CAÑADA COLLEGE SPARK CLUB MATH OER FOR RAVENSWOOD MS



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Cañada College SPARK Club



Cañada College

ZERO TEXTBOOK COST PROGRAM (ZTC)

Cañada College SPARK Club Math OER for Ravenswood HS

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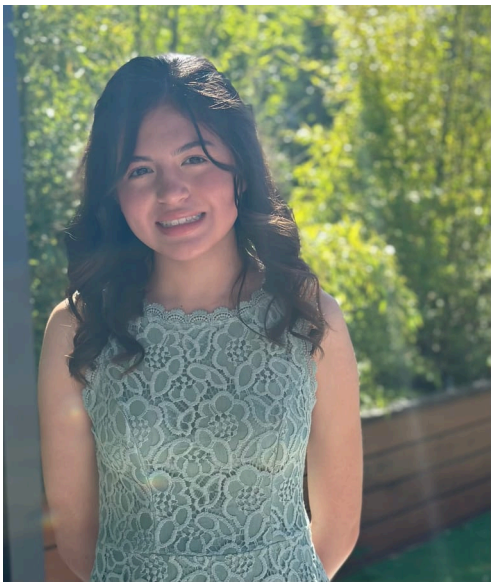
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About the Team

Remixing and Editing Personnel

This project is a collaborative effort between the SPARK Club at Cañada College and EPA Tutors. We thank everyone involved at both organizations, along with the students and faculty at Ravenswood High School.

Neve Willis, Principle Remixing Author



Neve Willis is a Middle College student and rising senior at Cañada College. She is passionate about tutoring, student advocacy, and volunteering. She created this resource to make math more accessible and help learners build confidence while supporting underprivileged students.

Harriette Huang, Advisor, EPA Tutors

Julie Manuel, SPARK Club Mentor

Sarah Harmon, Technical Editor and Project Manager



Dr. Sarah Harmon is an Adjunct Professor of Linguistics and the OER/ZTC Program Manager at Cañada College, where she has taught most of her courses since 2006. She earned a B.A. in Spanish and a M.A. in Linguistics from the University of California at Davis, and a Ph.D. in Romance Linguistics from the University of Texas at Austin. She has developed online and in-person courses in Spanish and Linguistics at the collegiate level since 2006, all of which have undergone the CVC-OEI review process. As a member of the Entrada Libre/Tarea Libre Team, she co-authored *Tarea Libre* (2023) for first-year collegiate Spanish courses, *Tarea Libre 2* (2024) for second-year collegiate Spanish courses, and the second edition of the popular second-year collegiate Spanish grammar manual, *Entrada Libre 2e* (2026). She has created a remix of *Entrada Libre plus Yepes* (2021) for second-year collegiate Spanish courses, and an adaptation of *Essentials of Linguistics* (2024) for introductory linguistics courses. For this project, she served as the technical editor and project manager, particularly with respect to integration and publication on LibreTexts.

What is SPARK Club?

The Sharing Passion through Acts of Radiant Kindness (SPARK) Club is a community service-focused club at Cañada College dedicated to making a tangible impact both on campus and in underserved local communities. Through a variety of intentional projects and collaborations, including delivering candy grams through SparkPoint, volunteering at EPA Tutors, making cards for veterans and hospital patients, and creating thoughtful gifts like handmade flowers, SPARK members work together to spread kindness, support those who are underprivileged, and inspire others to give back.

What is EPA Tutors?

East Palo Alto (EPA) Tutors is a student-led nonprofit that provides free, personalized online tutoring for sixth-grade students in East Palo Alto. Many students in the program face significant educational and economic barriers: 77% are economically disadvantaged, approximately 30% do not graduate from high school, and many perform several grade levels below standard. Recognizing these challenges, Harriete Huang, a sixth-grade math teacher at Ravenswood Middle School, helped recruit volunteer tutors to support her students. Through weekly one-on-one tutoring sessions, volunteers meet students where they are academically, helping them build foundational skills and gain confidence in the classroom.

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Intensions and Uses

General Intentions and Uses of the Text

Through volunteering and tutoring, we found that many students lack basic foundational mathematical skills. In fast-paced environments where teachers have limited time to cover a wide range of content and cannot always provide individualized support, many students fall behind and lack the foundational skills needed to understand more complex concepts. This Open Education Resource aims to provide a comprehensive understanding of fundamental mathematical concepts through interactive activities, practice resources, and a curated selection of videos with helpful strategies. It is intended for students in Grades 4-8 who want to learn, review, or strengthen their understanding of core mathematical concepts. We also aimed to make this resource accessible to all learners, especially those with learning challenges.

Layout of the Text and Specific Uses

This text remixes existing material with the addition of practice activities and video resources located under each lesson. In these pages, you will find GeoGebra graphic representations, videos, H5P practice activities, and extra practice resources. The GeoGebra graphic representations are designed to help students preview material and visualize concepts through interactive activities related to the lesson. There is also a selection of helpful YouTube videos intended to explain concepts and strategies, provide worked examples, and illustrate ideas in greater detail. These videos generally include captions, and some pages allow you to adjust the settings to view captions in different languages. Watching all videos is not necessary, but they can be useful for clarifying and strengthening understanding of core concepts. The H5P practice questions are designed to check your understanding of the lesson's content. There are four to five of these questions per lesson; some pages do not contain practice questions and instead include an activity. The extra practice resource section includes several free resources to gain additional exposure to and repetition of the concept. Most pages include links to IXL practice, and it is important to note that IXL only allows ten free practice questions before prompting users to sign up for an account.

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Revision History

On this page, the team will indicate when revisions are made. The initial text was published in July 2026.

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1: FOURTH GRADE MATH

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1.1: Place Value and Names for Whole Numbers

Learning Objectives

- Identify counting numbers and whole numbers
- Model whole numbers
- Identify the place value of a digit
- Use place value to name whole numbers
- Use place value to write whole numbers
- Round whole numbers

Identify Counting Numbers and Whole Numbers

Learning algebra is similar to learning a language. You start with a basic vocabulary and then add to it as you go along. You need to practice often until the vocabulary becomes easy to you. The more you use the vocabulary, the more familiar it becomes.

Algebra uses numbers and symbols to represent words and ideas. Let's look at the numbers first. The most basic numbers used in algebra are those we use to count objects: $1, 2, 3, 4, 5, \ldots$ and so on. These are called the **counting numbers**. The notation "..." is called an ellipsis, which is another way to show "and so on", or that the pattern continues endlessly. Counting numbers are also called natural numbers.

Definition: Counting Numbers

The counting numbers start with 1 and continue.

$$1, 2, 3, 4, 5 \ldots$$

Counting numbers and whole numbers can be visualized on a number line as shown in Figure $1.1.1$.

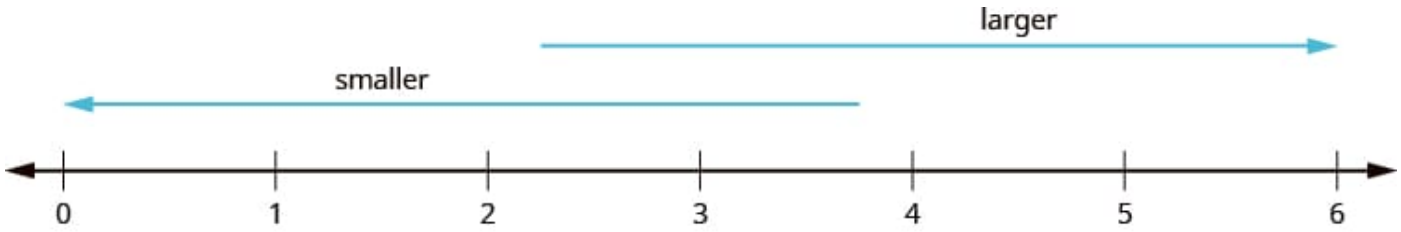


Figure $1.1.1$: The numbers on the number line increase from left to right, and decrease from right to left.

The point labeled 0 is called the **origin**. The points are equally spaced to the right of 0 and labeled with the counting numbers. When a number is paired with a point, it is called the **coordinate** of the point.

The discovery of the number zero was a big step in the history of mathematics. Including zero with the counting numbers gives a new set of numbers called the **whole numbers**.

Definition: Whole Numbers

The whole numbers are the counting numbers and zero.

$$0, 1, 2, 3, 4, 5 \ldots$$

We stopped at 5 when listing the first few counting numbers and whole numbers. We could have written more numbers if they were needed to make the patterns clear.

Example 1.1.1: Number Identification

Which of the following are

a. counting numbers
b. whole numbers

$$0, \frac{1}{4}, 3, 5.2, 15, 105$$

Solution

a. The counting numbers start at 1, so 0 is not a counting number. The numbers 3, 15, and 105 are all counting numbers.
b. Whole numbers are counting numbers and 0. The numbers $0, 3, 15,$ and 105 are whole numbers. The numbers $\frac{1}{4}$ and 5.2 are neither counting numbers nor whole numbers. We will discuss these numbers later.

Exercise 1.1.1

Which of the following are

a. whole numbers

$$0, \frac{2}{3}, 2, 9, 11.8, 241, 376$$

Answer a available at the end of this section.

Answer b available at the end of this section.

Exercise 1.1.2

Which of the following are

a. counting numbers
b. whole numbers

$$0, \frac{5}{3}, 7, 8.8, 13, 201$$

Answer a available at the end of this section.

Answer b available at the end of this section.

Model Whole Numbers

Our number system is called a **place value system** because the value of a digit depends on its position, or place, in a number. The number 537 has a different value than the number 735. Even though they use the same digits, their value is different because of the different placement of the 3 and the 7 and the 5.

Money gives us a familiar model of place value. Suppose a wallet contains three $100 bills, seven $10 bills, and four $1 bills. The amounts are summarized in Figure 1.1.2. How much money is in the wallet?



Figure 1.1.2

Find the total value of each kind of bill, and then add to find the total. The wallet contains $374.



Base-10 blocks provide another way to model place value, as shown in Figure 1.1.3. The blocks can be used to represent hundreds, tens, and ones. Notice that the tens rod is made up of 10 ones, and the hundreds square is made of 10 tens, or 100 ones.

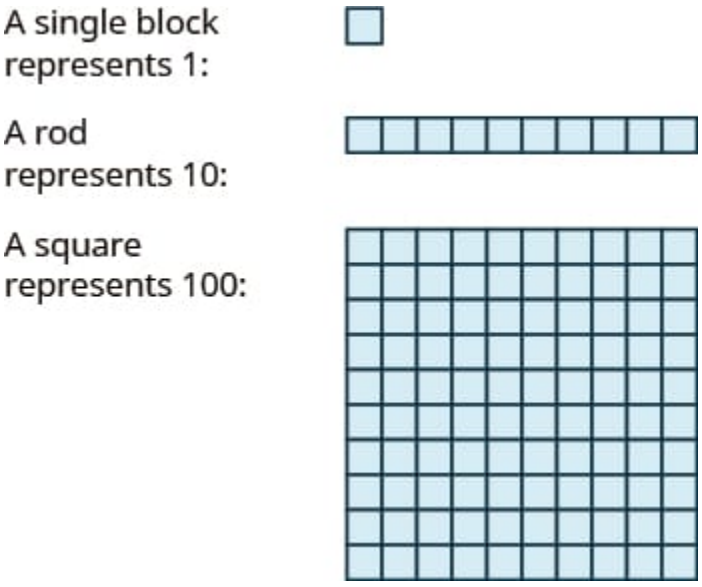


Figure 1.1.3

Figure 1.1.4 shows the number 138 modeled with base-10 blocks.

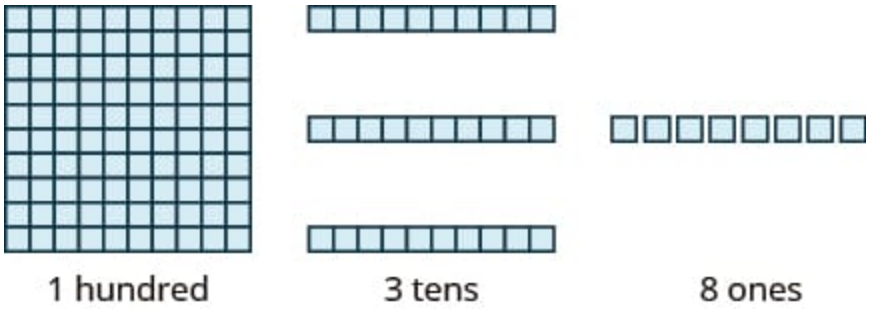
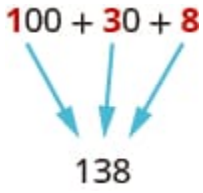


Figure 1.1.4: We use place value notation to show the value of the number 138.



Digit	Place value	Number	Value	Total value
1	hundreds	1	100	100
3	tens	3	10	30
8	ones	8	1	+8
			Sum =	138

Example 1.1.2: place value notation

Use place value notation to find the value of the number modeled by the base-10 blocks shown.

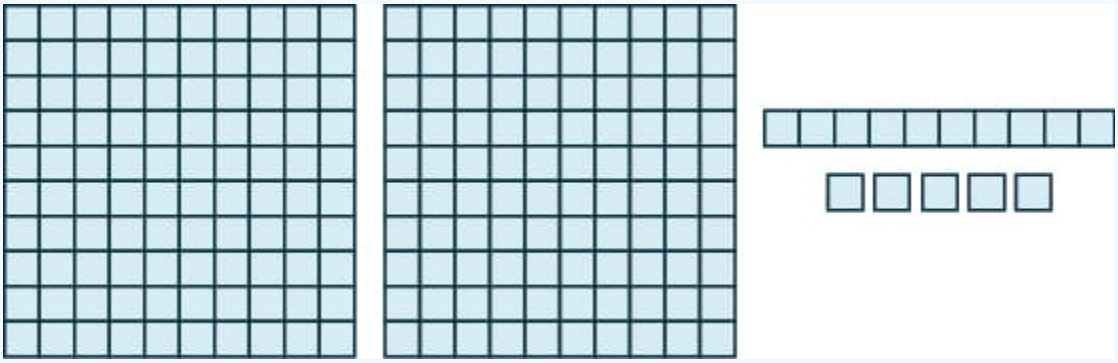


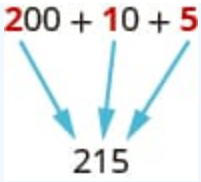
Figure 1.1.5

Solution

There are 2 hundreds squares, which is 200.

There is 1 tens rod, which is 10.

There are 5 ones blocks, which is 5.



Digit	Place value	Number	Value	Total value
2	hundreds	2	100	200
1	tens	1	10	10
5	ones	5	1	+5
				215

The base-10 blocks model the number 215.

Identify the Place Value of a Digit

By looking at money and base-10 blocks, we saw that each place in a number has a different value. A place value chart is a useful way to summarize this information. The place values are separated into groups of three, called periods. The periods are *ones, thousands, millions, billions, trillions*, and so on. In a written number, commas separate the periods.

Just as with the base-10 blocks, where the value of the tens rod is ten times the value of the ones block and the value of the hundreds square is ten times the tens rod, the value of each place in the place-value chart is ten times the value of the place to the right of it.

Figure 1.1.8 shows how the number $5,278,194$ is written in a place value chart.

Place Value														
Trillions			Billions			Millions			Thousands			Ones		
Hundred trillions	Ten trillions	Trillions	Hundred billions	Ten billions	Billions	Hundred millions	Ten millions	Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
								5	2	7	8	1	9	4

Figure 1.1.8

- The digit 5 is in the millions place. Its value is $5,000,000$.
- The digit 2 is in the hundred thousands place. Its value is $200,000$.
- The digit 7 is in the ten thousands place. Its value is $70,000$.
- The digit 8 is in the thousands place. Its value is $8,000$.
- The digit 1 is in the hundreds place. Its value is 100.

- The digit 9 is in the tens place. Its value is 90.
- The digit 4 is in the ones place. Its value is 4.

Example 1.1.3: place value

In the number $63,407,218$; find the place value of each of the following digits:

a. 7
b. 0
c. 1
d. 6
e. 3

Solution

Write the number in a place value chart, starting at the right.

Trillions			Billions			Millions			Thousands			Ones		
Hundred trillions	Ten trillions	Trillions	Hundred billions	Ten billions	Billions	Hundred millions	Ten millions	Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
							6	3	4	0	7	2	1	8

Figure 1.1.9

a. The 7 is in the thousands place.
b. The 0 is in the ten thousands place.
c. The 1 is in the tens place.
d. The 6 is in the ten millions place.
e. The 3 is in the millions place.

Exercise 1.1.6

For each number, find the place value of digits listed: $519,711,641,328$

a. 9
b. 4
c. 2
d. 6
e. 7

Answer a available at the end of this section.

Answer b available at the end of this section.

Answer c available at the end of this section.

Answer d available at the end of this section.

Answer e available at the end of this section.

Use Place Value to Name Whole Numbers

When you write a check, you write out the number in words as well as in digits. To write a number in words, write the number in each period followed by the name of the period without the 's' at the end. Start with the digit at the left, which has the largest place value. The commas separate the periods, so wherever there is a comma in the number, write a comma between the words. The ones period, which has the smallest place value, is not named.



So the number $37,519,248$ is written thirty-seven million, five hundred nineteen thousand, two hundred forty-eight. Notice that the word and is not used when naming a whole number.

How to: Name a Whole Number in Words.

Step 1. Starting at the digit on the left, name the number in each period, followed by the period name. Do not include the period name for the ones.

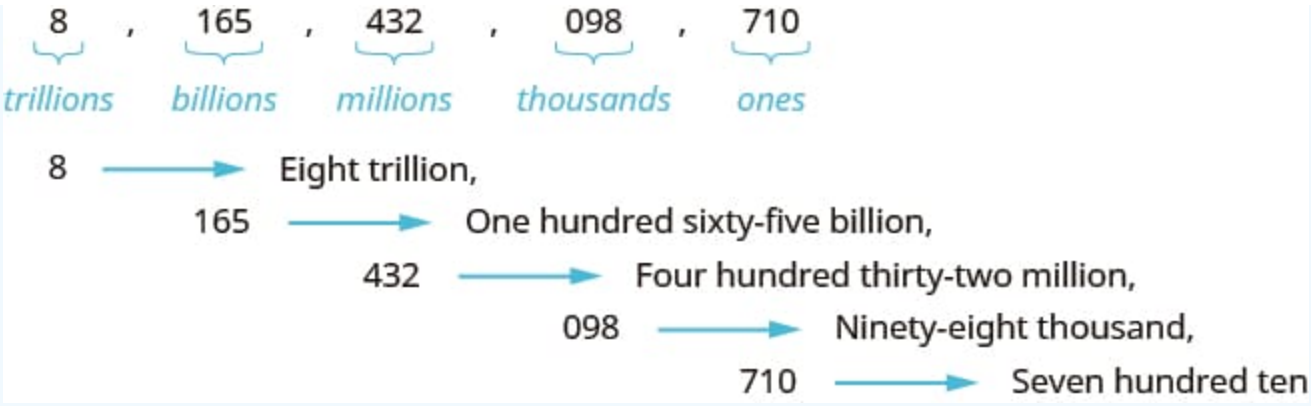
Step 2. Use commas in the number to separate the periods.

Example 1.1.4: name whole numbers

Name the number $8,165,432,098,710$ in words.

Solution

Begin with the leftmost digit, which is 8. It is in the trillions place.	eight trillion
The next period to the right is billions.	one hundred sixty-five billion
The next period to the right is millions.	four hundred thirty-two million
The next period to the right is thousands.	ninety-eight thousand
The rightmost period shows the ones.	seven hundred ten



Putting all of the words together, we write $8,165,432,098,710$ as eight trillion, one hundred sixty-five billion, four hundred thirty-two million, ninety-eight thousand, seven hundred ten.

Exercise 1.1.7

Name each number in words: $9,258,137,904,061$

Answer available at the end of this section.

Example 1.1.5: name whole numbers

A student conducted research and found that the number of mobile phone users in the United States during one month in 2014 was $327,577,529$. Name that number in words.

Solution

Identify the periods associated with the number.



Name the number in each period, followed by the period name. Put the commas in to separate the periods.

Millions period: three hundred twenty-seven million

Thousands period: five hundred seventy-seven thousand

Ones period: five hundred twenty-nine

So the number of mobile phone users in the Unites States during the month of April was three hundred twenty-seven million, five hundred seventy-seven thousand, five hundred twenty-nine.

Exercise 1.1.9

The population in a country is $316,128,839$. Name that number

Answer available at the end of this section.

Use Place Value to Write Whole Numbers

We will now reverse the process and write a number given in words as digits.

How to: Use Place Value to Write Whole Numbers

Step 1. Identify the words that indicate periods. (Remember the ones period is never named.)

Step 2. Draw three blanks to indicate the number of places needed in each period. Separate the periods by commas.

Step 3. Name the number in each period and place the digits in the correct place value position.

Example 1.1.6: write whole numbers

Write the following numbers using digits.

a. fifty-three million, four hundred one thousand, seven hundred forty-two
b. nine billion, two hundred forty-six million, seventy-three thousand, one hundred eighty-nine

Solution

a. Identify the words that indicate periods.

Except for the first period, all other periods must have three places. Draw three blanks to indicate the number of places needed in each period. Separate the periods by commas.

Then write the digits in each period.

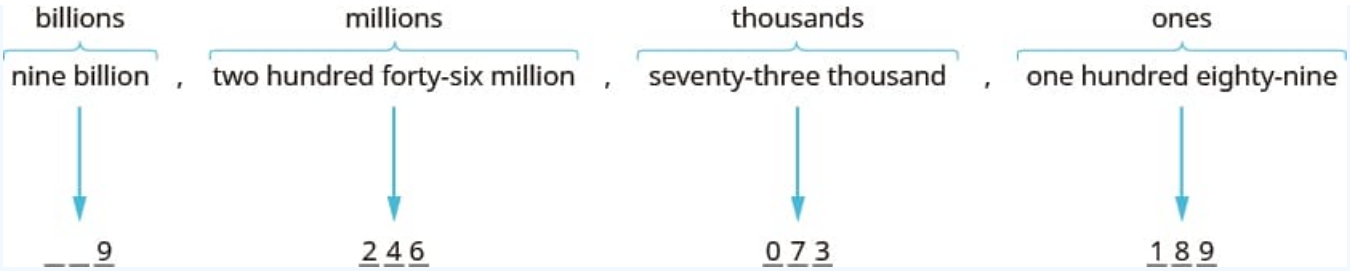


Put the numbers together, including the commas. The number is $53,401,742$.

b. Identify the words that indicate periods.

Except for the first period, all other periods must have three places. Draw three blanks to indicate the number of places needed in each period. Separate the periods by commas.

Then write the digits in each period.



The number is $9,246,073,189$.

Notice that in part (b), a zero was needed as a place-holder in the hundred thousands place. Be sure to write zeros as needed to make sure that each period, except possibly the first, has three places.

Exercise 1.1.11

Write each number in standard form:

fifty-three million, eight hundred nine thousand, fifty-one

Answer available at the end of this section.

Exercise 1.1.12

Write each number in standard form:

two billion, twenty-two million, seven hundred fourteen thousand, four hundred sixty-six

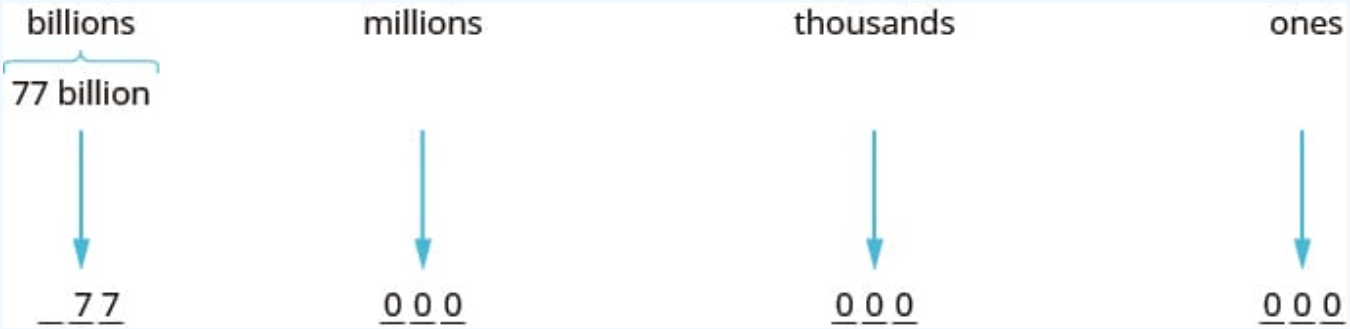
Answer available at the end of this section.

Example 1.1.7: write standard form

A state budget was about $\$77$ billion. Write the budget in standard form.

Solution

Identify the periods. In this case, only two digits are given and they are in the billions period. To write the entire number, write zeros for all of the other periods.



So the budget was about $\$77,000,000,000$.

Exercise 1.1.14

Write each number in standard form:

The total weight of an aircraft carrier is 204 million pounds.

Answer available at the end of this section.

Contributors and Attributions

- Lynn Marecek (Santa Ana College) and MaryAnne Anthony-Smith (formerly of Santa Ana College). This content produced by OpenStax and is licensed under a Creative Commons Attribution License 4.0 license.

Answers

Exercise 1.1.1
$2, 9, 241, 376$

Exercise 1.1.1
$0, 2, 9, 241, 376$

Exercise 1.1.2
$7, 13, 201$

Exercise 1.1.2
$0, 7, 13, 201$

Exercise 1.1.6
billions

Exercise 1.1.6
ten thousands

Exercise 1.1.6
tens

Exercise 1.1.6
hundred thousands

Exercise 1.1.6
hundred millions

Exercise 1.1.7
nine trillion, two hundred fifty-eight billion, one hundred thirty-seven million, nine hundred four thousand, sixty-one

Exercise 1.1.9
three hundred sixteen million, one hundred twenty-eight thousand, eight hundred thirty nine

Exercise 1.1.11
$53, 809, 051$

Exercise 1.1.12
$2, 022, 714, 466$

Exercise 1.1.14
$204, 000, 000\ \ pounds$

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1.1.1: Practice and Videos for Place Values and Names for Whole Numbers

Learning Objectives

- I can understand the definition of place value.
- I can use place value blocks, models and tables.
- I can determine the place value of a number.
- I can round using place value.

GeoGebra: Numbers up to the Thousands Place

Complete the activity below by dragging the sliders in the image, according to the instructions.

Videos on Place Value

Watch the following videos to learn about place value.

YouTube Video

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

YouTube Video

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

Practice!

Check your understanding of place value by answering these questions. Use the arrows at the bottom of the box to go to the next question.

Practice 1.1.1.1.1

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this Interactive Content online

Extra Practice Resources

For more practice on place value, visit these websites.

1. IXL: Place Value Models

2. IXL: Place Value Review

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1.2: Adding and Subtracting Whole Numbers

In the expression 3 + 4, which shows the sum of two whole numbers, the whole numbers 3 and 4 are called *addends* or *terms*. We can use a visual approach to find the sum of 3 and 4. First, construct a number line as shown in Figure 1.2.

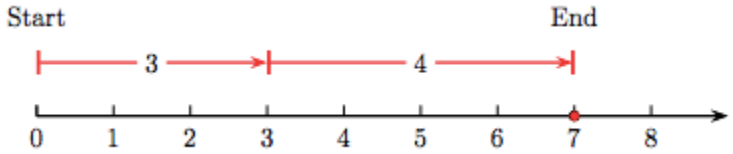


Figure 1.2: Adding whole numbers on the number line.

To add 3 and 4, proceed as follows.

1. Start at the number 0, then draw an arrow 3 units to the right, as shown in Figure 1.2. This arrow has magnitude (length) three and represents the whole number 3.
2. Draw a second arrow of length four, starting at the end of the first arrow representing the number 3. This arrow has magnitude (length) four and represents the whole number 4.
3. The sum of 3 and 4 could be represented by an arrow that starts at the number 0 and ends at the number 7. However, we prefer to mark this sum on the number line as a solid dot at the whole number 7. This number represents the sum of the whole numbers 3 and 4.

The Commutative Property of Addition

Let's change the order in which we add the whole numbers 3 and 4. That is, let's find the sum 4 + 3 instead.

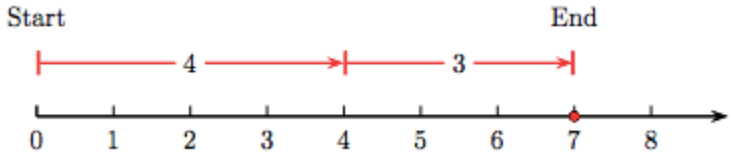


Figure 1.3: Addition is commutative; i.e., order doesn't matter.

As you can see in Figure 1.3, we start at zero then draw an arrow of length four, followed by an arrow of length three. However, the result is the same; i.e., 4 + 3 = 7.

Thus, the order in which we add three and four does not matter; that is,

$$3 + 4 = 4 + 3.$$

This property of addition of whole numbers is known as the *commutative property* of addition.

The Commutative Property of Addition

Let a and b represent two whole numbers. Then,

$$a + b = b + a.$$

Grouping Symbols

In mathematics, we use *grouping symbols* to affect the order in which an expression is evaluated. Whether we use parentheses, brackets, or curly braces, the expression inside any pair of grouping symbols must be evaluated first. For example, note how we

first evaluate the sum in the parentheses in the following calculation.

$$(3 + 4) + 5 = 7 + 5$$

$$= 12$$

The rule is simple: Whatever is inside the parentheses is evaluated first.

Writing Mathematics

When writing mathematical statements, follow the mantra:

One equal sign per line.

We can use brackets instead of parentheses.

$$5 + [7 + 9] = 5 + 16$$

$$= 21$$

Again, note how the expression inside the brackets is evaluated first.

We can also use curly braces instead of parentheses or brackets.

$$\{2+3\} + 4 = 5 + 4$$

$$= 9$$

Again, note how the expression inside the curly braces is evaluated first.

If grouping symbols are nested, we evaluate the innermost parentheses first.

For example,

$$2 + [3 + (4 + 5)] = 2 + [3 + 9]$$

$$= 2 + 12$$

$$= 14.$$

Grouping Symbols

Use parentheses, brackets, or curly braces to delimit the part of an expression you want evaluated first. If grouping symbols are nested, evaluate the expression in the innermost pair of grouping symbols first.

The Associative Property of Addition

Consider the evaluation of the expression (2+3)+4. We evaluate the expression in parentheses first.

$$(2 + 3) + 4 = 5 + 4$$

$$= 9$$

Now, suppose we change the order of addition to 2 + (3 + 4). Then,

$$2 + (3 + 4) = 2 + 7$$

$= 9.$

Although the grouping has changed, the result is the same. That is,

$$(2 + 3) + 4 = 2 + (3 + 4).$$

This property of addition of whole numbers is called the *associate property* of addition.

Associate Property of Addition

Let a, b, and c represent whole numbers. Then,

$$(a + b) + c = a + (b + c).$$

Because of the associate property of addition, when presented with a sum of three numbers, whether you start by adding the first two numbers or the last two numbers, the resulting sum is the same.

The Additive Identity

Imagine a number line visualization of the sum of four and zero; i.e., $4 + 0$.

In Figure 1.4, we start at zero, then draw an arrow of magnitude (length) four pointing to the right. Now, at the end of this arrow, attach a second arrow of length zero. Of course, that means that we remain right where we are, at 4. Hence the shaded dot at 4 is the sum. That is, $4 + 0 = 4$.

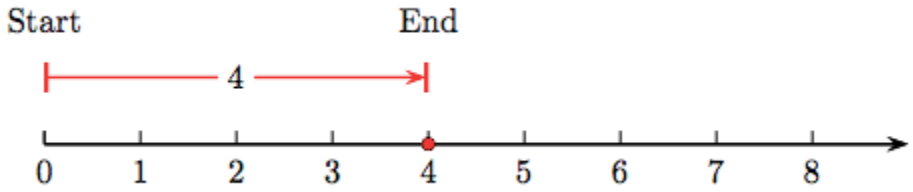


Figure 1.4: Adding zero to four.

The Additive Identity Property

The whole number zero is called the additive identity. If a is any whole number, the

$$a + 0 = a.$$

The number zero is called the additive identity because if you add zero to any number, you get the identical number back.

Adding Larger Whole Numbers

For completeness, we include two examples of adding larger whole numbers. Hopefully, the algorithm is familiar from previous coursework.

Example 1

Simplify: 1, 234 + 498.

Solution

Align the numbers vertically, then add, starting at the furthest column to the right. Add the digits in the ones column, 4 + 8 = 12. Write the 2, then carry a 1 to the tens column. Next, add the digits in the tens column, 3 + 9 = 12, add the carry to get 13, then write the 3 and carry a 1 to the hundreds column. Continue in this manner, working from right to left

$$\begin{array}{r} {\scriptstyle 11} \\ 1234 \\ +\ \ 498 \\ \hline 1732 \end{array}$$

Therefore, 1, 234 + 498 = 1, 732

Exercise

Simplify: 1,286 + 349.

Answer available at the end of this section.

Add three or more numbers in the same manner.

Example 2

Simplify: 256 + 322 + 418.

Solution

Align the numbers vertically, then add, starting at the furthest column to the right. Add the digits in the ones column, 6 + 2 + 8 = 16. Write the 6, then carry a 1 to the tens column. Continue in this manner, working from right to left.

$$\begin{array}{r} {\scriptstyle 11} \\ 256 \\ 322 \\ +418 \\ \hline 996 \end{array}$$

Therefore, 256 + 322 + 418 = 996.

Exercise

Simplify: 256 + 342 + 283

Answer available at the end of this section.

Subtraction of Whole Numbers

The key idea is this: *Subtraction is the opposite of addition*. For example, consider the difference 7 - 4 depicted on the number line in Figure 1.5.

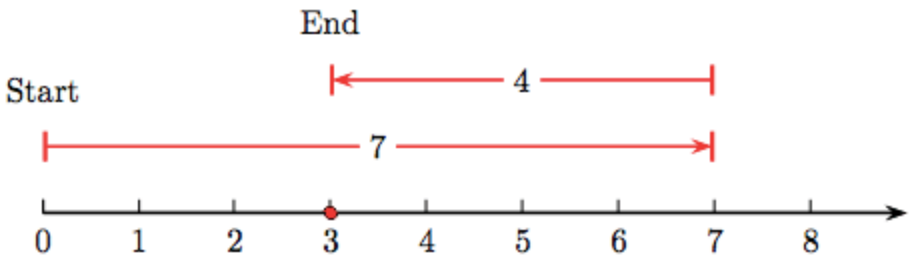


Figure 1.5: Subtraction means *add the opposite*.

If we were adding 7 and 4, we first draw an arrow starting at zero pointing to the right with magnitude (length) seven. Then, to add 4, we would draw a second arrow of magnitude (length) 4, attached to the end of the first arrow and pointing to the right.

However, because subtraction is the *opposite* of addition, in Figure 1.5 we attach an arrow of magnitude (length) four to the end of the first arrow, but *pointing in the opposite direction* (to the left). Note that this last arrow ends at the answer, which is a shaded dot on the number line at 3. That is, 7 – 4 = 3.

Note that subtraction is **not commutative**; that is, it make no sense to say that 7 – 5 is the same as 5 – 7.

Subtraction is **not associative**. It is not the case that (9 – 5) – 2 is the same as 9 – (5 – 2). On the one hand,

$$(9 - 5) - 2 = 4 - 2$$

$$= 2,$$

but

$$9 - (5 - 2) = 9 - 3$$

$$= 6.$$

Subtracting Larger Whole Numbers

Much as we did with adding larger whole numbers, to subtract two large whole numbers, align them vertically then subtract, working from right to left. You may have to “borrow” to complete the subtraction at any step.

Example 3

Simplify: 1, 755 – 328.

Solution

Align the numbers vertically, then subtract, starting at the ones column, then working right to left. At the ones column, we cannot subtract 8 from 5, so we borrow from the previous column. Now, 8 from 15 is 7. Continue in this manner, working from right to left.

$$\begin{array}{r} {\scriptstyle 4} \\ 1\,7\,\cancel{5}^{\,1}5 \\ -\quad 3\,2\,8 \\ \hline 1\,4\,2\,7 \end{array}$$

Therefore, 1, 755 – 328 = 1, 427.

Exercise

Simplify: 5,635 - 288.

Answer available at the end of this section.

Order of Operations

In the absence of grouping symbols, it is important to understand that addition holds no precedence over subtraction, and vice-versa.

> Perform all additions and subtractions in the order presented, moving left to right.

Let's look at an example.

Example 4

Simplify the expression $15 - 8 + 4$.

Solution

This example can be trickier than it seems. However, if we follow the rule (perform all additions and subtractions in the order presented, moving left to right), we should have no trouble. First comes fifteen minus eight, which is seven. Then seven plus four is eleven.

$$\begin{aligned} 15 - 8 + 4 &= 7 + 4 \\ &= 11. \end{aligned}$$

Exercise

Simplify: $25 - 10 + 8$.

Answer available at the end of this section.

Caution! Incorrect answer ahead!

Note that it is possible to arrive at a different (but incorrect) answer if we favor addition over subtraction in Example 4. If we first add eight and four, then 15 – 8 + 4 becomes 15 – 12, which is 3. However, note that **this is incorrect**, because it violates the rule "perform all additions and subtractions in the order presented, moving left to right."

Applications — Geometry

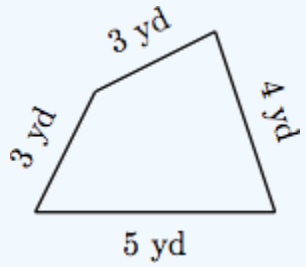
There are any number of applications that require a sum or difference of whole numbers. Let's examine a few from the world of geometry.

Perimeter of a Polygon

In geometry a polygon is a plane figure made up of a closed path of a finite sequence of segments. The segments are called the *edges* or *sides* of the polygon and the points where two edges meet are called the *vertices* of the polygon. The *perimeter* of any polygon is the sum of the lengths of its sides.

Example 5

A quadrilateral is a polygon with four sides. Find the perimeter of the quadrilateral shown below, where the sides are measured below.



Solution

To find the perimeter of the quadrilateral, find the sum of the lengths of the sides.

$$\text{Perimeter} = 3 + 3 + 4 + 5 = 15$$

Hence, the perimeter of the quadrilateral is 15 yards.

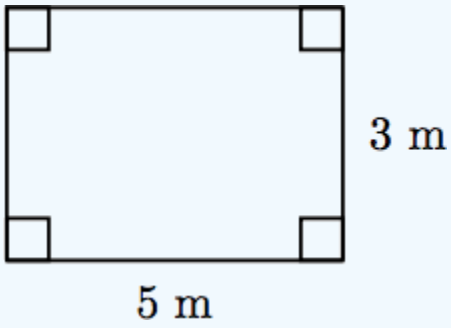
Exercise

A quadrilateral has sides that measure in 4 in., 3 in., 5 in., and 5 in. Find the perimeter.

Answer available at the end of this section.

Example 6

A quadrilateral (four sides) is a rectangle if all four of its angles are right angles. It can be shown that the opposite sides of a rectangle must be equal. Find the perimeter of the rectangle shown below, where the sides of the rectangle are measured in meters.



Solution

To find the perimeter of the rectangle, find the sum of the four sides. Because opposite sides have the same length, we have two sides of length 5 meters and two sides of length 3 meters. Hence,

$$\text{Perimeter} = 5 + 3 + 5 + 3 = 16.$$

Thus, the perimeter of the rectangle is 16 meters.

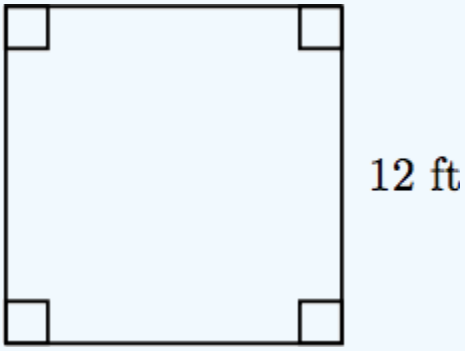
Exercise

A rectangle has length 12 meters and width 8 meters. Find its perimeter.

Answer available at the end of this section.

Example 7

A quadrilateral (four sides) is a *square* if all four of its sides measures 18 centimeters. Find its perimeter. are equal and all four of its angles are right angles. Pictured below is a square having a side of length 12 feet. Find the perimeter of the square.



Solution

Because the quadrilateral is a square, all four sides have the same length, namely 12 feet. To find the perimeter of the square, find the sum of the four sides. Perimeter = 12 + 12 + 12 + 12 = 48

Hence, the perimeter of the square is 48 feet.

Exercise

A square has a side that measures 18 centimeters. Find its perimeter.

Answer available at the end of this section.

Application — Alternative Fuels

Automobiles that run on alternative fuels (other than gasoline) have increased in the United States over the years.

Example 8

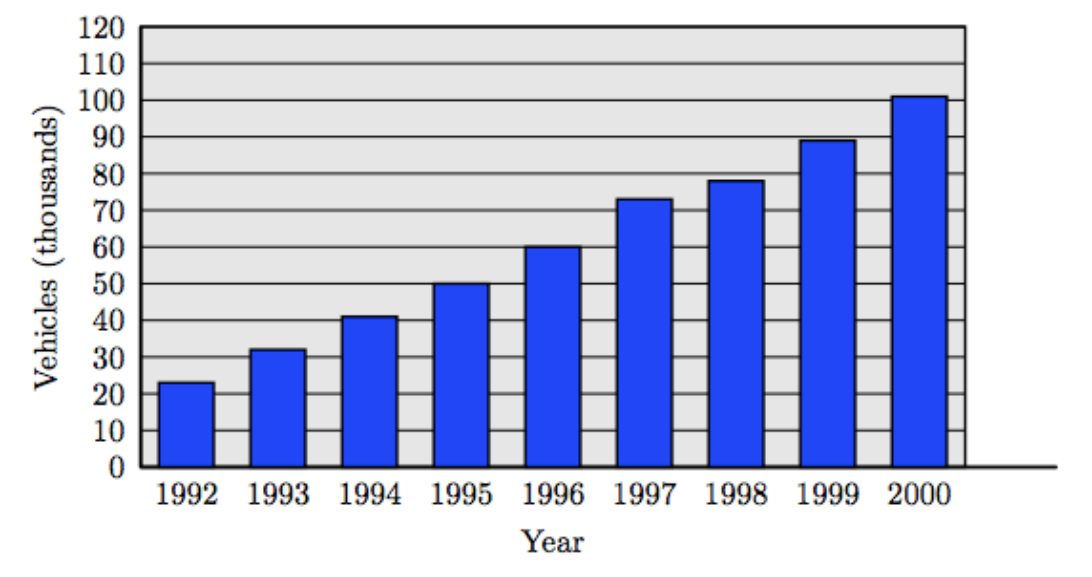
Table 1.2 show the number of cars (in thousands) running on compressed natural gas versus the year. Create a bar chart showing the number of cars running on compressed natural gas versus the year.

Year	1992	1993	1994	1995	1996	1997	1998	1999	2000
Number	23	32	4`	50	60	73	78	89	101

Table 1.2: Number of vehicles (in thousands) running on compressed natural gas.

Solution

Place the years on the horizontal axis. At each year, sketch a bar having height equal to the number of cars in that year that are running on compressed natural gas. Scale the vertical axis in thousands.



Exercise

The following table shows the number of hybrid cars (in thousands) by country.

Country	**Number**
U.S.	279
Japan	77
Canada	17
U.K.	14
Netherlands	11

Create a bar chart showing the number of cars versus the country of use.

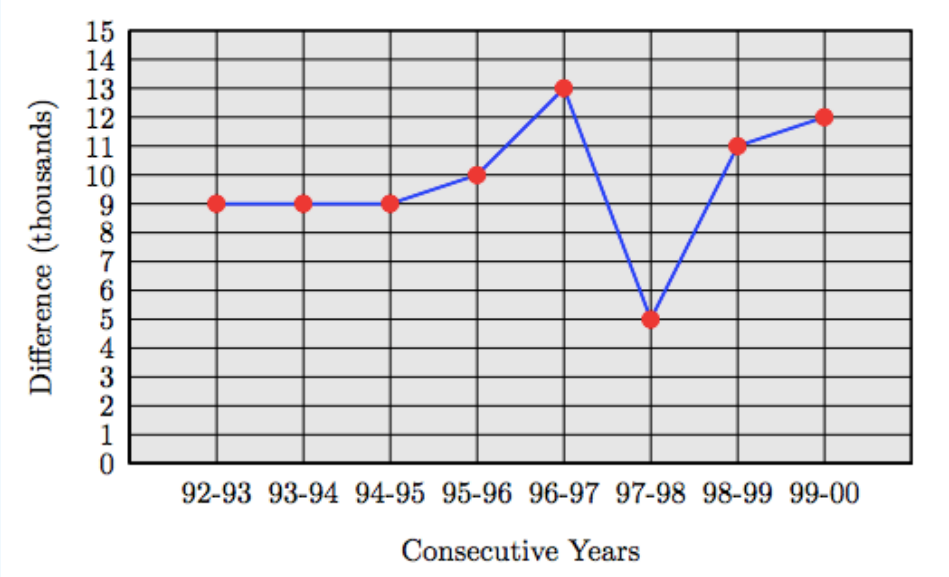
Example 9

Using the data in Table 1.2, crease a table that shows the differences in consecutive years, then create a line plot of the result. In what consecutive years did the United States see the greatest increase in cars powered by compressed natural gas?

Solution

Table 1.3 shows the differences in consecutive years.

Next, craft a line graph. Place consecutive years on the horizontal axis. At each consecutive year pair, plot a point at a height equal to the difference in alternative fuel vehicles. Connect the points with straight line segments.



Note how the line graph makes it completely clear that the greatest increase in vehicles powered by compressed natural gas occurred in the consecutive years 1996-1997, an increase of 13,000 vehicles.

Exercise

The following table show Alphonso's percentage scores on his examinations in mathematics.

Exam	Percentage
Exam #1	52
Exam #2	45
Exam #3	72
Exam #4	889
Exam #5	76

Construct a line graph of Alphonso's exam scores versus exam number.

Exercises

1. Sketch a number line diagram depicting the sum 3 + 2, as shown in Figure 1.2 in the narrative of this section.
2. Sketch a number line diagram depicting the sum 3 + 5, as shown in Figure 1.2 in the narrative of this section.
3. Sketch a number line diagram depicting the sum 3 + 4, as shown in Figure 1.2 in the narrative of this section.
4. Sketch a number line diagram depicting the sum 2 + 4, as shown in Figure 1.2 in the narrative of this section.
5. Sketch a number line diagram depicting the sum 4 + 2, as shown in Figure 1.2 in the narrative of this section.
6. Sketch a number line diagram depicting the sum 4 + 3, as shown in Figure 1.2 in the narrative of this section.

7. Sketch a number line diagram depicting the sum 2 + 5, as shown in Figure 1.2 in the narrative of this section.

8. Sketch a number line diagram depicting the sum 4 + 5, as shown in Figure 1.2 in the narrative of this section.

9. Sketch a number line diagram depicting the sum 4 + 4, as shown in Figure 1.2 in the narrative of this section.

10. Sketch a number line diagram depicting the sum 3 + 3, as shown in Figure 1.2 in the narrative of this section.

In Exercises 11-28, determine which property of addition is depicted by the given identity

11. 28 + 0 = 28

12. 53 + 0 = 53

13. 24 + 0 = 24

14. 93 + 0 = 93

15. (51 + 66) + 88 = 51 + (66 + 88)

16. (90 + 96) + 4 = 90 + (96 + 4)

17. 64 + 39 = 39 + 64

18. 68 + 73 = 73 + 68

19. (70 + 27) + 52 = 70 + (27 + 52)

20. (8 + 53) + 81 = 8 + (53 + 81)

21. 79 + 0 = 79

22. 42 + 0 = 42

23. 10 + 94 = 94 + 10

24. 55 + 86 = 86 + 55

25. 47 + 26 = 26 + 47

26. 62 + 26 = 26 + 62

27. (61 + 53) + 29 = 61 + (53 + 29)

28. (29 + 96) + 61 = 29 + (96 + 61)

---

29. Sketch a number line diagram depicting the difference 8–2, as shown in Figure 1.5 in the narrative of this section.

30. Sketch a number line diagram depicting the difference 8–4, as shown in Figure 1.5 in the narrative of this section.

31. Sketch a number line diagram depicting the difference 7–2, as shown in Figure 1.5 in the narrative of this section.

32. Sketch a number line diagram depicting the difference 9–5, as shown in Figure 1.5 in the narrative of this section.

33. Sketch a number line diagram depicting the difference 7–4, as shown in Figure 1.5 in the narrative of this section.

34. Sketch a number line diagram depicting the difference 6–4, as shown in Figure 1.5 in the narrative of this section.

35. Sketch a number line diagram depicting the difference 9–4, as shown in Figure 1.5 in the narrative of this section.

36. Sketch a number line diagram depicting the difference 6–5, as shown in Figure 1.5 in the narrative of this section.

37. Sketch a number line diagram depicting the difference 8–5, as shown in Figure 1.5 in the narrative of this section.

38. Sketch a number line diagram depicting the difference 9–3, as shown in Figure 1.5 in the narrative of this section.

In Exercises 39-50, simplify the given expression.

39. 16 – 8+2

40. 17 – 3+5

41. 20 – 5 + 14

42. 14 – 5+6

43. 15 – 2+5

44. 13 – 4+2

45. 12 – 5+4

46. 19 – 4 + 13

47. 12 – 6+4

48. 13 – 4 + 18

49. 15 – 5+8

50. 13 – 3 + 11

In Exercises 51-58, the width W and length L of a rectangle are given. Find the perimeter P of the rectangle.

51. W = 7 in, L = 9 in

52. W = 4 in, L = 6 in

53. W = 8 in, L = 9 in

54. W = 5 in, L = 9 in

55. W = 4 cm, L = 6 cm

56. W = 5 in, L = 8 in

57. W = 4 cm, L = 7 cm

58. W = 4 in, L = 9 in

In Exercises 59-66, the length s of a side of a square is given. Find the perimeter P of the square.

59. s = 25 cm

60. s = 21 in

61. s = 16 cm

62. s = 10 in

63. s = 18 in

64. s = 7 in

65. s = 3 in

66. s = 20 in

In Exercises 67-86, find the sum.

67. 3005 + 5217

68. 1870 + 5021

69. 575 + 354 + 759

70. 140 + 962 + 817

71. 472 + (520 + 575)

72. 318 + (397 + 437)

73. 274 + (764 + 690)

74. 638 + (310 + 447)

75. 8583 + 592

76. 5357 + 9936

77. 899 + 528 + 116

78. 841 + 368 + 919

79. (466 + 744) + 517

80. (899 + 996) + 295

81. 563 + 298 + 611 + 828

82. 789 + 328 + 887 + 729

83. 607 + 29 + 270 + 245

84. 738 + 471 + 876 + 469

85. (86 + 557) + 80

86. (435 + 124) + 132

In Exercises 87-104, find the difference.

87. 3493 – 2034 – 227

88. 3950 – 1530 – 2363

89. 8338 – 7366

90. 2157 – 1224

91. 2974 – 2374

92. 881 – 606

93. 3838 – (777 – 241)

94. 8695 – (6290 – 4233)

95. 5846 – 541 – 4577

96. 5738 – 280 – 4280

97. 3084 – (2882 – 614)

98. 1841 – (217 – 28)

99. 2103 – (1265 – 251)

100. 1471 – (640 – 50)

101. 9764 – 4837 – 150

102. 9626 – 8363 – 1052

103. 7095 – 226

104. 4826 – 1199

---

105. **Water Subsidies**. Since the drought began in 2007, California farms have received $79 million in water subsidies. California cotton and rice farmers received an additional $439 million. How much total water subsidies have farmers received? *Associated Press Times-Standard 4/15/09*

106. **War Budget**. The 2010 Federal budget allocates $534 billion for the Department of Defense base programs and an additional $130 billion for the nation's two wars. How much will the Department of Defense receive altogether? *Associated Press Times-Standard 5/8/09*

107. **Sun Frost**. Arcata, CA is home to Sun Frost, a manufacturer of highly efficient refrigerators and freezers. The AC model RF12 refrigerator/freezer costs $2,279 while an R16 model refrigerator/freezer costs $3,017. How much more does the R16 model cost? Source: www.sunfrost.com/retail pricelist.html

108. **Shuttle Orbit**. The space shuttle usually orbits at 250 miles above the surface of the earth. To service the Hubble Space Telescope, the shuttle had to go to 350 miles above the surface. How much higher did the shuttle have to orbit?

109. **Earth's Orbit**. Earth orbits the sun in an ellipse. When earth is at its closest to the sun, called *perihelion*, earth is about 147 million kilometers. When earth is at its furthest point from the sun, called *aphelion*, earth is about 152 million kilometers from the sun. What's the difference in millions of kilometers between aphelion and perihelion?

110. **Pluto's Orbit**. Pluto's orbit is highly eccentric. Find the difference between Pluto's closest approach to the sun and Pluto's furthest distance from the sun if Pluto's perihelion (closest point on its orbit about the sun) is about 7 billion kilometers and its aphelion (furthest point on its orbit about the sun) is about 30 billion kilometers.

111. **Sunspot Temperature**. The surface of the sun is about 10,000 degrees Fahrenheit. Sunspots are darker regions on the surface of the sun that have a relatively cooler temperature of 6,300 degrees Fahrenheit. How many degrees cooler are sunspots?

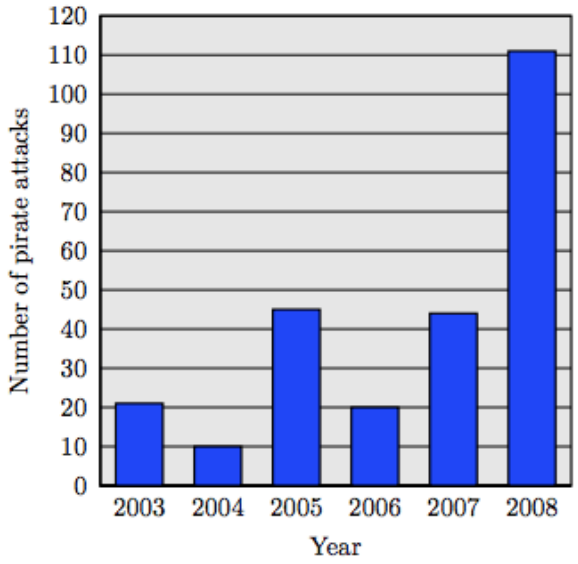
112. **Jobs**. The Times-Standard reports that over the next year, the credit- and debitcard processing business Humboldt Merchant Services expects to cut 36 of its 80 jobs, but then turn around and hire another 21. How many people will be

working for the company then? *Times-Standard 5/6/09*

113. **Wild tigers**. The chart shows the estimated wild tiger population, by region. According to this chart, what is the total wild tiger population worldwide? *Associated Press-Times-Standard 01/24/10 Pressure mounts to save the tiger.*

Region	Tiger population
India, Nepal and Bhutan	1650
China and Russia	450
Bangladesh	250
Sumatra (Indonesia)	400
Malaysia	500
other SE Asia	350

114. **Pirate Attacks**. The following bar chart tracks pirate attacks off the coast of Somalia.



Source: ICC International Maritime Bureau, AP Times-Standard, 4/15/2009

a) How many pirate attacks were there in 2003, 2004, and 2005 combined?

b) How many pirate attacks were there in 2006, 2007, and 2008 combined?

c) How many more pirate attacks were there in 2008 than in 2007?

115. Emily shows improvement on each successive examination throughout the term. Her exam scores are recorded in the following table.

Exam	Score
Exam #1	48
Exam #2	51
Exam #3	54
Exam #4	59
Exam #5	67

Exam #6	70

a) Create a bar plot for Emily's examination scores. Place the examination numbers on the horizontal axis in the same order shown in the table above.

b) Create a table that shows successive differences in examination scores. Make a line plot of these differences. Between which two exams did Emily show the greatest improvement?

116. Jason shows improvement on each successive examination throughout the term. His exam scores are recorded in the following table.

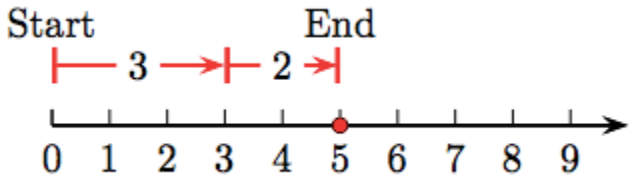
Exam	**Score**
Exam #1	34
Exam #2	42
Exam #3	45
Exam #4	50
Exam #5	57
Exam #6	62

a) Create a bar plot for Jason's examination scores. Place the examination numbers on the horizontal axis in the same order shown in the table above.

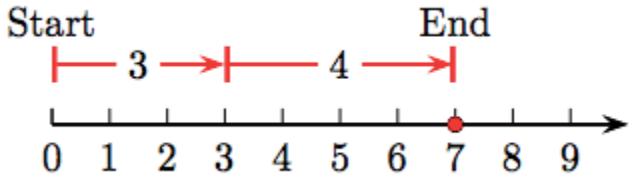
b) Create a table that shows successive differences in examination scores. Make a line plot of these differences. Between which two exams did Jason show the greatest improvement?

Answers

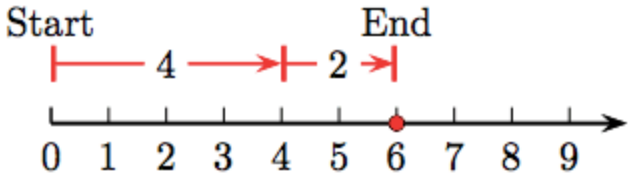
1. 3 + 2 = 5.



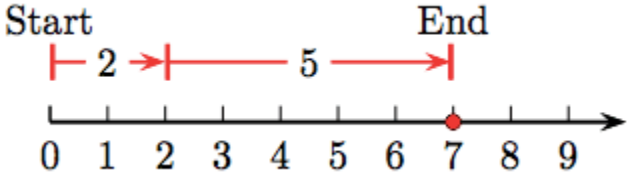
3. 3 + 4 = 7.



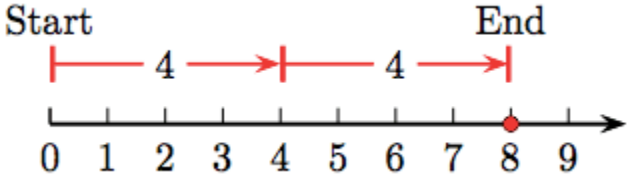
5. 4 + 2 = 6.



7. 2 + 5 = 7.



9. 4 + 4 = 8.



11. Additive identity property of addition.

13. Additive identity property of addition.

15. Associative property of addition

17. Commutative property of addition

19. Associative property of addition

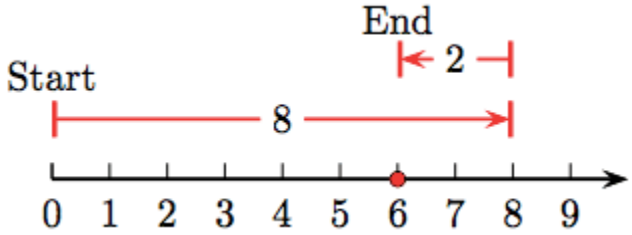
21. Additive identity property of addition.

23. Commutative property of addition

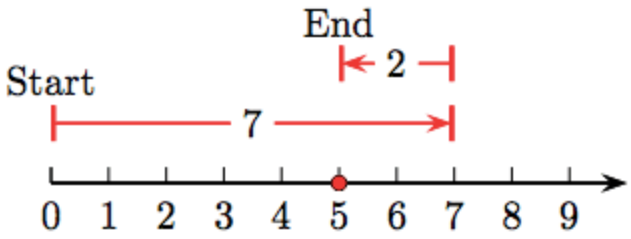
25. Commutative property of addition

27. Associative property of addition

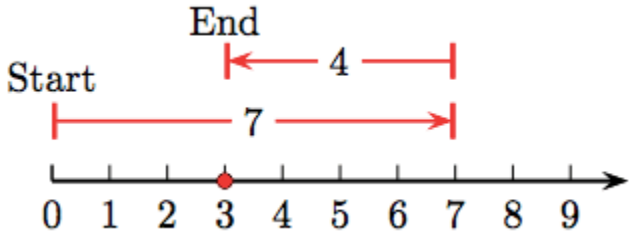
29. 8 – 2 = 6.



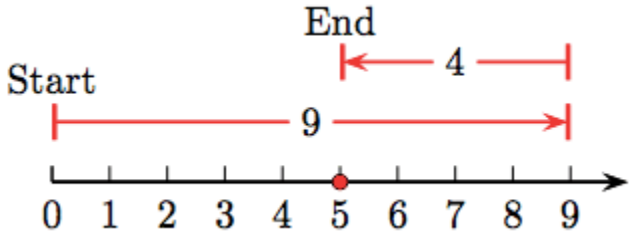
31. 7 – 2 = 5.



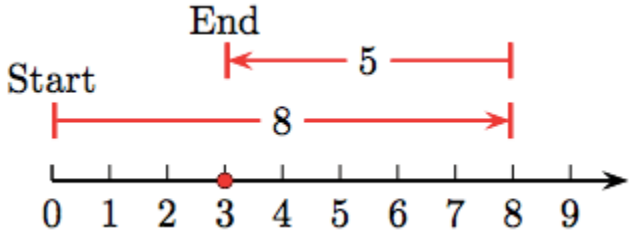
33. 7 – 4 = 3.



35. 9 – 4 = 5.



37. 8 – 5 = 3.



39. 10

41. 29

43. 18

45. 11

47. 10

49. 18

51. P = 32 in

53. P = 34 in

55. P = 20 cm

57. P = 22 cm

59. P = 100 cm

61. P = 64 cm

63. P = 72 in

65. P = 12 in

67. 8222

69. 1688

71. 1567

73. 1728

75. 9175

77. 1543

79. 1727

81. 2300

83. 1151

85. 723

87. 1232

89. 972

91. 600

93. 3302

95. 728

97. 816

99. 1089

101. 4777

103. 6869

105. $518 million

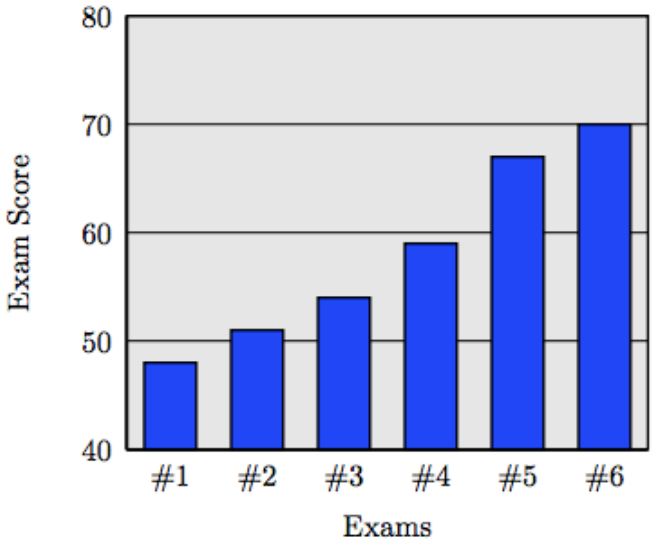
107. $738

109. 5 million kilometers

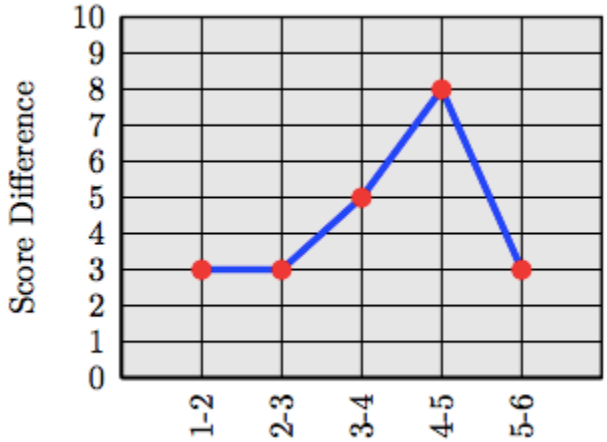
111. 3,700 degrees Fahrenheit

113. 3600

115. a) Bar chart.



b) Line plot of consecutive differences. The line plot of consecutive examination score differences.



The largest improvement was between Exam #4 and Exam #5, where Emily improved by 8 points.

Answers

Exercise
1635

Exercise
881

Exercise
5,347

Exercise
23

Exercise
17 inches

Exercise
40 meters

Exercise
72 centimeters

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1.2.1: Practice and Videos for Adding Whole Numbers

Learning Objectives

- I can use place value understanding to add multi-digit whole numbers.
- I can identify basic multi-digit addition strategies.

Geogebra: Addition Practice

Practice single-digit addition by completing the activity below. Insert the sum of each equation in the boxes to the right of the equals sign.

Videos on Multi-digit Addition

Watch the following two videos to learn about addition. The videos are appropriately captioned.

YouTube Video

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

YouTube Video

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

Practice!

Check your understanding of addition by answering these questions. Use the arrows at the bottom of the box to go to the next question.

Practice 1.2.1.1

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View this Interactive Content online

Extra Practice Resources

For more practice on place value, visit these websites.

1. Math is Fun: Who Wants to be a Mathionaire - Addition Quiz

2. IXL: Add two multi-digit numbers

3. IXL: Add two multi-digit numbers: word problems

4. Math playground: Alien Addition

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1.2.2: Practice and Videos for Subtracting Whole Numbers

Learning Objectives

- I can understand multi-digit subtraction of whole numbers.
- I can use place value understanding to subtract multi-digit whole numbers.

GeoGebra: Subtraction Number Line

Review your understanding of addition and start learning subtraction by completing the activity below. Enter any first and the second number of your choice. To change the calculation operation, click the drop-down menu. Observe the number line to formulate a solution. **Check your solution** by clicking the numerical expression that shows the calculation.

Videos on Subtraction

Watch the following videos to learn about multi-digit subtraction. All videos have appropriate captions in English; in some cases, there may also be appropriate captions in other languages.

YouTube Video

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

YouTube Video

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

YouTube Video

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

Practice!

Check your understanding of subtraction by answering these questions. Use the arrows at the bottom of the box to switch between questions.

Practice 1.2.2.1

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View this Interactive Content online

Extra Resources

For more practice on multi-digit subtraction, visit these websites.

1. ORYX: Multi-Digit Subtraction

2. IXL: Subtract two multi-digit numbers: word problems

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1.2.3: Common Mistakes for Subtracting Whole Numbers

Common Mistake 1: Switching the Order in Subtraction

Spot the mistake 1.2.3.1

The following problem has an error. Identify the mistake.

$5 - 17 = 12$

Click for explanation available at the end of this section.

Common Mistake 2: Subtracting the Smaller Digit from the Larger Digit

Spot the mistake 1.2.3.1

The following problem has an error. Identify the mistake.

$28 - 9 = 21$

$$\begin{array}{rr} 2 & 8 \\ - & 9 \\ \hline 2 & 1 \end{array} \tag{1.2.3.1}$$

Click for explanation available at the end of this section.

Answers

Spot the mistake 1.2.3.1

The answer is **not** 12.

The key is to recognize that $5 - 17$ and $17 - 5$ are **not** the same thing.

For example, if you have \$5. Then you spend \$17 on candy.

Since you did not have enough money, you **owe** money. You are \$12 below zero.

So the correct answer is -12, meaning you owe **\$12**.

View the number line below to visualize this concept:



(1.2.3.2)

Start at 5. Move left past 0. Land at -12.

$$5 - 17 = -12$$

Key Idea

Subtraction is **not commutative**.

That means changing the order can change the answer.

$$17 - 5 \neq 5 - 17 \tag{1.2.3.3}$$

Spot the mistake 1.2.3.1

The answer is **not** 21.

Explanation	Work

Step 1: Write out the problem with the numbers stacked.	$\begin{array}{r} 2\ 8 \\ -\ 9 \end{array}$
Step 2: Since we cannot subtract 9 ones from 8 ones, we need to regroup. We can borrow 1 ten from the 2 tens on the left. This means the 2 tens become 1 ten and the 8 ones become 18 ones.	$\begin{array}{r} \color{blue}{1}\ \color{blue}{18} \\ \cancel{2}\ \cancel{8} \\ -\ 9 \end{array}$
Step 3: Now subtract. Bring down the 1 ten. The correct answer is 19.	$\begin{array}{r} \color{blue}{1}\ \color{blue}{18} \\ \cancel{2}\ \cancel{8} \\ -\ 9 \\ \hline 1\ 9 \end{array}$

Key Idea

When subtracting, do not just subtract the smaller digit from the larger digit. The order matters. If the top digit is too small, you may need to regroup.

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1.3: Multiplying Whole-Digit Numbers

In the expression 3 + 4, which shows the sum of two whole numbers, the whole numbers 3 and 4 are called *addends* or *terms*. We can use a visual approach to find the sum of 3 and 4. First, construct a number line as shown in Figure 1.2.

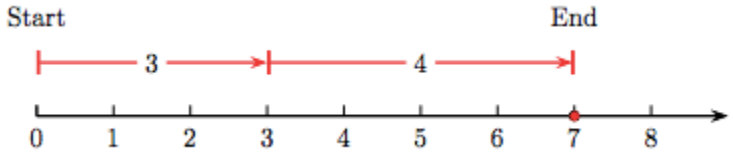


Figure 1.2: Adding whole numbers on the number line.

To add 3 and 4, proceed as follows.

1. Start at the number 0, then draw an arrow 3 units to the right, as shown in Figure 1.2. This arrow has magnitude (length) three and represents the whole number 3.
2. Draw a second arrow of length four, starting at the end of the first arrow representing the number 3. This arrow has magnitude (length) four and represents the whole number 4.
3. The sum of 3 and 4 could be represented by an arrow that starts at the number 0 and ends at the number 7. However, we prefer to mark this sum on the number line as a solid dot at the whole number 7. This number represents the sum of the whole numbers 3 and 4.

The Commutative Property of Addition

Let's change the order in which we add the whole numbers 3 and 4. That is, let's find the sum 4 + 3 instead.

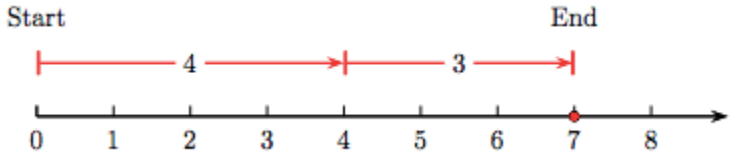


Figure 1.3: Addition is commutative; i.e., order doesn't matter.

As you can see in Figure 1.3, we start at zero then draw an arrow of length four, followed by an arrow of length three. However, the result is the same; i.e., 4 + 3 = 7.

Thus, the order in which we add three and four does not matter; that is,

$$3 + 4 = 4 + 3.$$

This property of addition of whole numbers is known as the *commutative property* of addition.

The Commutative Property of Addition

Let a and b represent two whole numbers. Then,

$$a + b = b + a.$$

Grouping Symbols

In mathematics, we use *grouping symbols* to affect the order in which an expression is evaluated. Whether we use parentheses, brackets, or curly braces, the expression inside any pair of grouping symbols must be evaluated first. For example, note how we

first evaluate the sum in the parentheses in the following calculation.

$$(3 + 4) + 5 = 7 + 5$$

$$= 12$$

The rule is simple: Whatever is inside the parentheses is evaluated first.

Writing Mathematics

When writing mathematical statements, follow the mantra:

One equal sign per line.

We can use brackets instead of parentheses.

$$5 + [7 + 9] = 5 + 16$$

$$= 21$$

Again, note how the expression inside the brackets is evaluated first.

We can also use curly braces instead of parentheses or brackets.

$$\{2+3\} + 4 = 5 + 4$$

$$= 9$$

Again, note how the expression inside the curly braces is evaluated first.

If grouping symbols are nested, we evaluate the innermost parentheses first.

For example,

$$2 + [3 + (4 + 5)] = 2 + [3 + 9]$$

$$= 2 + 12$$

$$= 14.$$

Grouping Symbols

Use parentheses, brackets, or curly braces to delimit the part of an expression you want evaluated first. If grouping symbols are nested, evaluate the expression in the innermost pair of grouping symbols first.

The Associative Property of Addition

Consider the evaluation of the expression (2+3)+4. We evaluate the expression in parentheses first.

$$(2 + 3) + 4 = 5 + 4$$

$$= 9$$

Now, suppose we change the order of addition to 2 + (3 + 4). Then,

$$2 + (3 + 4) = 2 + 7$$

$$= 9.$$

Although the grouping has changed, the result is the same. That is,

$$(2 + 3) + 4 = 2 + (3 + 4).$$

This property of addition of whole numbers is called the *associate property* of addition.

Associate Property of Addition

Let a, b, and c represent whole numbers. Then,

$$(a + b) + c = a + (b + c).$$

Because of the associate property of addition, when presented with a sum of three numbers, whether you start by adding the first two numbers or the last two numbers, the resulting sum is the same.

The Additive Identity

Imagine a number line visualization of the sum of four and zero; i.e., $4 + 0$.

In Figure 1.4, we start at zero, then draw an arrow of magnitude (length) four pointing to the right. Now, at the end of this arrow, attach a second arrow of length zero. Of course, that means that we remain right where we are, at 4. Hence the shaded dot at 4 is the sum. That is, $4 + 0 = 4$.

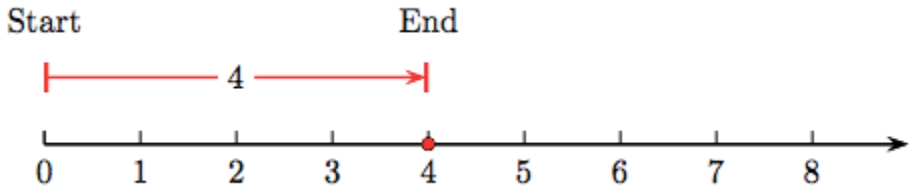


Figure 1.4: Adding zero to four.

The Additive Identity Property

The whole number zero is called the additive identity. If a is any whole number, the

$$a + 0 = a.$$

The number zero is called the additive identity because if you add zero to any number, you get the identical number back.

Adding Larger Whole Numbers

For completeness, we include two examples of adding larger whole numbers. Hopefully, the algorithm is familiar from previous coursework.

Example 1

Simplify: 1, 234 + 498.

Solution

Align the numbers vertically, then add, starting at the furthest column to the right. Add the digits in the ones column, 4 + 8 = 12. Write the 2, then carry a 1 to the tens column. Next, add the digits in the tens column, 3 + 9 = 12, add the carry to get 13, then write the 3 and carry a 1 to the hundreds column. Continue in this manner, working from right to left

$$\begin{array}{r} {\scriptstyle 11} \\ 1234 \\ +\ \ 498 \\ \hline 1732 \end{array}$$

Therefore, 1, 234 + 498 = 1, 732

Exercise

Simplify: 1,286 + 349.

Answer available at the end of this section.

Add three or more numbers in the same manner.

Example 2

Simplify: 256 + 322 + 418.

Solution

Align the numbers vertically, then add, starting at the furthest column to the right. Add the digits in the ones column, 6 + 2 + 8 = 16. Write the 6, then carry a 1 to the tens column. Continue in this manner, working from right to left.

$$\begin{array}{r} {\scriptstyle 11} \\ 256 \\ 322 \\ +418 \\ \hline 996 \end{array}$$

Therefore, 256 + 322 + 418 = 996.

Exercise

Simplify: 256 + 342 + 283

Answer available at the end of this section.

Subtraction of Whole Numbers

The key idea is this: *Subtraction is the opposite of addition*. For example, consider the difference 7 - 4 depicted on the number line in Figure 1.5.

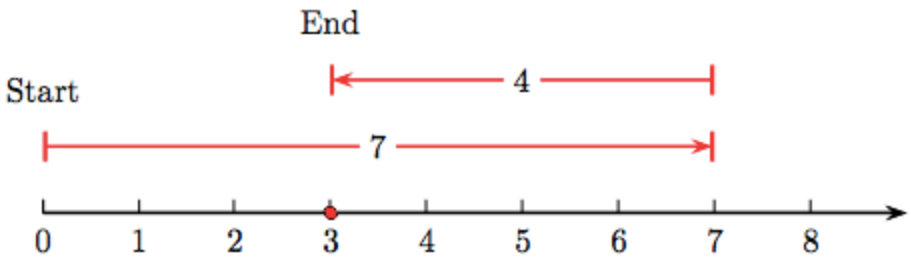


Figure 1.5: Subtraction means *add the opposite*.

If we were adding 7 and 4, we first draw an arrow starting at zero pointing to the right with magnitude (length) seven. Then, to add 4, we would draw a second arrow of magnitude (length) 4, attached to the end of the first arrow and pointing to the right.

However, because subtraction is the *opposite* of addition, in Figure 1.5 we attach an arrow of magnitude (length) four to the end of the first arrow, but *pointing in the opposite direction* (to the left). Note that this last arrow ends at the answer, which is a shaded dot on the number line at 3. That is, 7 – 4 = 3.

Note that subtraction is **not commutative**; that is, it make no sense to say that 7 – 5 is the same as 5 – 7.

Subtraction is **not associative**. It is not the case that (9 – 5) – 2 is the same as 9 – (5 – 2). On the one hand,

$$(9 - 5) - 2 = 4 - 2$$

$$= 2,$$

but

$$9 - (5 - 2) = 9 - 3$$

$$= 6.$$

Subtracting Larger Whole Numbers

Much as we did with adding larger whole numbers, to subtract two large whole numbers, align them vertically then subtract, working from right to left. You may have to “borrow” to complete the subtraction at any step.

Example 3

Simplify: 1, 755 – 328.

Solution

Align the numbers vertically, then subtract, starting at the ones column, then working right to left. At the ones column, we cannot subtract 8 from 5, so we borrow from the previous column. Now, 8 from 15 is 7. Continue in this manner, working from right to left.

$$\begin{array}{r} {\scriptstyle}\quad {\scriptstyle}\quad {\scriptstyle}\quad {\scriptstyle 4} \\ 1\,7\,\cancel{5}^{\,1}\,5 \\ -\quad 3\,2\,8 \\ \hline 1\,4\,2\,7 \end{array}$$

Therefore, 1, 755 – 328 = 1, 427.

Exercise

Simplify: 5,635 - 288.

Answer available at the end of this section.

Order of Operations

In the absence of grouping symbols, it is important to understand that addition holds no precedence over subtraction, and vice-versa.

> Perform all additions and subtractions in the order presented, moving left to right.

Let's look at an example.

Example 4

Simplify the expression $15 - 8 + 4$.

Solution

This example can be trickier than it seems. However, if we follow the rule (perform all additions and subtractions in the order presented, moving left to right), we should have no trouble. First comes fifteen minus eight, which is seven. Then seven plus four is eleven.

$$\begin{aligned} 15 - 8 + 4 &= 7 + 4 \\ &= 11. \end{aligned}$$

Exercise

Simplify: $25 - 10 + 8$.

Answer available at the end of this section.

Caution! Incorrect answer ahead!

Note that it is possible to arrive at a different (but incorrect) answer if we favor addition over subtraction in Example 4. If we first add eight and four, then 15 – 8 + 4 becomes 15 – 12, which is 3. However, note that **this is incorrect**, because it violates the rule "perform all additions and subtractions in the order presented, moving left to right."

Applications — Geometry

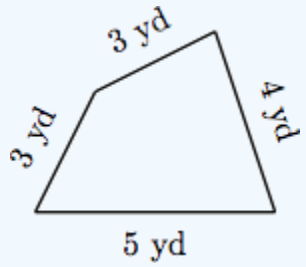
There are any number of applications that require a sum or difference of whole numbers. Let's examine a few from the world of geometry.

Perimeter of a Polygon

In geometry a polygon is a plane figure made up of a closed path of a finite sequence of segments. The segments are called the *edges* or *sides* of the polygon and the points where two edges meet are called the *vertices* of the polygon. The *perimeter* of any polygon is the sum of the lengths of its sides.

Example 5

A quadrilateral is a polygon with four sides. Find the perimeter of the quadrilateral shown below, where the sides are measured below.



Solution

To find the perimeter of the quadrilateral, find the sum of the lengths of the sides.

$$\text{Perimeter} = 3 + 3 + 4 + 5 = 15$$

Hence, the perimeter of the quadrilateral is 15 yards.

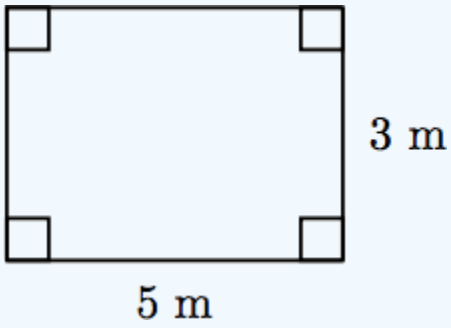
Exercise

A quadrilateral has sides that measure in 4 in., 3 in., 5 in., and 5 in. Find the perimeter.

Answer available at the end of this section.

Example 6

A quadrilateral (four sides) is a rectangle if all four of its angles are right angles. It can be shown that the opposite sides of a rectangle must be equal. Find the perimeter of the rectangle shown below, where the sides of the rectangle are measured in meters.



Solution

To find the perimeter of the rectangle, find the sum of the four sides. Because opposite sides have the same length, we have two sides of length 5 meters and two sides of length 3 meters. Hence,

$$\text{Perimeter} = 5 + 3 + 5 + 3 = 16.$$

Thus, the perimeter of the rectangle is 16 meters.

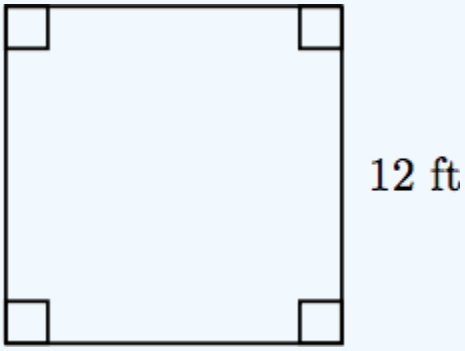
Exercise

A rectangle has length 12 meters and width 8 meters. Find its perimeter.

Answer available at the end of this section.

Example 7

A quadrilateral (four sides) is a *square* if all four of its sides measures 18 centimeters. Find its perimeter. are equal and all four of its angles are right angles. Pictured below is a square having a side of length 12 feet. Find the perimeter of the square.



Solution

Because the quadrilateral is a square, all four sides have the same length, namely 12 feet. To find the perimeter of the square, find the sum of the four sides. Perimeter = 12 + 12 + 12 + 12 = 48

Hence, the perimeter of the square is 48 feet.

Exercise

A square has a side that measures 18 centimeters. Find its perimeter.

Answer available at the end of this section.

Application — Alternative Fuels

Automobiles that run on alternative fuels (other than gasoline) have increased in the United States over the years.

Example 8

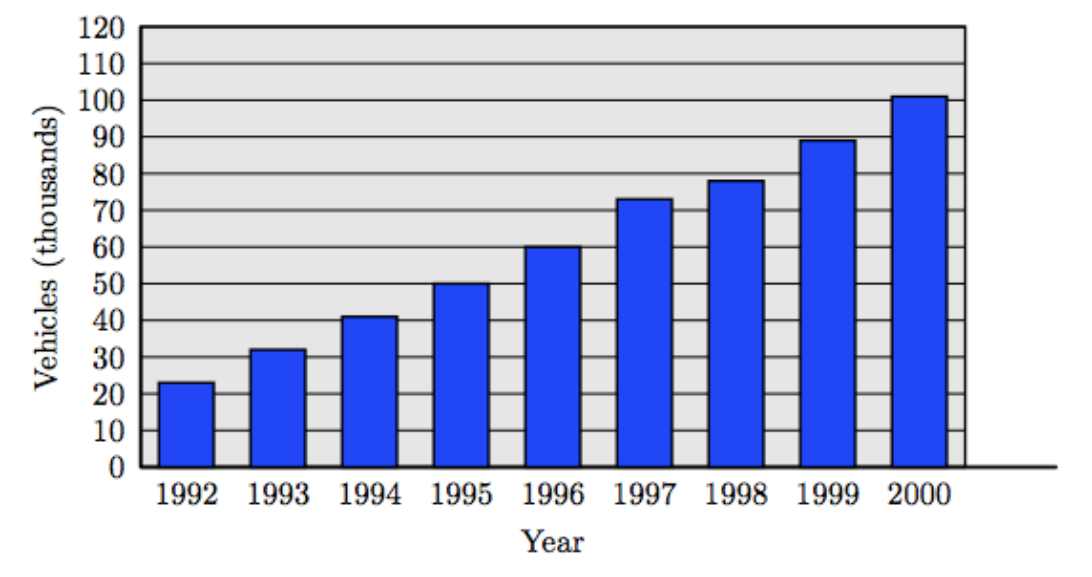
Table 1.2 show the number of cars (in thousands) running on compressed natural gas versus the year. Create a bar chart showing the number of cars running on compressed natural gas versus the year.

Year	1992	1993	1994	1995	1996	1997	1998	1999	2000
Number	23	32	4`	50	60	73	78	89	101

Table 1.2: Number of vehicles (in thousands) running on compressed natural gas.

Solution

Place the years on the horizontal axis. At each year, sketch a bar having height equal to the number of cars in that year that are running on compressed natural gas. Scale the vertical axis in thousands.



Exercise

The following table shows the number of hybrid cars (in thousands) by country.

Country	**Number**
U.S.	279
Japan	77
Canada	17
U.K.	14
Netherlands	11

Create a bar chart showing the number of cars versus the country of use.

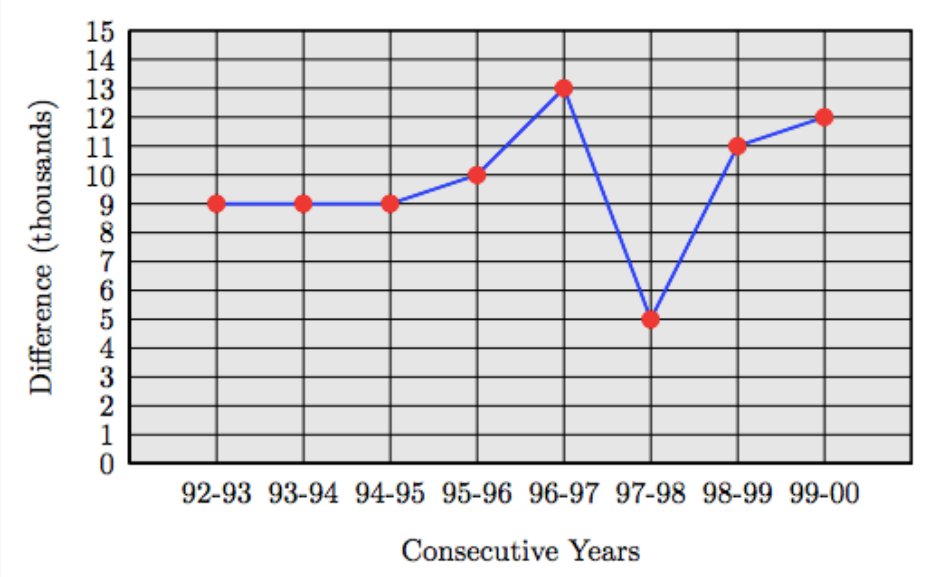
Example 9

Using the data in Table 1.2, crease a table that shows the differences in consecutive years, then create a line plot of the result. In what consecutive years did the United States see the greatest increase in cars powered by compressed natural gas?

Solution

Table 1.3 shows the differences in consecutive years.

Next, craft a line graph. Place consecutive years on the horizontal axis. At each consecutive year pair, plot a point at a height equal to the difference in alternative fuel vehicles. Connect the points with straight line segments.



Note how the line graph makes it completely clear that the greatest increase in vehicles powered by compressed natural gas occurred in the consecutive years 1996-1997, an increase of 13,000 vehicles.

Exercise

The following table show Alphonso's percentage scores on his examinations in mathematics.

Exam	Percentage
Exam #1	52
Exam #2	45
Exam #3	72
Exam #4	889
Exam #5	76

Construct a line graph of Alphonso's exam scores versus exam number.

Exercises

1. Sketch a number line diagram depicting the sum 3 + 2, as shown in Figure 1.2 in the narrative of this section.
2. Sketch a number line diagram depicting the sum 3 + 5, as shown in Figure 1.2 in the narrative of this section.
3. Sketch a number line diagram depicting the sum 3 + 4, as shown in Figure 1.2 in the narrative of this section.
4. Sketch a number line diagram depicting the sum 2 + 4, as shown in Figure 1.2 in the narrative of this section.
5. Sketch a number line diagram depicting the sum 4 + 2, as shown in Figure 1.2 in the narrative of this section.
6. Sketch a number line diagram depicting the sum 4 + 3, as shown in Figure 1.2 in the narrative of this section.

7. Sketch a number line diagram depicting the sum 2 + 5, as shown in Figure 1.2 in the narrative of this section.

8. Sketch a number line diagram depicting the sum 4 + 5, as shown in Figure 1.2 in the narrative of this section.

9. Sketch a number line diagram depicting the sum 4 + 4, as shown in Figure 1.2 in the narrative of this section.

10. Sketch a number line diagram depicting the sum 3 + 3, as shown in Figure 1.2 in the narrative of this section.

In Exercises 11-28, determine which property of addition is depicted by the given identity

11. 28 + 0 = 28

12. 53 + 0 = 53

13. 24 + 0 = 24

14. 93 + 0 = 93

15. (51 + 66) + 88 = 51 + (66 + 88)

16. (90 + 96) + 4 = 90 + (96 + 4)

17. 64 + 39 = 39 + 64

18. 68 + 73 = 73 + 68

19. (70 + 27) + 52 = 70 + (27 + 52)

20. (8 + 53) + 81 = 8 + (53 + 81)

21. 79 + 0 = 79

22. 42 + 0 = 42

23. 10 + 94 = 94 + 10

24. 55 + 86 = 86 + 55

25. 47 + 26 = 26 + 47

26. 62 + 26 = 26 + 62

27. (61 + 53) + 29 = 61 + (53 + 29)

28. (29 + 96) + 61 = 29 + (96 + 61)

---

29. Sketch a number line diagram depicting the difference 8–2, as shown in Figure 1.5 in the narrative of this section.

30. Sketch a number line diagram depicting the difference 8–4, as shown in Figure 1.5 in the narrative of this section.

31. Sketch a number line diagram depicting the difference 7–2, as shown in Figure 1.5 in the narrative of this section.

32. Sketch a number line diagram depicting the difference 9–5, as shown in Figure 1.5 in the narrative of this section.

33. Sketch a number line diagram depicting the difference 7–4, as shown in Figure 1.5 in the narrative of this section.

34. Sketch a number line diagram depicting the difference 6–4, as shown in Figure 1.5 in the narrative of this section.

35. Sketch a number line diagram depicting the difference 9–4, as shown in Figure 1.5 in the narrative of this section.

36. Sketch a number line diagram depicting the difference 6–5, as shown in Figure 1.5 in the narrative of this section.

37. Sketch a number line diagram depicting the difference 8–5, as shown in Figure 1.5 in the narrative of this section.

38. Sketch a number line diagram depicting the difference 9–3, as shown in Figure 1.5 in the narrative of this section.

In Exercises 39-50, simplify the given expression.

39. 16 – 8+2

40. 17 – 3+5

41. 20 – 5 + 14

42. 14 – 5+6

43. 15 – 2+5

44. 13 – 4+2

45. 12 – 5+4

46. 19 – 4 + 13

47. 12 – 6+4

48. 13 – 4 + 18

49. 15 – 5+8

50. 13 – 3 + 11

In Exercises 51-58, the width W and length L of a rectangle are given. Find the perimeter P of the rectangle.

51. W = 7 in, L = 9 in

52. W = 4 in, L = 6 in

53. W = 8 in, L = 9 in

54. W = 5 in, L = 9 in

55. W = 4 cm, L = 6 cm

56. W = 5 in, L = 8 in

57. W = 4 cm, L = 7 cm

58. W = 4 in, L = 9 in

In Exercises 59-66, the length s of a side of a square is given. Find the perimeter P of the square.

59. s = 25 cm

60. s = 21 in

61. s = 16 cm

62. s = 10 in

63. s = 18 in

64. s = 7 in

65. s = 3 in

66. s = 20 in

In Exercises 67-86, find the sum.

67. 3005 + 5217

68. 1870 + 5021

69. 575 + 354 + 759

70. 140 + 962 + 817

71. 472 + (520 + 575)

72. 318 + (397 + 437)

73. 274 + (764 + 690)

74. 638 + (310 + 447)

75. 8583 + 592

76. 5357 + 9936

77. 899 + 528 + 116

78. 841 + 368 + 919

79. (466 + 744) + 517

80. (899 + 996) + 295

81. 563 + 298 + 611 + 828

82. 789 + 328 + 887 + 729

83. 607 + 29 + 270 + 245

84. 738 + 471 + 876 + 469

85. (86 + 557) + 80

86. (435 + 124) + 132

In Exercises 87-104, find the difference.

87. 3493 – 2034 – 227

88. 3950 – 1530 – 2363

89. 8338 – 7366

90. 2157 – 1224

91. 2974 – 2374

92. 881 – 606

93. 3838 – (777 – 241)

94. 8695 – (6290 – 4233)

95. 5846 – 541 – 4577

96. 5738 – 280 – 4280

97. 3084 – (2882 – 614)

98. 1841 – (217 – 28)

99. 2103 – (1265 – 251)

100. 1471 – (640 – 50)

101. 9764 – 4837 – 150

102. 9626 – 8363 – 1052

103. 7095 – 226

104. 4826 – 1199

---

105. **Water Subsidies**. Since the drought began in 2007, California farms have received $79 million in water subsidies. California cotton and rice farmers received an additional $439 million. How much total water subsidies have farmers received? *Associated Press Times-Standard 4/15/09*

106. **War Budget**. The 2010 Federal budget allocates $534 billion for the Department of Defense base programs and an additional $130 billion for the nation's two wars. How much will the Department of Defense receive altogether? *Associated Press Times-Standard 5/8/09*

107. **Sun Frost**. Arcata, CA is home to Sun Frost, a manufacturer of highly efficient refrigerators and freezers. The AC model RF12 refrigerator/freezer costs $2,279 while an R16 model refrigerator/freezer costs $3,017. How much more does the R16 model cost? Source: www.sunfrost.com/retail pricelist.html

108. **Shuttle Orbit**. The space shuttle usually orbits at 250 miles above the surface of the earth. To service the Hubble Space Telescope, the shuttle had to go to 350 miles above the surface. How much higher did the shuttle have to orbit?

109. **Earth's Orbit**. Earth orbits the sun in an ellipse. When earth is at its closest to the sun, called *perihelion*, earth is about 147 million kilometers. When earth is at its furthest point from the sun, called *aphelion*, earth is about 152 million kilometers from the sun. What's the difference in millions of kilometers between aphelion and perihelion?

110. **Pluto's Orbit**. Pluto's orbit is highly eccentric. Find the difference between Pluto's closest approach to the sun and Pluto's furthest distance from the sun if Pluto's perihelion (closest point on its orbit about the sun) is about 7 billion kilometers and its aphelion (furthest point on its orbit about the sun) is about 30 billion kilometers.

111. **Sunspot Temperature**. The surface of the sun is about 10,000 degrees Fahrenheit. Sunspots are darker regions on the surface of the sun that have a relatively cooler temperature of 6,300 degrees Fahrenheit. How many degrees cooler are sunspots?

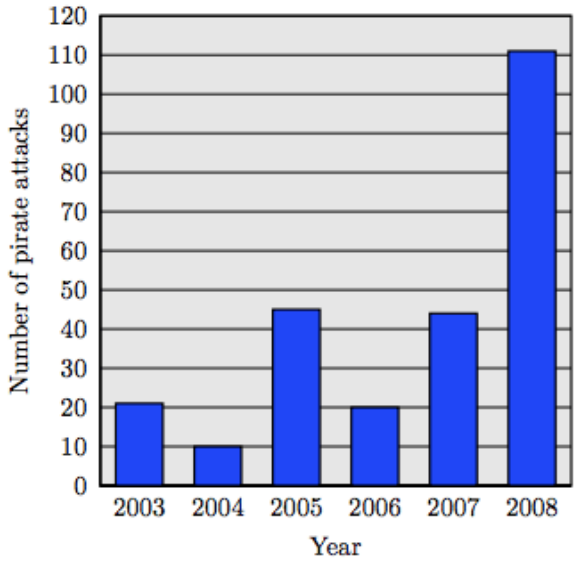
112. **Jobs**. The Times-Standard reports that over the next year, the credit- and debitcard processing business Humboldt Merchant Services expects to cut 36 of its 80 jobs, but then turn around and hire another 21. How many people will be

working for the company then? *Times-Standard 5/6/09*

113. **Wild tigers**. The chart shows the estimated wild tiger population, by region. According to this chart, what is the total wild tiger population worldwide? *Associated Press-Times-Standard 01/24/10 Pressure mounts to save the tiger.*

Region	Tiger population
India, Nepal and Bhutan	1650
China and Russia	450
Bangladesh	250
Sumatra (Indonesia)	400
Malaysia	500
other SE Asia	350

114. **Pirate Attacks**. The following bar chart tracks pirate attacks off the coast of Somalia.



Source: ICC International Maritime Bureau, AP Times-Standard, 4/15/2009

a) How many pirate attacks were there in 2003, 2004, and 2005 combined?

b) How many pirate attacks were there in 2006, 2007, and 2008 combined?

c) How many more pirate attacks were there in 2008 than in 2007?

115. Emily shows improvement on each successive examination throughout the term. Her exam scores are recorded in the following table.

Exam	Score
Exam #1	48
Exam #2	51
Exam #3	54
Exam #4	59
Exam #5	67

Exam #6	70

a) Create a bar plot for Emily's examination scores. Place the examination numbers on the horizontal axis in the same order shown in the table above.

b) Create a table that shows successive differences in examination scores. Make a line plot of these differences. Between which two exams did Emily show the greatest improvement?

116. Jason shows improvement on each successive examination throughout the term. His exam scores are recorded in the following table.

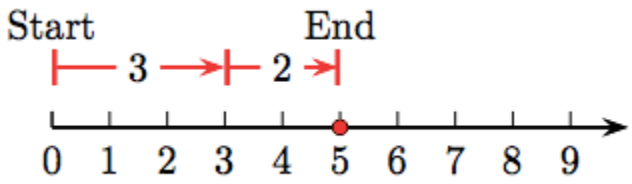
Exam	**Score**
Exam #1	34
Exam #2	42
Exam #3	45
Exam #4	50
Exam #5	57
Exam #6	62

a) Create a bar plot for Jason's examination scores. Place the examination numbers on the horizontal axis in the same order shown in the table above.

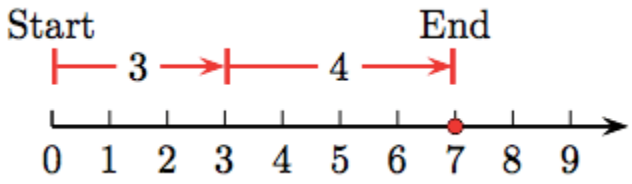
b) Create a table that shows successive differences in examination scores. Make a line plot of these differences. Between which two exams did Jason show the greatest improvement?

Answers

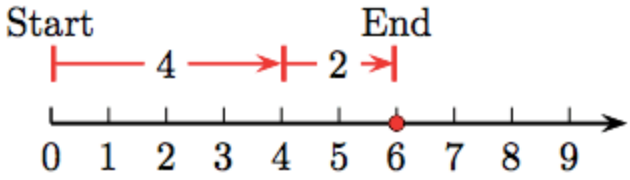
1. 3 + 2 = 5.



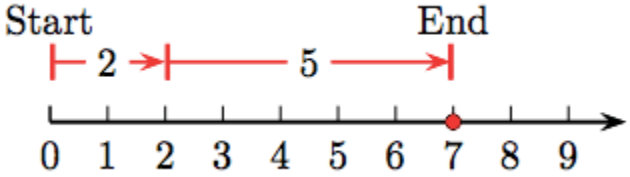
3. 3 + 4 = 7.



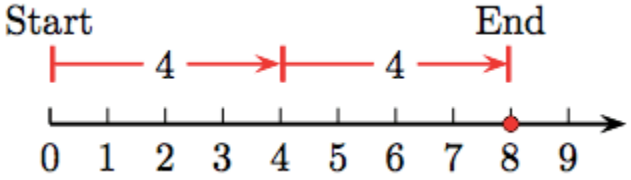
5. 4 + 2 = 6.



7. 2 + 5 = 7.



9. 4 + 4 = 8.



11. Additive identity property of addition.

13. Additive identity property of addition.

15. Associative property of addition

17. Commutative property of addition

19. Associative property of addition

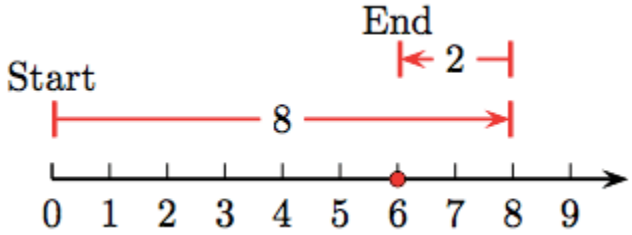
21. Additive identity property of addition.

23. Commutative property of addition

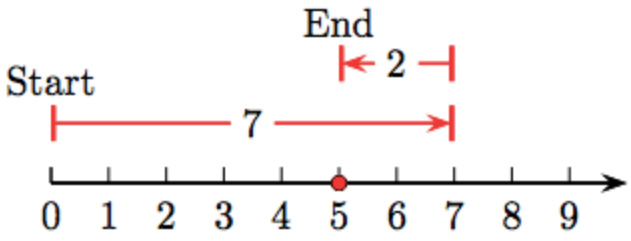
25. Commutative property of addition

27. Associative property of addition

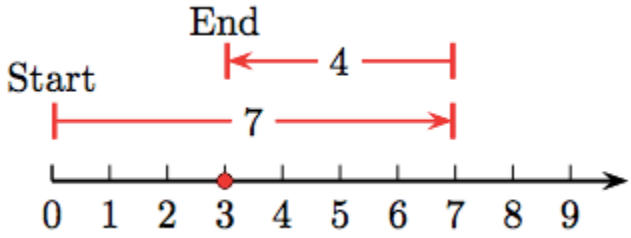
29. 8 – 2 = 6.



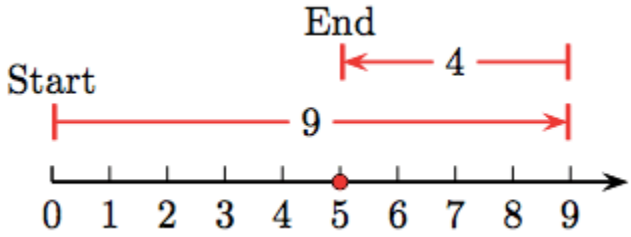
31. 7 – 2 = 5.



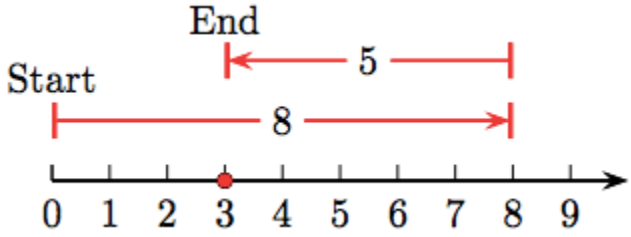
33. 7 – 4 = 3.



