. You should get help right away or you will quickly be overwhelmed. See your instructor as soon as you can to discuss your situation. Together you can come up with a plan to get you the help you need.

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3.2: Add Integers

Learning Objectives

By the end of this section, you will be able to:

- Model addition of integers
- Simplify expressions with integers
- Evaluate variable expressions with integers
- Translate word phrases to algebraic expressions
- Add integers in applications

Be Prepared 3.3

Before you get started, take this readiness quiz.

Evaluate $x + 8$ when $x = 6$.
If you missed this problem, review Example 2.13.

Simplify: $8 + 2\,(5 + 1)$.
If you missed this problem, review Example 2.8.

Translate *the sum of* 3 *and negative* 7 into an algebraic expression.
If you missed this problem, review Table 2.7

Model Addition of Integers

Now that we have located positive and negative numbers on the number line, it is time to discuss arithmetic operations with integers.

Most students are comfortable with the addition and subtraction facts for positive numbers. But doing addition or subtraction with both positive and negative numbers may be more difficult. This difficulty relates to the way the brain learns.

The brain learns best by working with objects in the real world and then generalizing to abstract concepts. Toddlers learn quickly that if they have two cookies and their older brother steals one, they have only one left. This is a concrete example of $2 - 1$. Children learn their basic addition and subtraction facts from experiences in their everyday lives. Eventually, they know the number facts without relying on cookies.

Addition and subtraction of negative numbers have fewer real world examples that are meaningful to us. Math teachers have several different approaches, such as number lines, banking, temperatures, and so on, to make these concepts real.

We will model addition and subtraction of negatives with two color counters. We let a blue counter represent a positive and a red counter will represent a negative.



If we have one positive and one negative counter, the value of the pair is zero. They form a neutral pair. The value of this neutral pair is zero as summarized in Figure 3.17.

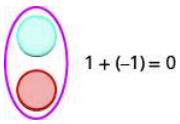


Figure 3.17 A blue counter represents $+1$. A red counter represents -1. Together they add to zero.

Manipulative Mathematics

Doing the Manipulative Mathematics activity "Addition of signed Numbers" will help you develop a better understanding of adding integers.

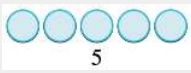
We will model four addition facts using the numbers $5, -5$ and $3, -3$.

$$5+3 \quad -5+(-3) \quad -5+3 \quad 5+(-3)$$

Example 3.14

Model: $5+3$.

Answer

Interpret the expression.	$5+3$ means the sum of 5 and 3.
Model the first number. Start with 5 positives.	
Model the second number. Add 3 positives.	
Count the total number of counters.	
The sum of 5 and 3 is 8.	$5+3=8$

Try It 3.27

Model the expression.

$2+4$

Try It 3.28

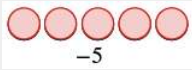
Model the expression.

$2+5$

Example 3.15

Model: $-5+(-3)$.

Answer

Interpret the expression.	$-5+(-3)$ means the sum of -5 and -3.
Model the first number. Start with 5 negatives.	
Model the second number. Add 3 negatives.	
Count the total number of counters.	
The sum of -5 and -3 is -8.	$-5+-3=-8$

Try It 3.29

Model the expression.

$-2+(-4)$

Try It 3.30

Model the expression.

$-2+(-5)$

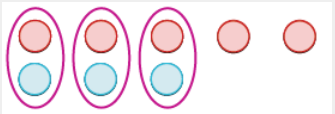
Example 3.14 and Example 3.15 are very similar. The first example adds 5 positives and 3 positives—both positives. The second example adds 5 negatives and 3 negatives—both negatives. In each case, we got a result of 8—either 8 positives or 8 negatives. When the signs are the same, the counters are all the same color.

Now let's see what happens when the signs are different.

Example 3.16

Model: $-5+3$.

Answer

Interpret the expression.	$-5+3$ means the sum of -5 and 3.
Model the first number. Start with 5 negatives.	
Model the second number. Add 3 positives.	
Remove any neutral pairs.	
Count the result.	 2 negatives
The sum of −5 and 3 is −2.	$-5+3=-2$

Notice that there were more negatives than positives, so the result is negative.

Try It 3.31

Model the expression, and then simplify:

$2+(-4)$

Try It 3.32

Model the expression, and then simplify:

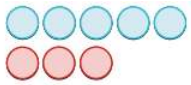
$2+(-5)$

Example 3.17

Model: $5+(-3)$.

Answer

Interpret the expression.	$5+(-3)$ means the sum of 5 and -3.
Model the first number. Start with 5 positives.	

Model the second number. Add 3 negatives.	
Remove any neutral pairs.	
Count the result.	 2 positives
The sum of 5 and −3 is 2.	$5+(-3)=2$

Try It 3.33

Model the expression, and then simplify:

$(-2)+4$

Try It 3.34

Model the expression:

$(-2)+5$

Example 3.18

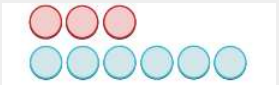
Modeling Addition of Positive and Negative Integers

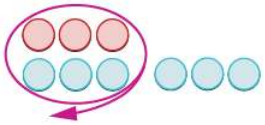
Model each addition.

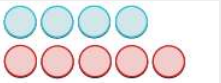
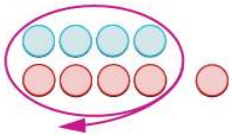
1. ⓐ 4 + 2
2. ⓑ −3 + 6
3. ⓒ 4 + (−5)
4. ⓓ -2 + (−3)

Answer

ⓐ	
	$4+2$
Start with 4 positives.	
Add two positives.	
How many do you have?	$4+2=6$

ⓑ	
	$-3+6$
Start with 3 negatives.	
Add 6 positives.	

Remove neutral pairs.	
How many are left?	
	3. $-3+6=3$

ⓒ	
	$4+(-5)$
Start with 4 positives.	
Add 5 negatives.	
Remove neutral pairs.	
How many are left?	
	-1. $4+(-5)=-1$

ⓓ	
	$-2+(-3)$
Start with 2 negatives.	
Add 3 negatives.	
How many do you have?	-5. $-2+(-3)=-5$

Try It 3.35

Model each addition.

1. ⓐ 3 + 4
2. ⓑ −1 + 4
3. ⓒ 4 + (−6)
4. ⓓ −2 + (−2)

Try It 3.36

1. ⓐ 5 + 1
2. ⓑ −3 + 7
3. ⓒ 2 + (−8)
4. ⓓ −3 + (−4)

Simplify Expressions with Integers

Now that you have modeled adding small positive and negative integers, you can visualize the model in your mind to simplify expressions with any integers.

For example, if you want to add $37+(-53)$, you don't have to count out 37 blue counters and 53 red counters.

Picture 37 blue counters with 53 red counters lined up underneath. Since there would be more negative counters than positive counters, the sum would be negative. Because $53-37=16$, there are 16 more negative counters.

$$37+(-53)=-16$$

Let's try another one. We'll add $-74+(-27)$. Imagine 74 red counters and 27 more red counters, so we have 101 red counters all together. This means the sum is -101.

$$-74+(-27)=-101$$

Look again at the results of Example 3.14 - Example 3.17.

$5+3$	$-5+(-3)$
both positive, sum positive	both negative, sum negative
When the signs are the same, the counters would be all the same color, so add them.	
$-5+3$	$5+(-3)$
different signs, more negatives	different signs, more positives
Sum negative	sum positive
When the signs are different, some counters would make neutral pairs; subtract to see how many are left.	

Table 3.1 Addition of Positive and Negative Integers

Example 3.19

Simplify:

1. ⓐ $19+(-47)$
2. ⓑ $-32+40$

Answer

ⓐ Since the signs are different, we subtract 19 from 47. The answer will be negative because there are more negatives than positives.

$$19+(-47)$$
$$-28$$

ⓑ The signs are different so we subtract 32 from 40. The answer will be positive because there are more positives than negatives

$$-32+40$$
$$8$$

Try It 3.37

Simplify each expression:

1. ⓐ $15+(-32)$
2. ⓑ $-19+76$

Try It 3.38

Simplify each expression:

1. ⓐ $-55+9$
2. ⓑ $43+(-17)$

Example 3.20

Simplify: $-14+(-36)$.

Answer

Since the signs are the same, we add. The answer will be negative because there are only negatives.

$$-14+(-36)$$
$$-50$$

Try It 3.39

Simplify the expression:

$-31+(-19)$

Try It 3.40

Simplify the expression:

$-42+(-28)$

The techniques we have used up to now extend to more complicated expressions. Remember to follow the order of operations.

Example 3.21

Simplify: $-5+3(-2+7)$.

Answer

	$-5+3\,(-2+7)$
Simplify inside the parentheses.	$-5+3\,(5)$
Multiply.	$-5+15$
Add left to right.	10

Try It 3.41

Simplify the expression:

$-2+5(-4+7)$

Try It 3.42

Simplify the expression:

$-4+2(-3+5)$

Evaluate Variable Expressions with Integers

Remember that to evaluate an expression means to substitute a number for the variable in the expression. Now we can use negative numbers as well as positive numbers when evaluating expressions.

Example 3.22

Evaluate $x + 7$ when

1. ⓐ $x = -2$
2. ⓑ $x = -11$.

Answer

ⓐ Evaluate $x + 7$ when $x = -2$	
	$-2 + 7$
Substitute -2 for x.	$-2 + 7$
Simplify.	5

ⓑ Evaluate $x + 7$ when $x = -11$	
	$x + 7$
Substitute -11 for x.	$-11 + 7$
Simplify.	-4

Try It 3.43

Evaluate each expression for the given values:

$x + 5$ when

1. ⓐ $x = -3$ and
2. ⓑ $x = -17$

Try It 3.44

Evaluate each expression for the given values: $y + 7$ when

1. ⓐ $y = -5$
2. ⓑ $y = -8$

Example 3.23

When $n = -5$, evaluate

1. ⓐ $n + 1$
2. ⓑ $-n + 1$.

Answer

ⓐ Evaluate $n + 1$ when $n = -5$	
	$n + 1$
Substitute -5 for n.	$-5 + 1$
Simplify.	-4

ⓑ Evaluate $-n + 1$ when $n = -5$	
	$-n + 1$
Substitute -5 for n.	$-(-5) + 1$

Simplify.	$5+1$
Add.	6

Try It 3.45

When $n=-8$, evaluate

1. ⓐ $n+2$
2. ⓑ $-n+2$

Try It 3.46

When $y=-9$, evaluate

1. ⓐ $y+8$
2. ⓑ $-y+8$.

Next we'll evaluate an expression with two variables.

Example 3.24

Evaluate $3a+b$ when $a=12$ and $b=-30$.

Answer

	$3a+b$
Substitute 12 for a and -30 for b.	$3(12)+(-30)$
Multiply.	$36+(-30)$
Add.	6

Try It 3.47

Evaluate the expression:

$a+2b$ when $a=-19$ and $b=14$.

Try It 3.48

Evaluate the expression:

$5p+q$ when $p=4$ and $q=-7$.

Example 3.25

Evaluate $(x+y)^2$ when $x=-18$ and $y=24$.

Answer

This expression has two variables. Substitute -18 for x and 24 for y.

	$(x+y)^2$
Substitute -18 for x and 24 for y.	$(-18+24)^2$
Add inside the parentheses.	$(6)^2$
Simplify	36

Try It 3.49

Evaluate:

$(x+y)^2$ when $x=-15$ and $y=29$.

Try It 3.50

Evaluate:

$(x+y)^3$ when $x=-8$ and $y=10$.

Translate Word Phrases to Algebraic Expressions

All our earlier work translating word phrases to algebra also applies to expressions that include both positive and negative numbers. Remember that the phrase *the sum* indicates addition.

Example 3.26

Translate and simplify: the sum of -9 and 5.

Answer

The sum of -9 and 5 indicates addition.	the sum of -9 and 5
Translate.	$-9+5$
Simplify.	-4

Try It 3.51

Translate and simplify the expression:

the sum of -7 and 4

Try It 3.52

Translate and simplify the expression:

the sum of -8 and -6

Example 3.27

Translate and simplify: the sum of 8 and -12, increased by 3.

Answer

The phrase *increased by* indicates addition.

	The sum of 8 and -12, increased by 3
Translate.	$[8+(-12)]+3$
Simplify.	$-4+3$
Add.	-1

Try It 3.53

Translate and simplify:

the sum of 9 and -16, increased by 4.

Try It 3.54

Translate and simplify:

the sum of -8 and -12, increased by 7.

Add Integers in Applications

Recall that we were introduced to some situations in everyday life that use positive and negative numbers, such as temperatures, banking, and sports. For example, a debt of $\$5$ could be represented as $-\$5$. Let's practice translating and solving a few applications.

Solving applications is easy if we have a plan. First, we determine what we are looking for. Then we write a phrase that gives the information to find it. We translate the phrase into math notation and then simplify to get the answer. Finally, we write a sentence to answer the question.

Example 3.28

The temperature in Buffalo, NY, one morning started at 7 degrees below zero Fahrenheit. By noon, it had warmed up 12 degrees. What was the temperature at noon?

Answer

We are asked to find the temperature at noon.

Write a phrase for the temperature.	The temperature warmed up 12 degrees from 7 degrees below zero.
Translate to math notation.	$-7 + 12$
Simplify.	5
Write a sentence to answer the question.	The temperature at noon was 5 degrees Fahrenheit.

Table 3.2

Try It 3.55

The temperature in Chicago at 5 A.M. was 10 degrees below zero Celsius. Six hours later, it had warmed up 14 degrees Celsius. What is the temperature at 11 A.M.?

Try It 3.56

A scuba diver was swimming 16 feet below the surface and then dove down another 17 feet. What is her new depth?

Example 3.29

A football team took possession of the football on their 42-yard line. In the next three plays, they lost 6 yards, gained 4 yards, and then lost 8 yards. On what yard line was the ball at the end of those three plays?

Answer

We are asked to find the yard line the ball was on at the end of three plays.

Write a word phrase for the position of the ball.	Start at 42, then lose 6, gain 4, lose 8.
Translate to math notation.	$42 - 6 + 4 - 8$
Simplify.	32
Write a sentence to answer the question.	At the end of the three plays, the ball is on the 32-yard line.

Table 3.3

Try It 3.57

The Bears took possession of the football on their 20-yard line. In the next three plays, they lost 9 yards, gained 7 yards, then lost 4 yards. On what yard line was the ball at the end of those three plays?

Try It 3.58

The Chargers began with the football on their 25-yard line. They gained 5 yards, lost 8 yards and then gained 15 yards on the next three plays. Where was the ball at the end of these plays?

Media

ACCESS ADDITIONAL ONLINE RESOURCES

- Adding Integers with Same Sign Using Color Counters
- Adding Integers with Different Signs Using Counters
- Ex1: Adding Integers
- Ex2: Adding Integers

Section 3.2 Exercises

Practice Makes Perfect

Model Addition of Integers

In the following exercises, model the expression to simplify.

63.

$7+4$

64.

$8+5$

65.

$-6+(-3)$

66.

$-5+(-5)$

67.

$-7+5$

68.

$-9+6$

69.

$8+(-7)$

70.

$9+(-4)$

Simplify Expressions with Integers

In the following exercises, simplify each expression.

71.

$-21+(-59)$

72.

$-35 + (-47)$

73.

$48 + (-16)$

74.

$34 + (-19)$

75.

$-200 + 65$

76.

$-150 + 45$

77.

$2 + (-8) + 6$

78.

$4 + (-9) + 7$

79.

$-14 + (-12) + 4$

80.

$-17 + (-18) + 6$

81.

$135 + (-110) + 83$

82.

$140 + (-75) + 67$

83.

$-32 + 24 + (-6) + 10$

84.

$-38 + 27 + (-8) + 12$

85.

$19 + 2(-3 + 8)$

86.

$24 + 3(-5 + 9)$

Evaluate Variable Expressions with Integers

In the following exercises, evaluate each expression.

87.

$x + 8$ when

1. ⓐ $x = -26$
2. ⓑ $x = -95$

88.

$y + 9$ when

1. ⓐ $y = -29$
2. ⓑ $y = -84$

89.

$y + (-14)$ when

1. ⓐ $y = -33$
2. ⓑ $y = 30$

90.

$x + (-21)$ when

1. ⓐ $x = -27$
2. ⓑ $x = 44$

91.

When $a = -7$, evaluate:

1. ⓐ $a + 3$
2. ⓑ $-a + 3$

92.

When $b = -11$, evaluate:

1. ⓐ $b + 6$
2. ⓑ $-b + 6$

93.

When $c = -9$, evaluate:

1. ⓐ $c + (-4)$
2. ⓑ $-c + (-4)$

94.

When $d = -8$, evaluate:

1. ⓐ $d + (-9)$
2. ⓑ $-d + (-9)$

95.

$m + n$ when, $m = -15$, $n = 7$

96.

$p + q$ when, $p = -9$, $q = 17$

97.

$r - 3s$ when, $r = 16$, $s = 2$

98.

$2t + u$ when, $t = -6$, $u = -5$

99.

$(a + b)^2$ when, $a = -7$, $b = 15$

100.

$(c + d)^2$ when, $c = -5$, $d = 14$

101.

$(x + y)^2$ when, $x = -3$, $y = 14$

102.

$(y + z)^2$ when, $y = -3$, $z = 15$

Translate Word Phrases to Algebraic Expressions

In the following exercises, translate each phrase into an algebraic expression and then simplify.

103.

The sum of -14 and 5

104.

The sum of -22 and 9

105.

8 more than -2

106.

5 more than -1

107.

-10 added to -15

108.

-6 added to -20

109.

6 more than the sum of -1 and -12

110.

3 more than the sum of -2 and -8

111.

the sum of 10 and -19, increased by 4

112.

the sum of 12 and -15, increased by 1

Add Integers in Applications

In the following exercises, solve.

113.

Temperature The temperature in St. Paul, Minnesota was $-19°\text{F}$ at sunrise. By noon the temperature had risen $26°\text{F}$. What was the temperature at noon?

114.

Temperature The temperature in Chicago was $-15°\text{F}$ at 6 am. By afternoon the temperature had risen $28°\text{F}$. What was the afternoon temperature?

115.

Credit Cards Lupe owes $\$73$ on her credit card. Then she charges $\$45$ more. What is the new balance?

116.

Credit Cards Frank owes $\$212$ on his credit card. Then he charges $\$105$ more. What is the new balance?

117.

Football A team lost 3 yards the first play. Then they lost 2 yards, gained 1 yard, and then lost 4 yards. What was the change in overall yardage over the four plays?

118.

Card Games April lost 5 cards the first turn. Over the next three turns, she lost 3 cards, gained 2 cards, and then lost 1 card. What was the change in cards over the four turns?

119.

Football The Rams took possession of the football on their own 35-yard line. In the next three plays, they lost 12 yards, gained 8 yards, then lost 6 yards. On what yard line was the ball at the end of those three plays?

120.

Football The Cowboys began with the ball on their own 20-yard line. They gained 15 yards, lost 3 yards and then gained 6 yards on the next three plays. Where was the ball at the end of these plays?

121.

Scuba Diving A scuba diver swimming 8 feet below the surface dove 17 feet deeper; the pressure got to them and they rose five feet. What is their new depth?

122.

Gas Consumption: Ozzie rode their motorcycle for 30 minutes, using 168 fluid ounces of gas. Then they stopped and got 140-fluid ounces of gas. Represent the change in gas amount as an integer.

Everyday Math

123.

Stock Market The week of September 15, 2008, was one of the most volatile weeks ever for the U.S. stock market. The change in the Dow Jones Industrial Average each day was:

Monday -504 Tuesday $+142$ Wednesday -449
Thursday $+410$ Friday $+369$

What was the overall change for the week?

124.

Stock Market During the week of June 22, 2009, the change in the Dow Jones Industrial Average each day was:

Monday -201 Tuesday -16 Wednesday -23
Thursday $+172$ Friday -34

What was the overall change for the week?

Writing Exercises

125.

Explain why the sum of -8 and 2 is negative, but the sum of 8 and -2 and is positive.

126.

Give an example from your life experience of adding two negative numbers.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
model addition of integers.			
simplify expressions with integers.			
evaluate variable expressions with integers.			
translate word phrases to algebraic expressions.			
add integers in applications.			

ⓑ After reviewing this checklist, what will you do to become confident for all objectives?

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3.3: Subtract Integers

Learning Objectives

By the end of this section, you will be able to:

- Model subtraction of integers
- Simplify expressions with integers
- Evaluate variable expressions with integers
- Translate words phrases to algebraic expressions
- Subtract integers in applications

Be Prepared 3.4

Before you get started, take this readiness quiz.

Simplify: $12-(8-1)$.
If you missed this problem, review Example 2.8.

Be Prepared 3.5

Translate *the difference of* 20 *and* -15 into an algebraic expression.
If you missed this problem, review Example 1.36.

Be Prepared 3.6

Add: $-18+7$.
If you missed this problem, review Example 3.20.

Model Subtraction of Integers

Remember the story in the last section about the toddler and the cookies? Children learn how to subtract numbers through their everyday experiences. Real-life experiences serve as models for subtracting positive numbers, and in some cases, such as temperature, for adding negative as well as positive numbers. But it is difficult to relate subtracting negative numbers to common life experiences. Most people do not have an intuitive understanding of subtraction when negative numbers are involved. Math teachers use several different models to explain subtracting negative numbers.

We will continue to use counters to model subtraction. Remember, the blue counters represent positive numbers and the red counters represent negative numbers.

Perhaps when you were younger, you read $5-3$ as *five take away three*. When we use counters, we can think of subtraction the same way.

Manipulative Mathematics

Doing the Manipulative Mathematics activity "Subtraction of Signed Numbers" will help you develop a better understanding of subtracting integers.

We will model four subtraction facts using the numbers 5 and 3.

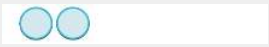
$$5-3 \quad -5-(-3) \quad -5-3 \quad 5-(-3)$$

Example 3.30

Model: $5-3$.

Answer

Interpret the expression.	$5-3$ means 5 take away 3.

Model the first number. Start with 5 positives.	
Take away the second number. So take away 3 positives.	
Find the counters that are left.	
	$5-3=2$. The difference between 5 and 3 is 2.

Try It 3.59

Model the expression:

$6-4$

Try It 3.60

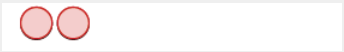
Model the expression:

$7-4$

Example 3.31

Model: $-5-(-3)$.

Answer

Interpret the expression.	$-5-(-3)$ means -5 take away -3.
Model the first number. Start with 5 negatives.	
Take away the second number. So take away 3 negatives.	
Find the number of counters that are left.	
	$-5-(-3)=-2$. The difference between -5 and -3 is -2.

Try It 3.61

Model the expression:

$-6-(-4)$

Try It 3.62

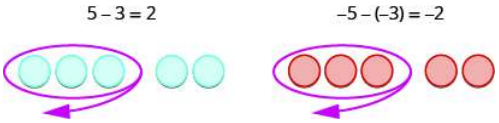
Model the expression:

$-7-(-4)$

Notice that Example 3.30 and Example 3.31 are very much alike.

- First, we subtracted 3 positives from 5 positives to get 2 positives.
- Then we subtracted 3 negatives from 5 negatives to get 2 negatives.

Each example used counters of only one color, and the “take away” model of subtraction was easy to apply.

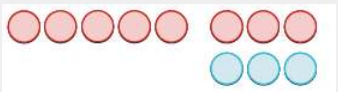
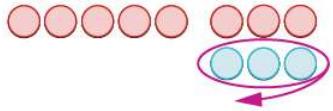


Now let’s see what happens when we subtract one positive and one negative number. We will need to use both positive and negative counters and sometimes some neutral pairs, too. Adding a neutral pair does not change the value.

Example 3.32

Model: $-5-3$.

Answer

Interpret the expression.	$-5-3$ means -5 take away 3.
Model the first number. Start with 5 negatives.	
Take away the second number. So we need to take away 3 positives.	
But there are no positives to take away. Add neutral pairs until you have 3 positives.	
Now take away 3 positives.	
Count the number of counters that are left.	
	$-5-3=-8$. The difference of -5 and 3 is -8.

Try It 3.63

Model the expression:

$-6-4$

Try It 3.64

Model the expression:

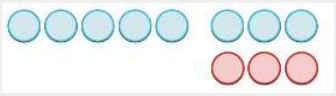
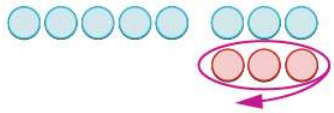
$-7-4$

Example 3.33

Model: $5-(-3)$.

Answer

Interpret the expression.	$5-(-3)$ means 5 take away -3.
Model the first number. Start with 5 positives.	
Take away the second number, so take away 3 negatives.	

But there are no negatives to take away. Add neutral pairs until you have 3 negatives.	
Then take away 3 negatives.	
Count the number of counters that are left.	 8 positives
	The difference of 5 and -3 is 8. $5-(-3)=8$

Try It 3.65

Model the expression:

$6-(-4)$

Try It 3.66

Model the expression:

$7-(-4)$

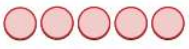
Example 3.34

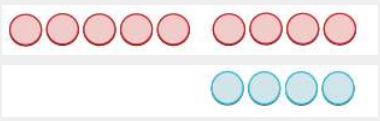
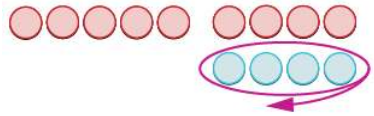
Model each subtraction.

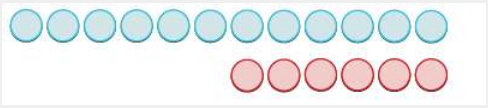
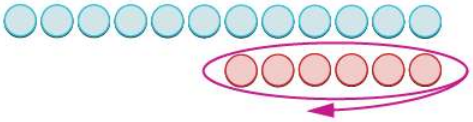
1. ⓐ $8-2$
2. ⓑ $-5-4$
3. ⓒ $6-(-6)$
4. ⓓ $-8-(-3)$

Answer

ⓐ	
	$8-2$ This means 8 take away 2.
Start with 8 positives.	
Take away 2 positives.	
How many are left?	6
	$8-2=6$

ⓑ	
	$-5-4$ This means -5 take away 4.
Start with 5 negatives.	

You need to take away 4 positives. Add 4 neutral pairs to get 4 positives.	
Take away 4 positives.	
How many are left?	
	-9
	$-5-4=-9$

ⓒ	
	$6-(-6)$ This means 6 take away -6.
Start with 6 positives.	
Add 6 neutrals to get 6 negatives to take away.	
Remove 6 negatives.	
How many are left?	
	12
	$6-(-6)=12$

ⓓ	
	$-8-(-3)$ This means -8 take away -3.
Start with 8 negatives.	
Take away 3 negatives.	
How many are left?	
	-5
	$-8-(-3)=-5$

Try It 3.67

Model each subtraction.

1. ⓐ 7 - (-8)
2. ⓑ -7 - (-2)
3. ⓒ 4 - 1
4. ⓓ -6 - 8

Try It 3.68

Model each subtraction.

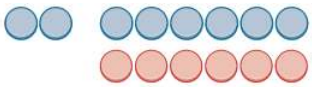
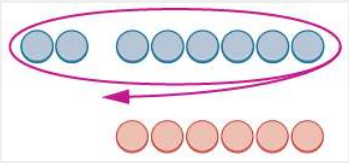
1. ⓐ 4 - (-6)
2. ⓑ -8 - (-1)
3. ⓒ 7 - 3
4. ⓓ -4 - 2

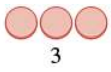
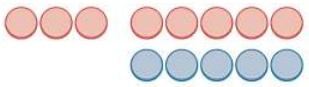
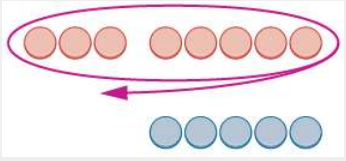
Example 3.35

Model each subtraction expression:

1. ⓐ $2-8$
2. ⓑ $-3-(-8)$

Answer

ⓐ We start with 2 positives.	
We need to take away 8 positives, but we have only 2.	
Add neutral pairs until there are 8 positives to take away.	
Then take away eight positives.	
Find the number of counters that are left. There are 6 negatives.	
	$2-8=-6$

ⓑ We start with 3 negatives.	
We need to take away 8 negatives, but we have only 3.	
Add neutral pairs until there are 8 negatives to take away.	
Then take away the 8 negatives.	

Find the number of counters that are left. There are 5 positives.	 5 positives
	$-3-(-8)=5$

Try It 3.69

Model each subtraction expression.

1. ⓐ $7-9$
2. ⓑ $-5-(-9)$

Try It 3.70

Model each subtraction expression.

1. ⓐ $4-7$
2. ⓑ $-7-(-10)$

Simplify Expressions with Integers

Do you see a pattern? Are you ready to subtract integers without counters? Let's do two more subtractions. We'll think about how we would model these with counters, but we won't actually use the counters.

- Subtract $-23-7$.
  Think: We start with 23 negative counters.
  We have to subtract 7 positives, but there are no positives to take away.
  So we add 7 neutral pairs to get the 7 positives. Now we take away the 7 positives.
  So what's left? We have the original 23 negatives plus 7 more negatives from the neutral pair. The result is 30 negatives.

  $$-23-7=-30$$

  Notice, that to subtract 7, we added 7 negatives.
- Subtract $30-(-12)$.
  Think: We start with 30 positives.
  We have to subtract 12 negatives, but there are no negatives to take away.
  So we add 12 neutral pairs to the 30 positives. Now we take away the 12 negatives.
  What's left? We have the original 30 positives plus 12 more positives from the neutral pairs. The result is 42 positives.

  $$30-(-12)=42$$

  Notice that to subtract -12, we added 12.

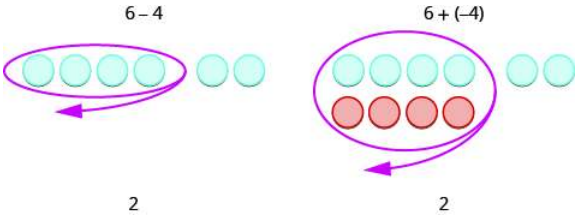
While we may not always use the counters, especially when we work with large numbers, practicing with them first gave us a concrete way to apply the concept, so that we can visualize and remember how to do the subtraction without the counters.

Have you noticed that subtraction of signed numbers can be done by adding the opposite? You will often see the idea, the Subtraction Property, written as follows:

Subtraction Property

$$a-b=a+(-b)$$

Look at these two examples.



We see that $6-4$ gives the same answer as $6+(-4)$.

Of course, when we have a subtraction problem that has only positive numbers, like the first example, we just do the subtraction. We already knew how to subtract $6-4$ long ago. But knowing that $6-4$ gives the same answer as $6+(-4)$ helps when we are subtracting negative numbers.

Example 3.36

Simplify:

1. ⓐ $13-8$ and $13+(-8)$
2. ⓑ $-17-9$ and $-17+(-9)$

Answer

ⓐ	
	$13-8$ and $13+(-8)$
Subtract to simplify.	$13-8=5$
Add to simplify.	$13+(-8)=5$
Subtracting 8 from 13 is the same as adding −8 to 13.	

ⓑ	
	$-17-9$ and $-17+(-9)$
Subtract to simplify.	$-17-9=-26$
Add to simplify.	$-17+(-9)=-26$
Subtracting 9 from −17 is the same as adding −9 to −17.	

Try It 3.71

Simplify each expression:

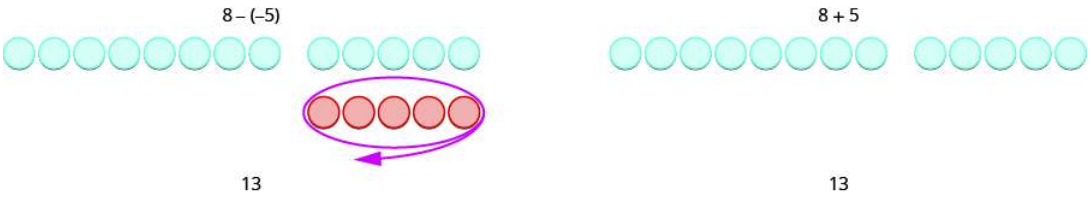
1. ⓐ $21-13$ and $21+(-13)$
2. ⓑ $-11-7$ and $-11+(-7)$

Try It 3.72

Simplify each expression:

1. ⓐ $15-7$ and $15+(-7)$
2. ⓑ $-14-8$ and $-14+(-8)$

Now look what happens when we subtract a negative.



We see that $8-(-5)$ gives the same result as $8+5$. Subtracting a negative number is like adding a positive.

Example 3.37

Simplify:

1. ⓐ $9-(-15)$ and $9+15$
2. ⓑ $-7-(-4)$ and $-7+4$

Answer

ⓐ	
	$9-(-15)$ and $9+15$
Subtract to simplify.	$9-(-15)=24$
Add to simplify.	$9+15=24$
Subtracting −15 from 9 is the same as adding 15 to 9.	

ⓑ	
	$-7-(-4)$ and $-7+4$
Subtract to simplify.	$-7-(-4)=-3$
Add to simplify.	$-7+4=-3$
Subtracting −4 from −7 is the same as adding 4 to −7	

Try It 3.73

Simplify each expression:

1. ⓐ $6-(-13)$ and $6+13$
2. ⓑ $-5-(-1)$ and $-5+1$

Try It 3.74

Simplify each expression:

1. ⓐ $4-(-19)$ and $4+19$
2. ⓐ $-4-(-7)$ and $-4+7$

Look again at the results of Example 3.30 - Example 3.33.

$5-3$	$-5-(-3)$
2	−2
2 positives	2 negatives
When there would be enough counters of the color to take away, subtract.	
$-5-3$	$5-(-3)$

-8	8
5 negatives, want to subtract 3 positives	5 positives, want to subtract 3 negatives
need neutral pairs	need neutral pairs
When there would not be enough of the counters to take away, add neutral pairs.	

Table 3.4 Subtraction of Integers

Example 3.38

Simplify: $-74-(-58)$.

Answer

We are taking 58 negatives away from 74 negatives.	$-74-(-58)$
Subtract.	-16

Try It 3.75

Simplify the expression:

$-67-(-38)$

Try It 3.76

Simplify the expression:

$-83-(-57)$

Example 3.39

Simplify: $7-(-4-3)-9$.

Answer

We use the order of operations to simplify this expression, performing operations inside the parentheses first. Then we subtract from left to right.

Simplify inside the parentheses first.	$7-(-4-3)-9$
Subtract from left to right.	$7-(-7)-9$
Subtract.	$14-9$
	5

Try It 3.77

Simplify the expression:

$8-(-3-1)-9$

Try It 3.78

Simplify the expression:

$12-(-9-6)-14$

Example 3.40

Simplify: $3 \cdot 7 - 4 \cdot 7 - 5 \cdot 8$.

Answer

We use the order of operations to simplify this expression. First we multiply, and then subtract from left to right.

Multiply first.	$3 \cdot 7 - 4 \cdot 7 - 5 \cdot 8$
Subtract from left to right.	$21 - 28 - 40$
Subtract.	$-7 - 40$
	-47

Try It 3.79

Simplify the expression:

$6 \cdot 2 - 9 \cdot 1 - 8 \cdot 9$.

Try It 3.80

Simplify the expression:

$2 \cdot 5 - 3 \cdot 7 - 4 \cdot 9$

Evaluate Variable Expressions with Integers

Now we'll practice evaluating expressions that involve subtracting negative numbers as well as positive numbers.

Example 3.41

Evaluate $x - 4$ when

1. ⓐ $x = 3$
2. ⓑ $x = -6$.

Answer

ⓐ To evaluate $x - 4$ when $x = 3$, substitute 3 for x in the expression.

	$x - 4$
Substitute 3 for x.	$3 - 4$
Subtract.	-1

ⓑ To evaluate $x - 4$ when $x = -6$, substitute -6 for x in the expression.

	$x - 4$
Substitute -6 for x.	$-6 - 4$
Subtract.	-10

Try It 3.81

Evaluate each expression:

$y - 7$ when

1. ⓐ $y = 5$

2. ⓑ $y = -8$

Try It 3.82

Evaluate each expression:

$m - 3$ when

1. ⓐ $m = 1$
2. ⓑ $m = -4$

Example 3.42

Evaluate $20 - z$ when

1. ⓐ $z = 12$
2. ⓑ $z = -12$

Answer

ⓐ To evaluate $20 - z$ when $z = 12$, substitute 12 for z in the expression.

	$20 - z$
Substitute 12 for z.	$20 - 12$
Subtract.	8

ⓑ To evaluate $20 - z$ when $z = -12$, substitute -12 for z in the expression.

	$20 - z$
Substitute -12 for z.	$20 - (-12)$
Subtract.	32

Try It 3.83

Evaluate each expression:

$17 - k$ when

1. ⓐ $k = 19$
2. ⓑ $k = -19$

Try It 3.84

Evaluate each expression:

$-5 - b$ when

1. ⓐ $b = 14$
2. ⓑ $b = -14$

Translate Word Phrases to Algebraic Expressions

When we first introduced the operation symbols, we saw that the expression $a - b$ may be read in several ways as shown below.

$a - b$
a minus b
the difference of a and b
subtract b from a
b subtracted from a
b less than a

Figure 3.18

Be careful to get a and b in the right order!

Example 3.43

Translate and then simplify:

1. ⓐ the difference of 13 and -21
2. ⓑ subtract 24 from -19

Answer

ⓐ A *difference* means subtraction. Subtract the numbers in the order they are given.

	the difference of 13 and -21
Translate.	$13-(-21)$
Simplify.	34

ⓑ *Subtract* means to take 24 away from -19.

	subtract 24 from -19
Translate.	$-19-24$
Simplify.	-43

Try It 3.85

Translate and simplify:

1. ⓐ the difference of 14 and -23
2. ⓑ subtract 21 from -17

Try It 3.86

Translate and simplify:

1. ⓐ the difference of 11 and -19
2. ⓑ subtract 18 from -11

Subtract Integers in Applications

It's hard to find something if we don't know what we're looking for or what to call it. So when we solve an application problem, we first need to determine what we are asked to find. Then we can write a phrase that gives the information to find it. We'll translate the phrase into an expression and then simplify the expression to get the answer. Finally, we summarize the answer in a sentence to make sure it makes sense.

How To

Solve Application Problems.

1. Step 1. Identify what you are asked to find.
2. Step 2. Write a phrase that gives the information to find it.
3. Step 3. Translate the phrase to an expression.
4. Step 4. Simplify the expression.
5. Step 5. Answer the question with a complete sentence.

Example 3.44

In the morning, the temperature in Urbana, Illinois was 11 degrees Fahrenheit. By mid-afternoon, the temperature had dropped to -9 degrees Fahrenheit. What was the difference between the morning and afternoon temperatures?

Answer

Step 1. Identify what we are asked to find.	the difference between the morning and afternoon temperatures
Step 2. Write a phrase that gives the information to find it.	the difference of 11 and -9
Step 3. Translate the phrase to an expression. The word *difference* indicates subtraction.	$11-(-9)$
Step 4. Simplify the expression.	20
Step 5. Write a complete sentence that answers the question.	The difference in temperature was 20 degrees Fahrenheit.

Try It 3.87

In the morning, the temperature in Anchorage, Alaska was 15 degrees Fahrenheit. By mid-afternoon the temperature had dropped to 30 degrees below zero. What was the difference between the morning and afternoon temperatures?

Try It 3.88

The temperature in Denver was -6 degrees Fahrenheit at lunchtime. By sunset the temperature had dropped to -15 degree Fahrenheit. What was the difference between the lunchtime and sunset temperatures?

Geography provides another application of negative numbers with the elevations of places below sea level.

Example 3.45

Dinesh hiked from Mt. Whitney, the highest point in California, to Death Valley, the lowest point. The elevation of Mt. Whitney is $14{,}497$ feet above sea level and the elevation of Death Valley is 282 feet below sea level. What is the difference in elevation between Mt. Whitney and Death Valley?

Answer

Step 1. What are we asked to find?	The difference in elevation between Mt. Whitney and Death Valley
Step 2. Write a phrase.	elevation of Mt. Whitney−elevation of Death Valley
Step 3. Translate.	$14{,}497-(-282)$
Step 4. Simplify.	$14{,}779$
Step 5. Write a complete sentence that answers the question.	The difference in elevation is $14{,}779$ feet.

Try It 3.89

One day, John hiked to the $10{,}023$ foot summit of Haleakala volcano in Hawaii. The next day, while scuba diving, he dove to a cave 80 feet below sea level. What is the difference between the elevation of the summit of Haleakala and the depth of the cave?

Try It 3.90

The submarine Nautilus is at 340 feet below the surface of the water and the submarine Explorer is 573 feet below the surface of the water. What is the difference in the position of the Nautilus and the Explorer?

Managing your money can involve both positive and negative numbers. You might have overdraft protection on your checking account. This means the bank lets you write checks for more money than you have in your account (as long as they know they can get it back from you!)

Example 3.46

Leslie has $\$25$ in her checking account and she writes a check for $\$8$.

1. ⓐ What is the balance after she writes the check?
2. ⓑ She writes a second check for $\$20$. What is the new balance after this check?
3. ⓒ Leslie's friend told her that she had lost a check for $\$10$ that Leslie had given her with her birthday card. What is the balance in Leslie's checking account now?

Answer

ⓐ	
What are we asked to find?	The balance of the account
Write a phrase.	$\$25$ minus $\$8$
Translate	$\$25 - \$8$
Simplify.	$\$17$
Write a sentence answer.	The balance is $\$17$.

ⓑ	
What are we asked to find?	The new balance
Write a phrase.	$\$17$ minus $\$20$
Translate	$\$17 - \$20$
Simplify.	$-\$3$
Write a sentence answer.	She is overdrawn by $\$3$.

ⓒ	
What are we asked to find?	The new balance
Write a phrase.	$\$10$ more than $-\$3$
Translate	$-\$3 + \$10$
Simplify.	$\$7$
Write a sentence answer.	The balance is now $\$7$.

Try It 3.91

Araceli has $\$75$ in her checking account and writes a check for $\$27$.

1. ⓐ What is the balance after she writes the check?
2. ⓑ She writes a second check for $\$50$. What is the new balance?
3. ⓒ The check for $\$20$ that she sent a charity was never cashed. What is the balance in Araceli's checking account now?

Try It 3.92

Genevieve's bank account was overdrawn and the balance is $-\$78$.

1. ⓐ She deposits a check for $\$24$ that she earned babysitting. What is the new balance?
2. ⓑ She deposits another check for $\$49$. Is she out of debt yet? What is her new balance?

Links To Literacy

The Links to Literacy activity "Elevator Magic" will provide you with another view of the topics covered in this section.

Media

ACCESS ADDITIONAL ONLINE RESOURCES

- Adding and Subtracting Integers
- Subtracting Integers with Color Counters
- Subtracting Integers Basics
- Subtracting Integers
- Integer Application

Section 3.3 Exercises

Practice Makes Perfect

Model Subtraction of Integers

In the following exercises, model each expression and simplify.

127.

$8-2$

128.

$9-3$

129.

$-5-(-1)$

130.

$-6-(-4)$

131.

$-5-4$

132.

$-7-2$

133.

$8-(-4)$

134.

$7-(-3)$

Simplify Expressions with Integers

In the following exercises, simplify each expression.

135.

1. ⓐ $15 - 6$
2. ⓑ $15 + (-6)$

136.

1. ⓐ $12 - 9$
2. ⓑ $12 + (-9)$

137.

1. ⓐ $44 - 28$
2. ⓑ $44 + (-28)$

138.

1. ⓐ $35 - 16$
2. ⓑ $35 + (-16)$

139.

1. ⓐ $8 - (-9)$
2. ⓑ $8 + 9$

140.

1. ⓐ $4 - (-4)$
2. ⓑ $4 + 4$

141.

1. ⓐ $27 - (-18)$
2. ⓑ $27 + 18$

142.

1. ⓐ $46 - (-37)$
2. ⓑ $46 + 37$

In the following exercises, simplify each expression.

143.

$15 - (-12)$

144.

$14 - (-11)$

145.

$10 - (-19)$

146.

$11 - (-18)$

147.

$48 - 87$

148.

$45 - 69$

149.

$31 - 79$

150.

$39 - 81$

151.

$-31-11$

152.

$-32-18$

153.

$-17-42$

154.

$-19-46$

155.

$-103-(-52)$

156.

$-105-(-68)$

157.

$-45-(-54)$

158.

$-58-(-67)$

159.

$8-3-7$

160.

$9-6-5$

161.

$-5-4+7$

162.

$-3-8+4$

163.

$-14-(-27)+9$

164.

$-15-(-28)+5$

165.

$71+(-10)-8$

166.

$64+(-17)-9$

167.

$-16-(-4+1)-7$

168.

$-15-(-6+4)-3$

169.

$(2-7)-(3-8)$

170.

$(1-8)-(2-9)$

171.

$-(6-8)-(2-4)$

172.

$-(4-5)-(7-8)$

173.

$25-[10-(3-12)]$

174.

$32-[5-(15-20)]$

175.

$6 \cdot 3-4 \cdot 3-7 \cdot 2$

176.

$5 \cdot 7-8 \cdot 2-4 \cdot 9$

177.

5^2-6^2

178.

6^2-7^2

Evaluate Variable Expressions with Integers

In the following exercises, evaluate each expression for the given values.

179.

$x-6$ when

1. ⓐ $x=3$
2. ⓑ $x=-3$

180.

$x-4$ when

1. ⓐ $x=5$
2. ⓑ $x=-5$

181.

$5-y$ when

1. ⓐ $y=2$
2. ⓑ $y=-2$

182.

$8-y$ when

1. ⓐ $y=3$
2. ⓑ $y=-3$

183.

$4x^2-15x+1$ when $x=3$

184.

$5x^2-14x+7$ when $x=2$

185.

$-12-5x^2$ when $x=6$

186.

$-19-4x^2$ when $x=5$

Translate Word Phrases to Algebraic Expressions

In the following exercises, translate each phrase into an algebraic expression and then simplify.

187.

1. ⓐ The difference of 3 and -10
2. ⓑ Subtract -20 from 45

188.

1. ⓐ The difference of 8 and -12
2. ⓑ Subtract -13 from 50

189.

1. ⓐ The difference of -6 and 9
2. ⓑ Subtract -12 from -16

190.

1. ⓐ The difference of -8 and 9
2. ⓑ Subtract -15 from -19

191.

1. ⓐ 8 less than -17
2. ⓑ -24 minus 37

192.

1. ⓐ 5 less than -14
2. ⓑ -13 minus 42

193.

1. ⓐ 21 less than 6
2. ⓑ 31 subtracted from -19

194.

1. ⓐ 34 less than 7
2. ⓑ 29 subtracted from -50

Subtract Integers in Applications

In the following exercises, solve the following applications.

195.

Temperature One morning, the temperature in Urbana, Illinois, was $28°$ Fahrenheit. By evening, the temperature had dropped $38°$ Fahrenheit. What was the temperature that evening?

196.

Temperature On Thursday, the temperature in Spincich Lake, Michigan, was $22°$ Fahrenheit. By Friday, the temperature had dropped $35°$ Fahrenheit. What was the temperature on Friday?

197.

Temperature On January 15, the high temperature in Anaheim, California, was $84°$ Fahrenheit. That same day, the high temperature in Embarrass, Minnesota was $-12°$ Fahrenheit. What was the difference between the temperature in Anaheim and the temperature in Embarrass?

198.

Temperature On January 21, the high temperature in Palm Springs, California, was $89°$, and the high temperature in Whitefield, New Hampshire was $-31°$. What was the difference between the temperature in Palm Springs and the temperature in Whitefield?

199.

Football At the first down, the Warriors football team had the ball on their 30-yard line. On the next three downs, they gained 2 yards, lost 7 yards, and lost 4 yards. What was the yard line at the end of the third down?

200.

Football At the first down, the Barons football team had the ball on their 20-yard line. On the next three downs, they lost 8 yards, gained 5 yards, and lost 6 yards. What was the yard line at the end of the third down?

201.

Checking Account John has $\$148$ in his checking account. He writes a check for $\$83$. What is the new balance in his checking account?

202.

Checking Account Ellie has $\$426$ in her checking account. She writes a check for $\$152$. What is the new balance in her checking account?

203.

Checking Account Gina has $\$210$ in her checking account. She writes a check for $\$250$. What is the new balance in her checking account?

204.

Checking Account Frank has $\$94$ in his checking account. He writes a check for $\$110$. What is the new balance in his checking account?

205.

Checking Account Bill has a balance of $-\$14$ in his checking account. He deposits $\$40$ to the account. What is the new balance?

206.

Checking Account Patty has a balance of $-\$23$ in her checking account. She deposits $\$80$ to the account. What is the new balance?

Everyday Math

207.

Camping Rene is on an Alpine hike. The temperature is $-7°$. Rene's sleeping bag is rated "comfortable to $-20°$". How much can the temperature change before it is too cold for Rene's sleeping bag?

208.

Scuba Diving Shelly's scuba watch is guaranteed to be watertight to -100 feet. She is diving at -45 feet on the face of an underwater canyon. By how many feet can she change her depth before her watch is no longer guaranteed?

Writing Exercises

209.

Explain why the difference of 9 and -6 is 15.

210.

Why is the result of subtracting $3-(-4)$ the same as the result of adding $3+4$?

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
model subtraction of integers.			
simplify expressions with integers.			
evaluate variable expressions with integers.			
translate word phrases to algebraic expressions.			
subtract integers in applications.			

ⓑ What does this checklist tell you about your mastery of this section? What steps will you take to improve?

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3.4: Multiply and Divide Integers

Learning Objectives

By the end of this section, you will be able to:

- Multiply integers
- Divide integers
- Simplify expressions with integers
- Evaluate variable expressions with integers
- Translate word phrases to algebraic expressions

Be Prepared 3.7

Before you get started, take this readiness quiz.

Translate the quotient of 20 and 13 into an algebraic expression.
If you missed this problem, review Example 1.67.

Be Prepared 3.8

Add: $-5+(-5)+(-5)$.
If you missed this problem, review Example 3.21.

Be Prepared 3.9

Evaluate $n+4$ when $n=-7$.
If you missed this problem, review Example 3.23.

Multiply Integers

Since multiplication is mathematical shorthand for repeated addition, our counter model can easily be applied to show multiplication of integers. Let's look at this concrete model to see what patterns we notice. We will use the same examples that we used for addition and subtraction.

We remember that Math input error

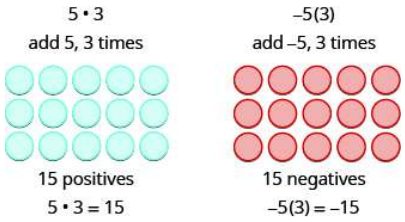


Figure 3.19

Now consider what it means to multiply Math input error

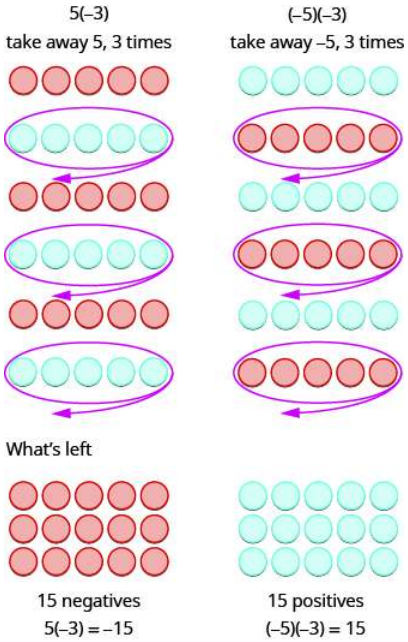


Figure 3.20

In both cases, we started with 15 neutral pairs. In the case on the left, we took away $5, 3$ times and the result was -15. To multiply $(-5)\,(-3)$, we took away $-5, 3$ times and the result was 15. So we found that

$$5 \cdot 3 = 15 \qquad -5(3) = -15$$
$$5(-3) = -15 \qquad (-5)(-3) = 15$$

Notice that for multiplication of two signed numbers, when the signs are the same, the product is positive, and when the signs are different, the product is negative.

Multiplication of Signed Numbers

The sign of the product of two numbers depends on their signs.

Same signs	Product
•Two positives •Two negatives	Positive Positive

Different signs	Product
•Positive • negative •Negative • positive	Negative Negative

Example 3.47

Multiply each of the following:

1. ⓐ $-9 \cdot 3$
2. ⓑ $-2(-5)$
3. ⓒ $4(-8)$
4. ⓓ $7 \cdot 6$

Answer

ⓐ

	$-9 \cdot 3$
Multiply, noting that the signs are different and so the product is negative.	-27

Table 3.5

ⓑ	
	$-2\,(-5)$
Multiply, noting that the signs are the same and so the product is positive.	10

Table 3.6

ⓒ	
	$4\,(-8)$
Multiply, noting that the signs are different and so the product is negative.	-32

Table 3.7

ⓓ	
	$7 \cdot 6$
The signs are the same, so the product is positive.	42

Table 3.8

Try It 3.93

Multiply:

1. ⓐ $-6 \cdot 8$
2. ⓑ $-4(-7)$
3. ⓒ $9(-7)$
4. ⓓ $5 \cdot 12$

Try It 3.94

Multiply:

1. ⓐ $-8 \cdot 7$
2. ⓑ $-6(-9)$
3. ⓒ $7(-4)$
4. ⓓ $3 \cdot 13$

When we multiply a number by 1, the result is the same number. What happens when we multiply a number by -1? Let's multiply a positive number and then a negative number by -1 to see what we get.

$$\begin{array}{ll} -1 \cdot 4 & -1(-3) \\ -4 & 3 \\ -4 \text{ is the opposite of } \mathbf{4} & \mathbf{3} \text{ is the opposite of } -3 \end{array}$$

Each time we multiply a number by -1, we get its opposite.

-1 Multiplication by

Multiplying a number by -1 gives its opposite.

$$-1a = -a$$

Example 3.48

Multiply each of the following:

1. ⓐ $-1 \cdot 7$
2. ⓑ $-1(-11)$

Answer

ⓐ	
The signs are different, so the product will be negative.	$-1 \cdot 7$
Notice that -7 is the opposite of 7.	-7

ⓑ	
The signs are the same, so the product will be positive.	$-1\,(-11)$
Notice that 11 is the opposite of -11.	11

Try It 3.95

Multiply.

1. ⓐ $-1 \cdot 9$
2. ⓑ $-1 \cdot (-17)$

Try It 3.96

Multiply.

1. ⓐ $-1 \cdot 8$
2. ⓑ $-1 \cdot (-16)$

Divide Integers

Division is the inverse operation of multiplication. So, $15 \div 3 = 5$ because $5 \cdot 3 = 15$ In words, this expression says that $\mathbf{15}$ can be divided into $\mathbf{3}$ groups of $\mathbf{5}$ each because adding five three times gives $\mathbf{15}$. If we look at some examples of multiplying integers, we might figure out the rules for dividing integers.

$$5 \cdot 3 = 15 \text{ so } 15 \div 3 = 5 \qquad -5(3) = -15 \text{ so } -15 \div 3 = -5$$
$$(-5)(-3) = 15 \text{ so } 15 \div (-3) = -5 \qquad 5(-3) = -15 \text{ so } -15 \div -3 = 5$$

Division of signed numbers follows the same rules as multiplication. When the signs are the same, the quotient is positive, and when the signs are different, the quotient is negative.

Division of Signed Numbers

The sign of the quotient of two numbers depends on their signs.

Same signs	Quotient
•Two positives •Two negatives	Positive Positive

Different signs	Quotient
•Positive & negative •Negative & positive	Negative Negative

Remember, you can always check the answer to a division problem by multiplying.

Example 3.49

Divide each of the following:

1. ⓐ $-27 \div 3$
2. ⓑ $-100 \div (-4)$

Answer

ⓐ	
	$-27 \div 3$
Divide, noting that the signs are different and so the quotient is negative.	-9

Table 3.9

ⓑ	
	$-100 \div (-4)$
Divide, noting that the signs are the same and so the quotient is positive.	25

Table 3.10

Try It 3.97

Divide:

1. ⓐ $-42 \div 6$
2. ⓑ $-117 \div (-3)$

Try It 3.98

Divide:

1. ⓐ $-63 \div 7$
2. ⓑ $-115 \div (-5)$

Just as we saw with multiplication, when we divide a number by 1, the result is the same number. What happens when we divide a number by -1? Let's divide a positive number and then a negative number by -1 to see what we get.

$8 \div (-1)$	$-9 \div (-1)$
-8	9
-8 is the opposite of 8	9 is the opposite of -9

When we divide a number by, -1 we get its opposite.

Division by -1

Dividing a number by -1 gives its opposite.

$$a \div (-1) = -a$$

Example 3.50

Divide each of the following:

1. ⓐ $16 \div (-1)$
2. ⓑ $-20 \div (-1)$

Answer

ⓐ	
	$16 \div (-1)$
The dividend, 16, is being divided by –1.	-16
Dividing a number by –1 gives its opposite.	
Notice that the signs were different, so the result was negative.	

Table 3.11

ⓑ	
	$-20 \div (-1)$
The dividend, –20, is being divided by –1.	20
Dividing a number by –1 gives its opposite.	

Table 3.12

Notice that the signs were the same, so the quotient was positive.

Try It 3.99

Divide:

1. ⓐ $6 \div (-1)$
2. ⓑ $-36 \div (-1)$

Try It 3.100

Divide:

1. ⓐ $28 \div (-1)$
2. ⓑ $-52 \div (-1)$

Simplify Expressions with Integers

Now we'll simplify expressions that use all four operations–addition, subtraction, multiplication, and division–with integers. Remember to follow the order of operations.

Example 3.51

Simplify: $7(-2) + 4(-7) - 6$.

Answer

We use the order of operations. Multiply first and then add and subtract from left to right.

	$7(-2)+4(-7)-6$
Multiply first.	$-14+(-28)-6$
Add.	$-42-6$
Subtract.	-48

Try It 3.101

Simplify:

$8(-3)+5(-7)-4$

Try It 3.102

Simplify:

$9(-3)+7(-8)-1$

Example 3.52

Simplify:

1. ⓐ $(-2)^4$
2. ⓑ -2^4

Answer

The exponent tells how many times to multiply the base.

ⓐ The exponent is 4 and the base is -2. We raise -2 to the fourth power.

	$(-2)^4$
Write in expanded form.	$(-2)(-2)(-2)(-2)$
Multiply.	$4(-2)(-2)$
Multiply.	$-8(-2)$
Multiply.	16

ⓑ The exponent is 4 and the base is 2. We raise 2 to the fourth power and then take the opposite.

	-2^4
Write in expanded form.	$-(2\cdot 2\cdot 2\cdot 2)$
Multiply.	$-(4\cdot 2\cdot 2)$
Multiply.	$-(8\cdot 2)$
Multiply.	-16

Try It 3.103

Simplify:

1. ⓐ $(-3)^4$
2. ⓑ -3^4

Try It 3.104

Simplify:

1. ⓐ $(-7)^2$
2. ⓑ -7^2

Example 3.53

Simplify: $12-3(9-12)$.

Answer

According to the order of operations, we simplify inside parentheses first. Then we will multiply and finally we will subtract.

	$12-3(9-12)$
Subtract the parentheses first.	$12-3(-3)$
Multiply.	$12-(-9)$
Subtract.	21

Try It 3.105

Simplify:

$17-4(8-11)$

Try It 3.106

Simplify:

$16-6(7-13)$

Example 3.54

Simplify: $8(-9)\div(-2)^3$.

Answer

We simplify the exponent first, then multiply and divide.

	$8(-9)\div(-2)^3$
Simplify the exponent.	$8(-9)\div(-8)$
Multiply.	$-72\div(-8)$
Divide.	9

Try It 3.107

Simplify:

$12(-9)\div(-3)^3$

Try It 3.108

Simplify:

$18(-4)\div(-2)^3$

Example 3.55

Simplify: $-30 \div 2 + (-3)(-7)$.

Answer

First we will multiply and divide from left to right. Then we will add.

	$-30 \div 2 + (-3)(-7)$
Divide.	$-15 + (-3)(-7)$
Multiply.	$-15 + 21$
Add.	6

Try It 3.109

Simplify:

$-27 \div 3 + (-5)(-6)$

Try It 3.110

Simplify:

$-32 \div 4 + (-2)(-7)$

Evaluate Variable Expressions with Integers

Now we can evaluate expressions that include multiplication and division with integers. Remember that to evaluate an expression, substitute the numbers in place of the variables, and then simplify.

Example 3.56

Evaluate $2x^2 - 3x + 8$ when $x = -4$.

Answer

	$2x^2 - 3x + 8$
Substitute -4 for x.	$2(-4)^2 - 3(-4) + 8$
Simplify exponents.	$2(16) - 3(-4) + 8$
Multiply.	$32 - (-12) + 8$
Subtract.	$44 + 8$
Add.	52

Keep in mind that when we substitute -4 for x, we use parentheses to show the multiplication. Without parentheses, it would look like $2 \cdot -4^2 - 3 \cdot -4 + 8$.

Try It 3.111

Evaluate:

$3x^2 - 2x + 6$ when $x = -3$

Try It 3.112

Evaluate:

$4x^2 - x - 5$ when $x = -2$

Example 3.57

Evaluate $3x + 4y - 6$ when $x = -1$ and $y = 2$.

Answer

	$3x + 4y - 6$
Substitute $x = -1$ and $y = 2$.	$3(-1) + 4(2) - 6$
Multiply.	$-3 + 8 - 6$
Simplify.	-1

Try It 3.113

Evaluate:

$7x + 6y - 12$ when $x = -2$ and $y = 3$

Try It 3.114

Evaluate:

$8x - 6y + 13$ when $x = -3$ and $y = -5$

Translate Word Phrases to Algebraic Expressions

Once again, all our prior work translating words to algebra transfers to phrases that include both multiplying and dividing integers. Remember that the key word for multiplication is *product* and for division is *quotient*.

Example 3.58

Translate to an algebraic expression and simplify if possible: the product of -2 and 14.

Answer

The word *product* tells us to multiply.

	the product of -2 and 14
Translate.	$(-2)(14)$
Simplify.	-28

Try It 3.115

Translate to an algebraic expression and simplify if possible:

the product of -5 and 12

Try It 3.116

Translate to an algebraic expression and simplify if possible:

the product of 8 and -13

Example 3.59

Translate to an algebraic expression and simplify if possible: the quotient of -56 and -7.

Answer

The word *quotient* tells us to divide.

	the quotient of −56 and −7
Translate.	$-56 \div (-7)$
Simplify.	8

Try It 3.117

Translate to an algebraic expression and simplify if possible:

the quotient of -63 and -9

Try It 3.118

Translate to an algebraic expression and simplify if possible:

the quotient of -72 and -9

Media

ACCESS ADDITIONAL ONLINE RESOURCES

- Multiplying Integers Using Color Counters
- Multiplying Integers Using Color Counters With Neutral Pairs
- Multiplying Integers Basics
- Dividing Integers Basics
- Ex. Dividing Integers
- Multiplying and Dividing Signed Numbers

Section 3.4 Exercises

Practice Makes Perfect

Multiply Integers

In the following exercises, multiply each pair of integers.

211.

$-4 \cdot 8$

212.

$-3 \cdot 9$

213.

$-5(7)$

214.

$-8(6)$

215.

$-18(-2)$

216.

$-10(-6)$

217.

$9(-7)$

218.

$13(-5)$

219.

$-1 \cdot 6$

220.

$-1 \cdot 3$

221.

$-1(-14)$

222.

$-1(-19)$

Divide Integers

In the following exercises, divide.

223.

$-24 \div 6$

224.

$-28 \div 7$

225.

$56 \div (-7)$

226.

$35 \div (-7)$

227.

$-52 \div (-4)$

228.

$-84 \div (-6)$

229.

$-180 \div 15$

230.

$-192 \div 12$

231.

$49 \div (-1)$

232.

$62 \div (-1)$

Simplify Expressions with Integers

In the following exercises, simplify each expression.

233.

$5(-6) + 7(-2) - 3$

234.

$8(-4) + 5(-4) - 6$

235.

$-8(-2)-3(-9)$

236.

$-7(-4)-5(-3)$

237.

$(-5)^3$

238.

$(-4)^3$

239.

$(-2)^6$

240.

$(-3)^5$

241.

-4^2

242.

-6^2

243.

$-3(-5)(6)$

244.

$-4(-6)(3)$

245.

$-4 \cdot 2 \cdot 11$

246.

$-5 \cdot 3 \cdot 10$

247.

$(8-11)(9-12)$

248.

$(6-11)(8-13)$

249.

$26-3(2-7)$

250.

$23-2(4-6)$

251.

$-10(-4) \div (-8)$

252.

$-8(-6) \div (-4)$

253.

$65 \div (-5)+(-28) \div (-7)$

254.

$52 \div (-4)+(-32) \div (-8)$

255.

$9 - 2[3 - 8(-2)]$

256.

$11 - 3[7 - 4(-2)]$

257.

$(-3)^2 - 24 \div (8 - 2)$

258.

$(-4)^2 - 32 \div (12 - 4)$

Evaluate Variable Expressions with Integers

In the following exercises, evaluate each expression.

259.

$-2x + 17$ when

1. ⓐ $x = 8$
2. ⓑ $x = -8$

260.

$-5y + 14$ when

1. ⓐ $y = 9$
2. ⓑ $y = -9$

261.

$10 - 3m$ when

1. ⓐ $m = 5$
2. ⓑ $m = -5$

262.

$18 - 4n$ when

1. ⓐ $n = 3$
2. ⓑ $n = -3$

263.

$p^2 - 5p + 5$ when $p = -1$

264.

$q^2 - 2q + 9$ when $q = -2$

265.

$2w^2 - 3w + 7$ when $w = -2$

266.

$3u^2 - 4u + 5$ when $u = -3$

267.

$6x - 5y + 15$ when $x = 3$ and $y = -1$

268.

$3p - 2q + 9$ when $p = 8$ and $q = -2$

269.

$9a - 2b - 8$ when $a = -6$ and $b = -3$

270.

$7m - 4n - 2$ when $m = -4$ and $n = -9$

Translate Word Phrases to Algebraic Expressions

In the following exercises, translate to an algebraic expression and simplify if possible.

271.

The product of -3 and 15

272.

The product of -4 and 16

273.

The quotient of -60 and -20

274.

The quotient of -40 and -20

275.

The quotient of -6 and the sum of a and b

276.

The quotient of -7 and the sum of m and n

277.

The product of -10 and the difference of p and q

278.

The product of -13 and the difference of c and d

Everyday Math

279.

Stock market Javier owns 300 shares of stock in one company. On Tuesday, the stock price dropped $\$12$ per share. What was the total effect on Javier's portfolio?

280.

Weight loss In the first week of a diet program, eight women lost an average of 3 pounds each. What was the total weight change for the eight women?

Writing Exercises

281.

In your own words, state the rules for multiplying two integers.

282.

In your own words, state the rules for dividing two integers.

283.

Why is $-2^4 \neq (-2)^4$?

284.

Why is $-4^2 \neq (-4)^2$?

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
multiply integers.			
divide integers.			
simplify expressions with integers.			
evaluate variable expressions with integers.			
translate word phrases to algebraic expressions.			

ⓑ On a scale of 1–10, how would you rate your mastery of this section in light of your responses on the checklist? How can you improve this?