35. 9 – 4 = 5.



37. 8 – 5 = 3.



39. 10

41. 29

43. 18

45. 11

47. 10

49. 18

51. P = 32 in

53. P = 34 in

55. P = 20 cm

57. P = 22 cm

59. P = 100 cm

61. P = 64 cm

63. P = 72 in

65. P = 12 in

67. 8222

69. 1688

71. 1567

73. 1728

75. 9175

77. 1543

79. 1727

81. 2300

83. 1151

85. 723

87. 1232

89. 972

91. 600

93. 3302

95. 728

97. 816

99. 1089

101. 4777

103. 6869

105. $518 million

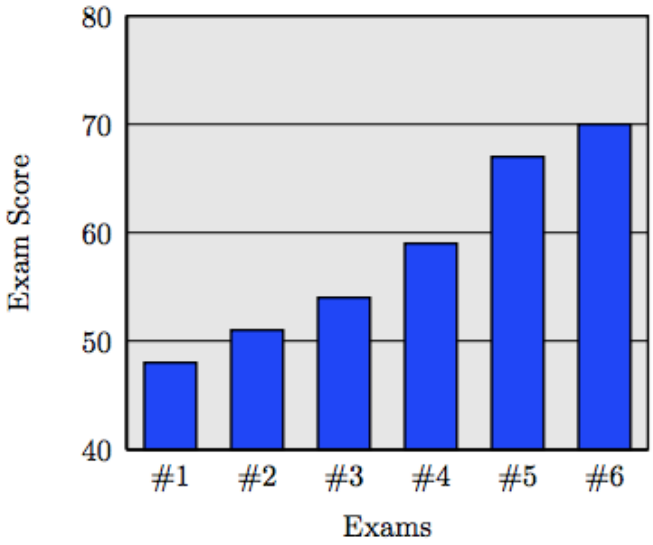
107. $738

109. 5 million kilometers

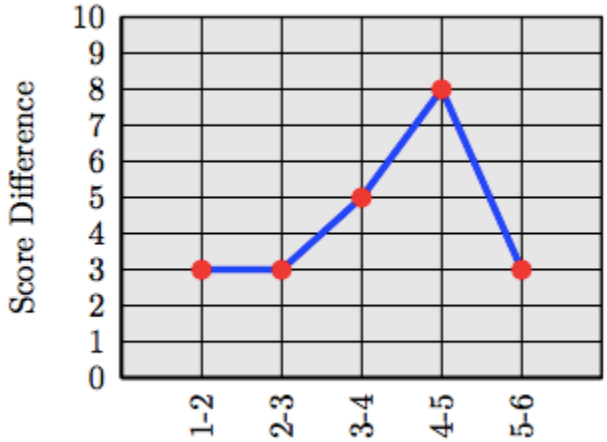
111. 3,700 degrees Fahrenheit

113. 3600

115. a) Bar chart.



b) Line plot of consecutive differences. The line plot of consecutive examination score differences.



The largest improvement was between Exam #4 and Exam #5, where Emily improved by 8 points.

Answers

Exercise
1635

Exercise
881

Exercise
5,347

Exercise
23

Exercise
17 inches

Exercise
40 meters

Exercise
72 centimeters

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1.3.1: Practice and Videos for Multiplying Whole Numbers

Learning Objectives

- I can understand how to multiply whole-digit numbers up to four digits by 1-digit numbers.
- I can use multiplication tables and area models.

GeoGebra: Multiplying 2-Digit Numbers Using Partial Products

Find the product of two-digit numbers using partial products and area models. Insert the numbers of each equation in the boxes as appropriate.

Videos on Mult-Digit Multiplication

Watch the following videos to learn about multi-digit multiplication.

YouTube Video

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

YouTube Video

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

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View this YouTube Video online

YouTube Video

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

Practice!

Check your understanding of multiplication by answering these questions. Use the arrows at the bottom of the box to go to the next question.

Practice 1.3.1.1

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this Interactive Content online

Extra Resources

For more practice on multi-digit multiplication, visit the following websites.

1. IXL: Multiply 1-digit numbers by 2-digit number
2. I Know It: Multiplying 3-digit numbers by 1-digit numbers
3. IXL: Multiply 1-digit numbers by 3-digit or 4-digit numbers: choose the area model
4. Math Playground: Grand Prix Multiplication
5. Math is Fun: Math Trainer - Multiplication

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1.3.2: Addition and Multiplication Strategies

Application

Exercise 1.3.2.1

Lucy works for 5 hours. She earns $13 each hour. To calculate how much money she earned, Lucy adds up her total earnings like this:

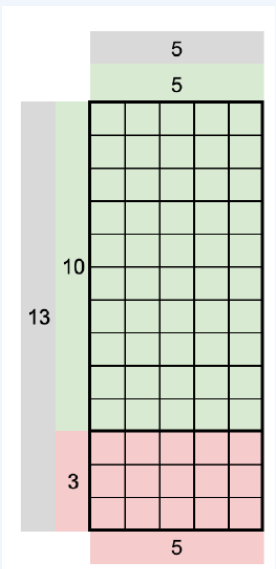
$13 + 13 + 13 + 13 + 13$

Question: What would be a more efficient way to express Lucy's earnings? What is another approach to adding up her total earnings?

Click for explanation available at the end of this section.

What if I don't know how to solve 13 x 5?

It is okay if you do not know 13×5 right away. You can use numbers you already know to figure it out.

Explanation	Work	Diagram
Step 1: Split 13 into manageable numbers you know how to multiply.	13 = 10 + 3	
Step 2: Instead of multiplying 13×5 all at once, we can think:	13 x 5 = (10 x 5) + (3 x 5)	
Step 3: Use your 5 times tables to multiply.	13 x 5 = (50) + (15)	
Step 4: Add the remaining numbers.	13 x 5 = 65	

Great job! View the diagram on the right to visualize this calculation.

The key idea is that repeated addition and multiplication are connected.

Answers

Exercise 1.3.2.1

This means Lucy earns:

- $13 for the first hour
- $13 for the second hour
- $13 for the third hour

- $13 for the fourth hour
- $13 for the fifth hour

To find how much Lucy earns in all, we could add:

$13 + 13 + 13 + 13 + 13$

This works, but it takes a while because we are adding the same number 5 times

Sometimes we can add the same number again and again. But when the same number is repeated, multiplication gives us a shorter way to write and solve the problem. A shorter way to write it is like this:

13×5

This means **5 groups of 13**.

So,

$13 \times 5 = 65$

Lucy earns $65.

Key idea: Repeated addition is the same as multiplication.

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1.4: Introduction to Division

Learning Objectives

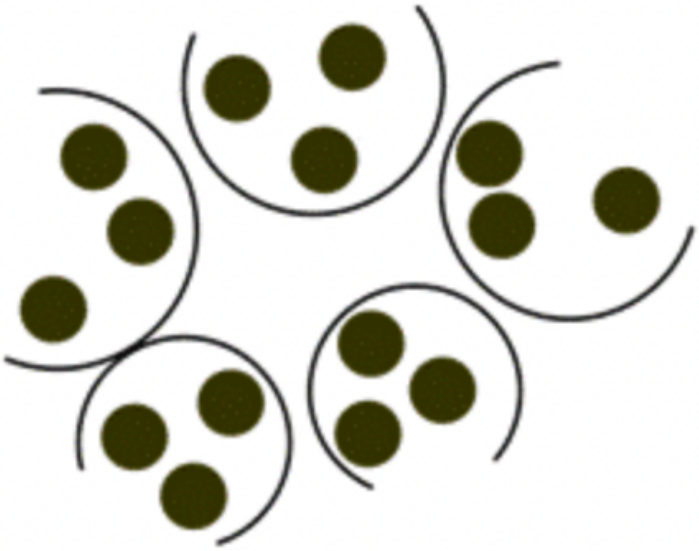
- Use three different ways to represent division.
- Divide whole numbers.
- Perform long division.
- Divide whole numbers by a power of 10.
- Recognize that division by 0 is not defined.
- Solve application problems using division.

Introduction

Some people think about division as “fair sharing” because when you divide a number you are trying to create equal parts. Division is also the **inverse operation** of multiplication because it “undoes” multiplication. In multiplication, you combine equal sets to create a total. In division, you separate a whole group into sets that have the same amount. For example, you could use division to determine how to share 40 empanadas among 12 guests at a party.

What is Division?

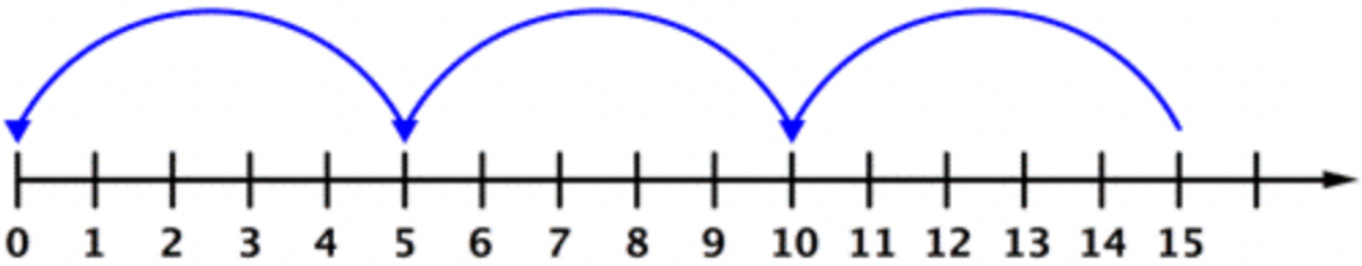
Division is splitting into equal parts or groups. For example, one might use division to determine how to share a plate of cookies evenly among a group. If there are 15 cookies to be shared among five people, you could divide 15 by 5 to find the “fair share” that each person would get. Consider the picture below.



15 cookies split evenly across 5 plates results in 3 cookies on each plate. You could represent this situation with the equation:

$$15 \div 5 = 3$$

You could also use a number line to model this division. Just as you can think of multiplication as repeated addition, you can think of division as repeated subtraction. Consider how many jumps you take by 5s as you move from 15 back to 0 on the number line.

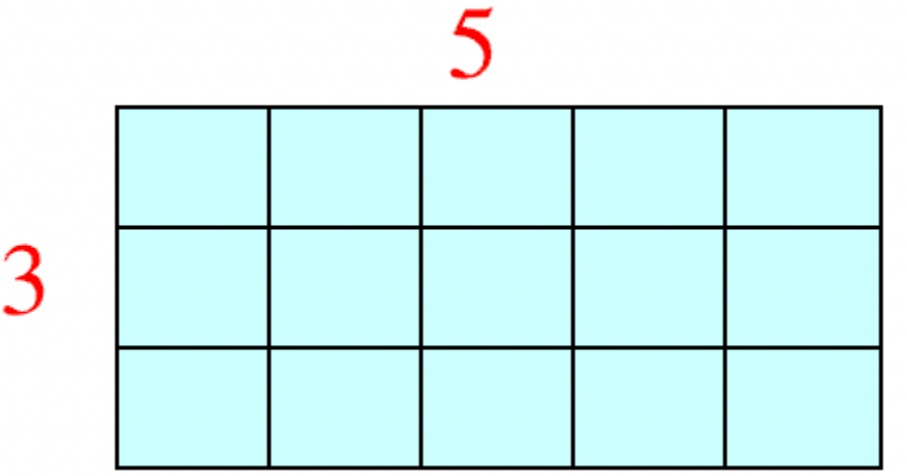


Notice that there are 3 jumps that you make when you skip count by 5 from 15 back to 0 on the number line. This is like subtracting 5 from 15 three times. This repeated subtraction can be represented by the equation: $15 \div 5 = 3$.

Finally, consider how an area model can show this division. Ask yourself, if you were to make a rectangle that contained 15 squares with 5 squares in a row, how many rows would there be in the rectangle? Start by making one row of 5 squares:



Then add two more rows of 5 squares so you have 15 squares.



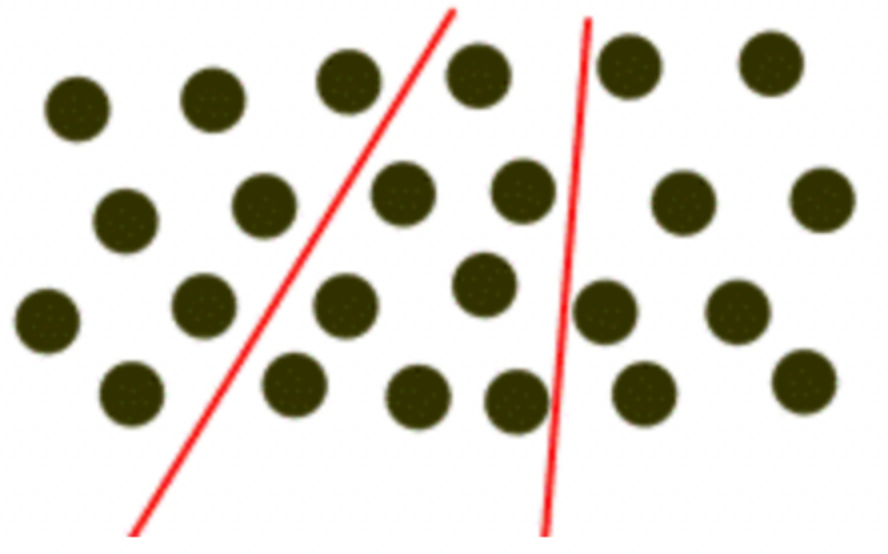
The number of rows is 3. So, 15 divided by 5 is equal to 3.

Example

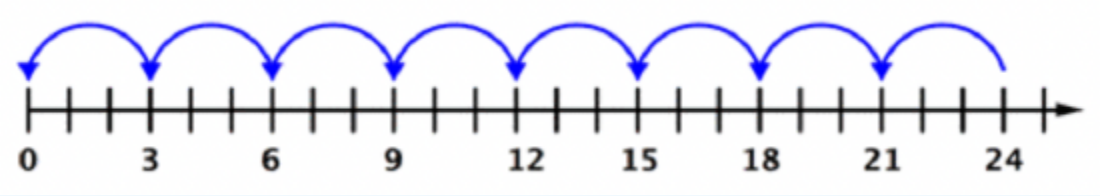
Find $24 \div 3$ using a set model and a number line model.

Solution

Set Model:



Number line model:



$24 \div 3 = 8$

Ways to Represent Division

As with multiplication, division can be written using a few different symbols. We showed this division written as $15 \div 5 = 3$, but it can also be written two other ways:

$$5\overline{)15}\,3$$

$\frac{15}{5} = 3$

Each part of a division problem has a name. The number that is being divided up, that is the total, is called the **dividend**. In the work in this topic, this number will be the larger number, but that is not always true in mathematics. The number that is dividing the dividend is called the **divisor**. The answer to a division problem is called the **quotient**.

The blue box below summarizes the terminology and common ways to represent division.

Three Ways to Represent Division

$12 \div 3 = 4$ (**with a division symbol**; this equation is read "12 divided by 3 equals 4."

$3\overline{)12}\,4$ (**with a division or long division symbol**; this expression is read "12 divided by 3 equals 4." Notice here, though, that you have to start with what is underneath the symbol. This may take some getting used to since you are reading from right to left and bottom to top!)

$\frac{12}{3} = 4$ (**with a fraction bar**; this expression can also be read "12 divided by 3 equals 4." In this format, you read from top to bottom.)

In the examples above, 12 is the **dividend**, 3 is the **divisor** and 4 is the **quotient.**

$$\text{Dividend} \div \text{Divisor} = \text{Quotient}$$

$$\begin{array}{r} \text{Quotient} \\ \text{Divisor}\,\overline{)\,\text{Dividend}} \end{array}$$

$$\frac{\text{Dividend}}{\text{Divisor}} = \text{Quotient}$$

Exercise

Which of the following expressions represent dividing $56 equally among 7 people?

#1: $\frac{7}{56}$

#2: $56 \div 7$

#3: $56\overline{)\,7}$

A. #2 represents the situation.
B. All three expressions represent the situation.
C. #1 represents the situation.
D. #3 represents the situation.

Answer available at the end of this section.

Dividing Whole Numbers

Once you understand how division is written, you are on your way to solving simple division problems. You will need your multiplication facts to perform division. If you do not have them memorized, you can guess and check or use a calculator.

Consider the following problems:

$10 \div 5 = ?$

$48 \div 2 = ?$

$30 \div 5 = ?$

In the first problem, $10 \div 5$, you could ask yourself, “how many fives are there in ten?” You can probably answer this easily. Another way to think of this is to consider breaking up 10 into 5 groups and picturing how many would be in each group.

$10 \div 5 = 2$

To solve $48 \div 2$, you might realize that dividing by 2 is like splitting into two groups or splitting the total in half. What number could you double to get 48?

$48 \div 2 = 24$

To figure out $30 \div 5$, you could ask yourself, how many times do you have to skip count by 5 to get from 0 to 30? 5, 10, 15, 20, 25, 30. You have to skip count 6 times to get to 30.

$30 \div 5 = 6$

Exercise

Compute $35 \div 5$.

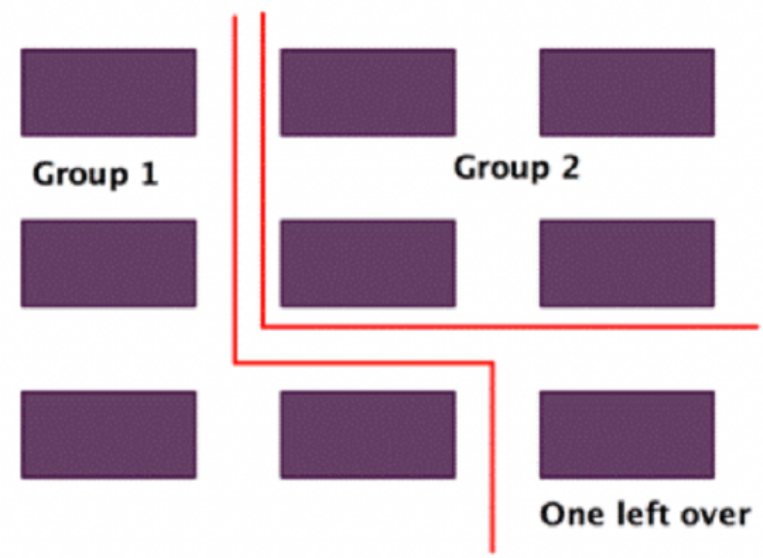
Answer available at the end of this section.

Exercise

Compute $32 \div 4$.

Answer available at the end of this section.

Sometimes when you are dividing, you cannot easily share the number equally. Think about the division problem $9 \div 2$. You could think of this problem as 9 pieces of chocolate being split between 2 people. You could make two groups of 4 chocolates, and you would have one chocolate left over.



In mathematics, this left over part is called the **remainder**. It is the part that *remains* after performing the division. In the example above, the remainder is 1. We can write this as:

$9 \div 2 = 4 \quad \text{R}1$

We read this equation: “Nine divided by two equals four with a remainder of 1."

You might be thinking you could split that extra piece of chocolate in parts to share it. This is great thinking! If you split the chocolate in half, you could give each person another half of a piece of chocolate. They would each get $4\frac{1}{2}$ pieces of chocolate. We are not going to worry about expressing remainders as fractions or decimals right now. We are going to use the remainder notation with the letter R. Here’s an example:

Example

$45 \div 6$

Solution

$6 \cdot 7 = 42$	How many sixes are there in 45? Try 7.
$45 - 42 = 3$	3 is not enough for another 6. So, 3 is the remainder.

$45 \div 6 = 7 \quad \text{R3}$

Since multiplication is the inverse of division, you can check your answer to a division problem with multiplication. To check the answer 7 R3, first multiply 6 by 7 and then add 3.

$6 \cdot 7 = 42$

$42 + 3 = 45$, so the quotient 7 R3 is correct.

Exercise

Compute $67 \div 7$.

A. 9
B. 9 R4
C. 60
D. 10

Answer available at the end of this section.

Performing Long Division

Long division is a method that is helpful when you are performing division that you cannot do easily in your head, such as division involving larger numbers. Below is an example of a way to write out the division steps.

Example

$68 \div 4$

Solution

$4\longdiv{68}$	Rewrite the division.
$\begin{array}{r} 1 \\ 4\longdiv{68} \\ -4 \\ 28 \end{array}$	Divide the tens. What is 6 divided by 4? $4 \cdot 1 = 4$, so write a 1 above the 6. Subtract 4 from 6 and bring down the next digit of the dividend, 8. What is 28 divided by 4?
$\begin{array}{r} 17 \\ 4\longdiv{68} \\ -4 \\ 28 \\ -28 \\ 0 \end{array}$	$7 \cdot 4 = 28$, so write a 7 above the 8. There is no remainder.

\(\ \begin{array}{r} 17 \cdot 4 \ ^217 \ \times 4 \ \hline 68 \end{array}\)	Check your answer using multiplication.

$68 \div 4 = 17$

Example

$6,707 \div 233$

Solution

$\begin{array}{r} 2 \\ 233\overline{)6707} \\ -466 \\ \hline \end{array}$	Examine the first 3 digits of the dividend and determine how many 233s are in it. Use guess and check. Try: $2 \cdot 233 = 466$ Try: $3 \cdot 233 = 699$ (too large)
$\begin{array}{r} 2 \\ 233\overline{)6707} \\ -466 \\ 2047 \end{array}$	Subtract 466 from 670 and bring down the next digit of the dividend, 7.
$\begin{array}{r} 28 \\ 233\overline{)6707} \\ -466 \\ 2047 \\ -1864 \\ 183 \end{array}$	How many 233s are in 2,047? It looks like close to 10 because $233 \cdot 10 = 2,330$. Try $9.233 \cdot 9$ equals 2,097. $\begin{array}{r} 22 \\ 233 \\ \times \quad 9 \\ \hline 2,097 \end{array}$ 2,097 (Too large) Must be 8! $\begin{array}{r} 22 \\ 233 \\ \times \quad 8 \\ \hline 1,864 \end{array}$

$233 \cdot 28 = 6,524$ $6,524 + 183 = 6,707$	Check your answer using multiplication. First, multiply $233 \cdot 28$. Then, add the remainder.

$6,707 \div 233 = 28 \quad \text{R}183$

Exercise

Compute $417 \div 34$.

A. 451
B. 12
C. 12 R9
D. 13

Answer available at the end of this section.

Dividing Whole Numbers by a Power of 10

Just as multiplication by powers of 10 results in a pattern, there is a pattern with division by powers of 10. Consider three quotients: $20 \div 10; 200 \div 10; 2,000 \div 10$.

Think about $20 \div 10$. There are 2 tens in twenty, so $20 \div 10 = 2$. The computations for $200 \div 10$ and $2,000 \div 10$ are shown below.

Example

$200 \div 10$

Solution

$10\longdiv{200}$	Rewrite the problem.
$10\longdiv{200}$	Divide the first digit of the dividend, 2, by the divisor. Since $2 \div 10$ does not give a whole number, go to the next digit, 0.
$\begin{array}{r} 2 \\ 10\longdiv{200} \end{array}$	$20 \div 10 = 2$
$\begin{array}{r} 2 \\ 10\longdiv{200} \\ -20 \\ 0 \end{array}$	$2 \cdot 10 = 20$ $20 - 20 = 0$
$\begin{array}{r} 2 \\ 10\longdiv{200} \\ -20 \\ 00 \end{array}$	Bring down the next digit of the dividend, which is 0.

$$\begin{array}{r} 20 \\ 10\overline{)200} \\ -20 \\ \hline 00 \\ 0 \\ \hline 0 \end{array}$$	Since 10 still does not go into 00 and we have nothing left to bring down, multiply the 0 by 10. $0 \cdot 10 = 0$ $0 - 0 = 0$ We have no remainder.

$200 \div 10 = 20$

Example

$2000 \div 10$

Solution

$10\overline{)2000}$	Rewrite the problem.
$10\overline{)2000}$	Divide the first digit of the dividend, 2, by the divisor. Since $2 \div 10$ does not give a whole number, go to the next digit, 0.
$$\begin{array}{r} 2 \\ 10\overline{)2000} \end{array}$$	$20 \div 10 = 2$
$$\begin{array}{r} 2 \\ 10\overline{)2000} \\ -20 \\ \hline 0 \end{array}$$	$2 \cdot 10 = 20$ $20 - 20 = 0$
$$\begin{array}{r} 2 \\ 10\overline{)2000} \\ -20 \\ \hline 00 \end{array}$$	Bring down the next digit, 0, of the dividend.
$$\begin{array}{r} 20 \\ 10\overline{)2000} \\ -20 \\ \hline 000 \end{array}$$	Since 10 does not go into 00, add a 0 to the quotient and bring down the next digit, 0.
$$\begin{array}{r} 200 \\ 10\overline{)2000} \\ -20 \\ \hline 000 \\ 0 \\ \hline 0 \end{array}$$	Since 10 still does not go into 000 and we have nothing left to bring down, add a 0 to the quotient, multiply the 0 by 10. $0 \cdot 10 = 0$ $0 - 0 = 0$ We have no remainder.

$2,000 \div 10 = 200$

Examine the results of these three problems to try to determine a pattern in division by 10.

$$20 \div 10 = 2$$
$$200 \div 10 = 20$$
$$2,000 \div 10 = 200$$

Notice that the number of zeros in the quotient decreases when a dividend is divided by 10: 20 becomes 2; 200 becomes 20 and 2,000 become 200. In each of the examples above, you can see that there is one fewer 0 in the quotient than there was in the dividend.

Continue another example of division by a power of 10.

Example

$2,000 \div 100$

Solution

$100\longdiv{2000}$	Rewrite the problem.
$100\longdiv{2000}$	Divide the first digit of the dividend, 2, by the divisor. Since $2 \div 100$ does not give a whole number, go to the next digit, 0.
$100\longdiv{2000}$	Divide the first two digits of the dividend, 20, by the divisor. Since $20 \div 100$ does not give a whole number, go to the next digit, 0.
$100\longdiv{2000}$	$200 \div 100 = 2$
$\begin{array}{r} 2 \\ 100\longdiv{2000} \\ -200 \\ 0 \end{array}$	$2 \cdot 100 = 200$ $200 - 200 = 0$
$\begin{array}{r} 2 \\ 100\longdiv{2000} \\ -200 \\ 00 \end{array}$	Bring down the next digit, 0, of the dividend.
$\begin{array}{r} 20 \\ 100\longdiv{2000} \\ -200 \\ 00 \\ 0 \\ 0 \end{array}$	Since 100 still does not go into 00 and we have nothing left to bring down, add a 0 to the quotient, multiply the 0 by 10. $0 \cdot 10 = 0$ $0 - 0 = 0$ We have no remainder.

$2,000 \div 100 = 20$

Consider this set of examples of division by powers of 10. What pattern do you see?

$$\begin{aligned} 20 &\div 10 = 2 \\ 200 &\div 10 = 20 \\ 2,000 &\div 10 = 200 \\ 2,000 &\div 100 = 20 \\ 2,000 &\div 1,000 = 2 \end{aligned}$$

Notice that when you divide a number by a power of 10, the quotient has fewer zeros. This is because division by a power of 10 has an effect on the place value. For example, when you perform the division $18,000 \div 100 = 180$, the quotient, 180, has two fewer zeros than the dividend, 18,000. This is because the power of 10 divisor, 100, has two zeros.

Exercise

Compute $135,000 \div 100$.

A. 13,500
B. 134,900
C. 13,500,000
D. 1,350

Answer available at the end of this section.

Division by Zero

You know what it means to divide by 2 or divide by 10, but what does it mean to divide a quantity by 0? Is this even possible? Can you divide 0 by a number? Consider the two problems written below.

$\frac{0}{8}$ and $\frac{8}{0}$

We can read the first expression, “zero divided by eight” and the second expression, “eight divided by zero.” Since multiplication is the inverse of division, we could rewrite these as multiplication problems.

$0 \div 8 = ?$

$? \cdot 8 = 0$

The quotient must be 0 because $0 \cdot 8 = 0$.

$\frac{0}{8} = 0$

Now let’s consider $\frac{8}{0}$.

$8 \div 0 = ?$

$? \cdot 0 = 8$

This is not possible. There is no number that you could multiply by zero and get eight. Any number multiplied by zero is always zero. There is no quotient for $\frac{8}{0}$. There is no quotient for any number when it is divided by zero.

Division by zero is an operation for which you cannot find an answer, so it is not allowed. We say that division by 0 is undefined.

Using Division in Problem Solving

Division is used in solving many types of problems. Below are three examples from real life that use division in their solutions.

Example

Luana made 40 empanadas for a party. If the empanadas are divided equally among 12 guests, how many will each guest have? Will there be any leftover empanadas?

Solution

$40 \div 12$	Since each guest will have an equal share, we can use division.
$12\overline{)40}$ -36 4	Use trial and error. Try 3. $12 \cdot 3 = 36$ When 40 empanadas are divided equally among 12 people, there are 4 left over.

Each guest will have 3 empanadas. There will be 4 empanadas left over.

Example

A case of floor tiles has 12 boxes in it. The case costs $384. How much does one box cost?

Solution

$384 \div 12$	Since the boxes each cost the same amount, you want to divide $384 into 12 equal parts.
$12\overline{)384}$	Perform the division. Try to divide the first digit in the dividend by the divisor. 12 will not divide into 3, so go to the next digit.
3 $12\overline{)384}$ -36	Perform $38 \div 12$. Pick a quotient and test it. Try 3. $3 \cdot 12 = 36$.
3 $12\overline{)384}$ -36 2	Subtract 36 from 38.
32 $12\overline{)384}$ -36 24 -24 0	Bring the next digit of the dividend down and perform division. $12 \cdot 2 = 24$ $24 - 24 = 0$

$\begin{array}{r} 32 \\ \times \quad 12 \\ \hline 64 \\ + \quad 320 \\ \hline 384 \end{array}$	Does $32 \cdot 12$ equal 384? Check your answer by multiplying. Yes! The answer is correct!

Each box of tiles costs $32

Example

A banana grower is shipping 4,644 bananas. There are 86 crates, each containing the same number of bananas. How many bananas are in each crate?

Solution

$4,644 \div 86$	Since each crate, or box, has the same number of bananas, you can take the total number of bananas and divide by the number of crates. Rewrite the division.
$86\overline{)4644}$	Use trial and error to determine what $464 \div 86$ equals. Try 5: $\begin{array}{r} {}^{3}86 \\ \times \quad 5 \\ \hline 430 \end{array}$
$\begin{array}{r} 5 \\ 86\overline{)4644} \\ -430 \\ \hline 344 \end{array}$	$464 - 430 = 34$ Then, bring down the next digit of the dividend, 4.
$\begin{array}{r} 54 \\ 86\overline{)4644} \\ -430 \\ \hline 344 \\ -344 \\ \hline 0 \end{array}$	Use trial and error to determine the quotient of 344 and 86. Try 4: $\begin{array}{r} {}^{2}86 \\ \times 4 \\ \hline 344 \end{array}$
$\begin{array}{r} 86 \\ \times 54 \\ \hline 344 \\ +4300 \\ \hline 4644 \end{array}$	Check your answer by multiplying. Yes! The answer is correct!

Each crate contains 54 bananas.

Exercise

A theater has 1,440 seats. The theater has 30 rows of seats. How many seats are in each row?

A. 1,410
B. 48
C. 43,200
D. 480

Answer available at the end of this section.

Summary

Division is the inverse operation of multiplication, and can be used to determine how to evenly share a quantity among a group. Division can be written in three different ways: using a fraction bar, using a division symbol, and using long division. Division can be represented as splitting a total quantity into sets of equal quantities, as skip subtracting on the number line, and as a dimension with an area model. Remainders may result when performing division and they can be represented with the letter R, followed by the number remaining. Since division is the inverse operation of multiplication, you need to know your multiplication facts in order to do division. For larger numbers, you can use long division to find the quotient.

Answers

Exercise

A. Correct. #2 is the only expression that represents 56 divided by 7.
B. Incorrect. #1 and #3 represent 7 divided by 56, not 56 divided by 7. The correct answer is #2 is the only expression that represents the situation.
C. Incorrect. This expression represents 7 divided by 56, not 56 divided by 7. The correct answer is #2 is the only expression that represents the situation.
D. Incorrect. This expression represents 7 divided by 56, not 56 divided by 7. The correct answer is #2 is the only expression that represents the situation.

Exercise

7

Exercise

8

Exercise

A. Incorrect. $9 \cdot 7 = 63$. There is a remainder of 4. The correct answer is 9 R4.
B. Correct. $9 \cdot 7 = 63$ and there are 4 left over.
C. Incorrect. This is a division, not subtraction, problem. The correct answer is 9 R4.
D. Incorrect. $70 \div 7 = 10$, so the answer to $67 \div 7$ cannot be; $9 \cdot 7 = 63$ and there are 4 left over. The correct answer is 9 R4.

Exercise

A. Incorrect. This is a division problem, not an addition problem. The correct answer is 12 R9.
B. Incorrect. $12 \cdot 34 = 408$. The correct answer is 12 R9.
C. Correct. $12 \cdot 34 = 408$ and $408 + 9 = 417$
D. Incorrect. $13 \cdot 34 = 442$. The correct answer is 12 R9.

Exercise

A. Incorrect. This answer is too large. $13,500 \cdot 100 = 1,350,000$. The correct answer is 1,350.
B. Incorrect. This is a division, not a subtraction, problem. The correct answer is 1,350.
C. Incorrect. This is a division, not a multiplication, problem. The correct answer is 1,350.
D. Correct. 1,350 \cdot 100=135,000.

Exercise

A. Incorrect. This answer is too large. Use division, $1440 \div 30$, not subtraction for this problem. The correct answer is 48.
B. Correct. $1440 \div 30 = 48$.
C. Incorrect. The answer is too large. Use division, $1440 \div 30$, not multiplication, for this problem. The correct answer is 48.
D. Incorrect. There is a place-value error. The correct answer is 48.

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1.4.1: Practice and Videos for Introduction to Division

Learning Objectives

- I can divide two-digit numbers by one-digit numbers.
- I can divide using place value understanding.
- I can divide using rectangular arrays and area models.

GeoGebra: Seeing Divisibility

Complete the activity. Start by entering a dividend (the total amount being divided). Adjust the slider to indicate how much you want to divide by. Select 'show division equation' or 'show related multiplication equation' to see how the visual connects to a mathematical equation.

GeoGebra: Seeing Division

Use this other division simulation to visualize division as repeated subtraction or sharing equally. Enter two numbers in the empty boxes, click start, select a visualization option, then watch the simulation! Once you've made observations, hit the reset button to start over.

Videos on Division

Watch the following videos to learn about division. All videos have appropriate captions in English; in some cases, there may also be appropriate captions in other languages.

YouTube Video

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

YouTube Video

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

Practice!

Check your understanding of division by answering these questions. Use the arrows at the bottom of the box to go to the next question.

Practice 1.4.1.1

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this Interactive Content online

Extra Resources

For more practice on division, visit the following websites.

1. ORYX: Divide (Three-Digit Quotients)
2. IXL: Divide 2-digit numbers by 1-digit numbers using area models
3. Virtual Nerd: How Do You Divide a Two-Digit Number by a One-Digit Number With No Remainder? (Video)
4. Virtual Nerd: How Do You Divide a Two-Digit Number by a One-Digit Number With a Remainder? (Video)

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1.4.2: Common Mistakes for Introduction to Division

Common Mistake: Switching the Order in Division

Spot the Mistake 1.4.2.1

The following problem has an error. Identify the mistake.

$7 \div 21 = 3$

Click for explanation available at the end of this section.

Answers

Spot the Mistake 1.4.2.1

The answer is **not** 3.

The key is to recognize that $7 \div 21$ and $21 \div 7$ are not the same thing.

Multiplication can be done in either order:

$$3 \times 7 = 21 \tag{1.4.2.1}$$

$$7 \times 3 = 21 \tag{1.4.2.2}$$

But division works differently. A division sign can be read like a fraction bar. So the correct answer would look like this:

$$7 \div 21 = \frac{7}{21} \tag{1.4.2.3}$$

Reduce the answer by using the common factor of 7 by dividing the numerator and denominator by the same number 7.

$$\frac{7}{21} = \frac{7 \div 7}{21 \div 7} = \frac{1}{3} \tag{1.4.2.4}$$

Why does this work?

This means 7 is being divided into **21 equal groups.** If there are 7 brownies and 21 children, we do not have enough brownies to give to everyone. So each child gets only $\frac{1}{3}$.

Key Idea

Division is not commutative, which means changing the order can change the answer.

$$21 \div 7 \neq 7 \div 21 \tag{1.4.2.5}$$

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1.5: More on Division of Whole Numbers

Learning Objectives

By the end of this section, you will be able to:

- Use division notation
- Model division of whole numbers
- Divide whole numbers
- Translate word phrases to math notation
- Divide whole numbers in applications

Be Prepared 1.7

Before you get started, take this readiness quiz.

Multiply: $27 \cdot 3$.
If you missed this problem, review Example 1.44.

Be Prepared 1.8

Subtract: $43 - 26$.
If you missed this problem, review Example 1.32

Be Prepared 1.9

Multiply: $62(87)$.
If you missed this problem, review Example 1.45.

Use Division Notation

So far we have explored addition, subtraction, and multiplication. Now let's consider division. Suppose you have the Math input error



Figure 1.13

You might put 4 cookies in first bag, 4 in the second bag, and so on until you run out of cookies. Doing it this way, you would fill 3 bags.



In other words, starting with the 12 cookies, you would take away, or subtract, 4 cookies at a time. Division is a way to represent repeated subtraction just as multiplication represents repeated addition.

Instead of subtracting 4 repeatedly, we can write

$$12 \div 4$$

We read this as *twelve divided by four* and the result is the quotient of 12 and 4. The quotient is 3 because we can subtract 4 from 12 exactly 3 times. We call the number being divided the dividend and the number dividing it the divisor. In this case, the dividend is 12 and the divisor is 4.

In the past you may have used the notation $4\overline{)12}$, but this division also can be written as $12 \div 4$, $12/4$, $\frac{12}{4}$. In each case the 12 is the dividend and the 4 is the divisor.

Operation Symbols for Division

To represent and describe division, we can use symbols and words.

Operation	Notation	Expression	Read as	Result
Division	$\div$ $\frac{a}{b}$ $b\overline{)a}$ a/b	$12 \div 4$ $\frac{12}{4}$ $4\overline{)12}$ $12/4$	Twelve divided by	the quotient of 12 and

Division is performed on two numbers at a time. When translating from math notation to English words, or English words to math notation, look for the words *of* and *and* to identify the numbers.

Example 1.56

Translate from math notation to words.

ⓐ $64 \div 8$ ⓑ $\frac{42}{7}$ ⓒ $4\overline{)28}$

Answer available at the end of this section.

Model Division of Whole Numbers

As we did with multiplication, we will model division using counters. The operation of division helps us organize items into equal groups as we start with the number of items in the dividend and subtract the number in the divisor repeatedly.

Manipulative Mathematics

Doing the Manipulative Math Worksheets activity "Model Division of Whole Numbers" will help you develop a better understanding of dividing whole numbers.

Example 1.57

Model the division: $24 \div 8$.

Answer available at the end of this section.

Divide Whole Numbers

We said that addition and subtraction are inverse operations because one undoes the other. Similarly, division is the inverse operation of multiplication. We know $12 \div 4 = 3$ because $3 \cdot 4 = 12$. Knowing all the multiplication number facts is very important when doing division.

We check our answer to division by multiplying the quotient by the divisor to determine if it equals the dividend. In Example 1.57, we know $24 \div 8 = 3$ is correct because $3 \cdot 8 = 24$.

Example 1.58

Divide. Then check by multiplying. ⓐ $42 \div 6$ ⓑ $\frac{72}{9}$ ⓒ $7\overline{)63}$

Answer available at the end of this section.

- ⓒ

	$7\overline{)63}$
Divide 63 by 7.	9
Check by multiplying. $9 \cdot 7$	
63✓	

What is the quotient when you divide a number by itself?

Division Properties of One

Any number (except 0) divided by itself is one.	$a \div a =$
Any number divided by one is the same number.	$a \div 1 = \mathrm{a}$

Table 1.6

Example 1.59

Divide. Then check by multiplying:

1. ⓐ $11 \div 11$
2. ⓑ $\frac{19}{1}$
3. ⓒ $1\overline{)7}$

Answer available at the end of this section.

ⓒ	
	$1\overline{)7}$
A number divided by 1 equals itself.	7
Check by multiplying. $7 \cdot 1$	
7✓	

Suppose we have $\$0$, and want to divide it among 3 people. How much would each person get? Each person would get $\$0$. Zero divided by any number is 0.

Now suppose that we want to divide $\$10$ by 0. That means we would want to find a number that we multiply by 0 to get 10. This cannot happen because 0 times any number is 0. Division by zero is said to be *undefined*.

These two ideas make up the Division Properties of Zero.

Division Properties of Zero

Zero divided by any number is 0.	$0 \div a = 0$
Dividing a number by zero is undefined.	$a \div 0$ undefined

Table 1.7

Another way to explain why division by zero is undefined is to remember that division is really repeated subtraction. How many times can we take away 0 from 10? Because subtracting 0 will never change the total, we will never get an answer. So we cannot divide a number by 0.

Example 1.60

Divide. Check by multiplying: ⓐ $0 \div 3$ ⓑ $10/0$.

Answer available at the end of this section.

When the divisor or the dividend has more than one digit, it is usually easier to use the $4\overline{)12}$ notation. This process is called long division. Let's work through the process by dividing 78 by 3.

Divide the first digit of the dividend, 7, by the divisor, 3.	
The divisor 3 can go into 7 two times since $2 \times 3 = 6$. Write the 2 above the 7 in the quotient.	$3\overline{)78}$ with 2 above 7
Multiply the 2 in the quotient by 3 and write the product, 6, under the 7.	$3\overline{)78}$ with 2 above 7, 6 under 7

Subtract that product from the first digit in the dividend. Subtract $7 - 6$. Write the difference, 1, under the first digit in the dividend.	$\begin{array}{r} 2 \\ 3\overline{)78} \\ \underline{6} \\ 1 \end{array}$
Bring down the next digit of the dividend. Bring down the 8.	$\begin{array}{r} 2 \\ 3\overline{)78} \\ \underline{6} \\ 18 \end{array}$
Divide 18 by the divisor, 3. The divisor 3 goes into 18 six times.	$\begin{array}{r} 26 \\ 3\overline{)78} \\ \underline{6} \\ 18 \end{array}$
Write 6 in the quotient above the 8.	
Multiply the 6 in the quotient by the divisor and write the product, 18, under the dividend. Subtract 18 from 18.	$\begin{array}{r} 26 \\ 3\overline{)78} \\ \underline{6} \\ 18 \\ \underline{18} \\ 0 \end{array}$

We would repeat the process until there are no more digits in the dividend to bring down. In this problem, there are no more digits to bring down, so the division is finished.

$$\text{So } 78 \div 3 = 26.$$

Check by multiplying the quotient times the divisor to get the dividend. Multiply 26×3 to make sure that product equals the dividend, 78.

$$\begin{array}{r} {\scriptstyle 1} \\ 26 \\ \times 3 \\ \hline \end{array}$$

It does, so our answer is correct.

How To

Divide whole numbers.

1. Step 1. Divide the first digit of the dividend by the divisor.
   If the divisor is larger than the first digit of the dividend, divide the first two digits of the dividend by the divisor, and so on.
2. Step 2. Write the quotient above the dividend.
3. Step 3. Multiply the quotient by the divisor and write the product under the dividend.
4. Step 4. Subtract that product from the dividend.
5. Step 5. Bring down the next digit of the dividend.
6. Step 6. Repeat from Step 1 until there are no more digits in the dividend to bring down.
7. Step 7. Check by multiplying the quotient times the divisor.

Example 1.61

Divide $2{,}596 \div 4$. Check by multiplying:

Answer available at the end of this section.

Example 1.62

Divide $4{,}506 \div 6$. Check by multiplying:

Answer available at the end of this section.

Example 1.63

Divide $7{,}263 \div 9$. Check by multiplying.

Answer available at the end of this section.

So far all the division problems have worked out evenly. For example, if we had Math input error



Figure 1.14

Try to put the cookies in groups of eight as in Figure 1.15.

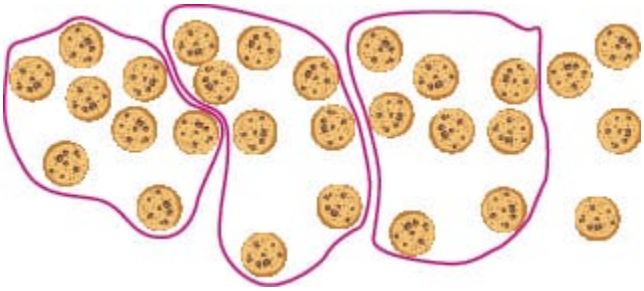


Figure 1.15

There are 3 groups of eight cookies, and 4 cookies left over. We call the 4 cookies that are left over the remainder and show it by writing R4 next to the 3. (The R stands for remainder.)

To check this division we multiply 3 times 8 to get 24, and then add the remainder of 4.

$$\begin{array}{r} 3 \\ \times 8 \\ \hline 24 \\ +4 \\ \hline 28 \end{array}$$

Example 1.64

Divide $1{,}439 \div 4$. Check by multiplying.

Answer available at the end of this section.

Example 1.65

Divide and then check by multiplying: $1{,}461 \div 13$.

Answer available at the end of this section.

Example 1.66

Divide and check by multiplying: $74{,}521 \div 241$.

Answer available at the end of this section.

Translate Word Phrases to Math Notation

Earlier in this section, we translated math notation for division into words. Now we'll translate word phrases into math notation. Some of the words that indicate division are given in Table 1.8.

Operation	Word Phrase	Example	Expression
Division	divided by quotient of divided into	12 divided by 4 the quotient of 12 and 4 4 divided into 12	$12 \div 4$ $\frac{12}{4}$ $12/4$ $4\overline{)12}$

Table 1.8

Example 1.67

Translate and simplify: the quotient of 51 and 17.

Answer available at the end of this section.

Divide Whole Numbers in Applications

We will use the same strategy we used in previous sections to solve applications. First, we determine what we are looking for. Then we write a phrase that gives the information to find it. We then translate the phrase into math notation and simplify it to get the answer. Finally, we write a sentence to answer the question.

Example 1.68

Cecelia bought a 160- box of oatmeal at the big box store. She wants to divide the 160 ounces of oatmeal into 8- servings. She will put each serving into a plastic bag so she can take one bag to work each day. How many servings will she get from the big box?

Answer available at the end of this section.

Media

ACCESS ADDITIONAL ONLINE RESOURCES

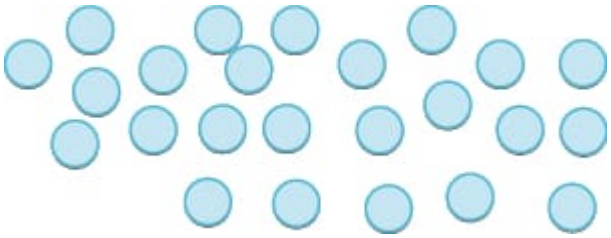
- Dividing Whole Numbers
- Dividing Whole Numbers No Remainder
- Dividing Whole Numbers With Remainder

Answers

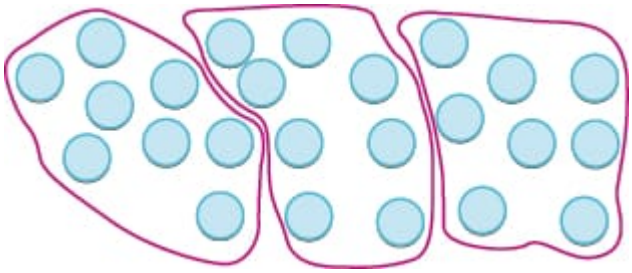
- ⓐ We read this as *sixty-four divided by eight* and the result is *the quotient of sixty-four and eight*.
- ⓑ We read this as *forty-two divided by seven* and the result is *the quotient of forty-two and seven*.
- ⓒ We read this as *twenty-eight divided by four* and the result is *the quotient of twenty-eight and four*.

To find the quotient $24 \div 8$, we want to know how many groups of 8 are in 24.

Model the dividend. Start with 24 counters.



The divisor tell us the number of counters we want in each group. Form groups of 8 counters.



Count the number of groups. There are 3 groups.

$24 \div 8 = 3$

ⓐ	
	$42 \div 6$
Divide 42 by 6.	7
Check by multiplying. $7 \cdot 6$	
42✓	

ⓑ	

	$\frac{72}{9}$
Divide 72 by 9.	8
Check by multiplying. $8 \cdot 9$	
$72 \checkmark$	

ⓐ	
	$11 \div 11$
A number divided by itself is 1.	1
Check by multiplying. $1 \cdot 11$	
$11 \checkmark$	

ⓑ	
	$\frac{19}{1}$
A number divided by 1 equals itself.	19
Check by multiplying. $19 \cdot 1$	
$19 \checkmark$	

ⓐ	
	$0 \div 3$
Zero divided by any number is zero.	0
Check by multiplying. $0 \cdot 3$	
$0 \checkmark$	

ⓑ	
	$10/0$
Division by zero is undefined.	undefined

Let's rewrite the problem to set it up for long division.	$4\overline{)2596}$
Divide the first digit of the dividend, 2, by the divisor, 4.	$4\overline{)2596}$
Since 4 does not go into 2, we use the first two digits of the dividend and divide 25 by 4. The divisor 4 goes into 25 six times.	
We write the 6 in the quotient above the 5.	$\begin{array}{r} 6 \\ 4\overline{)2596} \end{array}$
Multiply the 6 in the quotient by the divisor 4 and write the product, 24, under the first two digits in the dividend.	$\begin{array}{r} 6 \\ 4\overline{)2596} \\ \underline{24} \end{array}$

Subtract that product from the first two digits in the dividend. Subtract $25 - 24$. Write the difference, 1, under the second digit in the dividend.	$\begin{array}{r} 6 \\ 4\overline{)2596} \\ \underline{24} \\ 1 \end{array}$
Now bring down the 9 and repeat these steps. There are 4 fours in 19. Write the 4 over the 9. Multiply the 4 by 4 and subtract this product from 19.	$\begin{array}{r} 64 \\ 4\overline{)2596} \\ \underline{24} \\ 19 \\ \underline{16} \\ 3 \end{array}$
Bring down the 6 and repeat these steps. There are 9 fours in 36. Write the 9 over the 6. Multiply the 9 by 4 and subtract this product from 36.	$\begin{array}{r} 649 \\ 4\overline{)2596} \\ \underline{24} \\ 19 \\ \underline{16} \\ 36 \\ \underline{36} \\ 0 \end{array}$
So $2{,}596 \div 4 = 649$.	
Check by multiplying. $\begin{array}{r} {\scriptstyle 1\ 3} \\ 649 \\ \times \quad 4 \\ \hline 2{,}596 \checkmark \end{array}$	

It equals the dividend, so our answer is correct.

Let's rewrite the problem to set it up for long division.	$6\overline{)4506}$
First we try to divide 6 into 4.	$6\overline{)4506}$
Since that won't work, we try 6 into 45. There are 7 sixes in 45. We write the 7 over the 5.	$\begin{array}{r} 7 \\ 6\overline{)4506} \end{array}$
Multiply the 7 by 6 and subtract this product from 45.	$\begin{array}{r} 7 \\ 6\overline{)4506} \\ \underline{42} \\ 3 \end{array}$

Now bring down the 0 and repeat these steps. There are 5 sixes in 30. Write the 5 over the 0. Multiply the 5 by 6 and subtract this product from 30.	$\begin{array}{r} 75 \\ 6\overline{)4506} \\ \underline{42} \\ 30 \\ \underline{30} \\ 0 \end{array}$
Now bring down the 6 and repeat these steps. There is 1 six in 6. Write the 1 over the 6. Multiply 1 by 6 and subtract this product from 6.	$\begin{array}{r} 751 \\ 6\overline{)4506} \\ \underline{42} \\ 30 \\ \underline{30} \\ 06 \\ \underline{6} \\ 0 \end{array}$
Check by multiplying. $\begin{array}{r} {\scriptstyle 3}\,751 \\ \times\quad 6 \\ \hline 4{,}506\ \checkmark \end{array}$	

It equals the dividend, so our answer is correct.

Let's rewrite the problem to set it up for long division.	$9\overline{)7263}$
First we try to divide 9 into 7.	$9\overline{)7263}$
Since that won't work, we try 9 into 72. There are 8 nines in 72. We write the 8 over the 2.	$\begin{array}{r} 8 \\ 9\overline{)7263} \end{array}$
Multiply the 8 by 9 and subtract this product from 72.	$\begin{array}{r} 8 \\ 9\overline{)7263} \\ \underline{72} \\ 0 \end{array}$
Now bring down the 6 and repeat these steps. There are 0 nines in 6. Write the 0 over the 6. Multiply the 0 by 9 and subtract this product from 6.	$\begin{array}{r} 80 \\ 9\overline{)7263} \\ \underline{72} \\ 06 \\ \underline{0} \\ 6 \end{array}$

Now bring down the 3 and repeat these steps. There are 7 nines in 63. Write the 7 over the 3. Multiply the 7 by 9 and subtract this product from 63.	$\begin{array}{r} 807 \\ 9\overline{)7263} \\ 72 \\ \hline 06 \\ 0 \\ \hline 63 \\ 63 \\ \hline 0 \end{array}$
Check by multiplying. $\begin{array}{r} {\scriptstyle 6} \\ 807 \\ \times \quad 9 \\ \hline 7{,}263 \end{array}$ ✓	

It equals the dividend, so our answer is correct.

Let's rewrite the problem to set it up for long division.	$4\overline{)1439}$
First we try to divide 4 into 1. Since that won't work, we try 4 into 14. There are 3 fours in 14. We write the 3 over the 4.	$\begin{array}{r} 3 \\ 4\overline{)1439} \end{array}$
Multiply the 3 by 4 and subtract this product from 14.	$\begin{array}{r} 3 \\ 4\overline{)1439} \\ 12 \\ \hline 2 \end{array}$
Now bring down the 3 and repeat these steps. There are 5 fours in 23. Write the 5 over the 3. Multiply the 5 by 4 and subtract this product from 23.	$\begin{array}{r} 35 \\ 4\overline{)1439} \\ 12\downarrow \\ \hline 23 \\ 20 \\ \hline 3 \end{array}$
Now bring down the 9 and repeat these steps. There are 9 fours in 39. Write the 9 over the 9. Multiply the 9 by 4 and subtract this product from 39. There are no more numbers to bring down, so we are done. The remainder is 3.	$\begin{array}{r} 359\text{R}3 \\ 4\overline{)1439} \\ 12 \\ \hline 23 \\ 20\downarrow \\ \hline 39 \\ 36 \\ \hline 3 \end{array}$

Check by multiplying. $\begin{array}{rl} {\scriptstyle 2\ 3}\ 359 & \text{quotient} \\ \times\quad 4 & \text{divisor} \\ \hline 1{,}436 & \\ +\quad 3 & \text{remainder} \\ \hline 1{,}439\ \checkmark & \end{array}$	

So $1{,}439 \div 4$ is 359 with a remainder of 3. Our answer is correct.

Let's rewrite the problem to set it up for long division.	$13\overline{)1{,}461}$
First we try to divide 13 into 1. Since that won't work, we try 13 into 14. There is 1 thirteen in 14. We write the 1 over the 4.	$\begin{array}{r} 1 \\ 13\overline{)1461} \end{array}$
Multiply the 1 by 13 and subtract this product from 14.	$\begin{array}{r} 1 \\ 13\overline{)1461} \\ \underline{13} \\ 1 \end{array}$
Now bring down the 6 and repeat these steps. There is 1 thirteen in 16. Write the 1 over the 6. Multiply the 1 by 13 and subtract this product from 16.	$\begin{array}{r} 11 \\ 13\overline{)1461} \\ \underline{13}\downarrow \\ 16 \\ \underline{13} \\ 3 \end{array}$
Now bring down the 1 and repeat these steps. There are 2 thirteens in 31. Write the 2 over the 1. Multiply the 2 by 13 and subtract this product from 31. There are no more numbers to bring down, so we are done. The remainder is 5. $1{,}462 \div 13$ is 112 with a remainder of 5.	$\begin{array}{r} 112\text{R}5 \\ 13\overline{)1461} \\ \underline{13}\downarrow \\ 16 \\ \underline{13}\downarrow \\ 31 \\ \underline{26} \\ 5 \end{array}$
Check by multiplying. $\begin{array}{rl} 112 & \text{quotient} \\ \times\ 13 & \text{divisor} \\ \hline 336 & \\ 1{,}120 & \\ +\quad 5 & \text{remainder} \\ \hline 1{,}461 & \checkmark \end{array}$	

Our answer is correct.

Let's rewrite the problem to set it up for long division.	$241\overline{)74{,}521}$
First we try to divide 241 into 7. Since that won't work, we try 241 into 74. That still won't work, so we try 241 into 745. Since 2 divides into 7 three times, we try 3. Since $3 \times 241 = 723$, we write the 3 over the 5 in 745. Note that 4 would be too large because $4 \times 241 = 964$, which is greater than 745.	
Multiply the 3 by 241 and subtract this product from 745.	$\begin{array}{r} 3 \\ 241\overline{)74521} \\ 723 \\ \hline 22 \end{array}$
Now bring down the 2 and repeat these steps. 241 does not divide into 222. We write a 0 over the 2 as a placeholder and then continue.	$\begin{array}{r} 30 \\ 241\overline{)74521} \\ 723 \\ \hline 222 \end{array}$
Now bring down the 1 and repeat these steps. Try 9. Since $9 \times 241 = 2{,}169$, we write the 9 over the 1. Multiply the 9 by 241 and subtract this product from 2,221.	$\begin{array}{r} 309\text{ R}52 \\ 241\overline{)74521} \\ 723 \\ \hline 2221 \\ 2169 \\ \hline 52 \end{array}$
There are no more numbers to bring down, so we are finished. The remainder is 52. So $74{,}521 \div 241$ is 309 with a remainder of 52.	
Check by multiplying. $\begin{array}{rl} 3 & \\ 309 & \text{quotient} \\ \times\ 241 & \text{divisor} \\ \hline 309 & \\ 12{,}360 & \\ 61{,}800 & \\ \hline 72{,}469 & \\ +\ \ 52 & \text{remainder} \\ \hline 74{,}521 & \checkmark \end{array}$	

Sometimes it might not be obvious how many times the divisor goes into digits of the dividend. We will have to guess and check numbers to find the greatest number that goes into the digits without exceeding them.

The word *quotient* tells us to divide.

	the quotient of 51 and 17
Translate.	$51 \div 17$
Divide.	3

We could just as correctly have translated *the quotient of* 51 *and* 17 using the notation $17\overline{)51}$ or $\frac{51}{17}$.

We are asked to find the how many servings she will get from the big box.

Write a phrase.	160 ounces divided by 8 ounces

Translate to math notation.	$160 \div 8$
Simplify by dividing.	20
Write a sentence to answer the question.	Cecelia will get 20 servings from the big box.

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1.6: Extra Division Practice and Word Problems

- **Submit homework separately from this workbook and staple all pages together. (One staple for the entire submission of all the unit homework)**
- **Start a new module on the front side of a new page and write the module number on the top center of the page.**
- **Answers without supporting work will receive no credit.**
- **Some solutions are given in the solutions manual.**
- **You may work with classmates but do your own work.**

HW #1

Mae had 72 lollipops and wanted to sell packages that had 4 lollipops in each package. How many packages could she make?

a. What division problem do you need to do to find the answer?

b. What interpretation of division does this word problem represent?

c. Explain how to use the correct interpretation to draw a picture to find the answer.

d. Draw a picture using the correct interpretation to help find the answer.

e. Write the multiplication problem shown by the picture:

f. Explain how you use the picture to find the answer to the problem. Be specific.

g. What is the answer to this problem?

HW #2

Tina had 72 marbles and wanted to divide them up evenly amongst the 9 children in her class. How many marbles should each child receive?

a. What division problem do you need to do to find the answer? :

b. What interpretation of division does this word problem represent?

c. Explain how to use the correct interpretation to draw a picture to find the answer.

d. Draw a picture using the correct interpretation to help find the answer.

e. Write the multiplication problem shown by the picture:

f. Explain how you use the picture to find the answer to the problem. Be specific.

g. What is the answer to this problem?

HW #3

Joan had 12 cups of sugar. A cookie recipe required 2/3 of a cup of sugar. How many recipes could she make?

a. What division problem do you need to do to find the answer? :

b. What interpretation of division does this word problem represent?

c. Explain how to use the correct interpretation to draw a picture to find the answer.

d. Draw a picture using the correct interpretation to help find the answer.

e. Write the multiplication problem shown by the picture:

f. Explain how you use the picture to find the answer to the problem. Be specific.

g. What is the answer to this problem?

HW #4

a. Make up a word problem that would require using the repeated subtraction interpretation of division and the division problem: $20 \div 5$.

b. Draw a picture using this interpretation to find the answer.

c. Explain how you use the picture to find the answer to the problem. Be specific.

d. Write the multiplication problem shown by the picture.

e. What is the answer to this problem?

HW #5

a. Make up a word problem that would require using the repeated subtraction interpretation of division and the division problem: $6 \div 2/3$.

b. Draw a picture using this interpretation to find the answer.

c. Explain how you use the picture to find the answer to the problem. Be specific.

d. Write the multiplication problem shown by the picture.

e. What is the answer to this problem? _____

HW #6

Someone was solving this problem: “Mark had 42 pencils and wanted to divide them up equally amongst his 3 children. How many pencils should he give each child?”

Which picture best represents a picture he might draw to find the solution?



HW #7

Is division commutative?

Support your answer with either an example or a counterexample.

HW #8

Is division associative?

Support your answer with either an example or a counterexample

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1.6.1: Practice and Videos on Division and Word Problems

Learning Objectives

- I can use models to illustrate division word problems.
- I can match division expressions with the correct quotients.
- I can fluently divide using division strategies.

GeoGebra: Division Word Problems

Use models to solve division word problems in this visual activity. First, predict your answer to the question. Then, use the slider to animate the graphic and check if your answer was correct. Click on the 'new question' button when you are ready to move on.

Videos on Division Word Problems

Watch these videos to learn how to do division word problems and review dividing by two-digit numbers. All videos have appropriate captions in English; in some cases, there may also be appropriate captions in other languages.

YouTube Video

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

YouTube Video

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

Extra Practice Resources

1. IXL: Division facts to 12: word problems
2. IXL: Divide larger numbers by 1-digit numbers: word problems
3. Math Playground: Drag Race Division

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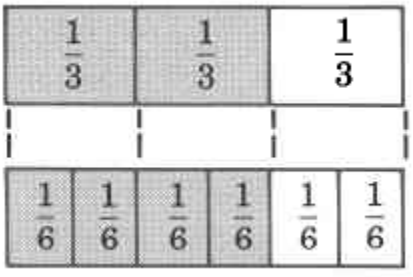
1.7: Equivalent Fractions, Reducing Fractions to Lowest Terms, and Raising Fractions to Higher Terms

Learning Objectives

- be able to recognize equivalent fractions
- be able to reduce a fraction to lowest terms
- be able to raise a fraction to higher terms

Equivalent Fractions

Let's examine the following two diagrams.



$\frac{2}{3}$ of the whole is shaded.

$\frac{4}{6}$ of the whole is shaded.

Notice that both $\frac{2}{3}$ and $\frac{4}{6}$ represent the *same part* of the whole, that is, they represent the same number.

Definition: Equivalent Fractions

Fractions that have the same value are called **equivalent fractions**. Equivalent fractions may look different, but they are still the same point on the number line.

There is an interesting property that equivalent fractions satisfy.

$$\frac{2}{3} \times \frac{4}{6}$$

A Test for Equivalent Fractions Using the Cross Product

Definition: cross products

These pairs of products are called **cross products**.

$$2 \cdot 6 \overset{?}{=} 3 \cdot 4$$
$$12 = 12$$

If the cross products are equal, the fractions are equivalent. If the cross products are not equal, the fractions are not equivalent.

Thus, $\frac{2}{3}$ and $\frac{4}{6}$ are equivalent, that is, $\frac{2}{3} = \frac{4}{6}$.

Sample Set A

Determine if the following pairs of fractions are equivalent.

$\frac{3}{4}$ and $\frac{6}{8}$. Test for equality of the cross products.

Solution

$$\frac{3}{4} \times \frac{6}{8} \quad 3 \cdot 8 \stackrel{?}{=} 6 \cdot 4$$
$$24 \stackrel{\checkmark}{=} 24$$

The cross products are equals.

The fractions $\frac{3}{4}$ and $\frac{6}{8}$ are equivalent, so $\frac{3}{4} = \frac{6}{8}$.

Sample Set A

$\frac{3}{8}$ and $\frac{9}{16}$. Test for equality of the cross products.

Solution

$$\frac{3}{8} \times \frac{9}{16} \quad 3 \cdot 16 \stackrel{?}{=} 9 \cdot 8$$
$$48 \neq 72$$

The cross products are equals.

The fractions $\frac{3}{8}$ and $\frac{9}{16}$ are not equivalent.

Practice Set A

Determine if the pairs of fractions are equivalent.

$\frac{1}{2}, \frac{3}{6}$

Answer available at the end of this section.

Practice Set A

$\frac{4}{5}, \frac{12}{15}$

Answer available at the end of this section.

Practice Set A

$\frac{2}{3}, \frac{8}{15}$

Answer available at the end of this section.

Practice Set A

$\frac{1}{8}, \frac{4}{50}$

Answer available at the end of this section.

Practice Set A

$\frac{3}{12}, \frac{1}{4}$

Answer available at the end of this section.

Reducing Fractions to Lowest Terms

It is often very useful to *convert* one fraction to an equivalent fraction that has reduced values in the numerator and denominator. We can suggest a method for doing so by considering the equivalent fractions $\frac{9}{15}$ and $\frac{3}{5}$. First, divide both the numerator and denominator of $\frac{9}{15}$ by 3. The fraction $\frac{9}{15}$ and $\frac{3}{5}$ are equivalent.

(Can you prove this?) So, $\frac{9}{15} = \frac{3}{5}$. We wish to convert $\frac{9}{15}$ to $\frac{3}{5}$. Now divide the numerator and denominator of $\frac{9}{15}$ by 3, and see what happens.

$$\frac{9 \div 3}{15 \div 3} = \frac{3}{5}$$

The fraction $\frac{9}{15}$ is converted to $\frac{3}{5}$.

A natural question is "Why did we choose to divide by 3?" Notice that

$$\frac{9}{15} = \frac{3 \cdot 3}{5 \cdot 3}$$

We can see that the *factor* 3 is common to both the numerator and denominator.

Definition: Reducing a Fraction

From these observations we can suggest the following method for converting one fraction to an equivalent fraction that has reduced values in the numerator and denominator. The method is called **reducing a fraction**.

A fraction can be **reduced** by dividing *both* the numerator and denominator by the *same* nonzero whole number.

$$\frac{9}{12}=\frac{9\div 3}{12\div 3}=\frac{3}{4} \qquad \frac{16}{30}=\frac{16\div 2}{30\div 2}=\frac{8}{15}$$

Notice that $\frac{3}{3}=1$ and $\frac{2}{2}=1$

Consider the collection of equivalent fractions

$$\frac{5}{20},\frac{4}{16},\frac{3}{12},\frac{2}{8},\frac{1}{4}$$

Reduced to Lowest Terms

Notice that each of the first four fractions can be *reduced* to the last fraction, $\frac{1}{4}$, by dividing both the numerator and denominator by, respectively, 5, 4, 3, and 2. When a fraction is converted to the fraction that has the smallest numerator and denominator in its collection of equivalent fractions, it is said to be **reduced to lowest terms**. The fractions $\frac{1}{4}$, $\frac{3}{8}$, $\frac{2}{5}$, and $\frac{7}{10}$ are all reduced to lowest terms.

Observe a very important property of a fraction that has been reduced to lowest terms. The *only* whole number that divides *both* the numerator and denominator without a remainder is the number 1. When 1 is the only whole number that divides two whole numbers, the two whole numbers are said to be **relatively prime**.

Relatively Prime
A fraction is reduced to lowest terms if its numerator and denominator are **relatively prime**.

Methods of Reducing Fractions to Lowest Terms

Method 1: Dividing Out Common Primes

1. Write the numerator and denominator as a product of primes.
2. Divide the numerator and denominator by each of the common prime factors. We often indicate this division by drawing a slanted line through each divided out factor. This process is also called **cancelling common factors**.
3. The product of the remaining factors in the numerator and the product of remaining factors of the denominator are relatively prime, and this fraction is reduced to lowest terms.

Sample Set B

$$\frac{6}{18}=\frac{\cancel{2}^{1}\cdot\cancel{3}^{1}}{\cancel{2}_{1}\cdot\cancel{3}_{1}\cdot 3}=\frac{1}{3}$$ 1 and 3 are relatively prime.

Sample Set B

$$\frac{16}{20}=\frac{\cancel{2}^{1}\cdot\cancel{2}^{1}\cdot 2\cdot 2}{\cancel{2}_{1}\cdot\cancel{2}_{1}\cdot 5}=\frac{4}{5}$$ 4 and 5 are relatively prime.

Sample Set B

$\frac{56}{104} = \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{3}} \cdot \overset{1}{\cancel{2}} \cdot 7}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot 13} = \frac{7}{13}$ 7 and 13 are relatively prime (and also truly prime)

Sample Set B

$\frac{315}{336} = \frac{\overset{1}{\cancel{3}} \cdot 3 \cdot 5 \cdot \overset{1}{\cancel{7}}}{2 \cdot 2 \cdot 2 \cdot 2 \cdot \underset{1}{\cancel{3}} \cdot \underset{1}{\cancel{7}}} = \frac{15}{16}$ 15 and 16 are relatively prime.

Sample Set B

$\frac{8}{15} = \frac{2 \cdot 2 \cdot 2}{3 \cdot 5}$ No common prime factors, so 8 and 15 are relatively prime.

The fraction $\frac{8}{15}$ is reduced to lowest terms.

Practice Set B

Reduce each fraction to lowest terms.

$\frac{4}{8}$

Answer available at the end of this section.

Practice Set B

$\frac{6}{15}$

Answer available at the end of this section.

Practice Set B

$\frac{6}{48}$

Answer available at the end of this section.

Practice Set B

$\frac{21}{48}$

Answer available at the end of this section.

Practice Set B

$\frac{72}{42}$

Answer available at the end of this section.

Practice Set B

$\frac{135}{243}$

Answer available at the end of this section.

Method 2: Dividing Out Common Factors

1. Mentally divide the numerator and the denominator by a factor that is common to each. Write the quotient above the original number.
2. Continue this process until the numerator and denominator are relatively prime.

Sample Set C

Reduce each fraction to lowest terms.

$\frac{25}{30}$. 5 divides into both 25 and 30.

$$\frac{\overset{5}{\cancel{25}}}{\underset{6}{\cancel{30}}} = \frac{5}{6}$$ 5 and 6 are relatively prime.

Sample Set C

$\frac{18}{24}$. Both numbers are even so we can divide by 2.

$\frac{\overset{9}{\cancel{18}}}{\underset{12}{\cancel{24}}}$ Now, both 9 and 12 are divisible by 3.

$$\frac{\overset{3}{\cancel{9}}\overset{}{\cancel{18}}}{\underset{4}{\cancel{12}}\underset{}{\cancel{24}}} = \frac{3}{4}$$ 3 and 4 are relatively prime.

Sample Set C

$\frac{\cancel{210}^{7}}{\cancel{150}_{5}} = \frac{7}{5}$ 7 and 5 are relatively prime.

Sample Set C

$\frac{36}{96} = \frac{18}{48} = \frac{9}{24} = \frac{3}{8}$. 3 and 8 are relatively prime.

Practice Set C

Reduce each fraction to lowest terms.

$\frac{12}{16}$

Answer available at the end of this section.

Practice Set C

$\frac{9}{24}$

Answer available at the end of this section.

Practice Set C

$\frac{21}{84}$

Answer available at the end of this section.

Practice Set C

$\frac{48}{64}$

Answer available at the end of this section.

Practice Set C

$\frac{63}{81}$

Answer available at the end of this section.

Practice Set C

$\frac{150}{240}$

Answer available at the end of this section.

Raising Fractions to Higher Terms

Equally as important as reducing fractions is raising fractions to higher terms. Raising a fraction to higher terms is the process of constructing an equivalent fraction that has higher values in the numerator and denominator than the original fraction.

The fractions $\frac{3}{5}$ and $\frac{9}{15}$ are equivalent, that is, $\frac{3}{5} = \frac{9}{15}$. Notice also,

$$\frac{3 \cdot 3}{5 \cdot 3} = \frac{9}{15}$$

Notice that $\frac{3}{3} = 1$ and that $\frac{3}{5} \cdot 1 = \frac{3}{5}$. We are not changing the value of $\frac{3}{5}$.

From these observations we can suggest the following method for converting one fraction to an equivalent fraction that has higher values in the numerator and denominator. This method is called **raising a fraction to higher terms**.

Raising a Fraction to Higher Terms
A fraction can be raised to an equivalent fraction that has higher terms in the numerator and denominator by multiplying both the numerator and denominator by the same nonzero whole number.

The fraction $\frac{3}{4}$ can be raised to $\frac{24}{32}$ by multiplying both the numerator and denominator by 8.

$$\frac{3}{4} = \frac{3 \cdot 8}{4 \cdot 8} = \frac{24}{32}$$

Notice that $\frac{8}{8} = 1$.

Most often, we will want to convert a given fraction to an equivalent fraction with a higher specified denominator. For example, we may wish to convert $\frac{5}{8}$ to an equivalent fraction that has denominator 32, that is,

$$\frac{5}{8} = \frac{?}{32}$$

This is possible to do because we know the process. We must multiply *both* the numerator and denominator of $\frac{5}{8}$ by the *same* nonzero whole number in order to 8 obtain an equivalent fraction.

We have some information. The denominator 8 was raised to 32 by multiplying it by some nonzero whole number. Division will give us the proper factor. Divide the original denominator into the new denominator.

$$32 \div 8 = 4$$

Now, multiply the numerator 5 by 4.

$$5 \cdot 4 = 20$$

Thus,

$$\frac{5}{8}=\frac{5\cdot 4}{8\cdot 4}=\frac{20}{32}$$

So,

$$\frac{5}{8}=\frac{20}{32}$$

Sample Set D

Determine the missing numerator or denominator.

$\frac{3}{7}=\frac{?}{35}$. Divide the original denominator into the new denominator.

$35\div 7=5$. The quotient is 5. Multiply the original numerator by 5.

$\frac{3}{7}=\frac{3\cdot 5}{7\cdot 5}=\frac{15}{35}$ The missing numerator is 15.

Sample Set D

$\frac{5}{6}=\frac{45}{?}$. Divide the original numerator into the new numerator.

$45\div 5=9$. The quotient is 9. Multiply the original denominator by 9.

$\frac{5}{6}=\frac{5\cdot 9}{6\cdot 9}=\frac{45}{54}$ The missing denominator is 45.

Practice Set D

Determine the missing numerator or denominator.

$$\frac{4}{5}=\frac{?}{40}$$

Answer available at the end of this section.

Practice Set D

$$\frac{3}{7}=\frac{?}{28}$$

Answer available at the end of this section.

Practice Set D

$$\frac{1}{6}=\frac{?}{24}$$

Answer available at the end of this section.

Practice Set D

$$\frac{3}{10} = \frac{45}{?}$$

Answer available at the end of this section.

Practice Set D

$$\frac{8}{15} = \frac{?}{165}$$

Answer available at the end of this section.

Exercises

For the following problems, determine if the pairs of fractions are equivalent.

Exercise 1.7.1

$$\frac{1}{2}, \frac{5}{10}$$

Answer available at the end of this section.

Exercise 1.7.2

$$\frac{2}{3}, \frac{8}{12}$$

Exercise 1.7.3

$$\frac{5}{12}, \frac{10}{24}$$

Answer available at the end of this section.

Exercise 1.7.4

$$\frac{1}{2}, \frac{3}{6}$$

Exercise 1.7.5

$$\frac{3}{5}, \frac{12}{15}$$

Answer available at the end of this section.

Exercise 1.7.6

$\frac{1}{6}, \frac{7}{42}$

Exercise 1.7.7

$\frac{16}{25}, \frac{49}{75}$

Answer available at the end of this section.

Exercise 1.7.8

$\frac{5}{28}, \frac{20}{112}$

Exercise 1.7.9

$\frac{3}{10}, \frac{36}{110}$

Answer available at the end of this section.

Exercise 1.7.10

$\frac{6}{10}, \frac{18}{32}$

Exercise 1.7.11

$\frac{5}{8}, \frac{15}{24}$

Answer available at the end of this section.

Exercise 1.7.12

$\frac{10}{16}, \frac{15}{24}$

Exercise 1.7.13

$\frac{4}{5}, \frac{3}{4}$

Answer available at the end of this section.

Exercise 1.7.14

$\frac{5}{7}, \frac{15}{21}$

Exercise 1.7.15

$\frac{9}{11}, \frac{11}{9}$

Answer available at the end of this section.

For the following problems, determine the missing numerator or denominator.

Exercise 1.7.16

$\frac{1}{3} = \frac{?}{12}$

Exercise 1.7.17

$\frac{1}{5} = \frac{?}{30}$

Answer available at the end of this section.

Exercise 1.7.18

$\frac{2}{3} = \frac{?}{9}$

Exercise 1.7.19

$\frac{3}{4} = \frac{?}{16}$

Answer available at the end of this section.

Exercise 1.7.20

$\frac{5}{6} = \frac{?}{18}$

Exercise 1.7.21

$\frac{4}{5} = \frac{?}{25}$

Answer available at the end of this section.

Exercise 1.7.22

$\frac{1}{2} = \frac{4}{?}$

Exercise 1.7.23

$\frac{9}{25} = \frac{27}{?}$

Answer available at the end of this section.

Exercise 1.7.24

$\frac{3}{2} = \frac{18}{?}$

Exercise 1.7.25

$\frac{5}{3} = \frac{80}{?}$

Answer available at the end of this section.

Exercise 1.7.26

$\frac{1}{8} = \frac{3}{?}$

Exercise 1.7.27

$\frac{4}{5} = \frac{?}{100}$

Answer available at the end of this section.

Exercise 1.7.28

$\frac{1}{2} = \frac{25}{?}$

Exercise 1.7.29

$\frac{3}{16} = \frac{?}{96}$

Answer available at the end of this section.

Exercise 1.7.30

$\frac{15}{16} = \frac{225}{?}$

Exercise 1.7.31

$\frac{11}{12} = \frac{?}{168}$

Answer available at the end of this section.

Exercise 1.7.32

$\frac{9}{13} = \frac{?}{286}$

Exercise 1.7.33

$\frac{32}{33} = \frac{?}{1518}$

Answer available at the end of this section.

Exercise 1.7.34

$\frac{19}{20} = \frac{1045}{?}$

Exercise 1.7.35

$\frac{37}{50} = \frac{1369}{?}$

Answer available at the end of this section.

For the following problems, reduce, if possible, each of the fractions to lowest terms.

Exercise 1.7.36

$\frac{6}{8}$

Exercise 1.7.37

$\frac{8}{10}$

Answer available at the end of this section.

Exercise 1.7.38

$\frac{5}{10}$

Exercise 1.7.39

$\frac{6}{14}$

Answer available at the end of this section.

Exercise 1.7.40

$\frac{3}{12}$

Exercise 1.7.41

$\frac{4}{14}$

Answer available at the end of this section.

Exercise 1.7.42

$\frac{1}{6}$

Exercise 1.7.43

$\frac{4}{6}$

Answer available at the end of this section.

Exercise 1.7.44

$\frac{18}{14}$

Exercise 1.7.45

$\frac{20}{8}$

Answer available at the end of this section.

Exercise 1.7.46

$\frac{4}{6}$

Exercise 1.7.47

$\frac{10}{6}$

Answer available at the end of this section.

Exercise 1.7.48

$\frac{6}{14}$

Exercise 1.7.49

$\frac{14}{6}$

Answer available at the end of this section.

Exercise 1.7.50

$\frac{10}{12}$

Exercise 1.7.51

$\frac{16}{70}$

Answer available at the end of this section.

Exercise 1.7.52

$\frac{40}{60}$

Exercise 1.7.53

$\frac{20}{12}$

Answer available at the end of this section.

Exercise 1.7.54

$\frac{32}{28}$

Exercise 1.7.55

$\frac{36}{10}$

Answer available at the end of this section.

Exercise 1.7.56

$\frac{36}{60}$

Exercise 1.7.57

$\frac{12}{18}$

Answer available at the end of this section.

Exercise 1.7.58

$\frac{18}{27}$

Exercise 1.7.59

$\frac{18}{24}$

Answer available at the end of this section.

Exercise 1.7.60

$\frac{32}{40}$

Exercise 1.7.61

$\frac{11}{22}$

Answer available at the end of this section.

Exercise 1.7.62

$\frac{27}{81}$

Exercise 1.7.63

$\frac{17}{51}$

Answer available at the end of this section.

Exercise 1.7.64

$\frac{16}{42}$

Exercise 1.7.65

$\frac{39}{13}$

Answer available at the end of this section.

Exercise 1.7.66

$\frac{44}{11}$

Exercise 1.7.67

$\frac{66}{33}$

Answer available at the end of this section.

Exercise 1.7.68

$\frac{15}{1}$

Exercise 1.7.69

$\frac{15}{16}$

Answer available at the end of this section.

Exercise 1.7.70

$\frac{15}{40}$

Exercise 1.7.71

$\frac{36}{100}$

Answer available at the end of this section.

Exercise 1.7.72

$\frac{45}{32}$

Exercise 1.7.73

$\frac{30}{75}$

Answer available at the end of this section.

Exercise 1.7.74

$\frac{121}{132}$

Exercise 1.7.75

$\frac{72}{64}$

Answer available at the end of this section.

Exercise 1.7.76

$\frac{30}{105}$

Exercise 1.7.77

$\frac{46}{60}$

Answer available at the end of this section.

Exercise 1.7.78

$\frac{75}{45}$

Exercise 1.7.79

$\frac{40}{18}$

Answer available at the end of this section.

Exercise 1.7.80

$\frac{108}{76}$

Exercise 1.7.81

$\frac{7}{21}$

Answer available at the end of this section.

Exercise 1.7.82

$\frac{6}{51}$

Exercise 1.7.83

$\frac{51}{12}$

Answer available at the end of this section.

Exercise 1.7.84

$\frac{8}{100}$

Exercise 1.7.85

$\frac{51}{54}$

Answer available at the end of this section.

Exercise 1.7.86

A ream of paper contains 500 sheets. What fraction of a ream of paper is 200 sheets? Be sure to reduce.

Exercise 1.7.87

There are 24 hours in a day. What fraction of a day is 14 hours?

Answer available at the end of this section.

Exercise 1.7.88

A full box contains 80 calculators. How many calculators are in $\frac{1}{4}$ of a box?

Exercise 1.7.89

There are 48 plants per flat. How many plants are there in $\frac{1}{3}$ of a flat?

Answer available at the end of this section.

Exercise 1.7.90

A person making $18,000 per year must pay $3,960 in income tax. What fraction of this person's yearly salary goes to the IRS?

For the following problems, find the mistake.

Exercise 1.7.91

$$\frac{3}{24}=\frac{\cancel{3}}{\cancel{3}\cdot 8}=\frac{0}{8}=0$$

Answer available at the end of this section.

Exercise 1.7.92

$$\frac{8}{10}=\frac{\cancel{2}+6}{\cancel{2}+8}=\frac{6}{8}=\frac{3}{4}$$

Exercise 1.7.93

$$\frac{7}{15}=\frac{\cancel{7}}{\cancel{7}+8}=\frac{1}{8}$$

Answer available at the end of this section.

Exercise 1.7.94

$$\frac{6}{7} = \frac{\cancel{5}+1}{\cancel{5}+2} = \frac{1}{2}$$

Exercise 1.7.95

$$\frac{\cancel{9}}{\cancel{9}} = \frac{0}{0} = 0$$

Answer available at the end of this section.

Exercises for Review

Exercise 1.7.96

Round 816 to the nearest thousand.

Exercise 1.7.97

Perform the division: $0 \div 6$.

Answer available at the end of this section.

Exercise 1.7.98

Find all the factors of 24.

Exercise 1.7.99

Find the greatest common factor of 12 and 18.

Answer available at the end of this section.

Exercise 1.7.100

Convert $\frac{15}{8}$ to a mixed number.

Answers

Practice Set A

$6 \leq 6$,

, yes

Practice Set A

$60 \leq 60$

, yes

Practice Set A

$30 \neq 24$, no

Practice Set A

$40 \not\leq 40$,

, yes

Practice Set A

$12 \not\leq 12$,

, yes

Practice Set B

$\frac{1}{2}$

Practice Set B

$\frac{2}{5}$

Practice Set B

$\frac{1}{8}$

Practice Set B

$\frac{7}{16}$

Practice Set B

$\frac{12}{7}$

Practice Set B

$\frac{5}{9}$

Practice Set C

$\frac{3}{4}$

Practice Set C

$\frac{3}{8}$

Practice Set C

$\frac{1}{4}$

Practice Set C

$\frac{3}{4}$

Practice Set C

$\frac{7}{9}$

Practice Set C

$\frac{5}{8}$

Practice Set D

32

Practice Set D

12

Practice Set D

4

Practice Set D

150

Practice Set D

88

Exercise 1.7.1

equivalent

Exercise 1.7.3

equivalent

Exercise 1.7.5

not equivalent

Exercise 1.7.7

not equivalent

Exercise 1.7.9

not equivalent

Exercise 1.7.11

equivalent

Exercise 1.7.13

not equivalent

Exercise 1.7.15

not equivalent

Exercise 1.7.17

6

Exercise 1.7.19

12

Exercise 1.7.21

20

Exercise 1.7.23

75

Exercise 1.7.25

48

Exercise 1.7.27

80

Exercise 1.7.29

18

Exercise 1.7.31

154

Exercise 1.7.33

1,472

Exercise 1.7.35

1,850

Exercise 1.7.37

$\frac{4}{5}$

Exercise 1.7.39

$\frac{3}{7}$

Exercise 1.7.41

$\frac{2}{7}$

Exercise 1.7.43

$\frac{2}{3}$

Exercise 1.7.45

$\frac{5}{2}$

Exercise 1.7.47

$\frac{5}{3}$

Exercise 1.7.49

$\frac{7}{3}$

Exercise 1.7.51

$\frac{8}{35}$

Exercise 1.7.53

$\frac{5}{3}$

Exercise 1.7.55

$\frac{18}{5}$

Exercise 1.7.57

$\frac{2}{3}$

Exercise 1.7.59

$\frac{3}{4}$

Exercise 1.7.61

$\frac{1}{2}$

Exercise 1.7.63

$\frac{1}{3}$

Exercise 1.7.65

3

Exercise 1.7.67

2

Exercise 1.7.69

already reduced

Exercise 1.7.71

$\frac{9}{25}$

Exercise 1.7.73

$\frac{2}{5}$

Exercise 1.7.75

$\frac{9}{8}$

Exercise 1.7.77

$\frac{23}{30}$

Exercise 1.7.79

$\frac{20}{9}$

Exercise 1.7.81

$\frac{1}{3}$

Exercise 1.7.83

$\frac{17}{4}$

Exercise 1.7.85

$\frac{17}{18}$

Exercise 1.7.87

$\frac{7}{12}$

Exercise 1.7.89

16

Exercise 1.7.91

Should be $\frac{1}{8}$; the cancellation is division, so the numerator should be 1.

Exercise 1.7.93

Cancel factors only, not addends; $\frac{7}{15}$ is already reduced.

Exercise 1.7.95

1

Exercise 1.7.97

0

Exercise 1.7.99

6

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1.7.1: Practice and Videos for Introduction to Equivalent Fractions

Learning Objectives

- I can understand fraction equivalence using visual models.
- I can compose and decompose fractions .

GeoGebra: Equivalent Fractions

Experiment with manipulating the blue sliders to change the left-hand fraction. Use the green sliders to perform an operation on the numerator and/or the denominator. Use the verify slider to test equivalence.

Reflection Questions

- What happens if you multiply or divide the numerator and denominator by the same value?
- What happens if you add or subtract the same value to the numerator and the denominator? Can you explain why?

Videos on Equivalent Fractions

Watch the following videos to learn about equivalent fractions. All videos have appropriate captions in English; in some cases, there may also be appropriate captions in other languages.

YouTube Video

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

YouTube Video

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

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YouTube Video

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

Practice!

Check your understanding of fraction equivalence by answering these questions. Use the arrows at the bottom of the box to go to the next question.

Practice 1.7.1.1

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this Interactive Content online

Extra Resources

For more practice on fraction equivalence, visit the following websites.

1. IXL: Find equivalent fractions using area models

2. IXL: Equivalent fractions: find the missing numerator or denominator

3. IXL: Fractions with denominators of 10 and 100

4. PhET Simulation Fraction Equality (see instructions below)

PhET Simulation: Fractions Equality Game Instructions

- Go to the PhET simulation fraction equality game here and select 'game.'
- Complete all levels of the game. Drag fraction diagrams or fractions to the scale. Only pick fractions that are equivalent to each other.
- Check your answer by clicking the check button.
- Answer all questions and move on to the next level. Aim to reach 3 stars on each level. Difficulty will increase as you progress.

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1.8: Equivalent Fractions

In this section we deal with fractions, numbers or expressions of the form a/b.

Definition: Fractions

A number of the form

$$\frac{a}{b}$$

where a and b are numbers is called a *fraction*. The number a is called the *numerator* of the fraction, while the number b is called the *denominator* of the fraction.

Near the end of this section, we'll see that the numerator and denominator of a fraction can also be algebraic expressions, but for the moment we restrict our attention to fractions whose numerators and denominators are integers. We start our study of fractions with the definition of *equivalent fractions*.

Equivalent Fractions

Two fractions are *equivalent* if they represent the same numerical value.

But how can we tell if two fractions represent the same number? Well, one technique involves some simple visualizations. Consider the image shown in Figure 4.1, where the shaded region represents 1/3 of the total area of the figure (one of three equal regions is shaded).



Figure 4.1: The shaded region is 1/3 of the whole region.

In Figure 4.2, we've shaded 2/6 of the entire region (two of six equal regions are shaded).



Figure 4.2: The shaded region is 2/6 of the whole region.

In Figure 4.3, we've shaded 4/12 of the entire region (four of twelve equal regions are shaded).

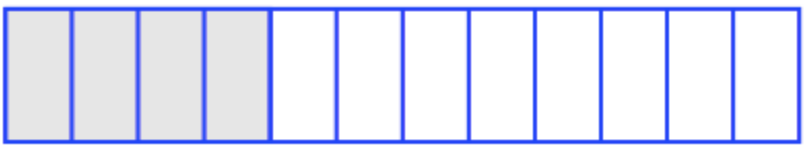


Figure 4.3: The shaded region is 4/12 of the whole region.

Let's take the diagrams from Figure 4.1, Figure 4.2, and Figure 4.3 and stack them one atop the other, as shown in Figure 4.4.

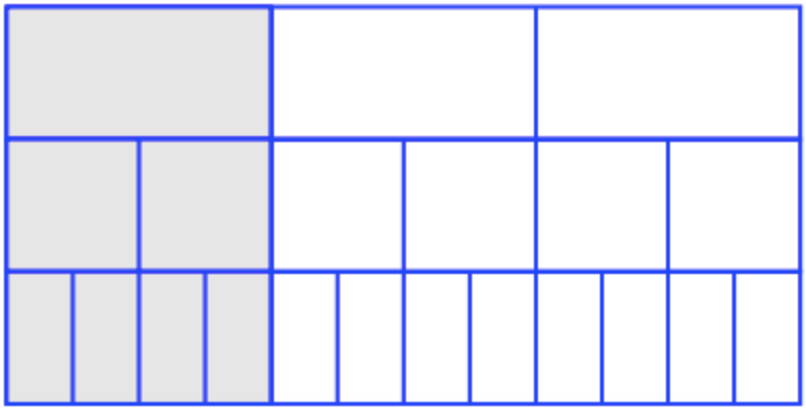


Figure 4.4: One of three equals two of six equals four of twelve.

Figure 4.4 provides solid visual evidence that the following fractions are equivalent.

$$\frac{1}{3} = \frac{2}{6} = \frac{4}{12}$$

Key Observations

1. If we start with the fraction 1/3, then multiply both numerator and denominator by 2, we get the following result.

$$\begin{aligned} \frac{1}{3} &= \frac{1 \cdot 2}{3 \cdot 2} && \text{Multiply the numerator and denominator by 2.} \\ &= \frac{2}{6} && \text{Simplify numerator and denominator.} \end{aligned}$$

This is precisely the same thing that happens going from Figure 4.1 to 4.2, where we double the number of available boxes (going from 3 available to 6 available) and double the number of shaded boxes (going from 1 shaded to 2 shaded).

2. If we start with the fraction 1/3, then multiply both numerator and denominator by 4, we get the following result.

$$\begin{aligned} = \frac{1}{3} &= \frac{1 \cdot 4}{3 \cdot 4} && \text{Multiply numerator and denominator by 4.} \\ &= \frac{4}{12} && \text{Simplify numerator and denominator.} \end{aligned}$$

This is precisely the same thing that happens going from Figure 4.1 to 4.3, where we multiply the number of available boxes by 4 (going from 3 available to 12 available) and multiply the number of shaded boxes by 4 (going from 1 shaded to 4 shaded).

The above discussion motivates the following fundamental result.

Creating Equivalent Fractions

If you start with a fraction, then multiply both its numerator and denominator by the same number, the resulting fraction is equivalent (has the same numerical value) to the original fraction. In symbols,

$$\frac{a}{b} = \frac{a \cdot x}{b \cdot x}$$

Arguing in Reverse

Reversing the above argument also holds true.

1. If we start with the fraction 2/6, then divide both numerator and denominator by 2, we get the following result.

$$\frac{2}{6} = \frac{2 \div 2}{6 \div 2} \quad \text{Divide numerator and denominator by 2.}$$
$$= \frac{1}{3} \quad \text{Simplify numerator and denominator.}$$

This is precisely the same thing that happens going backwards from Figure 4.2 to 4.1, where we divide the number of available boxes by 2 (going from 6 available to 3 available) and dividing the number of shaded boxes by 2 (going from 2 shaded to 1 shaded).

2. If we start with the fraction 4/12, then divide both numerator and denominator by 4, we get the following result.

$$\frac{4}{12} = \frac{4 \div 4}{12 \div 4} \quad \text{Multiply numerator and denominator by 4.}$$
$$= \frac{1}{3} \quad \text{Simplify numerator and denominator.}$$

This is precisely the same thing that happens going backwards from Figure 4.3 to 4.1, where we divide the number of available boxes by 4 (going from 12 available to 3 available) and divide the number alignof shaded boxes by 4 (going from 4 shaded to 1 shaded).

The above discussion motivates the following fundamental result.

Creating Equivalent Fractions

If you start with a fraction, then divide both its numerator and denominator by the same number, the resulting fraction is equivalent (has the same numerical value) to the original fraction. In symbols,

$$\frac{a}{b} = \frac{a \div x}{b \div x}.$$

The Greatest Common Divisor

We need a little more terminology.

Divisor

If d and a are natural numbers, we say that “d divides a” if and only if when a is divided by d, the remainder is zero. In this case, we say that “d is a divisor of a.”

For example, when 36 is divided by 4, the remainder is zero. In this case, we say that “4 is a divisor of 36.” On the other hand, when 25 is divided by 4, the remainder is not zero. In this case, we say that “4 is not a divisor of 25.”

Greatest Common Divisor

Let a and b be natural numbers. The common divisors of a and b are those natural numbers that divide both a and b. The *greatest common divisor* is the largest of these common divisors.

Example 1

Find the greatest common divisor of 18 and 24.

Solution

First list the divisors of each number, the numbers that divide each number with zero remainder.

Divisors of 18 : 1, 2, 3, 6, 9, and 18

Divisors of 24 : 1, 2, 3, 4, 6, 8, 12, and 24

The common divisors are:

Common Divisors : 1, 2, 3, and 6

The greatest common divisor is the largest of the common divisors. That is,

Greatest Common Divisor = 6.

That is, the largest number that divides both 18 and 24 is the number 6.

Exercise

Find the greatest common divisor of 12 and 18.

Answer available at the end of this section.

Reducing a Fraction to Lowest Terms

First, a definition.

Lowest Terms

A fraction is said to be *reduced to lowest terms* if the greatest common divisor of both numerator and denominator is 1.

Thus, for example, 2/3 is reduced to lowest terms because the greatest common divisior of 2 and 3 is 1. On the other hand, 4/6 is **not** reduced to lowest terms because the greatest common divisor of 4 and 6 is 2.

Example 2

Reduce the fraction 18/24 to lowest terms.

Solution

One technique that works well is dividing both numerator and denominator by the greatest common divisor of the numerator and denominator. In Example 1, we saw that the greatest common divisor of 18 and 24 is 6. We divide both numerator and denominator by 6 to get

$$\frac{18}{24} = \frac{18 \div 6}{24 \div 6} \quad \text{Divide numerator and denominator by 6.}$$
$$= \frac{3}{4} \quad \text{Simplify numerator and dice.}$$

Note that the greatest common divisor of 3 and 4 is now 1. Thus, 3/4 is reduced to lowest terms.

There is a second way we can show division of numerator and denominator by 6. First, factor both numerator and denominator as follows:

$$\frac{18}{24} = \frac{3 \cdot 6}{4 \cdot 6} \quad \text{Factor out a 6.}$$

You can then show “division” of both numerator and denominator by 6 by “crossing out” or “canceling” a 6 in the numerator for a 6 in the denominator, like this:

$$= \frac{3 \cdot \cancel{6}}{4 \cdot \cancel{6}} \quad \text{Cancel common factor.}$$
$$= \frac{3}{4}$$

Note that we get the same equivalent fraction, reduced to lowest terms, namely 3/4.

Exercise

Reduce the fraction 12/18 to lowest terms.

Answer available at the end of this section.

Important Point

In Example 2 we saw that 6 was both a *divisor* and a *factor* of 18. The words *divisor* and *factor* are equivalent.

We used the following technique in our second solution in Example 2.

Cancellation Rule

If you express numerator and denominator as a product, then you may cancel common factors from the numerator and denominator. The result will be an equivalent fraction.

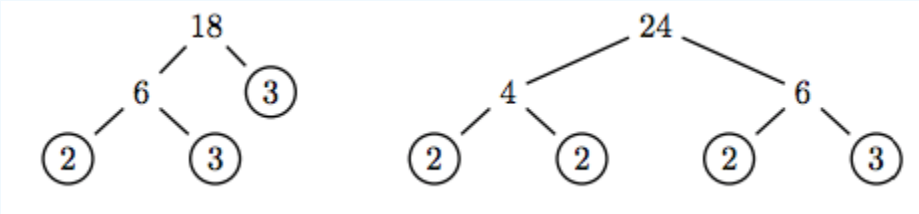
Because of the “Cancellation Rule,” one of the most effective ways to reduce a fraction to lowest terms is to first find prime factorizations for both numerator and denominator, then cancel all common factors.

Example 3

Reduce the fraction 18/24 to lowest terms.

Solution

Use factor trees to prime factor numerator and denominator.



Once we’ve factored the numerator and denominator, we cancel common factors.

$$\begin{aligned}\frac{18}{24} &= \frac{2\cdot 3\cdot 3}{2\cdot 2\cdot 2\cdot 3} && \text{Prime factor numerator and denominator.}\\ &= \frac{\cancel{2}\cdot\cancel{3}\cdot 3}{\cancel{2}\cdot 2\cdot 2\cdot\cancel{3}} && \text{Cancel common factors.}\\ &= \frac{3}{2\cdot 2} && \text{Remaining factors.}\\ &= \frac{3}{4} && \text{Simplify denominator.}\end{aligned}$$

Thus, 18/24 = 3/4.

Exercise 1.8.1

Reduce the fraction 28/35 to lowest terms.

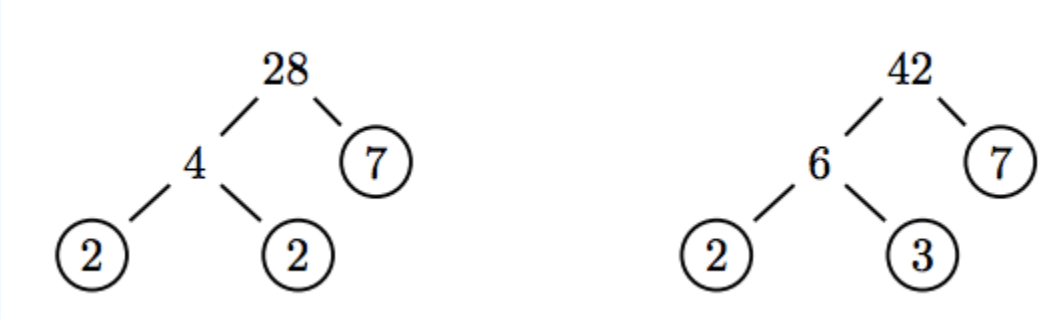
Answer available at the end of this section.

Example 4

Reduce the fraction 28/42 to lowest terms.

Solution

Use factor trees to prime factor numerator and denominator.



Now we can cancel common factors.

$$\begin{aligned}\frac{28}{42} &= \frac{2\cdot 2\cdot 7}{2\cdot 3\cdot 7} && \text{Prime factor numerator and denominator.}\\ &= \frac{\cancel{2}\cdot 2\cdot\cancel{7}}{\cancel{2}\cdot 3\cdot\cancel{7}} && \text{Cancel common factors.}\\ &= \frac{2}{3}\end{aligned}$$

Thus, 28/42 = 2/3.

Exercise

Reduce the fraction 36/60 to lowest terms.

Answer available at the end of this section.

Reducing Fractions with Variables

We use exactly the same technique to reduce fractions whose numerators and denominators contain variables.

Example 5

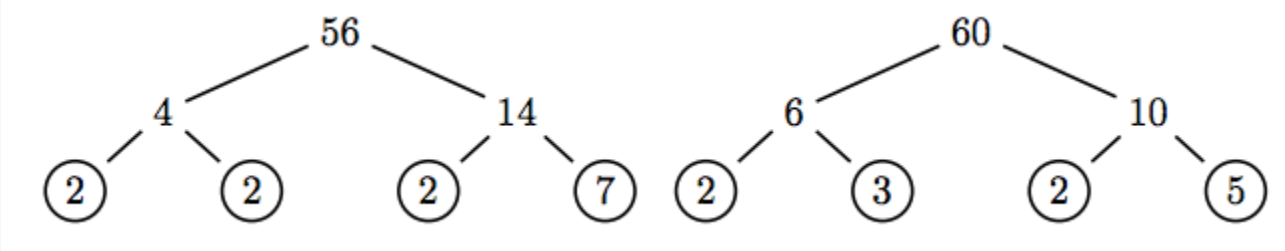
Reduce

$$\frac{56x^2y}{60xy^2}$$

to lowest terms.

Solution

Use factor trees to factor the coefficients of numerator and denominator.