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CHAPTER OVERVIEW

4: Fractions and Decimals

The sections in this module will help you to learn about fractions. Your goal is to demonstrate mastery on each of the following objectives.

- **Objective 4.1 - Representing Fractions and Mixed Numbers:** Represent fractions as simple fractions, improper fractions, and mixed numbers, identify the location of the fraction on a numberline, and calculate equivalent fractions.
- **Objective 4.2 - Multiplying and Dividing Fractions:** Perform addition, subtraction, multiplication, division, and simplification involving simple and improper fractions, including applications, without the use of a calculator.
- **Objective 4.3** - Writing, Ordering, and Rounding Decimals: Identify place value of decimals and use that information to round decimals and to identify the location of a decimal on a numberline.
- **Objective 4.4 - Operations with Decimals:** Perform addition, subtraction, multiplication, and division involving decimals, including applications, without the use of a calculator.
- **Objective 4.5 - Converting between Decimals and Fractions:** Convert fractions to decimals and terminating decimals to fractions.

4.1: Visualize Fractions
4.2: Multiply and Divide Fractions
4.3: Multiply and Divide Mixed Numbers and Complex Fractions
4.4: Add and Subtract Fractions with Common Denominators
4.5: Add and Subtract Fractions with Different Denominators
4.6: Add and Subtract Mixed Numbers
4.7: Decimals
4.8: Decimal Operations
4.9: Decimals and Fractions

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4.1: Visualize Fractions

Learning Objectives

By the end of this section, you will be able to:

- Understand the meaning of fractions
- Model improper fractions and mixed numbers
- Convert between improper fractions and mixed numbers
- Model equivalent fractions
- Find equivalent fractions
- Locate fractions and mixed numbers on the number line
- Order fractions and mixed numbers

Be Prepared 4.1

Before you get started, take this readiness quiz.

Simplify: $5 \cdot 2 + 1$.
If you missed this problem, review Example 2.8.

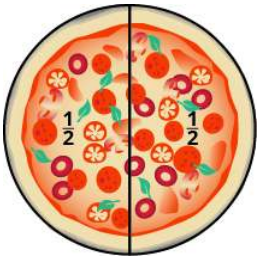
Be Prepared 4.2

Fill in the blank with $<$ or $>$: $-2 \_ -5$
If you missed this problem, review Example 3.2.

Understand the Meaning of Fractions

Andy and Bobby love pizza. On Monday night, they share a pizza equally. How much of the pizza does each one get? Are you thinking that each boy gets half of the pizza? That's right. There is one whole pizza, evenly divided into two parts, so each boy gets one of the two equal parts.

In math, we write $\frac{1}{2}$ to mean one out of two parts.



On Tuesday, Andy and Bobby share a pizza with their parents, Fred and Christy, with each person getting an equal amount of the whole pizza. How much of the pizza does each person get? There is one whole pizza, divided evenly into four equal parts. Each person has one of the four equal parts, so each has $\frac{1}{4}$ of the pizza.



On Wednesday, the family invites some friends over for a pizza dinner. There are a total of 12 people. If they share the pizza equally, each person would get $\frac{1}{12}$ of the pizza.



Fractions

A fraction is written $\frac{a}{b}$, where a and b are integers and $b \neq 0$. In a fraction, a is called the numerator and b is called the denominator.

A fraction is a way to represent parts of a whole. The denominator b represents the number of equal parts the whole has been divided into, and the numerator a represents how many parts are included. The denominator, b, cannot equal zero because division by zero is undefined.

In Figure 4.2, the circle has been divided into three parts of equal size. Each part represents $\frac{1}{3}$ of the circle. This type of model is called a fraction circle. Other shapes, such as rectangles, can also be used to model fractions.

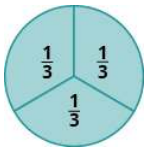
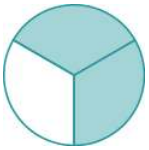


Figure 4.2

Manipulative Mathematics

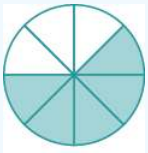
Doing the Manipulative Mathematics activity Model Fractions will help you develop a better understanding of fractions, their numerators and denominators.

What does the fraction $\frac{2}{3}$ represent? The fraction $\frac{2}{3}$ means two of three equal parts.

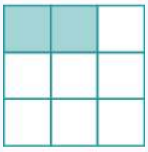


Example 4.1

Name the fraction of the shape that is shaded in each of the figures.



(a)



(b)

Answer

We need to ask two questions. First, how many equal parts are there? This will be the denominator. Second, of these equal parts, how many are shaded? This will be the numerator.

ⓐ

How many equal parts are there?	There are eight equal parts.
How many are shaded?	Five parts are shaded.

Five out of eight parts are shaded. Therefore, the fraction of the circle that is shaded is $\frac{5}{8}$.

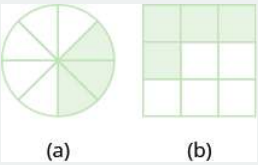
ⓑ

How many equal parts are there?	There are nine equal parts.
How many are shaded?	Two parts are shaded.

Two out of nine parts are shaded. Therefore, the fraction of the square that is shaded is $\frac{2}{9}$.

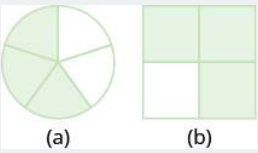
Try It 4.1

Name the fraction of the shape that is shaded in each figure:



Try It 4.2

Name the fraction of the shape that is shaded in each figure:



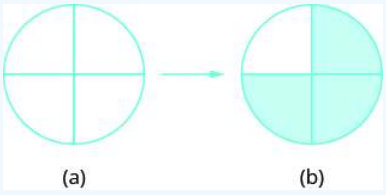
Example 4.2

Shade $\frac{3}{4}$ of the circle.

Answer

The denominator is 4, so we divide the circle into four equal parts ⓐ.

The numerator is 3, so we shade three of the four parts ⓑ.



$\frac{3}{4}$ of the circle is shaded.

Try It 4.3

Shade $\frac{6}{8}$ of the circle.

https://math.libretexts.org/@go/page/229546



Try It 4.4

Shade $\frac{2}{5}$ of the rectangle.

In Example 4.1 and Example 4.2, we used circles and rectangles to model fractions. Fractions can also be modeled as manipulatives called fraction tiles, as shown in Figure 4.3. Here, the whole is modeled as one long, undivided rectangular tile. Beneath it are tiles of equal length divided into different numbers of equally sized parts.

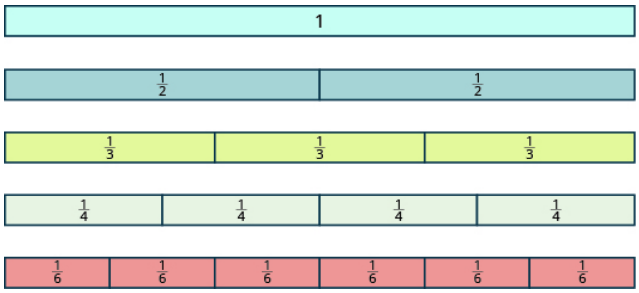


Figure 4.3

We'll be using fraction tiles to discover some basic facts about fractions. Refer to Figure 4.3 to answer the following questions:

How many $\frac{1}{2}$ tiles does it take to make one whole tile?	It takes two halves to make a whole, so two out of two is $\frac{2}{2}=1$.
How many $\frac{1}{3}$ tiles does it take to make one whole tile?	It takes three thirds, so three out of three is $\frac{3}{3}=1$.
How many $\frac{1}{4}$ tiles does it take to make one whole tile?	It takes four fourths, so four out of four is $\frac{4}{4}=1$.
How many $\frac{1}{6}$ tiles does it take to make one whole tile?	It takes six sixths, so six out of six is $\frac{6}{6}=1$.
What if the whole were divided into 24 equal parts? (We have not shown fraction tiles to represent this, but try to visualize it in your mind.) How many $\frac{1}{24}$ tiles does it take to make one whole tile?	It takes 24 twenty-fourths, so $\frac{24}{24}=1$.

It takes 24 twenty-fourths, so $\frac{24}{24}=1$.

This leads us to the *Property of One*.

Property of One

Any number, except zero, divided by itself is one.

$$\frac{a}{a}=1 \qquad (a\neq 0)$$

Manipulative Mathematics

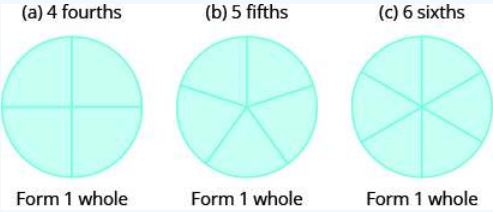
Example 4.3

Use fraction circles to make wholes using the following pieces:

1. ⓐ 4 fourths

2. ⓑ 5 fifths
3. ⓒ 6 sixths

Answer



Try It 4.5

Use fraction circles to make wholes with the following pieces: 3 thirds.

Try It 4.6

Use fraction circles to make wholes with the following pieces: 8 eighths.

What if we have more fraction pieces than we need for 1 whole? We'll look at this in the next example.

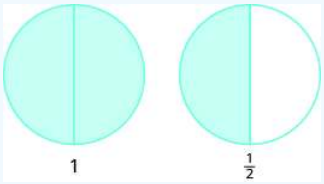
Example 4.4

Use fraction circles to make wholes using the following pieces:

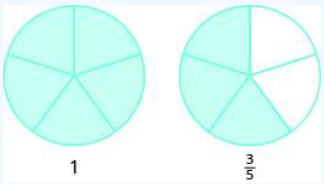
1. ⓐ 3 halves
2. ⓑ 8 fifths
3. ⓒ 7 thirds

Answer

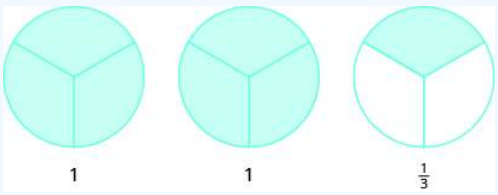
ⓐ 3 halves make 1 whole with 1 half left over.



ⓑ 8 fifths make 1 whole with 3 fifths left over.



ⓒ7 thirds make 2 wholes with 1 third left over.



Try It 4.7

Use fraction circles to make wholes with the following pieces: 5 thirds.

Try It 4.8

Use fraction circles to make wholes with the following pieces: 5 halves.

Model Improper Fractions and Mixed Numbers

In Example 4.4 (b), you had eight equal fifth pieces. You used five of them to make one whole, and you had three fifths left over. Let us use fraction notation to show what happened. You had eight pieces, each of them one fifth, $\frac{1}{5}$, so altogether you had eight fifths, which we can write as $\frac{8}{5}$. The fraction $\frac{8}{5}$ is one whole, 1, plus three fifths, $\frac{3}{5}$, or $1\frac{3}{5}$, which is read as *one and three-fifths*.

The number $1\frac{3}{5}$ is called a mixed number. A mixed number consists of a whole number and a fraction.

Mixed Numbers

A mixed number consists of a whole number a and a fraction $\frac{b}{c}$ where $c \neq 0$. It is written as follows.

$$a\frac{b}{c} \qquad c \neq 0$$

Fractions such as $\frac{5}{4}, \frac{3}{2}, \frac{5}{5}$, and $\frac{7}{3}$ are called improper fractions. In an improper fraction, the numerator is greater than or equal to the denominator, so its value is greater than or equal to one. When a fraction has a numerator that is smaller than the denominator, it is called a proper fraction, and its value is less than one. Fractions such as $\frac{1}{2}, \frac{3}{7}$, and $\frac{11}{18}$ are proper fractions.

Proper and Improper Fractions

The fraction $\frac{a}{b}$ is a proper fraction if $a < b$ and an improper fraction if $a \geq b$.

Manipulative Mathematics

Example 4.5

Name the improper fraction modeled. Then write the improper fraction as a mixed number.

Answer

Each circle is divided into three pieces, so each piece is $\frac{1}{3}$ of the circle. There are four pieces shaded, so there are four thirds or $\frac{4}{3}$. The figure shows that we also have one whole circle and one third, which is $1\frac{1}{3}$. So, $\frac{4}{3} = 1\frac{1}{3}$.

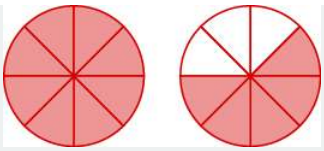
Try It 4.9

Name the improper fraction. Then write it as a mixed number.



Try It 4.10

Name the improper fraction. Then write it as a mixed number.

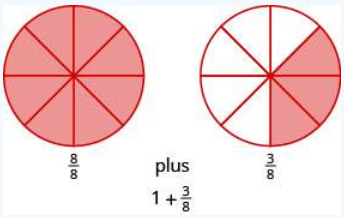


Example 4.6

Draw a figure to model $\frac{11}{8}$.

Answer

The denominator of the improper fraction is 8. Draw a circle divided into eight pieces and shade all of them. This takes care of eight eighths, but we have 11 eighths. We must shade three of the eight parts of another circle.



So, $\frac{11}{8}=1\frac{3}{8}$.

Try It 4.11

Draw a figure to model $\frac{7}{6}$.

Try It 4.12

Draw a figure to model $\frac{6}{5}$.

Example 4.7

Use a model to rewrite the improper fraction $\frac{11}{6}$ as a mixed number.

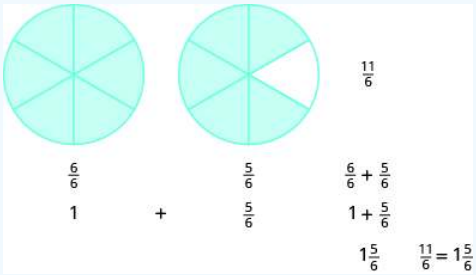
Answer

We start with 11 sixths $\left(\frac{11}{6}\right)$. We know that six sixths makes one whole.

$$\frac{6}{6}=1$$

That leaves us with five more sixths, which is $\frac{5}{6}$ $(11\text{ sixths minus }6\text{ sixths is }5\text{ sixths})$.

So, $\frac{11}{6}=1\frac{5}{6}$.



Try It 4.13

Use a model to rewrite the improper fraction as a mixed number: $\frac{9}{7}$.

Try It 4.14

Use a model to rewrite the improper fraction as a mixed number: $\frac{7}{4}$.

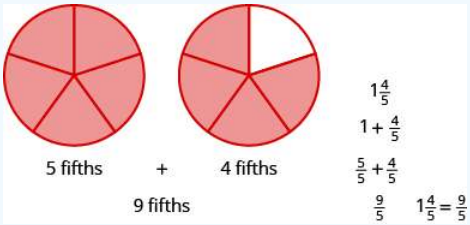
Example 4.8

Use a model to rewrite the mixed number $1\frac{4}{5}$ as an improper fraction.

Answer

The mixed number $1\frac{4}{5}$ means one whole plus four fifths. The denominator is 5, so the whole is $\frac{5}{5}$. Together five fifths and four fifths equals nine fifths.

So, $1\frac{4}{5} = \frac{9}{5}$.



Try It 4.15

Use a model to rewrite the mixed number as an improper fraction: $1\frac{3}{8}$.

Try It 4.16

Use a model to rewrite the mixed number as an improper fraction: $1\frac{5}{6}$.

Convert between Improper Fractions and Mixed Numbers

In Example 4.7, we converted the improper fraction $\frac{11}{6}$ to the mixed number $1\frac{5}{6}$ using fraction circles. We did this by grouping six sixths together to make a whole; then we looked to see how many of the 11 pieces were left. We saw that $\frac{11}{6}$ made one whole group of six sixths plus five more sixths, showing that $\frac{11}{6} = 1\frac{5}{6}$.

The division expression $\frac{11}{6}$ (which can also be written as $6\ 11$) tells us to find how many groups of 6 are in 11. To convert an improper fraction to a mixed number without fraction circles, we divide.

Example 4.9

Convert $\frac{11}{6}$ to a mixed number.

Answer

	$\frac{11}{6}$
Divide the denominator into the numerator.	Remember $\frac{11}{6}$ means $11 \div 6$.
	divisor → $6\overline{)11}$ ← quotient $\frac{6}{5}$ ← remainder
Identify the quotient, remainder and divisor.	
Write the mixed number as $\text{quotient}\ \frac{\text{remainder}}{\text{divisor}}$.	$1\frac{5}{6}$
So, $\frac{11}{6} = 1\frac{5}{6}$	

Try It 4.17

Convert the improper fraction to a mixed number: $\frac{13}{7}$.

Try It 4.18

Convert the improper fraction to a mixed number: $\frac{14}{9}$.

How To

Convert an improper fraction to a mixed number.

1. Step 1. Divide the denominator into the numerator.
2. Step 2. Identify the quotient, remainder, and divisor.
3. Step 3. Write the mixed number as quotient $\frac{\text{remainder}}{\text{divisor}}$.

Example 4.10

Convert the improper fraction $\frac{33}{8}$ to a mixed number.

Answer

	$\frac{33}{8}$
Divide the denominator into the numerator.	Remember, $\frac{33}{8}$ means $8\overline{)33}$.
Identify the quotient, remainder, and divisor.	$8\overline{)33}$ → divisor; 4 ← quotient; 32; 1 ← remainder
Write the mixed number as quotient $\frac{\text{remainder}}{\text{divisor}}$.	$4\frac{1}{8}$
	So, $\frac{33}{8} = 4\frac{1}{8}$

Try It 4.19

Convert the improper fraction to a mixed number: $\frac{23}{7}$.

Try It 4.20

Convert the improper fraction to a mixed number: $\frac{48}{11}$.

In Example 4.8, we changed $1\frac{4}{5}$ to an improper fraction by first seeing that the whole is a set of five fifths. So we had five fifths and four more fifths.

$$\frac{5}{5} + \frac{4}{5} = \frac{9}{5}$$

Where did the nine come from? There are nine fifths—one whole (five fifths) plus four fifths. Let us use this idea to see how to convert a mixed number to an improper fraction.

Example 4.11

Convert the mixed number $4\frac{2}{3}$ to an improper fraction.

Answer

	$4\frac{2}{3}$
Multiply the whole number by the denominator.	

The whole number is 4 and the denominator is 3.	$\frac{4 \cdot 3 + \square}{\square}$
Simplify.	$\frac{12 + \square}{\square}$
Add the numerator to the product.	
The numerator of the mixed number is 2.	$\frac{12 + 2}{\square}$
Simplify.	$\frac{14}{\square}$
Write the final sum over the original denominator.	
The denominator is 3.	$\frac{14}{3}$

Try It 4.21

Convert the mixed number to an improper fraction: $3\frac{5}{7}$.

Try It 4.22

Convert the mixed number to an improper fraction: $2\frac{7}{8}$.

How To

Convert a mixed number to an improper fraction.

1. Step 1. Multiply the whole number by the denominator.
2. Step 2. Add the numerator to the product found in Step 1.
3. Step 3. Write the final sum over the original denominator.

Example 4.12

Convert the mixed number $10\frac{2}{7}$ to an improper fraction.

Answer

	$10\frac{2}{7}$
Multiply the whole number by the denominator.	
The whole number is 10 and the denominator is 7.	$\frac{10 \cdot 7 + \square}{\square}$
Simplify.	$\frac{70 + \square}{\square}$
Add the numerator to the product.	
The numerator of the mixed number is 2.	$\frac{70 + 2}{\square}$
Simplify.	$\frac{72}{\square}$
Write the final sum over the original denominator.	
The denominator is 7.	$\frac{72}{7}$

Try It 4.23

Convert the mixed number to an improper fraction: $4\frac{6}{11}$.

Try It 4.24

Convert the mixed number to an improper fraction: $11\frac{1}{3}$.

Model Equivalent Fractions

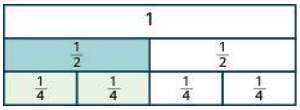
Let's think about Andy and Bobby and their favorite food again. If Andy eats $\frac{1}{2}$ of a pizza and Bobby eats $\frac{2}{4}$ of the pizza, have they eaten the same amount of pizza? In other words, does $\frac{1}{2} = \frac{2}{4}$? We can use fraction tiles to find out whether Andy and Bobby have eaten *equivalent* parts of the pizza.

Equivalent Fractions

Equivalent fractions are fractions that have the same value.

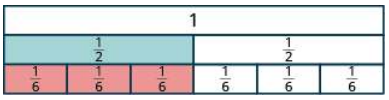
Fraction tiles serve as a useful model of equivalent fractions. You may want to use fraction tiles to do the following activity. Or you might make a copy of Figure 4.3 and extend it to include eighths, tenths, and twelfths.

Start with a $\frac{1}{2}$ tile. How many fourths equal one-half? How many of the $\frac{1}{4}$ tiles exactly cover the $\frac{1}{2}$ tile?



Since two $\frac{1}{4}$ tiles cover the $\frac{1}{2}$ tile, we see that $\frac{2}{4}$ is the same as $\frac{1}{2}$, or $\frac{2}{4} = \frac{1}{2}$.

How many of the $\frac{1}{6}$ tiles cover the $\frac{1}{2}$ tile?



Since three $\frac{1}{6}$ tiles cover the $\frac{1}{2}$ tile, we see that $\frac{3}{6}$ is the same as $\frac{1}{2}$.

So, $\frac{3}{6} = \frac{1}{2}$. The fractions are equivalent fractions.

Manipulative Mathematics

Doing the activity "Equivalent Fractions" will help you develop a better understanding of what it means when two fractions are equivalent.

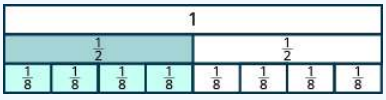
Example 4.13

Use fraction tiles to find equivalent fractions. Show your result with a figure.

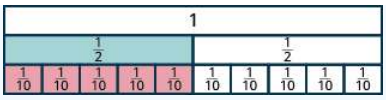
1. ⓐ How many eighths equal one-half?
2. ⓑ How many tenths equal one-half?
3. ⓒ How many twelfths equal one-half?

Answer

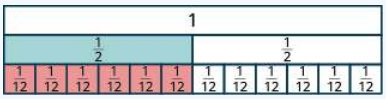
ⓐ It takes four $\frac{1}{8}$ tiles to exactly cover the $\frac{1}{2}$ tile, so $\frac{4}{8} = \frac{1}{2}$.



ⓑ It takes five $\frac{1}{10}$ tiles to exactly cover the $\frac{1}{2}$ tile, so $\frac{5}{10} = \frac{1}{2}$.



ⓒ It takes six $\frac{1}{12}$ tiles to exactly cover the $\frac{1}{2}$ tile, so $\frac{6}{12} = \frac{1}{2}$.



Suppose you had tiles marked $\frac{1}{20}$. How many of them would it take to equal $\frac{1}{2}$? Are you thinking ten tiles? If you are, you're right, because $\frac{10}{20} = \frac{1}{2}$.

We have shown that $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$, $\frac{5}{10}$, $\frac{6}{12}$, and $\frac{10}{20}$ are all equivalent fractions.

Try It 4.25

Use fraction tiles to find equivalent fractions: How many eighths equal one-fourth?

Try It 4.26

Use fraction tiles to find equivalent fractions: How many twelfths equal one-fourth?

Find Equivalent Fractions

We used fraction tiles to show that there are many fractions equivalent to Math input error

We can show this with pizzas, too. Figure 4.4(a) shows a single pizza, cut into two equal pieces with Math input error

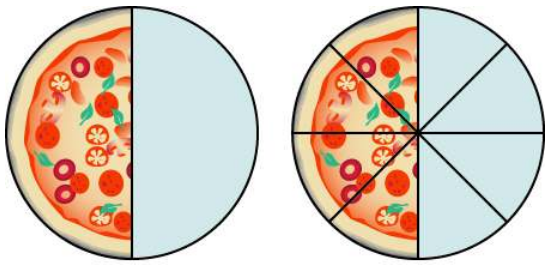


Figure 4.4

This is another way to show that $\frac{1}{2}$ is equivalent to $\frac{4}{8}$.

How can we use mathematics to change $\frac{1}{2}$ into $\frac{4}{8}$? How could you take a pizza that is cut into two pieces and cut it into eight pieces? You could cut each of the two larger pieces into four smaller pieces! The whole pizza would then be cut into eight pieces instead of just two. Mathematically, what we've described could be written as:

$$\frac{1 \cdot 4}{2 \cdot 4} = \frac{4}{8}$$

These models lead to the Equivalent Fractions Property, which states that if we multiply the numerator and denominator of a fraction by the same number, the value of the fraction does not change.

Equivalent Fractions Property

If a, b, and c are numbers where $b \neq 0$ and $c \neq 0$, then

$$\frac{a}{b} = \frac{a \cdot c}{b \cdot c}$$

When working with fractions, it is often necessary to express the same fraction in different forms. To find equivalent forms of a fraction, we can use the Equivalent Fractions Property. For example, consider the fraction one-half.

$\frac{1 \cdot 3}{2 \cdot 3} = \frac{3}{6}$ so $\frac{1}{2} = \frac{3}{6}$

$\frac{1 \cdot 2}{2 \cdot 2} = \frac{2}{4}$ so $\frac{1}{2} = \frac{2}{4}$

$\frac{1 \cdot 10}{2 \cdot 10} = \frac{10}{20}$ so $\frac{1}{2} = \frac{10}{20}$

So, we say that $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, and $\frac{10}{20}$ are equivalent fractions.

Example 4.14

Find three fractions equivalent to $\frac{2}{5}$.

Answer

To find a fraction equivalent to $\frac{2}{5}$, we multiply the numerator and denominator by the same number (but not zero). Let us multiply them by $2, 3,$ and 5.

$\frac{2 \cdot 2}{5 \cdot 2} = \frac{4}{10}$ $\frac{2 \cdot 3}{5 \cdot 3} = \frac{6}{15}$ $\frac{2 \cdot 5}{5 \cdot 5} = \frac{10}{25}$

So, $\frac{4}{10}$, $\frac{6}{15}$, and $\frac{10}{25}$ are equivalent to $\frac{2}{5}$.

Try It 4.27

Find three fractions equivalent to $\frac{3}{5}$.

Try It 4.28

Find three fractions equivalent to $\frac{4}{5}$.

Example 4.15

Find a fraction with a denominator of 21 that is equivalent to $\frac{2}{7}$.

Answer

To find equivalent fractions, we multiply the numerator and denominator by the same number. In this case, we need to multiply the denominator by a number that will result in 21.

Since we can multiply 7 by 3 to get 21, we can find the equivalent fraction by multiplying both the numerator and denominator by 3.

$\frac{2}{7} = \frac{2 \cdot 3}{7 \cdot 3} = \frac{6}{21}$

Try It 4.29

Find a fraction with a denominator of 21 that is equivalent to $\frac{6}{7}$.

Try It 4.30

Find a fraction with a denominator of 100 that is equivalent to $\frac{3}{10}$.

Locate Fractions and Mixed Numbers on the Number Line

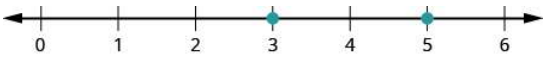
Now we are ready to plot fractions on a number line. This will help us visualize fractions and understand their values.

Manipulative Mathematics

Doing the Manipulative Mathematics activity "Number Line Part 3" will help you develop a better understanding of the location of fractions on the number line.

Let us locate $\frac{1}{5}, \frac{4}{5}, 3, 3\frac{1}{3}, \frac{7}{4}, \frac{9}{2}, 5$, and $\frac{8}{3}$ on the number line.

We will start with the whole numbers 3 and 5 because they are the easiest to plot.



The proper fractions listed are $\frac{1}{5}$ and $\frac{4}{5}$. We know proper fractions have values less than one, so $\frac{1}{5}$ and $\frac{4}{5}$ are located between the whole numbers 0 and 1. The denominators are both 5, so we need to divide the segment of the number line between 0 and 1 into five equal parts. We can do this by drawing four equally spaced marks on the number line, which we can then label as $\frac{1}{5}, \frac{2}{5}, \frac{3}{5}$, and $\frac{4}{5}$.

Now plot points at $\frac{1}{5}$ and $\frac{4}{5}$.



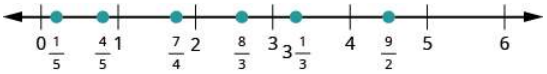
The only mixed number to plot is $3\frac{1}{3}$. Between what two whole numbers is $3\frac{1}{3}$? Remember that a mixed number is a whole number plus a proper fraction, so $3\frac{1}{3} > 3$. Since it is greater than 3, but not a whole unit greater, $3\frac{1}{3}$ is between 3 and 4. We need to divide the portion of the number line between 3 and 4 into three equal pieces (thirds) and plot $3\frac{1}{3}$ at the first mark.



Finally, look at the improper fractions $\frac{7}{4}, \frac{9}{2}$, and $\frac{8}{3}$. Locating these points will be easier if you change each of them to a mixed number.

$$\frac{7}{4} = 1\frac{3}{4}, \quad \frac{9}{2} = 4\frac{1}{2}, \quad \frac{8}{3} = 2\frac{2}{3}$$

Here is the number line with all the points plotted.

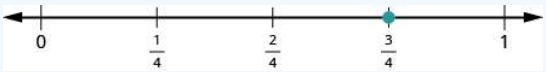


Example 4.16

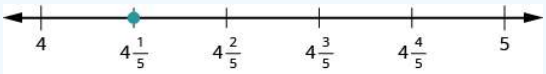
Locate and label the following on a number line: $\frac{3}{4}, \frac{4}{3}, \frac{5}{3}, 4\frac{1}{5}$, and $\frac{7}{2}$.

Answer

Start by locating the proper fraction $\frac{3}{4}$. It is between 0 and 1. To do this, divide the distance between 0 and 1 into four equal parts. Then plot $\frac{3}{4}$.



Next, locate the mixed number $4\frac{1}{5}$. It is between 4 and 5 on the number line. Divide the number line between 4 and 5 into five equal parts, and then plot $4\frac{1}{5}$ one-fifth of the way between 4 and 5.

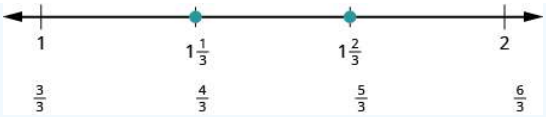


Now locate the improper fractions $\frac{4}{3}$ and $\frac{5}{3}$.

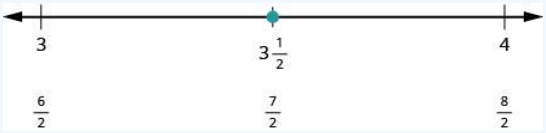
It is easier to plot them if we convert them to mixed numbers first.

$$\frac{4}{3} = 1\frac{1}{3}, \quad \frac{5}{3} = 1\frac{2}{3}$$

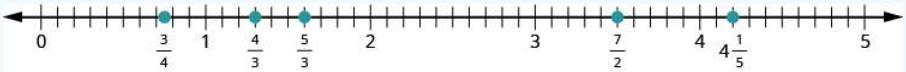
Divide the distance between 1 and 2 into thirds.



Next let us plot $\frac{7}{2}$. We write it as a mixed number, $\frac{7}{2} = 3\frac{1}{2}$. Plot it between 3 and 4.



The number line shows all the numbers located on the number line.



Try It 4.31

Locate and label the following on a number line: $\frac{1}{3}, \frac{5}{4}, \frac{7}{4}, 2\frac{3}{5}, \frac{9}{2}$.

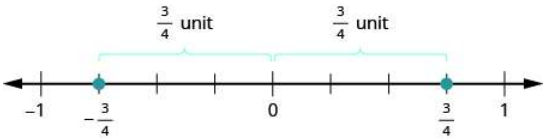
Try It 4.32

Locate and label the following on a number line: $\frac{2}{3}, \frac{5}{2}, \frac{9}{4}, \frac{11}{4}, 3\frac{2}{5}$.

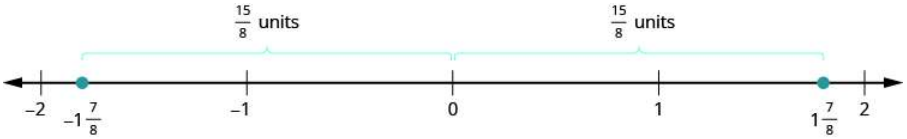
In Introduction to Integers, we defined the opposite of a number. It is the number that is the same distance from zero on the number line but on the opposite side of zero. We saw, for example, that the opposite of 7 is -7 and the opposite of -7 is 7.



Fractions have opposites, too. The opposite of $\frac{3}{4}$ is $-\frac{3}{4}$. It is the same distance from 0 on the number line, but on the opposite side of 0.



Thinking of negative fractions as the opposite of positive fractions will help us locate them on the number line. To locate $-\frac{15}{8}$ on the number line, first think of where $\frac{15}{8}$ is located. It is an improper fraction, so we first convert it to the mixed number $1\frac{7}{8}$ and see that it will be between 1 and 2 on the number line. So its opposite, $-\frac{15}{8}$, will be between -1 and -2 on the number line.



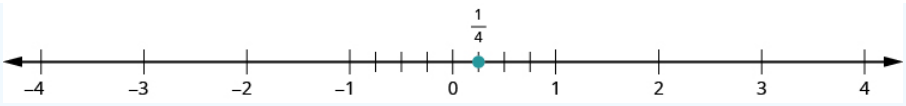
Example 4.17

Locate and label the following on the number line: $\frac{1}{4}, -\frac{1}{4}, 1\frac{1}{3}, -1\frac{1}{3}, \frac{5}{2}$, and $-\frac{5}{2}$.

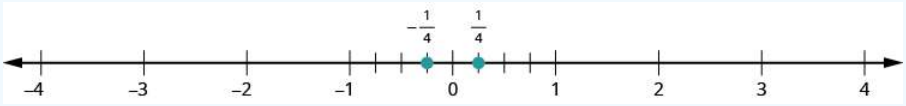
Answer

Draw a number line. Mark 0 in the middle and then mark several units to the left and right.

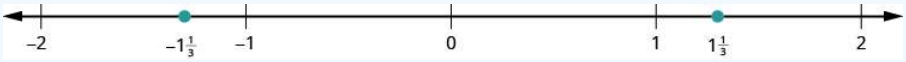
To locate $\frac{1}{4}$, divide the interval between 0 and 1 into four equal parts. Each part represents one-quarter of the distance. So plot $\frac{1}{4}$ at the first mark.



To locate $-\frac{1}{4}$, divide the interval between 0 and -1 into four equal parts. Plot $-\frac{1}{4}$ at the first mark to the left of 0.

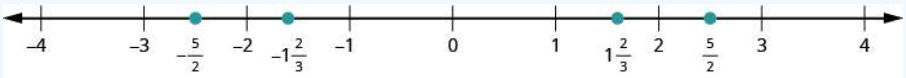


Since $1\frac{1}{3}$ is between 1 and 2, divide the interval between 1 and 2 into three equal parts. Plot $1\frac{1}{3}$ at the first mark to the right of 1. Then since $-1\frac{1}{3}$ is the opposite of $1\frac{1}{3}$ it is between -1 and -2. Divide the interval between -1 and -2 into three equal parts. Plot $-1\frac{1}{3}$ at the first mark to the left of -1.



To locate $\frac{5}{2}$ and $-\frac{5}{2}$, it may be helpful to rewrite them as the mixed numbers $2\frac{1}{2}$ and $-2\frac{1}{2}$.

Since $2\frac{1}{2}$ is between 2 and 3, divide the interval between 2 and 3 into two equal parts. Plot $\frac{5}{2}$ at the mark. Then since $-2\frac{1}{2}$ is between -2 and -3, divide the interval between -2 and -3 into two equal parts. Plot $-\frac{5}{2}$ at the mark.



Try It 4.33

Locate and label each of the given fractions on a number line:

$\frac{2}{3}, -\frac{2}{3}, 2\frac{1}{4}, -2\frac{1}{4}, \frac{3}{2}, -\frac{3}{2}$

Try It 4.34

Locate and label each of the given fractions on a number line:

$\frac{3}{4}, -\frac{3}{4}, 1\frac{1}{2}, -1\frac{1}{2}, \frac{7}{3}, -\frac{7}{3}$

Order Fractions and Mixed Numbers

We can use the inequality symbols to order fractions. Remember that $a > b$ means that a is to the right of b on the number line. As we move from left to right on a number line, the values increase.

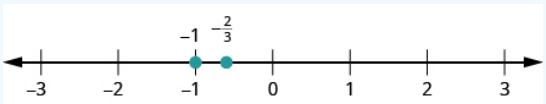
Example 4.18

Order each of the following pairs of numbers, using $<$ or $>$:

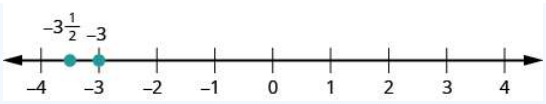
1. ⓐ $-\frac{2}{3}$ ___ -1
2. ⓑ $-3\frac{1}{2}$ ___ -3
3. ⓒ $-\frac{3}{7}$ ___ $-\frac{3}{8}$
4. ⓓ -2 ___ $\frac{-16}{9}$

Answer

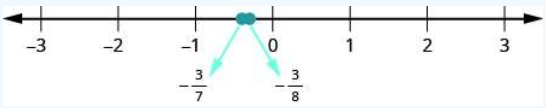
ⓐ $-\frac{2}{3} > -1$



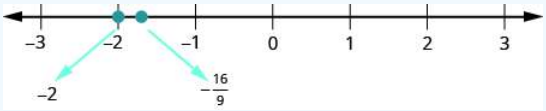
ⓑ $-3\frac{1}{2} < -3$



ⓒ $-\frac{3}{7} < -\frac{3}{8}$



ⓓ $-2 < \frac{-16}{9}$



Try It 4.35

Order each of the following pairs of numbers, using $<$ or $>$:

1. ⓐ $-\frac{1}{3}$ _ -1
2. ⓑ $-1\frac{1}{2}$ _ -2
3. ⓒ $-\frac{2}{3}$ _ $-\frac{1}{3}$
4. ⓓ -3 _ $-\frac{7}{3}$

Try It 4.36

Order each of the following pairs of numbers, using $<$ or $>$:

1. ⓐ -3 _ $-\frac{17}{5}$
2. ⓑ $-2\frac{1}{4}$ _ -2
3. ⓒ $-\frac{3}{5}$ _ $-\frac{4}{5}$
4. ⓓ -4 _ $-\frac{10}{3}$

Media

ACCESS ADDITIONAL ONLINE RESOURCES

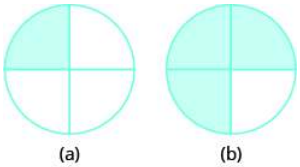
- Introduction to Fractions
- Identify Fractions Using Pattern Blocks

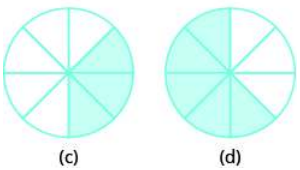
Section 4.1 Exercises

Practice Makes Perfect

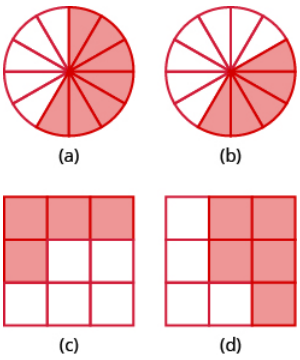
In the following exercises, name the fraction of each figure that is shaded.

1.





2.



In the following exercises, shade parts of circles or squares to model the following fractions.

3.

$\frac{1}{2}$

4.

$\frac{1}{3}$

5.

$\frac{3}{4}$

6.

$\frac{2}{5}$

7.

$\frac{5}{6}$

8.

$\frac{7}{8}$

9.

$\frac{5}{8}$

10.

$\frac{7}{10}$

In the following exercises, use fraction circles to make wholes using the following pieces.

11.

3 thirds

12.

8 eighths

13.

7 sixths

14.

4 thirds

15.

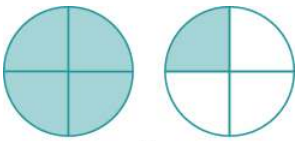
7 fifths

16.

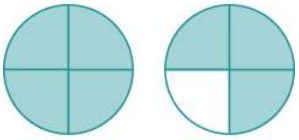
7 fourths

In the following exercises, name the improper fractions. Then write each improper fraction as a mixed number.

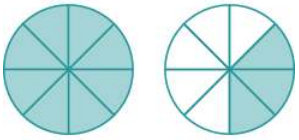
17.



(a)

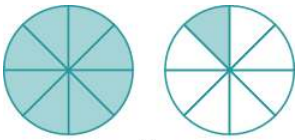


(b)

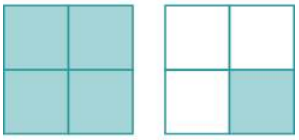


(c)

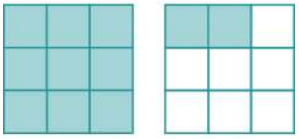
18.



(a)

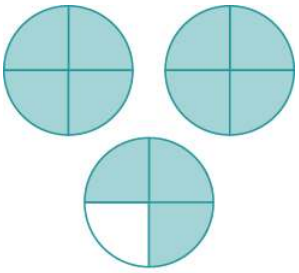


(b)

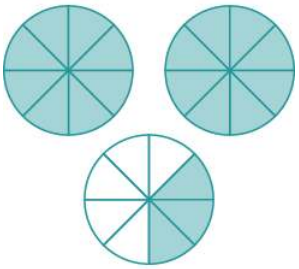


(c)

19.



(a)



(b)

In the following exercises, draw fraction circles to model the given fraction.

20.

$\frac{3}{3}$

21.

$\frac{4}{4}$

22.

$\frac{7}{4}$

23.

$\frac{5}{3}$

24.

$\frac{11}{6}$

25.

$\frac{13}{8}$

26.

$\frac{10}{3}$

27.

$\frac{9}{4}$

In the following exercises, rewrite the improper fraction as a mixed number.

28.

$\frac{3}{2}$

29.

$\frac{5}{3}$

30.

$\frac{11}{4}$

31.

$\frac{13}{5}$

32.

$\frac{25}{6}$

33.

$\frac{28}{9}$

34.

$\frac{42}{13}$

35.

$\frac{47}{15}$

In the following exercises, rewrite the mixed number as an improper fraction.

36.

$1\frac{2}{3}$

37.

$1\frac{2}{5}$

38.

$2\frac{1}{4}$

39.

$2\frac{5}{6}$

40.

$2\frac{7}{9}$

41.

$2\frac{5}{7}$

42.

$3\frac{4}{7}$

43.

$3\frac{5}{9}$

In the following exercises, use fraction tiles or draw a figure to find equivalent fractions.

44.

How many sixths equal one-third?

45.

How many twelfths equal one-third?

46.

How many eighths equal three-fourths?

47.

How many twelfths equal three-fourths?

48.

How many fourths equal three-halves?

49.

How many sixths equal three-halves?

In the following exercises, find three fractions equivalent to the given fraction. Show your work, using figures or algebra.

50.

$\frac{1}{4}$

51.

$\frac{1}{3}$

52.

$\frac{3}{8}$

53.

$\frac{5}{6}$

54.

$\frac{2}{7}$

55.

$\frac{5}{9}$

In the following exercises, plot the numbers on a number line.

56.

$\frac{2}{3}, \frac{5}{4}, \frac{12}{5}$

57.

$\frac{1}{3}, \frac{7}{4}, \frac{13}{5}$

58.

$\frac{1}{4}, \frac{9}{5}, \frac{11}{3}$

59.

$\frac{7}{10}, \frac{5}{2}, \frac{13}{8}, 3$

60.

$2\frac{1}{3}, -2\frac{1}{3}$

61.

$1\frac{3}{4}, -1\frac{3}{5}$

62.

$\frac{3}{4}, -\frac{3}{4}, 1\frac{2}{3}, -1\frac{2}{3}, \frac{5}{2}, -\frac{5}{2}$

63.

$\frac{2}{5}, -\frac{2}{5}, 1\frac{3}{4}, -1\frac{3}{4}, \frac{8}{3}, -\frac{8}{3}$

In the following exercises, order each of the following pairs of numbers, using $<$ or $>$.

64.

$-1 \_ -\frac{1}{4}$

65.

$-1 \_ -\frac{1}{3}$

66.

$-2\frac{1}{2} \_ -3$

67.

$-1\frac{3}{4} \_ -2$

68.

$-\frac{5}{12} \_ -\frac{7}{12}$

69.

$-\frac{9}{10}$ __ $-\frac{3}{10}$

70.

-3 __ $-\frac{13}{5}$

71.

-4 __ $-\frac{23}{6}$

Everyday Math

72.

Music Measures A choreographed dance is broken into counts. A $\frac{1}{1}$ count has one step in a count, a $\frac{1}{2}$ count has two steps in a count and a $\frac{1}{3}$ count has three steps in a count. How many steps would be in a $\frac{1}{5}$ count? What type of count has four steps in it?

73.

Music Measures Fractions are used often in music. In $\frac{4}{4}$ time, there are four quarter notes in one measure.

1. ⓐ How many measures would eight quarter notes make?
2. ⓑ The song "Happy Birthday to You" has 25 quarter notes. How many measures are there in "Happy Birthday to You?"

74.

Baking Nina is making five pans of fudge to serve after a music recital. For each pan, she needs $\frac{1}{2}$ cup of walnuts.

1. ⓐ How many cups of walnuts does she need for five pans of fudge?
2. ⓑ Do you think it is easier to measure this amount when you use an improper fraction or a mixed number? Why?

Writing Exercises

75.

Give an example from your life experience (outside of school) where it was important to understand fractions.

76.

Explain how you locate the improper fraction $\frac{21}{4}$ on a number line on which only the whole numbers from 0 through 10 are marked.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
understand the meaning of fractions.			
model improper fractions and mixed numbers.			
convert between improper fractions and mixed numbers.			
model equivalent fractions.			
find equivalent fractions.			
locate fractions and mixed numbers on the number line.			
order fractions and mixed numbers.			

ⓑ If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them. What did you do to become confident of your ability to do these things? Be specific.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is important to make sure you have a strong foundation before you move on. Whom can you ask for help? Your fellow classmates and instructor are good resources. Is there a place on campus where math tutors are available? Can your study skills be improved?

…no—I don't get it! This is a warning sign and you must not ignore it. You should get help right away or you will quickly be overwhelmed. See your instructor as soon as you can to discuss your situation. Together you can come up with a plan to get you the help you need.

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4.2: Multiply and Divide Fractions

Learning Objectives

By the end of this section, you will be able to:

- Simplify fractions
- Multiply fractions
- Find reciprocals
- Divide fractions

Be Prepared 4.3

Before you get started, take this readiness quiz.

Find the prime factorization of 48.
If you missed this problem, review Example 2.48.

Be Prepared 4.4

Draw a model of the fraction $\frac{3}{4}$.
If you missed this problem, review Example 4.2.

Be Prepared 4.5

Find two fractions equivalent to $\frac{5}{6}$.
If you missed this problem, review Example 4.14.

Simplify Fractions

In working with equivalent fractions, you saw that there are many ways to write fractions that have the same value, or represent the same part of the whole. How do you know which one to use? Often, we'll use the fraction that is in *simplified* form.

A fraction is considered simplified if there are no common factors, other than 1, in the numerator and denominator. If a fraction does have common factors in the numerator and denominator, we can reduce the fraction to its simplified form by removing the common factors.

Simplified Fraction

A fraction is considered simplified if there are no common factors in the numerator and denominator.

For example,

- $\frac{2}{3}$ is simplified because there are no common factors of 2 and 3.
- $\frac{10}{15}$ is not simplified because 5 is a common factor of 10 and 15.

The process of simplifying a fraction is often called *reducing the fraction*. In the previous section, we used the Equivalent Fractions Property to find equivalent fractions. We can also use the Equivalent Fractions Property in reverse to simplify fractions. We rewrite the property to show both forms together.

Equivalent Fractions Property

If a, b, c are numbers where $b \neq 0, c \neq 0$, then

$$\frac{a}{b} = \frac{a \cdot c}{b \cdot c} \quad \text{and} \quad \frac{a \cdot c}{b \cdot c} = \frac{a}{b}.$$

Notice that c is a common factor in the numerator and denominator. Anytime we have a common factor in the numerator and denominator, it can be removed.

How To

Simplify a fraction.

1. Step 1. Rewrite the numerator and denominator to show the common factors. If needed, factor the numerator and denominator into prime numbers.
2. Step 2. Simplify, using the equivalent fractions property, by removing common factors.
3. Step 3. Multiply any remaining factors.

Example 4.19

Simplify: $\frac{10}{15}$.

Answer

To simplify the fraction, we look for any common factors in the numerator and the denominator.

Notice that 5 is a factor of both 10 and 15.	$\frac{10}{15}$
Factor the numerator and denominator.	$\frac{2 \cdot 5}{3 \cdot 5}$
Remove the common factors.	$\frac{2 \cdot \cancel{5}}{3 \cdot \cancel{5}}$
Simplify.	$\frac{2}{3}$

Try It 4.37

Simplify: $\frac{8}{12}$.

Try It 4.38

Simplify: $\frac{12}{16}$.

To simplify a negative fraction, we use the same process as in Example 4.19. Remember to keep the negative sign.

Example 4.20

Simplify: $-\frac{18}{24}$.

Answer

We notice that 18 and 24 both have factors of 6.	$-\frac{18}{24}$
Rewrite the numerator and denominator showing the common factor.	$-\frac{3 \cdot 6}{4 \cdot 6}$
Remove common factors.	$-\frac{3 \cdot \cancel{6}}{4 \cdot \cancel{6}}$
Simplify.	$-\frac{3}{4}$

Try It 4.39

Simplify: $-\frac{21}{28}$.

Try It 4.40

Simplify: $-\frac{16}{24}$.

After simplifying a fraction, it is always important to check the result to make sure that the numerator and denominator do not have any more factors in common. Remember, the definition of a simplified fraction: *a fraction is considered simplified if there are no common factors in the numerator and denominator*.

When we simplify an improper fraction, there is no need to change it to a mixed number.

Example 4.21

Simplify: $-\frac{56}{32}$.

Answer

	$-\frac{56}{32}$
Rewrite the numerator and denominator, showing the common factors, 8.	$\frac{7 \cdot 8}{4 \cdot 8}$
Remove common factors.	$\frac{7 \cdot \cancel{8}}{4 \cdot \cancel{8}}$
Simplify.	$-\frac{7}{4}$

Try It 4.41

Simplify: $-\frac{54}{42}$.

Try It 4.42

Simplify: $-\frac{81}{45}$.

How To

Simplify a fraction.

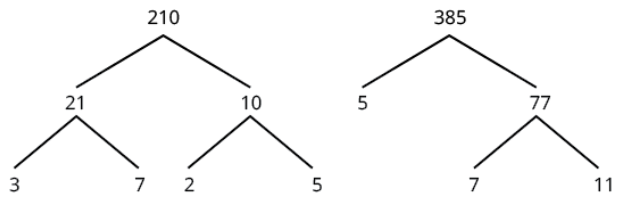
1. Step 1. Rewrite the numerator and denominator to show the common factors. If needed, factor the numerator and denominator into prime numbers.
2. Step 2. Simplify, using the equivalent fractions property, by removing common factors.
3. Step 3. Multiply any remaining factors

Sometimes it may not be easy to find common factors of the numerator and denominator. A good idea, then, is to factor the numerator and the denominator into prime numbers. (You may want to use the factor tree method to identify the prime factors.) Then divide out the common factors using the Equivalent Fractions Property.

Example 4.22

Simplify: $\frac{210}{385}$.

Answer

Use factor trees to factor the numerator and denominator.	$\frac{210}{385}$ 
Rewrite the numerator and denominator as the product of the primes.	$\frac{210}{385} = \frac{2 \cdot 3 \cdot 5 \cdot 7}{5 \cdot 7 \cdot 11}$

Remove the common factors.	$\frac{2 \cdot 3 \cdot \cancel{5} \cdot \cancel{7}}{\cancel{5} \cdot \cancel{7} \cdot 11}$
Simplify.	$\frac{2 \cdot 3}{11}$
Multiply any remaining factors.	$\frac{6}{11}$

Try It 4.43

Simplify: $\frac{69}{120}$.

Try It 4.44

Simplify: $\frac{120}{192}$.

We can also simplify fractions containing variables. If a variable is a common factor in the numerator and denominator, we remove it just as we do with an integer factor.

Example 4.23

Simplify: $\frac{5xy}{15x}$.

Answer

	$\frac{5xy}{15x}$
Rewrite numerator and denominator showing common factors.	$\frac{5 \cdot x \cdot y}{3 \cdot 5 \cdot x}$
Remove common factors.	$\frac{\cancel{5} \cdot \cancel{x} \cdot y}{3 \cdot \cancel{5} \cdot \cancel{x}}$
Simplify.	$\frac{y}{3}$

Try It 4.45

Simplify: $\frac{7x}{7y}$.

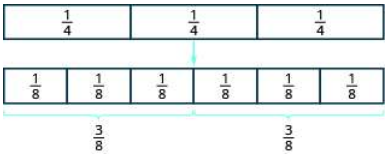
Try It 4.46

Simplify: $\frac{9a}{9b}$.

Multiply Fractions

A model may help you understand multiplication of fractions. We will use fraction tiles to model $\frac{1}{2} \cdot \frac{3}{4}$. To multiply $\frac{1}{2}$ and $\frac{3}{4}$, think $\frac{1}{2}$ of $\frac{3}{4}$.

Start with fraction tiles for three-fourths. To find one-half of three-fourths, we need to divide them into two equal groups. Since we cannot divide the three $\frac{1}{4}$ tiles evenly into two parts, we exchange them for smaller tiles.



We see $\frac{6}{8}$ is equivalent to $\frac{3}{4}$. Taking half of the six $\frac{1}{8}$ tiles gives us three $\frac{1}{8}$ tiles, which is $\frac{3}{8}$.

Therefore,

$$\frac{1}{2} \cdot \frac{3}{4} = \frac{3}{8}$$

Manipulative Mathematics

Example 4.24

Use a diagram to model $\frac{1}{2} \cdot \frac{3}{4}$.

Answer

First shade in $\frac{3}{4}$ of the rectangle.

We will take $\frac{1}{2}$ of this $\frac{3}{4}$, so we heavily shade $\frac{1}{2}$ of the shaded region.

Notice that 3 out of the 8 pieces are heavily shaded. This means that $\frac{3}{8}$ of the rectangle is heavily shaded.

Therefore, $\frac{1}{2}$ of $\frac{3}{4}$ is $\frac{3}{8}$, or $\frac{1}{2} \cdot \frac{3}{4} = \frac{3}{8}$.

Try It 4.47

Use a diagram to model: $\frac{1}{2} \cdot \frac{3}{5}$.

Try It 4.48

Use a diagram to model: $\frac{1}{2} \cdot \frac{5}{6}$.

Look at the result we got from the model in Example 4.24. We found that $\frac{1}{2} \cdot \frac{3}{4} = \frac{3}{8}$. Do you notice that we could have gotten the same answer by multiplying the numerators and multiplying the denominators?

	$\frac{1}{2} \cdot \frac{3}{4}$
Multiply the numerators, and multiply the denominators.	$\frac{1}{2} \cdot \frac{3}{4}$
Simplify.	$\frac{3}{8}$

This leads to the definition of fraction multiplication. To multiply fractions, we multiply the numerators and multiply the denominators. Then we write the fraction in simplified form.

Fraction Multiplication

If a, b, c, and d are numbers where $b \neq 0$ and $d \neq 0$, then

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

Example 4.25

Multiply, and write the answer in simplified form: $\frac{3}{4} \cdot \frac{1}{5}$.

Answer

	$\frac{3}{4} \cdot \frac{1}{5}$
Multiply the numerators; multiply the denominators.	$\frac{3 \cdot 1}{4 \cdot 5}$
Simplify.	$\frac{3}{20}$

There are no common factors, so the fraction is simplified.

Try It 4.49

Multiply, and write the answer in simplified form: $\frac{1}{3} \cdot \frac{2}{5}$.

Try It 4.50

Multiply, and write the answer in simplified form: $\frac{3}{5} \cdot \frac{7}{8}$.

When multiplying fractions, the properties of positive and negative numbers still apply. It is a good idea to determine the sign of the product as the first step. In Example 4.26 we will multiply two negatives, so the product will be positive.

Example 4.26

Multiply, and write the answer in simplified form: $-\frac{5}{8}\left(-\frac{2}{3}\right)$.

Answer

	$-\frac{5}{8}\left(-\frac{2}{3}\right)$
The signs are the same, so the product is positive. Multiply the numerators, multiply the denominators.	$\frac{5 \cdot 2}{8 \cdot 3}$
Simplify.	$\frac{10}{24}$
Look for common factors in the numerator and denominator. Rewrite showing common factors.	$\frac{5 \cdot 2}{12 \cdot 2}$
Remove common factors.	$\frac{5}{12}$

Another way to find this product involves removing common factors earlier.

	$-\frac{5}{8}\left(-\frac{2}{3}\right)$
Determine the sign of the product. Multiply.	$\frac{5 \cdot 2}{8 \cdot 3}$
Show common factors and then remove them.	$\frac{5 \cdot 2}{4 \cdot 2 \cdot 3}$
Multiply remaining factors.	$\frac{5}{12}$

We get the same result.

Try It 4.51

Multiply, and write the answer in simplified form: $-\frac{4}{7}\left(-\frac{5}{8}\right)$.

Try It 4.52

Multiply, and write the answer in simplified form: $-\frac{7}{12}\left(-\frac{8}{9}\right)$.

Example 4.27

Multiply, and write the answer in simplified form: $-\frac{14}{15} \cdot \frac{20}{21}$.

Answer

	$-\frac{14}{15} \cdot \frac{20}{21}$

Determine the sign of the product; multiply.	$-\frac{14}{15} \cdot \frac{20}{21}$
Are there any common factors in the numerator and the denominator? We know that 7 is a factor of 14 and 21, and 5 is a factor of 20 and 15.	
Rewrite showing common factors.	$-\frac{2 \cdot 7 \cdot 4 \cdot 5}{3 \cdot 5 \cdot 3 \cdot 7}$
Remove the common factors.	$-\frac{2 \cdot 4}{3 \cdot 3}$
Multiply the remaining factors.	$-\frac{8}{9}$

Try It 4.53

Multiply, and write the answer in simplified form: $-\frac{10}{28} \cdot \frac{8}{15}$.

Try It 4.54

Multiply, and write the answer in simplified form: $-\frac{9}{20} \cdot \frac{5}{12}$.

When multiplying a fraction by an integer, it may be helpful to write the integer as a fraction. Any integer, a, can be written as $\frac{a}{1}$. So, $3 = \frac{3}{1}$, for example.

Example 4.28

Multiply, and write the answer in simplified form:

ⓐ $\frac{1}{7} \cdot 56$

ⓑ $\frac{12}{5}(-20x)$

Answer

ⓐ	
	$\frac{1}{7} \cdot 56$
Write 56 as a fraction.	$\frac{1}{7} \cdot \frac{56}{1}$
Determine the sign of the product; multiply.	$\frac{56}{7}$
Simplify.	8

ⓑ	
	$\frac{12}{5}(-20x)$
Write −20x as a fraction.	$\frac{12}{5}\left(\frac{-20x}{1}\right)$
Determine the sign of the product; multiply.	$-\frac{12 \cdot 20 \cdot x}{5 \cdot 1}$
Show common factors and then remove them.	$-\frac{12 \cdot 4 \cdot 5x}{5 \cdot 1}$
Multiply remaining factors; simplify.	−48x

Try It 4.55

Multiply, and write the answer in simplified form:

1. ⓐ $\frac{1}{8} \cdot 72$
2. ⓑ $\frac{11}{3}(-9a)$

Try It 4.56

Multiply, and write the answer in simplified form:

1. ⓐ $\frac{3}{8} \cdot 64$
2. ⓑ $16x \cdot \frac{11}{12}$

Find Reciprocals

The fractions $\frac{2}{3}$ and $\frac{3}{2}$ are related to each other in a special way. So are $-\frac{10}{7}$ and $-\frac{7}{10}$. Do you see how? Besides looking like upside-down versions of one another, if we were to multiply these pairs of fractions, the product would be 1.

Reciprocal

The reciprocal of the fraction $\frac{a}{b}$ is $\frac{b}{a}$, where $a \neq 0$ and $b \neq 0$,

A number and its reciprocal have a product of 1.

$$\frac{a}{b} \cdot \frac{b}{a} = 1$$

To find the reciprocal of a fraction, we invert the fraction. This means that we place the numerator in the denominator and the denominator in the numerator.

To get a positive result when multiplying two numbers, the numbers must have the same sign. So reciprocals must have the same sign.

$$\frac{a}{b} \cdot \frac{b}{a} = 1 \text{ positive}$$

$3 \cdot \frac{1}{3} = 1$ and $-3 \cdot \left(-\frac{1}{3}\right) = 1$

both positive both negative

To find the reciprocal, keep the same sign and invert the fraction. The number zero does not have a reciprocal. Why? A number and its reciprocal multiply to 1. Is there any number r so that $0 \cdot r = 1$? No. So, the number 0 does not have a reciprocal.

Example 4.29

Find the reciprocal of each number. Then check that the product of each number and its reciprocal is 1.

1. ⓐ $\frac{4}{9}$
2. ⓑ $-\frac{1}{6}$
3. ⓒ $-\frac{14}{5}$
4. ⓓ 7

Answer

To find the reciprocals, we keep the sign and invert the fractions.

ⓐ	
Find the reciprocal of $\frac{4}{9}$.	The reciprocal of $\frac{4}{9}$ is $\frac{9}{4}$.
Check:	
Multiply the number and its reciprocal.	$\frac{4}{9} \cdot \frac{9}{4}$
Multiply numerators and denominators.	$\frac{36}{36}$
Simplify.	1✓

ⓑ	
Find the reciprocal of $-\frac{1}{6}$.	$-\frac{6}{1}$
Simplify.	-6
Check:	$-\frac{1}{6} \cdot (-6)$
	1✓

ⓒ	
Find the reciprocal of $-\frac{14}{5}$.	$-\frac{5}{14}$
Check:	$-\frac{14}{5} \cdot \left(-\frac{5}{14}\right)$
	$\frac{70}{70}$
	1✓

ⓓ	
Find the reciprocal of 7.	
Write 7 as a fraction.	$\frac{7}{1}$
Write the reciprocal of $\frac{7}{1}$.	$\frac{1}{7}$
Check:	$7 \cdot \left(\frac{1}{7}\right)$
	1✓

Try It 4.57

Find the reciprocal:

1. ⓐ $\frac{5}{7}$
2. ⓑ $-\frac{1}{8}$
3. ⓒ $-\frac{11}{4}$
4. ⓓ 14

Try It 4.58

Find the reciprocal:

1. ⓐ $\frac{3}{7}$
2. ⓑ $-\frac{1}{12}$
3. ⓒ $-\frac{14}{9}$
4. ⓓ 21

In a previous chapter, we worked with opposites and absolute values. Table 4.1 compares opposites, absolute values, and reciprocals.

Opposite	Absolute Value	Reciprocal
has opposite sign	is never negative	has same sign, fraction inverts

Table 4.1

Example 4.30

Fill in the chart for each fraction in the left column:

Number	Opposite	Absolute Value	Reciprocal
$-\frac{3}{8}$			
$\frac{1}{2}$			
$\frac{9}{5}$			
-5			

Answer

To find the opposite, change the sign. To find the absolute value, leave the positive numbers the same, but take the opposite of the negative numbers. To find the reciprocal, keep the sign the same and invert the fraction.

Number	Opposite	Absolute Value	Reciprocal
$-\frac{3}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$-\frac{8}{3}$
$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	2
$\frac{9}{5}$	$-\frac{9}{5}$	$\frac{9}{5}$	$\frac{5}{9}$
-5	5	5	$-\frac{1}{5}$

Try It 4.59

Fill in the chart for each number given:

Number	Opposite	Absolute Value	Reciprocal
$-\frac{5}{8}$			
$\frac{1}{4}$			
$\frac{8}{3}$			
-8			

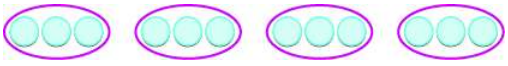
Try It 4.60

Fill in the chart for each number given:

Number	Opposite	Absolute Value	Reciprocal
$-\frac{4}{7}$			
$\frac{1}{8}$			
$\frac{9}{4}$			
-1			

Divide Fractions

Why is $12 \div 3 = 4$? We previously modeled this with counters. How many groups of 3 counters can be made from a group of 12 counters?



There are 4 groups of 3 counters. In other words, there are four 3s in 12. So, $12 \div 3 = 4$.

What about dividing fractions? Suppose we want to find the quotient: Math input error

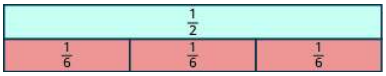


Figure 4.5

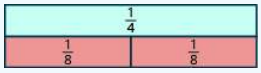
Manipulative Mathematics

Example 4.31

Model: $\frac{1}{4} \div \frac{1}{8}$.

Answer

We want to determine how many $\frac{1}{8}$s are in $\frac{1}{4}$. Start with one $\frac{1}{4}$ tile. Line up $\frac{1}{8}$ tiles underneath the $\frac{1}{4}$ tile.



There are two $\frac{1}{8}$s in $\frac{1}{4}$.

So, $\frac{1}{4} \div \frac{1}{8} = 2$.

Try It 4.61

Model: $\frac{1}{3} \div \frac{1}{6}$.

Try It 4.62

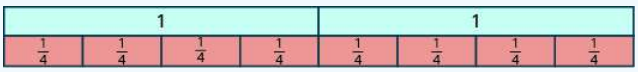
Model: $\frac{1}{2} \div \frac{1}{4}$.

Example 4.32

Model: $2 \div \frac{1}{4}$.

Answer

We are trying to determine how many $\frac{1}{4}$s there are in 2. We can model this as shown.



Because there are eight $\frac{1}{4}$s in 2, $2 \div \frac{1}{4} = 8$.

Try It 4.63

Model: $2 \div \frac{1}{3}$

Try It 4.64

Model: $3 \div \frac{1}{2}$

Let's use money to model Math input error



Figure 4.6 The U.S. coin called a quarter is worth one-fourth of a dollar.

Using fraction tiles, we showed that $\frac{1}{2} \div \frac{1}{6} = 3$. Notice that $\frac{1}{2} \cdot \frac{6}{1} = 3$ also. How are $\frac{1}{6}$ and $\frac{6}{1}$ related? They are reciprocals. This leads us to the procedure for fraction division.

Fraction Division

If a, b, c, and d are numbers where $b \neq 0, c \neq 0$, and $d \neq 0$, then

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$$

To divide fractions, multiply the first fraction by the reciprocal of the second.

We need to say $b \neq 0, c \neq 0$ and $d \neq 0$ to be sure we don't divide by zero.

Example 4.33

Divide, and write the answer in simplified form: $\frac{2}{5} \div \left(-\frac{3}{7}\right)$.

Answer

	$\frac{2}{5} \div \left(-\frac{3}{7}\right)$
Multiply the first fraction by the reciprocal of the second.	$\frac{2}{5}\left(-\frac{7}{3}\right)$
Multiply. The product is negative.	$-\frac{14}{15}$

Try It 4.65

Divide, and write the answer in simplified form: $\frac{3}{7} \div \left(-\frac{2}{3}\right)$.

Try It 4.66

Divide, and write the answer in simplified form: $\frac{2}{3} \div \left(-\frac{7}{5}\right)$.

Example 4.34

Divide, and write the answer in simplified form: $\frac{2}{3} \div \frac{n}{5}$.

Answer

	$\frac{2}{3} \div \frac{n}{5}$
Multiply the first fraction by the reciprocal of the second.	$\frac{2}{3} \cdot \frac{5}{n}$
Multiply.	$\frac{10}{3n}$

Try It 4.67

Divide, and write the answer in simplified form: $\frac{3}{5} \div \frac{p}{7}$.

Try It 4.68

Divide, and write the answer in simplified form: $\frac{5}{8} \div \frac{q}{3}$.

Example 4.35

Divide, and write the answer in simplified form: $-\frac{3}{4} \div \left(-\frac{7}{8}\right)$.

Answer

	$-\frac{3}{4} \div \left(-\frac{7}{8}\right)$
Multiply the first fraction by the reciprocal of the second.	$-\frac{3}{4} \cdot \left(-\frac{8}{7}\right)$
Multiply. Remember to determine the sign first.	$\frac{3 \cdot 8}{4 \cdot 7}$
Rewrite to show common factors.	$\frac{3 \cdot \cancel{4} \cdot 2}{\cancel{4} \cdot 7}$
Remove common factors and simplify.	$\frac{6}{7}$

Try It 4.69

Divide, and write the answer in simplified form: $-\frac{2}{3} \div \left(-\frac{5}{6}\right)$.

Try It 4.70

Divide, and write the answer in simplified form: $-\frac{5}{6} \div \left(-\frac{2}{3}\right)$.

Example 4.36

Divide, and write the answer in simplified form: $\frac{7}{18} \div \frac{14}{27}$.

Answer

	$\frac{7}{18} \div \frac{14}{27}$
Multiply the first fraction by the reciprocal of the second.	$\frac{7}{18} \cdot \frac{27}{14}$
Multiply.	$\frac{7 \cdot 27}{18 \cdot 14}$
Rewrite showing common factors.	$\frac{7 \cdot 9 \cdot 3}{9 \cdot 2 \cdot 7 \cdot 2}$
Remove common factors.	$\frac{3}{2 \cdot 2}$
Simplify.	$\frac{3}{4}$

Try It 4.71

Divide, and write the answer in simplified form: $\frac{7}{27} \div \frac{35}{36}$.

Try It 4.72

Divide, and write the answer in simplified form: $\frac{5}{14} \div \frac{15}{28}$.

Media

ACCESS ADDITIONAL ONLINE RESOURCES

- Simplifying Fractions
- Multiplying Fractions (Positive Only)
- Multiplying Signed Fractions
- Dividing Fractions (Positive Only)
- Dividing Signed Fractions

Section 4.2 Exercises

Practice Makes Perfect

Simplify Fractions

In the following exercises, simplify each fraction. Do not convert any improper fractions to mixed numbers.

77.

$\frac{7}{21}$

78.

$\frac{8}{24}$

79.

$\frac{15}{20}$

80.

$\frac{12}{18}$

81.

$-\frac{40}{88}$

82.

$-\frac{63}{99}$

83.

$-\frac{108}{63}$

84.

$-\frac{104}{48}$

85.

$\frac{120}{252}$

86.

$\frac{182}{294}$

87.

$-\frac{168}{192}$

88.

$-\frac{140}{224}$

89.

$\frac{11x}{11y}$

90.

$\frac{15a}{15b}$

91.

$-\frac{3x}{12y}$

92.

$-\frac{4x}{32y}$

93.

$\frac{14x^2}{21y}$

94.

$\frac{24a}{32b^2}$

Multiply Fractions

In the following exercises, use a diagram to model.

95.

$\frac{1}{2} \cdot \frac{2}{3}$

96.

$\frac{1}{2} \cdot \frac{5}{8}$

97.

$\frac{1}{3} \cdot \frac{5}{6}$

98.

$\frac{1}{3} \cdot \frac{2}{5}$

In the following exercises, multiply, and write the answer in simplified form.

99.

$\frac{2}{5} \cdot \frac{1}{3}$

100.

$\frac{1}{2} \cdot \frac{3}{8}$

101.

$\frac{3}{4} \cdot \frac{9}{10}$

102.

$\frac{4}{5} \cdot \frac{2}{7}$

103.

$-\frac{2}{3}\left(-\frac{3}{8}\right)$

104.

$-\frac{3}{4}\left(-\frac{4}{9}\right)$

105.

$-\frac{5}{9} \cdot \frac{3}{10}$

106.

$-\frac{3}{8} \cdot \frac{4}{15}$

107.

$\frac{7}{12}\left(-\frac{8}{21}\right)$

108.

$\frac{5}{12}\left(-\frac{8}{15}\right)$

109.

$\left(-\frac{14}{15}\right)\left(\frac{9}{20}\right)$

110.

$\left(-\frac{9}{10}\right)\left(\frac{25}{33}\right)$

111.

$\left(-\frac{63}{84}\right)\left(-\frac{44}{90}\right)$

112.

$\left(-\frac{33}{60}\right)\left(-\frac{40}{88}\right)$

113.

$4 \cdot \frac{5}{11}$

114.

$5 \cdot \frac{8}{3}$

115.

$\frac{3}{7} \cdot 21n$

116.

$\frac{5}{6} \cdot 30m$

117.

$-28p\left(-\frac{1}{4}\right)$

118.

$-51q\left(-\frac{1}{3}\right)$

119.

$-8\left(\frac{17}{4}\right)$

120.

$\frac{14}{5}(-15)$

121.

$-1\left(-\frac{3}{8}\right)$

122.

$(-1)\left(-\frac{6}{7}\right)$

123.

$\left(\frac{2}{3}\right)^3$

124.

$\left(\frac{4}{5}\right)^2$

125.

$\left(\frac{6}{5}\right)^4$

126.

$\left(\frac{4}{7}\right)^4$

Find Reciprocals

In the following exercises, find the reciprocal.

127.

$\frac{3}{4}$

128.

$\frac{2}{3}$

129.

$-\frac{5}{17}$

130.

$-\frac{6}{19}$

131.

$\frac{11}{8}$

132.

-13

133.

-19

134.

-1

135.

1

136.

Fill in the chart.

	Opposite	Absolute Value	Reciprocal
$-\frac{7}{11}$			
$\frac{4}{5}$			
$\frac{10}{7}$			
-8			

137.

Fill in the chart.

	Opposite	Absolute Value	Reciprocal
$-\frac{3}{13}$			
$\frac{9}{14}$			
$\frac{15}{7}$			
-9			

Divide Fractions

In the following exercises, model each fraction division.

138.

$\frac{1}{2} \div \frac{1}{4}$

139.

$\frac{1}{2} \div \frac{1}{8}$

140.

$2 \div \frac{1}{5}$

141.

$3 \div \frac{1}{4}$

In the following exercises, divide, and write the answer in simplified form.

142.

$\frac{1}{2} \div \frac{1}{4}$

143.

$\frac{1}{2} \div \frac{1}{8}$

144.

$\frac{3}{4} \div \frac{2}{3}$

145.

$\frac{4}{5} \div \frac{3}{4}$

146.

$-\frac{4}{5} \div \frac{4}{7}$

147.

$-\frac{3}{4} \div \frac{3}{5}$

148.

$-\frac{7}{9} \div \left(-\frac{7}{9}\right)$

149.

$-\frac{5}{6} \div \left(-\frac{5}{6}\right)$

150.

$\frac{3}{4} \div \frac{x}{11}$

151.

$\frac{2}{5} \div \frac{y}{9}$

152.

$\frac{5}{8} \div \frac{a}{10}$

153.

$\frac{5}{6} \div \frac{c}{15}$

154.

$\frac{5}{18} \div \left(-\frac{15}{24}\right)$

155.

$\frac{7}{18} \div \left(-\frac{14}{27}\right)$

156.

$\frac{7p}{12} \div \frac{21p}{8}$

157.

$\frac{5q}{12} \div \frac{15q}{8}$

158.

$\frac{8u}{15} \div \frac{12v}{25}$

159.

$\frac{12r}{25} \div \frac{18s}{35}$

160.

$-5 \div \frac{1}{2}$

161.

$-3 \div \frac{1}{4}$

162.

$\frac{3}{4} \div (-12)$

163.

$\frac{2}{5} \div (-10)$

164.

$-18 \div \left(-\frac{9}{2}\right)$

165.

$-15 \div \left(-\frac{5}{3}\right)$

166.

$\frac{1}{2} \div \left(-\frac{3}{4}\right) \div \frac{7}{8}$

167.

$\frac{11}{2} \div \frac{7}{8} \cdot \frac{2}{11}$

Everyday Math

168.

Baking A recipe for chocolate chip cookies calls for $\frac{3}{4}$ cup brown sugar. Imelda wants to double the recipe.

ⓐ How much brown sugar will Imelda need? Show your calculation. Write your result as an improper fraction and as a mixed number.

ⓑ Measuring cups usually come in sets of $\frac{1}{8}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}$, and 1 cup. Draw a diagram to show two different ways that Imelda could measure the brown sugar needed to double the recipe.

169.

Baking Nina is making 4 pans of fudge to serve after a music recital. For each pan, she needs $\frac{2}{3}$ cup of condensed milk.

1. ⓐ How much condensed milk will Nina need? Show your calculation. Write your result as an improper fraction and as a mixed number.
2. ⓑ Measuring cups usually come in sets of $\frac{1}{8}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}$, and 1 cup. Draw a diagram to show two different ways that Nina could measure the condensed milk she needs.

170.

Portions Don purchased a bulk package of candy that weighs 5 pounds. He wants to sell the candy in little bags that hold $\frac{1}{4}$ pound. How many little bags of candy can he fill from the bulk package?

171.

Portions Kristen has $\frac{3}{4}$ yards of ribbon. She wants to cut it into equal parts to make hair ribbons for her daughter's 6 dolls. How long will each doll's hair ribbon be?

Writing Exercises

172.

Explain how you find the reciprocal of a fraction.

173.

Explain how you find the reciprocal of a negative fraction.

174.

Rafael wanted to order half a medium pizza at a restaurant. The waiter told him that a medium pizza could be cut into 6 or 8 slices. Would he prefer 3 out of 6 slices or 4 out of 8 slices? Rafael replied that since he wasn't very hungry, he would prefer 3 out of 6 slices. Explain what is wrong with Rafael's reasoning.

175.

Give an example from everyday life that demonstrates how $\frac{1}{2} \cdot \frac{2}{3}$ is $\frac{1}{3}$.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
simplify fractions.			
multiply fractions.			
find reciprocals.			
divide fractions.			

ⓑ After reviewing this checklist, what will you do to become confident for all objectives?

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4.3: Multiply and Divide Mixed Numbers and Complex Fractions

Learning Objectives

By the end of this section, you will be able to:

- Multiply and divide mixed numbers
- Translate phrases to expressions with fractions
- Simplify complex fractions
- Simplify expressions written with a fraction bar

Be Prepared 4.6

Before you get started, take this readiness quiz.

Divide and reduce, if possible: $(4+5) \div (10-7)$.
If you missed this problem, review Example 3.21.

Be Prepared 4.7

Multiply and write the answer in simplified form: $\frac{1}{8} \cdot \frac{2}{3}$.
If you missed this problem, review Example 4.25.

Be Prepared 4.8

Convert $2\frac{3}{5}$ into an improper fraction.
If you missed this problem, review Example 4.11.

Multiply and Divide Mixed Numbers

In the previous section, you learned how to multiply and divide fractions. All of the examples there used either proper or improper fractions. What happens when you are asked to multiply or divide mixed numbers? Remember that we can convert a mixed number to an improper fraction. And you learned how to do that in Visualize Fractions.

Example 4.37

Multiply: $3\frac{1}{3} \cdot \frac{5}{8}$

Answer

	$3\frac{1}{3} \cdot \frac{5}{8}$
Convert $3\frac{1}{3}$ to an improper fraction.	$\frac{10}{3} \cdot \frac{5}{8}$
Multiply.	$\frac{10 \cdot 5}{3 \cdot 8}$
Look for common factors.	$\frac{2 \cdot 5 \cdot 5}{3 \cdot 2 \cdot 4}$
Remove common factors.	$\frac{5 \cdot 5}{3 \cdot 4}$
Simplify.	$\frac{25}{12}$

Notice that we left the answer as an improper fraction, $\frac{25}{12}$, and did not convert it to a mixed number. In algebra, it is preferable to write answers as improper fractions instead of mixed numbers. This avoids any possible confusion between $2\frac{1}{12}$ and $2 \cdot \frac{1}{12}$.

Try It 4.73

Multiply, and write your answer in simplified form: $5\frac{2}{3} \cdot \frac{6}{17}$.

Try It 4.74

Multiply, and write your answer in simplified form: $\frac{3}{7} \cdot 5\frac{1}{4}$.

How To

Multiply or divide mixed numbers.

1. Step 1. Convert the mixed numbers to improper fractions.
2. Step 2. Follow the rules for fraction multiplication or division.
3. Step 3. Simplify if possible.

Example 4.38

Multiply, and write your answer in simplified form: $2\frac{4}{5}\left(-1\frac{7}{8}\right)$.

Answer

	$2\frac{4}{5}\left(-1\frac{7}{8}\right)$
Convert mixed numbers to improper fractions.	$\frac{14}{5}\left(-\frac{15}{8}\right)$
Multiply.	$-\frac{14 \cdot 15}{5 \cdot 8}$
Look for common factors.	$-\frac{2 \cdot 7 \cdot 5 \cdot 3}{5 \cdot 2 \cdot 4}$
Remove common factors.	$-\frac{7 \cdot 3}{4}$
Simplify.	$-\frac{21}{4}$

Try It 4.75

Multiply, and write your answer in simplified form. $5\frac{5}{7}\left(-2\frac{5}{8}\right)$.

Try It 4.76

Multiply, and write your answer in simplified form. $-3\frac{2}{5} \cdot 4\frac{1}{6}$.

Example 4.39

Divide, and write your answer in simplified form: $3\frac{4}{7} \div 5$.

Answer

	$3\frac{4}{7} \div 5$
Convert mixed numbers to improper fractions.	$\frac{25}{7} \div \frac{5}{1}$
Multiply the first fraction by the reciprocal of the second.	$\frac{25}{7} \cdot \frac{1}{5}$
Multiply.	$\frac{25 \cdot 1}{7 \cdot 5}$
Look for common factors.	$\frac{5 \cdot 5 \cdot 1}{7 \cdot 5}$
Remove common factors.	$\frac{5 \cdot 1}{7}$
Simplify.	$\frac{5}{7}$

Try It 4.77

Divide, and write your answer in simplified form: $4\frac{3}{8} \div 7$.

Try It 4.78

Divide, and write your answer in simplified form: $2\frac{5}{8} \div 3$.

Example 4.40

Divide: $2\frac{1}{2} \div 1\frac{1}{4}$.

Answer

	$2\frac{1}{2} \div 1\frac{1}{4}$
Convert mixed numbers to improper fractions.	$\frac{5}{2} \div \frac{5}{4}$
Multiply the first fraction by the reciprocal of the second.	$\frac{5}{2} \cdot \frac{4}{5}$
Multiply.	$\frac{5 \cdot 4}{2 \cdot 5}$
Look for common factors.	$\frac{5 \cdot 2 \cdot 2}{2 \cdot 1 \cdot 5}$
Remove common factors.	$\frac{2}{1}$
Simplify.	2

Try It 4.79

Divide, and write your answer in simplified form: $2\frac{2}{3} \div 1\frac{1}{3}$.

Try It 4.80

Divide, and write your answer in simplified form: $3\frac{3}{4} \div 1\frac{1}{2}$.

Translate Phrases to Expressions with Fractions

The words *quotient* and *ratio* are often used to describe fractions. In Subtract Whole Numbers, we defined quotient as the result of division. The quotient of a and b is the result you get from dividing a by b, or $\frac{a}{b}$. Let's practice translating some phrases into algebraic expressions using these terms.

Example 4.41

Translate the phrase into an algebraic expression: "the quotient of $3x$ and 8 ."

Answer

The keyword is *quotient*; it tells us that the operation is division. Look for the words *of* and *and* to find the numbers to divide.

$$\text{The quotient of } 3x \text{ and } 8.$$

This tells us that we need to divide $3x$ by 8. $\frac{3x}{8}$

Try It 4.81

Translate the phrase into an algebraic expression: the quotient of $9s$ and 14.

Try It 4.82

Translate the phrase into an algebraic expression: the quotient of $5y$ and 6.

Example 4.42

Translate the phrase into an algebraic expression: the quotient of the difference of m and n, and p.

Answer

We are looking for the *quotient* of the *difference* of m and , and p. This means we want to divide the difference of m and n by p.

$$\frac{m-n}{p}$$

Try It 4.83

Translate the phrase into an algebraic expression: the quotient of the difference of a and b, and cd.

Try It 4.84

Translate the phrase into an algebraic expression: the quotient of the sum of p and q, and r.

Simplify Complex Fractions

Our work with fractions so far has included proper fractions, improper fractions, and mixed numbers. Another kind of fraction is called complex fraction, which is a fraction in which the numerator or the denominator contains a fraction.

Some examples of complex fractions are:

$$\frac{\frac{6}{7}}{3} \quad \frac{\frac{3}{4}}{\frac{5}{8}} \quad \frac{\frac{x}{2}}{\frac{5}{6}}$$

To simplify a complex fraction, remember that the fraction bar means division. So the complex fraction $\frac{\frac{3}{4}}{\frac{5}{8}}$ can be written as $\frac{3}{4} \div \frac{5}{8}$.

Example 4.43

Simplify: $\frac{\frac{3}{4}}{\frac{5}{8}}$.

Answer

	$\frac{\frac{3}{4}}{\frac{5}{8}}$
Rewrite as division.	$\frac{3}{4} \div \frac{5}{8}$
Multiply the first fraction by the reciprocal of the second.	$\frac{3}{4} \cdot \frac{8}{5}$
Multiply.	$\frac{3 \cdot 8}{4 \cdot 5}$
Look for common factors.	$\frac{3 \cdot \cancel{4} \cdot 2}{\cancel{4} \cdot 5}$
Remove common factors and simplify.	$\frac{6}{5}$

Try It 4.85

Simplify: $\frac{\frac{2}{3}}{\frac{5}{6}}$.

Try It 4.86

Simplify: $\frac{\frac{3}{7}}{\frac{6}{11}}$.

How To

Simplify a complex fraction.

1. Step 1. Rewrite the complex fraction as a division problem.
2. Step 2. Follow the rules for dividing fractions.
3. Step 3. Simplify if possible.

Example 4.44

Simplify: $\frac{-\frac{6}{7}}{3}$.

Answer

	$\frac{-\frac{6}{7}}{3}$
Rewrite as division.	$-\frac{6}{7} \div 3$
Multiply the first fraction by the reciprocal of the second.	$-\frac{6}{7} \cdot \frac{1}{3}$
Multiply; the product will be negative.	$-\frac{6 \cdot 1}{7 \cdot 3}$
Look for common factors.	$-\frac{3 \cdot 2 \cdot 1}{7 \cdot 3}$
Remove common factors and simplify.	$-\frac{2}{7}$

Try It 4.87

Simplify: $\frac{-\frac{8}{7}}{4}$.

Try It 4.88

Simplify: $-\frac{3}{\frac{9}{10}}$.

Example 4.45

Simplify: $\frac{\frac{x}{2}}{\frac{xy}{6}}$.

Answer

	$\frac{\frac{x}{2}}{\frac{xy}{6}}$
Rewrite as division.	$\frac{x}{2} \div \frac{xy}{6}$
Multiply the first fraction by the reciprocal of the second.	$\frac{x}{2} \cdot \frac{6}{xy}$
Multiply.	$\frac{x \cdot 6}{2 \cdot xy}$
Look for common factors.	$\frac{x \cdot 3 \cdot 2}{2 \cdot x \cdot y}$
Remove common factors and simplify.	$\frac{3}{y}$

Try It 4.89

Simplify: $\frac{\frac{a}{8}}{\frac{ab}{6}}$.

Try It 4.90

Simplify: $\frac{\frac{p}{2}}{\frac{pq}{8}}$.

Example 4.46

Simplify: $\frac{2\frac{3}{4}}{\frac{1}{8}}$.

Answer

	$\frac{2\frac{3}{4}}{\frac{1}{8}}$
Rewrite as division.	$2\frac{3}{4} \div \frac{1}{8}$
Change the mixed number to an improper fraction.	$\frac{11}{4} \div \frac{1}{8}$
Multiply the first fraction by the reciprocal of the second.	$\frac{11}{4} \cdot \frac{8}{1}$
Multiply.	$\frac{11 \cdot 8}{4 \cdot 1}$
Look for common factors.	$\frac{11 \cdot \cancel{4} \cdot 2}{\cancel{4} \cdot 1}$
Remove common factors and simplify.	22

Try It 4.91

Simplify: $\frac{\frac{5}{7}}{1\frac{2}{5}}$.

Try It 4.92

Simplify: $\frac{\frac{8}{5}}{3\frac{1}{5}}$.

Simplify Expressions with a Fraction Bar

Where does the negative sign go in a fraction? Usually, the negative sign is placed in front of the fraction, but you will sometimes see a fraction with a negative numerator or denominator. Remember that fractions represent division. The fraction $-\frac{1}{3}$ could be the result of dividing $\frac{-1}{3}$, a negative by a positive, or of dividing $\frac{1}{-3}$, a positive by a negative. When the numerator and denominator have different signs, the quotient is negative.

$$\frac{-1}{3} = -\frac{1}{3} \qquad \frac{\text{negative}}{\text{positive}} = \text{negative} \qquad \frac{1}{-3} = -\frac{1}{3} \qquad \frac{\text{positive}}{\text{negative}} = \text{negative}$$

If *both* the numerator and denominator are negative, then the fraction itself is positive because we are dividing a negative by a negative.

$$\frac{-1}{-3} = \frac{1}{3} \qquad \frac{\text{negative}}{\text{negative}} = \text{positive}$$

Placement of Negative Sign in a Fraction

For any positive numbers a and b,

$$\frac{-a}{b} = \frac{a}{-b} = -\frac{a}{b}$$

Example 4.47

Which of the following fractions are equivalent to $\frac{7}{-8}$?

$$\frac{-7}{-8}, \frac{-7}{8}, \frac{7}{8}, -\frac{7}{8}$$

Answer

The quotient of a positive and a negative is a negative, so $\frac{7}{-8}$ is negative. Of the fractions listed, $\frac{-7}{8}$ and $-\frac{7}{8}$ are also negative.

Try It 4.93

Which of the following fractions are equivalent to $\frac{-3}{5}$?

$\frac{-3}{-5}, \frac{3}{5}, -\frac{3}{5}, \frac{3}{-5}$

Try It 4.94

Which of the following fractions are equivalent to $-\frac{2}{7}$?

$\frac{-2}{-7}, \frac{-2}{7}, \frac{2}{7}, \frac{2}{-7}$

Fraction bars act as grouping symbols. The expressions above and below the fraction bar should be treated as if they were in parentheses. For example, $\frac{4+8}{5-3}$ means $(4+8) \div (5-3)$. The order of operations tells us to simplify the numerator and the denominator first—as if there were parentheses—before we divide.

We'll add fraction bars to our set of grouping symbols from Use the Language of Algebra to have a more complete set here.

Grouping Symbols

Parentheses	( )
Brackets	[ ]
Braces	{ }
Absolute value	| |
Fraction Bar	$\frac{\square}{\square}$

How To

Simplify an expression with a fraction bar.

1. Step 1. Simplify the numerator.
2. Step 2. Simplify the denominator.
3. Step 3. Simplify the fraction.

Example 4.48

Simplify: $\frac{4+8}{5-3}$.

Answer

	$\frac{4+8}{5-3}$
Simplify the expression in the numerator.	$\frac{12}{5-3}$
Simplify the expression in the denominator.	$\frac{12}{2}$
Simplify the fraction.	6

Try It 4.95

Simplify: $\frac{4+6}{11-2}$.

Try It 4.96

Simplify: $\frac{3+5}{18-2}$.

Example 4.49

Simplify: $\frac{4-2(3)}{2^2+2}$.

Answer

	$\frac{4-2(3)}{2^2+2}$
Use the order of operations. Multiply in the numerator and use the exponent in the denominator.	$\frac{4-6}{4+2}$
Simplify the numerator and the denominator.	$\frac{-2}{6}$
Simplify the fraction.	$-\frac{1}{3}$

Try It 4.97

Simplify: $\frac{6-3(5)}{3^2+3}$.

Try It 4.98

Simplify: $\frac{4-4(6)}{3^3+3}$.

Example 4.50

Simplify: $\frac{(8-4)^2}{8^2-4^2}$.

Answer

	$\frac{(8-4)^2}{8^2-4^2}$
Use the order of operations (parentheses first, then exponents).	$\frac{(4)^2}{64-16}$
Simplify the numerator and denominator.	$\frac{16}{48}$
Simplify the fraction.	$\frac{1}{3}$

Try It 4.99

Simplify: $\frac{(11-7)^2}{11^2-7^2}$.

Try It 4.100

Simplify: $\frac{(6+2)^2}{6^2+2^2}$.

Example 4.51

Simplify: $\frac{4(-3)+6(-2)}{-3(2)-2}$.

Answer

	$\frac{4(-3)+6(-2)}{-3(2)-2}$
Multiply.	$\frac{-12+(-12)}{-6-2}$
Simplify.	$\frac{-24}{-8}$
Divide.	3

Try It 4.101

Simplify: $\frac{8(-2)+4(-3)}{-5(2)+3}$.

Try It 4.102

Simplify: $\frac{7(-1)+9(-3)}{-5(3)-2}$.

Media

ACCESS ADDITIONAL ONLINE RESOURCES

- Division Involving Mixed Numbers
- Evaluate a Complex Fraction

Section 4.3 Exercises

Practice Makes Perfect

Multiply and Divide Mixed Numbers

In the following exercises, multiply and write the answer in simplified form.

176.

$4\frac{3}{8} \cdot \frac{7}{10}$

177.

$2\frac{4}{9} \cdot \frac{6}{7}$

178.

$\frac{15}{22} \cdot 3\frac{3}{5}$

179.

$\frac{25}{36} \cdot 6\frac{3}{10}$

180.

$4\frac{2}{3}\left(-1\frac{1}{8}\right)$

181.

$2\frac{2}{5}\left(-2\frac{2}{9}\right)$

182.

$-4\frac{4}{9} \cdot 5\frac{13}{16}$

183.

$-1\frac{7}{20} \cdot 2\frac{11}{12}$

In the following exercises, divide, and write your answer in simplified form.

184.

$5\frac{1}{3} \div 4$

185.

$13\frac{1}{2} \div 9$

186.

$-12 \div 3\frac{3}{11}$

187.

$-7 \div 5\frac{1}{4}$

188.

$6\frac{3}{8} \div 2\frac{1}{8}$

189.

$2\frac{1}{5} \div 1\frac{1}{10}$

190.

$-9\frac{3}{5} \div (-1\frac{3}{5})$

191.

$-18\frac{3}{4} \div (-3\frac{3}{4})$

Translate Phrases to Expressions with Fractions

In the following exercises, translate each English phrase into an algebraic expression.

192.

the quotient of $5u$ and 11

193.

the quotient of $7v$ and 13

194.

the quotient of p and q

195.

the quotient of a and b

196.

the quotient of r and the sum of s and 10

197.

the quotient of A and the difference of 3 and B

Simplify Complex Fractions

In the following exercises, simplify the complex fraction.

198.

$\frac{\frac{2}{3}}{\frac{8}{9}}$

199.

$\frac{\frac{4}{5}}{\frac{8}{15}}$

200.

$\frac{-\frac{8}{21}}{\frac{12}{35}}$

201.

$\frac{-\frac{9}{16}}{\frac{33}{40}}$

202.

$\frac{-\frac{4}{5}}{2}$

203.

$\frac{-\frac{9}{10}}{3}$

204.

$\frac{\frac{2}{5}}{8}$

205.

$\frac{\frac{5}{3}}{10}$

206.

$\frac{\frac{m}{3}}{\frac{n}{2}}$

207.

$\frac{\frac{r}{5}}{\frac{s}{3}}$

208.

$\frac{-\frac{x}{6}}{-\frac{8}{9}}$

209.

$\frac{-\frac{3}{8}}{-\frac{y}{12}}$

210.

$\frac{2\frac{4}{5}}{\frac{1}{10}}$

211.

$\frac{4\frac{2}{3}}{\frac{1}{6}}$

212.

$\frac{\frac{7}{9}}{-2\frac{4}{5}}$

213.

$\frac{\frac{3}{8}}{-6\frac{3}{4}}$

Simplify Expressions with a Fraction Bar

In the following exercises, identify the equivalent fractions.

214.

Which of the following fractions are equivalent to $\frac{5}{-11}$?
$\frac{-5}{-11}, \frac{-5}{11}, \frac{5}{11}, -\frac{5}{11}$

215.

Which of the following fractions are equivalent to $\frac{-4}{9}$?
$\frac{-4}{-9}, \frac{-4}{9}, \frac{4}{9}, -\frac{4}{9}$

216.

Which of the following fractions are equivalent to $-\frac{11}{3}$?
$\frac{-11}{3}, \frac{11}{3}, \frac{-11}{-3}, \frac{11}{-3}$

217.

Which of the following fractions are equivalent to $-\frac{13}{6}$?
$\frac{13}{6}, \frac{13}{-6}, \frac{-13}{-6}, \frac{-13}{6}$

In the following exercises, simplify.

218.

$\frac{4+11}{8}$

219.

$\frac{9+3}{7}$

220.

$\frac{22+3}{10}$

221.

$\frac{19-4}{6}$

222.

$\frac{48}{24-15}$

223.

$\frac{46}{4+4}$

224.

$\frac{-6+6}{8+4}$

225.

$\frac{-6+3}{17-8}$

226.

$\frac{22-14}{19-13}$

227.

$\frac{15+9}{18+12}$

228.

$\frac{5 \cdot 8}{-10}$

229.

$\frac{3 \cdot 4}{-24}$

230.

$\frac{4 \cdot 3}{6 \cdot 6}$

231.

$\frac{6 \cdot 6}{9 \cdot 2}$

232.

$\frac{4^2-1}{25}$

233.

$\frac{7^2+1}{60}$

234.

$\frac{8 \cdot 3+2 \cdot 9}{14+3}$

235.

$\frac{9 \cdot 6-4 \cdot 7}{22+3}$

236.

$\frac{15 \cdot 5-5^2}{2 \cdot 10}$

237.

$\frac{12 \cdot 9-3^2}{3 \cdot 18}$

238.

$\frac{5 \cdot 6-3 \cdot 4}{4 \cdot 5-2 \cdot 3}$

239.

$\frac{8 \cdot 9-7 \cdot 6}{5 \cdot 6-9 \cdot 2}$

240.

$\frac{5^2-3^2}{3-5}$

241.

$\frac{6^2-4^2}{4-6}$

242.

$\frac{2+4(3)}{-3-2^2}$

243.

$\frac{7+3(5)}{-2-3^2}$

244.

$\frac{7\cdot4-2(8-5)}{9\cdot3-3\cdot5}$

245.

$\frac{9\cdot7-3(12-8)}{8\cdot7-6\cdot6}$

246.

$\frac{9(8-2)-3(15-7)}{6(7-1)-3(17-9)}$

247.

$\frac{8(9-2)-4(14-9)}{7(8-3)-3(16-9)}$

Everyday Math

248.

Baking A recipe for chocolate chip cookies calls for $2\frac{1}{4}$ cups of flour. Graciela wants to double the recipe.

1. ⓐ How much flour will Graciela need? Show your calculation. Write your result as an improper fraction and as a mixed number.
2. ⓑ Measuring cups usually come in sets with cups for $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$, and 1 cup. Draw a diagram to show two different ways that Graciela could measure out the flour needed to double the recipe.

249.

Baking A booth at the county fair sells fudge by the pound. Their award winning "Chocolate Overdose" fudge contains $2\frac{2}{3}$ cups of chocolate chips per pound.

1. ⓐ How many cups of chocolate chips are in a half-pound of the fudge?
2. ⓑ The owners of the booth make the fudge in 10-pound batches. How many chocolate chips do they need to make a 10-pound batch? Write your results as improper fractions and as a mixed numbers.

Writing Exercises

250.

Explain how to find the reciprocal of a mixed number.

251.

Explain how to multiply mixed numbers.

252.

Randy thinks that $3\frac{1}{2}\cdot5\frac{1}{4}$ is $15\frac{1}{8}$. Explain what is wrong with Randy's thinking.

253.

Explain why $-\frac{1}{2}$, $\frac{-1}{2}$, and $\frac{1}{-2}$ are equivalent.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
multiply and divide mixed numbers.			
translate phrases to expressions with fractions.			
simplify complex fractions.			
simplify expressions written with a fraction bar.			

ⓑ What does this checklist tell you about your mastery of this section? What steps will you take to improve?

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4.4: Add and Subtract Fractions with Common Denominators

Learning Objectives

By the end of this section, you will be able to:

- Model fraction addition
- Add fractions with a common denominator
- Model fraction subtraction
- Subtract fractions with a common denominator

Be Prepared 4.9

Before you get started, take this readiness quiz.

Simplify: $2x + 9 + 3x - 4$.
If you missed this problem, review Example 2.22.

Be Prepared 4.10

Draw a model of the fraction $\frac{3}{4}$.
If you missed this problem, review Example 4.2.

Be Prepared 4.11

Simplify: $\frac{3+2}{6}$.
If you missed this problem, review Example 4.48.

Model Fraction Addition

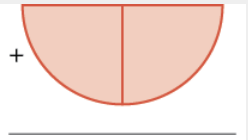
How many quarters are pictured? One quarter plus 2 quarters equals 3 quarters.

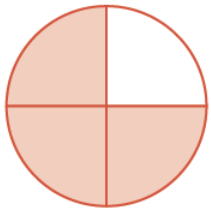


Remember, quarters are really fractions of a dollar. Quarters are another way to say fourths. So the picture of the coins shows that

$$\underset{\text{one quarter}}{\frac{1}{4}} + \underset{\text{two quarters}}{\frac{2}{4}} = \underset{\text{three}}{\frac{3}{4}}$$

Let's use fraction circles to model the same example, $\frac{1}{4} + \frac{2}{4}$.

Start with one $\frac{1}{4}$ piece.		$\frac{1}{4}$
Add two more $\frac{1}{4}$pieces.		$+\frac{2}{4}$

The result is $\frac{3}{4}$.		$\frac{3}{4}$

So again, we see that

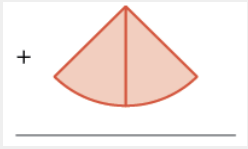
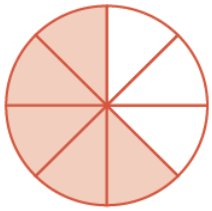
$$\frac{1}{4}+\frac{2}{4}=\frac{3}{4}$$

Manipulative Mathematics

Example 4.52

Use a model to find the sum $\frac{3}{8}+\frac{2}{8}$.

Answer

Start with three $\frac{1}{8}$ pieces.		$\frac{3}{8}$
Add two $\frac{1}{8}$pieces.		$+\frac{2}{8}$
How many $\frac{1}{8}$pieces are there?		$\frac{5}{8}$

There are five $\frac{1}{8}$ pieces, or five-eighths. The model shows that $\frac{3}{8}+\frac{2}{8}=\frac{5}{8}$.

Try It 4.103

Use a model to find each sum. Show a diagram to illustrate your model.

$\frac{1}{8}+\frac{4}{8}$

Try It 4.104

Use a model to find each sum. Show a diagram to illustrate your model.

$\frac{1}{6}+\frac{4}{6}$

Add Fractions with a Common Denominator

Example 4.52 shows that to add the same-size pieces—meaning that the fractions have the same denominator—we just add the number of pieces.

Fraction Addition

If a, b, and c are numbers where $c \neq 0$, then

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

To add fractions with a common denominator, add the numerators and place the sum over the common denominator.

Example 4.53

Find the sum: $\frac{3}{5} + \frac{1}{5}$.

Answer

	$\frac{3}{5} + \frac{1}{5}$
Add the numerators and place the sum over the common denominator.	$\frac{3+1}{5}$
Simplify.	$\frac{4}{5}$

Try It 4.105

Find each sum: $\frac{3}{6} + \frac{2}{6}$.

Try It 4.106

Find each sum: $\frac{3}{10} + \frac{7}{10}$.

Example 4.54

Find the sum: $\frac{x}{3} + \frac{2}{3}$.

Answer

	$\frac{x}{3} + \frac{2}{3}$
Add the numerators and place the sum over the common denominator.	$\frac{x+2}{3}$

Note that we cannot simplify this fraction any more. Since x and 2 are not like terms, we cannot combine them.

Try It 4.107

Find the sum: $\frac{x}{4} + \frac{3}{4}$.

Try It 4.108

Find the sum: $\frac{y}{8} + \frac{5}{8}$.

Example 4.55

Find the sum: $-\frac{9}{d} + \frac{3}{d}$.

Answer

We will begin by rewriting the first fraction with the negative sign in the numerator.

$-\frac{a}{b}=\frac{-a}{b}$

	$-\frac{9}{d}+\frac{3}{d}$
Rewrite the first fraction with the negative in the numerator.	$\frac{-9}{d}+\frac{3}{d}$
Add the numerators and place the sum over the common denominator.	$\frac{-9+3}{d}$
Simplify the numerator.	$\frac{-6}{d}$
Rewrite with negative sign in front of the fraction.	$-\frac{6}{d}$

Try It 4.109

Find the sum: $-\frac{7}{d}+\frac{8}{d}$.

Try It 4.110

Find the sum: $-\frac{6}{m}+\frac{9}{m}$.

Example 4.56

Find the sum: $\frac{2n}{11}+\frac{5n}{11}$.

Answer

	$\frac{2n}{11}+\frac{5n}{11}$
Add the numerators and place the sum over the common denominator.	$\frac{2n+5n}{11}$
Combine like terms.	$\frac{7n}{11}$

Try It 4.111

Find the sum: $\frac{3p}{8}+\frac{6p}{8}$.

Try It 4.112

Find the sum: $\frac{2q}{5}+\frac{7q}{5}$.

Example 4.57

Find the sum: $-\frac{3}{12}+\left(-\frac{5}{12}\right)$.

Answer

	$-\frac{3}{12}+\left(-\frac{5}{12}\right)$
Add the numerators and place the sum over the common denominator.	$\frac{-3+(-5)}{12}$
Add.	$\frac{-8}{12}$
Simplify the fraction.	$-\frac{2}{3}$

Try It 4.113

Find each sum: $-\frac{4}{15}+\left(-\frac{6}{15}\right)$.

Try It 4.114

Find each sum: $-\frac{5}{21}+\left(-\frac{9}{21}\right)$.

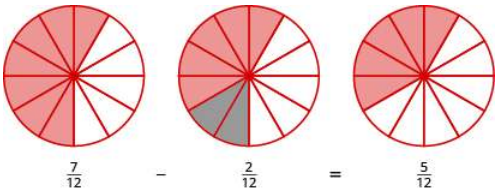
Model Fraction Subtraction

Subtracting two fractions with common denominators is much like adding fractions. Think of a pizza that was cut into 12 slices. Suppose five pieces are eaten for dinner. This means that, after dinner, there are seven pieces (or $\frac{7}{12}$ of the pizza) left in the box. If Leonardo eats 2 of these remaining pieces (or $\frac{2}{12}$ of the pizza), how much is left? There would be 5 pieces left (or $\frac{5}{12}$ of the pizza).

$$\frac{7}{12}-\frac{2}{12}=\frac{5}{12}$$

Let's use fraction circles to model the same example, $\frac{7}{12}-\frac{2}{12}$.

Start with seven $\frac{1}{12}$ pieces. Take away two $\frac{1}{12}$ pieces. How many twelfths are left?



Again, we have five twelfths, $\frac{5}{12}$.

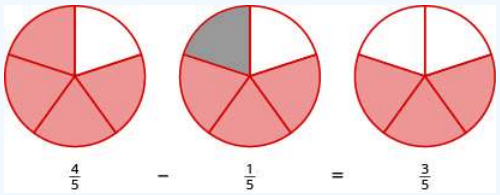
Manipulative Mathematics

Example 4.58

Use fraction circles to find the difference: $\frac{4}{5}-\frac{1}{5}$.

Answer

Start with four $\frac{1}{5}$ pieces. Take away one $\frac{1}{5}$ piece. Count how many fifths are left. There are three $\frac{1}{5}$ pieces left.



Try It 4.115

Use a model to find each difference. Show a diagram to illustrate your model.

$\frac{7}{8}-\frac{4}{8}$

Try It 4.116

Use a model to find each difference. Show a diagram to illustrate your model.

$\frac{5}{6}-\frac{4}{6}$

Subtract Fractions with a Common Denominator

We subtract fractions with a common denominator in much the same way as we add fractions with a common denominator.

Fraction Subtraction

If a, b, and c are numbers where $c \neq 0$, then

$$\frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$$

To subtract fractions with a common denominator, we subtract the numerators and place the difference over the common denominator.

Example 4.59

Find the difference: $\frac{23}{24} - \frac{14}{24}$.

Answer

	$\frac{23}{24} - \frac{14}{24}$
Subtract the numerators and place the difference over the common denominator.	$\frac{23-14}{24}$
Simplify the numerator.	$\frac{9}{24}$
Simplify the fraction by removing common factors.	$\frac{3}{8}$

Try It 4.117

Find the difference: $\frac{19}{28} - \frac{7}{28}$.

Try It 4.118

Find the difference: $\frac{27}{32} - \frac{11}{32}$.

Example 4.60

Find the difference: $\frac{y}{6} - \frac{1}{6}$.

Answer

	$\frac{y}{6} - \frac{1}{6}$
Subtract the numerators and place the difference over the common denominator.	$\frac{y-1}{6}$

The fraction is simplified because we cannot combine the terms in the numerator.

Try It 4.119

Find the difference: $\frac{x}{7} - \frac{2}{7}$.

Try It 4.120

Find the difference: $\frac{y}{14} - \frac{13}{14}$.

Example 4.61

Find the difference: $-\frac{10}{x}-\frac{4}{x}$.

Answer

Remember, the fraction $-\frac{10}{x}$ can be written as $\frac{-10}{x}$.

	$-\frac{10}{x}-\frac{4}{x}$
Subtract the numerators.	$\frac{-10-4}{x}$
Simplify.	$\frac{-14}{x}$
Rewrite with the negative sign in front of the fraction.	$-\frac{14}{x}$

Try It 4.121

Find the difference: $-\frac{9}{x}-\frac{7}{x}$.

Try It 4.122

Find the difference: $-\frac{17}{a}-\frac{5}{a}$.

Now lets do an example that involves both addition and subtraction.

Example 4.62

Simplify: $\frac{3}{8}+\left(-\frac{5}{8}\right)-\frac{1}{8}$.

Answer

	$\frac{3}{8}+\left(-\frac{5}{8}\right)-\frac{1}{8}$
Combine the numerators over the common denominator.	$\frac{3+(-5)-1}{8}$
Simplify the numerator, working left to right.	$\frac{-2-1}{8}$
Subtract the terms in the numerator.	$\frac{-3}{8}$
Rewrite with the negative sign in front of the fraction.	$-\frac{3}{8}$

Try It 4.123

Simplify: $\frac{2}{5}+\left(-\frac{4}{5}\right)-\frac{3}{5}$.

Try It 4.124

Simplify: $\frac{5}{9}+\left(-\frac{4}{9}\right)-\frac{7}{9}$.

Media

ACCESS ADDITIONAL ONLINE RESOURCES

- Adding Fractions With Pattern Blocks
- Adding Fractions With Like Denominators
- Subtracting Fractions With Like Denominators

Section 4.4 Exercises

Model Fraction Addition

In the following exercises, use a model to add the fractions. Show a diagram to illustrate your model.

254.

$\frac{2}{5}+\frac{1}{5}$

255.

$\frac{3}{10}+\frac{4}{10}$

256.

$\frac{1}{6}+\frac{3}{6}$

257.

$\frac{3}{8}+\frac{3}{8}$

Add Fractions with a Common Denominator

In the following exercises, find each sum.

258.

$\frac{4}{9}+\frac{1}{9}$

259.

$\frac{2}{9}+\frac{5}{9}$

260.

$\frac{6}{13}+\frac{7}{13}$

261.

$\frac{9}{15}+\frac{7}{15}$

262.

$\frac{x}{4}+\frac{3}{4}$

263.

$\frac{y}{3}+\frac{2}{3}$

264.

$\frac{7}{p}+\frac{9}{p}$

265.

$\frac{8}{q}+\frac{6}{q}$

266.

$\frac{8b}{9}+\frac{3b}{9}$

267.

$\frac{5a}{7}+\frac{4a}{7}$

268.

$\frac{-12y}{8}+\frac{3y}{8}$

269.

$\frac{-11x}{5}+\frac{7x}{5}$

270.

$-\frac{1}{8}+\left(-\frac{3}{8}\right)$

271.

$-\frac{1}{8}+\left(-\frac{5}{8}\right)$

272.

$-\frac{3}{16}+\left(-\frac{7}{16}\right)$

273.

$-\frac{5}{16}+\left(-\frac{9}{16}\right)$

274.

$-\frac{8}{17}+\frac{15}{17}$

275.

$-\frac{9}{19}+\frac{17}{19}$

276.

$\frac{6}{13}+\left(-\frac{10}{13}\right)+\left(-\frac{12}{13}\right)$

277.

$\frac{5}{12}+\left(-\frac{7}{12}\right)+\left(-\frac{11}{12}\right)$

Model Fraction Subtraction

In the following exercises, use a model to subtract the fractions. Show a diagram to illustrate your model.

278.

$\frac{5}{8}-\frac{2}{8}$

279.

$\frac{5}{6}-\frac{2}{6}$

Subtract Fractions with a Common Denominator

In the following exercises, find the difference.

280.

$\frac{4}{5}-\frac{1}{5}$

281.

$\frac{4}{5}-\frac{3}{5}$

282.

$\frac{11}{15}-\frac{7}{15}$

283.

$\frac{9}{13}-\frac{4}{13}$

284.

$\frac{11}{12}-\frac{5}{12}$

285.

$\frac{7}{12}-\frac{5}{12}$

286.

$\frac{4}{21}-\frac{19}{21}$

287.

$-\frac{8}{9}-\frac{16}{9}$

288.

$\frac{y}{17}-\frac{9}{17}$

289.

$\frac{x}{19} - \frac{8}{19}$

290.

$\frac{5y}{8} - \frac{7}{8}$

291.

$\frac{11z}{13} - \frac{8}{13}$

292.

$-\frac{8}{d} - \frac{3}{d}$

293.

$-\frac{7}{c} - \frac{7}{c}$

294.

$-\frac{23}{u} - \frac{15}{u}$

295.

$-\frac{29}{v} - \frac{26}{v}$

296.

$\frac{6c}{7} - \frac{5c}{7}$

297.

$\frac{12d}{11} - \frac{9d}{11}$

298.

$\frac{-4r}{13} - \frac{5r}{13}$

299.

$\frac{-7s}{3} - \frac{7s}{3}$

300.

$-\frac{3}{5} - \left(-\frac{4}{5}\right)$

301.

$-\frac{3}{7} - \left(-\frac{5}{7}\right)$

302.

$-\frac{7}{9} - \left(-\frac{5}{9}\right)$

303.

$-\frac{8}{11} - \left(-\frac{5}{11}\right)$

Mixed Practice

In the following exercises, perform the indicated operation and write your answers in simplified form.

304.

$-\frac{5}{18} \cdot \frac{9}{10}$

305.

$-\frac{3}{14} \cdot \frac{7}{12}$

306.

$\frac{n}{5} - \frac{4}{5}$

307.

$\frac{6}{11} - \frac{s}{11}$

308.

$-\frac{7}{24}+\frac{2}{24}$

309.

$-\frac{5}{18}+\frac{1}{18}$

310.

$\frac{8}{15}\div\frac{12}{5}$

311.

$\frac{7}{12}\div\frac{9}{28}$

Everyday Math

312.

Trail Mix Jacob is mixing together nuts and raisins to make trail mix. He has $\frac{6}{10}$ of a pound of nuts and $\frac{3}{10}$ of a pound of raisins. How much trail mix can he make?

313.

Baking Janet needs $\frac{5}{8}$ of a cup of flour for a recipe she is making. She only has $\frac{3}{8}$ of a cup of flour and will ask to borrow the rest from her next-door neighbor. How much flour does she have to borrow?

Writing Exercises

314.

Greg dropped his case of drill bits and three of the bits fell out. The case has slots for the drill bits, and the slots are arranged in order from smallest to largest. Greg needs to put the bits that fell out back in the case in the empty slots. Where do the three bits go? Explain how you know.
Bits in case: $\frac{1}{16}$, $\frac{1}{8}$, ___, ___, $\frac{5}{16}$, $\frac{3}{8}$, ___, $\frac{1}{2}$, $\frac{9}{16}$, $\frac{5}{8}$.
Bits that fell out: $\frac{7}{16}$, $\frac{3}{16}$, $\frac{1}{4}$.

315.

After a party, Lupe has $\frac{5}{12}$ of a cheese pizza, $\frac{4}{12}$ of a pepperoni pizza, and $\frac{4}{12}$ of a veggie pizza left. Will all the slices fit into 1 pizza box? Explain your reasoning.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
model fraction addition.			
add fractions with a common denominator.			
model fraction subtraction.			
subtract fractions with a common denominator.			

ⓑ On a scale of 1–10, how would you rate your mastery of this section in light of your responses on the checklist? How can you improve this?

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4.5: Add and Subtract Fractions with Different Denominators

Learning Objectives

By the end of this section, you will be able to:

- Find the least common denominator (LCD)
- Convert fractions to equivalent fractions with the LCD
- Add and subtract fractions with different denominators
- Identify and use fraction operations
- Use the order of operations to simplify complex fractions
- Evaluate variable expressions with fractions

Be Prepared 4.12

Before you get started, take this readiness quiz.

Find two fractions equivalent to $\frac{5}{6}$.
If you missed this problem, review Example 4.14.

Be Prepared 4.13

Simplify: $\frac{1+5\cdot 3}{2^2+4}$.
If you missed this problem, review Example 4.48.

Find the Least Common Denominator

In the previous section, we explained how to add and subtract fractions with a common denominator. But how can we add and subtract fractions with unlike denominators?

Let's think about coins again. Can you add one quarter and one dime? You could say there are two coins, but that's not very useful. To find the total value of one quarter plus one dime, you change them to the same kind of unit—cents. One quarter equals Math input error



Figure 4.7 Together, a quarter and a dime are worth 35 cents, or $\frac{35}{100}$ of a dollar.

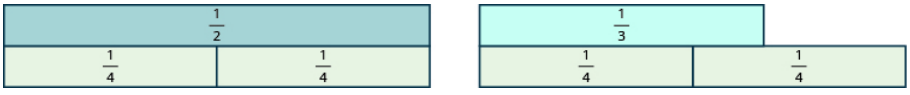
Similarly, when we add fractions with different denominators we have to convert them to equivalent fractions with a common denominator. With the coins, when we convert to cents, the denominator is 100. Since there are 100 cents in one dollar, 25 cents is $\frac{25}{100}$ and 10 cents is $\frac{10}{100}$. So we add $\frac{25}{100}+\frac{10}{100}$ to get $\frac{35}{100}$, which is 35 cents.

You have practiced adding and subtracting fractions with common denominators. Now let's see what you need to do with fractions that have different denominators.

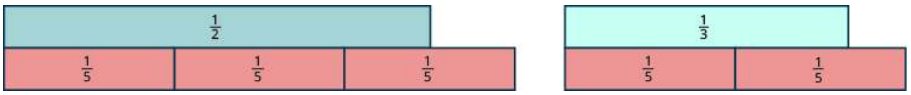
First, we will use fraction tiles to model finding the common denominator of $\frac{1}{2}$ and $\frac{1}{3}$.

We'll start with one $\frac{1}{2}$ tile and $\frac{1}{3}$ tile. We want to find a common fraction tile that we can use to match *both* $\frac{1}{2}$ and $\frac{1}{3}$ exactly.

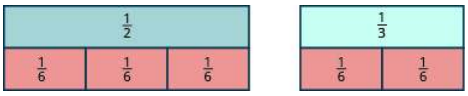
If we try the $\frac{1}{4}$ pieces, 2 of them exactly match the $\frac{1}{2}$ piece, but they do not exactly match the $\frac{1}{3}$ piece.



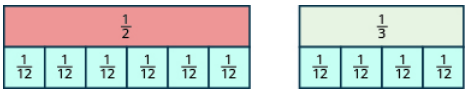
If we try the $\frac{1}{5}$ pieces, they do not exactly cover the $\frac{1}{2}$ piece or the $\frac{1}{3}$ piece.



If we try the $\frac{1}{6}$ pieces, we see that exactly 3 of them cover the $\frac{1}{2}$ piece, and exactly 2 of them cover the $\frac{1}{3}$ piece.



If we were to try the $\frac{1}{12}$ pieces, they would also work.



Even smaller tiles, such as $\frac{1}{24}$ and $\frac{1}{48}$, would also exactly cover the $\frac{1}{2}$ piece and the $\frac{1}{3}$ piece.

The denominator of the largest piece that covers both fractions is the least common denominator (LCD) of the two fractions. So, the least common denominator of $\frac{1}{2}$ and $\frac{1}{3}$ is 6.

Notice that all of the tiles that cover $\frac{1}{2}$ and $\frac{1}{3}$ have something in common: Their denominators are common multiples of 2 and 3, the denominators of $\frac{1}{2}$ and $\frac{1}{3}$. The least common multiple (LCM) of the denominators is 6, and so we say that 6 is the least common denominator (LCD) of the fractions $\frac{1}{2}$ and $\frac{1}{3}$.

Manipulative Mathematics

Least Common Denominator

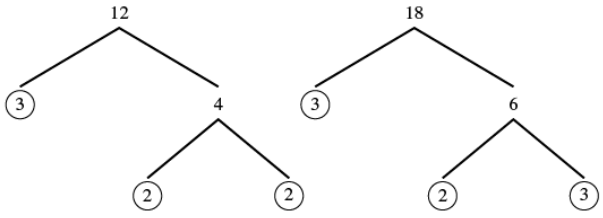
The least common denominator (LCD) of two fractions is the least common multiple (LCM) of their denominators.

To find the LCD of two fractions, we will find the LCM of their denominators. We follow the procedure we used earlier to find the LCM of two numbers. We only use the denominators of the fractions, not the numerators, when finding the LCD.

Example 4.63

Find the LCD for the fractions $\frac{7}{12}$ and $\frac{5}{18}$.

Answer

Factor each denominator into its primes.	
List the primes of 12 and the primes of 18 lining them up in columns when possible.	$12 = 2 \cdot 2 \cdot 3$ $18 = 2 \cdot \quad 3 \cdot 3$
Bring down the columns.	$12 = 2 \cdot 2 \cdot 3$ $18 = 2 \cdot \quad 3 \cdot 3$ $\text{LCM} = 2 \cdot 2 \cdot 3 \cdot 3$
Multiply the factors. The product is the LCM.	$\text{LCM} = 36$
The LCM of 12 and 18 is 36, so the LCD of $\frac{7}{12}$ and $\frac{5}{18}$ is 36.	LCD of $\frac{7}{12}$ and $\frac{5}{18}$ is 36.

Try It 4.125

Find the least common denominator for the fractions: $\frac{7}{12}$ and $\frac{11}{15}$.

Try It 4.126

Find the least common denominator for the fractions: $\frac{13}{15}$ and $\frac{17}{5}$.

To find the LCD of two fractions, find the LCM of their denominators. Notice how the steps shown below are similar to the steps we took to find the LCM.

How To

Find the least common denominator (LCD) of two fractions.

1. Step 1. Factor each denominator into its primes.
2. Step 2. List the primes, matching primes in columns when possible.
3. Step 3. Bring down the columns.
4. Step 4. Multiply the factors. The product is the LCM of the denominators.
5. Step 5. The LCM of the denominators is the LCD of the fractions.

Example 4.64

Find the least common denominator for the fractions $\frac{8}{15}$ and $\frac{11}{24}$.

Answer

To find the LCD, we find the LCM of the denominators.

Find the LCM of 15 and 24.

$$\begin{array}{ll} 15 = & 3 \cdot 5 \\ 24 = 2 \cdot 2 \cdot 2 \cdot & 3 \\ \hline \text{LCD} = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5 & \\ \text{LCD} = 120 & \end{array}$$

The LCM of 15 and 24 is 120. So, the LCD of $\frac{8}{15}$ and $\frac{11}{24}$ is 120.

Try It 4.127

Find the least common denominator for the fractions: $\frac{13}{24}$ and $\frac{17}{32}$.

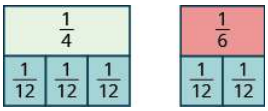
Try It 4.128

Find the least common denominator for the fractions: $\frac{9}{28}$ and $\frac{21}{32}$.

Convert Fractions to Equivalent Fractions with the LCD

Earlier, we used fraction tiles to see that the LCD of $\frac{1}{4}$ when $\frac{1}{6}$ is 12. We saw that three $\frac{1}{12}$ pieces exactly covered $\frac{1}{4}$ and two $\frac{1}{12}$ pieces exactly covered $\frac{1}{6}$, so

$$\frac{1}{4} = \frac{3}{12} \text{and} \frac{1}{6} = \frac{2}{12}.$$



We say that $\frac{1}{4}$ and $\frac{3}{12}$ are equivalent fractions and also that $\frac{1}{6}$ and $\frac{2}{12}$ are equivalent fractions.

We can use the Equivalent Fractions Property to algebraically change a fraction to an equivalent one. Remember, two fractions are equivalent if they have the same value. The Equivalent Fractions Property is repeated below for reference.

Equivalent Fractions Property

If a, b, c are whole numbers where $b \neq 0, c \neq 0$, then

$$\frac{a}{b} = \frac{a \cdot c}{b \cdot c} \quad \text{and} \quad \frac{a \cdot c}{b \cdot c} = \frac{a}{b}$$

To add or subtract fractions with different denominators, we will first have to convert each fraction to an equivalent fraction with the LCD. Let's see how to change $\frac{1}{4}$ and $\frac{1}{6}$ to equivalent fractions with denominator 12 without using models.

Example 4.65

Convert $\frac{1}{4}$ and $\frac{1}{6}$ to equivalent fractions with denominator 12, their LCD.

Answer

Find the LCD.	The LCD of $\frac{1}{4}$ and $\frac{1}{6}$ is 12.
Find the number to multiply 4 to get 12.	$4 \cdot 3 = 12$
Find the number to multiply 6 to get 12.	$6 \cdot 2 = 12$
Use the Equivalent Fractions Property to convert each fraction to an equivalent fraction with the LCD, multiplying both the numerator and denominator of each fraction by the same number.	$\frac{1}{4}$ $\quad$ $\frac{1}{6}$ $\frac{1 \cdot 3}{4 \cdot 3}$ $\quad$ $\frac{1 \cdot 2}{6 \cdot 2}$
Simplify the numerators and denominators.	$\frac{3}{12}$ $\quad$ $\frac{2}{12}$

We do not reduce the resulting fractions. If we did, we would get back to our original fractions and lose the common denominator.

Try It 4.129

Change to equivalent fractions with the LCD:

$\frac{3}{4}$ and $\frac{5}{6}$, LCD $= 12$

Try It 4.130

Change to equivalent fractions with the LCD:

$-\frac{7}{12}$ and $\frac{11}{15}$, LCD $= 60$

How To

Convert two fractions to equivalent fractions with their LCD as the common denominator.

1. Step 1. Find the LCD.
2. Step 2. For each fraction, determine the number needed to multiply the denominator to get the LCD.
3. Step 3. Use the Equivalent Fractions Property to multiply both the numerator and denominator by the number you found in Step 2.
4. Step 4. Simplify the numerator and denominator.

Example 4.66

Convert $\frac{8}{15}$ and $\frac{11}{24}$ to equivalent fractions with denominator 120, their LCD.

Answer

	The LCD is 120. We will start at Step 2.
Find the number that must multiply 15 to get 120.	$15 \cdot 8 = 120$
Find the number that must multiply 24 to get 120.	$24 \cdot 5 = 120$
Use the Equivalent Fractions Property.	$\frac{8 \cdot 8}{15 \cdot 8}$ $\frac{11 \cdot 5}{24 \cdot 5}$
Simplify the numerators and denominators.	$\frac{64}{120}$ $\frac{55}{120}$

Try It 4.131

Change to equivalent fractions with the LCD:

$\frac{13}{24}$ and $\frac{17}{32}$, LCD 96

Try It 4.132

Change to equivalent fractions with the LCD:

$\frac{9}{28}$ and $\frac{27}{32}$, LCD 224

Add and Subtract Fractions with Different Denominators

Once we have converted two fractions to equivalent forms with common denominators, we can add or subtract them by adding or subtracting the numerators.

How To

Add or subtract fractions with different denominators.

1. Step 1. Find the LCD.
2. Step 2. Convert each fraction to an equivalent form with the LCD as the denominator.
3. Step 3. Add or subtract the fractions.
4. Step 4. Write the result in simplified form.

Example 4.67

Add: $\frac{1}{2} + \frac{1}{3}$.

Answer

	$\frac{1}{2} + \frac{1}{3}$
Find the LCD of 2, 3. $2 = 2$ $3 = 3$ $\text{LCD} = 2 \cdot 3$ $\text{LCD} = 6$	
Change into equivalent fractions with the LCD 6.	$\frac{1 \cdot 3}{2 \cdot 3} + \frac{1 \cdot 2}{3 \cdot 2}$
Simplify the numerators and denominators.	$\frac{3}{6} + \frac{2}{6}$
Add.	$\frac{5}{6}$

Remember, always check to see if the answer can be simplified. Since 5 and 6 have no common factors, the fraction $\frac{5}{6}$ cannot be reduced.

Try It 4.133

Add: $\frac{1}{4}+\frac{1}{3}$.

Try It 4.134

Add: $\frac{1}{2}+\frac{1}{5}$.

Example 4.68

Subtract: $\frac{1}{2}-\left(-\frac{1}{4}\right)$.

Answer

	$\frac{1}{2}-\left(-\frac{1}{4}\right)$
Find the LCD of 2 and 4. $\begin{array}{r} 2=2 \\ 4=2\cdot 2 \\ \hline \text{LCD}=2\cdot 2 \\ \text{LCD}=4 \end{array}$	
Rewrite as equivalent fractions using the LCD 4.	$\frac{1\cdot 2}{2\cdot 2}-\left(-\frac{1}{4}\right)$
Simplify the first fraction.	$\frac{2}{4}-\left(-\frac{1}{4}\right)$
Subtract.	$\frac{2-(-1)}{4}$
Simplify.	$\frac{3}{4}$

One of the fractions already had the least common denominator, so we only had to convert the other fraction.

Try It 4.135

Simplify: $\frac{1}{2}-\left(-\frac{1}{8}\right)$.

Try It 4.136

Simplify: $\frac{1}{3}-\left(-\frac{1}{6}\right)$.

Example 4.69

Add: $\frac{7}{12}+\frac{5}{18}$.

Answer

	$\frac{7}{12}+\frac{5}{18}$
Find the LCD of 12 and 18. $\begin{array}{r} 12=2\cdot 2\cdot 3 \\ 18=2\cdot \quad 3\cdot 3 \\ \hline \text{LCD}=2\cdot 2\cdot 3\cdot 3 \\ \text{LCD}=36 \end{array}$	
Rewrite as equivalent fractions with the LCD.	$\frac{7\cdot 3}{12\cdot 3}+\frac{5\cdot 2}{18\cdot 2}$
Simplify the numerators and denominators.	$\frac{21}{36}+\frac{10}{36}$
Add.	$\frac{31}{36}$

Because 31 is a prime number, it has no factors in common with 36. The answer is simplified.

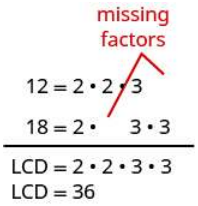
Try It 4.137

Add: $\frac{7}{12} + \frac{11}{15}$.

Try It 4.138

Add: $\frac{13}{15} + \frac{17}{20}$.

When we use the Equivalent Fractions Property, there is a quick way to find the number you need to multiply by to get the LCD. Write the factors of the denominators and the LCD just as you did to find the LCD. The "missing" factors of each denominator are the numbers you need.



The LCD, 36, has 2 factors of 2 and 2 factors of 3.

Twelve has two factors of 2, but only one of 3—so it is 'missing' one 3. We multiplied the numerator and denominator of $\frac{7}{12}$ by 3 to get an equivalent fraction with denominator 36.

Eighteen is missing one factor of 2—so you multiply the numerator and denominator $\frac{5}{18}$ by 2 to get an equivalent fraction with denominator 36. We will apply this method as we subtract the fractions in the next example.

Example 4.70

Subtract: $\frac{7}{15} - \frac{19}{24}$.

Answer

	$\frac{7}{15} - \frac{19}{24}$
Find the LCD. $15 = 3 \cdot 5$ $24 = 2 \cdot 2 \cdot 2 \cdot 3$ $LCD = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5$ $LCD = 120$ 15 is 'missing' three factors of 2 24 is 'missing' a factor of 5	
Rewrite as equivalent fractions with the LCD.	$\frac{7 \cdot 8}{15 \cdot 8} - \frac{19 \cdot 5}{24 \cdot 5}$
Simplify each numerator and denominator.	$\frac{56}{120} - \frac{95}{120}$
Subtract.	$-\frac{39}{120}$
Rewrite showing the common factor of 3.	$-\frac{13 \cdot 3}{40 \cdot 3}$
Remove the common factor to simplify.	$-\frac{13}{40}$

Try It 4.139

Subtract: $\frac{13}{24} - \frac{17}{32}$.

Try It 4.140

Subtract: $\frac{21}{32} - \frac{9}{28}$.

Example 4.71

Add: $-\frac{11}{30} + \frac{23}{42}$.

Answer

	$-\frac{11}{30} + \frac{23}{42}$
Find the LCD. $30 = 2 \cdot 3 \cdot 5$ $42 = 2 \cdot 3 \cdot 7$ $LCD = 2 \cdot 3 \cdot 5 \cdot 7$ $LCD = 210$	
Rewrite as equivalent fractions with the LCD.	$-\frac{11 \cdot 7}{30 \cdot 7} + \frac{23 \cdot 5}{42 \cdot 5}$
Simplify each numerator and denominator.	$-\frac{77}{210} + \frac{115}{210}$
Add.	$\frac{38}{210}$
Rewrite showing the common factor of 2.	$\frac{19 \cdot 2}{105 \cdot 2}$
Remove the common factor to simplify.	$\frac{19}{105}$

Try It 4.141

Add: $-\frac{13}{42} + \frac{17}{35}$.

Try It 4.142

Add: $-\frac{19}{24} + \frac{17}{32}$.

In the next example, one of the fractions has a variable in its numerator. We follow the same steps as when both numerators are numbers.

Example 4.72

Add: $\frac{3}{5} + \frac{x}{8}$.

Answer

The fractions have different denominators.