Now cancel common factors.

$$\begin{aligned}
\frac{56x^2y}{60xy^2} &= \frac{2\cdot 2\cdot 2\cdot 7\cdot x\cdot x\cdot y}{2\cdot 2\cdot 3\cdot 5\cdot x\cdot y\cdot y} && \text{Prime factor numerator and denominator.}\\
&= \frac{\cancel{2}\cdot\cancel{2}\cdot 2\cdot 7\cdot \cancel{x}\cdot x\cdot \cancel{y}}{\cancel{2}\cdot\cancel{2}\cdot 3\cdot 5\cdot \cancel{x}\cdot y\cdot \cancel{y}} && \text{Cancel cmmon factors.}\\
&= \frac{2\cdot 7\cdot x}{3\cdot 5\cdot y} && \text{Remaining factors.}\\
&= \frac{14x}{15y} && \text{Simplify numerator and denominator.}
\end{aligned}$$

Thus, $56x^2y/(60xy^2) = 14x/(15y)$.

Exercise

Reduce:

$$\frac{25a^3b}{40a^2b^3}$$

Answer available at the end of this section.

A Word on Mathematical Notation

There are two types of mathematical notation: (1) inline mathematical notation, and (2) displayed mathematical notation.

Inline Mathematical Notation

The notation 14*x*/(15*y*) is called **inline mathematical notation**. When the same expression is centered on its own line, as in

$$\frac{14x}{15y},$$

this type of notation is called displayed mathematical notation.

When you work a problem by hand, using pencil and paper calculations, the preferred format is displayed notation, like the displayed notation used to simplify the given expression in Example 5. However, computers and calculators require that you enter your expressions using inline mathematical notation. Therefore, it is extremely important that you are equally competent with either mathematical notation: displayed or inline.

By the way, order of operations, when applied to the inline expression 14x/(15y), requires that we perform the multiplication inside the parentheses first. Then we must perform multiplications and divisions as they occur, as we move from left to right through the expression. This is why the inline notation 14x/(15y) is equivalent to the displayed notation

$$\frac{14x}{15y}.$$

However, the expression 14x/15y is a different beast. There are no parentheses, so we perform multiplication and division as they occur, moving left to right through the expression. Thus, we must first take the product of 14 and x, divide the result by 15, then multiply by y. In displayed notation, this result is equivalent to

$$\frac{14x}{15} \cdot y,$$

which is a different result.

Some readers might wonder why we did not use the notation (14x)/(15y) to describe the solution in Example 5. After all, this inline notation is also equivalent to the displayed notation

$$\frac{14x}{15y}.$$

However, the point is that we do not need to, as order of operations already requires that we take the product of 14 and x before dividing by 15y. If this is hurting your head, know that it's quite acceptable to use the equivalent notation (14x)/(15y) instead of 14x/(15y). Both are correct.

Equivalent Fractions in Higher Terms

Sometimes the need arises to find an equivalent fraction with a different, larger denominator.

Example 6

Express 3/5 as an equivalent fraction having denominator 20.

Solution

The key here is to remember that multiplying numerator and denominator by the same number produces an equivalent fraction. To get an equivalent fraction with a denominator of 20, we'll have to multiply numerator and denominator of 3/5 by 4.

$$\frac{3}{5} \quad \text{Multiply numerator and denominator by 4.}$$
$$= \frac{12}{20} \quad \text{Simplify numerator and denominator.}$$

Therefore, 3/5 equals 12/20.

Exercise

Express 2/3 as an equivalent fraction having denominator 21.

Answer available at the end of this section.

Example 7

Express 8 as an equivalent fraction having denominator 5.

Solution

The key here is to note that

$$8 = \frac{8}{1} \quad \text{Understood denominator is 1.}$$

To get an equivalent fraction with a denominator of 5, we'll have to multiply numerator and denominator of 8/1 by 5.

$$= \frac{8 \cdot 5}{1 \cdot 5} \quad \text{Multiply numerator and denominator by 5.}$$
$$= \frac{40}{5} \quad \text{Simplify numerator and denominator.}$$

Therefore, 8 equals 40/5.

Exercise

Express 5 as an equivalent fraction having denominator 7.

Answer available at the end of this section.

Example 8

Express 2/9 as an equivalent fraction having denominator 18*a*.

Solution

To get an equivalent fraction with a denominator of 18*a*, we'll have to multiply numerator and denominator of 2/9 by 2*a*.

$$\frac{2}{9} = \frac{2 \cdot 2a}{9 \cdot 2a} \quad \text{Multiply numerator and denominator by } 2a.$$
$$= \frac{4a}{18a} \quad \text{Simplify numerator and denominator.}$$

Therefore, 2/9 equals 4*a*/(18*a*), or equivalently, (4*a*)/(18*a*).

Exercise

Express 3/8 as an equivalent fraction having denominator 24*a*.

Answer available at the end of this section.

Negative Fractions

We have to also deal with fractions that are negative. First, let's discuss placement of the negative sign.

- Positive divided by negative is negative, so

$$\frac{3}{-5} = -\frac{3}{5}.$$

- But it is also true that negative divided by positive is negative. Thus,

$$\frac{-3}{5} = \frac{-3}{5}.$$

These two observations imply that all three of the following fractions are equivalent (the same number):

$$\frac{3}{-5} = -\frac{3}{5} = \frac{-3}{5}.$$

Note that there are three possible placements for the negative sign: (1) the denominator, (2) the fraction bar, or (3) the numerator. Any one of these placements produces an equivalent fraction.

Fractions and Negative Signs

Let *a* and *b* be any integers. All three of the following fractions are equivalent (same number):

$$\frac{a}{-b} = -\frac{a}{b} = \frac{-a}{b}.$$

Mathematicians prefer to place the negative sign either in the numerator or on the fraction bar. The use of a negative sign in the denominator is discouraged.

Example 9

Reduce:

$$\frac{50x^3}{-75x^5}$$

to lowest terms.

Solution

Prime factor numerator and denominator and cancel.

$$\begin{aligned}\frac{50x^3}{-75x^5} &= \frac{2\cdot 5\cdot 5\cdot x\cdot x\cdot x}{-3\cdot 5\cdot 5\cdot x\cdot x\cdot x\cdot x\cdot x}\\ &= \frac{2\cdot \cancel{5}\cdot \cancel{5}\cdot \cancel{x}\cdot \cancel{x}\cdot \cancel{x}}{-3\cdot \cancel{5}\cdot \cancel{5}\cdot \cancel{x}\cdot \cancel{x}\cdot \cancel{x}\cdot x\cdot x}\\ &= \frac{2}{-3\cdot x\cdot x}\\ &= \frac{2}{-3x^2}\end{aligned}$$

However, it is preferred that there be no negative signs in the denominator, so let's place the negative sign on the fraction bar (the numerator would suit as well). Thus,

$$\frac{50x^3}{-75x^5} = -\frac{2}{3x^2}$$

We also have the following result.

Fractions and Negative Signs

Let a and b be any integers. Then,

$$\frac{-a}{-b} = \frac{a}{b}.$$

Example 10

Reduce:

$$\frac{-12xy^2}{-18x^2y}$$

Solution

Unlike Example 9, some like to take care of the sign of the answer first.

$$\frac{-12xy^2}{-18x^2y} = \frac{12xy^2}{18x^2y}$$

Now we can factor numerator and denominator and cancel common factors.

$$\begin{aligned}&= \frac{2\cdot 2\cdot 3\cdot x\cdot y\cdot y}{2\cdot 3\cdot 3\cdot x\cdot x\cdot y}\\ &= \frac{\cancel{2}\cdot 2\cancel{3}\cdot \cancel{x}\cdot y\cdot \cancel{y}}{\cancel{2}\cdot \cancel{3}\cdot 3\cdot \cancel{x}\cdot x\cdot \cancel{y}} \qquad = \frac{2y}{3x}\end{aligned}$$

Thus,

$$\frac{-12xy^2}{-18x^2y} = \frac{2y}{3x}.$$

Exercise

Reduce:

$$\frac{-21a^2b^3}{-56a^3b}$$

Answer available at the end of this section.

Exercises

In Exercises 1-12, find the GCD of the given numbers.

1. 72, 8
2. 76, 52
3. 52, 20
4. 56, 96
5. 36, 63
6. 63, 21
7. 72, 44
8. 10, 40
9. 16, 56
10. 54, 66
11. 84, 24
12. 75, 45

In Exercises 13-28, reduce the given fraction to lowest terms.

13. $\frac{22}{98}$
14. $\frac{28}{56}$
15. $\frac{93}{15}$
16. $\frac{90}{39}$
17. $\frac{69}{21}$
18. $\frac{74}{62}$
19. $\frac{74}{12}$
20. $\frac{66}{10}$
21. $\frac{66}{57}$

22. $\frac{34}{30}$

23. $\frac{33}{99}$

24. $\frac{20}{58}$

25. $\frac{69}{24}$

26. $\frac{18}{96}$

27. $\frac{46}{44}$

28. $\frac{92}{24}$

---

29. Express 3 as an equivalent fraction having denominator 24. 30. Express 3 as an equivalent fraction having denominator 8. 31. Express $\frac{25}{19}$ as an equivalent fraction having denominator 57. 32. Express $\frac{29}{22}$ as an equivalent fraction having denominator 44. 33. Express 2 as an equivalent fraction having denominator 2. 34. Express 2 as an equivalent fraction having denominator 8. 35. Express $\frac{18}{19}$ as an equivalent fraction having denominator 95. 36. Express $\frac{17}{22}$ as an equivalent fraction having denominator 44. 37. Express $\frac{1}{3}$ as an equivalent fraction having denominator 24. 38. Express $\frac{15}{19}$ as an equivalent fraction having denominator 95. 39. Express 16 as an equivalent fraction having denominator 4. 40. Express 5 as an equivalent fraction having denominator 2.

In Exercises 41-56, reduce the given fraction to lowest terms.

41. $\frac{34}{-86}$

42. $\frac{-48}{14}$

43. $\frac{-72}{-92}$

44. $\frac{27}{-75}$

45. $\frac{-92}{82}$

46. $\frac{-44}{-62}$

47. $\frac{-21}{33}$

48. $\frac{57}{-99}$

49. $\frac{22}{-98}$

50. $\frac{-33}{69}$

51. $\frac{42}{-88}$

52. $\frac{-100}{48}$

53. $\frac{94}{-6}$

54. $\frac{-36}{-38}$

55. $\frac{10}{-86}$

56. $\frac{-100}{-46}$

---

57. Express $\frac{3}{2}$ as an equivalent fraction having denominator 62n.

58. Express $\frac{6}{25}$ as an equivalent fraction having denominator 50a.

59. Express $\frac{13}{10}$ as an equivalent fraction having denominator 60m.

60. Express $\frac{1}{16}$ as an equivalent fraction having denominator 80p.

61. Express $\frac{3}{2}$ as an equivalent fraction having denominator 50n.

62. Express $\frac{43}{38}$ as an equivalent fraction having denominator 76a.

63. Express 11 as an equivalent fraction having denominator 4m. 64. Express 13 as an equivalent fraction having denominator 6n.

65. Express 3 as an equivalent fraction having denominator 10m.

66. Express 10 as an equivalent fraction having denominator 8b.

67. Express 6 as an equivalent fraction having denominator 5n.

68. Express 16 as an equivalent fraction having denominator 2y.

In Exercises 69-84, reduce the given fraction to lowest terms.

69. $\frac{82y^5}{-48y}$

70. $\frac{-40y^5}{-55y}$

71. $\frac{-77x^5}{44x^4}$

72. $\frac{-34x^6}{-80x}$

73. $\frac{-14y^5}{54y^2}$

74. $\frac{96y^4}{-40y^2}$

75. $\frac{42x}{81x^3}$

76. $\frac{26x^2}{32x^6}$

77. $\frac{-12x^5}{14x^6}$

78. $\frac{-28y^4}{72y^6}$

79. $\frac{-74x}{22x^2}$

80. $\frac{56x^2}{26x^3}$

81. $\frac{-12y^5}{98y^6}$

82. $\frac{96x^2}{14x^4}$

83. $\frac{18x^6}{-54x^2}$

84. $\frac{32x^6}{62x^2}$

In Exercises 85-100, reduce the given fraction to lowest terms.

85. $\frac{26y^2x^4}{-62y^6x^2}$

86. $\frac{6x^2y^3}{40x^3y^2}$

87. $\frac{-2y^6x^4}{-94y^2x^5}$

88. $\frac{90y^6x^3}{39y^3x^5}$

89. $\frac{30y^5x^5}{-26yx^4}$

90. $\frac{74x^6y^4}{-52xy^3}$

91. $\frac{36x^3y^2}{-98x^4y^5}$

92. $\frac{84x^3y}{16x^4y^2}$

93. $\frac{-8x^6y^3}{54x^3y^5}$

94. $\frac{70y^5x^2}{16y^4x^5}$

95. $\frac{34yx^6}{-58y^5x^4}$

96. $\frac{99y^2x^3}{88y^6x}$

97. $\frac{-36y^3x^5}{51y^2x}$

98. $\frac{44y^5x^5}{-88y^4x}$

99. $\frac{91y^3x^2}{-28y^5x^5}$

100. $\frac{-76y^2x}{-57y^5x^6}$

---

101. **Hurricanes**. According to the National Atmospheric and Oceanic Administration, in 2008 there were 16 named storms, of which 8 grew into hurricanes and 5 were major.

i) What fraction of named storms grew into hurricanes? Reduce your answer to lowest terms.

ii) What fraction of named storms were major hurricanes? Reduce your answer to lowest terms.

iii) What fraction of hurricanes were major? Reduce your answer to lowest terms.

102. **Tigers**. Tigers are in critical decline because of human encroachment, the loss of more than nine-tenths of their habitat, and the growing trade in tiger skins and body parts. *Associated Press-Times-Standard 01/24/10 Pressure mounts to save the tiger.*

i) Write the loss of habitat as a fraction.

ii) Describe in words what the numerator and denominator of this fraction represent.

iii) If the fraction represents the loss of the whole original habitat, how much of the original habitat remains?

Answers

1. 8

3. 4

5. 9

7. 4

9. 8

11. 12

13. $\frac{11}{49}$

15. $\frac{31}{5}$

17. $\frac{23}{7}$

19. $\frac{37}{6}$

21. $\frac{22}{19}$

23. $\frac{1}{3}$

25. $\frac{23}{8}$

27. $\frac{23}{22}$

29. $\frac{72}{24}$

31. $\frac{75}{57}$

33. $\frac{4}{2}$

35. $\frac{90}{95}$

37. $\frac{8}{24}$

39. $\frac{64}{4}$

41. $\frac{-17}{43}$

43. $\frac{18}{23}$

45. $\frac{-46}{41}$

47. $\frac{-7}{11}$

49. $\frac{-11}{49}$

51. $\frac{-21}{44}$

53. $\frac{-47}{3}$

55. $\frac{-5}{43}$

57. $\frac{93n}{62n}$

59. $\frac{78m}{60m}$

61. $\frac{75n}{50n}$

63. $\frac{44m}{4m}$

65. $\frac{30m}{10m}$

67. $\frac{30n}{5n}$

69. $\frac{-41y^4}{24}$

71. $\frac{-7x}{4}$

73. $-\frac{7y^3}{27}$

75. $\frac{14}{27x^2}$

77. $-\frac{6}{7x}$

79. $-\frac{37}{11x}$

81. $-\frac{6}{49y}$

83. $-\frac{x^4}{3}$

85. $-\frac{13x^2}{31y^4}$

87. $\frac{y^4}{47x}$

89. $-\frac{15y^4x}{13}$

91. $-\frac{18}{49xy^3}$

93. $-\frac{4x^3}{27y^2}$

95. $-\frac{17x^2}{29y^4}$

97. $-\frac{12yx^4}{17}$

99. $-\frac{13}{4y^2x^3}$

101.

i) $\frac{1}{2}$

ii) $\frac{5}{16}$

iii) $\frac{5}{8}$

Answers

Exercise

6

Exercise

2/3

Exercise 1.8.1

4/5

Exercise

3/5

Exercise

$$\frac{5a}{8b^2}$$

Exercise

14/21

Exercise

35/7

Exercise

$$\frac{9a}{24a}$$

Exercise

$$\frac{3b^2}{8a}$$

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1.9: Comparing Fractions

Learning Objectives

- Determine whether two fractions are equivalent.
- Use > or < to compare fractions.

Introduction

You often need to know when one fraction is greater or less than another fraction. Since a fraction is a part of a whole, to find the greater fraction you need to find the fraction that contains more of the whole. If the two fractions simplify to fractions with a **common denominator**, you can then compare numerators. If the denominators are different, you can find a common denominator first and then compare the numerators.

Determining Equivalent Fractions

Two fractions are **equivalent fractions** when they represent the same part of a whole. Since equivalent fractions do not always have the same numerator and denominator, one way to determine if two fractions are equivalent is to find a common denominator and rewrite each fraction with that denominator. Once the two fractions have the same denominator, you can check to see if the numerators are equal. If they are equal, then the two fractions are equal as well.

One way to find a common denominator is to check to see if one denominator is a factor of the other denominator. If so, the greater denominator can be used as the common denominator.

Example

Are $\frac{2}{6}$ and $\frac{8}{18}$ equivalent fractions?

Solution

Does $\frac{2}{6} = \frac{8}{18}$?	To solve this problem, find a common denominator for the two fractions. This will help you compare the two fractions. Since 6 is a factor of 18, you can write both fractions with 18 as the denominator.
$\frac{2 \cdot 3}{6 \cdot 3} = \frac{6}{18}$	Start with the fraction $\frac{2}{6}$. Multiply the denominator, 6, by 3 to get a new denominator of 18. Since you multiply the denominator by 3, you must also multiply the numerator by 3.
$\frac{8}{18}$	The fraction $\frac{8}{18}$ already has a denominator of 18, so you can leave it as is.
$\frac{6}{18}$ does not equal $\frac{8}{18}$	Compare the fractions. Now that both fractions have the same denominator, 18, you can compare numerators.

$\frac{2}{6}$ and $\frac{8}{18}$ are not equivalent fractions.

When one denominator is not a factor of the other denominator, you can find a common denominator by multiplying the denominators together.

Example

Determine whether $\frac{3}{6}$ and $\frac{5}{10}$ are equivalent fractions.

Solution

$6 \cdot 10 = 60$	Use 60 as a common denominator.
$\frac{3}{6} = \frac{3 \cdot 10}{6 \cdot 10} = \frac{30}{60}$	Multiply the numerator and denominator of $\frac{3}{6}$ by 10 to get 60 in the denominator.
$\frac{5}{10} = \frac{5 \cdot 6}{10 \cdot 6} = \frac{30}{60}$	Multiply the numerator and denominator of $\frac{5}{10}$ by 6.
$\frac{30}{60} = \frac{30}{60}$	Now that the denominators are the same, compare the numerators.
Yes, $\frac{3}{6}$ and $\frac{5}{10}$ are equivalent fractions.	Since 30 is the value of the numerator for both fractions, the two fractions are equal.

Notice in the above example that you can use 30 as the least common denominator since both 6 and 10 are factors of 30. Any common denominator will work.

In some cases, you can simplify one or both of the fractions, which can result in a common denominator.

Example

Determine whether $\frac{2}{3}$ and $\frac{40}{60}$ are equivalent fractions.

Solution

$\frac{40}{60} = \frac{40 \div 10}{60 \div 10} = \frac{4}{6}$	Simplify $\frac{40}{60}$. Divide the numerator and denominator by the common factor 10.
$\frac{4}{6} = \frac{4 \div 2}{6 \div 2} = \frac{2}{3}$	$\frac{4}{6}$ is still not in lowest terms, so divide the numerator and the denominator again, this time by the common factor 2.
$\frac{2}{3} = \frac{2}{3}$	Compare the fractions. The numerators and denominators are the same.

Yes, $\frac{2}{3}$ and $\frac{40}{60}$ are equivalent fractions.

Note

In the example above, you could have used the common factor of 20 to simplify $\frac{40}{60}$ directly to $\frac{2}{3}$.

Determining Equivalent Fractions

To determine whether or not two fractions are equivalent:

Step 1: Rewrite one or both of the fractions so that they have common denominators.

Step 2: Compare the numerators to see if they have the same value. If so, then the fractions are equivalent.

Exercise

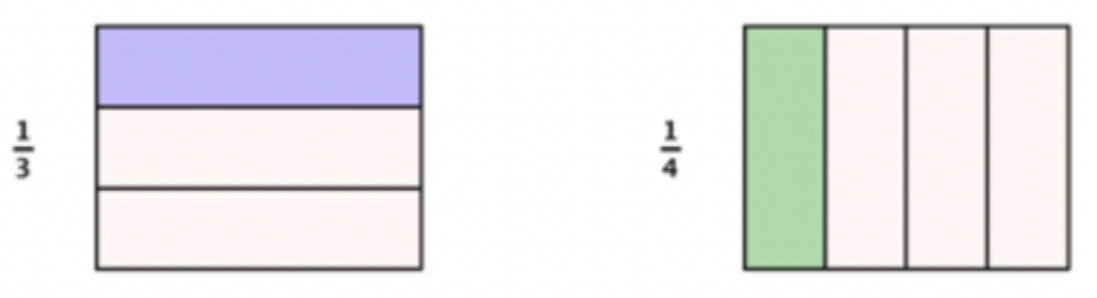
Which of the following fraction pairs are equivalent?

A. $\frac{5}{7}$ and $\frac{7}{5}$
B. $\frac{12}{30}$ and $\frac{6}{10}$
C. $\frac{4}{20}$ and $\frac{1}{5}$
D. $\frac{8}{11}$ and $\frac{8}{22}$

Answer available at the end of this section.

Comparing Fractions Using < and >

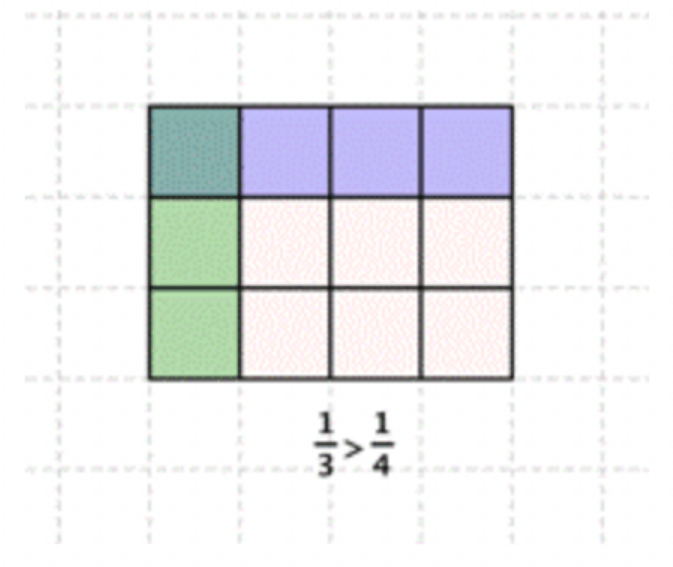
When given two or more fractions, it is often useful to know which fraction is greater than or less than the other. For example, if the discount in one store is $\frac{1}{3}$ off the original price and the discount in another store is $\frac{1}{4}$ off the original price, which store is offering a better deal? To answer this question, and others like it, you can compare fractions.



To determine which fraction is greater, you need to find a common denominator. You can then compare the fractions directly. Since 3 and 4 are both factors of 12, you will divide the whole into 12 parts, create equivalent fractions for $\frac{1}{3}$ and $\frac{1}{4}$, and then compare.



Now you see that $\frac{1}{3}$ contains 4 parts of 12, and $\frac{1}{4}$ contains 3 parts of 12. So, $\frac{1}{3}$ is greater than $\frac{1}{4}$.



As long as the denominators are the same, the fraction with the greater numerator is the greater fraction, as it contains more parts of the whole. The fraction with the lesser numerator is the lesser fraction, as it contains fewer parts of the whole.

Recall that the symbol < means “less than”, and the symbol > means “greater than”. These symbols are inequality symbols. So, the true statement 3<8 is read as "3 is less than 8" and the statement 5>3 is read as "5 is greater than 3". One way to help you remember the distinction between the two symbols is to think that the smaller end of the symbol points to the lesser number.

As with comparing whole numbers, the inequality symbols are used to show when one fraction is “greater than” or “less than” another fraction.

Comparing Fractions

To compare two fractions:

Step 1: Compare denominators. If they are different, rewrite one or both fractions with a common denominator.

Step 2: Check the numerators. If the denominators are the same, then the fraction with the greater numerator is the greater fraction. The fraction with the lesser numerator is the lesser fraction. And, as noted above, if the numerators are equal, the fractions are equivalent.

Example

Use < or > to compare the two fractions $\frac{4}{5}$ and $\frac{14}{20}$.

Solution

Is $\frac{4}{5} > \frac{14}{20}$ or is $\frac{4}{5} < \frac{14}{20}$?	You cannot compare the fractions directly because they have different denominators. You need to find a common denominator for the two fractions.
$\frac{4}{5} = \frac{?}{20}$	Since 5 is a factor of 20, you can use 20 as the common denominator.
$\frac{4 \cdot 4}{5 \cdot 4} = \frac{16}{20}$	Multiply the numerator and denominator by 4 to create an equivalent fraction with a denominator of 20.
$\frac{16}{20} > \frac{14}{20}$	Compare the two fractions. $\frac{16}{20}$ is greater than $\frac{14}{20}$.
$\frac{4}{5} > \frac{14}{20}$	If $\frac{16}{20} > \frac{14}{20}$, then $\frac{4}{5} > \frac{14}{20}$, since $\frac{4}{5} = \frac{16}{20}$.

Exercise

Which of the following is a true statement?

A. $\frac{5}{6} < \frac{24}{30}$
B. $\frac{75}{100} > \frac{9}{12}$
C. $\frac{4}{16} > \frac{1}{3}$
D. $\frac{3}{8} < \frac{20}{40}$

Answer available at the end of this section.

Summary

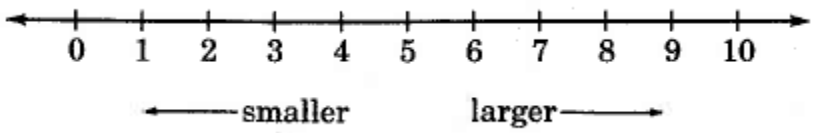
You can compare two fractions with like denominators by comparing their numerators. The fraction with the greater numerator is the greater fraction, as it contains more parts of the whole. The fraction with the lesser numerator is the lesser fraction as it contains fewer parts of the whole. If two fractions have the same denominator, then equal numerators indicate equivalent fractions.

Learning Objectives

- understand ordering of numbers and be familiar with grouping symbols
- be able to compare two or more fractions

Order and the Inequality Symbols

Our number system is called an **ordered number system** because the numbers in the system can be placed in order from smaller to larger. This is easily seen on the number line.



On the number line, a number that appears to the right of another number is larger than that other number. For example, 5 is greater than 2 because 5 is located to the right of 2 on the number line. We may also say that 2 is less than 5.

To make the inequality phrases "greater than" and "less than" more brief, mathematicians represent them with the symbols > and <, respectively.

Symbols for Greater Than > and Less Than <
> represents the phrase "greater than."
< represents the phrase "less than."

5 > 2 represents "5 is greater than 2."

2 < 5 represents "2 is less than 5."

Comparing Fractions

Recall that the fraction $\frac{4}{5}$ indicates that we have 4 of 5 parts of some whole quantity, and the fraction $\frac{3}{5}$ indicates that we have

3 of 5 parts. Since 4 of 5 parts is more than 3 of 5 parts, $\frac{4}{5}$ is greater than $\frac{3}{5}$; that is,

$$\frac{4}{5} > \frac{3}{5}$$

We have just observed that when two fractions have the same denominator, we can determine which is larger by comparing the numerators.

Comparing Fractions
If two fractions have the same denominators, the fraction with the larger numerator is the larger fraction.

Thus, to compare the sizes of two or more fractions, we need only convert each of them to equivalent fractions that have a common denominator. We then compare the numerators. It is convenient if the common denominator is the LCD. The fraction with the larger numerator is the larger fraction.

Sample Set A

Compare $\frac{8}{9}$ and $\frac{14}{15}$. Convert each fraction to an equivalent fraction with the LCD as the denominator. Find the LCD.

Solution

$$\left\{\begin{array}{rcl} 9 & = & 3^2 \\ 15 & = & 3 \cdot 5 \end{array}\right\} \text{ The LCD } = 3^2 \cdot 5 = 9 \cdot 5 = 45$$

$$\frac{8}{9} = \frac{8 \cdot 5}{45} = \frac{40}{45}$$

$$\frac{14}{15} = \frac{14 \cdot 3}{45} = \frac{42}{45}$$

Since $40 < 42$.

$$\frac{40}{45} < \frac{42}{45}$$

Thus $\frac{8}{9} < \frac{14}{15}$.

Sample Set A

Write $\frac{5}{6}$, $\frac{7}{10}$, and $\frac{13}{15}$ in order from smallest to largest.

Convert each fraction to an equivalent fraction with the LCD as the denominator.

Find the LCD.

Solution

$$\left\{\begin{array}{rcl} 6 & = & 2 \cdot 3 \\ 10 & = & 2 \cdot 5 \\ 15 & = & 3 \cdot 5 \end{array}\right\} \text{ The LCD } = 2 \cdot 3 \cdot 5 = 30$$

$$\frac{5}{6} = \frac{5 \cdot 5}{30} = \frac{25}{30}$$

$$\frac{7}{10} = \frac{7 \cdot 3}{30} = \frac{21}{30}$$

$\frac{13}{15} = \frac{13 \cdot 2}{30} = \frac{26}{30}$

Since $21 < 25 < 26$.

$\frac{21}{30} < \frac{25}{30} < \frac{26}{30}$

$\frac{7}{10} < \frac{5}{6} < \frac{13}{15}$

Writing these numbers in order from smallest to largest, we get $\frac{7}{10}$, $\frac{5}{6}$, $\frac{13}{15}$.

Sample Set A

Compare $8\frac{6}{7}$ and $6\frac{3}{4}$.

Solution

To compare mixed numbers that have different whole number parts, we need only compare whole number parts. Since 6 < 8, $6\frac{3}{4} < 8\frac{6}{7}$

Sample Set A

Compare $4\frac{5}{8}$ and $4\frac{7}{12}$

Solution

To compare mixed numbers that have the same whole number parts, we need only compare fractional parts.

$\left\{ \begin{array}{rcl} 8 & = & 2 \cdot 3 \\ 12 & = & 2^2 \cdot 3 \end{array} \right\}$ The LCD $= 2^3 \cdot 3 = 8 \cdot 3 = 24$

$\frac{5}{8} = \frac{5 \cdot 3}{24} = \frac{15}{24}$

$\frac{7}{12} = \frac{7 \cdot 2}{24} = \frac{14}{24}$

Since $14 < 15$.

$\frac{14}{24} < \frac{15}{24}$

$\frac{7}{12} < \frac{5}{8}$

Hence, $4\frac{7}{12} < 4\frac{5}{8}$

Practice Set A

Compare $\frac{3}{4}$ and $\frac{4}{5}$

Answer available at the end of this section.

Practice Set A

Compare $\frac{9}{10}$ and $\frac{13}{15}$

Answer available at the end of this section.

Practice Set A

Write $\frac{13}{16}$, $\frac{17}{20}$, and $\frac{33}{40}$ in order from smallest to largest.

Answer available at the end of this section.

Practice Set A

Compare $11\frac{1}{6}$ and $9\frac{2}{5}$.

Answer available at the end of this section.

Practice Set A

Compare $1\frac{9}{14}$ and $1\frac{11}{16}$.

Answer available at the end of this section.

Exercises

Arrange each collection of numbers in order from smallest to largest.

Exercise 1.9.1

$\frac{3}{5}, \frac{5}{8}$

Answer available at the end of this section.

Exercise 1.9.2

$\frac{3}{4}, \frac{5}{6}$

Answer available at the end of this section.

Exercise 1.9.3

$\frac{3}{8}, \frac{2}{5}$

Answer available at the end of this section.

Exercise 1.9.4

$\frac{1}{2}, \frac{3}{5}, \frac{4}{7}$

Answer available at the end of this section.

Exercise 1.9.5

$\frac{3}{4}, \frac{7}{9}, \frac{5}{4}$

Answer available at the end of this section.

Exercise 1.9.6

$\frac{3}{14}, \frac{2}{7}, \frac{3}{4}$

Answer available at the end of this section.

Exercise 1.9.7

$5\frac{3}{5}, 5\frac{4}{7}$

Answer available at the end of this section.

Exercise 1.9.8

$9\frac{2}{3}, 9\frac{4}{5}$

Answer available at the end of this section.

Exercise 1.9.9

$1\frac{9}{16}, 2\frac{1}{20}$

Answer available at the end of this section.

Exercise 1.9.10

$2\frac{2}{9}, 2\frac{3}{7}$

Answer available at the end of this section.

Exercises for Review

Exercise 1.9.11

Round 267,006,428 to the nearest ten million.

Answer available at the end of this section.

Exercise 1.9.12

Convert $3\frac{2}{7}$ to an improper fraction.

Answer available at the end of this section.

Exercise 1.9.13

Find the value of $8\frac{3}{8}+5\frac{1}{4}$.

Answer available at the end of this section.

Answers

Exercise

A. Incorrect. Although the same numbers, 5 and 7, are used in each fraction, the numerators and denominators are not equal, so the fractions cannot be equivalent. The correct answer is $\frac{4}{20}$ and $\frac{1}{5}$.
B. Incorrect. 30 is divisible by 10, and 12 is divisible by 6. However, they do not share a common multiple: $6 \cdot 2 = 12$, and $10 \cdot 3 = 30$. This means the fractions are not equivalent. The correct answer is $\frac{4}{20}$ and $\frac{1}{5}$.
C. Correct. Take the fraction $\frac{1}{5}$ and multiply both the numerator and denominator by 4. You are left with the fraction $\frac{4}{20}$. This means that the two fractions are equivalent.
D. Incorrect. The numerators of the two fractions are the same, but the denominators are different. This means the fractions are not equivalent. The correct answer is $\frac{4}{20}$ and $\frac{1}{5}$

Exercise

A. Incorrect. $\frac{5}{6}$ is equivalent to $\frac{25}{30}$, and since 25>24, $\frac{25}{30} > \frac{24}{30}$, which means $\frac{5}{6} > \frac{24}{30}$. The correct answer is $\frac{3}{8} < \frac{20}{40}$.
B. Incorrect. Both fractions simplify to $\frac{3}{4}$, so one isn't greater than or less than the other one. They are equivalent. The correct answer is $\frac{3}{8} < \frac{20}{40}$.
C. Incorrect. Finding a common denominator, you can compare $\frac{12}{48}$ to $\frac{16}{48}$, and see that $\frac{12}{48} < \frac{16}{48}$, which means $\frac{4}{16} < \frac{1}{3}$. The correct answer is $\frac{3}{8} < \frac{20}{40}$.
D. Correct. Simplifying $\frac{20}{40}$, you get the equivalent fraction $\frac{1}{2}$. Since you still don't have a common denominator, write $\frac{1}{2}$ as an equivalent fraction with a denominator of 8: $\frac{1}{2} = \frac{1 \cdot 4}{2 \cdot 4} = \frac{4}{8}$. You find that $\frac{3}{8} < \frac{4}{8}$, so $\frac{3}{8} < \frac{20}{40}$ as well.

Practice Set A

$\frac{3}{4} < \frac{4}{5}$

Practice Set A

$\frac{13}{15} < \frac{9}{10}$

Practice Set A

$\frac{13}{16}, \frac{33}{40}, \frac{17}{20}$

Practice Set A

$9\frac{2}{5} < 11\frac{1}{6}$

Practice Set A

$1\frac{9}{14} < 1\frac{11}{16}$

Exercise 1.9.1

$\frac{3}{5} < \frac{5}{8}$

Exercise 1.9.2

$\frac{3}{4} < \frac{5}{6}$

Exercise 1.9.3

$\frac{3}{8} < \frac{2}{5}$

Exercise 1.9.4

$\frac{1}{2} < \frac{4}{7} < \frac{3}{5}$

Exercise 1.9.5

$\frac{3}{4} < \frac{7}{9} < \frac{5}{4}$

Exercise 1.9.6

$\frac{3}{14} < \frac{2}{7} < \frac{3}{4}$

Exercise 1.9.7

$5\frac{4}{7} < 5\frac{3}{5}$

Exercise 1.9.8

$9\frac{2}{3} < 9\frac{4}{5}$

Exercise 1.9.9

$1\frac{9}{16} < 2\frac{1}{20}$

Exercise 1.9.10

$2\frac{2}{9} < 2\frac{3}{7}$

Exercise 1.9.11

270,000,000

Exercise 1.9.12

$\frac{23}{7}$

Exercise 1.9.13

$13\frac{5}{8}$ or $\frac{109}{8}$

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1.9.1: Practice and Videos for Comparing Fractions

Learning Objectives

- I can compare fractions with different numerators and different denominators.
- I can learn how to create common denominators or numerators.

GeoGebra: Comparing Fractions

Explore the graphic below to investigate how to compare two fractions. Move the sliders at the top to change the numerator and denominator. Move the sliders at the bottom to multiply the numerator and denominator by that value. What do you notice? When the denominators are not equal, can we still have equivalent fractions?

Videos for Comparing Fractions

Watch the following videos to review inequality symbols and learn how to compare fractions. All videos have appropriate captions in English; in some cases, there may also be appropriate captions in other languages.

YouTube Video

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Practice!

Check your understanding of fraction comparison by answering these questions. Use the arrows at the bottom of the box to go to the next question.

Practice 1.9.1.1

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this Interactive Content online

Extra Resources

For more practice on fraction comparison, visit these websites.

1. IXL: Compare fractions using models

2. IXL: Compare fractions: find the missing numerator or denominator

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1.10: Decimals

Learning Objectives

After completing this section, you should be able to:

1. Name decimals
2. Write decimals
3. Convert decimals to fractions or mixed numbers
4. Round decimals

Name Decimals

You probably already know quite a bit about decimals based on your experience with money. Suppose you buy a sandwich and a bottle of water for lunch. If the sandwich costs $\$3.45$, the bottle of water costs $\$1.25$, and the total sales tax is $\$0.33$, what is the total cost of your lunch?

$3.45	Sandwich
$1.25	Water
+ $0.33	Tax
$5.03	Total

The total is $\$5.03$. Suppose you pay with a $\$5$ bill and 3 pennies. Should you wait for change? No, $\$5$ and 3 pennies is the same as $\$5.03$.

Because $100 \text{ pennies} = \$1$, each penny is worth $\frac{1}{100}$ of a dollar. We write the value of one penny as $\$0.01$, since $0.01 = \frac{1}{100}$.

Writing a number with a decimal is known as decimal notation. It is a way of showing parts of a whole when the whole is a power of ten. In other words, decimals are another way of writing fractions whose denominators are powers of ten. Just as the counting numbers are based on powers of ten, decimals are based on powers of ten. Table 1.3.1 shows the counting numbers.

Counting number	Name
1	One
$10 = 10$	Ten
$10 \cdot 10 = 100$	One hundred
$10 \cdot 10 \cdot 10 = 1000$	One thousand
$10 \cdot 10 \cdot 10 \cdot 10 = 10{,}000$	Ten thousand

Table 1.3.1

How are decimals related to fractions? Table 1.3.2 shows the relation.

Decimal	Fraction	Name
0.1	$\frac{1}{10}$	One tenth
0.01	$\frac{1}{100}$	One hundredth
0.001	$\frac{1}{1{,}000}$	One thousandth

Decimal	Fraction	Name
0.0001	$\frac{1}{10,000}$	One ten-thousandth

Table 1.3.2

When we name a whole number, the name corresponds to the place value based on the powers of ten. We know to read $10,000$ as *ten thousand*. Likewise, the names of the decimal places correspond to their fraction values. Notice how the place value names in Figure 1.3.1 relate to the names of the fractions from Table 1.3.2.

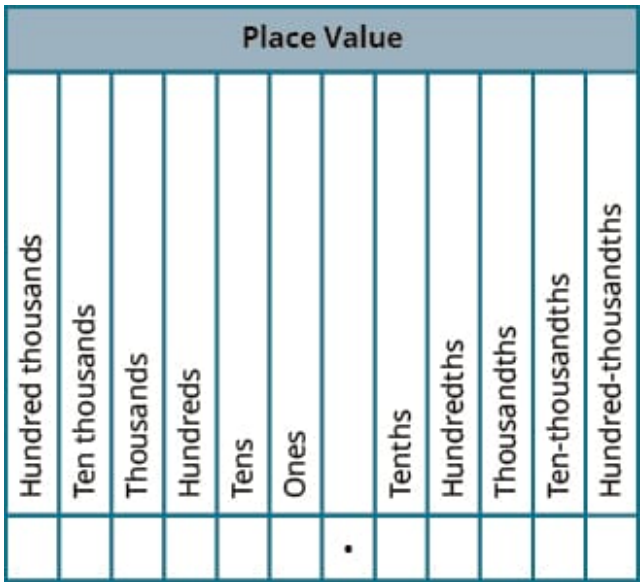


Figure 1.3.1 This chart illustrates place values to the left and right of the decimal point.

Notice two important facts shown in Figure 1.3.1.

- The "th" at the end of the name means the number is a fraction. "One thousand" is a number larger than one, but "one thousandth" is a number smaller than one.
- The tenths place is the first place to the right of the decimal, but the tens place is two places to the left of the decimal.

Remember that $\$5.03$ lunch? We read $\$5.03$ as *five dollars and three cents*. Naming decimals (those that don't represent money) is done in a similar way. We read the number 5.03 as *five and three hundredths*.

We sometimes need to translate a number written in decimal notation into words. As shown in Figure 1.3.2, we write the amount on a check in both words and numbers.



Figure 1.3.2: When we write a check, we write the amount as a decimal number as well as in words. The bank looks at the check to make sure both numbers match. This helps prevent errors.

Let's try naming a decimal, such as 15.68.	
We start by naming the number to the left of the decimal.	fifteen_____
We use the word "and" to indicate the decimal point.	fifteen and_____
Then we name the number to the right of the decimal point as if it were a whole number.	fifteen and sixty-eight_____
Last, name the decimal place of the last digit.	fifteen and sixty-eight hundredths

How To

Name a decimal number.

- Name the number to the left of the decimal point.
- Write "and" for the decimal point.
- Name the "number" part to the right of the decimal point as if it were a whole number.
- Name the decimal place of the last digit.

Example 1.3.1

Name each decimal: ⓐ 4.3 ⓑ 2.45 ⓒ 0.009 ⓓ -15.571.

Answer available at the end of this section.

Your Turn 1.3.1

Name each decimal:

ⓐ 6.7 ⓑ 19.58 ⓒ 0.018 ⓓ -2.053

Write Decimals

Now we will translate the name of a decimal number into decimal notation. We will reverse the procedure we just used.

Let's start by writing the number six and seventeen hundredths:

	six and seventeen hundredths
The word *and* tells us to place a decimal point.	___.___
The word before *and* is the whole number; write it to the left of the decimal point.	6._____
The decimal part is seventeen hundredths. Mark two places to the right of the decimal point for hundredths.	6._ _
Write the numerals for seventeen in the places marked.	6.17

Example 1.3.2

Write fourteen and thirty-seven hundredths as a decimal.

Answer available at the end of this section.

Your Turn 1.3.2

Write as a decimal: thirteen and sixty-eight hundredths.

How To

Write a decimal number from its name.

1. Look for the word “and”—it locates the decimal point.
2. Mark the number of decimal places needed to the right of the decimal point by noting the place value indicated by the last word.
   - Place a decimal point under the word “and.” Translate the words before “and” into the whole number and place it to the left of the decimal point.
   - If there is no “and,” write a “0” with a decimal point to its right.
3. Translate the words after “and” into the number to the right of the decimal point. Write the number in the spaces—putting the final digit in the last place.
4. Fill in zeros for place holders as needed.

The second bullet in Step 2 is needed for decimals that have no whole number part, like ‘nine thousandths’. We recognize them by the words that indicate the place value after the decimal – such as ‘tenths’ or ‘hundredths.’ Since there is no whole number, there is no ‘and.’ We start by placing a zero to the left of the decimal and continue by filling in the numbers to the right, as we did above.

Example 1.3.3

Write twenty-four thousandths as a decimal.

Answer available at the end of this section.

Your Turn 1.3.3

Write as a decimal: fifty-eight thousandths.

Before we move on to our next objective, think about money again. We know that $\$1$ is the same as $\$1.00$. The way we write $\$1$ $(\text{or } \$1.00)$ depends on the context. In the same way, integers can be written as decimals with as many zeros as needed to the right of the decimal.

$$\begin{aligned} 5 &= 5.0 & -2 &= -2.0 \\ 5 &= 5.00 & -2 &= -2.00 \\ 5 &= 5.000 & -2 &= -2.000 \end{aligned}$$

and so on...

Convert Decimals to Fractions or Mixed Numbers

We often need to rewrite decimals as fractions or mixed numbers. Let’s go back to our lunch order to see how we can convert decimal numbers to fractions. We know that $\$5.03$ means 5 dollars and 3 cents. Since there are 100 cents in one dollar, 3 cents means $\frac{3}{100}$ of a dollar, so $0.03 = \frac{3}{100}$.

We convert decimals to fractions by identifying the place value of the farthest right digit. In the decimal 0.03, the 3 is in the hundredths place, so 100 is the denominator of the fraction equivalent to 0.03.

$$0.03 = \frac{3}{100}$$

For our $\$5.03$ lunch, we can write the decimal 5.03 as a mixed number.

$$5.03 = 5\frac{3}{100}$$

Notice that when the number to the left of the decimal is zero, we get a proper fraction. When the number to the left of the decimal is not zero, we get a mixed number.

How To

Convert a decimal number to a fraction or mixed number.

1. Look at the number to the left of the decimal.
   - If it is zero, the decimal converts to a proper fraction.
   - If it is not zero, the decimal converts to a mixed number.
     - Write the whole number.
2. Determine the place value of the final digit.
3. Write the fraction.
   - numerator—the 'numbers' to the right of the decimal point
   - denominator—the place value corresponding to the final digit
4. Simplify the fraction, if possible.

Example 1.3.4

Write each of the following decimal numbers as a fraction or a mixed number:

ⓐ 4.09 ⓑ 3.7 ⓒ -0.286

Answer available at the end of this section.

Your Turn 1.3.4

Write as a fraction or mixed number. Simplify the answer if possible.

ⓐ 5.3 ⓑ 6.07 ⓒ -0.234

Equivalent Decimals

Two decimals are equivalent decimals if they convert to equivalent fractions.

Remember, writing zeros at the end of a decimal does not change its value. 0.31 and 0.310 are equivalent decimals. They have the same value.

Round Decimals

In the United States, gasoline prices are usually written with the decimal part as thousandths of a dollar. For example, a gas station might post the price of unleaded gas at $3.279.

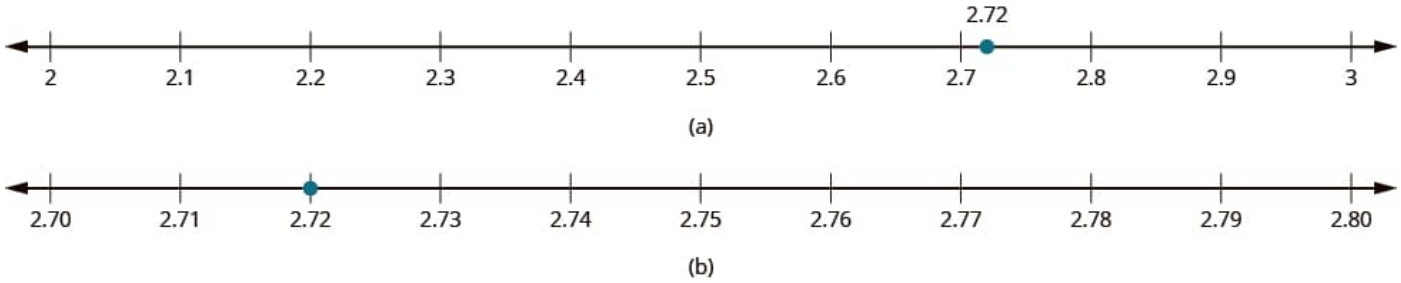


Figure 1.3.3 ⓐ We see that 2.72 is closer to 3 than to 2. So, 2.72 rounded to the nearest whole number is 3.
ⓑ We see that 2.72 is closer to 2.70 than 2.80. So we say that 2.72 rounded to the nearest tenth is 2.7.

Can we round decimals without number lines? Yes! We use a method based on the one we used to round whole numbers.

How To

Round a decimal.

1. Locate the given place value and mark it with an arrow.
2. Underline the digit to the right of the given place value.
3. Is this digit greater than or equal to 5?
   - Yes - add 1 to the digit in the given place value.
   - No - do not change the digit in the given place value
4. Rewrite the number, removing all digits to the right of the given place value.

Example 1.3.5

Round 18.379 to the nearest hundredth.

Answer available at the end of this section.

Your Turn 1.3.5

Round to the nearest hundredth: 1.047.

Example 1.3.6

Round 18.379 to the nearest ⓐ tenth ⓑ whole number.

Answer available at the end of this section.

Your Turn 1.3.6

Round 6.582 to the nearest ⓐ hundredth ⓑ tenth ⓒ whole number.

Media

ACCESS ADDITIONAL ONLINE RESOURCES

- Introduction to Decimal Notation
- Write a Number in Decimal Notation from Words
- Rounding Decimals
- Writing a Decimal as a Simplified Fraction

Answers

ⓐ	
	4.3
Name the number to the left of the decimal point.	four____
Write "and" for the decimal point.	four and____
Name the number to the right of the decimal point as if it were a whole number.	four and three____
Name the decimal place of the last digit.	four and three tenths

ⓑ	
	2.45
Name the number to the left of the decimal point.	two____
Write "and" for the decimal point.	two and____
Name the number to the right of the decimal point as if it were a whole number.	two and forty-five____
Name the decimal place of the last digit.	two and forty-five hundredths

ⓒ	
	0.009
Name the number to the left of the decimal point.	Zero is the number to the left of the decimal; it is not included in the name.
Name the number to the right of the decimal point as if it were a whole number.	nine____
Name the decimal place of the last digit.	nine thousandths

ⓓ	
	-15.571
Name the number to the left of the decimal point.	negative fifteen
Write "and" for the decimal point.	negative fifteen and____
Name the number to the right of the decimal point as if it were a whole number.	negative fifteen and five hundred seventy-one____
Name the decimal place of the last digit.	negative fifteen and five hundred seventy-one thousandths

	fourteen and thirty-seven hundredths
Place a decimal point under the word ‘and’.	_____. _____
Translate the words before ‘and’ into the whole number and place it to the left of the decimal point.	14. _____
Mark two places to the right of the decimal point for “hundredths”.	14.__ __

Translate the words after “and” and write the number to the right of the decimal point.	14.37
	Fourteen and thirty-seven hundredths is written 14.37.

	twenty-four thousandths
Look for the word "and".	There is no "and" so start with 0 0.
To the right of the decimal point, put three decimal places for thousandths.	0. _ _ _ tenths hundredths thousandths
Write the number 24 with the 4 in the thousandths place.	0. _ 2 4 tenths hundredths thousandths
Put zeros as placeholders in the remaining decimal places.	0.024
	So, twenty-four thousandths is written 0.024

ⓐ	
	4.09
There is a 4 to the left of the decimal point. Write "4" as the whole number part of the mixed number.	$4\frac{\square}{\square}$
Determine the place value of the final digit.	4. 0 9 tenths hundredths
Write the fraction. Write 9 in the numerator as it is the number to the right of the decimal point.	$4\frac{9}{\square}$
Write 100 in the denominator as the place value of the final digit, 9, is hundredth.	$4\frac{9}{100}$
The fraction is in simplest form.	So, $4.09 = 4\frac{9}{100}$

Did you notice that the number of zeros in the denominator is the same as the number of decimal places?

ⓑ	
	3.7
There is a 3 to the left of the decimal point. Write "3" as the whole number part of the mixed number.	$3\frac{\square}{\square}$
Determine the place value of the final digit.	3. 7 tenths

Write the fraction. Write 7 in the numerator as it is the number to the right of the decimal point.	$3\frac{7}{\square}$
Write 10 in the denominator as the place value of the final digit, 7, is tenths.	$3\frac{7}{10}$
The fraction is in simplest form.	So, $3.7 = 3\frac{7}{10}$

ⓒ	
	−0.286
There is a 0 to the left of the decimal point. Write a negative sign before the fraction.	$-\frac{\square}{\square}$
Determine the place value of the final digit and write it in the denominator.	−0. 2 8 6 tenths hundredths thousandths
Write the fraction. Write 286 in the numerator as it is the number to the right of the decimal point. Write 1,000 in the denominator as the place value of the final digit, 6, is thousandths.	$-\frac{286}{1000}$
We remove a common factor of 2 to simplify the fraction.	$-\frac{143}{500}$

	18.379
Locate the hundredths place and mark it with an arrow.	hundredths place ↓ 18.379
Underline the digit to the right of the 7.	hundredths place ↓ 18.379
Because 9 is greater than or equal to 5, add 1 to the 7.	18.379 delete add 1
Rewrite the number, deleting all digits to the right of the hundredths place.	18.38
	18.38 is 18.379 rounded to the nearest hundredth.

ⓐ Round 18.379 to the nearest tenth.	

	18.379
Locate the tenths place and mark it with an arrow.	
Underline the digit to the right of the tenths digit.	
Because 7 is greater than or equal to 5, add 1 to the 3.	
Rewrite the number, deleting all digits to the right of the tenths place.	18.4
	So, 18.379 rounded to the nearest tenth is 18.4.

ⓑ Round 18.379 to the nearest whole number.	
	18.379
Locate the ones place and mark it with an arrow.	
Underline the digit to the right of the ones place.	
Since 3 is not greater than or equal to 5, do not add 1 to the 8.	
Rewrite the number, deleting all digits to the right of the ones place.	18
	So 18.379 rounded to the nearest whole number is 18.

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1.10.1: Practice and Videos for Decimal Notation

Learning Objectives

- I can understand decimal notation.
- I can understand how to convert decimals to fractions.
- I can identify the connection between place value and decimals.

GeoGebra: The Meaning of Place Value in Decimal Numbers

Complete the activity below to investigate place value in decimals. Discover the value of each digit for numbers with decimals up to the thousandths place. Start by clicking on a digit to explore your options.

Videos on Decimal Notation

Watch the following videos on decimal notation. All videos have appropriate captions in English; in some cases, there may also be appropriate captions in other languages.

YouTube Video

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

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Practice!

Check your understanding of decimal notation by answering these questions. Use the arrows at the bottom of the box to go to the next question.

Practice 1.10.1.1

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View this Interactive Content online

Extra Practice Resources

For more practice on decimal notation, visit these websites.

1. IXL: Model decimals and fractions
2. IXL: Graph fractions as decimals on number lines

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1.10.2: Common Mistakes with Decimals and Place Value Understanding

Spot the Mistake 1.10.2.1

Spot the mistake.

357.28 – 3 = 357.25

$$\begin{array}{r} 3\ 5\ 7\ .\ 2\ 8 \\ -\qquad\qquad 3 \\ \hline 3\ 5\ 7\ .\ 2\ 5 \end{array} \tag{1.10.2.1}$$

Solution

Work	E
$$\begin{array}{r} 3\ 5\ 7\ .\ 2\ 8 \\ -\quad\ 3\ .\ 0\ 0 \\ \hline 3\ 5\ 4\ .\ 2\ 8 \end{array} \tag{1.10.2.2}$$	T p li S w n r 3 b b b o s n a c s is 3 3

You Try: Spot the Mistake 1.10.2.1

Spot the mistake and correct it. When you are finished, click the solution to reveal the answer.

589.76 - 5 = ?

$$\begin{array}{r} 5\ 8\ 9\ .\ 7\ 6 \\ -\qquad\qquad 5 \\ \hline 5\ 8\ 9\ .\ 7\ 1 \end{array} \tag{1.10.2.3}$$

Reveal Solution available at the end of this section.

Answers

You Try: Spot the Mistake 1.10.2.1

See how to solve this problem below.

Work

$$\begin{array}{r} 5\ 8\ 9\ .\ 7\ 6 \\ -\quad\ 5\ .\ 0\ 0 \\ \hline 5\ 8\ 4\ .\ 7\ 6 \end{array} \tag{1.10.2.4}$$

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1.11: Points, Lines, and Planes

Learning Objectives

1. Identify and describe points, lines, and planes.
2. Express points and lines using proper notation.
3. Determine union and intersection of sets.

In this section, we will begin our exploration of geometry by looking at the basic definitions as defined by Euclid. These definitions form the foundation of the geometric theories that are applied in everyday life.



Figure 1.11.1 : The lower right-hand corner of *The School of Athens* depicts a figure representing Euclid illustrating to students how to use a compass on a small chalkboard. (credit: modification of work "School of Athens" by Raphael (1483–1520), Vatican Museums/Wikimedia, Public Domain)

In *The Elements*, Euclid summarized the geometric principles discovered earlier and created an axiomatic system, a system composed of postulates. A *postulate* is another term for axiom, which is a statement that is accepted as truth without the need for proof or verification. There were no formal geometric definitions before Euclid, and when terms could not be defined, they could be described. In order to write his postulates, Euclid had to describe the terms he needed and he called the descriptions "definitions." Ultimately, we will work with theorems, which are statements that have been proved and can be proved.

Points and Lines

The first definition Euclid wrote was that of a point. He defined a point as "that which has no part." It was later expanded to "an indivisible location which has no width, length, or breadth." Here are the first two of the five postulates, as they are applicable to this first topic:

1. Postulate 1: A straight line segment can be drawn joining any two points.
2. Postulate 2: Any straight line segment can be extended indefinitely in a straight line.

Before we go further, we will define some of the symbols used in geometry in Figure 1.11.2

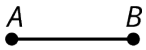
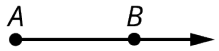
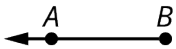
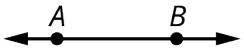
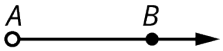
Symbol	Definition	Picture
•	Points are defined with capital letters, like point A	
$\overline{AB}$	A line segment from point A to point B	
$\overrightarrow{AB}$	A ray from point A in the direction of B	
$\overleftarrow{AB}$	A ray from point B in the direction of A	
$\overleftrightarrow{AB}$	A line that includes the points A and B goes off indefinitely in both directions	
$\circ\overrightarrow{AB}$	A half line from, but not including, point A in the direction of point B	
$\overleftarrow{AB}\circ$	A half line from, but not including, point B in the direction of A	

Figure 1.11.2 : Basic Geometric Symbols for Points and Lines

From Figure 10.3, we see the variations in lines, such as line segments, rays, or half-lines. What is consistent is that two collinear points (points that lie on the same line) are required to form a line. Notice that a line segment is defined by its two endpoints showing that there is a definite beginning and end to a line segment. A ray is defined by two points on the line; the first point is where the ray begins, and the second point gives the line direction. A half-line is defined by two points, one where the line starts and the other to give direction, but an open circle at the starting point indicates that the starting point is not part of the half-line. A regular line is defined by any two points on the line and extends infinitely in both directions. Regular lines are typically drawn with arrows on each end.

Example 1.11.1: Defining Lines

For the following exercises, use this line (Figure 1.11.3).



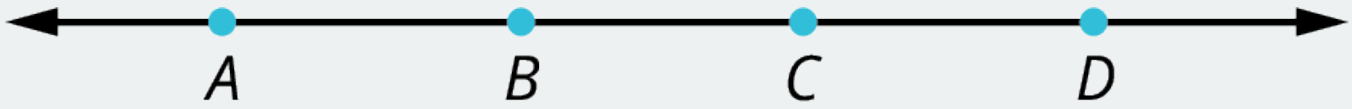
Figure 1.11.3

1. Define $\overline{DE}$.
2. Define F.
3. Define $\overleftrightarrow{DF}$.
4. Define $\overline{EF}$.

Answer available at the end of this section.

Your Turn 1.11.1

For the following exercises, use this line.



Figure

Define $\overrightarrow{BD}$

Define $\overline{AB}$

$\overleftarrow{BA}$

$\overleftrightarrow{AD}$

There are numerous applications of line segments in daily life. For example, airlines working out routes between cities, where each city's airport is a point, and the points are connected by line segments. Another example is a city map. Think about the intersection of roads, such that the center of each intersection is a point, and the points are connected by line segments representing the roads. See Figure 1.11.5.

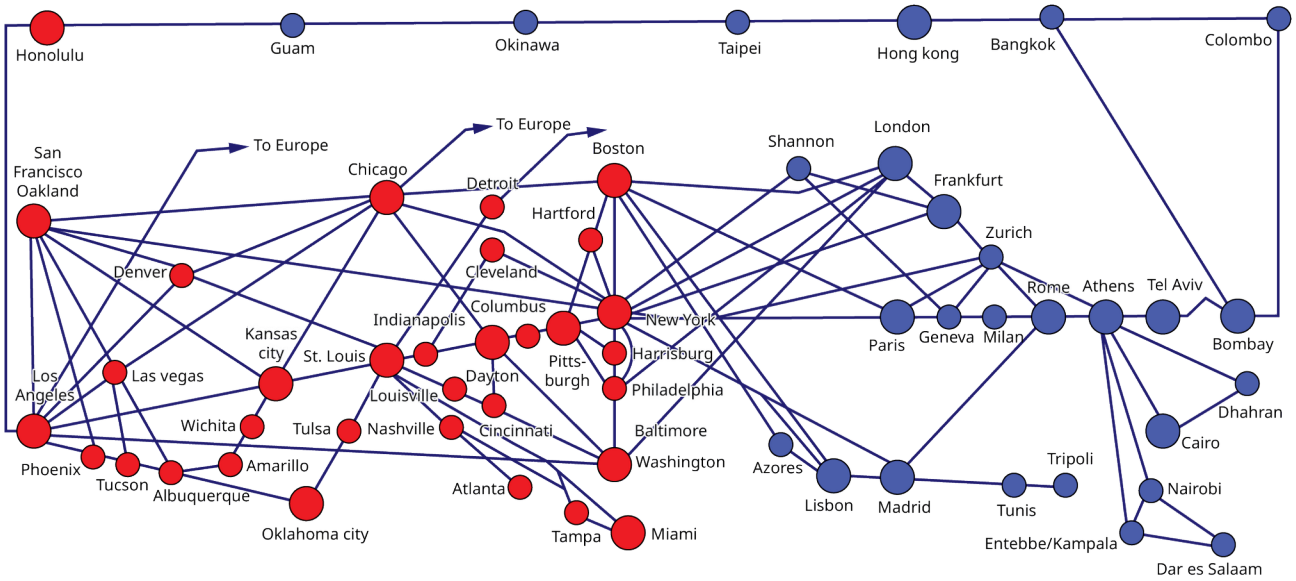


Figure 1.11.5: Air Line Routes

Example 1.11.2 : Determining the Best Route

View the street map (Figure 1.11.6) as a series of line segments from point to point. For example, we have vertical line segments $\overline{AB}$, $\overline{BC}$, and $\overline{CD}$ on the right. On the left side of the map, we have vertical line segments $\overline{HI}$, $\overline{FG}$. The horizontal line segments are $\overline{HA}$, $\overline{EI}$, $\overline{IE}$, $\overline{EB}$, $\overline{FC}$, $\overline{CF}$, and $\overline{GB}$. There are two diagonal line segments, $\overline{AE}$ and $\overline{EF}$. Assume that each location is on a corner and that you live next door to the library.

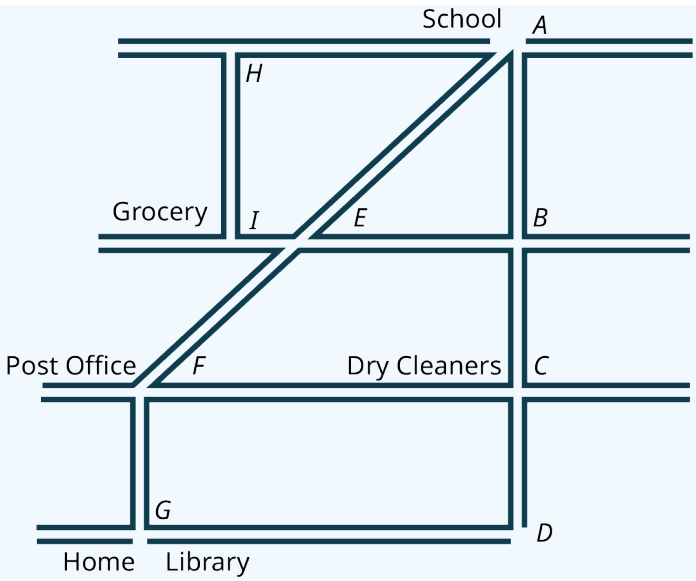


Figure 1.11.6 Street Map

1. Let's say that you want to stop at the grocery store on your way home from school. Come up with three routes you might take to do your errand and then go home. In other words, name the three ways by the line segments in the order you would walk, and which way do you think would be the most efficient route?
2. How about stopping at the library after school? Name four ways you might travel to the library and which way do you think is the most efficient?
3. Suppose you need to go to the post office and the dry cleaners on your way home from school. Name three ways you might walk to do your errands and end up at home. Which way do you think is the most efficient way to walk, get your errands done, and go home?

Answer available at the end of this section.

Your Turn 1.11.2

Using the street map in Figure 1.11.6, find two ways you would stop at the dry cleaners and the grocery store after school on your way home.

Parallel Lines

Parallel lines are lines that lie in the same plane and move in the same direction, but never intersect. To indicate that the line [redacted]l1 Figure 1.11.7.

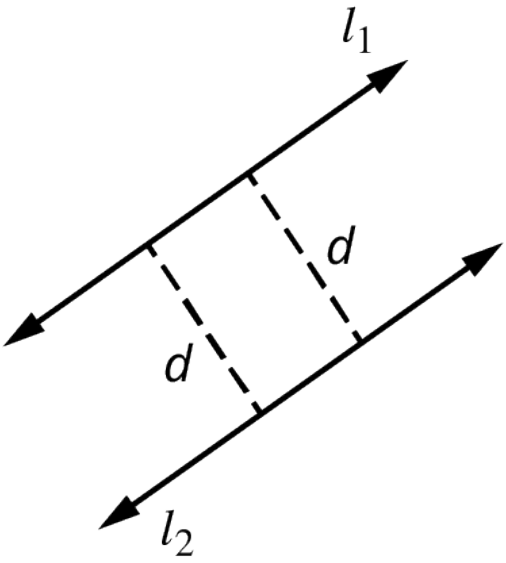


Figure 1.11.7: Parallel Lines

Perpendicular Lines

Two lines that intersect at a 90°

see Figure 1.11.8.

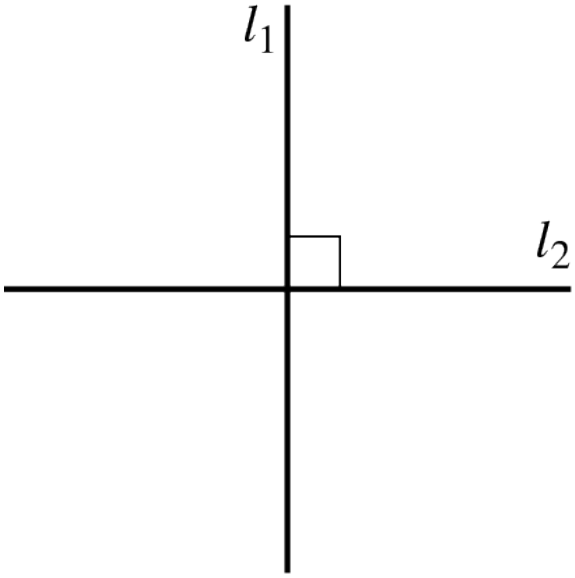
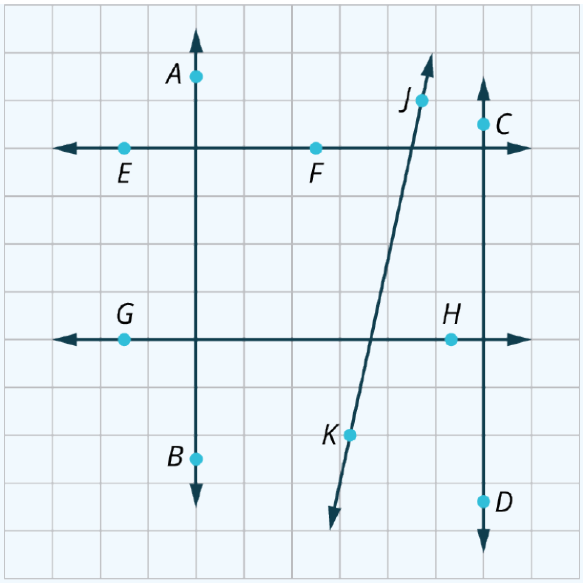


Figure 1.11.8: Perpendicular Lines

Example 1.11.3 : Identifying Parallel and Perpendicular Lines

Identify the sets of parallel and perpendicular lines in Figure 1.11.9.

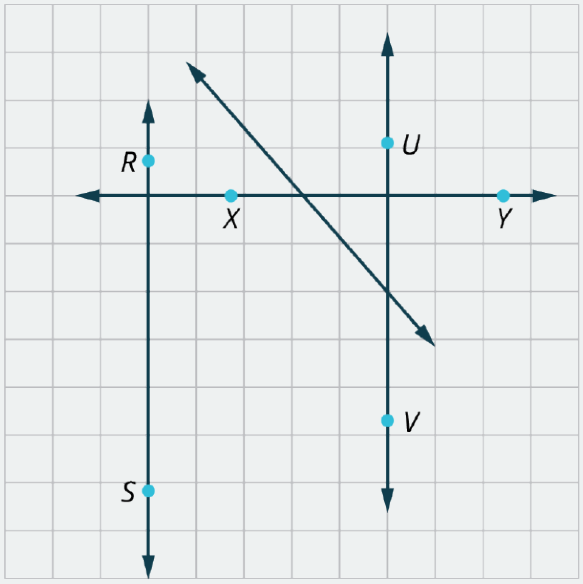


Figure

Answer available at the end of this section.

Your Turn 1.11.3

Identify the sets of parallel and perpendicular lines in the given figure.



Figure

Defining Union and Intersection of Sets

Union and intersection of sets is a topic from set theory that is often associated with points and lines. So, it seems appropriate to introduce a mini-version of set theory here. First, a **set** is a collection of objects joined by some common criteria. We usually name sets with capital letters. For example, the set of odd integers between 0 and 10 looks like this: $A = \{1, 3, 5, 7, 9\}$. When it involves sets of lines, line segments, or points, we are usually referring to the **union** or **intersection** of set.

The union of two or more sets contains all the elements in either one of the sets or elements in all the sets referenced, and is written by placing this symbol $\cup$ in between each of the sets. For example, let set $A = \{1, 2, 3\}$, and let set $B = \{4, 5, 6\}$. Then, the union of sets A and B is $A \cup B = \{1, 2, 3, 4, 5, 6\}$.

The intersection of two or more sets contains only the elements that are common to each set, and we place this symbol $\cap$ in between each of the sets referenced. For example, let's say that set $A = \{1, 3, 5\}$, and let set $B = \{5, 7, 9\}$. Then, the intersection of sets A and B is $A \cap B = \{5\}$.

Example 1.11.4: Defining Union and Intersection of Sets

Use the line (Figure 1.11.11) for the following exercises. Draw each answer over the main drawing.

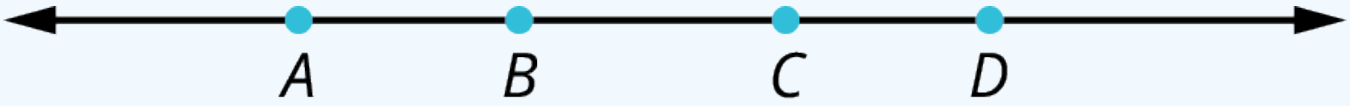


Figure 1.11.11

1. Find $\overrightarrow{BD} \cap \overleftarrow{CA}$.
2. Find $\overline{AB} \cup \overline{AD}$.
3. Find $\overleftrightarrow{AD} \cup \overline{BC}$.
4. Find $\overleftrightarrow{AD} \cap \overline{BC}$.
5. Find $\overleftarrow{BA} \cap \overline{CD}$.

Answer available at the end of this section.

Your Turn 1.11.4

For the following exercises, use the line shown to identify and draw the union or intersection of sets.



Figure

$\overline{AB} \cap \overline{BC}$

$\overrightarrow{BC} \cup \overleftarrow{CA}$

$\overline{BC} \cap \overline{AC}$

Planes

A **plane,** as defined by Euclid, is a "surface which lies evenly with the straight lines on itself." A plane is a two-dimensional surface with infinite length and width, and no thickness. We also identify a plane by three noncollinear points, or points that do not lie on the same line. Think of a piece of paper, but one that has infinite length, infinite width, and no thickness. However, not all planes must extend infinitely. Sometimes a plane has a limited area.

We usually label planes with a single capital letter, such as Plane [redacted]PFigure 10.19, or by all points that determine the edges of a plane. In the following figure, Plane P contains points A and B, which are on the same line, and point C, which is not on that line. By definition, P is a plane. We can move laterally in any direction on a plane.

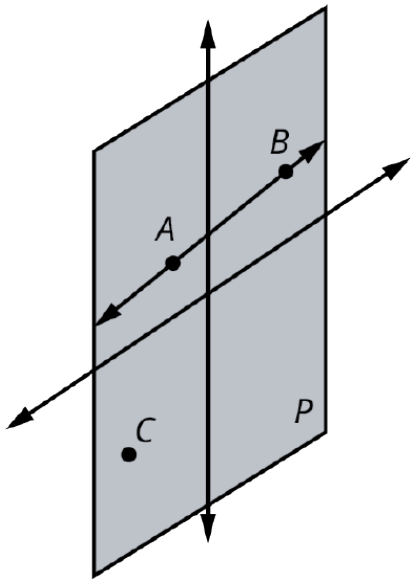


Figure 1.11.18: Plane P

One way to think of a plane is the Cartesian coordinate system with the [redacted]xFigure 1.11.19.

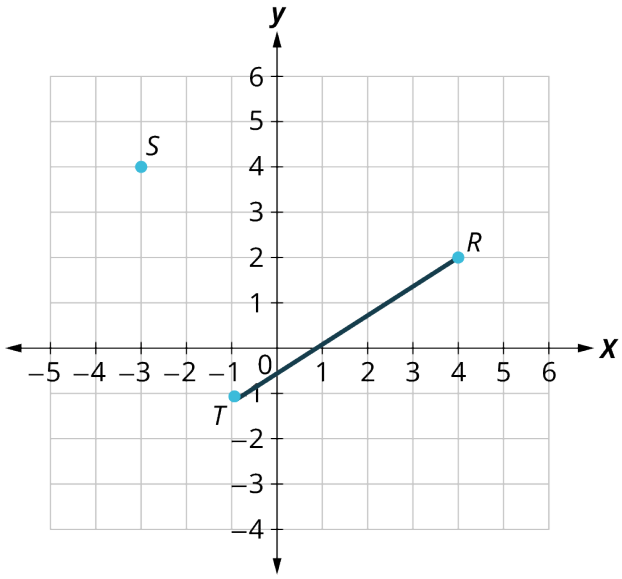


Figure 1.11.19: Cartesian Coordinate Plane

This plane contains points S, T, and R. Points T and R are colinear and form a line segment. Point S is not on that line segment. Therefore, this represents a plane.

To give the location of a point on the Cartesian plane, remember that the first number in the ordered pair is the horizontal move and the second number is the vertical move. Point R is located at $(4, 2)$; point S is located at $(-3, 4)$; and point T is located at $(-1, -1)$. We can also identify the line segment as $\overline{TR}$.

Two other concepts to note: Parallel planes do not intersect and the intersection of two planes is a straight line. The equation of that line of intersection is left to a study of three-dimensional space. See Figure 1.11.20.

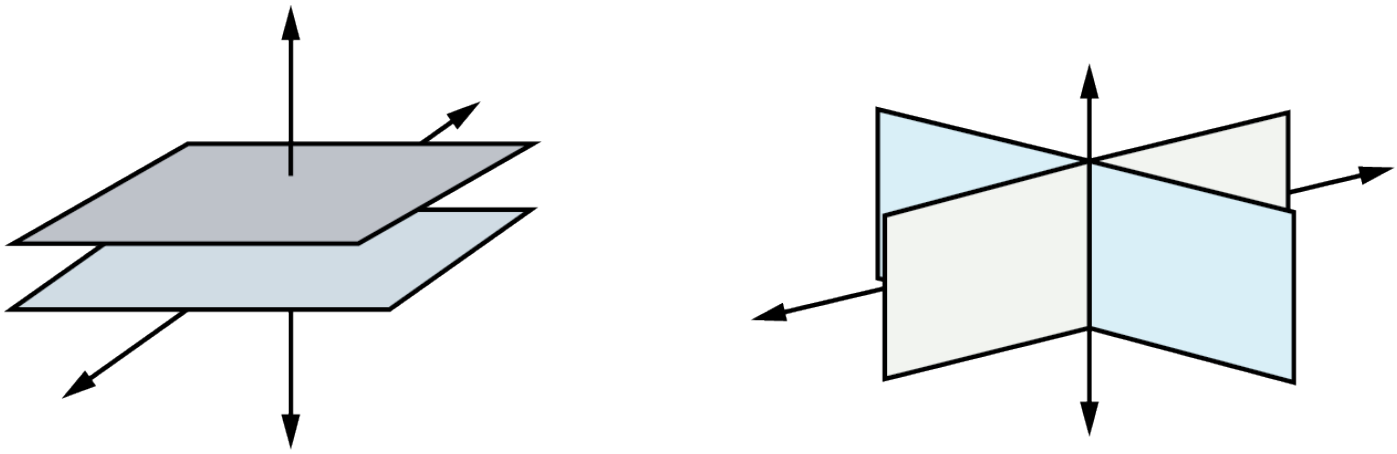


Figure 1.11.20: Parallel and Intersecting Planes

To summarize, some of the properties of planes include:

- Three points including at least one noncollinear point determine a plane.
- A line and a point not on the line determine a plane.
- The intersection of two distinct planes is a straight line.

Example 1.11.5: Identifying a Plane

For the following exercises, refer to Figure 1.11.21

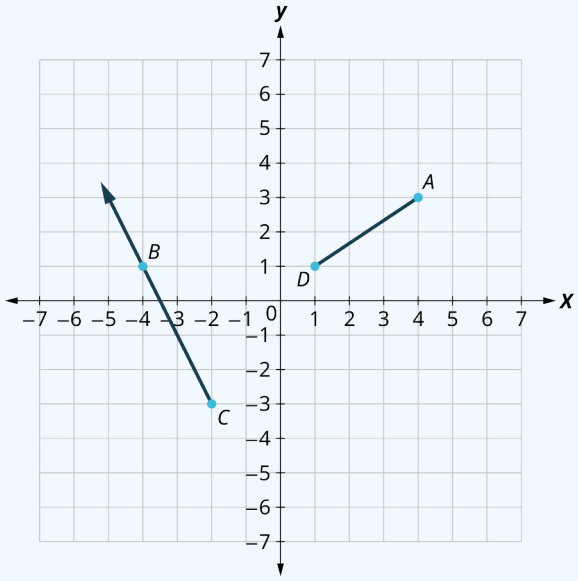


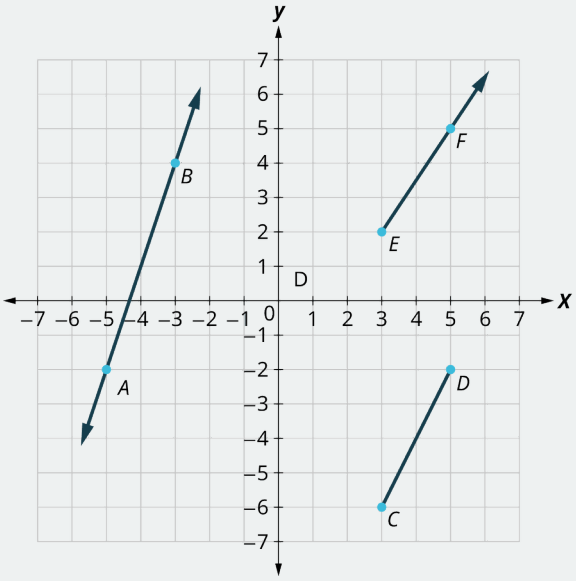
Figure 1.11.21

1. Identify the location of points A, B, C, and D.
2. Describe the line from point A to point D.
3. Describe the line from point C containing point B.
4. Does this figure represent a plane?

Answer available at the end of this section.

Your Turn 1.11.5

For the following exercises, refer to the given figure.



Figure

Identify the location of points A, B, C, D, E, and F.

Describe the line that includes point A and point B.

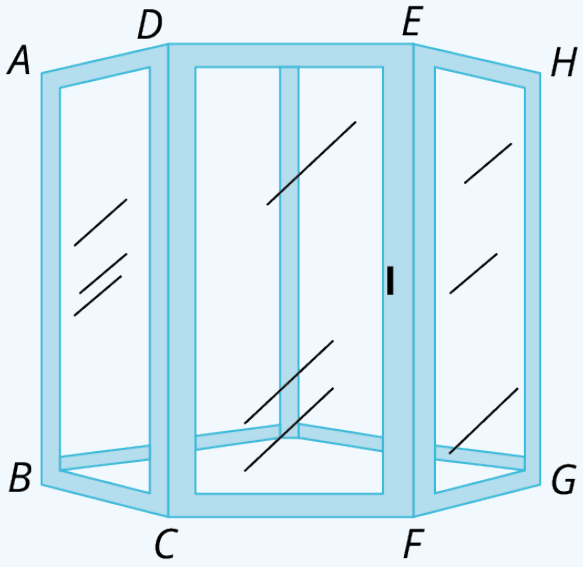
Describe the line from E to point F.

Describe the line from C to point D.

Does this figure represent a plane?

Example 1.11.6: Intersecting Planes

Name two pairs of intersecting planes on the shower enclosure illustration (Figure 1.11.23).

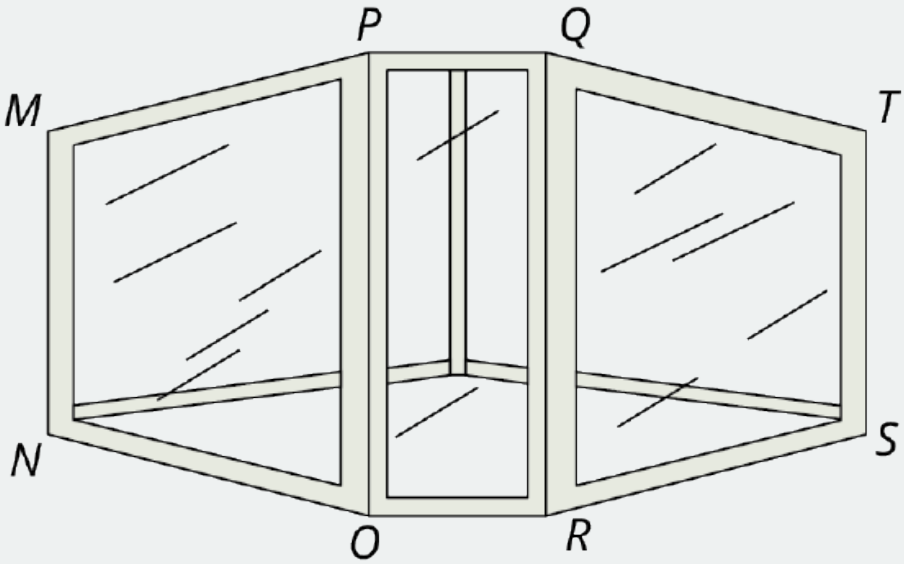


Figure

Answer available at the end of this section.

Your Turn 1.11.6

Name two pairs of intersecting planes on the shower enclosure shown.



Figure

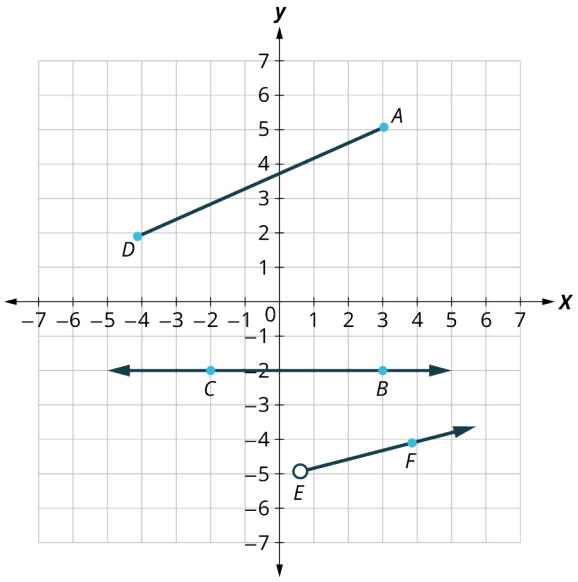
People in Mathematics: Plato

Part of a remarkable chain of Greek mathematicians, Plato (427–347 BC) is known as the teacher. He was responsible for shaping the development of Western thought perhaps more powerfully than anyone of his time. One of his greatest achievements was the founding of the Academy in Athens where he emphasized the study of geometry. Geometry was considered by the Greeks to be the "ultimate human endeavor." Above the doorway to the Academy, an inscription read, "Let no one ignorant of geometry enter here."

The curriculum of the Academy was a 15-year program. The students studied the exact sciences for the first 10 years. Plato believed that this was the necessary foundation for preparing students' minds to study relationships that require abstract thinking. The next 5 years were devoted to the study of the "dialectic." The dialectic is the art of question and answer. In Plato's view, this skill was critical to the investigation and demonstration of innate mathematical truths. By training young students how to prove propositions and test hypotheses, he created a culture in which the systematic process was guaranteed. The Academy was essentially the world's first university and held the reputation as the ultimate center of learning for more than 900 years.

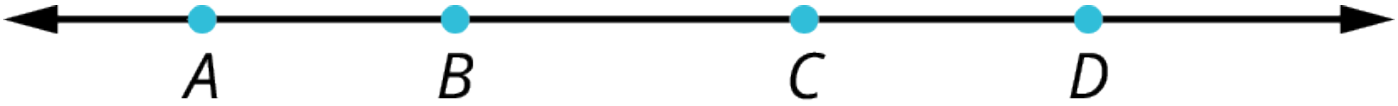
Check Your Understanding

Use the graph for the following exercises.



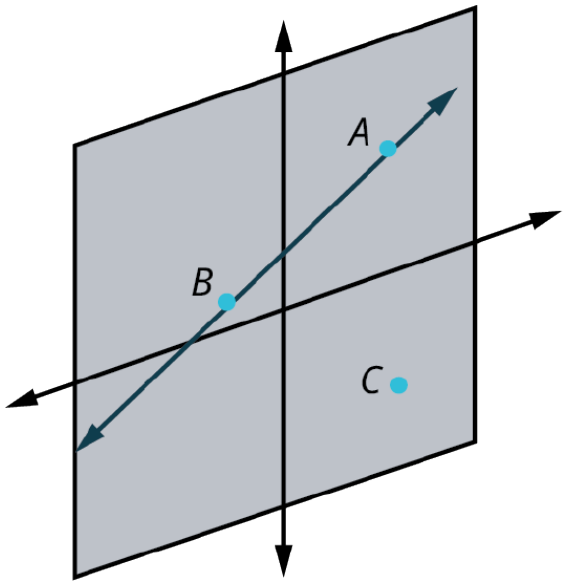
1. Identify the type of line containing point D and point A.
2. Identify the type of line containing points C and B.
3. Identify the type of line that references point E and contains point F.

Use the line for the following exercises.



4. Determine $\overline{AB} \cup \overline{BD}$.
5. Determine $\overleftarrow{BD} \cap \overline{BC}$.
6. Determine $\overleftarrow{BA} \cup \overrightarrow{BD}$.
7. How can you determine whether two lines are parallel?

8. How can you determine whether two lines are perpendicular?
9. Determine if the illustration represents a plane.



Answers

Example 1.11.1**: Defining Lines**

1. The symbol $\overline{DE}$, two letters with a straight line above, refers to the line segment that starts at point D and ends at point E.
2. The letter F alone refers to point F.
3. The symbol $\overleftrightarrow{DF}$, two letters with a line above containing arrows on both ends, refers to the line that extends infinitely in both directions and contains the points D and F.
4. The symbol $\overline{EF}$, two letters with a straight line above, refers to the line segment that starts at point E and ends at point F.

Example 1.11.2 **: Determining the Best Route**

1. From school to the grocery store and home: first way $\overline{AH}, \overline{HI}, \overline{IE}, \overline{EF}, \overline{FG}$; second way $\overline{AB}, \overline{BE}, \overline{EI}, \overline{IE}, \overline{EF}, \overline{FG}$; third way $\overline{AE}, \overline{EI}, \overline{IE}, \overline{EF}, \overline{FG}$. It seems that the third way is the most efficient way.
2. From school to the library: first way $\overline{AE}, \overline{EF}, \overline{FG}$; second way $\overline{AB}, \overline{BC}, \overline{CD}, \overline{DG}$; third way $\overline{AB}, \overline{BE}, \overline{EF}, \overline{FG}$; fourth way $\overline{AB}, \overline{BC}, \overline{CF}, \overline{FG}$. The first way should be the most efficient way.
3. From school to the post office or dry cleaners to home: first way $\overline{AE}, \overline{EF}, \overline{FC}, \overline{CD}, \overline{DG}$; second way $\overline{AB}, \overline{BE}, \overline{EF}, \overline{FC}, \overline{CD}, \overline{DG}$; third way $\overline{AB}, \overline{BC}, \overline{CF}, \overline{FG}$. The third way would be the most efficient way.

Example 1.11.3 **: Identifying Parallel and Perpendicular Lines**

Drawing these lines on a grid is the best way to distinguish which pairs of lines are parallel and which are perpendicular. Because they are on a grid, we assume all lines are equally spaced across the grid horizontally and vertically. The grid also tells us that the vertical lines are parallel and the horizontal lines are parallel. Additionally, all intersections form a 90° angle. Therefore, we can safely say the following:

$\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$, the line containing the points A and B is parallel to the line containing the points C and D.

$\overleftrightarrow{EF} \parallel \overleftrightarrow{GH}$, the line containing the points E and F is parallel to the line containing the points G and H.

$\overleftrightarrow{AB} \perp \overleftrightarrow{EF}$, the line containing the points A and B is perpendicular to the line containing the points E and F. We know this because both lines trace grid lines, and intersecting grid lines are perpendicular.

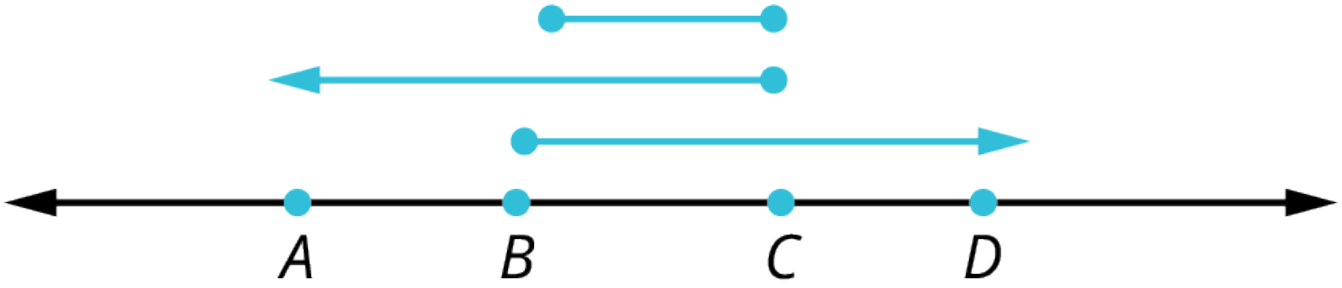
We can also state that $\overleftrightarrow{AB} \perp \overleftrightarrow{GH}$; the line containing the points A and B is perpendicular to the line containing the points G and H because both lines trace grid lines, which are perpendicular by definition.

We also have $\overleftrightarrow{CD} \perp \overleftrightarrow{EF}$; the line containing the points C and D is perpendicular to the line containing the points E and F because both lines trace grid lines, which are perpendicular by definition.

Finally, we see that $\overleftrightarrow{CD} \perp \overleftrightarrow{GH}$; the line containing the points C and D is perpendicular to the line containing the points G and H because both lines trace grid lines, which are perpendicular by definition.

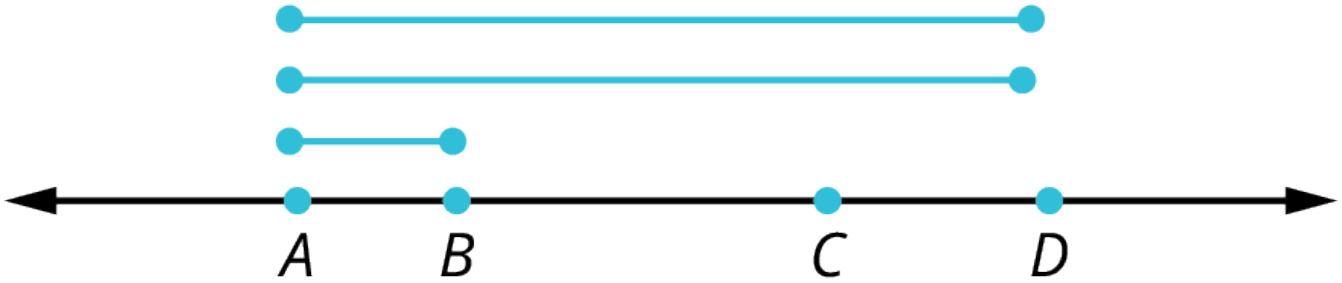
Example 1.11.4: Defining Union and Intersection of Sets

1. Find $\overrightarrow{BD} \cap \overleftarrow{CA}$. This is the intersection of the ray $\overrightarrow{BD}$ and the ray $\overleftarrow{CA}$. Intersection includes only the elements that are common to both lines. For this intersection, only the line segment $\overline{BC}$ is common to both rays. Thus, $\overrightarrow{BD} \cap \overleftarrow{CA} = \overline{BC}$.



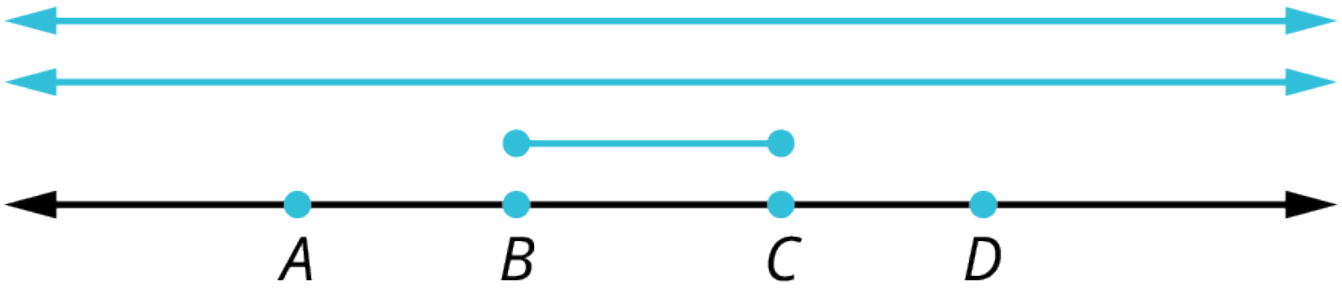
Figure

2. Find $\overline{AB} \cup \overline{AD}$. The problem is asking for the union of two line segments, $\overline{AB}$ and $\overline{AD}$. Union includes all elements in the first line and all elements in the second line. Since $\overline{AB}$ is part of $\overline{AD}$, $\overline{AB} \cup \overline{AD} = \overline{AD}$.



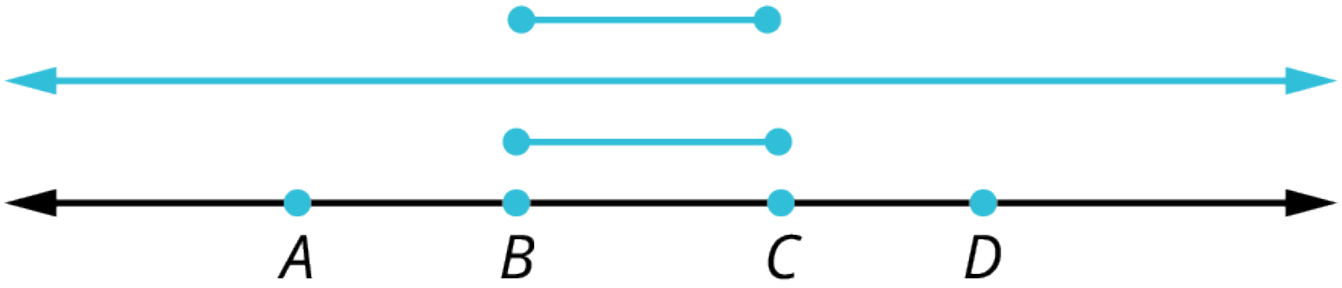
Figure

3. Find $\overleftrightarrow{AD} \cup \overline{BC}$. This is the union of the line $\overleftrightarrow{AD}$ with the line segment $\overline{BC}$. As the line segment $\overline{BC}$ is included on the line $\overleftrightarrow{AD}$, then the union of these two lines equals the line $\overleftrightarrow{AD}$.



Figure

4. Find $\overleftrightarrow{AD} \cap \overline{BC}$. The intersection of the line $\overleftrightarrow{AD}$ with line segment $\overline{BC}$ is the set of elements common to both lines. In this case, the only element in common is the line segment $\overline{BC}$.



Figure

5. Find Math input error

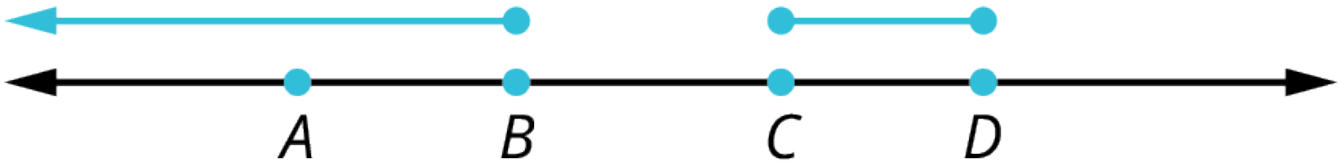


Figure 1.11.16

Example 1.11.5**: Identifying a Plane**

1. Point A is located at $(4, 3)$; point B is located at $(-4, 1)$; point C is located at $(-2, -3)$; point D is located at $(1, 1)$.
2. The line from point A to point D is a line segment $\overline{AD}$.
3. The line from point C containing point B is a ray $\overrightarrow{CB}$ starting at point C in the direction of B.
4. Yes, this figure represents a plane because it contains at least three points, points A and D form a line segment, and neither point B nor point C is on that line segment.

Example 1.11.6**: Intersecting Planes**

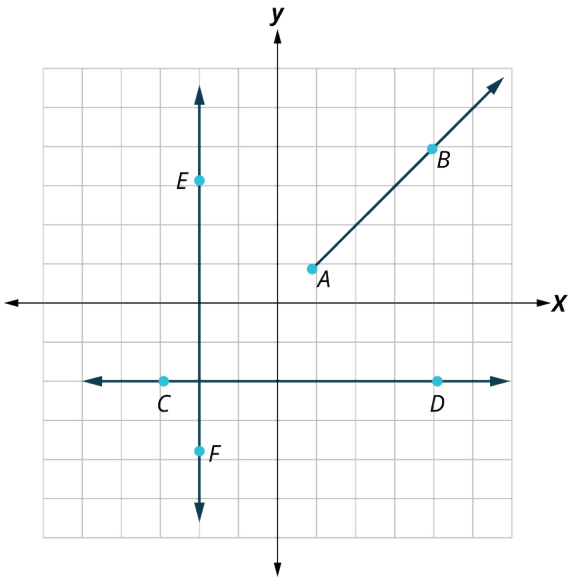
The plane [redacted]ABCDABCD intersects plane [redacted]CDEFCDEF, and plane [redacted]CDEFCDEF intersects plane [redacted]EFGHEFGH.

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1.11.1: Exercises

Use this graph for the following exercises.



Exercise 1.11.1.1

Identify the kind of line passing through the points A and B.

Exercise 1.11.1.2

Identify the kind of line passing through the points C and D.

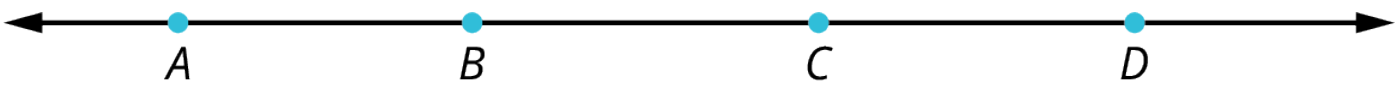
Exercise 1.11.1.3

Identify all sets of parallel lines.

Exercise 1.11.1.4

Identify all sets of perpendicular lines.

Use the line for the following exercises.



Exercise 1.11.1.5

Find $\overleftarrow{BA} \cup \overline{BC}$.

Exercise 1.11.1.6

Find $\overrightarrow{AD} \cup \overrightarrow{CD}$.

Exercise 1.11.1.7

Find $\overline{AC} \cap \overline{BD}$.

Exercise 1.11.1.8

Find $\overleftarrow{CA} \cap \overrightarrow{BD}$

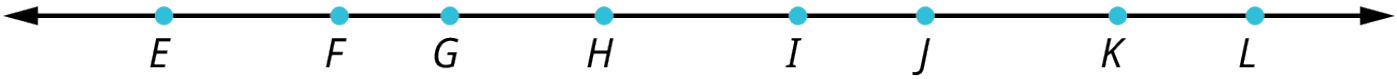
Exercise 1.11.1.9

Find $\overline{AB} \cup \overline{BD}$

Exercise 1.11.1.10

Find $\overleftrightarrow{BC} \cup \overline{BD}$

For the following exercises, reference this line.



Exercise 1.11.1.11

Name the points in the set $\overleftrightarrow{FH} \cap \overline{GH}$.

Exercise 1.11.1.12

Name the points contained in the set $\overrightarrow{HL} \cap \overline{JL}$.

Exercise 1.11.1.13

Name the points in the set $\overleftrightarrow{EL} \cup \overline{GH}$.

Exercise 1.11.1.14

Name the points in the set $\overleftarrow{GE} \cup \overrightarrow{KL}$.

Exercise 1.11.1.15

Name the points in the set $\overleftarrow{GE} \cap \overrightarrow{KL}$.

Exercise 1.11.1.16

Name the points in the set $\overrightarrow{HK} \cap \overleftarrow{JF}$.