	$\frac{3}{5} + \frac{x}{8}$
Find the LCD. $5 = 5$ $8 = 2 \cdot 2 \cdot 2$ $LCD = 2 \cdot 2 \cdot 2 \cdot 5$ $LCD = 40$	
Rewrite as equivalent fractions with the LCD.	$\frac{3 \cdot 8}{5 \cdot 8} + \frac{x \cdot 5}{8 \cdot 5}$
Simplify the numerators and denominators.	$\frac{24}{40} + \frac{5x}{40}$
Add.	$\frac{24 + 5x}{40}$

We cannot add 24 and $5x$ since they are not like terms, so we cannot simplify the expression any further.

Try It 4.143

Add: $\frac{y}{6} + \frac{7}{9}$.

Try It 4.144

Add: $\frac{x}{6} + \frac{7}{15}$.

Identify and Use Fraction Operations

By now in this chapter, you have practiced multiplying, dividing, adding, and subtracting fractions. The following table summarizes these four fraction operations. Remember: You need a common denominator to add or subtract fractions, but not to multiply or divide fractions

Summary of Fraction Operations

Fraction multiplication: Multiply the numerators and multiply the denominators.

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

Fraction division: Multiply the first fraction by the reciprocal of the second.

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$$

Fraction addition: Add the numerators and place the sum over the common denominator. If the fractions have different denominators, first convert them to equivalent forms with the LCD.

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

Fraction subtraction: Subtract the numerators and place the difference over the common denominator. If the fractions have different denominators, first convert them to equivalent forms with the LCD.

$$\frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$$

Example 4.73

Simplify:

1. ⓐ $-\frac{1}{4} + \frac{1}{6}$
2. ⓑ $-\frac{1}{4} \div \frac{1}{6}$

Answer

First we ask ourselves, “What is the operation?”

ⓐ The operation is addition.

Do the fractions have a common denominator? No.

	$-\frac{1}{4} + \frac{1}{6}$
Find the LCD. $4 = 2 \cdot 2$ $6 = 2 \cdot 3$ $\text{LCD} = 2 \cdot 2 \cdot 3$ $\text{LCD} = 12$	
Rewrite each fraction as an equivalent fraction with the LCD.	$-\frac{1 \cdot 3}{4 \cdot 3} + \frac{1 \cdot 2}{6 \cdot 2}$

Simplify the numerators and denominators.	$-\frac{3}{12}+\frac{2}{12}$
Add the numerators and place the sum over the common denominator.	$-\frac{1}{12}$
Check to see if the answer can be simplified. It cannot.	

ⓑ The operation is division. We do not need a common denominator.

	$-\frac{1}{4}\div\frac{1}{6}$
To divide fractions, multiply the first fraction by the reciprocal of the second.	$-\frac{1}{4}\cdot\frac{6}{1}$
Multiply.	$-\frac{6}{4}$
Simplify.	$-\frac{3}{2}$

Try It 4.145

Simplify each expression:

1. ⓐ $-\frac{3}{4}-\frac{1}{6}$
2. ⓑ $-\frac{3}{4}\cdot\frac{1}{6}$

Try It 4.146

Simplify each expression:

1. ⓐ $\frac{5}{6}\div\left(-\frac{1}{4}\right)$
2. ⓑ $\frac{5}{6}-\left(-\frac{1}{4}\right)$

Example 4.74

Simplify:

1. ⓐ $\frac{5x}{6}-\frac{3}{10}$
2. ⓑ $\frac{5x}{6}\cdot\frac{3}{10}$

Answer

ⓐ The operation is subtraction. The fractions do not have a common denominator.

	$\frac{5x}{6}-\frac{3}{10}$
Rewrite each fraction as an equivalent fraction with the LCD, 30.	$\frac{5x\cdot5}{6\cdot5}-\frac{3\cdot3}{10\cdot3}$
	$\frac{25x}{30}-\frac{9}{30}$
Subtract the numerators and place the difference over the common denominator.	$\frac{25x-9}{30}$

ⓑ The operation is multiplication; no need for a common denominator.

	$\frac{5x}{6}\cdot\frac{3}{10}$
To multiply fractions, multiply the numerators and multiply the denominators.	$\frac{5x\cdot3}{6\cdot10}$
Rewrite, showing common factors.	$\frac{\cancel{5}\cdot x\cdot\cancel{3}}{2\cdot\cancel{3}\cdot2\cdot\cancel{5}}$

Remove common factors to simplify.	$\frac{x}{4}$

Try It 4.147

Simplify:

1. ⓐ $\frac{(27a-32)}{36}$
2. ⓑ $\frac{2a}{3}$

Try It 4.148

Simplify:

1. ⓐ $\frac{(24k+25)}{30}$
2. ⓑ $\frac{24k}{5}$

Use the Order of Operations to Simplify Complex Fractions

In Multiply and Divide Mixed Numbers and Complex Fractions, we saw that a complex fraction is a fraction in which the numerator or denominator contains a fraction. We simplified complex fractions by rewriting them as division problems. For example,

$$\frac{\frac{3}{4}}{\frac{5}{8}} = \frac{3}{4} \div \frac{5}{8}$$

Now we will look at complex fractions in which the numerator or denominator can be simplified. To follow the order of operations, we simplify the numerator and denominator separately first. Then we divide the numerator by the denominator.

How To

Simplify complex fractions.

1. Step 1. Simplify the numerator.
2. Step 2. Simplify the denominator.
3. Step 3. Divide the numerator by the denominator.
4. Step 4. Simplify if possible.

Example 4.75

Simplify: $\frac{\left(\frac{1}{2}\right)^2}{4+3^2}$.

Answer

	$\frac{\left(\frac{1}{2}\right)^2}{4+3^2}$
Simplify the numerator.	$\frac{\frac{1}{4}}{4+3^2}$
Simplify the term with the exponent in the denominator.	$\frac{\frac{1}{4}}{4+9}$
Add the terms in the denominator.	$\frac{\frac{1}{4}}{13}$
Divide the numerator by the denominator.	$\frac{1}{4} \div 13$
Rewrite as multiplication by the reciprocal.	$\frac{1}{4} \cdot \frac{1}{13}$
Multiply.	$\frac{1}{52}$

Try It 4.149

Simplify: $\frac{\left(\frac{1}{3}\right)^2}{2^3+2}$.

Try It 4.150

Simplify: $\frac{1+4^2}{\left(\frac{1}{4}\right)^2}$.

Example 4.76

Simplify: $\frac{\frac{1}{2}+\frac{2}{3}}{\frac{3}{4}-\frac{1}{6}}$.

Answer

	$\frac{\frac{1}{2}+\frac{2}{3}}{\frac{3}{4}-\frac{1}{6}}$
Rewrite numerator with the LCD of 6 and denominator with LCD of 12.	$\frac{\frac{3}{6}+\frac{4}{6}}{\frac{9}{12}-\frac{2}{12}}$
Add in the numerator. Subtract in the denominator.	$\frac{\frac{7}{6}}{\frac{7}{12}}$
Divide the numerator by the denominator.	$\frac{7}{6} \div \frac{7}{12}$
Rewrite as multiplication by the reciprocal.	$\frac{7}{6} \cdot \frac{12}{7}$
Rewrite, showing common factors.	$\frac{7 \cdot 6 \cdot 2}{6 \cdot 7 \cdot 1}$
Simplify.	2

Try It 4.151

Simplify: $\frac{\frac{1}{3}+\frac{1}{2}}{\frac{3}{4}-\frac{1}{3}}$.

Try It 4.152

Simplify: $\frac{\frac{2}{3}-\frac{1}{2}}{\frac{1}{4}+\frac{1}{3}}$.

Evaluate Variable Expressions with Fractions

We have evaluated expressions before, but now we can also evaluate expressions with fractions. Remember, to evaluate an expression, we substitute the value of the variable into the expression and then simplify.

Example 4.77

Evaluate $x + \frac{1}{3}$ when

1. ⓐ $x = -\frac{1}{3}$
2. ⓑ $x = -\frac{3}{4}$.

Answer

ⓐ To evaluate $x + \frac{1}{3}$ when $x = -\frac{1}{3}$, substitute $-\frac{1}{3}$ for x in the expression.

	$x + \frac{1}{3}$
Substitute $-\frac{1}{3}$ for x.	$-\frac{1}{3} + \frac{1}{3}$
Simplify.	0

ⓑ To evaluate $x + \frac{1}{3}$ when $x = -\frac{3}{4}$, we substitute $-\frac{3}{4}$ for x in the expression.

	$x + \frac{1}{3}$

Substitute $-\frac{3}{4}$ for x.	$-\frac{3}{4}+\frac{1}{3}$
Rewrite as equivalent fractions with the LCD, 12.	$-\frac{3\cdot3}{4\cdot3}+\frac{1\cdot4}{3\cdot4}$
Simplify the numerators and denominators.	$-\frac{9}{12}+\frac{4}{12}$
Add.	$-\frac{5}{12}$

Try It 4.153

Evaluate: $x+\frac{3}{4}$ when

1. ⓐ $x=-\frac{7}{4}$
2. ⓑ $x=-\frac{5}{4}$

Try It 4.154

Evaluate: $y+\frac{1}{2}$ when

1. ⓐ $y=\frac{2}{3}$
2. ⓑ $y=-\frac{3}{4}$

Example 4.78

Evaluate $y-\frac{5}{6}$ when $y=-\frac{2}{3}$.

Answer

We substitute $-\frac{2}{3}$ for y in the expression.

	$y-\frac{5}{6}$
Substitute $-\frac{2}{3}$ for y.	$-\frac{2}{3}-\frac{5}{6}$
Rewrite as equivalent fractions with the LCD, 6.	$-\frac{4}{6}-\frac{5}{6}$
Subtract.	$-\frac{9}{6}$
Simplify.	$-\frac{3}{2}$

Try It 4.155

Evaluate: $y-\frac{1}{2}$ when $y=-\frac{1}{4}$.

Try It 4.156

Evaluate: $x-\frac{3}{8}$ when $x=-\frac{5}{2}$.

Example 4.79

Evaluate $2x^2y$ when $x=\frac{1}{4}$ and $y=-\frac{2}{3}$.

Answer

Substitute the values into the expression. In $2x^2y$, the exponent applies only to x.

	$2x^2y$

Substitute $\frac{1}{4}$ for x and $-\frac{2}{3}$ for y.	$2\left(\frac{1}{4}\right)^2\left(-\frac{2}{3}\right)$
Simplify exponents first.	$2\left(\frac{1}{16}\right)\left(-\frac{2}{3}\right)$
Multiply. The product will be negative.	$-\frac{2}{1}\cdot\frac{1}{16}\cdot\frac{2}{3}$
Simplify.	$-\frac{4}{48}$
Remove the common factors.	$-\frac{1\cdot\cancel{4}}{\cancel{4}\cdot 12}$
Simplify.	$-\frac{1}{12}$

Try It 4.157

Evaluate. $3ab^2$ when $a=-\frac{2}{3}$ and $b=-\frac{1}{2}$.

Try It 4.158

Evaluate. $4c^3d$ when $c=-\frac{1}{2}$ and $d=-\frac{4}{3}$.

Example 4.80

Evaluate $\frac{p+q}{r}$ when $p=-4, q=-2$, and $r=8$.

Answer

We substitute the values into the expression and simplify.

	$\frac{p+q}{r}$
Substitute -4 for p, -2 for q and 8 for r.	$\frac{-4+(-2)}{8}$
Add in the numerator first.	$-\frac{6}{8}$
Simplify.	$-\frac{3}{4}$

Try It 4.159

Evaluate: $\frac{a+b}{c}$ when $a=-8, b=-7$, and $c=6$.

Try It 4.160

Evaluate: $\frac{x+y}{z}$ when $x=9, y=-18$, and $z=-6$.

Section 4.5 Exercises

Practice Makes Perfect

Find the Least Common Denominator (LCD)

In the following exercises, find the least common denominator (LCD) for each set of fractions.

316.

$\frac{2}{3}$ and $\frac{3}{4}$

317.

$\frac{3}{4}$ and $\frac{2}{5}$

318.

$\frac{7}{12}$ and $\frac{5}{8}$

319.

$\frac{9}{16}$ and $\frac{7}{12}$

320.

$\frac{13}{30}$ and $\frac{25}{42}$

321.

$\frac{23}{30}$ and $\frac{5}{48}$

322.

$\frac{21}{35}$ and $\frac{39}{56}$

323.

$\frac{18}{35}$ and $\frac{33}{49}$

324.

$\frac{2}{3}$, $\frac{1}{6}$, and $\frac{3}{4}$

325.

$\frac{2}{3}$, $\frac{1}{4}$, and $\frac{3}{5}$

Convert Fractions to Equivalent Fractions with the LCD

In the following exercises, convert to equivalent fractions using the LCD.

326.

$\frac{1}{3}$ and $\frac{1}{4}$, LCD $= 12$

327.

$\frac{1}{4}$ and $\frac{1}{5}$, LCD $= 20$

328.

$\frac{5}{12}$ and $\frac{7}{8}$, LCD $= 24$

329.

$\frac{7}{12}$ and $\frac{5}{8}$, LCD $= 24$

330.

$\frac{13}{16}$ and $-\frac{11}{12}$, LCD $= 48$

331.

$\frac{11}{16}$ and $-\frac{5}{12}$, LCD $= 48$

332.

$\frac{1}{3}$, $\frac{5}{6}$, and $\frac{3}{4}$, LCD $= 12$

333.

$\frac{1}{3}$, $\frac{3}{4}$, and $\frac{3}{5}$, LCD $= 60$

Add and Subtract Fractions with Different Denominators

In the following exercises, add or subtract. Write the result in simplified form.

334.

$\frac{1}{3} + \frac{1}{5}$

335.

$\frac{1}{4}+\frac{1}{5}$

336.

$\frac{1}{2}+\frac{1}{7}$

337.

$\frac{1}{3}+\frac{1}{8}$

338.

$\frac{1}{3}-\left(-\frac{1}{9}\right)$

339.

$\frac{1}{4}-\left(-\frac{1}{8}\right)$

340.

$\frac{1}{5}-\left(-\frac{1}{10}\right)$

341.

$\frac{1}{2}-\left(-\frac{1}{6}\right)$

342.

$\frac{2}{3}+\frac{3}{4}$

343.

$\frac{3}{4}+\frac{2}{5}$

344.

$\frac{7}{12}+\frac{5}{8}$

345.

$\frac{5}{12}+\frac{3}{8}$

346.

$\frac{7}{12}-\frac{9}{16}$

347.

$\frac{7}{16}-\frac{5}{12}$

348.

$\frac{11}{12}-\frac{3}{8}$

349.

$\frac{5}{8}-\frac{7}{12}$

350.

$\frac{2}{3}-\frac{3}{8}$

351.

$\frac{5}{6}-\frac{3}{4}$

352.

$-\frac{11}{30}+\frac{27}{40}$

353.

$-\frac{9}{20}+\frac{17}{30}$

354.

$-\frac{13}{30}+\frac{25}{42}$

355.

$-\frac{23}{30}+\frac{5}{48}$

356.

$-\frac{39}{56}-\frac{22}{35}$

357.

$-\frac{33}{49}-\frac{18}{35}$

358.

$-\frac{2}{3}-\left(-\frac{3}{4}\right)$

359.

$-\frac{3}{4}-\left(-\frac{4}{5}\right)$

360.

$-\frac{9}{16}-\left(-\frac{4}{5}\right)$

361.

$-\frac{7}{20}-\left(-\frac{5}{8}\right)$

362.

$1+\frac{7}{8}$

363.

$1+\frac{5}{6}$

364.

$1-\frac{5}{9}$

365.

$1-\frac{3}{10}$

366.

$\frac{x}{3}+\frac{1}{4}$

367.

$\frac{y}{2}+\frac{2}{3}$

368.

$\frac{y}{4}-\frac{3}{5}$

369.

$\frac{x}{5}-\frac{1}{4}$

Identify and Use Fraction Operations

In the following exercises, perform the indicated operations. Write your answers in simplified form.

370.

1. ⓐ $\frac{3}{4}+\frac{1}{6}$
2. ⓑ $\frac{3}{4}\div\frac{1}{6}$

371.

1. ⓐ $\frac{2}{3}+\frac{1}{6}$
2. ⓑ $\frac{2}{3}\div\frac{1}{6}$

372.

1. ⓐ $-\frac{2}{5}-\frac{1}{8}$
2. ⓑ $-\frac{2}{5}\cdot\frac{1}{8}$

373.

1. ⓐ $-\frac{4}{5}-\frac{1}{8}$
2. ⓑ $-\frac{4}{5} \cdot \frac{1}{8}$

374.

1. ⓐ $\frac{5n}{6} \div \frac{8}{15}$
2. ⓑ $\frac{5n}{6}-\frac{8}{15}$

375.

1. ⓐ $\frac{3a}{8} \div \frac{7}{12}$
2. ⓑ $\frac{3a}{8}-\frac{7}{12}$

376.

1. ⓐ $\frac{9}{10} \cdot \left(-\frac{11d}{12}\right)$
2. ⓑ $\frac{9}{10}+\left(-\frac{11d}{12}\right)$

377.

1. ⓐ $\frac{4}{15} \cdot \left(-\frac{5q}{9}\right)$
2. ⓑ $\frac{4}{15}+\left(-\frac{5q}{9}\right)$

378.

$-\frac{3}{8} \div \left(-\frac{3}{10}\right)$

379.

$-\frac{5}{12} \div \left(-\frac{5}{9}\right)$

380.

$-\frac{3}{8}+\frac{5}{12}$

381.

$-\frac{1}{8}+\frac{7}{12}$

382.

$\frac{5}{6}-\frac{1}{9}$

383.

$\frac{5}{9}-\frac{1}{6}$

384.

$\frac{3}{8} \cdot \left(-\frac{10}{21}\right)$

385.

$\frac{7}{12} \cdot \left(-\frac{8}{35}\right)$

386.

$-\frac{7}{15}-\frac{y}{4}$

387.

$-\frac{3}{8}-\frac{x}{11}$

388.

$\frac{11}{12a} \cdot \frac{9a}{16}$

389.

$\frac{10y}{13} \cdot \frac{8}{15y}$

Use the Order of Operations to Simplify Complex Fractions

In the following exercises, simplify.

390.

$\frac{\left(\frac{1}{5}\right)^2}{2+3^2}$

391.

$\frac{\left(\frac{1}{3}\right)^2}{5+2^2}$

392.

$\frac{2^3+4^2}{\left(\frac{2}{3}\right)^2}$

393.

$\frac{3^3-3^2}{\left(\frac{3}{4}\right)^2}$

394.

$\frac{\left(\frac{3}{5}\right)^2}{\left(\frac{3}{7}\right)^2}$

395.

$\frac{\left(\frac{3}{4}\right)^2}{\left(\frac{5}{8}\right)^2}$

396.

$\frac{2}{\frac{1}{3}+\frac{1}{5}}$

397.

$\frac{5}{\frac{1}{4}+\frac{1}{3}}$

398.

$\frac{\frac{2}{3}+\frac{1}{2}}{\frac{3}{4}-\frac{2}{3}}$

399.

$\frac{\frac{3}{4}+\frac{1}{2}}{\frac{5}{6}-\frac{2}{3}}$

400.

$\frac{\frac{7}{8}-\frac{2}{3}}{\frac{1}{2}+\frac{3}{8}}$

401.

$\frac{\frac{3}{4}-\frac{3}{5}}{\frac{1}{4}+\frac{2}{5}}$

Mixed Practice

In the following exercises, simplify.

402.

$\frac{1}{2}+\frac{2}{3}\cdot\frac{5}{12}$

403.

$\frac{1}{3}+\frac{2}{5}\cdot\frac{3}{4}$

404.

$1-\frac{3}{5}\div\frac{1}{10}$

405.

$1-\frac{5}{6}\div\frac{1}{12}$

406.

$\frac{2}{3}+\frac{1}{6}+\frac{3}{4}$

407.

$\frac{2}{3}+\frac{1}{4}+\frac{3}{5}$

408.

$\frac{3}{8} - \frac{1}{6} + \frac{3}{4}$

409.

$\frac{2}{5} + \frac{5}{8} - \frac{3}{4}$

410.

$12\left(\frac{9}{20} - \frac{4}{15}\right)$

411.

$8\left(\frac{15}{16} - \frac{5}{6}\right)$

412.

$\frac{\frac{5}{8}+\frac{1}{6}}{\frac{19}{24}}$

413.

$\frac{\frac{1}{6}+\frac{3}{10}}{\frac{14}{30}}$

414.

$\left(\frac{5}{9} + \frac{1}{6}\right) \div \left(\frac{2}{3} - \frac{1}{2}\right)$

415.

$\left(\frac{3}{4} + \frac{1}{6}\right) \div \left(\frac{5}{8} - \frac{1}{3}\right)$

In the following exercises, evaluate the given expression. Express your answers in simplified form, using improper fractions if necessary.

416.

$x + \frac{1}{2}$ when

1. ⓐ $x = -\frac{1}{8}$
2. ⓑ $x = -\frac{1}{2}$

417.

$x + \frac{2}{3}$ when

1. ⓐ $x = -\frac{1}{6}$
2. ⓑ $x = -\frac{5}{3}$

418.

$x + \left(-\frac{5}{6}\right)$ when

1. ⓐ $x = \frac{1}{3}$
2. ⓑ $x = -\frac{1}{6}$

419.

$x + \left(-\frac{11}{12}\right)$ when

1. ⓐ $x = \frac{11}{12}$
2. ⓑ $x = \frac{3}{4}$

420.

$x - \frac{2}{5}$ when

1. ⓐ $x = \frac{3}{5}$
2. ⓑ $x = -\frac{3}{5}$

421.

$x - \frac{1}{3}$ when

1. ⓐ $x = \frac{2}{3}$

2. ⓑ $x = -\frac{2}{3}$

422.

$\frac{7}{10} - w$ when

1. ⓐ $w = \frac{1}{2}$
2. ⓑ $w = -\frac{1}{2}$

423.

$\frac{5}{12} - w$ when

1. ⓐ $w = \frac{1}{4}$
2. ⓑ $w = -\frac{1}{4}$

424.

$4p^2q$ when $p = -\frac{1}{2}$ and $q = \frac{5}{9}$

425.

$5m^2n$ when $m = -\frac{2}{5}$ and $n = \frac{1}{3}$

426.

$2x^2y^3$ when $x = -\frac{2}{3}$ and $y = -\frac{1}{2}$

427.

$8u^2v^3$ when $u = -\frac{3}{4}$ and $v = -\frac{1}{2}$

428.

$\frac{u+v}{w}$ when $u = -4, v = -8, w = 2$

429.

$\frac{m+n}{p}$ when $m = -6, n = -2, p = 4$

430.

$\frac{a+b}{a-b}$ when $a = -3, b = 8$

431.

$\frac{r-s}{r+s}$ when $r = 10, s = -5$

Everyday Math

432.

Decorating Laronda is making covers for the throw pillows on her sofa. For each pillow cover, she needs $\frac{3}{16}$ yard of print fabric and $\frac{3}{8}$ yard of solid fabric. What is the total amount of fabric Laronda needs for each pillow cover?

433.

Baking Vanessa is baking chocolate chip cookies and oatmeal cookies. She needs $1\frac{1}{4}$ cups of sugar for the chocolate chip cookies, and $1\frac{1}{8}$ cups for the oatmeal cookies How much sugar does she need altogether?

Writing Exercises

434.

Explain why it is necessary to have a common denominator to add or subtract fractions.

435.

Explain how to find the LCD of two fractions.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
add and subtract fractions with different denominators.			
identify and use fraction operations.			
use the order of operations to simplify complex fractions.			
evaluate variable expressions with fractions.			

ⓑ After looking at the checklist, do you think you are well prepared for the next section? Why or why not?

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4.6: Add and Subtract Mixed Numbers

Learning Objectives

By the end of this section, you will be able to:

- Model addition of mixed numbers with a common denominator
- Add mixed numbers with a common denominator
- Model subtraction of mixed numbers
- Subtract mixed numbers with a common denominator
- Add and subtract mixed numbers with different denominators

Be Prepared 4.14

Before you get started, take this readiness quiz.

Draw figure to model $\frac{7}{3}$.
If you missed this problem, review Example 4.6.

Be Prepared 4.15

Change $\frac{11}{4}$ to a mixed number.
If you missed this problem, review Example 4.9.

Be Prepared 4.16

Change $3\frac{1}{2}$ to an improper fraction.
If you missed this problem, review Example 4.11.

Model Addition of Mixed Numbers with a Common Denominator

So far, we've added and subtracted proper and improper fractions, but not mixed numbers. Let's begin by thinking about addition of mixed numbers using money.

If Ron has 1 dollar and 1 quarter, he has $1\frac{1}{4}$ dollars.

If Don has 2 dollars and 1 quarter, he has $2\frac{1}{4}$ dollars.

What if Ron and Don put their money together? They would have 3 dollars and 2 quarters. They add the dollars and add the quarters. This makes $3\frac{2}{4}$ dollars. Because two quarters is half a dollar, they would have 3 and a half dollars, or $3\frac{1}{2}$ dollars.

$$\begin{array}{r} 1\frac{1}{4} \\ +2\frac{1}{4} \\ \hline \end{array}$$

$$3\frac{2}{4} = 3\frac{1}{2}$$

When you added the dollars and then added the quarters, you were adding the whole numbers and then adding the fractions.

$$1\frac{1}{4} + 2\frac{1}{4}$$

We can use fraction circles to model this same example:

		$1\frac{1}{4} + 2\frac{1}{4}$

Start with $1\frac{1}{4}$.	one whole and one $\frac{1}{4}$ pieces		$1\frac{1}{4}$
Add $2\frac{1}{4}$ more.	two wholes and one $\frac{1}{4}$ pieces		$+\ 2\frac{1}{4}$
The sum is:	three wholes and two $\frac{1}{4}$'s		$3\frac{2}{4} = 3\frac{1}{2}$

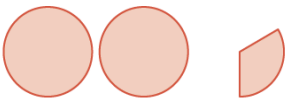
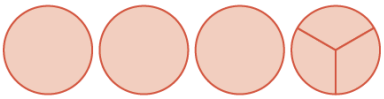
Manipulative Mathematics

Example 4.81

Model $2\frac{1}{3} + 1\frac{2}{3}$ and give the sum.

Answer

We will use fraction circles, whole circles for the whole numbers and $\frac{1}{3}$ pieces for the fractions.

two wholes and one $\frac{1}{3}$		$2\frac{1}{3}$
plus one whole and two $\frac{1}{3}$s		$+\ 1\frac{2}{3}$
sum is three wholes and three $\frac{1}{3}$s		$3\frac{3}{3} = 4$

This is the same as 4 wholes. So, $2\frac{1}{3} + 1\frac{2}{3} = 4$.

Try It 4.161

Use a model to add the following. Draw a picture to illustrate your model.

$1\frac{2}{5} + 3\frac{3}{5}$

Try It 4.162

Use a model to add the following. Draw a picture to illustrate your model.

$2\frac{1}{6} + 2\frac{5}{6}$

Example 4.82

Model $1\frac{3}{5}+2\frac{3}{5}$ and give the sum as a mixed number.

Answer

We will use fraction circles, whole circles for the whole numbers and $\frac{1}{5}$ pieces for the fractions.

one whole and three $\frac{1}{5}$s		$1\frac{3}{5}$
plus two wholes and three $\frac{1}{5}$s.		$+2\frac{3}{5}$
sum is three wholes and six $\frac{1}{5}$s		$3\frac{6}{5}=4\frac{1}{5}$

Adding the whole circles and fifth pieces, we got a sum of $3\frac{6}{5}$. We can see that $\frac{6}{5}$ is equivalent to $1\frac{1}{5}$, so we add that to the 3 to get $4\frac{1}{5}$.

Try It 4.163

Model, and give the sum as a mixed number. Draw a picture to illustrate your model.

$2\frac{5}{6}+1\frac{5}{6}$

Try It 4.164

Model, and give the sum as a mixed number. Draw a picture to illustrate your model.

$1\frac{5}{8}+1\frac{7}{8}$

Add Mixed Numbers

Modeling with fraction circles helps illustrate the process for adding mixed numbers: We add the whole numbers and add the fractions, and then we simplify the result, if possible.

How To

Add mixed numbers with a common denominator.

Step 1. Add the whole numbers.

Step 2. Add the fractions.

Step 3. Simplify, if possible.

Example 4.83

Add: $3\frac{4}{9}+2\frac{2}{9}$.

Answer

	$3\frac{4}{9}+2\frac{2}{9}$

Add the whole numbers.	$\begin{array}{r} 3\frac{4}{9} \\ +2\frac{2}{9} \\ \hline 5 \end{array}$
Add the fractions.	$\begin{array}{r} 3\frac{4}{9} \\ +2\frac{2}{9} \\ \hline 5\frac{6}{9} \end{array}$
Simplify the fraction.	$\begin{array}{r} 3\frac{4}{9} \\ +2\frac{2}{9} \\ \hline 5\frac{6}{9}=5\frac{2}{3} \end{array}$

Try It 4.165

Find the sum: $4\frac{4}{7}+1\frac{2}{7}$.

Try It 4.166

Find the sum: $2\frac{3}{11}+5\frac{6}{11}$.

In Example 4.83, the sum of the fractions was a proper fraction. Now we will work through an example where the sum is an improper fraction.

Example 4.84

Find the sum: $9\frac{5}{9}+5\frac{7}{9}$.

Answer

	$9\frac{5}{9}+5\frac{7}{9}$
Add the whole numbers and then add the fractions.	$\begin{array}{r} 9\frac{5}{9} \\ +5\frac{7}{9} \\ \hline 14\frac{12}{9} \end{array}$
Rewrite $\frac{12}{9}$ as an improper fraction.	$14+1\frac{3}{9}$
Add.	$15\frac{3}{9}$
Simplify.	$15\frac{1}{3}$

Try It 4.167

Find the sum: $8\frac{7}{8}+7\frac{5}{8}$.

Try It 4.168

Find the sum: $6\frac{7}{9}+8\frac{5}{9}$.

An alternate method for adding mixed numbers is to convert the mixed numbers to improper fractions and then add the improper fractions. This method is usually written horizontally.

Example 4.85

Add by converting the mixed numbers to improper fractions: $3\frac{7}{8}+4\frac{3}{8}$.

Answer

	$3\frac{7}{8}+4\frac{3}{8}$
Convert to improper fractions.	$\frac{31}{8}+\frac{35}{8}$
Add the fractions.	$\frac{31+35}{8}$
Simplify the numerator.	$\frac{66}{8}$
Rewrite as a mixed number.	$8\frac{2}{8}$
Simplify the fraction.	$8\frac{1}{4}$

Since the problem was given in mixed number form, we will write the sum as a mixed number.

Try It 4.169

Find the sum by converting the mixed numbers to improper fractions:

$5\frac{5}{9}+3\frac{7}{9}$.

Try It 4.170

Find the sum by converting the mixed numbers to improper fractions:

$3\frac{7}{10}+2\frac{9}{10}$.

Table 4.2 compares the two methods of addition, using the expression $3\frac{2}{5}+6\frac{4}{5}$ as an example. Which way do you prefer?

Mixed Numbers	Improper Fractions
$3\frac{2}{5}$ $\frac{+6\frac{4}{5}}{9\frac{6}{5}}$ $9+\frac{6}{5}$ $9+1\frac{1}{5}$ $10\frac{1}{5}$	$3\frac{2}{5}+6\frac{4}{5}$ $\frac{17}{5}+\frac{34}{5}$ $\frac{51}{5}$ $10\frac{1}{5}$

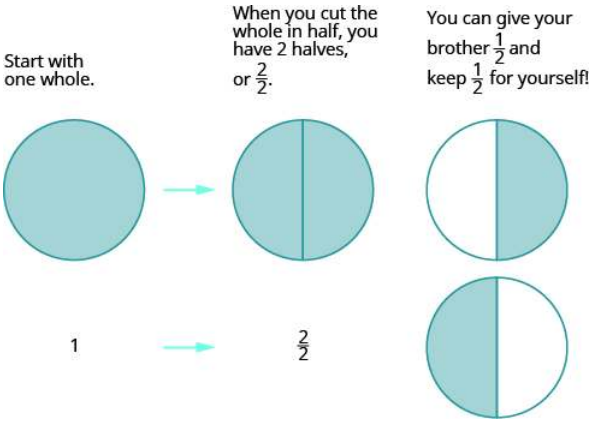
Table 4.2

Model Subtraction of Mixed Numbers

Let's think of pizzas again to model subtraction of mixed numbers with a common denominator. Suppose you just baked a whole pizza and want to give your brother half of the pizza. What do you have to do to the pizza to give him half? You have to cut it into at least two pieces. Then you can give him half.

We will use fraction circles (pizzas!) to help us visualize the process.

Start with one whole.



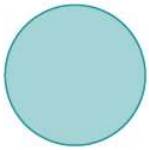
Algebraically, you would write:

$$1 \quad \frac{2}{2} \quad \frac{2}{2}$$
$$-\frac{1}{2} \rightarrow -\frac{1}{2} \rightarrow -\frac{1}{2}$$
$$\frac{1}{2}$$

Example 4.86

Use a model to subtract: $1 - \frac{1}{3}$.

Answer

	Model	Math Notation
Rewrite vertically. Start with one whole.		1 $-\frac{1}{3}$
Since $\frac{1}{3}$ has denominator 3, cut the whole into 3 pieces. The 1 whole becomes $\frac{3}{3}$.		$\frac{3}{3}$ $-\frac{1}{3}$
Take away $\frac{1}{3}$. There are $\frac{2}{3}$ left.		$\frac{3}{3}$ $-\frac{1}{3}$ $\frac{2}{3}$

Try It 4.171

Use a model to subtract: $1 - \frac{1}{4}$.

Try It 4.172

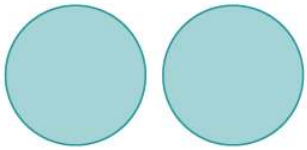
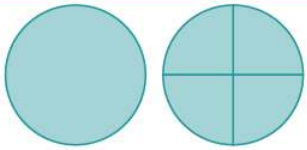
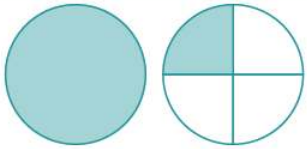
Use a model to subtract: $1 - \frac{1}{5}$.

What if we start with more than one whole? Let's find out.

Example 4.87

Use a model to subtract: $2 - \frac{3}{4}$.

Answer

	Model	Math Notation
Rewrite vertically. Start with two wholes.		$\begin{array}{r} 2 \\ -\frac{3}{4} \\ \hline \end{array}$
Since $\frac{3}{4}$ has denominator 4, cut one of the wholes into 4 pieces. You have 1 whole and $\frac{4}{4}$.		$\begin{array}{r} 1\frac{4}{4} \\ -\frac{3}{4} \\ \hline \end{array}$
Take away $\frac{3}{4}$. There is $1\frac{1}{4}$ left.		$\begin{array}{r} 1\frac{4}{4} \\ -\frac{3}{4} \\ \hline 1\frac{1}{4} \end{array}$

Try It 4.173

Use a model to subtract: $2 - \frac{1}{5}$.

Try It 4.174

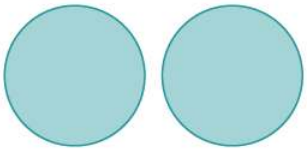
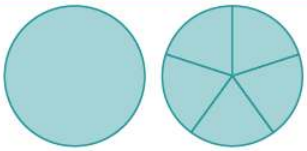
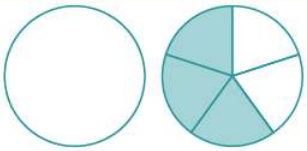
Use a model to subtract: $2 - \frac{1}{3}$.

In the next example, we'll subtract more than one whole.

Example 4.88

Use a model to subtract: $2 - 1\frac{2}{5}$.

Answer

	Model	Math Notation
Rewrite vertically. Start with two wholes.		$\begin{array}{r} 2 \\ -1\frac{2}{5} \\ \hline \end{array}$
Since $\frac{2}{5}$ has denominator 5, cut one of the wholes into 5 pieces. You have 1 whole and $\frac{5}{5}$.		$\begin{array}{r} 1\frac{5}{5} \\ -1\frac{2}{5} \\ \hline \end{array}$
Take away $1\frac{2}{5}$. There is $\frac{3}{5}$ left.		$\begin{array}{r} 1\frac{5}{5} \\ -1\frac{2}{5} \\ \hline \frac{3}{5} \end{array}$

Try It 4.175

Use a model to subtract: $2 - 1\frac{1}{3}$.

Try It 4.176

Use a model to subtract: $2 - 1\frac{1}{4}$.

What if you start with a mixed number and need to subtract a fraction? Think about this situation: You need to put three quarters in a parking meter, but you have only a $\$1$ bill and one quarter. What could you do? You could change the dollar bill into 4 quarters. The value of 4 quarters is the same as one dollar bill, but the 4 quarters are more useful for the parking meter. Now, instead of having a $\$1$ bill and one quarter, you have 5 quarters and can put 3 quarters in the meter.

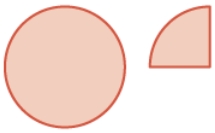
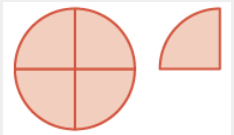
This models what happens when we subtract a fraction from a mixed number. We subtracted three quarters from one dollar and one quarter.

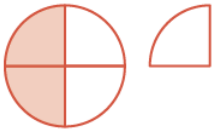
We can also model this using fraction circles, much like we did for addition of mixed numbers.

Example 4.89

Use a model to subtract: $1\frac{1}{4} - \frac{3}{4}$

Answer

Rewrite vertically. Start with one whole and one fourth.		$\begin{array}{r} 1\frac{1}{4} \\ -\frac{3}{4} \\ \hline \end{array}$
Since the fractions have denominator 4, cut the whole into 4 pieces. You now have $\frac{4}{4}$ and $\frac{1}{4}$ which is $\frac{5}{4}$.		$\begin{array}{r} \frac{5}{4} \\ -\frac{3}{4} \\ \hline \end{array}$

Take away $\frac{3}{4}$. There is $\frac{1}{2}$ left.		$\begin{array}{r} \frac{5}{4} \\ -\frac{3}{4} \\ \hline \frac{2}{4} = \frac{1}{2} \end{array}$

Try It 4.177

Use a model to subtract. Draw a picture to illustrate your model.

$1\frac{1}{3} - \frac{2}{3}$

Try It 4.178

Use a model to subtract. Draw a picture to illustrate your model.

$1\frac{1}{5} - \frac{4}{5}$

Subtract Mixed Numbers with a Common Denominator

Now we will subtract mixed numbers without using a model. But it may help to picture the model in your mind as you read the steps.

How To

Subtract mixed numbers with common denominators.

1. Step 1. Rewrite the problem in vertical form.
2. Step 2. Compare the two fractions.
   - If the top fraction is larger than the bottom fraction, go to Step 3.
   - If not, in the top mixed number, take one whole and add it to the fraction part, making a mixed number with an improper fraction.
3. Step 3. Subtract the fractions.
4. Step 4. Subtract the whole numbers.
5. Step 5. Simplify, if possible.

Example 4.90

Find the difference: $5\frac{3}{5} - 2\frac{4}{5}$.

Answer

	$5\frac{3}{5} - 2\frac{4}{5}$
Rewrite the problem in vertical form.	$\begin{array}{r} 5\frac{3}{5} \\ -2\frac{4}{5} \\ \hline \end{array}$
Since $\frac{3}{5}$ is less than $\frac{4}{5}$, take 1 from the 5 and add it to the $\frac{3}{5}$: $\left(\frac{5}{5} + \frac{3}{5} = \frac{8}{5}\right)$	$\begin{array}{r} 5\frac{3}{5} \\ -2\frac{4}{5} \\ \hline \end{array} \rightarrow \begin{array}{r} 4\frac{8}{5} \\ -2\frac{4}{5} \\ \hline \end{array}$
Subtract the fractions.	$\begin{array}{r} 4\frac{8}{5} \\ -2\frac{4}{5} \\ \hline \frac{4}{5} \end{array}$

Subtract the whole parts. The result is in simplest form.	$4\frac{8}{5}$ $-2\frac{4}{5}$ $2\frac{4}{5}$

Since the problem was given with mixed numbers, we leave the result as mixed numbers.

Try It 4.179

Find the difference: $6\frac{4}{9}-3\frac{7}{9}$.

Try It 4.180

Find the difference: $4\frac{4}{7}-2\frac{6}{7}$.

Just as we did with addition, we could subtract mixed numbers by converting them first to improper fractions. We should write the answer in the form it was given, so if we are given mixed numbers to subtract we will write the answer as a mixed number.

How To

Subtract mixed numbers with common denominators as improper fractions.

Step 1. Rewrite the mixed numbers as improper fractions.

Step 2. Subtract the numerators.

Step 3. Write the answer as a mixed number, simplifying the fraction part, if possible.

Example 4.91

Find the difference by converting to improper fractions:

$9\frac{6}{11}-7\frac{10}{11}$.

Answer

	$9\frac{6}{11}-7\frac{10}{11}$
Rewrite as improper fractions.	$\frac{105}{11}-\frac{87}{11}$
Subtract the numerators.	$\frac{18}{11}$
Rewrite as a mixed number.	$1\frac{7}{11}$

Try It 4.181

Find the difference by converting the mixed numbers to improper fractions:

$6\frac{4}{9}-3\frac{7}{9}$.

Try It 4.182

Find the difference by converting the mixed numbers to improper fractions:

$4\frac{4}{7}-2\frac{6}{7}$.

Add and Subtract Mixed Numbers with Different Denominators

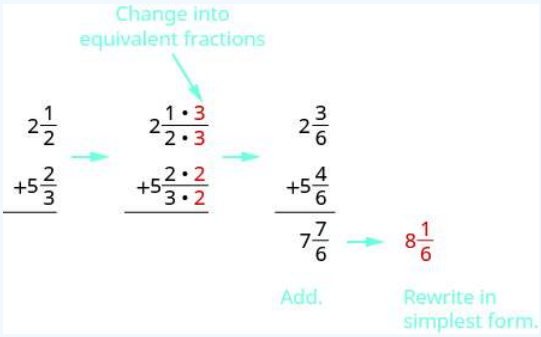
To add or subtract mixed numbers with different denominators, we first convert the fractions to equivalent fractions with the LCD. Then we can follow all the steps we used above for adding or subtracting fractions with like denominators.

Example 4.92

Add: $2\frac{1}{2}+5\frac{2}{3}$.

Answer

Since the denominators are different, we rewrite the fractions as equivalent fractions with the LCD, 6. Then we will add and simplify.



We write the answer as a mixed number because we were given mixed numbers in the problem.

Try It 4.183

Add: $1\frac{5}{6}+4\frac{3}{4}$.

Try It 4.184

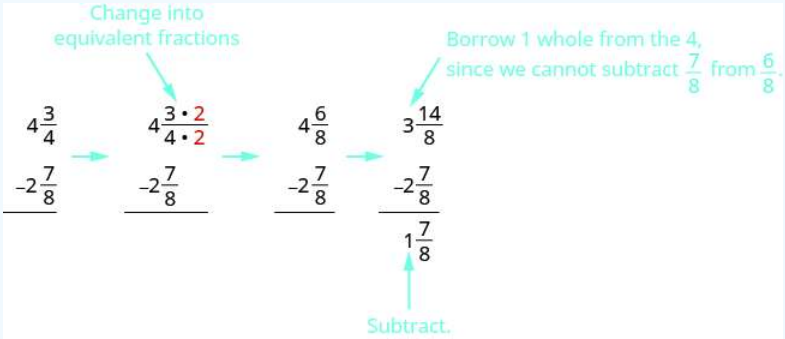
Add: $3\frac{4}{5}+8\frac{1}{2}$.

Example 4.93

Subtract: $4\frac{3}{4}-2\frac{7}{8}$.

Answer

Since the denominators of the fractions are different, we will rewrite them as equivalent fractions with the LCD 8. Once in that form, we will subtract. But we will need to borrow 1 first.



We were given mixed numbers, so we leave the answer as a mixed number.

Try It 4.185

Find the difference: $8\frac{1}{2}-3\frac{4}{5}$.

Try It 4.186

Find the difference: $4\frac{3}{4}-1\frac{5}{6}$.

Example 4.94

Subtract: $3\frac{5}{11}-4\frac{3}{4}$.

Answer

We can see the answer will be negative since we are subtracting 4 from 3. Generally, when we know the answer will be negative it is easier to subtract with improper fractions rather than mixed numbers.

	$3\frac{5}{11}-4\frac{3}{4}$
Change to equivalent fractions with the LCD.	$3\frac{5\cdot4}{11\cdot4}-4\frac{3\cdot11}{4\cdot11}$ $3\frac{20}{44}-4\frac{33}{44}$
Rewrite as improper fractions.	$\frac{152}{44}-\frac{209}{44}$
Subtract.	$-\frac{57}{44}$
Rewrite as a mixed number.	$-1\frac{13}{44}$

Try It 4.187

Subtract: $1\frac{3}{4}-6\frac{7}{8}$.

Try It 4.188

Subtract: $10\frac{3}{7}-22\frac{4}{9}$.

Media

ACCESS ADDITIONAL ONLINE RESOURCES

- Adding Mixed Numbers
- Subtracting Mixed Numbers

Section 4.6 Exercises

Practice Makes Perfect

Model Addition of Mixed Numbers

In the following exercises, use a model to find the sum. Draw a picture to illustrate your model.

436.

$1\frac{1}{5}+3\frac{1}{5}$

437.

$2\frac{1}{3}+1\frac{1}{3}$

438.

$1\frac{3}{8}+1\frac{7}{8}$

439.

$1\frac{5}{6}+1\frac{5}{6}$

Add Mixed Numbers with a Common Denominator

In the following exercises, add.

440.

$5\frac{1}{3}+6\frac{1}{3}$

441.

$2\frac{4}{9}+5\frac{1}{9}$

442.

$4\frac{5}{8}+9\frac{3}{8}$

443.

$7\frac{9}{10}+3\frac{1}{10}$

444.

$3\frac{4}{5}+6\frac{4}{5}$

445.

$9\frac{2}{3}+1\frac{2}{3}$

446.

$6\frac{9}{10}+8\frac{3}{10}$

447.

$8\frac{4}{9}+2\frac{8}{9}$

Model Subtraction of Mixed Numbers

In the following exercises, use a model to find the difference. Draw a picture to illustrate your model.

448.

$1\frac{1}{6}-\frac{5}{6}$

449.

$1\frac{1}{8}-\frac{5}{8}$

Subtract Mixed Numbers with a Common Denominator

In the following exercises, find the difference.

450.

$2\frac{7}{8}-1\frac{3}{8}$

451.

$2\frac{7}{12}-1\frac{5}{12}$

452.

$8\frac{17}{20}-4\frac{9}{20}$

453.

$19\frac{13}{15}-13\frac{7}{15}$

454.

$8\frac{3}{7}-4\frac{4}{7}$

455.

$5\frac{2}{9}-3\frac{4}{9}$

456.

$2\frac{5}{8}-1\frac{7}{8}$

457.

$2\frac{5}{12} - 1\frac{7}{12}$

Add and Subtract Mixed Numbers with Different Denominators

In the following exercises, write the sum or difference as a mixed number in simplified form.

458.

$3\frac{1}{4} + 6\frac{1}{3}$

459.

$2\frac{1}{6} + 5\frac{3}{4}$

460.

$1\frac{5}{8} + 4\frac{1}{2}$

461.

$7\frac{2}{3} + 8\frac{1}{2}$

462.

$9\frac{7}{10} - 2\frac{1}{3}$

463.

$6\frac{4}{5} - 1\frac{1}{4}$

464.

$2\frac{2}{3} - 3\frac{1}{2}$

465.

$2\frac{7}{8} - 4\frac{1}{3}$

Mixed Practice

In the following exercises, perform the indicated operation and write the result as a mixed number in simplified form.

466.

$2\frac{5}{8} \cdot 1\frac{3}{4}$

467.

$1\frac{2}{3} \cdot 4\frac{1}{6}$

468.

$\frac{2}{7} + \frac{4}{7}$

469.

$\frac{2}{9} + \frac{5}{9}$

470.

$1\frac{5}{12} \div \frac{1}{12}$

471.

$2\frac{3}{10} \div \frac{1}{10}$

472.

$13\frac{5}{12} - 9\frac{7}{12}$

473.

$15\frac{5}{8} - 6\frac{7}{8}$

474.

$\frac{5}{9} - \frac{4}{9}$

475.

$\frac{11}{15} - \frac{7}{15}$

476.

$4 - \frac{3}{4}$

477.

$6 - \frac{2}{5}$

478.

$\frac{9}{20} \div \frac{3}{4}$

479.

$\frac{7}{24} \div \frac{14}{3}$

480.

$9\frac{6}{11} + 7\frac{10}{11}$

481.

$8\frac{5}{13} + 4\frac{9}{13}$

482.

$3\frac{2}{5} + 5\frac{3}{4}$

483.

$2\frac{5}{6} + 4\frac{1}{5}$

484.

$\frac{8}{15} \cdot \frac{10}{19}$

485.

$\frac{5}{12} \cdot \frac{8}{9}$

486.

$6\frac{7}{8} - 2\frac{1}{3}$

487.

$6\frac{5}{9} - 4\frac{2}{5}$

488.

$5\frac{2}{9} - 4\frac{4}{5}$

489.

$4\frac{3}{8} - 3\frac{2}{3}$

Everyday Math

490.

Sewing Renata is sewing matching shirts for her husband and son. According to the patterns she will use, she needs $2\frac{3}{8}$ yards of fabric for her husband's shirt and $1\frac{1}{8}$ yards of fabric for her son's shirt. How much fabric does she need to make both shirts?

491.

Sewing Pauline has $3\frac{1}{4}$ yards of fabric to make a jacket. The jacket uses $2\frac{2}{3}$ yards. How much fabric will she have left after making the jacket?

492.

Printing Nishant is printing invitations on his computer. The paper is $8\frac{1}{2}$ inches wide, and he sets the print area to have a $1\frac{1}{2}$-inch border on each side. How wide is the print area on the sheet of paper?

493.

Framing a picture Tessa bought a picture frame for her son's graduation picture. The picture is 8 inches wide. The picture frame is $2\frac{5}{8}$ inches wide on each side. How wide will the framed picture be?

Writing Exercises

494.

Draw a diagram and use it to explain how to add $1\frac{5}{8}+2\frac{7}{8}$.

495.

Edgar will have to pay $\$3.75$ in tolls to drive to the city.

ⓐ Explain how he can make change from a $\$10$ bill before he leaves so that he has the exact amount he needs.

ⓑ How is Edgar's situation similar to how you subtract $10-3\frac{3}{4}$?

496.

Add $4\frac{5}{12}+3\frac{7}{8}$ twice, first by leaving them as mixed numbers and then by rewriting as improper fractions. Which method do you prefer, and why?

497.

Subtract $3\frac{7}{8}-4\frac{5}{12}$ twice, first by leaving them as mixed numbers and then by rewriting as improper fractions. Which method do you prefer, and why?

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
model addition of mixed numbers common with a denominator.			
add mixed numbers with a common denominator.			
model subtraction of mixed numbers.			
subtract mixed numbers with a common denominator.			
add and subtract mixed numbers with different denominators.			

ⓑ After reviewing this checklist, what will you do to become confident for all objectives?

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4.7: Decimals

Learning Objectives

By the end of this section, you will be able to:

- Name decimals
- Write decimals
- Convert decimals to fractions or mixed numbers
- Locate decimals on the number line
- Order decimals
- Round decimals

Be Prepared 5.1

Before you get started, take this readiness quiz.

Name the number $4{,}926{,}015$ in words.
If you missed this problem, review Example 1.4.

Be Prepared 5.2

Round 748 to the nearest ten.
If you missed this problem, review Example 1.9.

Be Prepared 5.3

Locate $\frac{3}{10}$ on a number line.
If you missed this problem, review Example 4.16.

Name Decimals

You probably already know quite a bit about decimals based on your experience with money. Suppose you buy a sandwich and a bottle of water for lunch. If the sandwich costs $\$3.45$, the bottle of water costs $\$1.25$, and the total sales tax is $\$0.33$, what is the total cost of your lunch?

$3.45	Sandwich
$1.25	Water
+ $0.33	Tax
$5.03	Total

The total is $\$5.03$. Suppose you pay with a $\$5$ bill and 3 pennies. Should you wait for change? No, $\$5$ and 3 pennies is the same as $\$5.03$.

Because $100 \text{ pennies} = \$1$, each penny is worth $\frac{1}{100}$ of a dollar. We write the value of one penny as $\$0.01$, since $0.01 = \frac{1}{100}$.

Writing a number with a decimal is known as decimal notation. It is a way of showing parts of a whole when the whole is a power of ten. In other words, decimals are another way of writing fractions whose denominators are powers of ten. Just as the counting numbers are based on powers of ten, decimals are based on powers of ten. Table 5.1 shows the counting numbers.

Counting number	Name
1	One
$10 = 10$	Ten
$10 \cdot 10 = 100$	One hundred
$10 \cdot 10 \cdot 10 = 1000$	One thousand
$10 \cdot 10 \cdot 10 \cdot 10 = 10{,}000$	Ten thousand

Table 5.1

How are decimals related to fractions? Table 5.2 shows the relation.

Decimal	Fraction	Name
0.1	$\frac{1}{10}$	One tenth
0.01	$\frac{1}{100}$	One hundredth
0.001	$\frac{1}{1,000}$	One thousandth
0.0001	$\frac{1}{10,000}$	One ten-thousandth

Table 5.2

When we name a whole number, the name corresponds to the place value based on the powers of ten. In Whole Numbers, we learned to read $10{,}000$ as *ten thousand*. Likewise, the names of the decimal places correspond to their fraction values. Notice how the place value names in Figure 5.2 relate to the names of the fractions from Table 5.2.

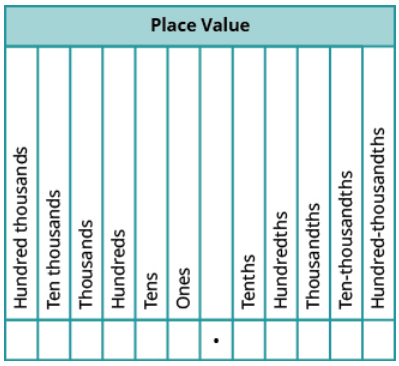


Figure 5.2 This chart illustrates place values to the left and right of the decimal point.

Notice two important facts shown in Figure 5.2.

- The “th” at the end of the name means the number is a fraction. “One thousand” is a number larger than one, but “one thousandth” is a number smaller than one.
- The tenths place is the first place to the right of the decimal, but the tens place is two places to the left of the decimal.

Remember that $\$5.03$ lunch? We read $\$5.03$ as *five dollars and three cents*. Naming decimals (those that don’t represent money) is done in a similar way. We read the number 5.03 as *five and three hundredths*.

We sometimes need to translate a number written in decimal notation into words. As shown in Figure 5.3, we write the amount on a check in both words and numbers.



Figure 5.3 When we write a check, we write the amount as a decimal number as well as in words. The bank looks at the check to make sure both numbers match. This helps prevent errors.

Let’s try naming a decimal, such as 15.68.	
We start by naming the number to the left of the decimal.	fifteen_____
We use the word “and” to indicate the decimal point.	fifteen and_____

Then we name the number to the right of the decimal point as if it were a whole number.	fifteen and sixty-eight_____
Last, name the decimal place of the last digit.	fifteen and sixty-eight hundredths

The number 15.68 is read *fifteen and sixty-eight hundredths*.

How To

Name a decimal number.

- Name the number to the left of the decimal point.
- Write “and” for the decimal point.
- Name the “number” part to the right of the decimal point as if it were a whole number.
- Name the decimal place of the last digit.

Example 5.1

Name each decimal: ⓐ 4.3 ⓑ 2.45 ⓒ 0.009 ⓓ -15.571.

Answer

ⓐ	
	4.3
Name the number to the left of the decimal point.	four_____
Write "and" for the decimal point.	four and_____
Name the number to the right of the decimal point as if it were a whole number.	four and three_____
Name the decimal place of the last digit.	four and three tenths

ⓑ	
	2.45
Name the number to the left of the decimal point.	two_____
Write "and" for the decimal point.	two and_____
Name the number to the right of the decimal point as if it were a whole number.	two and forty-five_____
Name the decimal place of the last digit.	two and forty-five hundredths

ⓒ	
	0.009
Name the number to the left of the decimal point.	Zero is the number to the left of the decimal; it is not included in the name.
Name the number to the right of the decimal point as if it were a whole number.	nine_____
Name the decimal place of the last digit.	nine thousandths

ⓓ	
	-15.571
Name the number to the left of the decimal point.	negative fifteen
Write "and" for the decimal point.	negative fifteen and_____
Name the number to the right of the decimal point as if it were a whole number.	negative fifteen and five hundred seventy-one_____
Name the decimal place of the last digit.	negative fifteen and five hundred seventy-one thousandths

Try It 5.1

Name each decimal:

ⓐ 6.7 ⓑ 19.58 ⓒ 0.018 ⓓ -2.053

Try It 5.2

Name each decimal:

ⓐ 5.8 ⓑ 3.57 ⓒ 0.005 ⓓ -13.461

Write Decimals

Now we will translate the name of a decimal number into decimal notation. We will reverse the procedure we just used.

Let's start by writing the number six and seventeen hundredths:

	six and seventeen hundredths
The word *and* tells us to place a decimal point.	___.___
The word before *and* is the whole number; write it to the left of the decimal point.	6._____
The decimal part is seventeen hundredths. Mark two places to the right of the decimal point for hundredths.	6._ _
Write the numerals for seventeen in the places marked.	6.17

Example 5.2

Write fourteen and thirty-seven hundredths as a decimal.

Answer

	fourteen and thirty-seven hundredths
Place a decimal point under the word 'and'.	_____. _____
Translate the words before 'and' into the whole number and place it to the left of the decimal point.	14. _____
Mark two places to the right of the decimal point for "hundredths".	14.__ __
Translate the words after "and" and write the number to the right of the decimal point.	14.37
	Fourteen and thirty-seven hundredths is written 14.37.

Try It 5.3

Write as a decimal: thirteen and sixty-eight hundredths.

Try It 5.4

Write as a decimal: five and eight hundred ninety-four thousandths.

How To

Write a decimal number from its name.

1. Step 1. Look for the word “and”—it locates the decimal point.
2. Step 2. Mark the number of decimal places needed to the right of the decimal point by noting the place value indicated by the last word.
   - Place a decimal point under the word “and.” Translate the words before “and” into the whole number and place it to the left of the decimal point.
   - If there is no “and,” write a “0” with a decimal point to its right.
3. Step 3. Translate the words after “and” into the number to the right of the decimal point. Write the number in the spaces—putting the final digit in the last place.
4. Step 4. Fill in zeros for place holders as needed.

The second bullet in Step 2 is needed for decimals that have no whole number part, like ‘nine thousandths’. We recognize them by the words that indicate the place value after the decimal – such as ‘tenths’ or ‘hundredths.’ Since there is no whole number, there is no ‘and.’ We start by placing a zero to the left of the decimal and continue by filling in the numbers to the right, as we did above.

Example 5.3

Write twenty-four thousandths as a decimal.

Answer

	twenty-four thousandths
Look for the word "and".	There is no "and" so start with 0 0.
To the right of the decimal point, put three decimal places for thousandths.	0. _ _ _ tenths hundredths thousandths
Write the number 24 with the 4 in the thousandths place.	0. _ 2 4 tenths hundredths thousandths
Put zeros as placeholders in the remaining decimal places.	0.024
	So, twenty-four thousandths is written 0.024

Try It 5.5

Write as a decimal: fifty-eight thousandths.

Try It 5.6

Write as a decimal: sixty-seven thousandths.

Before we move on to our next objective, think about money again. We know that $\$1$ is the same as $\$1.00$. The way we write $\$1$ $(\text{or } \$1.00)$ depends on the context. In the same way, integers can be written as decimals with as many zeros as needed to the right of the decimal.

$$5 = 5.0 \qquad -2 = -2.0$$
$$5 = 5.00 \qquad -2 = -2.00$$
$$5 = 5.000 \qquad -2 = -2.000$$

and so on...

Convert Decimals to Fractions or Mixed Numbers

We often need to rewrite decimals as fractions or mixed numbers. Let's go back to our lunch order to see how we can convert decimal numbers to fractions. We know that $\$5.03$ means 5 dollars and 3 cents. Since there are 100 cents in one dollar, 3 cents means $\frac{3}{100}$ of a dollar, so $0.03 = \frac{3}{100}$.

We convert decimals to fractions by identifying the place value of the farthest right digit. In the decimal 0.03, the 3 is in the hundredths place, so 100 is the denominator of the fraction equivalent to 0.03.

$$0.03 = \frac{3}{100}$$

For our $\$5.03$ lunch, we can write the decimal 5.03 as a mixed number.

$$5.03 = 5\frac{3}{100}$$

Notice that when the number to the left of the decimal is zero, we get a proper fraction. When the number to the left of the decimal is not zero, we get a mixed number.

How To

Convert a decimal number to a fraction or mixed number.

1. Step 1. Look at the number to the left of the decimal.
   - If it is zero, the decimal converts to a proper fraction.
   - If it is not zero, the decimal converts to a mixed number.
     - Write the whole number.
2. Step 2. Determine the place value of the final digit.
3. Step 3. Write the fraction.
   - numerator—the 'numbers' to the right of the decimal point
   - denominator—the place value corresponding to the final digit
4. Step 4. Simplify the fraction, if possible.

Example 5.4

Write each of the following decimal numbers as a fraction or a mixed number:

ⓐ 4.09 ⓑ 3.7 ⓒ -0.286

Answer

ⓐ	
	4.09
There is a 4 to the left of the decimal point. Write "4" as the whole number part of the mixed number.	$4\frac{\square}{\square}$
Determine the place value of the final digit.	4. 0 9 tenths hundredths
Write the fraction. Write 9 in the numerator as it is the number to the right of the decimal point.	$4\frac{9}{\square}$

Write 100 in the denominator as the place value of the final digit, 9, is hundredth.	$4\frac{9}{100}$
The fraction is in simplest form.	So, $4.09 = 4\frac{9}{100}$

Did you notice that the number of zeros in the denominator is the same as the number of decimal places?

ⓑ	
	3.7
There is a 3 to the left of the decimal point. Write "3" as the whole number part of the mixed number.	$3\frac{\square}{\square}$
Determine the place value of the final digit.	3. 7 tenths
Write the fraction. Write 7 in the numerator as it is the number to the right of the decimal point.	$3\frac{7}{\square}$
Write 10 in the denominator as the place value of the final digit, 7, is tenths.	$3\frac{7}{10}$
The fraction is in simplest form.	So, $3.7 = 3\frac{7}{10}$

ⓒ	
	−0.286
There is a 0 to the left of the decimal point. Write a negative sign before the fraction.	$-\frac{\square}{\square}$
Determine the place value of the final digit and write it in the denominator.	−0. 2 8 6 tenths hundredths thousandths
Write the fraction. Write 286 in the numerator as it is the number to the right of the decimal point. Write 1,000 in the denominator as the place value of the final digit, 6, is thousandths.	$-\frac{286}{1000}$
We remove a common factor of 2 to simplify the fraction.	$-\frac{143}{500}$

Try It 5.7

Write as a fraction or mixed number. Simplify the answer if possible.

ⓐ 5.3 ⓑ 6.07 ⓒ −0.234

Try It 5.8

Write as a fraction or mixed number. Simplify the answer if possible.

ⓐ 8.7 ⓑ 1.03 ⓒ −0.024

Locate Decimals on the Number Line

Since decimals are forms of fractions, locating decimals on the number line is similar to locating fractions on the number line.

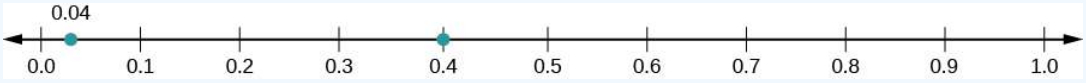
Example 5.5

Locate 0.4 on a number line.

Answer

The decimal 0.4 is equivalent to $\frac{4}{10}$, so 0.4 is located between 0 and 1. On a number line, divide the interval between 0 and 1 into 10 equal parts and place marks to separate the parts.

Label the marks $0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0$. We write 0 as 0.0 and 1 as 1.0, so that the numbers are consistently in tenths. Finally, mark 0.4 on the number line.



Try It 5.9

Locate 0.6 on a number line.

Try It 5.10

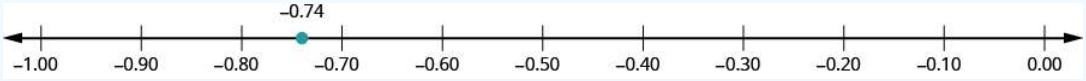
Locate 0.9 on a number line.

Example 5.6

Locate -0.74 on a number line.

Answer

The decimal -0.74 is equivalent to $-\frac{74}{100}$, so it is located between 0 and -1. On a number line, mark off and label the multiples of -0.10 in the interval between 0 and -1 (-0.10, -0.20, etc.) and mark -0.74 between -0.70 and -0.80, a little closer to -0.70.



Try It 5.11

Locate -0.63 on a number line.

Try It 5.12

Locate -0.25 on a number line.

Order Decimals

Which is larger, 0.04 or 0.40?

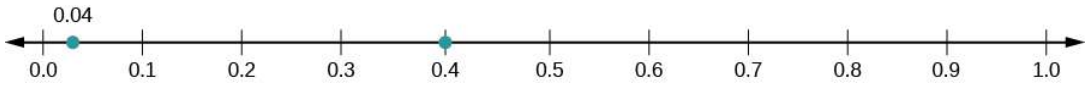
If you think of this as money, you know that $\$0.40$ (forty cents) is greater than $\$0.04$ (four cents). So,

$$0.40 > 0.04$$

In previous chapters, we used the number line to order numbers.

$$a < b \text{ `} a \text{ is less than } b' \text{ when } a \text{ is to the left of } b \text{ on the number line}$$
$$a > b \text{ `} a \text{ is greater than } b' \text{ when } a \text{ is to the right of } b \text{ on the number line}$$

Where are 0.04 and 0.40 located on the number line?



We see that 0.40 is to the right of 0.04. So we know $0.40 > 0.04$.

How does 0.31 compare to 0.308? This doesn't translate into money to make the comparison easy. But if we convert 0.31 and 0.308 to fractions, we can tell which is larger.

	0.31	0.308
Convert to fractions.	$\frac{31}{100}$	$\frac{308}{1000}$
We need a common denominator to compare them.	$\frac{31 \cdot 10}{100 \cdot 10}$	$\frac{308}{1000}$
	$\frac{310}{1000}$	$\frac{308}{1000}$

Because $310 > 308$, we know that $\frac{310}{1000} > \frac{308}{1000}$. Therefore, $0.31 > 0.308$.

Notice what we did in converting 0.31 to a fraction—we started with the fraction $\frac{31}{100}$ and ended with the equivalent fraction $\frac{310}{1000}$. Converting $\frac{310}{1000}$ back to a decimal gives 0.310. So 0.31 is equivalent to 0.310. Writing zeros at the end of a decimal does not change its value.

$$\frac{31}{100} = \frac{310}{1000} \text{ and } 0.31 = 0.310$$

If two decimals have the same value, they are said to be equivalent decimals.

$$0.31 = 0.310$$

We say 0.31 and 0.310 are equivalent decimals.

Equivalent Decimals

Two decimals are equivalent decimals if they convert to equivalent fractions.

Remember, writing zeros at the end of a decimal does not change its value.