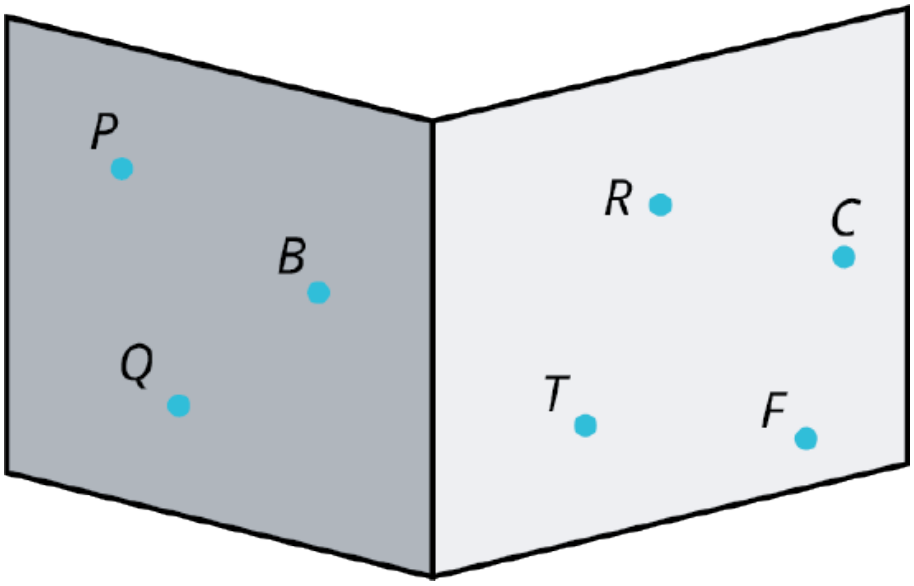
Exercise 1.11.1.17

Name the points in the set $\overrightarrow{GL} \cap K$.

Exercise 1.11.1.18

Name the points in the set $\overleftarrow{HE} \cup \overrightarrow{KL}$.

Use the given figure for the following exercises.



Exercise 1.11.1.19

Describe the lighter plane (right) in the drawing.

Exercise 1.11.1.20

Describe the darker plane (left) in the drawing.

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1.11.2: Practice and Videos for Points, Lines and Rays

Learning Objective

- I can identify points, lines, line segments, rays, and angles.

GeoGebra: Naming Points, Segments, Rays, and Lines

Complete the activity below by dragging the tiles to create objects using the given points. Build names of geometric objects to identify them in a figure. Explore the different ways an object can be named.

Videos on Points, Lines, and Rays

Watch the following videos to learn about points, lines, and rays. All videos have appropriate captions in English; in some cases, there may also be appropriate captions in other languages.

YouTube Video

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

YouTube Video

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

Practice!

Check your understanding of Points, Lines, and Rays by answering these questions. Use the arrows at the bottom of the box to go to the next question.

Query 1.11.2.1

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View this Interactive Content online

Extra Resources

For more practice on Points, Lines, and Rays, visit these websites.

1. IXL: Points, lines, line segments, rays, and angles

2. Khan Academy: Identify points, lines, line segments, rays, and angles

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1.12: Basic Geometric Concepts and Figures

Introduction

You use geometric terms in everyday language, often without thinking about it. For example, any time you say “walk along this line” or “watch out, this road quickly angles to the left” you are using geometric terms to make sense of the environment around you. You use these terms flexibly, and people generally know what you are talking about.

In the world of mathematics, each of these geometric terms has a specific definition. It is important to know these definitions—as well as how different figures are constructed—to become familiar with the language of geometry. Let’s start with a basic geometric figure: the plane.

Figures on a Plane

A **plane** is a flat surface that continues forever (or, in mathematical terms, infinitely) in every direction. It has two dimensions: length and width.

You can visualize a plane by placing a piece of paper on a table. Now imagine that the piece of paper stays perfectly flat and extends as far as you can see in two directions, left-to-right and front-to-back. This gigantic piece of paper gives you a sense of what a geometric plane is like: it continues infinitely in two directions. (Unlike the piece of paper example, though, a geometric plane has no height.)

A plane can contain a number of geometric figures. The most basic geometric idea is a **point**, which has no dimensions. A point is simply a location on the plane. It is represented by a dot. Three points that don’t lie in a straight line will determine a plane.

The image below shows four points, labeled A, B, C, D.



Figure 1.12.1: A set of points

Two points on a plane determine a line. A line is a one-dimensional figure that is made up of an infinite number of individual points placed side by side. In geometry, all lines are assumed to be straight; if they bend they are called a curve. A line continues infinitely in two directions.

Below is line *AB* or, in geometric notation, $\overleftrightarrow{AB}$. The arrows indicate that the line keeps going forever in the two directions. This line could also be called line *BA*. While the order of the points does not matter for a line, it is customary to name the two points in alphabetical order.

The image below shows the points A and B and the line $\overleftrightarrow{AB}$.

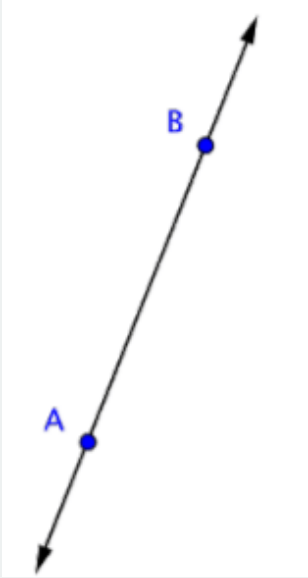


Figure 1.12.2: Line *AB*

There are two more figures to consider. The section between any two points on a line is called a line segment. A line segment can be very long, very short, or somewhere in between. The difference between a line and a **line segment** is that the line segment has two endpoints and a line goes on forever. A line segment is denoted by its two endpoints, as in $\overline{CD}$.

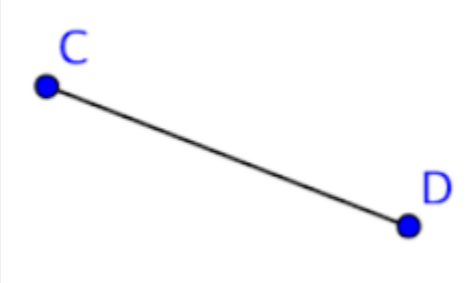


Figure 1.12.3: Line segment *CD*

A **ray** has one endpoint and goes on forever in one direction. Mathematicians name a ray with notation like $\overrightarrow{EF}$, where point *E* is the endpoint and *F* is a point on the ray. When naming a ray, we always say the endpoint first. Note that $\overrightarrow{FE}$ would have the endpoint at *F*, and continue through *E*, which is a different ray than $\overrightarrow{EF}$, which would have an endpoint at E, and continue through F.

The term “ray” may be familiar because it is a common word in English. “Ray” is often used when talking about light. While a ray of light resembles the geometric term “ray,” it does not go on forever, and it has some width. A geometric ray has no width; only length.

Below is an image of ray *EF* or $\overrightarrow{EF}$. Notice that the endpoint is *E*.

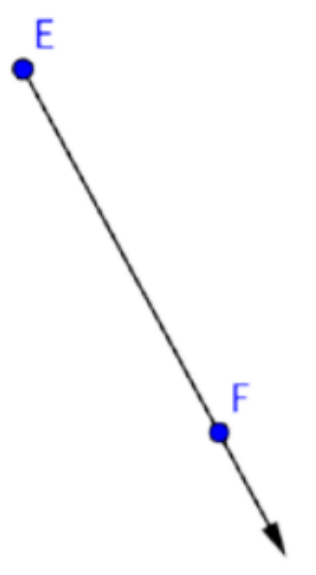


Figure 1.12.4: Ray *EF*

Example 1.12.1

Identify each line and line segment in the picture below.

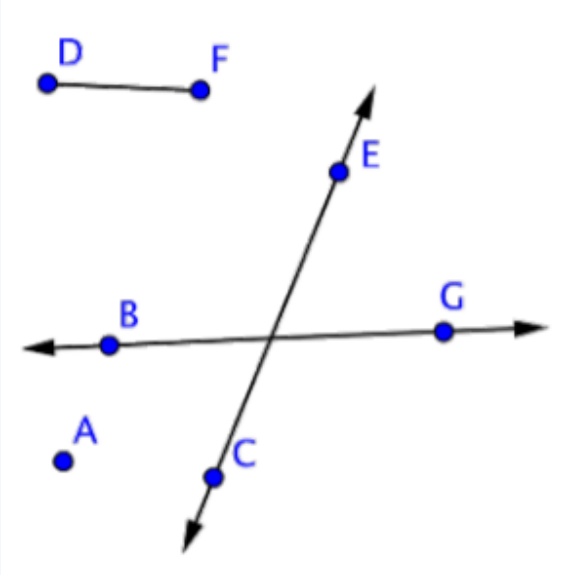


Figure 1.12.5: Line and Line Segment

Answer available at the end of this section.

Your Turn 1.12.1

Identify each point and ray in the picture below.

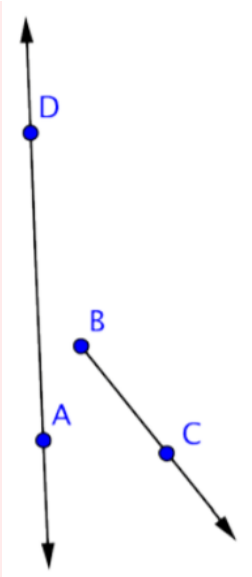


Figure 1.12.6: Point and Ray

Answer available at the end of this section.

Angles

Lines, line segments, points, and rays are the building blocks of other figures. For example, two rays with a common endpoint make up an **angle**. The common endpoint of the angle is called the **vertex**.

The angle ABC is shown below. This angle can also be called ∠ABC, ∠CBA or simply ∠B. When you are naming angles, be careful to include the vertex (here, point B) as the middle letter.

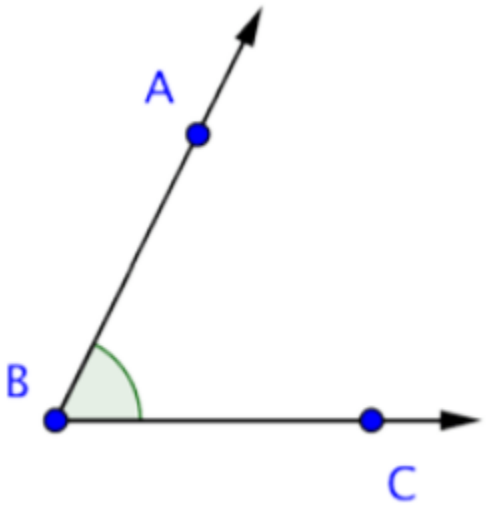


Figure 1.12.7: Angle

The image below shows a few angles on a plane. Notice that the label of each angle is written “point-vertex-point,” and the geometric notation is in the form ∠ABC.

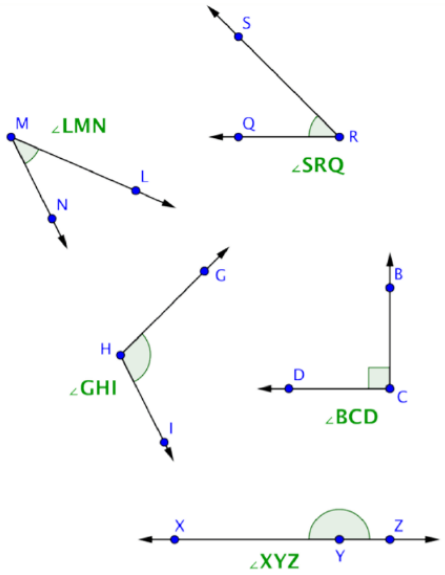


Figure 1.12.8: Naming Angles

Sometimes angles are very narrow; sometimes they are very wide. When people talk about the “size” of an angle, they are referring to the arc between the two rays. The length of the rays has nothing to do with the size of the angle itself. Drawings of angles will often include an arc (as shown above) to help the reader identify the correct ‘side’ of the angle.

Think about an analog clock face. The minute and hour hands are both fixed at a point in the middle of the clock. As time passes, the hands rotate around the fixed point, making larger and smaller angles as they go. The length of the hands does not impact the angle that is made by the hands.

An angle is measured in degrees, represented by the symbol °. A circle is defined as having 360°. (In skateboarding and basketball, “doing a 360” refers to jumping and doing one complete body rotation.

A **right angle** is any degree that measures exactly 90°. This represents exactly one-quarter of the way around a circle. Rectangles contain exactly four right angles. A corner mark is often used to denote a right angle, as shown in right angle DCB below.

Angles that are between 0° and 90° (smaller than right angles) are called **acute angles**. Angles that are between 90° and 180° (larger than right angles and less than 180°) are called **obtuse angles**. An angle that measures exactly 180° is called a **straight angle** because it forms a straight line.

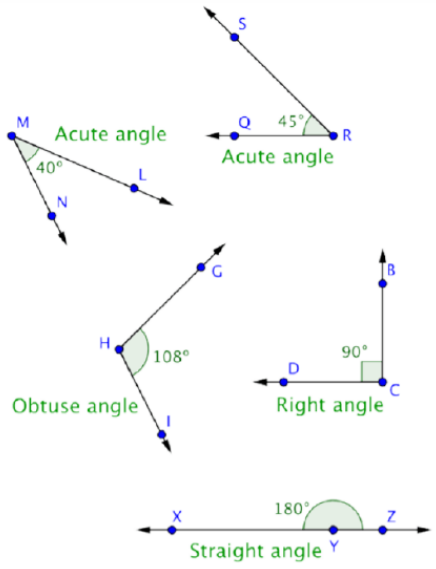


Figure 1.12.9: Different Type of Angles

Example 1.12.2

Label each angle below as acute, right, or obtuse.

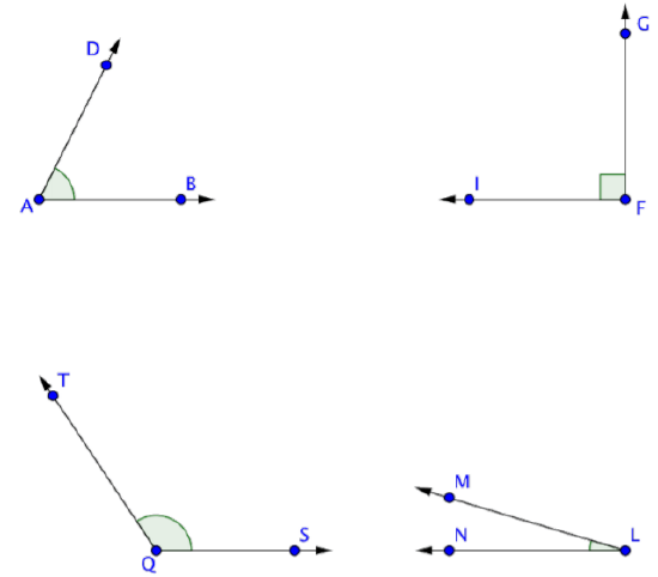


Figure 1.12.10: Identify the Angles

Answer available at the end of this section.

Your Turn 1.12.2: Identify the Angle

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Properties of Angles

Introduction

Imagine two separate and distinct lines on a plane. There are two possibilities for these lines: they will either intersect at one point, or they will never intersect. When two lines intersect, four angles are formed. Understanding how these angles relate to each other can help you figure out how to measure them, even if you only have information about the size of one angle.

Parallel and Perpendicular

Parallel lines are two or more lines that never intersect. Likewise, parallel line segments are two line segments that never intersect even if the line segments were turned into lines that continued forever. Examples of parallel line segments are all around you, in the two sides of this page and in the shelves of a bookcase. When you see lines or structures that seem to run in the same direction, never cross one another, and are always the same distance apart, there's a good chance that they are parallel.

Perpendicular lines are two lines that intersect at a 90° (right) angle. And perpendicular line segments also intersect at a 90° (right) angle. You can see examples of perpendicular lines everywhere as well—on graph paper, in the crossing pattern of roads at an intersection, to the colored lines of a plaid shirt. In our daily lives, you may be happy to call two lines perpendicular if they merely

seem to be at right angles to one another. When studying geometry, however, you need to make sure that two lines intersect at a 90° angle before declaring them to be perpendicular.

The image below shows some parallel and perpendicular lines. The geometric symbol for parallel is ||, so you can show that *AB* || *CD*. Parallel lines are also often indicated by the marking >> on each line (or just a single > on each line). Perpendicular lines are indicated by the symbol ⊥, so you can write $\overleftrightarrow{WX} \perp \overleftrightarrow{YZ}$.

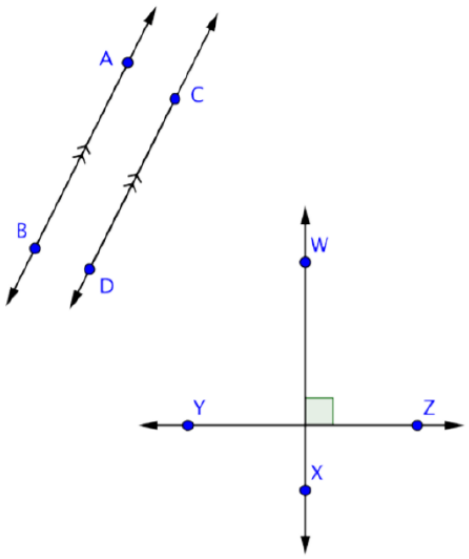


Figure 1.12.11: Parallel and Perpendicular Lines

If two lines are parallel, then any line that is perpendicular to one line will also be perpendicular to the other line. Similarly, if two lines are both perpendicular to the same line, then those two lines are parallel to each other. Let's take a look at one example and identify some of these types of lines.

Example 1.12.3

Identify a set of parallel lines and a set of perpendicular lines in the image below.

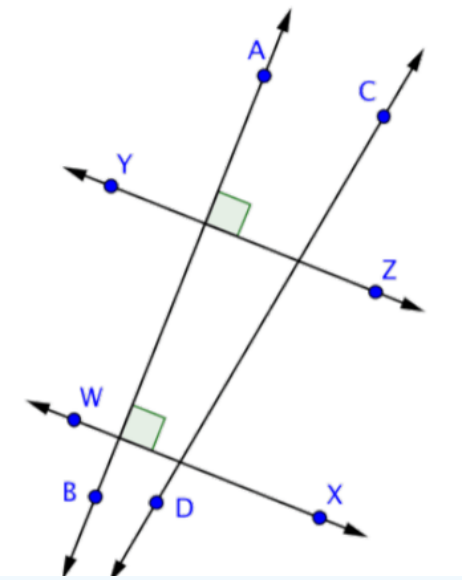


Figure 1.12.12: Intersecting Lines

Answer available at the end of this section.

Supplementary and Complementary

In the example above, m∠BAC and m∠DAC add up to 180°. Two angles whose measures add up to 180° are called **supplementary angles**. There's also a term for two angles whose measurements add up to 90°, they are called **complementary angles**.

One way to remember the difference between the two terms is that "corner" and "complementary" each begin with *c* (a 90° angle looks like a corner), while straight and "supplementary" each begin with *s* (a straight angle measures 180°).

If you can identify supplementary or complementary angles within a problem, finding missing angle measurements is often simply a matter of adding or subtracting.

Example 1.12.4

Two angles are supplementary. If one of the angles measures 48°, what is the measurement of the other angle?

Answer available at the end of this section.

Your Turn 1.12.4: Complementary or Supplementary?

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Example 1.12.5

Find the measurement of ∠AXZ.

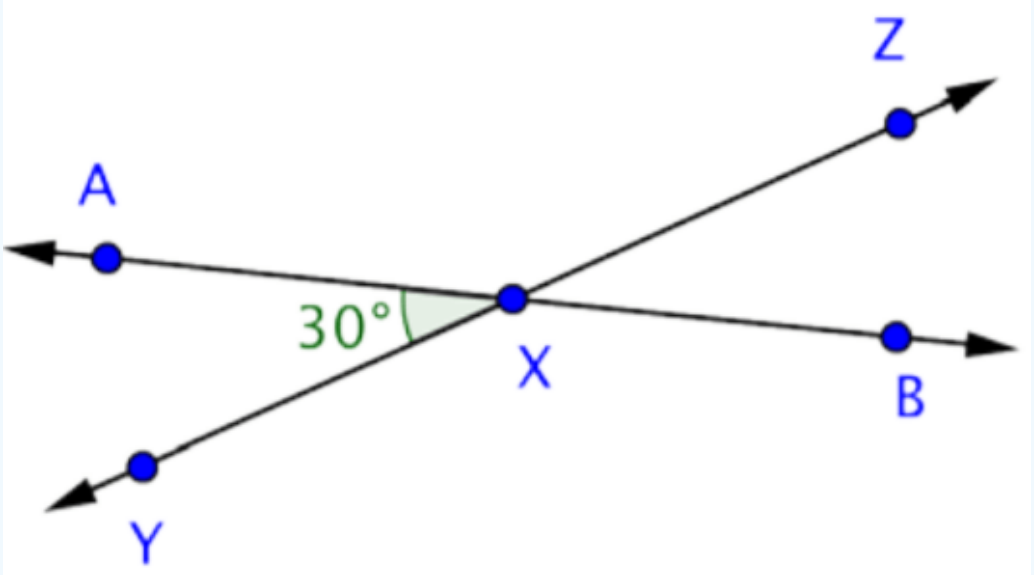


Figure 1.12.13

Answer available at the end of this section.

m∠AXZ = 150°

Your Turn 1.12.5: Complementary or Supplementary?

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Your Turn 1.12.6: Matching with Definition

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Vertical Angles

When two lines intersect, the opposite angles are called vertical angles, and vertical angles have equal measure. For example, Figure 1.12.15 shows two straight lines intersecting each other. One set of opposite angles shows angle markers; those angles have the same measure. The other two opposite angles have the same measure as well.

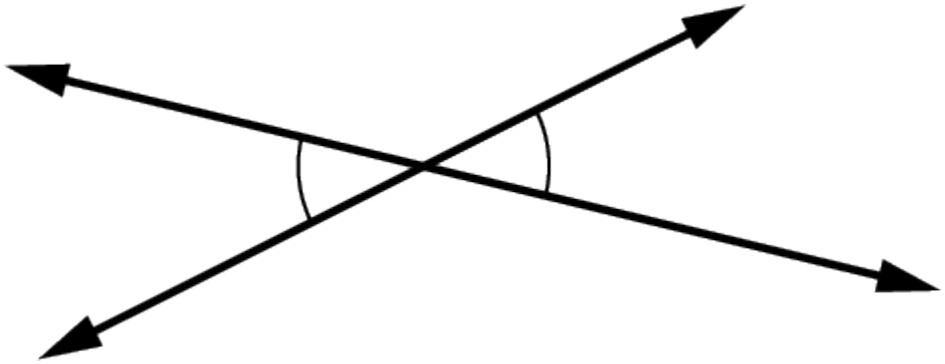


Figure 1.12.15: Vertical Angles

Example 1.12.7: Calculating Vertical Angles

In Figure 1.12.16, one angle measures 40°. Find the measures of the remaining angles.

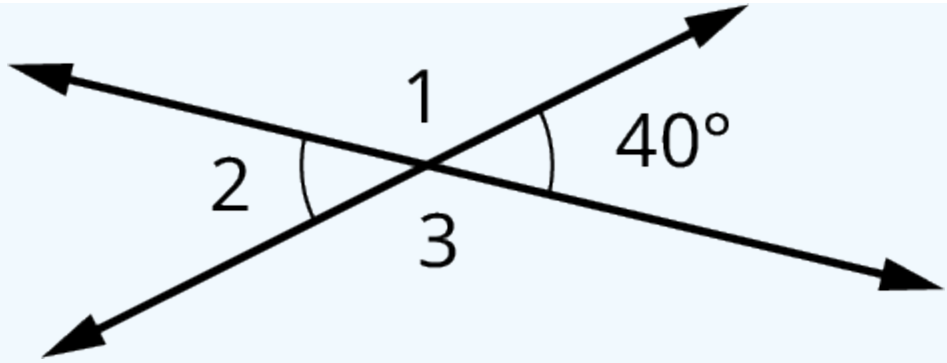


Figure 1.12.16: Vertical Angles

Answer available at the end of this section.

Finding Angle Measurements

In the example below, notice how you can use the same technique as shown above (using straight angles) to find the measurement of a missing angle.

Example 1.12.8: Evaluating Space

You live on the corner of First Avenue and Linton Street. You want to plant a garden in the far corner of your property (Figure 1.12.17) and fence off the area. However, the corner of your property does not form the traditional right angle. You learned from the city that the streets intersect at an angle of 150°. What is the measure of the angle that will border your garden?

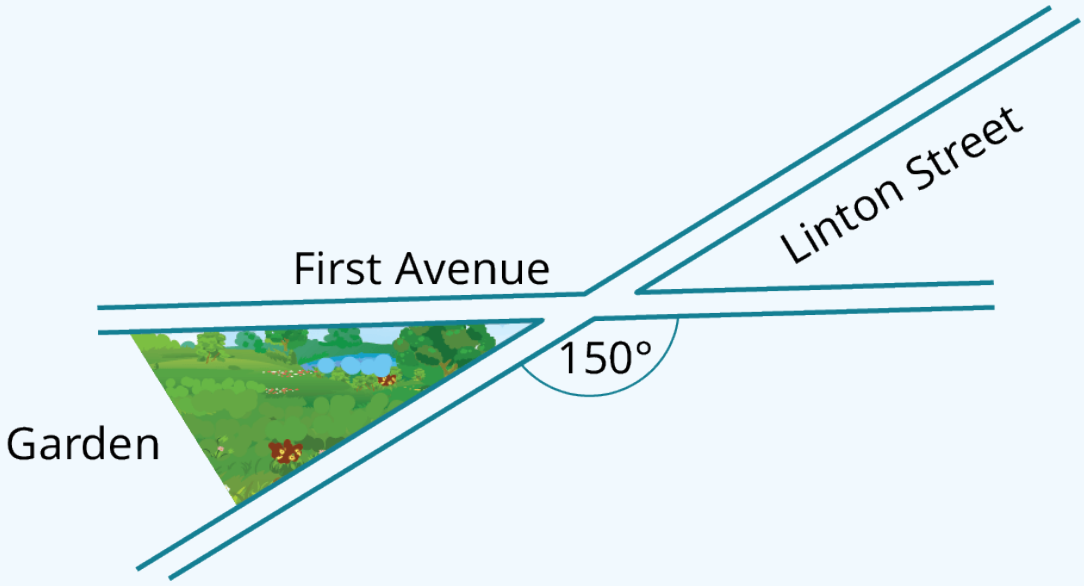


Figure 1.12.17: Intersection of Two Streets

Answer available at the end of this section.

Example 1.12.9: Solving for Angle Measurements and Supplementary Angles

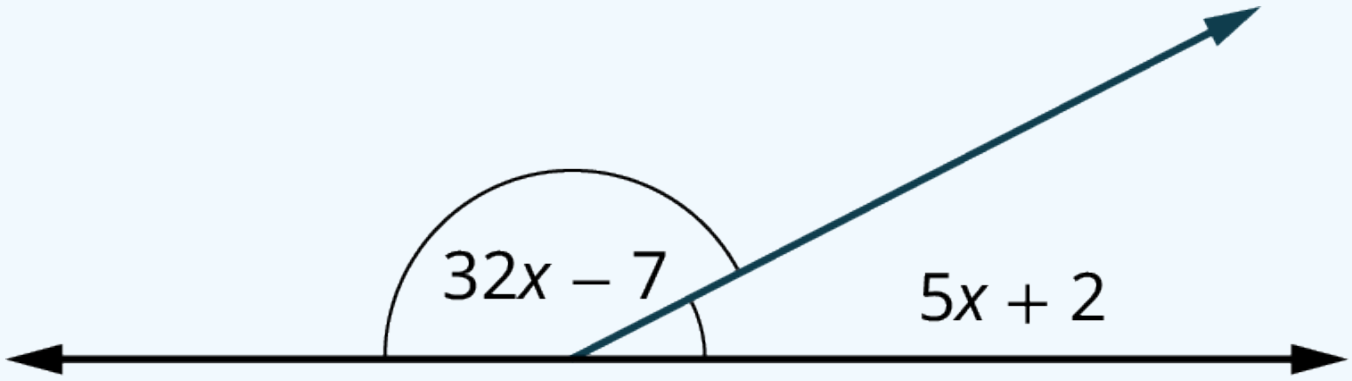


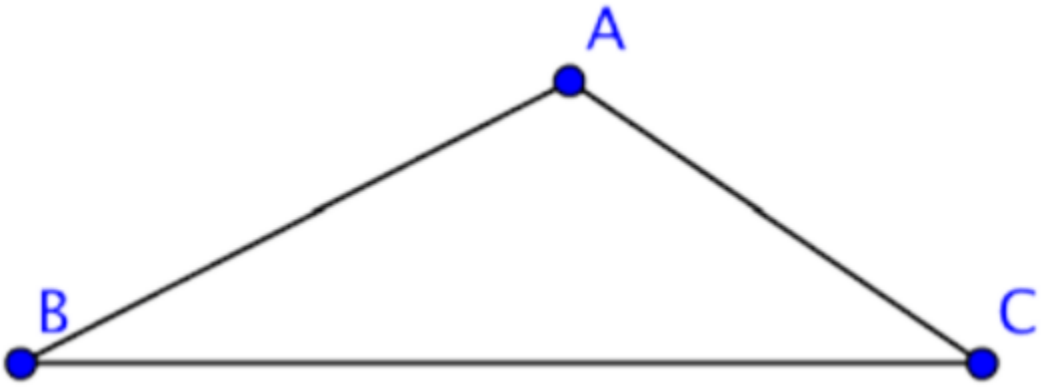
Figure 1.12.18: Supplement Angles

Answer available at the end of this section.

Triangles

Geometric shapes, also known as figures, are a crucial part of the study of geometry. The triangle is one of the basic shapes in geometry. It is the simplest shape within a classification of shapes called polygons. All triangles have three sides and three angles, but they come in many different shapes and sizes. Within the group of all triangles, the characteristics of a triangle's sides and angles are used to classify it even further. Triangles have some important characteristics, and understanding these characteristics allows you to apply the ideas in real-world problems.

There is an established convention for naming triangles. The labels of the vertices of the triangle, which are generally capital letters, are used to name a triangle.



You can call this triangle *ABC* or ΔABC since *A*, *B*, and *C* are the vertices of the triangle. When naming the triangle, you can begin with any vertex. Then keep the letters in order as you go around the polygon. The triangle above could be named in a variety of ways: ΔABC, or ΔCBA. The sides of the triangle are line segments *AB*, *AC*, and *CB*.

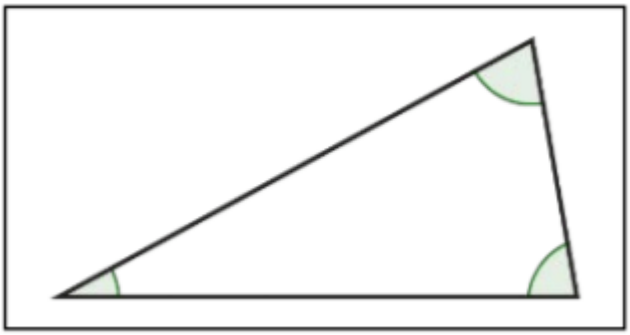
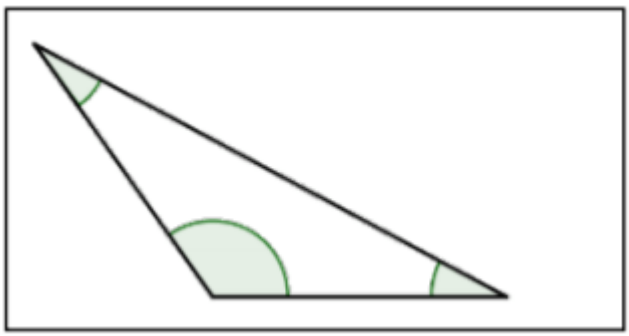
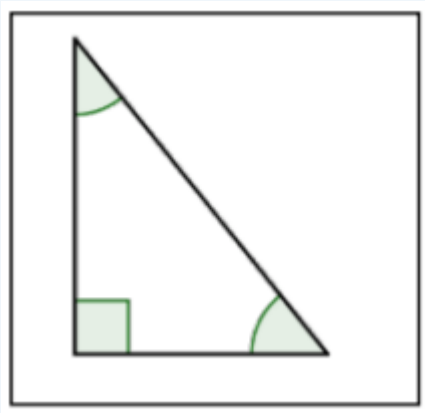
Just as triangles can be classified as acute, obtuse, or right based on their angles, they can also be classified by the length of their sides. Sides of equal length are called **congruent** sides. While we designate a segment joining points *A* and *B* by the notation $\overline{AB}$, we designate the length of a segment joining points *A* and *B* by the notation *AB* without a segment bar over it. The length *AB* is a number, and the segment $\overline{AB}$ is the collection of points that make up the segment.

Classifying and Naming Triangles

A polygon is a closed plane figure with three or more straight sides. Polygons each have a special name based on the number

of sides they have. For example, the polygon with three sides is called a triangle because “tri” is a prefix that means “three.” Its name also indicates that this polygon has three angles. The prefix “poly” means many.

The table below shows and describes three classifications of triangles. Notice how the types of angles in the triangle are used to classify the triangle.

Name of Triangle	Picture of a Triangle	Description
Acute Triangle		A triangle with 3 acute angles (3 angles measuring between 0° and90°).
Obtuse Triangle		A triangle with 1 obtuse angle (1 angle measuring between 90° and 180°).
Right Triangle		A triangle containing one right angle (1 angle that measures 90°). Note that the right angle is indicated by a corner mark and does not require labeling as 90°.

The sum of the measures of the three interior angles of a triangle is always 180°. This fact can be applied to find the measure of the third angle of a triangle, if you are given the other two. Consider the examples below.

Example 1.12.10

A triangle has two angles that measure 35° and 75°. Find the measure of the third angle.

Answer available at the end of this section.

Your Turn 1.12.9: Find Missing Angle of Triangle

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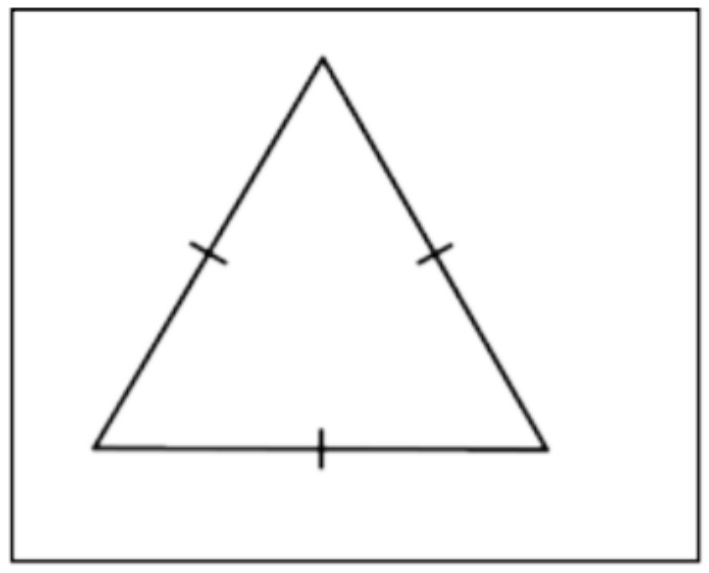
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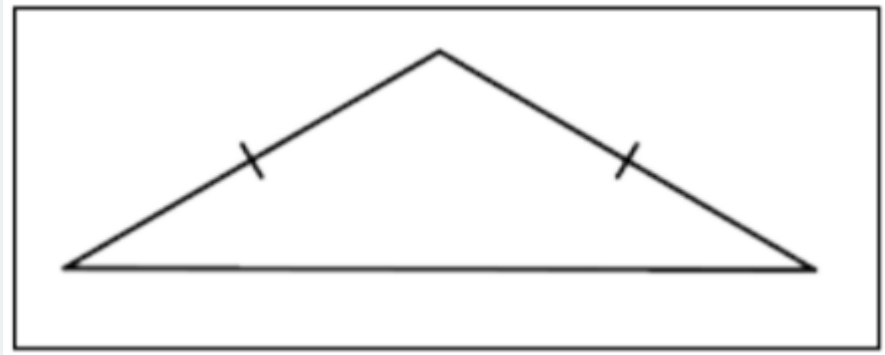
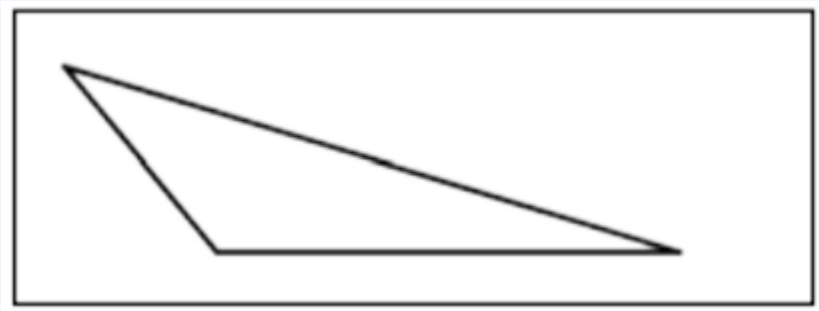
Your Turn 1.12.10: Find Missing Angles

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Mathematicians show congruency by putting a hash mark symbol through the middle of the sides of equal length. If the hash mark is the same on one or more sides, then those sides are congruent. If the sides have different hash marks, they are *not* congruent. The table below shows the classification of triangles by their side lengths.

Name of Triangle	Picture of a Triangle	Description
Equilateral Triangle		A triangle whose three sides have the same length. These sides of equal length are called congruent sides.

Isosceles Triangle		A triangle with exactly two congruent sides.
Scalene Triangle		A triangle in which all three sides are of different lengths.

Your Turn 1.12.11: Classify Triangle

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Polygons

A polygon is a closed, two-dimensional shape classified by the number of straight-line sides. See Figure 10.82 for some examples. We show only up to eight-sided polygons, but there are many, many more.

Number of Sides	Name	Shape
3	Triangle	
4	Quadrilateral	
5	Pentagon	
6	Hexagon	
7	Heptagon	
8	Octagon	

Figure 1.12.19: Types of Polygons

If all the sides of a polygon have equal lengths and all the angles are equal, they are called **regular polygons**. However, any shape with sides that are line segments can be classified as a polygon. For example, the first two shapes, shown in Figure 1.12.20, are both pentagons because they each have five sides and five vertices. The third shape Figure 1.12.20 is a hexagon because it has six sides and six vertices. We should note here that the hexagon in Figure 1.12.20 is a concave hexagon, as opposed to the first two shapes, which are convex pentagons. Technically, what makes a polygon concave is having an interior angle that measures greater than 180°. They are hollowed out, or cave in, so to speak. Convex refers to the opposite effect, where the shape is rounded out or pushed out.

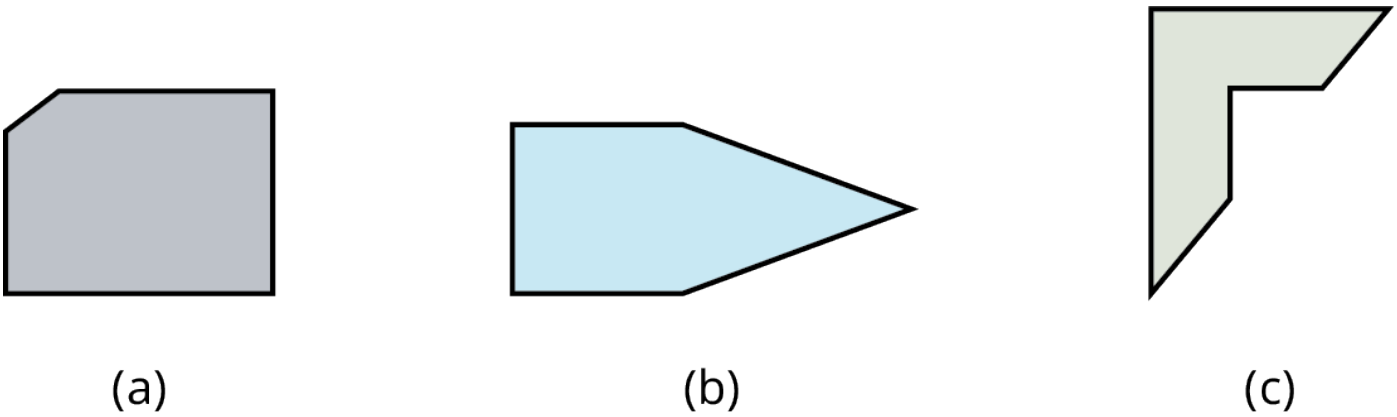


Figure 1.12.20: Not Regular Polygons

The formula is used to find the sum of all interior angles of a polygon.

Definition: Sum of Interior Angles

Sum of Interior Angles Formula for a polygon with n sides

$$\text{Sum} = (n-2) \times 180^\circ$$

If the polygon is regular (all sides and angles equal):

$$\text{Each interior angle} = \frac{\text{Sum}}{n}$$

For triangle $n = 3$

$$\text{sum} = (3-2) \times 180^\circ = 1 \times 180^\circ = 180^\circ$$

For quadrilateral $n = 4$

$$\text{sum} = (4-2) \times 180^\circ = 2 \times 180^\circ = 360^\circ$$

Your Turn 1.12.12: Find the Angel of Regular Polygon

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Quadrilaterals

Quadrilaterals are a special type of polygon. As with triangles and other polygons, quadrilaterals have special properties and can be classified by characteristics of their angles and sides. Understanding the properties of different quadrilaterals can help you in solving problems that involve this type of polygon. While there are variations of all polygons, quadrilaterals contain an additional set of figures classified by angles and whether there is one or more pairs of parallel sides. See Figure 1.12.21.

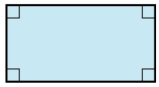
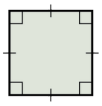
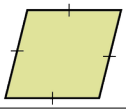
Description	Shape
A **trapezoid** has one pair of parallel sides.	
A **parallelogram** has two sets of parallel sides and no right angles.	
A **rectangle** is a parallelogram with four right angles and two sets of parallel sides.	
A **square** is a rectangle with four equal sides.	
A **rhombus** is a parallelogram with all sides equal, two sets of parallel sides.	

Figure 1.12.21: Types of Quadrilaterals

Example 1.12.13: Determining Multiple Polygons

What polygons make up Figure 1.12.22?

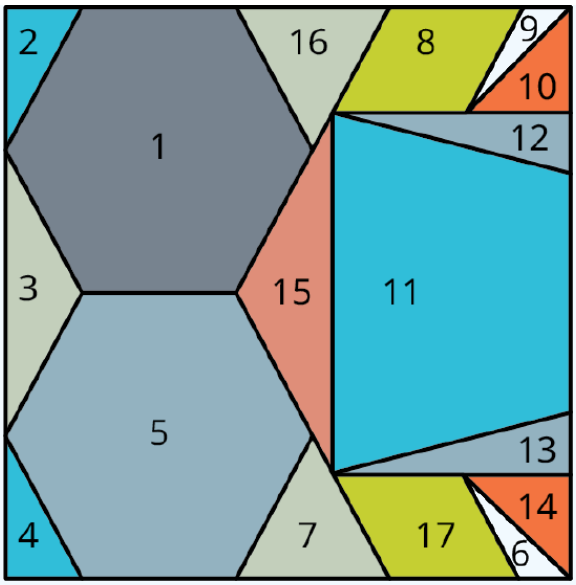


Figure 1.12.22: Naming Polygons

Answer available at the end of this section.

Interior Angles of a Quadrilateral

The sum of the interior angles of any quadrilateral is 360°. Consider the two examples below.

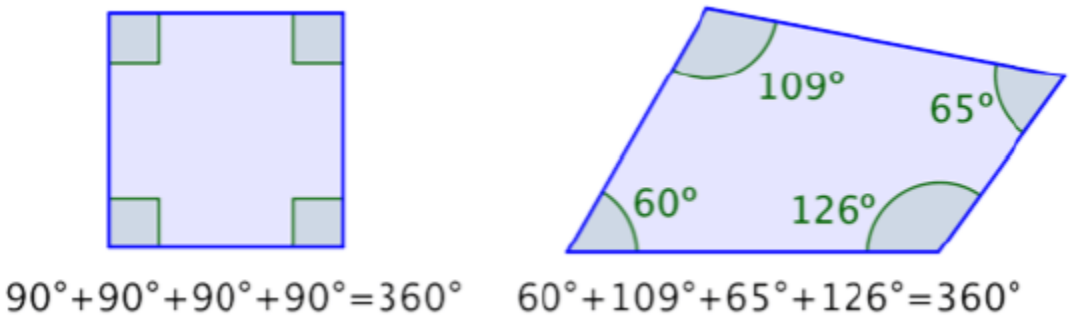


Figure 1.12.23: Sum of Interior Angles of Quadrilateral

You could draw many quadrilaterals, such as these, and carefully measure the four angles. You would find that for every quadrilateral, the sum of the interior angles will always be 360°.

Sum of the Interior Angles of a Quadrilateral

Since the sum of the interior angles of any triangle is 180° and there are two triangles in a quadrilateral, the sum of the angles for each quadrilateral is 360°.

You can use the properties of parallelograms to solve problems. Consider the example that follows.

Example 1.12.14: Find Missing Angle of a Quadrilateral

Determine the measures of three unknown angles.

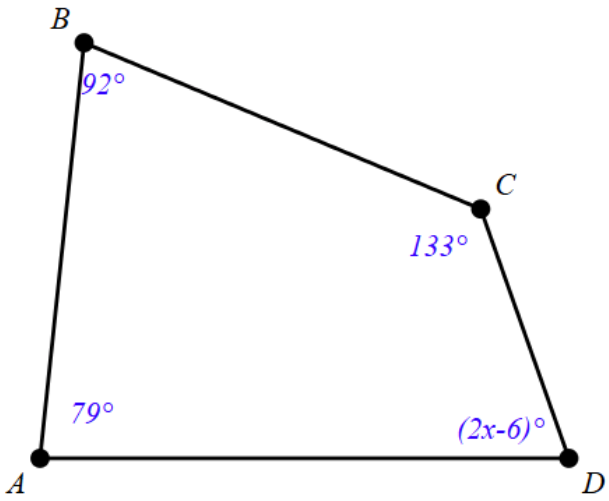


Figure 1.12.24: Missing Angle of Quadrilateral

Answer available at the end of this section.

Your Turn 1.12.15: Find Missing Angle of Quadrilateral

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The diagram below illustrates the relationship between the different types of quadrilaterals.

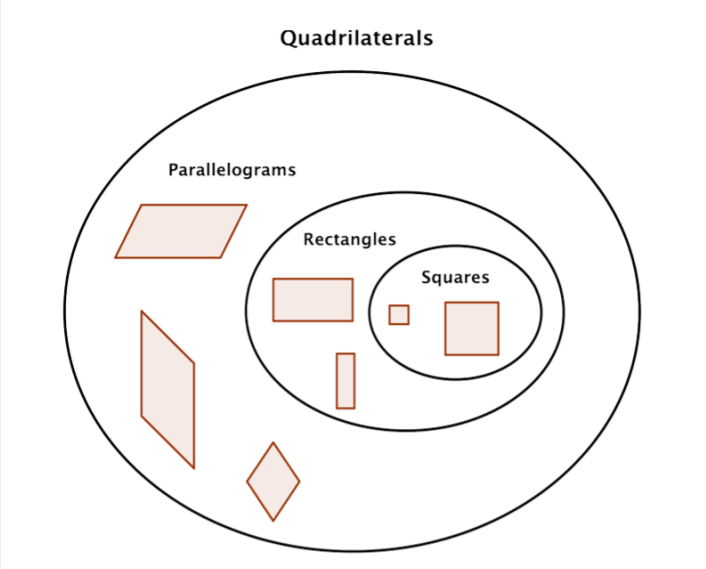


Figure 1.12.25: Relationship Between Quadrilaterals

Your Turn 1.12.16: Identify Quadrilateral

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Answers

Example 1.12.1

Two points define a line, and a line is denoted with arrows. There are two lines in this picture: $\overleftrightarrow{CE}$ and $\overleftrightarrow{BG}$.

A line segment is a section between two points. $\overline{DF}$ is a line segment. But there are also two more line segments on the lines themselves: $\overline{CE}$ and $\overline{BG}$.

Lines: $\overleftrightarrow{CE}$, $\overleftrightarrow{BG}$

Line segments: $\overline{DF}$, $\overline{CE}$, $\overline{BG}$.

Your Turn 1.12.1

There are four points: *A*, *B*, *C*, and *D*. There are also three rays, though only one may be obvious.

Ray $\overrightarrow{BC}$ begins at point *B* and goes through *C*. Two more rays exist on line $\overleftrightarrow{AD}$: they are $\overrightarrow{DA}$ and $\overrightarrow{AD}$.

Points: *A*, *B*, *C*, *D*

Rays: $\overrightarrow{BC}$, $\overrightarrow{AD}$, $\overrightarrow{DA}$

Example 1.12.2

You can start by identifying any right angles.

∠GFI is a right angle, as indicated by the corner mark at vertex F.

Acute angles will be smaller than ∠GFI (or less than 90°). This means that ∠DAB and ∠MLN are acute.

∠TQS is larger than ∠GFI, so it is an obtuse angle.

∠DAB and ∠MLN are acute angles. ∠GFI is a right angle. ∠TQS is an obtuse angle.

Example 1.12.3

Parallel lines never meet, and perpendicular lines intersect at a right angle. $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ do not intersect in this image, but if you imagine extending both lines, they will intersect soon. So, they are neither parallel nor perpendicular.

$\overleftrightarrow{AB}$ is perpendicular to both $\overleftrightarrow{WX}$ and $\overleftrightarrow{YZ}$, as indicated by the right-angle marks at the intersection of those lines.

Since $\overleftrightarrow{AB}$ is perpendicular to both lines, then $\overleftrightarrow{WX}$ and $\overleftrightarrow{YZ}$ are parallel.

$\overleftrightarrow{WX} \parallel \overleftrightarrow{YZ}$

$\overleftrightarrow{AB} \perp \overleftrightarrow{WX}$, $\overleftrightarrow{AB} \perp \overleftrightarrow{YZ}$

Example 1.12.4

Two supplementary angles make up a straight angle, so the measurements of the two angles will be 180°.

$$m\angle A + m\angle B = 180^\circ$$

You know the measurement of one angle. To find the measurement of the second angle, subtract 48° from 180°.

$$48^\circ + m\angle B = 180^\circ$$
$$m\angle B = 180^\circ - 48^\circ$$
$$m\angle B = 132^\circ$$

The measurement of the other angle is 132°.

Example 1.12.5

This image shows two intersecting lines, $\overleftrightarrow{AB}$ and $\overleftrightarrow{YZ}$. They intersect at point *X*, forming four angles. Angles ∠AXY and ∠AXZ are supplementary because together they make up the straight angle

$$m\angle AXY + m\angle AXZ = m\angle YXZ$$
$$30^\circ + m\angle AXZ = 180^\circ$$
$$m\angle AXZ = 150^\circ$$

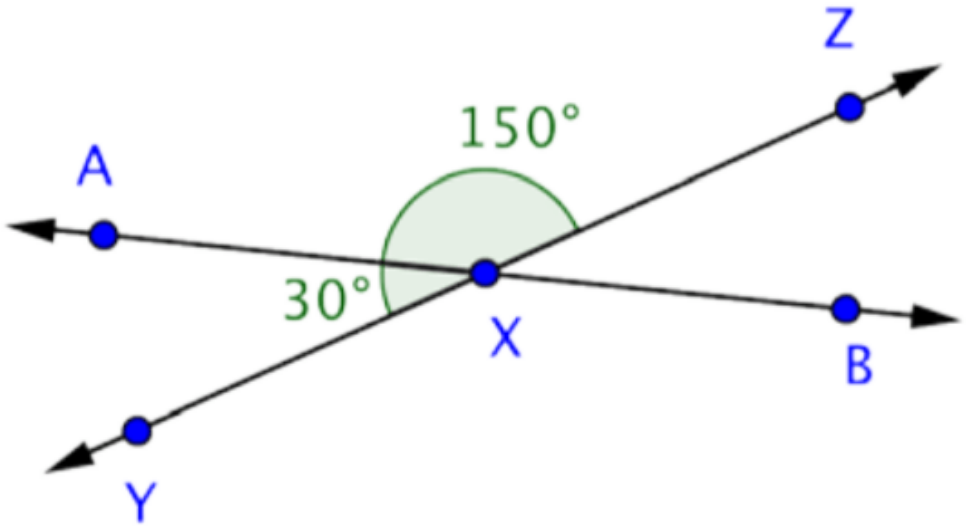


Figure 1.12.14

Example 1.12.7: Calculating Vertical Angles

The 40° angle and $\angle 2$ vertical angles. Therefore, $m\angle 2 = 40^\circ$

Notice that $\angle 2$ and $\angle 1$ are supplementary angles, meaning that the sum of $m\angle 2$ and $m\angle 2$ equals 180°.

Therefore, $m\angle 1 = 180^\circ - 40^\circ = 140^\circ$

Since $\angle 3$ and $\angle 1$ are vertical angles, then $m\angle 2 = 140^\circ$

Example 1.12.8: Evaluating Space

As the angle between Linton Street and First Avenue is150°, the supplementary angle is 30°.Therefore, the garden will from a 30° angle at the corner of your property.

Example 1.12.9: Solving for Angle Measurements and Supplementary Angles

Step 1: These are supplementary angles. We can see this because the two angles are part of a horizontal line, and a horizontal line represents 180°. Therefore, the sum of the two angles equals 180°.

Step 2:

$$\begin{aligned}(32x-7)+(5x+2) &= 180\\ 37x-5 &= 180\\ 37x &= 185\\ x &= 5\end{aligned}$$

Step 3: Find the measure of each angle:

$$\begin{aligned}32x-7 &= 32(5)-7\\ &= 153^\circ\\ 5x+2 &= 5(5)+2\\ &= 27^\circ\end{aligned}$$

Step 4: We check: $153^\circ + 27^\circ = 180^\circ$.

Example 1.12.10

The sum of the three interior angles of a triangle is 180°.

$$35^\circ + 75^\circ + x = 180^\circ$$

Find the value of x.

$$\begin{aligned}110^\circ + x &= 180^\circ\\ x &= 180^\circ - 110^\circ\\ x &= 70^\circ\end{aligned}$$

The third angle of the triangle measures 70°.

Example 1.12.13: Determining Multiple Polygons

Shapes 1 and 5 are hexagons; shapes $2, 3, 4, 6, 7, 9, 10, 12, 13, 14, 15,$ and 16 are triangles; shapes 8 and 17 are parallelograms; and shape 11 is a trapezoid.

Example 1.12.14: Find Missing Angle of a Quadrilateral

The total of the sum of the interior angles is 360°

$$\begin{aligned}92+79+133+(2x-6) &= 360\\ 292+2x &= 360\\ 2x &= 360-298\\ 2x &= 62\\ x &= \frac{62}{2}\\ x &= 31\end{aligned}$$

So the unknown angles are

$$2x-6 \quad = 2 \times 31 - 6 \quad = 56$$

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1.12.1: Practice and Videos for Acute, Right and Obtuse Angles

Learning Objectives:

- I can classify acute, right, and obtuse angles.
- I can use a protractor to measure angles.

GeoGebra: Exploring Types of Angles

Use the graphic below to investigate various types of angles by answering the task questions below.

Task Questions

1. Click the blue dot and move it until the angle measurement reads 90 degrees. What is this type of angle called?
2. Slide the blue dot to the right. What do you notice about the angle measurements as you move it to the right?
3. Slide the blue dot to the left. What do you notice about the angle measurements as you move it to the left?
4. When does the label change from "obtuse" to "acute"?
5. Explore more angles by sliding the blue dot, and then write your own definition for an obtuse angle, a right angle, and an acute angle.
6. Give an example measurement of an acute angle. Explain why it is acute, not obtuse.
7. Give an example measurement of an obtuse angle. Explain why it is obtuse, not acute.
8. What is the largest whole-number measurement an acute angle could have?
9. What is the smallest whole-number measurement an obtuse angle could have?
10. What makes a right angle unique?

Videos on Acute, Right, and Obtuse Angles

Watch the videos to learn about how to identify acute, right, and obtuse angles. All videos have appropriate captions in English; in some cases, there may also be appropriate captions in other languages.

YouTube Video

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Practice!

Check your understanding of acute, right, and obtuse angles by answering these questions. Use the arrows at the bottom of the box to go to the next question.

Practice 1.12.1.1

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Extra Practice Resources

1. IXL: Acute, right, obtuse, and straight angles
2. IXL: Measure angles with a protractor
3. IXL: Draw angles with a protractor

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1.13: Classifying Polygons

Categories of polygons based on the number of sides, and whether they are concave or convex.

Polygons

A **polygon** is any closed, 2-dimensional figure that is made entirely of line segments that intersect at their endpoints. Polygons can have any number of sides and angles, but the sides can never be curved. The segments are called the **sides** of the polygons, and the points where the segments intersect are called **vertices**.

Polygons can be either **convex** or **concave**. The term concave refers to a cave, or the polygon is "caving in". All stars are concave polygons.

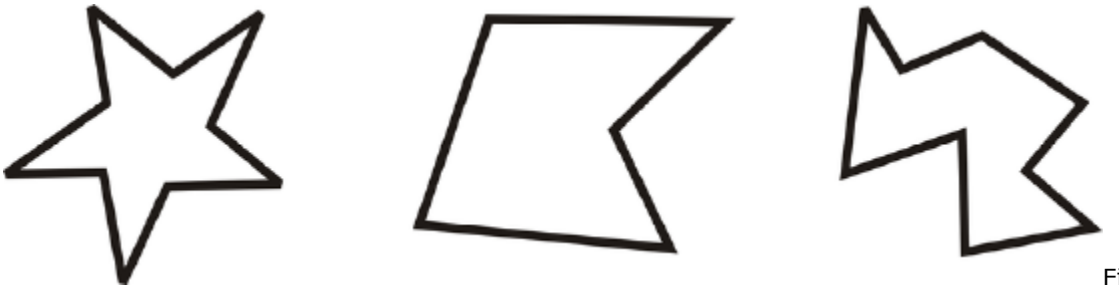


Figure 1.13.1

A convex polygon does not cave in. Convex polygons look like:

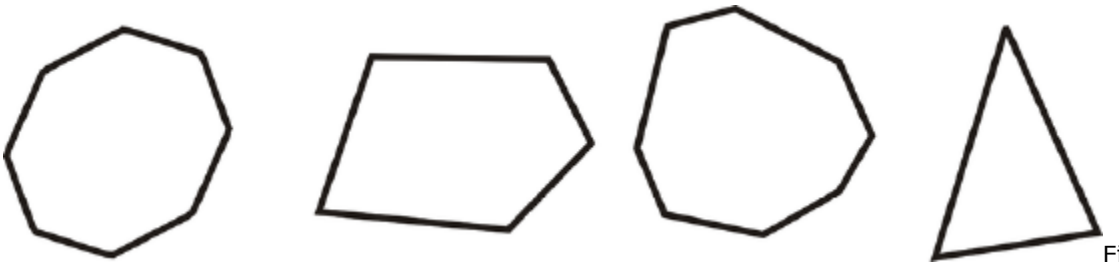


Figure 1.13.2

A **diagonal** is a non-side line segment that connects two vertices of a convex polygon.

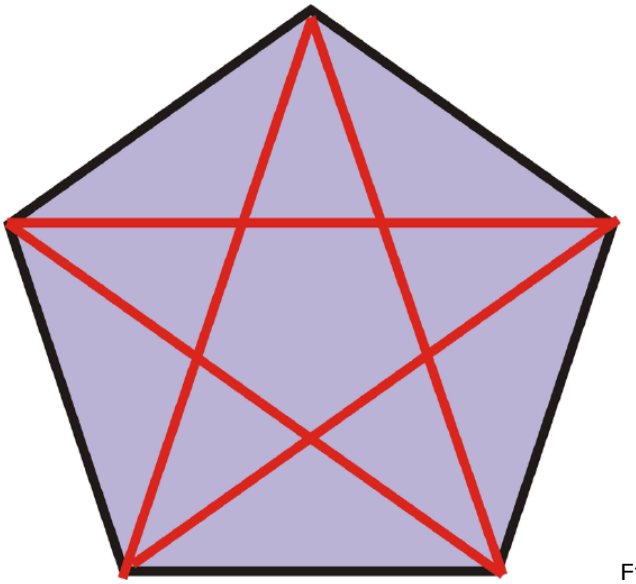
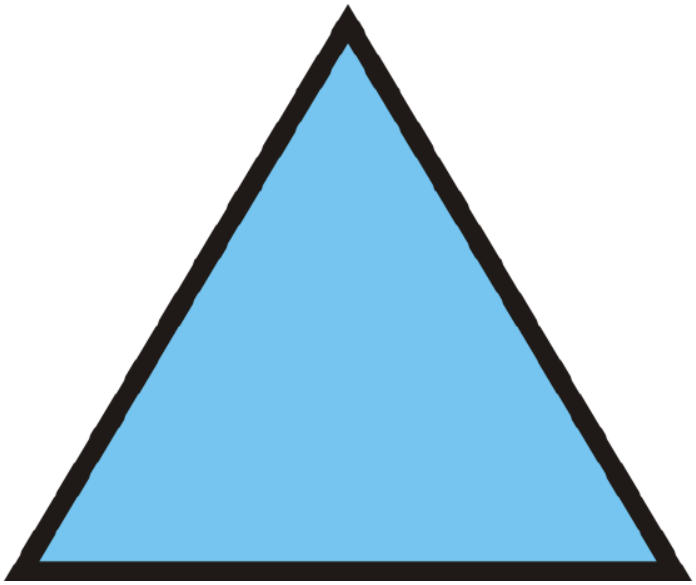
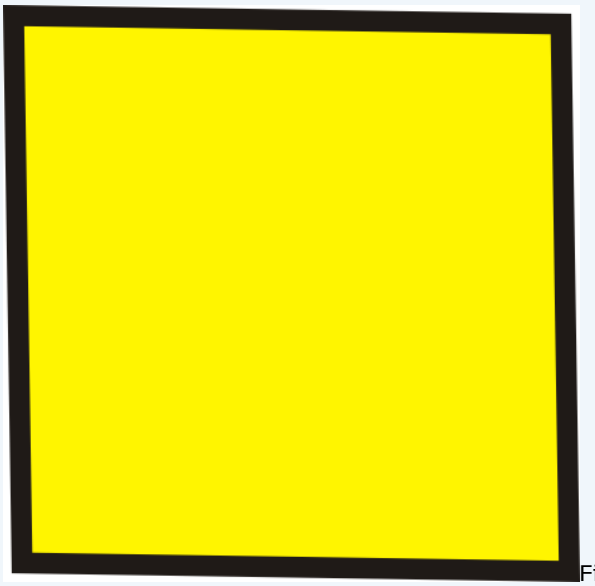
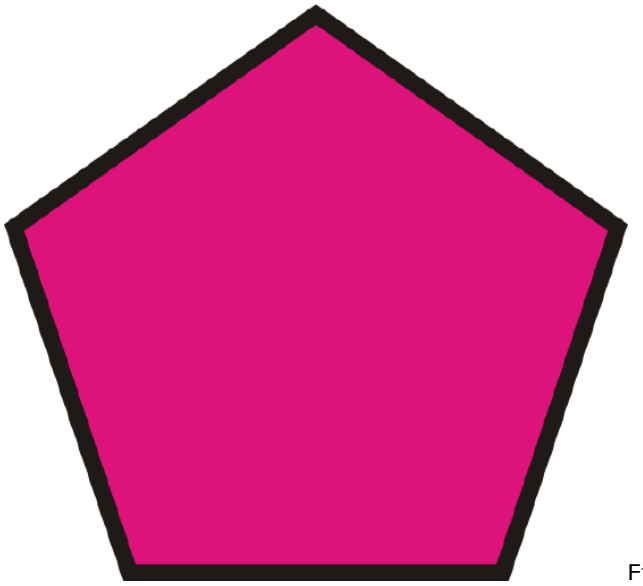


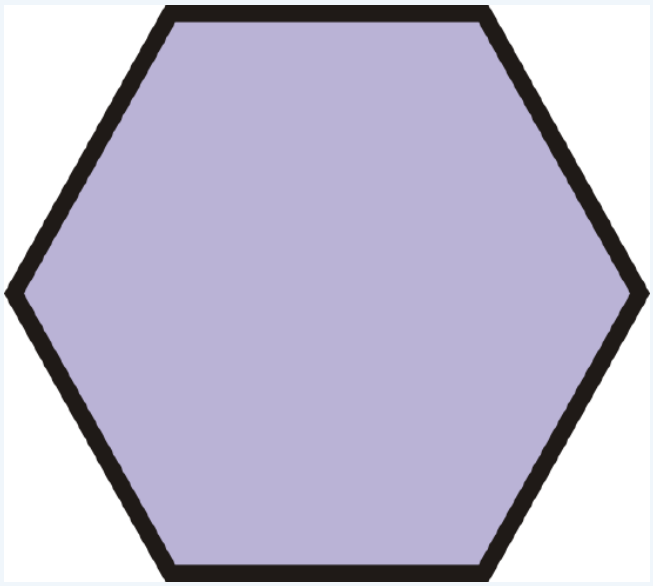
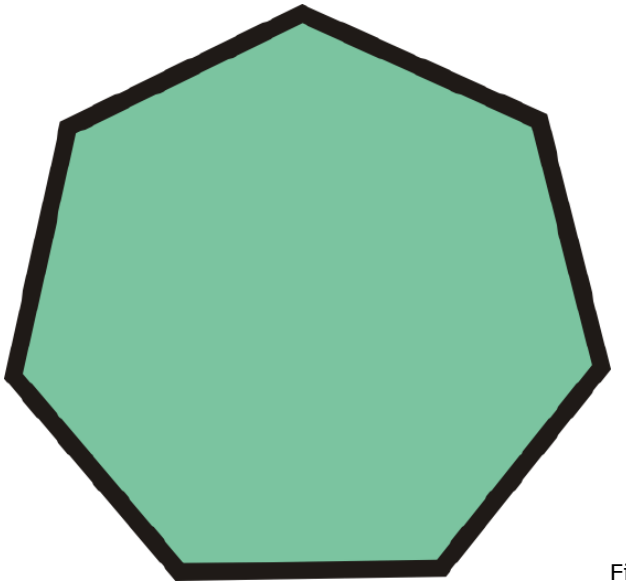
Figure 1.13.3

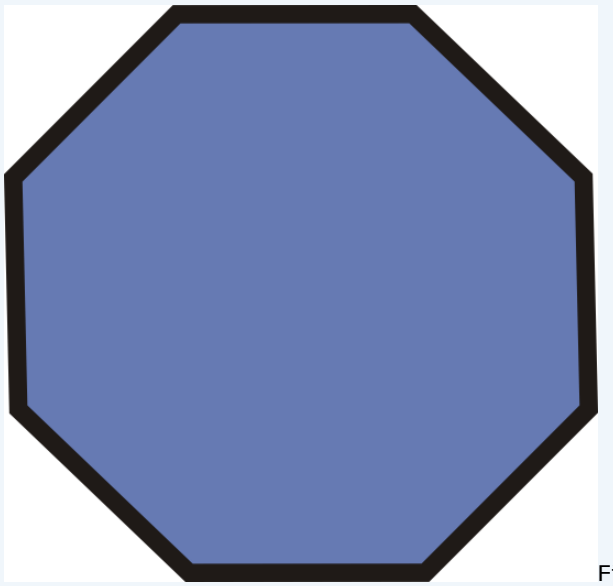
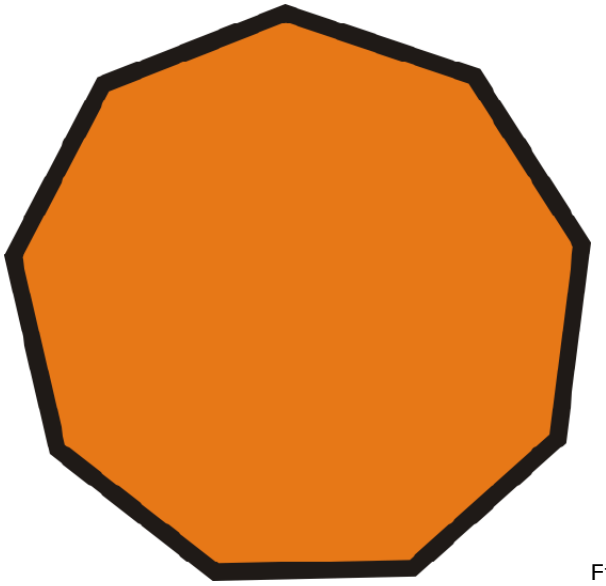
The red line segments are all diagonals. This pentagon has 5 diagonals.

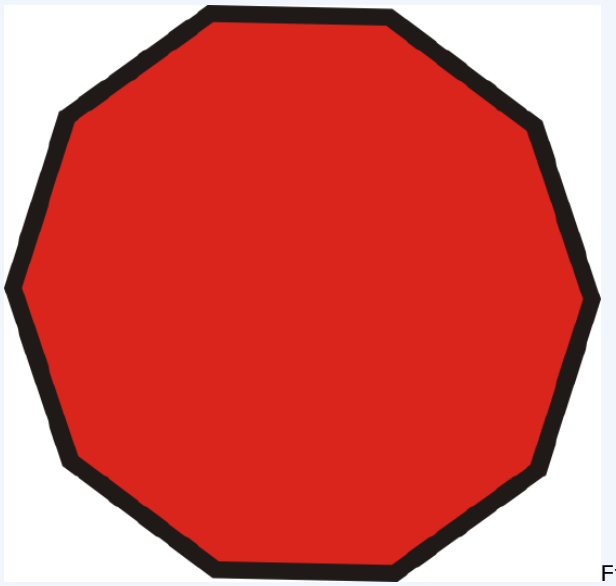
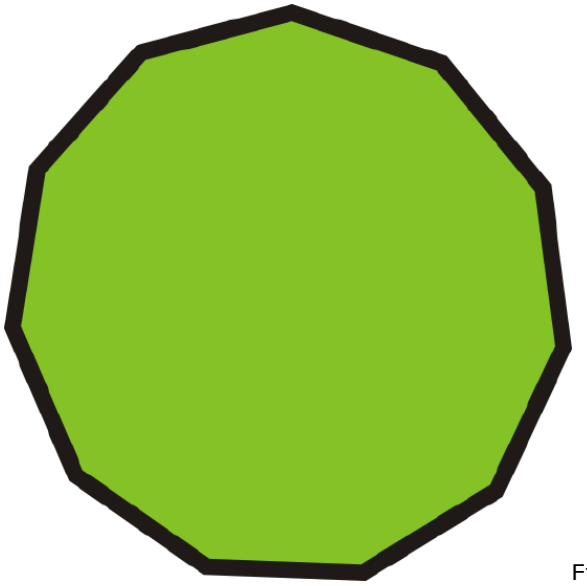
Whether a polygon is convex or concave, it is always named by the number of sides. Explore the relationship between the number of sides of a convex polygon and its diagonals. Can you complete the table?

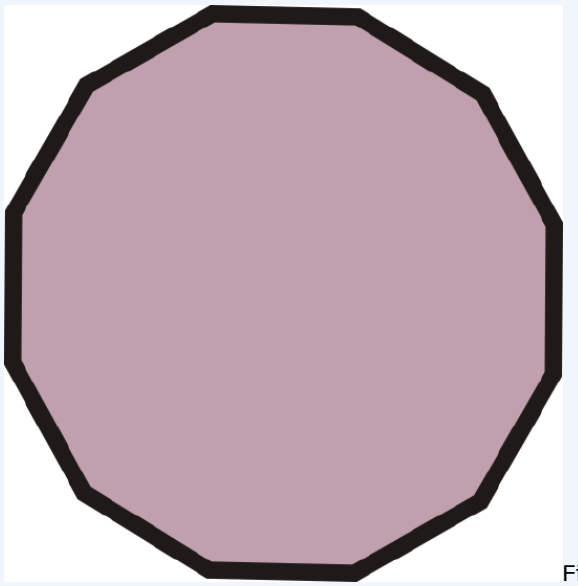
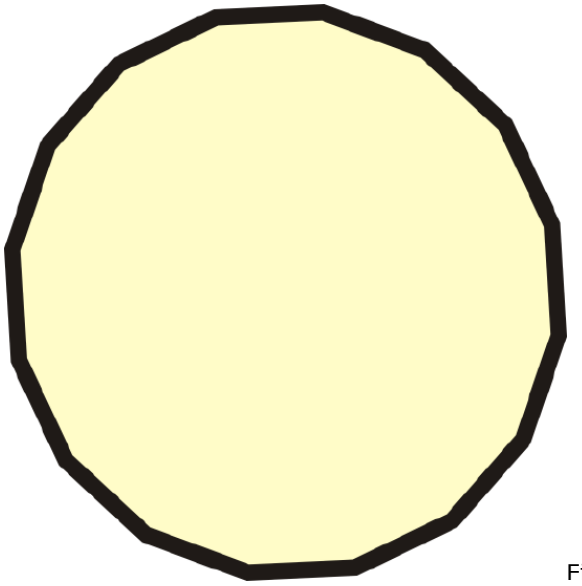
Polygon Name	Number of Sides	Number of Diagonals	Convex Example
Triangle	**3**	**0**	 Figure 1.13.4

Polygon Name	Number of Sides	Number of Diagonals	Convex Example
Quadrilateral	**4**	**2**	 Figure 1.13.5
Pentagon	**5**	**5**	 Figure 1.13.6

Polygon Name	Number of Sides	Number of Diagonals	Convex Example
Hexagon	**6**	**9**	 Figure 1.13.7
Heptagon	**7**	**?**	 Figure 1.13.8

Polygon Name	Number of Sides	Number of Diagonals	Convex Example
Octagon	**8**	**?**	 Figure 1.13.9
Nonagon	**9**	**?**	 Figure 1.13.10

Polygon Name	Number of Sides	Number of Diagonals	Convex Example
Decagon	**10**	**?**	 Figure 1.13.11
Undecagon or hendecagon	**11**	**?**	 Figure 1.13.12

Polygon Name	Number of Sides	Number of Diagonals	Convex Example
Dodecagon	**12**	**?**	 Figure 1.13.13
n-gon	**n (where n>12)**	**?**	 Figure 1.13.14

What if you were told how many sides a polygon has? How would you describe the polygon based on that information?

YouTube Video

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Example 1.13.1

Which of the figures below is a polygon?

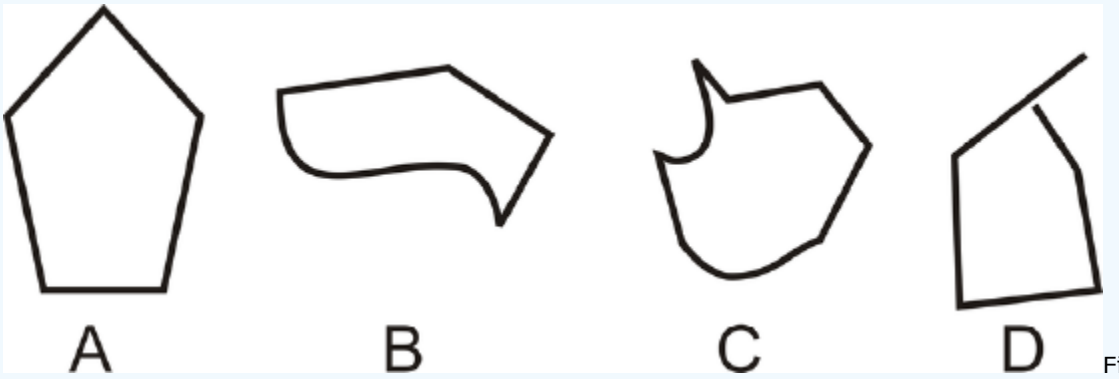


Figure 1.13.15

Solution

The easiest way to identify the polygon is to identify which shapes are **not** polygons. B and C each have at least one curved side, so they are not be polygons. D has all straight sides, but one of the vertices is not at the endpoint, so it is not a polygon. A is the only polygon.

Example 1.13.2

Determine if the shapes below are convex or concave.



Figure 1.13.16

Solution

To see if a polygon is concave, look at the polygons and see if any angle "caves in" to the interior of the polygon. The first polygon does not do this, so it is convex. The other two do, so they are concave.

Example 1.13.3

Name the three polygons below by their number of sides and if it is convex or concave.

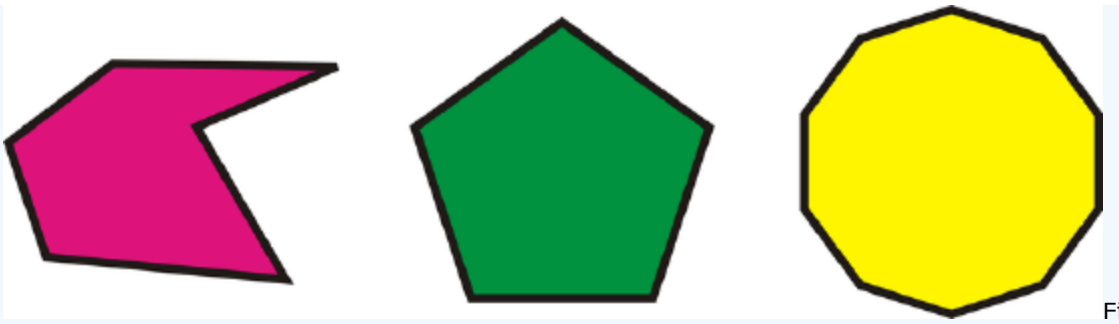


Figure 1.13.17

Solution

The pink polygon is a concave hexagon (6 sides).

The green polygon convex pentagon (5 sides).

The yellow polygon is a convex decagon (10 sides).

Example 1.13.4

Draw a 7-sided polygon, also called a heptagon. How many diagonals does a heptagon have?

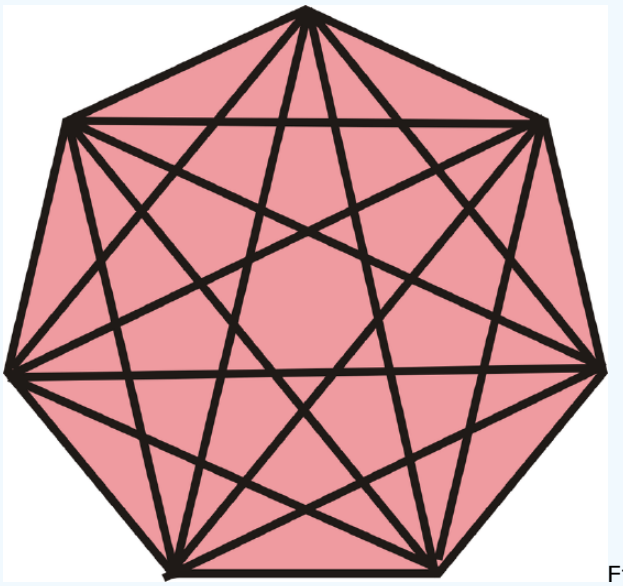


Figure 1.13.18

Solution

First, draw the heptagon. Drawing in all the diagonals and counting them, we see there are 14.

Example 1.13.5

True or false: A quadrilateral is always a square.

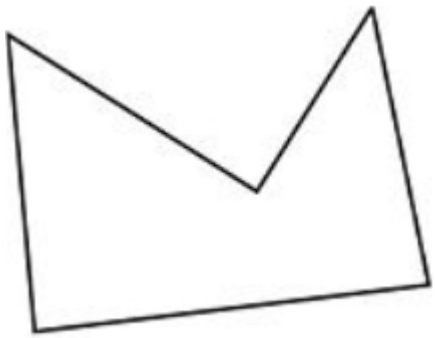
Solution

False. Only quadrilaterals with four congruent sides and four right angles will be squares. There are many quadrilaterals (such as rectangles, kites, parallelograms, trapezoids, etc.) that are not necessarily squares.

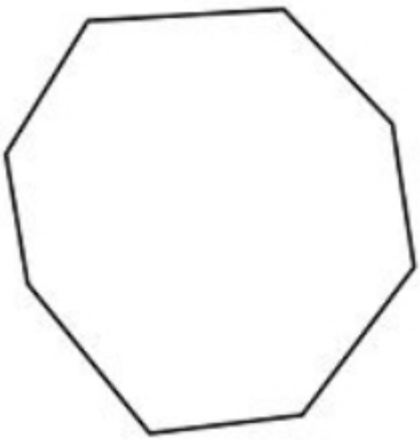
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Review

In problems 1-6, name each polygon in as much detail as possible.



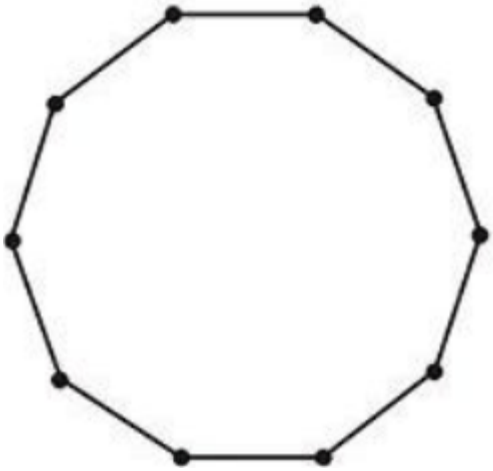
1. Figure 1.13.19

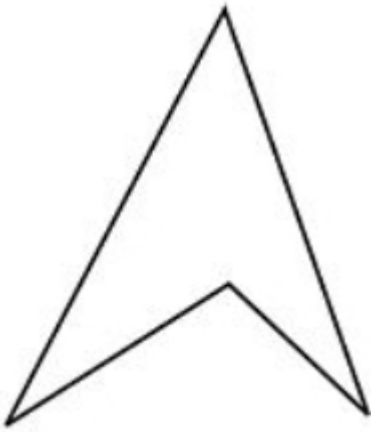


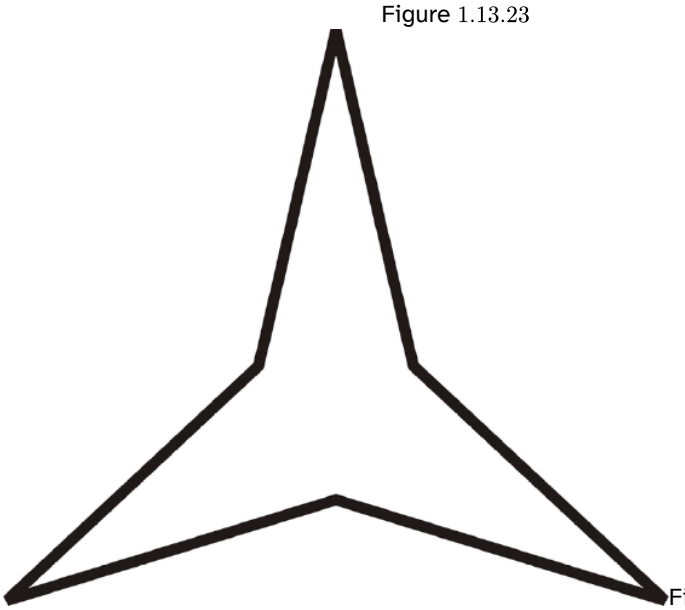
2. Figure 1.13.20



3. Figure 1.13.21

4. 
Figure 1.13.22

5. 
Figure 1.13.23

6. 
Figure 1.13.24

7. Explain why the following figures are NOT polygons:

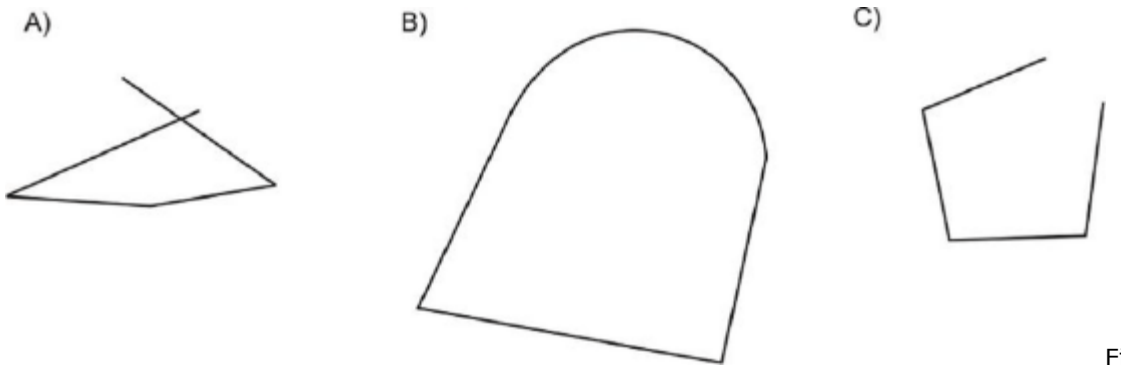


Figure 1.13.25

8. How many diagonals can you draw from **one vertex** of a pentagon? Draw a sketch of your answer.
9. How many diagonals can you draw from **one vertex** of an octagon? Draw a sketch of your answer.
10. How many diagonals can you draw from **one vertex** of a dodecagon?
11. Determine the number of total diagonals for an octagon, nonagon, decagon, undecagon, and dodecagon.

For 12-14, determine if the statement is true or false.

12. A polygon must be enclosed.
13. A star is a convex polygon.
14. A 5-point star is a decagon.

Review (Answers)

To see the Review answers, open this PDF file and look for section 1.12.

Resource

YouTube Video

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Vocabulary

Term	Definition
diagonal (of a polygon)	A non-side line segment that connect two vertices of a convex polygon.

Term	Definition
Polygon	A polygon is a simple closed figure with at least three straight sides.
Concave	A concave polygon has at least one interior angle greater than 180 degrees. A common way to identify a concave polygon is to look for a "caved-in" side of the polygon.
Convex	A convex polygon contains no interior angles greater than 180 degrees.
Diagonal	A diagonal is a line segment in a polygon that connects nonconsecutive vertices
Exterior angles	An exterior angle is the angle formed by one side of a polygon and the extension of the adjacent side.
Interior angles	Interior angles are the angles inside a figure.
Regular Polygon	A regular polygon is a polygon with all sides the same length and all angles the same measure.
Vertices	Vertices are points where line segments intersect.
Equilateral	A polygon is equilateral if all of its sides are the same length.

Additional Resource

Interactive Element

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Video: Sum of Interior Angles of a Polygon

Activities: Polygon Classification Discussion Questions

Study Aids: Polygons Study Guide

Practice: Classify Polygons

Real World: Nature's Polygons

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1.13.1: Practice and Videos for Classifying Polygons

Learning Objectives

- I can name polygons up to 10 sides.
- I can classify polygons and other 2D figures.

GeoGebra: Naming Polygons

Using the activity below, explore the connection between the name and the number of sides of a polygon. Use the + button to increase the number of sides and the - button to decrease the number of sides. Slide the purple slider at the bottom to count the sides of the polygon and see how they connect to its name.

Videos on Classifying Polygons

Watch these videos to learn how to classify polygons. All videos have appropriate captions in English; in some cases, there may also be appropriate captions in other languages.

YouTube Video

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Practice!

Check your understanding of polygons by answering these questions. Use the arrows at the bottom of the box to go to the next question.

Practice 1.13.1.1

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Extra Practice Resources

For more practice on classifying polygons, visit these websites.

1. IXL: Acute, obtuse, and right triangles
2. IXL: Scalene, isosceles, and equilateral triangles
3. IXL: Classify quadrilaterals

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1.14: Finding Area by Decomposing and Rearranging

Lesson

Let's create shapes and find their areas.

Exercise 1.14.1: What is Area?

You may recall that the term **area** tells us something about the number of squares inside a two-dimensional shape.

1. Here are four drawings that each show squares inside a shape. Select **all** drawings whose squares could be used to find the area of the shape. Be prepared to explain your reasoning.

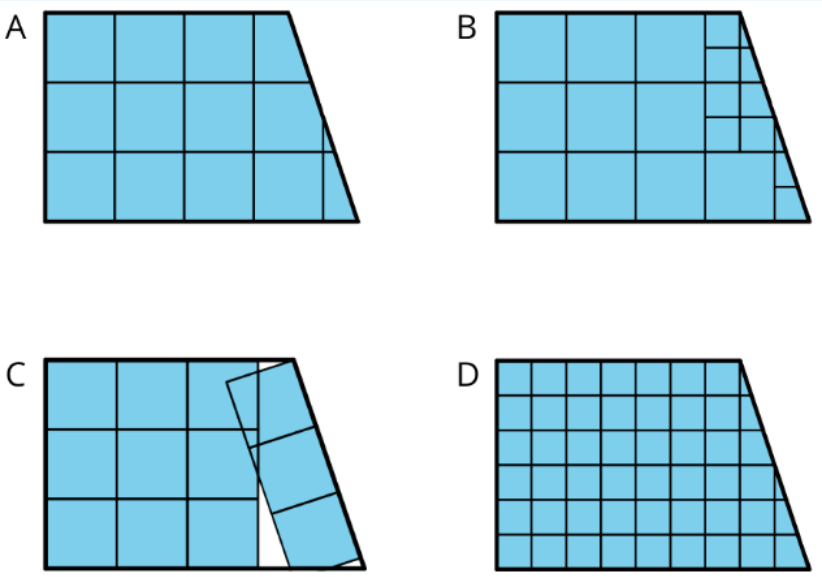


Figure 1.14.1: Four drawings that each show squares inside a shape. Shape A is broken up into large squares, Shape B is broken up into a combination of large and small squares, Shape C is broken up into a combination of large squares and white space, and Shape D is broken up into small squares.

2. Write a definition of area that includes all the information that you think is important.

Exercise 1.14.2: Composing Shapes

This applet has one square and some small, medium, and large right triangles. The area of the square is 1 square unit.

Click on a shape and drag to move it. Grab the point at the vertex and drag to turn it.

1. Notice that you can put together two small triangles to make a square. What is the area of the square composed of two small triangles? Be prepared to explain your reasoning.
2. Use your shapes to create a new shape with an area of 1 square unit that is not a square. Draw your shape on paper and label it with its area.
3. Use your shapes to create a new shape with an area of 2 square units. Draw your shape and label it with its area.
4. Use your shapes to create a *different* shape with an area of 2 square units. Draw your shape and label it with its area.
5. Use your shapes to create a new shape with an area of 4 square units. Draw your shape and label it with its area.

Are you ready for more?

Find a way to use all of your pieces to compose a single large square. What is the area of this large square?

Exercise 1.14.3: Tangram Triangles

Recall that the area of the square you saw earlier is 1 square unit. Complete each statement and explain your reasoning.

1. The area of the small triangle is _____ square units. I know this because . . .
2. The area of the medium triangle is _____ square units. I know this because . . .
3. The area of the large triangle is _____ square units. I know this because . . .

Summary

Here are two important principles for finding **area**:

1. If two figures can be placed one on top of the other so that they match up exactly, then they have the *same area*.
2. We can **decompose** a figure (break a figure into pieces) and **rearrange** the pieces (move the pieces around) to find its area.

Here are illustrations of the two principles.

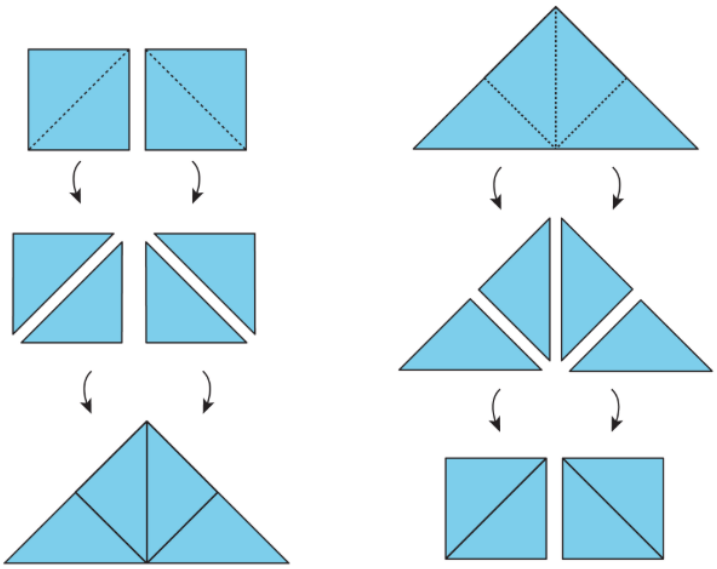


Figure 1.14.2: An image of two squares, which are then decomposed into four triangles, which are then rearranged into one large triangle. Another image of a large triangle that is decomposed into four smaller triangles, and then rearranged into two squares.

- Each square on the left can be decomposed into 2 triangles. These triangles can be rearranged into a large triangle. So the large triangle has the *same area* as the 2 squares.
- Similarly, the large triangle on the right can be decomposed into 4 equal triangles. The triangles can be rearranged to form 2 squares. If each square has an area of 1 square unit, then the area of the large triangle is 2 square units. We also can say that each small triangle has an area of $\frac{1}{2}$ square unit.

Glossary Entries

Definition: Area

Area is the number of square units that cover a two-dimensional region, without any gaps or overlaps.

For example, the area of region A is 8 square units. The area of the shaded region of B is $\frac{1}{2}$ square unit.

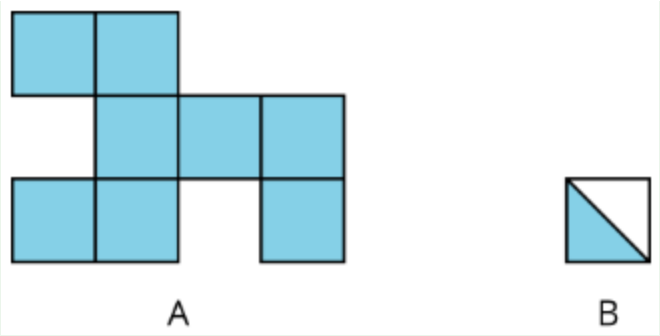


Figure 1.14.3

Definition: Compose

Compose means “put together.” We use the word *compose* to describe putting more than one figure together to make a new shape.

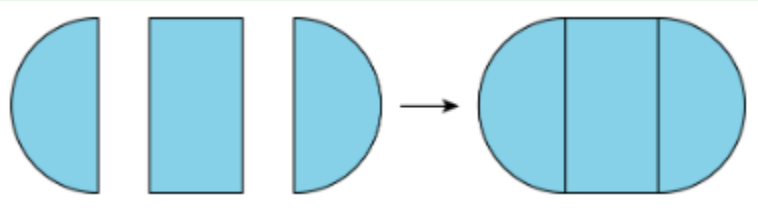


Figure 1.14.4

Definition: Decompose

Decompose means “take apart.” We use the word *decompose* to describe taking a figure apart to make more than one new shape.

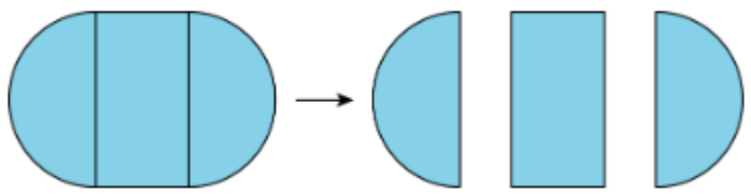


Figure 1.14.5

Definition: Region

A region is the space inside of a shape. Some examples of two-dimensional regions are inside a circle or inside a polygon. Some examples of three-dimensional regions are the inside of a cube or the inside of a sphere.

Practice

Exercise 1.14.4

The diagonal of a rectangle is shown.

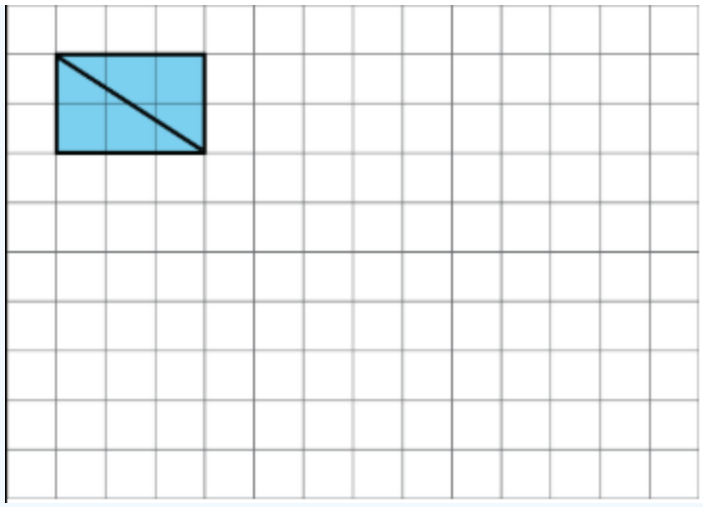


Figure 1.14.6

1. Decompose the rectangle along the diagonal, and recompose the two pieces to make a *different* shape.
2. How does the area of this new shape compare to the area of the original rectangle? Explain how you know.

Exercise 1.14.5

Priya decomposed a square into 16 smaller, equal-size squares and then cut out 4 of the small squares and attached them around the outside of original square to make a new figure.

How does the area of her new figure compare with that of the original square?

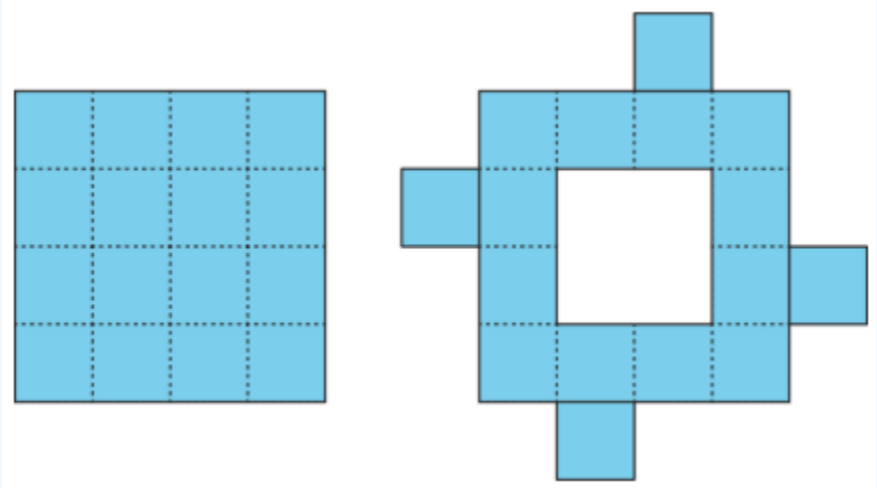


Figure 1.14.7

A. The area of the new figure is greater.
B. The two figures have the same area.
C. The area of the original square is greater.
D. We don't know because neither the side length nor the area of the original square is known.

Exercise 1.14.6

The area of the square is 1 square unit. Two small triangles can be put together to make a square or to make a medium triangle.

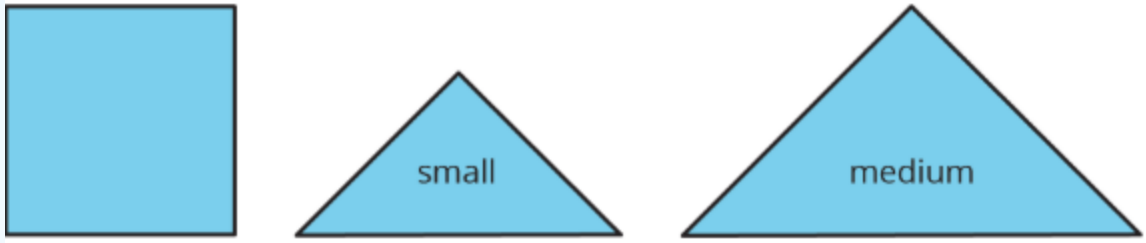


Figure 1.14.8

Which figure also has an area of $1\frac{1}{2}$ square units? Select **all** that apply.

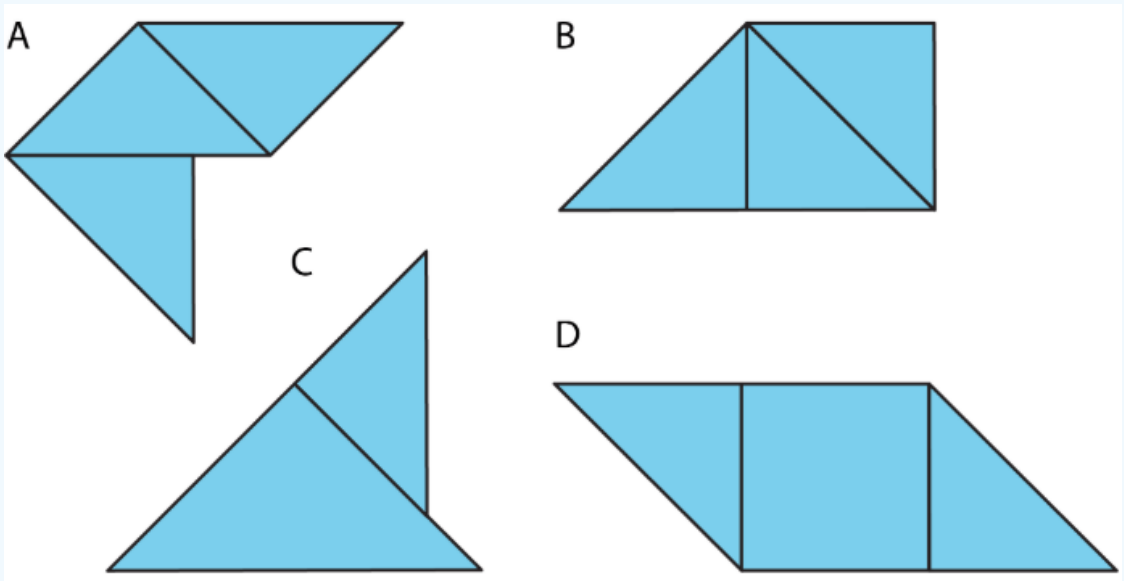


Figure A is composed of three small triangles, figure B is composed of three small triangles in a different arrangement, figure C is composed of one medium triangle and one small triangle, and figure D is composed of two small triangles and one square.

A. Figure A
B. Figure B
C. Figure C
D. Figure D

Exercise 1.14.7

The area of a rectangular playground is 78 square meters. If the length of the playground is 13 meters, what is its width?

(From Unit 1.1.1)

Exercise 1.14.8

A student said, “We can’t find the area of the shaded region because the shape has many different measurements, instead of just a length and a width that we could multiply.”

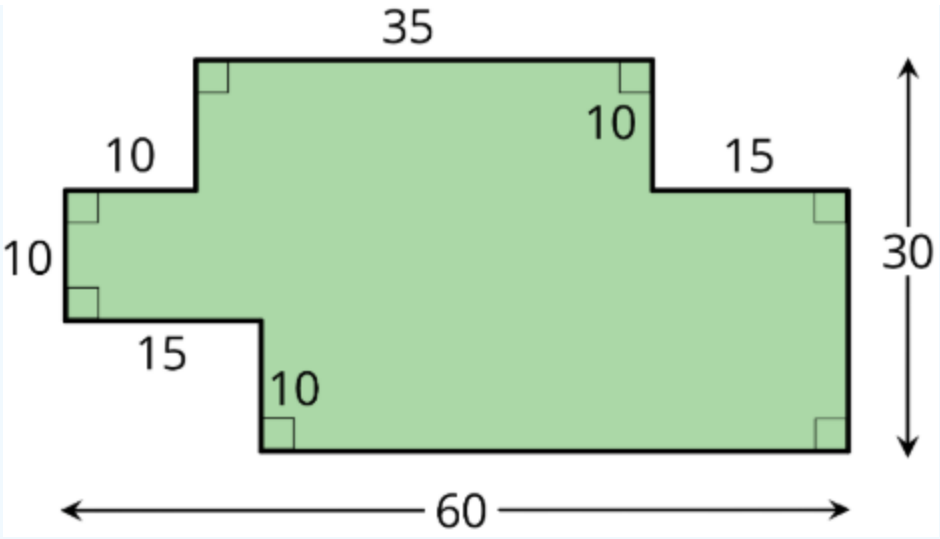


Figure 1.14.10: A multi-sided figure. The sides on top measure 10 units, 35 units, and 15 units. Two of the three sides on the left measure 10 units. One of the two sides on the right measures 10 units. One of the two sides on the bottom measure 15 units. The total width of the figure is 60 units, and the total height is 30 units. All angles are right angles.

Explain why the student's statement about area is incorrect.

(From Unit 1.1.1)

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1.14.1: Practice and Videos for Area and Perimeter

Learning Objectives

- I can understand how to calculate area and perimeter.
- I can apply the area and perimeter formulas for rectangles in real-world and mathematical problems.

GeoGebra: Area and Perimeter

Explore the graphic below to investigate area and perimeter. Click and drag on the corner dots to expand or compress the figure. Make observations about the area and perimeter. What do you notice?

Videos on Area and Perimeter

Watch the following videos to learn about area and perimeter. All videos have appropriate captions in English; in some cases, there may also be appropriate captions in other languages.

YouTube Video

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

YouTube Video

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

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YouTube Video

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View this YouTube Video online

Practice!

Check your understanding of area and perimeter by answering these questions. Use the arrows at the bottom of the box to go to

the next question.

Practice 1.14.1.1

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Extra Practice Resources

1. IXL: Find the perimeter of rectangles using formulas

2. IXL: Area and perimeter: word problems

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1.15: Multiplicative Comparison



Under Construction

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1.15.1: Practice and Videos for Multiplicative Comparison

Learning Objectives

- I can notice repeated groups and use multiplication language such as "times as long".
- I can recognize multiplicative relationships involving fractions.

GeoGebra: Fractions as Multiplicative Comparison

Complete the activity below to learn how to compare two quantities using multiplication and division operations. Use the gold slider to adjust the length of the gold bar. Use the green slider to adjust the length of the green bar. Complete the reflection questions below to guide your exploration.

Reflection Questions

- Set the gold bar to length 1. Move the green bar until it is exactly twice as long as the gold bar.
  - What do you notice about the relationship between the two bars?
  - Complete the sentence: "The green bar is ___ times as long as the gold bar."
- Keep the gold bar at length 1. Try making the green bar 3 times as long, then 4 times as long.
  - How does the green bar change each time?
  - What stays the same?
  - What does "times as long" mean in this model?
- Move the bars until the green bar is shorter than the gold bar.
  - Can you make the green bar half as long as the gold bar?

Videos on Multiplicative Comparison

Watch the following videos to learn about multiplicative comparison. All videos have appropriate captions in English; in some cases, there may also be appropriate captions in other languages.

YouTube Video

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View this YouTube Video online

YouTube Video

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

YouTube Video

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

Practice!

Check your understanding of multiplicative comparison by answering these questions. Use the arrows at the bottom of the box to go to the next question.

Practice 1.15.1.1

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this Interactive Content online

Extra Practice Resources

For more practice on multiplicative comparison, visit the following websites.

1. IXL: Compare numbers using multiplication

2. Khan Academy: Compare with multiplication

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1.16: Finding Multiples and Factors

Learning Objectives

By the end of this section, you will be able to:

- Identify multiples of numbers
- Use common divisibility tests
- Find all the factors of a number
- Identify prime and composite numbers

Be Prepared 2.10

Before you get started, take this readiness quiz.

Which of the following numbers are counting numbers (natural numbers)?
$0, 4, 215$
If you missed this problem, review Example 1.1.

Be Prepared 2.11

Find the sum of $3, 5,$ and 7.
If you missed the problem, review Example 2.1.

Identify Multiples of Numbers

Annie is counting the shoes in her closet. The shoes are matched in pairs, so she doesn't have to count each one. She counts by twos: $2, 4, 6, 8, 10, 12$. She has 12 shoes in her closet.

The numbers $2, 4, 6, 8, 10, 12$ are called multiples of 2. Multiples of 2 can be written as the product of a counting number and 2. The first six multiples of 2 are given below.

$$\begin{aligned} 1 \cdot 2 &= 2 \\ 2 \cdot 2 &= 4 \\ 3 \cdot 2 &= 6 \\ 4 \cdot 2 &= 8 \\ 5 \cdot 2 &= 10 \\ 6 \cdot 2 &= 12 \end{aligned}$$

A multiple of a number is the product of the number and a counting number. So a multiple of 3 would be the product of a counting number and 3. Below are the first six multiples of 3.

$$\begin{aligned} 1 \cdot 3 &= 3 \\ 2 \cdot 3 &= 6 \\ 3 \cdot 3 &= 9 \\ 4 \cdot 3 &= 12 \\ 5 \cdot 3 &= 15 \\ 6 \cdot 3 &= 18 \end{aligned}$$

We can find the multiples of any number by continuing this process. Table 2.8 shows the multiples of 2 through 9 for the first

twelve counting numbers.

Counting Number	1	2	3	4	5	6	7	8	9	10	11	12
Multiples	2	4	6	8	10	12	14	16	18	20	22	24
Multiples of 3	3	6	9	12	15	18	21	24	27	30	33	36
Multiples of 4	4	8	12	16	20	24	28	32	36	40	44	48
Multiples of 5	5	10	15	20	25	30	35	40	45	50	55	60
Multiples of 6	6	12	18	24	30	36	42	48	54	60	66	72
Multiples of 7	7	14	21	28	35	42	49	56	63	70	77	84
Multiples of 8	8	16	24	32	40	48	56	64	72	80	88	96
Multiples of 9	9	18	27	36	45	54	63	72	81	90	99	108

Table 2.8

Multiple of a Number

A number is a multiple of n if it is the product of a counting number and n.

Recognizing the patterns for multiples of $2, 5, 10, \text{and } 3$ will be helpful to you as you continue in this course.

Manipulative Mathematics

Doing the Manipulative Mathematics activity "Multiples" will help you develop a better understanding of multiples.

Figure 2.6 shows the counting numbers from 1 to 50. Multiples of 2 are highlighted. Do you notice a pattern?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

Figure 2.6 Multiples of 2 between 1 and 50

The last digit of each highlighted number in Figure 2.6 is either $0, 2, 4, 6, \text{or } 8$. This is true for the product of 2 and any counting number. So, to tell if any number is a multiple of 2 look at the last digit. If it is $0, 2, 4, 6, \text{ or } 8$, then the number is a multiple of 2.

Example 2.40

Determine whether each of the following is a multiple of 2:

1. ⓐ 489
2. ⓑ $3{,}714$

Answer available at the end of this section.

Try It 2.79

Determine whether each number is a multiple of 2:

1. ⓐ 678
2. ⓑ $21{,}493$

Try It 2.80

Determine whether each number is a multiple of 2:

1. ⓐ 979
2. ⓑ $17{,}780$

Now let's look at multiples of Math input error

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

Figure 2.7 Multiples of 5 between 1 and 50

All multiples of 5 end with either 5 or 0. Just like we identify multiples of 2 by looking at the last digit, we can identify multiples of 5 by looking at the last digit.

Example 2.41

Determine whether each of the following is a multiple of 5:

1. ⓐ 579
2. ⓑ 880

Answer available at the end of this section.

Try It 2.81

Determine whether each number is a multiple of 5.

1. ⓐ 675
2. ⓑ $1{,}578$

Try It 2.82

Determine whether each number is a multiple of 5.

1. ⓐ 421
2. ⓑ $2{,}690$

Figure 2.8 highlights the multiples of 10 between 1 and 50. All multiples of 10 all end with a zero.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

Figure 2.8 Multiples of 10 between 1 and 50

Example 2.42

Determine whether each of the following is a multiple of 10:

1. ⓐ 425
2. ⓑ 350

Answer available at the end of this section.

Try It 2.83

Determine whether each number is a multiple of 10:

1. ⓐ 179
2. ⓑ $3{,}540$

Try It 2.84

Determine whether each number is a multiple of 10:

1. ⓐ 110
2. ⓑ $7{,}595$

Figure 2.9 highlights multiples of 3. The pattern for multiples of 3 is not as obvious as the patterns for multiples of $2, 5, \text{and } 10$.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

Figure 2.9 Multiples of 3 between 1 and 50

Unlike the other patterns we've examined so far, this pattern does not involve the last digit. The pattern for multiples of Math input error

Multiple of 3	3	6	9	12	15	18	21	24
Sum of digits	3	6	9	$1+2$ 3	$1+5$ 6	$1+8$ 9	$2+1$ 3	$2+4$ 6

Table 2.9

Consider the number 42. The digits are 4 and 2, and their sum is $4+2=6$. Since 6 is a multiple of 3, we know that 42 is also a multiple of 3.

Example 2.43

Determine whether each of the given numbers is a multiple of 3:

1. ⓐ 645
2. ⓑ $10{,}519$

Answer available at the end of this section.

When we divide $10{,}519$ by 3, we do not get a counting number, so $10{,}519$ is not the product of a counting number and 3. It is not a multiple of 3.

Try It 2.85

Determine whether each number is a multiple of 3:

1. ⓐ 954
2. ⓑ $3{,}742$

Try It 2.86

Determine whether each number is a multiple of 3:

1. ⓐ 643
2. ⓑ $8{,}379$

Look back at the charts where you highlighted the multiples of 2, of 5, and of 10. Notice that the multiples of 10 are the numbers that are multiples of both 2 and 5. That is because $10=2\cdot 5$. Likewise, since $6=2\cdot 3$, the multiples of 6 are the numbers that are multiples of both 2 and 3.

Use Common Divisibility Tests

Another way to say that Math input error

Divisibility

If a number m is a multiple of n, then we say that m is divisible by n.

Since multiplication and division are inverse operations, the patterns of multiples that we found can be used as divisibility tests. Table 2.10 summarizes divisibility tests for some of the counting numbers between one and ten.

Divisibility Tests	
A number is divisible by	
2	if the last digit is $0,\ 2,\ 4,\ 6,$ or 8
3	if the sum of the digits is divisible by 3

Divisibility Tests A number is divisible by	
5	if the last digit is 5 or 0
6	if divisible by both 2 and 3
10	if the last digit is 0

Table 2.10

Example 2.44

Determine whether $1{,}290$ is divisible by $2, 3, 5, \text{and } 10$.

Answer available at the end of this section.

Thus, $1{,}290$ is divisible by $2, 3, 5, \text{and } 10$.

Try It 2.87

Determine whether the given number is divisible by $2, 3, 5, \text{and } 10$.

6240

Try It 2.88

Determine whether the given number is divisible by $2, 3, 5, \text{and } 10$.

7248

Example 2.45

Determine whether $5{,}625$ is divisible by $2, 3, 5, \text{and } 10$.

Answer available at the end of this section.

Thus, $5{,}625$ is divisible by 3 and 5, but not 2, or 10.

Try It 2.89

Determine whether the given number is divisible $\text{by } 2, 3, 5, \text{and } 10$.

4962

Try It 2.90

Determine whether the given number is divisible $\text{by } 2, 3, 5, \text{and } 10$.

3765

Find All the Factors of a Number

There are often several ways to talk about the same idea. So far, we've seen that if m is a multiple of n, we can say that m is divisible by n. We know that 72 is the product of 8 and 9, so we can say 72 is a multiple of 8 and 72 is a multiple of 9. We can also say 72 is divisible by 8 and by 9. Another way to talk about this is to say that 8 and 9 are factors of 72. When we write $72 = 8 \cdot 9$ we can say that we have factored 72.



Factors

In the expression $a \cdot b$, both *a* and *b* are called **factors**. If $a \cdot b = m$, and both *a* and *b* are integers, then a and b are factors of m, and m is the product of a and b.

In algebra, it can be useful to determine all of the factors of a number. This is called factoring a number, and it can help us solve many kinds of problems.

Manipulative Mathematics

Doing the Manipulative Mathematics activity "Model Multiplication and Factoring" will help you develop a better understanding of multiplication and factoring.

For example, suppose a choreographer is planning a dance for a ballet recital. There are 24 dancers, and for a certain scene, the choreographer wants to arrange the dancers in groups of equal sizes on stage.

In how many ways can the dancers be put into groups of equal size? Answering this question is the same as identifying the factors of 24. Table 2.13 summarizes the different ways that the choreographer can arrange the dancers.

Number of Groups	Dancers per Group	Total Dancers
1	24	$1 \cdot 24 = 24$
2	12	$2 \cdot 12 = 24$
3	8	$3 \cdot 8 = 24$
4	6	$4 \cdot 6 = 24$
6	4	$6 \cdot 4 = 24$
8	3	$8 \cdot 3 = 24$
12	2	$12 \cdot 2 = 24$
24	1	$24 \cdot 1 = 24$

Table 2.13

What patterns do you see in Table 2.13? Did you notice that the number of groups times the number of dancers per group is always 24? This makes sense, since there are always 24 dancers.

You may notice another pattern if you look carefully at the first two columns. These two columns contain the exact same set of numbers—but in reverse order. They are mirrors of one another, and in fact, both columns list all of the factors of 24, which are:

$$1, 2, 3, 4, 6, 8, 12, 24$$

We can find all the factors of any counting number by systematically dividing the number by each counting number, starting with 1. If the quotient is also a counting number, then the divisor and the quotient are factors of the number. We can stop when the quotient becomes smaller than the divisor.

How To

Find all the factors of a counting number.

1. Step 1. Divide the number by each of the counting numbers, in order, until the quotient is smaller than the divisor.
   - If the quotient is a counting number, the divisor and quotient are a pair of factors.
   - If the quotient is not a counting number, the divisor is not a factor.
2. Step 2. List all the factor pairs.
3. Step 3. Write all the factors in order from smallest to largest.

Example 2.46

Find all the factors of 72.

Answer available at the end of this section.

Try It 2.91

Find all the factors of the given number:

96

Try It 2.92

Find all the factors of the given number:

80

Identify Prime and Composite Numbers

Some numbers, like 72, have many factors. Other numbers, such as 7, have only two factors: 1 and the number. A number with only two factors is called a prime number. A number with more than two factors is called a composite number. The number 1 is neither prime nor composite. It has only one factor, itself.

Prime Numbers and Composite Numbers

A prime number is a counting number greater than 1 whose only factors are 1 and itself.

A composite number is a counting number that is not prime.

Figure 2.10 lists the counting numbers from 2 through 20 along with their factors. The highlighted numbers are prime, since each has only two factors.

Number	Factors	Prime or Composite?		Number	Factor	Prime or Composite?
2	1,2	Prime		12	1,2,3,4,6,12	Composite
3	1,3	Prime		13	1,13	Prime
4	1,2,4	Composite		14	1,2,7,14	Composite
5	1,5	Prime		15	1,3,5,15	Composite
6	1,2,3,6	Composite		16	1,2,4,8,16	Composite
7	1,7	Prime		17	1,17	Prime
8	1,2,4,8	Composite		18	1,2,3,6,9,18	Composite
9	1,3,9	Composite		19	1,19	Prime
10	1,2,5,10	Composite		20	1,2,4,5,10,20	Composite
11	1,11	Prime				

Figure 2.10 Factors of the counting numbers from 2 through 20, with prime numbers highlighted

The prime numbers less than 20 are $2, 3, 5, 7, 11, 13, 17, \text{and } 19$. There are many larger prime numbers too. In order to determine whether a number is prime or composite, we need to see if the number has any factors other than 1 and itself. To do this, we can test each of the smaller prime numbers in order to see if it is a factor of the number. If none of the prime numbers are factors, then that number is also prime.

How To

Determine if a number is prime.

1. Step 1. Test each of the primes, in order, to see if it is a factor of the number.
2. Step 2. Start with 2 and stop when the quotient is smaller than the divisor or when a prime factor is found.
3. Step 3. If the number has a prime factor, then it is a composite number. If it has no prime factors, then the number is prime.

Example 2.47

Identify each number as prime or composite:

1. ⓐ 83
2. ⓑ 77

Answer available at the end of this section.

Since 77 is divisible by 7, we know it is not a prime number. It is composite.

Try It 2.93

Identify the number as prime or composite:

91

Try It 2.94

Identify the number as prime or composite:

137

Links To Literacy

Media

ACCESS ADDITIONAL ONLINE RESOURCES

- Divisibility Rules
- Factors
- Ex 1: Determine Factors of a Number
- Ex 2: Determine Factors of a Number
- Ex 3: Determine Factors of a Number

Section 2.4 Exercises

Practice Makes Perfect

Identify Multiples of Numbers

In the following exercises, list all the multiples less than 50 for the given number.

215.

2

216.

3

217.

4

218.

5

219.

6

220.

7

221.

8

222.

9

223.

10

224.

12

Use Common Divisibility Tests

In the following exercises, use the divisibility tests to determine whether each number is divisible by $2, 3, 4, 5, 6, \text{and } 10$.

225.

84

226.

96

227.

75

228.

78

229.

168

230.

264

231.

900

232.

800

233.

896

234.

942

235.

375

236.

750

237.

350

238.

550

239.

1430

240.

1080

241.

22,335

242.

39,075

Find All the Factors of a Number

In the following exercises, find all the factors of the given number.

243.

36

244.

42

245.

60

246.

48

247.

144

248.

200

249.

588

250.

576

Identify Prime and Composite Numbers

In the following exercises, determine if the given number is prime or composite.

251.

43

252.

67

253.

39

254.

53

255.

71

256.

119

257.

481

258.

221

259.

209

260.

359

261.

667

262.

1771

Everyday Math

263.

Banking Frank's grandmother gave him $\$100$ at his high school graduation. Instead of spending it, Frank opened a bank account. Every week, he added $\$15$ to the account. The table shows how much money Frank had put in the account by the end of each week. Complete the table by filling in the blanks.

Weeks after graduation	Total number of dollars Frank put in the account	Simplified Total
0	100	100
1	$100+15$	115
2	$100+15\cdot 2$	130
3	$100+15\cdot 3$	
4	$100+15\cdot [\,]$	
5	$100+[\,]$	
6		
20		
x		

264.

Banking In March, Gina opened a Christmas club savings account at her bank. She deposited $\$75$ to open the account. Every week, she added $\$20$ to the account. The table shows how much money Gina had put in the account by the end of each week. Complete the table by filling in the blanks.

Weeks after opening the account	Total number of dollars Gina put in the account	Simplified Total
0	75	75
1	$75+20$	95
2	$75+20\cdot 2$	115
3	$75+20\cdot 3$	
4	$75+20\cdot [\,]$	
5	$75+[\,]$	
6		
20		
x		

Writing Exercises

265.

If a number is divisible by 2 and by 3, why is it also divisible by 6?

266.

What is the difference between prime numbers and composite numbers?

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
identify multiples of numbers.			
use common divisibility tests.			
find all the factors of a number.			
identify prime and composite numbers.			

ⓑ On a scale of 1–10, how would you rate your mastery of this section in light of your responses on the checklist? How can you improve this?

Answers

ⓐ	
Is 489 a multiple of 2?	
Is the last digit 0, 2, 4, 6, or 8?	No.
	489 is not a multiple of 2.

ⓑ	
Is 3,714 a multiple of 2?	
Is the last digit 0, 2, 4, 6, or 8?	Yes.
	3,714 is a multiple of 2.

ⓐ	
Is 579 a multiple of 5?	
Is the last digit 5 or 0?	No.
	579 is not a multiple of 5.

ⓑ	
Is 880 a multiple of 5?	
Is the last digit 5 or 0?	Yes.
	880 is a multiple of 5.

ⓐ	
Is 425 a multiple of 10?	
Is the last digit zero?	No.
	425 is not a multiple of 10.

ⓑ	
Is 350 a multiple of 10?	
Is the last digit zero?	Yes.
	350 is a multiple of 10.

ⓐ Is 645 a multiple of 3?

Find the sum of the digits.	$6+4+5=15$
Is 15 a multiple of 3?	Yes.
If we're not sure, we could add its digits to find out. We can check it by dividing 645 by 3.	$645 \div 3$
The quotient is 215.	$3 \cdot 215 = 645$

ⓑ Is $10{,}519$ a multiple of 3?

Find the sum of the digits.	$1+0+5+1+9=16$
Is 16 a multiple of 3?	No.
So 10,519 is not a multiple of 3 either..	$645 \div 3$
We can check this by dividing by 10,519 by 3.	$3\overline{)10{,}519}$ $3{,}506\text{R}1$

Table 2.11 applies the divisibility tests to $1{,}290$. In the far right column, we check the results of the divisibility tests by seeing if the quotient is a whole number.

Divisible by...?	Test	Divisible?	Check
2	Is last digit $0,\ 2,\ 4,\ 6,$ or 8? *Yes.*	yes	$1290 \div 2 = 645$
3	Is sum of digits divisible by 3? $1+2+9+0=12$ *Yes.*	yes	$1290 \div 3 = 430$
5	Is last digit 5 or 0? *Yes.*	yes	$1290 \div 5 = 258$
10	Is last digit 0? *Yes.*	yes	$1290 \div 10 = 129$

Table 2.11

Table 2.12 applies the divisibility tests to $5{,}625$ and tests the results by finding the quotients.

Divisible by...?	Test	Divisible?	Check
2	Is last digit $0,\ 2,\ 4,\ 6,$ or 8? *No.*	no	$5625 \div 2 = 2812.5$
3	Is sum of digits divisible by 3? $5+6+2+5=18$ *Yes.*	yes	$5625 \div 3 = 1875$
5	Is last digit is 5 or 0? *Yes.*	yes	$5625 \div 5 = 1125$
10	Is last digit 0? *No.*	no	$5625 \div 10 = 562.5$

Table 2.12

Divide 72 by each of the counting numbers starting with 1. If the quotient is a whole number, the divisor and quotient are a pair of factors.

Dividend	Divisor	Quotient	Factors
72	1	72	1, 72
72	2	36	2, 36
72	3	24	3, 24
72	4	18	4, 18
72	5	14.4	–
72	6	12	6, 12
72	7	~10.29	–
72	8	9	8, 9

The next line would have a divisor of 9 and a quotient of 8. The quotient would be smaller than the divisor, so we stop. If we continued, we would end up only listing the same factors again in reverse order. Listing all the factors from smallest to greatest, we have

$1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36,$ and 72

ⓐ Test each prime, in order, to see if it is a factor of 83, starting with 2, as shown. We will stop when the quotient is smaller than the divisor.

Prime	Test	Factor of 83?
2	Last digit of 83 is not $0, 2, 4, 6, \text{or } 8$.	No.
3	$8 + 3 = 11$, and 11 is not divisible by 3.	No.
5	The last digit of 83 is not 5 or 0.	No.
7	$83 \div 7 = 11.857....$	No.
11	$83 \div 11 = 7.545...$	No.

We can stop when we get to 11 because the quotient $(7.545...)$ is less than the divisor.
We did not find any prime numbers that are factors of 83, so we know 83 is prime.
ⓑ Test each prime, in order, to see if it is a factor of 77.

Prime	Test	Factor of 77?
2	Last digit is not $0, 2, 4, 6, \text{or } 8$.	No.
3	$7 + 7 = 14$, and 14 is not divisible by 3.	No.
5	the last digit is not 5 or 0.	No.
7	$77 \div 7 = 11$	Yes.

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1.16.1: Practice and Videos for Factors and Multiples

Learning Objectives

- I can find factors and multiples.
- I can develop different strategies for finding factors and multiples, such as factoring trees or the rainbow method.

GeoGebra: Factoring with Trees

Learn how to break down composite numbers into their prime factors using an interactive prime factorization tree.

What is prime factorization? It is sometimes necessary to break a number down into its prime factors. These prime factors multiply together to create the original number. This process is called **prime factorization.** We can use a factor tree to determine the prime factorization of a number. Using the factor tree below, think of any two numbers that multiply to make the big number at the top. Click on the drop-down menu and select your numbers. Keep splitting the branches until all the numbers at the ends of the branches are prime (numbers that can only be divided evenly by 1 and itself).

Stuck? Use the divisibility rules below.

Divisibility Rules

Use these divisibility rules to help you find factors of larger numbers.

A number is divisible by...

2	if the last digit is **even** or **zero.**
3	if the sum of the digits is divisible by **three.**
4	if the last two digits are divisible by **four.**
5	if the last digit is **zero** or **five**.
6	if the number is divisible by both **two** and **three**.
8	if the last three digits are divisible by **eight**.
9	if the sum of the digits is divisible by **nine.**
10	if the last digit is **zero.**

Videos on Factors and Multiples

Watch these videos to learn about factors and multiples.

YouTube Video

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Practice!

Check your understanding of factors and multiples by answering these questions. Use the arrows at the bottom of the box to go to the next question.

Practice 1.16.1.1

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Extra Practice Resources

For more practice on factors and multiples, visit these websites.

1. IXL: Choose the multiples of a given number up to 10
2. IXL: Identify factors
3. Math Playground: Match 10 Multiples

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2: FIFTH GRADE MATH

2.1: Ordering and Rounding Decimals

2.1.1: Practice and Videos for Comparing, Ordering, and Rounding Decimals

2.2: Exponents and Powers of 10

2.2.1: Practice and Videos for Exponents and Powers of 10

2.3: Comparing, Expanded and Written Form, and Rounding of Decimals

2.3.1: Practice and Videos for Comparing, Expanded and Written Form and Rounding of Decimals

2.4: Basic Multi-Digit Multiplication and Division

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2.5: Adding and Subtracting Fractions

2.5.1: Practice and Videos for Adding and Subtracting Fractions

2.6: Multiplying and Dividing Fractions

2.6.1: Practice and Videos for Multiplying and Dividing Fractions

2.7: Adding, Subtracting and Multiplying Decimals

2.7.1: Practice and Videos for Adding, Subtracting and Multiplying Decimals

2.7.2: Common Mistakes with Adding and Subtracting Decimals

2.8: Rounding Decimals

2.8.1: Practice and Videos for Rounding Decimals

2.9: Multiplication of Whole Numbers

2.9.1: Practice and Videos for Multiplication of Whole Numbers

2.10: Division of Whole Numbers

2.10.1: Practice and Videos for Division of Whole Numbers

2.11: Unit Conversion - American System

2.11.1: Practice and Videos for Unit Conversion American System

2.12: Area and Volume of Geometric Figures and Objects

2.12.1: Practice and Videos for Introduction to Volume

2.13: Order of Operations

2.13.1: Practice and Videos for Order of Operations

2.14: Cartesian Plane

2.14.1: Practice and Videos for Graphing on a Coordinate Plane

2.14P: Practice

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2.1: Ordering and Rounding Decimals

Learning Objectives

- Use a number line to assist with comparing decimals.
- Compare decimals, beginning with their digits from left to right.
- Use < or > to compare decimals.
- Round a given decimal to a specified place.

Introduction

Decimal numbers are a combination of whole numbers and numbers between whole numbers. It is sometimes important to be able to compare decimals to know which is greater. For example, if someone ran the 100-meter dash in 10.57 seconds, and someone else ran the same race in 10.67 seconds, you can compare the decimals to determine which time is faster. Knowing how to compare decimals requires an understanding of decimal place value, and is similar to comparing whole numbers.

When working with decimals, there are times when a precise number isn't needed. When that's true, rounding decimal numbers is helpful. For example, if the pump at the gas station shows that you filled a friend's car with 16.478 gallons of gasoline, you may want to round the number and just tell her that you filled it with 16.5 gallons.

Comparing Decimals

You can use a number line to compare decimals. The number that is further to the right is greater. Examine this method in the example below.

Example

Use < or > for [] to write a true sentence: 10.5 [] 10.7.

Solution



Here, the digits in the *tenths* places differ. The numbers are both plotted on a number line ranging from 10.0 to 11.0. Because 10.5 is to the left of 10.7 on the number line, 10.5 is less than (<) 10.7.

10.5 < 10.7

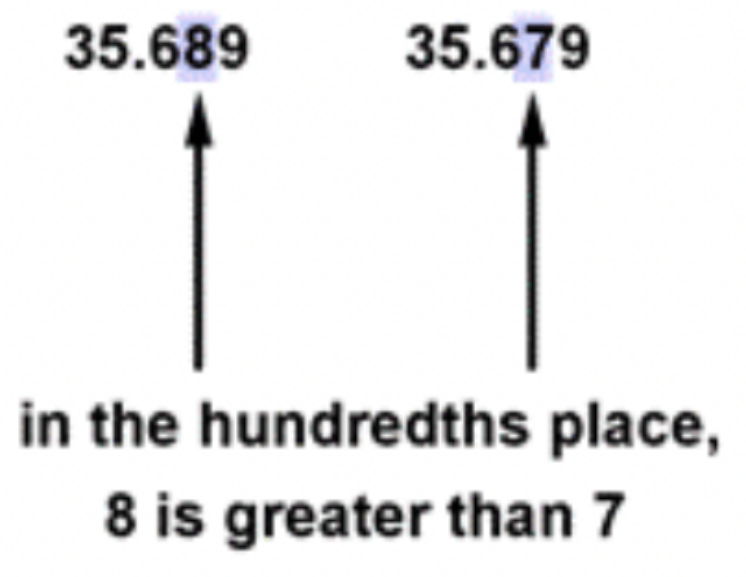
Another approach to comparing decimals is to compare the digits in each number, beginning with the greatest place value, which is on the left. When one digit in a decimal number is greater than the corresponding digit in the other number, then that decimal number is greater.

For example, first compare the tenths digits. If they are equal, move to the hundredths place. If these digits are not equal, the decimal with the greater digit is the greater decimal number. Observe how this is done in the examples below.

Example

Use < or > for [] to write a true sentence: 35.689 [] 35.679.

Solution



Here, the numbers in the tens, ones, and tenths places, 35.6, are the same. However, the digits in the hundredths places differ. Because 8 is greater than 7, 35.689 is greater than 35.679.

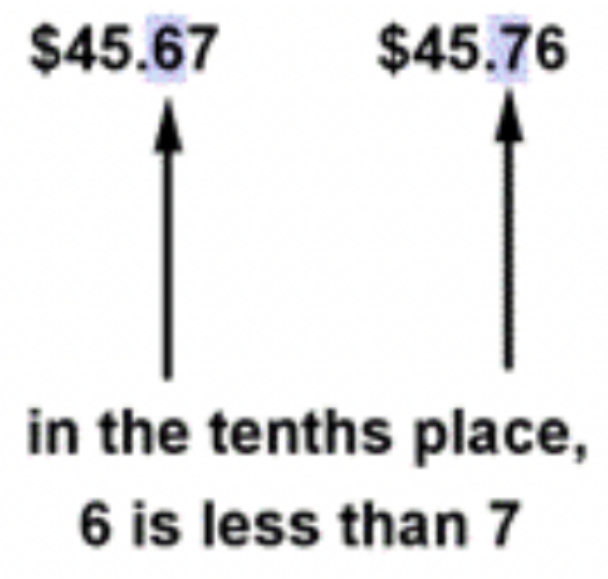
35.689 > 35.679

If more than two digits in the two numbers differ, focus on the digit in the greatest place value. Look at this example in which two sums of money are compared.

Example

Use < or > for [] to write a true sentence: $ 45.67 [] $ 45.76.

Solution



Here, the place value that determines which amount is greater is not the hundredths place, but the tenths place. Because 6 tenths is less than 7 tenths, $45.67 is less than $ 45.76.

$ 45.67 < $ 45.76

If one number has more decimal places than another, you may use 0s as placeholders in the number with fewer decimal places to help you compare. For example, if you are comparing 4.75 and 4.7, you may find it helpful to write 4.7 as 4.70 so that each number has three digits. Note that adding this extra 0 does not change the value of the decimal; you are adding 0 hundredths to the number. You can add placeholder 0s as long as you remember to add the zeros at the end of the number, to the right of the decimal point.

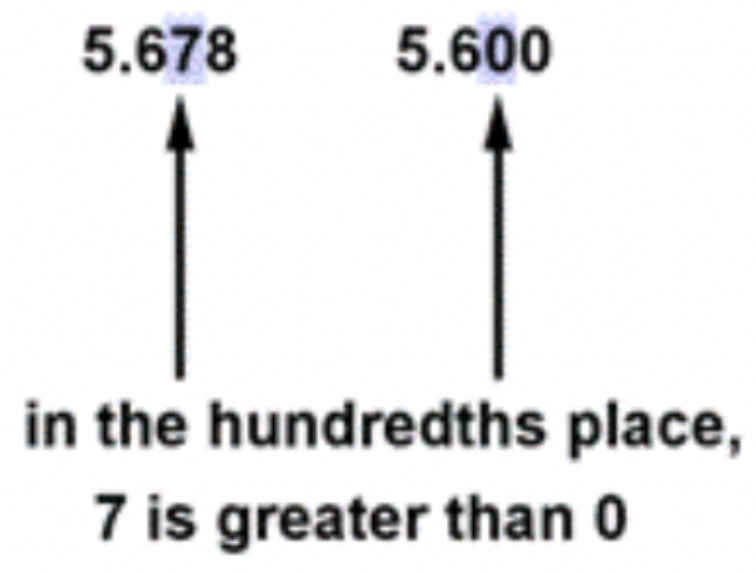
This is demonstrated in the example below.

Example

Use < or > for [] to write a true sentence: 5.678 [] 5.6.

Solution

Here, you can insert two zeros in the hundredths and thousandths places of 5.6 so that you have the same number of digits for each. Then, you can compare digits. Look for the greatest place value that has different values in each number. Here, it is the hundredths place.



Because 7 is greater than 0, 5.678 is greater than 5.600.

5.678 > 5.6

Strategies for Comparing Decimals

- Use a number line to assist with comparing decimals, as you did with whole numbers.
- Compare decimals beginning with their digits from left to right. When two digits are not equal, the one with the greater digit is the larger number.

Exercise

Use < or > for [] to write a true sentence: 45.675 [] 45.645.

A. 45.675 < 45.645
B. 45.675 > 45.645

Answer available at the end of this section.

Rounding Decimals

Rounding with decimals is like rounding with whole numbers. As with whole numbers, you round a number to a given place value. Everything to the right of the given place value becomes a zero, and the digit in the given place value either stays the same or increases by one.

With decimals, you can "drop off" the zeros at the end of a number without changing its value. For example, 0.20=0.2, as $\frac{20}{100}$ simplifies to $\frac{2}{10}$. Of course, you cannot drop zeros before the decimal point: $200 \neq 20$.

The zeros that occur at the end of a decimal number are called **trailing zeros**.

Dropping Zeros with Whole Numbers and Decimals

Dropping zeros at the end of a whole number changes the value of a number.	$7200 \neq 72$ $200,000 \neq 2$
Dropping zeros at the end of a decimal does not change the number's value.	$36.00 = 36$ $1.00000 = 1$

One way to think of it is to consider the number "thirty-six dollars." This amount can be written equally well one of two ways:

$\$36 = \$36.00$

Any zero at the very end of a decimal number can be dropped:

18.25000=18.2500=18.250=18.25

Example

A sprinter ran a race in 7.354 seconds. What was the sprinter's time, rounded to the nearest tenth of a second?

Solution

7.354	Look at the first digit to the right of the tenths digit.
7.354 $\rightarrow 7.400$	Since 5=5, round the 3 up to 4.
7.400	Change all digits to the right of the given place value into zeros. This is an intermediate step that you don't actually write down.
7.4	Since 0.400=0.4, the zeros are not needed and should be dropped.

7.354 rounded to the nearest tenth is 7.4

In the example above, the digit next to the selected place value is 5, so you round up. Let's look at a case in which the digit next to the selected place value is less than 5.

Example

Round 7.354 to the nearest hundredth.

Solution

7.354	Look at the first digit to the right of the hundredths digit.
$7.354 \rightarrow 7.350$	Since 4<5, leave 5 as is.
7.35	The zeros to the right of the given place value are not needed and should be dropped.

7.354 rounded to the nearest hundredth is 7.35.

Sometimes you're asked to round a decimal number to a place value that is in the whole number part. Remember that you may not drop zeros to the left of the decimal point.

Example

Round 1,294.6374 to the nearest hundred.

Solution

$1,294.6374$	Look at the first digit to the right of the hundreds place.
$1,294.6374 \rightarrow 1,300.000$	9 is greater than 5, so round the 2 up to 3.
$1,300$	Zeros to the left of the decimal must be included. Zeros to the right of the decimal can be dropped and should be dropped.

1,294.6374 rounded to the nearest hundred is 1,300.

Tip on Rounding Decimals

When you round, everything to the right of the given place value becomes a zero, and the digit in the given place value either stays the same or rounds up one. Trailing zeros after the decimal point should be dropped.

Exercise

Round 10.473 to the nearest tenth.

A. 10.47
B. 10.4
C. 10.473
D. 10.5

Answer available at the end of this section.

Summary

Ordering, comparing, and rounding decimals are important skills. You can figure out the relative sizes of two decimal numbers by using number lines and by comparing the digits in the same place value of the two numbers. To round a decimal to a specific place value, change all digits to the right of the given place value to zero, and then round the digit in the given place value either up or down. Trailing zeros after the decimal point should be dropped.

Answers

Exercise

A. Incorrect. Comparing the digits in the hundredths places shows that 7 is greater than 4. As a result, 45.675 > 45.645.
B. Correct. 45.675 > 45.645, because the 7 in 45.675 is greater than the corresponding 4 in 45.645.

Exercise

A. Incorrect. You rounded to the nearest hundredth, when you should have rounded to the nearest tenth. The correct answer is 10.5.
B. Incorrect. You rounded down when you should have rounded up. The correct answer is 10.5.
C. Incorrect. You did not round this number at all. If you had rounded to the nearest tenth, you would have found the correct answer, 10.5.
D. Correct. You rounded to the nearest tenth, and were right to round up, because the 7 in 10.473 is greater than 5.

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2.1.1: Practice and Videos for Comparing, Ordering, and Rounding Decimals

Learning Objectives

- I can compare decimals using inequality symbols.
- I can use place value understanding to round decimals to any place.
- I can order a given set of decimals from least to greatest, and vice versa.

GeoGebra: Comparing Decimals up to the Thousandths Place

Complete the activity below by comparing decimals. Examine each pair of digits in decreasing place values by clicking the purple arrows to move back and forth between each place value. Click "new numbers" to compare a new set of decimals.

Videos on Decimal Comparison

Watch the following videos to learn about how to compare, order, and round decimals. All videos have appropriate captions in English; in some cases, there may also be appropriate captions in other languages.

YouTube Video

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Practice!

Check your understanding of comparing, ordering, and rounding decimals by answering these questions. Use the arrows at the bottom of the box to switch between questions.

Practice 2.1.1.1

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Extra Practice Resources

For more practice on comparing, ordering, and rounding decimals, visit these websites.

1. IXL: Place values in decimal numbers
2. IXL: Relationship between place values
3. IXL: Relationship between decimal place values
4. IXL: Round decimals

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2.2: Exponents and Powers of 10

Lesson

Let's look at powers of powers of 10.

Exercise 2.2.1: Big Cube

What is the volume of a giant cube that measures 10,000 km on each side?

Exercise 2.2.2: Raising Powers of 10 to Another Power

1. a. Complete the table to explore patterns in the exponents when raising a power of 10 to a power. You may skip a single box in the table, but if you do, be prepared to explain why you skipped it.

   Table 2.2.1

expression	expanded	single power of 10
$(10^3)^2$	$(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10)$	10^6
$(10^2)^5$	$(10 \cdot 10)(10 \cdot 10)(10 \cdot 10)(10 \cdot 10)(10 \cdot 10)$	
	$(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10)$	
$(10^4)^2$		
$(10^8)^{11}$		

   b. If you chose to skip one entry in the table, which entry did you skip? Why?
2. Use the patterns you found in the table to rewrite $(10^m)^n$ as an equivalent expression with a single exponent, like 10^x.
3. you took the amount of oil consumed in 2 months in 2013 worldwide, you could make a cube of oil that measures 10^3 meters on each side. How many cubic meters of oil is this? Do you think this would be enough to fill a pond, a lake, or an ocean?

Exercise 2.2.3: How Do the Rules Work?

Andre and Elena want to write $10^2 \cdot 10^2 \cdot 10^2$ with a single exponent.

- Andre says, "When you multiply powers with the same **base**, it just means you add the exponents, so $10^2 \cdot 10^2 \cdot 10^2 = 10^{2+2+2} = 10^6$."
- Elena says, "10^2 is multiplied by itself 3 times, so $10^2 \cdot 10^2 \cdot 10^2 = (10^2)^3 = 10^{2+3} = 10^5$."

Do you agree with either of them? Explain your reasoning.

Are you ready for more?

$2^{12} = 4,096$. How many other whole numbers can you raise to a power and get 4,096? Explain or show your reasoning.

Summary

In this lesson, we developed a rule for taking a power of 10 to another power: Taking a power of 10 and raising it to another power is the same as multiplying the exponents. See what happens when raising 10^4 to the power of 3.

$$(10^4)^3 = 10^4 \cdot 10^4 \cdot 10^4 = 10^{12}$$

This works for any power of powers of 10. For example, $(10^6)^{11} = 10^{66}$. This is another rule that will make it easier to work with and make sense of expressions with exponents.

Glossary Entries

Definition: Base (of an exponent)

In expressions like 5^3 and 8^2, the 5 and the 8 are called bases. They tell you what factor to multiply repeatedly. For example, $5^3 = 5 \cdot 5 \cdot 5$, and $8^2 = 8 \cdot 8$.

Practice

Exercise 2.2.4

Write each expression with a single exponent:

1. $(10^7)^2$
2. $(10^9)^3$
3. $(10^6)^3$
4. $(10^2)^3$
5. $(10^3)^2$
6. $(10^5)^7$

Exercise 2.2.5

An amoeba divides to form two amoebas after one hour. One hour later, each of the two amoebas divides to form two more. Every hour, each amoeba divides to form two more.

1. How many amoebas are there after 1 hour?
2. How many amoebas are there after 2 hours?
3. Write an expression for the number of amoebas after 6 hours.
4. Write an expression for the number of amoebas after 24 hours.
5. Why might exponential notation be preferable to answer these questions?

(From Unit 7.1.1)

Learning Objectives

- understand and be able to read exponential notation
- understand the concept of root and be able to read root notation
- be able to use a calculator having the y^x key to determine a root

Exponential Notation

Definition: Exponential Notation

We have noted that multiplication is a description of repeated addition. **Exponential notation** is a description of repeated multiplication.

Suppose we have the repeated multiplication

$8 \cdot 8 \cdot 8 \cdot 8 \cdot 8$

Definition: Exponent

The factor 8 is repeated 5 times. Exponential notation uses a *superscript* for the number of times the factor is repeated. The superscript is placed on the repeated factor, 8^5, in this case. The superscript is called an **exponent**.

Definition: The Function of an Exponent

An **exponent** records the number of identical factors that are repeated in a multiplication.

Sample Set A

Write the following multiplication using exponents.

$3 \cdot 3$. Since the factor 3 appears 2 times, we record this as

Solution

3^2

Sample Set A

$62 \cdot 62 \cdot 62 \cdot 62 \cdot 62 \cdot 62 \cdot 62 \cdot 62 \cdot 62$. Since the factor 62 appears 9 times, we record this as

Solution

62^9

Expand (write without exponents) each number.

Sample Set A

12^4. The exponent 4 is recording 4 factors of 12 in a multiplication. Thus,

Solution

$12^4 = 12 \cdot 12 \cdot 12 \cdot 12$

Sample Set A

706^3. The exponent 3 is recording 3 factors of 706 in a multiplication. Thus,

Solution

$706^3 = 706 \cdot 706 \cdot 706$

Practice Set A

Write the following using exponents.

$37 \cdot 37$

Answer available at the end of this section.

Practice Set A

$16 \cdot 16 \cdot 16 \cdot 16 \cdot 16$

Answer available at the end of this section.

Practice Set A

$9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9$

Answer available at the end of this section.

Write each number without exponents.

Practice Set A

85^3

Answer available at the end of this section.

Practice Set A

4^7

Answer available at the end of this section.

Practice Set A

$1,739^2$

Answer available at the end of this section.

Reading Exponential Notation

In a number such as 8^5.

Base
8 is called the **base**.

Exponent, Power
5 is called the **exponent**, or **power**. 8^5 is read as "eight to the fifth power," or more simply as "eight to the fifth," or "the fifth power of eight."

Squared
When a whole number is raised to the second power, it is said to be **squared**. The number 5^2 can be read as

5 to the second power, or
5 to the second, or
5 squared.

Cubed
When a whole number is raised to the third power, it is said to be **cubed**. The number 5^3 can be read as

5 to the third power, or
5 to the third, or
5 cubed.

When a whole number is raised to the power of 4 or higher, we simply say that that number is raised to that particular power. The number 5^8 can be read as

5 to the eighth power, or just
5 to the eighth.

Roots

In the English language, the word "root" can mean a source of something. In mathematical terms, the word "root" is used to indicate that one number is the source of another number through repeated multiplication.

Square Root
We know that $49 = 7^2$, that is, $49 = 7 \cdot 7$. Through repeated multiplication, 7 is the source of 49. Thus, 7 is a root of 49. Since two 7's must be multiplied together to produce 49, the 7 is called the second or **square root** of 49.

Cube Root
We know that $8 = 2^3$, that is, $8 = 2 \cdot 2 \cdot 2$. Through repeated multiplication, 2 is the source of 8. Thus, 2 is a root of 8. Since three 2's must be multiplied together to produce 8, 2 is called the third or **cube root** of 8.

We can continue this way to see such roots as fourth roots, fifth roots, sixth roots, and so on.

Answers

Practice Set A
37^2

Practice Set A
16^5

Practice Set A
9^{10}

Practice Set A
$85 \cdot 85 \cdot 85$

Practice Set A
$4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$

Practice Set A
$1,739 \cdot 1,739$

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2.2.1: Practice and Videos for Exponents and Powers of 10

Learning Objectives

- I can understand powers of ten and place them in the context of real-world scenarios.
- I can understand what exponents are and how to evaluate them.

GeoGebra: Powers of Ten

Explore the scope and real-world application of exponents and powers of ten using this graphic for scaling pictures. Use the right and left arrows on the side of the page to toggle further or closer to Earth.

Videos on Exponents and Powers of 10

Watch the following videos to learn about exponents and powers of 10. All videos have appropriate captions in English; in some cases, there may also be appropriate captions in other languages.

YouTube Video

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Practice!

Check your understanding of powers of 10 and exponents by answering these questions. Use the arrows at the bottom of the box to switch between questions.

Practice 2.2.1.1

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Extra Practice Resources

1. IXL: Understanding powers of ten
2. IXL: Write powers of ten with exponents
3. Math Playground: Otter Rush

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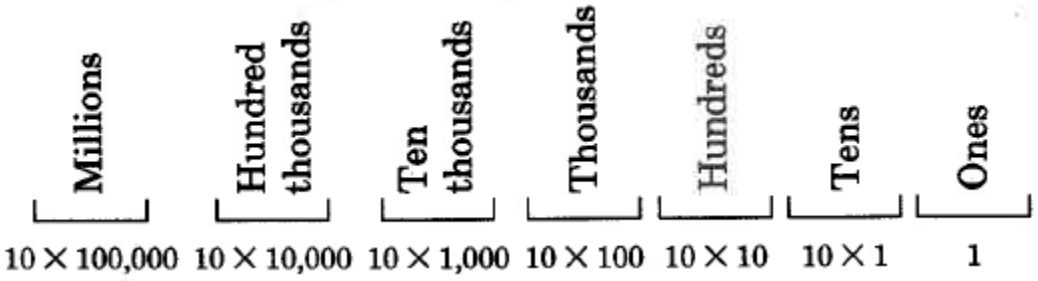
2.3: Comparing, Expanded and Written Form, and Rounding of Decimals

Learning Objectives

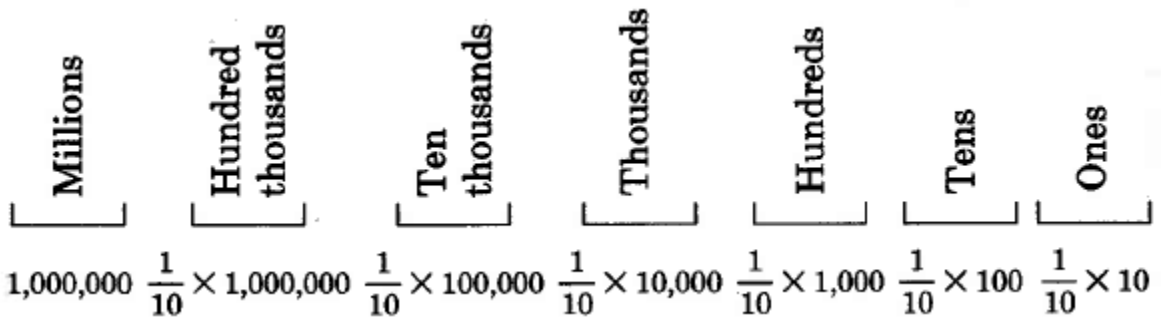
- understand the meaning of digits occurring to the right of the ones position
- be familiar with the meaning of decimal fractions
- be able to read and write a decimal fraction

Digits to the Right of the Ones Position

We began our study of arithmetic by noting that our number system is called a positional number system with base ten. We also noted that each position has a particular value. We observed that each position has ten times the value of the position to its right.



This means that each position has $\frac{1}{10}$ the value of the position to its left.



Thus, a digit written to the right of the units position must have a value of $\frac{1}{10}$ of 1. Recalling that the word "of" translates to multiplication $(\cdot)$, we can see that the value of the *first position* to the right of the units digit is $\frac{1}{10}$ of 1, or

$$\frac{1}{10} \cdot 1 = \frac{1}{10}$$

The value of the *second position* to the right of the units digit is $\frac{1}{10}$ of $\frac{1}{10}$, or

$$\frac{1}{10} \cdot \frac{1}{10} = \frac{1}{10^2} = \frac{1}{100}$$

The value of the third position to the right of the units digit is $\frac{1}{10}$ of $\frac{1}{100}$, or

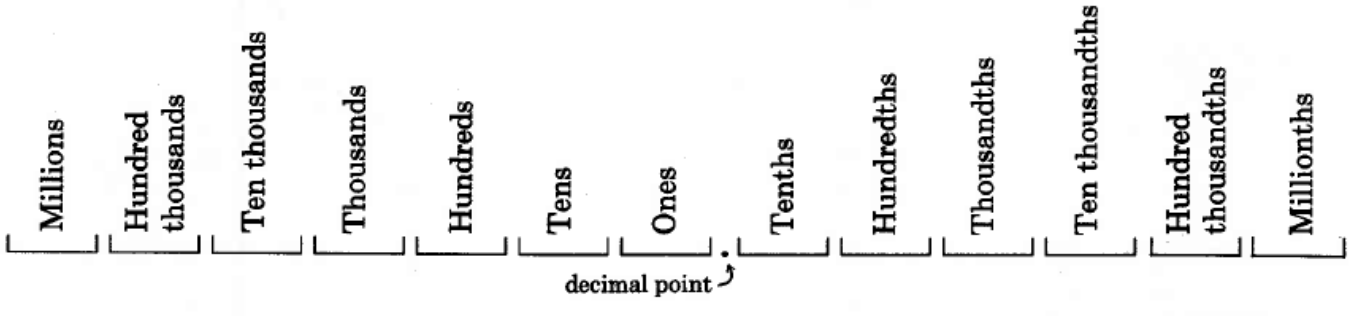
$$\frac{1}{10} \cdot \frac{1}{100} = \frac{1}{10^3} = \frac{1}{1000}$$

This pattern continues.

We can now see that if we were to write digits in positions to the right of the units positions, those positions have values that are fractions. Not only do the positions have fractional values, but the fractional values are all powers of 10 $(10, 10^2, 10^3, ...)$.

Decimal Fractions

Decimal Point, Decimal
If we are to write numbers with digits appearing to the right of the units digit, we must have a way of denoting where the whole number part ends and the fractional part begins. Mathematicians denote the separation point of the units digit and the tenths digit by writing a **decimal point**. The word *decimal* comes from the Latin prefix "deci" which means ten, and we use it because we use a base ten number system. Numbers written in this form are called **decimal fractions**, or more simply, **decimals**.



Notice that decimal numbers have the suffix "th."

Decimal Fraction
A **decimal fraction** is a fraction in which the denominator is a power of 10.

The following numbers are examples of decimals.

42.6

The 6 is in the tenths position.

$42.6 = 42\frac{6}{10}$

9.8014

The 8 is in the tenths position.
The 0 is in the hundredths position.
The 1 is in the thousandths position.
The 4 is in the ten thousandths position.

$9.8014 = 9\frac{8014}{10,000}$

0.93

The 9 is in the tenths position.
The 3 is in the hundredths position.

$0.93 = \frac{93}{100}$

Note

Quite often a zero is inserted in front of a decimal point (in the units position) of a decimal fraction that has a value less than one. This zero helps keep us from overlooking the decimal point.

0.7

The 7 is in the tenths position.

$$0.7 = \frac{7}{10}$$

Note

We can insert zeros to the right of the right-most digit in a decimal fraction without changing the value of the number.

$$\frac{7}{10} = 0.7 = 0.70 = \frac{70}{100} = \frac{7}{10}$$

Reading Decimal Fractions

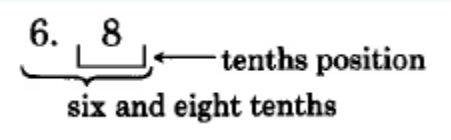
Reading a Decimal Fraction
To read a decimal fraction,

1. Read the whole number part as usual. (If the whole number is less than 1, omit steps 1 and 2.)
2. Read the decimal point as the word "and."
3. Read the number to the right of the decimal point as if it were a whole number.
4. Say the name of the position of the last digit.

Sample Set A

Read the following numbers.

6.8

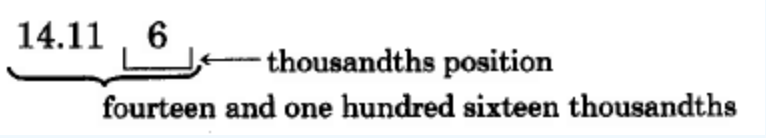


Note

Some people read this as "six point eight." This phrasing gets the message across, but technically, "six *and* eight tenths" is the correct phrasing.

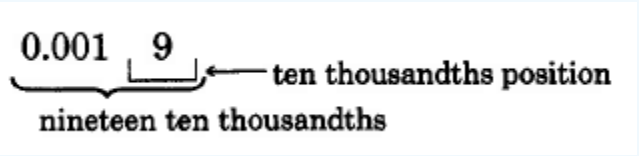
Sample Set A

14.116



Sample Set A

0.0019



Sample Set A

81

Eighty-one

In this problem, the indication is that any whole number is a decimal fraction. Whole numbers are often called *decimal numbers*.

$81 = 81.0$

Practice Set A

Read the following decimal fractions.

12.9

Answer available at the end of this section.

Practice Set A

4.86

Answer available at the end of this section.

Practice Set A

7.00002

Answer available at the end of this section.

Practice Set A

0.030405

Answer available at the end of this section.

Writing Decimal Fractions

Writing a Decimal Fraction
To write a decimal fraction,

Write the whole number part.
Write a decimal point for the word "and."
Write the decimal part of the number so that the right-most digit appears in the position indicated in the word name. If necessary, insert zeros to the right of the decimal point in order that the right-most digit appears in the correct position.

Sample Set B

Write each number.

Thirty-one and twelve hundredths.

Solution

The decimal position indicated is the hundredths position.

31.12

Sample Set B

Two and three hundred-thousandths.

Solution

The decimal position indicated is the hundred thousandths. We'll need to insert enough zeros to the immediate right of the decimal point in order to locate the 3 in the correct position.

2.00003

Sample Set B

Six thousand twenty-seven and one hundred four millionths.

Solution

The decimal position indicated is the millionths position. We'll need to insert enough zeros to the immediate right of the decimal point in order to locate the 4 in the correct position.

6,027.000104

Sample Set B

Seventeen hundredths.

Solution

The decimal position indicated is the hundredths position.

0.17

Practice Set B

Write each decimal fraction.

Three hundred six and forty-nine hundredths.

Answer available at the end of this section.

Practice Set B

Nine and four thousandths.

Answer available at the end of this section.

Practice Set B

Sixty-one millionths.

Answer available at the end of this section.

Exercises

For the following three problems, give the decimal name of the position of the given number in each decimal fraction.

Exercise 2.3.1

1. 3.941
9 is in the position.
4 is in the position.
1 is in the position.

Answer available at the end of this section.

Exercise 2.3.2

17.1085
1 is in the position.
0 is in the position.
8 is in the position.
5 is in the position.

Exercise 2.3.3

652.3561927
9 is in the position.
7 is in the position.

Answer available at the end of this section.

For the following 7 problems, read each decimal fraction by writing it.

Exercise 2.3.4

9.2

Exercise 2.3.5

8.1

Answer available at the end of this section.

Exercise 2.3.6

10.15

Exercise 2.3.7

55.06

Answer available at the end of this section.

Exercise 2.3.8

0.78

Exercise 2.3.9

1.904

Answer available at the end of this section.

Exercise 2.3.10

10.00011

For the following 10 problems, write each decimal fraction.

Exercise 2.3.11

Three and twenty one-hundredths.

Answer available at the end of this section.

Exercise 2.3.12

Fourteen and sixty seven-hundredths.

Exercise 2.3.13

One and eight tenths.

Answer available at the end of this section.

Exercise 2.3.14

Sixty-one and five tenths.

Exercise 2.3.15

Five hundred eleven and four thousandths.

Answer available at the end of this section.

Exercise 2.3.16

Thirty-three and twelve ten-thousandths.

Exercise 2.3.17

Nine hundred forty-seven thousandths.

Answer available at the end of this section.

Exercise 2.3.18

Two millionths.

Exercise 2.3.19

Seventy-one hundred-thousandths.

Answer available at the end of this section.

Exercise 2.3.20

One and ten ten-millionths.

Rounding Decimal Numbers

Learning Objectives

- be able to round a decimal number to a specified position

We first considered the concept of rounding numbers in [link] where our concern with rounding was related to whole numbers only. With a few minor changes, we can apply the same rules of rounding to decimals.

To round a decimal to a particular position:

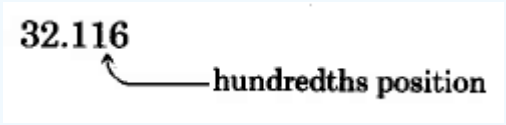
1. Mark the position of the round-off digit (with an arrow or check).
2. Note whether the digit to the immediate right of the marked digit is
   a. *less than 5*. If so, leave the round-off digit unchanged.
   b. *5 or greater*. If so, add 1 to the round-off digit.
3. If the round-off digit is
   a. to the right of the decimal point, eliminate all the digits to its right.
   b. to the left of the decimal point, replace all the digits between it and the decimal point with zeros and eliminate the decimal point and all the decimal digits.

Sample Set A

Round each decimal to the specified position. (The numbers in parentheses indicate which step is being used.)

Round 32.116 to the nearest hundredth.

Solution

1: 

2b: The digit immediately to the right is 6, and $6 > 5$, so we add 1 to the round-off digit:
$1 + 1 = 2$

3a: The round-off digit is to the right of the decimal point, so we eliminate all digits to its right.

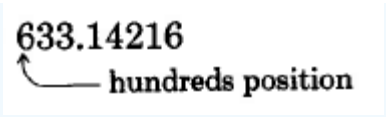
32.12

The number 32.116 rounded to the nearest hundredth is 32.12.

Sample Set A

Round 633.14216 to the nearest hundred.

Solution

1: 

2a: The digit immediately to the right is 3, and $3 < 5$ so we leave the round-off digit unchanged.

3b: The round-off digit is to the left of 0, so we replace all the digits between it and the decimal point with zeros and eliminate the decimal point and all the decimal digits.

600

The number 633.14216 rounded to the nearest hundred is 600.

Sample Set A

1,729.63 rounded to the nearest ten is 1,730.

Sample Set A

1.0144 rounded to the nearest tenth is 1.0.

Sample Set A

60.98 rounded to the nearest one is 61.

Sometimes we hear a phrase such as "round to three decimal places." This phrase means that the round-off digit is the third decimal digit (the digit in the thousandths position).

Sample Set A

67.129 rounded to the second decimal place is 67.13.

Sample Set A

67.129558 rounded to 3 decimal places is 67.130.

Practice Set A

Round each decimal to the specified position.

4.816 to the nearest hundredth.

Answer available at the end of this section.

Practice Set A

0.35928 to the nearest ten thousandths.

Answer available at the end of this section.

Practice Set A

82.1 to the nearest one.

Answer available at the end of this section.

Practice Set A

753.98 to the nearest hundred.

Answer available at the end of this section.

Practice Set A

Round 43.99446 to three decimal places.

Answer available at the end of this section.

Practice Set A

Round 105.019997 to four decimal places.

Answer available at the end of this section.

Practice Set A

Round 99.9999 to two decimal places.

Answer available at the end of this section.

Answers

Practice Set A
twelve and nine tenths

Practice Set A
four and eighty-six hundredths

Practice Set A
seven and two hundred thousandths

Practice Set A
thirty thousand four hundred five millionths

Practice Set B
306.49

Practice Set B
9.004

Practice Set B
0.000061

Exercise 2.3.1
Tenths; hundredths, thousandths

Exercise 2.3.3

Hundred thousandths; ten millionths

Exercise 2.3.5

eight and one tenth

Exercise 2.3.7

fifty-five and six hundredths

Exercise 2.3.9

one and nine hundred four thousandths

Exercise 2.3.11

3.20

Exercise 2.3.13

1.8

Exercise 2.3.15

511.004

Exercise 2.3.17

0.947

Exercise 2.3.19

0.00071

Practice Set A

4.82

Practice Set A

0.3593

Practice Set A

82

Practice Set A

800

Practice Set A

43.994

Practice Set A

105.0200

Practice Set A

100.00

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2.3.1: Practice and Videos for Comparing, Expanded and Written Form and Rounding of Decimals

Learning Objectives

- I can compare, expand, write, and round decimals.

GeoGebra: Writing Expanded Form of Decimals up to the Thousandths Place

Complete the activity below to learn how to expand decimals up to the thousandths place. Start from the left tiles and work your way to the right by dragging the colored lines up the tiles to make the given number.

Note: Click the colored line at the bottom of the tiles so it glows, then slide it up. After filling each column of tiles, check your answer. If your answer was incorrect, adjust the sliders to match the given number.

Videos on Comparing, Expanding, Writing, and Rounding Decimals

Watch the following videos to learn how to compare, expand, write, and round decimals. All videos have appropriate captions in English; in some cases, there may also be appropriate captions in other languages.

YouTube Video

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

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View this YouTube Video online

Practice!

Check your understanding of comparing, expanding, writing, and rounding decimals by answering these questions. Use the arrows at the bottom of the box to switch between questions.

Practice 2.3.1.1

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View this Interactive Content online

Extra Practice

For more practice on comparing, expanding, writing, and rounding decimals, visit these websites.

1. IXL: Convert decimals between standard and expanded form
2. IXL: Convert decimals between standard and expanded form using fractions

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2.4: Basic Multi-Digit Multiplication and Division

We begin this section by discussing multiplication of whole numbers. The first order of business is to introduce the various symbols used to indicate multiplication of two whole numbers.

Mathematical symbols that indicate multiplication

Symbol		Example
×	times symbol	3 × 4
·	dot	3 · 4
( )	parentheses	(3)(4) or 3(4) of (3)4

Products and Factors

In the expression $3 \cdot 4$, the whole numbers 3 and 4 are called the **factors** and $3 \cdot 4$ is called the **product**.

The key to understanding multiplication is held in the following statement.

> Multiplication is equivalent to repeated addition.

Suppose, for example, that we would like to evaluate the product $3 \cdot 4$. Because multiplication is equivalent to repeated addition, $3 \cdot 4$ is equivalent to adding three fours. That is,

$$3 \cdot 4 = \underbrace{4 + 4 + 4}_{\text{three fours}}$$

Thus, $3 \cdot 4 = 12$. You can visualize the product $3 \cdot 4$ as the sum of three fours on a number line, as shown in Figure 1.6.

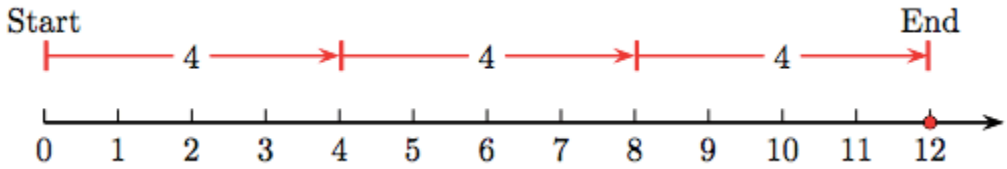


Figure 1.6: Note that 3 · 4 = 4 + 4 + 4. That is, 3 · 4 = 12.

Like addition, the order of the factors does not matter.

$$4 \cdot 3 = \underbrace{3 + 3 + 3 + 3}_{\text{four threes}}$$

Thus, $4 \cdot 3 = 12$. Consider the visualization of $4 \cdot 3$ in Figure 1.7.

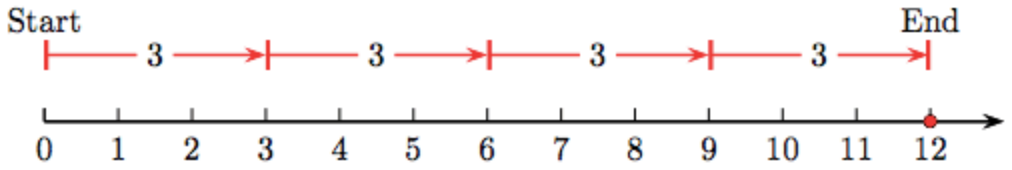


Figure 1.7: Note that 4 · 3 = 3 + 3 + 3 + 3. That is, 4 · 3 = 12.

The evidence in Figure 1.6 and Figure 1.7 show us that multiplication is commutative. That is,

$$3 \cdot 4 = 4 \cdot 3$$

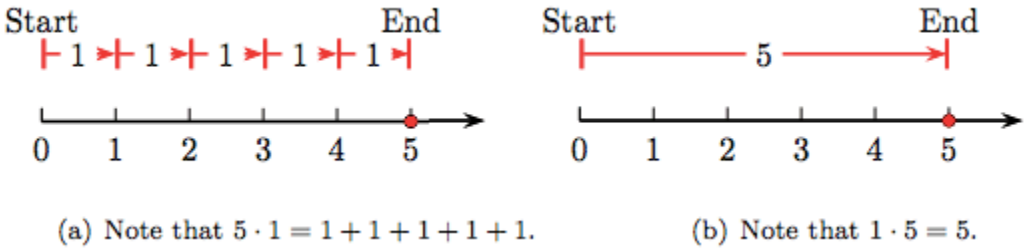
Commutative Property of Multiplication

If *a* and *b* are any whole numbers, then

$$a \cdot b = b \cdot a.$$

The Multiplicative Identity

In Figure 1.8(a), note that five ones equals 5; that is, $5 \cdot 1 = 5$. On the other hand, in Figure 1.8(b), we see that one five equals five; that is, 1 · 5 = 5.



(a) Note that $5 \cdot 1 = 1 + 1 + 1 + 1 + 1$. (b) Note that $1 \cdot 5 = 5$.

Figure 1.8: Note that 5 · 1 = 5 and 1 · 5 = 5.

Because multiplying a whole number by 1 equals that identical number, the whole number 1 is called the multiplicative identity.

The Multiplicative Identity Property

If a is any whole number, then

$a \cdot 1 = a$ and $1 \cdot a = a$.

Multiplication by Zero

Because $3 \cdot 4 = 4 + 4 + 4$, we can say that the product $3 \cdot 4$ represents "3 sets of 4," as depicted in Figure 1.9, where three groups of four boxes are each enveloped in an oval.

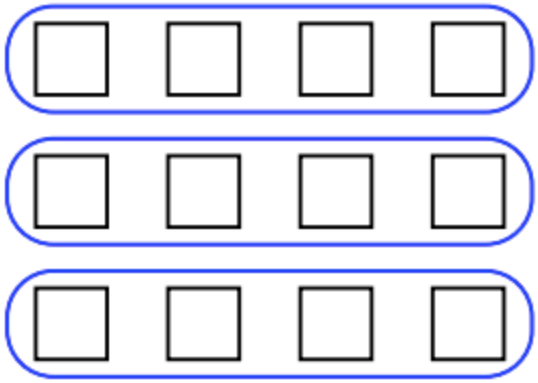


Figure 1.9: Three sets of four: 3 · 4 = 12.

Therefore, $0 \cdot 4$ would mean zero sets of four. Of course, zero sets of four is zero.

Multiplication by Zero.

If *a* represents any whole number, then

$$a \cdot 0 = 0 \text{ and } 0 \cdot a = 0.$$

The Associative Property of Multiplication

Like addition, multiplication of whole numbers is associative. Indeed,

$$\begin{aligned} 2 \cdot (3 \cdot 4) &= 2 \cdot 12 \\ &= 24 \end{aligned}$$

and

$$\begin{aligned} (2 \cdot 3) \cdot 4 &= 6 \cdot 4 \\ &= 24. \end{aligned}$$

The Associative Property of Multiplication.

If *a*, *b*, and *c* are any whole numbers, then

$$a \cdot (b \cdot c) = (a \cdot b) \cdot c.$$

Multiplying Larger Whole Numbers

Much like addition and subtraction of large whole numbers, we will also need to multiply large whole numbers. Again, we hope the algorithm is familiar from previous coursework.

Example 1

Simplify: $35 \cdot 127$.

Solution

Align the numbers vertically. The order of multiplication does not matter, but we'll put the larger of the two numbers on top of the smaller number. The first step is to multiply 5 times 127. Again, we proceed from right to left. So, 5 times 7 is 35. We write the 5, then carry the 3 to the tens column. Next, 5 times 2 is 10. Add the carry digit 3 to get 13. Write the 3 and carry the 1 to the hundreds column. Finally, 5 times 1 is 5. Add the carry digit to get 6.

$$\begin{array}{r} {\scriptstyle 1\ 3\ \ } \\ 1\,2\,7 \\ \times\ 3\,5 \\ \hline 6\,3\,5 \end{array}$$

The next step is to multiply 3 times 127. However, because 3 is in the tens place, its value is 30, so we actually multiply 30 times 127. This is the same as multiplying 127 by 3 and placing a 0 at the end of the result.

$$\begin{array}{r} {\scriptstyle \ \ 2\ \ } \\ 1\,2\,7 \\ \times\ 3\,5 \\ \hline 6\,3\,5 \\ 3\,8\,1\,0 \\ \hline \end{array}$$

After adding the 0, 3 times 7 is 21. We write the 1 and carry the 2 above the 2 in the tens column. Then, 3 times 2 is 6. Add the carry digit 2 to get 8. Finally, 3 times 1 is 1.

All that is left to do is to add the results.

$$\begin{array}{r} 1\,2\,7 \\ \times\ 3\,5 \\ \hline 6\,3\,5 \\ 3\,8\,1\,0 \\ \hline 4\,4\,4\,5 \end{array}$$

Thus, $35 \cdot 127 = 4,445$.

Alternate Format

It does not hurt to omit the trailing zero in the second step of the multiplication, where we multiply 3 times 127. The result would look like this:

$$\begin{array}{r} 1\,2\,7 \\ \times\ 3\,5 \\ \hline 6\,3\,5 \\ 3\,8\,1\,0 \\ \hline 4\,4\,4\,5 \end{array}$$

In this format, the zero is understood, so it is not necessary to have it physically present. The idea is that with each multiplication by a new digit, we indent the product one space from the right.

Exercise

Simplify: 56 · 335

Answer available at the end of this section.

Division of Whole Numbers

We now turn to the topic of division of whole numbers. We first introduce the various symbols used to indicate division of whole numbers.

Mathematical Symbols that Indicate Division

Symbol		Example
÷	division symbol	12 ÷ 4
-	fraction bar	$\frac{12}{4}$
$\longdiv{}$	division bar	$4\longdiv{12}$

Note that each of the following say the same thing; that is, “12 divided by 4 is 3.”

$12 \div 4 = 3$ or $\frac{12}{4} = 3$ or $4\overline{)12}$

Quotients, Dividends, and Divisors

In the statement

$$\frac{3}{4)12}$$

the whole number 12 is called the *dividend*, the whole number 4 is called the *divisor*, and the whole number 3 is called the *quotient*. Note that this division bar notation is equivalent to

$$12 \div 4 = 3 \quad \text{and} \quad \frac{12}{4} = 3.$$

The expression *a*/*b* means “*a* divided by *b*,” but this construct is also called a *fraction*.

Definition: Fraction

The expression

$$\frac{a}{b}$$

is called a **fraction**. The number a on top is called the **numerator** of the fraction; the number b on the bottom is called the **denominator** of the fraction.

The key to understanding division of whole numbers is contained in the following statement.

> Division is equivalent to repeated subtraction.

Suppose for example, that we would like to divide the whole number 12 by the whole number 4. This is equivalent to asking the question "how many fours can we subtract from 12?" This can be visualized in a number line diagram, such as the one in Figure 1.10.

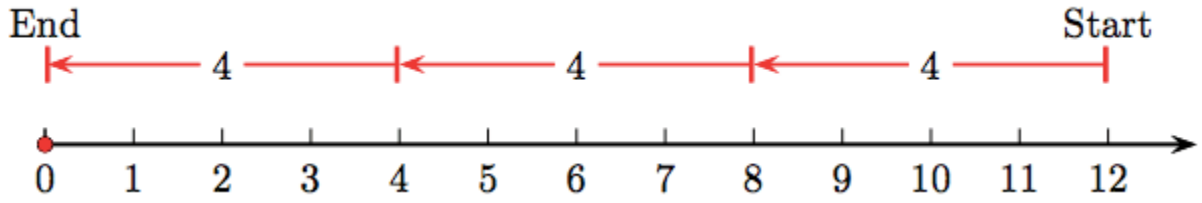


Figure 1.10: Division is repeated subtraction.

In Figure 1.10, note that we if we subtract three fours from twelve, the result is zero. In symbols,

$$12 - \underbrace{4 - 4 - 4}_{\text{three fours}} = 0.$$

Equivalently, we can also ask "How many groups of four are there in 12," and arrange our work as shown in Figure 1.11, where we can see that in an array of twelve objects, we can circle three groups of four ; i.e., 12 ÷ 4 = 3.

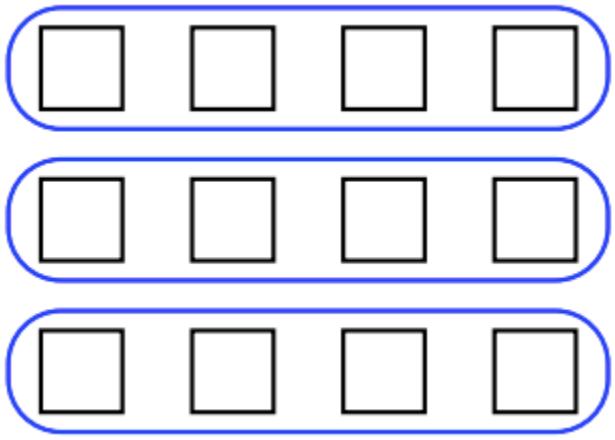


Figure 1.11: There are three groups of four in twelve.

In Figure 1.10 and Figure 1.11, note that the division (repeated subtraction) leaves no remainder. This is not always the case.

Example 2.

Divide 7 by 3.

Solution

In Figure 1.12, we can see that we can subtract two threes from seven, leaving a remainder of one.

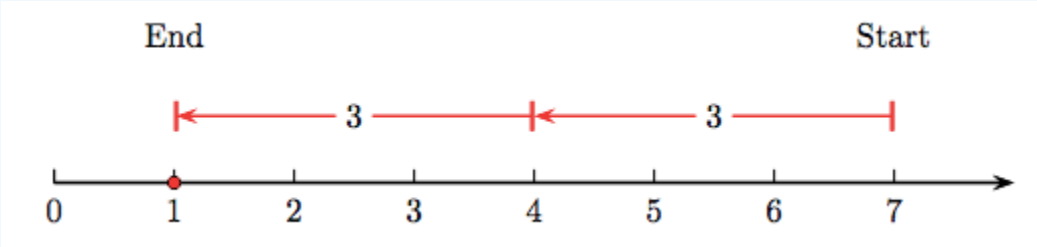


Figure 1.12: Division with a remainder.

Alternatively, in an array of seven objects, we can circle two groups of three, leaving a remainder of one.

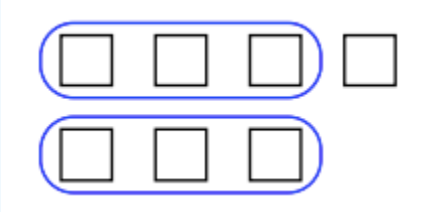


Figure 1.13: Dividing seven by three leaves a remainder of one.

Both Figure 1.12 and Figure1.13 show that there are two groups of three in seven, with one left over. We say “Seven divided by three is two, with a remainder of one.

Exercise

Use both the number line approach and the array of boxes approach to divide 12 by 5.

Division is not Commutative

When dividing whole numbers, the order matters. For example

$$12 \div 4 = 3,$$

but $4 \div 12$ is not even a whole number. Thus, if a and b are whole numbers, then $a \div b$ does **not** have to be the same as $b \div a$.

Division is not Associative

When you divide three numbers, the order in which they are grouped will usually affect the answer. For example,

$$(48 \div 8) \div 2 = 6 \div 2$$

$$= 3,$$

but

$$48 \div (8 \div 2) = 48 \div 4$$

= 12.

Thus, if *a*, *b*, and *c* are whole numbers, (*a* ÷ *b*) ÷ *c* does not have to be the same as *a* ÷ (*b* ÷ *c*).

Division by Zero is Undefined

Suppose that we are asked to divide six by zero; that is, we are asked to calculate 6 ÷ 0. In Figure 1.14, we have an array of six objects.

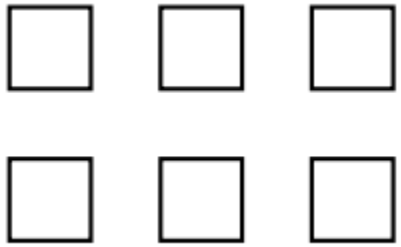


Figure 1.14: How many groups of zero do you see?

Now, to divide six by zero, we must answer the question "How many groups of zero can we circle in Figure 1.14?" Some thought will provide the answer: This is a meaningless request! It makes absolutely no sense to ask how many groups of zero can be circled in the array of six objects in Figure 1.14.

Division by Zero

Division by zero is **undefined**. Each of the expressions

$$6 \div 0 \quad \text{and} \quad \frac{6}{0} \quad \text{and} \quad 0\overline{)6}$$

is **undefined.**

On the other hand, it make sense to ask "What is zero divided by six?" If we create an array of zero objects, then ask how many groups of six we can circle, the answer is "zero groups of six." That is, zero divided by six is zero.

$$0 \div 6 = 0 \quad \text{and} \quad \frac{0}{6} = 0 \quad \text{and} \quad 6\overline{)0}$$

Exercises

In Exercises 1-4 use number line diagrams as shown in Figure 1.6 to depict the multiplication.

1. 2 · 4.
2. 3 · 4.
3. 4 · 2.
4. 4 · 3.

In Exercises 5-16, state the property of multiplication depicted by the given identity.

5. 9 · 8=8 · 9
6. 5 · 8=8 · 5

7. $8 \cdot (5 \cdot 6) = (8 \cdot 5) \cdot 6$

8. $4 \cdot (6 \cdot 5) = (4 \cdot 6) \cdot 5$

9. $6 \cdot 2 = 2 \cdot 6$

10. $8 \cdot 7 = 7 \cdot 8$

11. $3 \cdot (5 \cdot 9) = (3 \cdot 5) \cdot 9$

12. $8 \cdot (6 \cdot 4) = (8 \cdot 6) \cdot 4$

13. $21 \cdot 1 = 21$

14. $39 \cdot 1 = 39$

15. $13 \cdot 1 = 13$

16. $44 \cdot 1 = 44$

In Exercises 17-28, multiply the given numbers.

17. $78 \cdot 3$

18. $58 \cdot 7$

19. $907 \cdot 6$

20. $434 \cdot 80$

21. $128 \cdot 30$

22. $454 \cdot 90$

23. $799 \cdot 60$

24. $907 \cdot 20$

25. $14 \cdot 70$

26. $94 \cdot 90$

27. $34 \cdot 90$

28. $87 \cdot 20$

In Exercises 29-40, multiply the given numbers.

29. $237 \cdot 54$

30. $893 \cdot 94$

31. $691 \cdot 12$

32. $823 \cdot 77$

33. $955 \cdot 89$

34. $714 \cdot 41$

35. 266 · 61

36. 366 · 31

37. 365 · 73

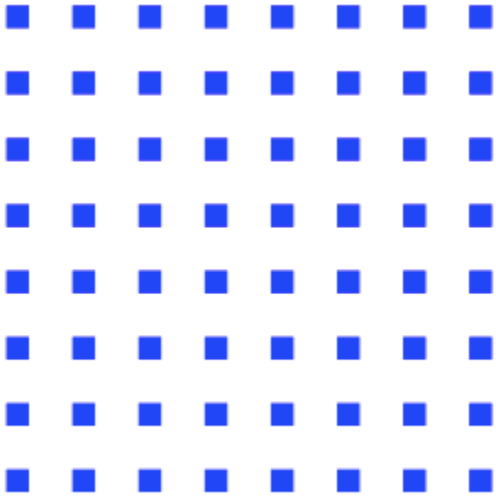
38. 291 · 47

39. 955 · 57

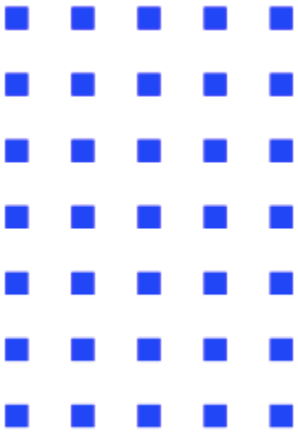
40. 199 · 33

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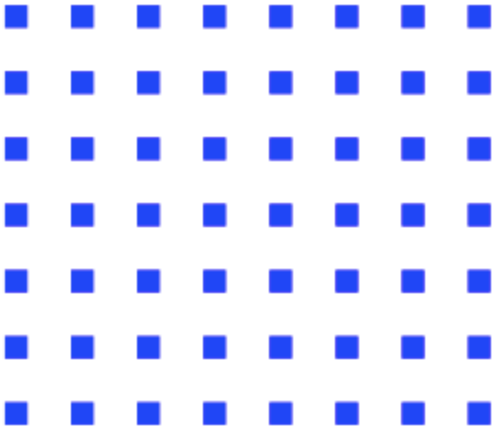
41. Count the number of objects in the array.



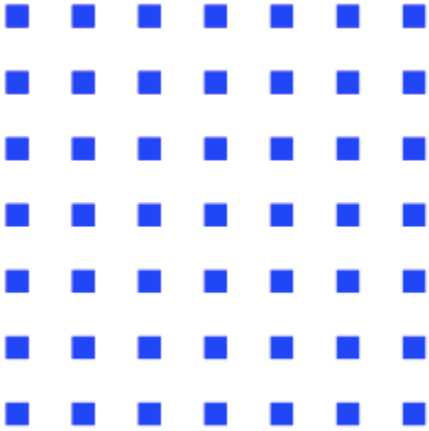
42. Count the number of objects in the array.



43. Count the number of objects in the array.



44. Count the number of objects in the array.



In Exercises 45-48, find the area of the rectangle having the given length and width.

45. L = 50 in, W = 25 in

46. L = 48 in, W = 24 in

47. L = 47 in, W = 13 in

48. L = 19 in, W = 10 in

In Exercises 49-52, find the perimeter of the rectangle having the given length and width.

49. L = 25 in, W = 16 in

50. L = 34 in, W = 18 in

51. L = 30 in, W = 28 in

52. L = 41 in, W = 25 in

---

53. A set of beads costs 50 cents per dozen. What is the cost (in dollars) of 19 dozen sets of beads?

54. A set of beads costs 60 cents per dozen. What is the cost (in dollars) of 7 dozen sets of beads?

55. If a math tutor worked for 47 hours and was paid $15 each hour, how much money would she have made?

56. If a math tutor worked for 46 hours and was paid $11 each hour, how much money would he have made?

57. There are 12 eggs in one dozen, and 12 dozen in one gross. How many eggs are in a shipment of 24 gross?

58. There are 12 eggs in one dozen, and 12 dozen in one gross. How many eggs are in a shipment of 11 gross?

59. If bricks weigh 4 kilograms each, what is the weight (in kilograms) of 5000 bricks?

60. If bricks weigh 4 pounds each, what is the weight (in pounds) of 2000 bricks?

In Exercises 61-68, which of the following four expressions differs from the remaining three?

61. $\frac{30}{5}$, 30 ÷ 5, $5\longdiv{30}$, 5 ÷ 30

62. $\frac{12}{2}$, 12 ÷ 2, $2\longdiv{12}$, 2 ÷ 12

63. $\frac{8}{2}$, 8 ÷ 2, $2\longdiv{8}$, $8\longdiv{2}$

64. $\frac{8}{4}$, 8 ÷ 4, $4\longdiv{8}$, $8\longdiv{4}$

65. $2\longdiv{14}$, $14\longdiv{2}$, $\frac{14}{2}$, 14 ÷ 2

66. $9\longdiv{54}$, $54\longdiv{9}$, $\frac{54}{9}$, 54 ÷ 9

67. $3\longdiv{24}$, 3 ÷ 24, $\frac{24}{3}$, 24 ÷ 3

68. $3\longdiv{15}$, 3 ÷ 15, $\frac{15}{3}$, 15 ÷ 3

In Exercises 69-82, simplify the given expression. If the answer doesn't exist or is undefined, write "undefined".

69. 0 ÷ 11

70. 0 ÷ 5

71. 17 ÷ 0

72. 24 ÷ 0

73. 10 · 0

74. 20 · 0

75. $\frac{7}{0}$

76. $\frac{23}{0}$

77. $16\longdiv{0}$

78. $25\longdiv{0}$

79. $\frac{0}{24}$

80. $\frac{0}{22}$

81. $0\longdiv{0}$

82. $0 \div 0$

In Exercises 83-94, divide the given numbers.

83. $\frac{2816}{44}$

84. $\frac{1998}{37}$

85. $\frac{2241}{83}$

86. $\frac{2716}{97}$

87. $\frac{3212}{73}$

88. $\frac{1326}{17}$

89. $\frac{8722}{98}$

90. $\frac{1547}{91}$

91. $\frac{1440}{96}$

92. $\frac{2079}{27}$

93. $\frac{8075}{85}$

94. $\frac{1587}{23}$

In Exercises 95-106, divide the given numbers.

95. $\frac{17756}{92}$

96. $\frac{46904}{82}$

97. $\frac{11951}{19}$

98. $\frac{22304}{41}$

99. $\frac{18048}{32}$

100. $\frac{59986}{89}$

101. $\frac{29047}{31}$

102. $\frac{33264}{86}$

103. $\frac{22578}{53}$

104. $\frac{18952}{46}$

105. $\frac{12894}{14}$

106. $\frac{18830}{35}$

---

107. A concrete sidewalk is laid in square blocks that measure 6 feet on each side. How many blocks will there be in a walk that is 132 feet long?

108. A concrete sidewalk is laid in square blocks that measure 5 feet on each side. How many blocks will there be in a walk that is 180 feet long?

109. One boat to the island can take 5 people. How many trips will the boat have to take in order to ferry 38 people to the island? (Hint: Round up your answer.)

110. One boat to the island can take 4 people. How many trips will the boat have to take in order to ferry 46 people to the island? (Hint: Round up your answer.)

111. If street lights are placed at most 145 feet apart, how many street lights will be needed for a street that is 4 miles long, assuming that there are lights at each end of the street? (Note: 1 mile = 5280 feet.)

112. If street lights are placed at most 70 feet apart, how many street lights will be needed for a street that is 3 miles long, assuming that there are lights at each end of the street? (Note: 1 mile = 5280 feet.)

113. A concrete sidewalk is laid in square blocks that measure 4 feet on each side. How many blocks will there be in a walk that is 292 feet long?

114. A concrete sidewalk is laid in square blocks that measure 5 feet on each side. How many blocks will there be in a walk that is 445 feet long?

115. One boat to the island can take 3 people. How many trips will the boat have to take in order to ferry 32 people to the island? (Hint: Round up your answer.)

116. One boat to the island can take 4 people. How many trips will the boat have to take in order to ferry 37 people to the island? (Hint: Round up your answer.)

117. If street lights are placed at most 105 feet apart, how many street lights will be needed for a street that is 2 miles long, assuming that there are lights at each end of the street? (Note: 1 mile = 5280 feet.)

118. If street lights are placed at most 105 feet apart, how many street lights will be needed for a street that is 3 miles long, assuming that there are lights at each end of the street? (Note: 1 mile = 5280 feet.)

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119. **Writing articles**. Eli writes an average of 4 articles a day, five days a week, to support product sales. How many articles does Eli write in one week?

120. **Machine gun**. A 0.50-caliber antiaircraft machine gun can fire 800 rounds each minute. How many rounds could fire in three minutes? *Associated Press Times-Standard 4/15/09*

121. **Laps**. The swimming pool at CalCourts is 25 yards long. If one lap is up and back again, how many yards has Wendell swam doing 27 laps?

122. **Refrigerator wattage**. A conventional refrigerator will run about 12 hours each day can use 150 Watts of power each hour. How many Watts of power will a refrigerator use over the day?

123. **Horse hay**. A full-grown horse should eat a minimum of 12 pounds of hay each day and may eat much more depending on their weight. How many pounds minimum would a horse eat over a year?

124. **College costs**. After a $662 hike in fees, California residents who want to attend the University of California as an undergraduate should expect to pay $8,700 in for the upcoming academic year 2009- 2010. If the cost were to remain the same for the next several years, how much should a student expect to pay for a four-year degree program at a UC school?

125. **Non-resident costs**. Nonresident undergraduates who want to attend a University of California college should expect to pay about $22,000 for the upcoming academic year. Assuming costs remain the same, what can a four-year degree cost?

126. **Student tax**. The mayer of Providence, Rhode Island wants to tax its 25,000 Brown University students $150 each to contribute to tax receipts saying students should pay for the resources they use just like the town residents. How many dollars would the mayer generate?

127. **New iceberg**. A new iceberg, shaved off a glacier after a collision with another iceberg, measures about 48 miles long and 28 miles wide. What's the approximate area of the new iceberg? Associated PressTimes-Standard 02/27/10 2 Huge icebergs set loose off Antarctica's coast.

128. **Solar panels**. One of the solar panels on the International Space Station is 34 meters long and 11 meters wide. If there are eight of these, what's the total area for solar collection?

129. **Sidewalk**. A concrete sidewalk is to be 80 foot long and 4 foot wide. How much will it cost to lay the sidewalk at $8 per square foot?

130. **Hay bales**. An average bale of hay weighs about 60 pounds. If a horse eats 12 pounds of hay a day, how many days will one bale feed a horse?

131. **Sunspots**. Sunspots, where the sun's magnetic field is much higher, usually occur in pairs. If the total count of sunspots is 72, how many pairs of sunspots are there?

Answers

1. $2 \cdot 4 = \underbrace{4+4}_{2 \text{ times}} = 8$

3. $4 \cdot 2 = \underbrace{2+2+2+2}_{4 \text{ times}} = 8$

5. Commutative property of multiplication

7. Associative property of multiplication

9. Commutative property of multiplication

11. Associative property of multiplication

13. Multiplicative identity property

15. Multiplicative identity property

17. 234

19. 5442

21. 3840

23. 47940

25. 980

27. 3060

29. 12798

31. 8292

33. 84995

35. 16226

37. 26645

39. 54435

41. 64

43. 56

45. 1250 in^2

47. 611 in^2

49. 82 in

51. 116 in

53. 9.50

55. 705

57. 3456

59. 20000

61. 5 ÷ 30

63. $8\sqrt{2}$

65. $14\sqrt{2}$

67. 3 ÷ 24

69. 0

71. Undefined

73. 0

75. Undefined

77. 0

79. 0

81. Undefined

83. 64

85. 27

87. 44

89. 89

91. 15

93. 95

95. 193

97. 629

99. 564

101. 937

103. 426

105. 921

107. 22

109. 8

111. 147

113. 73

115. 11

117. 102

119. 20 articles

121. 1350 yards

123. 4380 pounds of hay

125. $88,000

127. 1344 mi^2

129. $2,560

131. 36

Learning Objectives

- be able to divide a whole number by a single or multiple digit divisor
- be able to interpret a calculator statement that a division results in a remainder

Division with a Single Digit Divisor

Our experience with multiplication of whole numbers allows us to perform such divisions as $75 \div 5$. We perform the division by performing the corresponding multiplication, $5 \times Q = 75$. Each division we considered in [link] had a one-digit quotient. Now we will consider divisions in which the quotient may consist of two or more digits. For example, $75 \div 5$.

Let's examine the division $75 \div 5$. We are asked to determine how many 5's are contained in 75. We'll approach the problem in the following way.

1. Make an educated guess based on experience with multiplication.
2. Find how close the estimate is by multiplying the estimate by 5.
3. If the product obtained in step 2 is less than 75, find out how much less by subtracting it from 75.
4. If the product obtained in step 2 is greater than 75, decrease the estimate until the product is less than 75. Decreasing the estimate makes sense because we do not wish to exceed 75.

We can suggest from this discussion that the process of division consists of

The Four Steps in Division

1. an educated guess
2. a multiplication
3. a subtraction

4. bringing down the next digit (if necessary)

The educated guess can be made by determining how many times the divisor is contained in the dividend by using only one or two digits of the dividend.

Sample Set A

Find $75 \div 5$.

Solution

$5\overline{)75}$ Rewrite the problem using a division bracket.

$$\begin{array}{r} 10 \\ 5\overline{)75} \end{array}$$

Make an educated guess by noting that one 5 is contained in 75 at most 10 times.
Since 7 is the tens digit, we estimate that 5 goes into 75 at most 10 times.

$$\begin{array}{r} 10 \\ 5\overline{)75} \\ \underline{-50} \\ 25 \end{array}$$

Now determine how close the estimate is.
10 fives is $10 \times 5 = 50$. Subtract 50 from 75.
Estimate the number of 5's in 25.
There are exactly 5 fives in 25.

$$\begin{array}{r} 5 \\ 10 \\ 5\overline{)75} \\ \underline{-50} \\ 25 \\ \underline{-25} \\ 0 \end{array}$$

10 fives + 5 fives = 15 fives.
There are 15 fives contained in 75.

Check:

$$75 \stackrel{?}{=} 15 \times 5$$
$$75 = 75$$

Thus, $75 \div 5 = 15$.

The notation in this division can be shortened by writing.

$$\begin{array}{r} 15 \\ 5\overline{)75} \\ -5\downarrow \\ \hline 25 \\ -25 \\ \hline 0 \end{array}$$

$$\begin{cases} \text{Divide:} & 5 \text{ goes into } 7 \text{ at most } 1 \text{ time.} \\ \text{Multiply:} & 1 \times 5 = 5. \text{Write } 5 \text{ below } 7. \\ \text{Subtract:} & 7 - 5 = 2. \text{Bring down the } 5. \end{cases} \quad \begin{cases} \text{Divide:} & 5 \text{ goes into } 25 \text{ exactly } 5 \text{ times} \\ \text{Multiply:} & 5 \times 5 = 25. \text{Write } 25 \text{ below } 25. \\ \text{Subtract:} & 25 - 25 = 0 \end{cases}$$

Sample Set A

Find $4,944 \div 8$.

Solution

$$8\overline{)4944}$$

Rewrite the problem using a division bracket.

$$\begin{array}{r} 600 \\ 8\overline{)4944} \\ \underline{-4800} \\ 144 \end{array}$$

8 goes into 49 at most 6 times, and 9 is in the hundreds column. We'll guess 600.
Then, $8 \times 600 = 4800$.

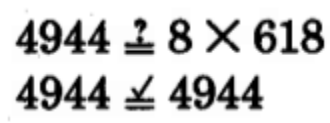
$$\begin{array}{r} 10 \\ 600 \\ 8\overline{)4944} \\ \underline{-4800} \\ 144 \\ \underline{-\ \ 80} \\ 64 \end{array}$$

8 goes into 14 at most 1 time, and 4 is in the tens column. We'll guess 10.

$$\begin{array}{r} 8 \\ 10 \\ 600 \\ 8\overline{)4944} \\ \underline{-4800} \\ 144 \\ \underline{-\ \ 80} \\ 64 \\ \underline{-64} \\ 0 \end{array}$$

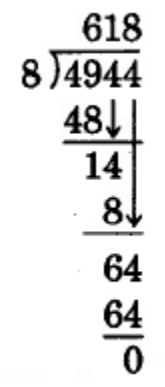
8 goes into 64 exactly 8 times.
600 eights + 10 eights + 8 eights = 618 eights.

Check:



Thus, $4,944 \div 8 = 618$.

As in the first problem, the notation in this division can be shortened by eliminating the subtraction signs and the zeros in each educated guess.



$$\begin{cases}\text{Divide:} & 8 \text{ goes into } 49 \text{ at most } 6 \text{ times.}\\ \text{Multiply:} & 6 \times 8 = 48. \text{Write } 48 \text{ below } 49.\\ \text{Subtract:} & 49 - 48 = 1. \text{Bring down the } 4.\end{cases}$$

$$\begin{cases}\text{Divide:} & 8 \text{ goes into } 14 \text{ at most } 1 \text{ time.}\\ \text{Multiply:} & 1 \times 8 = 8. \text{Write } 8 \text{ below } 14.\\ \text{Subtract:} & 14 - 8 = 6. \text{Bring down the } 4.\end{cases}$$

$$\begin{cases}\text{Divide:} & 8 \text{ goes into } 64 \text{ exactly } 8 \text{ times.}\\ \text{Multiply:} & 8 \times 8 = 64. \text{Write } 64 \text{ below } 64.\\ \text{Subtract:} & 64 - 64 = 0.\end{cases}$$

Note

Not all divisions end in zero. We will examine such divisions in a subsequent subsection.

Practice Set A

Perform the following divisions.

$126 \div 7$

Answer available at the end of this section.

Practice Set A

$324 \div 4$

Answer available at the end of this section.

Practice Set A

$2,559 \div 3$

Answer available at the end of this section.

Practice Set A

$5,645 \div 5$

Answer available at the end of this section.

Practice Set A

$757,125 \div 9$

Answer available at the end of this section.

Division with a Multiple Digit Divisor

The process of division also works when the divisor consists of two or more digits. We now make educated guesses using the first digit of the divisor and one or two digits of the dividend.

Sample Set B

Find $2,232 \div 36$.

Solution

$36\overline{)2232}$

Use the first digit of the divisor and the first two digits of the dividend to make the educated guess.

3 goes into 22 at most 7 times.

Try 7: $7 \times 36 = 252$ which is greater than 223. Reduce the estimate.

Try 6: $6 \times 36 = 216$ which is less than 223.

$$\begin{array}{r} 6 \\ 36\overline{)\ 2232} \\ -216\downarrow \\ \hline 72 \end{array}$$

$\begin{array}{ll} \text{Multiply:} & 6 \times 36 = 216. \text{Write 216 below 223.} \\ \text{Subtract:} & 223 - 216 = 7. \text{Bring down the 2.} \end{array}$

Divide 3 into 7 to estimate the number of times 36 goes into 72. The 3 goes into 7 at most 2 times.

Try 2: $2 \times 36 = 72$.

$$\begin{array}{r} 62 \\ 36\overline{)2232} \\ 216\downarrow \\ \hline 72 \\ -72 \\ \hline 0 \end{array}$$

Check:

$$2232 \stackrel{?}{=} 36 \times 62$$
$$2232 \checkmark 2232$$

Thus $2,232 \div 36 = 62$.

Sample Set B

Find $2,417,228 \div 802$.

Solution

$802\overline{)2417228}$

First, the educated guess: $24 \div 8 = 3$. Then $3 \times 802 = 2406$, which is less than 2417. Use 3 as the guess. Since $3 \times 802 = 2406$, and 2406 has four digits, place the 3 above the fourth digit of the dividend.

$$\begin{array}{r} 3 \\ 802\overline{)\ 2417228} \\ -2406\downarrow \\ \hline 112 \end{array}$$

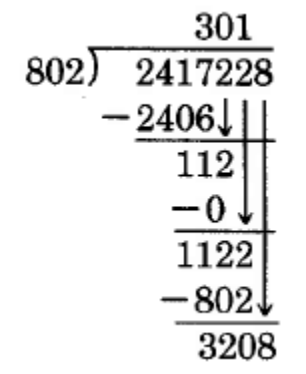
Subtract: 2417 - 2406 = 11.
Bring down the 2.

The divisor 802 goes into 112 at most 0 times. Use 0.

$$\begin{array}{r} 30 \\ 802\overline{)\ 2417228} \\ -2406\downarrow \\ \hline 112 \\ -0\downarrow \\ \hline 1122 \end{array}$$

Multiply:	$0 \times 802 = 0$.
Subtract:	$112 - 0 = 112$.
Bring down the 2.	

The 8 goes into 11 at most 1 time, and $1 \times 802 = 802$, which is less than 1122. Try 1.

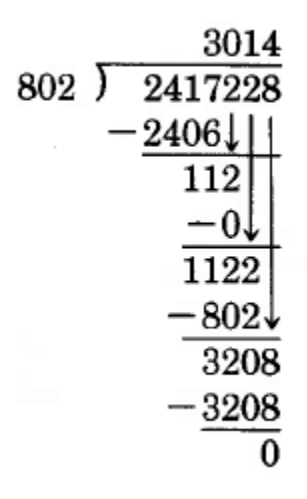


Subtract 1122 - 802 = 320
Bring down the 8.

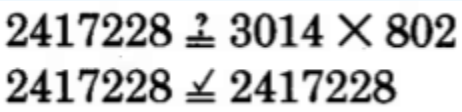
8 goes into 32 at most 4 times.

$4 \times 802 = 3208$.

Use 4.



Check:



Thus, $2,417,228 \div 802 = 3,014$.

Practice Set B

Perform the following divisions.

$1,376 \div 32$

Answer available at the end of this section.

Practice Set B

$6,160 \div 55$

Answer available at the end of this section.

Practice Set B

$18,605 \div 61$

Answer available at the end of this section.

Practice Set B

$144,768 \div 48$

Answer available at the end of this section.

Division with a Remainder

We might wonder how many times 4 is contained in 10. Repeated subtraction yields

$$\begin{array}{r} 10 \\ -\ 4 \\ \hline 6 \\ -4 \\ \hline 2 \end{array}$$

Since the remainder is less than 4, we stop the subtraction. Thus, 4 goes into 10 two times with 2 remaining. We can write this as a division as follows.

$$\begin{array}{r} 2 \\ \hline 4)10 \\ -\ 8 \\ \hline 2 \end{array}$$

Divide:	4 goes into 10 at most 2 times.
Multiply:	$2 \times 4 = 8$.Write 8 below 0.
Subtract:	$10 - 8 = 2$.

$$\begin{array}{r} 2\text{R}2 \\ 4\overline{)\ 10} \\ \underline{-8} \\ 2 \end{array} \quad \text{or} \quad 10 \div 4 = \underbrace{2\text{R}2}_{2 \text{ with remainder } 2}$$

Sample Set C

Find $85 \div 3$.