How To

Order decimals.

1. Step 1. Check to see if both numbers have the same number of decimal places. If not, write zeros at the end of the one with fewer digits to make them match.
2. Step 2. Compare the numbers to the right of the decimal point as if they were whole numbers.
3. Step 3. Order the numbers using the appropriate inequality sign.

Example 5.7

Order the following decimals using $<$ or $>$:

1. ⓐ $0.64 \_ 0.6$
2. ⓑ $0.83 \_ 0.803$

Answer

ⓐ	
	$0.64 \_ 0.6$

Check to see if both numbers have the same number of decimal places. They do not, so write one zero at the right of 0.6.	$0.64 \_ 0.60$
Compare the numbers to the right of the decimal point as if they were whole numbers.	$64 > 60$
Order the numbers using the appropriate inequality sign.	$0.64 > 0.60$ $0.64 > 0.6$

ⓑ	
	$0.83 \_ 0.803$
Check to see if both numbers have the same number of decimal places. They do not, so write one zero at the right of 0.83.	$0.830 \_ 0.803$
Compare the numbers to the right of the decimal point as if they were whole numbers.	$830 > 803$
Order the numbers using the appropriate inequality sign.	$0.830 > 0.803$ $0.83 > 0.803$

Try It 5.13

Order each of the following pairs of numbers, using $<$ or $>$:

ⓐ $0.42 \_ 0.4$ ⓑ $0.76 \_ 0.706$

Try It 5.14

Order each of the following pairs of numbers, using $<$ or $>$:

ⓐ $0.1 \_ 0.18$ ⓑ $0.305 \_ 0.35$

When we order negative decimals, it is important to remember how to order negative integers. Recall that larger numbers are to the right on the number line. For example, because -2 lies to the right of -3 on the number line, we know that $-2 > -3$. Similarly, smaller numbers lie to the left on the number line. For example, because -9 lies to the left of -6 on the number line, we know that $-9 < -6$.



If we zoomed in on the interval between 0 and -1, we would see in the same way that $-0.2 > -0.3$ and $-0.9 < -0.6$.

Example 5.8

Use $<$ or $>$ to order. $-0.1 \_ -0.8$.

Answer

	$-0.1 \_ -0.8$
Write the numbers one under the other, lining up the decimal points.	-0.1 -0.8

They have the same number of digits.	
Since $-1 > -8, -1$ tenth is greater than -8 tenths.	$-0.1 > -0.8$

Try It 5.15

Order each of the following pairs of numbers, using $<$ or $>$:

$-0.3 \_\_ -0.5$

Try It 5.16

Order each of the following pairs of numbers, using $<$ or $>$:

$-0.6 \_\_ -0.7$

Round Decimals

In the United States, gasoline prices are usually written with the decimal part as thousandths of a dollar. For example, a gas station might post the price of unleaded gas at Math input error

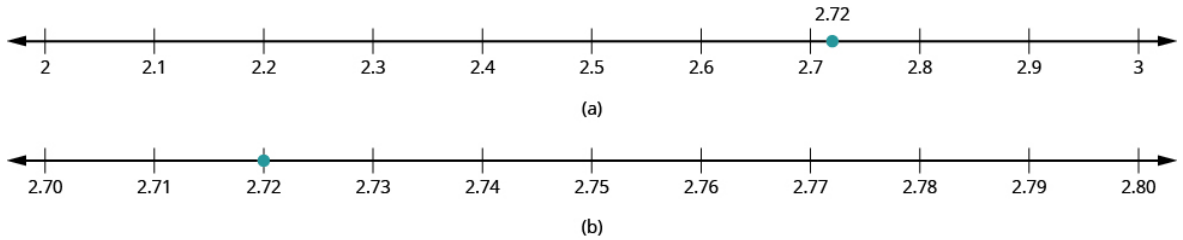


Figure 5.4 ⓐ We see that 2.72 is closer to 3 than to 2. So, 2.72 rounded to the nearest whole number is 3. ⓑ We see that 2.72 is closer to 2.70 than 2.80. So we say that 2.72 rounded to the nearest tenth is 2.7.

Can we round decimals without number lines? Yes! We use a method based on the one we used to round whole numbers.

How To

Round a decimal.

1. Step 1. Locate the given place value and mark it with an arrow.
2. Step 2. Underline the digit to the right of the given place value.
3. Step 3. Is this digit greater than or equal to 5?
   - Yes - add 1 to the digit in the given place value.
   - No - do not change the digit in the given place value
4. Step 4. Rewrite the number, removing all digits to the right of the given place value.

Example 5.9

Round 18.379 to the nearest hundredth.

Answer

	18.379
Locate the hundredths place and mark it with an arrow.	hundredths place ↓ 18.379
Underline the digit to the right of the 7.	hundredths place ↓ 18.379

Because 9 is greater than or equal to 5, add 1 to the 7.	
Rewrite the number, deleting all digits to the right of the hundredths place.	18.38
	18.38 is 18.379 rounded to the nearest hundredth.

Try It 5.17

Round to the nearest hundredth: 1.047.

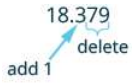
Try It 5.18

Round to the nearest hundredth: 9.173.

Example 5.10

Round 18.379 to the nearest ⓐ tenth ⓑ whole number.

Answer

ⓐ Round 18.379 to the nearest tenth.	
	
Locate the tenths place and mark it with an arrow.	
Underline the digit to the right of the tenths digit.	
Because 7 is greater than or equal to 5, add 1 to the 3.	
Rewrite the number, deleting all digits to the right of the tenths place.	
	So, 18.379 rounded to the nearest tenth is 18.4.

ⓑ Round 18.379 to the nearest whole number.	
	
Locate the ones place and mark it with an arrow.	
Underline the digit to the right of the ones place.	
Since 3 is not greater than or equal to 5, do not add 1 to the 8.	

Rewrite the number, deleting all digits to the right of the ones place.	18
	So 18.379 rounded to the nearest whole number is 18.

Try It 5.19

Round 6.582 to the nearest ⓐ hundredth ⓑ tenth ⓒ whole number.

Try It 5.20

Round 15.2175 to the nearest ⓐ thousandth ⓑ hundredth ⓒ tenth.

Media

ACCESS ADDITIONAL ONLINE RESOURCES

- Introduction to Decimal Notation
- Write a Number in Decimal Notation from Words
- Identify Decimals on the Number Line
- Rounding Decimals
- Writing a Decimal as a Simplified Fraction

Section 5.1 Exercises

Practice Makes Perfect

Name Decimals

In the following exercises, name each decimal.

1. 5.5
2. 7.8
3. 5.01
4. 14.02
5. 8.71
6. 2.64
7. 0.002
8. 0.005
9. 0.381
10.

0.479

11.

-17.9

12.

-31.4

Write Decimals

In the following exercises, translate the name into a decimal number.

13.

Eight and three hundredths

14.

Nine and seven hundredths

15.

Twenty-nine and eighty-one hundredths

16.

Sixty-one and seventy-four hundredths

17.

Seven tenths

18.

Six tenths

19.

One thousandth

20.

Nine thousandths

21.

Twenty-nine thousandths

22.

Thirty-five thousandths

23.

Negative eleven and nine ten-thousandths

24.

Negative fifty-nine and two ten-thousandths

25.

Thirteen and three hundred ninety-five ten thousandths

26.

Thirty and two hundred seventy-nine thousandths

Convert Decimals to Fractions or Mixed Numbers

In the following exercises, convert each decimal to a fraction or mixed number.

27.

1.99

28.

5.83

29.

15.7

30.

18.1

31.

0.239

32.

0.373

33.

0.13

34.

0.19

35.

0.011

36.

0.049

37.

-0.00007

38.

-0.00003

39.

6.4

40.

5.2

41.

7.05

42.

9.04

43.

4.006

44.

2.008

45.

10.25

46.

12.75

47.

1.324

48.

2.482

49.

14.125

50.

20.375

Locate Decimals on the Number Line

In the following exercises, locate each number on a number line.

51.

0.8

52.

0.3

53.

−0.2

54.

−0.9

55.

3.1

56.

2.7

57.

−2.5

58.

−1.6

Order Decimals

In the following exercises, order each of the following pairs of numbers, using $<$ or $>$.

59.

0.9 _ 0.6

60.

0.7 _ 0.8

61.

0.37 _ 0.63

62.

0.86 _ 0.69

63.

0.6 _ 0.59

64.

0.27 _ 0.3

65.

0.91 _ 0.901

66.

$0.415 \_ 0.41$

67.

$-0.5 \_ -0.3$

68.

$-0.1 \_ -0.4$

69.

$-0.62 \_ -0.619$

70.

$-7.31 \_ -7.3$

Round Decimals

In the following exercises, round each number to the nearest tenth.

71.

0.67

72.

0.49

73.

2.84

74.

4.63

In the following exercises, round each number to the nearest hundredth.

75.

0.845

76.

0.761

77.

5.7932

78.

3.6284

79.

0.299

80.

0.697

81.

4.098

82.

7.096

In the following exercises, round each number to the nearest ⓐ hundredth ⓑ tenth ⓒ whole number.

83.

5.781

84.

1.638

85.

63.479

86.

84.281

Everyday Math

87.

Salary Increase Danny got a raise and now makes $58,965.95 a year. Round this number to the nearest:

ⓐ dollar

ⓑ thousand dollars

ⓒ ten thousand dollars.

88.

New Car Purchase Selena's new car cost $23,795.95. Round this number to the nearest:

ⓐ dollar

ⓑ thousand dollars

ⓒ ten thousand dollars.

89.

Sales Tax Hyo Jin lives in San Diego. She bought a refrigerator for $1624.99 and when the clerk calculated the sales tax it came out to exactly $142.186625. Round the sales tax to the nearest ⓐ penny ⓑ dollar.

90.

Sales Tax Jennifer bought a $1,038.99 dining room set for her home in Cincinnati. She calculated the sales tax to be exactly $67.53435. Round the sales tax to the nearest ⓐ penny ⓑ dollar.

Writing Exercises

91.

How does your knowledge of money help you learn about decimals?

92.

Explain how you write "three and nine hundredths" as a decimal.

93.

Jim ran a 100-meter race in 12.32 seconds. Tim ran the same race in 12.3 seconds. Who had the faster time, Jim or Tim? How do you know?

94.

Gerry saw a sign advertising postcards marked for sale at "10 for 0.99¢." What is wrong with the advertised price?

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
name decimals.			
write decimals.			
convert decimals to fractions or mixed numbers.			
locate decimals on the number line.			
order decimals.			
round decimals.			

ⓑ If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them. What did you do to become confident of your ability to do these things? Be specific.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is important to make sure you have a strong foundation before you move on. Whom can you ask for help? Your fellow classmates and instructor are good resources. Is there a place on campus where math tutors are available? Can your study skills be improved?

…no—I don't get it! This is a warning sign and you must not ignore it. You should get help right away or you will quickly be overwhelmed. See your instructor as soon as you can to discuss your situation. Together you can come up with a plan to get you the help you need.

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4.8: Decimal Operations

Learning Objectives

By the end of this section, you will be able to:

- Add and subtract decimals
- Multiply decimals
- Divide decimals
- Use decimals in money applications

Be Prepared 5.4

Before you get started, take this readiness quiz.

Simplify $\frac{70}{100}$.
If you missed this problem, review Example 4.19.

Be Prepared 5.5

Multiply $\frac{3}{10} \cdot \frac{9}{10}$.
If you missed this problem, review Example 4.25.

Be Prepared 5.6

Divide $-36 \div (-9)$.
If you missed this problem, review Example 3.49.

Add and Subtract Decimals

Let's take one more look at the lunch order from the start of Decimals, this time noticing how the numbers were added together.

$3.45	Sandwich
$1.25	Water
+ $0.33	Tax
$5.03	Total

All three items (sandwich, water, tax) were priced in dollars and cents, so we lined up the dollars under the dollars and the cents under the cents, with the decimal points lined up between them. Then we just added each column, as if we were adding whole numbers. By lining up decimals this way, we can add or subtract the corresponding place values just as we did with whole numbers.

How To

Add or subtract decimals.

1. Step 1. Write the numbers vertically so the decimal points line up.
2. Step 2. Use zeros as place holders, as needed.
3. Step 3. Add or subtract the numbers as if they were whole numbers. Then place the decimal in the answer under the decimal points in the given numbers.

Example 5.11

Add: $3.7 + 12.4$.

Answer

	$3.7 + 12.4$
Write the numbers vertically so the decimal points line up.	$\begin{array}{r} 3.7 \\ +12.4 \end{array}$

Place holders are not needed since both numbers have the same number of decimal places.	
Add the numbers as if they were whole numbers. Then place the decimal in the answer under the decimal points in the given numbers.	$\begin{array}{r} {\scriptstyle 1}\\ 3.7\\ +12.4\\ \hline 16.1 \end{array}$

Try It 5.21

Add: $5.7 + 11.9$.

Try It 5.22

Add: $18.32 + 14.79$.

Example 5.12

Add: $23.5 + 41.38$.

Answer

	$23.5 + 41.38$
Write the numbers vertically so the decimal points line up.	$\begin{array}{r} 23.5\\ +\,41.38\\ \hline \end{array}$
Place 0 as a place holder after the 5 in 23.5, so that both numbers have two decimal places.	$\begin{array}{r} 23.50\\ +\,41.38\\ \hline \end{array}$
Add the numbers as if they were whole numbers. Then place the decimal in the answer under the decimal points in the given numbers.	$\begin{array}{r} 23.50\\ +\,41.38\\ \hline 64.88 \end{array}$

Try It 5.23

Add: $4.8 + 11.69$.

Try It 5.24

Add: $5.123 + 18.47$.

How much change would you get if you handed the cashier a $\$20$ bill for a $\$14.65$ purchase? We will show the steps to calculate this in the next example.

Example 5.13

Subtract: $20 - 14.65$.

Answer

	$20 - 14.65$
Write the numbers vertically so the decimal points line up. Remember 20 is a whole number, so place the decimal point after the 0.	$\begin{array}{r} 20.\\ -\,14.65\\ \hline \end{array}$
Place two zeros after the decimal point in 20, as place holders so that both numbers have two decimal places.	$\begin{array}{r} 20.00\\ -\,14.65\\ \hline \end{array}$

Subtract the numbers as if they were whole numbers. Then place the decimal in the answer under the decimal points in the given numbers.	$\begin{array}{r} {}^{1}\not{2}{}^{9}\not{0}.{}^{9}\not{0}{}^{10}\not{0} \\ -\ 14.65 \\ \hline 5.35 \end{array}$

Try It 5.25

Subtract:

$10 - 9.58.$

Try It 5.26

Subtract:

$50 - 37.42.$

Example 5.14

Subtract: $2.51 - 7.4.$

Answer

If we subtract 7.4 from 2.51, the answer will be negative since $7.4 > 2.51.$ To subtract easily, we can subtract 2.51 from $7.4.$ Then we will place the negative sign in the result.

	$2.51 - 7.4$
Write the numbers vertically so the decimal points line up.	$\begin{array}{r} 7.4 \\ -\ 2.51 \\ \hline \end{array}$
Place zero after the 4 in 7.4 as a place holder, so that both numbers have two decimal places.	$\begin{array}{r} 7.40 \\ -\ 2.51 \\ \hline \end{array}$
Subtract and place the decimal in the answer.	$\begin{array}{r} 7.40 \\ -\ 2.51 \\ \hline 4.89 \end{array}$
Remember that we are really subtracting $2.51 - 7.4$ so the answer is negative.	$2.51 - 7.4 = -4.89$

Try It 5.27

Subtract: $4.77 - 6.3.$

Try It 5.28

Subtract: $8.12 - 11.7.$

Multiply Decimals

Multiplying decimals is very much like multiplying whole numbers—we just have to determine where to place the decimal point. The procedure for multiplying decimals will make sense if we first review multiplying fractions.

Do you remember how to multiply fractions? To multiply fractions, you multiply the numerators and then multiply the denominators.

So let's see what we would get as the product of decimals by converting them to fractions first. We will do two examples side-by-side in Table 5.3. Look for a pattern.

	A	B
	$(0.3)(0.7)$	$(0.2)(0.46)$
Convert to fractions.	$\left(\frac{3}{10}\right)\left(\frac{7}{10}\right)$	$\left(\frac{2}{10}\right)\left(\frac{46}{100}\right)$
Multiply.	$\frac{21}{100}$	$\frac{92}{1000}$
Convert back to decimals.	0.21	0.092

Table 5.3

There is a pattern that we can use. In A, we multiplied two numbers that each had one decimal place, and the product had two decimal places. In B, we multiplied a number with one decimal place by a number with two decimal places, and the product had three decimal places.

How many decimal places would you expect for the product of $(0.01)(0.004)$? If you said “five”, you recognized the pattern. When we multiply two numbers with decimals, we count all the decimal places in the factors—in this case two plus three—to get the number of decimal places in the product—in this case five.

$$\underbrace{(0.01)}_{2\text{ places}}\ \underbrace{(0.004)}_{3\text{ places}} = \underbrace{0.00004}_{5\text{ places}}$$

$$\left(\frac{1}{100}\right)\left(\frac{4}{1000}\right) = \frac{4}{100{,}000}$$

Once we know how to determine the number of digits after the decimal point, we can multiply decimal numbers without converting them to fractions first. The number of decimal places in the product is the sum of the number of decimal places in the factors.

The rules for multiplying positive and negative numbers apply to decimals, too, of course.

Multiplying Two Numbers

When multiplying two numbers,

- if their signs are the same, the product is positive.
- if their signs are different, the product is negative.

When you multiply signed decimals, first determine the sign of the product and then multiply as if the numbers were both positive. Finally, write the product with the appropriate sign.

How To

Multiply decimal numbers.

1. Step 1. Determine the sign of the product.
2. Step 2. Write the numbers in vertical format, lining up the numbers on the right.
3. Step 3. Multiply the numbers as if they were whole numbers, temporarily ignoring the decimal points.
4. Step 4. Place the decimal point. The number of decimal places in the product is the sum of the number of decimal places in the factors. If needed, use zeros as placeholders.
5. Step 5. Write the product with the appropriate sign.

Example 5.15

Multiply: $(3.9)(4.075)$.

Answer

	$(3.9)(4.075)$
Determine the sign of the product. The signs are the same.	The product will be positive.

Write the numbers in vertical format, lining up the numbers on the right.	$\begin{array}{r} 4.075 \\ \times\ 3.9 \end{array}$
Multiply the numbers as if they were whole numbers, temporarily ignoring the decimal points.	$\begin{array}{r} 4.075 \\ \times\ 3.9 \\ \hline 36675 \\ 12225 \\ \hline 158925 \end{array}$
Place the decimal point. Add the number of decimal places in the factors $(1+3)$. Place the decimal point 4 places from the right.	$\begin{array}{r} 4.075 \quad 3 \text{ places} \\ \times\ 3.9 \quad 1 \text{ place} \\ \hline 36675 \\ 12225 \\ \hline 158925 \quad 4 \text{ places} \end{array}$
The product is positive.	$(3.9)(4.075) = 15.8925$

Try It 5.29

Multiply: $4.5(6.107)$.

Try It 5.30

Multiply: $10.79(8.12)$.

Example 5.16

Multiply: $(-8.2)(5.19)$.

Answer

	$(-8.2)(5.19)$
The signs are different.	The product will be negative.
Write in vertical format, lining up the numbers on the right.	$\begin{array}{r} 5.19 \\ \times 8.2 \end{array}$
Multiply.	$\begin{array}{r} 5.19 \\ \times 8.2 \\ \hline 1038 \\ 4152 \\ \hline 42558 \end{array}$
Place the decimal point 3 places from the right. $(-8.2)(5.19)$ 1 place 2 places	$\begin{array}{r} 5.19 \\ \times 8.2 \\ \hline 1038 \\ 4152 \\ \hline 42.558 \end{array}$
The product is negative.	$(-8.2)(5.19) = -42.558$

Try It 5.31

Multiply: $(4.63)(-2.9)$.

Try It 5.32

Multiply: $(-7.78)(4.9)$.

In the next example, we'll need to add several placeholder zeros to properly place the decimal point.

Example 5.17

Multiply: $(0.03)(0.045)$.

Answer

	$(0.03)(0.045)$
The product is positive.	
Write in vertical format, lining up the numbers on the right.	$\begin{array}{r} 0.045 \\ \times\ 0.03 \\ \hline \end{array}$
Multiply.	$\begin{array}{r} 0.045 \\ \times\ 0.03 \\ \hline 135 \end{array}$
The decimal point must be 5 places from the right. (0.03)(0.045) 2 places 3 places Add zeros as needed to get the 5 places.	$\begin{array}{r} 0.045 \\ \times\ 0.03 \\ \hline 0.00135 \end{array}$
The product is positive.	$(0.03)(0.045) = 0.00135$

Try It 5.33

Multiply: $(0.04)(0.087)$.

Try It 5.34

Multiply: $(0.09)(0.067)$.

Multiply by Powers of 10

In many fields, especially in the sciences, it is common to multiply decimals by powers of 10. Let's see what happens when we multiply 1.9436 by some powers of 10.

1.9436(10)

$$\begin{array}{r} 1.9436 \\ \times\ \ 10 \\ \hline 19.4360 \end{array}$$

1.9436(100)

$$\begin{array}{r} 1.9436 \\ \times\ 100 \\ \hline 194.3600 \end{array}$$

1.9436(1000)

$$\begin{array}{r} 1.9436 \\ \times\ 1000 \\ \hline 1943.6000 \end{array}$$

Look at the results without the final zeros. Do you notice a pattern?

$$\begin{aligned} 1.9436(10) &= 19.436 \\ 1.9436(100) &= 194.36 \\ 1.9436(1000) &= 1943.6 \end{aligned}$$

The number of places that the decimal point moved is the same as the number of zeros in the power of ten. Table 5.4 summarizes the results.

Multiply by	Number of zeros	Number of places decimal point moves
10	1	1 place to the right
100	2	2 places to the right
1,000	3	3 places to the right
10,000	4	4 places to the right

Table 5.4

We can use this pattern as a shortcut to multiply by powers of ten instead of multiplying using the vertical format. We can count the zeros in the power of 10 and then move the decimal point that same of places to the right.

So, for example, to multiply 45.86 by 100, move the decimal point 2 places to the right.

$45.86 \times 100 = 4586.$



Sometimes when we need to move the decimal point, there are not enough decimal places. In that case, we use zeros as placeholders. For example, let's multiply 2.4 by 100. We need to move the decimal point 2 places to the right. Since there is only one digit to the right of the decimal point, we must write a 0 in the hundredths place.

$2.4 \times 100 = 240.$



How To

Multiply a decimal by a power of 10.

1. Step 1. Move the decimal point to the right the same number of places as the number of zeros in the power of 10.
2. Step 2. Write zeros at the end of the number as placeholders if needed.

Example 5.18

Multiply 5.63 by factors of ⓐ 10 ⓑ 100 ⓒ 1000.

Answer

By looking at the number of zeros in the multiple of ten, we see the number of places we need to move the decimal to the right.

ⓐ	
	$56.3\,(10)$
There is 1 zero in 10, so move the decimal point 1 place to the right.	5.63 
	56.3

ⓑ	
	$5.63\,(100)$
There are 2 zeros in 100, so move the decimal point 2 places to the right.	5.63 
	563

ⓒ	
	$5.63\,(1000)$
There are 3 zeros in 1000, so move the decimal point 3 places to the right.	5.63 
A zero must be added at the end.	$5{,}630$

Try It 5.35

Multiply 2.58 by factors of ⓐ 10 ⓑ 100 ⓒ 1000.

Try It 5.36

Multiply 14.2 by factors of ⓐ 10 ⓑ 100 ⓒ 1000.

Divide Decimals

Just as with multiplication, division of decimals is very much like dividing whole numbers. We just have to figure out where the decimal point must be placed.

To understand decimal division, let's consider the multiplication problem

$$(0.2)(4) = 0.8$$

Remember, a multiplication problem can be rephrased as a division problem. So we can write

$$0.8 \div 4 = 0.2$$

We can think of this as "If we divide 8 tenths into four groups, how many are in each group?" Figure 5.5 shows that there are four groups of two-tenths in eight-tenths. So $0.8 \div 4 = 0.2$.

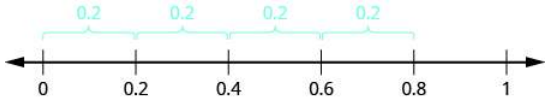


Figure 5.5

Using long division notation, we would write

$$4\overline{)0.8}\quad 0.2$$

Notice that the decimal point in the quotient is directly above the decimal point in the dividend.

To divide a decimal by a whole number, we place the decimal point in the quotient above the decimal point in the dividend and then divide as usual. Sometimes we need to use extra zeros at the end of the dividend to keep dividing until there is no remainder.

How To

Divide a decimal by a whole number.

1. Step 1. Write as long division, placing the decimal point in the quotient above the decimal point in the dividend.
2. Step 2. Divide as usual.

Example 5.19

Divide: $0.12 \div 3$.

Answer

	$0.12 \div 3$
Write as long division, placing the decimal point in the quotient above the decimal point in the dividend.	$3\overline{)0.12}$
Divide as usual. Since 3 does not go into 0 or 1 we use zeros as placeholders.	$3\overline{)0.12}\quad 0.04$ 12 0
	$0.12 \div 3 = 0.04$

Try It 5.37

Divide: $0.28 \div 4$.

Try It 5.38

Divide: $0.56 \div 7$.

In everyday life, we divide whole numbers into decimals—money—to find the price of one item. For example, suppose a case of 24 water bottles cost $\$3.99$. To find the price per water bottle, we would divide $\$3.99$ by 24, and round the answer to the nearest cent (hundredth).

Example 5.20

Divide: $\$3.99 \div 24$.

Answer

	$\$3.99 \div 24$
Place the decimal point in the quotient above the decimal point in the dividend.	$24\overline{)3.99}$
Divide as usual. When do we stop? Since this division involves money, we round it to the nearest cent (hundredth). To do this, we must carry the division to the thousandths place.	0.166 $24\overline{)3.990}$ 24 159 144 150 144 6
Round to the nearest cent.	$\$0.166 \approx \$0.17$
	$\$3.99 \div 24 \approx \$0.17$

This means the price per bottle is 17 cents.

Try It 5.39

Divide: $\$6.99 \div 36$.

Try It 5.40

Divide: $\$4.99 \div 12$.

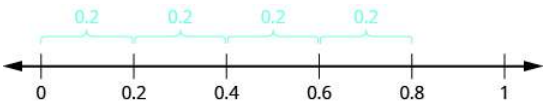
Divide a Decimal by Another Decimal

So far, we have divided a decimal by a whole number. What happens when we divide a decimal by another decimal? Let's look at the same multiplication problem we looked at earlier, but in a different way.

$$(0.2)(4) = 0.8$$

Remember, again, that a multiplication problem can be rephrased as a division problem. This time we ask, "How many times does 0.2 go into 0.8?" Because $(0.2)(4) = 0.8$, we can say that 0.2 goes into 0.8 four times. This means that 0.8 divided by 0.2 is 4.

$$0.8 \div 0.2 = 4$$



We would get the same answer, 4, if we divide 8 by 2, both whole numbers. Why is this so? Let's think about the division problem as a fraction.

$$\frac{0.8}{0.2}$$

$$\frac{(0.8)10}{(0.2)10}$$

$$\frac{8}{2}$$

$$4$$

We multiplied the numerator and denominator by 10 and ended up just dividing 8 by 2. To divide decimals, we multiply both the numerator and denominator by the same power of 10 to make the denominator a whole number. Because of the Equivalent Fractions Property, we haven't changed the value of the fraction. The effect is to move the decimal points in the numerator and denominator the same number of places to the right.

We use the rules for dividing positive and negative numbers with decimals, too. When dividing signed decimals, first determine the sign of the quotient and then divide as if the numbers were both positive. Finally, write the quotient with the appropriate sign.

It may help to review the vocabulary for division:

$a \div b$ — a dividend, b divisor

$\frac{a}{b}$ — a dividend, b divisor

$b\overline{)a}$ — b divisor, a dividend

How To

Divide decimal numbers.

1. Step 1. Determine the sign of the quotient.
2. Step 2. Make the divisor a whole number by moving the decimal point all the way to the right. Move the decimal point in the dividend the same number of places to the right, writing zeros as needed.
3. Step 3. Divide. Place the decimal point in the quotient above the decimal point in the dividend.
4. Step 4. Write the quotient with the appropriate sign.

Example 5.21

Divide: $-2.89 \div (3.4)$.

Answer

Determine the sign of the quotient.	The quotient will be negative.
Make the divisor the whole number by 'moving' the decimal point all the way to the right. 'Move' the decimal point in the dividend the same number of places to the right.	$3.4\overline{)2.89}$
Divide. Place the decimal point in the quotient above the decimal point in the dividend. Add zeros as needed until the remainder is zero.	$34\overline{)28.90}$ 0.85 272 170 170 0
Write the quotient with the appropriate sign.	$-2.89 \div (3.4) = -0.85$

Try It 5.41

Divide: $-1.989 \div 5.1$.

Try It 5.42

Divide: $-2.04 \div 5.1$.

Example 5.22

Divide: $-25.65 \div (-0.06)$.

Answer

	$-25.65 \div (-0.06)$
The signs are the same.	The quotient is positive.
Make the divisor a whole number by 'moving' the decimal point all the way to the right. 'Move' the decimal point in the dividend the same number of places.	$0.06\overline{)25.65}$
Divide. Place the decimal point in the quotient above the decimal point in the dividend.	$\begin{array}{r} 427.5 \\ 006.\overline{)2565.0} \\ -24 \\ \hline 16 \\ -12 \\ \hline 45 \\ -42 \\ \hline 30 \\ 30 \end{array}$
Write the quotient with the appropriate sign.	$-25.65 \div (-0.06) = 427.5$

Try It 5.43

Divide: $-23.492 \div (-0.04)$.

Try It 5.44

Divide: $-4.11 \div (-0.12)$.

Now we will divide a whole number by a decimal number.

Example 5.23

Divide: $4 \div 0.05$.

Answer

	$4 \div 0.05$
The signs are the same.	The quotient is positive.
Make the divisor a whole number by 'moving' the decimal point all the way to the right. Move the decimal point in the dividend the same number of places, adding zeros as needed.	$0.05\overline{)4.00}$
Divide. Place the decimal point in the quotient above the decimal point in the dividend.	$\begin{array}{r} 80. \\ 5\overline{)400.} \\ 40 \\ \hline 00 \\ 00 \end{array}$
Write the quotient with the appropriate sign.	$4 \div 0.05 = 80$

We can relate this example to money. How many nickels are there in four dollars? Because $4 \div 0.05 = 80$, there are 80 nickels in $\$4$.

Try It 5.45

Divide: $6 \div 0.03$.

Try It 5.46

Divide: $7 \div 0.02$.

Use Decimals in Money Applications

We often apply decimals in real life, and most of the applications involving money. The Strategy for Applications we used in The Language of Algebra gives us a plan to follow to help find the answer. Take a moment to review that strategy now.

Strategy for Applications

1. Identify what you are asked to find.
2. Write a phrase that gives the information to find it.
3. Translate the phrase to an expression.
4. Simplify the expression.
5. Answer the question with a complete sentence.

Example 5.24

Paul received $\$50$ for his birthday. He spent $\$31.64$ on a video game. How much of Paul's birthday money was left?

Answer

What are you asked to find?	How much did Paul have left?
Write a phrase.	\$50 less \$31.64
Translate.	$50 - 31.64$
Simplify.	18.36
Write a sentence.	Paul has \$18.36 left.

Try It 5.47

Nicole earned $\$35$ for babysitting her cousins, then went to the bookstore and spent $\$18.48$ on books and coffee. How much of her babysitting money was left?

Try It 5.48

Amber bought a pair of shoes for $\$24.75$ and a purse for $\$36.90$. The sales tax was $\$4.32$. How much did Amber spend?

Example 5.25

Jessie put 8 gallons of gas in her car. One gallon of gas costs $\$3.529$. How much does Jessie owe for the gas? (Round the answer to the nearest cent.)

Answer

What are you asked to find?	How much did Jessie owe for all the gas?
Write a phrase.	8 times the cost of one gallon of gas
Translate.	$8(\$3.529)$
Simplify.	\$28.232
Round to the nearest cent.	\$28.23

Write a sentence.	Jessie owes $28.23 for her gas purchase.

Try It 5.49

Hector put 13 gallons of gas into his car. One gallon of gas costs $\$3.175$. How much did Hector owe for the gas? Round to the nearest cent.

Try It 5.50

Christopher bought 5 pizzas for the team. Each pizza cost $\$9.75$. How much did all the pizzas cost?

Example 5.26

Four friends went out for dinner. They shared a large pizza and a pitcher of soda. The total cost of their dinner was $\$31.76$. If they divide the cost equally, how much should each friend pay?

Answer

What are you asked to find?	How much should each friend pay?
Write a phrase.	$31.76 divided equally among the four friends.
Translate to an expression.	$\$31.76 \div 4$
Simplify.	$7.94
Write a sentence.	Each friend should pay $7.94 for his share of the dinner.

Try It 5.51

Six friends went out for dinner. The total cost of their dinner was $\$92.82$. If they divide the bill equally, how much should each friend pay?

Try It 5.52

Chad worked 40 hours last week and his paycheck was $\$570$. How much does he earn per hour?

Be careful to follow the order of operations in the next example. Remember to multiply before you add.

Example 5.27

Marla buys 6 bananas that cost $\$0.22$ each and 4 oranges that cost $\$0.49$ each. How much is the total cost of the fruit?

Answer

What are you asked to find?	How much is the total cost of the fruit?
Write a phrase.	6 times the cost of each banana plus 4 times the cost of each orange
Translate to an expression.	$6(\$0.22) + 4(\$0.49)$
Simplify.	$\$1.32 + \$1.96$
Add.	$3.28
Write a sentence.	Marla's total cost for the fruit is $3.28.

Try It 5.53

Suzanne buys 3 cans of beans that cost $\$0.75$ each and 6 cans of corn that cost $\$0.62$ each. How much is the total cost of these groceries?

Try It 5.54

Lydia bought movie tickets for the family. She bought two adult tickets for $\$9.50$ each and four children's tickets for $\$6.00$ each. How much did the tickets cost Lydia in all?

Links To Literacy

The *Links to Literacy* activity "Alexander Who Used to be Rich Last Sunday" will provide you with another view of the topics covered in this section.

Media

ACCESS ADDITIONAL ONLINE RESOURCES

- Adding and Subtracting Decimals
- Multiplying Decimals
- Multiplying by Powers of Ten
- Dividing Decimals
- Dividing by Powers of Ten

Section 5.2 Exercises

Practice Makes Perfect

Add and Subtract Decimals

In the following exercises, add or subtract.

95.

$16.92 + 7.56$

96.

$18.37 + 9.36$

97.

$256.37 - 85.49$

98.

$248.25 - 91.29$

99.

$21.76 - 30.99$

100.

$15.35 - 20.88$

101.

$37.5 + 12.23$

102.

$38.6 + 13.67$

103.

$-16.53 - 24.38$

104.

$-19.47 - 32.58$

105.

$-38.69 + 31.47$

106.

$-29.83 + 19.76$

107.

$-4.2 + (-9.3)$

108.

$-8.6 + (-8.6)$

109.

$100 - 64.2$

110.

$100 - 65.83$

111.

$72.5 - 100$

112.

$86.2 - 100$

113.

$15 + 0.73$

114.

$27 + 0.87$

115.

$2.51 + 40$

116.

$9.38 + 60$

117.

$91.75 - (-10.462)$

118.

$94.69 - (-12.678)$

119.

$55.01 - 3.7$

120.

$59.08 - 4.6$

121.

$2.51 - 7.4$

122.

$3.84 - 6.1$

Multiply Decimals

In the following exercises, multiply.

123.

$(0.3)(0.4)$

124.

$(0.6)(0.7)$

125.

$(0.24)(0.6)$

126.

$(0.81)(0.3)$

127.

$(5.9)(7.12)$

128.

$(2.3)(9.41)$

129.

$(8.52)(3.14)$

130.

$(5.32)(4.86)$

131.

$(-4.3)(2.71)$

132.

$(-8.5)(1.69)$

133.

$(-5.18)(-65.23)$

134.

$(-9.16)(-68.34)$

135.

$(0.09)(24.78)$

136.

$(0.04)(36.89)$

137.

$(0.06)(21.75)$

138.

$(0.08)(52.45)$

139.

$(9.24)(10)$

140.

$(6.531)(10)$

141.

$(55.2)(1{,}000)$

142.

$(99.4)(1{,}000)$

Divide Decimals

In the following exercises, divide.

143.

$0.15 \div 5$

144.

$0.27 \div 3$

145.

$4.75 \div 25$

146.

$12.04 \div 43$

147.

$\$8.49 \div 12$

148.

$\$16.99 \div 9$

149.

$\$117.25 \div 48$

150.

$\$109.24 \div 36$

151.

$0.6 \div 0.2$

152.

$0.8 \div 0.4$

153.

$1.44 \div (-0.3)$

154.

$1.25 \div (-0.5)$

155.

$-1.75 \div (-0.05)$

156.

$-1.15 \div (-0.05)$

157.

$5.2 \div 2.5$

158.

$6.5 \div 3.25$

159.

$12 \div 0.08$

160.

$5 \div 0.04$

161.

$11 \div 0.55$

162.

$14 \div 0.35$

Mixed Practice

In the following exercises, simplify.

163.

$6(12.4 - 9.2)$

164.

$3(15.7 - 8.6)$

165.

$24(0.5) + (0.3)^2$

166.

$35(0.2) + (0.9)^2$

167.

$1.15(26.83 + 1.61)$

168.

$1.18(46.22 + 3.71)$

169.

$\$45 + 0.08(\$45)$

170.

$\$63 + 0.18(\$63)$

171.

$18 \div (0.75 + 0.15)$

172.

$27 \div (0.55 + 0.35)$

173.

$(1.43 + 0.27) \div (0.9 - 0.05)$

174.

$(1.5 - 0.06) \div (0.12 + 0.24)$

175.

$[\$75.42 + 0.18(\$75.42)] \div 5$

176.

$[\$56.31 + 0.22(\$56.31)] \div 4$

Use Decimals in Money Applications

In the following exercises, use the strategy for applications to solve.

177.

Spending money Brenda got $\$40$ from the ATM. She spent $\$15.11$ on a pair of earrings. How much money did she have left?

178.

Spending money Marissa found $\$20$ in her pocket. She spent $\$4.82$ on a smoothie. How much of the $\$20$ did she have left?

179.

Shopping Adam bought a t-shirt for $\$18.49$ and a book for $\$8.92$ The sales tax was $\$1.65$. How much did Adam spend?

180.

Restaurant Roberto's restaurant bill was $\$20.45$ for the entrée and $\$3.15$ for the drink. He left a $\$4.40$ tip. How much did Roberto spend?

181.

Coupon Emily bought a box of cereal that cost $\$4.29$. She had a coupon for $\$0.55$ off, and the store doubled the coupon. How much did she pay for the box of cereal?

182.

Coupon Diana bought a can of coffee that cost $\$7.99$. She had a coupon for $\$0.75$ off, and the store doubled the coupon. How much did she pay for the can of coffee?

183.

Diet Leo took part in a diet program. He weighed 190 pounds at the start of the program. During the first week, he lost 4.3 pounds. During the second week, he had lost 2.8 pounds. The third week, he gained 0.7 pounds. The fourth week, he lost 1.9 pounds. What did Leo weigh at the end of the fourth week?

184.

Snowpack On April 1, the snowpack at the ski resort was 4 meters deep, but the next few days were very warm. By April 5, the snow depth was 1.6 meters less. On April 8, it snowed and added 2.1 meters of snow. What was the total depth of the snow?

185.

Coffee Noriko bought 4 coffees for herself and her co-workers. Each coffee was $\$3.75$. How much did she pay for all the coffees?

186.

Subway Fare Arianna spends $\$4.50$ per day on subway fare. Last week she rode the subway 6 days. How much did she spend for the subway fares?

187.

Income Mayra earns $\$9.25$ per hour. Last week she worked 32 hours. How much did she earn?

188.

Income Peter earns $\$8.75$ per hour. Last week he worked 19 hours. How much did he earn?

189.

Hourly Wage Alan got his first paycheck from his new job. He worked 30 hours and earned $\$382.50$. How much does he earn per hour?

190.

Hourly Wage Maria got her first paycheck from her new job. She worked 25 hours and earned $\$362.50$. How much does she earn per hour?

191.

Restaurant Jeannette and her friends love to order mud pie at their favorite restaurant. They always share just one piece of pie among themselves. With tax and tip, the total cost is $\$6.00$. How much does each girl pay if the total number sharing the mud pie is

ⓐ 2?

ⓑ 3?

ⓒ 4?

ⓓ 5?

ⓔ 6?

192.

Pizza Alex and his friends go out for pizza and video games once a week. They share the cost of a $\$15.60$ pizza equally. How much does each person pay if the total number sharing the pizza is

ⓐ 2?

ⓑ 3?

ⓒ 4?

ⓓ 5?

ⓔ 6?

193.

Fast Food At their favorite fast food restaurant, the Carlson family orders 4 burgers that cost $3.29 each and 2 orders of fries at $2.74 each. What is the total cost of the order?

194.

Home Goods Chelsea needs towels to take with her to college. She buys 2 bath towels that cost $9.99 each and 6 washcloths that cost $2.99 each. What is the total cost for the bath towels and washcloths?

195.

Zoo The Lewis and Chousmith families are planning to go to the zoo together. Adult tickets cost $29.95 and children's tickets cost $19.95. What will the total cost be for 4 adults and 7 children?

196.

Ice Skating Jasmine wants to have her birthday party at the local ice skating rink. It will cost $8.25 per child and $12.95 per adult. What will the total cost be for 12 children and 3 adults?

Everyday Math

197.

Paycheck Annie has two jobs. She gets paid $14.04 per hour for tutoring at City College and $8.75 per hour at a coffee shop. Last week she tutored for 8 hours and worked at the coffee shop for 15 hours.

ⓐ How much did she earn?

ⓑ If she had worked all 23 hours as a tutor instead of working both jobs, how much more would she have earned?

198.

Paycheck Jake has two jobs. He gets paid $7.95 per hour at the college cafeteria and $20.25 at the art gallery. Last week he worked 12 hours at the cafeteria and 5 hours at the art gallery.

ⓐ How much did he earn?

ⓑ If he had worked all 17 hours at the art gallery instead of working both jobs, how much more would he have earned?

Writing Exercises

199.

At the 2010 winter Olympics, two skiers took the silver and bronze medals in the Men's Super-G ski event. Miller's time was 1 minute 30.62 seconds and Weibrecht's time was 1 minute 30.65 seconds. Find the difference in their times and then write the name of that decimal.

200.

Find the quotient of $0.12 \div 0.04$ and explain in words all the steps taken.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
add and subtract decimals.			
multiply decimals.			
divide decimals.			
use decimals in money applications.			

ⓑ After reviewing this checklist, what will you do to become confident for all objectives?

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4.9: Decimals and Fractions

Learning Objectives

By the end of this section, you will be able to:

- Convert fractions to decimals
- Order decimals and fractions
- Simplify expressions using the order of operations
- Find the circumference and area of circles

Be Prepared 5.7

Before you get started, take this readiness quiz.

Divide: $0.24 \div 8$.
If you missed this problem, review Example 5.19.

Be Prepared 5.8

Order $0.64 \_ 0.6$ using $<$ or $>$.
If you missed this problem, review Example 5.7.

Be Prepared 5.9

Order $-0.2 \_ -0.1$ using $<$ or $>$.
If you missed this problem, review Example 5.8.

Convert Fractions to Decimals

In Decimals, we learned to convert decimals to fractions. Now we will do the reverse—convert fractions to decimals. Remember that the fraction bar indicates division. So $\frac{4}{5}$ can be written $4 \div 5$ or $5\ \overline{)\ 4}$. This means that we can convert a fraction to a decimal by treating it as a division problem.

Convert a Fraction to a Decimal

To convert a fraction to a decimal, divide the numerator of the fraction by the denominator of the fraction.

Example 5.28

Write the fraction $\frac{3}{4}$ as a decimal.

Answer

A fraction bar means division, so we can write the fraction $\frac{3}{4}$ using division.	$4\overline{)3}$
Divide.	$4\overline{)3.00}$ 28 20 20 0
	So the fraction $\frac{3}{4}$ is equal to 0.75.

Try It 5.55

Write the fraction as a decimal: $\frac{1}{4}$.

Try It 5.56

Write the fraction as a decimal: $\frac{3}{8}$.

Example 5.29

Write the fraction $-\frac{7}{2}$ as a decimal.

Answer

The value of this fraction is negative. After dividing, the value of the decimal will be negative. We do the division ignoring the sign, and then write the negative sign in the answer.	$-\frac{7}{2}$
Divide 7 by 2.	$2\overline{)7.0}$ $\underline{6}$ 10 $\underline{10}$ 0
	So, $-\frac{7}{2} = -3.5$.

Try It 5.57

Write the fraction as a decimal: $-\frac{9}{4}$.

Try It 5.58

Write the fraction as a decimal: $-\frac{11}{2}$.

Repeating Decimals

So far, in all the examples converting fractions to decimals the division resulted in a remainder of zero. This is not always the case. Let's see what happens when we convert the fraction $\frac{4}{3}$ to a decimal. First, notice that $\frac{4}{3}$ is an improper fraction. Its value is greater than 1. The equivalent decimal will also be greater than 1.

We divide 4 by 3.

$$
\begin{array}{r}
1.333\ldots \\
3\overline{)4.000} \\
\underline{3} \\
10 \\
\underline{9} \\
10 \\
\underline{9} \\
10 \\
\underline{9} \\
1
\end{array}
$$

No matter how many more zeros we write, there will always be a remainder of 1, and the threes in the quotient will go on forever. The number $1.333\ldots$ is called a repeating decimal. Remember that the "…" means that the pattern repeats.

Repeating Decimal

A repeating decimal is a decimal in which the last digit or group of digits repeats endlessly.

How do you know how many 'repeats' to write? Instead of writing $1.333\ldots$ we use a shorthand notation by placing a line over the digits that repeat. The repeating decimal $1.333\ldots$ is written $1.\overline{3}$. The line above the 3 tells you that the 3 repeats endlessly. So $1.333\ldots = 1.\overline{3}$

For other decimals, two or more digits might repeat. Table 5.5 shows some more examples of repeating decimals.

$1.333... = 1.\overline{3}$	3 is the repeating digit
$4.1666... = 4.1\overline{6}$	6 is the repeating digit
$4.161616... = 4.\overline{16}$	16 is the repeating block
$0.271271271... = 0.\overline{271}$	271 is the repeating block

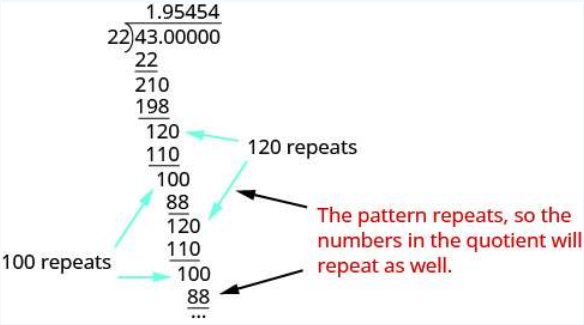
Table 5.5

Example 5.30

Write $\frac{43}{22}$ as a decimal.

Answer

Divide 43 by 22.



Notice that the differences of 120 and 100 repeat, so there is a repeat in the digits of the quotient; 54 will repeat endlessly. The first decimal place in the quotient, 9, is not part of the pattern. So,

$$\frac{43}{22} = 1.9\overline{54}$$

Try It 5.59

Write as a decimal: $\frac{27}{11}$.

Try It 5.60

Write as a decimal: $\frac{51}{22}$.

It is useful to convert between fractions and decimals when we need to add or subtract numbers in different forms. To add a fraction and a decimal, for example, we would need to either convert the fraction to a decimal or the decimal to a fraction.

Example 5.31

Simplify: $\frac{7}{8} + 6.4$.

Answer

		$\frac{7}{8} + 6.4$
Change $\frac{7}{8}$ to a decimal.	0.875 8)7.000 64 60 56 40 40 0	$0.875 + 6.4$

Add.		7.275

Try It 5.61

Simplify: $\frac{3}{8} + 4.9$.

Try It 5.62

Simplify: $5.7 + \frac{13}{20}$.

Order Decimals and Fractions

In Decimals, we compared two decimals and determined which was larger. To compare a decimal to a fraction, we will first convert the fraction to a decimal and then compare the decimals.

Example 5.32

Order $\frac{3}{8} \_ 0.4$ using $<$ or $>$.

Answer

	$\frac{3}{8} \_ 0.4$
Convert $\frac{3}{8}$ to a decimal.	$0.375 \_ 0.4$
Compare 0.375 to 0.4	$0.375 < 0.4$
Rewrite with the original fraction.	$\frac{3}{8} < 0.4$

Try It 5.63

Order each of the following pairs of numbers, using $<$ or $>$.

$\frac{17}{20} \_ 0.82$

Try It 5.64

Order each of the following pairs of numbers, using $<$ or $>$.

$\frac{3}{4} \_ 0.785$

When ordering negative numbers, remember that larger numbers are to the right on the number line and any positive number is greater than any negative number.

Example 5.33

Order $-0.5 \_\_ -\frac{3}{4}$ using $<$ or $>$.

Answer

	$-0.5 \_\_ -\frac{3}{4}$
Convert $-\frac{3}{4}$ to a decimal.	$-0.5 \_\_ -0.75$
Compare -0.5 to -0.75.	$-0.5 > -0.75$
Rewrite the inequality with the original fraction.	$-0.5 > -\frac{3}{4}$

Try It 5.65

Order each of the following pairs of numbers, using $<$ or $>$:

$-\frac{5}{8}$ _ -0.58

Try It 5.66

Order each of the following pairs of numbers, using $<$ or $>$:

-0.53 _ $-\frac{11}{20}$

Example 5.34

Write the numbers $\frac{13}{20}, 0.61, \frac{11}{16}$ in order from smallest to largest.

Answer

	$\frac{13}{20}, 0.61, \frac{11}{16}$
Convert the fractions to decimals.	$0.65, 0.61, 0.6875$
Write the smallest decimal number first.	$0.61, \_\_\_, \_\_\_$
Write the next larger decimal number in the middle place.	$0.61, 0.65, \_\_\_$
Write the last decimal number (the larger) in the third place.	$0.61, 0.65, 0.6875$
Rewrite the list with the original fractions.	$0.61, \frac{13}{20}, \frac{11}{16}$

Try It 5.67

Write each set of numbers in order from smallest to largest: $\frac{7}{8}, \frac{4}{5}, 0.82$.

Try It 5.68

Write each set of numbers in order from smallest to largest: $0.835, \frac{13}{16}, \frac{3}{4}$.

Simplify Expressions Using the Order of Operations

The order of operations introduced in Use the Language of Algebra also applies to decimals. Do you remember what the phrase "Please excuse my dear Aunt Sally" stands for?

Example 5.35

Simplify the expressions:

1. ⓐ $7\,(18.3 - 21.7)$
2. ⓑ $\frac{2}{3}(8.3 - 3.8)$

Answer

ⓐ	
	$7(18.3 - 21.7)$
Simplify inside parentheses.	$7(-3.4)$
Multiply.	-23.8

ⓑ	
	$\frac{2}{3}(8.3 - 3.8)$
Simplify inside parentheses.	$\frac{2}{3}(4.5)$

Write 4.5 as a fraction.	$\frac{2}{3}(\frac{4.5}{1})$
Multiply.	$\frac{9}{3}$
Simplify.	3

Try It 5.69

Simplify: ⓐ $8(14.6-37.5)$ ⓑ $\frac{3}{5}(9.6-2.1)$.

Try It 5.70

Simplify: ⓐ $25(25.69-56.74)$ ⓑ $\frac{2}{7}(11.9-4.2)$.

Example 5.36

Simplify each expression:

1. ⓐ $6\div 0.6+(0.2)4-(0.1)^2$
2. ⓑ $(\frac{1}{10})^2+(3.5)(0.9)$

Answer

ⓐ	
	$6\div 0.6+(0.2)4-(0.1)^2$
Simplify exponents.	$6\div 0.6+(0.2)4-0.01$
Divide.	$10+(0.2)4-0.01$
Multiply.	$10+0.8-0.01$
Add.	$10.8-0.01$
Subtract.	10.79

ⓑ	
	$(\frac{1}{10})^2+(3.5)(0.9)$
Simplify exponents.	$\frac{1}{100}+(3.5)(0.9)$
Multiply.	$\frac{1}{100}+3.15$
Convert $\frac{1}{100}$ to a decimal.	$0.01+3.15$
Add.	3.16

Try It 5.71

Simplify: $9\div 0.9+(0.4)3-(0.2)^2$.

Try It 5.72

Simplify: $(\frac{1}{2})^2+(0.3)(4.2)$.

Find the Circumference and Area of Circles

The properties of circles have been studied for over Math input error

The size of a circle can be measured in two ways. The distance around a circle is called its circumference.

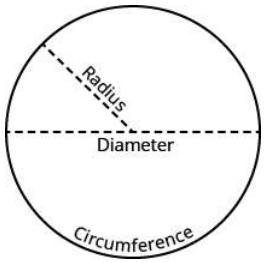


Figure 5.6

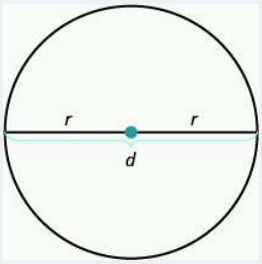
Archimedes discovered that for circles of all different sizes, dividing the circumference by the diameter always gives the same number. The value of this number is pi, symbolized by Greek letter Math input error

Manipulative Mathematics

Doing the Manipulative Mathematics activity Pi Lab will help you develop a better understanding of pi.

If we want the exact circumference or area of a circle, we leave the symbol π in the answer. We can get an approximate answer by substituting 3.14 as the value of π. We use the symbol $\approx$ to show that the result is approximate, not exact.

Properties of Circles



r is the length of the radius.
d is the length of the diameter.

The circumference is $2\pi r$.	$C = 2\pi r$
The area is πr^2.	$A = \pi r^2$

Since the diameter is twice the radius, another way to find the circumference is to use the formula $C = \pi d$.

Suppose we want to find the exact area of a circle of radius 10 inches. To calculate the area, we would evaluate the formula for the area when $r = 10$ inches and leave the answer in terms of π.

$$\begin{aligned} A &= \pi r^2 \\ A &= \pi(10^2) \\ A &= \pi \cdot 100 \end{aligned}$$

We write π after the 100. So the exact value of the area is $A = 100\pi$ square inches.

To approximate the area, we would substitute $\pi \approx 3.14$.

$$\begin{aligned} A &= 100\pi \\ &\approx 100 \cdot 3.14 \\ &\approx 314 \text{ square inches} \end{aligned}$$

Remember to use square units, such as square inches, when you calculate the area.

Example 5.37

A circle has radius 10 centimeters. Approximate its ⓐ circumference and ⓑ area.

Answer

ⓐ Find the circumference when $r = 10$.	
Write the formula for circumference.	$C = 2\pi r$
Substitute 3.14 for π and 10 for ,r.	$C \approx 2(3.14)(10)$
Multiply.	$C \approx 62.8$ centimeters

ⓑ Find the area when $r = 10$.	
Write the formula for area.	$A = \pi r^2$
Substitute 3.14 for π and 10 for r.	$A \approx (3.14)(10)^2$
Multiply.	$A \approx 314$ square centimeters

Try It 5.73

A circle has radius 50 inches. Approximate its ⓐ circumference and ⓑ area.

Try It 5.74

A circle has radius 100 feet. Approximate its ⓐ circumference and ⓑ area.

Example 5.38

A circle has radius 42.5 centimeters. Approximate its ⓐ circumference and ⓑ area.

Answer

ⓐ Find the circumference when $r = 42.5$.	
Write the formula for circumference.	$C = 2\pi r$
Substitute 3.14 for π and 42.5 for r	$C \approx 2(3.14)(42.5)$
Multiply.	$C \approx 266.9$ centimeters

ⓑ Find the area when $r = 42.5$.	
Write the formula for area.	$A = \pi r^2$
Substitute 3.14 for π and 42.5 for r.	$A \approx (3.14)(42.5)^2$
Multiply.	$A \approx 5671.625$ square centimeters

Try It 5.75

A circle has radius 51.8 centimeters. Approximate its ⓐ circumference and ⓑ area.

Try It 5.76

A circle has radius 26.4 meters. Approximate its ⓐ circumference and ⓑ area.

Approximate π with a Fraction

Convert the fraction $\frac{22}{7}$ to a decimal. If you use your calculator, the decimal number will fill up the display and show 3.14285714. But if we round that number to two decimal places, we get 3.14, the decimal approximation of π. When we have a circle with radius

given as a fraction, we can substitute $\frac{22}{7}$ for π instead of 3.14. And, since $\frac{22}{7}$ is also an approximation of π, we will use the $\approx$ symbol to show we have an approximate value.

Example 5.39

A circle has radius $\frac{14}{15}$ meter. Approximate its ⓐ circumference and ⓑ area.

Answer

ⓐ Find the circumference when $r = \frac{14}{15}$.	
Write the formula for circumference.	$C = 2\pi r$
Substitute $\frac{22}{7}$ for π and $\frac{14}{15}$ for r.	$C \approx 2(\frac{22}{7})(\frac{14}{15})$
Multiply.	$C \approx \frac{88}{15}$ meters

ⓑ Find the area when $r = \frac{14}{15}$.	
Write the formula for area.	$A = \pi r^2$
Substitute $\frac{22}{7}$ for π and $\frac{14}{15}$ for r.	$A \approx (\frac{22}{7})(\frac{14}{15})^2$
Multiply.	$A \approx \frac{616}{225}$ square meters

Try It 5.77

A circle has radius $\frac{5}{21}$ meters. Approximate its ⓐ circumference and ⓑ area.

Try It 5.78

A circle has radius $\frac{10}{33}$ inches. Approximate its ⓐ circumference and ⓑ area.

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ACCESS ADDITIONAL ONLINE RESOURCES

- Converting a Fraction to a Decimal - Part 2
- Convert a Fraction to a Decimal (repeating)
- Compare Fractions and Decimals using Inequality Symbols
- Determine the Area of a Circle
- Determine the Circumference of a Circle

Section 5.3 Exercises

Practice Makes Perfect

Convert Fractions to Decimals

In the following exercises, convert each fraction to a decimal.

201.

$\frac{2}{5}$

202.

$\frac{4}{5}$

203.

$-\frac{3}{8}$

204.

$-\frac{5}{8}$

205.

$\frac{17}{20}$

206.

$\frac{13}{20}$

207.

$\frac{11}{4}$

208.

$\frac{17}{4}$

209.

$-\frac{310}{25}$

210.

$-\frac{284}{25}$

211.

$\frac{5}{9}$

212.

$\frac{2}{9}$

213.

$\frac{15}{11}$

214.

$\frac{18}{11}$

215.

$\frac{15}{111}$

216.

$\frac{25}{111}$

In the following exercises, simplify the expression.

217.

$\frac{1}{2} + 6.5$

218.

$\frac{1}{4} + 10.75$

219.

$2.4 + \frac{5}{8}$

220.

$3.9 + \frac{9}{20}$

221.

$9.73 + \frac{17}{20}$

222.

$6.29 + \frac{21}{40}$

Order Decimals and Fractions

In the following exercises, order each pair of numbers, using $<$ or $>$.

223.

$\frac{1}{8}$ __ 0.8

224.

$\frac{1}{4}$ __ 0.4

225.

$\frac{2}{5}$ __0.25

226.

$\frac{3}{5}$ __ 0.35

227.

0.725 __ $\frac{3}{4}$

228.

0.92 __ $\frac{7}{8}$

229.

0.66 __ $\frac{2}{3}$

230.

0.83 __ $\frac{5}{6}$

231.

-0.75 __ $-\frac{4}{5}$

232.

-0.44 __ $-\frac{9}{20}$

233.

$-\frac{3}{4}$ __ -0.925

234.

$-\frac{2}{3}$ __ -0.632

In the following exercises, write each set of numbers in order from least to greatest.

235.

$\frac{3}{5}, \frac{9}{16}, 0.55$

236.

$\frac{3}{8}, \frac{7}{20}, 0.36$

237.

$0.702, \frac{13}{20}, \frac{5}{8}$

238.

$0.15, \frac{3}{16}, \frac{1}{5}$

239.

$-0.3, -\frac{1}{3}, -\frac{7}{20}$

240.

$-0.2, -\frac{3}{20}, -\frac{1}{6}$

241.

$-\frac{3}{4}, -\frac{7}{9}, -0.7$

242.

$-\frac{8}{9}, -\frac{4}{5}, -0.9$

Simplify Expressions Using the Order of Operations

In the following exercises, simplify.

243.

$10(25.1 - 43.8)$

244.

$30(18.1 - 32.5)$

245.

$62(9.75 - 4.99)$

246.

$42(8.45 - 5.97)$

247.

$\frac{3}{4}(12.4 - 4.2)$

248.

$\frac{4}{5}(8.6 + 3.9)$

249.

$\frac{5}{12}(30.58 + 17.9)$

250.

$\frac{9}{16}(21.96 - 9.8)$

251.

$10 \div 0.1 + (1.8)4 - (0.3)^2$

252.

$5 \div 0.5 + (3.9)6 - (0.7)^2$

253.

$(37.1 + 52.7) \div (12.5 \div 62.5)$

254.

$(11.4 + 16.2) \div (18 \div 60)$

255.

$\left(\frac{1}{5}\right)^2 + (1.4)(6.5)$

256.

$\left(\frac{1}{2}\right)^2 + (2.1)(8.3)$

257.

$-\frac{9}{10} \cdot \frac{8}{15} + 0.25$

258.

$-\frac{3}{8} \cdot \frac{14}{15} + 0.72$

Mixed Practice

In the following exercises, simplify. Give the answer as a decimal.

259.

$3\frac{1}{4} - 6.5$

260.

$5\frac{2}{5} - 8.75$

261.

$10.86 \div \frac{2}{3}$

262.

$5.79 \div \frac{3}{4}$

263.

$\frac{7}{8}(103.48) + 1\frac{1}{2}(361)$

264.

$\frac{5}{16}(117.6) + 2\frac{1}{3}(699)$

265.

$3.6\left(\frac{9}{8} - 2.72\right)$

266.

$5.1\left(\frac{12}{5} - 3.91\right)$

Find the Circumference and Area of Circles

In the following exercises, approximate the ⓐ circumference and ⓑ area of each circle. If measurements are given in fractions, leave answers in fraction form.

267.

radius $= 5$ in.

268.

radius $= 20$ in.

269.

radius $= 9$ ft.

270.

radius $= 4$ ft.

271.

radius $= 46$ cm

272.

radius $= 38$ cm

273.

radius $= 18.6$ m

274.

radius $= 57.3$ m

275.

radius $= \frac{7}{10}$ mile

276.

radius $= \frac{7}{11}$ mile

277.

radius $= \frac{3}{8}$ yard

278.

radius $= \frac{5}{12}$ yard

279.

$\text{diameter} = \frac{5}{6}\text{ m}$

280.

$\text{diameter} = \frac{3}{4}\text{ m}$

Everyday Math

281.

Kelly wants to buy a pair of boots that are on sale for $\frac{2}{3}$ of the original price. The original price of the boots is $\$84.99$. What is the sale price of the shoes?

282.

An architect is planning to put a circular mosaic in the entry of a new building. The mosaic will be in the shape of a circle with radius of 6 feet. How many square feet of tile will be needed for the mosaic? (Round your answer up to the next whole number.)

Writing Exercises

283.

Is it easier for you to convert a decimal to a fraction or a fraction to a decimal? Explain.

284.

Describe a situation in your life in which you might need to find the area or circumference of a circle.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
convert fractions to decimals.			
order decimals and fractions.			
simplify expressions using the order of operations.			
find the circumference and area of circles.			

ⓑ What does this checklist tell you about your mastery of this section? What steps will you take to improve?

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CHAPTER OVERVIEW

5: Proportions and Percents

The sections in this module will help you to learn about proportions and percentage. Your goal is to demonstrate mastery on each of the following objectives.

- **Objective 5.1 - Applications of Ratios and Rates:** Apply the concept of ratio and rate to solve application problems, with the aid of a calculator.
- **Objective 5.2 - Applications of Percents:** Apply the concept of percents to solve application problems, with the aid of a calculator.
- **Objective 5.3 - Applications of Proportions:** Apply the concept of proportions to solve application problems, with the aid of a calculator.

5.1: Ratios and Rate

5.2: Understand Percent

5.3: Solve General Applications of Percent

5.4: Solve Sales Tax, Commission, and Discount Applications

5.5: Solve Proportions and their Applications

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5.1: Ratios and Rate

Learning Objectives

By the end of this section, you will be able to:

- Write a ratio as a fraction
- Write a rate as a fraction
- Find unit rates
- Find unit price
- Translate phrases to expressions with fractions

Be Prepared 5.16

Before you get started, take this readiness quiz.

Simplify: $\frac{16}{24}$.
If you missed this problem, review Example 4.19.

Be Prepared 5.17

Divide: $2.76 \div 11.5$.
If you missed this problem, review Example 5.19.

Be Prepared 5.18

Simplify: $\frac{1\frac{1}{2}}{2\frac{3}{4}}$.
If you missed this problem, review Example 4.43.

Write a Ratio as a Fraction

When you apply for a mortgage, the loan officer will compare your total debt to your total income to decide if you qualify for the loan. This comparison is called the debt-to-income ratio. A ratio compares two quantities that are measured with the same unit. If we compare a and b, the ratio is written as a to b, $\frac{a}{b}$, or $a{:}b$.

Ratios

A ratio compares two numbers or two quantities that are measured with the same unit. The ratio of a to b is written a to b, $\frac{a}{b}$, or $a{:}b$.

In this section, we will use the fraction notation. When a ratio is written in fraction form, the fraction should be simplified. If it is an improper fraction, we do not change it to a mixed number. Because a ratio compares two quantities, we would leave a ratio as $\frac{4}{1}$ instead of simplifying it to 4 so that we can see the two parts of the ratio.

Example 5.58

Write each ratio as a fraction: ⓐ 15 to 27 ⓑ 45 to 18.

Answer

ⓐ	
	15 to 27
Write as a fraction with the first number in the numerator and the second in the denominator.	$\frac{15}{27}$
Simplify the fraction.	$\frac{5}{9}$

ⓑ	
	45 to 18
Write as a fraction with the first number in the numerator and the second in the denominator.	$\frac{45}{18}$
Simplify.	$\frac{5}{2}$

We leave the ratio in ⓑ as an improper fraction.

Try It 5.115

Write each ratio as a fraction: ⓐ 21 to 56 ⓑ 48 to 32.

Try It 5.116

Write each ratio as a fraction: ⓐ 27 to 72 ⓑ 51 to 34.

Ratios Involving Decimals

We will often work with ratios of decimals, especially when we have ratios involving money. In these cases, we can eliminate the decimals by using the Equivalent Fractions Property to convert the ratio to a fraction with whole numbers in the numerator and denominator.