Solution

$$\begin{array}{r} 28 \\ 3\overline{)85} \\ \underline{6\downarrow} \\ 25 \\ \underline{24} \\ 1 \end{array}$$

$\begin{cases} \text{Divide:} & 3 \text{ goes into } 8 \text{ at most } 2 \text{ times.} \\ \text{Multiply:} & 2 \times 3 = 6. \text{ Write } 6 \text{ below } 8. \\ \text{Subtract:} & 8 - 6 = 2. \text{ Bring down the } 5. \end{cases}$ $\begin{cases} \text{Divide:} & 3 \text{ goes into } 25 \text{ at most } 8 \text{ times.} \\ \text{Multiply:} & 3 \times 8 = 24. \text{ Write } 24 \text{ below } 25. \\ \text{Subtract:} & 25 - 24 = 1 \end{cases}$

Sample Set C

Find $726 \div 23$.

Solution

$$\begin{array}{r} 31 \\ 23\overline{)726} \\ \underline{69\downarrow} \\ 36 \\ \underline{23} \\ 13 \end{array}$$

Check: Multiply 31 by 23, then add 13.

$$\begin{array}{r} 31 \\ \times\ 23 \\ \hline 93 \\ 62 \\ \hline 713 \\ +\ 13 \\ \hline 726 \end{array}$$

Thus, $726 \div 23 = 31R13$.

Practice Set C

Perform the following divisions.

$75 \div 4$

Answer available at the end of this section.

Practice Set C

$346 \div 8$

Answer available at the end of this section.

Practice Set C

$489 \div 21$

Answer available at the end of this section.

Practice Set C

$5,016 \div 82$

Answer available at the end of this section.

Practice Set C

$41,196 \div 67$

Answer available at the end of this section.

Calculators

The calculator can be useful for finding quotients with single and multiple digit divisors. If, however, the division should result

in a remainder, the calculator is unable to provide us with the particular value of the remainder. Also, some calculators (most nonscientific) are unable to perform divisions in which one of the numbers has more than eight digits.

Sample Set D

Use a calculator to perform each division.

$328 \div 8$.

Solution

Type	328
Press	÷
Type	8
Press	=

The display now reads 41.

Sample Set D

$53,136 \div 82$.

Solution

Type	53136
Press	÷
Type	82
Press	=

The display now reads 648.

Sample Set D

$730,019,001 \div 326$

Solution

We first try to enter 730,019,001 but find that we can only enter 73001900. If our calculator has only an eight-digit display (as most nonscientific calculators do), we will be unable to use the calculator to perform this division.

Sample Set D

$3727 \div 49$.

Solution

Type	3727
Press	÷

Type	49
Press	=

The display now reads 76.061224.

This number is an example of a decimal number (see [link]). When a decimal number results in a calculator division, we can conclude that the division produces a remainder.

Practice Set D

Use a calculator to perform each division.

$3,330 \div 74$

Answer available at the end of this section.

Practice Set D

$63,365 \div 115$

Answer available at the end of this section.

Practice Set D

$21,996,385,287 \div 53$

Answer available at the end of this section.

Practice Set D

$4,558 \div 67$

Answer available at the end of this section.

Exercises

For the following problems, perform the divisions.

The first 38 problems can be checked with a calculator by multiplying the divisor and quotient then adding the remainder.

Exercise 2.4.1

$52 \div 4$

Answer available at the end of this section.

Exercise 2.4.2

$776 \div 8$

Exercise 2.4.3

$603 \div 9$

Answer available at the end of this section.

Exercise 2.4.4

$240 \div 8$

Exercise 2.4.5

$208 \div 4$

Answer available at the end of this section.

Exercise 2.4.6

$576 \div 6$

Exercise 2.4.7

$21 \div 7$

Answer available at the end of this section.

Exercise 2.4.8

$0 \div 0$

Exercise 2.4.9

$140 \div 2$

Answer available at the end of this section.

Exercise 2.4.10

$528 \div 8$

Exercise 2.4.11

$244 \div 4$

Answer available at the end of this section.

Exercise 2.4.12

$0 \div 7$

Exercise 2.4.13

$177 \div 3$

Answer available at the end of this section.

Exercise 2.4.14

$96 \div 8$

Exercise 2.4.15

$67 \div 1$

Answer available at the end of this section.

Exercise 2.4.16

$896 \div 56$

Exercise 2.4.17

$1,044 \div 12$

Answer available at the end of this section.

Exercise 2.4.18

$988 \div 19$

Exercise 2.4.19

$5,238 \div 97$

Answer available at the end of this section.

Exercise 2.4.20

$2530 \div 55$

Exercise 2.4.21

$4,264 \div 82$

Answer available at the end of this section.

Exercise 2.4.22

$637 \div 13$

Exercise 2.4.23

$3,420 \div 90$

Answer available at the end of this section.

Exercise 2.4.24

$5,655 \div 87$

Exercise 2.4.25

$2,115 \div 47$

Answer available at the end of this section.

Exercise 2.4.26

$9,328 \div 22$

Exercise 2.4.27

$55,167 \div 71$

Answer available at the end of this section.

Exercise 2.4.28

$68,356 \div 92$

Exercise 2.4.29

$27,702 \div 81$

Answer available at the end of this section.

Exercise 2.4.30

$6,510 \div 31$

Exercise 2.4.31

$60,536 \div 94$

Answer available at the end of this section.

Exercise 2.4.32

$31,844 \div 38$

Exercise 2.4.33

$23,985 \div 45$

Answer available at the end of this section.

Exercise 2.4.34

$60,606 \div 74$

Exercise 2.4.35

$2,975,400 \div 285$

Answer available at the end of this section.

Exercise 2.4.36

$1,389,660 \div 795$

Exercise 2.4.37

$7,162,060 \div 879$

Answer available at the end of this section.

Exercise 2.4.38

$7,561,060 \div 909$

Exercise 2.4.39

$38 \div 9$

Answer available at the end of this section.

Exercise 2.4.40

$97 \div 4$

Exercise 2.4.41

$199 \div 3$

Answer available at the end of this section.

Exercise 2.4.42

$573 \div 6$

Exercise 2.4.43

$10,701 \div 13$

Answer available at the end of this section.

Exercise 2.4.44

$13,521 \div 53$

Exercise 2.4.45

$3,628 \div 90$

Answer available at the end of this section.

Exercise 2.4.46

$10,592 \div 43$

Exercise 2.4.47

$19,965 \div 30$

Answer available at the end of this section.

Exercise 2.4.48

$8,320 \div 21$

Exercise 2.4.49

$61,282 \div 64$

Answer available at the end of this section.

Exercise 2.4.50

$1,030 \div 28$

Exercise 2.4.51

$7,319 \div 11$

Answer available at the end of this section.

Exercise 2.4.52

$3,628 \div 90$

Exercise 2.4.53

$35,279 \div 77$

Answer available at the end of this section.

Exercise 2.4.54

$52,196 \div 68$

Exercise 2.4.55

$67,751 \div 68$

Answer available at the end of this section.

For the following 5 problems, use a calculator to find the quotients.

Exercise 2.4.56

$4,346 \div 53$

Exercise 2.4.57

$3,234 \div 77$

Answer available at the end of this section.

Exercise 2.4.58

$6,771 \div 37$

Exercise 2.4.59

$4,272,320 \div 520$

Answer available at the end of this section.

Exercise 2.4.60

$7,558,110 \div 651$

Exercise 2.4.61

A mathematics instructor at a high school is paid $17,775 for 9 months. How much money does this instructor make each month?

Answer available at the end of this section.

Exercise 2.4.62

A couple pays $4,380 a year for a one-bedroom apartment. How much does this couple pay each month for this apartment?

Exercise 2.4.63

Thirty-six people invest a total of $17,460 in a particular stock. If they each invested the same amount, how much did each person invest?

Answer available at the end of this section.

Exercise 2.4.64

Each of the 28 students in a mathematics class buys a textbook. If the bookstore sells $644 worth of books, what is the price of each book?

Exercise 2.4.65

A certain brand of refrigerator has an automatic ice cube maker that makes 336 ice cubes in one day. If the ice machine makes ice cubes at a constant rate, how many ice cubes does it make each hour?

Answer available at the end of this section.

Exercise 2.4.66

A beer manufacturer bottles 52,380 ounces of beer each hour. If each bottle contains the same number of ounces of beer, and the manufacturer fills 4,365 bottles per hour, how many ounces of beer does each bottle contain?

Exercise 2.4.67

A computer program consists of 68,112 bits. 68,112 bits equals 8,514 bytes. How many bits in one byte?

Answer available at the end of this section.

Exercise 2.4.68

A 26-story building in San Francisco has a total of 416 offices. If each floor has the same number of offices, how many floors does this building have?

Exercise 2.4.69

A college has 67 classrooms and a total of 2,546 desks. How many desks are in each classroom if each classroom has the same number of desks?

Answer available at the end of this section.

Exercises for Review

Exercise 2.4.70

What is the value of 4 in the number 124,621?

Exercise 2.4.71

Round 604,092 to the nearest hundred thousand.

Answer available at the end of this section.

Exercise 2.4.72

What whole number is the additive identity?

Exercise 2.4.73

Find the product. $6,256 \times 100$.

Answer available at the end of this section.

Exercise 2.4.74

Find the quotient. $0 \div 11$

Answers

Exercise
18,760

Practice Set A
18

Practice Set A
81

Practice Set A
853

Practice Set A
1,129

Practice Set A
84,125

Practice Set B
43

Practice Set B
112

Practice Set B
305

Practice Set B
3,016

Practice Set C
18 R3

Practice Set C
43 R2

Practice Set C
23 R6

Practice Set C
61 R14

Practice Set C
614 R58

Practice Set D
45

Practice Set D
551

Practice Set D
Since the dividend has more than eight digits, this division cannot be performed on most nonscientific calculators. On others, the answer is 415,026,137.4

Practice Set D
This division results in 68.02985075, a decimal number, and therefore, we cannot, at this time, find the value of the remainder. Later, we will discuss decimal numbers.

Exercise 2.4.1
13

Exercise 2.4.3
67

Exercise 2.4.5
52

Exercise 2.4.7
3

Exercise 2.4.9
70

Exercise 2.4.11
61

Exercise 2.4.13
59

Exercise 2.4.15
67

Exercise 2.4.17
87

Exercise 2.4.19
54

Exercise 2.4.21
52

Exercise 2.4.23
38

Exercise 2.4.25
45

Exercise 2.4.27
777

Exercise 2.4.29
342

Exercise 2.4.31
644

Exercise 2.4.33
533

Exercise 2.4.35
10,440

Exercise 2.4.37
8,147 remainder 847

Exercise 2.4.39
4 remainder 2

Exercise 2.4.41
66 remainder 1

Exercise 2.4.43
823 remainder 2

Exercise 2.4.45
40 remainder 28

Exercise 2.4.47
665 remainder 15

Exercise 2.4.49
957 remainder 34

Exercise 2.4.51
665 remainder 4

Exercise 2.4.53
458 remainder 13

Exercise 2.4.55
996 remainder 23

Exercise 2.4.57
42

Exercise 2.4.59
8,216

Exercise 2.4.61

$1,975 per month

Exercise 2.4.63

$485 each person invested

Exercise 2.4.65

14 cubes per hour

Exercise 2.4.67

8 bits in each byte

Exercise 2.4.69

38

Exercise 2.4.71

600,000

Exercise 2.4.73

625,600

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2.4.1: Practice and Videos for Basic Multiplication and Division

Learning Objectives

- I can demonstrate my understanding of basic multiplication facts ranging from 0 to 12.
- I can divide single-digit numbers.
- I can understand division visually.

GeoGebra: Deciding Division

Complete the activity below to see how division works visually. Start by entering two whole numbers into the empty boxes. Then, move the slider to see what happens when each is the dividend and divisor.

Note: The simulation works better with smaller numbers.

Videos on Basic Multiplication and Division

Watch the following videos to learn about multiplication and division. All videos have appropriate captions in English; in some cases, there may also be appropriate captions in other languages.

YouTube Video

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

YouTube Video

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

Practice: Math Trainer - Multiplication

Check your understanding of basic multiplication facts by visiting this website: Math is Fun: Math Trainer - Multiplication

How to play:

- Adjust the duration of the practice by clicking the drop-down menu next to Workout Time.
- Select your question cutoff time. This determines how much time you will have to answer each question. Start at 4 seconds and decrease the cutoff each game.

- Next, click on the "new game" button at the top and play until the timer runs out. Use the keyboard or on-screen keypad to type your answers.

Extra Practice Resources

1. IXL: Relate multiplication and division

2. Spark Studio Jeopardy: Basic Multiplication and Division

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2.5: Adding and Subtracting Fractions

Paul and Tony order a pizza which has been cut into eight equal slices. Thus, each slice is 1/8 of the whole pizza. Paul eats two slices (shaded in light gray in Figure 2.5.1), or 2/8 of the whole pizza. Tony eats three slices (shaded in light red (or a darker shade of gray in black-and-white printing) in Figure 2.5.1), or 3/8 of the whole pizza.

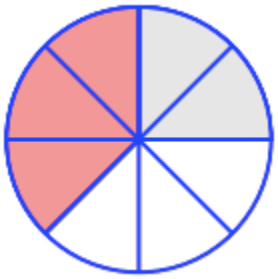


Figure 2.5.1: Paul eats two slices (2/8) and Tony eats three slices (3/8).

It should be clear that together Paul and Tony eat five slices, or 5/8 of the whole pizza. This reflects the fact that

$$\frac{2}{8}+\frac{3}{8}=\frac{5}{8}.$$

This demonstrates how to add two fractions with a common (same) denominator. Keep the common denominator and add the numerators. That is,

$$\begin{aligned}\frac{2}{8}+\frac{3}{8}&=\frac{2+3}{8} && \text{Keep denominator; add numerators.}\\ &=\frac{5}{8} && \text{Simplify numerator.}\end{aligned}$$

Adding Fractions with Common Denominators

Let *a/c* and *b/c* be two fractions with a common (same) denominator. Their sum is defined as

$$\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c}$$

That is, to add two fractions having common denominators, keep the common denominator and add their numerators.

A similar rule holds for subtraction.

Subtracting Fractions with Common Denominators

Let *a/c* and *b/c* be two fractions with a common (same) denominator. Their difference is defined as

$$\frac{a}{c}-\frac{b}{c}=\frac{a-b}{c}.$$

That is, to subtract two fractions having common denominators, keep the common denominator and subtract their numerators.

Example 2.5.1

Find the sum of 4/9 and 3/9.

Solution

Keep the common denominator and add the numerators.

$$\frac{4}{9}+\frac{3}{9}=\frac{4+3}{9} \qquad \text{Keep denominator; add numerators.}$$
$$=\frac{7}{9} \qquad \text{Simplify numerator.}$$

Exercise 2.5.1

Add:

$$\frac{1}{8}+\frac{2}{8}$$

Answer available at the end of this section.

Example 2.5.2

Subtract 5/16 from 13/16.

Solution

Keep the common denominator and subtract the numerators.

$$\frac{13}{16}-\frac{5}{16}=\frac{13-5}{16} \qquad \text{Keep denominator; subtract numerators.}$$
$$=\frac{8}{16} \qquad \text{Simplify numerator.}$$

Of course, as we learned in Section 4.1, we should always reduce our final answer to lowest terms. One way to accomplish that in this case is to divide numerator and denominator by 8, the greatest common divisor of 8 and 16.

$$=\frac{8\div 8}{16\div 8} \quad \text{Divide numerator and denominator by 8.}$$
$$=\frac{1}{2} \quad \text{Simplify numerator and denominator.}$$

Exercise 2.5.2

Subtract:

$$\frac{11}{12}-\frac{7}{12}$$

Answer available at the end of this section.

Example 2.5.3

Simplify:

$$\frac{3}{x}-\left(-\frac{7}{x}\right).$$

Solution

Both fractions share a common denominator.

$$\begin{aligned}\frac{3}{x}-\left(-\frac{7}{x}\right)&=\frac{3}{x}+\frac{7}{x} && \text{Add the opposite.}\\ &=\frac{3+7}{x} && \text{Keep denominator, add numerators.}\\ &=\frac{10}{x} && \text{Simplify.}\end{aligned}$$

Adding Fractions with Different Denominators

Consider the sum

$$\frac{4}{9}+\frac{1}{6}.$$

We cannot add these fractions because they do not have a common denominator. So, what to do?

Goals

In order to add two fractions with different denominators, we need to:

1. Find a common denominator for the given fractions.
2. Make fractions with the common denominator that are equivalent to the original fractions.

If we accomplish the two items in the "Goal," we will be able to find the sum of the given fractions.

So, how to start? We need to find a common denominator, but not just any common denominator. Let's agree that we want to keep the numbers as small as possible and find a *least common denominator*.

Definition: Least Common Denominator

The *least common denominator* (LCD) for a set of fractions is the smallest number divisible by each of the denominators of the given fractions.

Consider again the sum we wish to find:

$$\frac{4}{9}+\frac{1}{6}.$$

The denominators are 9 and 6. We wish to find a least common denominator, the smallest number that is divisible by both 9 and 6. A number of candidates come to mind: 36, 54, and 72 are all divisible by 9 and 6, to name a few. But the smallest number that is divisible by both 9 and 6 is 18. This is the least common denominator for 9 and 6.

We now proceed to the second item in "Goal." We need to make fractions having 18 as a denominator that are equivalent to 4/9 and 1/6. In the case of 4/9, if we multiply both numerator and denominator by 2, we get

$$\begin{aligned}\frac{4}{9}&=\frac{4\cdot 2}{9\cdot 2} && \text{Multiply numerator and denominator by 2.}\\ &=\frac{8}{18}. && \text{Simplify numerator and denominator.}\end{aligned}$$

In the case of 1/6, if we multiply both numerator and denominator by 3, we get

$$\begin{aligned} \frac{1}{6} &= \frac{1 \cdot 3}{6 \cdot 3} && \text{Multiply numerator and denominator by 3.} \\ &= \frac{3}{18}. && \text{Simplify numerator and denominator.} \end{aligned}$$

Typically, we'll arrange our work as follows.

$$\begin{aligned} \frac{4}{+}\frac{1}{6} &= \frac{4 \cdot 2}{9 \cdot 2} + \frac{1 \cdot 3}{6 \cdot 3} && \text{Equivalent fractions with LCD} = 18. \\ &= \frac{8}{18} + \frac{3}{18} && \text{Simplify numerators and denominators.} \\ &= \frac{8+3}{18} && \text{Keep common denominator; add numerators.} \\ &= \frac{11}{18} && \text{Simplify numerator.} \end{aligned}$$

Let's summarize the procedure.

Adding or Subtracting Fractions with Different Denominators

1. Find the LCD, the smallest number divisible by all the denominators of the given fractions.
2. Create fractions using the LCD as the denominator that are equivalent to the original fractions.
3. Add or subtract the resulting equivalent fractions. Simplify, including reducing the final answer to lowest terms.

Example 2.5.4

Simplify: $\frac{3}{5} - \frac{2}{3}$.

Solution

The smallest number divisible by both 5 and 3 is 15.

$$\begin{aligned} \frac{3}{5} - \frac{2}{3} &= \frac{3 \cdot 3}{5 \cdot 3} - \frac{2 \cdot 5}{3 \cdot 5} && \text{Equivalent fractions with LCD} = 15. \\ &= \frac{9}{15} - \frac{10}{15} && \text{Simplify numerators and denominators.} \\ &= \frac{9-10}{15} && \text{Keep LCD; subtract numerators.} \\ &= \frac{-1}{15} && \text{Simplify numerator.} \end{aligned}$$

Although this answer is perfectly acceptable, negative divided by positive gives us a negative answer, so we could also write

$$= -\frac{1}{15}.$$

Exercise 2.5.4

Subtract:

$$\frac{3}{4} - \frac{7}{5}$$

Answer available at the end of this section.

Example 2.5.5

Simplify: $-\frac{1}{4}-\frac{5}{6}$.

Solution

The smallest number divisible by both 4 and 6 is 12.

$$\begin{aligned} -\frac{1}{4}-\frac{5}{6} &= -\frac{1\cdot 3}{4\cdot 3}-\frac{5\cdot 2}{6\cdot 2} && \text{Equivalent fractions with LCD } =12. \\ &= -\frac{3}{12}-\frac{10}{12} && \text{Simplify numerators and denominators.} \\ &= \frac{-3-10}{12} && \text{Keep LCD; subtract numerators.} \\ &= \frac{-13}{12} && \text{Simplify numerator.} \end{aligned}$$

Exercise 2.5.5

Subtract: $-\frac{3}{8}-\frac{1}{12}$

Answer available at the end of this section.

Example 2.5.6

Simplify: $\frac{5}{x}+\frac{3}{4}$.

Solution

The smallest number divisible by both 4 and x is 4x.

$$\begin{aligned} \frac{5}{x}+\frac{3}{4} &= \frac{5\cdot 4}{x\cdot 4}+\frac{3\cdot x}{4\cdot x} && \text{Equivalent fractions with LCD } = 4x. \\ &== \frac{20}{4x}+\frac{3x}{4x} && \text{Simplify numerators and denominators.} \\ &= \frac{20+3x}{4x} && \text{Keep LCD; add numerators.} \end{aligned}$$

Exercise 2.5.6

Add:

$$\frac{5}{z}+\frac{2}{3}$$

Answer available at the end of this section.

Example 2.5.7

Simplify: $\frac{2}{3}-\frac{x}{5}$.

Solution

The smallest number divisible by both 3 and 5 is 15.

$$\begin{aligned}\frac{2}{3}-\frac{x}{5} &= \frac{2\cdot 5}{3\cdot 5}-\frac{x\cdot 3}{5\cdot 3} && \text{Equivalent fractions with LCD} = 15.\\ &= \frac{10}{15}-\frac{3x}{15} && \text{Simplify numerators and denominators.}\\ &= \frac{10-3x}{15} && \text{Keep LCD; subtract numerators.}\end{aligned}$$

Least Common Multiple

First we define the multiple of a number.

Definition: Multiples

The **multiples** of a number *d* are 1*d*, 2*d*, 3*d*, 4*d*, etc. That is, the multiples of *d* are the numbers *nd*, where n is a natural number.

For example, the multiples of 8 are $1\cdot 8$, $2\cdot 8$, $3\cdot 8$, $4\cdot 8$, etc., or equivalently, 8, 16, 24, 32, etc.

Definition: Least Common Multiple

The **least common multiple** (LCM) of a set of numbers is the smallest number that is a multiple of each number of the given set. The procedure for finding an LCM follows:

1. List all of the multiples of each number in the given set of numbers.
2. List the multiples that are in common.
3. Pick the least of the multiples that are in common.

Example 2.5.7

Find the least common multiple (LCM) of 12 and 16.

Solution

List the multiples of 12 and 16.

Multiples of 12 : 12, 24, 36, 48, 60, 72, 84, 96,...

Multiples of 16 : 16, 32, 48, 64, 80, 96, 112,...

Pick the common multiples.

Common Multiples : 48, 96,...

The LCM is the least of the common multiples.

LCM(12,16) = 48

Exercise 2.5.7

Find the least common denominator of 6 and 9.

Answer available at the end of this section.

Important Observation

The *least common denominator* is the *least common multiple* of the denominators.

For example, suppose your problem is 5/12 + 5/16. The LCD is the smallest number divisible by both 12 and 16. That number is 48, which is also the LCM of 12 and 16. Therefore, the procedure for finding the LCM can also be used to find the LCD.

Least Common Multiple Using Prime Factorization

You can also find the LCM using prime factorization.

LCM By Prime Factorization

To find an LCM for a set of numbers, follow this procedure:

1. Write down the prime factorization for each number in compact form using exponents.
2. The LCM is found by writing down every factor that appears in step 1 to the highest power of that factor that appears.

Example 2.5.8

Use prime factorization to find the least common multiple find the least common denominator of 18 and 24. (LCM) of 12 and 16.

Solution

Prime factor 12 and 16.

$$12 = 2 \cdot 2 \cdot 3$$
$$16 = 2 \cdot 2 \cdot 2 \cdot 2$$

Write the prime factorizations in compact form using exponents.

$$12 = 2^2 \cdot 3^1$$
$$16 = 2^4$$

To find the LCM, write down each factor that appears to the highest power of that factor that appears. The factors that appear are 2 and 3. The highest power of 2 that appears is 2^4. The highest power of 3 that appears is 3^1.

$$\text{LCM} = 2^4 \cdot 3^1 \quad \text{Keep highest power of each factor.}$$

Now we expand this last expression to get our LCM.

$$= 16 \cdot 3 \quad \text{Expand: } 2^4 = 16 \text{ and } 3^1 = 3.$$
$$= 48. \quad \text{Multiply.}$$

Note that this answer is identical to the LCM found in Example 8 that was found by listing multiples and choosing the smallest multiple in common.

Exercise 2.5.8

Use prime factorization to find the least common denominator of 18 and 24.

Answer available at the end of this section.

Example 2.5.10

Simplify: $\frac{5}{28}+\frac{11}{42}$.

Solution

Prime factor the denominators in compact form using exponents.

$$28 = 2 \cdot 2 \cdot 7 = 2^2 \cdot 7$$

$$42 = 2 \cdot 3 \cdot 7 = 2^1 \cdot 3^1 \cdot 7^1$$

To find the LCD, write down each factor that appears to the highest power of that factor that appears. The factors that appear are 2, 3, and 7. The highest power of 2 that appears is 2^2. The highest power of 3 that appears is 3^1. The highest power of 7 that appears is 7^1.

$$\begin{aligned} \text{LCM} &= 2^2 \cdot 3^1 \cdot 7^1 && \text{Keep highest power of each factor.} \\ &= 4 \cdot 3 \cdot 7 && \text{Expand: } 2^2 = 4,\ 3^1 = 3,\ 7^1 = 7. \\ &= 84 && \text{Multiply.} \end{aligned}$$

Create equivalent fractions with the new LCD, then add.

$$\begin{aligned} \frac{5}{28}+\frac{11}{42} &= \frac{5 \cdot 3}{28 \cdot 3}+\frac{11 \cdot 2}{42 \cdot 2} && \text{Equivalent fractions with LCD} = 84. \\ &= \frac{15}{84}+\frac{22}{84} && \text{Simplify numerators and denominators.} \\ &= \frac{37}{84} && \text{Keep LCD; add numerators.} \end{aligned}$$

Exercise 2.5.10

Simplify: $\frac{5}{24}+\frac{5}{36}$

Answer available at the end of this section.

Example 2.5.11

Simplify: $-\frac{11}{24}-\frac{1}{18}$.

Solution

Prime factor the denominators in compact form using exponents.

$$24 = 2 \cdot 2 \cdot 2 \cdot 3 = 2^3 \cdot 3^1$$

$$18 = 2 \cdot 3 \cdot 3 = 2^1 \cdot 3^2$$

To find the LCD, write down each factor that appears to the highest power of that factor that appears. The factors that appear are 2 and 3. The highest power of 2 that appears is 2^3. The highest power of 3 that appears is 3^2.

$$\begin{aligned} \text{LCM} &= 2^3 \cdot 3^2 && \text{Keep highest power of each factor.} \\ &= 8 \cdot 9 && \text{Expand: } 2^3 = 8 \text{ and } 3^2 = 9. \\ &= 72. && \text{Multiply.} \end{aligned}$$

Create equivalent fractions with the new LCD, then subtract.

$$\begin{aligned} -\frac{11}{24}-\frac{1}{18} &= -\frac{11\cdot 3}{24\cdot 3}-\frac{1\cdot 4}{18\cdot 4} && \text{Equivalent fractions with LCD} = 72. \\ &= -\frac{33}{72}-\frac{4}{72} && \text{Simplify numerators and denominators.} \\ &= \frac{-33-4}{72} && \text{Keep LCD; subtract numerators.} \\ &= \frac{-37}{72} && \text{Simplify numerator.} \end{aligned}$$

Of course, negative divided by positive yields a negative answer, so we can also write our answer in the form

$$-\frac{11}{24}-\frac{1}{18}=-\frac{37}{72}.$$

Exercise 2.5.11

Simplify: $-\frac{5}{24}-\frac{11}{36}$

Answer available at the end of this section.

Comparing Fractions

The simplest way to compare fractions is to create equivalent fractions.

Example 2.5.12

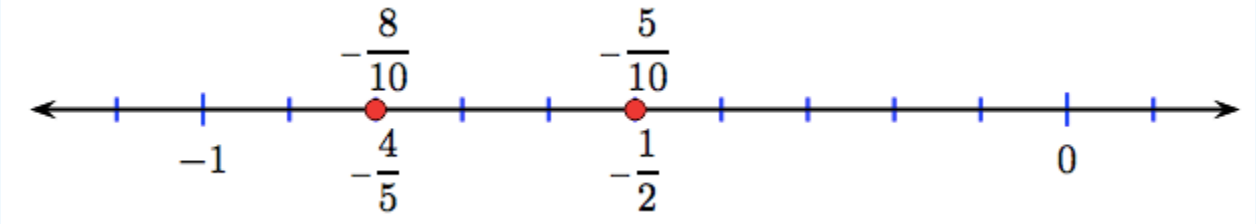
Arrange the fractions –1/2 and –4/5 on a number line, then compare them by using the appropriate inequality symbol.

Solution

The least common denominator for 2 and 5 is the number 10. First, make equivalent fractions with a LCD equal to 10.

$$-\tfrac{1}{2}=-\tfrac{1\cdot 5}{2\cdot 5}=-\tfrac{5}{10}$$
$$-\tfrac{4}{5}=-\tfrac{4\cdot 2}{5\cdot 2}=-\tfrac{8}{10}$$

To plot tenths, subdivide the interval between –1 and 0 into ten equal increments.



Because –4/5 lies to the left of –1/2, we have that –4/5 is less than –1/2, so we write

$$-\frac{4}{5}<-\frac{1}{2}.$$

Exercise 2.5.12

Compare –3/8 and –1/2.

Answer available at the end of this section.

Exercises

In Exercises 1-10, list the multiples the given numbers, then list the common multiples. Select the LCM from the list of common multiples.

1. 9 and 15
2. 15 and 20
3. 20 and 8
4. 15 and 6
5. 16 and 20
6. 6 and 10
7. 20 and 12
8. 12 and 8
9. 10 and 6
10. 10 and 12

In Exercises 11-20, for the given numbers, calculate the LCM using prime factorization.

11. 54 and 12
12. 108 and 24
13. 18 and 24
14. 36 and 54
15. 72 and 108
16. 108 and 72
17. 36 and 24
18. 18 and 12
19. 12 and 18
20. 12 and 54

In Exercises 21-32, add or subtract the fractions, as indicated, and simplify your result.

21. $\frac{7}{12} - \frac{1}{12}$

22. $\frac{3}{7} - \frac{5}{7}$
23. $\frac{1}{9} + \frac{1}{9}$
24. $\frac{1}{7} + \frac{3}{7}$
25. $\frac{1}{5} - \frac{4}{5}$
26. $\frac{3}{5} - \frac{2}{5}$
27. $\frac{3}{7} - \frac{4}{7}$
28. $\frac{6}{7} - \frac{2}{7}$
29. $\frac{4}{11} + \frac{9}{11}$
30. $\frac{10}{11} + \frac{4}{11}$
31. $\frac{3}{11} + \frac{4}{11}$
32. $\frac{3}{7} + \frac{2}{7}$

In Exercises 33-56, add or subtract the fractions, as indicated, and simplify your result.

33. $\frac{1}{6} - \frac{1}{8}$
34. $\frac{7}{9} - \frac{2}{3}$
35. $\frac{1}{5} + \frac{2}{3}$
36. $\frac{7}{9} + \frac{2}{3}$
37. $\frac{2}{3} + \frac{5}{8}$
38. $\frac{3}{7} + \frac{5}{9}$
39. $\frac{4}{7} - \frac{5}{9}$
40. $\frac{3}{5} - \frac{7}{8}$
41. $\frac{2}{3} - \frac{3}{8}$
42. \(\frac{2}{5} – \frac{1}{8\)
43. $\frac{6}{7} - \frac{1}{6}$
44. $\frac{1}{2} - \frac{1}{4}$
45. $\frac{1}{6} + \frac{2}{3}$
46. $\frac{4}{9} + \frac{7}{8}$
47. $\frac{7}{9} + \frac{1}{8}$
48. $\frac{1}{6} + \frac{1}{7}$
49. $\frac{1}{3} + \frac{1}{7}$
50. $\frac{5}{6} + \frac{1}{4}$

51. $\frac{1}{2} - \frac{2}{7}$

52. $\frac{1}{3} - \frac{1}{8}$

53. $\frac{5}{6} - \frac{4}{5}$

54. $\frac{1}{2} - \frac{1}{9}$

55. $\frac{1}{3} + \frac{1}{8}$

56. $\frac{1}{6} + \frac{7}{9}$

In Exercises 57-68, add or subtract the fractions, as indicated, by first using prime factorization to find the least common denominator.

57. $\frac{7}{36} + \frac{11}{54}$

58. $\frac{7}{54} + \frac{7}{24}$

59. $\frac{7}{18} - \frac{5}{12}$

60. $\frac{5}{54} - \frac{7}{12}$

61. $\frac{7}{36} + \frac{7}{54}$

62. $\frac{5}{72} + \frac{5}{108}$

63. $\frac{7}{24} - \frac{5}{36}$

64. $\frac{11}{54} + \frac{7}{72}$

65. $\frac{11}{12} + \frac{5}{18}$

66. $\frac{11}{24} + \frac{11}{108}$

67. $\frac{11}{54} - \frac{5}{24}$

68. $\frac{7}{54} - \frac{5}{24}$

In Exercises 69-80, add or subtract the fractions, as indicated, and simplify your result.

69. $\frac{-3}{7} + \left(\frac{-3}{7}\right)$

70. $\frac{-5}{9} + \left(\frac{-1}{9}\right)$

71. $\frac{7}{9} - \left(\frac{-1}{9}\right)$

72. $\frac{8}{9} - \left(\frac{-4}{9}\right)$

73. $\frac{7}{9} + \left(\frac{-2}{9}\right)$

74. $\frac{2}{3} + \left(\frac{-1}{3}\right)$

75. $\frac{-3}{5} - \frac{4}{5}$

76. $\frac{-7}{9} - \frac{1}{9}$

77. $\frac{-7}{8} + \frac{1}{8}$

78. $\frac{-2}{3} + \(\frac{1}{3}$

79. $\frac{-1}{3} - \left(\frac{-2}{3}\right)$

80. $\frac{-7}{8} - \left(\frac{-5}{8}\right)$

In Exercises 81-104, add or subtract the fractions, as indicated, and simplify your result.

81. $\frac{-2}{7}$ + \frac{4}{5}\)

82. $\frac{-1}{4} + \frac{2}{7}$

83. $\frac{-1}{4} - \left(\frac{-4}{9}\right)$

84. Missing \left or extra \right

85. $\frac{-2}{7} + \frac{3}{4}$

86. $\frac{-1}{3} + \frac{5}{8}$

87. $\frac{-4}{9} - \frac{1}{3}$

88. $\frac{-5}{6} - \frac{1}{3}$

89. $\frac{-5}{7} - \left(\frac{-1}{5}\right)$

90. $\frac{-6}{7} - \left(\frac{-1}{8}\right)$

91. $\frac{1}{9} + \left(\frac{-1}{3}\right)$

92. $\frac{1}{8} + \left(\frac{-1}{2}\right)$

93. $\frac{2}{3} + \left(\frac{-1}{9}\right)$

94. $\frac{3}{4} + \left(\frac{-2}{3}\right)$

95. $\frac{-1}{2} + \left(\frac{-6}{7}\right)$

96. $\frac{-4}{5} + \left(\frac{-1}{2}\right)$

97. $\frac{-1}{2} + \left(\frac{-3}{4}\right)$

98. $\frac{-3}{5} + \left(\frac{-1}{2}\right)$

99. $\frac{-1}{4} - \frac{1}{2}$

100. $\frac{-8}{9} - \frac{2}{3}$

101. $\frac{5}{8} - \left(\frac{-3}{4}\right)$

102. $\frac{3}{4} - \left(\frac{-3}{8}\right)$

103. $\frac{1}{8} - \left(\frac{-1}{3}\right)$

104. $\frac{1}{2} - \left(\frac{-4}{9}\right)$

In Exercises 105-120, add or subtract the fractions, as indicated, and write your answer is lowest terms.

105. $\frac{1}{2}+\frac{3q}{5}$

106. $\frac{4}{7}-\frac{b}{3}$

107. $\frac{4}{9}-\frac{3a}{4}$

108. $\frac{4}{9}-\frac{b}{2}$

109. $\frac{2}{s}+\frac{1}{3}$

110. $\frac{2}{s}+\frac{3}{7}$

111. $\frac{1}{3}-\frac{7}{b}$

112. $\frac{1}{2}-\frac{9}{s}$

113. $\frac{4b}{7}+\frac{2}{3}$

114. $\frac{2a}{5}+\frac{5}{8}$

115. $\frac{2}{3}-\frac{9}{t}$

116. $\frac{4}{7}-\frac{1}{y}$

117. $\frac{9}{s}+\frac{7}{8}$

118. $\frac{6}{t}-\frac{1}{9}$

119. $\frac{7b}{8}-\frac{5}{9}$

120. $\frac{3p}{4}-\frac{1}{8}$

In Exercises 121-132, determine which of the two given statements is true.

121. $\frac{-2}{3}<\frac{-8}{7}$ or $\frac{-2}{3}>\frac{-8}{7}$

122. $\frac{-1}{7}<\frac{-8}{9}$ or $\frac{-1}{7}>\frac{-8}{9}$

123. $\frac{6}{7}<\frac{7}{3}$ or $\frac{6}{7}>\frac{7}{3}$

124. $\frac{1}{2}<\frac{2}{7}$ or $\frac{1}{2}>\frac{2}{7}$

125. $\frac{-9}{4}<\frac{-2}{3}$ or \frac{– 9}{4} > \frac{–2}{3}\)

126. $\frac{-3}{7}<\frac{-9}{2}$ or $\frac{-3}{7}>\frac{-9}{2}$

127. $\frac{5}{7}<\frac{5}{9}$ or \frac{5}{7} > \frac{5}{9}\)

128. $\frac{1}{2}<\frac{1}{3}$ or $\frac{1}{2}>\frac{1}{3}$

129. $\frac{-7}{2}<\frac{-1}{5}$ or $\frac{-7}{2}>\frac{-1}{5}$

130. $\frac{-3}{4}<\frac{-5}{9}$ or $\frac{-3}{4}>\frac{-5}{9}$

131. $\frac{5}{9}<\frac{6}{5}$ or $\frac{5}{9}>\frac{6}{5}$

132. $\frac{3}{2}<\frac{7}{9}$ or $\frac{3}{2}>\frac{7}{9}$

Answers

1. 45

3. 40

5. 80

7. 60

9. 30

11. 108

13. 72

15. 216

17. 72

19. 36

21. $\frac{1}{2}$

23. $\frac{2}{9}$

25. $\frac{-3}{5}$

27. $\frac{-1}{7}$

29. $\frac{13}{11}$

31. $\frac{7}{11}$

33. $\frac{1}{24}$

35. $\frac{13}{15}$

37. $\frac{31}{24}$

39. $\frac{1}{63}$

41. $\frac{7}{24}$

43. $\frac{29}{42}$

45. $\frac{5}{6}$

47. $\frac{65}{72}$

49. $\frac{10}{21}$

51. $\frac{3}{14}$

53. $\frac{1}{30}$

55. $\frac{11}{24}$

57. $\frac{43}{108}$

59. $\frac{-1}{36}$

61. $\frac{35}{108}$

63. $\frac{11}{72}$

65. $\frac{43}{36}$

67. $\frac{-1}{216}$

69. $\frac{-6}{7}$

71. $\frac{8}{9}$

73. $\frac{5}{9}$

75. $\frac{-7}{5}$

77. $\frac{-3}{4}$

79. $\frac{1}{3}$

81. $\frac{18}{35}$

83. $\frac{7}{36}$

85. $\frac{13}{28}$

87. $\frac{-7}{9}$

89. $\frac{-18}{35}$

91. $\frac{-2}{9}$

93. $\frac{5}{9}$

95. $\frac{-19}{14}$

97. $\frac{-5}{4}$

99. $\frac{-3}{4}$

101. $\frac{11}{8}$

103. $\frac{11}{24}$

105. $\frac{5+6q}{10}$

107. $\frac{16-27a}{36}$

109. $\frac{6+s}{3s}$

111. $\frac{b-21}{3b}$

113. $\frac{12b+14}{21}$

115. $\frac{2t-27}{3t}$

117. $\frac{72+7s}{8s}$

119. $\frac{63b-40}{72}$

121. $\frac{-2}{3} > \backslash(\frac{-8}{7}$

123. $\frac{6}{7} < \frac{7}{3}$

125. $\frac{-9}{4} < \frac{-2}{3}$

127. $\frac{5}{7} > \frac{5}{9}$

129. $\frac{-7}{2} < \frac{-1}{5}$

131. $\frac{5}{9} < \frac{6}{5}$

Answers

Exercise 2.5.1

3/8

Exercise 2.5.2

1/3

Exercise 2.5.4

-13/20

Exercise 2.5.5

-11/24

Exercise 2.5.6

$$\frac{15+2z}{3z}$$

Exercise 2.5.7

18

Exercise 2.5.8

72

Exercise 2.5.10

25/72

Exercise 2.5.11

–37/72

Exercise 2.5.12

$$-\frac{1}{2} < -\frac{3}{8}$$

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2.5.1: Practice and Videos for Adding and Subtracting Fractions

Learning Objectives

- I can add fractions with unlike numerators and denominators.
- I can subtract fractions with unlike numerators and denominators.

GeoGebra: Subtracting Fractions

Complete the activity below to explore how to visually subtract fractions. Start by typing a number in the empty boxes. Make sure the number you type in the first 3 boxes to the left are larger than the numbers you type in the last three boxes to the right. Some examples of numbers you can type are:

- $5\frac{3}{8} - 2\frac{1}{8}$ (2.5.1.1)
- $4\frac{5}{8} - 2\frac{1}{16}$ (2.5.1.2)

After typing in the numbers, click "start." Make observations. Click and drag on the blue and green dots to rearrange the figure. Check the box that says "show numbers" to see how the subtraction expression you typed matches the visual representation. Click reset at the upper left corner to experiment with new numbers.

Videos on Adding and Subtracting Fractions

Watch the following videos to learn how to add and subtract fractions. All videos have appropriate captions in English; in some cases, there may also be appropriate captions in other languages.

YouTube Video

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

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View this YouTube Video online

Practice!

Check your understanding of adding and subtracting fractions by answering these questions. Use the arrows at the bottom of the box to go to the next question.

Practice 2.5.1.1

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View this Interactive Content online

Extra Practice Resources

For more practice on adding and subtracting fractions, visit the following websites.

1. IXL: Add fractions with unlike denominators using models
2. IXL: Subtract fractions with unlike denominators using models

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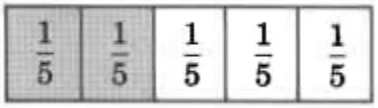
2.6: Multiplying and Dividing Fractions

Learning Objectives

- understand the concept of multiplication of fractions
- be able to multiply one fraction by another
- be able to multiply mixed numbers
- be able to find powers and roots of various fractions

Fractions of Fractions

We know that a fraction represents a part of a whole quantity. For example, two fifths of one unit can be represented by

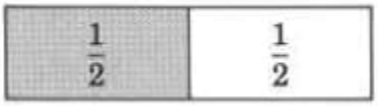


$\frac{2}{5}$ of the whole is shaded.

A natural question is, what is a fractional part of a fractional quantity, or, what is a fraction of a fraction? For example, what $\frac{2}{3}$ of $\frac{1}{2}$?

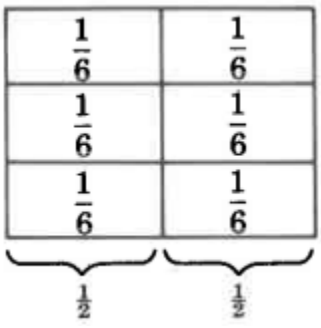
We can suggest an answer to this question by using a picture to examine $\frac{2}{3}$ of $\frac{1}{2}$.

First, let's represent $\frac{1}{2}$.



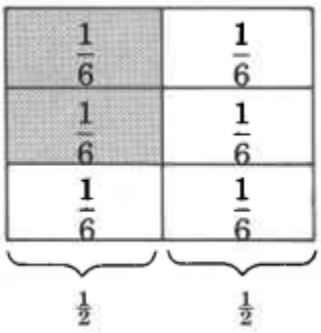
$\frac{1}{2}$ of the whole is shaded.

Then divide each of the $\frac{1}{2}$ parts into 3 equal parts.



Each part is $\frac{1}{6}$ of the whole.

Now we'll take $\frac{2}{3}$ of the $\frac{1}{2}$ unit.



$\frac{2}{3}$ of $\frac{1}{2}$ is $\frac{2}{6}$, which reduces to $\frac{1}{3}$.

Multiplication of Fractions

Now we ask, what arithmetic operation $(+, -, \times, \div)$ will produce $\frac{2}{6}$ from $\frac{2}{3}$ of $\frac{1}{2}$?

Notice that, if in the fractions $\frac{2}{3}$ and $\frac{1}{2}$, we multiply the numerators together and the denominators together, we get precisely $\frac{2}{6}$.

$$\frac{2 \cdot 1}{3 \cdot 2} = \frac{2}{6}$$

This reduces to $\frac{1}{3}$ as before.

Using this observation, we can suggest the following:

> The word "of" translates to the arithmetic operation "times."
> To multiply two or more fractions, multiply the numerators together and then multiply the denominators together. Reduce if necessary.

$$\frac{\text{numerator 1}}{\text{denominator 1}} \cdot \frac{\text{numerator 2}}{\text{denominator 2}} = \frac{\text{numerator 1}}{\text{denominator 1}} \cdot \frac{\text{numerator 2}}{\text{denominator 2}}$$

Sample Set A

Perform the following multiplications.

$\frac{3}{4} \cdot \frac{1}{6} = \frac{3 \cdot 1}{4 \cdot 6} = \frac{3}{24}$ Now, reduce,

$$= \frac{\overset{1}{\cancel{3}}}{\underset{8}{\cancel{24}}} = \frac{1}{8}$$

Thus

$$\frac{3}{4} \cdot \frac{1}{6} = \frac{1}{8}$$

This means that $\frac{3}{4}$ of $\frac{1}{6}$ is $\frac{1}{8}$, that is, $\frac{3}{4}$ of $\frac{1}{6}$ of a unit is $\frac{1}{8}$ of the original unit.

Sample Set A

$\frac{3}{8} \cdot 4$. Write 4 as a fraction by writing $\frac{4}{1}$

$$\frac{3}{8} \cdot \frac{4}{1} = \frac{3 \cdot 4}{8 \cdot 1} = \frac{12}{8} = \frac{\cancel{12}^{3}}{\cancel{8}_{2}} = \frac{3}{2}$$

$$\frac{3}{8} \cdot 4 = \frac{3}{2}$$

This means that $\frac{3}{8}$ of 4 whole units is $\frac{3}{2}$ of one whole unit.

Sample Set A

$$\frac{2}{5} \cdot \frac{5}{8} \cdot \frac{1}{4} = \frac{2 \cdot 5 \cdot 1}{5 \cdot 8 \cdot 4} = \frac{\cancel{10}^{1}}{\cancel{160}_{16}} = \frac{1}{16}$$

This means that $\frac{2}{5}$ of $\frac{5}{8}$ of $\frac{1}{4}$ of a whole unit is $\frac{1}{16}$ of the original unit.

Practice Set A

Perform the following multiplications.

$\frac{2}{5} \cdot \frac{1}{6}$

Answer available at the end of this section.

Practice Set A

$\frac{1}{4} \cdot \frac{8}{9}$

Answer available at the end of this section.

Practice Set A

$\frac{4}{9} \cdot \frac{15}{16}$

Answer available at the end of this section.

Practice Set A

$(\frac{2}{3})(\frac{2}{3})$

Answer available at the end of this section.

Practice Set A

$(\frac{7}{4})(\frac{8}{5})$

Answer available at the end of this section.

Practice Set A

$\frac{5}{6} \cdot \frac{7}{8}$

Answer available at the end of this section.

Practice Set A

$\frac{2}{3} \cdot 5$

Answer available at the end of this section.

Practice Set A

$(\frac{3}{10})(10)$

Answer available at the end of this section.

Practice Set A

$\frac{3}{4} \cdot \frac{8}{9} \cdot \frac{5}{12}$

Answer available at the end of this section.

Multiplying Fractions by Dividing Out Common Factors

We have seen that to multiply two fractions together, we multiply numerators together, then denominators together, then reduce to lowest terms, if necessary. The reduction can be tedious if the numbers in the fractions are large. For example,

$$\frac{9}{16} \cdot \frac{10}{21} = \frac{9 \cdot 10}{16 \cdot 21} = \frac{90}{336} = \frac{45}{168} = \frac{15}{28}$$

We avoid the process of reducing if we divide out common factors *before* we multiply.

$$\frac{9}{16}\cdot\frac{10}{21}=\frac{\overset{3}{\cancel{9}}}{\underset{8}{\cancel{16}}}\cdot\frac{\overset{5}{\cancel{10}}}{\underset{7}{\cancel{21}}}=\frac{3\cdot 5}{8\cdot 7}=\frac{15}{56}$$

Divide 3 into 9 and 21, and divide 2 into 10 and 16. The product is a fraction that is reduced to lowest terms.

How To: The Process of Multiplication by Dividing Out Common Factors

To multiply fractions by dividing out common factors, divide out factors that are common to both a numerator and a denominator. The factor being divided out can appear in any numerator and any denominator.

Sample Set A

Perform the following multiplications.

$\frac{4}{5}\cdot\frac{5}{6}$

$$\frac{\overset{2}{\cancel{4}}}{\underset{1}{\cancel{5}}}\cdot\frac{\overset{1}{\cancel{5}}}{\underset{1}{\cancel{6}}}=\frac{2\cdot 1}{1\cdot 3}=\frac{2}{3}$$

Divide 4 and 6 by 2
Divide 5 and 5 by 5

Sample Set A

$\frac{8}{12}\cdot\frac{8}{10}$

$$\frac{\overset{4}{\cancel{8}}}{\underset{3}{\cancel{12}}}\cdot\frac{\overset{2}{\cancel{8}}}{\underset{5}{\cancel{10}}}=\frac{4\cdot 2}{3\cdot 5}=\frac{8}{15}$$

Divide 8 and 10 by 2.
Divide 8 and 12 by 4.

Sample Set A

$$8\cdot\frac{5}{12}=\frac{\overset{2}{\cancel{8}}}{1}\cdot\frac{5}{\underset{3}{\cancel{12}}}=\frac{2\cdot 5}{1\cdot 3}=\frac{10}{3}$$

Sample Set A

$$\frac{35}{18} \cdot \frac{63}{105}$$

$$\frac{\cancel{35}^{1}}{\cancel{18}_{2}} \cdot \frac{\cancel{63}^{7}}{\cancel{105}_{3}\cancel{21}} = \frac{1 \cdot 7}{2 \cdot 3} = \frac{7}{6}$$

Sample Set A

$$\frac{13}{9} \cdot \frac{6}{39} \cdot \frac{1}{12}$$

$$\frac{\cancel{13}^{1}}{9} \cdot \frac{\cancel{6}^{2}}{\cancel{39}_{3}} \cdot \frac{1}{\cancel{12}_{6}} = \frac{1 \cdot 1 \cdot 1}{9 \cdot 1 \cdot 6} = \frac{1}{54}$$

Practice Set B

Perform the following multiplications.

$$\frac{2}{3} \cdot \frac{7}{8}$$

Answer available at the end of this section.

Practice Set B

$$\frac{25}{12} \cdot \frac{10}{45}$$

Answer available at the end of this section.

Practice Set B

$$\frac{40}{48} \cdot \frac{72}{90}$$

Answer available at the end of this section.

Practice Set B

$7 \cdot \frac{2}{49}$

Answer available at the end of this section.

Practice Set B

$12 \cdot \frac{3}{8}$

Answer available at the end of this section.

Practice Set B

$(\frac{13}{7})(\frac{14}{26})$

Answer available at the end of this section.

Practice Set B

$\frac{16}{10} \cdot \frac{22}{6} \cdot \frac{21}{44}$

Answer available at the end of this section.

Multiplication of Mixed Numbers

Multiplying Mixed Numbers
To perform a multiplication in which there are mixed numbers, it is convenient to first convert each mixed number to an improper fraction, then multiply.

Sample Set C

Perform the following multiplications. Convert improper fractions to mixed numbers.

$1\frac{1}{8} \cdot 4\frac{2}{3}$

Convert each mixed number to an improper fraction.

$1\frac{1}{8} = \frac{8 \cdot 1 + 1}{8} = \frac{9}{8}$

$4\frac{2}{3} = \frac{4 \cdot 3 + 2}{3} = \frac{14}{3}$

$\frac{\cancel{9}^{3}}{\cancel{8}_{4}} \cdot \frac{\cancel{14}^{7}}{\cancel{3}_{1}} = \frac{3 \cdot 7}{4 \cdot 1} = \frac{21}{4} = 5\frac{1}{4}$

Sample Set C

$16 \cdot 8\frac{1}{5}$

Convert $8\frac{1}{5}$ to an improper fraction.

$8\frac{1}{5} = \frac{5 \cdot 8 + 1}{5} = \frac{41}{5}$

$\frac{16}{1} \cdot \frac{41}{5}$.

There are no common factors to divide out.

$\frac{16}{1} \cdot \frac{41}{5} = \frac{16 \cdot 41}{1 \cdot 5} = \frac{656}{5} = 131\frac{1}{5}$

Sample Set C

$9\frac{1}{6} \cdot 12\frac{3}{5}$

Convert to improper fractions.

$9\frac{1}{6} = \frac{6 \cdot 9 + 1}{6} = \frac{55}{6}$

$12\frac{3}{5} = \frac{5 \cdot 12 + 3}{5} = \frac{63}{5}$

$\frac{\overset{11}{\cancel{55}}}{\underset{2}{\cancel{6}}} \cdot \frac{\overset{21}{\cancel{63}}}{\underset{1}{\cancel{5}}} = \frac{11 \cdot 21}{2 \cdot 1} = \frac{231}{2} = 115\frac{1}{2}$

Sample Set C

$\frac{11}{8} \cdot 4\frac{1}{2} \cdot 3\frac{1}{8} = \frac{11}{8} \cdot \frac{\overset{3}{\cancel{9}}}{\underset{1}{\cancel{2}}} \cdot \frac{\overset{5}{\cancel{10}}}{\underset{1}{\cancel{8}}}$

$= \frac{11 \cdot 3 \cdot 5}{8 \cdot 1 \cdot 1} = \frac{165}{8} = 20\frac{5}{8}$

Practice Set C

Perform the following multiplications. Convert improper fractions to mixed numbers.

$2\frac{2}{3} \cdot 2\frac{1}{4}$

Answer available at the end of this section.

Practice Set C

$6\frac{2}{3} \cdot 3\frac{3}{10}$

Answer available at the end of this section.

Practice Set C

$7\frac{1}{8} \cdot 12$

Answer available at the end of this section.

Practice Set C

$2\frac{2}{5} \cdot 3\frac{3}{4} \cdot 3\frac{1}{3}$

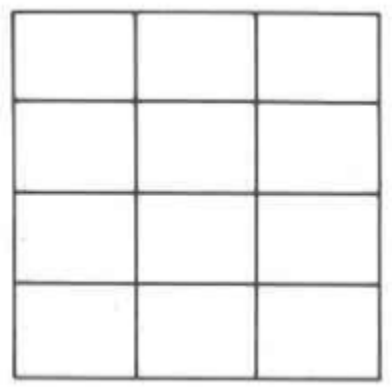
Answer available at the end of this section.

Exercises

For the following six problems, use the diagrams to find each of the following parts. Use multiplication to verify your result.

Exercise 2.6.1

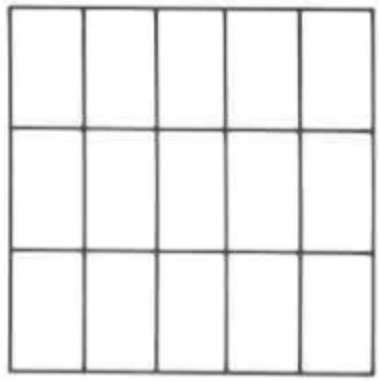
$\frac{3}{4}$ of $\frac{1}{3}$



Answer available at the end of this section.

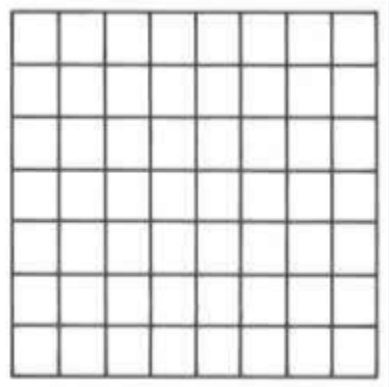
Exercise 2.6.2

$\frac{2}{3}$ of $\frac{3}{5}$



Exercise 2.6.3

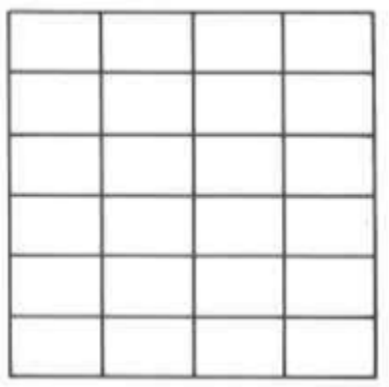
$\frac{2}{7}$ of $\frac{7}{8}$



Answer available at the end of this section.

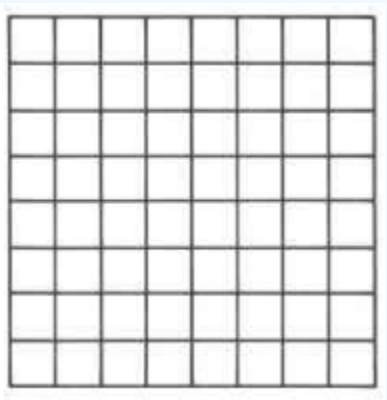
Exercise 2.6.4

$\frac{5}{6}$ of $\frac{3}{4}$



Exercise 2.6.5

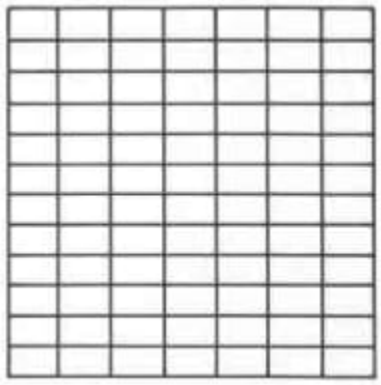
$\frac{1}{8}$ of $\frac{1}{8}$



Answer available at the end of this section.

Exercise 2.6.6

$\frac{7}{12}$ of $\frac{6}{7}$



For the following problems, find each part without using a diagram.

Exercise 2.6.7

$\frac{1}{2}$ of $\frac{4}{5}$

Answer available at the end of this section.

Exercise 2.6.8

$\frac{3}{5}$ of $\frac{5}{12}$

Exercise 2.6.9

$\frac{1}{4}$ of $\frac{8}{9}$

Answer available at the end of this section.

Exercise 2.6.10

$\frac{3}{16}$ of $\frac{12}{15}$

Exercise 2.6.11

$\frac{2}{9}$ of $\frac{6}{5}$

Answer available at the end of this section.

Exercise 2.6.12

$\frac{1}{8}$ of $\frac{3}{8}$

Exercise 2.6.13

$\frac{2}{3}$ of $\frac{9}{10}$

Answer available at the end of this section.

Exercise 2.6.14

$\frac{18}{19}$ of $\frac{38}{54}$

Exercise 2.6.15

$\frac{5}{6}$ of $2\frac{2}{5}$

Answer available at the end of this section.

Exercise 2.6.16

$\frac{3}{4}$ of $3\frac{3}{5}$

Exercise 2.6.17

$\frac{3}{2}$ of $2\frac{2}{9}$

Answer available at the end of this section.

Exercise 2.6.18

$\frac{15}{4}$ of $4\frac{4}{5}$

Exercise 2.6.19

$5\frac{1}{3}$ of $9\frac{3}{4}$

Answer available at the end of this section.

Exercise 2.6.20

$1\frac{13}{15}$ of $8\frac{3}{4}$

Exercise 2.6.21

$\frac{8}{9}$ of $\frac{3}{4}$ of $\frac{2}{3}$

Answer available at the end of this section.

Exercise 2.6.22

$\frac{1}{6}$ of $\frac{12}{13}$ of $\frac{26}{36}$

Exercise 2.6.23

$\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{1}{4}$

Answer available at the end of this section.

Exercise 2.6.24

$1\frac{3}{7}$ of $5\frac{1}{5}$ of $8\frac{1}{3}$

Exercise 2.6.25

$2\frac{4}{5}$ of $5\frac{5}{6}$ of $7\frac{5}{7}$

Answer available at the end of this section.

For the following problems, find the products. Be sure to reduce.

Exercise 2.6.26

$\frac{1}{3} \cdot \frac{2}{3}$

Exercise 2.6.27

$\frac{1}{2} \cdot \frac{1}{2}$

Answer available at the end of this section.

Exercise 2.6.28

$\frac{3}{4} \cdot \frac{3}{8}$

Exercise 2.6.29

$\frac{2}{5} \cdot \frac{5}{6}$

Answer available at the end of this section.

Exercise 2.6.30

$\frac{3}{8} \cdot \frac{8}{9}$

Exercise 2.6.31

$\frac{5}{6} \cdot \frac{14}{15}$

Answer available at the end of this section.

Exercise 2.6.32

$\frac{4}{7} \cdot \frac{7}{4}$

Exercise 2.6.33

$\frac{3}{11} \cdot \frac{11}{3}$

Answer available at the end of this section.

Exercise 2.6.34

$\frac{9}{16} \cdot \frac{20}{27}$

Exercise 2.6.35

$\frac{35}{36} \cdot \frac{48}{55}$

Answer available at the end of this section.

Exercise 2.6.36

$\frac{21}{25} \cdot \frac{15}{14}$

Exercise 2.6.37

$\frac{76}{99} \cdot \frac{66}{38}$

Answer available at the end of this section.

Exercise 2.6.38

$\frac{3}{7} \cdot \frac{14}{18} \cdot \frac{6}{2}$

Exercise 2.6.39

$\frac{4}{15} \cdot \frac{10}{3} \cdot \frac{27}{2}$

Answer available at the end of this section.

Exercise 2.6.40

$\frac{14}{15} \cdot \frac{21}{28} \cdot \frac{45}{7}$

Exercise 2.6.41

$\frac{8}{3} \cdot \frac{15}{4} \cdot \frac{16}{21}$

Answer available at the end of this section.

Exercise 2.6.42

$\frac{18}{14} \cdot \frac{21}{35} \cdot \frac{36}{7}$

Exercise 2.6.43

$\frac{3}{5} \cdot 20$

Answer available at the end of this section.

Exercise 2.6.44

$\frac{8}{9} \cdot 18$

Exercise 2.6.45

$\frac{6}{11} \cdot 33$

Answer available at the end of this section.

Exercise 2.6.46

$\frac{18}{19} \cdot 38$

Exercise 2.6.47

$\frac{5}{6} \cdot 10$

Answer available at the end of this section.

Exercise 2.6.48

$\frac{1}{9} \cdot 3$

Exercise 2.6.49

$5 \cdot \frac{3}{8}$

Answer available at the end of this section.

Exercise 2.6.50

$16 \cdot \frac{1}{4}$

Exercise 2.6.51

$\frac{2}{3} \cdot 12 \cdot \frac{3}{4}$

Answer available at the end of this section.

Exercise 2.6.52

$\frac{3}{8} \cdot 24 \cdot \frac{2}{3}$

Exercise 2.6.53

$\frac{5}{18} \cdot 10 \cdot \frac{2}{5}$

Answer available at the end of this section.

Exercise 2.6.54

$\frac{16}{15} \cdot 50 \cdot \frac{3}{10}$

Exercise 2.6.55

$5\frac{1}{3} \cdot \frac{27}{32}$

Answer available at the end of this section.

Exercise 2.6.56

$2\frac{6}{7} \cdot 5\frac{3}{5}$

Exercise 2.6.57

$6\frac{1}{4} \cdot 2\frac{4}{15}$

Answer available at the end of this section.

Exercise 2.6.58

$9\frac{1}{3} \cdot \frac{9}{16} \cdot 1\frac{1}{3}$

Exercise 2.6.59

$3\frac{5}{9} \cdot 1\frac{13}{14} \cdot 10\frac{1}{2}$

Answer available at the end of this section.

Exercise 2.6.60

$20\frac{1}{4} \cdot 8\frac{2}{3} \cdot 16\frac{4}{5}$

Exercise 2.6.61

$(\frac{2}{3})^2$

Answer available at the end of this section.

Exercise 2.6.62

$(\frac{3}{8})^2$

Exercise 2.6.63

$(\frac{2}{11})^2$

Answer available at the end of this section.

Exercise 2.6.64

$(\frac{8}{9})^2$

Exercise 2.6.65

$(\frac{1}{2})^2$

Answer available at the end of this section.

Exercise 2.6.66

$(\frac{3}{5})^2 \cdot \frac{20}{3}$

Exercise 2.6.67

$(\frac{1}{4})^2 \cdot \frac{16}{15}$

Answer available at the end of this section.

Exercise 2.6.68

$(\frac{1}{2})^2 \cdot \frac{8}{9}$

Exercise 2.6.69

$(\frac{1}{2})^2 \cdot (\frac{2}{5})^2$

Answer available at the end of this section.

Exercise 2.6.70

$(\frac{3}{7})^2 \cdot (\frac{1}{9})^2$

Lesson

Let's use blocks and diagrams to understand more about division with fractions.

Exercise 2.6.1: Reasoning with Fraction Strips

Write a fraction or whole number as an answer for each question. If you get stuck, use the fraction strips. Be prepared to share your reasoning.

1. How many $\frac{1}{2}$s are in 2?
2. How many $\frac{1}{5}$s are in 3?
3. How many $\frac{1}{8}$s are in $1\frac{1}{4}$?
4. $1 \div \frac{2}{6} = ?$
5. $2 \div \frac{2}{9} = ?$

6. $4 \div \frac{2}{10} = ?$

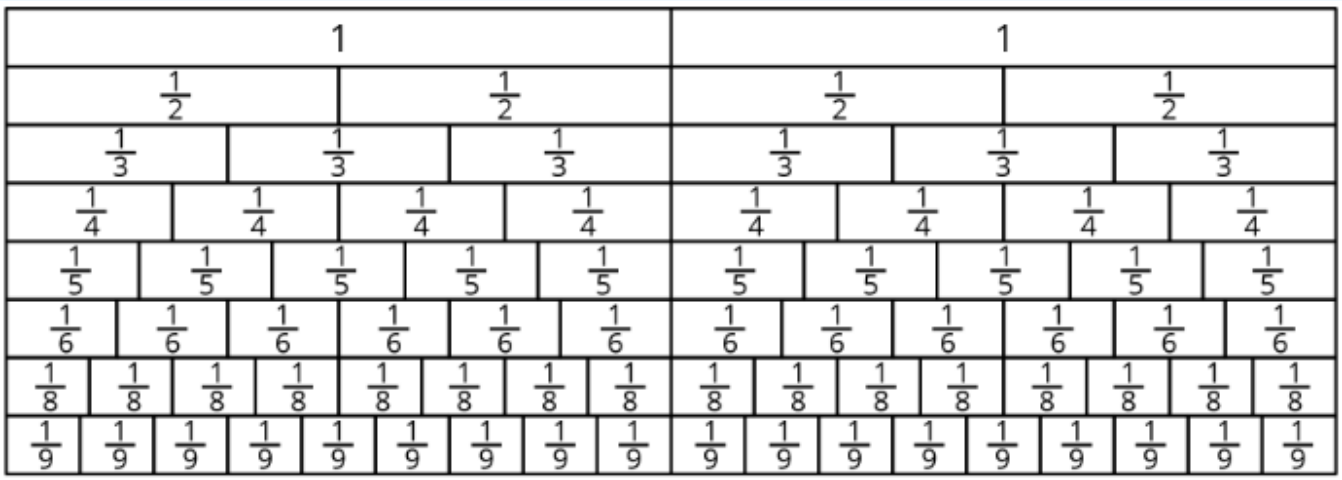


Figure 2.6.1: Fraction strips depicting 2 in 8 different ways, by rows. First row, two 1s. Second row, 4 of the fraction one over two. Third row, 6 of the fraction one over three. Fourth row, 8 of the fraction one over four. Fifth row, 10 of the fraction one over five. Sixth row, 12 of the fraction one over six. Seventh row, 16 of the fraction one over eight. Eighth row, 18 of the fraction one over nine.

Exercise 2.6.2: More Reasoning with Pattern Blocks

Use the pattern blocks in the applet to answer the questions. (If you need help aligning the pieces, you can turn on the grid.)

1. If the trapezoid represents 1 whole, what do each of these other shapes represent? Be prepared to explain or show your reasoning.
   a. 1 triangle
   b. 1 rhombus
   c. 1 hexagon
2. Use pattern blocks to represent each multiplication equation. Use the trapezoid to represent 1 whole.
   a. $3 \cdot \frac{1}{3} = 1$
   b. $3 \cdot \frac{2}{3} = 2$
3. Diego and Jada were asked "How many rhombuses are in a trapezoid?"
   - Diego says, "$1\frac{1}{3}$. If I put 1 rhombus on a trapezoid, the leftover shape is a triangle, which is $\frac{1}{3}$ of the trapezoid."
   - Jada says, "I think it's $1\frac{1}{2}$. Since we want to find out 'how many rhombuses,' we should compare the leftover triangle to a rhombus. A triangle is $\frac{1}{2}$ of a rhombus."

   Do you agree with either of them? Explain or show your reasoning.
4. Select **all** the equations that can be used to answer the question: "How many rhombuses are in a trapezoid?"
   - $\frac{2}{3} \div ? = 1$
   - $? \cdot \frac{2}{3} = 1$
   - $1 \div \frac{2}{3} = ?$
   - $1 \cdot \frac{2}{3} = ?$
   - $? \div \frac{2}{3} = 1$

Exercise 2.6.3: Drawing Diagrams to Show Equal-sized Groups

For each situation, draw a diagram for the relationship of the quantities to help you answer the question. Then write a multiplication equation or a division equation for the relationship. Be prepared to share your reasoning.

1. The distance around a park is $\frac{3}{2}$ miles. Noah rode his bicycle around the park for a total of 3 miles. How many times around the park did he ride?
2. You need $\frac{3}{4}$ yard of ribbon for one gift box. You have 3 yards of ribbon. How many gift boxes do you have ribbon for?
3. The water hose fills a bucket at $\frac{1}{3}$ gallon per minute. How many minutes does it take to fill a 2-gallon bucket?

> Are you ready for more?
>
> How many heaping teaspoons are in a heaping tablespoon? How would the answer depend on the shape of the spoons?

Summary

Suppose one batch of cookies requires $\frac{2}{3}$ cup flour. How many batches can be made with 4 cups of flour?

We can think of the question as being: "How many $\frac{2}{3}$ are in 4?" and represent it using multiplication and division equations.

$$? \cdot \frac{2}{3} = 4$$

$$4 \div \frac{2}{3} = ?$$

Let's use pattern blocks to visualize the situation and say that a hexagon is 1 whole.

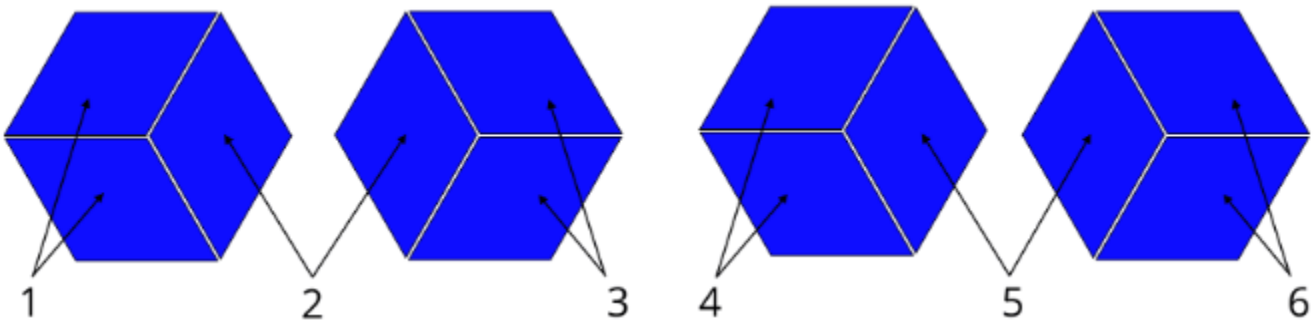


Figure 2.6.2: Diagram of 4 hexagons. Each hexagon is made up of 3 rhombuses using pattern blocks. 6 pairs of rhombuses are shown by pairs of arrows and labeled 1, 2, 3, 4, 5, and 6.

Since 3 rhombuses make a hexagon, 1 rhombus represents $\frac{1}{3}$ and 2 rhombuses represent $\frac{2}{3}$. We can see that 6 pairs of rhombuses make 4 hexagons, so there are 6 groups of $\frac{2}{3}$ in 4.

Other kinds of diagrams can also help us reason about equal-sized groups involving fractions. This example shows how we might reason about the same question from above: "How many $\frac{2}{3}$-cups are in 4 cups?"

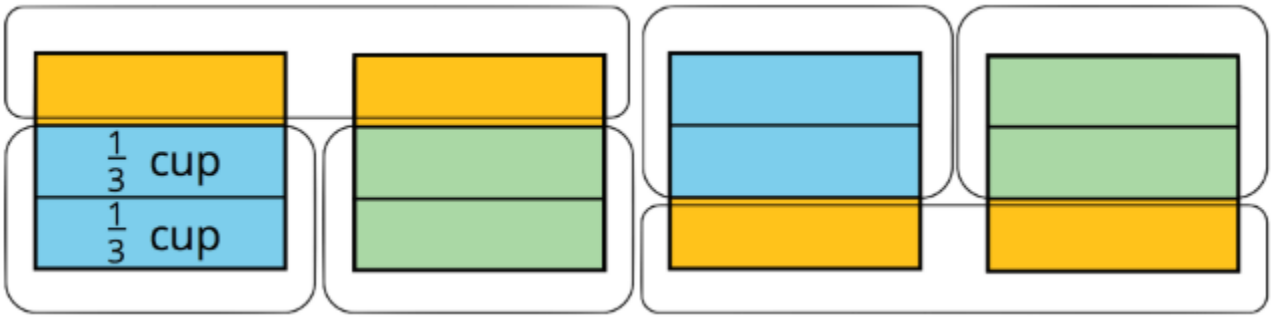


Figure 2.6.3: Diagram with 4 rectangles, partitioned into thirds. In the first rectangle two parts are each labeled one third. Across all 4 rectangles, 6 pairs of one third pieces are circled.

We can see each "cup" partitioned into thirds, and that there are 6 groups of $\frac{2}{3}$-cup in 4 cups. In both diagrams, we see that the unknown value (or the "?" in the equations) is 6. So we can now write:

$$6 \cdot \frac{2}{3} = 4$$

$$4 \div \frac{2}{3} = 6$$

Practice

Exercise 2.6.4

Use the tape diagram to find the value of $\frac{1}{2} \div \frac{1}{3}$. Show your reasoning.

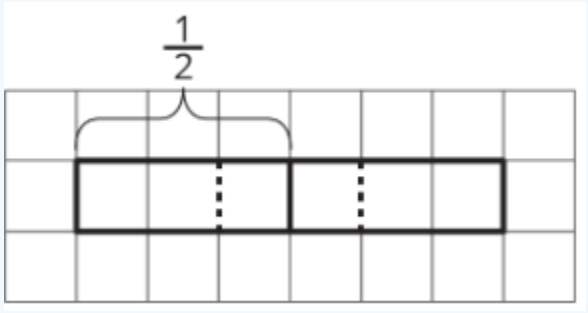


Figure 2.6.4: A tape diagram on a square grid, composed of 6 squares and is partitioned by a solid line into two equal parts. The tape diagram is also partitioned by two vertical dashed lines resulting in three equal parts. A brace extends over the first 3 squares and is labeled one half.

Exercise 2.6.5

What is the value of $\frac{1}{2} \div \frac{1}{3}$? Use pattern blocks to represent and find this value. The yellow hexagon represents 1 whole. Explain or show your reasoning.



Figure 2.6.5

Exercise 2.6.6

Use a standard inch ruler to answer each question. Then, write a multiplication equation and a division equation that answer the question.

1. How many $\frac{1}{2}$s are in 7?
2. How many $\frac{3}{8}$s are in 6?
3. How many $\frac{5}{16}$s are in $1\frac{7}{8}$?



Figure 2.6.6

Exercise 2.6.7

Use the tape diagram to answer the question: How many $\frac{2}{5}$s are in $1\frac{1}{2}$? Show your reasoning.

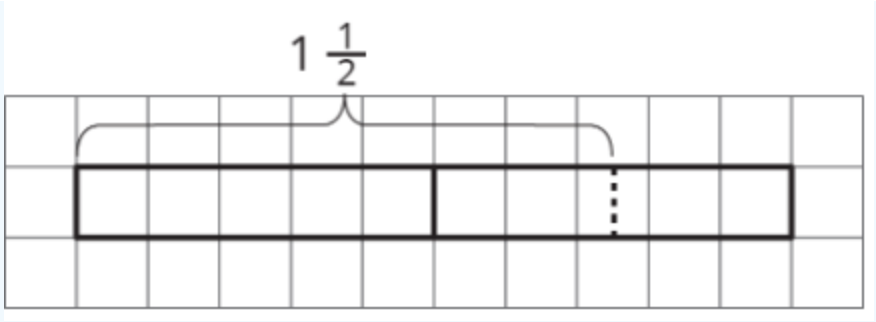


Figure 2.6.7

Exercise 2.6.8

Write a multiplication equation and a division equation to represent each sentence or diagram.

1. There are 12 fourths in 3.
2.

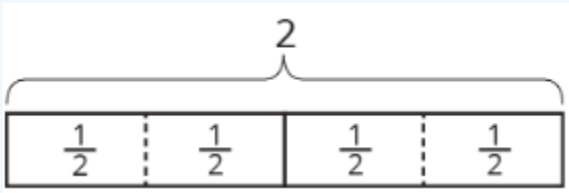


Figure 2.6.8

3. How many $\frac{2}{3}$s are in 6?
4.

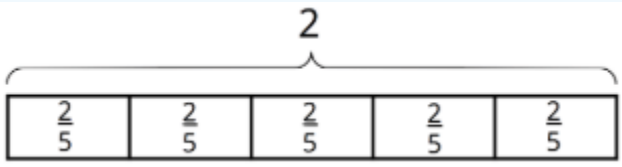


Figure 2.6.9

(From Unit 4.2.1)

Exercise 2.6.9

At a farmer's market, two vendors sell fresh milk. One vendor sells 2 liters for $3.80, and another vendor sells 1.5 liters for $2.70. Which is the better deal? Explain your reasoning.

(From Unit 3.3.1)

Exercise 2.6.10

A recipe uses 5 cups of flour for every 2 cups of sugar.

1. How much sugar is used for 1 cup of flour?
2. How much flour is used for 1 cup of sugar?
3. How much flour is used with 7 cups of sugar?

4. How much sugar is used with 6 cups of flour?

(From Unit 3.3.2)

Answers

Practice Set A

$\frac{1}{15}$

Practice Set A

$\frac{2}{9}$

Practice Set A

$\frac{5}{12}$

Practice Set A

$\frac{4}{9}$

Practice Set A

$\frac{14}{5}$

Practice Set A

$\frac{35}{48}$

Practice Set A

$\frac{10}{3}$

Practice Set A

$\frac{15}{2}$

Practice Set A

$\frac{5}{18}$

Practice Set B

$\frac{7}{12}$

Practice Set B

$\frac{25}{54}$

Practice Set B

$\frac{2}{3}$

Practice Set B

$\frac{2}{7}$

Practice Set B

$\frac{9}{2}$

Practice Set B

1

Practice Set B

$\frac{14}{5}$

Practice Set C

6

Practice Set C

22

Practice Set C

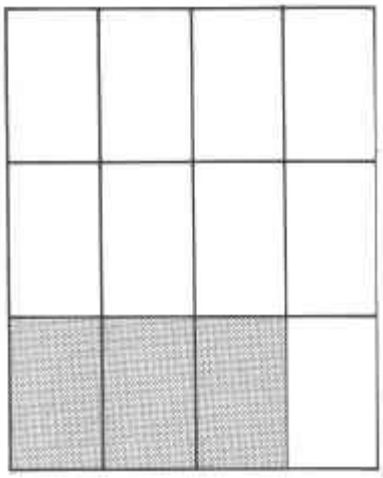
$85\frac{1}{2}$

Practice Set C

30

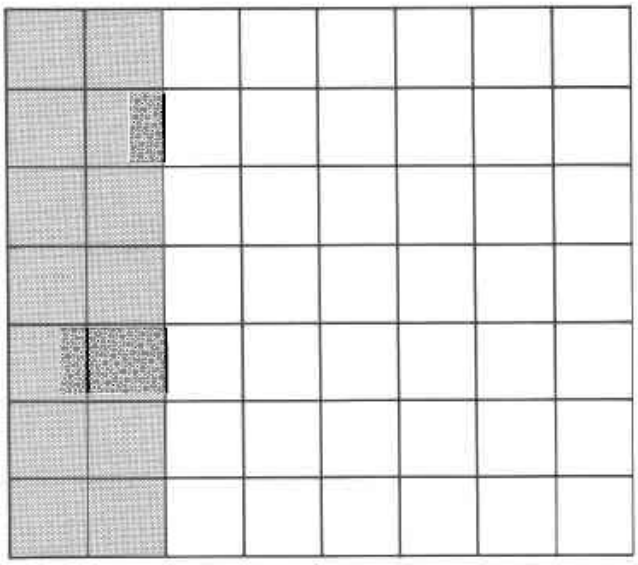
Exercise 2.6.1

$\frac{1}{4}$



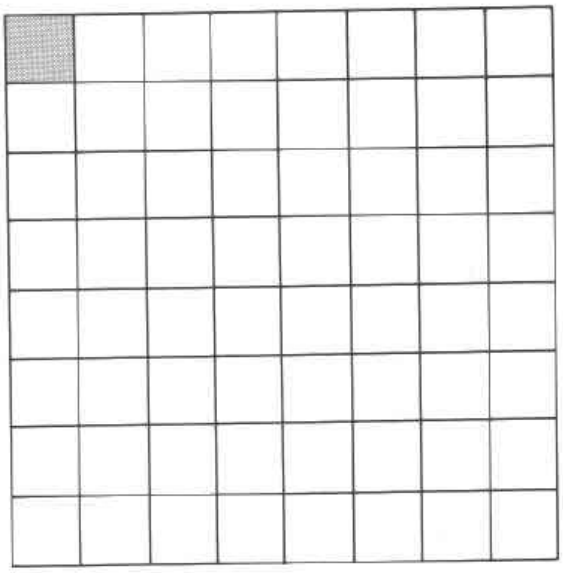
Exercise 2.6.3

$\frac{1}{4}$



Exercise 2.6.5

$\frac{1}{64}$



Exercise 2.6.7

$\frac{2}{5}$

Exercise 2.6.9

$\frac{2}{9}$

Exercise 2.6.11

$\frac{4}{15}$

Exercise 2.6.13

$\frac{3}{5}$

Exercise 2.6.15

2

Exercise 2.6.17

$\frac{10}{3}$ or $3\frac{1}{3}$

Exercise 2.6.19

52

Exercise 2.6.21

$\frac{4}{9}$

Exercise 2.6.23

$\frac{1}{24}$

Exercise 2.6.25

126

Exercise 2.6.27

$\frac{1}{4}$

Exercise 2.6.29

$\frac{1}{3}$

Exercise 2.6.31

$\frac{7}{9}$

Exercise 2.6.33

1

Exercise 2.6.35

$\frac{28}{33}$

Exercise 2.6.37

$\frac{4}{3}$

Exercise 2.6.39

12

Exercise 2.6.41

$7\frac{13}{21}$ or $\frac{160}{21}$

Exercise 2.6.43

12

Exercise 2.6.45

18

Exercise 2.6.47

$\frac{25}{3}$ or $8\frac{1}{3}$

Exercise 2.6.49

$\frac{15}{8} = 1\frac{7}{8}$

Exercise 2.6.51

6

Exercise 2.6.53

$\frac{10}{9} = 1\frac{1}{9}$

Exercise 2.6.55

$\frac{9}{2} = 4\backslash\text{drac}12$

Exercise 2.6.57

$\frac{85}{6} = 14\backslash\text{drac}16$

Exercise 2.6.59

72

Exercise 2.6.61

$\frac{4}{9}$

Exercise 2.6.63

$\frac{4}{121}$

Exercise 2.6.65

$\frac{1}{4}$

Exercise 2.6.67

$\frac{1}{15}$

Exercise 2.6.69

$\frac{1}{25}$

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- 22.2: How Many Groups? (Part 2) is licensed CC BY 4.0.

2.6.1: Practice and Videos for Multiplying and Dividing Fractions

Learning Objectives

- I can multiply fractions.
- I can divide fractions.
- I can use visual techniques to multiply and divide fractions.

GeoGebra: Multiplying Fractions

Complete the activity below to visually understand how to multiply fractions. Start by typing any fraction in the empty boxes where it says "fraction #1" and "fraction #2." Some examples of fractions you can type are:

- $\frac{1}{2} \times \frac{2}{3}$ (2.6.1.1)
- $\frac{3}{4} \times \frac{2}{8}$ (2.6.1.2)
- $\frac{1}{2} \times \frac{2}{5}$ (2.6.1.3)

After typing in your numbers, click "Go!" in the middle. Watch how the fractions overlap. The answer to the multiplication expression is the overlapping tiles over the total number of tiles. Click the "Reset" button to test a new set of fractions.

Videos on Fraction Multiplication and Division

Watch the following videos to learn how to multiply and divide fractions. All videos have appropriate captions in English; in some cases, there may also be appropriate captions in other languages.

YouTube Video

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

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Practice!

Check your understanding of fraction multiplication and division by answering these questions. Use the arrows at the bottom of the box to go to the next question.

Practice 2.6.1.1

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this Interactive Content online

Extra Practice Resources

For more practice on multiplying and dividing fractions, visit the following websites.

1. IXL: Multiply two fractions
2. IXL: Divide two fractions
3. Math Playground: Snow Sprint Fractions

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2.7: Adding, Subtracting and Multiplying Decimals

Adding Decimals

Addition of decimal numbers is quite similar to addition of whole numbers. For example, suppose that we are asked to add 2.34 and 5.25. We could change these decimal numbers to **mixed fractions** and add.

$$\begin{aligned} 2.34 + 5.25 &= 2\frac{34}{100} + 5\frac{25}{100} \\ &= 7\frac{59}{100} \end{aligned}$$

However, we can also line the decimal numbers on their decimal points and add vertically, as follows.

$$\begin{array}{r} 2.34 \\ +5.25 \\ \hline 7.59 \end{array}$$

Note that this alignment procedure produces the same result, “seven and fifty nine hundredths.” This motivates the following procedure for adding decimal numbers.

Adding Decimals

To add decimal numbers, proceed as follows:

1. Place the numbers to be added in vertical format, aligning the decimal points.
2. Add the numbers as if they were whole numbers.
3. Place the decimal point in the answer in the same column as the decimal points above it.

Example 1

Add 3.125 and 4.814.

Solution

Place the numbers in vertical format, aligning on their decimal points. Add, then place the decimal point in the answer in the same column as the decimal points that appear above the answer.

$$\begin{array}{r} 3.125 \\ +4.814 \\ \hline 7.939 \end{array}$$

Thus, 3.125 + 4.814 = 7.939.

Exercise

Add: 2.864 + 3.029

Answer available at the end of this section.

Example 2

Jane has $4.35 in her purse. Jim has $5.62 in his wallet. If they sum their money, what is the total?

Solution

Arrange the numbers in vertical format, aligning decimal points, then add.

$$\begin{array}{r} \$4.35 \\ +\$5.62 \\ \hline \$9.97 \end{array}$$

Exercise

Alice has $8.63 in her purse and Joanna has $2.29. If they combine sum their money, what is the total?

Answer available at the end of this section.

Before looking at another example, let's recall an important observation.

Important Observation

Adding zeros to the end of the fractional part of a decimal number does not change its value. Similarly, deleting trailing zeros from the end of a decimal number does not change its value.

For example, we could add two zeros to the end of the fractional part of 7.25 to obtain 7.2500. The numbers 7.25 and 7.2500 are identical as the following argument shows:

$$\begin{aligned} 7.2500 &= 7\frac{2500}{10000} \\ &= 7\frac{25}{100} \\ &= 7.25 \end{aligned}$$

Example 3

Add 7.5 and 12.23.

Solution

Arrange the numbers in vertical format, aligning their decimal points in a column. Note that we add a trailing zero to improve columnar alignment.

$$\begin{array}{r} 7.50 \\ +12.23 \\ \hline 19.73 \end{array}$$

Hence, 7.5 + 12.23 = 19.73.

Exercise

Add: 9.7 + 15.86

Answer available at the end of this section.

Example 4

Find the sum: 12.2+8.352 + 22.44.

Solution

Arrange the numbers in vertical format, aligning their decimal points in a column. Note that we add trailing zeros to improve the columnar alignment.

$$\begin{array}{r} 12.200 \\ 8.352 \\ +22.440 \\ \hline 42.992 \end{array}$$

Hence, 12.2+8.352 + 22.44 = 42.992.

Exercise

Add: 12.9+4.286 + 33.97

Answer available at the end of this section.

Subtracting Decimals

Subtraction of decimal numbers proceeds in much the same way as addition of decimal numbers.

Subtracting Decimals

To subtract decimal numbers, proceed as follows:

1. Place the numbers to be subtracted in vertical format, aligning the decimal points.
2. Subtract the numbers as if they were whole numbers.
3. Place the decimal point in the answer in the same column as the decimal points above it.

Example 5

Subtract 12.23 from 33.57.

Solution

Arrange the numbers in vertical format, aligning their decimal points in a column, then subtract. Note that we subtract 12.23 **from** 33.57.

$$\begin{array}{r} 33.57 \\ -12.23 \\ \hline 21.34 \end{array}$$

Hence, 33.57 – 12.23 = 21.34.

Exercise

Subtract: 58.76 – 38.95

Answer available at the end of this section.

As with addition, we add trailing zeros to the fractional part of the decimal numbers to help columnar alignment.

Example 6

Find the difference: 13.3 – 8.572.

Solution

Arrange the numbers in vertical format, aligning their decimal points in a column. Note that we add trailing zeros to the fractional part of 13.3 to improve columnar alignment.

$$\begin{array}{r} 13.300 \\ -8.572 \\ \hline 4.728 \end{array}$$

Hence, 13.3 – 8.572 = 4.728.

Exercise

Subtract: 15.2 – 8.756

Answer available at the end of this section.

Exercises

In Exercises 1-12, add the decimals.

1. $31.9 + 84.7$
2. $9.39 + 7.7$
3. $4 + 97.18$
4. $2.645 + 2.444$
5. $4 + 87.502$
6. $23.69 + 97.8$
7. $95.57 + 7.88$
8. $18.7 + 7$
9. $52.671 + 5.97$
10. $9.696 + 28.2$
11. $4.76 + 2.1$

12. $1.5 + 46.4$

In Exercises 13-24, subtract the decimals.

13. $9 - 2.261$

14. $98.14 - 7.27$

15. $80.9 - 6$

16. $9.126 - 6$

17. $55.672 - 3.3$

18. $4.717 - 1.637$

19. $60.575 - 6$

20. $8.91 - 2.68$

21. $39.8 - 4.5$

22. $8.210 - 3.7$

23. $8.1 - 2.12$

24. $7.675 - 1.1$

---

25. **Big Banks**. Market capitalization of nation's four largest banks (as of April 23, 2009)

JPMorgan Chase & Co	$124.8 billion
Wells Fargo & Co	$85.3 billion
Goldman Sachs Group Inc.	$61.8 billion
Bank of America	$56.4 billion

What is the total value of the nation's four largest banks? *Associated Press Times-Standard 4/22/09*

26. **Telescope Mirror.** The newly launched Herschel Telescope has a mirror 11.5 feet in diameter while Hubble's mirror is 7.9 feet in diameter. How much larger is Herschel's mirror in diameter than Hubble's?

27. **Average Temperature**. The average temperatures in Sacramento, California in July are a high daytime temperature of 93.8 degrees Fahrenheit and a low nighttime temperature of 60.9 degrees Fahrenheit. What is the change in temperature from day to night? Hint: See Section 2.3 for the formula for comparing temperatures.

28. **Average Temperature**. The average temperatures in Redding, California in July are a high daytime temperature of 98.2 degrees Fahrenheit and a low nighttime temperature of 64.9 degrees Fahrenheit. What is the change in temperature from day to night? Hint: See Section 2.3 for the formula for comparing temperatures.

29. **Net Worth**. Net worth is defined as assets minus liabilities. Assets are everything of value that can be converted to cash while liabilities are the total of debts. General Growth Properties, the owners of the Bayshore Mall, have $29.6 billion in assets and $27 billion in liabilities, and have gone bankrupt. What was General Growth Properties net worth before bankruptcy? *Times-Standard 4/17/2009*

30. **Grape crush**. The California Department of Food and Agriculture's preliminary grape crush report shows that the state produced 3.69 million tons of wine grapes in 2009. That's just shy of the record 2005 crush of 3.76 million tons. By how many tons short of the record was the crush of 2009? *Associated Press-Times-Standard Calif. winegrapes harvest jumped*

23% in '09.

31. **Turnover**. The Labor Department's Job Openings and Labor Turnover Survey claims that employers hired about 4.08 million people in January 2010 while 4.12 million people were fired or otherwise left their jobs. How many more people lost jobs than were hired? Convert your answer to a whole number. *Associated Press-Times-Standard 03/10/10 Job openings up sharply in January to 2.7M.*

Answers

1. 116.6

3. 101.18

5. 91.502

7. 103.45

9. 58.641

11. 6.86

13. 6.739

15. 74.9

17. 52.372

19. 54.575

21. 35.3

23. 5.98

25. $328.3 billion

26. −32.9 degrees Fahrenheit

28. $2.6 billion

28. 40, 000

Learning Objectives

By the end of this section, you will be able to:

- Name and write decimals
- Round decimals
- Add and subtract decimals
- Multiply and divide decimals
- Convert decimals, fractions, and percents

Note

A more thorough introduction to the topics covered in this section can be found in the *Prealgebra*chapter, **Decimals**.

Name and Write Decimals

Decimals are another way of writing **fraction**s whose denominators are powers of 10.

$$\begin{aligned} 0.1 &= \tfrac{1}{10} && \text{0.1 is "one tenth"} \\ 0.01 &= \tfrac{1}{100} && \text{0.01 is "one hundredth} \\ 0.001 &= \tfrac{1}{1{,}000} && \text{0.001 is "one thousandth} \\ 0.0001 &= \tfrac{1}{10{,}000} && \text{0.0001 is "one ten-thousandth"} \end{aligned} \tag{2.7.1}$$

Notice that “ten thousand” is a number larger than one, but “one ten-thousand**th**” is a number smaller than one. The “th” at the end of the name tells you that the number is smaller than one.

When we name a whole number, the name corresponds to the place value based on the powers of ten. We read 10,000 as “ten thousand” and 10,000,000 as “ten million.” Likewise, the names of the **decimal** places correspond to their fraction values. Figure 2.7.1 shows the names of the **place values** to the left and right of the decimal point.

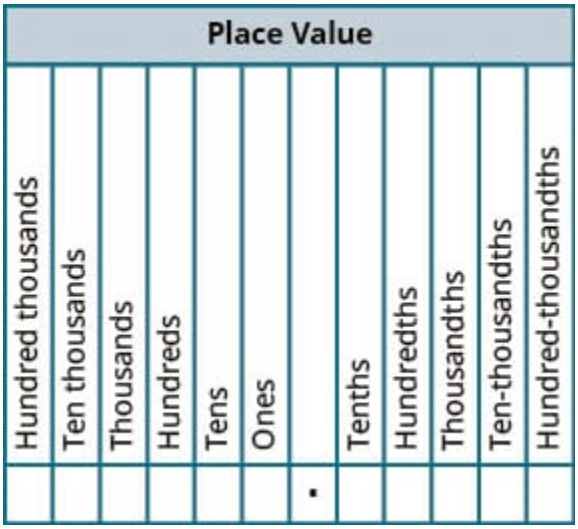


Figure 2.7.1: Place value of decimal numbers are shown to the left and right of the decimal point.

Exercise 2.7.1

Name the decimal 4.3.

Answer available at the end of this section.

Exercise 2.7.2

Name the decimal 6.7.

Answer available at the end of this section.

Exercise 2.7.3

Name the decimal 5.8.

Answer available at the end of this section.

We summarize the steps needed to name a **decimal** below.

NAME A DECIMAL.

1. Name the number to the left of the decimal point.
2. Write “and” for the decimal point.
3. Name the “number” part to the right of the decimal point as if it were a whole number.
4. Name the decimal place of the last digit.

Exercise 2.7.4

Name the decimal: -15.571.

Answer available at the end of this section.

Exercise 2.7.5

Name the decimal: -13.461.

Answer available at the end of this section.

Exercise 2.7.6

Name the decimal: -2.053.

Answer available at the end of this section.

When we write a check we write both the numerals and the name of the number. Let’s see how to write the decimal from the name.

Exercise 2.7.7: How to Write decimals

Write “fourteen and twenty-four thousandths” as a decimal.

Answer available at the end of this section.

Exercise 2.7.8

Write as a decimal: thirteen and sixty-eight thousandths.

Answer available at the end of this section.

Exercise 2.7.9

Write as a decimal: five and ninety-four thousandths.

Answer available at the end of this section.

We summarize the steps to writing a **decimal**.

WRITE A DECIMAL.

1. Look for the word “and”—it locates the decimal point.
   - Place a decimal point under the word “and.” Translate the words before “and” into the whole number and place it to the left of the decimal point.
   - If there is no “and,” write a “0” with a decimal point to its right.
2. Mark the number of decimal places needed to the right of the decimal point by noting the place value indicated by the last word.
3. Translate the words after “and” into the number to the right of the decimal point. Write the number in the spaces—putting the final digit in the last place.
4. Fill in zeros for place holders as needed.

Round Decimals

Rounding decimals is very much like rounding whole numbers. We will round decimals with a method based on the one we used to round whole numbers.

Exercise 2.7.10

Round 18.379 to the nearest hundredth.

Answer available at the end of this section.

Exercise 2.7.11

Round to the nearest hundredth: 1.047.

Answer available at the end of this section.

Exercise 2.7.12

Round to the nearest hundredth: 9.173.

Answer available at the end of this section.

We summarize the steps for rounding a **decimal** here.

ROUND DECIMALS.

1. Locate the given place value and mark it with an arrow.
2. Underline the digit to the right of the place value.
3. Is this digit greater than or equal to 5?
   - Yes—add 1 to the digit in the given place value.
   - No—do not change the digit in the given place value.
4. Rewrite the number, deleting all digits to the right of the rounding digit.

Exercise 2.7.13

Round 18.379 to the nearest

1. tenth
2. whole number.

Answer available at the end of this section.

Exercise 2.7.14

Round 6.582 to the nearest

1. hundredth
2. tenth
3. whole number.

Answer available at the end of this section.

Exercise 2.7.15

Round 15.2175 to the nearest

1. thousandth
2. hundredth
3. tenth.

Answer available at the end of this section.

Add and Subtract Decimals

To add or subtract decimals, we line up the decimal points. By lining up the decimal points this way, we can add or subtract the corresponding **place values**. We then add or subtract the numbers as if they were whole numbers and then place the decimal point in the sum.

ADD OR SUBTRACT DECIMALS.

1. Write the numbers so the decimal points line up vertically.
2. Use zeros as place holders, as needed.
3. Add or subtract the numbers as if they were whole numbers. Then place the decimal point in the answer under the decimal points in the given numbers.

Exercise 2.7.16

Add: $23.5 + 41.38$.

Answer available at the end of this section.

Exercise 2.7.17

Add: $4.8 + 11.69$.

Answer available at the end of this section.

Exercise 2.7.18

Add: $5.123 + 18.47$.

Answer available at the end of this section.

Exercise 2.7.19

Subtract: $20 - 14.65$.

Answer available at the end of this section.

Exercise 2.7.20

Subtract: $10 - 9.58$.

Answer available at the end of this section.

Exercise 2.7.21

Subtract: $50 - 37.42$.

Answer available at the end of this section.

Multiply Decimals

Multiplying decimals is very much like multiplying whole numbers—we just have to determine where to place the decimal point. The procedure for multiplying decimals will make sense if we first convert them to fractions and then multiply.

So let's see what we would get as the **product** of decimals by converting them to fractions first. We will do two examples side-by-side. Look for a pattern!

Table 2.7.1

	$\underbrace{(0.3)}_{1\text{ place}}\ \underbrace{(0.7)}_{1\text{ place}}$	$\underbrace{(0.2)}_{1\text{ place}}\ \underbrace{(0.46)}_{2\text{ places}}$
Convert to fractions.	$\frac{3}{10} \cdot \frac{7}{10}$	$\frac{2}{10} \cdot \frac{46}{100}$
Multiply.	$\frac{21}{100}$	$\frac{92}{1000}$
Convert to decimals.	$\underbrace{0.21}_{2\text{ places}}$	$\underbrace{0.092}_{3\text{ places}}$

Notice, in the first example, we multiplied two numbers that each had one digit after the decimal point and the product had two decimal places. In the second example, we multiplied a number with one decimal place by a number with two decimal places and the product had three decimal places.

We multiply the numbers just as we do whole numbers, temporarily ignoring the decimal point. We then count the number of decimal points in the factors and that sum tells us the number of decimal places in the product.

The rules for multiplying positive and negative numbers apply to decimals, too, of course!

When *multiplying* two numbers,

- if their signs are the *same* the product is *positive*.
- if their signs are *different* the product is *negative*.

When we multiply signed decimals, first we determine the sign of the product and then multiply as if the numbers were both positive. Finally, we write the product with the appropriate sign.

MULTIPLY DECIMALS.

1. Determine the sign of the product.
2. Write in vertical format, lining up the numbers on the right. Multiply the numbers as if they were whole numbers, temporarily ignoring the decimal points.
3. Place the decimal point. The number of decimal places in the product is the sum of the number of decimal places in the factors.
4. Write the product with the appropriate sign.

Exercise 2.7.22

Multiply: $(-3.9)(4.075)$.

Answer available at the end of this section.

Exercise 2.7.23

Multiply: $-4.5(6.107)$.

Answer available at the end of this section.

Exercise 2.7.24

Multiply: –10.79(8.12).

Answer available at the end of this section.

In many of your other classes, especially in the sciences, you will multiply decimals by powers of 10 (10, 100, 1000, etc.). If you multiply a few products on paper, you may notice a pattern relating the number of zeros in the power of 10 to number of decimal places we move the decimal point to the right to get the product.

MULTIPLY A DECIMAL BY A POWER OF TEN.

1. Move the decimal point to the right the same number of places as the number of zeros in the power of 10.
2. Add zeros at the end of the number as needed.

Exercise 2.7.25

Multiply 5.63

1. by 10
2. by 100
3. by 1,000.

Answer available at the end of this section.

Exercise 2.7.26

Multiply 2.58

1. by 10
2. by 100
3. by 1,000.

Answer available at the end of this section.

Exercise 2.7.27

Multiply 14.2

1. by 10
2. by 100
3. by 1,000.

Answer available at the end of this section.

Convert Decimals, Fractions, and Percents

We convert decimals into fractions by identifying the place value of the last (farthest right) digit. In the decimal 0.03 the 3 is in the hundredths place, so 100 is the denominator of the fraction equivalent to 0.03.

$$00.03 = \frac{3}{100} \tag{2.7.2}$$

Notice, when the number to the left of the decimal is zero, we get a fraction whose numerator is less than its denominator. Fractions like this are called **proper fractions**.

The steps to take to convert a decimal to a fraction are summarized in the procedure box.

CONVERT A DECIMAL TO A PROPER FRACTION.

1. Determine the place value of the final digit.
2. Write the fraction.
   - numerator—the “numbers” to the right of the decimal point
   - denominator—the place value corresponding to the final digit

Exercise 2.7.34

Write 0.374 as a fraction.

Answer available at the end of this section.

Exercise 2.7.35

Write 0.234 as a fraction.

Answer available at the end of this section.

Exercise 2.7.36

Write 0.024 as a fraction.

Answer available at the end of this section.

We've learned to convert decimals to fractions. Now we will do the reverse—convert fractions to decimals. Remember that the fraction bar means division. So $\frac{4}{5}$ can be written $4 \div 5$ or $5\overline{)4}$. This leads to the following method for converting a fraction to a decimal.

CONVERT A FRACTION TO A DECIMAL.

To convert a fraction to a decimal, divide the numerator of the fraction by the denominator of the fraction.

Exercise 2.7.37

Write $-\frac{5}{8}$ as a decimal.

Answer available at the end of this section.

Exercise 2.7.38

Write $-\frac{7}{8}$ as a decimal.

Answer available at the end of this section.

Exercise 2.7.39

Write $-\frac{3}{8}$ as a decimal.

Answer available at the end of this section.

When we divide, we will not always get a zero remainder. Sometimes the quotient ends up with a decimal that repeats. A **repeating decimal** is a decimal in which the last digit or group of digits repeats endlessly. A bar is placed over the repeating block of digits to indicate it repeats.

REPEATING DECIMAL

A **repeating decimal** is a decimal in which the last digit or group of digits repeats endlessly.

A bar is placed over the repeating block of digits to indicate it repeats.

Exercise 2.7.40

Write $\frac{43}{22}$ as a decimal.

Answer available at the end of this section.

Exercise 2.7.41

Write $\frac{27}{11}$ as a decimal.

Answer available at the end of this section.

Exercise 2.7.42

Write $\frac{51}{22}$ as a decimal.

Answer available at the end of this section.

Sometimes we may have to simplify expressions with fractions and decimals together.

Exercise 2.7.43

Simplify: $\frac{7}{8} + 6.4$.

Answer available at the end of this section.

Exercise 2.7.44

Simplify: $\frac{3}{8} + 4.9$.

Answer available at the end of this section.

Exercise 2.7.45

Simplify: $5.7 + \frac{13}{20}$.

Answer available at the end of this section.

A **percent** is a ratio whose denominator is 100. Percent means per hundred. We use the percent symbol, %, to show percent.

PERCENT

A **percent** is a ratio whose denominator is 100.

Since a percent is a ratio, it can easily be expressed as a fraction. Percent means per 100, so the denominator of the fraction is 100. We then change the fraction to a decimal by dividing the numerator by the denominator.

	6	78	135
Write as a ratio with denominator 100.	$\frac{6}{100}$	$\frac{78}{100}$	$\frac{135}{100}$
Change the fraction to a decimal by dividing the numerator by the denominator.	0.06	0.78	1.35

(2.7.3)

Do you see the pattern? *To convert a percent number to a decimal number, we move the decimal point two places to the left.*

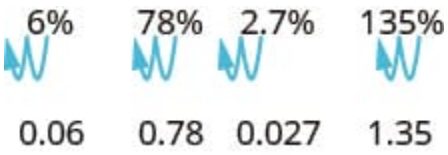


Figure 2.7.2

Exercise 2.7.46

Convert each percent to a decimal:

1. 62%
2. 135%
3. 35.7%.

Answer available at the end of this section.

Exercise 2.7.47

Convert each percent to a decimal:

1. 9%
2. 87%
3. 3.9%.

Answer available at the end of this section.

Exercise 2.7.48

Convert each percent to a decimal:

1. 3%
2. 91%
3. 8.3%.

Answer available at the end of this section.

Converting a decimal to a percent makes sense if we remember the definition of percent and keep place value in mind.

To convert a decimal to a percent, remember that percent means per hundred. If we change the decimal to a fraction whose denominator is 100, it is easy to change that fraction to a percent.

$$\begin{array}{lccc} & 0.83 & 1.05 & 0.075 \\ \text{Write as a fraction} & \frac{83}{100} & 1\frac{5}{100} & \frac{75}{1000} \\ \text{The denominator is 100.} & & \frac{105}{100} & \frac{7.5}{100} \\ \text{Write the ratio as a percent.} & 83 & 105 & 7.5 \end{array} \tag{2.7.4}$$

Recognize the pattern? *To convert a decimal to a percent, we move the decimal point two places to the right and then add the percent sign.*

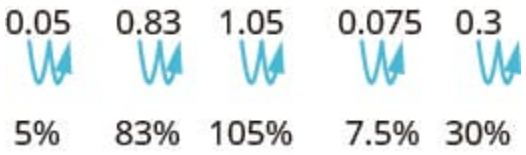


Figure 2.7.3

Exercise 2.7.49

Convert each decimal to a percent:

1. 0.51
2. 1.25
3. 0.093.

Answer available at the end of this section.

Exercise 2.7.50

Convert each decimal to a percent:

1. 0.17
2. 1.75
3. 0.0825

Answer available at the end of this section.

Exercise 2.7.51

Convert each decimal to a percent:

1. 0.41
2. 2.25
3. 0.0925.

Answer available at the end of this section.

Key Concepts

- **Name a Decimal**
  1. Name the number to the left of the decimal point.
  2. Write "and" for the decimal point.
  3. Name the "number" part to the right of the decimal point as if it were a whole number.

4. Name the decimal place of the last digit.

- **Write a Decimal**
  1. Look for the word ‘and’—it locates the decimal point. Place a decimal point under the word ‘and.’ Translate the words before ‘and’ into the whole number and place it to the left of the decimal point. If there is no “and,” write a “0” with a decimal point to its right.
  2. Mark the number of decimal places needed to the right of the decimal point by noting the place value indicated by the last word.
  3. Translate the words after ‘and’ into the number to the right of the decimal point. Write the number in the spaces—putting the final digit in the last place.
  4. Fill in zeros for place holders as needed.
- **Round a Decimal**
  1. Locate the given place value and mark it with an arrow.
  2. Underline the digit to the right of the place value.
  3. Is this digit greater than or equal to 5? Yes—add 1 to the digit in the given place value. No—do not change the digit in the given place value.
  4. Rewrite the number, deleting all digits to the right of the rounding digit.
- **Add or Subtract Decimals**
  1. Write the numbers so the decimal points line up vertically.
  2. Use zeros as place holders, as needed.
  3. Add or subtract the numbers as if they were whole numbers. Then place the decimal in the answer under the decimal points in the given numbers.
- **Multiply Decimals**
  1. Determine the sign of the product.
  2. Write in vertical format, lining up the numbers on the right. Multiply the numbers as if they were whole numbers, temporarily ignoring the decimal points.
  3. Place the decimal point. The number of decimal places in the product is the sum of the decimal places in the factors.
  4. Write the product with the appropriate sign.
- **Multiply a Decimal by a Power of Ten**
  1. Move the decimal point to the right the same number of places as the number of zeros in the power of 10.
  2. Add zeros at the end of the number as needed.
- **Divide Decimals**
  1. Determine the sign of the quotient.
  2. Make the divisor a whole number by “moving” the decimal point all the way to the right. “Move” the decimal point in the dividend the same number of places - adding zeros as needed.
  3. Divide. Place the decimal point in the quotient above the decimal point in the dividend.
  4. Write the quotient with the appropriate sign.
- **Convert a Decimal to a Proper Fraction**
  1. Determine the place value of the final digit.
  2. Write the fraction: numerator—the ‘numbers’ to the right of the decimal point; denominator—the place value corresponding to the final digit.
- **Convert a Fraction to a Decimal** Divide the numerator of the fraction by the denominator.

Answers

Exercise

5.893

Exercise

$10.91

Exercise

25.56

Exercise

51.156

Exercise

19.81

Exercise

6.444

Exercise 2.7.1

Step 1. Name the number to the left of the decimal point.	4 is to the left of the decimal point.	4.3 four _____
Step 2. Write 'and' for the decimal point.		four and _____
Step 3. Name the 'number' part to the right of the decimal point as if it were a whole number.	3 is to the right of the decimal point.	four and three _____
Step 4. Name the decimal place.		four and three tenths

Exercise 2.7.2

six and seven tenths

Exercise 2.7.3

five and eight tenths

Exercise 2.7.4

	-15.571
Name the number to the left of the decimal point.	negative fifteen _____
Write "and" for the decimal point.	negative fifteen and _____
Name the number to the right of the decimal point.	negative fifteen and five hundred seventy-one _____
The 1 is in the thousandths place.	negative fifteen and five hundred seventy-one thousandths

Exercise 2.7.5

negative thirteen and four hundred sixty-one thousandths

Exercise 2.7.6

negative two and fifty-three thousandths

Exercise 2.7.7**: How to Write decimals**

Step 1. Look for the word 'and'; it locates the decimal point. Place a decimal point under the word 'and'. Translate the words before 'and' into the whole number and place to the left of the decimal point.		fourteen and twenty-four thousandths fourteen and twenty-four thousandths _____._____ 14._____
Step 2. Mark the number of decimal places needed to the right of the decimal point by noting the place value indicated by the last word.	The last word is 'thousandths'.	14._____ _____ _____ tenths hundredths thousandths
Step 3. Translate the words after 'and' into the number to the right of the decimal point. Write the number in the spaces – putting the final digit in the last place.		14._____ 2 4

Step 4. Fill in zeros for empty place holders as needed.	Zeros are needed in the tenths place.	14. 0 2 4 Fourteen and twenty-four thousandths is written 14.024.

Exercise 2.7.8

13.068

Exercise 2.7.9

5.094

Exercise 2.7.10

Step 1. Locate the given place value and mark it with an arrow.		hundredths place ↓ 18.379
Step 2. Underline the digit to the right of the given place value.		hundredths place ↓ 18.379
Step 3. Is this digit greater than or equal to 5? **Yes:** Add 1 to the digit in the given place value. **No:** Do not change the digit in the given place value.	Because 9 is greater than or equal to 5, add 1 to the 7.	18.37 9 add 1 ↗ delete
Step 4. Rewrite the number, removing all digits to the right of the rounding digit.		18.38 18.38 is 18.379 rounded to the nearest hundredth.

Exercise 2.7.11

1.05

Exercise 2.7.12

9.17

Exercise 2.7.13

Round 18.379

1. to the nearest tenth

Locate the tenths place with an arrow.	tenths place ↓ 18.379
Underline the digit to the right of the given place value.	tenths place ↓ 18.379

Because 7 is greater than or equal to 5, add 1 to the 3.	
Rewrite the number, deleting all digits to the right of the rounding digit.	18.4
Notice that the deleted digits were NOT replaced with zeros.	So, 18.379 rounded to the nearest tenth is 18.4.

2. to the nearest whole number

Locate the ones place with an arrow.	
Underline the digit to the right of the given place value.	
Since 3 is not greater than or equal to 5, do not add 1 to the 8.	
Rewrite the number, deleting all digits to the right of the rounding digit.	18
	So, 18.379 rounded to the nearest whole number is 18.

Exercise 2.7.14

1. 6.58
2. 6.6
3. 7

Exercise 2.7.15

1. 15.218
2. 15.22
3. 15.2

Exercise 2.7.16

Write the numbers so that the decimal points line up vertically. $\begin{array}{r} 23.50 \\ +41.38 \\ \hline \end{array}$ (2.7.5)

Put 0 as a placeholder after the 5 in 23.5. Remember, $\frac{5}{10} = \frac{50}{100}$, so $0.5 = 0.50$ $\begin{array}{r} 23.50 \\ +41.38 \\ \hline \end{array}$ (2.7.6)

Add the numbers as if they were whole numbers . Then place the decimal point in the sum. $\begin{array}{r} 23.50 \\ +41.38 \\ \hline 64.88 \end{array}$ (2.7.7)

Exercise 2.7.17

16.49

Exercise 2.7.18

23.593

Exercise 2.7.19

Erroneous nesting of equation structures

Erroneous nesting of equation structures

Erroneous nesting of equation structures

Exercise 2.7.20

0.42

Exercise 2.7.21

12.58

Exercise 2.7.22

	$(-3.9)(4.075)$
The signs are different. The product will be negative.	
Write in vertical format, lining up the numbers on the right.	4.075 × 3.9
Multiply.	4.075 × 3.9 36675 12225 158925
Add the number of decimal places in the factors $(1+3)$. (–3.9) (4.075) 1 place 3 places Place the decimal point 4 places from the right.	4.075 × 3.9 36675 12225 15.8925 4 places
The signs are different, so the product is negative.	$(-3.9)(4.075) = -15.8925$

Exercise 2.7.23

-27.4815

Exercise 2.7.24

-87.6148

Exercise 2.7.25

By looking at the number of zeros in the multiple of ten, we see the number of places we need to move the decimal to the right.

ⓐ

	$5.63(10)$
There is 1 zero in 10, so move the decimal point 1 place to the right.	5.63 56.3

ⓑ

	$5.63(100)$
There are 2 zeros in 100, so move the decimal point 2 places to the right.	There are 2 zeros in 100, so move the decimal point 2 places to the right. 5.63 (100) 5.63 563

ⓒ

There are 3 zeros in 1,000, so move the decimal point 3 places to the right.	5.63
A zero must be added at the end.	5,630

Exercise 2.7.26

1. 25.8
2. 258
3. 2,580

Exercise 2.7.27

1. 142
2. 1,420
3. 14,200

Exercise 2.7.34

	0.374
Determine the place value of the final digit.	0.3 (tenths) 7 (hundredths) 4 (thousandths)
Write the fraction for 0.374: • The numerator is 374. • The denominator is 1,000.	$\frac{374}{1000}$
Simplify the fraction.	$\frac{2 \cdot 187}{2 \cdot 500}$
Divide out the common factors.	$\frac{187}{500}$ so, $0.374 = \frac{187}{500}$

Did you notice that the number of zeros in the denominator of $\frac{374}{1000}$ is the same as the number of decimal places in 0.374?

Exercise 2.7.35

$\frac{117}{500}$

Exercise 2.7.36

$\frac{3}{125}$

Exercise 2.7.37

Since a fraction bar means division, we begin by writing $\frac{5}{8}$ as $8\overline{)5}$. Now divide.

$$\begin{array}{r} 0.625 \\ 8\overline{)5.000} \\ \underline{48} \\ 20 \\ \underline{16} \\ 40 \\ \underline{40} \\ \end{array}$$

$$\text{so, } -\frac{5}{8} = -0.625$$

Exercise 2.7.38

−0.875

Exercise 2.7.39

−0.375

Exercise 2.7.40

Divide 43 by 22.

$$\frac{43}{22}$$

$$22\overline{)43.00000}\quad 1.95454$$

$$\begin{array}{r} 22 \\ \hline 210 \\ 198 \\ \hline 120 \\ 110 \\ \hline 100 \\ 88 \\ \hline 120 \\ 110 \\ \hline 100 \\ 88 \\ \hline \cdots \end{array}$$

120 repeats

100 repeats

The pattern repeats, so the numbers in the quotient will repeat as well.

so, $\frac{43}{22} = 1.95\overline{54}$

Exercise 2.7.41

$2.\overline{45}$

Exercise 2.7.42

$2.3\overline{18}$

Exercise 2.7.43

First we must change one number so both numbers are in the same form. We can change the fraction to a decimal, or change the decimal to a fraction. Usually it is easier to change the fraction to a decimal.

		$\frac{7}{8} + 6.4$
Change $\frac{7}{8}$ to a decimal.	$8\overline{)7.000}\quad 0.875$ 64 60 56 40 40 0	
Add.		$0.875 + 6.4$
		7.275
		So, $\frac{7}{8} + 6.4 = 7.275$

Exercise 2.7.44

5.275

Exercise 2.7.45

6.35

Exercise 2.7.46

1.	
	62%
Move the decimal point two places to the left.	0.62

2.	
	135%
Move the decimal point two places to the left.	1.35

3.	
	5.7%
Move the decimal point two places to the left.	0.057

Exercise 2.7.47

1. 0.09
2. 0.87
3. 0.039

Exercise 2.7.48

1. 0.03
2. 0.91
3. 0.083

Exercise 2.7.49

1.	
	0.51
Move the decimal point two places to the right.	51

2.	
	1.25
Move the decimal point two places to the right.	125

3.	
	0.093
Move the decimal point two places to the right.	9.3

Exercise 2.7.50

1. 17%
2. 175%
3. 8.25%

Exercise 2.7.51

1. 41%
2. 225%
3. 9.25%

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2.7.1: Practice and Videos for Adding, Subtracting and Multiplying Decimals

Learning Objectives

- I can add decimals algebraically and using base ten blocks.
- I can subtract decimals.
- I can multiply decimals.

GeoGebra: Adding Decimals with Base 10 Blocks

Complete the activity below to learn how to add decimals up to the hundredths place using a visual. Use the purple sliders to adjust the decimals. Pick any random numbers you desire for each place value. Next, slide the purple slider on the bottom left to model the addition expression. What do you notice?

Press "Start Again" to try new decimal numbers.

Videos on Adding, Subtracting, and Multiplying Decimals

Watch the following videos to learn how to add, subtract, and multiply decimals. All videos have appropriate captions in English; in some cases, there may also be appropriate captions in other languages.

YouTube Video

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Practice!

Check your understanding of adding, subtracting, and multiplying decimals by answering these questions. Use the arrows at the bottom of the box to go to the next question.

Practice 2.7.1.1

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Extra Practice Resources

For more practice on adding, subtracting, and multiplying decimals, visit these websites.

1. IXL: Add decimal numbers
2. IXL: Subtract decimal numbers
3. IXL: Multiplying decimals

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2.7.2: Common Mistakes with Adding and Subtracting Decimals

Adding Decimals

Spot the Mistake 2.7.2.1

The following problem has an error. Identify the mistake.

$$9.087 + 15.31 = 10.618 \tag{2.7.2.1}$$

Incorrect setup:

$$\begin{array}{r} {\scriptstyle 1} \\ 9.0\ 8\,7 \\ +\ 1\ \ 5.3\,1 \\ \hline 10.6\ \ 1\,8 \end{array} \tag{2.7.2.2}$$

Click for explanation available at the end of this section.

Subtracting Decimals

Spot the Mistake 2.7.2.1

The following problem has an error. Identify the mistake.

$758 - 0.3 = 75.5$

Incorrect setup:

$$\begin{array}{r} 7\,58 \\ -0.3 \\ \hline 75.5 \end{array} \tag{2.7.2.3}$$

Click for explanation available at the end of this section.

Answers

Spot the Mistake 2.7.2.1

The decimal points must line up. Rewrite 15.31 as 15.310 so both numbers have three decimal places. Then add from right to left and bring the decimal point straight down.

Explanation	Work

Step 1: Line up the decimal points. The decimal point in 9.087 must be directly above the decimal point in 15.31.	$$\begin{array}{r} 9.087 \\ +\ 15.31 \\ \hline \end{array}$$
Step 2: Add a zero as a placeholder. Since 15.31 has two decimal places after the decimal and 9.087 has three decimal places after the decimal, rewrite 15.31 as 15.310. This does not change the value of the number.	$$\begin{array}{r} 9.087 \\ +\ 15.310 \\ \hline \end{array}$$
Step 3: Add from right to left. Bring the decimal point straight down. The correct answer is 24.397.	$$\begin{array}{r} 9.087 \\ +\ 15.310 \\ \hline 24.397 \end{array}$$

Key Idea

When adding decimals, always line up the decimal points so their place values match.

Spot the Mistake $2.7.2.1$

The mistake is that the student ignored the decimal points and lined up the digits from the right. This made 0.3 act like 3 ones instead of 3 tenths. When subtracting decimals, the decimal points must line up. See the explanation below for how to solve this problem.

Explanation	Work

Step 1: Line up the decimal points. The 758 can be written as 758.0. These are equivalent. This setup ensures the tenths, ones, tens, and hundreds place values are aligned.	$$\begin{array}{rrrcr} & 7 & 5 & 8 & .\ 0 \\ - & & & 0 & .\ 3 \\ \hline \end{array}$$
Step 2: Since we cannot subtract 3 tenths from 0 tenths, we need to regroup. Borrow 1 whole from the 8 ones. The 8 ones become 7 ones, and the 0 tenths become 10 tenths.	$$\begin{array}{rrrcr} & 7 & 5 & 7 & .\ 10 \\ & 7 & 5 & \cancel{8} & .\ \cancel{0} \\ - & & & 0 & .\ 3 \\ \hline \end{array}$$
Step 3: Now subtract. 10 tenths minus 3 tenths is 7 tenths. Bring down the decimal point. The correct answer is 757.7, not 75.5.	$$\begin{array}{rrrcr} & 7 & 5 & 7 & .\ 10 \\ & 7 & 5 & \cancel{8} & .\ \cancel{0} \\ - & & & 0 & .\ 3 \\ \hline & 7 & 5 & 7 & .\ 7 \end{array}$$

Key Idea

When subtracting decimals, always line up the decimal points so their place values match.

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2.8: Rounding Decimals

Rounding Decimal Numbers

We previously considered the concept of rounding numbers where our concern with rounding was related to positive integers only. With a few minor changes, we can apply the same rules of rounding to decimals (non-integers) and negative integers. To round a value to any integer (positive or negative), we simply apply the same method as before while keeping in mind what it means for negative numbers to be less than or greater than other numbers. We have not discussed negative numbers yet, however. So, we will not focus on this at this time. So, we will focus on non-integers. The method outlined below applies to positive and negative values, but all the examples and exercises will involve positive values to keep things simple for now.

To round a decimal to a particular position:

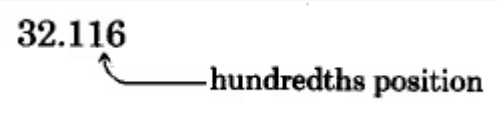
1. Mark the position of the round-off digit.
2. Note whether the digit to the immediate right of the marked digit is
   a. *less than 5*. If so, leave the round-off digit unchanged.
   b. *5 or greater*. If so, add 1 to the round-off digit.
3. If the round-off digit is
   a. to the right of the decimal point, eliminate all the digits to its right.
   b. to the left of the decimal point, replace all the digits between it and the decimal point with zeros and eliminate the decimal point and all the decimal digits.

Example 2.8.1

Round each decimal to the specified position. (The numbers in parentheses indicate which step is being used.)

Round 32.116 to the nearest hundredth.

Solution

1. 

2. The digit immediately to the right is 6, and $6 > 5$, so we add 1 to the round-off digit: $1 + 1 = 2$.
3. The round-off digit is to the right of the decimal point, so we eliminate all digits to its right.

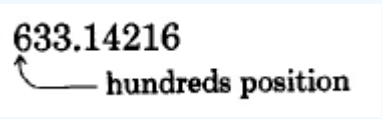
The number 32.116 rounded to the nearest hundredth is 32.12.

Note that the only real change to the method we saw in the previous section is that we need to consider what happens to the right of the round-off digit based on where it is in relation to the decimal point.

Example 2.8.1

Round 633.14216 to the nearest hundred.

Solution

1. 

2. The digit immediately to the right is 3, and $3 < 5$ so we leave the round-off digit unchanged.
3. The round-off digit is to the left of 0, so we replace all the digits between it and the decimal point with zeros and eliminate the decimal point and all the decimal digits.

The number 633.14216 rounded to the nearest hundred is 600.

We can actually combine 3a and 3b to the following: 3* Replace all digits to the right of the round-off digit with 0. This is because we could write literally any number as a number followed by infinitely many 0s. For example, 4.5 = 4.5000... and 670 = 670.000... The reason we usually do not do this is because by using the method with two cases for Step 3, the reader can tell exactly where our round-off digit is. Let's see what we mean by this.

Example 2.8.1

Round 84.996 to the nearest hundredth using both methods (one with Steps 3a and 3b and one with Step 3*).

Solution

1. The round-off digit is the 9 in the hundredths place.
2. The digit to the right of that is 6, so we add 1 to 9. That gives us 10, so we replace the 9 with 0 and add the 1 to the digit in the tenths place. Since the digit in the tenths place is also 9, we get 10 again. So, we replace that 9 with 0 and add the 1 to the digit in the ones place. The digit in the ones place is 4, so we get 5.
3. In the first method, we eliminate the digits to the right of the round-off digit.
   Thus, 84.996 rounded to the nearest hundredth is 85.00.
   In the second method, we replace all digits to the right of the round-off digit with 0.
   Thus, 84.996 rounded to the nearest hundredth is 85.000...

Notice that in the first method, we got 85.00, and the last written 0 is in the hundredths place, which was the round-off digit! In the second method, we get 85.000..., which is equivalent in value to 85.00. But by writing it this way, the person reading our answer does not know that the hundredths place was where we rounded to unless we explicitly state that in words.

The purpose of that previous example is to give a reason for why we use the usual rounding method in which Step 3 is split into 3a and 3b. It is common for people to think, "This is unnecessarily complicated" about rules in mathematics. Sometimes it's because it has to be (in these cases, the "rules" are actually "properties"). Sometimes it's because the rule is based on a reason that may not be evident at first (like what we just saw). But there definitely are times when the "rule" is just a convention with no particular reason behind it. In those cases, there is often disagreement between mathematicians about that rule. Yes, mathematicians argue about math. They do it a lot!

Example 2.8.1

Round each of the following numbers.

- 1,729.63 to the nearest ten
- 1.0144 to the nearest tenth
- 60.98 to the nearest one

Solution

- 1,729.63 rounded to the nearest ten is 1,730.
- 1.0144 rounded to the nearest tenth is 1.0.
- 60.98 rounded to the nearest one is 61.

Sometimes we hear a phrase such as "round to the fourth decimal place" or "round to three decimal places." This phrase means that the round-off digit is the fourth decimal digit (7the digit in the ten-thousandths position) or third decimal digit (the digit in the thousandths position), respectively.

Example 2.8.1

Round each of the following numbers.

- 67.129 to the second decimal place
- 67.129558 to 3 decimal places

Solution

- 67.129 rounded to the second decimal place is 67.13.
- 67.129558 rounded to 3 decimal places is 67.130.

Now, try these on your own.

Exercise 2.8.1

Round each decimal to the specified position.

1. 4.816 to the nearest hundredth.
2. 0.35928 to the nearest ten thousandths.
3. 82.1 to the nearest one.
4. 753.98 to the hundreds place.
5. Round 43.99446 to three decimal places.
6. Round 105.019997 to four decimal places.
7. Round 99.9999 to the second decimal place.

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2.8.1: Practice and Videos for Rounding Decimals

Learning Objectives

- I can round decimals to the nearest whole number, tenth, hundredth, thousandth, and so on.
- I can round decimals using visual representations such as number lines.

GeoGebra: Rounding Decimals on a Number Line

Learn how to round decimals on a number line by using a bucket to catch a falling ball in this activity. Using the given number, follow the instructions to round the number. Then, drag the brown bucket to that rounded number. Click the purple check box in the left corner. Keep trying and checking until your answer is correct. For more practice, click "new number" in the right corner to practice with a new example.

Videos on Rounding Decimals

Watch the following videos to learn how to round decimals. All videos have appropriate captions in English; in some cases, there may also be appropriate captions in other languages.

YouTube Video

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Practice!

Check your understanding of rounding decimals by answering these questions. Use the arrows at the bottom of the box to go to the next question.

Practice 2.8.1.1

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Extra Practice Resources

1. IXL: Round decimals

2. Math Playground: Tandem Turtles

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2.9: Multiplication of Whole Numbers

Multiplication of Whole Numbers

- understand the process of multiplication
- be able to multiply whole numbers
- be able to simplify multiplications with numbers ending in zero
- be able to use a calculator to multiply one whole number by another

Multiplication

Definition: Multiplication

Multiplication is a description of repeated addition.

In the addition of

$5+5+5$

the number 5 is repeated 3 times. Therefore, we say we have three times five and describe it by writing

3×5

Thus,

$3 \times 5 = 5+5+5$

Definition: Multiplicand

In a multiplication, the repeated addend (number being added) is called the **multiplicand**. In 3×5, the 5 is the multiplicand.

Definition: Multiplier

Also, in a multiplication, the number that records the number of times the multiplicand is used is called the **multiplier**. In 3×5, the 3 is the multiplier.

Sample Set A

Express each repeated addition as a multiplication. In each case, specify the multiplier and the multiplicand.

$7+7+7+7+7+7$

Solution

6×7, Multiplier is 6. Multiplicand is 7.

Sample Set A

$18 + 18 + 18$

Solution

3×18. Multiplier is 3. Multiplicand is 18.

Practice Set A

Express each repeated addition as a multiplication. In each case, specify the multiplier and the multiplicand.

$12 + 12 + 12 + 12$

. Multiplier is . Multiplicand is .

Answer available at the end of this section.

Practice Set A

$36 + 36 + 36 + 36 + 36 + 36 + 36 + 36$

. Multiplier is . Multiplicand is .

Answer available at the end of this section.

Practice Set A

$0 + 0 + 0 + 0 + 0$

. Multiplier is . Multiplicand is .

Answer available at the end of this section.

Practice Set A

$\underbrace{1847 + 1847 + \cdots + 1847}_{12{,}000 \text{ times}}$

s. Multiplier is . Multiplicand is .me

Answer available at the end of this section.

Definition: Factors

In a multiplication, the numbers being multiplied are also called **factors**.

Definition: Products

The result of a multiplication is called the **product**. In $3 \times 5 = 15$, the 3 and 5 are not only called the multiplier and multiplicand, but they are also called factors. The product is 15.

Indicators of Multiplication $\times, \cdot, ()$

The multiplication symbol $(\times)$ is not the only symbol used to indicate multiplication. Other symbols include the dot $(\cdot)$ and pairs of parentheses ( ). The expressions

$$3 \times 5, 3 \cdot 5, 3(5), (3)5, (3)(5)$$

all represent the same product.

The Multiplication Process With a Single Digit Multiplier

Since multiplication is repeated addition, we should not be surprised to notice that **carrying** can occur. Carrying occurs when we find the product of 38 and 7:

$$\begin{array}{r} {\scriptstyle 5} \\ 38 \\ \times\ \ 7 \\ \hline 266 \end{array}$$

First, we compute $7 \times 8 = 56$. Write the 6 in the ones column. Carry the 5. Then take $7 \times 3 = 21$. Add to 21 the 5 that was carried: $21 + 5 = 26$. The product is 266.

Sample Set B

Find the following products.

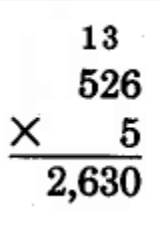
$$\begin{array}{r} {\scriptstyle 1} \\ 64 \\ \times\ \ 3 \\ \hline 192 \end{array}$$

Solution

$3 \times 4 = 12$	Write the 2, carry the 1.
$3 \times 6 = 18$	Add to 18 the 1 that was carried: $18 + 1 = 19$.

The product is 192.

Sample Set B

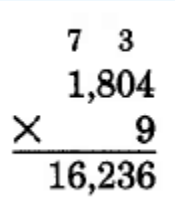


Solution

$5 \times 6 = 30$	Write the 0, carry the 3.
$5 \times 2 = 10$	Add to 10 the 3 that was carried: $10 + 3 = 13$. Write the 3, carry the 1.
$5 \times 5 = 25$	Add to 25 the 1 that was carried: $25 + 1 = 6$.

The product is 2,630.

Sample Set B



Solution

$9 \times 4 = 36$	Write the 6, carry the 3.
$9 \times 0 = 0$	Add to 0 the 3 that was carried: $0 + 3 = 13$. Write the 3.
$9 \times 8 = 72$	Write the 2, carry the 7.
	Since there are no more multiplications to perform, write both the 1 and 6.

The product is 16,236.

Practice Set B

Find the following products.

$$\begin{array}{r} 37 \\ \times\ 5 \\ \hline \end{array}$$

Answer available at the end of this section.

Practice Set B

Find the following products.

$$\begin{array}{r} 78 \\ \times\ 8 \\ \hline \end{array}$$

Answer available at the end of this section.

Practice Set B

Find the following products.

$$\begin{array}{r} 537 \\ \times \quad 7 \\ \hline \end{array}$$

Answer available at the end of this section.

Practice Set B

Find the following products.

$$\begin{array}{r} 40,019 \\ \times \quad 8 \\ \hline \end{array}$$

Answer available at the end of this section.

Practice Set B

Find the following products.

$$\begin{array}{r} 301,599 \\ \times \quad 3 \\ \hline \end{array}$$

Answer available at the end of this section.

The Multiplication Process With a Multiple Digit Multiplier

In a multiplication in which the multiplier is composed of two or more digits, the *multiplication must take place in parts*. The process is as follows:

- **First Partial Product** Multiply the multiplicand by the ones digit of the multiplier. This product is called the **first partial product**.
- **Second Partial Product** Multiply the multiplicand by the tens digit of the multiplier. This product is called the **second partial product**. Since the tens digit is used as a factor, the second partial product is written below the first partial product so that its rightmost digit appears in the tens column.
- If necessary, continue this way finding partial products. Write each one below the previous one so that the rightmost digit appears in the column directly below the digit that was used as a factor.
- **Total Product** Add the partial products to obtain the **total product**.

Note

It may be necessary to carry when finding each partial product.

Sample Set C

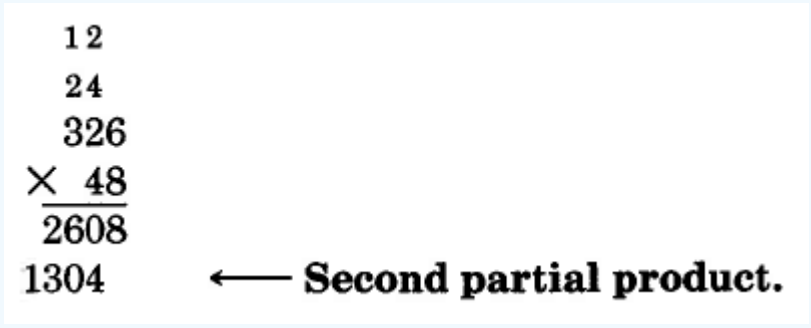
Multiply 326 by 48.

Solution

Part 1:

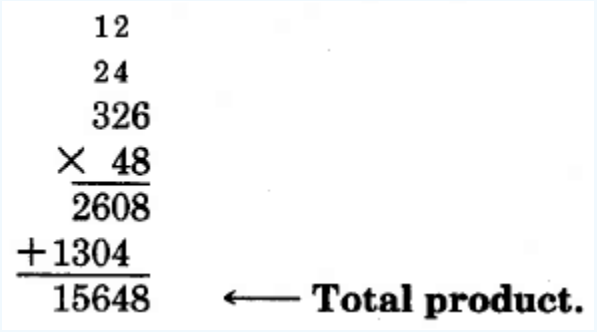


Part 2:



Part 3: This step is unnecessary since all of the digits in the multiplier have been used.

Part 4: Add the partial products to obtain the total product.



The product is 15,648.

Sample Set C

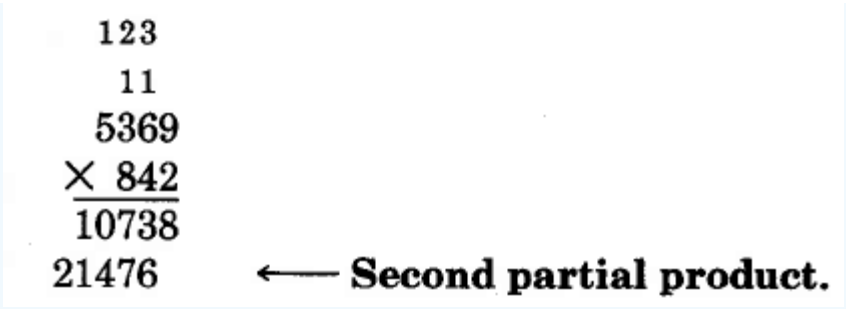
Multiply 5,369 by 842.

Solution

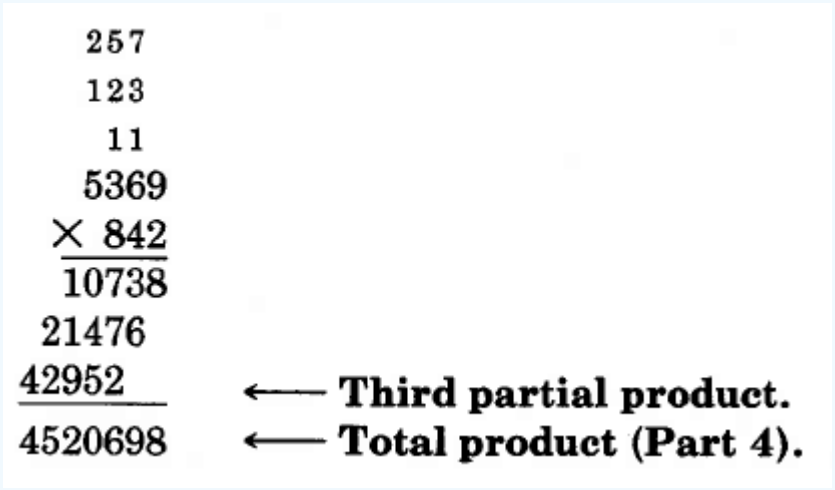
Part 1:



Part 2:



Part 3:



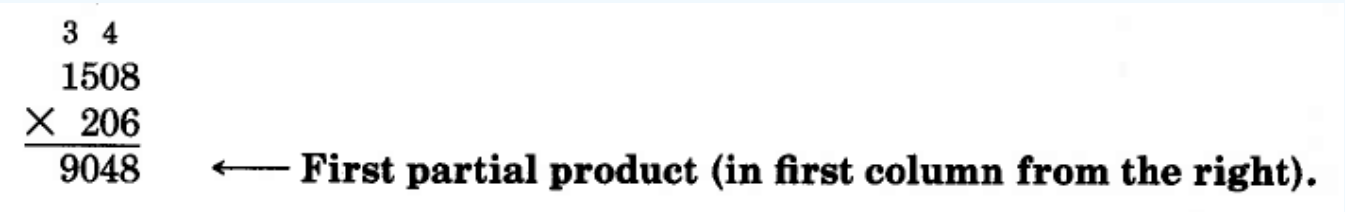
The product is 4,520,698.

Sample Set C

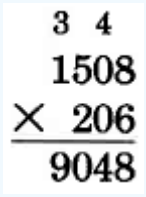
Multiply 1,508 by 206.

Solution

Part 1:

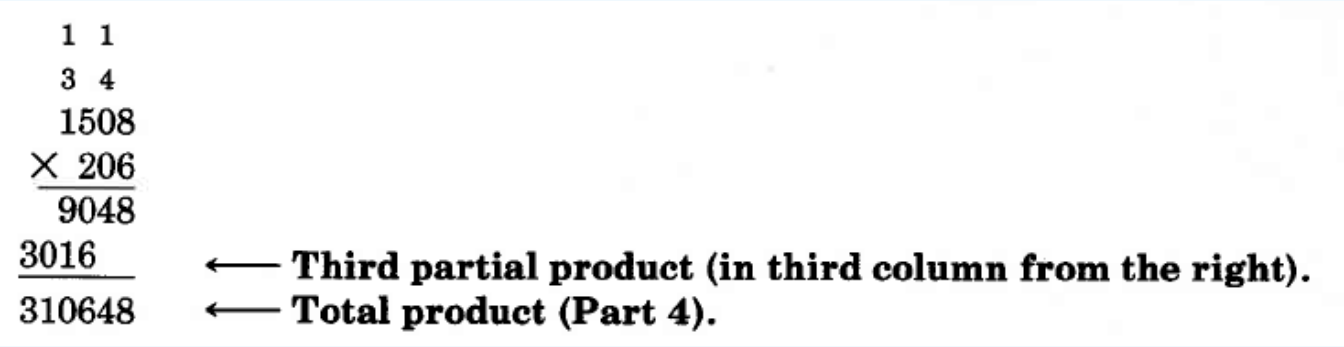


Part 2:



Since 0 times 1508 is 0, the partial product will not change the identity of the total product (which is obtained by addition). Go to the next partial product.

Part 3:



The product is 310,648

Practice Set C

Multiply 73 by 14.

Answer available at the end of this section.

Practice Set C

Multiply 86 by 52.

Answer available at the end of this section.

Practice Set C

Multiply 419 by 85.

Answer available at the end of this section.

Practice Set C

Multiply 2,376 by 613.

Answer available at the end of this section.

Practice Set C

Multiply 8,107 by 304.

Answer available at the end of this section.

Practice Set C

Multiply 66,260 by 1,008.

Answer available at the end of this section.

Practice Set C

Multiply 209 by 501.

Answer available at the end of this section.

Practice Set C

Multiply 24 by 10.

Answer available at the end of this section.

Practice Set C

Multiply 3,809 by 1,000.

Answer available at the end of this section.

Practice Set C

Multiply 813 by 10,000.

Answer available at the end of this section.

Multiplications With Numbers Ending in Zero

Often, when performing a multiplication, one or both of the factors will end in zeros. Such multiplications can be done quickly by aligning the numbers so that the rightmost nonzero digits are in the same column.

Sample Set D

Perform the multiplication (49,000)(1,200).

$$(49,000)(1,200) = \begin{array}{r} 49000 \\ \times\ \ 1200 \\ \hline \end{array}$$

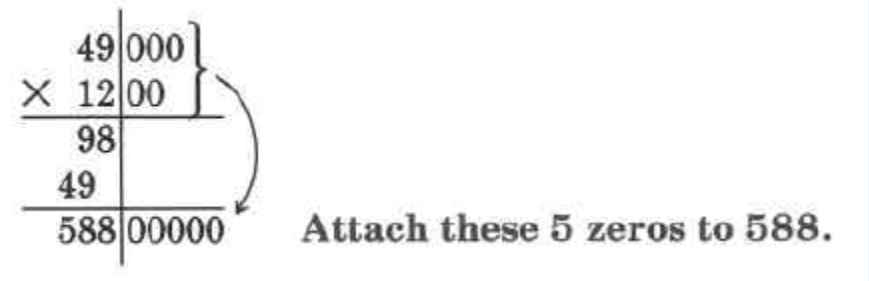
Since 9 and 2 are the rightmost nonzero digits, put them in the same column.

$$\begin{array}{r} 49000 \\ \times 1200 \\ \hline \end{array}$$

Draw (perhaps mentally) a vertical line to separate the zeros from the nonzeros.

$$\begin{array}{r} 49\,|\,000 \\ \times 12\,|\,00 \\ \hline \end{array}$$

Multiply the numbers to the left of the vertical line as usual, then attach to the right end of this product the total number of zeros.



The product is 58,800,000

Practice Set D

Multiply 1,800 by 90.

Answer available at the end of this section.

Practice Set D

Multiply 420,000 by 300.

Answer available at the end of this section.

Practice Set D

Multiply 20,500,000 by 140,000.

Answer available at the end of this section.

Calculators

Most multiplications are performed using a calculator.

Sample Set E

Multiply 75,891 by 263.

Solution

Display Reads		
Type	75891	75891
Press	×	75891
Type	263	263
Press	=	19959333

The product is 19,959,333.

Sample Set E

Multiply 4,510,000,000,000 by 1,700.

Solution

Display Reads		
Type	451	451
Press	×	451
Type	17	17
Press	=	7667

The display now reads 7667. We'll have to add the zeros ourselves. There are a total of 12 zeros. Attaching 12 zeros to 7667, we get 7,667,000,000,000,000.

The product is 7,667,000,000,000,000.

Sample Set E

Multiply 57,847,298 by 38,976.

Solution

Display Reads		
Type	57847298	57847298
Press	×	57847298
Type	38976	38976
Press	=	2.2546563 12

The display now reads 2.2546563 12. What kind of number is this? This is an example of a whole number written in **scientific notation**. We'll study this concept when we get to decimal numbers.

Practice Set E

Use a calculator to perform each multiplication.

52×27

Answer available at the end of this section.

Practice Set E

$1,448 \times 6,155$

Answer available at the end of this section.

Practice Set E

$8,940,000 \times 205,000$

Answer available at the end of this section.

Exercises

For the following problems, perform the multiplications. You may check each product with a calculator.

Exercise 2.9.1

$$\begin{array}{r} 8 \\ \times 3 \\ \hline \end{array}$$

Answer available at the end of this section.

Exercise 2.9.2

$$\begin{array}{r} 3 \\ \times 5 \\ \hline \end{array}$$

Exercise 2.9.3

$$\begin{array}{r} 8 \\ \times 6 \\ \hline \end{array}$$

Answer available at the end of this section.

Exercise 2.9.4

$$\begin{array}{r} 5 \\ \times 7 \\ \hline \end{array}$$

Exercise 2.9.5

6×1

Answer available at the end of this section.

Exercise 2.9.6

4×5

Exercise 2.9.7

75×3

Answer available at the end of this section.

Exercise 2.9.8

35×5

Exercise 2.9.9

$$\begin{array}{r} 45 \\ \times\ \ 6 \\ \hline \end{array}$$

Answer available at the end of this section.

Exercise 2.9.10

$$\begin{array}{r} 31 \\ \times\ \ 7 \\ \hline \end{array}$$

Exercise 2.9.11

$$\begin{array}{r} 97 \\ \times\ \ 6 \\ \hline \end{array}$$

Answer available at the end of this section.

Exercise 2.9.12

$$\begin{array}{r} 75 \\ \times 57 \\ \hline \end{array}$$

Exercise 2.9.13

$$\begin{array}{r} 64 \\ \times 15 \\ \hline \end{array}$$

Answer available at the end of this section.

Exercise 2.9.14

$$\begin{array}{r} 73 \\ \times 15 \\ \hline \end{array}$$

Exercise 2.9.15

$$\begin{array}{r} 81 \\ \times 95 \\ \hline \end{array}$$

Answer available at the end of this section.

Exercise 2.9.16

$$\begin{array}{r} 31 \\ \times 33 \\ \hline \end{array}$$

Exercise 2.9.17

57×64

Answer available at the end of this section.

Exercise 2.9.18

76×42

Exercise 2.9.19

894×52

Answer available at the end of this section.

Exercise 2.9.20

684×38

Exercise 2.9.21

$$\begin{array}{r} 115 \\ \times \ \ 22 \\ \hline \end{array}$$

Answer available at the end of this section.

Exercise 2.9.22

$$\begin{array}{r} 706 \\ \times \ \ 81 \\ \hline \end{array}$$

Exercise 2.9.23

$$\begin{array}{r} 328 \\ \times\ \ 21 \\ \hline \end{array}$$

Answer available at the end of this section.

Exercise 2.9.24

$$\begin{array}{r} 550 \\ \times\ \ 94 \\ \hline \end{array}$$

Exercise 2.9.25

930×26

Answer available at the end of this section.

Exercise 2.9.26

318×63

Exercise 2.9.27

$$\begin{array}{r} 582 \\ \times 127 \\ \hline \end{array}$$

Answer available at the end of this section.

Exercise 2.9.28

$$\begin{array}{r} 247 \\ \times 116 \\ \hline \end{array}$$

Exercise 2.9.29

$$\begin{array}{r} 305 \\ \times 225 \\ \hline \end{array}$$

Answer available at the end of this section.

Exercise 2.9.30

$$\begin{array}{r} 782 \\ \times 547 \\ \hline \end{array}$$

Exercise 2.9.31

$$\begin{array}{r} 771 \\ \times 663 \\ \hline \end{array}$$

Answer available at the end of this section.

Exercise 2.9.32

$$\begin{array}{r} 638 \\ \times 516 \\ \hline \end{array}$$

Exercise 2.9.33

$1,905 \times 710$

Answer available at the end of this section.

Exercise 2.9.34

$5,757 \times 5,010$

Exercise 2.9.35

$$\begin{array}{r} 3,106 \\ \times 1,752 \\ \hline \end{array}$$

Answer available at the end of this section.

Exercise 2.9.36

$$\begin{array}{r} 9,300 \\ \times 1,130 \\ \hline \end{array}$$

Exercise 2.9.37

$$\begin{array}{r} 7,057 \\ \times 5,229 \\ \hline \end{array}$$

Answer available at the end of this section.

Exercise 2.9.38

$$\begin{array}{r} 8,051 \\ \times 5,580 \\ \hline \end{array}$$

Exercise 2.9.39

$$\begin{array}{r} 5,804 \\ \times 4,300 \\ \hline \end{array}$$

Answer available at the end of this section.

Exercise 2.9.40

$$\begin{array}{r} 357 \\ \times\ \ 16 \\ \hline \end{array}$$

Exercise 2.9.41

$$\begin{array}{r} 724 \\ \times\ \ \ 0 \\ \hline \end{array}$$

Answer available at the end of this section.

Exercise 2.9.42

$$\begin{array}{r} 2,649 \\ \times\ \ \ 41 \\ \hline \end{array}$$

Exercise 2.9.43

$$\begin{array}{r} 5,173 \\ \times\ \ \ \ 8 \\ \hline \end{array}$$

Answer available at the end of this section.

Exercise 2.9.44

$$\begin{array}{r} 1,999 \\ \times\ \ \ \ 0 \\ \hline \end{array}$$

Exercise 2.9.45

$$\begin{array}{r} 1,666 \\ \times\ \ \ \ 0 \\ \hline \end{array}$$

Answer available at the end of this section.

Exercise 2.9.46

$$\begin{array}{r} 51,730 \\ \times\ \ \ 142 \\ \hline \end{array}$$

Exercise 2.9.47

$$\begin{array}{r} 387 \\ \times 190 \\ \hline \end{array}$$

Answer available at the end of this section.

Exercise 2.9.48

$$\begin{array}{r} 3,400 \\ \times \quad 70 \\ \hline \end{array}$$

Exercise 2.9.49

$$\begin{array}{r} 460,000 \\ \times \ 14,000 \\ \hline \end{array}$$

Answer available at the end of this section.

Exercise 2.9.50

$$\begin{array}{r} 558,000,000 \\ \times \quad 81,000 \\ \hline \end{array}$$

Exercise 2.9.51

$$\begin{array}{r} 37,000 \\ \times \quad 120 \\ \hline \end{array}$$

Answer available at the end of this section.

Exercise 2.9.52

$$\begin{array}{r} 498,000 \\ \times \quad 0 \\ \hline \end{array}$$

Exercise 2.9.53

$$\begin{array}{r} 4,585,000 \\ \times \quad 140 \\ \hline \end{array}$$

Answer available at the end of this section.

Exercise 2.9.54

$$\begin{array}{r} 30,700,000 \\ \times \quad 180 \\ \hline \end{array}$$

Exercise 2.9.55

$$\begin{array}{r} 8,000 \\ \times \quad 10 \\ \hline \end{array}$$

Answer available at the end of this section.

Exercise 2.9.56

Suppose a theater holds 426 people. If the theater charges $4 per ticket and sells every seat, how much money would they take in?

Exercise 2.9.57

In an English class, a student is expected to read 12 novels during the semester and prepare a report on each one of them. If there are 32 students in the class, how many reports will be prepared?

Answer available at the end of this section.

Exercise 2.9.58

In a mathematics class, a final exam consists of 65 problems. If this exam is given to 28 people, how many problems must the instructor grade?

Exercise 2.9.59

A business law instructor gives a 45 problem exam to two of her classes. If each class has 37 people in it, how many problems will the instructor have to grade?

Answer available at the end of this section.

Exercise 2.9.60

An algebra instructor gives an exam that consists of 43 problems to four of his classes. If the classes have 25, 28, 31, and 35 students in them, how many problems will the instructor have to grade?

Exercise 2.9.61

In statistics, the term "standard deviation" refers to a number that is calculated from certain data. If the data indicate that one standard deviation is 38 units, how many units is three standard deviations?

Answer available at the end of this section.

Exercise 2.9.62

Soft drinks come in cases of 24 cans. If a supermarket sells 857 cases during one week, how many individual cans were sold?

Exercise 2.9.63

There are 60 seconds in 1 minute and 60 minutes in 1 hour. How many seconds are there in 1 hour?

Answer available at the end of this section.

Exercise 2.9.64

There are 60 seconds in 1 minute, 60 minutes in one hour, 24 hours in one day, and 365 days in one year. How many seconds are there in 1 year?

Exercise 2.9.65

Light travels 186,000 miles in one second. How many miles does light travel in one year? (*Hint:* Can you use the result of the previous problem?)

Answer available at the end of this section.

Exercise 2.9.66

An elementary school cafeteria sells 328 lunches every day. Each lunch costs $1. How much money does the cafeteria bring in in 2 weeks?

Exercise 2.9.67

A computer company is selling stock for $23 a share. If 87 people each buy 55 shares, how much money would be brought in?

Answer available at the end of this section.

Exercises for Review

In the number 421,998, how may ten thousands are there?

Round 448,062,187 to the nearest hundred thousand.

Answer available at the end of this section.

Find the sum. 22,451 + 18,976.

Subtract 2,289 from 3,001.

Answer available at the end of this section.

Specify which property of addition justifies the fact that (a first whole number + a second whole number) = (the second whole number + the first whole number)

Answers

Practice Set A

4×12. Multiplier is 4. Multiplicand is 12.

Practice Set A

8×36. Multiplier is 8. Multiplicand is 36.

Practice Set A

5×0. Multiplier is 5. Multiplicand is 0.

Practice Set A

$12,000 \times 1,847$. Multiplier is 12,000. Multiplicand is 1,847.

Practice Set B

185

Practice Set B

624

Practice Set B

3,752

Practice Set B

320,152

Practice Set B

904,797

Practice Set C

1,022

Practice Set C

4,472

Practice Set C

35,615

Practice Set C

1,456,488

Practice Set C

2,464,528

Practice Set C

66,790,080

Practice Set C

104,709

Practice Set C

240

Practice Set C

3,809,000

Practice Set C

8,130,000

Practice Set D

162,000

Practice Set D

126,000,000

Practice Set D

2,870,000,000,000

Practice Set E

1,404

Practice Set E

8,912,440

Practice Set E

1,832,700,000,000

Exercise 2.9.1

24

Exercise 2.9.3

48

Exercise 2.9.5

6

Exercise 2.9.7
225

Exercise 2.9.9
270

Exercise 2.9.11
582

Exercise 2.9.13
960

Exercise 2.9.15
7,695

Exercise 2.9.17
3,648

Exercise 2.9.19
46,488

Exercise 2.9.21
2,530

Exercise 2.9.23
6,888

Exercise 2.9.25
24,180

Exercise 2.9.27
73,914

Exercise 2.9.29
68,625

Exercise 2.9.31
511,173

Exercise 2.9.33
1,352,550

Exercise 2.9.35
5,441,712

Exercise 2.9.37
36,901,053

Exercise 2.9.39
24,957,200

Exercise 2.9.41
0

Exercise 2.9.43
41,384

Exercise 2.9.45
0

Exercise 2.9.47
73,530

Exercise 2.9.49
6,440,000,000

Exercise 2.9.51
4,440,000

Exercise 2.9.53
641,900,000

Exercise 2.9.55
80,000

Exercise 2.9.57
384 reports

Exercise 2.9.59
3,330 problems

Exercise 2.9.61
114 units

Exercise 2.9.63

3,600 seconds

Exercise 2.9.65

5,865,696,000,000 miles per year

Exercise 2.9.67

$110,055

448,100,000

712

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2.9.1: Practice and Videos for Multiplication of Whole Numbers

Learning Objectives

- I can multiply multi-digit numbers.
- I can understand multi-digit multiplication using visuals such as area models.

GeoGebra: Multiplying Multi-Digit and 1-Digit Numbers Using Partial Products

Explore multiplication of multi-digit whole numbers by 1-digit whole numbers with partial product and area models in this activity. Click the empty boxes to reveal the answer for each partial product in each box. Keep clicking until you get to the last box. Observe the graphic to see the connection between the diagram and the addition expression on the left. Click the purple "try another" button on the bottom left for a new problem.

Videos on Multiplying Whole Numbers

Watch the following videos to learn how to multiply whole numbers. All videos have appropriate captions in English; in some cases, there may also be appropriate captions in other languages.

YouTube Video

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

YouTube Video

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

YouTube Video

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

Practice: Math Trainer - Multiplication

Check your understanding of basic multiplication facts by visiting this website: Math is Fun: Math Trainer - Multiplication

How to play:

- Adjust the duration of the practice by clicking the drop-down menu next to Workout Time.
- Select your question cutoff time. This determines how much time you will have to answer each question. Start at 4 seconds and decrease the cutoff each game.
- Next, click on the "new game" button at the top and play until the timer runs out. Use the keyboard or on-screen keypad to type your answers.

Extra Practice Resources

1. IXL: Multiply 1-digit numbers by 3-digit or 4-digit numbers using area models

2. IXL: Multiply 2-digit numbers by 2-digit numbers using area models

3. PhET: Area Model Multiplication

Instructions for PhET area model multiplication activity:

- Go to this website.
- Select Game.
- In the upper left corner, start the first level by clicking the purple box.
- Find the total area of the given expression by using the diagram displayed or another method you like.
- Type the answer in the box using the keypad and click check. If you got it right, there should be a smiley face. If you got it incorrect, try again until you get it right.
- Complete the remaining problems.

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2.10: Division of Whole Numbers

Learning Objectives

- be able to divide a whole number by a single or multiple digit divisor
- be able to interpret a calculator statement that a division results in a remainder

Division with a Single Digit Divisor

Our experience with multiplication of whole numbers allows us to perform such divisions as $75 \div 5$. We perform the division by performing the corresponding multiplication, $5 \times Q = 75$. Each division we considered in [link] had a one-digit quotient. Now we will consider divisions in which the quotient may consist of two or more digits. For example, $75 \div 5$.

Let's examine the division $75 \div 5$. We are asked to determine how many 5's are contained in 75. We'll approach the problem in the following way.

1. Make an educated guess based on experience with multiplication.
2. Find how close the estimate is by multiplying the estimate by 5.
3. If the product obtained in step 2 is less than 75, find out how much less by subtracting it from 75.
4. If the product obtained in step 2 is greater than 75, decrease the estimate until the product is less than 75. Decreasing the estimate makes sense because we do not wish to exceed 75.

We can suggest from this discussion that the process of division consists of

The Four Steps in Division

1. an educated guess
2. a multiplication
3. a subtraction
4. bringing down the next digit (if necessary)

The educated guess can be made by determining how many times the divisor is contained in the dividend by using only one or two digits of the dividend.

Sample Set A

Find $75 \div 5$.

Solution

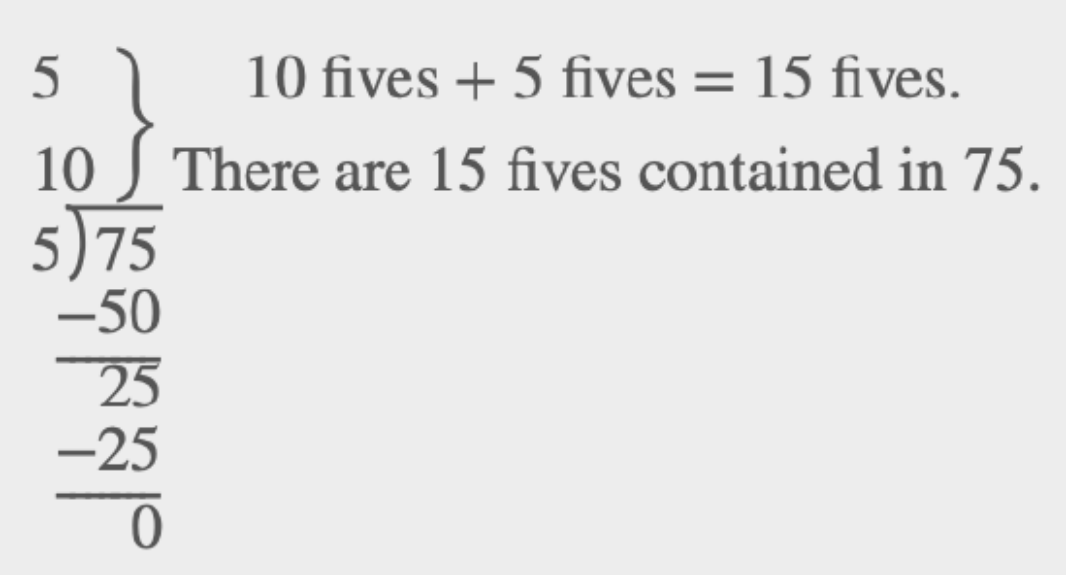
$5\overline{)75}$ Rewrite the problem using a division bracket.

$$\begin{array}{r} 10 \\ 5\overline{)75} \end{array}$$

Make an educated guess by noting that one 5 is contained in 75 at most 10 times.
Since 7 is the tens digit, we estimate that 5 goes into 75 at most 10 times.

$$\begin{array}{r} 10 \\ 5\overline{)75} \\ \underline{-50} \\ 25 \end{array}$$

Now determine how close the estimate is.
10 fives is $10 \times 5 = 50$. Subtract 50 from 75.
Estimate the number of 5's in 25.
There are exactly 5 fives in 25.



Check:

$$75 \stackrel{?}{=} 15 \times 5$$
$$75 \checkmark 75$$

Thus, $75 \div 5 = 15$.

The notation in this division can be shortened by writing.

$$\begin{array}{r} 15 \\ 5\overline{)75} \\ -5\downarrow \\ \hline 25 \\ -25 \\ \hline 0 \end{array}$$

Divide:	5 goes into 7 at most 1 time.	Divide:	5 goes into 25 exactly 5 times
Multiply:	$1 \times 5 = 5$.Write 5 below 7.	Multiply:	$5 \times 5 = 25$.Write 25 below 25.
Subtract:	$7 - 5 = 2$.Bring down the 5.	Subtract:	$25 - 25 = 0$

Sample Set A

Find $4,944 \div 8$.

Solution

$8\overline{)4944}$

Rewrite the problem using a division bracket.

$$
\begin{array}{r}
600 \\
8\overline{)4944} \\
-4800 \\
\hline
144
\end{array}
$$

8 goes into 49 at most 6 times, and 9 is in the hundreds column. We'll guess 600.
Then, $8 \times 600 = 4800$.

$$
\begin{array}{r}
10 \\
600 \\
8\overline{)4944} \\
-4800 \\
\hline
144 \\
-\ 80 \\
\hline
64
\end{array}
$$

8 goes into 14 at most 1 time, and 4 is in the tens column. We'll guess 10.

$$
\begin{array}{r}
8 \\
10 \\
600 \\
8\overline{)4944} \\
-4800 \\
\hline
144 \\
-\ 80 \\
\hline
64 \\
-64 \\
\hline
0
\end{array}
$$

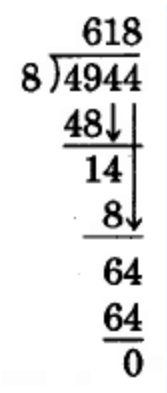
8 goes into 64 exactly 8 times.
600 eights + 10 eights + 8 eights = 618 eights.

Check:

$$4944 \stackrel{?}{=} 8 \times 618$$
$$4944 \stackrel{\checkmark}{=} 4944$$

Thus, $4,944 \div 8 = 618$.

As in the first problem, the notation in this division can be shortened by eliminating the subtraction signs and the zeros in each educated guess.



$\begin{cases} \text{Divide:} & 8 \text{ goes into } 49 \text{ at most } 6 \text{ times.} \\ \text{Multiply:} & 6 \times 8 = 48. \text{Write } 48 \text{ below } 49. \\ \text{Subtract:} & 49 - 48 = 1. \text{Bring down the } 4. \end{cases}$

$\begin{cases} \text{Divide:} & 8 \text{ goes into } 14 \text{ at most } 1 \text{ time.} \\ \text{Multiply:} & 1 \times 8 = 8. \text{Write } 8 \text{ below } 14. \\ \text{Subtract:} & 14 - 8 = 6. \text{Bring down the } 4. \end{cases}$

$\begin{cases} \text{Divide:} & 8 \text{ goes into } 64 \text{ exactly } 8 \text{ times.} \\ \text{Multiply:} & 8 \times 8 = 64. \text{Write } 64 \text{ below } 64. \\ \text{Subtract:} & 64 - 64 = 0. \end{cases}$

Note

Not all divisions end in zero. We will examine such divisions in a subsequent subsection.

Practice Set A

Perform the following divisions.

$126 \div 7$

Answer available at the end of this section.

Practice Set A

$324 \div 4$

Answer available at the end of this section.

Practice Set A

$2,559 \div 3$

Answer available at the end of this section.

Practice Set A

$5,645 \div 5$

Answer available at the end of this section.

Practice Set A

$757,125 \div 9$

Answer available at the end of this section.

Division with a Multiple Digit Divisor

The process of division also works when the divisor consists of two or more digits. We now make educated guesses using the first digit of the divisor and one or two digits of the dividend.

Sample Set B

Find $2,232 \div 36$.

Solution

$36\overline{)2232}$

Use the first digit of the divisor and the first two digits of the dividend to make the educated guess.

3 goes into 22 at most 7 times.

Try 7: $7 \times 36 = 252$ which is greater than 223. Reduce the estimate.

Try 6: $6 \times 36 = 216$ which is less than 223.

$$\begin{array}{r} 6 \\ 36\overline{)\ 2232} \\ -216\downarrow \\ \hline 72 \end{array}$$

$\begin{array}{ll} \text{Multiply:} & 6 \times 36 = 216. \text{Write 216 below 223.} \\ \text{Subtract:} & 223 - 216 = 7. \text{Bring down the 2.} \end{array}$

Divide 3 into 7 to estimate the number of times 36 goes into 72. The 3 goes into 7 at most 2 times.

Try 2: $2 \times 36 = 72$.

$$\begin{array}{r} 62 \\ 36\overline{)2232} \\ 216\downarrow \\ \hline 72 \\ -72 \\ \hline 0 \end{array}$$

Check:

$$2232 \stackrel{?}{=} 36 \times 62$$
$$2232 \checkmark 2232$$

Thus $2,232 \div 36 = 62$.

Sample Set B

Find $2,417,228 \div 802$.

Solution

$802\overline{)2417228}$

First, the educated guess: $24 \div 8 = 3$. Then $3 \times 802 = 2406$, which is less than 2417. Use 3 as the guess. Since $3 \times 802 = 2406$, and 2406 has four digits, place the 3 above the fourth digit of the dividend.

$$\begin{array}{r} 3 \\ 802\overline{)\ 2417228} \\ -2406\downarrow \\ \hline 112 \end{array}$$

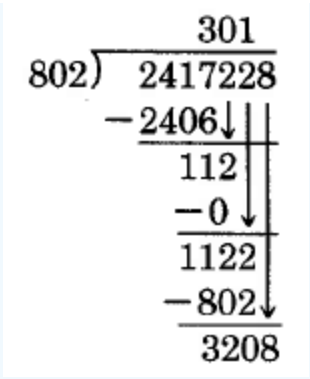
Subtract: 2417 - 2406 = 11.
Bring down the 2.

The divisor 802 goes into 112 at most 0 times. Use 0.

$$\begin{array}{r} 30 \\ 802\overline{)\ 2417228} \\ -2406\downarrow\ \ \ \\ \hline 112\ \ \ \\ -0\downarrow\ \ \\ \hline 1122 \end{array}$$

Multiply: $0 \times 802 = 0$.
Subtract: $112 - 0 = 112$.
Bring down the 2.

The 8 goes into 11 at most 1 time, and $1 \times 802 = 802$, which is less than 1122. Try 1.

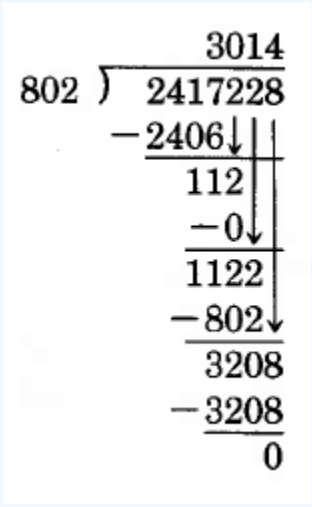


Subtract 1122 - 802 = 320
Bring down the 8.

8 goes into 32 at most 4 times.

$4 \times 802 = 3208.$

Use 4.



Check:

$$2417228 \stackrel{?}{=} 3014 \times 802$$
$$2417228 \checkmark 2417228$$

Thus, $2,417,228 \div 802 = 3,014.$

Practice Set B

Perform the following divisions.

$1,376 \div 32$

Answer available at the end of this section.

Practice Set B

$6,160 \div 55$

Answer available at the end of this section.

Practice Set B

$18,605 \div 61$

Answer available at the end of this section.

Practice Set B

$144,768 \div 48$

Answer available at the end of this section.

Division with a Remainder

We might wonder how many times 4 is contained in 10. Repeated subtraction yields

$$\begin{array}{r} 10 \\ -\ 4 \\ \hline 6 \\ -4 \\ \hline 2 \end{array}$$

Since the remainder is less than 4, we stop the subtraction. Thus, 4 goes into 10 two times with 2 remaining. We can write this as a division as follows.

$$\begin{array}{r} 2 \\ \hline 4)10 \\ -\ 8 \\ \hline 2 \end{array}$$

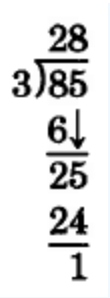
Divide:	4 goes into 10 at most 2 times.
Multiply:	$2 \times 4 = 8$. Write 8 below 0.
Subtract:	$10 - 8 = 2$.

$$\begin{array}{r} 2\text{R}2 \\ \hline 4)\ 10 \\ \underline{-8} \\ 2 \end{array} \quad \text{or} \quad 10 \div 4 = \underbrace{2\text{R}2}_{2 \text{ with remainder } 2}$$

Sample Set C

Find $85 \div 3$.

Solution

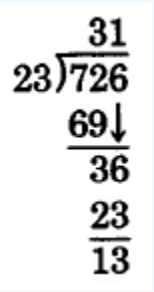


$$\begin{cases}\text{Divide:} & 3 \text{ goes into } 8 \text{ at most } 2 \text{ times.}\\ \text{Multiply:} & 2 \times 3 = 6. \text{ Write } 6 \text{ below } 8.\\ \text{Subtract:} & 8 - 6 = 2. \text{ Bring down the } 5.\end{cases} \quad \begin{cases}\text{Divide:} & 3 \text{ goes into } 25 \text{ at most } 8 \text{ times.}\\ \text{Multiply:} & 3 \times 8 = 24. \text{ Write } 24 \text{ below } 25.\\ \text{Subtract:} & 25 - 24 = 1\end{cases}$$

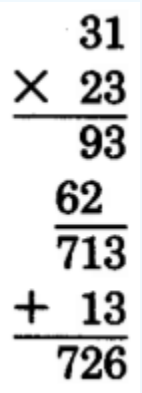
Sample Set C

Find $726 \div 23$.

Solution



Check: Multiply 31 by 23, then add 13.



Thus, $726 \div 23 = 31R13$.

Practice Set C

Perform the following divisions.

$75 \div 4$

Answer available at the end of this section.

Practice Set C

$346 \div 8$

Answer available at the end of this section.

Practice Set C

$489 \div 21$

Answer available at the end of this section.

Practice Set C

$5,016 \div 82$

Answer available at the end of this section.

Practice Set C

$41,196 \div 67$

Answer available at the end of this section.

Calculators

The calculator can be useful for finding quotients with single and multiple digit divisors. If, however, the division should result in a remainder, the calculator is unable to provide us with the particular value of the remainder. Also, some calculators (most nonscientific) are unable to perform divisions in which one of the numbers has more than eight digits.

Sample Set D

Use a calculator to perform each division.

$328 \div 8$.

Solution

Type	328
Press	÷
Type	8
Press	=

The display now reads 41.

Sample Set D

$53,136 \div 82$.

Solution

Type	53136
Press	÷
Type	82
Press	=

The display now reads 648.

Sample Set D

$730,019,001 \div 326$

Solution

We first try to enter 730,019,001 but find that we can only enter 73001900. If our calculator has only an eight-digit display (as most nonscientific calculators do), we will be unable to use the calculator to perform this division.

Sample Set D

$3727 \div 49$.

Solution

Type	3727
Press	÷
Type	49
Press	=

The display now reads 76.061224.

This number is an example of a decimal number (see [link]). When a decimal number results in a calculator division, we can conclude that the division produces a remainder.

Practice Set D

Use a calculator to perform each division.

$3,330 \div 74$

Answer available at the end of this section.

Practice Set D

$63,365 \div 115$

Answer available at the end of this section.

Practice Set D

$21,996,385,287 \div 53$

Answer available at the end of this section.

Practice Set D

$4,558 \div 67$

Answer available at the end of this section.

Exercises

For the following problems, perform the divisions.

The first 38 problems can be checked with a calculator by multiplying the divisor and quotient then adding the remainder.

Exercise 2.10.1

$52 \div 4$

Answer available at the end of this section.

Exercise 2.10.2

$776 \div 8$

Exercise 2.10.3

$603 \div 9$

Answer available at the end of this section.

Exercise 2.10.4

$240 \div 8$

Exercise 2.10.5

$208 \div 4$

Answer available at the end of this section.

Exercise 2.10.6

$576 \div 6$

Exercise 2.10.7

$21 \div 7$

Answer available at the end of this section.

Exercise 2.10.8

$0 \div 0$

Exercise 2.10.9

$140 \div 2$

Answer available at the end of this section.

Exercise 2.10.10

$528 \div 8$

Exercise 2.10.11

$244 \div 4$

Answer available at the end of this section.

Exercise 2.10.12

$0 \div 7$

Exercise 2.10.13

$177 \div 3$

Answer available at the end of this section.

Exercise 2.10.14

$96 \div 8$

Exercise 2.10.15

$67 \div 1$

Answer available at the end of this section.

Exercise 2.10.16

$896 \div 56$

Exercise 2.10.17

$1,044 \div 12$

Answer available at the end of this section.

Exercise 2.10.18

$988 \div 19$

Exercise 2.10.19

$5,238 \div 97$

Answer available at the end of this section.

Exercise 2.10.20

$2530 \div 55$

Exercise 2.10.21

$4,264 \div 82$

Answer available at the end of this section.

Exercise 2.10.22

$637 \div 13$

Exercise 2.10.23

$3,420 \div 90$

Answer available at the end of this section.

Exercise 2.10.24

$5,655 \div 87$

Exercise 2.10.25

$2,115 \div 47$

Answer available at the end of this section.

Exercise 2.10.26

$9,328 \div 22$

Exercise 2.10.27

$55,167 \div 71$

Answer available at the end of this section.

Exercise 2.10.28

$68,356 \div 92$

Exercise 2.10.29

$27,702 \div 81$

Answer available at the end of this section.

Exercise 2.10.30

$6,510 \div 31$

Exercise 2.10.31

$60,536 \div 94$

Answer available at the end of this section.

Exercise 2.10.32

$31,844 \div 38$

Exercise 2.10.33

$23,985 \div 45$

Answer available at the end of this section.

Exercise 2.10.34

$60,606 \div 74$

Exercise 2.10.35

$2,975,400 \div 285$

Answer available at the end of this section.

Exercise 2.10.36

$1,389,660 \div 795$

Exercise 2.10.37

$7,162,060 \div 879$

Answer available at the end of this section.

Exercise 2.10.38

$7,561,060 \div 909$

Exercise 2.10.39

$38 \div 9$

Answer available at the end of this section.

Exercise 2.10.40

$97 \div 4$

Exercise 2.10.41

$199 \div 3$

Answer available at the end of this section.

Exercise 2.10.42

$573 \div 6$

Exercise 2.10.43

$10,701 \div 13$

Answer available at the end of this section.

Exercise 2.10.44

$13,521 \div 53$

Exercise 2.10.45

$3,628 \div 90$

Answer available at the end of this section.

Exercise 2.10.46

$10,592 \div 43$

Exercise 2.10.47

$19,965 \div 30$

Answer available at the end of this section.

Exercise 2.10.48

$8,320 \div 21$

Exercise 2.10.49

$61,282 \div 64$

Answer available at the end of this section.

Exercise 2.10.50

$1,030 \div 28$

Exercise 2.10.51

$7,319 \div 11$

Answer available at the end of this section.

Exercise 2.10.52

$3,628 \div 90$

Exercise 2.10.53

$35,279 \div 77$

Answer available at the end of this section.

Exercise 2.10.54

$52,196 \div 68$

Exercise 2.10.55

$67,751 \div 68$

Answer available at the end of this section.

For the following 5 problems, use a calculator to find the quotients.

Exercise 2.10.56

$4,346 \div 53$

Exercise 2.10.57

$3,234 \div 77$

Answer available at the end of this section.

Exercise 2.10.58

$6,771 \div 37$

Exercise 2.10.59

$4,272,320 \div 520$

Answer available at the end of this section.

Exercise 2.10.60

$7,558,110 \div 651$

Exercise 2.10.61

A mathematics instructor at a high school is paid $17,775 for 9 months. How much money does this instructor make each month?

Answer available at the end of this section.

Exercise 2.10.62

A couple pays $4,380 a year for a one-bedroom apartment. How much does this couple pay each month for this apartment?

Exercise 2.10.63

Thirty-six people invest a total of $17,460 in a particular stock. If they each invested the same amount, how much did each person invest?

Answer available at the end of this section.

Exercise 2.10.64

Each of the 28 students in a mathematics class buys a textbook. If the bookstore sells $644 worth of books, what is the price of each book?

Exercise 2.10.65

A certain brand of refrigerator has an automatic ice cube maker that makes 336 ice cubes in one day. If the ice machine makes ice cubes at a constant rate, how many ice cubes does it make each hour?

Answer available at the end of this section.

Exercise 2.10.66

A beer manufacturer bottles 52,380 ounces of beer each hour. If each bottle contains the same number of ounces of beer, and the manufacturer fills 4,365 bottles per hour, how many ounces of beer does each bottle contain?

Exercise 2.10.67

A computer program consists of 68,112 bits. 68,112 bits equals 8,514 bytes. How many bits in one byte?

Answer available at the end of this section.

Exercise 2.10.68

A 26-story building in San Francisco has a total of 416 offices. If each floor has the same number of offices, how many floors does this building have?

Exercise 2.10.69

A college has 67 classrooms and a total of 2,546 desks. How many desks are in each classroom if each classroom has the same number of desks?

Answer available at the end of this section.

Exercises for Review

Exercise 2.10.70

What is the value of 4 in the number 124,621?

Exercise 2.10.71

Round 604,092 to the nearest hundred thousand.

Answer available at the end of this section.

Exercise 2.10.72

What whole number is the additive identity?

Exercise 2.10.73

Find the product. $6,256 \times 100$.

Answer available at the end of this section.

Exercise 2.10.74

Find the quotient. $0 \div 11$

Answers

Practice Set A
18

Practice Set A
81

Practice Set A
853

Practice Set A
1,129

Practice Set A
84,125

Practice Set B
43

Practice Set B
112

Practice Set B
305

Practice Set B
3,016

Practice Set C
18 R3

Practice Set C
43 R2

Practice Set C
23 R6

Practice Set C
61 R14

Practice Set C
614 R58

Practice Set D
45

Practice Set D
551

Practice Set D
Since the dividend has more than eight digits, this division cannot be performed on most nonscientific calculators. On others, the answer is 415,026,137.4

Practice Set D
This division results in 68.02985075, a decimal number, and therefore, we cannot, at this time, find the value of the remainder. Later, we will discuss decimal numbers.

Exercise 2.10.1
13

Exercise 2.10.3
67

Exercise 2.10.5
52

Exercise 2.10.7
3

Exercise 2.10.9
70

Exercise 2.10.11
61

Exercise 2.10.13
59

Exercise 2.10.15
67

Exercise 2.10.17
87

Exercise 2.10.19
54

Exercise 2.10.21
52

Exercise 2.10.23
38

Exercise 2.10.25
45

Exercise 2.10.27
777

Exercise 2.10.29
342

Exercise 2.10.31
644

Exercise 2.10.33
533

Exercise 2.10.35
10,440

Exercise 2.10.37
8,147 remainder 847

Exercise 2.10.39
4 remainder 2

Exercise 2.10.41
66 remainder 1

Exercise 2.10.43
823 remainder 2

Exercise 2.10.45
40 remainder 28

Exercise 2.10.47
665 remainder 15

Exercise 2.10.49
957 remainder 34

Exercise 2.10.51
665 remainder 4

Exercise 2.10.53
458 remainder 13

Exercise 2.10.55
996 remainder 23

Exercise 2.10.57
42

Exercise 2.10.59
8,216

Exercise 2.10.61
$1,975 per month

Exercise 2.10.63
$485 each person invested

Exercise 2.10.65
14 cubes per hour

Exercise 2.10.67
8 bits in each byte

Exercise 2.10.69
38

Exercise 2.10.71
600,000

Exercise 2.10.73
625,600

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2.10.1: Practice and Videos for Division of Whole Numbers

Learning Objectives

- I can divide multi-digit whole numbers using long division.
- I can visually understand long division and its connection to place value.

GeoGebra: Long Division With One-Digit Divisors and Remainders

Complete the activity below to explore division of multi-digit numbers by one-digit divisors with the standard algorithm. Use the purple slider in the bottom left corner to observe the steps for performing long division. For a new problem, click "try another" in the bottom right corner.

Videos on Multi-Digit Division

Watch the following videos to learn about multi-digit division. All videos have appropriate captions in English; in some cases, there may also be appropriate captions in other languages.

YouTube Video

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

YouTube Video

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

YouTube Video

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

YouTube Video

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

Division Practice

1. Go to MM Practice Long Division.

2. Complete the 6 long-division problems by typing numbers into the empty boxes. It is okay to have remainders.

3. Check your answer for each problem and click next to move on to the next problem!

Extra Practice Resources

1. Divide multi-digit numbers by 1-digit numbers: word problems

2. SPARK Studio: Jeopardy: Dividing by Multi-Digit Numbers

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2.11: Unit Conversion - American System

In this section we will develop a technique for converting units used in the American system. We begin with a discussion of common measurements of length in the United States.

Units of Length

The most common units of length are the inch, foot, yard, and mile. Our focus will be on the technique used to convert from one unit of length to another.

American Units of Length

Facts relating common units of length.

- 1 foot (ft) = 12 inches (in)
- 1 yard (yd) = 3 feet (ft)
- 1 mile (mi) = 5280 feet (ft)

Take for example, the fact that there are 3 feet in 1 yard, which can be stated as an equation, using the common abbreviations for feet (ft) and yards (yd).

$$3 \text{ ft} = 1 \text{ yd}$$

If we divide both sides of the equation by 3 ft,

$$\frac{3 \text{ ft}}{3 \text{ ft}} = \frac{1 \text{ yd}}{3 \text{ ft}},$$

or equivalently,

$$1 = \frac{1 \text{ yd}}{3 \text{ ft}}.$$

The key observation is the fact that the ratio 1 yd/3 ft equals the number 1. Consequently, multiplying by the "conversion factor" 1 yd/3 ft is equivalent to multiplying by 1. This can be used to change a measurement in feet to yards.

Example 1

Change 36 feet to yards.

Solution

Multiply by the conversion factor 1 yd/3 ft.

$$\begin{aligned}
36 \text{ ft} &= 36 \text{ ft} \cdot 1 && \text{Multiplicative Identity Property.} \\
&= 36 \text{ ft} \cdot \frac{1 \text{ yd}}{3 \text{ ft}} && \text{Replace 1 with 1 yd/3 ft.} \\
&= 36 \cancel{\text{ft}} \cdot \frac{1 \text{ yd}}{3 \cancel{\text{ft}}} && \text{Cancel common unit.} \\
&= \frac{36 \cdot 1}{3} \text{ yd} && \text{Multiply fractions.} \\
&= \frac{36}{3} \text{ yd} && \text{Simplify.} \\
&= 12 \text{ yd} && \text{Divide.}
\end{aligned}$$

Hence, 36 feet equals 12 yards.

Exercise

Change 81 feet to yards.

Answer available at the end of this section.

On the other hand, we can start again with

$$3 \text{ ft} = 1 \text{ yd}$$

and divide both sides of the equation by 1 yd.

$$\frac{3 \text{ ft}}{1 \text{ yd}} = \frac{1 \text{ yd}}{1 \text{ yd}}$$

This gives the conversion factor

$$\frac{3 \text{ ft}}{1 \text{ yd}} = 1.$$

The key observation is the fact that the ratio 3 ft/1 yd equals the number 1. Consequently, multiplying by the "conversion factor" 3 ft/1 yd is equivalent to multiplying by 1. This can be used to change a measurement in yards to feet.

Example 2

Change 18 yards to feet.

Solution

Multiply by the conversion factor 3 ft/1 yd.

\[\begin{aligned} 18 \text{ yd} = 18 \text{ yd} \cdot 1 ~ & \textcolor{red}{ \text{ Multiplicative Identity Property.}} \ = 18 \text{ yd} \cdot \frac{1 \text{ ft}}{1 \text{ yd}} ~ & \textcolor{red}{ \text{ Replace 1 with 3 ft/1 yd.} } \ = 18 \cancel{ \text{ yd}} \cdot \frac{3 \text{ ft}}{1 \cancel{ \text{ yd}}} ~ & \textcolor{red}{ \text{ Cancel common unit.}} \ = \frac{18 \cdot 3}{1} \text{ ft} ~ & \textcolor{red}{ \text{ Multiply fractions.}} \ = 54 \text{ ft} ~ & \textcolor{red}{ \text{ Simplify.}} \end{aligned}\nonumber \]

Hence, 18 yards equals 54 feet.

Exercise

Change 15 yards to feet.

Answer available at the end of this section.

Another common comparison is the fact that there are 12 inches in 1 foot. This can be represented as an equation using the common abbreviation for inches (in) and feet (ft).

$$12 \text{ in} = 1 \text{ ft}$$

Dividing both sides by 12 in

$$\frac{12 \text{ in}}{12 \text{ in}} = \frac{1 \text{ ft}}{12 \text{ in}},$$

yields the conversion factor

$$1 = \frac{1 \text{ ft}}{12 \text{ in}}.$$

The key observation is the fact that the ratio 1 ft/12 in equals the number 1. Consequently, multiplying by the "conversion factor" 1 ft/12 in is equivalent to multiplying by 1. This can be used to change a measurement in inches to feet.

Example 3

Change 24 inches to feet.

Solution

Multiply by the conversion factor 1 ft/12 in.

$$\begin{aligned} 24 \text{ in} &= 24 \text{ in} \cdot 1 && \text{Multiplicative Identity Property.} \\ &= 24 \text{ in} \cdot \frac{1 \text{ ft}}{12 \text{ in}} && \text{Replace 1 with 1 ft/12 in.} \\ &= 24 \cancel{\text{in}} \cdot \frac{1 \text{ in}}{12 \cancel{\text{in}}} && \text{Cancel common unit.} \\ &= \frac{24 \cdot 1}{12} \text{ ft} && \text{Multiply fractions.} \\ &= 2 \text{ ft} && \text{Simplify.} \end{aligned}$$

Hence, 24 inches equals 2 feet.

Exercise

Change 48 inches to feet.

Answer available at the end of this section.

We provide a summary of conversion factors for units of length in Table 6.1.

Table 6.1: Conversion factors for units of length.

Convert	Conversion Factor	Convert	Conversion Factor
feet to inches	12 in/1 ft	inches to feet	1 ft/12 in
yards to feet	3 ft/1 yd	feet to yards	1 yd/3 ft
miles to feet	5280 ft/1 mi	feet to miles	1 mi/5280 ft

Some conversions require more than one application of a conversion factor.

Example 4

Change 4 yards to inches.

Solution

We multiply by a chain of conversion factors, the first to change yards to feet, the second to change feet to inches.

$$\begin{aligned} 4 \text{ yd} &= 4 \text{ yd} \cdot \frac{3 \text{ ft}}{1 \text{ yd}} \cdot \frac{12 \text{ in}}{1 \text{ ft}} && \text{Multiply by conversion factors.} \\ &= 4 \cancel{\text{ yd}} \cdot \frac{3 \cancel{\text{ ft}}}{1 \cancel{\text{ yd}}} \cdot \frac{12 \text{ in}}{1 \cancel{\text{ ft}}} && \text{Cancel common units.} \\ &= \frac{4 \cdot 3 \cdot 12}{1 \cdot 1} \text{ in} && \text{Multiply fractions.} \\ &= 144 \text{ in} && \text{Simplify.} \end{aligned}$$

Hence, 4 yards equals 144 inches.

Exercise

Change 8 yards to inches.

Answer available at the end of this section.

Example 5

Change 2 miles to yards.

Solution

We multiply by a chain of conversion factors, the first to change miles to feet, the second to change feet to yards.

$$\begin{aligned} 2 \text{ mi} &= 2 \text{ mi} \cdot \frac{5280 \text{ ft}}{1 \text{ mi}} \cdot \frac{1 \text{ yd}}{3 \text{ ft}} && \text{Multiply by conversion factors.} \\ &= 2 \cancel{\text{ mi}} \cdot \frac{5280 \cancel{\text{ ft}}}{1 \cancel{\text{ mi}}} \cdot \frac{1 \text{ yd}}{3 \cancel{\text{ ft}}} && \text{Cancel common units.} \\ &= \frac{2 \cdot 5280 \cdot 1}{1 \cdot 3} \text{ yd} && \text{Multiply fractions.} \\ &= 3520 \text{ yd} && \text{Simplify.} \end{aligned}$$

Hence, 2 miles equals 3,520 yards.

Exercise

Change 5 miles to yards.

Answer available at the end of this section.

Units of Weight

The most common units of weight are the ounce, pound, and ton. Our focus will remain on how to convert from one unit to another.

American Units of Weight

Facts relating common units of weight.

- 1 pound (lb) = 16 ounces (oz)
- 1 ton = 2000 pounds (lb)

The above facts lead to the conversion factors in Table 6.2.

Table 6.2: Conversion factors for units of weight.

Convert	Conversion Factor	Convert	Conversion Factor
pounds to ounces	16 oz/1 lb	ounces to pounds	1 lb/16 oz
tons to pounds	2000 lb/1 ton	pounds to tons	1 ton/2000 lb

Example 6

Change $2\frac{1}{2}$ pounds to ounces.

Solution

Multiply by the appropriate conversion factor.

$$\begin{aligned} 2\frac{1}{2}\text{ lb} &= 2\frac{1}{2}\text{ lb} \cdot \frac{16\text{ oz}}{1\text{ lb}} && \text{Multiply by conversion factor.} \\ &= 2\frac{1}{2}\cancel{\text{lb}} \cdot \frac{16\text{ oz}}{1\cancel{\text{ lb}}} && \text{Cancel common units.} \\ &= \left(2\frac{1}{2} \cdot 16\right)\text{ oz} && \text{Multiply fractions.} \\ &= \left(\frac{5}{2} \cdot 16\right)\text{ oz} && \text{Mixed to improper fraction.} \\ &= \frac{80}{2}\text{ oz} && \text{Multiply.} \\ &= 40\text{ oz} && \text{Divide.} \end{aligned}$$

Hence, $2\frac{1}{2}$ pounds equals 40 ounces.

Exercise

Change $6\frac{1}{4}$ pounds to ounces.

Answer available at the end of this section.

Example 7

Change 3.2 tons to ounces.

Solution

This exercise requires multiplying by a chain of conversion factors.

$$\begin{aligned} 3.2\text{ ton} &= 3.2\text{ ton} \cdot \frac{2000\text{ lb}}{1\text{ ton}} \cdot \frac{16\text{ oz}}{1\text{ lb}} && \text{Multiply by conversion factors.} \\ &= 3.2\cancel{\text{ ton}} \cdot \frac{2000\cancel{\text{ lb}}}{1\cancel{\text{ ton}}} \cdot \frac{16\text{ oz}}{1\cancel{\text{ lb}}} && \text{Cancel common units.} \\ &= \frac{3.2 \cdot 2000 \cdot 16}{1 \cdot 1}\text{ oz} && \text{Multiply fractions.} \\ &= 102,400\text{ oz} && \text{Simplify.} \end{aligned}$$

Hence, 3.2 tons equals 102,400 ounces.

Exercise

Change 4.1 tons to ounces.

Answer available at the end of this section.

Units of Volume

The most common units of volume are fluid ounces, cups, pints, quarts, and gallons. We will focus on converting from one unit to another.

American Units of Volume

Facts relating common units of volume.

- 1 cup (c) = 8 fluid ounces (fl oz)
- 1 pint (pt) = 2 cups (c)
- 1 quart (qt) = 2 pints (pt)
- 1 gallon (gal) = 4 quarts (qt)

These facts lead to the conversion factors listed in Table 6.3.

Table 6.3: Conversion factors for units of volume.

Convert	Conversion Factor	Convert	Conversion Factor
cups to ounces	8 fl oz/1 c	ounces to cups	1c/8 fl oz
pints to cups	2 c/1 pt	cups to pints	1 pt/2 c
quarts to pints	2 pt/1 qt	pints to quarts	1 qt/2 pt
gallons to quarts	4 qt/1 gal	quarts to gallons	1 gal/4 qt

Example 8

Change 5.6 gallons to pints.

Solution

This exercise requires multiplying by a chain of conversion factors.

$$\begin{aligned} 5.6\text{ gal} &= 5.6\text{ gal} \cdot \frac{4\text{ qt}}{1\text{ gal}} \cdot \frac{2\text{ pt}}{1\text{ qt}} && \text{Multiply by conversion factors.} \\ &= 5.6\ \cancel{\text{gal}} \cdot \frac{4\ \cancel{\text{qt}}}{1\ \cancel{\text{gal}}} \cdot \frac{2\text{ pt}}{1\ \cancel{\text{qt}}} && \text{Cancel common units.} \\ &= \frac{5.6 \cdot 4 \cdot 2}{1 \cdot 1}\text{ pt} && \text{Multiply fractions.} \\ &= 44.8\text{ pt} && \text{Simplify.} \end{aligned}$$

Hence, 5.6 gallons equals 44.8 pints.

Exercise

Change 3.2 gallons to pints.

Answer available at the end of this section.

Units of Time

The most common units of time are seconds, minutes, hours, days, and years.

American Units of Time

Facts relating common units of time.

- 1 minute (min) = 60 seconds (s)
- 1 hour (hr) = 60 minutes (min)
- 1 day (day) = 24 hours (hr)
- 1 year (yr) = 365 days (day)

These facts lead to the conversion factors in Table 6.4.

Table 6.4: Conversion factors for units of time.

Convert	Conversion Factor	Convert	Conversion Factor
minutes to seconds	60 s/1 min	seconds to minutes	1 min/60 s
hours to minutes	60 min/1 hr	minutes to hours	1 hr/60 min
days to hours	24 hr/1 day	hours to days	1 day/24 hr
years to days	365 day/1 yr	days to years	1 yr/365 day

Example 9

How many seconds in a year?

Solution

A chain of conversion factors is needed.

$$1\text{ yr} = 1\text{ yr} \cdot \frac{365\text{ day}}{1\text{ yr}} \cdot \frac{24\text{ hr}}{1\text{ day}} \cdot \frac{60\text{ min}}{1\text{ hr}} \cdot \frac{60\text{ s}}{1\text{ min}} \quad \text{Conversion factors.}$$

$$= 1\ \cancel{\text{yr}} \cdot \frac{365\ \cancel{\text{day}}}{1\ \cancel{\text{yr}}} \cdot \frac{24\ \cancel{\text{hr}}}{1\ \cancel{\text{day}}} \cdot \frac{60\ \cancel{\text{min}}}{1\ \cancel{\text{hr}}} \cdot \frac{60\text{ s}}{1\ \cancel{\text{min}}} \quad \text{Cancel common units.}$$

$$= \frac{1 \cdot 365 \cdot 24 \cdot 60 \cdot 60}{1 \cdot 1 \cdot 1 \cdot 1}\text{ s} \quad \text{Multiply fractions.}$$

$$= 31,536,000\text{ s} \quad \text{Simplify.}$$

Thus, 1 year equals 31,536,000 seconds.

Exercise

How many seconds in a day?

Answer available at the end of this section.

Converting Units of Speed

Ever wonder how fast a baseball is moving?

Example 10

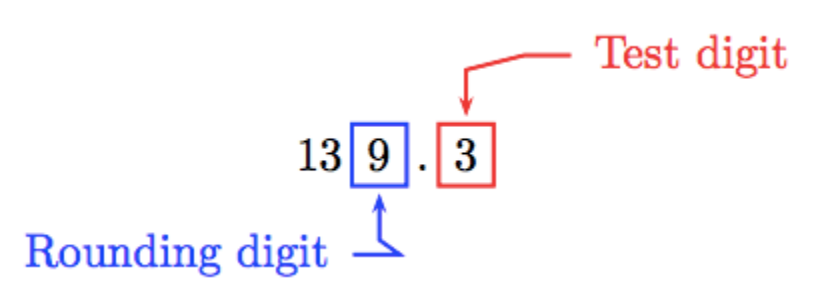
A professional pitcher can throw a baseball at 95 miles per hour. How fast is this in feet per second? Round your answer to the nearest foot per second.

Solution

There are 5280 feet in a mile, 60 minutes in an hour, and 60 seconds in a minute.

$$
\begin{aligned}
95\frac{\text{mi}}{\text{h}} &\approx 95\frac{\text{mi}}{\text{h}} \cdot \frac{5280\text{ ft}}{1\text{ mi}} \cdot \frac{1\text{ h}}{60\text{ min}} \cdot \frac{1\text{ min}}{60\text{ s}} && \text{Conversion factors.}\\
&\approx 95\frac{\cancel{\text{mi}}}{\cancel{\text{h}}} \cdot \frac{5280\text{ ft}}{1\cancel{\text{ mi}}} \cdot \frac{1\cancel{\text{ h}}}{60\cancel{\text{ min}}} \cdot \frac{1\cancel{\text{ min}}}{60\text{ s}} && \text{Cancel common units.}\\
&\approx \frac{95 \cdot 5280 \cdot 1 \cdot 1}{1 \cdot 60 \cdot 60}\frac{\text{ft}}{\text{s}} && \text{Multiply fractions.}\\
&\approx 139.3\frac{\text{ft}}{\text{s}} && \text{Multiply and divide.}
\end{aligned}
$$

To round to the nearest foot per second, identify the rounding and test digits.



Because the test digit is less than 5, leave the rounding digit alone and truncate. Thus, to the nearest foot per second, the speed is approximately 139 feet per second.

Whew! Since the batter stands at home plate, which is about 60 feet from where the pitch is delivered, the batter has less than 1/2 a second to react to the pitch!

Exercise

A women's softball pitcher can throw her fastball at 60 miles per hour. How fast is this in feet per second? Round your answer to the nearest foot per second.

Answer available at the end of this section.

Exercises

1. Change 8 yards to feet.

2. Change 60 yards to feet.

3. Change 261 feet to yards.

4. Change 126 feet to yards.

5. Change 235 inches to yards. Round your answer to the nearest tenth of a yard.

6. Change 244 inches to yards. Round your answer to the nearest tenth of a yard.

7. Change 141 feet to yards.

8. Change 78 feet to yards.

9. Change 2.8 miles to feet.

10. Change 4.9 miles to feet.

11. Change 104 inches to yards. Round your answer to the nearest tenth of a yard.

12. Change 101 inches to yards. Round your answer to the nearest tenth of a yard.

13. Change 168,372 inches to miles, correct to the nearest tenth of a mile.

14. Change 198,550 inches to miles, correct to the nearest tenth of a mile.

15. Change 82 feet to inches.

16. Change 80 feet to inches.

17. Change 2.9 yards to inches. Round your answer to the nearest inch.

18. Change 4.5 yards to inches. Round your answer to the nearest inch.

19. Change 25,756 feet to miles. Round your answer to the nearest tenth of a mile.

20. Change 19,257 feet to miles. Round your answer to the nearest tenth of a mile.

21. Change 5 yards to feet.

22. Change 20 yards to feet.

23. Change 169,312 inches to miles, correct to the nearest tenth of a mile.

24. Change 162,211 inches to miles, correct to the nearest tenth of a mile.

25. Change 1.5 yards to inches. Round your answer to the nearest inch.

26. Change 2.1 yards to inches. Round your answer to the nearest inch.

27. Change 360 inches to feet.

28. Change 768 inches to feet.

29. Change 48 inches to feet.

30. Change 528 inches to feet.

31. Change 15,363 feet to miles. Round your answer to the nearest tenth of a mile.

32. Change 8,540 feet to miles. Round your answer to the nearest tenth of a mile.

33. Change 1.7 miles to inches.

34. Change 4.7 miles to inches.

35. Change 3.1 miles to inches.

36. Change 1.8 miles to inches.

37. Change 3.6 miles to feet.

38. Change 3.1 miles to feet.

39. Change 18 feet to inches.

40. Change 33 feet to inches.

---

41. Change $5\frac{1}{8}$ pounds to ounces.

42. Change $3\frac{1}{16}$ pounds to ounces.

43. Change 2.4 tons to ounces.

44. Change 3.4 tons to ounces.

45. Change 34 ounces to pounds. Express your answer as a fraction reduced to lowest terms.

46. Change 78 ounces to pounds. Express your answer as a fraction reduced to lowest terms.

47. Change 2.2 tons to pounds.

48. Change 4.8 tons to pounds.

49. Change 70 ounces to pounds. Express your answer as a fraction reduced to lowest terms.

50. Change 20 ounces to pounds. Express your answer as a fraction reduced to lowest terms.

51. Change 9,560 pounds to tons. Round your answer to the nearest tenth of a ton.

52. Change 9,499 pounds to tons. Round your answer to the nearest tenth of a ton.

53. Change $2\frac{1}{2}$ pounds to ounces.

54. Change $7\frac{1}{16}$ pounds to ounces.

55. Change 5.9 tons to pounds.

56. Change 2.1 tons to pounds.

57. Change 2.5 tons to ounces.

58. Change 5.3 tons to ounces.

59. Change 8,111 pounds to tons. Round your answer to the nearest tenth of a ton.

60. Change 8,273 pounds to tons. Round your answer to the nearest tenth of a ton.

---

61. Change 4.5625 pints to fluid ounces.

62. Change 2.9375 pints to fluid ounces.

63. Change 32 fluid ounces to pints.

64. Change 160 fluid ounces to pints.

65. Change 3.7 gallons to pints.

66. Change 2.4 gallons to pints.

67. Change 216 pints to gallons.

68. Change 96 pints to gallons.

69. Change 544 fluid ounces to pints.

70. Change 432 fluid ounces to pints.

71. Change 112 pints to gallons.

72. Change 200 pints to gallons.

73. Change 7.7 gallons to pints.

74. Change 5.7 gallons to pints.

75. Change 3.875 pints to fluid ounces.

76. Change 3 pints to fluid ounces.

---

77. Change 7.8 years to hours.

78. Change 4.7 years to hours.

79. Change 7.6 years to hours.

80. Change 6.6 years to hours.

81. Change 4,025,005 seconds to days. Round your answer to the nearest tenth of a day.

82. Change 4,672,133 seconds to days. Round your answer to the nearest tenth of a day.

83. Change 37,668 hours to years.

84. Change 40,296 hours to years.

85. Change 22,776 hours to years.

86. Change 29,784 hours to years.

87. Change 96 days to seconds.

88. Change 50 days to seconds.

89. Change 40 days to seconds.

90. Change 10 days to seconds.

91. Change 3,750,580 seconds to days. Round your answer to the nearest tenth of a day.

92. Change 4,493,469 seconds to days. Round your answer to the nearest tenth of a day.

---

93. Change 367 feet per second to miles per hour. Round your answer to the nearest mile per hour.

94. Change 354 feet per second to miles per hour. Round your answer to the nearest mile per hour.

95. Change 442 feet per second to miles per hour. Round your answer to the nearest mile per hour.

96. Change 388 feet per second to miles per hour. Round your answer to the nearest mile per hour.

97. Change 30 miles per hour to feet per second. Round your answer to the nearest foot per second.

98. Change 99 miles per hour to feet per second. Round your answer to the nearest foot per second.

99. Change 106 miles per hour to feet per second. Round your answer to the nearest foot per second.

100. Change 119 miles per hour to feet per second. Round your answer to the nearest foot per second.

---

101. **Strong man.** Famed strongman Joe Rollino, who was still bending quarters with his fingers at age 104, once lifted 3,200 pounds at Coney Island Amusement Park. How many tons did Joe lift that day? *Associated Press-Times-Standard 01/12/10 NYC amusement park strongman, 104, killed by van.*

102. **Earth day.** The amount of time it takes the Earth to rotate once around its axis is one day. How many seconds is that?