For example, consider the ratio 0.8 to 0.05. We can write it as a fraction with decimals and then multiply the numerator and denominator by 100 to eliminate the decimals.

$$\frac{0.8}{0.05}$$

$$\frac{(0.8)100}{(0.05)100}$$

$$\frac{80}{5}$$

Do you see a shortcut to find the equivalent fraction? Notice that $0.8 = \frac{8}{10}$ and $0.05 = \frac{5}{100}$. The least common denominator of $\frac{8}{10}$ and $\frac{5}{100}$ is 100. By multiplying the numerator and denominator of $\frac{0.8}{0.05}$ by 100, we 'moved' the decimal two places to the right to get the equivalent fraction with no decimals. Now that we understand the math behind the process, we can find the fraction with no decimals like this:

	$\frac{0.80}{0.05}$
"Move" the decimal 2 places.	$\frac{80}{5}$
Simplify.	$\frac{16}{1}$

You do not have to write out every step when you multiply the numerator and denominator by powers of ten. As long as you move both decimal places the same number of places, the ratio will remain the same.

Example 5.59

Write each ratio as a fraction of whole numbers:

1. ⓐ 4.8 to 11.2
2. ⓑ 2.7 to 0.54

Answer

ⓐ 4.8 to 11.2	

Write as a fraction.	$\frac{4.8}{11.2}$
Rewrite as an equivalent fraction without decimals, by moving both decimal points 1 place to the right.	$\frac{48}{112}$
Simplify.	$\frac{3}{7}$

So 4.8 to 11.2 is equivalent to $\frac{3}{7}$.

ⓑ The numerator has one decimal place and the denominator has 2. To clear both decimals we need to move the decimal 2 places to the right. 2.7 to 0.54	
Write as a fraction.	$\frac{2.7}{0.54}$
Move both decimals right two places.	$\frac{270}{54}$
Simplify.	$\frac{5}{1}$

So 2.7 to 0.54 is equivalent to $\frac{5}{1}$.

Try It 5.117

Write each ratio as a fraction: ⓐ 4.6 to 11.5 ⓑ 2.3 to 0.69.

Try It 5.118

Write each ratio as a fraction: ⓐ 3.4 to 15.3 ⓑ 3.4 to 0.68.

Some ratios compare two mixed numbers. Remember that to divide mixed numbers, you first rewrite them as improper fractions.

Example 5.60

Write the ratio of $1\frac{1}{4}$ to $2\frac{3}{8}$ as a fraction.

Answer

	$1\frac{1}{4}$ to $2\frac{3}{8}$
Write as a fraction.	$\frac{1\frac{1}{4}}{2\frac{3}{8}}$
Convert the numerator and denominator to improper fractions.	$\frac{\frac{5}{4}}{\frac{19}{8}}$
Rewrite as a division of fractions.	$\frac{5}{4} \div \frac{19}{8}$
Invert the divisor and multiply.	$\frac{5}{4} \cdot \frac{8}{19}$
Simplify.	$\frac{10}{19}$

Try It 5.119

Write each ratio as a fraction: $1\frac{3}{4}$ to $2\frac{5}{8}$.

Try It 5.120

Write each ratio as a fraction: $1\frac{1}{8}$ to $2\frac{3}{4}$.

Applications of Ratios

One real-world application of ratios that affects many people involves measuring cholesterol in blood. The ratio of total cholesterol to HDL cholesterol is one way doctors assess a person's overall health. A ratio of less than 5 to 1 is considered good.

Example 5.61

Hector's total cholesterol is 249 mg/dl and his HDL cholesterol is 39 mg/dl. ⓐ Find the ratio of his total cholesterol to his HDL cholesterol. ⓑ Assuming that a ratio less than 5 to 1 is considered good, what would you suggest to Hector?

Answer

ⓐ First, write the words that express the ratio. We want to know the ratio of Hector's total cholesterol to his HDL cholesterol.

Write as a fraction.	$\frac{\text{total cholesterol}}{\text{HDL cholesterol}}$
Substitute the values.	$\frac{249}{39}$
Simplify.	$\frac{83}{13}$

ⓑ Is Hector's cholesterol ratio ok? If we divide 83 by 13 we obtain approximately 6.4, so $\frac{83}{13} \approx \frac{6.4}{1}$. Hector's cholesterol ratio is high! Hector should either lower his total cholesterol or raise his HDL cholesterol.

Try It 5.121

Find the patient's ratio of total cholesterol to HDL cholesterol using the given information.

Total cholesterol is 185 mg/dL and HDL cholesterol is 40 mg/dL.

Try It 5.122

Find the patient's ratio of total cholesterol to HDL cholesterol using the given information.

Total cholesterol is 204 mg/dL and HDL cholesterol is 38 mg/dL.

Ratios of Two Measurements in Different Units

To find the ratio of two measurements, we must make sure the quantities have been measured with the same unit. If the measurements are not in the same units, we must first convert them to the same units.

We know that to simplify a fraction, we divide out common factors. Similarly in a ratio of measurements, we divide out the common unit.

Example 5.62

The Americans with Disabilities Act (ADA) Guidelines for wheel chair ramps require a maximum vertical rise of 1 inch for every 1 foot of horizontal run. What is the ratio of the rise to the run?

Answer

In a ratio, the measurements must be in the same units. We can change feet to inches, or inches to feet. It is usually easier to convert to the smaller unit, since this avoids introducing more fractions into the problem.

Write the words that express the ratio.

	Ratio of the rise to the run
Write the ratio as a fraction.	$\frac{\text{rise}}{\text{run}}$
Substitute in the given values.	$\frac{1 \text{ inch}}{1 \text{ foot}}$
Convert 1 foot to inches.	$\frac{1 \text{ inch}}{12 \text{ inches}}$

Simplify, dividing out common factors and units.	$\frac{1}{12}$

So the ratio of rise to run is 1 to 12. This means that the ramp should rise 1 inch for every 12 inches of horizontal run to comply with the guidelines.

Try It 5.123

Find the ratio of the first length to the second length: 32 inches to 1 foot.

Try It 5.124

Find the ratio of the first length to the second length: 1 foot to 54 inches.

Write a Rate as a Fraction

Frequently we want to compare two different types of measurements, such as miles to gallons. To make this comparison, we use a rate. Examples of rates are 120 miles in 2 hours, 160 words in 4 minutes, and $\$5$ dollars per 64 ounces.

Rate

A rate compares two quantities of different units. A rate is usually written as a fraction.

When writing a fraction as a rate, we put the first given amount with its units in the numerator and the second amount with its units in the denominator. When rates are simplified, the units remain in the numerator and denominator.

Example 5.63

Bob drove his car 525 miles in 9 hours. Write this rate as a fraction.

Answer

	525 miles in 9 hours
Write as a fraction, with 525 miles in the numerator and 9 hours in the denominator.	$\frac{525 \text{ miles}}{9 \text{ hours}}$
	$\frac{175 \text{ miles}}{3 \text{ hours}}$

So 525 miles in 9 hours is equivalent to $\frac{175 \text{ miles}}{3 \text{ hours}}$.

Try It 5.125

Write the rate as a fraction: 492 miles in 8 hours.

Try It 5.126

Write the rate as a fraction: 242 miles in 6 hours.

Find Unit Rates

In the last example, we calculated that Bob was driving at a rate of $\frac{175 \text{ miles}}{3 \text{ hours}}$. This tells us that every three hours, Bob will travel 175 miles. This is correct, but not very useful. We usually want the rate to reflect the number of miles in one hour. A rate that has a denominator of 1 unit is referred to as a unit rate.

Unit Rate

A unit rate is a rate with denominator of 1 unit.

Unit rates are very common in our lives. For example, when we say that we are driving at a speed of 68 miles per hour we mean that we travel 68 miles in 1 hour. We would write this rate as 68 miles/hour (read 68 miles per hour). The common abbreviation for this is 68 mph. Note that when no number is written before a unit, it is assumed to be 1.

So 68 miles/hour really means $68 \text{ miles}/1 \text{ hour}$.

Two rates we often use when driving can be written in different forms, as shown:

Example	Rate	Write	Abbreviate	Read
68 miles in 1 hour	$\frac{68 \text{ miles}}{1 \text{ hour}}$	68 miles/hour	68 mph	68 miles per hour
36 miles to 1 gallon	$\frac{36 \text{ miles}}{1 \text{ gallon}}$	36 miles/gallon	36 mpg	36 miles per gallon

Another example of unit rate that you may already know about is hourly pay rate. It is usually expressed as the amount of money earned for one hour of work. For example, if you are paid $12.50 for each hour you work, you could write that your hourly (unit) pay rate is $12.50/hour (read $12.50 per hour.)

To convert a rate to a unit rate, we divide the numerator by the denominator. This gives us a denominator of 1.

Example 5.64

Anita was paid $384 last week for working 32 hours. What is Anita's hourly pay rate?

Answer

Start with a rate of dollars to hours. Then divide.	$384 last week for 32 hours
Write as a rate.	$\frac{\$384}{32 \text{ hours}}$
Divide the numerator by the denominator.	$\frac{\$12}{1 \text{ hour}}$
Rewrite as a rate.	$12/hour

Anita's hourly pay rate is $12 per hour.

Try It 5.127

Find the unit rate: $630 for 35 hours.

Try It 5.128

Find the unit rate: $684 for 36 hours.

Example 5.65

Sven drives his car 455 miles, using 14 gallons of gasoline. How many miles per gallon does his car get?

Answer

Start with a rate of miles to gallons. Then divide.

	455 miles to 14 gallons of gas
Write as a rate.	$\frac{455 \text{ miles}}{14 \text{ gallons}}$
Divide 455 by 14 to get the unit rate.	$\frac{32.5 \text{ miles}}{1 \text{ gallon}}$

Sven's car gets 32.5 miles/gallon, or 32.5 mpg.

Try It 5.129

Find the unit rate: 423 miles to 18 gallons of gas.

Try It 5.130

Find the unit rate: 406 miles to 14.5 gallons of gas.

Find Unit Price

Sometimes we buy common household items 'in bulk', where several items are packaged together and sold for one price. To compare the prices of different sized packages, we need to find the unit price. To find the unit price, divide the total price by the number of items. A unit price is a unit rate for one item.

Unit price

A unit price is a unit rate that gives the price of one item.

Example 5.66

The grocery store charges $3.99 for a case of 24 bottles of water. What is the unit price?

Answer

What are we asked to find? We are asked to find the unit price, which is the price per bottle.

Write as a rate.	$\frac{\$3.99}{24 \text{ bottles}}$
Divide to find the unit price.	$\frac{\$0.16625}{1 \text{ bottle}}$
Round the result to the nearest penny.	$\frac{\$0.17}{1 \text{ bottle}}$

The unit price is approximately $0.17 per bottle. Each bottle costs about $0.17.

Try It 5.131

Find the unit price. Round your answer to the nearest cent if necessary.

24-pack of juice boxes for $6.99

Try It 5.132

Find the unit price. Round your answer to the nearest cent if necessary.

24-pack of bottles of ice tea for $12.72

Unit prices are very useful if you comparison shop. The *better buy* is the item with the lower unit price. Most grocery stores list the unit price of each item on the shelves.

Example 5.67

Paul is shopping for laundry detergent. At the grocery store, the liquid detergent is priced at $14.99 for 64 loads of laundry and the same brand of powder detergent is priced at $15.99 for 80 loads.

Which detergent has the lowest cost per load?

Answer

To compare the prices, we first find the unit price for each type of detergent.

	Liquid	Powder
Write as a rate.	$\frac{\$14.99}{64 \text{ loads}}$	$\frac{\$15.99}{80 \text{ loads}}$

Find the unit price.	$\frac{\$0.234\ldots}{1 \text{ load}}$	$\frac{\$0.199\ldots}{1 \text{ load}}$
Round to the nearest cent.	$\$0.23/\text{load}$ (23 cents per load.)	$\$0.20/\text{load}$ (20 cents per load)

Now we compare the unit prices. The unit price of the liquid detergent is about $\$0.23$ per load and the unit price of the powder detergent is about $\$0.20$ per load. The powder is the better buy.

Try It 5.133

Find each unit price and then determine the better buy. Round to the nearest cent if necessary.

Brand A Storage Bags, $\$4.59$ for 40 count, or Brand B Storage Bags, $\$3.99$ for 30 count

Try It 5.134

Find each unit price and then determine the better buy. Round to the nearest cent if necessary.

Brand C Chicken Noodle Soup, $\$1.89$ for 26 ounces, or Brand D Chicken Noodle Soup, $\$0.95$ for 10.75 ounces

Notice in Example 5.67 that we rounded the unit price to the nearest cent. Sometimes we may need to carry the division to one more place to see the difference between the unit prices.

Translate Phrases to Expressions with Fractions

Have you noticed that the examples in this section used the comparison words *ratio of*, *to*, *per*, *in*, *for*, *on*, and *from*? When you translate phrases that include these words, you should think either ratio or rate. If the units measure the same quantity (length, time, etc.), you have a ratio. If the units are different, you have a rate. In both cases, you write a fraction.

Example 5.68

Translate the word phrase into an algebraic expression:

1. ⓐ 427 miles per h hours
2. ⓑ x students to 3 teachers
3. ⓒ y dollars for 18 hours

Answer

ⓐ	
	427 miles per h hours
Write as a rate.	$\frac{427 \text{ miles}}{h \text{ hours}}$

ⓑ	
	x students to 3 teachers
Write as a rate.	$\frac{x \text{ students}}{3 \text{ teachers}}$

ⓒ	
	y dollars for 18 hours
Write as a rate.	$\frac{\$y}{18 \text{ hours}}$

Try It 5.135

Translate the word phrase into an algebraic expression.

ⓐ 689 miles per h hours ⓑ y parents to 22 students ⓒ d dollars for 9 minutes

Try It 5.136

Translate the word phrase into an algebraic expression.

ⓐ m miles per 9 hours ⓑ x students to 8 buses ⓒ y dollars for 40 hours

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ACCESS ADDITIONAL ONLINE RESOURCES

- Ratios
- Write Ratios as a Simplified Fractions Involving Decimals and Fractions
- Write a Ratio as a Simplified Fraction
- Rates and Unit Rates
- Unit Rate for Cell Phone Plan

Section 5.6 Exercises

Practice Makes Perfect

Write a Ratio as a Fraction

In the following exercises, write each ratio as a fraction.

403.

20 to 36

404.

20 to 32

405.

42 to 48

406.

45 to 54

407.

49 to 21

408.

56 to 16

409.

84 to 36

410.

6.4 to 0.8

411.

0.56 to 2.8

412.

1.26 to 4.2

413.

$1\frac{2}{3}$ to $2\frac{5}{6}$

414.

$1\frac{3}{4}$ to $2\frac{5}{8}$

415.

$4\frac{1}{6}$ to $3\frac{1}{3}$

416.

$5\frac{3}{5}$ to $3\frac{3}{5}$

417.

$18 to $63

418.

$16 to $72

419.

$1.21 to $0.44

420.

$1.38 to $0.69

421.

28 ounces to 84 ounces

422.

32 ounces to 128 ounces

423.

12 feet to 46 feet

424.

15 feet to 57 feet

425.

246 milligrams to 45 milligrams

426.

304 milligrams to 48 milligrams

427.

total cholesterol of 175 to HDL cholesterol of 45

428.

total cholesterol of 215 to HDL cholesterol of 55

429.

27 inches to 1 foot

430.

28 inches to 1 foot

Write a Rate as a Fraction

In the following exercises, write each rate as a fraction.

431.

140 calories per 12 ounces

432.

180 calories per 16 ounces

433.

8.2 pounds per 3 square inches

434.

9.5 pounds per 4 square inches

435.

488 miles in 7 hours

436.

527 miles in 9 hours

437.

$\$595$ for 40 hours

438.

$\$798$ for 40 hours

Find Unit Rates

In the following exercises, find the unit rate. Round to two decimal places, if necessary.

439.

140 calories per 12 ounces

440.

180 calories per 16 ounces

441.

8.2 pounds per 3 square inches

442.

9.5 pounds per 4 square inches

443.

488 miles in 7 hours

444.

527 miles in 9 hours

445.

$\$595$ for 40 hours

446.

$\$798$ for 40 hours

447.

576 miles on 18 gallons of gas

448.

435 miles on 15 gallons of gas

449.

43 pounds in 16 weeks

450.

57 pounds in 24 weeks

451.

46 beats in 0.5 minute

452.

54 beats in 0.5 minute

453.

The bindery at a printing plant assembles $96{,}000$ magazines in 12 hours. How many magazines are assembled in one hour?

454.

The pressroom at a printing plant prints $540{,}000$ sections in 12 hours. How many sections are printed per hour?

Find Unit Price

In the following exercises, find the unit price. Round to the nearest cent.

455.

Soap bars at 8 for $\$8.69$

456.

Soap bars at 4 for $\$3.39$

457.

Women's sports socks at 6 pairs for $\$7.99$

458.

Men's dress socks at 3 pairs for $\$8.49$

459.

Snack packs of cookies at 12 for $\$5.79$

460.

Granola bars at 5 for $\$3.69$

461.

CD-RW discs at 25 for $\$14.99$

462.

CDs at 50 for $\$4.49$

463.

The grocery store has a special on macaroni and cheese. The price is $\$3.87$ for 3 boxes. How much does each box cost?

464.

The pet store has a special on cat food. The price is $\$4.32$ for 12 cans. How much does each can cost?

In the following exercises, find each unit price and then identify the better buy. Round to three decimal places.

465.

Mouthwash, 50.7-ounce size for $\$6.99$ or 33.8-ounce size for $\$4.79$

466.

Toothpaste, 6 ounce size for $\$3.19$ or 7.8-ounce size for $\$5.19$

467.

Breakfast cereal, 18 ounces for $\$3.99$ or 14 ounces for $\$3.29$

468.

Breakfast Cereal, 10.7 ounces for $\$2.69$ or 14.8 ounces for $\$3.69$

469.

Ketchup, 40-ounce regular bottle for $\$2.99$ or 64-ounce squeeze bottle for $\$4.39$

470.

Mayonnaise 15-ounce regular bottle for $\$3.49$ or 22-ounce squeeze bottle for $\$4.99$

471.

Cheese $\$6.49$ for 1 lb. block or $\$3.39$ for $\frac{1}{2}$ lb. block

472.

Candy $\$10.99$ for a 1 lb. bag or $\$2.89$ for $\frac{1}{4}$ lb. of loose candy

Translate Phrases to Expressions with Fractions

In the following exercises, translate the English phrase into an algebraic expression.

473.

793 miles per p hours

474.

78 feet per r seconds

475.

$\$3$ for 0.5 lbs.

476.

j beats in 0.5 minutes

477.

105 calories in x ounces

478.

400 minutes for m dollars

479.

the ratio of y and $5x$

480.

the ratio of $12x$ and y

Everyday Math

481.

One elementary school in Ohio has 684 students and 45 teachers. Write the student-to-teacher ratio as a unit rate.

482.

The average American produces about $1{,}600$ pounds of paper trash per year (365 days). How many pounds of paper trash does the average American produce each day? (Round to the nearest tenth of a pound.)

483.

A popular fast food burger weighs 7.5 ounces and contains 540 calories, 29 grams of fat, 43 grams of carbohydrates, and 25 grams of protein. Find the unit rate of ⓐ calories per ounce ⓑ grams of fat per ounce ⓒ grams of carbohydrates per ounce ⓓ grams of protein per ounce. Round to two decimal places.

484.

A 16-ounce chocolate mocha coffee with whipped cream contains 470 calories, 18 grams of fat, 63 grams of carbohydrates, and 15 grams of protein. Find the unit rate of ⓐ calories per ounce ⓑ grams of fat per ounce ⓒ grams of carbohydrates per ounce ⓓ grams of protein per ounce.

Writing Exercises

485.

Would you prefer the ratio of your income to your friend's income to be $3/1$ or $1/3$? Explain your reasoning.

486.

The parking lot at the airport charges $\$0.75$ for every 15 minutes. ⓐ How much does it cost to park for 1 hour? ⓑ Explain how you got your answer to part ⓐ. Was your reasoning based on the unit cost or did you use another method?

487.

Kathryn ate a 4-ounce cup of frozen yogurt and then went for a swim. The frozen yogurt had 115 calories. Swimming burns 422 calories per hour. For how many minutes should Kathryn swim to burn off the calories in the frozen yogurt? Explain your reasoning.

488.

Mollie had a 16-ounce cappuccino at her neighborhood coffee shop. The cappuccino had 110 calories. If Mollie walks for one hour, she burns 246 calories. For how many minutes must Mollie walk to burn off the calories in the cappuccino? Explain your reasoning.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
write a ratio as a fraction.			
write a rate as a fraction.			
find unit rates.			
find unit price.			
translate phrases to expressions with fractions.			

ⓑ After reviewing this checklist, what will you do to become confident for all objectives?

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5.2: Understand Percent

Learning Objectives

By the end of this section, you will be able to:

- Use the definition of percent
- Convert percents to fractions and decimals
- Convert decimals and fractions to percents

Be Prepared 6.1

Before you get started, take this readiness quiz.

Translate “the ratio of 33 to 5” into an algebraic expression.
If you missed this problem, review Table 2.7.

Be Prepared 6.2

Write $\frac{3}{5}$ as a decimal.
If you missed this problem, review Example 5.28.

Be Prepared 6.3

Write 0.62 as a fraction.
If you missed this problem, review Example 5.4.

Use the Definition of Percent

How many cents are in one dollar? There are 100 cents in a dollar. How many years are in a century? There are 100 years in a century. Does this give you a clue about what the word “percent” means? It is really two words, “per cent,” and means per one hundred. A percent is a ratio whose denominator is 100. We use the percent symbol $\%$, to show percent.

Percent

A percent is a ratio whose denominator is 100.

According to data from the American Association of Community Colleges Math input error

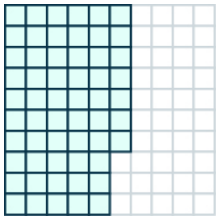


Figure 6.2 Among every 100 community college students, 57 are female.

Similarly, 25% means a ratio of $\frac{25}{100}$, 3% means a ratio of $\frac{3}{100}$ and 100% means a ratio of $\frac{100}{100}$. In words, "one hundred percent" means the total 100% is $\frac{100}{100}$, and since $\frac{100}{100} = 1$, we see that 100% means 1 whole.

Example 6.1

According to the Public Policy Institute of California (2010), 44% of parents of public school children would like their youngest child to earn a graduate degree. Write this percent as a ratio.

Answer

The amount we want to convert is 44%.	44%

Write the percent as a ratio. Remember that *percent* means per 100.	$\frac{44}{100}$

Try It 6.1

Write the percent as a ratio.

According to a survey, 89% of college students have a smartphone.

Try It 6.2

Write the percent as a ratio.

A study found that 72% of U.S. teens send text messages regularly.

Example 6.2

In 2007, according to a U.S. Department of Education report, 21 out of every 100 first-time freshmen college students at 4-year public institutions took at least one remedial course. Write this as a ratio and then as a percent.

Answer

The amount we want to convert is 21 out of 100.	21 out of 100
Write as a ratio.	$\frac{21}{100}$
Convert the 21 per 100 to percent.	21%

Try It 6.3

Write as a ratio and then as a percent: The American Association of Community Colleges reported that 62 out of 100 full-time community college students balance their studies with full-time or part time employment.

Try It 6.4

Write as a ratio and then as a percent: In response to a student survey, 41 out of 100 Santa Ana College students expressed a goal of earning an Associate's degree or transferring to a four-year college.

Convert Percents to Fractions and Decimals

Since percents are ratios, they can easily be expressed as fractions. Remember that percent means per 100, so the denominator of the fraction is 100.

How To

Convert a percent to a fraction.

1. Step 1. Write the percent as a ratio with the denominator 100.
2. Step 2. Simplify the fraction if possible.

Example 6.3

Convert each percent to a fraction:

1. ⓐ 36%
2. ⓑ 125%

Answer

ⓐ	
	36%
Write as a ratio with denominator 100.	$\frac{36}{100}$
Simplify.	$\frac{9}{25}$

ⓑ	
	125%
Write as a ratio with denominator 100.	$\frac{125}{100}$
Simplify.	$\frac{5}{4}$

Try It 6.5

Convert each percent to a fraction:

1. ⓐ 48%
2. ⓑ 110%

Try It 6.6

Convert each percent to a fraction:

1. ⓐ 64%
2. ⓑ 150%

The previous example shows that a percent can be greater than 1. We saw that 125% means $\frac{125}{100}$, or $\frac{5}{4}$. These are improper fractions, and their values are greater than one.

Example 6.4

Convert each percent to a fraction:

1. ⓐ 24.5%
2. ⓑ $33\frac{1}{3}\%$

Answer

ⓐ	
	24.5%
Write as a ratio with denominator 100.	$\frac{24.5}{100}$
Clear the decimal by multiplying numerator and denominator by 10.	$\frac{24.5(10)}{100(10)}$
Multiply.	$\frac{245}{1000}$
Rewrite showing common factors.	$\frac{5 \cdot 49}{5 \cdot 200}$
Simplify.	$\frac{49}{200}$

ⓑ	
	$33\frac{1}{3}\%$
Write as a ratio with denominator 100.	$\frac{33\frac{1}{3}}{100}$

Write the numerator as an improper fraction.	$\frac{\frac{100}{3}}{100}$
Rewrite as fraction division, replacing 100 with $\frac{100}{1}$.	$\frac{100}{3} \div \frac{100}{1}$
Multiply by the reciprocal.	$\frac{100}{3} \cdot \frac{1}{100}$
Simplify.	$\frac{1}{3}$

Try It 6.7

Convert each percent to a fraction:

1. ⓐ 64.4%
2. ⓑ $66\frac{2}{3}\%$

Try It 6.8

Convert each percent to a fraction:

1. ⓐ 42.5%
2. ⓑ $8\frac{3}{4}\%$

In Decimals, we learned how to convert fractions to decimals. To convert a percent to a decimal, we first convert it to a fraction and then change the fraction to a decimal.

How To

Convert a percent to a decimal.

1. Step 1. Write the percent as a ratio with the denominator 100.
2. Step 2. Convert the fraction to a decimal by dividing the numerator by the denominator.

Example 6.5

Convert each percent to a decimal:

1. ⓐ 6%
2. ⓑ 78%

Answer

Because we want to change to a decimal, we will leave the fractions with denominator 100 instead of removing common factors.

ⓐ	
	6%
Write as a ratio with denominator 100.	$\frac{6}{100}$
Change the fraction to a decimal by dividing the numerator by the denominator.	0.06

ⓑ	
	78%
Write as a ratio with denominator 100.	$\frac{78}{100}$
Change the fraction to a decimal by dividing the numerator by the denominator.	0.78

Try It 6.9

Convert each percent to a decimal:

1. ⓐ 9%
2. ⓑ 87%

Try It 6.10

Convert each percent to a decimal:

1. ⓐ 3%
2. ⓑ 91%

Example 6.6

Convert each percent to a decimal:

1. ⓐ 135%
2. ⓑ 12.5%

Answer

ⓐ	
	135%
Write as a ratio with denominator 100.	$\frac{135}{100}$
Change the fraction to a decimal by dividing the numerator by the denominator.	1.35

ⓑ	
	12.5%
Write as a ratio with denominator 100.	$\frac{12.5}{100}$
Change the fraction to a decimal by dividing the numerator by the denominator.	0.125

Try It 6.11

Convert each percent to a decimal:

1. ⓐ 115%
2. ⓑ 23.5%

Try It 6.12

Convert each percent to a decimal:

1. ⓐ 123%
2. ⓑ 16.8%

Let's summarize the results from the previous examples in Table 6.1, and look for a pattern we could use to quickly convert a percent number to a decimal number.

Percent	Decimal

Percent	Decimal
6%	0.06
78%	0.78
135%	1.35
12.5%	0.125

Table 6.1

Do you see the pattern?

To convert a percent number to a decimal number, we move the decimal point two places to the left and remove the % sign. (Sometimes the decimal point does not appear in the percent number, but just like we can think of the integer 6 as 6.0, we can think of 6% as 6.0%.) Notice that we may need to add zeros in front of the number when moving the decimal to the left.

Figure 6.3 uses the percents in Table 6.1 and shows visually how to convert them to decimals by moving the decimal point two places to the left.

Percent	Decimal
006.%	0.06
078.%	0.78
135.%	1.35
012.5%	0.125

Figure 6.3

Example 6.7

Among a group of business leaders, 77% believe that poor math and science education in the U.S. will lead to higher unemployment rates.

Convert the percent to: ⓐ a fraction ⓑ a decimal

Answer

ⓐ	
	77%
Write as a ratio with denominator 100.	$\frac{77}{100}$

ⓑ	
	$\frac{77}{100}$
Change the fraction to a decimal by dividing the numerator by the denominator.	0.77

Try It 6.13

Convert the percent to: ⓐ a fraction and ⓑ a decimal

Twitter's share of web traffic jumped 24% when one celebrity tweeted live on air.

Try It 6.14

Convert the percent to: ⓐ a fraction and ⓑ a decimal

The U.S. Census estimated that in 2013, 44% of the population of Boston age 25 or older have a bachelor's or higher degrees.

Example 6.8

There are four suits of cards in a deck of cards—hearts, diamonds, clubs, and spades. The probability of randomly choosing a heart from a shuffled deck of cards is 25%. Convert the percent to:

1. ⓐ a fraction
2. ⓑ a decimal



Figure 6.4 (credit: Riles32807, Wikimedia Commons)

Answer

ⓐ	
	25%
Write as a ratio with denominator 100.	$\frac{25}{100}$
Simplify.	$\frac{1}{4}$

ⓑ	$\frac{1}{4}$
Change the fraction to a decimal by dividing the numerator by the denominator.	0.25

Try It 6.15

Convert the percent to: ⓐ a fraction, and ⓑ a decimal

The probability that it will rain Monday is 30%.

Try It 6.16

Convert the percent to: ⓐ a fraction, and ⓑ a decimal

The probability of getting heads three times when tossing a coin three times is 12.5%.

Convert Decimals and Fractions to Percents

To convert a decimal to a percent, remember that percent means per hundred. If we change the decimal to a fraction whose denominator is 100, it is easy to change that fraction to a percent.

How To

Convert a decimal to a percent.

1. Step 1. Write the decimal as a fraction.
2. Step 2. If the denominator of the fraction is not 100, rewrite it as an equivalent fraction with denominator 100.

3. Step 3. Write this ratio as a percent.

Example 6.9

Convert each decimal to a percent: ⓐ 0.05 ⓑ 0.83

Answer

ⓐ	
	0.05
Write as a fraction. The denominator is 100.	$\frac{5}{100}$
Write this ratio as a percent.	5%

ⓑ	
	0.83
The denominator is 100.	$\frac{83}{100}$
Write this ratio as a percent.	83%

Try It 6.17

Convert each decimal to a percent: ⓐ 0.01 ⓑ 0.17.

Try It 6.18

Convert each decimal to a percent: ⓐ 0.04 ⓑ 0.41

To convert a mixed number to a percent, we first write it as an improper fraction.

Example 6.10

Convert each decimal to a percent: ⓐ 1.05 ⓑ 0.075

Answer

ⓐ	
	0.05
Write as a fraction.	$1\frac{5}{100}$
Write as an improper fraction. The denominator is 100.	$\frac{105}{100}$
Write this ratio as a percent.	105%

Notice that since $1.05 > 1$, the result is more than 100%.

ⓑ	
	0.075
Write as a fraction. The denominator is 1,000.	$\frac{75}{1{,}000}$
Divide the numerator and denominator by 10, so that the denominator is 100.	$\frac{7.5}{100}$
Write this ratio as a percent.	7.5%

Try It 6.19

Convert each decimal to a percent: ⓐ 1.75 ⓑ 0.0825

Try It 6.20

Convert each decimal to a percent: ⓐ 2.25 ⓑ 0.0925

Let's summarize the results from the previous examples in Table 6.2 so we can look for a pattern.

Decimal	Percent
0.05	5%
0.83	83%
1.05	105%
0.075	7.5%

Table 6.2

Do you see the pattern? To convert a decimal to a percent, we move the decimal point two places to the right and then add the percent sign.

Figure 6.5 uses the decimal numbers in Table 6.2 and shows visually to convert them to percents by moving the decimal point two places to the right and then writing the % sign.

Percent	Decimal
006.%	0.06
078.%	0.78
135.%	1.35
012.5%	0.125

Figure 6.5

In Decimals, we learned how to convert fractions to decimals. Now we also know how to change decimals to percents. So to convert a fraction to a percent, we first change it to a decimal and then convert that decimal to a percent.

How To

Convert a fraction to a percent.

1. Step 1. Convert the fraction to a decimal.
2. Step 2. Convert the decimal to a percent.

Example 6.11

Convert each fraction or mixed number to a percent: ⓐ $\frac{3}{4}$ ⓑ $\frac{11}{8}$ ⓒ $2\frac{1}{5}$

Answer

To convert a fraction to a decimal, divide the numerator by the denominator.

ⓐ	
Change to a decimal.	$\frac{3}{4}$
Write as a percent by moving the decimal two places.	0.75
	75%

ⓑ	
Change to a decimal.	$\frac{11}{8}$
Write as a percent by moving the decimal two places.	1.375
	137.5%

ⓒ	
Write as an improper fraction.	$2\frac{1}{5}$
Change to a decimal.	$\frac{11}{5}$
Write as a percent.	2.20
	220%

Notice that we needed to add zeros at the end of the number when moving the decimal two places to the right.

Try It 6.21

Convert each fraction or mixed number to a percent: ⓐ $\frac{5}{8}$ ⓑ $\frac{11}{4}$ ⓒ $3\frac{2}{5}$

Try It 6.22

Convert each fraction or mixed number to a percent: ⓐ $\frac{7}{8}$ ⓑ $\frac{9}{4}$ ⓒ $1\frac{3}{5}$

Sometimes when changing a fraction to a decimal, the division continues for many decimal places and we will round off the quotient. The number of decimal places we round to will depend on the situation. If the decimal involves money, we round to the hundredths place. For most other cases in this book we will round the number to the nearest thousandth, so the percent will be rounded to the nearest tenth.

Example 6.12

Convert $\frac{5}{7}$ to a percent.

Answer

To change a fraction to a decimal, we divide the numerator by the denominator.

	$\frac{5}{7}$
Change to a decimal—rounding to the nearest thousandth.	0.714
Write as a percent.	71.4%

Try It 6.23

Convert the fraction to a percent: $\frac{3}{7}$

Try It 6.24

Convert the fraction to a percent: $\frac{4}{7}$

When we first looked at fractions and decimals, we saw that some fractions converted to a repeating decimal. For example, when we converted the fraction $\frac{4}{3}$ to a decimal, we wrote the answer as $1.\overline{3}$. We will use this same notation, as well as fraction notation, when we convert fractions to percents in the next example.

Example 6.13

An article in a medical journal claimed that approximately $\frac{1}{3}$ of American adults are obese. Convert the fraction $\frac{1}{3}$ to a percent.

Answer

	$\frac{1}{3}$
Change to a decimal.	$3\overline{)1.00}$ $\quad 0.33\ldots$, 9, 10, 9, 1
Write as a repeating decimal.	$0.333\ldots$
Write as a percent.	$33\frac{1}{3}\%$

We could also write the percent as $33.\overline{3}\%$.

Try It 6.25

Convert the fraction to a percent:

According to the U.S. Census Bureau, about $\frac{1}{9}$ of United States housing units have just 1 bedroom.

Try It 6.26

Convert the fraction to a percent:

According to the U.S. Census Bureau, about $\frac{1}{6}$ of Colorado residents speak a language other than English at home.

Section 6.1 Exercises

Practice Makes Perfect

Use the Definition of Percents

In the following exercises, write each percent as a ratio.

1.

In 2014, the unemployment rate for those with only a high school degree was 6.0%.

2.

In 2015, among the unemployed, 29% were long-term unemployed.

3.

The unemployment rate for those with Bachelor's degrees was 3.2% in 2014.

4.

The unemployment rate in Michigan in 2014 was 7.3%.

In the following exercises, write as

1. ⓐ a ratio and
2. ⓑ a percent

5.

57 out of 100 nursing candidates received their degree at a community college.

6.

80 out of 100 firefighters and law enforcement officers were educated at a community college.

7.

42 out of 100 first-time freshmen students attend a community college.

8.

71 out of 100 full-time community college faculty have a master's degree.

Convert Percents to Fractions and Decimals

In the following exercises, convert each percent to a fraction and simplify all fractions.

9.

4%

10.

8%

11.

17%

12.

19%

13.

52%

14.

78%

15.

125%

16.

135%

17.

37.5%

18.

42.5%

19.

18.4%

20.

46.4%

21.

$9\frac{1}{2}\%$

22.

$8\frac{1}{2}\%$

23.

$5\frac{1}{3}\%$

24.

$6\frac{2}{3}\%$

In the following exercises, convert each percent to a decimal.

25.

5%

26.

9%

27.

1%

28.

2%

29.

63%

30.

71%

31.

40%

32.

50%

33.

115%

34.

125%

35.

150%

36.

250%

37.

21.4%

38.

39.3%

39.

7.8%

40.

6.4%

In the following exercises, convert each percent to

1. ⓐ a simplified fraction and
2. ⓑ a decimal

41.

In 2010, 1.5% of home sales had owner financing. (*Source:* Bloomberg Businessweek, 5/23–29/2011)

42.

In 2000, 4.2% of the United States population was of Asian descent. (*Source:* www.census.gov)

43.

According to government data, in 2013 the number of cell phones in India was 70.23% of the population.

44.

According to the U.S. Census Bureau, among Americans age 25 or older who had doctorate degrees in $2014, 37.1\%$ are women.

45.

A couple plans to have two children. The probability they will have two girls is 25%.

46.

Javier will choose one digit at random from 0 through 9. The probability he will choose 3 is 10%.

47.

According to the local weather report, the probability of thunderstorms in New York City on July 15 is 60%.

48.

A club sells 50 tickets to a raffle. Osbaldo bought one ticket. The probability he will win the raffle is 2%.

Convert Decimals and Fractions to Percents

In the following exercises, convert each decimal to a percent.

49.

0.01

50.

0.03

51.

0.18

52.

0.15

53.

1.35

54.

1.56

55.

3

56.

4

57.

0.009

58.

0.008

59.

0.0875

60.

0.0625

61.

1.5

62.

2.2

63.

2.254

64.

2.317

In the following exercises, convert each fraction to a percent.

65.

$\frac{1}{4}$

66.

$\frac{1}{5}$

67.

$\frac{3}{8}$

68.

$\frac{5}{8}$

69.

$\frac{7}{4}$

70.

$\frac{9}{8}$

71.

$6\frac{4}{5}$

72.

$5\frac{1}{4}$

73.

$\frac{5}{12}$

74.

$\frac{11}{12}$

75.

$2\frac{2}{3}$

76.

$1\frac{2}{3}$

77.

$\frac{3}{7}$

78.

$\frac{6}{7}$

79.

$\frac{5}{9}$

80.

$\frac{4}{9}$

In the following exercises, convert each fraction to a percent.

81.

$\frac{1}{4}$ of washing machines needed repair.

82.

$\frac{1}{5}$ of dishwashers needed repair.

In the following exercises, convert each fraction to a percent.

83.

According to the National Center for Health Statistics, in 2012, $\frac{7}{20}$ of American adults were obese.

84.

The U.S. Census Bureau estimated that in 2013, 85% of Americans lived in the same house as they did 1 year before.

In the following exercises, complete the table.

85.

Fraction	Decimal	Percent
$\frac{1}{2}$		
	0.45	
		18%
$\frac{1}{3}$		
	0.008	
2		

Table 6.3

86.

Fraction	Decimal	Percent
$\frac{1}{4}$		
	0.65	
		22%
$\frac{2}{3}$		
	0.004	
3		

Table 6.4

Everyday Math

87.

Sales tax Felipa says she has an easy way to estimate the sales tax when she makes a purchase. The sales tax in her city is 9.05%. She knows this is a little less than 10%.

1. ⓐ Convert 10% to a fraction.
2. ⓑ Use your answer from ⓐ to estimate the sales tax Felipa would pay on a $\$95$ dress.

88.

Savings Ryan has 25% of each paycheck automatically deposited in his savings account.

1. ⓐ Write 25% as a fraction.
2. ⓑ Use your answer from ⓐ to find the amount that goes to savings from Ryan's $\$2{,}400$ paycheck.

Amelio is shopping for textbooks online. He found three sellers that are offering a book he needs for the same price, including shipping. To decide which seller to buy from he is comparing their customer satisfaction ratings. The ratings are given in the chart.

Use the chart to answer the following questions

Seller	Rating
A	$4/5$
B	$3.5/4$
C	85%

89.

Write seller C's rating as a fraction and a decimal.

90.

Write seller B's rating as a percent and a decimal.

91.

Write seller A's rating as a percent and a decimal.

92.

Which seller should Amelio buy from and why?

Writing Exercises

93.

Convert $25\%, 50\%, 75\%,$ and 100% to fractions. Do you notice a pattern? Explain what the pattern is.

94.

Convert $\frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \frac{4}{10}, \frac{5}{10}, \frac{6}{10}, \frac{7}{10}, \frac{8}{10}$, and $\frac{9}{10}$ to percents. Do you notice a pattern? Explain what the pattern is.

95.

When the Szetos sold their home, the selling price was 500% of what they had paid for the house 30 years ago. Explain what 500% means in this context.

96.

According to cnn.com, cell phone use in 2008 was 600% of what it had been in 2001. Explain what 600% means in this context.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
use the definition of percent.			
convert percents to fractions and decimals.			
convert decimals and fractions to percents.			

ⓑ If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them. What did you do to become confident of your ability to do these things? Be specific.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is important to make sure you have a strong foundation before you move on. Whom can you ask for help? Your fellow classmates and instructor are good resources. Is there a place on campus where math tutors are available? Can your study skills be improved?

…no—I don't get it! This is a warning sign and you must not ignore it. You should get help right away or you will quickly be overwhelmed. See your instructor as soon as you can to discuss your situation. Together you can come up with a plan to get you the help you need.

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5.3: Solve General Applications of Percent

Learning Objectives

By the end of this section, you will be able to:

- Translate and solve basic percent equations
- Solve applications of percent
- Find percent increase and percent decrease

Be Prepared 6.4

Before you get started, take this readiness quiz.

Translate and solve: $\frac{3}{4}$ of x is 24.
If you missed this problem, review Example 4.105.

Be Prepared 6.5

Simplify: $(4.5)\,(2.38)$.
If you missed this problem, review Example 5.15.

Be Prepared 6.6

Solve: $3.5 = 0.7n$.
If you missed this problem, review Example 5.43.

Translate and Solve Basic Percent Equations

We will solve percent equations by using the methods we used to solve equations with fractions or decimals. In the past, you may have solved percent problems by setting them up as proportions. That was the best method available when you did not have the tools of algebra. Now as a prealgebra student, you can translate word sentences into algebraic equations, and then solve the equations.

We'll look at a common application of percent—tips to a server at a restaurant—to see how to set up a basic percent application.

When Aolani and her friends ate dinner at a restaurant, the bill came to $\$80$. They wanted to leave a 20% tip. What amount would the tip be?

To solve this, we want to find what *amount* is Math input error

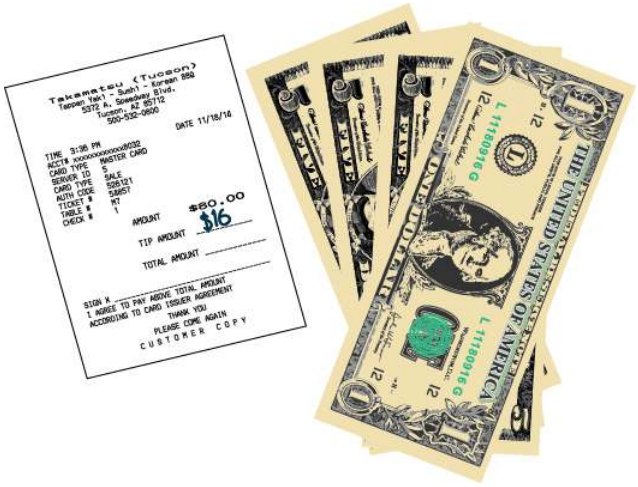


Figure 6.6 A 20% tip for an $\$80$ restaurant bill comes out to $\$16$.

In the next examples, we will find the amount. We must be sure to change the given percent to a decimal when we translate the words into an equation.

Example 6.14

What number is 35% of 90?

Answer

Translate into algebra. Let $n=$ the number.	$\underbrace{\text{What number}}_{n} \quad \underbrace{\text{is}}_{=} \quad \underbrace{35\%}_{0.35} \quad \underbrace{\text{of}}_{\cdot} \quad \underbrace{90?}_{90}$
Multiply.	$n = 31.5$
	31.5 is 35% of 90

Try It 6.27

What number is 45% of 80?

Try It 6.28

What number is 55% of 60?

Example 6.15

125% of 28 is what number?

Answer

Translate into algebra. Let $a =$ the number.	$\underbrace{125\%}_{1.25} \quad \underbrace{\text{of}}_{\cdot} \quad \underbrace{28}_{28} \quad \underbrace{\text{is}}_{=} \quad \underbrace{\text{what number?}}_{a}$
Multiply.	$35 = a$
	125% of 28 is 35.

Remember that a percent over 100 is a number greater than 1. We found that 125% of 28 is 35, which is greater than 28.

Try It 6.29

150% of 78 is what number?

Try It 6.30

175% of 72 is what number?

In the next examples, we are asked to find the base.

Example 6.16

Translate and solve: 36 is 75% of what number?

Answer

Translate. Let $b=$ the number.	$\underbrace{36}_{36} \quad \underbrace{\text{is}}_{=} \quad \underbrace{75\%}_{0.75} \quad \underbrace{\text{of}}_{\cdot} \quad \underbrace{\text{what number?}}_{b}$

Divide both sides by 0.75.	$\frac{36}{0.75} = \frac{0.75b}{0.75}$
Simplify.	$48 = b$ 36 is 75% of 48.

Try It 6.31

17 is 25% of what number?

Try It 6.32

40 is 62.5% of what number?

Example 6.17

6.5% of what number is $1.17?

Answer

Translate. Let b= the number.	$\underbrace{0.065}_{6.5\%} \underbrace{\cdot}_{\text{of}} \underbrace{b}_{\text{what number}} \underbrace{=}_{\text{is}} \underbrace{1.17}_{\$1.17?}$
Divide both sides by 0.065.	$\frac{0.065n}{0.065} = \frac{1.17}{0.065}$
Simplify.	$n = 18$ 6.5% of $18 is $1.17.

Try It 6.33

7.5% of what number is $1.95?

Try It 6.34

8.5% of what number is $3.06?

In the next examples, we will solve for the percent.

Example 6.18

What percent of 36 is 9?

Answer

Translate into algebra. Let p= the percent.	$\underbrace{p}_{\text{What percent}} \underbrace{\cdot}_{\text{of}} \underbrace{36}_{36} \underbrace{=}_{\text{is}} \underbrace{9}_{9?}$
Divide by 36.	$\frac{36p}{36} = \frac{9}{36}$
Simplify.	$p = \frac{1}{4}$
Convert to decimal form.	$p = 0.25$
Convert to percent.	$p = 25\%$ 25% of 36 is 9.

Try It 6.35

What percent of 76 is 57?

Try It 6.36

What percent of 120 is 96?

Example 6.19

144 is what percent of 96?

Answer

Translate into algebra. Let p= the percent.	$\underbrace{144}_{144}\ \underbrace{\text{is}}_{=}\ \underbrace{\text{what percent}}_{p}\ \underbrace{\text{of}}_{\cdot}\ \underbrace{96?}_{96}$
Divide by 96.	$\frac{144}{96} = \frac{96p}{96}$
Simplify.	$1.5 = p$
Convert to percent.	$150\% = p$ 144 is 150% of 96.

Try It 6.37

110 is what percent of 88?

Try It 6.38

126 is what percent of 72?

Solve Applications of Percent

Many applications of percent occur in our daily lives, such as tips, sales tax, discount, and interest. To solve these applications we'll translate to a basic percent equation, just like those we solved in the previous examples in this section. Once you translate the sentence into a percent equation, you know how to solve it.

We will update the strategy we used in our earlier applications to include equations now. Notice that we will translate a sentence into an equation.

How To

Solve an application

1. Step 1. Identify what you are asked to find and choose a variable to represent it.
2. Step 2. Write a sentence that gives the information to find it.
3. Step 3. Translate the sentence into an equation.
4. Step 4. Solve the equation using good algebra techniques.
5. Step 5. Check the answer in the problem and make sure it makes sense.
6. Step 6. Write a complete sentence that answers the question.

Now that we have the strategy to refer to, and have practiced solving basic percent equations, we are ready to solve percent applications. Be sure to ask yourself if your final answer makes sense—since many of the applications we'll solve involve everyday situations, you can rely on your own experience.

Example 6.20

Dezohn and his girlfriend enjoyed a dinner at a restaurant, and the bill was $68.50. They want to leave an 18% tip. If the tip will be 18% of the total bill, how much should the tip be?

Answer

What are you asked to find?	the amount of the tip
Choose a variable to represent it.	Let t= amount of tip.
Write a sentence that give the information to find it.	The tip is 18% of the total bill.
Translate the sentence into an equation.	$\underbrace{\text{The tip}}_{t}\ \underbrace{\text{is}}_{=}\ \underbrace{18\%}_{0.18}\ \underbrace{\text{of}}_{.}\ \underbrace{\$68.50.}_{\$68.50}$
Multiply.	$t = 12.33$
Check. Is this answer reasonable?	
If we approximate the bill to $70 and the percent to 20%, we would have a tip of $14. So a tip of $12.33 seems reasonable.	
Write a complete sentence that answers the question.	The couple should leave a tip of $12.33.

Try It 6.39

Cierra and her sister enjoyed a special dinner in a restaurant, and the bill was $81.50. If she wants to leave 18% of the total bill as her tip, how much should she leave?

Try It 6.40

Kimngoc had lunch at her favorite restaurant. She wants to leave 15% of the total bill as her tip. If her bill was $14.40, how much will she leave for the tip?

Example 6.21

The label on Masao's breakfast cereal said that one serving of cereal provides 85 milligrams (mg) of potassium, which is 2% of the recommended daily amount. What is the total recommended daily amount of potassium?

Nutrition Facts

Serving Size: 1 cup (47g)
Servings Per Container: About 7

Amount Per Serving	Cereal	With Milk
Calories	180	230
Calories from Fat	10	20
	% Daily Value*	
Total Fat 1g	**2%**	**2%**
Saturated Fat 0g	**0%**	**0%**
Trans Fat 0g		
Polyunsaturated Fat 0.5g		
Monounsaturated Fat 0.5g		
Cholesterol 0mg	**0%**	**0%**
Sodium 190mg	**8%**	**11%**
Potassium 85mg	**2%**	**8%**
Total Carbohydrate 40g	**13%**	**15%**
Dietary Fiber 1g	**4%**	**4%**
Sugars 8g		
Protein 3g		

Answer

What are you asked to find?	the total amount of potassium recommended
Choose a variable to represent it.	Let a= total amount of potassium.
Write a sentence that gives the information to find it.	85 mg is 2% of the total amount.
Translate the sentence into an equation.	$\underbrace{85\text{ mg}}_{85}\ \underbrace{\text{is}}_{=}\ \underbrace{2\%}_{0.02}\ \underbrace{\text{of}}_{\cdot}\ \underbrace{a}_{a}\ ?$
Divide both sides by 0.02.	$\frac{85}{0.02}=\frac{0.02a}{0.02}$
Simplify.	$4{,}250=a$
Check: Is this answer reasonable?	
Yes. 2% is a small percent and 85 is a small part of 4,250.	
Write a complete sentence that answers the question.	The amount of potassium that is recommended is 4250 mg.

Try It 6.41

One serving of wheat square cereal has 7 grams of fiber, which is 29% of the recommended daily amount. What is the total recommended daily amount of fiber?

Try It 6.42

One serving of rice cereal has 190 mg of sodium, which is 8% of the recommended daily amount. What is the total recommended daily amount of sodium?

Example 6.22

Mitzi received some gourmet brownies as a gift. The wrapper said each brownie was 480 calories, and had 240 calories of fat. What percent of the total calories in each brownie comes from fat?

Answer

What are you asked to find?	the percent of the total calories from fat
Choose a variable to represent it.	Let p= percent from fat.
Write a sentence that gives the information to find it.	What percent of 480 is 240?
Translate the sentence into an equation.	$\underbrace{\text{What percent}}_{p}\ \underbrace{\text{of}}_{\cdot}\ \underbrace{480}_{480}\ \underbrace{\text{is}}_{=}\ \underbrace{240?}_{240}$
Divide both sides by 480.	$\frac{p\cdot 480}{480}=\frac{240}{480}$
Simplify.	$p=0.5$
Convert to percent form.	$p=50\%$
Check. Is this answer reasonable?	
Yes. 240 is half of 480, so 50% makes sense.	
Write a complete sentence that answers the question.	Of the total calories in each brownie, 50% is fat.

Try It 6.43

Veronica is planning to make muffins from a mix. The package says each muffin will be 230 calories and 60 calories will be from fat. What percent of the total calories is from fat? (Round to the nearest whole percent.)

Try It 6.44

The brownie mix Ricardo plans to use says that each brownie will be 190 calories, and 70 calories are from fat. What percent of the total calories are from fat?

Find Percent Increase and Percent Decrease

People in the media often talk about how much an amount has increased or decreased over a certain period of time. They usually express this increase or decrease as a percent.

To find the percent increase, first we find the amount of increase, which is the difference between the new amount and the original amount. Then we find what percent the amount of increase is of the original amount.

How To

Find Percent Increase.

Step 1. Find the amount of increase.

- increase = new amount − original amount

Step 2. Find the percent increase as a percent of the original amount.

Example 6.23

In 2011, the California governor proposed raising community college fees from $26 per unit to $36 per unit. Find the percent increase. (Round to the nearest tenth of a percent.)

Answer

What are you asked to find?	the percent increase
Choose a variable to represent it.	Let p= percent.
Find the amount of increase.	$\underbrace{36}_{\text{new amount}} - \underbrace{26}_{\text{original amount}} = \underbrace{10}_{\text{increase}}$
Find the percent increase.	The increase is what percent of the original amount?
Translate to an equation.	$\underbrace{10}_{10} \underbrace{\text{is}}_{=} \underbrace{\text{what percent}}_{p} \underbrace{\text{of}}_{\cdot} \underbrace{26?}_{26}$
Divide both sides by 26.	$\frac{10}{26} = \frac{26p}{26}$
Round to the nearest thousandth.	$0.385 = p$
Convert to percent form.	$38.5\% = p$
Write a complete sentence.	The new fees represent a 38.5% increase over the old fees.

Try It 6.45

In 2011, the IRS increased the deductible mileage cost to 55.5 cents from 51 cents. Find the percent increase. (Round to the nearest tenth of a percent.)

Try It 6.46

In 1995, the standard bus fare in Chicago was $1.50. In 2008, the standard bus fare was $2.25. Find the percent increase. (Round to the nearest tenth of a percent.)

Finding the percent decrease is very similar to finding the percent increase, but now the amount of decrease is the difference between the original amount and the final amount. Then we find what percent the amount of decrease is of the original amount.

How To

Find percent decrease.

1. Step 1. Find the amount of decrease.
   - decrease = original amount − new amount
2. Step 2. Find the percent decrease as a percent of the original amount.

Example 6.24

The average price of a gallon of gas in one city in June 2014 was $3.71. The average price in that city in July was $3.64. Find the percent decrease.

Answer

What are you asked to find?	the percent decrease
Choose a variable to represent it.	Let p= percent.
Find the amount of decrease.	$\underbrace{3.71}_{\text{original amount}} - \underbrace{3.64}_{\text{new amount}} = \underbrace{0.07}_{\text{decrease}}$
Find the percent of decrease.	The decrease is what percent of the original amount?
Translate to an equation.	$\underbrace{0.07}_{0.07} \underbrace{\text{is}}_{=} \underbrace{\text{what percent}}_{p} \underbrace{\text{of}}_{\cdot} \underbrace{3.71}_{3.71}$
Divide both sides by 3.71.	$\frac{0.07}{3.71} = \frac{3.71p}{3.71}$
Round to the nearest thousandth.	$0.019 = p$
Convert to percent form.	$1.9\% = p$
Write a complete sentence.	The price of gas decreased 1.9%.

Try It 6.47

The population of one city was about 672,000 in 2010. The population of the city is projected to be about 630,000 in 2020. Find the percent decrease. (Round to the nearest tenth of a percent.)

Try It 6.48

Last year Sheila's salary was $42,000. Because of furlough days, this year her salary was $37,800. Find the percent decrease. (Round to the nearest tenth of a percent.)

Media

ACCESS ADDITIONAL ONLINE RESOURCES

- Percent Increase and Percent Decrease Visualization

Section 6.2 Exercises

Practice Makes Perfect

Translate and Solve Basic Percent Equations

In the following exercises, translate and solve.

97.

What number is 45% of 120?

98.

What number is 65% of 100?

99.

What number is 24% of 112?

100.

What number is 36% of 124?

101.

250% of 65 is what number?

102.

150% of 90 is what number?

103.

800% of $2{,}250$ is what number?

104.

600% of $1{,}740$ is what number?

105.

28 is 25% of what number?

106.

36 is 25% of what number?

107.

81 is 75% of what number?

108.

93 is 75% of what number?

109.

8.2% of what number is $\$2.87$?

110.

6.4% of what number is $\$2.88$?

111.

11.5% of what number is $\$108.10$?

112.

12.3% of what number is $\$92.25$?

113.

What percent of 260 is 78?

114.

What percent of 215 is 86?

115.

What percent of $1{,}500$ is 540?

116.

What percent of $1{,}800$ is 846?

117.

30 is what percent of 20?

118.

50 is what percent of 40?

119.

840 is what percent of 480?

120.

790 is what percent of 395?

Solve Applications of Percents

In the following exercises, solve the applications of percents.

121.

Geneva treated her parents to dinner at their favorite restaurant. The bill was $\$74.25$. She wants to leave 16% of the total bill as a tip. How much should the tip be?

122.

When Hiro and his co-workers had lunch at a restaurant the bill was $\$90.50$. They want to leave 18% of the total bill as a tip. How much should the tip be?

123.

Trong has 12% of each paycheck automatically deposited to his savings account. His last paycheck was $\$2{,}165$. How much money was deposited to Trong's savings account?

124.

Cherise deposits 8% of each paycheck into her retirement account. Her last paycheck was $\$1{,}485$. How much did Cherise deposit into her retirement account?

125.

One serving of oatmeal has 8 grams of fiber, which is 33% of the recommended daily amount. What is the total recommended daily amount of fiber?

126.

One serving of trail mix has 67 grams of carbohydrates, which is 22% of the recommended daily amount. What is the total recommended daily amount of carbohydrates?

127.

A bacon cheeseburger at a popular fast food restaurant contains $2{,}070$ milligrams (mg) of sodium, which is 86% of the recommended daily amount. What is the total recommended daily amount of sodium?

128.

A grilled chicken salad at a popular fast food restaurant contains 650 milligrams (mg) of sodium, which is 27% of the recommended daily amount. What is the total recommended daily amount of sodium?

129.

The nutrition fact sheet at a fast food restaurant says the fish sandwich has 380 calories, and 171 calories are from fat. What percent of the total calories is from fat?

130.

The nutrition fact sheet at a fast food restaurant says a small portion of chicken nuggets has 190 calories, and 114 calories are from fat. What percent of the total calories is from fat?

131.

Emma gets paid $3,000 per month. She pays $750 a month for rent. What percent of her monthly pay goes to rent?

132.

Dimple gets paid $3,200 per month. She pays $960 a month for rent. What percent of her monthly pay goes to rent?

Find Percent Increase and Percent Decrease

In the following exercises, find the percent increase or percent decrease.

133.

Tamanika got a raise in her hourly pay, from $15.50 to $17.55. Find the percent increase.

134.

Ayodele got a raise in her hourly pay, from $24.50 to $25.48. Find the percent increase.

135.

Annual student fees at the University of California rose from about $4,000 in 2000 to about $9,000 in 2014. Find the percent increase.

136.

The price of a share of one stock rose from $12.50 to $50. Find the percent increase.

137.

According to Time magazine (7/19/2011) annual global seafood consumption rose from 22 pounds per person in 1960 to 38 pounds per person today. Find the percent increase. (Round to the nearest tenth of a percent.)

138.

In one month, the median home price in the Northeast rose from $225,400 to $241,500. Find the percent increase. (Round to the nearest tenth of a percent.)

139.

A grocery store reduced the price of a loaf of bread from $2.80 to $2.73. Find the percent decrease.

140.

The price of a share of one stock fell from $8.75 to $8.54. Find the percent decrease.

141.

Hernando's salary was $49,500 last year. This year his salary was cut to $44,055. Find the percent decrease.

142.

From 2000 to 2010, the population of Detroit fell from about 951,000 to about 714,000. Find the percent decrease. (Round to the nearest tenth of a percent.)

143.

In one month, the median home price in the West fell from $203,400 to $192,300. Find the percent decrease. (Round to the nearest tenth of a percent.)

144.

Sales of video games and consoles fell from $1,150 million to $1,030 million in one year. Find the percent decrease. (Round to the nearest tenth of a percent.)

Everyday Math

145.

Tipping At the campus coffee cart, a medium coffee costs $1.65. MaryAnne brings $2.00 with her when she buys a cup of coffee and leaves the change as a tip. What percent tip does she leave?

146.

Late Fees Alison was late paying her credit card bill of $\$249$. She was charged a 5% late fee. What was the amount of the late fee?

Writing Exercises

147.

Without solving the problem "44 is 80% of what number", think about what the solution might be. Should it be a number that is greater than 44 or less than 44? Explain your reasoning.

148.

Without solving the problem "What is 20% of 300 ?" think about what the solution might be. Should it be a number that is greater than 300 or less than 300? Explain your reasoning.

149.

After returning from vacation, Alex said he should have packed 50% fewer shorts and 200% more shirts. Explain what Alex meant.

150.

Because of road construction in one city, commuters were advised to plan their Monday morning commute to take 150% of their usual commuting time. Explain what this means.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
translate and solve basic percent equations.			
solve applications of percent.			
find percent increase and percent decrease.			

ⓑ After reviewing this checklist, what will you do to become confident for all objectives?

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5.4: Solve Sales Tax, Commission, and Discount Applications

Learning Objectives

By the end of this section, you will be able to:

- Solve sales tax applications
- Solve commission applications
- Solve discount applications
- Solve mark-up applications

Be Prepared 6.7

Before you get started, take this readiness quiz.

Solve $0.0875(720)$ through multiplication.
If you missed this problem, review Example 5.17.

Be Prepared 6.8

Solve $12.96 \div 0.04$ through division.
If you missed this problem, review Example 5.22.

Solve Sales Tax Applications

Sales tax and commissions are applications of percent in our everyday lives. To solve these applications, we will follow the same strategy we used in the section on decimal operations. We show it again here for easy reference.

How To

Solve an application

1. Step 1. Identify what you are asked to find and choose a variable to represent it.
2. Step 2. Write a sentence that gives the information to find it.
3. Step 3. Translate the sentence into an equation.
4. Step 4. Solve the equation using good algebra techniques.
5. Step 5. Check the answer in the problem and make sure it makes sense.
6. Step 6. Write a complete sentence that answers the question.

Remember that whatever the application, once we write the sentence with the given information (Step 2), we can translate it to a percent equation and then solve it.

Do you pay a tax when you shop in your city or state? In many parts of the United States, sales tax is added to the purchase price of an item. See Figure 6.7. The sales tax is determined by computing a percent of the purchase price.

To find the sales tax multiply the purchase price by the sales tax rate. Remember to convert the sales tax rate from a percent to a decimal number. Once the sales tax is calculated, it is added to the purchase price. The result is the total cost—this is what the customer pays.


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1 D SOUP	13,75
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1 D LAMB	32,00
1 1/2 GL SAUV BLANC	4,25
1 D BANANAS FOSTER	9,50
SUBTOTAL	78,25
Tax (8%)	6,26
Total	84,51

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Figure 6.7 The sales tax is calculated as a percent of the purchase price.

Sales Tax

The sales tax is a percent of the purchase price.

$$\begin{aligned} \text{Sales Tax} &= \text{Tax Rate} \cdot \text{Purchase Price} \\ \text{Total Cost} &= \text{Purchase Price} + \text{Sales Tax} \end{aligned}$$

Example 6.25

Cathy bought a bicycle in Washington, where the sales tax rate was 6.5% of the purchase price. What was

1. ⓐ the sales tax and
2. ⓑ the total cost of a bicycle if the purchase price of the bicycle was $\$392$?

Answer

ⓐ	
Identify what you are asked to find.	What is the sales tax?
Choose a variable to represent it.	Let t= sales tax.
Write a sentence that gives the information to find it.	The sales tax is 6.5% of the purchase price.
Translate into an equation. (Remember to change the percent to a decimal).	$\underbrace{\text{The sales tax}}_{t}\ \underbrace{\text{is}}_{=}\ \underbrace{6.5\%}_{0.065}\ \underbrace{\text{of}}_{\cdot}\ \underbrace{\text{the \$392 purchase price.}}_{392}$
Simplify.	$t = 25.48$
Check: Is this answer reasonable?	
Yes, because the sales tax amount is less than 10% of the purchase price.	
Write a complete sentence that answers the question.	The sales tax is $25.48.

ⓑ	

Identify what you are asked to find.	What is the total cost of the bicycle?
Choose a variable to represent it.	Let c= total cost of bicycle.
Write a sentence that gives the information to find it.	The total cost is the purchase price plus the sales tax.
Translate into an equation.	$\underbrace{\text{The total cost}}_{c} \quad \underbrace{\text{is}}_{=} \quad \underbrace{\$392}_{392} \quad \underbrace{\text{plus}}_{+} \quad \underbrace{\$25.48}_{25.48}$
Simplify.	$c = 417.48$
Check: Is this answer reasonable?	
Yes, because the total cost is a little more than the purchase price.	
Write a complete sentence that answers the question.	The total cost of the bicycle is $417.48.

Try It 6.49

Find ⓐ the sales tax and ⓑ the total cost: Alexandra bought a television set for $\$724$ in Boston, where the sales tax rate was 6.25% of the purchase price.

Try It 6.50

Find ⓐ the sales tax and ⓑ the total cost: Kim bought a winter coat for $\$250$ in St. Louis, where the sales tax rate was 8.2% of the purchase price.

Example 6.26

Evelyn bought a new smartphone for $\$499$ plus tax. She was surprised when she got the receipt and saw that the tax was $\$42.42$. What was the sales tax rate for this purchase?

Answer

Identify what you are asked to find.	What is the sales tax rate?
Choose a variable to represent it.	Let r= sales tax.
Write a sentence that gives the information to find it.	What percent of the price is the sales tax?
Translate into an equation.	$\underbrace{\text{What percent}}_{r} \quad \underbrace{\text{of}}_{\cdot} \quad \underbrace{\text{the \$499 price}}_{499} \quad \underbrace{\text{is}}_{=} \quad \underbrace{\text{the \$42.42 tax?}}_{42.42}$
Divide.	$\frac{499r}{499} = \frac{42.42}{499}$
Simplify.	$r = 0.085$
Check. Is this answer reasonable?	
Yes, because 8.5% is close to 10%. 10% of $500 is $50, which is close to $42.42.	
Write a complete sentence that answers the question.	The sales tax rate is 8.5%.