103. **Water break.** "The average age of Washington, DC's water pipes is 76 years, and they are not alone. Every two minutes, somewhere in the country, a pipe breaks." How many pipes break each year in the US? *New York Times 03/14/10 Saving U.S. water and sewer systems could be costly.*

Answers

1. 24 feet

3. 87 yards

5. 6.5 yards

7. 47 yards

9. 14,784 feet

11. 2.9 yards

13. 2.7 miles

15. 984 inches

17. 104 inches

19. 4.9 miles

21. 15 feet

23. 2.7 miles

25. 54 inches

27. 30 feet

29. 4 feet

31. 2.9 miles

33. 107,712 inches

35. 196,416 inches

37. 19,008 feet

39. 216 inches

41. 82 ounces

43. 76,800 ounces

45. $2\frac{1}{8}$ pounds

47. 4,400 pounds

49. $4\frac{3}{8}$ pounds

51. 4.8 tons

53. 40 ounces

55. 11,800 pounds

57. 80,000 ounces

59. 4.1 tons

61. 73 fluid ounces

63. 2 pints

65. 29.6 pints

67. 27 gallons

69. 34 pints

71. 14 gallons

73. 61.6 pints

75. 62 fluid ounces

77. 68,328 hours

79. 66,576 hours

81. 46.6 days

83. 4.3 years

85. 2.6 years

87. 8,294,400 seconds

89. 3,456,000 seconds

91. 43.4 days

93. 250 mi/hr

95. 301 mi/hr

97. 44 ft/s

99. 155 ft/s

101. 1.6 tons

103. 262, 800

Answers

Exercise
27 yards

Exercise
45 feet

Exercise
4 feet

Exercise
288 inches

Exercise
8,800 yards

Exercise
100 ounces

Exercise
128,000 ounces

Exercise
25.6 pints

Exercise
86,400 seconds

Exercise
88 feet per second

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2.11.1: Practice and Videos for Unit Conversion American System

Learning Objectives

- I can convert between different units of measurement, such as minutes to hours or gallons to cups.
- I can arrange measurements written in different unit measures from least to greatest and vice versa.

GeoGebra: Estimating Liquid Volumes

Complete the activity below to learn how to estimate volumes of liquids using comparison strategies. First, use the given measurement on the left to estimate the amount of water on the right.

Note: Divide the given amount of water by the number of ticks to find how much water each tick represents. Next, write your estimation in the empty box and check your answer. Review the solution and animation to check your understanding. Click the purple new problem button to try again with new numbers.

Videos on Unit Conversion

Watch the following videos to learn how to convert between units. All videos have appropriate captions in English; in some cases, there may also be appropriate captions in other languages.

YouTube Video

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

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View this YouTube Video online

Practice!

Check your understanding of converting units by answering these questions. Use the arrows at the bottom of the box to go to the next question.

Practice 2.11.1.1

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View this Interactive Content online

Extra Practice Resources

For more practice on unit conversion, visit the following websites.

1. IXL: Convert metric units involving decimals
2. IXL: Convert measurement units
3. IXL: Convert customary and metric units using proportions
4. PhET: Unit Rates

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2.12: Area and Volume of Geometric Figures and Objects

Learning Objectives

- know the meaning and notation for area
- know the area formulas for some common geometric figures
- be able to find the areas of some common geometric figures
- know the meaning and notation for volume
- know the volume formulas for some common geometric objects
- be able to find the volume of some common geometric objects

Quite often it is necessary to multiply one denominate number by another. To do so, we multiply the number parts together and the unit parts together. For example,

$$\begin{aligned} 8 \text{ in.} \cdot 8 \text{ in.} &= 8 \cdot 8 \cdot \text{in.} \cdot \text{in.} \\ &= 64 \text{ in.}^2 \end{aligned}$$

$$\begin{aligned} 4 \text{ mm} \cdot 4 \text{ mm} \cdot 4 \text{ mm} &= 4 \cdot 4 \cdot 4 \cdot \text{mm} \cdot \text{mm} \cdot \text{mm} \\ &= 64 \text{ mm}^3 \end{aligned}$$

Sometimes the product of units has a physical meaning. In this section, we will examine the meaning of the products $(\text{length unit})^2$ and $(\text{length unit})^3$

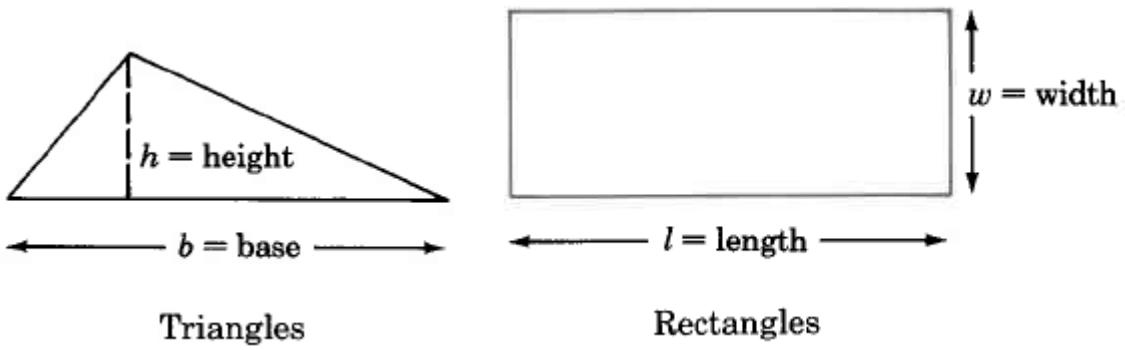
The Meaning and Notation for Area

The product $(\text{length unit}) \cdot (\text{length unit}) = (\text{length unit})^2$, or, square length unit (sq length unit), can be interpreted physically as the *area* of a surface.

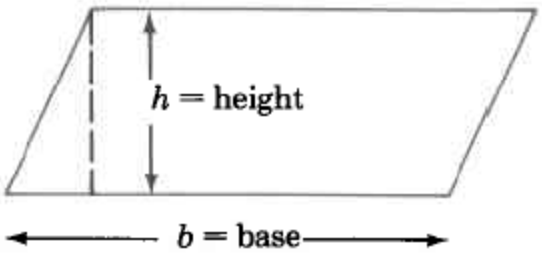
Area
The **area** of a surface is the amount of square length units contained in the surface.

For example, 3 sq in. means that 3 squares, 1 inch on each side, can be placed precisely on some surface. (The squares may have to be cut and rearranged so they match the shape of the surface.)

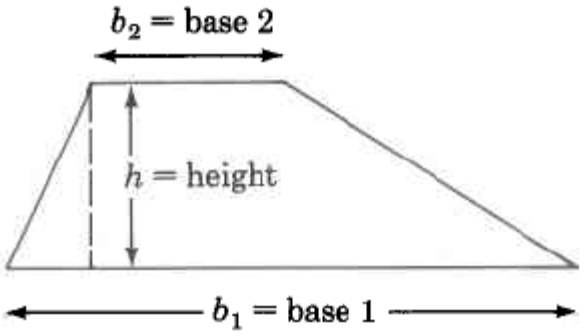
We will examine the area of the following geometric figures.



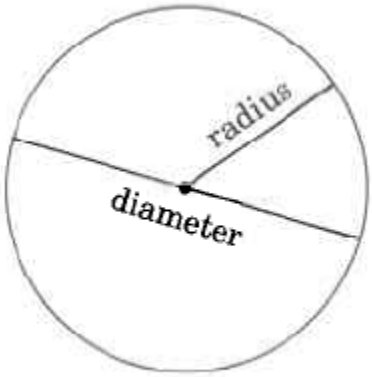
Triangles Rectangles



Parallelograms



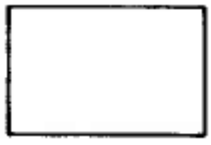
Trapezoids

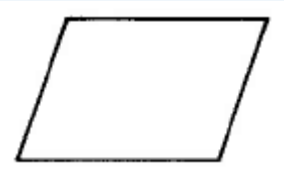
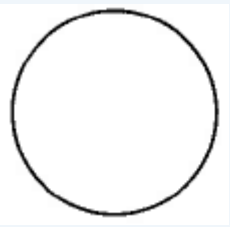


Circles

Area Formulas

We can determine the areas of these geometric figures using the following formulas.

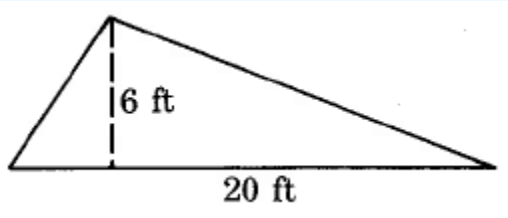
	Figure	Area Formula	Statement
	Triangle	$A_T = \frac{1}{2} \cdot b \cdot h$	Area of a triangle is one half the base times the height.
	Rectangle	$A_R = l \cdot w$	Area of a rectangle is the length times the width.

	Parallelogram	$A_P = b \cdot h$	Area of a parallelogram is base times the height.
	Trapezoid	$A_{Trap} = \frac{1}{2} \cdot (b_1 + b_2) \cdot h$	Area of a trapezoid is one half the sum of the two bases times the height.
	Circle	$A_c = \pi r^2$	Area of a circle is π times the square of the radius.

Finding Areas of Some Common Geometric Figures

Sample Set A

Find the area of the triangle.



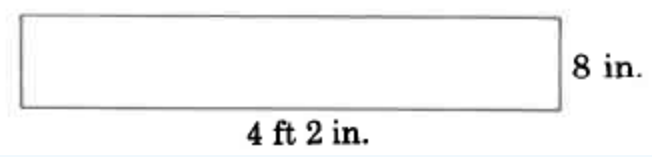
Solution

$$\begin{aligned} A_T &= \frac{1}{2} \cdot b \cdot h \\ &= \frac{1}{2} \cdot 20 \cdot 5 \text{ sq ft} \\ &= 10 \cdot 6 \text{ sq ft} \\ &= 60 \text{ sq ft} \\ &= 60 \text{ ft}^2 \end{aligned}$$

The area of this triangle is 60 sq ft, which is often written as 60 ft^2.

Sample Set A

Find the area of the rectangle.



Solution

Let's first convert 4 ft 2 in. to inches. Since we wish to convert to inches, we'll use the unit fraction $\frac{12 \text{ in.}}{1 \text{ ft}}$ since it has inches in the numerator. Then,

$$\begin{aligned} 4 \text{ ft} &= \frac{4 \text{ ft}}{1} \cdot \frac{12 \text{ in.}}{1 \text{ ft}} \\ &= \frac{4 \cancel{\text{ ft}}}{1} \cdot \frac{12 \text{ in.}}{1 \cancel{\text{ ft}}} \\ &= 48 \text{ in.} \end{aligned}$$

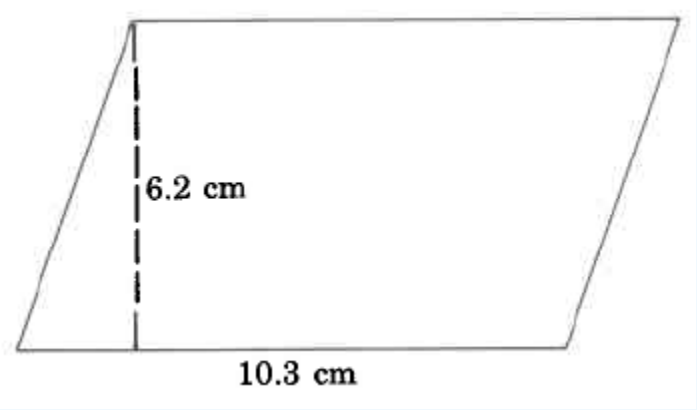
Thus, $4 \text{ ft } 2 \text{ in.} = 48 \text{ in.} + 2 \text{ in.} = 50 \text{ in.}$

$$\begin{aligned} A_R &= l \cdot w \\ &= 50 \text{ in.} \cdot 8 \text{ in.} \\ &= 400 \text{ sq in.} \end{aligned}$$

The area of this rectangle is 400 sq in.

Sample Set A

Find the area of the parallelogram.



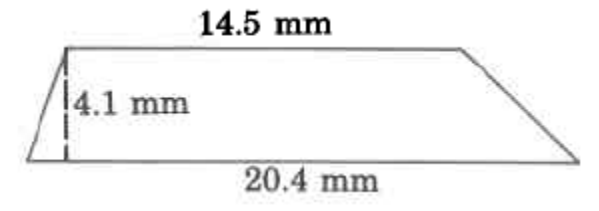
Solution

$$\begin{aligned} A_P &= b \cdot h \\ &= 10.3 \text{ cm} \cdot 6.2 \text{ cm} \\ &= 63.86 \text{ sq cm} \end{aligned}$$

The area of this parallelogram is 63.86 sq cm.

Sample Set A

Find the area of the trapezoid.



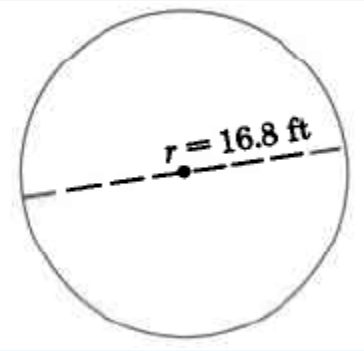
Solution

$$\begin{aligned} A_{Trap} &= \frac{1}{2} \cdot (b_1 + b_2) \cdot h \\ &= \frac{1}{2} \cdot (14.5 \text{ mm} + 20.4 \text{ mm}) \cdot (4.1 \text{ mm}) \\ &= \frac{1}{2} \cdot (34.9 \text{ mm}) \cdot (4.1 \text{ mm}) \\ &= \frac{1}{2} \cdot (143.09 \text{ sq mm}) \\ &= 71.545 \text{ sq mm} \end{aligned}$$

The area of this trapezoid is 71.545 sq mm.

Sample Set A

Find the approximate area of the circle.



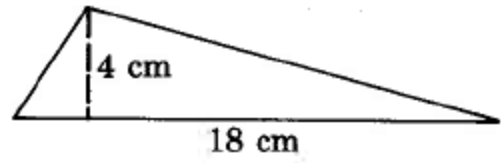
Solution

$$\begin{aligned} A_c &= \pi \cdot r^2 \\ &\approx (3.14) \cdot (16.8 \text{ ft})^2 \\ &\approx (3.14) \cdot (282.24 \text{ sq ft}) \\ &\approx 888.23 \text{ sq ft} \end{aligned}$$

The area of this circle is approximately 886.23 sq ft.

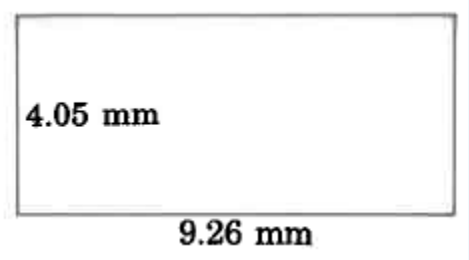
Practice Set A

Find the area of each of the following geometric figures.



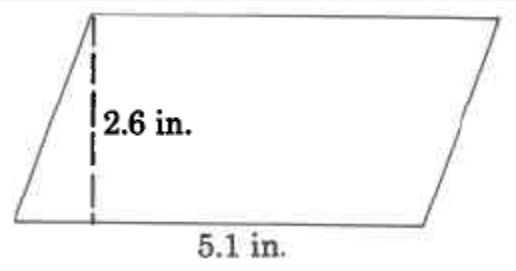
Answer available at the end of this section.

Practice Set A



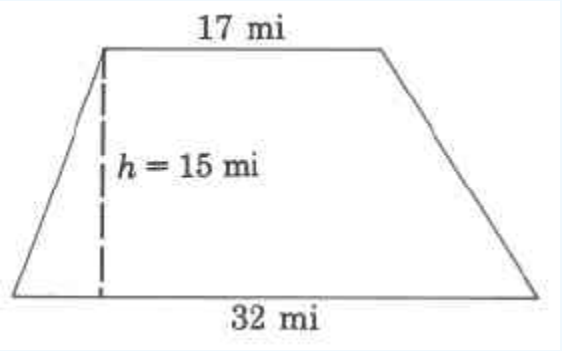
Answer available at the end of this section.

Practice Set A



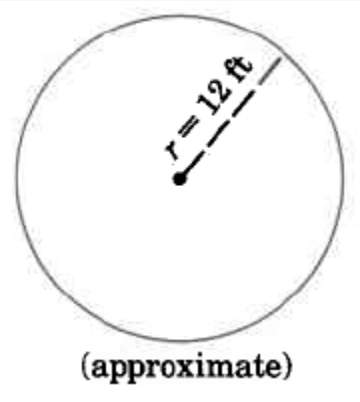
Answer available at the end of this section.

Practice Set A



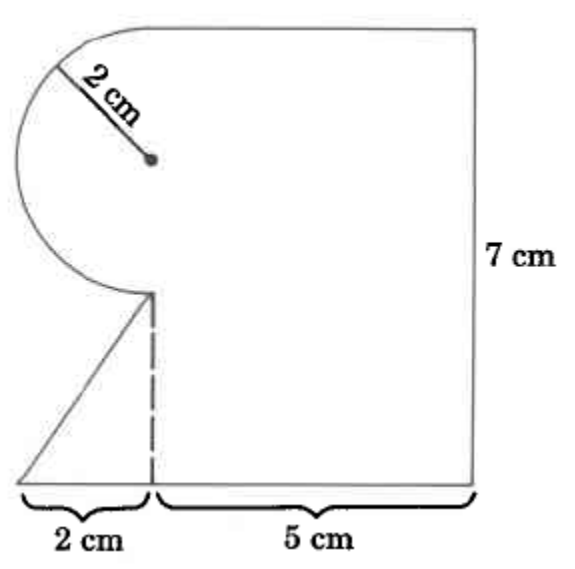
Answer available at the end of this section.

Practice Set A



Answer available at the end of this section.

Practice Set A



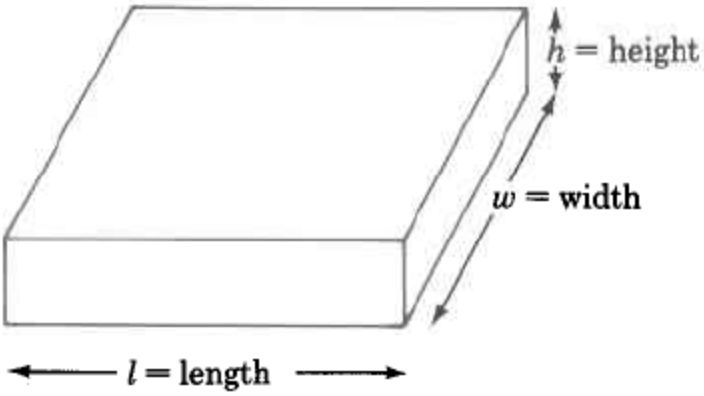
Answer available at the end of this section.

The Meaning and Notation for Volume

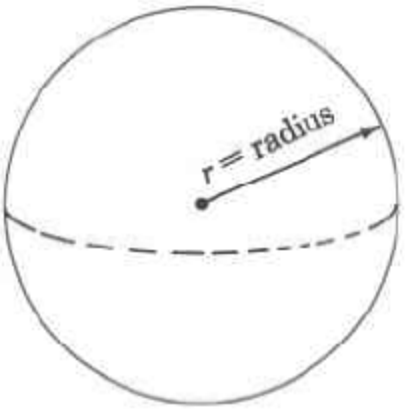
The product $(\text{length unit})(\text{length unit})(\text{length unit}) = (\text{length unit})^3$, or cubic length unit (cu length unit), can be interpreted physically as the *volume* of a three-dimensional object.

Volume
The **volume** of an object is the amount of cubic length units contained in the object.

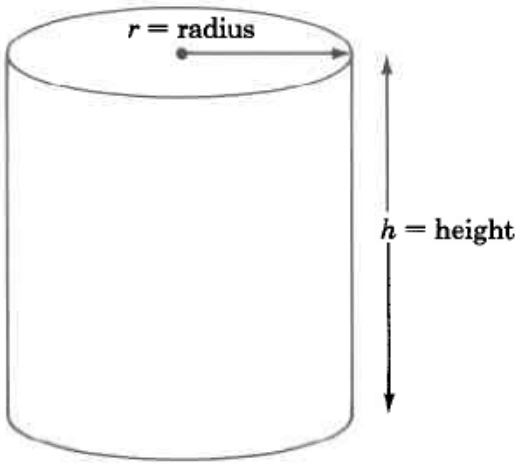
For example, 4 cu mm means that 4 cubes, 1 mm on each side, would precisely fill some three-dimensional object. (The cubes may have to be cut and rearranged so they match the shape of the object.)



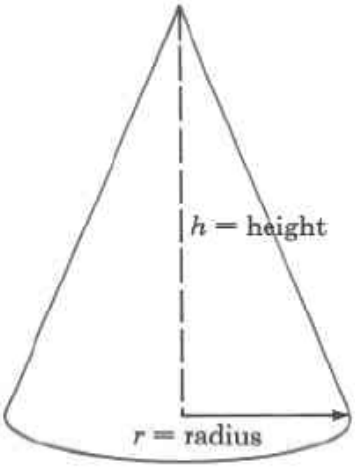
Rectangular solid



Sphere

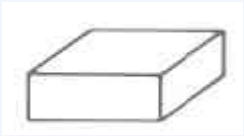
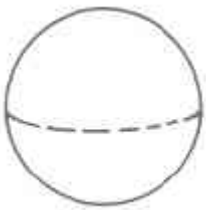


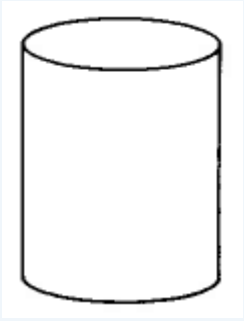
Cylinder



Cone

Volume Formulas

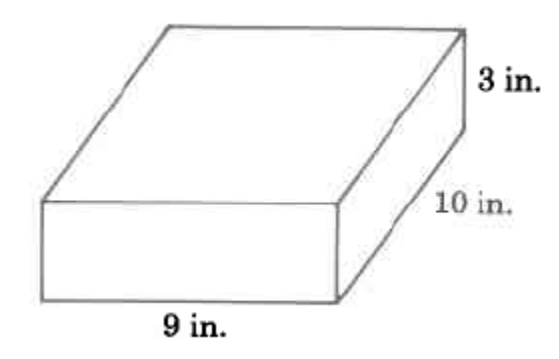
	Figure	Volume Formula	Statement
	Rectangular solid	$V_R = l \cdot w \cdot h$ $= (\text{area of base}) \cdot (\text{height})$	The volume of a rectangular solid is the length times the width times the height.
	Sphere	$V_s = \frac{4}{3} \cdot \pi \cdot r^3$	The volume of a sphere is $\frac{4}{3}$ times π times the cube of the radius.

	Cylinder	$V_{Cyl} = \pi \cdot r^2 \cdot h$ $= (\text{area of base}) \cdot (\text{height})$	The volume of a cylinder is π times the square of the radius times the height.
	Cone	$V_c = \frac{1}{3} \cdot \pi \cdot r^2 \cdot h$ $= (\text{area of base}) \cdot (\text{height})$	The volume of a cone is $\frac{1}{3}$ times π times the square of the radius times the height.

Finding Volumes of Some Common Geometric Objects

Sample Set B

Find the volume of the rectangular solid.



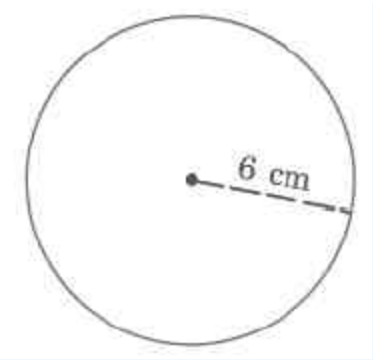
Solution

$$\begin{aligned} V_R &= l \cdot w \cdot h \\ &= 9 \text{ in.} \cdot 10 \text{ in.} \cdot 3 \text{ in.} \\ &= 270 \text{ cu in.} \\ &= 270 \text{ in.}^3 \end{aligned}$$

The volume of this rectangular solid is 270 cu in.

Sample Set B

Find the approximate volume of the sphere.



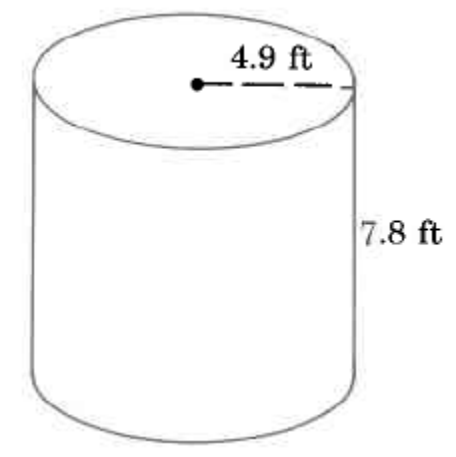
Solution

$$\begin{aligned} V_S &= \frac{4}{3} \cdot \pi \cdot r^3 \\ &\approx (\frac{4}{3}) \cdot (3.14) \cdot (6 \text{ cm})^3 \\ &\approx (\frac{4}{3}) \cdot (3.14) \cdot (216 \text{ cu cm}) \\ &\approx 904.32 \text{ cu cm} \end{aligned}$$

The approximate volume of this sphere is 904.32 cu cm, which is often written as 904.32 cm^3.

Sample Set B

Find the approximate volume of the cylinder.



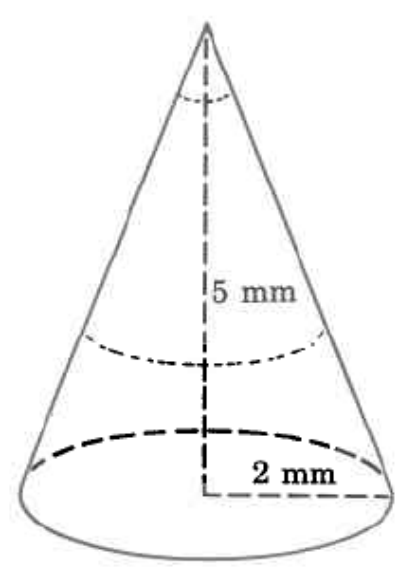
Solution

$$\begin{aligned} V_{Cyl} &= \pi \cdot r^2 \cdot h \\ &\approx (3.14) \cdot (4.9 \text{ ft})^2 \cdot (7.8 \text{ ft}) \\ &\approx (3.14) \cdot (24.01 \text{ sq ft}) \cdot (7.8 \text{ ft}) \\ &\approx (3.14) \cdot (187.278 \text{ cu ft}) \\ &\approx 588.05292 \text{ cu ft} \end{aligned}$$

The volume of this cylinder is approximately 588.05292 cu ft. The volume is approximate because we approximated π with 3.14.

Sample Set B

Find the approximate volume of the cone. Round to two decimal places.



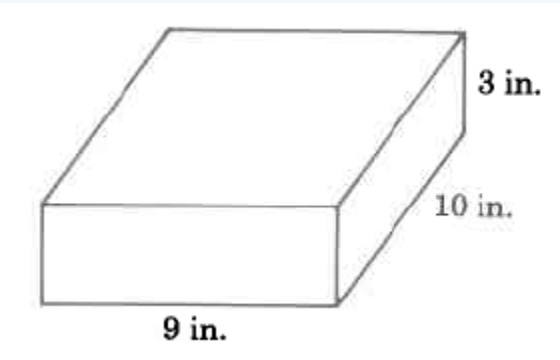
Solution

$$\begin{aligned} V_c &= \frac{1}{3} \cdot \pi \cdot r^2 \cdot h \\ &\approx (\frac{1}{3}) \cdot (3.14) \cdot (2 \text{ mm})^2 \cdot (5 \text{ mm}) \\ &\approx (\frac{1}{3}) \cdot (3.14) \cdot (4 \text{ sq mm}) \cdot (5 \text{ mm}) \\ &\approx (\frac{1}{3}) \cdot (3.14) \cdot (20 \text{ cu mm}) \\ &\approx 20.9\overline{3} \text{ cu mm} \\ &\approx 20.93 \text{ cu mm} \end{aligned}$$

The volume of this cone is approximately 20.93 cu mm. The volume is approximate because we approximated π with 3.14.

Practice Set B

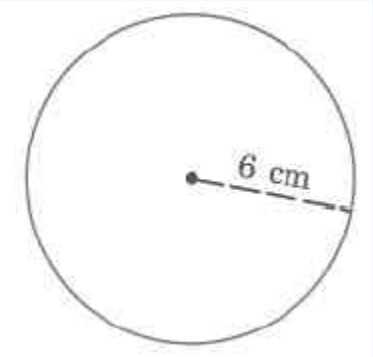
Find the volume of each geometric object. If π is required, approximate it with 3.14 and find the approximate volume.



Answer available at the end of this section.

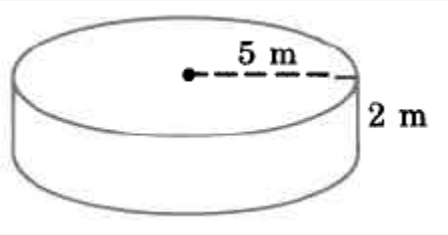
Practice Set B

Sphere



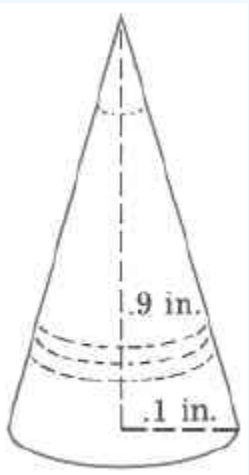
Answer available at the end of this section.

Practice Set B



Answer available at the end of this section.

Practice Set B



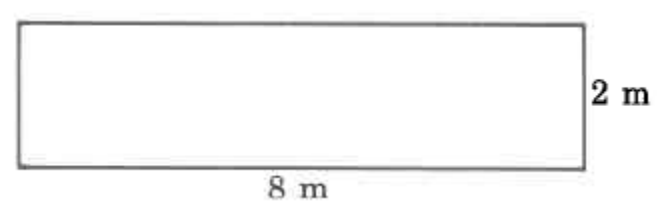
Answer available at the end of this section.

Exercises

Find each indicated measurement.

Exercise 2.12.1

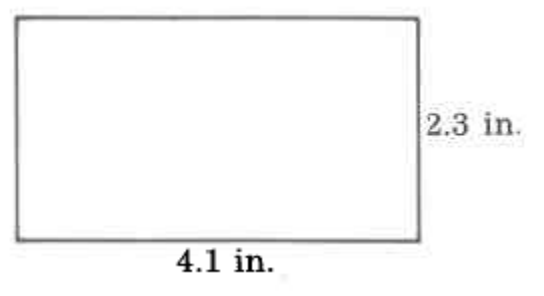
Area



Answer available at the end of this section.

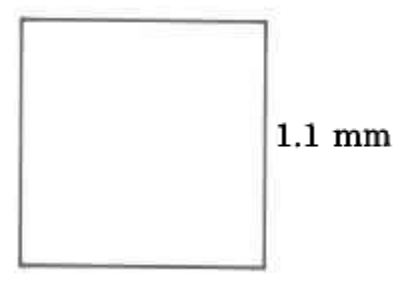
Exercise 2.12.2

Area



Exercise 2.12.3

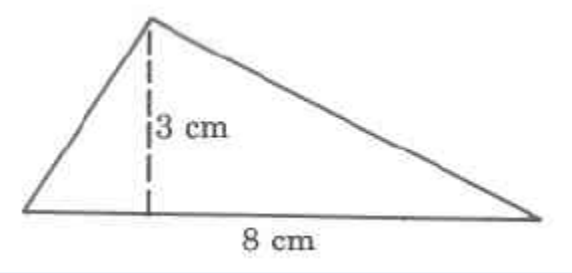
Area



Answer available at the end of this section.

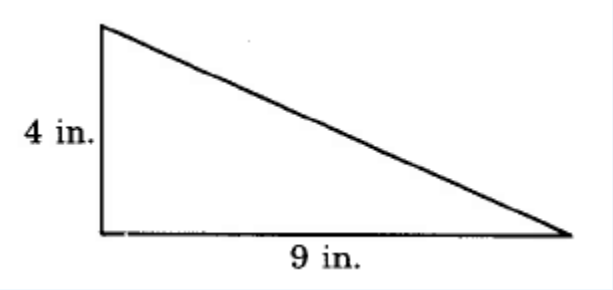
Exercise 2.12.4

Area



Exercise 2.12.5

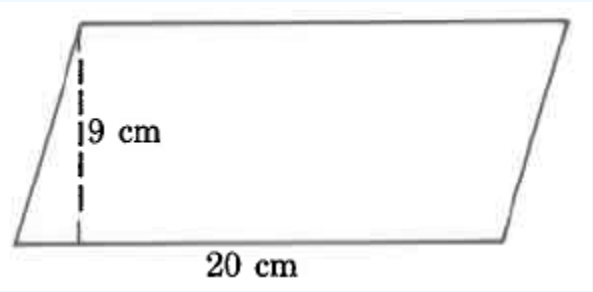
Area



Answer available at the end of this section.

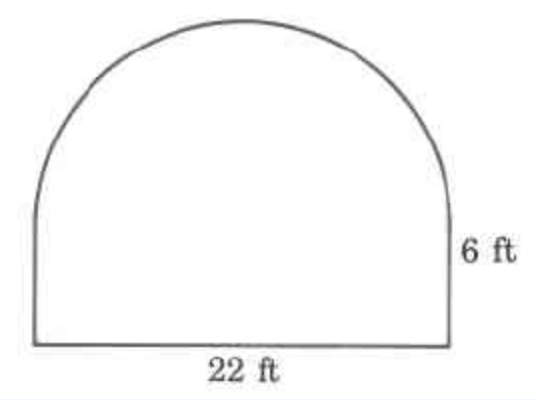
Exercise 2.12.6

Area



Exercise 2.12.7

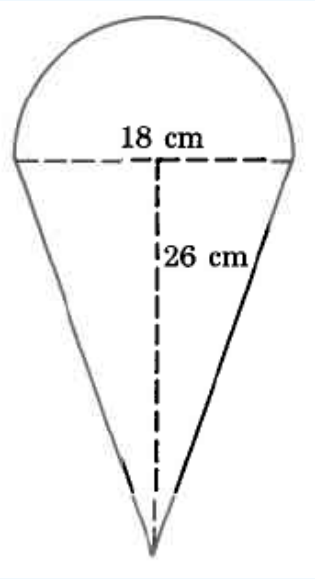
Exact area



Answer available at the end of this section.

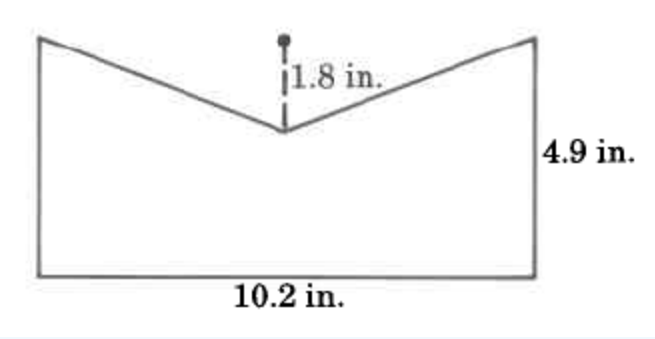
Exercise 2.12.8

Approximate area



Exercise 2.12.9

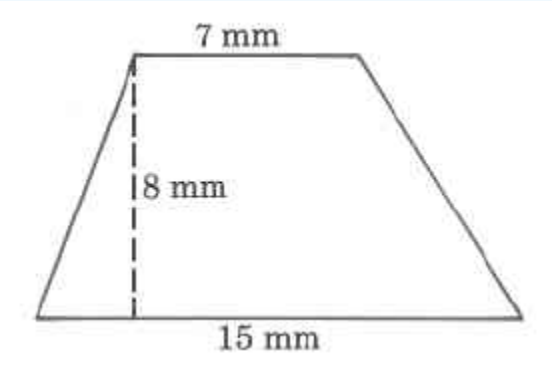
Area



Answer available at the end of this section.

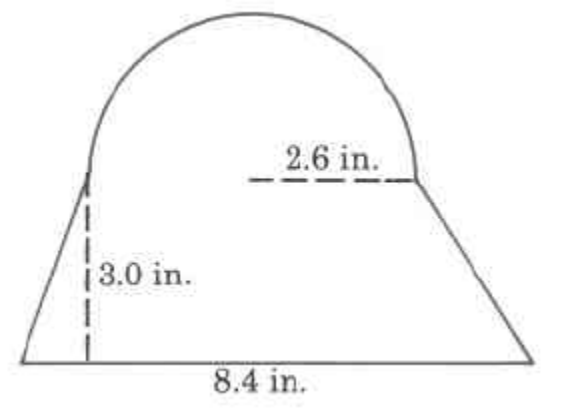
Exercise 2.12.10

Area



Exercise 2.12.11

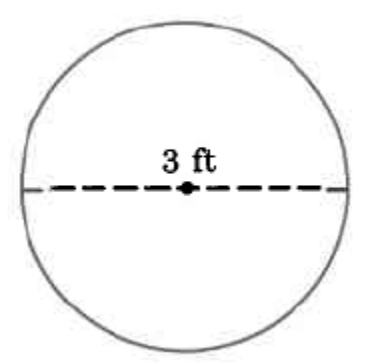
Approximate area



Answer available at the end of this section.

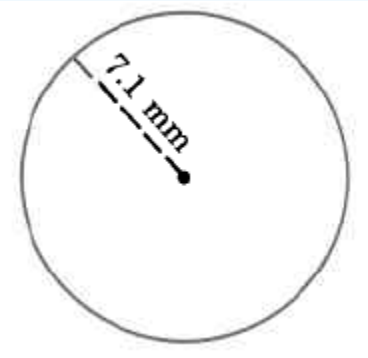
Exercise 2.12.12

Exact area



Exercise 2.12.13

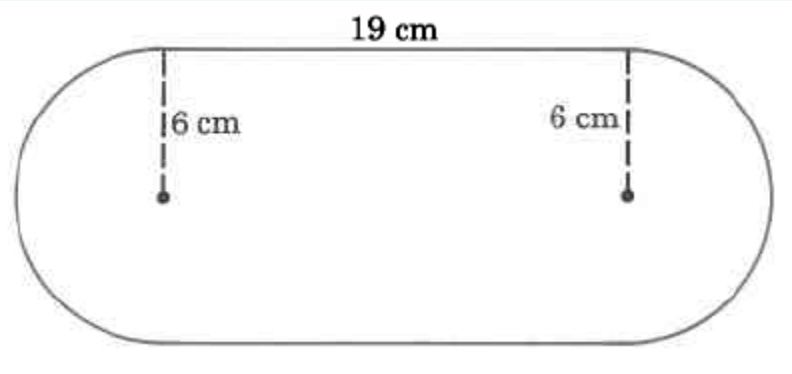
Approximate area



Answer available at the end of this section.

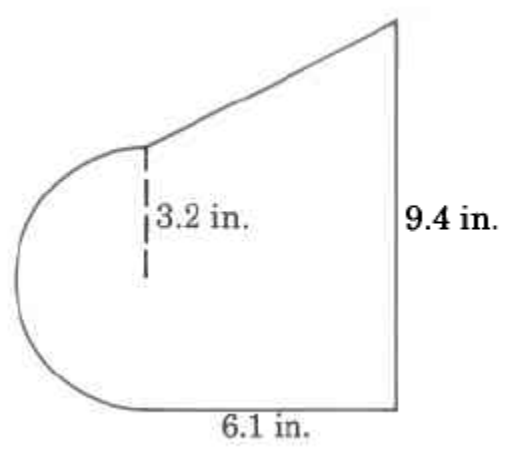
Exercise 2.12.14

Exact area



Exercise 2.12.15

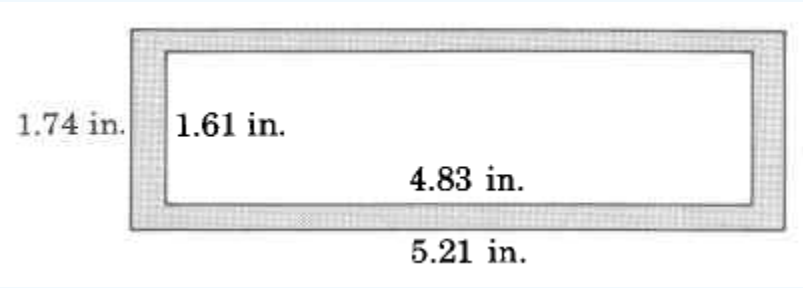
Approximate area



Answer available at the end of this section.

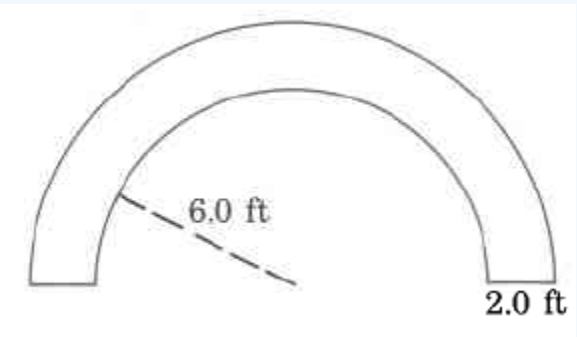
Exercise 2.12.16

Area



Exercise 2.12.17

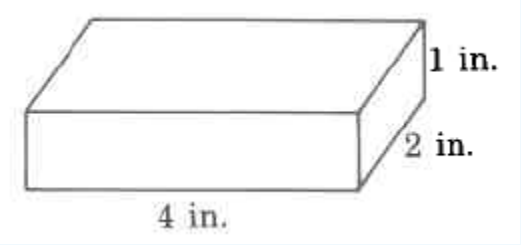
Approximate area



Answer available at the end of this section.

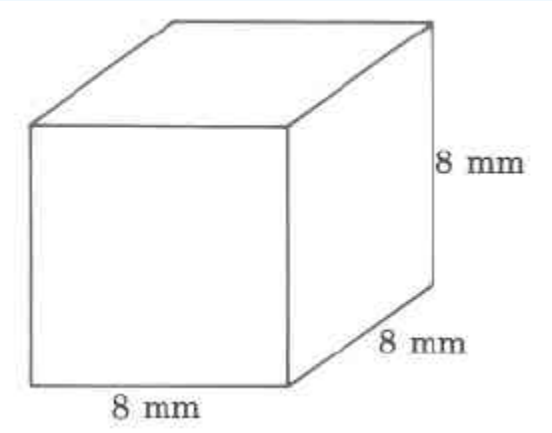
Exercise 2.12.18

Volume



Exercise 2.12.19

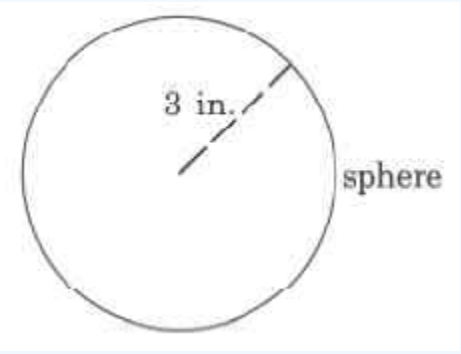
Volume



Answer available at the end of this section.

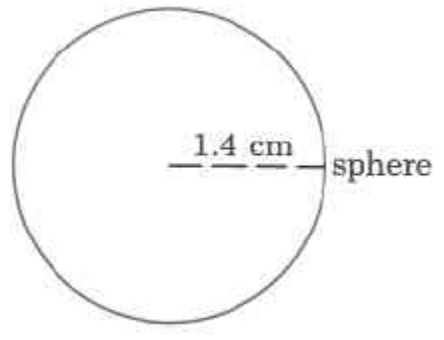
Exercise 2.12.20

Exact volume



Exercise 2.12.21

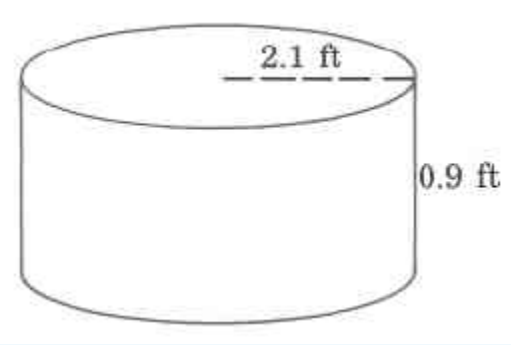
Approximate volume



Answer available at the end of this section.

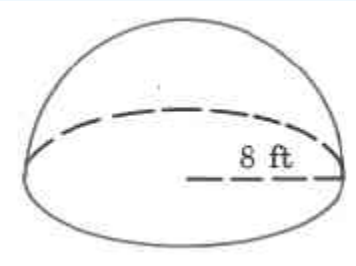
Exercise 2.12.22

Approximate volume



Exercise 2.12.23

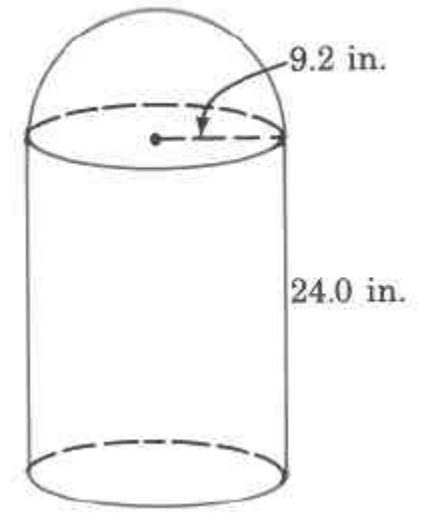
Exact volume



Answer available at the end of this section.

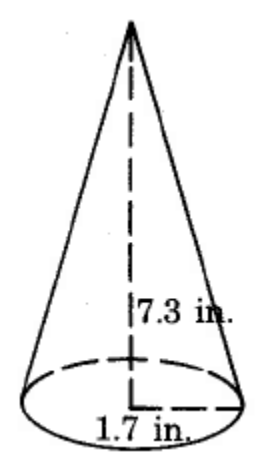
Exercise 2.12.24

Approximate volume



Exercise 2.12.25

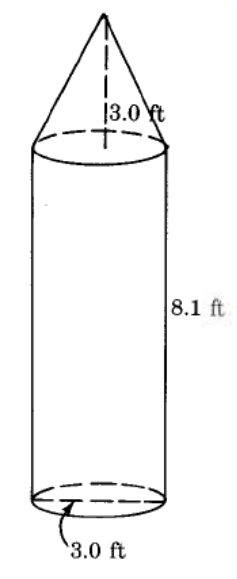
Approximate volume



Answer available at the end of this section.

Exercise 2.12.26

Approximate volume



Exercises for Review

Exercise 2.12.27

In the number 23,426, how many hundreds are there?

Answer available at the end of this section.

Exercise 2.12.28

List all the factors of 32.

Exercise 2.12.29

Find the value of $4\frac{3}{4} - 3\frac{5}{6} + 1\frac{2}{3}$.

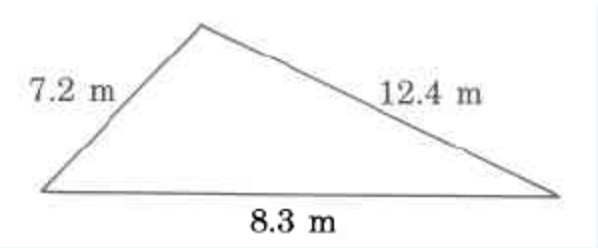
Answer available at the end of this section.

Exercise 2.12.30

Find the value of $\dfrac{5+\frac{1}{3}}{2+\frac{2}{15}}$.

Exercise 2.12.31

Find the perimeter.



Answer available at the end of this section.

Answers

Practice Set A
36 sq cm

Practice Set A
37.503 sq mm

Practice Set A
13.26 sq in.

Practice Set A
367.5 sq mi

Practice Set A
452.16 sq ft

Practice Set A
44.28 sq cm

Practice Set B
21 cu in.

Practice Set B
904.32 cu ft

Practice Set B
157 cu m

Practice Set B
0.00942 cu in.

Exercise 2.12.1
16 sq m

Exercise 2.12.3
1.21 sq mm

Exercise 2.12.5
18 sq in.

Exercise 2.12.7
$(60.5\pi + 132)$ sq ft

Exercise 2.12.9
40.8 sq in.

Exercise 2.12.11
31.0132 sq in.

Exercise 2.12.13
158.2874 sq mm

Exercise 2.12.15
64.2668 sq in.

Exercise 2.12.17
43.96 sq ft

Exercise 2.12.19
512 cu cm

Exercise 2.12.21
11.49 cu cm

Exercise 2.12.23

$\frac{1024}{3}\pi$ cu ft

Exercise 2.12.25

22.08 cu in.

Exercise 2.12.27

4

Exercise 2.12.29

$\frac{31}{12} = 2\frac{7}{12} = 2.58$

Exercise 2.12.31

27.9m

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2.12.1: Practice and Videos for Introduction to Volume

Learning Objectives

- I can calculate volume using visual techniques and formulas.
- I can understand the concept of volume with 3 dimensions.

GeoGebra: Calculating Volume with Invisible Cubes

Complete the activity below to investigate the volume of prisms with invisible cubic units. Develop patterns for finding volume without counting each cubic unit. When you have an answer, type it in the empty box and check it. Keep trying until you get the answer correct. Click "new prism" to calculate the volume of a new figure.

GeoGebra: Composing Prisms by Stacking Cubes

In this additional GeoGebra activity, explore how the volume of a 3-dimensional prism is composed layer by layer. Use the purple slider to reveal how the prism is composed. To try on another prism, click "new prism."

Videos on Volume

Watch the following videos to learn about volume. All videos have appropriate captions in English; in some cases, there may also be appropriate captions in other languages.

YouTube Video

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View this YouTube Video online

Practice!

Check your understanding of volume by answering these questions. Use the arrows at the bottom of the box to go to the next question.

Practice 2.12.1.1

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Extra Practice Resources

1. IXL: Volume of rectangular prisms made of unit cubes
2. IXL: Volume of rectangular prisms made of unit cubes: expressions
3. IXL: Volume of rectangular prisms made of unit cubes: word problems

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2.13: Order of Operations

Learning Objectives

- Use the order of operations to simplify expressions.
- Simplify expressions containing absolute values.

Introduction

People need a common set of rules for performing basic calculations. What does $3+5\cdot 2$ equal? Is it 16 or 13? Your answer depends on how you understand the **order of operations** — a set of rules that tell you the order in which addition, subtraction, multiplication, and division are performed in any calculation.

Mathematicians have developed a standard order of operations that tells you which calculations to make first in an expression with more than one operation. Without a standard procedure for making calculations, two people could get two different answers to the same problem.

The Four Basic Operations

The building blocks of the order of operations are the **arithmetic operations**: addition, subtraction, multiplication, and division. The order of operations states:

- multiply or divide first, going from left to right
- then add or subtract in order from left to right

What is the correct answer for the expression $3+5\cdot 2$? Use the order of operations listed above.

Multiply first. $3+5\cdot 2=3+10$

Then add. $3+10=13$

This order of operations is true for all real numbers.

Example

Simplify $7-5+3\cdot 8$**.**

Solution

$7-5+3\cdot 8$	According to the order of operations, multiplication comes before addition and subtraction. Multiply $3\cdot 8$.
$7-5+24$	Now, add and subtract from left to right. $7-5$ comes first.
$2+24=26$	Finally, add $2+24$.

$7-5+3\cdot 8=26$

When you are applying the order of operations to expressions that contain fractions, decimals, and negative numbers, you will need to recall how to do these computations as well.

Example

Simplify: $3 \cdot \frac{1}{3} - 8 \div \frac{1}{4}$

Solution

$3 \cdot \frac{1}{3} - 8 \div \frac{1}{4}$	According to the order of operations, multiplication comes before addition and subtraction. Multiply $3 \cdot \frac{1}{3}$ first.
$1 - 8 \div \frac{1}{4}$	Now, divide $8 \div \frac{1}{4}$.
$8 \div \frac{1}{4} = \frac{8}{1} \cdot \frac{4}{1} = 32$	
$1 - 32 = -31$	Subtract.

$3 \cdot \frac{1}{3} - 8 \div \frac{1}{4} = -31$

Exponents

When you are evaluating expressions, you will sometimes see exponents used to represent repeated multiplication. Recall that an expression such as 7^2 is **exponential notation** for $7 \cdot 7$. (Exponential notation has two parts: the **base** and the **exponent** or the **power**. In 7^2, 7 is the base and 2 is the exponent; the exponent determines how many times the base is multiplied by itself.)

Exponents are a way to represent repeated multiplication; the order of operations places it *before* any other multiplication, division, subtraction, and addition is performed.

Example

Simplify: $3^2 \cdot 2^3$

Solution

$3^2 \cdot 2^3$	This problem has exponents and multiplication in it. According to the order of operations, simplifying 3^2 and 2^3 comes before multiplication.
$9 \cdot 2^3$	3^2 is $3 \cdot 3$, which equals 9.
$9 \cdot 8$	2^3 is $2 \cdot 2 \cdot 2$, which equals 8.
$9 \cdot 8 = 72$	Multiply.

$3^2 \cdot 2^3 = 72$

Example

Simplify: $\left(\frac{1}{2}\right)^2 + \left(\frac{1}{4}\right)^3 \cdot 32$

Solution

$\left(\frac{1}{2}\right)^2 + \left(\frac{1}{4}\right)^3 \cdot 32$	This problem has exponents, multiplication, and addition in it. According to the order of operations, simplify the terms with the exponents first, then multiply, then add.

$\frac{1}{4}+\left(\frac{1}{4}\right)^3 \cdot 32$	Evaluate: $\left(\frac{1}{2}\right)^2=\frac{1}{2} \cdot \frac{1}{2}=\frac{1}{4}$
$\frac{1}{4}+\frac{1}{64} \cdot 32$	Evaluate: $\left(\frac{1}{4}\right)^3=\frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4}=\frac{1}{64}$
$\frac{1}{4}+\frac{32}{64}$	Multiply.
$\frac{1}{4}+\frac{1}{2}=\frac{3}{4}$	Simplify. $\frac{32 \div 32}{64 \div 32}=\frac{1}{2}$, so you can add $\frac{1}{4}+\frac{1}{2}$.

$\left(\frac{1}{2}\right)^2+\left(\frac{1}{4}\right)^3 \cdot 32=\frac{3}{4}$

Exercise

Simplify: $100-5^2 \cdot 4$

A. -300
B. 0
C. 100
D. 300

Answer available at the end of this section.

Grouping Symbols

The final piece that you need to consider in the order of operations is **grouping symbols**. These include parentheses ( ), brackets [ ], braces { }, and even fraction bars. These symbols are often used to help organize mathematical expressions (you will see them a lot in algebra).

Grouping symbols are used to clarify which operations to do first, especially if a specific order is desired. If there is an expression to be simplified within the grouping symbols, follow the order of operations.

The Order of Operations

- Perform all operations within grouping symbols first. Grouping symbols include parentheses ( ), brackets [ ], braces { }, and fraction bars.
- Evaluate exponents or square roots.
- Multiply or divide, from left to right.
- Add or subtract, from left to right.

When there are grouping symbols within grouping symbols, calculate from the inside to the outside. That is, begin simplifying within the innermost grouping symbols first.

Remember that parentheses can also be used to show multiplication. In the example that follows, both uses of parentheses—as a way to represent a group, as well as a way to express multiplication—are shown.

Example

Simplify: $(3+4)^2+(8)(4)$

Solution

$(3+4)^2$ $+(8)(4)$	This problem has parentheses, exponents, multiplication, and addition in it. The first set of parentheses is a grouping symbol. The second set indicates multiplication.
$(3+4)^2$ $+(8)(4)$	Grouping symbols are handled first. Add numbers in parentheses.
7^2 $+(8)(4)$	Simplify 7^2.
49 $+(8)(4)$	Perform multiplication.
$49+32$ $=81$	Perform addition.

$(3+4)^2+(8)(4)=81$

Example

Simplify: $(1.5+3.5)-2(0.5\cdot 6)^2$

Solution

$(1.5+3.5)-2(0.5\cdot 6)^2$	This problem has parentheses, exponents, multiplication, subtraction, and addition in it.
$5-2(0.5\cdot 6)^2$	Grouping symbols are handled first. Add numbers in the first set of parentheses. Multiply numbers in the second set of parentheses.
$5-2(3)^2$	Evaluate exponents.
$5-2\cdot 9$	Multiply.
$5-18=-13$	Subtract.

$(1.5+3.5)-2(0.5\cdot 6)^2=-13$

Example

Simplify: $\frac{5-[3+(2\cdot(-6))]}{3^2+2}$

Solution

$\frac{5-[3+(2\cdot(-6))]}{3^2+2}$	This problem has brackets, parentheses, fractions, exponents, multiplication, subtraction, and addition in it. Grouping symbols are handled first. The parentheses around the -6 aren't a grouping symbol, they are simply making it clear that the negative sign belongs to the 6. Start with the innermost set of parentheses that are a grouping symbol, here it is in the numerator of the fraction, ($2\cdot -6$), and begin working out. (The fraction line acts as a type of grouping symbol, too; you simplify the numerator and denominator independently, and then divide the numerator by the denominator at the end.)
$\frac{5-[3+(-12)]}{3^2+2}$	Add the values in the brackets.
$\frac{5-[-9]}{3^2+2}$	Subtract $5-[-9]=5+9=14$.

$\frac{14}{3^2+2}$	The top of the fraction is all set, but the bottom (denominator) has remained untouched. Apply the order of operations to that as well. Begin by evaluating $3^2 = 9$.
$\frac{14}{9+2}$ $\frac{14}{11}$	Now add. $9 + 2 = 11$.

$\frac{5-[3+(2\cdot(-6))]}{3^2+2} = \frac{14}{11}$

Exercise

Simplify $\left[\frac{3^3+3}{(-2)(-3)}\right]^2 + 1$.

A. 25
B. 26
C. 151
D. $\frac{11}{6}$

Answer available at the end of this section.

Remembering the Order of Operations

The Order of Operations

- Perform all operations within grouping symbols first. Grouping symbols include parentheses ( ), brackets [ ], braces { }, and fraction bars.
- Evaluate exponents or square roots.
- Multiply or divide, from left to right.
- Add or subtract, from left to right.

The order of operations is important to know, but it is sometimes hard to remember. Some people use a saying to help them remember the order of operations. This saying is “Please Excuse My Dear Aunt Sally,” or PEMDAS for short. The first letter of each word begins with the same letter of an arithmetic operation.

The P in Please stands for Parentheses (and other grouping symbols).

The E in Excuse stands for Exponents.

The M and D in My Dear stand for Multiplication and Division (from left to right).

The A and S in Aunt Sally stand for Addition and Subtraction (from left to right).

Note: Even though multiplication comes before division in the saying, division could be performed first. Whether multiplication or division is performed first is determined by which comes first when reading from left to right. The same is true of addition and subtraction. Don’t let the saying confuse you about this!

Absolute Value Expressions

Absolute value expressions are one final method of grouping that you may see. Recall that the absolute value of a quantity is always positive or 0.

When you see an absolute value expression included within a larger expression, follow the regular order of operations and evaluate the expression within the absolute value sign. Then take the absolute value of that expression. The example below shows how this is done.

Example

Simplify: $\frac{3+|2-6|}{2|3 \cdot 1.5|-(-3)}$

Solution

$\frac{3+\|2-6\|}{2\|3 \cdot 1.5\|-(-3)}$	This problem has absolute values, decimals, multiplication, subtraction, and addition in it. Grouping symbols, including absolute value, are handled first. Simplify the numerator, then the denominator. Evaluate $\|2-6\|$.
$\frac{3+\|-4\|}{2\|3 \cdot 1.5\|-(-3)}$	Take the absolute value of $\|-4\|$.
$\frac{3+4}{2\|3 \cdot 1.5\|-(-3)}$	Add the numbers in the numerator.
$\frac{7}{2\|3 \cdot 1.5\|-(-3)}$	Now that the numerator is simplified, turn to the denominator. Evaluate the absolute value expression first.
$\frac{7}{2\|4.5\|-(-3)}$ $\frac{7}{2 \cdot 4.5-(-3)}$	The expression " $2\|4.5\|$" reads "2 times the absolute value of 4.5." Multiply 2 times 4.5.
$\frac{7}{9-(-3)}$ $\frac{7}{12}$	Subtract.

$\frac{3+|2-6|}{2|3 \cdot 1.5|-(-3)} = \frac{7}{12}$

Exercise

Simplify: $(5|3-4|)^3$.

A. -125
B. 1331
C. -49
D. 125

Answer available at the end of this section.

Summary

The order of operations gives us a standard, consistent method to use when simplifying strings of numbers and algebraic expressions. Without the order of operations, different people could come up with different answers to the same computation problem. Some people remember the order of operations by using the phrase “Please Excuse My Dear Aunt Sally” or, more simply, PEMDAS.

Answers

Exercise

A. Incorrect. You may have found $4 \cdot 5 = 20$, squared 20, and then subtracted 400 from 100. The order of operations states that you should simplify the term with the exponent first, then multiply, then subtract. $5^2 = 25$, and $25 \cdot 4 = 100$, and $100 - 100 = 0$. The correct answer is 0.
B. Correct. To simplify this expression, simplify the term with the exponent first, then multiply, then subtract. $5^2 = 25$, and $25 \cdot 4 = 100$, and $100 - 100 = 0$.
C. Incorrect. The order of operations states that you should simplify the term with the exponent first, then multiply, then subtract. $5^2 = 25$, and $25 \cdot 4 = 100$, and $100 - 100 = 0$. The correct answer is 0.
D. Incorrect. You may have found that $5^2 = 25$, subtracted that from 100, and multiplied by 4. The order of operations states that you should simplify the term with the exponent first, then multiply, then subtract. $5^2 = 25$, and $25 \cdot 4 = 100$, and $100 - 100 = 0$. The correct answer is 0.

Exercise

A. Incorrect. You may have forgotten to add the 1! Simplify the expressions within the grouping symbols first (5), then square that expression (25), then add 1. The correct answer is 26.
B. Correct. The entire quantity within the brackets is $5 \cdot 5^2$ is 25, and $25 + 1 = 26$.
C. Incorrect. You may have squared the numerator of the fraction without simplifying the entire fraction first! Simplify the entire expression within the grouping symbols first (5), then square that expression (25), then add 1. The correct answer is 26.
D. Incorrect. You may have squared the denominator of the fraction without simplifying the entire fraction first! Simplify the entire expression within the grouping symbols first (5), then square that expression (25), then add 1. The correct answer is 26.

Exercise

A. Incorrect. You may have forgotten to take the absolute value of the expression $|3 - 4|$. Remember that the absolute value will be either 0 or a positive number. $|3 - 4| = |-1| = 1$ and 5 times 1 is 5. The correct answer is 125.
B. Incorrect. You may have ignored the absolute value expression and found 11^3. Evaluate $|3 - 4|$, multiply that by 5, and then cube that number: $|3 - 4| = |-1| = 1$ and 5 times 1 is 5; $5^3 = 125$. The correct answer is 125.
C. Incorrect. You may have subtracted 4^3 from $5(3)$. Evaluate $|3 - 4|$, multiply that by 5, and then cube that number: $|3 - 4| = |-1| = 1$ and 5 times 1 is 5; $5^3 = 125$ The correct answer is 125.
D. Correct. $|3 - 4| = |-1| = 1$ and 5 times 1 is 5. 5 cubed is 125.

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2.13.1: Practice and Videos for Order of Operations

Learning Objectives

- I can apply the order of operations when evaluating expressions.
- I can spot errors in the evaluation of expressions involving the order of operations.

GeoGebra: Correcting Errors in an Order of Operations

Complete the activity below. Find the error made in applying the order of operations and correct it to ensure the expression is calculated accurately. First, identify the error, then use the purple slider to highlight where the error was made. Next, use the keypad to correct the error by typing in the correct next step. Check your answer and review the explanation. Click "new question" to evaluate another error problem.

Videos on Order of Operations

Watch the following videos to learn more about the order of operations. All videos have appropriate captions in English; in some cases, there may also be appropriate captions in other languages.

YouTube Video

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YouTube Video

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Practice!

Check your understanding of order of operations by answering these questions. Use the arrows at the bottom of the box to go to the next question.

Practice 2.13.1.1

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Extra Practice Resources

For more practice on order of operations, visit the following websites.

1. IXL: Evaluate numerical expressions involving integers

2. IXL: Identify mistakes involving the order of operations

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2.14: Cartesian Plane

Be Prepared

We will rely heavily on these skills throughout this section.

- Simplify: $-3+7$
- Identify the opposite of -5
- Solve: $x-4=9$
- Evaluate $2x+1$ when $x=-2$

Learning Objectives

Motivating Problem

You're using a map app to meet a friend. They tell you they're 3 blocks east and 2 blocks north of the coffee shop. If the shop is at $0,0$, how can you describe exactly where they are—and in what quadrant of the coordinate plane their location falls?

Fun Fact

The Cartesian Plane is named after **René Descartes**, a French philosopher and mathematician. Legend says he developed the idea while watching a fly crawl across his ceiling and wondering how to describe its location with numbers!

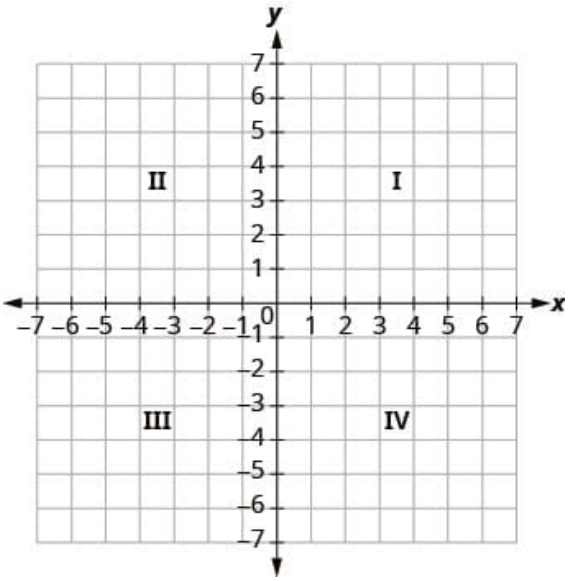
The Goal

In this section, we will learn how to locate and label points on a coordinate plane using ordered pairs. We'll also practice identifying quadrants, understanding axes, and interpreting visual information, which is foundational for graphing equations.

Plot Points on a Rectangular Coordinate System

Just like maps use a grid system to identify locations, a grid system is used in algebra to show a relationship between two variables in a **rectangular coordinate system**. The rectangular coordinate system is also called the *xy*-plane, the coordinate plane, or the Cartesian Plane.

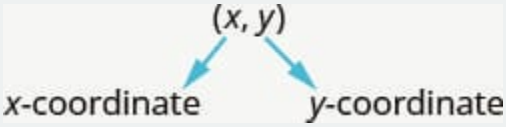
The horizontal number line is called the *x-axis,* and the vertical number line is called the *y-axis*. Together, the *x*-axis and the *y*-axis form the rectangular coordinate system. These axes divide a plane into four regions, called **quadrants**. The quadrants are identified by Roman numerals, beginning on the upper right and proceeding counterclockwise.



In the **rectangular coordinate system**, every point is represented by an *ordered pair*. The first number in the ordered pair is the ***x*-coordinate** of the point, and the second number is the ***y*-coordinate** of the point.

ORDERED PAIR

An **ordered pair**, (x, y), gives the coordinates of a point in a rectangular coordinate system.



The first number is the *x*-coordinate.

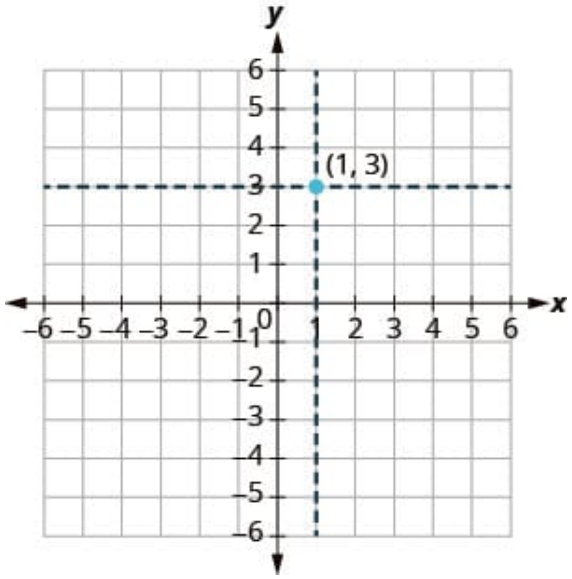
The second number is the *y*-coordinate.

The phrase 'ordered pair' means the order is important. What is the ordered pair of the point where the axes cross? At that point both coordinates are zero, so its ordered pair is $(0, 0)$. The point $(0, 0)$ has a special name. It is called the **origin**.

THE ORIGIN

The point $(0, 0)$ is called the **origin**. It is the point where the *x*-axis and *y*-axis intersect.

We use the coordinates to locate a point on the *xy*-plane. Let's plot the point $(1, 3)$ as an example. First, locate 1 on the *x*-axis and lightly sketch a vertical line through $x = 1$. Then, locate 3 on the *y*-axis and sketch a horizontal line through $y = 3$. Now, find the point where these two lines meet — that is the point with coordinates $(1, 3)$.



Notice that the vertical line through $x = 1$ and the horizontal line through $y = 3$ are not part of the graph. We just used them to help us locate the point $(1, 3)$.

Example 1

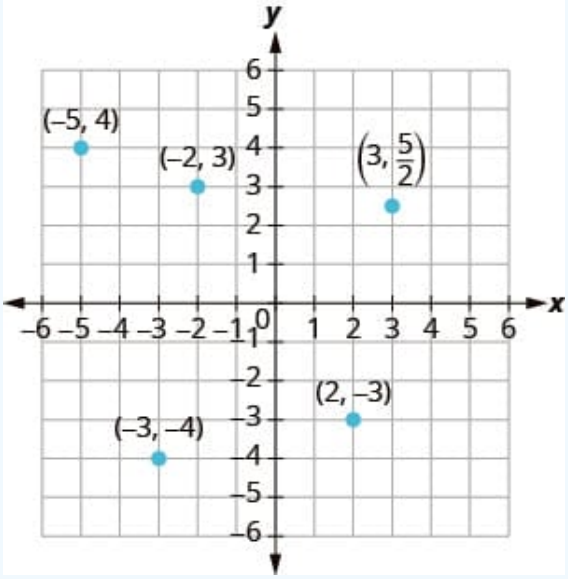
Plot each point in the rectangular coordinate system and identify the **quadrant** in which the point is located:

a. (−5,4)
b. (−3,−4)
c. (2,−3)
d. (−2,3)
e. $(3, \frac{5}{2})$

Solution

The first number of the coordinate pair is the *x*-coordinate, and the second number is the *y*-coordinate.

a. Since x=−5, the point is to the left of the *y*-axis. Also, since y=4, the point is above the *x*-axis. The point (−5,4) is in Quadrant II.
b. Since x=−3, the point is to the left of the *y*-axis. Also, since y=−4, the point is below the *x*-axis. The point (−3,−4) is in Quadrant III.
c. Since x=2, the point is to the right of the *y*-axis. Since y=−3, the point is below the *x*-axis. The point (2,−3) is in Quadrant IV.
d. Since x=−2, the point is to the left of the *y*-axis. Since y=3, the point is above the *x*-axis. The point (−2,3) is in Quadrant II.
e. Since x=3, the point is to the right of the *y*-axis. Since $y = \frac{5}{2}$, the point is above the *x*-axis. (It may be helpful to write $\frac{5}{2}$ as a mixed number or decimal.) The point $(3, \frac{5}{2})$ is in Quadrant I.



Try It 1

Plot each point in a rectangular coordinate system and identify the quadrant in which the point is located:

a. (–2,1)
b. (–3,–1)
c. (4,–4)
d. (–4,4)
e. $(-4, \frac{3}{2})$

Answer available at the end of this section.

How do the signs affect the location of the points? As you graphed the points in the previous example, you may have noticed some patterns.

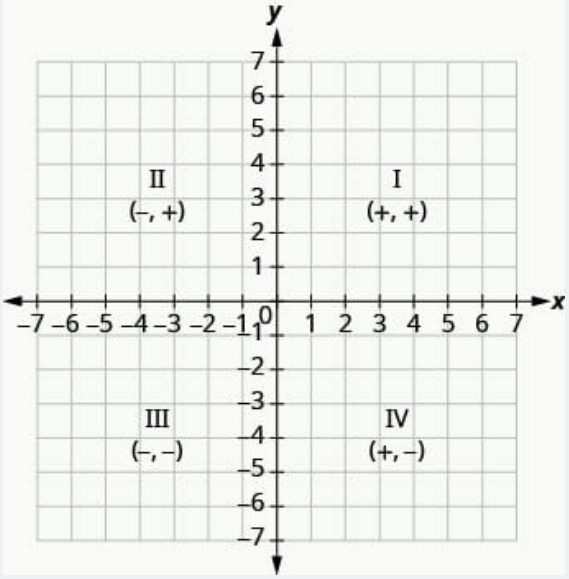
For the point in Quadrant IV, what do you notice about the signs of the coordinates? What about the signs of the coordinates of points in the third quadrant? The second quadrant? The first quadrant?

Can you tell just by looking at the coordinates in which quadrant the point (–2,5) is located? In which quadrant is (2,–5) located?

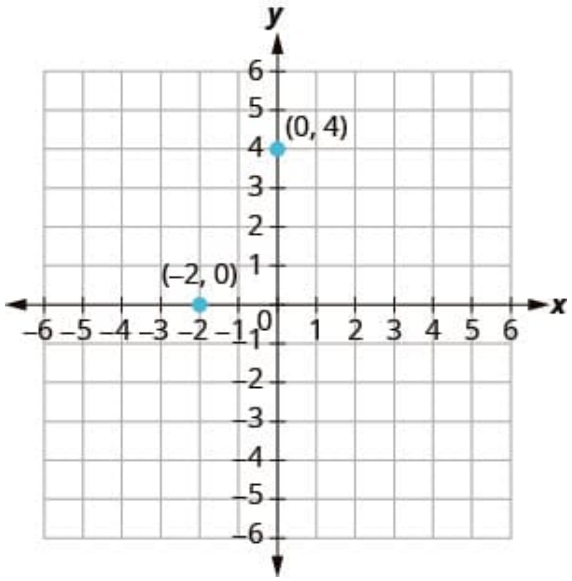
QUADRANTS

We can summarize the sign patterns of the quadrants in this way.

Quadrant I	Quadrant II	Quadrant III	Quadrant IV
(x, y)	(x, y)	(x, y)	(x, y)
$(+, +)$	$(-, +)$	$(-, -)$	$(+, -)$



What if one coordinate is zero, as shown in the figure below? Where is the point (0,4) located? Where is the point (–2,0) located?



The point (0,4) is on the *y*-axis and the point (–2,0) is on the *x*-axis.

POINTS ON THE AXES

Points with a ***y*-coordinate** equal to 0 are on the *x*-axis, and have coordinates (a,0).

Points with an ***x*-coordinate** equal to 0 are on the *y*-axis, and have coordinates (0,b).

Example 2

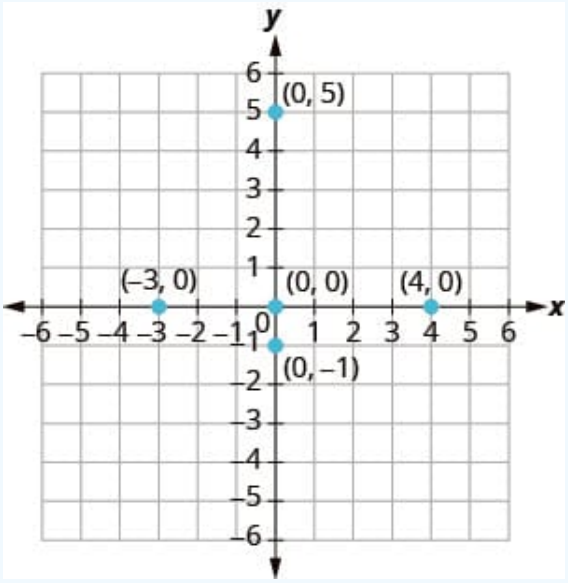
Plot each point:

1. (0,5)
2. (4,0)

3. (−3,0)
4. (0,0)
5. (0,−1)

Solution

1. Since x=0, the point whose coordinates are (0,5) is on the *y*-axis.
2. Since y=0, the point whose coordinates are (4,0) is on the *x*-axis.
3. Since y=0, the point whose coordinates are (−3,0) is on the *x*-axis.
4. Since x=0 and y=0, the point whose coordinates are (0,0) is the origin.
5. Since x=0, the point whose coordinates are (0,−1) is on the *y*-axis.



Try It 2

Plot each point:

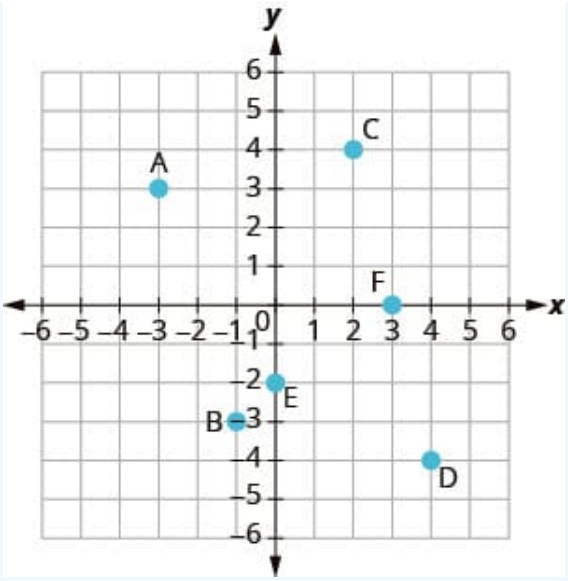
1. (−5,0)
2. (3,0)
3. (0,0)
4. (0,−1)
5. (0,4).

Answer available at the end of this section.

In algebra, being able to identify the coordinates of a point shown on a graph is just as important as being able to plot points. To identify the *x*-coordinate of a point on a graph, read the number on the *x*-axis directly above or below the point. To identify the *y*-coordinate of a point, read the number on the *y*-axis directly to the left or right of the point. Remember, when you write the **ordered pair,** use the correct order, (x,y).

Example 3

Name the ordered pair of each point shown in the rectangular coordinate system.



Solution

Point A: Point A is above −3 on the *x*-axis, so the *x*-coordinate is −3. It is to the left of 3 on the *y*-axis, so the *y*-coordinate is 3. The coordinates of the point are (−3, 3).

Point B: Point B is below −1 on the *x*-axis, so the *x*-coordinate is −1. It is to the left of −3 on the *y*-axis, so the *y*-coordinate is −3. The coordinates of the point are (−1, −3).

Point C: Point C is above 2 on the *x*-axis, so the *x*-coordinate is 2. It is to the right of 4 on the *y*-axis, so the *y*-coordinate is 4. The coordinates of the point are (2, 4).

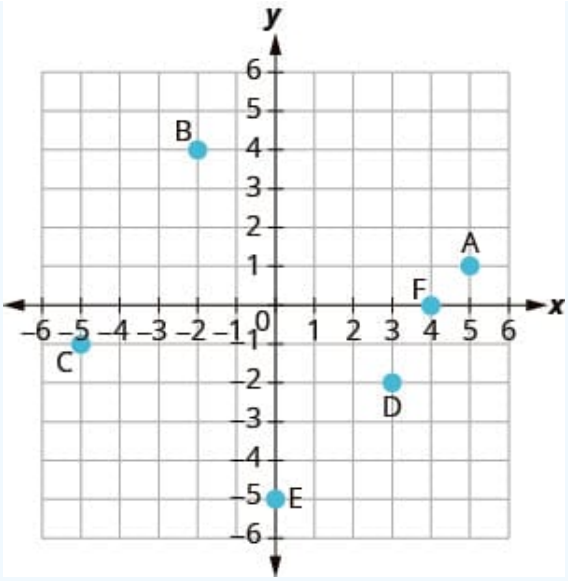
Point D: Point D is below 4 on the *x*-axis, so the *x*-coordinate is 4. It is to the right of −4 on the *y*-axis, so the *y*-coordinate is −4. The coordinates of the point are (4, −4).

Point E: Point E is on the *y*-axis at y = −2. The coordinates of point E are (0, −2).

Point F: Point F is on the *x*-axis at x = 3. The coordinates of point F are (3, 0).

Try It 3

Name the ordered pair of each point shown in the rectangular coordinate system.



Answer available at the end of this section.

Verify Solutions to an Equation in Two Variables

Up to now, all the equations you have solved were equations with just one variable. In almost every case, when you solved the equation you got exactly one solution. The process of solving an equation ended with a statement like x=4. (Then, you checked the solution by substituting back into the equation.)

Here's an example of an equation in one variable, and its one solution.

$$\begin{aligned} 3x + 5 &= 17 \\ 3x &= 12 \\ x &= 4 \end{aligned} \tag{2.14.1}$$

But equations can have more than one variable. Equations with two variables may be of the form Ax+By=C. Equations of this form are called **linear equations in two variables**.

LINEAR EQUATION

An equation of the form Ax+By=C, where A and B are not both zero, is called a **linear equation in two variables**.

Notice the word *line* in **linear**. Here is an example of a linear equation in two variables, x and y.

$$Ax + By = C$$

$$x + 4y = 8$$

$$A = 1, B = 4, C = 8$$

The equation $y = -3x + 5$ is also a **linear equation**. But it does not appear to be in the form Ax+By=C. We can use the Addition Property of Equality and rewrite it in Ax+By=C form.

	y	$=$	$-3x+5$
Add to both sides.	$y+3x$	$=$	$-3x+5+3x$
Simplify.	$y+3x$	$=$	5
Use the Commutative Property to put it in $Ax+By=C$ form.	$3x+y$	$=$	5

By rewriting $y=-3x+5$ as $3x+y=5$, we can easily see that it is a linear equation in two variables because it is of the form Ax +By=C. When an equation is in the form Ax+By=C, we say it is in *standard form*.

STANDARD FORM OF LINEAR EQUATION

A linear equation is in standard form when it is written Ax+By=C.

Linear equations have infinitely many solutions. For every number that is substituted for x there is a corresponding y value. This pair of values is a *solution* to the linear equation and is represented by the ordered pair (x,y). When we substitute these values of x and y into the equation, the result is a true statement, because the value on the left side is equal to the value on the right side.

Solution OF A LINEAR EQUATION IN TWO VARIABLES

An **ordered pair** (x,y) is a **solution** of the linear equation Ax+By=C, if the equation is a true statement when the x- and y -values of the ordered pair are substituted into the equation.

Example 4

Determine which ordered pairs are solutions to the equation $x+4y=8$.

(a) (0,2)

(b) (2,–4)

(c) (–4,3)

Solution

Substitute the *x*- and *y*-values from each ordered pair into the equation and determine if the result is a true statement.

(a)	(b)	(c)
(0, 2)	(2, –4)	(–4, 3)
$x=0, y=2$	$x=2, y=-4$	$x=-4, y=3$
$x+4y=8$	$x+4y=8$	$x+4y=8$
$0+4\cdot 2 \stackrel{?}{=} 8$	$2+4(-4) \stackrel{?}{=} 8$	$-4+4\cdot 3 \stackrel{?}{=} 8$
$0+8 \stackrel{?}{=} 8$	$2+(-16) \stackrel{?}{=} 8$	$-4+12 \stackrel{?}{=} 8$
$8=8$ ✓	$-14 \neq 8$	$8=8$ ✓
(0, 2) is a solution.	(2, –4) is not a solution.	(–4, 3) is a solution.

Try It 4

Which of the following ordered pairs are solutions to the equation $4x - y = 8$?

1. (0,8)
2. (2,0)
3. (1,−4)

Answer available at the end of this section.

Example 5

Which of the following ordered pairs are solutions to the equation $y = 5x - 1$?

(a) (0,−1)

(b) (1,4)

(c) (−2,−7)

Solution

Substitute the *x*- and *y*-values from each **ordered pair** into the equation and determine if it results in a true statement.

(a)	(b)	(c)
$(0, -1)$	$(1, 4)$	$(-2, -7)$
$x = 0, y = -1$	$x = 1, y = 4$	$x = -2, y = -7$
$y = 5x - 1$	$y = 5x - 1$	$y = 5x - 1$
$-1 \stackrel{?}{=} 5(0) - 1$	$4 \stackrel{?}{=} 5(1) - 1$	$-7 \stackrel{?}{=} 5(-2) - 1$
$-1 \stackrel{?}{=} 0 - 1$	$4 \stackrel{?}{=} 5 - 1$	$-7 \stackrel{?}{=} -10 - 1$
$-1 = -1$ ✓	$4 = 4$ ✓	$-7 \neq -11$
$(0, -1)$ is a solution.	$(1, 4)$ is a solution.	$(-2, -7)$ is not a solution.

Try It 5

Which of the following ordered pairs are solutions to the equation $y = 4x - 3$?

1. (0,3)
2. (1,1)
3. (−1,−1)

Answer available at the end of this section.

Complete a Table of Solutions to a Linear Equation in Two Variables

In the examples above, we substituted the *x*- and *y*-values of a given ordered pair to determine whether or not it was a solution to

a linear equation. But how do you find the ordered pairs if they are not given? It's easier than you might think—you can just pick a value for x and then solve the equation for y. Or, pick a value for y and then solve for x.

We'll start by looking at the solutions to the equation $y = 5x - 1$ that we found in Example 5. We can summarize this information in the table of solutions below.

$y = 5x - 1$		
x	y	(x, y)
0	–1	(0,–1)
1	4	(1,4)

To find a third solution, we'll select another value for x. Let's let $x = 2$, and solve for y.

	$y = 5x - 1$
Substitute $x = 2$.	$y = 5(2) - 1$
Multiply.	$y = 10 - 1$
Simplify.	$y = 9$

The ordered pair (2,9) is a solution to $y = 5x - 1$. We will add it to our table.

$y = 5x - 1$		
x	y	(x, y)
0	–1	(0,–1)
1	4	(1,4)
2	9	(2,9)

We can find more solutions to the equation by substituting in any value of x or any value of y and solving the resulting equation to get another ordered pair that is a solution. There are infinitely many solutions of this equation.

Example 6

Complete the table to find three solutions to the equation $y = 4x - 2$.

$y = 4x - 2$		
x	y	(x, y)
0		
–1		
2		

Solution

Substitute $x = 0$, $x = -1$, and $x = 2$ into $y = 4x - 2$.

Answer available at the end of this section.

Try It 6

Complete the table to find three solutions to the equation $y = 3x - 1$.

$y = 3x - 1$		
x	y	(x, y)
0		
–1		
2		

Answer available at the end of this section.

Example 7

Complete the following table to find three solutions to the equation $5x - 4y = 20$.

$5x - 4y = 20$		
x	y	(x, y)
0		
	0	
	5	

Solution

Substitute the given value into the equation $5x - 4y = 20$ and solve for the other variable. Then, fill in the values in the table.

Answer available at the end of this section.

Try It 7

Complete the table to find three solutions to the equation $2x - 5y = 20$.

$2x - 5y = 20$		
x	y	(x, y)
	-4	
	0	
-5		

Answer available at the end of this section.

Find Solutions to a Linear Equation

To find a solution to a linear equation, you really can pick *any* number you want to substitute into the equation for x or y. But since you'll need to use that number to solve for the other variable, choosing a number that's easy to work with is a good idea.

When the equation is in *y*-form, with *y* by itself on one side of the equation, it is usually easier to choose values of x and then solve for y.

Example 8

Find three solutions to the equation $y = -3x + 2$.

Solution

We can substitute any value we want for x or any value for y. Since the equation is in y-form, it will be easier to substitute in values of x. Let's pick $x = 0$, $x = 1$, and $x = -1$.

			$x = 0$	$x = 1$	$x = -1$
			$y = -3x + 2$	$y = -3x + 2$	$y = -3x + 2$
Substitute the value into the equation.			$y = -3 \cdot 0 + 2$	$y = -3 \cdot 1 + 2$	$y = -3(-1) + 2$
Simplify.			$y = 0 + 2$	$y = -3 + 2$	$y = 3 + 2$
Simplify.			$y = 2$	$y = -1$	$y = 5$
Write the ordered pair.			(0, 2)	(1, -1)	(-1, 5)
Check.					
y=−3x+2	y=−3x+2	y=−3x+2			
$2 \stackrel{?}{=} -3 \cdot 0 + 2$	$-1 \stackrel{?}{=} -3 \cdot 1 + 2$	$5 \stackrel{?}{=} -3(-1) + 2$			
$2 \stackrel{?}{=} 0 + 2$	$-1 \stackrel{?}{=} -3 + 2$	$5 \stackrel{?}{=} -3 + 2$			
$2 = 2$✓	$-1 = -1$✓	$5 = 5$✓			

Answer available at the end of this section.

Try It 8

Find three solutions to the equation $y = -2x + 3$.

Answer available at the end of this section.

We have seen how using zero as one value of x makes finding the value of y easy. When an equation is in standard form, with both the x and y on the same side of the equation, it is usually easier to first find one solution when $x = 0$, find a second solution when y=0, and then find a third solution.

Example 9

Find three solutions to the equation $3x + 2y = 6$.

Solution

We can substitute any value we want for x or any value for y. Since the equation is in standard form, let's pick first $x = 0$, then $y = 0$, and then find a third point.

	$x=0$	$y=0$	$x=1$
Substitute the value into the equation.	$3(0)+2y=6$	$3x+2(0)=6$	$3(1)+2y=6$
Simplify.	$0+2y=6$	$3x+0=6$	$3+2y=6$
Solve.	$2y=6$	$3x=6$	$2y=3$
	$y=3$	$x=2$	$y=\frac{3}{2}$
Write the ordered pair.	(0, 3)	(2, 0)	$(1,\frac{3}{2})$
Check.			

$3x+2y=6$	$3x+2y=6$	$3x+2y=6$
$3\cdot 0+2\cdot 3\stackrel{?}{=}6$	$3\cdot 2+2\cdot 0\stackrel{?}{=}6$	$3\cdot 1+2\cdot\frac{3}{2}\stackrel{?}{=}6$
$0+6\stackrel{?}{=}6$	$6+0\stackrel{?}{=}6$	$3+3\stackrel{?}{=}6$
$6=6$✓	$6=6$✓	$6=6$✓

So (0,3), (2,0), and $(1,\frac{3}{2})$ are all solutions to the equation $3x+2y=6$. These solutions are summarized in the table below.

$3x+2y=6$		
x	y	(x,y)
0	3	(0,3)
2	0	(2,0)
1	$\frac{3}{2}$	$(1,\frac{3}{2})$

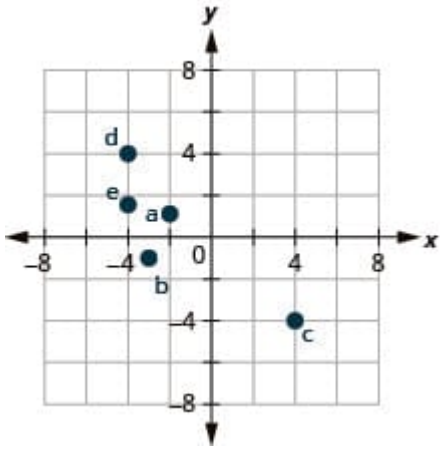
Try It 9

Find three solutions to the equation $2x+3y=6$.

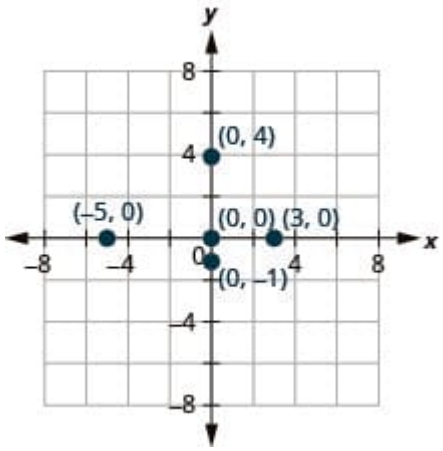
Answer available at the end of this section.

Answers

Try It 1

The point a (-2,1) is in Quadrant II. The point b (-3,-1) is in Quadrant III. The point c (4,-4) is in Quadrant IV. The point d (-4,4) is in Quadrant II. The point e $(-4, \frac{3}{2})$ is in Quadrant II.	

Try It 2



Try It 3

A: (5,1) B: (–2,4) C: (–5,–1) D: (3,–2) E: (0,–5) F: (4,0)

Try It 4

$(2, 0)$ and $(1, -4)$ are solutions to the equation $4x - y = 8$.

Try It 5

$(1, 1)$ is a solution to the equation $y = 4x - 3$.

Example 6

$x = 0$	$x = -1$	$x = 2$
$y = 4x - 2$	$y = 4x - 2$	$y = 4x - 2$
$y = 4 \cdot 0 - 2$	$y = 4(-1) - 2$	$y = 4 \cdot 2 - 2$
$y = 0 - 2$	$y = -4 - 2$	$y = 8 - 2$
$y = -2$	$y = -6$	$y = 6$
$(0, -2)$	$(-1, -6)$	$(2, 6)$

The results are summarized in the following table.

$y = 4x - 2$		
x	y	(x, y)
0	–2	(0,–2)

–1	–6	(–1,–6)
2	6	(2,6)

Try It 6

$y = 3x - 1$		
x	y	(x, y)
0	-1	(0, -1)
–1	-4	(-1, -4)
2	5	(2, 5)

Example 7

$$\begin{array}{rrr} x = 0 & y = 0 & y = 5 \\ 5x - 4y = 20 & 5x - 4y = 20 & 5x - 4y = 20 \\ 5 \cdot 0 - 4y = 20 & 5x - 4 \cdot 0 = 20 & 5x - 4 \cdot 5 = 20 \\ 0 - 4y = 20 & 5x - 0 = 20 & 5x - 20 = 20 \\ -4y = 20 & 5x = 20 & 5x = 40 \\ y = -5 & x = 4 & x = 8 \\ (0, -5) & (4, 0) & (8, 5) \end{array}$$

The results are summarized in the table below.

$5x - 4y = 20$		
x	y	(x, y)
0	–5	(0,–5)
4	0	(4,0)
8	5	(8,5)

Try It 7

$2x - 5y = 20$		
x	y	(x, y)
0	–4	(0,–4)
10	0	(10,0)
–5	–6	(–5,–6)

Example 8

So, (0,2), (1,–1) and (–1,5) are all solutions to $y = -3x + 2$. These solutions are summarized in the table below.

$y = -3x + 2$		
x	y	(x, y)
0	2	(0,2)
1	–1	(1,–1)

–1	5	(–1,5)

Try It 8

Answers will vary. Answers might include (0,3), (1,1) and (–1,5).

Try It 9

Answers will vary. Answers might include (0,2), (3,0), and (6, –2).

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2.14.1: Practice and Videos for Graphing on a Coordinate Plane

Learning Objectives

- I can graph points on a coordinate plane.
- I can identify patterns and notice relationships when graphing points on a coordinate plane.

GeoGebra: Plotting Points Practice: 4 quadrants, integers -4 to 4

Complete the activity below. Move point A (blue) to the position identified by the coordinate pair on the left. Click on "Check" to see if your placement is accurate. If it is accurate, you will receive a new coordinate pair to plot; if it is inaccurate, you can try again with the same coordinate pair. To start over, click "Reset".

Videos on Graphing on a Coordinate Plane

Watch the following videos to learn how to graph on a coordinate plane. All videos have appropriate captions in English; in some cases, there may also be appropriate captions in other languages.

YouTube Video

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

YouTube Video

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

Practice!

Check your understanding of order of operations by answering these questions. Use the arrows at the bottom of the box to go to the next question.

Practice 2.14.1.1

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View this Interactive Content online

Extra Practice Resources

For more practice on order of operations, visit the following websites.

1. IXL: Graph points on a coordinate plane

2. Math Playground: Space Graph Jr...

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2.14P: Practice

Practice Makes Progress

Plot Points in a Rectangular Coordinate System

In the following exercises, plot each point in a rectangular coordinate system.

Exercise 1

(–4,2)

Exercise 2

(–1,–2)

Exercise 3

(3,–5)

Exercise 4

(–3,5)

Exercise 5

$(\frac{5}{3}, 2)$

Exercise 6

(–2,0)

Exercise 7

(–3,0)

Exercise 8

(0,0)

Exercise 9

(0,4)

Exercise 10

(0,2)

In the following exercises, identify the quadrant in which the point is located.

Exercise 11

(3,–1)

Exercise 12

(–3,1)

Exercise 13

(–2,2)

Exercise 14

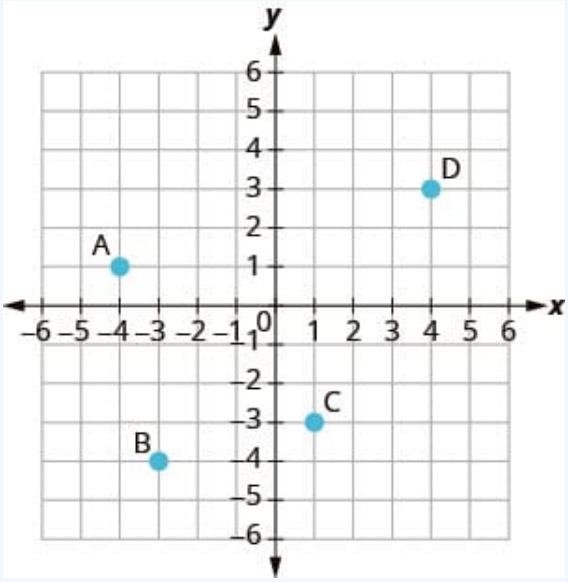
(–4,–3)

Exercise 15

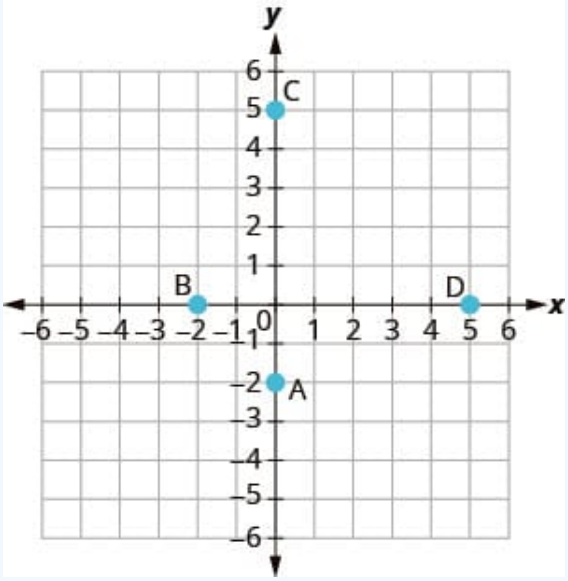
$\left(1, \frac{14}{5}\right)$

In the following exercises, name the ordered pair of each point shown in the rectangular coordinate system.

Exercise 16



Exercise 17



Verify Solutions to an Equation in Two Variables

In the following exercises, determine if the ordered pair is a solution to $2x + y = 6$.

Exercise 18

(1,4)

Exercise 19

(3,0)

Exercise 20

(2,3)

Determine if the ordered pair is a solution to $4 - 2y = 8$.

Exercise 21

(3,2)

Exercise 22

(1,4)

Exercise 23

(0,–4)

Determine if the ordered pair is a solution to $y = 4x + 3$.

Exercise 24

(4,3)

Exercise 25

(–1,–1)

Exercise 26

$\left(\frac{1}{2}, 5\right)$

Finding and Representing Solutions to a Linear Equation

Use the given x value to find the corresponding y value and then state the point.

Exercise 27

$y = 2x - 4$; $x = -1$

Exercise 28

$y = -x + 5$; $x = 3$

Exercise 29

$y = \frac{1}{3}x + 1$; $x = 6$

Use the given y value to find the corresponding x value and state the point.

Exercise 30

$y = -\frac{3}{2}x - 2$; $y = 1$

Exercise 31

$x + 3y = 6$; $y = 0$

Exercise 32

$2x - 5y = 10$; $y = \frac{1}{2}$

Everyday Math

Exercise 33

Weight of a baby. Mackenzie recorded her baby's weight every two months. The baby's age (in months) and weight (in pounds) are listed as ordered pairs below.

- (0, 7)
- (2, 11)
- (4, 15)
- (6, 16)
- (8, 19)
- (10, 20)
- (12, 21)

Plot the points on a coordinate plane and explain why, in this context, only Quadrant I is needed.

Writing Exercises

Exercise 34

Explain in words how you plot the point (4,–2) in a rectangular coordinate system.

Exercise 35

Is the point $(-3,0)$ on the x-axis or y-axis? How do you know?

Self Check

Use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
plot points in a rectangular coordinate system.			
identify points on a graph.			
verify solutions to an equation in two variables.			
complete a table of solutions to a linear equation.			
find solutions to a linear equation.			

If most of your checks were:

...confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them.

...with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is crucial to establish a strong foundation before proceeding. Who can you ask for help? Your classmates, tutors, and class instructor are good resources.

...no - I don't get it! This is a warning sign, and you must not ignore it. You should get help right away, or you will quickly be overwhelmed. Meet with your instructor during their drop-in hours as soon as you can to discuss your situation. Together, you can devise a plan to get the help you need.

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Sample Word 1 | Sample Definition 1

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