Try It 6.51

Diego bought a new car for $26,525. He was surprised that the dealer then added $2,387.25. What was the sales tax rate for this purchase?

Try It 6.52

What is the sales tax rate if a $7,594 purchase will have $569.55 of sales tax added to it?

Solve Commission Applications

Sales people often receive a commission, or percent of total sales, for their sales. Their income may be just the commission they earn, or it may be their commission added to their hourly wages or salary. The commission they earn is calculated as a certain percent of the price of each item they sell. That percent is called the **rate of commission**.

Commission

A commission is a percentage of total sales as determined by the rate of commission.

$$\text{commission} = \text{rate of commission} \cdot \text{total sales}$$

To find the commission on a sale, multiply the rate of commission by the total sales. Just as we did for computing sales tax, remember to first convert the rate of commission from a percent to a decimal.

Example 6.27

Helene is a realtor. She receives 3% commission when she sells a house. How much commission will she receive for selling a house that costs $260,000?

Answer

Identify what you are asked to find.	What is the commission?
Choose a variable to represent it.	Let c= the commission.
Write a sentence that gives the information to find it.	The commission is 3% of the price.
Translate into an equation.	$\underbrace{\text{The commission}}_{c}\ \underbrace{\text{is}}_{=}\ \underbrace{3\%}_{0.03}\ \underbrace{\text{of}}_{\cdot}\ \underbrace{\text{the \$260,000 price.}}_{260,000}$
Simplify.	$c = 7800$
Check. Is this answer reasonable?	
Yes. 1% of $260,000 is $2,600, and $7,800 is three times $2,600.	
Write a complete sentence that answers the question.	Helene will receive a commission of $7,800.

Try It 6.53

Bob is a travel agent. He receives 7% commission when he books a cruise for a customer. How much commission will he receive for booking a $3,900 cruise?

Try It 6.54

Fernando receives 18% commission when he makes a computer sale. How much commission will he receive for selling a computer for $2,190?

Example 6.28

Rikki earned $87 commission when she sold a $1,450 stove. What rate of commission did she get?

Answer

Identify what you are asked to find.	What is the rate of commission?
Choose a variable to represent it.	Let r= the rate of commission.
Write a sentence that gives the information to find it.	The commission is what percent of the sale?
Translate into an equation.	$\underbrace{\text{The \$87 commission}}_{87}\ \underbrace{\text{is}}_{=}\ \underbrace{\text{what percent}}_{r}\ \underbrace{\text{of}}_{\cdot}\ \underbrace{\text{the \$1450 sale}}_{1450}\ ?$
Divide.	$\frac{87}{1450} = \frac{1450r}{1450}$
Simplify.	$0.06 = r$
Change to percent form.	$r = 6\%$
Check if this answer is reasonable.	
Yes. A 10% commission would have been $145. The 6% commission, $87, is a little more than half of that.	
Write a complete sentence that answers the question.	The commission was 6% of the price of the stove.

Try It 6.55

Homer received $1,140 commission when he sold a car for $28,500. What rate of commission did he get?

Try It 6.56

Bernice earned $451 commission when she sold an $8,200 living room set. What rate of commission did she get?

Solve Discount Applications

Applications of discount are very common in retail settings Figure 6.8. When you buy an item on sale, the **original price** of the item has been reduced by some dollar amount. The **discount rate**, usually given as a percent, is used to determine the amount of the discount. To determine the **amount of discount**, we multiply the discount rate by the original price. We summarize the discount model in the box below.



Figure 6.8 Applications of discounts are common in everyday life. (credit: Charleston's TheDigitel, Flickr)

Discount

An amount of discount is a percent off the original price.

$$\begin{aligned} \text{amount of discount} &= \text{discount rate} \cdot \text{original price} \\ \text{sale price} &= \text{original price} - \text{discount} \end{aligned}$$

The sale price should always be less than the original price. In some cases, the amount of discount is a fixed dollar amount. Then we just find the sale price by subtracting the amount of discount from the original price.

Example 6.29

Jason bought a pair of sunglasses that were on sale for $10 off. The original price of the sunglasses was $39. What was the sale price of the sunglasses?

Answer

Identify what you are asked to find.	What is the sale price?
Choose a variable to represent it.	Let s= the sale price.
Write a sentence that gives the information to find it.	The sale price is the original price minus the discount.
Translate into an equation.	$\underbrace{\text{The sale price}}_{s}\ \underbrace{\text{is}}_{=}\ \underbrace{\text{the original \$39 price}}_{39}\ \underbrace{\text{minus}}_{-}\ \underbrace{\text{the \$10 discount.}}_{10}$
Simplify.	$s = 29$
Check if this answer is reasonable.	
Yes. The sale price, $29, is less than the original price, $39.	
Write a complete sentence that answers the question.	The sale price of the sunglasses was $29.

Try It 6.57

Marta bought a dishwasher that was on sale for $75 off. The original price of the dishwasher was $525. What was the sale price of the dishwasher?

Try It 6.58

Orlando bought a pair of shoes that was on sale for $30 off. The original price of the shoes was $112. What was the sale price of the shoes?

In Example 6.29, the amount of discount was a set amount, Math input error

Example 6.30

Elise bought a dress that was discounted 35% off of the original price of $140. What was ⓐ the amount of discount and ⓑ the sale price of the dress?

Answer

ⓐ Before beginning, you may find it helpful to organize the information in a list.
Original price = $140
Discount rate = 35%
Amount of discount = ?

Identify what you are asked to find.	What is the amount of discount?
Choose a variable to represent it.	Let d= the amount of discount.
Write a sentence that gives the information to find it.	The discount is 35% of the original price.

Translate into an equation.	$\underbrace{\text{The discount}}_{s} \underbrace{\text{is}}_{=} \underbrace{\text{35\%}}_{0.35} \underbrace{\text{of}}_{\cdot} \underbrace{\text{the \$140 original price.}}_{140}$
Simplify.	$d = 49$
Check if this answer is reasonable.	
Yes. A \$49 discount is reasonable for a \$140 dress.	
Write a complete sentence that answers the question.	The amount of discount was \$49.

ⓑ
Original price = \$140
Amount of discount = \$49
Sale price = ?

Identify what you are asked to find.	What is the sale price of the dress?
Choose a variable to represent it.	Let s= the sale price.
Write a sentence that gives the information to find it.	The sale price is the original price minus the discount.
Translate into an equation.	$\underbrace{\text{The sale price}}_{s} \underbrace{\text{is}}_{=} \underbrace{\text{the \$140}}_{140} \underbrace{\text{minus}}_{-} \underbrace{\text{the \$49 discount.}}_{49}$
Simplify.	$s = 91$
Check if this answer is reasonable.	
Yes. The sale price, \$91, is less than the original price, \$140.	
Write a complete sentence that answers the question.	The sale price of the dress was \$91.

Try It 6.59

Find ⓐ the amount of discount and ⓑ the sale price: Sergio bought a belt that was discounted 40% from an original price of $\$29$.

Try It 6.60

Find ⓐ the amount of discount and ⓑ the sale price: Oscar bought a barbecue grill that was discounted 65% from an original price of $\$395$.

There may be times when you buy something on sale and want to know the discount rate. The next example will show this case.

Example 6.31

Jeannette bought a swimsuit at a sale price of $\$13.95$. The original price of the swimsuit was $\$31$. Find the ⓐ amount of discount and ⓑ discount rate.

Answer

ⓐ Before beginning, you may find it helpful to organize the information in a list.
Original price = \$31
Amount of discount = ?
Sale price = \$13.95

Identify what you are asked to find.	What is the amount of discount?

Choose a variable to represent it.	Let d= the amount of discount.
Write a sentence that gives the information to find it.	The discount is the original price minus the sale price.
Translate into an equation.	$\underbrace{\text{The discount}}_{d}\ \underbrace{\text{is}}_{=}\ \underbrace{\text{the \$31 original price}}_{31}\ \underbrace{\text{minus}}_{-}\ \underbrace{\text{the \$13.95 sale price}}_{13.95}.$
Simplify.	$d = 17.05$
Check if this answer is reasonable.	
Yes. The $17.05 discount is less than the original price.	
Write a complete sentence that answers the question.	The amount of discount was $17.05.

ⓑ Before beginning, you may find it helpful to organize the information in a list.
Original price = $31
Amount of discount = $17.05
Discount rate = ?

Identify what you are asked to find.	What is the discount rate?
Choose a variable to represent it.	Let r= the discount rate.
Write a sentence that gives the information to find it.	The discount is what percent of the original price?
Translate into an equation.	$\underbrace{\text{The discount of \$17.05}}_{d}\ \underbrace{\text{is}}_{=}\ \underbrace{\text{what percent}}_{r}\ \underbrace{\text{of}}_{\cdot}\ \underbrace{\text{the \$31 original price}}_{31}.$
Divide.	$\frac{17.05}{31} = \frac{r(31)}{31}$
Simplify.	$0.55 = r$
Check if this answer is reasonable.	
The rate of discount was a little more than 50% and the amount of discount is a little more than half of $31.	
Write a complete sentence that answers the question.	The rate of discount was 55%.

Try It 6.61

Find ⓐ the amount of discount and ⓑ the discount rate: Lena bought a kitchen table at the sale price of $375.20. The original price of the table was $560.

Try It 6.62

Find ⓐ the amount of discount and ⓑ the discount rate: Nick bought a multi-room air conditioner at a sale price of $340. The original price of the air conditioner was $400.

Solve Mark-up Applications

Applications of mark-up are very common in retail settings. The price a retailer pays for an item is called the **wholesale price**. The retailer then adds a **mark-up** to the wholesale price to get the **list price**, the price he sells the item for. The mark-up is usually calculated as a percent of the wholesale price. The percent is called the **mark-up rate**. To determine the amount of mark-up, multiply the mark-up rate by the wholesale price. We summarize the mark-up model in the box below.

Mark-up

The mark-up is the amount added to the wholesale price.

$$\begin{aligned} \text{amount of mark-up} &= \text{mark-up rate} \cdot \text{wholesale price} \\ \text{list price} &= \text{wholesale price} + \text{mark up} \end{aligned}$$

The list price should always be more than the wholesale price.

Example 6.32

Adam's art gallery bought a photograph at the wholesale price of $\$250$. Adam marked the price up 40%. Find the ⓐ amount of mark-up and ⓑ the list price of the photograph.

Answer

ⓐ	
Identify what you are asked to find.	What is the amount of mark-up?
Choose a variable to represent it.	Let $m=$ the amount of each mark-up.
Write a sentence that gives the information to find it.	The mark-up is 40% of the wholesale price.
Translate into an equation.	$\underbrace{\text{The mark-up}}_{m}\ \underbrace{\text{is}}_{=}\ \underbrace{40\%}_{0.40}\ \underbrace{\text{of}}_{\cdot}\ \underbrace{\text{the \$250 wholesale price.}}_{250}$
Simplify.	$m = 100$
Check if this answer is reasonable.	
Yes. The markup rate is less than 50% and \$100 is less than half of \$250.	
Write a complete sentence that answers the question.	The mark-up on the photograph was \$100.

ⓑ	
Identify what you are asked to find.	What is the list price?
Choose a variable to represent it.	Let $p=$ the list price.
Write a sentence that gives the information to find it.	The list price is the wholesale price plus the mark-up.
Translate into an equation.	$\underbrace{\text{The list price}}_{p}\ \underbrace{\text{is}}_{=}\ \underbrace{\text{the \$250 wholesale price}}_{250}\ \underbrace{\text{plus}}_{+}\ \underbrace{\text{the \$100 mark-up.}}_{100}$
Simplify.	$p = 350$
Check if this answer is reasonable.	
Yes. The list price, \$350, is more than the wholesale price, \$250.	
Write a complete sentence that answers the question.	The list price of the photograph was \$350.

Try It 6.63

Jim's music store bought a guitar at wholesale price $\$1{,}200$. Jim marked the price up 50%. Find the ⓐ amount of mark-up and ⓑ the list price.

Try It 6.64

The Auto Resale Store bought Pablo's Toyota for $8,500. They marked the price up 35%. Find the ⓐ amount of mark-up and ⓑ the list price.

Section 6.3 Exercises

Practice Makes Perfect

Solve Sales Tax Applications

In the following exercises, find ⓐ the sales tax and ⓑ the total cost.

151.

The cost of a pair of boots was $84. The sales tax rate is 5% of the purchase price.

152.

The cost of a refrigerator was $1,242. The sales tax rate is 8% of the purchase price.

153.

The cost of a microwave oven was $129. The sales tax rate is 7.5% of the purchase price.

154.

The cost of a tablet computer is $350. The sales tax rate is 8.5% of the purchase price.

155.

The cost of a file cabinet is $250. The sales tax rate is 6.85% of the purchase price.

156.

The cost of a luggage set $400. The sales tax rate is 5.75% of the purchase price.

157.

The cost of a 6-drawer dresser $1,199. The sales tax rate is 5.125% of the purchase price.

158.

The cost of a sofa is $1,350. The sales tax rate is 4.225% of the purchase price.

In the following exercises, find the sales tax rate.

159.

Shawna bought a mixer for $300. The sales tax on the purchase was $19.50.

160.

Orphia bought a coffee table for $400. The sales tax on the purchase was $38.

161.

Bopha bought a bedroom set for $3,600. The sales tax on the purchase was $246.60.

162.

Ruth bought a washer and dryer set for $2,100. The sales tax on the purchase was $152.25.

Solve Commission Applications

In the following exercises, find the commission.

163.

Christopher sold his dinette set for $225 through an online site, which charged him 9% of the selling price as commission. How much was the commission?

164.

Michele rented a booth at a craft fair, which charged her 8% commission on her sales. One day her total sales were $193. How much was the commission?

165.

Farrah works in a jewelry store and receives 12% commission when she makes a sale. How much commission will she receive for selling a $\$8{,}125$ ring?

166.

Jamal works at a car dealership and receives 9% commission when he sells a car. How much commission will he receive for selling a $\$32{,}575$ car?

167.

Hector receives 17.5% commission when he sells an insurance policy. How much commission will he receive for selling a policy for $\$4{,}910$?

168.

Denise receives 10.5% commission when she books a tour at the travel agency. How much commission will she receive for booking a tour with total cost $\$7{,}420$?

In the following exercises, find the rate of commission.

169.

Dontay is a realtor and earned $\$11{,}250$ commission on the sale of a $\$375{,}000$ house. What is his rate of commission?

170.

Nevaeh is a cruise specialist and earned $\$364$ commission after booking a cruise that cost $\$5{,}200$. What is her rate of commission?

171.

As a waitress, Emily earned $\$420$ in tips on sales of $\$2{,}625$ last Saturday night. What was her rate of commission?

172.

Alejandra earned $\$1{,}393.74$ commission on weekly sales of $\$15{,}486$ as a salesperson at the computer store. What is her rate of commission?

173.

Maureen earned $\$7{,}052.50$ commission when she sold a $\$45{,}500$ car. What was the rate of commission?

174.

Lucas earned $\$4{,}487.50$ commission when he brought a $\$35{,}900$ job to his office. What was the rate of commission?

Solve Discount Applications

In the following exercises, find the sale price.

175.

Perla bought a cellphone that was on sale for $\$50$ off. The original price of the cellphone was $\$189$.

176.

Sophie saw a dress she liked on sale for $\$15$ off. The original price of the dress was $\$96$.

177.

Rick wants to buy a tool set with original price $\$165$. Next week the tool set will be on sale for $\$40$ off.

178.

Angelo's store is having a sale on TV sets. One set, with an original price of $\$859$, is selling for $\$125$ off.

In the following exercises, find ⓐ the amount of discount and ⓑ the sale price.

179.

Janelle bought a beach chair on sale at 60% off. The original price was $\$44.95$

180.

Errol bought a skateboard helmet on sale at 40% off. The original price was $\$49.95$.

181.

Kathy wants to buy a camera that lists for $389. The camera is on sale with a 33% discount.

182.

Colleen bought a suit that was discounted 25% from an original price of $245.

183.

Erys bought a treadmill on sale at 35% off. The original price was $949.95.

184.

Jay bought a guitar on sale at 45% off. The original price was $514.75.

In the following exercises, find ⓐ the amount of discount and ⓑ the discount rate. (Round to the nearest tenth of a percent if needed.)

185.

Larry and Donna bought a sofa at the sale price of $1,344. The original price of the sofa was $1,920.

186.

Hiroshi bought a lawnmower at the sale price of $240. The original price of the lawnmower is $300.

187.

Patty bought a baby stroller on sale for $301.75. The original price of the stroller was $355.

188.

Bill found a book he wanted on sale for $20.80. The original price of the book was $32.

189.

Nikki bought a patio set on sale for $480. The original price was $850.

190.

Stella bought a dinette set on sale for $725. The original price was $1,299.

Solve Mark-up Applications

In the following exercises, find ⓐ the amount of the mark-up and ⓑ the list price.

191.

Daria bought a bracelet at wholesale cost $16 to sell in her handicraft store. She marked the price up 45%.

192.

Regina bought a handmade quilt at wholesale cost $120 to sell in her quilt store. She marked the price up 55%.

193.

Tom paid $0.60 a pound for tomatoes to sell at his produce store. He added a 33% mark-up.

194.

Flora paid her supplier $0.74 a stem for roses to sell at her flower shop. She added an 85% mark-up.

195.

Alan bought a used bicycle for $115. After re-conditioning it, he added 225% mark-up and then advertised it for sale.

196.

Michael bought a classic car for $8,500. He restored it, then added 150% mark-up before advertising it for sale.

Everyday Math

197.

Coupons Yvonne can use two coupons for the same purchase at her favorite department store. One coupon gives her $20 off and the other gives her 25% off. She wants to buy a bedspread that sells for $195.

1. ⓐ Calculate the discount price if Yvonne uses the $20 coupon first and then takes 25% off.
2. ⓑ Calculate the discount price if Yvonne uses the 25% off coupon first and then uses the $20 coupon.

3. ⓒ In which order should Yvonne use the coupons?

198.

Cash Back Jason can buy a bag of dog food for $35 at two different stores. One store offers 6% cash back on the purchase plus $5 off his next purchase. The other store offers 20% cash back.

1. ⓐ Calculate the total savings from the first store, including the savings on the next purchase.
2. ⓑ Calculate the total savings from the second store.
3. ⓒ Which store should Jason buy the dog food from? Why?

Writing Exercises

199.

Priam bought a jacket that was on sale for 40% off. The original price of the jacket was $150. While the sales clerk figured the price by calculating the amount of discount and then subtracting that amount from $150, Priam found the price faster by calculating 60% of $150.

1. ⓐ Explain why Priam was correct.
2. ⓑ Will Priam's method work for any original price?

200.

Roxy bought a scarf on sale for 50% off. The original price of the scarf was $32.90. Roxy claimed that the price she paid for the scarf was the same as the amount she saved. Was Roxy correct? Explain.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
solve sales tax applications.			
solve commission applications.			
solve discount applications.			
solve mark-up applications.			

ⓑ What does this checklist tell you about your mastery of this section? What steps will you take to improve?

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5.5: Solve Proportions and their Applications

Learning Objectives

By the end of this section, you will be able to:

- Use the definition of proportion
- Solve proportions
- Solve applications using proportions
- Write percent equations as proportions
- Translate and solve percent proportions

Be Prepared 6.11

Before you get started, take this readiness quiz.

Simplify: $\frac{\frac{1}{3}}{4}$.
If you missed this problem, review Example 4.44.

Be Prepared 6.12

Solve: $\frac{x}{4} = 20$.
If you missed this problem, review Example 4.99.

Be Prepared 6.13

Write as a rate: Sale rode his bike 24 miles in 2 hours.
If you missed this problem, review Example 5.63.

Use the Definition of Proportion

In the section on Ratios and Rates we saw some ways they are used in our daily lives. When two ratios or rates are equal, the equation relating them is called a proportion.

Proportion

A proportion is an equation of the form $\frac{a}{b} = \frac{c}{d}$, where $b \neq 0, d \neq 0$.

The proportion states two ratios or rates are equal. The proportion is read "a is to b, as c is to d".

The equation $\frac{1}{2} = \frac{4}{8}$ is a proportion because the two fractions are equal. The proportion $\frac{1}{2} = \frac{4}{8}$ is read "1 is to 2 as 4 is to 8".

If we compare quantities with units, we have to be sure we are comparing them in the right order. For example, in the proportion mfrac = mfrac we compare the number of students to the number of teachers. We put students in the numerators and teachers in the denominators.

Example 6.40

Write each sentence as a proportion:

1. ⓐ 3 is to 7 as 15 is to 35.
2. ⓑ 5 hits in 8 at bats is the same as 30 hits in 48 at-bats.
3. ⓒ $\$1.50$ for 6 ounces is equivalent to $\$2.25$ for 9 ounces.

Answer

ⓐ	
	3 is to 7 as 15 is to 35.

Write as a proportion.	$\frac{3}{7} = \frac{15}{35}$

ⓑ	
	5 hits in 8 at-bats is the same as 30 hits in 48 at-bats.
Write each fraction to compare hits to at-bats.	$\frac{\text{hits}}{\text{at-bats}} = \frac{\text{hits}}{\text{at-bats}}$
Write as a proportion.	$\frac{5}{8} = \frac{30}{48}$

ⓒ	
	\$1.50 for 6 ounces is equivalent to \$2.25 for 9 ounces.
Write each fraction to compare dollars to ounces.	$\frac{\$}{\text{ounces}} = \frac{\$}{\text{ounces}}$
Write as a proportion.	$\frac{1.50}{6} = \frac{2.25}{9}$

Try It 6.79

Write each sentence as a proportion:

1. ⓐ 5 is to 9 as 20 is to 36.
2. ⓑ 7 hits in 11 at-bats is the same as 28 hits in 44 at-bats.
3. ⓒ $\$2.50$ for 8 ounces is equivalent to $\$3.75$ for 12 ounces.

Try It 6.80

Write each sentence as a proportion:

1. ⓐ 6 is to 7 as 36 is to 42.
2. ⓑ 8 adults for 36 children is the same as 12 adults for 54 children.
3. ⓒ $\$3.75$ for 6 ounces is equivalent to $\$2.50$ for 4 ounces.

Look at the proportions $\frac{1}{2} = \frac{4}{8}$ and $\frac{2}{3} = \frac{6}{9}$. From our work with equivalent fractions we know these equations are true. But how do we know if an equation is a proportion with equivalent fractions if it contains fractions with larger numbers?

To determine if a proportion is true, we find the **cross products** of each proportion. To find the cross products, we multiply each denominator with the opposite numerator (diagonally across the equal sign). The results are called a cross product because of the cross formed. If, and only if, the given proportion is true, that is, the two sides are equal, then the cross products of a proportion will be equal.

$8 \cdot 1 = 8 \quad 2 \cdot 4 = 8 \qquad 9 \cdot 2 = 18 \quad 3 \cdot 6 = 18$

$\frac{1}{2} = \frac{4}{8} \qquad \frac{2}{3} = \frac{6}{9}$

Cross Products of a Proportion

For any proportion of the form $\frac{a}{b} = \frac{c}{d}$, where $b \neq 0, d \neq 0$, its cross products are equal.

$a \cdot d = b \cdot c$

$\frac{a}{b} = \frac{c}{d}$

Cross products can be used to test whether a proportion is true. To test whether an equation makes a proportion, we find the cross products. If they are both equal, we have a proportion.

Example 6.41

Determine whether each equation is a proportion:

1. ⓐ $\frac{4}{9} = \frac{12}{28}$
2. ⓑ $\frac{17.5}{37.5} = \frac{7}{15}$

Answer

To determine if the equation is a proportion, we find the cross products. If they are equal, the equation is a proportion.

ⓐ	
	$\frac{4}{9} = \frac{12}{28}$
Find the cross products.	$28 \cdot 4 = 112 \qquad 9 \cdot 12 = 108$ $\frac{4}{9} \neq \frac{12}{28}$

Since the cross products are not equal, $28 \cdot 4 \neq 9 \cdot 12$, the equation is not a proportion.

ⓑ	
	$\frac{17.5}{37.5} = \frac{7}{15}$
Find the cross products.	$15 \cdot 17.5 = 262.5 \qquad 37.5 \cdot 7 = 262.5$ $\frac{17.5}{37.5} = \frac{7}{15}$

Since the cross products are equal, $15 \cdot 17.5 = 37.5 \cdot 7$, the equation is a proportion.

Try It 6.81

Determine whether each equation is a proportion:

1. ⓐ $\frac{7}{9} = \frac{54}{72}$
2. ⓑ $\frac{24.5}{45.5} = \frac{7}{13}$

Try It 6.82

Determine whether each equation is a proportion:

1. ⓐ $\frac{8}{9} = \frac{56}{73}$
2. ⓑ $\frac{28.5}{52.5} = \frac{8}{15}$

Solve Proportions

To solve a proportion containing a variable, we remember that the proportion is an equation. All of the techniques we have used so far to solve equations still apply. In the next example, we will solve a proportion by multiplying by the Least Common Denominator (LCD) using the Multiplication Property of Equality.

Example 6.42

Solve: $\frac{x}{63} = \frac{4}{7}$.

Answer

	$\frac{x}{63} = \frac{4}{7}$
To isolate x, multiply both sides by the LCD, 63.	$63\left(\frac{x}{63}\right) = 63\left(\frac{4}{7}\right)$
Simplify.	$x = \frac{9 \cdot 7 \cdot 4}{7}$

Divide the common factors.		$x = 36$
Check: To check our answer, we substitute into the original proportion.		
	$\frac{x}{63} = \frac{4}{7}$	
Substitute $x = 36$	$\frac{36}{63} \stackrel{?}{=} \frac{4}{7}$	
Show common factors.	$\frac{4 \cdot 9}{7 \cdot 9} \stackrel{?}{=} \frac{4}{7}$	
Simplify.	$\frac{4}{7} = \frac{4}{7}$ ✓	

Try It 6.83

Solve the proportion: $\frac{n}{84} = \frac{11}{12}$.

Try It 6.84

Solve the proportion: $\frac{y}{96} = \frac{13}{12}$.

When the variable is in a denominator, we'll use the fact that the cross products of a proportion are equal to solve the proportions.

We can find the cross products of the proportion and then set them equal. Then we solve the resulting equation using our familiar techniques.

Example 6.43

Solve: $\frac{144}{a} = \frac{9}{4}$.

Answer

Notice that the variable is in the denominator, so we will solve by finding the cross products and setting them equal.

	$\frac{144}{a} = \frac{9}{4}$
Find the cross products and set them equal.	$4 \cdot 144 = a \cdot 9$
Simplify.	$576 = 9a$
Divide both sides by 9.	$\frac{576}{9} = \frac{9a}{9}$
Simplify.	$64 = a$
Check your answer.	
	$\frac{144}{a} = \frac{9}{4}$
Substitute $a = 64$	$\frac{144}{64} \stackrel{?}{=} \frac{9}{4}$
Show common factors..	$\frac{9 \cdot 16}{4 \cdot 16} \stackrel{?}{=} \frac{9}{4}$
Simplify.	$\frac{9}{4} = \frac{9}{4}$ ✓

Another method to solve this would be to multiply both sides by the LCD, $4a$. Try it and verify that you get the same solution.

Try It 6.85

Solve the proportion: $\frac{91}{b} = \frac{7}{5}$.

Try It 6.86

Solve the proportion: $\frac{39}{c} = \frac{13}{8}$.

Example 6.44

Solve: $\frac{52}{91} = \frac{-4}{y}$.

Answer

Find the cross products and set them equal.	$\frac{52}{91} = \frac{-4}{y}$
	$y \cdot 52 = 91(-4)$
Simplify.	$52y = -364$
Divide both sides by 52.	$\frac{52y}{52} = \frac{-364}{52}$
Simplify.	$y = -7$
Check:	
	$\frac{52}{91} = \frac{-4}{y}$
Substitute $y = -7$	$\frac{52}{91} \stackrel{?}{=} \frac{-4}{-7}$
Show common factors.	$\frac{13 \cdot 4}{13 \cdot 4} \stackrel{?}{=} \frac{-4}{-7}$
Simplify.	$\frac{4}{7} = \frac{4}{7}$ ✓

Try It 6.87

Solve the proportion: $\frac{84}{98} = \frac{-6}{x}$.

Try It 6.88

Solve the proportion: $\frac{-7}{y} = \frac{105}{135}$.

Solve Applications Using Proportions

The strategy for solving applications that we have used earlier in this chapter, also works for proportions, since proportions are equations. When we set up the proportion, we must make sure the units are correct—the units in the numerators match and the units in the denominators match.

Example 6.45

When pediatricians prescribe acetaminophen to children, they prescribe 5 milliliters (ml) of acetaminophen for every 25 pounds of the child's weight. If Zoe weighs 80 pounds, how many milliliters of acetaminophen will her doctor prescribe?

Answer

Identify what you are asked to find.	How many ml of acetaminophen the doctor will prescribe
Choose a variable to represent it.	Let a = ml of acetaminophen.
Write a sentence that gives the information to find it.	If 5 ml is prescribed for every 25 pounds, how much will be prescribed for 80 pounds?
Translate into a proportion.	$\frac{\text{ml}}{\text{pounds}} = \frac{\text{ml}}{\text{pounds}}$
Substitute given values—be careful of the units.	$\frac{5}{25} = \frac{a}{80}$
Multiply both sides by 80.	$80 \cdot \frac{5}{25} = 80 \cdot \frac{a}{80}$
Multiply and show common factors.	$\frac{16 \cdot 5 \cdot 5}{5 \cdot 5} = \frac{80a}{80}$
Simplify.	$16 = a$
Check if the answer is reasonable.	
Yes. Since 80 is about 3 times 25, the medicine should be about 3 times 5.	
Write a complete sentence.	The pediatrician would prescribe 16 ml of acetaminophen to Zoe.

You could also solve this proportion by setting the cross products equal.

Try It 6.89

Pediatricians prescribe 5 milliliters (ml) of acetaminophen for every 25 pounds of a child's weight. How many milliliters of acetaminophen will the doctor prescribe for Emilia, who weighs 60 pounds?

Try It 6.90

For every 1 kilogram (kg) of a child's weight, pediatricians prescribe 15 milligrams (mg) of a fever reducer. If Isabella weighs 12 kg, how many milligrams of the fever reducer will the pediatrician prescribe?

Example 6.46

One brand of microwave popcorn has 120 calories per serving. A whole bag of this popcorn has 3.5 servings. How many calories are in a whole bag of this microwave popcorn?

Answer

Identify what you are asked to find.	How many calories are in a whole bag of microwave popcorn?
Choose a variable to represent it.	Let c = number of calories.
Write a sentence that gives the information to find it.	If there are 120 calories per serving, how many calories are in a whole bag with 3.5 servings?
Translate into a proportion.	$\frac{\text{calories}}{\text{serving}} = \frac{\text{calories}}{\text{serving}}$
Substitute given values.	$\frac{120}{1} = \frac{c}{3.5}$
Multiply both sides by 3.5.	$(3.5)\left(\frac{120}{1}\right) = (3.5)\left(\frac{c}{3.5}\right)$

Multiply.	$420 = c$
Check if the answer is reasonable.	
Yes. Since 3.5 is between 3 and 4, the total calories should be between 360 (3·120) and 480 (4·120).	
Write a complete sentence.	The whole bag of microwave popcorn has 420 calories.

Try It 6.91

Marissa loves the Caramel Macchiato at the coffee shop. The 16 oz. medium size has 240 calories. How many calories will she get if she drinks the large 20 oz. size?

Try It 6.92

Yaneli loves Starburst candies, but wants to keep her snacks to 100 calories. If the candies have 160 calories for 8 pieces, how many pieces can she have in her snack?

Example 6.47

Josiah went to Mexico for spring break and changed $\$325$ dollars into Mexican pesos. At that time, the exchange rate had $\$1$ U.S. is equal to 12.54 Mexican pesos. How many Mexican pesos did he get for his trip?

Answer

Identify what you are asked to find.	How many Mexican pesos did Josiah get?
Choose a variable to represent it.	Let $p =$ number of pesos.
Write a sentence that gives the information to find it.	If $1 U.S. is equal to 12.54 Mexican pesos, then $325 is how many pesos?
Translate into a proportion.	$\frac{\$}{\text{pesos}} = \frac{\$}{\text{pesos}}$
Substitute given values.	$\frac{1}{12.54} = \frac{325}{p}$
The variable is in the denominator, so find the cross products and set them equal.	$p \cdot 1 = 12.54(325)$
Simplify.	$c = 4{,}075.5$
Check if the answer is reasonable.	
Yes, $100 would be $1,254 pesos. $325 is a little more than 3 times this amount.	
Write a complete sentence.	Josiah has 4075.5 pesos for his spring break trip.

Try It 6.93

Yurianna is going to Europe and wants to change $\$800$ dollars into Euros. At the current exchange rate, $\$1$ US is equal to 0.738 Euro. How many Euros will she have for her trip?

Try It 6.94

Corey and Nicole are traveling to Japan and need to exchange $\$600$ into Japanese yen. If each dollar is 94.1 yen, how many yen will they get?

Write Percent Equations As Proportions

Previously, we solved percent equations by applying the properties of equality we have used to solve equations throughout this text. Some people prefer to solve percent equations by using the proportion method. The proportion method for solving percent problems involves a percent proportion. A **percent proportion** is an equation where a percent is equal to an equivalent ratio.

For example, $60\% = \frac{60}{100}$ and we can simplify $\frac{60}{100} = \frac{3}{5}$. Since the equation $\frac{60}{100} = \frac{3}{5}$ shows a percent equal to an equivalent ratio, we call it a percent proportion. Using the vocabulary we used earlier:

$$\frac{\text{amount}}{\text{base}} = \frac{\text{percent}}{100}$$

$$\frac{3}{5} = \frac{60}{100}$$

Percent Proportion

The amount is to the base as the percent is to 100.

$$\frac{\text{amount}}{\text{base}} = \frac{\text{percent}}{100}$$

If we restate the problem in the words of a proportion, it may be easier to set up the proportion:

The amount is to the base as the percent is to one hundred.

We could also say:

The amount out of the base is the same as the percent out of one hundred.

First we will practice translating into a percent proportion. Later, we'll solve the proportion.

Example 6.48

Translate to a proportion. What number is 75% of 90?

Answer

If you look for the word "of", it may help you identify the base.

Identify the parts of the percent proportion.	What number (amount) is 75% (percent) of 90? (base)
Restate as a proportion.	What number out of 90 is the same as 75 out of 100?
Set up the proportion. Let $n = \text{number}$.	$\frac{n}{90} = \frac{75}{100}$

Try It 6.95

Translate to a proportion: What number is 60% of 105?

Try It 6.96

Translate to a proportion: What number is 40% of 85?

Example 6.49

Translate to a proportion. 19 is 25% of what number?

Answer

Identify the parts of the percent proportion.	$\underbrace{19}_{\text{amount}}$ is $\underbrace{25\%}_{\text{percent}}$ of $\underbrace{\text{what number?}}_{\text{base}}$
Restate as a proportion.	19 out of what number is the same as 25 out of 100?
Set up the proportion. Let $n = \text{number}$.	$\frac{19}{n} = \frac{25}{100}$

Try It 6.97

Translate to a proportion: 36 is 25% of what number?

Try It 6.98

Translate to a proportion: 27 is 36% of what number?

Example 6.50

Translate to a proportion. What percent of 27 is 9?

Answer

Identify the parts of the percent proportion.	$\underbrace{\text{What percent}}_{\text{percent}}$ of $\underbrace{27}_{\text{base}}$ is $\underbrace{9?}_{\text{amount}}$
Restate as a proportion.	9 out of 27 is the same as what number out of 100?
Set up the proportion. Let $p = \text{percent}$.	$\frac{9}{27} = \frac{p}{100}$

Try It 6.99

Translate to a proportion: What percent of 52 is 39?

Try It 6.100

Translate to a proportion: What percent of 92 is 23?

Translate and Solve Percent Proportions

Now that we have written percent equations as proportions, we are ready to solve the equations.

Example 6.51

Translate and solve using proportions: What number is 45% of 80?

Answer

Identify the parts of the percent proportion.	$\underbrace{\text{What number}}_{\text{amount}}$ is $\underbrace{45\%}_{\text{percent}}$ of $\underbrace{80?}_{\text{base}}$
Restate as a proportion.	What number out of 80 is the same as 45 out of 100?
Set up the proportion. Let $n = \text{number}$.	$\frac{n}{80} = \frac{45}{100}$
Find the cross products and set them equal.	$100 \cdot n = 80 \cdot 45$
Simplify.	$100n = 3{,}600$

Divide both sides by 100.	$\frac{100n}{100} = \frac{3,600}{100}$
Simplify.	$n = 36$
Check if the answer is reasonable.	
Yes. 45 is a little less than half of 100 and 36 is a little less than half 80.	
Write a complete sentence that answers the question.	36 is 45% of 80.

Try It 6.101

Translate and solve using proportions: What number is 65% of 40?

Try It 6.102

Translate and solve using proportions: What number is 85% of 40?

In the next example, the percent is more than 100, which is more than one whole. So the unknown number will be more than the base.

Example 6.52

Translate and solve using proportions: 125% of 25 is what number?

Answer

Identify the parts of the percent proportion.	$\underbrace{125\%}_{\text{percent}}$ is $\underbrace{25}_{\text{base}}$ of $\underbrace{\text{what number}?}_{\text{amount}}$
Restate as a proportion.	What number out of 25 is the same as 125 out of 100?
Set up the proportion. Let $n =$ number.	$\frac{n}{25} = \frac{125}{100}$
Find the cross products and set them equal.	$100 \cdot n = 25 \cdot 125$
Simplify.	$100n = 3,125$
Divide both sides by 100.	$\frac{100n}{100} = \frac{3,125}{100}$
Simplify.	$n = 31.25$
Check if the answer is reasonable.	
Yes. 125 is more than 100 and 31.25 is more than 25.	
Write a complete sentence that answers the question.	125% of 25 is 31.25.

Try It 6.103

Translate and solve using proportions: 125% of 64 is what number?

Try It 6.104

Translate and solve using proportions: 175% of 84 is what number?

Percents with decimals and money are also used in proportions.

Example 6.53

Translate and solve: 6.5% of what number is $\$1.56$?

Answer

Identify the parts of the percent proportion.	$\underbrace{6.5\%}_{\text{percent}}$ of $\underbrace{\text{what number}}_{\text{base}}$ is $\underbrace{\$1.56}_{\text{amount}}$?
Restate as a proportion.	1.56 out of what number is the same as 6.5 out of 100?
Set up the proportion. Let $n =$ number.	$\frac{1.56}{n} = \frac{6.5}{100}$
Find the cross products and set them equal.	$100(1.56) = n \cdot 6.5$
Simplify.	$156 = 6.5n$
Divide both sides by 6.5 to isolate the variable.	$\frac{156}{6.5} = \frac{6.5n}{6.5}$
Simplify.	$24 = n$
Check if the answer is reasonable.	
Yes. 6.5% is a small amount and $1.56 is much less than $24.	
Write a complete sentence that answers the question.	6.5% of $24 is $1.56.

Try It 6.105

Translate and solve using proportions: 8.5% of what number is $\$3.23$?

Try It 6.106

Translate and solve using proportions: 7.25% of what number is $\$4.64$?

Example 6.54

Translate and solve using proportions: What percent of 72 is 9?

Answer

Identify the parts of the percent proportion.	$\underbrace{\text{What percent}}_{\text{percent}}$ of $\underbrace{72}_{\text{base}}$ is $\underbrace{9}_{\text{amount}}$?
Restate as a proportion.	9 out of 72 is the same as what number out of 100?
Set up the proportion. Let $n =$ number.	$\frac{9}{72} = \frac{n}{100}$
Find the cross products and set them equal.	$72 \cdot n = 100 \cdot 9$
Simplify.	$72n = 900$
Divide both sides by 72.	$\frac{72n}{72} = \frac{900}{72}$
Simplify.	$n = 12.5$
Check if the answer is reasonable.	
Yes. 9 is $\frac{1}{8}$ of 72 and $\frac{1}{8}$ is 12.5%.	
Write a complete sentence that answers the question.	12.5% of 72 is 9.

Try It 6.107

Translate and solve using proportions: What percent of 72 is 27?

Try It 6.108

Translate and solve using proportions: What percent of 92 is 23?

Section 6.5 Exercises

Practice Makes Perfect

Use the Definition of Proportion

In the following exercises, write each sentence as a proportion.

243.

4 is to 15 as 36 is to 135.

244.

7 is to 9 as 35 is to 45.

245.

12 is to 5 as 96 is to 40.

246.

15 is to 8 as 75 is to 40.

247.

5 wins in 7 games is the same as 115 wins in 161 games.

248.

4 wins in 9 games is the same as 36 wins in 81 games.

249.

8 campers to 1 counselor is the same as 48 campers to 6 counselors.

250.

6 campers to 1 counselor is the same as 48 campers to 8 counselors.

251.

$\$9.36$ for 18 ounces is the same as $\$2.60$ for 5 ounces.

252.

$\$3.92$ for 8 ounces is the same as $\$1.47$ for 3 ounces.

253.

$\$18.04$ for 11 pounds is the same as $\$4.92$ for 3 pounds.

254.

$\$12.42$ for 27 pounds is the same as $\$5.52$ for 12 pounds.

In the following exercises, determine whether each equation is a proportion.

255.

$\frac{7}{15} = \frac{56}{120}$

256.

$\frac{5}{12} = \frac{45}{108}$

257.

$\frac{11}{6} = \frac{21}{16}$

258.

$\frac{9}{4} = \frac{39}{34}$

259.

$\frac{12}{18} = \frac{4.99}{7.56}$

260.

$\frac{9}{16} = \frac{2.16}{3.89}$

261.

$\frac{13.5}{8.5} = \frac{31.05}{19.55}$

262.

$\frac{10.1}{8.4} = \frac{3.03}{2.52}$

Solve Proportions

In the following exercises, solve each proportion.

263.

$\frac{x}{56} = \frac{7}{8}$

264.

$\frac{n}{91} = \frac{8}{13}$

265.

$\frac{49}{63} = \frac{z}{9}$

266.

$\frac{56}{72} = \frac{y}{9}$

267.

$\frac{5}{a} = \frac{65}{117}$

268.

$\frac{4}{b} = \frac{64}{144}$

269.

$\frac{98}{154} = \frac{-7}{p}$

270.

$\frac{72}{156} = \frac{-6}{q}$

271.

$\frac{a}{-8} = \frac{-42}{48}$

272.

$\frac{b}{-7} = \frac{-30}{42}$

273.

$\frac{2.6}{3.9} = \frac{c}{3}$

274.

$\frac{2.7}{3.6} = \frac{d}{4}$

275.

$\frac{2.7}{j} = \frac{0.9}{0.2}$

276.

$\frac{2.8}{k} = \frac{2.1}{1.5}$

277.

$\frac{\frac{1}{2}}{1} = \frac{m}{8}$

278.

$\frac{\frac{1}{3}}{3} = \frac{9}{n}$

Solve Applications Using Proportions

In the following exercises, solve the proportion problem.

279.

Pediatricians prescribe 5 milliliters (ml) of acetaminophen for every 25 pounds of a child's weight. How many milliliters of acetaminophen will the doctor prescribe for Jocelyn, who weighs 45 pounds?

280.

Brianna, who weighs 6 kg, just received her shots and needs a pain killer. The pain killer is prescribed for children at 15 milligrams (mg) for every 1 kilogram (kg) of the child's weight. How many milligrams will the doctor prescribe?

281.

At the gym, Carol takes her pulse for 10 sec and counts 19 beats. How many beats per minute is this? Has Carol met her target heart rate of 140 beats per minute?

282.

Kevin wants to keep his heart rate at 160 beats per minute while training. During his workout he counts 27 beats in 10 seconds. How many beats per minute is this? Has Kevin met his target heart rate?

283.

A new energy drink advertises 106 calories for 8 ounces. How many calories are in 12 ounces of the drink?

284.

One 12 ounce can of soda has 150 calories. If Josiah drinks the big 32 ounce size from the local mini-mart, how many calories does he get?

285.

Karen eats $\frac{1}{2}$ cup of oatmeal that counts for 2 points on her weight loss program. Her husband, Joe, can have 3 points of oatmeal for breakfast. How much oatmeal can he have?

286.

An oatmeal cookie recipe calls for $\frac{1}{2}$ cup of butter to make 4 dozen cookies. Hilda needs to make 10 dozen cookies for the bake sale. How many cups of butter will she need?

287.

Janice is traveling to Canada and will change $\$250$ US dollars into Canadian dollars. At the current exchange rate, $\$1$ US is equal to $\$1.01$ Canadian. How many Canadian dollars will she get for her trip?

288.

Todd is traveling to Mexico and needs to exchange $\$450$ into Mexican pesos. If each dollar is worth 12.29 pesos, how many pesos will he get for his trip?

289.

Steve changed $\$600$ into 480 Euros. How many Euros did he receive per US dollar?

290.

Martha changed $\$350$ US into 385 Australian dollars. How many Australian dollars did she receive per US dollar?

291.

At the laundromat, Lucy changed $\$12.00$ into quarters. How many quarters did she get?

292.

When she arrived at a casino, Gerty changed $20 into nickels. How many nickels did she get?

293.

Jesse's car gets 30 miles per gallon of gas. If Las Vegas is 285 miles away, how many gallons of gas are needed to get there and then home? If gas is $3.09 per gallon, what is the total cost of the gas for the trip?

294.

Danny wants to drive to Phoenix to see his grandfather. Phoenix is 370 miles from Danny's home and his car gets 18.5 miles per gallon. How many gallons of gas will Danny need to get to and from Phoenix? If gas is $3.19 per gallon, what is the total cost for the gas to drive to see his grandfather?

295.

Hugh leaves early one morning to drive from his home in Chicago to go to Mount Rushmore, 812 miles away. After 3 hours, he has gone 190 miles. At that rate, how long will the whole drive take?

296.

Kelly leaves her home in Seattle to drive to Spokane, a distance of 280 miles. After 2 hours, she has gone 152 miles. At that rate, how long will the whole drive take?

297.

Phil wants to fertilize his lawn. Each bag of fertilizer covers about 4,000 square feet of lawn. Phil's lawn is approximately 13,500 square feet. How many bags of fertilizer will he have to buy?

298.

April wants to paint the exterior of her house. One gallon of paint covers about 350 square feet, and the exterior of the house measures approximately 2000 square feet. How many gallons of paint will she have to buy?

Write Percent Equations as Proportions

In the following exercises, translate to a proportion.

299.

What number is 35% of 250?

300.

What number is 75% of 920?

301.

What number is 110% of 47?

302.

What number is 150% of 64?

303.

45 is 30% of what number?

304.

25 is 80% of what number?

305.

90 is 150% of what number?

306.

77 is 110% of what number?

307.

What percent of 85 is 17?

308.

What percent of 92 is 46?

309.

What percent of 260 is 340?

310.

What percent of 180 is 220?

Translate and Solve Percent Proportions

In the following exercises, translate and solve using proportions.

311.

What number is 65% of 180?

312.

What number is 55% of 300?

313.

18% of 92 is what number?

314.

22% of 74 is what number?

315.

175% of 26 is what number?

316.

250% of 61 is what number?

317.

What is 300% of 488?

318.

What is 500% of 315?

319.

17% of what number is $\$7.65$?

320.

19% of what number is $\$6.46$?

321.

$\$13.53$ is 8.25% of what number?

322.

$\$18.12$ is 7.55% of what number?

323.

What percent of 56 is 14?

324.

What percent of 80 is 28?

325.

What percent of 96 is 12?

326.

What percent of 120 is 27?

Everyday Math

327.

Mixing a concentrate Sam bought a large bottle of concentrated cleaning solution at the warehouse store. He must mix the concentrate with water to make a solution for washing his windows. The directions tell him to mix 3 ounces of concentrate with 5 ounces of water. If he puts 12 ounces of concentrate in a bucket, how many ounces of water should he add? How many ounces of the solution will he have altogether?

328.

Mixing a concentrate Travis is going to wash his car. The directions on the bottle of car wash concentrate say to mix 2 ounces of concentrate with 15 ounces of water. If Travis puts 6 ounces of concentrate in a bucket, how much water must he mix with the concentrate?

Writing Exercises

329.

To solve "what number is 45% of 350" do you prefer to use an equation like you did in the section on Decimal Operations or a proportion like you did in this section? Explain your reason.

330.

To solve "what percent of 125 is 25" do you prefer to use an equation like you did in the section on Decimal Operations or a proportion like you did in this section? Explain your reason.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
use definition of proportion.			
solve proportions.			
solve applications using proportions.			
write percent equations as proportions.			
translate and solve percent proportions.			

ⓑ Overall, after looking at the checklist, do you think you are well-prepared for the next Chapter? Why or why not?

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CHAPTER OVERVIEW

6: Unit Conversions

The sections in this module will help you to learn about unit conversions. Your goal is to demonstrate mastery on each of the following objectives.

- **Objective 6.1 - Standard US Conversions:** Convert values in one unit of length, mass/weight, volume and time into another unit within the US system of units, with the aid of a calculator.
- **Objective 6.2 - Metric Conversions:** Convert values in one unit of length, mass/weight, volume and time into another unit within the metric system of units, with the aid of a calculator.
- **Objective 6.3 - Converting Between Standard US and Metric:** Convert values in one unit of length, mass/weight, volume and time into another unit between the metric system and the US system, with the aid of a calculator.

6.1: Systems of Measurement

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6.1: Systems of Measurement

Learning Objectives

By the end of this section, you will be able to:

- Make unit conversions in the U.S. system
- Use mixed units of measurement in the U.S. system
- Make unit conversions in the metric system
- Use mixed units of measurement in the metric system
- Convert between the U.S. and the metric systems of measurement
- Convert between Fahrenheit and Celsius temperatures

Be Prepared 7.13

Before you get started, take this readiness quiz.

Multiply: $4.29\,(1000)$.
If you missed this problem, review Example 5.18.

Be Prepared 7.14

Simplify: $\frac{30}{54}$.
If you missed this problem, review Example 4.20.

Be Prepared 7.15

Multiply: $\frac{7}{15} \cdot \frac{25}{28}$.
If you missed this problem, review Example 4.27.

In this section we will see how to convert among different types of units, such as feet to miles or kilograms to pounds. The basic idea in all of the unit conversions will be to use a form of 1, the multiplicative identity, to change the units but not the value of a quantity.

Make Unit Conversions in the U.S. System

There are two systems of measurement commonly used around the world. Most countries use the metric system. The United States uses a different system of measurement, usually called the U.S. system. We will look at the U.S. system first.

The U.S. system of measurement uses units of inch, foot, yard, and mile to measure length and pound and ton to measure weight. For capacity, the units used are cup, pint, quart and gallons. Both the U.S. system and the metric system measure time in seconds, minutes, or hours.

The equivalencies among the basic units of the U.S. system of measurement are listed in Table 7.2. The table also shows, in parentheses, the common abbreviations for each measurement.

U.S. System Units

Length	**Volume**
1 foot (ft) = 12 inches (in) 1 yard (yd) = 3 feet (ft) 1 mile (mi) = 5280 feet (ft)	3 teaspoons (t) = 1 tablespoon (T) 16 Tablespoons (T) = 1 cup (C) 1 cup (C) = 8 fluid ounces (fl oz) 1 pint (pt) = 2 cups (C) 1 quart (qt) = 2 pints (pt) 1 gallon (gal) = 4 quarts (qt)
Weight	**Time**

U.S. System Units	
1 pound (lb) = 16 ounces (oz) 1 ton = 2000 pounds (lb)	1 minute (min) = 60 seconds (s) 1 hour (h) = 60 minutes (min) 1 day = 24 hours (h) 1 week (wk) = 7 days 1 year (yr) = 365 days

Table 7.2

In many real-life applications, we need to convert between units of measurement. We will use the identity property of multiplication to do these conversions. We'll restate the Identity Property of Multiplication here for easy reference.

$$\text{For any real number } a, \qquad a \cdot 1 = a \qquad 1 \cdot a = a$$

To use the identity property of multiplication, we write 1 in a form that will help us convert the units. For example, suppose we want to convert inches to feet. We know that 1 foot is equal to 12 inches, so we can write 1 as the fraction $\frac{1 \text{ ft}}{12 \text{ in}}$. When we multiply by this fraction, we do not change the value but just change the units.

But $\frac{12 \text{ in}}{1 \text{ ft}}$ also equals 1. How do we decide whether to multiply by $\frac{1 \text{ ft}}{12 \text{ in}}$ or $\frac{12 \text{ in}}{1 \text{ ft}}$? We choose the fraction that will make the units we want to convert *from* divide out. For example, suppose we wanted to convert 60 inches to feet. If we choose the fraction that has inches in the denominator, we can eliminate the inches.

$$60 \cancel{\text{ in}} \cdot \frac{1 \text{ ft}}{12 \cancel{\text{ in}}} = 5 \text{ ft}$$

On the other hand, if we wanted to convert 5 feet to inches, we would choose the fraction that has feet in the denominator.

$$5 \text{ ft} \cdot \frac{12 \text{ in}}{1 \cancel{\text{ ft}}} = 60 \text{ in}$$

We treat the unit words like factors and 'divide out' common units like we do common factors.

How To

Make unit conversions.

1. Step 1. Multiply the measurement to be converted by 1; write 1 as a fraction relating the units given and the units needed.
2. Step 2. Multiply.
3. Step 3. Simplify the fraction, performing the indicated operations and removing the common units.

Example 7.44

Mary Anne is 66 inches tall. What is her height in feet?

Answer

Convert 66 inches into feet.	
Multiply the measurement to be converted by 1.	$66 \text{ inches} \cdot 1$
Write 1 as a fraction relating the units given and the units needed.	$66 \text{ inches} \cdot \frac{1 \text{ foot}}{12 \text{ inches}}$
Multiply.	$\frac{66 \text{ inches} \cdot 1 \text{ foot}}{12 \text{ inches}}$
Simplify the fraction.	$\frac{66 \cancel{\text{ inches}} \cdot 1 \text{ foot}}{12 \cancel{\text{ inches}}}$
	$\frac{66 \text{ feet}}{12}$
	5.5 feet

Notice that the when we simplified the fraction, we first divided out the inches.

Mary Anne is 5.5 feet tall.

Try It 7.87

Lexie is 30 inches tall. Convert her height to feet.

Try It 7.88

Rene bought a hose that is 18 yards long. Convert the length to feet.

When we use the Identity Property of Multiplication to convert units, we need to make sure the units we want to change from will divide out. Usually this means we want the conversion fraction to have those units in the denominator.

Example 7.45

Ndula, an elephant at the San Diego Safari Park, weighs almost 3.2 tons. Convert her weight to pounds.



Figure 7.5 (credit: Guldo Da Rozze, Flickr)

Answer

We will convert Math input error

	3.2 tons
Multiply the measurement to be converted by 1.	$3.2 \text{ tons} \cdot 1$
Write 1 as a fraction relating tons and pounds.	$3.2 \text{ tons} \cdot \frac{2000 \text{ lbs}}{1 \text{ ton}}$
Simplify.	$\frac{3.2 \cancel{\text{tons}} \cdot 2000 \text{ lbs}}{1 \cancel{\text{ton}}}$
Multiply.	6400 lbs
	Ndula weighs almost 6,400 pounds.

Try It 7.89

Arnold's SUV weighs about 4.3 tons. Convert the weight to pounds.

Try It 7.90

A cruise ship weighs $51{,}000$ tons. Convert the weight to pounds.

Sometimes to convert from one unit to another, we may need to use several other units in between, so we will need to multiply several fractions.

Example 7.46

Juliet is going with her family to their summer home. She will be away for 9 weeks. Convert the time to minutes.

Answer

To convert weeks into minutes, we will convert weeks to days, days to hours, and then hours to minutes. To do this, we will multiply by conversion factors of 1.

	9 weeks
Write 1 as $\frac{7 \text{ days}}{1 \text{ week}}, \frac{24 \text{ hours}}{1 \text{ day}}, \frac{60 \text{ minutes}}{1 \text{ hour}}$.	$\frac{9 \text{ wk}}{1} \cdot \frac{7 \text{ days}}{1 \text{ wk}} \cdot \frac{24 \text{ hr}}{1 \text{ day}} \cdot \frac{60 \text{ min}}{1 \text{ hr}}$
Cancel common units.	$\frac{9 \cancel{\text{wk}}}{1} \cdot \frac{7 \cancel{\text{days}}}{1 \cancel{\text{wk}}} \cdot \frac{24 \cancel{\text{hr}}}{1 \cancel{\text{day}}} \cdot \frac{60 \text{ min}}{1 \cancel{\text{hr}}}$
Multiply.	$\frac{9 \cdot 7 \cdot 24 \cdot 60 \text{ min}}{1 \cdot 1 \cdot 1 \cdot 1} = 90,720 \text{ min}$
	Juliet will be away for 90,720 minutes.

Try It 7.91

The distance between Earth and the moon is about $250{,}000$ miles. Convert this length to yards.

Try It 7.92

A team of astronauts spends 15 weeks in space. Convert the time to minutes.

Example 7.47

How many fluid ounces are in 1 gallon of milk?



Figure 7.6 (credit: www.bluewaikiki.com, Flickr)

Answer

Use conversion factors to get the right units: convert gallons to quarts, quarts to pints, pints to cups, and cups to fluid ounces.

	1 gallon
Multiply the measurement to be converted by 1.	$\frac{1 \text{ gal}}{1} \cdot \frac{4 \text{ qt}}{1 \text{ gal}} \cdot \frac{2 \text{ pt}}{1 \text{ qt}} \cdot \frac{2 \text{ C}}{1 \text{ pt}} \cdot \frac{8 \text{ fl oz}}{1 \text{ C}}$
Simplify.	$\frac{1 \cancel{\text{gal}}}{1} \cdot \frac{4 \cancel{\text{qt}}}{1 \cancel{\text{gal}}} \cdot \frac{2 \cancel{\text{pt}}}{1 \cancel{\text{qt}}} \cdot \frac{2 \cancel{\text{C}}}{1 \cancel{\text{pt}}} \cdot \frac{8 \text{ fl oz}}{1 \cancel{\text{C}}}$
Multiply.	$\frac{1 \cdot 4 \cdot 2 \cdot 2 \cdot 8 \text{ fl oz}}{1 \cdot 1 \cdot 1 \cdot 1 \cdot 1}$
Simplify.	128 fluid ounces
	There are 128 fluid ounces in a gallon.

Try It 7.93

How many cups are in 1 gallon?

Try It 7.94

How many teaspoons are in 1 cup?

Use Mixed Units of Measurement in the U.S. System

Performing arithmetic operations on measurements with mixed units of measures requires care. Be sure to add or subtract like units.

Example 7.48

Charlie bought three steaks for a barbecue. Their weights were 14 ounces, 1 pound 2 ounces, and 1 pound 6 ounces. How many total pounds of steak did he buy?



Figure 7.7 (credit: Helen Penjam, Flickr)

Answer

We will add the weights of the steaks to find the total weight of the steaks.

Add the ounces. Then add the pounds.	14 ounces 1 pound 2 ounces + 1 pound 6 ounces 2 pounds 22 ounces
Convert 22 ounces to pounds and ounces.	
Add the pounds.	2 pounds + 1 pound, 6 ounces 3 pounds, 6 ounces
	Charlie bought 3 pounds 6 ounces of steak.

Try It 7.95

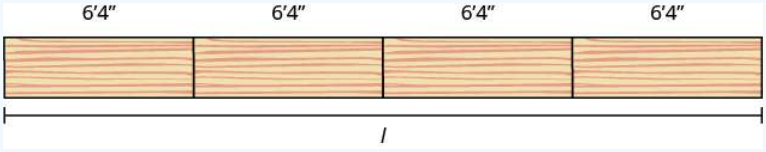
Laura gave birth to triplets weighing 3 pounds 12 ounces, 3 pounds 3 ounces, and 2 pounds 9 ounces. What was the total birth weight of the three babies?

Try It 7.96

Seymour cut two pieces of crown molding for his family room that were 8 feet 7 inches and 12 feet 11 inches. What was the total length of the molding?

Example 7.49

Anthony bought four planks of wood that were each 6 feet 4 inches long. If the four planks are placed end-to-end, what is the total length of the wood?



Answer

We will multiply the length of one plank by 4 to find the total length.

Multiply the inches and then the feet.	$\begin{array}{r} 6 \text{ feet} \quad 4 \text{ inches} \\ \times \qquad\qquad 4 \\ \hline 24 \text{ feet} \quad 16 \text{ inches} \end{array}$
Convert 16 inches to feet.	24 feet + 1 foot 4 inches
Add the feet.	25 feet 4 inches
	Anthony bought 25 feet 4 inches of wood.

Try It 7.97

Henri wants to triple his spaghetti sauce recipe, which calls for 1 pound 8 ounces of ground turkey. How many pounds of ground turkey will he need?

Try It 7.98

Joellen wants to double a solution of 5 gallons 3 quarts. How many gallons of solution will she have in all?

Make Unit Conversions in the Metric System

In the metric system, units are related by powers of 10. The root words of their names reflect this relation. For example, the basic unit for measuring length is a meter. One kilometer is 1000 meters; the prefix *kilo-* means thousand. One centimeter is $\frac{1}{100}$ of a meter, because the prefix *centi-* means one one-hundredth (just like one cent is $\frac{1}{100}$ of one dollar).

The equivalencies of measurements in the metric system are shown in Table 7.3. The common abbreviations for each measurement are given in parentheses.

Metric Measurements		
Length	Mass	Volume/Capacity
1 kilometer (km) = 1000 m 1 hectometer (hm) = 100 m 1 dekameter (dam) = 10 m 1 meter (m) = 1 m 1 decimeter (dm) = 0.1 m 1 centimeter (cm) = 0.01 m 1 millimeter (mm) = 0.001 m	1 kilogram (kg) = 1000 g 1 hectogram (hg) = 100 g 1 dekagram (dag) = 10 g 1 gram (g) = 1 g 1 decigram (dg) = 0.1 g 1 centigram (cg) = 0.01 g 1 milligram (mg) = 0.001 g	1 kiloliter (kL) = 1000 L 1 hectoliter (hL) = 100 L 1 dekaliter (daL) = 10 L 1 liter (L) = 1 L 1 deciliter (dL) = 0.1 L 1 centiliter (cL) = 0.01 L 1 milliliter (mL) = 0.001 L
1 meter = 100 centimeters 1 meter = 1000 millimeters	1 gram = 100 centigrams 1 gram = 1000 milligrams	1 liter = 100 centiliters 1 liter = 1000 milliliters

Table 7.3

To make conversions in the metric system, we will use the same technique we did in the U.S. system. Using the identity property of multiplication, we will multiply by a conversion factor of one to get to the correct units.

Have you ever run a 5 k or 10 k race? The lengths of those races are measured in kilometers. The metric system is commonly used in the United States when talking about the length of a race.

Example 7.50

Nick ran a 10-kilometer race. How many meters did he run?



Figure 7.8 (credit: William Warby, Flickr)

Answer

We will convert kilometers to meters using the Identity Property of Multiplication and the equivalencies in Table 7.3.

	10 kilometers
Multiply the measurement to be converted by 1.	$10\text{ km} \cdot 1$
Write 1 as a fraction relating kilometers and meters.	$10\text{ km} \cdot \frac{1000\text{ m}}{1\text{ km}}$
Simplify.	$\frac{10\cancel{\text{km}} \cdot 1000\text{ m}}{1\cancel{\text{km}}}$
Multiply.	10,000 m
	Nick ran 10,000 meters.

Try It 7.99

Sandy completed her first 5-km race. How many meters did she run?

Try It 7.100

Herman bought a rug 2.5 meters in length. How many centimeters is the length?

Example 7.51

Eleanor's newborn baby weighed 3200 grams. How many kilograms did the baby weigh?

Answer

We will convert grams to kilograms.

	3200 grams
Multiply the measurement to be converted by 1.	$3200\text{ g} \cdot 1$
Write 1 as a fraction relating kilograms and grams.	$3200\text{ g} \cdot \frac{1\text{ kg}}{1000\text{ g}}$
Simplify.	$3200\cancel{\text{g}} \cdot \frac{1\text{ kg}}{1000\cancel{\text{g}}}$
Multiply.	$\frac{3200\text{ kilograms}}{1000}$
Divide.	3.2 kilograms
	The baby weighed 3.2 kilograms.

Try It 7.101

Kari's newborn baby weighed 2800 grams. How many kilograms did the baby weigh?

Try It 7.102

Anderson received a package that was marked 4500 grams. How many kilograms did this package weigh?

Since the metric system is based on multiples of ten, conversions involve multiplying by multiples of ten. In Decimal Operations, we learned how to simplify these calculations by just moving the decimal.

To multiply by $10, 100, \text{or } 1000,$ we move the decimal to the right $1, 2, \text{or } 3$ places, respectively. To multiply by $0.1, 0.01, \text{or } 0.001$ we move the decimal to the left $1, 2, \text{or } 3$ places respectively.

We can apply this pattern when we make measurement conversions in the metric system.

In Example 7.51, we changed 3200 grams to kilograms by multiplying by $\frac{1}{1000}$ $(\text{or } 0.001)$. This is the same as moving the decimal 3 places to the left.

$3200 \cdot \frac{1}{1000}$ $\qquad$ $3200.$

3.2 $\qquad$ 3.2

Example 7.52

Convert:

1. ⓐ 350 liters to kiloliters
2. ⓑ 4.1 liters to milliliters.

Answer

ⓐ We will convert liters to kiloliters. In Table 7.3, we see that $1 \text{ kiloliter} = 1000 \text{ liters}.$

	350 L
Multiply by 1, writing 1 as a fraction relating liters to kiloliters.	$350 \text{ L} \cdot \frac{1 \text{ kL}}{1000 \text{ L}}$
Simplify.	$350 \cancel{\text{L}} \cdot \frac{1 \text{ kL}}{1000 \cancel{\text{L}}}$
Move the decimal 3 units to the left.	$350 \cancel{\text{L}} \cdot \frac{1 \text{ kL}}{1000 \cancel{\text{L}}}$
	0.35 kL

ⓑ We will convert liters to milliliters. In Table 7.3, we see that $1 \text{ liter} = 1000 \text{ milliliters}.$

	4.1 L
Multiply by 1, writing 1 as a fraction relating milliliters to liters.	$4.1 \text{ L} \cdot \frac{1000 \text{ mL}}{1 \text{ L}}$
Simplify.	$4.1 \cancel{\text{L}} \cdot \frac{1000 \text{ mL}}{1 \cancel{\text{L}}}$
Move the decimal 3 units to the left.	4.100 mL
	4100 mL

Try It 7.103

Convert: ⓐ 7.25 L to kL ⓑ 6.3 L to mL.

Try It 7.104

Convert: ⓐ 350 hL to L ⓑ 4.1 L to cL.

Use Mixed Units of Measurement in the Metric System

Performing arithmetic operations on measurements with mixed units of measures in the metric system requires the same care we used in the U.S. system. But it may be easier because of the relation of the units to the powers of 10. We still must make sure to add or subtract like units.

Example 7.53

Ryland is 1.6 meters tall. His younger brother is 85 centimeters tall. How much taller is Ryland than his younger brother?

Answer

We will subtract the lengths in meters. Convert 85 centimeters to meters by moving the decimal 2 places to the left; 85 cm is the same as 0.85 m.

Now that both measurements are in meters, subtract to find out how much taller Ryland is than his brother.

$$\begin{array}{r} 1.60\text{ m} \\ -0.85\text{ m} \\ \hline 0.75\text{ m} \end{array}$$

Ryland is 0.75 meters taller than his brother.

Try It 7.105

Mariella is 1.58 meters tall. Her daughter is 75 centimeters tall. How much taller is Mariella than her daughter? Write the answer in centimeters.

Try It 7.106

The fence around Hank's yard is 2 meters high. Hank is 96 centimeters tall. How much shorter than the fence is Hank? Write the answer in meters.

Example 7.54

Dena's recipe for lentil soup calls for 150 milliliters of olive oil. Dena wants to triple the recipe. How many liters of olive oil will she need?

Answer

We will find the amount of olive oil in milliliters then convert to liters.

	Triple 150 mL
Translate to algebra.	$3 \cdot 150\text{ mL}$
Multiply.	450 mL
Convert to liters.	$450\text{ mL} \cdot \frac{0.001\text{ L}}{1\text{ mL}}$
Simplify.	0.45 L
	Dena needs 0.45 liter of olive oil.

Try It 7.107

A recipe for Alfredo sauce calls for 250 milliliters of milk. Renata is making pasta with Alfredo sauce for a big party and needs to multiply the recipe amounts by 8. How many liters of milk will she need?

Try It 7.108

To make one pan of baklava, Dorothea needs 400 grams of filo pastry. If Dorothea plans to make 6 pans of baklava, how many kilograms of filo pastry will she need?

Convert Between U.S. and Metric Systems of Measurement

Many measurements in the United States are made in metric units. A drink may come in 2-liter bottles, calcium may come in 500-mg capsules, and we may run a 5-K race. To work easily in both systems, we need to be able to convert between the two systems.

Table 7.4 shows some of the most common conversions.

Conversion Factors Between U.S. and Metric Systems		
Length	Weight	Volume
1 in = 2.54 cm 1 ft = 0.305 m 1 yd = 0.914 m 1 mi = 1.61 km 1 m = 3.28 ft	1 lb = 0.45 kg 1 oz = 28 g 1 kg = 2.2 lb	1 qt = 0.95 L 1 fl oz = 30 mL 1 L = 1.06 qt

Table 7.4

We make conversions between the systems just as we do within the systems—by multiplying by unit conversion factors.

Example 7.55

Lee's water bottle holds 500 mL of water. How many fluid ounces are in the bottle? Round to the nearest tenth of an ounce.

Answer

	500 mL
Multiply by a unit conversion factor relating mL and ounces.	$500\text{ mL} \cdot \frac{1\text{ fl oz}}{30\text{ mL}}$
Simplify.	$\frac{500\text{ fl oz}}{30}$
Divide.	16.7 fl. oz.
	The water bottle holds 16.7 fluid ounces.

Try It 7.109

How many quarts of soda are in a 2-liter bottle?

Try It 7.110

How many liters are in 4 quarts of milk?

The conversion factors in Table 7.4 are not exact, but the approximations they give are close enough for everyday purposes. In Example 7.55, we rounded the number of fluid ounces to the nearest tenth.

Example 7.56

Soleil lives in Minnesota but often travels in Canada for work. While driving on a Canadian highway, she passes a sign that says the next rest stop is in 100 kilometers. How many miles until the next rest stop? Round your answer to the nearest mile.

Answer

	100 kilometers
Multiply by a unit conversion factor relating kilometers and miles.	$100 \text{ kilometers} \cdot \frac{1 \text{ mile}}{1.61 \text{ kilometers}}$ $100 \cdot \frac{1 \text{ mi}}{1.61 \text{ km}}$
Simplify.	$\frac{100 \text{ mi}}{1.61}$
Divide.	62 mi
	It is about 62 miles to the next rest stop.

Try It 7.111

The height of Mount Kilimanjaro is $5{,}895$ meters. Convert the height to feet. Round to the nearest foot.

Try It 7.112

The flight distance from New York City to London is $5{,}586$ kilometers. Convert the distance to miles. Round to the nearest mile.

Convert Between Fahrenheit and Celsius Temperatures

Have you ever been in a foreign country and heard the weather forecast? If the forecast is for $22°\text{C}$. What does that mean?

The U.S. and metric systems use different scales to measure temperature. The U.S. system uses degrees Fahrenheit, written Math input error

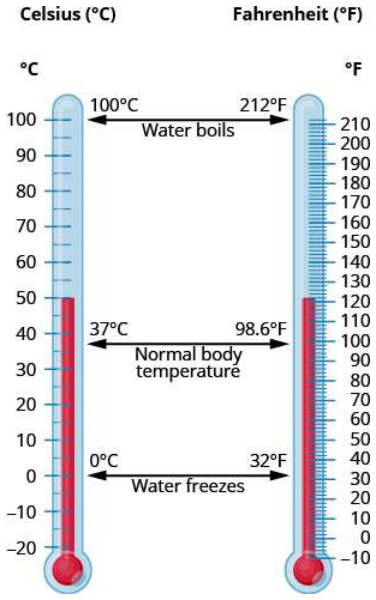


Figure 7.9 A temperature of $37°\text{C}$ is equivalent to $98.6°\text{F}$.

If we know the temperature in one system, we can use a formula to convert it to the other system.

Temperature Conversion

To convert from Fahrenheit temperature, F, to Celsius temperature, C, use the formula

$$C = \frac{5}{9}(F - 32)$$

To convert from Celsius temperature, C, to Fahrenheit temperature, F, use the formula

$$F = \frac{9}{5}C + 32$$

Example 7.57

Convert $50°F$ into degrees Celsius.

Answer

We will substitute $50°F$ into the formula to find C.

Use the formula for converting °F to °C	$C = \frac{5}{9}(F - 32)$
Substitute 50 for F.	$C = \frac{5}{9}(50 - 32)$
Simplify in parentheses.	$C = \frac{5}{9}(18)$
Multiply.	$C = 10$
	A temperature of 50°F is equivalent to 10°C.

Try It 7.113

Convert the Fahrenheit temperatures to degrees Celsius: $59°F$.

Try It 7.114

Convert the Fahrenheit temperatures to degrees Celsius: $41°F$.

Example 7.58

The weather forecast for Paris predicts a high of $20°C$. Convert the temperature into degrees Fahrenheit.

Answer

We will substitute $20°C$ into the formula to find F.

Use the formula for converting °F to °C	$F = \frac{9}{5}C + 32$
Substitute 20 for C.	$F = \frac{9}{5}(20) + 32$
Multiply.	$F = 36 + 32$
Add.	$F = 68$
	So 20°C is equivalent to 68°F.

Try It 7.115

Convert the Celsius temperatures to degrees Fahrenheit:

The temperature in Helsinki, Finland was $15°C$.

Try It 7.116

Convert the Celsius temperatures to degrees Fahrenheit:

The temperature in Sydney, Australia was $10°C$.

Media

ACCESS ADDITIONAL ONLINE RESOURCES

- American Unit Conversion
- Time Conversions
- Metric Unit Conversions
- American and Metric Conversions
- Convert from Celsius to Fahrenheit
- Convert from Fahrenheit to Celsius

Section 7.5 Exercises

Practice Makes Perfect

Make Unit Conversions in the U.S. System

In the following exercises, convert the units.

214.

A park bench is 6 feet long. Convert the length to inches.

215.

A floor tile is 2 feet wide. Convert the width to inches.

216.

A ribbon is 18 inches long. Convert the length to feet.

217.

Carson is 45 inches tall. Convert his height to feet.

218.

Jon is 6 feet 4 inches tall. Convert his height to inches.

219.

Faye is 4 feet 10 inches tall. Convert her height to inches.

220.

A football field is 160 feet wide. Convert the width to yards.

221.

On a baseball diamond, the distance from home plate to first base is 30 yards. Convert the distance to feet.

222.

Ulises lives 1.5 miles from school. Convert the distance to feet.

223.

Denver, Colorado, is $5{,}183$ feet above sea level. Convert the height to miles.

224.

A killer whale weighs 4.6 tons. Convert the weight to pounds.

225.

Blue whales can weigh as much as 150 tons. Convert the weight to pounds.

226.

An empty bus weighs $35{,}000$ pounds. Convert the weight to tons.

227.

At take-off, an airplane weighs $220{,}000$ pounds. Convert the weight to tons.

228.

The voyage of the *Mayflower* took 2 months and 5 days. Convert the time to days (30 days = 1 month).

229.

Lynn's cruise lasted 6 days and 18 hours. Convert the time to hours.

230.

Rocco waited $1\frac{1}{2}$ hours for his appointment. Convert the time to seconds.

231.

Misty's surgery lasted $2\frac{1}{4}$ hours. Convert the time to seconds.

232.

How many teaspoons are in a pint?

233.

How many tablespoons are in a gallon?

234.

JJ's cat, Posy, weighs 14 pounds. Convert her weight to ounces.

235.

April's dog, Beans, weighs 8 pounds. Convert his weight to ounces.

236.

Baby Preston weighed 7 pounds 3 ounces at birth. Convert his weight to ounces.

237.

Baby Audrey weighed 6 pounds 15 ounces at birth. Convert her weight to ounces.

238.

Crista will serve 20 cups of juice at her son's party. Convert the volume to gallons.

239.

Lance needs 500 cups of water for the runners in a race. Convert the volume to gallons.

Use Mixed Units of Measurement in the U.S. System

In the following exercises, solve and write your answer in mixed units.

240.

Eli caught three fish. The weights of the fish were 2 pounds 4 ounces, 1 pound 11 ounces, and 4 pounds 14 ounces. What was the total weight of the three fish?

241.

Judy bought 1 pound 6 ounces of almonds, 2 pounds 3 ounces of walnuts, and 8 ounces of cashews. What was the total weight of the nuts?

242.

One day Anya kept track of the number of minutes she spent driving. She recorded trips of $45, 10, 8, 65, 20, \text{and } 35$ minutes. How much time (in hours and minutes) did Anya spend driving?

243.

Last year Eric went on 6 business trips. The number of days of each was $5, 2, 8, 12, 6, \text{and } 3$. How much time (in weeks and days) did Eric spend on business trips last year?

244.

Renee attached a 6-foot-6-inch extension cord to her computer's 3-foot-8-inch power cord. What was the total length of the cords?

245.

Fawzi's SUV is 6 feet 4 inches tall. If he puts a 2-foot-10-inch box on top of his SUV, what is the total height of the SUV and the box?

246.

Leilani wants to make 8 placemats. For each placemat she needs 18 inches of fabric. How many yards of fabric will she need for the 8 placemats?

247.

Mireille needs to cut 24 inches of ribbon for each of the 12 girls in her dance class. How many yards of ribbon will she need altogether?

Make Unit Conversions in the Metric System

In the following exercises, convert the units.

248.

Ghalib ran 5 kilometers. Convert the length to meters.

249.

Kitaka hiked 8 kilometers. Convert the length to meters.

250.

Estrella is 1.55 meters tall. Convert her height to centimeters.

251.

The width of the wading pool is 2.45 meters. Convert the width to centimeters.

252.

Mount Whitney is $3{,}072$ meters tall. Convert the height to kilometers.

253.

The depth of the Mariana Trench is $10{,}911$ meters. Convert the depth to kilometers.

254.

June's multivitamin contains $1{,}500$ milligrams of calcium. Convert this to grams.

255.

A typical ruby-throated hummingbird weights 3 grams. Convert this to milligrams.

256.

One stick of butter contains 91.6 grams of fat. Convert this to milligrams.

257.

One serving of gourmet ice cream has 25 grams of fat. Convert this to milligrams.

258.

The maximum mass of an airmail letter is 2 kilograms. Convert this to grams.

259.

Dimitri's daughter weighed 3.8 kilograms at birth. Convert this to grams.

260.

A bottle of wine contained 750 milliliters. Convert this to liters.

261.

A bottle of medicine contained 300 milliliters. Convert this to liters.

Use Mixed Units of Measurement in the Metric System

In the following exercises, solve and write your answer in mixed units.

262.

Matthias is 1.8 meters tall. His son is 89 centimeters tall. How much taller, in centimeters, is Matthias than his son?

263.

Stavros is 1.6 meters tall. His sister is 95 centimeters tall. How much taller, in centimeters, is Stavros than his sister?

264.

A typical dove weighs 345 grams. A typical duck weighs 1.2 kilograms. What is the difference, in grams, of the weights of a duck and a dove?

265.

Concetta had a 2-kilogram bag of flour. She used 180 grams of flour to make biscotti. How many kilograms of flour are left in the bag?

266.

Harry mailed 5 packages that weighed 420 grams each. What was the total weight of the packages in kilograms?

267.

One glass of orange juice provides 560 milligrams of potassium. Linda drinks one glass of orange juice every morning. How many grams of potassium does Linda get from her orange juice in 30 days?

268.

Jonas drinks 200 milliliters of water 8 times a day. How many liters of water does Jonas drink in a day?

269.

One serving of whole grain sandwich bread provides 6 grams of protein. How many milligrams of protein are provided by 7 servings of whole grain sandwich bread?

Convert Between U.S. and Metric Systems

In the following exercises, make the unit conversions. Round to the nearest tenth.

270.

Bill is 75 inches tall. Convert his height to centimeters.

271.

Frankie is 42 inches tall. Convert his height to centimeters.

272.

Marcus passed a football 24 yards. Convert the pass length to meters.

273.

Connie bought 9 yards of fabric to make drapes. Convert the fabric length to meters.

274.

Each American throws out an average of $1{,}650$ pounds of garbage per year. Convert this weight to kilograms (2.20 pounds = 1 kilogram).

275.

An average American will throw away $90{,}000$ pounds of trash over his or her lifetime. Convert this weight to kilograms (2.20 pounds = 1 kilogram).

276.

A 5K run is 5 kilometers long. Convert this length to miles.

277.

Kathryn is 1.6 meters tall. Convert her height to feet.

278.

Dawn's suitcase weighed 20 kilograms. Convert the weight to pounds.

279.

Jackson's backpack weighs 15 kilograms. Convert the weight to pounds.

280.

Ozzie put 14 gallons of gas in his truck. Convert the volume to liters.

281.

Bernard bought 8 gallons of paint. Convert the volume to liters.

Convert between Fahrenheit and Celsius

In the following exercises, convert the Fahrenheit temperature to degrees Celsius. Round to the nearest tenth.

282.

$86°\text{F}$

283.

$77°\text{F}$

284.

$104°\text{F}$

285.

$14°\text{F}$

286.

$72°\text{F}$

287.

$4°\text{F}$

288.

$0°\text{F}$

289.

$120°\text{F}$

In the following exercises, convert the Celsius temperatures to degrees Fahrenheit. Round to the nearest tenth.

290.

$5°\text{C}$

291.

$25°\text{C}$

292.

$-10°\text{C}$

293.

$-15°\text{C}$

294.

$22°\text{C}$

295.

$8°\text{C}$

296.

$43°\text{C}$

297.

$16°C$

Everyday Math

298.

Nutrition Julian drinks one can of soda every day. Each can of soda contains 40 grams of sugar. How many kilograms of sugar does Julian get from soda in 1 year?

299.

Reflectors The reflectors in each lane-marking stripe on a highway are spaced 16 yards apart. How many reflectors are needed for a one-mile-long stretch of highway?

Writing Exercises

300.

Some people think that $65°$ to $75°$ Fahrenheit is the ideal temperature range.

1. ⓐ What is your ideal temperature range? Why do you think so?
2. ⓑ Convert your ideal temperatures from Fahrenheit to Celsius.

301.

ⓐ Did you grow up using the U.S. customary or the metric system of measurement? ⓑ Describe two examples in your life when you had to convert between systems of measurement. ⓒ Which system do you think is easier to use? Explain.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
make unit conversions in the U.S. system.			
use mixed units of measurement in the U.S. system.			
use mixed units of measurement in the metric system.			
convert between the U.S. and the metric systems of measurement.			
convert between Fahrenheit and Celsius temperatures.			

ⓑ Overall, after looking at the checklist, do you think you are well-prepared for the next chapter? Why or why not?

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CHAPTER OVERVIEW

7: Roots, Exponents, and Scientific Notation

The sections in this module will help you to learn about exponents and roots. Your goal is to demonstrate mastery on each of the following objectives.

- **Objective 7.1 - Roots for Signed Numbers:** Simplify and estimate root values for whole numbers and integers and demonstrate the relationship between roots and exponents.
- **Objective 7.2 - Exponents for Signed Numbers:** Calculate values for expressions involving integer exponents.
- **Objective 7.3 -: Writing Numbers in Scientific Notation:** Convert between numbers written in decimal form and scientific notation.
- **Objective 7.4 - Operations using Scientific Notation:** Perform addition, subtraction, multiplication, and division involving scientific notation, including applications, with the aid of a calculator.

7.1: Simplify and Use Square Roots
7.2: Integer Exponents and Scientific Notation (Part 1)
7.3: Integer Exponents and Scientific Notation (Part 2)

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7.1: Simplify and Use Square Roots

Learning Objectives

By the end of this section, you will be able to:

- Simplify expressions with square roots
- Estimate square roots
- Approximate square roots
- Simplify variable expressions with square roots
- Use square roots in applications

Be Prepared 5.19

Before you get started, take this readiness quiz.

Simplify: $(-9)^2$.
If you missed this problem, review Example 3.52.

Be Prepared 5.20

Round 3.846 to the nearest hundredth.
If you missed this problem, review Example 5.9.

Be Prepared 5.21

Evaluate $12d$ for $d = 80$.
If you missed this problem, review Example 2.14.

Simplify Expressions with Square Roots

To start this section, we need to review some important vocabulary and notation.

Remember that when a number n is multiplied by itself, we can write this as n^2, which we read aloud as "n squared." For example, 8^2 is read as "8 squared."

We call 64 the *square* of 8 because $8^2 = 64$. Similarly, 121 is the square of 11, because $11^2 = 121$.

Square of a Number

If $n^2 = m$, then m is the square of n.

Modeling Squares

Do you know why we use the word *square*? If we construct a square with three tiles on each side, the total number of tiles would be nine.

This is why we say that the square of three is nine.

$$3^2 = 9$$

The number 9 is called a perfect square because it is the square of a whole number.

Manipulative Mathematics

Doing the Manipulative Mathematics activity Square Numbers will help you develop a better understanding of perfect square numbers

The chart shows the squares of the counting numbers 1 through 15. You can refer to it to help you identify the perfect squares.

Number	n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Square	n^2	1	4	9	16	25	36	49	64	81	100	121	144	169	196	225

Perfect Squares

A perfect square is the square of a whole number.

What happens when you square a negative number?

$$\begin{aligned}(-8)^2 &= (-8)(-8)\\ &= 64\end{aligned}$$

When we multiply two negative numbers, the product is always positive. So, the square of a negative number is always positive.

The chart shows the squares of the negative integers from -1 to -15.

Number	n	–1	–2	–3	–4	–5	–6	–7	–8	–9	–10	–11	–12	–13	–14	–15
Square	n^2	1	4	9	16	25	36	49	64	81	100	121	144	169	196	225

Did you notice that these squares are the same as the squares of the positive numbers?

Square Roots

Sometimes we will need to look at the relationship between numbers and their squares in reverse. Because $10^2 = 100$, we say 100 is the square of 10. We can also say that 10 is a square root of 100.

Square Root of a Number

A number whose square is m is called a square root of m.

If $n^2 = m$, then n is a square root of m.

Notice $(-10)^2 = 100$ also, so -10 is also a square root of 100. Therefore, both 10 and -10 are square roots of 100.

So, every positive number has two square roots: one positive and one negative.

What if we only want the positive square root of a positive number? The *radical sign*, $\sqrt{\ }$, stands for the positive square root. The positive square root is also called the principal square root.

Square Root Notation

$\sqrt{m}$ is read as "the square root of m."

If $m = n^2$, then $\sqrt{m} = n$ for $n \geq 0$.



We can also use the radical sign for the square root of zero. Because $0^2 = 0$, $\sqrt{0} = 0$. Notice that zero has only one square root.

The chart shows the square roots of the first 15 perfect square numbers.

$\sqrt{1}$	$\sqrt{4}$	$\sqrt{9}$	$\sqrt{16}$	$\sqrt{25}$	$\sqrt{36}$	$\sqrt{49}$	$\sqrt{64}$	$\sqrt{81}$	$\sqrt{100}$	$\sqrt{121}$	$\sqrt{144}$	$\sqrt{169}$	$\sqrt{196}$	$\sqrt{225}$
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Example 5.69

Simplify: ⓐ $\sqrt{25}$ ⓑ $\sqrt{121}$.

Answer

ⓐ

	$\sqrt{25}$
Since $5^2 = 25$	5

ⓑ	
	$\sqrt{121}$
Since $11^2 = 121$	11

Try It 5.137

Simplify: ⓐ $\sqrt{36}$ ⓑ $\sqrt{169}$.

Try It 5.138

Simplify: ⓐ $\sqrt{16}$ ⓑ $\sqrt{196}$.

Every positive number has two square roots and the radical sign indicates the positive one. We write $\sqrt{100} = 10$. If we want to find the negative square root of a number, we place a negative in front of the radical sign. For example, $-\sqrt{100} = -10$.

Example 5.70

Simplify. ⓐ $-\sqrt{9}$ ⓑ $-\sqrt{144}$.

Answer

ⓐ	
	$-\sqrt{9}$
The negative is in front of the radical sign.	-3

ⓑ	
	$-\sqrt{144}$
The negative is in front of the radical sign.	-12

Try It 5.139

Simplify: ⓐ $-\sqrt{4}$ ⓑ $-\sqrt{225}$.

Try It 5.140

Simplify: ⓐ $-\sqrt{81}$ ⓑ $-\sqrt{64}$.

Square Root of a Negative Number

Can we simplify $\sqrt{-25}$? Is there a number whose square is -25?

$$(\quad)^2 = -25?$$

None of the numbers that we have dealt with so far have a square that is -25. Why? Any positive number squared is positive, and any negative number squared is also positive. In the next chapter we will see that all the numbers we work with are called the real numbers. So we say there is no real number equal to $\sqrt{-25}$. If we are asked to find the square root of any negative number, we say that the solution is not a real number.

Example 5.71

Simplify: ⓐ $\sqrt{-169}$ ⓑ $-\sqrt{121}$.

Answer

ⓐ There is no real number whose square is -169. Therefore, $\sqrt{-169}$ is not a real number.

ⓑ The negative is in front of the radical sign, so we find the opposite of the square root of 121.

	$-\sqrt{121}$
The negative is in front of the radical.	-11

Try It 5.141

Simplify: ⓐ $\sqrt{-196}$ ⓑ $-\sqrt{81}$.

Try It 5.142

Simplify: ⓐ $\sqrt{-49}$ ⓑ $-\sqrt{121}$.

Square Roots and the Order of Operations

When using the order of operations to simplify an expression that has square roots, we treat the radical sign as a grouping symbol. We simplify any expressions under the radical sign before performing other operations.

Example 5.72

Simplify: ⓐ $\sqrt{25}+\sqrt{144}$ ⓑ $\sqrt{25+144}$.

Answer

ⓐ Use the order of operations.	
	$\sqrt{25}+\sqrt{144}$
Simplify each radical.	$5+12$
Add.	17

ⓑ Use the order of operations.	
	$\sqrt{25+144}$
Add under the radical sign.	$\sqrt{169}$
Simplify.	13

Try It 5.143

Simplify: ⓐ $\sqrt{9}+\sqrt{16}$ ⓑ $\sqrt{9+16}$.

Try It 5.144

Simplify: ⓐ $\sqrt{64+225}$ ⓑ $\sqrt{64}+\sqrt{225}$.

Notice the different answers in parts ⓐ and ⓑ of Example 5.72. It is important to follow the order of operations correctly. In ⓐ, we took each square root first and then added them. In ⓑ, we added under the radical sign first and then found the square root.

Estimate Square Roots

So far we have only worked with square roots of perfect squares. The square roots of other numbers are not whole numbers.

Number	Square root
4	$\sqrt{4}=2$
5	$\sqrt{5}$
6	$\sqrt{6}$
7	$\sqrt{7}$
8	$\sqrt{8}$
9	$\sqrt{9}=3$

We might conclude that the square roots of numbers between 4 and 9 will be between 2 and 3, and they will not be whole numbers. Based on the pattern in the table above, we could say that $\sqrt{5}$ is between 2 and 3. Using inequality symbols, we write

$$2 < \sqrt{5} < 3$$

Example 5.73

Estimate $\sqrt{60}$ between two consecutive whole numbers.

Answer

Think of the perfect squares closest to 60. Make a small table of these perfect squares and their squares roots.

Number	Square root
36	6
49	7
64	8
81	9

60 — $\sqrt{60}$

Locate 60 between two consecutive perfect squares.	$49 < 60 < 64$
$\sqrt{60}$ is between their square roots.	$7 < \sqrt{60} < 8$

Try It 5.145

Estimate $\sqrt{38}$ between two consecutive whole numbers.

Try It 5.146

Estimate $\sqrt{84}$ between two consecutive whole numbers.

Approximate Square Roots with a Calculator

There are mathematical methods to approximate square roots, but it is much more convenient to use a calculator to find square roots. Find the $\sqrt{\ }$ or $\sqrt{x}$ key on your calculator. You will to use this key to approximate square roots. When you use your calculator to find the square root of a number that is not a perfect square, the answer that you see is not the exact number. It is an approximation, to the number of digits shown on your calculator's display. The symbol for an approximation is $\approx$ and it is read *approximately*.

Suppose your calculator has a 10-digit display. Using it to find the square root of 5 will give 2.236067977. This is the approximate square root of 5. When we report the answer, we should use the "approximately equal to" sign instead of an equal sign.

$$\sqrt{5} \approx 2.236067978$$

You will seldom use this many digits for applications in algebra. So, if you wanted to round $\sqrt{5}$ to two decimal places, you would write

$$\sqrt{5} \approx 2.24$$

How do we know these values are approximations and not the exact values? Look at what happens when we square them.

$$\begin{aligned} 2.236067978^2 &= 5.000000002 \\ 2.24^2 &= 5.0176 \end{aligned}$$

The squares are close, but not exactly equal, to 5.

Example 5.74

Round $\sqrt{17}$ to two decimal places using a calculator.

Answer

	$\sqrt{17}$
Use the calculator square root key.	4.123105626
Round to two decimal places.	4.12
	$\sqrt{17} \approx 4.12$

Try It 5.147

Round $\sqrt{11}$ to two decimal places.

Try It 5.148

Round $\sqrt{13}$ to two decimal places.

Simplify Variable Expressions with Square Roots

Expressions with square root that we have looked at so far have not had any variables. What happens when we have to find a square root of a variable expression?

Consider $\sqrt{9x^2}$, where $x \geq 0$. Can you think of an expression whose square is $9x^2$?

$$\begin{aligned} (?)^2 &= 9x^2 \\ (3x)^2 &= 9x^2 \qquad \text{so } \sqrt{9x^2} = 3x \end{aligned}$$

When we use a variable in a square root expression, for our work, we will assume that the variable represents a non-negative number. In every example and exercise that follows, each variable in a square root expression is greater than or equal to zero.

Example 5.75

Simplify: $\sqrt{x^2}$.

Answer

Think about what we would have to square to get x^2. Algebraically, $(?)^2 = x^2$

	$\sqrt{x^2}$
Since $(x)^2 = x^2$	x

Try It 5.149

Simplify: $\sqrt{y^2}$.

Try It 5.150

Simplify: $\sqrt{m^2}$.

Example 5.76

Simplify: $\sqrt{16x^2}$.

Answer

	$\sqrt{16x^2}$
Since $(4x)^2 = 16x^2$	$4x$

Try It 5.151

Simplify: $\sqrt{64x^2}$.

Try It 5.152

Simplify: $\sqrt{169y^2}$.

Example 5.77

Simplify: $-\sqrt{81y^2}$.

Answer

	$-\sqrt{81y^2}$
Since $(9y)^2 = 81y^2$	$-9y$

Try It 5.153

Simplify: $-\sqrt{121y^2}$.

Try It 5.154

Simplify: $-\sqrt{100p^2}$.

Example 5.78

Simplify: $\sqrt{36x^2y^2}$.

Answer

	$\sqrt{36x^2y^2}$
Since $(6xy)^2 = 36x^2y^2$	$6xy$

Try It 5.155

Simplify: $\sqrt{100a^2b^2}$.

Try It 5.156

Simplify: $\sqrt{225m^2n^2}$.

Use Square Roots in Applications

As you progress through your college courses, you'll encounter several applications of square roots. Once again, if we use our strategy for applications, it will give us a plan for finding the answer!

How To

Use a strategy for applications with square roots.

1. Step 1. Identify what you are asked to find.
2. Step 2. Write a phrase that gives the information to find it.
3. Step 3. Translate the phrase to an expression.
4. Step 4. Simplify the expression.
5. Step 5. Write a complete sentence that answers the question.

Square Roots and Area

We have solved applications with area before. If we were given the length of the sides of a square, we could find its area by squaring the length of its sides. Now we can find the length of the sides of a square if we are given the area, by finding the square root of the area.

If the area of the square is Math input error

Area (square units)	Length of side (units)
9	$\sqrt{9}=3$
144	$\sqrt{144}=12$
A	$\sqrt{A}$

Table 5.7

Example 5.79

Mike and Lychelle want to make a square patio. They have enough concrete for an area of 200 square feet. To the nearest tenth of a foot, how long can a side of their square patio be?

Answer

We know the area of the square is 200 square feet and want to find the length of the side. If the area of the square is A square units, the length of a side is $\sqrt{A}$ units.

What are you asked to find?	The length of each side of a square patio
Write a phrase.	The length of a side
Translate to an expression.	$\sqrt{A}$
Evaluate $\sqrt{A}$ when $A=200$.	$\sqrt{200}$
Use your calculator.	$14.142135\ldots$
Round to one decimal place.	14.1 feet
Write a sentence.	Each side of the patio should be 14.1 feet.

Try It 5.157

Katie wants to plant a square lawn in her front yard. She has enough sod to cover an area of 370 square feet. To the nearest tenth of a foot, how long can a side of her square lawn be?

Try It 5.158

Sergio wants to make a square mosaic as an inlay for a table he is building. He has enough tile to cover an area of 2704 square centimeters. How long can a side of his mosaic be?

Square Roots and Gravity

Another application of square roots involves gravity. On Earth, if an object is dropped from a height of h feet, the time in seconds it will take to reach the ground is found by evaluating the expression $\frac{\sqrt{h}}{4}$. For example, if an object is dropped from a height of 64 feet, we can find the time it takes to reach the ground by evaluating $\frac{\sqrt{64}}{4}$.

	$\frac{\sqrt{64}}{4}$
Take the square root of 64.	$\frac{8}{4}$
Simplify the fraction.	2

It would take 2 seconds for an object dropped from a height of 64 feet to reach the ground.

Example 5.80

Christy dropped her sunglasses from a bridge 400 feet above a river. How many seconds does it take for the sunglasses to reach the river?

Answer

What are you asked to find?	The number of seconds it takes for the sunglasses to reach the river
Write a phrase.	The time it will take to reach the river
Translate to an expression.	$\frac{\sqrt{h}}{4}$
Evaluate $\frac{\sqrt{h}}{4}$ when $h = 400$.	$\frac{\sqrt{400}}{4}$
Find the square root of 400.	$\frac{20}{4}$
Simplify.	5
Write a sentence.	It will take 5 seconds for the sunglasses to reach the river.

Try It 5.159

A helicopter drops a rescue package from a height of 1296 feet. How many seconds does it take for the package to reach the ground?

Try It 5.160

A window washer drops a squeegee from a platform 196 feet above the sidewalk. How many seconds does it take for the squeegee to reach the sidewalk?

Square Roots and Accident Investigations

Police officers investigating car accidents measure the length of the skid marks on the pavement. Then they use square roots to determine the speed, in miles per hour, a car was going before applying the brakes. According to some formulas, if the length of the skid marks is d feet, then the speed of the car can be found by evaluating $\sqrt{24d}$.

Example 5.81

After a car accident, the skid marks for one car measured 190 feet. To the nearest tenth, what was the speed of the car (in mph) before the brakes were applied?

Answer

What are you asked to find?	The speed of the car before the brakes were applied
Write a phrase.	The speed of the car

Translate to an expression.	$\sqrt{24d}$
Evaluate $\sqrt{24d}$ when $d = 190$.	$\sqrt{24 \cdot 190}$
Multiply.	$\sqrt{4{,}560}$
Use your calculator.	$67.527772\ldots$
Round to tenths.	67.5
Write a sentence.	The speed of the car was approximately 67.5 miles per hour.

Try It 5.161

An accident investigator measured the skid marks of a car and found their length was 76 feet. To the nearest tenth, what was the speed of the car before the brakes were applied?

Try It 5.162

The skid marks of a vehicle involved in an accident were 122 feet long. To the nearest tenth, how fast had the vehicle been going before the brakes were applied?

Links To Literacy

The *Links to Literacy* activity "Sea Squares" will provide you with another view of the topics covered in this section.

Media

ACCESS ADDITIONAL ONLINE RESOURCES

- Introduction to Square Roots
- Estimating Square Roots with a Calculator

Section 5.7 Exercises

Practice Makes Perfect

Simplify Expressions with Square Roots

In the following exercises, simplify.

489.

$\sqrt{36}$

490.

$\sqrt{4}$

491.

$\sqrt{64}$

492.

$\sqrt{144}$

493.

$-\sqrt{4}$

494.

$-\sqrt{100}$

495.

$-\sqrt{1}$

496.

$-\sqrt{121}$

497.

$\sqrt{-121}$

498.

$\sqrt{-36}$

499.

$\sqrt{-9}$

500.

$\sqrt{-49}$

501.

$\sqrt{9+16}$

502.

$\sqrt{25+144}$

503.

$\sqrt{9}+\sqrt{16}$

504.

$\sqrt{25}+\sqrt{144}$

Estimate Square Roots

In the following exercises, estimate each square root between two consecutive whole numbers.

505.

$\sqrt{70}$

506.

$\sqrt{55}$

507.

$\sqrt{200}$

508.

$\sqrt{172}$

Approximate Square Roots with a Calculator

In the following exercises, use a calculator to approximate each square root and round to two decimal places.

509.

$\sqrt{19}$

510.

$\sqrt{21}$

511.

$\sqrt{53}$

512.

$\sqrt{47}$

Simplify Variable Expressions with Square Roots

In the following exercises, simplify. (Assume all variables are greater than or equal to zero.)

513.

$\sqrt{y^2}$

514.

$\sqrt{b^2}$

515.

$\sqrt{49x^2}$

516.

$\sqrt{100y^2}$

517.

$-\sqrt{64a^2}$

518.

$-\sqrt{25x^2}$

519.

$\sqrt{144x^2y^2}$

520.

$\sqrt{196a^2b^2}$

Use Square Roots in Applications

In the following exercises, solve. Round to one decimal place.

521.

Landscaping Reed wants to have a square garden plot in his backyard. He has enough compost to cover an area of 75 square feet. How long can a side of his garden be?

522.

Landscaping Vince wants to make a square patio in his yard. He has enough concrete to pave an area of 130 square feet. How long can a side of his patio be?

523.

Gravity An airplane dropped a flare from a height of $1{,}024$ feet above a lake. How many seconds did it take for the flare to reach the water?

524.

Gravity A hang glider dropped his cell phone from a height of 350 feet. How many seconds did it take for the cell phone to reach the ground?

525.

Gravity A construction worker dropped a hammer while building the Grand Canyon skywalk, $4{,}000$ feet above the Colorado River. How many seconds did it take for the hammer to reach the river?

526.

Accident investigation The skid marks from a car involved in an accident measured 54 feet. What was the speed of the car before the brakes were applied?

527.

Accident investigation The skid marks from a car involved in an accident measured 216 feet. What was the speed of the car before the brakes were applied?

528.

Accident investigation An accident investigator measured the skid marks of one of the vehicles involved in an accident. The length of the skid marks was 175 feet. What was the speed of the vehicle before the brakes were applied?

529.

Accident investigation An accident investigator measured the skid marks of one of the vehicles involved in an accident. The length of the skid marks was 117 feet. What was the speed of the vehicle before the brakes were applied?

Everyday Math

530.

Decorating Denise wants to install a square accent of designer tiles in her new shower. She can afford to buy 625 square centimeters of the designer tiles. How long can a side of the accent be?

531.

Decorating Morris wants to have a square mosaic inlaid in his new patio. His budget allows for $2{,}025$ tiles. Each tile is square with an area of one square inch. How long can a side of the mosaic be?

Writing Exercises

532.

Why is there no real number equal to $\sqrt{-64}$?

533.

What is the difference between 9^2 and $\sqrt{9}$?

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
simplify expressions with square roots.			
estimate square roots.			
approximate square roots.			
simplify variable expressions with square roots.			
use square roots in applications.			

ⓑ Overall, after looking at the checklist, do you think you are well-prepared for the next Chapter? Why or why not?

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7.2: Integer Exponents and Scientific Notation (Part 1)

Learning Objectives

- Use the definition of a negative exponent
- Simplify expressions with integer exponents
- Convert from decimal notation to scientific notation
- Convert scientific notation to decimal form
- Multiply and divide using scientific notation

be prepared!

Before you get started, take this readiness quiz.

1. What is the place value of the 6 in the number 64,891? If you missed this problem, review Example 1.1.3.
2. Name the decimal 0.0012. If you missed this problem, review Exercise 5.1.1.
3. Subtract: 5 − (−3). If you missed this problem, review Example 3.5.8.

Use the Definition of a Negative Exponent

The Quotient Property of Exponents, introduced in Divide Monomials, had two forms depending on whether the exponent in the numerator or denominator was larger.

Definition: Quotient Property of Exponents

If a is a real number, a ≠ 0, and m, n are whole numbers, then

$$\frac{a^m}{a^n} = a^{m-n},\ m > n \quad and \quad \frac{a^m}{a^n} = \frac{1}{a^{n-m}},\ n > m \tag{7.2.1}$$

What if we just subtract exponents, regardless of which is larger? Let's consider $\frac{x^2}{x^5}$. We subtract the exponent in the denominator from the exponent in the numerator.

$$\begin{array}{c} \frac{x^2}{x^5} \\ x^{2-5} \\ x^{-3} \end{array} \tag{7.2.2}$$

We can also simplify $\frac{x^2}{x^5}$ by dividing out common factors: $\frac{x^2}{x^5}$.

$$\begin{array}{c} \frac{\cancel{x} \cdot \cancel{x}}{\cancel{x} \cdot \cancel{x} \cdot x \cdot x \cdot x} \\ \frac{1}{x^3} \end{array} \tag{7.2.3}$$

This implies that $x^{-3} = \frac{1}{x^3}$ and it leads us to the definition of a **negative exponent**.

Definition: negative exponent

If n is a positive integer and a ≠ 0, then $a^{-n} = \frac{1}{a^n}$.

The negative exponent tells us to re-write the expression by taking the reciprocal of the base and then changing the sign of the exponent. Any expression that has negative exponents is not considered to be in simplest form. We will use the definition of a negative exponent and other properties of exponents to write an expression with only positive exponents.

Example 7.2.1:

Simplify: (a) 4^{-2} (b) 10^{-3}

Solution

(a) 4^{-2}

Use the definition of a negative exponent, $a^{-n} = \frac{1}{a^n}$.	$\frac{1}{4^2}$	(7.2.4)
Simplify.	$\frac{1}{16}$	(7.2.5)

(b) 10^{-3}

Use the definition of a negative exponent, $a^{-n} = \frac{1}{a^n}$.	$\frac{1}{10^3}$	(7.2.6)
Simplify.	$\frac{1}{1000}$	(7.2.7)

Exercise 7.2.1:

Simplify: (a) 2^{-3} (b) 10^{-2}

Answer a

$\frac{1}{8}$

Answer b

$\frac{1}{100}$

Exercise 7.2.2:

Simplify: (a) 3^{-2} (b) 10^{-4}

Answer a

$\frac{1}{9}$

Answer b

$\frac{1}{10,000}$

When simplifying any expression with exponents, we must be careful to correctly identify the base that is raised to each exponent.

Example 7.2.2:

Simplify: (a) $(-3)^{-2}$ (b) -3^{-2}

Solution

The negative in the exponent does not affect the sign of the base.

a) $(-3)^{-2}$

The exponent applies to the base, −3.	$(-3)^{-2}$	(7.2.8)
Take the reciprocal of the base and change the sign of the exponent.	$\frac{1}{(-3)^2}$	(7.2.9)
Simplify.	$\frac{1}{9}$	(7.2.10)

(b) -3^{-2}

The expression -3^{-2} means "find the opposite of 3^{-2}. The exponent applies only to the base, 3.	-3^{-2}	(7.2.11)
Rewrite as a product with −1.	$-1 \cdot 3^{-2}$	(7.2.12)
Take the reciprocal of the base and change the sign of the exponent.	$-1 \cdot \frac{1}{3^2}$	(7.2.13)
Simplify.	$-\frac{1}{9}$	(7.2.14)

? Exercise 7.2.3:

Simplify: (a) $(-5)^{-2}$ (b) -5^{-2}

Answer a

$\frac{1}{25}$

Answer b

$-\frac{1}{25}$

? Exercise 7.2.4:

Simplify: (a) $(-2)^{-2}$ (b) -2^{-2}

Answer a

$\frac{1}{4}$

Answer b

$-\frac{1}{4}$

We must be careful to follow the order of operations. In the next example, parts (a) and (b) look similar, but we get different results.

✓ Example 7.2.3:

Simplify: (a) $4 \cdot 2^{-1}$ (b) $(4 \cdot 2)^{-1}$

Solution

Remember to always follow the order of operations.

(a) $4 \cdot 2^{-1}$

Do exponents before multiplication.	$4 \cdot 2^{-1}$	(7.2.15)
Use $a^{-n} = \frac{1}{a^n}$.	$4 \cdot \frac{1}{2^1}$	(7.2.16)
Simplify.	2	(7.2.17)

(b) $(4 \cdot 2)^{-1}$

Simplify inside the parentheses first.	$(8)^{-1}$	(7.2.18)
Use $a^{-n} = \frac{1}{a^n}$.	$\frac{1}{8^1}$	(7.2.19)
Simplify.	$\frac{1}{8}$	(7.2.20)

? Exercise 7.2.5:

Simplify: (a) $6 \cdot 3^{-1}$ (b) $(6 \cdot 3)^{-1}$

Answer a

2

Answer b

$\frac{1}{18}$

? Exercise 7.2.6:

Simplify: (a) $8 \cdot 2^{-2}$ (b) $(8 \cdot 2)^{-2}$

Answer a

2

Answer b

$\frac{1}{256}$

When a variable is raised to a negative exponent, we apply the definition the same way we did with numbers.

✓ Example 7.2.4:

Simplify: x^{-6}.

Solution

Use the definition of a negative exponent, $a^{-n} = \frac{1}{a^n}$.	$\frac{1}{x^6}$	(7.2.21)

Exercise 7.2.7:

Simplify: y^{-7}.

Answer

$\frac{1}{y^7}$

Exercise 7.2.8:

Simplify: z^{-8}.

Answer

$\frac{1}{z^8}$

When there is a product and an exponent we have to be careful to apply the exponent to the correct quantity. According to the order of operations, expressions in parentheses are simplified before exponents are applied. We'll see how this works in the next example.

Example 7.2.5:

Simplify: (a) $5y^{-1}$ (b) $(5y)^{-1}$ (c) $(-5y)^{-1}$

Solution

(a) $5y^{-1}$

Notice the exponent applies to just the base y .	$5y^{-1}$	(7.2.22)
Take the reciprocal of y and change the sign of the exponent.	$5 \cdot \frac{1}{y^1}$	(7.2.23)
Simplify.	$\frac{5}{y}$	(7.2.24)

(b) $(5y)^{-1}$

Here the parentheses make the exponent apply to the base 5y.	$(5y)^{-1}$	(7.2.25)
Take the reciprocal of 5y and change the sign of the exponent.	$\frac{1}{(5y)^1}$	(7.2.26)
Simplify.	$\frac{1}{5y}$	(7.2.27)

(c) $(-5y)^{-1}$

The base is $-5y$. Take the reciprocal of $-5y$ and change the sign of the exponent.	$\frac{1}{(-5y)^1}$	(7.2.28)
Simplify.	$\frac{1}{-5y}$	(7.2.29)

Use $\frac{a}{-b} = -\frac{a}{b}$.	$-\frac{1}{5y}$	(7.2.30)

Exercise 7.2.9:

Simplify: (a) $8p^{-1}$ (b) $(8p)^{-1}$ (c) $(-8p)^{-1}$

Answer a

$\frac{8}{p}$

Answer b

$\frac{1}{8p}$

Answer c

$-\frac{1}{8p}$

Exercise 7.2.10:

Simplify: (a) $11q^{-1}$ (b) $(11q)^{-1}$ (c) $(-11q)^{-1}$

Answer a

$\frac{11}{q}$

Answer b

$\frac{1}{11q}$

Answer c

$-\frac{1}{11q}$

Now that we have defined negative exponents, the Quotient Property of Exponents needs only one form, $\frac{a^m}{a^n} = a^{m-n}$, where a ≠ 0 and m and n are integers.

When the exponent in the denominator is larger than the exponent in the numerator, the exponent of the quotient will be negative. If the result gives us a negative exponent, we will rewrite it by using the definition of negative exponents, $a^{-n} = \frac{1}{a^n}$.

Simplify Expressions with Integer Exponents

All the exponent properties we developed earlier in this chapter with whole number exponents apply to integer exponents, too. We restate them here for reference.

Summary of Exponent Properties

If a, b are real numbers and m, n are integers, then

Product Property	$a^m \cdot a^n = a^{m+n}$
Power Property	$(a^m)^n = a^{m \cdot n}$
Product to a Power Property	$(ab)^m = a^m b^m$
Quotient Property	$\frac{a^m}{a^n} = a^{m-n}$, a ≠ 0, m > n
	$\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$, a ≠ 0, n > m

Zero Exponent Property	$a^0 = 1, a \neq 0$
Quotient to a Power Property	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$
Definition of a Negative Exponent	$a^{-n} = \frac{1}{a^n}$

Example 7.2.6:

Simplify: (a) $x^{-4} \cdot x^6$ (b) $y^{-6} \cdot y^4$ (c) $z^{-5} \cdot z^{-3}$

Solution

(a) $x^{-4} \cdot x^6$

Use the Product Property, $a^m \cdot a^n = a^{m+n}$.	x^{-4+6}	(7.2.31)
Simplify.	x^2	(7.2.32)

(b) $y^{-6} \cdot y^4$

The bases are the same, so add the exponents.	y^{-6+4}	(7.2.33)
Simplify.	y^{-2}	(7.2.34)
Use the definition of a negative exponent, $a^{-n} = \frac{1}{a^n}$.	$\frac{1}{y^2}$	(7.2.35)

(c) $z^{-5} \cdot z^{-3}$

The bases are the same, so add the exponents.	z^{-5-3}	(7.2.36)
Simplify.	z^{-8}	(7.2.37)
Use the definition of a negative exponent, $a^{-n} = \frac{1}{a^n}$.	$\frac{1}{z^8}$	(7.2.38)

Exercise 7.2.11:

Simplify: (a) $x^{-3} \cdot x^7$ (b) $y^{-7} \cdot y^2$ (c) $z^{-4} \cdot z^{-5}$

Answer a

x^4

Answer b

$\frac{1}{y^5}$

Answer c

$\frac{1}{z^9}$

Exercise 7.2.12:

Simplify: (a) $a^{-1} \cdot a^{6}$ (b) $b^{-6} \cdot b^{4}$ (c) $c^{-8} \cdot c^{-7}$

Answer a

a^{5}

Answer b

$\frac{1}{b^{4}}$

Answer c

$\frac{1}{c^{15}}$

In the next two examples, we'll start by using the Commutative Property to group the same variables together. This makes it easier to identify the like bases before using the Product Property of Exponents.

Example 7.2.7:

Simplify: $(m^{4}n^{-3})(m^{-5}n^{-2})$.

Solution

Use the Commutative Property to get like bases together.	$m^{4}m^{-5} \cdot n^{-2}n^{-3}$	(7.2.39)
Add the exponents for each base.	$m^{-1} \cdot n^{-5}$	(7.2.40)
Take reciprocals and change the signs of the exponents.	$\frac{1}{m^{1}} \cdot \frac{1}{n^{5}}$	(7.2.41)
Simplify.	$\frac{1}{mn^{5}}$	(7.2.42)

Exercise 7.2.13:

Simplify: $(p^{6}q^{-2})(p^{-9}q^{-1})$.

Answer

$\frac{1}{p^{3}q^{3}}$

Exercise 7.2.14:

Simplify: $(r^{5}s^{-3})(r^{-7}s^{-5})$.

Answer

$\frac{1}{r^{2}s^{8}}$

If the monomials have numerical coefficients, we multiply the coefficients, just as we did in Use Multiplication Properties of Exponents.

Example 7.2.8:

Simplify: $(2x^{-6}y^{8})(-5x^{5}\,y^{-3})$.

Solution

Rewrite with the like bases together.	$2(-5)\cdot(x^{-6}x^{5})\cdot(y^{8}y^{-3})$	(7.2.43)
Simplify.	$-10\cdot x^{-1}\cdot y^{5}$	(7.2.44)
Use the definition of a negative exponent, $a^{-n}=\frac{1}{a^{n}}$.	$-10\cdot\frac{1}{x^{1}}\cdot y^{5}$	(7.2.45)
Simplify.	$\frac{-10y^{5}}{x}$	(7.2.46)

Exercise 7.2.15:

Simplify: $(3u^{-5}v^{7})(-4u^{4}v^{-2})$.

Answer

$-\frac{12v^{5}}{u}$

Exercise 7.2.16:

Simplify: $(-6c^{-6}d^{4})(-5c^{-2}d^{-1})$.

Answer

$\frac{30d^{3}}{c^{8}}$

In the next two examples, we'll use the Power Property and the Product to a Power Property.

Example 7.2.9:

Simplify: $(k^{3})^{-2}$.

Solution

Use the Product to a Power Property, $(ab)^{m}=a^{m}b^{m}$.	$k^{3(-2)}$	(7.2.47)
Simplify.	k^{-6}	(7.2.48)
Rewrite with a positive exponent.	$\frac{1}{k^{6}}$	(7.2.49)

Exercise 7.2.17:

Simplify: $(x^{4})^{-1}$.

Answer

$\frac{1}{x^{4}}$

Exercise 7.2.18:

Simplify: $(y^2)^{-2}$.

Answer

$\frac{1}{y^4}$

Example 7.2.10:

Simplify: $(5x^{-3})^2$.

Solution

Use the Product to a Power Property, $(ab)^m = a^m b^m$.	$5^2(x^{-3})^2$	(7.2.50)
Simplify 5^2 and multiply the exponents of x using the Power Property, $(a^m)^n = a^{m \cdot n}$.	$25k^{-6}$	(7.2.51)
Rewrite x^{-6} by using the definition of a negative exponent, $a^{-n} = \frac{1}{a^n}$.	$25 \cdot \frac{1}{x^6}$	(7.2.52)
Simplify.	$\frac{25}{x^6}$	(7.2.53)

Exercise 7.2.19:

Simplify: $(8a^{-4})^2$.

Answer

$\frac{64}{a^8}$

Exercise 7.2.20:

Simplify: $(2c^{-4})^3$.

Answer

$\frac{8}{c^12}$

To simplify a fraction, we use the Quotient Property.

Example 7.2.11:

Simplify: $\frac{r^5}{r^{-4}}$.

Solution

Use the Quotient Property, $\frac{a^m}{a^n} = a^{m-n}$.	$r^{5-(-4)}$	(7.2.54)
Be careful to subtract 5 - (\textcolor{red}{-4}).		
Simplify.	r^9	(7.2.55)

Exercise 7.2.21:

Simplify: $\frac{x^8}{x^{-3}}$.

Answer

x^{11}

Exercise 7.2.22:

Simplify: $\frac{y^7}{y^{-6}}$.

Answer

y^{13}

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7.3: Integer Exponents and Scientific Notation (Part 2)

Convert from Decimal Notation to Scientific Notation

Remember working with place value for whole numbers and decimals? Our number system is based on powers of 10. We use tens, hundreds, thousands, and so on. Our decimal numbers are also based on powers of tens—tenths, hundredths, thousandths, and so on.

Consider the numbers 4000 and 0.004. We know that 4000 means 4×1000 and 0.004 means $4 \times \frac{1}{1000}$. If we write the 1000 as a power of ten in exponential form, we can rewrite these numbers in this way:

$$\begin{array}{ll} 4000 & 0.004 \\ 4 \times 1000 & 4 \times \frac{1}{1000} \\ 4 \times 10^3 & 4 \times \frac{1}{10^3} \\ & 4 \times 10^{-3} \end{array} \tag{7.3.1}$$

When a number is written as a product of two numbers, where the first factor is a number greater than or equal to one but less than 10, and the second factor is a power of 10 written in exponential form, it is said to be in *scientific notation*.

Definition: Scientific Notation

A number is expressed in scientific notation when it is of the form $a \times 10^n$ where $a \geq 1$ and $a < 10$ and n is an integer.

It is customary in scientific notation to use × as the multiplication sign, even though we avoid using this sign elsewhere in algebra.

Scientific notation is a useful way of writing very large or very small numbers. It is used often in the sciences to make calculations easier.

If we look at what happened to the decimal point, we can see a method to easily convert from decimal notation to scientific notation.

$4000. = 4 \times 10^3$	$0.004 = 4 \times 10^{-3}$
$4000. = 4 \times 10^3$	$0.004 = 4 \times 10^{-3}$
Moved the decimal point 3 places to the left.	Moved the decimal point 3 places to the right.

In both cases, the decimal was moved 3 places to get the first factor, 4, by itself.

- The power of 10 is positive when the number is larger than 1: $4000 = 4 \times 10^3$.
- The power of 10 is negative when the number is between 0 and 1: $0.004 = 4 \times 10^{-3}$.

Example 7.3.12:

Write 37,000 in scientific notation.

Solution

Step 1: Move the decimal point so that the first factor is greater than or equal to 1 but less than 10.	37000.
Step 2: Count the number of decimal places, n, that the decimal point was moved.	3.70000 4 places
Step 3: Write the number as a product with a power of 10.	3.7×10^4
If the original number is: • greater than 1, the power of 10 will be 10^n • between 0 and 1, the power of 10 will be 10^{-n}	

Step 4: Check.10^4 is 10,000 and 10,000 times 3.7 will be 37,000.	$37{,}000 = 3.7\times10^4$

Exercise 7.3.23:

Write in scientific notation: 96,000.

Answer

9.6×10^4

Exercise 7.3.24:

Write in scientific notation: 48,300.

Answer

4.83×10^4

HOW TO: CONVERT FROM DECIMAL NOTATION TO SCIENTIFIC NOTATION

Step 1. Move the decimal point so that the first factor is greater than or equal to 1 but less than 10.

Step 2. Count the number of decimal places, n, that the decimal point was moved.

Step 3. Write the number as a product with a power of 10.

If the original number is:

- greater than 1, the power of 10 will be 10^n.
- between 0 and 1, the power of 10 will be 10^{-n}.

Step 4. Check.

Example 7.3.13:

Write in scientific notation: 0.0052.

Solution

Move the decimal point to get 5.2, a number between 1 and 10.	0.0052
Count the number of decimal places the point was moved.	3 places
Write as a product with a power of 10.	5.2×10^{-3}
Check your answer:	5.2×10^{-3} $5.2 \times \frac{1}{10^3}$ $5.2 \times \frac{1}{1000}$ 5.2×0.001 0.0052 (7.3.2)
	$0.0052 = 5.2 \times 10^{-3}$

Exercise 7.3.25:

Write in scientific notation: 0.0078.

Answer

7.8×10^{-3}

Exercise 7.3.26:

Write in scientific notation: 0.0129.

Answer

1.29×10^{-2}

Convert Scientific Notation to Decimal Form

How can we convert from scientific notation to decimal form? Let's look at two numbers written in scientific notation and see.

$$\begin{array}{ll} 9.12 \times 10^{4} & 9.12 \times 10^{-4} \\ 9.12 \times 10,000 & 9.12 \times 0.0001 \\ 91,200 & 0.000912 \end{array} \tag{7.3.3}$$

If we look at the location of the decimal point, we can see an easy method to convert a number from scientific notation to decimal form.

$$\begin{array}{ll} 9.12 \times 10^{4} = 91{,}200 & 9.12 \times 10^{-4} = 0.000912 \\ 9.12\_\_\times 10^{4} = 91{,}200 & \_\_\_9.12 \times 10^{-4} = 0.000912 \end{array}$$

In both cases the decimal point moved 4 places. When the exponent was positive, the decimal moved to the right. When the exponent was negative, the decimal point moved to the left.

Example 7.3.14:

Convert to decimal form: 6.2×10^{3}.

Solution

Step 1: Determine the exponent, n, on the factor 10.	6.2×10^{3}
Step 2: Move the decimal point n places, adding zeros if needed.	6,200.
• If the exponent is positive, move the decimal point n places to the right. • If the exponent is negative, move the decimal point |n| places to the left.	6,200
Step 3: Check to see if your answer makes sense.	
10^{3} is 1000 and 1000 times 6.2 will be 6,200.	$6.2 \times 10^{3} = 6{,}200$

Exercise 7.3.27:

Convert to decimal form: 1.3×10^{3}.

Answer

1,300

Exercise 7.3.28:

Convert to decimal form: 9.25×10^4.

Answer

92,500

HOW TO: CONVERT SCIENTIFIC NOTATION TO DECIMAL FORM

Step 1. Determine the exponent, n, on the factor 10.

Step 2. Move the decimal n places, adding zeros if needed.

- If the exponent is positive, move the decimal point n places to the right.
- If the exponent is negative, move the decimal point |n| places to the left.

Step 3. Check.

Example 7.3.15:

Convert to decimal form: 8.9×10^{-2}.

Solution

Determine the exponent n, on the factor 10.	The exponent is −2.
Move the decimal point 2 places to the left.	$\underset{\smile}{.}89$
Add zeros as needed for placeholders.	0.089
	$8.9 \times 10^{-2} = 0.089$
The Check is left to you.	

Exercise 7.3.29:

Convert to decimal form: 1.2×10^{-4}.

Answer

0.00012

Exercise 7.3.30:

Convert to decimal form: 7.5×10^{-2}.

Answer

0.075

Multiply and Divide Using Scientific Notation

We use the Properties of Exponents to multiply and divide numbers in scientific notation.

Example 7.3.16:

Multiply. Write answers in decimal form: $(4 \times 10^5)(2 \times 10^{-7})$.

Solution

Use the Commutative Property to rearrange the factors.	$4 \cdot 2 \cdot 10^5 \cdot 10^{-7}$
Multiply 4 by 2 and use the Product Property to multiply 10^5 by 10^{-7}.	8×10^{-2}
Change to decimal form by moving the decimal two places left.	0.08

Exercise 7.3.31:

Multiply. Write answers in decimal form: $(3 \times 10^6)(2 \times 10^{-8})$.

Answer

0.06

Exercise 7.3.32:

Multiply. Write answers in decimal form: $(3 \times 10^{-2})(3 \times 10^{-1})$.

Answer

0.009

Example 7.3.17:

Divide. Write answers in decimal form: $\frac{9 \times 10^3}{3 \times 10^{-2}}$.

Solution

Separate the factors.	$\frac{9}{3} \times \frac{10^3}{10^{-2}}$	(7.3.4)
Divide 9 by 3 and use the Quotient Property to divide 10^3 by 10^{-2}.	3×10^5	
Change to decimal form by moving the decimal five places right.	300,000	

Exercise 7.3.33:

Divide. Write answers in decimal form:\dfrac{8 \times 10^{4}}{2 \times 10^{-1}}.

Answer

400,000

Exercise 7.3.34:

Divide. Write answers in decimal form:\dfrac{8 \times 10^{2}}{4 \times 10^{-2}}.

Answer

20,000

ACCESS ADDITIONAL ONLINE RESOURCES

Negative Exponents

Examples of Simplifying Expressions with Negative Exponents

Scientific Notation

Practice Makes Perfect

Use the Definition of a Negative Exponent

In the following exercises, simplify.

316. 5^{-3}
317. 8^{-2}
318. 3^{-4}
319. 2^{-5}
320. 7^{-1}
321. 10^{-1}
322. $2^{-3} + 2^{-2}$
323. $3^{-2} + 3^{-1}$
324. $3^{-1} + 4^{-1}$
325. $10^{-1} + 2^{-1}$
326. $10^{0} - 10^{-1} + 10^{-2}$
327. $2^{0} - 2^{-1} + 2^{-2}$
328. (a) $(-6)^{-2}$ (b) -6^{-2}
329. (a) $(-8)^{-2}$ (b) -8^{-2}
330. (a) $(-10)^{-4}$ (b) -10^{-4}
331. (a) $(-4)^{-6}$ (b) -4^{-6}
332. (a) $5 \cdot 2^{-1}$ (b) $(5 \cdot 2)^{-1}$
333. (a) $10 \cdot 3^{-1}$ (b) $(10 \cdot 3)^{-1}$
334. (a) $4 \cdot 10^{-3}$ (b) $(4 \cdot 10)^{-3}$
335. (a) $3 \cdot 5^{-2}$ (b) $(3 \cdot 5)^{-2}$
336. n^{-4}
337. p^{-3}
338. c^{-10}
339. m^{-5}
340. (a) $4x^{-1}$ (b) $(4x)^{-1}$ (c) $(-4x)^{-1}$
341. (a) $3q^{-1}$ (b) $(3q)^{-1}$ (c) $(-3q)^{-1}$
342. (a) $6m^{-1}$ (b) $(6m)^{-1}$ (c) $(-6m)^{-1}$
343. (a) $10k^{-1}$ (b) $(10k)^{-1}$ (c) $(-10k)^{-1}$

Simplify Expressions with Integer Exponents

In the following exercises, simplify.

344. $p^{-4} \cdot p^{8}$
345. $r^{-2} \cdot r^{5}$
346. $n^{-10} \cdot n^{2}$

347. $q^{-8} \cdot q^{3}$
348. $k^{-3} \cdot k^{-2}$
349. $z^{-6} \cdot z^{-2}$
350. $a \cdot a^{-4}$
351. $m \cdot m^{-2}$
352. $p^{5} \cdot p^{-2} \cdot p^{-4}$
353. $x^{4} \cdot x^{-2} \cdot x^{-3}$
354. $a^{3}b^{-3}$
355. $u^{2}\, v^{-2}$
356. $(x^{5}y^{-1})(x^{-10}\, y^{-3})$
357. $(a^{3}b^{-3})(a^{-5}b^{-1})$
358. $(uv^{-2})(u^{-5}v^{-4})$
359. $(pq^{-4})(p^{-6}q^{-3})$
360. $(-2r^{-3}s^{9})(6r^{4}s^{-5})$
361. $(-3p^{-5}q^{8})(7p^{2}q^{-3})$
362. $(-6m^{-8}n^{-5})(-9m^{4}n^{2})$
363. $(-8a^{-5}b^{-4})(-4a^{2}b^{3})$
364. $(a^{3})^{-3}$
365. $(q^{10})^{-10}$
366. $(n^{2})^{-1}$
367. $(x^{4})^{-1}$
368. $(y^{-5})^{4}$
369. $(p^{-3})^{2}$
370. $(q^{-5})^{-2}$
371. $(m^{-2})^{-3}$
372. $(4y^{-3})^{2}$
373. $(3q^{-5})^{2}$
374. $(10p^{-2})^{-5}$
375. $(2n^{-3})^{-6}$
376. $u^{9}u^{-2}$
377. $b^{5}b^{-3}$
378. $x^{-6}x^{4}$
379. $m^{5}m^{-2}$
380. $q^{3}q^{12}$
381. $r^{6}r^{9}$
382. $n^{-4}n^{-10}$
383. $p^{-3}p^{-6}$

Convert from Decimal Notation to Scientific Notation

In the following exercises, write each number in scientific notation.

384. 45,000
385. 280,000
386. 8,750,000
387. 1,290,000
388. 0.036
389. 0.041
390. 0.00000924
391. 0.0000103
392. The population of the United States on July 4, 2010 was almost 310,000,000.
393. The population of the world on July 4, 2010 was more than 6,850,000,000.
394. The average width of a human hair is 0.0018 centimeters.
395. The probability of winning the 2010 Megamillions lottery is about 0.0000000057.

Convert Scientific Notation to Decimal Form

In the following exercises, convert each number to decimal form.

396. 4.1×10^{2}
397. 8.3×10^{2}
398. 5.5×10^{8}
399. 1.6×10^{10}
400. 3.5×10^{-2}
401. 2.8×10^{-2}
402. $1.93 \times 10-5$
403. $6.15 \times 10-8$
404. In 2010, the number of Facebook users each day who changed their status to 'engaged' was 2×10^{4}.
405. At the start of 2012, the US federal budget had a deficit of more than $\$1.5 \times 10^{13}$.
406. The concentration of carbon dioxide in the atmosphere is 3.9×10^{-4}.
407. The width of a proton is 1×10^{-5} of the width of an atom.

Multiply and Divide Using Scientific Notation

In the following exercises, multiply or divide and write your answer in decimal form.

408. $(2 \times 10^{5})(2 \times 10^{-9})$
409. $(3 \times 10^{2})(1 \times 10^{-5})$
410. $(1.6 \times 10^{-2})(5.2 \times 10^{-6})$
411. $(2.1 \times 10^{-4})(3.5 \times 10^{-2})$
412. $\frac{6 \times 10^{4}}{3 \times 10^{-2}}$
413. $\frac{8 \times 10^{6}}{4 \times 10^{-1}}$
414. $\frac{7 \times 10^{-2}}{1 \times 10^{-8}}$
415. $\frac{5 \times 10^{-3}}{1 \times 10^{-10}}$

Everyday Math

416. **Calories** In May 2010 the Food and Beverage Manufacturers pledged to reduce their products by 1.5 trillion calories by the end of 2015.
    1. Write 1.5 trillion in decimal notation.
    2. Write 1.5 trillion in scientific notation.
417. **Length of a year** The difference between the calendar year and the astronomical year is 0.000125 day.
    1. Write this number in scientific notation.
    2. How many years does it take for the difference to become 1 day?
418. **Calculator display** Many calculators automatically show answers in scientific notation if there are more digits than can fit in the calculator's display. To find the probability of getting a particular 5-card hand from a deck of cards, Mario divided 1 by 2,598,960 and saw the answer 3.848×10^{-7}. Write the number in decimal notation.
419. **Calculator display** Many calculators automatically show answers in scientific notation if there are more digits than can fit in the calculator's display. To find the number of ways Barbara could make a collage with 6 of her 50 favorite photographs, she multiplied $50 \cdot 49 \cdot 48 \cdot 47 \cdot 46 \cdot 45$. Her calculator gave the answer 1.1441304×10^{10}. Write the number in decimal notation.

Writing Exercises

420. (a) Explain the meaning of the exponent in the expression 2^{3}. (b) Explain the meaning of the exponent in the expression 2^{-3}.
421. When you convert a number from decimal notation to scientific notation, how do you know if the exponent will be positive or negative?

Self Check

(a) After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
use the definition of a negative exponent.			
simplify expressions with integer exponents.			
convert from decimal notation to scientific notation.			
convert scientific notation to decimal form.			
multiply and divide using scientific notation.			

(b) After looking at the checklist, do you think you are well prepared for the next section? Why or why not?

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